Title: Matrix Inflation

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Abstract: Abstract: In this talk a model of inflation is presented where the inflaton fields are non-commutative matrices. The spectrum of adiabatic and iso-curvature perturbations and their implications on CMB are studied. It is argued that our model of matrix inflation can naturally be embedded in string theory.

Matrix Inflation (M-flation)

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arXiv: 0903.1481

In collaborations with

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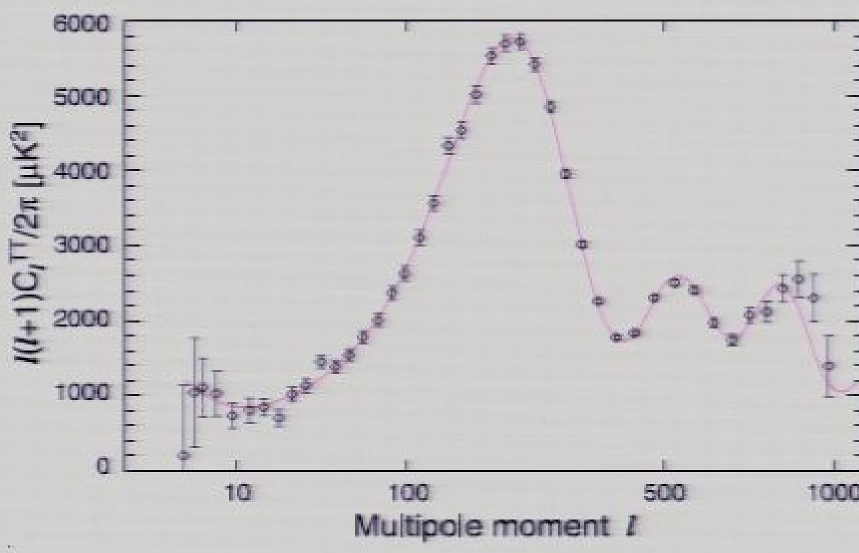
Outline

- M-flation set up
- Various inflation models from M-flation
- Adiabatic and entropic modes
- Power spectra for adiabatic and entropic perturbations
- Motivations from string theory
- Preheating
- Conclusion

WMAP 2003-08

- All observations, specially WMAP 2003-2008, strongly support inflation.
- Different inflationary models predict different values for cosmological parameters like the scalar spectrum index n_s which can be measured in CMB.



WMAP 08

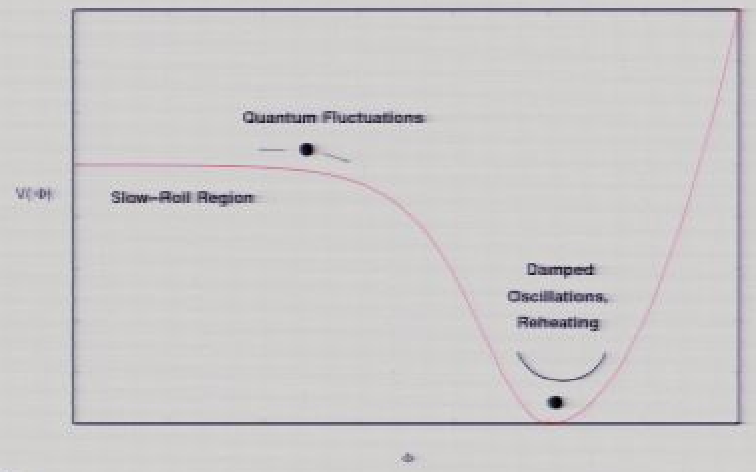
- There is no compelling and theoretically well-motivated model of inflation. There have been many attempts to embed inflation within the context of string theory.
- If it works, this would provide a unique chance to test the relevance of string theory to the real world.

Slow Roll Inflation

In most models, inflation is derived by a scalar field, the inflaton.
This creates a negative pressure required for acceleration.

For a scalar field $\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi)$ $p = \frac{1}{2}\dot{\phi}^2 - V(\phi)$

$a(t) \sim e^{Ht}$, $H^2 = \frac{8\pi G}{3}V$



- Simple models of chaotic inflation suffers from fine-tuning and issues with super-Planckian field values.

$V = \frac{1}{2}m^2\phi^2 \longrightarrow \phi_i \sim 15\,M_P$

$V = \frac{\lambda}{4}\phi^4 \longrightarrow \lambda \sim 10^{-14}$ $\phi_i \sim 22\,M_P$

M-Flation

Suppose inflation is driven by non-commutative matrices:

$$S = \int d^4x \sqrt{-g} \left(\frac{M_P^2}{2} R - \frac{1}{2} \sum_i \mathrm{Tr} \left(\partial_\mu \Phi_i \partial^\mu \Phi_i \right) - V(\Phi_i, [\Phi_i, \Phi_j]) \right)$$

Example :

$$V = \mathrm{Tr} \left(-\frac{\lambda}{4} [\Phi_i, \Phi_j][\Phi_i, \Phi_j] + \frac{i\kappa}{3} \epsilon_{jkl} [\Phi_k, \Phi_l] \Phi_j + \frac{m^2}{2} \Phi_i^2 \right)$$

where Φ_i are $N \times N$ matrices.

The equations of motion are

$$H^2 = \frac{1}{3M_P^2} \left(-\frac{1}{2} \mathrm{Tr} \left(\partial_\mu \Phi_i \partial^\mu \Phi_i \right) + V(\Phi_i, [\Phi_i, \Phi_j]) \right)$$

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Truncation to SU(2)sector

$$\Phi_i = \hat{\phi}(t) J_i \ , \qquad i = 1, 2, 3$$

where J_i are the N-dimensional irreducible representation of SU(2) algebra.

$$[J_i, J_j] = i\,\epsilon_{ijk} J_k \ , \qquad \mathrm{Tr}(J_i\, J_j) = \frac{N}{12}(N^2 - 1)\,\delta_{ij} \, .$$

Plugging this ansatz into the action, we obtain

$$S = \int d^4x \sqrt{-g} \left[\frac{M_P^2}{2} R + \mathrm{Tr}J^2 \left(-\frac{1}{2} \partial_\mu \hat{\phi} \partial^\mu \hat{\phi} - \frac{\lambda}{2} \hat{\phi}^4 + \frac{2\kappa}{3} \hat{\phi}^3 - \frac{m^2}{2} \hat{\phi}^2 \right) \right]$$

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Examples:

I- Chaotic Inflation $V = \frac{1}{4}\lambda_{eff}\phi^4$: $m = \kappa = 0$

To fit the CMB observation, we need

$$\lambda_{eff} \sim 10^{-14} \qquad \Delta\phi \sim 10 M_P.$$

On the other hand $\lambda_{eff} \sim \lambda N^{-3}$ $\Delta\hat{\phi} \sim N^{-3/2}\Delta\phi$

One obtains $N \sim 10^5$ $\Delta\hat{\phi} \sim 10^{-7} M_P$

Due to large running of field values, a considerable amount of gravity waves can be produced. $r \simeq 0.26$

The scalar spectral index is $n_{\mathcal{R}} \simeq 0.949$

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2- Symmetry breaking potential:

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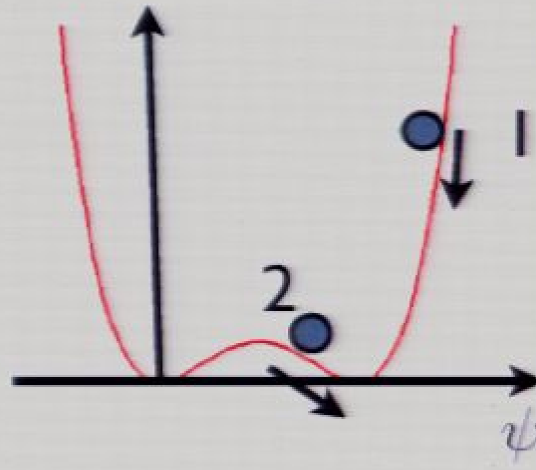


I: $\phi_i > \mu$

To fit the observational constraints

$$\phi_i \simeq 43.57 M_P \quad , \quad \phi_f \simeq 27.07 M_P \quad , \quad \mu \simeq 26 M_P.$$

$$\lambda_{eff} \simeq 4.91 \times 10^{-14}, \quad m \simeq 4.07 \times 10^{-6} M_P, \quad \kappa_{eff} \simeq 9.57 \times 10^{-13} M_P.$$

II: $\mu/2 < \phi_i < \mu$

$$\phi_i \simeq 23.5 M_P \quad , \quad \phi_f \simeq 35.03 M_P \quad , \quad \mu \simeq 36 M_P.$$

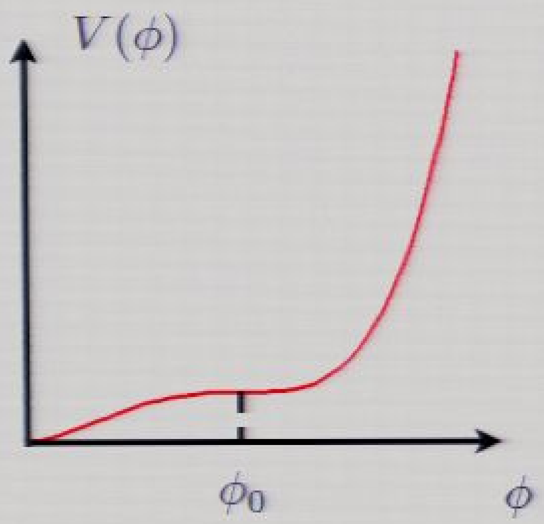
$$\lambda_{eff} \simeq 7.18 \times 10^{-14}, \quad m \simeq 6.82 \times 10^{-6} M_P, \quad \kappa_{eff} \simeq 1.94 \times 10^{-12} M_P.$$

One finds $N \sim 10^5.$ $\Delta\hat{\phi} \sim 10^{-7} M_P$

3-Saddle-point Inflation $\kappa = \sqrt{2\lambda}m$

$$V(\phi) \simeq V(\phi_0) + \frac{1}{3!}V'''(\phi_0)(\phi - \phi_0)^3$$

$$V(\phi_0) = \frac{m^2}{12}\phi_0^2 \quad , \quad V'''(\phi_0) = \frac{2m^2}{\phi_0}.$$



The CMB observables are given by

$$n_s \simeq 1 - \frac{4}{N_e} \quad , \quad \delta_H \simeq \frac{2}{5\pi}\frac{\lambda_{eff}M_P}{m}N_e^2.$$

$$\lambda_{eff} = (\frac{9\,r}{32})^{1/3}\left(\frac{5\pi}{8}\delta_H\right)^2(1 - n_s)^{8/3}.$$

The upper bound r < 0.2 from WMAP5, and ns=0.96, gives

$$\lambda_{eff} \lesssim 10^{-13} \text{ and } N \gtrsim 10^5$$

Consistency of truncation to SU(2) sector

Φ_i are hermitian matrices, so we have $3N^2$ real scalar fields.

We have considered ϕ as the inflaton field and turned off the remaining $3N^2-1$ fields. How consistent is this truncation?

Suppose $\Psi_i = \Phi_i - \hat{\phi} J_i$ where $\hat{\phi} = \frac{4}{N(N^2-1)} \mathrm{Tr}(\Phi_i J_i)$

So

$$\mathrm{Tr}(\Psi_i J_i) = 0.$$

Then

$$V = V_0(\hat{\phi}) + V_{(2)}(\hat{\phi}, \Psi_i)$$

with

$$V_{(2)}(\hat{\phi}, \Psi_i = 0) = 0 \,, \qquad \left(\frac{\delta V_{(2)}}{\delta \Psi_i}\right)_{\Psi_i=0} = 0.$$

So there is no linear term of Ψ_i in potential.

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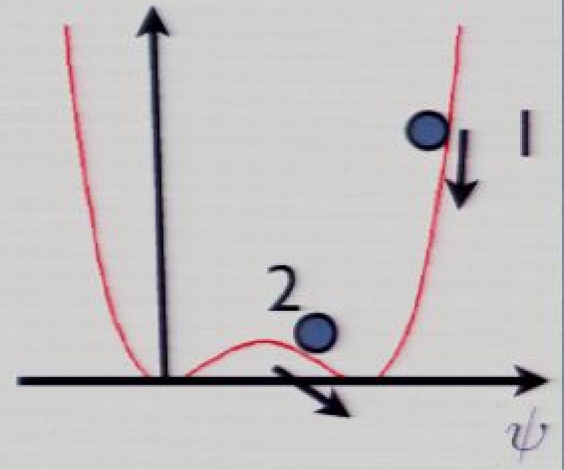

