Title: The "in-in" Formalism and Cosmology: Inflation at Large N

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Abstract: TBA

The "in-in" Formalism and Cosmology: Inflation at Large N

Peter J. Adshead, Richard Easther and Eugene Lim

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Department of Physics
Yale University

Effective Field Theories In Inflation,
Perimeter Institute
May 21, 2009

Inflation at Zeroth Order

Zeroth order scalar field inflation very successful, solves all of the classic cosmological problems;

- Horizon
- Monopole
- Flatness
- Entropy

Quantum fluctuations about the classical trajectory provide the (nearly) scale invariant spectrum of perturbations that seed structure/ CMB anisotropies.

Unfortunately, implementation is not unique;

- Many ways of implementing an inflationary scenario,
- Nearly scale invariant spectrum of (almost) gaussian fluctuations is generic.

Inflationary Models at Lowest Order.

At lowest order, in field theory language, we think of the power spectrum, or 2-pt correlation function as the propagator;

$$P(k) \sim$$ +

- Generated by QM fluctuations of inflaton during inflation
- Amplitude and shape constrained by CMB data

Gravity couples to all forms of energy density

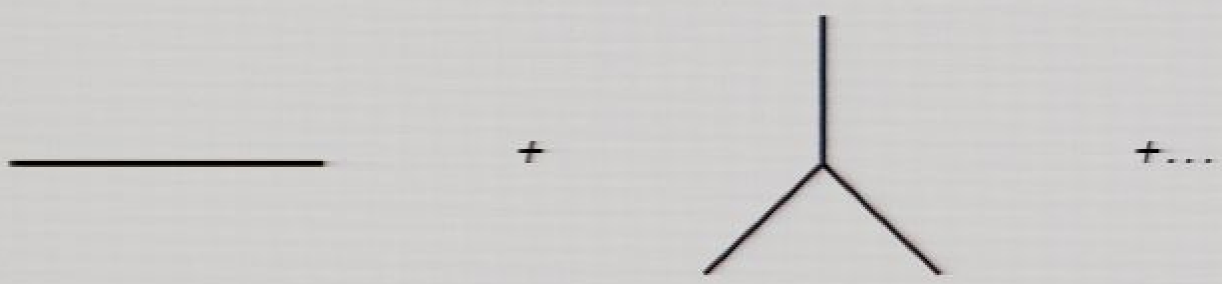
- Beyond lowest order, modes will couple, evolve non-linearly...

Diagrammatically:

$+\ldots$

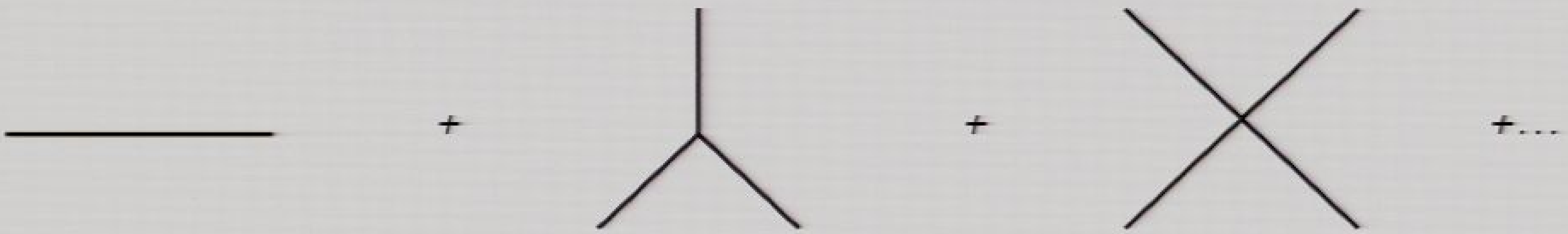
1980's

Diagrammatically:



1990's - 2002 (Maldacena, ...)

Diagrammatically:



2006 (Seery, Sloth, Lidsey, ...)

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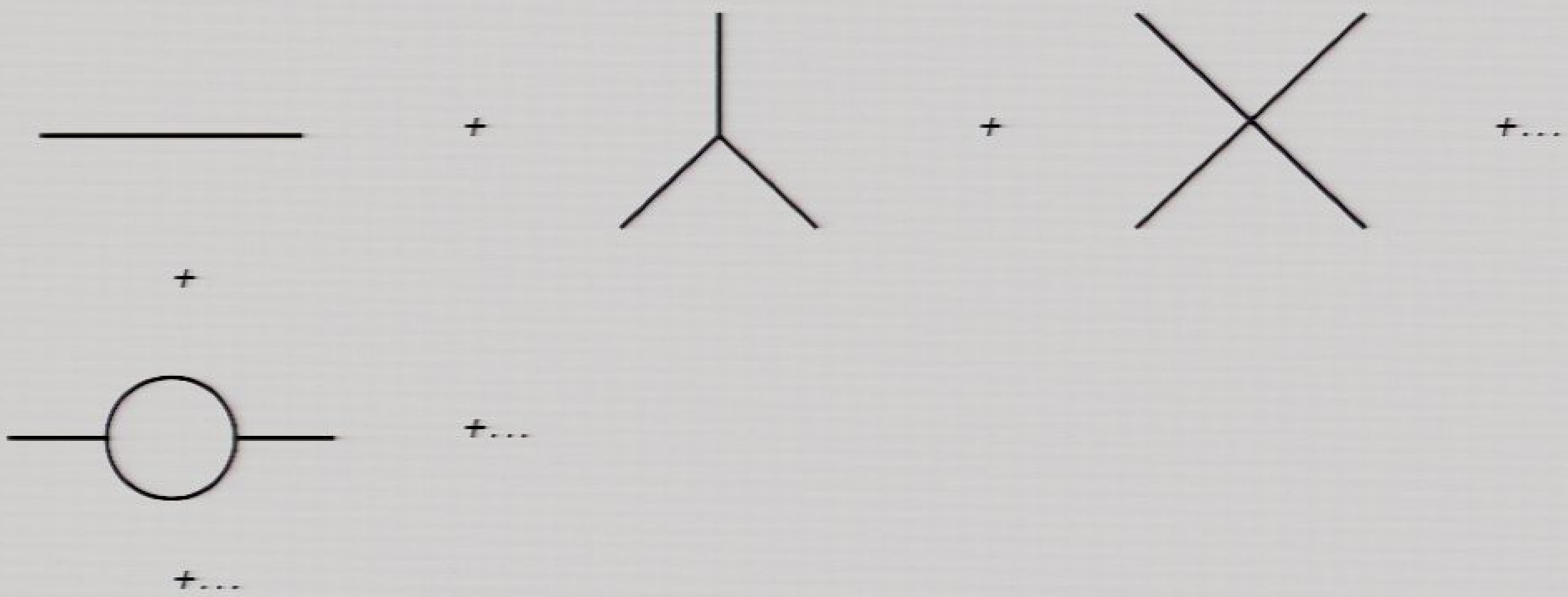

(Seery, Sloth, Weinberg...)

