

Title: Non-Gaussianity, Loops and the Stability of de Sitter Space

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Abstract: TBA

Introduction

de Sitter space and eternity
Non-linear perturbations and loops
Non-Gaussianity and stability of de Sitter space
Conclusions

Inflation

Non-Gaussianity and loops
Outline of talk

Inflation

- Inflation is a very successful paradigm
- However, we know little about the theory of Inflation itself
- New experiments may soon change this situation

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Inflation

Inflation provides an exiting window to physics at energies \gg
TeV scale!

Non-Gaussianity and loops

- Non-Gaussian signatures are one of our main windows into the underlying model of inflation
- The study of non-linear perturbations also enables us to study loop corrections to Inflation
- The IR divergences of de Sitter might lead to large loop corrections, which should be properly understood.

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Non-Gaussianity and loops

What we can learn about the systematics of n -point correlation functions and their contribution to non-Gaussianity by studying the stability of de Sitter against loop corrections?

Outline of talk

The plan for my talk will be as follows

- Introduction
- **de Sitter space and eternity**
- **Non-linear perturbations and loops**
- **Non-Gaussianity and the stability of de Sitter space**
- Conclusion

The IR divergences in de Sitter

- The stability issue of de Sitter may teach us about the cosmological constant problem.
- The two-point function of a massless scalar field in de Sitter space is IR divergent

$$\langle \phi^2(x) \rangle = U.V. + \frac{H^2}{4\pi^2} \ln(aH/L)$$

- The IR divergency has prompted many to believe that pure de Sitter could by itself be unstable [see f.ex. Polyakov 2007].

Spectroscopy of de Sitter

Proposal: a universe with $\Lambda > 0$ is described by a finite number of *d.o.f.*

$$N = \ln \tilde{n}, \quad \tilde{n} = e^S \quad [\text{Banks 2000}]$$

- The entropy of pure de Sitter space

$$S = \frac{1}{4}A = \frac{3\pi}{\Lambda}$$

- which implies

$$N = \frac{3\pi}{\Lambda}$$

- If we quantize the horizon in terms of the fundamental area l_{pl}^2

$$A = 8\pi n, \quad n = 0, 1, 2, \dots$$

this implies $N = \ln g(n)$, where $g(n)$ is the degeneracy factor
i.e. $g(n) \sim \exp(2n)$ [Bekenstein, Mukhanov 1995].

Correspondence principle

Classical relations are valid in the limit of large quantum numbers

- Consider the quantum harmonic oscillator,

$$E = (n + 1/2)\hbar\omega, \quad n = 0, 1, 2, \dots$$

- The classical relation between energy and amplitude

$$E = \frac{m\omega^2 A^2}{2}$$

is valid in the the limit of large quantum numbers.

- From the de Sitter spectroscopy

$$\frac{1}{4}A = 2\pi n, \quad n = 0, 1, 2, \dots$$

we see that de Sitter space with a small cosmological constant is a macroscopic system described by large quantum numbers

- The classical relation between the horizon area and the cosmological constant

$$\frac{1}{4}A = \frac{3\pi}{\Lambda}$$

should be valid in the limit of small cosmological constant

Thus, the correspondence principle dictates that de Sitter with a small cosmological constant is (quasi) stable!

Wavefunction of the Universe

As a fun application of this line of thinking, we may use this argument to fix the Wavefunction of the Universe

- Consider the tunneling probability from nothing to de Sitter

Wavefunction of the Universe	Hartle-Hawking	Linde	Vilenkin
Tunneling probability from nothing	$P_{HH} = e^{3\pi/G\Lambda}$	$P_L = e^{-3\pi/G\Lambda}$	$P_V = e^{-3\pi/G\Lambda}$

- It has been debated which of these wavefunctions yield the correct answer for the tunneling probability?

