Title: Inflation with a Random Potential: Fluctuations in the CMB Power Spectrum

Date: May 21, 2009 09:00 AM

URL: http://pirsa.org/09050055

Abstract: TBA

Inflation with a Random Potential : Fluctuations in the CMB Power Spectrum

Henry Tye

with Jiajun Xu, in progress

with J. Xu and Y. Zhang, 0812.1944

Cornell University

Perimeter Institute 5/21/09

Cosmic Landscape

- Treat the cosmic landscape as a d-dimensional effective potential.
- Due to the vacuum energy, it is reasonable to assume inflation in the landscape.
- If the universe is trapped at a local minimum, we'll have eternal inflation with a scaling power spectrum. But WMAP etc shows that is not the case.
- The inflaton is mobile $\Rightarrow$ $d > 2$.
- What features can we expect to see for an inflaton moving in a complicated multi-dimensional potential ?

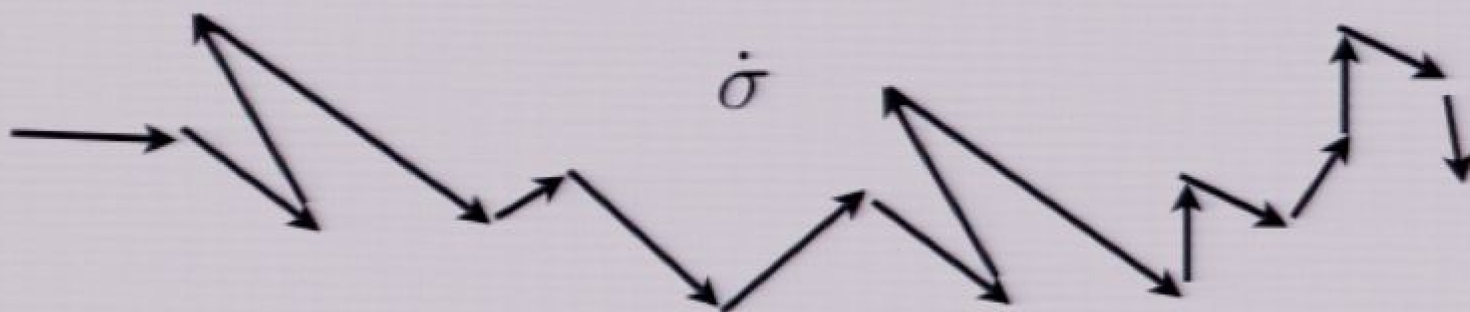
Cosmic Landscape

- Treat the cosmic landscape as a d-dimensional effective potential.
- Due to the vacuum energy, it is reasonable to assume inflation in the landscape.
- If the universe is trapped at a local minimum, we'll have eternal inflation with a scaling power spectrum. But WMAP etc shows that is not the case.
- The inflaton is mobile ➡ $d > 2$.
- What features can we expect to see for an inflaton moving in a complicated multi-dimensional potential ?

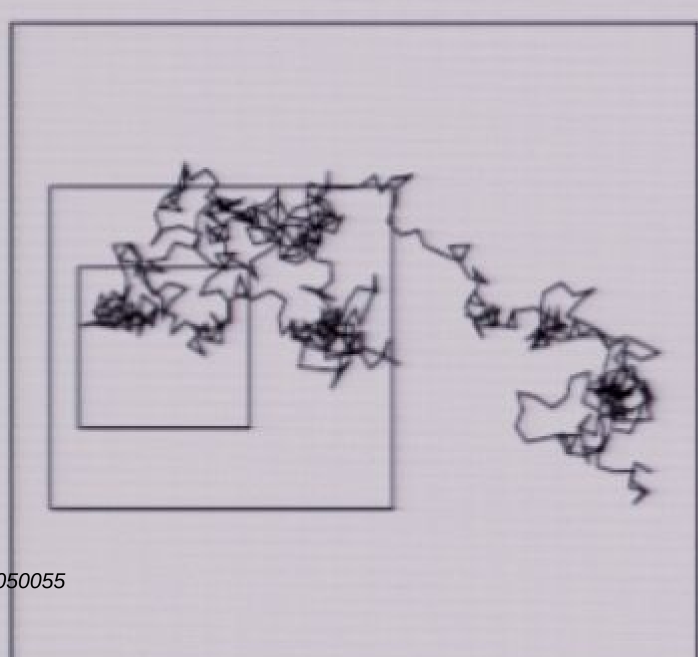
Likely Scenario

- Treat the landscape as a random effective potential.
- If the multi-field inflaton scatters many times during inflation: it is like a random walk with a drift velocity.
- The density perturbation comes from the quantum fluctuation.
- Find some distinctive signatures.
- A statistical treatment can be useful.

Inflation in the cosmic landscape

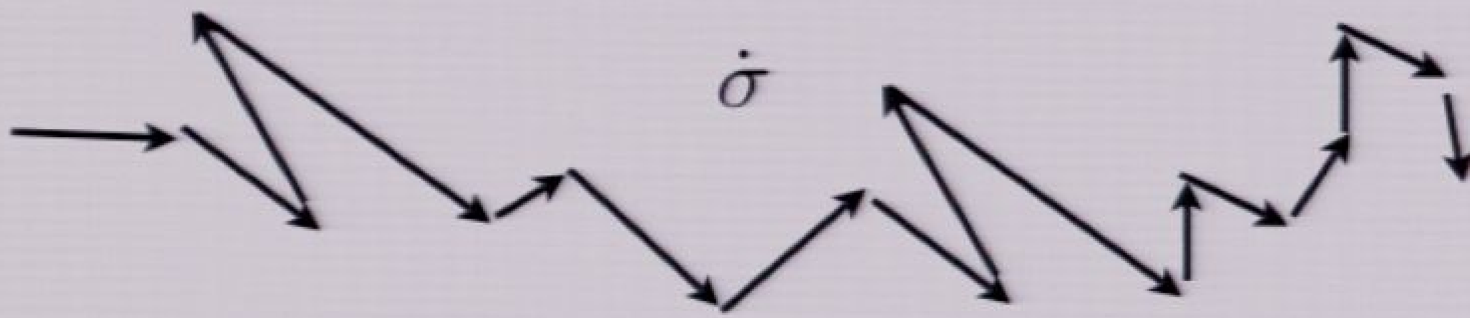


$v = < \dot{\sigma} >$ drift velocity ⟶

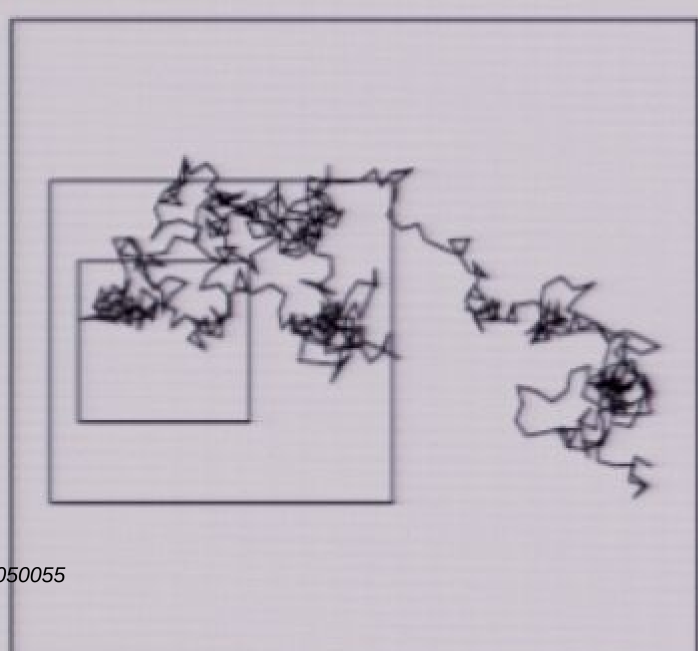


Does this help ameliorating the fine-tuning of the flatness of the inflaton potential ?

