Title: Dilution of the Cosmological Constant: Higher Codimension Branes and Higher Curvature Terms

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Abstract: TBA

Dilution of the Cosmological Constant

Higher Codimension Branes and Higher Curvature Terms

Alberto Iglesias

University of California, Davis

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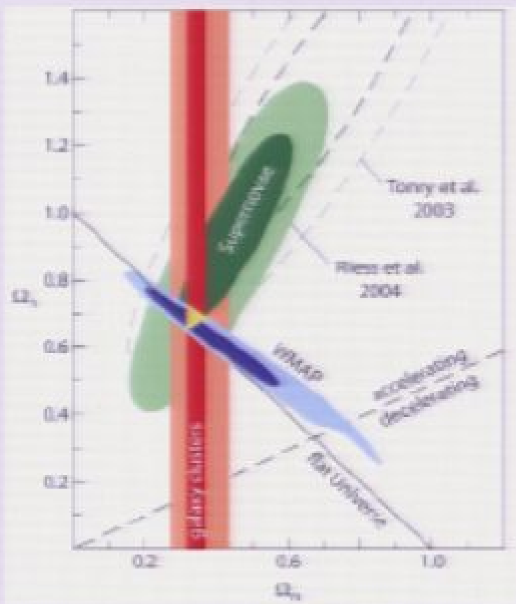
Outline

- Cosmological Constant Problem
- Infinite Volume Extra Dimensions
  Setup: higher curvature terms → Gauss-Bonnet
- Codimension 2 case
- Higher Codimension
  Singularities and smoothing
- Codimension 3 case
- Conclusions

Cosmological Constant Problem

$$\mathcal{L} = M_{pl}^2 \int d^4x \sqrt{-g}(R - 2\Lambda) \;+\; \mathcal{L}_{SM}$$

Observations



$$M_{pl}^2 \Lambda_{obs} \sim (10^{-3} eV)^4$$

QFT with cutoff

$$\langle \mathcal{E} \rangle = \int_0^{M_{cutoff}} k^2 dk \, \frac{\omega_k}{2}$$

$$M_{pl}^2 \Lambda_{QFT} \sim M_{cutoff}{}^4$$

Goal

- Since $\Lambda_{obs} \ll \Lambda_{QFT}$
- Find a way to make geometry of our $3+1$ spacetime *insensitive* to Λ_{QFT} $\rightarrow$ embed in extra dimensions
- Use $\langle \mathcal{E} \rangle$ to deform extra dimensions
  - Codimension 2: create a deficit angle
  - Higher codimension: create curvature
- In particular: find compatibility of Minkowski $3+1$ for a wide range of values of Λ

Setup

Brane of codimension d in bulk of dimension D

$$S = \hat{M}^{D-4} \int_{\Sigma_{D-d}} (\hat{R} - \hat{\Lambda}) + M^{D-2} \int_{\mathcal{M}_D} [R + \xi(R^2 - 4R_{MN}^2 + R_{MNRS}^2)]$$

Codimension 2

Codimension 2

$$S = -\int_{\Sigma_{D-2}} f + M^{D-2}\int_{\mathcal{M}_D}[R + \xi(R^2 - 4R_{MN}^2 + R_{MNRS}^2)]$$

Ansatz:

$$ds^2 = \eta_{\mu\nu}dx^\mu dx^\nu + e^{2\omega}\delta_{ij}dx^i dx^j$$

EoM:

$$\partial^j\partial_j\omega = -\frac{f}{2M^{D-2}}\delta^{(2)}(x^i)$$

$$\omega = -\frac{f}{8\pi M^{D-2}}\log\left(\frac{\rho^2}{a^2}\right)$$

$$ds_2^2 = dr^2 + e^{-2\beta}r^2 d\phi^2$$

$$r \sim \rho^{1-\theta/2\pi}$$

$$e^{-\beta} = 1 - \frac{f}{4\pi M^{D-2}}$$

$$\theta = 2\pi[1 - e^{-\beta}] = \frac{f}{2M^{D-2}}$$

$$0 < f < 4\pi M^{D-2}$$

Linearized Gravity

- At linear level EoM coincide with those for

$$S = \hat{M}^{D-4}\int_{\Sigma_{D-2}}(\hat{R}-\hat{\Lambda}) + M^{D-2}\int_{\mathcal{M}_D} R + S_{int}$$

$$\hat{M}^{D-4} = 2\xi f\,, \qquad \hat{\Lambda} = 1/2\xi$$

- Some of the diffs are broken (tension)

$$\begin{aligned} \delta h_{MN} &= \nabla_M \xi_N + \nabla_N \xi_M \\ 0 &= \partial_i \xi_\mu |_{r=0} \\ 0 &= \partial^i [\xi_i \delta^{(2)}(x^j)] \end{aligned}$$

