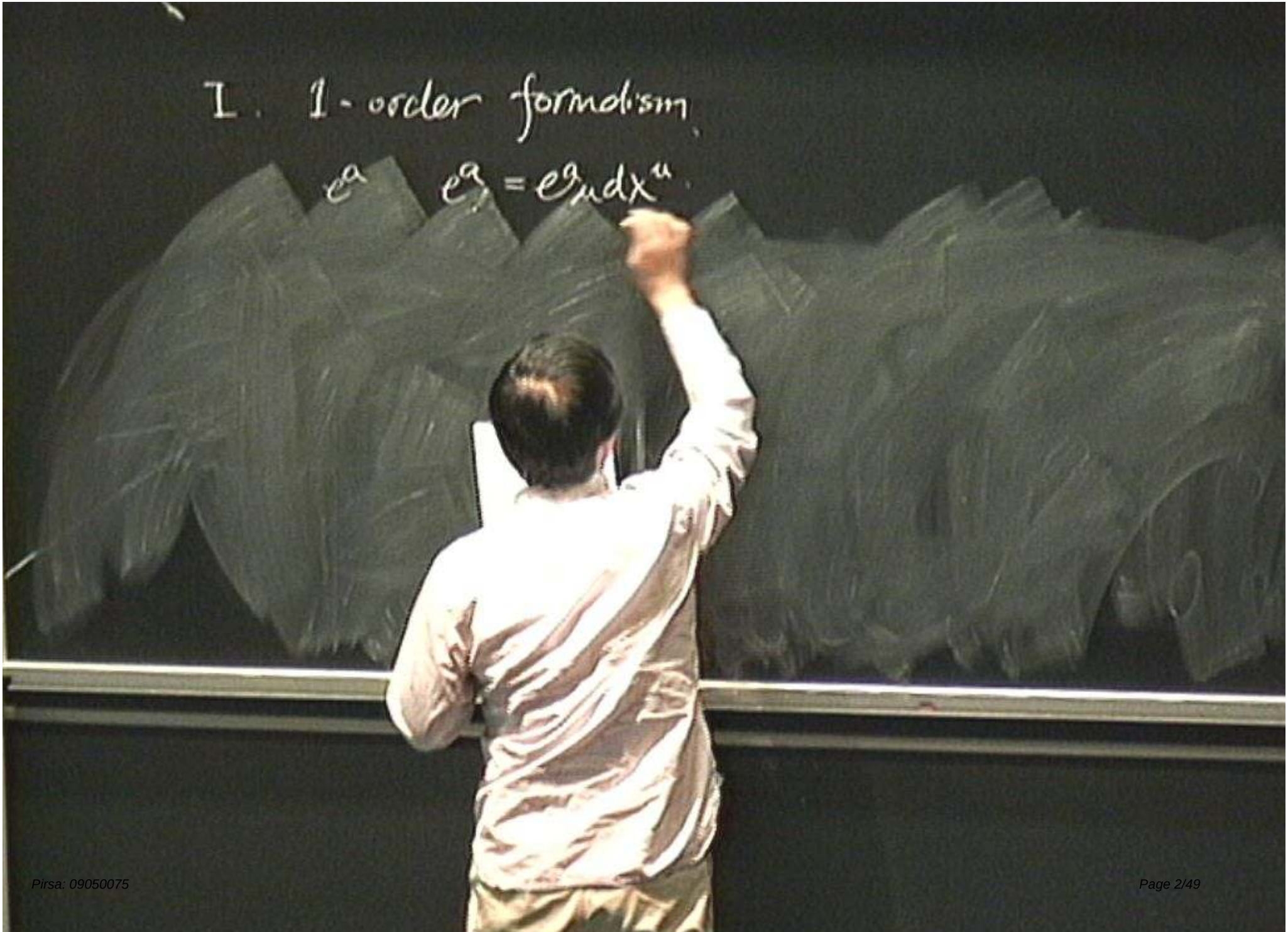
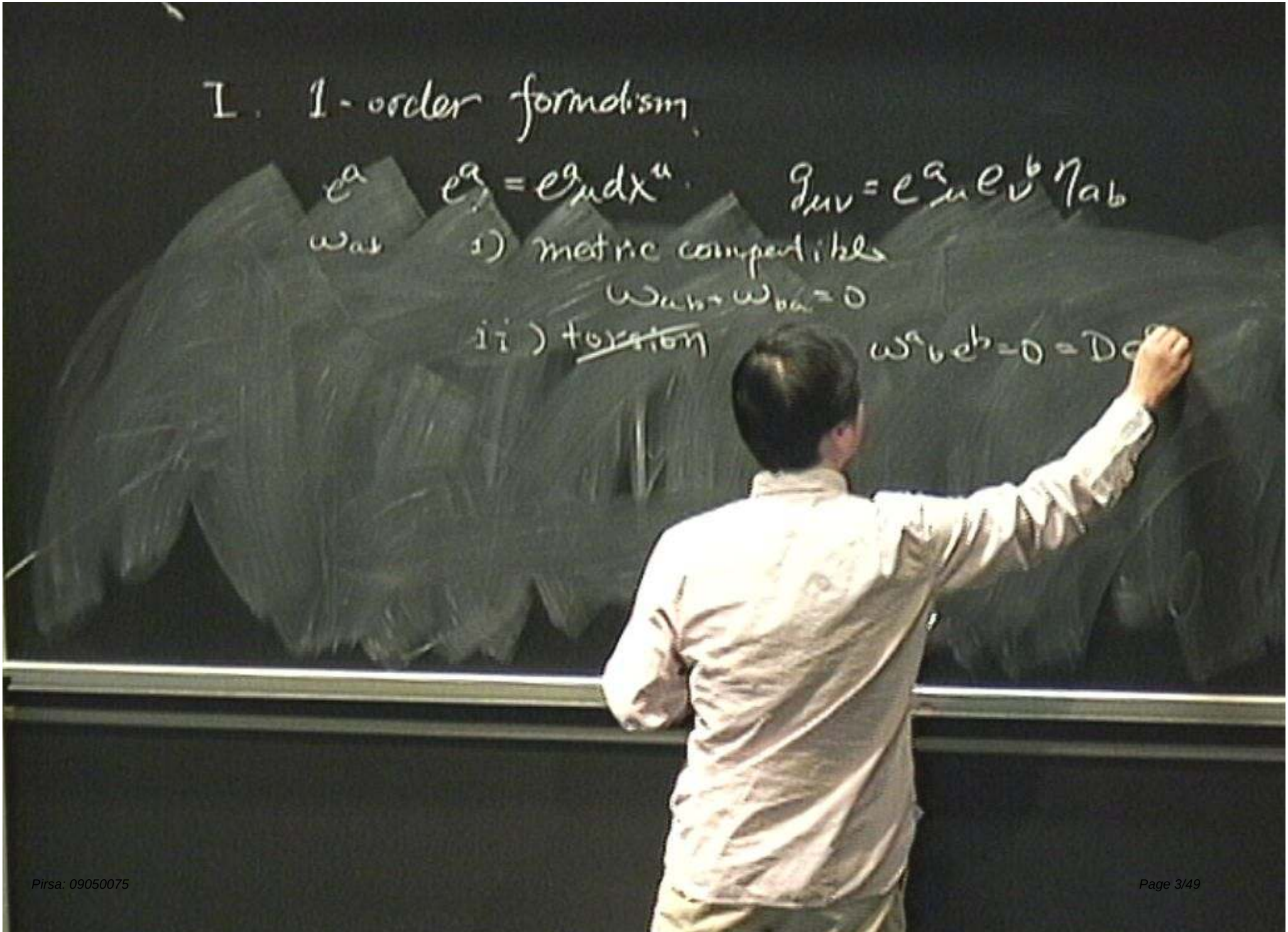
Title: Massive gravity in 3-D and the Chern-Simons-Proca theory

Date: May 26, 2009 02:45 PM

URL: http://pirsa.org/09050075

Abstract: TBA


I. 1-order formalism
e^a $e^a_\mu = e^a_\mu dx^\mu$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e^a $e^a = e^a_\mu dx^\mu$ $g_{\mu\nu} = e^a_\mu e^b_\nu \eta_{ab}$
ω_{ab} i) metric compatible
$\omega_{ab\mu} + \omega_{ba\mu} = 0$
ii) torsion $\omega^a{}_{b\mu} = 0 = D e^a$