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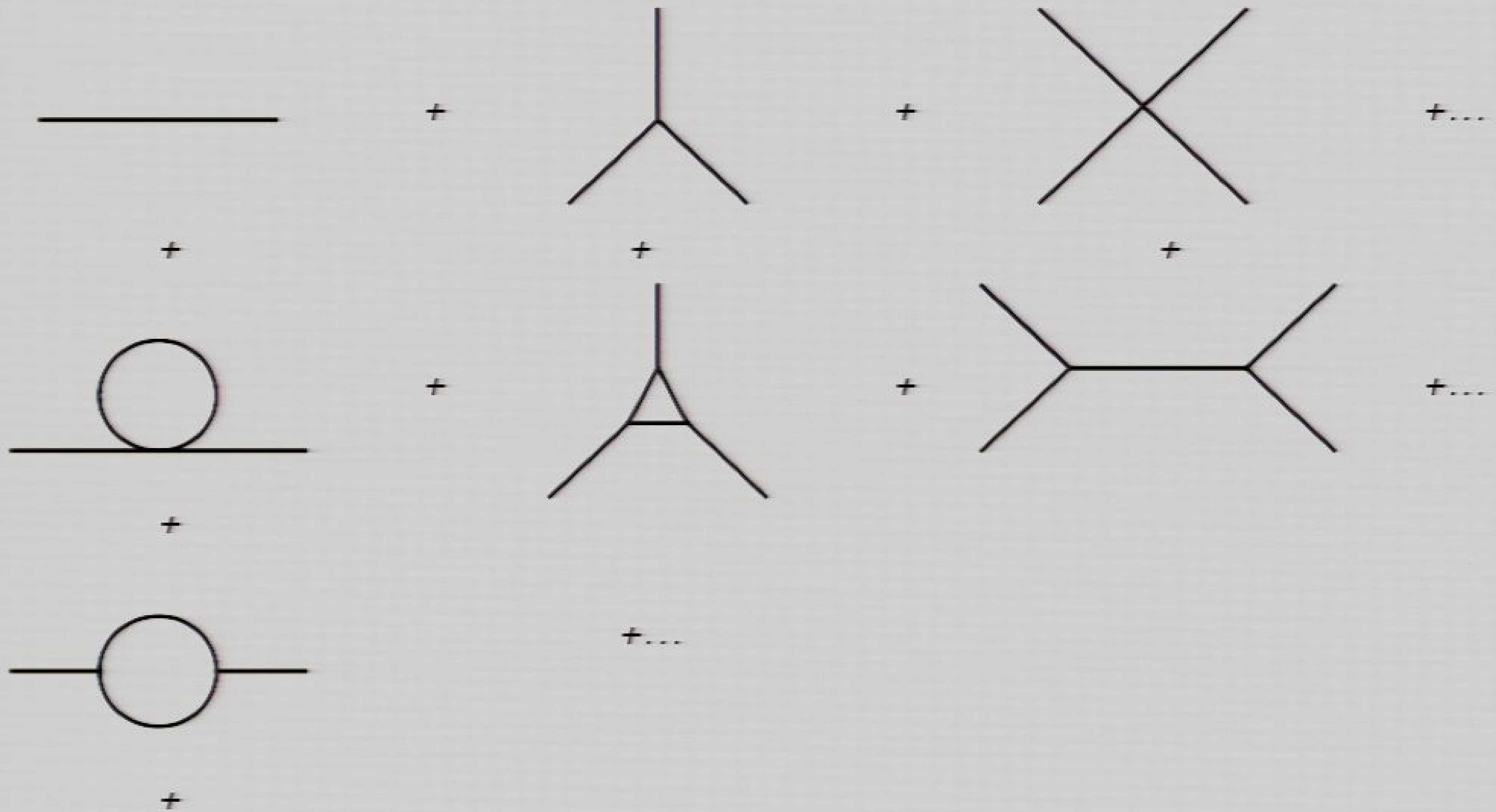


Outline

1. The ADM Formulation of GR and the “in-in” Formalism
   - Operator Formalism
2. Loop Corrections in N-Field Inflation: Bounds on N?
   - N-Field Inflation
   - Radiative Stability and Loop Corrections
   - Inflation with N-Spectator Fields
   - Coherent Field Description
3. Conclusions

The ADM Formulation of GR and the “in-in” Formalism

The ADM Formulation of GR

Perturbing fields in the ADM metric:

$$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

N and N^i Lagrange multipliers, h_{ij} metric on spatial hypersurface

- Not all $\{h_{ij}, N^i, N\}$ lead to unique field configurations
- Specify a gauge, i.e. a spatial slicing and a threading

Spatially flat gauge;

$$h_{ij} = a^2(t)(\delta_{ij} + \gamma_{ij}), \quad \phi(\mathbf{x}, t) = \bar{\phi}(t) + \delta\phi(\mathbf{x}, t)$$

$a(t)$ scale factor.

ADM action:

$$S = \int d^3x dt \sqrt{h} N \Big[\mathcal{R}^{(3)} - 2NV_I(\phi_I) + N^{-1}(E^{ij}E_{ij} - E^2 + \pi^I \pi_I) + h^{ij}(\partial_i \phi_I \partial_j \phi_I) \Big] ,$$

'Gravitational momentum:'

$$E_{ij} = \dot{h}_{ij} - \nabla_{(i} N_{j)}$$

Field momentum:

$$\pi^I = \dot{\phi}^I - N^i \partial_i \phi^I$$

- N and N^i have no dynamics; they do not propagate, and are constraints.
- Once known, substituted back into the action.
- Action contains only dynamical degrees of freedom.

The "In-In" Formalism

Calculation of cosmological correlation functions differs from usual QFT:

- Not interested in elements of a S-matrix, or transition amplitudes, but in expectation values of fields at fixed times,
- Conditions are imposed on the fields at very early times - only have "in-states,"

Can formulate as a path integral (Seery, Collins, Holman) or using operators (Weinberg).

Use the operator formulation of the "in-in" formalism of Schwinger;

Set up:

- Expand the action in powers of the fluctuations $\delta\phi$ and γ_{ij} and discard the zeroth and first order pieces.
- Define conjugate momenta, e.g. $\pi_{\delta\phi} = \frac{\partial \mathcal{L}}{\partial \delta\dot{\phi}}$, and construct the Hamiltonian.
- Work in an interaction picture, divide the Hamiltonian into a quadratic piece, H_0 and a higher order piece, H_{int}.
- H_0 evolves the fields.
- H_I evolves the states.

The interaction picture fields are free fields;

$$\delta\phi_I(\mathbf{x},\tau) \;=\; \int d^3k\, e^{i\mathbf{k}\cdot\mathbf{x}} \left[a_{\mathbf{k}} U_k(\tau) + a^{\dagger}_{-\mathbf{k}} U_k^*(\tau)\right]$$

$U_k(\tau)$ are solutions to the equation of motion:

$$\partial_\tau^2(aU_k) + \left[k^2 - a^2H^2\left(2+\epsilon - m'^2\right)\right] aU_k \;=\; 0$$

$$m' = \frac{V''}{H^2} \sim \eta$$

- de-Sitter limit (and taking the fields to be massless):

$$U_k \;=\; \sqrt{\frac{H^2}{2(2\pi)^3 k^3}}(1+ik\tau)e^{-ik\tau}$$

Quantization of Theories with Derivative Interactions

- The interactions generically contain derivatives of the fields
- Schematically;

$$\mathcal{L} = \frac{1}{2}\dot{\delta\phi}^2 - V(\delta\phi) + \left(\sqrt{\epsilon}\delta\phi^2 + \delta\phi^3\right)\dot{\delta\phi} + \frac{1}{2}\left(\sqrt{\epsilon}\delta\phi + \delta\phi^2\right)\dot{\delta\phi}^2 + \frac{1}{3}\delta\phi\dot{\delta\phi}^3 + \mathcal{O}(\epsilon\delta\phi^3) + \mathcal{O}(\epsilon\delta\phi^4) + \mathcal{O}(\delta\phi^5)$$

