Title: Lagrangian mean curvature flow for entire Lipschitz graphs

Date: May 09, 2009 03:15 PM

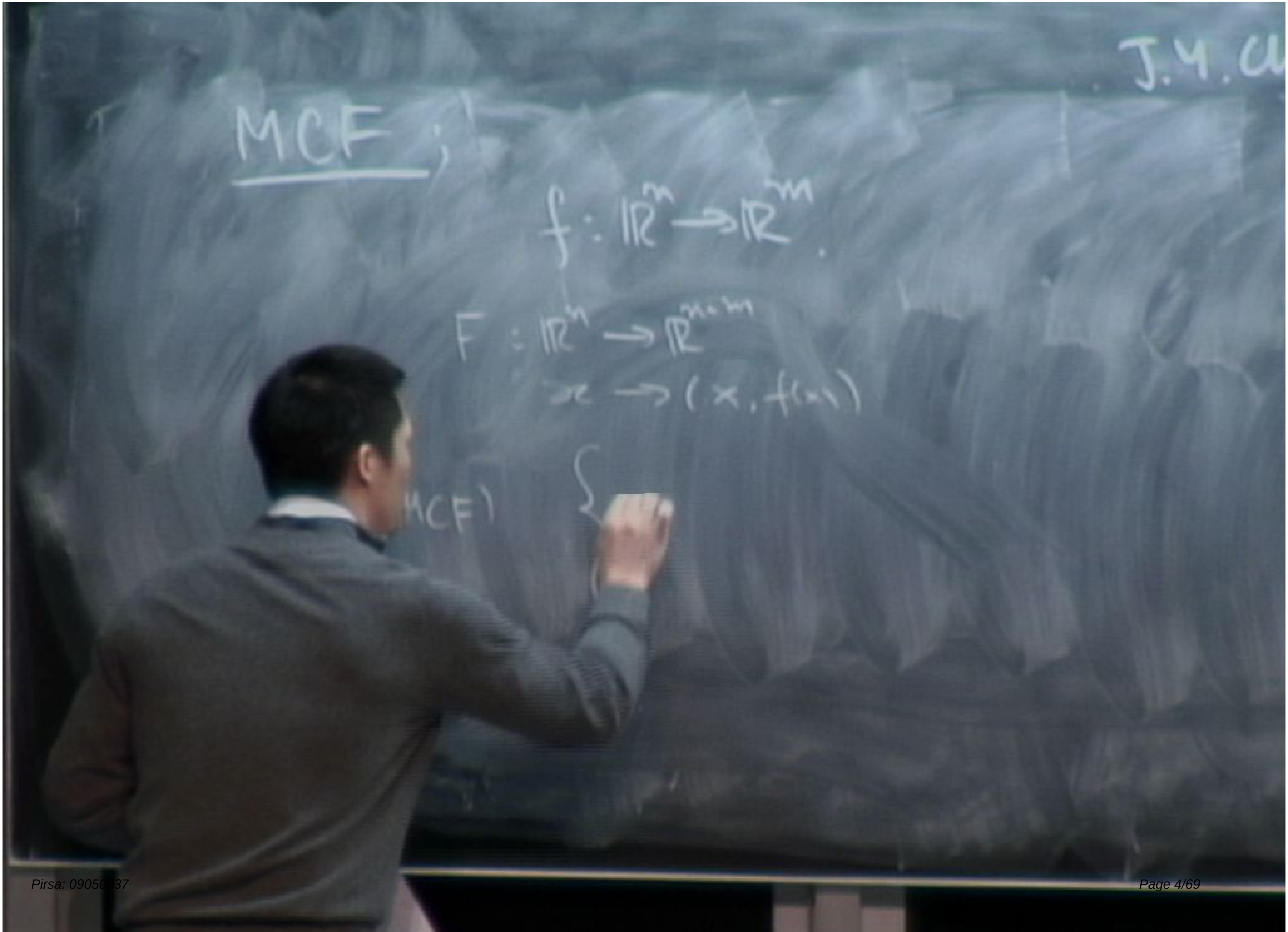
URL: http://pirsa.org/09050037

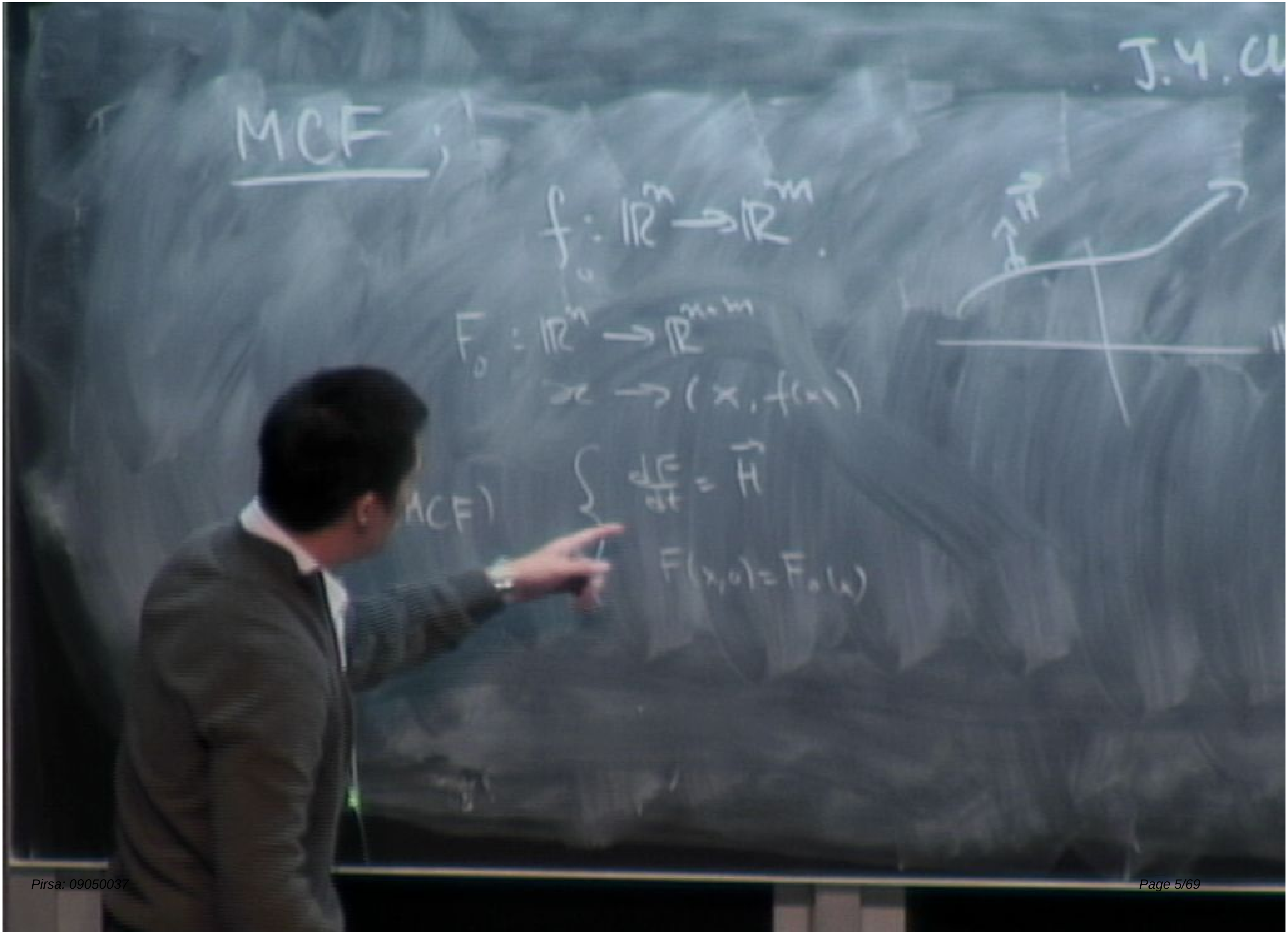
Abstract: "In this joint work with Jingyi Chen and Weiyong He, we prove existence of longtime smooth solutions to mean curvature flow of entire Lipschitz Lagrangian graphs. A Bernstein type result for translating solitons is also obtained."

J. Y. Chen, W. He.

- MCF
- Lagrangian (LMCF)
- Results
- Main Results

prof.


J. Y.
MCF ;
$f : \mathbb{R}^n \to \mathbb{R}^m$.
$F : \mathbb{R}^n \to \mathbb{R}^{n+m}$
$x \to (x, f(x))$
(MCF)


MCF ;
f0 : R^n → R^m
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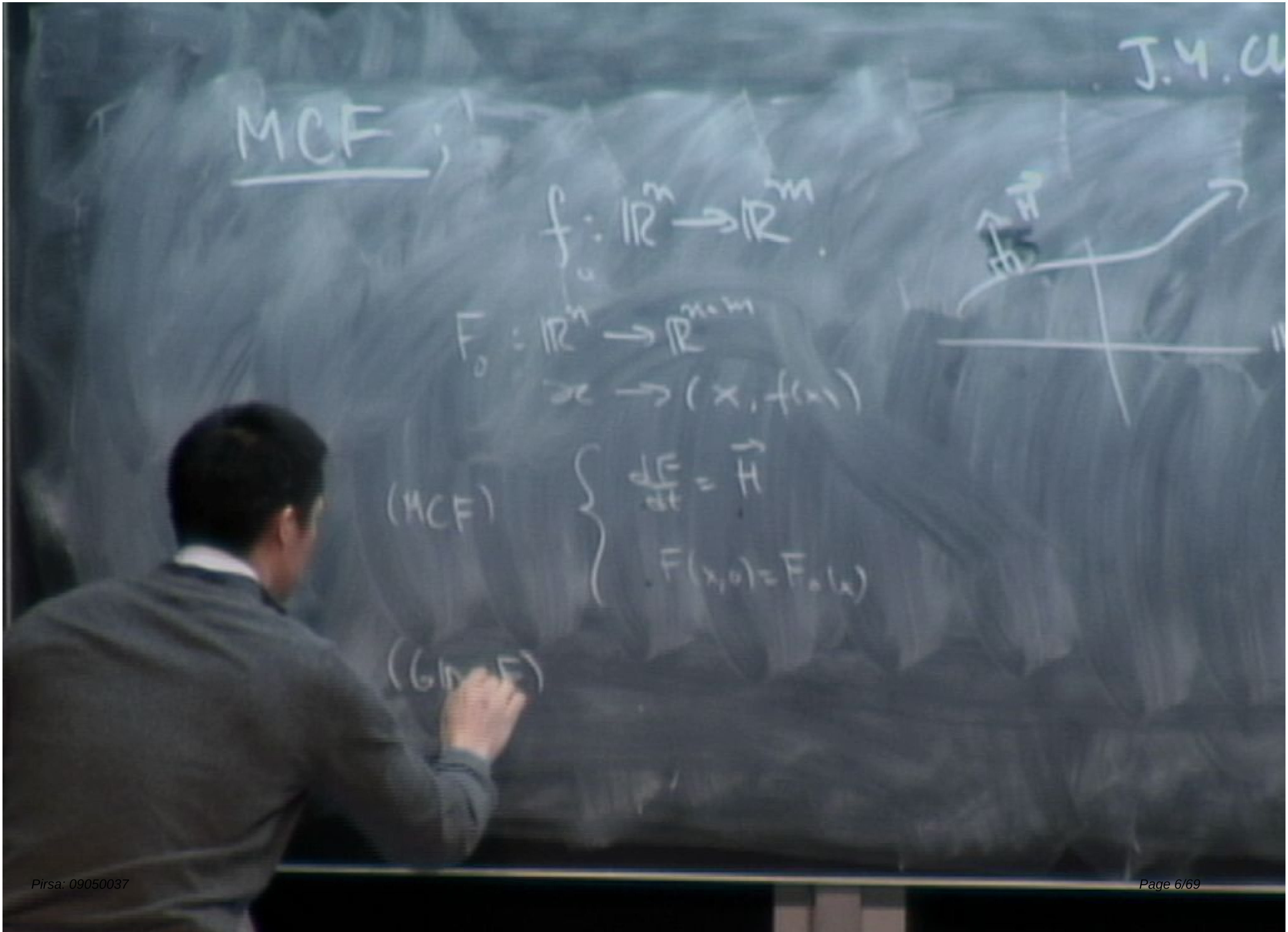
x → (x, f(x))
MCF)
dF/dt = H
F(x,0) = F0(x)