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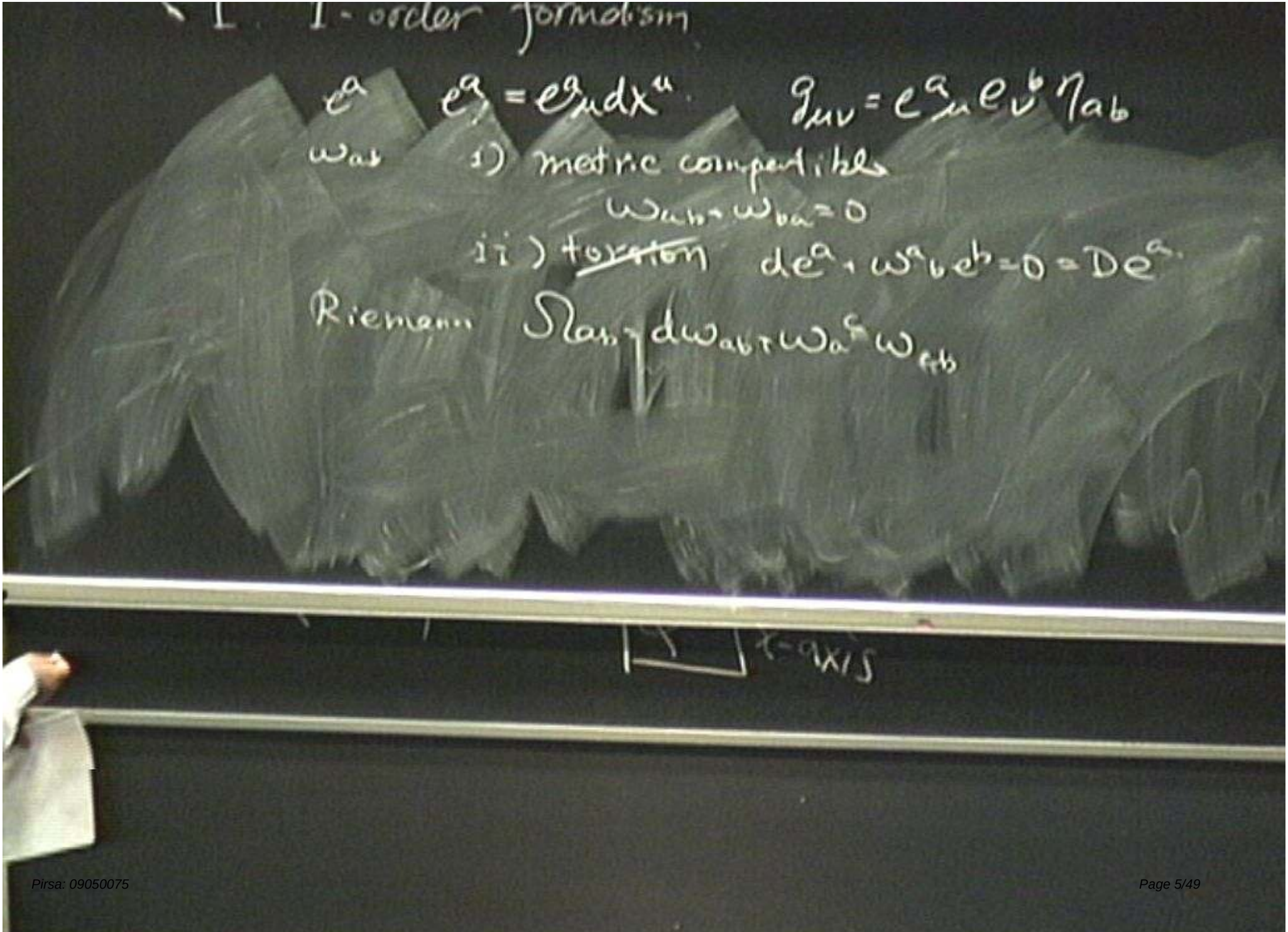
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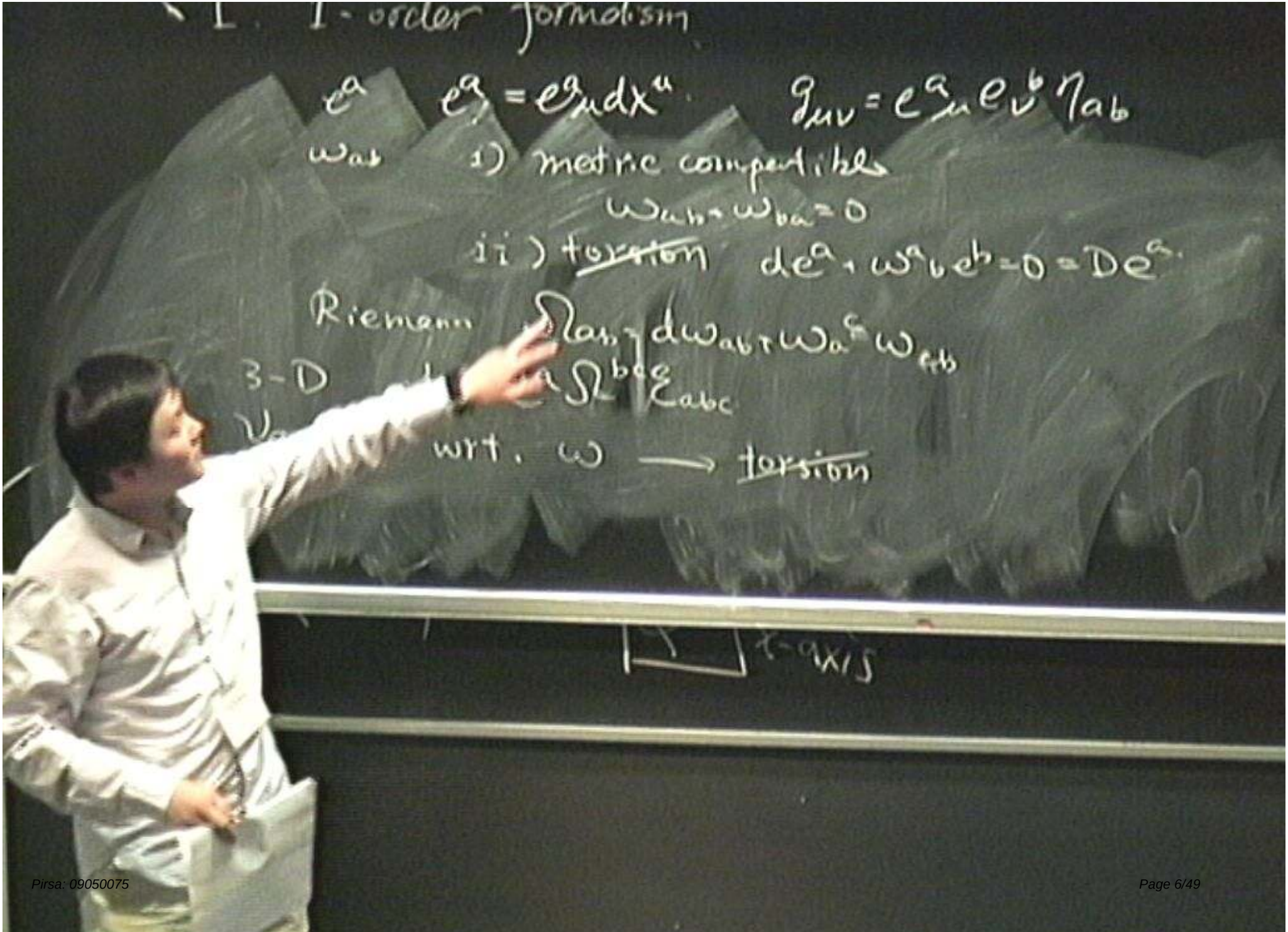
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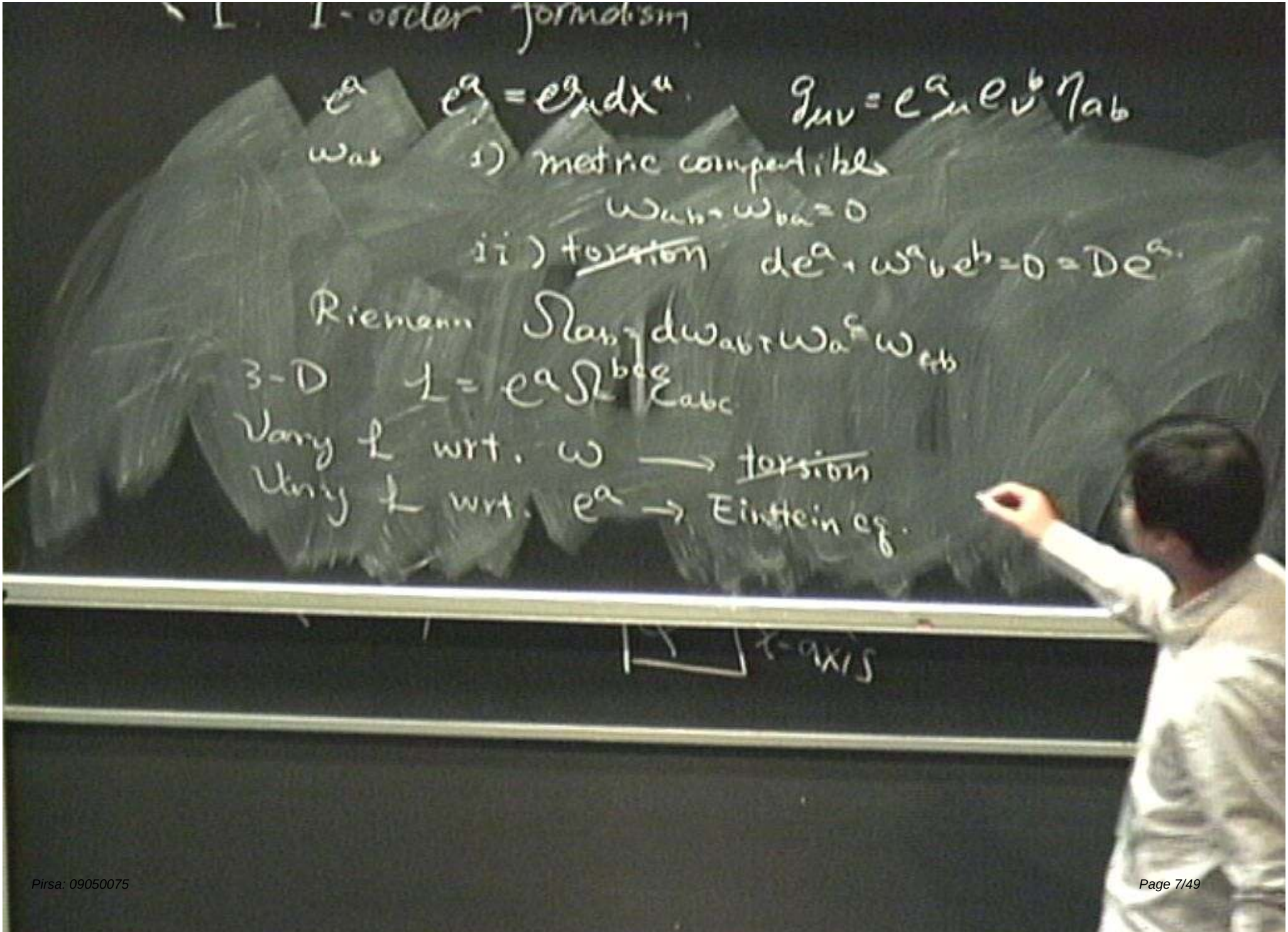
3-D $\Omega^{bc} \varepsilon_{abc}$
wrt. $\omega \longrightarrow$ torsion
z-axis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3-D $\mathcal{L} = e^a \Omega^{bc} \varepsilon_{abc}$
Vary $\mathcal{L}$ wrt. $\omega \longrightarrow$ ~~torsion~~
Vary $\mathcal{L}$ wrt. $e^a \rightarrow$ Einstein eq.
z-axis

