Title: Is there Eternal Inflation in the Cosmic Landscape ?

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Abstract: TBA

Is there eternal inflation in the cosmic landscape ?

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hep-th/0611148
ArXiv:0708.4374 [hep-th]
with Qing-Guo Huang, ArXiv:0803.0663 [hep-th]
with Jiajun Xu and Yang Zhang, ArXiv:0812.1944 [hep-th]

Perimeter May 27, 2009

Eternal Inflation

$$a(t) \simeq e^{Ht} \rightarrow V = a(t)^3 \simeq e^{3Ht}$$

Suppose the universe is sitting at a local minimum, with a lifetime longer than the Hubble time: $\tau > 1/H$
Then the number of Hubble patches will increase exponentially. Even after some Hubble patches have decayed, there would be many Hubble patches that remain and continue to inflate.

Eternal inflation implies that somewhere in the universe (outside our horizon), inflation is still happening today.

Flux compactification in String theory

where all moduli of the 6-dim. "Calabi-Yau" manifold are stabilized

- There are many meta-stable manifolds/vacua, 10^{500} or more, probably infinite, with positive, zero, as well as negative cosmological constants.
- There are many (tens to hundreds or more) moduli (scalar modes) that are dynamically stabilized.

KKLT vacua

Giddings, Kachru, Polchinski, Kachru, Kallosh, Linde, Trivedi and many others, 2001....

Bousso, Polchinski

Pros : It seems that eternal inflation can be quite generic (unavoidable) in the cosmic landscape.
Its presence leads to a non-zero probability for every single site in the landscape, including our very own universe.

Cons : Since there are many (10^{500} or more) vacua, and some parts of the universe are still inflating, it is difficult to see why we end up where we are without invoking some strong version of the anthropic principle.

It is very difficult to make any testable prediction.

There is also a measure problem.

Scenarios with or without eternal inflation are very different

The wavefunction of the universe

may be crudely approximated by that of a D3-brane

moduli — open string modes

$$\Psi(a, \ldots, \varphi_j, \phi_i, \ldots)$$

cosmic scale factor

The behavior of the wavefunction in the landscape is
a quantum diffusion and percolation problem.

Outline

- Tunneling in the cosmic landscape is quite different from tunneling between 2 sites.
- Comments : (1) it is harder to trap a wavefunction in higher dimensions; (2) the barriers in some directions in the landscape are exponentially low; (3) the vacuum energy in the landscape plays the role of a finite (Gibbons-Hawking) temperature, enabling mobility.
- Treating the landscape as a random potential, we can estimate the mobility of the wavefunction in the landscape, suggesting a plausible solution to the CC problem.
- Signatures in the CMB power spectrum.

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A simple question :

Consider tunneling from A to C :

ssume for simplicity, consider WKB tunneling rate :

$$\Gamma_{A\to B} = \Gamma_{B\to C} = \Gamma_0 \sim e^{-S}$$

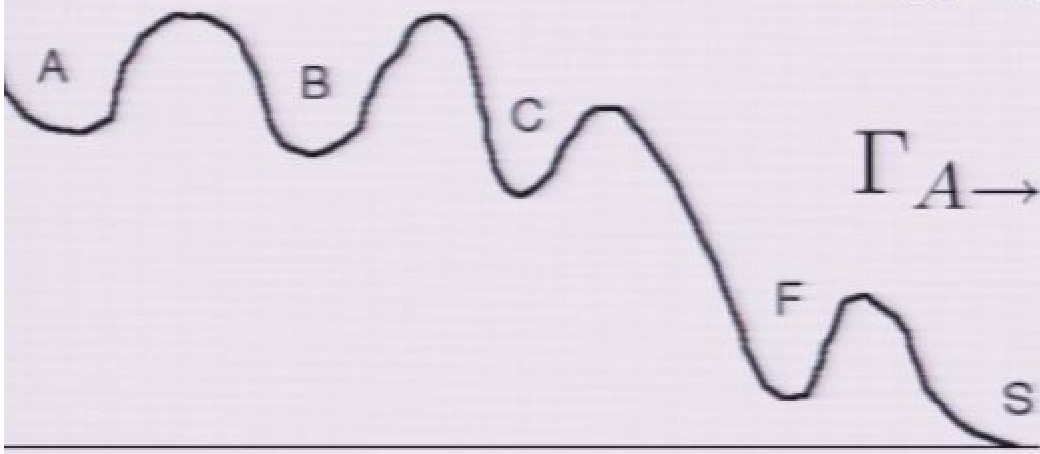
$$\Gamma_{A\to C} \sim \Gamma_{A\to B}\Gamma_{B\to C} = \Gamma_0^2$$

$$\sim e^{-2S}$$

?

A simple question :

Consider tunneling from A to C :



$$\Gamma_{A\to B} = \Gamma_{B\to C} = \Gamma_0 \sim e^{-S}$$

$$\Gamma_{A\to C} \sim \Gamma_{A\to B}\Gamma_{B\to C} = \Gamma_0^2$$

?

Life time :

$$t_{A\to C} = t_{A\to B} + t_{B\to C} = \frac{1}{\Gamma_{A\to B}} + \frac{1}{\Gamma_{B\to C}} \sim \frac{2}{\Gamma_0}$$

$$\Gamma_{A\to C} = \frac{\Gamma_0}{2}$$?

Which is correct ?

?

$$\Gamma(2) \sim e^{-2S} \Rightarrow \Gamma(n) \sim e^{-nS}$$ negligibl

$$\Gamma(2) \sim e^{-S} \Rightarrow \Gamma(n) \sim e^{-S}$$

$$\Gamma(1) = \Gamma_0 \sim e^{-S} \sim T_0$$ Transmission coefficient

Consider the transmission coefficient for the second case :

