Title: Massive gravity in 3-D and the Chern-Simons-Proca theory

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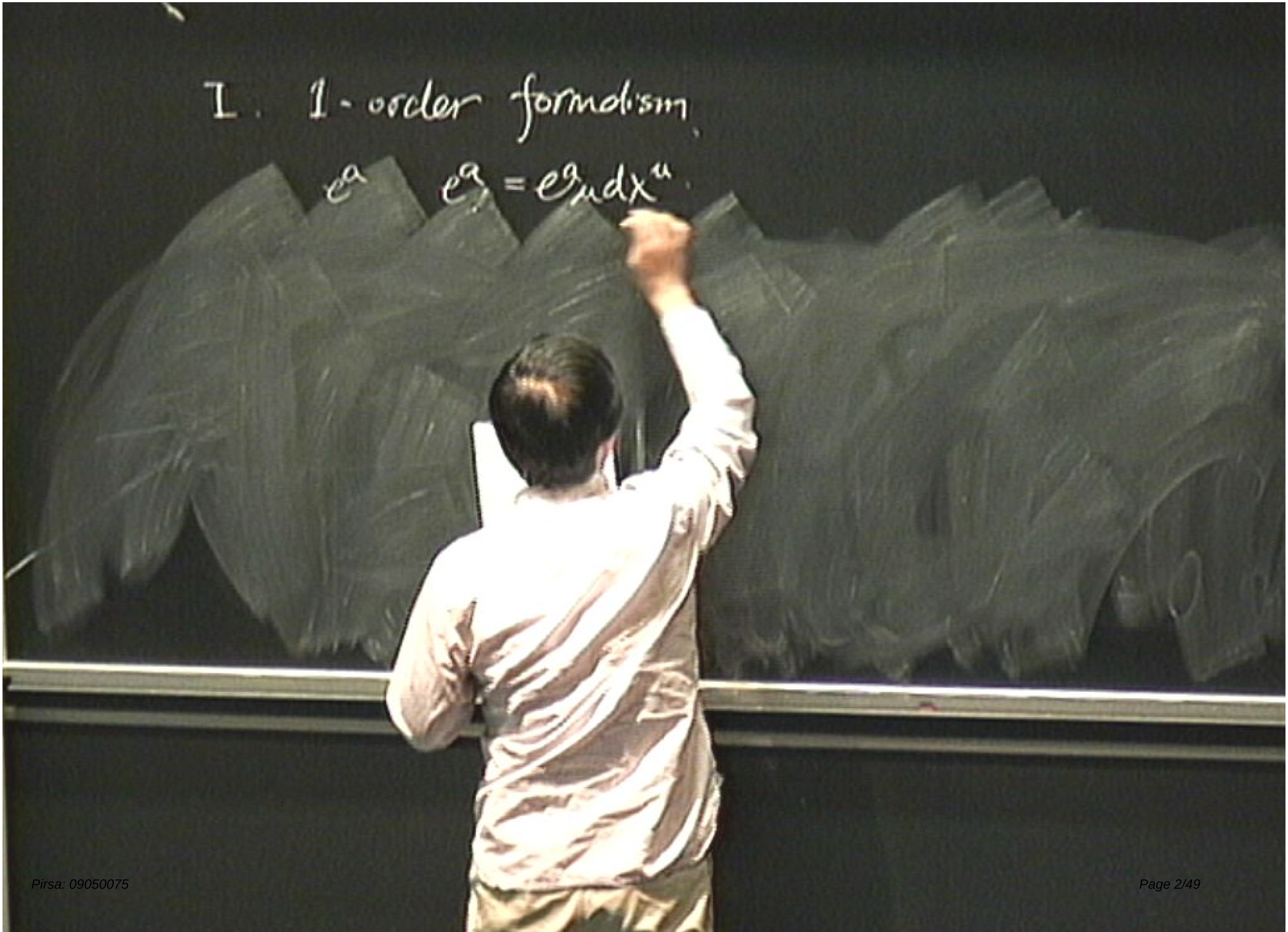
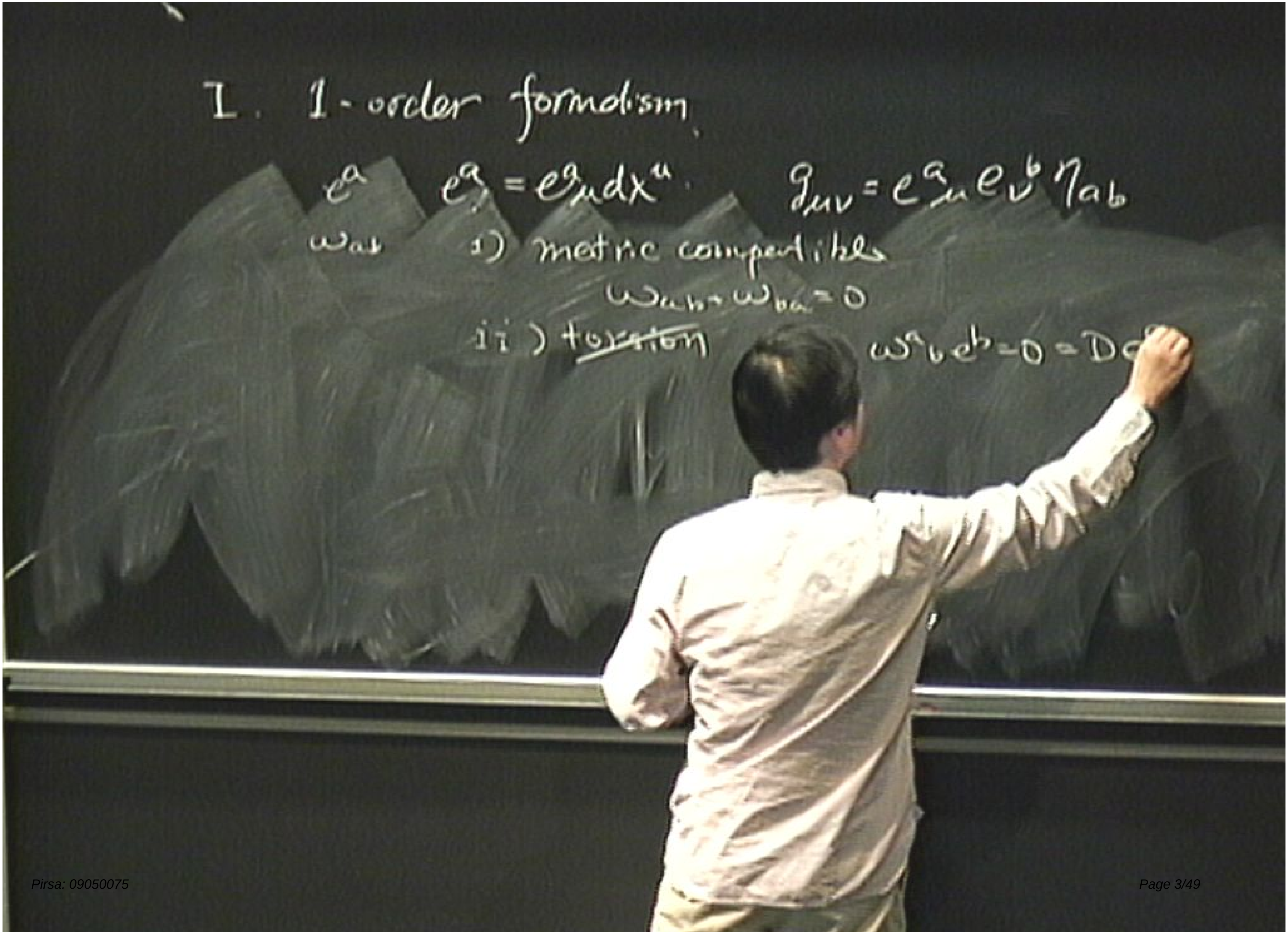
Abstract: TBA


