Title: Perturbative Bounds in Inflation & Quantum Loops

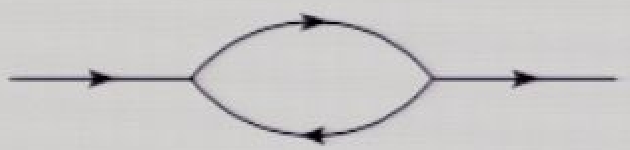
Date: May 22, 2009 11:30 AM

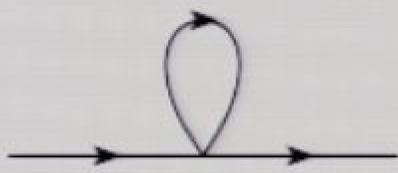
URL: http://pirsa.org/09050062

Abstract: We estimate the size of loop corrections in various inflationary systems and determine the region of parameter space where the perturbation theory around a quasi de Sitter background is strongly coupled. In some models, we argue that backreaction to the inflatonary background become important before the erturbations become strongly coupled while in others, there seems to exist a legitimate strongly coupled but still inflating regime. We also demonstrate that loop effects could be dominant in the bispectrum while still having a well controlled perturbation theory and we explore the phenomenological implications.

The Perturbative Regime of Inflation and Loop Corrections

Louis Leblond
Texas A&M





arXiv:0802.2290
arXiv:0805.1229
work in progress

L.L. and Sarah Shandera
Bhaskar Dutta, Jason Kumar, L.L.
Jason Kumar, L.L., Arvind Rajaraman

Motivation

$$\langle \zeta_k^2 \rangle =$$

- From observation: Planck and other data set are coming. We want to calculate more precisely the primordial curvature 2-pt and higher.

- From theory: Holographic description of the early universe (dS/CFT), eternal inflation... Anomalous dimensions of some operators in quasi-de Sitter.

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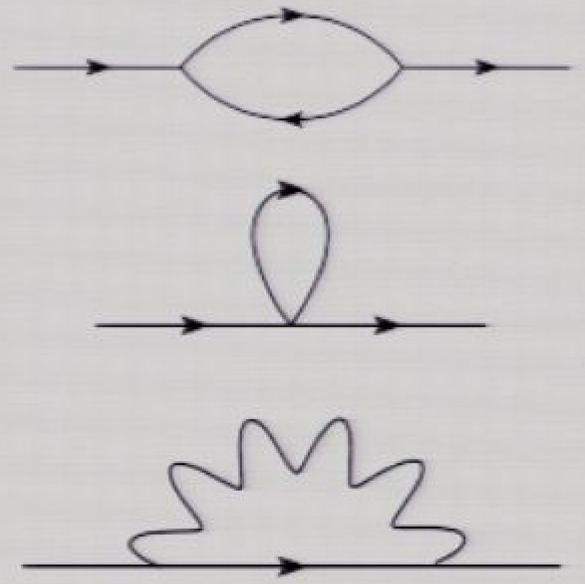
$$\langle \zeta_k^2 \rangle =$$  $+$ 

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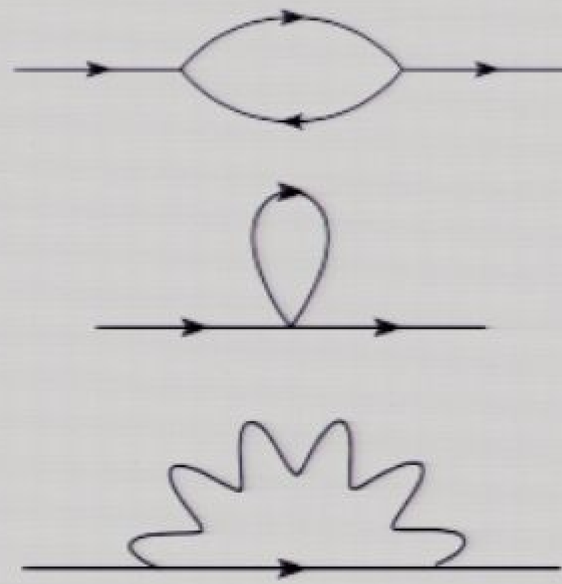

$$2.74 \times 10^{-9}$$

(on CMB scale)

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Precision cosmology = Loops?

- Dissecting the 1-loop, there are 3 main parts

IR log

$$\mathcal{P}_\zeta = \mathcal{P}_\zeta^{tree}\left(1 + A(\ln(kL) + \beta)\right)$$

Amplitude

UV constant

- Q: When does the perturbative regime breaks down? For log of order 1, A~1.
- Q: Can we have large loops effect?
- Q: Renormalization? Resummation of Logs?

Outline

- Set-up: 2 examples & 4 numbers
- Backreaction/perturbative bounds
- Loop dominated higher point functions.

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2 examples, 4 numbers:

- Single field, general sound speed. (e.g DBI & Ghost inflation)

$$S = \frac{1}{2}\int d^4x\sqrt{-g}\left(M_p^2 R + 2P(X,\phi)\right)$$

$$X = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi \qquad c_s^2 = \frac{dP}{d\rho} = \frac{P_{,X}}{P_{,X} + 2XP_{,XX}}$$

- Many uncoupled fields (e.g. N-flation type or 1 inflaton and N spectators massless fields)

$$S = \int d^4x\sqrt{g}\left(\sum_i \frac{1}{2}\dot{\phi}_i^2 - V(\phi_i)\right)$$

not going to look at most general EFT theory

Effective Field Theory of Perturbation

- Expand around a FRW inflating background. (set the tensor modes to zero).

$$\delta\phi(\vec{x},t) \quad \zeta(\vec{x},t)$$

$$S = S_0 + S_2 + S_3 + \ldots$$

$$S_0 \quad > \quad S_2$$ Gradient Energy Condition

$$S_2 \quad > \quad S_n,\ n \geq 3$$ Perturbative Regime

c.f. Nicolis

very simplified, power counting

Burgess, Lee & Trott

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Acquaviva, Bartolo, Mattarese, Riotto
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Neglecting interactions and solving EoM

$$\langle \zeta^2 \rangle^{1/2} \sim \frac{H}{\sqrt{\epsilon c_s} M_p}$$

over time
and size

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$$\Delta x \sim \frac{c_s}{aH}$$

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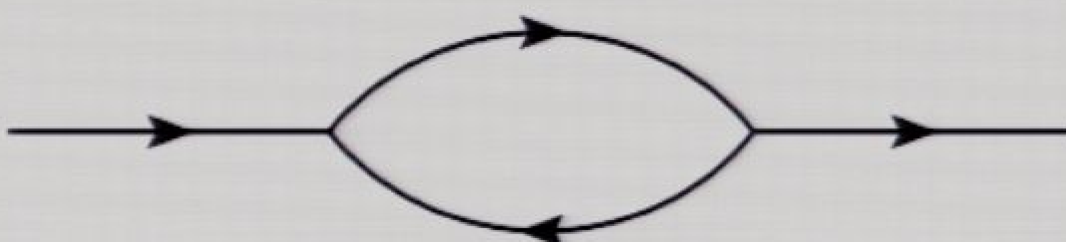

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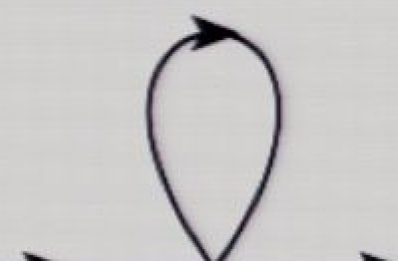

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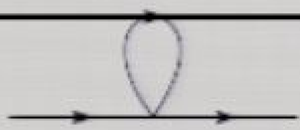

Seery/Sloth

Gao & Xu

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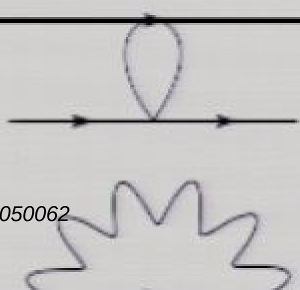