J.Y. C
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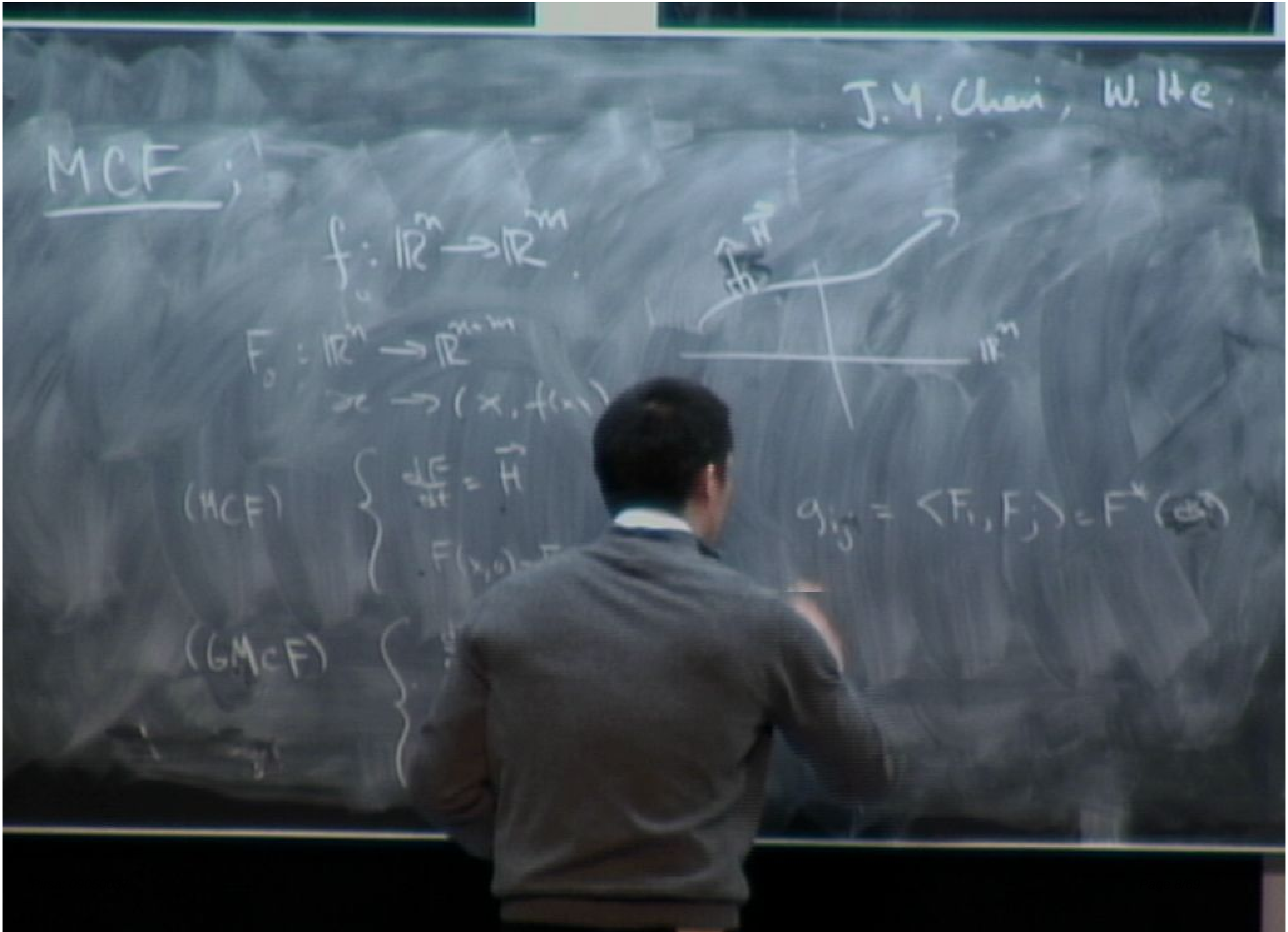
$$(MCF) \quad \begin{cases} \dfrac{dF}{dt} = \vec{H} \\ F(x,0) = F_0(x) \end{cases}$$

$$(GMCF) \quad \begin{cases} \dfrac{df}{dt} = g^{ij} f_{,ij} \\ f(x,0) = f_0 \end{cases}$$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g_ij = ⟨F_i, F_j⟩ = F*(ds²)

J.Y. Chen, W. He.

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J.Y. Chen, W. He.

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(GMCF) $\begin{cases} \frac{df}{dt} = g^{ij} f_{ij} \\ f(x,0) = f_0 \end{cases}$


$\exists\ r_\epsilon : \mathbb{R}^n \to \mathbb{R}^m$
$F($

$\exists\ r_t : \mathbb{R}^m \to \mathbb{R}^m$

✓ $F(x,t) = (r_t(x), f(r_t(x),t))$

$\exists\ r_t : \mathbb{R}^n \to \mathbb{R}^m$

$F(x,t) := (r_t(x), f(r_t(x),t))$

<u>Lagrangian</u>;

$\mathbb{R}^n \oplus \mathbb{R}^n = \mathbb{C}^n$

$\exists\ r_t : \mathbb{R}^m \to \mathbb{R}^m$

✓ $F(x,t) \doteq (r_t(x), f(r_t(x),t))$

<u>Lagrangian</u> ;

$\mathbb{R}^n \oplus \mathbb{R}^n = \mathbb{C}^n$; $\quad dz_j = x_j + i y_j$ (hol. coord.)

$\omega = \sum dx^j \wedge dy^j$ (symplectic form)

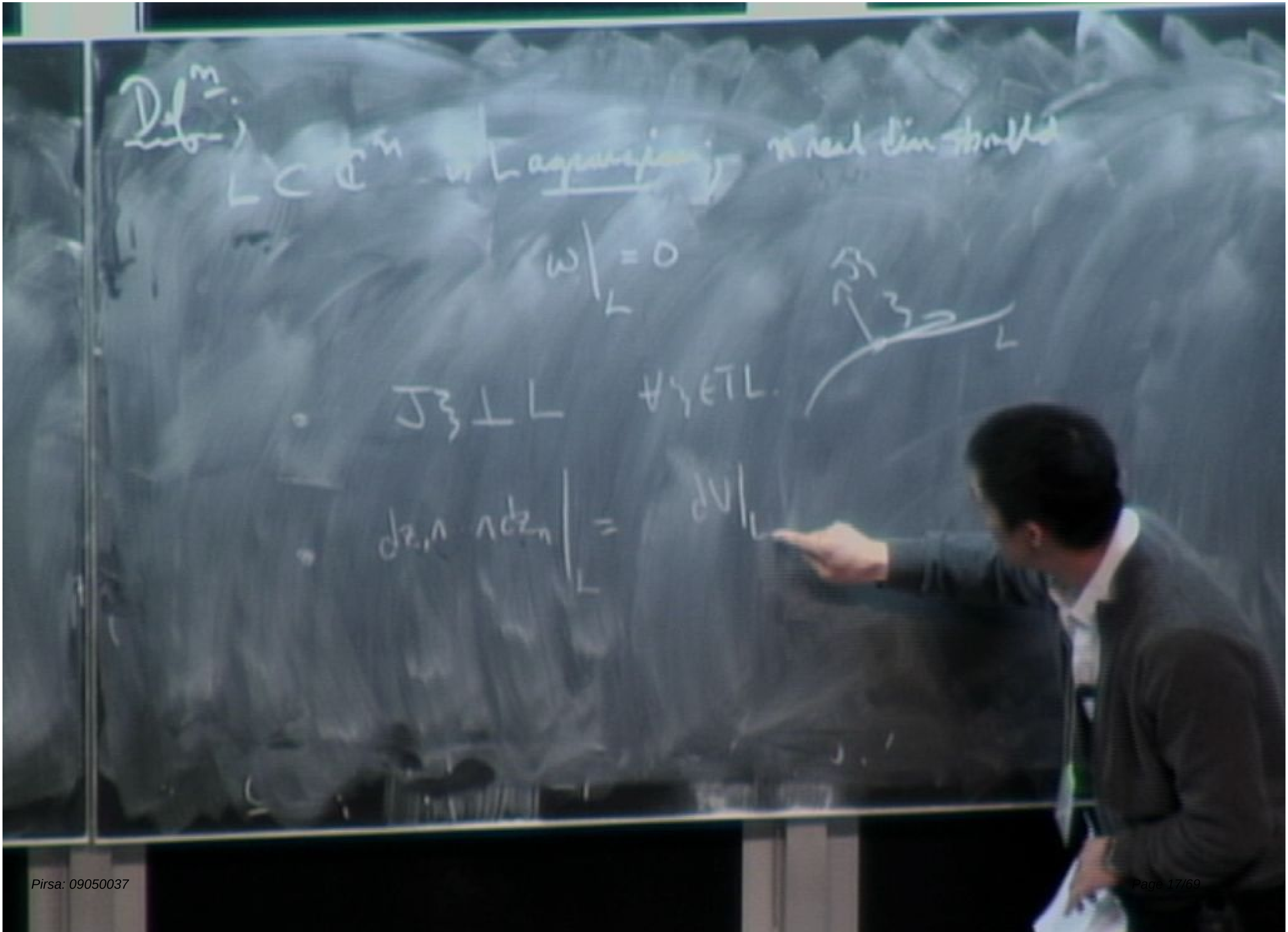
$J : T_p\mathbb{C}^n \to T_p\mathbb{C}^n$

$$\begin{cases} \partial x^i \to \partial y^i \\ \partial y^j \to -\partial x^j \end{cases}$$

Defn:

$L \subset \mathbb{C}^n$ is Lagrangian; n real dim submfld

$$\omega\big|_L = 0$$


Defn:
L ⊂ ℂ^n is Lagrangian if n real dim submfd
ω|_L = 0
Jξ ⊥ L ∀ξ ∈ TL.
Jξ
ξ
L
dz_1 ∧ ∧ dz_n |_L = dV|_L

$\mathrm{Def}^{\underline{n}}$: $L \subset \mathbb{C}^n$ is Lagrangian, n real dim submfld

$$\omega\big|_L = 0$$

- $J\xi \perp L \quad \forall \xi \in TL.$
- $dz_1 \wedge \dots \wedge dz_n\big|_L = e^{i\theta}\, dV\big|_L$

on L;

$$H = J\nabla\theta$$

$\left\{ \begin{array}{l} L : \mathrm{SLag} \\ (\text{minimal}, \mathrm{Lag}) \end{array} \right. \Longleftrightarrow \theta = \text{const}$

entire:

$\mathrm{Lag};\ (x, f(x)) \subseteq \mathbb{C}^n \Longleftrightarrow f = Du$

$f : \mathbb{R}^n_x \to \mathbb{R}^n_y \qquad u : \mathbb{R}^n \to \mathbb{R}.$

$$\begin{cases} \dfrac{du}{dt} = \dfrac{1}{t}\log\dfrac{\det(I D_{11} D^2 u)}{\sqrt{\det(I D \cdot D^2 u))}} \\ u(x,0) = u_0 \end{cases}$$

$$\begin{cases} \dfrac{du}{dt} = \dfrac{1}{i}\log \dfrac{\det(I + i D^2 u)}{\sqrt{\det(I + (D^2u)^2)}} \\ u(x,0) = u_0 \end{cases} \Longleftrightarrow \begin{array}{ll} f(x,t) = Du(x,t) & \text{GMCF} \\ (x, f(x,t)) \subseteq \mathbb{C}^n & \text{"MCF"} \end{array}$$

$$\begin{cases} \dfrac{du}{dt} = \dfrac{1}{i}\log \dfrac{\det(I + i D^2 u)}{\sqrt{\det(I + (D^2 u)^2)}} \\ u(y,0) = u_0 \end{cases} \iff \begin{array}{ll} f(y,t) = Du(y,t) & \text{GMCF} \\ (y, f(y,t)) \subseteq \mathbb{C}^n & \text{"MCF"} \end{array}$$

$$\begin{cases} \dfrac{du}{dt} = \dfrac{1}{i}\log \underbrace{\dfrac{\det(I + iD^2u)}{\sqrt{\det(I + (D^2u)^2)}}}_{F(D^2u)} \\ u(x,0) = u_0 \end{cases} \iff \begin{array}{l} f(x,t) = Du(x,t) \quad \text{GMCF} \\ (x, f(x,t)) \subseteq \mathbb{C}^n \quad \text{"MCF"} \end{array}$$

$$\begin{cases} \dfrac{du}{dt} = \dfrac{1}{i}\log \dfrac{\det(ID + iD^2u)}{\sqrt{\det(ID + (D^2u)^2)}} \\ u(x,0) = u_0 \end{cases} \qquad F(D^2u)$$

$\longleftrightarrow$ $f(x,t) = Du(x,t)$ GMCF

$(x, f(x,t)) \subseteq \mathbb{C}^n$ "MCF"

$$\begin{cases} F(D^2u) = \text{const} \end{cases}$$

$\longleftrightarrow$ $(x, Du(x,t)) \subseteq \mathbb{C}^n$ minimal.

