Title: Inhomogeneities in Loop Quantum Cosmology: The Gowdy Model

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Abstract: As a necessary step towards the extraction of realistic results from Loop Quantum Cosmology, we analyze the physical consequences of including inhomogeneities. We consider a gravitational model in vacuo which possesses local degrees of freedom, namely, the linearly polarized Gowdy cosmologies. We carry out a hybrid quantization which combines loop and Fock techniques. We discuss the main results of this hybrid quantization, which include the resolution of the cosmological singularity, the construction of the Hilbert space of physical states, and the recovery of a conventional quantization for the inhomogeneities. In addition, an analysis of the model at the effective level confirms the robustness of the Big Bounce scenario, with preservation -or partial amplification- of the amplitudes of the inhomogeneous modes through the bounce in a statistical average.

INHOMOGENEITIES IN LOOP QUANTUM COSMOLOGY: THE GOWDY MODEL

Motivation

- The loop quantization of homogeneous cosmologies has been studied in detail. Besides, a satisfactory Fock quantization of inhomogeneous (Gowdy) cosmologies has been achieved.

- The simplest possibility to incorporate inhomogeneities is a hybrid quantization. The Gowdy T^3 model is a natural test bed.

- An initial singularity appears in the homogeneous solutions of the model (Bianchi I). How do inhomogeneities affect its resolution? Does the loop quantization of the zero modes suffice to resolve the singularity? (Different from the "BKL" approach).

- Does the Big Bounce persist? How are the inhomogeneities affected?

- Questions in mind are **internal time**, semiclassical behavior, validity of the Fock quantization, perturbative approaches, primordial fluctuations...

Classical Gowdy model: properties

- The Gowdy cosmologies are vacuum spacetimes with compact spatial topology and two commuting Killing vector fields.

- We consider the case of a T^3 topology and linear polarization.

- The classical metric is

$$ds^2 = e^{\tilde{\Gamma}[\phi]}\left(-dt^2 + d\theta^2\right) + t^2 e^{-\phi(t,\theta)} d\sigma^2 + e^{\phi(t,\theta)} d\delta^2 .$$

$\theta, \sigma, \delta \in S^1$. $\partial_\sigma, \partial_\delta$ are Killing vectors.

$$\phi(t,\theta) = \alpha + \beta \ln t + \sum_n [c_n J_0(nt) \sin(n\theta + \epsilon_n) + d_n N_0(nt) \sin(n\theta + \varepsilon_n)].$$

- Generically, $t=0$ is a curvature singularity.

- For vanishing inhomogeneities, we get Bianchi I with compact spatial sections.

Metric and degrees of freedom

- With a convenient gauge fixing, the reduced metric is

$$ds^2 = \frac{|p_1 p_2 p_3|}{4\pi^2}\left[e^{\Gamma[\xi,\, p_1]}\left(-\frac{N^2}{(2\pi)^4} dt^2 + \frac{d\theta^2}{(p_1)^2} \right) + e^{-\frac{2\pi\xi}{\sqrt{|p_1|}}} \frac{d\sigma^2}{(p_2)^2} + e^{\frac{2\pi\xi}{\sqrt{|p_1|}}} \frac{d\delta^2}{(p_3)^2} \right].$$

The spatial dependence of the metric is on θ via the field ξ, which has no zero mode. Γ is determined by $(\xi,\, p_1)$ and has no zero mode.

- Two "global" constraints remain in the system: the zero modes of the Hamiltonian and θ -difeomorphisms constraints.

- The zero modes can be seen as the degrees of freedom of a Bianchi I model with T^3 compact sections.

Variables and constraints

- ξ can be understood as a (gravitational wave) field propagating on the Bianchi I background.

- The Bianchi I background is described by the Ashtekar variables

$$(\tilde{E}^{BI})_i^a = \frac{p_i}{4\pi^2}\delta_i^a, \quad (A^{BI})_a^i = \frac{c^i}{2\pi}\delta_a^i, \quad \{c^i, p_j\} = 8\pi G\gamma\delta_j^i.$$

- We expand ξ and its momentum in Fourier modes, (ξ_m, P_ξ^m), and introduce the (free-field) creation and annihilation-like variables

$$(a_m, a_m^\star), \qquad a_m = \frac{|m|\xi_m + iK^2 P_\xi^m}{\sqrt{2|m|}K}, \qquad m \in \mathbb{Z} - \{0\}; \quad K = \sqrt{\frac{4G}{\pi}}.$$

Constraints

- The diffeomorphisms constraint generates S^1 translations.

$$C_\theta = \sum_{m>0} m\left(a_m^* a_m - a_{-m}^* a_{-m}\right).$$

It does not depend on the zero modes.

- Scalar constraint: Bianchi I plus the inhomogeneous Hamiltonian.

$$C_G := -\left(\frac{C_{BI}}{\gamma^2} + C_\xi\right),$$

$$C_{BI} = 2\frac{c^1 p_1 c^2 p_2 + c^1 p_1 c^3 p_3 + c^2 p_2 c^3 p_3}{\sqrt{|p_1 p_2 p_3|}}.$$

$$C_\xi = -\frac{4\pi^3 |p_1|}{\sqrt{|p_1 p_2 p_3|}}\left[\frac{(c^2 p_2 + c^3 p_3)^2}{16\pi^2\gamma^2 (p_1)^2}\sum |\xi_m|^2 + \sum\left\{\left(\frac{4G}{\pi}\right)^2 |P_\xi^m|^2 + m^2|\xi_m|^2\right\}\right].$$

Fock quantization: summary

- We adopt the complex structure naturally associated with the identification of $\{(a_m, a_m^*)\}$ as creation and annihilation operators.