Solution to Linearized EoM

$p^2 \neq 0$

$$H_{\mu\nu} = \chi = 0 \text{ for } \rho \neq 0$$

$$\begin{aligned} H_{\mu\nu}(p, \rho = 0) &= \frac{\hat{M}^{4-D}}{p^2}[T_{\mu\nu}(p) - \frac{1}{D-4}\eta_{\mu\nu}T(p) + \frac{2}{D-4}\frac{p_\mu p_\nu}{p^2}T(p)] \\ \chi(p, \rho = 0) &= \frac{2}{D-4}\frac{\hat{M}^{4-D}}{p^2}T(p) \end{aligned}$$

$p^2 = 0$

$$\begin{aligned} \chi(\rho) &= \frac{M^{2-D}}{2\pi(D-2)}T(p)\log\left(\frac{\rho^2+\epsilon^2}{b^2}\right) \\ H_{\mu\nu}(\rho) &= -\frac{M^{2-D}}{4\pi}[T_{\mu\nu}(p) - \frac{1}{D-2}\eta_{\mu\nu}T(p) + \hat{M}^{D-4}p_\mu p_\nu\chi(0)]\log\left(\frac{\rho^2+\epsilon^2}{b^2}\right) \end{aligned}$$

Higher Codimension

Singularities and smoothing

Tension → Background singularities

- Origin
- Finite distance (codim 3 and higher)

$(D-d)$ worldvolume (codimension d)

Replace brane worldvolume $R^{D-d-1,1}$ by $R^{D-d-1,1} \times S_\epsilon^{d-1}$

$$S = \tilde{M}^{D-3} \int_{R^{D-d-1,1} \times S_\epsilon^{d-1}} (\tilde{R} - \tilde{\Lambda}) + M^{D-2} \int_{\mathcal{M}_D} [R + \xi (GB)]$$

$$S = \tilde{M}^{D-3} \int_{\Sigma_{D-1}} (\tilde{R} - \tilde{\Lambda}) + M^{D-2} \int_{\mathcal{M}_D} [R + \xi(R^2 - 4R_{MN}^2 + R_{MNRS}^2)]$$

Ansatz:

$$ds^2 = e^{2A(r)}\eta_{\mu\nu}dx^\mu dx^\nu + e^{2B(r)}(dr^2 + r^2 d\Omega_{d-1}^2)$$

$$\downarrow$$

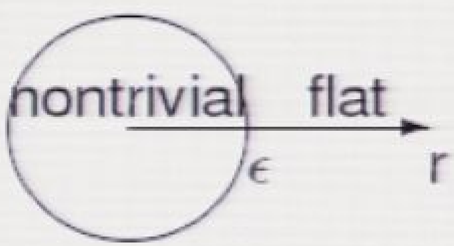
$$R^{D-d-1,1}$$

- EoM have no third or fourth derivatives
- Specialize to $d = 3$: simplification of EoM (GB)
- Want to find solutions with $\tilde{\Lambda}$ not determined by the other parameters

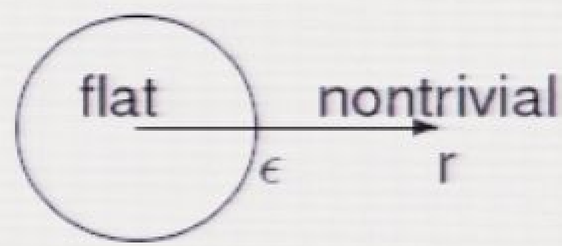
Structure of the EoM

$$
\begin{aligned}
(rr) \qquad & \{A', B, B', B''\} = 0 \\
(others) \qquad & \{A', A'', B, B', B''\} = \delta(r - \epsilon)
\end{aligned}
$$

- Flat Solution: $A,\ B$ constant
- Nontrivial solutions
- Impose junction conditions at $r = \epsilon$



