Title: Cascading Gravity and the Cosmological Constant

Date: May 26, 2009 11:30 AM

URL: http://pirsa.org/09050080

Abstract: TBA

Cascading Gravity and Λ

Justin Khoury

with N. Afshordi, C. de Rham, G. Geshnizjani, A. Tolley, M. Wyman

G. Dvali, S. Hofmann and JK, Phys. Rev. D76, 084006 (2007)

C. de Rham, S. Hofmann, JK and A. Tolley JCAP 0802:011 (2008)

C. de Rham, G. Dvali, S. Hofmann, JK, O. Pujolas, M. Redi and A. Tolley, Phys. Rev. Lett. 100, 251603 (2008)

N. Afshordi, G. Geshnizjani and JK, astro-ph/0812.2244

JK and Mark Wyman, astro-ph/0903.1292

Cascading Gravity and Λ

Justin Khoury

with N. Afshordi, C. de Rham, G. Geshnizjani, A. Tolley, M. Wyman

G. Dvali, S. Hofmann and JK, Phys. Rev. D76, 084006 (2007)

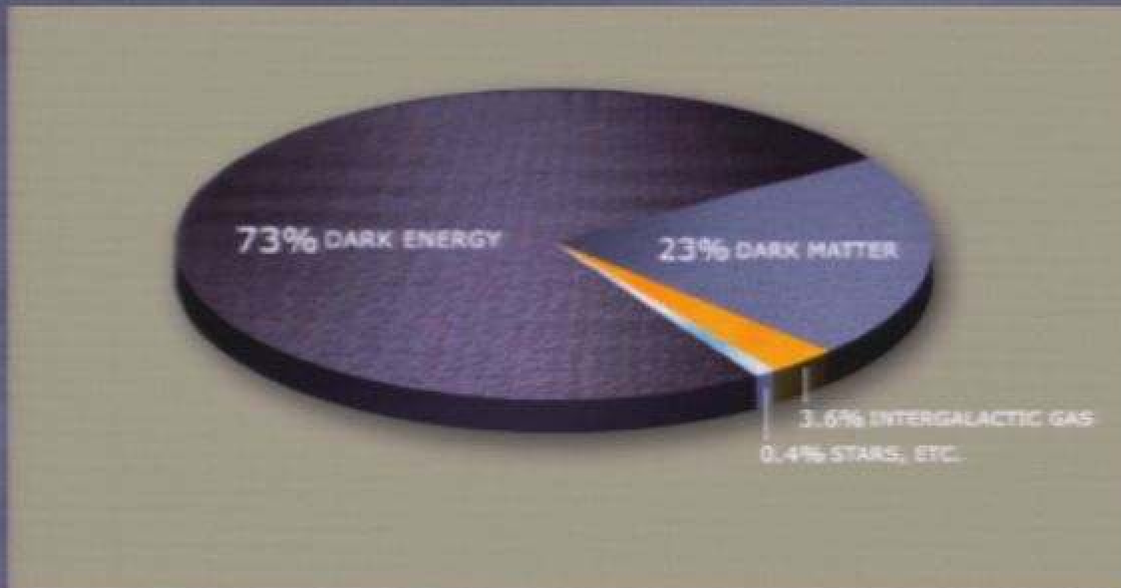
C. de Rham, S. Hofmann, JK and A. Tolley JCAP 0802:011 (2008)

