Title: Can we tell whether Stochastic Gravitational Waves are really primordial?

Date: May 20, 2009 02:00 PM

URL: http://pirsa.org/09050052

Abstract: TBA

Can we ever tell whether stochastic gravity waves are really primordial?

Eugene A Lim (Columbia)
with Peter Adshead (Yale) and Lam Hui (Columbia)

EFT Conference,
Perimeter Institute 20-22 May 2009

(And LISA/LIGO webpage for letting me steal their images)

Detecting non-Gaussianities in the stochastic gravity wave background

Eugene A Lim (Columbia)
with Peter Adshead (Yale) and Lam Hui (Columbia)

EFT Conference,
Perimeter Institute 20-22 May 2009

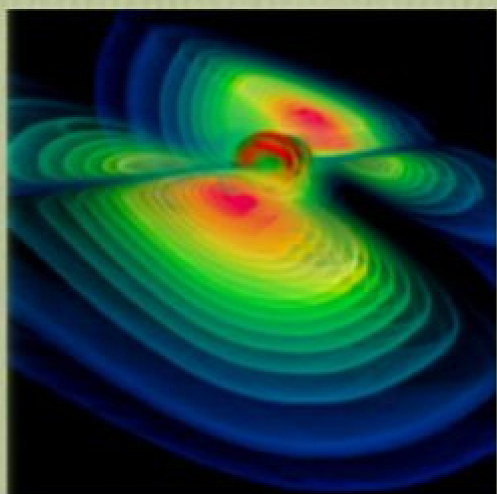
(And LISA/LIGO webpage for letting me steal their images)

Outline

- Introduction : why should we care?
- Review : Direct Detection of Stochastic Gravitational Wave Power Spectrum
- Constructing the estimator for SGW : from detector data streams to 3-pt correlation functions
- What can see?
- Summary

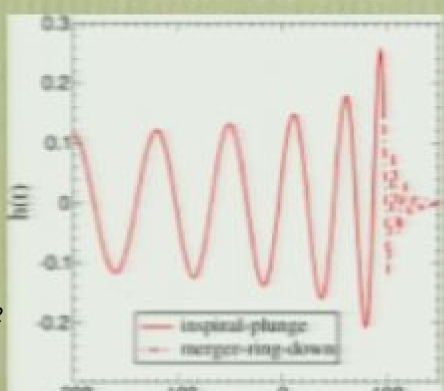
GW : The Final Frontier

- Two Targets for direction detection of Gravity Waves



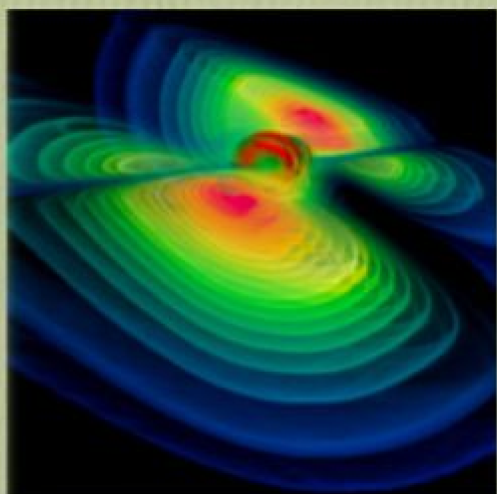
Resolved sources

Black hole mergers, compact binaries etc.



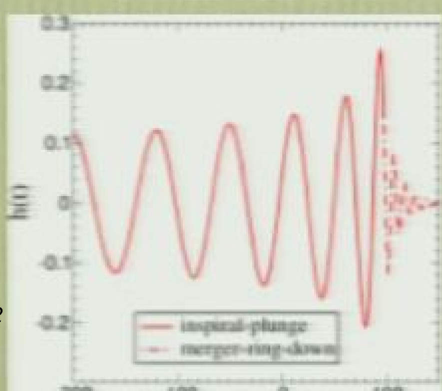
GW : The Final Frontier

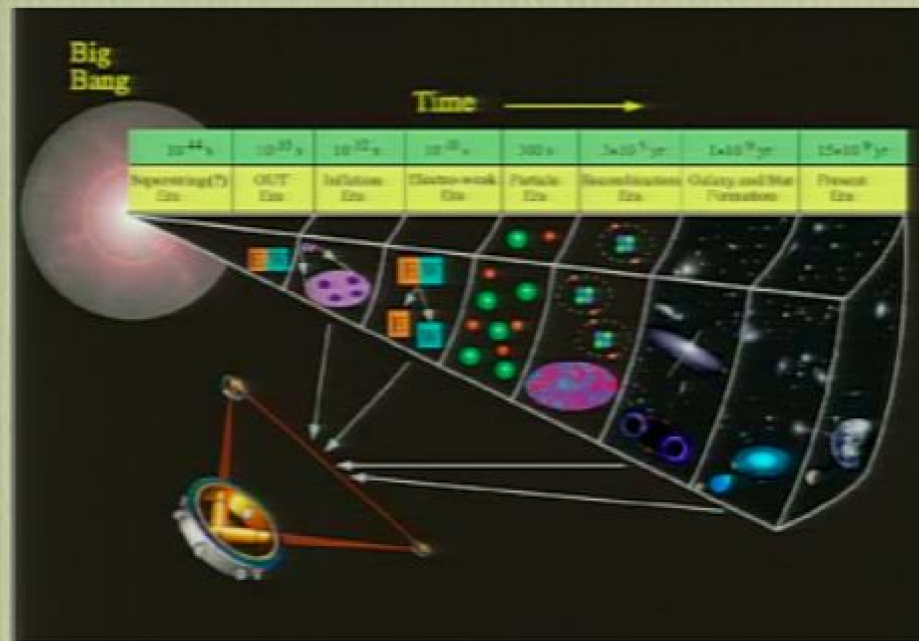
- Two Targets for direction detection of Gravity Waves



Resolved sources

Black hole mergers, compact binaries etc.



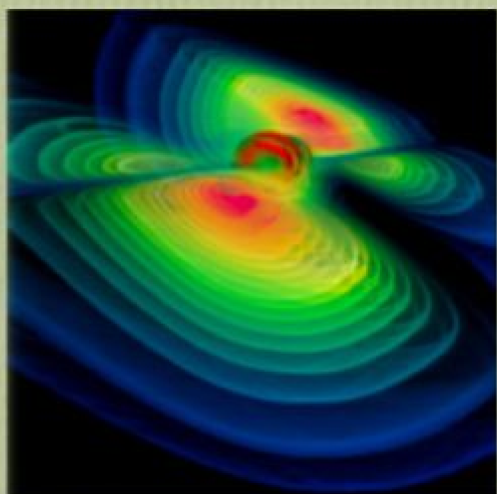


Unresolved sources

Primordial GW

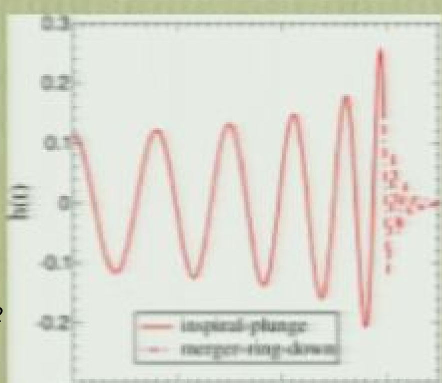
GW : The Final Frontier

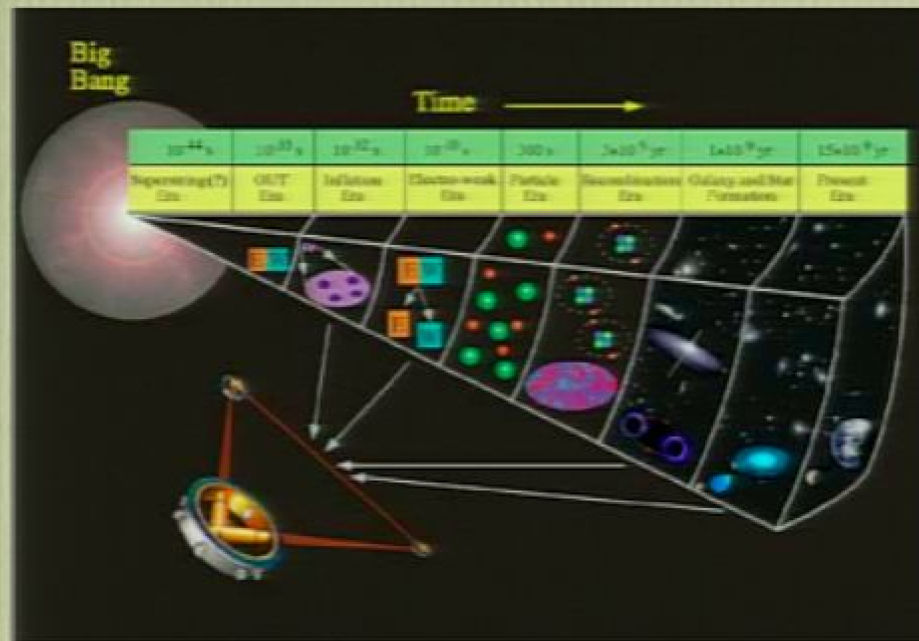
- Two Targets for direction detection of Gravity Waves



Resolved sources

Black hole mergers, compact binaries etc.



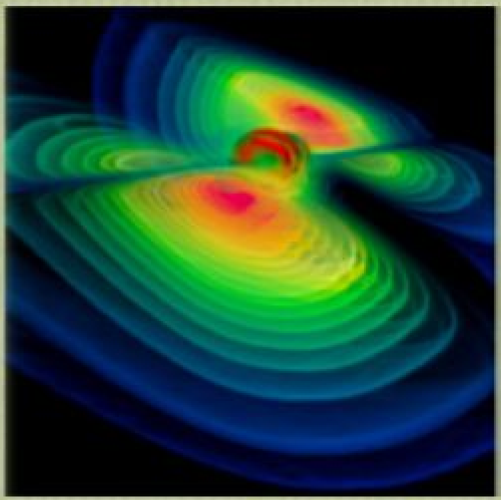


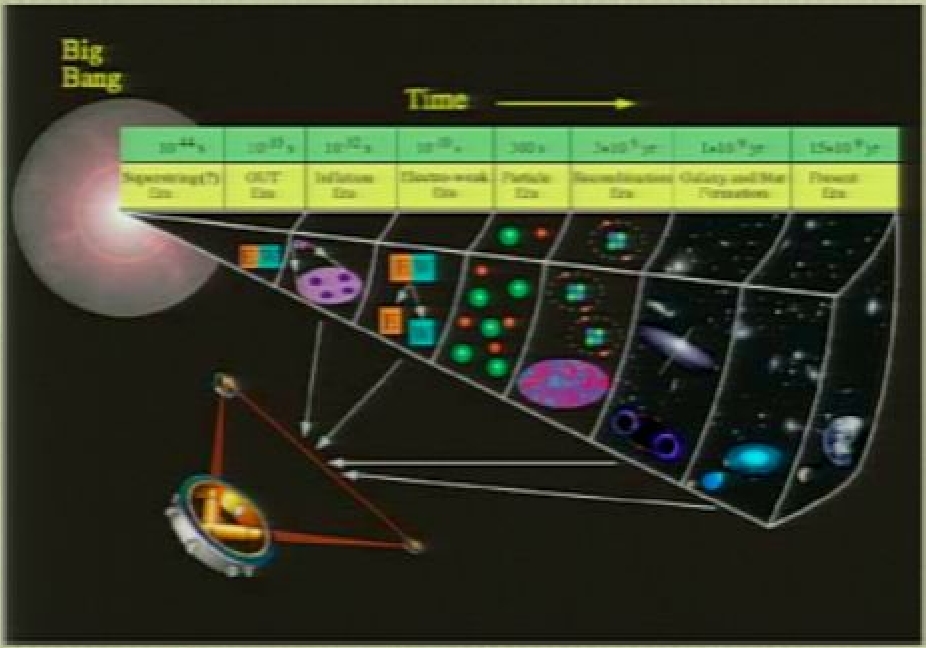
Unresolved sources

Primordial GW
Unresolved astrophysical sources
(binaries, supernovae etc)

GW : The Final Frontier

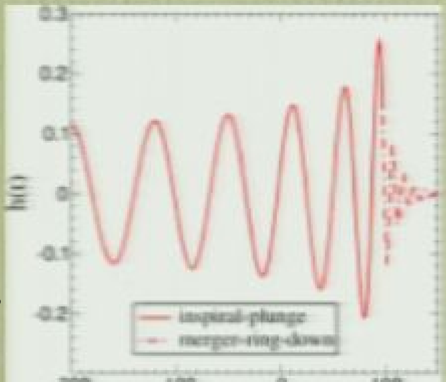
- Two Targets for direction detection of Gravity Waves





Resolved sources

Black hole mergers, compact binaries etc.

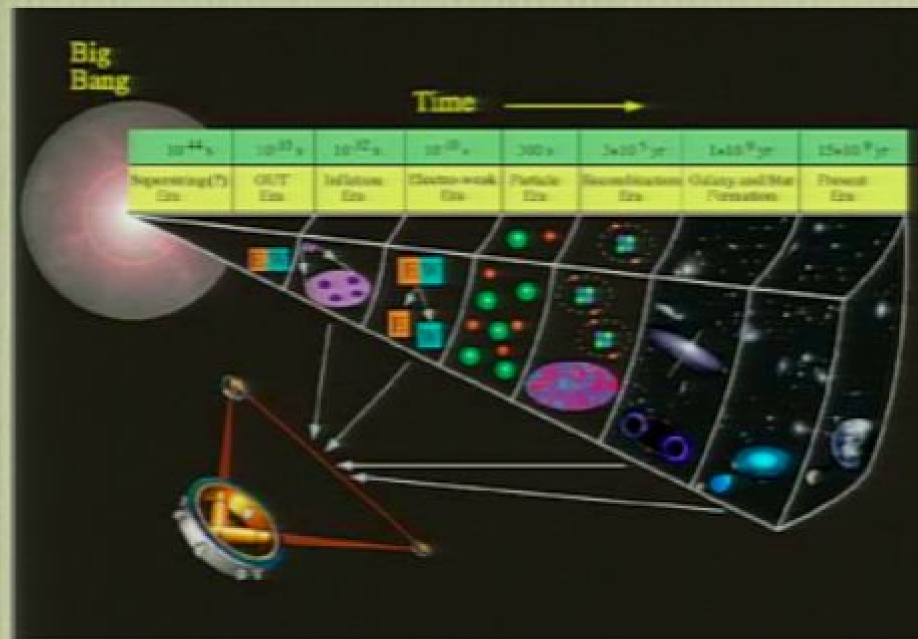


Unresolved sources

Primordial GW
Unresolved astrophysical sources
(binaries, supernovae etc)
Cosmological Turbulence
(preheating, phase transitions)

Thanks Thorsten!

GW : The Final Frontier



Unresolved sources

Primordial GW
Unresolved astrophysical sources
(binaries, supernovae etc)
Cosmological Turbulence
(preheating, phase transitions)
Strings/Defects? New physics?

If we detect SGW, which one is it??

Thanks Thorsten!

Unresolved Sources = Stochastic GW

Stochastic : they "look" just like noise

Characterize them via *correlation functions*

Unresolved Sources = Stochastic GW

Stochastic : they "look" just like noise

Characterize them via *correlation functions*

Power Spectrum : $\langle h(f)h(f)\rangle$

Convention : *energy density* $\Omega_{gw} = \dfrac{\rho_{gw}}{\rho_g}$ $\rho_{gw} = \dfrac{M_p^2}{32\pi}\langle \dot{h}_{ij}\dot{h}^{ij}\rangle$

Fourier space $\langle h(f)h(f')\rangle = \dfrac{3H_0^2}{20\pi^2} f^{-3}\Omega_{gw}(f)\delta(f-f')$

Given a fixed *strain*, it is generally *harder* to detect correlations of it at *higher frequencies*

Unresolved Sources = Stochastic GW

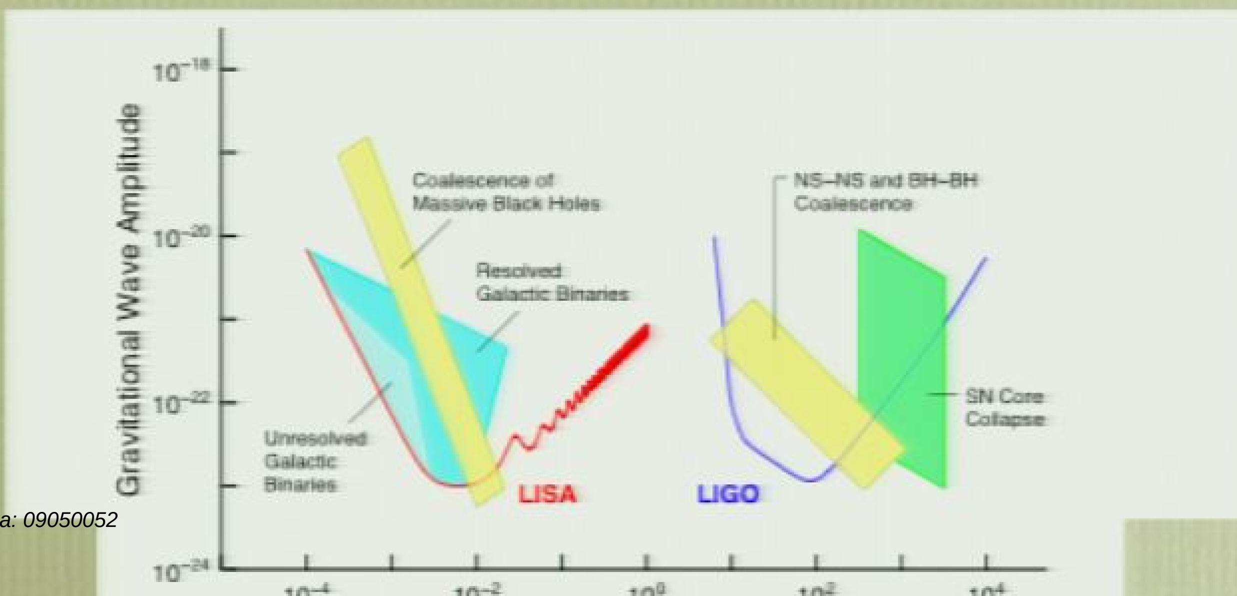
Stochastic : they "look" just like noise

Characterize them via *correlation functions*

Power Spectrum : $\langle h(f)h(f)\rangle$

Convention : *energy density* $\Omega_{gw} = \dfrac{\rho_{gw}}{\rho_g}$ $\rho_{gw} = \dfrac{M_p^2}{32\pi}\langle \dot{h}_{ij}\dot{h}^{ij}\rangle$

Fourier space $\langle h(f)h(f')\rangle = \dfrac{3H_0^2}{20\pi^2} f^{-3}\Omega_{gw}(f)\delta(f-f')$



Given a fixed *strain*, it is generally *harder* to detect correlations of it at *higher frequencies*

Unresolved Sources = Stochastic GW

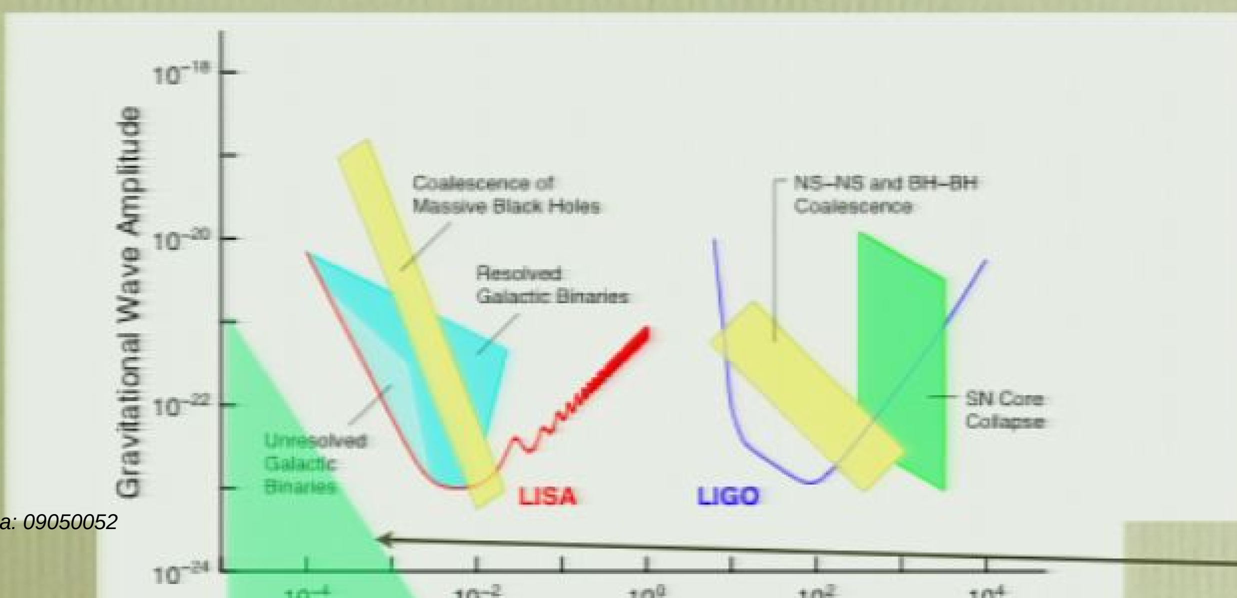
Stochastic : they "look" just like noise

Characterize them via *correlation functions*

Power Spectrum : $\langle h(f)h(f)\rangle$

Convention : *energy density* $\Omega_{gw} = \dfrac{\rho_{gw}}{\rho_g}$ $\rho_{gw} = \dfrac{M_p^2}{32\pi}\langle \dot{h}_{ij}\dot{h}^{ij}\rangle$

Fourier space $\langle h(f)h(f')\rangle = \dfrac{3H_0^2}{20\pi^2} f^{-3}\Omega_{gw}(f)\delta(f-f')$

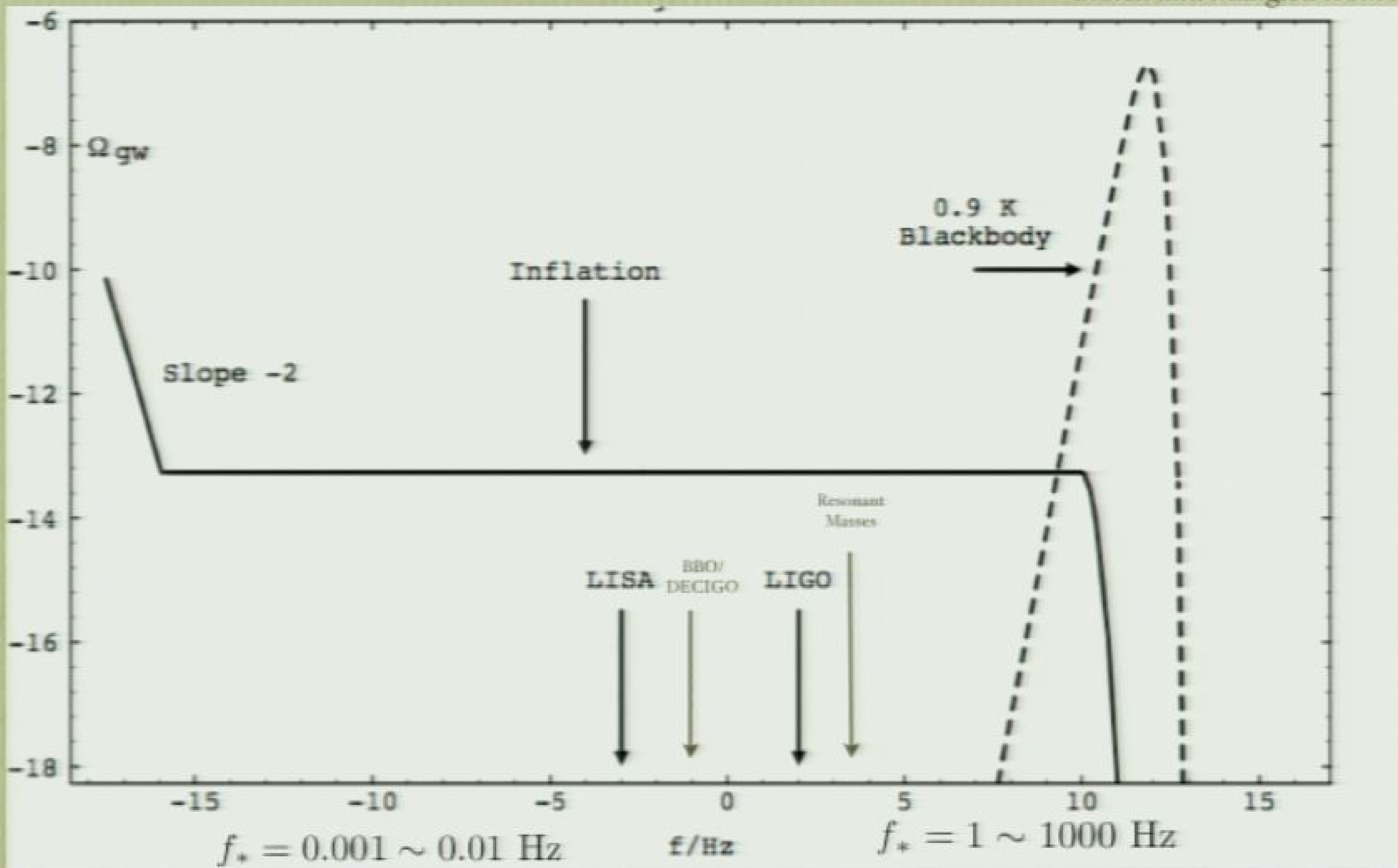


Given a fixed *strain*, it is generally *harder* to detect correlations of it at *higher frequencies*

GUT scale best case

The Frequency Range : an orientation

Stolen and Mangled from Allen (1996)



"Golden Range"

Unresolved Sources = Stochastic GW

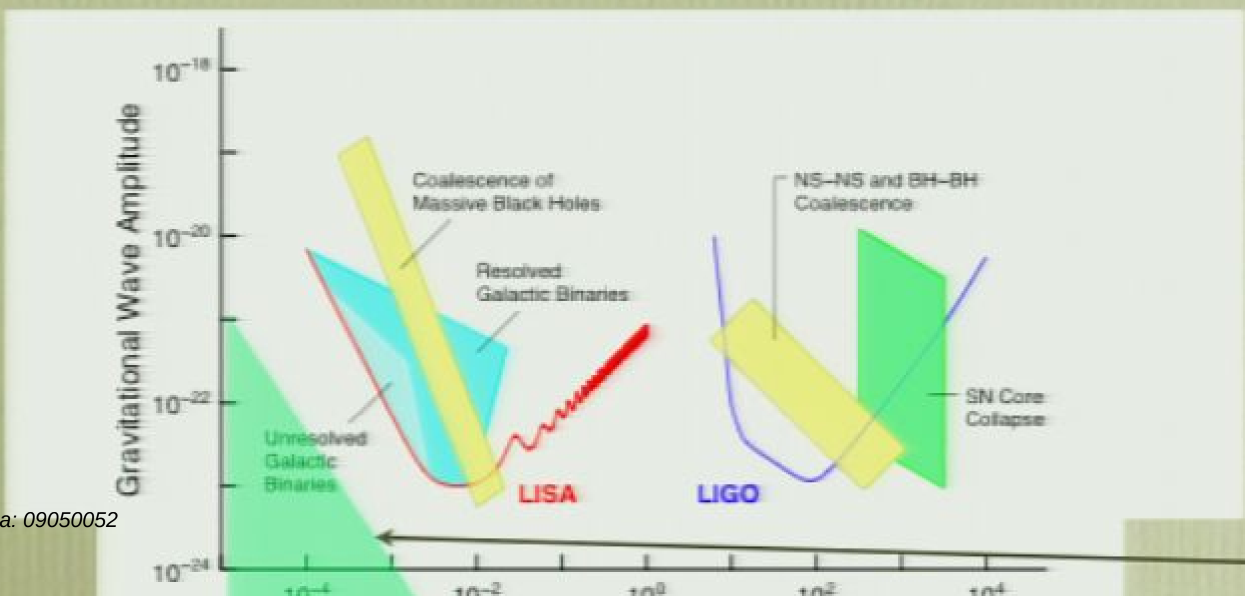
Stochastic : they "look" just like noise

Characterize them via *correlation functions*

Power Spectrum : $\langle h(f)h(f)\rangle$

Convention : *energy density* $\Omega_{gw} = \dfrac{\rho_{gw}}{\rho_g}$ $\rho_{gw} = \dfrac{M_p^2}{32\pi}\langle \dot{h}_{ij}\dot{h}^{ij}\rangle$

Fourier space $\langle h(f)h(f')\rangle = \dfrac{3H_0^2}{20\pi^2} f^{-3}\Omega_{gw}(f)\delta(f-f')$