Wavefunction of the Universe

The Linde-Vilenkin proposals give the correct tunneling probability!

de Sitter with a small cosmological constant is a macroscopic object described by large quantum numbers \Rightarrow the tunneling probability into a de Sitter space with small cosmological constant must be exponentially suppressed

Non-linearity parameters

Non-Gaussianity is a new observable that can give us insight into the complexity of inflation

- The size of non-Gaussianity from the bispectrum is parameterized in terms of f_{NL} is defined by

$$\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle = (2\pi)^3 \delta\left(\sum_i \vec{k}_i\right) \frac{6}{5} f_{NL} [P_\zeta(k_1)P_\zeta(k_2) + 2 \text{ perm.}]$$

- Similarly for the trispectrum, one can define a τ_{NL}

$$\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \zeta_{k_4} \rangle = (2\pi)^3 \delta\left(\sum_i \vec{k}_i\right) \frac{1}{2} \tau_{NL} [P_\zeta(k_1)P_\zeta(k_2)P_\zeta(k_3) + 11 \text{ perm.}]$$

- So, an order of magnitude estimate gives

$$f_{NL} \approx \frac{\langle \zeta^3 \rangle}{P_\zeta^2}, \quad \tau_{NL} \approx \frac{\langle \zeta^4 \rangle}{P_\zeta^3}, \quad f_{NL}^{(n)} \sim \frac{\langle \zeta^n \rangle}{P_\zeta^{(n-1)}}$$

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Second order action of perturbations

The bispectrum of non-Gaussianity can be obtained from the third order action

- Second order action in uniform curvature gauge [Mukhanov]

$$S_2 = \frac{1}{2} \int a^3 \left[\delta\dot{\phi}^2 - \partial^i \delta\phi \partial_i \delta\phi - V'' \delta\phi^2 + \frac{\dot{\phi}_c^2}{H^2} V \delta\phi^2 + 2 \frac{\dot{\phi}_c^2}{H^2} \left(\frac{\ddot{\phi}_c^2}{H \dot{\phi}_c} + \frac{\dot{\phi}_c^2}{2H^2} \right) \delta\phi^2 \right]$$

Third order action of perturbations

- Third order action [Maldacena, 2002]

$$\begin{aligned}
 S_3 = \int a^3 & \left[-\frac{1}{4} \frac{\dot{\phi}_c}{H} \delta\dot{\phi}^2 \delta\phi - \frac{1}{4} \frac{\dot{\phi}_c}{H} \delta\phi (\partial\delta\phi)^2 - \delta\dot{\phi} \partial^i \chi_1 \partial_i \delta\phi \right. \\
 & + \frac{3}{8} \frac{\dot{\phi}_c^3}{H} \delta\phi^3 - \frac{1}{4} \frac{\dot{\phi}_c}{H} V_{,\phi\phi} \delta\phi^3 - \frac{1}{6} V_{,\phi\phi\phi} \delta\phi^3 + \frac{1}{4} \frac{\dot{\phi}_c^3}{H^2} \delta\phi^2 \delta\dot{\phi} + \frac{1}{4} \frac{\dot{\phi}_c^2}{H} \delta\phi^2 \partial^2 \chi_1 \\
 & \left. + \frac{1}{4} \frac{\dot{\phi}_c}{H} (-\delta\phi \partial^i \partial^j \chi_1 \partial_i \partial_j \chi_1 + \delta\phi \partial^2 \chi_1 \partial^2 \chi_1) \right]
 \end{aligned}$$

$$\alpha_1 = \frac{1}{2} \frac{\dot{\phi}_c}{H} \delta\phi, \quad \partial^2 \chi_1 = -\frac{1}{2} \frac{\dot{\phi}_c}{H} \delta\dot{\phi} - \frac{1}{2} \frac{\dot{\phi}_c}{H} \frac{\dot{H}}{H^2} \delta\phi + \frac{1}{2} \frac{\ddot{\phi}}{H} \delta\phi$$

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 & + \frac{3}{8} \frac{\dot{\phi}_c^3}{H} \delta\phi^3 - \frac{1}{4} \frac{\dot{\phi}_c}{H} V_{,\phi\phi} \delta\phi^3 - \frac{1}{6} V_{,\phi\phi\phi} \delta\phi^3 + \frac{1}{4} \frac{\dot{\phi}_c^3}{H^2} \delta\phi^2 \delta\dot{\phi} + \frac{1}{4} \frac{\dot{\phi}_c^2}{H} \delta\phi^2 \partial^2 \chi_1 \\
 & \left. + \frac{1}{4} \frac{\dot{\phi}_c}{H} (-\delta\phi \partial^i \partial^j \chi_1 \partial_i \partial_j \chi_1 + \delta\phi \partial^2 \chi_1 \partial^2 \chi_1) \right]
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Fourth order action of perturbations