$$T_{A\to C} = T_0/2 \quad \to \quad T(n) \simeq T_0/n$$

$$\Gamma(1) = \Gamma_0 \sim e^{-S}$$

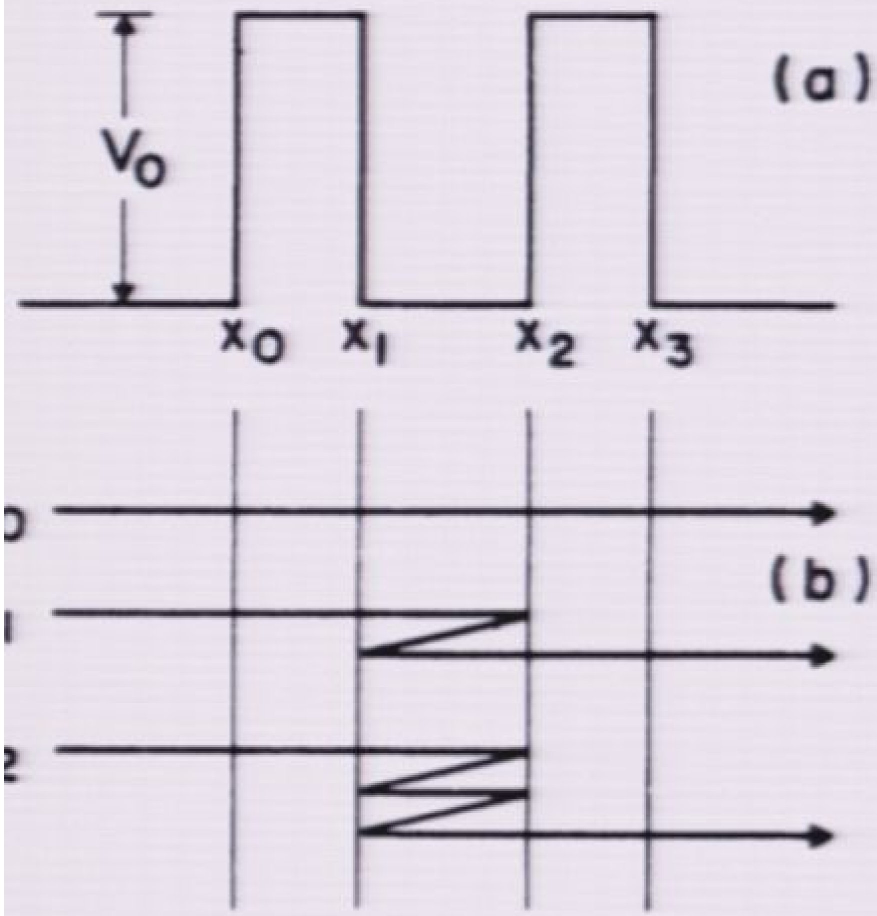
$$\Gamma(2) \sim e^{-2S} \quad \text{✗} \quad \Rightarrow \quad \Gamma(n) \quad \text{negligible}$$

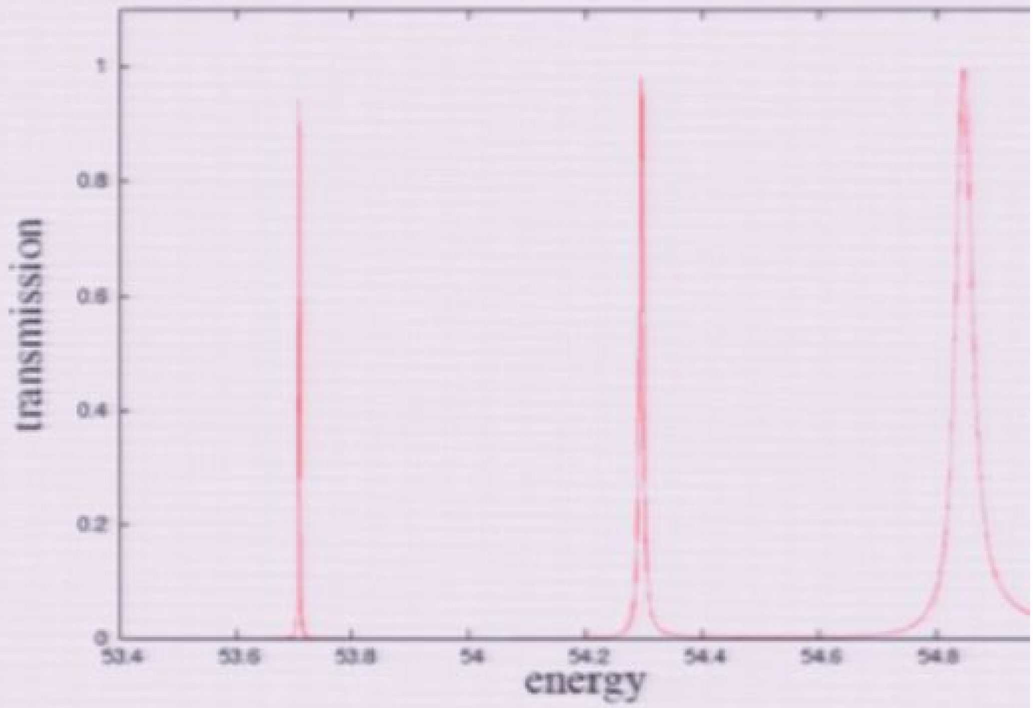
$$\Gamma(2) \sim e^{-S} \quad \Rightarrow \quad \Gamma(n) \sim e^{-S}$$

correct

$$T_{A \to C} = T_0/2 \qquad T(n) \simeq T_0/n$$

To understand why : resonance tunneling





Y. Zhota, PRB 41 (1990) 7879.

Resonance tunneling in semi-conductors

Wikipedia

A **resonant tunnel diode** (RTD) is a device which uses quantum effects to produce negative differential resistance (NDR). As an RTD is capable of generating a terahertz wave at room temperature, it can be used in ultra high-speed circuitry. Therefore the RTD is extensively studied.

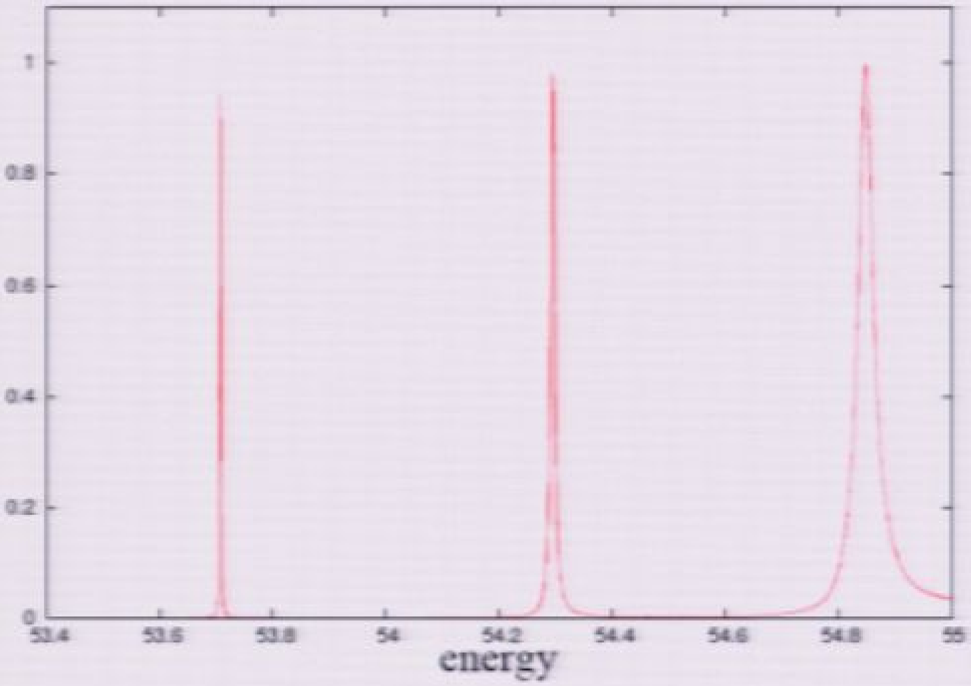
Chang, Esaki, Tsu, 1974

Generic wavefunction spread in energy :