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3-D $\mathcal{L} = e^a \Omega^{bc} \varepsilon_{abc} + \Lambda e^a e^b e^c \varepsilon_{abc}$

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Vary $\mathcal{L}$ wrt. ω $\longrightarrow$ ~~torsion~~

Vary $\mathcal{L}$ wrt. e^a $\rightarrow$ Einstein eq.

$\Omega_{ab} + 2\Lambda e_a e_b = 0$

3-D $\Omega_{ab} = 2(R_a e_b - R_b e_a) - R e_a e_b$

$R_a = R_{ab} e^b \quad R = R_{ab} \eta^{ab}$

$\Omega_{ab} + 2\Lambda e_a e_b = 0$

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$G_{ab} = G_{abc} e^c \quad G = G_{ab}\eta^{ab}$

$\Omega_{ab} = 2(G_a e_b - G_b e_a - G e_a e_b)$

$\Omega_{ab} + 2\Lambda e_a e_b = 0$

$G_a - \Lambda e_a = 0$ ← Einst

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$\Omega_{ab} = 2(G_{ab} e_b - G_b e_a - G e_b e_c)$

$\Omega_{ab} + 2\Lambda e_a e_b = 0$

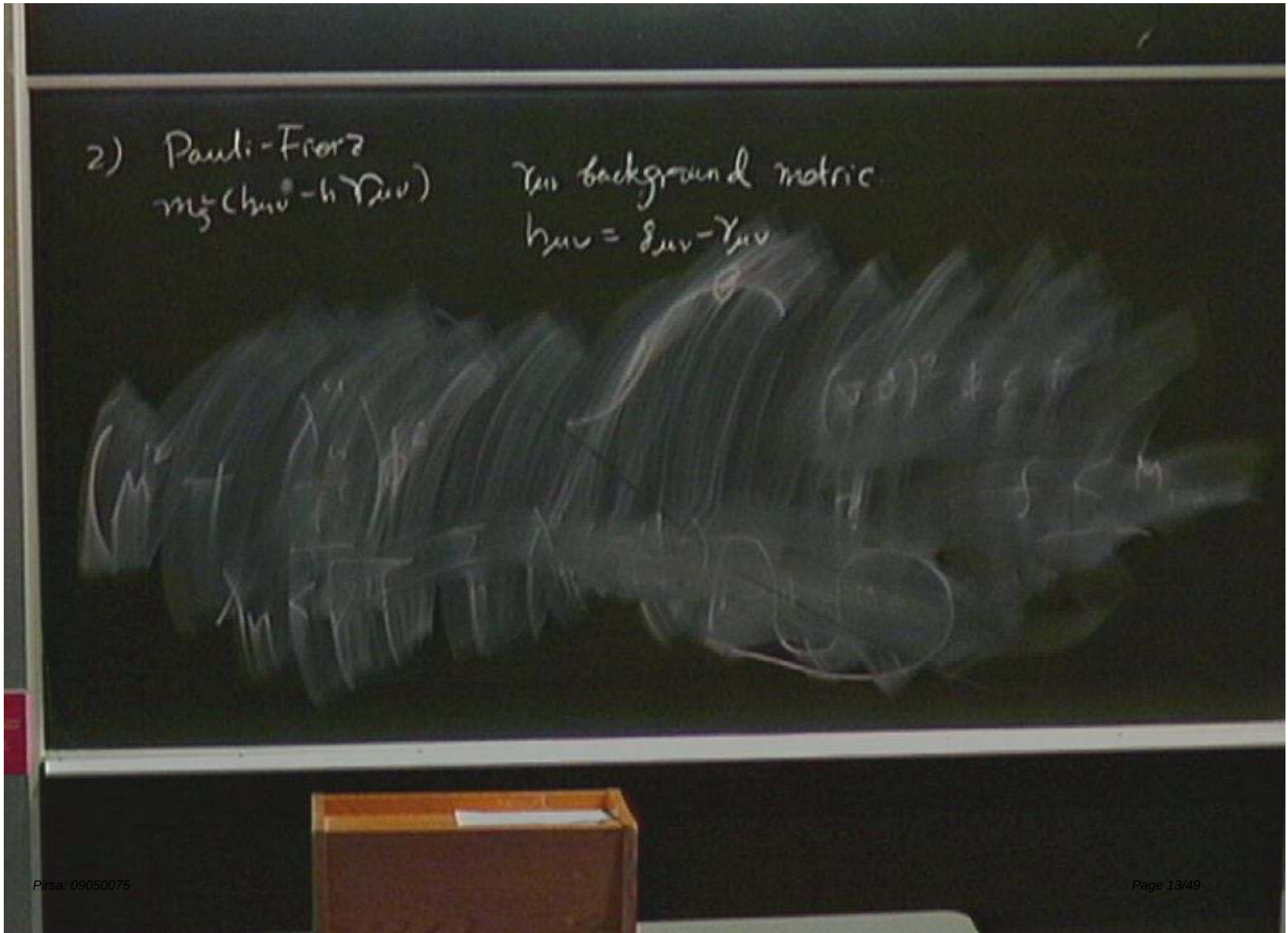
$R_a - \Lambda e_a = 8\pi T_a$ ← Einstein eqs.

2) Pauli-Fierz

$m_g^2 (h_{\mu\nu}^2 - h\gamma_{\mu\nu})$

$\gamma_{\mu\nu}$ background metric

$h_{\mu\nu} = g_{\mu\nu} - \gamma_{\mu\nu}$



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$h = h_{\mu\nu}\gamma^{\mu\nu}$

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$h = h_{\mu\nu}\gamma^{\mu\nu} \approx h_{\mu\nu}g^{\mu\nu}$ $h\gamma_{\mu\nu} \approx hg_{\mu\nu}$

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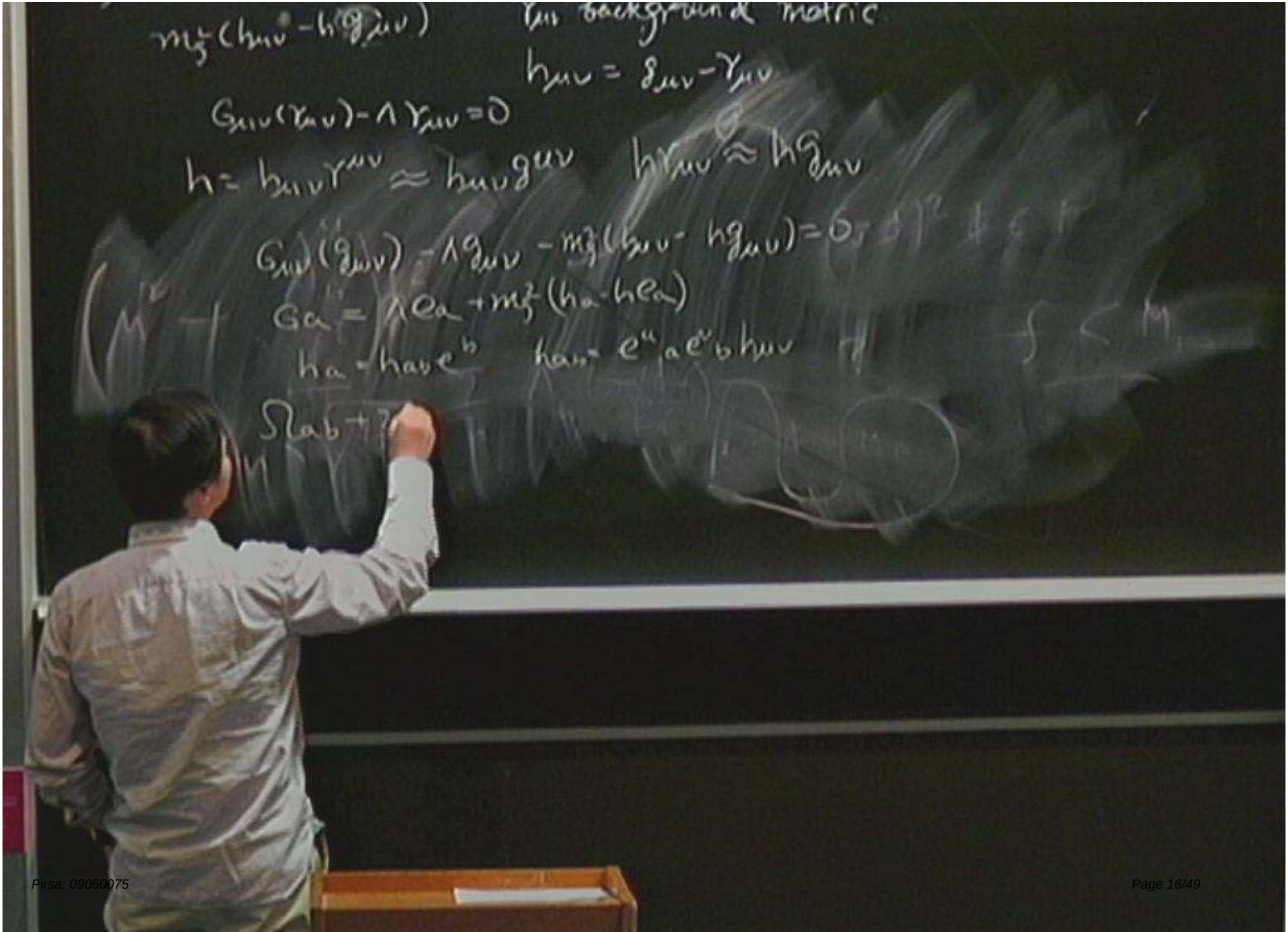
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background metric
$h_{\mu\nu} = g_{\mu\nu} - \gamma_{\mu\nu}$
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$h\gamma_{\mu\nu} \approx h g_{\mu\nu}$
$G_{\mu\nu}(g_{\mu\nu}) - \Lambda g_{\mu\nu} - m_g^2(h_{\mu\nu} - h g_{\mu\nu}) = 0$
$G_a = \Lambda e_a + m_g^2(h_a - h e_a)$
$h_a = h_{av} e^b$
$h_{ab} = e^{\mu}{}_a e^{\nu}{}_b h_{\mu\nu}$

