Title: The quantization of unimodular gravity and the cosmological constant problem

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Abstract: TBA

Unimodular quantum gravity as a solution to the cosmological constant problem

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May 09

1. The basic idea of unimodular gravity
2. A better formulation of it due to Henneaux and Teitelboim
3. Hamiltonian formulation
4. The path integral quantum theory is unimodular
5. Why is the cosmo constant is so small
6. Hamiltonian quantization, time and the inner product

arXiv:0904.4841

he cosmological constant problems

1. Why the cosmo constant is not enormous.
2. Why it has the particular value it has
3. Coincidence problems.

lany people have worked on unimodular gravity and commented nat it might have something to do with the cosmological onstant problems

Einstein	1919	
Zee	1985	
Sorkin		
Unruh		
Weinberg		
Ng and van Dam		
Henneaux and Teitelboim		1991
Bombelli, Couch, Torrence		1991

The basic idea:

The basic idea of unimodular gravity:

$$S^{uni} = \int_{\mathcal{M}} \epsilon_0 \left(-\frac{1}{8\pi G} \bar{g}^{ab} R_{ab} + \mathcal{L}^{matter}(\bar{g}_{ab}, \psi) \right)$$

let(g) has been constrained to be equal to a fixed volume element:

$$\sqrt{-g} = \epsilon_0$$

The diffeomorphism group is reduced to volume preserving diffeo's:

$$\partial_a(\epsilon_0 v^a) = 0$$

The eq's of motion are just the tracefree part of Einstein

$$R_{ab} - \frac{1}{4}\bar{g}_{ab} R = 4\pi G \left(T_{ab} - \frac{1}{4}\bar{g}_{ab} T \right)$$

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This has a decoupling symmetry:

$$T_{ab} \rightarrow T'_{ab} = T_{ab} + g_{ab}C$$

This means that contributions to the energy-momentum tensor proportional to the metric don't couple to gravity!

$$R_{ab} - \frac{1}{4}\bar{g}_{ab}R = 4\pi G\left(T_{ab} - \frac{1}{4}\bar{g}_{ab}T\right)$$

The divergence of this yields

$$\partial_a\left(R + 4\pi GT\right) = 0$$

which implies that there is a constant $-4\Lambda = R + 4\pi GT$

so one gets the Einstein equations with an arbitrary Λ

$$G_{ab} - \Lambda g_{ab} = 4\pi GT_{ab}$$

The decoupling symmetry is still present:

$$T_{ab} \to T'_{ab} = T_{ab} + g_{ab}C$$

now implies also

$$\Lambda \to \Lambda - 4\pi GC$$

Unimodular gravity is not a new theory, it is a reformulation of GR.

Why isn't this the solution to the first cosmological constant problem? Or, why isn't the fact that Λ is not Planck scale evidence that this is the right formulation of GR for coupling to quantum physics?

Weinberg discussed this in his 1989 review and said:

> *"In my view, the key question in deciding whether this is a plausible classical theory of gravitation is whether it can be obtained as the classical limit of any physically satisfactory [quantum] theory of gravitation."*

We will study this problem and see that the answer is YES.

The Henneaux-Teitelboim formulation

The Henneaux-Teitelboim reformulation of unimodular gravity

They introduce a new field, which is a three form a_{bcd}

$$\tilde{a}^a = \frac{1}{3!}\epsilon^{abcd} a_{bcd}$$

There is then a four form

$$b_{abcd} = da_{abcd} \qquad \tilde{b} = \frac{1}{4!}\epsilon^{abcd} b_{abcd} = \partial_a \tilde{a}^a$$

$$S^{HT} = \int_{\mathcal{M}} \sqrt{-g}\left(-\frac{1}{8\pi G}(\bar{g}^{ab} R_{ab} + \phi) + \mathcal{L}^{matter}\right) + \frac{1}{8\pi G}\phi\tilde{b}$$

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This has full spacetime diffeo invariance and an additional gauge invariance: a→ a + dr and also an additional field ϕ