$$T(A \to C) = \frac{T(A \to B)T(B \to C)}{T(A \to B) + T(B \to C)} \sim T_0/2$$

naive

$$T(n) \simeq T_0/n$$



In any meta-stable site in the cosmic landscape, we cannot focus only at its nearest neighbors.

Resonance tunneling in QFT :

Saffin, Padilla and Copeland, arXiv:0804.3801 [hep-th]

Sarangi, Shiu, Shlaer, arXiv:0708.4375 [hep-th]

In a d-dimensional potential :

naive : $\Gamma_t^{nr} \sim 2d\ \Gamma_0$

The time for 1 e-fold of inflation is Hubble time 1/H, so the lifetime of a typical site seems to be much longer than the Hubble scale, and eternal inflation is unavoidable.

Very roughly : $\Gamma_t \sim n^{d-1} \Gamma_0$

$n \sim 1/Hs$

For large enough d (and maybe n) the tunneling can be fast.

Message : The landscape has a big impact on the tunneling properties

Outline

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The QM wavefunction is harder to be trapped in higher dimensions:

$$\eta(r)'' + \left(2m[E - V(r)] - \frac{(d-1)(d-3)}{4r^2}\right)\eta(r) = 0$$

↑ Repulsive

Recall : A 1-dim. attractive delta-function potential always has a bound state but not a 3-dim one.

$$\psi(r) = r^{-(d-1)/2}\eta(r)$$

Harder to trap in higher dimensions

$$P_c \simeq 0.58d + 0.5$$

Spherical square well :

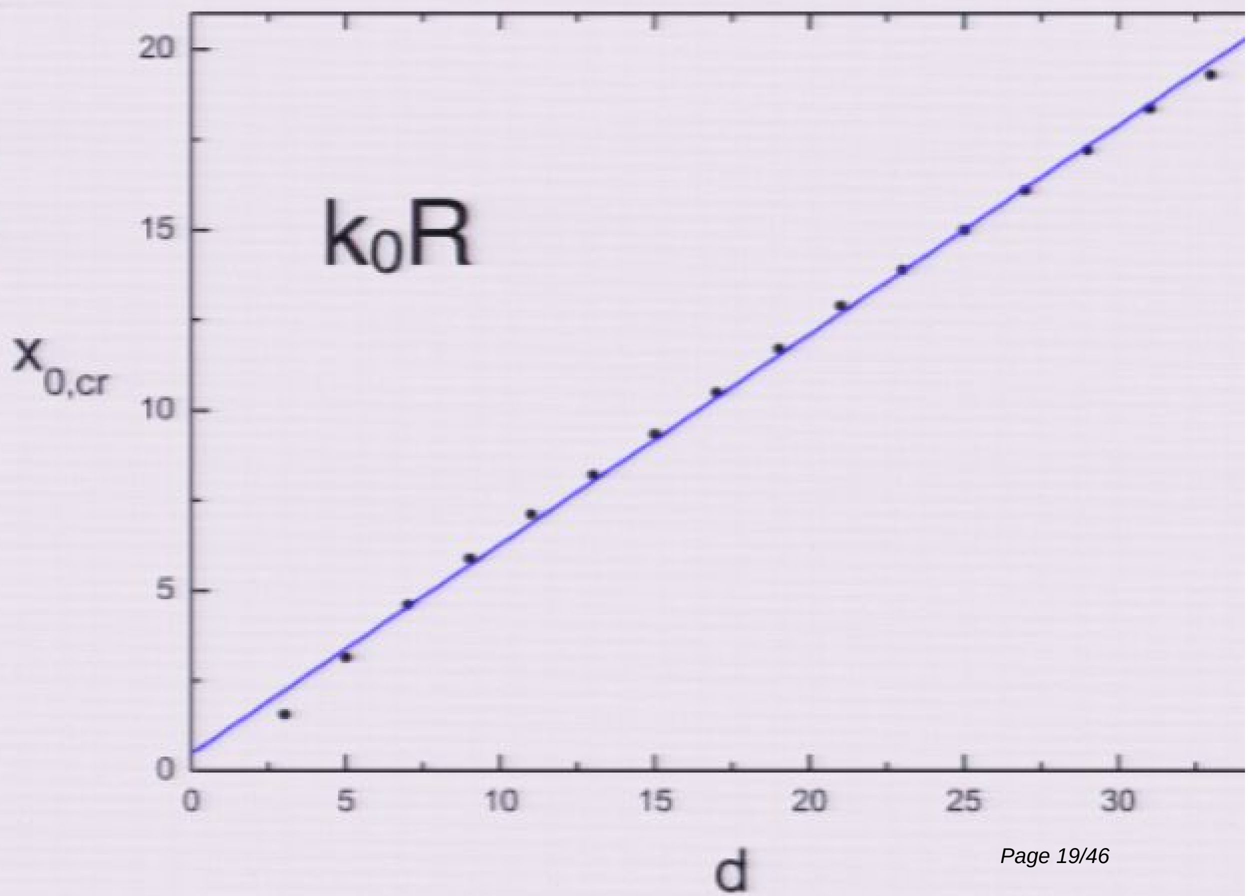
$V(r) = -V_0 \quad r < R$

$\quad\;\; = 0 \quad\quad r > R$

$k_0^2 = 2mV_0$

$\psi(r) \sim e^{-ar}$

$a^2 = 2m|E|$



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Axionic potentials

$$V = V_0 + \sum_i \alpha_i \cos(2\pi\phi_i) + \sum_{i,j} \beta_{ij} \cos(2\pi\phi_i - 2\pi\phi_j)$$

$$\phi_i = \varphi_i / f_i \qquad \alpha_i \sim e^{-S_i}$$

$$|\beta_{ij}| < \alpha_i$$

For light axions, the potential heights are very low. In some directions, the barriers can easily be exponentially low.