$m_g^2(h_{\mu\nu} - h g_{\mu\nu})$

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$G_{\mu\nu}(g_{\mu\nu}) - \Lambda g_{\mu\nu} - m_g^2(h_{\mu\nu} - h g_{\mu\nu}) = 0$

$G_a = \Lambda e_a + m_g^2(h_a - h e_a)$

$h_a = h_{ab} e^b$ $\quad h_{ab} = e^\mu{}_a e^\nu{}_b h_{\mu\nu}$

$\Omega_{ab} + 2\Lambda e_a e_b + m_g^2(h_a e_b - h_b e_a) = 0$

$h_a = \delta e_a + e^{\mu}{}_a \delta e_{\mu b}$

Not nice.

"If Only" $e^{\mu}{}_a \delta e_{\mu b} = e^{\mu}{}_b \delta e_{\mu a}$ ⇐ "Wishful thinking"

$h_a = 2 \delta e_a$

$h_a = \delta e_a + e^{\mu}{}_a \delta e_{\mu b}$

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II Chern-Simons-Proca

$h_a = \delta e_a + e^{\mu}{}_a \delta e_{\mu b} \ldots$

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II Chern - Simons - Proca

3DGR $\xleftrightarrow{\text{Classically}}$ CS

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II Chern - Simons - Proca

3DGR $\xleftrightarrow{\text{Classically}}$ CS

$A dA + \frac{2}{3} A A A + m A \wedge * A$

$h_a = \delta e_a + e^\mu{}_a \delta e_{\mu b}$

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II Chern - Simons - Proca

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$h_a = \delta e_a \cdot e^{\mu}{}_a \delta e_{\mu b}$

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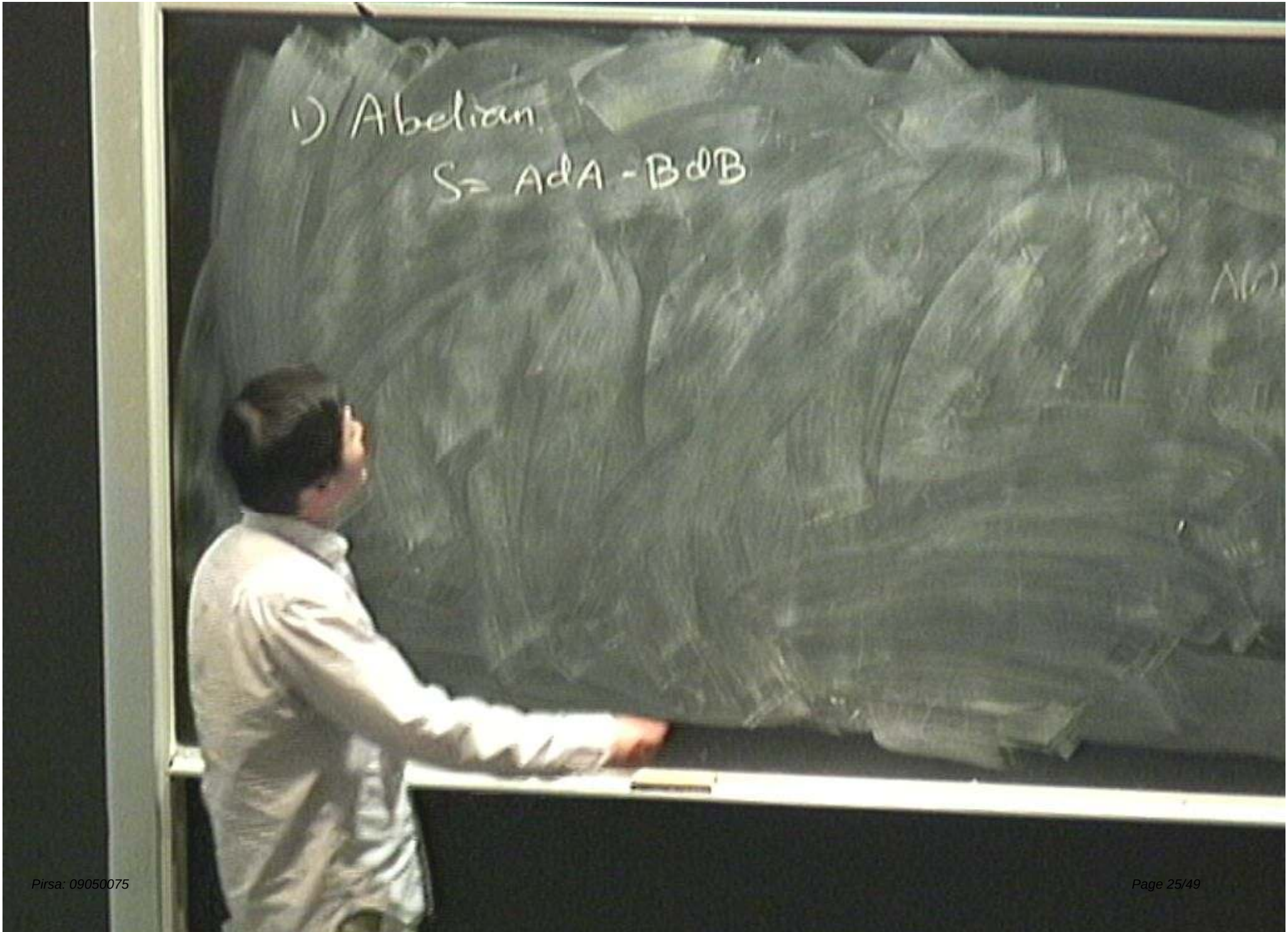
$h_a = 2\delta e_a$

