Title: Inflation on a codimension-two brane

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Abstract: We consider a codimension-2 brane inflation scenario in a warped flux compactification of 6D gauged supergravity. The volume modulus of the model is stabilized by means of potentials localized on the regularized background branes. We discuss the cosmological evolution of the world-volume of a probe codimension-2 brane when it moves along the radial direction of the internal space. In order to have slow-roll inflation, we find that the warping of the internal space is required to be weak, in contrast to the string inflation constructions with strong warping. We discuss the parameter range that the inflation is in agreement with the observationally inferred parameters and which furthermore is consistent with the probe brane approximation. We argue that from a multi-brane solution, the backreaction of the probe brane on the weak warp factor is ignorable

Inflation on a codimension-two brane

Hyun Min Lee

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based on

A. Papazoglou & HML, arXiv:0901.4962 [hep-th]

New Prospects for CCP, Perimeter Institute, May 25, 2009

Outline

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1. Motivation
2. 6D supergravity
3. The probe brane inflation
4. Backreaction issues - Conclusion

Cosmological constant problem

- Cosmological observations show [Perlmutter et al; Riess et al(1997); Bahcall et al(1999); Spergel et al(2003)]

$$|\rho_\Lambda^{(\mathrm{obs})}| \sim (10^{-3} eV)^4 \sim 10^{-120} M_P^4. \qquad (1)$$

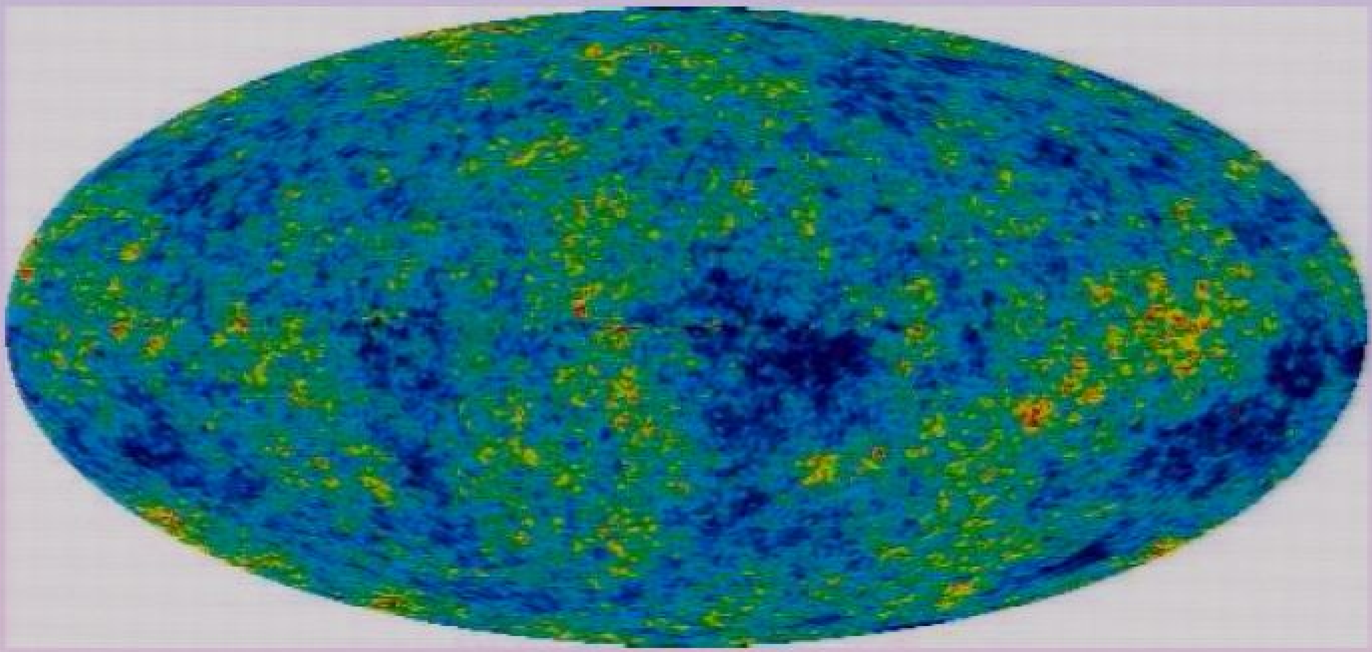
However, there is no known symmetry which could enforce a vanishing vacuum energy and remain consistent with other observations. One needs an enormous fine-tuning between all contributions to the vacuum energy. [Weinberg(1989); Carroll(2000)]

- Why is the cosmological constant comparable to the present energy density?

$$\Omega_\Lambda = 0.75, \quad \Omega_m = 0.25, \qquad (2)$$

with $\Omega_i \equiv \rho_i/\rho_c$ and ρ_c the critical energy density.

Cosmic Microwave Background



- Basic observed quantities

$$\delta_H = (1.91 \pm 0.17) \cdot 10^{-5}, \quad n_s = 0.960 \pm 0.013$$
$$\text{(COBE)} \qquad\qquad \text{(WMAP, BAO, SN)}$$

Cosmic inflation

- Cosmic inflation asserts a period that a slowly varying scalar field dominates the dynamics of the universe.
- Cosmic inflation solves the problems in Standard Big Bang cosmology such as the horizon, flatness, relic problems.
- Inflation predicts scale-invariant and Gaussian spectrum of density perturbations.
- Quantum fluctuations of inflaton provide the seeds for structure formation.
- How can we obtain a period of slow-roll inflation with large vacuum energy? How does inflation end?

Codimension-two brane

- Codimension-two branes

$$\text{Conical brane} \quad : \quad \delta - \text{function distribution}$$
$$\text{Non} - \text{conical brane} \quad : \quad \text{more singular than } \delta - \text{function}$$

- The tension(T) of a conical brane deforms the geometry of extra dimensions by a deficit angle without curving the 4D spacetime: locally, $ds^2 \simeq dr^2 + \beta^2 r^2 d\theta^2$, with a deficit angle $\Delta \equiv 2\pi(1-\beta) = T$. [Chen,Luty,Ponton(2000)]
- For the Standard Model fields living on a conical brane, one may make brane-localized vacuum energy not to gravitate the 4D spacetime by adjusting a deficit angle such that a late cosmological constant is made small.