C. de Rham, G. Dvali, S. Hofmann, JK, O. Pujolas, M. Redi and A. Tolley, Phys. Rev. Lett. 100, 251603 (2008)

N. Afshordi, G. Geshnizjani and JK, astro-ph/0812.2244

JK and Mark Wyman, astro-ph/0903.1292

Concordance ΛCDM model



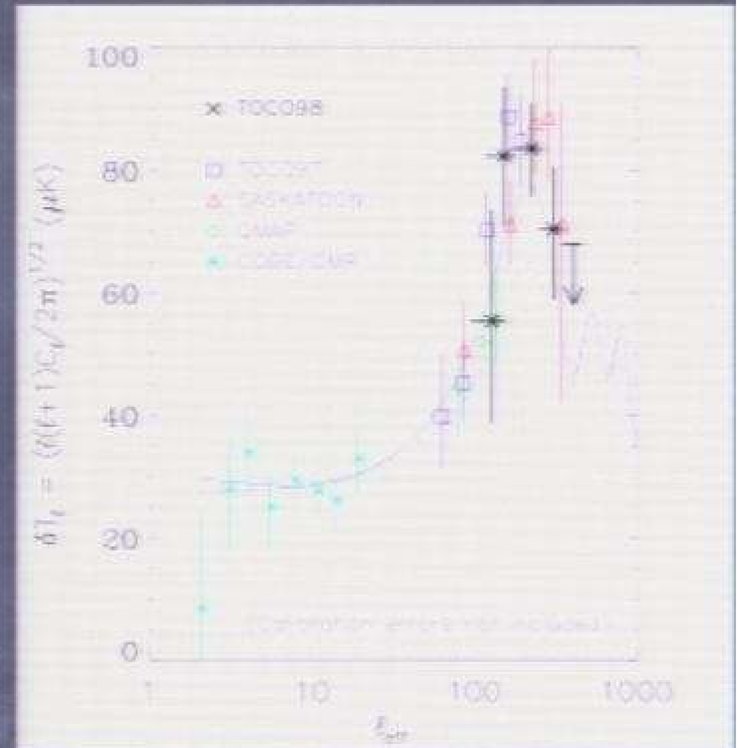
- Cosmology has a standard model

- Remarkably predictive

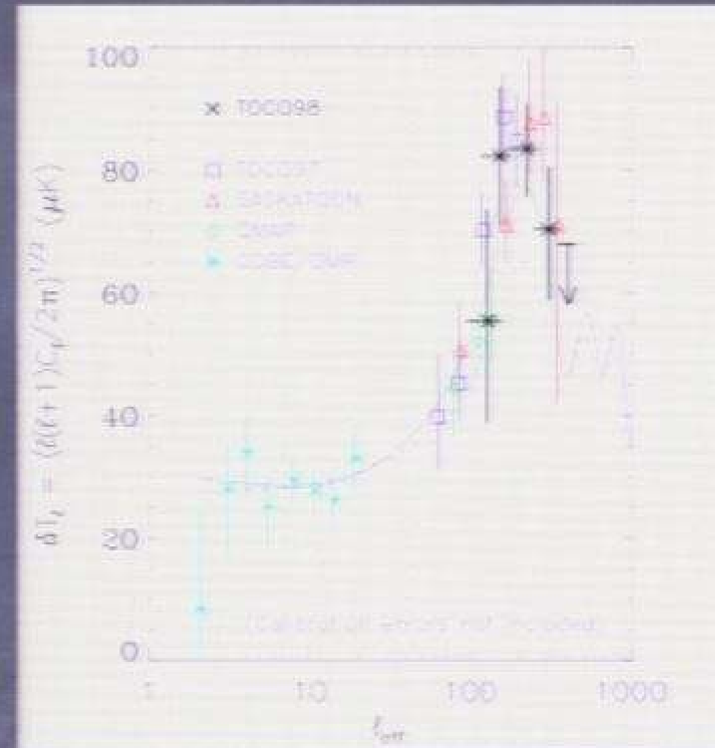
Λ → expansion history

Λ → growth of structure

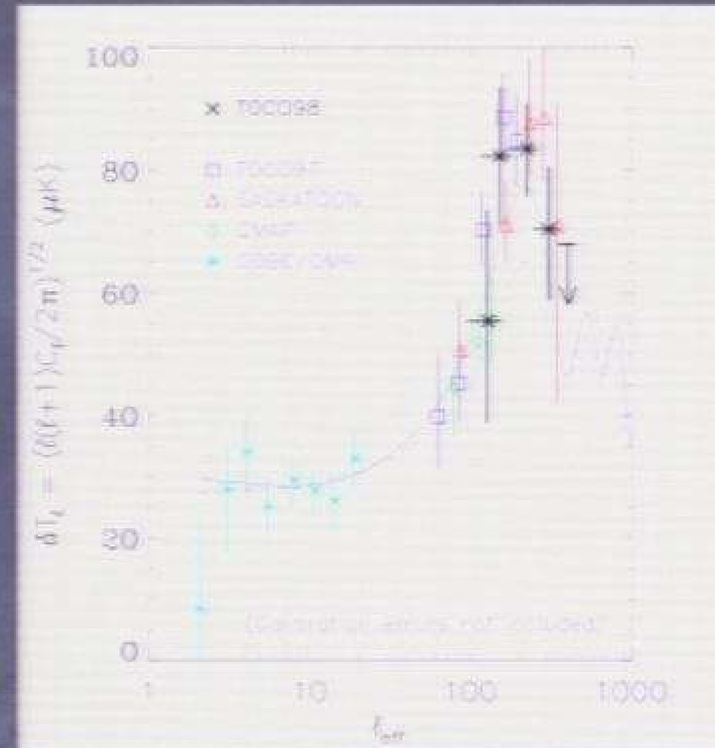
TOCO
June 1999



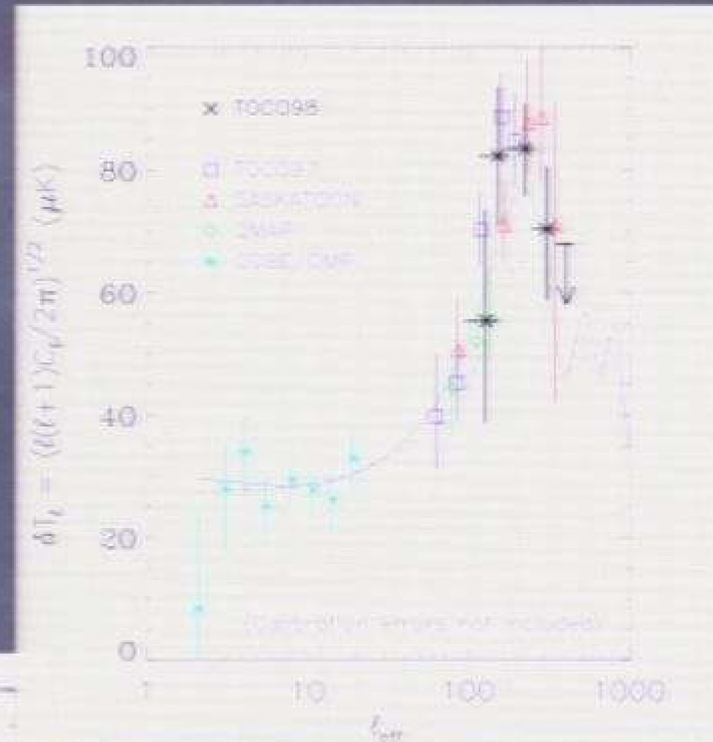
TOCO
June 1999

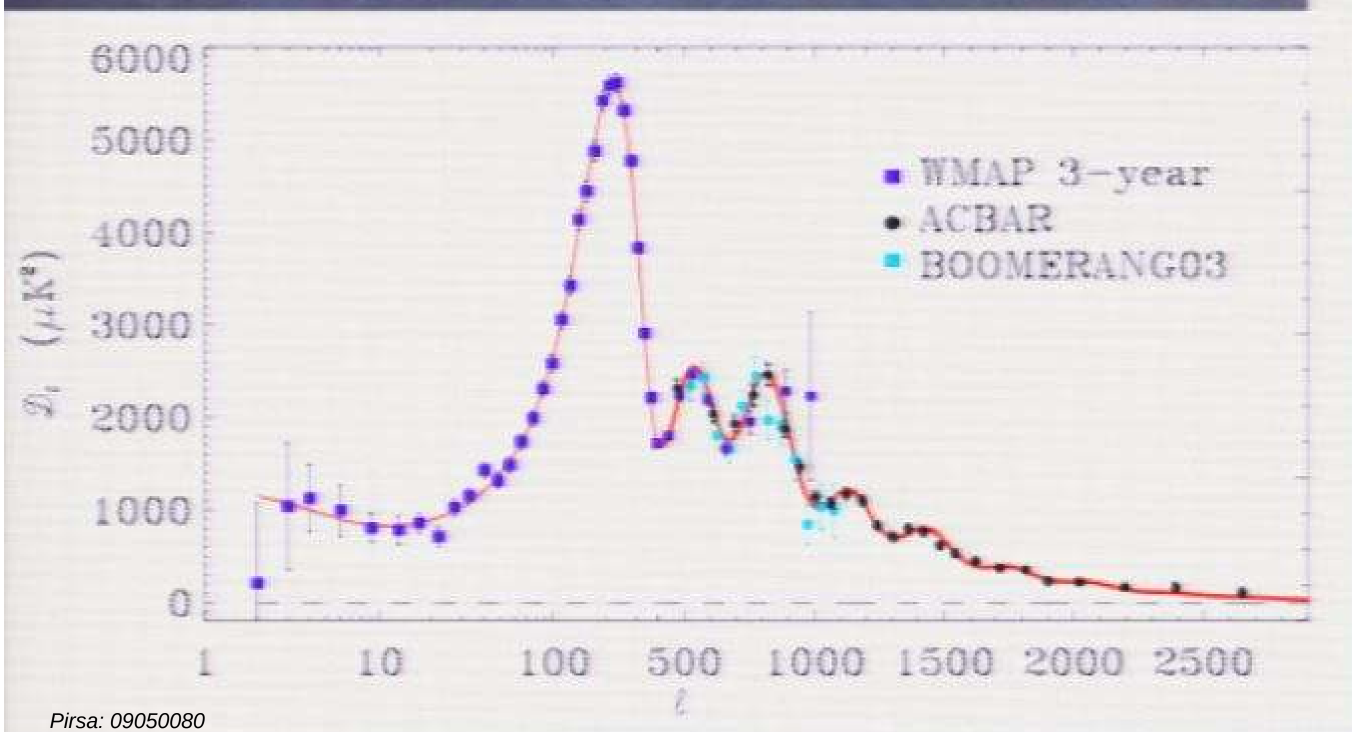


TOCO
June 1999



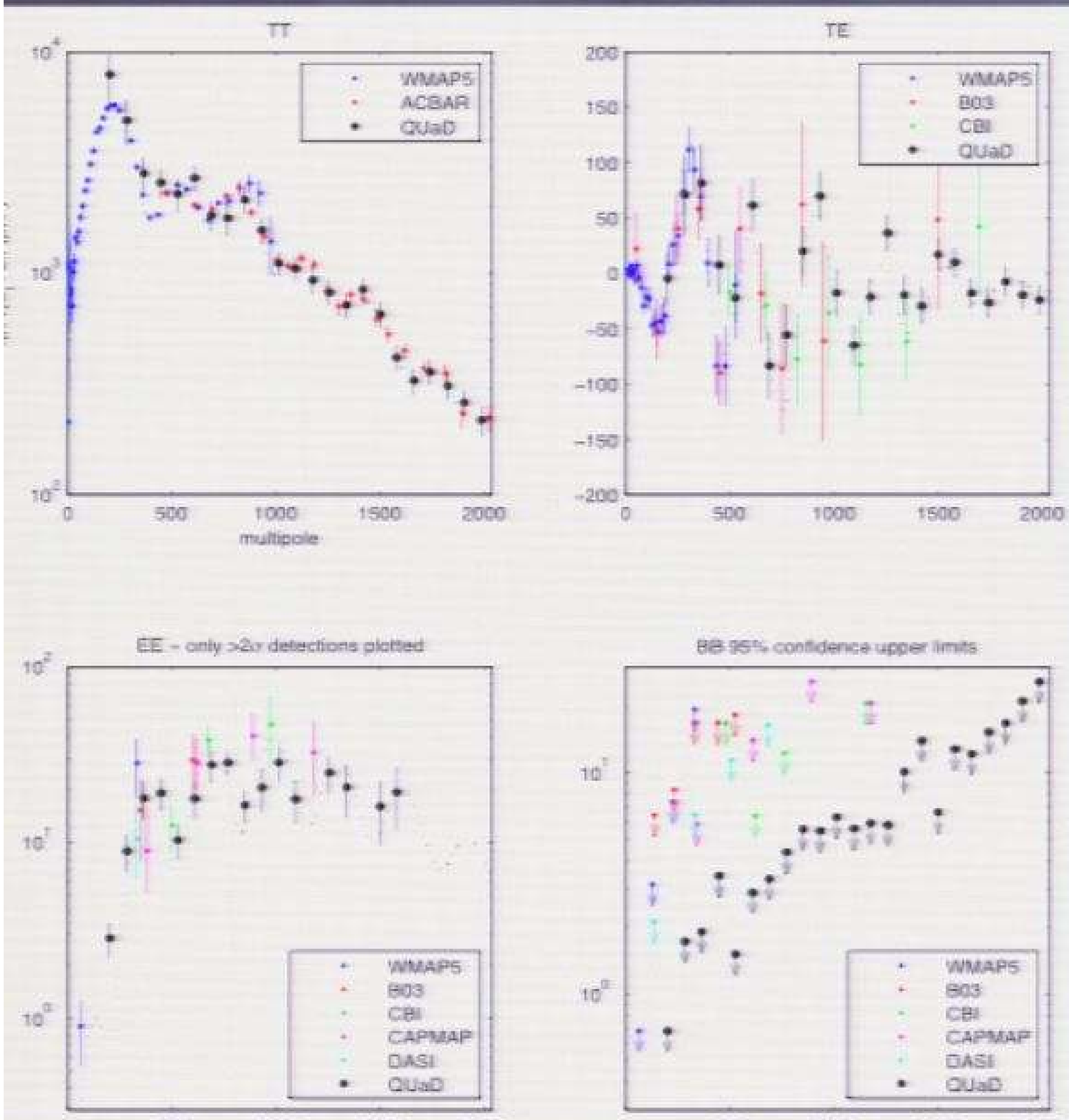
TOCO
June 1999



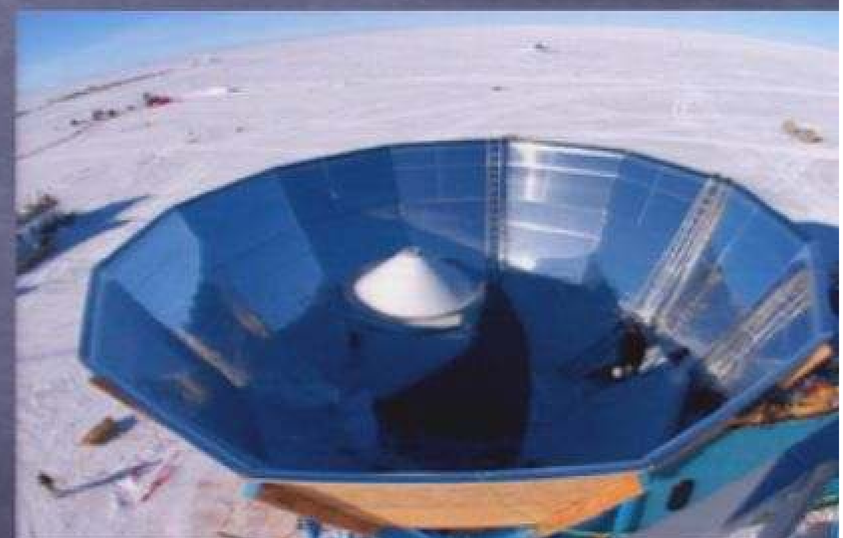


ACBAR
Jan 2008

DASI, Sept. 2002

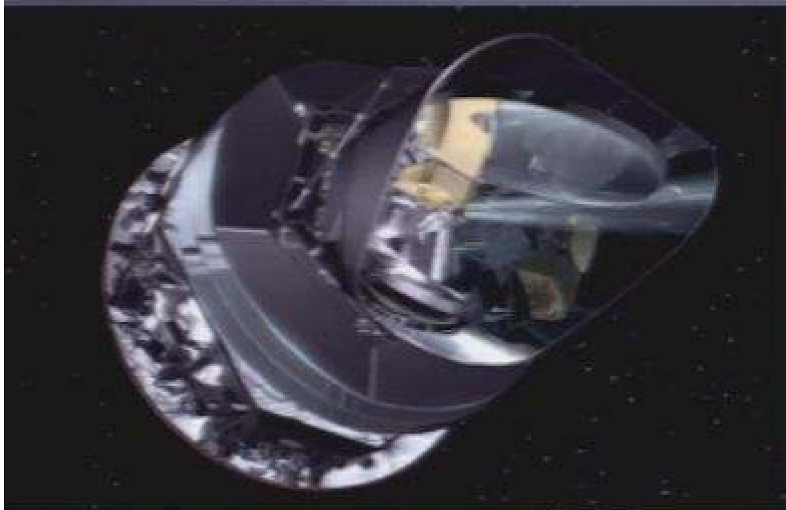


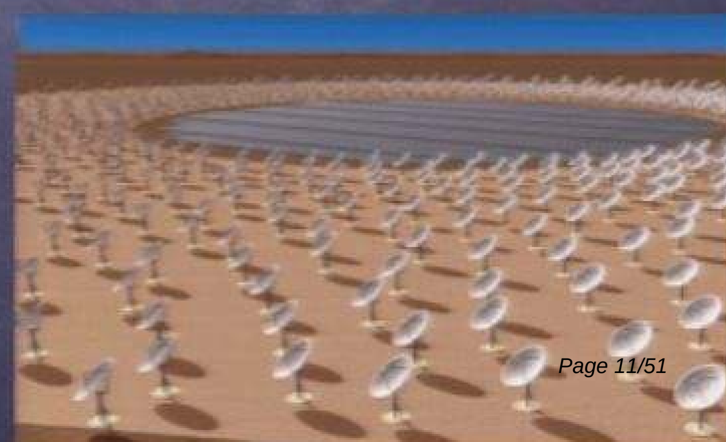
DASI, Sept. 2002



QUaD, May 2008
Pryke et al.

Tomorrow Never Dies...





The end of cosmology?