Interior Solutions



Exterior Solutions

Studied cases

No Einstein-Hilbert bulk term

- String in 5D
- Membrane in 6D
- 3-brane in 7D

EH plus GB bulk term

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String in 5D

$$S = \tilde{M}^2 \int_{R^{1,1}\times S^2} (\tilde{R} - \tilde{\Lambda}) + M^3 \int_{\mathcal{M}_3} \xi (R^2 - 4R_{MN}^2 + R_{MNRS}^2)$$

$$\begin{aligned}
(A')^2[3B'^2 + 6\frac{B'}{r} + \frac{2}{r^2}] &= 0 \\
(A'' + A'^2 - A'B')(B'^2 + 2\frac{B'}{r}) + 2A'(B' + \frac{1}{r})(B'' + \frac{B'}{r}) &= \frac{e^{3B}}{8\xi}(\tilde{\Lambda} - 2\lambda)L\delta(r - \\
A'^2(B'' + \frac{B'}{r}) + 2(A'' + A'^2 - A'B')A'(B' + \frac{1}{r}) &= \frac{\tilde{\Lambda}}{8\xi} e^{3B} L\delta(r - \epsilon)
\end{aligned}$$

Solutions

- A, B constant
- $A = \log(r_A/r)^{\alpha-1}, \quad B = \log(r_B/r)^{\alpha}, \quad \alpha_\pm = 1 \pm 1/\sqrt{3}$

- Singularities

$$ds^2 = (r_A/r)^{2(\alpha-1)}\eta_{\mu\nu}dx^\mu dx^\nu + (r_B/r)^{2\alpha}(dr^2 + r^2 d\Omega_2^2)$$

$$R \sim r^{2(\alpha-1)} = r^{\pm 2/\sqrt{3}} \quad \text{for } \alpha_\pm$$

$$\sigma \sim r^{\mp 2/\sqrt{3}} + \text{finite} \quad (\text{radial geodesics})$$

Interior sol: Choose α_+ Exterior sol: Choose α_-
Geod. Complete, $R \to 0$ as $r \to 0$ (idem $r \to \infty$)

- Junction conditions

$$\pm\frac{\alpha(\alpha-1)(\alpha-2)}{\epsilon} = -\frac{\kappa-2}{8\xi}L(r_B/\epsilon)^\alpha$$

$$\pm\frac{(\alpha-1)^3}{\epsilon} = -\frac{\kappa}{8\xi}L(r_B/\epsilon)^\alpha$$

$$\xi = \frac{\sqrt{3}L\epsilon}{4(\alpha-1)}(r_B/\epsilon)^\alpha\,, \qquad \tilde{\Lambda} = \frac{8L^2}{\xi^2} \quad \text{Fixed!}$$

String in 5D

$$S = \tilde{M}^2 \int_{R^{1,1}\times S^2} (\tilde{R} - \tilde{\Lambda}) + M^3 \int_{\mathcal{M}_3} \xi (R^2 - 4R_{MN}^2 + R_{MNRS}^2)$$

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$$\xi = \frac{\sqrt{3}L\epsilon}{4(\alpha-1)}(r_B/\epsilon)^\alpha\,, \qquad \tilde{\Lambda} = \frac{8L^2}{\xi^2}$$ Fixed!