I. 1-order formalism
$e^a \quad e^a = e^a_\mu dx^\mu$


I. 1-order formalism
e^a $e^a = e^a_\mu dx^\mu$. $g_{\mu\nu} = e^a_\mu e^b_\nu \eta_{ab}$
ω_{ab} i) metric compatible
$\omega_{ab} + \omega_{ba} = 0$
ii) ~~torsion~~ $\omega^a{}_b e^b = 0 = De^a$

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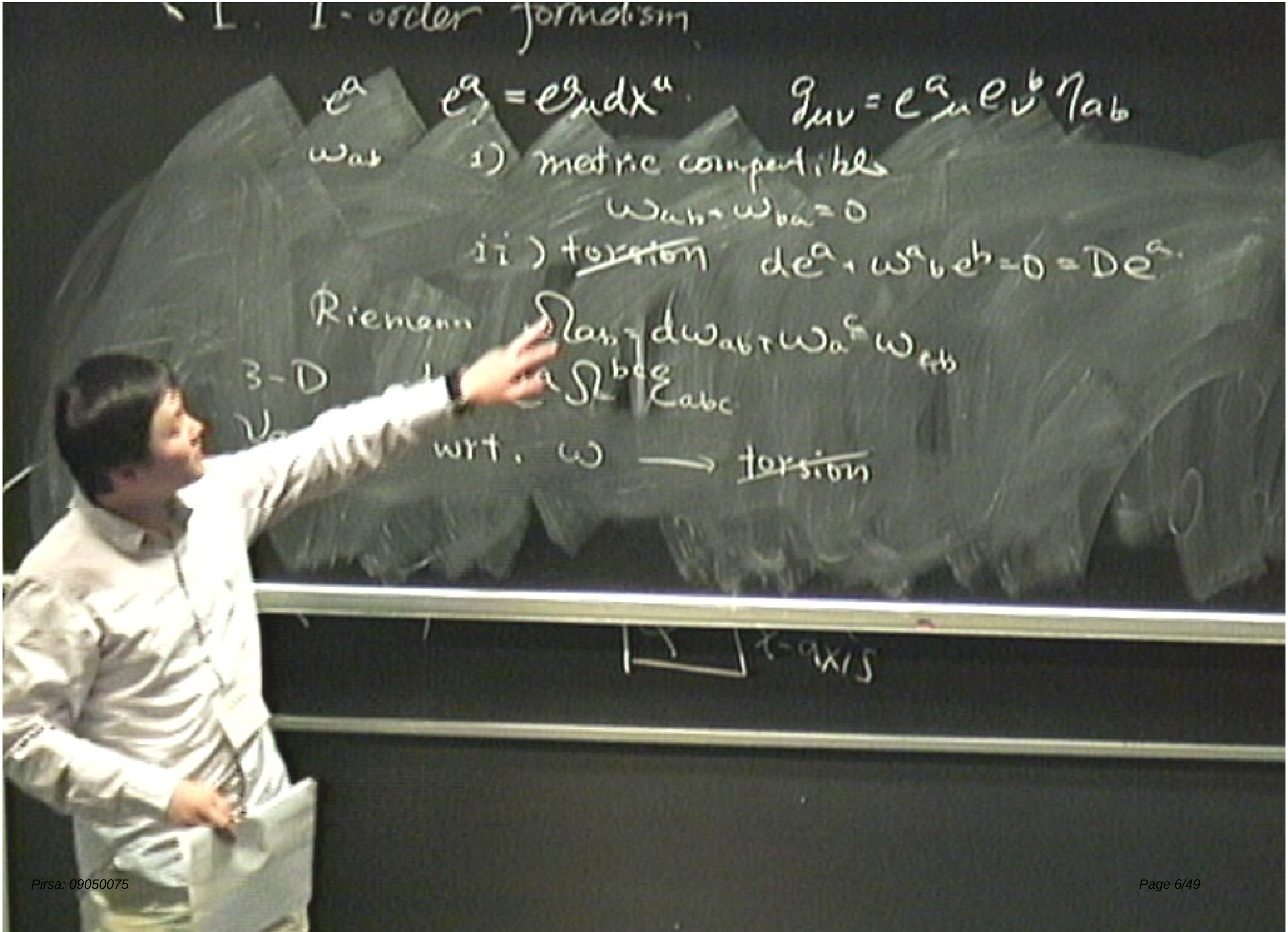
Riemann $\Omega_{ab} = d\omega_{ab} + \omega_a{}^c \omega_{cb}$