Equations of motion:

$$G_{ab} - \phi g_{ab} = 4\pi G T_{ab}$$

$$\sqrt{-g} = \tilde{b}$$

$$\partial_a \phi = 0$$

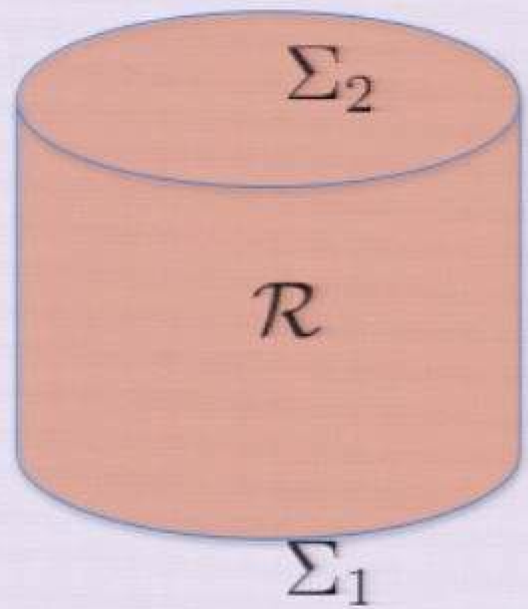
so $\phi = \Lambda$ a constant

Decoupling symmetry:

$$T_{ab} \rightarrow T_{ab} + g_{ab}C, \qquad \phi \rightarrow \phi - 4\pi G C$$

The a_{bcd} field measures a global time:

$$\int_{\Sigma_2} a - \int_{\Sigma_1} a = Vol = \int_{\mathcal{R}} \sqrt{-g}$$



he canonical route to quantum gauge theories:

;tart with the classical action

Vork out Hamiltonian formulation

Gauge symmetries imply constraints

;auge fix to get deterministic dynamics in phase space.

:onstruct fully gauge fixed path integral in phase space

"Faddeev-Poppov"

Vork backwards to configuration space path integral

:onstruct quantum effective action for averaged fields.

Question: is the resulting quantum effective action unimodular?

so, the decoupling symmetry is present quantum mechanically!

The answer is **YES.**

Hamiltonian formulation

amiltonian formulation of Henneaux-Teitelboim:

$$S^{HT} = \int_{\mathcal{M}} \sqrt{-g}\left(-\frac{1}{8\pi G}(\bar{g}^{ab}R_{ab} + \phi) + \mathcal{L}^{matter}\right) + \frac{1}{8\pi G}\phi\tilde{b}$$

Canonical momenta: $\tilde{\pi}^{ij} = \sqrt{q}(k^{ij} - q^{ij}K)$

Primary constraints: $P_i = P_\phi = \pi_N = \pi_{N^i} = 0$

$$\mathcal{E} = P_0 + \phi = 0$$

Secondary constraints:

$$\mathcal{H} = h_0 + \phi + 4\pi G\rho = 0$$

$$\mathcal{D}^i = \nabla_j \tilde{\pi}^{ij} \qquad h_o = \frac{1}{q}\left[\tilde{\pi}^{jk}\tilde{\pi}_{jk} - \frac{1}{2}\tilde{\pi}^2\right] + {}^3R$$

$$\mathcal{G}_c = \partial_c \phi = 0 \qquad \mathcal{C} = N\sqrt{q} - \partial_i \tilde{a}^i$$

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C is new and with π_N are second class: $\{\mathcal{C}, \pi_N\} = \sqrt{q}$

We solve ***C*** and eliminate them with N by setting: $N = \dfrac{\partial_i \tilde{a}^i}{\sqrt{q}}$

We then eliminate ϕ and its momenta by solving ***E***.

As a result ***H*** turns into: $\mathcal{W} = P_0 - h_0 - 4\pi G\rho = 0$

G_c turns into $\mathcal{G}_c = \partial_c P_0 = 0$

combining them: $\mathcal{S}_i = \partial_i (h_0 + 4\pi G\rho) = 0$

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The final canonical system

Canonical pairs: $(q_{jk}, \tilde{\pi}^{ij}), (\tilde{a}^a, P_a)$

Constraints: $\mathcal{W} = P_0 - h_0 - 4\pi G\rho = 0$

$$\mathcal{S}_i = \partial_i(h_0 + 4\pi G\rho) = 0$$

$$\mathcal{D}^i = \nabla_j \tilde{\pi}^{ij}$$

Hamiltonian:

$$H = \int_\Sigma (-(\partial_i \tilde{a}^i)(h_0 + 4\pi G\rho) + N^i \mathcal{D}_i + \tilde{w}^i \mathcal{S}_i + \tilde{u}^i P_i$$

