

Title: Massive gravity in 3-D and the Chern-Simons-Proca theory

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Abstract: TBA

I. 1-order formalism

$$e^a = e^a_\mu dx^\mu$$

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e^a
 ω_{ab}

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$$g_{\mu\nu} = e^a_\mu e^b_\nu \eta_{ab}$$

i) metric compatible

$$\omega_{ab} + \omega_{ba} = 0$$

ii) ~~torsion~~

$$\omega^a_b e^b = 0 = D e^a$$

I. 1-order formalism

$$e^a \quad e^a = e^a_\mu dx^\mu \quad g_{\mu\nu} = e^a_\mu e^b_\nu \eta_{ab}$$

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1. 1-order formalism

$$e^a = e^a_\mu dx^\mu \quad g_{\mu\nu} = e^a_\mu e^b_\nu \eta_{ab}$$

was

i) metric compatible

$$\omega_{ab} + \omega_{ba} = 0$$

ii) ~~torsion~~ $de^a + \omega^a_b e^b = 0 = D e^a$

Riemann $\nabla_{[a} \omega_{bc]} + \omega^d_{[a} \omega_{bc]} = 0$

x-axis

1. 1-order formalism

$e^a = e^a_\mu dx^\mu$ $g_{\mu\nu} = e^a_\mu e^b_\nu \eta_{ab}$

ω_{ab} i) metric compatible

$\omega_{ab} + \omega_{ba} = 0$

ii) ~~torsion~~ $de^a + \omega^a_b e^b = 0 = D e^a$

Riemann $\Omega^a_b = d\omega^a_b + \omega^a_c \omega^c_b$
 $\Omega^a_b = \Omega^c_{bc} e^c$

3-D
 ω_{ab}

wrt. $\omega \rightarrow$ ~~torsion~~



1. 1-order formalism

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Riemann $\Omega^a_b = d\omega^a_b + \omega^a_c \omega^c_b$

3-D $L = e^a \Omega^b_{abc}$

Vary L wrt. ω \rightarrow torsion

Vary L wrt. e^a \rightarrow Einstein eq.



1. 1-order formalism

e^a was $e^a = e^a_\mu dx^\mu$ $g_{\mu\nu} = e^a_\mu e^b_\nu \eta_{ab}$

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Riemann $\Omega^a_b = d\omega^a_b + \omega^a_c \omega^c_b$

3-D $L = e^a \Omega^b_{abc} + \Lambda e^a e^b e^c$

Vary L wrt. $\omega \rightarrow$ ~~torsion~~

Vary L wrt. $e^a \rightarrow$ Einstein eq.



I. 1-order formalism

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Riemann $\Omega_{ab} = d\omega_{ab} + \omega_a^c \omega_{cb}$

3-D $L = e^a \Omega^{bc} \epsilon_{abc} + \Lambda e^a e^b e^c \epsilon_{abc}$

Vary L wrt. $\omega \rightarrow$ ~~torsion~~

Vary L wrt. $e^a \rightarrow$ Einstein eq.

$$S_{lab} + 2A e_a e_b = 0$$

$$3-0 \quad S_{lab} = 2(R_a e_b - R_b e_a) - R e_a e_b$$

$$R_a = R_a e^b \quad R = R_a \eta_{ab}$$

$$\sigma_{ab} + 2\lambda \epsilon_a \epsilon_b = 0$$

$$3-0 \quad \sigma_{ab} = 2(R_a \epsilon_b - R_b \epsilon_a) - R \epsilon_a \epsilon_b$$