We construct with it the symmetric Fock space F^{ξ}.

- The field dynamics and the gauge group of S^1 translations are unitarily implemented in this quantization.

- Given these unitary requirements the Fock quantization is unique, as well as the field parametrization of the (gauge-fixed) model.

- With a complete deparametrization, the physical Hilbert space is the subspace F^{ξ}_{phys} annihilated by the constraint

$$\hat{C}_{\theta} = \sum_{m>0} m(\hat{a}_m^{\dagger}\hat{a}_m - \hat{a}_{-m}^{\dagger}\hat{a}_{-m}).$$

- A basis of this physical space is provided by those "n-particle" states $|\{n_m\}\rangle$ with $\sum_{m>0} m(n_m - n_{-m}) = 0$.

Bianchi I: representation

- For $\{I\}=\{1,2,3\}$, we consider the algebra of almost periodic functions of c^I. We call Cyl_S^I the corresponding vector space. The Hilbert space H_{kin}^{BI} is the tensor product of $H_{kin}^{(I)}=L^2(\mathbb{R}_{Bohr}, d\mu_{Bohr}^I)$.

- Defining $N_{(\mu_I)}:=\langle c^I|\mu_I\rangle=\exp(i\mu_I c^I/2)$, the action of fluxes and holonomies is given by $(l_p^2=\hbar G)$

$$\hat{p}_I|\mu_I\rangle=4\pi\gamma l_p^2\mu_I|\mu_I\rangle, \quad \hat{N}_{(\mu_I^0)}|\mu_I\rangle=|\mu_I+\mu_I^0\rangle.$$

- Using standard methods of LQC, we get

$$\hat{C}_{BI}=\sum_{(I,J,K)=\pi(1,2,3)}\hat{\Delta}_I\hat{\Delta}_J\left[\widehat{\frac{1}{\sqrt{|p_K|}}}\right], \quad \Delta_I:=\frac{\sqrt{|p_I|}}{\bar{\mu}_I}sgn(p_I)\sin(\bar{\mu}_I c^I).$$

$$\hat{\Delta}_I=\frac{1}{2}\left[\widehat{\frac{|p_I|^{1/4}}{\sqrt{\bar{\mu}_I}}}\right]\left[\widehat{\sin(\bar{\mu}_I c^I)}\widehat{sgn(p_I)}+\widehat{sgn(p_I)}\widehat{\sin(\bar{\mu}_I c^I)}\right]\left[\widehat{\frac{|p_I|^{1/4}}{\sqrt{\bar{\mu}_I}}}\right].$$

Singularity resolution

- We adopt an improved dynamics proposal $\bar{\mu}_I^{-1} \propto \widehat{\sqrt{|p_I|}}$.

- We change basis:

$$\hat{p}_I |v_I\rangle = 4\pi\gamma l_p^2 \, sgn(\mu_I) (D|v_I|)^{2/3} |v_I\rangle, \quad v_I := \frac{sgn(\mu_I)}{D} |\mu_I|^{3/2}, \quad D = \frac{3^{5/4}}{2^{3/2}}.$$

$$\left[\widehat{\frac{1}{\sqrt{|p_I|}}}\right] |v_I\rangle = \frac{1}{\sqrt{\gamma} l_p} B(v_I) |v_I\rangle, \quad B(v) := \sqrt{\frac{3^{7/6}}{2^3 \pi}} |v|^{1/3} \left| |v+1|^{1/3} - |v-1|^{1/3} \right|.$$

- With our factorization: $\hat{C}_{BI} = \sum \hat{\Delta}_I \hat{\Delta}_J \left[\widehat{\frac{1}{\sqrt{|p_K|}}}\right]$,

$$\hat{\Delta}_I = a \widehat{\sqrt{|p_I|}} \left[\widehat{\sin(\bar{\mu}_I c^I)} \, \widehat{sgn(p_I)} + \widehat{sgn(p_I)} \, \widehat{\sin(\bar{\mu}_I c^I)} \right] \widehat{\sqrt{|p_I|}}, \quad a = (8\sqrt{3}\pi\gamma l_p^2)^{-1/2}$$

- $\hat{C}^{BI}$ annihilates all the "zero volume" states: states in the basis $\{|v_1\rangle \otimes |v_2\rangle \otimes |v_3\rangle\}$ with any $v_I = 0$.

- These zero volume states decouple. The singularity is resolved.

Densitized constraint

- We call $\bar{Cyl}_S^I$ and $\bar{H}_{kin}^{(I)}$ the space of cylindrical functions and the kinematical Hilbert space without zero volume states. We restrict to them in the following.
- The operator $\left[\widehat{\frac{1}{\sqrt{|p_I|}}}\right]$ is invertible in $\bar{H}_{kin}^{(I)}$. In the dual $\bar{Cyl}_S^{I\star}$ we can introduce the map:

$$(\psi| \rightarrow (\psi| \left[\widehat{\frac{1}{\sqrt{|p_I|}}}\right]^{1/2}.$$

Solutions to the Bianchi I constraint (in the dual) are now annihilated by

$$\hat{\hat{C}}_{BI} = \left[\widehat{\frac{1}{\sqrt{|p_1 p_2 p_3|}}}\right]^{-1/2} \hat{C}_{BI} \left[\widehat{\frac{1}{\sqrt{|p_1 p_2 p_3|}}}\right]^{-1/2}.$$

We do not change the representation, and continue in $\bar{H}_{kin}^{BI}$.

