Title: 6D Brane Models and their Perturbations

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Abstract: Models with two supersymmetric large extra dimensions (SLED) may provide a way to approach both the cosmological constant problem and dark energy.
After reviewing these ideas, I shall discuss warped brane world solutions in 6D supergravity, as a laboratory in which to explore SLED and codimension two branes in general. Solving the linearized perturbations for all the bosonic fields, and some of the fermions, we can observe how the corresponding physics compares with 5D models and standard Kaluza-Klein compactifications. These results should help to better our understanding of the cancellations in the 4D effective vacuum energy with SLED.

6D Brane Models and their Perturbations

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Nucl.Phys.B767:54-81 (2007)
JHEP 0801:051 (2008)
JHEP 0903:136 (2009)

The 6D Brane World

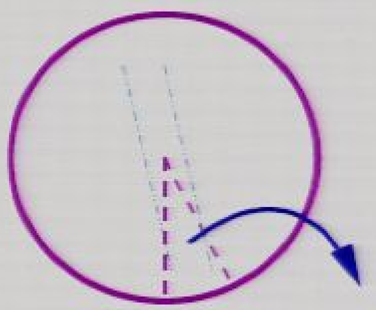
Vacuum energy on brane worlds can curve the extra dimensions rather than their instrinsic 4D spacetime.

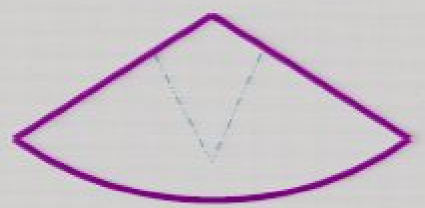
Rubakov & Shaposhnikov '83
...
Arkani-Hamed, Dimopoulos, Kaloper & Sundrum '05
Kachru, Schulz & Silverstein '05

A 3-brane in 6D induces a conical singularity in the transverse dimensions with $\Delta\varphi = \frac{T_3}{M_6^4}$:

Sundrum '98
Chen, Luty & Ponton '00

$$R_2 = R_2^{smth} + 2\sum_i T_3^i \frac{\delta^{(2)}(y - y_i)}{\sqrt{g_2}}$$





Tantalizing Numerology

Sundrum '98

Chen, Luty & Ponton '00

Observed scale of Dark Energy:

$$\Lambda \sim \left(\frac{M_W^2}{M_{Pl}}\right)^4 \sim \frac{1}{r^4}$$

with:

$$M_W \sim 10\,TeV$$

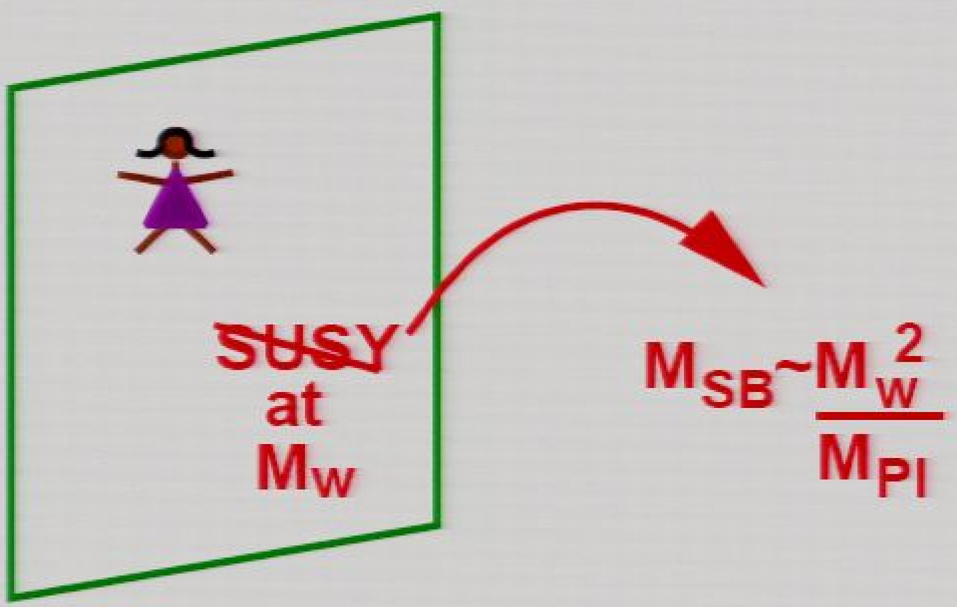
and:

$$r \sim 10\,\mu m$$

Supersymmetric Large Extra Dimensions I

Aghababaie, Burgess, S.L.P & Quevedo '02
Burgess '04

- Change gravity at the scale of Λ
- Separation of SUSY breaking scales



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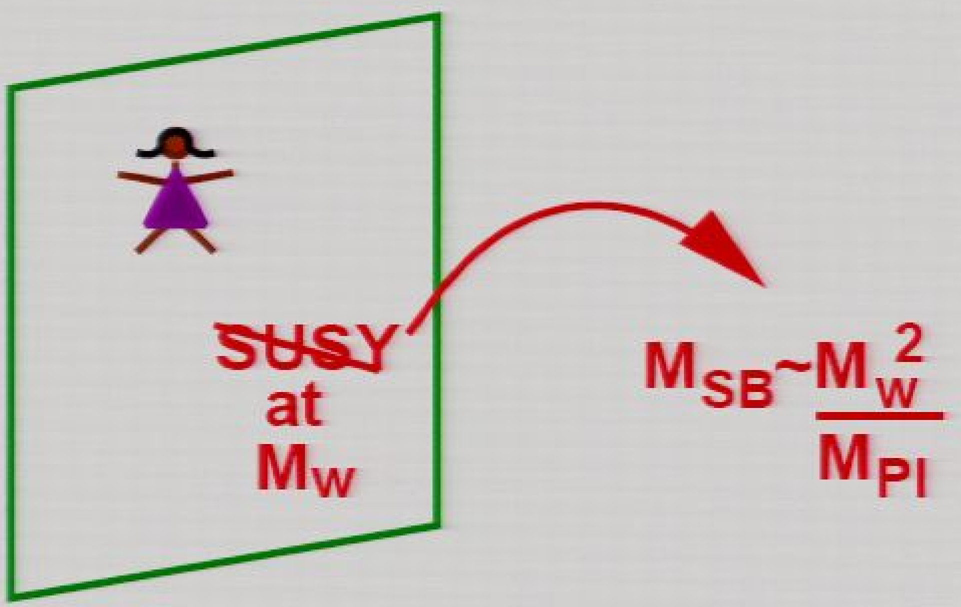


Supersymmetric Large Extra Dimensions II

Explain not only why the cosmological constant is zero, but why it is $(10^{-120} M_{Pl})^4$!

- Supersymmetry is badly broken on the brane:

$$T_3 \sim M_W^4$$

This localized vacuum energy is absorbed by the bulk curvature.

- Bulk susy-breaking is gravitationally suppressed:

$$M_{SB} = \frac{M_W^2}{M_{Pl}}$$

If bulk contribution to vacuum energy is $\mathcal{O}(M_{SB}^4)$ then we are there...

Overview

- Brane Models in 6D Supergravity
- The Perturbations
- Codimension 2 and the KK/Braneworld Lores
- Conclusions
- Cosmology at Colliders!

6D Supergravity + Branes

6D chiral gauged supergravity in the bulk:

$$S_{bulk} = \int d^6X \sqrt{-g} \left[\frac{1}{4} R - \frac{1}{4} \partial_M \sigma \, \partial^M \sigma - \frac{e^{-2\sigma}}{12} G_{MNP} G^{MNP} - \frac{e^{-\sigma}}{4} F^I_{MN} F^{I\,MN} - G_{\alpha\beta} D_M \Phi^\alpha D^M \Phi^\beta - 2 g_1^2 \, v(\Phi) \, e^\sigma + \text{fermions} \right]$$

Points to note:

Nishino & Sezgin '84

- *Gauged R-symmetry ⇒ positive-definite scalar potential, with minimum at $\Phi = 0$ where $v(0) = 1$.*
- *Chiral fermions ⇒ in general anomolous ($n_H = n_V + 244$).*

3-brane sources:

$$S_{brane} = -T_3 \int d^4y \sqrt{-\det \left(g_{MN} \partial_\alpha Y^M \, \partial_\beta Y^N \right)}$$

Explicit Solutions

Gibbons, Guven & Pope '03
Aghababaie, Burgess, Cline, Firouzjahi, S.L.P. Quevedo, Tasinato & Zavala '03

A general class of solutions with:

1. Maximal symmetry in 4D
2. Axial symmetry in 2D
3. At most conical singularities

$$ds^2 = e^{A(r)}\,\eta_{\mu\nu}\,dx^\mu\,dx^\nu + dr^2 + e^{B(r)}\,d\varphi^2$$
$$\mathcal{A} = \mathcal{A}_\varphi(r)\,Q\,d\varphi\,, \qquad \sigma = \sigma(r)$$
$$G_{MNP} = 0\,, \qquad \Phi^\alpha = 0$$

Solution has conical singularities at $r = 0$ and $r = \bar{r}$ with deficit angles:

$$\delta = 2\pi\left(1 - \frac{1}{\omega}\right) \qquad \text{and} \qquad \bar{\delta} = 2\pi\left(1 - \frac{1}{\bar{\omega}}\right)$$

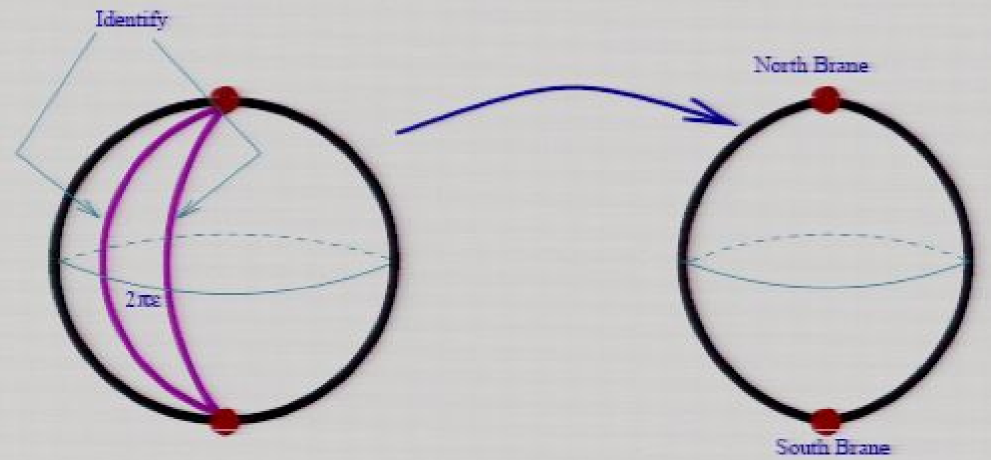
where $\omega, \bar{\omega} > 0$ are integration constants.

Sphere Limit

As $\omega \to \bar{\omega}$ the warp factor goes to one $\Rightarrow$ **unwarped rugby ball**:

$$ds_2^2 = \left(\frac{r_0}{2}\right)^2 \left(d\theta^2 + \left(1 - \frac{\delta}{2\pi}\right)^2 \sin^2\theta d\varphi^2\right)$$

Carroll & Guica '02
Navarro '02
Aghababaie, Burgess, S.L.P & Quevedo '02

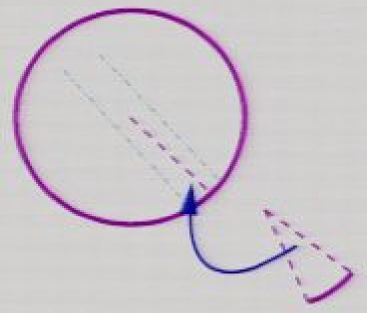


As both $\omega \to 1$ and $\bar{\omega} \to 1$ the deficit angles go to zero $\Rightarrow$ **classic sphere-monopole compactification.**

Randjbar-Daemi, Salam & Strathdee '83
Salam & Sezgin '84

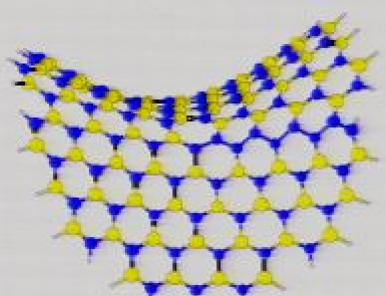
Negative Tension Branes

Negative deficit angles are also possible:

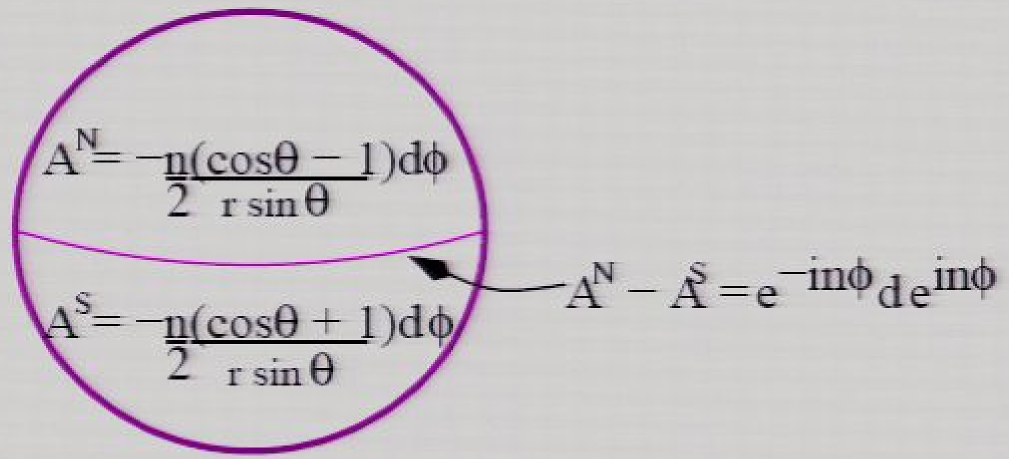


Like nanocones with negative disclination angles in condensed matter physics:

Azevedo, Mazzoni, Chacham & Nunes '04



Topology Rules



Dirac Quantization Condition in terms of Monopole Numbers, N^i:

$$e^i \frac{g_{bk}}{g_1} \frac{1}{(\omega\bar{\omega})^{1/2}} = N^i$$

Laboratory for SLED and 6D Brane Worlds

Having established the background solutions:

- Are they stable to small perturbations?
- What are the symmetries & particle content of the low energy EFT?
- What are the modifications to 4D Newtonian gravity?
- What is the effective vacuum energy measured by a 4D observer?

What role to the branes play, and how are they different to branes in 5D or traditional KK models?

The Fluctuations

Bosonic Perturbations (we'll truncate the branons):

$$G_{MN} \to G_{MN} + h_{MN}, \quad \mathcal{A}_M \to \mathcal{A}_M + V_M,$$
$$\sigma \to \sigma + \tau, \quad B_{MN} \to B_{MN} + b_{MN}, \quad Y^M \to Y^M + \xi^M$$

lead to bilinear action:

$$S(h,h) + S(V,V) + S(h,V) + S(\tau,\tau) + S(h,\tau)$$
$$+S(V,\tau) + S(V_2,V_2) + S(V,V_2) + S(\xi,\xi) + S(h,\xi)$$

E.g. $S(h,h) =$

$$\int d^DX\sqrt{-G}\left\{\frac{1}{2\kappa^2}\left[\left(h^{MN}{}_{;M} - \frac{1}{2}h^{;N}\right)^2 - \frac{1}{2}h^{NP}{}_{;M}h_{NP}{}^{;M} + \frac{1}{4}h^{;M}h_{;M} - \frac{1}{2}R_1 h^2\right]\right.$$
$$-\frac{1}{2}h_{PM}h^P{}_N\left(\frac{1}{2}e^{\sigma/2}F^{MR}F^N{}_R + \frac{1}{4\kappa^2}\partial^M\sigma\,\partial^N\sigma\right)$$
$$-\frac{1}{2}h^{MN}h^{PR}\left(\frac{1}{\kappa^2}R_{PMNR} - \frac{1}{2}e^{\sigma/2}F_{PM}F_{NR}\right) + brane\ contributions$$

Light Cone Gauge

Linearized theory enjoys a number of symmetries descending from:

- 6D coordinate invariance $x^M \to x^M + \eta^M$
- 6D gauge symmetry $A \to A + d\chi$
- Kalb-Ramond symmetry $B_2 \to B_2 + d\lambda$

$$\begin{aligned} \delta h_{MN} &= -\eta_{N;M} - \eta_{M;N} \\ \delta V_M &= -\eta^L F_{LM} - D_M \chi \\ \delta \tau &= -\eta^M \partial_M \sigma, \\ \delta V_{MN} &= 2\kappa \chi F_{MN} + \lambda_{N;M} - \lambda_{M;N} \end{aligned}$$

Use to fix light cone gauge $(x^{(\pm)} \equiv \frac{1}{2}(x^3 \pm x^0))$:

$$V_{(-)} = 0, \quad h_{(-)M} = 0, \quad V_{(-)M} = 0, \ \forall M$$

Then $V_{(+)}$, $h_{(+)M}$ and $V_{(+)M}$ can be eliminated *via* their *e.o.m's,* leaving only physical degrees of freedom.