$$R_a = R \epsilon_a \epsilon_b \quad R = R_{ab} \eta^{ab}$$

$$G_{ab} = G \epsilon_a \epsilon_b \quad G = G_{ab} \eta^{ab}$$

$$\sigma_{ab} = 2(G_a \epsilon_b - G_b \epsilon_a - G \epsilon_a \epsilon_b)$$

$$\sigma_{ab} + 2\lambda \epsilon_a \epsilon_b = 0$$

$$\underline{G_a - \lambda \epsilon_a = 0} \quad \leftarrow \text{Eins}$$

$$S_{ab} + 2\Lambda e_a e_b = 0$$

$$3-0 \quad S_{ab} = 2(R_{ab} - R_{ba}) - R e_a e_b$$

$$R_a = R_{ab} e^b \quad R = R_{ab} \eta^{ab}$$

$$G_{ab} = G_{ba} e^b \quad G = G_{ab} \eta^{ab}$$

$$S_{ab} = 2(G_{ab} - G_{ba} - G e_a e_b)$$

$$S_{ab} + 2\Lambda e_a e_b = 0$$

$$\underline{G_a = \Lambda e_a} \leftarrow \text{Einstein's}$$

2) Pauli-Fierz
 $m_2^2 (h_{\mu\nu} - \frac{1}{2} \gamma_{\mu\nu})$

$\gamma_{\mu\nu}$ background metric

$$h_{\mu\nu} = \delta_{\mu\nu} - \gamma_{\mu\nu}$$

2) Pauli-Fierz
 $m_g^2 (h_{\mu\nu} - \frac{1}{2} \gamma_{\mu\nu})$

$\gamma_{\mu\nu}$ background metric

$$h_{\mu\nu} = \delta_{\mu\nu} - \gamma_{\mu\nu}$$

$$G_{\mu\nu}(\gamma_{\mu\nu}) - \frac{1}{2} \gamma_{\mu\nu} = 0$$

$$h = h_{\mu\nu} \gamma^{\mu\nu}$$

2) Pauli-Fierz

$$m_2^2 (h_{\mu\nu} - \eta_{\mu\nu} \gamma_{\mu\nu})$$

$\gamma_{\mu\nu}$ background metric.

$$h_{\mu\nu} = \delta_{\mu\nu} - \gamma_{\mu\nu}$$

$$G_{\mu\nu}(\gamma_{\mu\nu}) - \Lambda \gamma_{\mu\nu} = 0$$

$$h = h_{\mu\nu} \gamma^{\mu\nu} \approx h_{\mu\nu} g^{\mu\nu}$$

$$h \gamma_{\mu\nu} \approx h g_{\mu\nu}$$

$$G_{\mu\nu}(g_{\mu\nu}) - \Lambda g_{\mu\nu} - m_2^2 (h_{\mu\nu} - h g_{\mu\nu}) = 0$$

$$m_g^2 (h_{\mu\nu} - h^{\alpha\beta} g_{\alpha\beta})$$

in background metric

$$h_{\mu\nu} = \delta_{\mu\nu} - \gamma_{\mu\nu}$$

$$G_{\mu\nu}(\gamma_{\mu\nu}) - \Lambda \gamma_{\mu\nu} = 0$$

$$h = h_{\mu\nu} \gamma^{\mu\nu} \approx h_{\mu\nu} g^{\mu\nu} \quad h^{\mu\nu} \approx h^{\alpha\beta} g_{\alpha\beta}$$

$$G_{\mu\nu}(g_{\mu\nu}) - \Lambda g_{\mu\nu} - m_g^2 (h_{\mu\nu} - h^{\alpha\beta} g_{\alpha\beta}) = 0, \quad \delta \in F$$

$$G_a = \Lambda e_a + m_g^2 (h_a - h^b e_b)$$

$$h_a = h_{ab} e^b \quad h_{ab} = e^c{}_a e^d{}_b h_{cd}$$

Slab + ?