Densitized constraint (2)

- We get $\hat{\hat{C}}_{BI}=2\left(\hat{\Theta}_1\hat{\Theta}_2+\hat{\Theta}_1\hat{\Theta}_3+\hat{\Theta}_2\hat{\Theta}_3\right),$

$$\hat{\Theta}_I=a\left[\widehat{\frac{1}{\sqrt{|p_I|}}}\right]^{-1/2}\widehat{\sqrt{|p_I|}}\left[\widehat{\sin(\bar{\mu}_I c^I)}\,\widehat{sgn(p_I)}+\widehat{sgn(p_I)}\,\widehat{\sin(\bar{\mu}_I c^I)}\right]\widehat{\sqrt{|p_I|}}\left[\widehat{\frac{1}{\sqrt{|p_I|}}}\right]^{-1/2}$$

$$\hat{\Theta}_I|v_I\rangle=-\pi i\gamma l_p^2\left(f_+(v_I)|v_I+2\rangle-f_-(v_I)|v_I-2\rangle\right),$$

$$f_\pm(v):=g(v\pm 2)s_\pm(v)g(v),\quad g(v):=\left|\left|1+\frac{1}{v}\right|^{1/3}-\left|1-\frac{1}{v}\right|^{1/3}\right|^{-1/2},$$

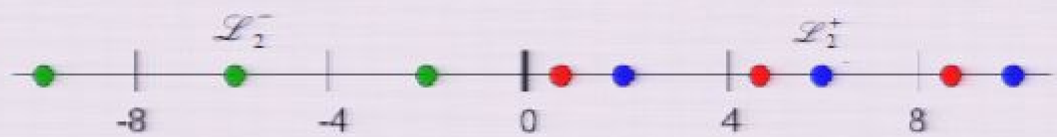
$$g(0):=0,\quad s_\pm(v):=sgn(v\pm 2)+sgn(v).$$

- The $\hat{\Theta}_I$ are observables in Bianchi I.
- Owing to the presence of the signs, $f_+(v_I)$ $\left(f_-(v_I)\right)$ vanishes in the whole interval $[-2,0]$ $\left([0,2]\right)$.

Superselection and no-boundary

- $$\hat{\Theta}_I|v_I\rangle = -\pi i \gamma l_p^2 \left(f_+(v_I)|v_I+2\rangle - f_-(v_I)|v_I-2\rangle \right),$$

$\hat{\Theta}_I$ does not mix the semilattices $\mathscr{L}^{\mp}_{\epsilon_I} := \{ \mp(\epsilon_I + 2n),\ \ n \in \mathbb{N} \}$, with ϵ_I in (0,2].



- We call $\bar{H}^{\mp}_{\epsilon_I}$ the subspaces of states with support in these semilattices. Each of them provides a superselection sector.

- The WDW analog of $\hat{\Theta}_I$ would be $8\pi i \gamma G p_I \partial_{p_I}$.

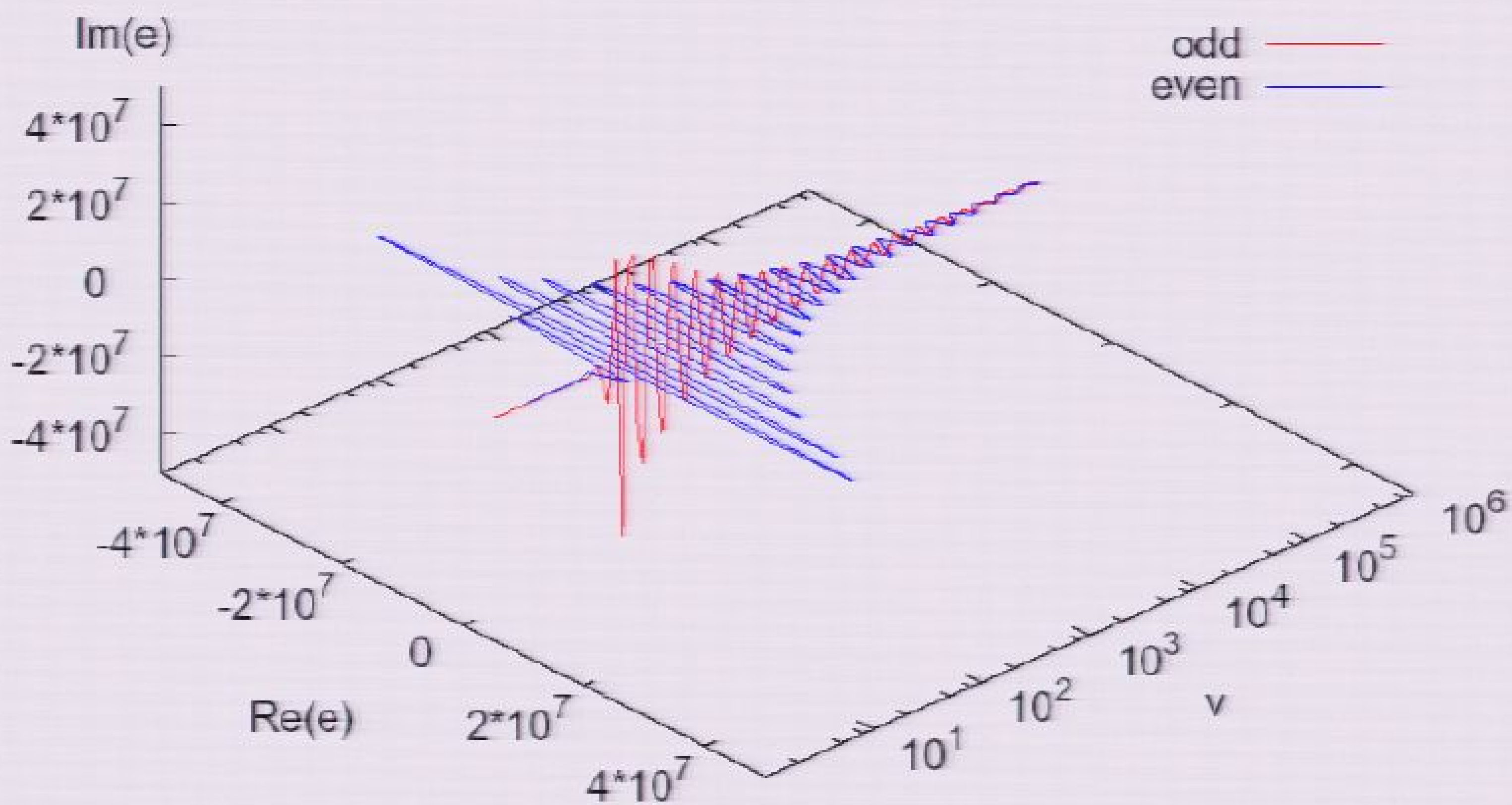
- Although $\hat{\Theta}_I$ is instead a "second-order" difference operator, when one hits the origin one gets a consistency relation. In this sense, the constraint equation provides a no-boundary.