Inflation in the cosmic landscape

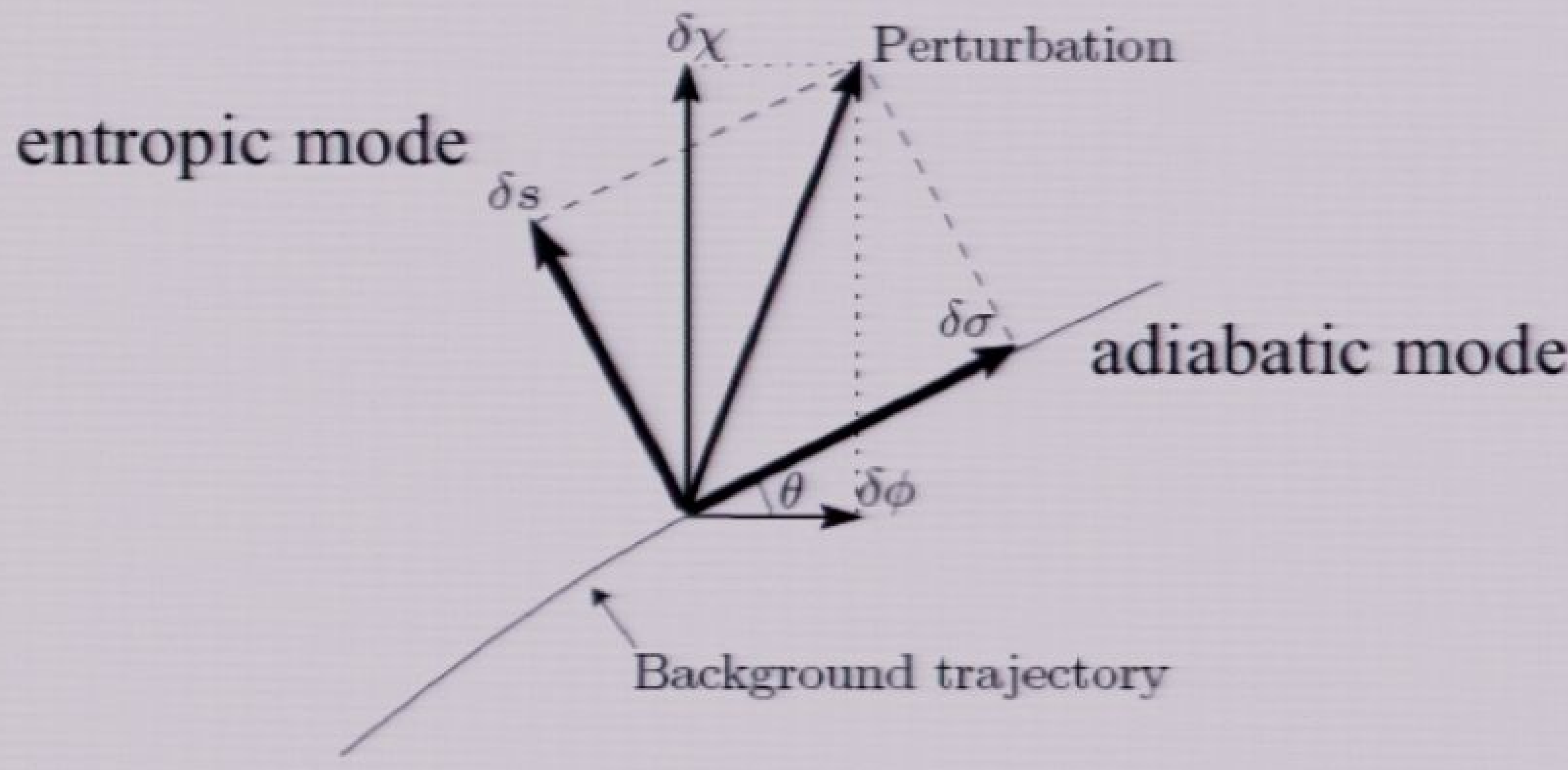


$v = < \dot{\sigma} >$ drift velocity ⟶



Does this help ameliorating the fine-tuning of the flatness of the inflaton potential ?

Decomposition of an arbitrary perturbation during inflation



Multi-field Inflation

$V\left(\phi^I\right)$

$$\ddot{\phi}_I + 3H\dot{\phi}_I + V_I = 0\ , \quad V_I \equiv \frac{\partial V}{\partial \phi^I}$$

$$\ddot{\sigma} + 3H\dot{\sigma} + V_\sigma = 0\ , \quad V_\sigma \equiv V_I \mathbf{e}^I_\sigma$$

curvature perturbation $\quad \zeta = \delta N$

$$\delta N = N_I\, Q^I + \frac{1}{2} N_{IJ}\left(Q^I Q^J - \langle Q^I Q^J \rangle\right) + \dots$$

$N_I \equiv \partial N / \partial \phi^I$

Q^I is the perturbation of ϕ^I in the spatially flat gauge

Power Spectrum

$$\delta N = -\frac{H}{\dot{\sigma}}\Big|_{t^*} Q^\sigma - \int_{t^*}^{t_E} \mathrm{d}t\, \frac{2H}{\dot{\sigma}} \dot{\mathrm{e}}^I_\sigma Q_I$$

$$\dot{\zeta} = -\frac{2H}{\dot{\sigma}}\, \dot{\mathrm{e}}^I_\sigma\, Q_I$$

$$P_\zeta = \frac{k^3}{2\pi^2} \langle N_I Q^I N_J Q^J \rangle = \frac{k^3}{2\pi^2} \left(\langle N_\sigma Q^\sigma N_\sigma Q^\sigma \rangle + \langle N_s Q^s N_{s'} Q^{s'} \rangle \right)$$

$$P_\zeta = \frac{H^2}{4\pi^2} \left(N_\sigma^2 + \langle N_s N_{s'} \rangle \delta^{ss'} \right)$$

$$P_\zeta = P_{adiabatic} + P_{entropic}$$

Predictions

$$\chi \equiv \frac{\dot{\sigma}^2}{\langle\dot{\sigma}\rangle^2} \qquad \tilde{P}_\zeta(k) = \frac{H^2}{8\pi^2\epsilon\,\bar{\chi}}\left[1+\vartheta N^*_{\rm eff}\right]\Big|_{k=aH}$$

$$n_s - 1 = \frac{\mathrm{d}\ln P_\zeta}{\mathrm{d}\ln k} \approx -2\epsilon - \eta - \frac{\vartheta}{1+N_e^*\vartheta}$$

$$\epsilon \equiv -\frac{\bar{\dot{H}}}{H^2} \qquad \eta \equiv \dot{\epsilon}/(H\epsilon)$$

$$P_T = 8\left(\frac{H}{2\pi}\right)^2 , \quad r = \frac{P_T}{\tilde{P}_\zeta} = \frac{16\,\epsilon\,\bar{\chi}}{1+N_e^*\vartheta}$$

Power Spectrum

$$\delta N = -\frac{H}{\dot\sigma}\Big|_{t^*} Q^\sigma - \int_{t^*}^{t_E} \mathrm{d}t\, \frac{2H}{\dot\sigma} \dot{\mathrm{e}}_\sigma^I Q_I$$

$$\dot\zeta = -\frac{2H}{\dot\sigma}\, \dot{\mathrm{e}}_\sigma^I\, Q_I$$

$$P_\zeta = \frac{k^3}{2\pi^2}\langle N_I Q^I N_J Q^J\rangle = \frac{k^3}{2\pi^2}\left(\langle N_\sigma Q^\sigma N_\sigma Q^\sigma\rangle + \langle N_s Q^s N_{s'} Q^{s'}\rangle\right)$$

$$P_\zeta = \frac{H^2}{4\pi^2}\left(N_\sigma^2 + \langle N_s N_{s'}\rangle \delta^{ss'}\right)$$

$$P_\zeta = P_{adiabatic} + P_{entropic}$$

Predictions

$$\chi \equiv \frac{\dot{\sigma}^2}{\langle\dot{\sigma}\rangle^2} \qquad \tilde{P}_\zeta(k) = \frac{H^2}{8\pi^2\epsilon\,\bar{\chi}}\left[1+\vartheta N^*_{\rm eff}\right]\Big|_{k=aH}$$

$$n_s - 1 = \frac{\mathrm{d}\ln P_\zeta}{\mathrm{d}\ln k} \approx -2\epsilon - \eta - \frac{\vartheta}{1+N_e^*\vartheta}$$

$$\epsilon \equiv -\frac{\bar{\dot{H}}}{H^2} \qquad \eta \equiv \dot{\epsilon}/(H\epsilon)$$

$$P_T = 8\left(\frac{H}{2\pi}\right)^2 , \quad r = \frac{P_T}{\tilde{P}_\zeta} = \frac{16\,\epsilon\,\bar{\chi}}{1+N_e^*\vartheta}$$

Predictions

- The power spectrum has an extra red tilt of up to 2%
- The non-Gaussianity due to the randomness is typically very small
- The tensor to scalar mode ratio r can be bigger by as much as a factor of χ
- The randomness can show up in the power spectrum as a random oscillation

Comment

We may treat both the classical random walk fluctuation and the quantum fluctuation at the same time :

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = \xi(t) + \eta(t)$$

$$< \xi(t)\xi(t') > \sim P_{entropic}\delta(t - t')$$

$$< \eta(t)\eta(t') > \sim P_{adiabatic}\delta(t - t')$$

$$P_\zeta = P_{adiabatic} + P_{entropic}$$

Power Spectrum

$$\delta N = -\frac{H}{\dot{\sigma}}\Big|_{t^*} Q^\sigma - \int_{t^*}^{t_E} \mathrm{d}t \, \frac{2H}{\dot{\sigma}} \dot{\mathrm{e}}_\sigma^I Q_I$$

$$\dot{\zeta} = -\frac{2H}{\dot{\sigma}} \, \dot{\mathrm{e}}_\sigma^I \, Q_I$$

$$P_\zeta = \frac{k^3}{2\pi^2} \langle N_I Q^I N_J Q^J \rangle = \frac{k^3}{2\pi^2} \left(\langle N_\sigma Q^\sigma N_\sigma Q^\sigma \rangle + \langle N_s Q^s N_{s'} Q^{s'} \rangle \right)$$

$$P_\zeta = \frac{H^2}{4\pi^2} \left(N_\sigma^2 + \langle N_s N_{s'} \rangle \delta^{ss'} \right)$$

$$P_\zeta = P_{adiabatic} + P_{entropic}$$

Langevin-Fokker-Planck

$$\epsilon \equiv -\frac{\dot{\bar{H}}}{H^2} \qquad \chi \equiv \frac{\dot{\sigma}^2}{\langle\dot{\sigma}\rangle^2}$$

Slow-Roll : $\frac{\mathrm{d}\vec{\phi}}{\mathrm{d}t} = -\frac{\nabla_\phi \bar{V}}{3H} + \vec{\xi}(t) = \vec{v}(\vec{\phi}) + \vec{\xi}(t)$