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Weinberg/Adshead,
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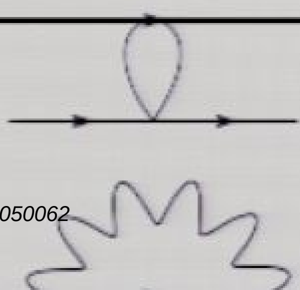

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if all fields participate in inflati

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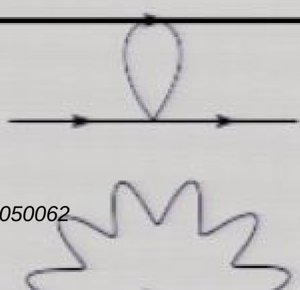	Seery/Sloth Dimastrogiovanni & Bartolo	Gao & Xu	Weinberg/Adshead, Easther & Lim

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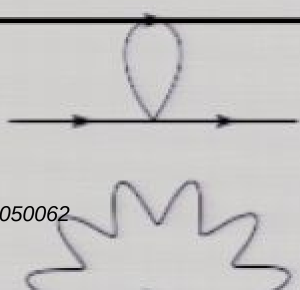

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Dimastrogiovanni & Bartolo

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Shandera

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Cheung, Creminelli, Fitzpatric
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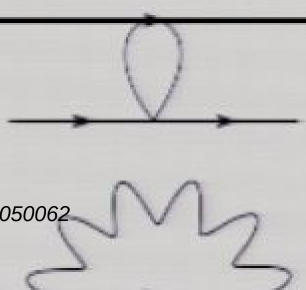

Seery/Sloth Dimastrogiovanni & Bartolo	Gao & Xu	Weinberg/Adsheac Easther & Lim

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Comments on Eternal Inflation

- Eternal inflation occur when the quantum fluctuations of the inflaton are of the same order as the classical motion. This translate to order 1 curvature perturbations $\zeta \sim 1 \quad \mathcal{P}^\zeta \sim 1$

Creminelli, Dubovsky, Nicc
Senatore, Zaldarriaga

- For slow-roll, you can locally reach order one curvature in the weak coupling regime

$$\mathcal{P}^\zeta \sim \frac{1}{\epsilon}$$

- But for small sound speed

$$\mathcal{P}^\zeta < c_s^4 < 1$$

c.f Shander
c.f. Nicolis

- There is no point locally, where one can eternal inflate in the perturbative regime if the sound speed is small.

Tolley, Wyman
Helmer Winitzk

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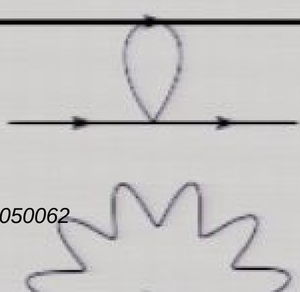	Seery/Sloth Dimastrogiovanni & Bartolo	Gao & Xu	Weinberg/Adshead, Easther & Lim

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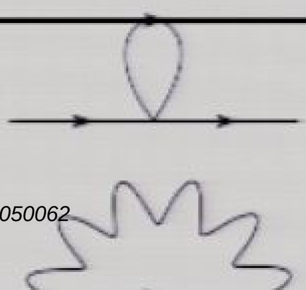
Perturbative (S4/S2 < 1)	$\frac{H^2}{M_P^2} < 1$	$\frac{\mathcal{P}_\zeta}{c_s^4} < 1$	??
Loop Calculation (A)	$\frac{H^2}{M_P^2}$	$\frac{\mathcal{P}_\zeta}{c_s^4}$	$\frac{H^2}{M_p^2}$



Seery/Sloth

Dimastrogiovanni

& Bartolo

Gao & Xu

Weinberg/Adsheac

Easther & Lim

if all fields participate in inflati

These bounds are showing up everywhere, in every talk...

- These are order of magnitude estimates which are not true in general. No huge IR effect included, no cancellation of diagrams.

- For small sound speed, there might be higher derivative interaction besides X but no new more dominant term seem to appear.

Shandera

- Generic action at quadratic order also gets the same bound (including gravity terms) for single field.

$$\frac{\mathcal{P}_\zeta}{c_s^4} < 1$$

Cheung, Creminelli, Fitzpatric
Kaplan, Senatore

Comments on Eternal Inflation

- Eternal inflation occur when the quantum fluctuations of the inflaton are of the same order as the classical motion. This translate to order 1 curvature perturbations $\zeta \sim 1 \quad \mathcal{P}^{\zeta} \sim 1$

Creminelli, Dubovsky, Nicc
Senatore, Zaldarriaga

- For slow-roll, you can locally reach order one curvature in the weak coupling regime

$$\mathcal{P}^{\zeta} \sim \frac{1}{\epsilon}$$

- But for small sound speed

$$\mathcal{P}^{\zeta} < c_s^4 < 1$$

c.f Shander
c.f. Nicolis

- There is no point locally, where one can eternal inflate in the perturbative regime if the sound speed is small.

Tolley, Wyman
Helmer Winitzk

- eternal inflation is naively easier (sound speed enhancement)

Large Loops Effect?

- All the A are very small. $\frac{H^2}{M_p^2} < 10^{-10}$
- The Gaussianity of CMB tells us that these effect have to be small

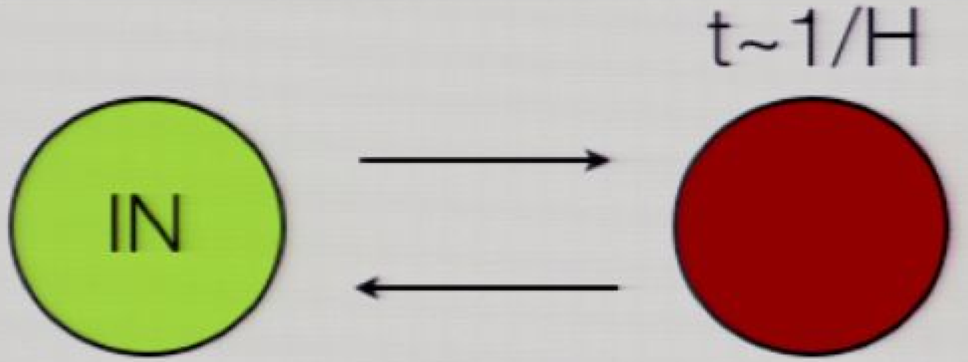
$$|f_{NL}| \sim \frac{1}{c_s^2} < 10^2 \qquad \frac{\mathcal{P}^\zeta}{c_s^4} < 10^{-5}$$

L.L. Shandera
Armendaritz-Picon, Lim
Khoury, Piazza

- could get large loops on other scales than CMB. For example running of cs
- large logs?? Sloth, Riotto & Sloth
- most sensitive probe may be the running $n_s - 1 \sim \frac{A}{1 + A\ln(kL)}$

Switch to a completely different way of getting loop like effects

Loops from δN



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$$N = \int_{\phi_*}^{\phi_c} \frac{H}{\dot{\phi}} d\phi \qquad \zeta = \delta N = -\frac{H}{\dot{\phi}}\delta\phi\Big|_*$$

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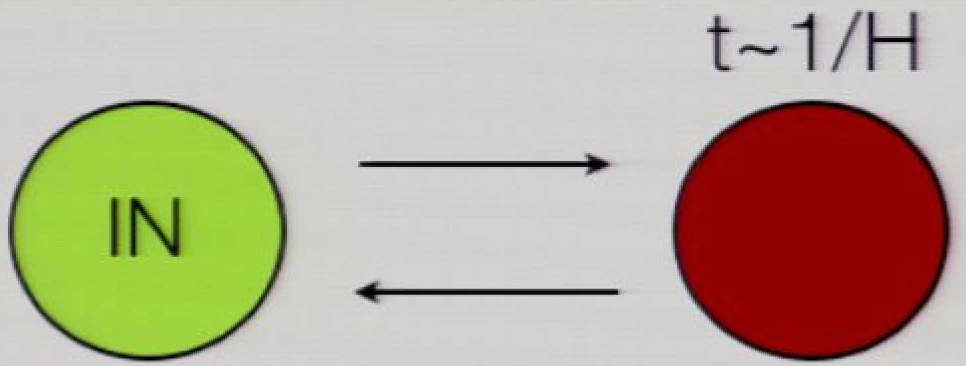

