Title: NEC Violation and String Inspired Alternatives to Inflation

Date: May 22, 2009 09:00 AM

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Abstract: Inflationary cosmology provides a causal mechanism for the generation of super-Hubble cosmological fluctuations. There have been many alternative proposals suggested to accomplish this feat, however these all seem to share the need to violate at least the Null Energy Condition. I will attempt to make this statement more precise, and focusing on the case of string motivated models that contain a gravi-scalar in their spectrum (such as the string theoretic dilaton) we will find a "no-go" theorem. This provides an important challenge for models such as String Gas Cosmology and the Pre-Big Bang, if such models are to become more predictive.

NEC Violation and String Inspired Alternatives to Inflation

Scott Watson
Michigan Center for Theoretical Physics

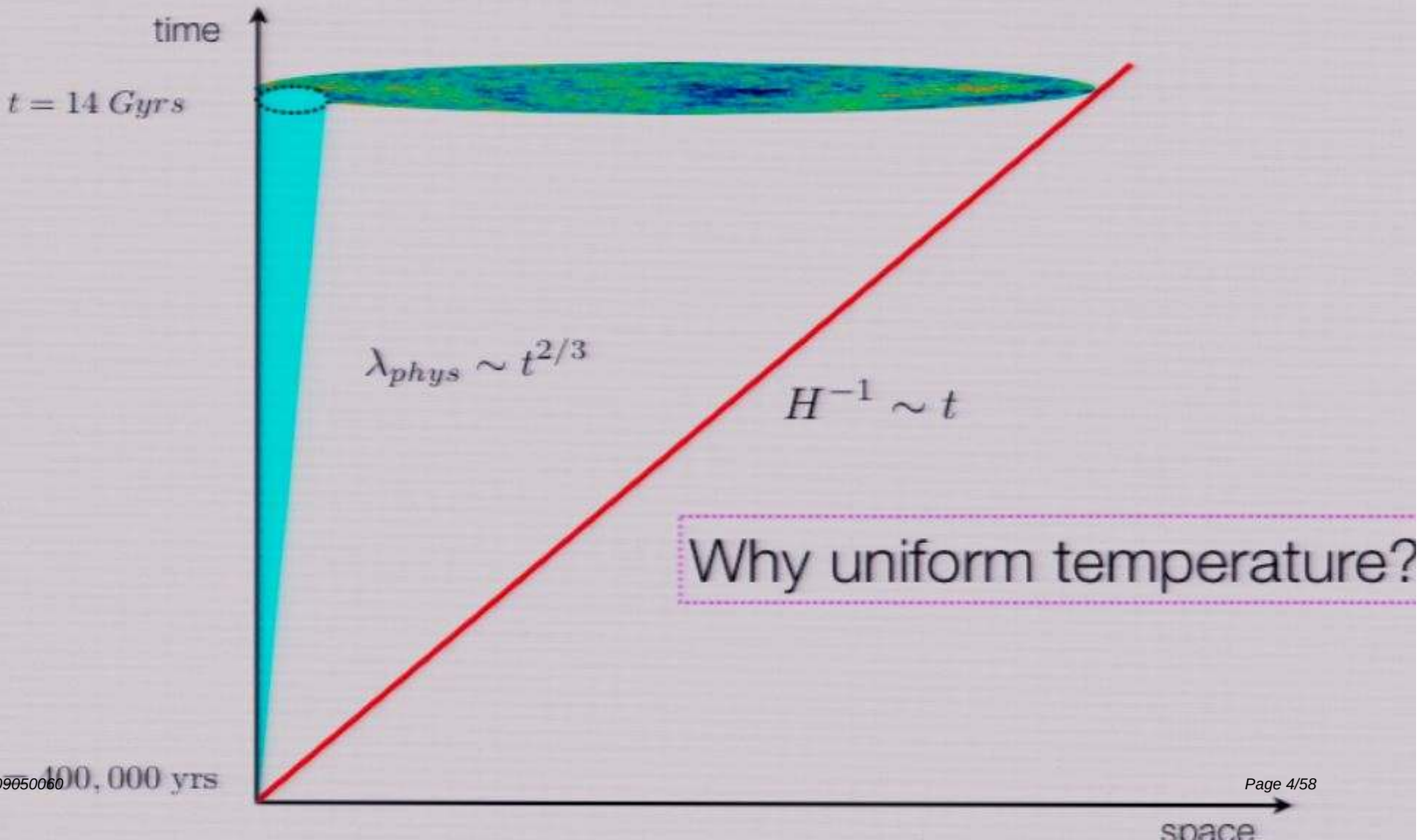
Based on work with Nemanja Kaloper

Spring workshop -- Part II

Effective Field Theory and Cosmology

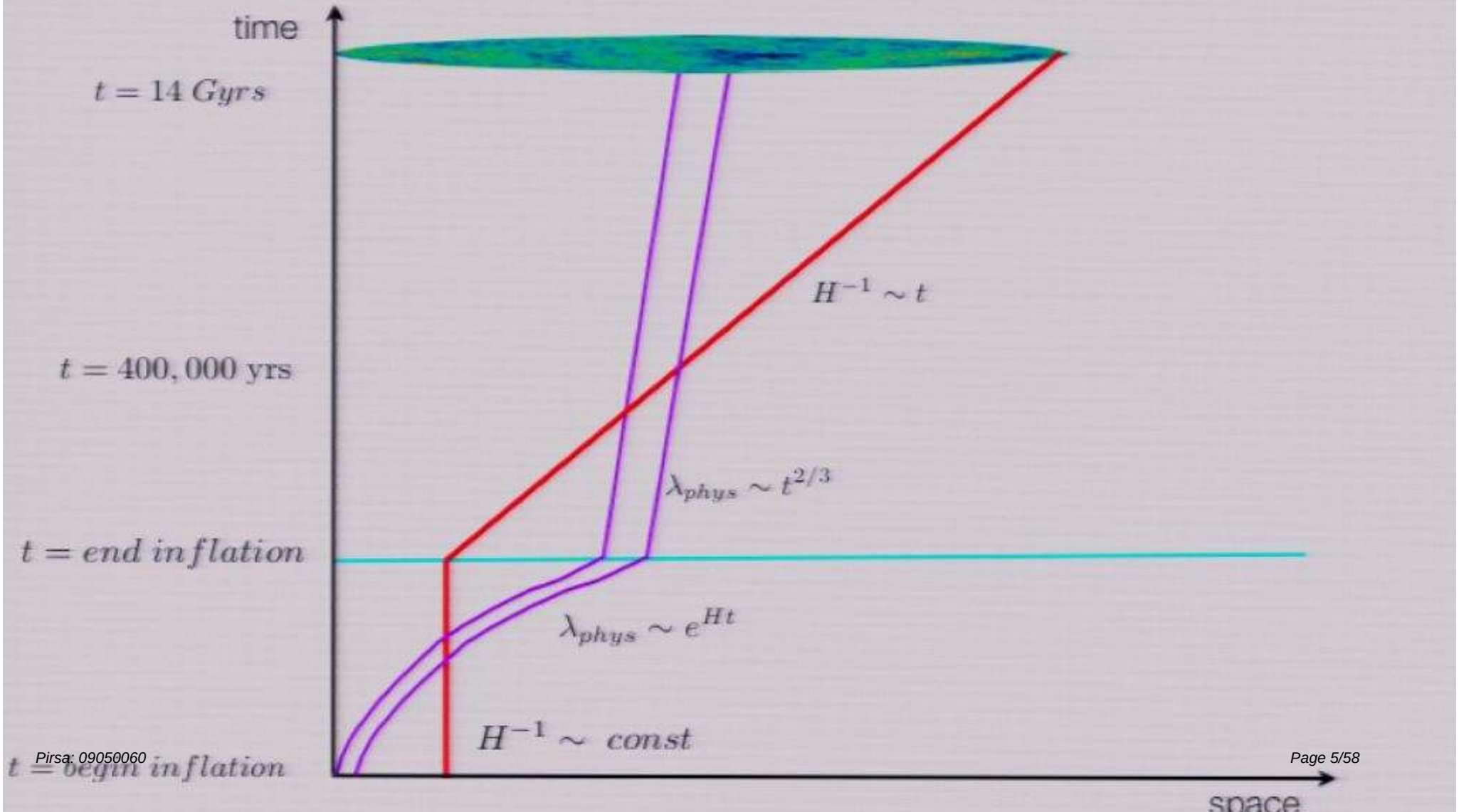
Michigan Center for Theoretical Physics
Ann Arbor, Michigan
Spring 2010

Causality and Structure



Why uniform temperature?

Inflationary Cosmology

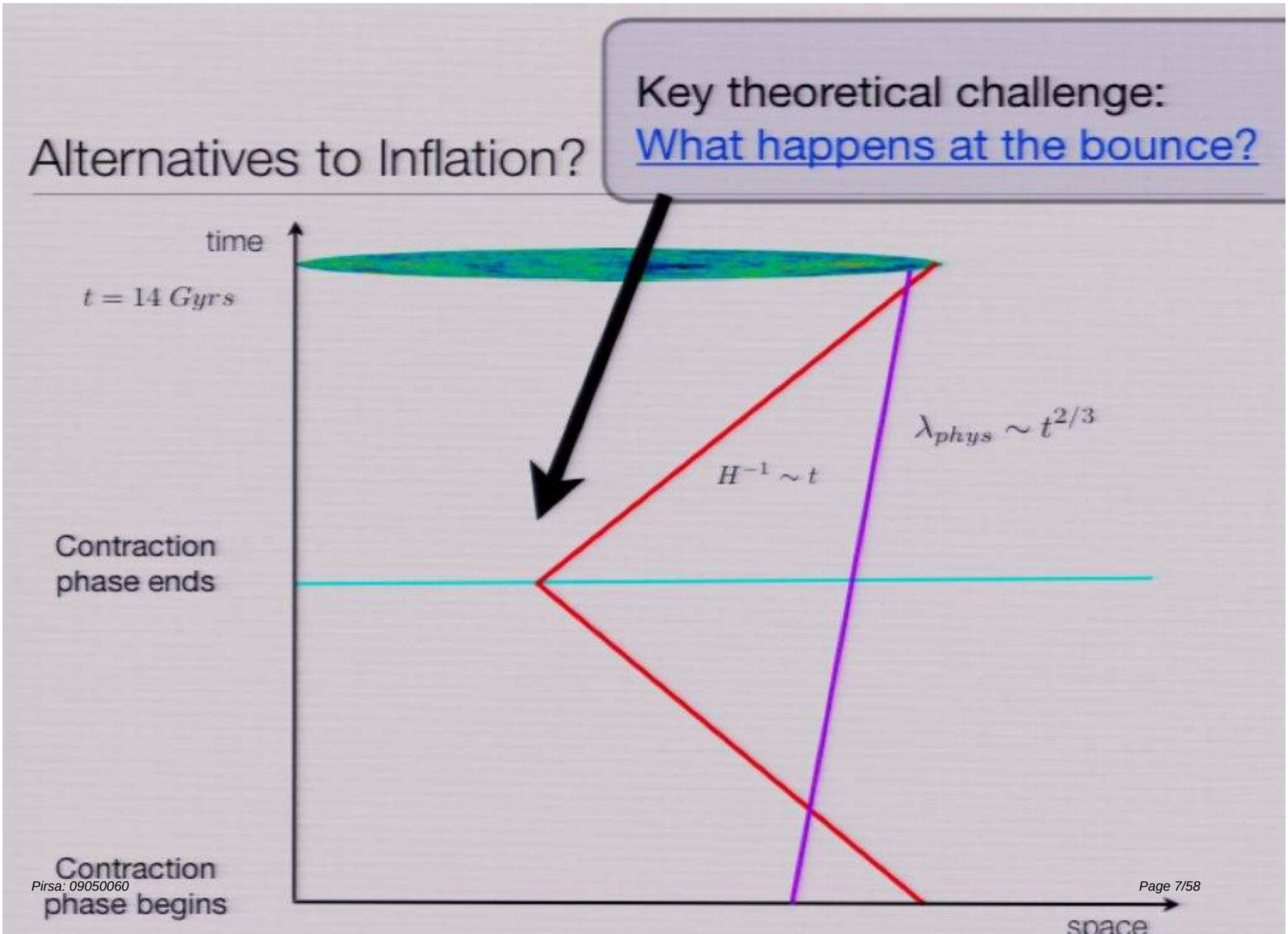


Is Inflation unique in making this prediction?