- The trispectrum is obtainable from the fourth order action [MSS, 2006], [Seery, Lidsey, MSS, 2006], [Jarnhus, MSS, 2007]

$$S_4 = \int a^3 \left[-\frac{1}{24} V_{,\phi\phi\phi\phi} \delta\phi^4 + \frac{1}{2} \partial_j \chi_1 \partial^j \delta\phi \partial_m \chi_1 \partial^m \delta\phi - \delta\dot{\phi} \partial_j \chi_2 \partial^j \delta\phi \right. \\ \left. + (\alpha_1^2 \alpha_2 - \frac{1}{2} \alpha_2^2) (-6H^2 + \dot{\phi}^2) + \frac{\alpha_1}{2} \left\{ -\frac{1}{3} V_{,\phi\phi\phi} \delta\phi^3 - 2\alpha_1^2 V_{,\phi} \delta\phi \right. \right. \\ \left. \left. + \alpha_1 \left(-\partial^i \delta\phi \partial_i \delta\phi - V_{,\phi\phi} \delta\phi^2 \right) - 2\partial_i \partial_j \chi_2 \partial^i \partial^j \chi_1 + 2\partial^2 \chi_2 \partial^2 \chi_1 \right. \right. \\ \left. \left. + 2\dot{\phi} \partial_j \chi_2 \partial^j \delta\phi + 2\delta\dot{\phi} \partial_j \chi_1 \partial^j \delta\phi \right\} \right]$$

$$\alpha_2 = \frac{\dot{\phi}_c^2}{8H^2} \delta\phi^2 + F(\delta\phi, \dot{\phi}),$$

$$\partial^2 \chi_2 = \frac{3}{8} \frac{\dot{\phi}_c^2}{H} \delta\phi^2 + \frac{3}{4} \frac{\ddot{\phi}_c}{\dot{\phi}_c} \delta\phi^2 - \frac{a^2}{4H} (\partial\delta\phi)^2 - \frac{1}{4H} \delta\dot{\phi}^2 + \frac{\dot{\phi}_c}{2H} \partial_i \chi_1 \partial_i \delta\phi \\ + \frac{1}{4H} \left((\partial^2 \chi_1)^2 - (\partial_i \partial_j \chi_1)^2 \right) - \frac{V}{H} F(\delta\phi, \dot{\phi})$$

$$F(\delta\phi, \dot{\phi}) = \frac{1}{2H} \partial^{-2} \left[\partial^2 \alpha_1 \partial^2 \chi_1 - \partial_i \partial_j \alpha_1 \partial_i \partial_j \chi_1 + \partial_i \delta\phi \partial_i \delta\phi + \delta\dot{\phi} \partial^2 \delta\phi \right]$$

Tree-level stuff

From these results the Bispectrum and Trispectrum has been calculated in single field slow-roll inflation

- $S_2 \sim \mathcal{O}(1) \Rightarrow \langle \delta\phi^2 \rangle \propto H^2$ and
 $\zeta = \epsilon^{-1/2} \delta\phi / M_p \Rightarrow \langle \zeta^2 \rangle \propto \epsilon^{-1} (H/M_p)^2$
- $S_3 \sim \mathcal{O}(\epsilon^{1/2}) \Rightarrow \langle \delta\phi^3 \rangle \propto \epsilon^{1/2} H^4 / M_p \Rightarrow$
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Perhaps slightly surprisingly

$$f_{NL} \sim \tau_{NL}$$

Loops and IR divergences

The third and fourth order actions of perturbations also enables us to calculate the loop corrections to the power-spectrum