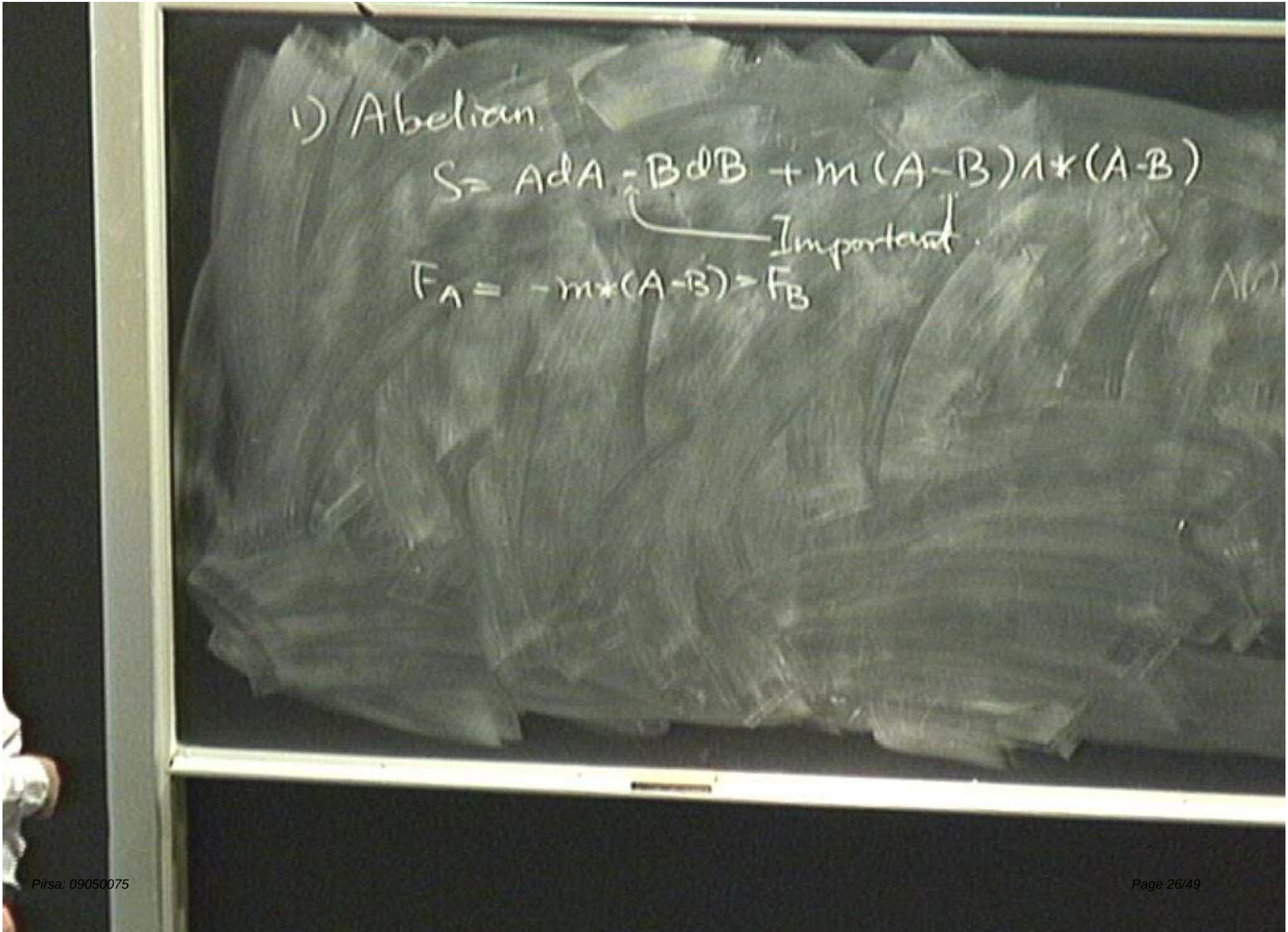
II Chern - Simons - Proca

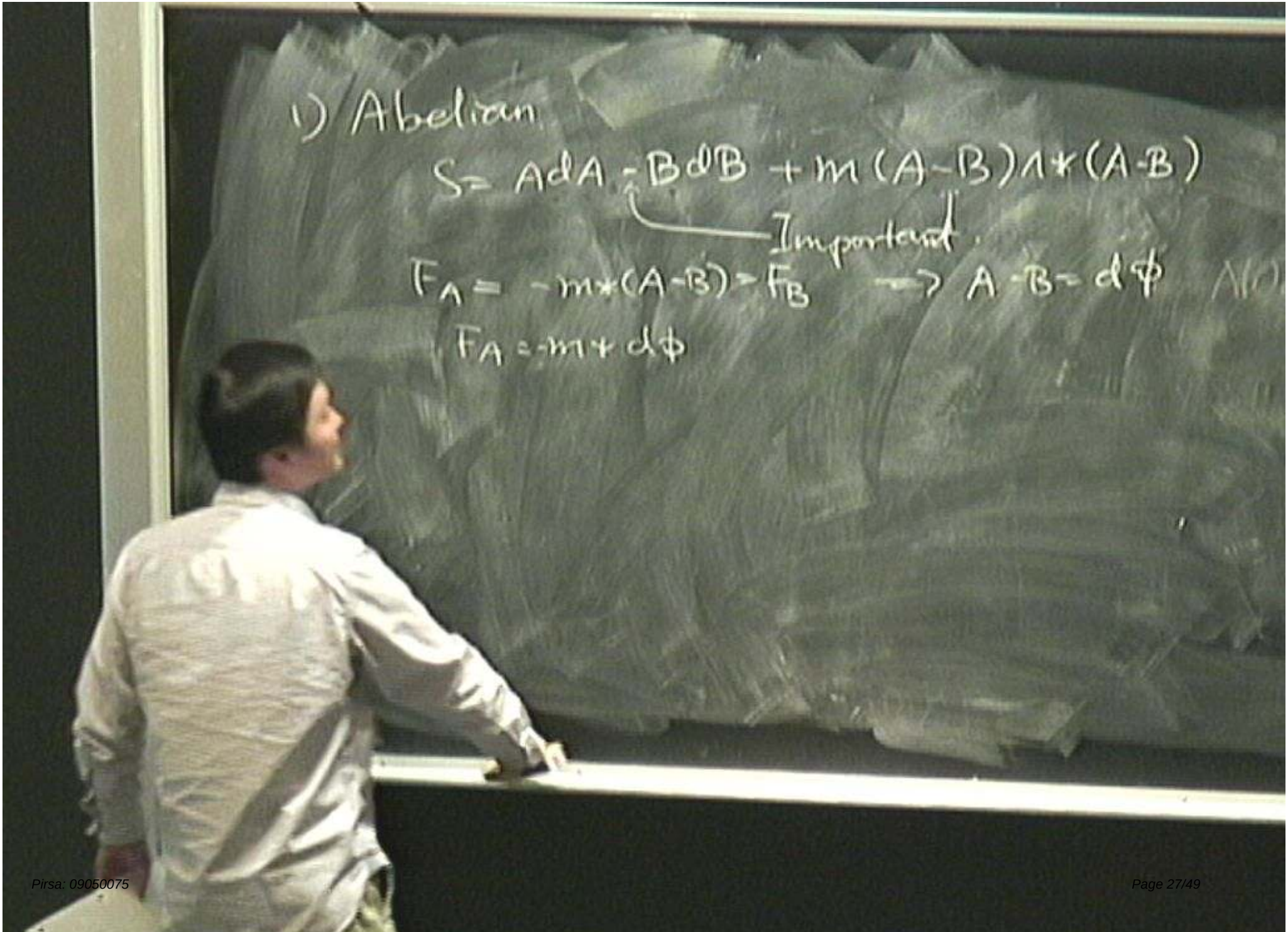
3DGR $\xleftrightarrow{\text{Classically}}$ CS

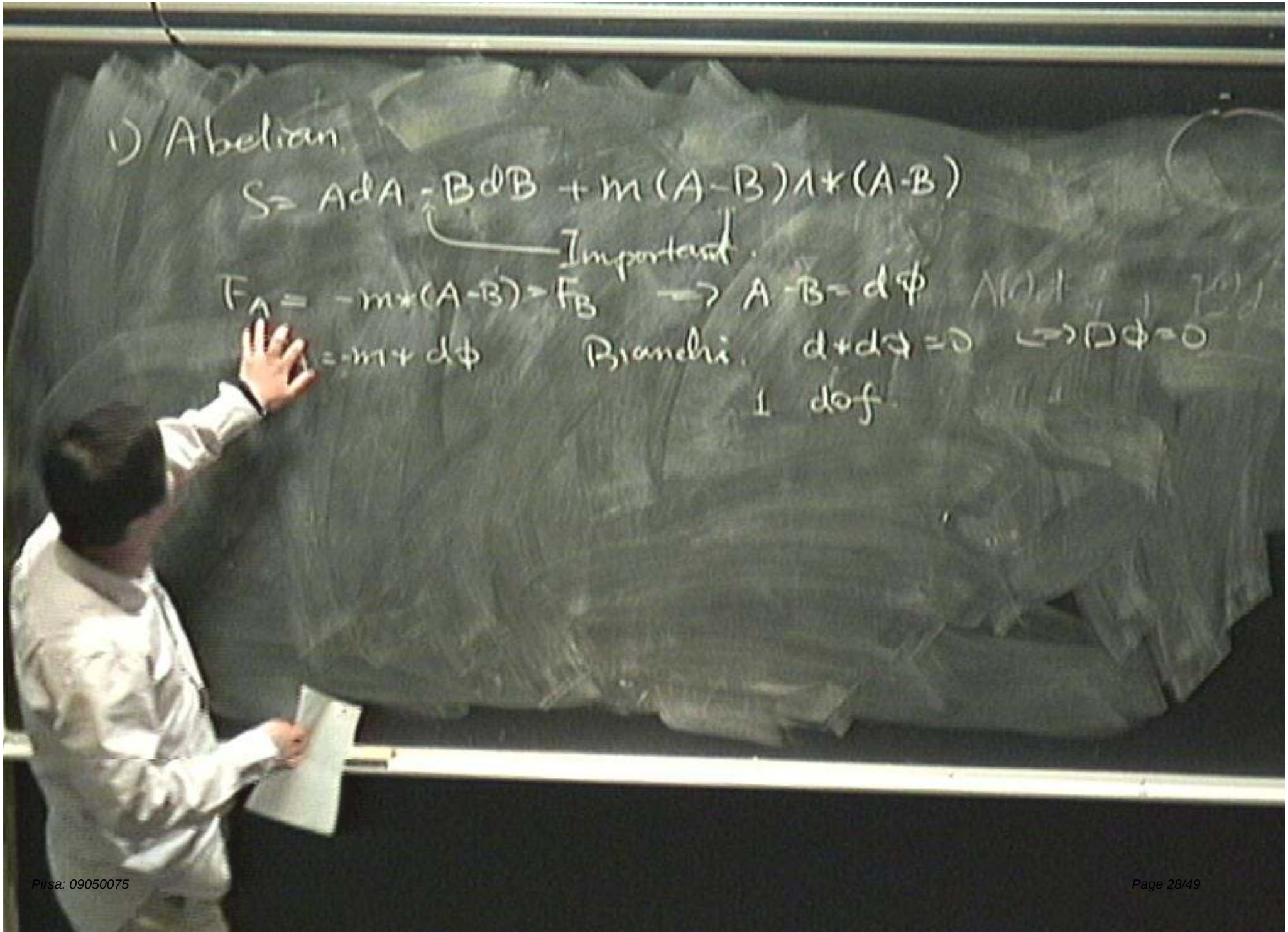
$AdA + \frac{2}{3} AAA + m A \wedge * A$

1) What if : 2 copies of CS


1) Abelian
S= AdA - BdB


1) Abelian.
$S = AdA - BdB + m(A-B) \wedge *(A-B)$
Important.
$F_A = -m*(A-B) = F_B$


1) Abelian
S = AdA - BdB + m(A-B)∧*(A-B)
Important
$F_A = -m*(A-B) = F_B$ → $A - B = d\phi$
$F_A = -m*d\phi$


1) Abelian.
S= AdA - BdB + m(A-B)∧*(A-B)
Important.
F_A = -m*(A-B) = F_B → A-B = dφ
F_A = -m*dφ
Bianchi. d*dφ = 0 → □φ = 0
1 dof.