- What is $\mathcal{H}$? Is $\mathcal{H}_{\rm int} = -\mathcal{L}_{\rm int}$?
- Recall:

$$\mathcal{H}(\pi,\delta\phi) = \dot{\delta\phi}(\pi)\pi - \mathcal{L}(\pi,\delta\phi).$$

- But, $\pi = \dot{\delta\phi} + \mathcal{O}(\sqrt{\epsilon}\delta\phi^2) + \mathcal{O}(\delta\phi^2)\dot{\delta\phi} + \ldots$
- So,

$$\mathcal{H} = \mathcal{H}_0 - \mathcal{L}_{\rm int} + \mathcal{O}(\epsilon\,\delta\phi^4) + \mathcal{O}(\delta\phi^5).$$

Correlation functions

$$\langle Q(t)\rangle = \left\langle \left[Te^{-i\int_{t_0}^{t} H_{\rm int}(t')dt'}\right]^{\dagger} Q_I(t)\left[Te^{-i\int_{t_0}^{t} H_{\rm int}(t'')dt''}\right]\right\rangle,$$

- $Q_I(t)$ is some product of fields.

Nothing mysterious about "in-in,"

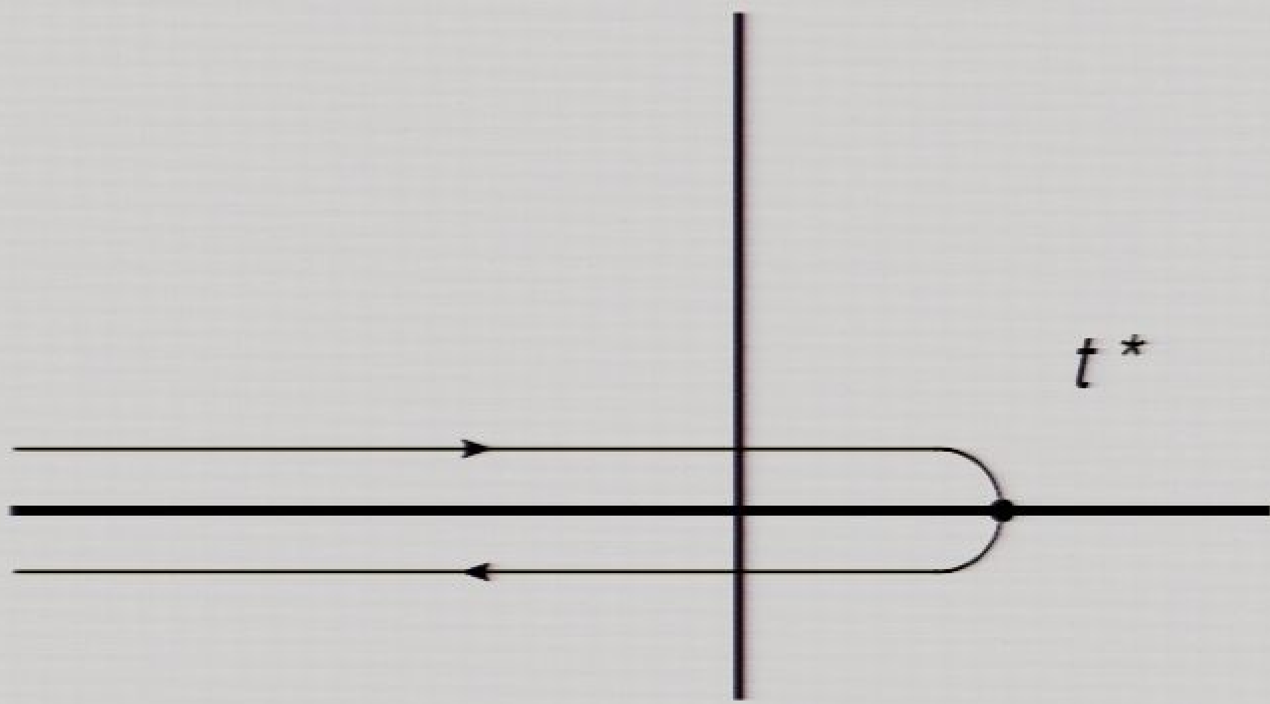
$$= \int d\alpha\, d\beta \langle 0| \left(Te^{-i\int_{t_0}^{t} H_{\rm int}(t')dt'}\right)^{\dagger} |\alpha\rangle\langle\alpha|Q(t)|\beta\rangle\langle\beta| \left(Te^{-i\int_{t_0}^{t} H_{\rm int}(t'')dt''}\right)|0\rangle$$

$$= \int d\alpha\, d\beta \langle\alpha|Q(t)|\beta\rangle\langle\beta|\, Te^{-i\int_{t_0}^{t} H_{\rm int}(t'')dt''}|0\rangle \left(\langle\alpha|\, Te^{-i\int_{t_0}^{t} H_{\rm int}(t'')dt''}|0\rangle\right)^{\dagger}$$

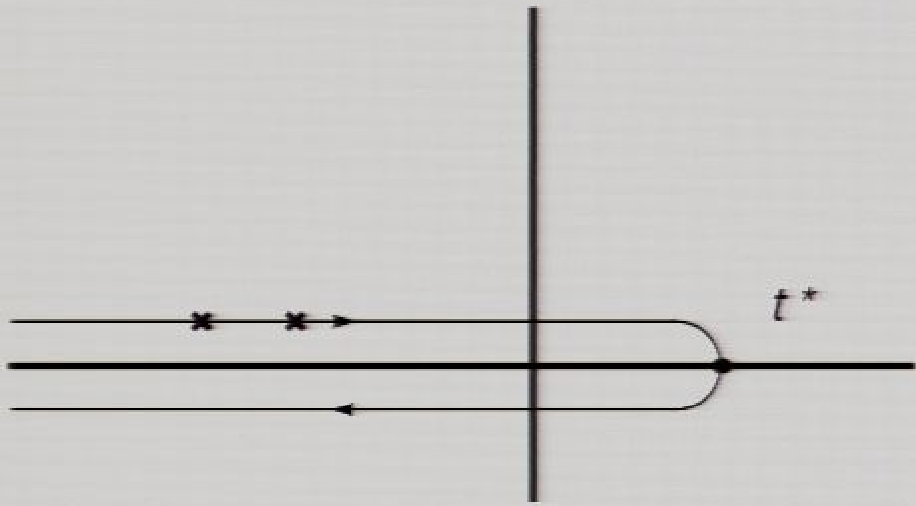
N-pt function $\langle\delta\phi^N\rangle$ is simply the sum over ways of obtaining a final state with $\alpha+\beta=N$

Time Path Interpretation:

$$\langle Q(t^*)\rangle = \left\langle \left[T e^{-i\int_{t_0}^{t^*} H_{\rm int}(t')dt'}\right]^\dagger Q_I(t^*) \left[T e^{-i\int_{t_0}^{t^*} H_{\rm int}(t'')dt''}\right] \right\rangle,$$



