Title: Integrability and spectrum of field theories

Date: May 06, 2009 11:00 AM

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Abstract: Recently methods of integrability were shown to be useful for solving gauge theories in various dimensions. I will make an introduction into integrability in two dimensions and demonstrate how the integrability works also for some three and four dimensional gauge theories.

Integrability in field theories

Nikolay Gromov

DESY & Hamburg University/PNPI

- Integrability $\simeq$ infinitely many integrals of motion
- Many physical quantities could be computed in these theories exactly
- Most studied in 2D
- Since recently in 3D and 4D

Plan

- Integrability in 2D
- Classical integrability of S3 sigma model
- Integrability in 4D via AdS/CFT

Integrability in 2D

Operator corresponding to an integral of motion $\widehat{C}_n$

$$\widehat{C}_n|k\rangle = \omega_n(k)|k\rangle$$

Between in and out states

$$_{out}\langle p_1,\ldots,p_m|\widehat{C}_n|k_1,\ldots,k_{m'}\rangle_{in}$$

The outgoing momenta are constrained:

$$A_n = \sum_i \omega_n(k_i) = \sum_i \omega_n(p_i)\ ,\ \ n = 1,\ldots$$

The only solution:

$$m = m' \quad \{k_i\} = \{p_i\}$$

Factorization of S-matrix

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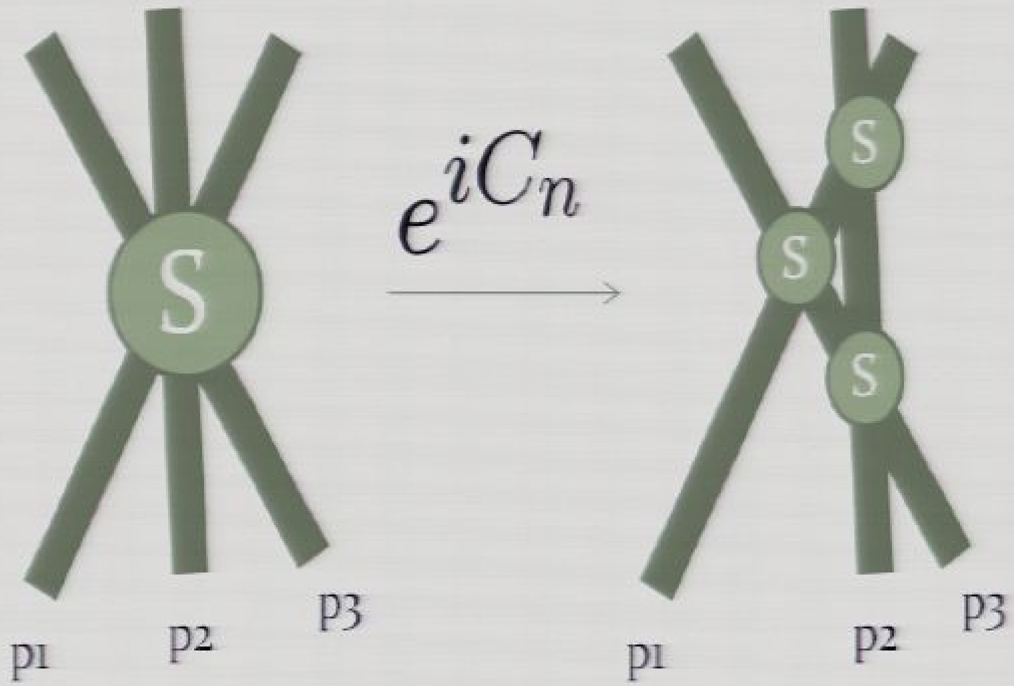

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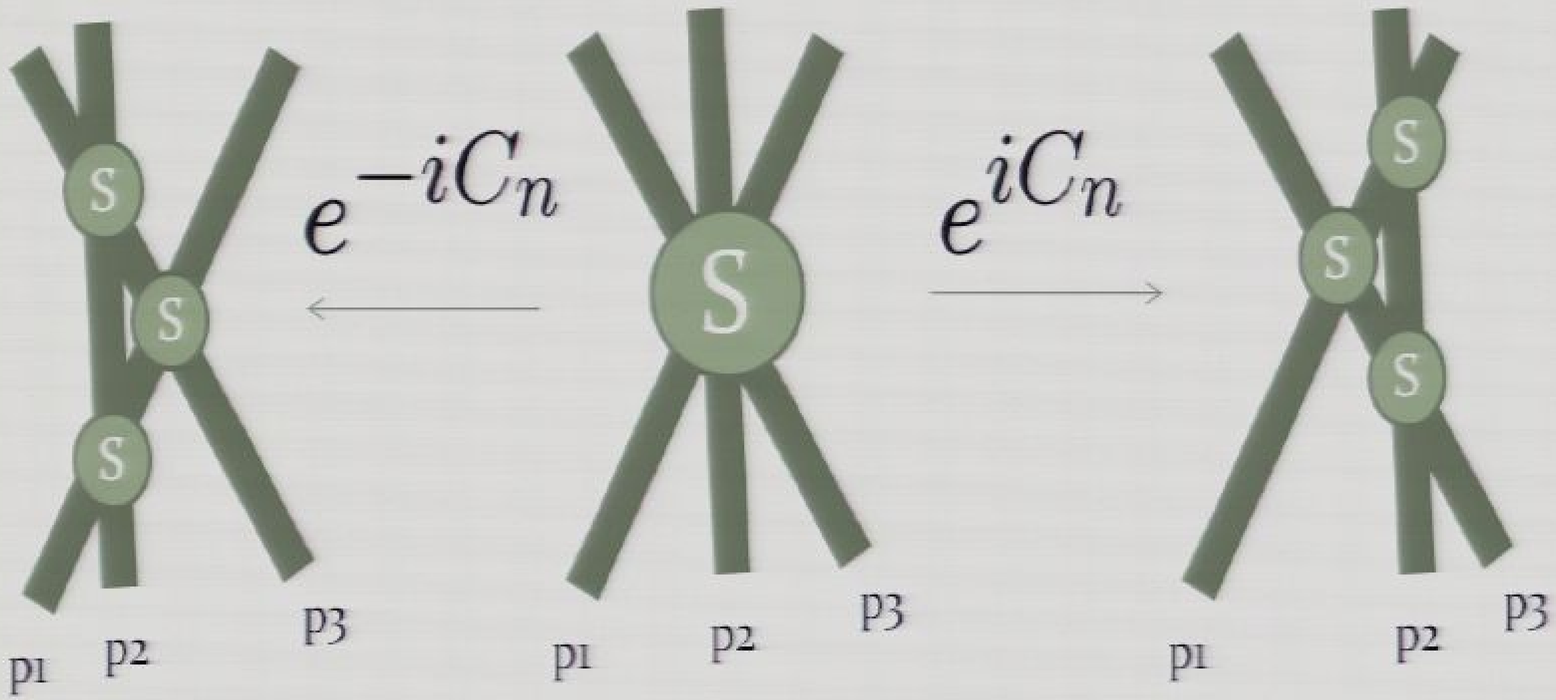

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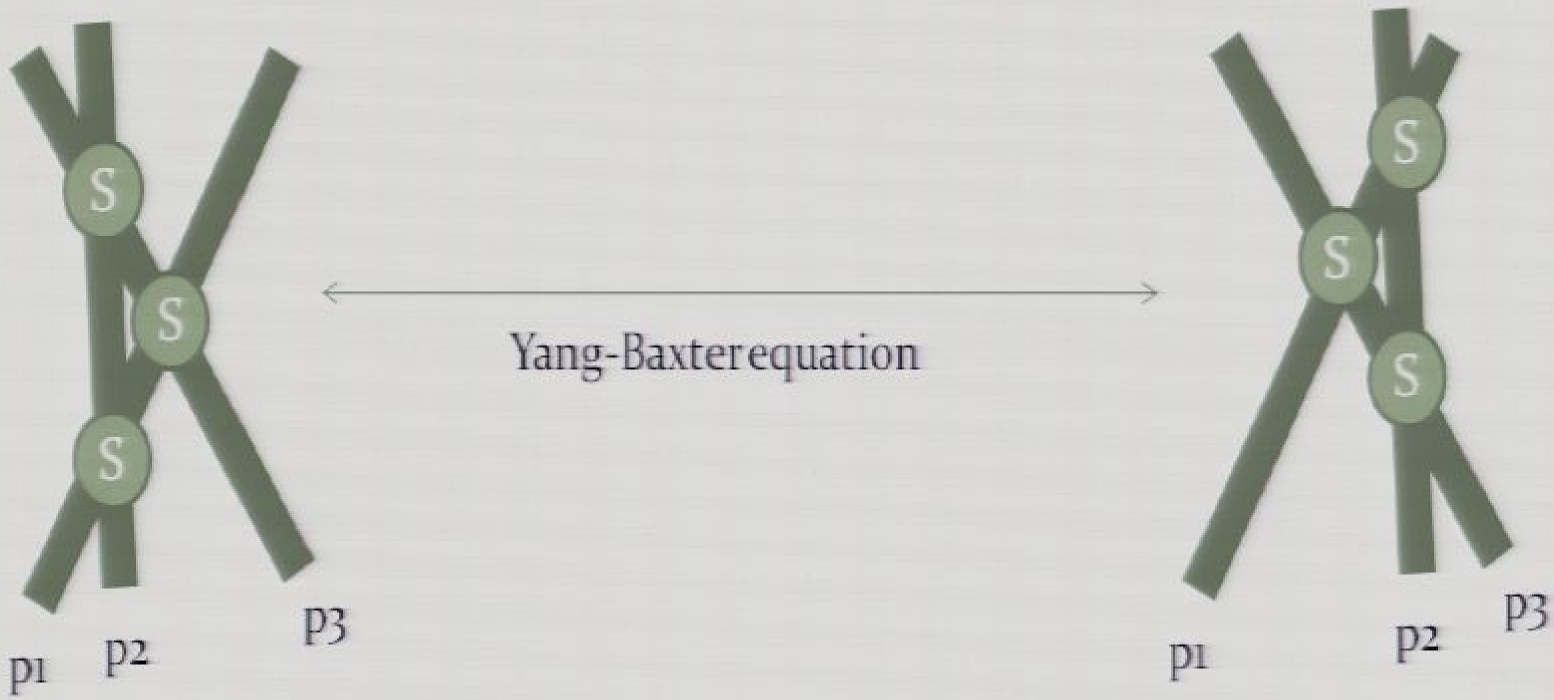
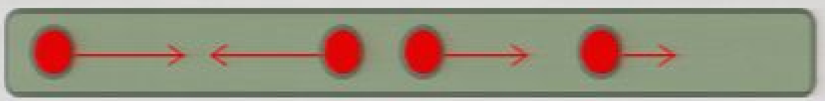


Asymptotic Spectrum

- For spectral density we need finite volume

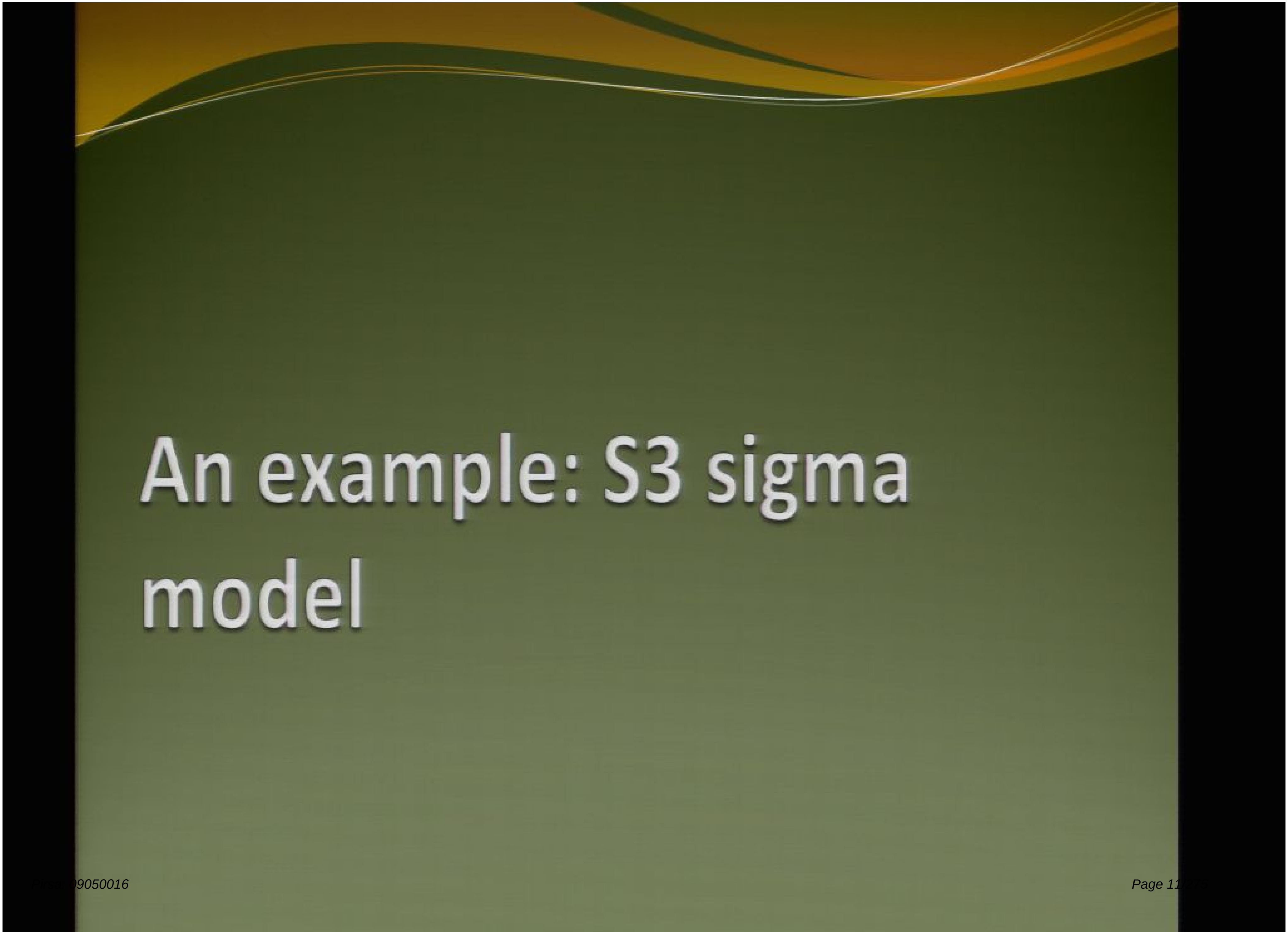


$$\Psi(x_1+L,x_2,\ldots)=e^{ip_1L}S(p_1,p_2)\ldots S(p_1,p_n)\Psi(x_1,x_2,\ldots)$$

- From periodicity of the wave function

$$ip_iL=2\pi i n_i+\sum_j \log S(p_i,p_j)$$

-Bethe equations


An example: S3 sigma
model

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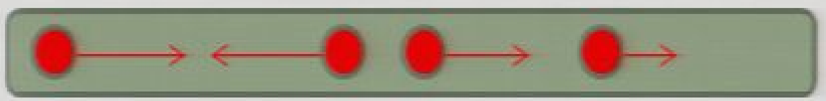

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Example – S^3 sigma model

$$S = \int d\tau d\sigma \sum_{a=1}^{4} \left((\partial_\tau X_a)^2 - (\partial_\sigma X_a)^2 \right)$$

The scalar fields are living on a sphere

$$X_1^2 + X_2^2 + X_3^2 + X_4^2 = 1$$

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Equations of motion

$$\partial_\mu \partial^\mu X_a + (\partial_\nu X_b \partial^\nu X_b) X_a = 0 \qquad \partial^\mu j_\mu = 0$$

Example – S³ sigma model

$$A_\mu(z) = \frac{j_\mu + z\ \ \epsilon_{\mu\nu} j^\nu}{z^2 - 1} \qquad \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu] = 0$$

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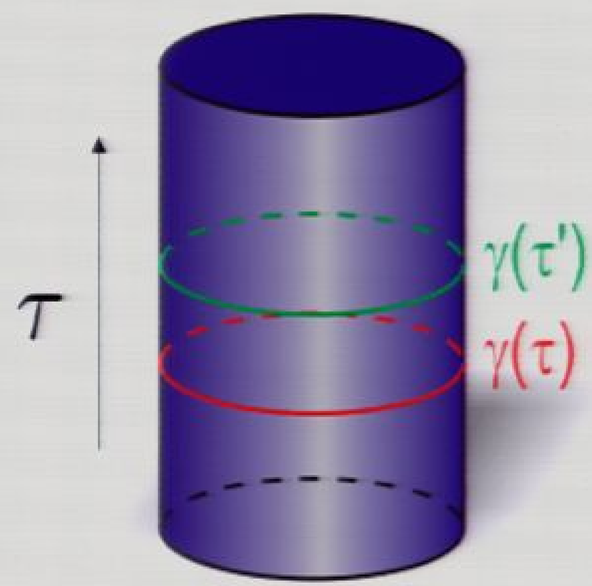

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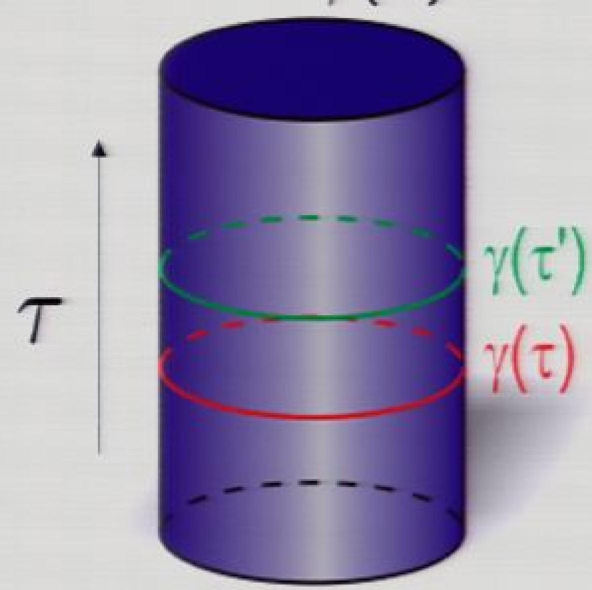

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Bethe ansatz equations

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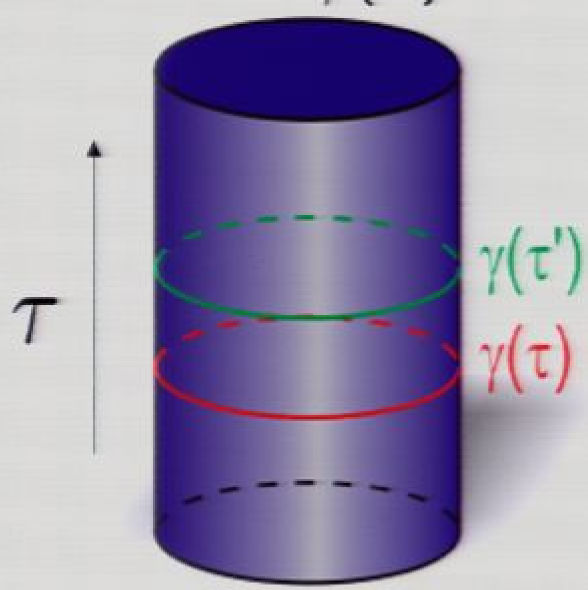

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Symmetry:

$$O(4) \simeq SU_R(2) \times SU_L(2)$$

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Bethe ansatz:

Zamolodchikov x2
Faddeev, Reshetikhin

$$1 = \prod_{\beta}^{J_u} \frac{u_j - \theta_\beta - i/2}{u_j - \theta_\beta + i/2} \prod_{i \neq j}^{J_u} \frac{u_j - u_i + i}{u_j - u_i - i},$$

$$e^{-imL \sinh \pi\theta_\alpha} = \prod_{\beta \neq \alpha}^{L} S_0^2\left(\theta_\alpha - \theta_\beta\right) \prod_{j}^{J_u} \frac{\theta_\alpha - u_j + i/2}{\theta_\alpha - u_j - i/2} \prod_{k}^{J_v} \frac{\theta_\alpha - v_k + i/2}{\theta_\alpha - v_k - i/2},$$

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e^{-imL \sinh \pi\theta_\alpha} &= \prod_{\beta \neq \alpha}^{L} S_0^2\left(\theta_\alpha - \theta_\beta\right) \prod_{j}^{J_u} \frac{\theta_\alpha - u_j + i/2}{\theta_\alpha - u_j - i/2} \prod_{k}^{J_v} \frac{\theta_\alpha - v_k + i/2}{\theta_\alpha - v_k - i/2}, \\
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$$E = \sum m \cosh \pi\theta_\alpha \ , \ \ P = \sum m \sinh \pi\theta_\alpha$$

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Integrability in higher dimensions via AdS/CFT correspondence

Integrability in higher dimensions

N=4 Super Yang-Mills: $S = \frac{1}{g_{YM}^2} \int d^4x \, \mathrm{tr} \left\{ \frac{1}{2} F_{\mu\nu}^2 + (D_\mu \Phi_i)^2 - \frac{1}{2} [\Phi_i, \Phi_j]^2 + \mathrm{fermions} \right\}$

$$\mathcal{O}_i(x) = \mathrm{tr} \Phi_1 \Phi_2 \Phi_1 \Phi_1 \Phi_1 \Phi_2 \Phi_2 \Phi_1 \Phi_1 \Phi_1$$

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$\mathcal{O}_i^{\mathrm{ren}} = Z_{ij}(\Lambda)\mathcal{O}_j^{\mathrm{bare}}$ $\quad \Gamma = Z^{-1}\frac{dZ}{d\log\Lambda}$ - Mixing matrix – integrable Hamiltonian

[Minahan, Zarembo 2002&2008]

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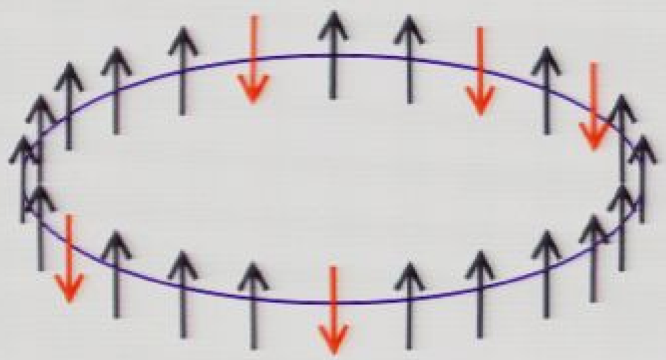

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- Mixing matrix – integrable Hamiltonian

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The problem of diagonalization of the huge matrix can be solved by Bethe ansatz

YM: One-loop

1) Solve polynomial equation

$$\left(\frac{u_j + i/2}{u_j - i/2}\right)^L = \prod_{k=1 (k \neq j)}^{J} \frac{u_j - u_k + i}{u_j - u_k - i}$$

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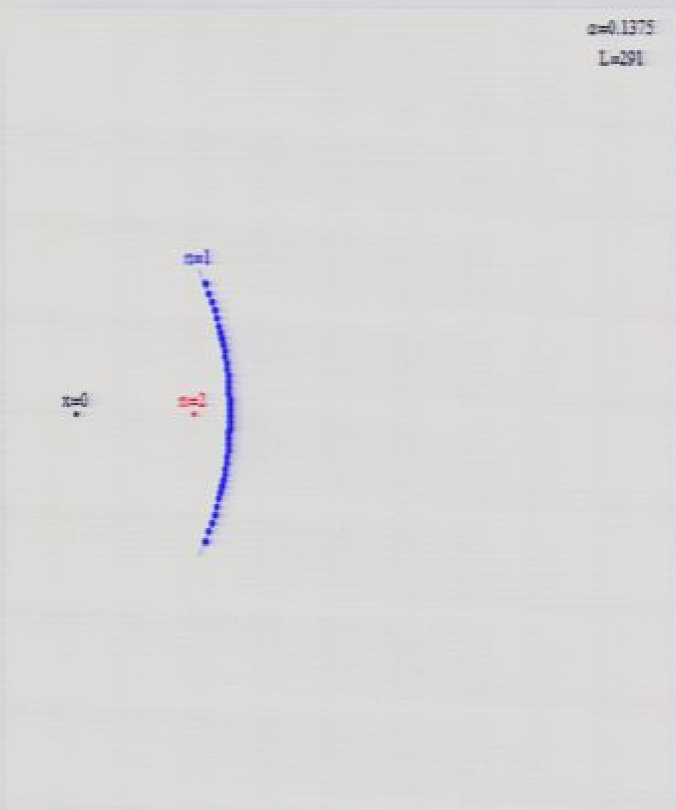
Numerical Solution



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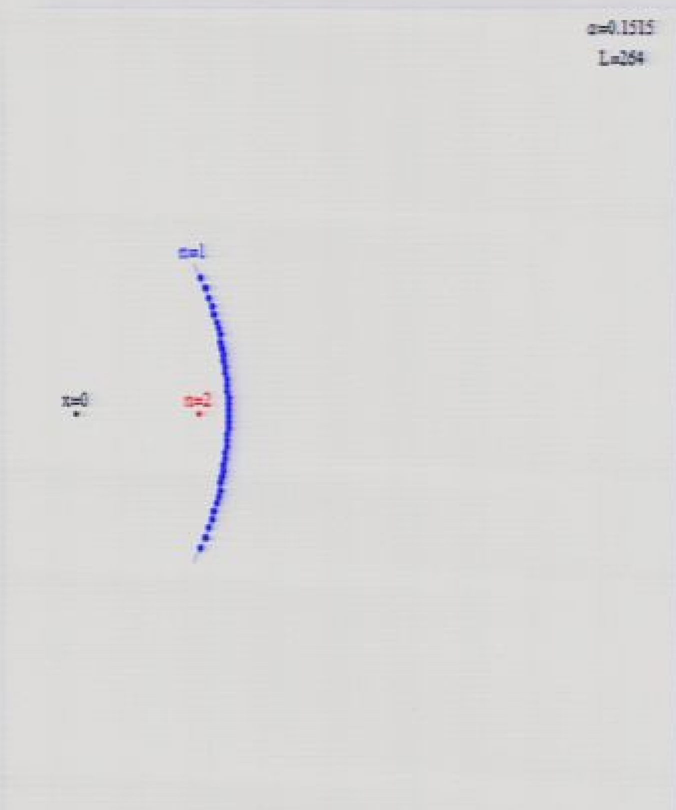

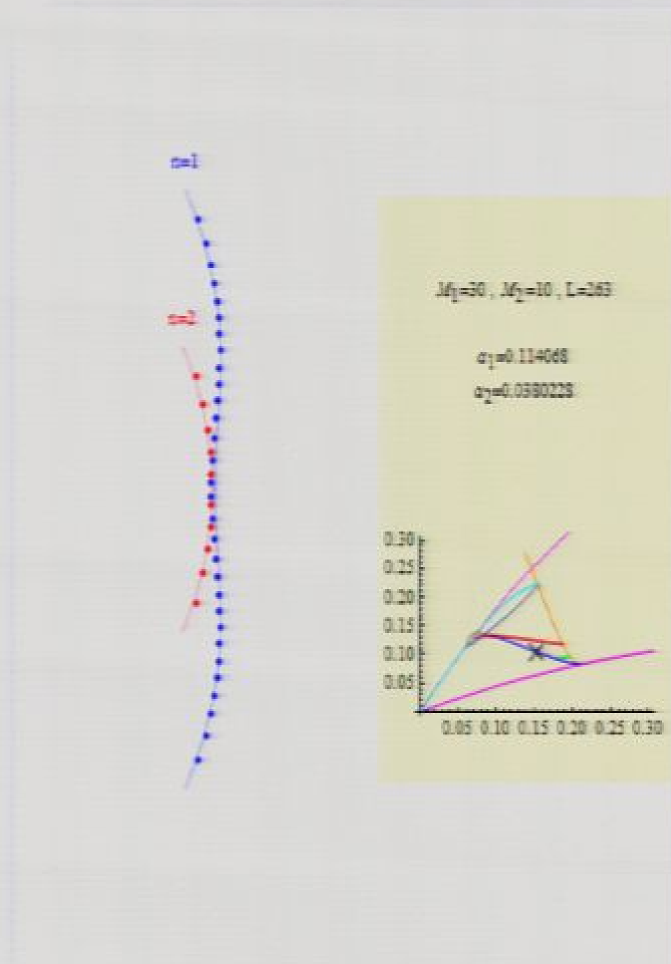

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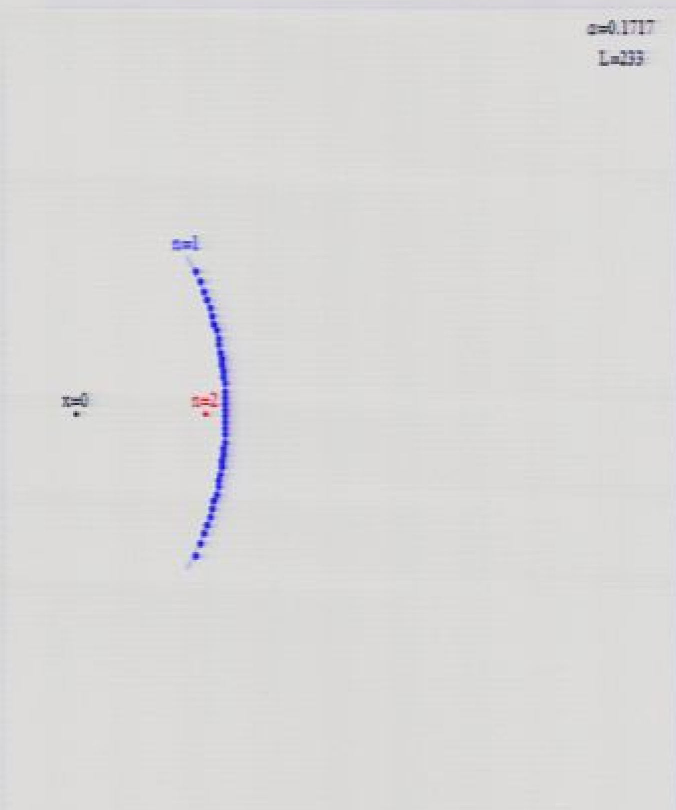

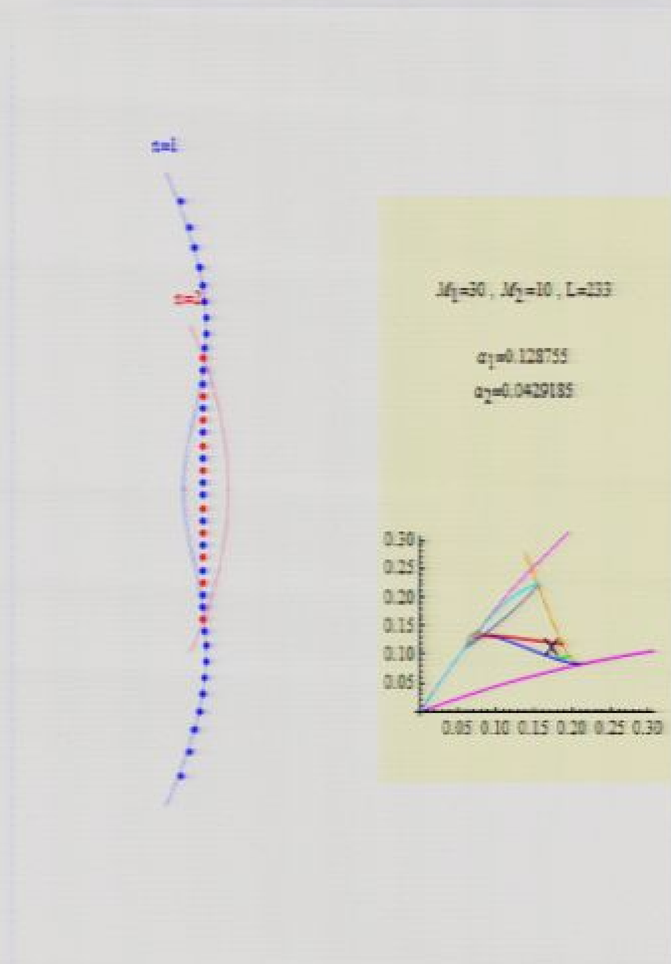

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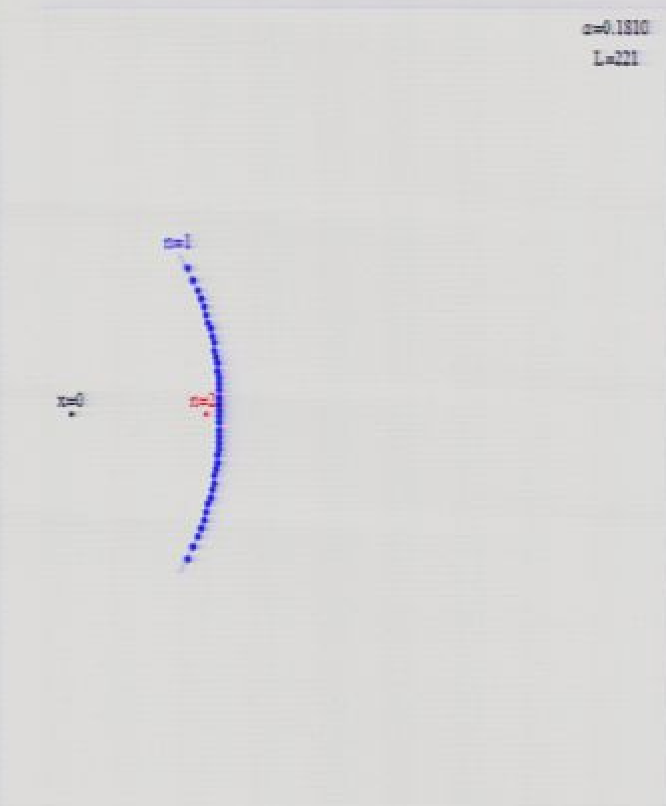

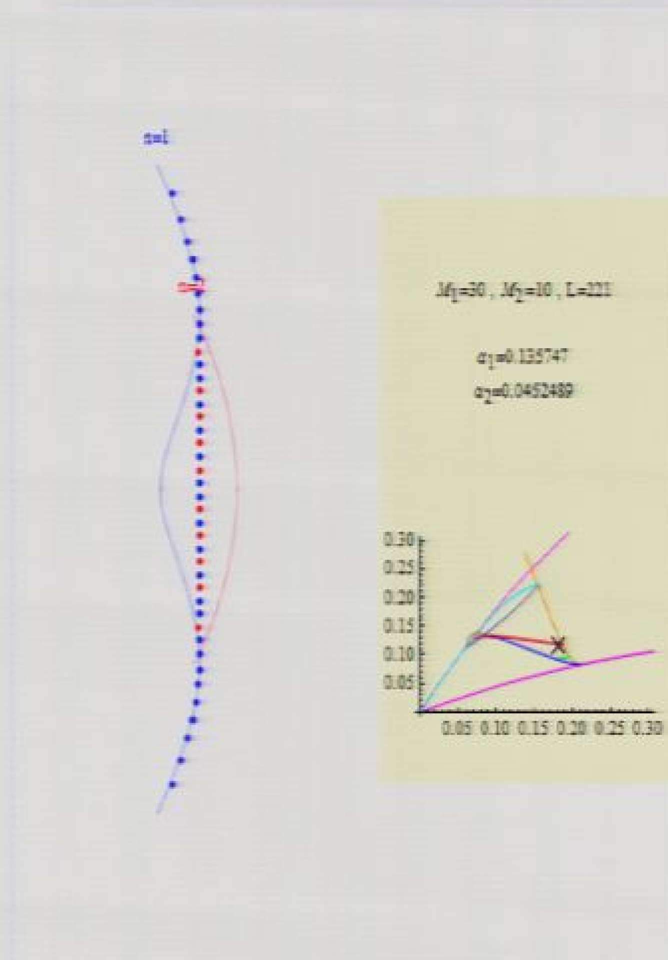

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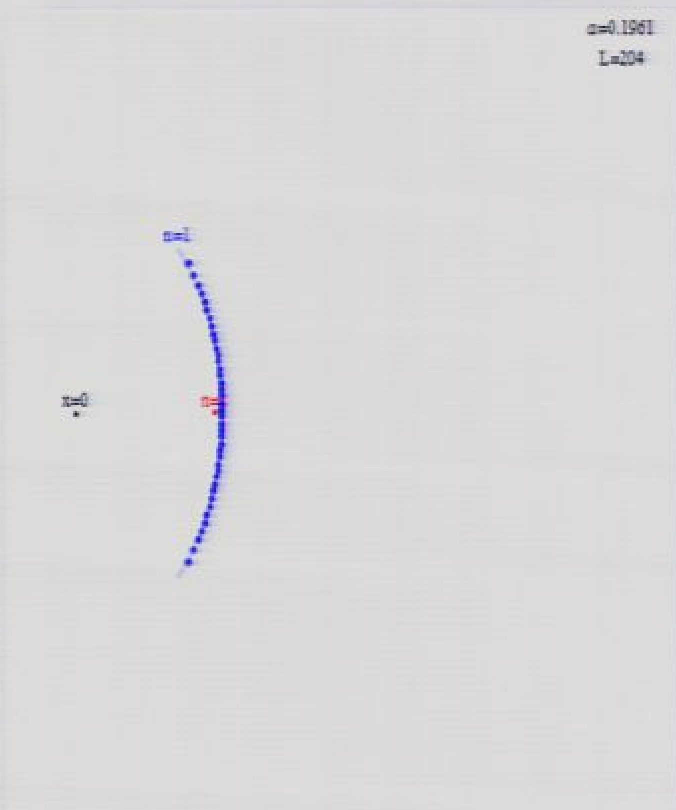

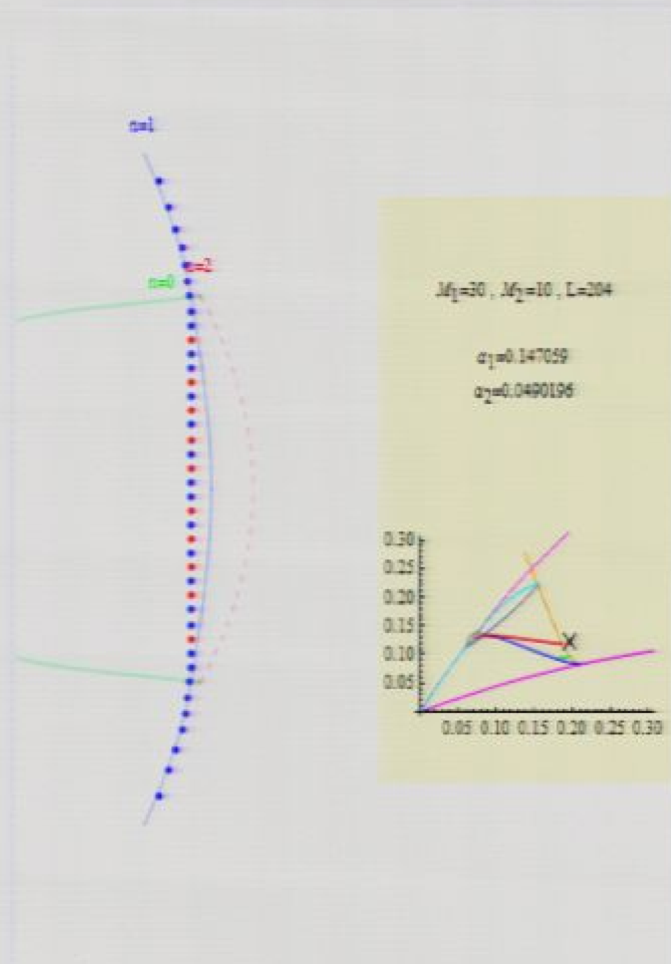

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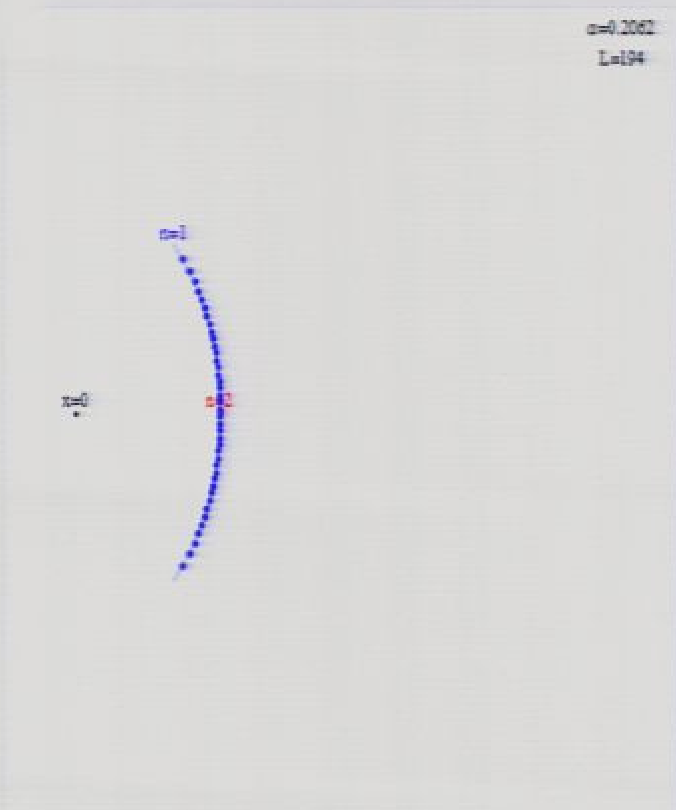

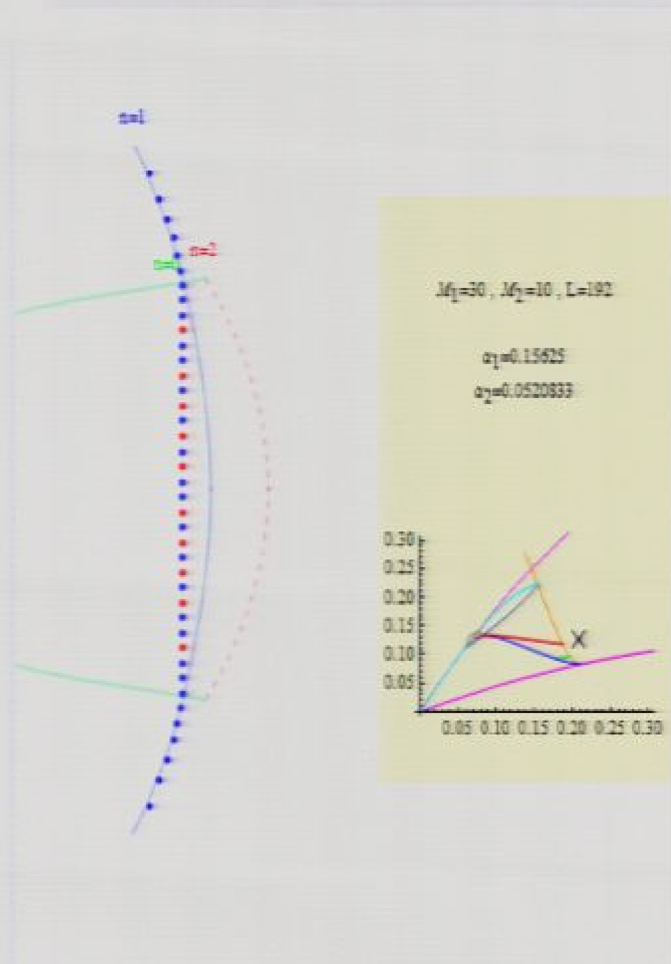

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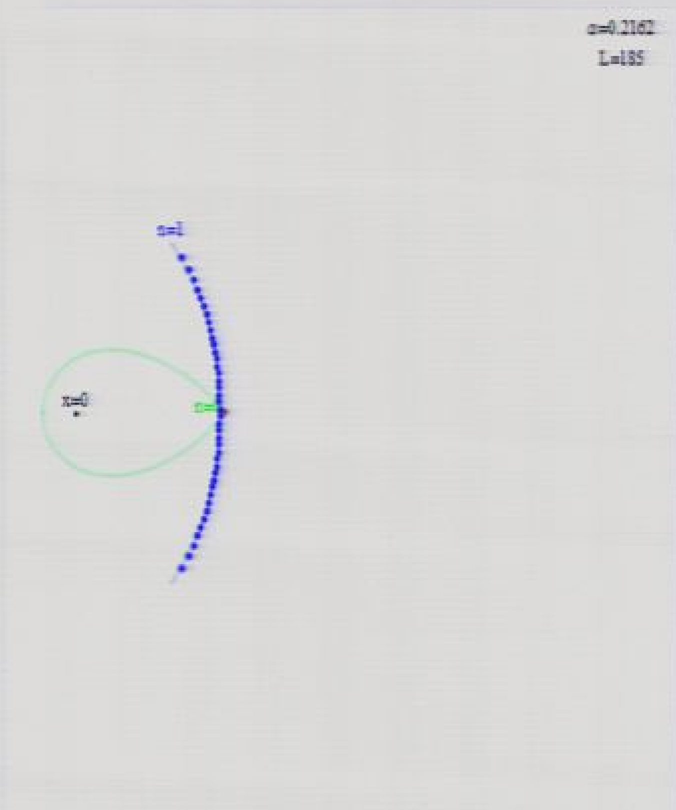

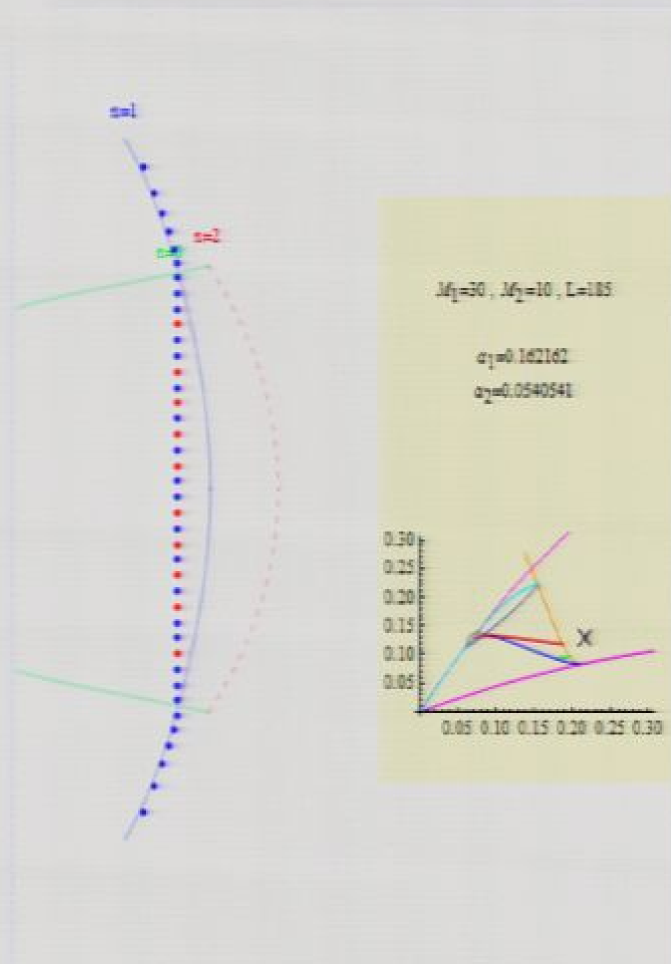

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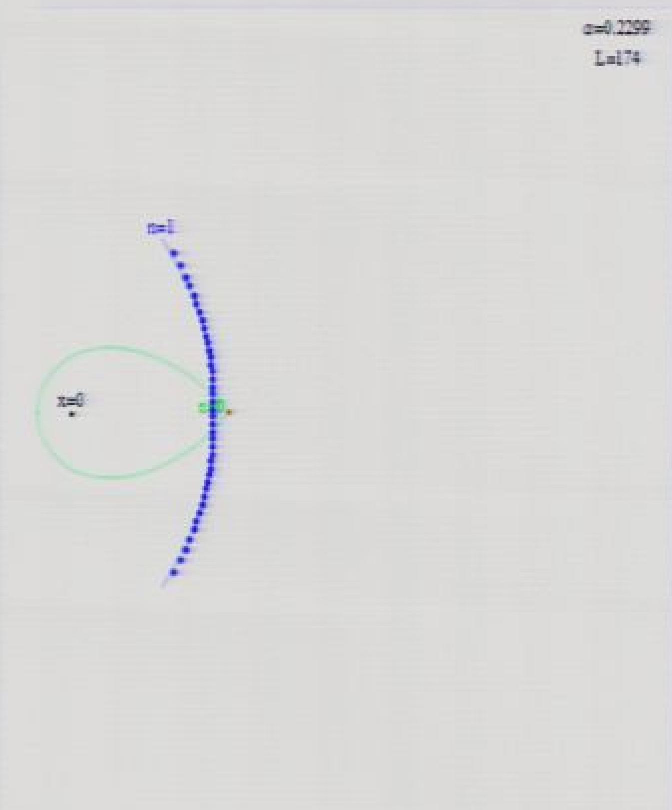

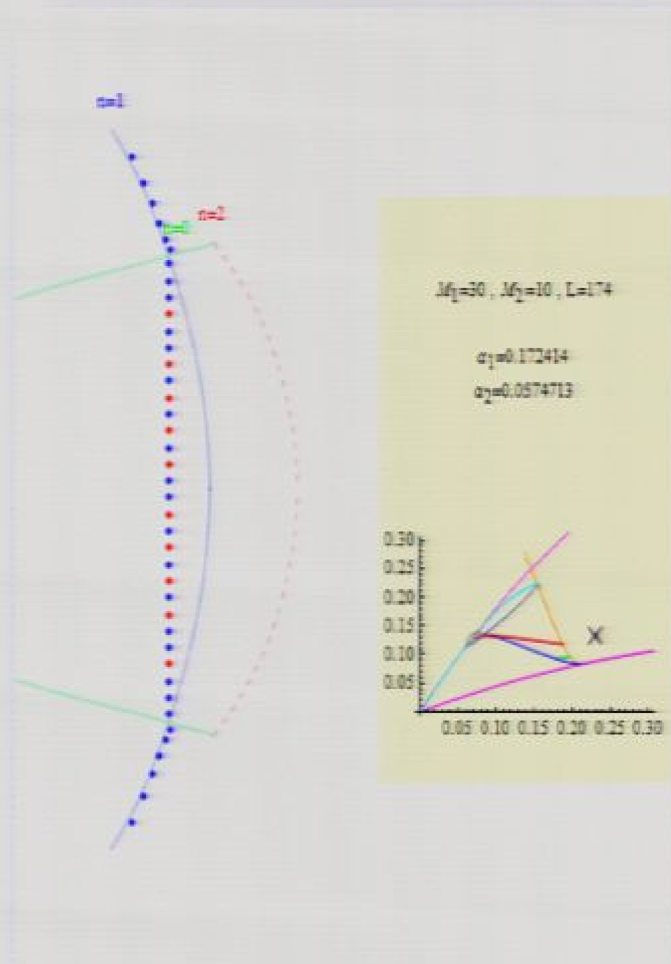

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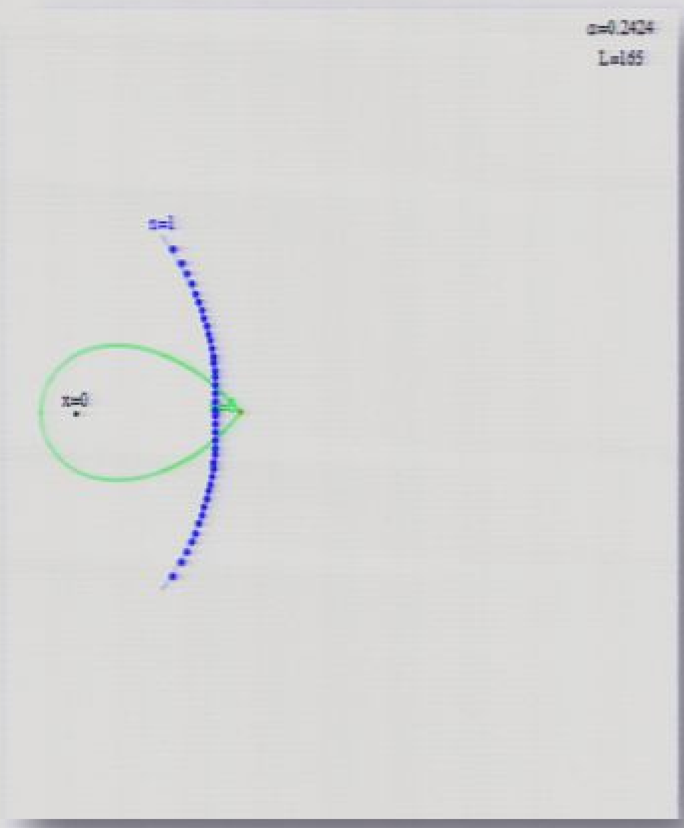

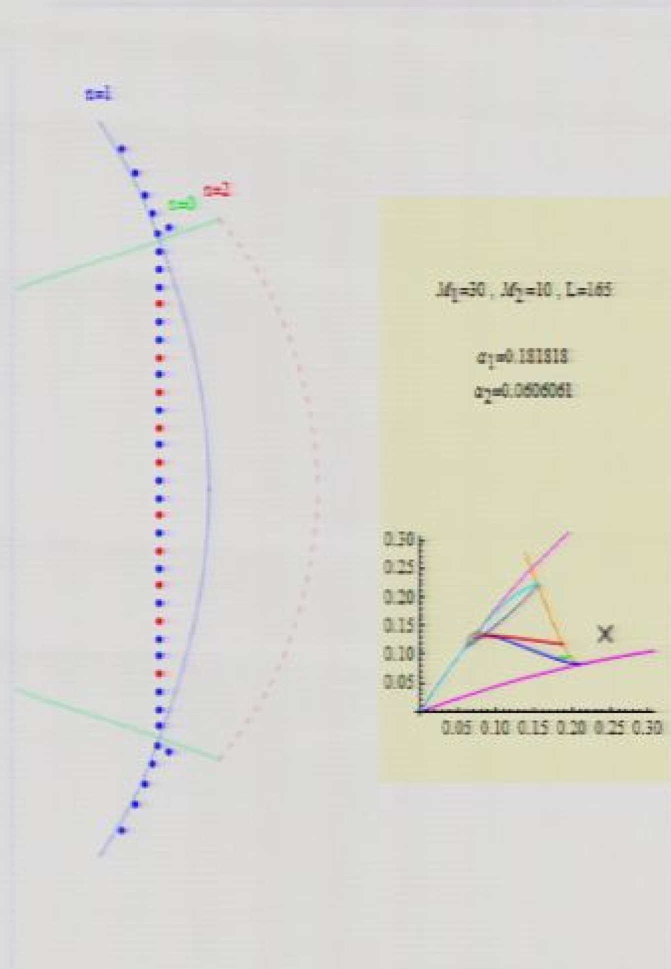

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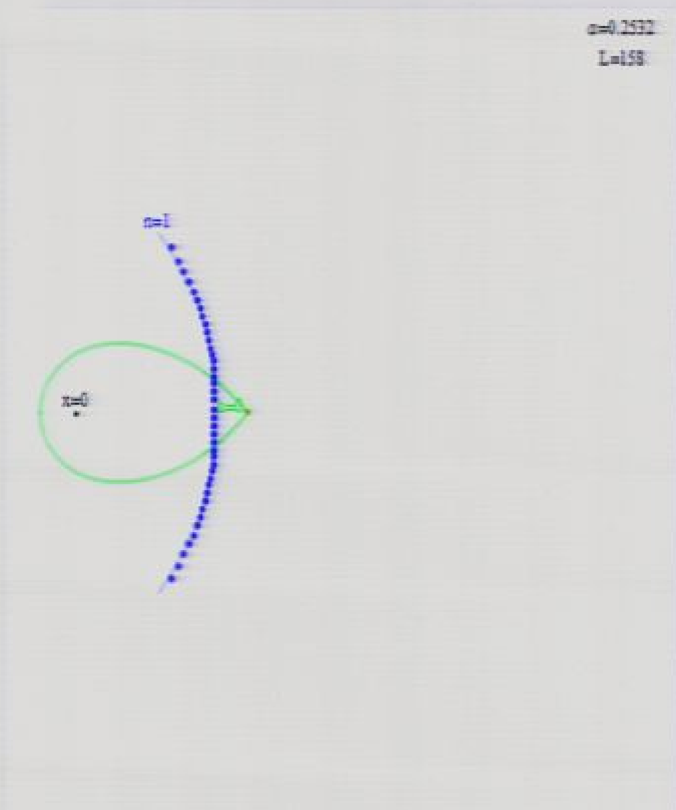

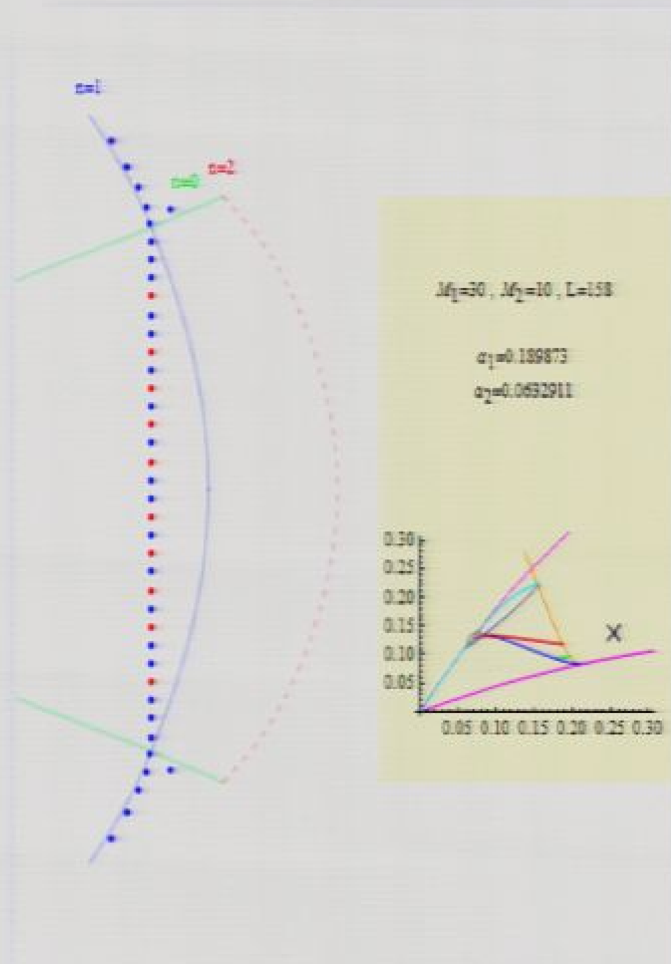

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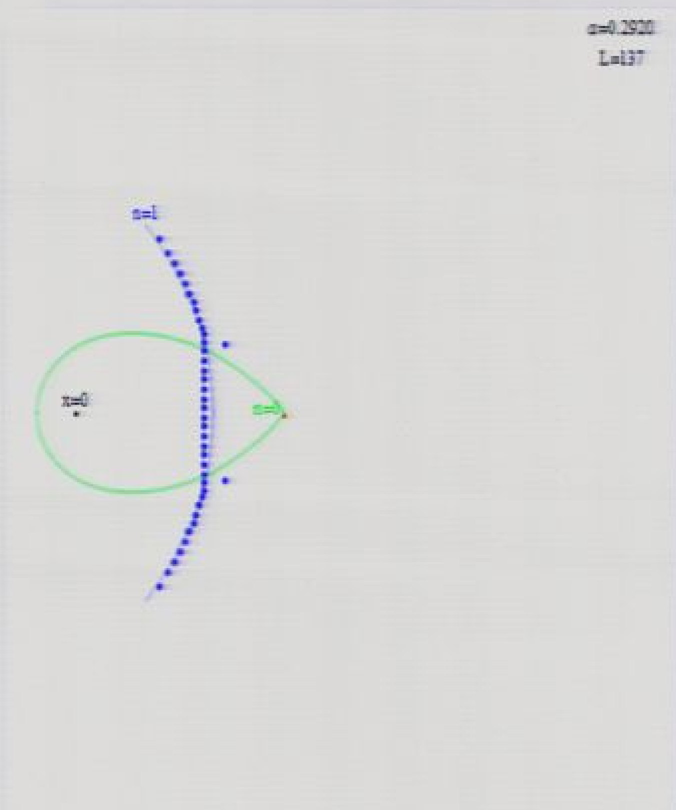

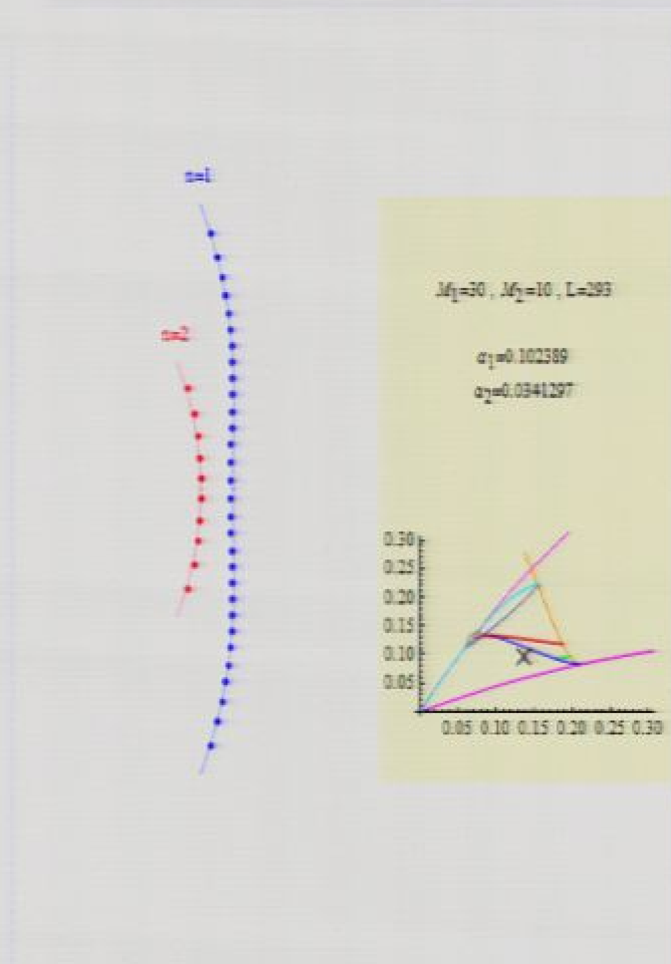

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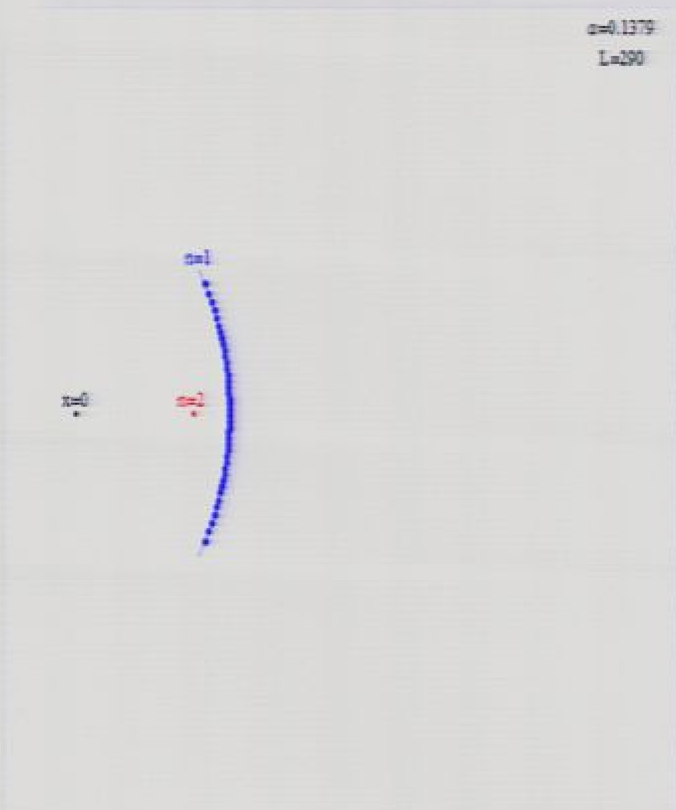

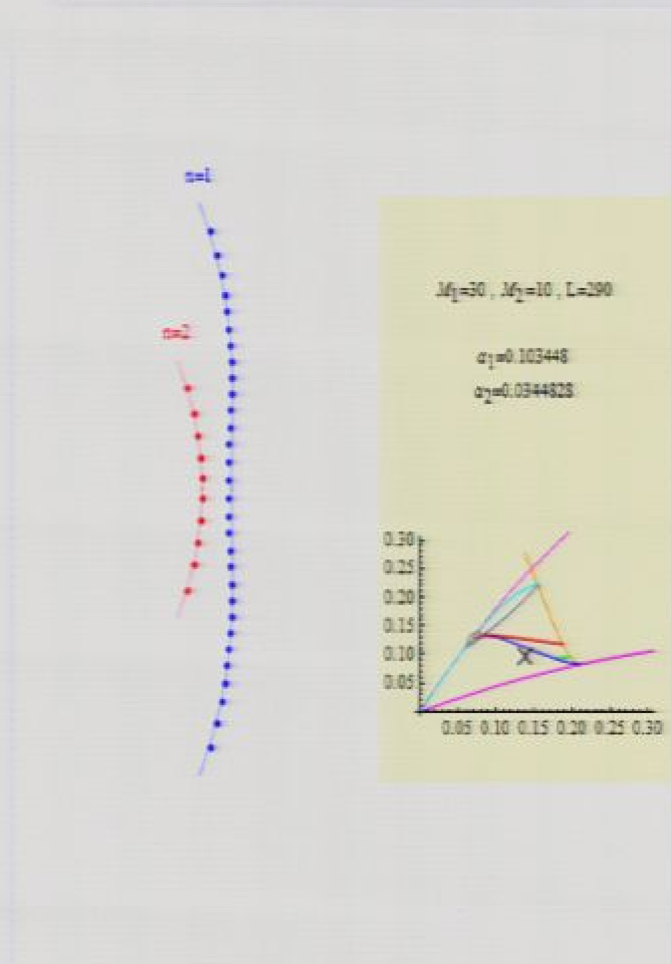

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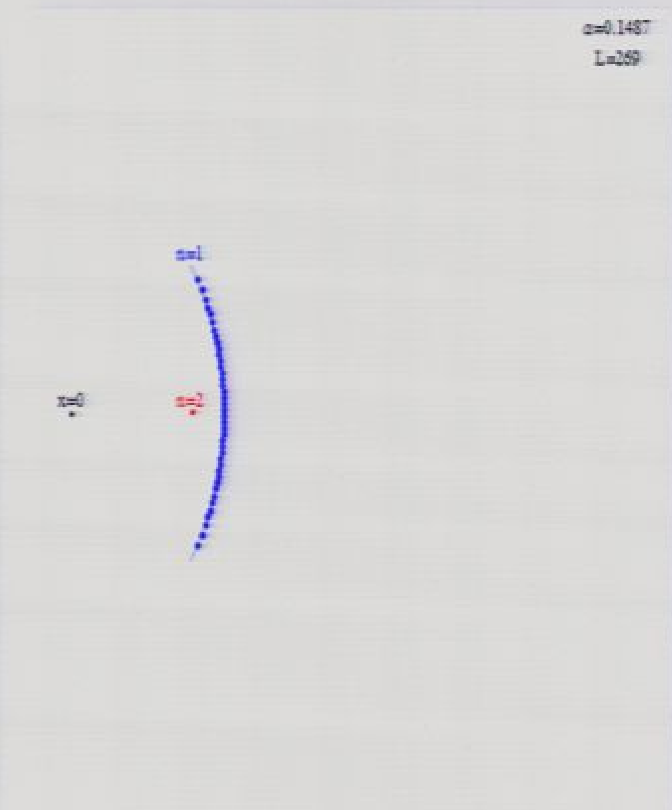