Svrcek and Witten, hep-th/0605206

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Tunneling is much faster at higher C.C.

CDL thin wall : $$B \simeq \frac{27\pi^2\tau^4}{2\epsilon^3} \to \frac{2\pi^2\tau}{H^3}$$

Hawking-Moss : $$B_{HM} = \frac{8\pi^2}{3}\frac{\delta U}{H_+^2 H_t^2}$$

Gibbons-Hawking (GH) temperature $$T_H = H/2\pi$$

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The vastness of cosmic landscape

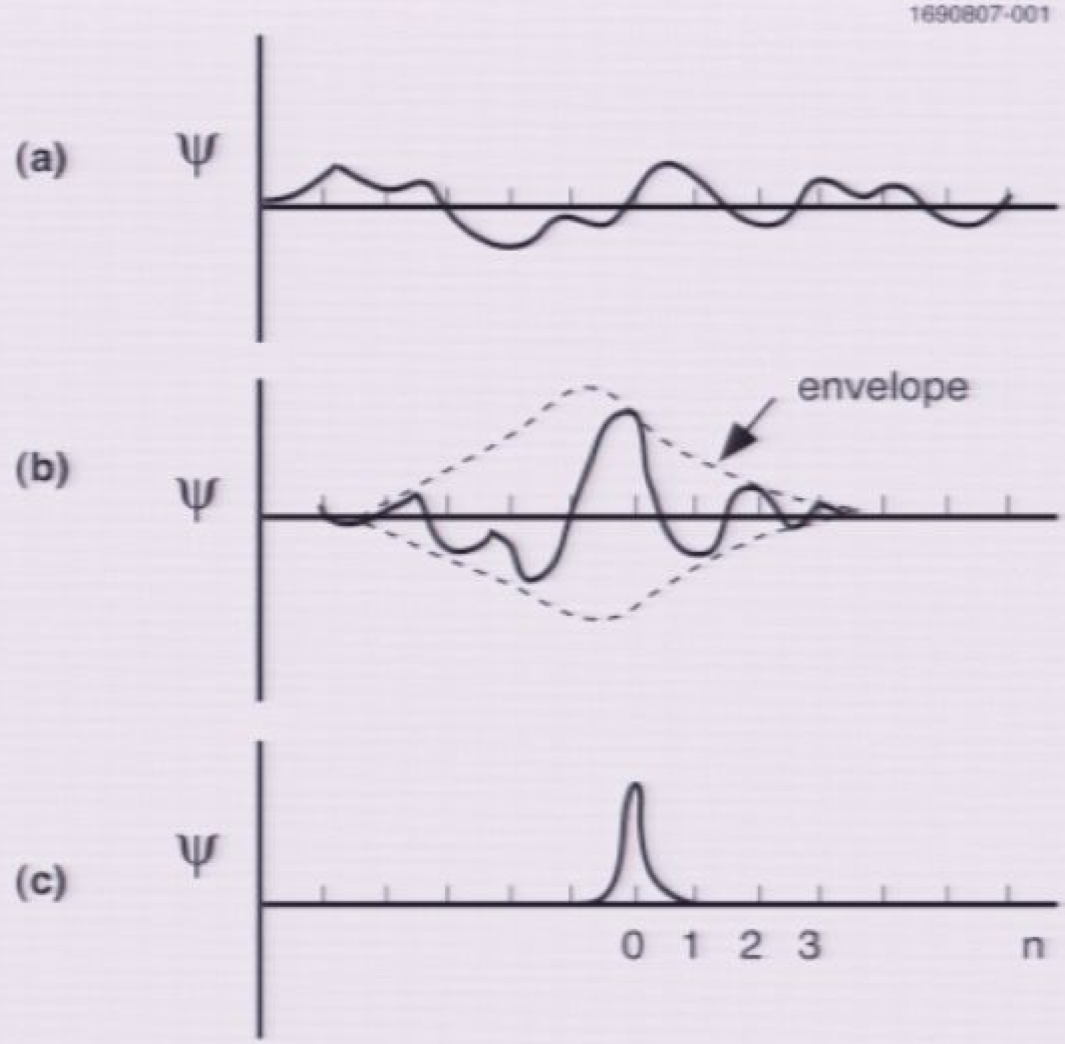
- At a typical meta-stable site, count the number of parameters or the number of light scalar fields. This gives the number of moduli, or directions in the field space.
- This number at any neighborhood in the landscape may be taken as the dimension d of the landscape around that neighborhood.
- The landscape potential is not periodic (though it may be close to periodic along some axion directions). In general, it is very complicated.
- Let us treat it as a random potential.

Anderson localization transition

- random potential/disorder medium
- insulation-superconductivity transition
- quantum mesoscopic systems
- conductivity-insulation in disordered systems
- percolation
- strongly interacting electronic systems
- doped systems, alloys, ...

Some references :

- P.W. Anderson, *Absence of Diffusion in Certain Random Lattices*, Phys. Rev. Lett. 109, 1492 (1958).
- E. Abrahams, P.W. Anderson, D. C. Licciardello and T.V. Ramakrishnan, *Scaling Theory of Localization :Absence of Quantum Diffusion in Two Dimensions*, Phys. Rev. Lett. 42, 673 (1979).
- B. Shapiro, *Renormalization-Group Transformation for Anderson Transition*, Phys. Rev. Lett. 48, 823 (1982).
- P. A. Lee and T.V. Ramakrishnan, Disordered electronic systems, Rev. Mod. Phys. 57, 287 (1985).
- M.V. Sadovskii, Superconductivity and Localization," (World Scientific, 2000).
- Les Houches 1994, Mesoscopic Quantum Physics.



$$|\psi(\mathbf{r})| \sim \exp(-|\mathbf{r} - \mathbf{r}_0|/\xi)$$

$$\Gamma_0 \sim |\psi(a)|^2 \sim e^{-2a/\xi}$$

Define a dimensionless conductance g in a d-dim. hypercubic region of size L

$$g_d(L) = \sigma L^{d-2}$$

conductivity

$$d = 3,\ g = \sigma(\text{Area})/L \sim \sigma L$$

Thouless $g(L) \sim (L/a)^{(d-2)}$

Conducting/mobile (metalic) with finite conductivity

Conductance = Mobileness

Conductivity = Mobility

How does g scale ?
Given g at scale a, what is g at scale L as L becomes large ?

