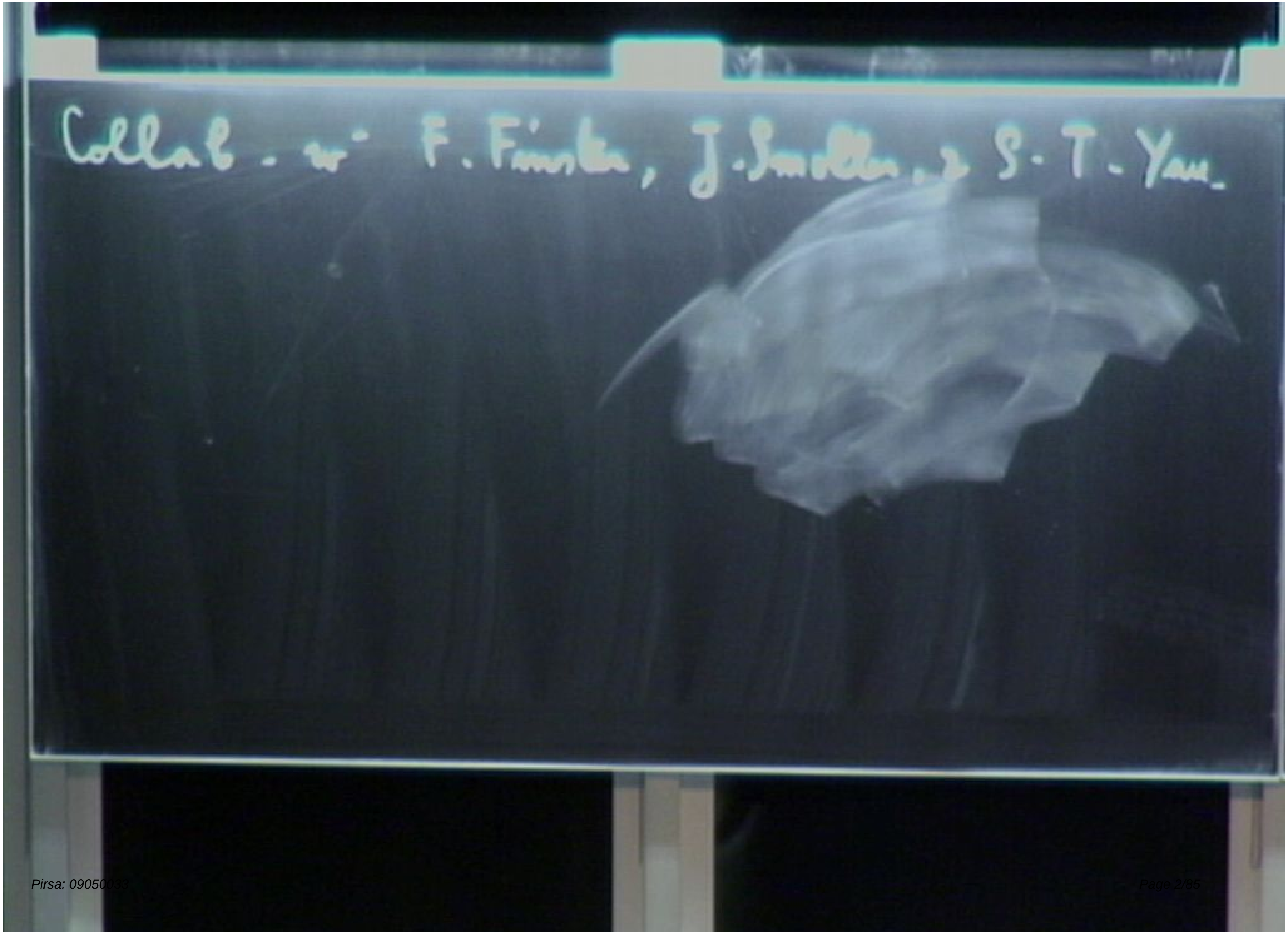
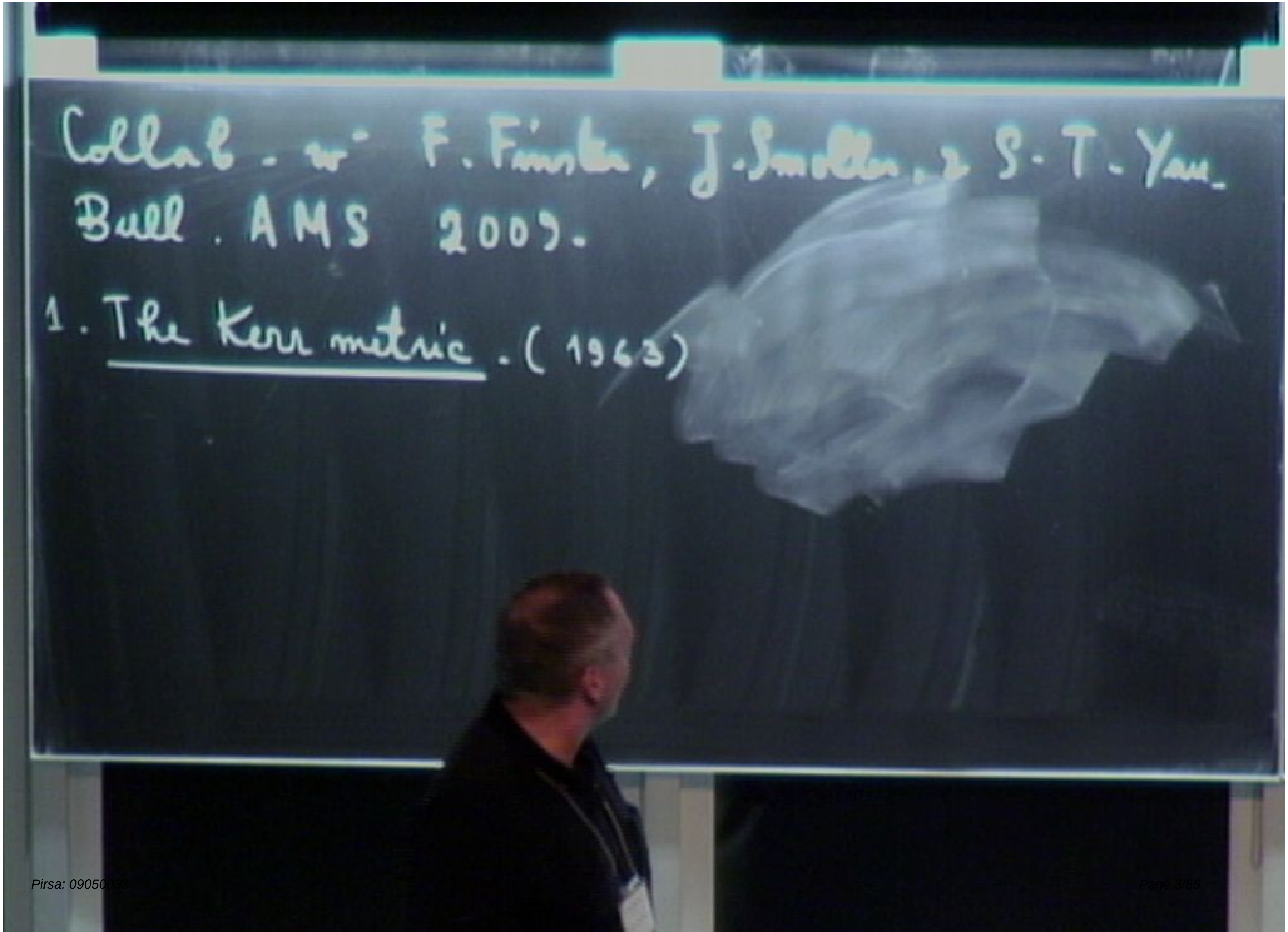
Title: Wave equations in Kerr geometry

Date: May 09, 2009 10:00 AM

URL: http://pirsa.org/09050033

Abstract: Quite a bit of progress has been achieved over the past seven years in understanding from a rigorous mathematical perspective the long time dynamics of waves in the Kerr geometry of a rotating black hole in equilibrium. A proof of the Penrose process for scalar waves has notably been given in this context. I will review some of these results, obtained in collaboration with Felix Finster, Joel Smoller and Shing-Tung Yau. I will also indicate a number of open problems.


Collab. w/ F. Finster, J. Smoller, & S.-T. Yau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Bull. AMS 2009.
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$$ds^2 = \frac{\Delta}{U}\left(dt - a\sin^2\theta\, d\varphi\right)^2 - U\left(\frac{dr^2}{\Delta} + d\theta^2\right) - \frac{\sin^2\theta}{U}\left(a\,dt - (r^2+a^2)\,d\varphi\right)^2$$

$$\Delta(r) := r^2 - 2Mr + a^2, \quad \Sigma(r,\theta) := r^2 + a^2\cos^2\theta$$

$$M > 0, \quad a^2 < M^2$$

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$$Ric(g) = 0$$

M: mass of the black hole

a: ang. mom. per unit mass.

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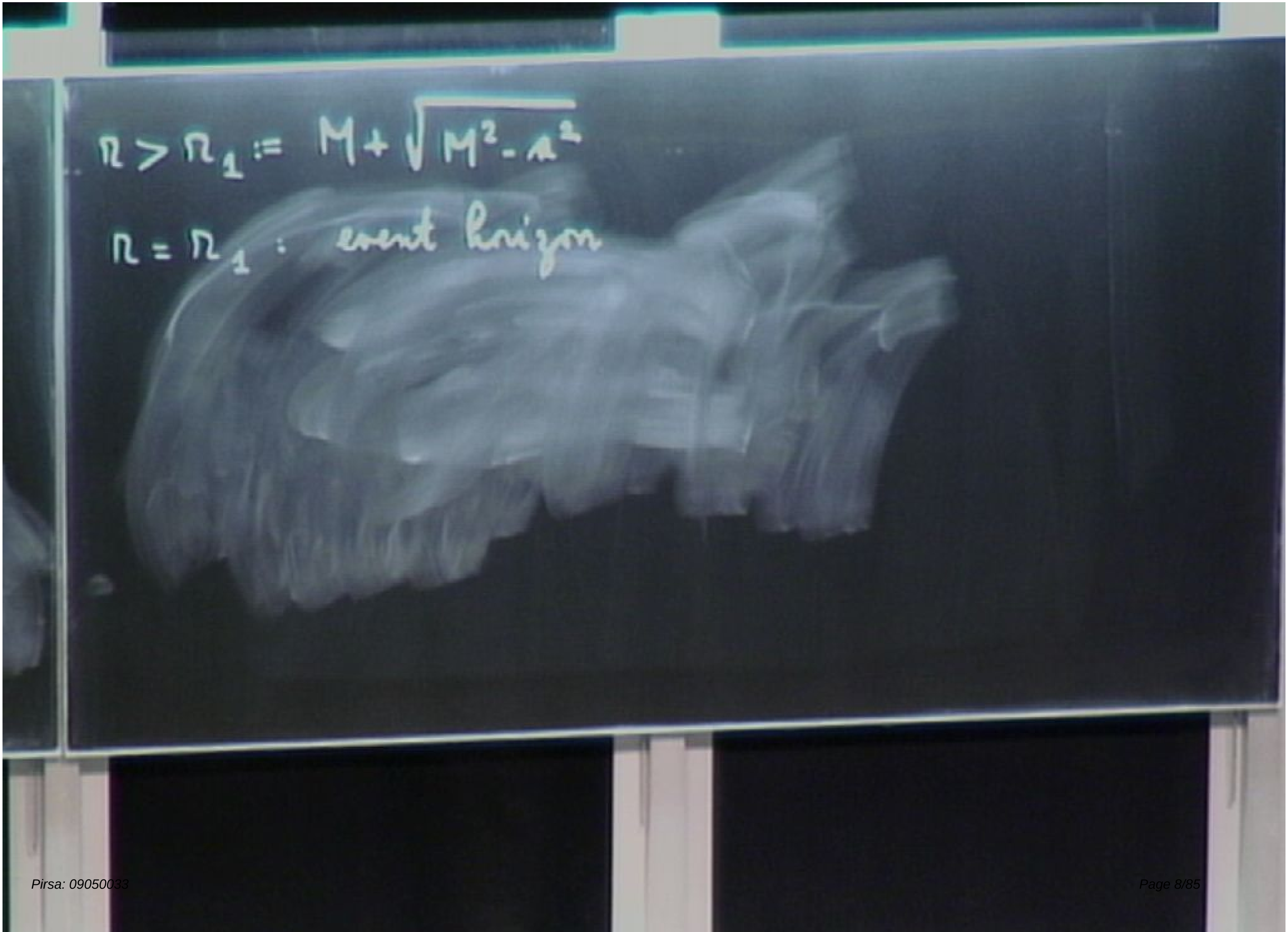
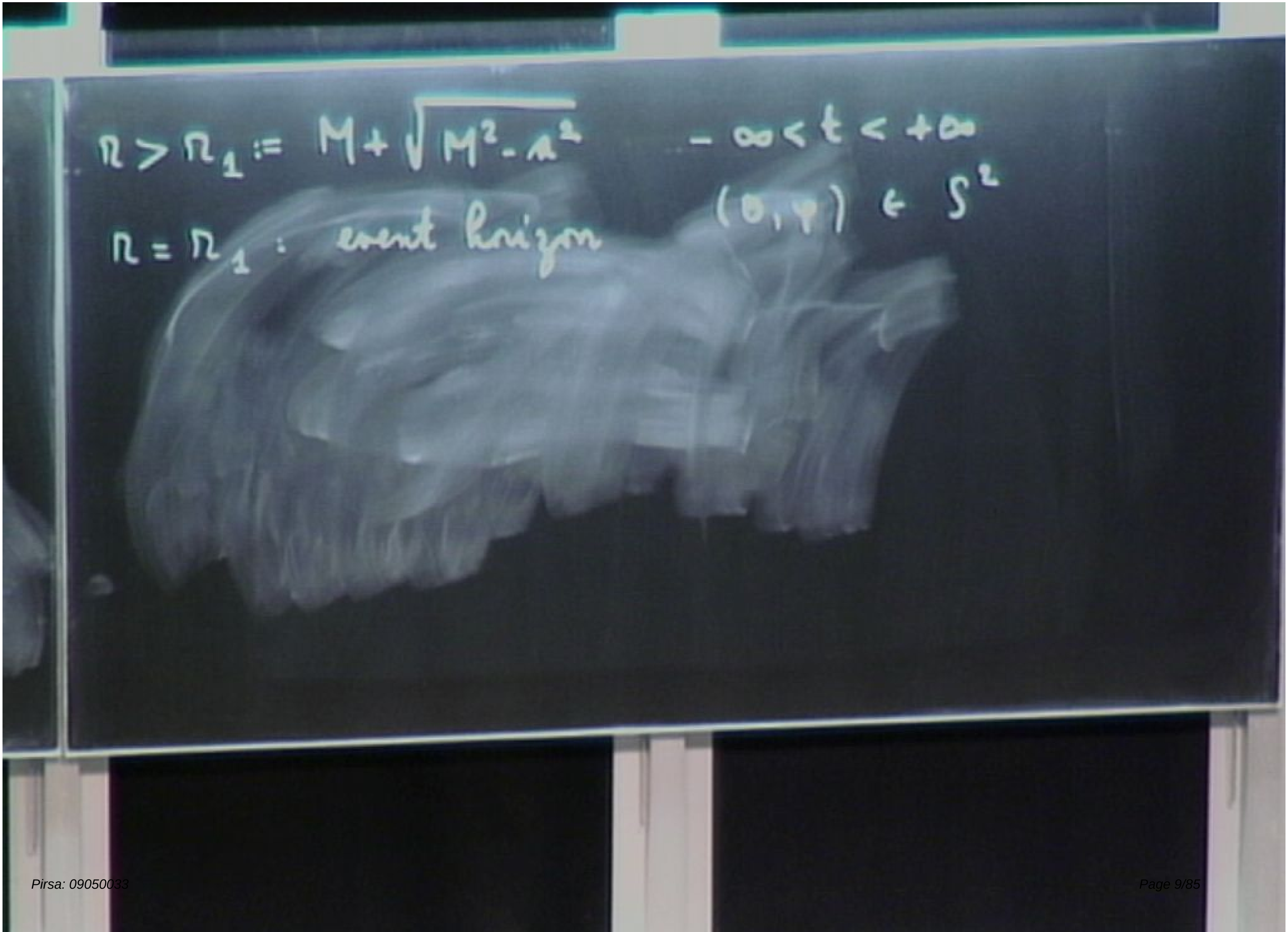

$r > r_1 := M + \sqrt{M^2 - a^2}$
$r = r_1$: event horizon


r > r_1 := M + √(M² - a²)
- ∞ < t < + ∞
(θ, φ) ∈ S²
r = r_1 : event horizon

$$r > r_1 := M + \sqrt{M^2 - a^2} \qquad -\infty < t < +\infty$$

$r = r_1$: event horizon $\qquad (\theta, \varphi) \in S^2$

$M \simeq \mathbb{R}^2 \times S^2$, event horizon $\simeq \mathbb{R} \times S^2$.

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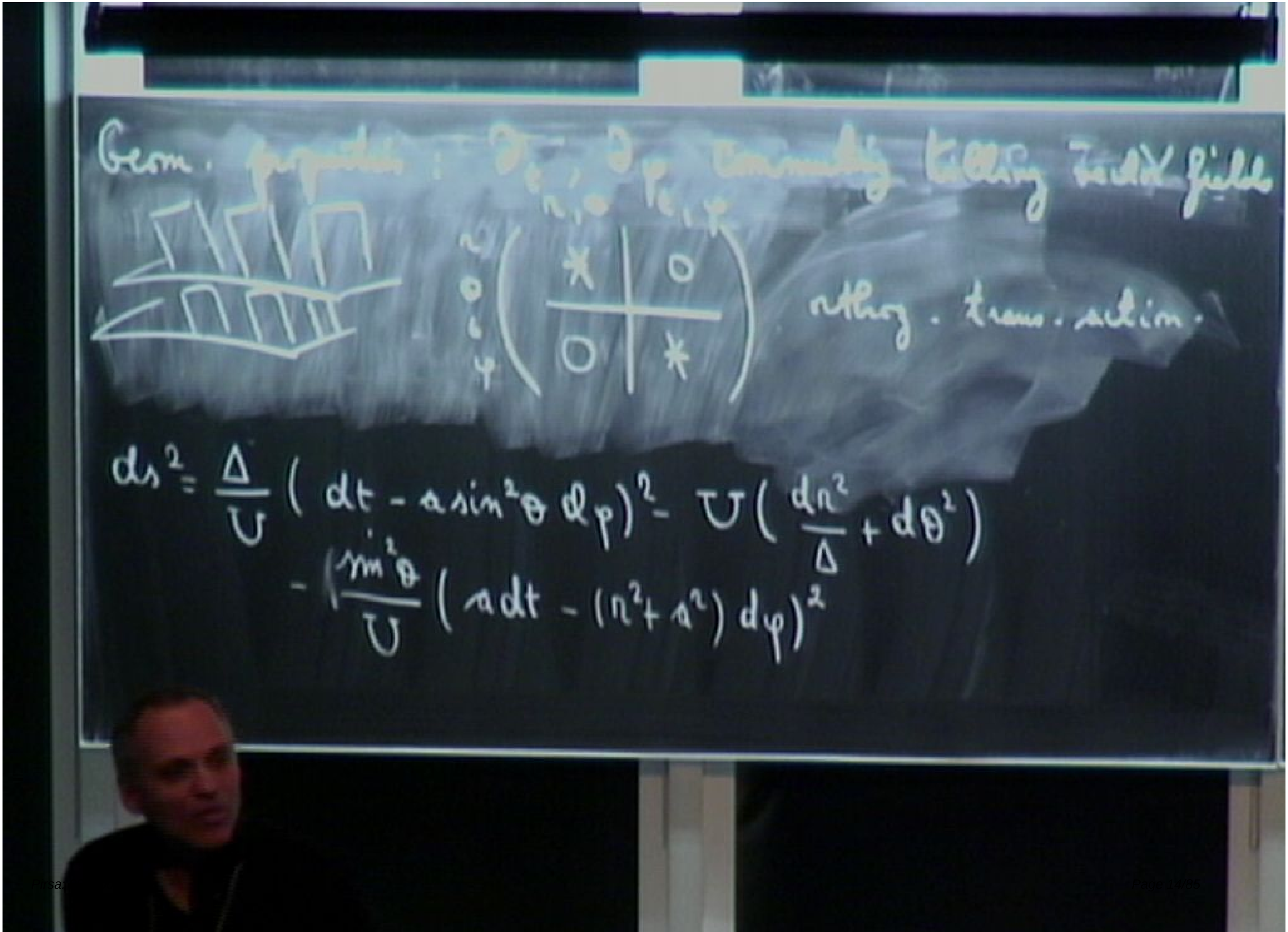
Geom. properties: $\partial_t, \partial_\varphi$ commuting Killing vector fields
r, θ t, φ
orthog. trans. action
$ds^2 = \frac{\Delta}{U}\left(dt - a\sin^2\theta\, d\varphi\right)^2 - U\left(\frac{dr^2}{\Delta} + d\theta^2\right) - \frac{\sin^2\theta}{U}\left(a\,dt - (r^2+a^2)\,d\varphi\right)^2$

Geom. properties: $\partial_t, \partial_\varphi$ commuting Killing vector fields

$$\begin{array}{c} r \\ \theta \\ t \\ \varphi \end{array}\left(\begin{array}{c|c} * & 0 \\ \hline 0 & * \end{array}\right) \quad \text{orthog. trans. subm.}$$

