Title: Modified gravity, degravitation and equivalence principle

Date: May 26, 2009 02:00 PM

URL: http://pirsa.org/09050083

Abstract: TBA

Equivalence Principle and Cosmic Acceleration

Lam Hui & Alberto Nicolis
Columbia University

Chris Stubbs
Harvard University

Outline

- Long distance modification of gravity - the generic nature of scalar-tensor theory.
- 2 screening mechanisms - mandatory suppression of scalar on small scales.
- The problem of motion - how do things move? Do they really all fall at the same rate under gravity (i.e. equivalence principle)?
- Observational tests - look for O(1) violation!

Examples of IR modification of GR:

- f(R) and generalizations - scalar-tensor (Chiba).
- DGP - brane bending mode (Luty, Porrati, Rattazzi).
- massive gravity - Stueckelberg (Arkani-Hamed, Georgi, Schwartz).
- resonance gravity/filtering/degravitation - Stueckelberg (Dvali, Hofmann, Khoury).
- galileon (Nicolis, Rattazzi, Trincherini).
- ghost condensate (Arkani-Hamed, Cheng, Luty, Mukohyama; Dubovsky).
- cucuston (Afshordi, Chung, Geshnizjani).

Side remark 1 (on Box alpha):

$$S_{\text{filtering}} = \int d^4x h^{\mu\nu} m^2(\Box)(h_{\mu\nu} - \eta_{\mu\nu} h) \qquad , \qquad m^2(\Box) = \mu^{2-2\alpha}(-\Box)^\alpha$$

Stueckelberg: $h_{\mu\nu} = \tilde{h}_{\mu\nu} + 2\partial_\mu\partial_\nu \frac{\phi}{m^2} + \partial_\mu\partial_\alpha \frac{\phi}{m^2}\partial_\nu\partial^\alpha \frac{\phi}{m^2}$

$$S_{\text{int}} = \int d^4x \frac{\partial^{6-4\alpha}\phi^3}{\Lambda^{5-4\alpha}} \quad , \quad \Lambda^{5-4\alpha} = M_P \mu^{4-4\alpha}$$

Dvali

Problem: do not obtain local interaction (even for DGP), i.e. predictive power in doubt.

Side remark 2 (no-go theorem?)

$$G^{(1)}_{\mu\nu} - m^2(\Box)(h_{\mu\nu} - \eta_{\mu\nu} h) = T_{\mu\nu}$$

zero curvature: $R^{(1)} = \partial^\mu \partial^\nu (h_{\mu\nu} - \eta_{\mu\nu} h) = 0$

Stueckelberg: $S_{\rm Jordan} = \int d^4x\, \phi R^{(1)} + h^{\mu\nu} T_{\mu\nu}$ i.e. Brans-Dicke w = 0

conformal transf.:

$$h_{\mu\nu} = \tilde{h}_{\mu\nu} + \eta_{\mu\nu}\phi \;\longrightarrow\; S_{\rm Einstein} \sim \int d^4x\, \tilde{R}^{(1)} - (\partial\phi)^2 + (\tilde{h}^{\mu\nu} + \eta^{\mu\nu}\phi) T_{\mu\nu}$$

$$\Box\phi = T \quad \longrightarrow \quad \phi \propto x^2 \quad \text{for} \quad T = \text{constant}$$

Need screening via Vainshtein to satisfy solar system tests. But for such screening to not to kill the x^2 profile, need more than two derivatives i.e. ghost.

Weinberg's theorem: at low energy, a Lorentz invariant theory of massless spin-2 particle must be GR.

Therefore, to modify gravity, either add new degrees of freedom (e.g. scalar) or make the graviton massive (which via Stueckelberg also contains scalar) or violate Lorentz invariance (e.g. ghost condensate).

Some form of scalar-tensor theory seems generic.

Side remark 2 (no-go theorem?)

$$G^{(1)}_{\mu\nu} - m^2(\Box)(h_{\mu\nu} - \eta_{\mu\nu}h) = T_{\mu\nu}$$

zero curvature: $R^{(1)} = \partial^\mu\partial^\nu(h_{\mu\nu} - \eta_{\mu\nu}h) = 0$

Stueckelberg: $S_{\rm Jordan} = \int d^4x\,\phi R^{(1)} + h^{\mu\nu}T_{\mu\nu}$ i.e. Brans-Dicke w = 0

conformal transf.:

$$h_{\mu\nu} = \bar{h}_{\mu\nu} + \eta_{\mu\nu}\phi \;\longrightarrow\; S_{\rm Einstein} \sim \int d^4x\,\bar{R}^{(1)} - (\partial\phi)^2 + (\bar{h}^{\mu\nu} + \eta^{\mu\nu}\phi)T_{\mu\nu}$$

$$\Box\phi = T \quad \longrightarrow \quad \phi \propto x^2 \quad \text{for} \quad T = \text{constant}$$

Need screening via Vainshtein to satisfy solar system tests. But for such screening to not to kill the x^2 profile, need more than two derivatives i.e. ghost.

Examples of IR modification of GR:

- f(R) and generalizations - scalar-tensor (Chiba).
- DGP - brane bending mode (Luty, Porrati, Rattazzi).
- massive gravity - Stueckelberg (Arkani-Hamed, Georgi, Schwartz).
- resonance gravity/filtering/degravitation - Stueckelberg (Dvali, Hofmann, Khoury).
- galileon (Nicolis, Rattazzi, Trincherini).
- ghost condensate (Arkani-Hamed, Cheng, Luty, Mukohyama; Dubovsky).
- cucuston (Afshordi, Chung, Geshnizjani).