IS COSMOLOGY SOLVED?
An Astrophysical Cosmologist's Viewpoint

P. J. E. Peebles
Joseph Henry Laboratories, Princeton University, and Princeton Institute for Advanced Study

30 Oct 1998

ABSTRACT

We have fossil evidence from the thermal background radiation that our universe expanded from a considerably hotter denser state. We have a well defined, testable, and so far quite successful theoretical description of the expansion: the relativistic Friedmann-

"Does ΛCDM signify completion of the fundamental physics that will be needed in the analysis of … future generations of observational cosmology? Or might we only have arrived at the simplest approximation we can get away with at the present level of evidence?"

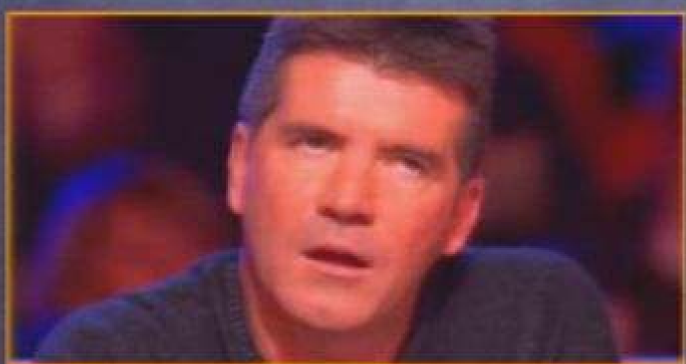
More bluntly...

...will convergence to ΛCDM continue?

More bluntly...

...will convergence to ΛCDM continue?

Or are we in for a BIG surprise?