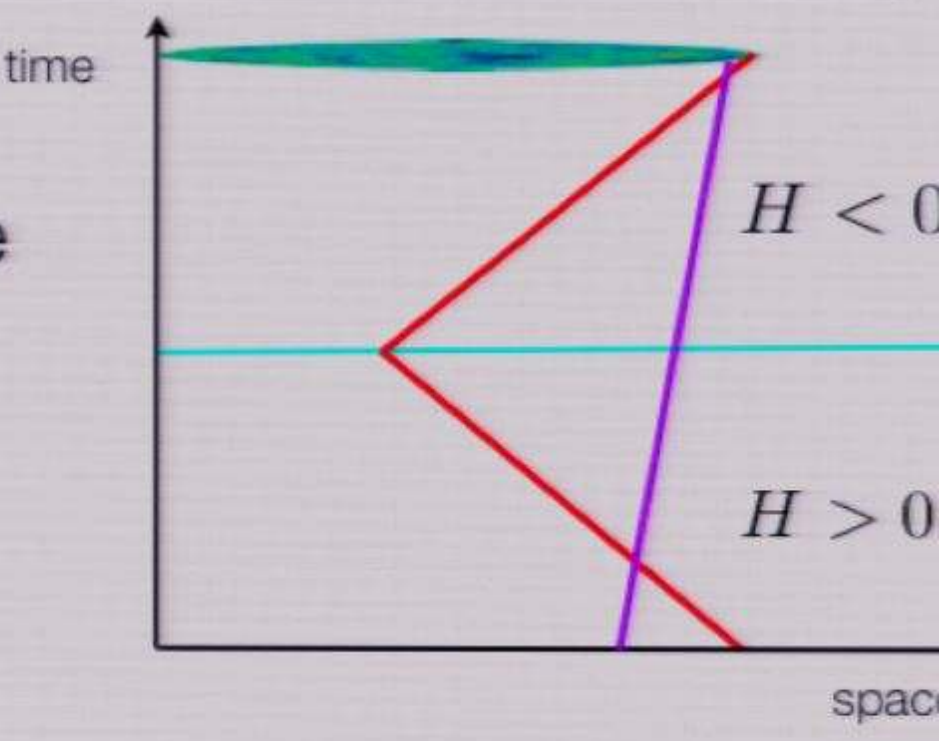
(e.g. Does a detection of B-mode polarization mean inflation took place

Alternatives to Inflation?

Key theoretical challenge:
What happens at the bounce?



NEC Violation and the Bounce



Bounce requires sign change

$$\dot{H} = -4\pi G\left(\rho + p\right)$$

NEC must be violated

$$T_{\mu\nu}n^{\mu}n^{\nu} \geq 0$$

$$\rho + p \geq 0$$

Bounce requires:

$$\dot{H} > 0$$

Dark Energy and the NEC

NEC for Fluids

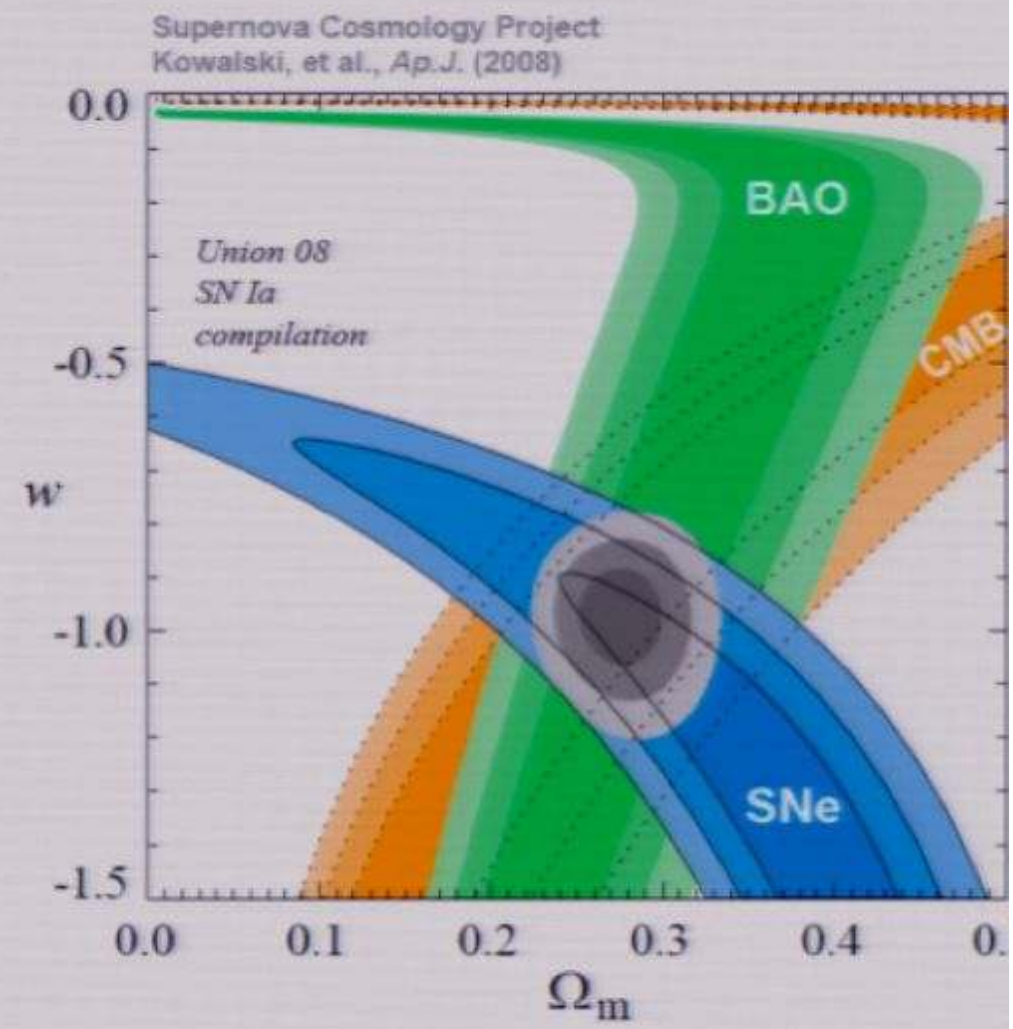
$$T_{\mu\nu}n^\mu n^\nu \geq 0$$

$$\rho + p \geq 0$$

Equation of State bound

$$p = w\rho$$

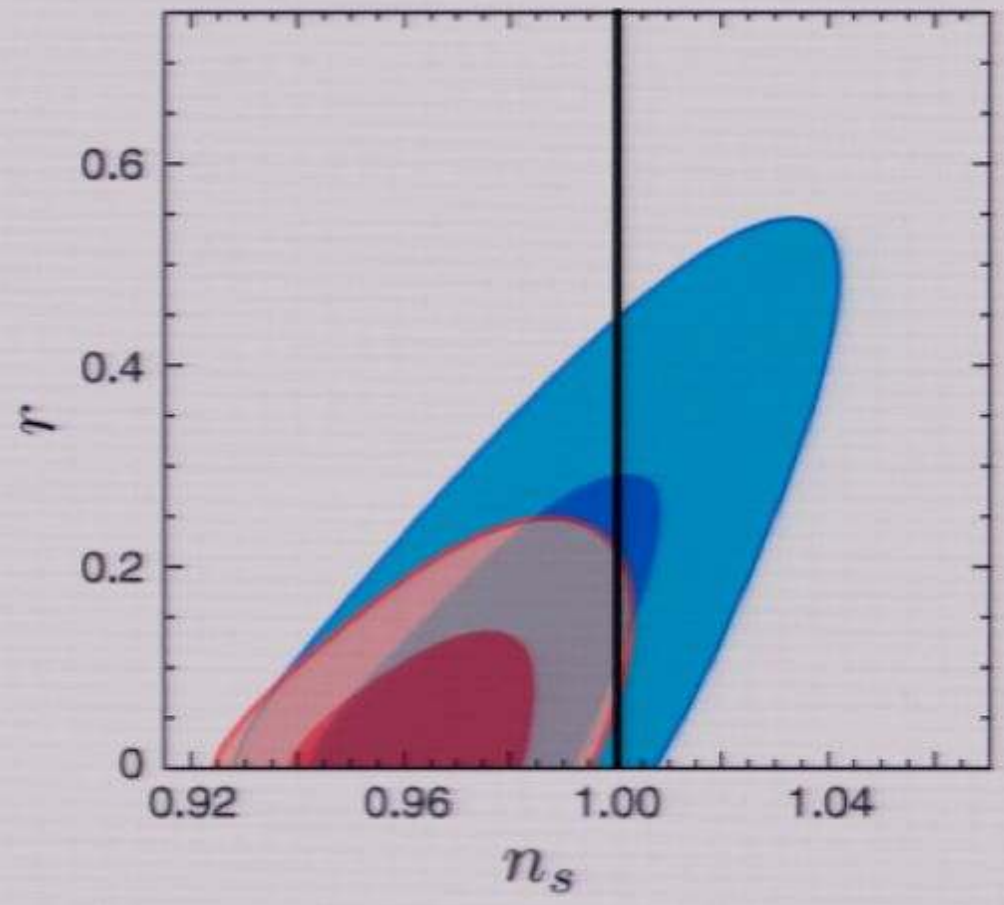
$$w \geq -1$$



Not ruled out by data

Inflation and the NEC

$$\dot{H} = -4\pi G\dot{\varphi}^2$$

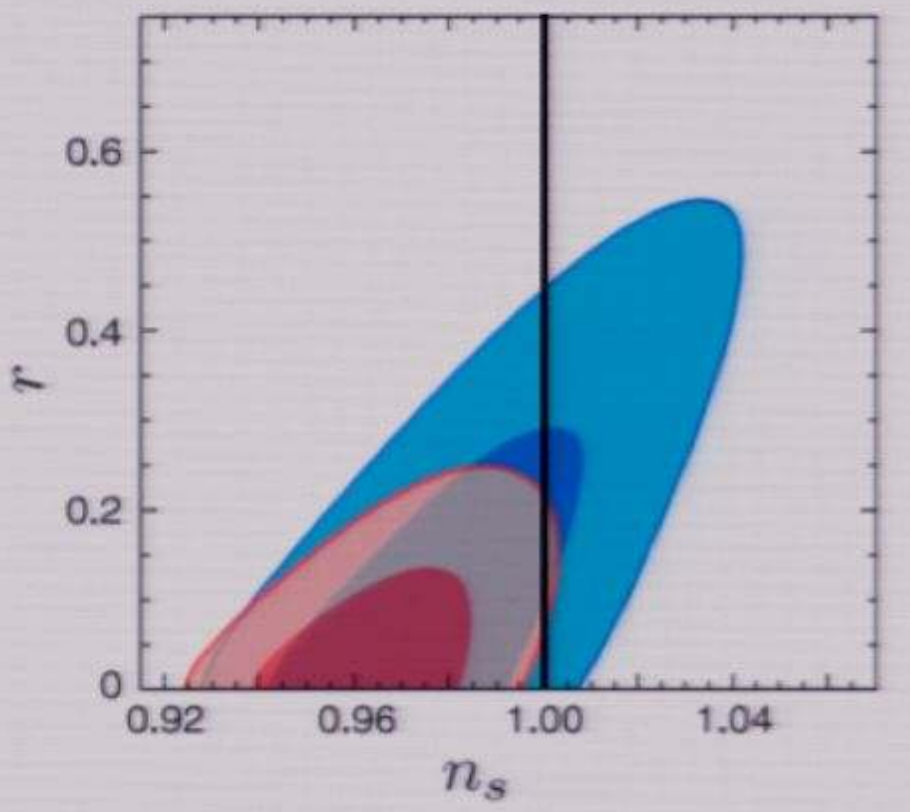


Inflation never violates the NEC

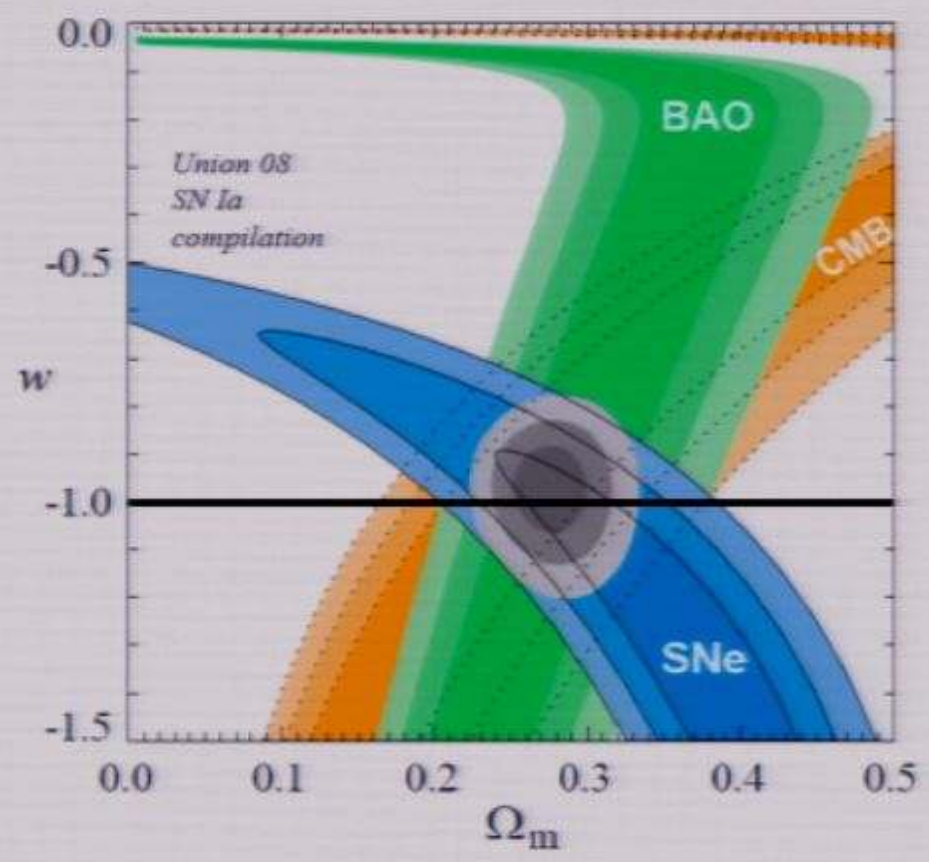
Thus always a red spectrum

Summary of Experimental Results

Early Universe



Late Universe



Small window for NEC violation

Theoretical Perspective: Why is violating the NEC bad?

- Violation of NEC typically implies the presence of ghosts (negative kinetic term) and/or tachyons (negative mass^2)
- When coupled to other matter leads to catastrophic instabilities
- Consequences explored by a number of authors
  - Carroll, Hofman, Trodden (astro-ph/0301273)
  - Cline, Jeon, and Moore (hep-ph/0311312)
  - Holdom (hep-th/0404109)
  - Hsu, Jenkins, Wise (astro-ph/0406043)

Goldstone Approach (recall Alberto's talk yesterday)

Creminelli, et. al. hep-th/060609

Summary

- Specify FRW background (fluids, scalars, etc...)

$$a(t)$$

- Focus on adiabatic mode and gauge fix time diffs (unitary gauge)

$$\phi(t,\vec{x}) = \phi_0(t) \qquad \delta\phi = 0$$

- Construct action (all terms that respect symmetries -- space diffs)

$$g^{00} \qquad E_{ij} = NK_{ij} = \frac{1}{2}\left[\partial_t g_{ij} - \nabla_i N_j - \nabla_j N_i\right]$$

- Perform broken gauge transformation and introduce Goldstone-Stuckelberg field

$$t \to t + \xi^0(t,x) \qquad \xi^0 \to \pi(t,x)$$

Goldstone Action

$$S_\pi = d^4x\sqrt{-g}\left[\frac{1}{2}M_p^2 R - M_p^2\dot{H}\left(\dot{\pi}^2 - (\nabla\pi)^2\right)\right.$$

Corrections

$\dot{H} > 0$ Ghosts!

Goldstone Action

$$S_\pi = d^4x\sqrt{-g}\left[\frac{1}{2}M_p^2 R - M_p^2\dot{H}\left(\dot{\pi}^2 - (\nabla\pi)^2\right) + 2M_2^4\left(\dot{\pi}^2 + \dot{\pi}^3 - \dot{\pi}(\nabla\pi)^2\right) - \frac{4}{3}M_3^4\dot{\pi}^3 + \ldots\right]$$

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Higher derivatives can eliminate ghosts

$$2M_2^4 - M_p^2\dot{H} > 0 \qquad c_s^2 = \frac{1}{1 - \frac{2M_2^4}{M_p^2\dot{H}}}$$

Higher spatial derivatives can tame gradient instabilities

Creminelli, et. al. hep-th/0606090

$$\Delta S_\pi = -\frac{\bar{M}^2}{2}(\nabla^2\pi)^2 \qquad \omega^2 = -\frac{2M_p^2\dot{H}}{M_2^4}k^2 + \frac{\bar{M}^2}{M_2^4}k^4$$

Allows for small and fast NEC violation $\Delta t < H^{-1}$

Spatial Gradients and UV Completions

Adams, et. al. hep-th/0606090

NEC can be violated, BUT...

$$c_s^2 = \frac{1}{1 - \frac{2M_2^4}{M_p^2 \dot{H}}} \qquad \Delta S_\pi = -\frac{\bar{M}^2}{2}(\nabla^2 \pi)^2$$

Superluminal propagation

Not UV Complete(able)

(S-matrix in the UV is sick (not analytic))

"SWAMPLAND"

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Partial Motivation for this work

Want to construct consistent low energy effective field theory that could separate the issue of the origin of the expanding universe from need for full theory of quantum gravity

AND

Perhaps provide an alternative to inflation

String Cosmology

These goals have also been pursued in the context of theories with a UV completion

NEC Violation and String Inspired Alternatives to Inflation

I still don't know what the idea is!

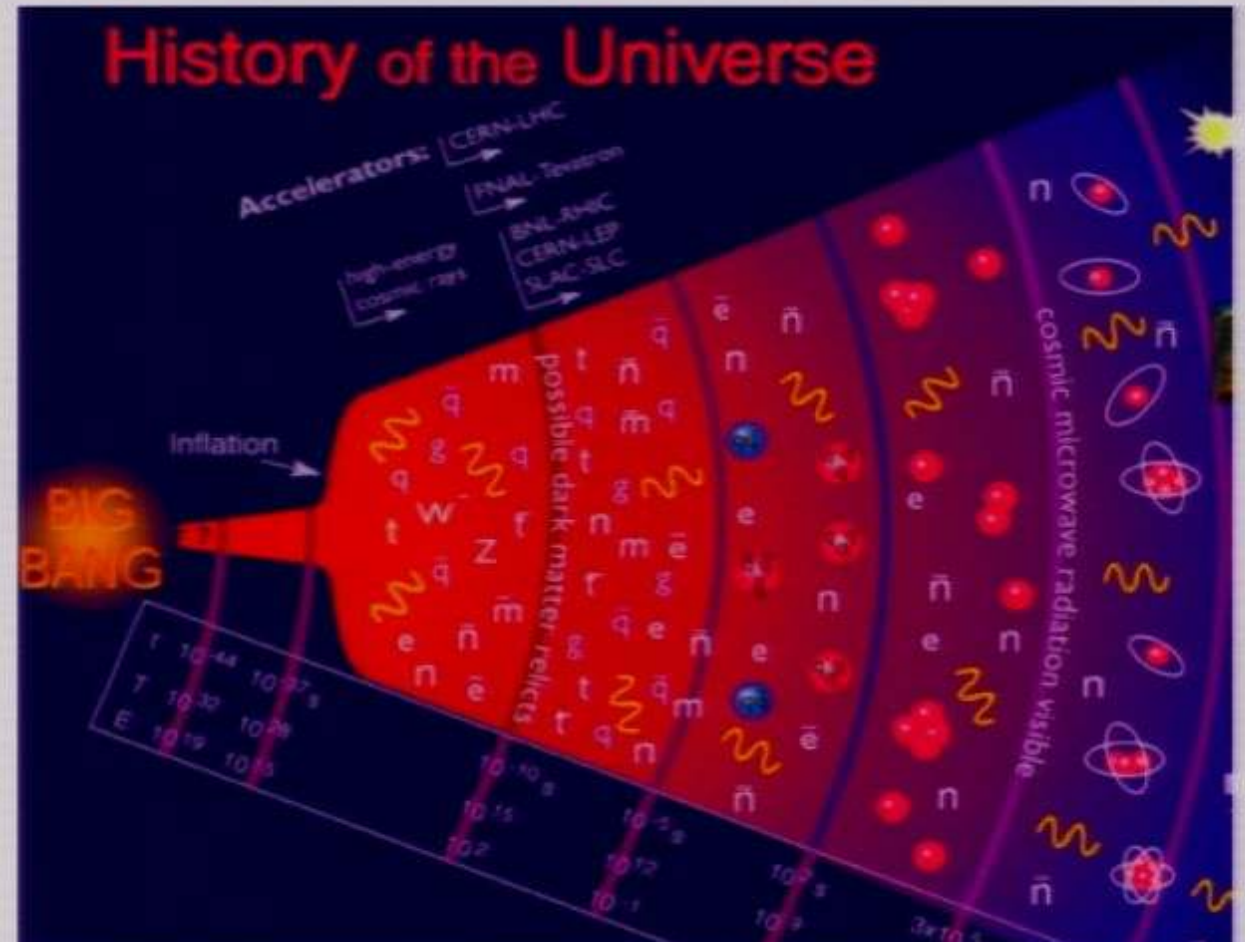
It's about nothing!