Side remark 1 (on Box alpha):

$$S_{\text{filtering}} = \int d^4x h^{\mu\nu} m^2(\Box)(h_{\mu\nu} - \eta_{\mu\nu} h) \quad , \quad m^2(\Box) = \mu^{2-2\alpha}(-\Box)^\alpha$$

Stueckelberg: $h_{\mu\nu} = \tilde{h}_{\mu\nu} + 2\partial_\mu\partial_\nu \frac{\phi}{m^2} + \partial_\mu\partial_\alpha \frac{\phi}{m^2}\partial_\nu\partial^\alpha \frac{\phi}{m^2}$

$$S_{\text{int}} = \int d^4x \frac{\partial^{6-4\alpha}\phi^3}{\Lambda^{5-4\alpha}} \quad , \quad \Lambda^{5-4\alpha} = M_P \mu^{4-4\alpha}$$ Dvali

Problem: do not obtain local interaction (even for DGP), i.e. predictive power in doubt.

Side remark 2 (no-go theorem?)

$$G^{(1)}_{\mu\nu} - m^2(\Box)(h_{\mu\nu} - \eta_{\mu\nu}h) = T_{\mu\nu}$$

zero curvature: $R^{(1)} = \partial^\mu\partial^\nu(h_{\mu\nu} - \eta_{\mu\nu}h) = 0$

Stueckelberg: $S_{\text{Jordan}} = \int d^4x\, \phi R^{(1)} + h^{\mu\nu}T_{\mu\nu}$ i.e. Brans-Dicke w = 0

conformal transf.:

$$h_{\mu\nu} = \tilde{h}_{\mu\nu} + \eta_{\mu\nu}\phi \;\longrightarrow\; S_{\text{Einstein}} \sim \int d^4x\, \tilde{R}^{(1)} - (\partial\phi)^2 + (\tilde{h}^{\mu\nu} + \eta^{\mu\nu}\phi)T_{\mu\nu}$$

$$\Box\phi = T \quad\longrightarrow\quad \phi \propto x^2 \quad \text{for} \quad T = \text{constant}$$

Need screening via Vainshtein to satisfy solar system tests. But for such screening to not to kill the x^2 profile, need more than two derivatives i.e. ghost.

Weinberg's theorem: at low energy, a Lorentz invariant theory of massless spin-2 particle must be GR.

Therefore, to modify gravity, either add new degrees of freedom (e.g. scalar) or make the graviton massive (which via Stueckelberg also contains scalar) or violate Lorentz invariance (e.g. ghost condensate).

Some form of scalar-tensor theory seems generic.

Also: modified gravity is in a sense no more exotic than quintessence. Absent symmetries, quintessence should be coupled to matter at gravitational strength i.e. scalar-tensor theory yet again.

Screening

We generally want the scalar to be alive on large scales i.e. induce O(1) modification on Hubble scale.

But the scalar must be screened on small scales to match solar system tests.

Two known screening mechanisms:

chameleon and strong coupling/Vainshsten.

Both make use of scalar self-interactions, one uses potential, the other uses derivatives.

Chameleon screening: give scalar a large mass in high density environment (Khoury & Weltman).

- Jordan frame (General)

$$S = M_P^2 \int d^4x \sqrt{-g}\left[\frac{1}{2}\Omega^2(\varphi)R - \frac{1}{2}h(\partial\varphi)^2 - \Omega^4(\varphi)V\right] + \int d^4x \mathcal{L}_m(\psi_m, g_{\mu\nu})$$

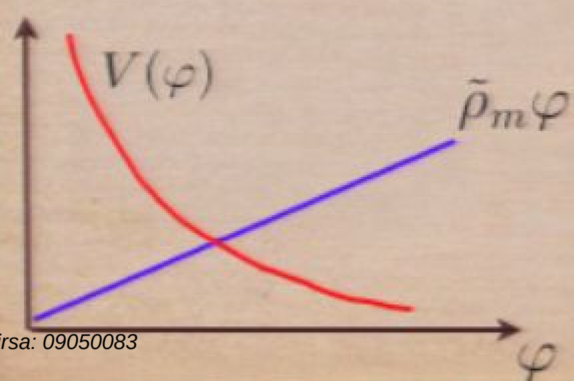
$$\Omega(\varphi) \equiv \exp\left[-\alpha\varphi\right]$$

matter minimally coupled to $g_{\mu\nu}$

- Einstein frame $\tilde{g}_{\mu\nu} = \Omega^2(\varphi) g_{\mu\nu}$

$$S = M_P^2 \int d^4x \sqrt{-\tilde{g}}\left[\frac{1}{2}\tilde{R} - \frac{1}{2}(\tilde{\partial}\varphi)^2 - V\right] + \int d^4x \mathcal{L}_m(\psi_m, \Omega^{-2}(\varphi)\tilde{g}_{\mu\nu})$$

φ mediates a fifth force



Idea : make m_φ density dependent

$$\tilde{\Box}\varphi = [V + \alpha M_P^{-2}\tilde{\rho}_m\varphi]_{,\varphi}$$

Screening by strong coupling (Vainshtein).

Einstein frame, expansion around Minkowski (DGP):

$$\text{scalar action } = \int d^4x \left[-3M_P^2(\partial\varphi)^2 - 2\frac{M_P^2}{m^2}(\partial\varphi)^2\Box\varphi + \varphi T_m{}^{\mu}{}_{\mu} \right]$$

$$\text{equation of motion}: \Box\varphi + \frac{2}{3m^2}\left[(\Box\varphi)^2 - \partial^\mu\partial^\nu\varphi\partial_\mu\partial_\nu\varphi\right] = \frac{\rho_m}{6M_P^2}$$

Scalar profile around mass M:

$$\text{large } r: \ \varphi \sim \frac{M}{M_P^2}\frac{1}{r} \quad , \quad \text{small } r: \ \varphi \sim \left[\frac{Mm^2}{M_P^2}\right]^{1/2}\sqrt{r}$$

$$r_{\text{Vainshtein}} \sim \left(r_{\text{Schwarz.}} m^{-2}\right)^{1/3}$$

How do objects move under these screening mechanisms?