t-axis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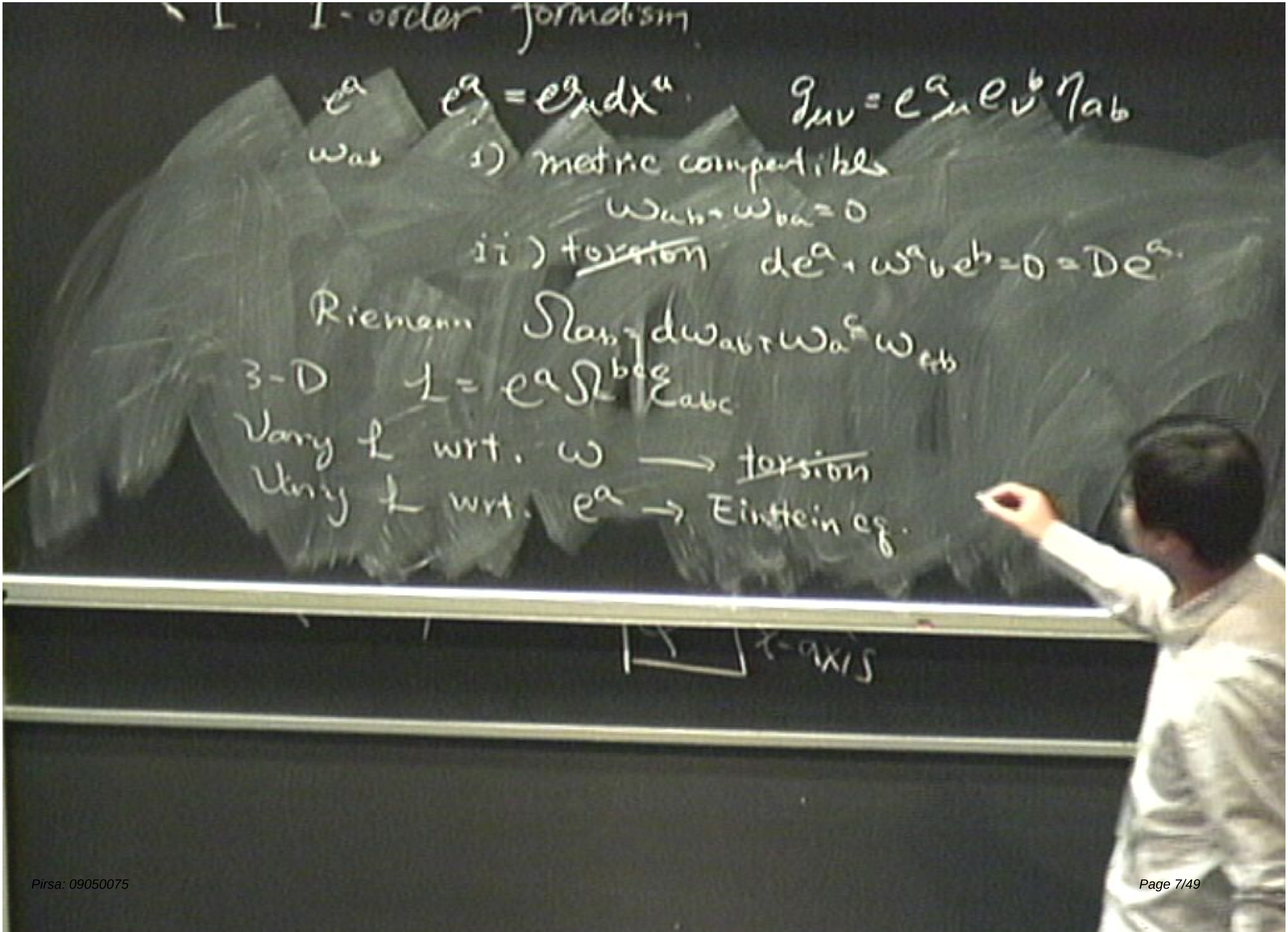
$e^a \Omega^{bc} \epsilon_{abc}$
wrt. $\omega \longrightarrow$ ~~torsion~~
z-axis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ii) torsion   de^a + ω^a_b e^b = 0 = De^a
Riemann   Ω_ab = dω_ab + ω_a^c ω_cb
3-D   L = e^a Ω^bc ε_abc
Vary L wrt. ω → torsion
Vary L wrt. e^a → Einstein eq.
t-axis

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3-D $\mathcal{L} = e^a \Omega^{bc} \varepsilon_{abc} + \Lambda e^a e^b e^c \varepsilon_{abc}$

Vary $\mathcal{L}$ wrt. $\omega \longrightarrow$ ~~torsion~~

Vary $\mathcal{L}$ wrt. $e^a \rightarrow$ Einstein eq.

t-axis

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Vary $\mathcal{L}$ wrt. $e^a \rightarrow$ Einstein eq.

$$\Omega_{ab} + 2\Lambda e_a e_b = 0$$

3-D $$\Omega_{ab} = 2(R_a e_b - R_b e_a) - R e_a e_b$$

$$R_a = R_{ab} e^b \qquad R = R_{ab}\eta^{ab}$$

$$\Omega_{ab} + 2\Lambda e_a e_b = 0$$

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$$R_a = R_{ab} e^b \qquad R = R_{ab}\eta^{ab}$$

$$G_a = G_{ab} e^b \qquad G = G_{ab}\eta^{ab}$$

$$\Omega_{ab} = 2(G_a e_b - G_b e_a - G e_a e_b)$$

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$\underline{G_a - \Lambda e_a = 0}$ ← Eins

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2) Pauli-Fierz

$m_g^2 (h_{\mu\nu} - h\,\gamma_{\mu\nu})$

$\gamma_{\mu\nu}$ background metric

$h_{\mu\nu} = g_{\mu\nu} - \gamma_{\mu\nu}$

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$G_{\mu\nu}(\gamma_{\mu\nu}) - \Lambda \gamma_{\mu\nu} = 0$

