Title: Dealing with derivative interactions

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Abstract: Single field inflation with derivative interactions provides a class of scenarios with interesting theoretical and observational properties. I will discuss properties of correlation functions in generic single field models and the implications of those relationships for inflationary observables, as well as for eternal inflation

DEALING WITH DERIVATIVES

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Columbia University

arXiv:0802.2290 (L. Leblond, S.S.)
arXiv:0812.0818 (S.S.)
work in progress...

THE PLAN

I. Generic single field inflation: the action

* What is relevant for observation?

II. Small sound speed and the Fokker-Planck equation (eternal inflation)

SINGLE FIELD APPROACHES

- Fluid (k-inflation; first derivatives) (Mukhanov and Co.)
- Effective theory of fluctuations (Cheung et al)
- Effective theory of the inflaton (Weinberg, 4-derivatives)
- Special cases: DBI inflation (Silverstein, Tong)

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↓

Extend these

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WHY WE CARE

- Theoretical: Fundamental description of inflation
- Observational: Non-Gaussianity will distinguish between different inflationary physics

SINGLE FIELD FLUID

- Single field models with first-derivative interactions

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}M_p^2 R + P(X,\phi)\right]$$
$$X = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

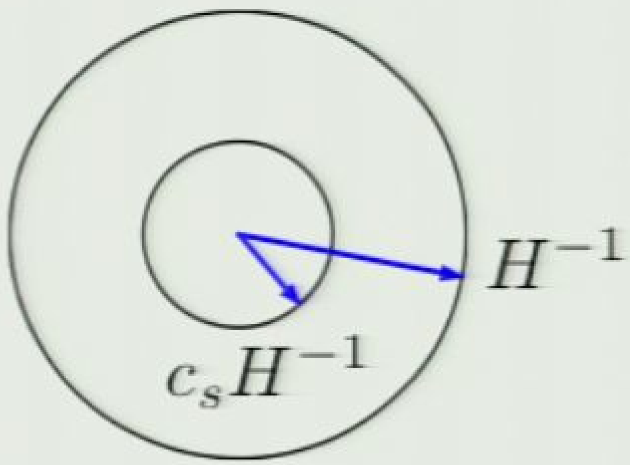
Armendariz-Picon, Damour, Mukhanov;
Garriga, Mukhanov;
Alishahiha, Silverstein, Tong (DBI); Chen;

SOUND SPEED

- Importance of the sum of kinetic terms = small sound speed:

$$c_s^2 = \frac{P_{,X}}{P_{,X} + 2XP_{,XX}}$$

- Scalar fluctuations see sound horizon



(FIRST) DERIVATIVE INTERACTIONS

$$P(X,\phi) = X - V(\phi) + \frac{1}{2}\frac{X^2}{M^4} + \frac{1}{2}\frac{X^3}{M^8} + \ldots$$

(Creminelli)

$$X = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

M = scale of new physics

$M \ll M_p \Rightarrow$ Terms may be important

Brane Action: String theory (symmetries) sums the terms...

$$P(X,\phi) = -M^4\sqrt{1-\frac{2X}{M^4}} - V(\phi) + M^4$$

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MOST GENERAL SCALAR FIELD

- Expand in number of derivatives
- Write down all (unique) terms;
- Use Eq. of Motion
  - * Treat as perturbations

AT 4 DERIVATIVES

- Three (unique of 7) possible terms (+ grav.):

$$X^2,\ X\Box\phi,\ (\Box\phi)^2$$

- Eq. of motion eliminates $\Box\phi$ in terms of the potential

- Only remaining term: $f_1(\phi)\dfrac{X^2}{M^4}$

S. Weinberg

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CONTINUE THIS PROCEDURE

- Goals:
  - Perturbative conditions for generic actions
  - Uncover structure of correlation functions

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AT SIX DERIVATIVES

- Two (unique of 23) terms (scalar field only)

$$X^3\ ,\ g^{\mu\nu}g^{\alpha\beta}(\partial_\mu\partial_\alpha\phi)(\partial_\nu\partial_\beta\phi)X \equiv Y$$

- Gravity / mixed terms suppressed by Planck scale
- Alternative approach: Extrinsic curvature terms (Cheung et al)

$$K^\mu_\mu \propto a^{-2}\partial_i^2(\delta\phi)$$

FOR FLUCTUATIONS...

- Taylor expand $S = S_0 + S_2 + S_3 + \ldots$ $S_0 \approx V(\phi_0)$
- Substitute $\zeta \to \langle \zeta^2 \rangle^{1/2} = \frac{H}{2\pi M_p \sqrt{2\epsilon c_s}}$ $\zeta = -\frac{H}{\dot{\phi}}\delta\phi$
- Two comparisons:
  - Gradient energy small? $\frac{S_2}{S_0} < 1$
  - Interactions perturbative?