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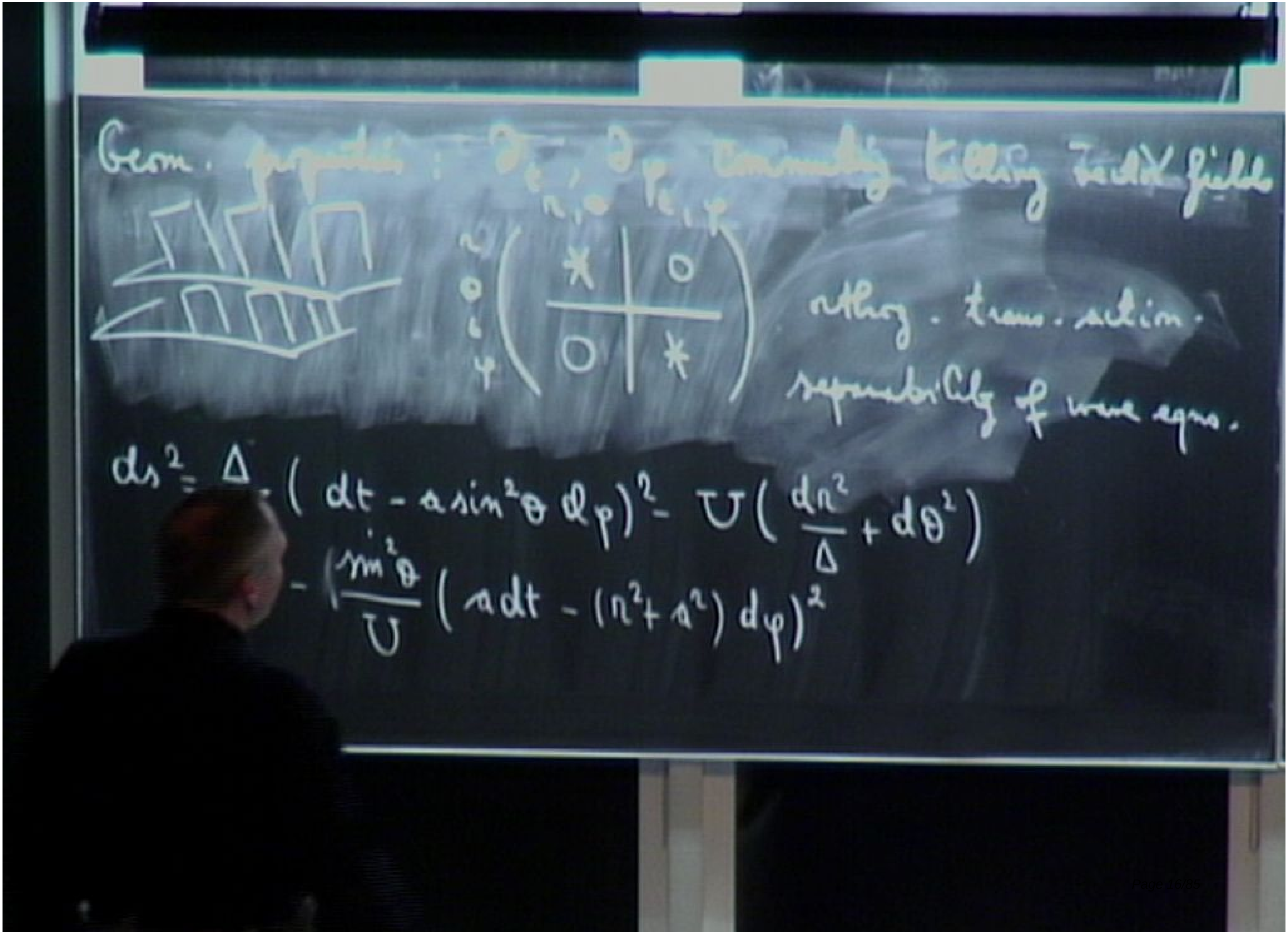
Geom. properties: ∂t, ∂φ commuting Killing vector fields
orthog. trans. action.
separability of wave eqns.
ds² = Δ/U (dt − a sin²θ dφ)² − U (dr²/Δ + dθ²)
− sin²θ/U (a dt − (r² + a²) dφ)²

Geom. properties: ∂_t, ∂_φ commuting Killing vector fields

$\perp_{r,\theta}$ $\quad$ t, φ

$$\begin{matrix} r \\ \theta \\ t \\ \varphi \end{matrix} \left(\begin{array}{c|c} * & 0 \\ \hline 0 & * \end{array} \right)$$

orthog. trans. action.

$\left\{ \begin{array}{l} \text{separability of wave eqns.} \\ \text{geod. flow. integrable} \end{array} \right\}$

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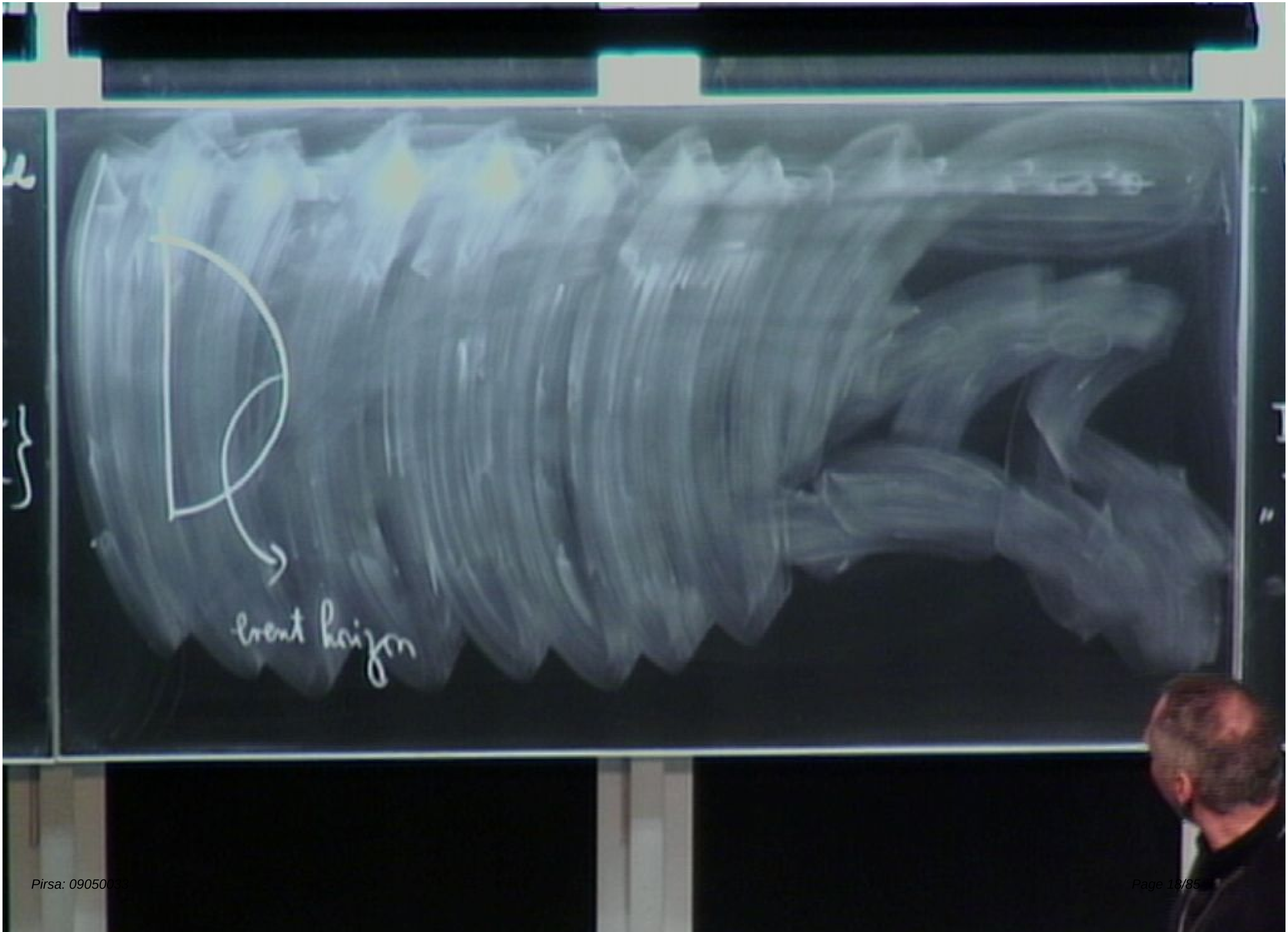
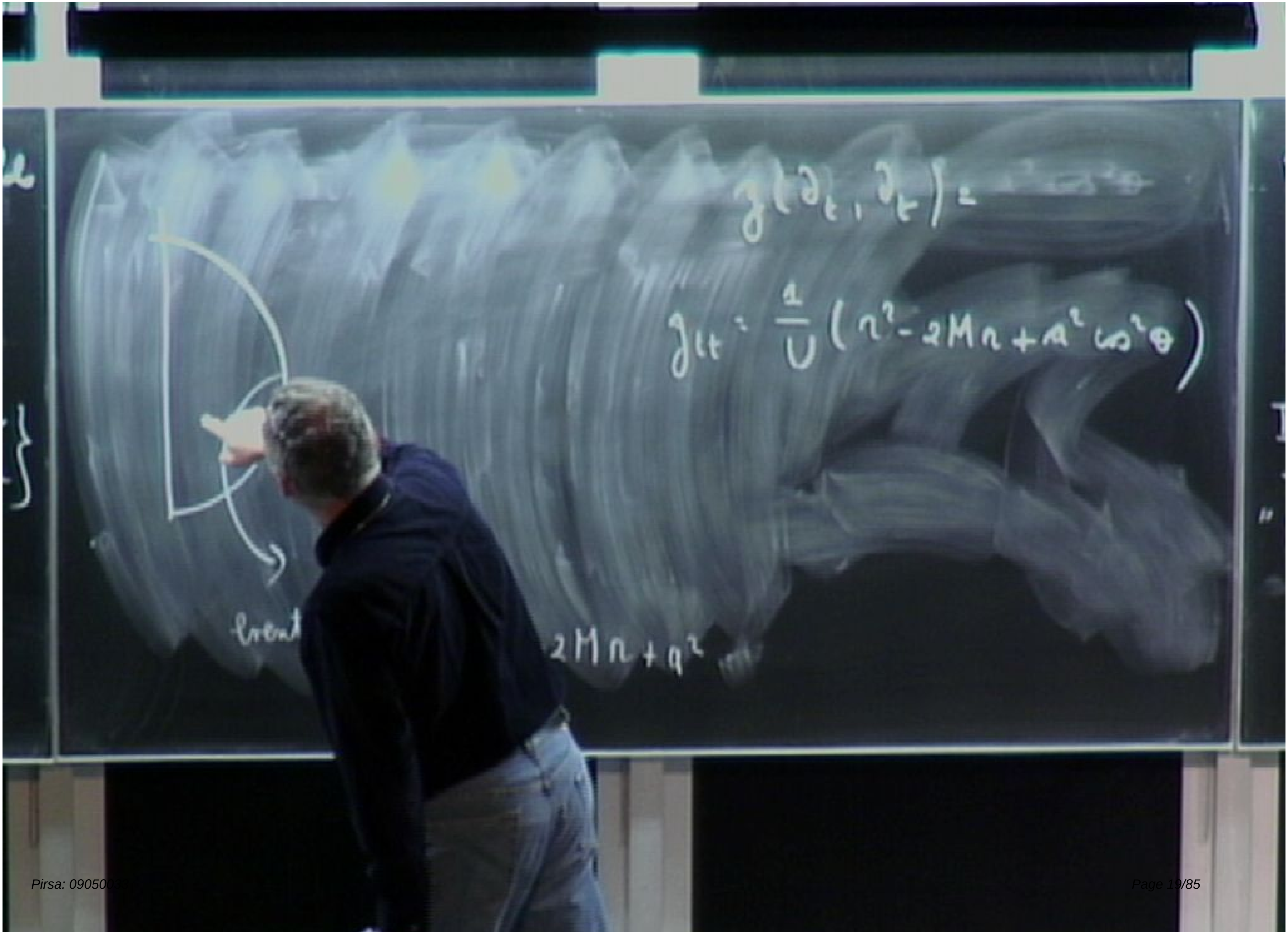
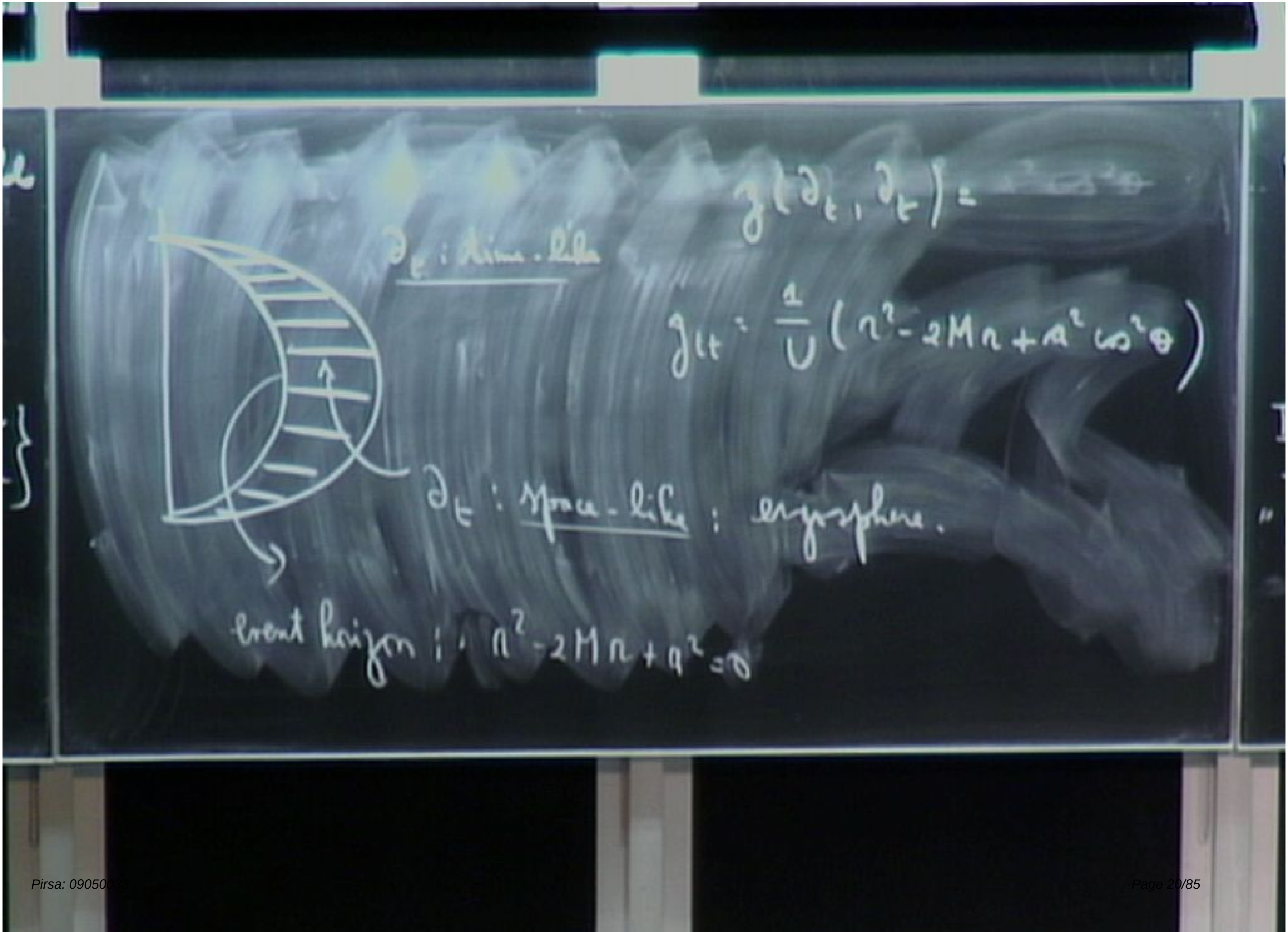
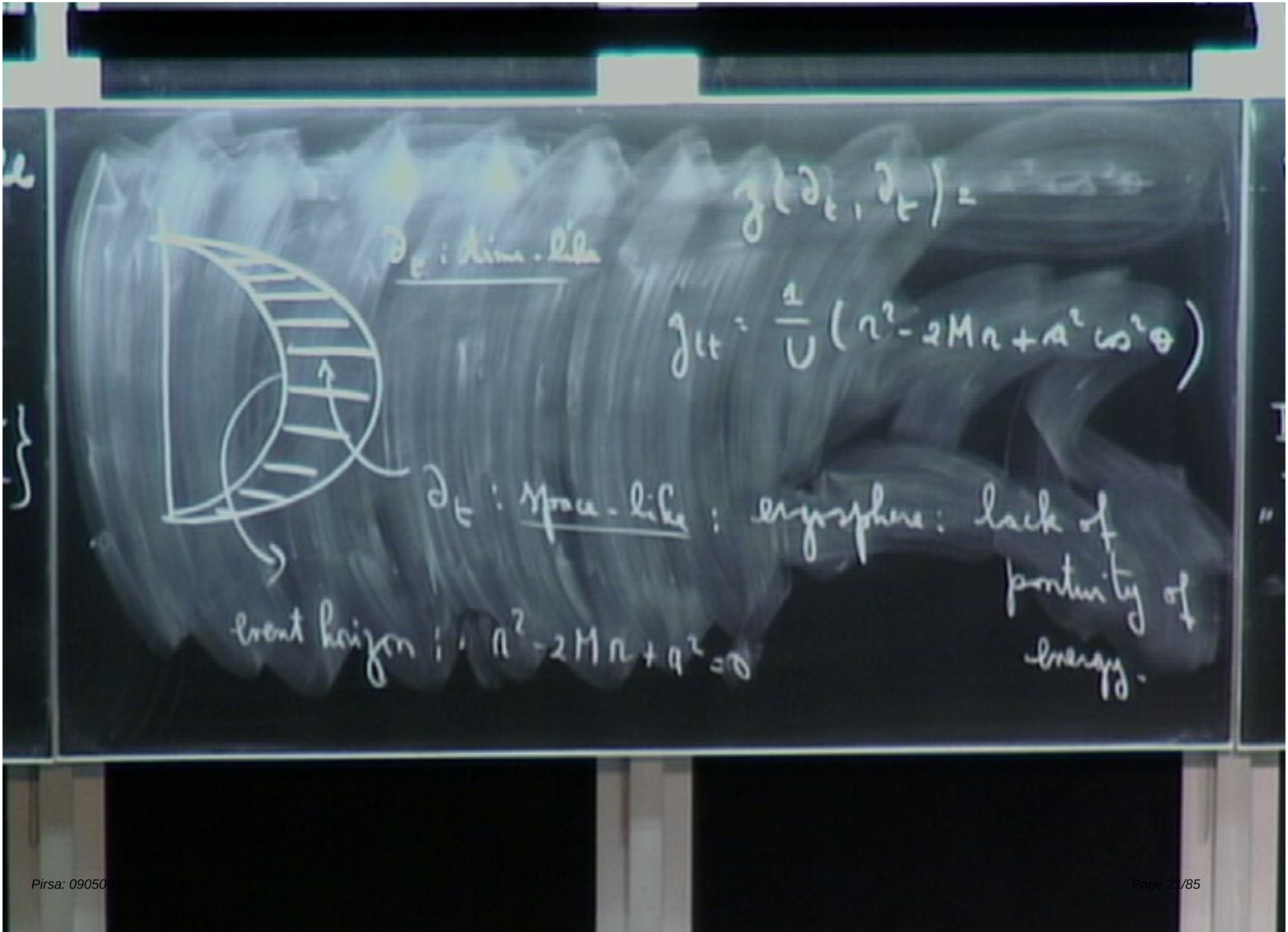

Event horizon


$g(\partial_t, \partial_t) =$
$g_{tt} = \frac{1}{U}(r^2 - 2Mr + a^2\cos^2\theta)$
$2Mr + q^2$