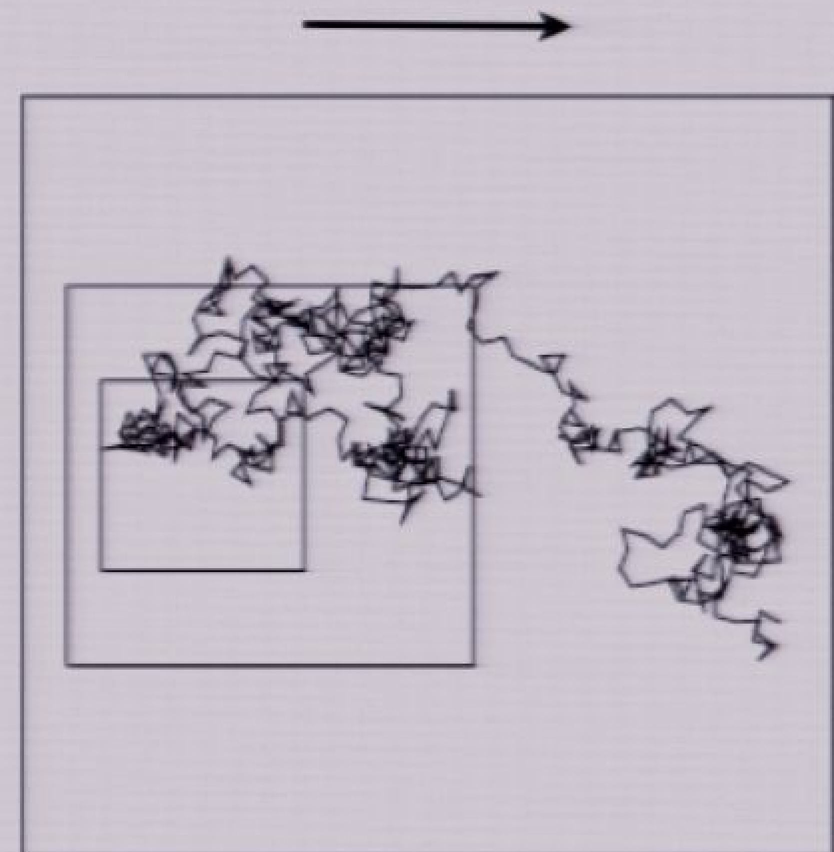
$$\frac{\partial P}{\partial t} = -\nabla \cdot \left[\vec{v}(\vec{\phi})\, P\right] + \partial_I \partial_J \left[D^{IJ}(\vec{\phi})\, P\right]$$

$\vec{v}$ is almost a constant vector and $D^{IJ} \equiv \lambda\, \delta^{IJ}$

HT, J. Xu and Y. Zhang, arXiv:0812.1944 [hep-th]

Does the random walk in multi-field space help ameliorating the fine-tuning of the flatness of the inflaton potential ?

$v = < \dot{\sigma} >$ drift velocity



Langevin

$$\ddot{\phi} = -3H\dot{\phi} - V'(\phi) + \xi(t)$$

$$V(\phi) = 3H^2 M_{\rm pl}^2$$

$$\langle \xi(t)\xi(t') \rangle = 2D\delta(t - t')$$

$$V(\phi) = V_0 - b\,\phi$$

$$\phi(t) = \frac{b}{3H}t + \frac{1}{3H}\left(1 - e^{-3Ht}\right)\left(\dot{\phi}_0 - \frac{b}{3H}\right) + \int_0^t \mathrm{d}s\, \frac{\xi(s)}{3H}\left[1 - e^{3H(s-t)}\right]$$

$$(\text{with } \phi(0) = 0 \text{ and } \dot{\phi}(0) = \dot{\phi}_0)$$

Variance

$$\langle\dot\phi\rangle = e^{-3Ht}\dot\phi_0 + \frac{b}{3H}\left(1-e^{-3Ht}\right)\ ,$$

$$\langle(\dot\phi-\langle\dot\phi\rangle)^2\rangle = \frac{D}{3H}\left(1-e^{-6Ht}\right)\ .$$

$$\langle\phi\rangle = \phi_0 + \frac{b\,t}{3H} + \frac{1-e^{-3Ht}}{3H}\left(\dot\phi_0 - \frac{b}{3H}\right)$$

$$\langle(\phi-\langle\phi\rangle)^2\rangle = \frac{2D}{9H^2}\left(t+\frac{1-e^{-6Ht}}{6H}-\frac{2-2e^{-3Ht}}{3H}\right)$$

$$\langle\dot\phi(t)\rangle \sim b/3H,\ \langle[\dot\phi(t)-\langle\dot\phi(t)\rangle]^2\rangle \sim D/3H.$$

Probability distribution

$$P(\dot{\phi},t) = \sqrt{\frac{3H}{2\pi D\,(1-e^{-6Ht})}}\exp\left(-\frac{3H(\dot{\phi}-b/3H-\dot{\phi}_0 e^{-3Ht})^2}{2D(1-e^{-6Ht})}\right)$$
$$\to \sqrt{\frac{3H}{2\pi D}}\exp\left(-\frac{3H(\dot{\phi}-b/3H)^2}{2D}\right)$$

The drift speed is close to the slow-roll speed.

Little or no improvement on the fine-tuning problem.

Variance

$$\langle\dot\phi\rangle = e^{-3Ht}\dot\phi_0 + \frac{b}{3H}\left(1-e^{-3Ht}\right) \ ,$$

$$\langle(\dot\phi - \langle\dot\phi\rangle)^2\rangle = \frac{D}{3H}\left(1-e^{-6Ht}\right) \ .$$

$$\langle\phi\rangle = \phi_0 + \frac{bt}{3H} + \frac{1-e^{-3Ht}}{3H}\left(\dot\phi_0 - \frac{b}{3H}\right)$$

$$\langle(\phi - \langle\phi\rangle)^2\rangle = \frac{2D}{9H^2}\left(t + \frac{1-e^{-6Ht}}{6H} - \frac{2-2e^{-3Ht}}{3H}\right)$$

$$\langle\dot\phi(t)\rangle \sim b/3H, \ \langle[\dot\phi(t) - \langle\dot\phi(t)\rangle]^2\rangle \sim D/3H.$$

Langevin

$$\ddot{\phi} = -3H\dot{\phi} - V'(\phi) + \xi(t)$$

$$V(\phi) = 3H^2 M_{\rm pl}^2$$

$$\langle \xi(t)\xi(t') \rangle = 2D\delta(t - t')$$

$$V(\phi) = V_0 - b\,\phi$$

$$\phi(t) = \frac{b}{3H}t + \frac{1}{3H}\left(1 - e^{-3Ht}\right)\left(\dot{\phi}_0 - \frac{b}{3H}\right) + \int_0^t {\rm d}s\, \frac{\xi(s)}{3H}\left[1 - e^{3H(s-t)}\right]$$

(with $\phi(0) = 0$ and $\dot{\phi}(0) = \dot{\phi}_0$)

Variance

$$\langle\dot\phi\rangle = e^{-3Ht}\dot\phi_0 + \frac{b}{3H}\left(1-e^{-3Ht}\right)\ ,$$

$$\langle(\dot\phi - \langle\dot\phi\rangle)^2\rangle = \frac{D}{3H}\left(1-e^{-6Ht}\right)\ .$$

$$\langle\phi\rangle = \phi_0 + \frac{b\,t}{3H} + \frac{1-e^{-3Ht}}{3H}\left(\dot\phi_0 - \frac{b}{3H}\right)$$

$$\langle(\phi - \langle\phi\rangle)^2\rangle = \frac{2D}{9H^2}\left(t + \frac{1-e^{-6Ht}}{6H} - \frac{2-2e^{-3Ht}}{3H}\right)$$

$$\langle\dot\phi(t)\rangle \sim b/3H,\ \langle[\dot\phi(t) - \langle\dot\phi(t)\rangle]^2\rangle \sim D/3H.$$

Probability distribution

$$P(\dot{\phi},t) = \sqrt{\frac{3H}{2\pi D\,(1-e^{-6Ht})}}\exp\left(-\frac{3H(\dot{\phi}-b/3H-\dot{\phi}_0 e^{-3Ht})^2}{2D(1-e^{-6Ht})}\right)$$

$$\rightarrow \sqrt{\frac{3H}{2\pi D}}\exp\left(-\frac{3H(\dot{\phi}-b/3H)^2}{2D}\right)$$

The drift speed is close to the slow-roll speed.

Little or no improvement on the fine-tuning problem.

a subtlety

$$\ddot{x} + \gamma \dot{x} + V'(x) = \xi(t)$$

$$\begin{aligned} \frac{\mathrm{d}x}{\mathrm{d}t} &= v \\ \frac{\mathrm{d}v}{\mathrm{d}t} &= -(\gamma\, v + V') + \xi(t) \end{aligned}$$

Klein–Kramers equation for $P(x, v, t)$

$$\frac{\partial P}{\partial t} = \left[-\frac{\partial}{\partial x} v - \frac{\partial}{\partial v} [-(\gamma\, v + V')] + D \frac{\partial^2}{\partial v^2} \right] P$$

If entropic perturbation is negligible

$$\delta = (\delta P_{adiabatic} + \delta P_{entropic})/P_\zeta$$

strength of scatterings → $\dot{\phi}^2 g$; measures frequency → ΔN_e

$$D = 9H^2\lambda = 9H\dot{\phi}^2 g\,\Delta N_e/4$$

Diffusion constant :

$$D = \delta\frac{b^2}{3H} = 96\sqrt{2}\pi^3\delta\,\epsilon^{5/2}\Delta^3 M_P^5$$

$$\Delta = 5.4\times 10^{-5} \qquad \epsilon = -\dot{H}/H^2 < 1$$

$$\Delta N_e = \frac{4\delta}{3g}$$

WMAP bound $\delta \lesssim 10\%$.

$$\Delta l \sim \frac{50}{g}$$

Otherwise, features can be much closer.

$$\delta = (\delta P_{adiabatic} + \delta P_{entropic})/P_\zeta$$

Modeling the scatterings :

$$v_k'' + \left(k^2 - \frac{z''}{z}\right) v_k = 0$$

where τ is the conformal time, $a\mathrm{d}\tau = \mathrm{d}t$

$$z \equiv a\dot{\phi}/H \qquad\qquad \frac{z''}{z} \approx \frac{a''}{a} + 3a^2H\frac{\ddot{\phi}}{\dot{\phi}}$$

$$v_k'' + \left(k^2 - \frac{a''}{a} - \sum_{i=1}^{N} 3aH\Delta_i\delta(\tau - \tau_i)\right) v_k = 0$$

A. A. Starobinsky, "Spectrum Of Adiabatic Perturbations In The Universe When There Are Singularities In The Inflation Potential," JETP Lett. **55**, 489 (1992) [Pisma Zh. Eksp. Teor. Fiz. **55**, 477 (1992)].