3-brane in 7D with EH + GB bulk terms

Define

$V = rA' \,, \quad U = rB' \,, \quad z = \xi^{-1} r^2 e^{2B}$

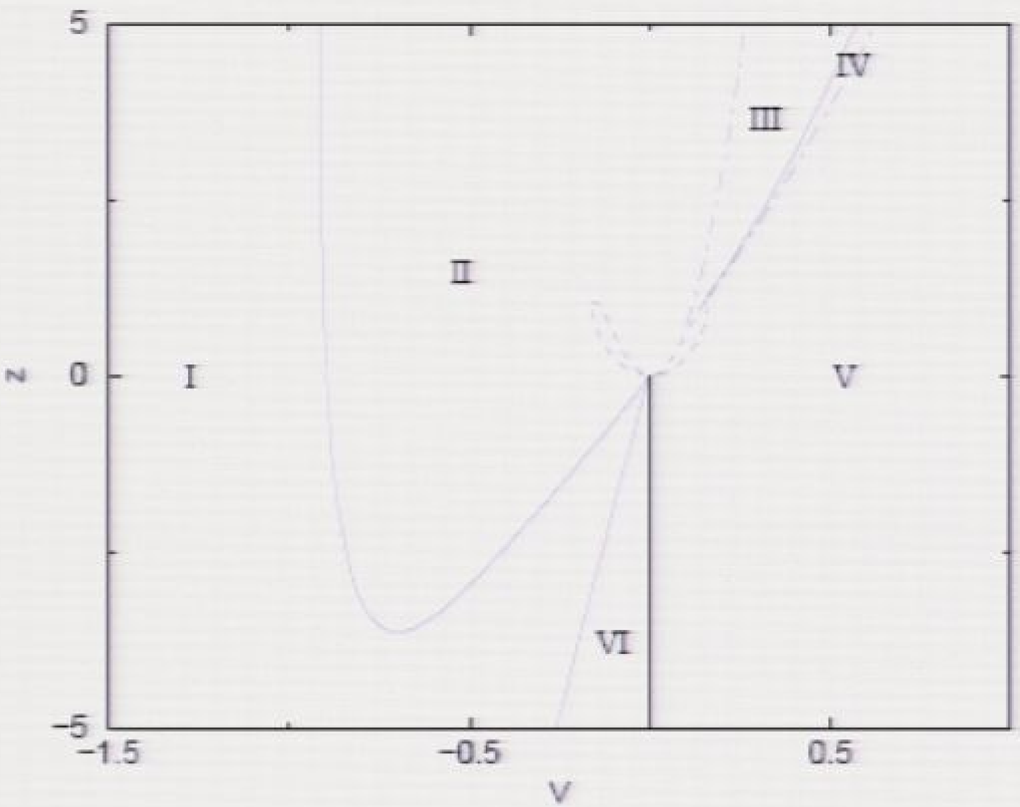
$$(rr) \qquad 0 = U^2(72V^2 - z) - 8UV(z - 12V^2) - z(6V^2 - 1) - 12V^2(2 - V^2$$

$$U \equiv U_\pm(V, z)$$

$$(rr - \alpha\beta) \qquad 2zU\frac{dV}{dz} = \frac{g_1}{g_2} \,, \quad g_{1,2} \text{ pol in } V, z$$

- Impose matching conditions
- Study flow of solutions in (V, z) plane
- Approximate equations in regions of possible divergences
- Identify coordinate/true singularities

3-brane in 7D EH+GB bulk terms: + case



I,II,V,VI $\rightarrow \quad \tilde{\Lambda} < 0$
III,IV $\rightarrow \quad \tilde{\Lambda} > 0$

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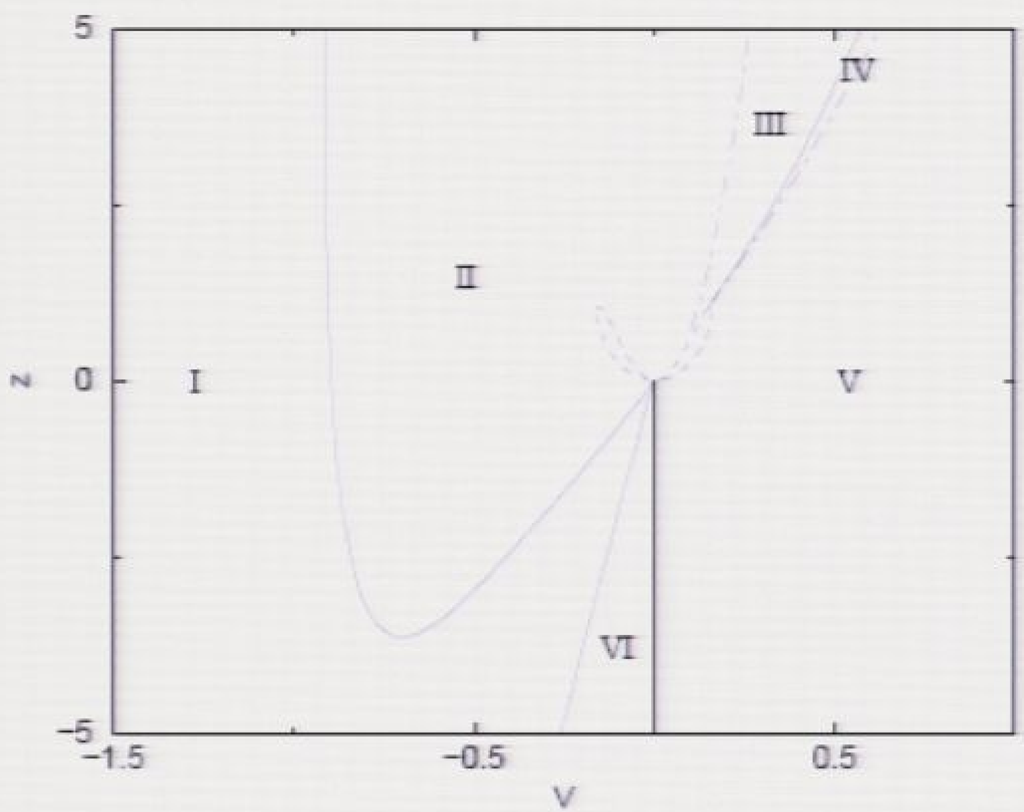