I think you may have something there...

String Gas Cosmology

T. Battefeld, and S.W. (skeptical review)
Reviews of Modern Physics 78:435-454,2006

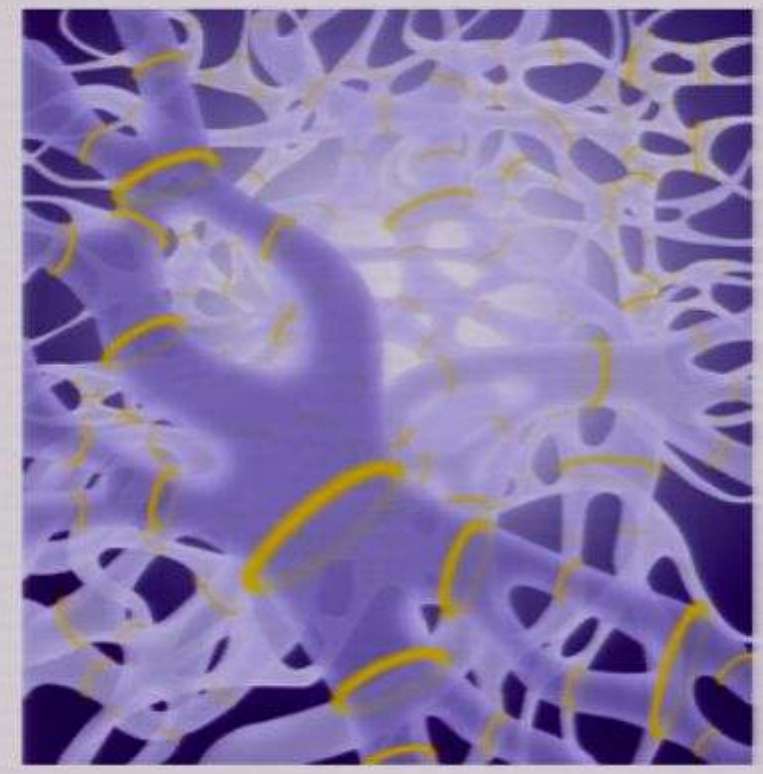
As we go back in time all observations suggest hotter, denser universe, and particle constituents become important



String Gas Cosmology

As we approach the scale of quantum gravity, new physics should become important:

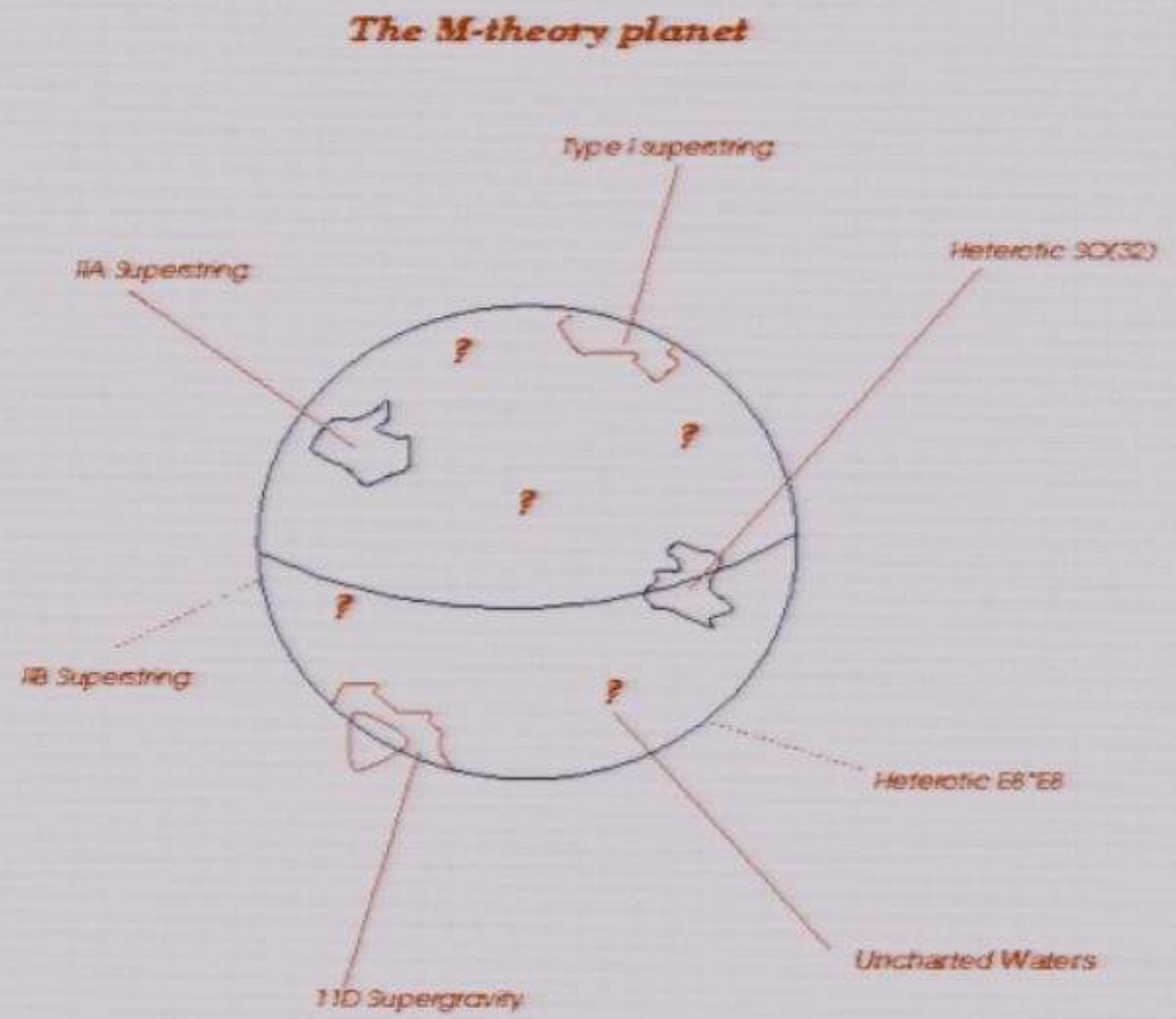
- Extra Dimensions
- Strings / Branes
- New Gauge Fields
- New Symmetries



Consider all ingredients and ask:

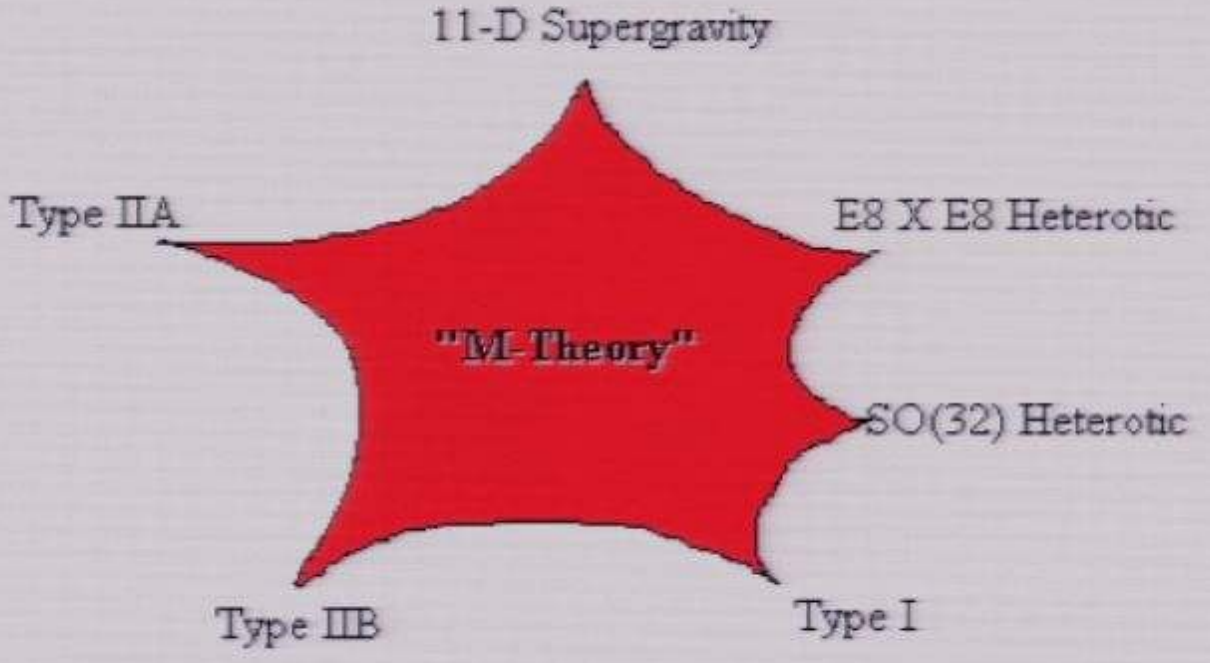
What are the cosmological consequences?