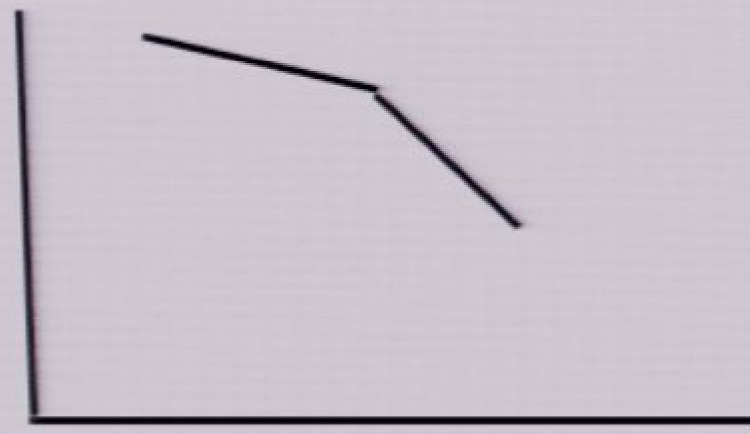
multiple scatterings

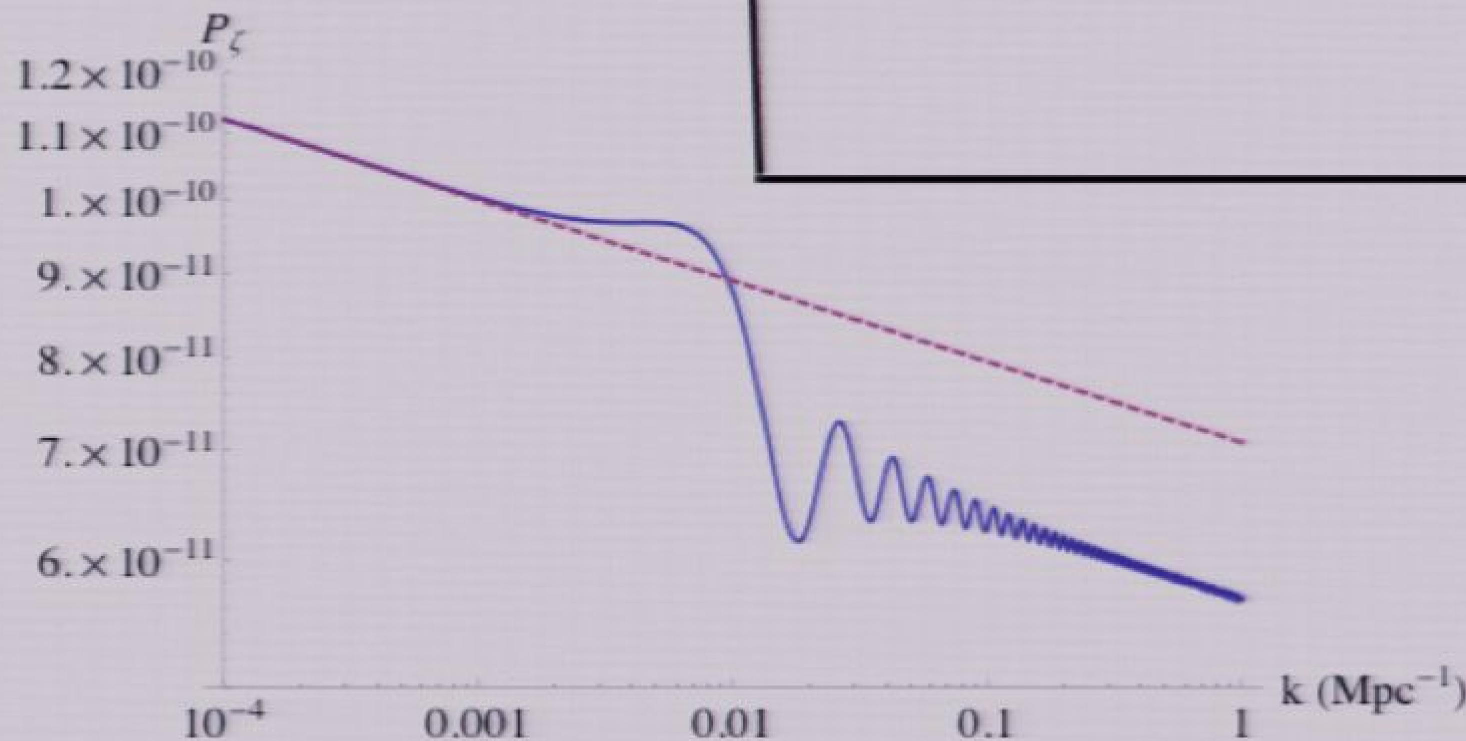
$$v_i(k,\tau) = \alpha_i v^-(k,\tau) + \beta_i v^+(k,\tau)$$

$$v^\pm(k,\tau) = \frac{1}{\sqrt{2k}} e^{\pm ik\tau}\left(1 \pm \frac{i}{k\tau}\right)$$

$$v(k,\tau_i^+) = v(k,\tau_i^-)\ , \quad v'(k,\tau_i^+) - v'(k,\tau_i^-) = 3aH\Delta_i$$

$$\begin{pmatrix} \alpha_i \\ \beta_i \end{pmatrix} = T_{i,i-1}\begin{pmatrix} \alpha_{i-1} \\ \beta_{i-1} \end{pmatrix}$$