$g(\partial_t, \partial_t) =$
∂_t : time-like
$g_{tt} = \frac{1}{U}(r^2 - 2Mr + a^2\cos^2\theta)$
∂_t : space-like : ergosphere.
event horizon : $r^2 - 2Mr + a^2 = 0$


g(∂_t, ∂_t) =
∂_t : time-like
g_tt = 1/U (r^2 - 2Mr + a^2 cos^2 θ)
∂_t : space-like : ergosphere : lack of positivity of energy.
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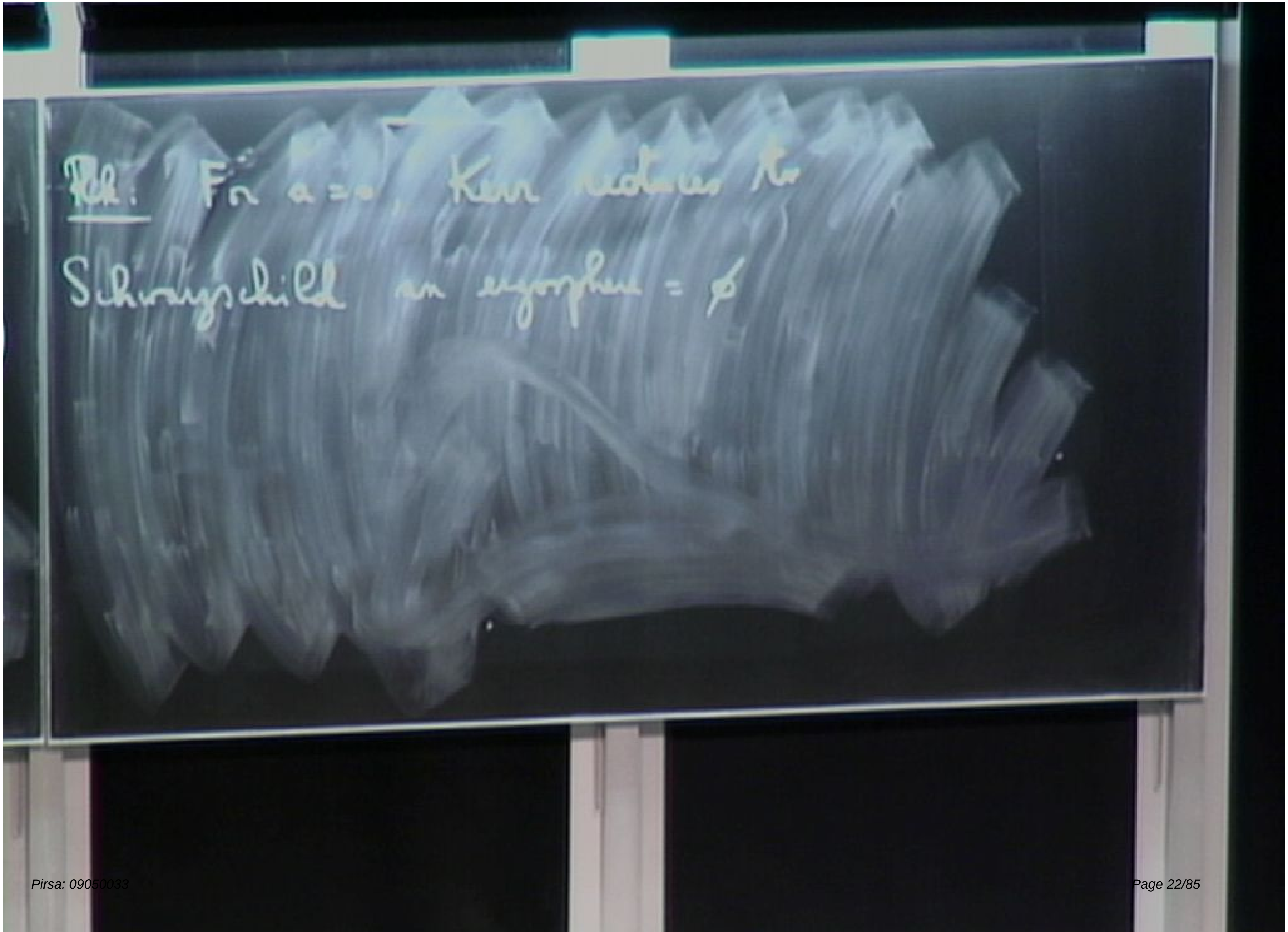
Rk: For a = 0, Kerr reduces to
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Rk: For $a = 0$, Kerr reduces to Schwarzschild and ergosphere $= \emptyset$

General Question: Consider a wave equation

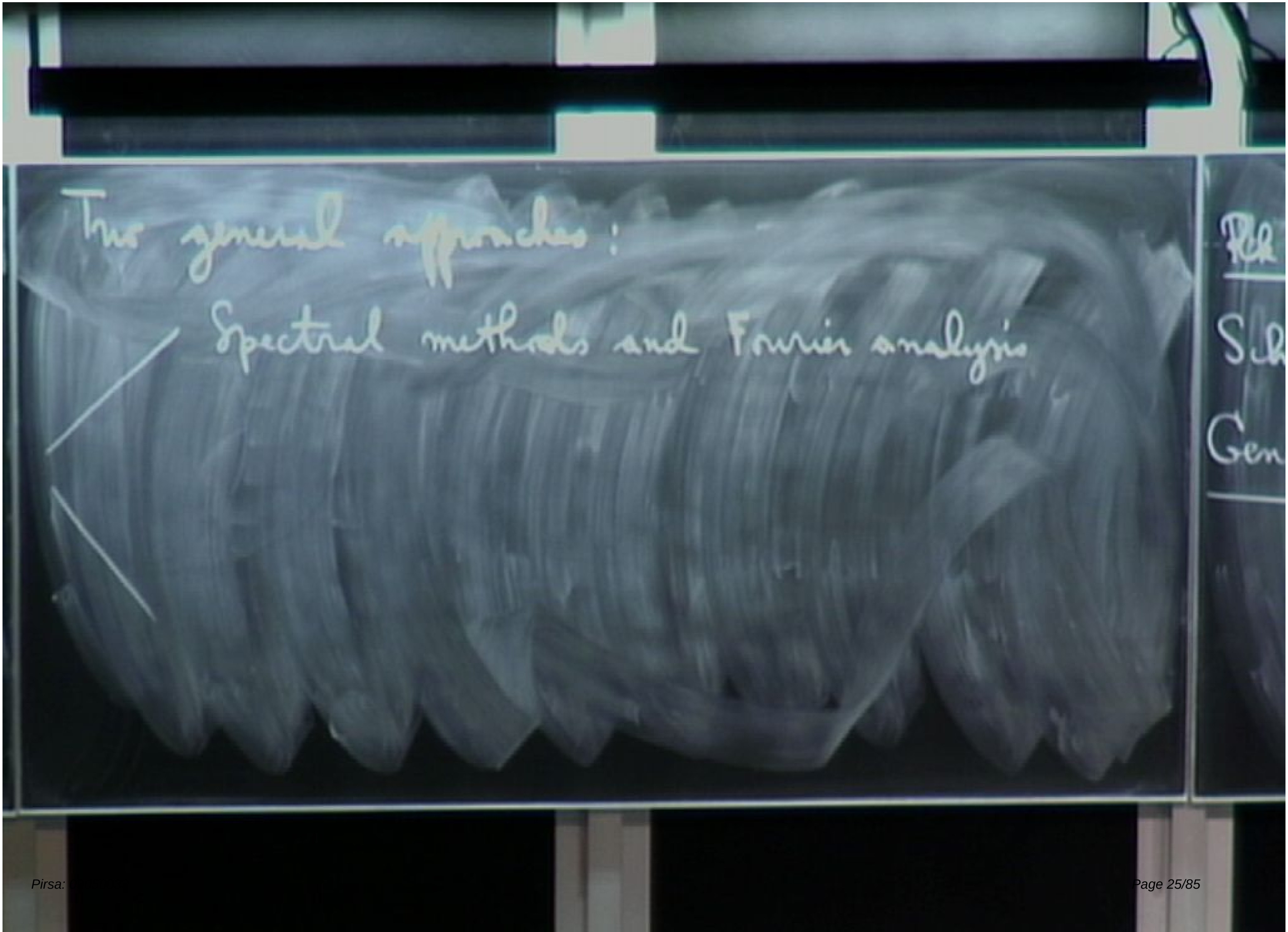
$$\Box_g \Psi = 0.$$

where

$$T_g = \begin{cases} \not{D}_g & \text{Dirac} \\ \Box_g & \\ M_g & \text{Maxwell} \\ E_g & \text{linearized Einstein} \\ \vdots & \end{cases}$$

Understand the dynamics of the solutions as $t \longrightarrow \infty$

$$\begin{cases} \text{decay (rates)} \\ \text{scattering (Penrose fusion)} \\ \vdots \end{cases}$$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Spectral methods and Fourier analysis

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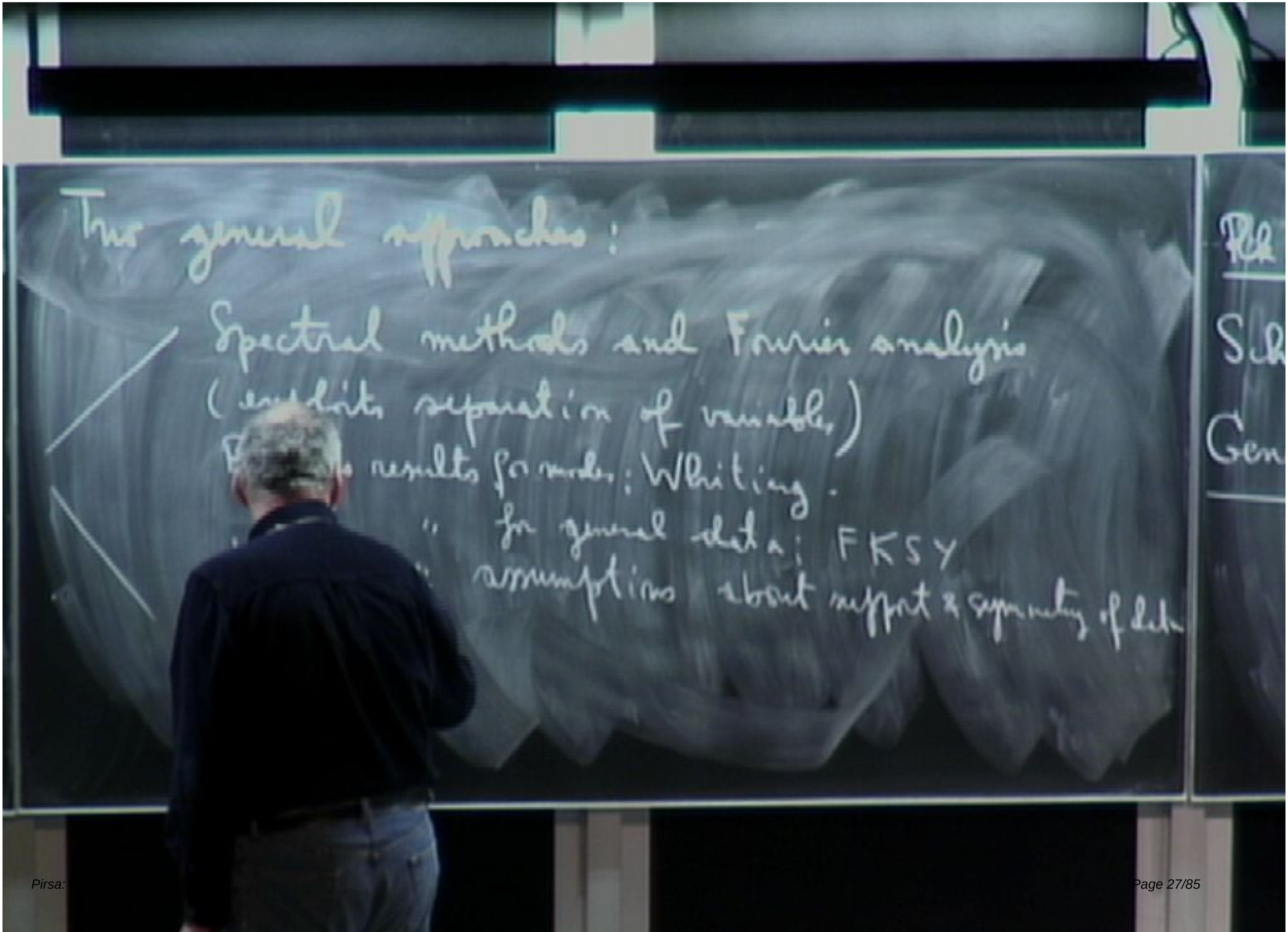
- Rigorous results: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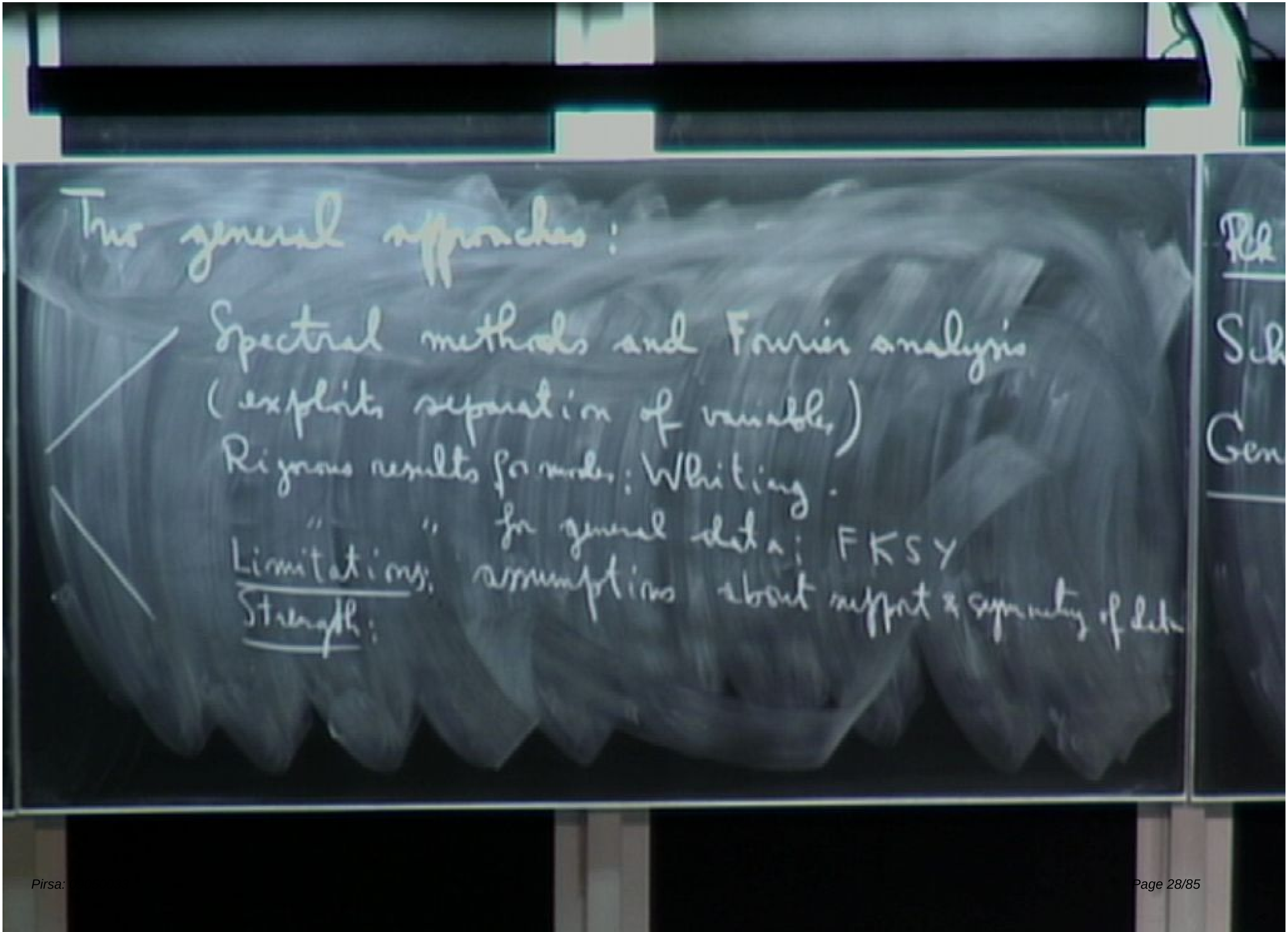
results for modes: Whiting.
" for general data: FKSY
" assumptions about support & symmetry of data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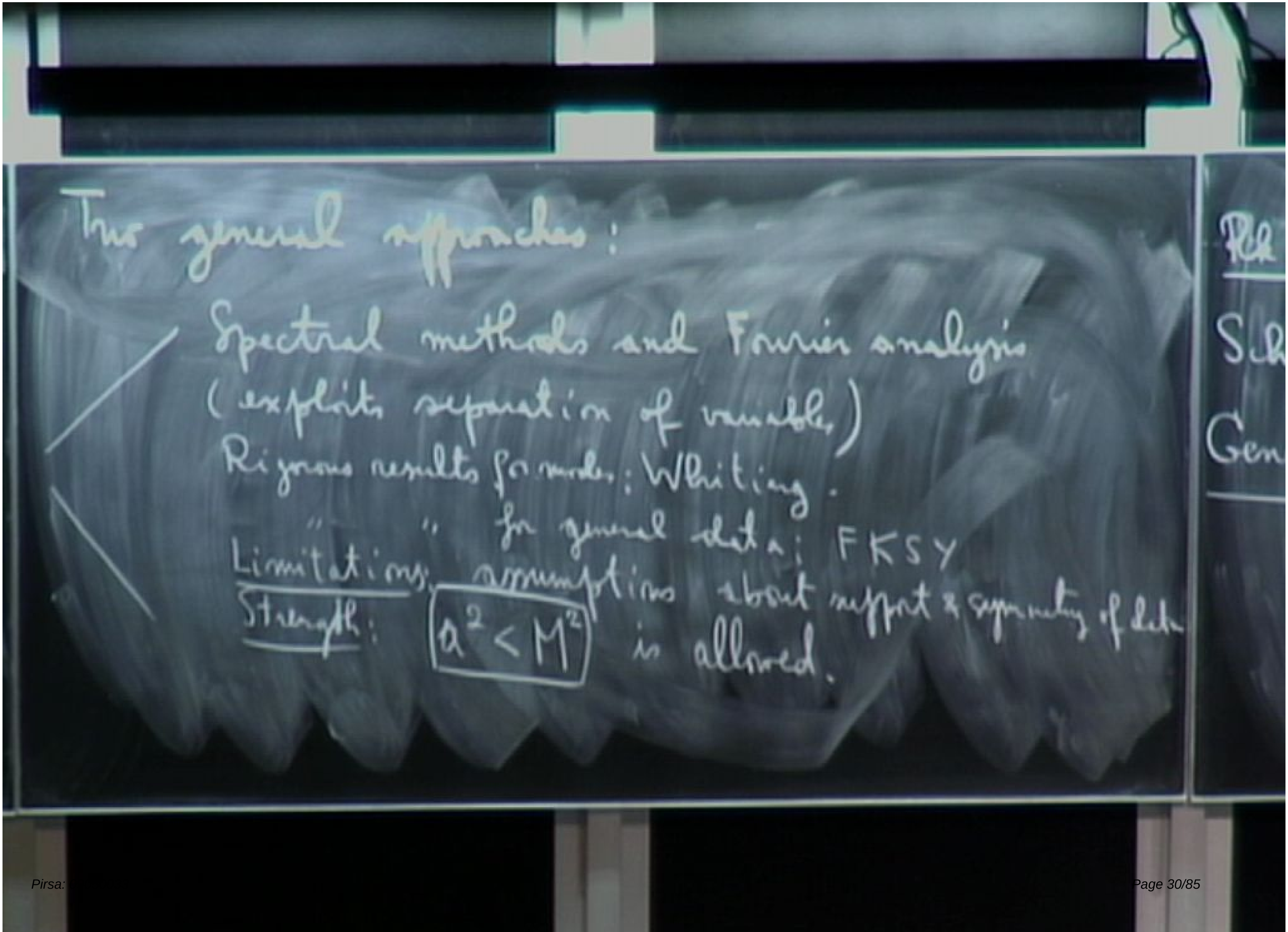
- Strength: $a^2 < M^2$ is allowed.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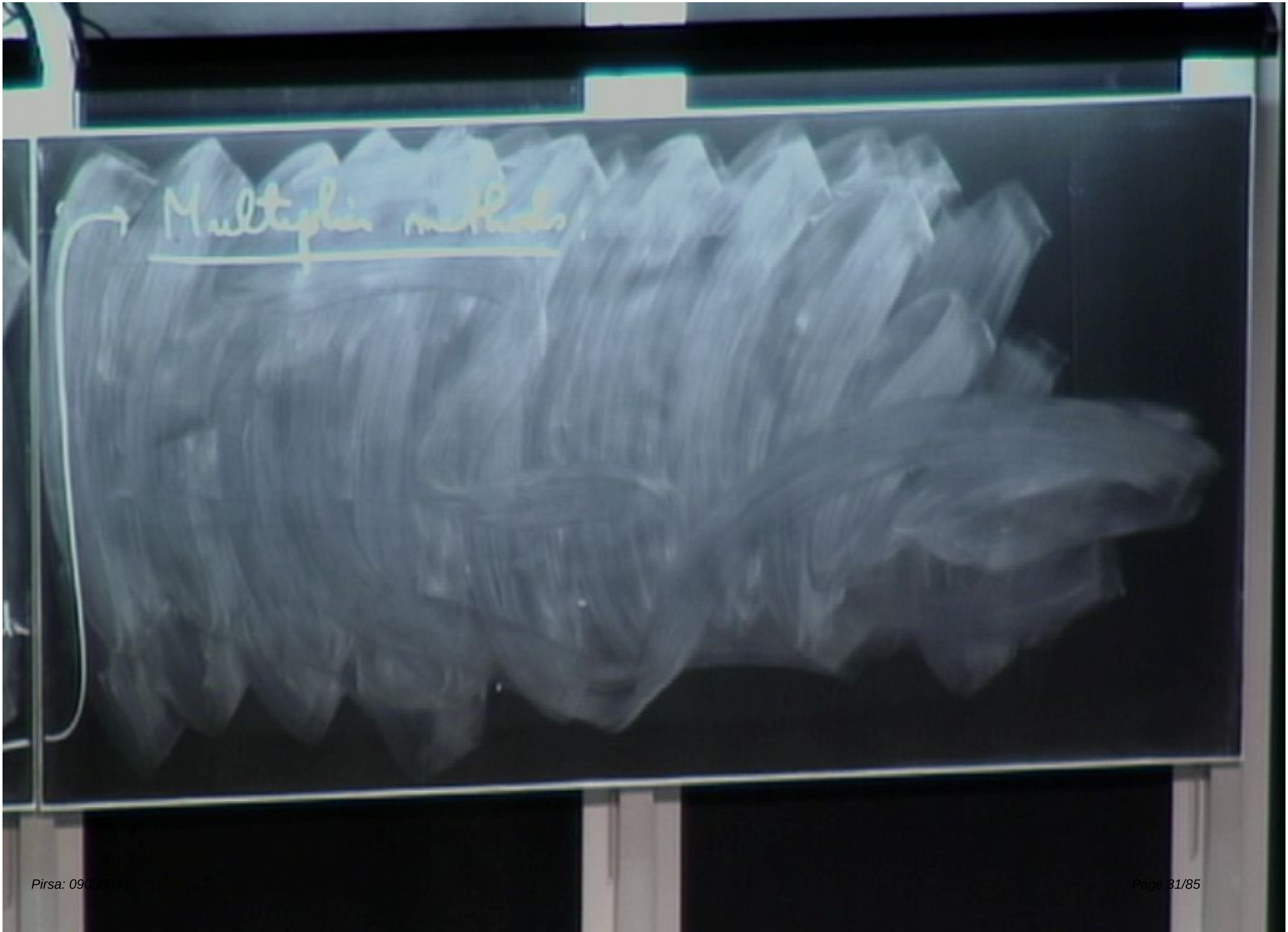
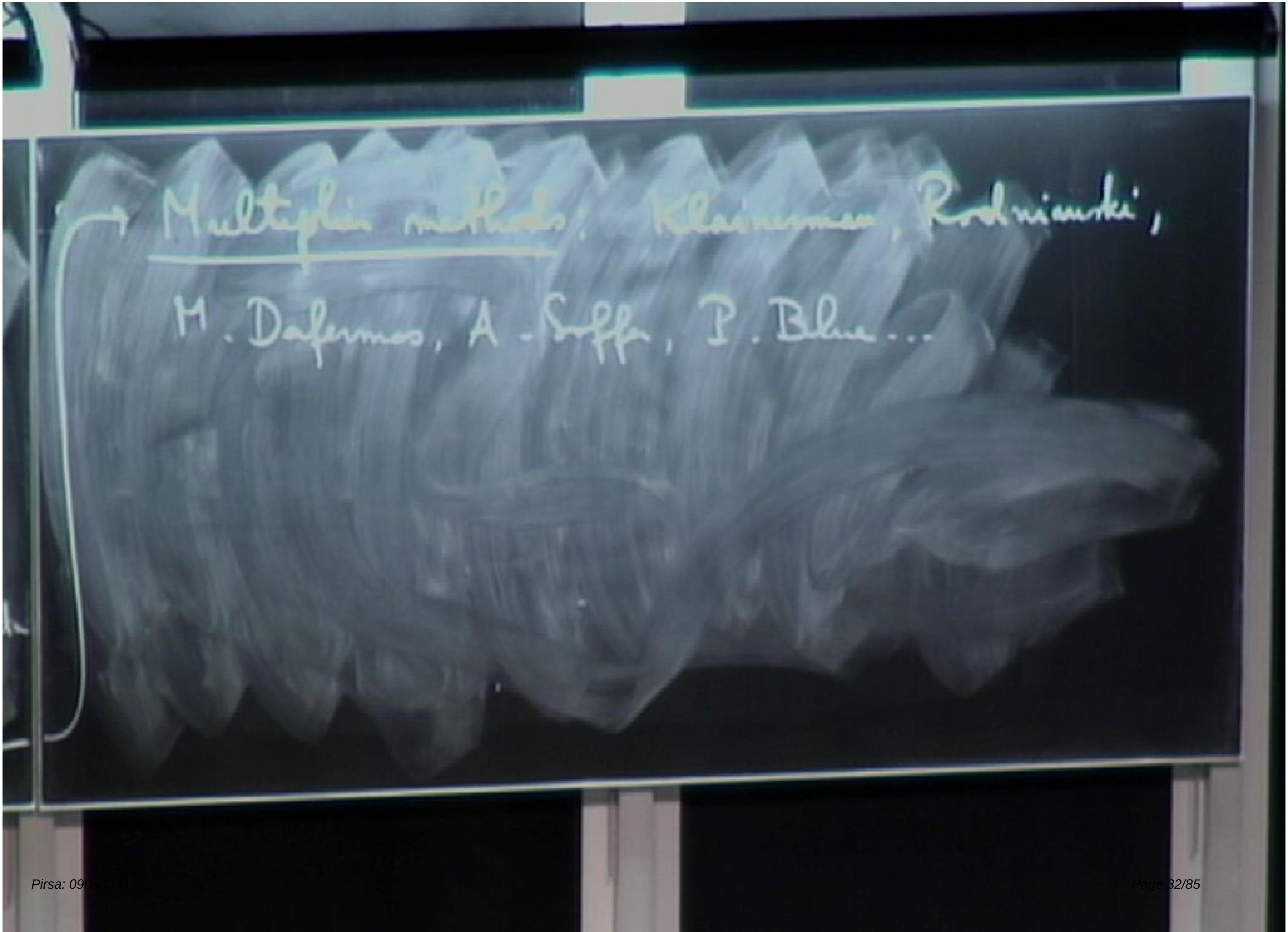