[Cliff's talk]

- A natural question is then "Is it possible to have an early inflation on a codimension-two brane?"
- An answer is that inflation may occur on "a non-conical brane", the localized vacuum energy of which gravitates along four dimensions.

  [Aghababaie,Burgess,Hoover,Tolley(2005)]
- We consider a possibility of having an early inflation on a codimension-two brane in a 6D chiral gauged supergravity.
- We also address how the early inflation period is linked to the late universe with a small cosmological constant.

6D chiral gauged supergravity

[Nishino,Sezgin(1984); Salam,Sezgin(1984)]

- The 6D chiral gauged supergravity is composed of

$$\begin{aligned} -\text{gravity} &: \quad e_M^A, \quad \psi_M, \quad B^+_{MN}, \\ -\text{tensor} &: \quad \phi, \quad \chi, \quad B^-_{MN}, \\ -\text{vector} &: \quad A_M, \quad \lambda. \end{aligned}$$

- The R symmetry ($U(1)_R$) is gauged.
- The bulk anomalies are cancelled for $n_H = n_V + 244$.
- Anomaly-free models: $U(1)_R$ with hyperino 245_0; $E_7 \times E_6 \times U(1)_R$ with hyperino $(912, 1)_0$; $E_7 \times G_2 \times U(1)_R$ with hyperino $(56, 14)_0$; $F_4 \times Sp(9) \times U(1)_R$ with hyperino $(52, 18)_0$; models with products of $U(1)$ and $SU(2)$.

[Randjbar-Daemi et al(1985); Avramis et al(2005); Suzuki et al(2005)]

[Lee,Papazoglou(2007)]

- Without hyperscalars, the bosonic part of the 6D supergravity action with supersymmetric branes is $S=\int d^6x\sqrt{-g}\mathcal{L}$ with

$$S = \int d^6x\sqrt{-g}\Big(R-\frac{1}{4}(\partial_M\phi)^2-4g^2e^{-\frac{1}{2}\phi}-\frac{1}{4}e^{\frac{1}{2}\phi}\hat{F}_{MN}\hat{F}^{MN} -\frac{1}{12}e^{\phi}\hat{G}_{MNP}\hat{G}^{MNP}-T_i\frac{\delta^2(y-y_i)}{e_2}\Big)$$

with $\hat{F}_{MN}=\partial_MA_N-\partial_NA_M-\delta^m_M\delta^n_N\xi_i\delta^i_{mn}$ and $\hat{G}_{MNP}=3\partial_{[M}B_{NP]}+\frac{3}{2}F_{[MN}A_{P]}-3\delta^{\mu}_{[M}\delta^m_N\delta^n_{P]}\xi_iA_\mu\delta^i_{mn}$. Here, $\delta^i_{mn}\equiv\epsilon_{mn}\frac{\delta^2(y-y_i)}{e_2}$ and the localized Fayet-Iliopoulos term (or magnetic couplings) are $\xi_i=\eta_i\frac{T_i}{4g}$ with $\eta_i=\pm1$ depending on the 4D chirality of the brane SUSY.

General warped solutions

[Gibbons et al(2003); Aghababaie et al(2003); Lee,Papazoglou(2007)]

- Setting $B_{MN} = 0$ and assuming the axial symmetry of extra dimensions, the general regular solution with 4D maximal symmetry is

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\begin{aligned}
ds^2 &= e^{\frac{1}{2}\phi_0}\Big(W^2(r)\eta_{\mu\nu}dx^\mu dx^\nu + A^2(r)(dr^2 + B^2(r)d\theta^2)\Big), \\
\hat{F}_{mn} &= \lambda q e^{-\frac{1}{2}\phi} W^{-4}\epsilon_{mn}, \quad \phi = \phi_0 + 4\ln W,
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with

$$W^4 = \frac{1+\frac{r^2}{r_1^2}}{1+\frac{r^2}{r_0^2}}; \quad r_0^2 = \frac{1}{2g^2}, \quad r_1^2 = \frac{8}{q^2}.$$

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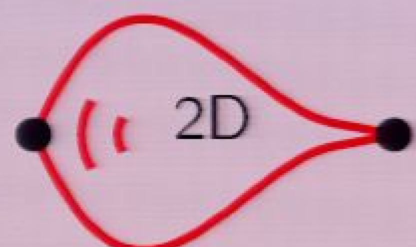
T_1  $T_2 = f(T_1, r_1/r_0)$

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- The localized FI terms with $\xi_i = T_i / 4g (i = 1, 2)$ modify the gauge potentials at the brane positions, being crucial for showing that the football solution is supersymmetric.

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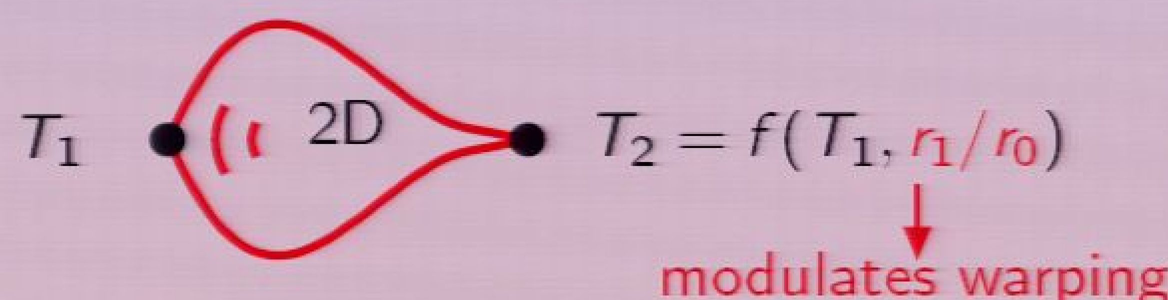