- From S_4 we can calculate the one-loop corrected power spectrum of inflaton perturbations in the "in-in" formalism

$$\langle \zeta^4(t) \rangle = \left\langle U_{int}^{-1} \zeta^4(t) U_{int}(t, t_0) \right\rangle, \quad U_{int} = T e^{-i \int_{t_0}^t H_{int}(t') dt'}$$

which gives

$$\mathcal{P}(\eta_0, k) \approx \frac{H^2}{4\pi^2} \left[1 - \left(\frac{1}{16} \epsilon + \frac{1}{2} (2\epsilon - \eta) - \frac{3}{8} (2\epsilon - \eta) \text{Ci}(-2k\eta_0) \right) \langle \delta\phi^2 \rangle \right]$$

- With $V(\phi) = \lambda M_p^{4-\alpha} \phi^\alpha$ and assuming chaotic inflation

$$\Delta\phi = \dot{\phi} \Delta t < \delta\phi \Rightarrow \phi_i < \lambda^{-1/6} M_p, \quad \Delta t = 1/H$$

$$\langle \delta\phi \rangle = \int_{a_i H_i}^{a\Lambda} \frac{dk}{k} \mathcal{P}_{\delta\phi}(k) \propto N^{(4+\alpha)/2}, \quad N \approx \frac{1}{2\alpha M_p^2} \phi_i^2$$

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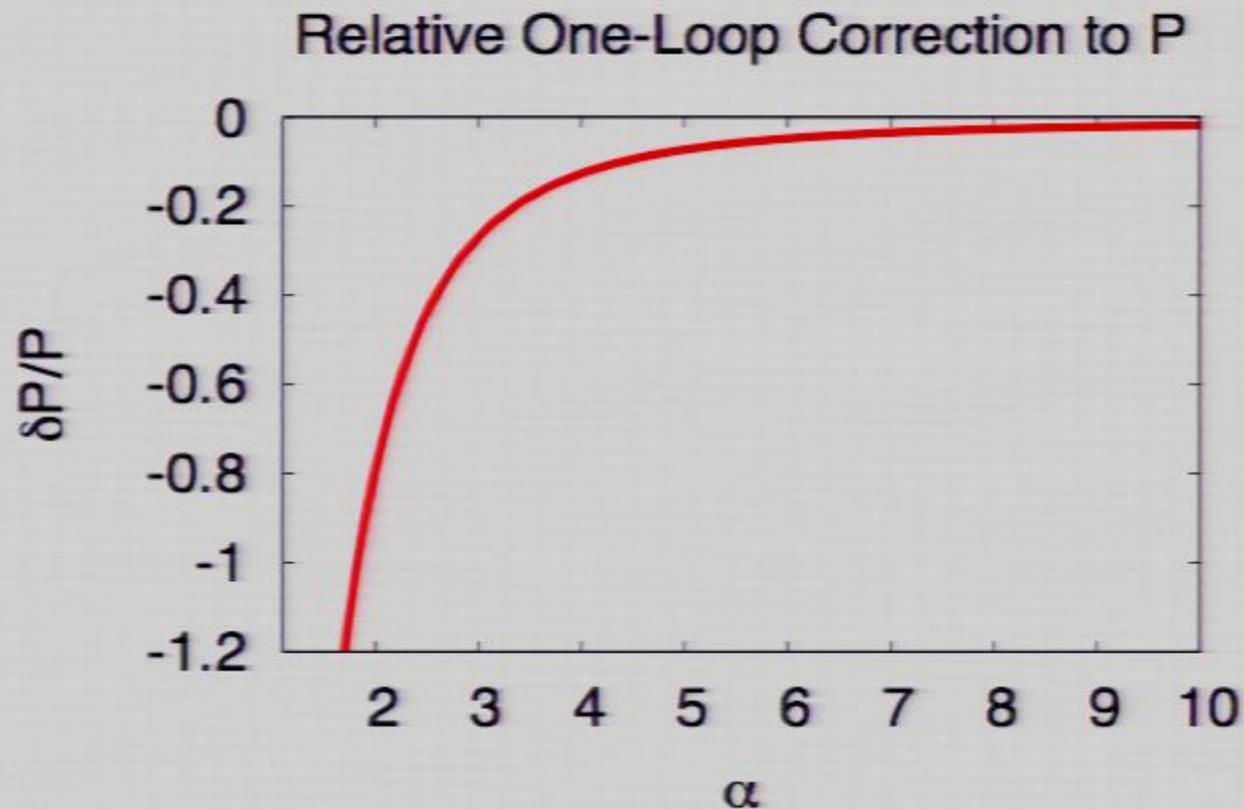
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Size of one-loop corrections