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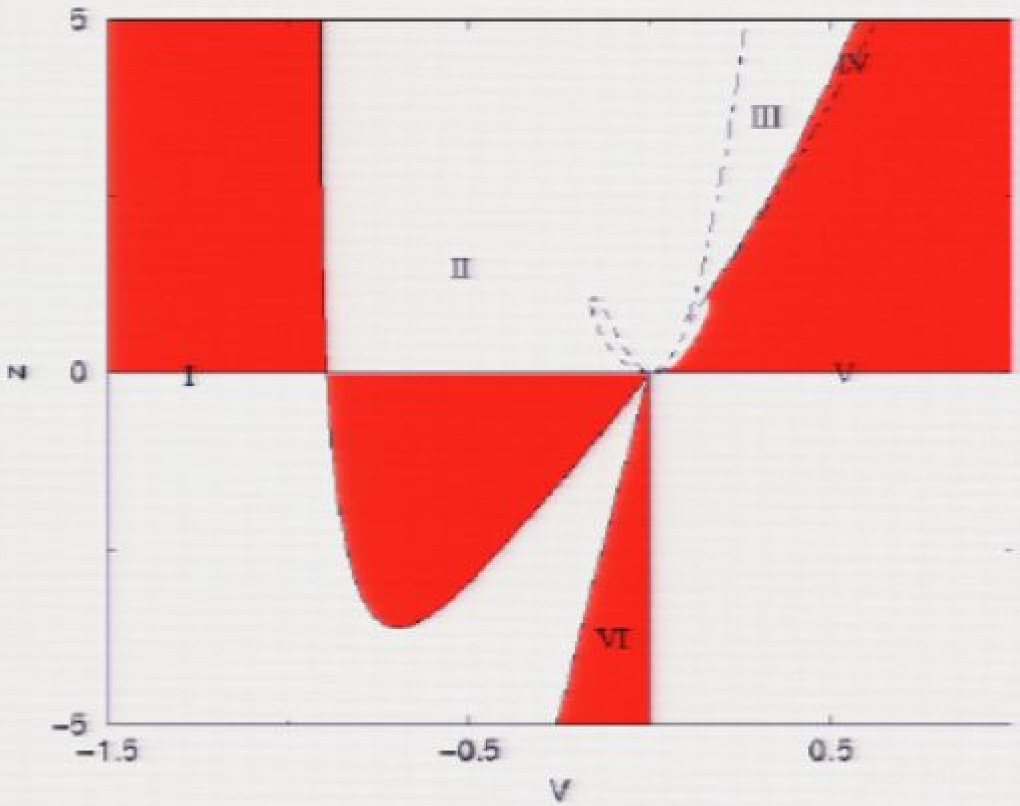
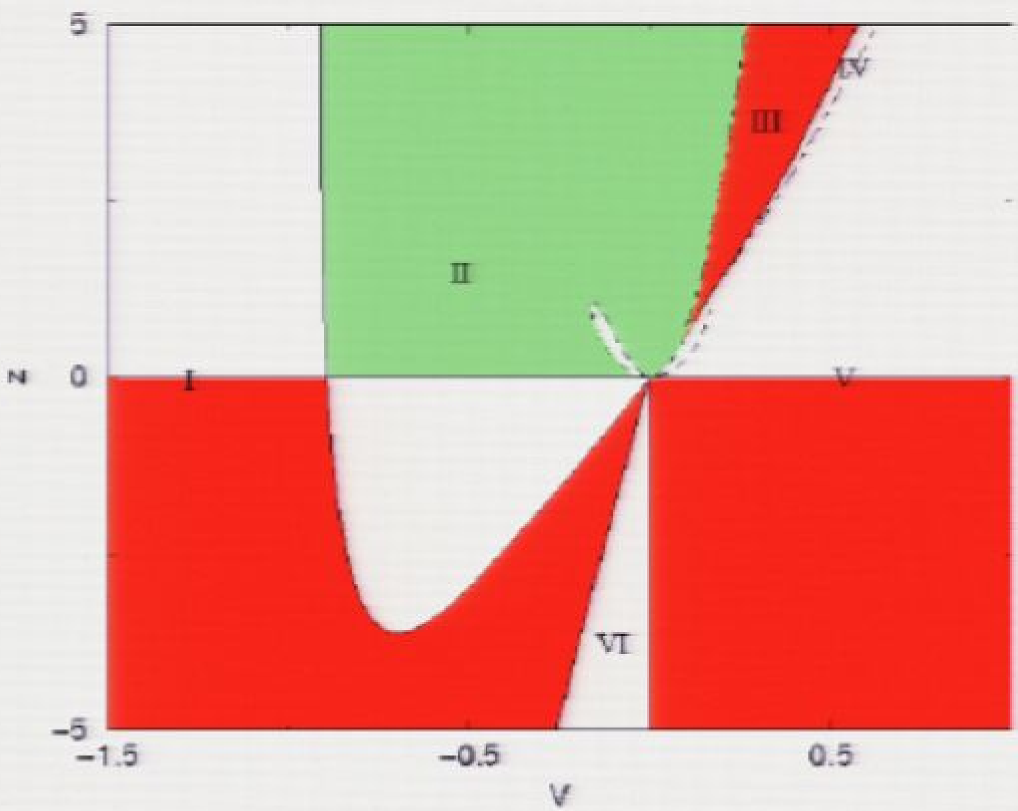