- $\hat{\Theta}_I$ (with the natural domain) is essentially self-adjoint on $\bar{H}^{\mp}_{\epsilon_I}$.

Spectrum and eigenfunctions

- $\hat{\Theta}_I$ has an absolutely continuous spectrum.

$$1^{\mp}_{\epsilon_I} = \int_{-\infty}^{\infty} d\lambda \left| e_\lambda^{\mp\epsilon_I} \right\rangle \left\langle e_\lambda^{\mp\epsilon_I} \right|, \quad \hat{\Theta}_I \left| e_\lambda^{\mp\epsilon_I} \right\rangle = \lambda \gamma l_p^2 \left| e_\lambda^{\mp\epsilon_I} \right\rangle.$$



Physical states for Bianchi I

- $$\hat{\tilde{C}}_{BI}=2\left(\hat{\Theta}_1\hat{\Theta}_2+\hat{\Theta}_1\hat{\Theta}_3+\hat{\Theta}_2\hat{\Theta}_3\right).$$

Since $\hat{\Theta}_I$ are observables and we know their associated resolution of the identity, it is straightforward to solve the constraint.

- The same results can be obtained with group averaging. Physical states have the form

$$\psi(v_1,v_2,v_3)=\int_{-\infty}^{\infty}d\lambda_2\int_{-\infty}^{\infty}d\lambda_3\,\tilde{\psi}(\lambda_2,\lambda_3)\,e^{\epsilon_1}_{\lambda_1[\lambda]}(v_1)\,e^{\epsilon_2}_{\lambda_2}(v_2)\,e^{\epsilon_3}_{\lambda_3}(v_3)$$

with the Hilbert structure $\tilde{\psi}\in H^{BI}:=L^2(\mathbb{R}^2,d\lambda_2 d\lambda_3|\lambda_2+\lambda_3|)$ and

$$\lambda_1[\lambda]=-\lambda_2\lambda_3/(\lambda_2+\lambda_3).$$

- A complete set of observables is given by $\hat{\Theta}_2,\hat{\Theta}_3,\hat{v}_2|_{v_1^0},\hat{v}_3|_{v_1^0},$ for any given section v_1^0.

Hybrid model

- The kinematical Hilbert space is $H^{BI}_{kin} \otimes F^{\xi}_{phys}$.

- Including the inhomogeneities in the constraint, the zero volume states still decouple. We restrict ourselves to $\bar{H}_{kin} := \bar{H}^{BI}_{kin} \otimes F^{\xi}_{phys}$.

Proceeding as in Bianchi I, we arrive at the densitized constraint:

$$\hat{\tilde{C}}_G = -\frac{\hat{\tilde{C}}_{BI}}{\gamma^2} - \hat{\tilde{C}}_{\xi}, \quad \hat{\tilde{C}}_{\xi} = -l_p^2 \left\{ \frac{(\hat{\Theta}_2 + \hat{\Theta}_3)^2}{\gamma^2} \left[\widehat{\frac{1}{\sqrt{|p_1|}}} \right]^2 \hat{\mathrm{H}}^{\xi}_{Inter} + 32\pi^2 \widehat{|p_1|} \hat{\mathrm{H}}^{\xi}_0 \right\},$$

$$\hat{\mathrm{H}}^{\xi}_{Inter} := \sum \frac{1}{2|m|} \left(2\hat{a}^{\dagger}_m \hat{a}_m + \hat{a}^{\dagger}_m \hat{a}^{\dagger}_{-m} + \hat{a}_m \hat{a}_{-m} \right), \quad \hat{\mathrm{H}}^{\xi}_0 := \sum |m| \hat{a}^{\dagger}_m \hat{a}_m .$$

- $\hat{\Theta}_2$ and $\hat{\Theta}_3$ are **observables,** but $\hat{\Theta}_1$ is not.

If we view the constraint as an evolution equation, p_1 plays the role of **internal time**. Essentially, this was the time chosen to deparametrize the system in previous quantizations.