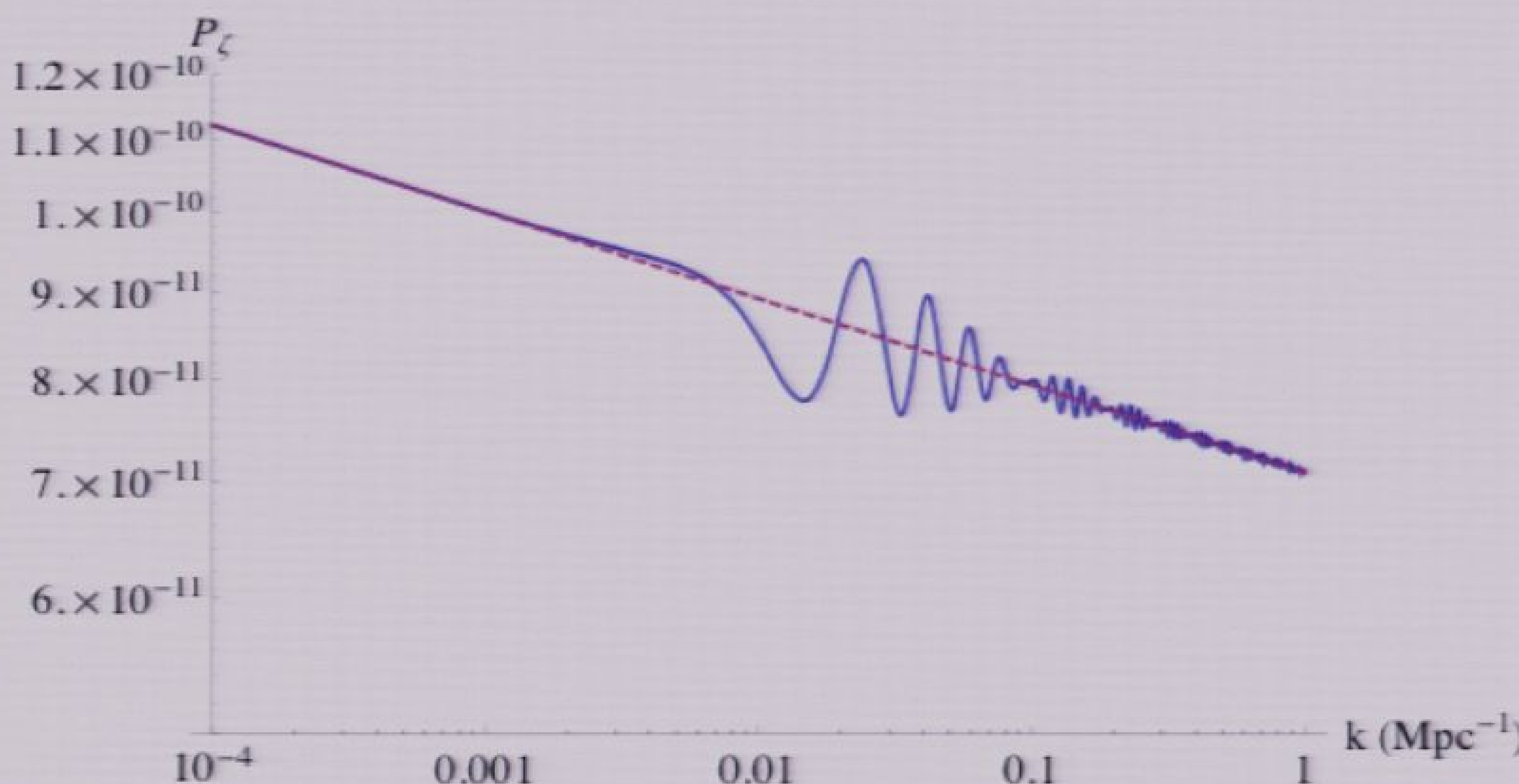




One scattering with $\Delta_1 = 0.1$, $k_1 = 0.005 Mpc^{-1}$

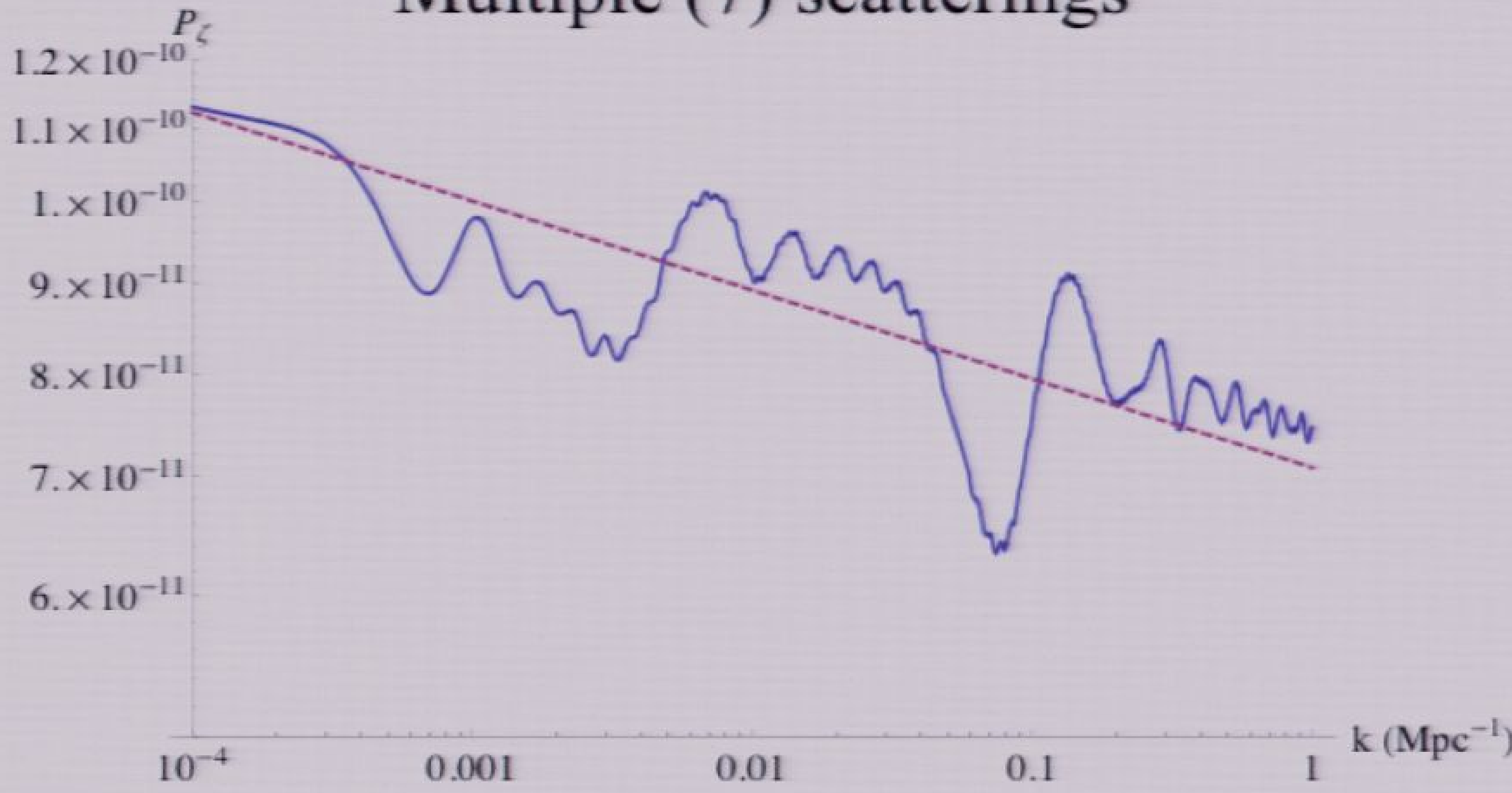
The dashed line is $n_s = 0.95$.

a pair of scatterings that mimic a step



$\Delta_1 = 0.1,\ k_1 = 0.005Mpc^{-1},\ \Delta_2 = -0.1,\ k_1 = 0.006Mpc^{-1}$

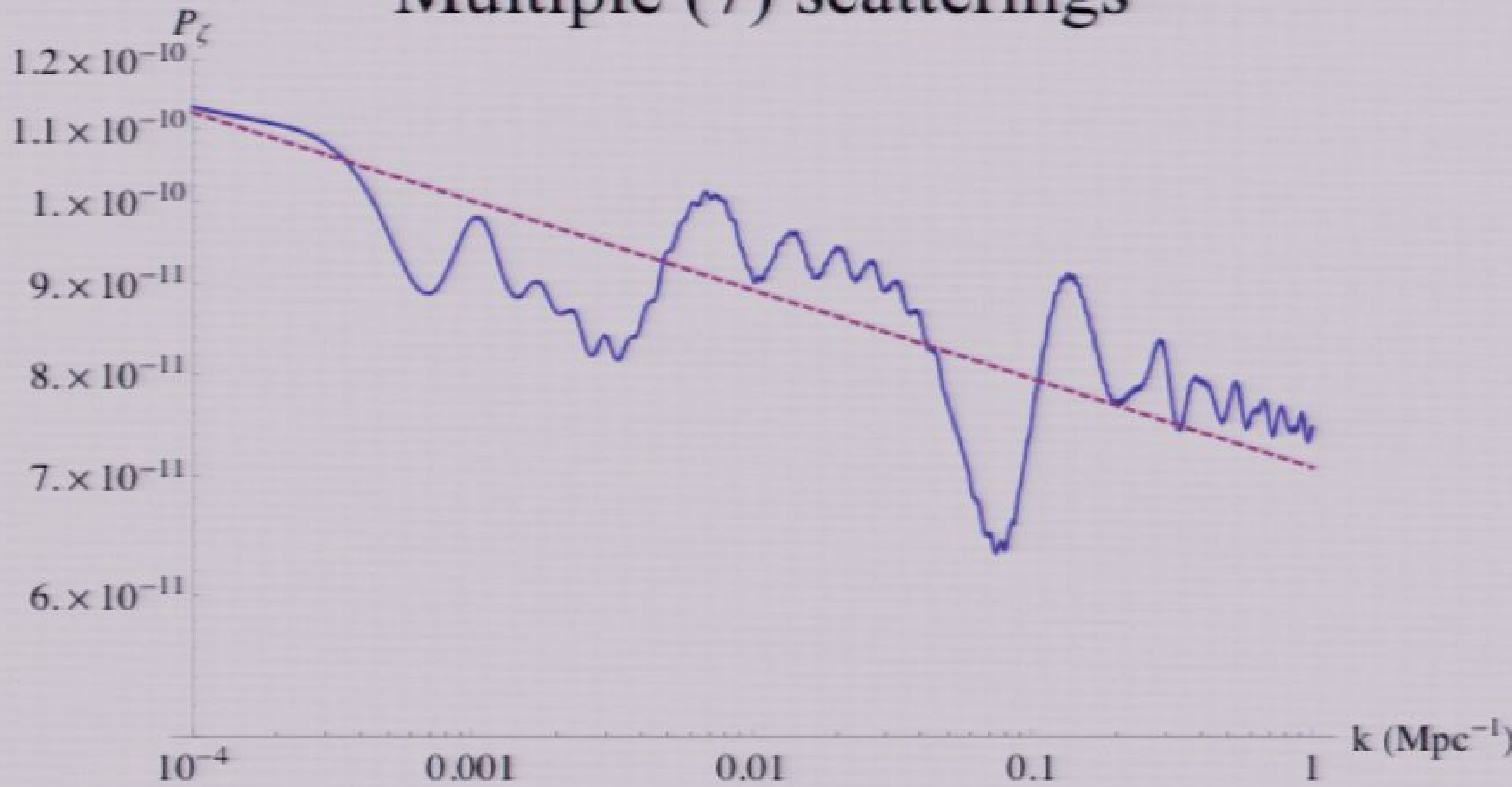
Multiple (7) scatterings



$$\Delta_i = \{0.05, -0.02, 0.015, -0.07, 0.03, 0.07, -0.1\}$$

$$k_i = \{2\times10^{-4}, 3\times10^{-4}, 10^{-3}, 2\times10^{-3}, 0.02, 0.025, 0.04\}Mpc^{-1}$$

Multiple (7) scatterings



$$\Delta_i = \{0.05, -0.02, 0.015, -0.07, 0.03, 0.07, -0.1\}$$

$$k_i = \{2\times10^{-4}, 3\times10^{-4}, 10^{-3}, 2\times10^{-3}, 0.02, 0.025, 0.04\} Mpc^{-1}$$