- One would think the answer is simple: objects move on geodesics in Jordan frame, where matter is minimally coupled to metric.

$$S_m \sim \int d^4x \, \underbrace{h_{\mu\nu}}_{\text{Jordan}} T^{\mu\nu} \sim \int d^4x \, \left[\underbrace{\tilde{h}_{\mu\nu}}_{\text{Einstein}} + \eta_{\mu\nu}\varphi\right] T^{\mu\nu}$$

- Not so fast: this might be true for infinitesimal test particles, but is it true for extended objects?

Even in Newtonian gravity, extended objects do not necessarily move like a test particle.
They only do if we ignore tides.

e.g. the Earth's motion is well approximated by that of a test particle because the Earth is small compared to the scale on which the Sun's grav. field varies (Principia).

We will work within the same zero-tide approximation.

Intuitive reasoning in Einstein frame:

$$S_m \sim \int d^4x \left[\tilde{h}_{\mu\nu} + \eta_{\mu\nu}\varphi\right] T^{\mu\nu}$$

Scalar mediates a fifth force.

- Scalar is universally coupled: no apparent equivalence principle violation in microscopic action.
- Macroscopic object interacts with scalar via charge:

$$S_{\rm int} \sim Q \int d\tau \varphi$$

- A well-known effect (Nordvedt): $Q \sim \int d^3x T_\mu{}^\mu$
  Therefore, relativistic object has $Q/M \to 0$.
  A black hole and a star would therefore fall at different rates because star has Q/M=1, while black hole has Q/M = 0 (no hair).

- Nordvedt effect is $O(1/c^2)$ in sense of post-Newtonian expansion i.e. $(1 - Q/M)$ is roughly equal to fraction of M from gravitational binding energy.
- Chameleon screening adds a new twist: an $O(1)$ equivalence principle violation, from classical renormalization of Q.
- The scalar field interior to a screened object is massive and Yukawa suppressed. From outside's perspective, only a small fraction of the screened object's mass (that at the surface) contributes to its exterior scalar profile i.e. effective Q is small.
- Screened and unscreened objects have $O(1)$ difference in Q/M, and therefore $O(1)$ equivalence principle violation.

Intuitive reasoning in Einstein frame:

$$S_m \sim \int d^4x \, [\tilde{h}_{\mu\nu} + \eta_{\mu\nu}\varphi] \, T^{\mu\nu}$$

Scalar mediates a fifth force.

- Scalar is universally coupled: no apparent equivalence principle violation in microscopic action.
- Macroscopic object interacts with scalar via charge:

$$S_{\rm int} \sim Q \int d\tau \varphi$$

- A well-known effect (Nordvedt): $Q \sim \int d^3x T_\mu{}^\mu$

  Therefore, relativistic object has $Q/M \rightarrow 0$.

  A black hole and a star would therefore fall at different rates because star has $Q/M=1$, while black hole has $Q/M = 0$ (no hair).

- Nordvedt effect is $O(1/c^2)$ in sense of post-Newtonian expansion i.e. $(1 - Q/M)$ is roughly equal to fraction of M from gravitational binding energy.
- Chameleon screening adds a new twist: an $O(1)$ equivalence principle violation, from classical renormalization of Q.
- The scalar field interior to a screened object is massive and Yukawa suppressed. From outside's perspective, only a small fraction of the screened object's mass (that at the surface) contributes to its exterior scalar profile i.e. effective Q is small.
- Screened and unscreened objects have $O(1)$ difference in Q/M, and therefore $O(1)$ equivalence principle violation.

Intuitive reasoning in Einstein frame:

$$S_m \sim \int d^4x \left[\tilde{h}_{\mu\nu} + \eta_{\mu\nu}\varphi\right] T^{\mu\nu}$$

Scalar mediates a fifth force.

- Scalar is universally coupled: no apparent equivalence principle violation in microscopic action.
- Macroscopic object interacts with scalar via charge:

$$S_{\rm int} \sim Q \int d\tau \varphi$$

- A well-known effect (Nordvedt): $Q \sim \int d^3x T_\mu{}^\mu$

  Therefore, relativistic object has $Q/M \longrightarrow 0$.

  A black hole and a star would therefore fall at different rates because star has Q/M=1, while black hole has Q/M = 0 (no hair).

- Nordvedt effect is $O(1/c^2)$ in sense of post-Newtonian expansion i.e. $(1 - Q/M)$ is roughly equal to fraction of M from gravitational binding energy.
- Chameleon screening adds a new twist: an $O(1)$ equivalence principle violation, from classical renormalization of Q.
- The scalar field interior to a screened object is massive and Yukawa suppressed. From outside's perspective, only a small fraction of the screened object's mass (that at the surface) contributes to its exterior scalar profile i.e. effective Q is small.
- Screened and unscreened objects have $O(1)$ difference in Q/M, and therefore $O(1)$ equivalence principle violation.