$S_2 > S_n,\ n \geq 3$ $\frac{S_3}{S_2} < 1$

6 DERIVATIVE TERMS IN THE THREE-POINT

$$H_I^{(3)} \propto \int dt\, a^3 (\delta\phi)^3 \frac{\dot{\phi}_0}{M^4} \frac{H^3}{c_s^2} \left[f_1 + f_2 \frac{H^2}{M^2 c_s^2} + f_3 \frac{X}{M^4} \right]$$

$$f_1 \frac{X^2}{M^4} \qquad f_2 \frac{Y}{M^6} \qquad f_3 \frac{X^3}{M^8}$$

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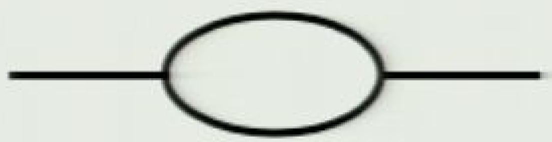
Check loop corrections:



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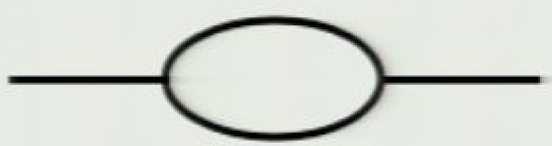

In practice:

$$\frac{S_3}{S_2} < 1$$

(S.S.)

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PERTURBATIVE?

- Original terms in the action: $\frac{X}{M^4} < 1$
- Slow-roll:
- First derivatives only:
- Higher derivatives:

(S.S.)

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No:

$$\frac{X}{M^4} < 1,\ P_\zeta < c_s^4 \Rightarrow \frac{H}{M} < c_s$$

(S.S.)

CONSEQUENCE...

- Eternal inflation is outside the perturbative regime:

$$\frac{(\Delta\phi)_q}{(\Delta\phi)_c} = P_\zeta^{1/2} \sim 1$$

$$P_\zeta < c_s^4$$

(Leblond, Shandera; Creminelli et al)

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STRUCTURE OF CORRELATION FUNCTIONS

- Hierarchical structure:

$$\langle \zeta^n \rangle \propto \langle \zeta^2 \rangle^{(n-1)}$$

  - Generated by non-linearities from gravity, local ansatz
  - Multi-field, cosmic strings, etc don't need to obey this...

STRUCTURE OF CORRELATION FUNCTIONS

- Calculating correlations

$$\langle \delta\phi(k_1,t)\dots\delta\phi(k_n,t)\rangle = i\int_{-\infty}^{t} dt' \,\langle [H_I^{(n)}(t'), \delta\phi(k_1,t)\dots\delta\phi(k_n,t)]\rangle$$

$$H_I^{(n)} \sim A_n \int d^3x\, a^3 \mathcal{L}_2 \left(\frac{\mathcal{P}_\zeta^{1/2}}{c_s^2}\right)^{n-2}$$

- Estimate integral: $(\Delta x)^3\, \Delta t \sim (c_s/aH)^3 H^{-1}$

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(S.S.)

STRUCTURE, CONT'D

- Hierarchical, up to sound speed

$$\langle\zeta^n\rangle \propto \frac{(\langle\zeta^2\rangle)^{n-1}}{(c_s^2)^{n-2}}$$

- Dimensionless combination (Expand Prob. Dist.)

$$\frac{\langle\zeta^n\rangle}{\langle\zeta^2\rangle^{(n-1)}}\langle\zeta^2\rangle^{(n-2)/2} \propto A_n \left(\frac{P_\zeta^{1/2}}{c_s^2}\right)^{n-2}$$

- Perturbative condition

$$c_s^4 > \frac{H^2}{M_p^2 \epsilon c_s} \sim P_\zeta \quad \checkmark$$

(S.S.)

SUMMARIZE:

- While $\frac{X}{M^4} < 1$

  1. First derivatives dominate $\frac{H}{M} < c_s$

  2. Correlation functions are hierarchical

- When $P_\zeta \sim c_s^4$

  1. Higher derivatives as important as first derivatives

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SUMMARIZE:

- While $\frac{H}{M_p} < 1$ $\frac{X}{M^4} < 1$

  1. First derivatives dominate

  2. Correlation functions are hierarchical

  $\frac{H}{M} < c_s$

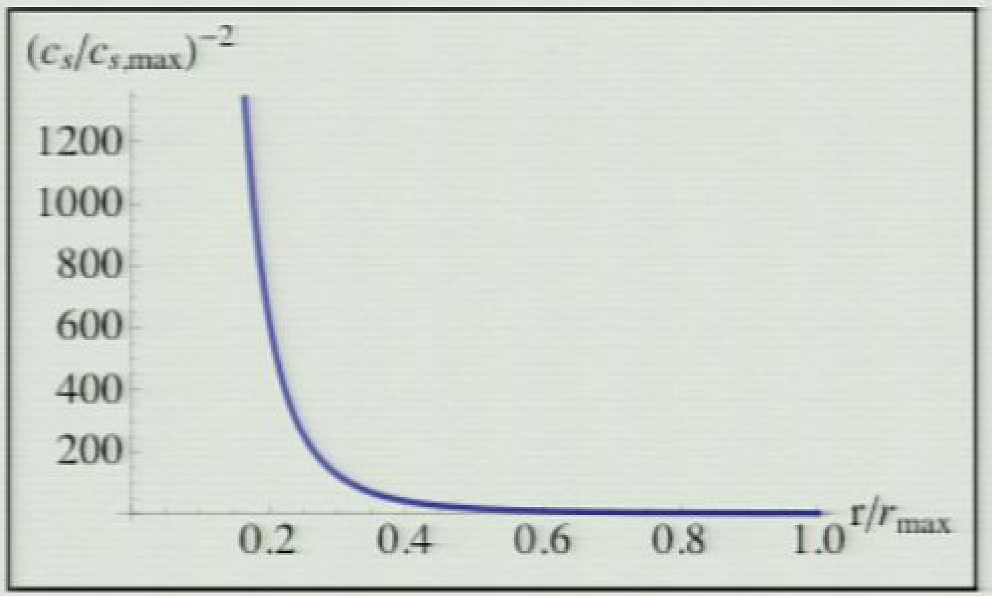
- When $P_\zeta \sim c_s^4$

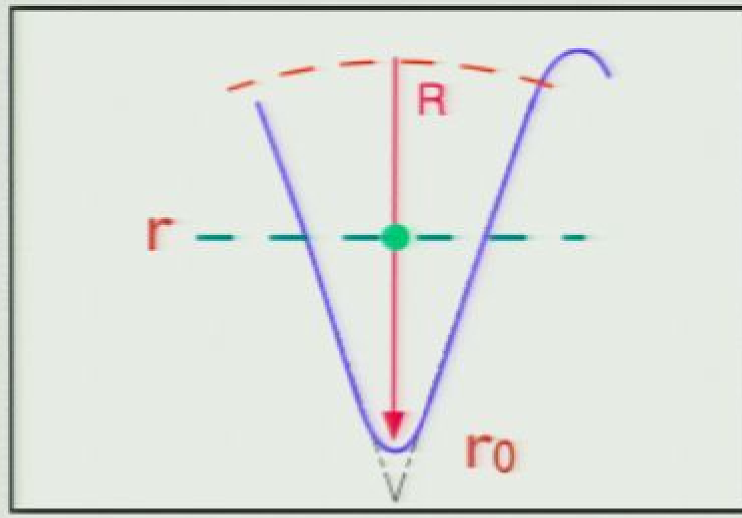
  1. Higher derivatives as important as first derivatives

BRANE INFLATION SUGGESTS:

1. Sound speed can be very small
2. Warped geometry: *M* decreases as inflation proceeds

$$M^4 = (m_s^w)^4 = m_s^4 \left(\frac{r}{R}\right)^4$$





$$\frac{\dot{c}_s}{c_s H} \to \text{constant}$$

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BRANE ACTION

- Lowest order:

$$S = -\int d^4x\, a^3 \left[f(\phi)\sqrt{1 - 2Xf^{-1}(\phi)} - f(\phi) + V(\phi) \right]$$

$$f(\phi) \equiv Sh^{-1}(\phi)\ (= T_3 h^{-1}(\phi)) = M^4(\phi)$$

$$ds_{10}^2 = h^{-1/2}(y) g_{\mu\nu}\, dx^\mu dx^\nu + h^{1/2}(y) g_{mn} dy^m dy^n$$

CURVATURE TERMS?

- Extrinsic Curvature of the brane

$$K^\mu_\mu \approx \frac{\partial_i^2(\delta\phi)}{a^2 M^2} < M$$

√ (Under control)

- Corrections to the action? Guess...

$$S_{DBI,\mathrm{guess}} = -\int d^4x\, a^3\, f(\phi)\sqrt{1-2X/M^4}$$
$$\times\left[1+\mathcal{F}^{mnkl}(X/M^4)(\partial_m\partial_n\phi)(\partial_k\partial_l\phi)+\dots\right]$$

Guess from: Andreev, Tseytlin: field strength corrections for bosonic case (SUSY case); T-dual

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Ib. IMPLICATIONS FOR OBSERVATION

HOW NON-GAUSSIAN?