$m_f^2 (h_{\mu\nu} - h^g_{\mu\nu})$ the background metric

$$h_{\mu\nu} = \delta_{\mu\nu} - \gamma_{\mu\nu}$$

$$G_{\mu\nu}(\gamma_{\mu\nu}) - \Lambda \gamma_{\mu\nu} = 0$$

$$h = h_{\mu\nu} \gamma^{\mu\nu} \approx h_{\mu\nu} g^{\mu\nu} \quad h^g_{\mu\nu} \approx h^g_{\mu\nu}$$

$$G_{\mu\nu}(g_{\mu\nu}) - \Lambda g_{\mu\nu} - m_f^2 (h_{\mu\nu} - h^g_{\mu\nu}) = 0, \quad \phi \in \mathbb{R}$$

$$G_a = \Lambda e_a + m_f^2 (h_a - h^g_a)$$

$$h_a = h_{ab} e^b \quad h^g_a = e^b{}_a e^c{}_b h_{\mu\nu}$$

$$\Lambda e_b + 2\Lambda e^c{}_b + m_f^2 (h_a e^c{}_b - h^g_a e^c{}_b) = 0$$

$$h_a = \frac{\delta l_a + e}{m} \frac{a}{b} \delta l_b$$

Not nice.

"If Only" $\frac{e^u a \delta l_b}{h_a = 2 \delta l_a} = \frac{e^u b \delta l_a}{\text{"Wishful thinking"}}$

$$h_a = \frac{\delta l_{a.} + e}{\frac{a}{m}}$$

Not nice.

"If Only" $e^u a \delta l_{ub} = e^u b \delta l_{ua} \leftarrow$ "Wishful thinking"

$$\underline{h_a = 2 \delta l_a}$$

$$h_a = \frac{\delta l_{a,b} + e^{\mu} a \delta l_{b,c}}{m}$$

Not nice.

"If Only" $e^{\mu} a \delta l_{b,c} = e^{\mu} b \delta l_{c,a}$ ← "Wishful thinking"

$$h_a = 2 \delta l_{a,b}$$

II Chern - Simons - Proca

$$h_a = \frac{\delta l_a + e^{\mu} a \delta l_b}{3}$$

Not nice.

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II Chern - Simons - Proca
 3DGR Classically CS

$$h_a = \frac{\delta l_a + e^{\mu} a \delta l_b}{3}$$

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$$h_a = 2 \delta l_a$$

II Chern - Simons - Proca
 3DGR Classically CS
 $H dA + \frac{2}{3} AAA + m A A * A$

$$h_a = \frac{\delta l_a}{2} + \frac{a \delta l_a}{2}$$

Not nice.

"If Only" $e^{\mu} a \delta l_{\mu b} = e^{\mu} b \delta l_{\mu a} \leftarrow$ "Wishful thinking"

$$h_a = 2 \delta l_a$$

II Chern - Simons - Proca
 3DGR Classically CS
 $H dA + \frac{2}{3} AAA + mAA * A$

$$h_a = \frac{\delta l_a}{\dots} \quad \frac{a \dots}{\dots}$$

↑
Not nice.

"If Only" $e^{\mu} a \delta l_{\mu b} = e^{\mu} b \delta l_{\mu a} \leftarrow$ "Wishful thinking"

$h_a = 2 \delta l_a$

II Chern - Simons - Proca
3DGR Classically CS

$$A dA + \frac{2}{3} AAA + m A A * A$$

1) What if : 2 copies of CS

1) Abelian

$$S = A \circ A - B \circ B$$

1) Abelian

$$S = A \partial A - B \partial B + m(A-B) A * (A-B)$$

Important

$$F_A = -m * (A-B) = F_B$$

1) Abelian

$$S = A dA - B dB + m(A-B) \wedge (A-B)$$

Important.

$$F_A = -m \wedge (A-B) = F_B \quad \rightarrow \quad A-B = d\phi$$

$$F_A = -m \wedge d\phi$$

1) Abelian

$$S = \int A dA - B dB + m(A-B) \wedge *(A-B)$$

Important.