$h = h_{\mu\nu}\gamma^{\mu\nu}$

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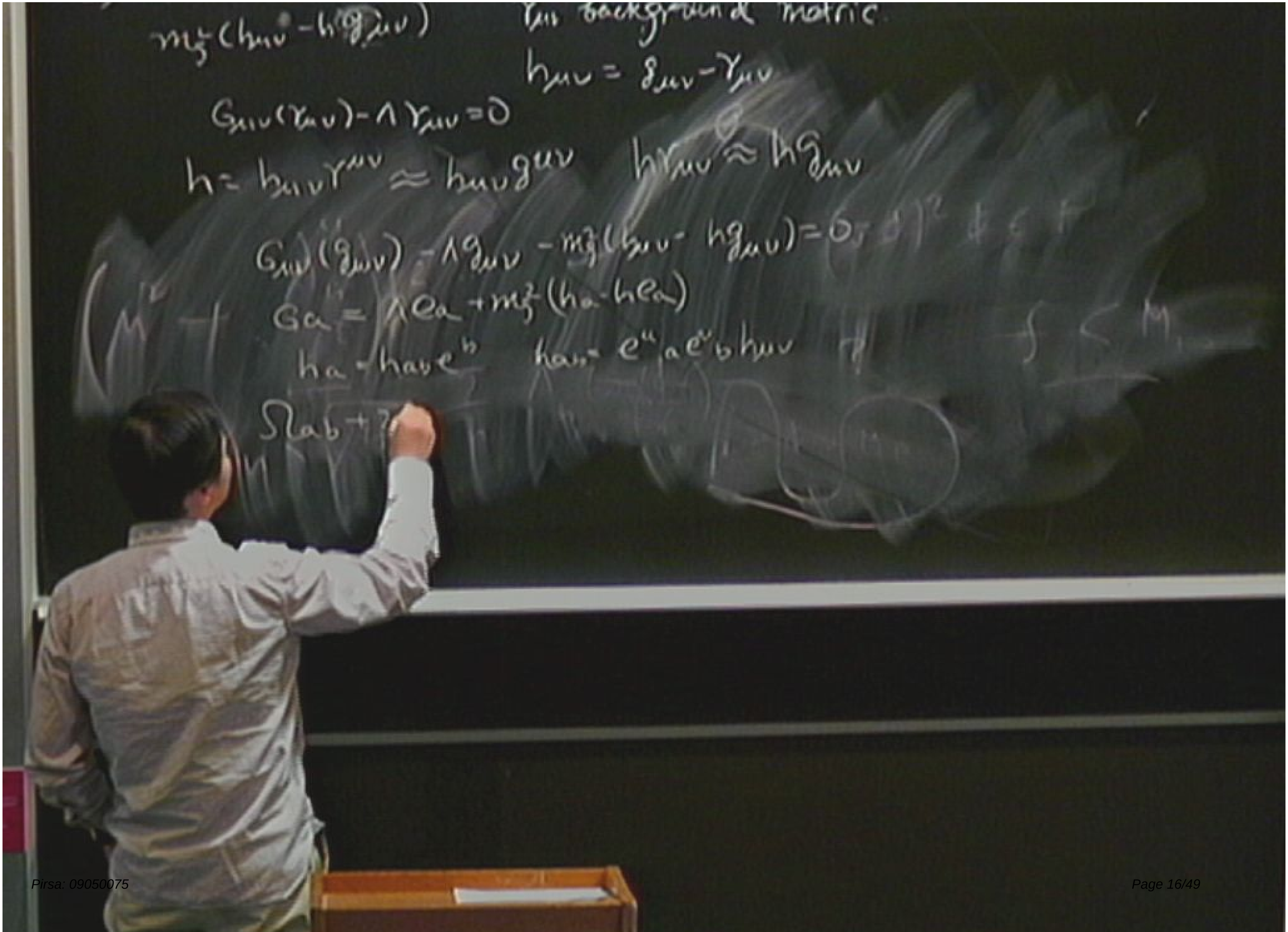
$$G_{\mu\nu}(\gamma_{\mu\nu}) - \Lambda \gamma_{\mu\nu} = 0$$

$$h = h_{\mu\nu}\gamma^{\mu\nu} \approx h_{\mu\nu} g^{\mu\nu} \qquad h\gamma_{\mu\nu} \approx h g_{\mu\nu}$$

$$G_{\mu\nu}(g_{\mu\nu}) - \Lambda g_{\mu\nu} - m_g^2(h_{\mu\nu} - h g_{\mu\nu}) = 0$$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$G_{\mu\nu}(g_{\mu\nu}) - \Lambda g_{\mu\nu} - m_g^2(h_{\mu\nu} - h g_{\mu\nu}) = 0$
$G_a = \Lambda e_a + m_g^2(h_a - h e_a)$
$h_a = h_{a\nu}e^{\nu}$
$h_{a\nu} = e^{\mu}{}_a e^{\nu}{}_b h_{\mu\nu}$
$\Omega_{ab} + ?$

$m_\xi^2 (h_{\mu\nu} - h g_{\mu\nu})$

$\gamma_{\mu\nu}$ background metric

$h_{\mu\nu} = g_{\mu\nu} - \gamma_{\mu\nu}$

$G_{\mu\nu}(\gamma_{\mu\nu}) - \Lambda \gamma_{\mu\nu} = 0$

$h = h_{\mu\nu}\gamma^{\mu\nu} \approx h_{\mu\nu} g^{\mu\nu} \quad h\gamma_{\mu\nu} \approx h g_{\mu\nu}$

$G_{\mu\nu}(g_{\mu\nu}) - \Lambda g_{\mu\nu} - m_\xi^2 (h_{\mu\nu} - h g_{\mu\nu}) = 0$

$G_a = \Lambda e_a + m_\xi^2 (h_a - h e_a)$

$h_a = h_{ab} e^b \quad h_{ab} = e^\mu{}_a e^\nu{}_b h_{\mu\nu}$

$\Omega_{ab} + 2\Lambda e_a e_b + m_\xi^2 (h_a e_b - h_b e_a) = 0$

$h_a = \delta e_a + \ldots$

Not nice.

"If Only" $e^{\mu}{}_a \delta e_{\mu b} = e^{\mu}{}_b \delta e_{\mu a}$ ← "Wishful thinking"

$h_a = 2\delta e_a$.