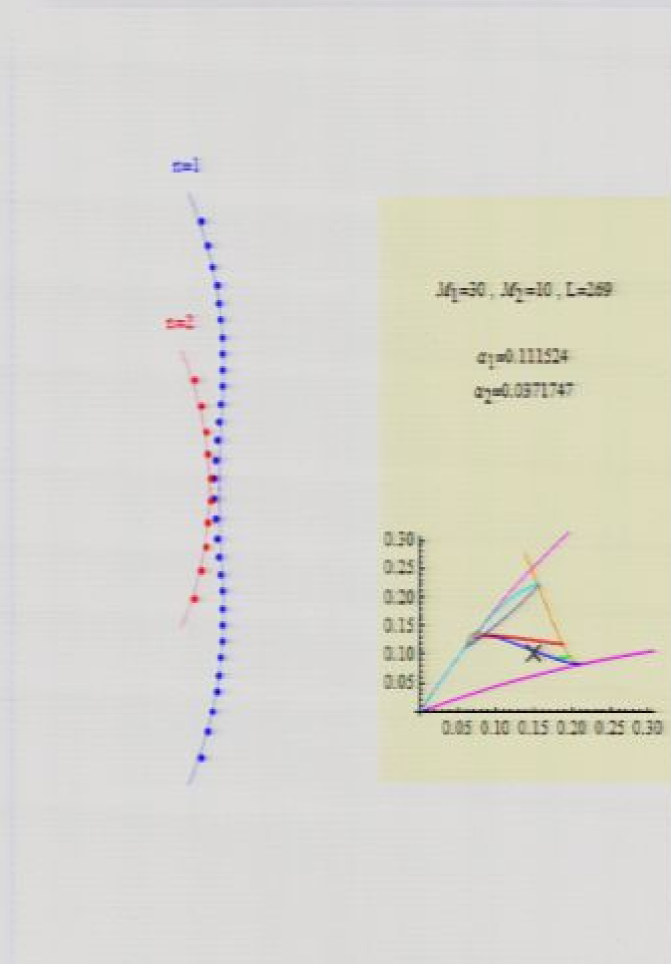

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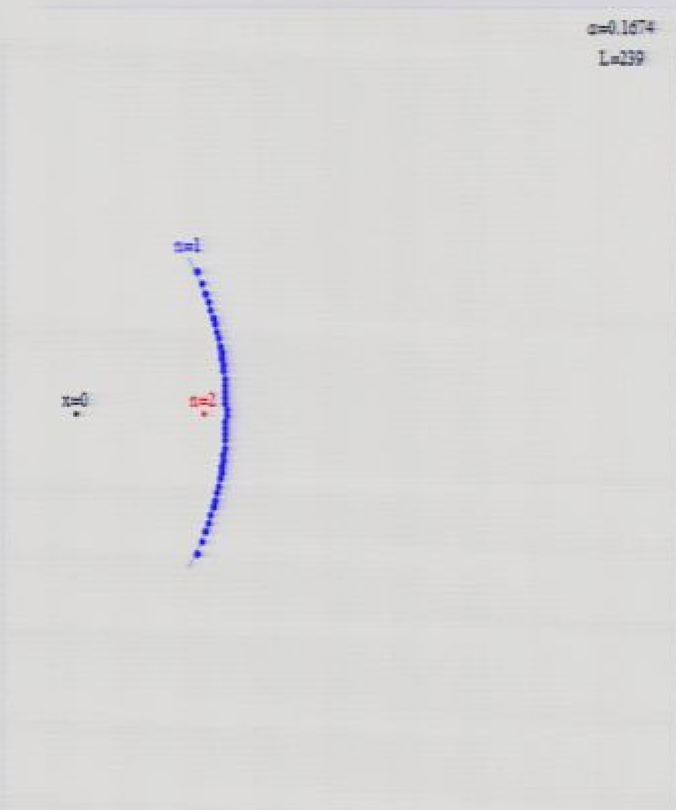

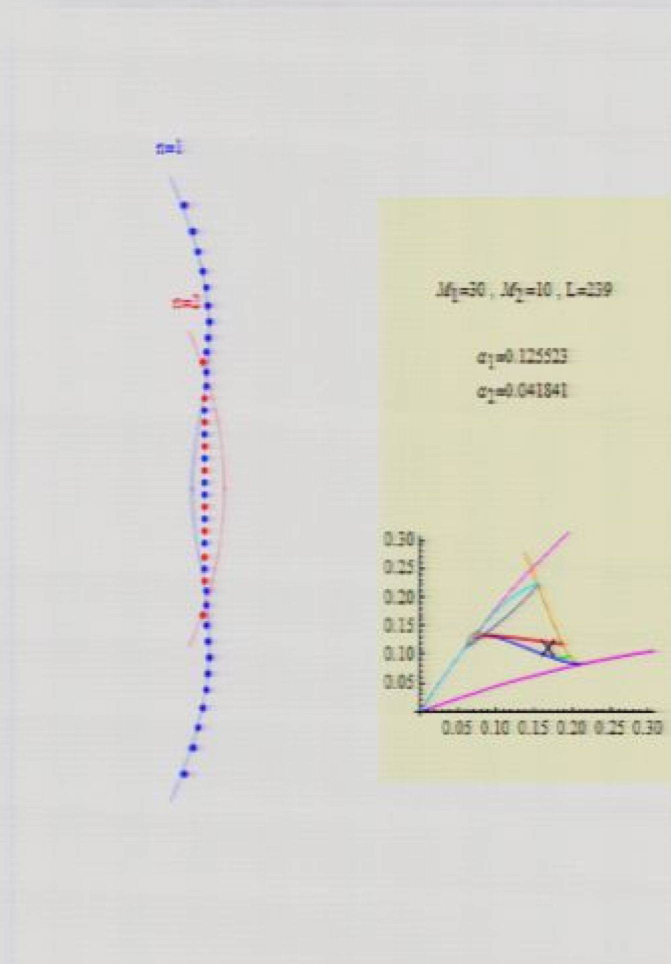

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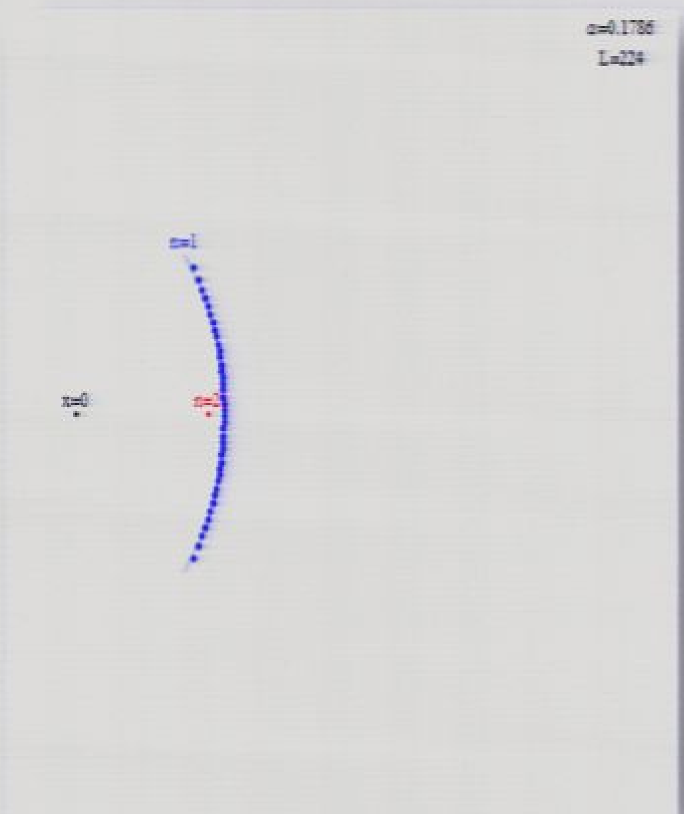

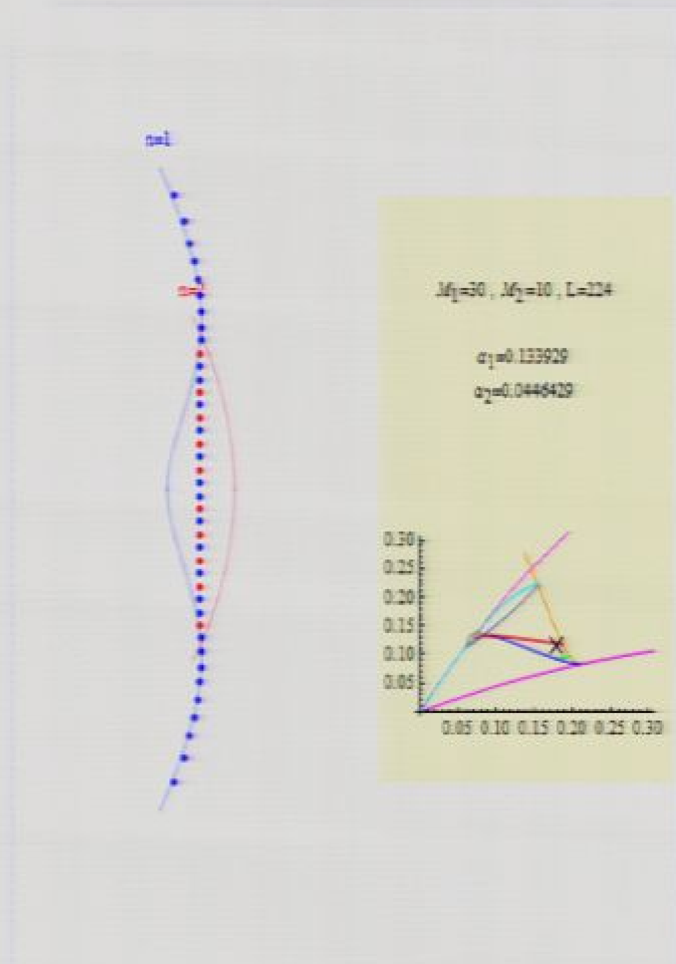

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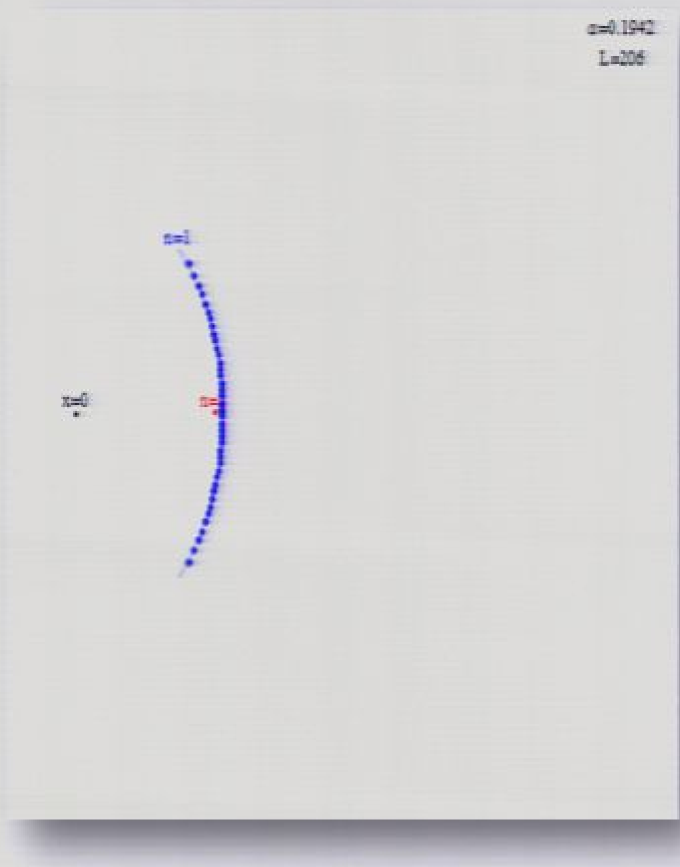

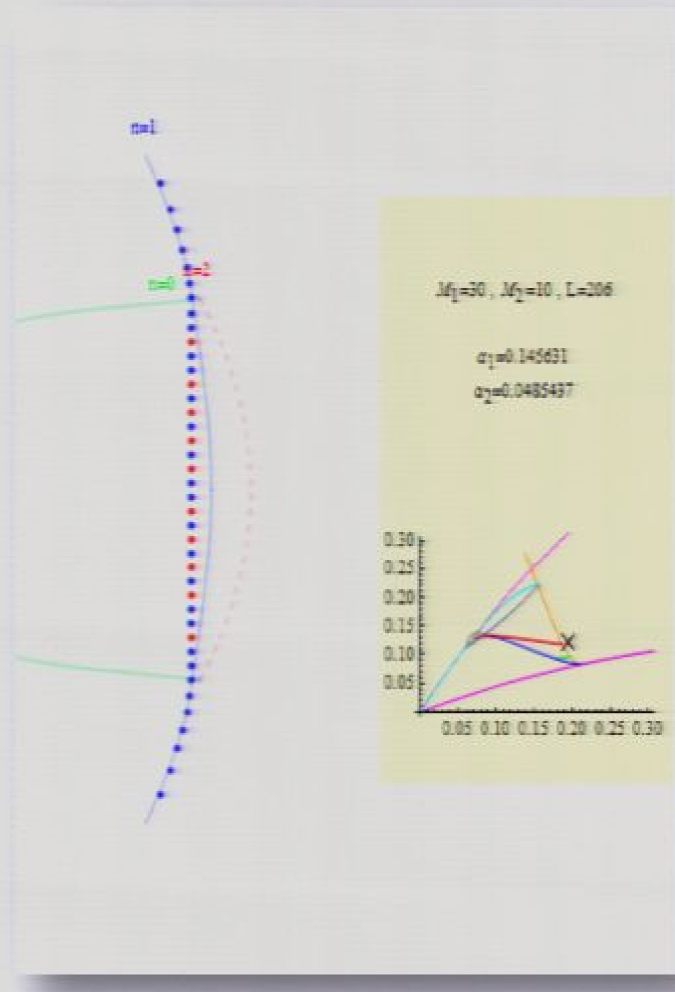

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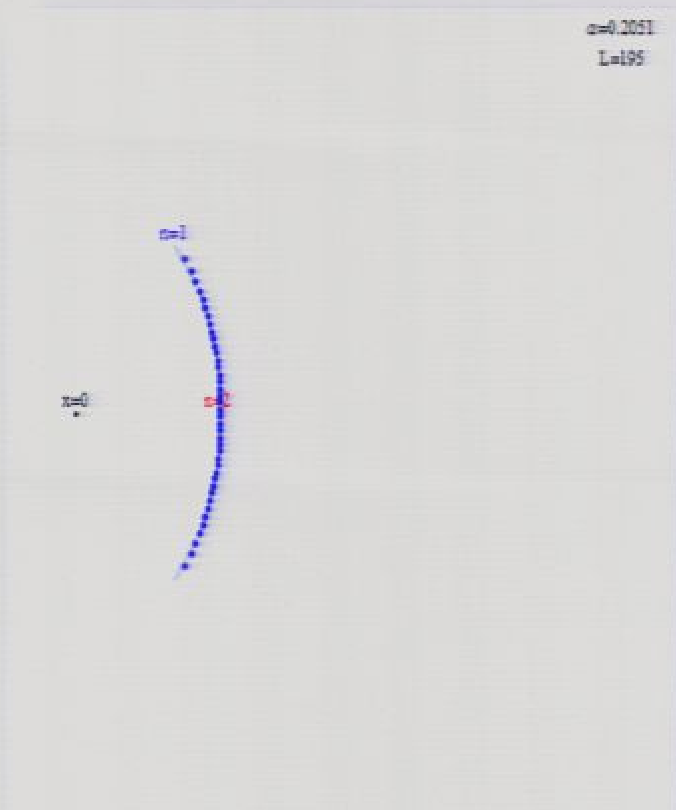

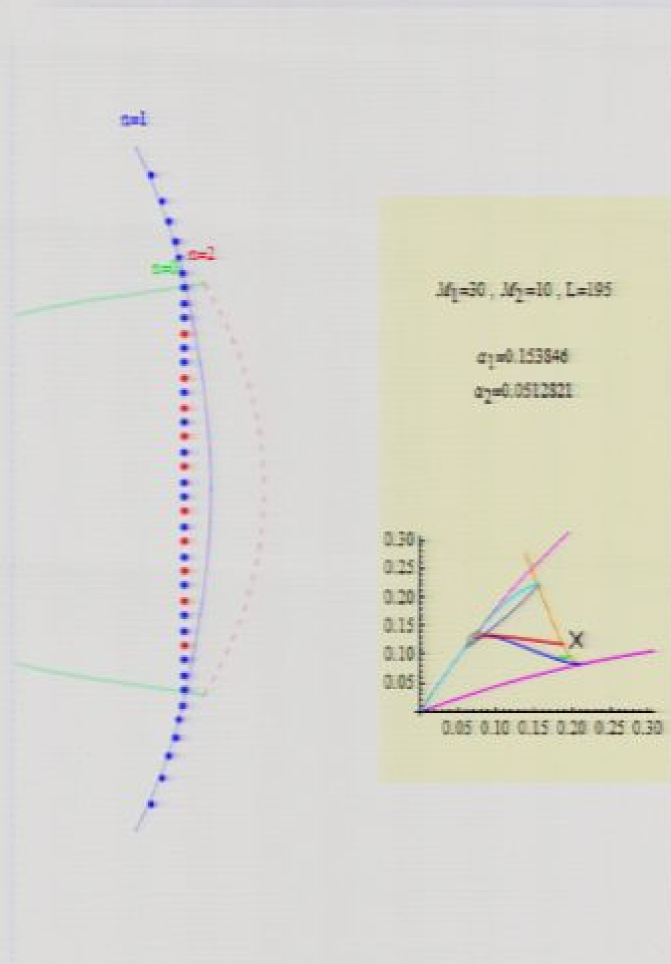

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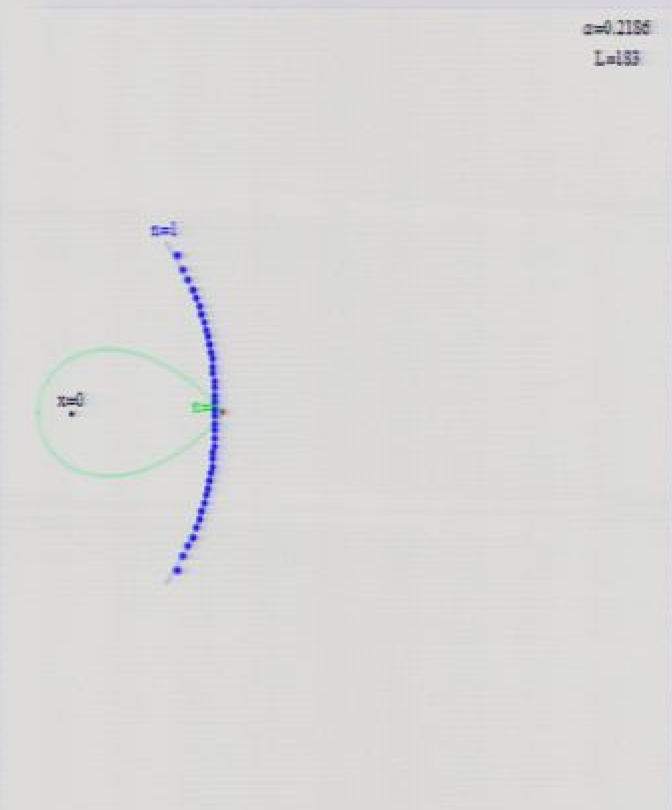

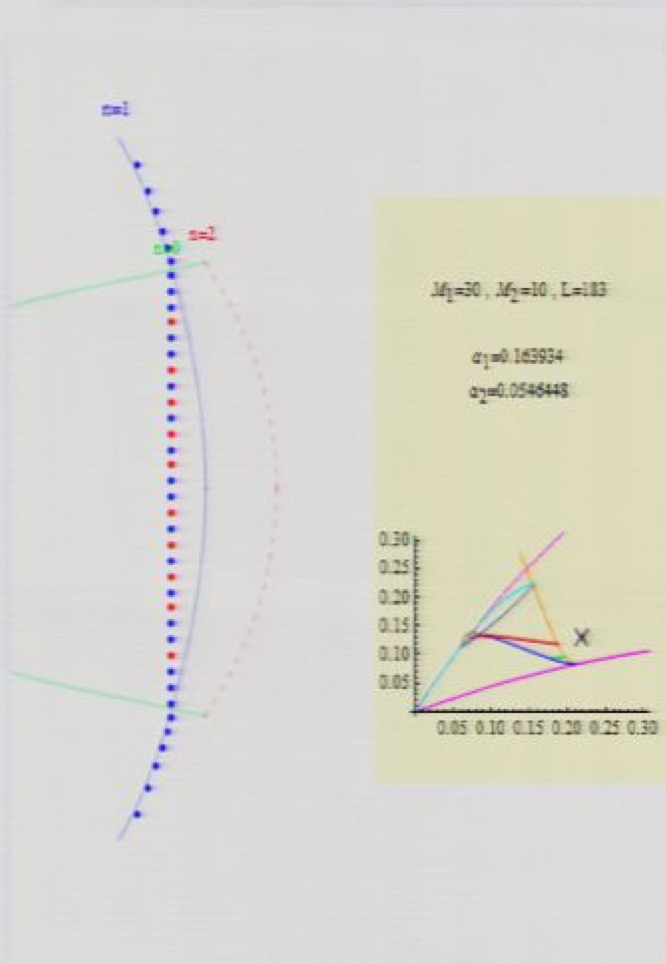

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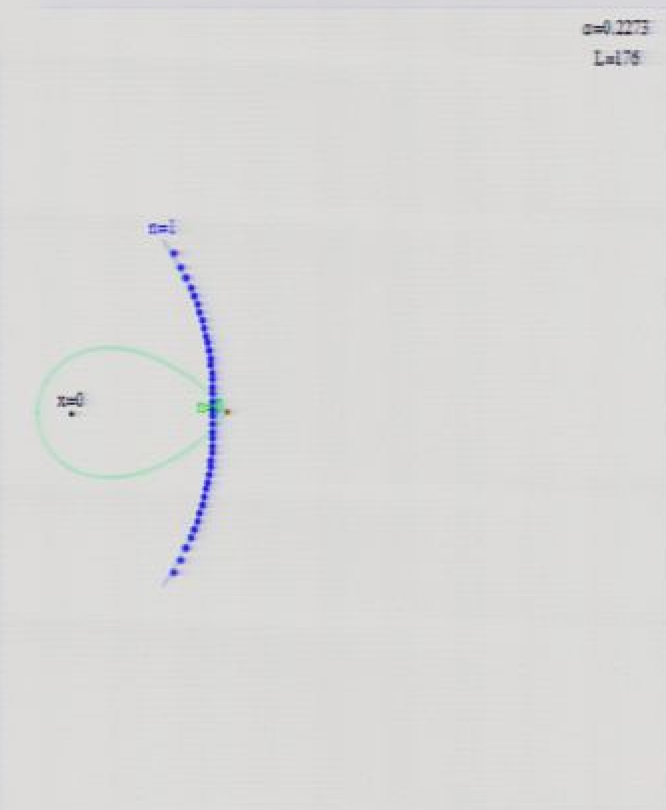

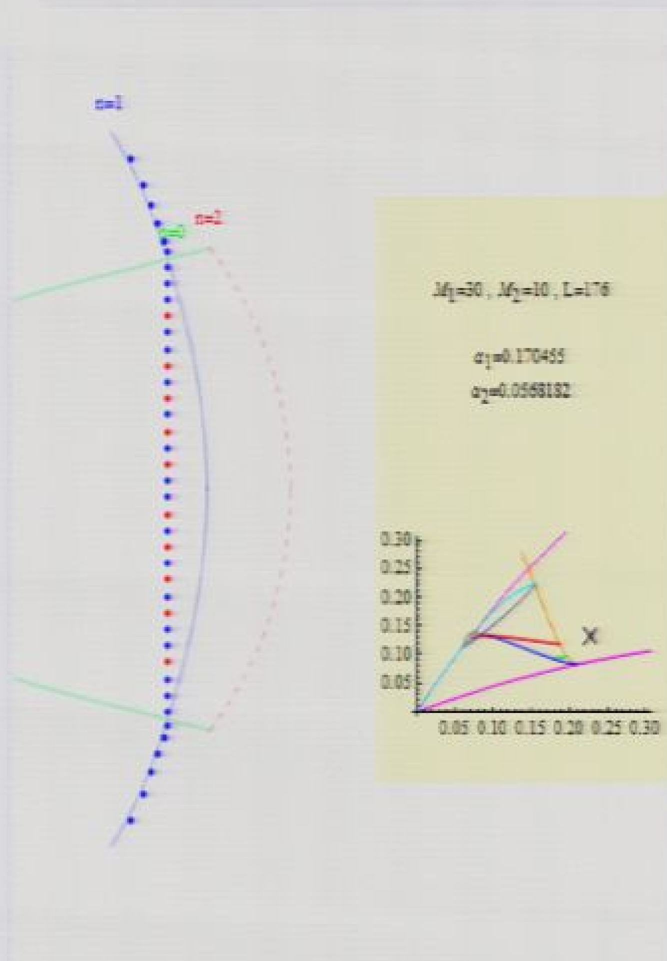

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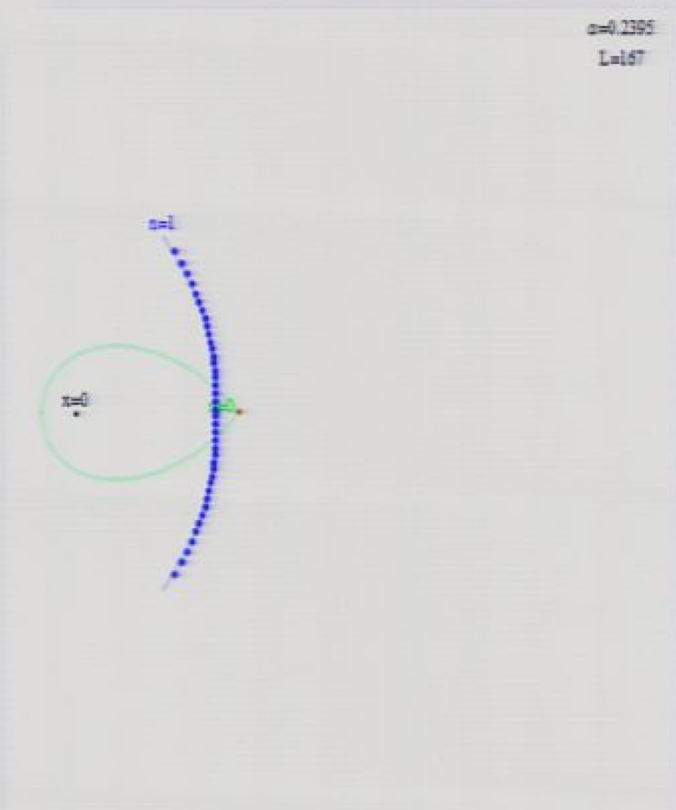

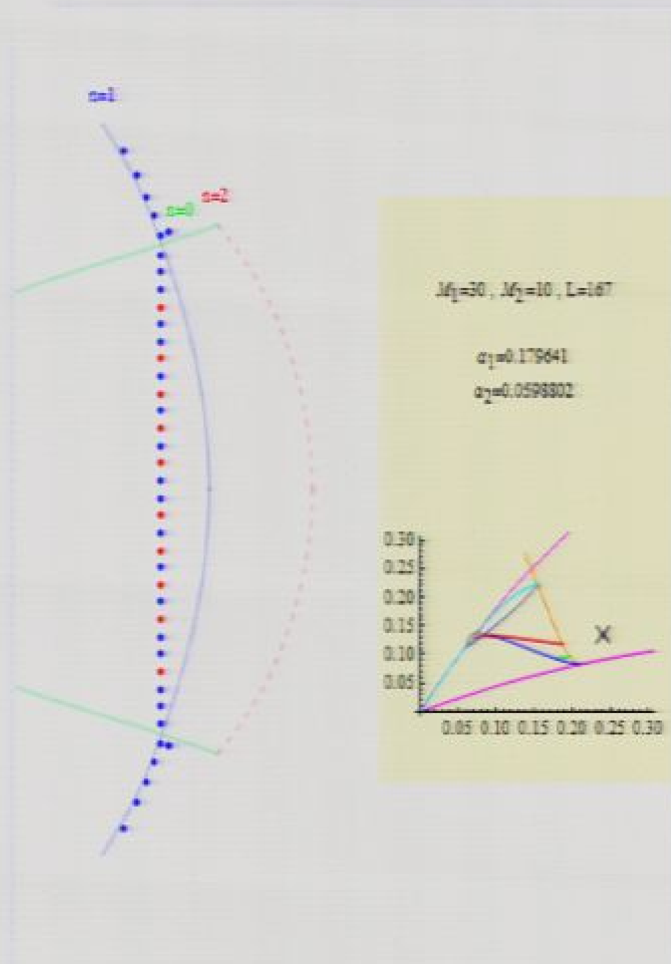

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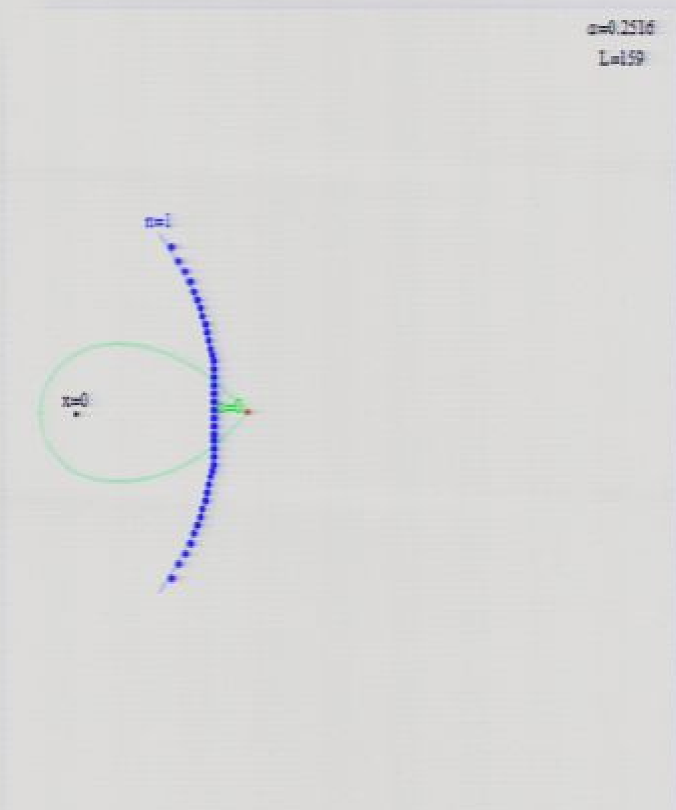

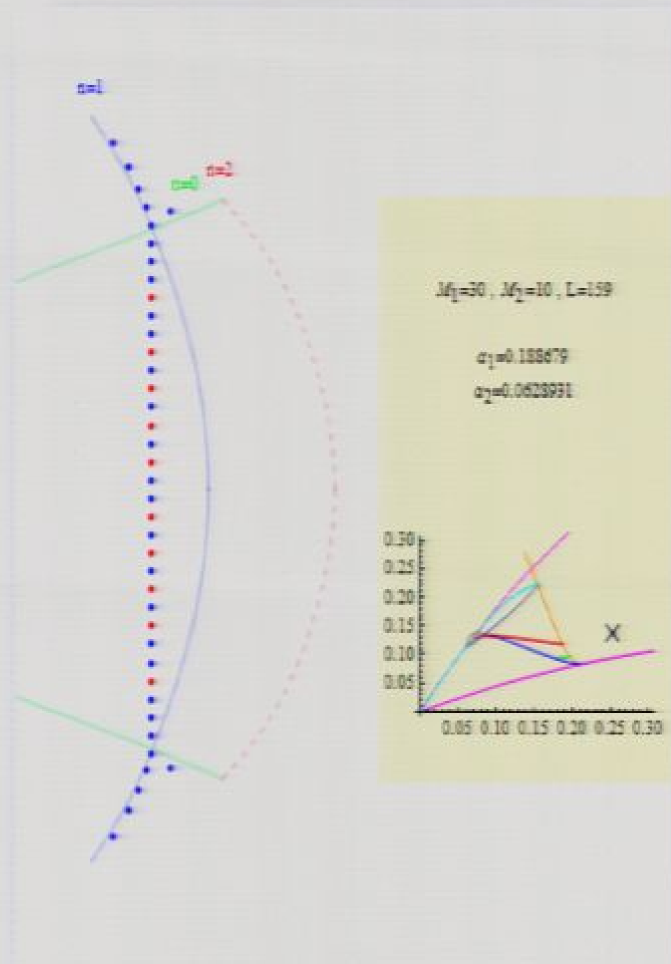

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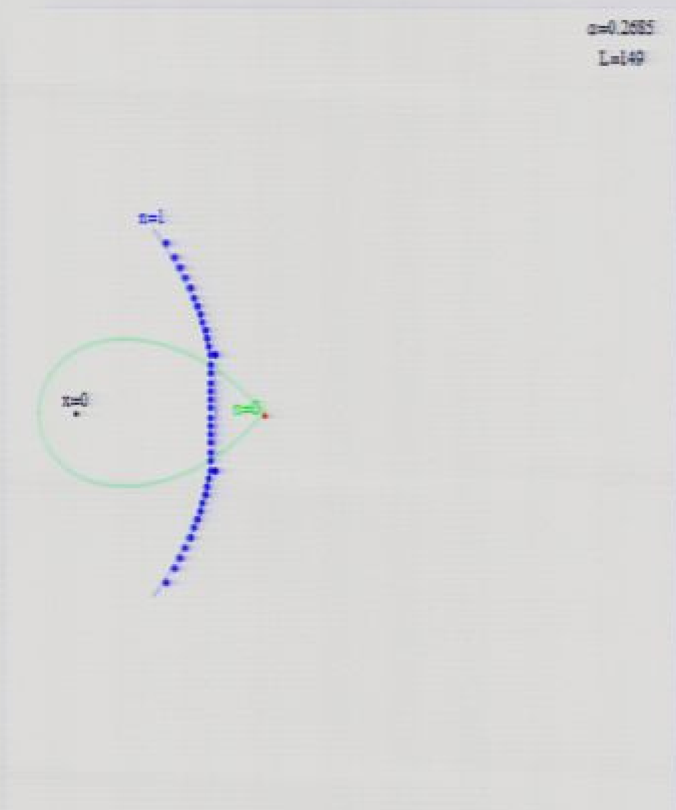

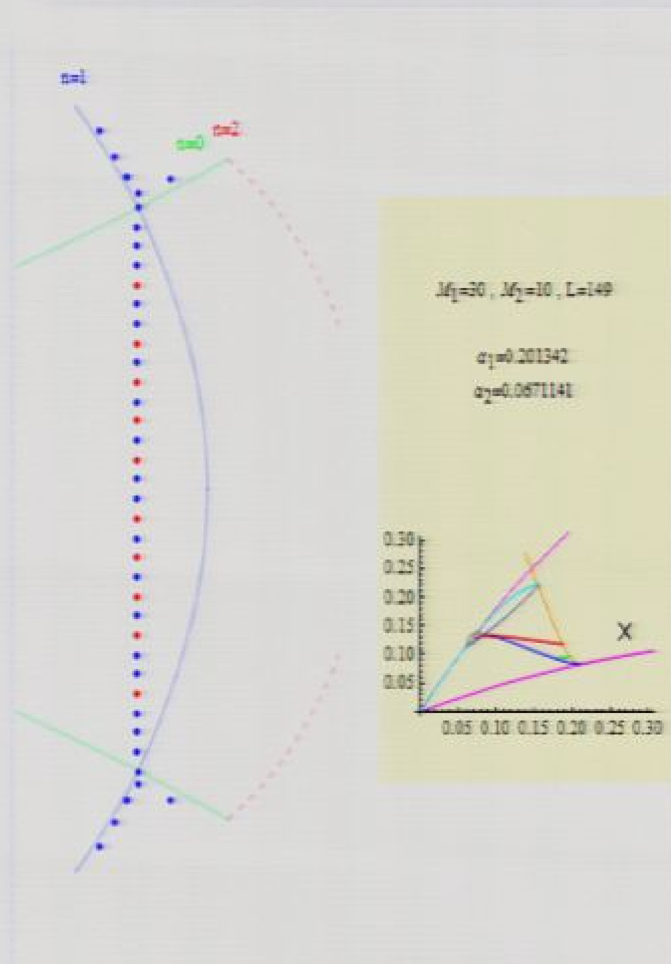

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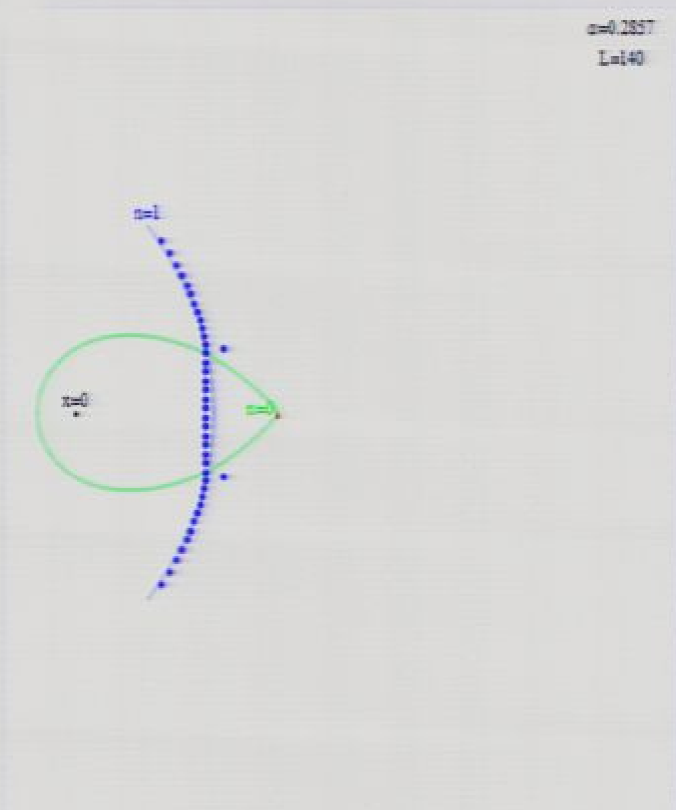
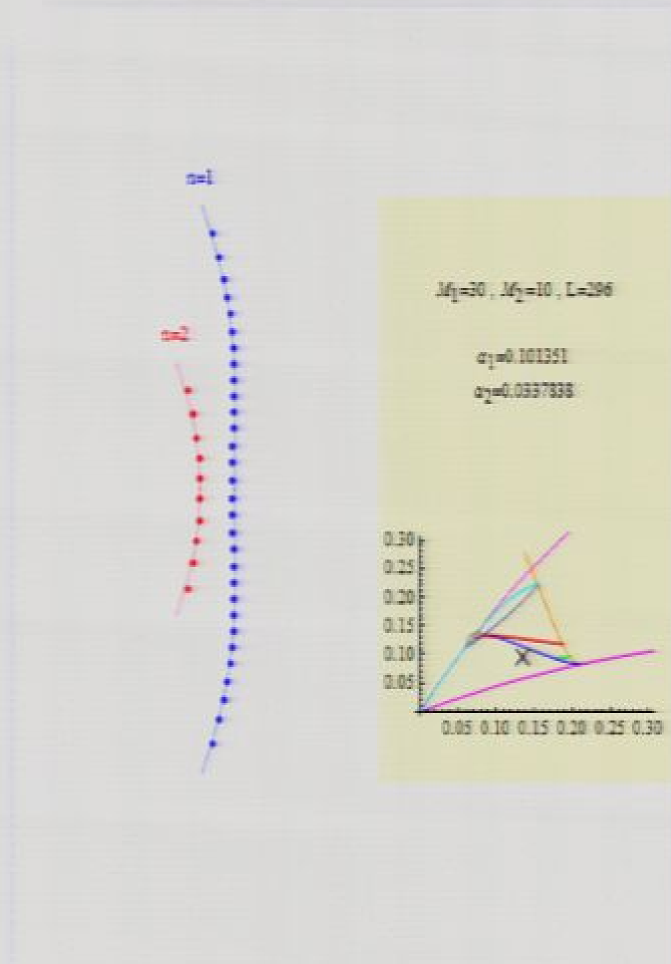
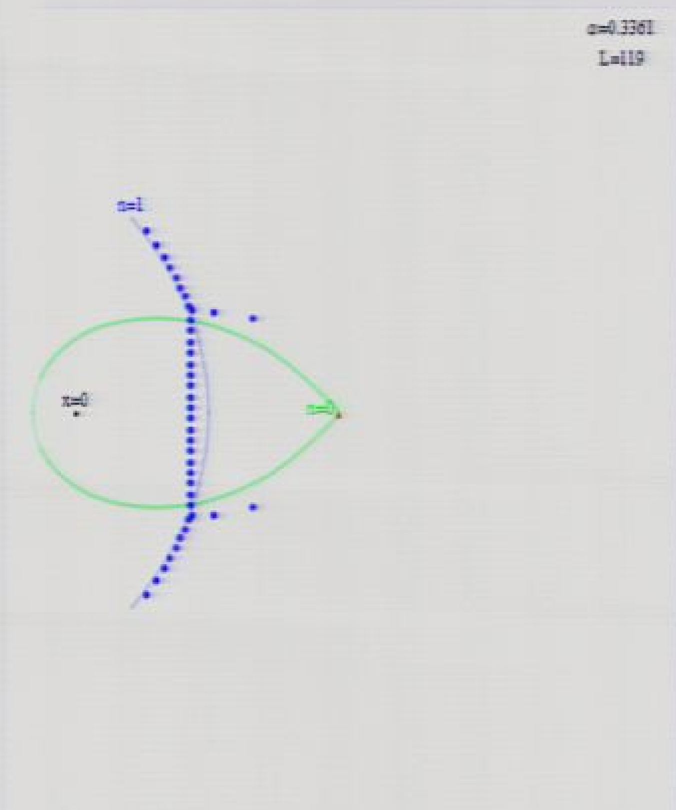
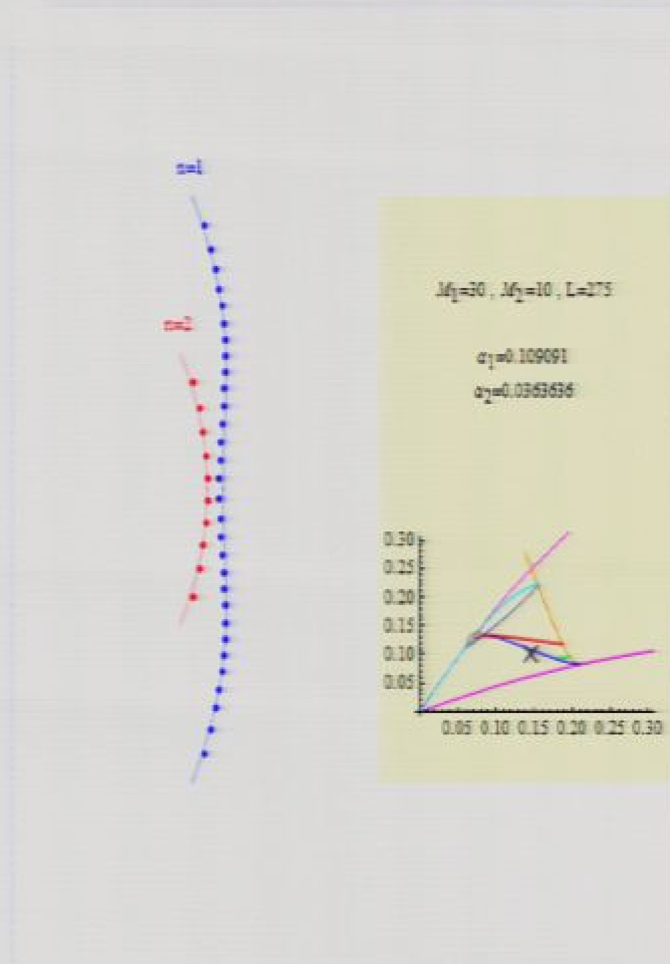




Numerical Solution

$$\left(\frac{u_j + i/2}{u_j - i/2}\right)^L = \prod_{k=1(k\neq j)}^{J} \frac{u_j - u_k + i}{u_j - u_k - i}$$





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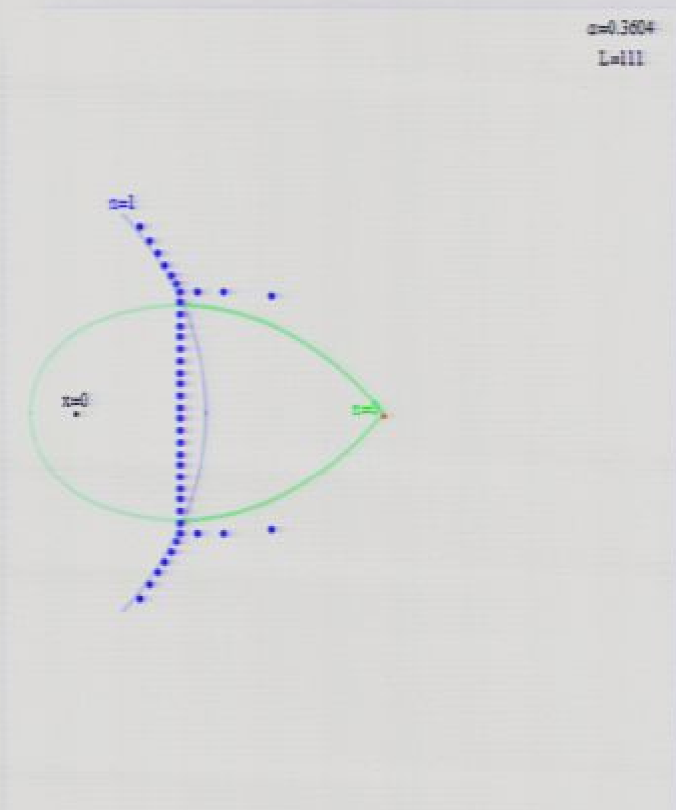

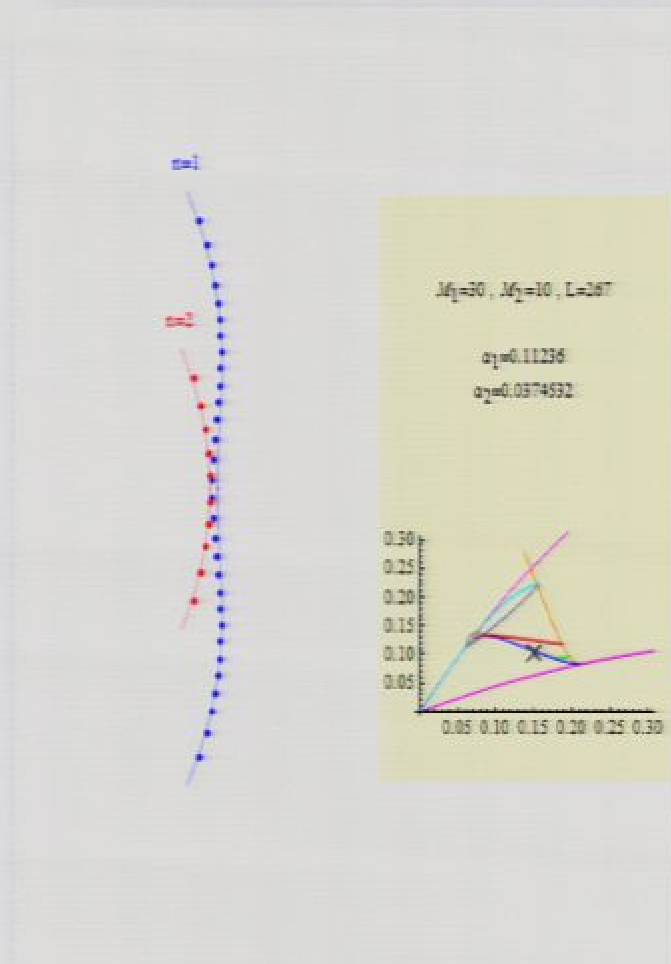

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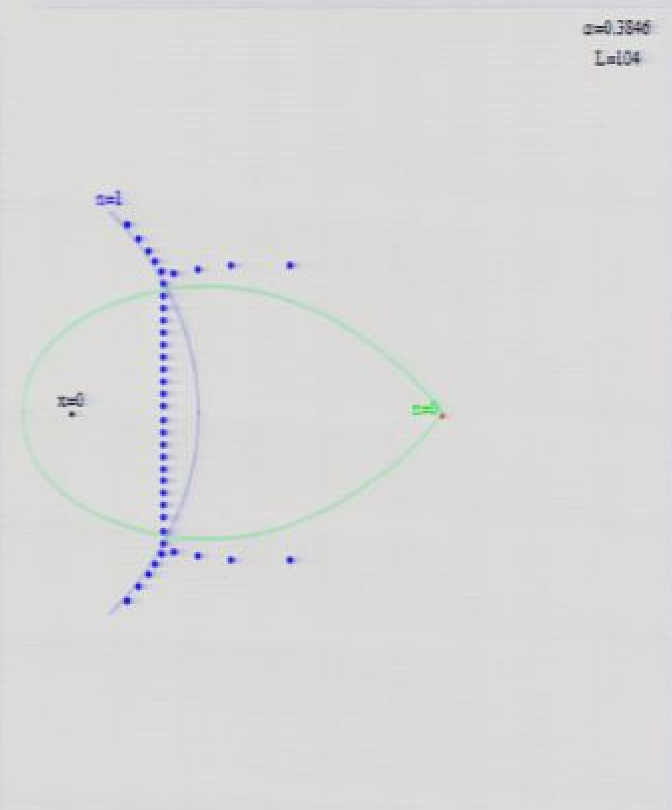

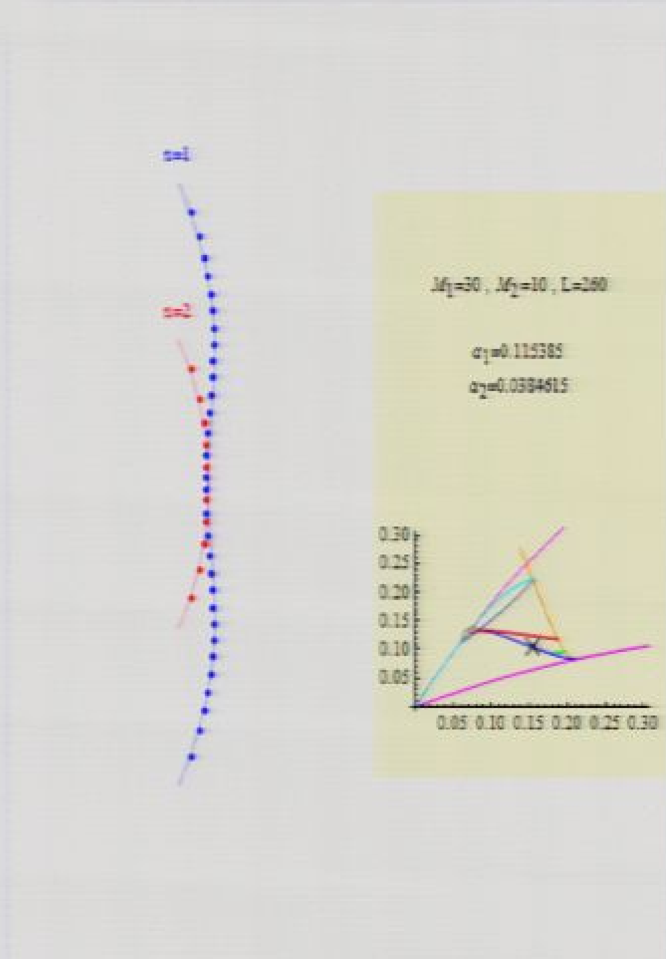

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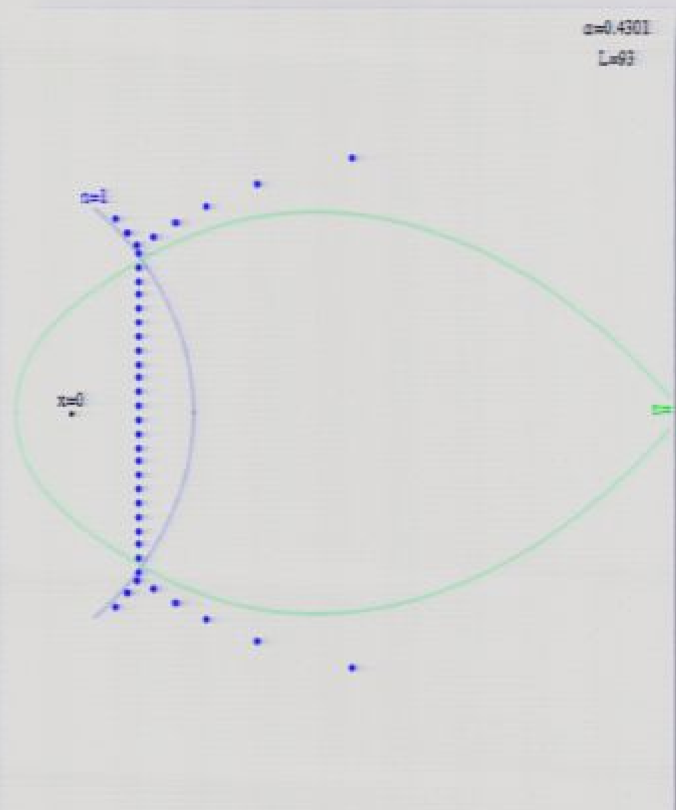

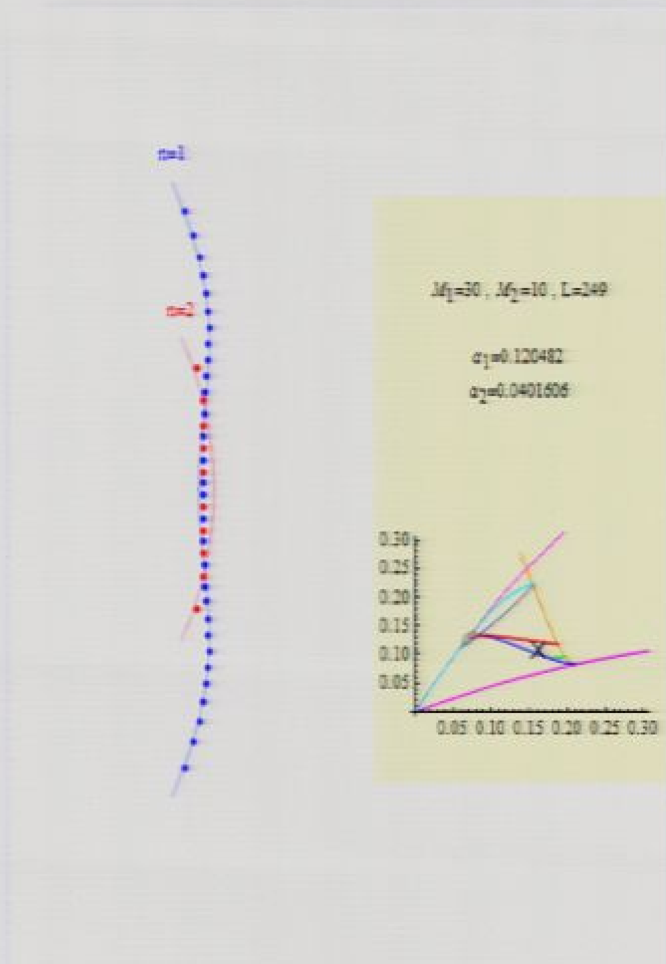

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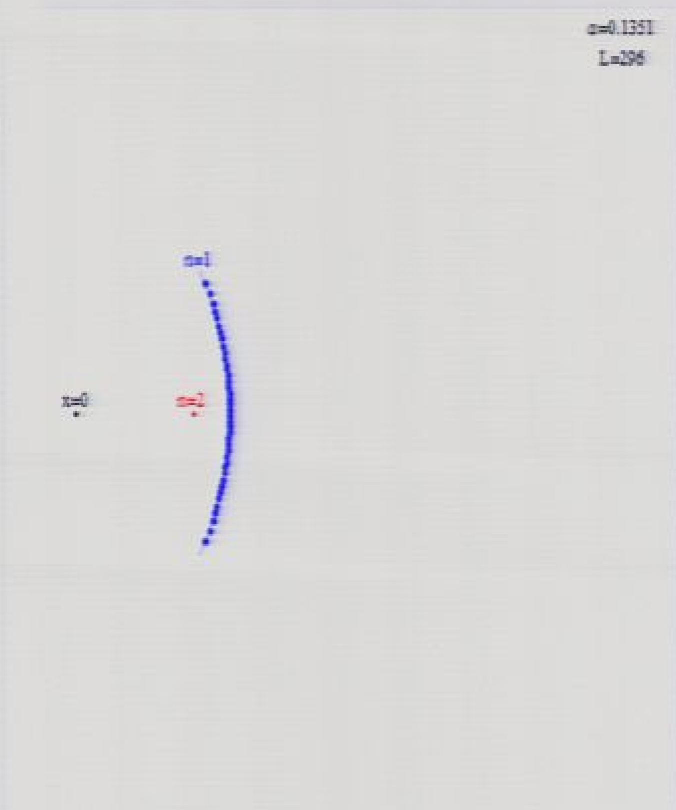

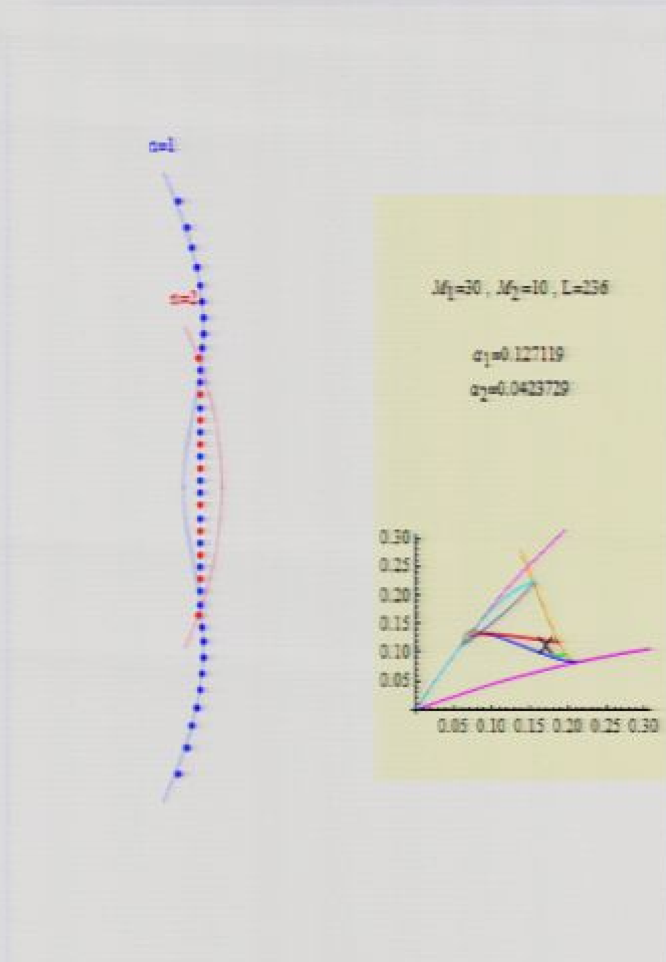

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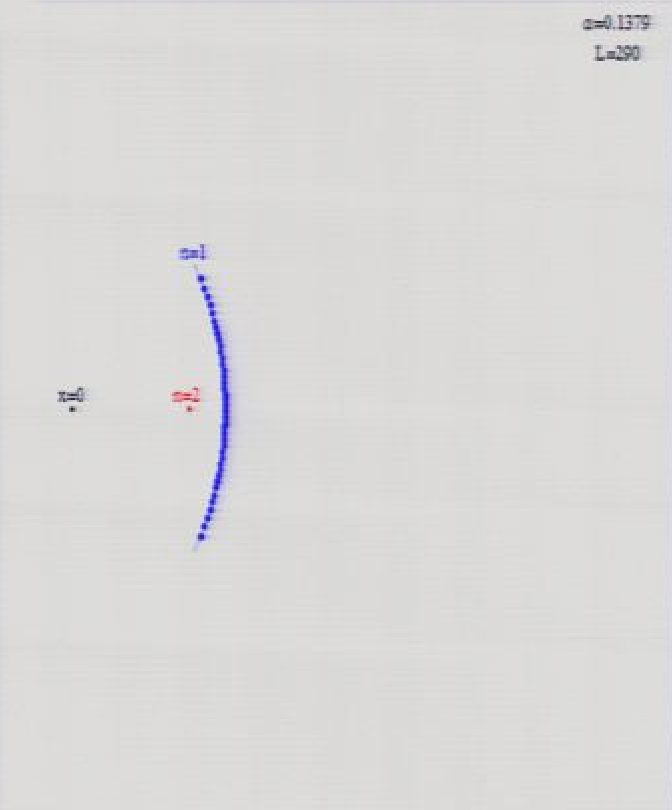

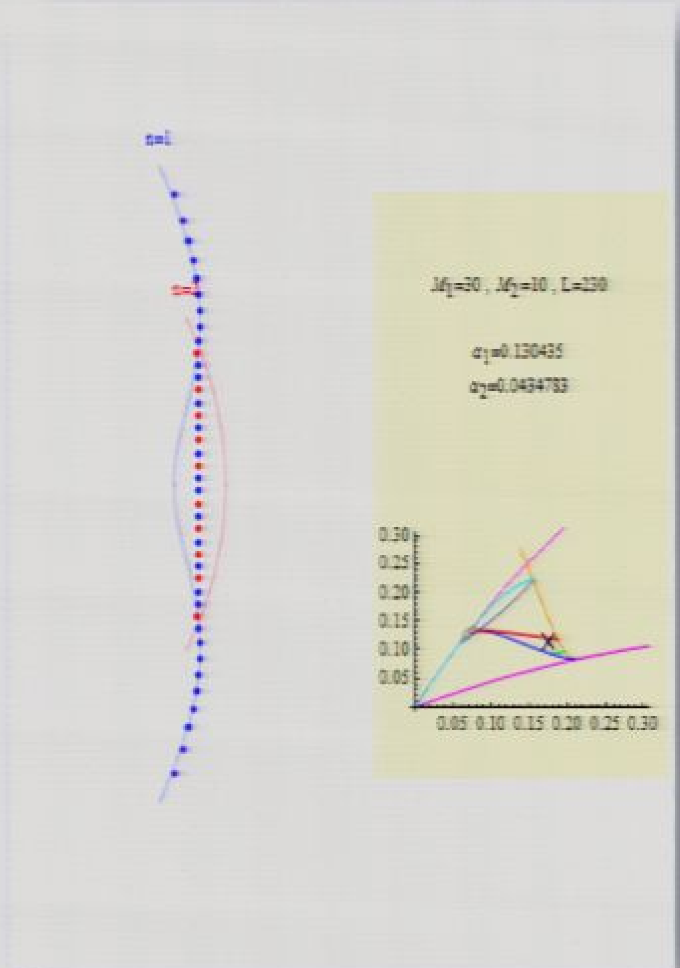

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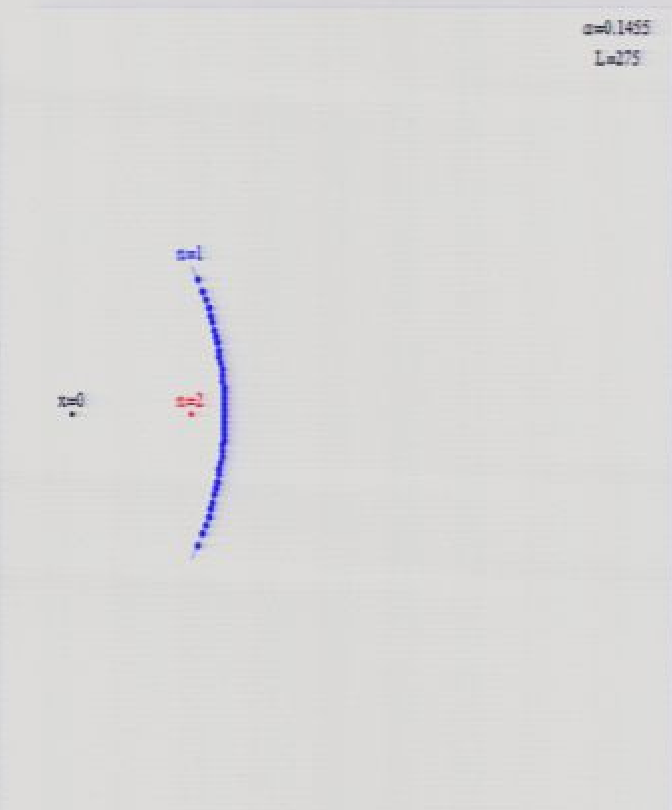

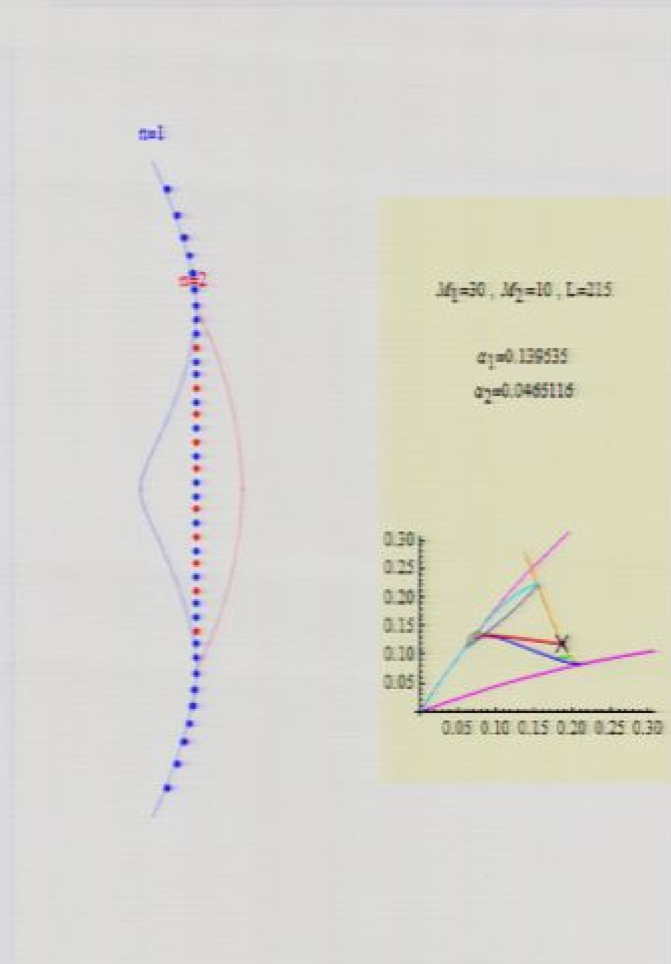

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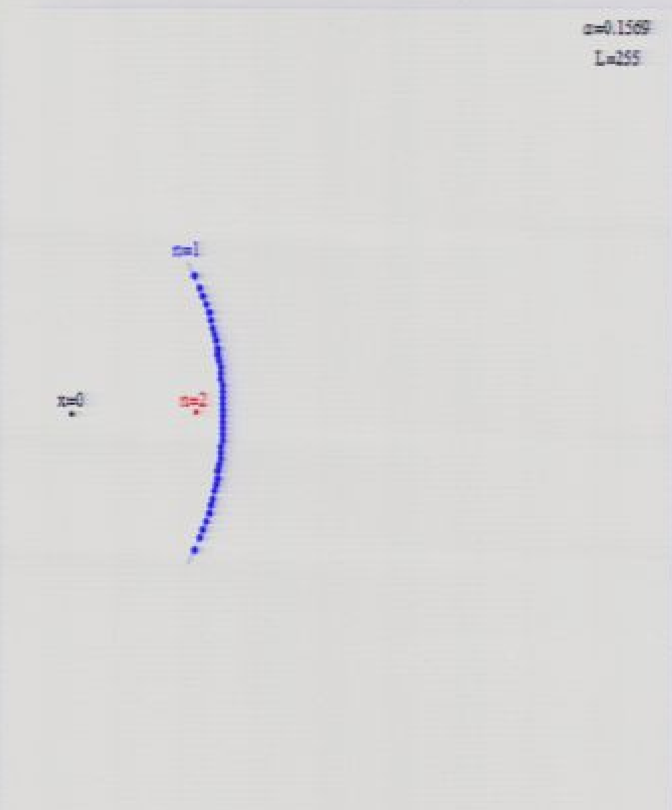