$$F_A = -m*(A-B) = F_B$$

$$\rightarrow A-B = d\phi$$

$$= -m + d\phi$$

Bianchi

$$d + d\eta = 0$$

$$\Leftrightarrow D\phi = 0$$

1 dof

1) Abelian

$$S = \int A dA - B dB + m(A-B) \wedge *(A-B)$$

Important.

$$F_A = -m*(A-B) = F_B \quad \rightarrow \quad A-B = d\phi$$

$$F_A = -m + d\phi \quad \text{Bianchi} \quad d + d\eta = 0 \quad \rightarrow \quad D\phi = 0$$

1 dof

identical
to Stückelberg
in Proca in the
gauge $d\wedge A = 0$

1) Abelian

$$S = \int A dA - B dB + m(A-B) \wedge (A-B)$$

Important.

$$F_A = -m \wedge (A-B) = F_B \quad \rightarrow \quad A-B = d\phi$$

$$F_A = -m \wedge d\phi \quad \text{Bianchi} \quad d + d \circlearrowleft = 0 \quad \rightarrow \quad \square \phi = 0$$

$m > 0$ ghost

1 dof

identical
to Stückelberg
in Proca in the
gauge $d \wedge A = 0$

Important.

$$F_A = -m*(A-B) \rightarrow F_B \rightarrow A-B = d\phi$$

$$F_A = -m + d\phi$$

Bianchi. $d+d\phi = 0 \rightarrow D\phi = 0$

1 dof. \implies

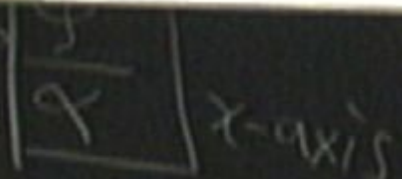
$m > 0$ ghost

2) Non-Abelian

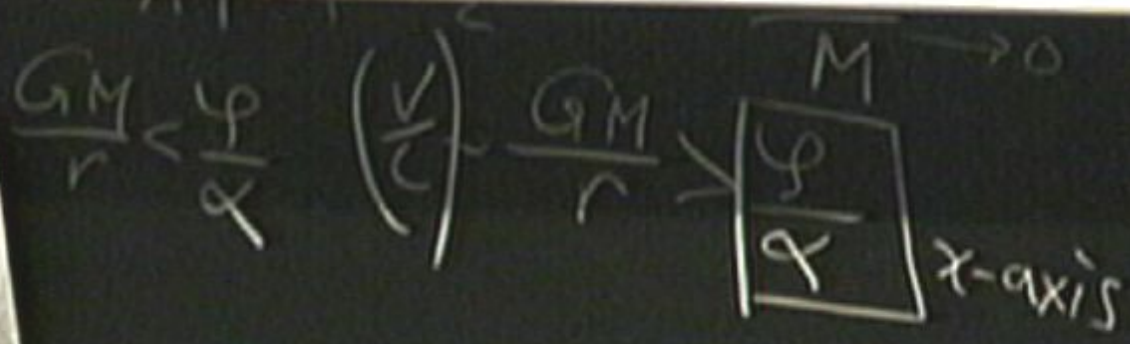
$$S_{CS}(A) = \text{Tr} (A dA + \frac{2}{3} AAA)$$

$$S = S(A) - S_{CS}(B) + m$$

identical to Stückelberg in Proca in the gauge $d+A=0$



$F_A = -m + d\phi$ Bianchi $d+d\phi = 0 \Rightarrow 1+4=0$
 $m > 0$ ~~ghost~~ 1 dof $\}} \Rightarrow$
 2) Non-Abelian identical to Stückelberg
 $S_{CS}(A) = \text{Tr} (A dA + \frac{2}{3} AAA)$ in Proca in the gauge $d+A=0$
 $S = S_{CS}(A) - S_{CS}(B) + m(A-B)A + (A-B)$
 $F_A = -m(A-B) = F_B$



1) Abelian

$$S = \int d^4x \left(-\frac{1}{2} F_{\mu\nu} F^{\mu\nu} + m(A-B) \right) \star (A-B)$$