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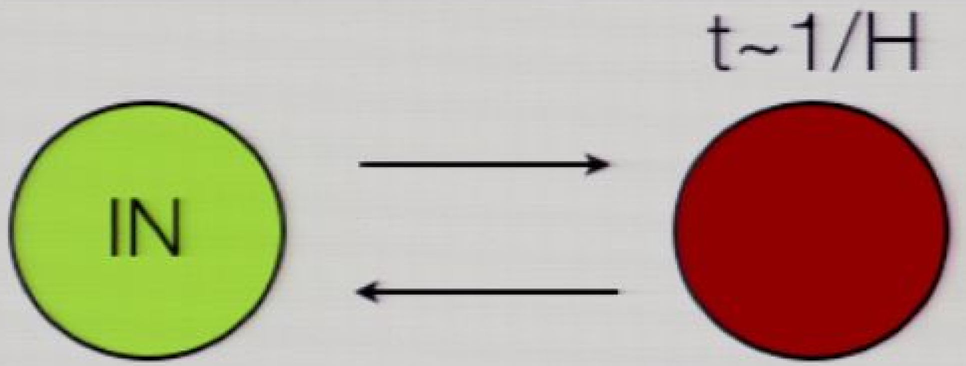

classical evolution
$\longrightarrow$

$$\zeta(t,\vec{x}) = \delta N(t,\vec{x})$$

Sasaki & Stewart

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$$\zeta = \alpha\delta\phi + \beta\delta\phi^2$$

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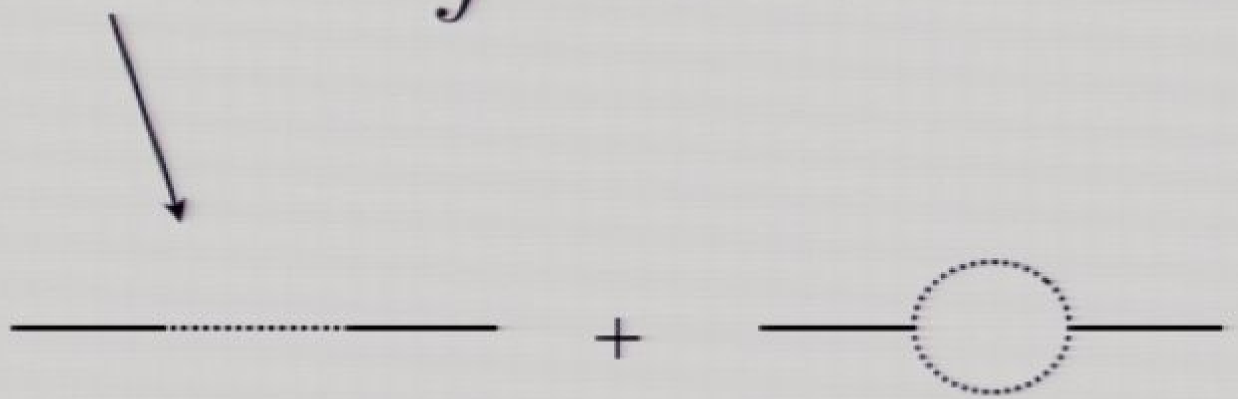
ζ^2



$$\begin{aligned}\langle\zeta_{k_1}\zeta_{k_2}\rangle_{loop} &= \beta^2\int d^3k' d^3k''\,\langle\delta\phi_{k_1-k'}\delta\phi_{k'}\delta\phi_{k_2-k''}\delta\phi_k \\ &= \beta^2\delta^3(\sum_i \vec{k}_i)\int d^3\vec{k}'\,\frac{(2)(2\pi^2\mathcal{P}_*)^2}{|\vec{k}-\vec{k}'|^3 k'^3}\end{aligned}$$

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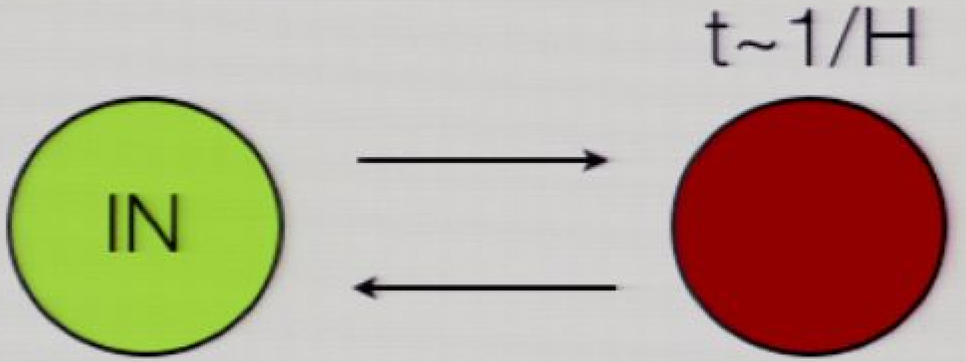

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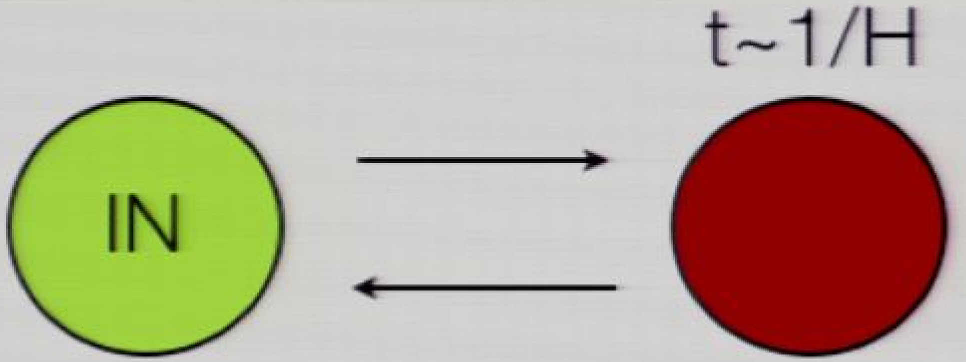

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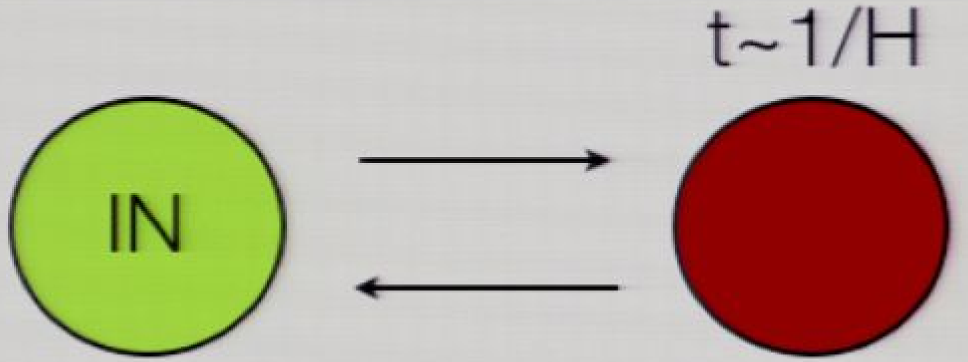

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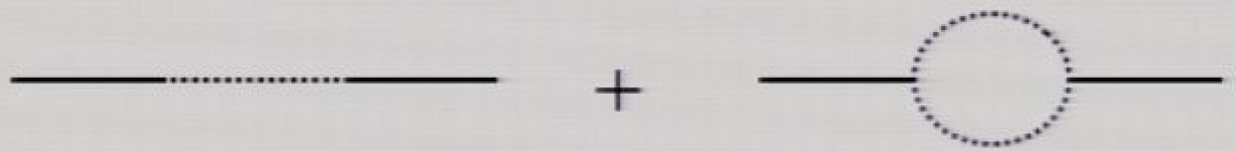