Resumming the loops

Large one-loop corrections requires resummation of the loops to all orders in perturbation theory

- Consider an $O(N)$ invariant toy model in the large N limit

$$\mathcal{L} = \frac{1}{2} \left(\partial_\mu \phi_i \partial^\mu \phi_i - m^2 \phi_i \phi_i - \frac{\lambda}{4N} (\phi_i \phi_i)^2 \right)$$

- From the 2PI effective action we can derive a *gap* equation for the two-point function $\langle \phi(x) \phi(x') \rangle$ valid to all orders
- The gap equation can be solved self-consistently for the variance $\langle \phi^2(x) \rangle$ yielding

$$\langle \phi^2(x) \rangle = H^2 \frac{\tanh\left(\frac{\sqrt{\lambda}}{4\pi^2} \ln a\right)}{\sqrt{\lambda}}$$

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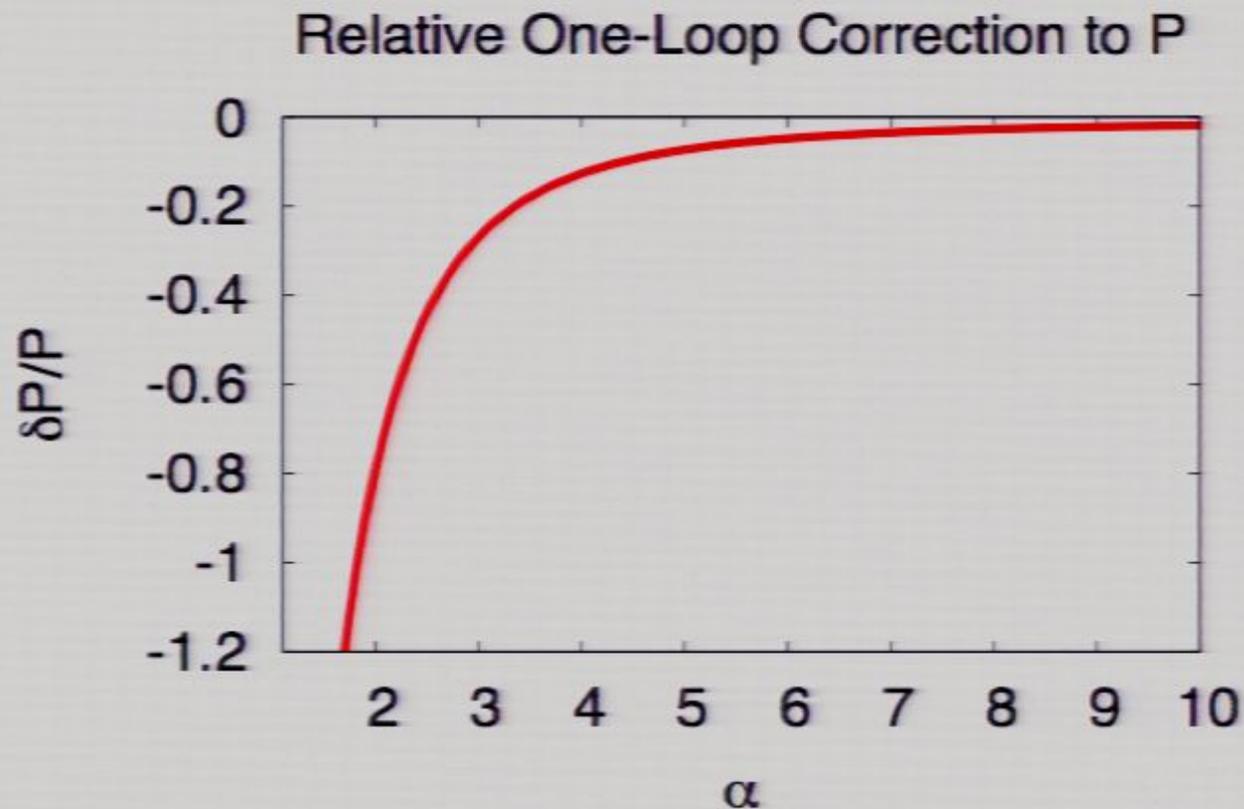
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Size of one-loop corrections