How chameleon mechanism works in detail:

Drop ~ and time derivatives

$\alpha =$ scalar-matter coupling
$\epsilon =$ screening factor

$$\nabla^2 \varphi = V_{,\varphi} + \alpha M_P^{-2} \rho_m$$

$$\frac{1}{r_c}\frac{\Delta\varphi}{\Delta r_c} \sim \alpha M_P^{-2}\rho_m \quad \text{implies} \quad \frac{\Delta r_c}{r_c} \sim \frac{\Delta\varphi}{\alpha G M / r_c}$$

Therefore $\varphi_{\text{out}}(r) \sim \epsilon \dfrac{\alpha G M}{r}$

unscreened $\longrightarrow$ with $\epsilon \sim 1$ if $\Delta r_c / r_c \geq 1$

screened $\longrightarrow$ and $\epsilon \sim \dfrac{\varphi}{\alpha G M / r_c}$ if $\Delta r_c / r_c < 1$

Can think of $\epsilon\alpha M$ as scalar charge Q.

Depth of potential determines screened vs unscreened.

Motion of an extended object:

momentum $P_i = \int d^3x\, t_i{}^0$

momentum flux

$$\dot{P}_i = \int d^3x\, \partial_0 t_i{}^0 = -\int d^3x\, \partial_j t_i{}^j = -\oint dS \hat{x}_j t_i{}^j$$

What $t_\mu{}^\nu$ to use ?

Einstein frame: $G_\mu{}^\nu = 8\pi G T_\mu{}^\nu$

Let $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ $\quad$ ($h_{\mu\nu}$ not small)

$$G^{(1)}{}_\mu{}^\nu = 8\pi G \left[T_\mu{}^\nu - \frac{G^{(2)}{}_\mu{}^\nu}{8\pi G} \right] \equiv 8\pi G t_\mu{}^\nu$$

$\partial_\nu G^{(1)}{}_\mu{}^\nu = 0$ implies $\partial_\nu t_\mu{}^\nu = 0$

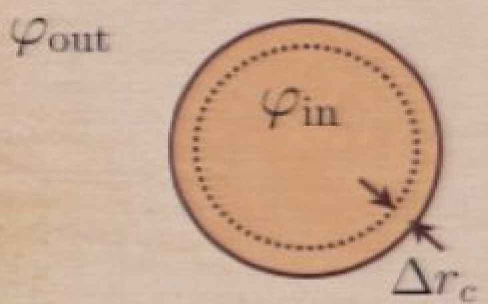
Einstein, Hofmann, Infeld; Damour

How chameleon mechanism works in detail:

Drop ~ and time derivatives

α = scalar-matter coupling
ϵ = screening factor

$$\nabla^2\varphi = V_{,\varphi} + \alpha M_P^{-2}\rho_m$$



$$\frac{1}{r_c}\frac{\Delta\varphi}{\Delta r_c} \sim \alpha M_P^{-2}\rho_m \quad \text{implies} \quad \frac{\Delta r_c}{r_c} \sim \frac{\Delta\varphi}{\alpha GM/r_c}$$

Therefore $\varphi_{\rm out}(r) \sim \epsilon\dfrac{\alpha GM}{r}$

unscreened $\longrightarrow$ with $\epsilon \sim 1$ if $\Delta r_c/r_c \geq 1$

screened $\longrightarrow$ and $\epsilon \sim \dfrac{\varphi}{\alpha GM/r_c}$ if $\Delta r_c/r_c < 1$

Can think of $\epsilon\alpha M$ as scalar charge Q.

Depth of potential determines screened vs unscreened.

Motion of an extended object:

momentum $P_i = \int d^3x\, t_i{}^0$ momentum flux

$$\dot{P}_i = \int d^3x\, \partial_0 t_i{}^0 = -\int d^3x\, \partial_j t_i{}^j = -\oint dS\, \hat{x}_j t_i{}^j$$

What $t_\mu{}^\nu$ to use ?

Einstein frame: $G_\mu{}^\nu = 8\pi G T_\mu{}^\nu$

Let $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ $\quad$ ($h_{\mu\nu}$ not small)

$$G^{(1)}{}_\mu{}^\nu = 8\pi G \left[T_\mu{}^\nu - \frac{G^{(2)}{}_\mu{}^\nu}{8\pi G} \right] \equiv 8\pi G t_\mu{}^\nu$$

$$\partial_\nu G^{(1)}{}_\mu{}^\nu = 0 \quad \text{implies} \quad \partial_\nu t_\mu{}^\nu = 0$$

Einstein, Hofmann, Infeld; Damour

Motion of an extended object:

momentum $P_i = \int d^3x \, t_i{}^0$ momentum flux

$$\dot{P}_i = \int d^3x \, \partial_0 t_i{}^0 = -\int d^3x \, \partial_j t_i{}^j = -\oint dS \hat{x}_j t_i{}^j$$

What $t_\mu{}^\nu$ to use ?

Jordan frame extra

Jordan frame: $G_\mu{}^\nu + \Delta_\mu{}^\nu = 8\pi G T_\mu{}^\nu$

Let $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ $\quad$ ($h_{\mu\nu}$ not small)

$$G^{(1)}{}_\mu{}^\nu = 8\pi G \left[T_\mu{}^\nu - \frac{G^{(2)}{}_\mu{}^\nu}{8\pi G} - \frac{\Delta_\mu{}^\nu}{8\pi G} \right] \equiv 8\pi G t_\mu{}^\nu$$

$$\partial_\nu G^{(1)}{}_\mu{}^\nu = 0 \quad \text{implies} \quad \partial_\nu t_\mu{}^\nu = 0$$

Einstein, Hofmann, Infeld; Damour

Motion of an extended object:

momentum $P_i = \int d^3x\, t_i{}^0$

momentum flux

$$\dot{P}_i = \int d^3x\, \partial_0 t_i{}^0 = -\int d^3x\, \partial_j t_i{}^j = -\oint dS\, \hat{x}_j t_i{}^j$$

What $t_\mu{}^\nu$ to use ?

