

Title: Cascading Gravity and the Cosmological Constant

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Abstract: TBA

# Cascading Gravity and $\Lambda$

Justin Khoury

with N. Afshordi, C. de Rham, G. Geshnizjani, A. Tolley, M. Wyman

G. Dvali, S. Hofmann and JK, Phys. Rev. D76, 084006 (2007)

C. de Rham, S. Hofmann, JK and A. Tolley JCAP 0802:011 (2008)

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Phys. Rev. Lett. 100, 251603 (2008)

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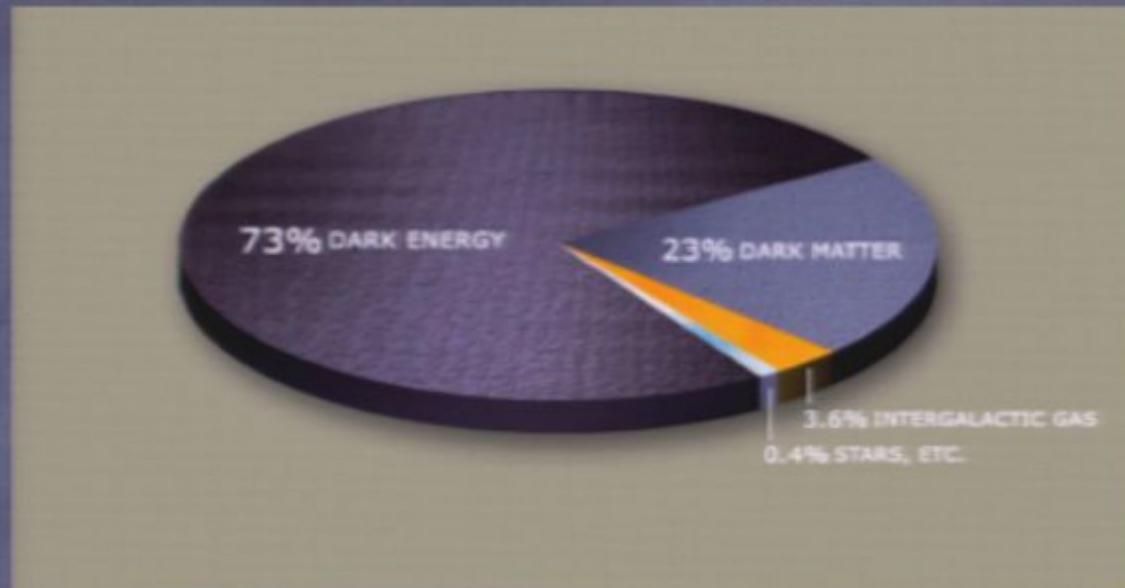
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# Concordance $\Lambda$ CDM model