Slow roll	$f_{NL} \propto -(n_s - 1) \approx 0.04$
vs	
Observation	$\|f_{NL}^{eff}(k_{CMB})\| < \mathcal{O}(100)$
vs	
Theoretical Bound	$\|f_{NL}^{eff}\| < 10^{9/2}$

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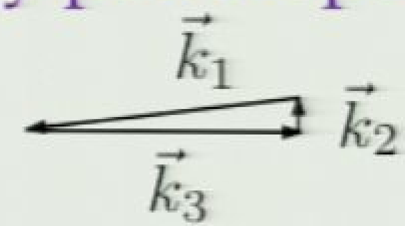
COMPARING STATISTICS

Correlation functions

vs

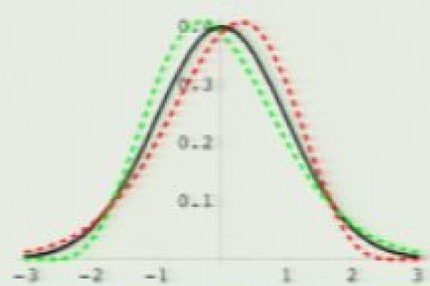
Anything else

CMB bispectrum, galaxy power spectrum



Minkowski functionals

Cluster number counts



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III. FOKKER-PLANCK AND INTERACTIONS (ETERNAL INFLATION)

STOCHASTIC INFLATION

- Global picture
- Eternal inflation
- Useful for understanding IR-dependence in loop corrections?
- Interactions :: Non-Gaussianity :: deviations from exact dS

w/ Sash Sarangi; Tolley, Wyman

FOKKER-PLANCK

- Quadratic Fokker-Planck

$$\frac{\partial f(\phi,t)}{\partial t} = \frac{1}{8\pi^2}\frac{\partial^2}{\partial\phi^2}[H^3 f(\phi,t)] + \frac{\partial}{\partial\phi}\left[2M_p^2 H' f(\phi,t)\right]$$

- Probability conservation + locally peaked jump distribution (Master Equation)

$$\frac{\partial f(\phi,t)}{\partial t} = \int_{-\infty}^{\infty}[-f(\phi,t)W(\phi;\phi') + f(\phi',t)W(\phi';\phi)]d\phi'$$

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Diffusion term

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Diffusion term

Drift term

- Probability conservation + locally peaked jump distribution (Master Equation)

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A STEP BACK...

- Taylor expand: $\xi \equiv \phi - \phi'$

$$\frac{\partial f(\phi,t)}{\partial t} = -\frac{\partial}{\partial \phi}[\mu_1(\phi) f(\phi,t)] + \frac{1}{2}\frac{\partial^2}{\partial \phi^2}[\mu_2(\phi) f(\phi,t)] + \dots$$

- Jump Moments

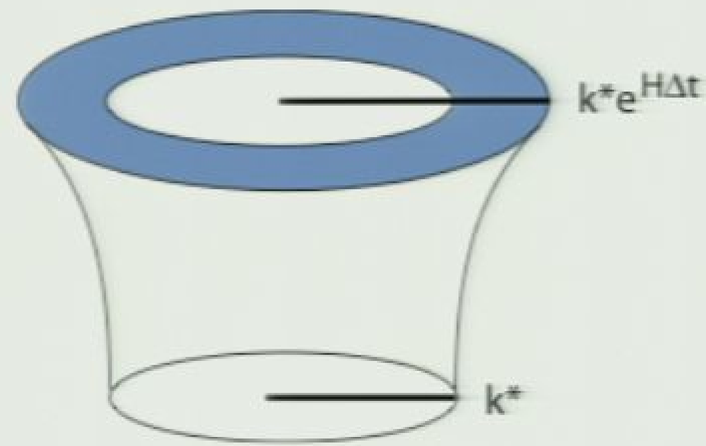
$$\mu_1(\phi) = \int_{-\infty}^{\infty} \xi W(\phi; \phi - \xi) d\xi = \lim_{\delta t \to 0} \frac{\delta\phi}{\delta t} = v_\phi$$

$$\mu_2(\phi) = \int_{-\infty}^{\infty} \xi^2 W(\phi; \phi - \xi) d\xi = \lim_{\delta t \to 0} \frac{(\delta\phi)^2}{\delta t}$$