Status of Effective Field Theories with UV Completions prior to 1995



Relating EFTs in the UV

- "Islands" in the Landscape are related by dualities
- Dualities connect different effective field theories by exchange of couplings and solitonic degrees of freedom

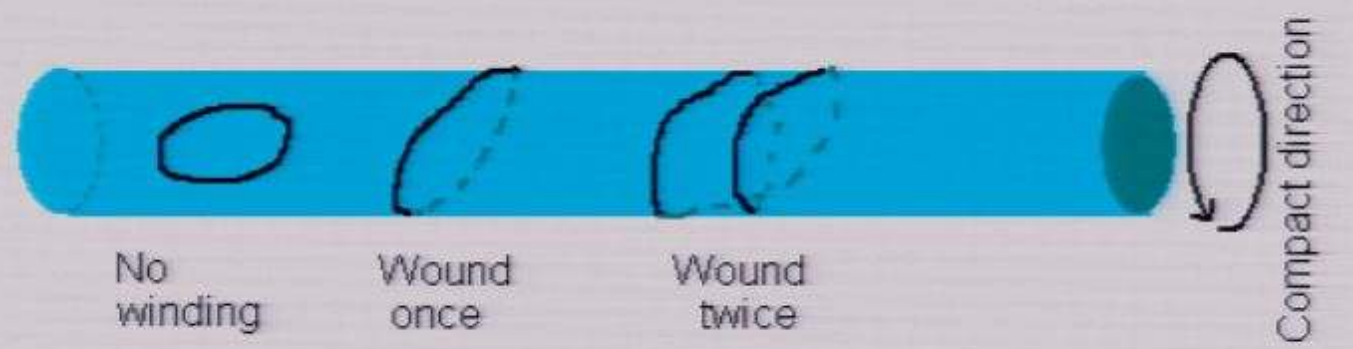


Example: T-duality

Theory of closed string on circle of radius R

Duality

Theory of wound string on circle of radius 1/R

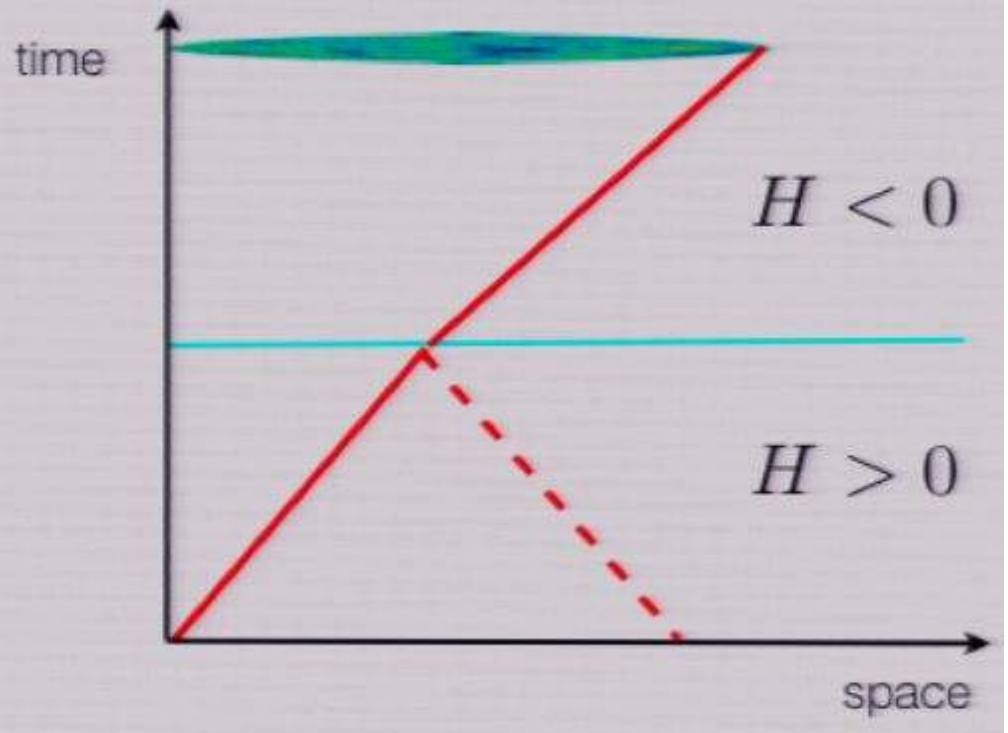


Duality and Cosmology

Brandenberger, Vafa, 1991.

T-duality relates scale factors as well

$$a \longrightarrow \frac{1}{a}$$



Strings in a collapsing universe described mathematically by their dual in an expanding universe

String scale plays crucial role

(can be substantially lower than Planck scale)

Elements of String Cosmology

Tseytlin and Vafa hep-th/9109048

Effective theory in N spatial dimensions

$$S_e = \frac{1}{2\kappa_N}\int d^{N+1}x\sqrt{-g_s}e^{-\phi_s}\left(R_s + (\partial\phi_s)^2 - \mathcal{L}_m\right) \qquad g_s^2 = e^{\langle\phi_s\rangle}$$

Space-time Metric

$$ds^2 = -n(t)^2dt^2 + e^{2\lambda(t)}d\vec{x}^2 \qquad \lambda = \ln a(t)$$

Mini-Superspace (Quantum mechanics)

$$S_{1D} = \int dt\, n\left\{\frac{e^{-\varphi_s}}{M_s}\left(N\frac{\dot{\lambda}^2}{n} - \frac{\dot{\varphi}_s^2}{n}\right) + F(\lambda, n\beta)\right\} \qquad \varphi = \phi - N\lambda$$

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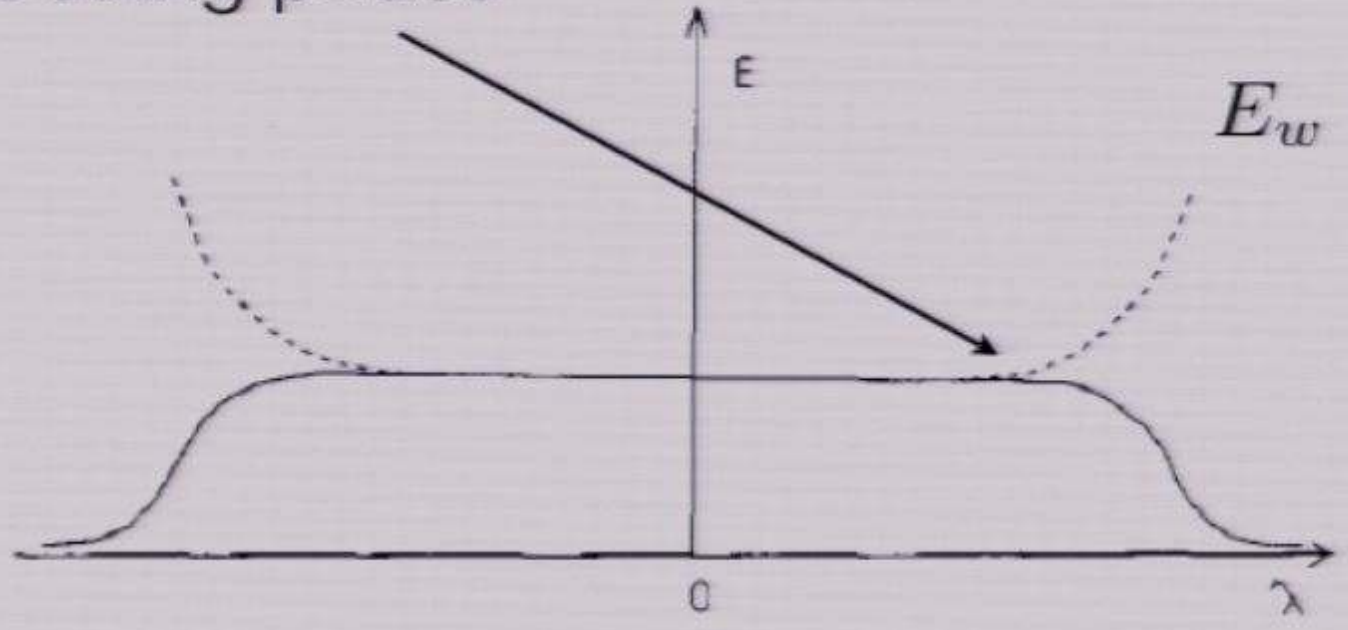
Hagedorn Regime

Exit string phase



$$E_w \sim e^{\lambda} \sim a(t)$$

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As universe cools we exit the "Hagedorn" phase, into a standard radiation dominated universe

$$E \sim a \sim const. \qquad H^{-1} \sim diverge$$

"Geometric Precipices in String Cosmology"

N. Kaloper and S.W. 0712.1820

Equations of motion

$$\begin{aligned}
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Solutions come on four branches

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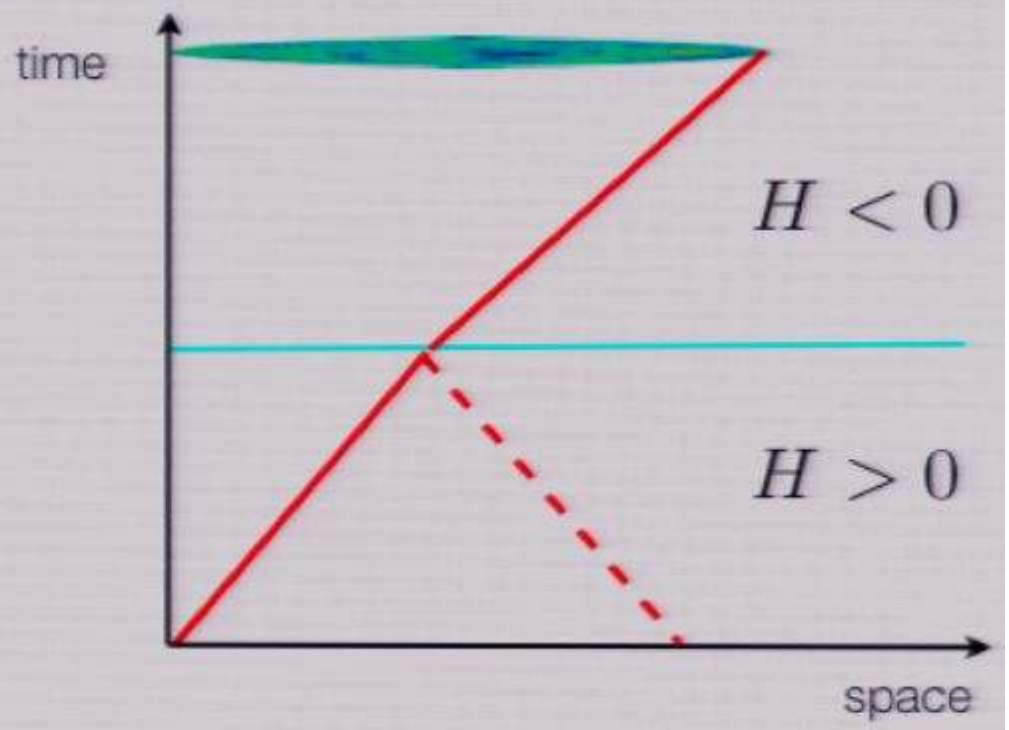

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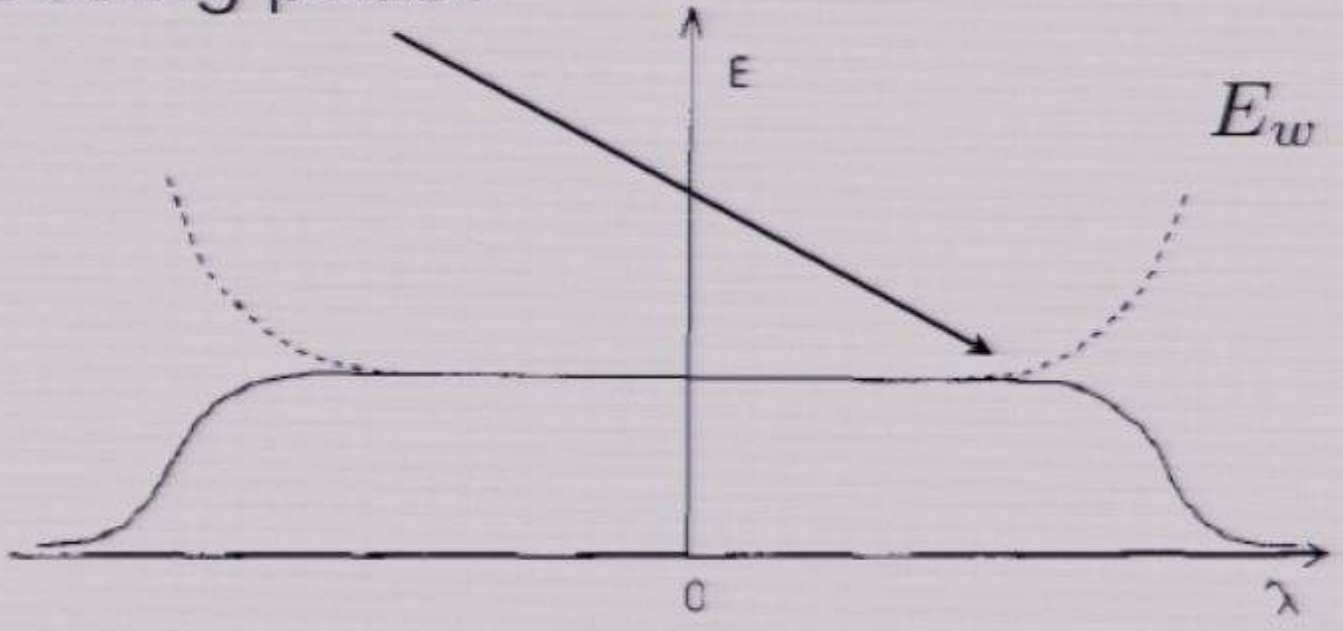