Densitized constraint

$$\hat{\tilde{C}}_G = -\frac{2}{\gamma^2}\left(\hat{\Theta}_1\hat{\Theta}_2 + \hat{\Theta}_1\hat{\Theta}_3 + \hat{\Theta}_2\hat{\Theta}_3\right) + l_p^2\left\{\frac{\left(\hat{\Theta}_2+\hat{\Theta}_3\right)^2}{\gamma^2}\left[\widehat{\frac{1}{|p_1|}}\right]\hat{\mathrm{H}}^{\xi}_{Inter} + 32\pi^2\widehat{|p_1|}\hat{\mathrm{H}}^{\xi}_0\right\},$$

$$\hat{\Theta}_I|v_I\rangle = -\pi i\gamma l_p^2\left(f_+(v_I)|v_I+2\rangle - f_-(v_I)|v_I-2\rangle\right),$$

$$\hat{\mathrm{H}}^{\xi}_{Inter} := \sum\frac{1}{2|m|}\left(2\hat{a}^\dagger_m\hat{a}_m + \hat{a}^\dagger_m\hat{a}^\dagger_{-m} + \hat{a}_m\hat{a}_{-m}\right), \quad \hat{\mathrm{H}}^{\xi}_0 := \sum|m|\hat{a}^\dagger_m\hat{a}_m.$$

- We restrict to (superselection) $\bar{H}^{\mp}_{\epsilon_1}\otimes\bar{H}^{\mp}_{\epsilon_2}\otimes\bar{H}^{\mp}_{\epsilon_3}\otimes F^{\xi}_{phys}$.
- We define $\hat{\tilde{C}}_G$ with domain the span of

$$\left\{|v_1\rangle\otimes|v_2\rangle\otimes|v_3\rangle\otimes|\{n_m\}\rangle = |v_1,v_2,v_3,\{n_m\}\rangle;\, v_I\in\mathscr{L}^{\mp}_{\epsilon_I},\ |\{n_m\}\rangle\in F^{\xi}_{phys}\right\}.$$

The operator is well-defined and symmetric.

- The (complex) eigenvalue equation for $\hat{\tilde{C}}_G$ leads to

$$\left(\Psi\middle|\hat{\tilde{C}}_G\middle|v_1,v_2,v_3\{n_m\}\right\rangle = \rho\gamma^2 l_p^4\left(\Psi\middle|v_1,v_2,v_3\{n_m\}\right\rangle, \quad \rho\in\mathbb{C}.$$

Eigenvalue equation: solutions

- Substituting $(\Psi| = \sum_{v_1} \int_{\mathbb{R}^2} d\lambda_2 d\lambda_3 \langle v_1| \otimes \langle e^{\epsilon_2}_{\lambda_2}| \otimes \langle e^{\epsilon_3}_{\lambda_3}| \otimes (\Psi[v_1, \lambda_2, \lambda_3]|,$

we get
$$(\Psi[\epsilon_1+2M]|\{n_m\}\rangle = (\Psi[\epsilon_1]| \sum_{\{r_i\}\cup\{s_j\}\in O(M)} \left[\Pi_{r_i} F(\epsilon_1+2r_i+2)\right] \times P\left[\Pi_{s_j} \hat{\mathrm{H}}^{\xi}_{\rho}[\epsilon_1+2s_j]\right]|\{n_m\}\rangle,$$

$$F(v_1) := \frac{f_-(v_1)}{f_+(v_1)}, \qquad \hat{\mathrm{H}}^{\xi}_{\rho}[v_1] := \frac{i}{2\pi(\lambda_2+\lambda_3) f_+(v_1)} \times\left[\rho + 2\lambda_2\lambda_3 - \frac{(\lambda_2+\lambda_3)^2}{\gamma} B^2(v_1)\hat{\mathrm{H}}^{\xi}_{Inter} - 2^6 3^{5/6} \pi^3 \gamma |v_1|^{2/3} \hat{\mathrm{H}}^{\xi}_0\right].$$

$O(M)$ is the set of paths from 0 to M with jumps of 1 or 2 steps.
$\{s_j\}$ are the points followed by a jump of 1 step.
P denotes path ordering.

Physical states

- Solutions to the constraint correspond (formally) to $\rho=0$.

They are **determined** by the initial data $(\Psi[\epsilon_1]|$.

- If we identify solutions to the constraint with initial data, observables are operators acting on these data.

A complete set is provided by the observables introduced for Bianchi I and a complete set acting on the inhomogenous modes. It is easier to relax the S^1 symmetry and consider, e.g., the operators representing

$$\left\{\left(\xi_m+\xi_{-m}, i\xi_m-i\xi_{-m}, P_\xi^m+P_\xi^{-m}, iP_\xi^m-iP_\xi^{-m}\right);\; m\in\mathbb{N}^+\right\}.$$

Reality conditions select the space $L^2(\mathbb{R}^2, d\lambda_2 d\lambda_3|\lambda_2+\lambda_3|)\otimes F^\xi$.
Restoring the S^1 symmetry, we finally arrive at

$$L^2(\mathbb{R}^2, d\lambda_2 d\lambda_3|\lambda_2+\lambda_3|)\otimes F^\xi_{phys}.$$

Effective dynamics

- In order to extract physics, we appeal to the effective dynamics.

$$\tilde{C}_G = -\frac{2}{\gamma^2}\left(\Theta_1\Theta_2 + \Theta_1\Theta_3 + \Theta_2\Theta_3\right) + l_p^2\left\{\frac{\left(\Theta_2+\Theta_3\right)^2}{\gamma^2}\frac{1}{|p_1|}\mathrm{H}^{\xi}_{Inter} + 32\pi^2|p_1|\mathrm{H}^{\xi}_0\right\},$$

$$\Theta_I = \frac{p_I\sqrt{|p_I|}}{M}\sin\left(\frac{M c^I}{\sqrt{|p_I|}}\right), \qquad M = l_p\sqrt{2\sqrt{3}\pi\gamma}$$

$$\mathrm{H}^{\xi}_{Inter} := \sum \frac{1}{2|m|}\left(2a^{\dagger}_m a_m + a^{\dagger}_m a^{\dagger}_{-m} + a_m a_{-m}\right), \quad \mathrm{H}^{\xi}_0 := \sum |m| a^{\dagger}_m a_m .$$

- Θ_2 and Θ_3 are constants of motion.
- A bounce in p_I is characterized by $|p_I|^3 = M^2\Theta_I^2$.
- Bounces for p_2 and p_3 are not affected by inhomogeneities.