Multiplier methods


Multiplier methods: Klainerman, Rodnianski,
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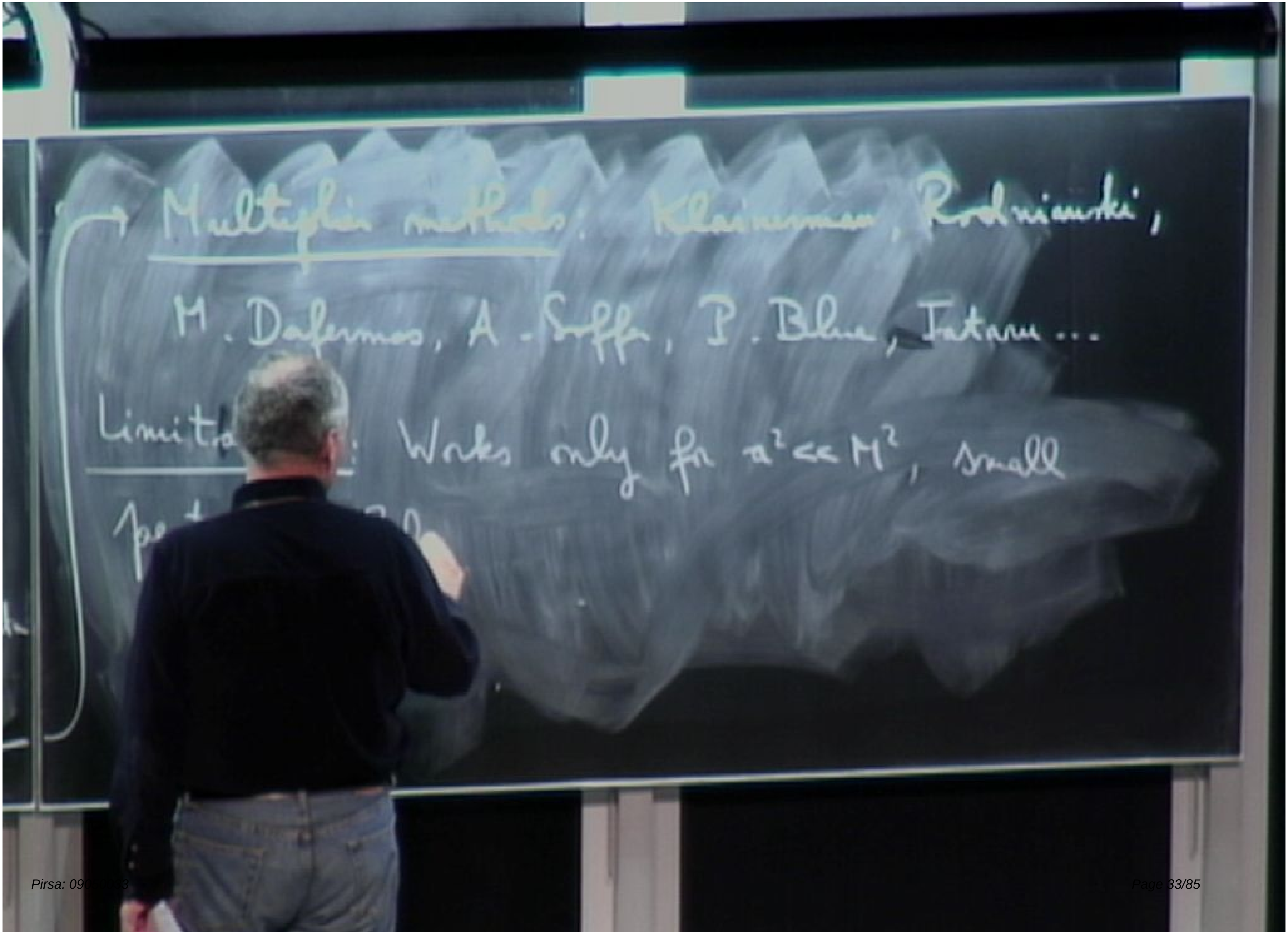
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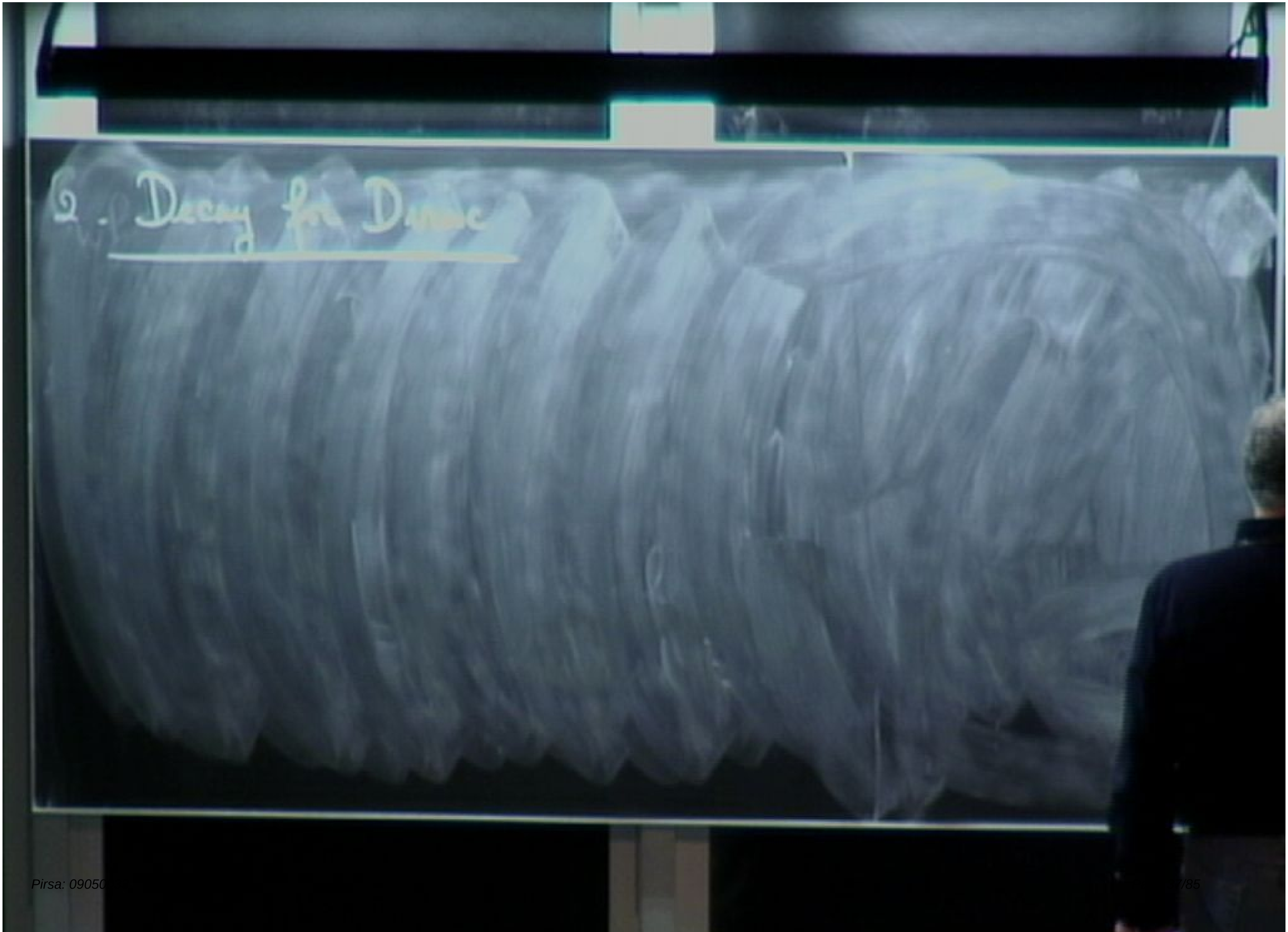
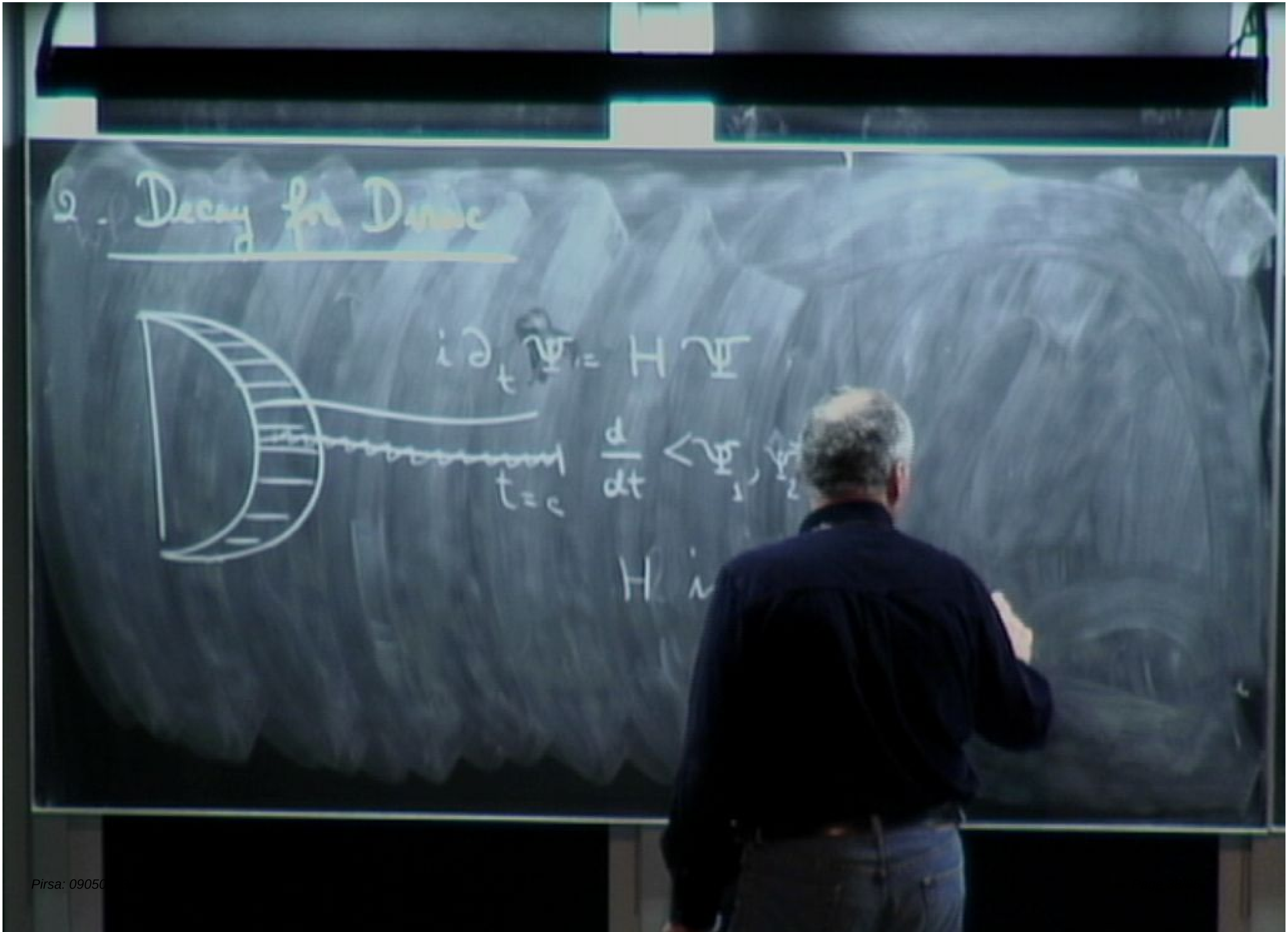

2. Decay for Dirac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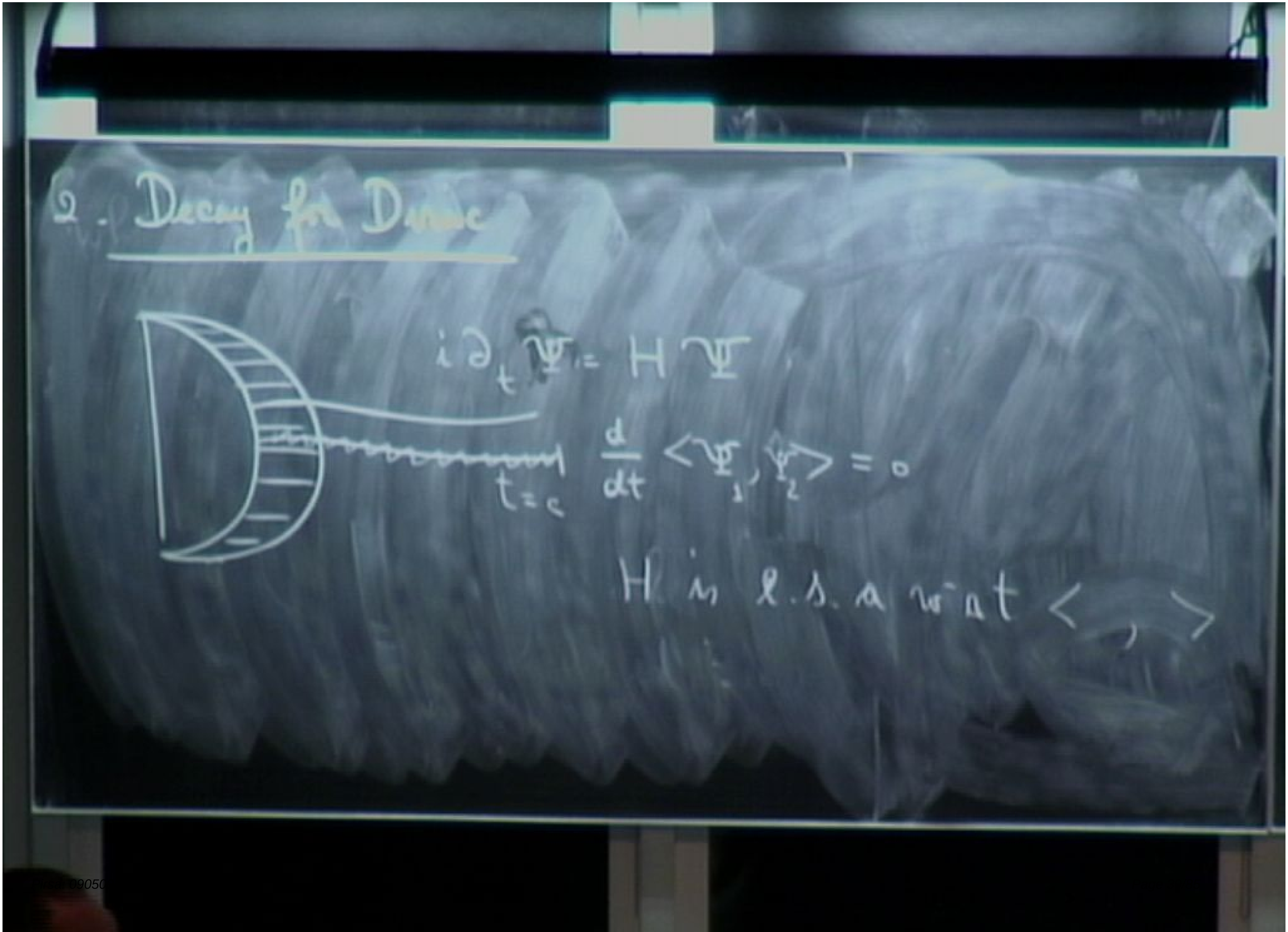
i∂_t Ψ = H Ψ
d/dt <Ψ_1, Ψ_2
t=c
H i