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Assuming no tilt, loop integral has two poles

$$\int d^3\vec{k}' \frac{1}{|\vec{k}-\vec{k}'|^3 k'^3} \qquad \sim \ln(kL)$$

And converge rapidly for k' > k. It is UV finite. (good since this formalism is not valid inside the horizon)

- The series can truncate and in general each coefficient could be completely independent of each other. If the higher order term are not there (or much smaller) one can consistently have large 1-loop coefficient while neglecting 2-loops and higher

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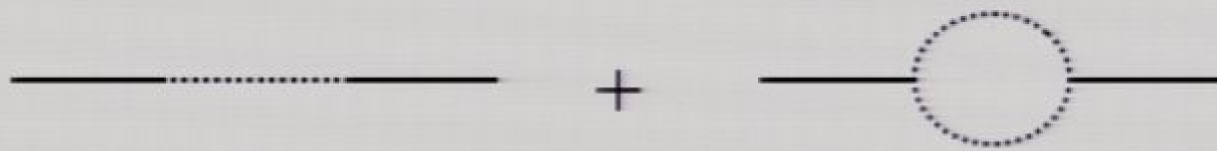

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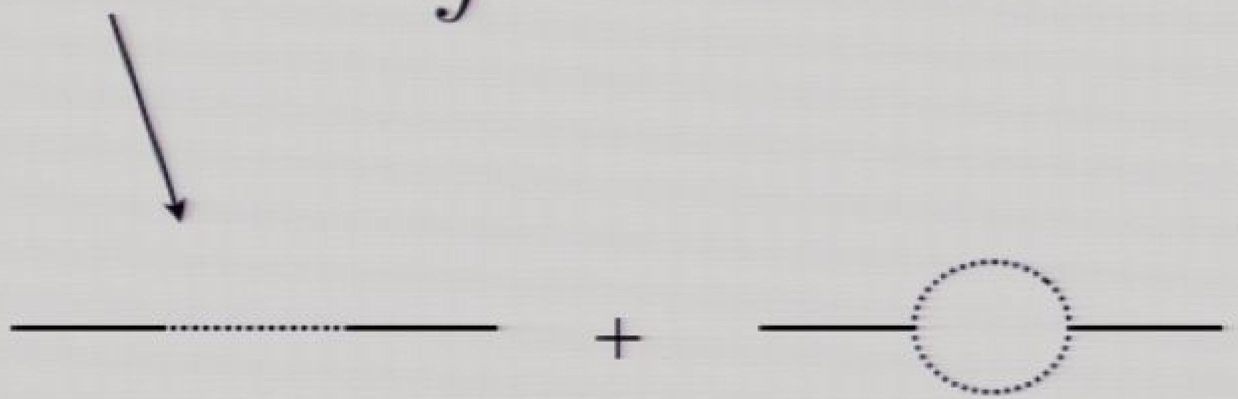

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What is the IR cutoff?

- For these delta N loops, one can argue that the cutoff is limited by observations.

Lyth/
Bartolo, Matarrese, Pietr
Riotto & Seery/
Enqvist, Nurmi, Podolsk
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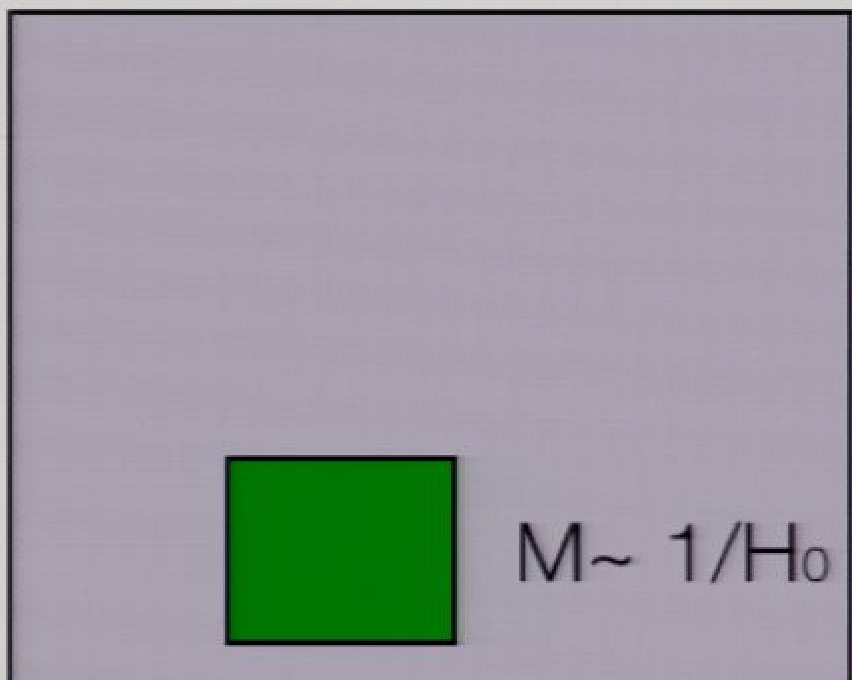
$$\delta\phi = \phi(\vec{x}, t) - \phi\ \ (t)$$



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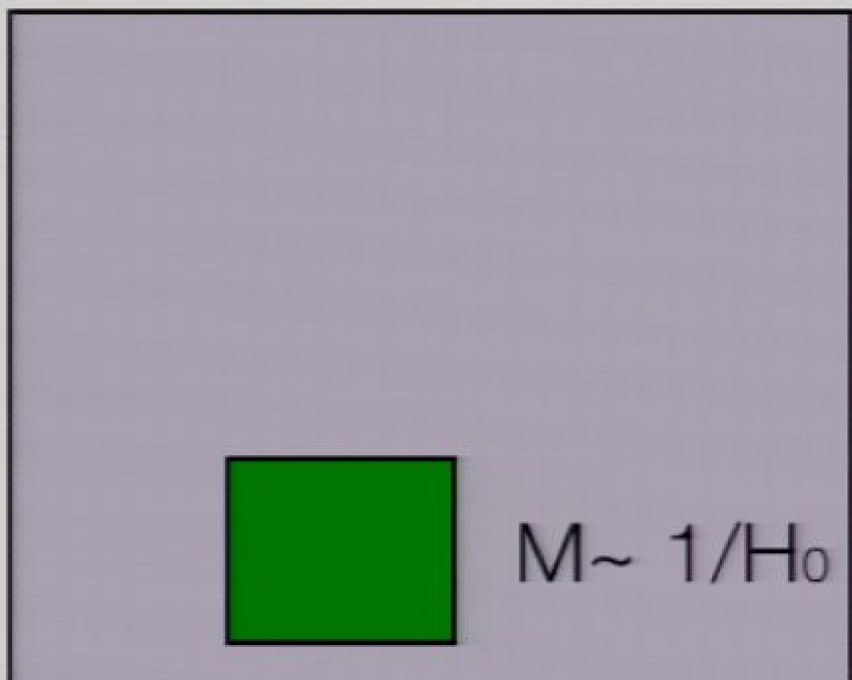