The Coupled 2nd Order Differential Problems

After the gauge fixings the bosonic sectors are:

Spin	Dimension of System	Fields
2	1×1	h_{ij}
1	5×5	$h_{im}, V_{im}, V_i^{\|\|}$
1	1×1	$V_i^{\perp}$
0	8×8	$h_{mn}, V_{mn}, V_{ij}, \tau, V_m^{\|\|}$
0	2×2	$V_m^{\perp}$

We have also considered some fermionic sectors:

Spin	Dimension of System	Fields
$\frac{1}{2}$	1×1	$\lambda^{\perp}$
$\frac{1}{2}$	1×1	ψ^a

Example: Spin-1 Sector

Here I will present the 5×5 spin-1 sectors for the unwarped rugbyball.

Their linearized dynamics are governed by the action:

$$
\begin{aligned}
S^{(1)}(h,h) &= -\frac{1}{2\kappa^2}\int d^6X\sqrt{-G}\left(\partial_\mu h_{mi}\partial^\mu h^{mi} + h_{mi;n}h^{mi;n} + \frac{R}{2}h_{mi}h^{mi}\right) \\
S^{(1)}(V,V) &= -\frac{1}{2}\int d^6X\sqrt{-G}\left[\partial_\mu V_i\partial^\mu V^i + \partial_m V_i\partial^m V^i + \frac{\kappa^2}{2}F^2 V_i V^i\right] \\
S^{(1)}(V_2,V_2) &= -\frac{1}{8}\int d^6X\sqrt{-G}\left(\partial_\mu V_{mi}\partial^\mu V^{mi} + V_{mi;n}V^{mi;n} + \frac{R}{2}V_{mi}V^{mi}\right) \\
S^{(1)}(h,V) &= \int d^6X\sqrt{-G}\,(-\partial_m V_i h_{ni}F^{nm}) \\
S^{(1)}(V,V_2) &= \int d^6X\sqrt{-G}\left(-\frac{\kappa}{2}\partial_m V^i V_{ni}F^{nm}\right)
\end{aligned}
$$

EOMs and Boundary Conditions

The action gives rise to the following EOMs:

$$\left(\partial^2 + D^2 - \frac{R}{2}\right) h_{mi} - \kappa^2 F_m{}^l \partial_l V_i = 0$$

$$\left(\partial^2 + D^2 - \frac{R}{2}\right) V_{mi} - 2\kappa F_m{}^l \partial_l V_i = 0$$

$$\left(\partial^2 + D^2 - \frac{\kappa^2}{2} F^2\right) V_i + F^{nm} h_{ni;m} + \frac{\kappa}{2} F^{nm} V_{ni;m} = 0$$

and Boundary Conditions:

$$\int d^6X \sqrt{-G} \left(\delta h_{mi} h^{mi;n}\right)_{;n} = 0, \quad \int d^6X \sqrt{-G} \left(\delta V_{mi} V^{mi;n}\right)_{;n} = 0,$$

$$\int d^6X \sqrt{-G} \left(\delta V^i h_{li} F^{lm}\right)_{;m} = 0, \quad \int d^6X \sqrt{-G} \left(\delta V^i V_{li} F^{lm}\right)_{;m} = 0,$$

$$\int d^6X \sqrt{-G} \left(\delta V_i \partial^m V^i\right)_{;m} = 0$$

Also impose finite kinetic energy (Normalizability Condition)

Rugby Ball Harmonics

- Perform KK decomposition for fields $(h_{\pm i} \equiv \frac{1}{r_0}\left(e^{B/4}h_{\theta i} \pm i e^{-B/4}h_{\varphi i}\right))$:

$$V_i(X) = \sum_{\mathbf{n},\mathbf{m}} V_{i\,\mathbf{nm}}(x) f_{\mathbf{nm}}(\theta) e^{i\mathbf{m}\varphi}, \quad h_{\pm i}(X) = \sum_{\mathbf{n},\mathbf{m}} h_{\pm i\ \mathbf{n}\,\mathbf{m}}(x) f^{\pm}_{\mathbf{n}\,\mathbf{m}}(\theta) e^{i\mathbf{m}\varphi}$$

