Title: Power Counting vs Higgs Inflation

Date: May 21, 2009 02:00 PM

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Abstract: We use the power-counting formalism of effective field theory to study the size of loop corrections in theories of slow-roll inflation, with the aim of more precisely identifying the limits of validity of the usual classical inflationary treatments. Although most slow-roll models lie within the semiclassical domain, we find the consistency of the Higgs-Inflaton scenario to be more delicate due to the proximity between the Hubble scale during inflation and the upper bound allowed by unitarity on the new-physics scale.

T.B.A.
:Taking eft Basics and Applying them to slow roll inflation models

Michael Trott, PI

Based on arXiv:0902.4465 with C.P Burgess and Hyun Min Lee

Can the Higgs Boson be the Inflaton?

Hint: Remember Hinchliffe's rule

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Outline:

1. Background basics.

2. A power counting tool for GR+scalars inflation models such as this one.

3. Higgs Inflation basics, the good and bad of the proposal of Shaposhnikov and Bezrukov that revives non-minimal coupling in Higgsflation.

4. Power counting insights into Higgs inflation and higher curvature inflation and their severe limitations as effective theories.

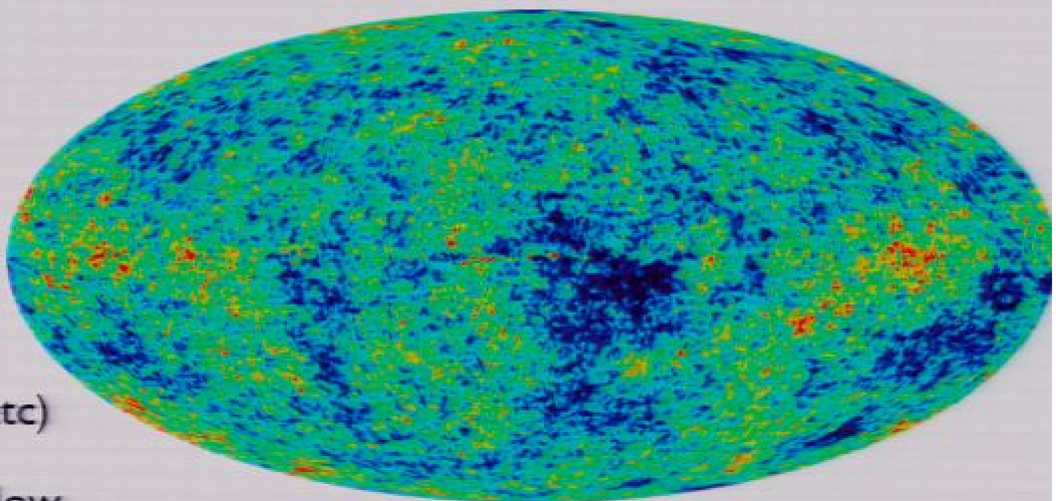
5. Conclusions and lamentations.

Quick summary: An EFT power counting morality tale.

Inflation basics

Most are confident that inflation occurred
as it is consistent with CMB measurements.
It efficiently addresses the:

- flatness,
- homogeneity, isotropy,
- horizon and
- undesired relic problems.
  (string modulii, monopoles, gravitinos,etc)



WMAP 05

This talk will be about the most vanilla, of vanilla slow roll inflation models, we will examine using the one scalar field we actually have evidence for so far , the higgs, to get slow roll inflation.

Setting notation:

$$\epsilon(\phi) = \frac{M_{pl}^2}{2}\left(\frac{V'(\phi)}{V(\phi)}\right)^2 \qquad \eta(\phi) = M_{pl}^2\left(\frac{V''(\phi)}{V(\phi)}\right)$$

$$\zeta(\phi) = M_{pl}^4 \frac{(V''')(V')}{V^2}$$

Also recall the defns:

$$H \equiv \frac{\dot{a}}{a} = \frac{\sqrt{V(\phi)/3}}{M_{pl}} \qquad M_{pl} = (8\pi G)^{-1/2} = 2.4\times 10^{18}\,\mathrm{GeV}$$

Do any two people at this workshop agree how Inflation is coming about

Data supports the inflation paradigm but doesn't specify the details (yet)
so we engage in fun & inciting speculation:

- slow roll scalar field(s) of various origins,
- fields in various (S)GUTS,
- alternate brane inflation scenarios,
- various embeddings of inflation in string theory,
- multi field inflation models, higher derivative gravity, etc, etc, etc.

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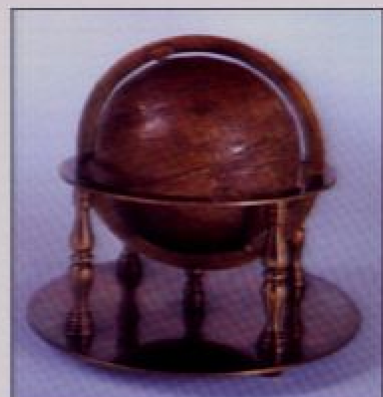

Hunt-Lenox Globe 1510
(third oldest globe known)

Hic sunt dracones

"Here there be dragons"

19th century Japanese map
'Jishin-no-ben'

Dragons thought to be everywhere where
the mapmaker hasn't been personally.
Does this sound familiar?