Loops and IR divergences

The third and fourth order actions of perturbations also enables us to calculate the loop corrections to the power-spectrum

- From S_4 we can calculate the one-loop corrected power spectrum of inflaton perturbations in the "in-in" formalism

$$\langle \zeta^4(t) \rangle = \left\langle U_{int}^{-1} \zeta^4(t) U_{int}(t, t_0) \right\rangle, \quad U_{int} = T e^{-i \int_{t_0}^t H_{int}(t') dt'}$$

which gives

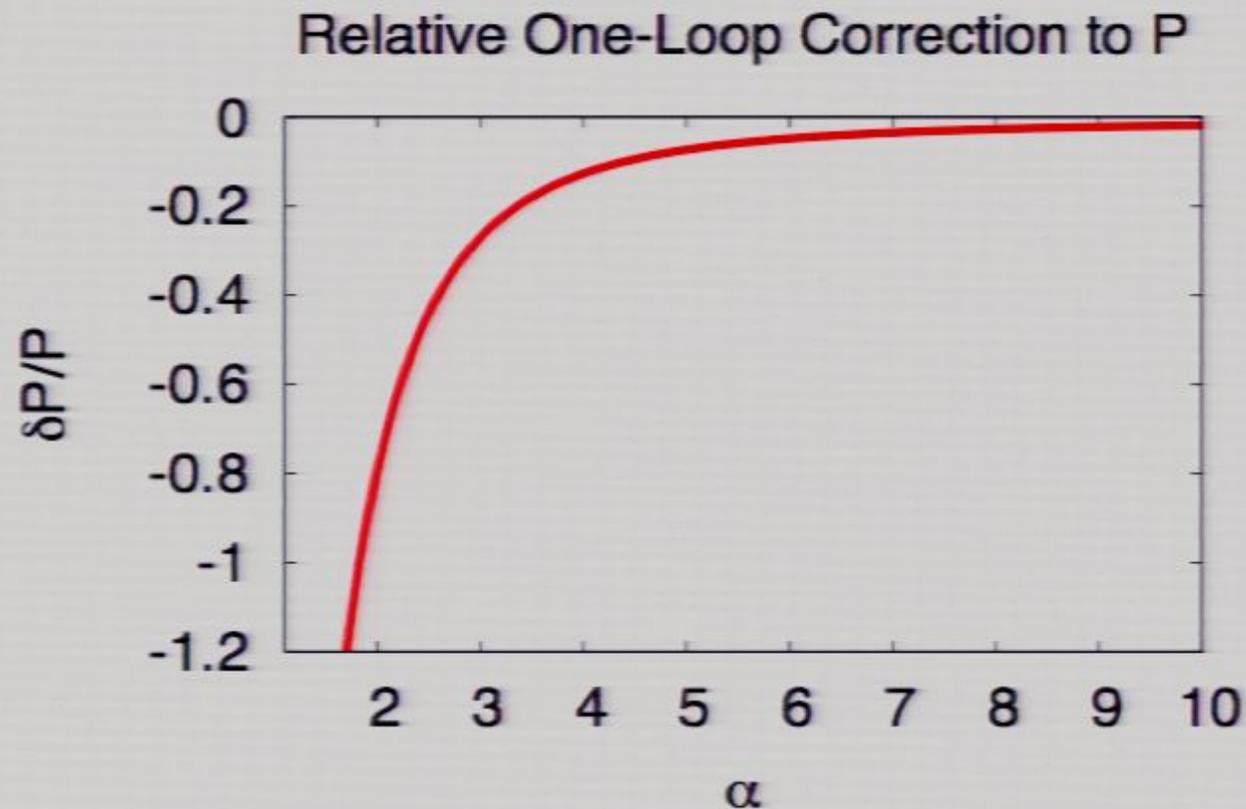
$$\mathcal{P}(\eta_0, k) \approx \frac{H^2}{4\pi^2} \left[1 - \left(\frac{1}{16} \epsilon + \frac{1}{2} (2\epsilon - \eta) - \frac{3}{8} (2\epsilon - \eta) \text{Ci}(-2k\eta_0) \right) \langle \delta\phi^2 \rangle \right]$$