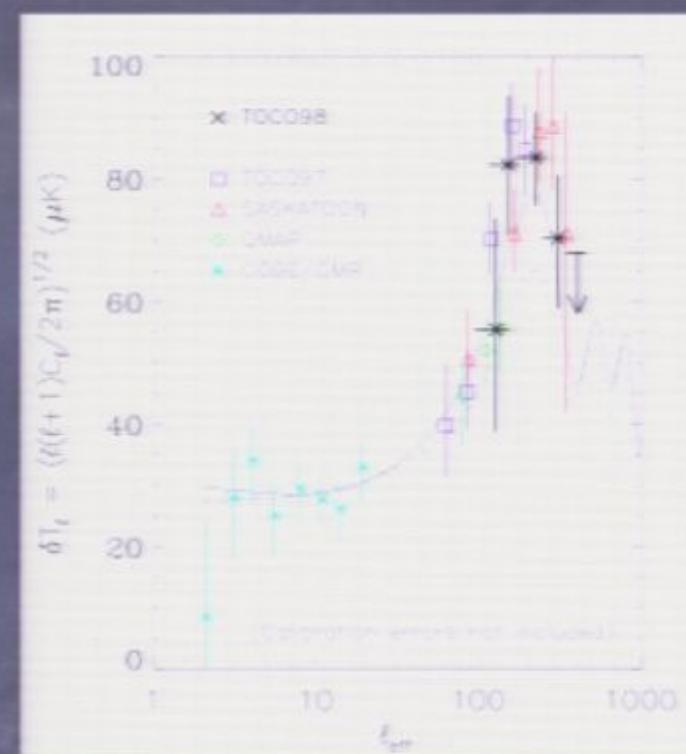


- Cosmology has a standard model
- Remarkably predictive

$\Lambda$  → expansion history  
→ growth of structure

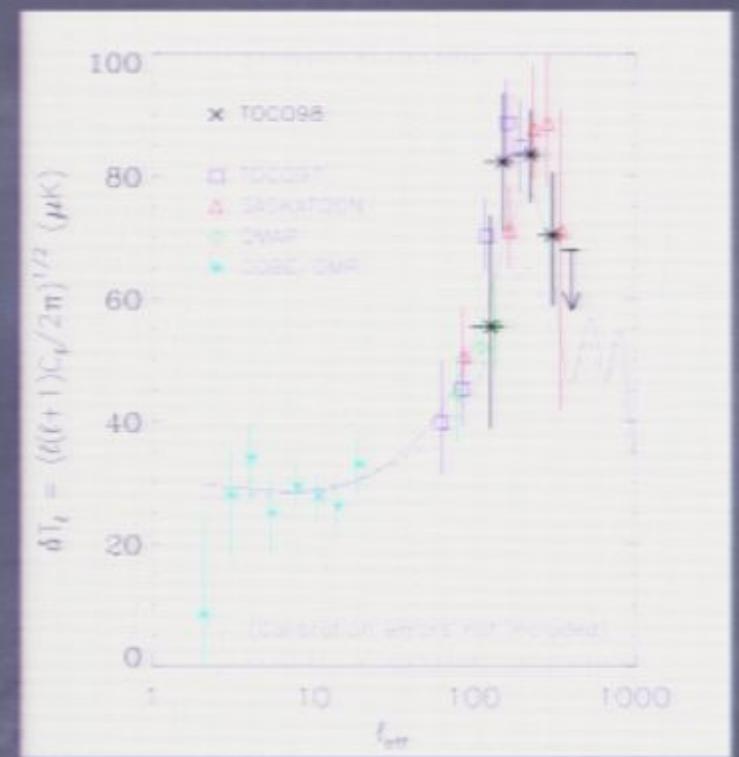
# TOCO

## June 1999



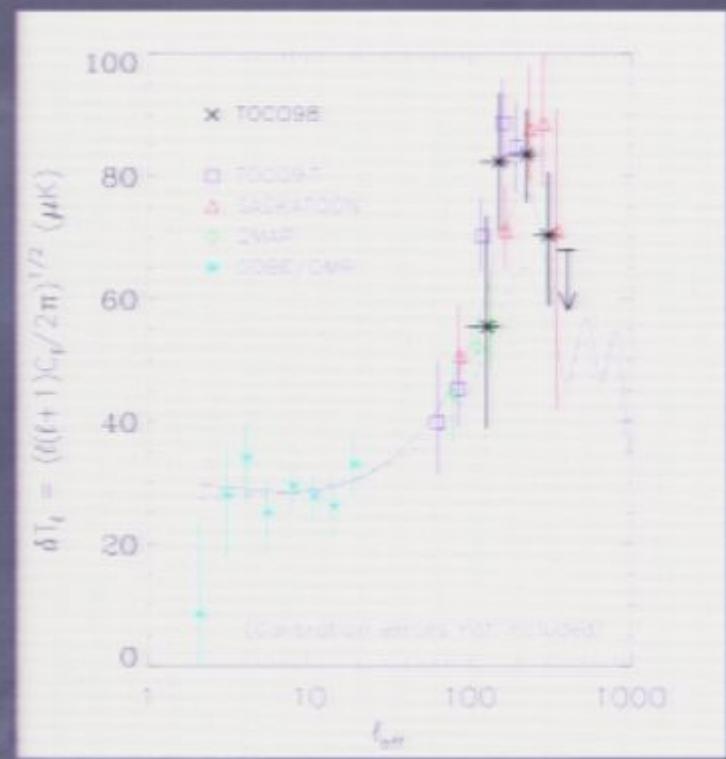
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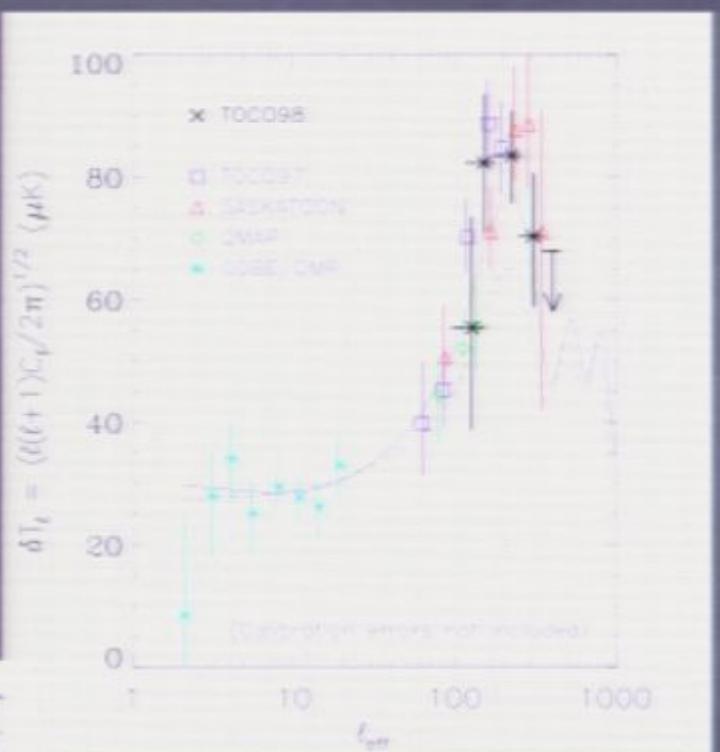