Relaxation mechanisms not possible with
4d massless graviton S. Weinberg, Rev. Mod. Phys. (1989)

$$\Lambda$$

UV modif

New physics at $E > 1\ mm^{-1}$ modify expected radiative corrections
(SLED)

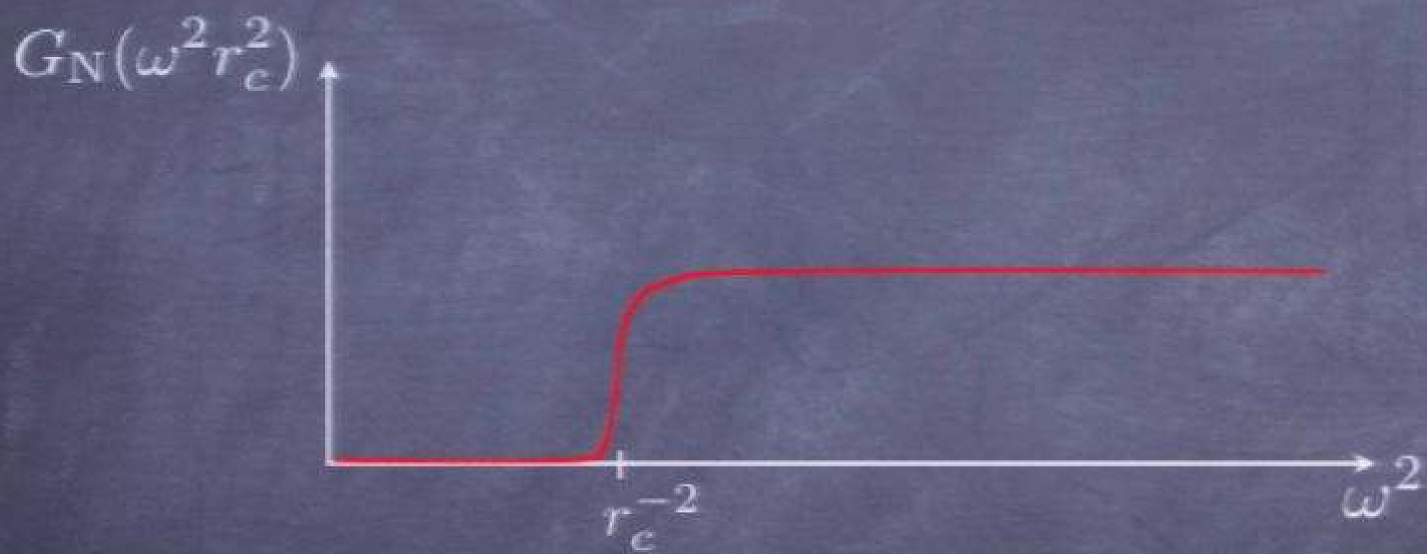
IR modif

Λ is large but gravitates weakly
(Degravitation)

Gravity as a Filter

Arkani-Hamed, Dvali,
Dimopoulos & Gabadadze (2002)
Dvali, Hofmann & JK (2007)

$$\underbrace{G_{\rm N}^{-1}(\Box r_c^2)}_{\text{high-pass filter}} G_{\mu\nu} = 8\pi T_{\mu\nu} \qquad ; \qquad \Box \equiv \nabla^\mu \nabla_\mu$$



Sources with wavelength $\gg r_c$ gravitate normally, whereas those with wavelength $\ll r_c$ (including vacuum energy) degravitate.

De-electrification analogue:

A classical analogue of the vacuum energy problem in E&M is to consider a uniform charge density

$$J_\mu = \delta_\mu^{\,0} \rho$$

(analogue of $T_{\mu\nu} = -\Lambda g_{\mu\nu}$)

Solving Maxwell's equations yields

$$\underbrace{A_0 = \frac{1}{2}\rho t^2\,;}\qquad \vec{A} = \frac{\vec{x}}{3}\rho t \implies$$

E-field grows unbounded

(like $h_{00} \sim \Lambda t^2$ for de Sitter)

De-electrification analogue:

A classical analogue of the vacuum energy problem in E&M is to consider a uniform charge density

$$J_\mu = \delta_\mu^{\ 0}\rho$$ (analogue of $T_{\mu\nu} = -\Lambda g_{\mu\nu}$)

Solving Maxwell's equations yields

$$\underbrace{A_0 = \frac{1}{2}\rho t^2\,;}\qquad \vec{A} = \frac{\vec{x}}{3}\rho t \implies$$ E-field grows unbounded

(like $h_{00} \sim \Lambda t^2$ for de Sitter)

2 possible solns: * Give photon a mass: $m^2 A_\mu A^\mu$

* Introduce high-pass filter: $\mu^{-1}(\Box r_c^2)\partial^\mu F_{\mu\nu} = J_\nu$

These 2 are equivalent!

Start with Proca theory: $\partial^\mu F_{\mu\nu} + m^2 A_\nu = J_\nu$

Restore gauge inv. using Goldstone-Stuckelberg field

$$A_\mu = \tilde{A}_\mu + \frac{1}{m}\partial_\mu \phi$$

where, under gauge transfn, $\tilde{A}_\mu \to \tilde{A}_\mu + \partial_\mu \omega \; ; \quad \phi \to \phi - m\omega$

Thus A_μ is gauge invariant (physical observable)

Solve for Goldstone and substitute in equation of motion:

$$\underbrace{\left(1 - \frac{m^2}{\Box}\right)}_{\text{high-pass } \mu^{-1}(\Box r_c^2)} \partial^\mu \tilde{F}_{\mu\nu} = J_\nu$$

Hence, mass <=> high-pass filter!

Adding mass solves analogue of "C.C. problem" in E&M:

$$\partial^\mu F_{\mu\nu} + m^2 A_\nu = \delta_\nu^{\,0} \rho$$

with solution $$A_0 = \frac{\rho}{m^2}\,; \quad \vec{A} = 0$$

With mass term, vacuum is superconducting, and uniform sources are screened

$\Longrightarrow$ Degravitation of Λ is analogous to screening of uniform charge density in the Higgs (superconducting) phase of Maxwell theory

Generalize to gravity...

$$(\mathcal{E}h)_{\mu\nu} + \frac{m^2(\Box)}{2}(h_{\mu\nu} - \eta_{\mu\nu}h) = T_{\mu\nu}$$

where $(\mathcal{E}h)_{\mu\nu} = -\frac{1}{2}\Box h_{\mu\nu} + \ldots$ is the linearized Einstein tensor

Generalize to gravity...

$$(\mathcal{E}h)_{\mu\nu} + \frac{m^2(\Box)}{2}(h_{\mu\nu} - \eta_{\mu\nu}h) = T_{\mu\nu}$$

where $(\mathcal{E}h)_{\mu\nu} = -\frac{1}{2}\Box h_{\mu\nu} + \ldots$ is the linearized Einstein tensor

Stuckelberize this equation, and solve for Stuckelbergs:

$$\underbrace{\left(1 - \frac{m^2(\Box)}{\Box}\right)}_{\text{high-pass } G_N^{-1}(\Box r_c^2)}(\mathcal{E}\tilde{h})_{\mu\nu} = T_{\mu\nu}$$

$\Longrightarrow$ Degravitation = Massive/Resonance Graviton !

Generalize to gravity...

$$(\mathcal{E}h)_{\mu\nu} + \frac{m^2(\Box)}{2}(h_{\mu\nu} - \eta_{\mu\nu}h) = T_{\mu\nu}$$

where $(\mathcal{E}h)_{\mu\nu} = -\frac{1}{2}\Box h_{\mu\nu} + \ldots$ is the linearized Einstein tensor

Stuckelberize this equation, and solve for Stuckelbergs:

$$\underbrace{\left(1 - \frac{m^2(\Box)}{\Box}\right)}_{\text{high-pass } G_N^{-1}(\Box r_c^2)}(\mathcal{E}\tilde{h})_{\mu\nu} = T_{\mu\nu}$$

$\Longrightarrow$ Degravitation = Massive/Resonance Graviton !