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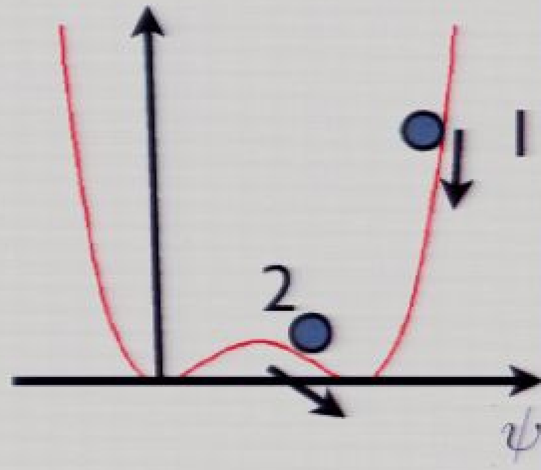
where $\mathrm{Tr} J^2 = \mathrm{Tr}(J_i^2) = N(N^2 - 1)/4.$

2- Symmetry breaking potential:

$$V_0 = \frac{\lambda_{eff}}{4}\phi^2\,(\phi-\mu)^2 \qquad\qquad \mu \equiv \sqrt{2}m/\sqrt{\lambda_{eff}}.$$



I: $\phi_i > \mu$

To fit the observational constraints

$$\phi_i \simeq 43.57M_P \quad , \quad \phi_f \simeq 27.07M_P \quad , \quad \mu \simeq 26M_P.$$

$$\lambda_{eff} \simeq 4.91\times 10^{-14}, \quad m \simeq 4.07\times 10^{-6}M_P, \quad \kappa_{eff} \simeq 9.57\times 10^{-13}M_P.$$

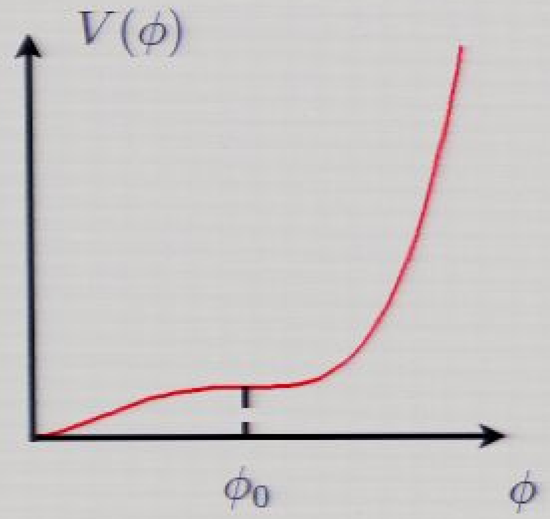
II: $\mu/2 < \phi_i < \mu$

$$\phi_i \simeq 23.5M_P \quad , \quad \phi_f \simeq 35.03M_P \quad , \quad \mu \simeq 36M_P.$$

$$\lambda_{eff} \simeq 7.18\times 10^{-14}, \quad m \simeq 6.82\times 10^{-6}M_P, \quad \kappa_{eff} \simeq 1.94\times 10^{-12}M_P.$$

One finds $\quad N \sim 10^5. \qquad \Delta\hat{\phi} \sim 10^{-7}M_P$

3-Saddle-point Inflation $\kappa = \sqrt{2\lambda} m$



$$V(\phi) \simeq V(\phi_0) + \frac{1}{3!} V'''(\phi_0)(\phi - \phi_0)^3$$

$$V(\phi_0) = \frac{m^2}{12}\phi_0^2 \quad , \quad V'''(\phi_0) = \frac{2m^2}{\phi_0} .$$

The CMB observables are given by

$$n_s \simeq 1 - \frac{4}{N_e} \quad , \quad \delta_H \simeq \frac{2}{5\pi} \frac{\lambda_{eff} M_P}{m} N_e^2 .$$

$$\lambda_{eff} = (\frac{9\,r}{32})^{1/3} \left(\frac{5\pi}{8} \delta_H \right)^2 (1 - n_s)^{8/3} .$$

The upper bound r < 0.2 from WMAP5, and ns=0.96, gives

$$\lambda_{eff} \lesssim 10^{-13} \text{ and } N \gtrsim 10^5$$

Consistency of truncation to SU(2) sector

Φ_i are hermitian matrices, so we have $3N^2$ real scalar fields.

We have considered ϕ as the inflaton field and turned off the remaining $3N^2-1$ fields. How consistent is this truncation?

Suppose $\Psi_i = \Phi_i - \hat{\phi} J_i$ where $\hat{\phi} = \frac{4}{N(N^2-1)} \mathrm{Tr}(\Phi_i J_i)$

So

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Then

$$V = V_0(\hat{\phi}) + V_{(2)}(\hat{\phi}, \Psi_i)$$

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Upon field re-definition

$$\hat{\phi} = \left(\mathrm{Tr}J^2\right)^{-1/2}\phi = \left[\frac{N}{4}(N^2-1)\right]^{-1/2}\phi$$

The effective potential is

$$V_0(\phi) = \frac{\lambda_{eff}}{4}\phi^4 - \frac{2\kappa_{eff}}{3}\phi^3 + \frac{m^2}{2}\phi^2$$

Where

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Examples:

I- Chaotic Inflation $V = \frac{1}{4}\lambda_{eff}\phi^4$: $m = \kappa = 0$

To fit the CMB observation, we need

$$\lambda_{eff} \sim 10^{-14} \qquad \Delta\phi \sim 10 M_P.$$

On the other hand $\lambda_{eff} \sim \lambda N^{-3}$ $\Delta\hat{\phi} \sim N^{-3/2}\Delta\phi$

One obtains $N \sim 10^5$ $\Delta\hat{\phi} \sim 10^{-7} M_P$

Due to large running of field values, a considerable amount of gravity waves can be produced. $r \simeq 0.26$

The scalar spectral index is $n_{\mathcal{R}} \simeq 0.949$

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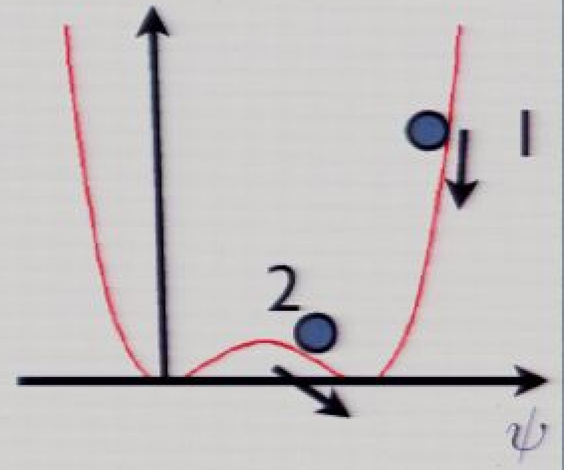

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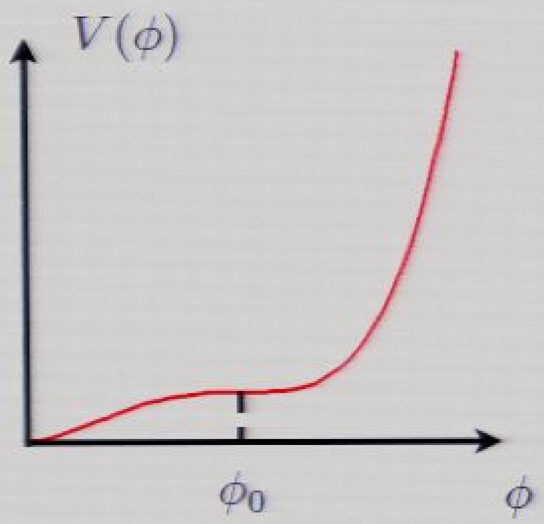

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How special are the SU(2) sectors?