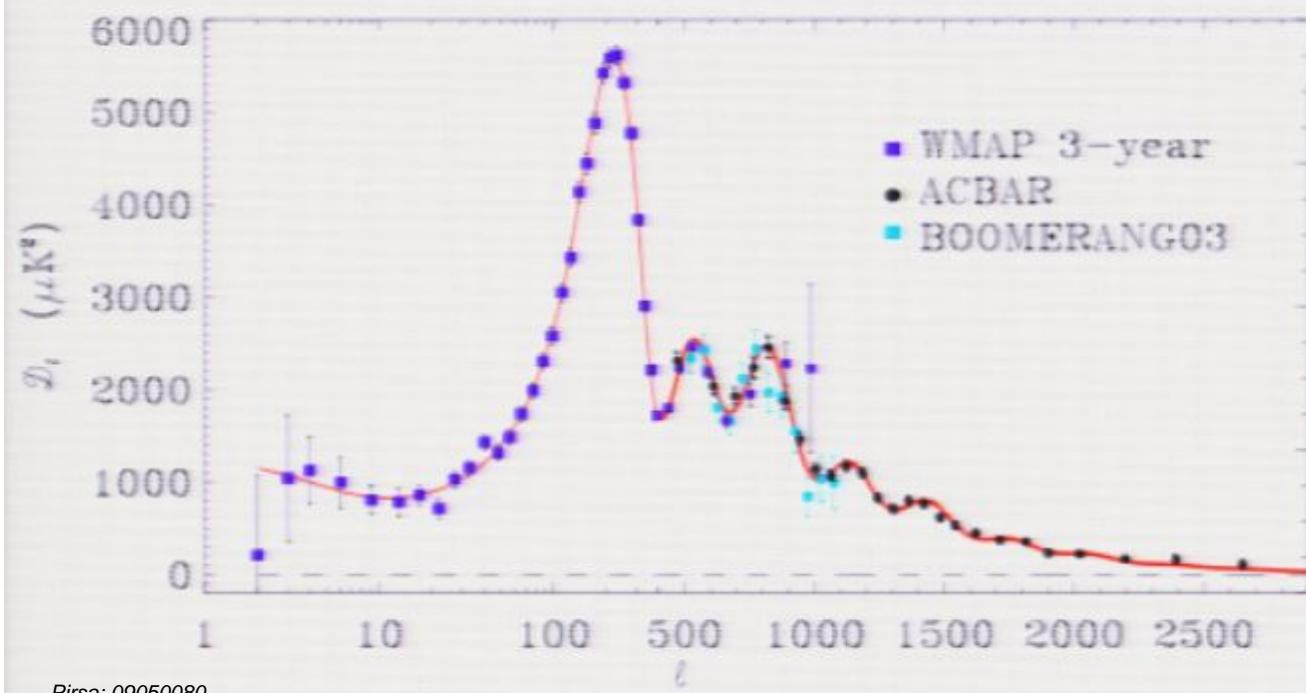
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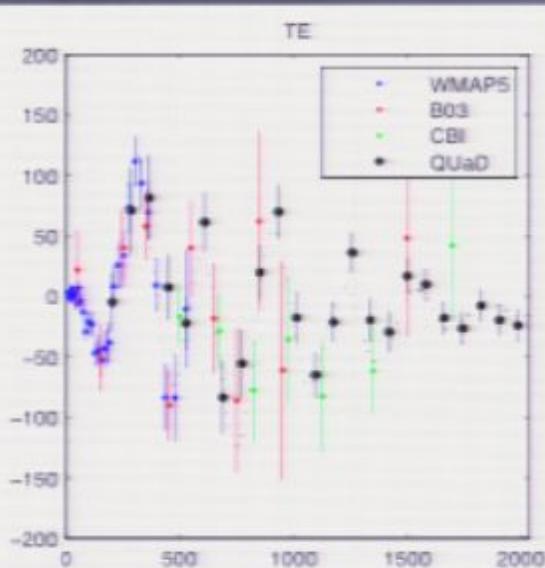
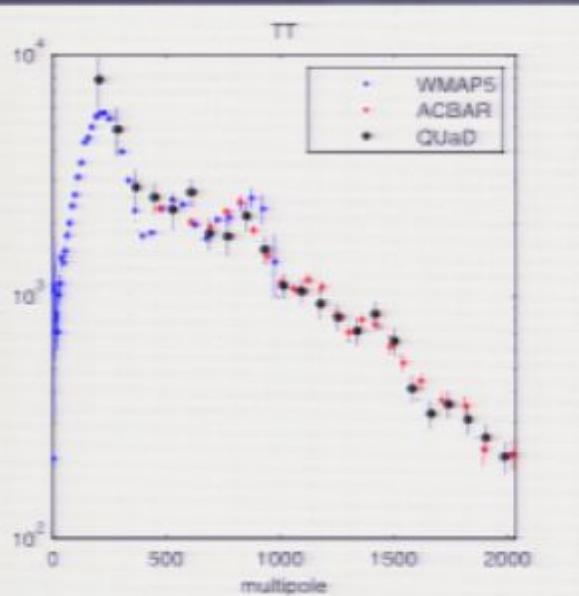
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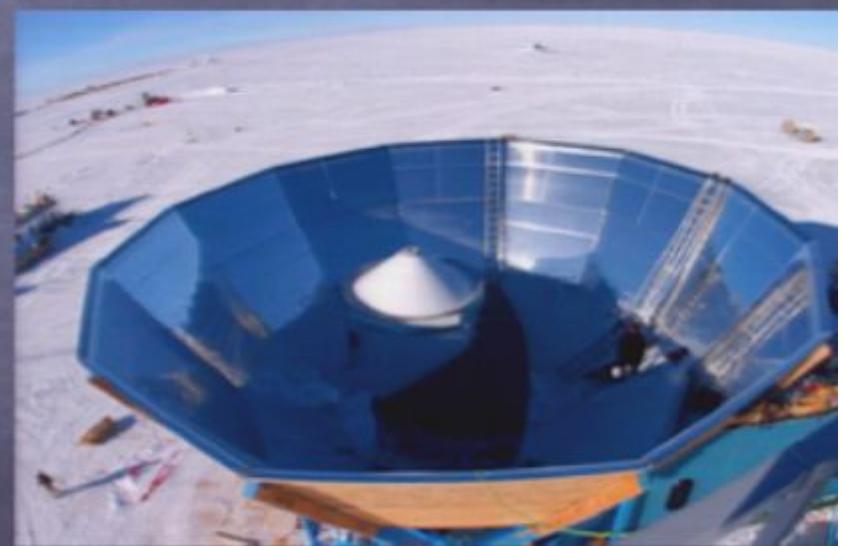
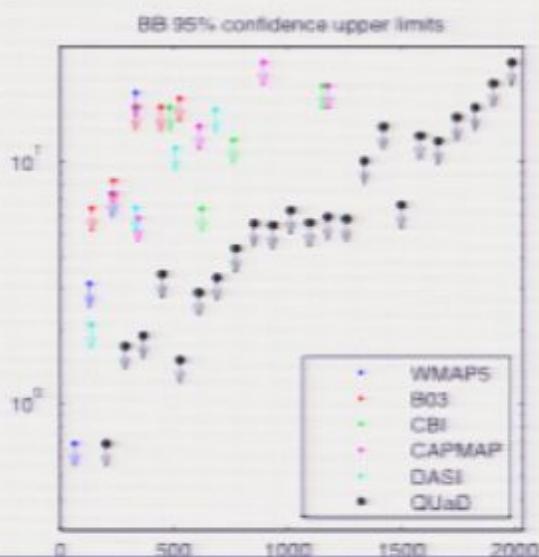
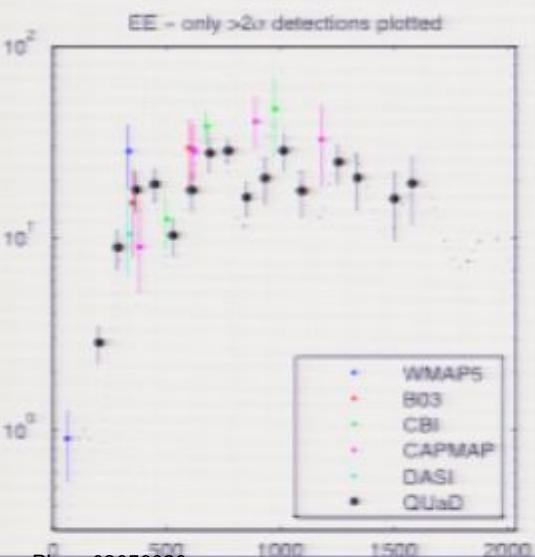
ACBAR  
Jan 2008