$$\begin{cases} \dfrac{du}{dt} = \dfrac{1}{i}\log \underbrace{\dfrac{\det(I + i D^2 u)}{\sqrt{\det(I + (D^2 u)^2)}}}_{F(D^2u)} \\ u(x,0) = u_0 \end{cases} \longleftrightarrow \begin{array}{l} f(x,t) = Du(x,t) \quad \text{GMCF} \\ (x, f(x,t)) \subseteq \mathbb{C}^n \quad \text{"MCF"} \end{array}$$

$$\begin{cases} F(D^2 u) = \text{const} \end{cases} \longleftrightarrow (x, Du(x,t)) \subseteq \mathbb{C}^n \text{ minimal.}$$

$$F(D^2 u) = \frac{1}{i}\log \prod_j \left(\frac{1 + i\lambda_j}{\sqrt{1+\lambda_j^2}}\right)$$

$$D^2 u = \begin{pmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{pmatrix}$$

$$\begin{cases} \dfrac{du}{dt} = \underbrace{\dfrac{1}{i}\log \dfrac{\det(I + i D^2 u)}{\sqrt{\det(I + (D^2 u)^2)}}}_{F(D^2 u)} \\ u(x,0) = u_0 \end{cases} \quad \longleftrightarrow \quad f(x,t) = Du(x,t) \quad \text{GMCF}$$

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$$F(D^2 u) = \frac{1}{i}\log \prod_j \left(\frac{1+i\lambda_j}{\sqrt{1+\lambda_j^2}}\right) \qquad \frac{1+i\lambda_j}{\sqrt{1+\lambda_j^2}} = e^{i \arctan \lambda_j}$$

$$D^2 u = \begin{pmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{pmatrix} \qquad = \sum_j \arctan \lambda_j$$

$$\begin{cases} \dfrac{du}{dt} = \dfrac{1}{i}\operatorname{Log}\dfrac{\det(I D + i D^2 u)}{\sqrt{\det(I D + (D^2 u)^2)}} \\ u(x,0) = u_0 \end{cases} \qquad F(D^2u) \qquad \Longleftrightarrow \qquad f(x,t) = Du(x,t) \quad \text{GMCF}$$

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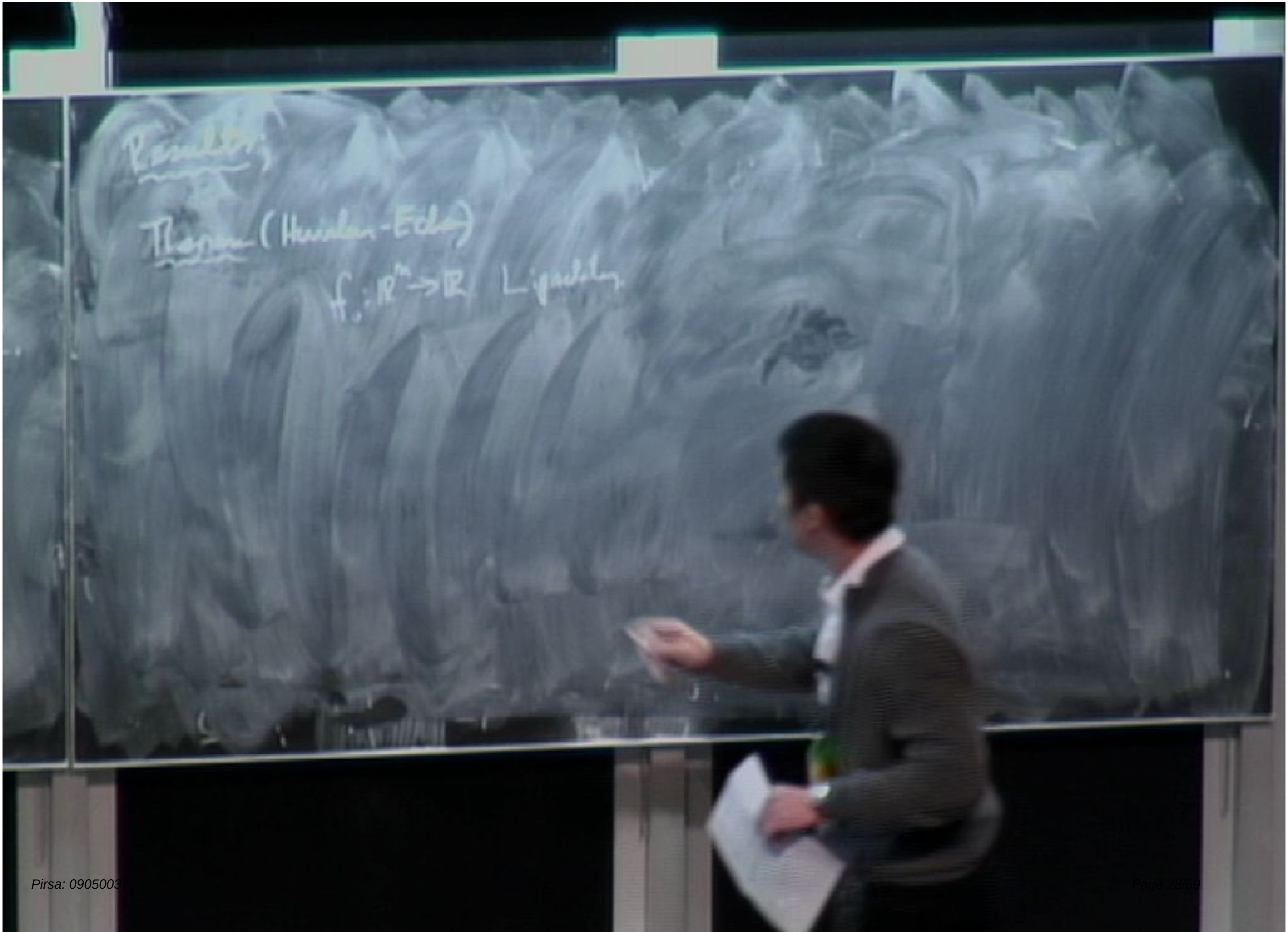
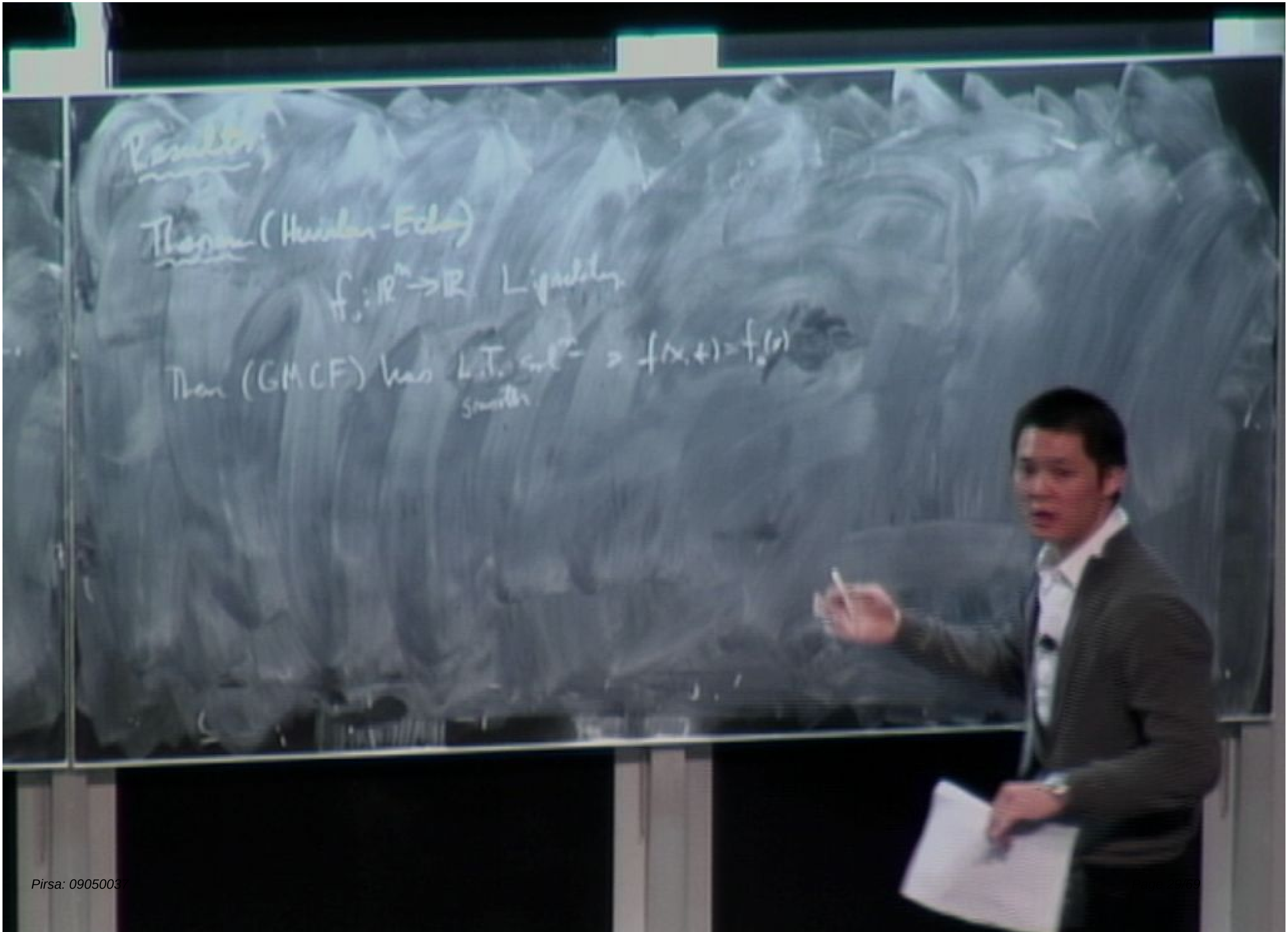
$$\left\{ F(D^2u) = \text{const} \right. \qquad \Longleftrightarrow \qquad (x, Du(x,t)) \subseteq \mathbb{C}^n \ \text{minimal.}$$

$$F(D^2u) = \frac{1}{i}\operatorname{Log}\prod_j \left(\frac{1+i\lambda_j}{\sqrt{1+\lambda_j^2}}\right) \to e^{i\arctan\lambda_j} \qquad \frac{\partial F}{\partial \lambda_i} = \frac{1}{1+\lambda_i^2} > 0$$

$$D^2u = \begin{pmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{pmatrix} \qquad = \sum_j \arctan\lambda_j$$


Results
Theorem
$f_i : \mathbb{R}^n \to \mathbb{R}$ Lipschitz


Results
Theorem (Huisken-Ecker)
$f_0 : \mathbb{R}^n \to \mathbb{R}$ Lipschitz
Then (GMCF) has L.T. sol^n $\ni f(x,t) \to f_0(x)$
smooth

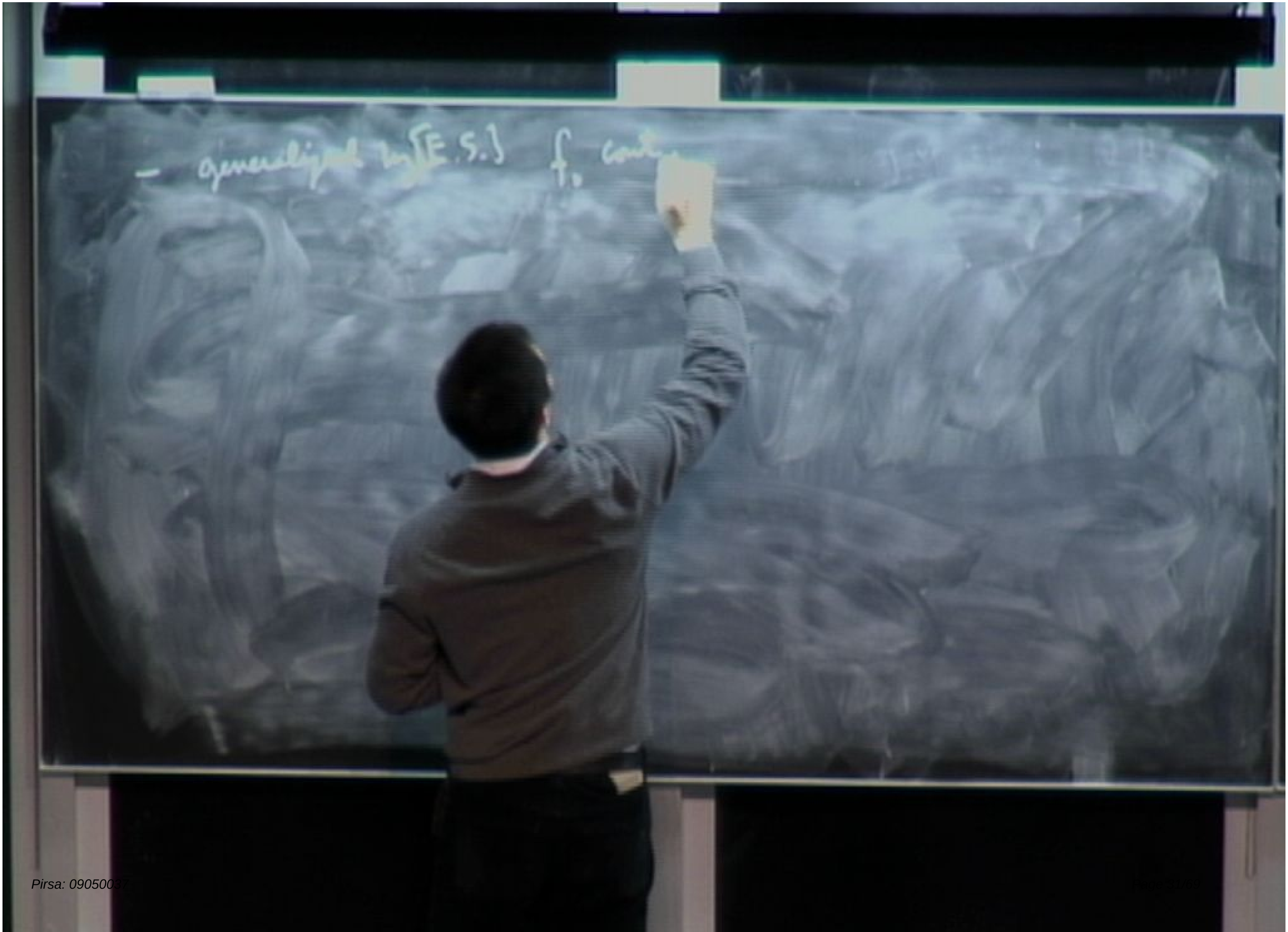
Results

Theorem (Huisken-Ecker)

$f_0 : \mathbb{R}^n \to \mathbb{R}$ Lipschitz

Then (GMCF) has L.T. sol^n (smooth) $\to f(x,t) \to f_0(x)$

Moreover, $|Df_0| \le C$ then $|D^l f|^2 \le \frac{C_l}{t^{l-1}}$ $\quad l \ge 2$


– generalized by [E.S.] f_0 cont

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– $\exists$ minimal Lipschitz $(x, f(x)) \subset \mathbb{R}^7$

$f : \mathbb{R}^4 \to \mathbb{R}^3$

[L.O.].