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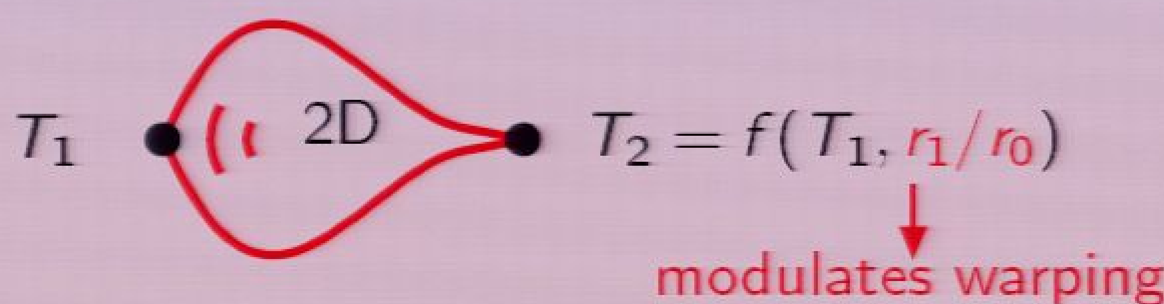

Supersymmetric football

[Papazoglou,Lee(2007)]

- Warping breaks the bulk SUSY completely.
- For a constant warping, the geometry becomes a football:

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu + \frac{1}{(1+\frac{r^2}{r_0^2})^2}(dr^2 + \lambda^2 r^2 d\theta^2).$$

That is, for $q = 4g$, we get $n = 1$ and $T_1 = T_2 = 2\pi(1-\lambda)$ with arbitrary λ.

- 4D $\mathcal{N} = 1$ SUSY is given by $P_L\varepsilon$ satisfying

$$\begin{aligned} 0 = \delta\lambda &= i2\sqrt{2}g(P_R\varepsilon), \\ 0 = \delta\psi_\theta &= \left[\partial_\theta + \frac{i}{2}\left\{1 + \lambda\left(1 - \frac{2}{f_0}\right)\right\}\gamma^5 + i\lambda\left(\frac{1}{f_0} - 1\right) - i\frac{g\xi_0}{2\pi}\right. \\ &= \partial_\theta(P_L\varepsilon). \end{aligned}$$

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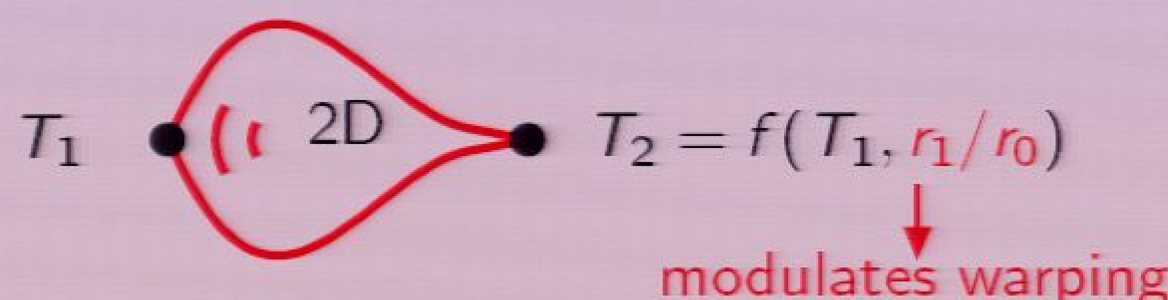

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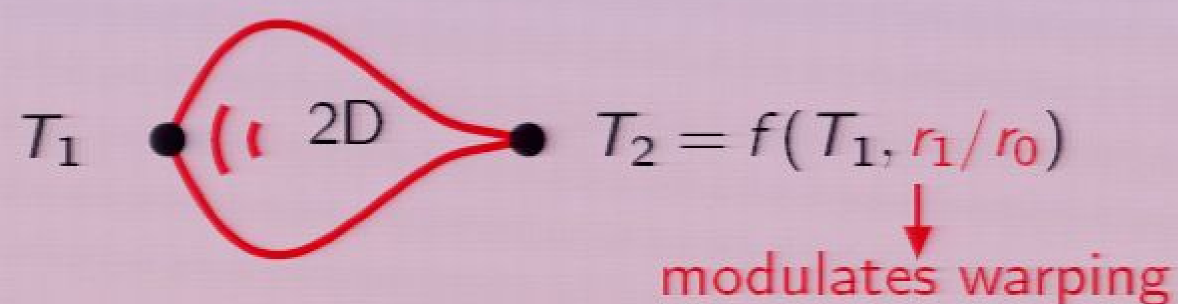

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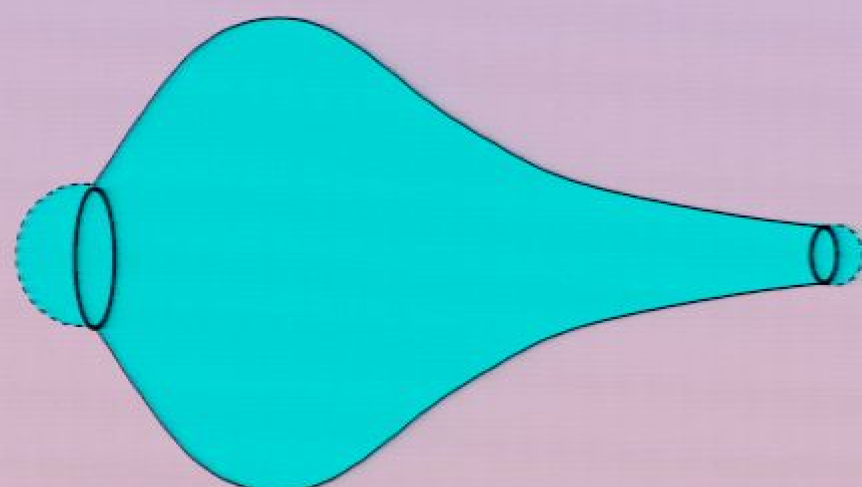
Modulus stabilization with regularized branes

- Bulk EOMs with the conical branes are scale invariant under $g_{MN} \to e^{\phi_0/2} g_{MN}$ and $\phi \to \phi + \phi_0 \Rightarrow$ a volume modulus
- Replace the conical 3-branes by ring-like codimension-1 branes capped with a regular sphere and introduce the dilaton potentials on the ring-like branes.