Fast tunneling in the landscape

$$\gamma = \Gamma / H^4$$

$$t_F \simeq \frac{3}{4\pi H \gamma} = \frac{3H^3}{4\pi\Gamma}$$

$$\zeta = \delta N \simeq H \delta t \sim H t_F$$

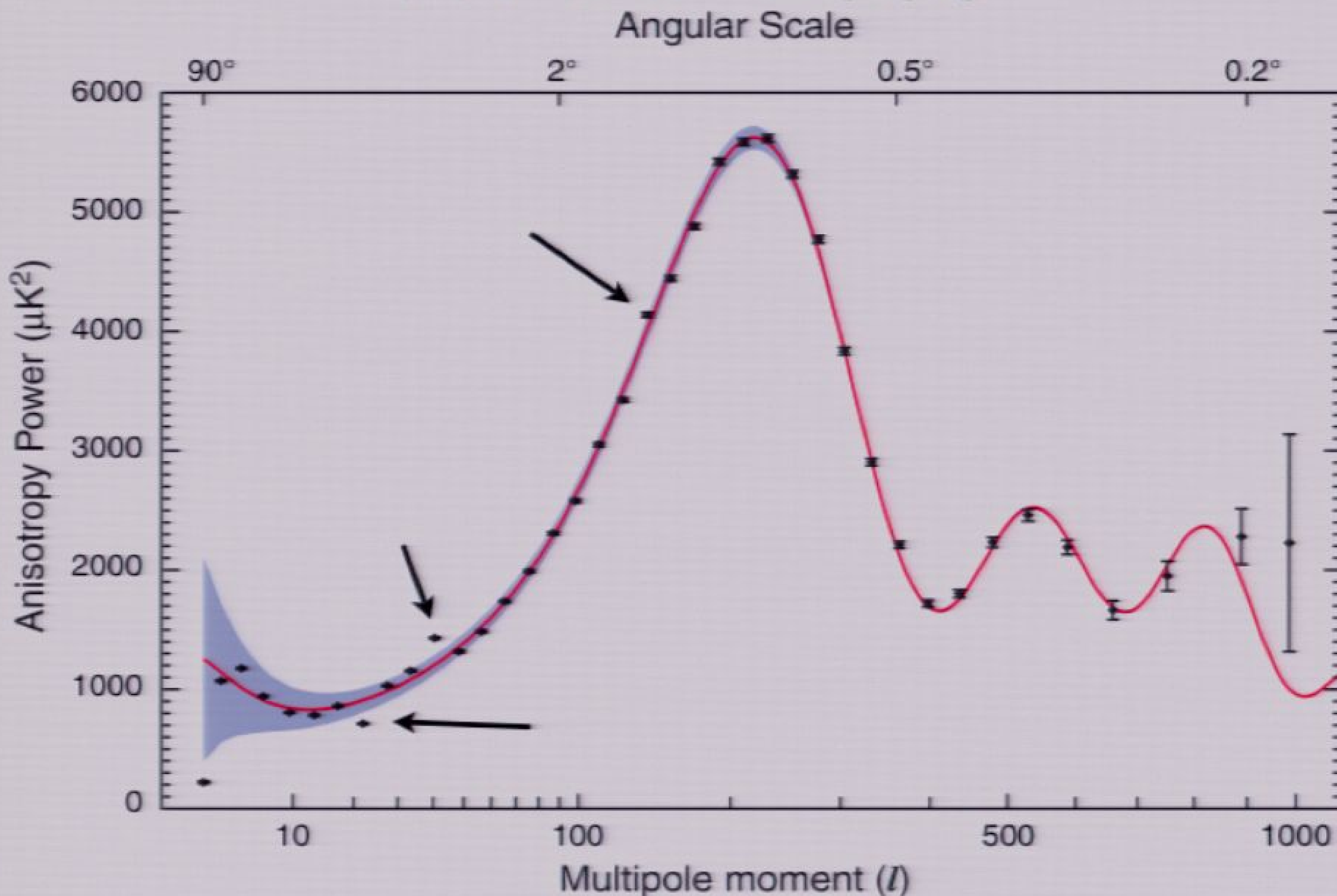
$$t_F \sim 10^{-5} H^{-1}$$

H.Q. Huang

quantum ↘

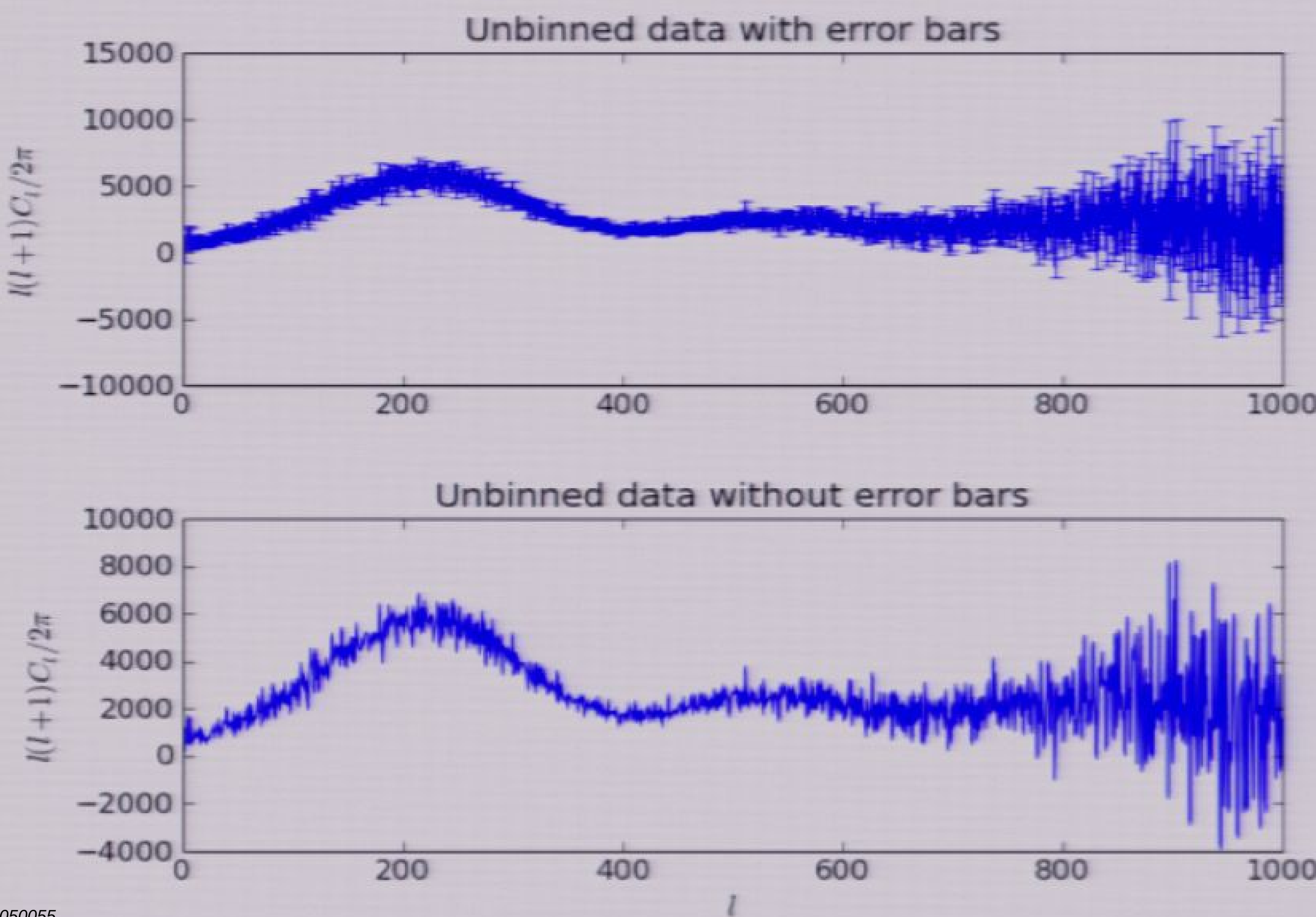
Another likely source for features in the power spectrum

WMAP 2006



Are there features in the power spectrum ?

WMAP 5 years



Unbinned data with error bars
15000
10000
5000
0
−5000
−10000
0
200
400
600
800
1000
$l(l+1)C_l/2\pi$
Unbinned data without error bars
10000
8000
6000
4000
2000
0
−2000
−4000
0
200
400
600
800
1000
$l(l+1)C_l/2\pi$
l