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EW sym breaking ← *Hic sunt scalares* → **Inflation**

"Here there be scalars"

Maybe it's the same scalar gets you to Higgs Inflation

Scalars are thought to be everywhere?
Could be a sign of trouble.... they could be dragons.

Do any two people at this workshop agree how Inflation is coming abou

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Don't take my word for it :

Gratuitous name drop! (count 1)

As Dr. Nima Arkani-Hamed, a Princeton particle theorist, puts it, due to the probabilistic nature of quantum physics, "There is some minuscule probability that the **Large Hadron Collider** **might make dragons** that might eat us up."

General EFT construction, bottom up

Consider the general derivative expansion Lagrangian of scalers θ^i coupled to the metric:

$$-\frac{L_{eff}}{\sqrt{-g}} = v^4 V(\theta) + \frac{M_p^2}{2} g^{\mu\nu}\left[W(\theta)\, R_{\mu\nu} + G_{ij}(\theta)\,\partial_\mu\theta^i\partial_\nu\theta^j\right]$$
$$+A(\theta)(\partial\theta)^4 + B(\theta)\, R^2 + C(\theta)\, R\,(\partial\theta)^2 + \frac{E(\theta)}{M^2}(\partial\theta)^6 + \frac{F(\theta)}{M^2}\, R^3 + \cdots$$

All possible invariants involving one Riemann tensor and two derivatives acting on θ^i

M that makes up the dimensions is characteristic of whatever underlying microscopic physics has been integrated out, generally $M \ll M_p = (8\pi G)^{-1/2}$

All possible 3 Riemann tensor invariants, or two Riemann tensors and two covariant derivatives

A particular model in an EFT will also have a cut off on E above which unitarity will be violated.

This constraint on E will supply an upper bound on M for unitarity to be preserved.

Unitarity constraints.

General EFT construction, bottom up

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Expand around a classical background solution

$$\theta^i(x) = \vartheta^i(x) + \frac{\phi^i(x)}{M_p} \quad \text{and} \quad g_{\mu\nu}(x) = \hat{g}_{\mu\nu}(x) + \frac{h_{\mu\nu}(x)}{M_p}.$$

Perform semi-classical perturbation theory as follows

$$L_{eff} = (\hat{L}_{eff}(\vartheta, \hat{g}_{\mu\nu}) + L_{mass}) + M^2 M_p^2 \sum_n \frac{c_n}{M^{d_n}}\, O_n\left(\frac{\phi}{M_p}, \frac{h_{\mu\nu}}{M_p}\right)$$

the 1PI generator is

$$\Gamma[\theta, g_{\mu\nu}] = \int d^4x\,(\hat{L}_{eff}(\vartheta, \hat{g}_{\mu\nu}) + L_{mass}) + L_{int}$$

Power counting makes manifest the nature of the expansion in the scales of the problem.

Power Counting

Power counting is simple for $L_{int} = M^2 M_{pl}^2 \sum_n \frac{c_n}{M^{d_n}} O_n\left(\frac{\phi}{M_{pl}}, \frac{h_{\mu\nu}}{M_{pl}}\right)$

- Number of loops in a connected graph $L = 1 + I - \sum_n V_n$
- Conservation of ends in a connected graph $2I + \xi = \sum_n N_n V_n$

just topology

- For a graph with V_n vertices get a factor $M_{pl}^{2-2L-\xi} \prod_n \left[c_n M^{2-d_n}\right]^{V_n}$

Some dimensionless ratios are buried in the c_n

$$c_n = \left(\frac{v^4}{M^2 M_{pl}^2}\right) \lambda_n$$ no derivative scalar potential terms

$$c_n = \left(\frac{M^2}{M_{pl}^2}\right) g_n$$ more than 2 derivative terms

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just topology

- For a graph with V_n vertices get a factor $\quad M_{pl}^{2-2L-\xi} \prod_n \left[c_n M^{2-d_n}\right]^{V_n}$

For an arbitrary graph regulated with dim reg:

$$A_\xi(E) \simeq M_{pl}^{2-2L-\xi} \prod_n \left[c_n M^{2-d_n}\right]^{V_n} \int \cdots \int \left(\frac{d^d p}{(2\pi)^d}\right)^L \frac{\prod_n p^{d_n V_n}}{(P^2 - m_i^2)^I}$$

$$\simeq E^2 M^2 \left(\frac{1}{M_{pl}}\right)^\xi \left(\frac{E}{4\pi M_{pl}}\right)^{2L} \prod_{d_n=2} (c_n)^{V_n} \prod_{d_n=0} \left[\lambda_n \left(\frac{v^4}{E^2 M_{pl}^2}\right)\right]^{V_n} \prod_{d_n \geq 4} \left[g_n \left(\frac{E}{M_{pl}}\right)^2 \left(\frac{E}{M}\right)^{d_n-4}\right]^{V}$$

This is a powerfull eqn for studying scalar field inflation models.