[Peloso et al(2006); Papantonopoulos et al(2006); Burgess et al(2007)]



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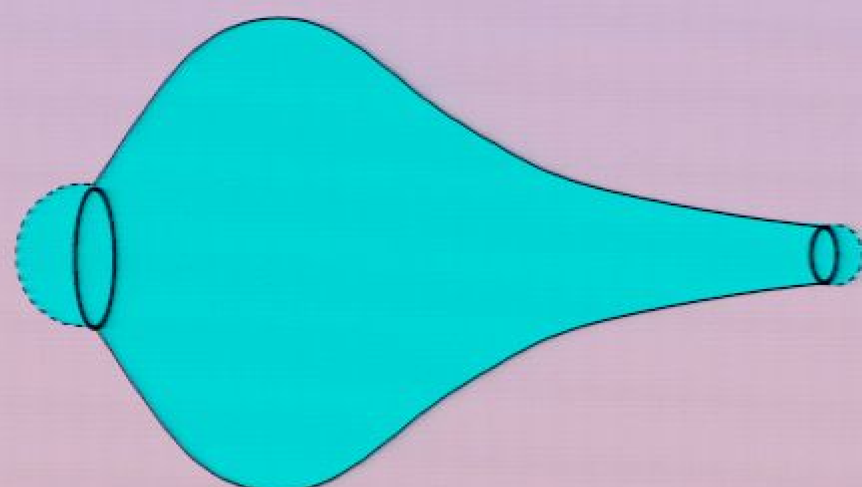

Supersymmetric football

[Papazoglou,Lee(2007)]

- Warping breaks the bulk SUSY completely.
- For a constant warping, the geometry becomes a football:

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu + \frac{1}{(1+\frac{r^2}{r_0^2})^2}(dr^2 + \lambda^2 r^2 d\theta^2).$$

That is, for $q = 4g$, we get $n = 1$ and $T_1 = T_2 = 2\pi(1-\lambda)$ with arbitrary λ.

- 4D $\mathcal{N} = 1$ SUSY is given by $P_L\varepsilon$ satisfying

$$\begin{aligned} 0 = \delta\lambda &= i2\sqrt{2}g(P_R\varepsilon), \\ 0 = \delta\psi_\theta &= \Big[\partial_\theta + \frac{i}{2}\Big\{1 + \lambda\Big(1 - \frac{2}{f_0}\Big)\Big\}\gamma^5 + i\lambda\Big(\frac{1}{f_0} - 1\Big) - i\frac{g\xi_0}{2\pi} \\ &= \partial_\theta(P_L\varepsilon). \end{aligned}$$

General warped solutions

[Gibbons et al(2003); Aghababaie et al(2003); Lee,Papazoglou(2007)]

- Setting $B_{MN}=0$ and assuming the axial symmetry of extra dimensions, the general regular solution with 4D maximal symmetry is

$$ds^2 = e^{\frac{1}{2}\phi_0}\Big(W^2(r)\eta_{\mu\nu}dx^\mu dx^\nu + A^2(r)(dr^2+B^2(r)d\theta^2)\Big),$$
$$\hat{F}_{mn} = \lambda q e^{-\frac{1}{2}\phi} W^{-4}\epsilon_{mn}, \quad \phi=\phi_0+4\ln W,$$

with

$$W^4 = \frac{1+\frac{r^2}{r_1^2}}{1+\frac{r^2}{r_0^2}}; \quad r_0^2=\frac{1}{2g^2}, \quad r_1^2=\frac{8}{q^2}.$$

Here $A=\frac{W}{1+r^2/r_0^2}$, $B=\frac{\lambda r}{W^4}$, and λ, q, ϕ_0 are constant.

- Two brane tensions must be located at the conical singularities, $r = 0$ and $r = \infty$:

$$T_1 = 2\pi M_*^4(1-\lambda),$$
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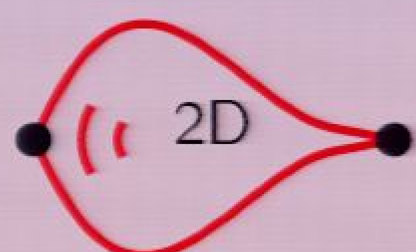
- The localized FI terms with $\xi_i = T_i/4g(i = 1, 2)$ modify the gauge potentials at the brane positions, being crucial for showing that the football solution is supersymmetric.

[Lee,Papazoglou(2007)]

T_1  $T_2 = f(T_1, r_1/r_0)$

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Modulus stabilization with regularized branes

- Bulk EOMs with the conical branes are scale invariant under $g_{MN} \to e^{\phi_0/2} g_{MN}$ and $\phi \to \phi + \phi_0 \Rightarrow$ a volume modulus
- Replace the conical 3-branes by ring-like codimension-1 branes capped with a regular sphere and introduce the dilaton potentials on the ring-like branes.