DASI, Sept. 2002

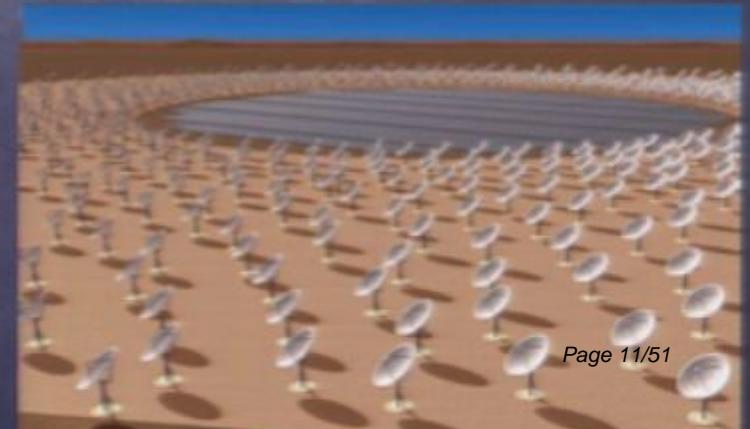
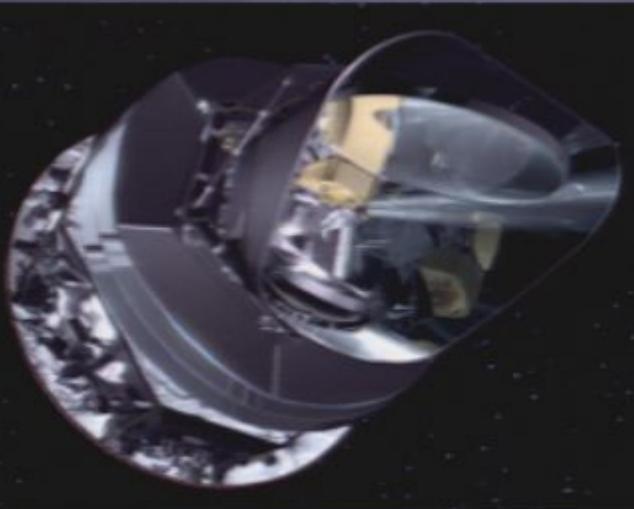


DASI, Sept. 2002



QUaD, May 2008  
Pryke et al.

# Tomorrow Never Dies...



# The end of cosmology?

30 Oct 1998

## IS COSMOLOGY SOLVED? An Astrophysical Cosmologist's Viewpoint

P. J. E. Peebles

*Joseph Henry Laboratories, Princeton University,  
and Princeton Institute for Advanced Study*

### ABSTRACT

We have fossil evidence from the thermal background radiation that our universe expanded from a considerably hotter denser state. We have a well defined, testable, and so far quite successful theoretical description of the expansion: the relativistic Friedmann-

**"Does  $\Lambda$ CDM signify completion of the fundamental physics that will be needed in the analysis of ... future generations of observational cosmology? Or might we only have arrived at the simplest approximation we can get away with at the present level of evidence?"**



More bluntly...

...will convergence to  $\Lambda$ CDM continue?

More bluntly...

...will convergence to  $\Lambda$ CDM continue?

Or are we in for a BIG surprise?



Relaxation mechanisms not possible with  
4d massless graviton

S. Weinberg, Rev. Mod. Phys. (1989)



$$\Lambda$$



UV modif

New physics at  $E > 1 \text{ mm}^{-1}$  modify  
expected radiative corrections  
(SLED)