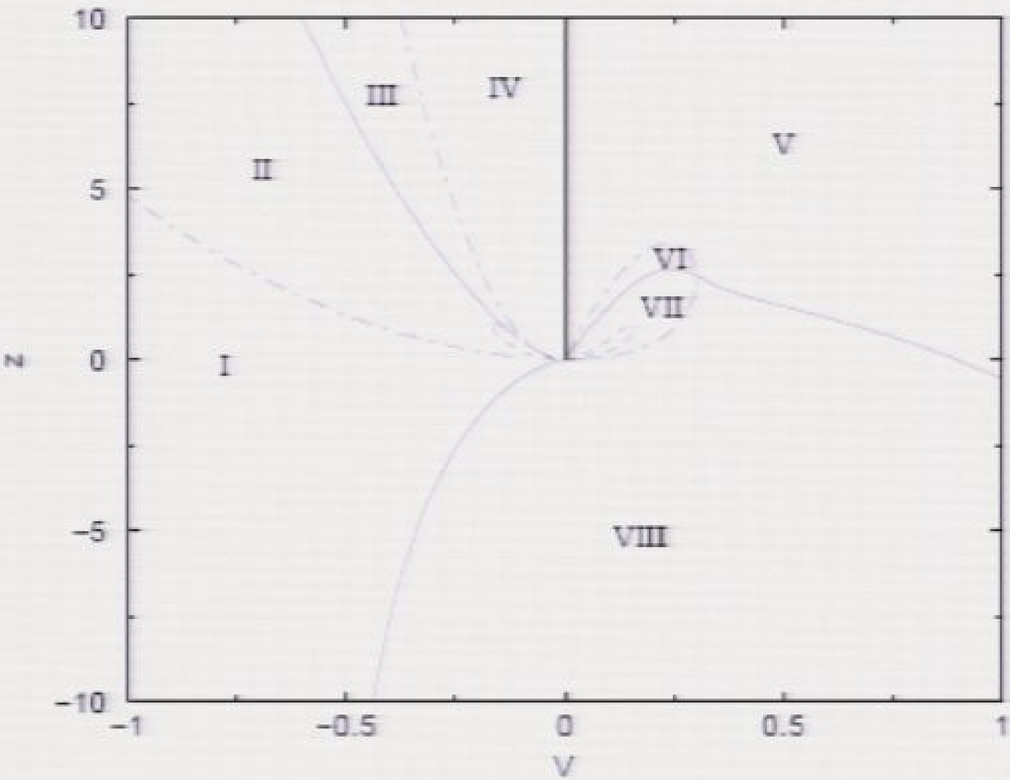
Exterior solutions all singular

3-brane in 7D EH+GB bulk terms: + case



Interior solutions
Region II
$A \sim \text{const}$, $B \sim -2 \log r/r_0$
coordinate singularity
$\tilde{\Lambda} < 0$