1) Abelian.

$$S = A\,dA - B\,dB + m(A-B)\wedge *(A-B)$$

Imported.

$$F_A = -m*(A-B) = F_B \longrightarrow A - B = d\phi$$

$$F_A = -m*d\phi \qquad \text{Bianchi} \quad d*d\phi = 0 \hookrightarrow \Box\phi = 0$$

1 dof

$\Updownarrow$

identical to Stückelberg in Proca in the gauge $d*A = 0$

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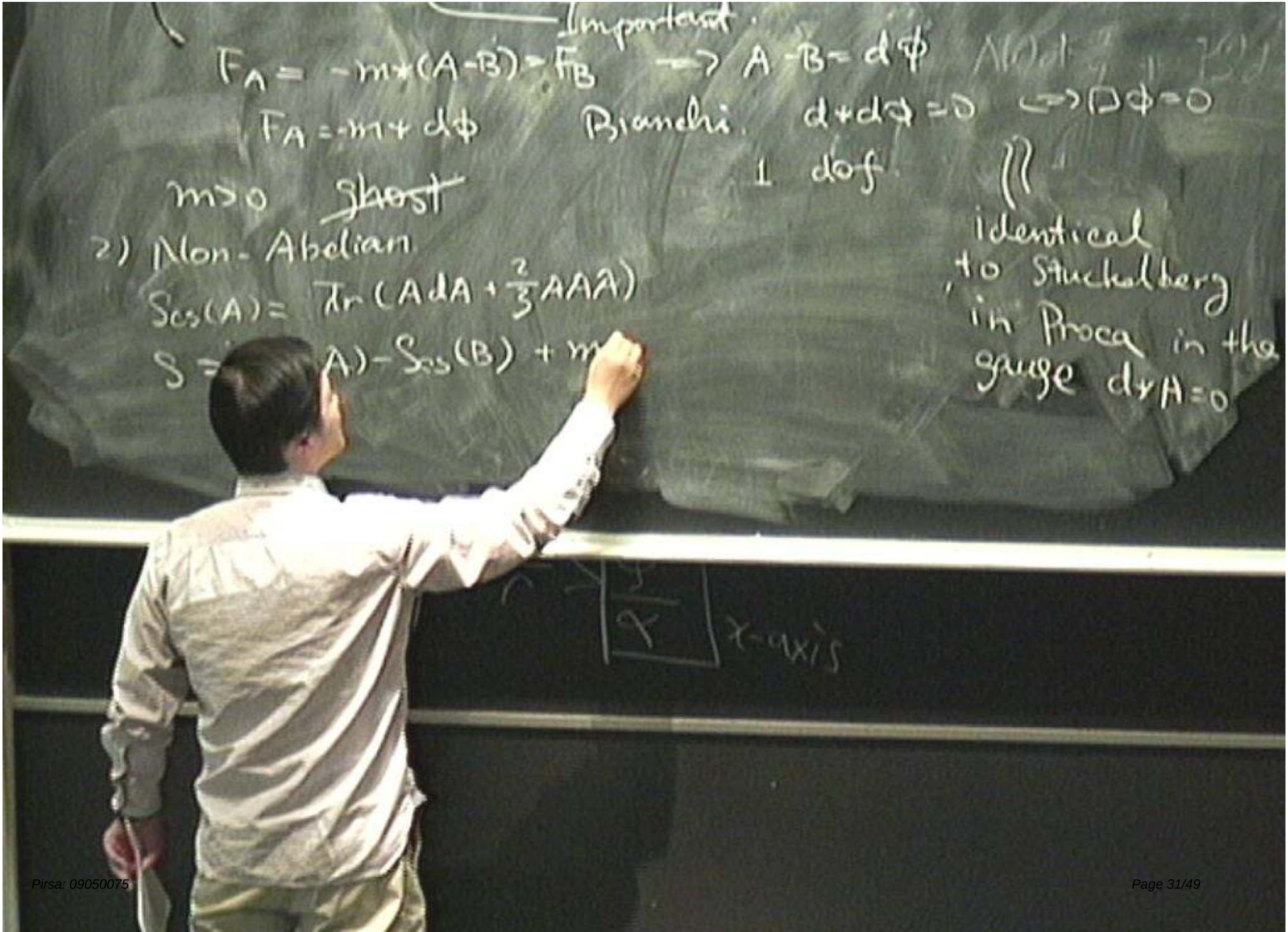
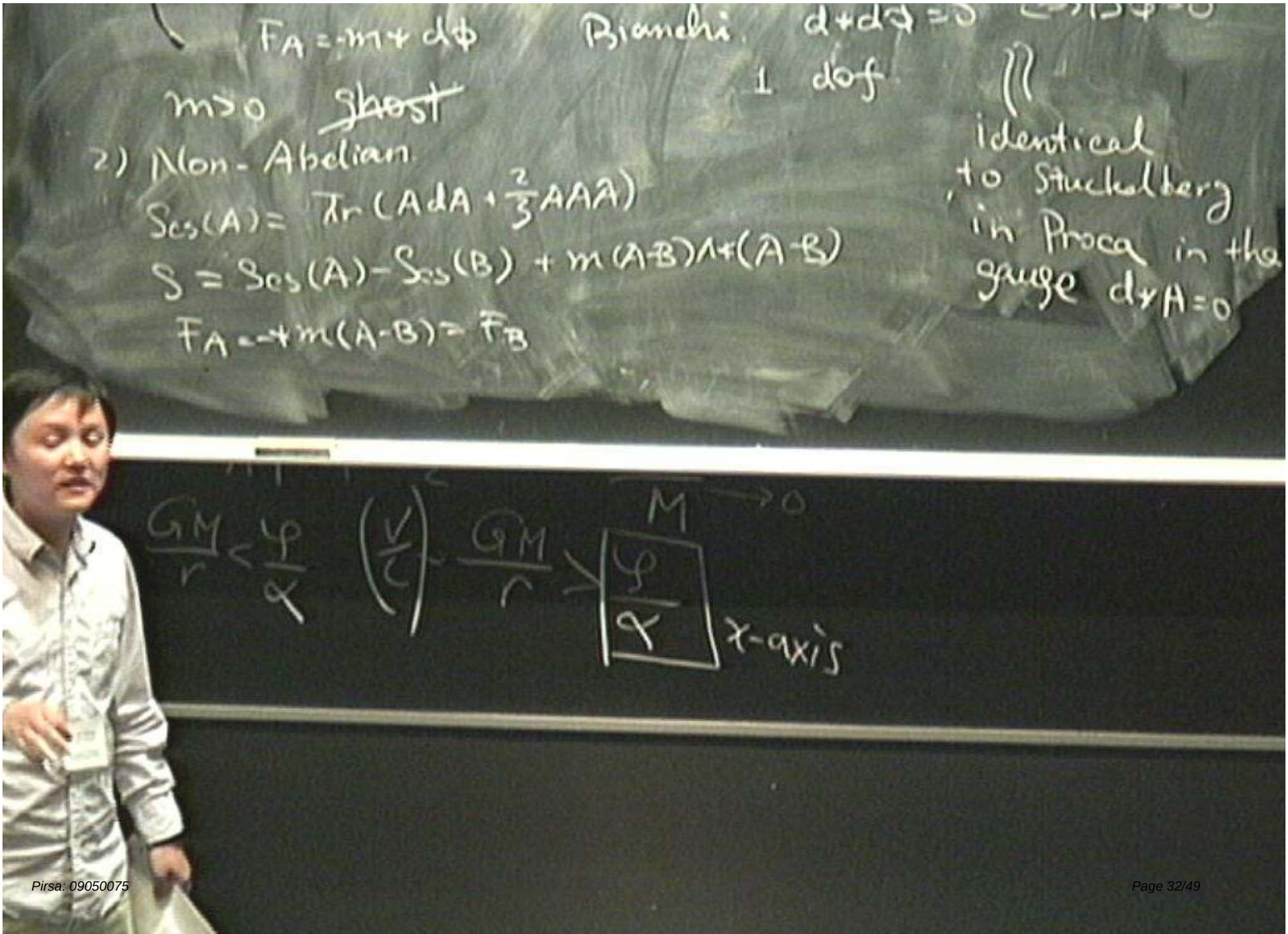
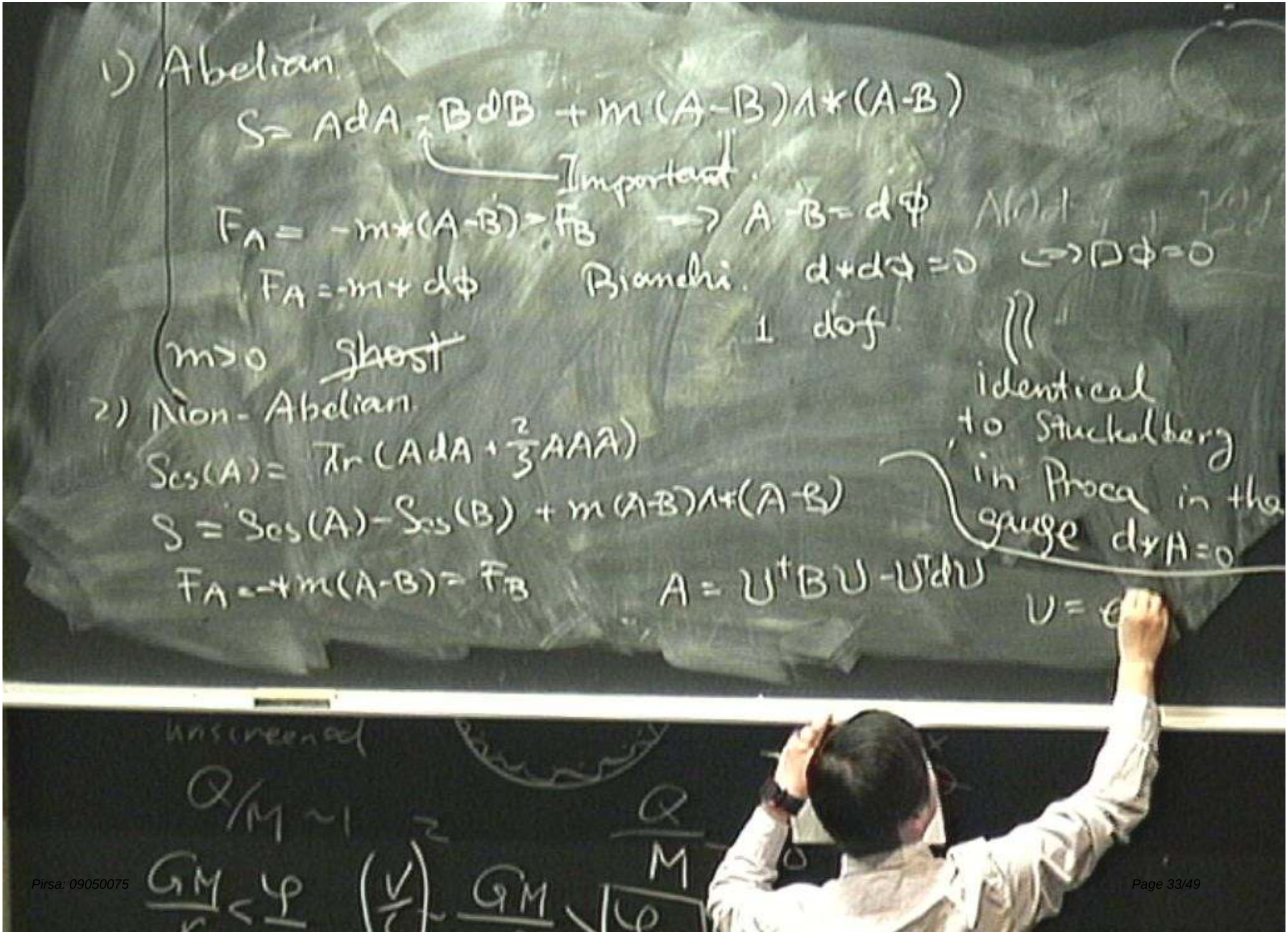
$m > 0$ ~~Ghost~~ 1 dof