At second order: $\langle Q(t^*)\rangle_2 =$



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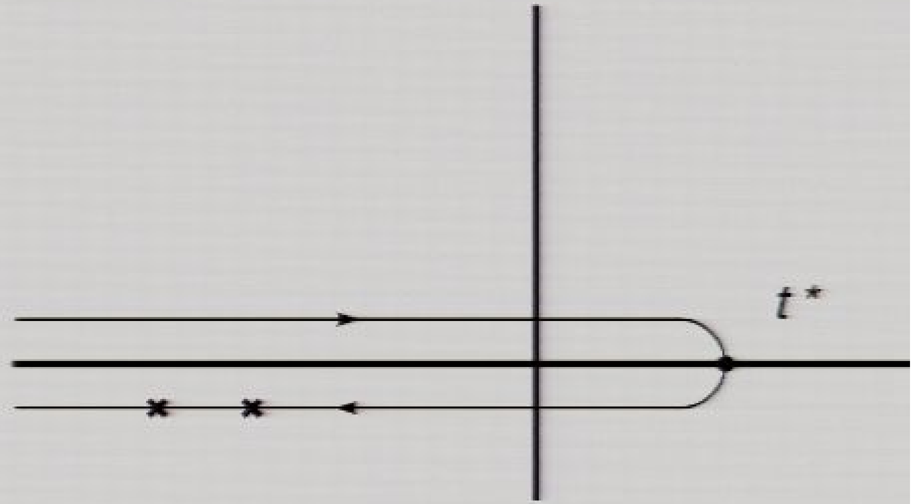

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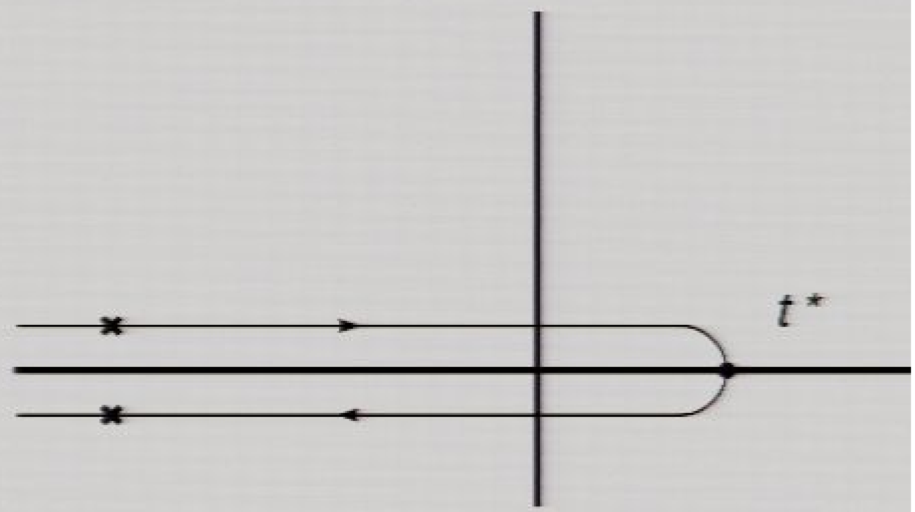

Rather than inserting states explicitly, use the Dyson solution;

$$Te^{-i\int_{t_0}^{t} H(t'')dt''} = \sum_{N=0}^{\infty}(-i)^N \int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2 ... \int_{t_0}^{t_{N-1}} dt_N H(t_1)H(t_2)...H(t_N)$$

$$\left(Te^{-i\int_{t_0}^{t} H(t'')dt''}\right)^{\dagger} = \sum_{N=0}^{\infty}(i)^N \int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2 ... \int_{t_0}^{t_{N-1}} dt_N H(t_N)...H(t_2)H(t_1)$$

Then, expanding

$$\langle Q(t)\rangle = \langle Q(t)\rangle_0 + \langle Q(t)\rangle_1 + \langle Q(t)\rangle_2 + ...,$$

where

$$\langle Q(t)\rangle_1 = -2\Im \int_{t_0}^{t} dt_1 \ \langle H_{\rm int}(t_1)Q(t)\rangle ,$$

$$\langle Q(t)\rangle_2 = -2\Re\left[\left\langle \int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2 H_{\rm int}(t_2)H_{\rm int}(t_1)Q(t)\right\rangle\right] + \left\langle \int_{t_0}^{t} dt_1 H_{\rm int}(t_1)Q(t) \int_{t_0}^{t} dt_2 H_{\rm int}(t_2)\right\rangle .$$

Time ordering has been taken care of!

At tree level, the two point correlation function is:

$$\langle \delta\phi_I \delta\phi_I \rangle = U_k U^*_{k'} \delta(\mathbf{k} - \mathbf{k}')$$

- Contraction of two fields:

$$\overline{\delta\phi^I_{\mathbf{k}} \delta\phi^J_{\mathbf{p}}} = \delta\phi^I_{\mathbf{k}} \delta\phi^J_{\mathbf{p}} - :\delta\phi^I_{\mathbf{k}} \delta\phi^J_{\mathbf{p}}:$$

- Propagator:

$$\langle \delta\phi^I_{\mathbf{k}}(\tau) \delta\phi^J_{\mathbf{p}}(\tau') \rangle = U_k(\tau) U^*_p(\tau') \delta^{IJ} \delta(\mathbf{k} + \mathbf{p})$$

- *Operator ordering matters.*
- Wightman functions instead of Feynman propagators
- Wick's theorem follows in the usual way.
  - Disconnected diagrams cancel by unitarity:

$$\left\langle \left[T e^{-i \int_{t_0}^{t} H_{\rm int}(t) dt} \right]^\dagger \left[T e^{-i \int_{t_0}^{t} H_{\rm int}(t) dt} \right] \right\rangle = 1$$

Subtleties:

$$\langle Q(t)\rangle = \left\langle \left[Te^{-i\int_{t_0}^{t} H_{\rm int}(t)dt}\right]^{\dagger} Q_I(t) \left[Te^{-i\int_{t_0}^{t} H_{\rm int}(t)dt}\right] \right\rangle ,$$

To calculate:

- Assume the initial (infinite past) conditions are adiabatic vacuum,
- Computationally this amounts to allowing a small amount of evolution in imaginary time in the far past: $-\infty \to -\infty(1+i\epsilon)$
- Left and right time integrations (vertices) no longer equivalent, but conjugates of each other.
- Implementation:
  - Active: Redefine integrations to run over a complex interval
  - Passive: Analytically continue the time variable to include a small imaginary piece.

Subtleties:

Temptation: use

$$\langle Q(t)\rangle = \sum_{N=0}^{\infty} i^N \int_{t_0}^{t} dt_N \int_{t_0}^{t_N} dt_{N-1} \cdots \int_{t_0}^{t_2} dt_1 \langle [H_{\rm int}(t_1), [H_{\rm int}(t_2), \ldots [H_{\rm int}(t_N), Q_I(t)] \ldots]]]\rangle .$$

- Physical terms are broken up into unphysical pieces

At 2nd order:

$$\int_{t_0}^{t} dt' \int_{t_0}^{t} dt'' \langle H_{\rm int}(t'') Q(t) H_{\rm int}(t')\rangle \equiv \int_{t_0}^{t} dt' \int_{t_0}^{t} dt'' \int d\alpha d\beta \langle 0| H_{\rm int}(t'')|\alpha\rangle \langle \alpha| Q(t)|\beta\rangle \langle \beta|$$

$$\rightarrow \int_{t_0}^{t} dt' \int_{t_0}^{t'} dt'' (H_{\rm int}(t') Q(t) H_{\rm int}(t'') + H_{\rm int}(t'') Q(t) H_{\rm int}(t'))$$

$$= 2\Re \int_{t_0}^{t} dt' \int_{t_0}^{t'} dt'' \langle H_{\rm int}(t'') Q(t) H_{\rm int}(t')\rangle$$

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Summary - Operator Formalism

- Nothing mysterious about "in-in" formalism:
  - Simple interpretation via transition amplitudes.
  - Just ordinary QFT rigged to compute correlation functions.
- Operator Formalism:
  - Fast, transparent way of doing "in-in" calculations.
  - Only one contraction.
  - One must be careful with derivative couplings.
  - One should avoid artificially splitting up diagrams.