Power Counting is indeed powerfull

The purpose of this digression is to illustrate how power counting systematically leads you to understanding where the problems come from. Consider a low energy expansion:

$$A_\xi(E) \simeq E^2 M^2 \left(\frac{1}{M_{pl}}\right)^\xi \left(\frac{E}{4\pi M_{pl}}\right)^{2L} \prod_{d_n=2} (c_n)^{V_n} \prod_{d_n=0} \left[\lambda_n \left(\frac{v^4}{E^2 M_{pl}^2}\right)\right]^{V_n} \prod_{d_n \geq 4} \left[g_n \left(\frac{E}{M_{pl}}\right)^2 \left(\frac{E}{M}\right)^{d_n-4}\right]$$

One might worry about this term in a low energy expansion, (you should) but watch what the power counting does for you.

Recall that for this equation $L_{int} = M^2 M_{pl}^2 \sum_n \frac{c_n}{M^{d_n}} O_n \left(\frac{\phi}{M_{pl}}, \frac{h_{\mu\nu}}{M_{pl}}\right)$

Focus on the scalar potential (zero derivative terms):

$\tilde{N}_n$ the number of fields that have to be rescal

One would worry even more if the potential was of the form

$V = v^4 f(\phi/v)$ rescale potential terms as $\lambda_n = \left(\frac{M_{pl}}{v}\right)^{\tilde{N}_n} \tilde{\lambda}_n$

The worst case scenario is when all the scalars are rescaled $\tilde{N}_n = N_n$ and only $d_n = 0$ terms.

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$$A_\xi(E) \simeq E^2 M^2 \left(\frac{1}{M_{pl}}\right)^{\xi} \left(\frac{E}{4\pi M_{pl}}\right)^{2L} \prod_{d_n=2} (c_n)^{V_n} \prod_{d_n=0}\left[\lambda_n\left(\frac{v^4}{E^2 M_{pl}^2}\right)\right]^{V_n} \prod_{d_n\geq 4}\left[g_n\left(\frac{E}{M_{pl}}\right)^{2}\left(\frac{E}{M}\right)^{d_n-4}\right]$$

For $V = v^4 f(\phi/v)$ with $\lambda_n = \left(\frac{M_{pl}}{v}\right)^{\tilde{N}_n} \tilde{\lambda}_n$ (for $m_\phi \sim v$) in the worst case scenario $\tilde{N}_n = N_n$

$d_n = 0$

However then topology enforces $\sum_n (\tilde{N}_n - 2)\tilde{V}_n = \tilde{\xi} - 2 + 2L$

And in power counting one can see that all the M_{pl} cancel out as they should:

$$A_\xi(E) \simeq E^2 v^2 \left(\frac{1}{v}\right)^{\tilde{\xi}} \left(\frac{E}{4\pi v}\right)^{2L} \prod_{d_n=2}\left[\tilde{\lambda}_n\left(\frac{v^2}{E^2}\right)\right]^{\tilde{V}_n} \quad \Rightarrow \quad \left(\frac{E^2}{v^2}\right)^{4+\sum_n(\tilde{N}_n-4)\tilde{V}_n}$$

(provided $\phi \simeq E$)

This tells you directly that terms you have to worry about in a low energy expansion are cubics

$\lambda_3\, v^4 (\phi/M_{pl})^3 \simeq \tilde{\lambda}_3\, v\, \phi$ This makes sense as depending on the relative size of E, v in a low energy expansion you can get into trouble

Constraints

Power counting formula

$$A_\xi(E) \simeq E^2 M^2 \left(\frac{1}{M_{pl}}\right)^{\xi} \left(\frac{E}{4\pi M_{pl}}\right)^{2L} \prod_{d_n=2} (c_n)^{V_n} \prod_{d_n=0} \left[\lambda_n \left(\frac{v^4}{E^2 M_{pl}^2}\right)\right]^{V_n} \prod_{d_n \geq 4} \left[g_n \left(\frac{E}{M_{pl}}\right)^2 \left(\frac{E}{M}\right)^{d_n-4}\right.$$

Power counting constraints:

$$\boxed{\begin{aligned} \frac{E}{4\pi M_{pl}} &\ll 1 \\ g_n \left(\frac{E}{M_{pl}}\right)^2 \left(\frac{E}{M}\right)^{d_n-4} &\ll 1 \\ \lambda_n \left(\frac{v^4}{E^2 M_{pl}^2}\right) &\ll 1 \end{aligned}}$$

Power counting constraints for a sane expansion

A particular model in an EFT will also have a cut off on E that above which unitarity will be violated.