MASSLESS FIELD IN FIXED dS

- Modes:

$$\psi_k(\eta) = \frac{\sqrt{\pi}}{2} H \eta^{3/2} H^{(1)}_{3/2}(k\eta)$$



- Shift:

$$\lim_{\delta t \to 0} \frac{\langle \phi^2 \rangle}{\delta t} = \lim_{\delta t \to 0} \frac{1}{\delta t} \int_H^{He^{H\delta t}} \frac{d^3k}{(2\pi)^3} |\psi_k|^2$$

reviewed by: A. Linde, hep-th/0503203

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MASSLESS FIELD CONT'D

- Diffusion term only

$$\frac{\partial f(\phi,t)}{\partial t} = \frac{H^3}{8\pi^2}\frac{\partial^2}{\partial\phi^2}[f(\phi,t)]$$

- Gaussian distribution, variance grows with time

$$\sigma^2 = H^3 t/4\pi^2$$

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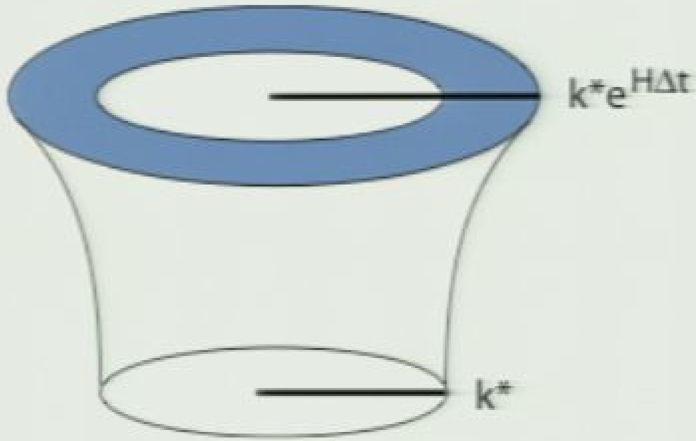

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MASSIVE FIELD IN FIXED dS

- Conditions: $m^2 \ll H^2$

$$V = V_0 + \frac{1}{2}m^2\phi^2 \qquad V_0 \gg \frac{1}{2}m^2\phi^2$$

- Mode function shifts $H_{3/2} \to H_\nu$ $\qquad \nu \approx \sqrt{\frac{9}{4} - \frac{m^2}{H^2}}$
- Integrating:

$$\langle\phi^2\rangle = \frac{3H^4}{8\pi^2 m^2}\left[1 - \exp\left(\frac{-2m^2}{3H}\delta t\right)\right]$$

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MASSIVE FIELD CONT'D

- Jump moments:

$$\lim_{\delta t \to 0} \frac{\langle \phi^2 \rangle}{\delta t} = \frac{H^3}{4\pi^2}$$

$$\dot{\phi}_0 = -\frac{V'}{3H}$$

- Fokker-Planck

$$\frac{\partial f(\phi, t)}{\partial t} = \frac{H^3}{8\pi^2} \frac{\partial^2}{\partial \phi^2} [f(\phi, t)] + \frac{1}{3H} \frac{\partial}{\partial \phi} [V' f(\phi, t)]$$

MASSIVE FIELD, CONT'D

- Some checks:

$$\lim_{\delta t\to 0}\frac{\langle\phi_{cl}^2\rangle}{\delta t} = \lim_{\delta t\to 0}\dot{\phi}_0^2\times\delta t$$

$$\lim_{\delta t\to 0}\frac{\langle\phi_{cl}^n\rangle}{\delta t} = \lim_{\delta t\to 0}\dot{\phi}_0^n\times(\delta t)^{n-1}$$

$$\lim_{\delta t\to 0}\frac{\langle\phi_q^4\rangle}{\delta t} = \lim_{\delta t\to 0}3\frac{\langle\phi_q^2\rangle^2}{\delta t} = 0$$

MASSIVE FIELD, CONT'D

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Quadratic FP √

MASSIVE FIELD, CONT'D

- Taylor expand, integrate by parts:

$$\frac{d\langle\phi\rangle}{dt} = -\frac{m^2}{3H}\langle\phi\rangle$$

$$\langle\phi\rangle(t) = \phi(0)e^{-(m^2/3H)t}$$

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Classical EoM √

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Classical EoM √

$$\frac{d\langle\phi^2\rangle}{dt} = \frac{2H^3}{8\pi^2} - \frac{2m^2}{3H}\langle\phi^2\rangle$$

MASSIVE FIELD, CONT'D

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Classical EoM √

$$\frac{d\langle\phi^2\rangle}{dt} = \frac{2H^3}{8\pi^2} - \frac{2m^2}{3H}\langle\phi^2\rangle$$

Stationary solution: $\langle\phi^2\rangle = \frac{3H^4}{8\pi^2 m^2}$ √

MASSIVE FIELD, CONT'D

- Check higher order terms...