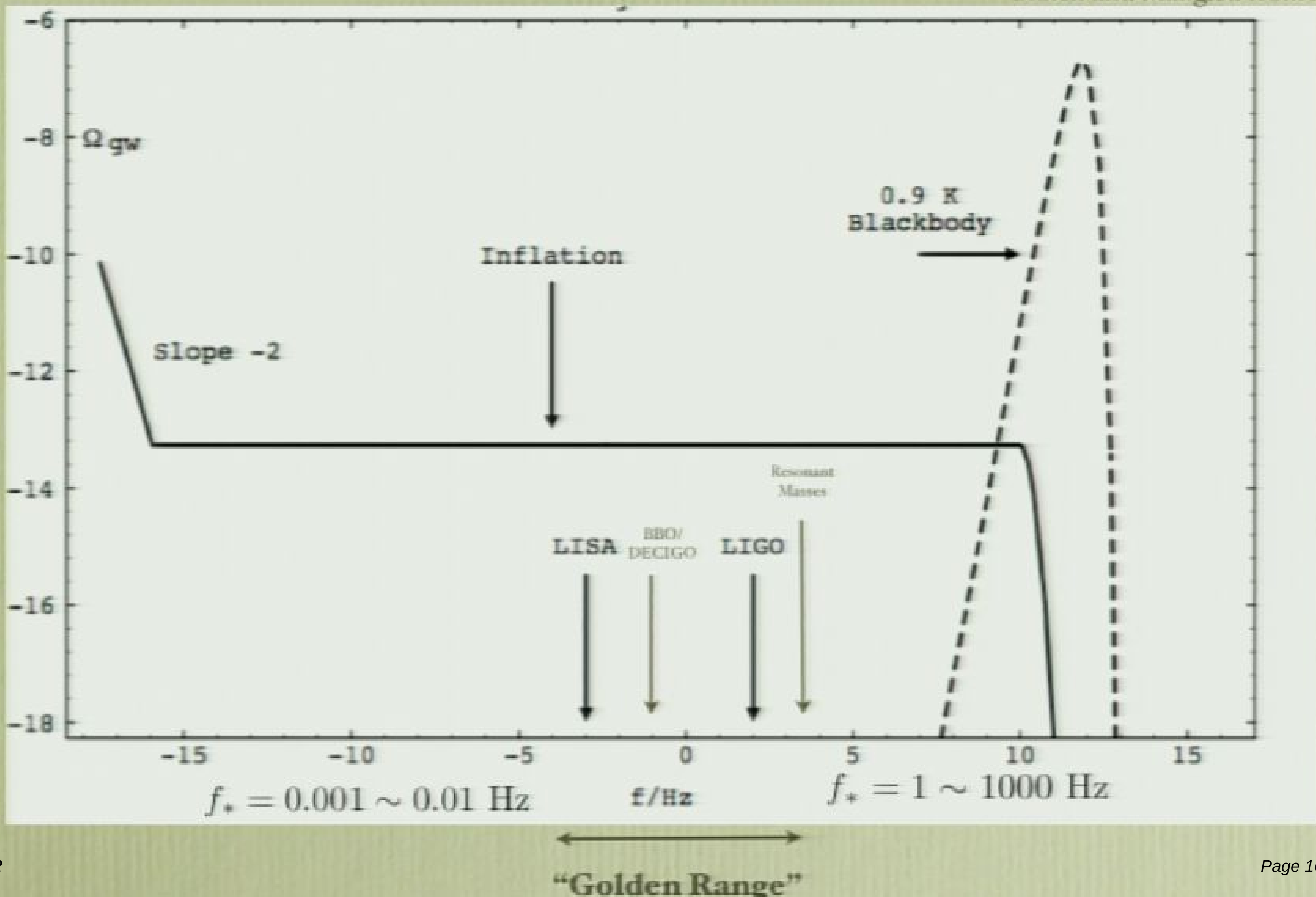


Given a fixed *strain*, it is generally *harder* to detect correlations of it at *higher frequencies*

GUT scale best case

The Frequency Range : an orientation

Stolen and Mangled from Allen (1996)



Unresolved Sources = Stochastic GW

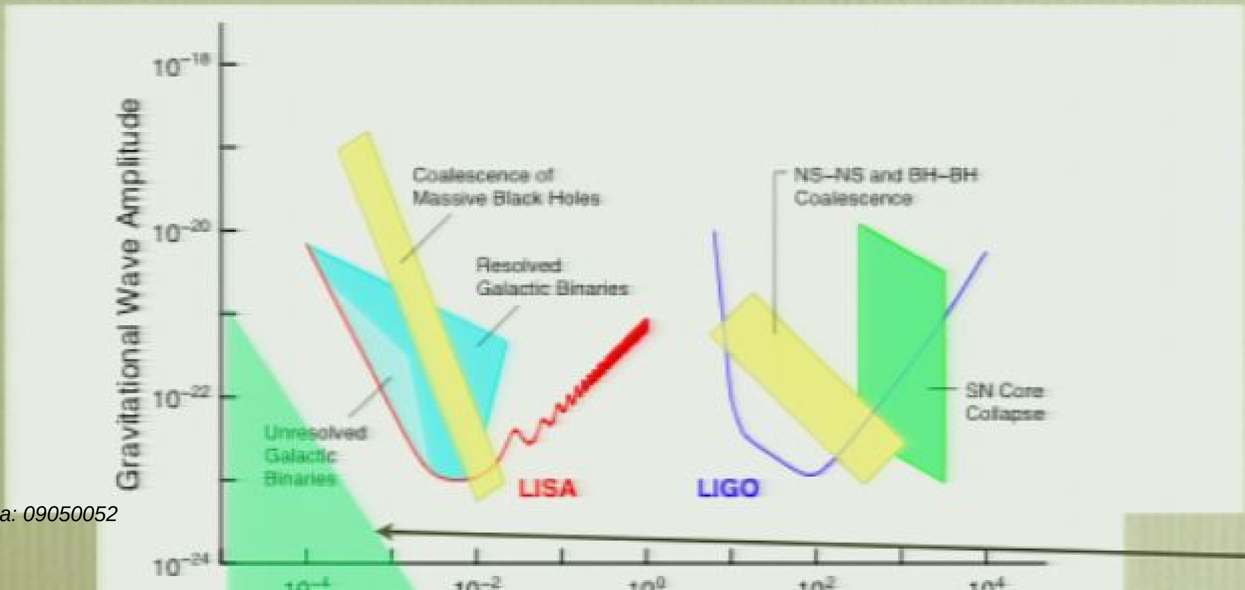
Stochastic : they "look" just like noise

Characterize them via *correlation functions*

Power Spectrum : $\langle h(f)h(f)\rangle$

Convention : *energy density* $\Omega_{gw} = \frac{\rho_{gw}}{\rho_g}$ $\rho_{gw} = \frac{M_p^2}{32\pi}\langle \dot{h}_{ij}\dot{h}^{ij}\rangle$

Fourier space $\langle h(f)h(f')\rangle = \frac{3H_0^2}{20\pi^2} f^{-3}\Omega_{gw}(f)\delta(f-f')$

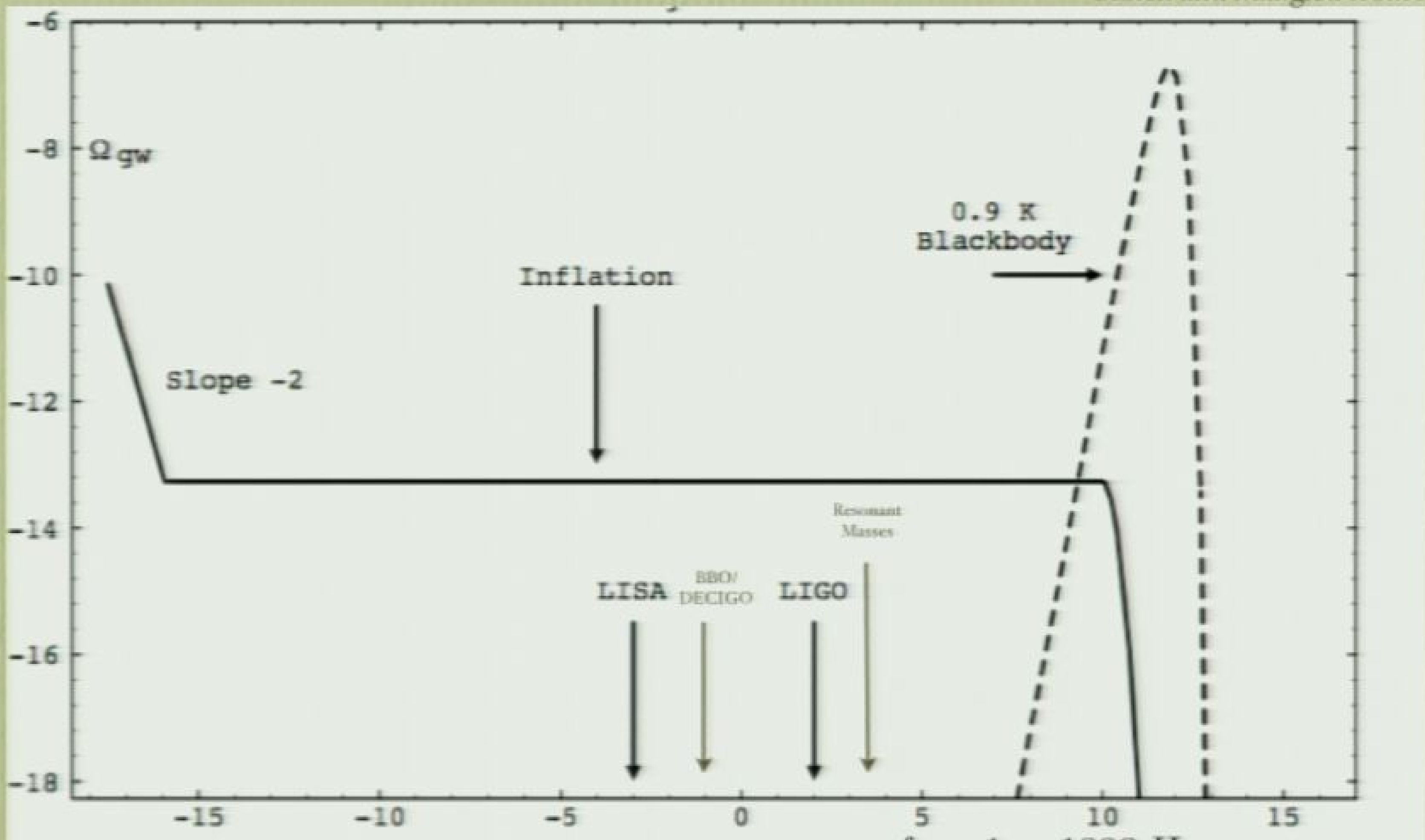


Given a fixed *strain*, it is generally *harder* to detect correlations of it at *higher frequencies*

GUT scale best case

The Frequency Range : an orientation

Stolen and Mangled from Allen (1996)



$f_* = 0.001 \sim 0.01$ Hz

$f_* = 1 \sim 1000$ Hz

"Golden Range"

Unresolved Sources = Stochastic GW

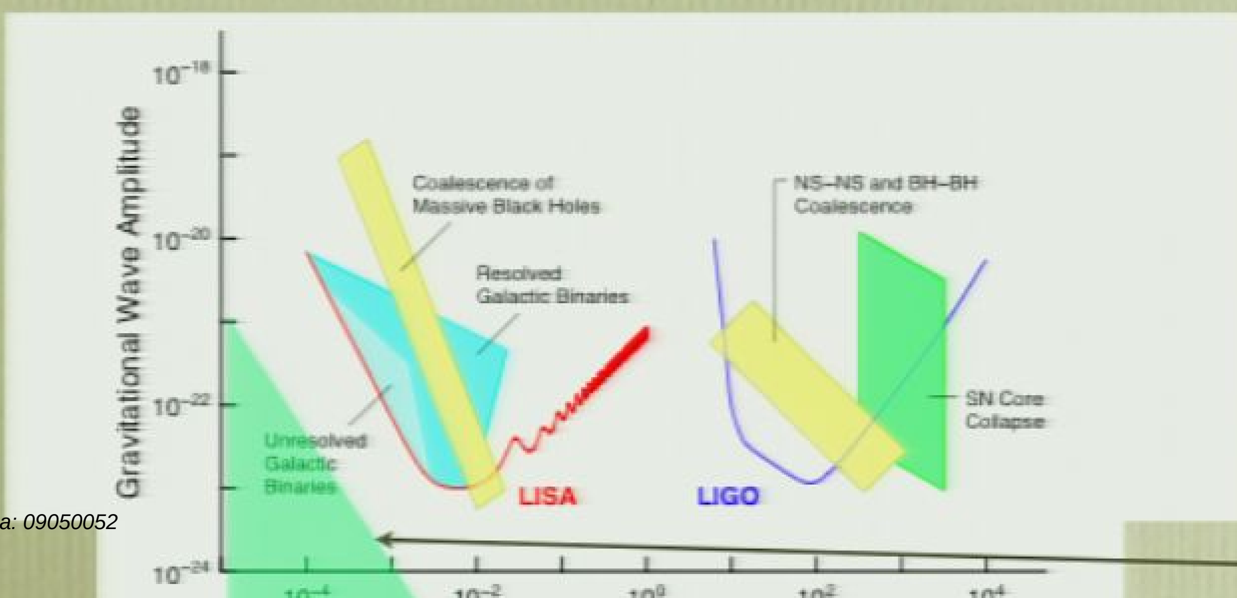
Stochastic : they "look" just like noise

Characterize them via *correlation functions*

Power Spectrum : $\langle h(f)h(f)\rangle$

Convention : *energy density* $\Omega_{gw} = \frac{\rho_{gw}}{\rho_g}$ $\rho_{gw} = \frac{M_p^2}{32\pi}\langle \dot{h}_{ij}\dot{h}^{ij}\rangle$

Fourier space $\langle h(f)h(f')\rangle = \frac{3H_0^2}{20\pi^2} f^{-3}\Omega_{gw}(f)\delta(f-f')$

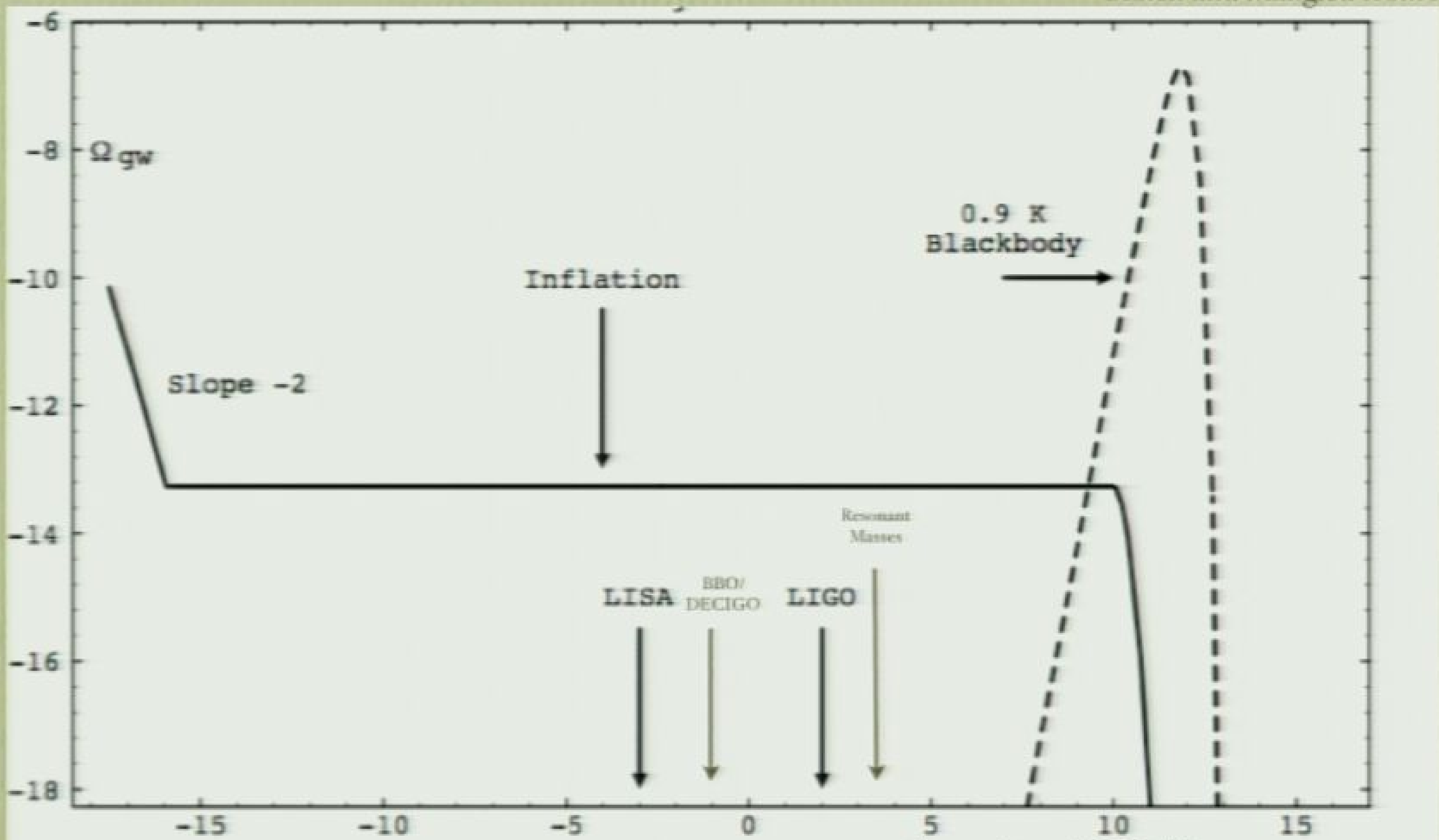


Given a fixed *strain*, it is generally *harder* to detect correlations of it at *higher frequencies*

GUT scale best case

The Frequency Range : an orientation

Stolen and Mangled from Allen (1996)



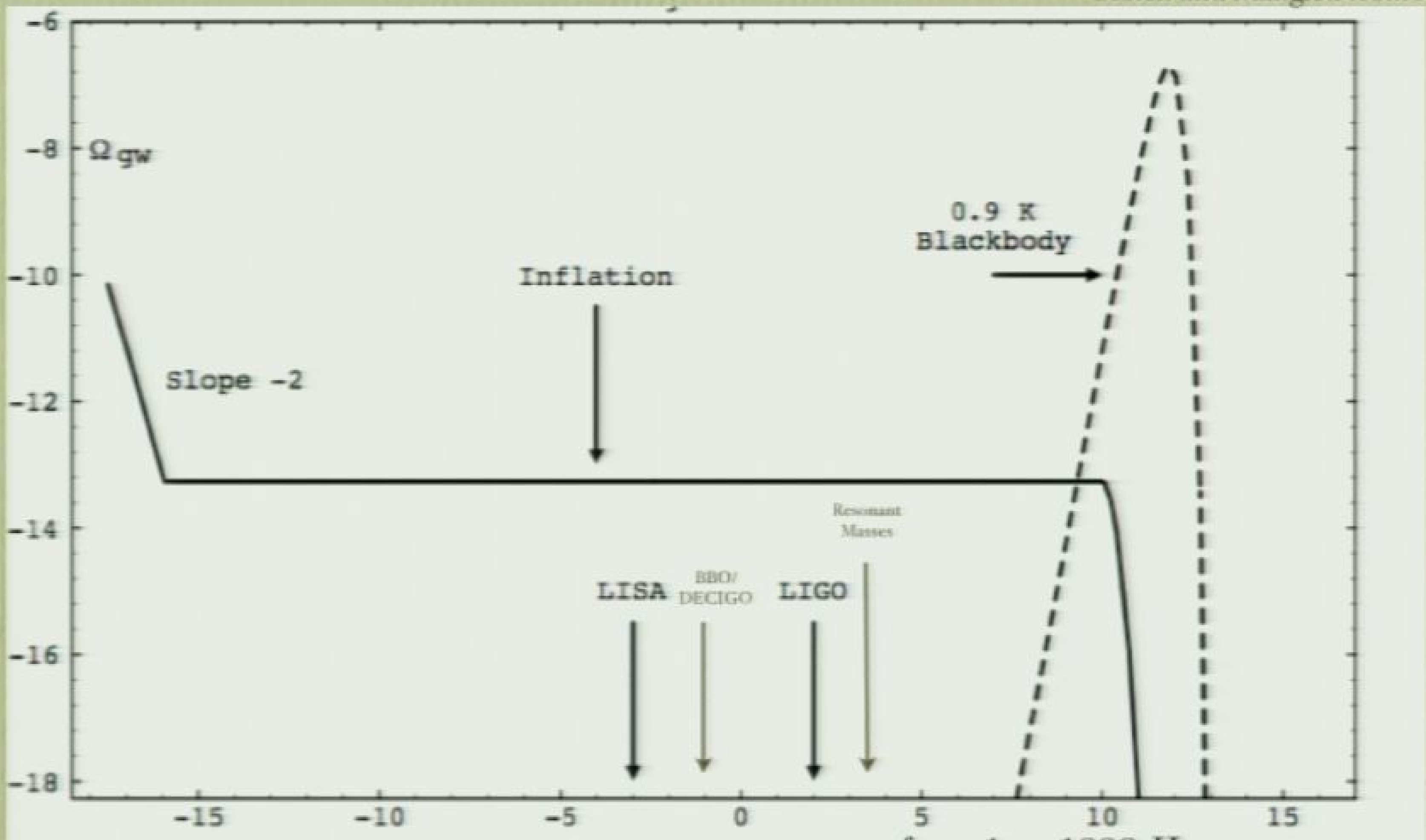
$f_* = 0.001 \sim 0.01$ Hz

$f_* = 1 \sim 1000$ Hz

"Golden Range"

The Frequency Range : an orientation

Stolen and Mangled from Allen (1996)



$f_* = 0.001 \sim 0.01$ Hz

$f_* = 1 \sim 1000$ Hz

Solar System Scales ⟷ Harder to construct small interferometers (maybe new tech : AGIS?)

"Golden Range"

Harder to detect correlations at higher

Unresolved Sources = Stochastic GW

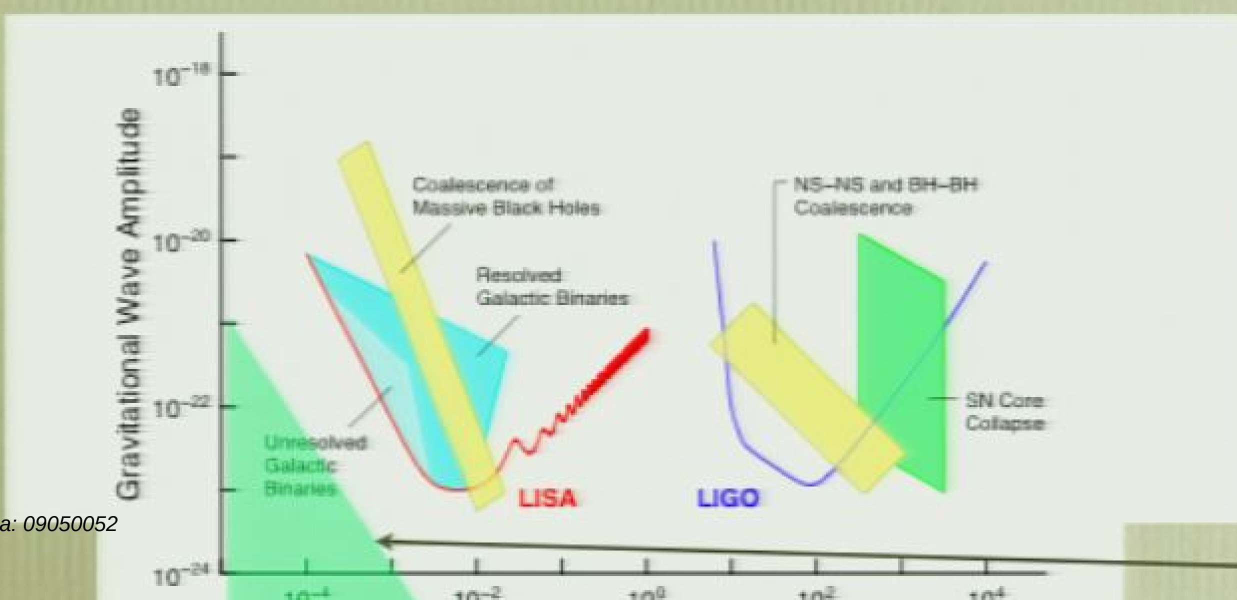
Stochastic : they "look" just like noise

Characterize them via *correlation functions*

Power Spectrum : $\langle h(f)h(f)\rangle$

Convention : *energy density* $\Omega_{gw} = \dfrac{\rho_{gw}}{\rho_g}$ $\rho_{gw} = \dfrac{M_p^2}{32\pi}\langle \dot{h}_{ij}\dot{h}^{ij}\rangle$

Fourier space $\langle h(f)h(f')\rangle = \dfrac{3H_0^2}{20\pi^2} f^{-3}\Omega_{gw}(f)\delta(f-f')$

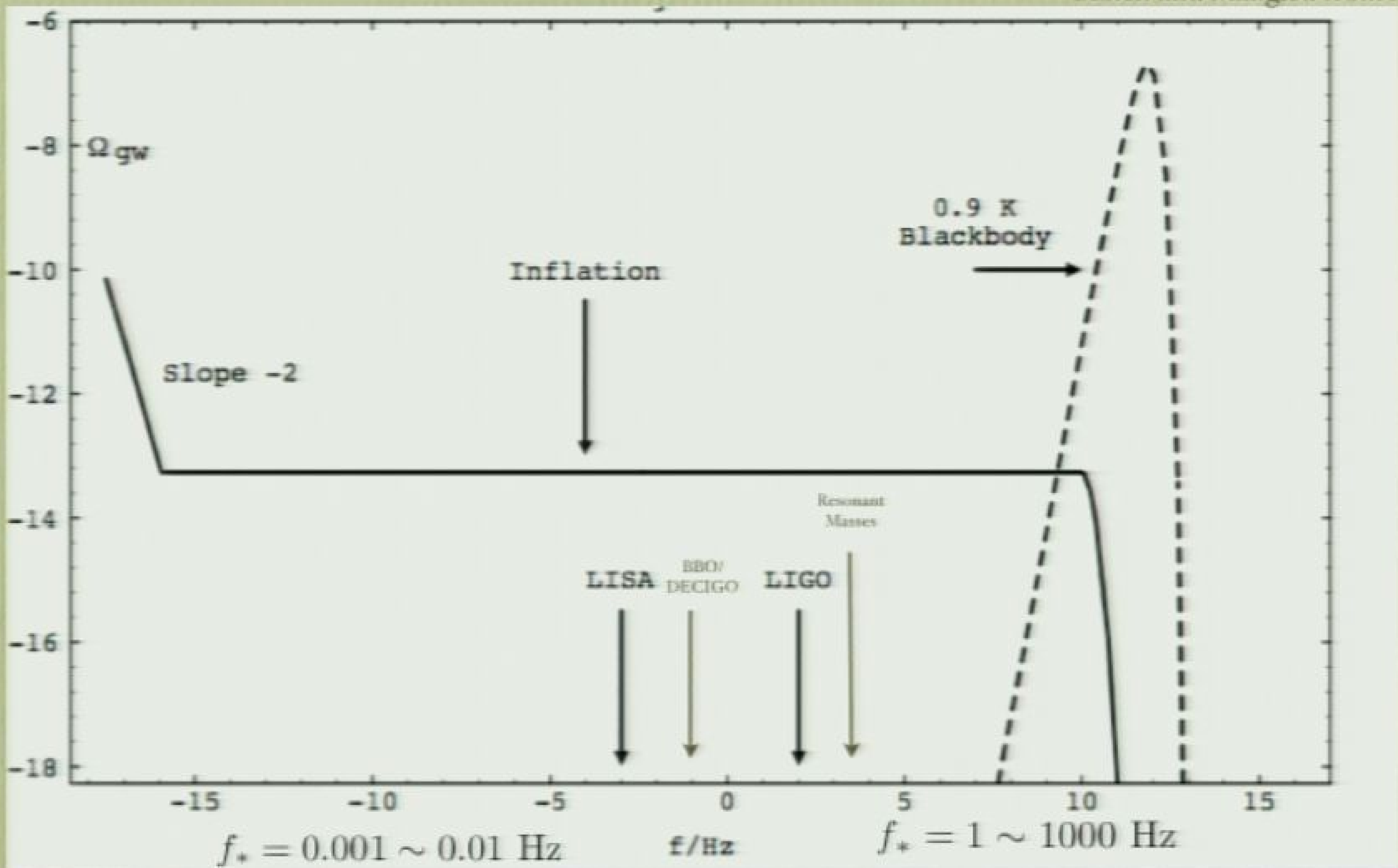


Given a fixed *strain*, it is generally *harder* to detect correlations of it at *higher frequencies*

GUT scale best case

The Frequency Range : an orientation

Stolen and Mangled from Allen (1996)



"Golden Range"