Immediate implication: Extra degrees of freedom

2 helicity-2 + 2 helicity-1 + 1 helicity-0

Allowed modifications:

$$(\mathcal{E}h)_{\mu\nu} + \frac{m^2(\Box)}{2}(h_{\mu\nu} - \eta_{\mu\nu}h) = T_{\mu\nu}$$

Assume a power-law form

$$m^2(p^2) = r_c^{-2(1-\alpha)} p^{2\alpha}$$

* To be an IR modification: $\alpha < 1$
* Absence of ghosts: $\alpha \geq 0$
* Degravitation in "decoupling limit": $\alpha < 1/2$

Hence allowed range: $0 \leq \alpha < 1/2$

Note:
* Massive gravity corresponds to $\alpha = 0$
* DGP corresponds to $\alpha = 1/2$

Gravity as Filter $\Longrightarrow$ Massive/Resonance Graviton

$$G_{\mathrm{N}}^{-1}(\Box r_c^2) G_{\mu\nu} = 8\pi T_{\mu\nu}$$

$$\frac{1}{p^2 + m^2(p^2)} = \int_0^\infty \mathrm{d}M^2 \frac{\rho(M^2)}{p^2 + M^2}$$

Gravity as Filter $\Longrightarrow$ Massive/Resonance Graviton

$$G_{\mathrm{N}}^{-1}(\Box r_c^2)G_{\mu\nu} = 8\pi T_{\mu\nu}$$

$$\frac{1}{p^2 + m^2(p^2)} = \int_0^\infty \mathrm{d}M^2 \frac{\rho(M^2)}{p^2 + M^2}$$

$\Longrightarrow$ Infinite-Volume extra dimensions?

Gravity as Filter $\Longrightarrow$ Massive/Resonance Graviton

$$G_{\mathrm{N}}^{-1}(\Box r_c^2)G_{\mu\nu} = 8\pi T_{\mu\nu}$$

$$\frac{1}{p^2 + m^2(p^2)} = \int_0^\infty \mathrm{d}M^2 \frac{\rho(M^2)}{p^2 + M^2}$$

$\Longrightarrow$ Infinite-Volume extra dimensions?

If it walks like extra dimensions,
talks like extra dimensions, then...

Gravity as Filter $\Longrightarrow$ Massive/Resonance Graviton

$$G_{\mathrm{N}}^{-1}(\Box r_c^2)G_{\mu\nu} = 8\pi T_{\mu\nu}$$

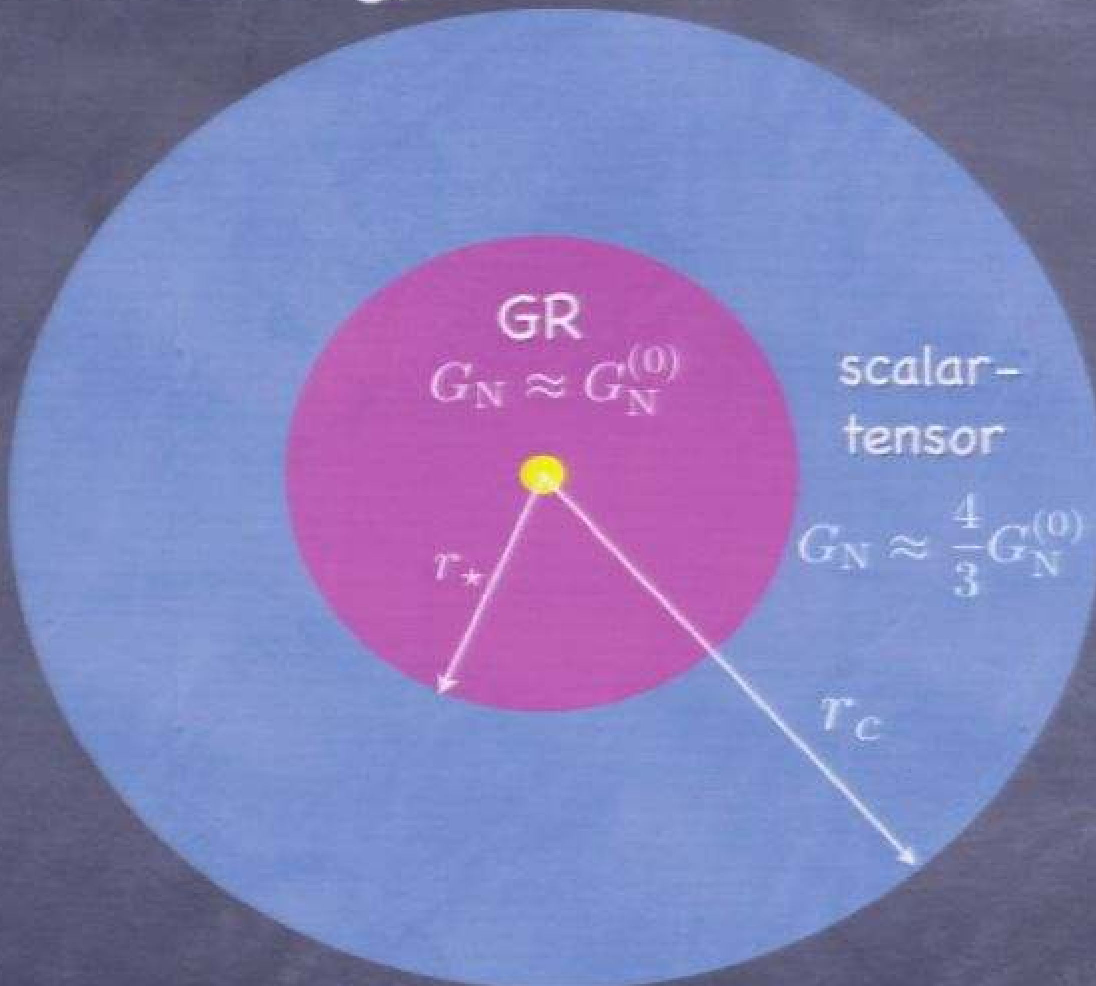
$$\frac{1}{p^2 + m^2(p^2)} = \int_0^\infty \mathrm{d}M^2 \frac{\rho(M^2)}{p^2 + M^2}$$

$\Longrightarrow$ Infinite-Volume extra dimensions?

If it walks like extra dimensions, talks like extra dimensions, then...

$\Longrightarrow$ Old idea of Rubakov & Saposhnikov?

Helicity-0 Phenomenology:



$$r_\star = \left(r_c^{4(1-\alpha)} r_{\rm Sch} \right)^{\frac{1}{1+4(1-\alpha)}}$$

Sun: $r_\star \sim \mathrm{kpc}$

Galaxy: $r_\star \sim \mathrm{Mpc}$

Cluster: $r_\star \sim 10\ \mathrm{Mpc}$

$\Longrightarrow$ Helicity-0 mediates 5th force which is stronger at larger distances.

Evidence of stronger gravity on $\gtrsim$ 100 Mpc scales?