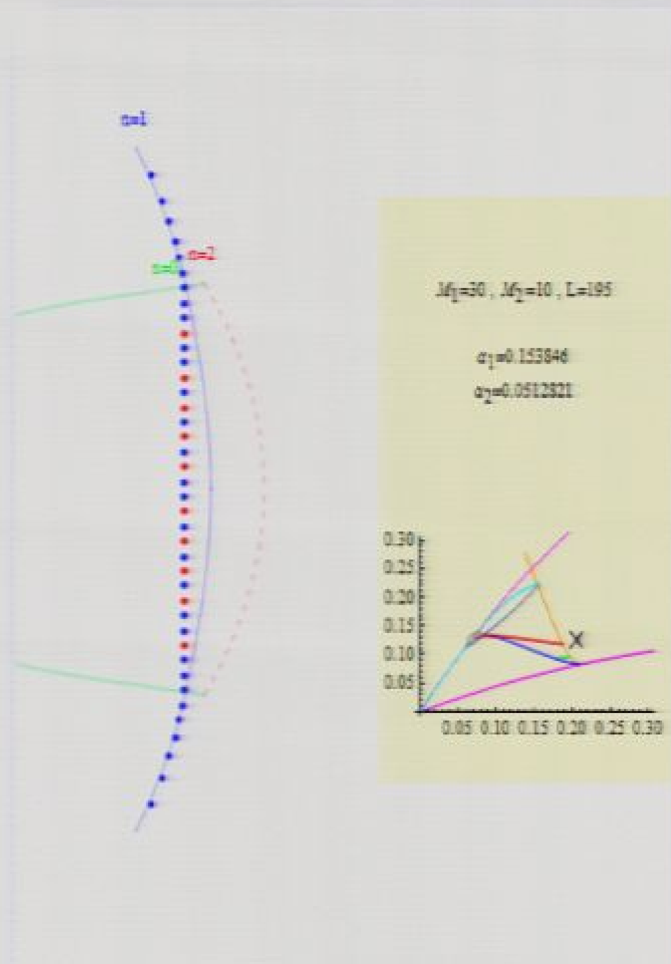

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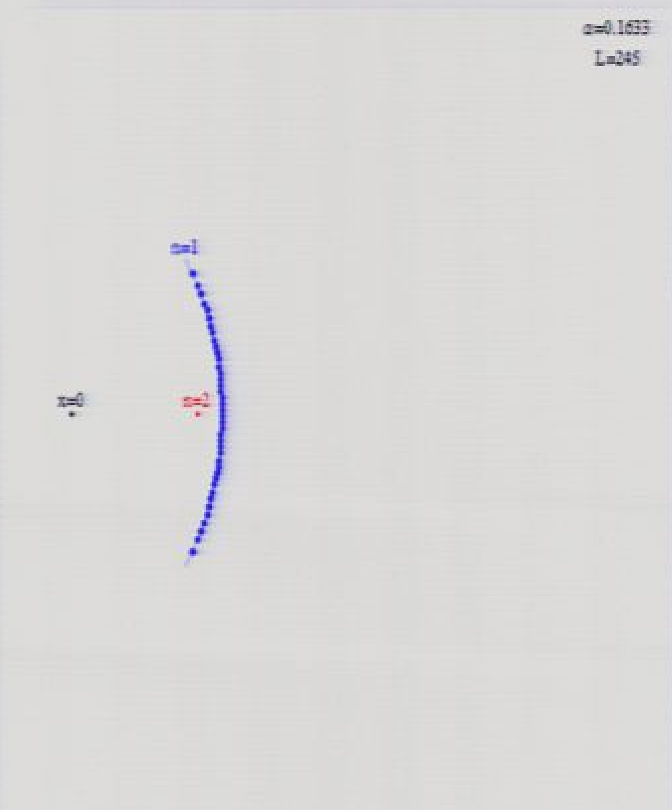

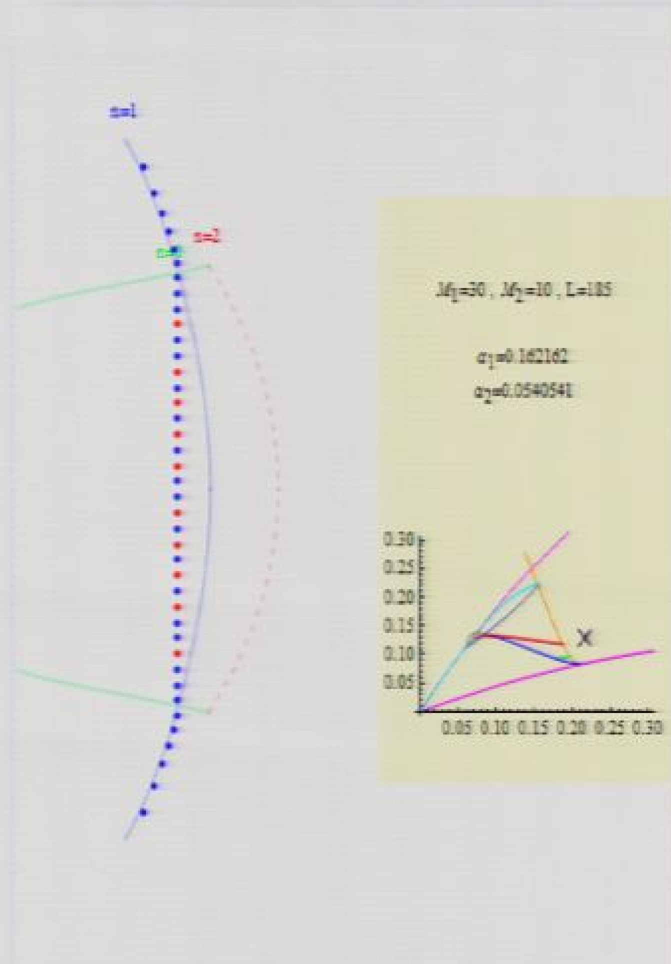

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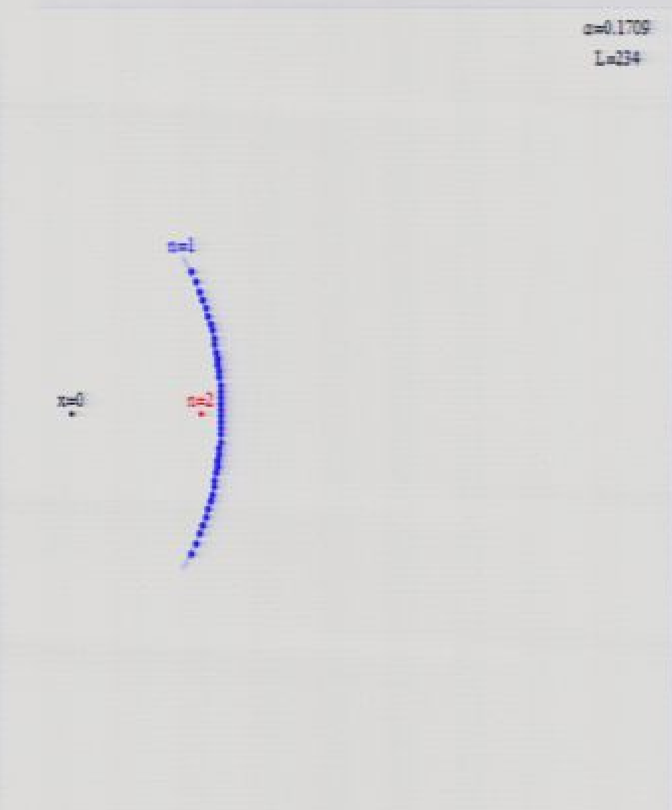

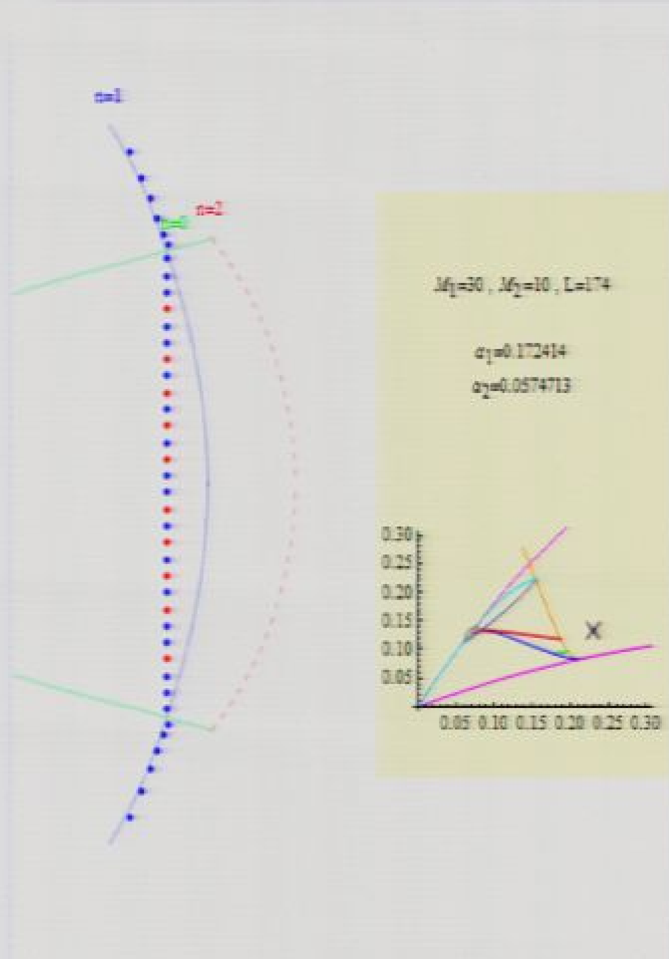

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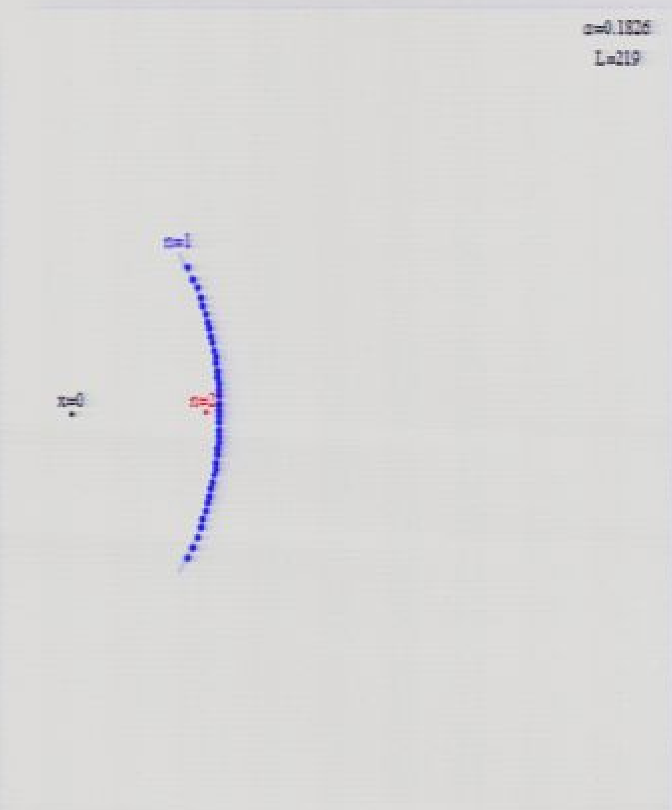

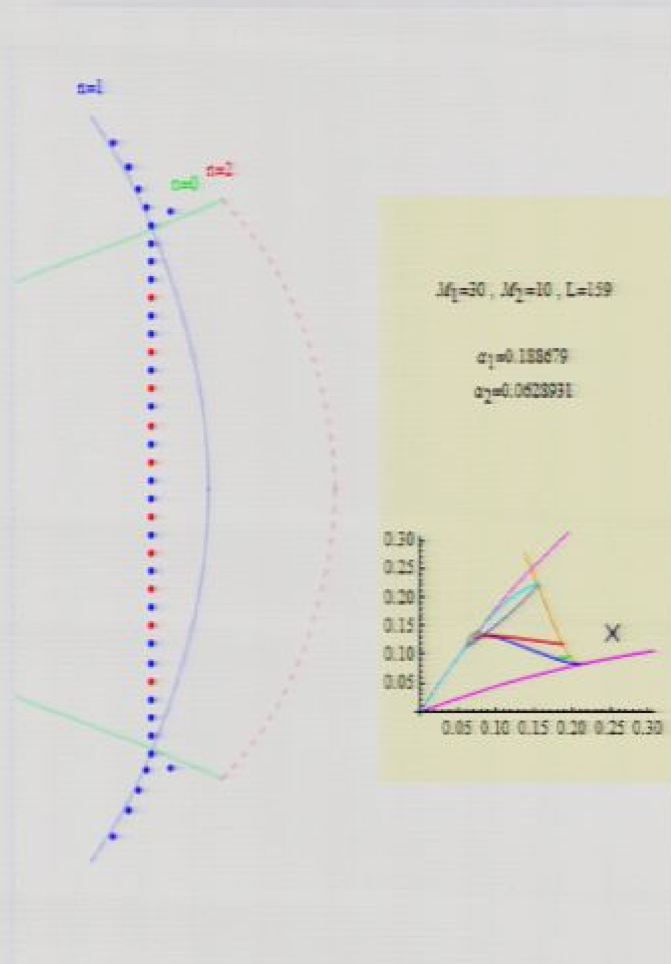

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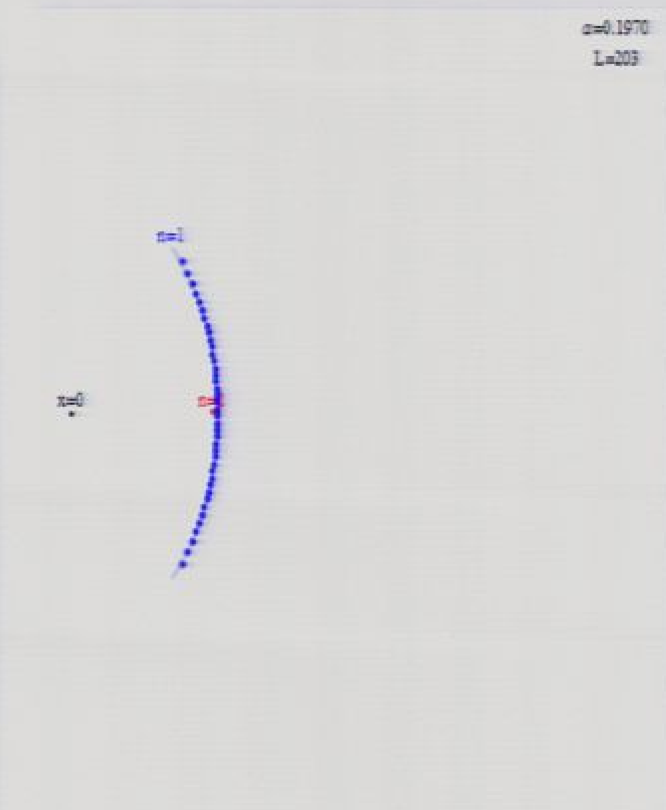

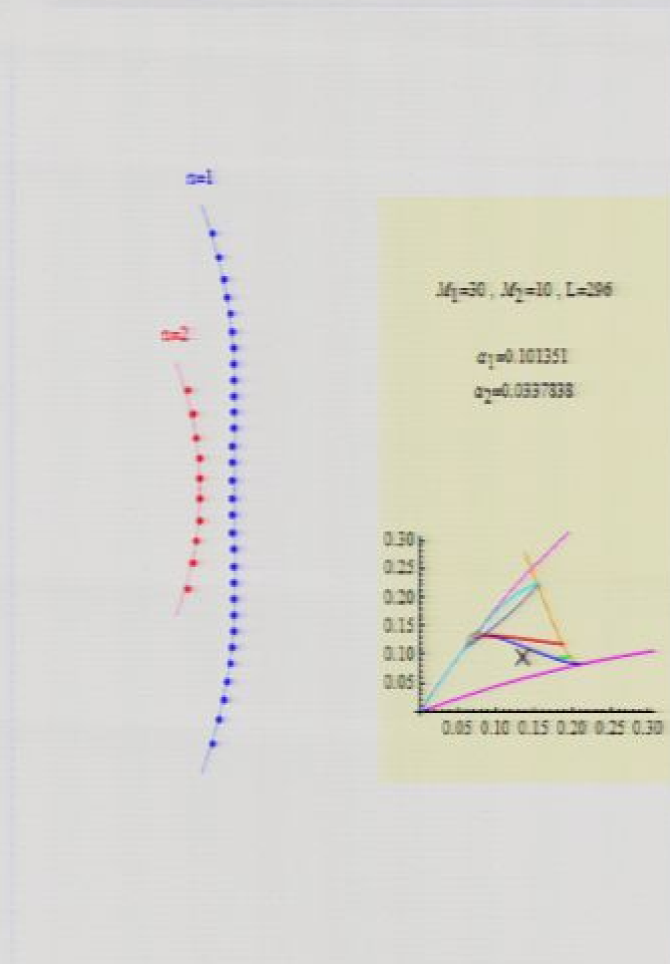

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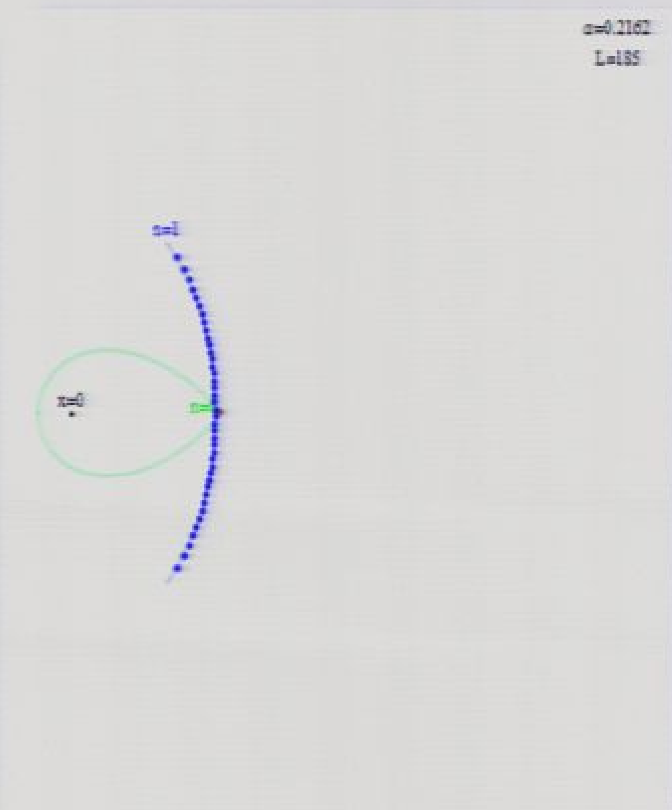

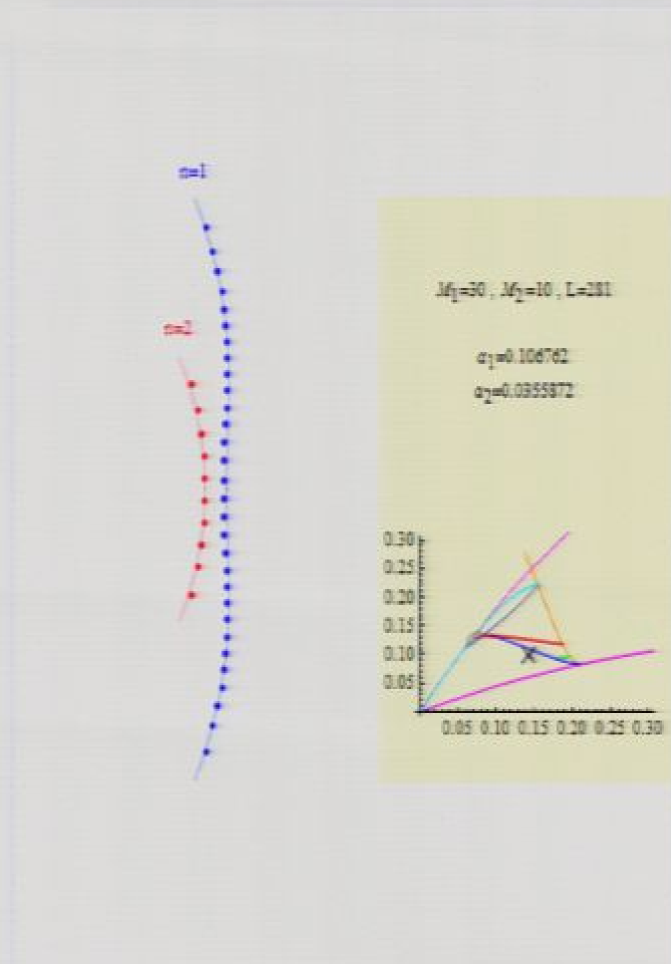

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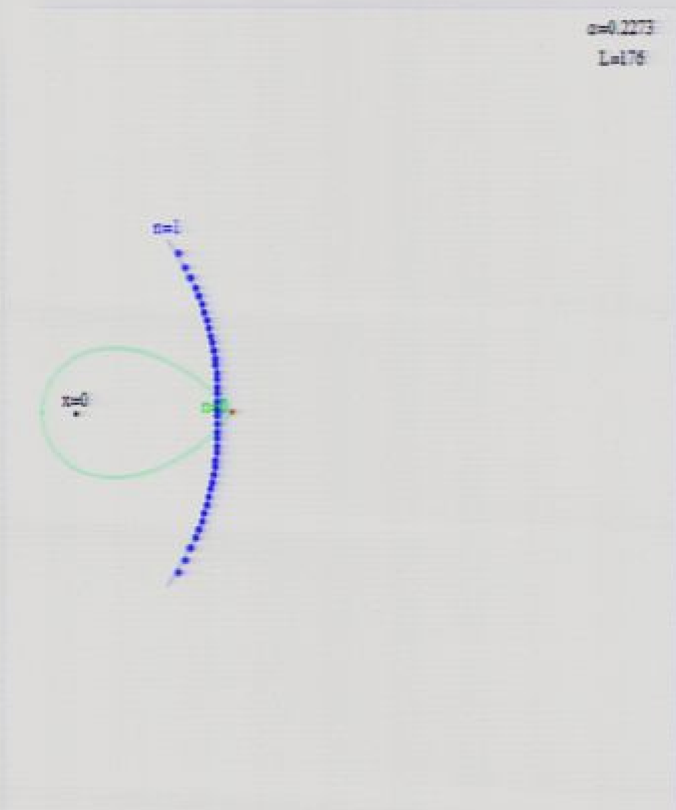

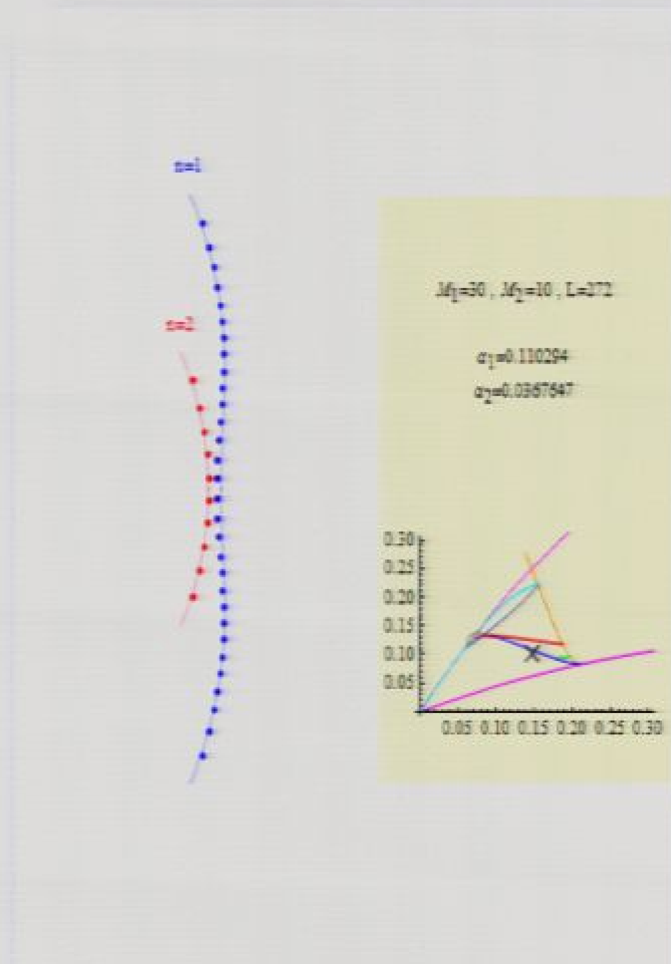

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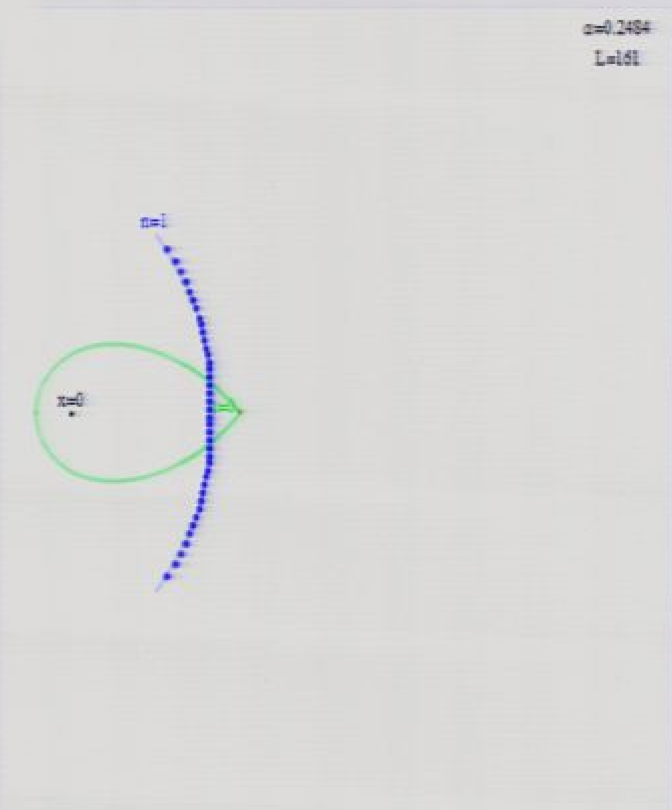

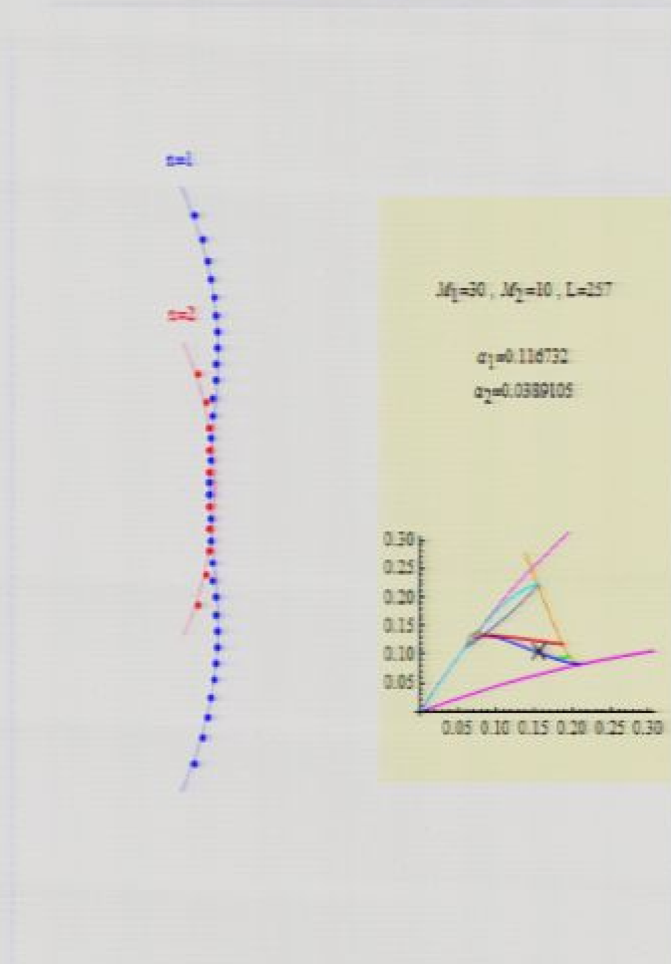

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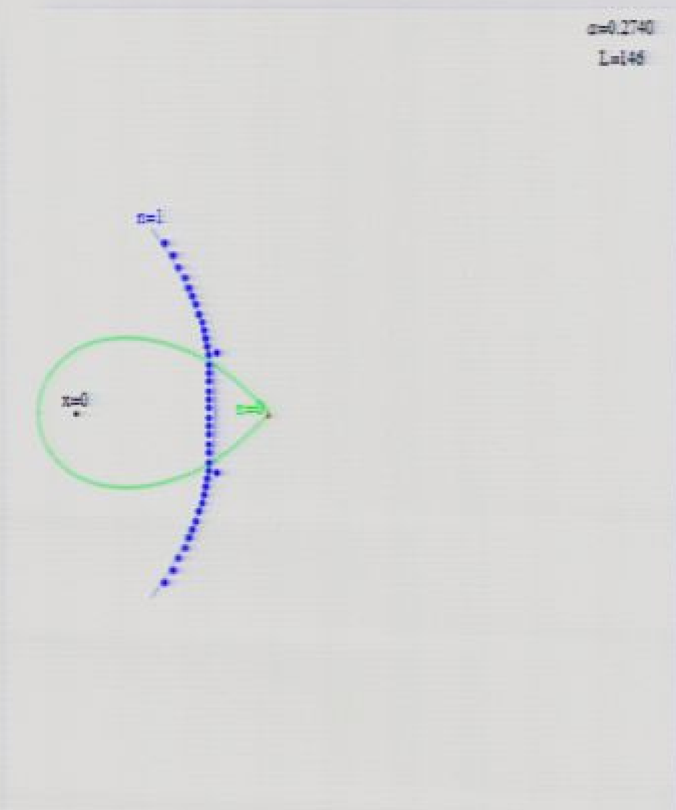

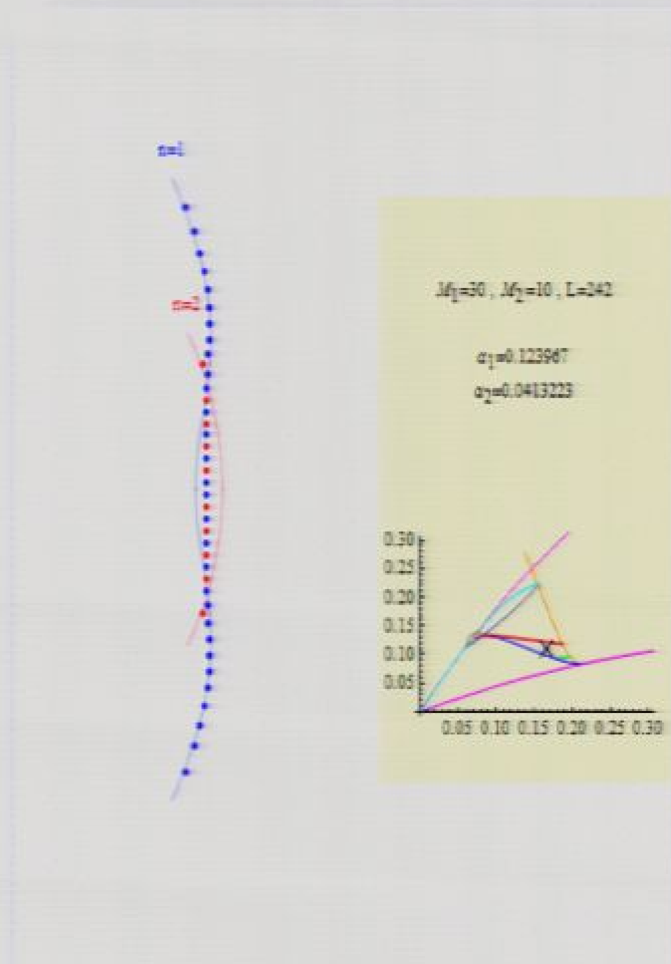

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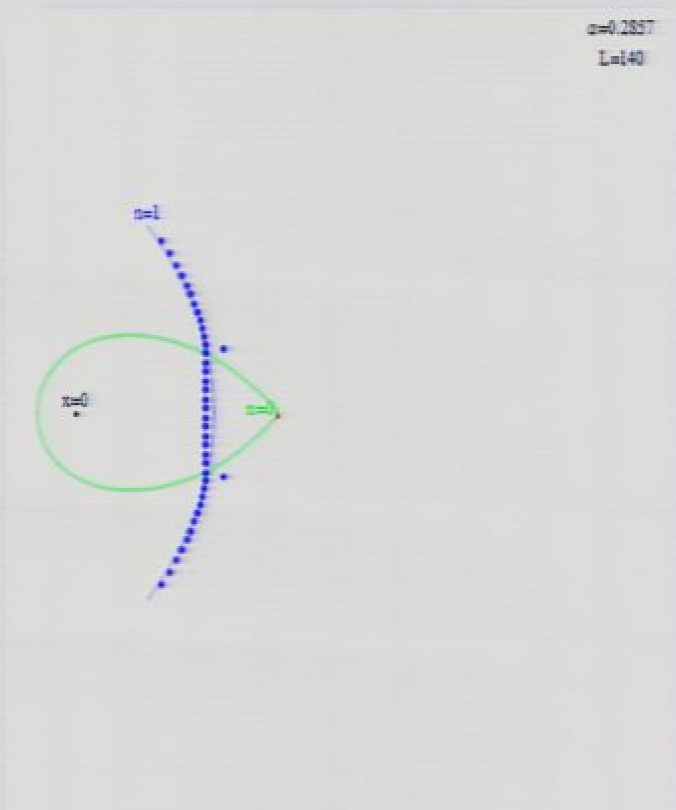

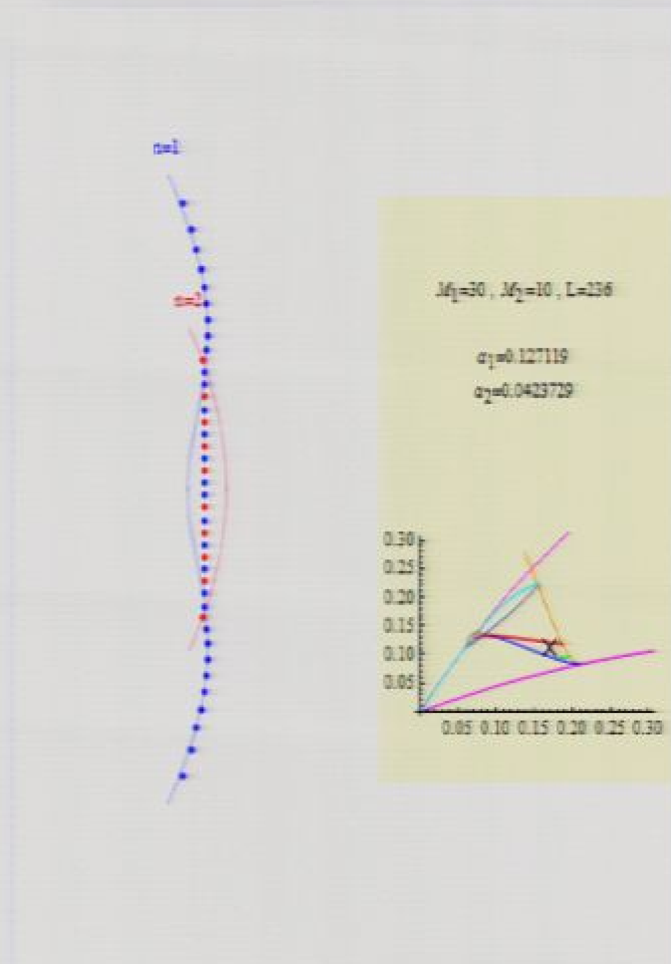

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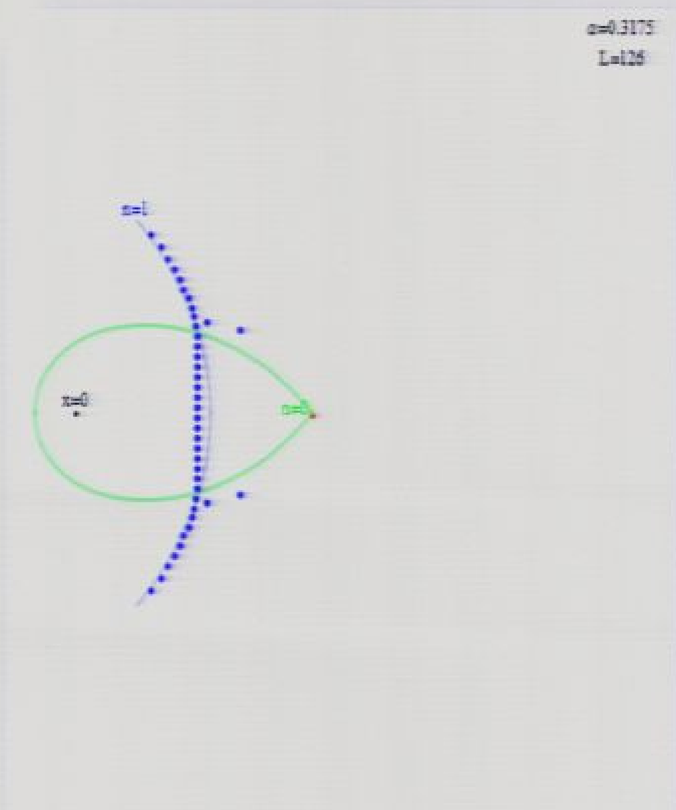

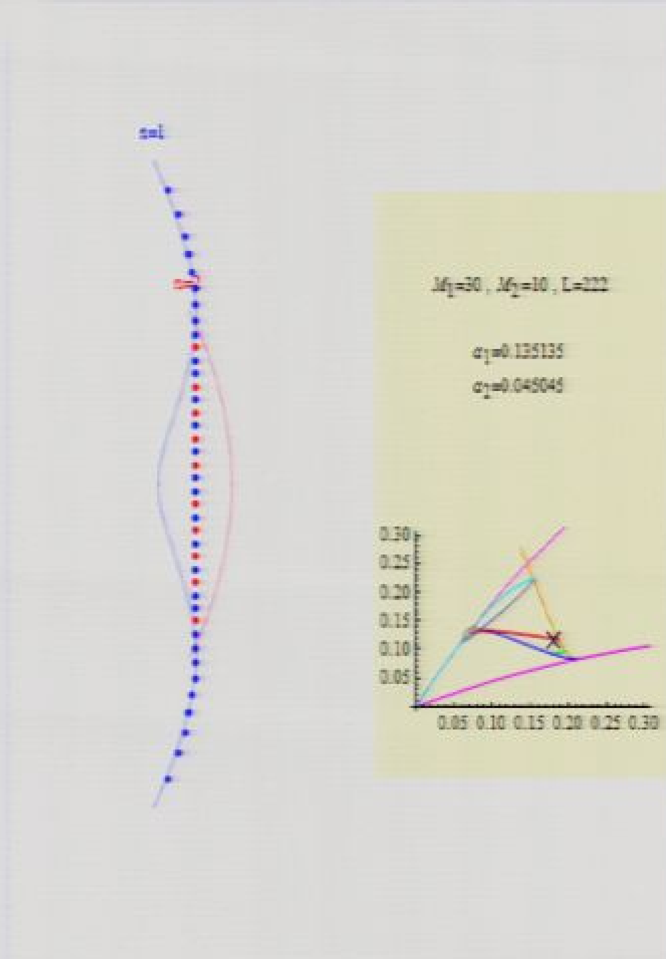

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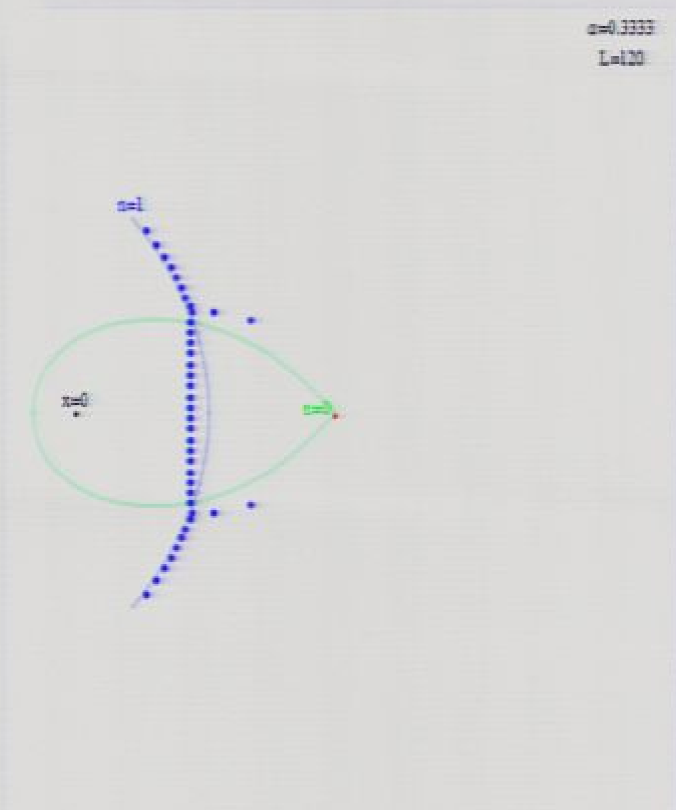

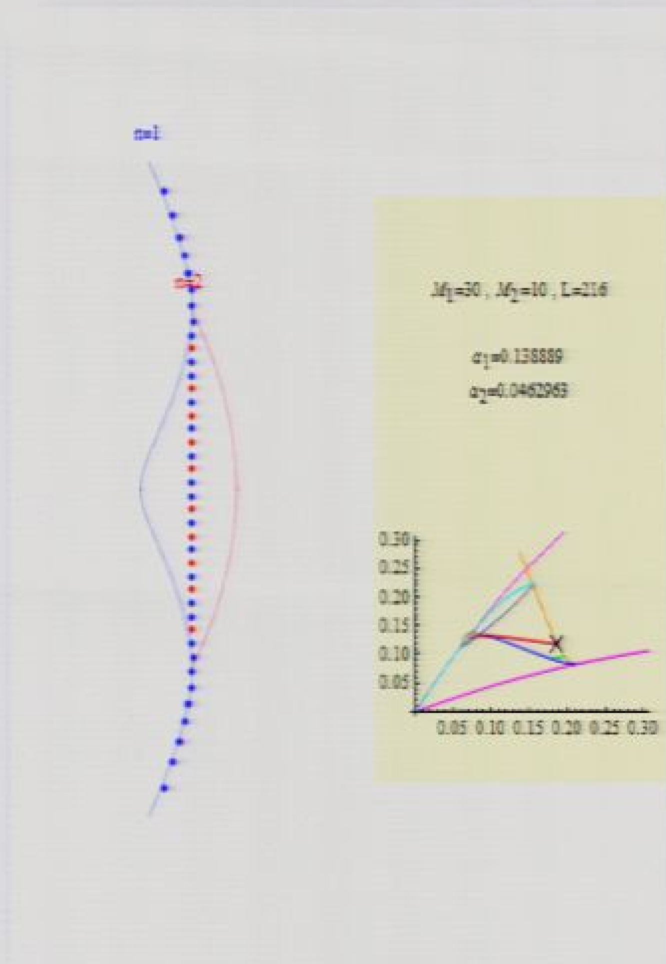

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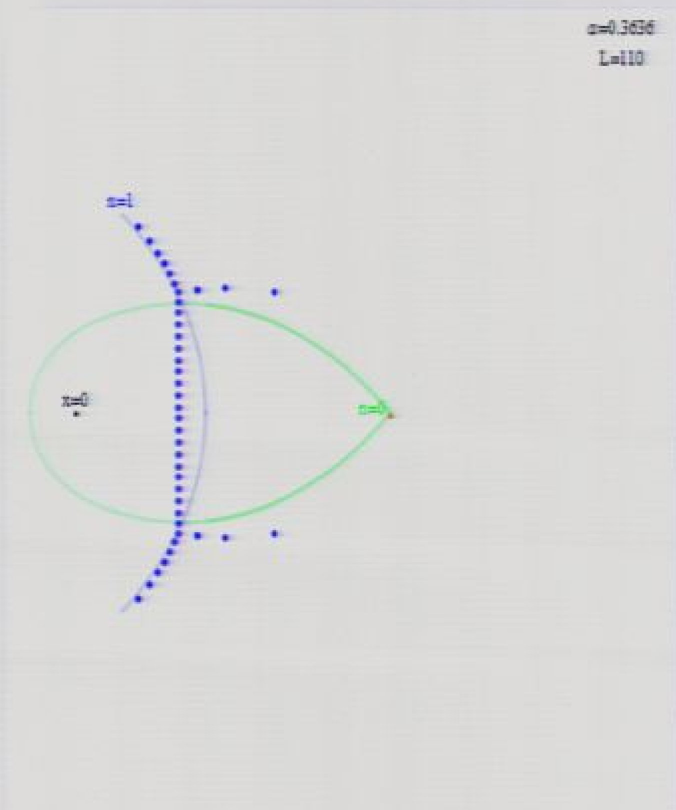

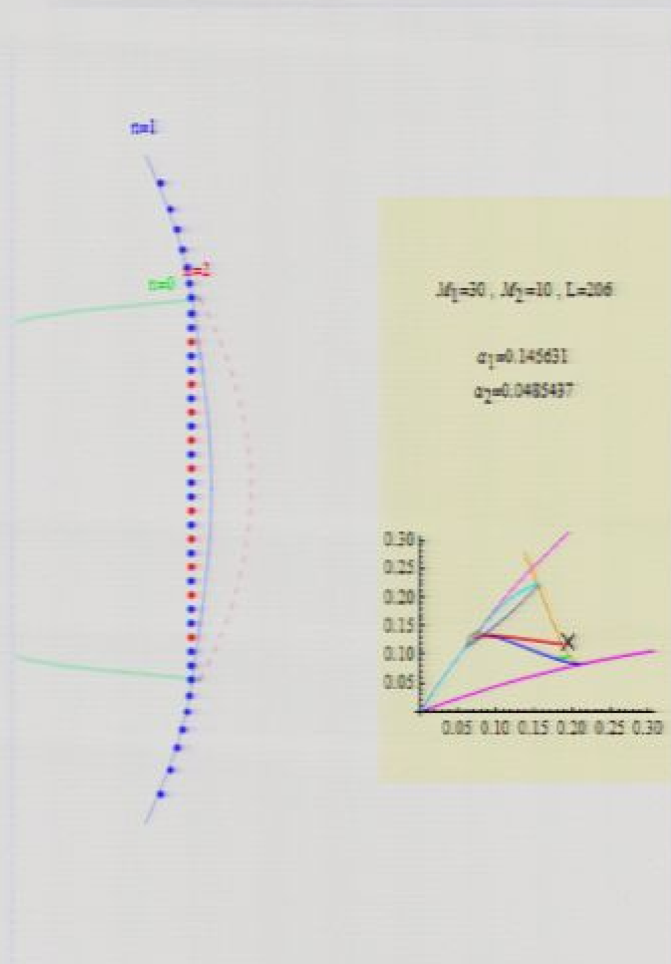

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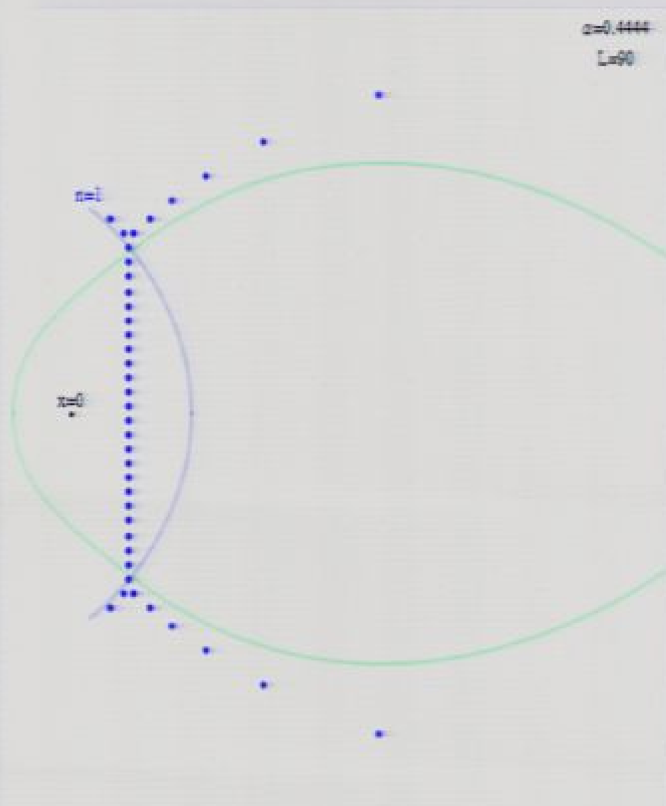

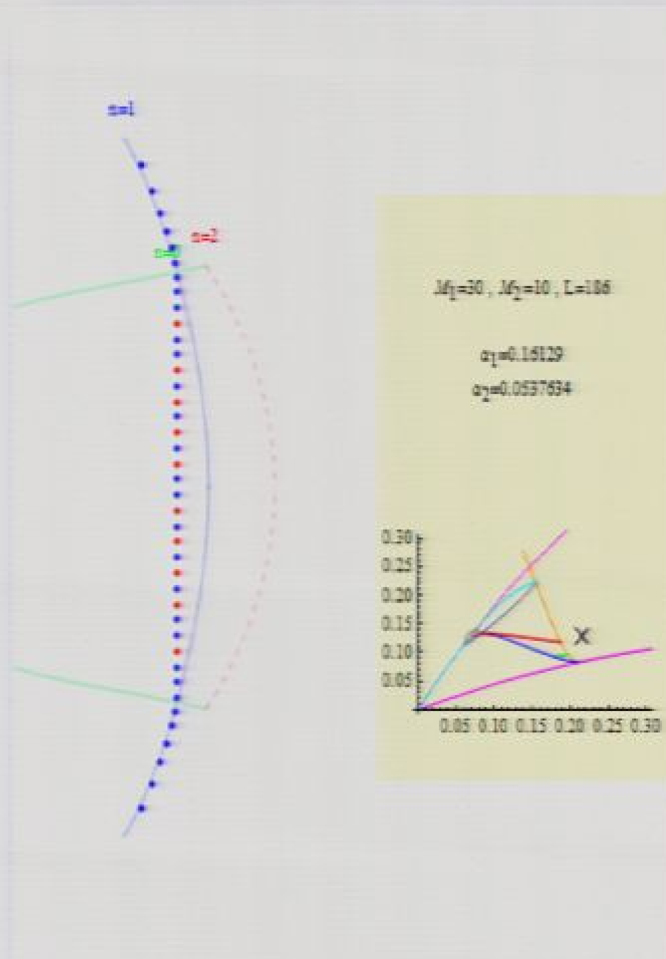

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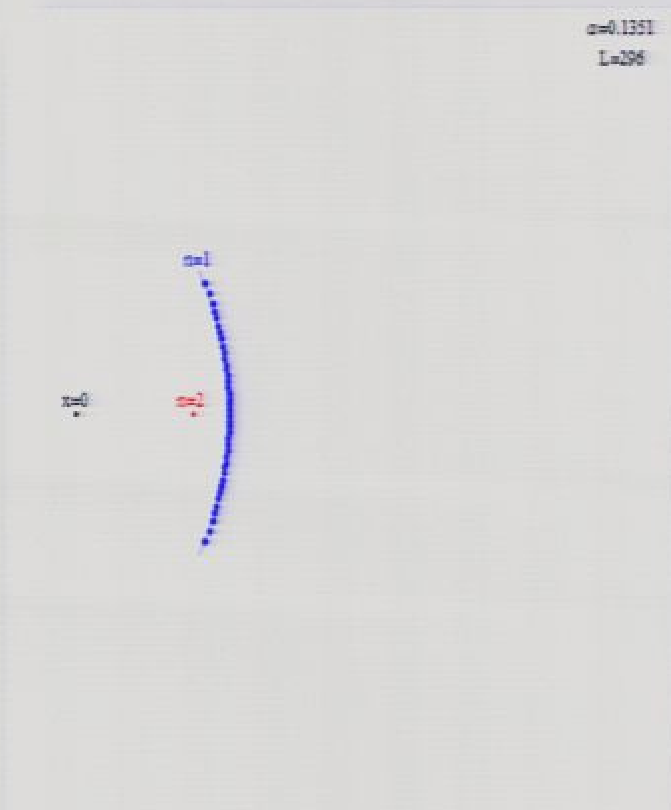

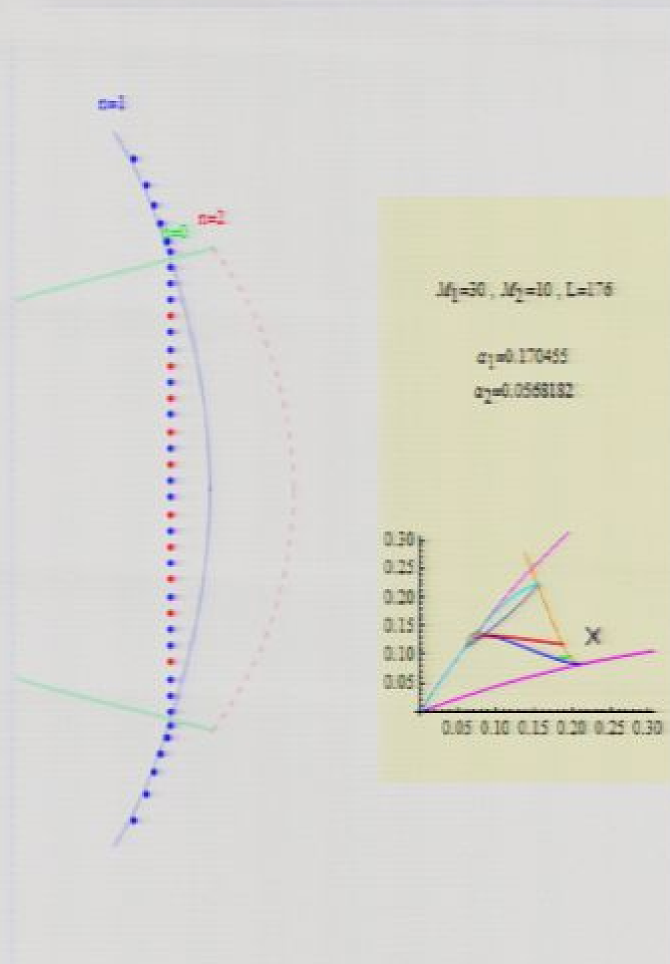

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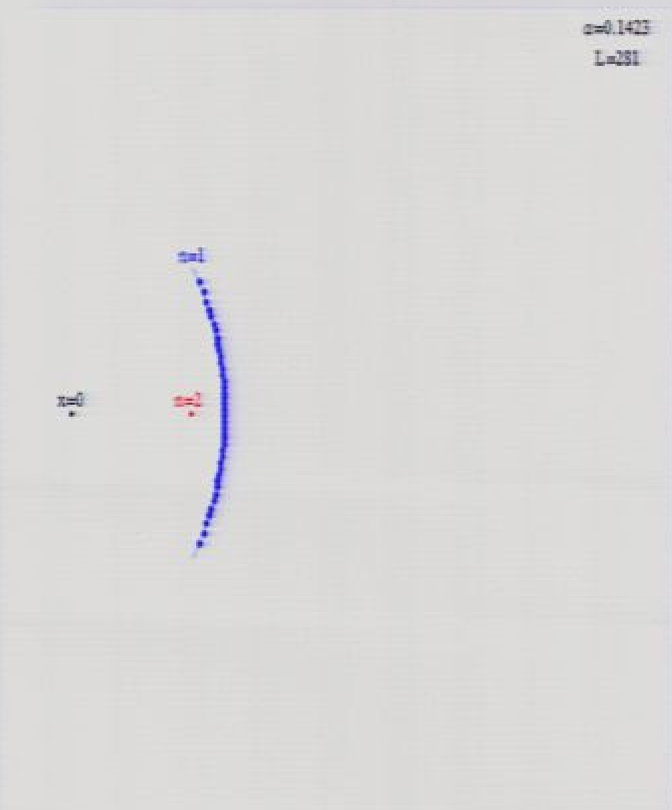

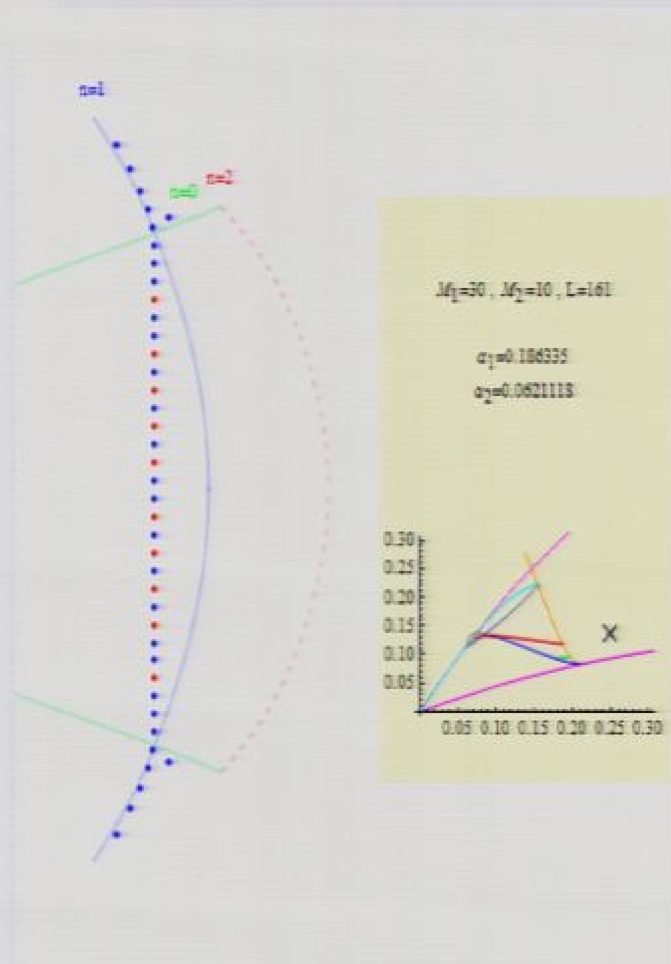

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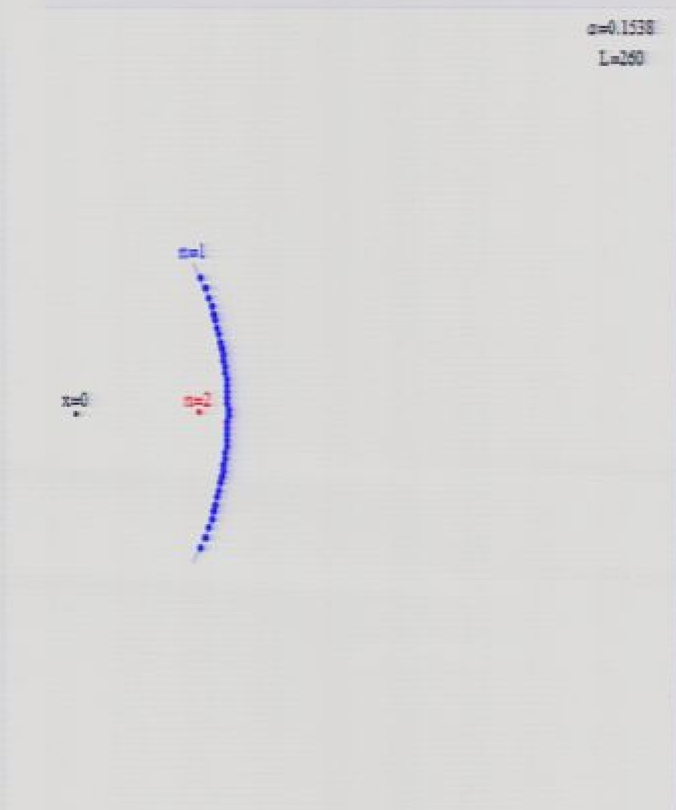

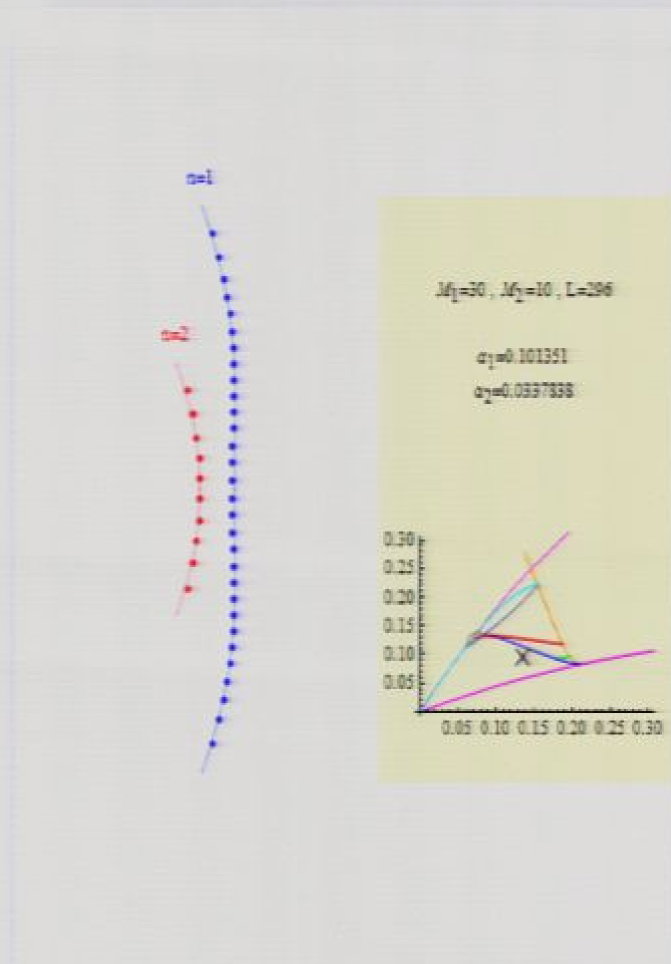

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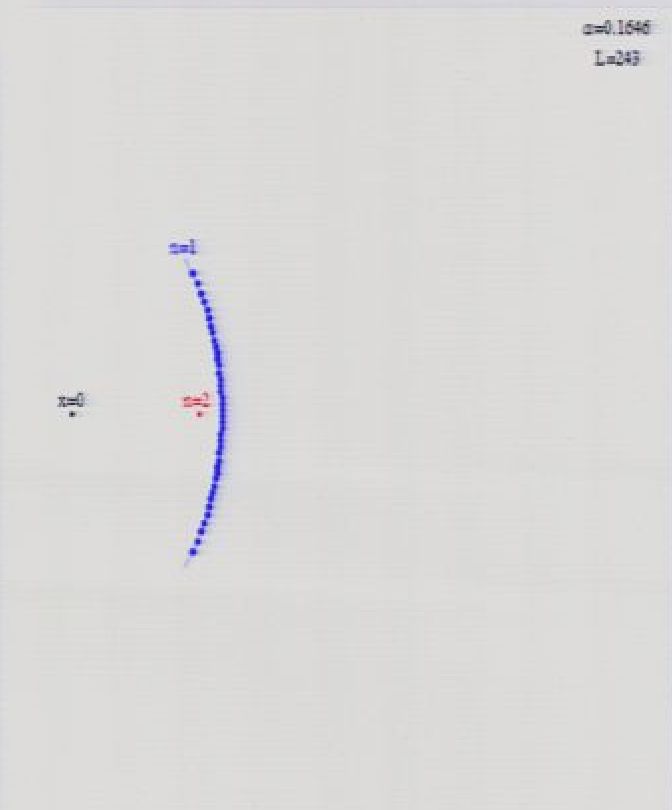

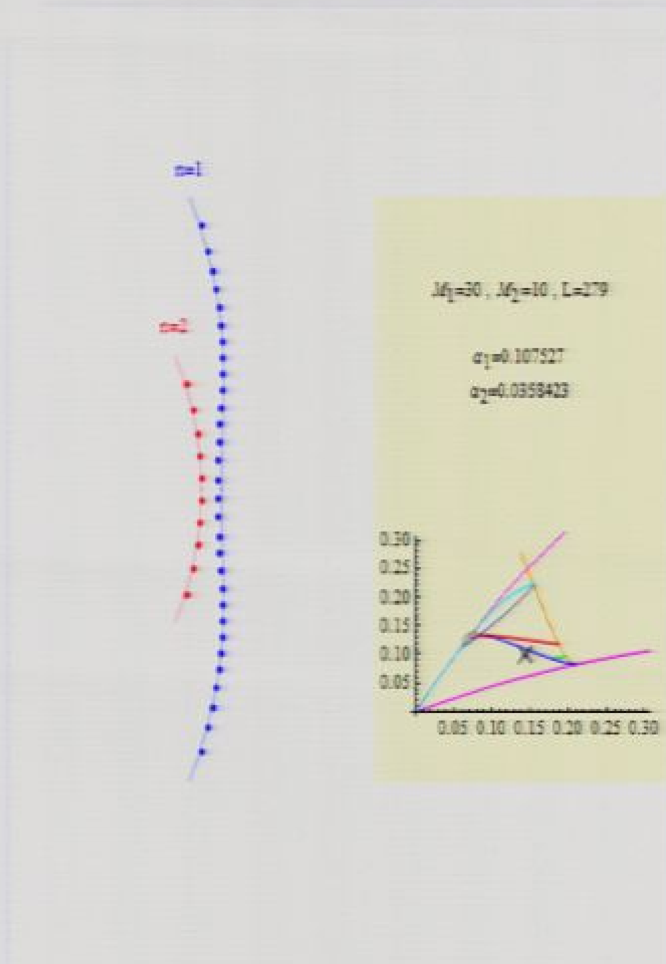

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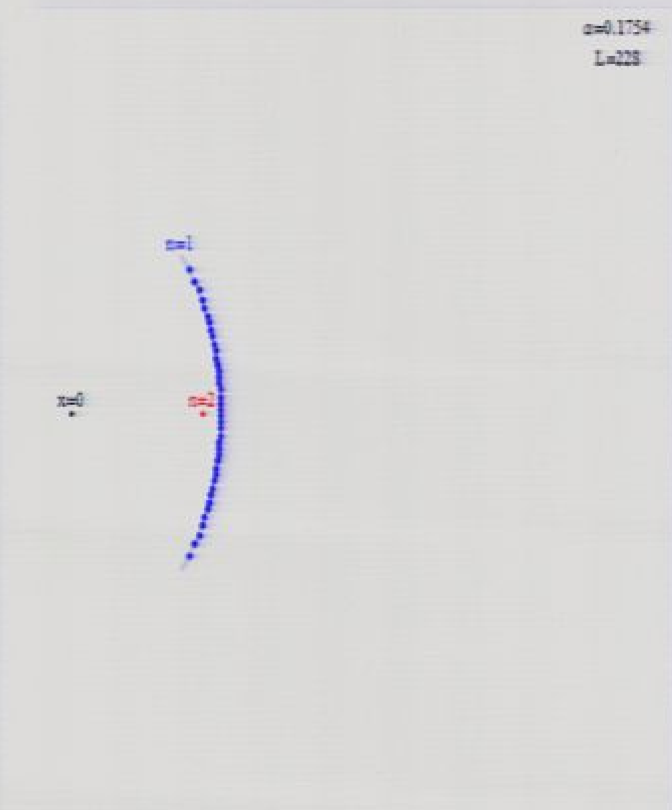