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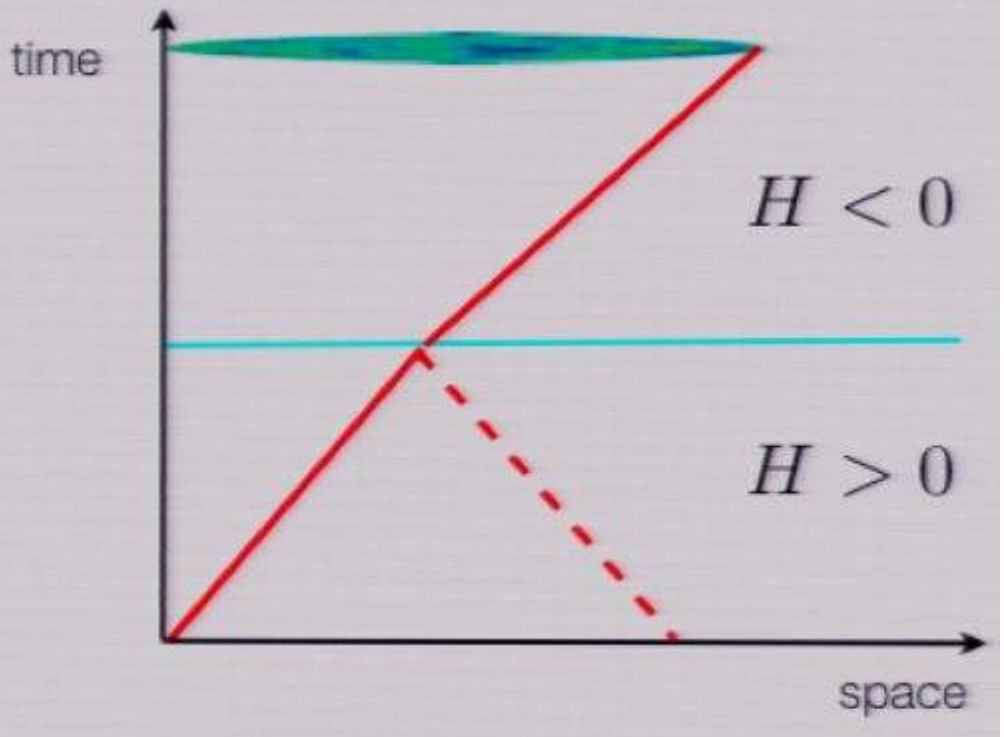

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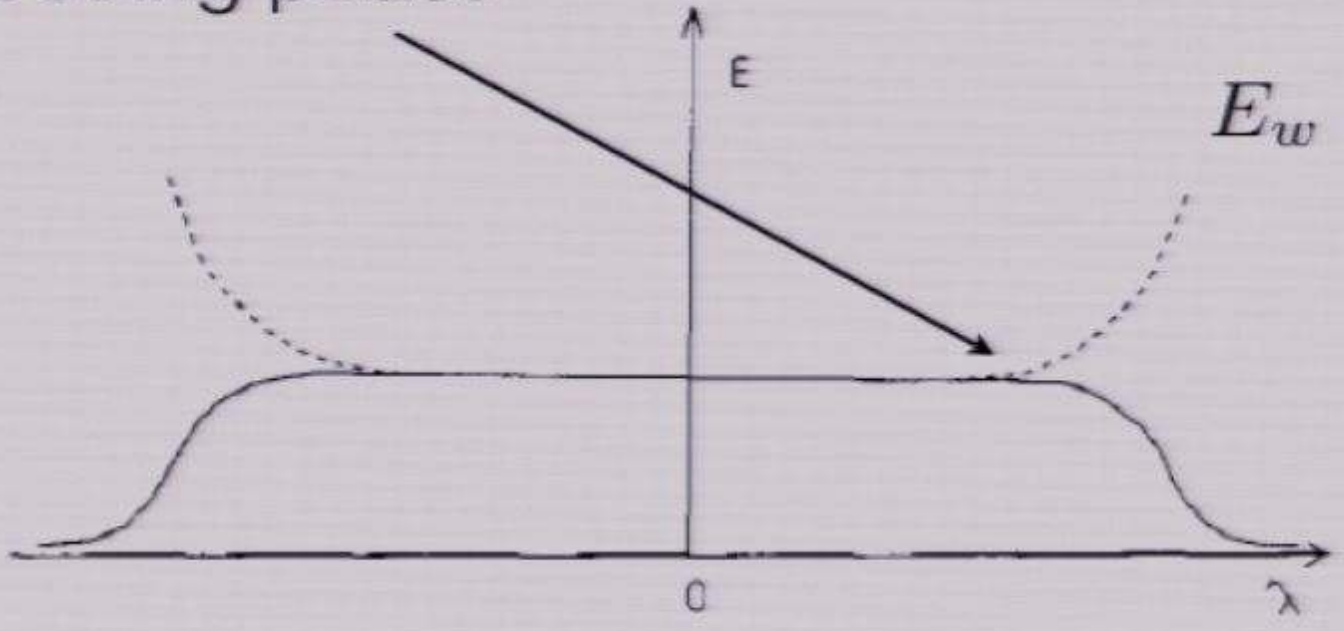

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Momentum mode gas

Our Universe

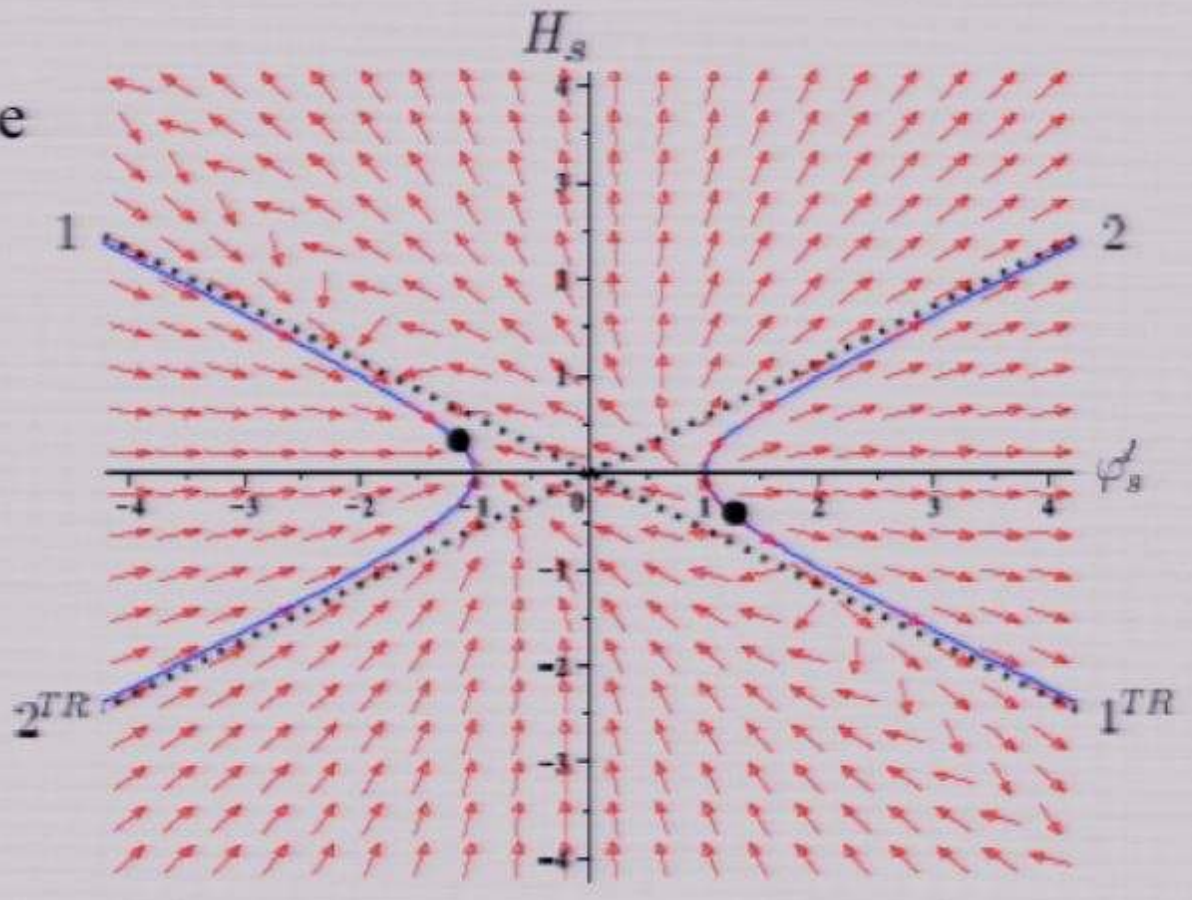


Figure 5: The (φ_s', H_s) phase diagram of momentum mode cosmologies, with phase space flow and the limiting envelopes (dotted lines) describing the case $E_0 = 0$.