The path integral quantization

onstruction of the path integral from the constrained amiltonian theory:

The starting point is the fully gauge fixed and constrained integral

$$= \int dq_{ij} d\tilde{\pi}^{kl} d\tilde{a}^a dP_a d\Psi dP_\Psi \delta(\mathcal{D}_i)\delta(\mathcal{S}_i)\delta(\mathcal{W})\delta(P_i)\delta(\text{ gauge fixing}) Det_{FP}$$

$$\times exp\, \imath \int dt \int_\Sigma (\tilde{\pi}^{jk}\dot{q}_{jk} + P_a\dot{\tilde{a}}^a + (\partial_i \tilde{a}^i)(h_0 + 4\pi G\rho))$$

his can be transformed into:

$$Z = \int dg_{ab} d\Psi \delta(\sqrt{-g} - \epsilon_0)\delta(\text{ gauge fixing}) Det_{FP}\sqrt{-g}$$

$$\times exp\, \imath \int dt \int_\Sigma \left(\epsilon_0 (\frac{1}{8\pi G} R + \mathcal{L}^{matter}) \right)$$

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So the final path integral is:

$$Z = \int dg_{ab} d\Psi \delta(\sqrt{-g} - \epsilon_0)\delta(\text{ gauge fixing}) Det_{FP}\sqrt{-g}$$

$$\times exp\, \imath \int dt \int_{\Sigma} \left(\epsilon_0 (\frac{1}{8\pi G} R + \mathcal{L}^{matter}) \right)$$

So Weinberg's challenge is met: *the semi-classical limit is unimodular gravity.* So if we define the quantum effective action, it is a function of the determinant-fixed metric. Hence the quantum effective equations of motion have the decoupling symmetry.

$$T_{ab} \rightarrow T'_{ab} = T_{ab} + g_{ab} C$$

ɔ make this more precise we define the quantum effective action

ҳpand around flat spacetime, pick coordinates $\epsilon_0 = 1$

$$g_{ab} = [exp(h_{..})]_{ab} \quad \delta^{ab} h_{ab} = 0 \quad \mathcal{F}_b = \partial^a h_{ab} = 0$$

Introduce external current:

$$Z[J^{ab}] = e^{W[J]^{ab}]} = \int dh_{ab} d\Psi \delta(\text{ gauge fixing}) Det_{FP} e^{\imath(S^{uni} + \int_{\mathcal{M}} h_{ab} J^{ab}}$$

Define expectation value:

$$< h_{ab} >= \frac{\delta W}{\delta J^{ab}}|_{J=0} \qquad \delta^{ab} < h_{ab} >= 0$$

$$< \bar{g}_{ab} >= exp < h_{ab} >$$ **is unimodular**

In perturbation theory

$$S^{eff}[< \bar{g}_{ab} >] = S^{uni}(< \bar{g}_{ab} >, \phi, \psi) + \hbar \Delta S(< \bar{g}_{ab} >, \phi, \psi)$$

Details of the construction of the path integral

Why is the cosmological constant so small?

Could this be a quantum effect?

/e can rework the partition function into a form conjectured
y Ng & van Dam

$$Z = \int d\Lambda \int \prod_{x^a} dg_{ab} d\Psi \delta(\text{ gauge fixing}) Det_{FP}$$

$$\times exp\, \imath \int_{\mathcal{M}} \sqrt{-g}\,(R + 2\Lambda + \mathcal{L}^{matter})$$

ı the semi-classical approximation:

$$Z \approx \int d\Lambda \sum_{g_{ab},\Psi} exp\, \imath \int_{\mathcal{M}} \sqrt{-g}\left(-\frac{\Lambda}{4\pi G} + (\mathcal{L}^{matter} - \frac{T}{2})\right)$$

his is dominated by histories for which

$$\frac{\Lambda}{4\pi G} \approx \frac{\int_{\mathcal{M}} \sqrt{-g}(\mathcal{L}^{matter} - T)}{Vol} = < (\mathcal{L}^{matter} - \frac{T}{2}) >$$