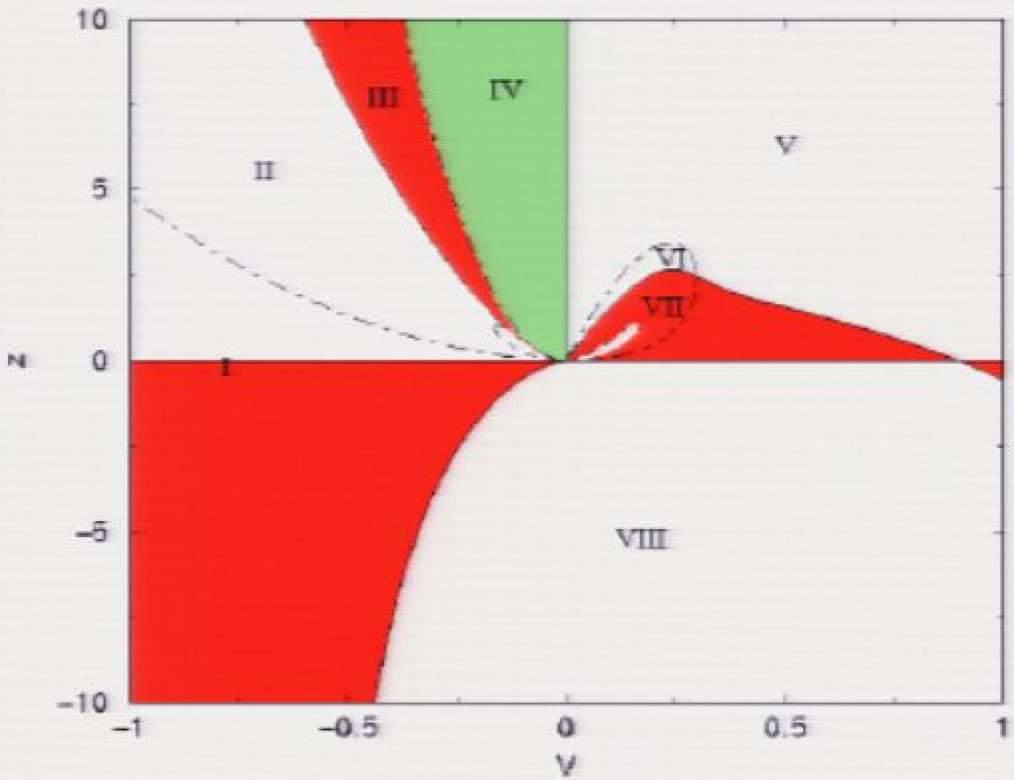
3-brane in 7D EH+GB bulk terms: - case



I,III,V,VII $\rightarrow \quad \tilde{\Lambda} > 0$

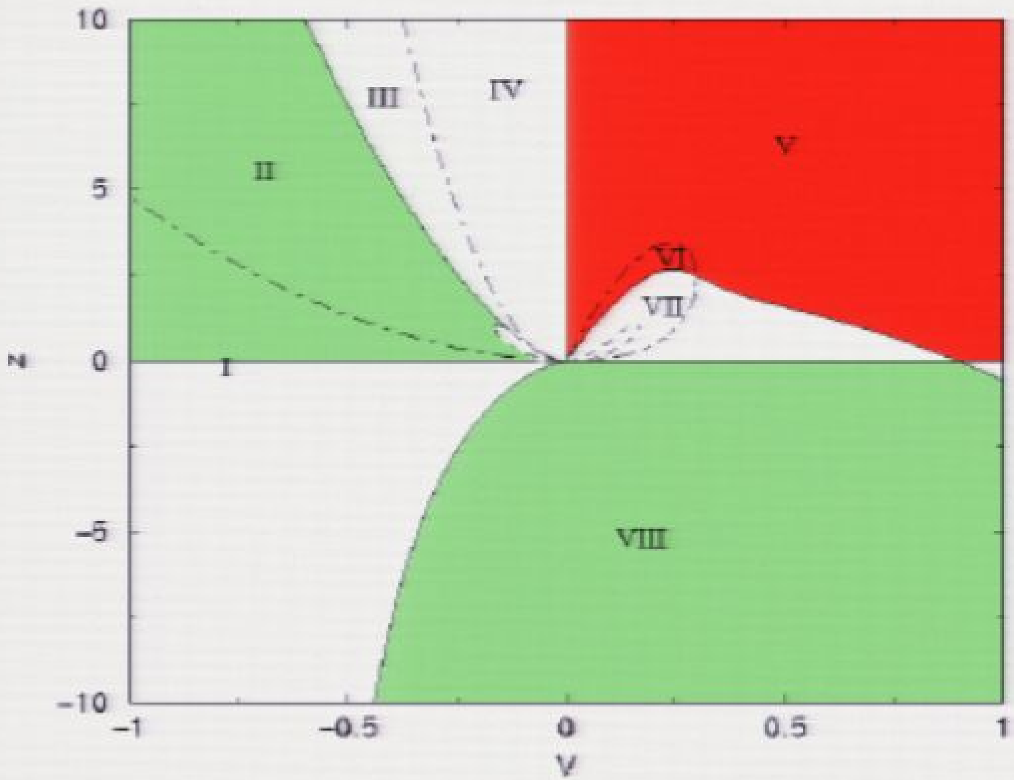
II,IV,VI,VIII $\rightarrow \quad \tilde{\Lambda} < 0$