Unresolved Sources = Stochastic GW

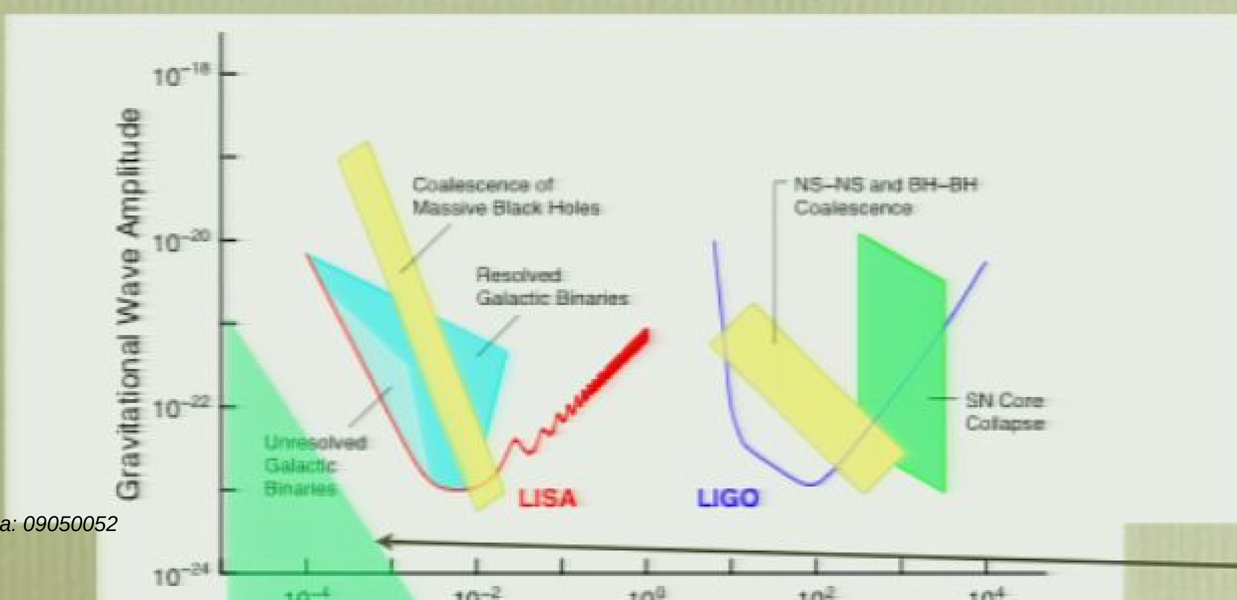
Stochastic : they "look" just like noise

Characterize them via *correlation functions*

Power Spectrum : $\langle h(f)h(f)\rangle$

Convention : *energy density* $\Omega_{gw} = \dfrac{\rho_{gw}}{\rho_g}$ $\rho_{gw} = \dfrac{M_p^2}{32\pi}\langle \dot{h}_{ij}\dot{h}^{ij}\rangle$

Fourier space $\langle h(f)h(f')\rangle = \dfrac{3H_0^2}{20\pi^2} f^{-3}\Omega_{gw}(f)\delta(f-f')$

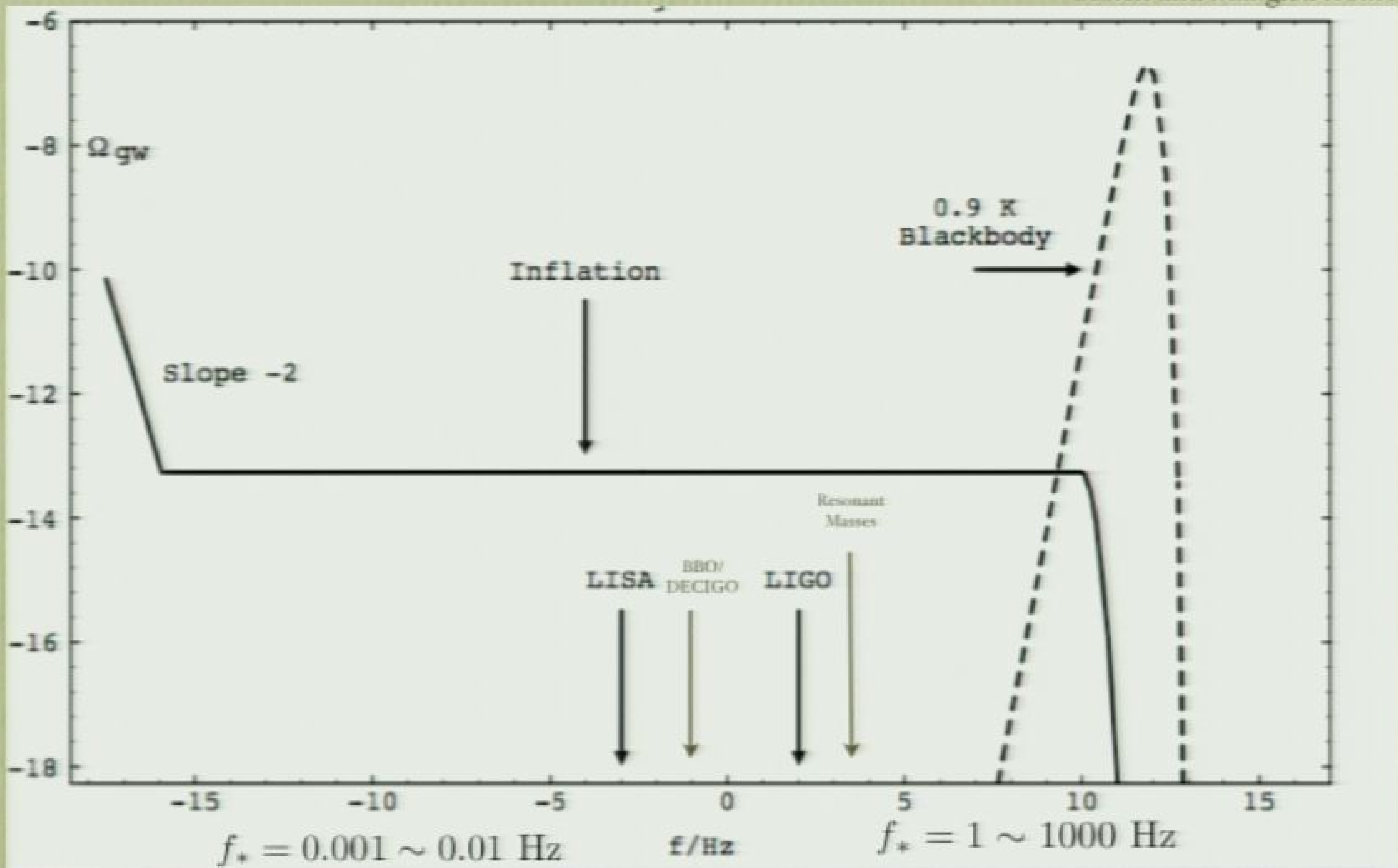


Given a fixed *strain*, it is generally *harder* to detect correlations of it at *higher frequencies*

GUT scale best case

The Frequency Range : an orientation

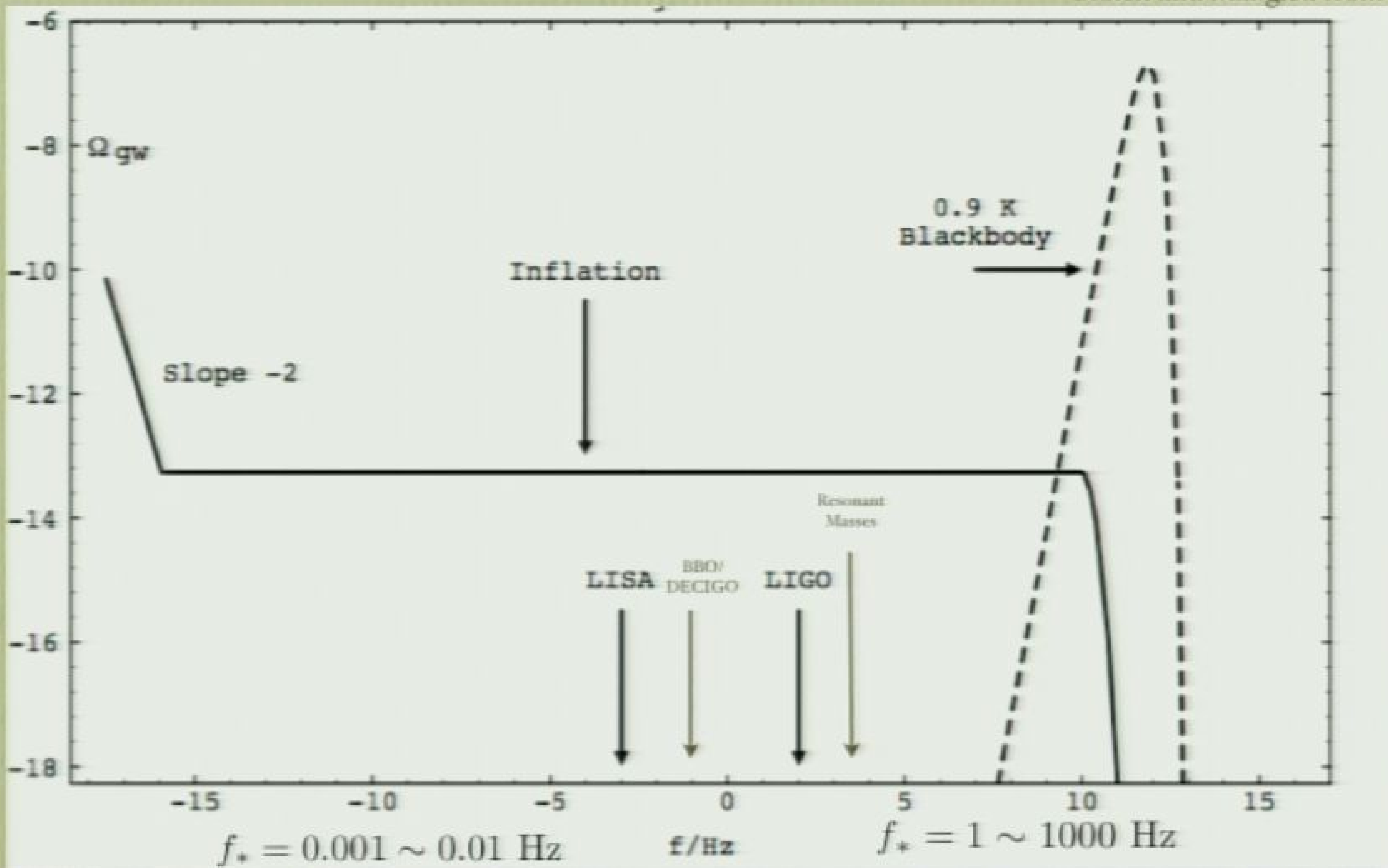
Stolen and Mangled from Allen (1996)



"Golden Range"

The Frequency Range : an orientation

Stolen and Mangled from Allen (1996)



Solar System Scales ⟷ Harder to construct small interferometers

(maybe new tech : AGIS?)

"Golden Range"

Harder to detect correlations at higher

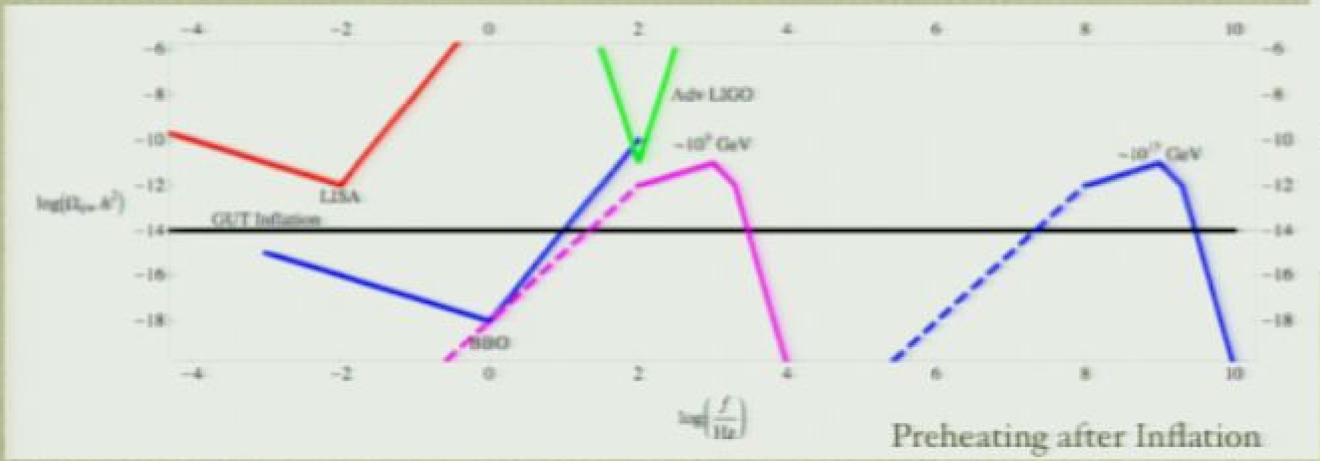
Question

If we detect the SGW *power spectrum*, how do we know what is its source?

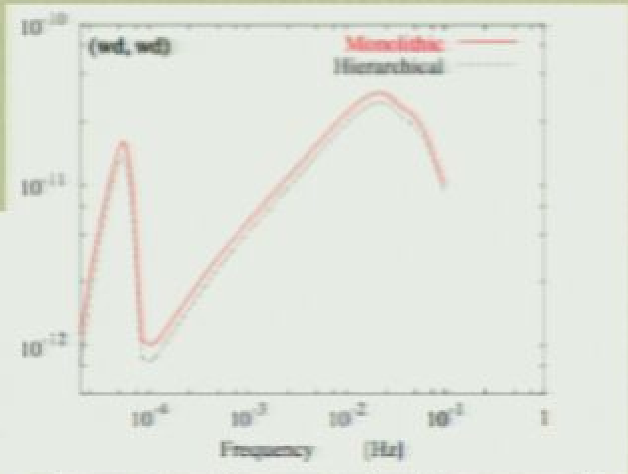
Question

If we detect the SGW *power spectrum*, how do we know what is its source?

- Distinguish by spectra :



Preheating after Inflation
Easther, Giblin Lim (2007)



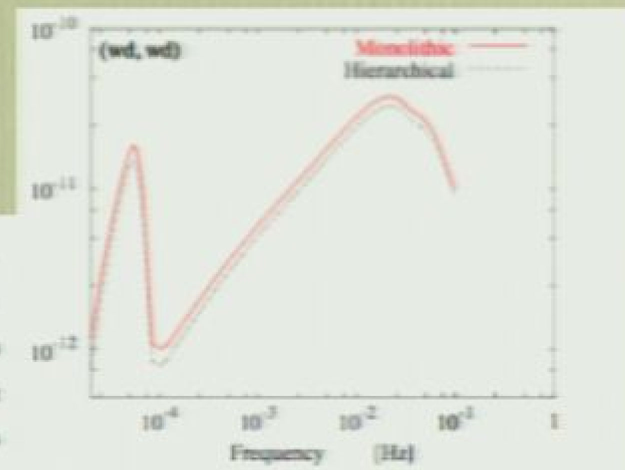
Unresolved Cosmological Compact Binaries
Schneider et al. (2000)

Question

If we detect the SGW *power spectrum*, how do we know what is its source?

- Distinguish by spectra :

Preheating after Inflation
Easther, Giblin Lim (2007)



Unresolved Cosmological Compact Binaries
Schneider et al. (2000)

- Distinguish by higher correlation functions :

“Non-gaussianities of SGW”

Small detector ranges (“pencil-dips” sensitivities)

Lesson from CMB : higher correlations are *extra information*.

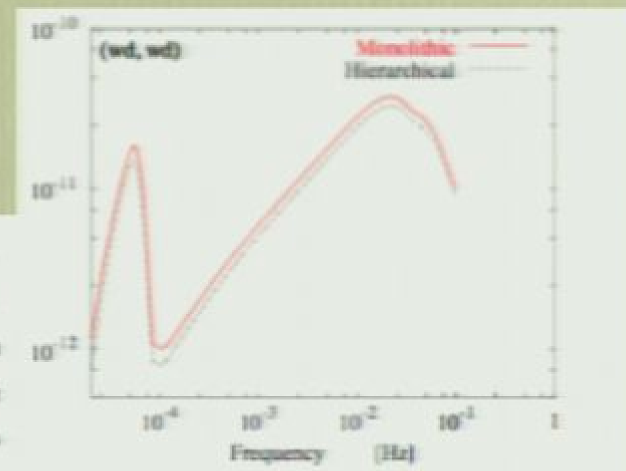
Question

If we detect the SGW *power spectrum*, how do we know what is its source?

- Distinguish by spectra :

Preheating after Inflation

Easther, Giblin Lim (2007)



Unresolved Cosmological Compact Binaries

Schneider et al. (2000)

- Distinguish by higher correlation functions :

"Non-gaussianities of SGW"

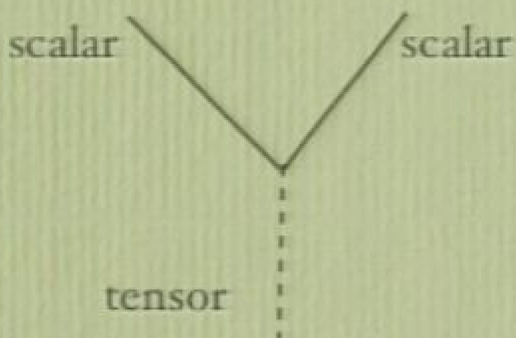
Small detector ranges ("pencil-dips" sensitivities)

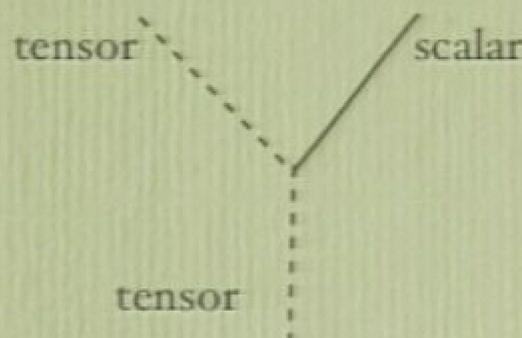
Lesson from CMB : higher correlations are *extra information*.

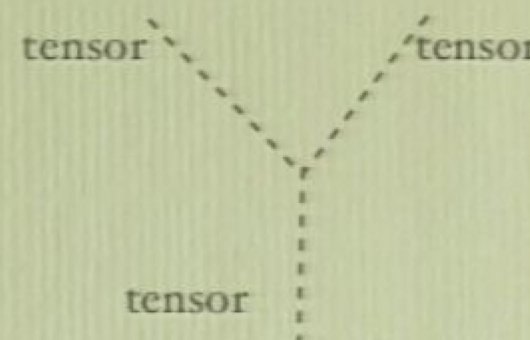
IS HE CRAZY?

What are 3-pt correlations?

Higher-order interactions







Gravitational Bremsstrahlung

$\langle \zeta\zeta h \rangle$ $\quad$ $\langle \zeta h h \rangle$ $\quad$ $\langle h h h \rangle$

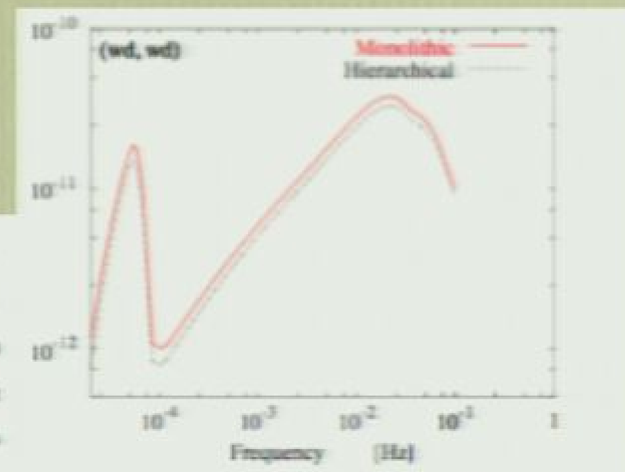
See Peter Adshead's talk

Question

If we detect the SGW *power spectrum*, how do we know what is its source?

- Distinguish by spectra :

Preheating after Inflation
Easther, Giblin Lim (2007)



Unresolved Cosmological Compact Binaries
Schneider et al. (2000)

- Distinguish by higher correlation functions :

“Non-gaussianities of SGW”

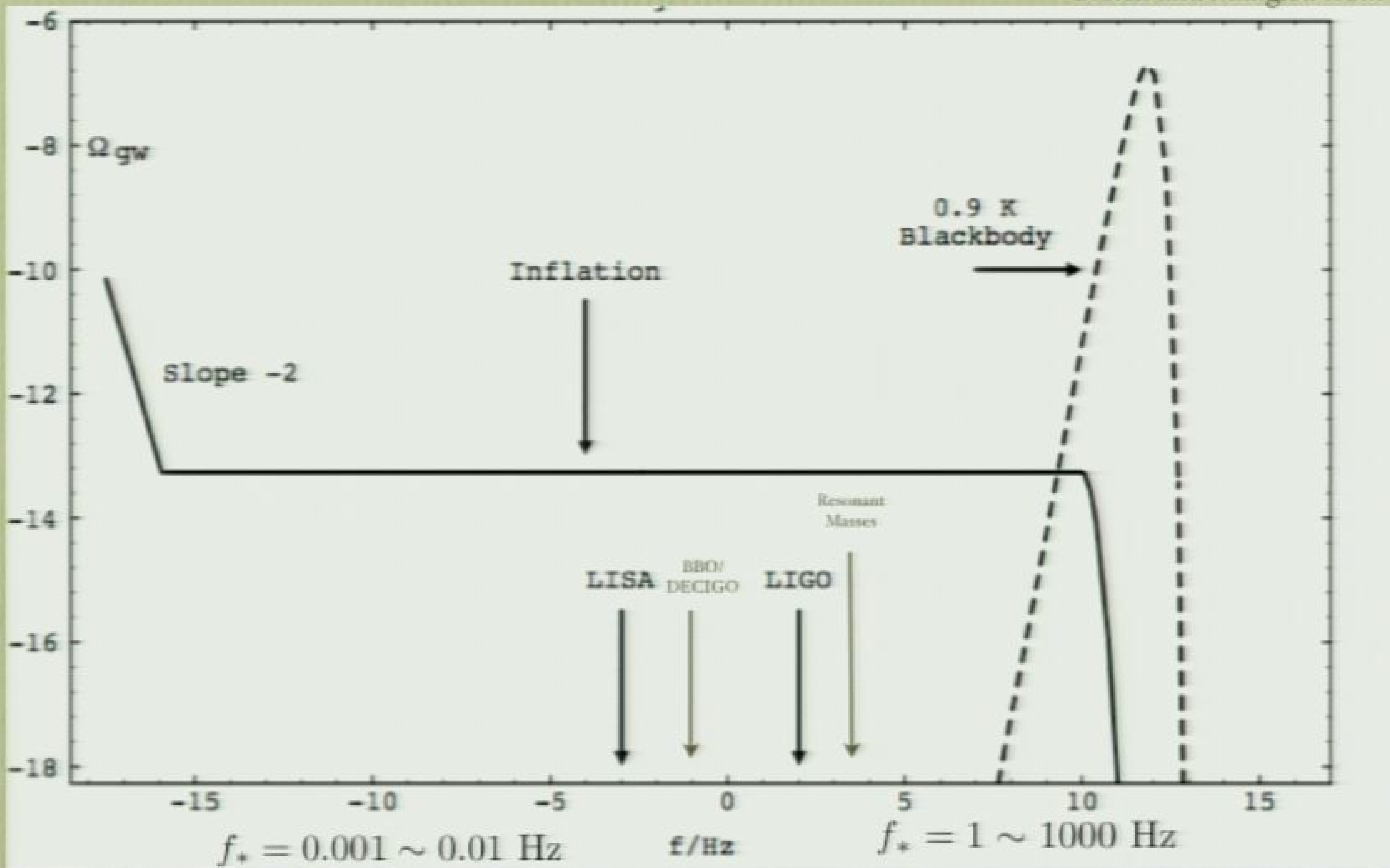
Small detector ranges (“pencil-dips” sensitivities)

Lesson from CMB : higher correlations are *extra information.*

IS HE CRAZY?

The Frequency Range : an orientation

Stolen and Mangled from Allen (1996)



Solar System Scales ⟷ Harder to construct small interferometers

(maybe new tech : AGIS?)

"Golden Range"

Harder to detect correlations at higher

Unresolved Sources = Stochastic GW

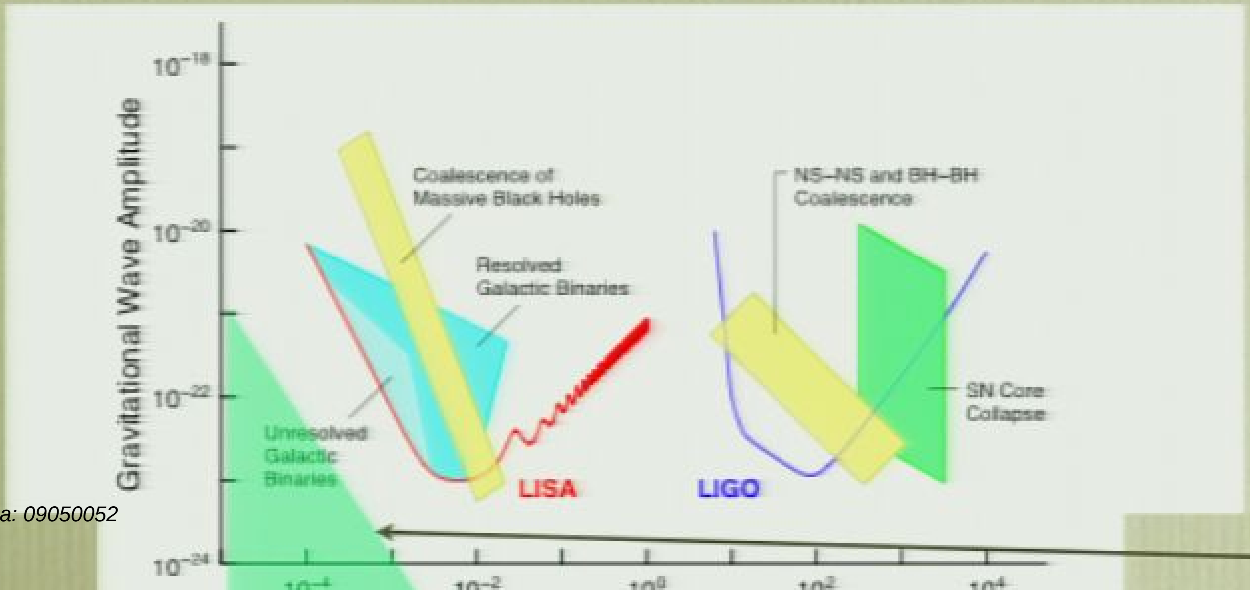
Stochastic : they "look" just like noise

Characterize them via *correlation functions*

Power Spectrum : $\langle h(f)h(f)\rangle$

Convention : *energy density* $\Omega_{gw} = \frac{\rho_{gw}}{\rho_g}$ $\rho_{gw} = \frac{M_p^2}{32\pi}\langle \dot{h}_{ij}\dot{h}^{ij}\rangle$

Fourier space $\langle h(f)h(f')\rangle = \frac{3H_0^2}{20\pi^2} f^{-3}\Omega_{gw}(f)\delta(f-f')$

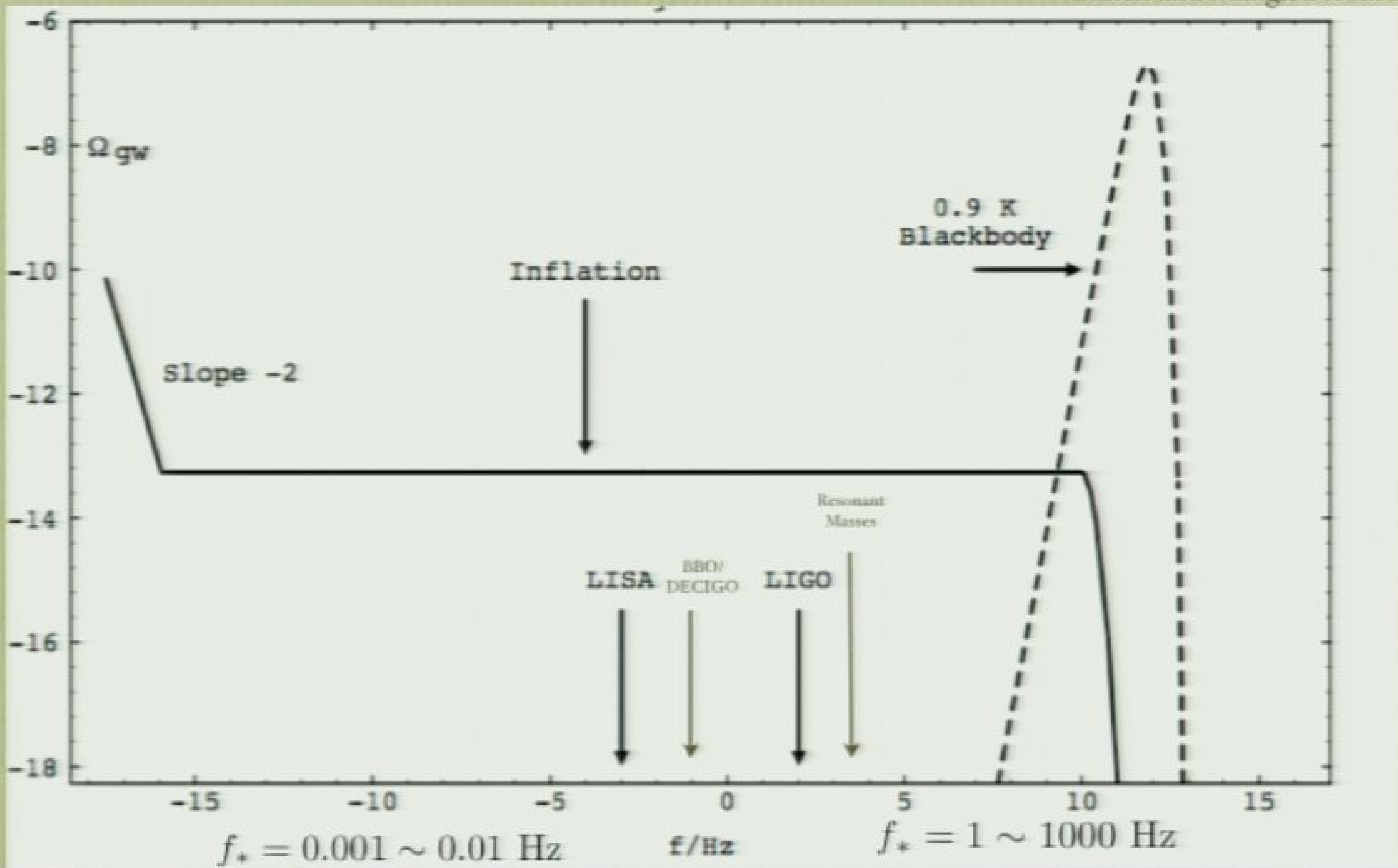


Given a fixed *strain*, it is generally *harder* to detect correlations of it at *higher frequencies*

GUT scale best case

The Frequency Range : an orientation

Stolen and Mangled from Allen (1996)



Solar System Scales ⟷ Harder to construct small interferometers

(maybe new tech : AGIS?)