What is the meaning of:

$$\frac{\Lambda}{4\pi G} \approx \frac{\int_{\mathcal{M}} \sqrt{-g}(\mathcal{L}^{matter} - T)}{Vol} = < (\mathcal{L}^{matter} - \frac{T}{2}) >$$

For perfect fluids $\mathcal{L}^{matter} = P$

So we find, roughly, neglecting P:

$$\boxed{\frac{\Lambda}{2\pi G} \approx \frac{\int_{\mathcal{M}} \sqrt{-g}\rho}{Vol}}$$

To make more progress and to address the issue of time, we work in the loop representation, where there are exact results about the quantum theory.

Unimodular gravity in the Plebanski formalism:

anonical pairs: $(A_a^i, \tilde{E}_i^a), (\tilde{a}^a, \pi_a), (\tilde{a}^0, \pi_0)$

onstraints:

$$\mathcal{H} = \epsilon^{ijk} \tilde{E}_i^a \tilde{E}_j^b F_{abk} - \pi_0 det(\tilde{E}_i^a) = 0 \quad \mathcal{G}^i = \mathcal{D}_a \tilde{E}^{ai} =$$

$$\mathcal{D}_a = \tilde{E}_i^b F_{ab}^i = 0 \qquad G_c = \partial_c \pi_0 = 0$$

$$\pi_c = 0$$

ully constrained momentum space path integral:

$$Z = \int dA_a^i d\tilde{E}_i^a d\tilde{a}^0 d\tilde{a}^a d\pi_a d\pi_o \delta(\mathcal{H}) \delta(\mathcal{D}_a) \delta(\mathcal{G}^i) \delta(\mathcal{G}_c) \delta(\pi_c) \delta(\text{ gauge fixing}) Det_{FP}$$

$$\times exp\, \imath \int dt \int_\Sigma \left(\tilde{E}_i^a \dot{A}_a^i + \pi_0 \dot{\tilde{a}}^0 + \pi_c \dot{\tilde{a}}^c \right)$$

Momentum space path integral

$$Z = \int dA^i_a d\tilde{E}^a_i d\tilde{a}^0 d\tilde{a}^a d\pi_a d\pi_o \delta(\mathcal{H})\delta(\mathcal{D}_a)\delta(\mathcal{G}^i)\delta(\mathcal{G}_c)\delta(\pi_c)\delta(\text{ gauge fixing})Det_F$$

$$\times exp\, \imath \int dt \int_\Sigma \left(\tilde{E}^a_i \dot{A}^i_a + \pi_0 \dot{\tilde{a}}^0 + \pi_c \dot{\tilde{a}}^c \right)$$

becomes unimodular configuration space path integral:

$$Z = \int dA^i_\mu de^\mu \delta\left(det(e) - \epsilon_0\right) \delta(\text{ gauge fixing}) Det'_{FP}$$

$$\times exp\, \imath \int dt \int_\Sigma \left(e^\mu \wedge e^\nu \wedge F^+_{\mu\nu}\right)$$

The canonical quantization of unimodular gravity.

1) The connection representation.
2) The spin network representation.
3) Infrared regularization and finite temperature

ey results of loop quantum gravity:

- The Hilbert space of spatially diffeomorphism invariant states, H^{diff} is precisely defined.
- The volume operator is precisely defined on H^{diff}.
- The hamiltonian constraint can be precisely defined on H^{diff}
  - These and other operators are uv finite

ey open issues:

- The inner product on physical states, ie solutions also to the Hamiltonian constraint
- The issue of physical observables.
- The issue of time and evolution.

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Might unimodular gravity's global time provide a new approach to these?

Some first thoughts....