$h_a = \underline{\delta e_a} + \underline{e^{\mu}{}_a \,\delta e_{\mu a}}$

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$\underline{h_a = 2\,\delta e_a}$

$h_a = \underline{\delta e_a} + \underline{e^{\mu}{}_{a}\,\delta e_{\mu b}\ldots}$

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$h_a = 2\,\delta e_a$

II Chern - Simons - Proca

$h_a = \underline{\delta e_a} + \underline{e \; a \; \delta e \ldots}$

Not nice.

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$h_a = 2 \delta e_a$

II Chern - Simons - Proca

3DGR $\xleftrightarrow{\text{Classically}}$ CS

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II Chern-Simons-Proca

3DGR $\xleftrightarrow{\text{Classically}}$ CS

$A dA + \frac{2}{3} AAA + m A \wedge * A$

$h_a = \underline{\delta e_a} + \ldots \underline{\qquad}$

Not nice.

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II Chern - Simons - Proca

3DGR $\xleftrightarrow{\text{Classically}}$ CS

$A\,dA + \frac{2}{3} AAA + m A \wedge {*A}$

1) What if : 2 copies of CS

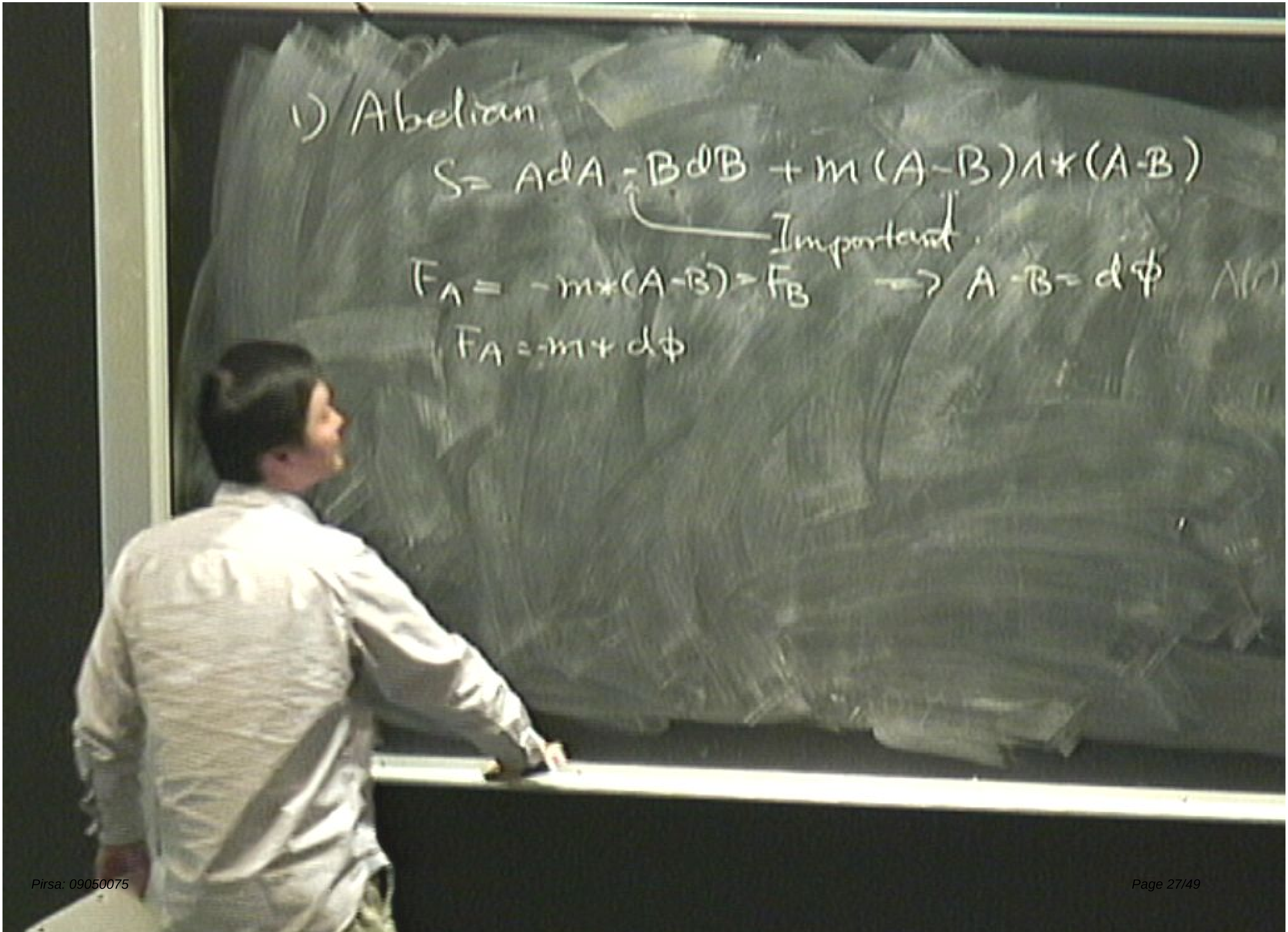

1) Abelian
S= AdA - BdB

1) Abelian

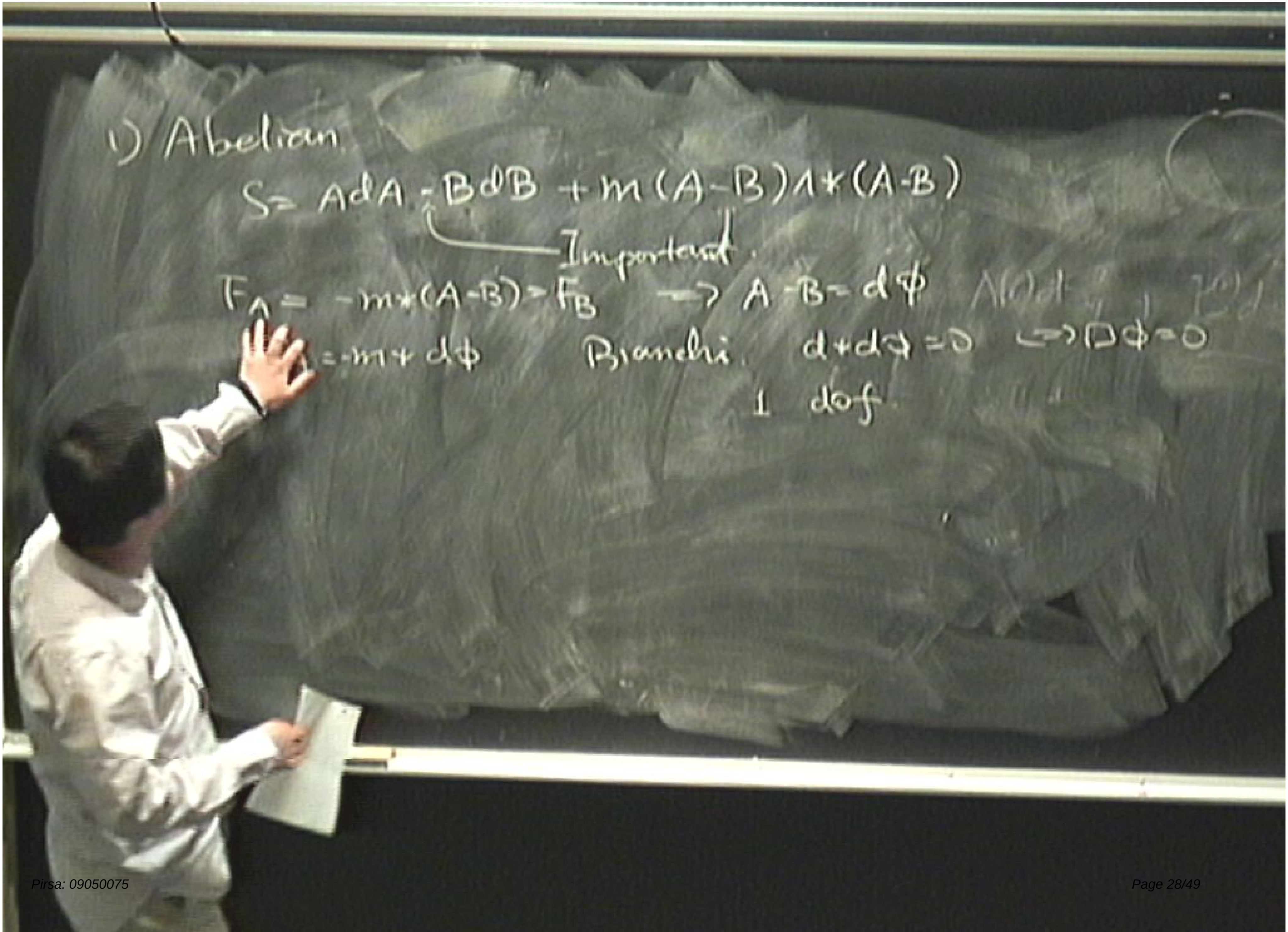
$$S = A\,dA - B\,dB + m\,(A-B)\wedge *(A-B)$$

Important

$$F_A = -m*(A-B) = F_B$$



1) Abelian
$S = AdA - BdB + m(A-B)\wedge *(A-B)$
Important
$F_A = -m*(A-B) = F_B \longrightarrow A-B = d\phi$
$F_A = -m * d\phi$


1) Abelian
S = AdA - BdB + m(A-B)∧*(A-B)
Important
F_A = -m*(A-B) = F_B → A-B = dφ
= -m*dφ
Bianchi d*dφ = 0 → □φ = 0
1 dof

1) Abelian

$$S = A\,dA - B\,dB + m(A-B)\wedge *(A-B)$$

Important

$$F_A = -m*(A-B) = F_B \longrightarrow A - B = d\phi$$

$F_A = -m * d\phi$ Bianchi $d*d\phi = 0 \Longrightarrow \Box\phi = 0$

1 dof

$\parallel$

identical to Stuckelberg in Proca in the gauge $d*A=0$

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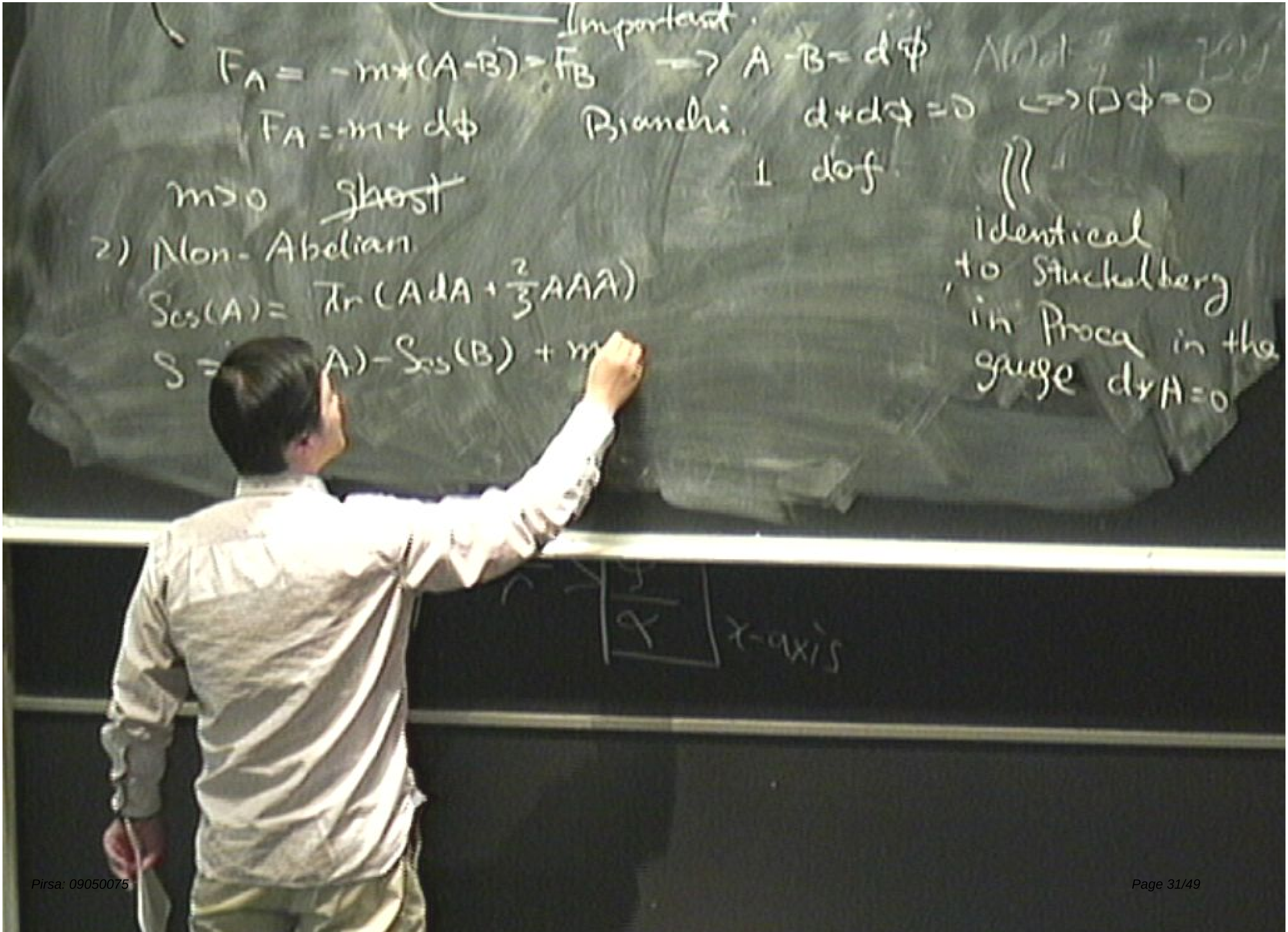
$$F_A = -m*(A-B) = F_B \longrightarrow A - B = d\phi$$