Einstein frame: $G_\mu{}^\nu = 8\pi G T_\mu{}^\nu$

Let $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ $\quad$ ($h_{\mu\nu}$ not small)

$$G^{(1)}{}_\mu{}^\nu = 8\pi G \left[T_\mu{}^\nu - \frac{G^{(2)}{}_\mu{}^\nu}{8\pi G} \right] \equiv 8\pi G t_\mu{}^\nu$$

$$\partial_\nu G^{(1)}{}_\mu{}^\nu = 0 \quad \text{implies} \quad \partial_\nu t_\mu{}^\nu = 0$$

Einstein, Hofmann, Infeld; Damour

Motion of an extended object:

momentum $P_i = \int d^3x\, t_i{}^0$

momentum flux

$$\dot{P}_i = \int d^3x\, \partial_0 t_i{}^0 = -\int d^3x\, \partial_j t_i{}^j = -\oint dS \hat{x}_j t_i{}^j$$

What $t_\mu{}^\nu$ to use ?

Jordan frame extra

Jordan frame: $G_\mu{}^\nu + \Delta_\mu{}^\nu = 8\pi G T_\mu{}^\nu$

Let $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ $\quad$ ($h_{\mu\nu}$ not small)

$$G^{(1)}{}_\mu{}^\nu = 8\pi G \left[T_\mu{}^\nu - \frac{G^{(2)}{}_\mu{}^\nu}{8\pi G} - \frac{\Delta_\mu{}^\nu}{8\pi G} \right] \equiv 8\pi G t_\mu{}^\nu$$

$\partial_\nu G^{(1)}{}_\mu{}^\nu = 0$ implies $\partial_\nu t_\mu{}^\nu = 0$

Einstein, Hofmann, Infeld; Damour

Trick: choose S so that $h_{\mu\nu}$ is small at S, but not necessarily at object.
Bypass consideration of self-forces.

e.g. $-\frac{1}{4\pi G}\oint dS\hat{x}^j\partial_i\Phi\partial_j\Phi$ where $\Phi =$ grav. potential

split: $\Phi = \Phi_{\rm ext} + \Phi_{\rm obj}$ where $\partial_i\Phi_{\rm ext} \sim$ const. and $\Phi_{\rm obj} = -GM/r$

$$-\frac{1}{4\pi G}\oint dS\hat{x}^j\partial_i\Phi\partial_j\Phi = -M\partial_i\Phi_{\rm ext}$$

Motion of an extended object:

momentum $P_i = \int d^3x\, t_i{}^0$

momentum flux

$$\dot{P}_i = \int d^3x\, \partial_0 t_i{}^0 = -\int d^3x\, \partial_j t_i{}^j = -\oint dS \hat{x}_j t_i{}^j$$

What $t_\mu{}^\nu$ to use ?

Jordan frame extra

Jordan frame: $G_\mu{}^\nu + \Delta_\mu{}^\nu = 8\pi G T_\mu{}^\nu$

Let $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ ($h_{\mu\nu}$ not small)

$$G^{(1)}{}_\mu{}^\nu = 8\pi G \left[T_\mu{}^\nu - \frac{G^{(2)}{}_\mu{}^\nu}{8\pi G} - \frac{\Delta_\mu{}^\nu}{8\pi G} \right] \equiv 8\pi G t_\mu{}^\nu$$

$\partial_\nu G^{(1)}{}_\mu{}^\nu = 0$ implies $\partial_\nu t_\mu{}^\nu = 0$

Einstein, Hofmann, Infeld; Damour

Trick: choose S so that $h_{\mu\nu}$ is small at S,
but not necessarily at object.
Bypass consideration of self-forces.

e.g. $-\frac{1}{4\pi G}\oint dS\hat{x}^j\partial_i\Phi\partial_j\Phi$ where $\Phi =$ grav. potential

split: $\Phi = \Phi_{\text{ext}} + \Phi_{\text{obj}}$ where $\partial_i\Phi_{\text{ext}} \sim$ const. and $\Phi_{\text{obj}} = -GM/r$

$$-\frac{1}{4\pi G}\oint dS\hat{x}^j\partial_i\Phi\partial_j\Phi = -M\partial_i\Phi_{\text{ext}}$$

Jordan frame summary for chameleon:

$$M\ddot{X}_i = -M\left[\frac{1+2\epsilon\alpha^2}{1+2\alpha^2}\right]\partial_i\Phi_{\rm ext}$$

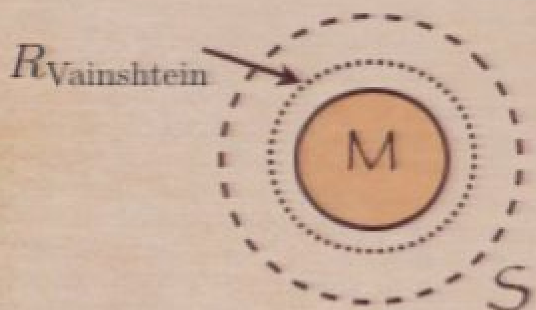
Recall $\epsilon \sim 1$ for unscreened objects and $\epsilon \sim 0$ for screened objects

For $f(R)$ $\alpha = 1/\sqrt{6}$: unscreened grav. mass $= 4/3$ screened grav. mass

Generically $\alpha \sim 1$, so expect O(1) violation of equivalence principle between screened and unscreened objects. Only unscreened objects move on geodesics.

Interestingly, for Vainshtein mechanism , there's no such O(1) violation of equivalence principle.

$$\text{Eqt for } \varphi: \ \partial_\mu J^\mu \sim \frac{\rho_m}{M_P^2} \quad \text{where} \quad J^\mu \sim \partial^\mu\varphi + \frac{1}{m^2}\partial^\mu\varphi\partial^2\varphi$$



Scalar charge is conserved.