Suppose $\Phi_i = \Upsilon_i + \Xi_i$

such that $\mathrm{Tr}(\Upsilon_i \Xi_i) = 0$, $\mathrm{Tr}(\Upsilon_i) = 0$

The potential becomes

$$V = V_0(\Upsilon_i) + V_{(1)}(\Upsilon, \Xi) \quad \text{with} \quad V_0(\Upsilon_i) = V(\Xi_i = 0)$$

$$V_{(1)}(\Upsilon_i, \Xi_i) = \mathrm{Tr}\left[\left(-\lambda[\Upsilon_i, [\Upsilon_i, \Upsilon_k]] + i\kappa\epsilon_{ijk}[\Upsilon_i, \Upsilon_j]\right)\Xi_k\right] + \mathcal{O}(\Xi^2)\ .$$

The linear term can be killed if

$$[\Upsilon_i, \Upsilon_j] = f_{ijk}\Upsilon_k$$

Mass spectrum of the Ψ_i modes

Expanding the potential up to second order in Ψ_i one obtains

$$V_{(2)} = \mathrm{Tr}\left[\frac{\lambda}{2}\hat{\phi}^2\,\Omega_i\Omega_i + \frac{m^2}{2}\,\Psi_i\Psi_i + \left(-\frac{\lambda}{2}\hat{\phi}^2 + \kappa\hat{\phi}\right)\Psi_i\Omega_i\right]$$

where $$\Omega_k \equiv i\epsilon_{ijk}[J_i, \Psi_j]\ .$$

The eigen-values problem: $\Omega_i = \omega\Psi_i$

$$V_{(2)} = \left(\frac{\lambda_{eff}}{4}\phi^2(\omega^2-\omega) + \kappa_{eff}\,\omega\,\phi + \frac{m^2}{2}\right)\ \mathrm{Tr}\,\Psi_i\Psi_i\ .$$

The mass of Ψ_i modes are

$$\begin{aligned} M^2 &= \frac{\lambda_{eff}}{2}\phi^2(\omega^2-\omega) + 2\kappa_{eff}\,\omega\phi + m^2 \\ &= V_0''(\omega+1)^2 - \frac{V_0'}{\phi}(4\omega+3)(\omega+2) + \frac{6V_0}{\phi^2}(\omega+1)(\omega+2) \end{aligned}$$

The classification of the Ψ_i modes

The modes are classified in three categories:

- *" The zero modes"* $\omega = -1$ $\qquad M^2 = \frac{V_0'}{\phi}$.

- *"The α modes"*: $\omega = -(l+1)$, $l \in \mathbb{Z}$, $0 \le l < N$, with the mass

$$M_l^2 = \frac{\lambda_{eff}}{2}(l+1)(l+2)\phi^2 - 2\kappa_{eff}(l+1)\phi + m^2 \,.$$

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$$M_l^2 = \frac{\lambda_{eff}}{2} l(l-1)\phi^2 - 2\kappa_{eff} l\phi + m^2 .$$

The light modes:

$$\frac{M^2}{3H^2} = \left[\eta(\omega+1)^2 - sgn(V_0')\sqrt{2\epsilon}\,\frac{M_P}{\phi}(4\omega+3)(\omega+2) + 6\frac{M_P^2}{\phi^2}(\omega+1)(\omega+2)\right]$$

For the zero modes with $\omega = -1$

$$M^2/3H^2 \sim \sqrt{\epsilon}M_P/\phi \sim 0.01$$

so there are N^2 of such light zero modes.

For α and β modes, only those with $l \lesssim \epsilon^{-1/2}, \eta^{-1/2} \sim 10$

are light, while the modes with higher values of l are heavy.

Adiabatic and entropic power spectra

Our Lagrangian is

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}\partial_\mu\psi^{(i)}_{mn}\partial^\mu\psi^{(i)}_{nm} - V_0(\phi) - \frac{1}{2}M^2(\phi)\psi^{(i)}_{mn}\psi^{(i)}_{nm}\,.$$

Define $$Q_\phi \equiv \delta\phi + \frac{\dot{\phi}}{H}\Phi\,.$$

The equations of motion are

$$\ddot{Q}_\phi + 3H\dot{Q}_\phi + \frac{k^2}{a^2}Q_\phi + \left[V_{0,\phi\phi} - \frac{1}{a^3M_P^2}\left(\frac{a^3}{H}\dot{\phi}^2\right)^{\cdot}\right]Q_\phi = 0$$

$$\ddot{\Psi}_{r,lm} + 3H\dot{\Psi}_{r,lm} + \left(\frac{k^2}{a^2} + M^2_{r,l}(\phi)\right)\Psi_{r,lm} = 0\,.$$

Interestingly enough, the adiabatic and the iso-curvature modes decouple.