$$g(a) \sim |\psi(a)| \sim e^{-a/\xi}$$

↙ ↘

$$g_d(L) \sim e^{-L/\xi}$$

↓

Insulating, localized,
rapped, eternal inflation
zero conductivity

$$g_d(L) = \sigma L^{d-2}$$

↓

Conducting, mobile

$$\beta_d(g_d(L)) = \frac{d \ln g_d(L)}{d \ln L}$$

$$g_d(L) \sim e^{-L/\xi}$$

$$g_d(L) = \sigma L^{d-2}$$

$$\mathrm{n}\, g \sim -L = -e^{\ln L}$$

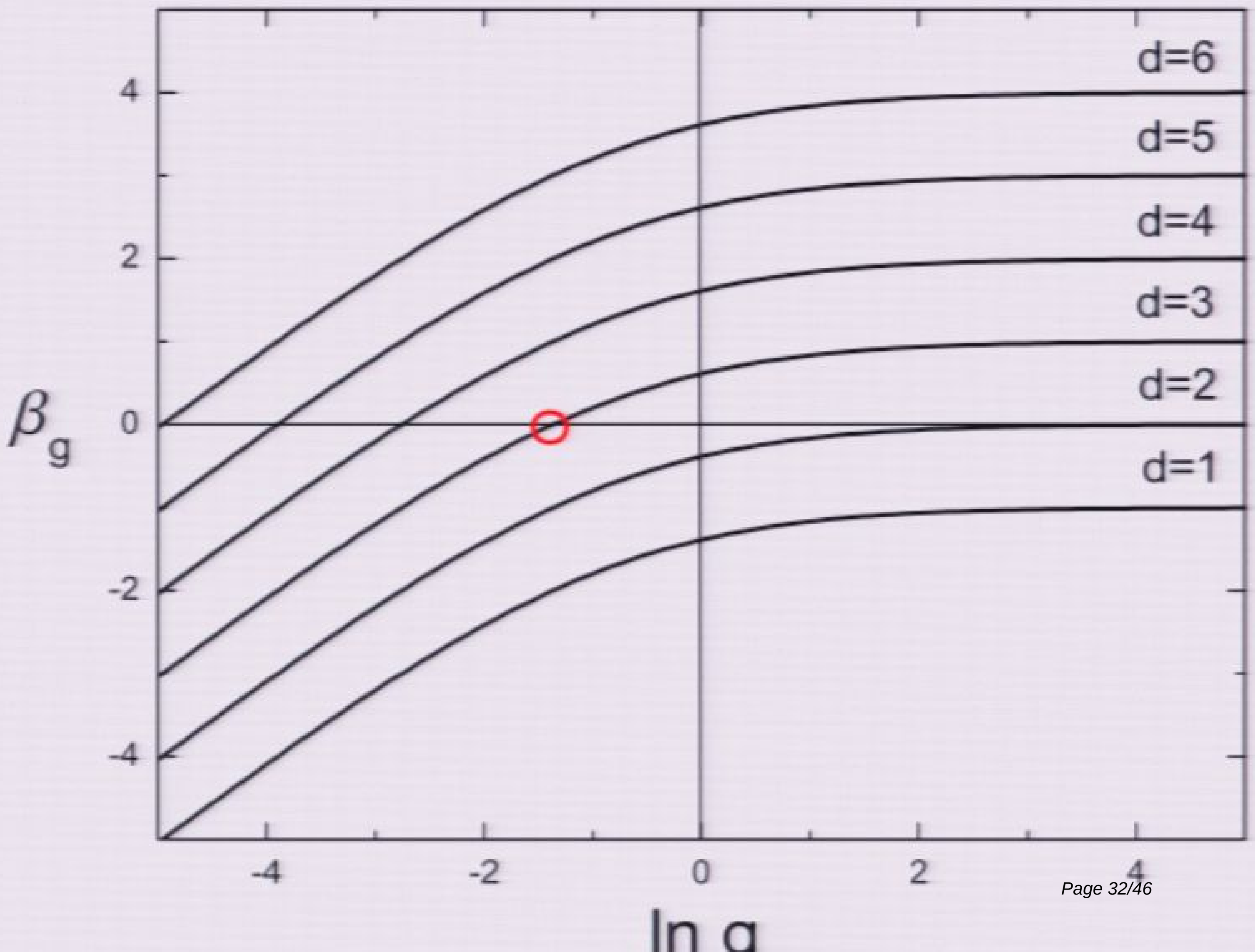
$$\lim_{g \to 0} \beta_d(g) \to \ln \frac{g}{g_c}$$

$$\lim_{g \to \infty} \beta_d(g) \to d - 2$$

Insulating, localized, trapped, eternal inflation

Conducting, mobile

$$\beta_d(g_d(L)) = \frac{d \ln g_d(L)}{d \ln L}$$



$$\beta_d(g_c) = 0$$

$$\beta_d(g) \approx \frac{1}{\nu} \ln \frac{g}{g_c} \approx \frac{1}{\nu} \frac{g - g_c}{g_c}$$

this zero of $\beta_d(g)$ corresponds to an unstable fixed point

$g(a) < g_c$	$g(a) > g_c$
↓	↓
$g_d(L) \sim e^{-L/\xi}$	$g_d(L) = \sigma L^{d-2}$
↓	↓
Insulating, localized, trapped, eternal inflation	Conducting, mobile

Condition for mobility

$$g_c = e^{-(d-1)} \qquad g(a) \sim |\psi(a)| \sim e^{-a/\xi}$$

$$d > \frac{a}{\xi} + 1$$

$$\Gamma_0 \sim |\psi(a)|^2 \sim e^{-2a/\xi}$$

d ~ 100

Generic decay rate to nearest neighbor in the landscape :

$$\Gamma_0 > e^{-2(d-1)}$$

Mobile

What is the critical g_c ?

$$\Delta \ln g_c = \ln g_c(d) - \ln g_c(d+1) = k > 0$$

$$g_c(d) \simeq e^{-(d-3)k} g_c(3)$$

Shapiro : $$\beta_d(g) = (d-1) - (g+1)\ln(1+1/g)$$

$$g_c = e^{-(d-1)}$$

$$\nu \to 1$$

d=1 :

Anderson, Thouless, Abraham, Fisher, Phys. Rev. B 22, 3519 (1980)