Theorem (Wang)

$\Sigma \subset M$, suff. small Lipschitz norm.

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(MCF) has s.t. smooth sol^n.

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then (MCF) has s.t. smooth sol$^{\underline{n}}$.

Theorem (Yuan)

If $u: \mathbb{R}^n \to \mathbb{R}$ smooth $\Rightarrow -I_D \le D^2 u \le I_D$

$F(D^2 u) = \text{const.}$

- generalized by [E.S.] f_0 cont.

- $\exists$ minimal Lipschitz $(x, f(x)) \subseteq \mathbb{R}^n$
  $f : \mathbb{R}^4 \to \mathbb{R}^3$
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Theorem (Wang)

$\Sigma \subset M$ compact, suff. small Lipschitz norm.

then (MCF) has s.t. smooth sol^{n}.

Theorem (Yuan)

If $u : \mathbb{R}^n \to \mathbb{R}$ smooth, $\Rightarrow -ID \le D^2u \le ID$

$F(D^2u) = $ const.

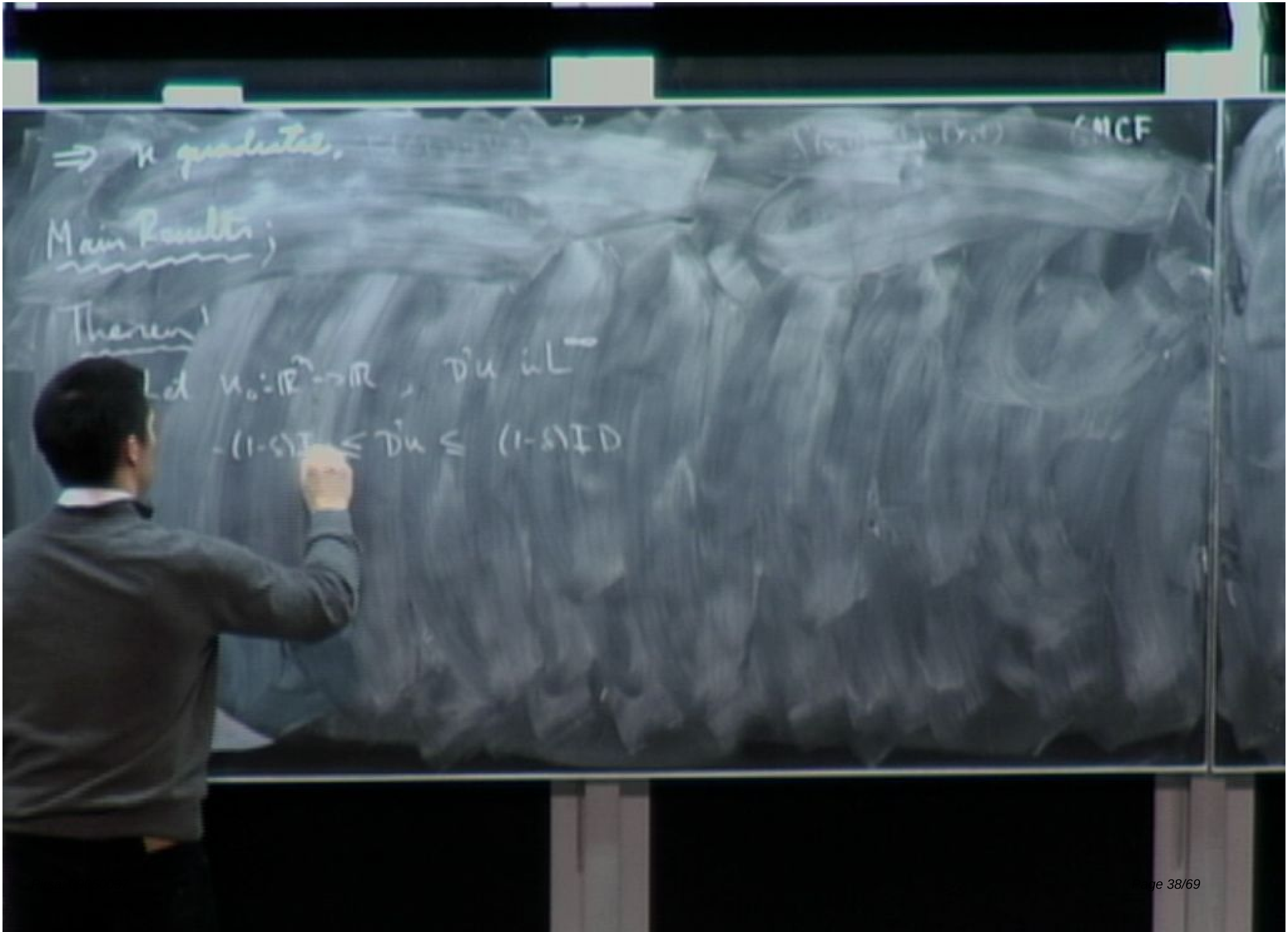
$\Rightarrow$ M quadratic, MCF

Main Resul

$\Rightarrow$ u quadratic, ... (MCF

Main Results;

Theorem:


⇒ u quadratic,
GMCF
Main Results;
Theorem
Let u0 : R^n → R , D²u in L^∞
-(1-δ)I ≤ D²u ≤ (1-δ)I D

$\Rightarrow$ u graduated,

MCF

Main Results;

Theorem:

Let $u_0 : \mathbb{R}^n \to \mathbb{R}$, $D^2 u$ in L^∞

$\delta \in (0,1)$; $-(1-\delta) I D \le D^2 u \le (1-\delta) I D$

Then $\begin{cases} \frac{du}{dt} = F(D^2 u) \\ u(x,0) = u_0 \end{cases}$

has L.T. sol'n $u(x,t) \in C^\infty((0,\infty)\times\mathbb{R}^n)$

$\Rightarrow$ u quadratic.

Main Results;

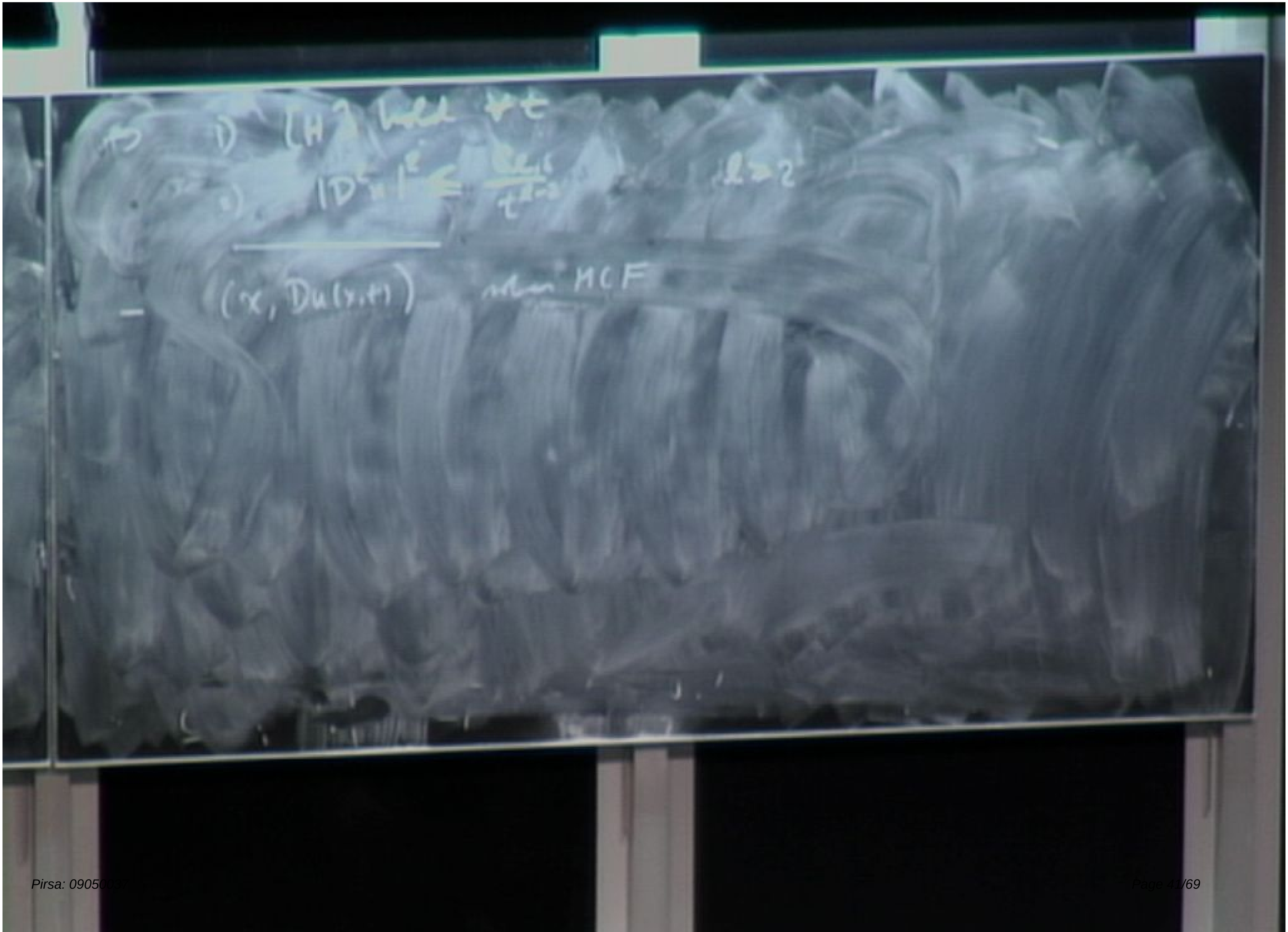
Theorem:

Let $u_0 : \mathbb{R}^n \to \mathbb{R}$, $D^2 u$ in L^∞

$\delta \in (0,1)$; $\quad -(1-\delta) ID \leq D^2 u \leq (1-\delta) ID$

Then
$$\begin{cases} \frac{\partial u}{\partial t} = F(D^2 u) \\ u(x,0) = u_0 \end{cases}$$

has C.T. sol'n $u(x,t) \in C^\infty((0,\infty)\times\mathbb{R}^n) \cap C^{1,1,1}_{loc}([0,\infty)\times\mathbb{R}^n)$


1) [H] hold ∀t
2) |D^ℓ u|^2 ≤ … ℓ ≥ 2
(x, Du(x,t)) … MCF

1) $\{H\}$ hold $\forall t$

2) $|D^\ell u|^2 \leq \frac{C_{\ell,s}}{t^{\ell-2}}$ $\ell \geq 2$

- $(x, Du(x,t))$ soln MCF
- T.Y. Chen - Chao. P. established existence unique cont. visc. sol $\leq$.