“Golden Range”

Harder to detect correlations at higher

Question

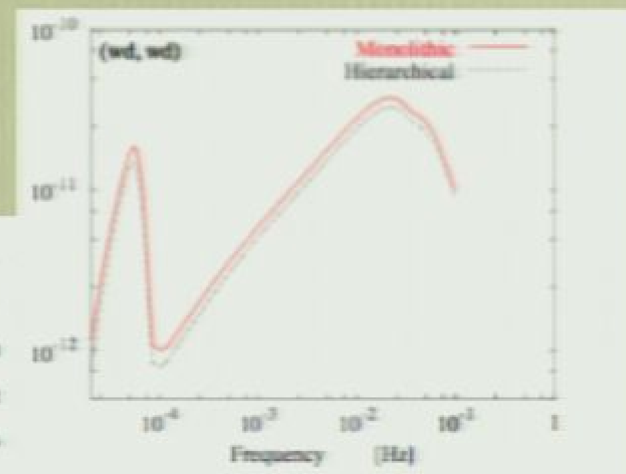
If we detect the SGW *power spectrum*, how do we know what is its source?

Question

If we detect the SGW *power spectrum*, how do we know what is its source?

- Distinguish by spectra :

Preheating after Inflation
Easther, Giblin Lim (2007)



Unresolved Cosmological Compact Binaries
Schneider et al. (2000)

- Distinguish by higher correlation functions :

  "Non-gaussianities of SGW"

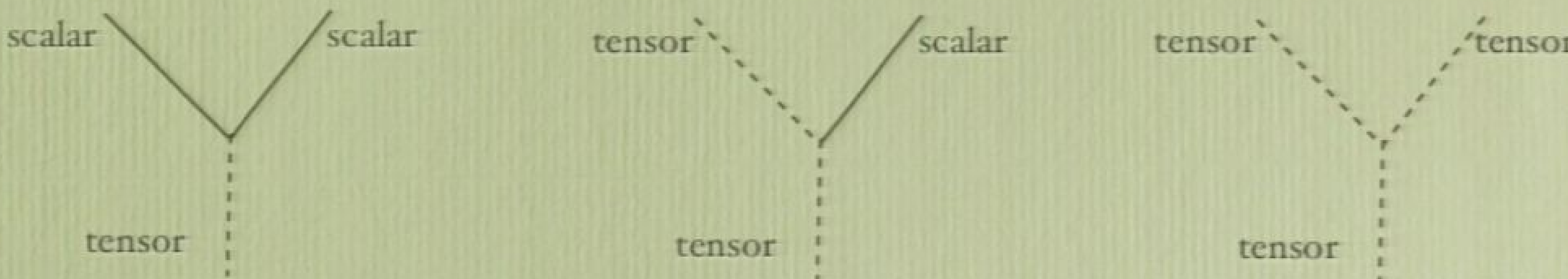
Small detector ranges ("pencil-dips" sensitivities)

Lesson from CMB : higher correlations are *extra information*.

IS HE CRAZY?

What are 3-pt correlations?

Higher-order interactions



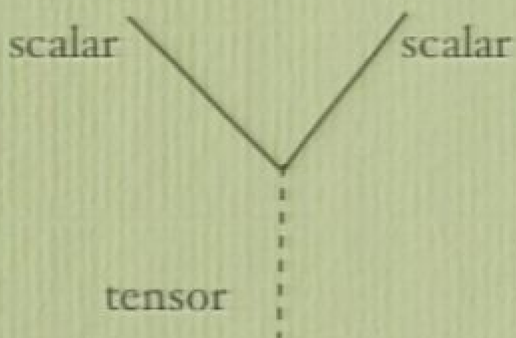
Gravitational Bremsstrahlung

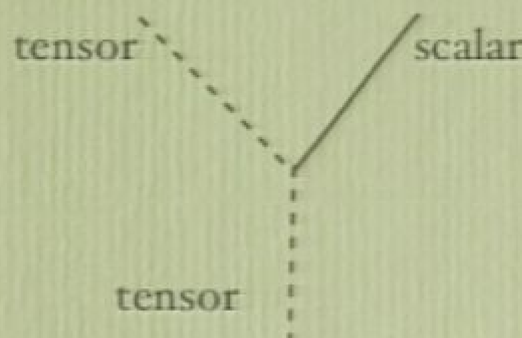
$\langle \zeta\zeta h \rangle$ $\langle \zeta h h \rangle$ $\langle hhh \rangle$

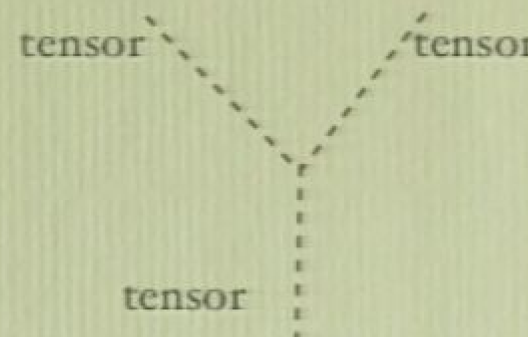
See Peter Adshead's talk

What are 3-pt correlations?

Higher-order interactions







Gravitational Bremsstrahlung

$\langle \zeta\zeta h \rangle$

See Peter Adshead's talk

$\langle \zeta h h \rangle$

e.g. : BBT crosscorrelation?

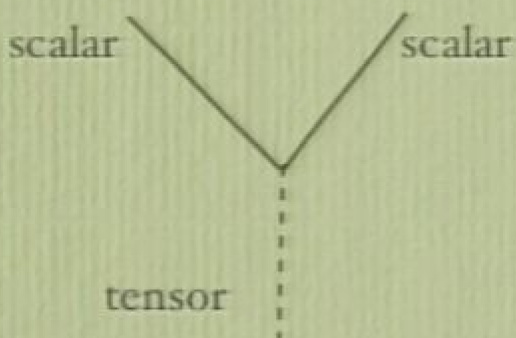
SGW-21 cm lines?

(Work in progress)

$\langle h h h \rangle$

What are 3-pt correlations?

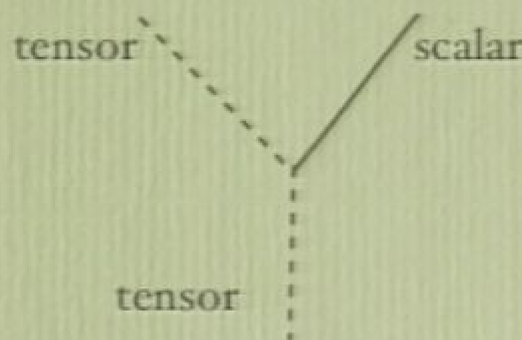
Higher-order interactions



Gravitational Bremsstrahlung

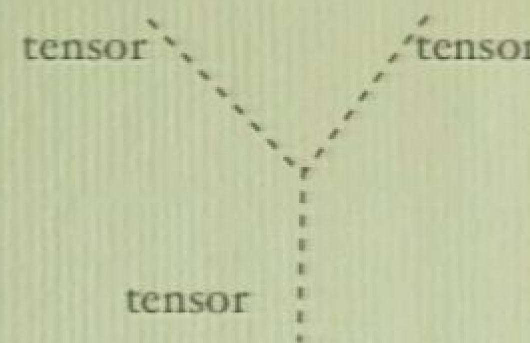
$\langle \zeta\zeta h \rangle$

See Peter Adshead's talk



$\langle \zeta h h \rangle$

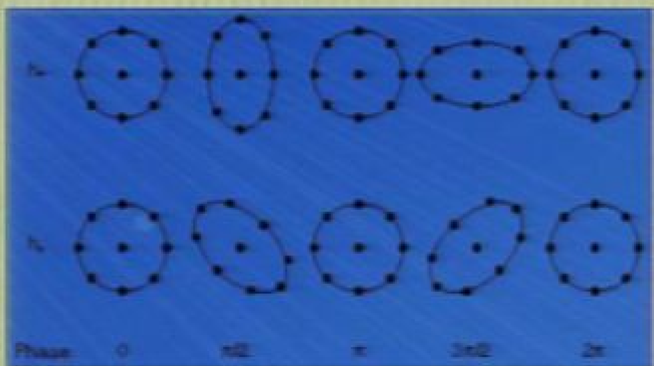
e.g. : BBT crosscorrelation?
SGW-21 cm lines?
(Work in progress)



$\langle h h h \rangle$

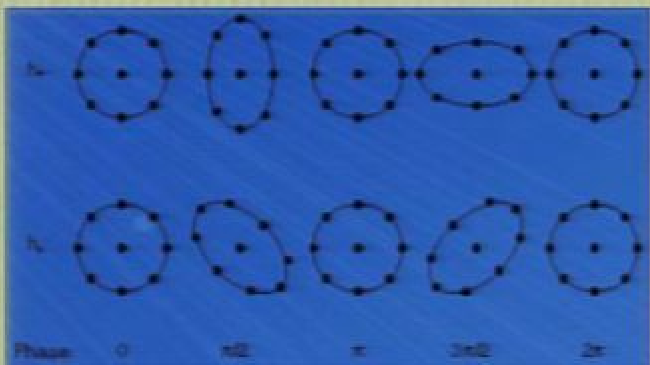
"Non-Gaussianities in SGW"

How do we detect SGW power spectrum?

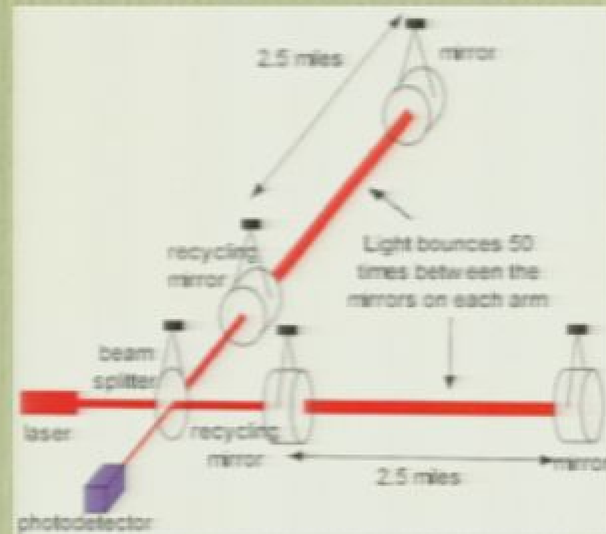


$$h_{ij}(\mathbf{x},t) = \int d\hat{\mathbf{\Omega}} \int df \sum_{A} \tilde{h}(\hat{\mathbf{\Omega}}, f) e^{-2\pi i f(t-\hat{\mathbf{\Omega}}\cdot\mathbf{x})} e_{ij}^{A}(\hat{\mathbf{\Omega}})$$

How do we detect SGW power spectrum?



assume isotropy →

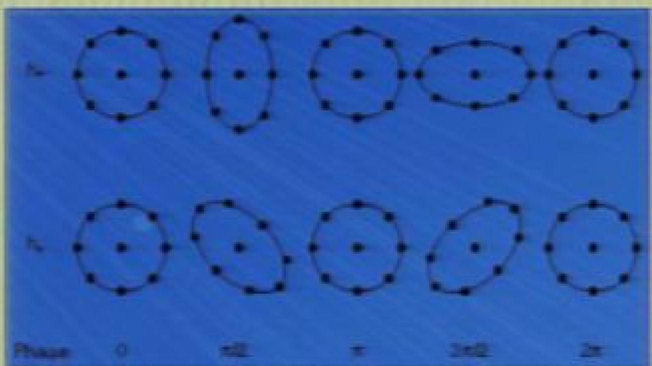


$$F_A(\hat{\Omega}) = D_{ij} e^{ij}(\hat{\Omega})_A$$

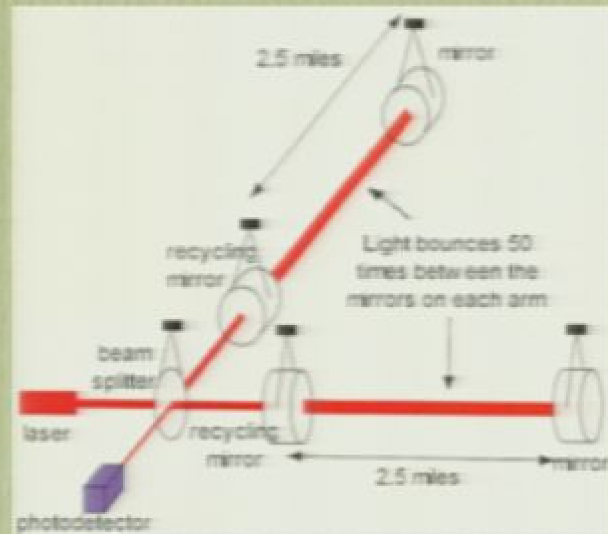
Detector Pattern Tensor

$$h_{ij}(\mathbf{x},t) = \int d\hat{\mathbf{\Omega}} \int df \sum_A \tilde{h}(\hat{\mathbf{\Omega}}, f) e^{-2\pi i f(t - \hat{\mathbf{\Omega}}\cdot\mathbf{x})} e^A_{ij}(\hat{\mathbf{\Omega}})$$

How do we detect SGW power spectrum?



assume isotropy



$$F_A(\hat{\Omega}) = D_{ij} e^{ij}(\hat{\Omega})_A$$

Detector Pattern Tensor

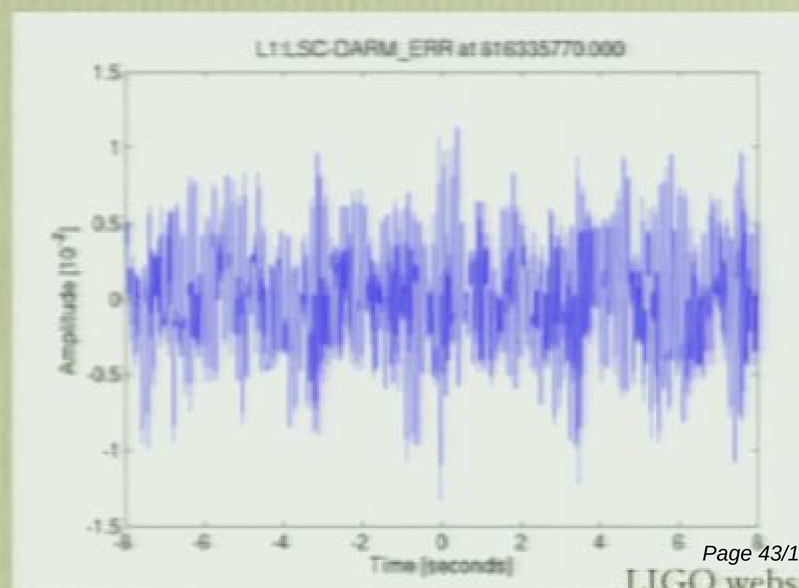
$$h_{ij}(\mathbf{x}, t) = \int d\hat{\mathbf{\Omega}} \int df \sum_A \tilde{h}(\hat{\mathbf{\Omega}}, f) e^{-2\pi i f(t - \hat{\mathbf{\Omega}}\cdot\mathbf{x})} e^A_{ij}(\hat{\mathbf{\Omega}})$$

Time Series $S(t)$

Looks like detector noise!

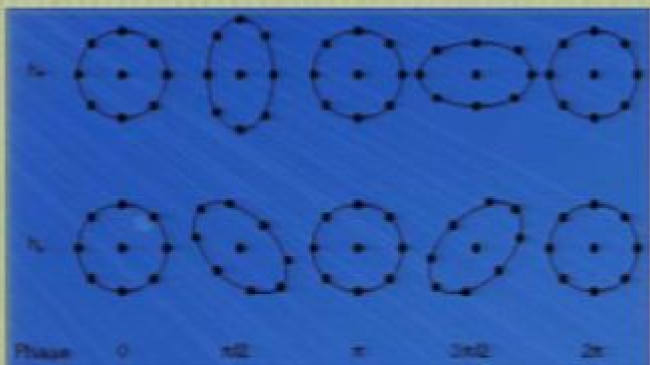
$$S(t) = s(t) + n(t) \qquad n(t) \gg s(t)$$

noise dominated

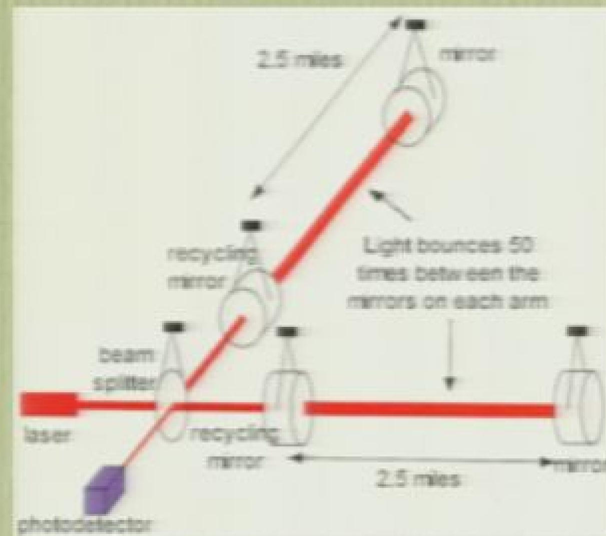


LIGO website

How do we detect SGW power spectrum?



assume isotropy

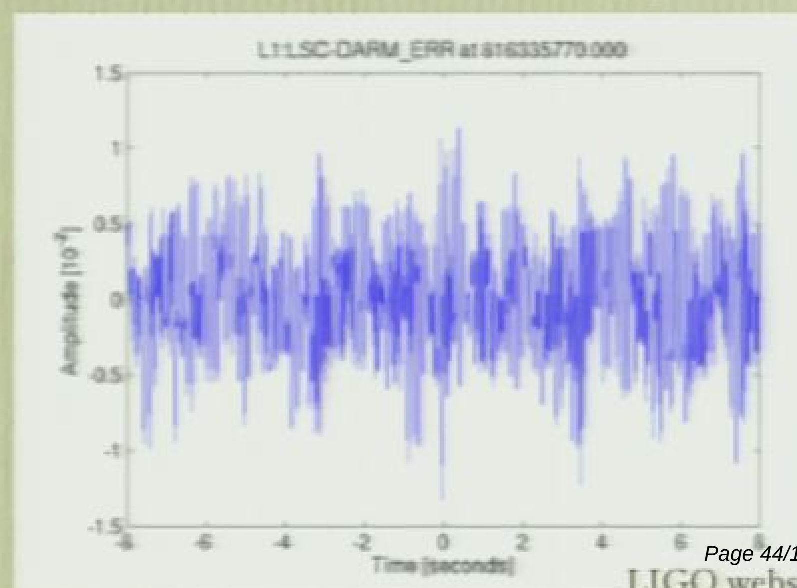


$$F_A(\hat{\Omega}) = D_{ij} e^{ij}(\hat{\Omega})_A$$

Detector Pattern Tensor

$$h_{ij}(\mathbf{x},t) = \int d\hat{\mathbf{\Omega}} \int df \sum_A \tilde{h}(\hat{\mathbf{\Omega}}, f) e^{-2\pi i f(t-\hat{\mathbf{\Omega}}\cdot\mathbf{x})} e^A_{ij}(\hat{\mathbf{\Omega}})$$

Time Series $S(t)$



LIGO website

Looks like detector noise!

$$S(t) = s(t) + n(t) \qquad n(t) \gg s(t)$$

noise dominated

With *one* detector

Filter

$$\int df P^2(f) e^{-2\pi i f t} = \int dt' S(t) S(t') W(t-t')$$

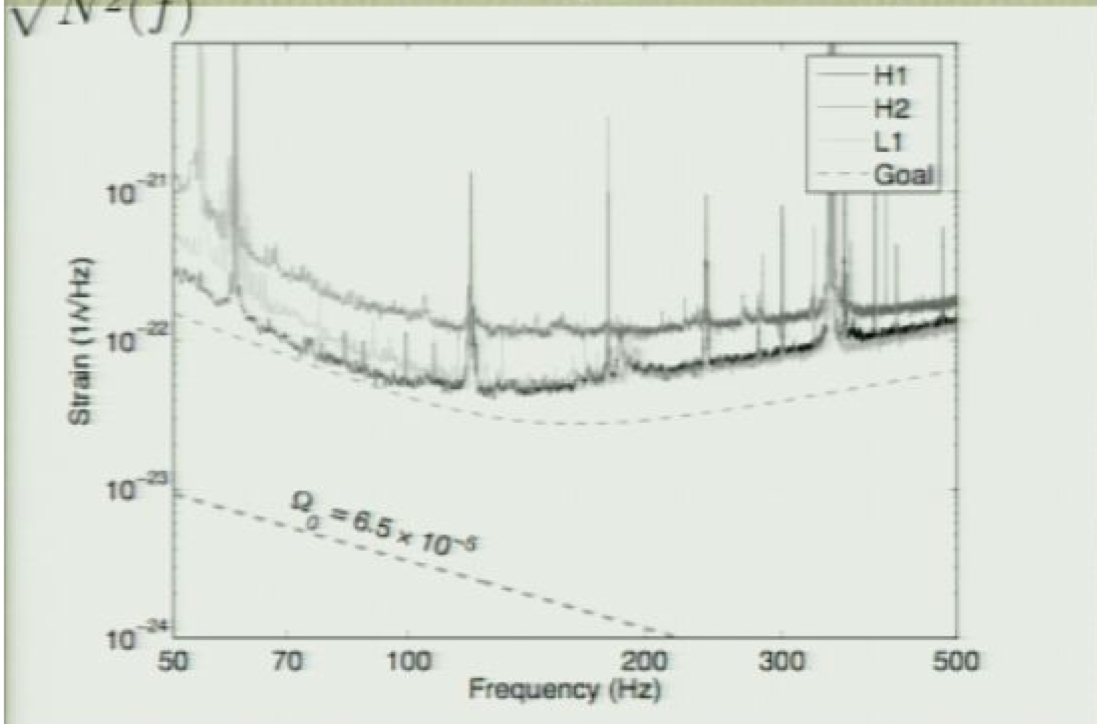
Noise Power

$$\int df N^2(f) e^{-2\pi i f t} = \int dt' n(t) n(t') W(t-t')$$

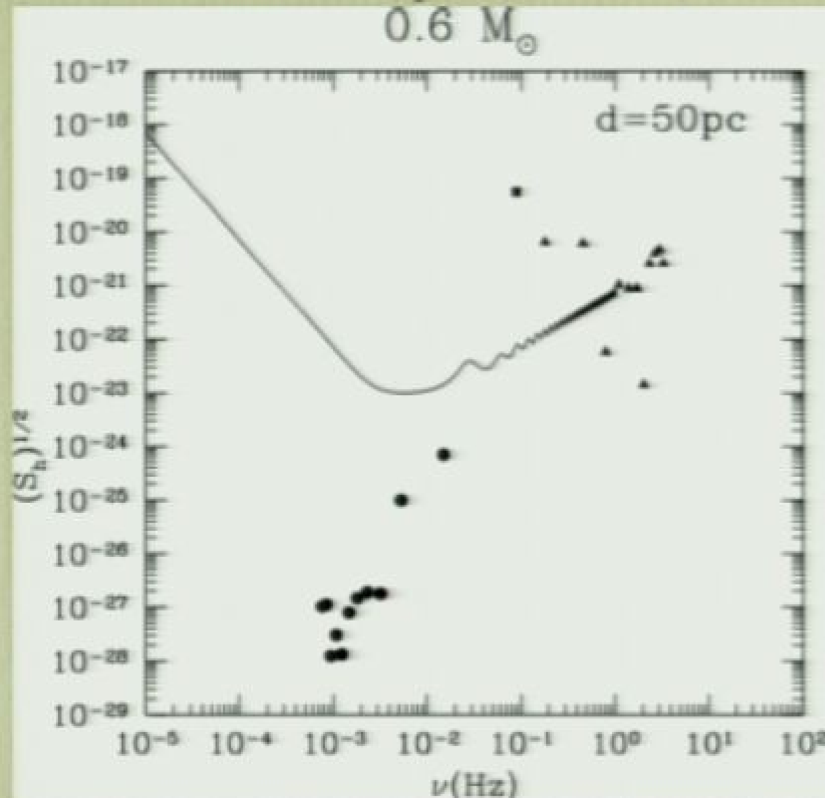
Noise Properties of Detectors

LIGO noise spectrum $N(f)$

$\sqrt{N^2(f)}$



LISA noise spectrum $N(f)$



Sources : Thermal, Shot, radiation pressure, seismic, etc.

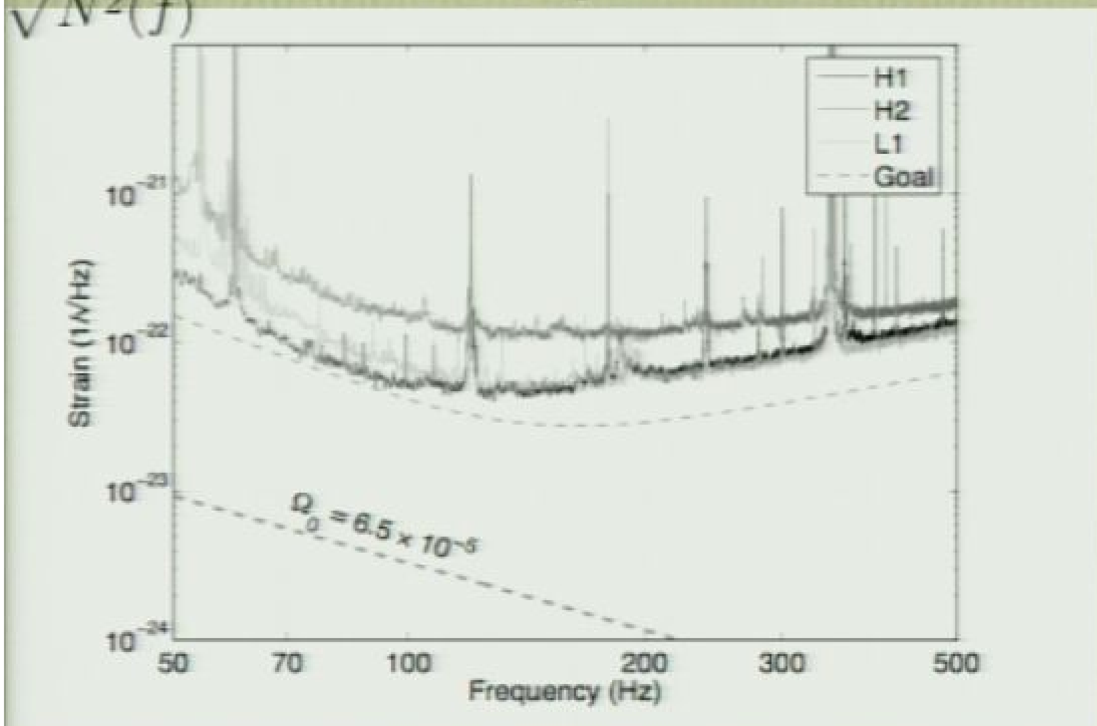
Same $N(f)$ at lower f = better

"Pencil-dips" structure : interferometer arm lengths

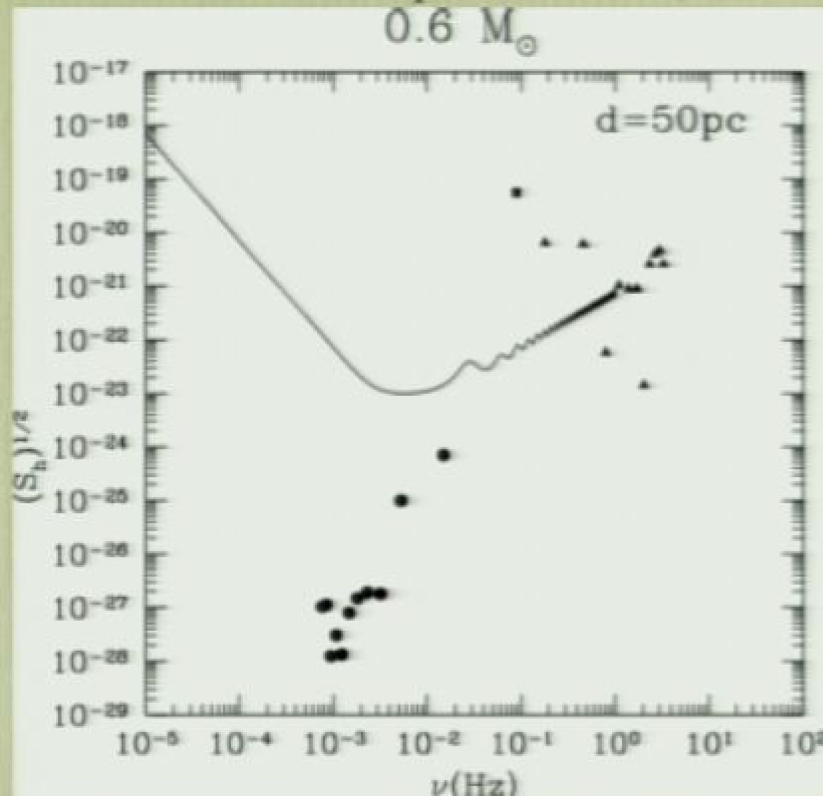
Noise Properties of Detectors

LIGO noise spectrum $N(f)$

$\sqrt{N^2(f)}$



LISA noise spectrum $N(f)$



Sources : Thermal, Shot, radiation pressure, seismic, etc.