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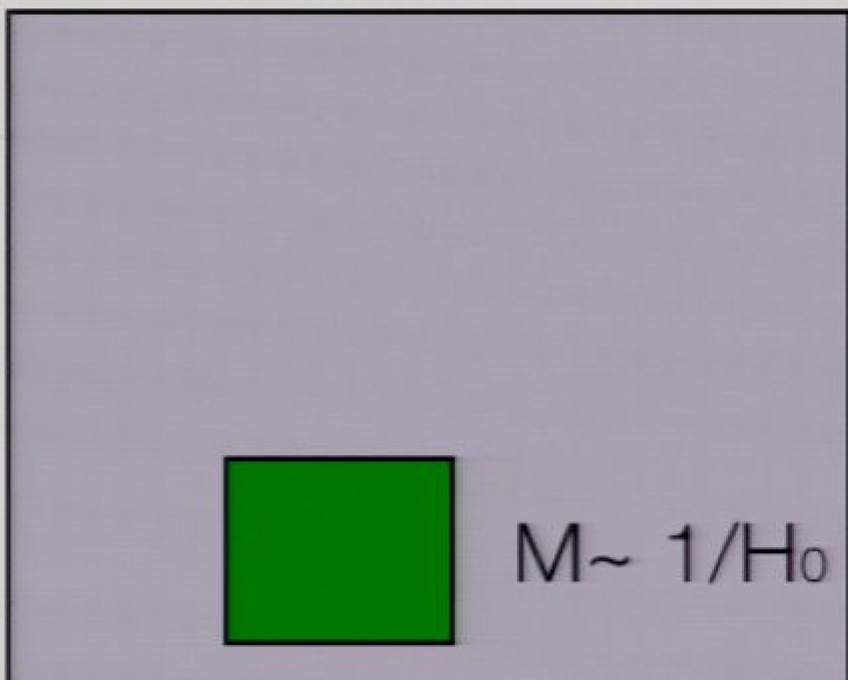

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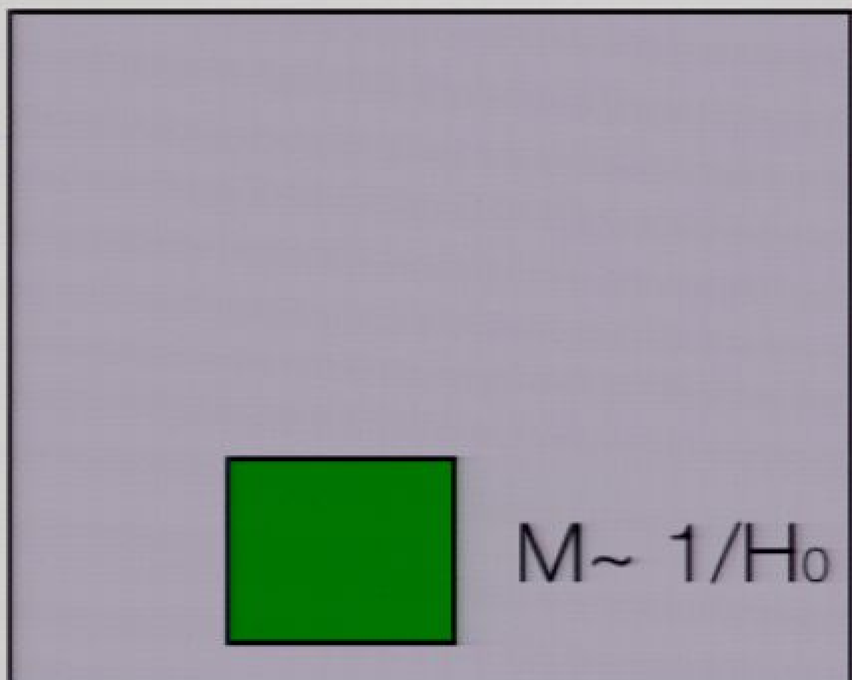

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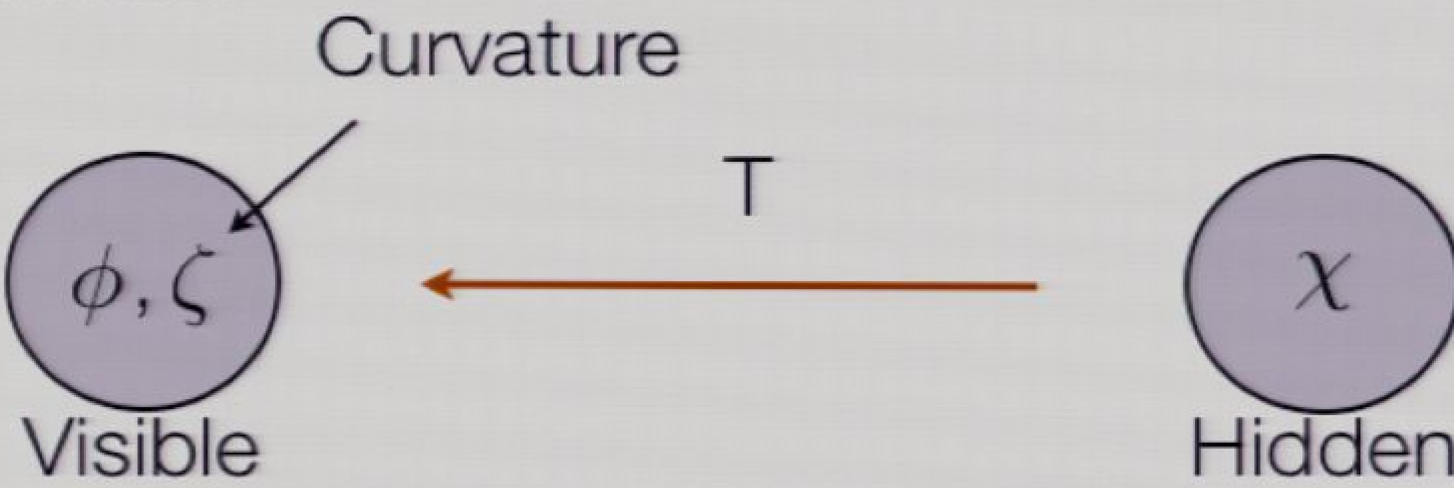
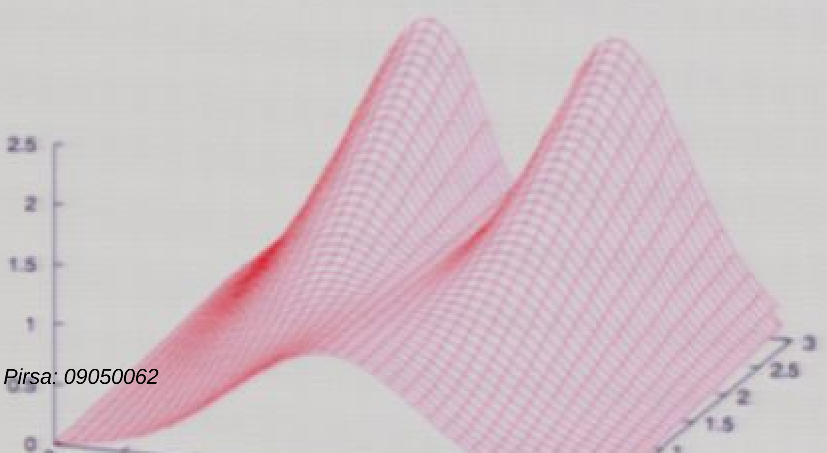
Explicit Example
Tachyon Mediated Density Perturbations

- In Hybrid inflation





We are not generating
density perturbations from the tachyon.
(assume that contribution from tachyon
preheating for example are small)

Lyth
Alabidi & Lyth

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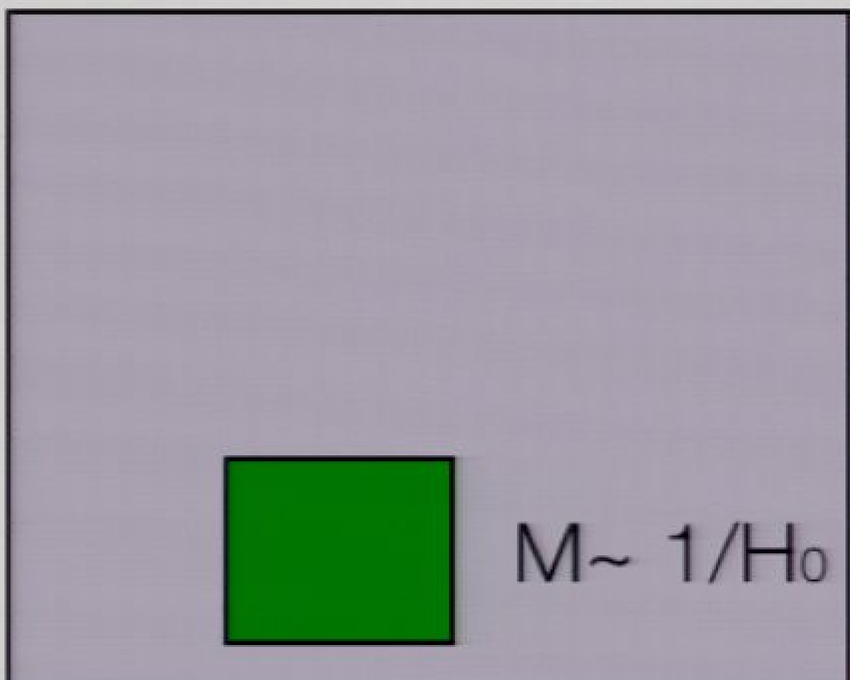