This constraint on E will supply an upper bound on M for unitarity to be preserved. ***Unitarity constrain***

EFT is it applicable?

Ipso Facto everyone here thinks it is, but it is a legitimate to ask if EFT is appropriate in curved space times.

Wisdom from Burgess's review gr-qc/0311082

Papers exist that claim no EFT treatment is appropriate for inflation (and black holes).
The claim is energies measured by different observers will be totally different.

- All EFT's have a certain amount of frame dependence, energies are used to classify what is in or out of the EFT. You only need the existence of low energy observers, not that all observers are low energy.

- What is special is that the physics not depend on possibly problematic observers near the horizon. The "nice slice" criterion of Polchinski (9507094) needs to be satisfied.

  This means that one should be able to foliate space time such that slices with only adiabatic background fields and small curvatures. *One only needs such slices to exist.*

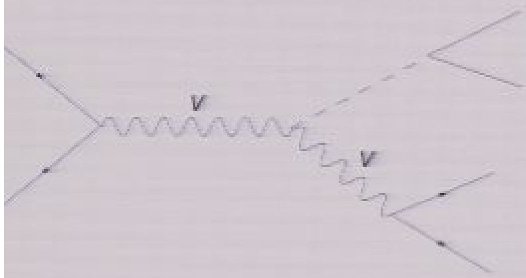
ratuitous name op! (count2)

This means, $H, \mu_\phi \ll M$ where the time scales are $\mu_\phi = \frac{\dot{\phi}}{\phi}$ $H = \frac{\dot{a}}{a}$

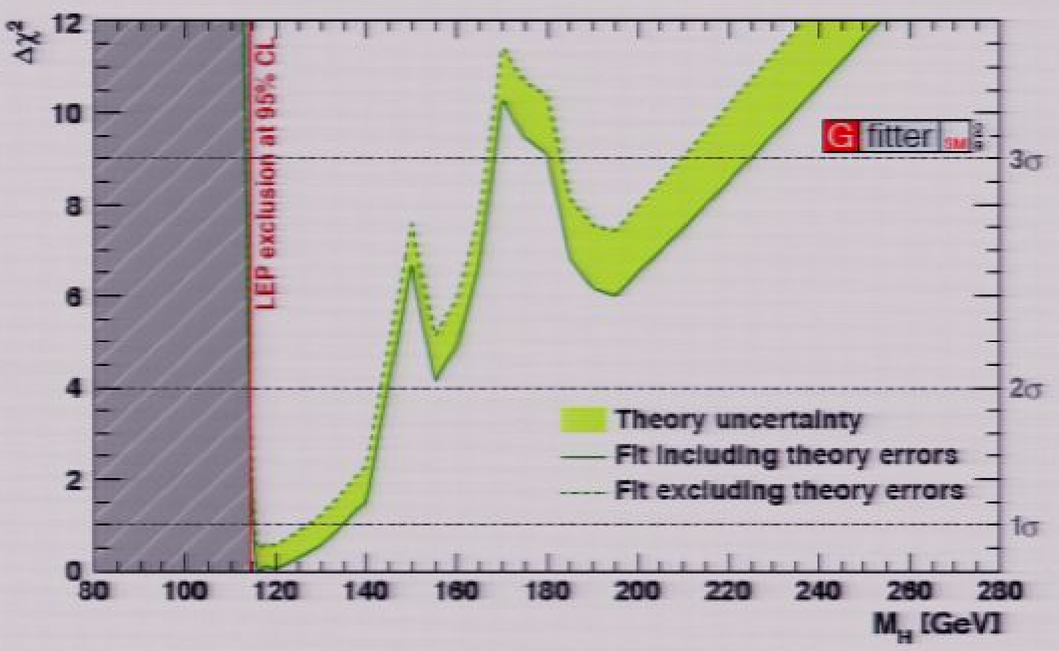
The EFT is an expansion in $\boxed{\frac{R}{M^2} \simeq \left(\frac{H}{M}\right)^2 \qquad \frac{(\partial\theta)^2}{M^2} \simeq \left(\frac{\mu_\phi}{M}\right)^2}$ ***Adiabatic constraint***

What is the status of the Higgs?

~~LHC~~, TeVatron has data, the latest fit results including EWPD, ~3 fb^-1 TeVatron data are:



From J. Haller for Gfitter group 0810.3664v1



If the Higgs exists and it's the SM (~ alone) it's probably light. Any NP ca change this of course....

So the ***only fundamental (?) scalar field*** we have been dragged (kicking and screaming) to believing actually exists due to experiment is probably:

- Disturbingly light (makes no sense) and with unknown properties other than it couples to W's and Z's in the manner expected for it to be involved with EWSB.