Solutions

$$E = \frac{dx}{dt}$$

$$\lambda_s = \lambda_{s0} + \frac{\gamma}{\alpha}\ln\left[x(x-x_*)\right] + \frac{1}{\alpha\sqrt{N}}\ln\left(1-\frac{x_*}{x}\right),$$

$$\varphi_s = \varphi_{s0} - \frac{1}{\alpha}\ln\left[x(x-x_*)\right] - \frac{\gamma\sqrt{N}}{\alpha}\ln\left(1-\frac{x_*}{x}\right),$$

$$\phi_s = \phi_{s0} - \frac{1-N\gamma}{\alpha}\ln\left[x(x-x_*)\right] - \frac{(\gamma-1)\sqrt{N}}{\alpha}\ln\left(1-\frac{x_*}{x}\right),$$

- **The 4 classes of solutions are parameterized by the signs of x and x_* for fixed external parameters γ and N (note that $\alpha = 1 - N\gamma^2$)**

Momentum mode gas

Our Universe

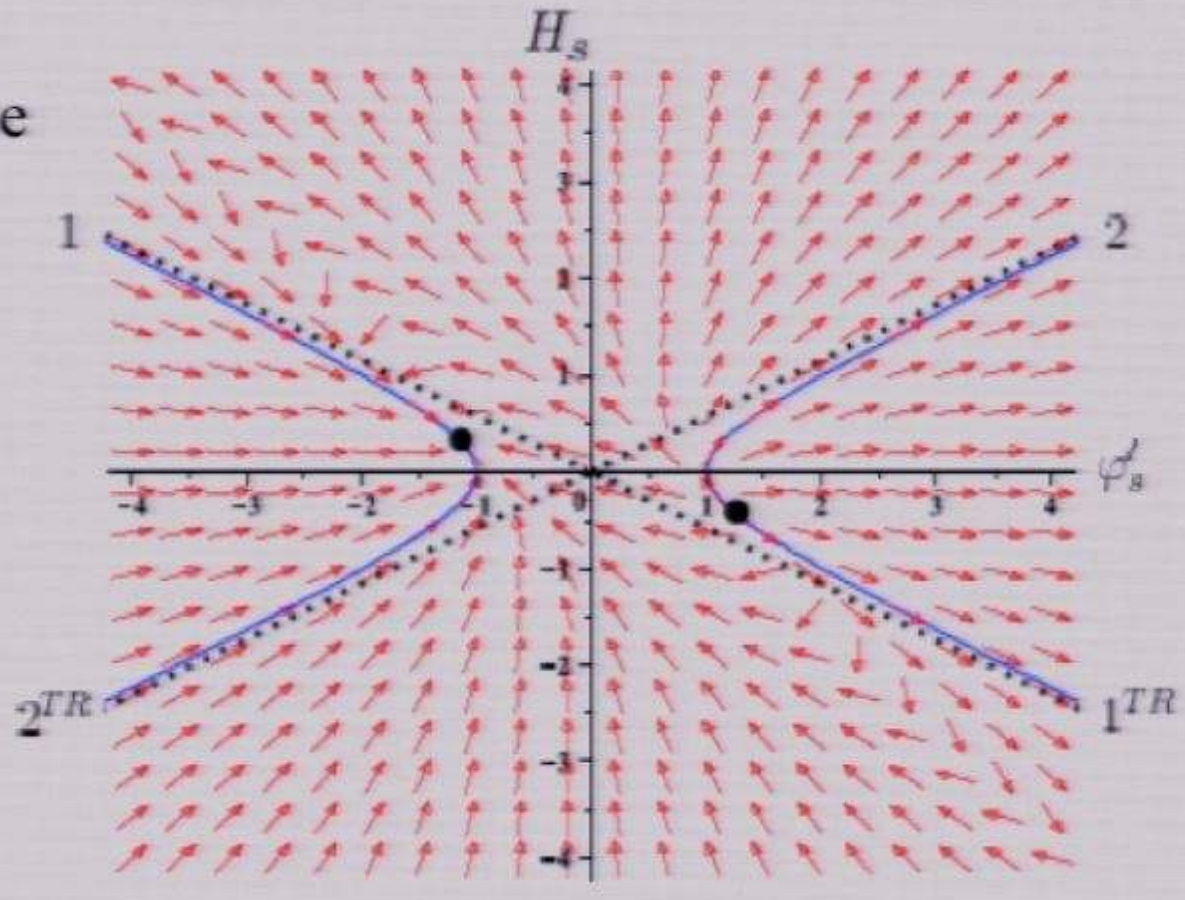


Figure 5: The (φ_s', H_s) phase diagram of momentum mode cosmologies, with phase space flow and the limiting envelopes (dotted lines) describing the case $E_0 = 0$.

Hagedorn gas

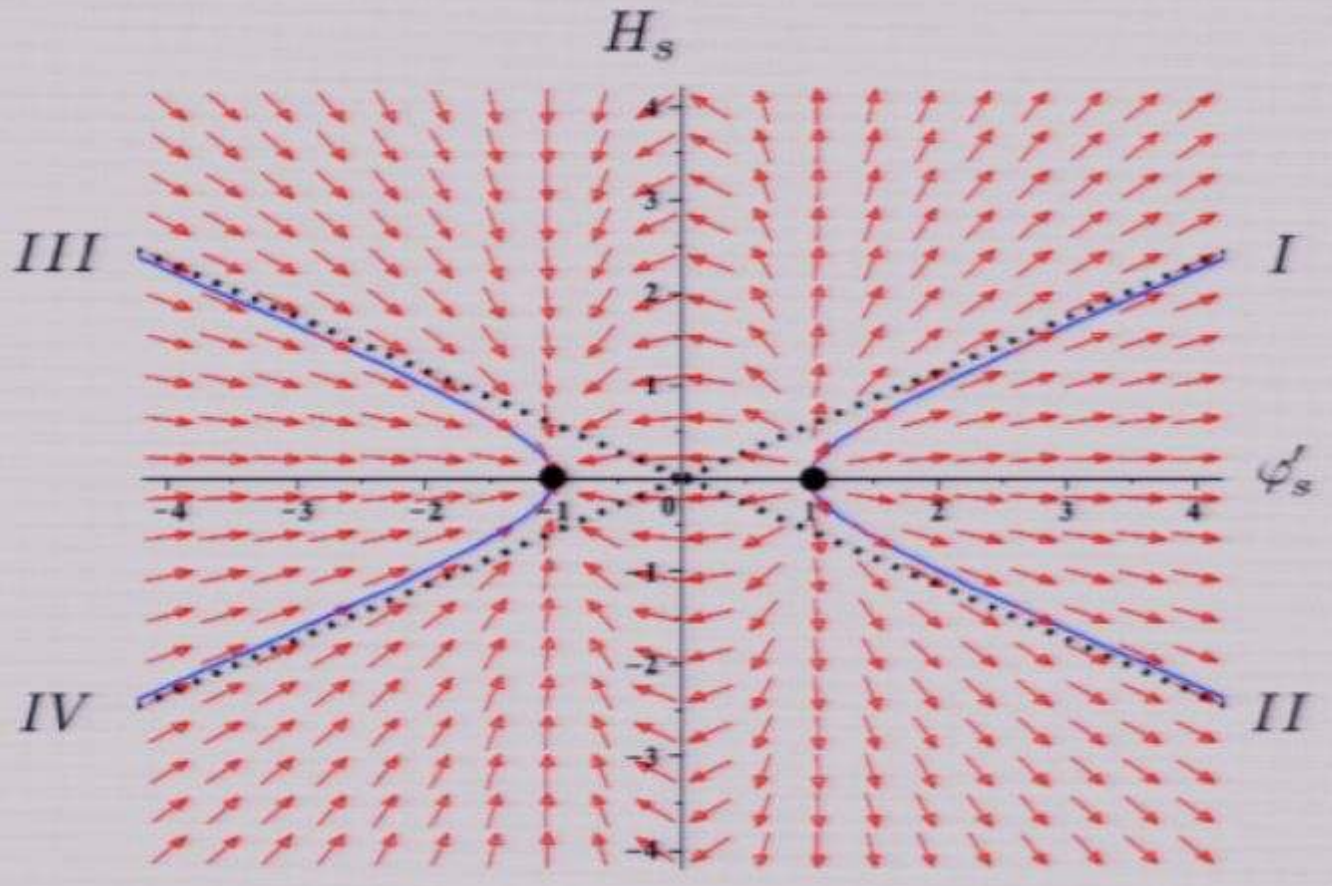


Figure 4: The (φ'_s, H_s) phase diagram of Hagedorn cosmologies, with phase space flow and the limiting envelopes (dotted lines) describing the case $E_0 = 0$.