Same $N(f)$ at lower f = better

"Pencil-dips" structure : interferometer arm lengths

Almost Gaussian : 3 pt vanishes

$$\langle n(t)n(t)n(t)\rangle \approx 0$$

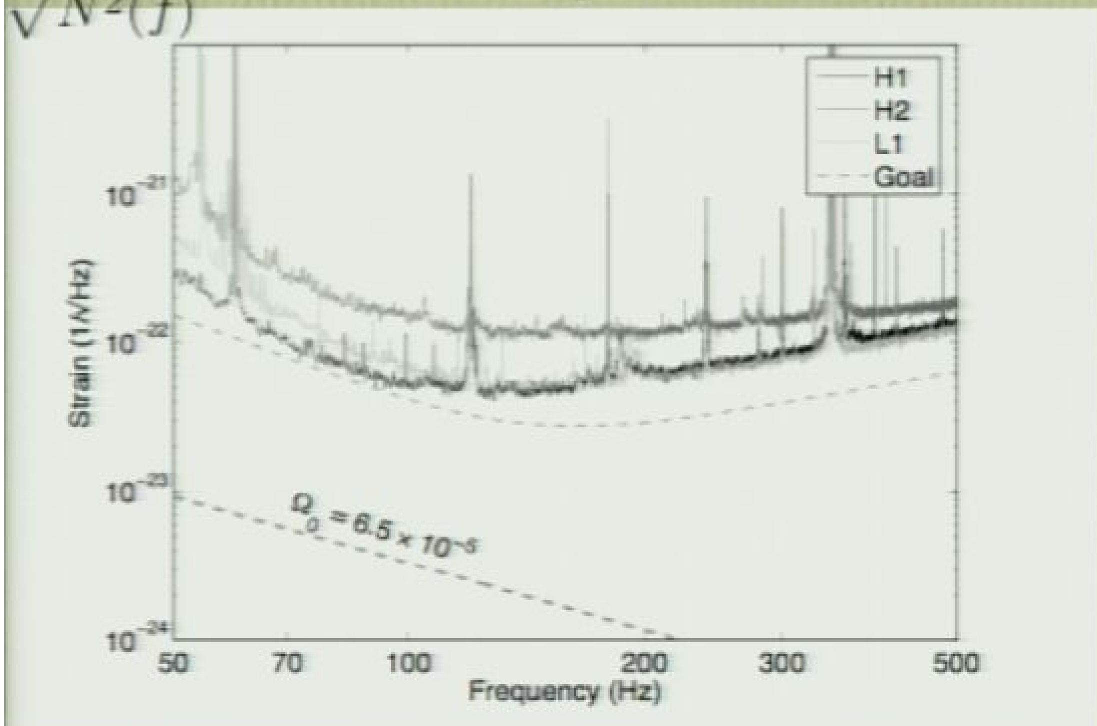
Localized : cross-correlated noise is smaller

$$\langle n_1(t)n_2(t)\rangle \ll \langle n_1(t)n_1(t)\rangle$$

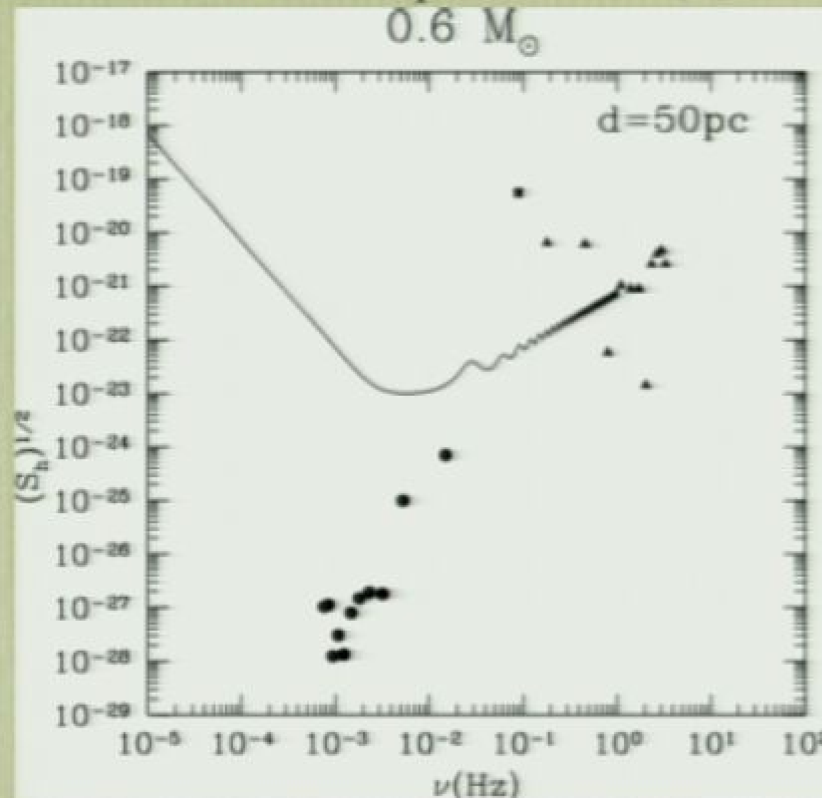
Noise Properties of Detectors

LIGO noise spectrum $N(f)$

$\sqrt{N^2(f)}$



LISA noise spectrum $N(f)$



Sources : Thermal, Shot, radiation pressure, seismic, etc.

Same $N(f)$ at lower f = better

"Pencil-dips" structure : interferometer arm lengths

Almost Gaussian : 3 pt vanishes

$$\langle n(t)n(t)n(t)\rangle \approx 0$$

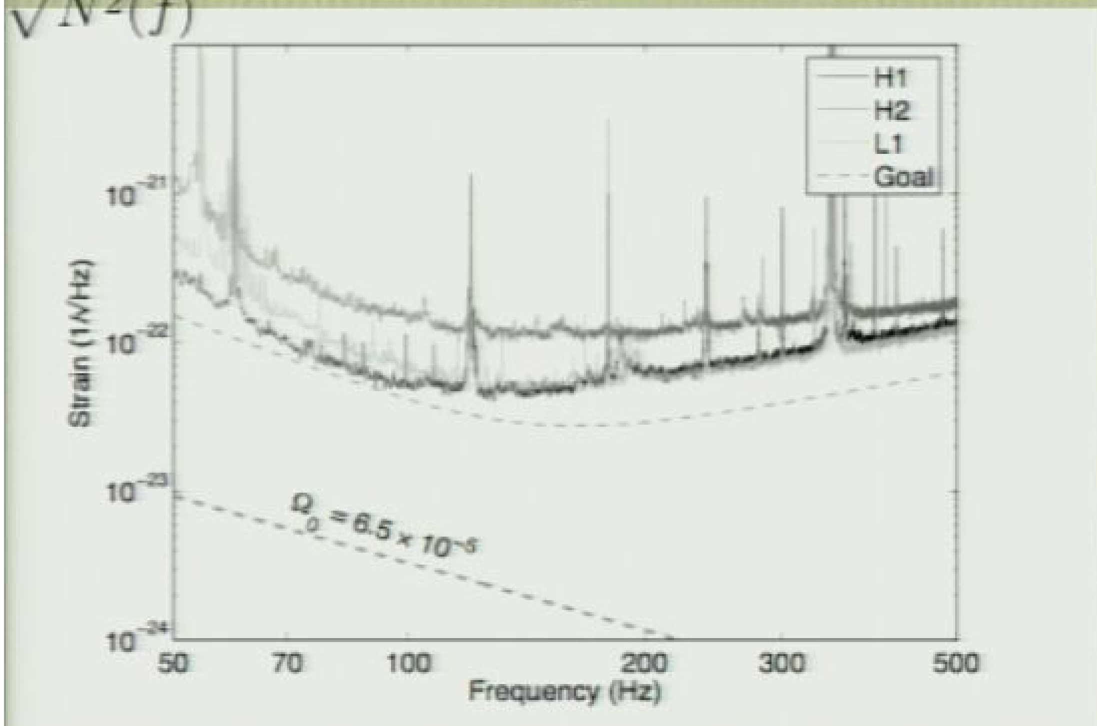
Localized : cross-correlated noise is smaller

$$\langle n_1(t)n_2(t)\rangle \ll \langle n_1(t)n_1(t)\rangle$$

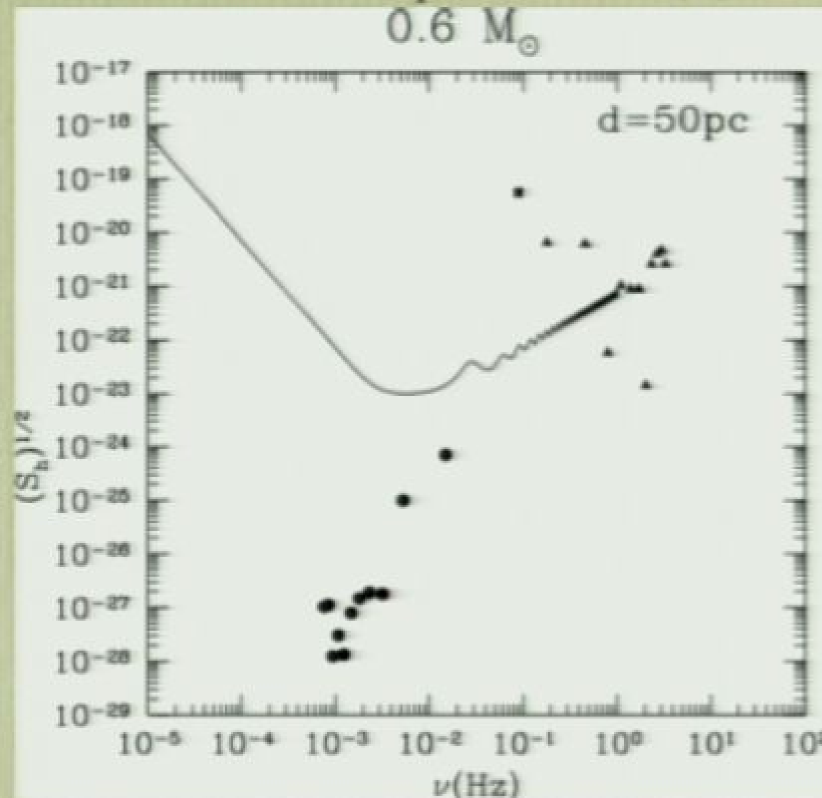
Noise Properties of Detectors

LIGO noise spectrum $N(f)$

$\sqrt{N^2(f)}$



LISA noise spectrum $N(f)$



Sources : Thermal, Shot, radiation pressure, seismic, etc.

Same $N(f)$ at lower f = better

"Pencil-dips" structure : interferometer arm lengths

Almost Gaussian : 3 pt vanishes

$$\langle n(t)n(t)n(t)\rangle \approx 0$$

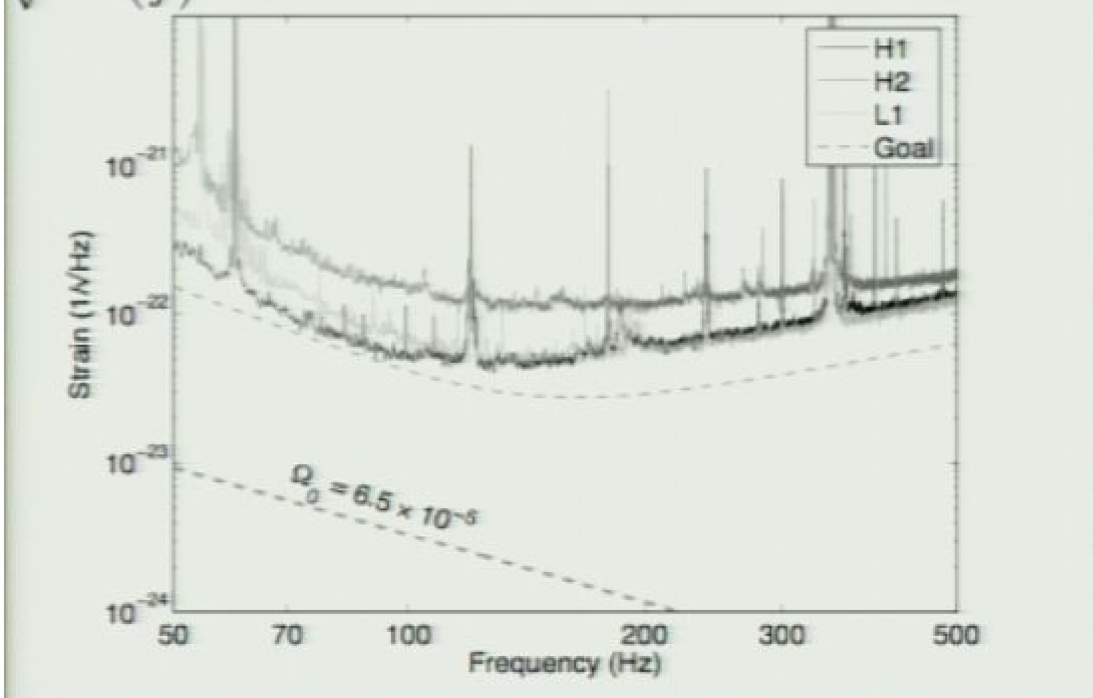
Localized : cross-correlated noise is smaller

$$\langle n_1(t)n_2(t)\rangle \ll \langle n_1(t)n_1(t)\rangle$$

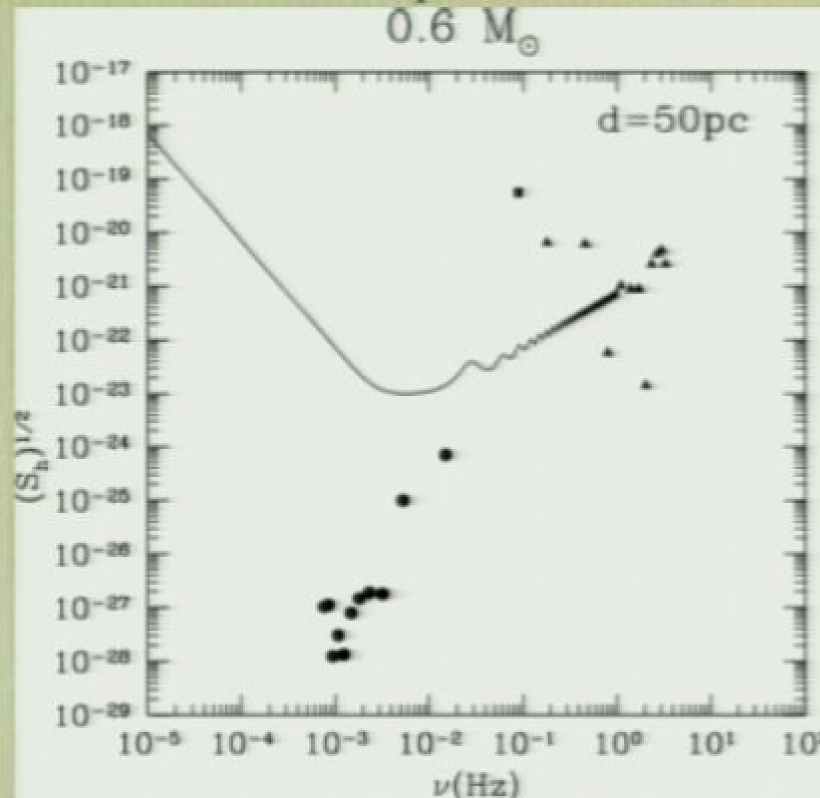
Noise Properties of Detectors

LIGO noise spectrum $N(f)$

$\sqrt{N^2(f)}$



LISA noise spectrum $N(f)$



Sources : Thermal, Shot, radiation pressure, seismic, etc.

Same $N(f)$ at lower f = better

"Pencil-dips" structure : interferometer arm lengths

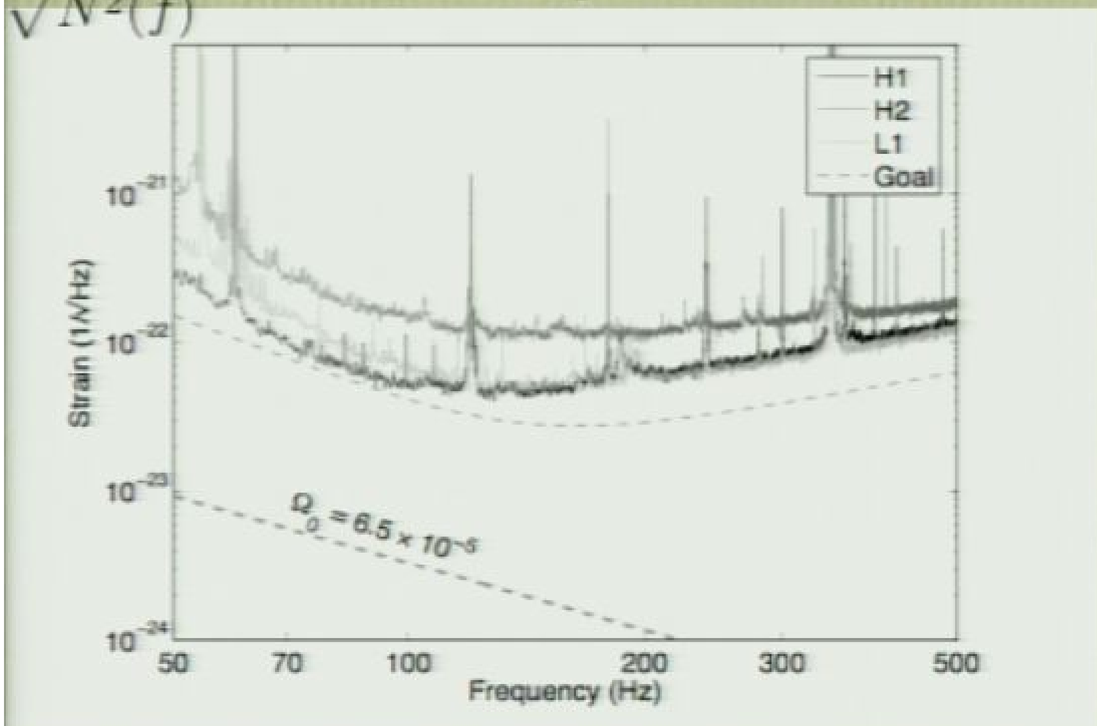
Almost Gaussian : 3 pt vanishes

$$\langle n(t)n(t)n(t)\rangle \approx 0$$

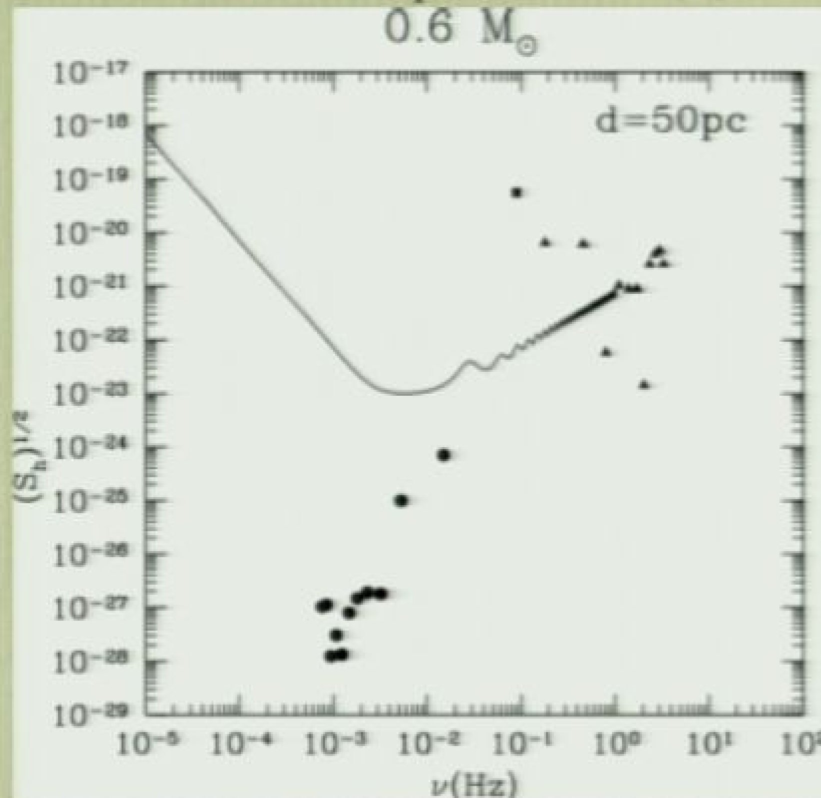
Noise Properties of Detectors

LIGO noise spectrum $N(f)$

$\sqrt{N^2(f)}$



LISA noise spectrum $N(f)$



Sources : Thermal, Shot, radiation pressure, seismic, etc.

Same $N(f)$ at lower f = better

"Pencil-dips" structure : interferometer arm lengths

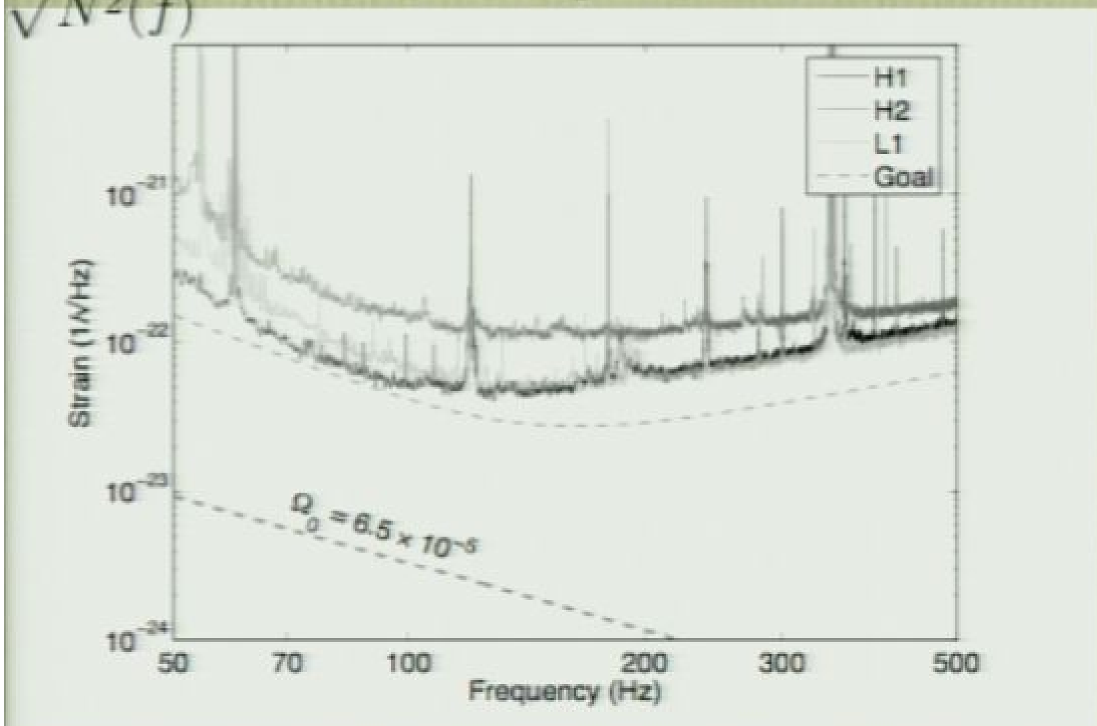
Almost Gaussian : 3 pt vanishes

$$\langle n(t)n(t)n(t)\rangle \approx 0$$

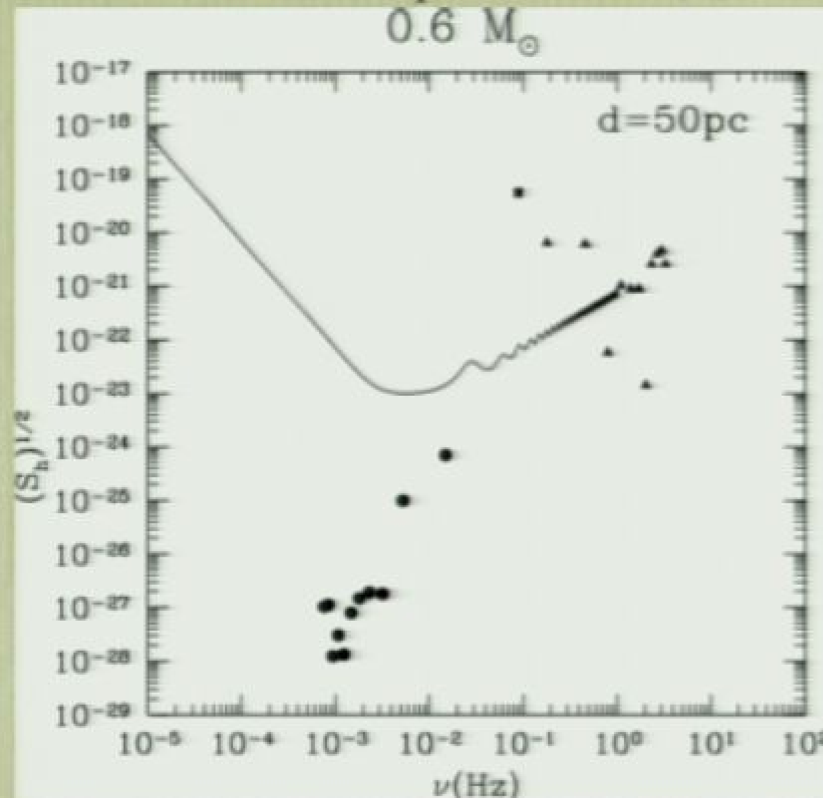
Noise Properties of Detectors

LIGO noise spectrum $N(f)$

$\sqrt{N^2(f)}$



LISA noise spectrum $N(f)$



Sources : Thermal, Shot, radiation pressure, seismic, etc.

Same $N(f)$ at lower f = better

"Pencil-dips" structure : interferometer arm lengths

Almost Gaussian : 3 pt vanishes

$$\langle n(t)n(t)n(t)\rangle \approx 0$$

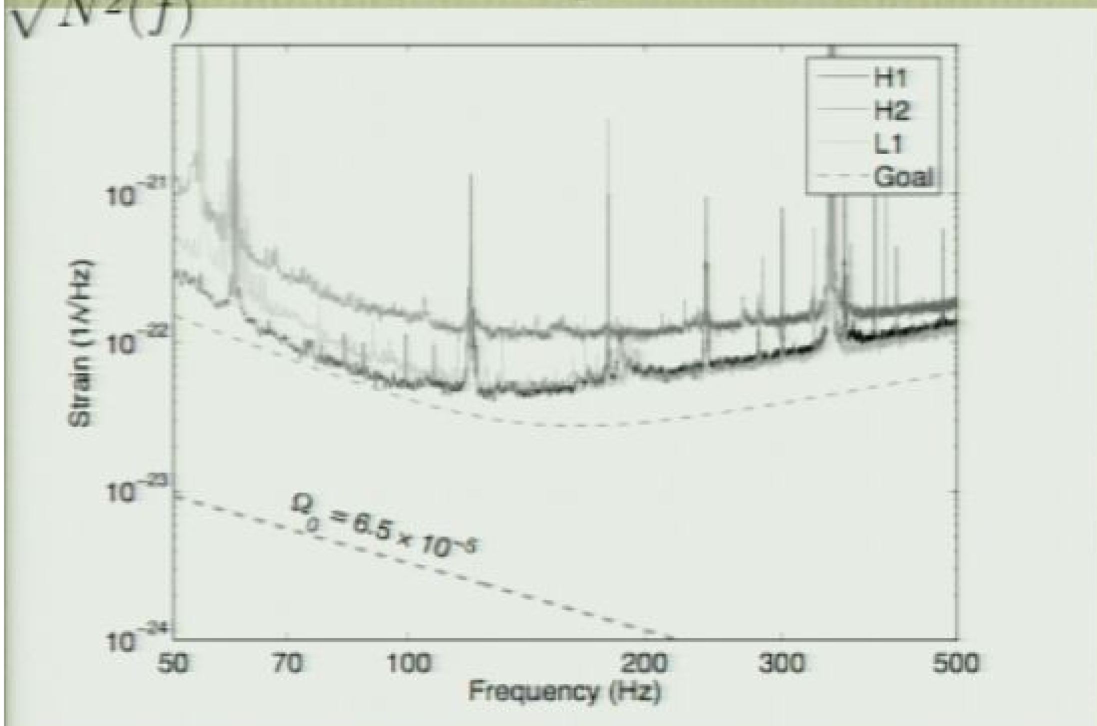
Localized : cross-correlated noise is smaller

$$\langle n_1(t)n_2(t)\rangle \ll \langle n_1(t)n_1(t)\rangle$$

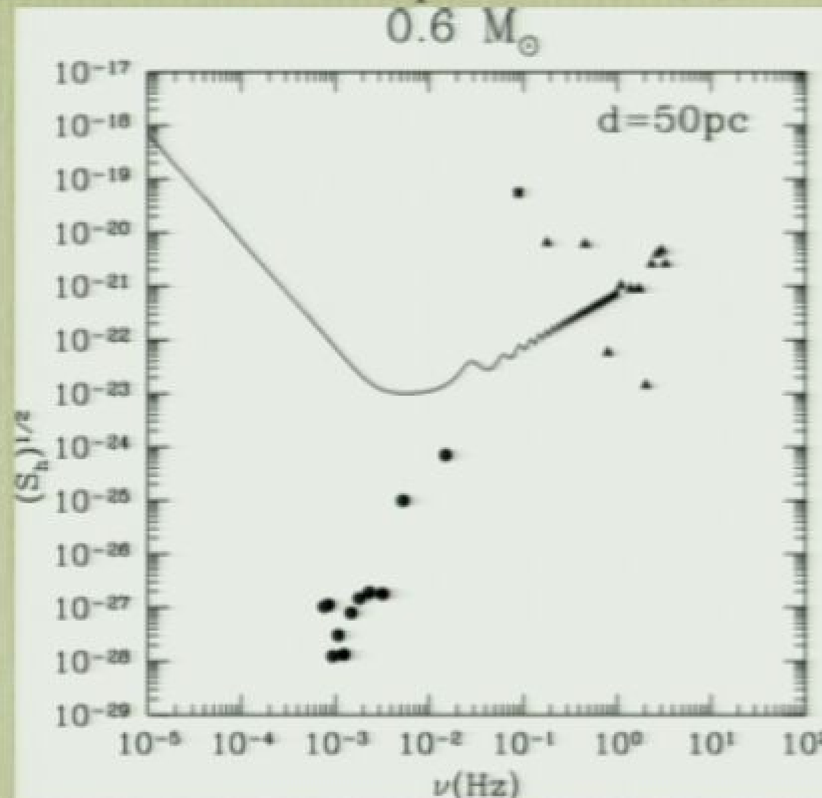
Noise Properties of Detectors

LIGO noise spectrum $N(f)$

$\sqrt{N^2(f)}$



LISA noise spectrum $N(f)$



Sources : Thermal, Shot, radiation pressure, seismic, etc.