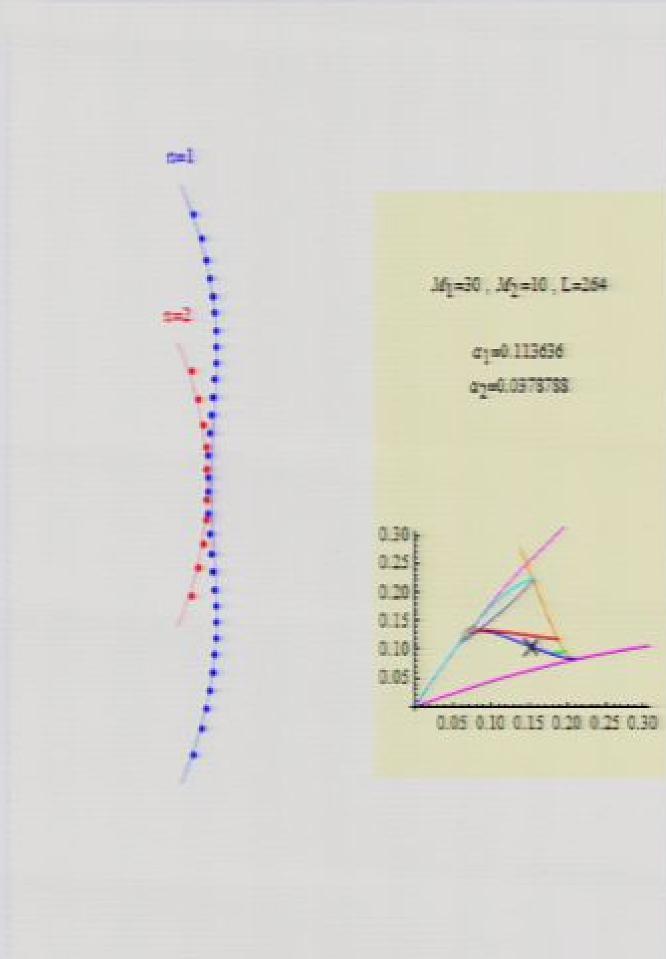

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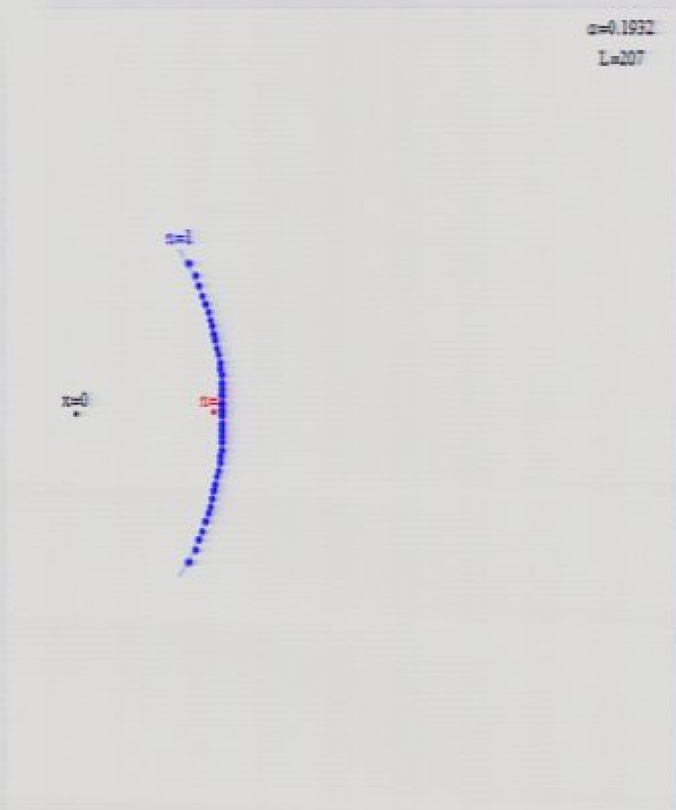

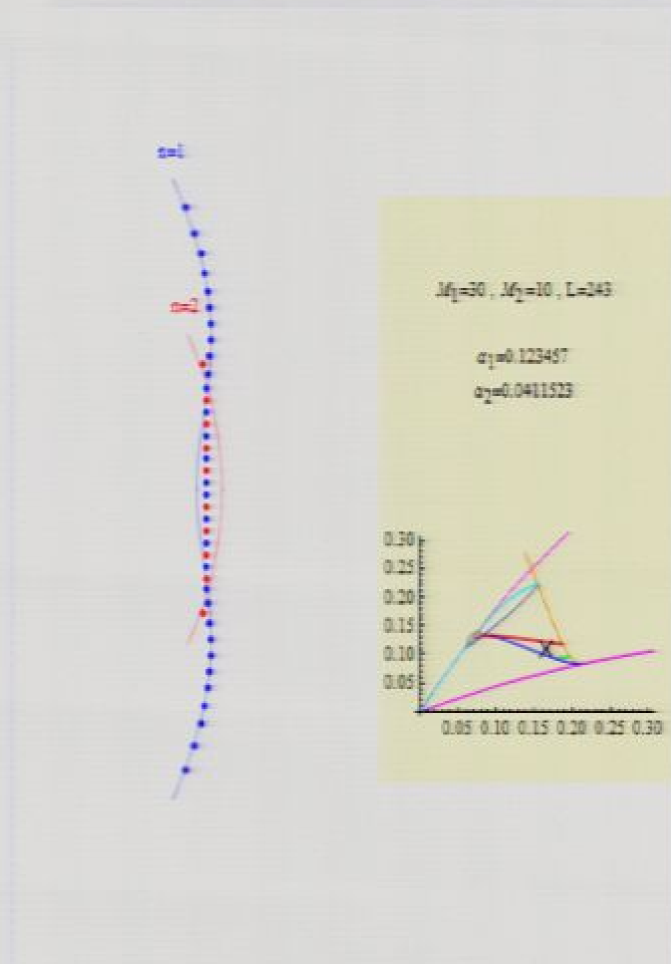

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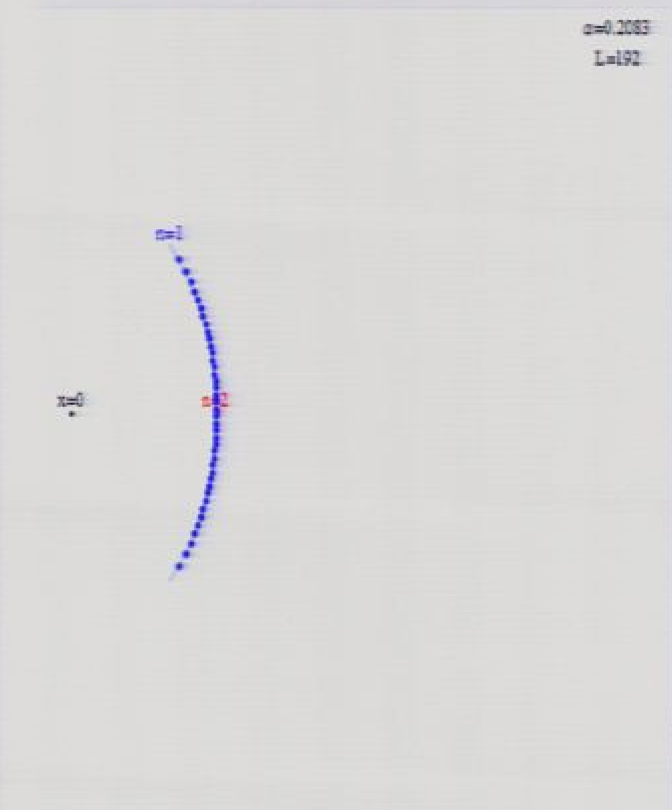

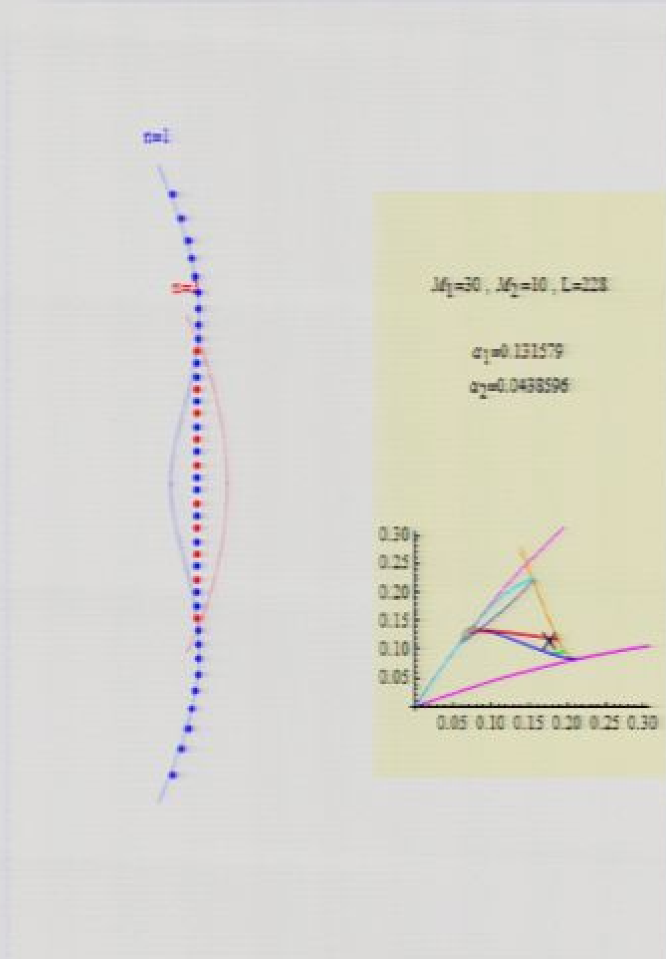

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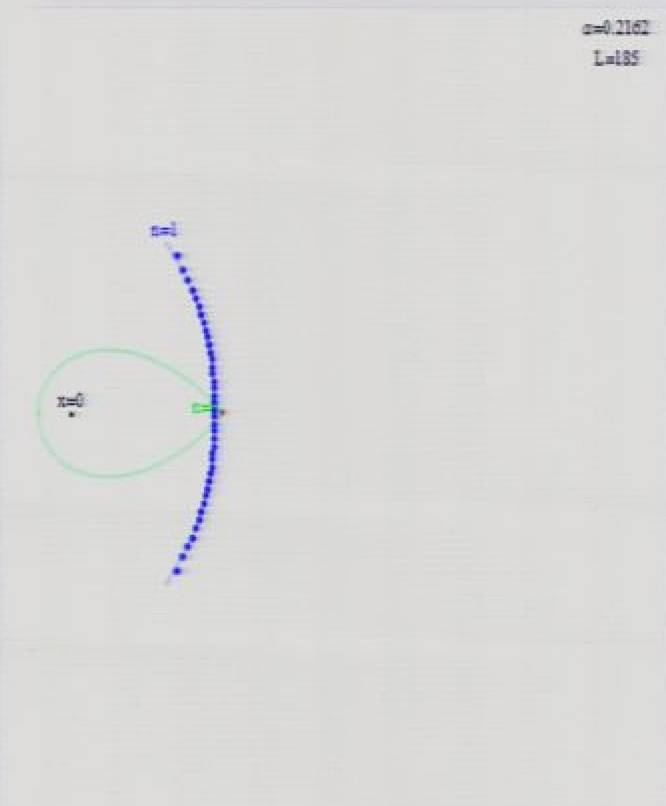

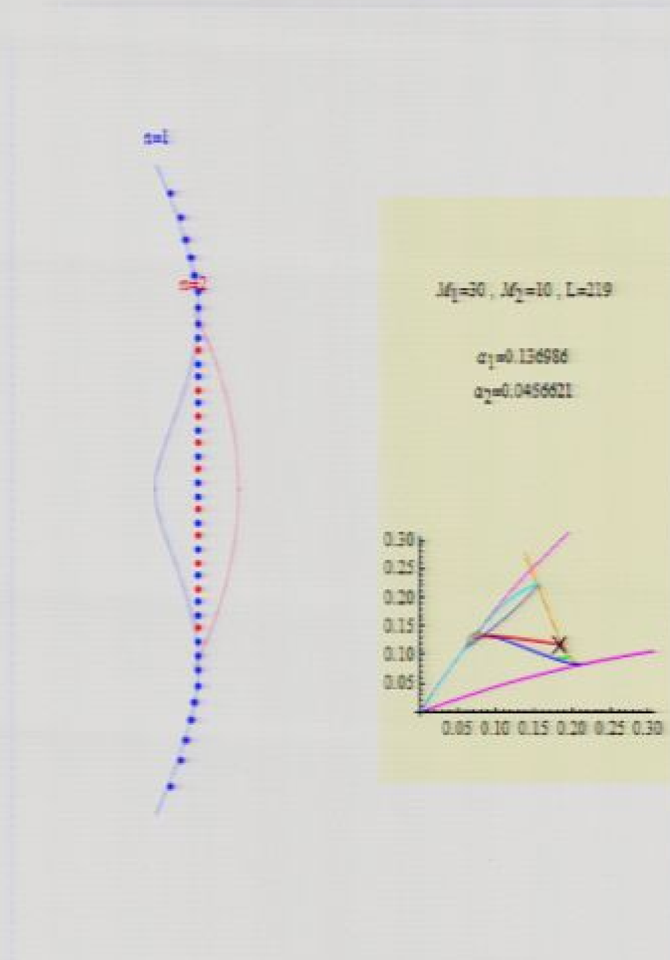

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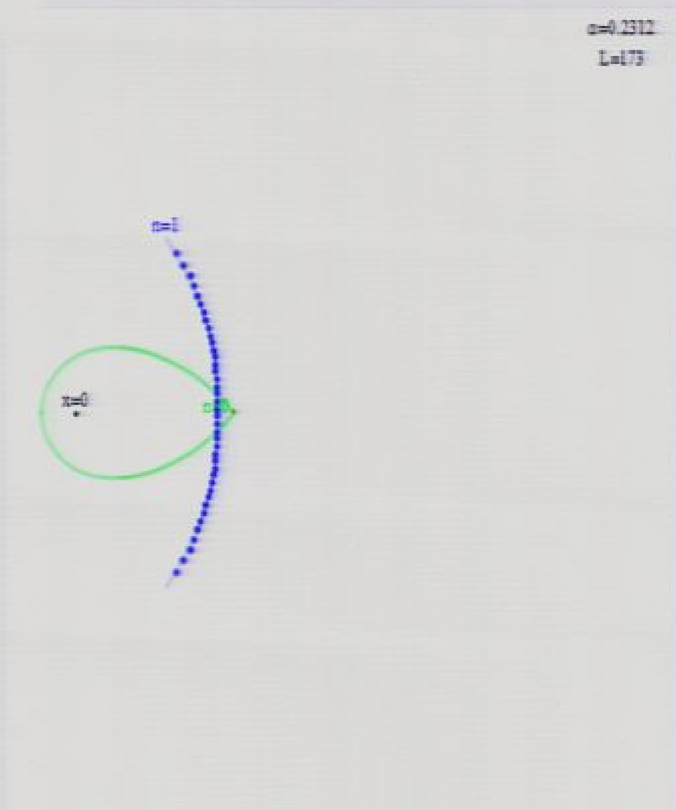

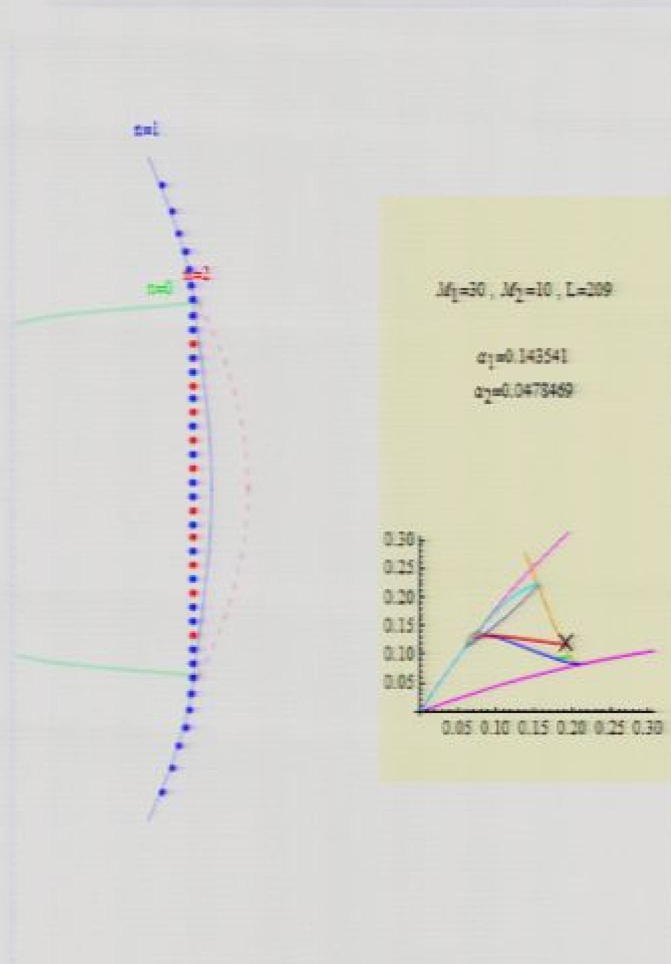

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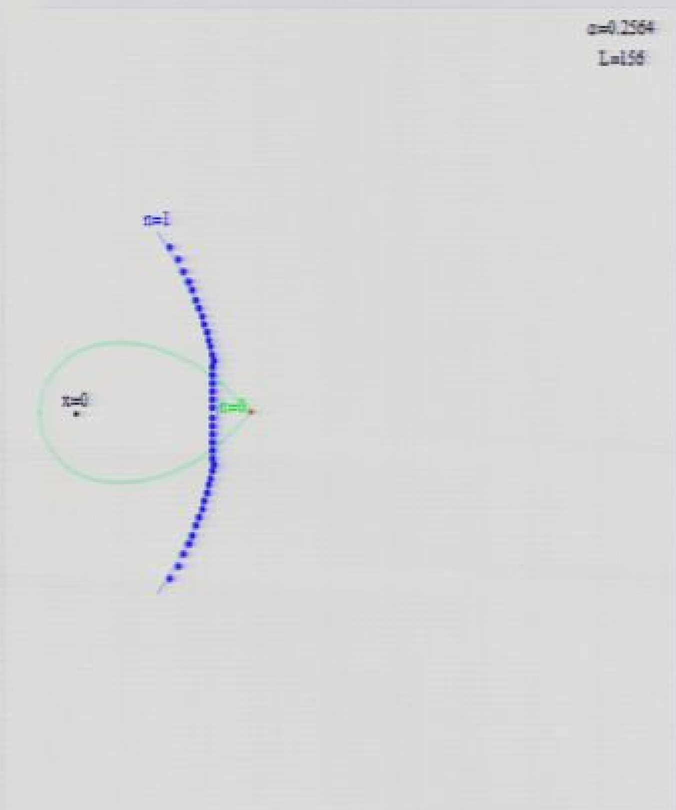

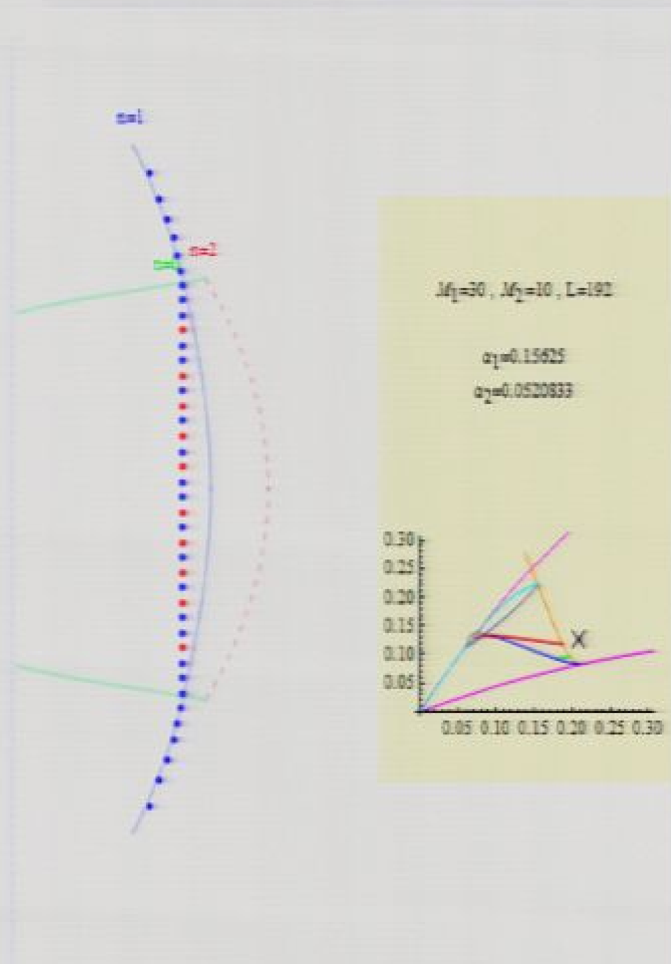

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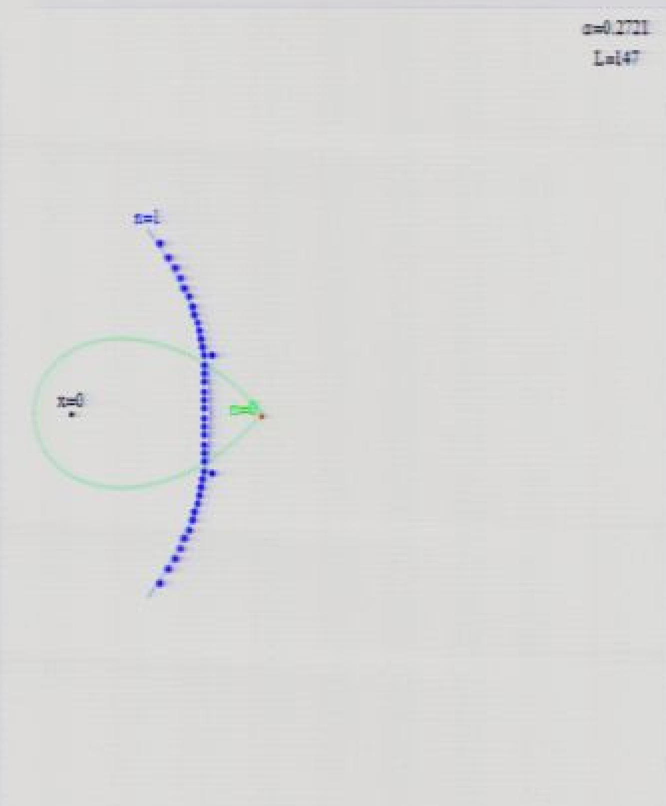

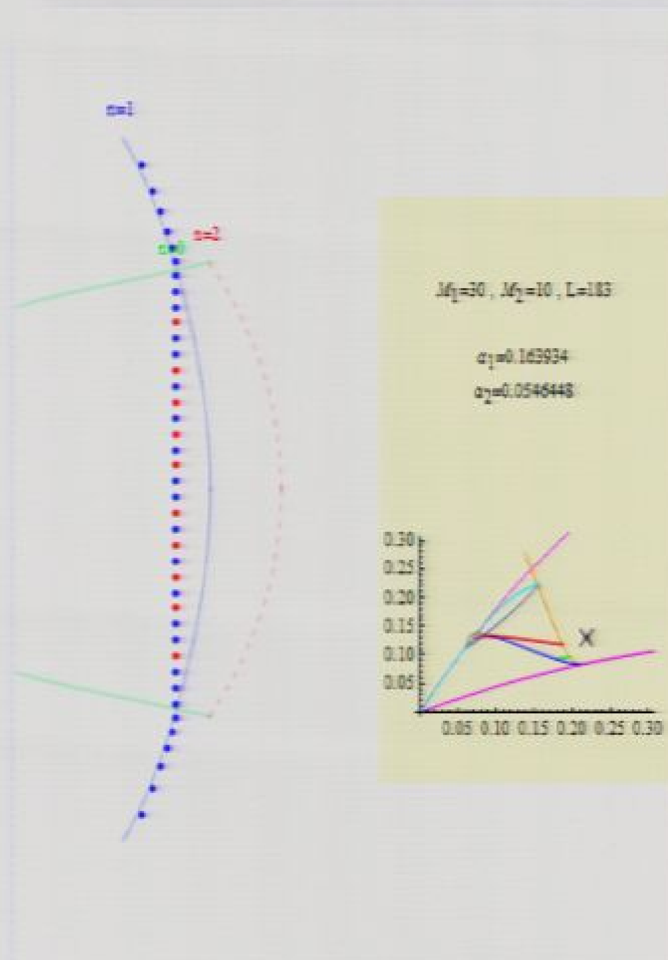

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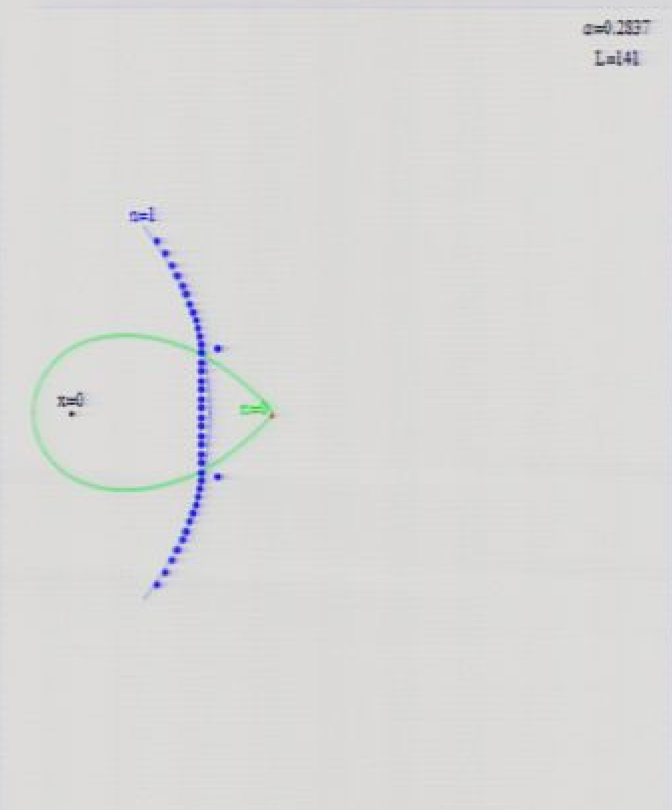

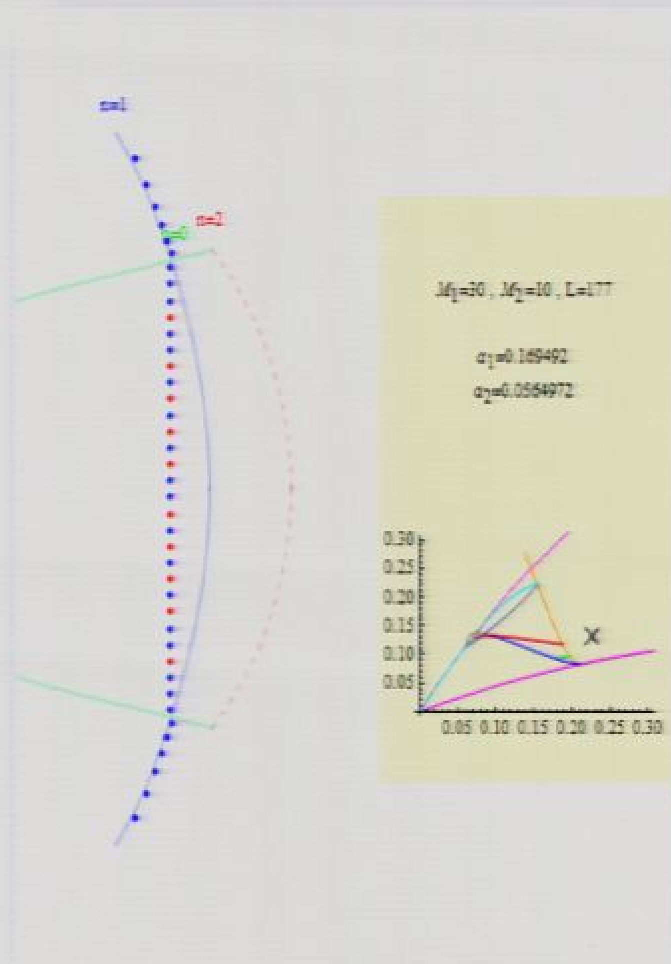

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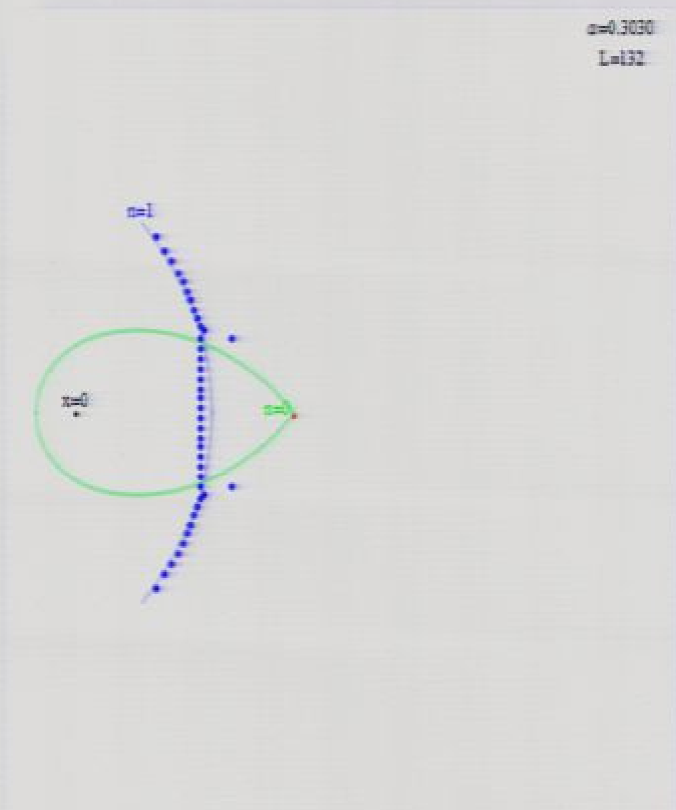

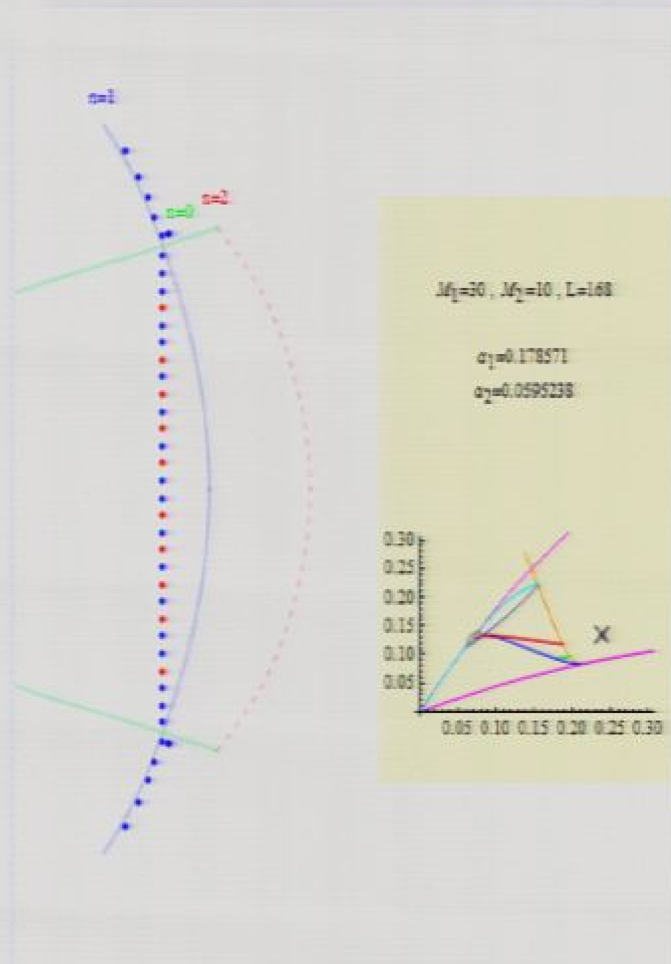

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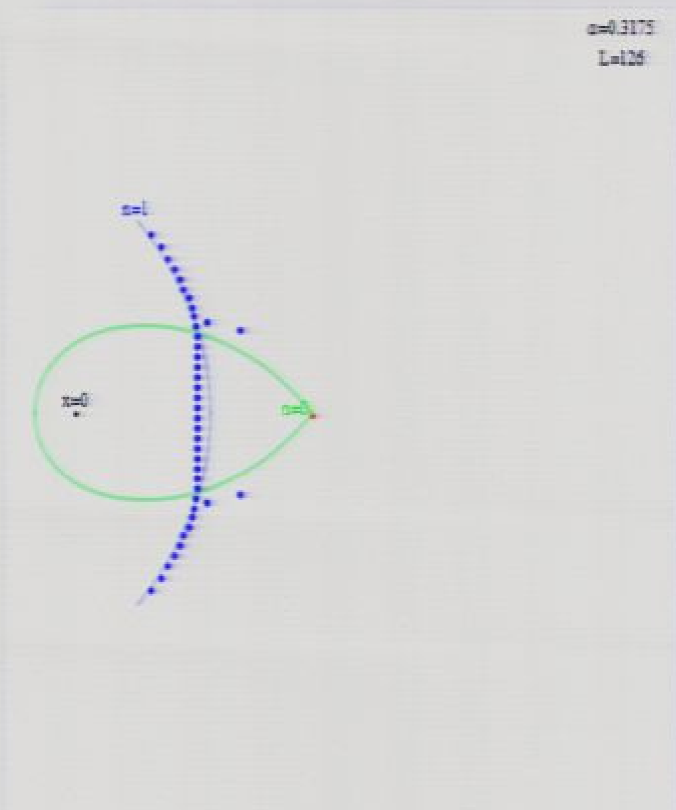

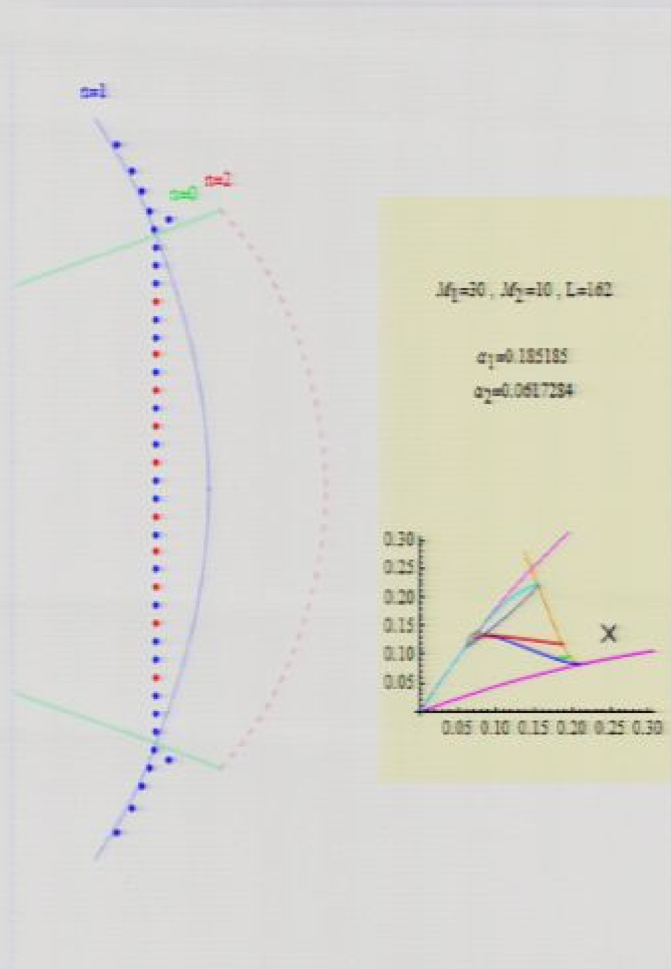

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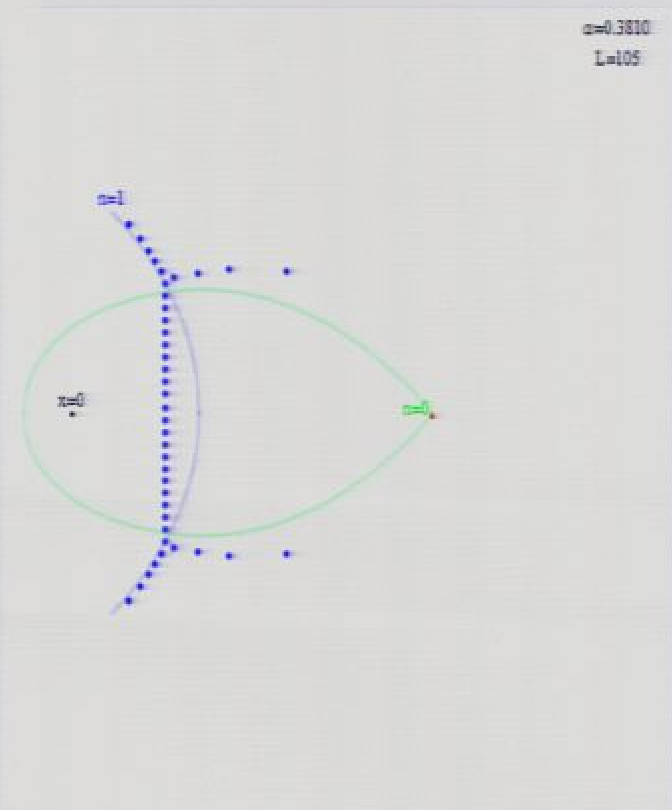

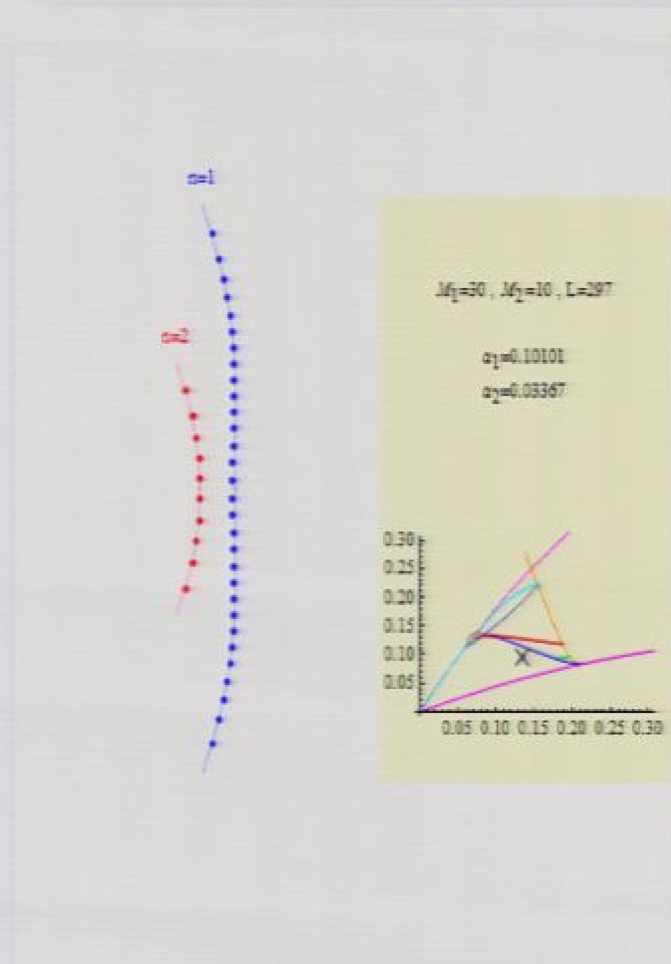

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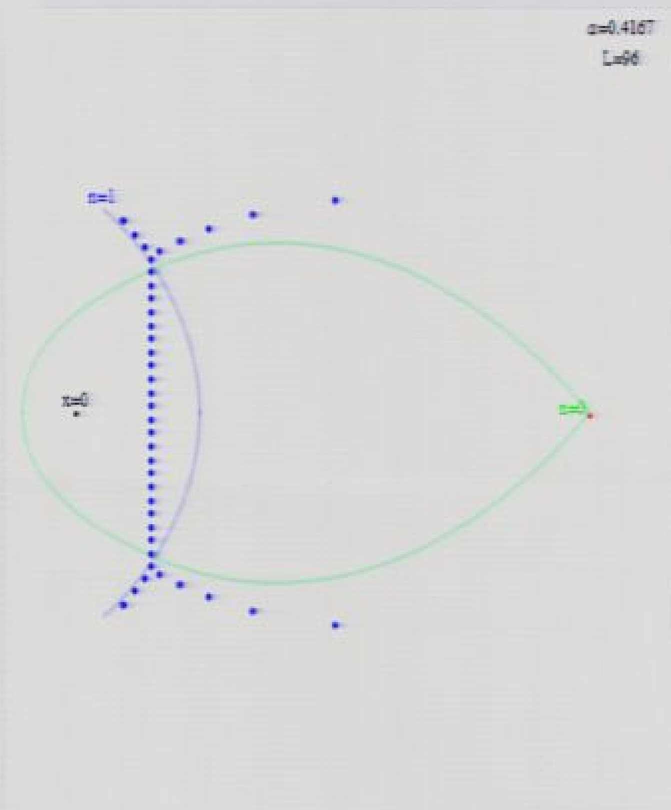

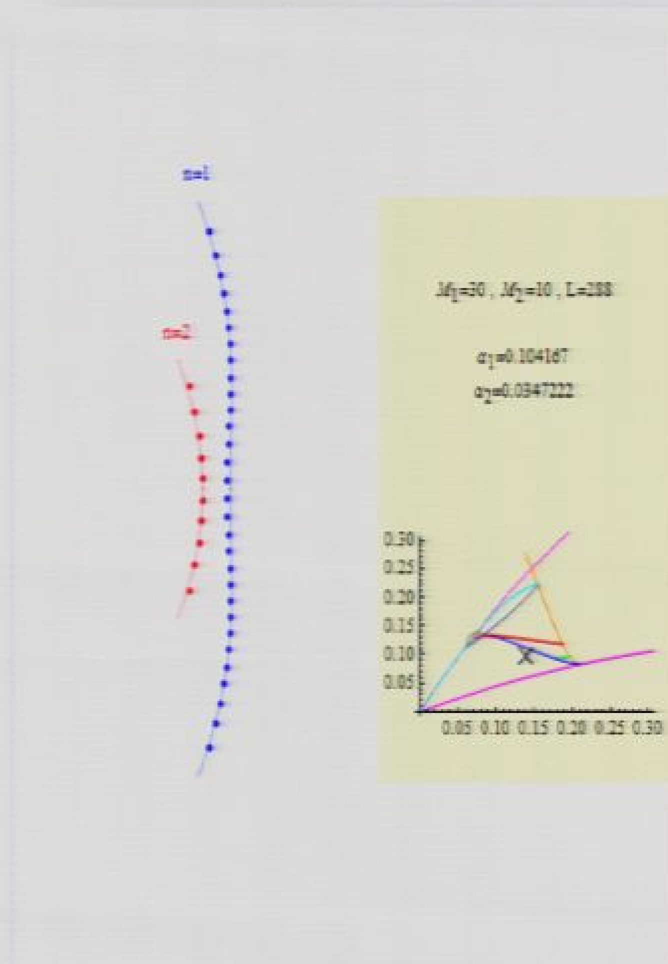

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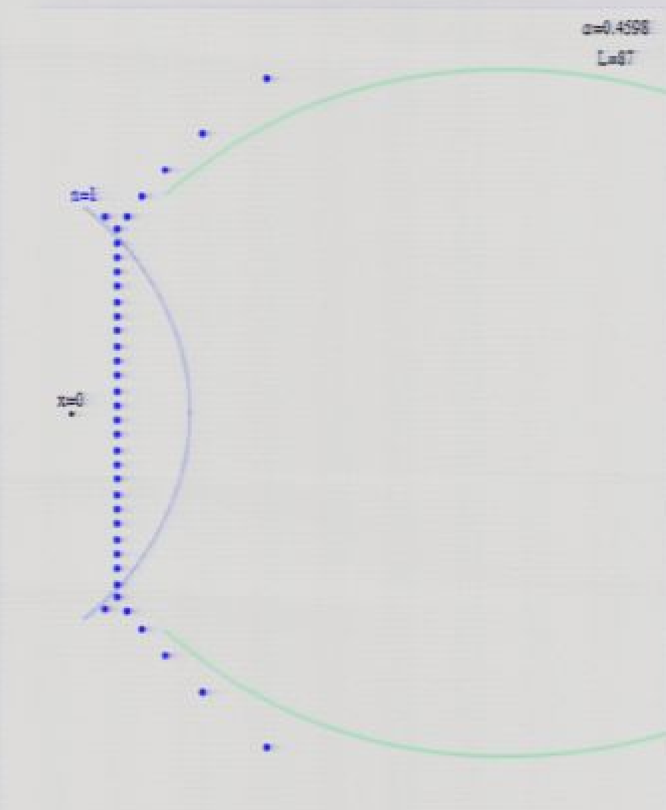

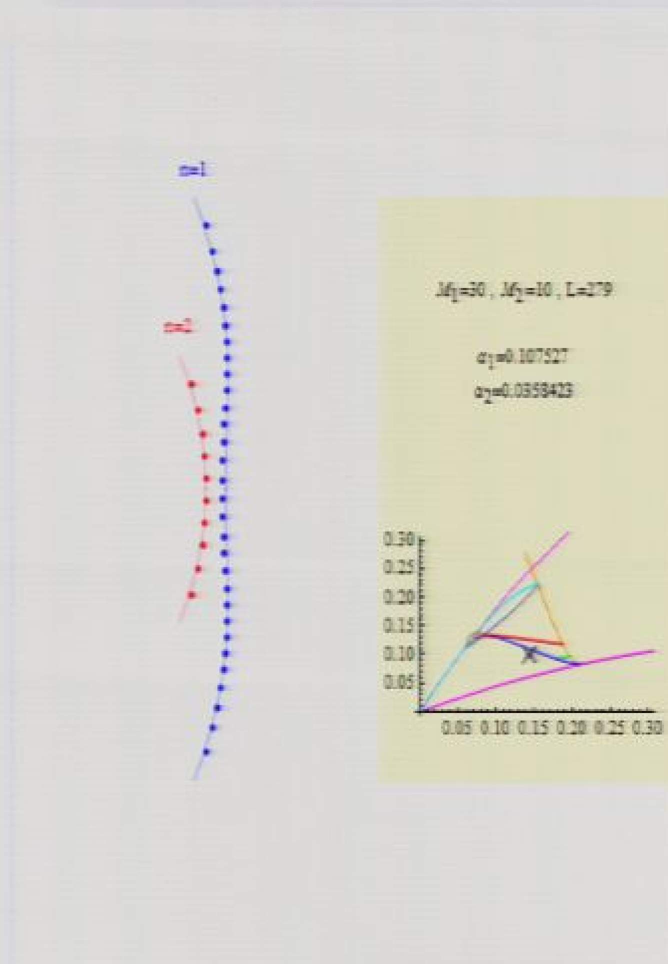

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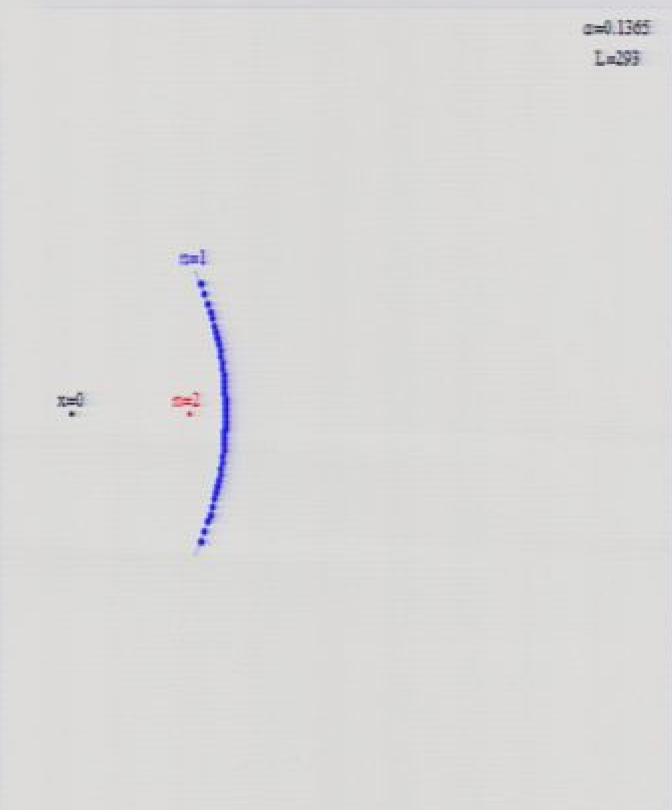

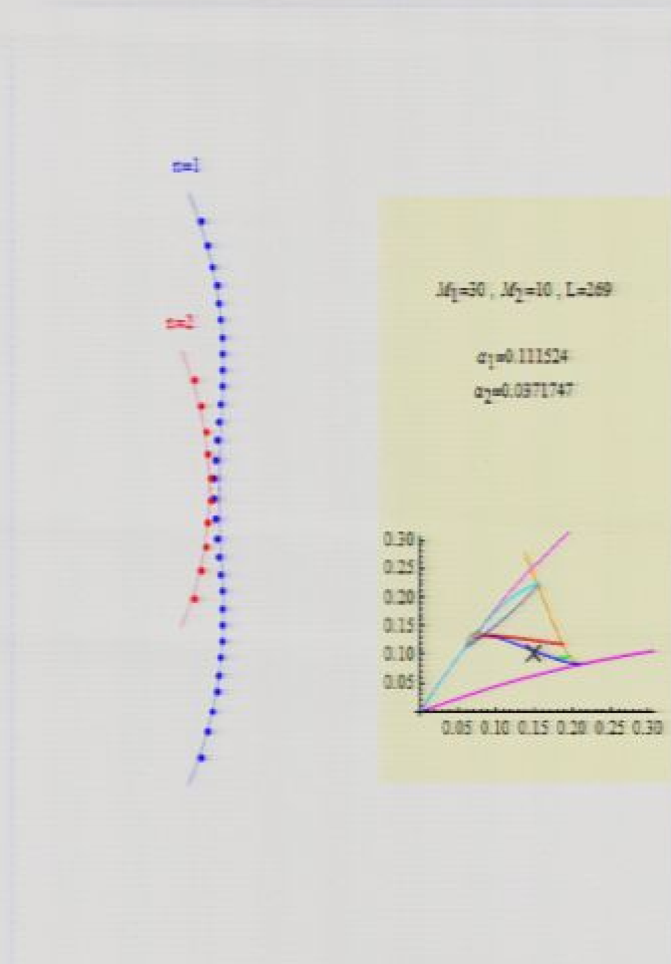

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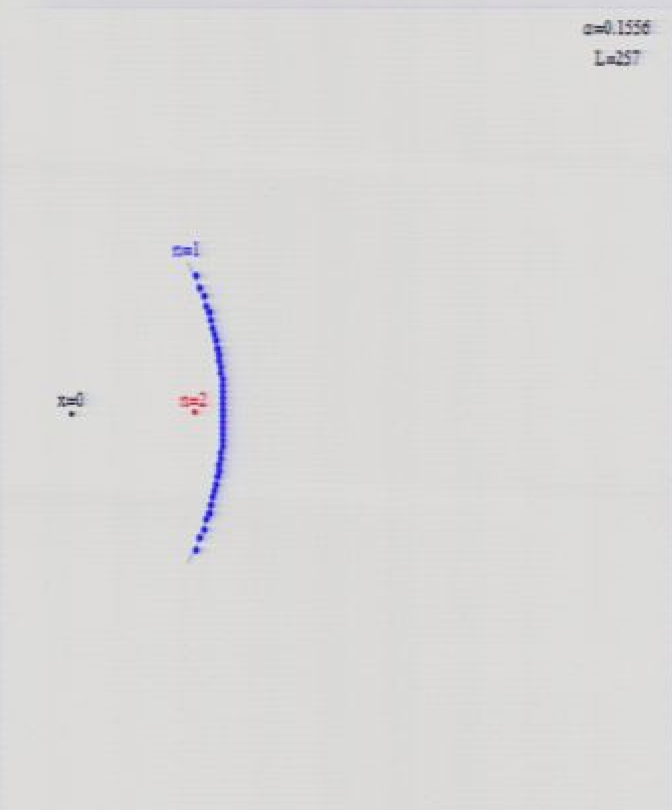

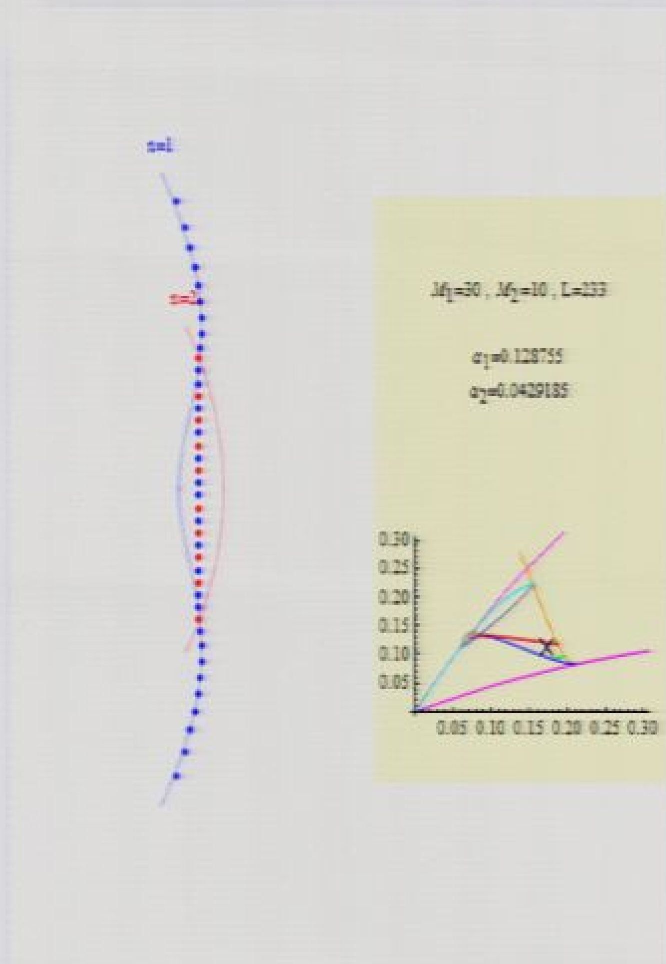

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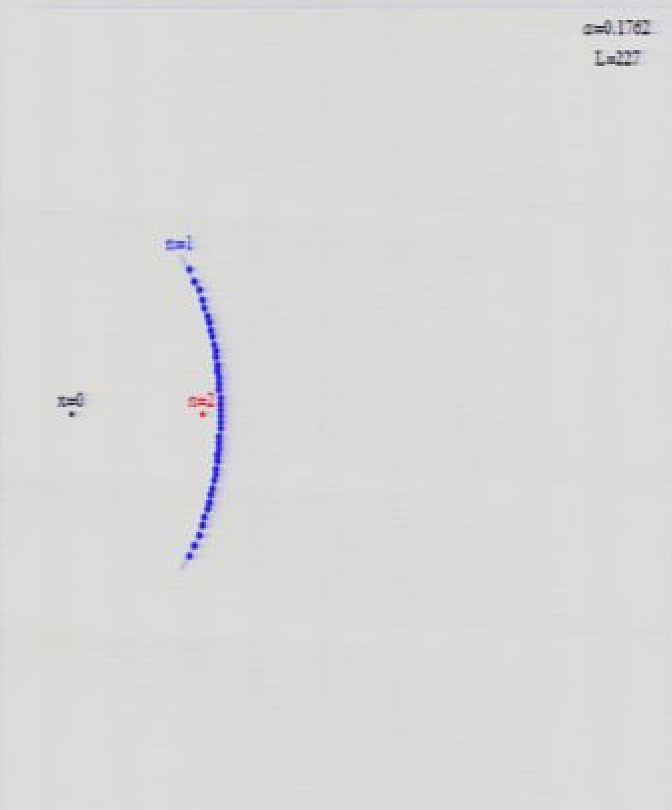

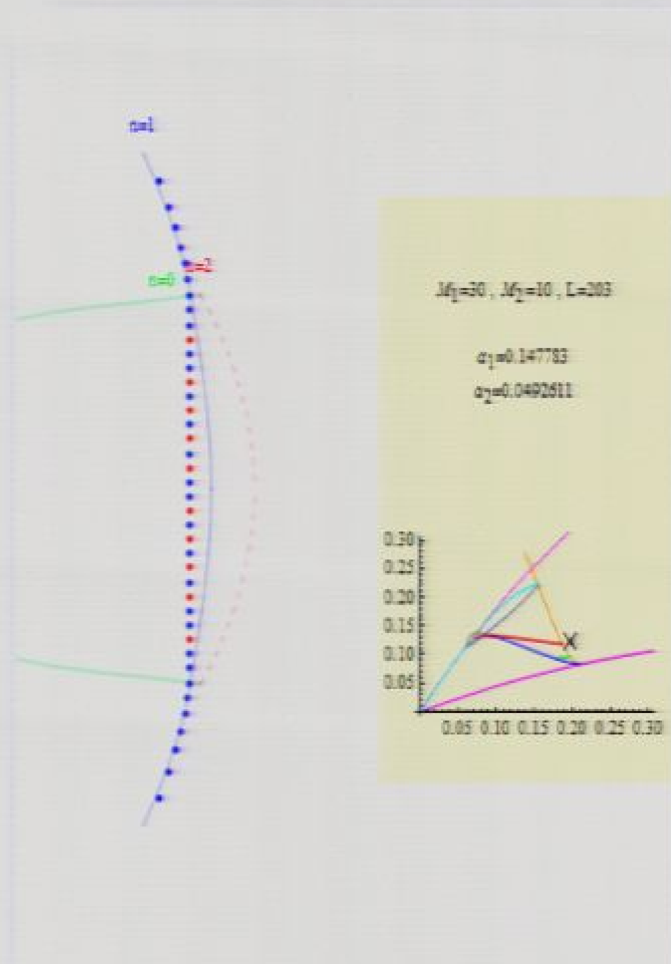

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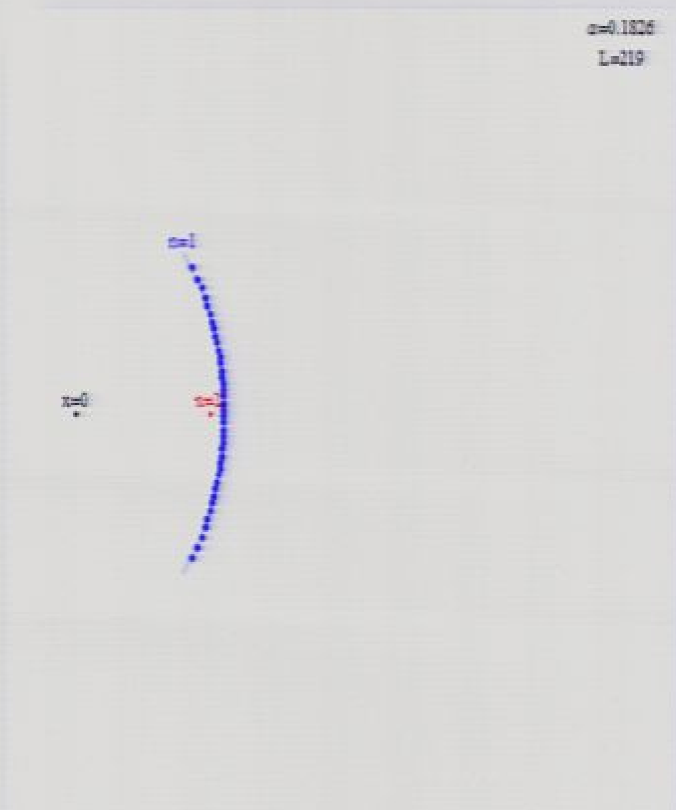

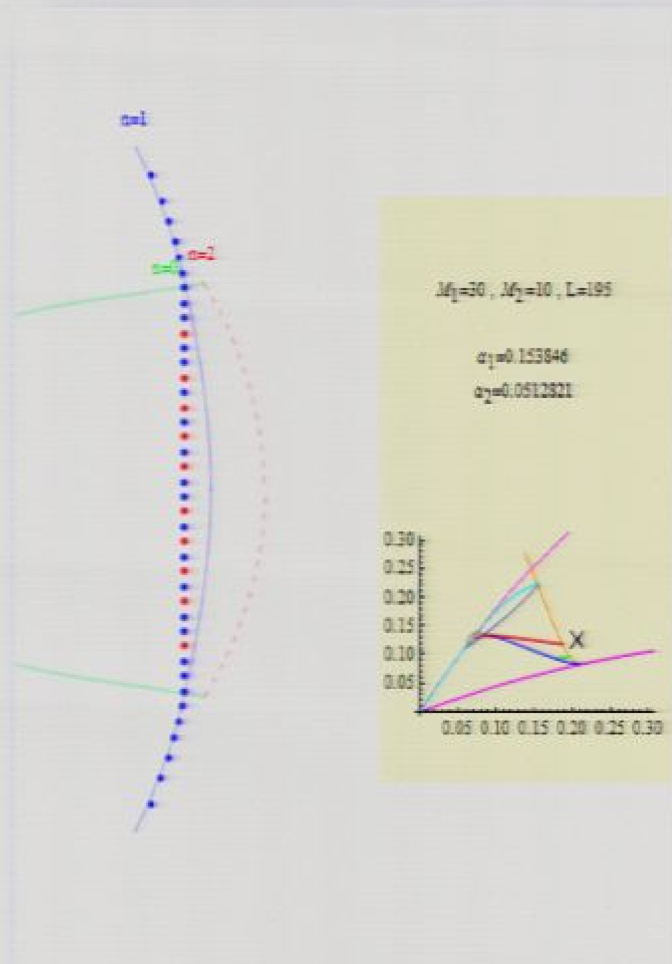

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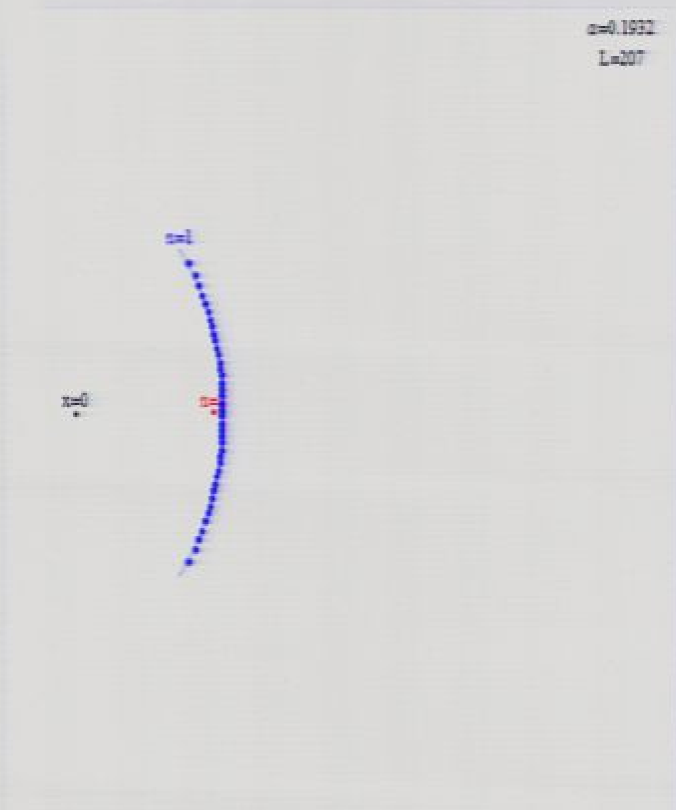