2. Decay for Dirac
i∂t Ψ = H Ψ
t=0
d/dt <Ψ1, Ψ2> = 0
H is l.s.a. w.r.t. < , >

2. Decay for Dirac

$$i\partial_t \Psi = H\Psi$$

$$\frac{d}{dt}\langle \Psi_1, \Psi_2 \rangle = 0$$

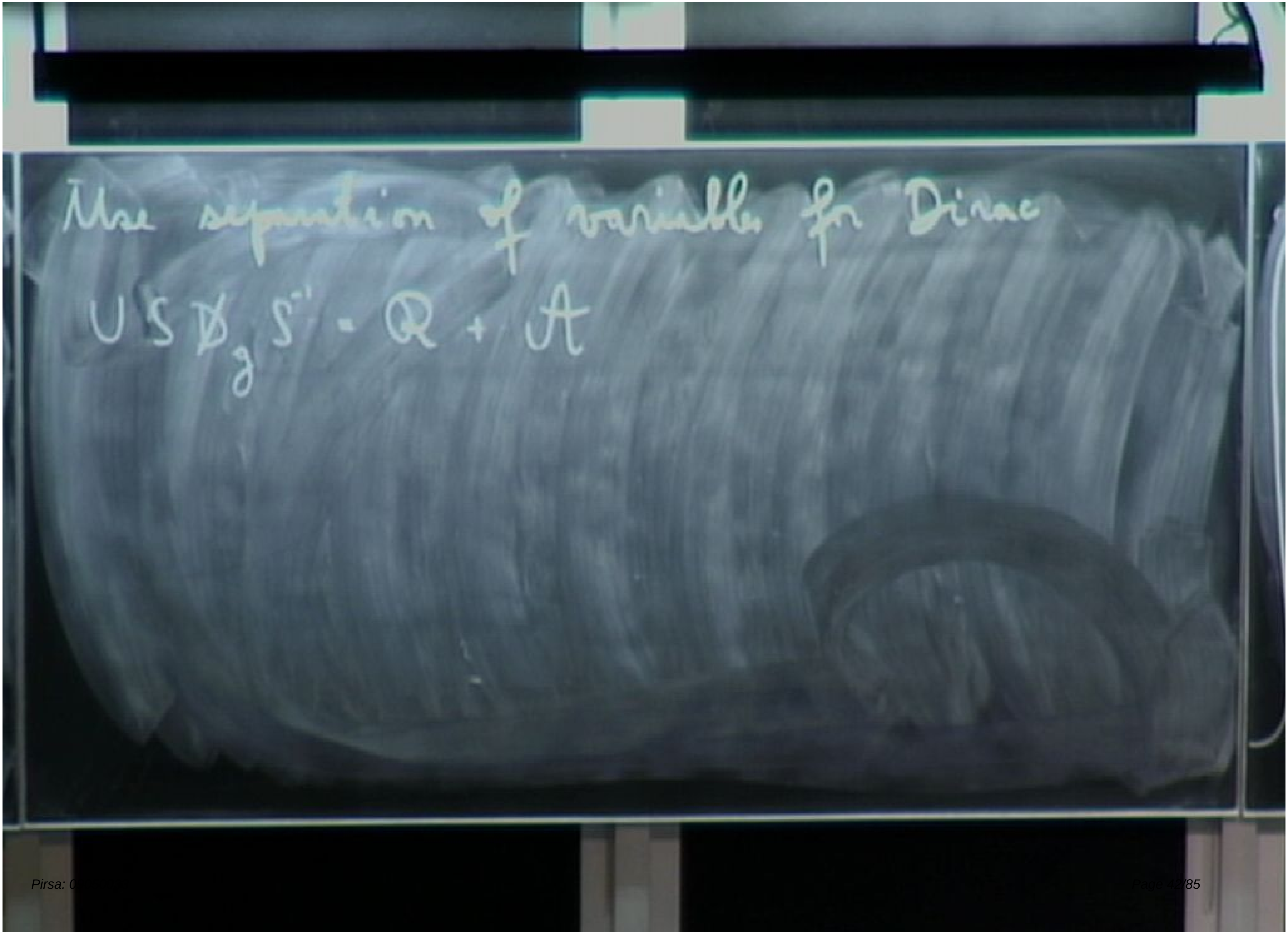
$$\nabla^j(\bar{\Psi}\gamma_j\Psi) = 0$$

H is e.s.a. wrt $\langle \, , \, \rangle$

$$\Psi(t) = e^{itH}\Psi_0$$

Use separation of variables for Dirac

$U S \not{D}_g S$


Use separation of variables for Dirac
$U S \not{D}_g S^{-1} = Q + A$

Use separation of variables for Dirac

$$U S \not{D}_g S^{-1} = \underset{r,t,\varphi}{Q} + \underset{\theta,t,\varphi}{A}$$

Use separation of variables for Dirac

$$U S \not{D}_g S^{-1} = \mathcal{R}_{r,t,\varphi} + \mathcal{A}_{\theta,t,\varphi}$$

$$\Psi = e^{-i(k_\varphi + \frac{1}{2})\varphi - i\omega t}\,\hat{\Psi}(r,\theta)$$

Use separation of variables for Dirac

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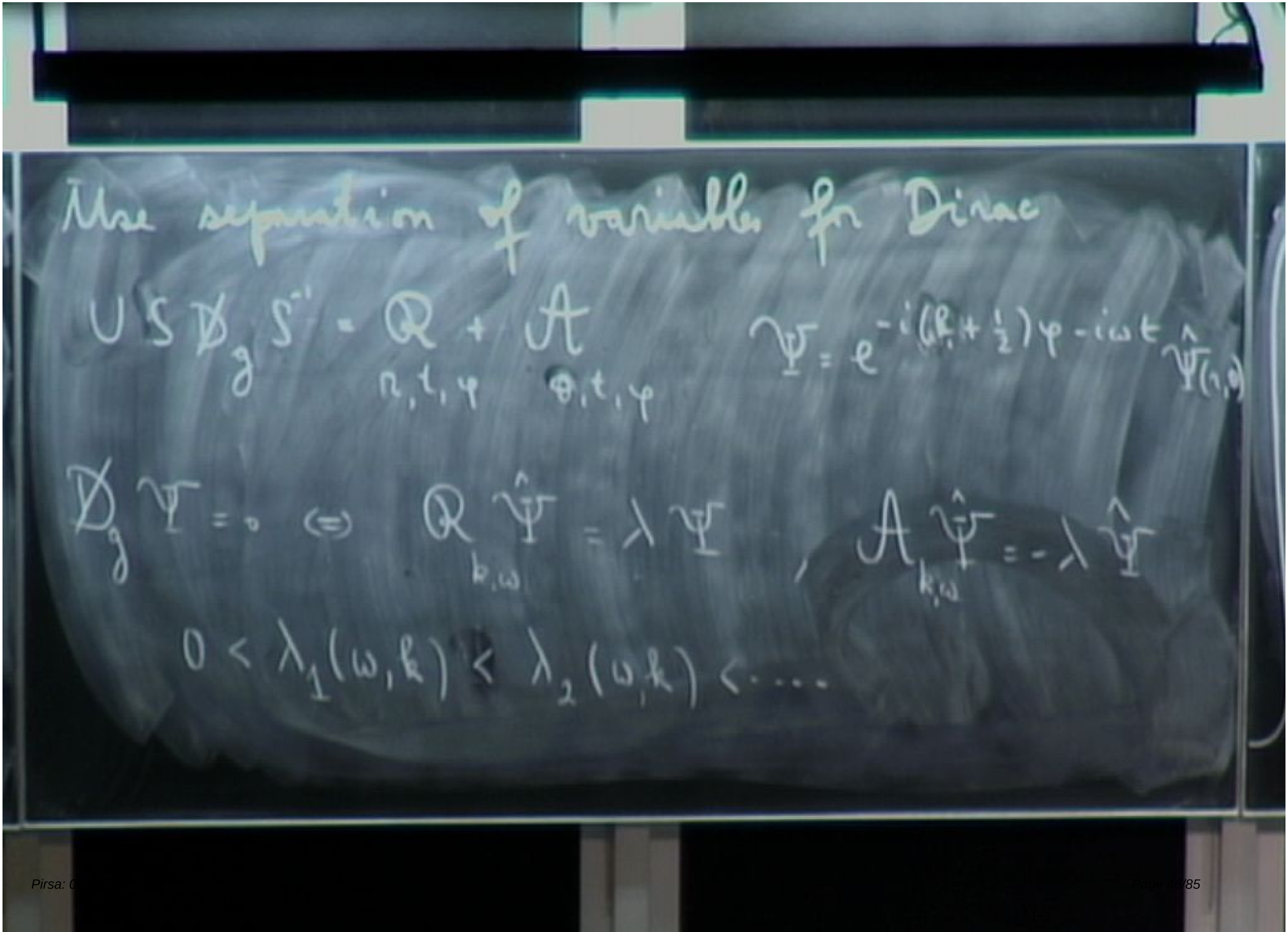
$$\not{D}_g \Psi = 0 \iff \mathcal{Q}_{k,\omega}\hat{\Psi} = \lambda \hat{\Psi}, \quad \mathcal{A}_{k,\omega}\hat{\Psi} = -\lambda \hat{\Psi}$$


Use separation of variables for Dirac
$U S \not{D}_g S^{-1} = Q_{r,t,\varphi} + A_{\theta,t,\varphi}$
$\Psi = e^{-i(k+\frac{1}{2})\varphi - i\omega t}\,\hat{\Psi}(r,\theta)$
$\not{D}_g \Psi = 0 \iff Q_{k,\omega}\hat{\Psi} = \lambda \hat{\Psi}, \quad A_{k,\omega}\hat{\Psi} = -\lambda\hat{\Psi}$
$0 < \lambda_1(\omega,k) < \lambda_2(\omega,k) < \dots$

Use separation of variables for Dirac

$$U S \not{D}_g S^{-1} = \mathcal{R}_{r,t,\varphi} + \mathcal{A}_{\theta,t,\varphi} \qquad \Psi = e^{-i(k+\frac{1}{2})\varphi - i\omega t}\,\hat{\Psi}(r,\theta)$$

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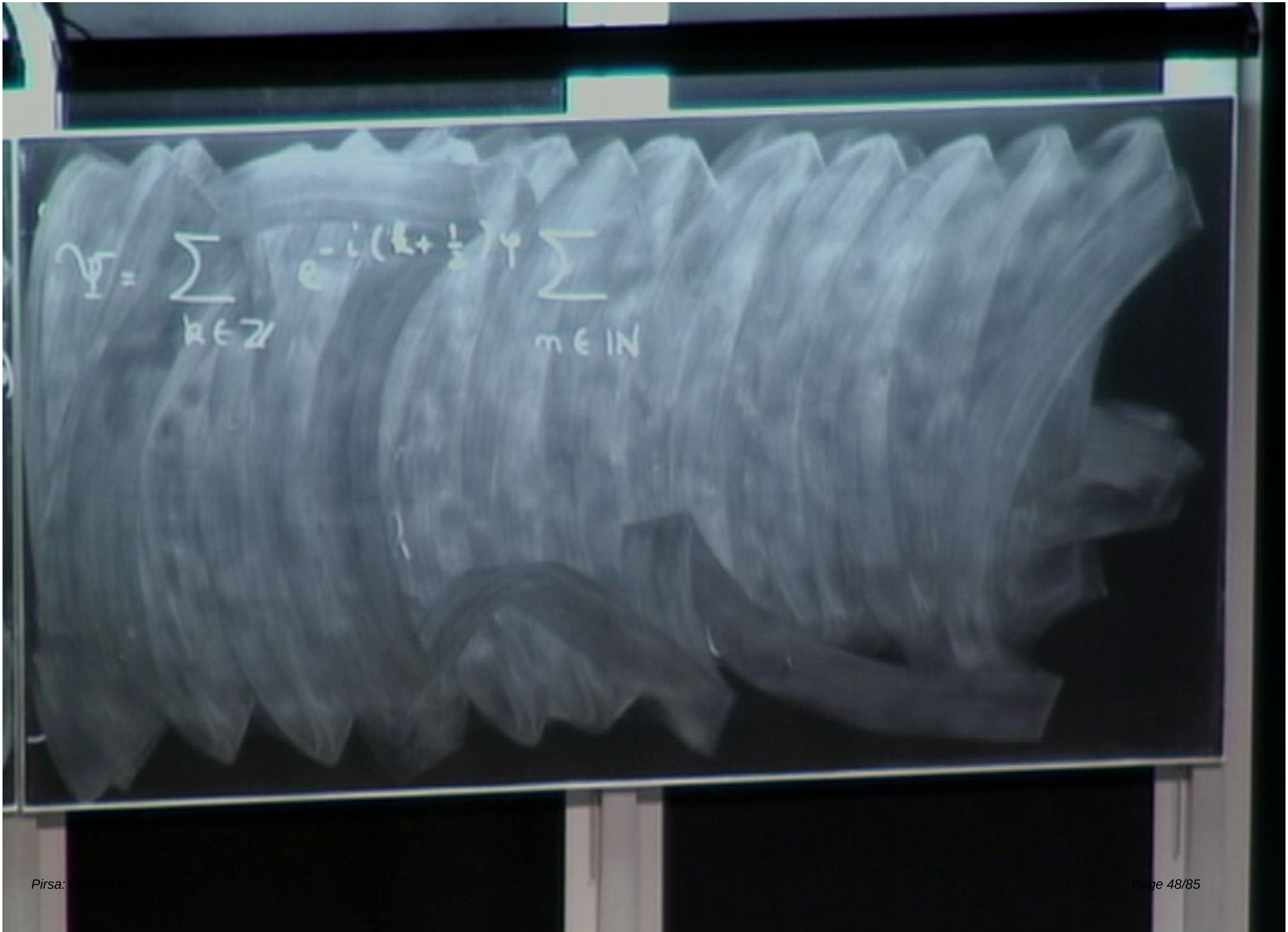
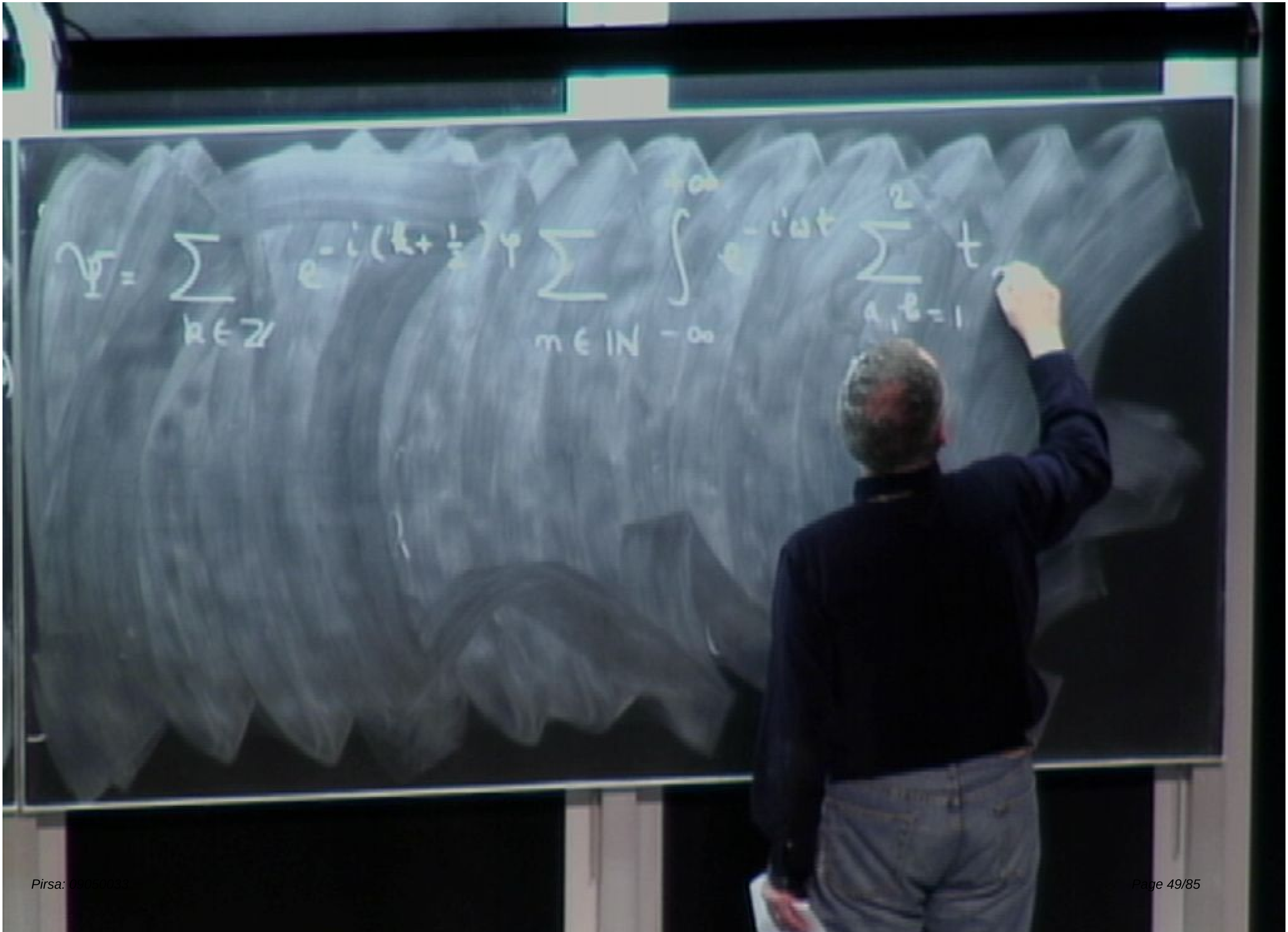
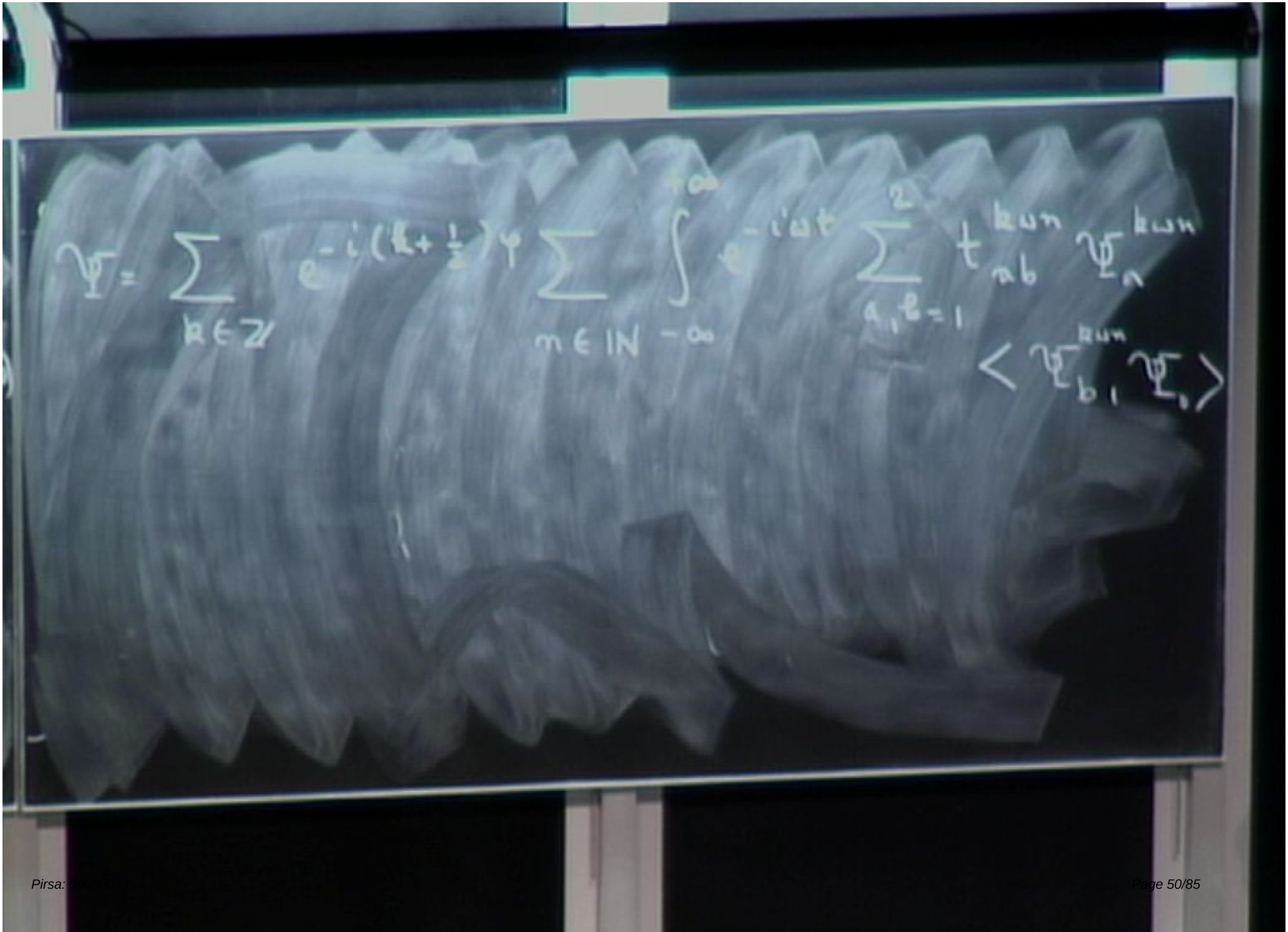

$\Psi = \sum_{k \in \mathbb{Z}} e^{-i(k+\frac{1}{2})\varphi} \sum_{m \in \mathbb{N}}$


$\Psi = \sum_{k \in \mathbb{Z}} e^{-i(k+\frac{1}{2})\varphi} \sum_{m \in \mathbb{N}} \int_{-\infty}^{+\infty} e^{-i\omega t} \sum_{a,b=1}^{2} t$