The normalized curvature and entropy perturbations are

$$\mathcal{R} \equiv \frac{H}{\dot\phi} Q_\phi \quad , \quad \mathcal{S}^{(i)}_{mn} \equiv \frac{H}{\dot\phi} \psi^{(i)}_{mn} \, .$$

with the initial conditions

$$\begin{aligned}
\langle Q^\star_{\phi \mathbf{k}} Q_{\phi \mathbf{k}'} \rangle &= \frac{2\pi^2}{k^3} P_{Q_\phi} \delta^3(\mathbf{k} - \mathbf{k}') \\
\langle \Psi^\star_{r,lm \ \mathbf{k}} \Psi_{r',l'm' \ \mathbf{k}'} \rangle &= \frac{2\pi^2}{k^3} P_{\Psi_{r,l}} \delta_{rr'} \delta_{ll'} \delta_{mm'} \delta^3(\mathbf{k} - \mathbf{k}') \\
\langle Q^\star_{\phi \mathbf{k}} \Psi_{r,lm \ \mathbf{k}'} \rangle &= 0 \, ,
\end{aligned}$$

Using our equations of motion one obtains $\quad \dot{\mathcal{R}} = \frac{H}{\dot H} \frac{k^2}{a^2} \Phi \, .$

Compare this to general multiple-field case

$$\dot{\mathcal{R}} = \frac{H}{\dot H} \frac{k^2}{a^2} \Phi + 2 \sum_{\alpha=1}^{3N^2-1} \dot\theta_\alpha \mathcal{S}_\alpha \, .$$

Power Spectra:

Define $$u \equiv aQ_\phi \quad , \quad v_{r,lm} \equiv a\Psi_{r,lm} \, .$$

Then the e.o.m. are

$$\frac{d^2u}{d\tau^2} + \left[k^2 - \frac{2 - 3\eta + 9\epsilon}{\tau^2}\right] u = 0$$

$$\frac{d^2 v_{r,lm}}{d\tau^2} + \left[k^2 - \frac{2 - 3\eta_{r,l} + 3\epsilon}{\tau^2}\right] v_{r,lm} = 0 \qquad \eta_{r,l} = M^2_{r,l}(\phi)/3H^2$$

At the time of Horizon crossing

$$P_{\mathcal{R}}|_\star \simeq \left(\frac{H^2}{2\pi\dot{\phi}}\right)^2_\star [1 + (-2 + 6C)\epsilon - 2C\eta]_\star$$

$$P_{\mathcal{S}^{(s)}_{mn}}|_\star \simeq \left(\frac{H^2}{2\pi\dot{\phi}}\right)^2_\star [1 + (-2 + 2C)\epsilon - 2C\eta_{ss}]_\star$$

when the mode leaves the horizon till Ne before the end of inflation

$$P_{\mathcal{R}}(N_e) \simeq P_{\mathcal{R}}|_\star$$

$$P_{\mathcal{S}^{(i)}_{mn}}(N_e) \simeq P_{\mathcal{S}^{(i)}_{mn}}|_* \exp\left[-2\int_0^{N_e} dN'_e B(N'_e)\right]$$

where $B_{r,l}(N_e) \equiv 2\epsilon - \eta + \eta_{r,l}$

$$B(N_e) \simeq 2\epsilon + (2\omega + \omega^2)\eta - sgn(V'_0)\sqrt{2\epsilon}\frac{M_P}{\phi}(4\omega+3)(\omega+2) + 6\frac{M_P^2}{\phi^2}(\omega+1)(\omega+2)$$

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I. Chaotic inflation $\frac{m^2}{2}\phi^2$

$$S = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}\partial_\mu\psi^{(i)}_{mn}\partial^\mu\psi^{(i)}_{nm} - \frac{1}{2}m^2\left[\phi^2 + \psi^{(i)}_{mn}\psi^{(i)}_{nm}\right] .$$

The system has SO $(3N^2)$ symmetry which is a specific realization of N-flation.

Physical observables: $n_R \simeq 0.966$ and $n_\psi \simeq 0.9998$ $\quad r \simeq 0.132.$

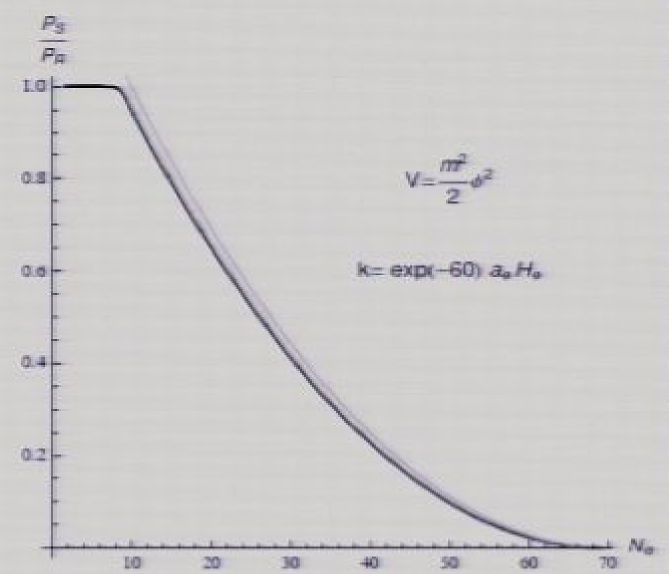
To fit the WMAP bound one requires

$$m \simeq 6.304 \times 10^{-6} M_P.$$

At the end of inflation Ps ~ 10^(-14) for mode of horizon scales.

Using our analytical formula

$$\frac{P_{S^{(i)}_{mn}}(N_e)}{P_{\mathcal{R}}|_*} \simeq (1 - N_e/60)^2$$



II. Chaotic inflation: $\frac{\lambda_{eff}}{4}\phi^4$

The potential is
$$V = \frac{\lambda_{eff}}{4}\phi^4 + \frac{\lambda_{eff}}{4}(\omega^2 - \omega)\,\phi^2\,\psi^{(i)}_{mn}\psi^{(i)}_{nm}\,.$$

The mass of entropy modes are different:

- *" The zero modes"* $\quad \lambda_{eff}\phi^2$
- *"The α modes":* $\quad M_l^2 = \frac{\lambda_{eff}}{2}(l+1)(l+2)\phi^2 \qquad 0 \le l < N$
- *"The β modes":* $\quad M_l^2 = \frac{\lambda_{eff}}{2}l(l-1)\phi^2 \qquad 0 < l < N$

The lowest mass states

- $l = 1$ β–mode $\qquad M^2_{\beta,1}(\phi) = 0.$

$$Ps_{\beta,1m} \simeq 3.949 \times 10^{-11} \qquad n_{\Psi_{\beta,1m}} \simeq 0.966$$

- zero mode, $l = 0$ $\alpha-$mode and $l = 2$ $\beta-$mode # $N^2 + 6$

$$M^2_{\beta,2}(\phi) = \lambda_{eff}\phi^2$$

$$P_{S_{\beta,2m}} \simeq 4.449 \times 10^{-13} \qquad n_{\Psi_{\beta,1m}} \simeq 0.9828.$$