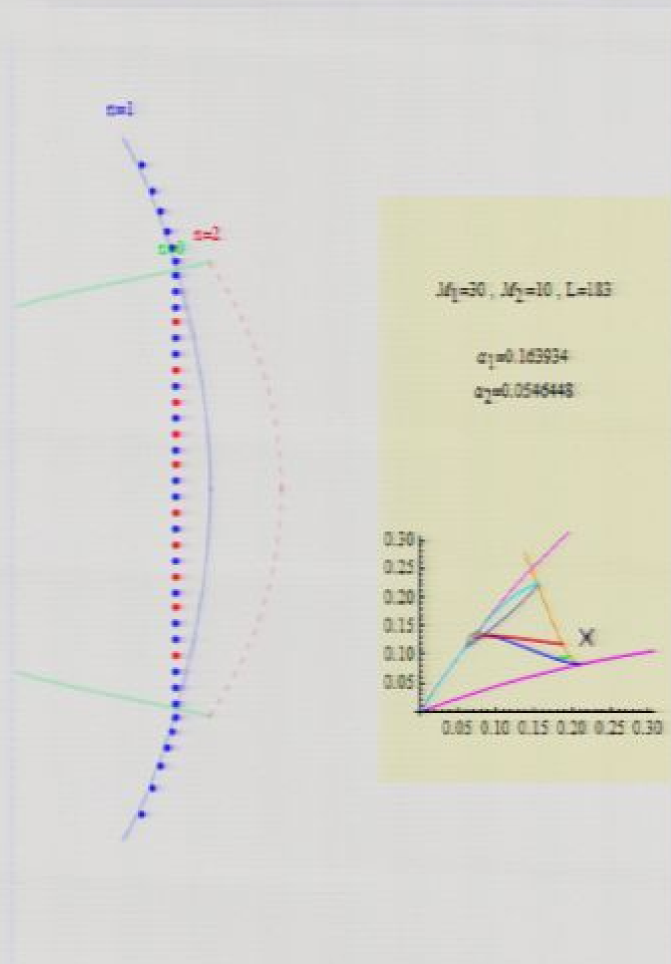

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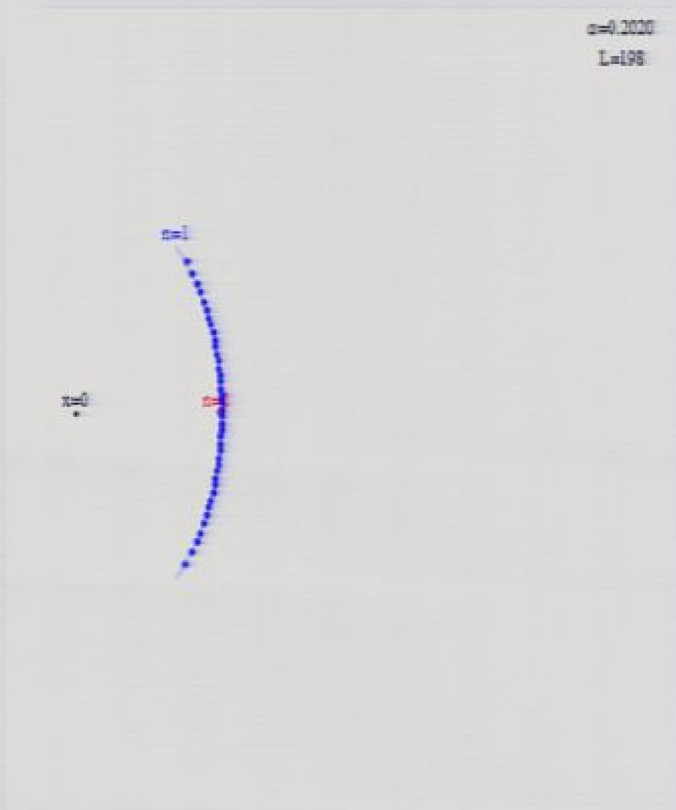

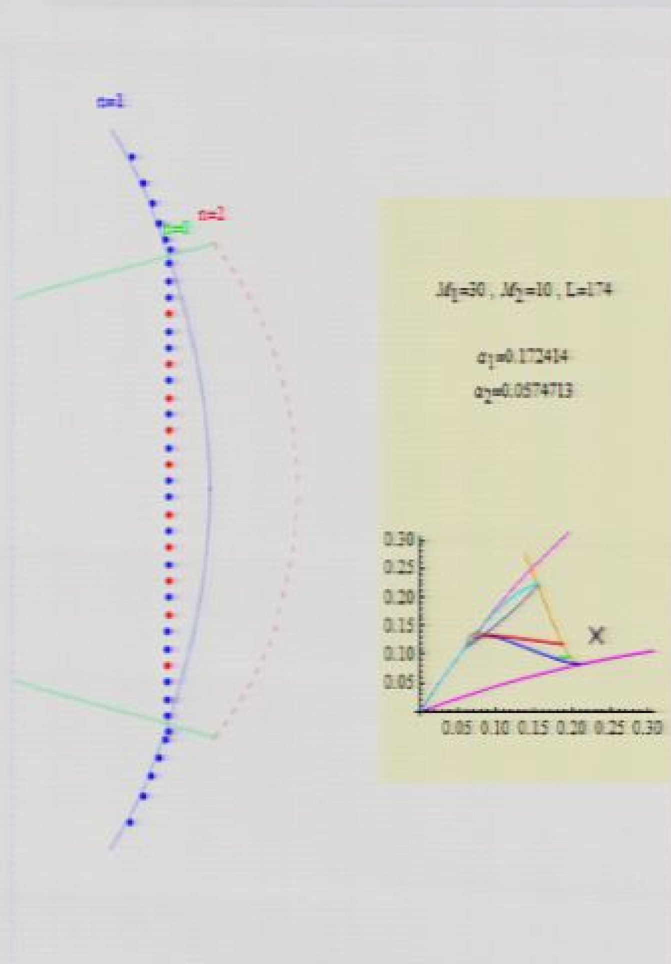

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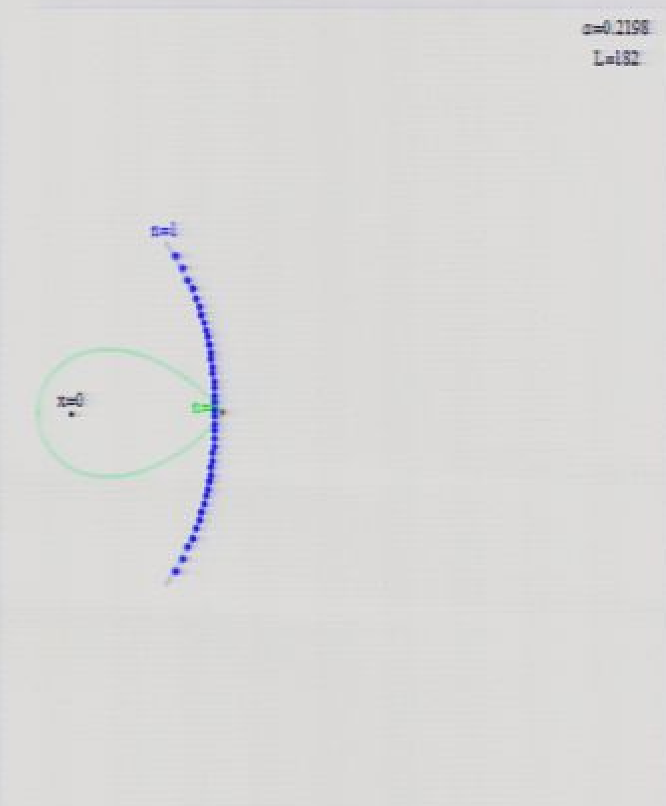

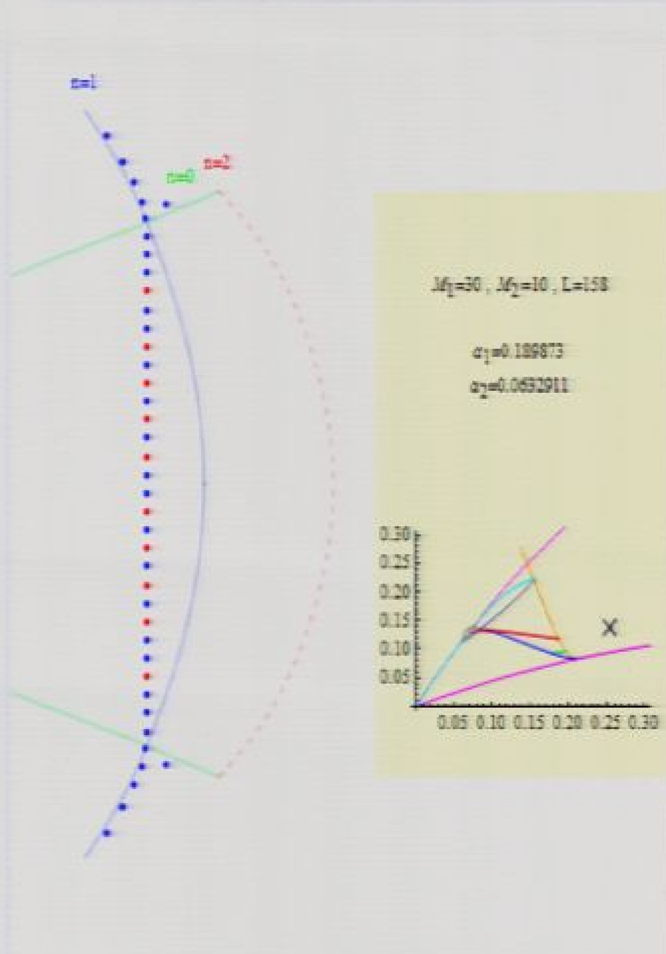

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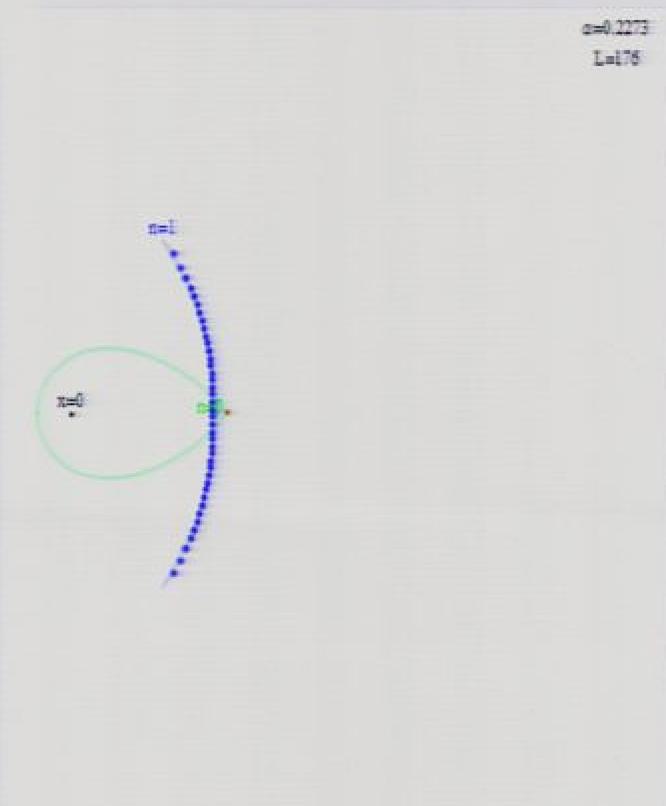

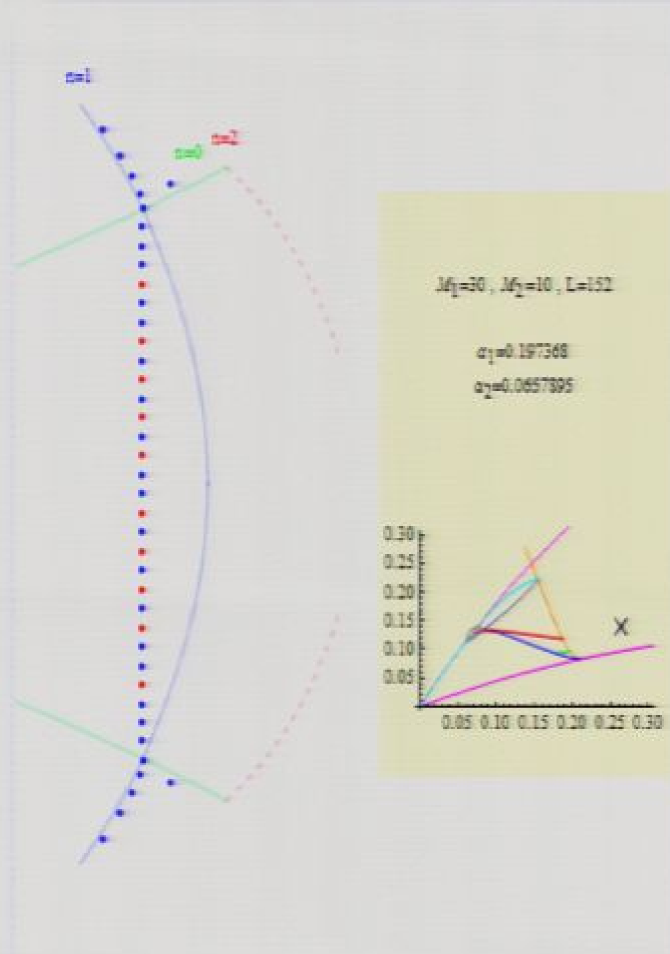

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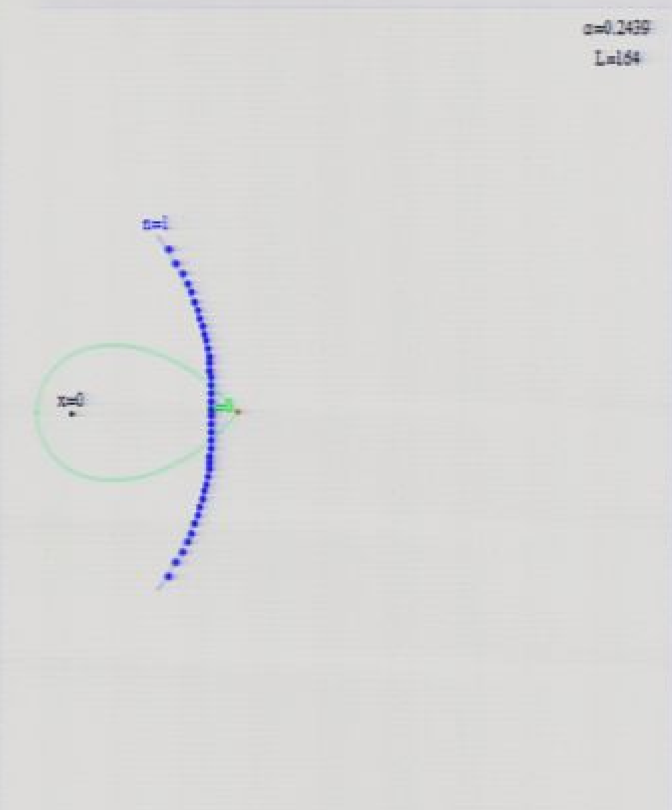

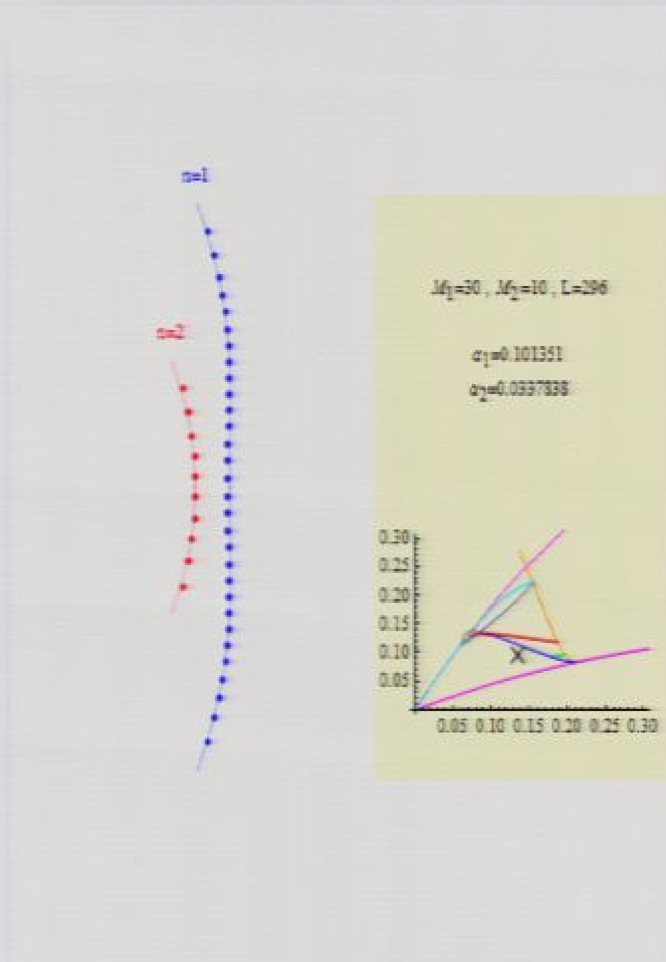

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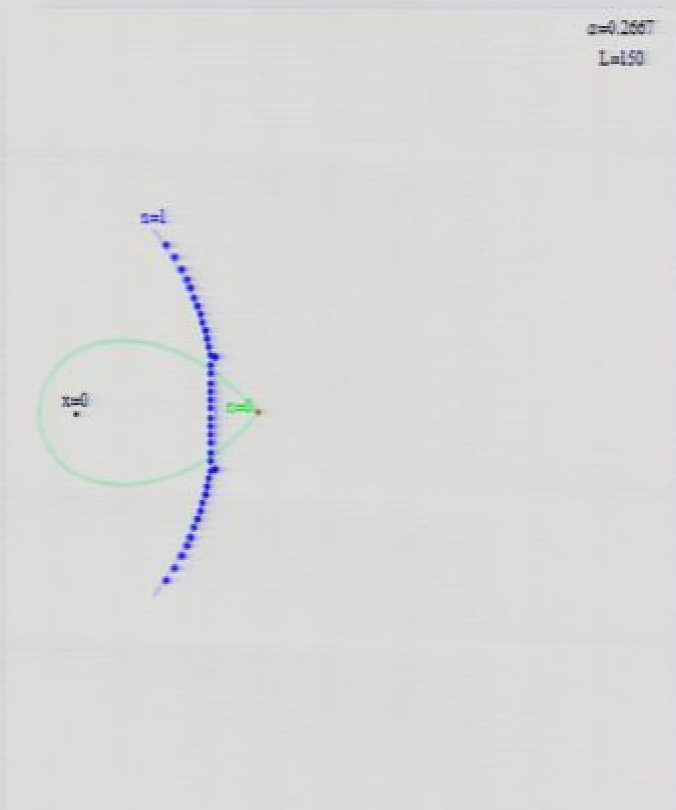

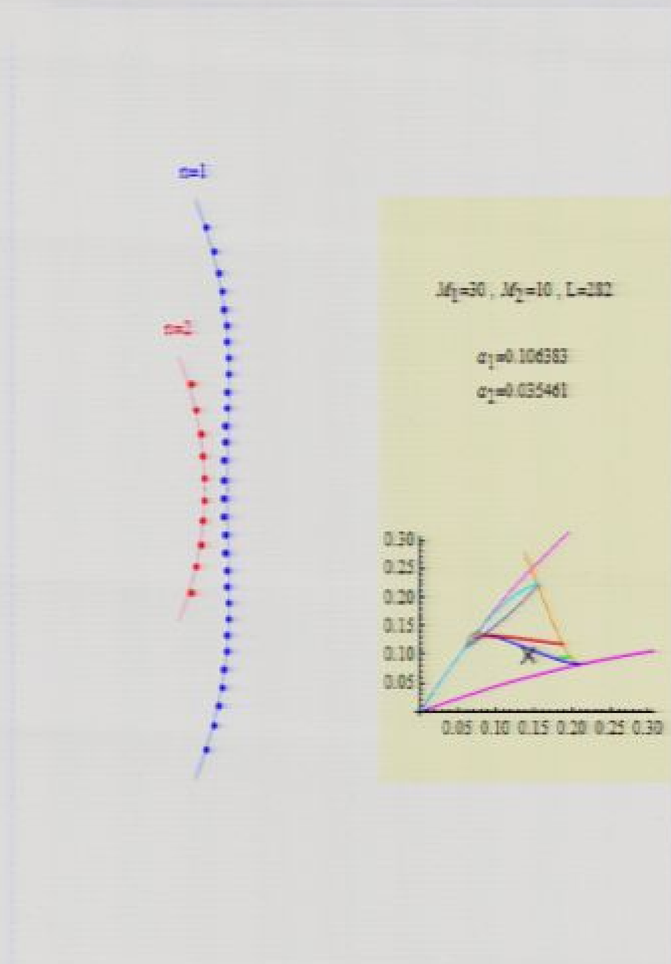

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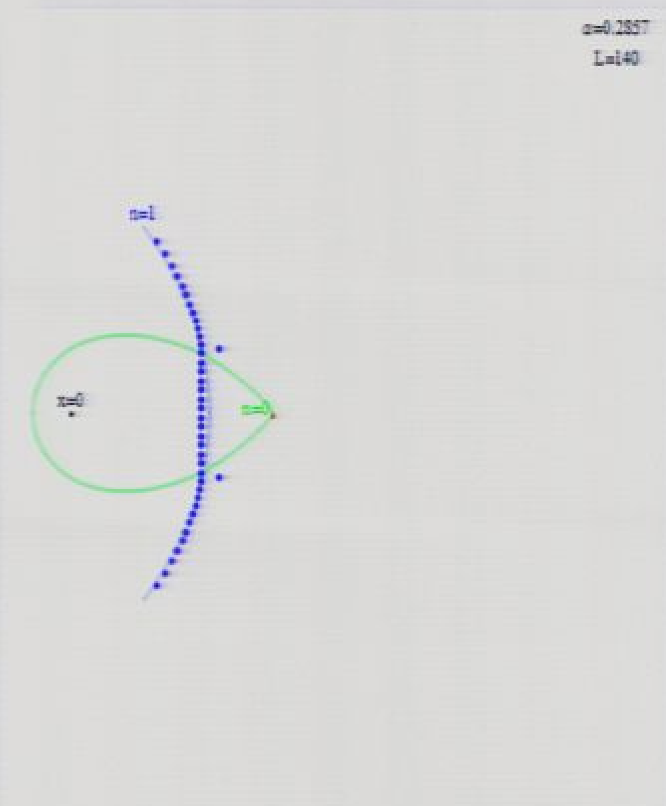

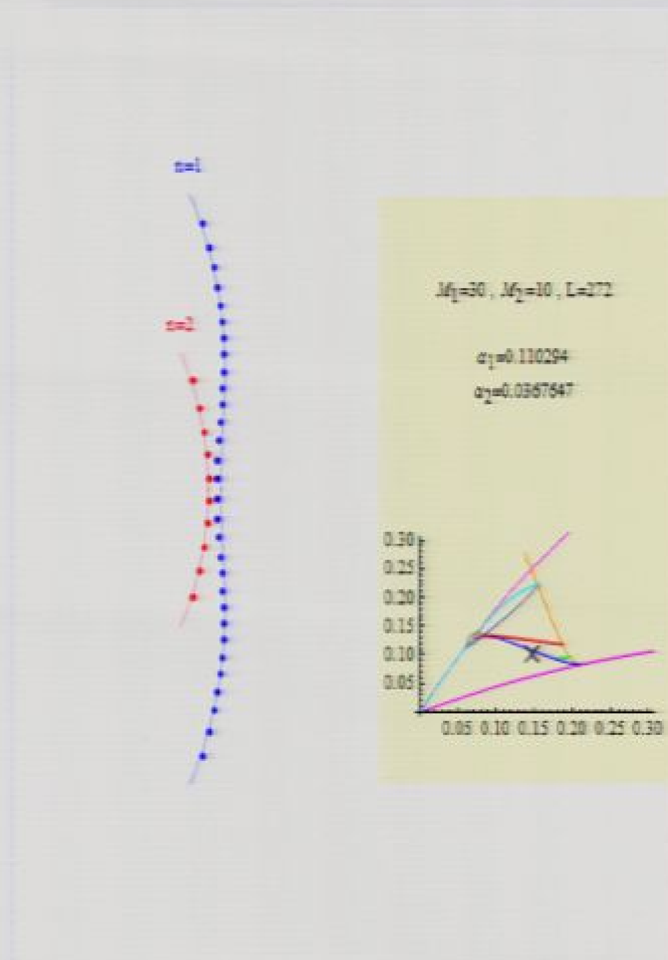

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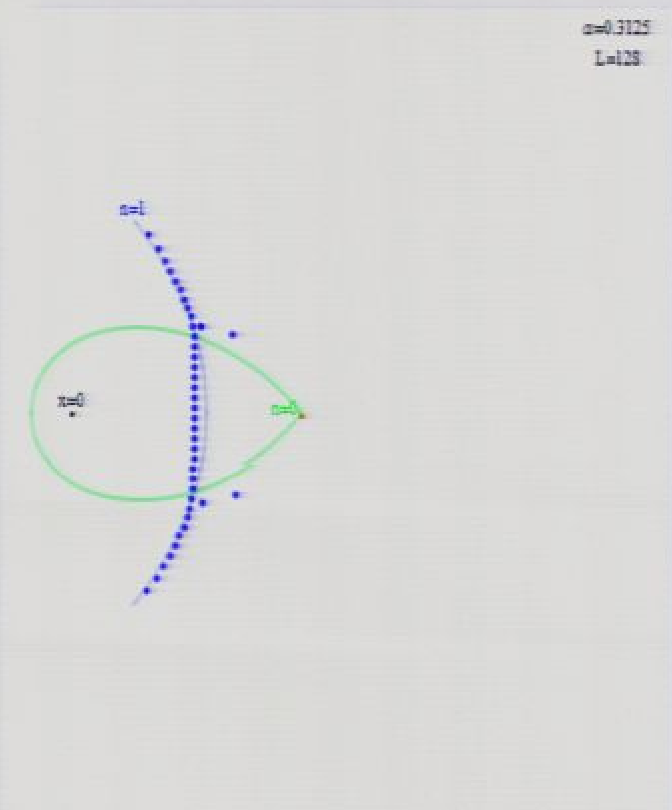

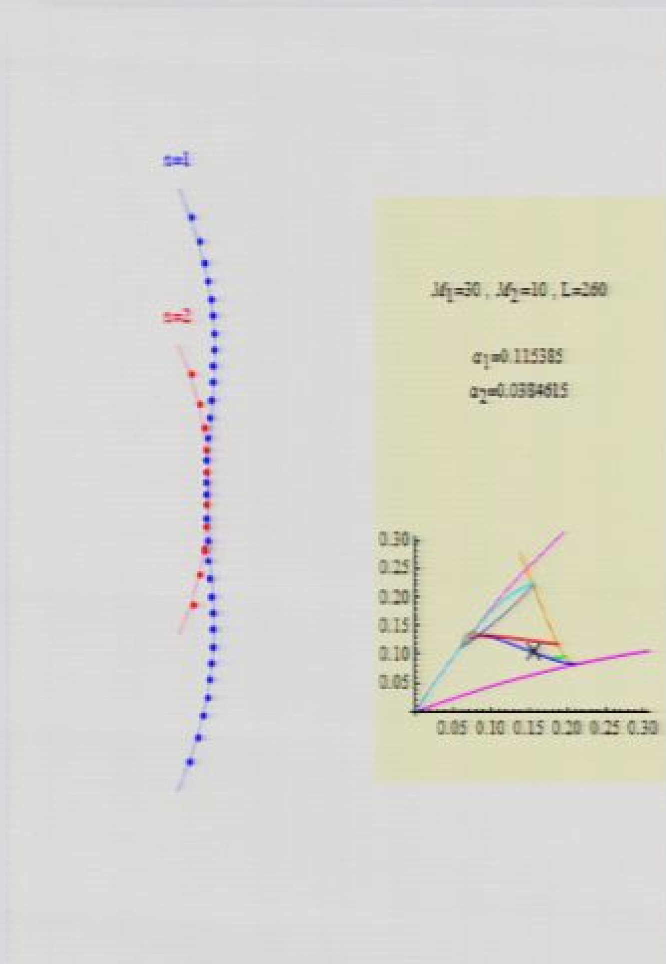

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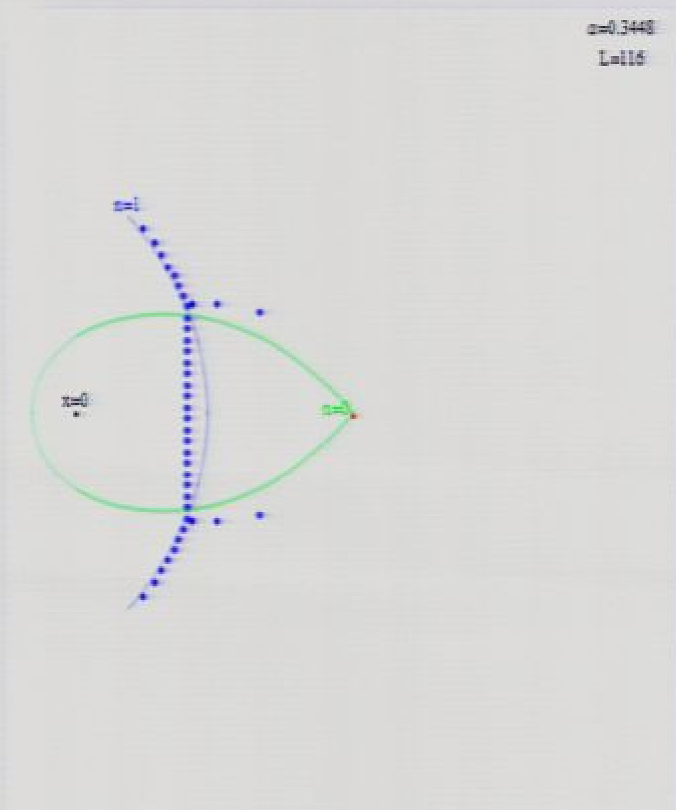

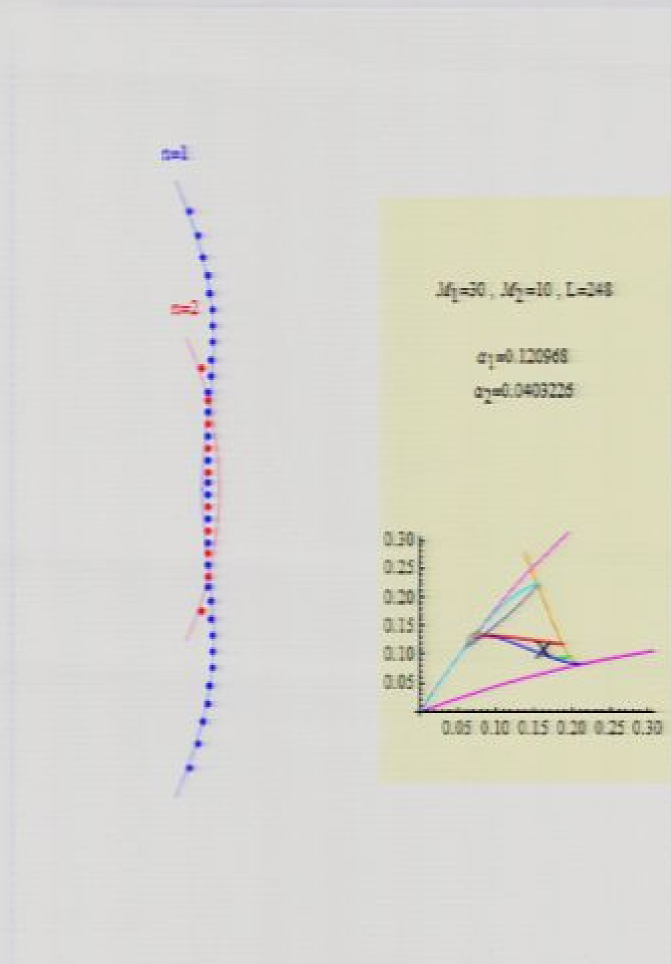

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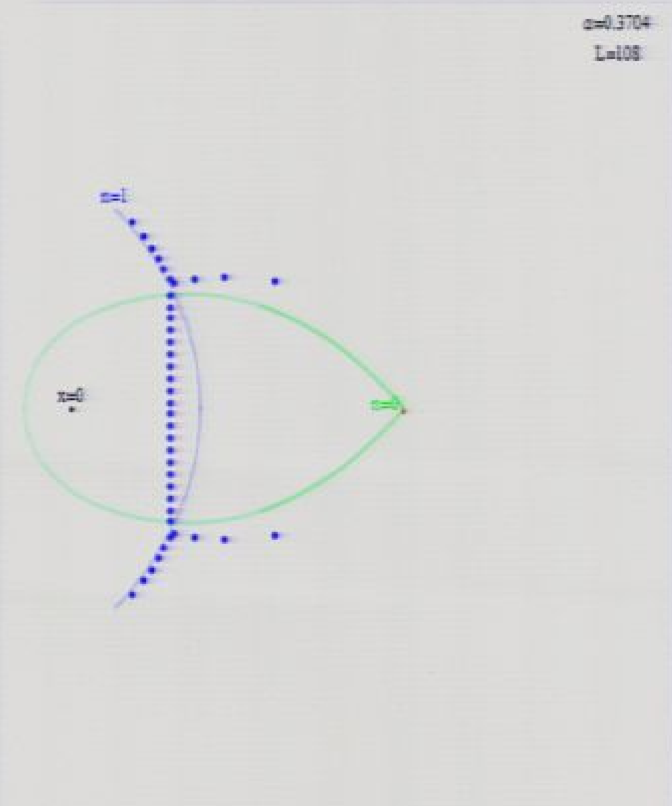

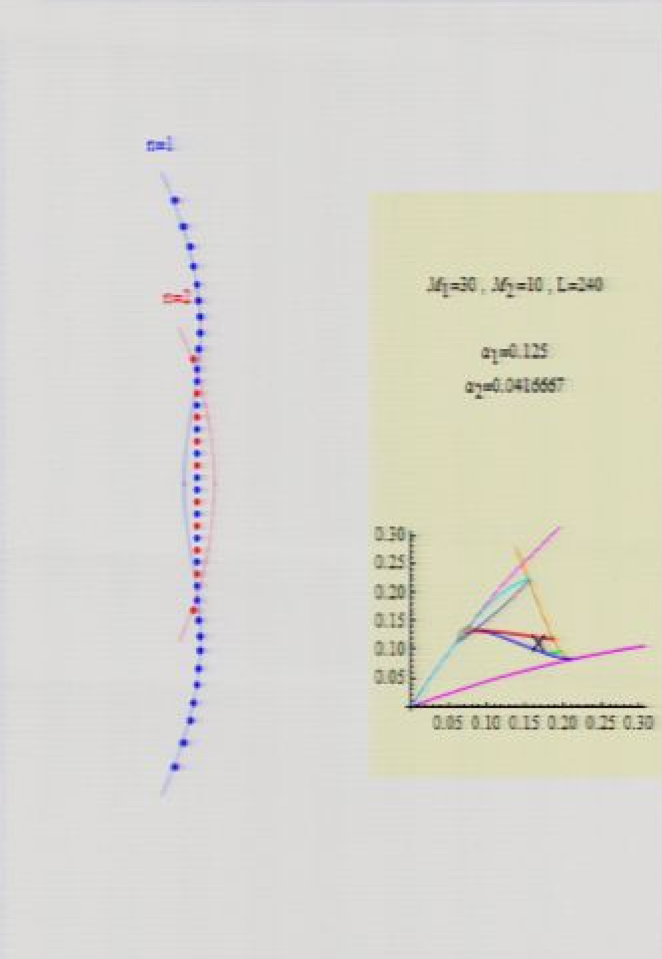

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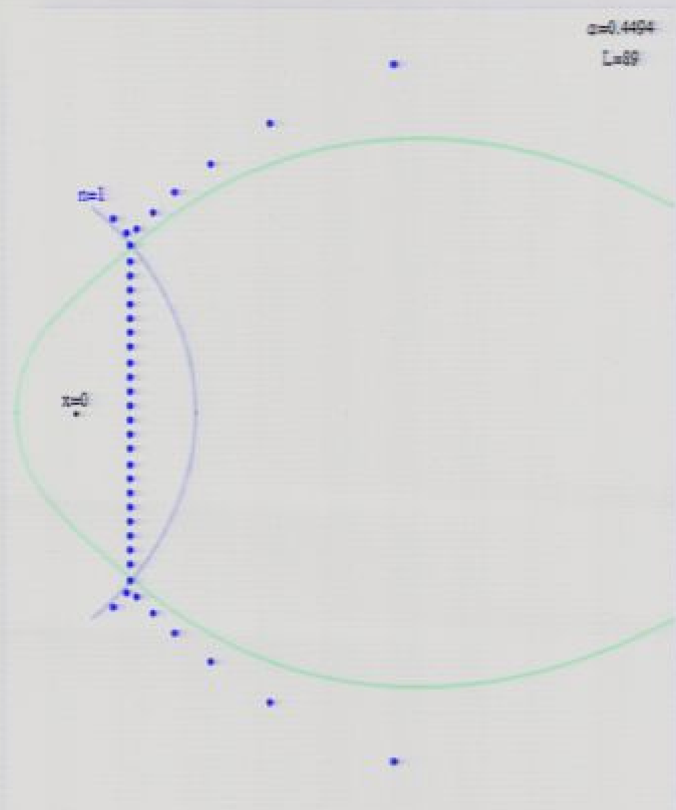

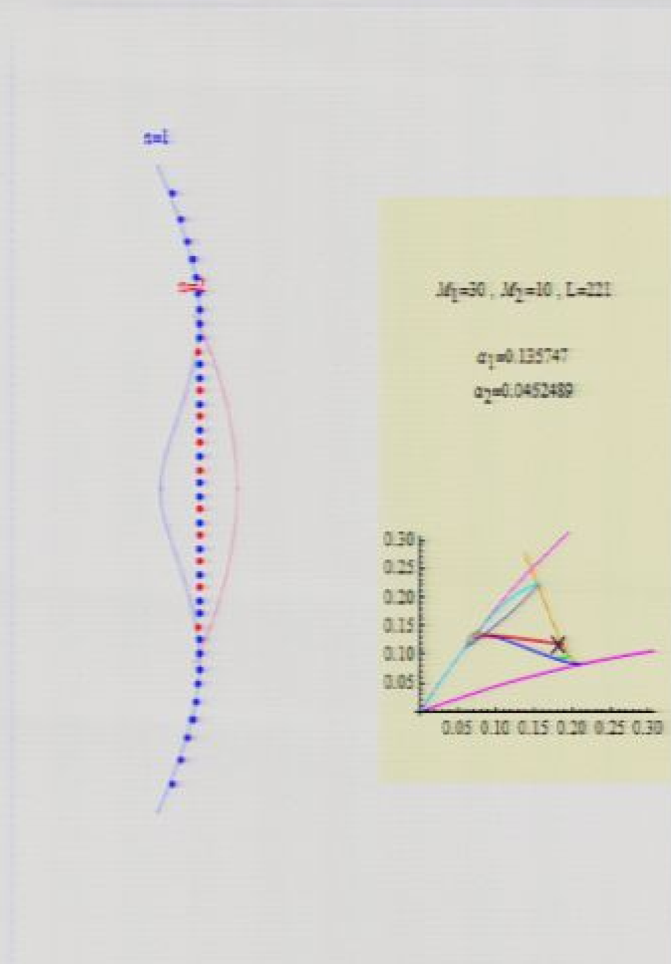

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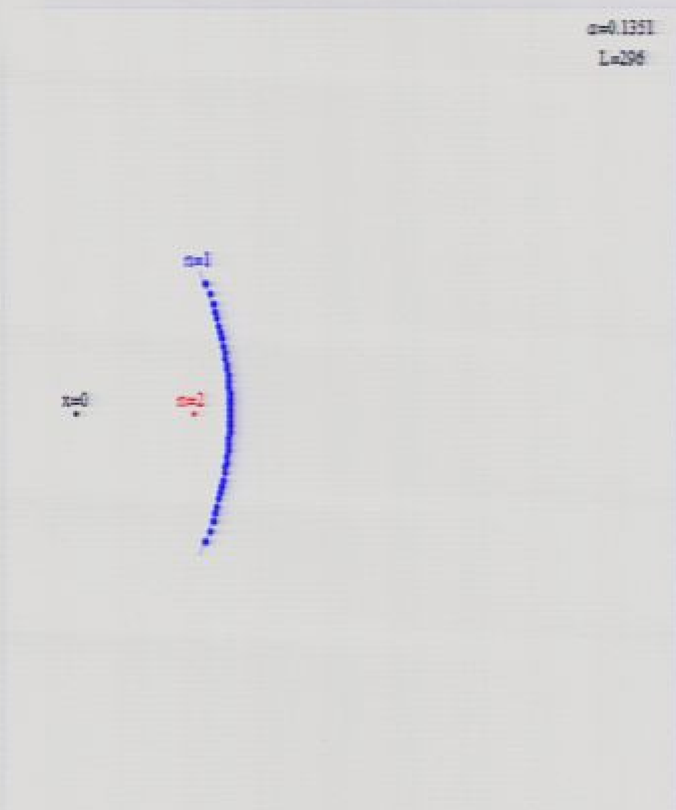

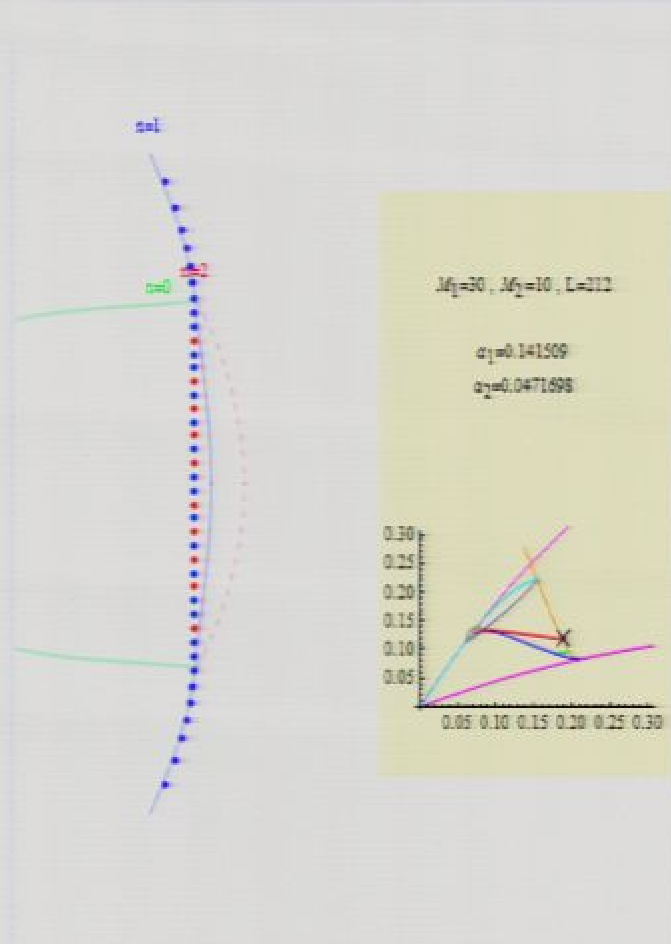

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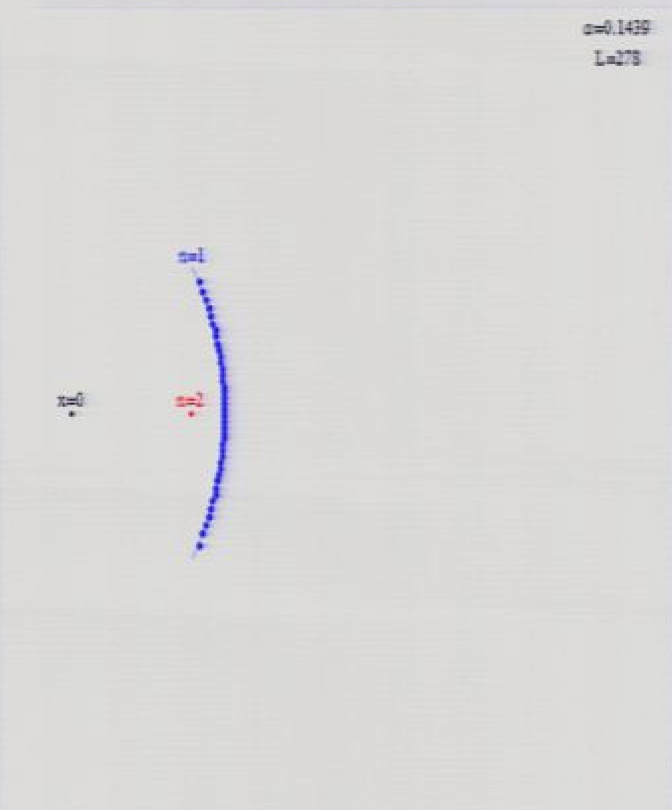

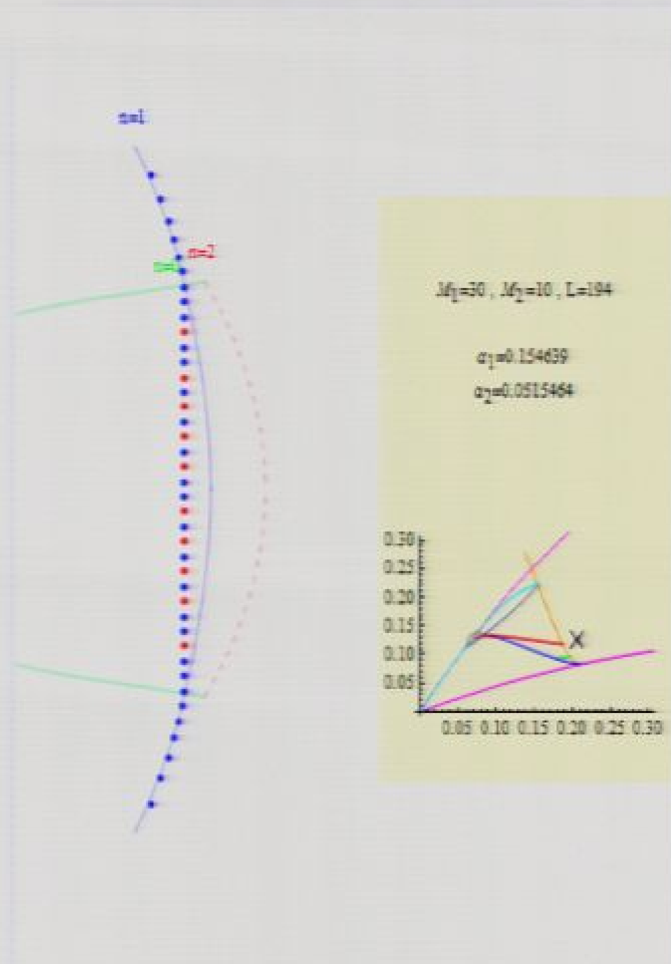

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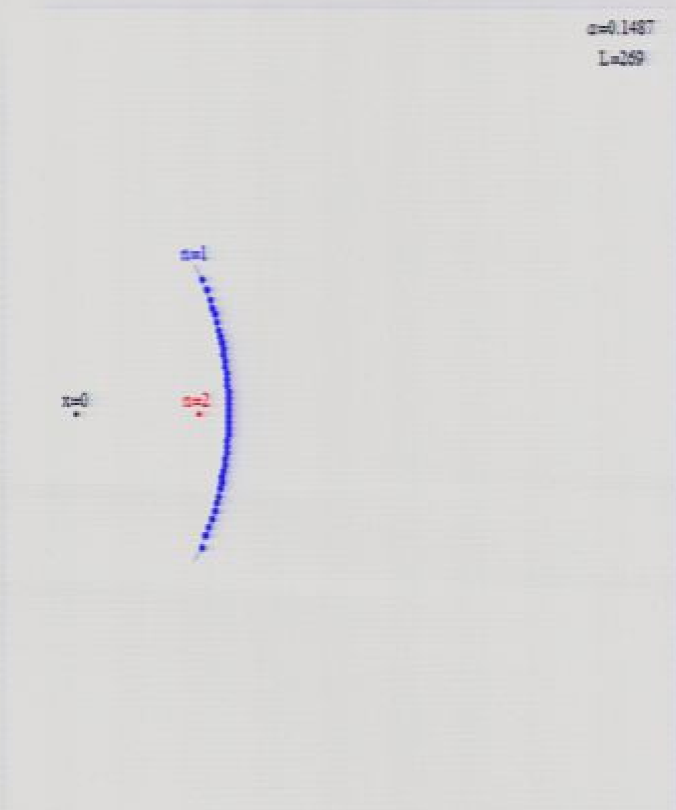

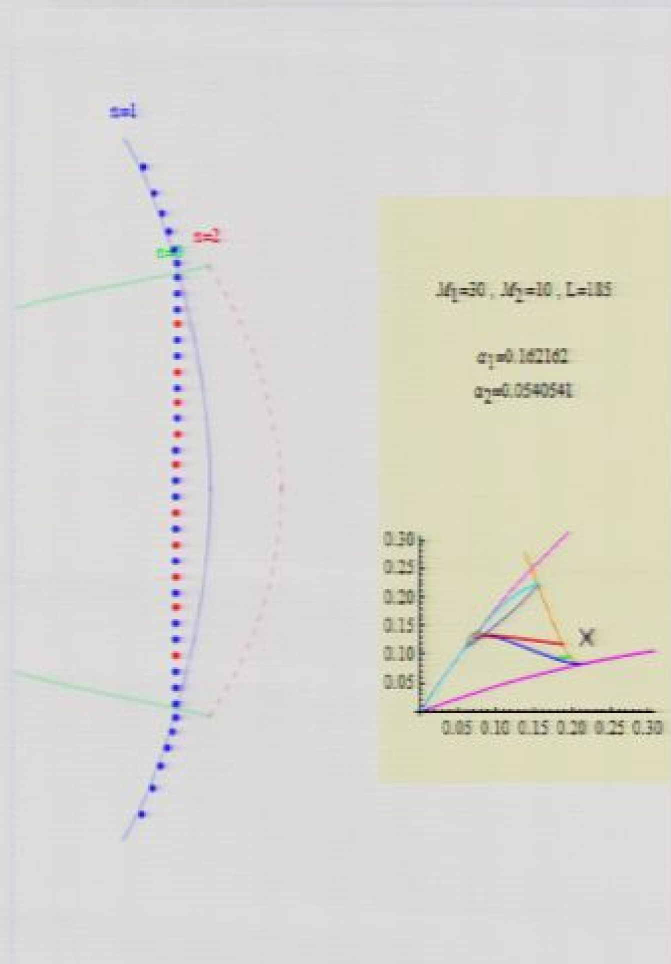

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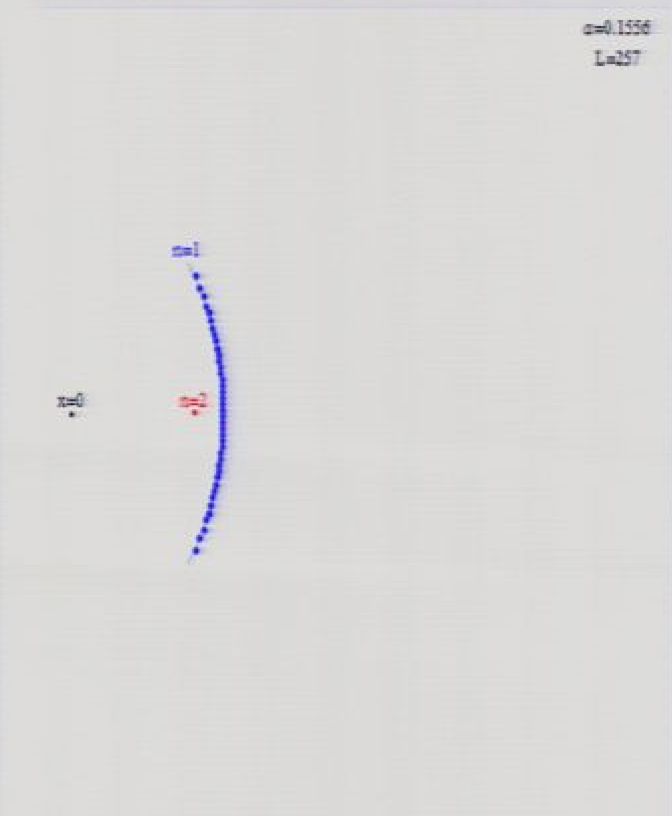

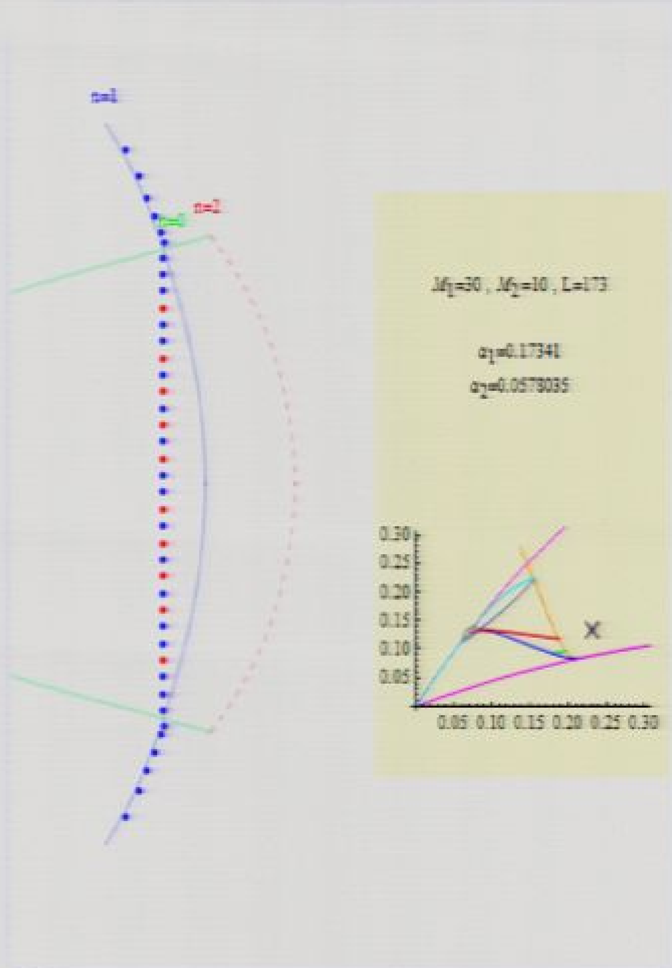

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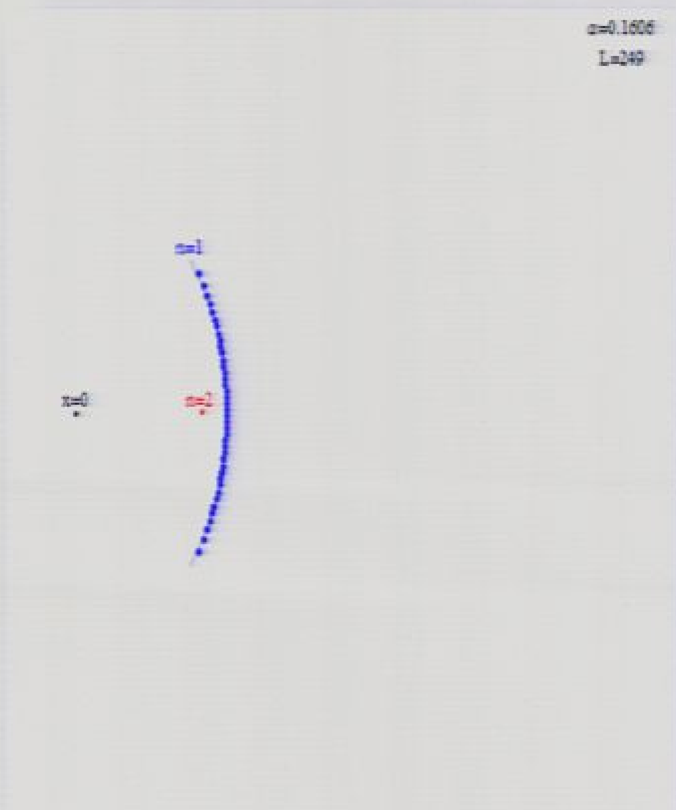

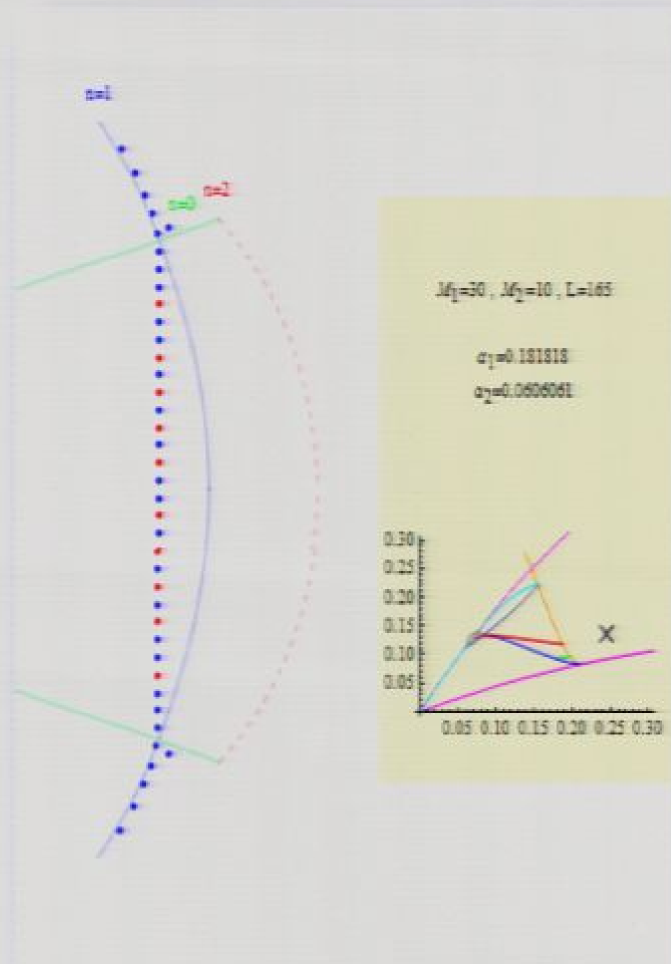

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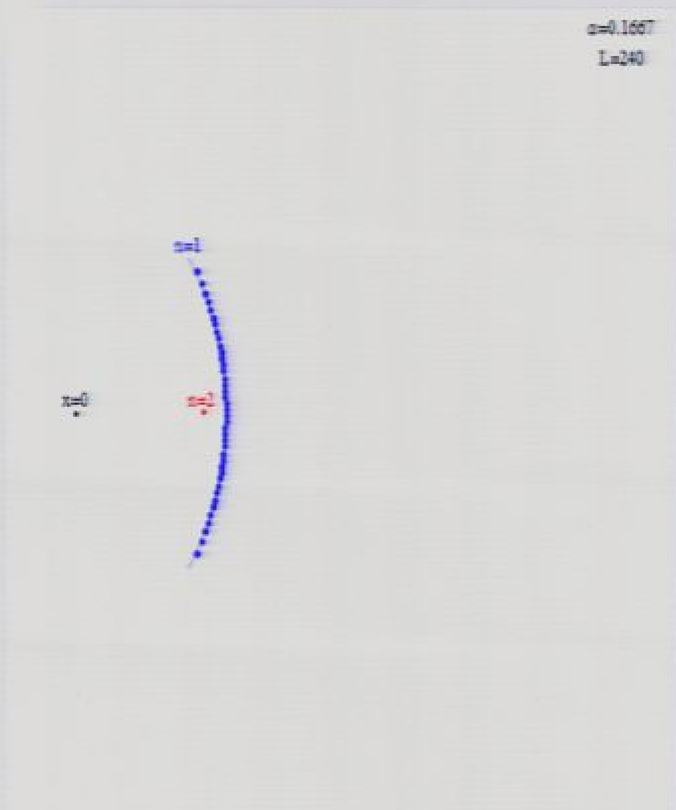

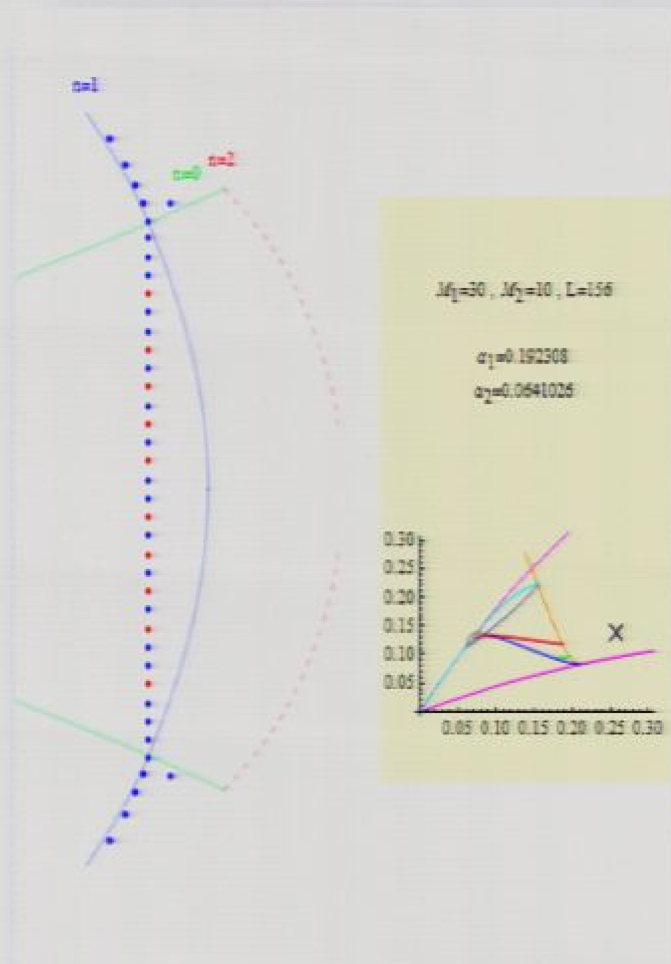

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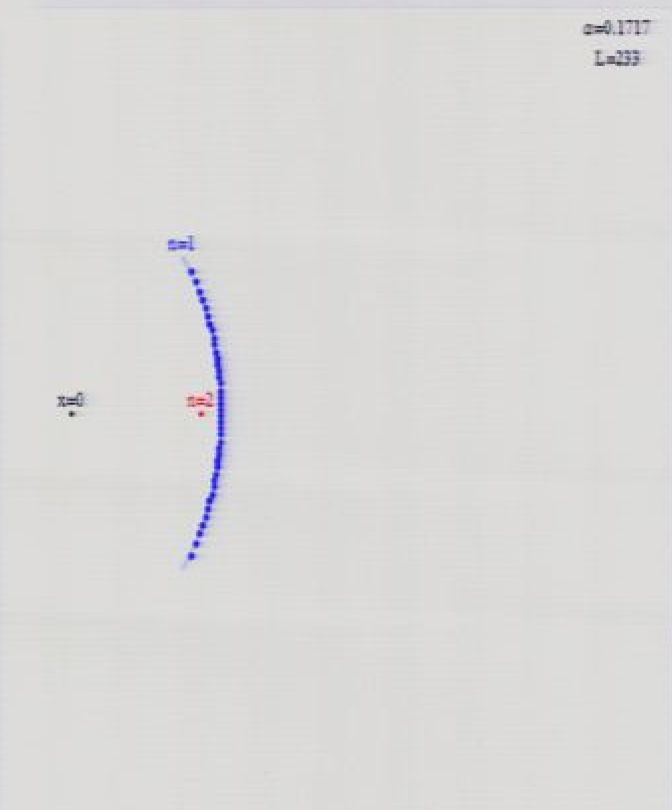

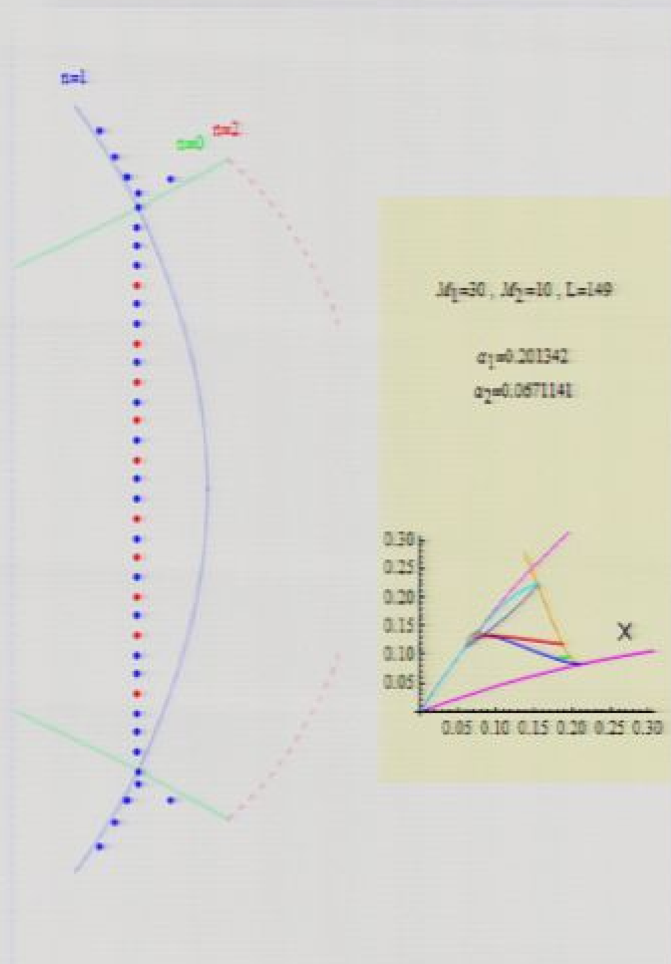

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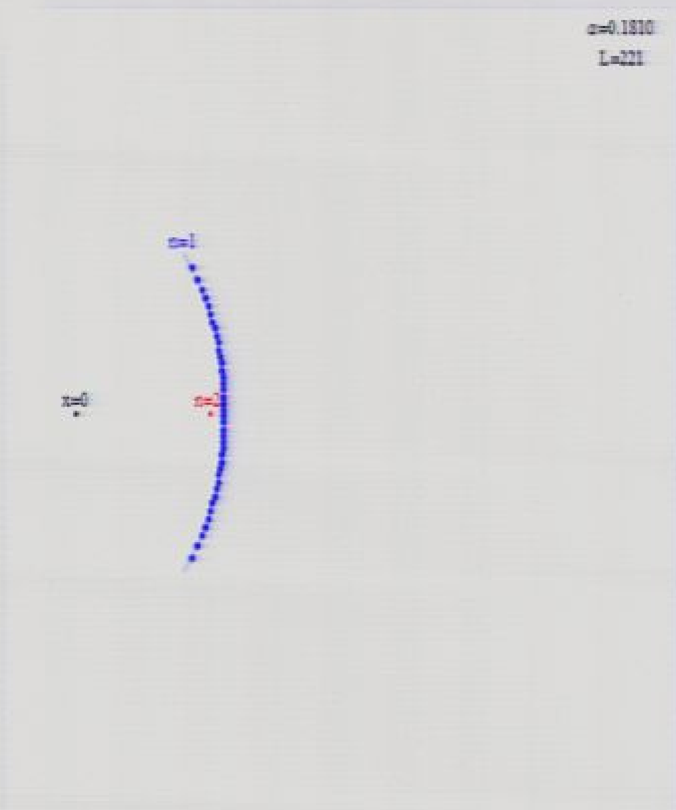