1) $[H]$ hold $\forall t$

2) $|D^l u|^2 \leq \frac{C_{l,1}}{t^{l-2}}$, $l \geq 2$

— $(x, Du(x,t))$ solves MCF

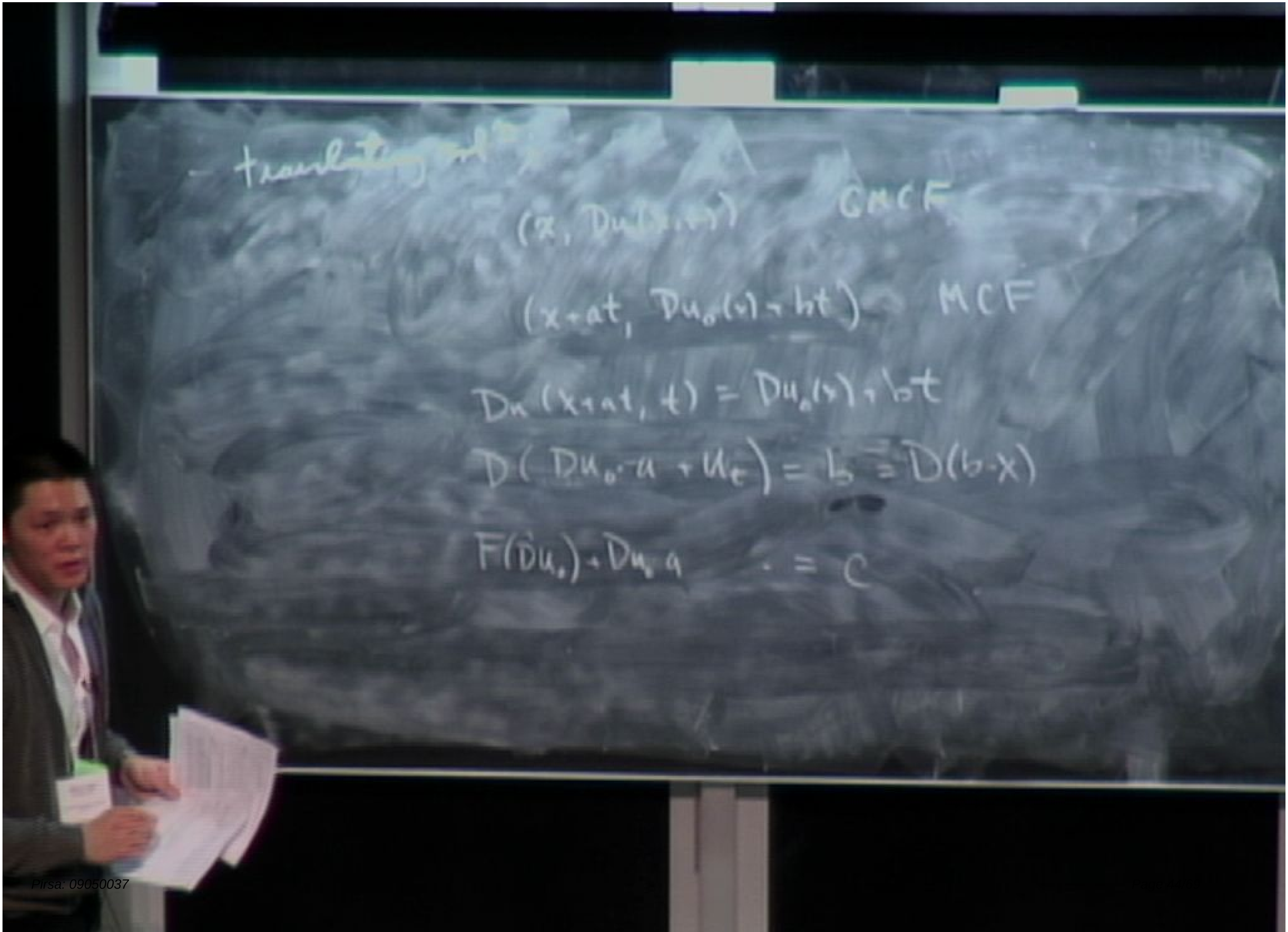
— T.Y. Chen - Chao. P. established existence & uniqueness cont. visc. sol^{n}s.

Theorem 2

If $u: \mathbb{R}^n \to \mathbb{R}$, some a, b, c.

$$F(D^2 u) + Du \cdot a - b \cdot x = c$$

then u quadratic.



- translating sol
(x, Du(x, t)) GMCF
(x + at, Du_0(x) + bt) MCF
Du(x + at, t) = Du_0(x) + bt
D(Du_0 · a + u_t) = b = D(b · x)
F(Du_0) + Du_0 · a = C