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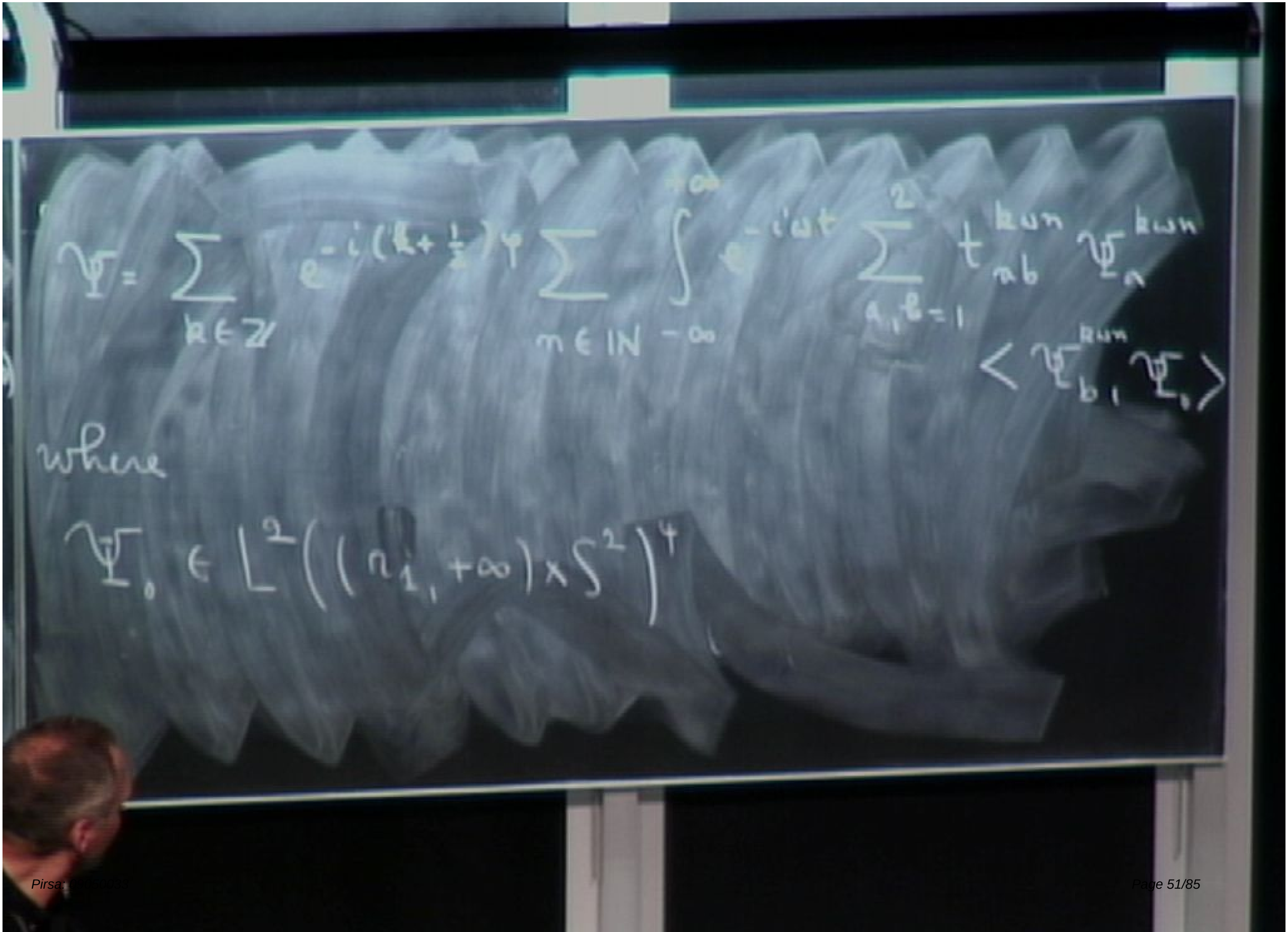
$\langle \Psi_{b}^{k\omega n}, \Psi \rangle$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where
$\Psi_0 \in L^2((r_1, +\infty) \times S^2)^4$

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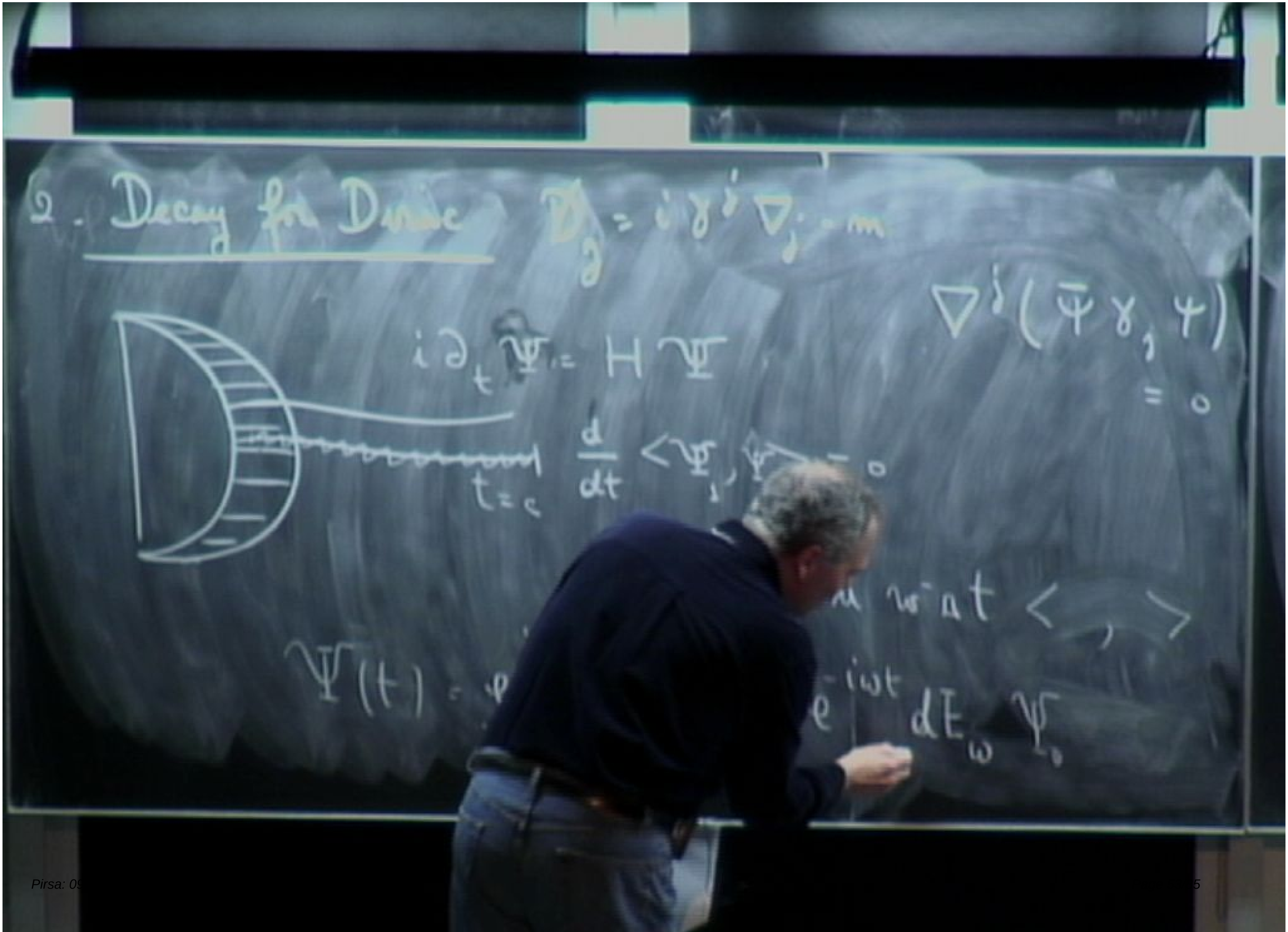
where

$$\Psi_0 \in L^2\left((r_1, +\infty) \times S^2\right)^4 \cap L^\infty_{loc}(\text{event horizon})$$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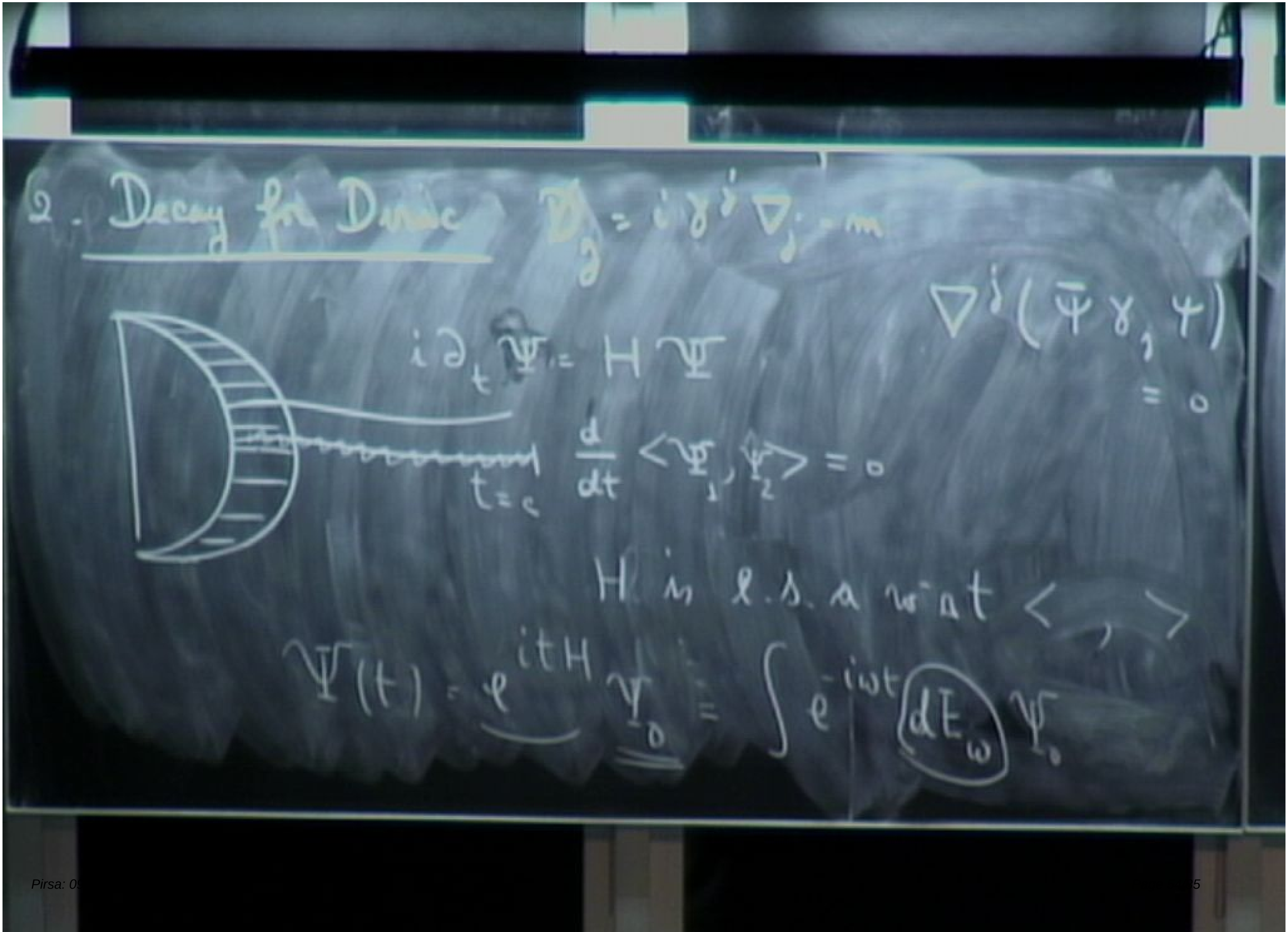
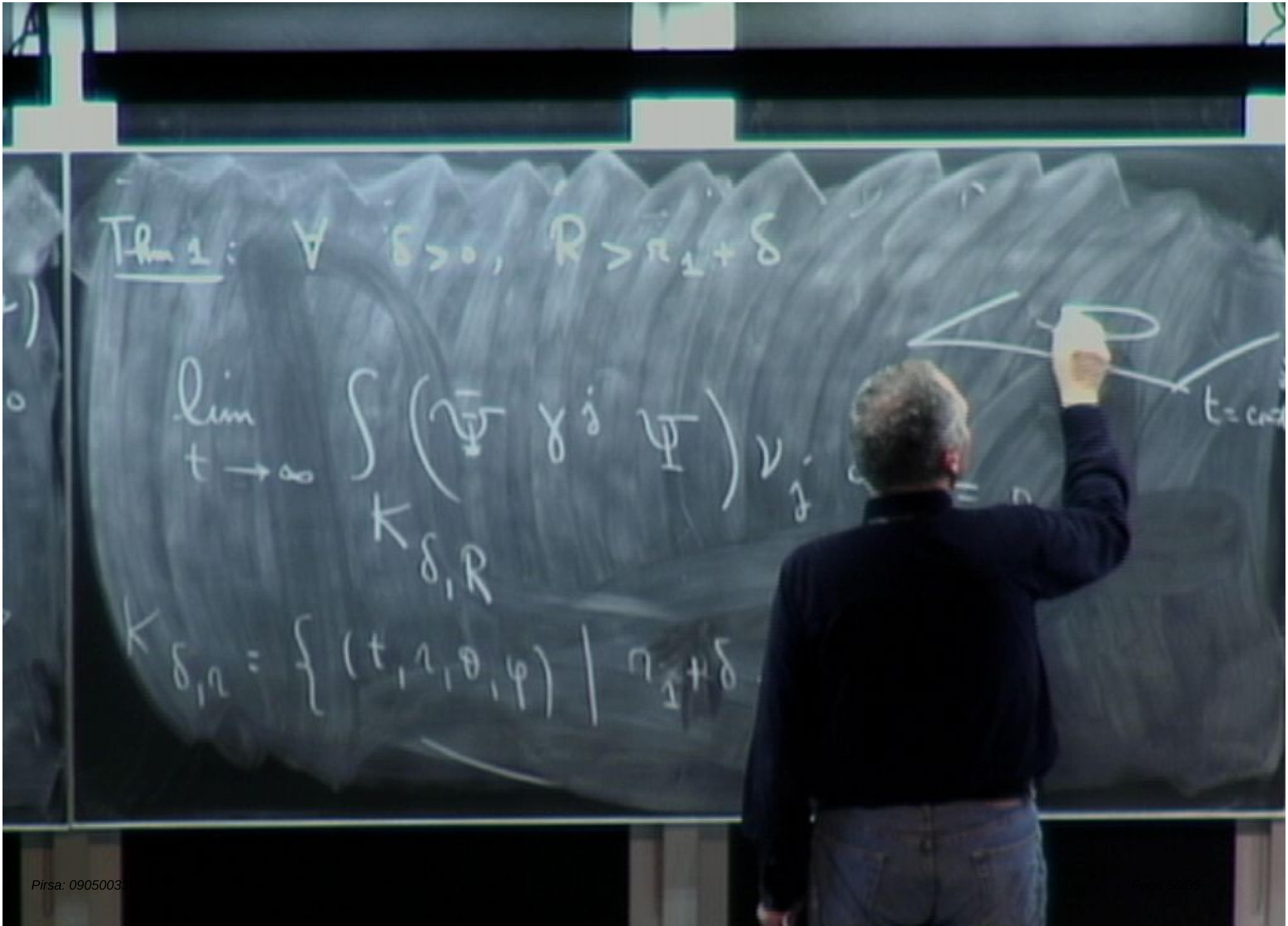
d/dt ⟨Ψ, Ψ⟩ = 0
t = c
= 0


2. Decay for Dirac
$\not{D} = i\gamma^j \nabla_j - m$
$\nabla^j(\bar{\Psi}\gamma_j \Psi) = 0$
$i\partial_t \Psi = H\Psi$
$\frac{d}{dt}\langle \Psi_1, \Psi_2 \rangle = 0$
$t = c$
H is e.s.a. w.r.t. $\langle\ ,\ \rangle$
$\Psi(t) = e^{itH}\Psi_0 = \int e^{-i\omega t} dE_\omega \Psi_0$

Thm 1: $\forall\ \delta > 0,\ R > R_1$

$$\lim_{t \to \infty} \int_{K_{\delta, R}}$$


Thm 1: ∀ δ > 0, R > r1 + δ
lim t → ∞ ∫ K δ,R (Ψ̄ γ j Ψ) ν j
K δ,r = { (t, r, θ, φ) | r1 + δ

Thm 1: $\forall\ \delta > 0,\ R > r_1 + \delta$

$$\lim_{t \to \infty} \int_{K_{\delta,R}} \left(\bar{\Psi} \gamma^j \Psi \right) \nu_j \, d\mu = 0$$

$$K_{\delta,r} = \left\{ (t, r, \theta, \varphi) \mid r_1 + \delta < r < R \right\}$$

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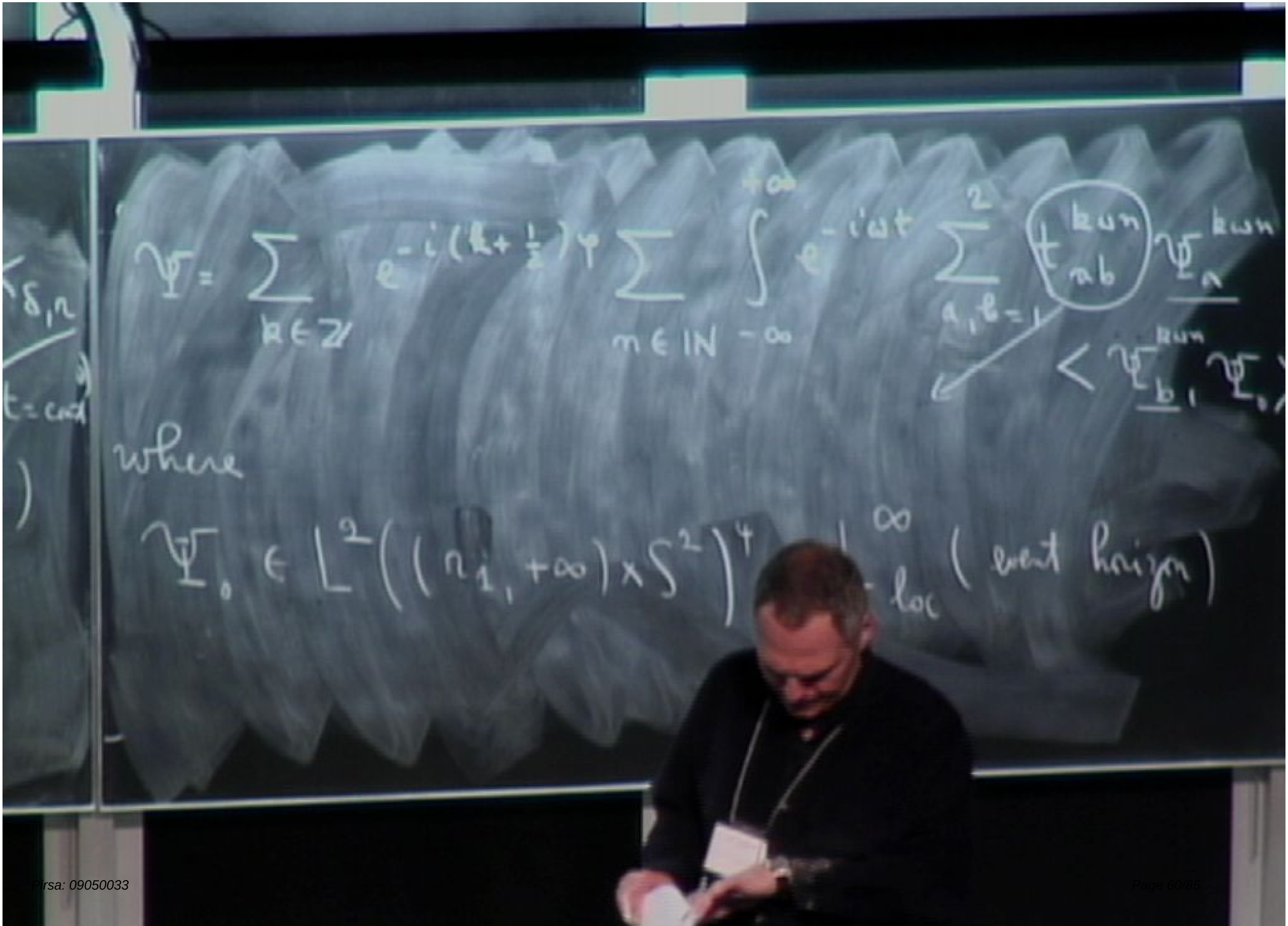
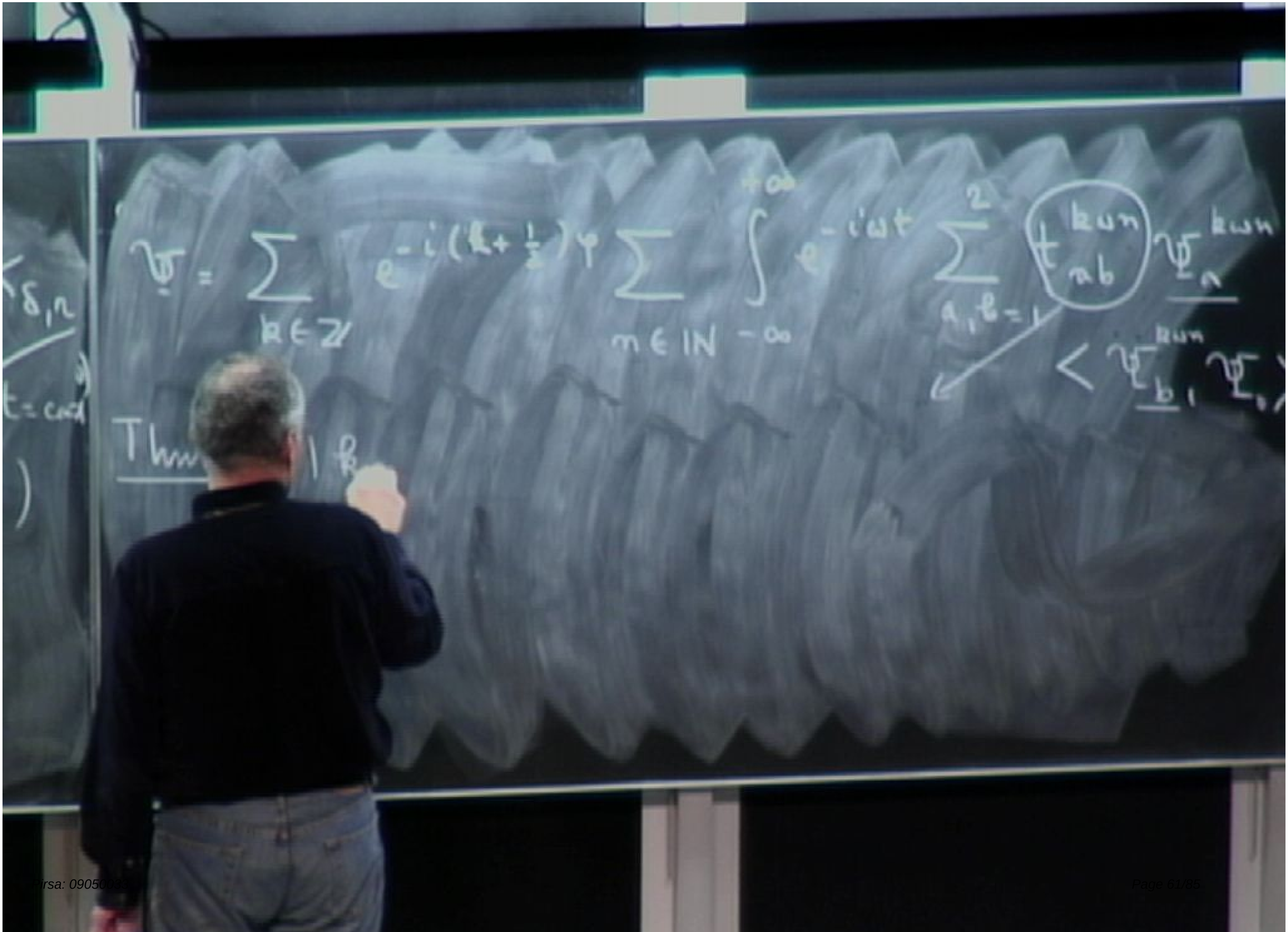
$$K_{\delta,r} = \{ (t, r, \theta, \varphi) \mid r_1 + \delta < r < R \}$$