- With $V(\phi) = \lambda M_p^{4-\alpha} \phi^\alpha$ and assuming chaotic inflation

$$\Delta\phi = \dot{\phi} \Delta t < \delta\phi \Rightarrow \phi_i < \lambda^{-1/6} M_p, \quad \Delta t = 1/H$$

$$\langle \delta\phi \rangle = \int_{\alpha_i H_i}^{a\Lambda} \frac{dk}{k} \mathcal{P}_{\delta\phi}(k) \propto N^{(4+\alpha)/2}, \quad N \approx \frac{1}{2\alpha M_p^2} \phi_i^2$$

Size of one-loop corrections



Resumming the loops

Large one-loop corrections requires resummation of the loops to all orders in perturbation theory

- Consider an $O(N)$ invariant toy model in the large N limit

$$\mathcal{L} = \frac{1}{2} \left(\partial_\mu \phi_i \partial^\mu \phi_i - m^2 \phi_i \phi_i - \frac{\lambda}{4N} (\phi_i \phi_i)^2 \right)$$

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de Sitter Limit

We saw that $S_3 = O(\epsilon^{1/2})$ while $S_2 = O(1)$, $S_4 = O(1)$ implying

$$f_{NL} \sim \tau_{NL}$$

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- We may wonder what requires the third order action, $S_3(\delta\phi)$, to be slow-roll suppressed, when there is nothing which forces the second and fourth order actions $S_2(\delta\phi)$, $S_4(\delta\phi)$ to be slow-roll suppressed

A toy example

- Assume that we have unsuppressed third order terms. The interaction Hamiltonian for the perturbations will have the form

$$H_I = \int d^3x a^3 \left[\delta\phi(\ddot{\phi}_c + 3H\dot{\phi}_c + V') + gO(\delta\phi^3) + \dots \right]$$

with the $O(\delta\phi^3)$ toy term being any operator of the type $\delta\phi^3$, $\delta\dot{\phi}^2\delta\phi$, $\delta\dot{\phi}\partial^{-2}(\partial_i\delta\phi\partial^i\delta\phi)$

- The term $O(\delta\phi^3)$ will give a contribution to the tadpole diagram in the figure.

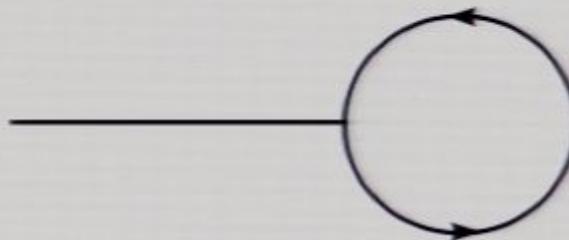


Figure: Tadpole diagram

Tadpole destabilization of de Sitter?

- The tadpole condition yields the one-loop correction to the equation of motion of the classical background field

$$0 = \langle \delta\phi \rangle = \ddot{\phi}_c + 3H\dot{\phi}_c + V' + g\Gamma_t ,$$

where $g\Gamma_t$ is the amputated tadpole contribution

- In the simplest case of a massless scalar field and with $O(\delta\phi^3) = H\delta\phi^3$, the tadpole contribution would become

$$g\Gamma_t = 3gH \langle \delta\phi^2 \rangle = \frac{3g}{4\pi^2} H^4 t$$

- In this case, the time-independent de Sitter solution is destabilized by the tadpole.

Tadpole destabilization of de Sitter?

- If the toy term has a more complicated form involving derivatives, the infrared divergency will generally approach a constant
- In this case the solution to with $V' = 0$ and $g\Gamma_t = \text{const.}$ at late times is

$$\phi_c(t) = -\frac{g\Gamma_t}{3H}t$$

- This is inconsistent with a time-independent de Sitter solution.