Reason: shift symmetry.

Recall important chameleon parameters: α & $\frac{\varphi}{\alpha}$

scalar-matter coulping   controls screening

$$M\ddot{X}_i = -M\left[\frac{1+2\epsilon\alpha^2}{1+2\alpha^2}\right]\partial_i\Phi_{\rm ext}$$

$$\epsilon \sim 0 \text{ if } \frac{\varphi}{\alpha GM/r_c} < 1 \text{ (screened)} \quad , \quad \epsilon \sim 1 \text{ if } \frac{\varphi}{\alpha GM/r_c} \geq 1 \text{ (unscreened)}$$

Current observational bound:

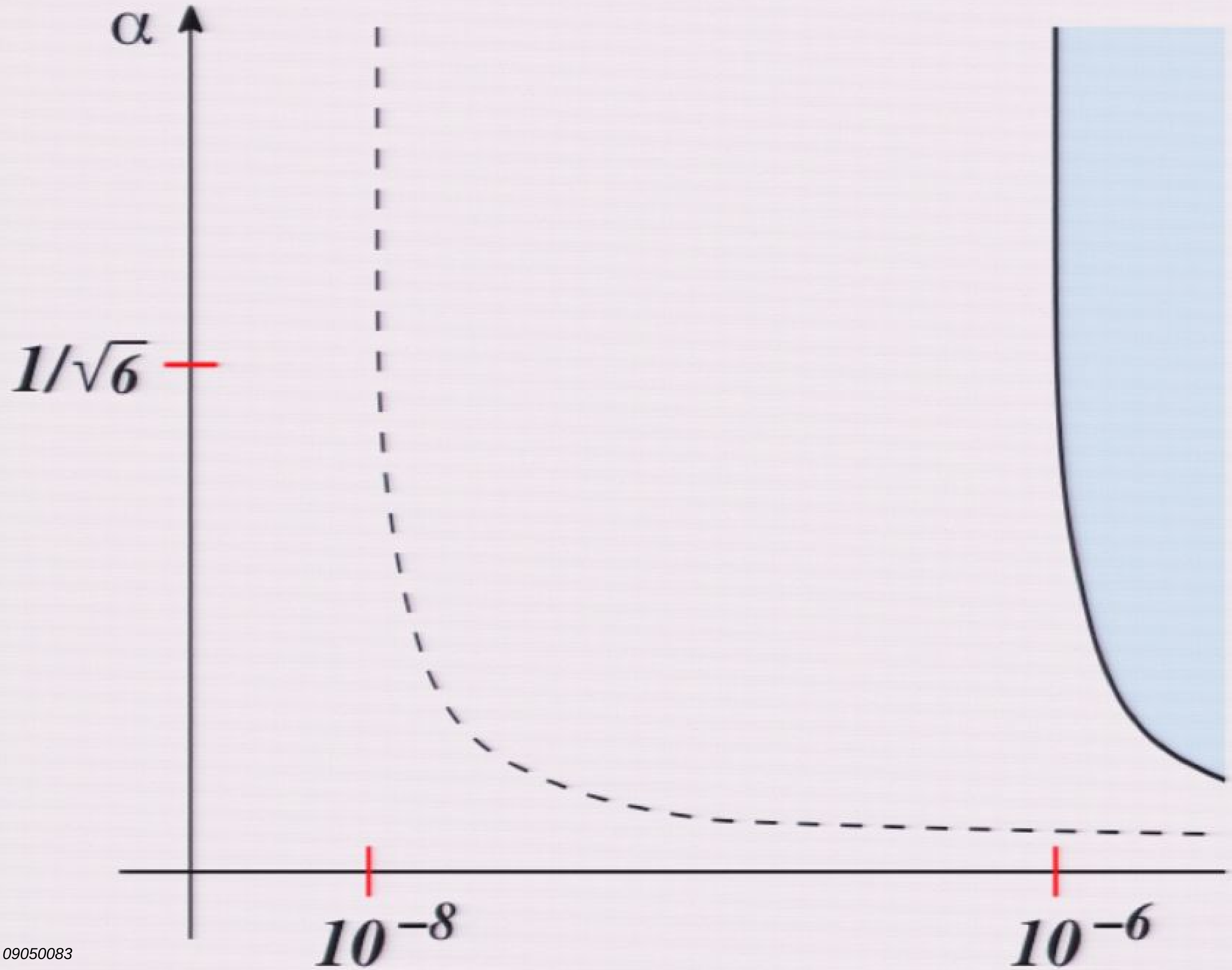
$$\frac{GM}{r_c} \text{ for Milky Way \& Sun } \sim 10^{-6}\left[\frac{v}{300\,\text{km/s}}\right]^2$$

Therefore $\frac{\varphi}{\alpha} \lesssim 10^{-6}$.

Observationally interesting region: $10^{-8} \lesssim \frac{\varphi}{\alpha} \lesssim 10^{-6}$

unscreened gal.   screened gal.

note : $\alpha\varphi \sim f_R$


α
$1/\sqrt{6}$
10^{-8}
10^{-6}

Bulk motion tests:

1. Small galaxies should move faster than large galaxies. Must avoid blanket screening in overdense regions. Also need to avoid Yukawa suppression.

2. Small galaxies should stream out of voids faster than large galaxies creating larger than expected voids.

Internal motion tests:

3. Diffuse gas (e.g. HI) should move faster than stars in small galaxies even if they are on the same orbit.

4. Gravitational lensing mass should agree with dynamical mass from stars, but disagree with that from HI in small galaxies.