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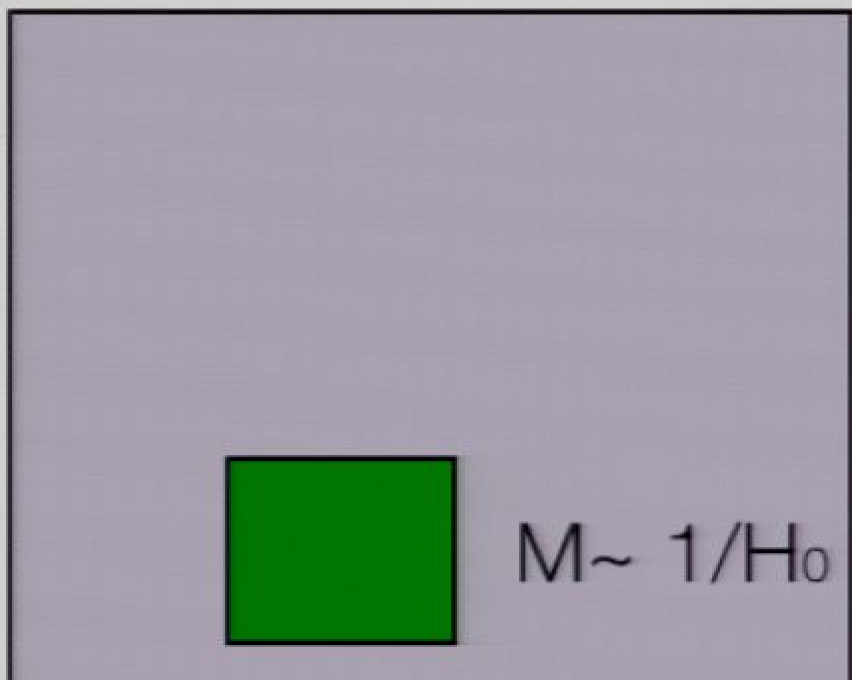

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What is the IR cutoff?

- For these delta N loops, one can argue that the cutoff is limited by observations.

Lyth/
Bartolo, Matarrese, Pietr
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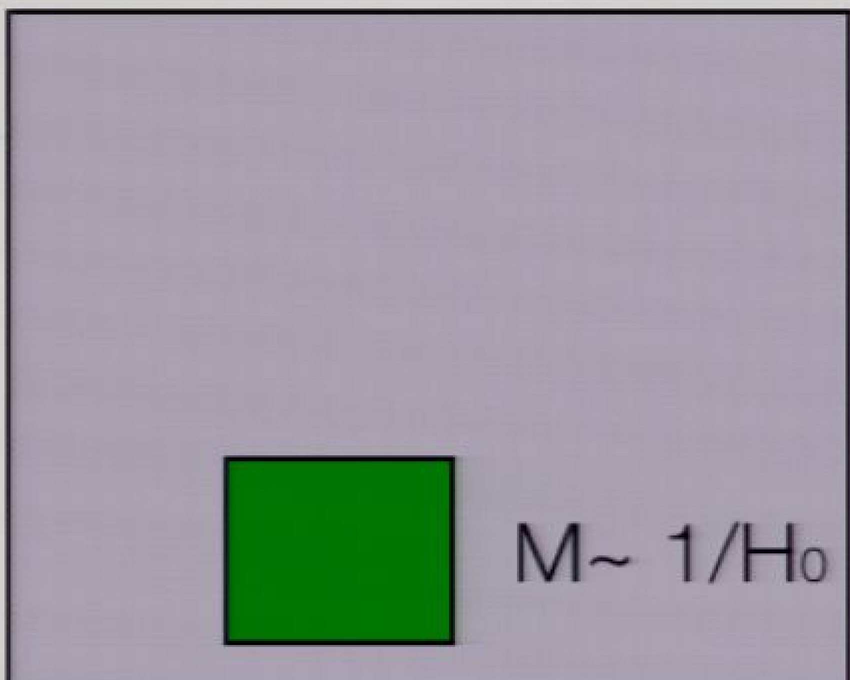
$$\delta\phi = \phi(\vec{x},t) - \phi\ \ (t)$$



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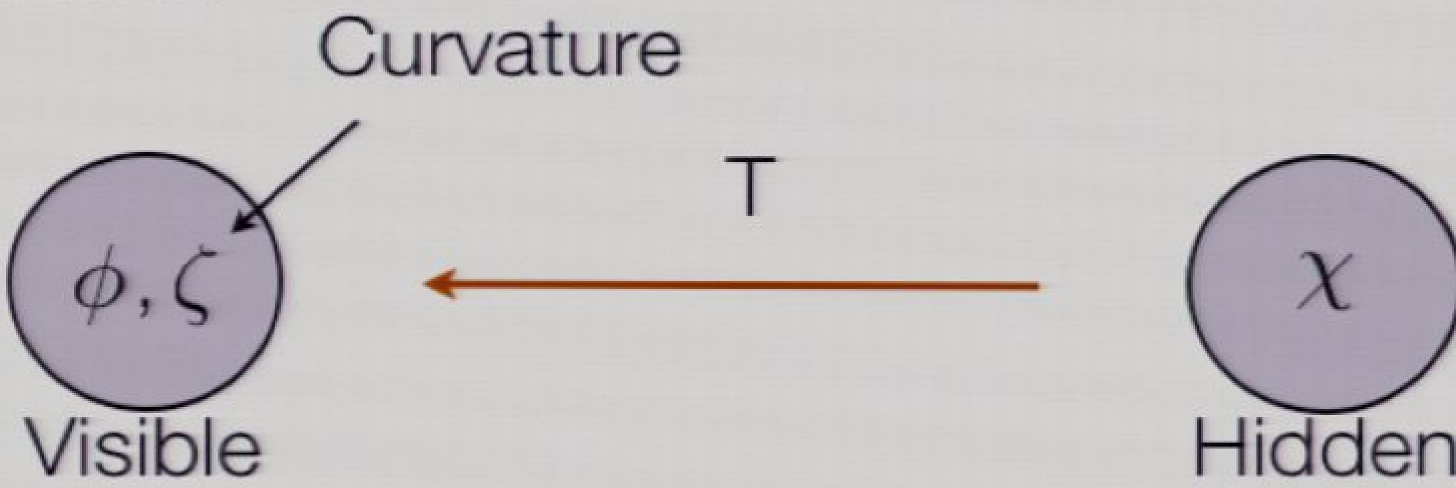