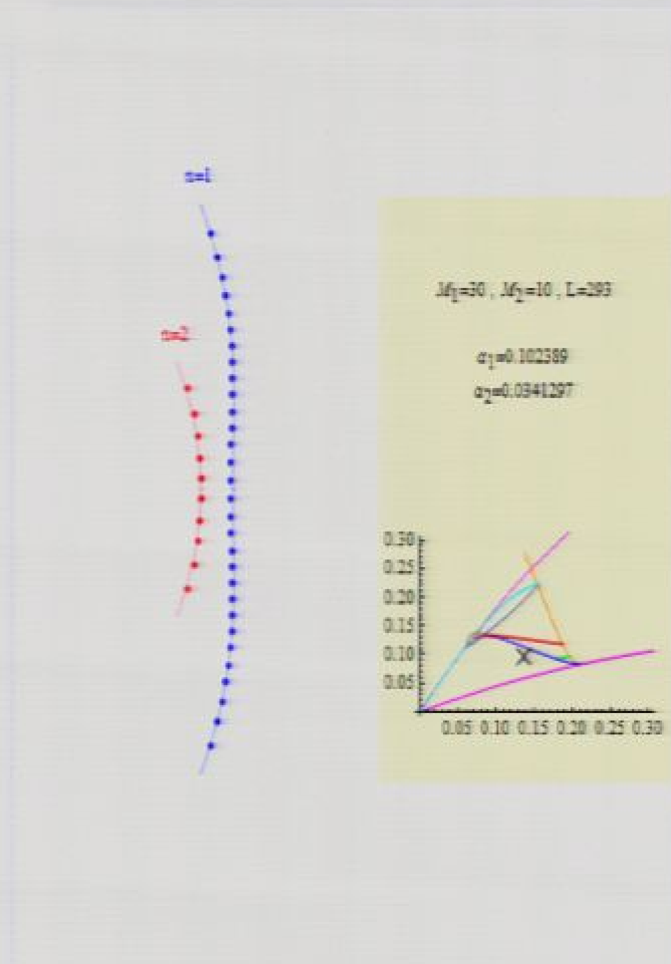

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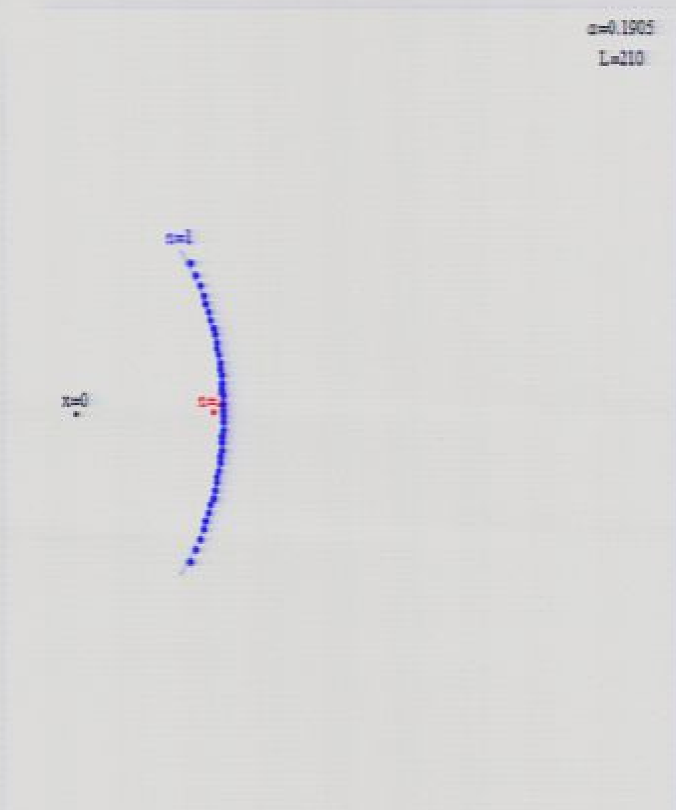

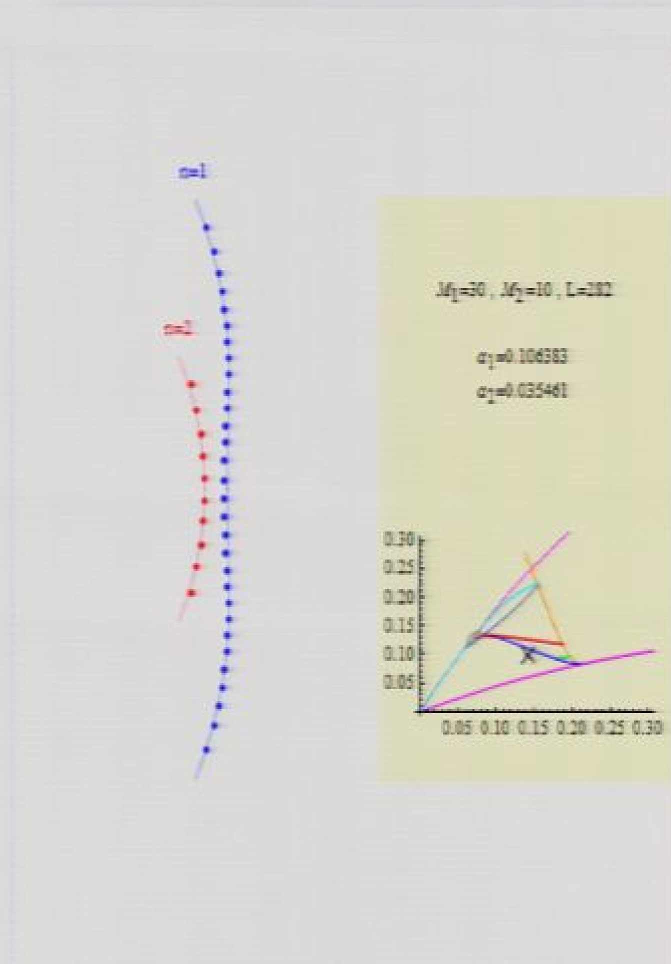

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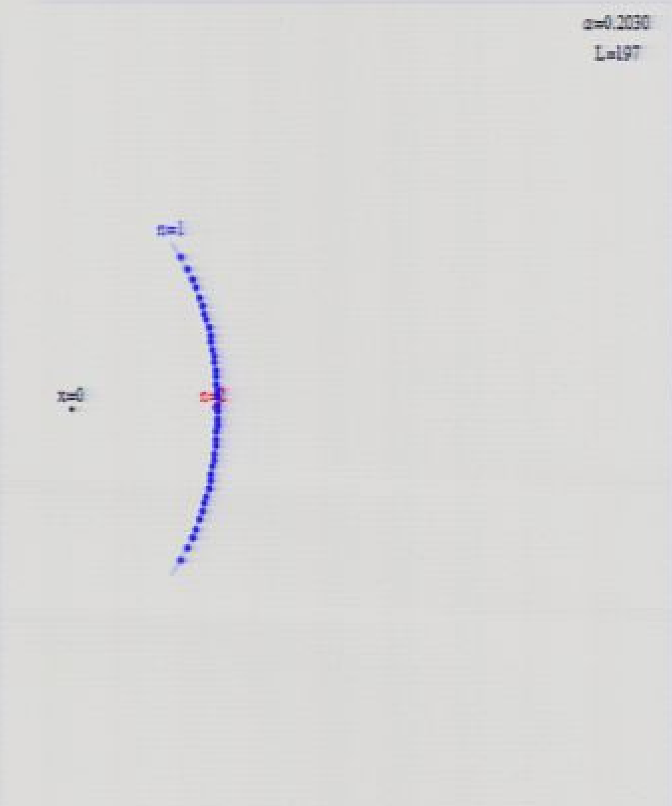

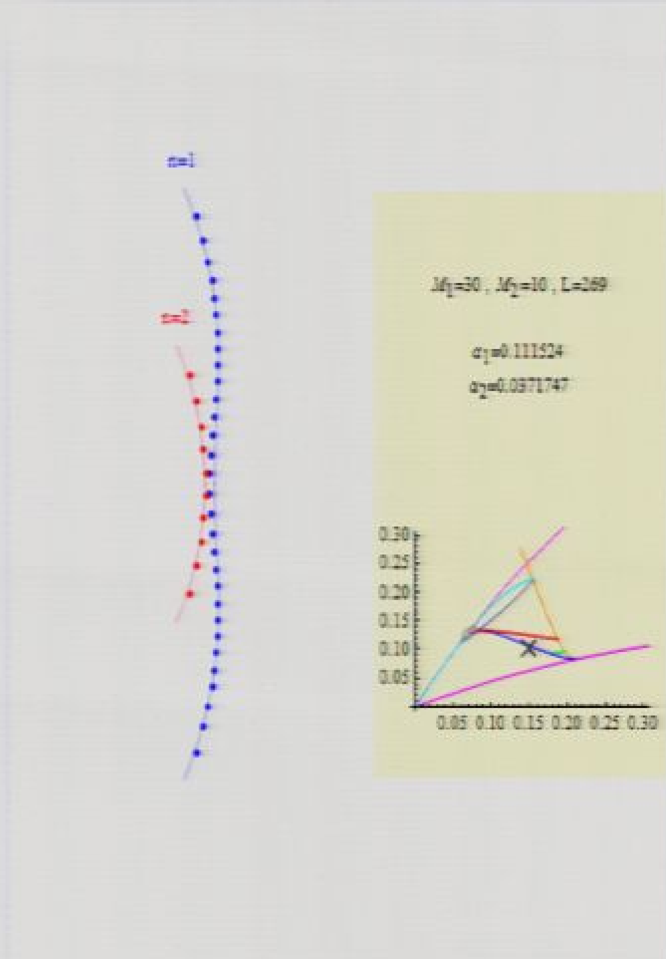

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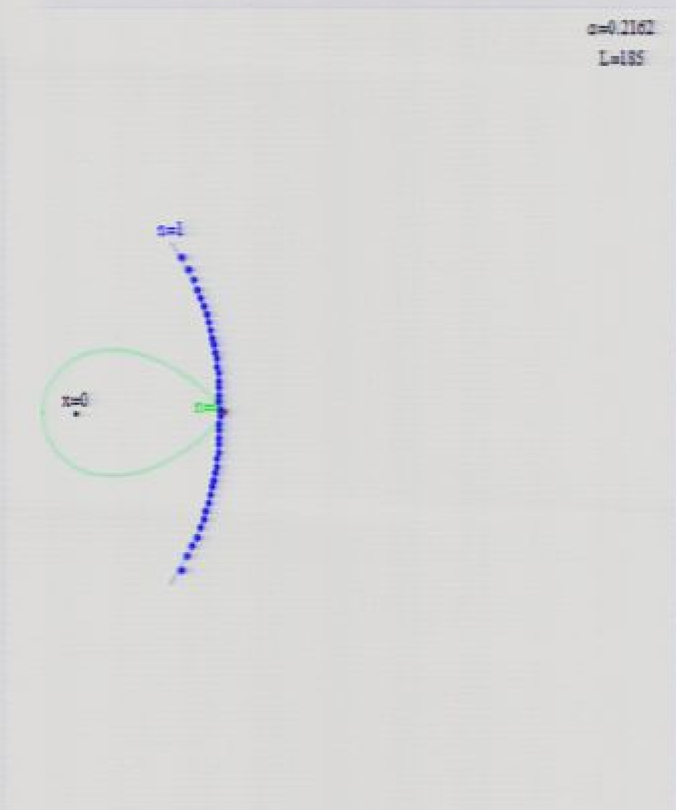

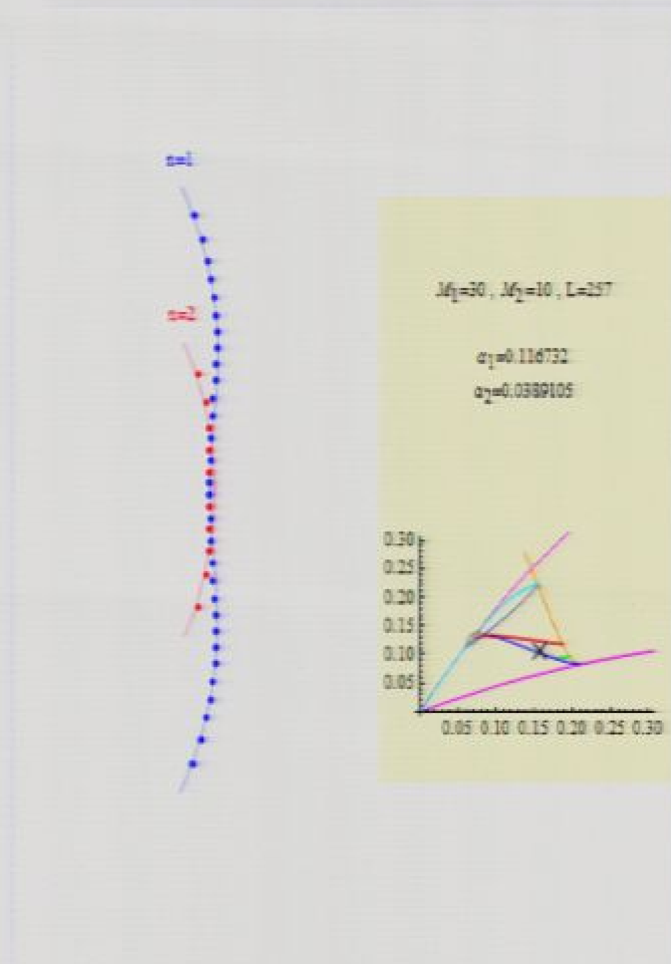

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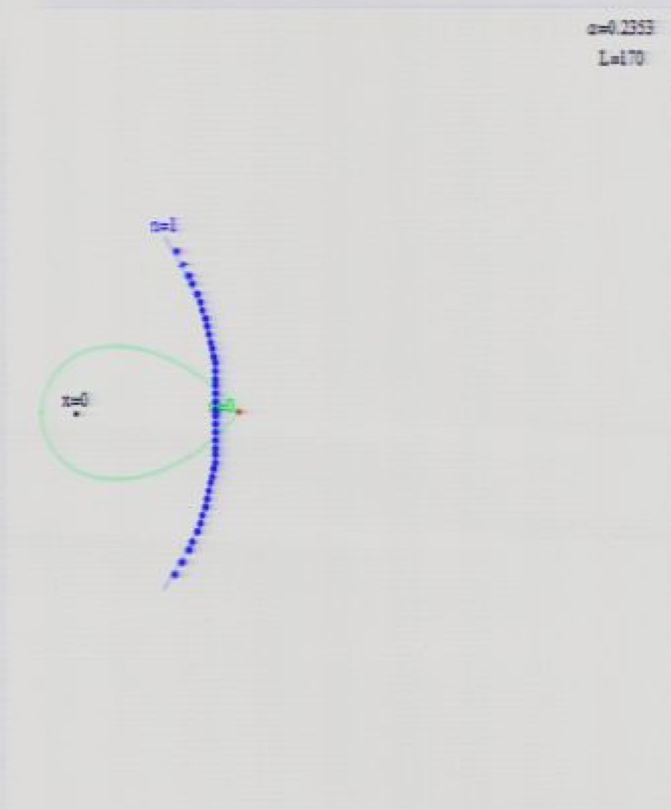

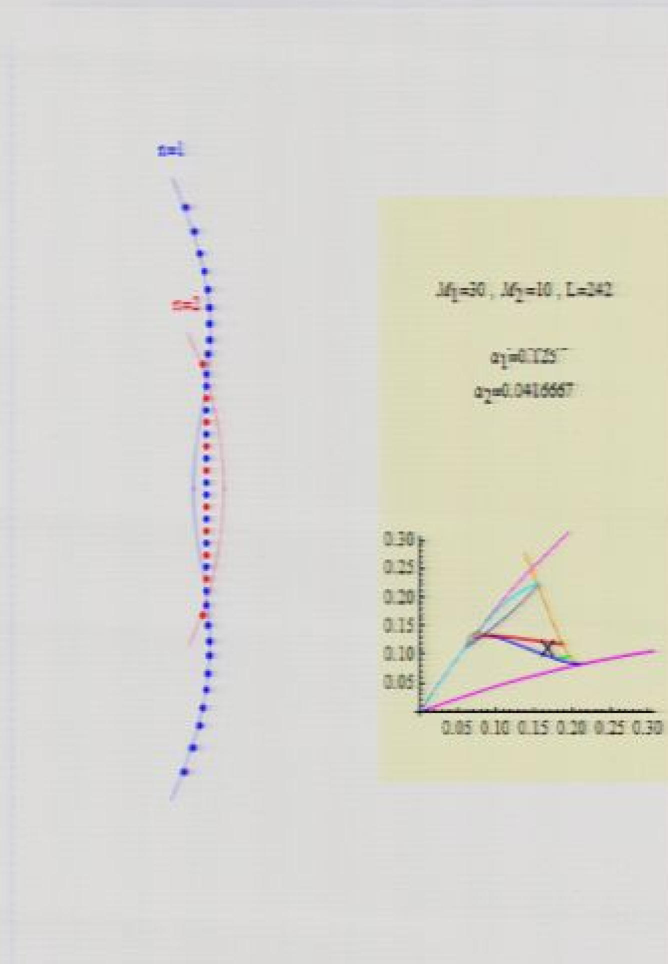

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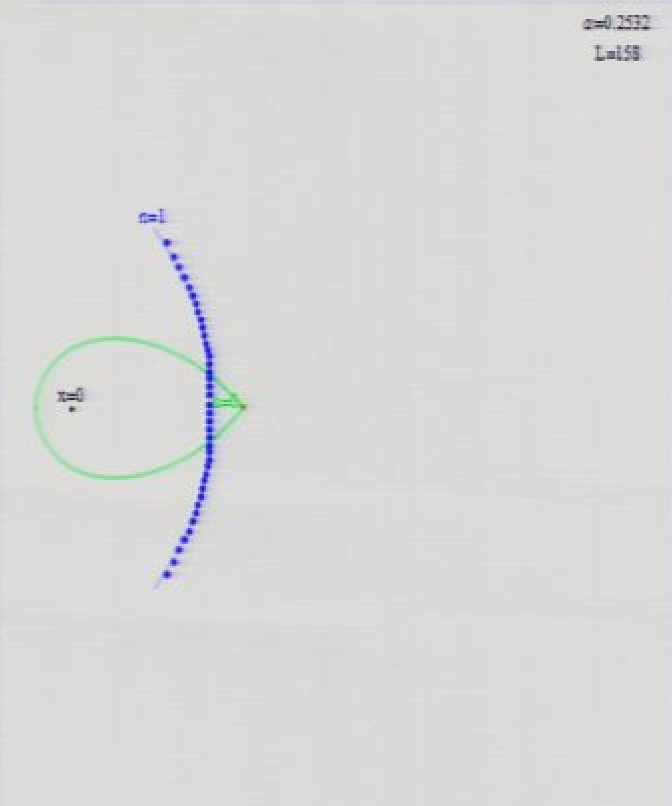

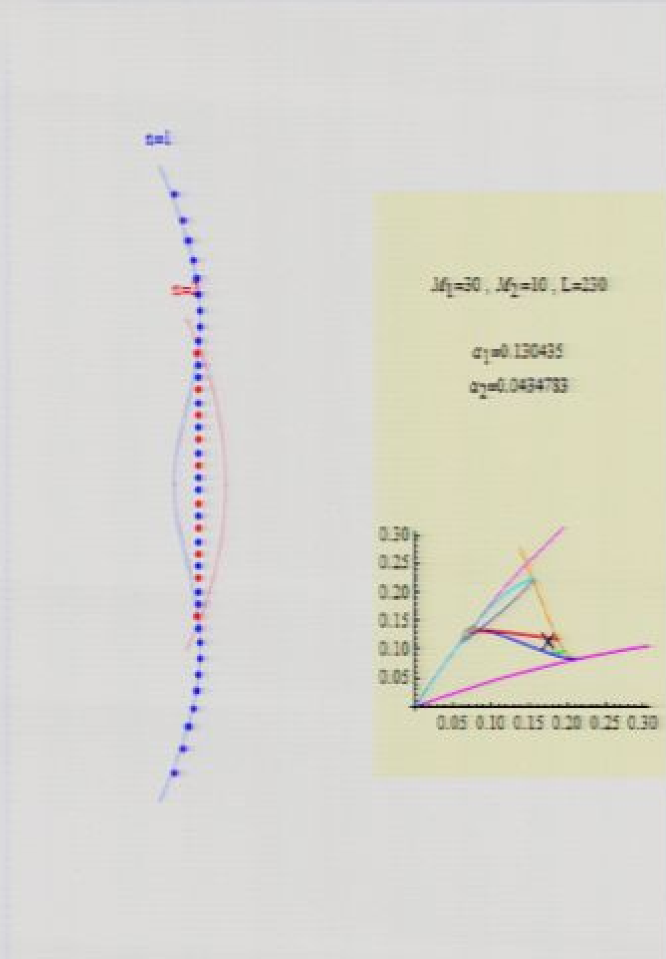

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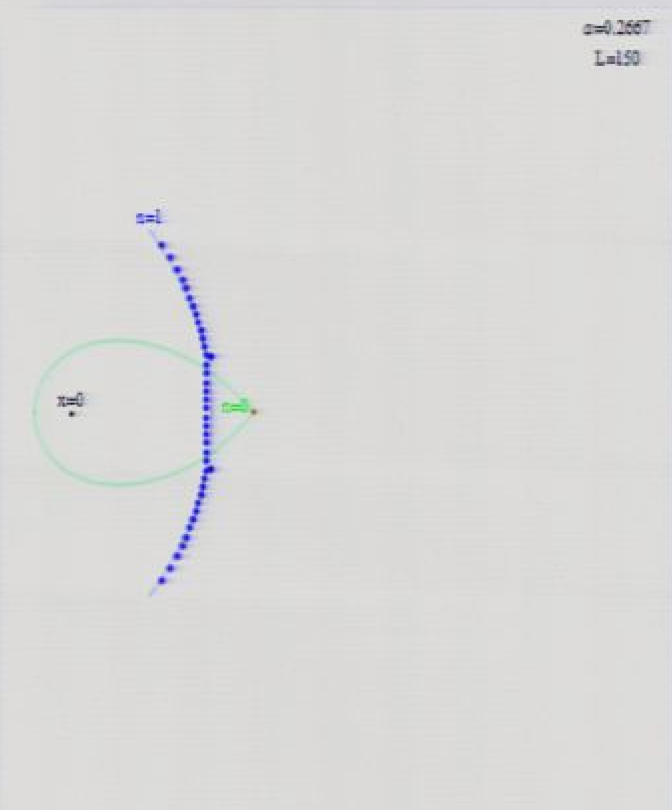

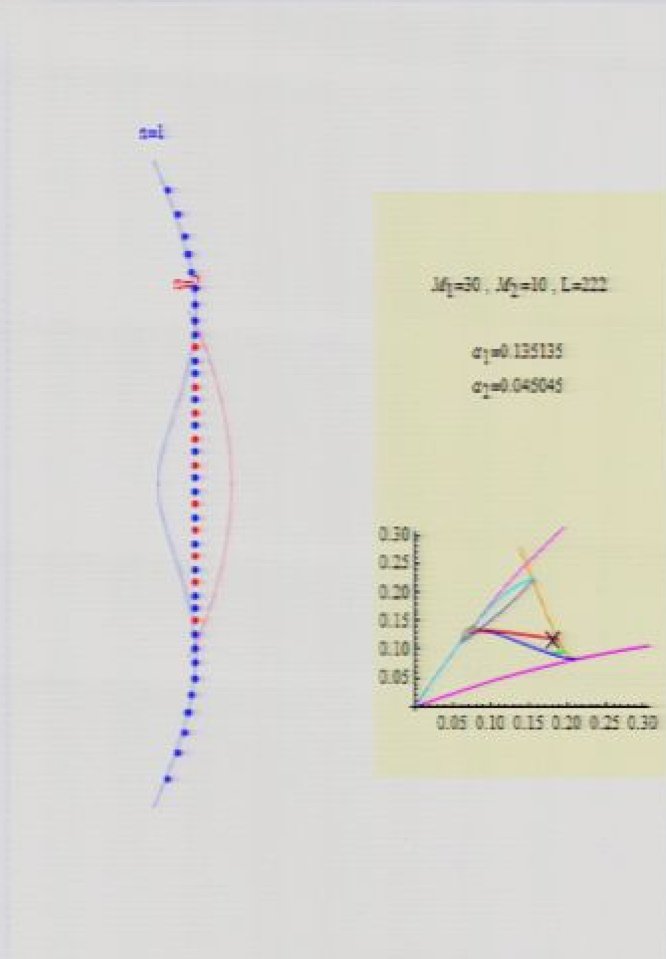

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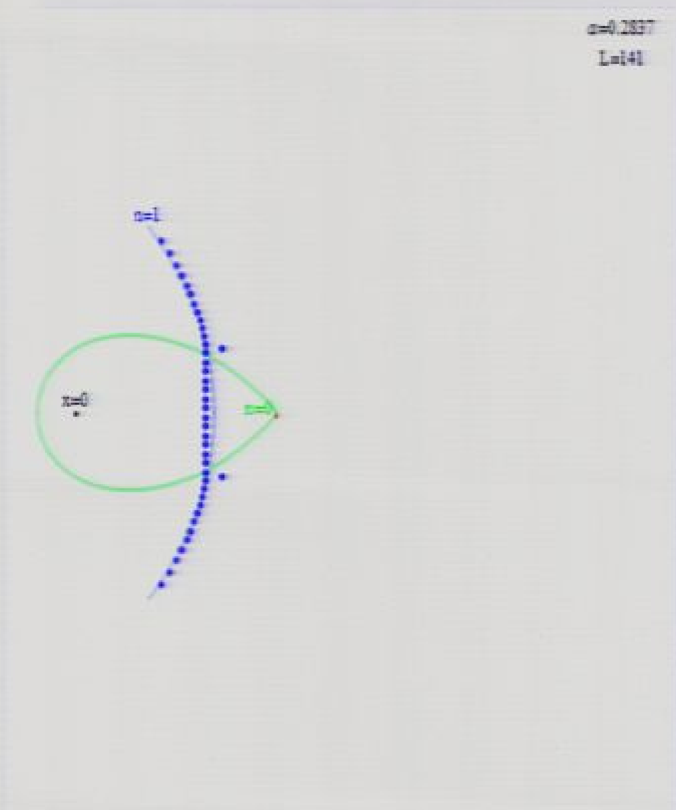

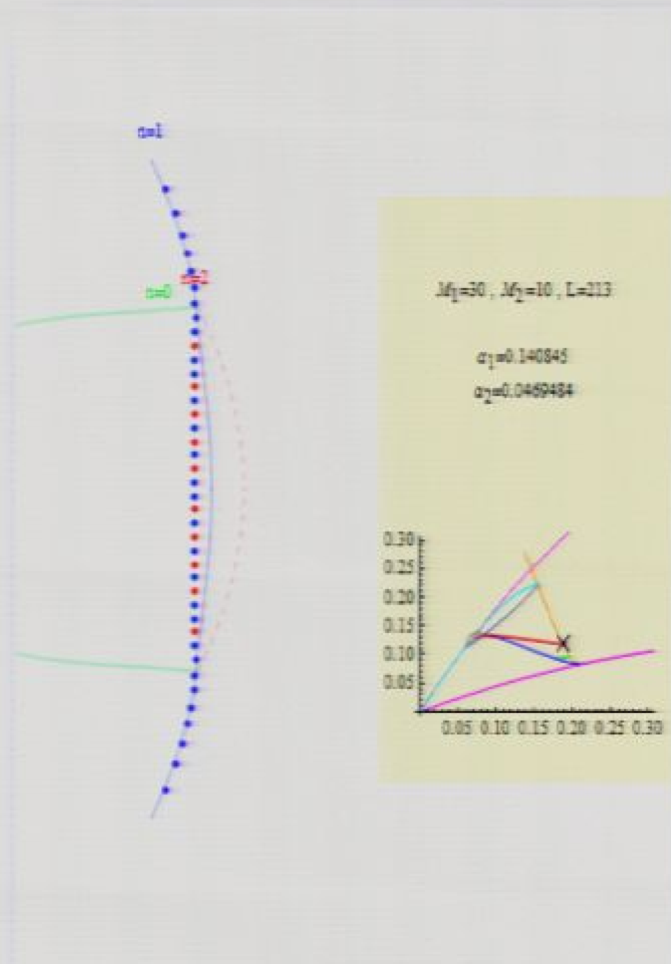

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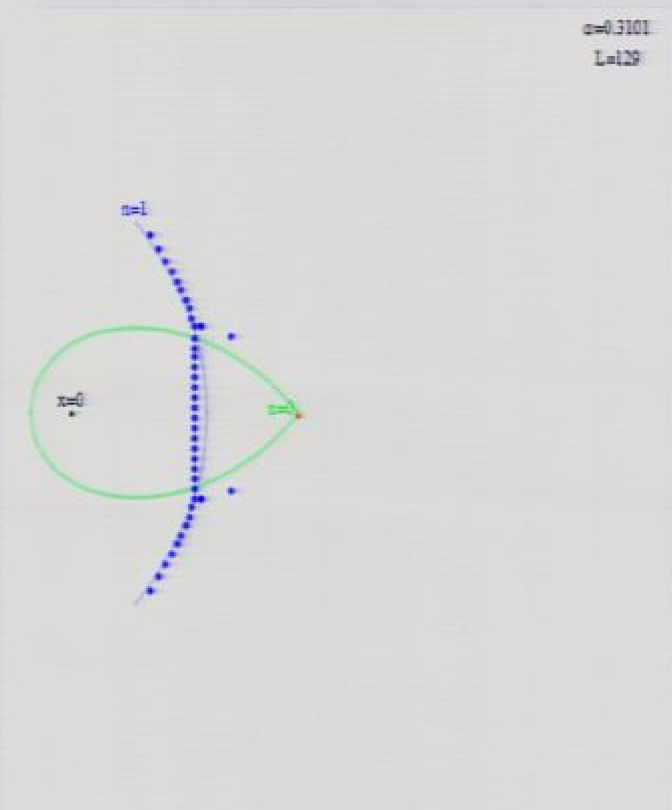

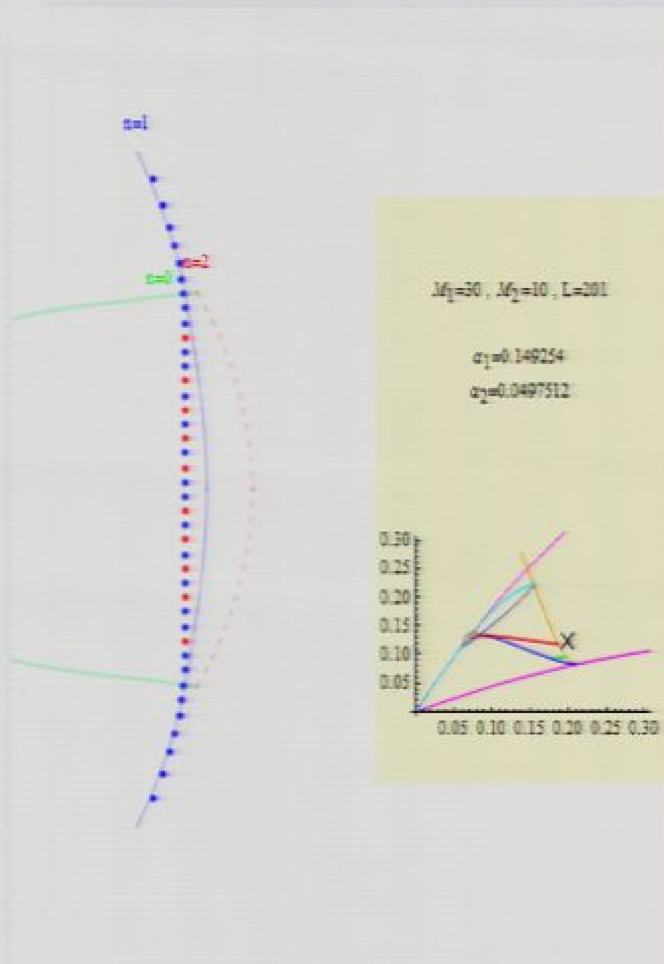

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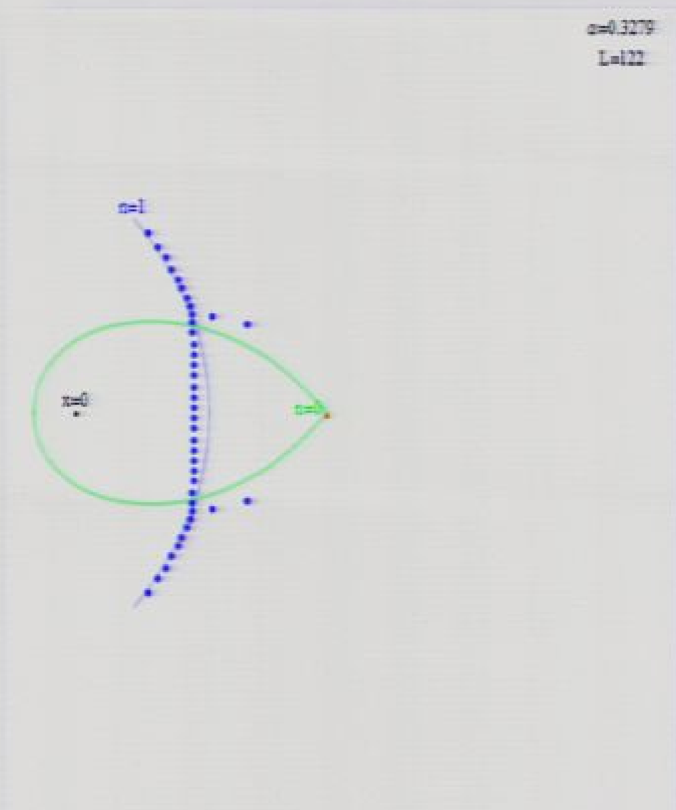

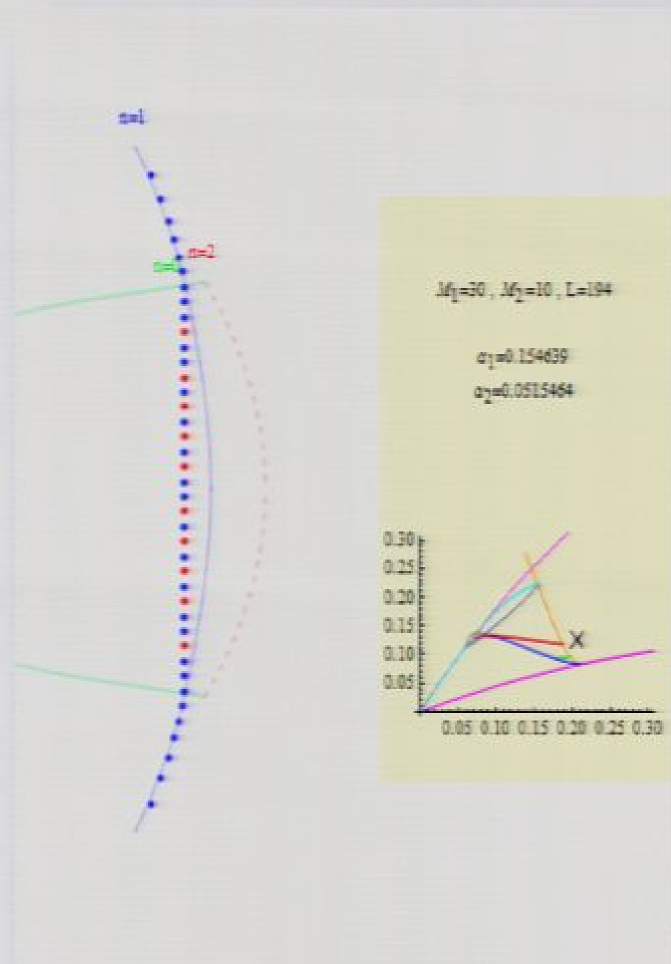

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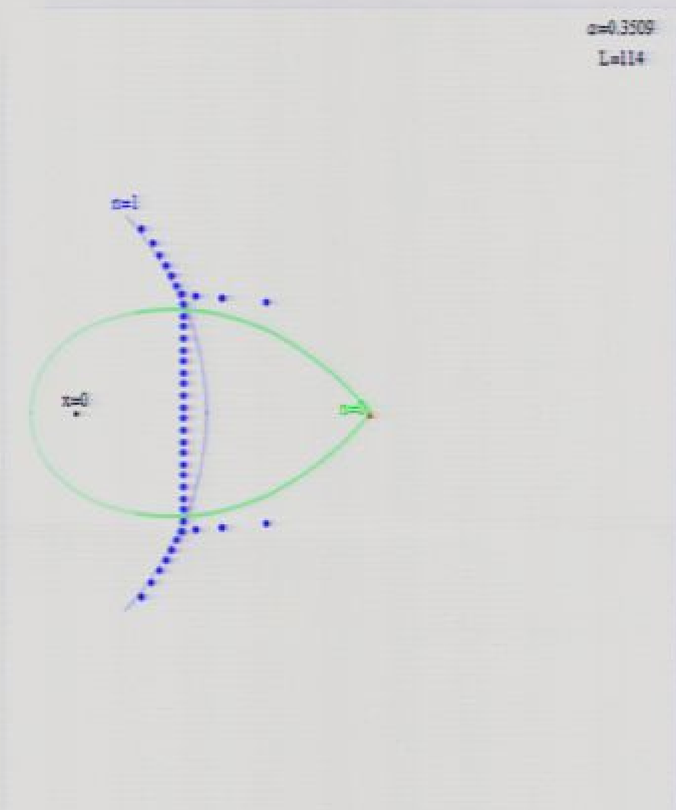

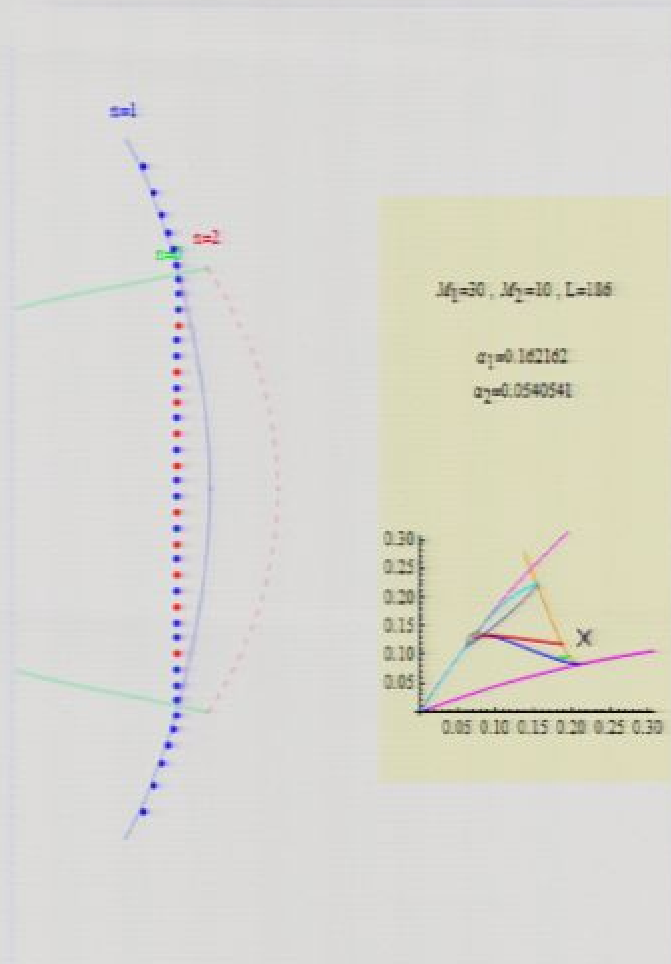

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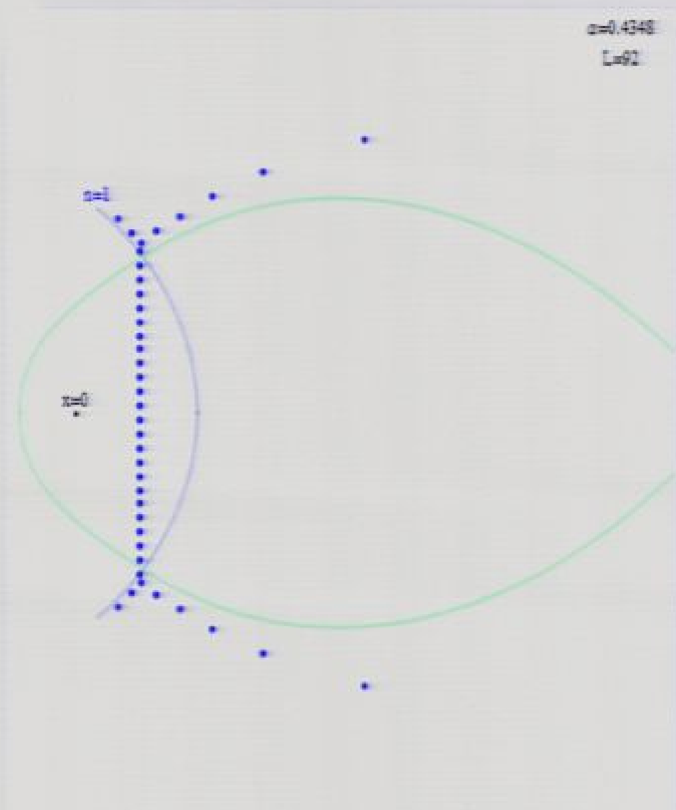

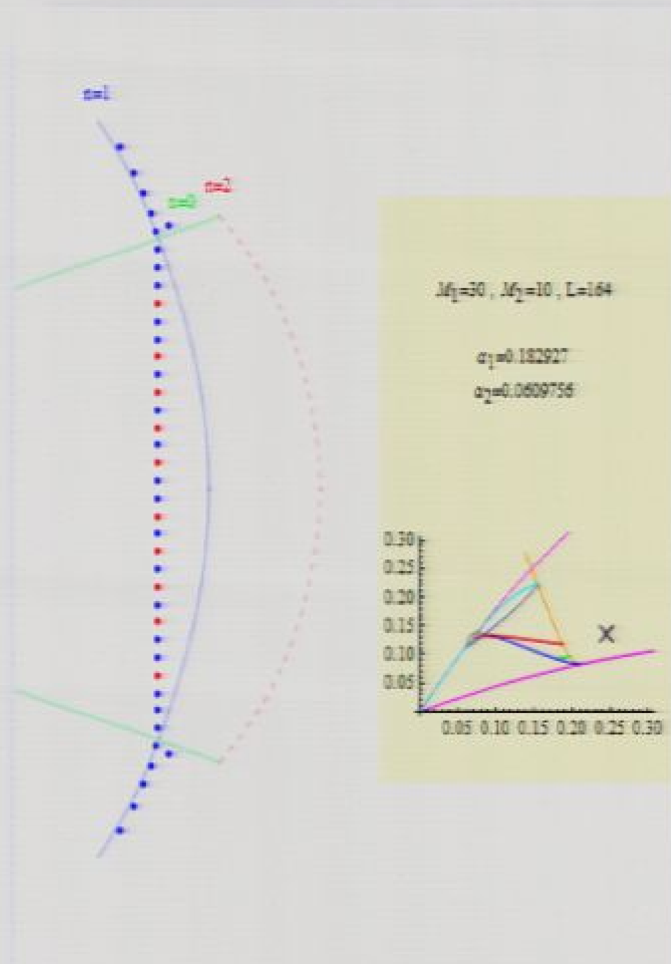

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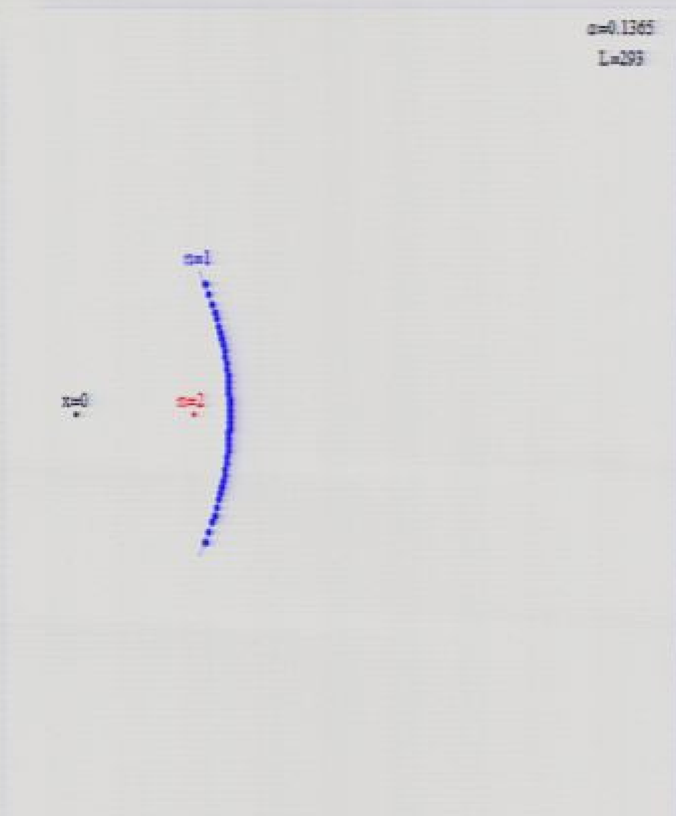

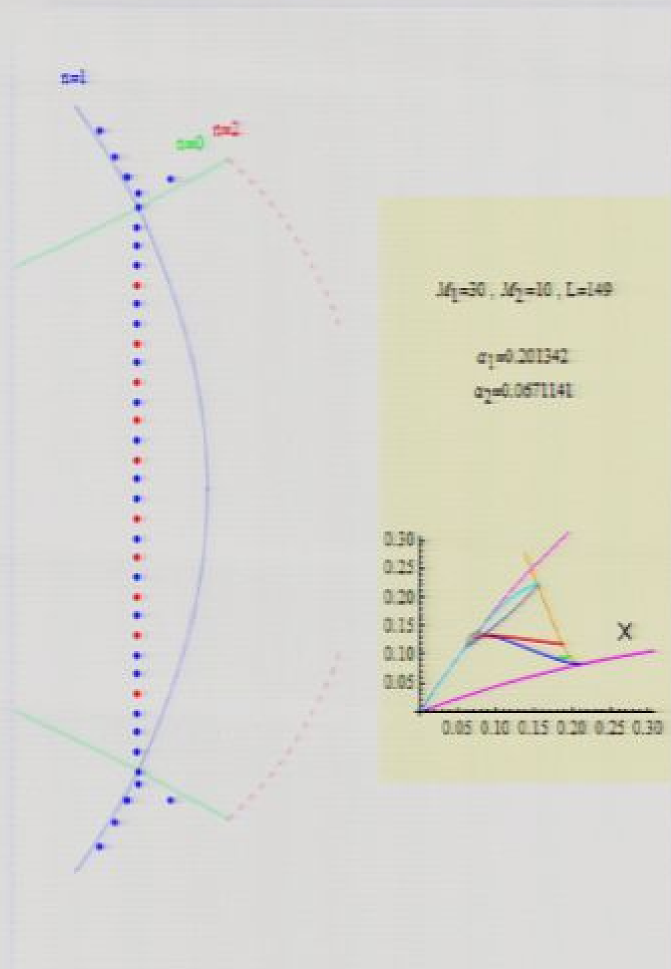

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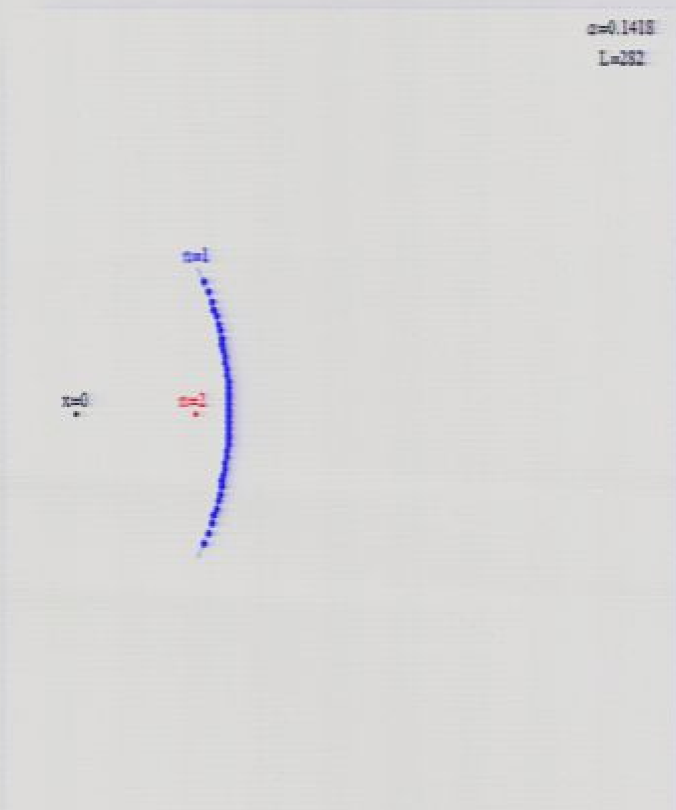

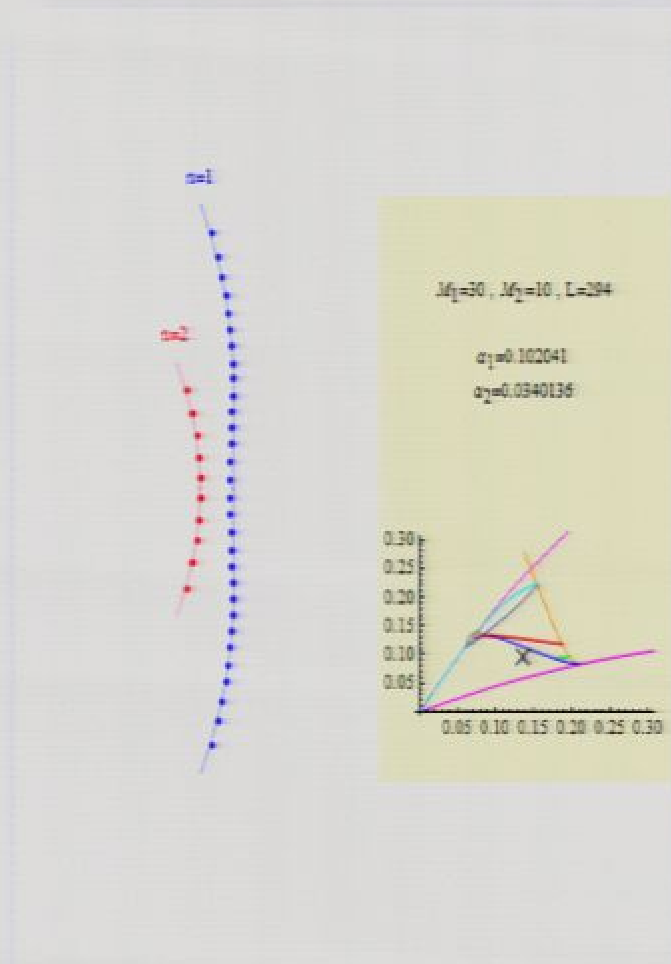

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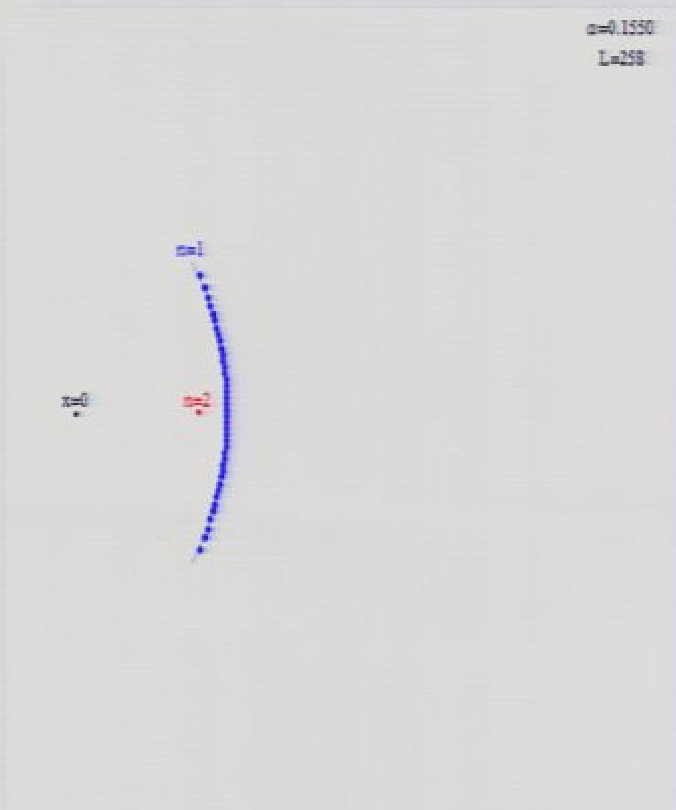

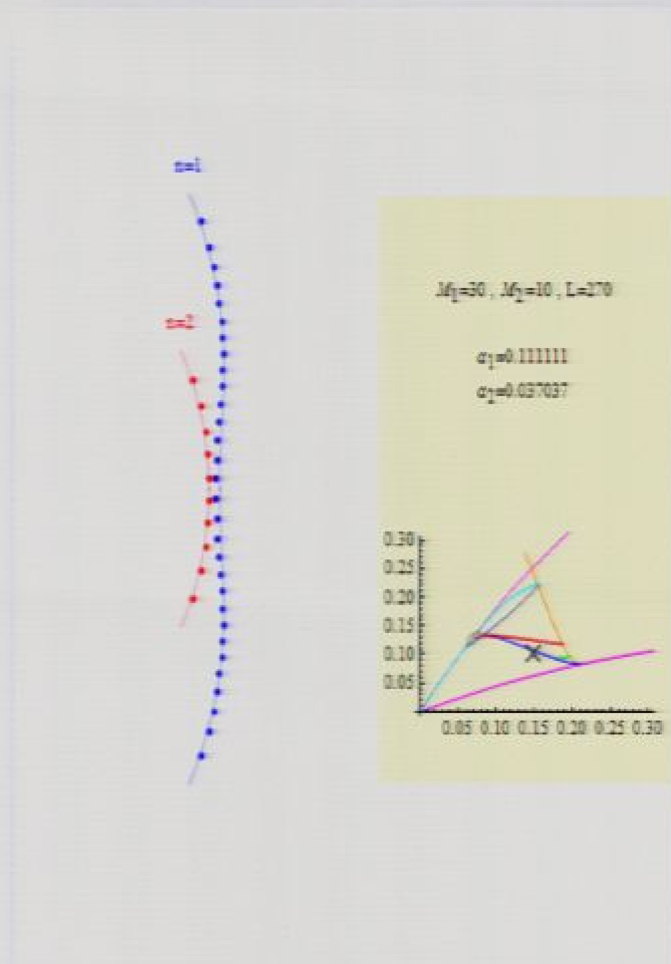

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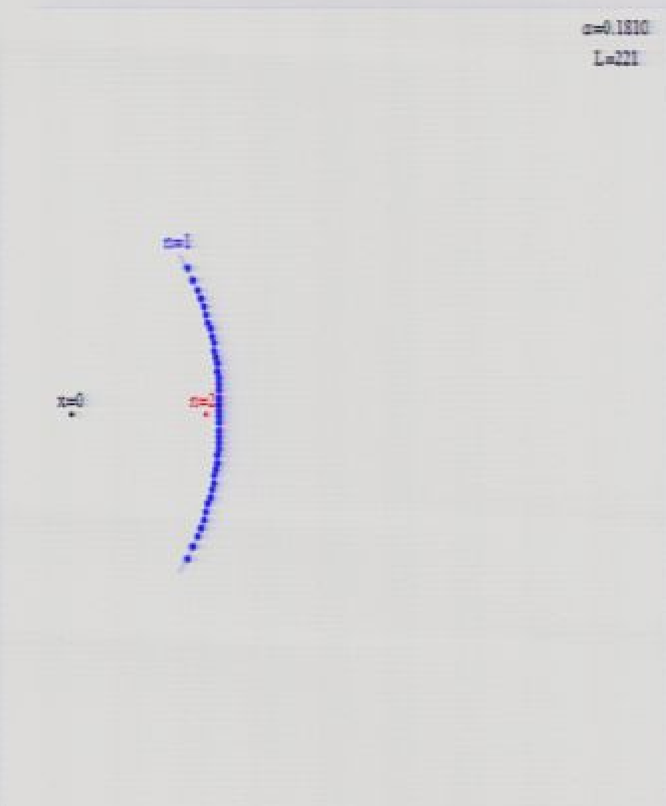

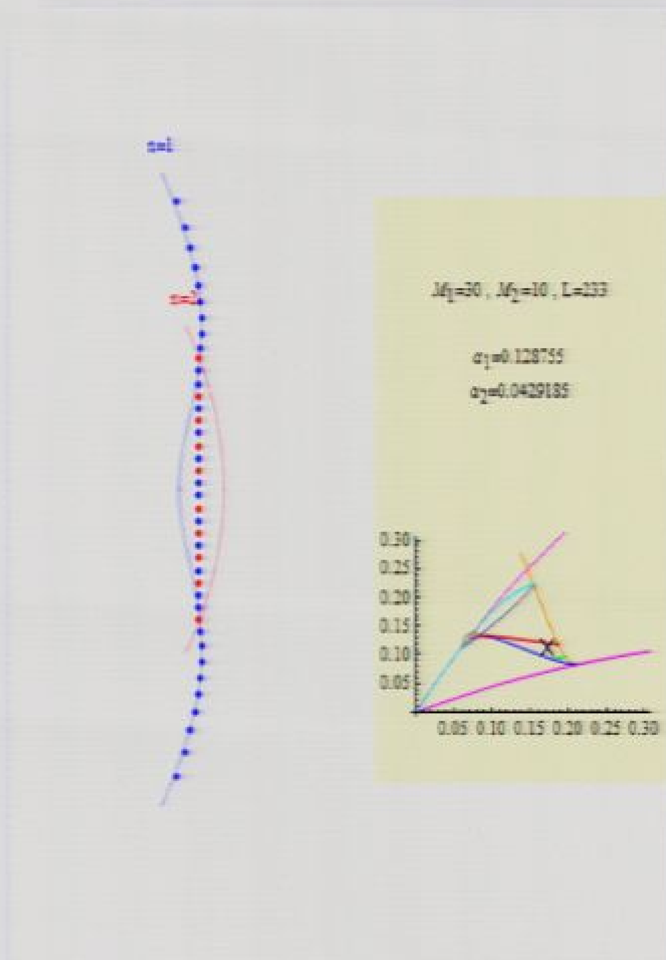

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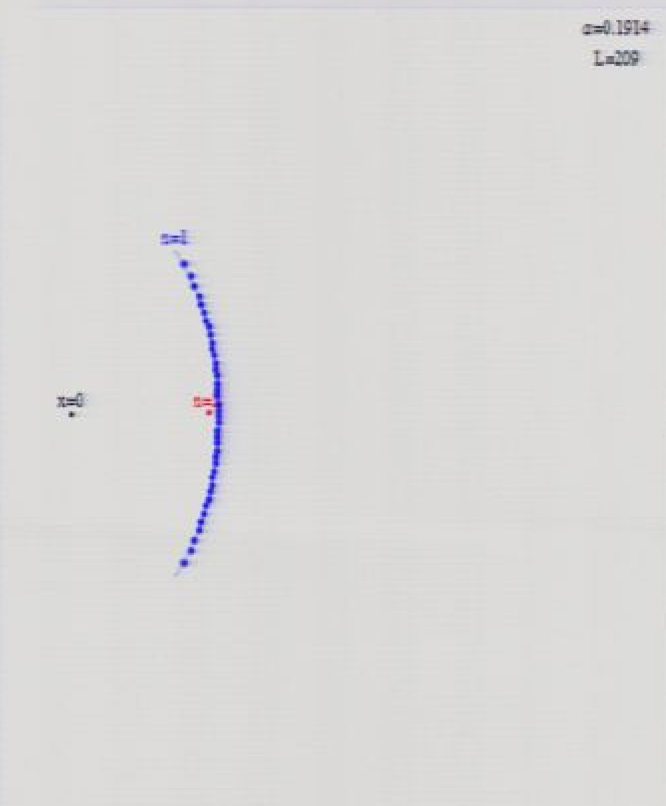

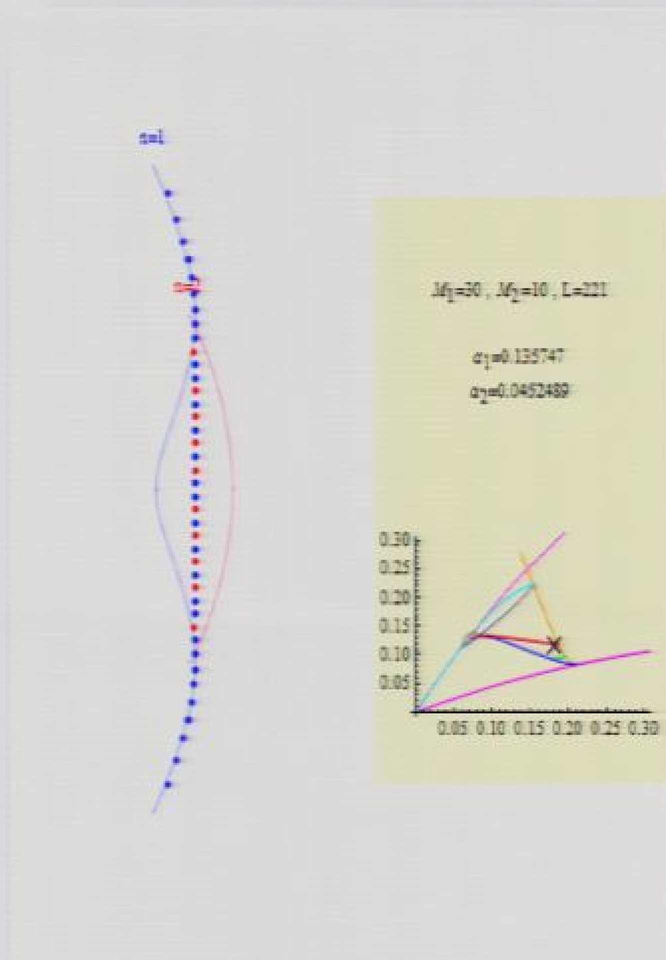

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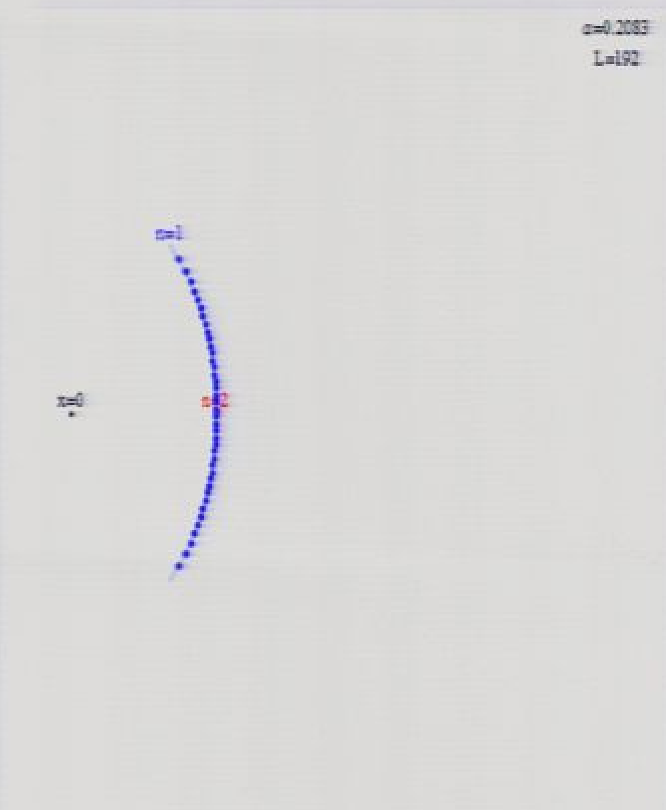

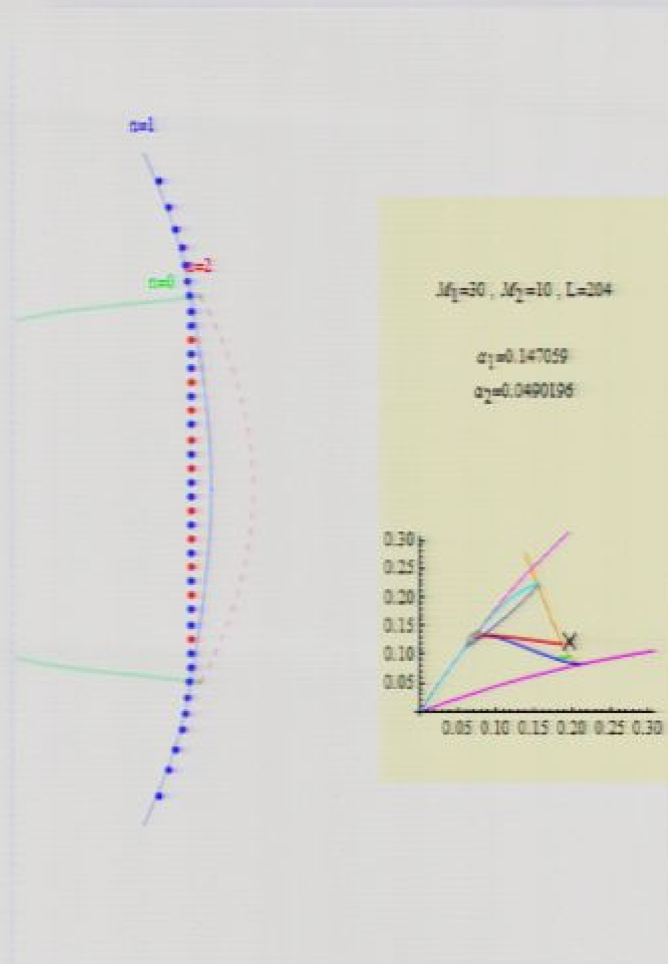

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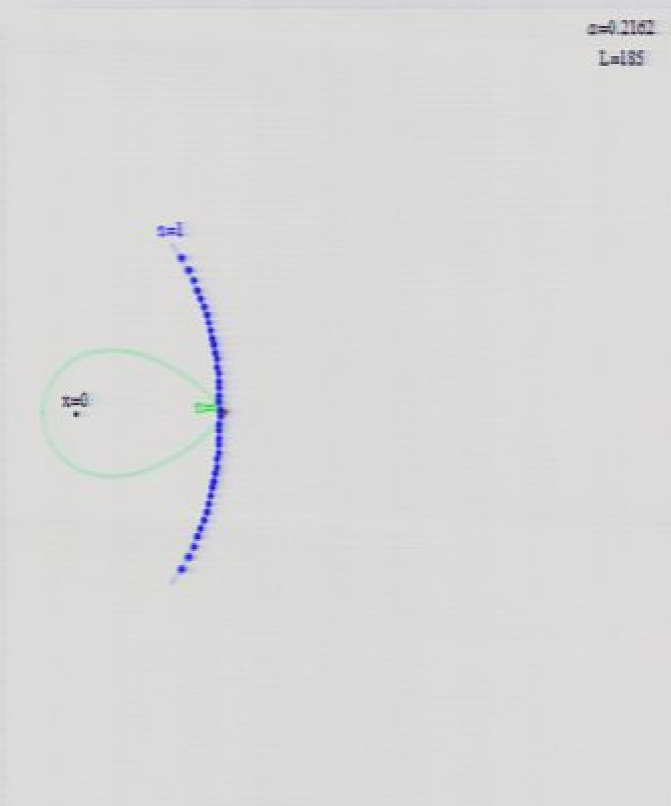