Same $N(f)$ at lower f = better

"Pencil-dips" structure : interferometer arm lengths

Almost Gaussian : 3 pt vanishes

$$\langle n(t)n(t)n(t)\rangle \approx 0$$

Localized : cross-correlated noise is smaller

$$\langle n_1(t)n_2(t)\rangle \ll \langle n_1(t)n_1(t)\rangle$$

Power spectrum : cross-correlating 2 detectors

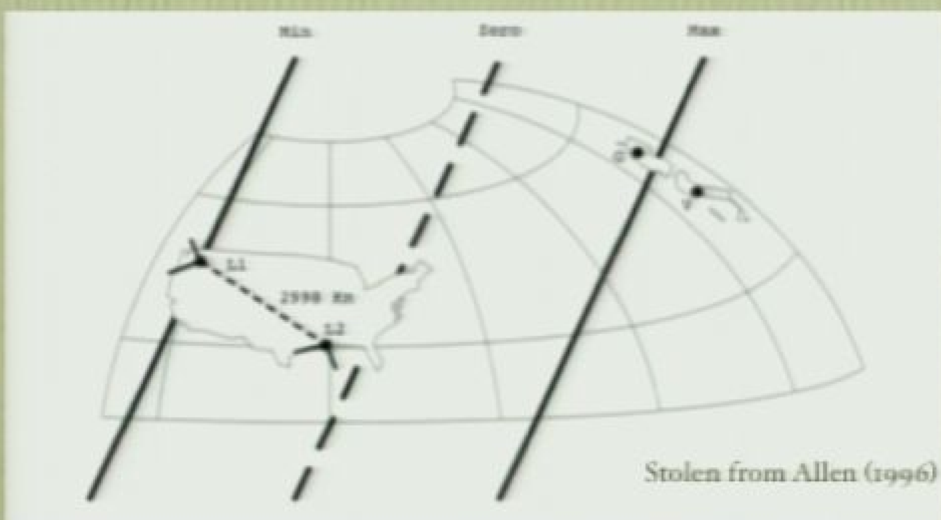
- Reminder : one detector = SGW > noise
- Two detectors = *cross-correlate* to reduce noise

$$\langle S_1(t)S_2(t)\rangle = \langle s_1(t)s_2(t)\rangle + \langle n_1(t)n_2(t)\rangle + ...$$

Power spectrum : cross-correlating 2 detectors

- Reminder : one detector = SGW > noise
- Two detectors = *cross-correlate* to reduce noise

$$\langle S_1(t)S_2(t)\rangle = \langle s_1(t)s_2(t)\rangle + \langle n_1(t)n_2(t)\rangle + ...$$



Stolen from Allen (1996)

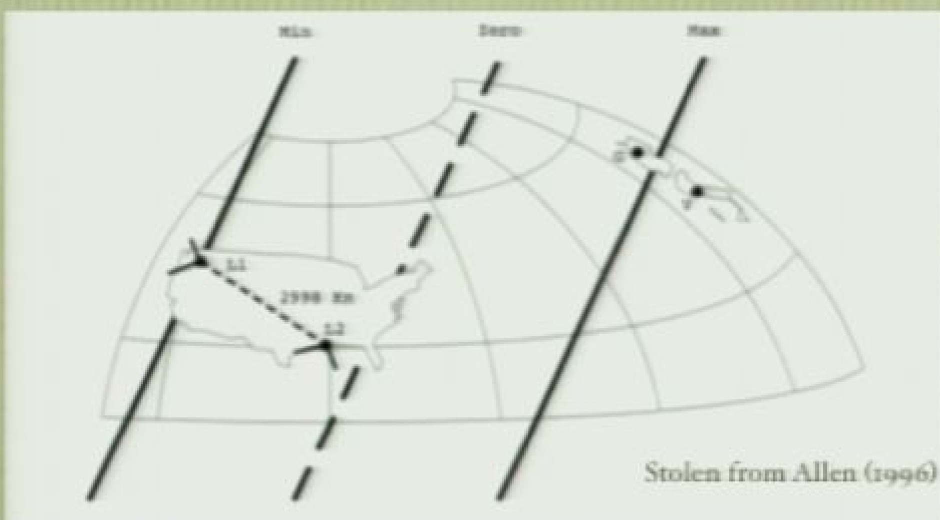
$$\langle S_1(t)S_2(t)\rangle \equiv \langle S_{12}\rangle = \int_{-T/2}^{T/2} dt \int_{-T/2}^{T/2} dt' S_1(t)S_2(t)W(t-t'$$

$T =$ total integration time

Power spectrum : cross-correlating 2 detectors

- Reminder : one detector = SGW > noise
- Two detectors = *cross-correlate* to reduce noise

$$\langle S_1(t)S_2(t)\rangle = \langle s_1(t)s_2(t)\rangle + \langle n_1(t)n_2(t)\rangle + ...$$



Stolen from Allen (1996)

$$\langle S_1(t)S_2(t)\rangle \equiv \langle S_{12}\rangle = \int_{-T/2}^{T/2} dt \int_{-T/2}^{T/2} dt' S_1(t)S_2(t)W(t-t'$$

$T =$ total integration time

If we choose filter $W(t,t')$ carefully :

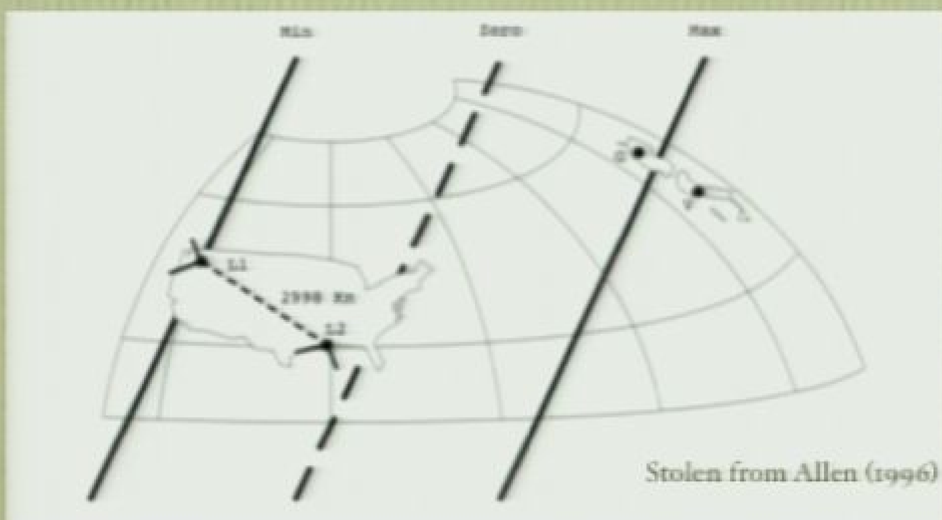
$$\langle s_1(t)s_2(t)\rangle \equiv \langle s_{12}\rangle \propto T$$

$$\langle n_1(t)n_2(t)\rangle \equiv \langle n_{12}\rangle \propto \sqrt{T}$$

Power spectrum : cross-correlating 2 detectors

- Reminder : one detector = SGW > noise
- Two detectors = *cross-correlate* to reduce noise

$$\langle S_1(t)S_2(t)\rangle = \langle s_1(t)s_2(t)\rangle + \langle n_1(t)n_2(t)\rangle + ...$$



Stolen from Allen (1996)

$$\langle S_1(t)S_2(t)\rangle \equiv \langle S_{12}\rangle = \int_{-T/2}^{T/2} dt \int_{-T/2}^{T/2} dt' S_1(t)S_2(t)W(t-t'$$

$T =$ total integration time

If we choose filter $W(t,t')$ carefully :

$$\langle s_1(t)s_2(t)\rangle \equiv \langle s_{12}\rangle \propto T$$

$$\langle n_1(t)n_2(t)\rangle \equiv \langle n_{12}\rangle \propto \sqrt{T}$$

Wait long enough, we will get Signal to Noise $\mathrm{SNR} = \dfrac{\langle s_{12}\rangle}{\langle n_{12}\rangle} \propto \sqrt{T} > 1$

e.g. Advanced LIGO : 4 months T, SNR = 1.65 for $\Omega_{gw}h^2 = 10^{-8} = 10^6\,\Omega_{gw}^{GUT,\,Inf}$

Beatin' down the noise, yo

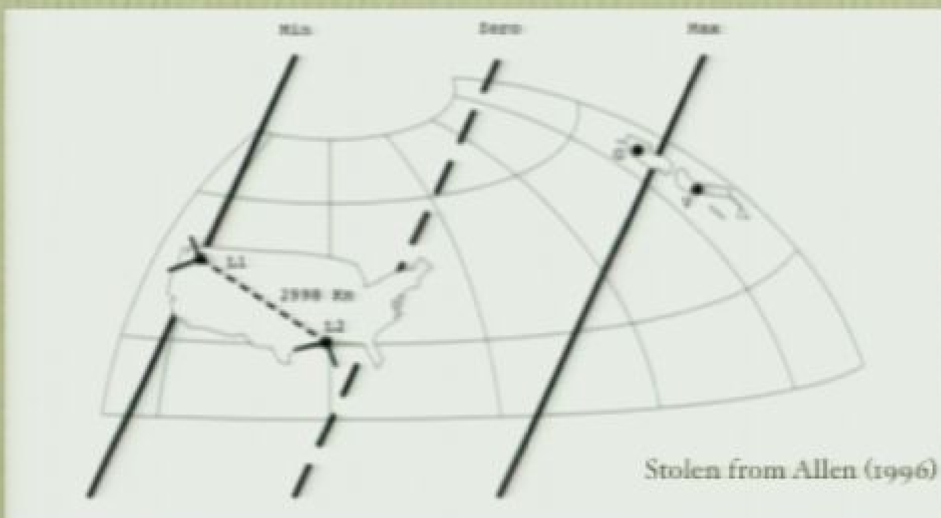
Effective SNR $\mathrm{SNR_{eff}} = \mathrm{SNR} \times \sqrt{M}$, $M =$ uncorrelated measurement of same object

Detectors best sensitivity $f_* \gg \frac{1}{T}$ e.g. $f_*^{LIGO} \sim 150\ \mathrm{Hz} = \frac{1}{150}\ \mathrm{sec}$

Power spectrum : cross-correlating 2 detectors

- Reminder : one detector = SGW > noise
- Two detectors = *cross-correlate* to reduce noise

$$\langle S_1(t)S_2(t)\rangle = \langle s_1(t)s_2(t)\rangle + \langle n_1(t)n_2(t)\rangle + \ldots$$



Stolen from Allen (1996)

$$\langle S_1(t)S_2(t)\rangle \equiv \langle S_{12}\rangle = \int_{-T/2}^{T/2} dt \int_{-T/2}^{T/2} dt' S_1(t)S_2(t)W(t-t'$$

$T =$ total integration time

If we choose filter $W(t,t')$ carefully :

$$\langle s_1(t)s_2(t)\rangle \equiv \langle s_{12}\rangle \propto T$$

$$\langle n_1(t)n_2(t)\rangle \equiv \langle n_{12}\rangle \propto \sqrt{T}$$

Wait long enough, we will get Signal to Noise $\mathrm{SNR} = \dfrac{\langle s_{12}\rangle}{\langle n_{12}\rangle} \propto \sqrt{T} > 1$

e.g. Advanced LIGO : 4 months T, SNR = 1.65 for $\Omega_{gw}h^2 = 10^{-8} = 10^6\Omega_{gw}^{GUT,\,Inf}$

Beatin' down the noise, yo

Effective SNR $\text{SNR}_{\text{eff}} = \text{SNR} \times \sqrt{M}$, $M =$ uncorrelated measurement of same object

Detectors best sensitivity $f_* \gg \frac{1}{T}$ e.g. $f_*^{LIGO} \sim 150 \text{ Hz} = \frac{1}{150}$ sec

Beatin' down the noise, yo

Effective SNR $\mathrm{SNR_{eff}} = \mathrm{SNR} \times \sqrt{M}$, $M =$ uncorrelated measurement of same object

Detectors best sensitivity $\quad f_* \gg \frac{1}{T}$ e.g. $f_*^{LIGO} \sim 150\ \mathrm{Hz} = \frac{1}{150}\ \mathrm{sec}$

Chop up T into M chunks : $\quad T = M \times \Delta T$, $\Delta T \sim \frac{1}{f_*}$

Make M measurements of $\quad \langle s_{12}(f_*) \rangle \approx P_h^{inf}(f_*) \propto T$

Noise is uncorrelated across chunks (random walk) $\quad \langle n_{12} \rangle \propto \sqrt{T}$

Beatin' down the noise, yo

Effective SNR $\mathrm{SNR}_{\mathrm{eff}} = \mathrm{SNR} \times \sqrt{M}$, $M =$ uncorrelated measurement of same object

Detectors best sensitivity $f_* \gg \frac{1}{T}$ e.g. $f_*^{LIGO} \sim 150$ Hz $= \frac{1}{150}$ sec

Chop up T into M chunks : $T = M \times \Delta T$, $\Delta T \sim \frac{1}{f_*}$

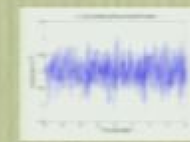
Make M measurements of $\langle s_{12}(f_*)\rangle \approx P_h^{inf}(f_*) \propto T$

Noise is uncorrelated across chunks (random walk) $\langle n_{12}\rangle \propto \sqrt{T}$

This is achieved operationally by choosing a clever filter

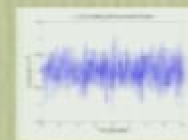
$W(t,t') = W(t-t')$, such that $\tilde{W}(f_*)$ have support around f_*

Orientation, location of detectors

Detector data stream $S(t) = s(t) + n(t)$ 

$$s(t, \mathbf{x}) = \sum_A \int df \int d\hat{\Omega}\; h^A(f, \hat{\Omega}) e^{-2\pi i (ft - \hat{\Omega}\cdot\mathbf{x})} D^{ij} e^A_{ij}(\hat{\Omega})$$

Orientation, location of detectors

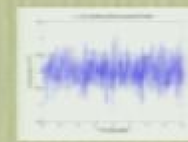
Detector data stream $S(t) = s(t) + n(t)$ 

$$s(t, \mathbf{x}) = \sum_A \int df \int d\hat{\Omega}\; h^A(f, \hat{\Omega}) e^{-2\pi i (ft - \hat{\Omega}\cdot\mathbf{x})} D^{ij} e^{A}_{ij}(\hat{\Omega})$$

- Detector Pattern Tensor $F^A(\hat{\Omega}) = D^{ij} e^{A}_{ij}(\hat{\Omega})$

Orientation, location of detectors

Detector data stream $S(t) = s(t) + n(t)$



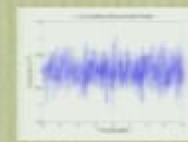
$$s(t,\mathbf{x}) = \sum_A \int df \int d\hat{\Omega}\; h^A(f,\hat{\Omega}) e^{-2\pi i(ft - \hat{\Omega}\cdot\mathbf{x})} D^{ij} e^{A}_{ij}(\hat{\Omega})$$

- Detector Pattern Tensor $F^A(\hat{\Omega}) = D^{ij} e^{A}_{ij}(\hat{\Omega})$

- Relative location of detectors

$$\begin{aligned} \textit{Signal} \quad \langle s_{12} \rangle &= \int df\; \langle s_1(f) s_2(f) \rangle W(f) \\ &= \int df\, d\hat{\Omega}\, d\hat{\Omega}' e^{2\pi f(\hat{\Omega}\cdot\mathbf{x}_1 - \hat{\Omega}\cdot\mathbf{x}_2)} \sum_{A,A'} F_1^A(\hat{\Omega}) F_2^{A'}(\hat{\Omega}') \langle h^A(f,\hat{\Omega}) h^{A'}(f,\hat{\Omega}') \rangle W(f) \end{aligned}$$

Orientation, location of detectors

Detector data stream $\quad S(t) = s(t) + n(t)$ 

$$s(t,\mathbf{x}) = \sum_A \int df \int d\hat{\Omega}\; h^A(f,\hat{\Omega}) e^{-2\pi i (ft - \hat{\Omega}\cdot\mathbf{x})} D^{ij} e^{A}_{ij}(\hat{\Omega})$$

- Detector Pattern Tensor $\quad F^A(\hat{\Omega}) = D^{ij} e^A_{ij}(\hat{\Omega})$

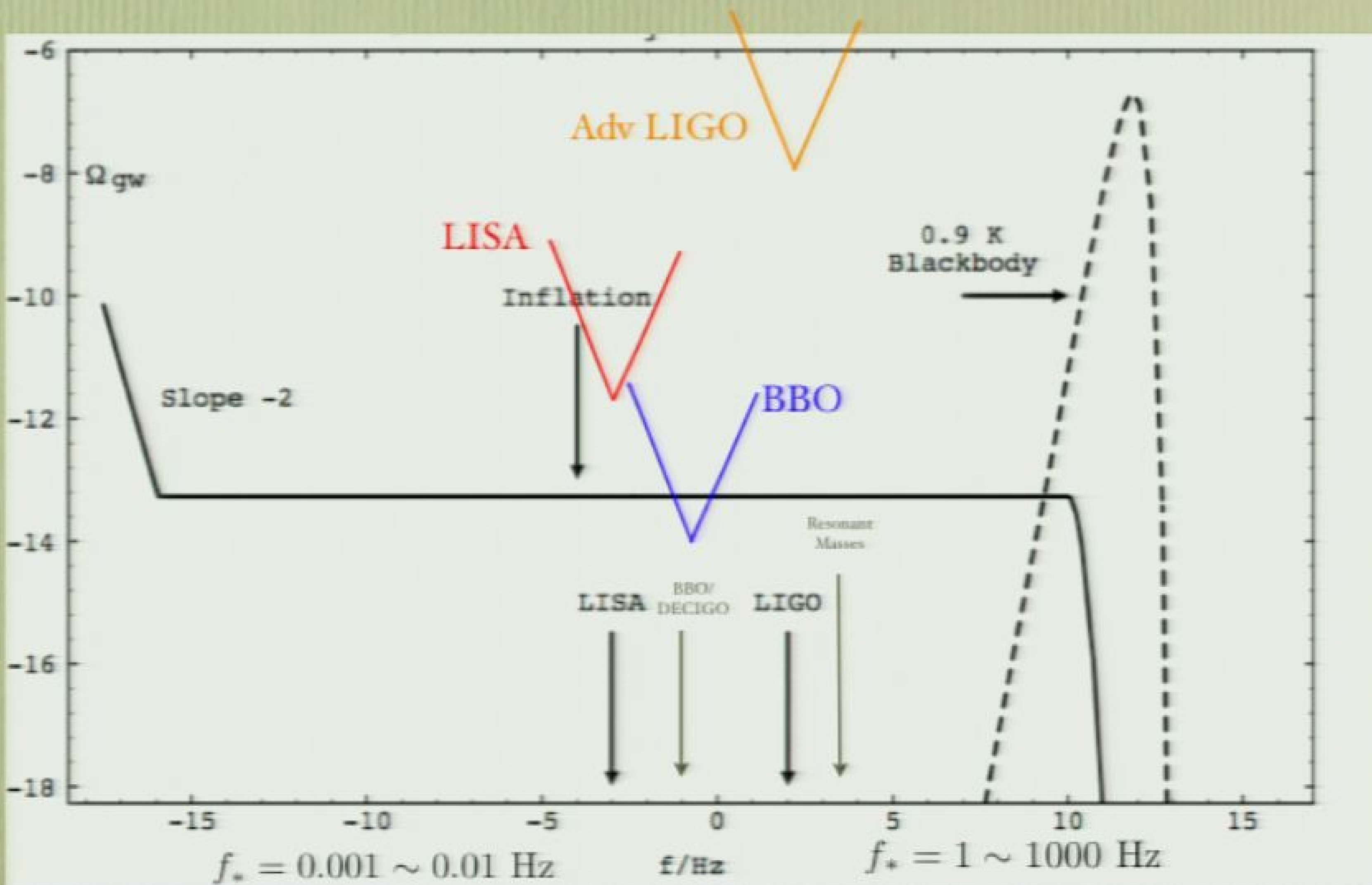
- Relative location of detectors

$$\begin{aligned} \textit{Signal} \quad \langle s_{12} \rangle &= \int df\; \langle s_1(f) s_2(f) \rangle W(f) \\ &= \int df \boxed{d\hat{\Omega} d\hat{\Omega}' e^{2\pi f(\hat{\Omega}\cdot\mathbf{x}_1 - \hat{\Omega}\cdot\mathbf{x}_2)} \sum_{A,A'} F_1^A(\hat{\Omega}) F_2^{A'}(\hat{\Omega}')} \langle h^A(f,\hat{\Omega}) h^{A'}(f,\hat{\Omega}') \rangle W(f) \end{aligned}$$

Oscillatory (bad) $\qquad\qquad$ orientation effects

Overlap Reduction factor $\gamma(f)$

2-Pt Expectations

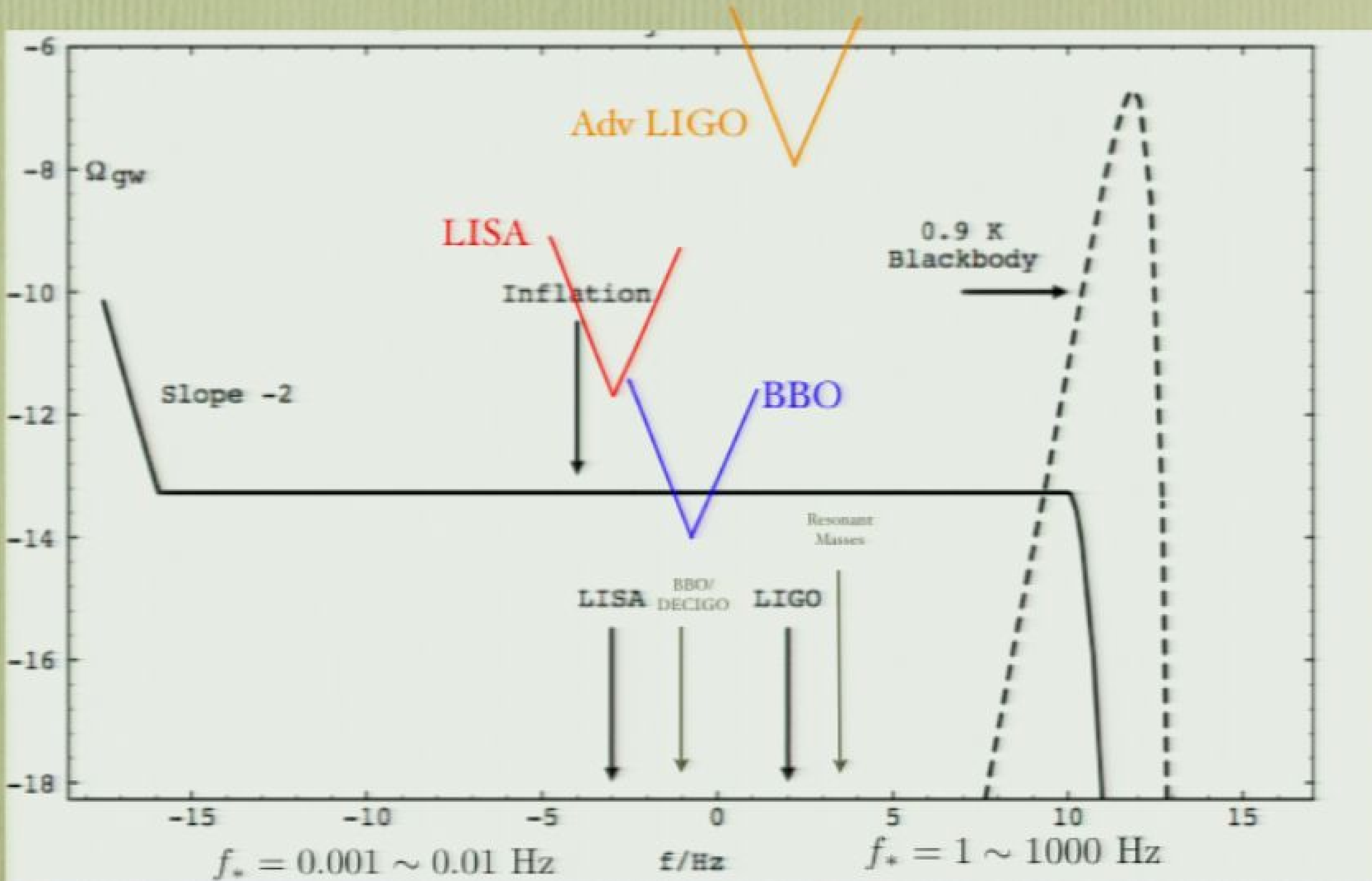


Constructing 3-pt Estimators

- Use 3 detectors $S_i(t) = s_i(t) + n_i(t)$

$$\langle S_{123} \rangle \equiv \langle S_1(t) S_2(t) S_3(t) \rangle \quad \langle (s_1(t) + n_1(t))(s_2(t) + n_2(t))(s_3(t) + n_3(t)) \rangle = \langle s_1 s_2 s_3 \rangle + (\langle s_1 s_2 n_3 \rangle + \cancel{\langle s_1 n_2 n_3 \rangle} + \text{sym}$$

2-Pt Expectations



Constructing 3-pt Estimators

- Use 3 detectors $\quad S_i(t) = s_i(t) + n_i(t)$

$$\langle S_{123} \rangle \equiv \langle S_1(t) S_2(t) S_3(t) \rangle \quad \langle (s_1(t) + n_1(t))(s_2(t) + n_2(t))(s_3(t) + n_3(t)) \rangle = \langle s_1 s_2 s_3 \rangle + (\langle s_1 s_2 n_3 \rangle \cancel{+ \langle s_1 n_2 n_3 \rangle} + \text{sym}$$

Constructing 3-pt Estimators

- Use 3 detectors $S_i(t) = s_i(t) + n_i(t)$

$$\langle S_{123}\rangle \equiv \langle S_1(t)S_2(t)S_3(t)\rangle \quad \langle (s_1(t)+n_1(t))(s_2(t)+n_2(t))(s_3(t)+n_3(t))\rangle = \langle s_1s_2s_3\rangle + (\langle s_1s_2n_3\rangle + \cancel{\langle s_1n_2n_3\rangle} + \mathrm{sym}$$

- Key point : 3-pt noise vanishes $\langle n_1(t)n_2(t)n_3(t)\rangle \approx 0$

Constructing 3-pt Estimators

- Use 3 detectors $\quad S_i(t) = s_i(t) + n_i(t)$

$$\langle S_{123}\rangle \equiv \langle S_1(t) S_2(t) S_3(t)\rangle \quad \langle (s_1(t)+n_1(t))(s_2(t)+n_2(t))(s_3(t)+n_3(t))\rangle = \langle s_1 s_2 s_3\rangle + (\langle s_1 s_2 n_3\rangle + \cancel{\langle s_1 n_2 n_3\rangle} + \text{sym}$$

- Key point : 3-pt noise vanishes $\langle n_1(t) n_2(t) n_3(t)\rangle \approx 0$

- Next to leading order noise :

$$\langle N_{123}^2\rangle \equiv \langle n_1(t) n_1(t)\rangle \langle n_2(t) n_2(t)\rangle \langle n_3(t) n_3(t)\rangle$$

detector noise *power* $N_i(f)$

Constructing 3-pt Estimators

Signal

$$\equiv \langle s_{123} \rangle$$

- Use 3 detectors $\quad S_i(t) = s_i(t) + n_i(t)$

$$\langle S_{123} \rangle \equiv \langle S_1(t) S_2(t) S_3(t) \rangle \quad \langle (s_1(t)+n_1(t))(s_2(t)+n_2(t))(s_3(t)+n_3(t)) \rangle = \langle s_1 s_2 s_3 \rangle + \cancel{\langle s_1 s_2 n_3 \rangle + \langle s_1 n_2 n_3 \rangle} + \mathrm{sym}$$