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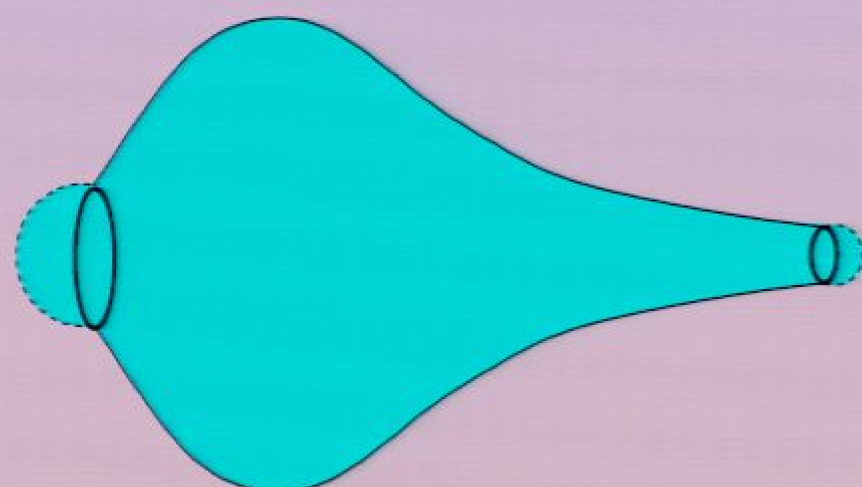

- The ring brane action is given by

$$S_i = -\int d^5x\sqrt{-\gamma_i}\Big[V_i(\phi) + \frac{1}{2}U_i(\phi)(D_{\hat{\mu}}\sigma_i D^{\hat{\mu}}\sigma_i)\Big]$$

with $\gamma^i_{\hat{\mu}\hat{\nu}}$ the induced metric on the branes, V_i, U_i dilaton couplings to the branes and σ_i brane Goldstone fields.

- The modulus of the ring radius is stabilized by the $U(1)$ current induced by the brane Goldstone field.

[See "superconducting string" by Witten(1985)]

- The volume modulus can be stabilized as seen from 4D effective action, [Burgess et al(2007)]

$$S_{\rm eff} = \pi\int d^4x\sqrt{-\gamma_i}\Big[\Big(\frac{V_i}{2}+2\frac{dV_i}{d\phi}\Big)-\frac{1}{2}\Big(\frac{U_i}{2}-2\frac{dU_i}{d\phi}\Big)g^{\theta\theta}\tilde{k}_i^2\Big]\,,$$

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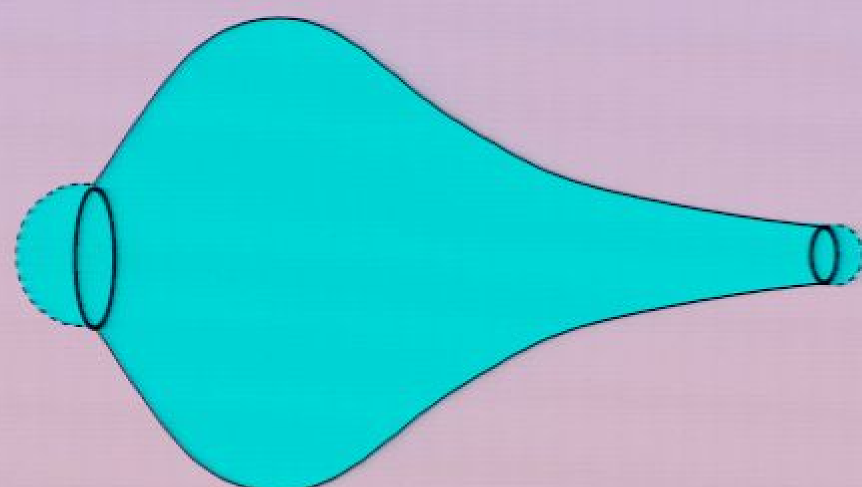

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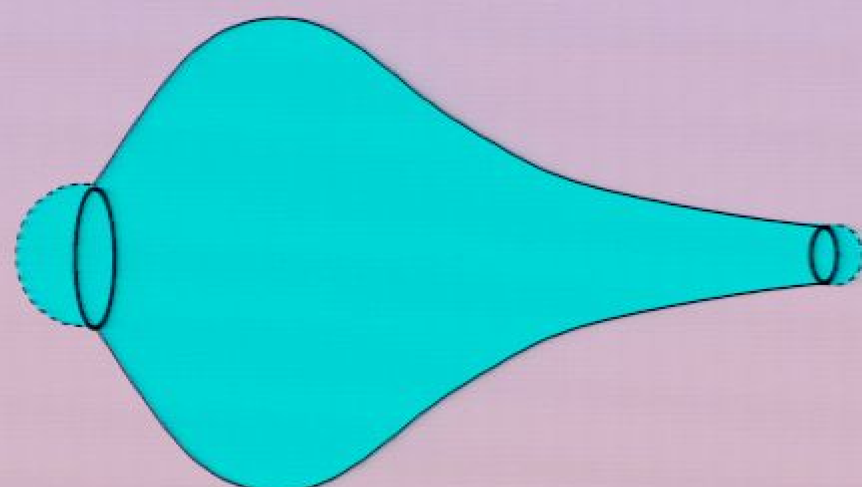

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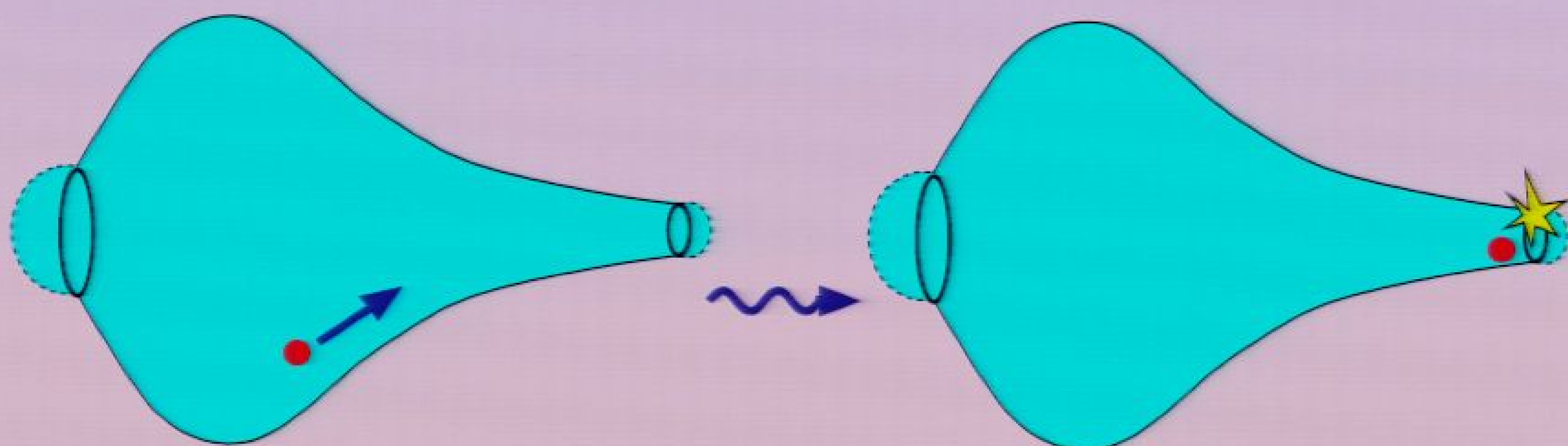
Outline