- Lyman-alpha forest

- Bulk flows

- CBI/ACBAR excess

- ISW cross-correlation

See Ghazal's talk this afternoon

Explicit Constructions

Realizing degravitation: DGP model

$$S = \frac{M_5^3}{2}\int_{\text{bulk}} \mathrm{d}^5x\sqrt{-g_5}R_5 + \frac{M_4^2}{2}\int_{\text{brane}} \mathrm{d}^4x\sqrt{-g_4}R_4$$

* Force law goes from $1/r^2$ at short distances to $1/r^3$ at large distances

* Because extra dimn is infinite, graviton is resonance of width $r_c^{-1} = \frac{M_5^3}{M_4^2}$

* Linearized eqn has degravitation form with $\alpha = 1/2$

$$(\mathcal{E}h)_{\mu\nu} + r_c^{-1}\sqrt{-\Box}(h_{\mu\nu} - \eta_{\mu\nu}h) = T_{\mu\nu}$$

Realizing degravitation: DGP model

$$S = \frac{M_5^3}{2}\int_{\text{bulk}} \mathrm{d}^5x\sqrt{-g_5}R_5 + \frac{M_4^2}{2}\int_{\text{brane}} \mathrm{d}^4x\sqrt{-g_4}R_4$$

* Force law goes from $1/r^2$ at short distances to $1/r^3$ at large distances

* Because extra dimn is infinite, graviton is resonance of width $r_c^{-1} = \frac{M_5^3}{M_4^2}$

* Linearized eqn has degravitation form with $\alpha = 1/2$

$$(\mathcal{E}h)_{\mu\nu} + r_c^{-1}\sqrt{-\Box}(h_{\mu\nu} - \eta_{\mu\nu}h) = T_{\mu\nu}$$

* However, at non-linear level, only glimmers of degravitation...

$$H^2 = \frac{8\pi G}{3}\rho - \frac{H}{r_c}$$

Post-Mortem of Degravitation in DGP:

* Weakening of gravity (from $1/r^2$ to $1/r^3$) not "steep" enough

* Consistent with earlier conclusion that need $\alpha < 1/2$

Post-Mortem of Degravitation in DGP:

* Weakening of gravity (from $1/r^2$ to $1/r^3$) not "steep" enough

* Consistent with earlier conclusion that need $\alpha < 1/2$

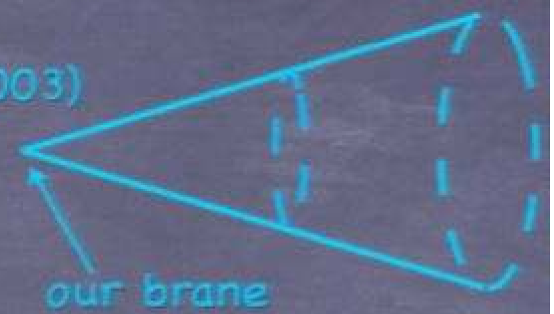
$\Longrightarrow$ Move on to higher-codimension models

  * Steeper weakening (from $1/r^2$ to $1/r^D$)

  * All higher codn DGP correspond to $\alpha \approx 0$

Codim-2 DGP:

Dvali & Gabadadze, PRD (2001)
Kolanovic, Porrati & Rombouts, PRD (2003)



$$S = \frac{M_6^4}{2}\int_{\rm bulk} {\rm d}^6x\sqrt{-g_6}R_6 + \frac{M_4^2}{2}\int_{\rm brane} {\rm d}^4x\sqrt{-g_4}R_4$$

2 issues:

* Brane-to-brane propagator diverges in the thin-brane limit

$$G(p) \sim \int_0^\Lambda \frac{d\omega\omega}{p^2+\omega^2} \sim \log\left(\frac{\Lambda}{p}\right)$$

* 

Gabadadze & Shifman, PRD (2004)
Dubovsky & Rubakov, PRD (2003)

⟹ Higher-codimension DGP was thought to be inconsistent

Cascading Gravity:

de Rham, Dvali, Hofmann, JK, Pujolas, Redi & Tolley, PRL (2008)
de Rham, Hofmann, JK & Tolley, JCAP (2008)
See also, Kaloper & Kiley (2007)

Idea: Put codim-2 brane on a codim-1 brane

$$S = \frac{M_6^4}{2}\int_{\rm bulk} \mathrm{d}^6x\sqrt{-g_6}R_6 + \frac{M_5^3}{2}\int_{\rm cod-1} \mathrm{d}^5x\sqrt{-g_5}R_5 + \frac{M_4^2}{2}\int_{\rm cod-2} \mathrm{d}^4x\sqrt{-g_4}R_4$$

* Force law goes from $1/r^2$ to $1/r^3$ to $1/r^4$ as we probe larger distances

Cascading Gravity:

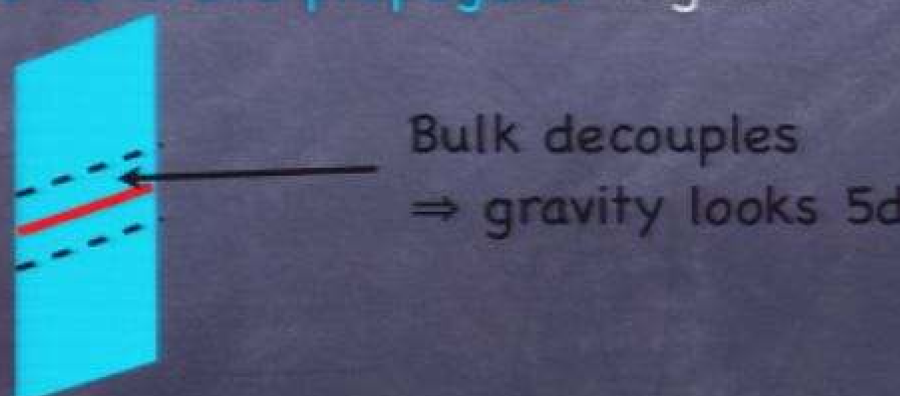
de Rham, Dvali, Hofmann, JK, Pujolas, Redi & Tolley, PRL (2008)
de Rham, Hofmann, JK & Tolley, JCAP (2008)
See also, Kaloper & Kiley (2007)

Idea: Put codim-2 brane on a codim-1 brane

$$S = \frac{M_6^4}{2}\int_{\rm bulk} d^6x\sqrt{-g_6}R_6 + \frac{M_5^3}{2}\int_{\rm cod-1} d^5x\sqrt{-g_5}R_5 + \frac{M_4^2}{2}\int_{\rm cod-2} d^4x\sqrt{-g_4}R_4$$

* Force law goes from $1/r^2$ to $1/r^3$ to $1/r^4$ as we probe larger distances

* Parent brane makes brane-to-brane propagator regular!



Get: $G(p) \sim \log\left(\frac{L_6^{-1}}{p}\right)$, where $L_6 = M_5^3/M_6^4$ is 5D to 6D crossover scale.

Cascading Gravity (cont'd):

* By adding a small tension on the (flat) cod-2 brane, resulting spectrum is ghost-free

⟹ Whole new family of modified gravity theories

(Can easily generalize to arbitrary codimension.)

⟹ Offers a promising framework to realize the screening of vacuum energy

Cascading Gravity:

de Rham, Dvali, Hofmann, JK, Pujolas, Redi & Tolley, PRL (2008)
de Rham, Hofmann, JK & Tolley, JCAP (2008)
See also, Kaloper & Kiley (2007)

Idea: Put codim-2 brane on a codim-1 brane

$$S = \frac{M_6^4}{2}\int_{\rm bulk} \mathrm{d}^6x\sqrt{-g_6}R_6 + \frac{M_5^3}{2}\int_{\rm cod-1} \mathrm{d}^5x\sqrt{-g_5}R_5 + \frac{M_4^2}{2}\int_{\rm cod-2} \mathrm{d}^4x\sqrt{-g_4}R_4$$

* Force law goes from $1/r^2$ to $1/r^3$ to $1/r^4$ as we probe larger distances

Cascading Gravity:

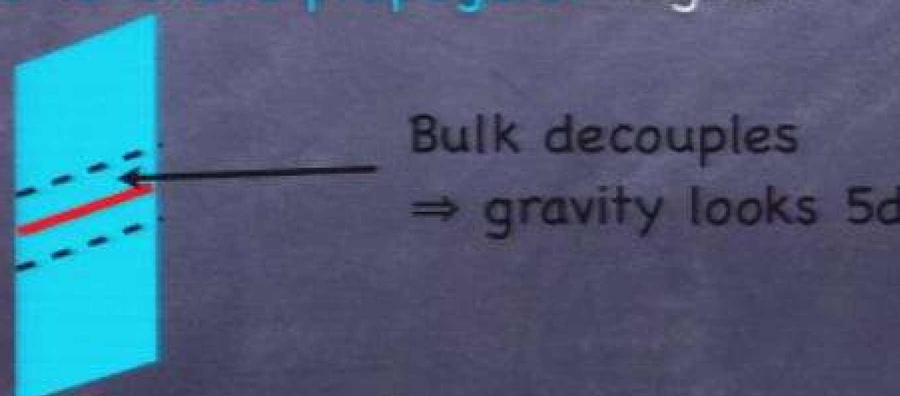
de Rham, Dvali, Hofmann, JK, Pujolas, Redi & Tolley, PRL (2008)
de Rham, Hofmann, JK & Tolley, JCAP (2008)
See also, Kaloper & Kiley (2007)

Idea: Put codim-2 brane on a codim-1 brane

$$S = \frac{M_6^4}{2}\int_{\rm bulk} d^6x\sqrt{-g_6}R_6 + \frac{M_5^3}{2}\int_{\rm cod-1} d^5x\sqrt{-g_5}R_5 + \frac{M_4^2}{2}\int_{\rm cod-2} d^4x\sqrt{-g_4}R_4$$

* Force law goes from $1/r^2$ to $1/r^3$ to $1/r^4$ as we probe larger distances

* Parent brane makes brane-to-brane propagator regular!