$\Big\|$

identical to Stückelberg in Proca in the gauge $d*A = 0$


Important:
$F_A = -m*(A-B) = F_B \rightarrow A - B = d\phi$
$F_A = -m * d\phi$
Bianchi: $d*d\phi = 0 \hookrightarrow \Box\phi = 0$
1 dof
$m > 0$ ~~Ghost~~
2) Non-Abelian
$S_{CS}(A) = Tr(A dA + \frac{2}{3} AAA)$
$S = S_{CS}(A) - S_{CS}(B) + m$
identical to Stuckelberg in Proca in the gauge $d*A = 0$
x-axis


$F_A = -m * d\phi$
Bianchi
$d * d\phi = 0$
1 dof
$m > 0$ ~~ghost~~
2) Non-Abelian
$S_{cs}(A) = Tr(AdA + \frac{2}{3}AAA)$
$S = S_{cs}(A) - S_{cs}(B) + m(A-B) \wedge *(A-B)$
$F_A = -*m(A-B) = F_B$
identical to Stuckelberg in Proca in the gauge $d*A = 0$
$\frac{GM}{r} < \frac{\varphi}{\alpha}$
$\left(\frac{v}{c}\right)^2 \sim \frac{GM}{r} > \boxed{\frac{\varphi}{\alpha}}$
x-axis
$M \to 0$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$S_{CS}(A) = Tr(AdA + \frac{2}{3}AAA)$
$S = S_{CS}(A) - S_{CS}(B) + m(A-B) \wedge *(A-B)$
$F_A = -*m(A-B) = F_B$
$A = U^{\dagger}BU - U^{\dagger}dU$
identical to Stückelberg in Proca in the gauge $d*A = 0$
$U =$
unscreened
$Q/M \sim 1$
$\frac{GM}{r} < \Phi$
$\left(\frac{v}{c}\right)^2 \sim \frac{GM}{r} \sim \Phi$
$\frac{Q}{M}$

1) Abelian

$$S = AdA - BdB + m(A-B)\wedge *(A-B)$$

Important.

$$F_A = -m*(A-B) = F_B \quad \rightarrow \quad A - B = d\phi$$

$$F_A = -m*d\phi \qquad \text{Bianchi} \quad d*d\phi = 0 \quad \hookrightarrow \Box\phi = 0$$

$m > 0$ ~~Ghost~~ 1 dof

2) Non-Abelian

$$S_{CS}(A) = Tr\left(AdA + \tfrac{2}{3}AAA\right)$$

$$S = S_{CS}(A) - S_{CS}(B) + m(A-B)\wedge *(A-B)$$

$$F_A = -*m(A-B) = F_B \qquad A = U^\dagger B U - U^\dagger dU = U^\dagger D_B U$$

$U = e^{\pi^a t_a}$ t_a - Gauge ge

identical to Stuckelberg in Proca in the gauge $d*A = 0$

$$F_B = *(U^\dagger D_B U - B)$$

$F_B = *(U^\dagger D_B U - B)$ $\quad D_B F_B = 0$

$D_A * D U - D * B = 0$

$F_B = *(U^\dagger D_B U - B)$ $\quad D_B F_B = 0$

$D * D U - D * B = 0$ $\quad \Longleftrightarrow$ eom of the Stuckelberg in Proca, in the gauge $D * B = 0$

$F_B = *(U^\dagger D_B U - B)$

$D_\mu + DU - D*B = 0$

$\dim(G)$ dof.

$D_B F_B = 0$

$\Leftrightarrow$ eom of the Stuckelberg in Proca, in the gauge $D*B = 0$

$F_B = *(U^\dagger D_B U - B)$

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$D_\mu + DU - D*B = 0$

dim(G) dof.

Q: 1) ghost?

2) massive mode?

$F_B = *(U^* D_B U - B)$ $\quad D_B F_B = 0$

$D_0 * D U - D * B = 0$ $\quad \Leftrightarrow$ eom of the Stuckelberg in Proca, in the gauge $D * B = 0$

$\dim(G)$ dof.

Q: 1) ghost?

2) massive mode?

III) 3D GR with $\Lambda < 0$

$S = S_{CS}(A_+) - S_{CS}(A_-)$

$A_+ = (\omega^a + \frac{1}{l} e^a) T^a \qquad A_- = (\omega^a - \frac{1}{l} e^a) T^a \quad T^a \in$

$F_B = *(U^T D_B U - B)$ $\quad D_B F_B = 0$

$D_0 + DU - D*B = 0$ $\quad \Leftrightarrow$ eom of the Stuckelberg in Proca, in the gauge $D*B = 0$

dim(G) dof.

Q: 1) ghost?

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$S = S_{CS}(A_+) - S_{CS}(A_-)$

$A_+ = (\omega^a + \frac{1}{l} e^a) T^a$ $\quad A_- = (\omega^a - \frac{1}{l} e^a) T^a$ $\quad T^a \in sl(2,R)$

$F_B = +(U^t D_B U - B)$ $\quad D_B F_B = 0$

$D_0 + DU - D*B = 0$ $\quad \Leftrightarrow$ eom of the Stückelberg in Proca, in the gauge $D*B = 0$

dim(G) dof.

Q: 1) ghost?

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III) 3D GR with $\Lambda < 0$

$S = S_{CS}(A_+) - S_{CS}(A_-) + m\,(A_+ - A_-)\wedge *(A_+ - A_-)$

$A_+ = (\omega^a + \frac{1}{\ell} e^a) T^a$ $\quad A_- = (\omega^a - \frac{1}{\ell} e^a) T^a$ $\quad T^a \in sl(2,R)$

$\rightarrow \frac{2}{\ell} e^a T^a$

$F_B = *(U^* D_B U - B)$ $\quad D_B F_B = 0$

$D_0 + DU - D*B = 0$ $\quad \Leftrightarrow$ eom of the Stuckelberg in Proca, in the gauge $D*B = 0$

dim(G) def.

Q: 1) Ghost?

2) massive mode?