Important

$$F_A = -m \star (A-B) = F_B \quad \rightarrow \quad A-B = d\phi$$

$$F_A = -m + d\phi$$

Bianchi $d + d\star = 0 \quad \Leftrightarrow \quad \square\phi = 0$

1 dof

$m > 0$ ghost

2) Non-Abelian

$$S_{\text{CS}}(A) = \int \text{Tr} \left(A dA + \frac{2}{3} A A A \right)$$

$$S = S_{\text{CS}}(A) - S_{\text{CS}}(B) + m(A-B) \star (A-B)$$

$$F_A = -m \star (A-B) = F_B$$

$$A = U^\dagger B U - U^\dagger dU$$

$$U = e^{i\phi}$$

identical to Stückelberg in Proca in the gauge $d\star A = 0$

1) Abelian

$$S = \int d^4x \left[-\frac{1}{2} (d_\mu A - d_\mu B)^2 + m(A-B) \right] \wedge *(A-B)$$

Important

$$F_A = -m*(A-B) = F_B \quad \rightarrow \quad A-B = d\phi \quad \text{Add } \int d^4x \frac{1}{2} (d\phi)^2$$

$$F_A = -m + d\phi \quad \text{Bianchi} \quad d+d\phi = 0 \quad \Leftrightarrow \quad \square\phi = 0$$

1 dof

$m > 0$ ghost

2) Non-Abelian

$$S_{\text{CS}}(A) = \int \text{Tr} \left(A dA + \frac{2}{3} A A A \right)$$

$$S = S_{\text{CS}}(A) - S_{\text{CS}}(B) + m(A-B) \wedge *(A-B)$$

$$F_A = -m*(A-B) = F_B$$

$$A = U^\dagger B U - U^\dagger dU \\ = U^\dagger D_B U$$

identical to Stückelberg in Proca in the gauge $d_\mu A = 0$

$$U = e^{i\pi t a} \\ \text{t a - Gagne ga}$$

$$F_B \rightarrow (U^T D_B U - B)$$

$$F_B = +(U^T D_B U - B)$$

$$D_B + D U - D + B = 0$$

$$D_B F_B = 0$$

$$F_B = +(U^\dagger D_B U - B)$$
$$D_\mu + D U - D + B = 0$$

$D_B F_B = 0$
 \Leftrightarrow eom of the Stueckelberg
in Proca, in the gauge $D + B = 0$

$$F_B = +(U^T D_B U - B)$$
$$D_\mu + D U - D + B = 0$$

$\dim(\mathfrak{g})$ dof.

$$D_B F_B = 0$$

\Leftrightarrow eom of the Stückelberg
in Proca, in the gauge $D+B=0$

$$F_B = +(V^\dagger D_B V - B)$$

$$D_\mu + D V - D + B = 0$$

$\dim(\mathfrak{g})$ dof.

Q: 1) ghost?

2) massive mode?

$$D_B F_B = 0$$

\Leftrightarrow eom of the Stückelberg
in Proca, in the gauge $D+B=0$

$$F_B = +(U^\dagger D_B U - B)$$

$$D_\mu + D U - D + B = 0$$

$\dim(\mathfrak{g})$ dof.

Q: Dghost?

2) massive mode?

III) 3D GR with $\Lambda < 0$

$$S = S_{\text{CS}}(A_+) - S_{\text{CS}}(A_-)$$

$$A_+ = (\omega^a + \frac{1}{\ell} e^a) T^a$$

$$A_- = (\omega^a - \frac{1}{\ell} e^a) T^a \quad T^a \in \mathfrak{so}(2,1)$$

$D_B F_B = 0$
 \Leftrightarrow eom of the Stückelberg
in Proca, in the gauge $D+B=0$

$$F_B = +(V^T D_B V - B)$$

$$D_\mu + D V - D + B = 0$$

dim(g) dof.

$$D_B F_B = 0$$

\Leftrightarrow eom of the Stueckelberg
in Proca, in the gauge $D+B=0$

Q: Dghost?