What is the nature of the Higgs beast?

When the Higgs is found its (NP?) context and properties will be crucial to test.

- Is it also leading to the mass of the Fermions as well as EWSB? Can we be sure (Not really.)

  - only hope is to measure this in $W^+ W^- \to h \to \tau^+ \tau^-$ at LHC (very tough going)

- What's it's self coupling? This matters for the phase transition and Baryogenesis schemes.

  - only hope is to measure this in $g\, g \to h\, h$ at LHC (this is even worse)

Here is a question that one might have thought (i did) we could probably NEVER address:

- What is the coupling of the Higgs to gravity?

  What was exciting about the Higgs Inflation ***claim*** was the prospect that adding this term to the lagrangian

$$\delta\, \mathcal{L} = \xi\, H^{\dagger} H\, R$$

Could lead to inflation if

$$\xi \simeq 5 \times 10^4 \left(\frac{m_h}{\sqrt{2}\, v} \right)$$

We would know in our lifetime, LHC + PLANCK (even the lifetime of the older members in the audience.....probably)

Genealogy of Higgs Inflation

Non-minimal coupling models proposed/developed

Spokoiny Phys Lett B 147B 39 (1984)

Salopek, Bond, Bardeen Phys Rev D 40 1753 (1989)

Fakir, Unruh Phys Rev D 41 1783 (1990)

A second look and updated constraints

Kaiser Phys Rev D 52 4295 (1995) astro-ph/9408044

Komatsu, Futamase Phys Rev D 59 064029 (1999) atro-ph/9901127

Higgs inflation (re)proposed and (re)developed

Bezrukov, Shaposhnikov Phys Lett B 659, 703 (2008) arXiv:0710.3755

Barvinsky, Kamenshchik, Starobinsky JCAP 0811, 021 (2008) arXiv:0809.2104

Garcia-Bellido, Figueroa, Rubio arXiv:0812.4624

De Simone, Hertzberg, Wilczek arXiv:0812.4946

Bezrukov, Shaposhnikov arXiv:0812.4950

Higgs Inflation Basics

How does Higgs inflation work?

1) Minimal coupling case ($\xi = 0$) doesn't work for the SM.

$$\mathcal{L}_H = (D_\mu H)^\dagger (D^\mu H) - \lambda_H \left(H^\dagger H - \frac{v^2}{2} \right)^2$$

Gauge rotation $H^T = \frac{1}{\sqrt{2}}(0, v + \phi)$

$$\mathcal{L}_\phi = (\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2}(2\,\lambda_H\, v^2)\,\phi^2 - (\lambda_H\, v)\,\phi^3 - (\frac{\lambda_H}{4})\phi^4$$

As $\langle \phi \rangle \gg v$ during inflation this term is totally dominant

Slow roll conditions and amplitude of density pert for these field values (N efolds):

$$\epsilon \sim \frac{1}{N} \qquad \eta \sim \frac{3}{2\,N} \qquad \text{and} \qquad \Delta_R^2 = \frac{V}{24\,\pi^2\, M_{pl}^4\, \epsilon} \quad \text{at } k^* = 0.002\,\mathrm{Mpc}^{-1}$$

WMAP gives $\Delta_R^2 = (2.445\ \pm 0.096) \times 10^{-9}$ trivially one finds:

$$\frac{2\,\lambda_H\, N^3}{3\,\pi^2} = (2.445 \pm 0.096) \times 10^{-9} \Rightarrow \lambda_H \sim 10^{-13},\ m_H \sim 10^{-4}\,\mathrm{GeV}$$

Certainly NOT the SM Higgs sitting in EWPD

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$$S = \int d^4x\sqrt{-g}\left[(\partial_\mu\phi)(\partial^\mu\phi) - \frac{1}{2}(2\lambda_H v^2)\phi^2 - (\lambda_H v)\phi^3 - (\frac{\lambda_4}{4})\phi^4 - \underline{\frac{\xi}{2}\phi^2 R} - \frac{M_{pl}^2}{2}R\right]$$

all things considered, not too offensive a term, not wild specula

To directly analyze this is a total pain:

- the stress energy tensor has extra terms,
- the usual equal-time commutation relations are screwed up,

  ie R has second derivatives of the metric so the $\phi^2 R$ term and integration by parts leads to extra phi derivatives....

all discussed i
Salopek, Bond,
Bardeen '89

To minimize suffering everyone uses the Weyl rescaling to the Einstein frame:

$$\hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu} \quad \text{where} \quad \Omega^2 = 1 + \frac{\xi\phi^2}{M_{pl}^2} \quad \text{and} \quad V_E(\phi) = \frac{V(\phi)}{(1+\xi\frac{\phi^2}{M_{pl}^2})^2}$$