Get: $G(p) \sim \log\left(\frac{L_6^{-1}}{p}\right)$, where $L_6 = M_5^3/M_6^4$ is 5D to 6D crossover scale.

Cascading Gravity (cont'd):

* By adding a small tension on the (flat) cod-2 brane, resulting spectrum is ghost-free

⟹ Whole new family of modified gravity theories

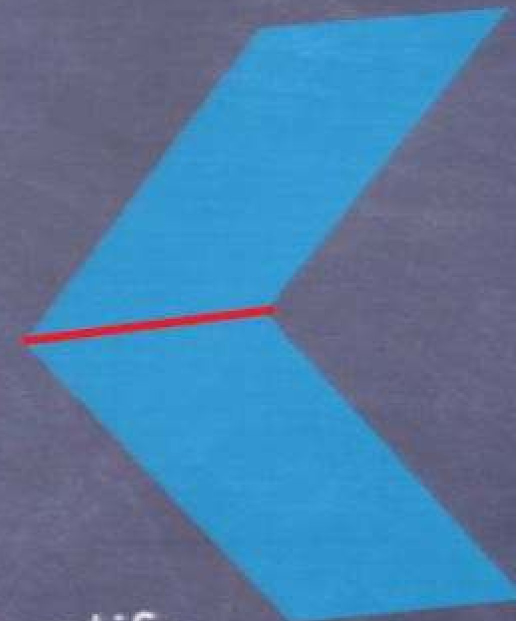
(Can easily generalize to arbitrary codimension.)

⟹ Offers a promising framework to realize the screening of vacuum energy

Cod-2 (Wedge) Solution:

de Rham, JK & Tolley, to appear

$$ds^2 = dz^2 + (dy - \beta\epsilon(y)\epsilon(z))^2 + \eta_{\mu\nu}dx^\mu dx^\nu$$



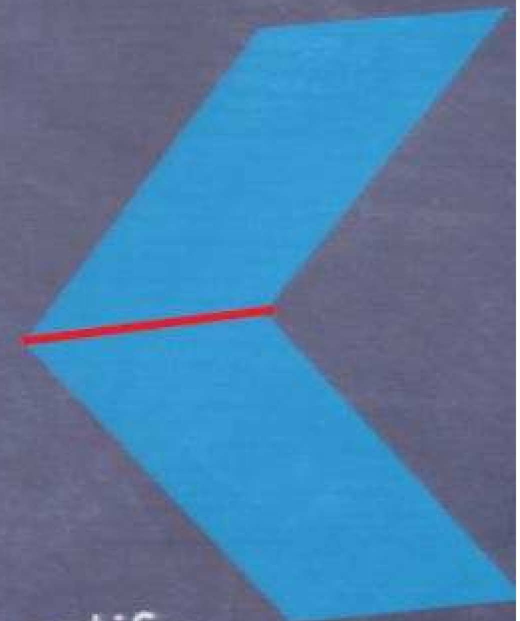
- Solution non-singular everywhere
- 3-brane is flat and has 4d gravity on relevant scales
- Crucial that extra dims have ∞-volume: if compactify, need to cancel tension against other branes or fluxes

Gibbons, Kallosh & Linde (2001)

Cod-2 (Wedge) Solution:

de Rham, JK & Tolley, to appear

$$\mathrm{d}s^2 = \mathrm{d}z^2 + (\mathrm{d}y - \beta\epsilon(y)\epsilon(z))^2 + \eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu$$



- Solution non-singular everywhere
- 3-brane is flat and has 4d gravity on relevant scales
- Crucial that extra dims have ∞-volume: if compactify, need to cancel tension against other branes or fluxes

Gibbons, Kallosh & Linde (2001)

But...

- Wedge angle is bounded $\Longrightarrow$ $\Lambda < M_6^4$
- Degravitation or usual cosmic string?

What about codimension N ≥ 3?:

Dvali, Gabadadze & Shifman, hep-th/0202174

$$S = \frac{M_{4+N}^{N+2}}{2}\int_{\rm bulk} {\rm d}^{4+N}x\sqrt{-g_{4+N}}R_{4+N} + \frac{M_4^2}{2}\int_{\rm cod-N} {\rm d}^4x\sqrt{-g_4}R_4$$

Look for static soln, with 4d flat space:

$$ {\rm d}s^2 = A^2(r)\eta_{\mu\nu}{\rm d}x^\mu {\rm d}x^\nu + B^2(r){\rm d}r^2 + C^2(r)r^2{\rm d}\Omega_{N-1}^2$$

Find: $A(r) = f^{-\frac{1}{2\sqrt{10}}}$; $B(r) = f^{\frac{2}{\sqrt{10}}}$; $C(r) = f^{\frac{1}{2}+\frac{2}{\sqrt{10}}}$

where $f(r) = 1 + \dfrac{r_0}{r}$

Charmousis, Emparan & Gregory hep-th/0101198

$\Longrightarrow$ highly distorted naked singularity

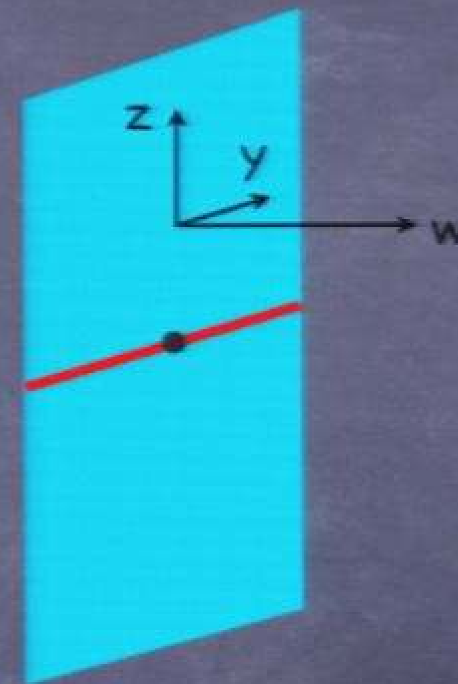
Dvali et al.: Singularity is artefact of static ansatz. Brane must inflate with

$$H \sim M_{4+N}\left(\frac{M_{4+N}^4}{\Lambda}\right)^{\frac{1}{N-2}}$$

Singularity-free Cod-3 solution:

de Rham, JK & Tolley, to appear

In weak-field approximation:



$$
\begin{aligned}
ds^2 &= (1+\Phi(y,z,w))\left(dw^2+dz^2+dy^2\right) \\
&- \frac{\Theta(w)}{2m_7}\partial_\alpha\Phi_0(y,z)dx^\alpha dw \\
&+ \left(1-\frac{\Phi(y,z,w)}{4}\right)\eta_{\mu\nu}dx^\mu dx^\nu
\end{aligned}
$$

Solving cascading equations of motion:

$$\Phi(y,z,w) = \frac{8\Lambda}{5M_6^4}\int\frac{d\omega dq_y}{(2\pi)^2}\frac{e^{-|w|\sqrt{\omega^2+q_y^2}}e^{i\omega z}e^{iq_y y}}{\omega^2+q_y^2+m_7\sqrt{\omega^2+q_y^2}}\cdot\frac{g(q_y)}{\frac{3}{5}q_y^2-g(q_y)}$$

$\Longrightarrow$ finite and perturbative everywhere!