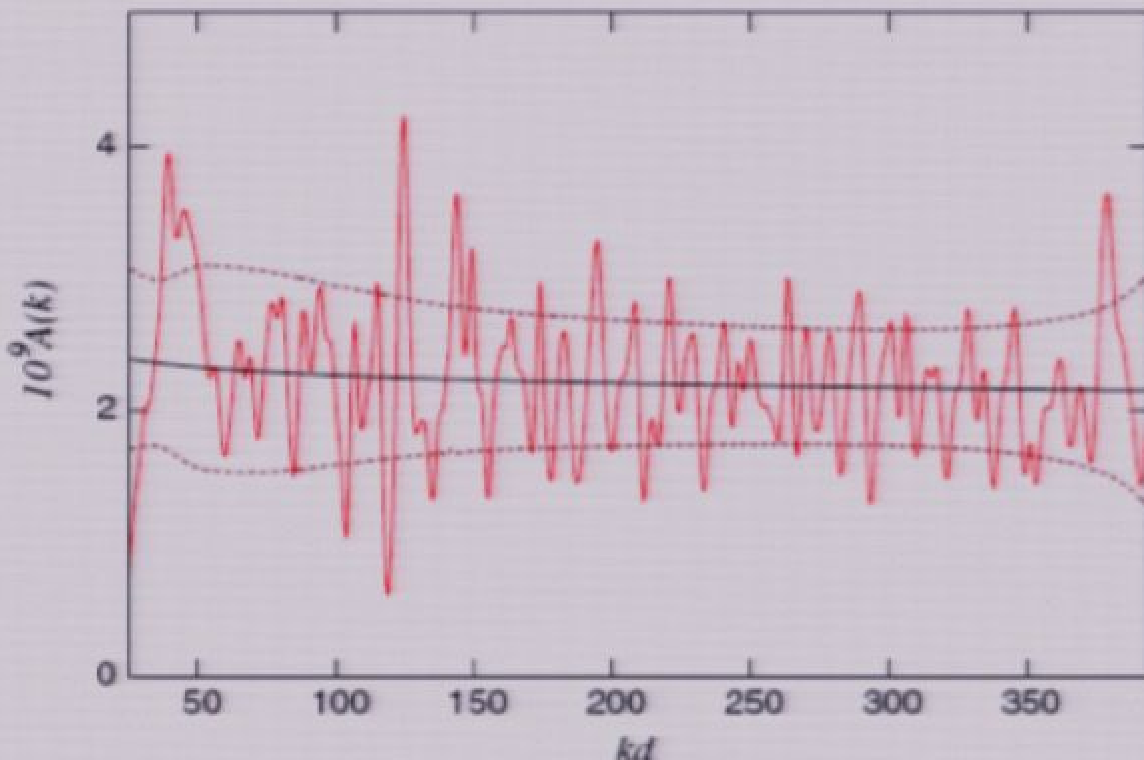
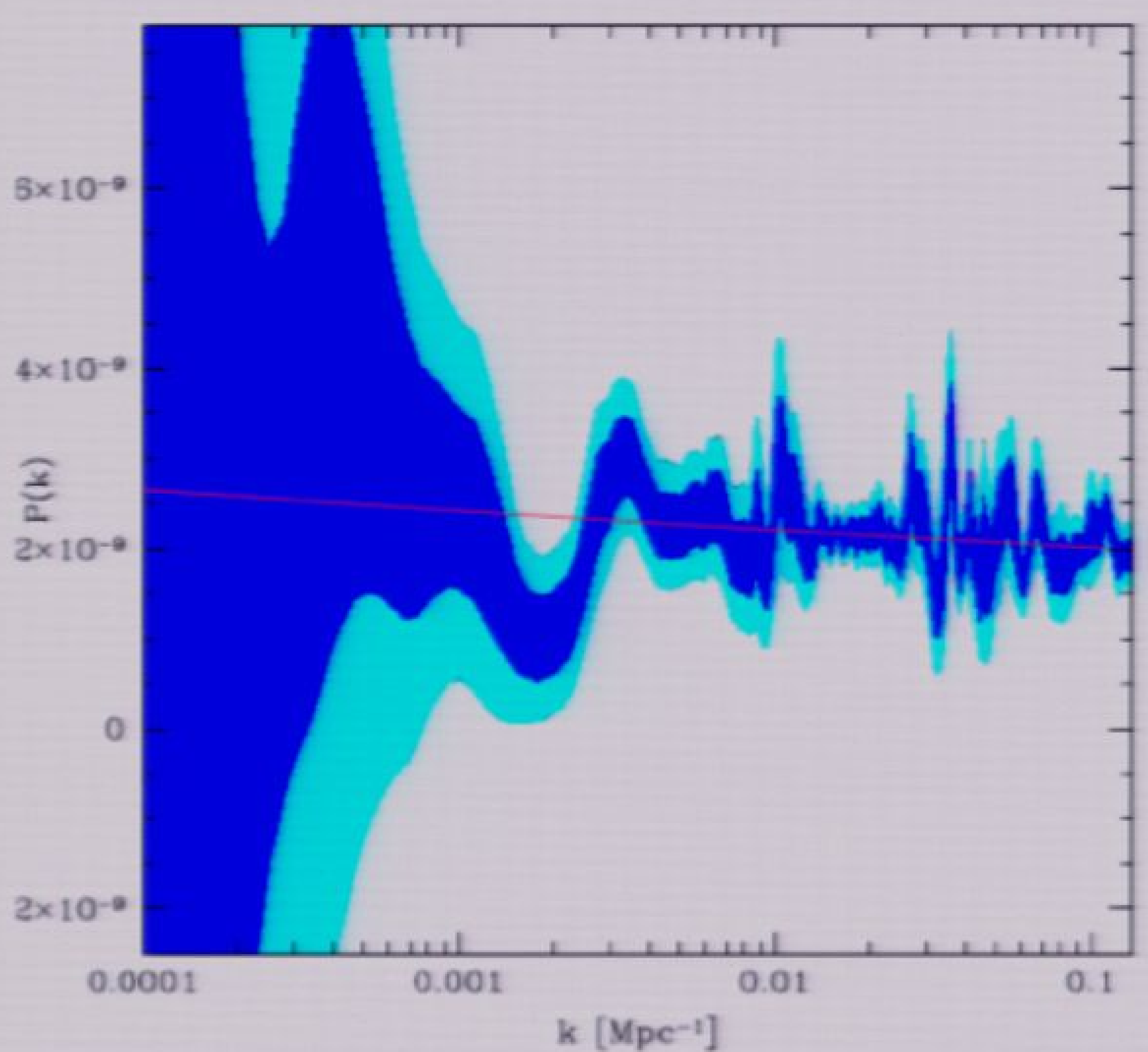


FIG. 1 (color online). The reconstruction of the primordial spectrum by the maximum likelihood reconstruction method from the five-year WMAP data. The solid wavy curve is the result of reconstruction and the straight line is the best-fit power-law spectrum. Dotted lines are the standard deviation around the best-fit power law.

a 3.3σ deviation

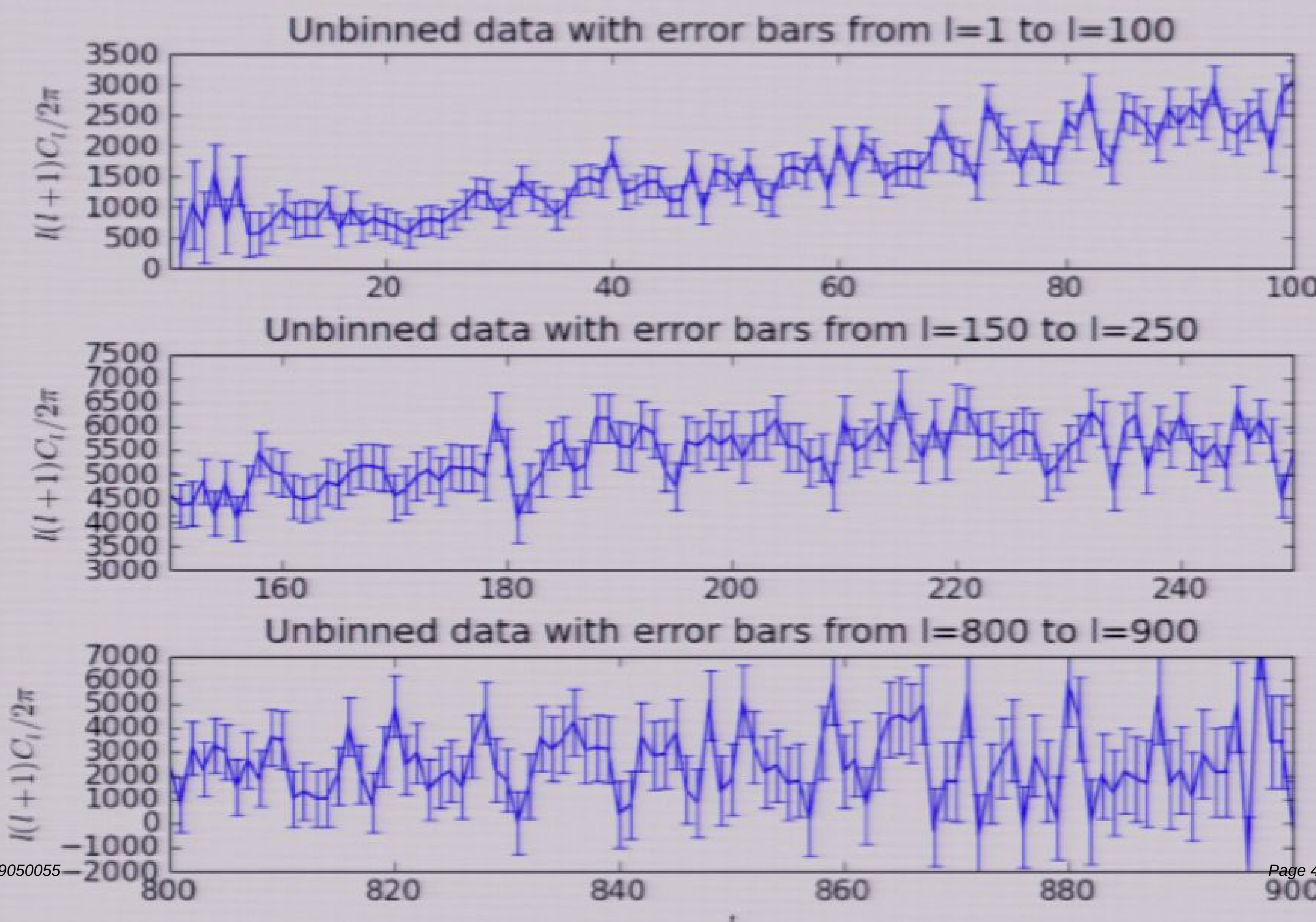
Nagata and Yokoyama, 0812.4585



Nicholson and Contaldi 0903.1106

FIG. 3: Current limits from a combination of CMB data sets (WMAP, ACBAR, QUaD, BOOMERanG and CBI). There is some evidence of a dip in power at around $k \approx 0.002$ below the best fit power law model. Shaded regions are defined as in Fig. 2.