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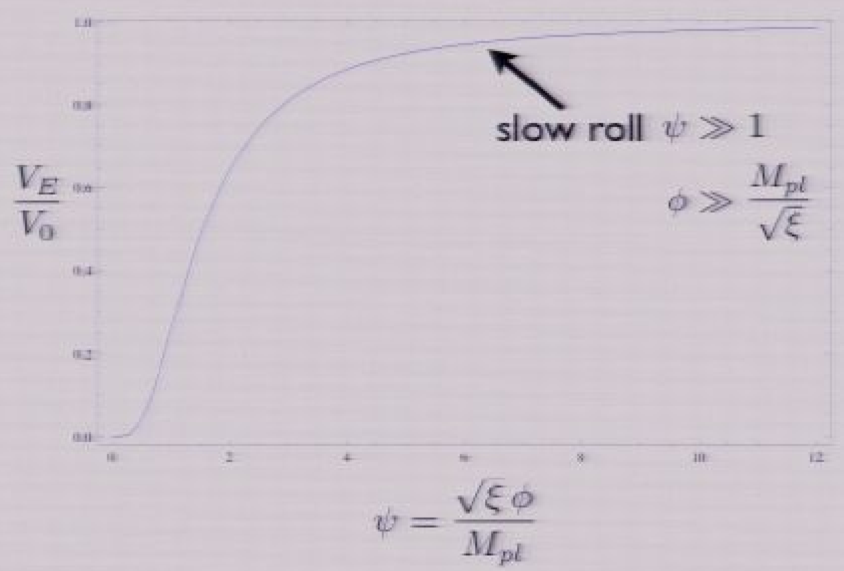

EXPONENTIALLY FLAT
potential "naturally" comes about

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Weyl rescaling to the Einstein frame:

$$\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu} \quad \text{where} \quad \Omega^2 = 1 + \frac{\xi\,\phi^2}{M_{pl}^2} \quad \text{and} \quad V_E(\phi) = \frac{V(\phi)}{(1+\frac{\xi\,\phi^2}{M_{pl}^2})^2}$$

The Higgs field has a non-canonical kinetic term so perform a field redefinition:

$$\frac{d\chi}{d\phi} = \sqrt{\frac{\Omega^2 + 6\,\xi^2\,\phi^2/M_{pl}^2}{\Omega^4}} \quad \xrightarrow[\xi \gg 1]{\phi \gg M_{pl}/\sqrt{\xi}} \quad \frac{d\chi}{d\phi} = \frac{\sqrt{6}M_{pl}}{\phi}$$

$$\phi \simeq \frac{M_{pl}}{\sqrt{\xi}}\exp\left(\frac{\chi}{\sqrt{6}\,M_{pl}}\right)$$

The Einstein frame potential in terms of the canonical field makes the exponential flatness manifest:

$$V_E(\chi) \simeq \frac{\lambda\,M_{pl}^4}{\xi^2}\left[1 + A\exp(-a\,\chi/M_{pl})\right]^{-2}$$
$$\simeq \frac{\lambda\,M_{pl}^4}{\xi^2}\left[1 - 2A\exp(-a\,\chi/M_{pl})\right]$$

EXPONENTIALLY FLAT
potential "naturally" comes abou

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2) Non-Minimal coupling case:

$$S = \int d^4x \sqrt{\hat{g}} \left[(\partial_\mu \chi)(\partial^\mu \chi) - \frac{M_{pl}^2}{2} \hat{R} - V_E(\chi) \right]$$ where $$V_E(\chi) \simeq \frac{\lambda_H M_{pl}^4}{\xi^2} [1 - 2A \exp(-a\chi/M_{pl})]$$

The slow roll parameters for $\psi = \frac{\sqrt{\xi}\,\phi}{M_{pl}}$ in the large ξ limit:

$$\epsilon \simeq \frac{4}{3\,\psi^4} \qquad \eta \simeq -\frac{4}{3\,\psi^2}(1 - \frac{1}{\psi^2}) \qquad \zeta \simeq \frac{16}{9\,\psi^4}(1 - \frac{3}{\psi^2})$$

Trade $N = 60$ for ψ via $N \simeq \frac{3}{4}\left[\psi^2 - \psi_{end}^2 - \log(\frac{1+\psi^2}{1+\psi_{end}^2})\right]$

Higgs inflation	WMAP05/BAO/SN
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$r \simeq 3.0 \times 10^{-3}$	$r < 0.22$
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and $\Delta_R^2 = \frac{V}{24\,\pi^2\, M_{pl}^4\,\epsilon}$ $\quad \xi \simeq 5 \times 10^4 \left(\frac{m_h}{\sqrt{2}\,v}\right)$

Condition for this to work classically

Higgs Inflation Basics

How does Higgs inflation work?

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Inflation is happening at energies much higher than the SM energies, we should run our parameters.