⇒ if unsuppressed third order terms were allowed, classical de Sitter space with a massless scalar field would be destabilized

The systematics of non-Gaussianity

Since any unsuppressed odd order terms in the action would lead to a non-vanishing tadpole contribution

⇒ all odd order terms in the action should be slow-roll suppressed, while even order terms are not slow-roll suppressed

- We can then extrapolate our results from $n \leq 4$ to any n , and conjecture the order of magnitude of any n -point correlation function in single field inflation
- We can also generalize the nonlinearity parameter f_{NL} to n 'th order up to a numerical factor of order one

$$\langle \zeta^n \rangle \approx f_{NL}^{(n)} \mathcal{P}_\zeta^{n-1}$$

The systematics of non-Gaussianity

Order	$S(\delta\phi)$	$S(\zeta_n)$	$S(\zeta)$
2nd	$\mathcal{O}(1)$	$\mathcal{O}(\epsilon)$	$\mathcal{O}(\epsilon)$
3rd	$\mathcal{O}(\epsilon^{1/2})$	$\mathcal{O}(\epsilon^2)$	$\mathcal{O}(\epsilon)$
4rd	$\mathcal{O}(1)$	$\mathcal{O}(\epsilon^2)$	$\mathcal{O}(\epsilon)$
2nth	$\mathcal{O}(1)$	$\mathcal{O}(\epsilon^n)$	$\mathcal{O}(\epsilon)$
(2n + 1)th	$\mathcal{O}(\epsilon^{1/2})$	$\mathcal{O}(\epsilon^{n+1})$	$\mathcal{O}(\epsilon)$

Table: Slow-roll order of the action to n 'th order.

[Jarnhus, MSS, 2007]

The systematics of non-Gaussianity

p	$\langle \delta\phi^p \rangle$	$\langle \zeta^p \rangle$	$f_{NL}^{(p)} \approx \langle \zeta^p \rangle / \mathcal{P}_\zeta^{p-1}$
2	$\mathcal{O}(H^2)$	$\mathcal{O}(\epsilon^{-1} H^2)$	$\mathcal{O}(1)$
3	$\mathcal{O}(\epsilon^{1/2} H^4)$	$\mathcal{O}(\epsilon^{-1} H^4)$	$\mathcal{O}(\epsilon)$
4	$\mathcal{O}(H^6)$	$\mathcal{O}(\epsilon^{-2} H^6)$	$\mathcal{O}(\epsilon)$
$2n$	$\mathcal{O}(H^{2p-2})$	$\mathcal{O}(\epsilon^{-p/2} H^{2p-2})$	$\mathcal{O}(\epsilon^{p/2-1})$
$2n+1$	$\mathcal{O}(\epsilon^{1/2} H^{2p-2})$	$\mathcal{O}(\epsilon^{(1-p)/2} H^{2p-2})$	$\mathcal{O}(\epsilon^{(p-1)/2})$

Table: Slow-roll order of the n -point functions and generalized nonlinearity parameter.

The systematics of non-Gaussianity

- It is easy to verify that the action $S(\delta\phi)$ to any even order, $2n$, in perturbations will be unsuppressed in the slow-roll parameters, as it will contain contributions from $\alpha^{(2)2n}$
- Similarly, to odd orders $\alpha^{(2)}$ will always appear in combination with some $\alpha^{(n)}$ or $\chi^{(n)}$ to odd order, which are slow-roll suppressed
- As an example we predict that the nonlinearity parameter related to the 5- and 6-point function is $f_{NL}^{(5)} = f_{NL}^{(6)} = \epsilon^2$
- Including gravitational wave modes one can have unsuppressed odd n -terms in the action with gravitational waves, γ_{ij} , of the form $\partial_i\delta\phi\partial_j\delta\phi\gamma^{ij}$
- But terms like $\delta\phi\dot{\gamma}_{ij}\dot{\gamma}^{ij}$ have to be slow-roll suppressed. This agrees with [Maldacena, 2002]

Conclusions

- The quasi-stability of de Sitter can be used to make systematic predictions for the order of magnitude of non-Gaussianity.
- On the other hand, there is nothing so far in our studies of non-linear perturbation theory that seems to vindicate the point of view that de Sitter is unstable
- The IR divergences that appears order by order in perturbation theory, which could be a problem for the stability of de Sitter, go away when they are resummed to all orders in perturbation theory
- However, the IR divergences do lead to a non-perturbative (but finite) enhancement of correlation functions

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