3-brane in 7D EH+GB bulk terms: - case



Exterior solutions
Region IV
$A, B \sim$ const
$\tilde{\Lambda} < 0$

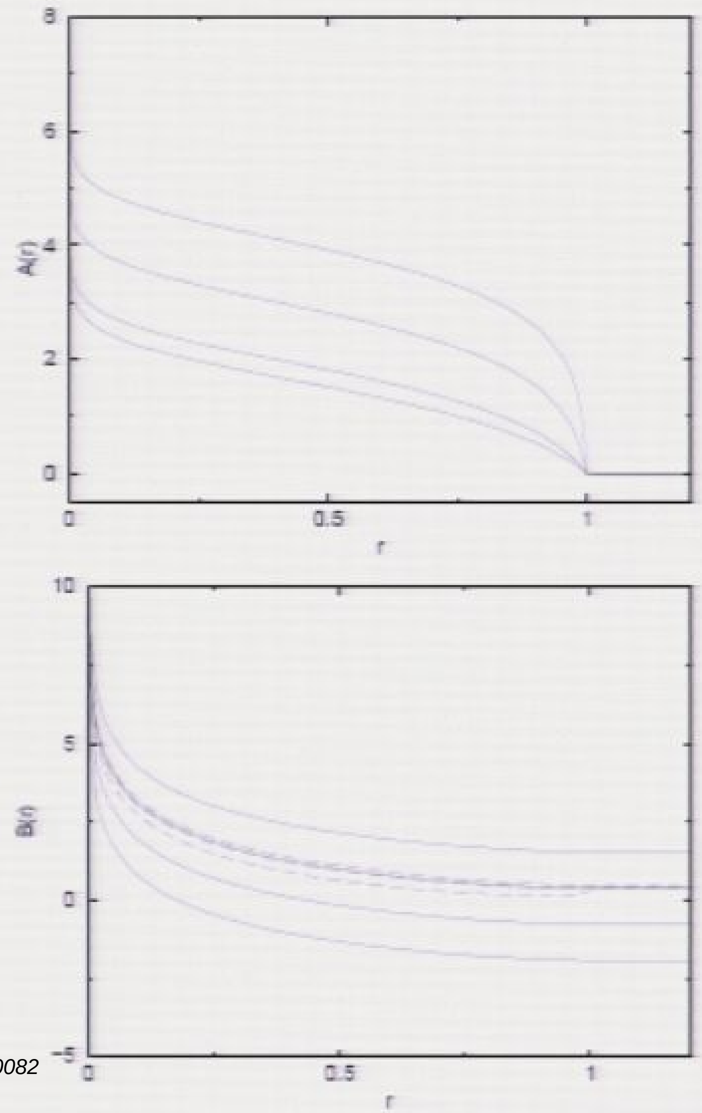
3-brane in 7D EH+GB bulk terms: - case



Interior solutions
Region VIII
$A \sim \text{const}$, $B \sim -2 \log r/r_0$
coordinate singularity
$\tilde{\Lambda} < 0$

Regions I, II
$A \sim -\log(\log r/r_0)$,
$e^{-B} \sim r \log r/r_0$
coordinate singularity
$\tilde{\Lambda} > 0$ in I

3-brane in 7D EH+GB bulk terms: - case interior solutions $\tilde{\Lambda} > 0$



$$\tilde{M}^4/M^2\xi = 10^8,\ 10^6,\ 10^4,\ 1$$

$$\tilde{\Lambda} = .1,\ 1,\ 10,\ 100$$

Conclusions

- Found smooth interior infinite volume 3-brane solutions with flat worldvolume for a wide range of positive (as well as negative) values of the brane tension that could account for zero point energy of SM fields
- Stability should be further analyzed
- Viability of 4D gravity in this setups needs to be carefully addressed
- These setups provide a fruitful arena to study the phenomenon of dilution of the CC and might hint on a possible way towards a solution to the CC Problem