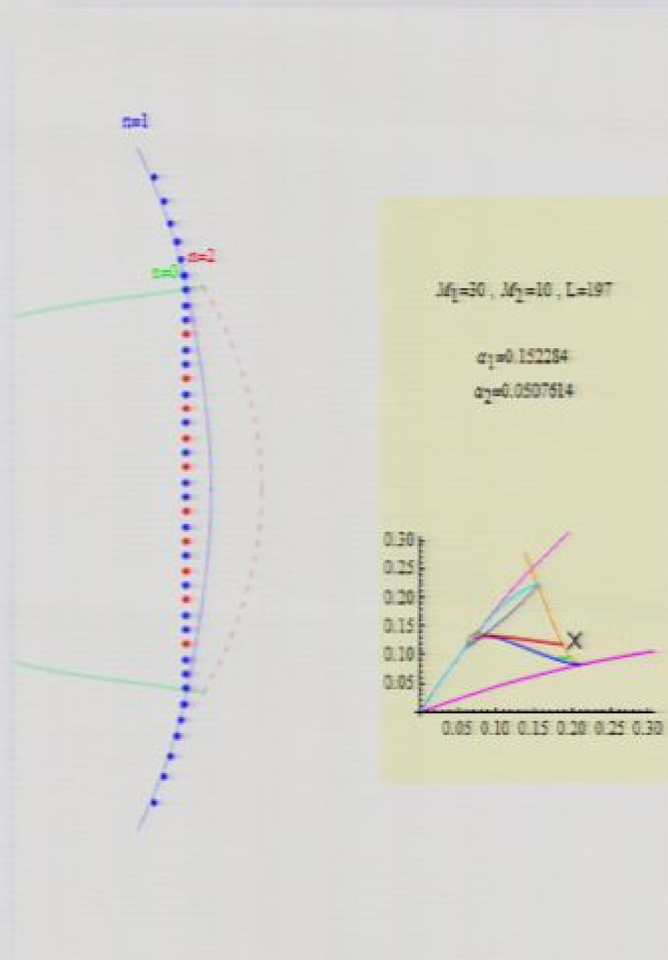

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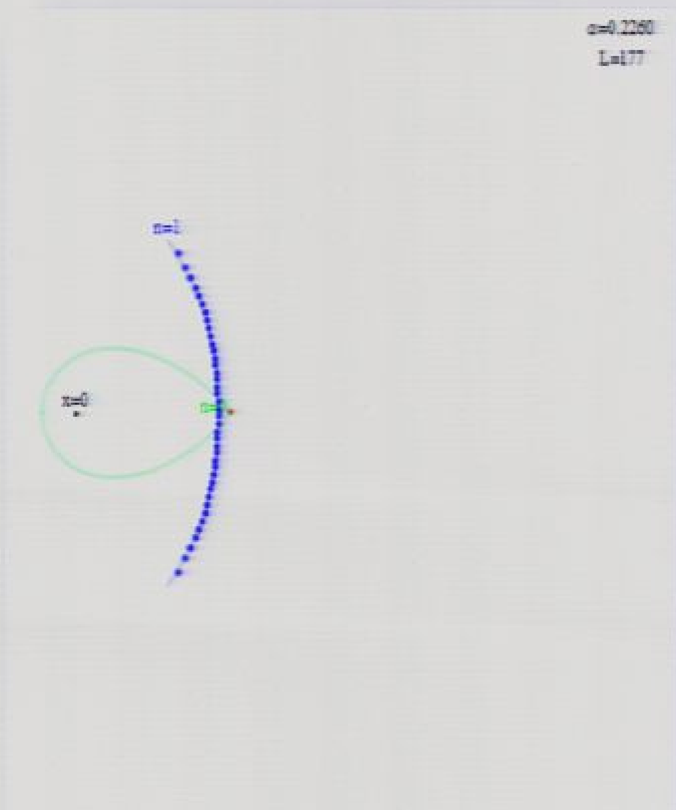

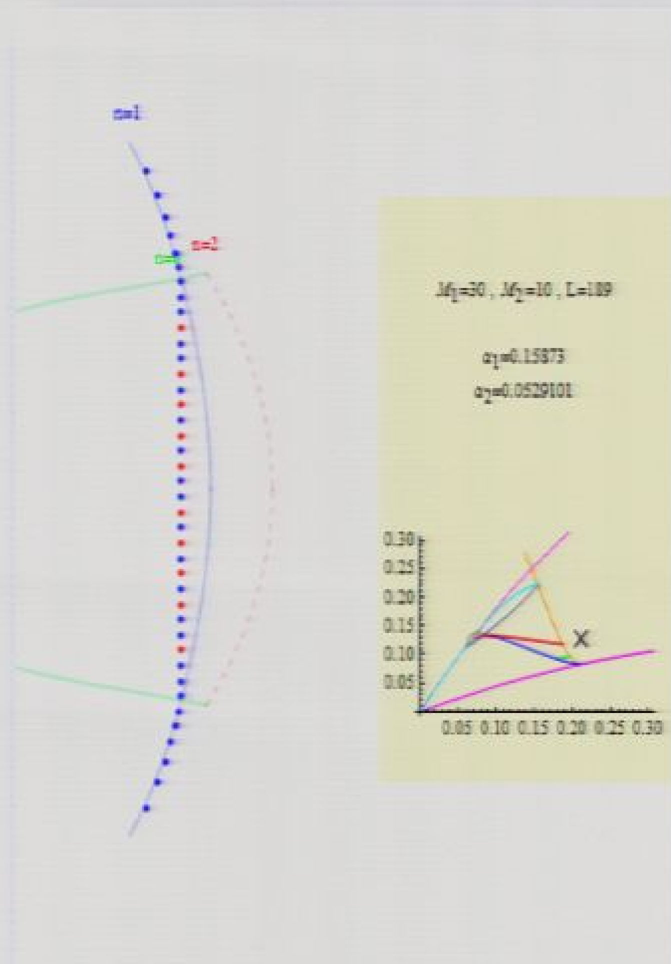

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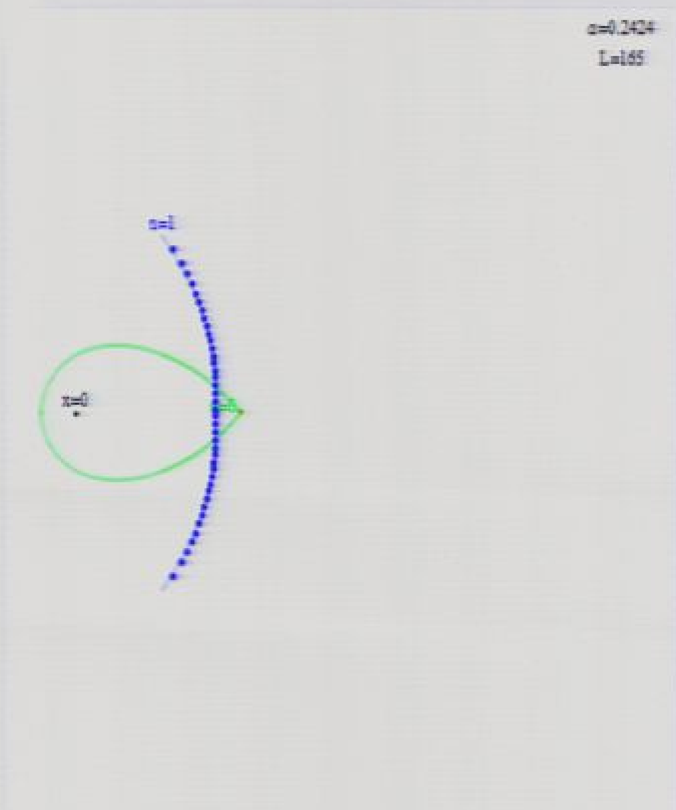
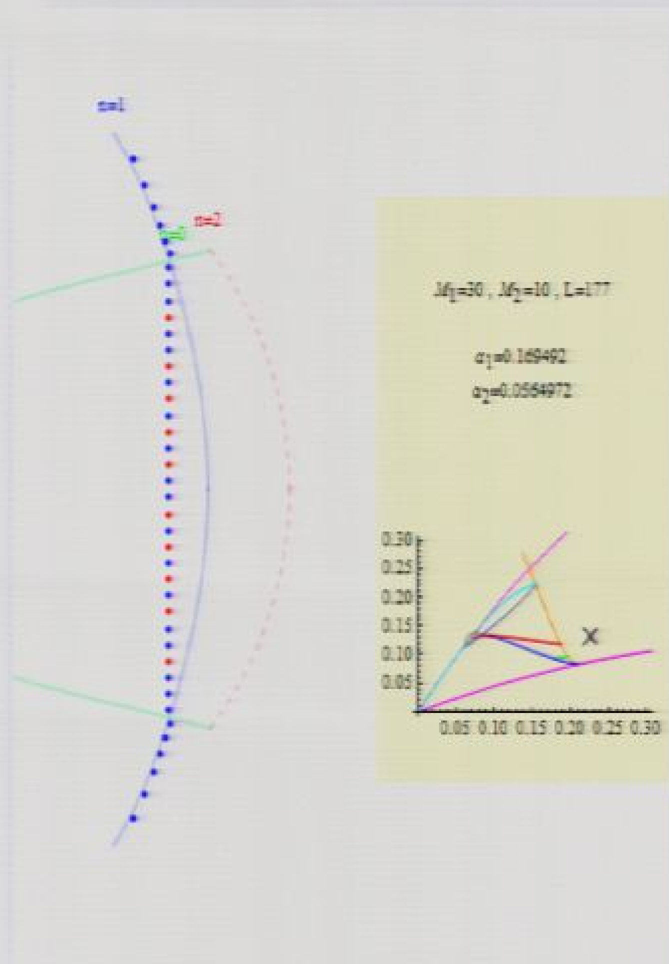
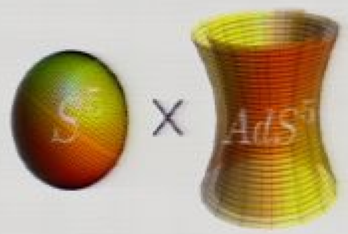




AdS/CFT correspondence

AdS/CFT duality:

$$S = \frac{T}{2} \int \partial_\mu \vec{u} \cdot \partial^\mu \vec{u} \;\, d\sigma d\tau$$



String tension $T = \dfrac{\sqrt{\lambda}}{2\pi}$

`t Hooft coupling $\lambda = g_{YM}^2 N$

String coupling $g_s = \dfrac{\lambda}{4\pi N}$

Number of colors N

Anomalous dimensions = spectrum of 2D integrable field theories

AdS/CFT correspondence

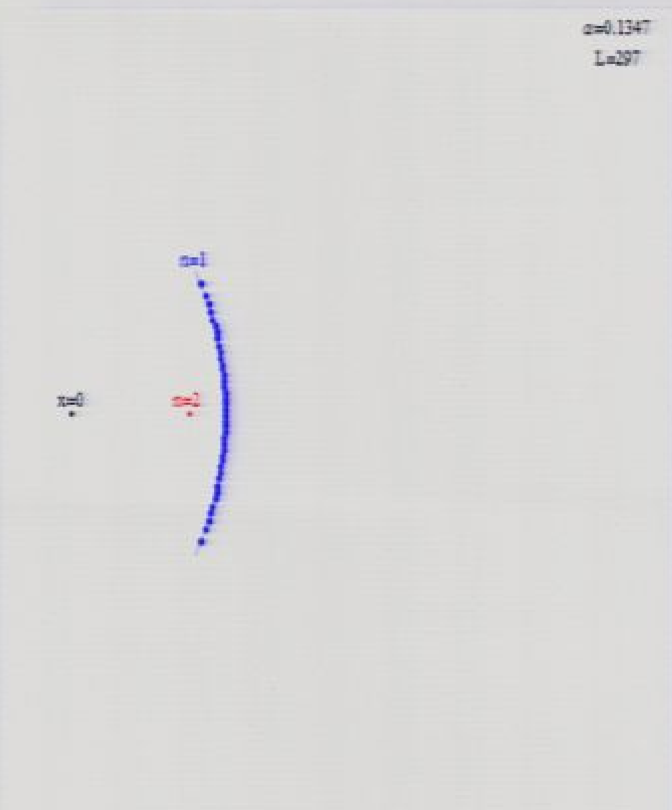
AdS/CFT duality: $S = \frac{T}{2} \int \partial_\mu \vec{u} \cdot \partial^\mu \vec{u} \;\; d\sigma d\tau$

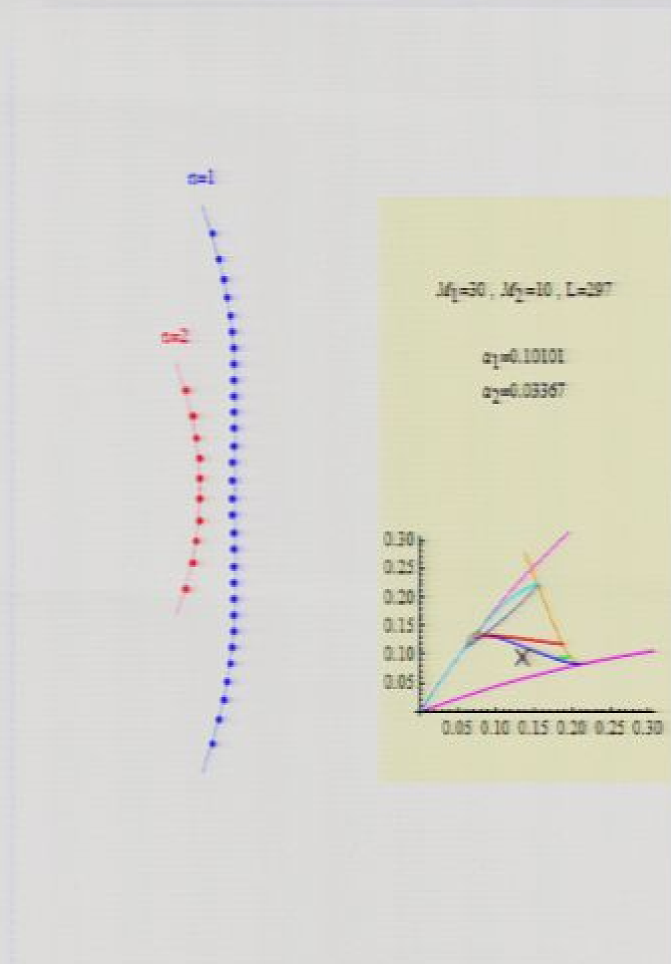
String tension $T = \frac{\sqrt{\lambda}}{2\pi}$ **`t Hooft coupling** $\lambda = g_{YM}^2 N$

String coupling $g_s = \frac{\lambda}{4\pi N}$ **Number of colors** N

Numerical Solution

$$\left(\frac{u_j + i/2}{u_j - i/2}\right)^L = \prod_{k=1(k\neq j)}^{J} \frac{u_j - u_k + i}{u_j - u_k - i}$$





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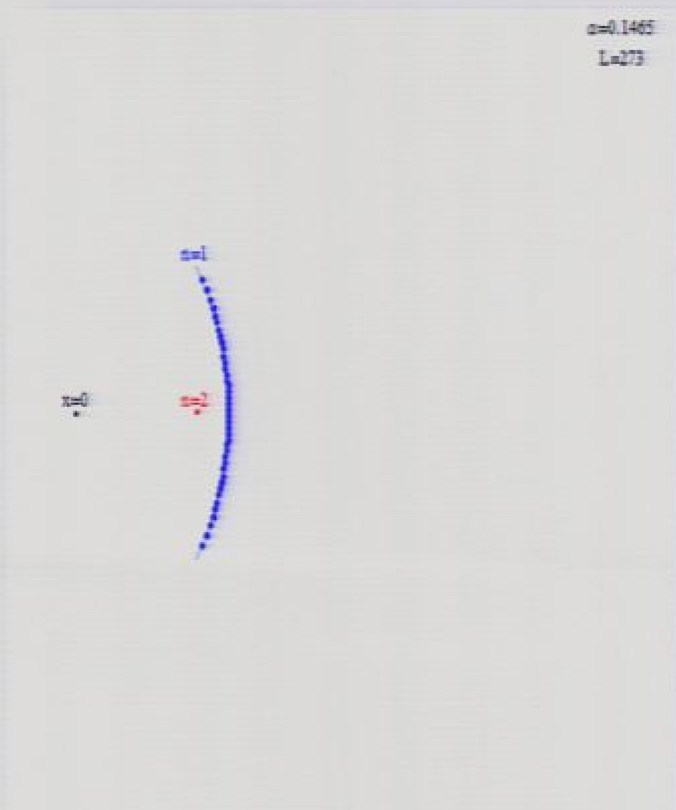

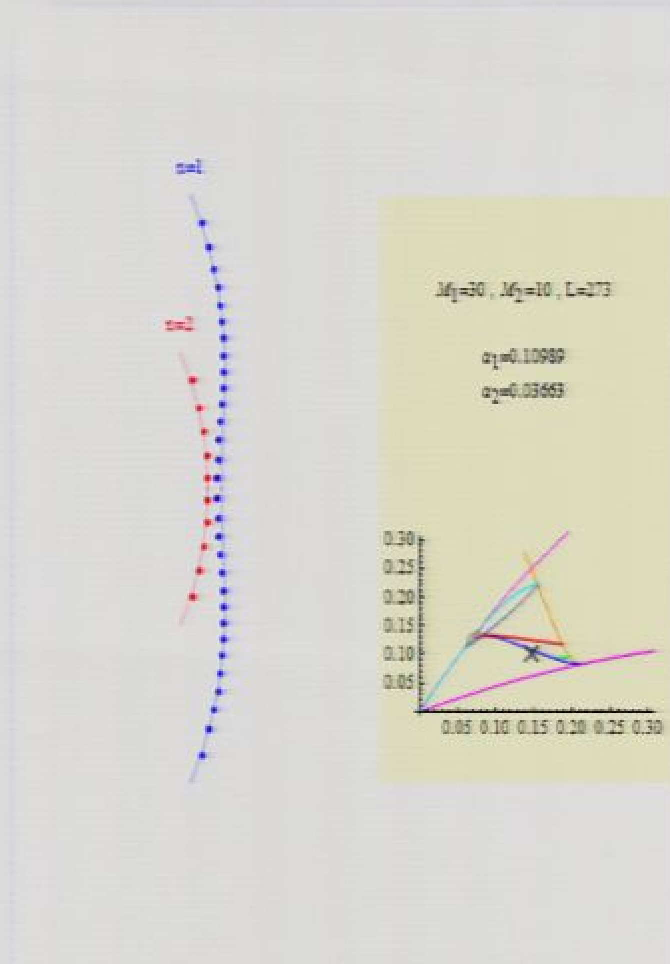

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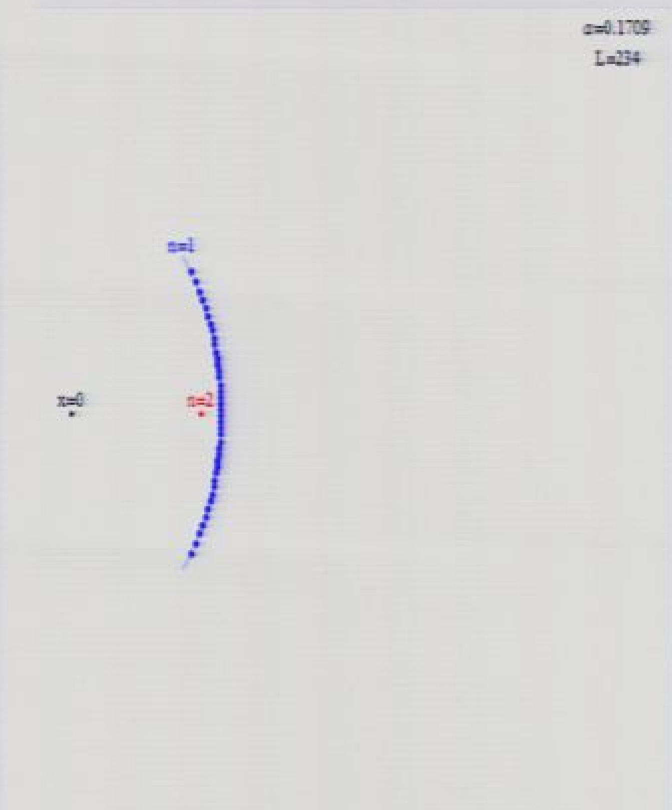

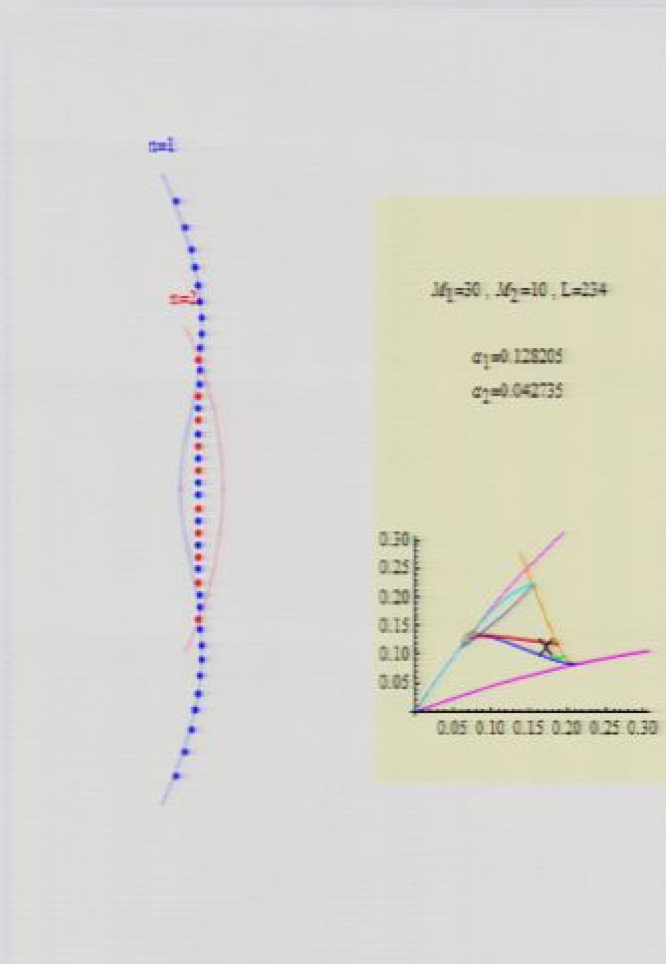

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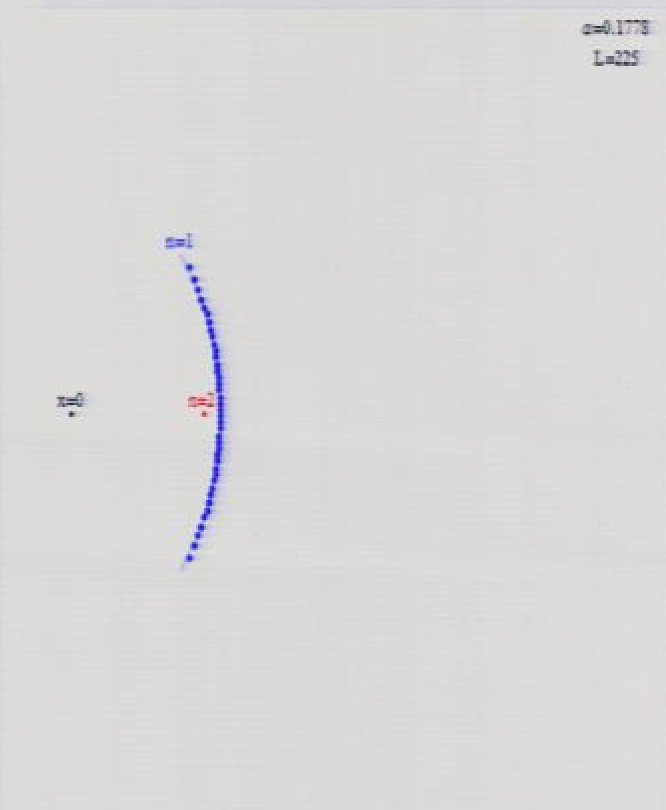

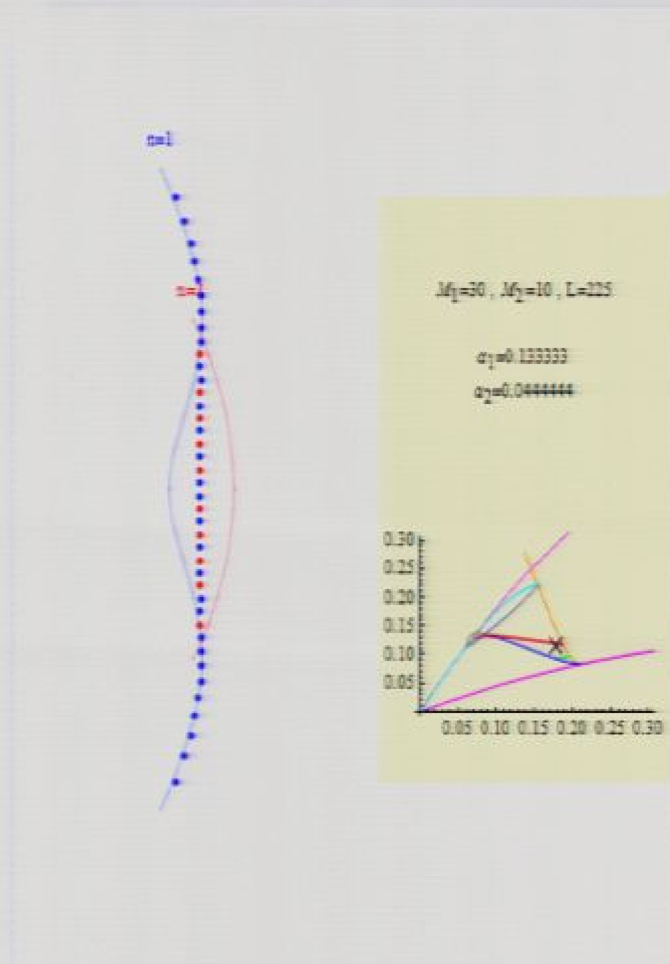

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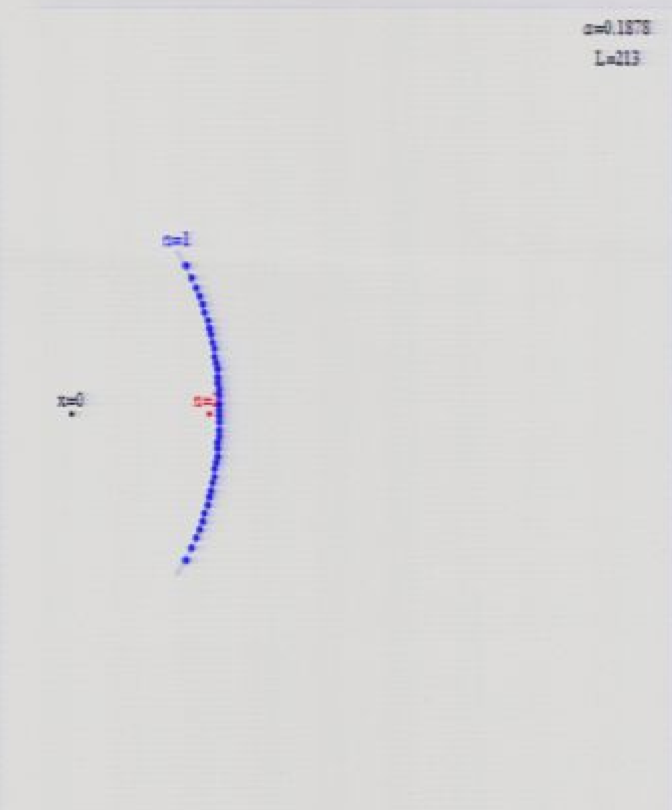

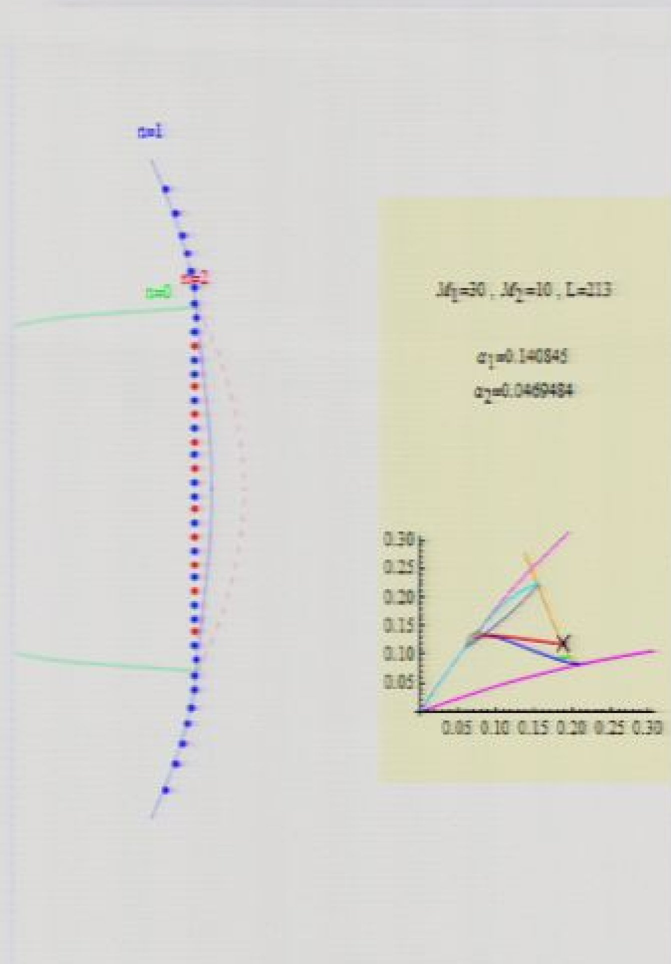

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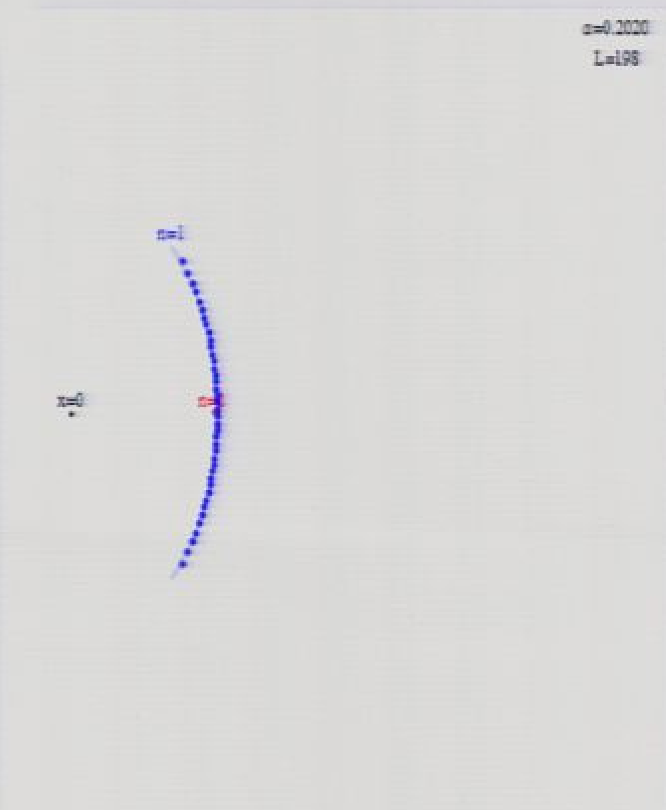

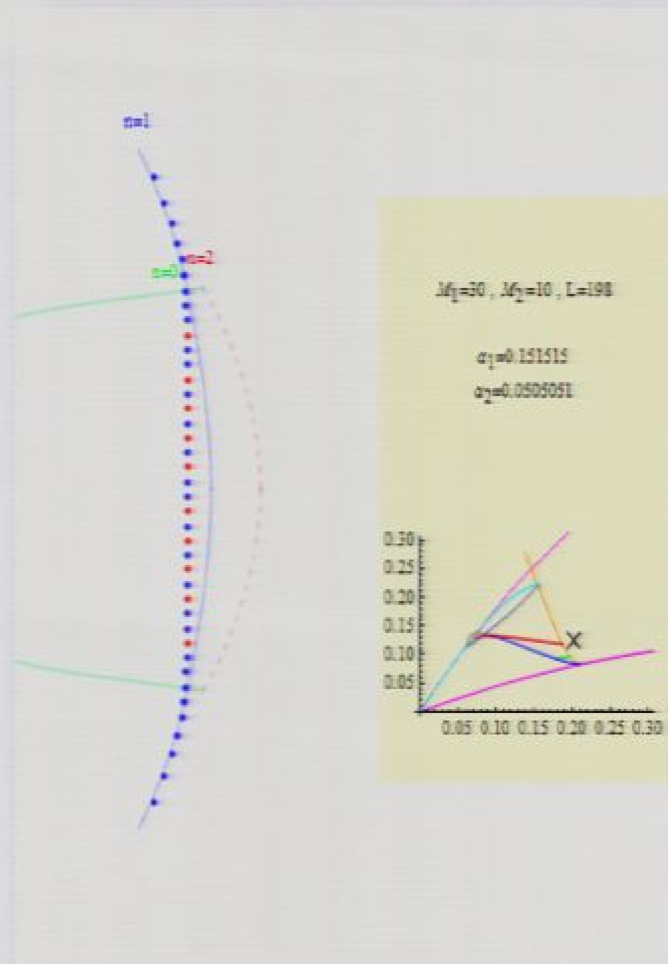

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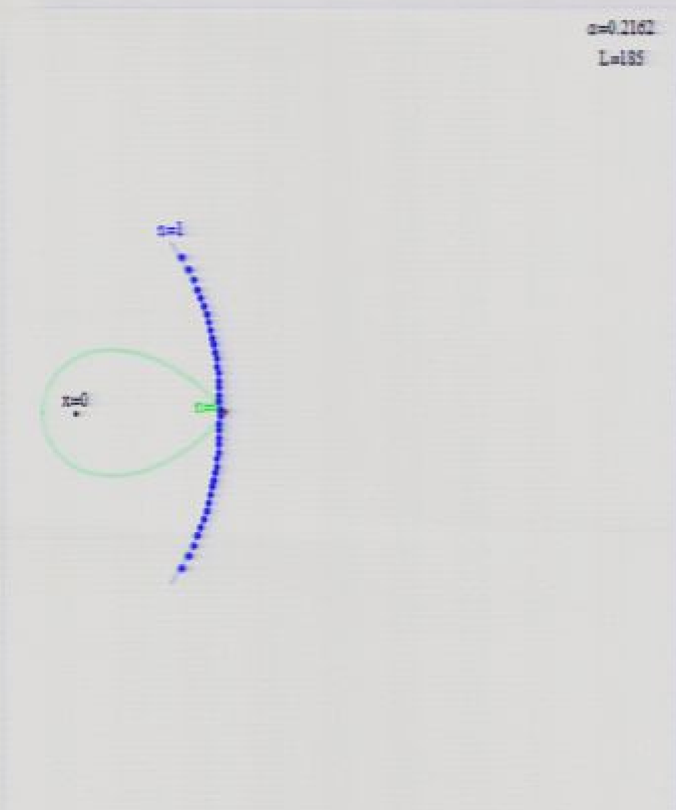

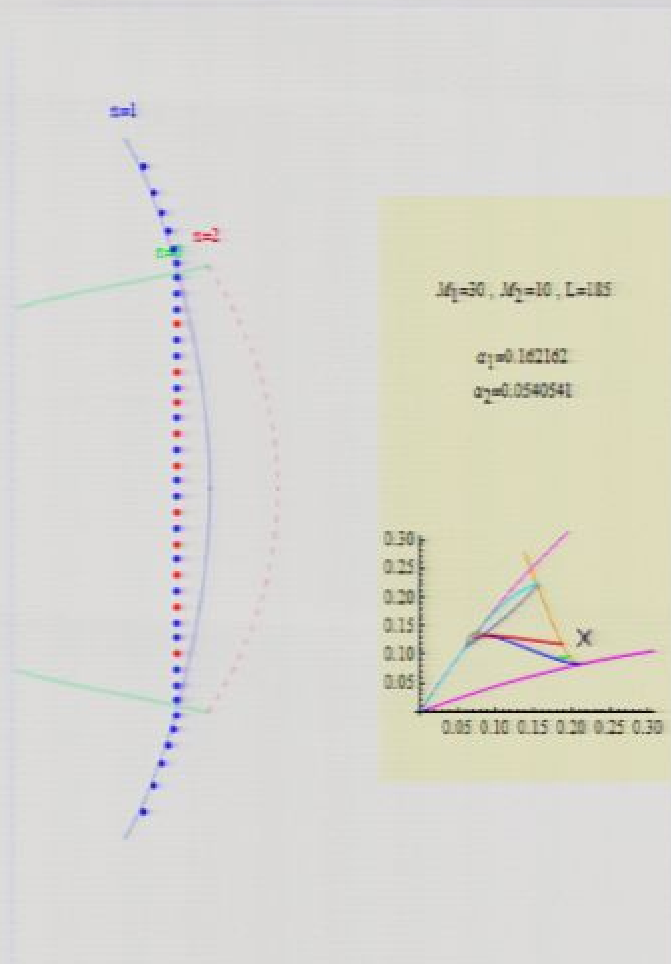

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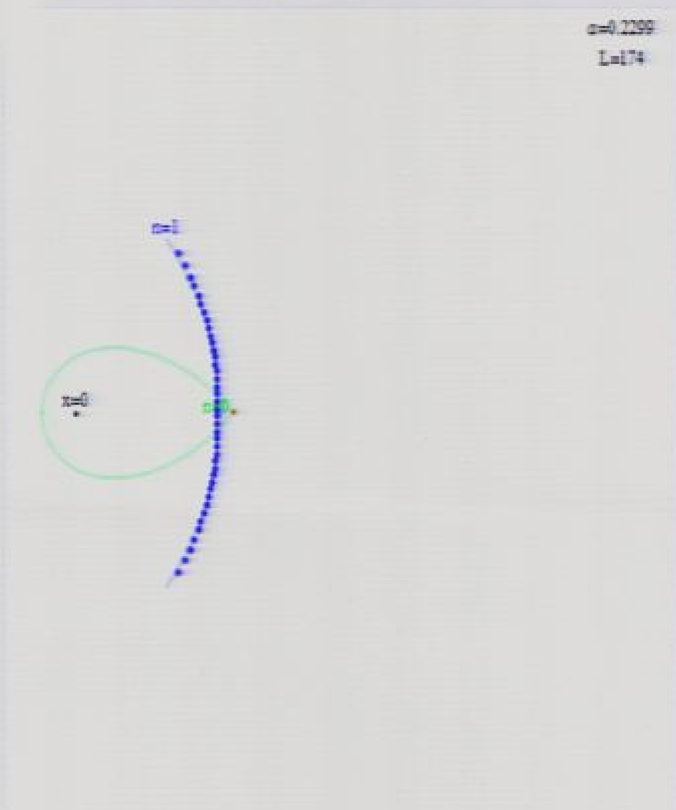

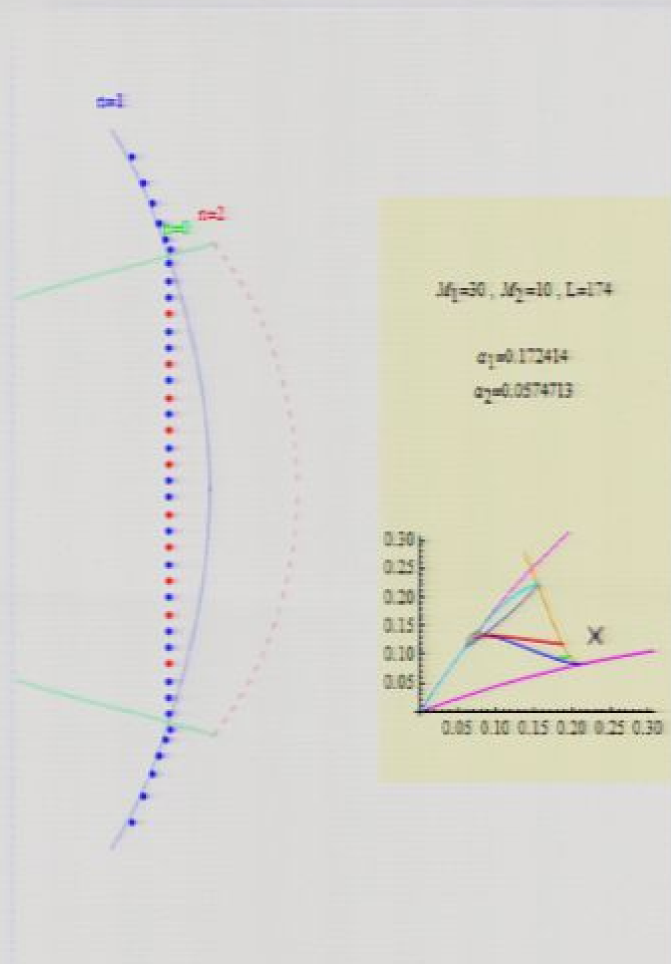

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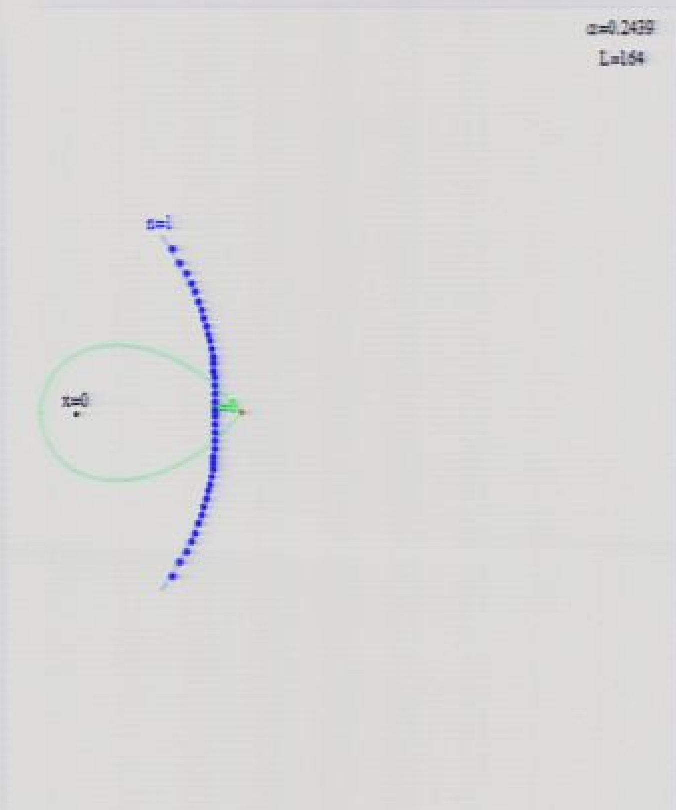

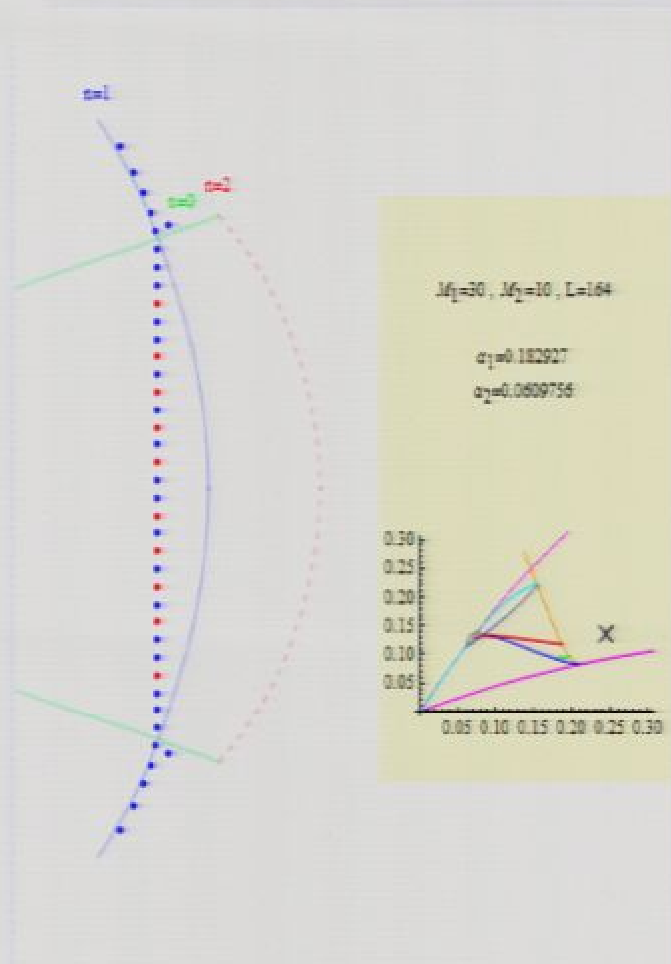

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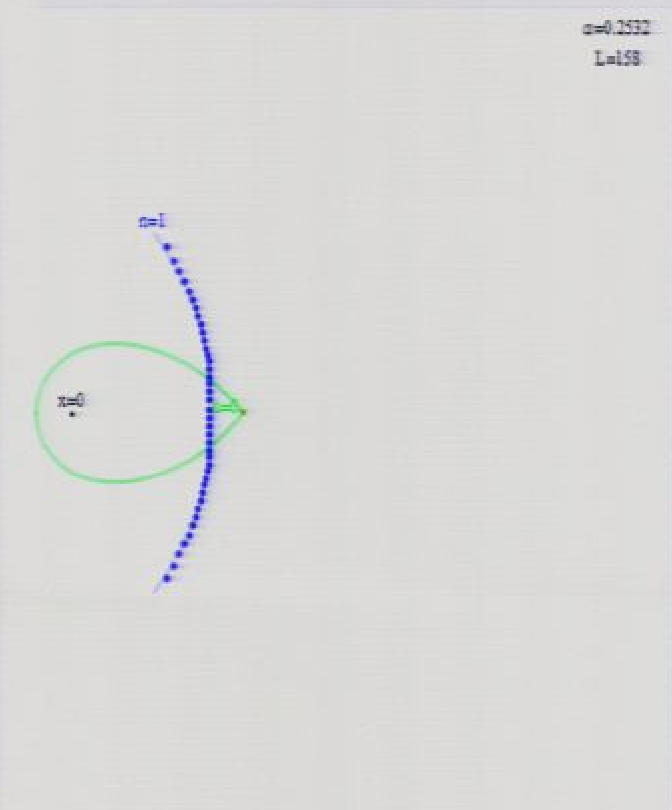

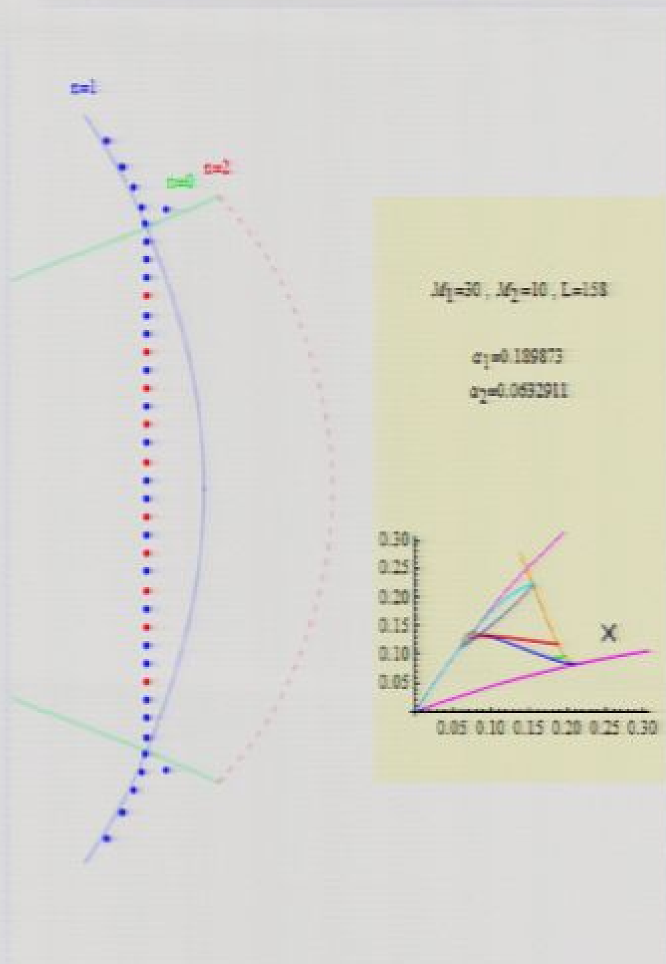

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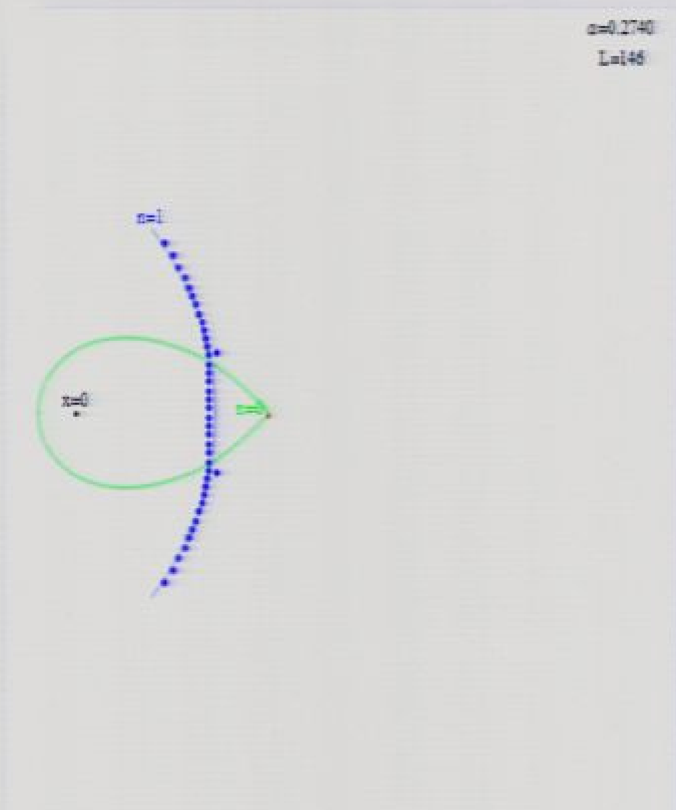

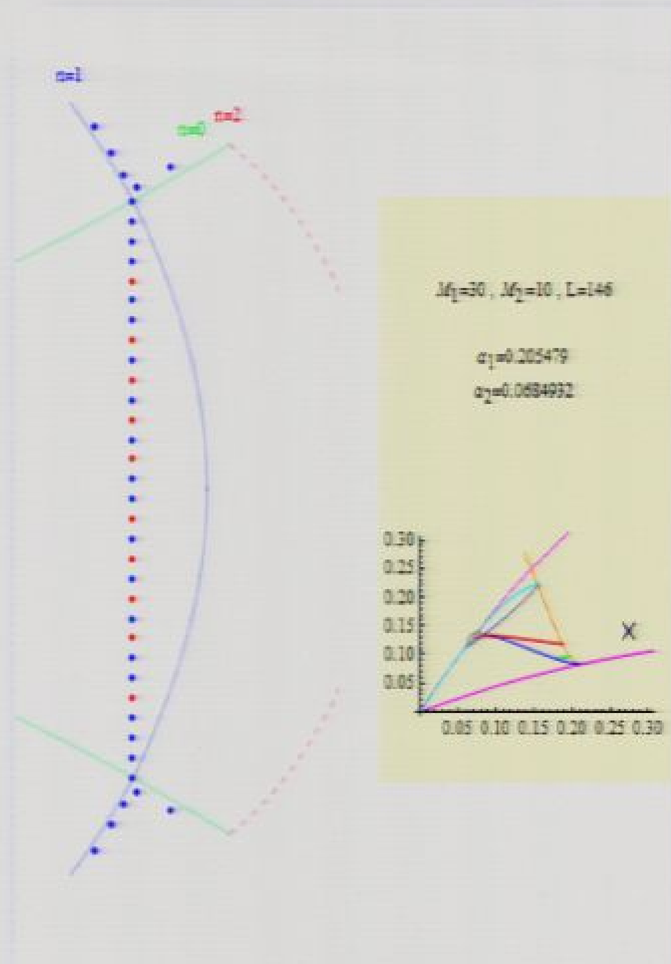

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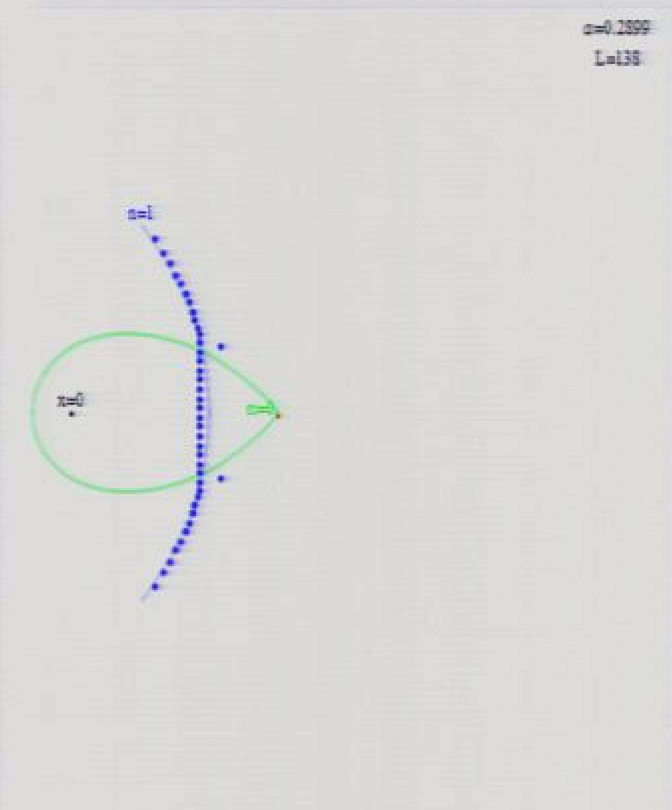

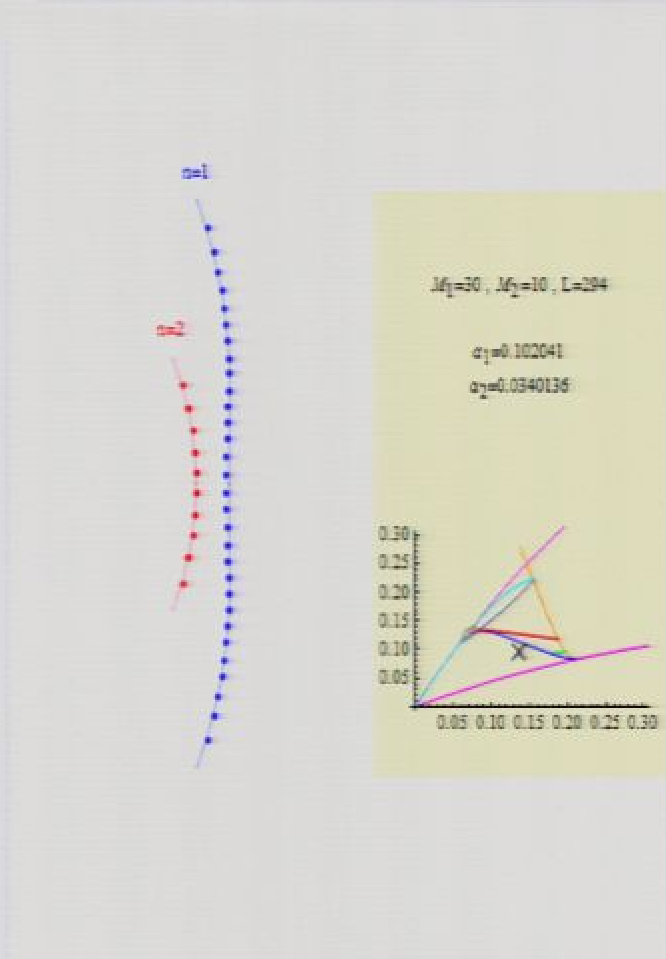

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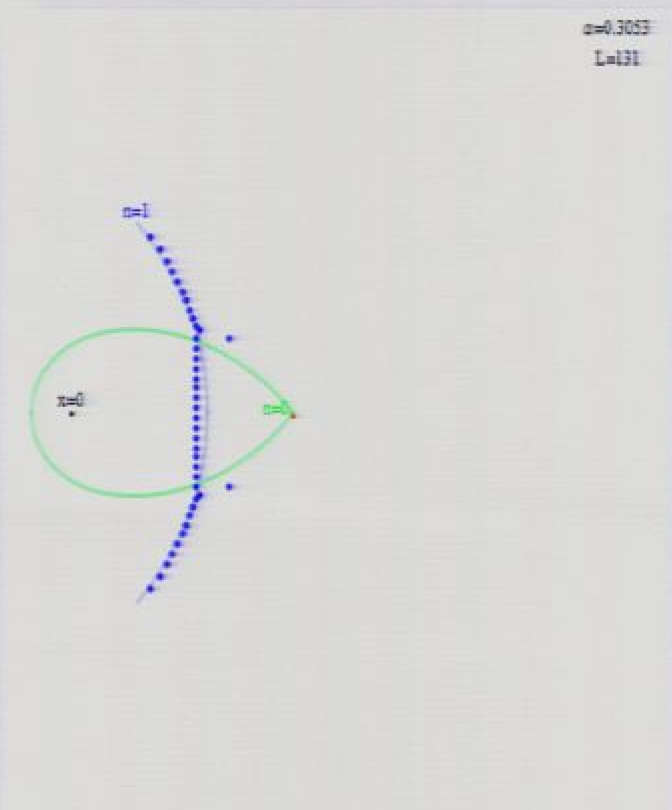

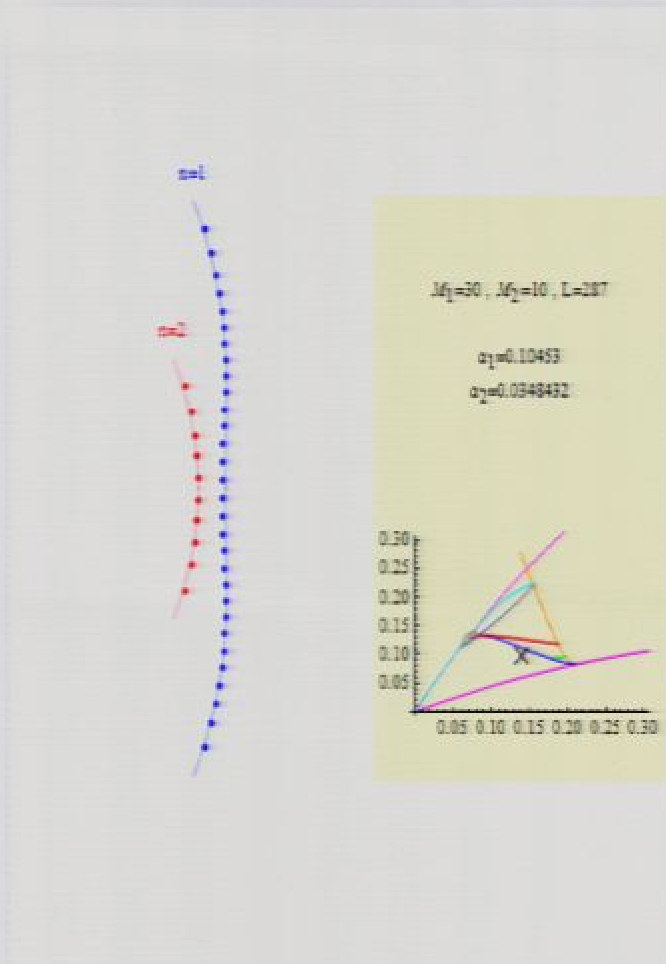

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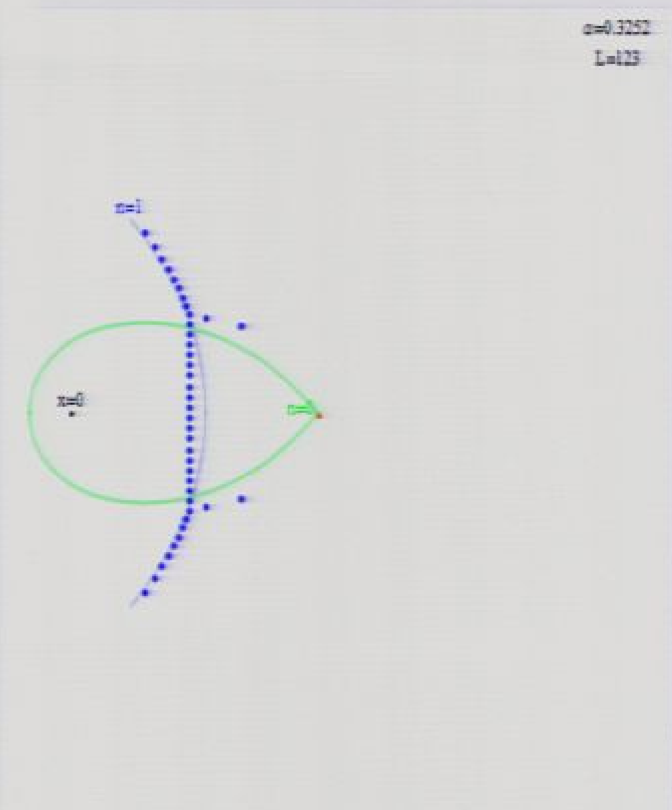

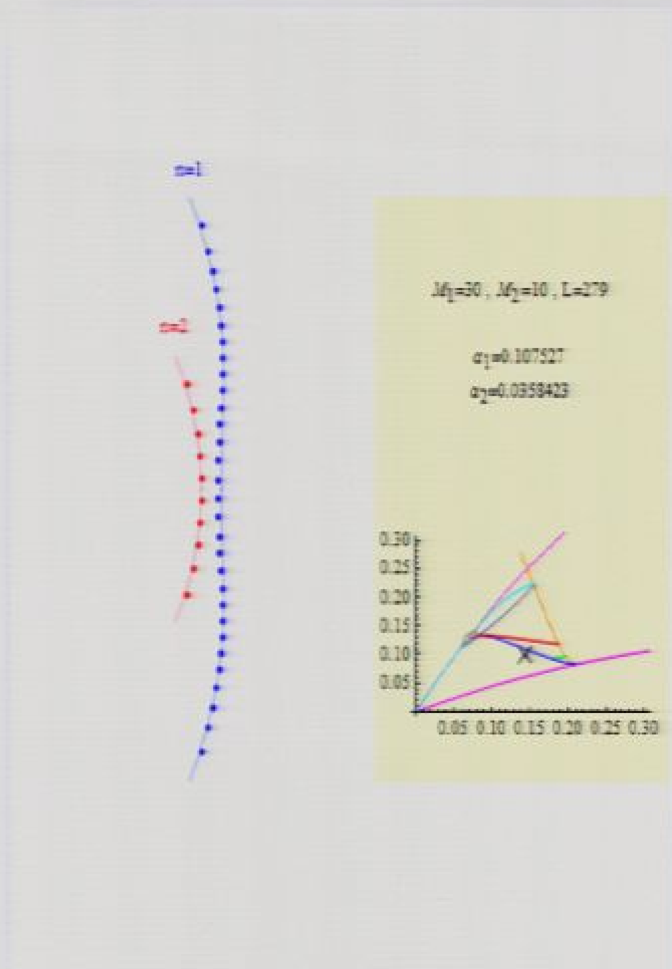

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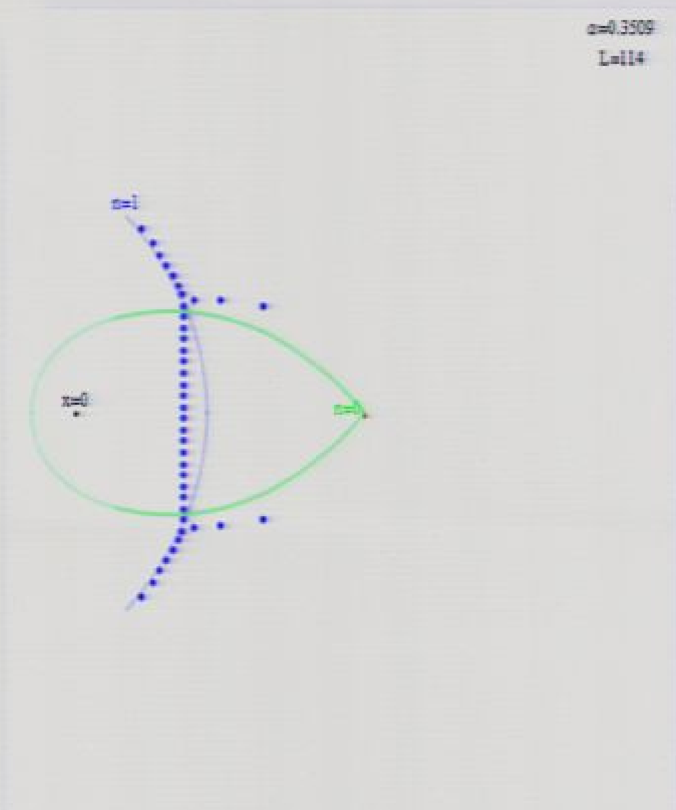

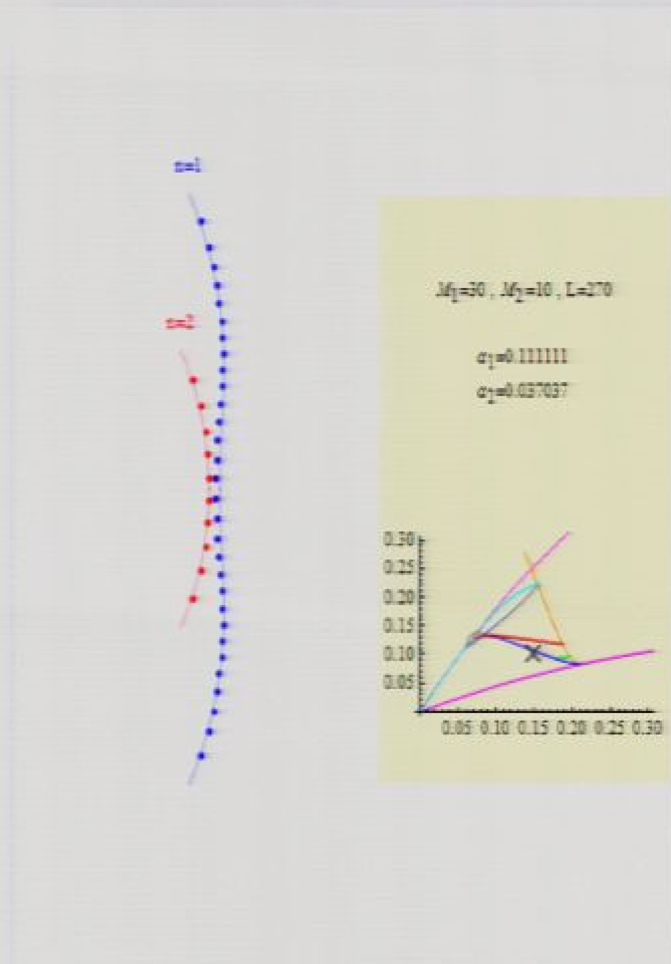

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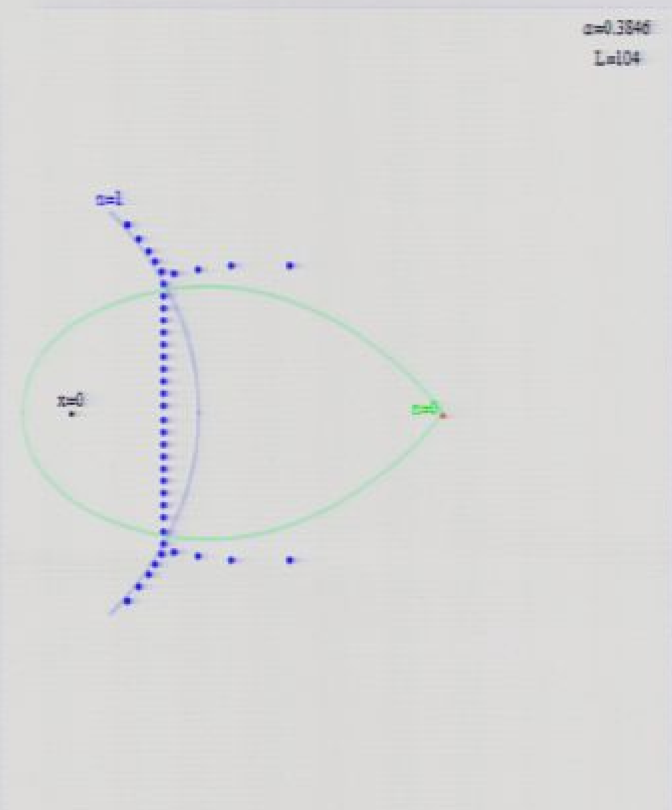