Effective dynamics (2)

- For a bounce in p_1,

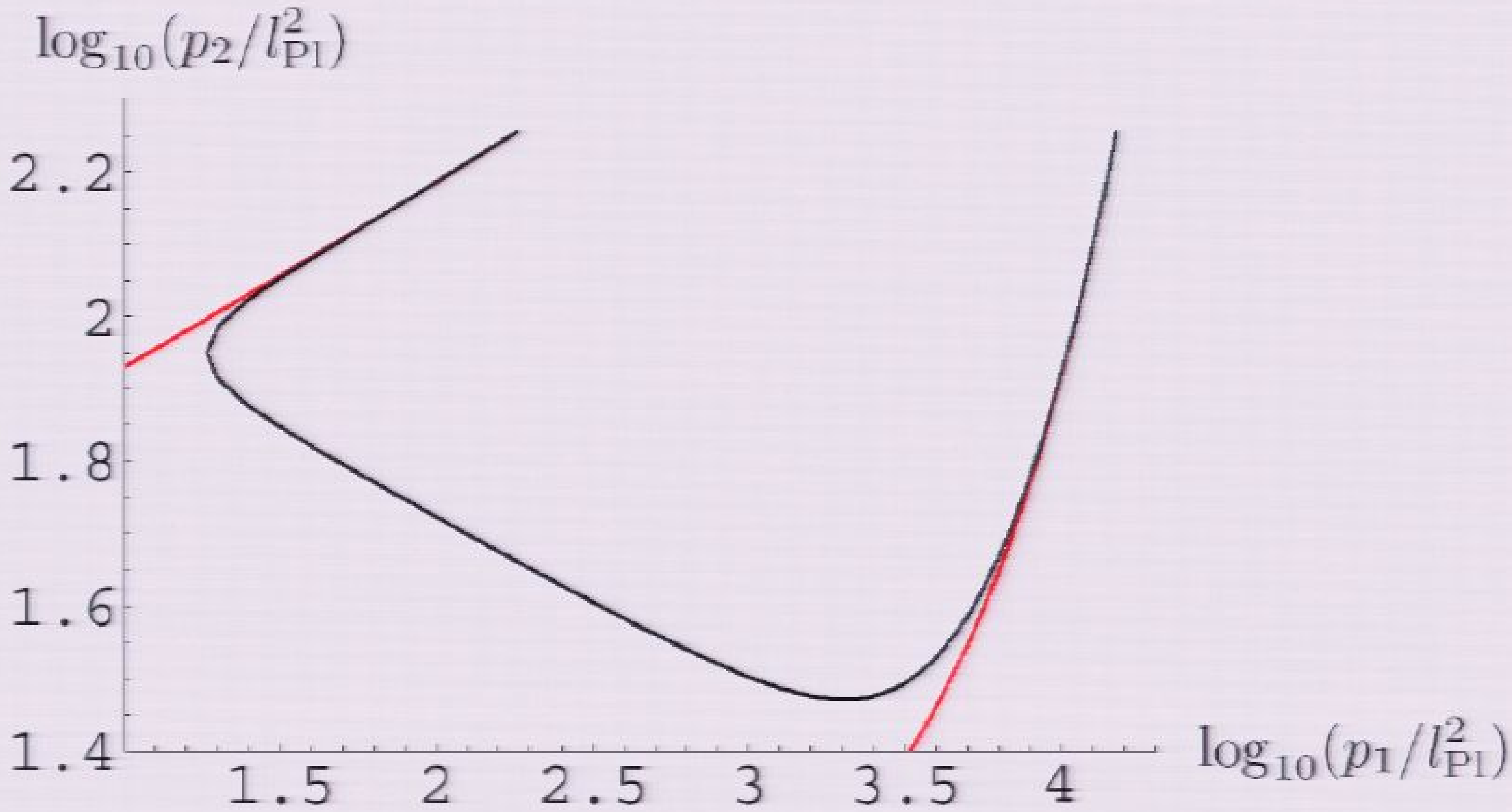
$$|p_1^3| = \frac{M^2}{(\Theta_2+\Theta_3)^2}\left[l_p^2 \frac{(\Theta_2+\Theta_3)^2 \mathrm{H}^{\xi}_{Inter}}{2|p_1|} + 16\pi^2\gamma^2 l_p^2 |p_1| \mathrm{H}^{\xi}_0 - \Theta_2\Theta_3\right]^2,$$

Since $\mathrm{H}^{\xi}_{Inter}, \mathrm{H}^{\xi}_0 \geq 0$, the inhomogeneities increase the value of p_1 when $\Theta_2\Theta_3 < 0$.

- The dynamics was probed using Monte-Carlo methods, for a population of 10^4 points, with a_m generated with Gaussian probability except for the S^1 translation constraint.
- The number of modes was restricted to $m \leq m_{max} = 5, 10, 20$.
- The equations of motion were integrated with Mathematica.
- Special attention was paid to the case $\Theta_2\Theta_3 > 0$.