Generic consequences of a light scalar (scalar-tensor theories: DGP, f(R), etc):

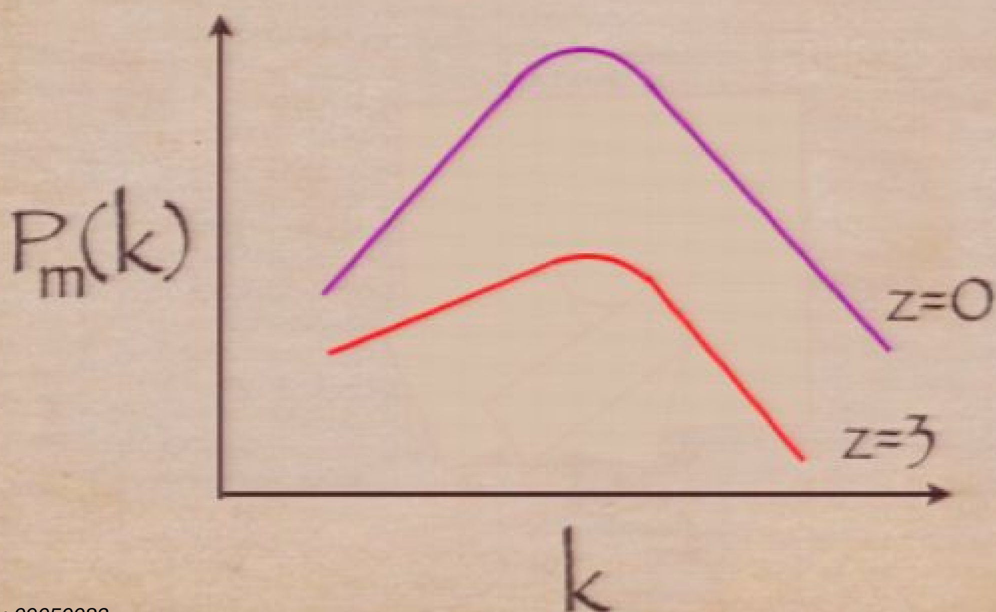
1. Scale-dependent growth leads to a large scale scale-dependent galaxy bias (with Parfrey).

2. O(1) violation of equivalence principle: large and small galaxies fall at different rates (with Nicolis).

Classic argument due to Jim Fry: ①

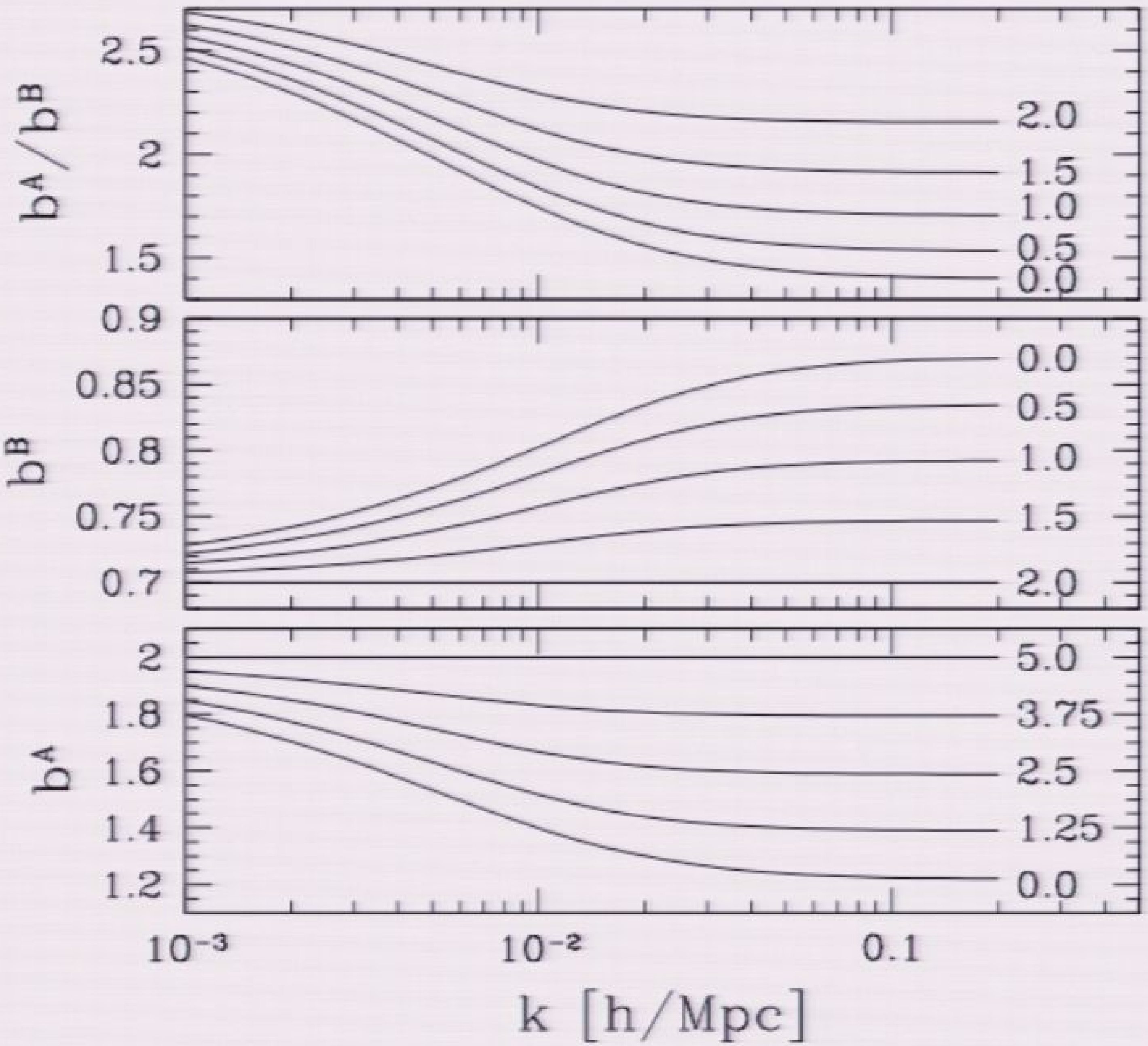
Galaxy bias likes to relax towards unity. $b(z) = 1 + \frac{\#}{D(z)}$

This is robust against modifications of gravity.