WMAP 5 years



Unbinned data with error bars from l=1 to l=100
$l(l+1)C_l/2\pi$
3500
3000
2500
2000
1500
1000
500
0
20
40
60
80
100
Unbinned data with error bars from l=150 to l=250
$l(l+1)C_l/2\pi$
7500
7000
6500
6000
5500
5000
4500
4000
3500
3000
160
180
200
220
240
Unbinned data with error bars from l=800 to l=900
$l(l+1)C_l/2\pi$
7000
6000
5000
4000
3000
2000
1000
0
−1000
−2000
800
820
840
860
880
900

Uncertainties

$$\delta C_l \;=\; \sqrt{\frac{2}{(2l+1)f_{\rm sky}}}\left(C_l + \omega^{-1}\, e^{l^2\sigma_b^2}\right)\;,$$

$$\sigma_b \;=\; \frac{\pi}{180}\frac{1}{\sqrt{8\ln 2}}\frac{\Delta\theta}{1^\circ}\;,\quad \omega^{-1} = (\Delta T_{\rm pix})^2\,\Delta\theta^2$$

Gaussian beam width

pixel noise $\Delta T_{\rm pix}$

Table 1. Experiment Specifications for WMAP and PLANCK

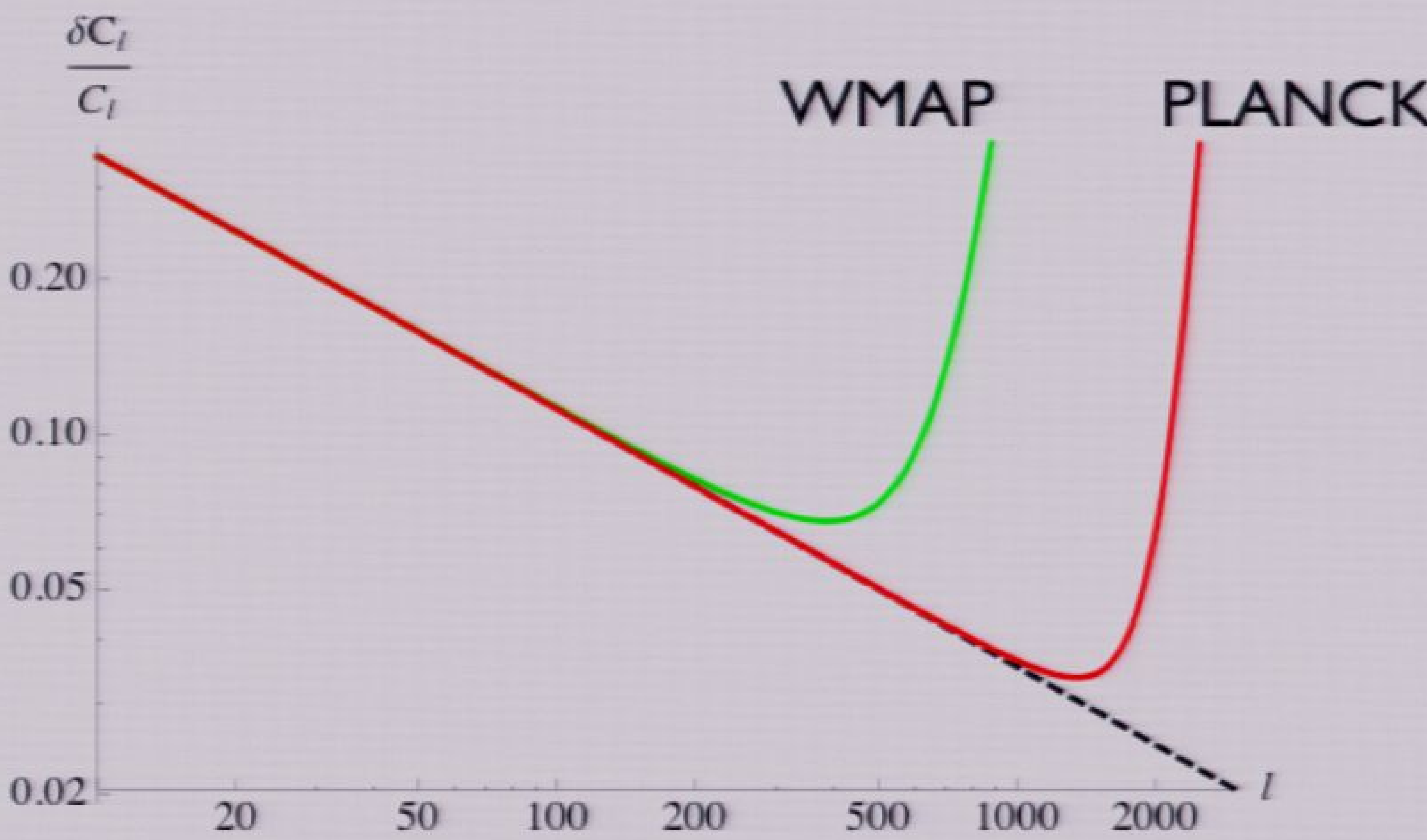
	WMAP		PLANCK		
ν (GHz)	61	94	70	100	143
$\theta_{\rm FWHM}$ (arcmin)	19.8	12.6	14	9.5	7.1
$\Delta T_{\rm pix}$ (μK)	21.1	31.9	12.8	6.8	6.0

$$C_l \sim 10^4 l^{-2} (\mu K)^2$$

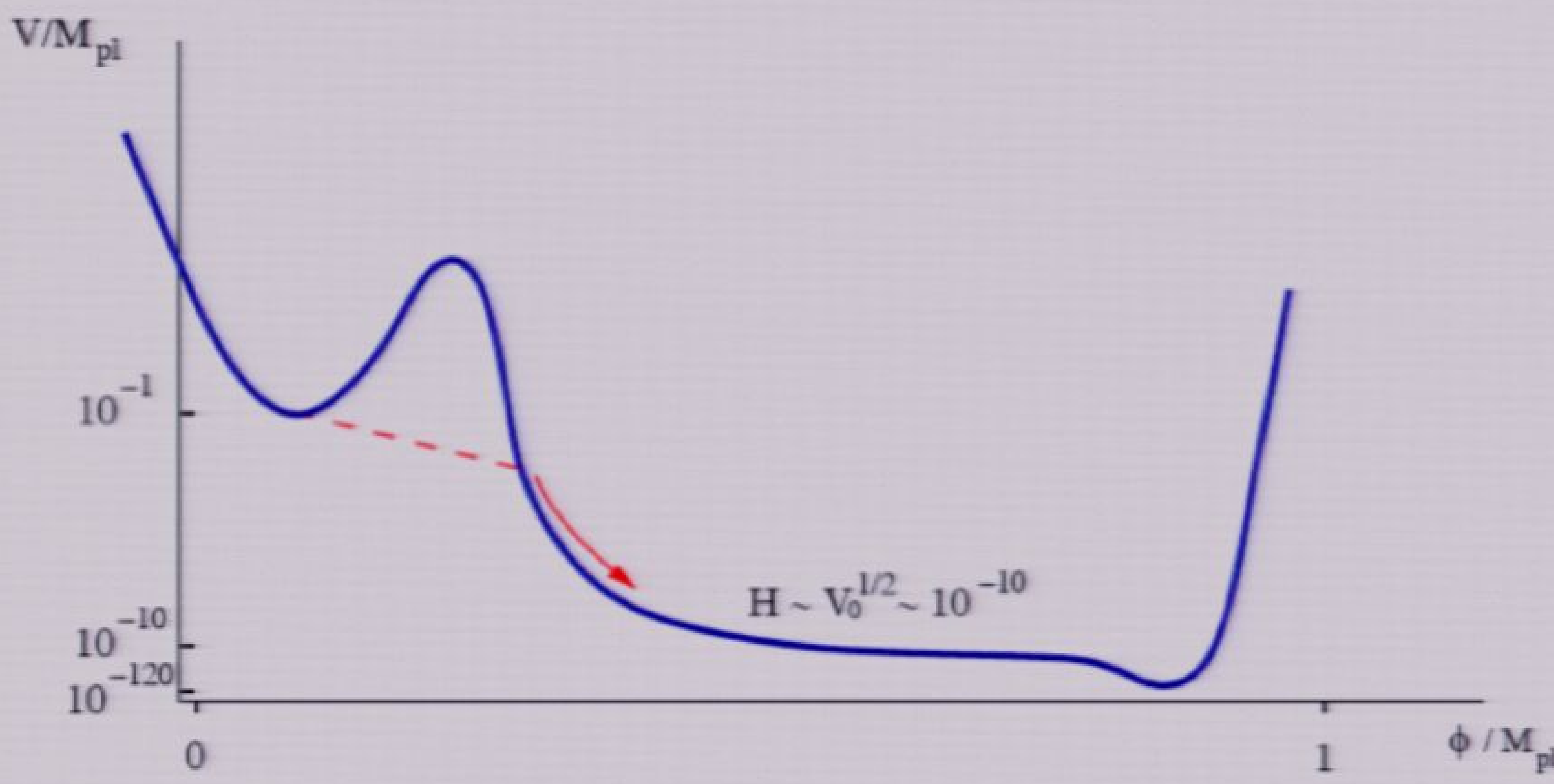
Table 2. Error Estimation in C_l for WMAP and PLANCK

l	20	200	500	800	1000	1500	2000
Cosmic Variance (%)	19.8	6.3	4.0	3.2	2.8	2.3	2.0
WMAP $\delta C_l/C_l$ (%)	19.8	6.5	5.9	18.1	65.5	-	-
PLANCK $\delta C_l/C_l$ (%)	19.8	6.3	4.0	3.2	2.9	2.8	5.1

Sensitivities to fluctuations



Eternal inflation and observed inflation



Remarks

- Multi-field inflation with a random potential naturally leads to fluctuations in the CMB power spectrum.
- Enough such features in the CMB power spectrum will support that the inflaton is mobile in the landscape, at least for the 5 to 8 e-folds covered by CMB data.
- Such features will be strong evidence against eternal inflation.

    - Another possible signature : Steps in the warped throat (due to duality cascade) can show up as features in the power spectrum.

String theory can be tested via inflation