$$\frac{d\langle\phi^3\rangle}{dt} = \frac{6H^3}{8\pi^2}\langle\phi\rangle - \frac{3m^2}{H}\langle\phi^3\rangle$$

$$\frac{d\langle\phi^4\rangle}{dt} = \frac{3H^3}{2\pi^2}\langle\phi^2\rangle - \frac{4m^2}{3H}\langle\phi^4\rangle$$

MASSIVE FIELD, CONT'D

- Check higher order terms...

$$\frac{d\langle\phi^3\rangle}{dt} = \frac{6H^3}{8\pi^2}\langle\phi\rangle - \frac{3m^2}{H}\langle\phi^3\rangle$$

$$\langle\phi^3\rangle \to 0,\ Ht > \frac{3H^2}{m^2}$$ √

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TIME SCALES

- H constant: $$H\delta t \ll \frac{1}{\epsilon} = \frac{3H^2}{m^2}\left[\frac{V_0}{\frac{1}{2}m^2\phi^2}\right]$$
- Stationary solution reached: $$H\delta t > \frac{3H^2}{2m^2}$$

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TIME SCALES, CONT'D**

- If $H \neq$ constant
  - Small kick to velocity

$$\frac{(\delta\dot{\phi})_q}{\dot{\phi}_0} \ll 1 \quad \Rightarrow H\delta t \ll \frac{8\pi^2 M_p^2 \epsilon}{H^2\eta^2} \equiv \frac{1}{P_\zeta \eta^2}$$

  - small kick to acceleration

$$\frac{\delta(\dot{\phi}_0)_q}{\delta t} = \left(\frac{\delta\phi_q}{\delta t}\right)\left(\frac{\partial\dot{\phi}_0}{\partial\phi}\right) \ll \ddot{\phi}_0 < H\dot{\phi}_0 \Rightarrow P_\zeta \eta^2 < H\delta t$$

TIME SCALES, CONT'D**

- Use an interval of order H^{-1}

$$P_\zeta \eta^2 \ll \delta t H \ll \frac{1}{P_\zeta \eta^2}$$

- Quantum contributions dominant?
  - Eternal inflation condition

WITH (CLASSICAL) GRAVITY: CHAOTIC INFLATION

- Allow H to vary $V = V_0 + \frac{1}{2}m^2\phi^2$ $V_0 \ll \frac{1}{2}m^2\phi^2$
- Cannot assume the late time stationary solution:

$$H\delta t \ll \frac{1}{\epsilon} \qquad \epsilon = \frac{2M_p^2}{\phi^2} = \frac{m^2}{3H^2}$$

- Fokker-Planck:

$$\frac{\partial f(\phi,t)}{\partial t} = \frac{\partial^2}{\partial \phi^2}\left[\frac{H^3}{8\pi^2} f(\phi,t)\right] + \frac{\partial}{\partial \phi}\left[\frac{\sqrt{6} m M_p}{3} f(\phi,t)\right]$$

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GENERICALLY NON-GAUSSIAN

$$\frac{d\langle\phi\rangle}{dt} = -\frac{\sqrt{6}mM_p}{3}$$ Classical EoM √

$$\frac{d\langle\phi^2\rangle}{dt} = \frac{m^3}{24\sqrt{6}\pi^2 M_p^3}\langle\phi^3\rangle - \frac{2\sqrt{6}mM_p}{3}\langle\phi\rangle$$