- Find a complete basis for 2D scalar fields by solving (with BCs):

$$-D^2\left(f_{\mathbf{nm}}(\theta)e^{i\mathbf{m}\varphi}\right) = \mu^2_{\mathbf{nm}} f_{\mathbf{nm}}(\theta) e^{i\mathbf{m}\varphi}$$

and a complete basis for 2D vector fields by solving (with BCs):

$$\left(-D^2 + R/2\right)\left(f^{\pm}_{\mathbf{n}\,\mathbf{m}}(\theta)e^{i\mathbf{m}\varphi}\right) = \mu^2_{\mathbf{nm}} f^{\pm}_{\mathbf{n}\,\mathbf{m}}(\theta) e^{i\mathbf{m}\varphi}$$

- The scalar and vector harmonics obey a derivative relation:

$$\partial_{\pm}\left(f_{\mathbf{nm}}(\theta)e^{i\mathbf{m}\varphi}\right) = \frac{\mu_{\mathbf{nm}}}{\sqrt{2}} f^{\pm}_{\mathbf{nm}}(\theta) e^{i\mathbf{m}\varphi}$$

provided $\mathbf{m} = 0$ or $\mathbf{m} \geq 1/\omega$.

The Squared Mass Operator

- Plug in rugbyball harmonic expansions
- Apply derivative relation
- Integrate out extra dimensions

$$S^{(1)}(h, V, V_2) = \int d^4x \sum_{\mathbf{n},\mathbf{m}} \Big\{ \frac{1}{2} (V_{i\,\mathbf{nm}})^* \left(\partial^2 - \mu^2_{\mathbf{nm}} - \frac{8}{r_0^2} \right) V_{i\,\mathbf{nm}}$$

$$+ \frac{1}{2\kappa^2} (h_{+i\,\mathbf{nm}})^* \left(\partial^2 - \mu^2_{\mathbf{nm}} \right) h_{+i\,\mathbf{nm}} + \frac{1}{2\kappa^2} (h_{-i\,\mathbf{nm}})^* \left(\partial^2 - \mu^2_{\mathbf{nm}} \right) h_{-i\,\mathbf{nm}}$$

$$+ \frac{1}{8} (V_{+i\,\mathbf{nm}})^* \left(\partial^2 - \mu^2_{\mathbf{nm}} \right) V_{+i\,\mathbf{nm}} + \frac{1}{8} (V_{-i\,\mathbf{nm}})^* \left(\partial^2 - \mu^2_{\mathbf{nm}} \right) V_{-i\,\mathbf{nm}}$$

$$- \frac{2\mu_{\mathbf{nm}} i}{r_0 \kappa} \Big[(h_{+i\,\mathbf{nm}})^* V_{i\,\mathbf{nm}} - (h_{-i\,\mathbf{nm}})^* V_{i\,\mathbf{nm}}$$

$$+ \frac{\kappa}{2} \left((V_{+i\,\mathbf{nm}})^* V_{i\,\mathbf{nm}} - (V_{-i\,\mathbf{nm}})^* V_{i\,\mathbf{nm}} \right) \Big] \Big\}$$

$\Rightarrow$ the squared mass operator is transformed into an algebraic matrix with constant entries

The KK Mass Towers

The KK towers for physical spin-1 perturbations about the rugbyball:

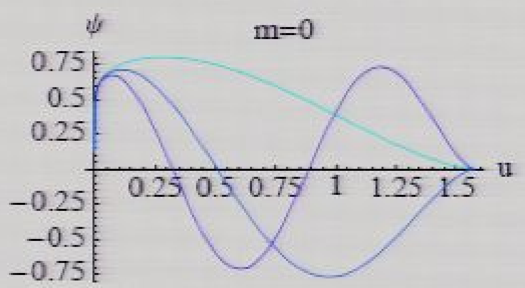
Squared mass	Multiplicity
$\frac{8}{r_0^2}$	1
$\mu^2_{\mathbf{n}\,\mathbf{m}} \equiv \frac{4}{r_0^2}\left(\mathbf{n} + \lvert\mathbf{m}\rvert\omega\right)\left(\mathbf{n} + \lvert\mathbf{m}\rvert\omega + 1\right)$	2
$\frac{4}{r_0^2}\left(1 + \frac{r_0^2}{4}\mu^2_{\mathbf{n}\,\mathbf{m}} + \sqrt{1 + r_0^2\mu^2_{\mathbf{n}\,\mathbf{m}}}\right)$	1
$\frac{4}{r_0^2}\left(1 + \frac{r_0^2}{4}\mu^2_{\mathbf{n}\,\mathbf{m}} - \sqrt{1 + r_0^2\mu^2_{\mathbf{n}\,\mathbf{m}}}\right)$	1