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$$180\,\mathrm{GeV} \gtrsim m_h \gtrsim 115\,\mathrm{GeV}$$

landau pole

vacuum metastability + experimental lower bound

Full 2 loop beta fcn running by Wilczek deSimone,Hertzberg in 0812.4946 in Higgs Inflation

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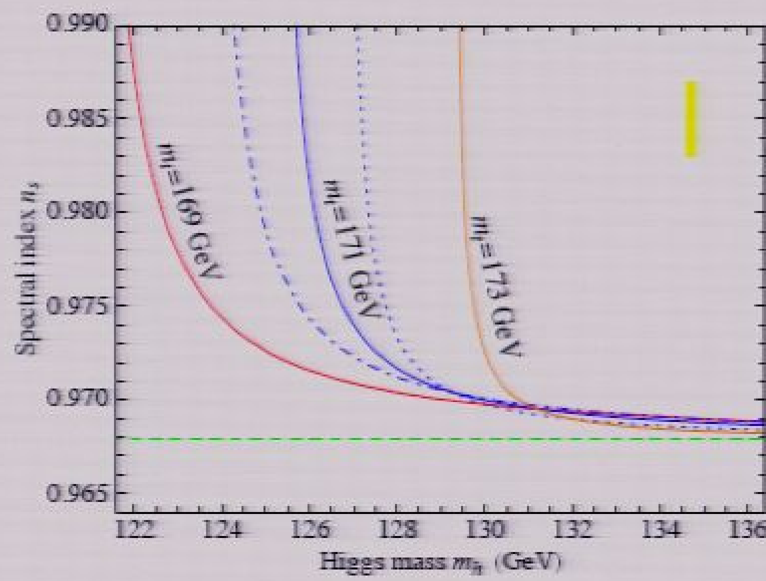
In Higgs Inflation the running gives a relationship between the following parameters:

Real science, predictions!



Wilczek, deSimone, Hertzberg 0812.4946

Similar results for running of spectral index and the tensor to scalar ratio.

Assessment

Higgs inflation is very interesting:

- it is truly minimal only the SM degrees of freedom and gravity.
- that term might exist and isn't totally wild speculation
- it is realistically testable and falsifiable (PLANCK+LHC how cool is that? no ILC required).
- the extremely flat slow roll potential emerges "naturally".

But many pretty ideas fail in the details in a second look (ex SM Baryogenesis).

Now lets try to kill it with EFT constraints.

(or at least we will make clear its severe issues)

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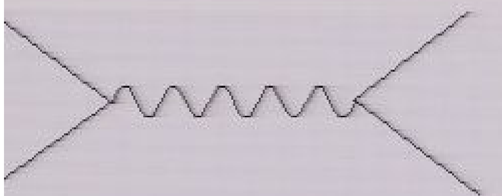
Power Counting vs Higgs Inflation

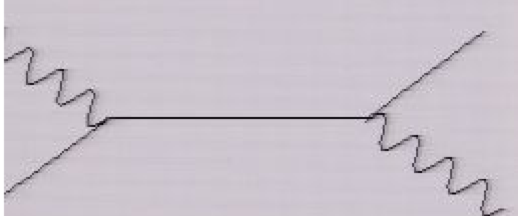
For Higgs inflation the Hubble scale is $H \equiv \frac{\dot{a}}{a} = \frac{\sqrt{V(\phi)/3}}{M_{pl}}$ ➡ $H \simeq \sqrt{\lambda_H}\frac{M_{pl}}{\xi}$

As $\xi \simeq 5 \times 10^4 \left(\frac{m_h}{\sqrt{2}\,v}\right)$ largest dependence on ξ is probably the origin of scattering that violates unitarity.



The power counting gives $A_4(E) \simeq \left(\frac{E}{M_{pl}}\right)^2 \left(\frac{E}{4\pi M_{pl}}\right)^{2L} \prod_n \xi^{V_n}$

Also it gives $\sum_n (N_n - 2)V_n = 2 + 2L$



So strongest dependence on ξ comes from multiple 3 field insertions

$$A_4(E) \simeq \left(\frac{\xi E}{M_{pl}}\right)^2 \left(\frac{\xi E}{4\pi M_{pl}}\right)^{2L}$$

$h\,h \to h\,h$

$g\,h \to g\,h$

Insisting on unitarity ie $\sigma \propto 1/E^2$ we find

$$E < E_{max} \simeq \frac{M_{pl}}{\xi} \qquad M < \frac{M_{pl}}{\xi}$$

People got the cut off wrong when they didn't power count!