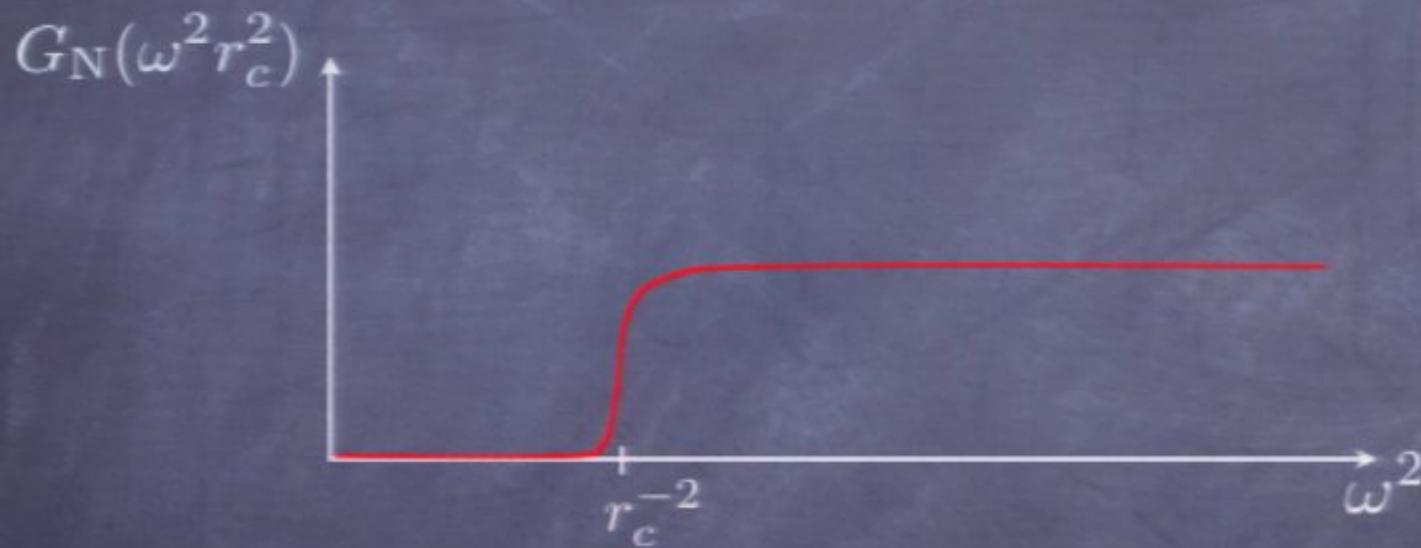
IR modif

$\Lambda$  is large but gravitates  
weakly  
(Degravitation)

# Gravity as a Filter

Arkani-Hamed, Dvali,  
Dimopoulos & Gabadadze (2002)  
Dvali, Hofmann & JK (2007)

$$\underbrace{G_N^{-1}(\square r_c^2)}_{\text{high-pass filter}} G_{\mu\nu} = 8\pi T_{\mu\nu} \quad ; \quad \square \equiv \nabla^\mu \nabla_\mu$$



Sources with wavelength  $\gg r_c$  gravitate normally, whereas those with wavelength  $\ll r_c$  (including vacuum energy) degravitate.

## De-electrification analogue:

A classical analogue of the vacuum energy problem in E&M is to consider a uniform charge density

$$J_\mu = \delta_\mu^0 \rho \quad (\text{analogue of } T_{\mu\nu} = -\Lambda g_{\mu\nu})$$

Solving Maxwell's equations yields

$$\underbrace{A_0 = \frac{1}{2}\rho t^2}_{(\text{like } h_{00} \sim \Lambda t^2 \text{ for de Sitter})}; \quad \vec{A} = \frac{\vec{x}}{3}\rho t \implies \text{E-field grows unbounded}$$

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(like  $h_{00} \sim \Lambda t^2$  for de Sitter)

- 2 possible solns:
- \* Give photon a mass:  $m^2 A_\mu A^\mu$
  - \* Introduce high-pass filter:  $\mu^{-1}(\square r_c^2) \partial^\mu F_{\mu\nu} = J_\nu$

These 2 are equivalent!

Start with Proca theory:  $\partial^\mu F_{\mu\nu} + m^2 A_\nu = J_\nu$

Restore gauge inv. using Goldstone-Stuckelberg field

$$A_\mu = \tilde{A}_\mu + \frac{1}{m} \partial_\mu \phi$$

where, under gauge transfn,  $\tilde{A}_\mu \rightarrow \tilde{A}_\mu + \partial_\mu \omega$ ;  $\phi \rightarrow \phi - m\omega$

Thus  $A_\mu$  is gauge invariant (physical observable)

Solve for Goldstone and substitute in equation of motion:

$$\underbrace{\left(1 - \frac{m^2}{\square}\right)}_{\text{high-pass } \mu^{-1}(\square r_c^2)} \partial^\mu \tilde{F}_{\mu\nu} = J_\nu$$

Hence, mass  $\Leftrightarrow$  high-pass filter!

Adding mass solves analogue of “C.C. problem” in E&M:

$$\partial^\mu F_{\mu\nu} + m^2 A_\nu = \delta_\nu^0 \rho$$

with solution  $A_0 = \frac{\rho}{m^2}$ ;  $\vec{A} = 0$

With mass term, vacuum is **superconducting**, and uniform sources are **screened**

⇒ Degravitation of  $\Lambda$  is analogous to screening of uniform charge density in the Higgs (superconducting) phase of Maxwell theory

Generalize to gravity...

$$(\mathcal{E}h)_{\mu\nu} + \frac{m^2(\square)}{2}(h_{\mu\nu} - \eta_{\mu\nu}h) = T_{\mu\nu}$$

where  $(\mathcal{E}h)_{\mu\nu} = -\frac{1}{2}\square h_{\mu\nu} + \dots$  is the linearized Einstein tensor

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Stuckelberize this equation, and solve for Stuckelbergs:

$$\underbrace{\left(1 - \frac{m^2(\square)}{\square}\right)}_{\text{high-pass } G_N^{-1}(\square r_c^2)} (\mathcal{E}\tilde{h})_{\mu\nu} = T_{\mu\nu}$$

⇒ Degravitation = Massive/Resonance Graviton !

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Immediate implication: Extra degrees of freedom

2 helicity-2 + 2 helicity-1 + 1 helicity-0

Allowed modifications:

$$(\mathcal{E}h)_{\mu\nu} + \frac{m^2(\square)}{2}(h_{\mu\nu} - \eta_{\mu\nu}h) = T_{\mu\nu}$$

Assume a power-law form

$$m^2(p^2) = r_c^{-2(1-\alpha)} p^{2\alpha}$$

- \* To be an IR modification:  $\alpha < 1$
- \* Absence of ghosts:  $\alpha \geq 0$
- \* Degravitation in “decoupling limit”:  $\alpha < 1/2$

Hence allowed range:

$$0 \leq \alpha < 1/2$$

- Note:
- \* Massive gravity corresponds to  $\alpha = 0$
  - \* DGP corresponds to  $\alpha = 1/2$

Gravity as Filter

$\implies$  Massive/Resonance Graviton

$$G_N^{-1}(\square r_c^2)G_{\mu\nu} = 8\pi T_{\mu\nu}$$

$$\frac{1}{p^2 + m^2(p^2)} = \int_0^\infty dM^2 \frac{\rho(M^2)}{p^2 + M^2}$$

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$\implies$  Infinite-Volume extra dimensions?

Gravity as Filter

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$\rightarrow$  Infinite-Volume extra dimensions?