Ψ = Σ_{k∈Z} e^{-i(k+1/2)φ} Σ_{n∈IN} ∫_{-∞}^{+∞} e^{-iωt} Σ_{a,b=1}^{2} t_{ab}^{kωn} Ψ_a^{kωn}
< Ψ_b^{kωn}, Ψ_0 >
where
Ψ_0 ∈ L^2((r_1, +∞) × S^2)^4 ∩ L^∞_loc (event horizon)


$\Psi = \sum_{k\in\mathbb{Z}} e^{-i(k+\frac{1}{2})\varphi} \sum_{n\in\mathbb{N}} \int_{-\infty}^{+\infty} e^{-i\omega t} \sum_{a,b=1}^{2} t_{ab}^{k\omega n}\, \Psi_a^{k\omega n}$
$< \Psi_b^{k\omega n}, \Psi_a >$
Thm

δ, n

$t = \text{const}$

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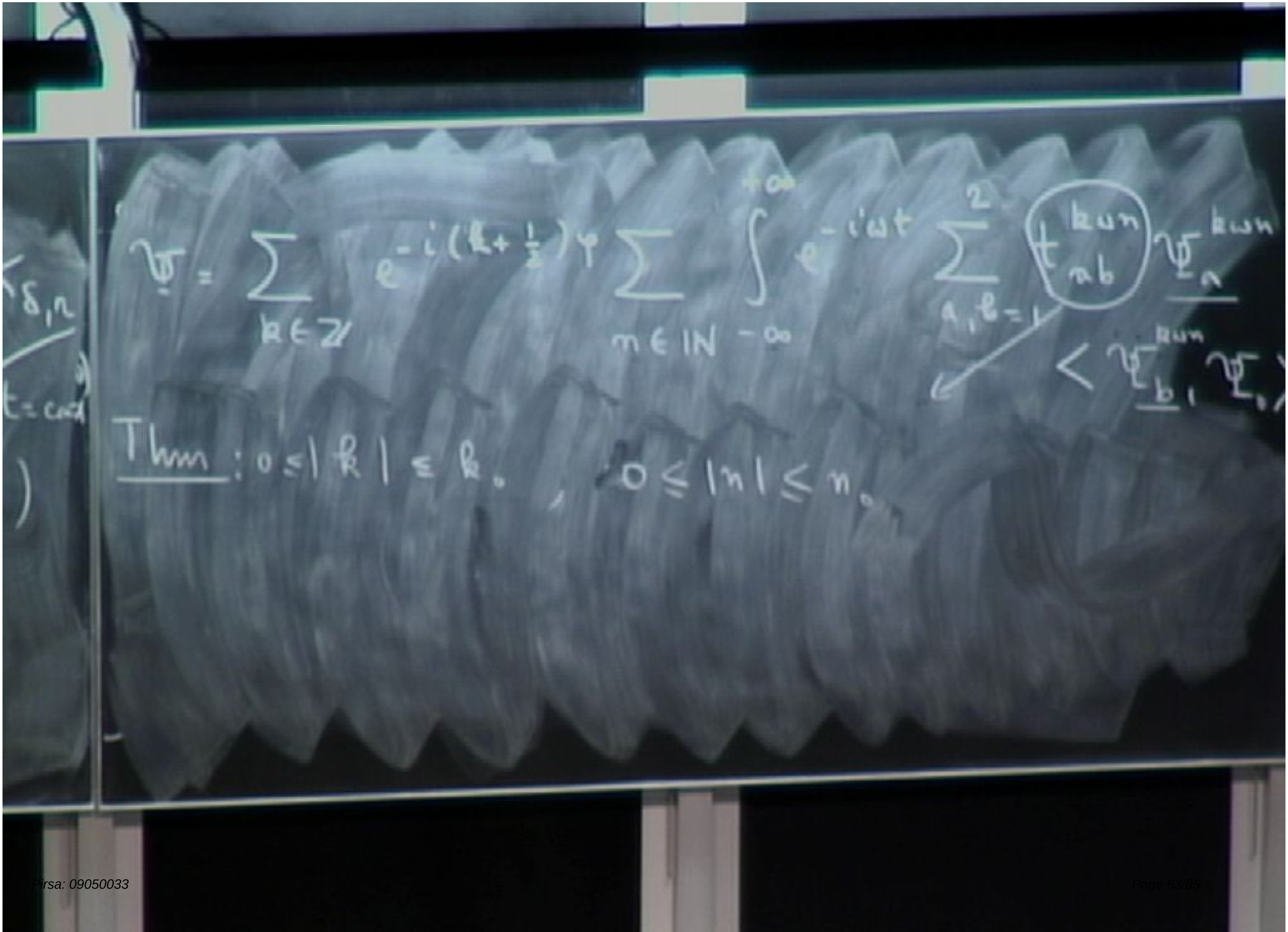
$$t^{k\omega n}_{ab} \rightarrow \langle \Psi^{k\omega n}_{b}, \Psi_{0} \rangle$$

Thm: $0 \le |k| \le k_0$, $0 \le |n| \le n_0$



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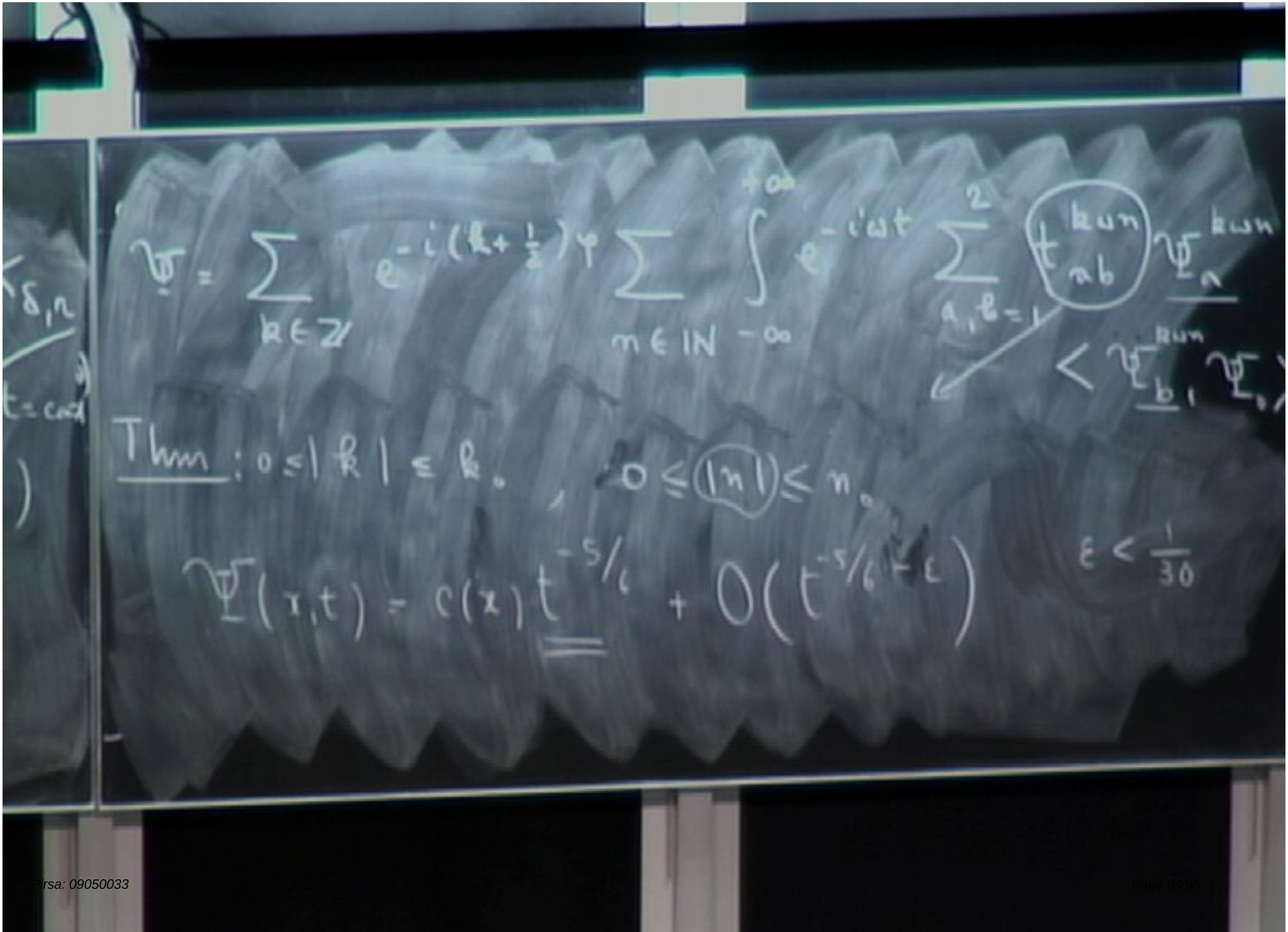
Ψ = Σ_{k∈ℤ} e^{-i(k+1/2)φ} Σ_{n∈ℕ} ∫_{-∞}^{+∞} e^{-iωt} Σ_{a,b=1}^{2} t_{ab}^{kωn} Ψ_a^{kωn}
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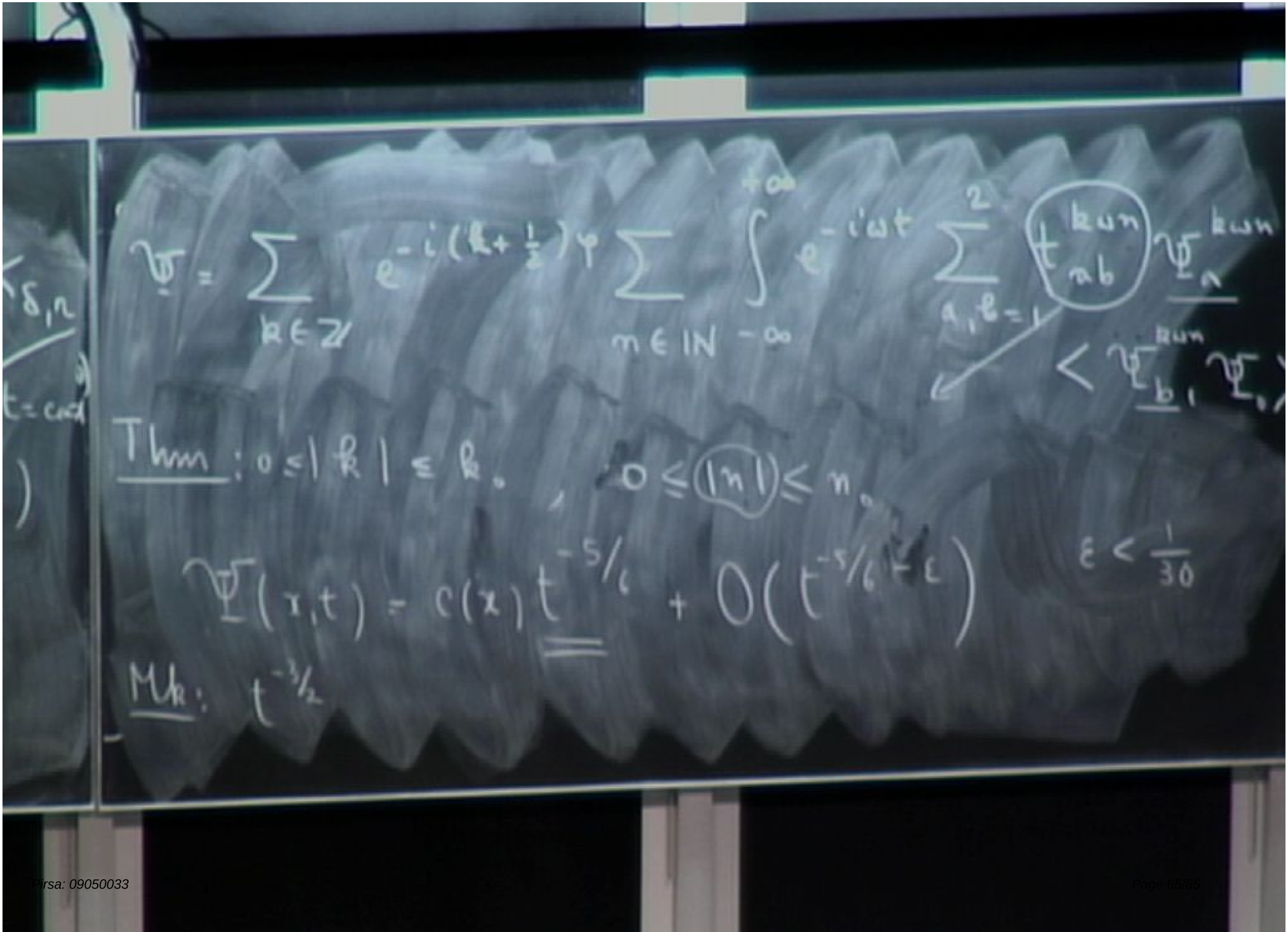
Thm: 0 ≤ |k| ≤ k_0 , 0 ≤ |n| ≤ n_0
Ψ(x,t) = c(x) t^{-5/6} + O(t^{-5/6+ε})
ε < 1/30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Thm: 0 ≤ |k| ≤ k₀ , 0 ≤ |n| ≤ n₀
Ψ(x,t) = c(x) t^{-5/6} + O(t^{-5/6-ε})
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Mk: t^{-3/2}

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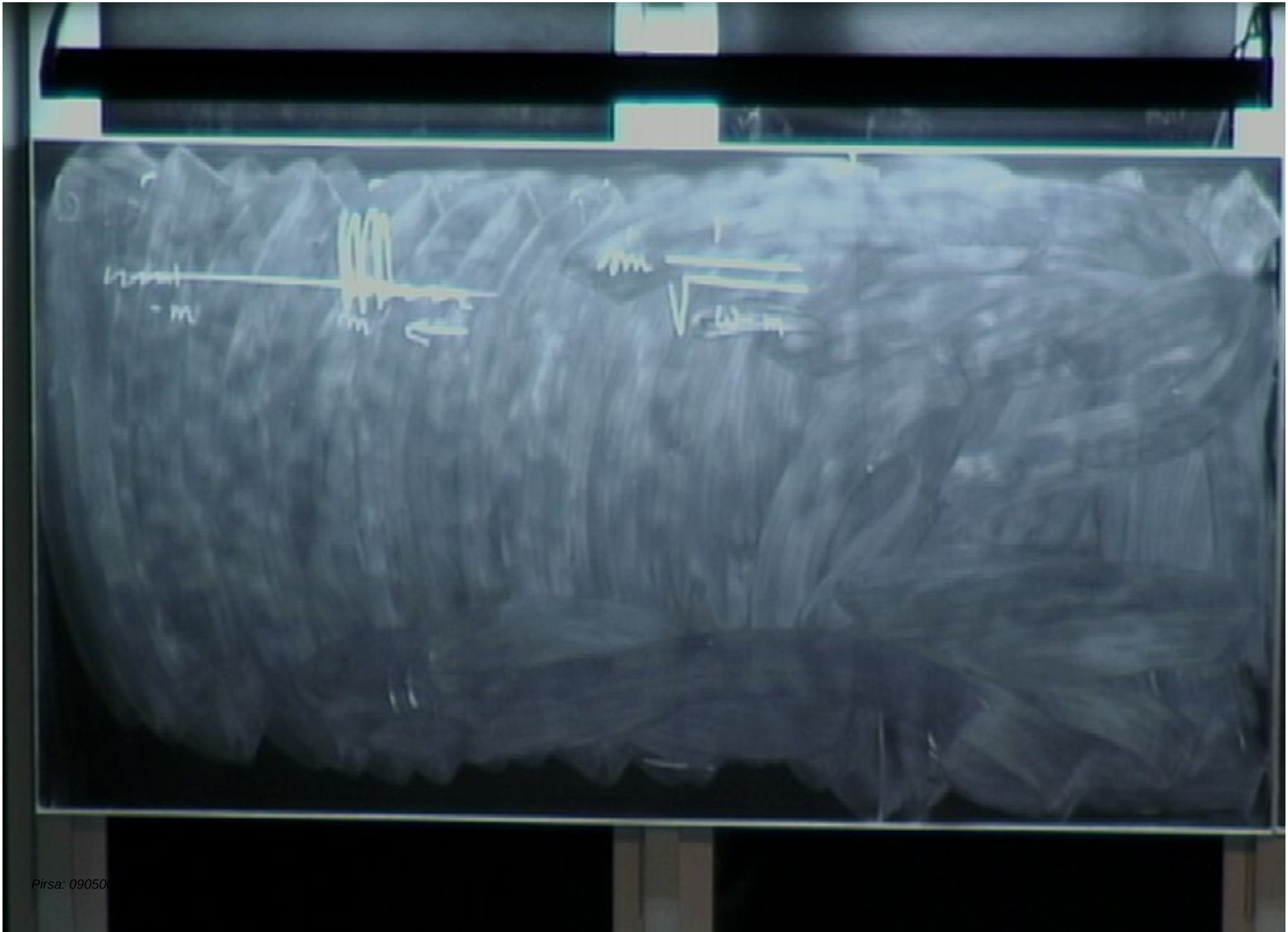
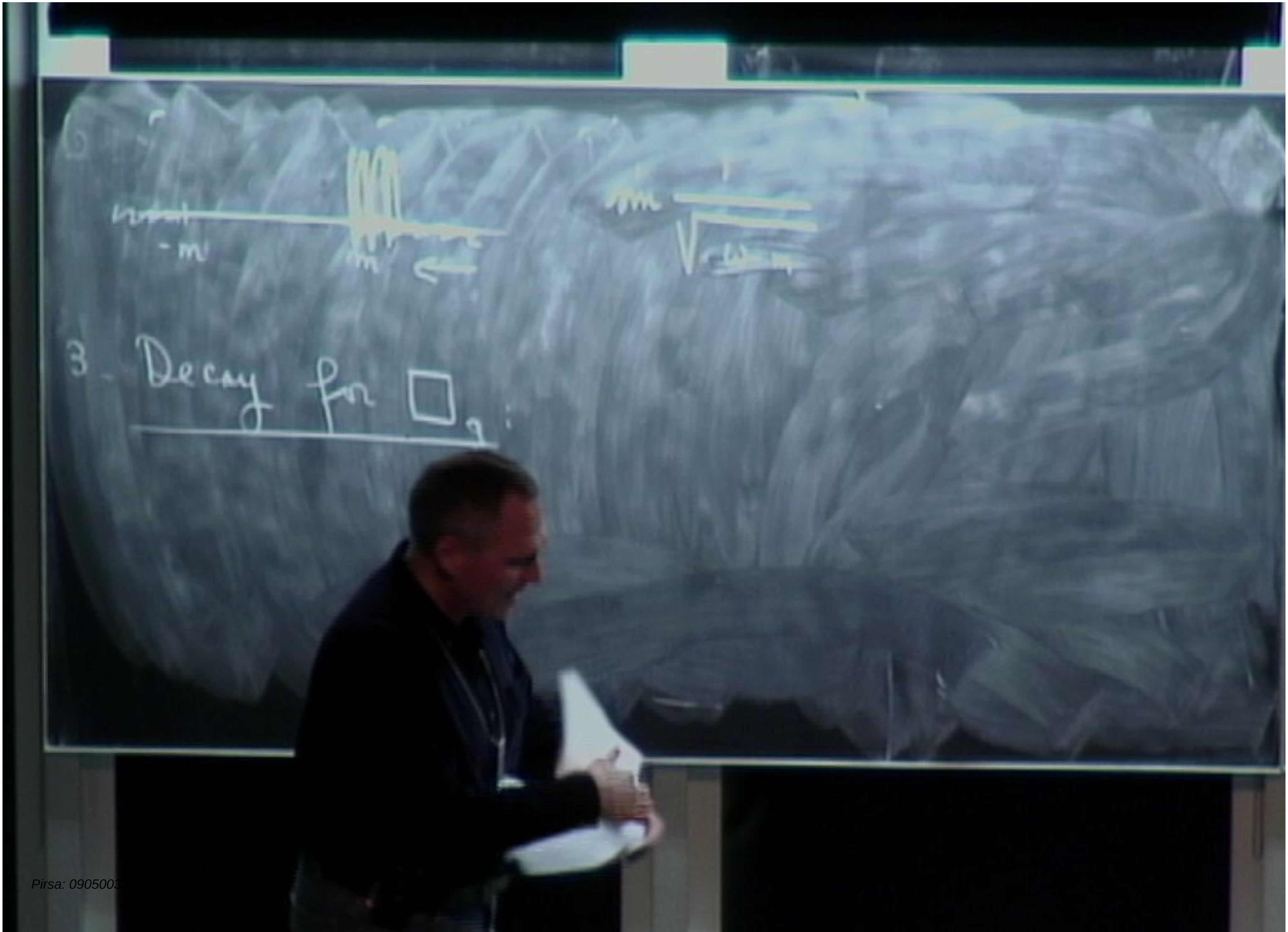
Thm: $0 \le |k| \le k_0$, $0 \le |n| \le n_0$

$$\Psi(x,t) = c(x)\, t^{-5/6} + O(t^{-5/6-\varepsilon}) \qquad \varepsilon < \frac{1}{30}$$

Rk: $t^{-3/2}$ Slower in Kerr



3. Decay for $\Box_g$:


3. Decay for □g :
Pirsa: 0905003

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$$\mathcal{E} = \cdots \left(\frac{1}{m^2_\phi} - \frac{a^2}{\Delta} \right) |\partial_\varphi \bar{\Phi}|^2$$

< 0 inside ergosphere.