III) 3D GR with $\Lambda < 0$

$S = S_{CS}(A_+) - S_{CS}(A_-) + m\,(A_+ - A_-) \wedge *(A_+ - A_-)$

$A_+ = (\omega^a + \frac{1}{\ell} e^a) T^a$ $\quad$ $A_- = (\omega^a - \frac{1}{\ell} e^a) T^a$ $\quad$ $T^a \in sl(2,R)$

$\hookrightarrow \frac{2}{\ell} e^a T^a$

"*"

$F_B = *(U^* D_B U - B)$ $\quad D_B F_B = 0$

$D_0 + DU - D*B = 0$ $\quad \Leftrightarrow$ eom of the Stückelberg in Proca, in the gauge $D*B = 0$

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III) 3D GR with $\Lambda < 0$

$S = S_{CS}(A_+) - S_{CS}(A_-) + m\,(A_+ - A_-) \wedge *(A_+ - A_-)$

$A_+ = (\omega^a + \frac{1}{\ell} e^a) T^a$ $\quad$ $A_- = (\omega^a - \frac{1}{\ell} e^a) T^a$ $\quad$ $T^a \in sl(2,R)$

$\rightarrow \frac{2}{\ell} e^a T^a$

"*" $\quad g_{\mu\nu} = e^a{}_\mu e^b{}_\nu \eta_{ab}$

$F_B = *(U^{-1} D_B U - B)$ $\quad D_B F_B = 0$

$D_* * D U - D * B = 0 \iff$ eom of the Stuckelberg in Proca, in the gauge $D * B = 0$

dim(G) dof.

Q: 1) ghost?

2) massive mode?

III) 3D GR with $\Lambda < 0$

$S = S_{CS}(A_+) - S_{CS}(A_-) + m\,(A_+ - A_-) \wedge *(A_+ - A_-)$

$A_+ = (\omega^a + \frac{1}{\ell} e^a) T^a$ $\quad A_- = (\omega^a - \frac{1}{\ell} e^a) T^a$ $\quad T^a \in sl(2,\mathbb{R})$

$\to \frac{2}{\ell} e^a T^a$

"*" $\quad g_{\mu\nu} = e^a{}_\mu e^b{}_\nu \eta_{ab}$

↳ choose a background. $\gamma_{\mu\nu} = \bar{e}^a{}_\mu \bar{e}^b{}_\nu \eta_{ab}$

$F_B = *(U^T D_B U - B)$ $\quad D_B F_B = 0$

$D_+ + D U - D * B = 0$ $\quad \Leftrightarrow$ eom of the Stucked berg in Proca, in the gauge $D * B = 0$

$\dim(G)$ dof.

Q: 1) ghost?

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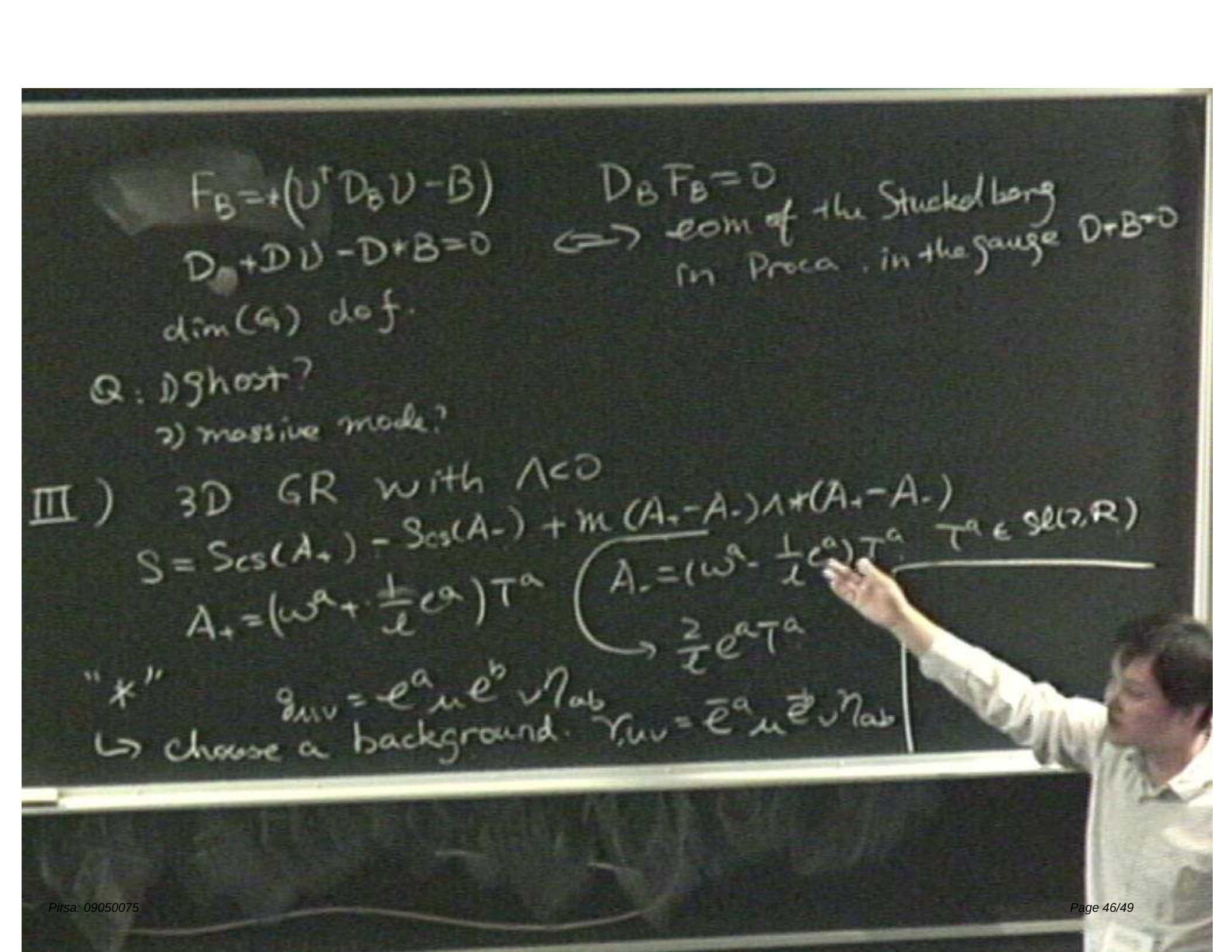
$\hookrightarrow \frac{2}{\ell} e^a T^a$

"*" $\quad g_{\mu\nu} = e^a{}_\mu e^b{}_\nu \eta_{ab}$ $\quad \gamma_{\mu\nu} = \bar{e}^a{}_\mu \bar{e}^b{}_\nu \eta_{ab}$

$\hookrightarrow$ choose a background.



$F_B = +(U^\dagger D_B U - B)$ $\quad D_B F_B = 0$

$D_0 + DU - D^\dagger B = 0$ $\quad \Leftrightarrow$ eom of the Stückelberg in Proca, in the gauge $D^\dagger B = 0$

$\dim(G)$ dof.

Q. 1) ghost?

2) massive mode?

III) 3D GR with $\Lambda < 0$

$S = S_{CS}(A_+) - S_{CS}(A_-) + m(A_+ - A_-)$

$A_+ = (\omega^a + \frac{1}{\ell} e^a) T^a$ $\quad A_- = (\omega^a - \frac{1}{\ell} e^a) T^a$

$\to \frac{2}{\ell} e^a T^a$

$T^a \in sl(2,R)$

$\frac{m}{\ell}(h_{\mu\nu} - \frac{1}{2} h^2)$

"*"

$g_{\mu\nu} = e^a{}_\mu e^b{}_\nu \eta_{ab}$ $\quad \bar{g}_{\mu\nu} = \bar{e}^a{}_\mu \bar{e}^b{}_\nu \eta_{ab}$

"$\to$ choose a background.

$F_B = +(U^\dagger D_B U - B)$ $\quad D_B F_B = 0$

$D_0 + DU - D^\dagger B = 0$ $\quad \Leftrightarrow$ eom of the Stuckelberg in Proca, in the gauge $D^\dagger B = 0$

$\dim(G)$ dof.

Q: 1) ghost?

2) massive mode?

III) 3D GR with $\Lambda < 0$

$S = S_{CS}(A_+) - S_{CS}(A_-) + m(A_+ - A_-)\Lambda + (A_+ - A_-)$

$A_+ = (\omega^a + \frac{1}{\ell} e^a) T^a$ $\quad A_- = (\omega^a - \frac{1}{\ell} e^a) T^a$

$\hookrightarrow \frac{2}{\ell} e^a T^a$ $\quad T^a \in sl(2,R)$

"$*$" $\quad g_{\mu\nu} = e^a{}_\mu e^b{}_\nu \eta_{ab}$ $\quad \gamma_{\mu\nu} = \bar{e}^a{}_\mu \bar{e}^b{}_\nu \eta_{ab}$

$\hookrightarrow$ choose a background.

$\frac{m}{\ell}(h_{\mu\nu} - \frac{1}{2} h \gamma_{\mu\nu})$

Screwsmy story

"If Only" $e^{\mu}{}_a \delta e_{\mu b} = e^{\mu}{}_b \delta e_{\mu a}$ ← "Wishful thinking"

$h_{ab} = 2\delta e_{ab}$

II Chern-Simons-Proca

3DGR $\xrightarrow{\text{Classically}}$ CS

$AdA + \frac{2}{3}AAA + mA\wedge *A$

1) What if: 2 copies of CS

$R_{\mu\nu}^2 - \frac{3}{8}R^2$

$R_{\mu\nu}^2 - \frac{1}{4}R^2$