The probe brane action

- The background solution is 4D flat before adding another brane.
- Add one non-BPS codimension-two brane moving in the warped background with regularized branes.
- Consider the added brane to inflate on its worldvolume without backreaction.



- The DBI action for a probe brane is

$$S_{3\mathrm{b}} = -T_3 \int d^4x\, e^{\zeta_3\phi}\sqrt{-\gamma_{\mu\nu}}; \quad S_{4\mathrm{b}} = -T_4 \int d^5x\, e^{\zeta_4\phi}\sqrt{-\gamma_{ab}}.$$

- We consider a 3-brane or a ring 4-brane with small radius for slow-roll inflation.
- We choose scale-invariant dilaton couplings: $\zeta_3 = 0$ and $\zeta_4 = -\frac{1}{4}$.
- A non-conical probe 3-brane should be taken as a small radius limit of a ring 4-brane. When scaling symmetry is violated slightly at a ring 4-brane, the non-conical probe brane can be approximated by a conical brane.

- Take the metric and dilaton ansätz as

$$ds^2 = e^{-\psi(x)} W^2(r)\tilde{g}_{\mu\nu}(x)dx^\mu dx^\nu + e^{\psi(x)} A^2(r)(dr^2 + B^2(r)d\theta$$
$$\phi = 4\ln W + 2\psi(x)\ .$$

- Choosing the embedding coordinates for the probe 3-brane, $X^M = [x^\mu, R(t), \Theta(t)]$, the induced metric is

$$\gamma_{00} = g_{00} + g_{rr}\dot{R} + g_{\theta\theta}\dot{\Theta}, \quad \gamma_{ij} = g_{ij}.$$

- For $v^2 \equiv W^2(R(t))|\tilde{g}_{mn}\tilde{g}^{00}\dot{X}^m\dot{X}^n| \ll 1$, we get

$$S_{\rm probe} \approx \int d^4x\sqrt{-\tilde{g}}\left[\frac{1}{2}T_3 v^2 - V_{\rm probe}\right];$$

$$V_{\rm probe} = T_3 W^4(R)e^{-2\psi} = T_3\left(\frac{1+\frac{R^2}{r_1^2}}{1+\frac{R^2}{r_0^2}}\right)e^{-2\psi}.$$

- Integrating out heavy modes and considering the slow radial motion only, we get the effective action as

$$S_{\rm eff} = \int d^4x\sqrt{-\tilde{g}}\Big[\frac{1}{2}M_P^2 R_4(\tilde{g}_4) - M_P^2(\partial_\mu\psi)^2 - \frac{1}{2}m_\psi^2(\psi-\psi_0)^2 \\ -\frac{1}{2}T_3 A^2(R)W^2(R)(\partial_\mu R)^2 - T_3 W^4(R)e^{-2\psi}\Big].$$

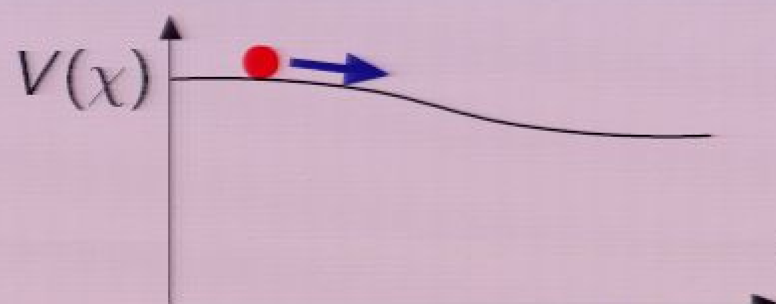
- When the inflation scale is lower than the scale of the modulus stabilization, we can consider ψ to be fixed during inflation.

Slow-roll inflation

- The slow-roll parameters are

$$\epsilon \sim \frac{M_P^2}{2\chi_0^2}\delta^2, \quad \eta \sim -\frac{2M_P^2}{\chi_0^2}\delta; \quad \delta \equiv \frac{r_0^2}{r_1^2} - 1 = \frac{T_2 - T_1}{2\pi M_*^4 - T_2}.$$

- For weak warping $|\delta| \ll 1$, i.e. close to SUSY football vacuum, we get a slow-rolling.
- For canonical field, $\chi \simeq \chi_0 \arctan\frac{R}{r_0}$ with $\chi_0 \equiv r_0\sqrt{T_3}$, the probe brane potential becomes $V_{\rm probe} \simeq T_3\Big[1 + \delta \sin^2\Big(\frac{\chi}{\chi_0}\Big)\Big]$.