2) massive mode?

III) 3D GR with $\Lambda < 0$

$$S = S_{CS}(A_+) - S_{CS}(A_-)$$

$$A_+ = (\omega^a \quad e^a) T^a$$

$$A_- = (\omega^a \quad \frac{1}{\ell} e^a) T^a \quad T^a \in \mathfrak{sl}(2, \mathbb{R})$$

$$F_B = +(V^T D_B V - B)$$

$$D_\mu + D V - D + B = 0$$

dim(g) dof.

$D_B F_B = 0$
 \Leftrightarrow eom of the Stückelberg
 in Proca, in the gauge $D+B=0$

Q: 1) ghost?

2) massive mode?

III) 3D GR with $\Lambda < 0$

$$S = S_{CS}(A_+) - S_{CS}(A_-) + m(A_+ - A_-) \wedge (A_+ - A_-)$$

$$A_\pm = (\omega^a \pm \frac{1}{\ell} e^a) T^a \quad \left(\begin{array}{l} A_- = (\omega^a - \frac{1}{\ell} e^a) T^a \\ \rightarrow \frac{2}{\ell} e^a T^a \end{array} \right) \quad T^a \in \mathfrak{sl}(2, \mathbb{R})$$

$$F_B = +(V^T D_B V - B)$$

$$D_+ + D V - D + B = 0$$

dim(g) dof.

$D_B F_B = 0$
 \Leftrightarrow eom of the Stueckelberg
 in Proca, in the gauge $D+B=0$

Q: 1) ghost?

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III) 3D GR with $\Lambda < 0$

$$S = S_{CS}(A_+) - S_{CS}(A_-) + m(A_+ - A_-) \wedge (A_+ - A_-)$$

$$A_+ = (\omega^a + \frac{1}{\ell} e^a) T^a \quad \left(\begin{array}{l} A_- = (\omega^a - \frac{1}{\ell} e^a) T^a \\ \rightarrow \frac{2}{\ell} e^a T^a \end{array} \right. \quad T^a \in \mathfrak{sl}(2, \mathbb{R})$$

"*"

$$F_B = +(U^\dagger D_B U - B)$$

$$D_+ + D U - D + B = 0$$

dim(g) dof.

$D_B F_B = 0$
 \Leftrightarrow eom of the Stueckelberg
 in Proca. in the gauge $D+B=0$

Q: Dghost?

2) massive mode?

III) 3D GR with $\Lambda < 0$

$$S = S_{CS}(A_+) - S_{CS}(A_-) + m \frac{(A_+ - A_-) \wedge (A_+ - A_-)}{2}$$

$$A_+ = (\omega^a + \frac{1}{l} e^a) T^a$$

$$A_- = (\omega^a - \frac{1}{l} e^a) T^a \quad T^a \in \mathfrak{sl}(2, \mathbb{R})$$

"*"

$$g_{\mu\nu} = e^a_\mu e^b_\nu \eta_{ab}$$

$$F_B = + (D_\mu D_\nu D^\mu - D^\mu D_\nu D_\mu)$$

$$D_\mu + D_\nu D^\mu - D^\mu D_\nu = 0$$

dim(g) dof.

$D_\mu B = 0$
 \Leftrightarrow eom of the Stückelberg
 in Proca, in the gauge $D_\mu B = 0$

Q: 1) ghost?

2) massive mode?