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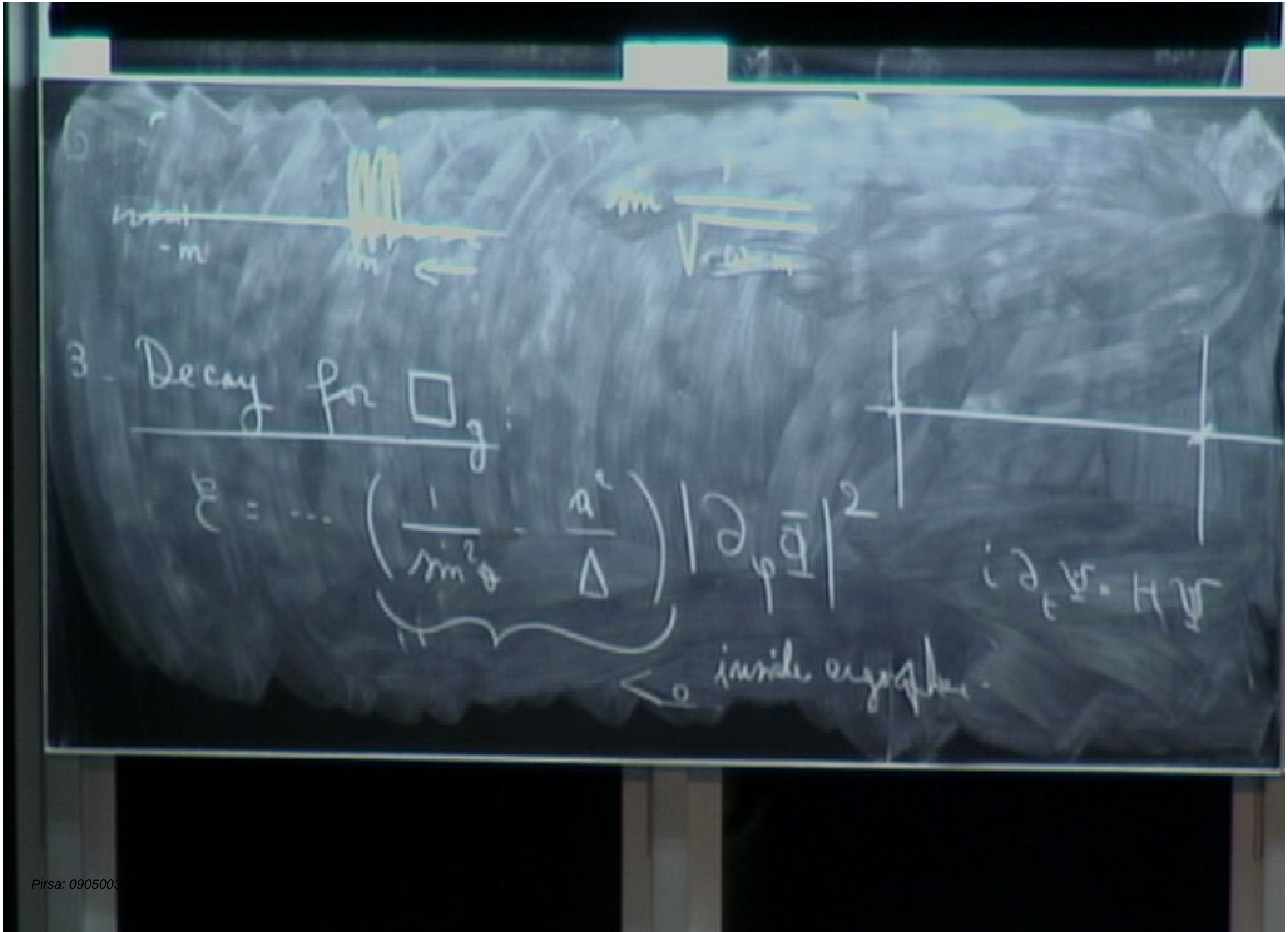
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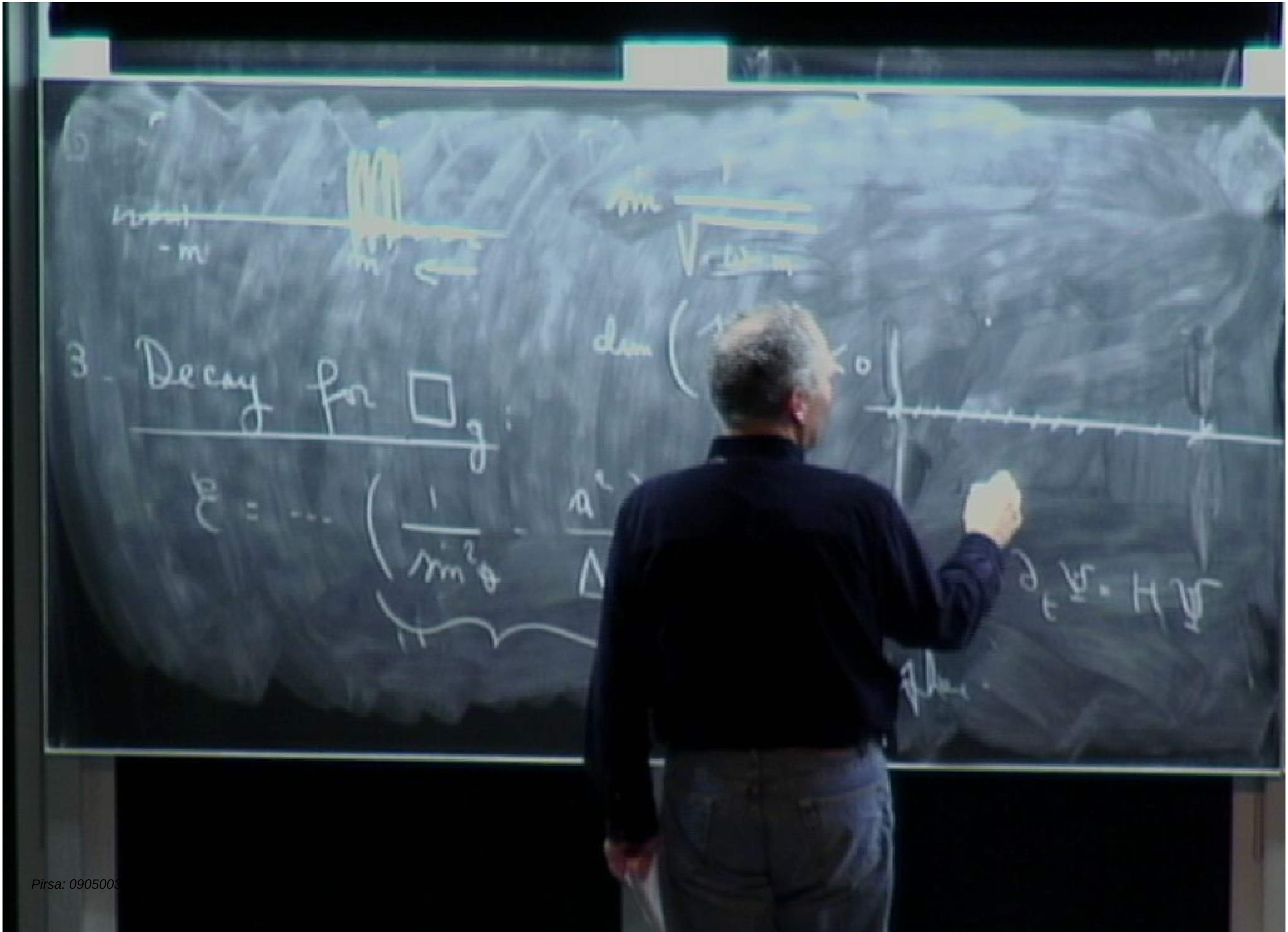
E = ... (1/m²φ - a²/Δ) |∂φ Φ|²
inside ergosphere.
i ∂t Ψ = H Ψ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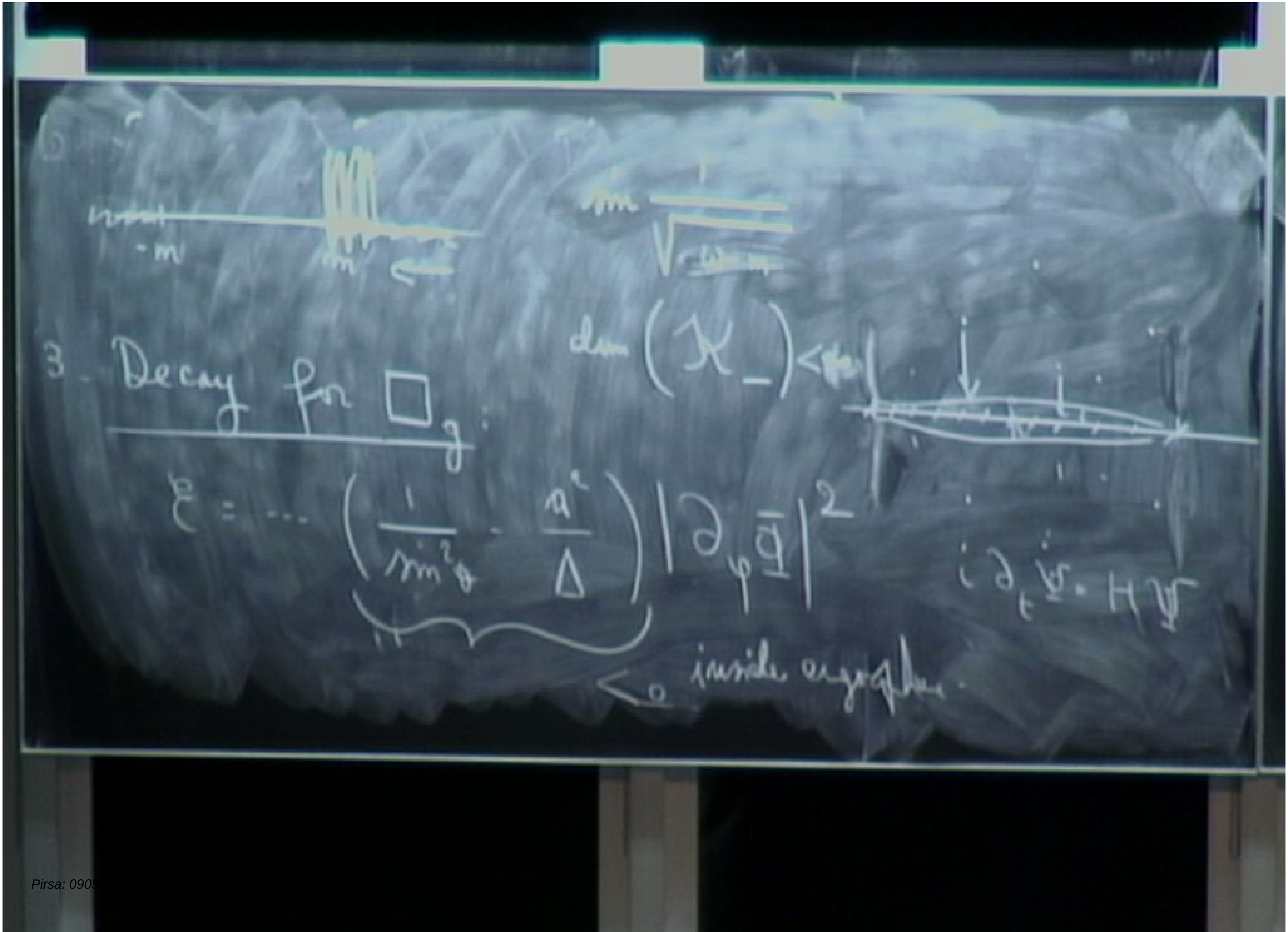
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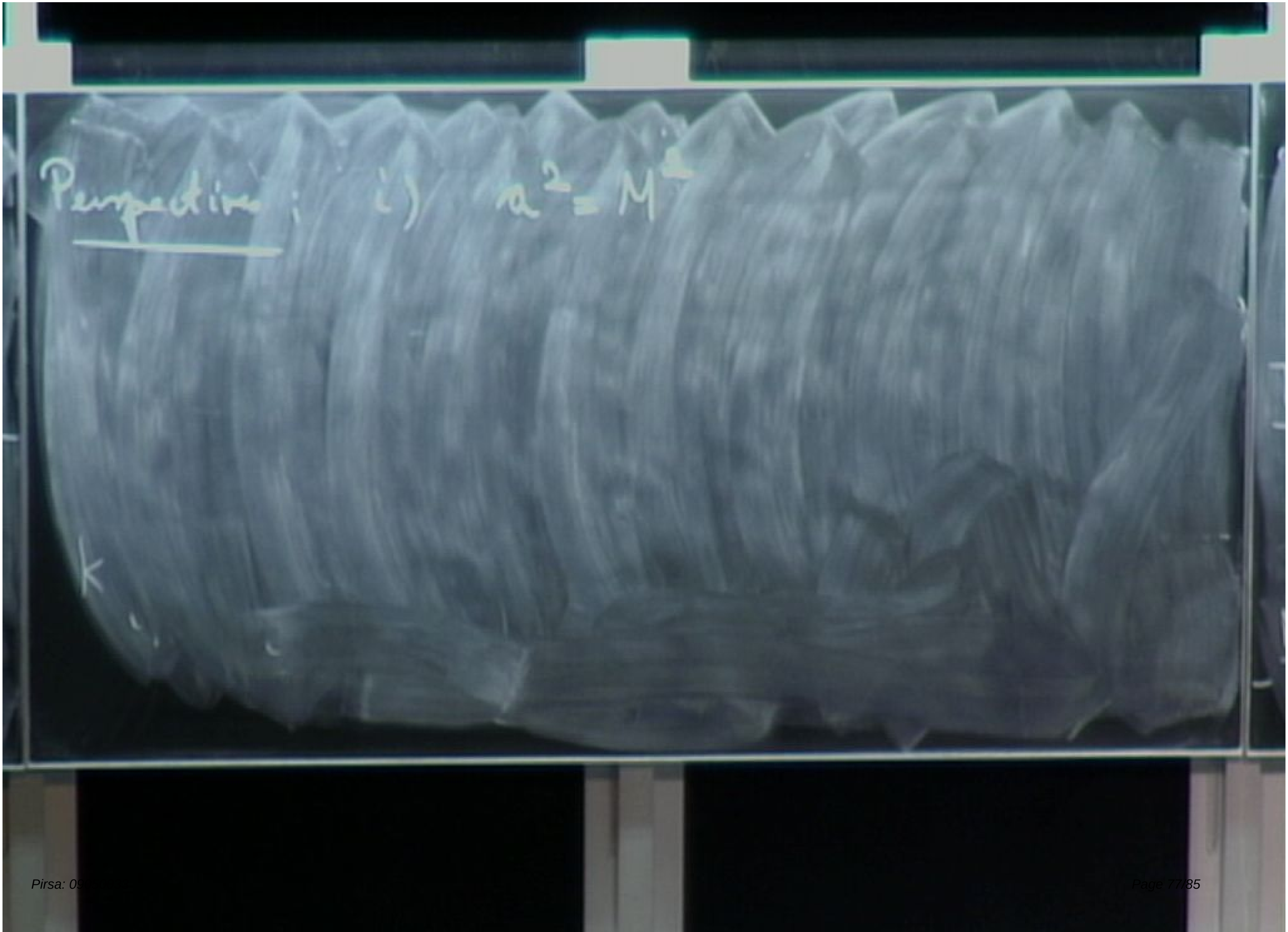
Perspectives : i) a^2 = M^2

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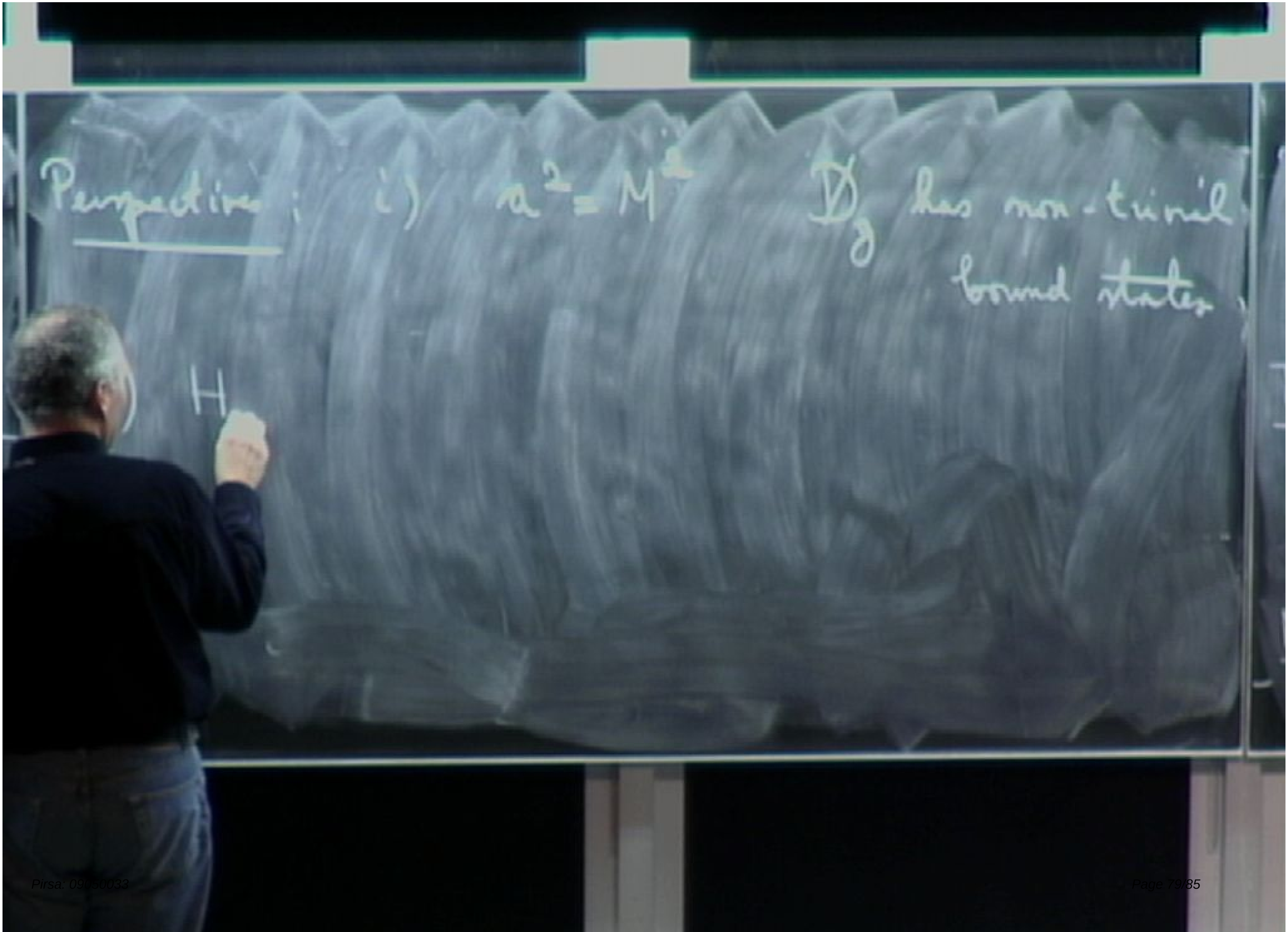
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H

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ii) Higher-dim'l BH's Myers-Perry solutions-

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iv)