– translating sol

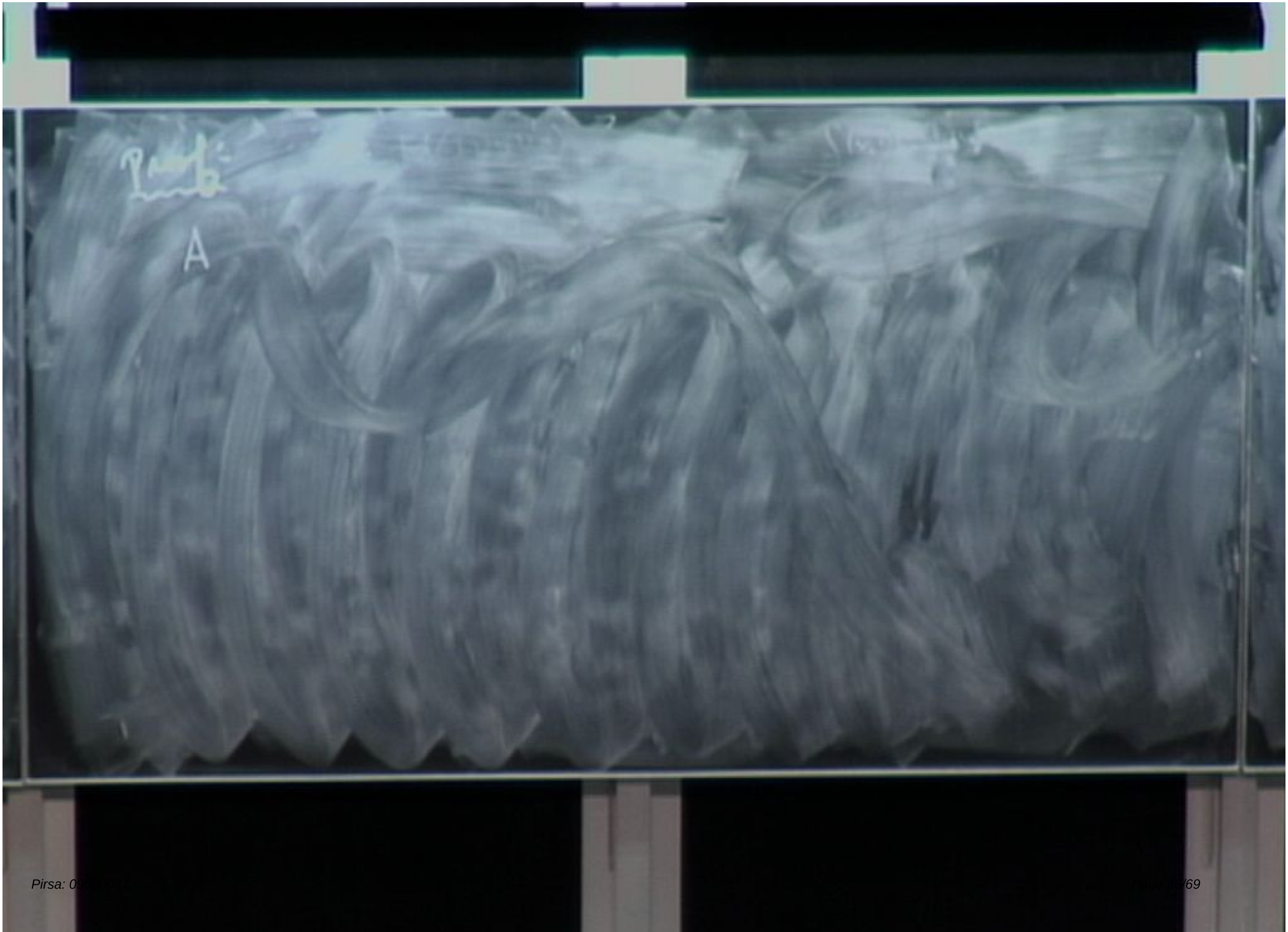
$(x, Du(x,t))$ GMCF

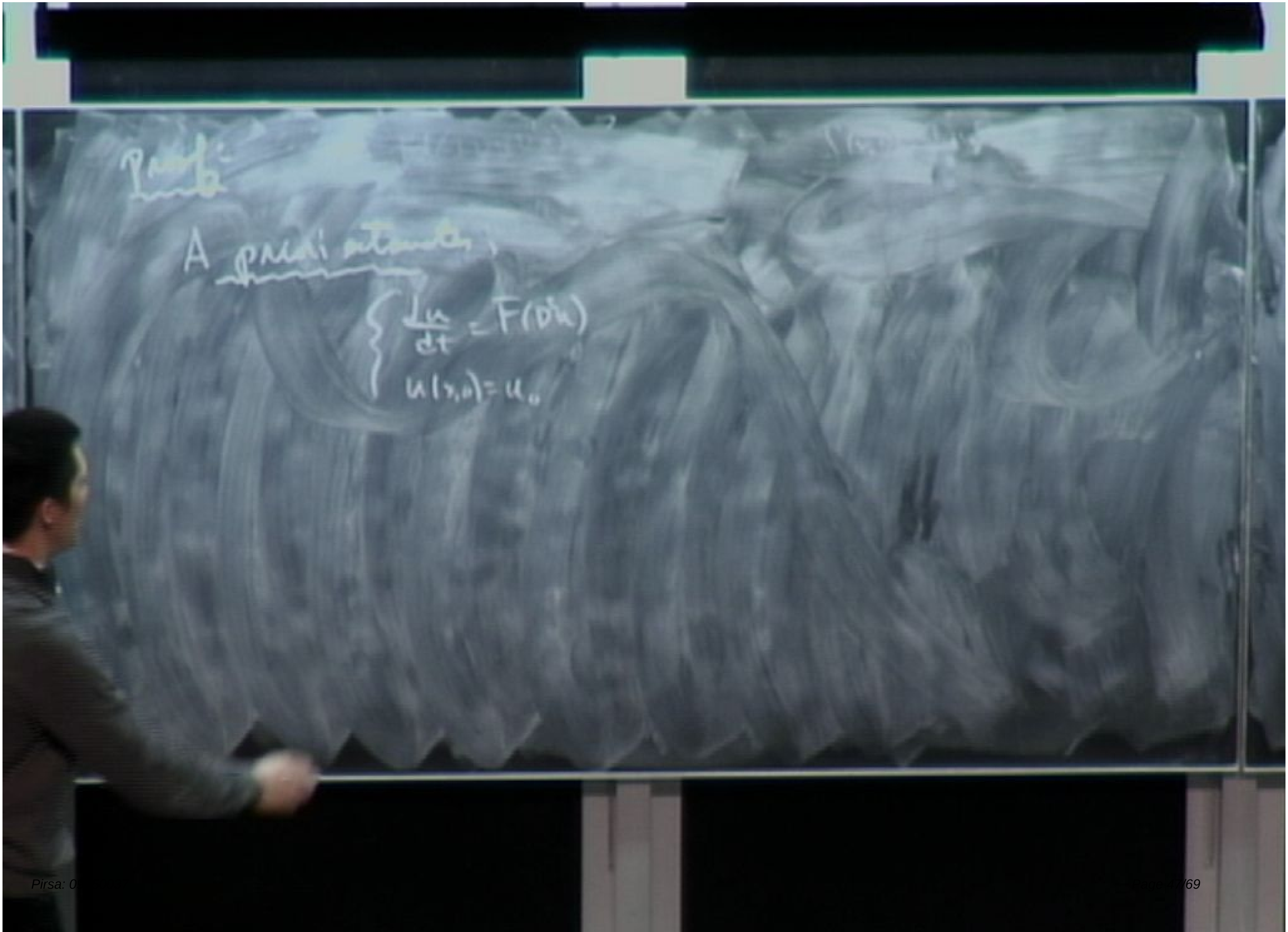
$(x+at, Du_0(x)+bt)$ MCF

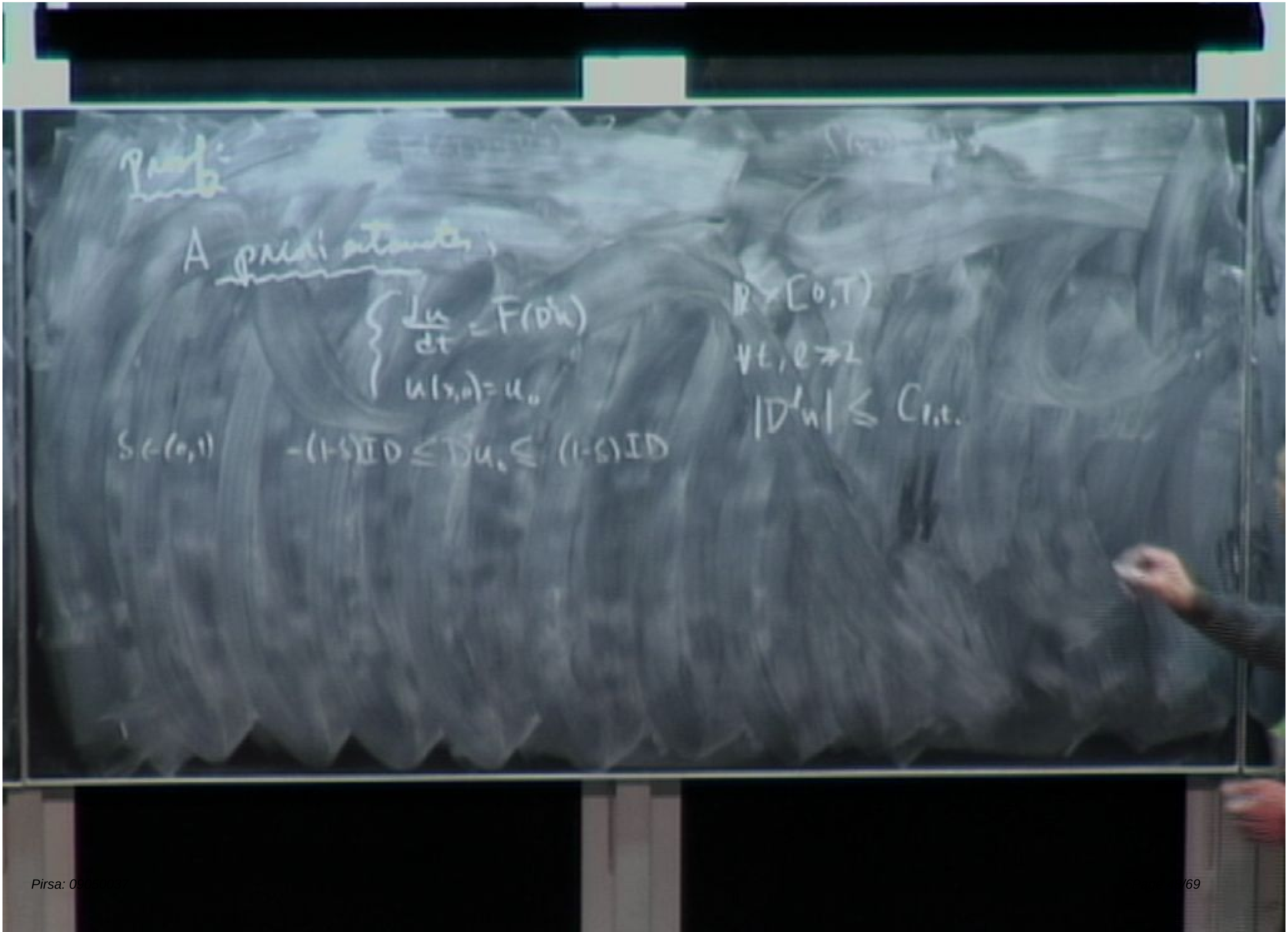
$$Du(x+at, t) = Du_0(x) + bt$$

$$D(Du_0 \cdot a + u_t) = b x = D(b \cdot x)$$

$$\boxed{F(Du_0) + Du_0\, a - b \cdot x = C}$$


Proof:
A


Proof:
A priori estimates,
$\begin{cases} \frac{du}{dt} = F(D^2u) \\ u(x,0) = u_0 \end{cases}$


Proof:
A priori estimates:
$\begin{cases} \frac{du}{dt} = F(D^2u) \\ u(x,0) = u_0 \end{cases}$
$\mathbb{R} \times [0,T)$
$\forall t, l \geq 2$
$|D^l u| \leq C_{l,t}$
$\delta \in (0,1)$
$-(1-\delta)ID \leq Du_0 \leq (1-\delta)ID$

Proof:

A priori estimates:

$$\begin{cases} \frac{du}{dt} = F(D^2u) \\ u(x,0) = u_0 \end{cases}$$

$\mathbb{R}^n \times [0,T)$

$\forall t,\ \ell \geq 2$

$|D^\ell u| \leq C_{\ell,t}$

$\delta \in (0,1)$ $\quad -(1-\delta)\mathbb{I}_D \leq D^2u_0 \leq (1-\delta)\mathbb{I}_D$

Proof:

A priori estimates:

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$\delta \in (0,1)$ $\quad -(1-\delta)\mathrm{ID} \leq D^2u_0 \leq (1-\delta)\mathrm{ID}$

$F_t(x) = (x, Du(x,t))$

$: \mathbb{R}^n \to \mathbb{C}^n$

g_{ij}

Proof:

A priori estimates:

$$\begin{cases} \dfrac{du}{dt} = F(D^2 u) \\ u(x,0) = u_0 \end{cases} \qquad \mathbb{R}^n \times [0,T)$$

$\forall t,\ l \geq 2$

$$|D^l u| \leq C_{l,t}$$

$\delta \in (0,1)$ $\qquad -(1-\delta)\,\mathrm{ID} \leq D^2 u_0 \leq (1-\delta)\,\mathrm{ID}$

$$F_t(x) = (x, Du(x,t)) : \mathbb{R}^n \to \mathbb{C}^n$$

$$g_{ij} = F^*(ds^2) = F^*(\langle F_i, F_j \rangle)$$

$$S_{ij} = F^*(\langle \bar{v}, w \rangle)$$

Proof:

A priori estimates:

$$\begin{cases} \frac{du}{dt} = F(D^2u) \\ u(x,0) = u_0 \end{cases}$$

$\mathbb{R}^n \times [0,T)$

$\forall t, \ell \geq 2$

$|D^\ell u| \leq C_{\ell,t}$

$\delta \in (0,1)$ $\quad -(1-\delta)\mathbb{I}D \leq D^2u_0 \leq (1-\delta)\mathbb{I}D$

$$F_t(x) = (x, Du(x,t))$$

$$: \mathbb{R}^n \to \mathbb{C}^n$$

$$g_{ij} = F^*(ds^2)$$

$$= F^*(\langle F_i, F_j \rangle)$$

$$S_{ij} = F^*(\langle \bar{v}, w \rangle)$$

$$= F^*(\langle \bar{F}_i, F_j \rangle)$$

Proof:

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$$\begin{cases} \frac{du}{dt} = F(D^2u) \\ u(x,0) = u_0 \end{cases}$$

$\mathbb{R}^n \times [0,T)$

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$\delta \in (0,1)$ $\quad -(1-\delta)\mathbb{I}D \leq D^2u_0 \leq (1-\delta)\mathbb{I}D$

$F_t(x) = (x, Du(x,t))$

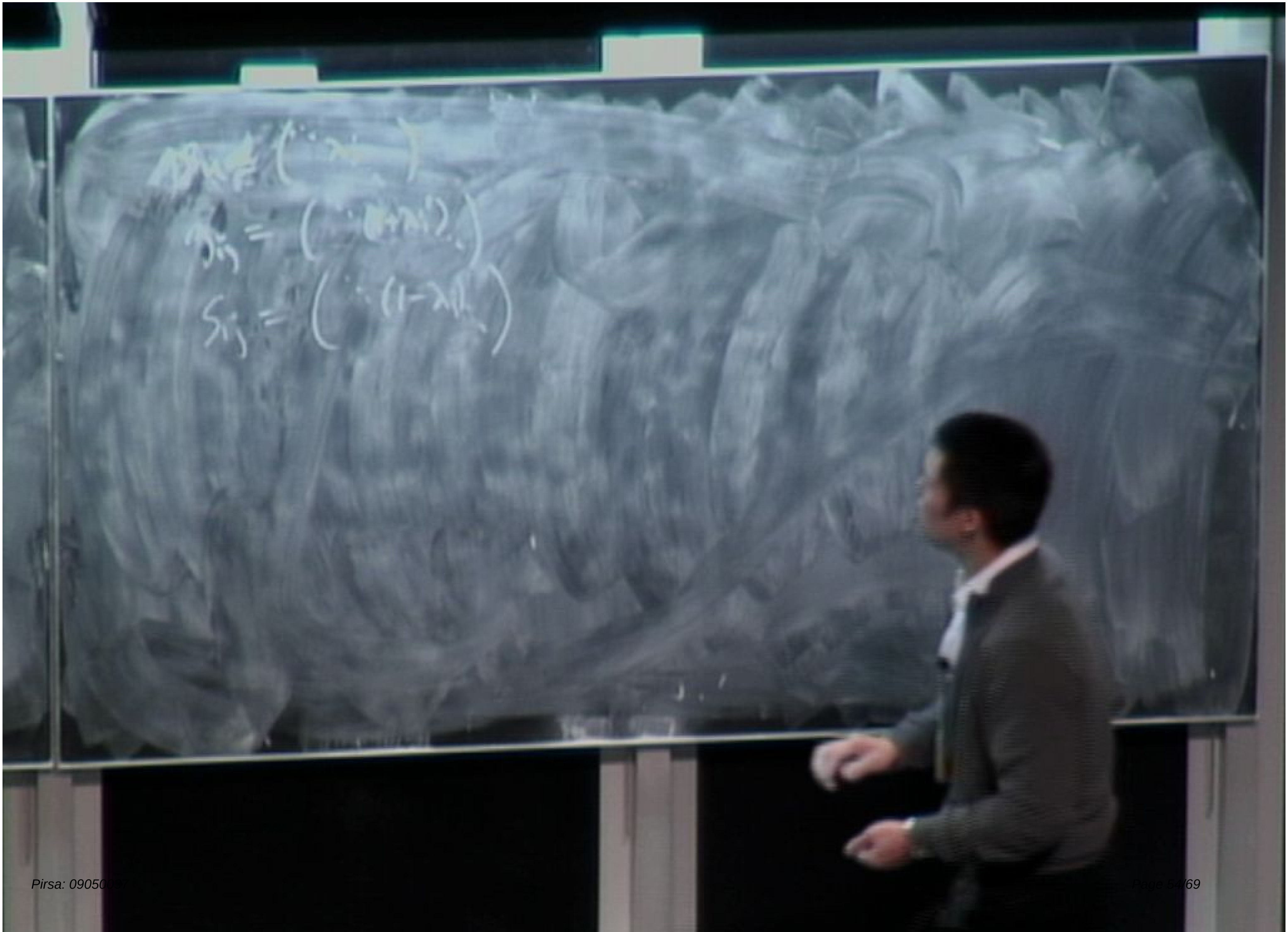
$: \mathbb{R}^n \to \mathbb{C}^n$

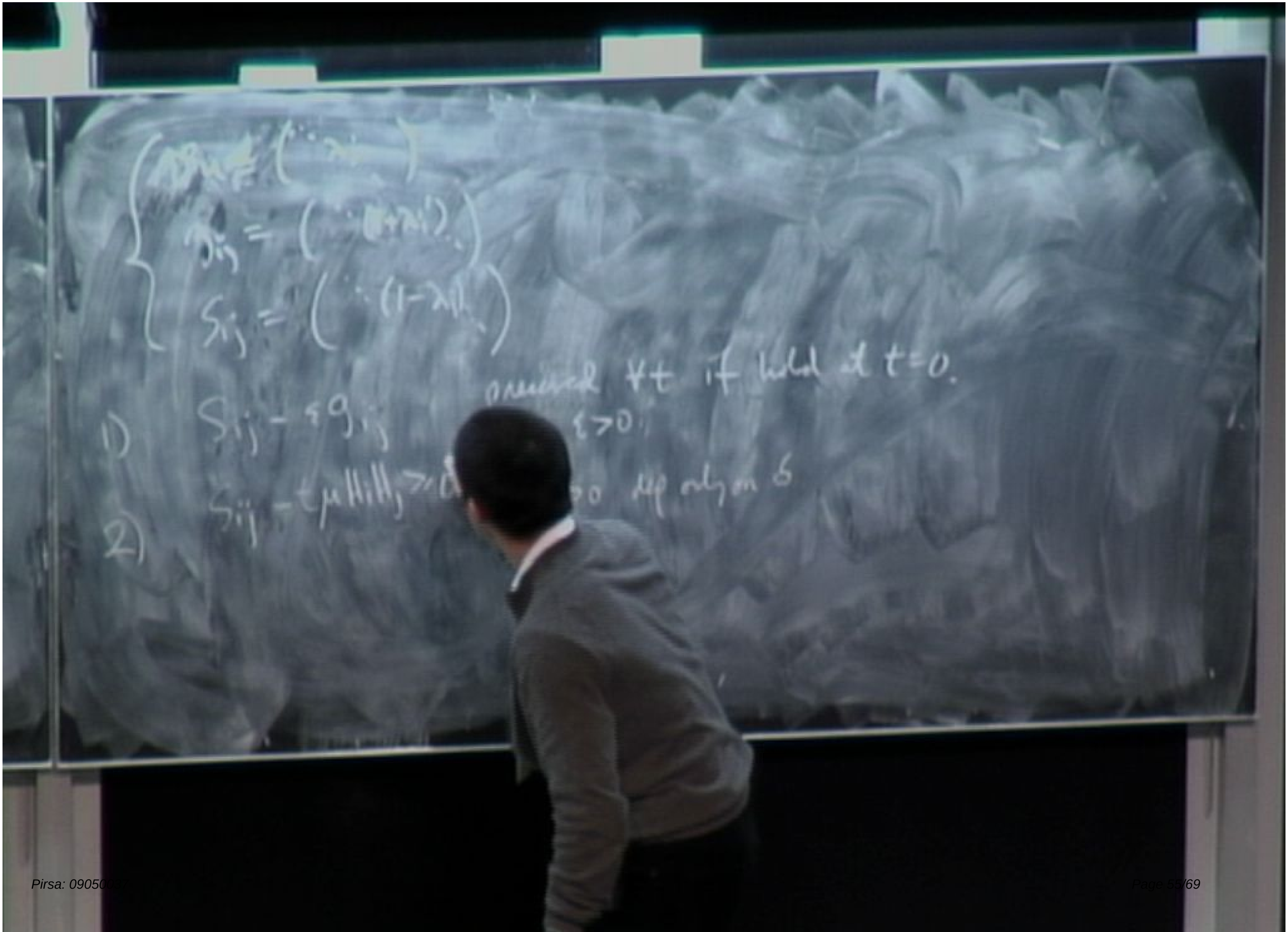
$g_{ij} = F^*(ds^2)$

$= F^*(\langle F_i, F_j \rangle)$

$S_{ij} = F^*(\langle \bar{V}, W \rangle)$

$= F^*(\langle \bar{F}_i, F_j \rangle)$


$S_{ij} = \left(\ \ (1+\lambda_i) \ \ \right)$
$S_{ij} = \left(\ \ (1-\lambda_i) \ \ \right)$