- Key point : 3-pt noise vanishes $\langle n_1(t) n_2(t) n_3(t) \rangle \approx 0$

- Next to leading order noise :

$$\langle N_{123}^2 \rangle \equiv \langle n_1(t) n_1(t) \rangle \langle n_2(t) n_2(t) \rangle \langle n_3(t) n_3(t) \rangle$$

detector noise *power* $N_i(f)$

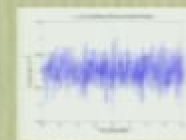
- Chopping up integration time T into M chunks

$$T = M \times \Delta T \ , \ \Delta T \sim \frac{1}{f_*}$$

$$\mathrm{SNR}_{eff} = \frac{\langle s_{123} \rangle_M}{\sqrt{\langle N_{123}^2 \rangle}} \times \sqrt{M} \propto \sqrt{T}$$

Wait long enough to see signal : *how long??*

Orientation, location of detectors

Detector data stream $S(t) = s(t) + n(t)$ 

$$s(t,\mathbf{x}) = \sum_A \int df \int d\hat{\Omega}\; h^A(f,\hat{\Omega}) e^{-2\pi i (ft - \hat{\Omega}\cdot\mathbf{x})} D^{ij} e^{A}_{ij}(\hat{\Omega})$$

- Detector Pattern Tensor $F^A(\hat{\Omega}) = D^{ij} e^{A}_{ij}(\hat{\Omega})$

- Relative location of detectors

$$\begin{aligned} \textit{Signal} \quad \langle s_{12}\rangle &= \int df\; \langle s_1(f) s_2(f)\rangle W(f) \\ &= \int df \boxed{d\hat{\Omega} d\hat{\Omega}' e^{2\pi f(\hat{\Omega}\cdot\mathbf{x}_1 - \hat{\Omega}\cdot\mathbf{x}_2)} \sum_{A,A'} F_1^A(\hat{\Omega}) F_2^{A'}(\hat{\Omega}')} \langle h^A(f,\hat{\Omega}) h^{A'}(f,\hat{\Omega}')\rangle W(f) \end{aligned}$$

Oscillatory (bad) orientation effects

Overlap Reduction factor $\gamma(f)$

Constructing 3-pt Estimators

- Signal : given theory $\langle h_1^A(\mathbf{k}) h_2^{A'}(\mathbf{k}') h_3^{A''}(\mathbf{k}')\rangle$, $\mathbf{k} = 2\pi f \hat{\Omega}$

Example : slow-roll inflation (Maldacena 2002)

$$\langle h^A(\mathbf{k}) h^{A'}(\mathbf{k}') h^{A''}(\mathbf{k}'')\rangle = \left(-K + \frac{kk' + k'k'' + kk''}{K} + \frac{kk'k''}{K}\right)(2\pi)^3 \delta\left(\sum \mathbf{k}\right)$$
$$\times \frac{H^4}{M_p^4} \frac{-4}{8(kk'k'')} \left(e^A_{ii'}(\mathbf{k}) e^{A'}_{jj'}(\mathbf{k}') e^{A''}_{ll'}(\mathbf{k}'') t_{ijl} t_{i'j'l'}\right)$$

$$t_{ijk} \equiv k'^i \delta_{jl} + k''^j \delta_{il} + k^l \delta_{ij}$$

- Our estimator is

$$\langle S_{123}\rangle_M = \int_{\Delta T} dt \int_{\Delta T} dt' \int_{\Delta T} dt''\ S_1(t) S_2(t') S_3(t'') W(t, t', t'')$$

3-pt Filter

Factorizable choice : $W(t, t', t'') = W_1(t) W_2(t') W_3(t'')$

Constructing 3-pt Estimators

- Signal : given theory $\langle h_1^A(\mathbf{k}) h_2^{A'}(\mathbf{k}') h_3^{A''}(\mathbf{k}')\rangle$, $\mathbf{k} = 2\pi f \hat{\Omega}$

Example : slow-roll inflation (Maldacena 2002)

$$\langle h^A(\mathbf{k}) h^{A'}(\mathbf{k}') h^{A''}(\mathbf{k}'')\rangle = \left(-K + \frac{kk' + k'k'' + kk''}{K} + \frac{kk'k''}{K}\right)(2\pi)^3 \delta(\sum \mathbf{k})$$

$$\times \frac{H^4}{M_p^4} \frac{-4}{8(kk'k'')} (e^A_{ii'}(\mathbf{k}) e^{A'}_{jj'}(\mathbf{k}') e^{A''}_{ll'}(\mathbf{k}'') t_{ijl} t_{i'j'l'})$$

$$t_{ijk} \equiv k'^i \delta_{jl} + k''^j \delta_{il} + k^l \delta_{ij}$$

Constructing 3-pt Estimators

Signal $\equiv \langle s_{123} \rangle$

- Use 3 detectors $\quad S_i(t) = s_i(t) + n_i(t)$

$$\langle S_{123} \rangle \equiv \langle S_1(t) S_2(t) S_3(t) \rangle \quad \langle (s_1(t)+n_1(t))(s_2(t)+n_2(t))(s_3(t)+n_3(t)) \rangle = \langle s_1 s_2 s_3 \rangle + \cancel{\langle s_1 s_2 n_3 \rangle + \langle s_1 n_2 n_3 \rangle + \mathrm{sym}}$$

- Key point : 3-pt noise vanishes $\langle n_1(t) n_2(t) n_3(t) \rangle \approx 0$

- Next to leading order noise :

$$\langle N_{123}^2 \rangle \equiv \langle n_1(t) n_1(t) \rangle \langle n_2(t) n_2(t) \rangle \langle n_3(t) n_3(t) \rangle$$

detector noise *power* $N_i(f)$

- Chopping up integration time T into M chunks

$$T = M \times \Delta T \ , \ \Delta T \sim \frac{1}{f_*}$$

$$\mathrm{SNR}_{eff} = \frac{\langle s_{123} \rangle_M}{\sqrt{\langle N_{123}^2 \rangle}} \times \sqrt{M} \propto \sqrt{T}$$

Wait long enough to see signal : *how long??*

Constructing 3-pt Estimators

- Signal : given theory $\langle h_1^A(\mathbf{k}) h_2^{A'}(\mathbf{k}') h_3^{A''}(\mathbf{k}')\rangle$, $\mathbf{k} = 2\pi f \hat{\Omega}$

Example : slow-roll inflation (Maldacena 2002)

$$\langle h^A(\mathbf{k}) h^{A'}(\mathbf{k}') h^{A''}(\mathbf{k}'')\rangle = \left(-K + \frac{kk' + k'k'' + kk''}{K} + \frac{kk'k''}{K}\right)(2\pi)^3 \delta(\sum \mathbf{k})$$

$$\times \frac{H^4}{M_p^4} \frac{-4}{8(kk'k'')} (e_{ii'}^A(\mathbf{k}) e_{jj'}^{A'}(\mathbf{k}') e_{ll'}^{A''}(\mathbf{k}'') t_{ijl} t_{i'j'l'})$$

$$t_{ijk} \equiv k'^i \delta_{jl} + k''^j \delta_{il} + k^l \delta_{ij}$$

Constructing 3-pt Estimators

- Signal : given theory $\langle h_1^A(\mathbf{k}) h_2^{A'}(\mathbf{k}') h_3^{A''}(\mathbf{k}')\rangle$, $\mathbf{k} = 2\pi f \hat{\Omega}$

Example : slow-roll inflation (Maldacena 2002)

$$\langle h^A(\mathbf{k}) h^{A'}(\mathbf{k}') h^{A''}(\mathbf{k}'')\rangle = \left(-K + \frac{kk' + k'k'' + kk''}{K} + \frac{kk'k''}{K}\right)(2\pi)^3 \delta(\sum \mathbf{k})$$

$$\times \frac{H^4}{M_p^4} \frac{-4}{8(kk'k'')} (e^A_{ii'}(\mathbf{k}) e^{A'}_{jj'}(\mathbf{k}') e^{A''}_{ll'}(\mathbf{k}'') t_{ijl} t_{i'j'l'})$$

$$t_{ijk} \equiv k'^i \delta_{jl} + k''^j \delta_{il} + k^l \delta_{ij}$$

- Our estimator is

$$\boxed{\begin{array}{c} \langle S_{123}\rangle_M = \int_{\Delta T} dt \int_{\Delta T} dt' \int_{\Delta T} dt'' \; S_1(t) S_2(t') S_3(t'') \underbrace{W(t,t',t'')}_{\textit{3-pt Filter}} \\ \textit{Factorizable choice :}\; W(t,t',t'') = W_1(t) W_2(t') W_3(t'') \end{array}}$$

Expected Signal $\langle s_{123} \rangle_M$

$$\langle s_{123} \rangle_M \;=\; \int d\hat{\Omega} d\hat{\Omega}' d\hat{\Omega}'' df\, df'\, df'' \sum_{A,A',A''} \langle h^A(f,\hat{\Omega}) h^{A'}(f',\hat{\Omega}') h^{A''}(f'',\hat{\Omega}'') \rangle$$

$$F_1^A(\hat{\Omega}) F_2^{A'}(\hat{\Omega}') F_3^{A''}(\hat{\Omega}'') e^{i(2\pi(f\hat{\Omega}\cdot\mathbf{x}_1 + f'\hat{\Omega}'\cdot\mathbf{x}_2 + f''\hat{\Omega}''\cdot\mathbf{x}_3)} W_1(f) W_2(f') W_3(f'')$$

Constructing 3-pt Estimators

- Signal : given theory $\langle h_1^A(\mathbf{k}) h_2^{A'}(\mathbf{k}') h_3^{A''}(\mathbf{k}')\rangle$, $\mathbf{k} = 2\pi f \hat{\Omega}$

Example : slow-roll inflation (Maldacena 2002)

$$\langle h^A(\mathbf{k}) h^{A'}(\mathbf{k}') h^{A''}(\mathbf{k}'')\rangle = \left(-K + \frac{kk' + k'k'' + kk''}{K} + \frac{kk'k''}{K}\right)(2\pi)^3 \delta\left(\sum \mathbf{k}\right)$$
$$\times \frac{H^4}{M_p^4} \frac{-4}{8(kk'k'')} \left(e_{ii'}^A(\mathbf{k}) e_{jj'}^{A'}(\mathbf{k}') e_{ll'}^{A''}(\mathbf{k}'') t_{ijl} t_{i'j'l'}\right)$$

$$t_{ijk} \equiv k'^i \delta_{jl} + k''^j \delta_{il} + k^l \delta_{ij}$$

- Our estimator is

$$\langle S_{123}\rangle_M = \int_{\Delta T} dt \int_{\Delta T} dt' \int_{\Delta T} dt'' \; S_1(t) S_2(t') S_3(t'') W(t, t', t'')$$

3-pt Filter

Factorizable choice : $W(t, t', t'') = W_1(t) W_2(t') W_3(t'')$

Expected Signal $\langle s_{123} \rangle_M$

$$\langle s_{123} \rangle_M = \int d\hat{\Omega} d\hat{\Omega}' d\hat{\Omega}'' df\, df'\, df'' \sum_{A,A',A''} \langle h^A(f,\hat{\Omega}) h^{A'}(f',\hat{\Omega}') h^{A''}(f'',\hat{\Omega}'') \rangle$$
$$F_1^A(\hat{\Omega}) F_2^{A'}(\hat{\Omega}') F_3^{A''}(\hat{\Omega}'') e^{i(2\pi(f\hat{\Omega}\cdot\mathbf{x}_1 + f'\hat{\Omega}'\cdot\mathbf{x}_2 + f''\hat{\Omega}''\cdot\mathbf{x}_3)} W_1(f) W_2(f') W_3(f'')$$

Expected Signal $\langle s_{123} \rangle_M$

Momentum Conservation

$\propto \delta^3(f\hat{\Omega} + f'\hat{\Omega}' + f''\hat{\Omega}'')$

$$\langle s_{123} \rangle_M = \int d\hat{\Omega} d\hat{\Omega}' d\hat{\Omega}'' df\, df'\, df'' \sum_{A,A',A''} \langle h^A(f,\hat{\Omega}) h^{A'}(f',\hat{\Omega}') h^{A''}(f'',\hat{\Omega}'') \rangle$$
$$F_1^A(\hat{\Omega}) F_2^{A'}(\hat{\Omega}') F_3^{A''}(\hat{\Omega}'') e^{i(2\pi(f\hat{\Omega}\cdot\mathbf{x}_1 + f'\hat{\Omega}'\cdot\mathbf{x}_2 + f''\hat{\Omega}''\cdot\mathbf{x}_3)} W_1(f) W_2(f') W_3(f'')$$

Expected Signal $\langle s_{123} \rangle_M$

$$\langle s_{123} \rangle_M = \int d\hat{\Omega} d\hat{\Omega}' d\hat{\Omega}'' df\, df'\, df'' \sum_{A,A',A''} \langle h^A(f,\hat{\Omega}) h^{A'}(f',\hat{\Omega}') h^{A''}(f'',\hat{\Omega}'') \rangle$$

$$F_1^A(\hat{\Omega}) F_2^{A'}(\hat{\Omega}') F_3^{A''}(\hat{\Omega}'') e^{i(2\pi(f\hat{\Omega}\cdot\mathbf{x}_1 + f'\hat{\Omega}'\cdot\mathbf{x}_2 + f''\hat{\Omega}''\cdot\mathbf{x}_3)} W_1(f) W_2(f') W_3(f'')$$

Constructing 3-pt Estimators

- Signal : given theory $\langle h_1^A(\mathbf{k}) h_2^{A'}(\mathbf{k}') h_3^{A''}(\mathbf{k}') \rangle$, $\mathbf{k} = 2\pi f \hat{\Omega}$

Example : slow-roll inflation (Maldacena 2002)

$$\langle h^A(\mathbf{k}) h^{A'}(\mathbf{k}') h^{A''}(\mathbf{k}'') \rangle = \left(-K + \frac{kk' + k'k'' + kk''}{K} + \frac{kk'k''}{K} \right) (2\pi)^3 \delta(\sum \mathbf{k})$$
$$\times \frac{H^4}{M_p^4} \frac{-4}{8(kk'k'')} (e^A_{ii'}(\mathbf{k}) e^{A'}_{jj'}(\mathbf{k}') e^{A''}_{ll'}(\mathbf{k}'') t_{ijl} t_{i'j'l'})$$

$$t_{ijk} \equiv k'^i \delta_{jl} + k''^j \delta_{il} + k^l \delta_{ij}$$

- Our estimator is

$$\langle S_{123} \rangle_M = \int_{\Delta T} dt \int_{\Delta T} dt' \int_{\Delta T} dt''\ S_1(t) S_2(t') S_3(t'') W(t, t', t'')$$

3-pt Filter

Factorizable choice : $W(t, t', t'') = W_1(t) W_2(t') W_3(t'')$

Expected Signal $\langle s_{123} \rangle_M$

$$\langle s_{123} \rangle_M = \int d\hat{\Omega} d\hat{\Omega}' d\hat{\Omega}'' df\, df'\, df'' \sum_{A,A',A''} \langle h^A(f,\hat{\Omega}) h^{A'}(f',\hat{\Omega}') h^{A''}(f'',\hat{\Omega}'') \rangle$$

$$F_1^A(\hat{\Omega}) F_2^{A'}(\hat{\Omega}') F_3^{A''}(\hat{\Omega}'') e^{i(2\pi(f\hat{\Omega}\cdot\mathbf{x}_1 + f'\hat{\Omega}'\cdot\mathbf{x}_2 + f''\hat{\Omega}''\cdot\mathbf{x}_3)} W_1(f) W_2(f') W_3(f'')$$

Expected Signal $\langle s_{123}\rangle_M$

Momentum Conservation

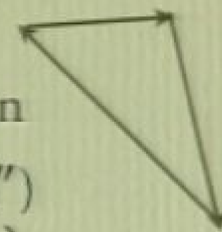
$\propto \delta^3(f\hat{\Omega} + f'\hat{\Omega}' + f''\hat{\Omega}'')$

$$\langle s_{123}\rangle_M = \int d\hat{\Omega}d\hat{\Omega}'d\hat{\Omega}''df\,df'\,df'' \sum_{A,A',A''} \langle h^A(f,\hat{\Omega})h^{A'}(f',\hat{\Omega}')h^{A''}(f'',\hat{\Omega}'')\rangle$$
$$F_1^A(\hat{\Omega})F_2^{A'}(\hat{\Omega}')F_3^{A''}(\hat{\Omega}'')e^{i(2\pi(f\hat{\Omega}\cdot\mathbf{x}_1+f'\hat{\Omega}'\cdot\mathbf{x}_2+f''\hat{\Omega}''\cdot\mathbf{x}_3)}W_1(f)W_2(f')W_3(f'')$$

Expected Signal $\langle s_{123}\rangle_M$

Momentum Conservation

$\propto \delta^3(f\hat{\Omega} + f'\hat{\Omega}' + f''\hat{\Omega}'')$



Integrated over all angles and frequencies

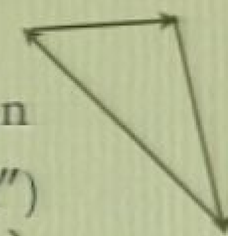
$$\langle s_{123}\rangle_M = \int d\hat{\Omega}d\hat{\Omega}'d\hat{\Omega}''df\,df'\,df'' \sum_{A,A',A''} \langle h^A(f,\hat{\Omega})h^{A'}(f',\hat{\Omega}')h^{A''}(f'',\hat{\Omega}'')\rangle$$

$$F_1^A(\hat{\Omega})F_2^{A'}(\hat{\Omega}')F_3^{A''}(\hat{\Omega}'')e^{i(2\pi(f\hat{\Omega}\cdot\mathbf{x}_1+f'\hat{\Omega}'\cdot\mathbf{x}_2+f''\hat{\Omega}''\cdot\mathbf{x}_3)}W_1(f)W_2(f')W_3(f'')$$

Expected Signal $\langle s_{123} \rangle_M$

Momentum Conservation

$\propto \delta^3(f\hat{\Omega} + f'\hat{\Omega}' + f''\hat{\Omega}'')$



Integrated over all angles and frequencies

$$\langle s_{123} \rangle_M = \int d\hat{\Omega} d\hat{\Omega}' d\hat{\Omega}'' df\, df'\, df'' \sum_{A,A',A''} \langle h^A(f,\hat{\Omega}) h^{A'}(f',\hat{\Omega}') h^{A''}(f'',\hat{\Omega}'') \rangle$$

$$F_1^A(\hat{\Omega}) F_2^{A'}(\hat{\Omega}') F_3^{A''}(\hat{\Omega}'') e^{i(2\pi(f\hat{\Omega}\cdot\mathbf{x}_1 + f'\hat{\Omega}'\cdot\mathbf{x}_2 + f''\hat{\Omega}''\cdot\mathbf{x}_3)} W_1(f) W_2(f') W_3(f'')$$

Orientation and relative-location effects

(*3-pt Overlap Reduction*)

Expected Signal $\langle s_{123} \rangle_M$

Integrated over all angles and frequencies

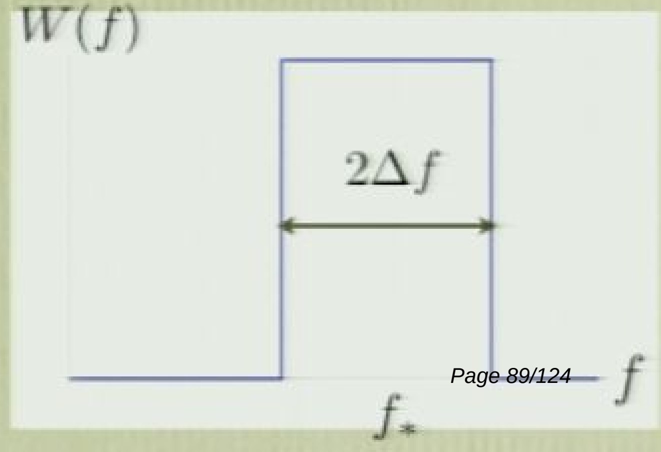
Momentum Conservation

$\propto \delta^3(f\hat{\Omega} + f'\hat{\Omega}' + f''\hat{\Omega}'')$

$$\langle s_{123} \rangle_M = \int d\hat{\Omega} d\hat{\Omega}' d\hat{\Omega}'' df\, df'\, df'' \sum_{A,A',A''} \langle h^A(f,\hat{\Omega}) h^{A'}(f',\hat{\Omega}') h^{A''}(f'',\hat{\Omega}'') \rangle$$
$$F_1^A(\hat{\Omega}) F_2^{A'}(\hat{\Omega}') F_3^{A''}(\hat{\Omega}'') e^{i(2\pi(f\hat{\Omega}\cdot\mathbf{x}_1 + f'\hat{\Omega}'\cdot\mathbf{x}_2 + f''\hat{\Omega}''\cdot\mathbf{x}_3)} W_1(f) W_2(f') W_3(f'')$$

Choose filters

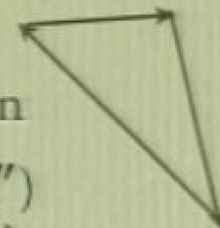
Orientation and relative-location effects
(3-pt Overlap Reduction)



Expected Signal $\langle s_{123}\rangle_M$

Momentum Conservation

$\propto \delta^3(f\hat{\Omega} + f'\hat{\Omega}' + f''\hat{\Omega}'')$



Integrated over all angles and frequencies

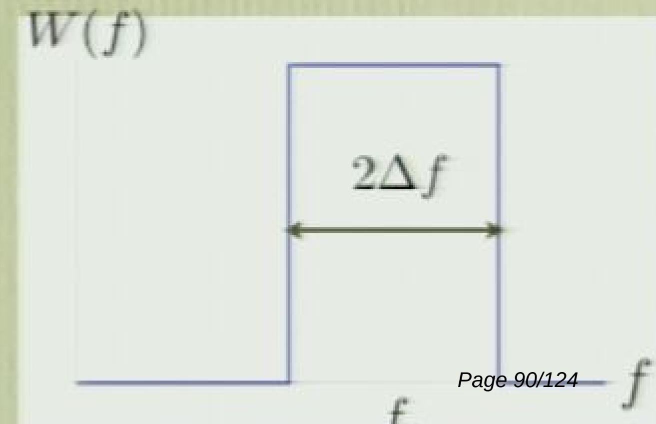
$$\langle s_{123}\rangle_M = \int d\hat{\Omega}d\hat{\Omega}'d\hat{\Omega}''df\,df'\,df'' \sum_{A,A',A''} \langle h^A(f,\hat{\Omega})h^{A'}(f',\hat{\Omega}')h^{A''}(f'',\hat{\Omega}'')\rangle$$

$$F_1^A(\hat{\Omega})F_2^{A'}(\hat{\Omega}')F_3^{A''}(\hat{\Omega}'')e^{i(2\pi(f\hat{\Omega}\cdot\mathbf{x}_1+f'\hat{\Omega}'\cdot\mathbf{x}_2+f''\hat{\Omega}''\cdot\mathbf{x}_3)}W_1(f)W_2(f')W_3(f'')$$

Choose filters

Orientation and relative-location effects

(3-pt Overlap Reduction)



Drive-by Estimation

$$\langle s_{123}\rangle_M = \int d\hat{\Omega} d\hat{\Omega}' d\hat{\Omega}'' df\, df'\, df'' \sum_{A,A',A''} \langle h^A(f,\hat{\Omega}) h^{A'}(f',\hat{\Omega}') h^{A''}(f'',\hat{\Omega}'')\rangle$$

$$F_1^A(\hat{\Omega}) F_2^{A'}(\hat{\Omega}') F_3^{A''}(\hat{\Omega}'') e^{i(2\pi(f\hat{\Omega}\cdot\mathbf{x}_1 + f'\hat{\Omega}'\cdot\mathbf{x}_2 + f''\hat{\Omega}''\cdot\mathbf{x}_3)} W_1(f) W_2(f') W_3(f'')$$

Drive-by Estimation

hacked inflation 3pt

$$\langle h^A(\mathbf{k}) h^{A'}(\mathbf{k}') h^{A''}(\mathbf{k}'')\rangle \;\propto\; \left(-K + \frac{kk' + k'k'' + kk''}{K} + \frac{kk'k''}{K}\right)(2\pi)^3\delta(\sum \mathbf{k})$$
$$\times \frac{-4}{8(kk'k'')}(kk' + k'k'' + kk'')$$

$$\langle s_{123}\rangle_M \;=\; \int d\hat{\Omega} d\hat{\Omega}' d\hat{\Omega}'' df\, df'\, df'' \sum_{A,A',A''} \langle h^A(f,\hat{\Omega}) h^{A'}(f',\hat{\Omega}') h^{A''}(f'',\hat{\Omega}'')\rangle$$
$$F_1^A(\hat{\Omega}) F_2^{A'}(\hat{\Omega}') F_3^{A''}(\hat{\Omega}'') e^{i(2\pi(f\hat{\Omega}\cdot\mathbf{x}_1 + f'\hat{\Omega}'\cdot\mathbf{x}_2 + f''\hat{\Omega}''\cdot\mathbf{x}_3)} W_1(f) W_2(f') W_3(f'')$$

Drive-by Estimation

hacked inflation 3pt

$$\langle h^A(\mathbf{k})h^{A'}(\mathbf{k}')h^{A''}(\mathbf{k}'')\rangle \propto \left(-K + \frac{kk' + k'k'' + kk''}{K} + \frac{kk'k''}{K}\right)(2\pi)^3\delta(\sum \mathbf{k})$$
$$\times \frac{-4}{8(kk'k'')}(kk' + k'k'' + kk'')$$

$$\langle s_{123}\rangle_M = \int d\hat{\Omega}d\hat{\Omega}'d\hat{\Omega}''df\,df'\,df'' \sum_{A,A',A''} \langle h^A(f,\hat{\Omega})h^{A'}(f',\hat{\Omega}')h^{A''}(f'',\hat{\Omega}'')\rangle$$
$$F_1^A(\hat{\Omega})F_2^{A'}(\hat{\Omega}')F_3^{A''}(\hat{\Omega}'')e^{i(2\pi(f\hat{\Omega}\cdot\mathbf{x}_1 + f'\hat{\Omega}'\cdot\mathbf{x}_2 + f''\hat{\Omega}''\cdot\mathbf{x}_3)}W_1(f)W_2(f')W_3(f'')$$

Set to 1
(ideal isotropic detectors)

Set to 1
(co-located detectors)

Drive-by Estimation

hacked inflation 3pt

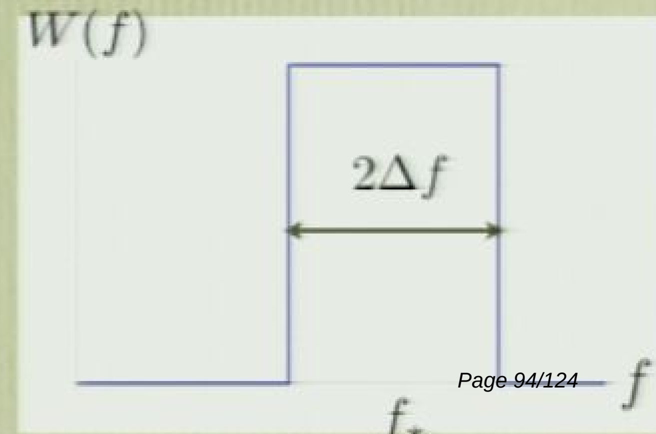
$$\langle h^{A}(\mathbf{k})h^{A'}(\mathbf{k}')h^{A''}(\mathbf{k}'')\rangle \;\propto\; \left(-K+\frac{kk'+k'k''+kk''}{K}+\frac{kk'k''}{K}\right)(2\pi)^3\delta\left(\sum\mathbf{k}\right)$$
$$\times\frac{-4}{8(kk'k'')}(kk'+k'k''+kk'')$$

$$\langle s_{123}\rangle_M \;=\; \int d\hat{\Omega}d\hat{\Omega}'d\hat{\Omega}''df\,df'\,df''\sum_{A,A',A''}\langle h^{A}(f,\hat{\Omega})h^{A'}(f',\hat{\Omega}')h^{A''}(f'',\hat{\Omega}'')\rangle$$
$$F_1^{A}(\hat{\Omega})F_2^{A'}(\hat{\Omega}')F_3^{A''}(\hat{\Omega}'')e^{i(2\pi(f\hat{\Omega}\cdot\mathbf{x}_1+f'\hat{\Omega}'\cdot\mathbf{x}_2+f''\hat{\Omega}''\cdot\mathbf{x}_3)}W_1(f)W_2(f')W_3(f'')$$

Set to 1
(ideal isotropic detectors)

Set to 1
(co-located detectors)

Choose filters



Drive-by Estimation

amplitude information has been factored out

$$\langle h^A(\mathbf{k}) h^{A'}(\mathbf{k}') h^{A''}(\mathbf{k}'')\rangle \propto \Omega_{gw}^4$$

hacked inflation 3pt

$$\langle h^A(\mathbf{k}) h^{A'}(\mathbf{k}') h^{A''}(\mathbf{k}'')\rangle \propto \left(-K + \frac{kk' + k'k'' + kk''}{K} + \frac{kk'k''}{K}\right)(2\pi)^3 \delta\left(\sum \mathbf{k}\right) \times \frac{-4}{8(kk'k'')}(kk' + k'k'' + kk'')$$

$$\langle s_{123}\rangle_M = \int d\hat{\Omega} d\hat{\Omega}' d\hat{\Omega}'' df\, df'\, df'' \sum_{A,A',A''} \langle h^A(f,\hat{\Omega}) h^{A'}(f',\hat{\Omega}') h^{A''}(f'',\hat{\Omega}'')\rangle$$

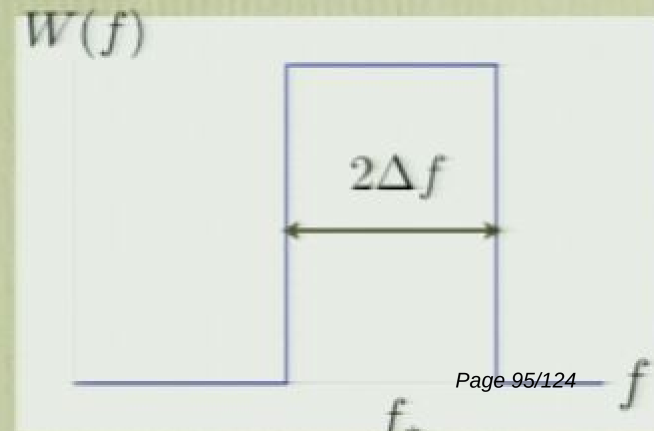
$$F_1^A(\hat{\Omega}) F_2^{A'}(\hat{\Omega}') F_3^{A''}(\hat{\Omega}'') e^{i(2\pi(f\hat{\Omega}\cdot\mathbf{x}_1 + f'\hat{\Omega}'\cdot\mathbf{x}_2 + f''\hat{\Omega}''\cdot\mathbf{x}_3)} W_1(f) W_2(f') W_3(f'')$$

Set to 1
(ideal isotropic detectors)

Set to 1
(co-located detectors)

Choose filters

"Shape Factor" γ



Drive-by Estimation

amplitude information
has been factored out

$$\langle h^A(\mathbf{k}) h^{A'}(\mathbf{k}') h^{A''}(\mathbf{k}'')\rangle \propto \Omega_{gw}^4$$

hacked inflation 3pt

$$\langle h^A(\mathbf{k}) h^{A'}(\mathbf{k}') h^{A''}(\mathbf{k}'')\rangle \propto \left(-K + \frac{kk' + k'k'' + kk''}{K} + \frac{kk'k''}{K}\right)(2\pi)^3 \delta\left(\sum \mathbf{k}\right) \times \frac{-4}{8(kk'k'')}(kk' + k'k'' + kk'')$$

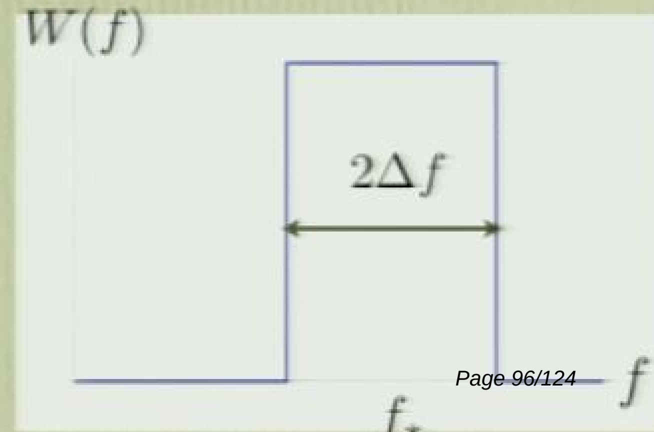
$$\langle s_{123}\rangle_M = \int d\hat{\Omega} d\hat{\Omega}' d\hat{\Omega}'' df\, df'\, df'' \sum_{A,A',A''} \langle h^A(f,\hat{\Omega}) h^{A'}(f',\hat{\Omega}') h^{A''}(f'',\hat{\Omega}'')\rangle$$

$$F_1^A(\hat{\Omega}) F_2^{A'}(\hat{\Omega}') F_3^{A''}(\hat{\Omega}'') e^{i(2\pi(f\hat{\Omega}\cdot\mathbf{x}_1 + f'\hat{\Omega}'\cdot\mathbf{x}_2 + f''\hat{\Omega}''\cdot\mathbf{x}_3)} W_1(f) W_2(f') W_3(f'')$$

Set to 1
(ideal isotropic detectors)

Set to 1
(co-located detectors)

Choose filters



"Shape Factor" γ

Over-estimate : isotropic/co-located assumptions

Under-estimate : sub-optimal filter, dropped polarization sum

How long do we need to wait?