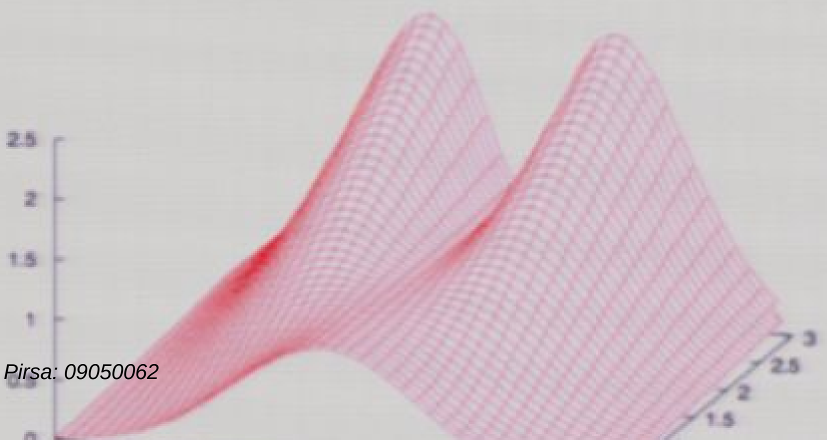

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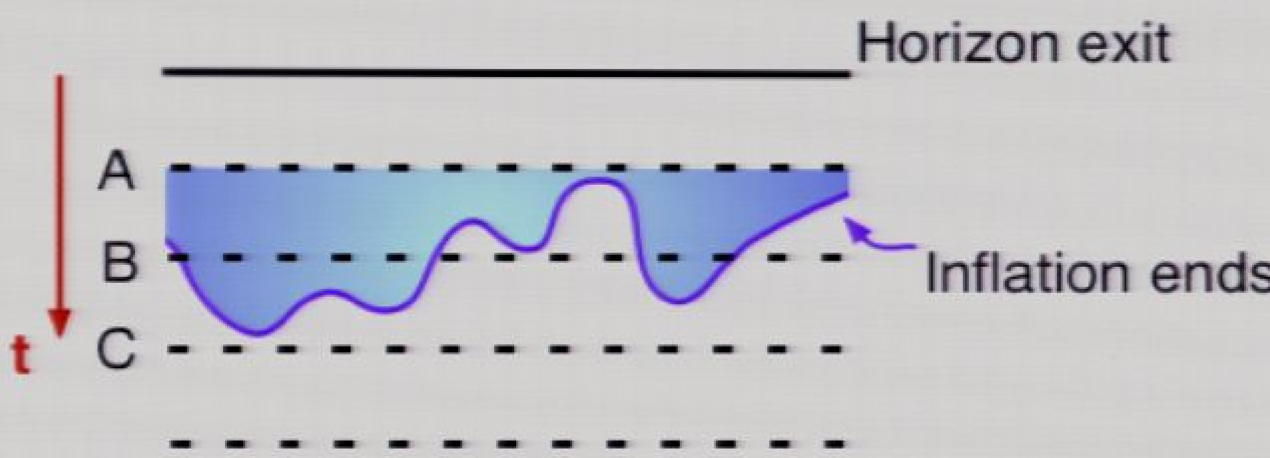

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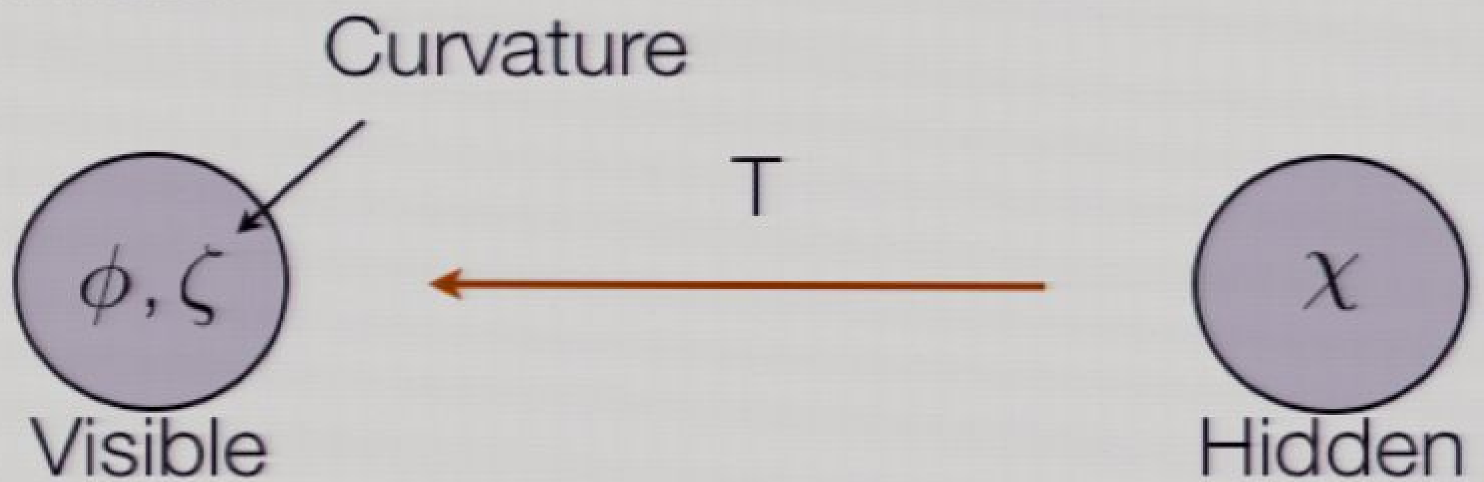

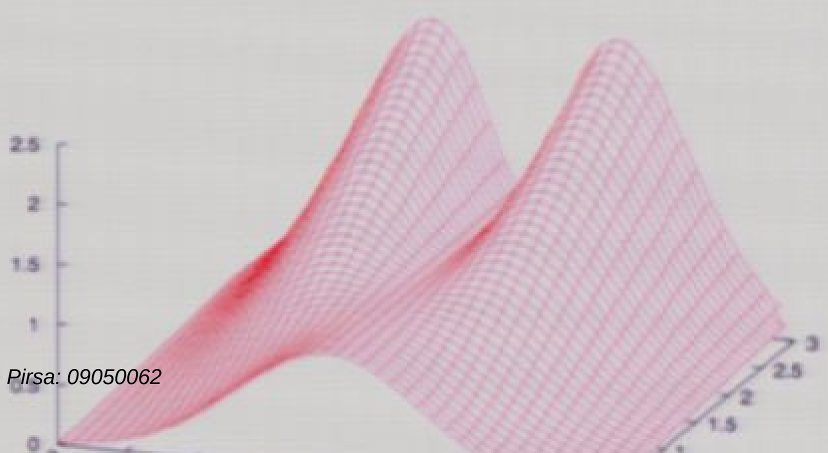

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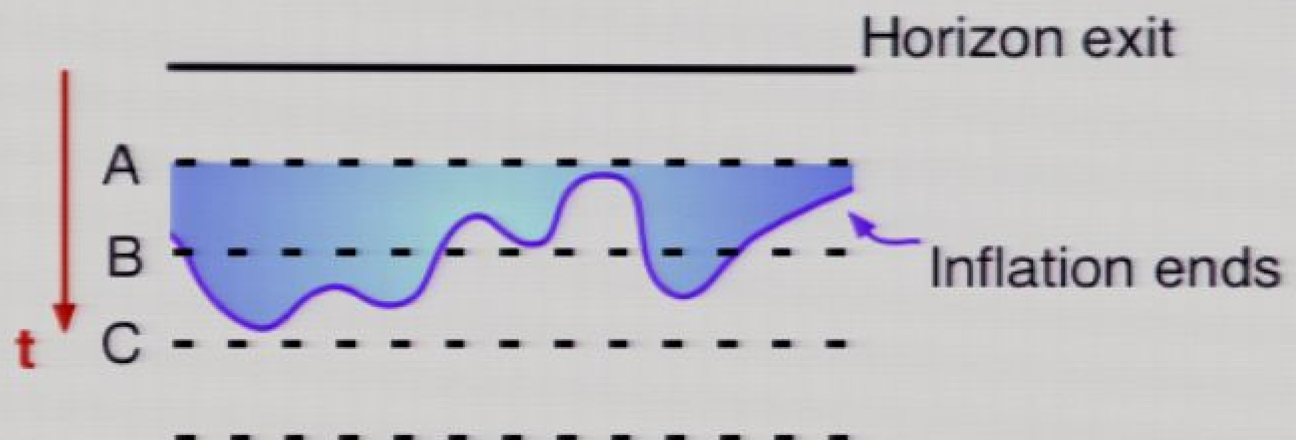