The connection representation:

Rename $\tilde{a}^0 \rightarrow \tilde{T}$ $\quad \pi_0 \rightarrow \pi$

Accumulated four volume: $\tau = \int_\Sigma \tilde{T}$

Canonical pairs:

$$\{\tilde{T}(x), \pi(y)\} = \delta^3(x, y)$$

$$\{A_a^i(x), \tilde{E}_j^b(y)\} = \delta^3(x, y)\delta_a^b \delta_j^i$$

Wavefunctionals: $\Psi[A, \tilde{T}]$

The hamiltonian constraint:

$$\imath\hbar \frac{\partial}{\partial \tilde{T}} \hat{det}(e) \Psi(A, \tilde{T}) = \hat{\tilde{h}} \Psi(A, \tilde{T})$$

conventional hamiltonian constrai

he full set of quantum constraints:

$$\tilde{h} = \frac{1}{\sqrt{q}}\epsilon^{ijk}\tilde{E}^a_i\tilde{E}^b_j F_{abk}$$

$$\imath\hbar\frac{\partial}{\partial\tilde{T}}\hat{det}(e)\Psi(A,\tilde{T}) = \hat{\tilde{h}}\Psi(A,\tilde{T})$$

$$\imath\hbar\partial_c\frac{\partial}{\partial\tilde{T}}\Psi(A,\tilde{T}) = 0$$

lus SU(2) gauge and spatial diffeomorphism constraints.

hysical observables are correlations between A and T.

o solve these as usual we go to the spin network representation:

$$\Psi(A,\tilde{T}) \rightarrow \Psi(\Gamma,\tilde{T})$$

spin network basis

olving the Hamiltonian constraint, starting with a graph Γ

artition space into regions, R_i, each containing one vertex (w volume)

$$\int_{\mathcal{R}_i} \tilde{T} = \tau_i \qquad \sum_i \tau_i = \tau$$

his defines a partition of the elapsed four volume time.

ssociate each τ_i to the vertex v_i $\quad \Psi(\Gamma, \tau_i)$

ach region has a volume operator and hamiltonian operator

$$V_i = \int_{\mathcal{R}_i} \sqrt{q} \qquad h_i = \int_{\mathcal{R}_i} \tilde{h}$$

he Hamiltonian constraint is now:

$$\imath\hbar \frac{\partial}{\partial \tau_i} \hat{V}_i \Psi(\Gamma, \tau_i) = \hat{h}_i \Psi(\Gamma, \tau_i)$$

lulti-time evolution by the time of each node.

TEP1: Euclideanize and compactify the N time coordinates:

$$0 \leq \tau_i \leq 2\pi\beta$$

TEP 2: Work in **H**^{diff}, the hilbert space of gauge and spatially diffeomorphism invariant states times $[L^2(s^1)]^N$

TEP 3: Fourier transform to discrete E's

$$E_i^n = \frac{\pi n}{\beta}$$

$$\Psi(\Gamma, \{E_i\}) = \oint \prod_i d\tau_i e^{-\imath E_i \tau_i} \hat{h}_i \Psi(\Gamma, \tau_i)$$

which solve time independent Schrodinger equations

$$\hat{\tilde{h}}(x)\Psi(\Gamma, \{E_i\}) = E_i \hat{det}(e)\Psi(\Gamma, \{E_i\})$$

TEP 4: For each set of discrete E's solutions to this define a subspace of **H**^{diff}

$$H^{diff}_{\{E_i\}}$$

STEP 5: Solve the remaining constraint, which is now in the form

$$(E_i - E_j)\Psi(\Gamma, \{E_i\}) = 0$$

The solutions of this live in a subspace of H^{diff} defined by

$$H^{phys} = \sum_n H^{diff}_{\{E_1=\Lambda_n, E_2=\Lambda_n, \ldots\}}$$

$$\Lambda_n = \frac{G\pi n}{\beta}$$

onclusions:

Jnimodular gravity can be quantized via path integrals and the resulting quantum theory is also unimodular.

hus, the quantum equations of motion have the decoupling symmetry

$$T_{ab} \rightarrow T'_{ab} = T_{ab} + g_{ab}C$$

Hence the first cosmological constant problem is solved.

he second, why so small, problem and third, coincidence problem are also addressed at least at a hand-waving, semiclassical level, a la Ng and van Dam.

here is a physical time coordinate, which is elapsed four volume. The hamiltonian quantization can be carried out in LQG and this time *might* be used to give a new approach to the physical inner product and physical observables.

he path integral can be constructed for Plebanski opening up

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