*If it walks like extra dimensions,  
talks like extra dimensions, then...*

Gravity as Filter

$\Rightarrow$

Massive/Resonance Graviton

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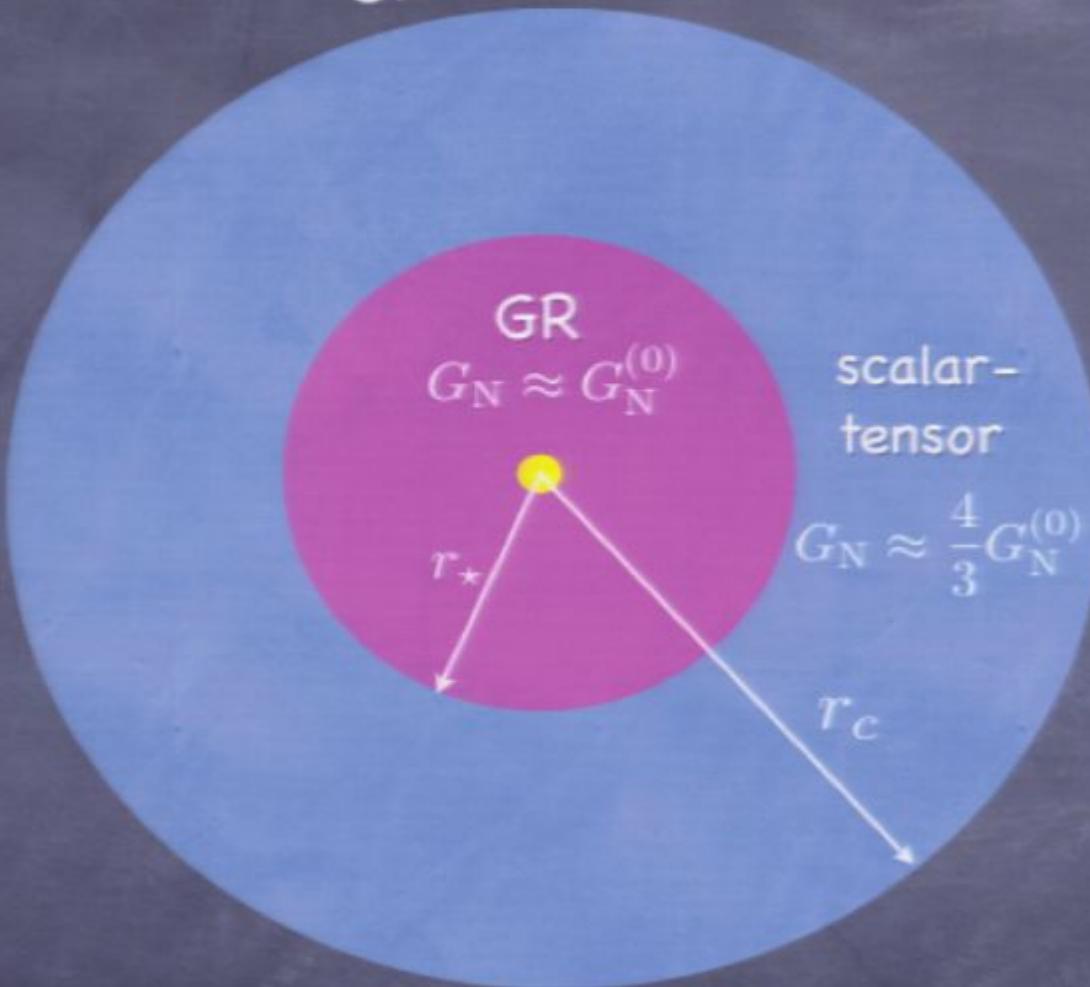
$\Rightarrow$  Infinite-Volume extra dimensions?



*If it walks like extra dimensions,  
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$\Rightarrow$  Old idea of Rubakov & Saposhnikov?

## Helicity-0 Phenomenology:



$$r_\star = \left( r_c^{4(1-\alpha)} r_{\text{Sch}} \right)^{\frac{1}{1+4(1-\alpha)}}$$

Sun:  $r_\star \sim \text{kpc}$   
Galaxy:  $r_\star \sim \text{Mpc}$   
Cluster:  $r_\star \sim 10 \text{ Mpc}$

# Evidence of stronger gravity on $\gtrsim 100$ Mpc scales?

- Lyman-alpha forest



- Bulk flows



- CBI/ACBAR excess



- ISW cross-correlation

# Explicit Constructions

## Realizing degravitation: DGP model



$$S = \frac{M_5^3}{2} \int_{\text{bulk}} d^5x \sqrt{-g_5} R_5 + \frac{M_4^2}{2} \int_{\text{brane}} d^4x \sqrt{-g_4} R_4$$

\* Force law goes from  $1/r^2$  at short distances to  $1/r^3$  at large distances

\* Because extra dimn is infinite, graviton is resonance of width  $r_c^{-1} = \frac{M_5^3}{M_4^2}$

\* Linearized eqn has degravitation form with  $\alpha = 1/2$

$$(\mathcal{E}h)_{\mu\nu} + r_c^{-1} \sqrt{-\square} (h_{\mu\nu} - \eta_{\mu\nu} h) = T_{\mu\nu}$$

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$$(\mathcal{E}h)_{\mu\nu} + r_c^{-1} \sqrt{-\square} (h_{\mu\nu} - \eta_{\mu\nu} h) = T_{\mu\nu}$$

\* However, at non-linear level, only glimmers of degravitation...

$$H^2 = \frac{8\pi G}{3} \rho - \frac{H}{r_c}$$

## Post-Mortem of Degravitation in DGP:

- \* Weakening of gravity (from  $1/r^2$  to  $1/r^3$ ) not "steep" enough
- \* Consistent with earlier conclusion that need  $\alpha < 1/2$

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⇒ Move on to higher-codimension models

- \* Steeper weakening (from  $1/r^2$  to  $1/r^D$ )
- \* All higher codn DGP correspond to  $\alpha \approx 0$