$$F_A = -m * d\phi \qquad \text{Bianchi} \quad d*d\phi = 0 \Longrightarrow \Box\phi = 0$$

$m>0$ ~~ghost~~ 1 dof

identical to Stuckelberg in Proca in the gauge $d*A=0$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F_A = -m*(A-B) = F_B ⟹ A - B = dφ
F_A = -m*dφ   Bianchi d*dφ = 0 ⟺ □φ = 0
1 dof
m>0 ghost
identical to Stuckelberg in Proca in the gauge d*A=0
2) Non-Abelian
S_CS(A) = Tr(AdA + 2/3 AAA)
S = (A) - S_CS(B) + m
x-axis

$F_A = -m * d\phi$ Bianchi $d * d\phi = 0 \Leftrightarrow \Box\phi = 0$

$m > 0$ ~~ghost~~

1 dof

identical to Stuckelberg in Proca in the gauge $d * A = 0$

2) Non-Abelian

$S_{cs}(A) = \mathrm{Tr}(A\,dA + \frac{2}{3} AAA)$

$S = S_{cs}(A) - S_{cs}(B) + m(A-B) \wedge *(A-B)$

$F_A = -*m(A-B) = F_B$

$\frac{GM}{r} < \frac{\varphi}{\alpha}$

$\left(\frac{v}{c}\right)^2 \sim \frac{GM}{r} > \boxed{\frac{\varphi}{\alpha}}$ x-axis

$M \to 0$

1) Abelian

$$S = A\,dA - B\,dB + m(A-B)\wedge *(A-B)$$

Important

$$F_A = -m*(A-B) = F_B \quad \longrightarrow \quad A - B = d\phi$$

$$F_A = -m*d\phi \qquad \text{Bianchi} \quad d*d\phi = 0 \iff \Box\phi = 0$$

1 dof

$m > 0$ ~~ghost~~

$\wr\wr$ identical to Stuckelberg in Proca in the gauge $d*A=0$

2) Non-Abelian

$$S_{cs}(A) = \mathrm{Tr}\left(A\,dA + \tfrac{2}{3}AAA\right)$$

$$S = S_{cs}(A) - S_{cs}(B) + m(A-B)\wedge *(A-B)$$

$$F_A = -*m(A-B) = F_B \qquad A = U^{\dagger}BU - U^{\dagger}dU \qquad U = e$$

unscreened

$Q/M \sim 1$

$\frac{GM}{r} < \varphi$

1) Abelian

$$S = AdA - BdB + m(A-B)\wedge *(A-B)$$

Important.

$F_A = -m*(A-B) = F_B \quad \Longrightarrow \quad A - B = d\phi$

$F_A = -m * d\phi \qquad$ Bianchi. $\quad d*d\phi = 0 \quad \Longleftrightarrow \Box\phi = 0$

1 dof. $\quad \parallel$

$m > 0 \quad$ ~~ghost~~

identical to Stuckelberg in Proca in the gauge $d*A = 0$

2) Non-Abelian

$S_{cs}(A) = \mathrm{Tr}\,(AdA + \frac{2}{3}AAA)$

$S = S_{cs}(A) - S_{cs}(B) + m(A-B)\wedge *(A-B)$

$F_A = -*m(A-B) = F_B \qquad A = U^{+}BU - U^{+}dU$

$= U^{+}D_B U \qquad U = e^{\pi_a t_a}$

t_a - Gauge ge

$$F_B = -i\left(U^\dagger D_B U - B\right)$$

$$F_B = \star\left(U^\dagger D_B U - B\right) \qquad D_B F_B = 0$$

$$D \star D U - D \star B = 0$$

$$F_B = *(U^\dagger D_B U - B) \qquad D_B F_B = 0$$

$$D * D U - D * B = 0$$

$\Longleftrightarrow$ eom of the Stuckelberg in Proca, in the gauge $D*B=0$

$F_B = *(U^\dagger D_B U - B)$ $\quad$ $D_B F_B = 0$

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dim(G) d.o.f.

$$F_B = *\left(U^\dagger D_B U - B\right) \qquad D_B F_B = 0$$

$$D_B * D U - D * B = 0 \iff$$ eom of the Stuckelberg in Proca, in the gauge $D*B=0$

dim(G) d.o.f.

Q: 1) ghost?

2) massive mode?