Powerful technique for calculating correlation functions.

Subtleties:

Temptation: use

$$\langle Q(t)\rangle = \sum_{N=0}^{\infty} i^N \int_{t_0}^{t} dt_N \int_{t_0}^{t_N} dt_{N-1} \cdots \int_{t_0}^{t_2} dt_1 \langle [H_{\rm int}(t_1), [H_{\rm int}(t_2), \ldots [H_{\rm int}(t_N), Q_I(t)] \ldots]]]\rangle .$$

- Physical terms are broken up into unphysical pieces

At 2nd order:

$$\int_{t_0}^{t} dt' \int_{t_0}^{t} dt'' \langle H_{\rm int}(t'')Q(t)H_{\rm int}(t')\rangle \equiv \int_{t_0}^{t} dt' \int_{t_0}^{t} dt'' \int d\alpha d\beta \langle 0|H_{\rm int}(t'')|\alpha\rangle\langle\alpha|Q(t)|\beta\rangle\langle\beta|$$

$$\rightarrow \int_{t_0}^{t} dt' \int_{t_0}^{t'} dt'' (H_{\rm int}(t')Q(t)H_{\rm int}(t'') + H_{\rm int}(t'')Q(t)H_{\rm int}(t'))$$

$$= 2\Re \int_{t_0}^{t} dt' \int_{t_0}^{t'} dt'' \langle H_{\rm int}(t'')Q(t)H_{\rm int}(t')\rangle$$

Summary - Operator Formalism

- Nothing mysterious about "in-in" formalism:
  - Simple interpretation via transition amplitudes.
  - Just ordinary QFT rigged to compute correlation functions.
- Operator Formalism:
  - Fast, transparent way of doing "in-in" calculations.
  - Only one contraction.
  - One must be careful with derivative couplings.
  - One should avoid artificially splitting up diagrams.

Powerful technique for calculating correlation functions.

Gravitationally Induced Loop Corrections in N-Field Inflation: Bounds on N?

Action: N-Field Inflation

Consider an action of the form with N scalar fields (*participator fields*), M massless scalars (*spectator fields*):

$$S = \frac{1}{2}\int d^4x\sqrt{g}\left[M_{\rm pl}^2\mathcal{R} + \sum_{I=1}^{N}\left((\partial\phi_I)^2 - 2V(\phi_I)\right) + \sum_{J=1}^{M}(\partial\sigma_J)^2\right],$$

Potential:

$$V(\phi_I) = \sum_{I=1}^{N} V_I(\phi_I)$$

Each V_I depends on a single ϕ_I.
(Canonical example, considered here N copies of $m^2\phi^2$.)

N-Participators: N-Field Inflation

Friedmann equation:

$$3H^2 = \sum_I \left(\frac{1}{2}\dot{\phi}_I^2 + V_I(\phi_I)\right)$$

Homogeneous Klein-Gordon equation:

$$\ddot{\phi}_I + 3H\dot{\phi}_I + \frac{dV(\phi)}{d\phi_I} = 0$$

- Each field feels gradient of its own potential.
- Feels the Hubble friction of *all* fields.
- Obtain inflation from a collection of potentials for which inflation cannot occur individually.

Slow Roll Params:

$$\epsilon \;=\; 2M_{\rm pl}^2\left(\frac{H'}{H}\right)^2 = \frac{1}{2}\sum_{I=1}^{N}\left(\frac{\dot{\phi}_I}{HM_{\rm pl}^2}\right)^2 = \sum_{I=1}^{N}\epsilon_I$$

Why N Fields?

- Many candidate theories of the early universe contain many additional degrees of freedom, e.g. string theory
- N-field inflation provides (theoretically!) a way of realizing chaotic inflation consistently within an effective field theory.
    - i.e. It is a way of side-stepping the problem of Planckian vevs,
        - $\epsilon \to \epsilon_I = \epsilon/N$,
        - $\Delta\phi \to \Delta\phi/\sqrt{N}$.
    - Get significant gravity waves while respecting the Lyth bound.
- N-copies of the Standard Model might solve the hierarchy problem
    - Novel solution to hierarchy problem if $N \sim 10^{32}$ (Dvali)

Simple Bounds on N

All approximately massless fields fluctuate with an amplitude set by the Hubble scale;

$$\delta\phi_i \sim \frac{H}{2\pi}$$

- Fluctuations freeze out on scales larger than $1/H$,
- Each field contributes gradient energy, $(\nabla\phi)^2/2$.

Gradient energy scales like

$$\frac{N}{2}\left(\frac{\delta\phi}{\delta x}\right)^2 \sim N\frac{H^4}{8\pi^2}$$

Given H, $\rho = 3M_{\rm pl}^2 H^2$. For self consistency:

$$N \ll \frac{M_{\rm pl}^2}{H^2}$$

Radiative Stability and Loop Corrections

Assume the form of the potential is radiatively stable for this work.
What about gravitationally induced loop corrections?

- Graviton couples to everything
- Loop corrections from the potential $\rightarrow$ radiative corrections to the slow roll parameters
- Gravitationally induced loop corrections $\rightarrow$ radiative corrections to the power spectrum.
    - N-degrees of freedom to run round the loops.

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Density fluctuations:

$$\mathcal{P}_k = \frac{1}{N^2}\sum_{I=1}^{N}\left(\frac{H}{\dot{\phi}_I}\right)^2 \langle\delta\phi_I\delta\phi_I\rangle$$

- Can bounds be put on N from loop corrections to the power spectrum?
- One might expect an m-loop correction to scale like N^m.

To one loop order

$$\langle\delta\phi_n\delta\phi_n\rangle \sim \frac{H^2}{2(2\pi)^3 M_{\rm pl}^2}\left(1 + N\frac{H^2}{M_{\rm pl}^2}\right)$$

So might expect $N \ll \frac{M_{\rm pl}^2}{H^2}$

Leading order third and fourth order actions are, respectively,

$$S^{(3)} = -\int dt d^3x \left[\frac{a^3}{4}\sqrt{2\epsilon_I}\delta\phi^I \dot{\delta\phi}^J \dot{\delta\phi}^J + \frac{a^3}{2}\sqrt{2\epsilon_I}\partial^{-2}\dot{\delta\phi}^I \dot{\delta\phi}^J \partial^2\delta\phi^J\right],$$

- Coupling: $\epsilon_I \equiv \frac{1}{2}\frac{\dot{\phi}_I^2}{H^2}$,

$$\begin{aligned} S^{(4)} \;=\; \int dt d^3x\, a^3 \Bigg[& \frac{1}{4Ha^2}\partial_i\delta\phi^J\partial_i\delta\phi^J\partial^{-2}(\partial_j\dot{\delta\phi}^I\partial_j\delta\phi^I + \dot{\delta\phi}^I\partial^2\delta\phi^I) \\ & + \frac{1}{4H}\dot{\delta\phi}^J\dot{\delta\phi}^J\partial^{-2}(\partial_i\dot{\delta\phi}^I\partial_j\delta\phi^I + \dot{\delta\phi}^I\partial^2\delta\phi^I) \\ & + \frac{3}{4H}\partial^{-2}(\partial_j\dot{\delta\phi}^J\partial_j\delta\phi^J + \dot{\delta\phi}^J\partial^2\delta\phi^J)\partial^{-2}(\partial_j\dot{\delta\phi}^I\partial_j\delta\phi^I + \dot{\delta\phi}^I\partial^2\delta\phi^I) \\ & + \frac{1}{4}\beta_{2,j}\partial^2\beta_{2,j} + \dot{\delta\phi}^I\beta_{2,i}\partial_i\delta\phi^I \Bigg], \end{aligned}$$

$$\frac{1}{2}\beta_{2,j} \simeq \partial^{-4}\left(\partial_j\partial_k\dot{\delta\phi}^I\partial_k\delta\phi^I + \partial_j\dot{\delta\phi}^I\partial^2\delta\phi^I - \partial^2\dot{\delta\phi}^I\partial_j\delta\phi^I - \partial_m\dot{\delta\phi}^I\partial_j\partial_m\delta\phi^I\right).$$