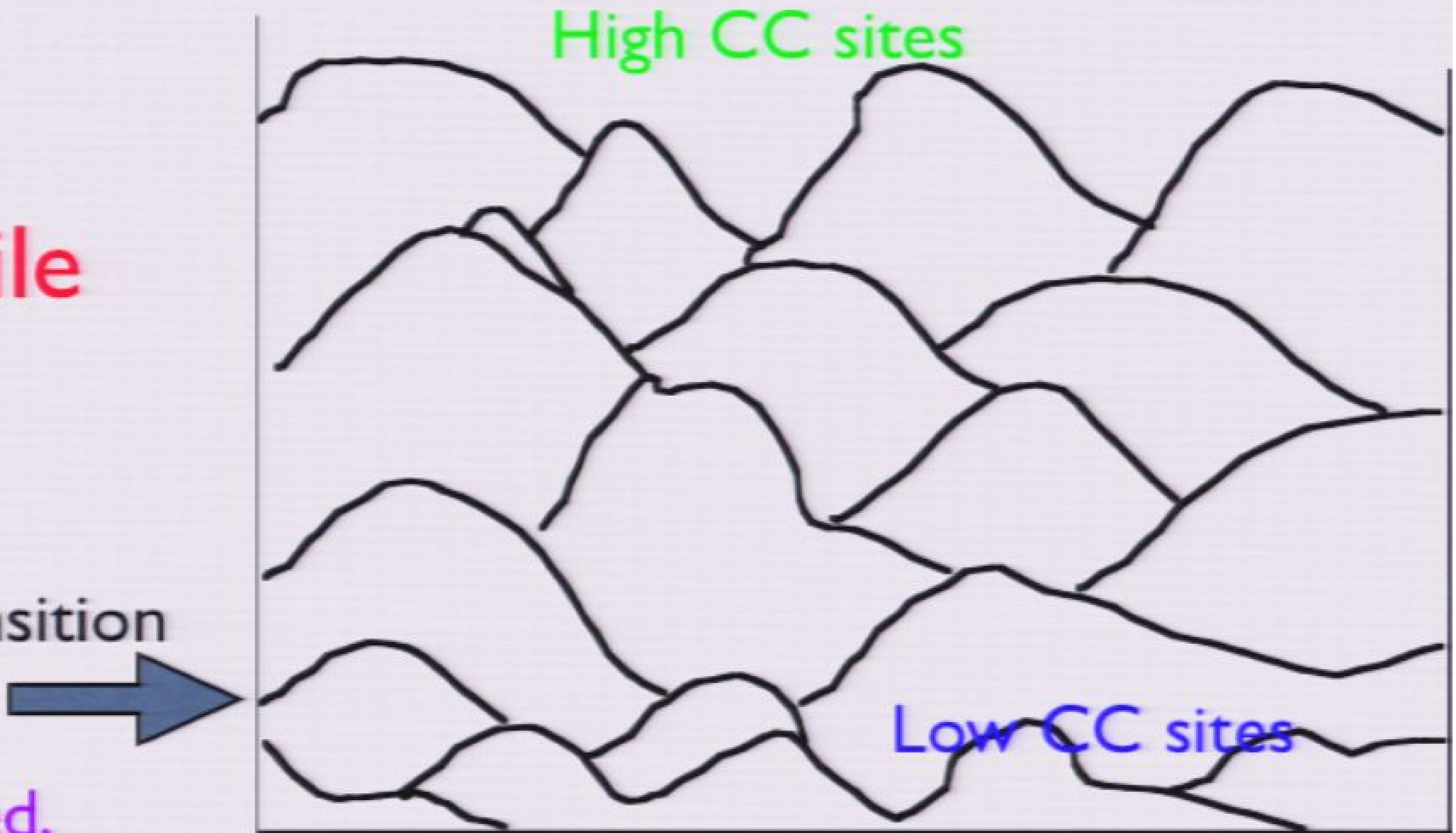
Properties

- No mobility in d=1,2.
- For d>2, there is a phase transition between mobility and "trapped".
- Mobility in the landscape implies no eternal inflation.
- The critical conductance is $g_c = e^{-(d-1)}$
- I like to translate this to a critical cosmological constant for dimension d.

The picture from the scaling theory

mobile

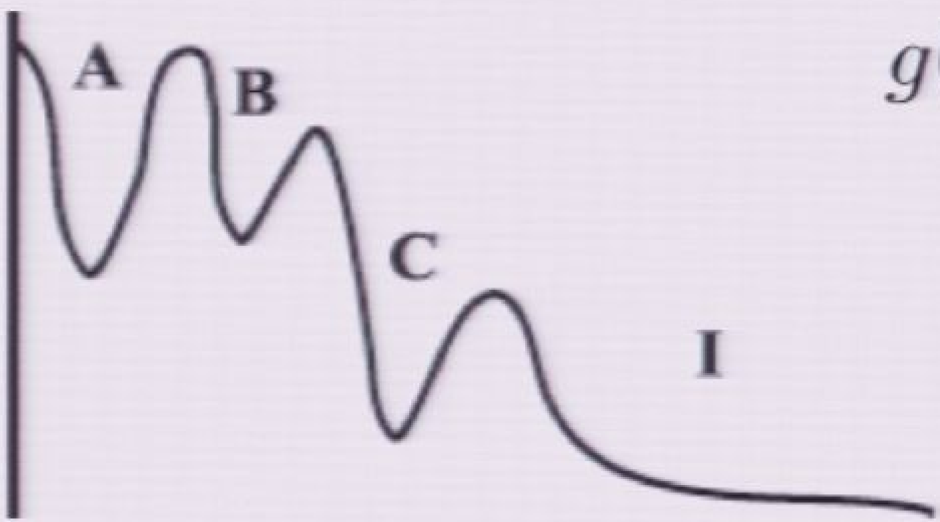
Sharp transition



trapped, exponentially long lifetimes

To estimate the transition CC in the cosmic landscape:

- Tunneling from a positive CC site to a negative CC site is ignored (CDL crunch).
- Tunneling from a dS site to another dS site with a larger CC is ignored.



$$g(a) \sim |\psi(a)| \sim e^{-a/\xi}$$

At string (Planck) scale : everything is string (Planck) scale.