Codim-2 DGP:

Dvali & Gabadadze, PRD (2001)

Kolanovic, Porrati & Rombouts, PRD (2003)



$$S = \frac{M_6^4}{2} \int_{\text{bulk}} d^6x \sqrt{-g_6} R_6 + \frac{M_4^2}{2} \int_{\text{brane}} d^4x \sqrt{-g_4} R_4$$

2 issues:

- \* Brane-to-brane propagator *diverges* in the thin-brane limit

$$G(p) \sim \int_0^\Lambda \frac{d\omega \omega}{p^2 + \omega^2} \sim \log \left( \frac{\Lambda}{p} \right)$$

- \*



Gabadadze & Shifman, PRD (2004)  
Dubovsky & Rubakov, PRD (2003)

→ Higher-codimension DGP was thought to be inconsistent

## Cascading Gravity:

de Rham, Dvali, Hofmann, JK, Pujolas, Redi & Tolley, PRL (2008)

de Rham, Hofmann, JK & Tolley, JCAP (2008)

See also, Kaloper & Kiley (2007)



Idea: Put codim-2 brane on a codim-1 brane

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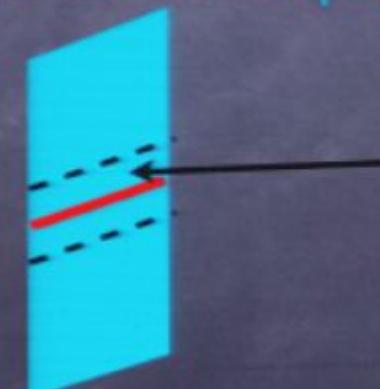
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- \* Parent brane makes brane-to-brane propagator regular!



Bulk decouples  
⇒ gravity looks 5d

Get  $G(p) \sim \log \left( \frac{L_6^{-1}}{p} \right)$ , where  $L_6 = M_5^3/M_6^4$  is 5D to 6D crossover scale.

## Cascading Gravity (cont'd):

- \* By adding a small tension on the (flat) cod-2 brane, resulting spectrum is ghost-free
- Whole new family of modified gravity theories  
(Can easily generalize to arbitrary codimension.)
- Offers a promising framework to realize the screening of vacuum energy



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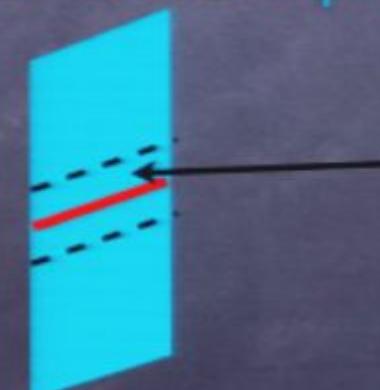
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## Cod-2 (Wedge) Solution:

de Rham, JK & Tolley, to appear

$$ds^2 = dz^2 + (dy - \beta\epsilon(y)\epsilon(z))^2 + \eta_{\mu\nu}dx^\mu dx^\nu$$

- Solution non-singular everywhere
- 3-brane is flat and has 4d gravity on relevant scales
- Crucial that extra dims have  $\infty$ -volume: if compactify, need to cancel tension against other branes or fluxes

Gibbons, Kallosh & Linde (2001)

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But...

- Wedge angle is bounded  $\implies \Lambda < M_6^4$
- Degravitation or usual cosmic string?

What about codimension  $N \geq 3$ :

Dvali, Gabadadze & Shifman,  
hep-th/0202174

$$S = \frac{M_{4+N}^{N+2}}{2} \int_{\text{bulk}} d^{4+N}x \sqrt{-g_{4+N}} R_{4+N} + \frac{M_4^2}{2} \int_{\text{cod-N}} d^4x \sqrt{-g_4} R_4$$

Look for static soln, with 4d flat space:

$$ds^2 = A^2(r) \eta_{\mu\nu} dx^\mu dx^\nu + B^2(r) dr^2 + C^2(r) r^2 d\Omega_{N-1}^2$$

Find:  $A(r) = f^{-\frac{1}{2\sqrt{10}}}$ ;  $B(r) = f^{\frac{2}{\sqrt{10}}}$ ;  $C(r) = f^{\frac{1}{2} + \frac{2}{\sqrt{10}}}$

where  $f(r) = 1 + \frac{r_0}{r}$

Charmousis, Emparan & Gregory  
hep-th/0101198

⇒ highly distorted naked singularity

Dvali et al.: Singularity is artefact of static ansatz. Brane must inflate with

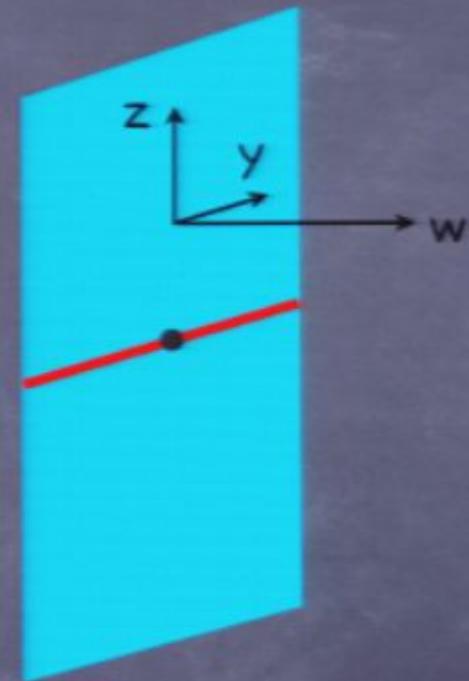
$$H \sim M_{4+N} \left( \frac{M_{4+N}^4}{\Lambda} \right)^{\frac{1}{N-2}}$$

Singularity-free Cod-3 solution:

de Rham, JK & Tolley, to appear

In weak-field approximation:

$$\begin{aligned} ds^2 = & (1 + \Phi(y, z, w)) (dw^2 + dz^2 + dy^2) \\ & - \frac{\Theta(w)}{2m_7} \partial_\alpha \Phi_0(y, z) dx^\alpha dw \\ & + \left(1 - \frac{\Phi(y, z, w)}{4}\right) \eta_{\mu\nu} dx^\mu dx^\nu \end{aligned}$$



Solving cascading equations of motion:

$$\Phi(y, z, w) = \frac{8\Lambda}{5M_6^4} \int \frac{d\omega dq_y}{(2\pi)^2} \frac{e^{-|w|\sqrt{\omega^2+q_y^2}} e^{i\omega z} e^{iq_y y}}{\omega^2 + q_y^2 + m_7 \sqrt{\omega^2 + q_y^2}} \cdot \frac{g(q_y)}{\frac{3}{5}q_y^2 - g(q_y)}$$

⇒ finite and perturbative everywhere!