Effective bounce

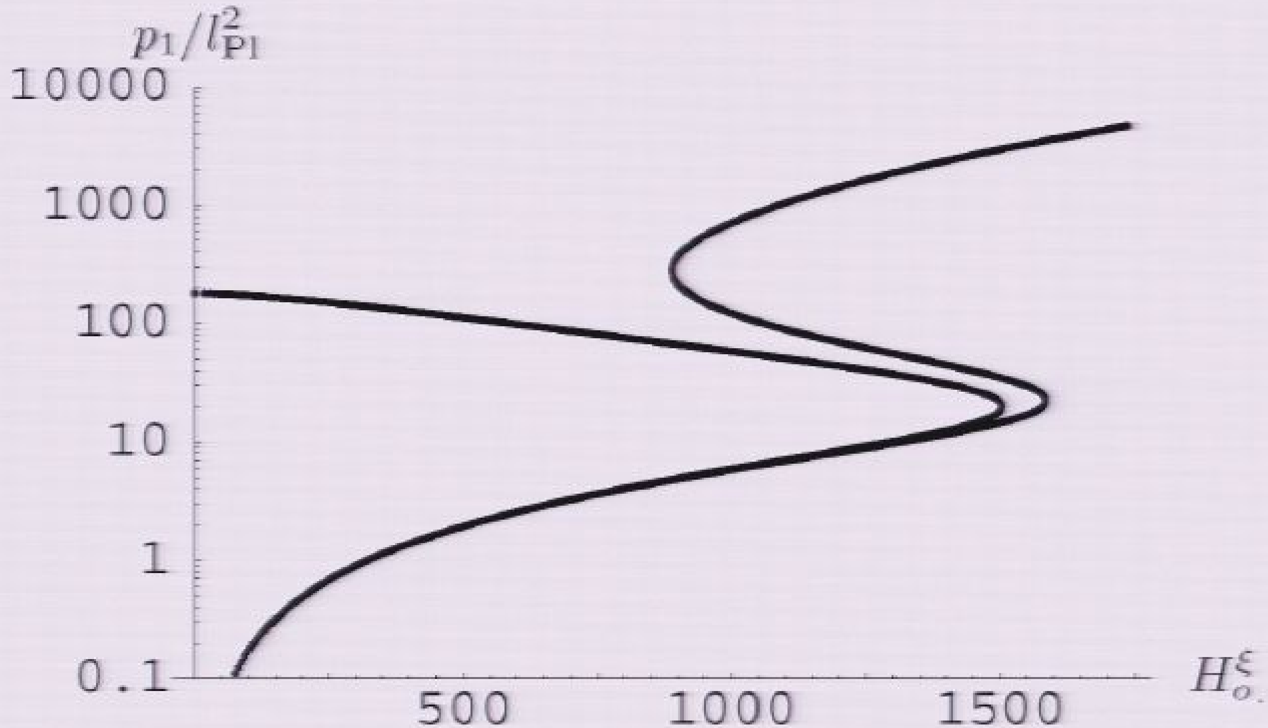
- The numerical analysis confirms the bounce in all three directions.



Initially $p_1 = p_2 = p_3 = 180\, l_p^2$, $\Theta_2 = \Theta_3 = 100\, l_p^2$, $\mathrm{H}_0^\xi = 0.657$,
$\mathrm{H}_{Inter}^\xi = 0.0507$. $m_{max} = 5$. $\gamma = 0.2375...$

Big Bounce in p_1

- Possible values of p_1 at a bounce for the case $\Theta_2\Theta_3>0$.



Here,

$\Theta_2=3750\,l_p^2$,

$\Theta_3=2500\,l_p^2$,

$H_{Inter}^\xi/H_0^\xi=2\ 10^{-4}$.

There might exist critical trajectories with no bounce, but they correspond to a zero mesaure set of initial data.

- Besides, the bounce happens typically at a value of p_1 which is at least 10% that found in the absence of inhomogeneities.

Big Bounce and inhomogeneities $(\Theta_2 \Theta_3 > 0)$

- We have studied the behavior of $\Delta|a_m| = \lim_{t\to\infty}|a_m| - \lim_{t\to-\infty}|a_m|$, which is always well defined.

- We take an statistical average, disregarding phases.

- $\Delta|a_m|$ is antisymmetric in the phase of a_m in the sector where the *inhomogeneities dominate* the bounce dynamics.

  Then, the amplitudes of the gravitational waves are statistically preserved through the bounce.

- In the sector where the vacuum dynamics is approximately valid around the bounce, $\Delta|a_m|$ is positive in average, so that the bounce pumps energy into the inhomogeneities.

- The results are valid for all values of m_{max}.