Estimate of critical C.C.

$$g(a) \sim |\psi(a)| \sim e^{-a/\xi}$$

$$g_c = e^{-(d-1)}$$

$$d(\Lambda) > \frac{a(\Lambda)}{\xi(\Lambda)} \quad \rightarrow \quad d > \frac{a(\hat{\Lambda})}{\xi}$$

$$\xi \sim s(\Lambda_s) \sim \frac{1}{m_s} \sim \Lambda_s^{-1/4}$$

assume a random distribution of vacua: $\sim \Lambda^{q-1}$

Estimate of critical C.C.

fraction of sites : $f(\Lambda) = (\Lambda/\Lambda_s)^q$

$$\xi \sim s(\Lambda_s) \sim \frac{1}{m_s} \sim \Lambda_s^{-1/4}$$

$$N(\Lambda) = f(\Lambda) N_T = f(\Lambda) \left(\frac{L}{s(\Lambda_s)} \right)^d = \left(\frac{L}{s(\Lambda)} \right)^d$$

$$s(\Lambda) = s(\Lambda_s) \left(\frac{\Lambda}{\Lambda_s} \right)^{-q/d}$$

$$d = s(\Lambda_c)/\xi + 1$$

For flat distribution : $\Lambda_c \sim d^{-d} M_s^4$

$$d > 60$$

$$\Lambda_c \sim d^{-d/q} M_s^4$$

Including percolation effects

g also depends on a percolation probability p $(0 \leq p \leq 1$

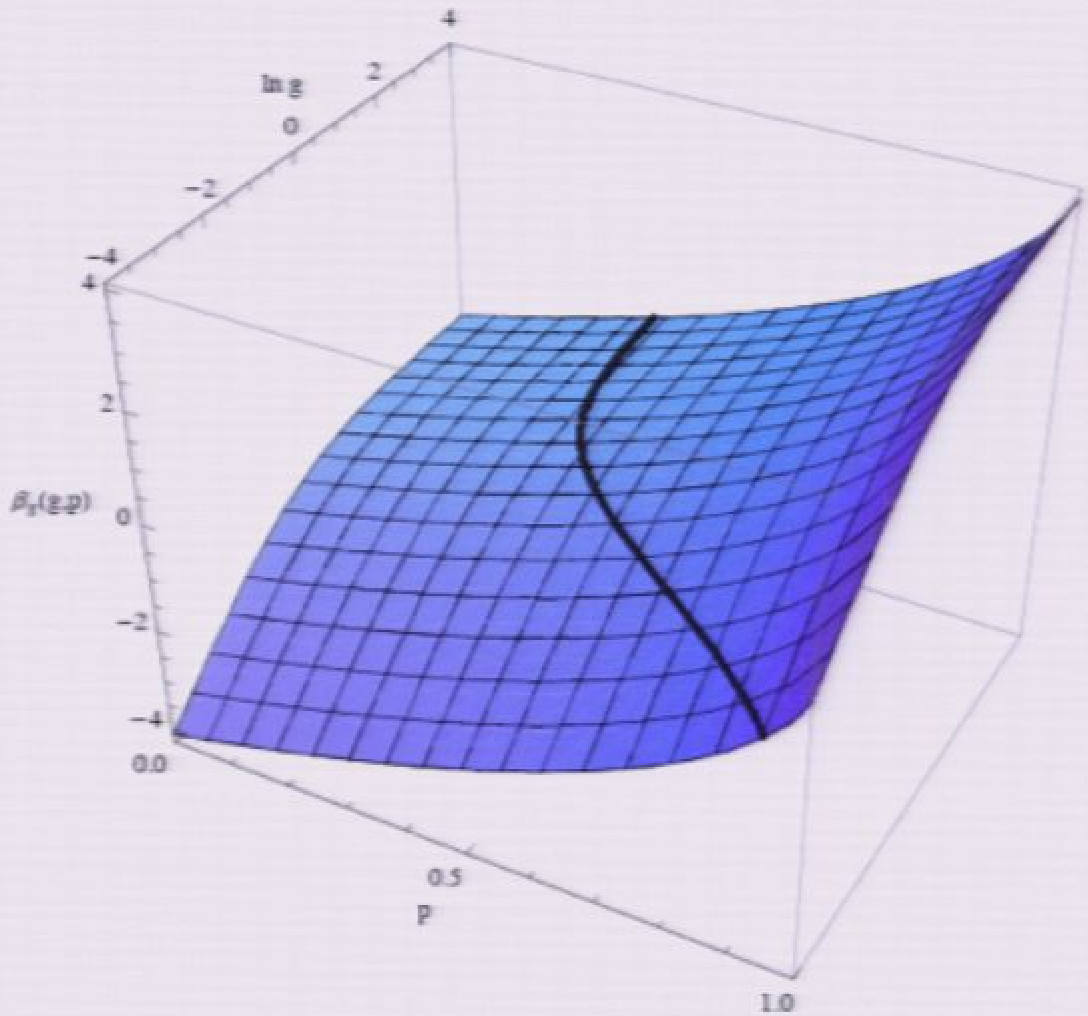
$$\beta_p(p) = \frac{\partial p}{\partial \ln L} = p \ln p - (d-1)(1-p)\ln(1-p)$$