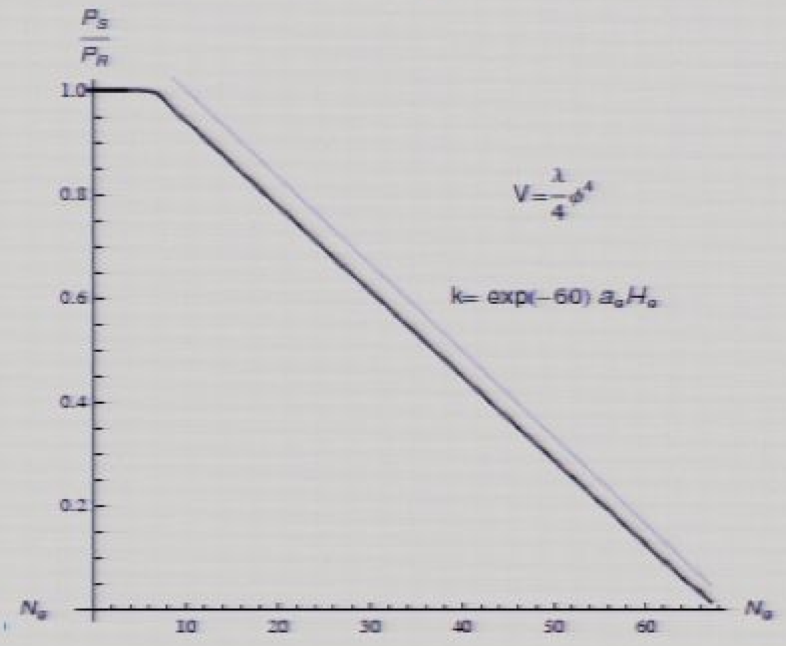
- $l = 1$ $\alpha-$mode $l = 3$ $\beta-$mode # 10

$$M^2_{\beta,3}(\phi) = 3\lambda_{eff}\phi^2$$

$$P_{S_{\beta,3m}} \simeq 3.967 \times 10^{-18} \qquad n_{\Psi_{\beta,3m}} \simeq 1.016$$

In general mass of a $l \geq 1$ $\alpha-$mode is identical to the $l+2$ $\beta-$mode.

Therefore, there are $4l+6$ iso-curvature modes with identical spectra all of which have a blue tilt.



From our analytical solution

$$\frac{P_{\mathcal{S}^{(i)}_{mn}}(N_e)}{P_{\mathcal{R}}|_*} \simeq (1-N_e/60)^{1+\frac{\omega^2-\omega}{2}} = \begin{cases} (1-N_e/60)^2 & \text{zero modes} \\ (1-N_e/60)^{(l^2+3l+4)/2} & \alpha-\text{modes} \\ (1-N_e/60)^{(l^2-l+2)/2} & \beta-\text{modes,} \end{cases}$$

III. Symmetry breaking potential:

$\phi > \mu$

l	M	Ps	ns
$l = 0\ \alpha$	$\lambda_{eff}\phi^2 - 2\kappa_{eff}\phi + m^2$	10^{-11}	0.981
$l = 1\ \alpha$	$3\lambda_{eff}\phi^2 - 4\kappa\phi + m^2$	10^{-15}	1.01
$l = 1\ \beta$	$2\kappa_{eff}\phi + m^2$	10^{-18}	1.002

$\mu/2 < \phi < \mu$

l	M	Ps	ns
$l = 0\ \alpha$		10^{-11}	0.987
$l = 1\ \alpha$		10^{-12}	0.988
$l = 1\ \beta$		10^{-15}	1.054

Motivation from string theory

- When N D-branes are located on top of each other the gauge symmetry enhances to U(N)

$$A_a = A_a^{(n)} T_n \quad , \quad F_{ab} = \partial_a A_b - \partial_b A_a + i[A_a, A_b]$$

$$D_a \Phi^i = \partial_a \Phi^i + i[A_a, \Phi^i]$$

The action for N coincident brane is

$$S = -T_3 \int d^4x \, \mathrm{STr}\left(\sqrt{-|g_{ab}|}\sqrt{|Q^i_j|}\right) + \frac{\mu_3}{2} \int d^4x \, \mathrm{STr}\left([\Phi_i, \Phi_j] C^{(6)}_{ij0123}\right)$$

Where

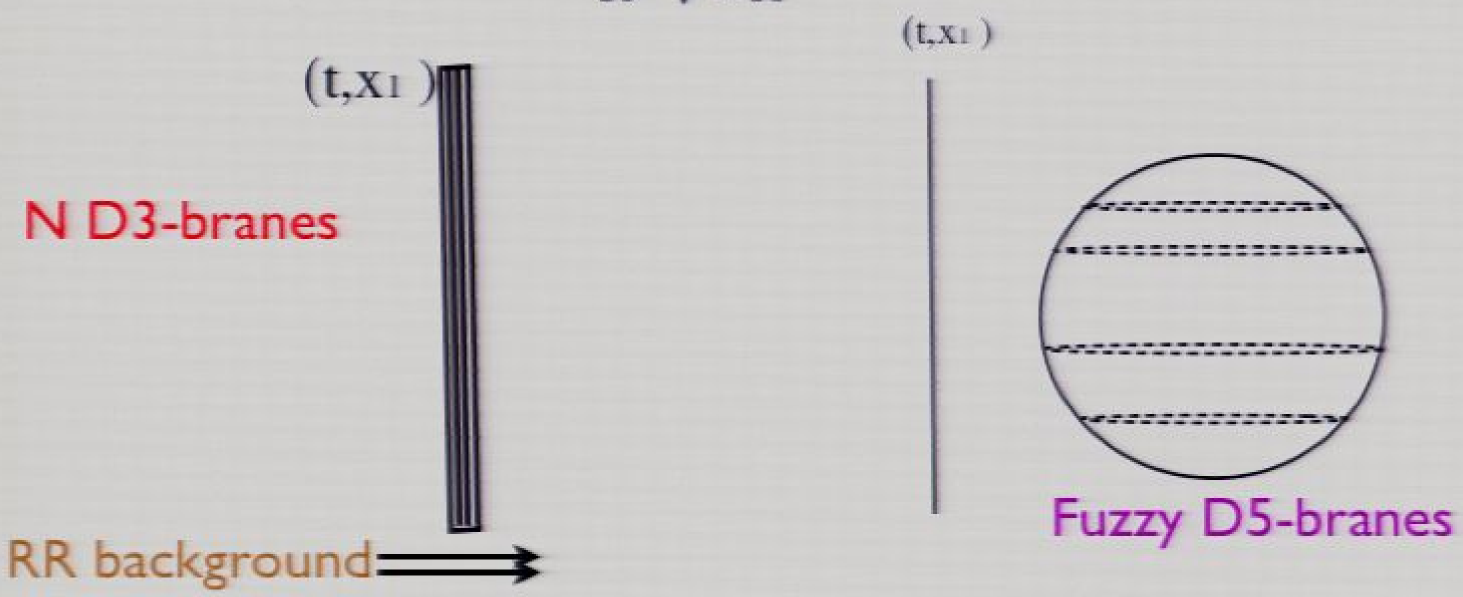
$$Q^j_k = \delta^i_j + 2\pi i \, \alpha' \, [\Phi_j, \Phi_k]$$