Winding mode gas

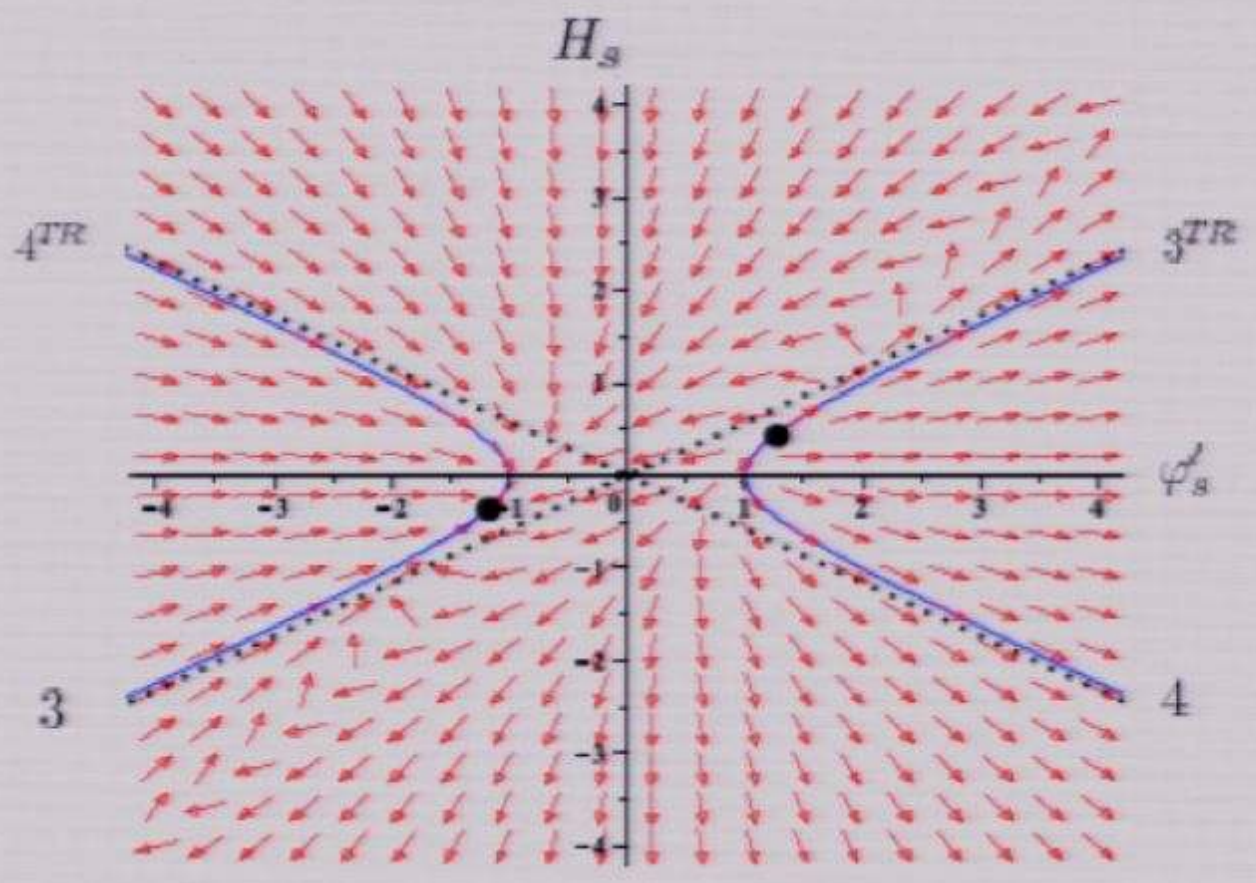
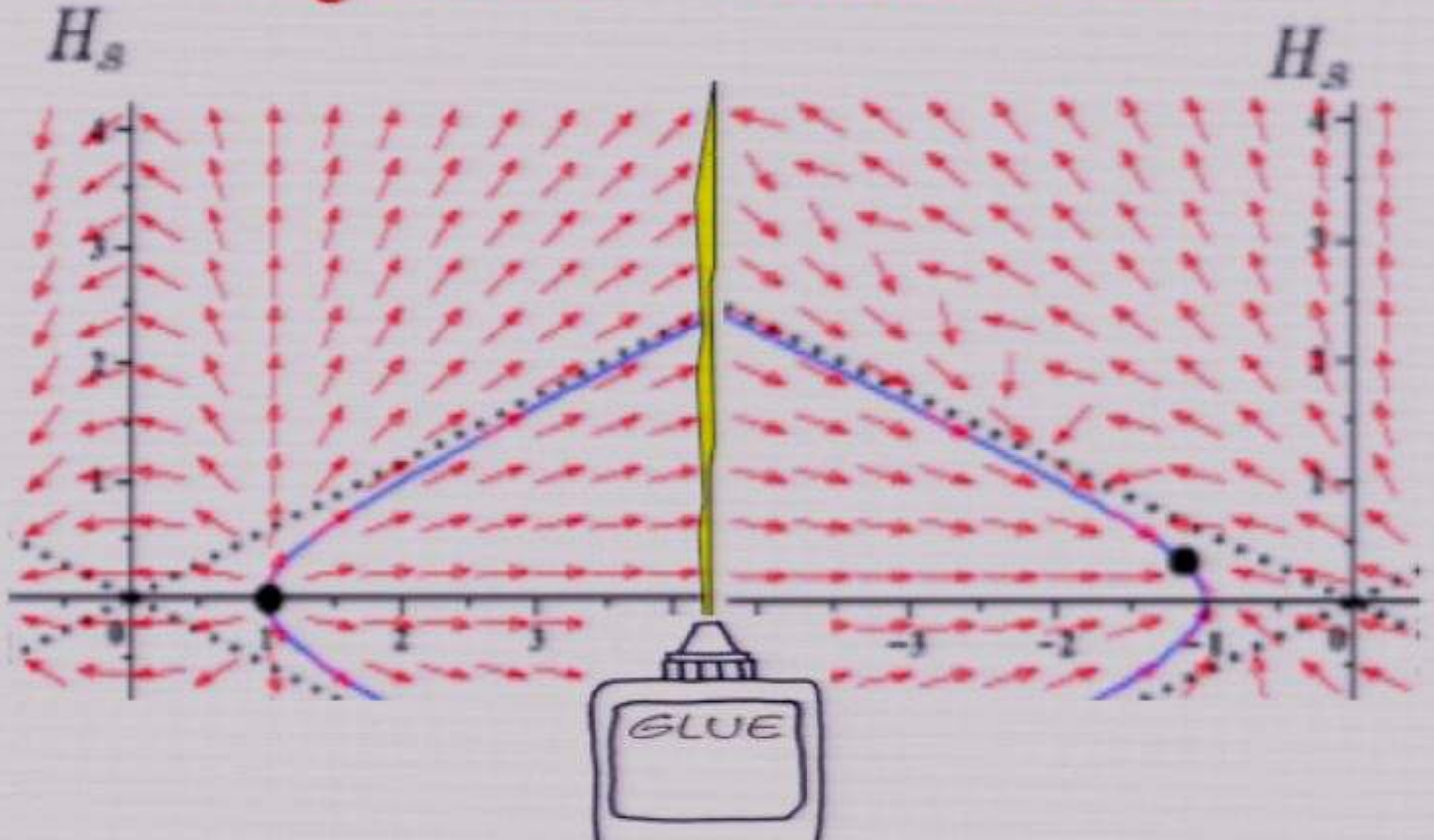


Figure 6: The (φ_s', H_s) phase diagram of winding mode cosmologies, with phase space flow and the limiting envelopes (dotted lines) describing the case $E_0 = 0$.

Cosmological Transitions?

- The stringy gas cosmology wants to link different solutions and avoid singularity; eg. of Tseytlin-Vafa:

Hagedorn I → Momentum 1.



No-go & Null Energy

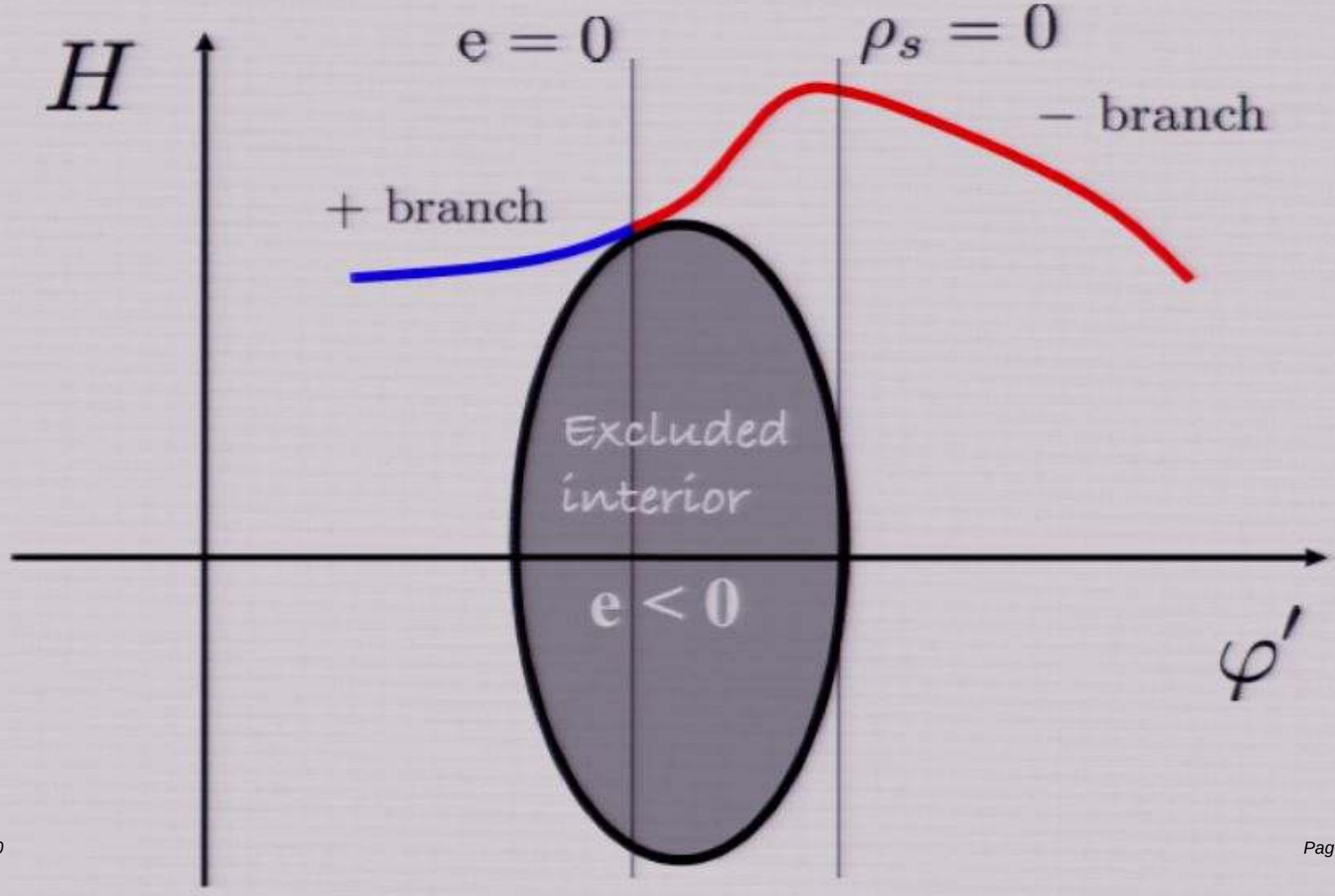
$$\dot{\phi}_s = NH_s \pm \sqrt{NH_s^2 + e^{\phi_s}\rho_s}\,,$$

- These solutions are on different branches: (+) and (-)!
- For matching to occur, dilaton velocity must reverse: there must be region where the `egg function' vanishes!

$$\mathrm{e} = NH_s^2 + e^{\phi_s}\rho_s$$

- For this to happen we must ensure:
  - Negative energy density somewhere
  - Collision between the trajectory and this region
  - Subsequent escape without further collision

Negative energy density somewhere: put in a potential which dips below zero for some values of the dilaton. But that in itself is NOT ENOUGH! What is required is something like:



- Consider evolution of e along a trajectory:

$$\pm \frac{d}{dt}\left(\sqrt{\mathsf{e}}\right) = -\dot{H}_s + H_s\dot{\phi}_s + \frac{1}{2}e^{\phi_s}\left(\rho_s + p_s\right)$$

- (+/-) refers to the branch on which this evolves.
- Assume that a trajectory hits the egg, and escapes and integrate between the hit and the escape time:

$$\int_{t_h}^{t_e} dt \left[\frac{d}{dt}\left(\pm\sqrt{\mathsf{e}} + H_s\right) - H_s\,\dot{\phi}_s\right] = \frac{1}{2}\int_{t_h}^{t_e} dt\, e^{\phi_s}\left(\rho_s + p_s\right)$$

- The 3rd term on the LHS is the negative area between the curve and the horizontal axis. Rewrite as

$$\left(\mp\sqrt{N} - 1\right)H_s(t_e) + H_s(t_h) + \mathcal{A} = -\frac{1}{2}\int_{t_h}^{t_e} dt\, e^{\phi_s}\left(\rho_s + p_s\right)$$

Same idea as in Kaloper, Madden, Olive, 1995.

- If you start on (+) branch:

$$\left(\sqrt{N}-1\right) H_s(t_e) + H_s(t_h) + \mathcal{A} = -\frac{1}{2}\int_{t_h}^{t_e} dt\, e^{\phi_s}\,(\rho_s + p_s)$$

- LHS positive definite: to have the transition from + to - must get RHS to be negative.
- If you start on (-) branch:

$$\left(\sqrt{N}+1\right) H_s(t_e) = H_s(t_h) + \mathcal{A} + \frac{1}{2}\int_{t_h}^{t_e} dt\, e^{\phi_s}\,(\rho_s + p_s)$$

- Branch change from - to + can occur with normal matter.
- But then all the - branch solutions are past-singular and + ones are future singular.
- This excludes the transitions proposed by Tseytlin and Vafa

- If energy is strictly positive, it is impossible to smoothly connect different branches.
- Impossible to remove the singularity from the EFT picture UNLESS one violates the null energy condition. Only then + to - transitions
- In the standard framework where null energy condition is maintained such evolution does not happen and singularity is present somewhere.
- A variant of the Hawking-Penrose singularity theorem, which one expects in the Einstein frame!
- Other transitions are possible! (work in progress)

Summary

- Still no viable alternative to inflation for generation of cosmological perturbations
- Key issue is the "bounce" and/or the NEC
- Interesting to combine String Gas approach and Ghost Condensate ideas
- Is B-mode detection really a smoking gun for inflation?