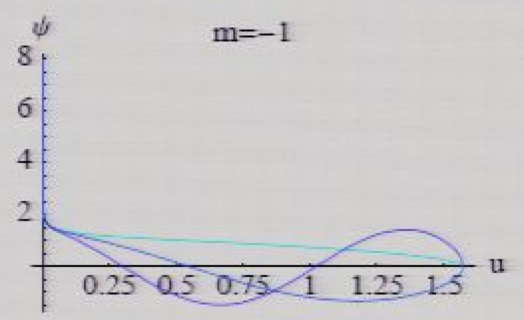
with wavefunctions in terms of particular hypergeometric functions.

- Free of tachyonic instabilities
- Massless modes:
  - $\mathbf{n} = 1,\ \mathbf{m} = 0$; $\mathbf{n} = 0,\ \mathbf{m} = 1/\omega$ and $\mathbf{n} = 0,\ \mathbf{m} = -1/\omega$
  - 1 massless 4D vector enhanced to 3 when $\delta = 0, -2\pi, \ldots$
  - correspond to 3 well-defined Killing vectors in 2D.
  - only one is gauge field in full 4D EFT but all are in Low Energy EFT

Summary of all KK towers

- Massless modes – *massless 4D graviton, one or three massless 4D vectors, two massless 4D scalars* – all separated from KK tower by mass gap $\mathcal{O}(1/r_0)$.
- Tachyonic instabilities can arise only from internal components of 6D gauge fields charged under monopole background – for monopole numbers $|N^I| > 1$ Beware of non-Abelian fluxes!
- Universal behaviour for wavefunction localization, *e.g.* for fermions:





$(r_0, \omega, \bar{\omega}, e) = (1, \frac{1}{4}, 1, 0)$

$n = 0, 1, 2$ modes for a single negative tension brane at $u = 0$.

$(m, n) = (-1, 0)$ mode is massless.

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$$+\frac{1}{2\kappa^2}(h_{+i\ \mathbf{nm}})^* \left(\partial^2 - \mu_{\mathbf{nm}}^2\right) h_{+i\ \mathbf{nm}} + \frac{1}{2\kappa^2}(h_{-i\ \mathbf{nm}})^* \left(\partial^2 - \mu_{\mathbf{nm}}^2\right) h_{-i\ \mathbf{nm}}$$

$$+\frac{1}{8}(V_{+i\ \mathbf{nm}})^* \left(\partial^2 - \mu_{\mathbf{nm}}^2\right) V_{+i\ \mathbf{nm}} + \frac{1}{8}(V_{-i\ \mathbf{nm}})^* \left(\partial^2 - \mu_{\mathbf{nm}}^2\right) V_{-i\ \mathbf{nm}}$$

$$-\frac{2\mu_{\mathbf{nm}} i}{r_0 \kappa}\left[(h_{+i\ \mathbf{nm}})^* V_{i\ \mathbf{nm}} - (h_{-i\ \mathbf{nm}})^* V_{i\ \mathbf{nm}}\right.$$

$$\left.\left. +\frac{\kappa}{2}\left((V_{+i\ \mathbf{nm}})^* V_{i\ \mathbf{nm}} - (V_{-i\ \mathbf{nm}})^* V_{i\ \mathbf{nm}}\right)\right]\right\}$$

$\Rightarrow$ the squared mass operator is transformed into an algebraic matrix with constant entries

The KK Mass Towers

The KK towers for physical spin-1 perturbations about the rugbyball:

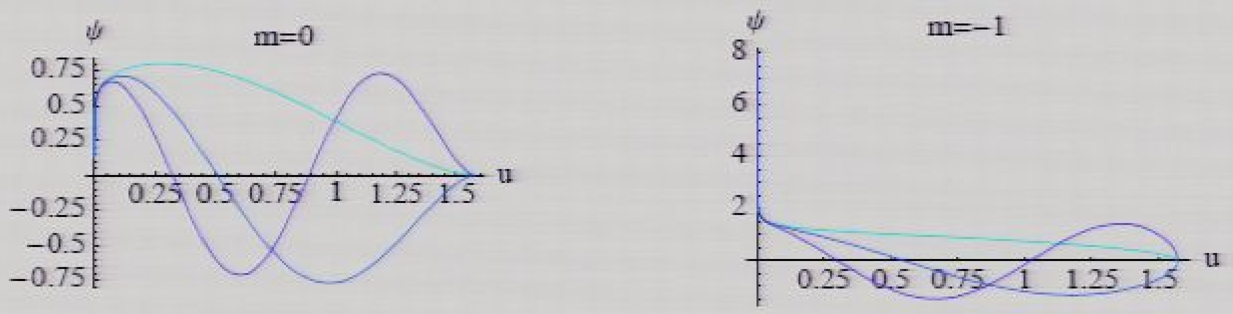
Squared mass	Multiplicity
$\frac{8}{r_0^2}$	1
$\mu^2_{\mathbf{n}\,\mathbf{m}} \equiv \frac{4}{r_0^2}\left(\mathbf{n}+\lvert\mathbf{m}\rvert\omega\right)\left(\mathbf{n}+\lvert\mathbf{m}\rvert\omega+1\right)$	2
$\frac{4}{r_0^2}\left(1+\frac{r_0^2}{4}\mu^2_{\mathbf{n}\,\mathbf{m}}+\sqrt{1+r_0^2\mu^2_{\mathbf{n}\,\mathbf{m}}}\right)$	1
$\frac{4}{r_0^2}\left(1+\frac{r_0^2}{4}\mu^2_{\mathbf{n}\,\mathbf{m}}-\sqrt{1+r_0^2\mu^2_{\mathbf{n}\,\mathbf{m}}}\right)$	1

with wavefunctions in terms of particular hypergeometric functions.

- Free of tachyonic instabilities
- Massless modes:
  - $\mathbf{n}=1,\ \mathbf{m}=0$; $\mathbf{n}=0,\ \mathbf{m}=1/\omega$ and $\mathbf{n}=0,\ \mathbf{m}=-1/\omega$
  - 1 massless 4D vector enhanced to 3 when $\delta=0,-2\pi,\ldots$
  - correspond to 3 well-defined Killing vectors in 2D.
  - only one is gauge field in full 4D EFT but all are in Low Energy EFT

Summary of all KK towers

- Massless modes – *massless 4D graviton, one or three massless 4D vectors, two massless 4D scalars* – all separated from KK tower by mass gap $\mathcal{O}(1/r_0)$.
- Tachyonic instabilities can arise only from internal components of 6D gauge fields charged under monopole background – for monopole numbers $|N^I| > 1$ Beware of non-Abelian fluxes!
- Universal behaviour for wavefunction localization, *e.g.* for fermions:



$(r_0, \omega, \bar{\omega}, e) = (1, \frac{1}{4}, 1, 0)$

$n = 0, 1, 2$ *modes for a single negative tension brane at* $u = 0$.
$(m, n) = (-1, 0)$ *mode is massless.*

Codimension 2 and the KK/Brane World Lore

Having analyzed (almost) all bosonic fluctuations and some fermionic fluctuations we suggest:

see also:
Lee & Papazoglou '06
Burgess, de Rham, Hoover, Mason & Tolley '07
Lee & Papazoglou '08

Models with only positive tension branes have qualitatively same physics as standard KK compactifications

- Power law warping induced by codimension two branes has qualitatively same physics as unwarped models

Novel behaviour can arise in presence of negative tension branes:

- Volume of extra dimensions $V_2 = 4\pi\,(1 - \delta/2\pi)\,(r_0/2)^2$ can be made large keeping large mass gap $\mathcal{O}(1/r_0)$.
- Number of massless vector fields can be enhanced beyond isometries thanks to infinitesimal isometries.
- Low Energy EFT for special saddle-spheres with $\delta = -2\pi, -4\pi, \ldots$ indistinguishable from that of smooth sphere.

Conclusions

- Branes and SLED may provide an interesting approach to the Cosmological Constant Problem and Dark Energy.
- Branes in 6D Supergravity provide a laboratory to explore these ideas and codimension 2 branes in general.
- Models with positive tension branes only have similar physics to standard KK compactifications. Negative tension branes give rise to new physics.
- A step towards understanding how vacuum energy contributions cancel amongst a 6D supermultiplet with brane susy breaking.
- SLED has many and diverse predictions within reach of upcoming experiments.