Beyond weak-field: requires numerical analysis

de Rham, Giblin, JK & Tolley (in progress)

What about codimension N ≥ 3?:

Dvali, Gabadadze & Shifman, hep-th/0202174

$$S = \frac{M_{4+N}^{N+2}}{2}\int_{\text{bulk}} \mathrm{d}^{4+N}x\sqrt{-g_{4+N}}R_{4+N} + \frac{M_4^2}{2}\int_{\text{cod}-\text{N}} \mathrm{d}^4x\sqrt{-g_4}R_4$$

Look for static soln, with 4d flat space:

$$\mathrm{d}s^2 = A^2(r)\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + B^2(r)\mathrm{d}r^2 + C^2(r)r^2\mathrm{d}\Omega_{N-1}^2$$

Find: $A(r) = f^{-\frac{1}{2\sqrt{10}}}$; $B(r) = f^{\frac{2}{\sqrt{10}}}$; $C(r) = f^{\frac{1}{2}+\frac{2}{\sqrt{10}}}$

where $f(r) = 1 + \dfrac{r_0}{r}$

Charmousis, Emparan & Gregory hep-th/0101198

$\Longrightarrow$ highly distorted naked singularity

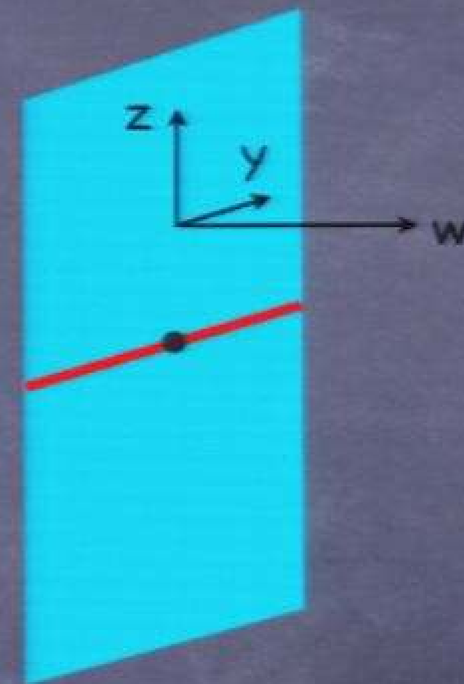
Dvali et al.: Singularity is artefact of static ansatz. Brane must inflate with

$$H \sim M_{4+N}\left(\frac{M_{4+N}^4}{\Lambda}\right)^{\frac{1}{N-2}}$$

Singularity-free Cod-3 solution:

de Rham, JK & Tolley, to appear

In weak-field approximation:



$$
\begin{aligned}
ds^2 &= (1+\Phi(y,z,w))\left(dw^2+dz^2+dy^2\right) \\
&- \frac{\Theta(w)}{2m_7}\partial_\alpha\Phi_0(y,z)dx^\alpha dw \\
&+ \left(1-\frac{\Phi(y,z,w)}{4}\right)\eta_{\mu\nu}dx^\mu dx^\nu
\end{aligned}
$$

Solving cascading equations of motion:

$$
\Phi(y,z,w) = \frac{8\Lambda}{5M_6^4}\int\frac{d\omega dq_y}{(2\pi)^2}\frac{e^{-|w|\sqrt{\omega^2+q_y^2}}e^{i\omega z}e^{iq_y y}}{\omega^2+q_y^2+m_7\sqrt{\omega^2+q_y^2}}\cdot\frac{g(q_y)}{\frac{3}{5}q_y^2-g(q_y)}
$$

$\Longrightarrow$ finite and perturbative everywhere!

Beyond weak-field: requires numerical analysis

de Rham, Giblin, JK & Tolley (in progress)

What about codimension N ≥ 3?:

Dvali, Gabadadze & Shifman, hep-th/0202174

$$S = \frac{M_{4+N}^{N+2}}{2}\int_{\text{bulk}} \mathrm{d}^{4+N}x\sqrt{-g_{4+N}}R_{4+N} + \frac{M_4^2}{2}\int_{\text{cod}-\text{N}} \mathrm{d}^4x\sqrt{-g_4}R_4$$

Look for static soln, with 4d flat space:

$$\mathrm{d}s^2 = A^2(r)\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + B^2(r)\mathrm{d}r^2 + C^2(r)r^2\mathrm{d}\Omega_{N-1}^2$$

Find: $A(r) = f^{-\frac{1}{2\sqrt{10}}}$; $B(r) = f^{\frac{2}{\sqrt{10}}}$; $C(r) = f^{\frac{1}{2}+\frac{2}{\sqrt{10}}}$

where $f(r) = 1 + \frac{r_0}{r}$

Charmousis, Emparan & Gregory hep-th/0101198

$\Longrightarrow$ highly distorted naked singularity

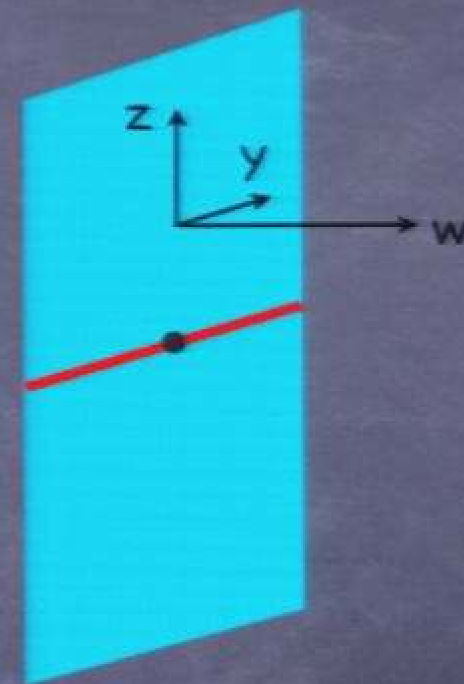
Dvali et al.: Singularity is artefact of static ansatz. Brane must inflate with

$$H \sim M_{4+N}\left(\frac{M_{4+N}^4}{\Lambda}\right)^{\frac{1}{N-2}}$$

Singularity-free Cod-3 solution:

de Rham, JK & Tolley, to appear

In weak-field approximation:



$$\begin{aligned} ds^2 &= (1+\Phi(y,z,w))\left(dw^2+dz^2+dy^2\right) \\ &- \frac{\Theta(w)}{2m_7}\partial_\alpha \Phi_0(y,z)dx^\alpha dw \\ &+ \left(1-\frac{\Phi(y,z,w)}{4}\right)\eta_{\mu\nu}dx^\mu dx^\nu \end{aligned}$$

Solving cascading equations of motion:

$$\Phi(y,z,w) = \frac{8\Lambda}{5M_6^4}\int \frac{d\omega dq_y}{(2\pi)^2}\frac{e^{-|w|\sqrt{\omega^2+q_y^2}}e^{i\omega z}e^{iq_y y}}{\omega^2+q_y^2+m_7\sqrt{\omega^2+q_y^2}}\cdot\frac{g(q_y)}{\frac{3}{5}q_y^2-g(q_y)}$$

$\Longrightarrow$ finite and perturbative everywhere!

Beyond weak-field: requires numerical analysis

de Rham, Giblin, JK & Tolley (in progress)

Conclusions

- C.C. problem requires giving up something: here 4d GR not recovered in IR.
- Filter gravity => Massive/Resonance Graviton => Extra dimns
- Cascading gravity offers explicit framework to realize degravitation
- Has testable observational consequences

See Ghazal's talk