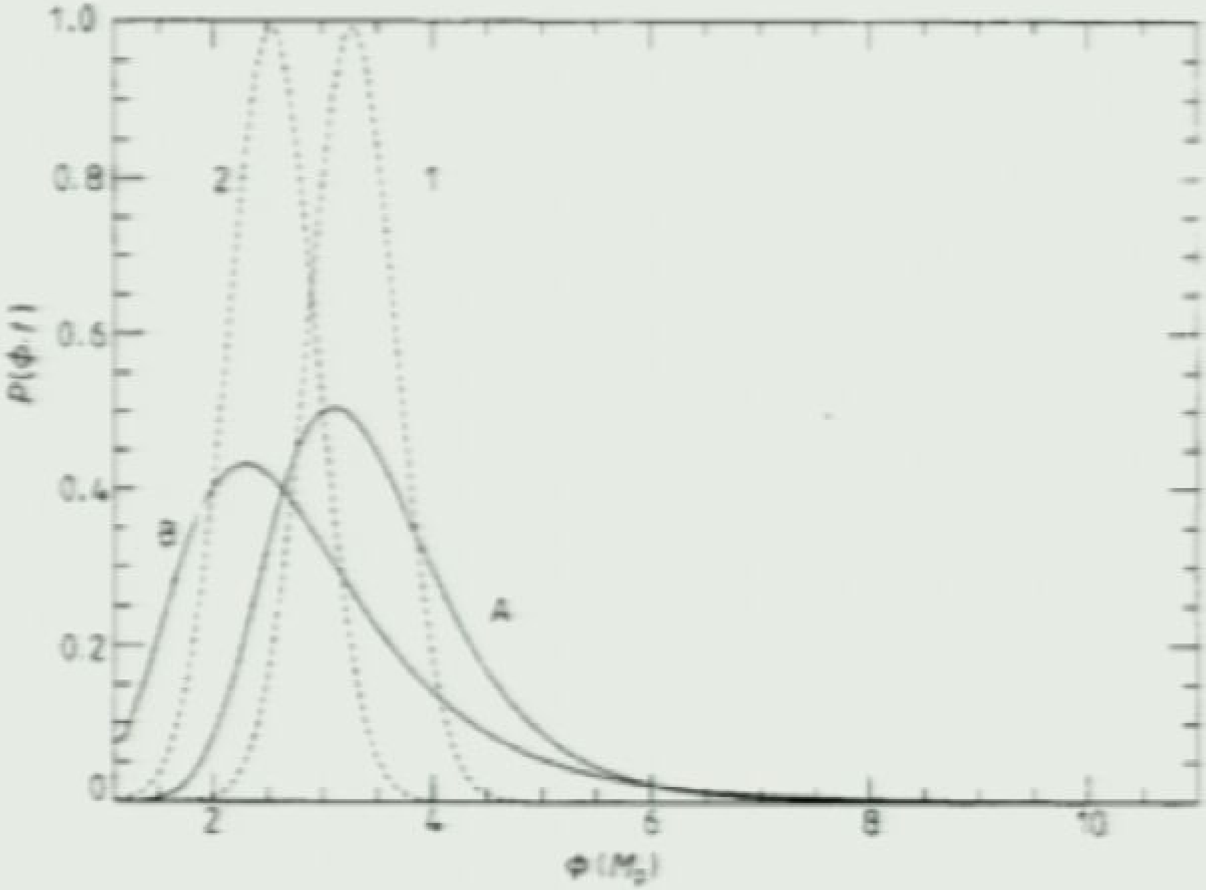

Figure 1. Evolution of the probability density in the chaotic inflationary scenario with a Gaussian initial probability centred at $\phi = 4M_p$ with a standard deviation of $0.4M_p$. Full curves, with quantum term; broken curves, classical evolution. 1 and A, $t = 45M_p^{-1}$; 2 and B, $t = 90M_p^{-1}$.

Bardeen, Bublik (1987)

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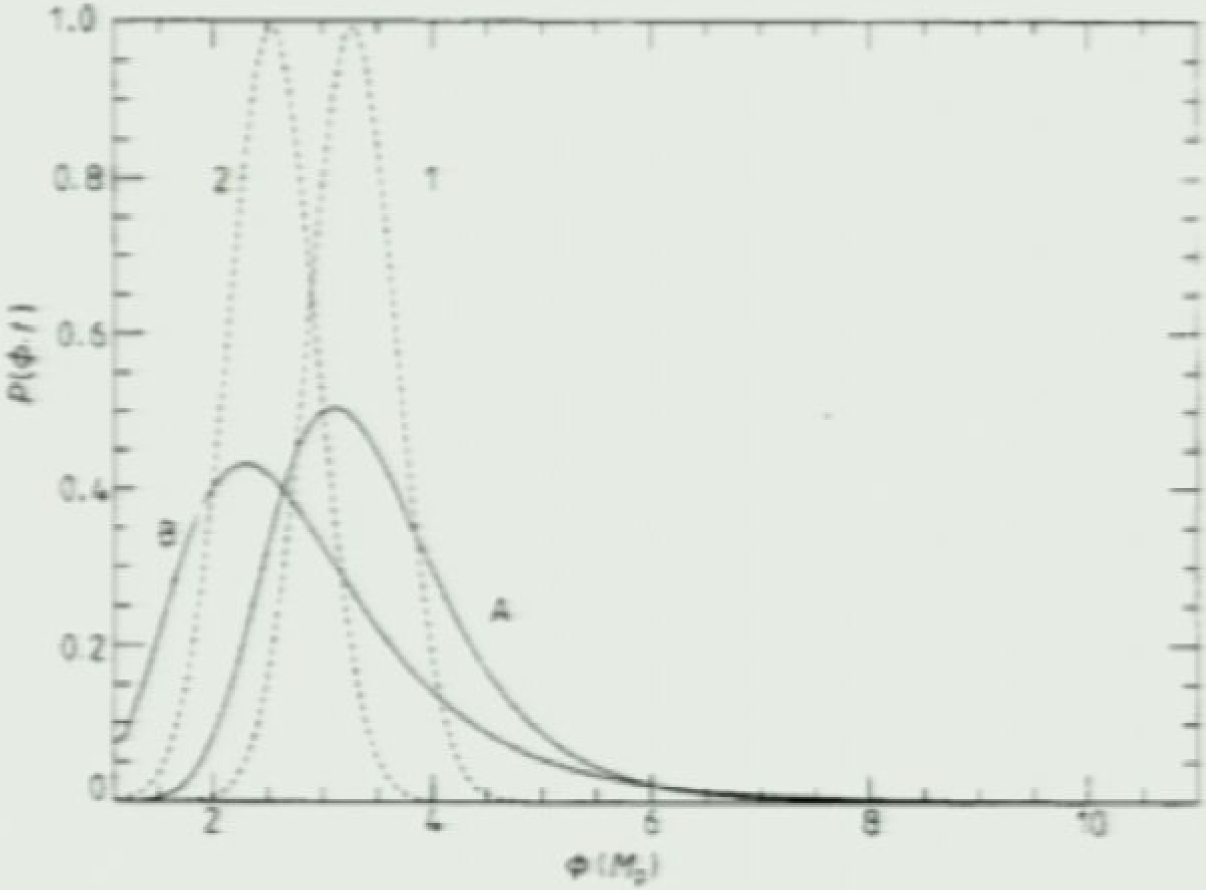