Scale dependent galaxy bias!

with Kyle Parfrey

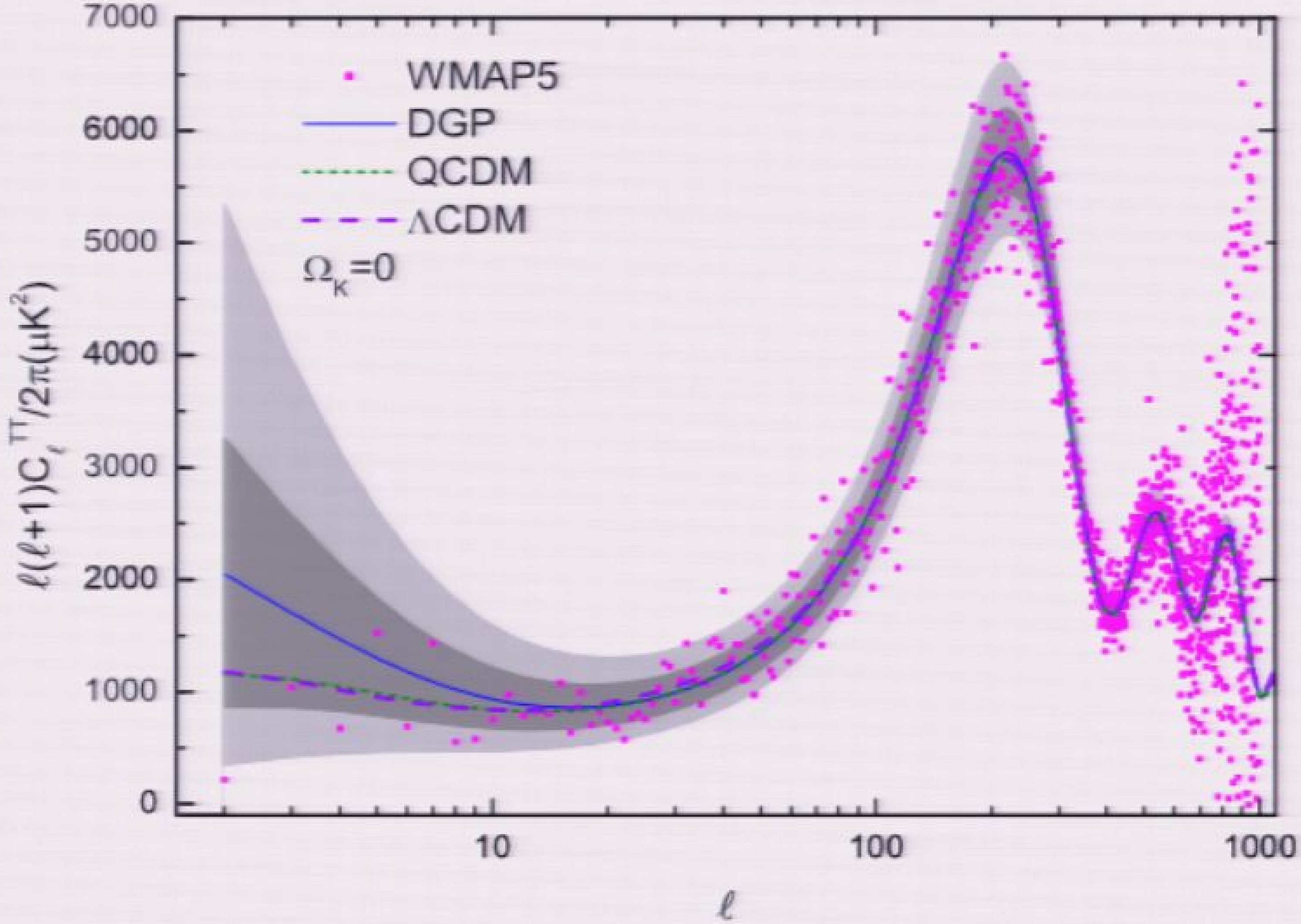


2.5
2
1.5
b^A/b^B
2.0
1.5
1.0
0.5
0.0
0.9
0.85
0.8
0.75
0.7
b^B
0.0
0.5
1.0
1.5
2.0
2
1.8
1.6
1.4
1.2
b^A
5.0
3.75
2.5
1.25
0.0
10^{-3}
10^{-2}
0.1
k [h/Mpc]

self-acc. DGP fit worse than LCDM by $\Delta\chi^2 = 28$.

(1/3 from growth, 2/3 from geometry)

Fang, Hu, Haiman, LH, May


7000
6000
5000
4000
3000
2000
1000
0
10
100
1000
WMAP5
DGP
QCDM
ΛCDM
$\Omega_K=0$
$\ell(\ell+1)C_\ell^{TT}/2\pi(\mu K^2)$
ℓ

self-acc. DGP fit worse than LCDM by $\Delta\chi^2 = 28$.

(1/3 from growth, 2/3 from geometry)

Fang, Hu, Haiman, LH, May