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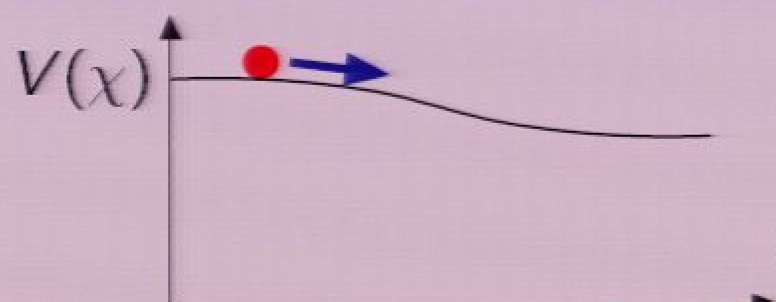

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$$n = 1 - 6\epsilon + 2\eta \simeq 0.960, \quad \delta_H = \frac{1}{\sqrt{150}\pi M_P^2} \frac{V^{1/2}}{\epsilon^{1/2}} \simeq 1.91 \times 10^{-5}.$$

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$$r_0^{-1} \simeq 10^{13}\mathrm{GeV}, \quad M_* \simeq 10^{15}\mathrm{GeV}, \quad T_3^{1/4} \simeq |\delta|^{1/4} 10^{16}\mathrm{GeV}.$$

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$$\eta_{\mathrm{he}} \simeq -0.02, \quad \epsilon_{\mathrm{he}} \simeq 0.005|\delta|, \quad N_{COBE} \simeq 50 \ln(R_f/R_i).$$

- Other observables:

$$r \simeq 0.06|\delta|, \qquad \frac{dn}{d\ln k} \sim \eta^2 \sim 10^{-4}.$$

Graceful exit

- When the probe brane hits the background brane at the other end, the background solution should settle to another flat solution reducing the vacuum energy to zero and reheating the brane.
- Close to collision, assume the inflation to be altered to a hybrid type with (χ, φ). Adding a (bulk) real scalar field φ with a coupling to the probe brane, the effective potential should be

$$V_{\text{eff}} = (T_3 + \alpha^2\varphi^2)W^4(\chi) + h(\varphi^2 - \beta^2)^2 - h\beta^4.$$

- For $\alpha^2 W^4(\chi) < 2h\beta^2$, the waterfall field starts rolling down to a minimum with nonzero VEV, thus ending the inflation as well as reheating the brane.

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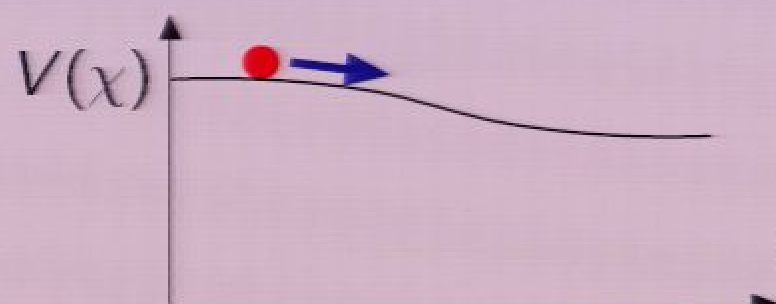

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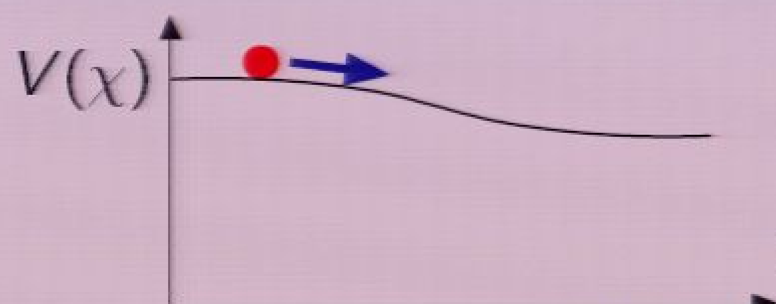

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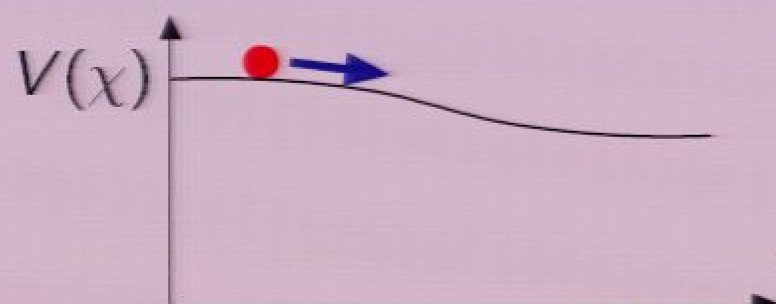

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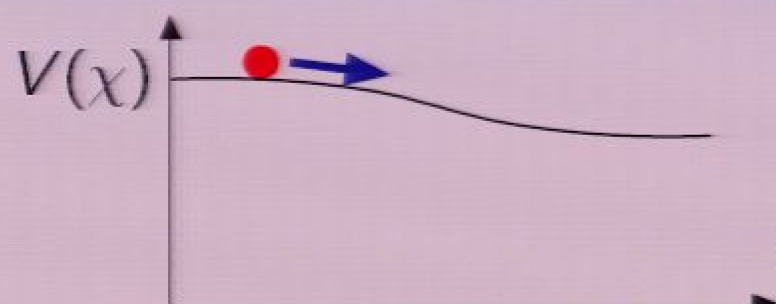

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- Close to collision, assume the inflation to be altered to a hybrid type with (χ, φ). Adding a (bulk) real scalar field φ with a coupling to the probe brane, the effective potential should be

$$V_{\rm eff} = (T_3 + \alpha^2\varphi^2)W^4(\chi) + h(\varphi^2 - \beta^2)^2 - h\beta^4.$$

- For $\alpha^2 W^4(\chi) < 2h\beta^2$, the waterfall field starts rolling down to a minimum with nonzero VEV, thus ending the inflation as well as reheating the brane.