Beyond weak-field: requires numerical analysis

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Dvali, Gabadadze & Shifman,  
hep-th/0202174

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Look for static soln, with 4d flat space:

$$ds^2 = A^2(r) \eta_{\mu\nu} dx^\mu dx^\nu + B^2(r) dr^2 + C^2(r) r^2 d\Omega_{N-1}^2$$

Find:  $A(r) = f^{-\frac{1}{2\sqrt{10}}} ; B(r) = f^{\frac{2}{\sqrt{10}}} ; C(r) = f^{\frac{1}{2} + \frac{2}{\sqrt{10}}}$

where  $f(r) = 1 + \frac{r_0}{r}$

Charmousis, Emparan & Gregory  
hep-th/0101198

⇒ highly distorted naked singularity

Dvali et al.: Singularity is artefact of static ansatz. Brane must inflate with

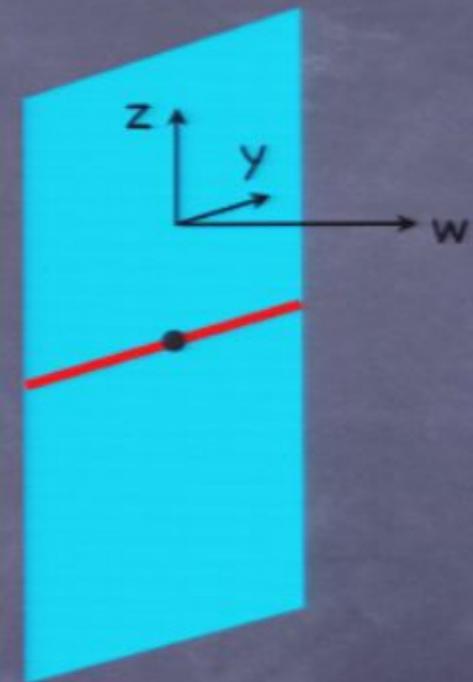
$$H \sim M_{4+N} \left( \frac{M_{4+N}^4}{\Lambda} \right)^{\frac{1}{N-2}}$$

Singularity-free Cod-3 solution:

de Rham, JK & Tolley, to appear

In weak-field approximation:

$$\begin{aligned} ds^2 = & (1 + \Phi(y, z, w)) (dw^2 + dz^2 + dy^2) \\ & - \frac{\Theta(w)}{2m_7} \partial_\alpha \Phi_0(y, z) dx^\alpha dw \\ & + \left(1 - \frac{\Phi(y, z, w)}{4}\right) \eta_{\mu\nu} dx^\mu dx^\nu \end{aligned}$$



Solving cascading equations of motion:

$$\Phi(y, z, w) = \frac{8\Lambda}{5M_6^4} \int \frac{d\omega dq_y}{(2\pi)^2} \frac{e^{-|w|\sqrt{\omega^2+q_y^2}} e^{i\omega z} e^{iq_y y}}{\omega^2 + q_y^2 + m_7 \sqrt{\omega^2 + q_y^2}} \cdot \frac{g(q_y)}{\frac{3}{5}q_y^2 - g(q_y)}$$

⇒ finite and perturbative everywhere!

Beyond weak-field: requires numerical analysis

What about codimension  $N \geq 3$ :

Dvali, Gabadadze & Shifman,  
[hep-th/0202174](#)

$$S = \frac{M_{4+N}^{N+2}}{2} \int_{\text{bulk}} d^{4+N}x \sqrt{-g_{4+N}} R_{4+N} + \frac{M_4^2}{2} \int_{\text{cod-N}} d^4x \sqrt{-g_4} R_4$$

Look for static soln, with 4d flat space:

$$ds^2 = A^2(r) \eta_{\mu\nu} dx^\mu dx^\nu + B^2(r) dr^2 + C^2(r) r^2 d\Omega_{N-1}^2$$

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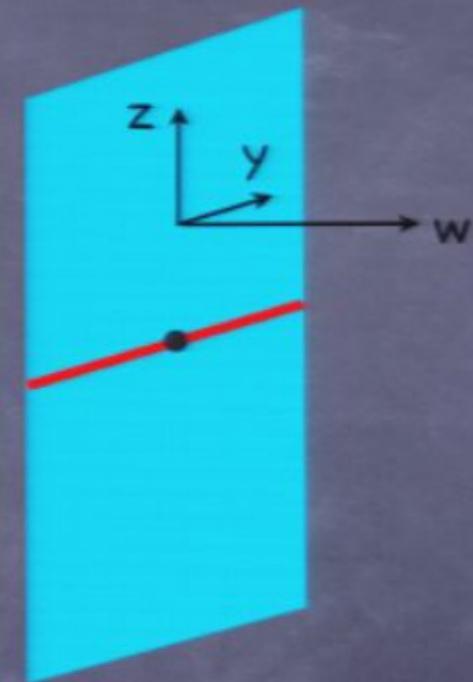
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## Conclusions

- C.C. problem requires giving up something: here 4d GR not recovered in IR.
- Filter gravity => Massive/Resonance Graviton => Extra dimns
- Cascading gravity offers explicit framework to realize degravitation
- Has testable observational consequences

See Ghazal's talk