III) 3D GR with $\Lambda < 0$

$$S = S_{CS}(A_+) - S_{CS}(A_-) + m \frac{(A_+ - A_-)_\mu (A_+ - A_-)^\mu}{2}$$

$$A_\pm = \left(\omega^a \pm \frac{1}{l} e^a \right) T^a \quad \left(A_- = \left(\omega^a - \frac{1}{l} e^a \right) T^a \right) \quad T^a \in \mathfrak{sl}(2, \mathbb{R})$$

"*"

$$g_{\mu\nu} = e^a_\mu e^b_\nu \eta_{ab}$$

\hookrightarrow choose a background. $\gamma_{\mu\nu} = \bar{e}^a_\mu \bar{e}^b_\nu \eta_{ab}$

$$F_B = +(U^\dagger D_B U - B)$$

$$D_\mu + D U - D + B = 0$$

dim(G) dof.

$D_B F_B = 0$
 \Leftrightarrow eom of the Stückelberg
 in Proca, in the gauge $D+B=0$

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$$A_+ = (\omega^a + \frac{1}{\ell} e^a) T^a$$

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$T^a \in \mathfrak{sl}(2, \mathbb{R})$

"*" η_{ab}

$$g_{\mu\nu} = e^a_\mu e^b_\nu \eta_{ab}$$

\hookrightarrow choose a background. $\gamma_{\mu\nu} = \bar{e}^a_\mu \bar{e}^b_\nu \eta_{ab}$

$$F_B = +(U^\dagger D_B U - B)$$

$$D_\mu + D U - D + B = 0$$

dim(G) dof.

$D_B F_B = 0$
 \Leftrightarrow eom of the Stückelberg
 in Proca, in the gauge $D+B=0$

Q: 1) ghost?

2) massive mode?

III) 3D GR with $\Lambda < 0$

$$S = S_{CS}(A_+) - S_{CS}(A_-) + m \frac{(A_+ - A_-) \wedge * (A_+ - A_-)}{2}$$

$$A_+ = (\omega^a + \frac{1}{l} e^a) T^a$$

$$A_- = (\omega^a - \frac{1}{l} e^a) T^a$$

$T^a \in \mathfrak{sl}(2, \mathbb{R})$

$$\frac{m}{l} (h_{uv}^2 - \frac{1}{2} h^2)$$

"*" η_{ab}

$$g_{\mu\nu} = e^a_\mu e^b_\nu \eta_{ab}$$

$$\gamma_{\mu\nu} = \bar{e}^a_\mu \bar{e}^b_\nu \eta_{ab}$$

\rightarrow choose a background.

$$F_B = +(U^T D_B U - B)$$

$$D_\mu + D U - D + B = 0$$

$\dim(\mathfrak{g})$ dof.

$D_B F_B = 0$
 \Leftrightarrow eom of the Stückelberg
 in Proca, in the gauge $D+B=0$

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$$S = S_{CS}(A_+) - S_{CS}(A_-) + m \frac{(A_+ - A_-) \wedge (A_+ - A_-)}{2}$$

$$A_+ = (\omega^a + \frac{1}{\ell} e^a) T^a$$

$$A_- = (\omega^a - \frac{1}{\ell} e^a) T^a$$

$T^a \in \mathfrak{sl}(2, \mathbb{R})$

$$\rightarrow \frac{2}{\ell} e^a T^a$$

$$\frac{m}{\ell} (h_{uv}^2 - \frac{1}{2} h^2)$$

Screw symmetry

"*" $g_{\mu\nu} = e^a_\mu e^b_\nu \eta_{ab}$

\hookrightarrow choose a background. $\gamma_{\mu\nu} = \bar{e}^a_\mu \bar{e}^b_\nu \eta_{ab}$

"If Only" $e^{\mu} a \delta_{\mu b} = e^{\mu} b \delta_{\mu a} \leftarrow$ "Wishful thinking"

$$h_a = 2 \delta_{la}$$

II Chern - Simons - Proca
3DGR Classically CS

$$A dA + \frac{2}{3} AAA + m A \wedge *A$$

1) What if : 2 copies of CS

$$R_{\mu\nu}^2 - \frac{3}{8} R^2$$

$$R_{\mu\nu}^2 - \frac{1}{4} R^2$$