Consider the RR background $C^{(6)}_{jk0123} = -\frac{2i}{3}\kappa\,\epsilon_{jkl}\,\Phi_l$

Expanding the action up to leading terms, one obtains

$$S = -\frac{1}{2}\sum_i \mathrm{Tr}\,(\partial_\mu\Phi_i\partial^\mu\Phi_i) \;-\frac{\lambda}{4}[\Phi_i,\Phi_j][\Phi_i,\Phi_j] + \frac{i\kappa}{3}\epsilon_{jkl}[\Phi_k,\Phi_l]\Phi_j$$

with $\lambda = 2\pi g_s$, $\hat{\kappa} = \frac{\kappa}{g_s\cdot\sqrt{2\pi g_s}}$

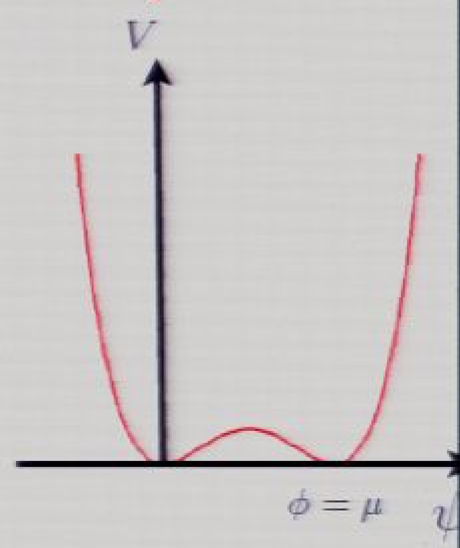


As mentioned the potential is

$$V_0(\phi) = \frac{\lambda_{eff}}{4}\phi^4 - \frac{2\kappa_{eff}}{3}\phi^3 + \frac{m^2}{2}\phi^2$$

The condition $\lambda m^2 = 4\kappa^2/9$ is required for background to be susy.

$$V_0 = \frac{\lambda_{eff}}{4}\phi^2(\phi - \mu)^2$$



The minimum $\phi = \mu$ is the susy vacuum.

This corresponds to the solution where N D-3 branes blow up into a fuzzy D5-branes.

Geometrically, ϕ is the radius of the fuzzy two-sphere.

Preheating

M-flation has a natural mechanism of preheating.

The time-dependence of $M(\phi)$, can leads to pair creation of ψ_{mn}

To be specific, let's consider $\lambda_{eff}\phi^4/4$ theory.

The preheating for $\lambda_{eff}\phi^4/4$ is studied by Greene et al, 1997

$$V_{\rm eff}(\phi,\chi) = \frac{1}{4}\lambda\phi^4 + \frac{1}{2}g^2\phi^2\chi^2$$

The structure of parametric resonance is completely determined by $g^2/\lambda.$

for our model $g^2/\lambda = n(n+1)/2$ $n = 1, l, l-1$

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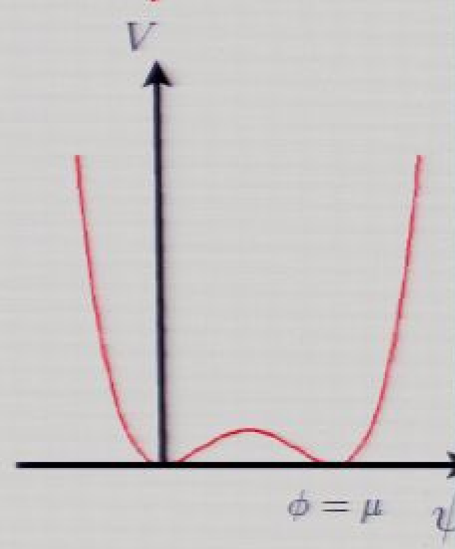

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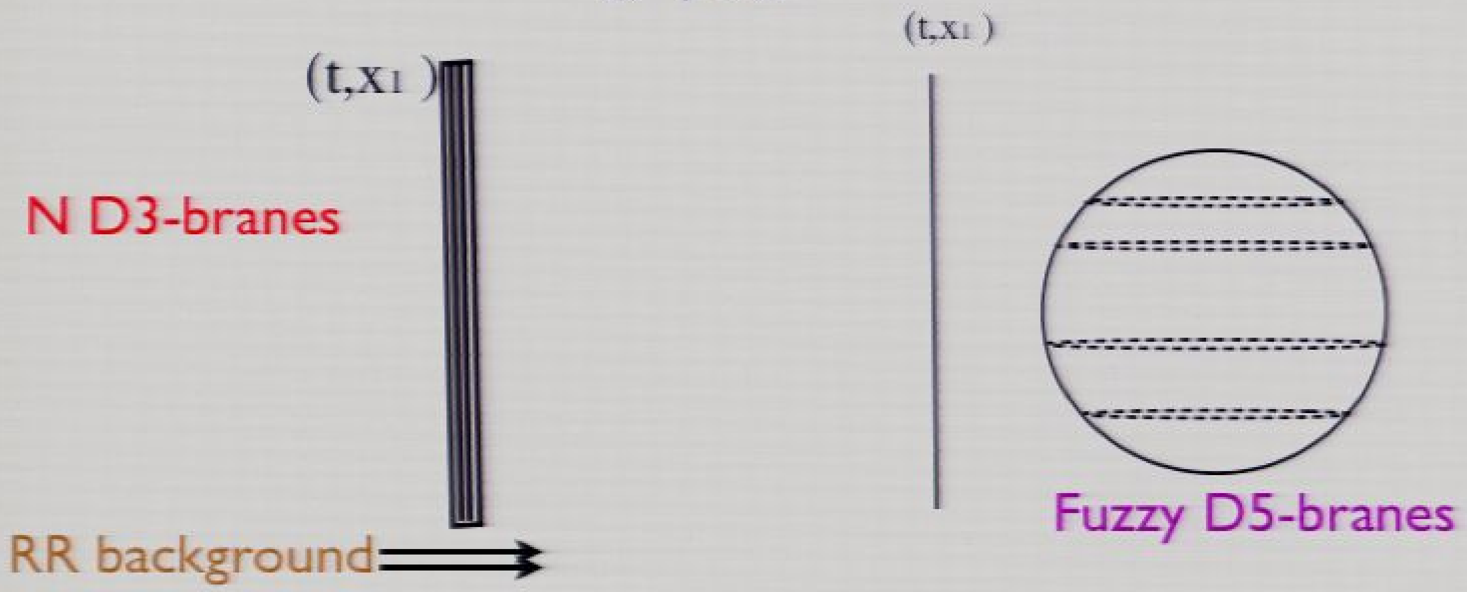