$$\beta_g(g,p) = (d-1)\left(1 + \frac{1-p}{p}\ln(1-p)\right) - (g+1)\ln(1+1/g)$$

$$\beta_d(g) = (d-1) - (g+1)\ln(1+1/g)$$

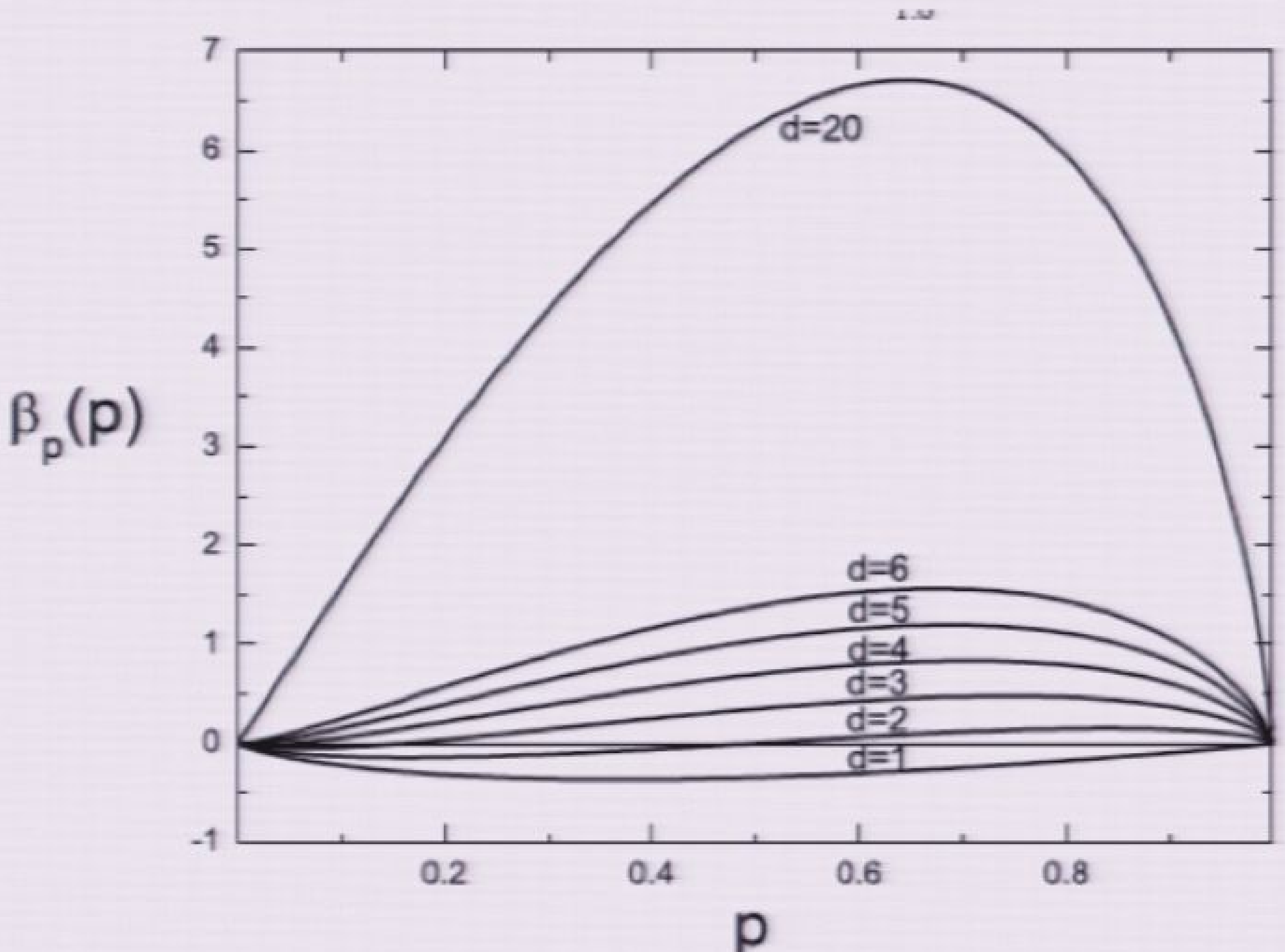
$p \rightarrow 1$

$\beta_g(g, p)$

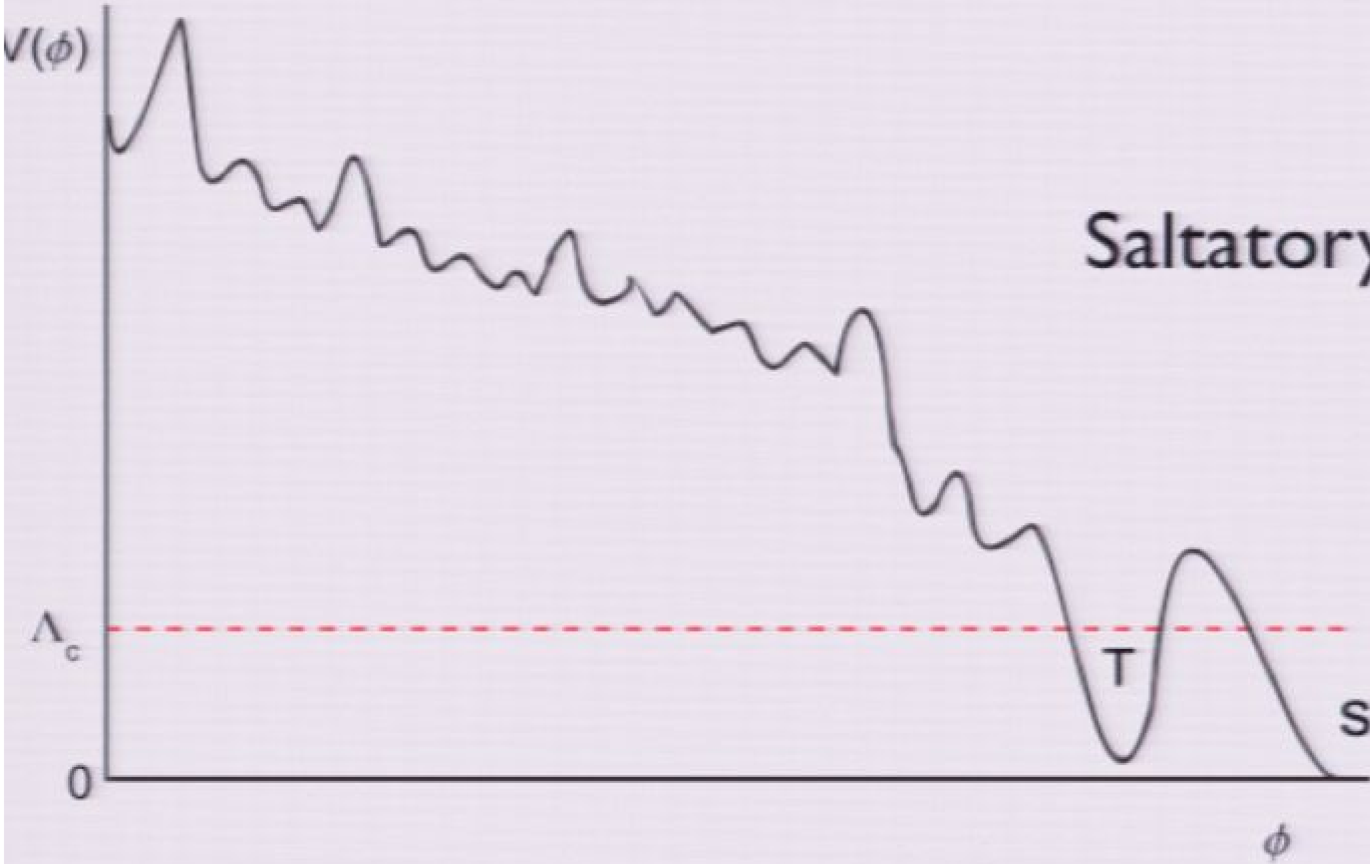


The β-function $\beta_g(g, p)$ with inclusion of percolation for $d = 6$

$$\beta_p(p) = \frac{\partial p}{\partial \ln L}$$



The scenario



Saltatory relaxation

Abbott, Brown and Teitelboim
Feng, March-Russell, Sethi and Wilczek

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- Signatures (fluctuations) in the CMB power spectrum.

Summary

- The universe is freely moving in the stringy landscape wher the vacuum energy density is above a critical value. Because of mobility, there is probably no eternal inflation.
- When the universe drops below the critical C.C. value, it stays there. Its lifetime there is exponentially long.
- The critical C.C. value is exponentially small compared to the string/Planck scale.
- This scenario suggests an inflationary scenario in the landscape, which predicts (hopefully observable) fluctuations in the CMBR power spectrum.