$F_B = *(U^{\dagger} D_B U - B)$ $\qquad$ $D_B F_B = 0$

eom of the Stuckelberg

$D_* * DU - D * B = 0$ $\Longleftrightarrow$ in Proca, in the gauge $D * B = 0$

dim(G) dof.

Q: 1) ghost?

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III) 3D GR with $\Lambda < 0$

$S = S_{CS}(A_+) - S_{CS}(A_-)$

$A_+ = (\omega^a + \frac{1}{l} e^a) T^a$ $\qquad$ $A_- = (\omega^a - \frac{1}{l} e^a) T^a$ $\qquad$ $T^a \in$

$F_B = *(U^\dagger D_B U - B)$ $\quad$ $D_B F_B = 0$

$D * D U - D * B = 0$ $\quad\Longleftrightarrow\quad$ eom of the Stuckelberg in Proca, in the gauge $D * B = 0$

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$A_+ = (\omega^a + \frac{1}{\ell} e^a) T^a$ $\quad$ $A_- = (\omega^a - \frac{1}{\ell} e^a) T^a$ $\quad$ $T^a \in sl(2,R)$

$F_B = *(U^\dagger D_B U - B)$ $\quad D_B F_B = 0$

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III) 3D GR with $\Lambda < 0$

$S = S_{CS}(A_+) - S_{CS}(A_-) + m\,(A_+ - A_-) \wedge *(A_+ - A_-)$

$A_+ = (\omega^a + \frac{1}{\ell} e^a) T^a$ $\quad A_- = (\omega^a - \frac{1}{\ell} e^a) T^a$ $\quad T^a \in sl(2,R)$

$\to \frac{2}{\ell} e^a T^a$

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$S = S_{CS}(A_+) - S_{CS}(A_-) + m\,(A_+ - A_-) \wedge *(A_+ - A_-)$ $\quad T^a \in sl(2,\mathbb{R})$

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$\frac{2}{\ell} e^a T^a$

"$*$"

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"$*$" $\quad g_{\mu\nu} = e^a{}_\mu e^b{}_\nu \eta_{ab}$

$F_B = *(D^{\ }D_B D - B)$ $\quad D_B F_B = 0$

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$$S = S_{CS}(A_+) - S_{CS}(A_-) + m\,(A_+ - A_-) \wedge *(A_+ - A_-)$$

$$A_+ = (\omega^a + \tfrac{1}{\ell} e^a)\,T^a \qquad A_- = (\omega^a - \tfrac{1}{\ell} e^a)\,T^a \qquad T^a \in sl(2,R)$$

$$\to \tfrac{2}{\ell} e^a T^a$$

"$*$" $\quad g_{\mu\nu} = e^a{}_\mu e^b{}_\nu \eta_{ab}$

$\hookrightarrow$ choose a background. $\gamma_{\mu\nu} = \bar{e}^a{}_\mu \bar{e}^b{}_\nu \eta_{ab}$

$F_B = *(U^\dagger D_B U - B)$ $D_B F_B = 0$

$D_* * D U - D * B = 0$ $\Longleftrightarrow$ eom of the Stuckelberg in Proca, in the gauge $D*B=0$

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$S = S_{cs}(A_+) - S_{cs}(A_-) + m\,(A_+ - A_-)\wedge *(A_+ - A_-)$ $T^a \in sl(2,R)$

$A_+ = (\omega^a + \frac{1}{\ell} e^a)\,T^a$ $A_- = (\omega^a - \frac{1}{\ell} e^a)\,T^a$

$\frac{2}{\ell} e^a T^a$

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$F_B = *(U^{\dagger} D_B U - B)$ $\quad D_B F_B = 0$

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$S = S_{CS}(A_+) - S_{CS}(A_-) + m\,(A_+ - A_-) \wedge *(A_+ - A_-)$ $\quad T^a \in sl(2,R)$

$A_+ = (\omega^a + \frac{1}{\ell} e^a) T^a$ $\quad A_- = (\omega^a - \frac{1}{\ell} e^a) T^a$

$\frac{2}{\ell} e^a T^a$

$\frac{m}{\ell}(h_{\mu\nu}^2 - \frac{1}{2} h^2)$

"$*$" $\quad g_{\mu\nu} = e^a{}_\mu e^b{}_\nu \eta_{ab}$

$\rightarrow$ choose a background. $\gamma_{\mu\nu} = \bar{e}^a{}_\mu \bar{e}^b{}_\nu \eta_{ab}$

$F_B = *(U^{\dagger} D_B U - B) \qquad D_B F_B = 0$

$\Longleftrightarrow$ eom of the Stuckelberg in Proca, in the gauge $D*B=0$

$D_{\cdot} * D U - D * B = 0$

dim(G) d.o.f.

Q: 1) ghost?

2) massive mode?

III) 3D GR with $\Lambda < 0$

$S = S_{cs}(A_+) - S_{cs}(A_-) + m\,(A_+ - A_-) \wedge *(A_+ - A_-) \qquad T^a \in sl(2,\mathbb{R})$

$A_+ = (\omega^a + \frac{1}{\ell} e^a) T^a \qquad A_- = (\omega^a - \frac{1}{\ell} e^a) T^a \to \frac{2}{\ell} e^a T^a$

$\frac{m}{\ell} (h_{\mu\nu}^2 - \frac{1}{2} h^2)$

Screwsmy story

"*" $\quad g_{\mu\nu} = e^a{}_\mu e^b{}_\nu \eta_{ab}$

$\hookrightarrow$ choose a background. $\gamma_{\mu\nu} = \bar{e}^a{}_\mu \bar{e}_\nu \eta_{ab}$

"If Only" $e^{\mu}{}_{a}\,\delta e_{\mu b} = e^{\mu}{}_{b}\,\delta e_{\mu a}$ $\Leftarrow$ "Wishful thinking"

$h_{a} = 2\,\delta e_{a}$.

II Chern – Simons – Proca

3D GR $\xleftrightarrow{\text{Classically}}$ CS

$A\,dA + \frac{2}{3} A \wedge A \wedge A + m A \wedge * A$

1) What if: 2 copies of CS

$R_{\mu\nu}^{2} - \frac{3}{8} R^{2}$

$R_{\mu\nu}^{2} - \frac{1}{4} R^{2}$