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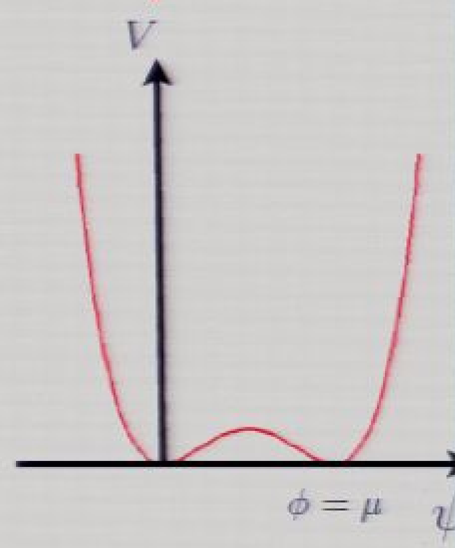

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Review of Greene et al:

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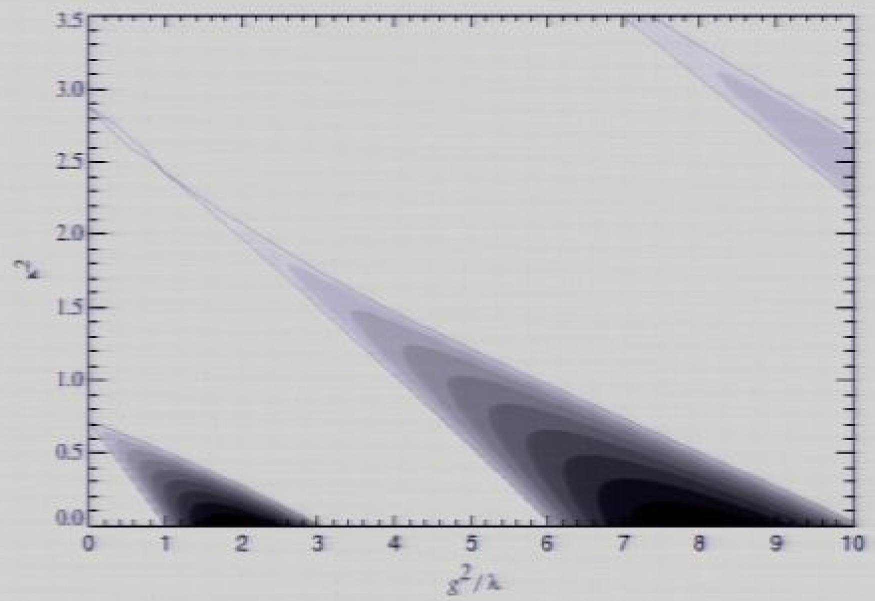

The analytical results are only known for

$$g^2/\lambda = n(n+1)/2$$

Some examples:

- $\frac{g^2}{\lambda} = 1$

  $$0 < \kappa^2 < \frac{1}{2}$$

  $$\mu_{max} \approx 0.1470 \text{ at } \kappa^2 \approx 0.228$$

- $\frac{g^2}{\lambda} = 3$

  $$\frac{3}{2} < \kappa^2 < \sqrt{3}$$

  $$\mu_{max} \approx 0.03598 \text{ at } \kappa^2 \approx 1.615$$

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- zero mode, $l = 0$ $\alpha-$mode and $l = 2$ $\beta-$mode # $N^2 + 6$

$$M^2_{\beta,2}(\phi) = \lambda_{eff}\phi^2$$

$$P_{S_{\beta,2m}} \simeq 4.449 \times 10^{-13} \qquad n_{\Psi_{\beta,1m}} \simeq 0.9828.$$

- $l = 1$ $\alpha-$mode $l = 3$ $\beta-$mode # 10

$$M^2_{\beta,3}(\phi) = 3\lambda_{eff}\phi^2$$

$$P_{S_{\beta,3m}} \simeq 3.967 \times 10^{-18} \qquad n_{\Psi_{\beta,3m}} \simeq 1.016$$

As mentioned the potential is

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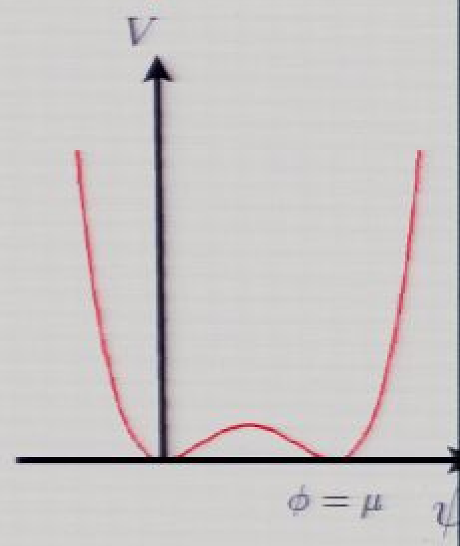

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Reheating ?

We have not provided a mechanism of reheating where the energy from the ψ_{mn} particles are transferred into SM particles.

One scenario in M-flation in string theory: We may imagine that SM fields are localized on branes as open strings gauge fields $A_\mu^{(a)}$

This can naturally be embedded in model noting that $D_a\Phi^i = \partial_a\Phi^i + i[A_a, \Phi^i]$

As an estimate of Preheat temperature, suppose we have an instant preheating

$$N^2T^4 \sim 3H^2M_P^2$$

for large N one can get sufficiently small reheat temperature.

Non-Gaussianity?

Due to multiple-field nature of the model, there would be plenty of NG produced. It would be interesting to calculate primordial NGs and compare it with observation.

Conclusion

- All observations strongly support inflation as a theory of early Universe and structure formation. But there is no deep theoretical understanding of its origin.
- M-flation is an interesting realization of inflation which is strongly supported from string theory. M-flation, like N-flation, can solve the fine-tunings associated with chaotic inflation and produce super-Planckian field during inflation.
- Due to Matrix nature of the fields there would be many scalar fields in the model. This leads to novel effects such as entropy productions, and no-Gaussianities which both are under intense observational investigations.
- M-flation has a natural built-in mechanism of preheating to end inflation. However, a mechanism of reheating has yet to be implemented.