Interactions

- Four point interaction:

$$H^{(4)}(t) \sim \int d^3x \, \frac{1}{aH^2} \partial^{-n}(\delta\phi^I \delta\phi^I)\partial^{-m}(\delta\phi^J \delta\phi^J)$$

- Three point interaction

$$H^{(3)}(t) \sim \int d^3x \, \frac{1}{aH} \sqrt{2\epsilon_I} \delta\phi^I \delta\phi^J \delta\phi^J$$

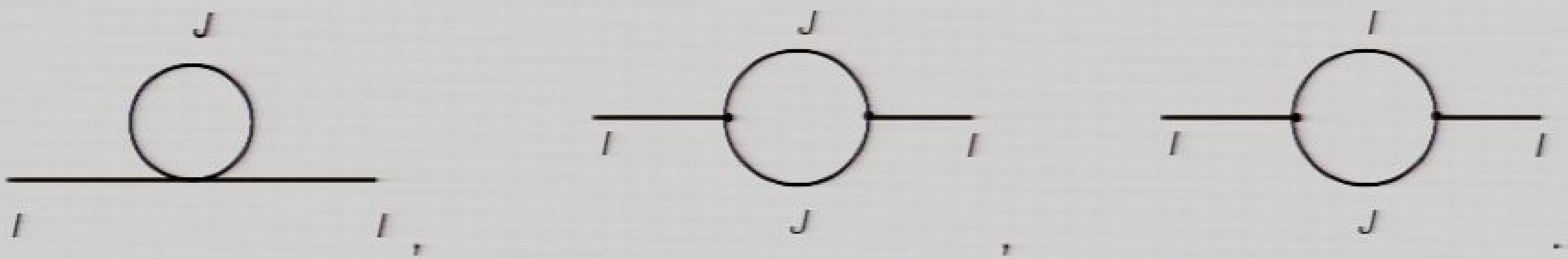
- Loop corrections given by:

$$\langle \delta\phi^I(t)\delta\phi^I(t)\rangle_{1L,1V} = -2\Im \int_{-\infty}^{t} dt_1 \left\langle H^{(4)}(t_1)\phi^I(t)\delta\phi^I(t) \right\rangle ,$$

and

$$\begin{aligned}\langle \delta\phi^I(t)\delta\phi^I(t)\rangle_{1L,2V} &= -2\Re \left[\left\langle \int_{-\infty}^{t} dt_2 \int_{-\infty}^{t_2} dt_1 H^{(3)}(t_1) H^{(3)}(t_2) \delta\phi^I(t)\delta\phi^I(t) \right\rangle \right] \\ &\quad + \left\langle \int_{-\infty}^{t} dt_1 H_I^{(3)}(t_1)\delta\phi^I(t)\delta\phi^I(t) \int_{-\infty}^{t} dt_2 H_I^{(3)}(t_2) \right\rangle .\end{aligned}$$

Diagrams:



- Not typical Feynman diagrams.
- Time doesn't flow through the diagrams - propagators have only 3-momenta.
- Times associated with vertices.
- Diagrams useful for visualization.
- Feynman rules can be constructed, but are cumbersome.

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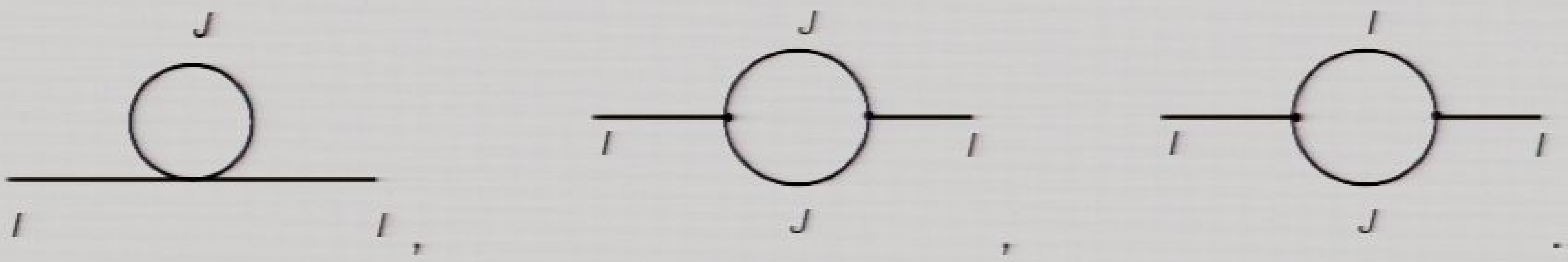

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- Diagrams useful for visualization.
- Feynman rules can be constructed, but are cumbersome.

N-Field Inflation: One Vertex One Loop

- Biggest possible effect from I propagator corrected by J other fields.
- Contribution of a loop of this form is given by:

$$\langle \delta\phi^I_{\mathbf{q}}(t)\delta\phi^I_{\mathbf{q}'}(t)\rangle_{1L,1V} \supset \Im \int_{-\infty}^{t} \frac{dt_1}{aH^2} \langle \partial^{-m}(\delta\phi^I_{\mathbf{p}}(t_1)\delta\phi^I_{\mathbf{p}'}(t_1))\delta\phi^I_{\mathbf{q}}(t)\delta\phi^I_{\mathbf{q}'}(t)$$
$$\times \sum_{J=1}^{N} \int d^3k \int d^3k' \langle \partial^{-n}(\delta\phi^J_{\mathbf{k}}(t_1)\delta\phi^J_{\mathbf{k}'}(t_1))\rangle$$

- Loop integral scale free - independent of the external momentum: *does not* make a physical contribution.

Can any of the one-loop one vertex loops contribute?

- Unlike $\lambda\phi^4$, can sneak the external scale into the integral:
- In Fourier space:

$$\partial^{-n}(\delta\phi^J(t_1)\delta\phi^J(t_1)) \sim \frac{1}{(\mathbf{k}+\mathbf{p})^n}\delta\phi^J_{\mathbf{k}}(t_1)\delta\phi^J_{\mathbf{p}}(t_1)$$

- Contract I fields with J fields, obtain

$$\begin{aligned}\langle\delta\phi^I_{\mathbf{q}}(t)\delta\phi^I_{\mathbf{q}}(t)\rangle_{1L,1V} \supset & \sum_{J=1}^{N}\Im\int_{-\infty}^{t}\frac{dt_1}{aH^2}\langle\delta\phi^I_{\mathbf{p}'}(t_1)\delta\phi^J_{\mathbf{k}'}(t_1)\delta\phi^I_{\mathbf{q}}(t)\delta\phi^I_{\mathbf{q}}(t)\rangle \\ & \times\int d^3k\int d^3p\frac{1}{(\mathbf{k}+\mathbf{k}')^n}\frac{1}{(\mathbf{p}+\mathbf{p}')^m}\langle\delta\phi^J_{\mathbf{k}}(t_1)\delta\phi^I_{\mathbf{p}}(t_1)\rangle \\ \sim & \sum_{J=1}^{N}\delta^{IJ}\Im\left(\frac{H^2}{M_{\rm pl}^2}\right)^2\int d^3k\frac{1}{k^3(\mathbf{k}+\mathbf{q})^{n+m}}\end{aligned}$$

- ∂^{-n} contracted *across* two fields yields an integral with a scale.