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ADDING INTERACTIONS**

1. Dynamical evolution of H √
2. Higher terms in the Fokker-Planck equation
3. Quantum calculation of quantities to evolve

SLOW-ROLL

- A typical term in the (slow-roll) action

$$\mathcal{L}_3 \supset \left(\frac{-\dot{\phi}_0}{4HM_p^2} \right) \delta\phi(\dot{\delta}\phi)^2$$

- Gives

$$\frac{\langle(\delta\phi(x,t')^3\rangle}{\delta t} \approx \frac{\sqrt{\epsilon}H^5}{M_p}$$

(S.S.)

SLOW-ROLL, CONT'D

- Fokker-Planck is modified:

$$\frac{\partial f(\phi,t)}{\partial t} = \frac{1}{8\pi^2}\frac{\partial^2}{\partial\phi^2}[H^3 f(\phi,t)] + \frac{\partial}{\partial\phi}\left[2M_p^2 H' f(\phi,t)\right] - \frac{\partial^3}{\partial\phi^3}\left[\frac{\sqrt{\epsilon}H^5}{16\pi^4 M_p} f(\phi,t)\right]$$

- Require modifications are small:

$$\frac{H}{M_p} < 1$$

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- Require modifications are small:

$$\frac{H}{M_p} < 1 \qquad \frac{S_2}{S_0} < 1 \quad \checkmark$$

(S.S.)

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SMALL SOUND SPEED

- When $\epsilon < 1 - c_s^2$
- Dominant terms are modified by powers of sound speed:

$$\mathcal{L}_3 \supset \frac{\delta\phi(\dot{\delta\phi})^2}{2M_p\sqrt{2\epsilon c_s}c_s^3}$$

- Which gives: $$\frac{\langle(\delta\phi)^3\rangle}{\delta t} \propto \frac{H^5}{M_p\sqrt{2\epsilon c_s}c_s^3}$$

(S.S.)

SMALL SOUND SPEED

- Keep corrections to quadratic Fokker-Planck small:

$$\frac{H^2}{M_p^2} < c_s^3$$

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QUANTUM CORRECTIONS

- One-loop correction to the two point, small sound speed:

$$(\delta\phi(x))^2 = \frac{H^2(H\delta t)}{4\pi^2}\int_H^{He^{H\delta t}} \frac{dp}{p}\left[1 + \frac{bH^2}{16\pi^2 M_p^2 \epsilon c_s^5}\log\left(1+\frac{\Lambda_{UV}}{p}\right)+\ldots\right]$$

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Perturbative bound

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CAN WE GET AROUND THIS?

- Fokker-Planck can't handle eternal inflation with small sound speed?
- Still useful in other contexts?
- What implications for tunneling/landscape?

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CONCLUSIONS I

- Non-Gaussianity: fundamental inflationary physics
- Need to understand the predictions of fundamental models
- For a *generic single-field*:
  - Sound speed perturbatively bounded
  - Structure of correlation functions is hierarchical
- Add modification to initial state? (Jan Pieter's talk)

CONCLUSIONS II

- Stochastic inflation is sensitive to the gradient and perturbative bounds
- Corrections due to small sound speed cannot be calculated in the eternally inflating regime
- How to see the global picture for the small sound speed case?