Power Counting vs Higgs Inflation

A death blow to Higgs inflation?

$$E < E_{max} \simeq \frac{M_{pl}}{\xi} \qquad M < \frac{M_{pl}}{\xi} \qquad H \simeq \sqrt{\lambda_H}\frac{M_{pl}}{\xi}$$

So we have a small window of validity of this as an EFT

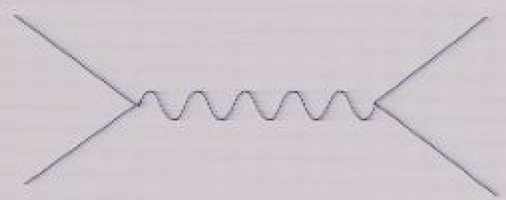
$$1 \gg \frac{H}{M} \gg \sqrt{\lambda_H} \quad \text{all approximations valid}$$

$$1 \gg \frac{H}{M} \gg 0.02 \quad \text{including the 2 loop running to the high scale}$$

Wait maybe the unknown number in the cut off is a big or a small one?

$$M < N\,\frac{M_{pl}}{\xi} \quad \text{then} \quad 1 \gg \frac{H}{M} \gg \frac{\sqrt{\lambda_H}}{N}$$

No. Han and Willenbrock hep-ph/0404182 calculated explicitly with partial wave expansions



$$M < \sqrt{\frac{\pi}{6}}\,\frac{M_{pl}}{\xi}$$

This doesn't help. Once you have one calc in the EFT this is an upper bound!

Power Counting vs Higgs Inflation

So we have a small window, so what ? $1 \gg \frac{H}{M} \gg \sqrt{\lambda_H}$

It gets worse:

- Higgs inflation relies on $V \propto (H^\dagger H)^2$ and a non minimal coupling $(H^\dagger H)\, R$

  True in Higgs inflation when $H^\dagger H \gg \frac{M_{pl}^2}{\xi} \gg v^2$

  Can't forbid by internal symmetries: $g(H^\dagger H)\,(\chi^\dagger \chi)$

  Integrate out χ: $\delta V \simeq \frac{g^3 (H^\dagger H)^3}{(4\pi M_\chi)^2}$ $\quad \delta f \simeq \frac{g^2 (H^\dagger H)^2}{(4\pi M_\chi)^2}$

  The window is very sensitive to any massive particles that are integrated out.

  So for the small window to be maintained $\frac{g^2 (H^\dagger H)}{(4\pi M_\chi)^2} \ll \xi$ $\quad \frac{g^3 (H^\dagger H)}{(4\pi M_\chi)^2} \ll \lambda_H$

- The cut off is $\frac{M_{pl}}{\xi}$ but the exponentially flat potential was for $\phi \simeq \frac{M_{pl}}{\sqrt{\xi}} \exp\left(\frac{\chi}{\sqrt{6}\, M_{pl}}\right)$

  Recall $V_E(\chi) \simeq \frac{\lambda_H M_{pl}^4}{\xi^2}\left[1 - 2A \exp(-a\chi/M_{pL}) + \cdots\right]$

  So we don't know the potential for the field values where inflation is said to occur

s + Espinosa & Bardon
couple days later

Power Counting vs R^2

Power counting is powerful, now we take on Higher curvature inflation (ie Starobinsky inflation)

$$\mathcal{L} = \sqrt{-g}\left[-\frac{M_{pl}^2}{2}R + \zeta R^2\right]$$

Metrics of this form have inflating solutions, imagine "integrating in" a scalar

$$\mathcal{L} = \sqrt{-g}\left[-\frac{M_{pl}^2}{2}R + 2\,\alpha\,\Phi\,R - \Phi^2\right] \qquad \text{(eqn motion } \Phi = \alpha\,R\text{)}$$

Then this looks similar to the sort of model we have been investigating.

Make the identification: $\xi h^2 = 2\alpha\,\Phi$, $\lambda h^4 = \Phi^2$ $\Rightarrow$ $\xi^2 = 4\zeta\,\lambda$ and $\zeta \sim 10^8$

Inflation occurs for $\xi h^2 \gg M_{pl}^2$ again and $V_{EF} \simeq \frac{\lambda M_{pl}^4}{\xi^2} \simeq \frac{M_{pl}^4}{4\,\zeta}$ while $H \simeq \frac{M_{pl}}{\sqrt{\zeta}}$

Work out the unitarity constraints for $g\,g \to g\,g$ the cut off is $E_{\max} = M_p/\zeta^{1/3}$

Constraint band $1 \gg \frac{H}{M} \gg \zeta^{-1/6} \sim 1/20$

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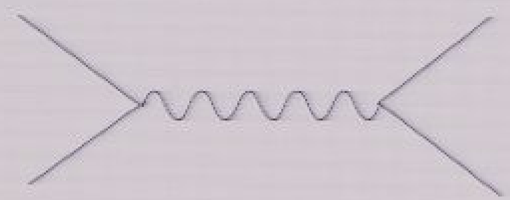

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On the plus side:

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- The Higgs inflation proposal (or something like it) has the potential to relate WMAP data to the properties of the scalars measured in accelerators.

On the minus side:

- The large non minimal coupling constants lead to the theories of this type having cut offs far below the planck scale and below the slow roll region of the potential.
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