Interactions

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$$H^{(4)}(t) \sim \int d^3x \, \frac{1}{aH^2} \partial^{-n}(\delta\phi^I \delta\phi^I) \partial^{-m}(\delta\phi^J \delta\phi^J)$$

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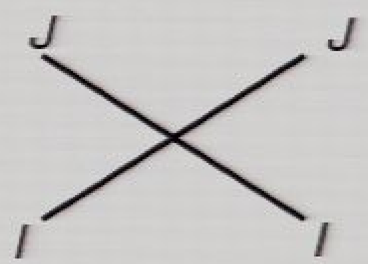
- ∂^{-n} contracted *across* two fields yields an integral with a scale.

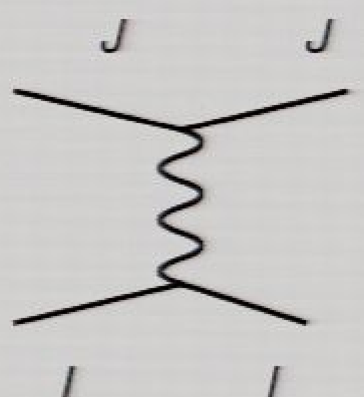
Hidden Gravitons

Non-appearance of the diagrams scaling with N can be understood clearly as follows:

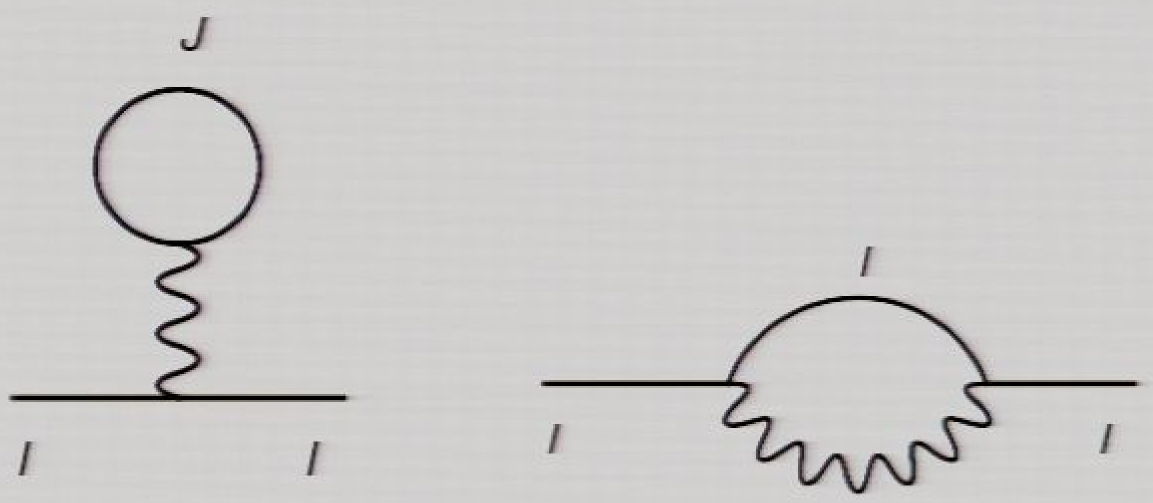
- The one loop, one vertex diagrams considered above really have gravitons secretly hidden inside them:
- The four point interaction:



is really mediated by a graviton:



In this gauge, the two one-loop one-vertex diagrams we drew above look like:



- Diagram that might scale like N^2, is a “balloon” diagram
- The propagator can't change species in the 2nd diagram.

What about the two vertex loop?

- Expect to scale as N due to N species which can appear in the loop.
- Cannot be cheated out of this loop, due to topology the external momenta *must* flow through the loop.
- Six distinct diagrams which must be summed:

$$\langle \delta\phi^I(t)\delta\phi^I(t)\rangle_{1L,2V} = \frac{H^2}{2(2\pi)^3 q^3} N\epsilon_I \left[\frac{2017}{120}\ln(q)\right]$$

- Note: ϵ_I is the slow roll parameter of one of the fields.
- The global slow roll parameter is:

$$\epsilon = N\epsilon_I$$

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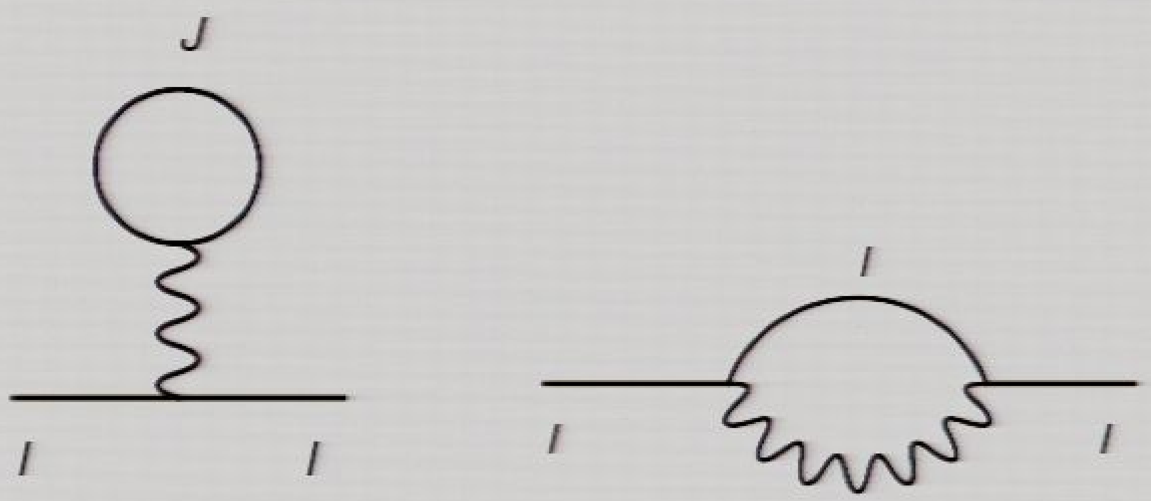

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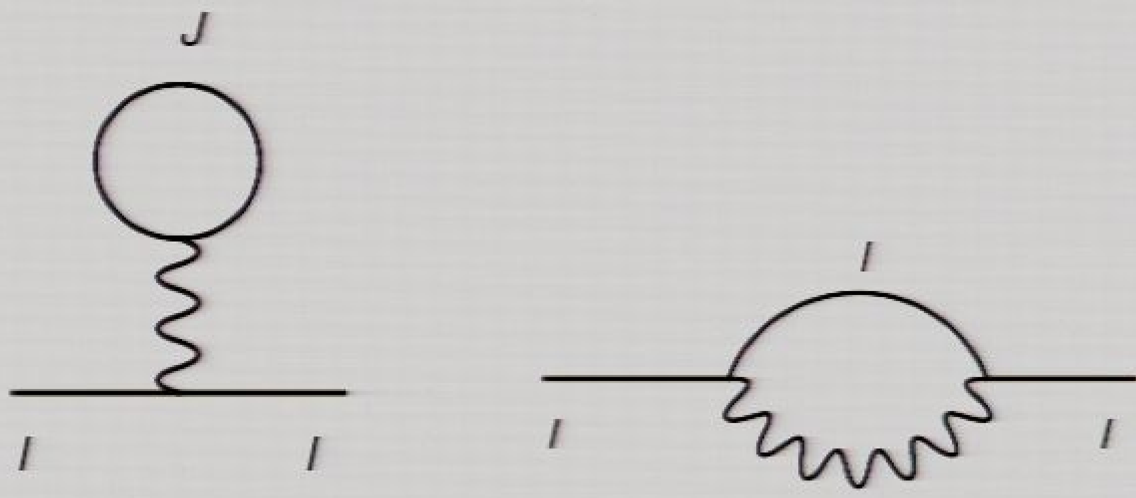

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What about more loops?