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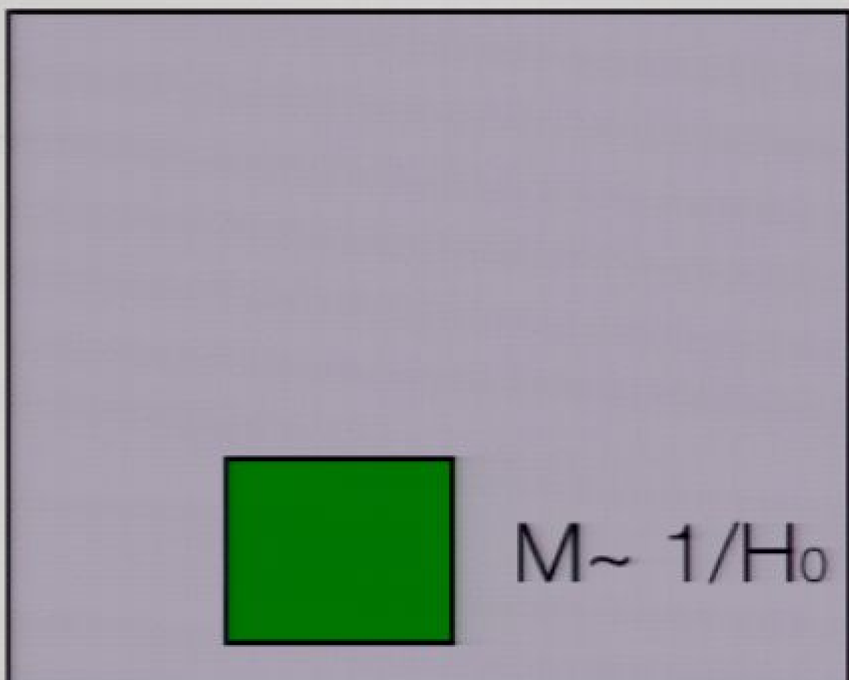

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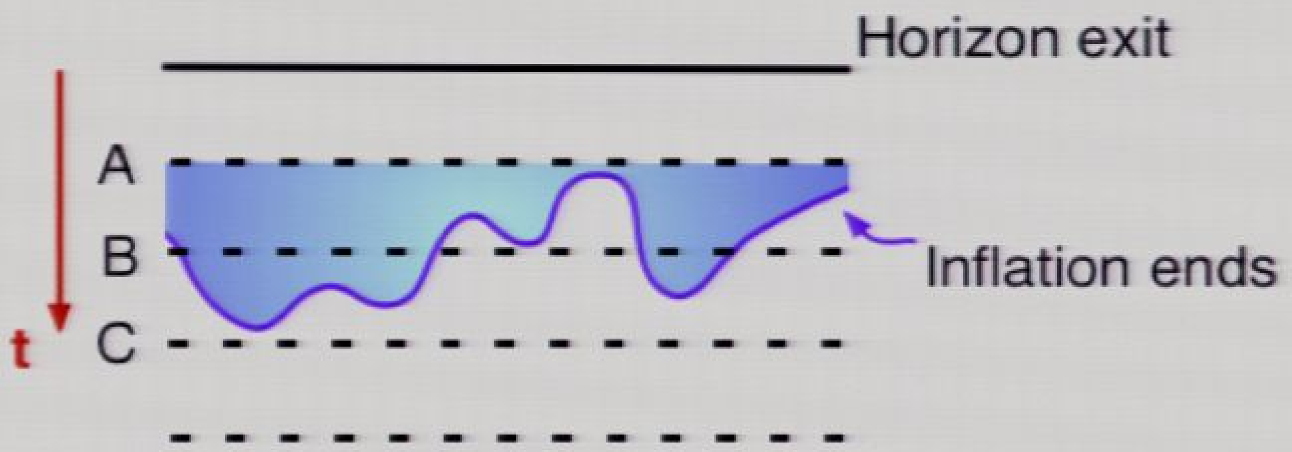

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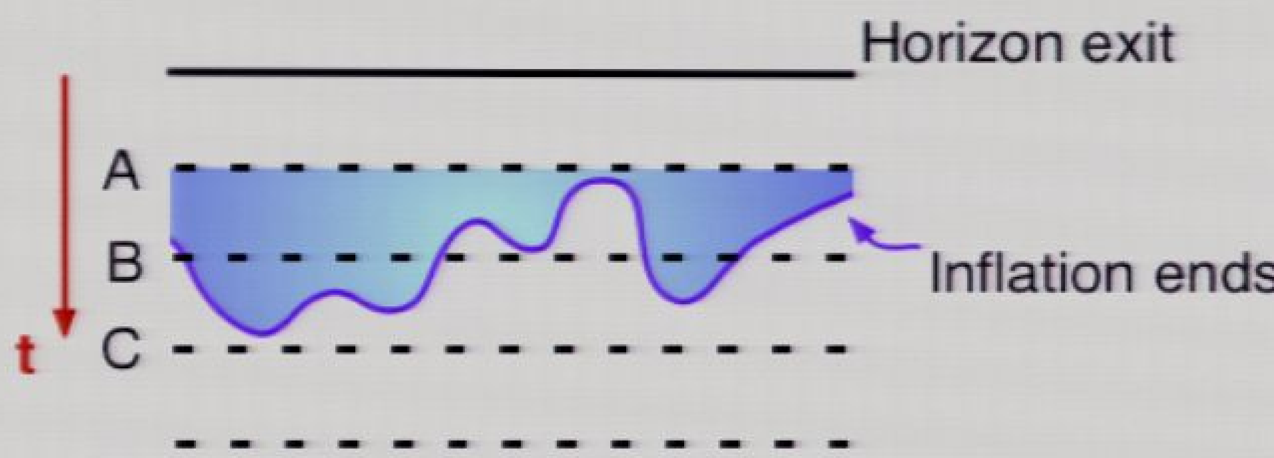

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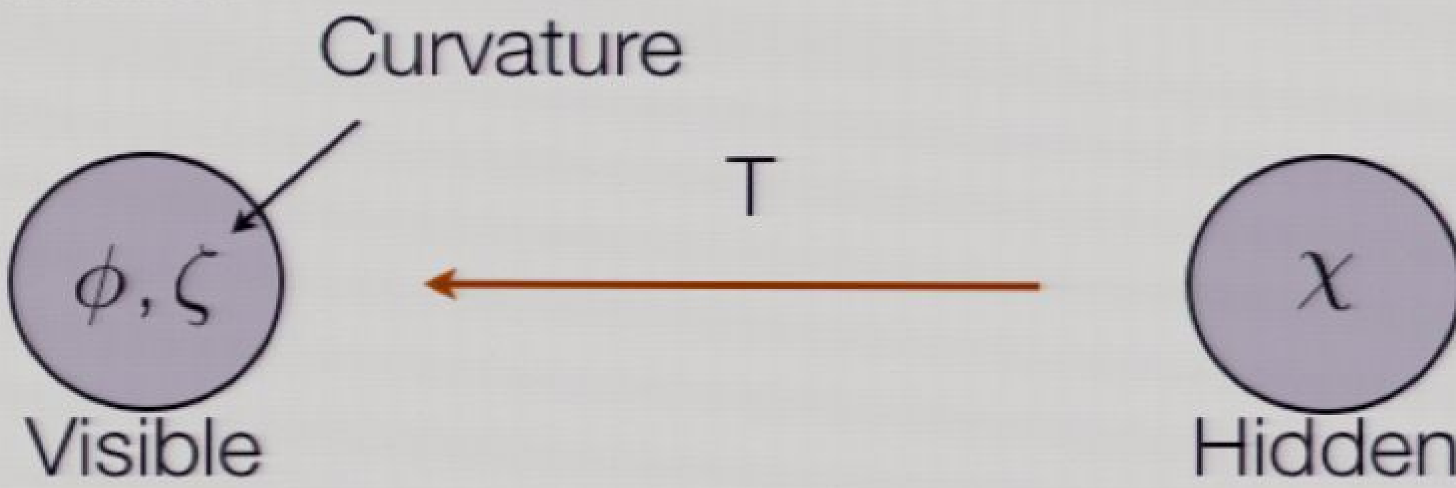

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