$S_{ij} = (\cdots (1-\lambda_i) \cdots)$
preserved ∀t if hold at t=0.
1) $S_{ij} - \varepsilon g_{ij}$
$\varepsilon > 0$
2) $S_{ij} - t\mu \|l\|_j \geq 0$
dep only on δ

$$\begin{cases} \cancel{g_{ij}} = \begin{pmatrix} \ddots & & \\ & \lambda_i & \\ & & \ddots \end{pmatrix} \\ g_{ij} = \begin{pmatrix} \ddots & & \\ & (1+\lambda_i) & \\ & & \ddots \end{pmatrix} \\ S_{ij} = \begin{pmatrix} \ddots & & \\ & (1-\lambda_i) & \\ & & \ddots \end{pmatrix} \end{cases}$$

1) $S_{ij} - \varepsilon g_{ij} \geq 0$ preserved $\forall t$ if held at $t=0$. some $\varepsilon > 0$;

2) $S_{ij} - t\mu \|\|\|\|_j \geq 0$ $\exists\, \mu > 0$ dep only on δ

$$\begin{cases} \cdots = \left(\ddots\ \lambda_i\ \ddots\right) \\ g_{ij} = \left(\ddots\ (1+\lambda_i)\ \ddots\right) \\ S_{ij} = \left(\ddots\ (1-\lambda_i)\ \ddots\right) \end{cases}$$

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some $\varepsilon > 0$.

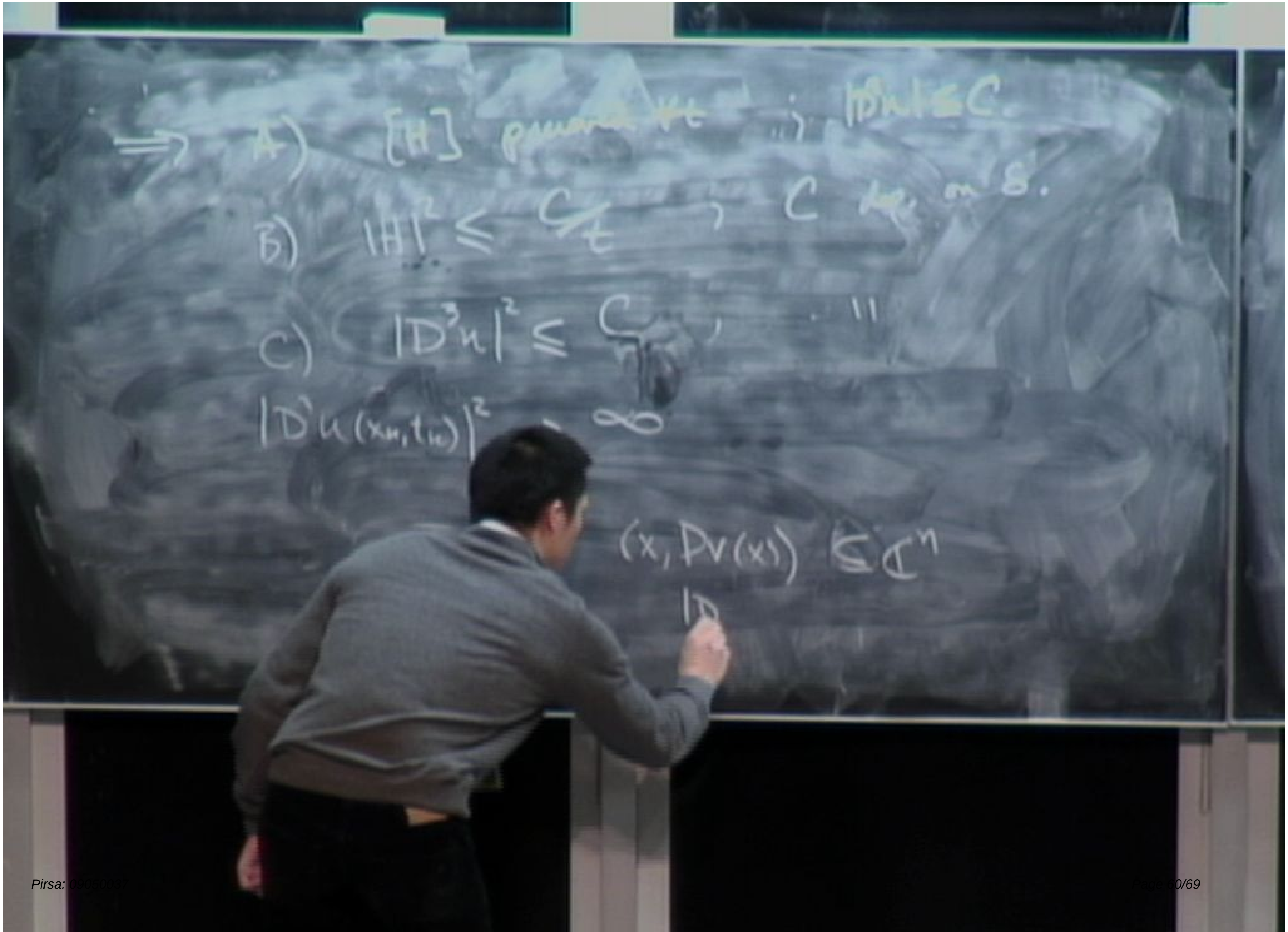
2) $S_{ij} - t\mu \|\!\|_{ij} \geq 0$ $\exists\, \mu > 0$ dep only on δ

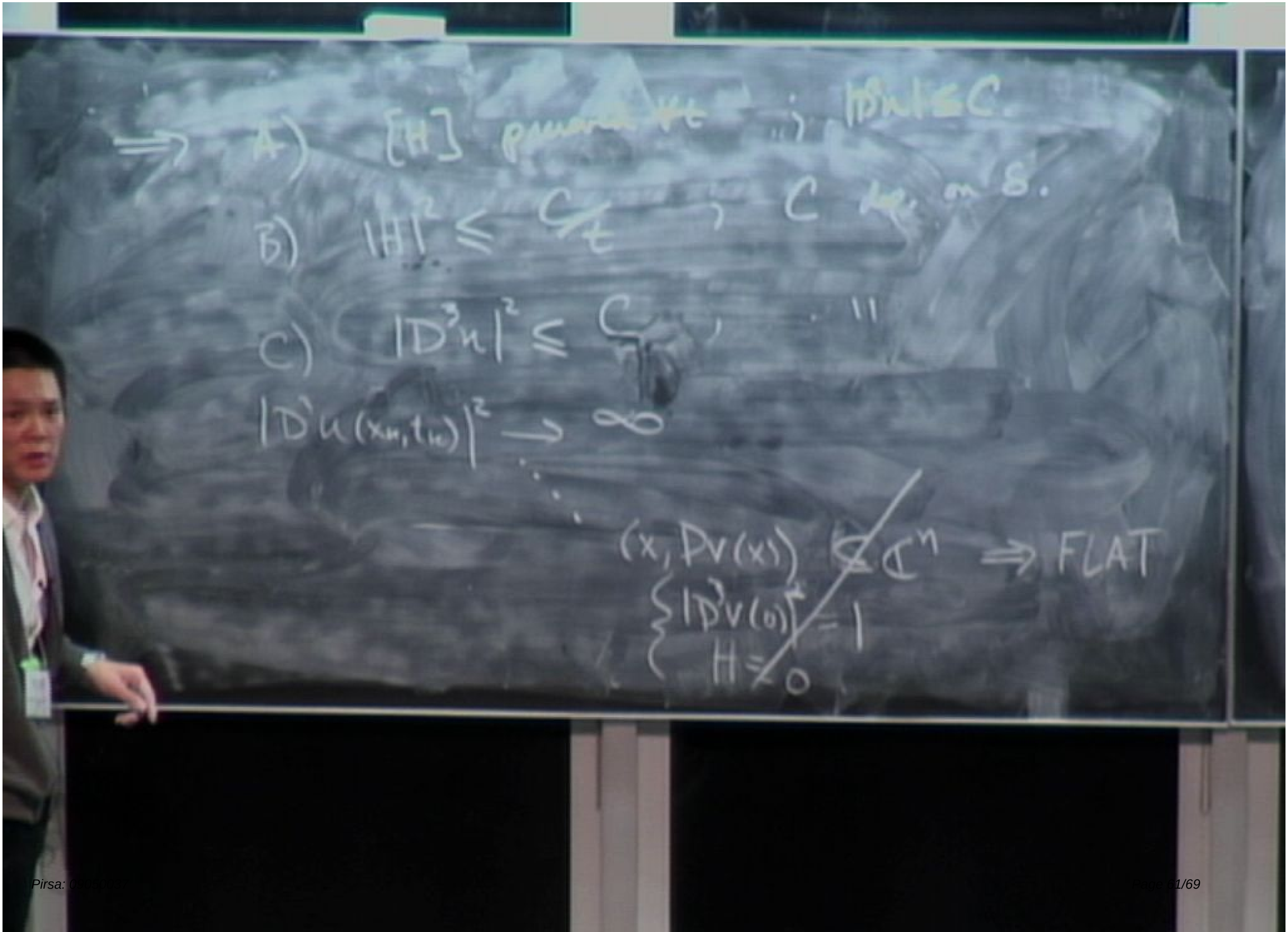
$\Rightarrow$ A) $[H]$ preserved $\forall t$; $|B_t| \leq C$.

B) $|H|^2 \leq C/t$; C

$\Rightarrow$ A) $[H]$ preserved $\forall t$; $|\nabla^n h| \leq C$.

B) $|H|^2 \leq C/t$; C dep. on g.


⟹ A) [H] preserved ∀t ; |D²u| ≤ C.
B) |H|² ≤ C/t , C dep. on δ.
C) |D³u|² ≤ C/t² , "
|D²u(x_k, t_k)|² → ∞
(x, Dv(x)) ⊆ ℂⁿ
|D


⇒ A) [H]
B) |H|² ≤ C/t ; C
C) |D³u|² ≤ C
|D³u(xₖ, tₖ)|² → ∞
(x, Dv(x)) ⇒ FLAT
|D³v(0)|² = 1
H = 0

$\Rightarrow$ A) $[H]$ preserved ; $|D^k u| \le C$.

B) $|H|^2 \le C/t$; C dep. on ε.

C) $|D^2 u|^2 \le C/t$; "

$|D^2 u(x_k, t_k)|^2 \to \infty$

$\vdots$

$(x, Dv(x)) \subset \mathbb{R}^n \Rightarrow$ FLAT

$\begin{cases} |D^2 v(0)|^2 = 1 \\ H = 0 \end{cases}$

$\Longrightarrow$ A) $[H]$ preserved $\forall t$; $|D^2 u| \le C$.

B) $|H|^2 \le C/t$, C dep. on δ.