Outline

1. Motivation
2. 6D supergravity
3. The probe brane inflation
4. Backreaction issues - Conclusion

4D dS space vs non-conical probe brane

- There must be at least one non-conical 3-brane in 6D solutions with de-Sitter 4D foliations:

$$\sum_i \rho_i \frac{\partial}{\partial \rho_i}\left(\ln W - \frac{1}{4}\phi\right)\Big|_{\rho_{i,0}} = \frac{3}{2\pi} H^2 V_2$$

where ρ_i are the local radial coordinates adapted to the brane. For $\phi_i \simeq 4 \ln W_i + 2\lambda_{3,i} \ln \rho_i$, a non-conical brane contributes to H^2. [Aghababaie,Burgess,Hoover,Tolley(2005)]

- Take a probe ring 4-brane with small radius ρ_0 centered at $R(t)$, the DBI action of the conical probe brane is a good approximation,

$$V_{\rm probe}(R) \simeq T_{3,{\rm eff}} W^4(R)(1 - \gamma \ln W(R))$$

where $T_{3,{\rm eff}} = \gamma T_4 \rho_0$ and $\gamma \ll 1$ for a small scale violation.

Backreactions

- The volume modulus must be stabilized at a scale higher than the inflation scale:

$$H^2 \ll m_\psi^2 \Rightarrow T_3 \ll M_*^4 \Rightarrow |\delta| \ll 10^{-2}.$$

- Slow rolling requires a tuning of the background brane tensions, $|T_1 - T_2| \sim |\delta| T_{1,2}$. On the other hand, the probe brane tension is determined to be $T_3 \sim |\delta| M_{\rm GUT}^4$. For $T_{1,2} \sim M_*^4 < M_{\rm GUT}^4$, we require $T_3 > |T_1 - T_2|$, so a dangerous backreaction on the warp factor would be expected.

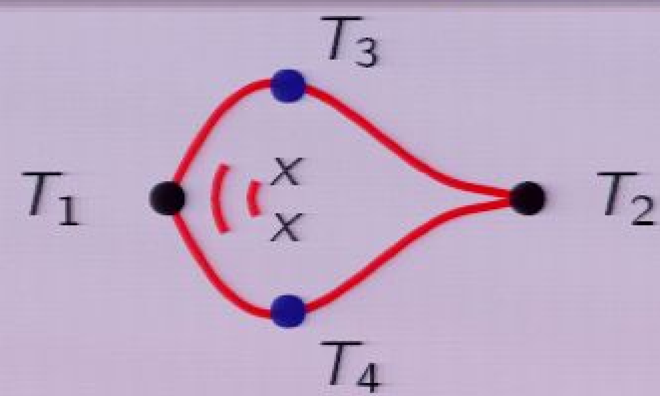
Multi-brane solutions with backreactions

[Lee,Lüdeling(2005)]

- The general static multi-brane solution has the 4D Minkowski space and it is determined by a holomorphic function $V(z)$ to be

$$W^4 = \frac{1}{2}(W_1^4 + W_2^4) + \frac{1}{2}(W_1^4 - W_2^4)\tanh\left[\frac{1}{2}(W_1^4 - W_2^4)\chi\right]$$

  with $e^{W_1^4\chi} + e^{W_2^4\chi} = (W_1^4 - W_2^4)e^{\frac{1}{2}g^2\zeta}$ and $\zeta(z) = \frac{1}{2}\int^z d\omega/V(\omega) + \text{c.c.}$ in complex coordinate z.
- Conical branes are located at either zeros or poles of $V(z)$; nontrivial tensions are allowed only at zeros.
- The warp factor lies in the finite range, $W_2 < W < W_1$, independent of $V(z)$.



- E.g. for $V(z) = -\frac{z}{|c|}(1 + \frac{i\beta}{z+z^{-1}})$ with $\beta > 0$, additional tensions are two fixed brane tensions $T_{\pm i} = -2\pi M_*^4$ at $z = \pm i$ and two nontrivial tensions at $z = -\frac{i}{2}(\beta \pm \sqrt{\beta^2+4})$:

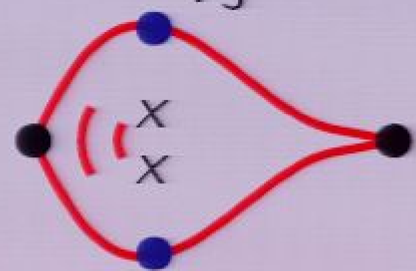
$$T_3 = 2\pi M_*^4(1-|b|\lambda), \quad T_4 = 2\pi M_*^4\left(1-|b|\lambda\frac{r_1^2}{r_0^2}\right); \; |b| \equiv \frac{|\beta|}{\sqrt{\beta^2+4}}.$$

- The relation between the additional brane tensions is

$$\frac{2\pi M_*^4 - T_4}{2\pi M_*^4 - T_3} = \frac{r_0^2}{r_1^2} = 1 + \delta.$$

- T_3 does not change $T_1 - T_2$ needed for $|\delta| \ll 1$.

- The probe brane inflation ends when it hits the background brane at $z = 0$ and $z = \infty$.
- Then the background brane solution settles into another flat solution with two brane tensions only,
  $T_1' = T_1 + T_3 = 2\pi M_*^4(1 - \lambda')$ and
  $T_2' = T_2 + T_4 = 2\pi M_*^4(1 - \lambda' \frac{r_1^2}{r_0^2})$ with $\lambda' = 2\lambda - 1$.



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Conclusion

- We identified the position of a probe codimension-two brane moving in 6D warped background as the inflaton.
- Slow-roll inflation is realized, being close to SUSY football vacuum with $|\delta| < 10^{-2}$.
- COBE normalization fixes the compactification scale to be $r_0^{-1} \simeq 10^{13}$ GeV, in turn the 6D fundamental scale at $M_* \simeq 10^{15}$ GeV.
- Backreaction on volume modulus can be avoided by lowering the bound on the warping to $|\delta| \ll 10^{-2}$. Thus we need to tune the background brane tensions, $|\Delta T| \sim |\delta| T_{1,2}$.

- Even if the probe brane tension is larger than $|\Delta T|$, multi-brane solutions may guarantee a small backreaction on the warp factor.
- Angular motion might lead to an interesting effect on the observations, e.g. isocurvature perturbations.

  [Langlois,Renaux-Petel,Steer,Tanaka(2008)]
- DBI inflation may be possible for a relativistic codimension-two brane moving in the strongly warped background.

  [Lee,Papazoglou(in progress)]