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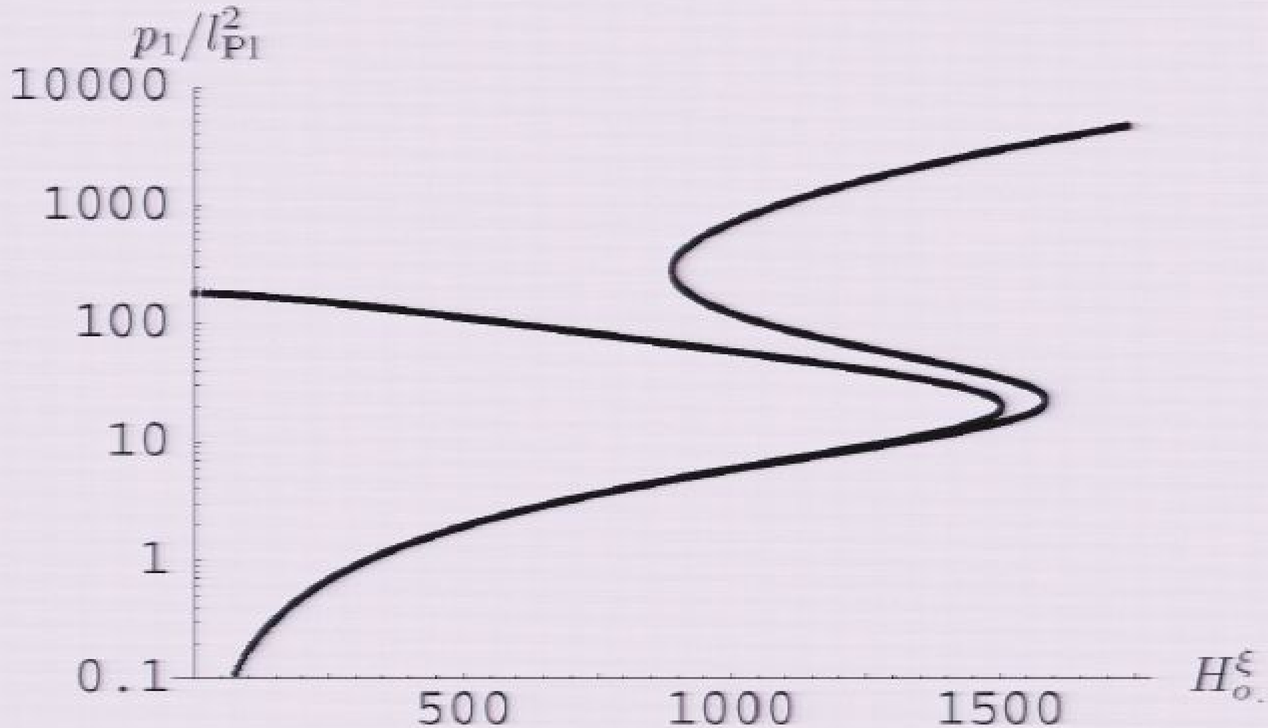
- Possible values of p_1 at a bounce for the case $\Theta_2\Theta_3>0$.



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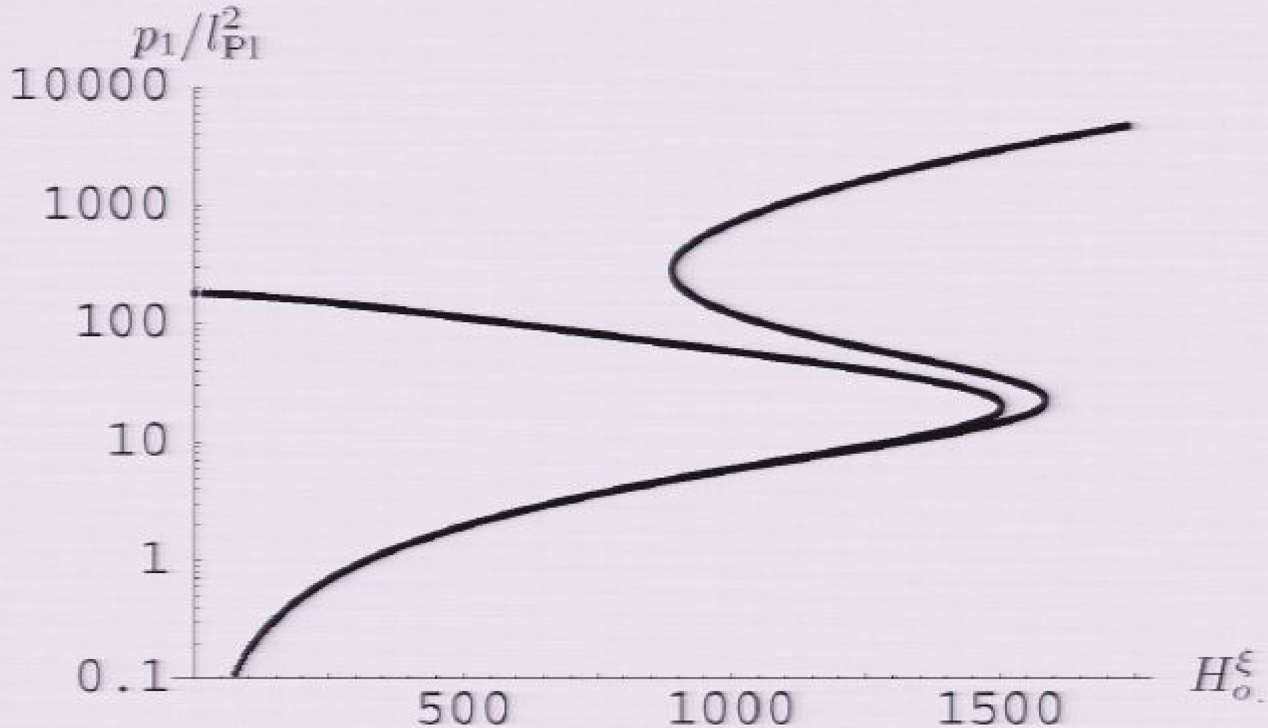

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Conclusions

- By combining the loop quantization of Bianchi I and the Fock quantization of the Gowdy model, we have constructed a hybrid quantization of these cosmologies in **vacuo**.

- We have obtained a well-defined constraint, found the solutions, and proceeded to determine the **physical states** and observables.

- The physical Hilbert space is equivalent to that of the **Fock** quantization.

- The initial singularity is avoided owing to the loop quantization of the **zero modes**.

- In the effective dynamics, the **Big Bounce** persists. The universe size at the bounce is not radically changed by the inhomogeneities.

- For each gravitational wave mode, the amplitude at early and late times is either preserved or increased in norm in statistical average.