C) $|D^3 u|^2 \le C/t$, "

$|D^3 u(x_k, t_k)|^2 \longrightarrow \infty$

$\Longrightarrow$ D) $|D^\ell u|^2 \le C_\ell / t^{\ell-2}$

$(x, Dv(x)) \subset \mathbb{C}^n \Longrightarrow$ FLAT

$\begin{cases} |D^3 v(0)|^2 = 1 \\ H = 0 \end{cases}$

Proof:

A priori estimates

$$\begin{cases} \frac{du}{dt} = F(D^2u) \\ u(x,0) = u_0 \end{cases} \qquad \mathbb{R}^n \times [0,T)$$

$\forall t,\ \ell \geq 2$

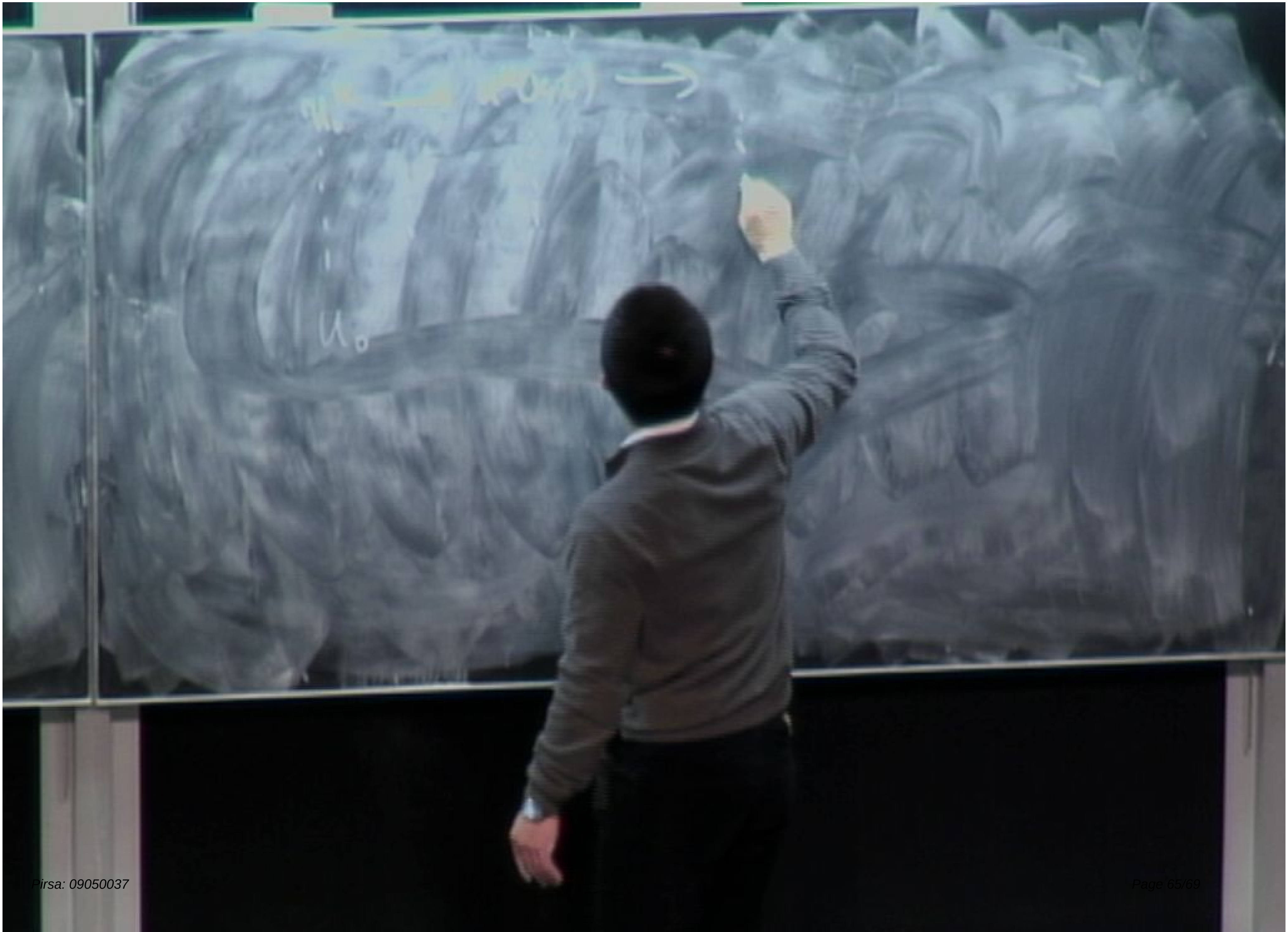
$$|D^\ell u| \leq C_{\ell,t}$$

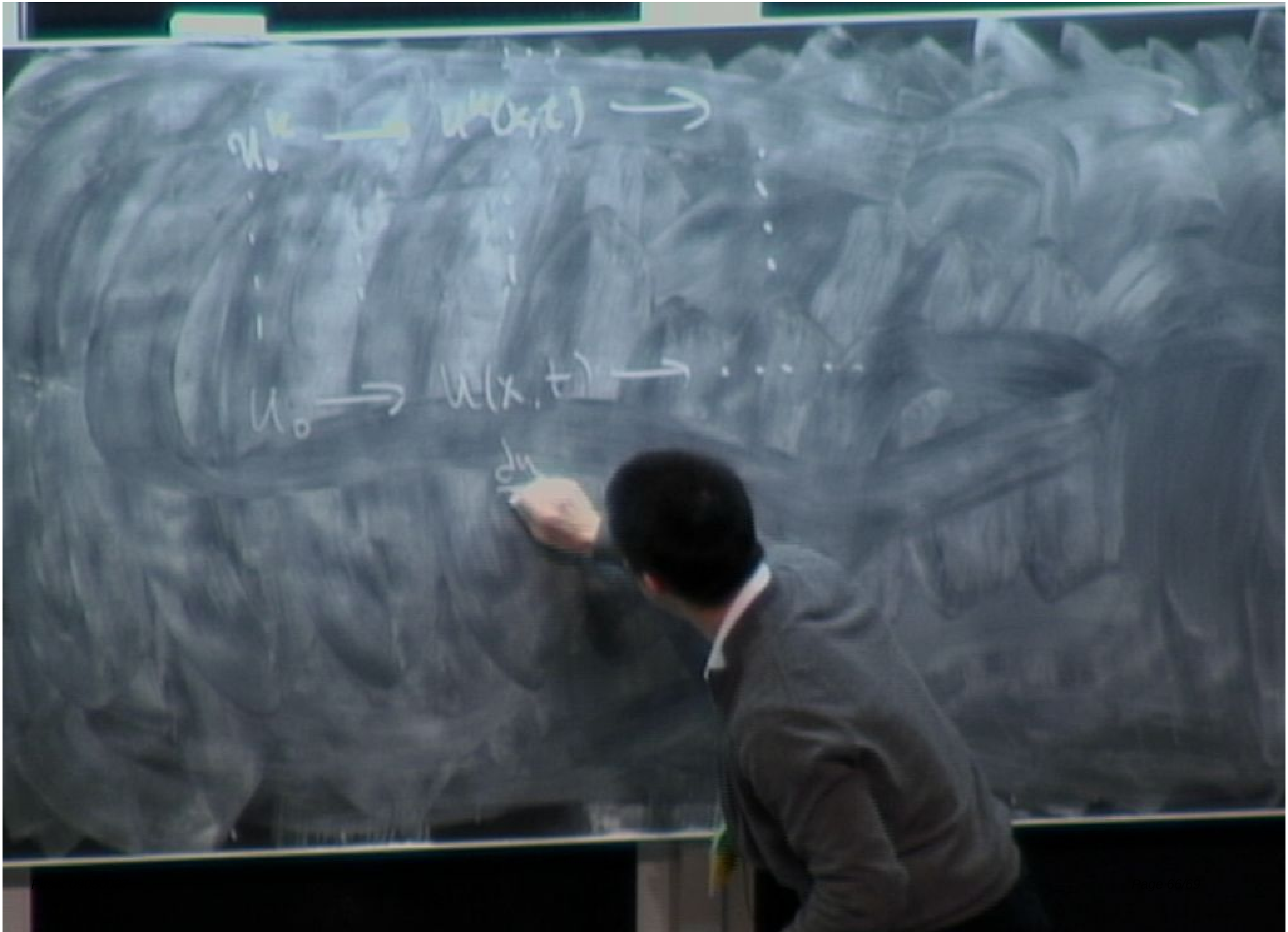
$\delta \in (0,1)$ $\quad -(1-\delta)\mathbb{I}D \leq D^2u_0 \leq (1-\delta)\mathbb{I}D$

$$F_t(x) = (x, Du(x,t)) : \mathbb{R}^n \to \mathbb{C}^n$$

$$g_{ij} = F^*(ds^2) = F^*(\langle F_i, F_j \rangle)$$

$$S_{ij} = F_t^*(\langle \bar{v}, w \rangle) = F_t^*(\langle \bar{F}_i, F_j \rangle)$$




u_0^k → u^k(x,t) →
u_0 → u(x,t) → . . . . . .
du

$$u_0^k \longrightarrow u^k(x,t) \longrightarrow$$

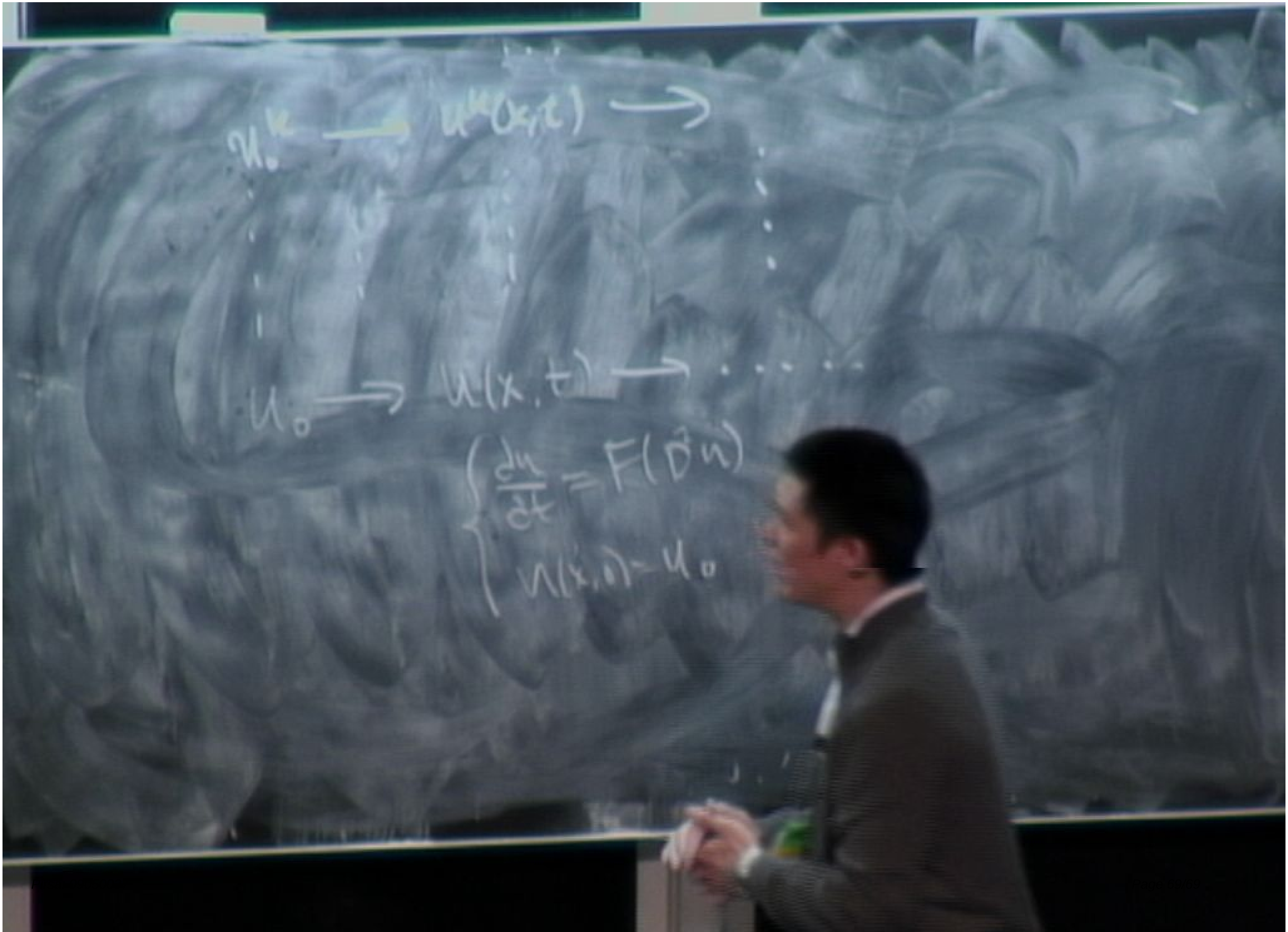
$$u_0 \longrightarrow u(x,t) \longrightarrow \cdots\cdots$$

$$\begin{cases} \dfrac{\partial u}{\partial t} = F(D^2 u) \\ u(x,0) = u_0 \end{cases}$$

$$u_0^k \longrightarrow u^k(x,t) \longrightarrow$$

$$u_0 \longrightarrow u(x,t) \longrightarrow \cdots$$

$$\begin{cases} \dfrac{\partial u}{\partial t} = F(\vec{D} u) \\ u(x,0) = u_0 \end{cases}$$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