Integration time needed for 90% detection given signal

$$T = (1.65)^2 \times \frac{10}{f_*} \times \frac{(N^2(f)\Delta f)^3}{(8\pi^2\gamma)^2}\left[12\left(\frac{H_{100}}{2\pi f_*}\right)^2 \Omega_{gw}\bar{h}^2\right]^{-4}$$

$$\propto f_*^7(\Omega_{gw}\bar{h}^2)^{-4}N^6(f)$$

Detector
best sensitivity
frequency

How long do we need to wait?

Integration time needed for 90% detection given signal

$$T = (1.65)^2 \times \frac{10}{f_*} \times \frac{(N^2(f)\Delta f)^3}{(8\pi^2\gamma)^2}\left[12\left(\frac{H_{100}}{2\pi f_*}\right)^2 \Omega_{gw}\bar{h}^2\right]^{-4}$$

$$\propto f_*^7(\Omega_{gw}\bar{h}^2)^{-4} N^6(f)$$

Detector best sensitivity frequency

Amplitude of metric perturbations

How long do we need to wait?

Integration time needed for 90% detection given signal

$$T = (1.65)^2 \times \frac{10}{f_*} \times \frac{(N^2(f)\Delta f)^3}{(8\pi^2\gamma)^2}\left[12\left(\frac{H_{100}}{2\pi f_*}\right)^2 \Omega_{gw}\bar{h}^2\right]^{-4}$$

$$\propto f_*^7(\Omega_{gw}\bar{h}^2)^{-4}N^6(f)$$

compare $T_{2pt} \propto f_*^5 N^4(f)(\Omega_{gw}\bar{h})^{-2}$

Noise Power

Detector
best sensitivity
frequency

Amplitude of metric
perturbations

How long do we need to wait?

Integration time needed for 90% detection given signal

$$T = (1.65)^2 \times \frac{10}{f_*} \times \frac{(N^2(f)\Delta f)^3}{(8\pi^2\gamma)^2}\left[12\left(\frac{H_{100}}{2\pi f_*}\right)^2 \Omega_{gw}\bar{h}^2\right]^{-4}$$

$$\propto f_*^7(\Omega_{gw}\bar{h}^2)^{-4}N^6(f)$$

compare $T_{2pt} \propto f_*^5 N^4(f)(\Omega_{gw}\bar{h})^{-2}$

Detector best sensitivity frequency

Amplitude of metric perturbations

Noise Power

Lesson

Lower frequency is GOOD

What can we see with BBO/DECIGO?

- Not going to see inflationary GW bispectrum with LISA/LIGO. (Not even power spectrum.)

How long do we need to wait?

Integration time needed for 90% detection given signal

$$T = (1.65)^2 \times \frac{10}{f_*} \times \frac{(N^2(f)\Delta f)^3}{(8\pi^2\gamma)^2}\left[12\left(\frac{H_{100}}{2\pi f_*}\right)^2 \Omega_{gw}\bar{h}^2\right]^{-4}$$

$$\propto f_*^7(\Omega_{gw}\bar{h}^2)^{-4}N^6(f)$$

compare $T_{2pt} \propto f_*^5 N^4(f)(\Omega_{gw}\bar{h})^{-2}$

Noise Power

Detector
best sensitivity
frequency

Amplitude of metric
perturbations

Lesson

Lower frequency is GOOD

What can we see with BBO/DECIGO?

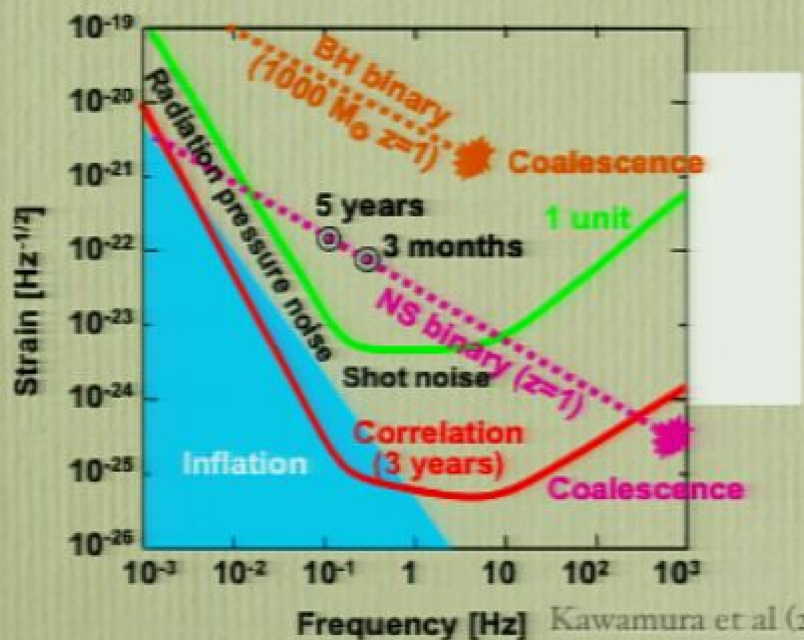
- Not going to see inflationary GW bispectrum with LISA/LIGO. (Not even power spectrum.)

What can we see with BBO/DECIGO?

- Not going to see inflationary GW bispectrum with LISA/LIGO. (Not even power spectrum.)

- BBO/DECIGO : designed to look for inflationary GW power spectrum

$$f_* = 0.1 - 1 \text{ Hz} \,, \; N(f_*) \approx 0.6 \times 10^{-25} \text{ Hz}^{-1/2}$$



Kawamura et al (2008)

What do we need to see Inflationary 3-pt?

Recall $T \propto f_*^7 N^6(f_*)(\Omega_{gw})^{-4}$

Lower Frequency
Lower Noise

What do we need to see Inflationary 3-pt?

Recall $T \propto f_*^7 N^6(f_*)(\Omega_{gw})^{-4}$

Lower Frequency
Lower Noise

NCC-1701 "Super-BBO"

$$f_* = 0.01 \text{ Hz}\ ,\ \ N(f_*) \approx 0.3 \times 10^{-27}\ \text{Hz}^{-1/2}$$

What do we need to see Inflationary 3-pt?

Recall $T \propto f_*^7 N^6(f_*)(\Omega_{gw})^{-4}$

Lower Frequency
Lower Noise

NCC-1701 "Super-BBO"

$$f_* = 0.01 \text{ Hz}\ , \ \ N(f_*) \approx 0.3 \times 10^{-27} \text{ Hz}^{-1/2}$$

LISA best frequency

2 orders of magnitude better sensitivity

What about other sources?

"LISA" $f_* = 0.01$ Hz , $N(f_*) \approx 0.5 \times 10^{-25}$ Hz$^{-1/2}$ "BBO/DECIGO"

What do we need to see Inflationary 3-pt?

Recall $T \propto f_*^7 N^6(f_*)(\Omega_{gw})^{-4}$ Lower Frequency Lower Noise

NCC-1701 "Super-BBO"

$$f_* = 0.01 \text{ Hz} \; , \; N(f_*) \approx 0.3 \times 10^{-27} \text{ Hz}^{-1/2}$$

LISA best frequency

2 orders of magnitude better sensitivity

$$T = 3.6 \text{ years}$$

What about other sources?

"LISA" $f_* = 0.01$ Hz , $N(f_*) \approx 0.5 \times 10^{-25}$ Hz$^{-1/2}$ "BBO/DECIGO"

What do we need to see Inflationary 3-pt?

Recall $T \propto f_*^7 N^6(f_*)(\Omega_{gw})^{-4}$

Lower Frequency
Lower Noise

NCC-1701 "Super-BBO"

$$f_* = 0.01 \text{ Hz} \ , \ N(f_*) \approx 0.3 \times 10^{-27} \text{ Hz}^{-1/2}$$

LISA best frequency

2 orders of magnitude better sensitivity

$$T = 3.6 \text{ years}$$

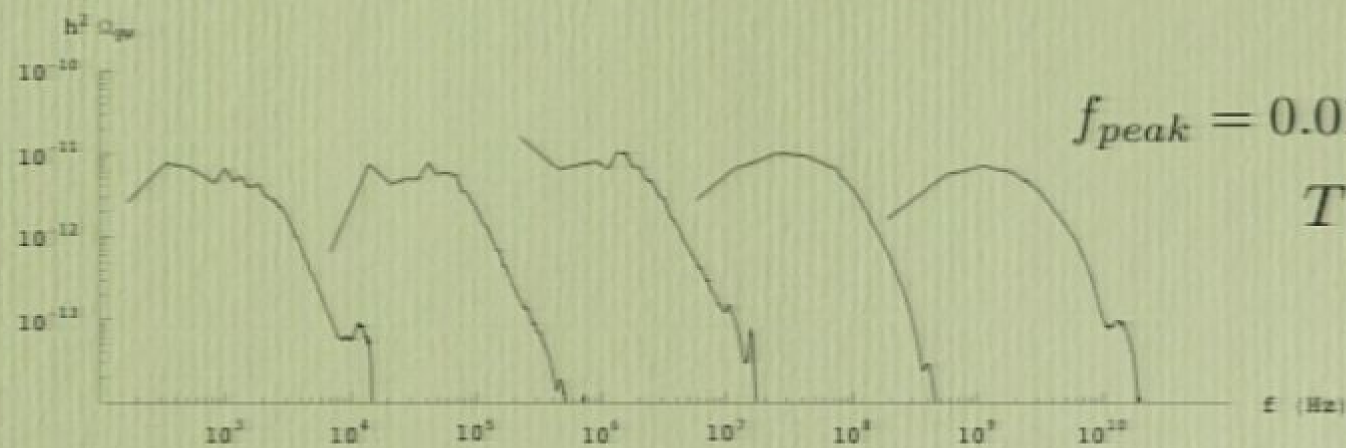
What about other sources?

"LISA" $f_* = 0.01\ \mathrm{Hz}\ ,\ N(f_*) \approx 0.5 \times 10^{-25}\ \mathrm{Hz}^{-1/2}$ "BBO/DECIGO"

What about other sources?

"LISA" $f_* = 0.01$ Hz , $N(f_*) \approx 0.5 \times 10^{-25}$ Hz$^{-1/2}$ "BBO/DECIGO"

TeV scale Inflation Preheating



Easther, Giblin, Lim (2007)

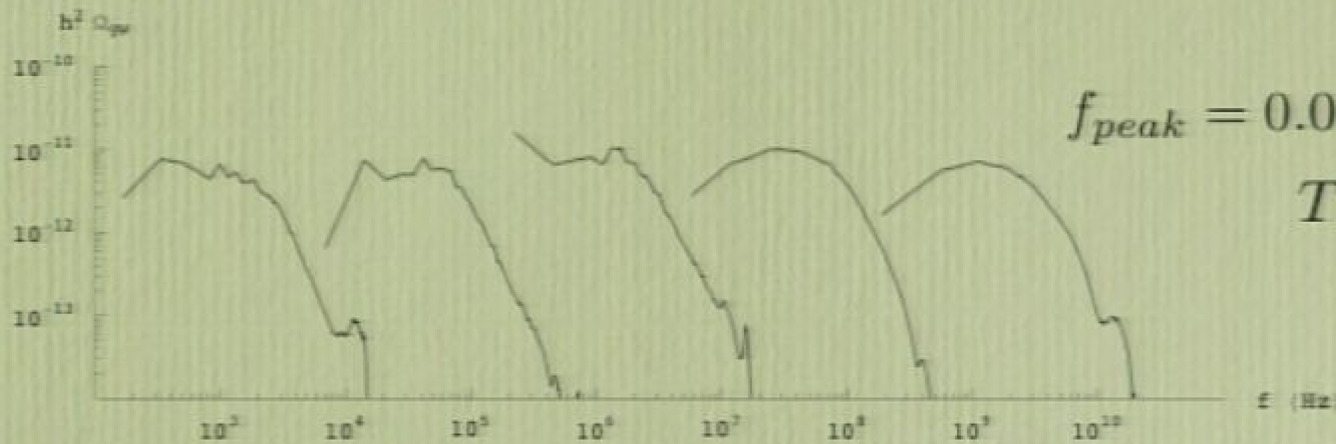
$$f_{peak} = 0.01 \text{ Hz} \ , \ \Omega_{gw}\bar{h}^2 \approx 10^{-11}$$

$$T = 7.8 \text{ years}$$

What about other sources?

"LISA" $f_* = 0.01$ Hz , $N(f_*) \approx 0.5 \times 10^{-25}$ Hz$^{-1/2}$ "BBO/DECIGO"

TeV scale Inflation Preheating



Easther, Giblin, Lim (2007)

$$f_{peak} = 0.01 \text{ Hz}\ ,\ \Omega_{gw}\bar{h}^2 \approx 10^{-11}$$

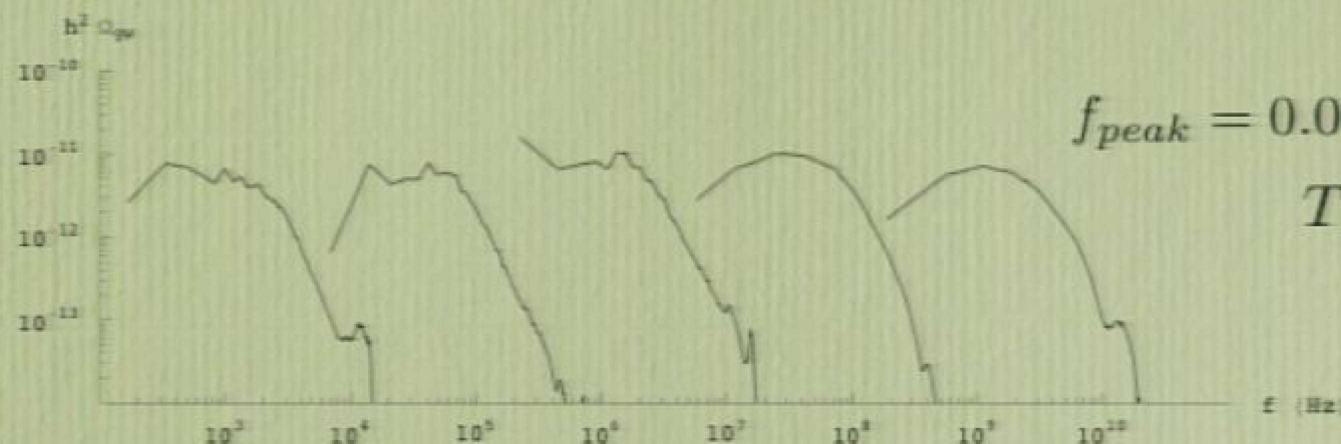
$$T = 7.8 \text{ years}$$

Similar for 1st order electro-weak breaking (Grojean et al, 2006)
Extra-dimensions (Servant+Randall, 2007)
SUSY breaking at TeV (Craig, 2009, Dufaux, 2009)

What about other sources?

"LISA" $f_* = 0.01$ Hz , $N(f_*) \approx 0.5 \times 10^{-25}$ Hz$^{-1/2}$ "BBO/DECIGO"

TeV scale Inflation Preheating

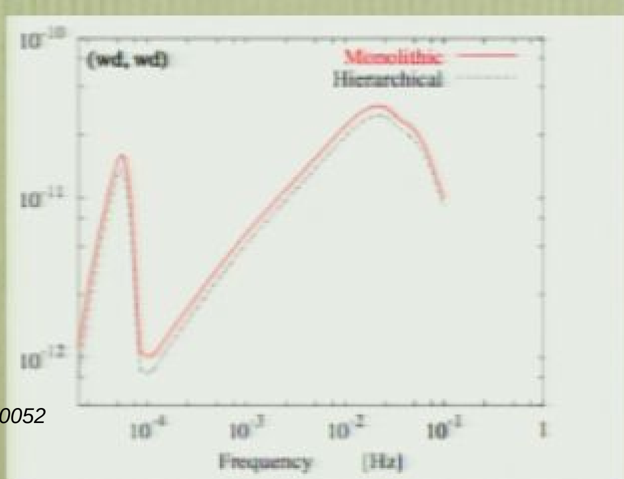


$$f_{peak} = 0.01 \text{ Hz} \; , \; \Omega_{gw}\bar{h}^2 \approx 10^{-11}$$

$$T = 7.8 \text{ years}$$

Easther, Giblin, Lim (2007)

Compact Binaries



Similar for 1st order electro-weak breaking (Grojean et al, 2006)
Extra-dimensions (Servant+Randall, 2007)
SUSY breaking at TeV (Craig, 2009, Dufaux, 2009)

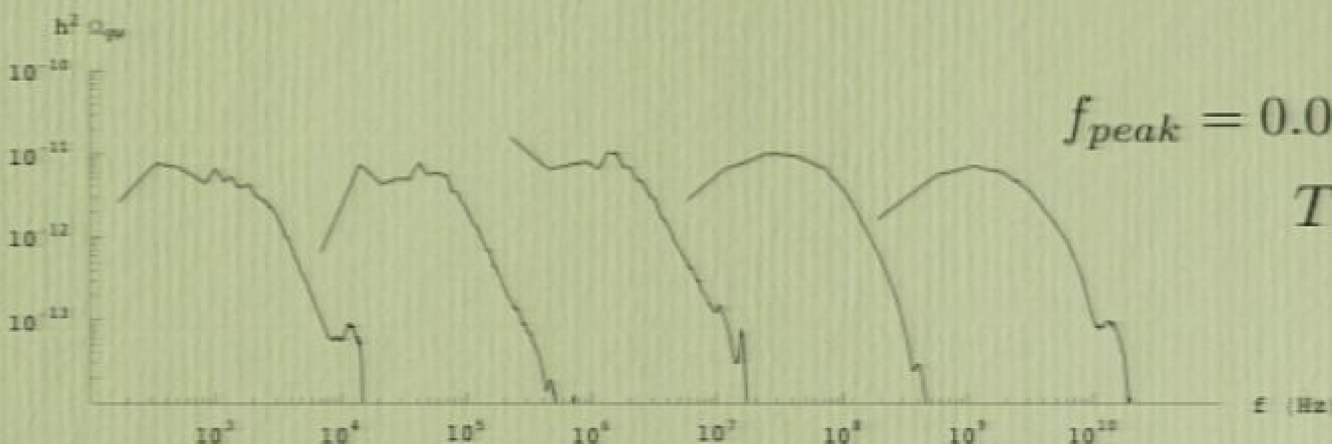
$$f_{peak} = 0.01 \text{ Hz} \; , \; \Omega_{gw}\bar{h}^2 \approx 5 \times 10^{-11}$$

$$T = 1 \text{ month}$$

What about other sources?

"LISA" $f_* = 0.01$ Hz , $N(f_*) \approx 0.5 \times 10^{-25}$ Hz$^{-1/2}$ "BBO/DECIGO"

TeV scale Inflation Preheating



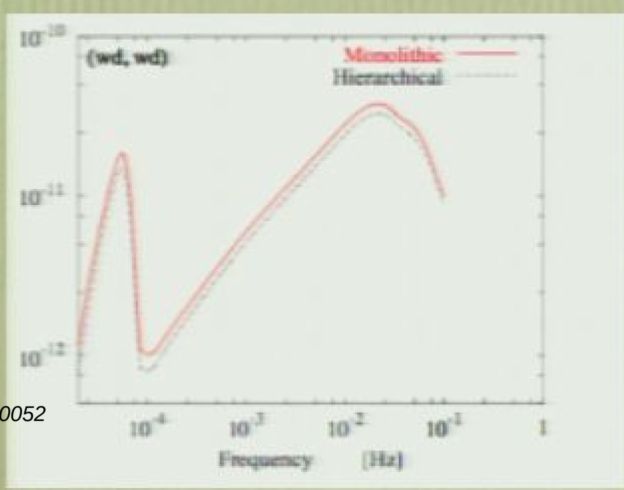
$$f_{peak} = 0.01 \text{ Hz} \ , \ \Omega_{gw}\bar{h}^2 \approx 10^{-11}$$

$$T = 7.8 \text{ years}$$

Easther, Giblin, Lim (2007)

Compact Binaries

Similar for 1st order electro-weak breaking (Grojean et al, 2006)
Extra-dimensions (Servant+Randall, 2007)
SUSY breaking at TeV (Craig, 2009, Dufaux, 2009)



$$f_{peak} = 0.01 \text{ Hz} \ , \ \Omega_{gw}\bar{h}^2 \approx 5 \times 10^{-11}$$

$$T = 1 \text{ month}$$

Caveat : I have used "inflationary" shape
Expect : "Local" shape

A test of General Relativity?

- The co-linear filter $W(f, f', f'') = \delta(f + f' + f'')$

$$\delta(f + f' + f'')\delta(\mathbf{k} + \mathbf{k}' + \mathbf{k}'') \ , \ \mathbf{k} = 2\pi f\hat{\Omega}$$

A test of General Relativity?

- The co-linear filter $W(f, f', f'') = \delta(f + f' + f'')$

$$\delta(f + f' + f'')\delta(\mathbf{k} + \mathbf{k}' + \mathbf{k}'') \ , \ \mathbf{k} = 2\pi f \hat{\Omega}$$

The filter picks up the "Co-linear" triangle

A test of General Relativity?

- The co-linear filter $W(f, f', f'') = \delta(f + f' + f'')$

$$\delta(f + f' + f'')\delta(\mathbf{k} + \mathbf{k}' + \mathbf{k}'') \ , \ \mathbf{k} = 2\pi f \hat{\Omega}$$

The filter picks up the "Co-linear" triangle

$$\langle h^{A}(\mathbf{k}) h^{A'}(\mathbf{k}') h^{A''}(\mathbf{k}'')\rangle \propto e^{A}_{ii'}(\mathbf{k}) e^{A'}_{jj'}(\mathbf{k}') e^{A''}_{ll'}(\mathbf{k}'') t_{ijl} t_{i'j'l'} = 0 \text{ if co} - \text{linear}$$

Thus signal is *always zero* if spin-2 is massless.

A test of General Relativity?

- The co-linear filter $W(f, f', f'') = \delta(f + f' + f'')$

$$\delta(f + f' + f'')\delta(\mathbf{k} + \mathbf{k}' + \mathbf{k}'') \ , \ \mathbf{k} = 2\pi f \hat{\Omega}$$

Massless

The filter picks up the "Co-linear" triangle

$$\langle h^A(\mathbf{k}) h^{A'}(\mathbf{k}') h^{A''}(\mathbf{k}'') \rangle \propto e^A_{ii'}(\mathbf{k}) e^{A'}_{jj'}(\mathbf{k}') e^{A''}_{ll'}(\mathbf{k}'') t_{ijl} t_{i'j'l'} = 0 \text{ if co} - \text{linear}$$

Thus signal is *always zero* if spin-2 is massless.

Can we use it as a "zero test" of massive gravity?

For the observers : this can be used to characterize non-G of instrument noise.

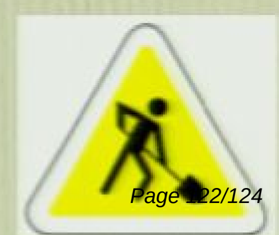
Summary

- 3-pt correlation functions of SGW are powerful discriminant of sources.
- We can potentially detect inflationary 3-pt with the next-to-next-to generation SGW.
- We can detect astrophysical and non-inflationary cosmological sources of SGW 3-pt with tweaked BBO/DECIGO.
- Maybe we can use it as a "zero-test" of massive gravity.

Summary

- 3-pt correlation functions of SGW are powerful discriminant of sources.
- We can potentially detect inflationary 3-pt with the next-to-next-to generation SGW.
- We can detect astrophysical and non-inflationary cosmological sources of SGW 3-pt with tweaked BBO/DECIGO.
- Maybe we can use it as a "zero-test" of massive gravity.

I am not crazy. (Maybe a little bit)



A test of General Relativity?

- The co-linear filter $W(f, f', f'') = \delta(f + f' + f'')$

$$\delta(f + f' + f'')\delta(\mathbf{k} + \mathbf{k}' + \mathbf{k}'') \ , \ \mathbf{k} = 2\pi f\hat{\Omega}$$

Massless

The filter picks up the "Co-linear" triangle

$$\langle h^A(\mathbf{k})h^{A'}(\mathbf{k}')h^{A''}(\mathbf{k}'')\rangle \propto e^A_{ii'}(\mathbf{k})e^{A'}_{jj'}(\mathbf{k}')e^{A''}_{ll'}(\mathbf{k}'')t_{ijl}t_{i'j'l'} = 0 \text{ if co} - \text{linear}$$

Thus signal is *always zero* if spin-2 is massless.

Can we use it as a "zero test" of massive gravity?

For the observers : this can be used to characterize non-G of instrument noise.

How long do we need to wait?

Integration time needed for 90% detection given signal

$$T = (1.65)^2 \times \frac{10}{f_*} \times \frac{(N^2(f)\Delta f)^3}{(8\pi^2\gamma)^2}\left[12\left(\frac{H_{100}}{2\pi f_*}\right)^2 \Omega_{gw}\bar{h}^2\right]^{-4}$$

$$\propto f_*^7(\Omega_{gw}\bar{h}^2)^{-4}N^6(f)$$

compare $T_{2pt} \propto f_*^5 N^4(f)(\Omega_{gw}\bar{h})^{-2}$

Noise Power

Amplitude of metric perturbations

Detector best sensitivity frequency