No matter how many loops one goes to, no factors of N;

- Leading order 4-pt interaction is only non-zero for self interactions.
- Coupling in the 3-pt interaction has a $1/\sqrt{N}$ hidden inside of it.
- 3-pt interactions must occur in pairs

What about higher order terms?

What about higher order terms?

- Interactions must appear in the action as scalars with respect to the field indices.
- With flat target space, fields pair with other fields (same index) or with background fields, i.e. $\dot{\phi}^I \delta\phi^I \delta\phi^J \delta\phi^J$ or $\dot{\phi}^I \dot{\phi}^J \ddot{\phi}^K \delta\phi^I \delta\phi^J \delta\phi^K$
- Interaction like $\delta\phi^I \delta\phi^J \delta\phi^K$, scaling like N^3 is forbidden.
- In terms of the background properties, $\ddot{\phi}^I, \dot{\phi}^I \sim 1/\sqrt{N}$

We can't do any better than the leading order scaling.

Inflation with M-Spectator Fields: Loop Corrections

What about the other extreme:

- Four point interaction:

$$\mathcal{H}_4(t) \sim \int d^3x \, \frac{1}{aH^2} \Big[\partial^{-n}(\delta\phi\delta\phi + \sum_{J=1}^{M} \delta\sigma^J \delta\sigma^J) \partial^{-m}(\delta\phi\delta\phi + \sum_{K=1}^{M} \delta\sigma^K \delta\sigma^K)$$

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- 4-pt generates only one loop

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- One finds (Weinberg)

$$\mathcal{P}_k \sim \frac{1}{\epsilon}\frac{H^2}{M_{\rm pl}^2}\left(1 + M\epsilon\frac{\pi}{10}\frac{H^2}{M_{\rm pl}^2}\ln(k)\right)$$

- Gives a bound:

$$M < \frac{M_{\rm pl}^2}{H^2}\frac{1}{\epsilon}$$

- Weaker than the gradient energy bound by ϵ

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A Coherent Field?

- Non appearance of any scaling of N in N-field inflation; really only one effective degree of freedom.
- Suggests that, effective degree of freedom: $\psi^2 = \sum_{J=1}^{N} \phi_J^2$
- For $m^2\phi^2$ potentials; Lagrangian is:

$$\mathcal{L} = \frac{1}{2}(\partial\psi)^2 - \frac{1}{2}m^2\psi^2 + \frac{1}{2}\psi^2(\partial\Omega)^2,$$

- Looks like one inflaton, ψ, and $N-1$ massless scalars, Ω.
- Why don't we recover Weinberg's result?

$$\mathcal{P}_k \sim \frac{1}{\epsilon}\frac{H^2}{M_{\rm pl}^2}\left(1 + N\epsilon\frac{\pi}{10}\frac{H^2}{M_{\rm pl}^2}\ln(k)\right)$$

- Short answer: this isn't *quite* the same case as Weinberg
- The fields Ω_i are not completely free; they satisfy

$$\sum_{i=1}^{N-1} \Omega_i = 1$$

- $\langle\psi^2\rangle \sim (N\epsilon_I)^{-1}(H^2/M_{\rm pl}^2)$
- Ω_i are quickly damped to attractor; $\partial\Omega = 0$

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Loop Corrections

$$\mathcal{L} = \frac{1}{2}(\partial\psi)^2 - \frac{1}{2}m^2\psi^2 + \frac{1}{2}\psi^2(\partial\Omega)^2,$$

- Perturb:

$$\psi \rightarrow \bar{\psi} + Q$$
$$\Omega_i \rightarrow \bar{\Omega}_i + \omega_i$$

- Three new interactions generated: $\bar{\Omega}_I QQ\partial\omega_I$, $QQ\partial\omega_I\partial\omega_I$ and $\bar{\psi}Q\partial\omega_I\partial\omega_I$
  - Choose, $\bar{\Omega}_I = \{1, 0, ..., 0\}$; $\bar{\Omega}_I QQ\partial\omega_I$ gives at most one loop
  - $QQ\partial\omega_I\partial\omega_I$ is scale free
  - Easily shown that $\omega_i \propto a^{-3}$; loops quickly redshifted away

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- Three new interactions generated: $\bar{\Omega}_I QQ\partial\omega_I$, $QQ\partial\omega_I\partial\omega_I$ and $\bar{\psi}Q\partial\omega_I\partial\omega_I$
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Loop Corrections

$$\mathcal{L} = \frac{1}{2}(\partial\psi)^2 - \frac{1}{2}m^2\psi^2 + \frac{1}{2}\psi^2(\partial\Omega)^2,$$

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- Short answer: this isn't *quite* the same case as Weinberg
- The fields Ω_i are not completely free; they satisfy

$$\sum_{i=1}^{N-1} \Omega_i \;=\; 1$$

- $\langle\psi^2\rangle \sim (N\epsilon_I)^{-1}(H^2/M_{\rm pl}^2)$
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What about loop corrections to perturbations?

A Coherent Field?

- Non appearance of any scaling of N in N-field inflation; really only one effective degree of freedom.
- Suggests that, effective degree of freedom: $\psi^2 = \sum_{J=1}^{N} \phi_J^2$
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Inflation with M-Spectator Fields: Loop Corrections

What about the other extreme:

- Four point interaction:

$$\mathcal{H}_4(t) \sim \int d^3x \, \frac{1}{aH^2}\Big[\partial^{-n}(\delta\phi\delta\phi + \sum_{J=1}^{M} \delta\sigma^J \delta\sigma^J)\partial^{-m}(\delta\phi\delta\phi + \sum_{K=1}^{M} \delta\sigma^K \delta\sigma^K)$$

- Three point interaction

$$\mathcal{H}_3(t) \sim \int d^3x \, \frac{1}{aH}\sqrt{2\epsilon}\, \delta\phi \sum_{J=1}^{M} \delta\sigma^J \delta\sigma^J$$

- 4-pt generates only one loop

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Summary

- Bounds on N:
  - Gradient energy bounds provide a constraint on the number of degrees of freedom in the early universe of:

$$N \ll \frac{M_{\rm pl}^2}{H^2}$$

  - One loop quantum corrections to the power spectrum in N-flation provide no bound on N
  - N-field inflation can be recast as a coherent single scalar field with one effective degree of freedom.
  - On the other extreme, single field field inflation with N spectator fields yields a bound on N which is weaker than the bound obtained from gradient energy considerations by ϵ:

$$N \ll \frac{M_{\rm pl}^2}{H^2}\frac{1}{\epsilon}$$

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