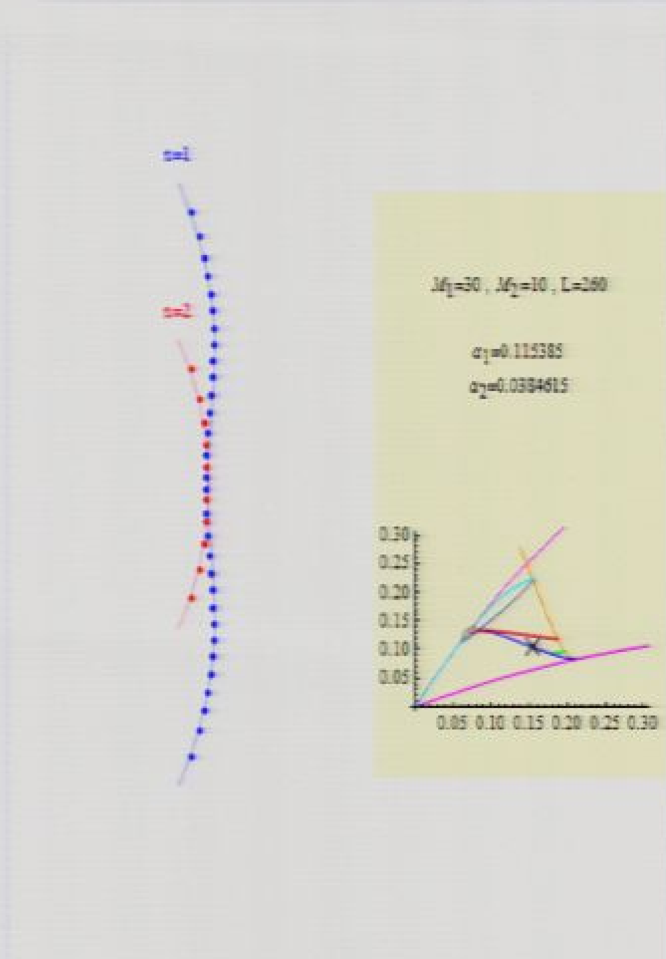

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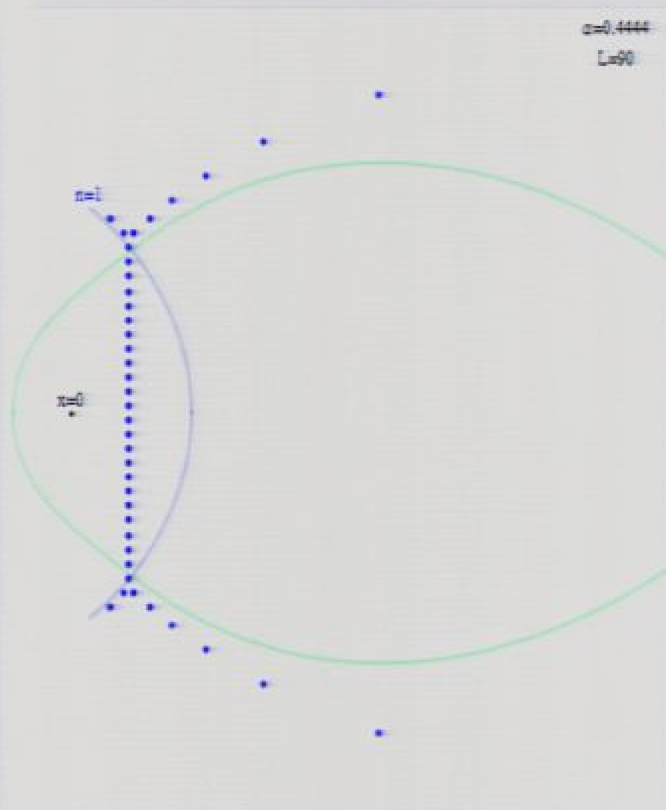

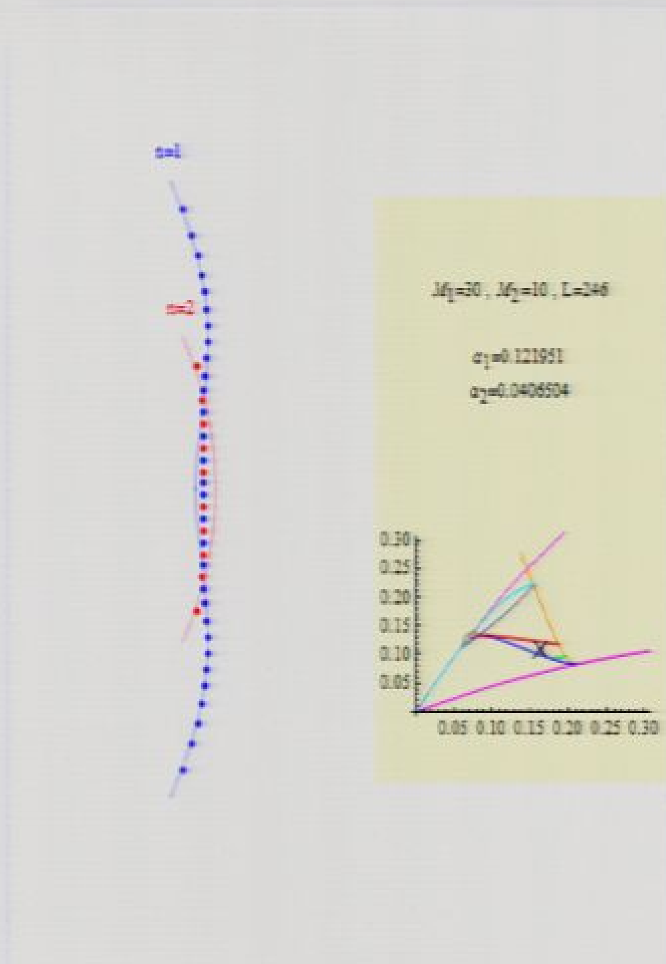

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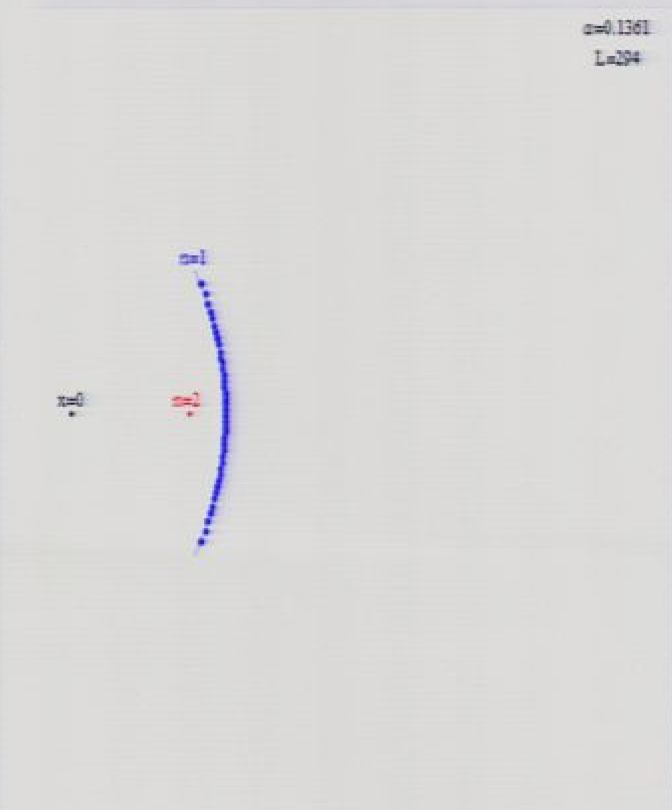

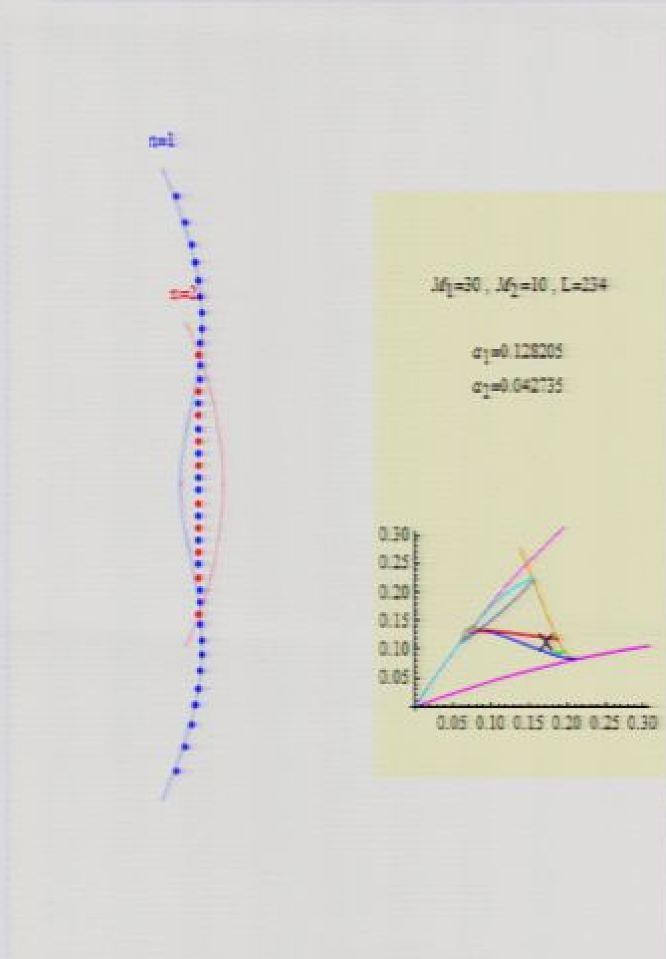

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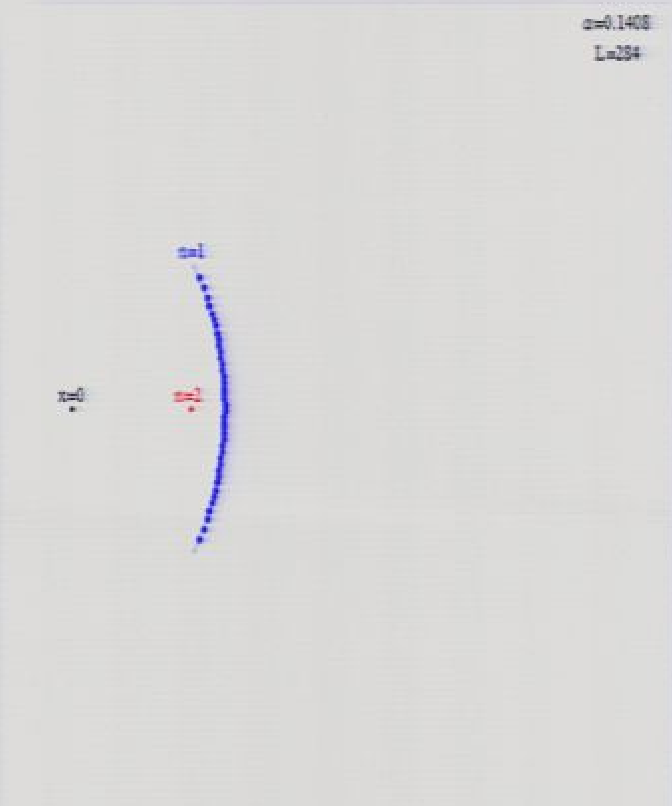

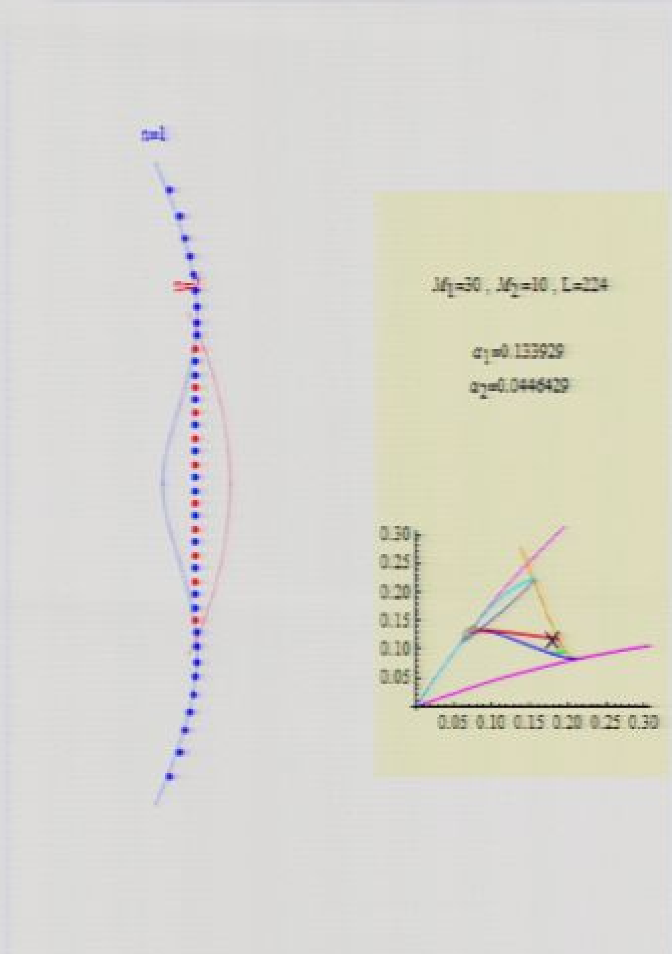

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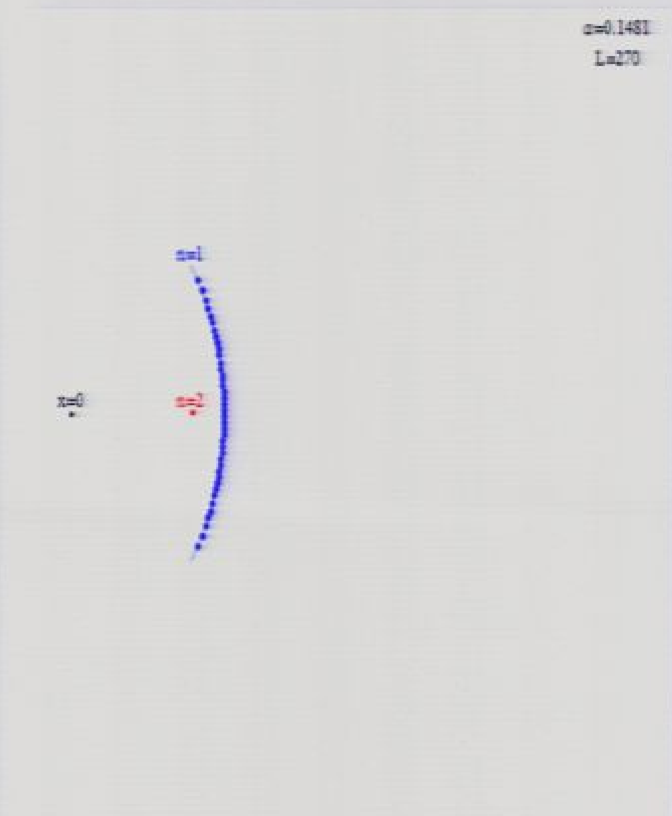

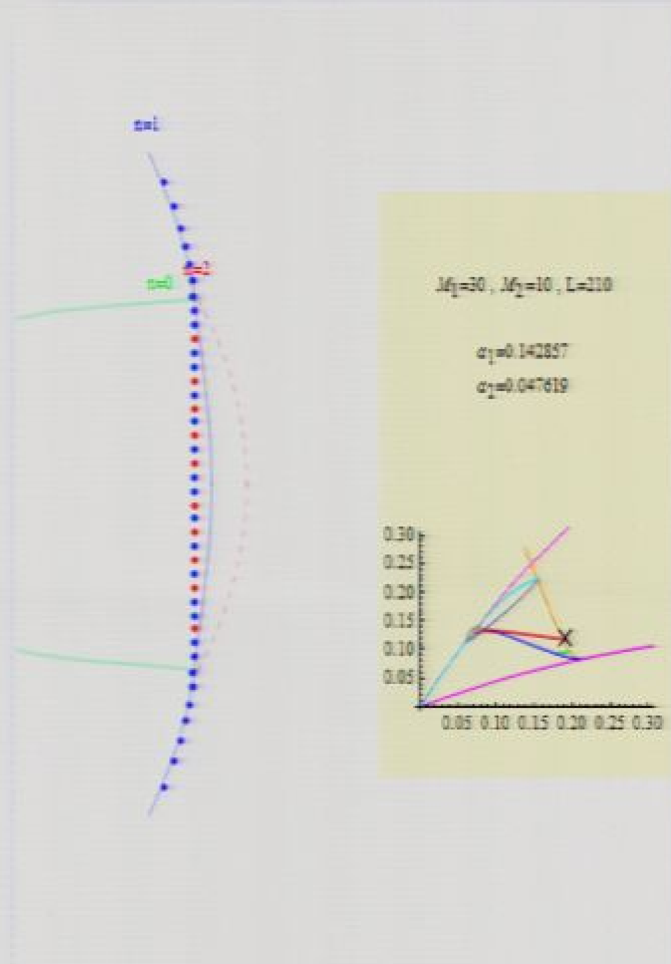

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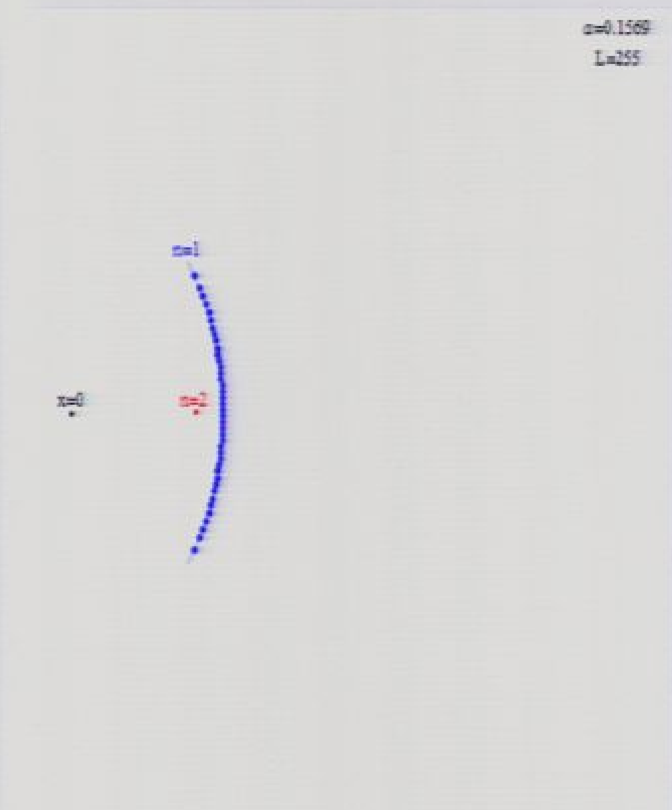

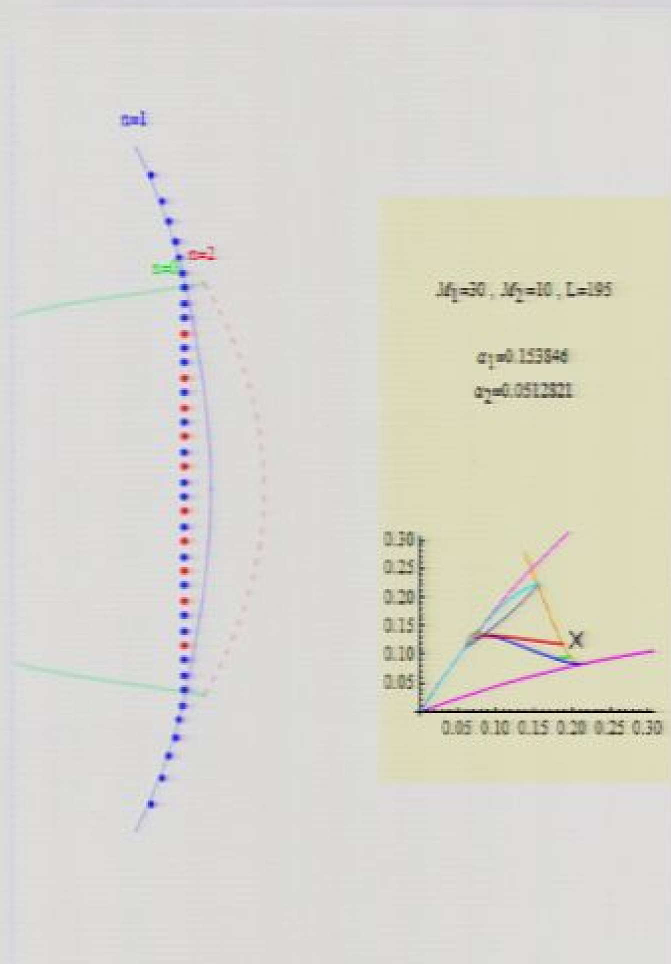

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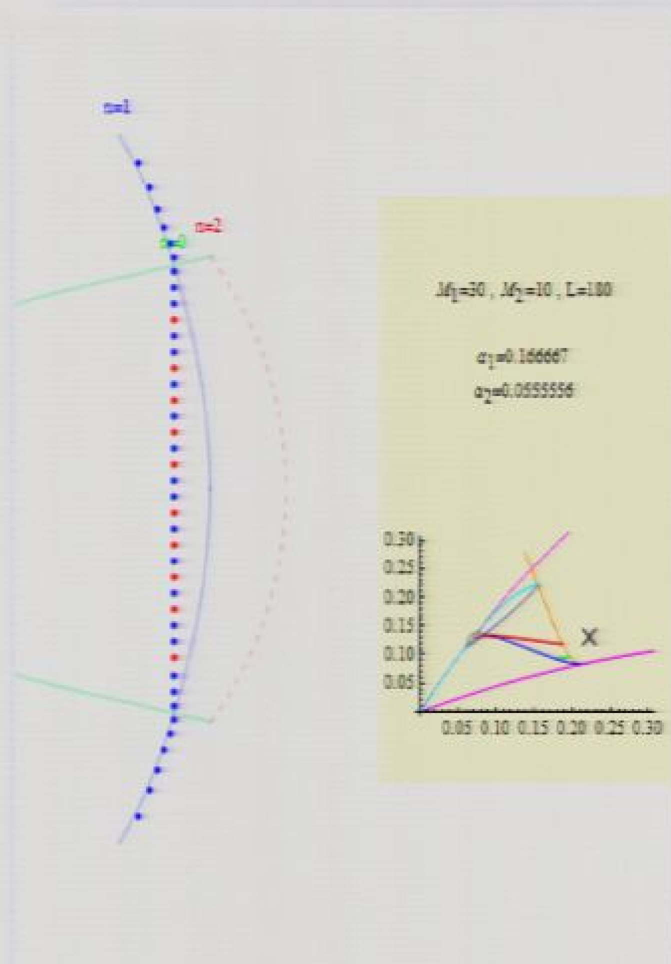

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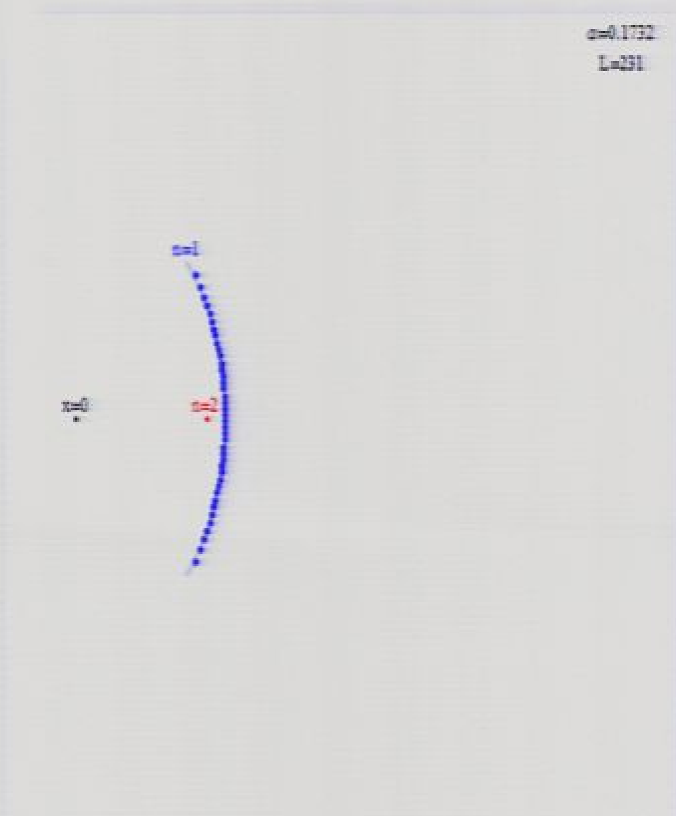

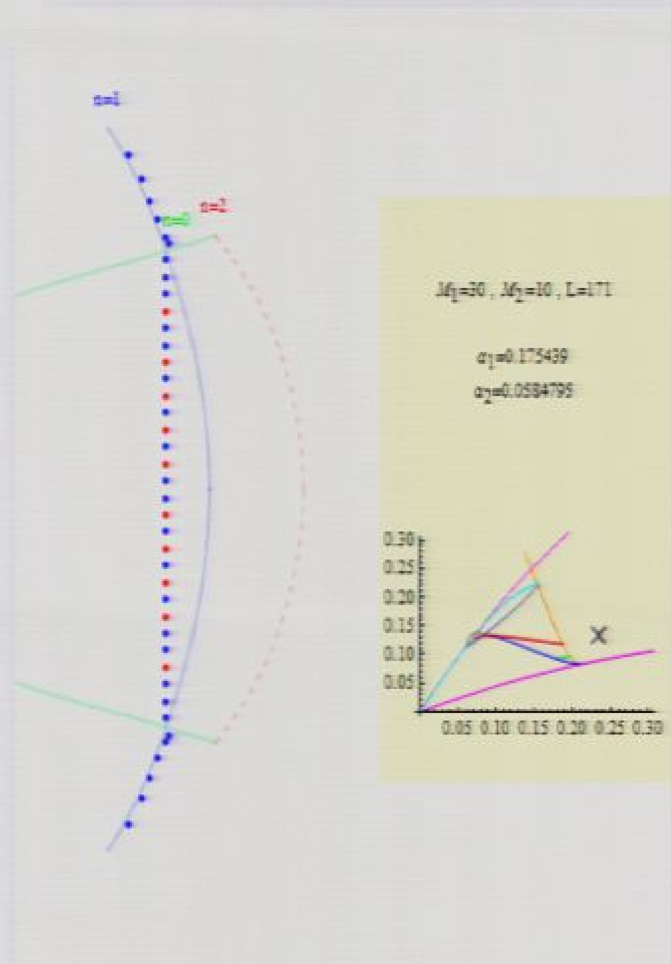

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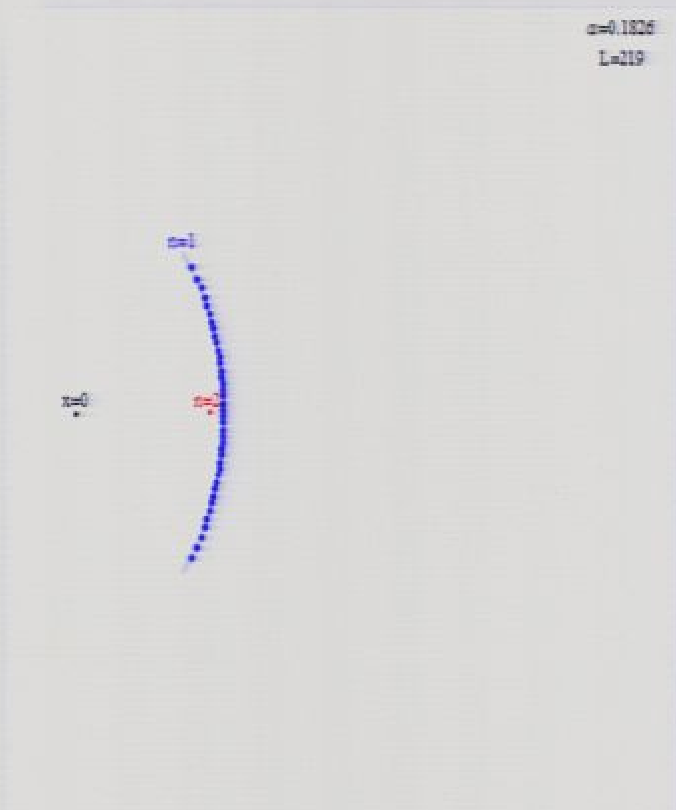

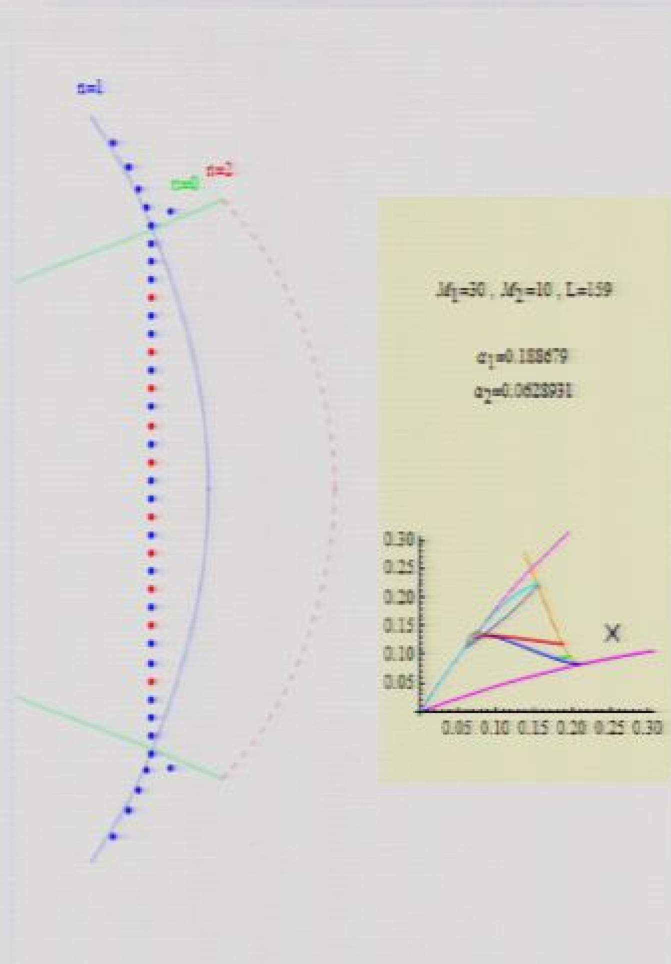

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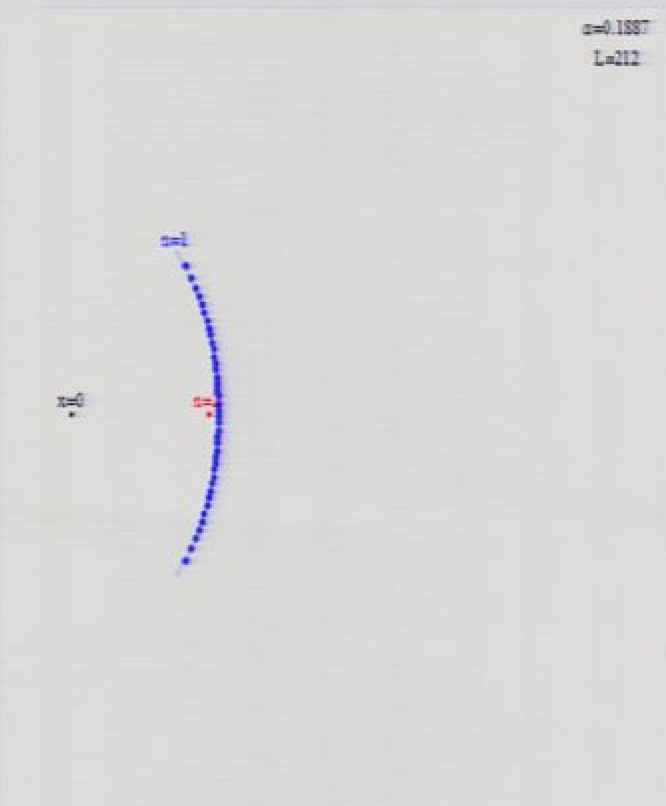

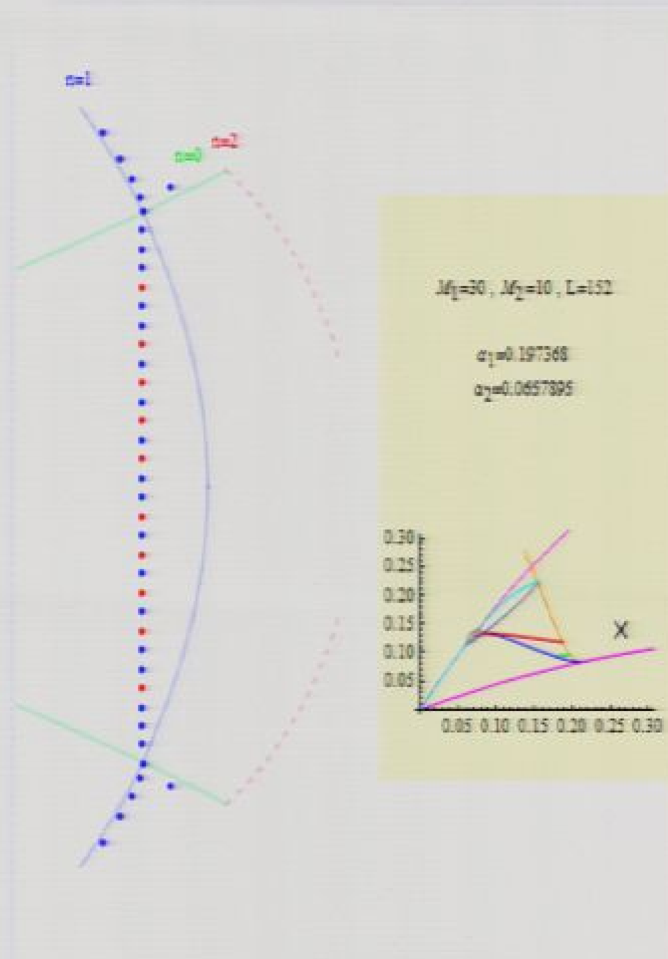

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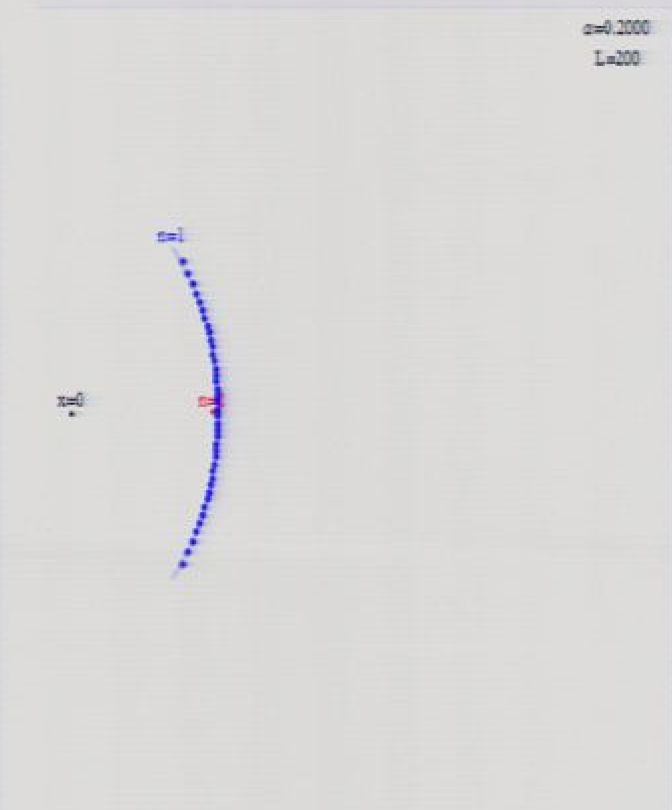

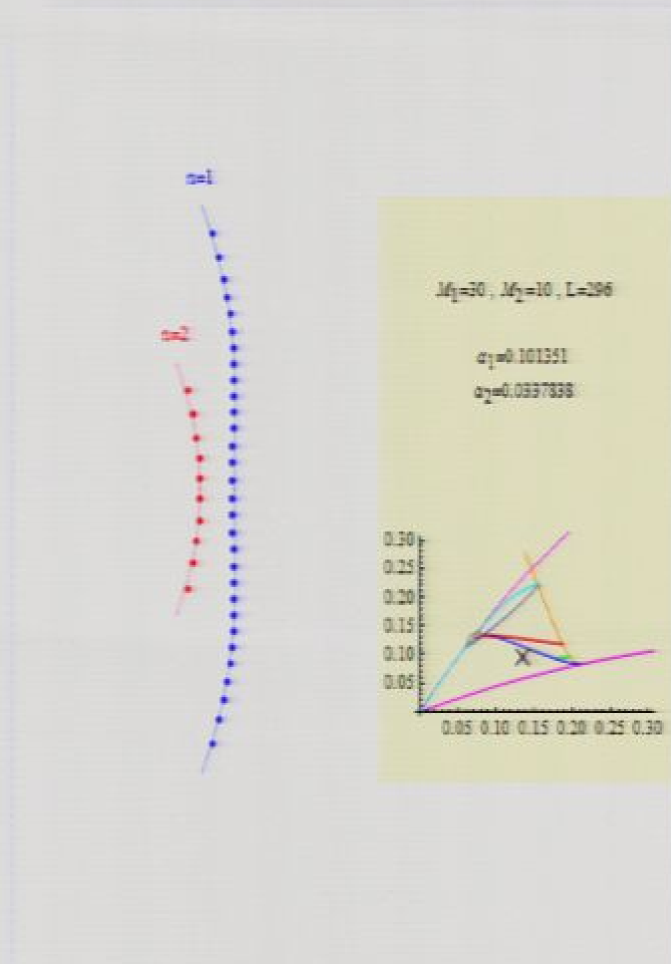

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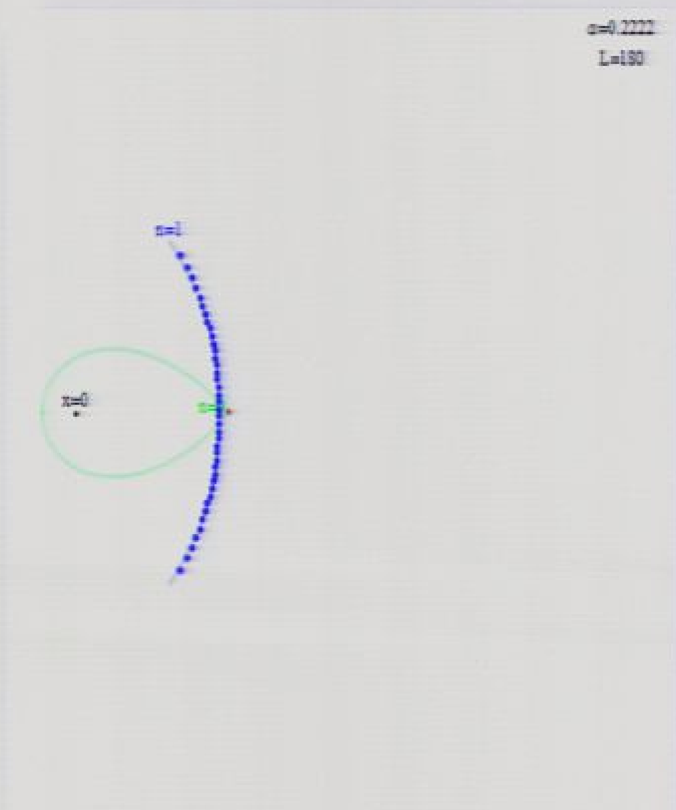

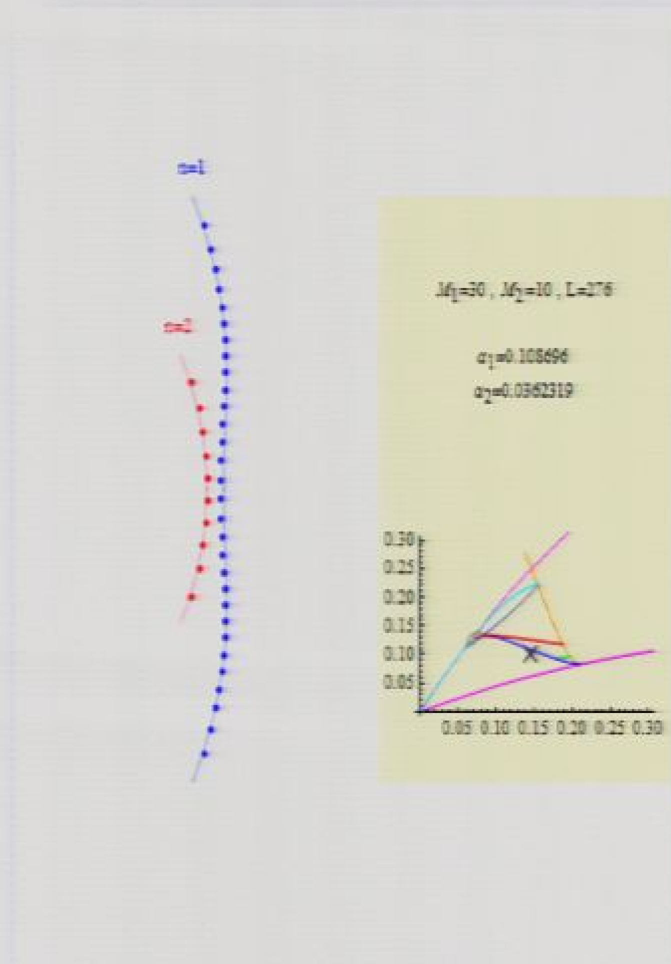

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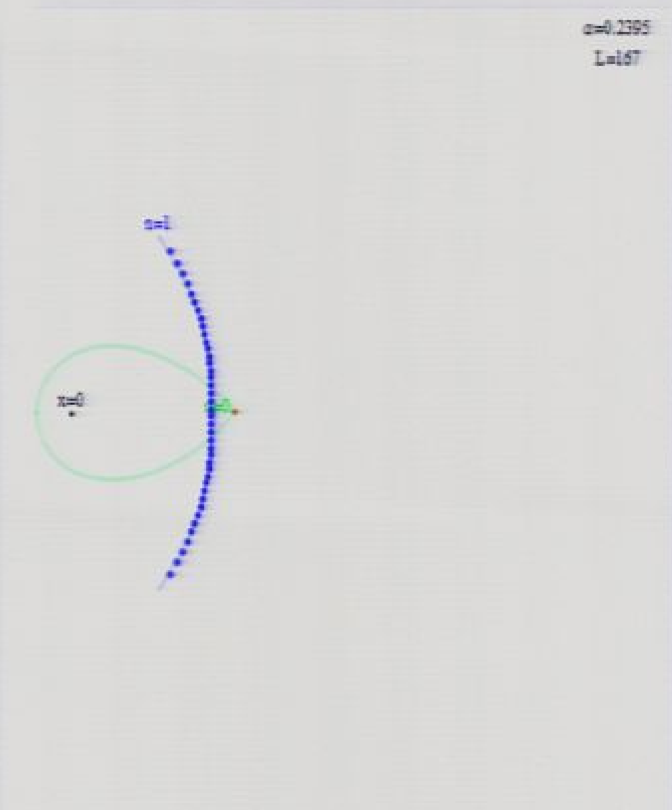

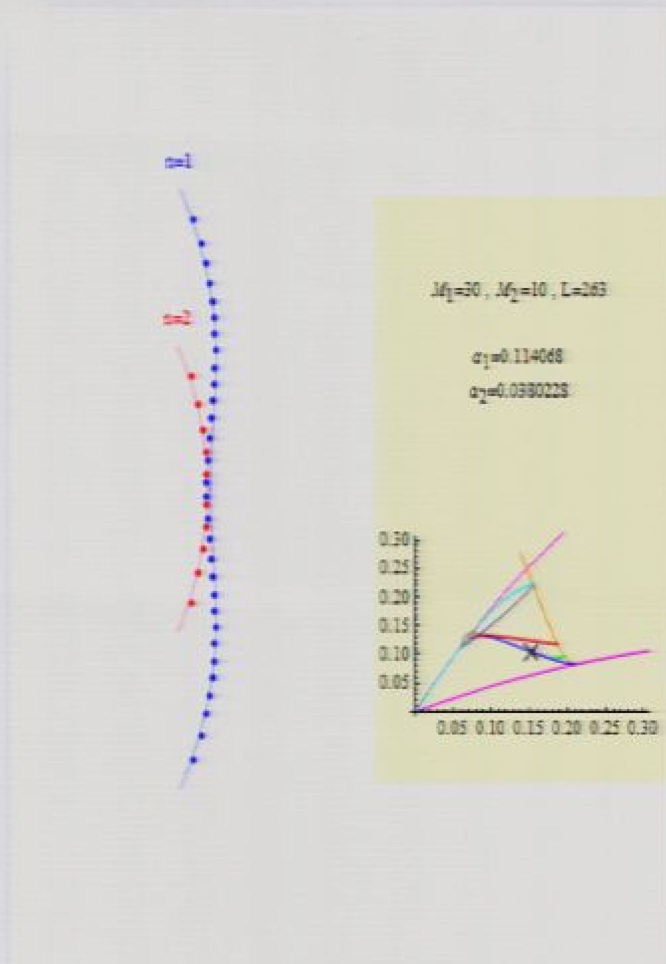

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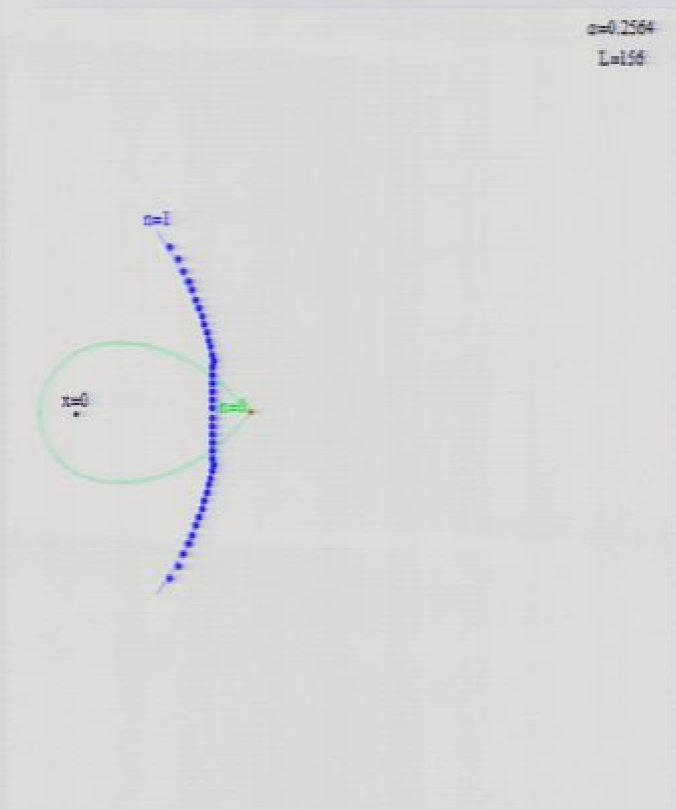

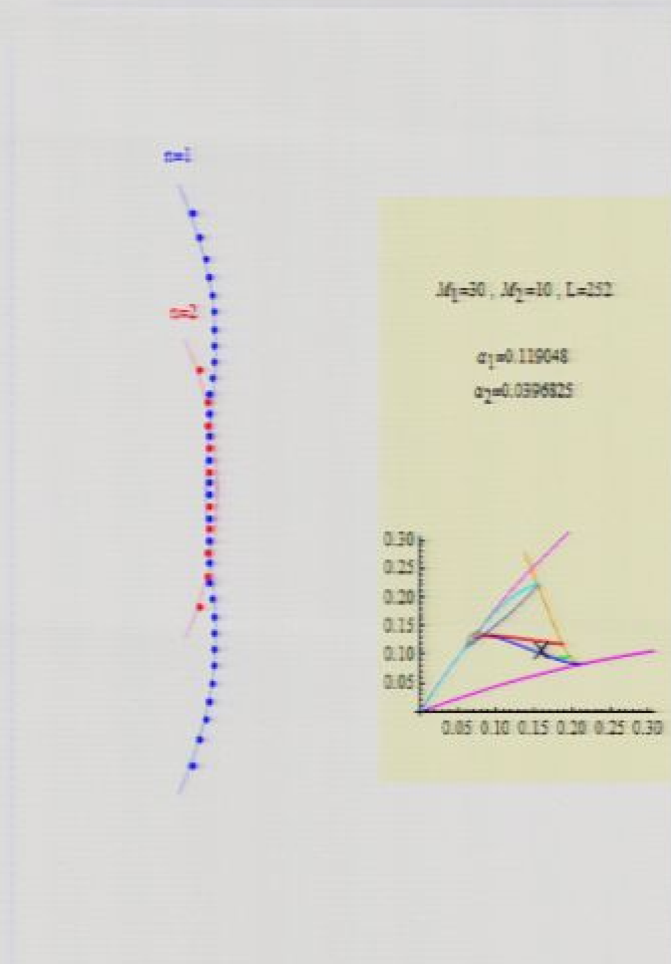

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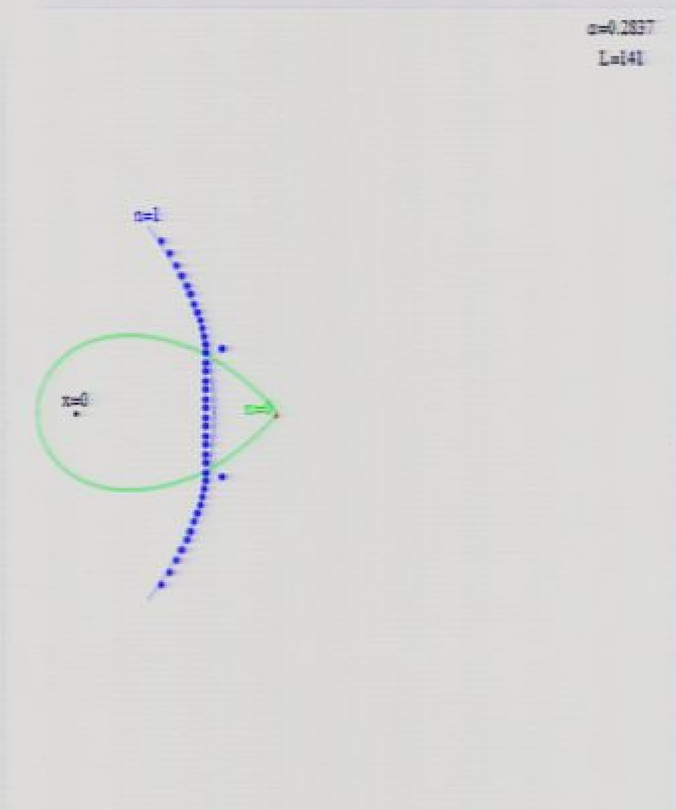
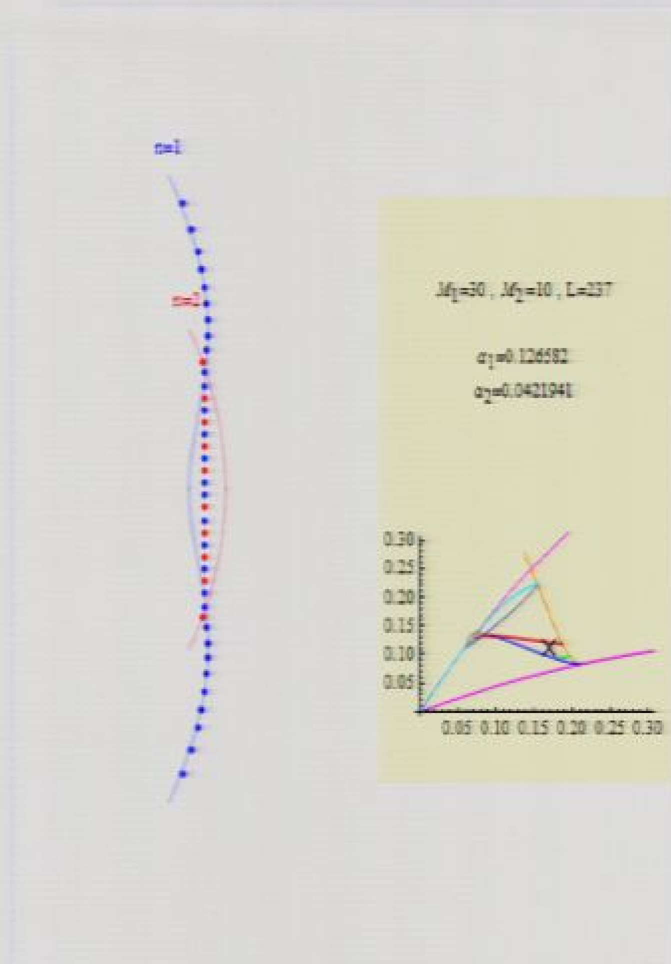
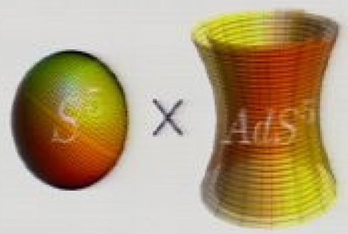




AdS/CFT correspondence

AdS/CFT duality:

$$S = \frac{T}{2}\int \partial_\mu \vec{u}\cdot\partial^\mu \vec{u}\;d\sigma d\tau$$



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Number of colors N

Anomalous dimensions = spectrum of 2D integrable field theories

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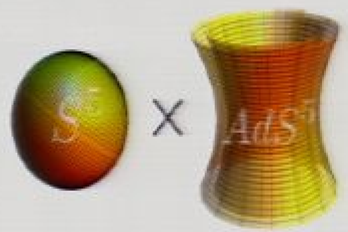

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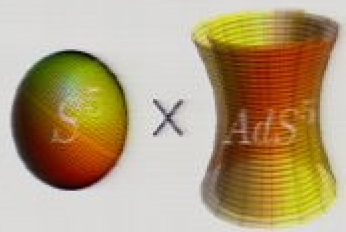

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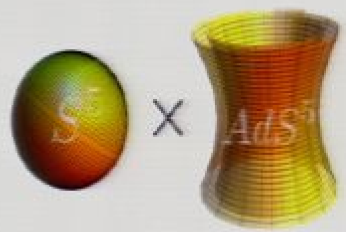

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$su_L(2|2)$

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S3 sigma model

Bethe ansatz:

$$\boxed{1 = \prod_{\beta}^{J_u} \frac{u_j-\theta_\beta-i/2}{u_j-\theta_\beta+i/2} \prod_{i\neq j}^{J_u} \frac{u_j-u_i+i}{u_j-u_i-i}, \qquad su_R(2)}$$

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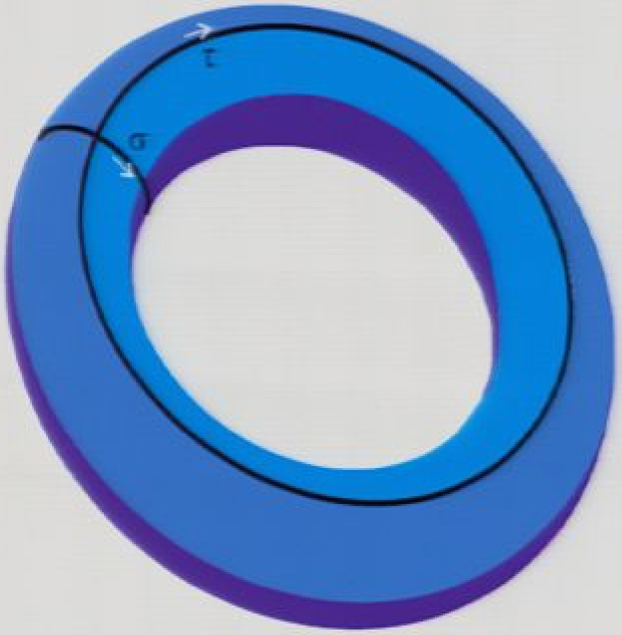
Vacuum



$$Z(\tau,\sigma) = Z(\sigma,\tau)$$

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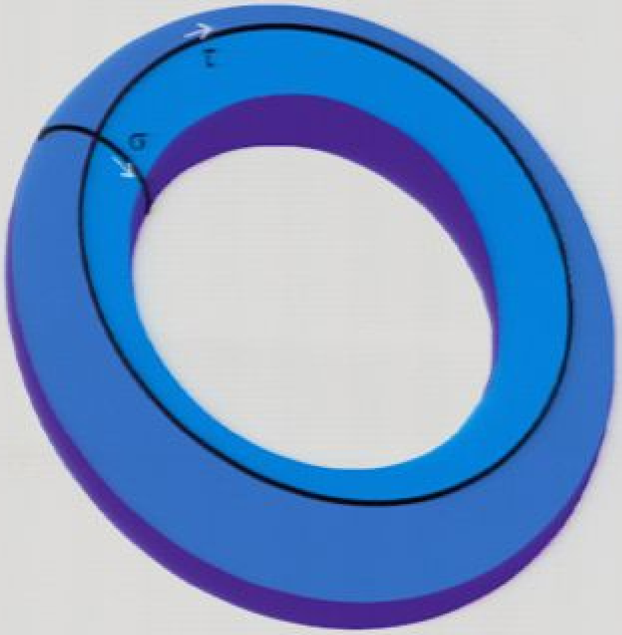

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$$\Downarrow \qquad\qquad \Downarrow$$

$$\sum e^{-E_n(L)R} \qquad \sum e^{-E_n(R)L}$$

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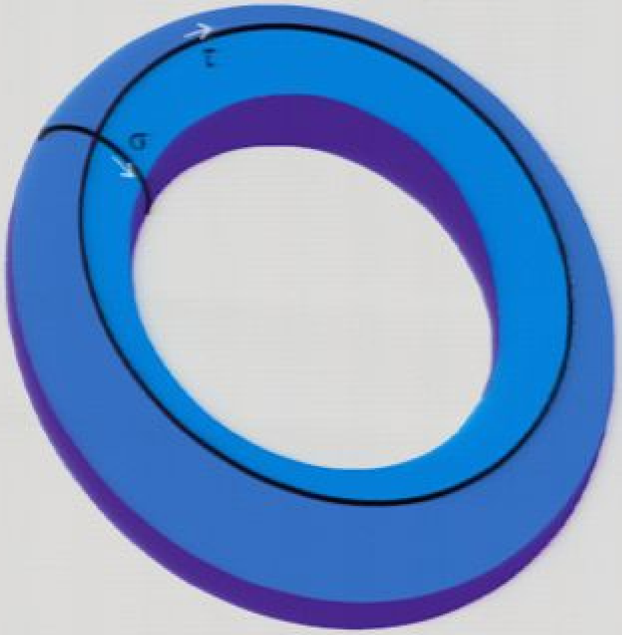

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I.e. from the asymptotical spectrum (infinite R) we can compute the Ground state energy for ANY finite volume!

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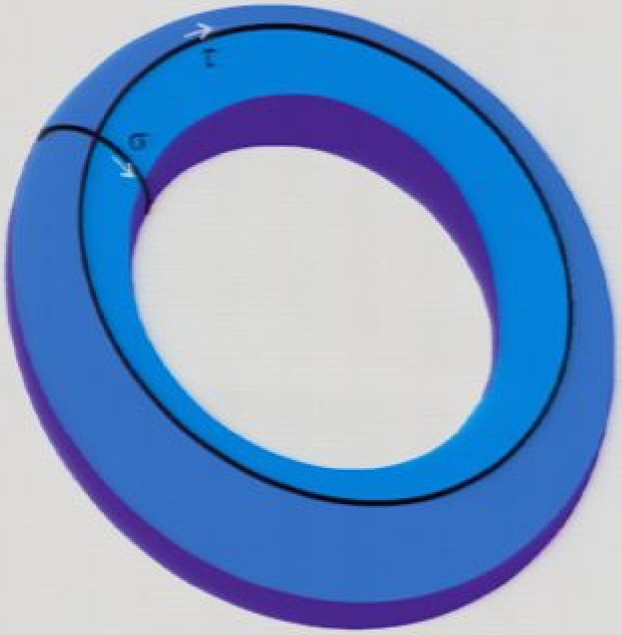

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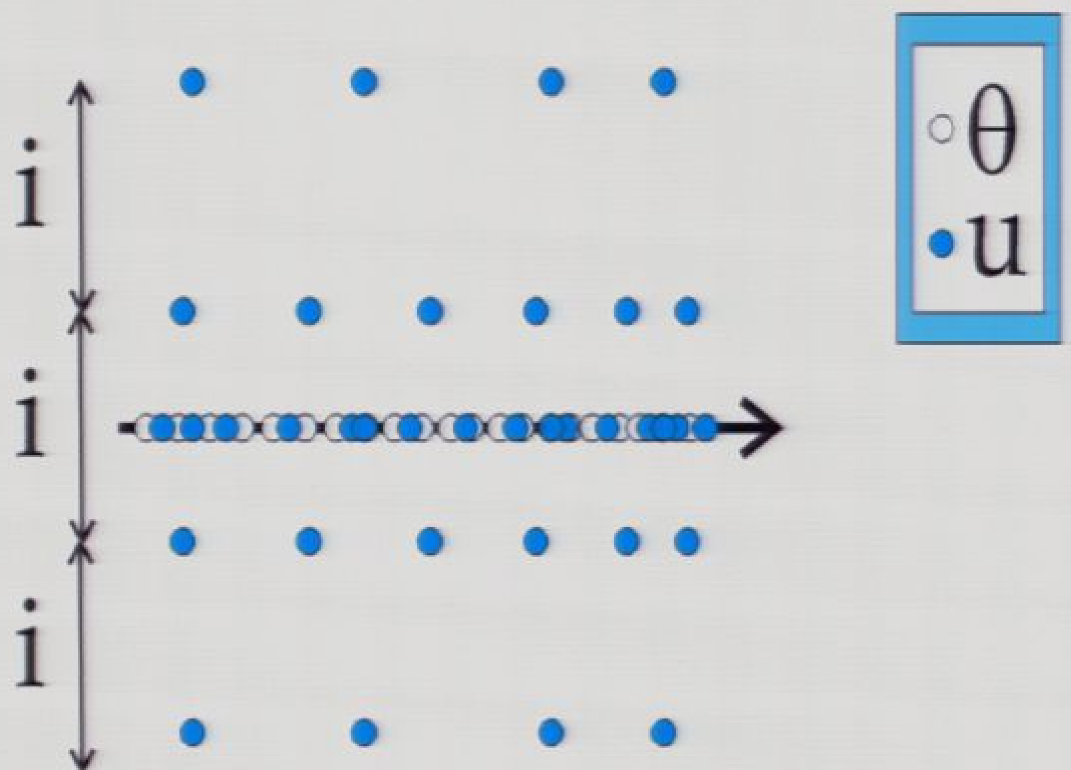
Ground state from ABA

- The typical configuration of roots is



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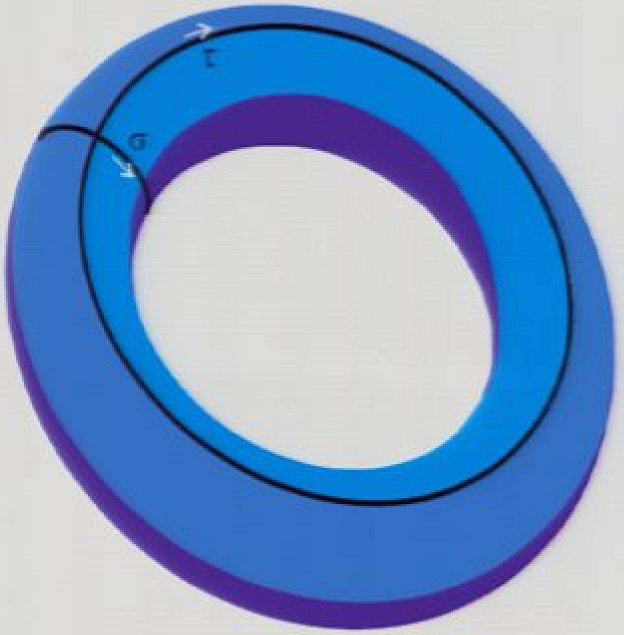

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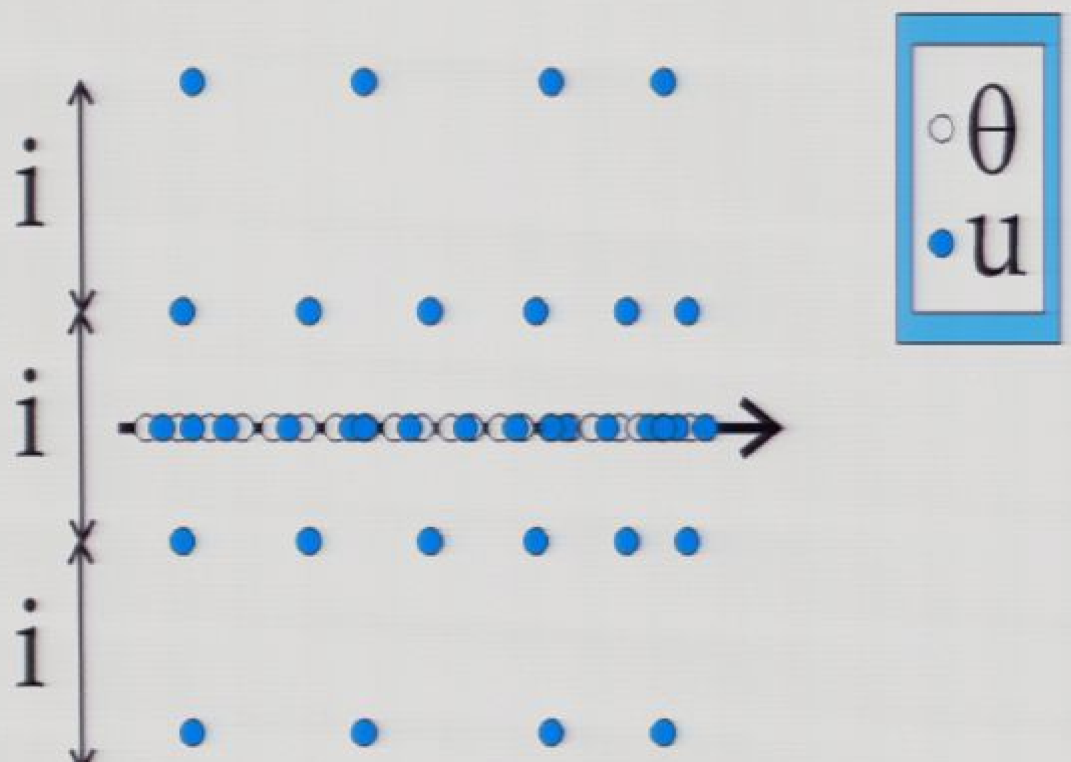

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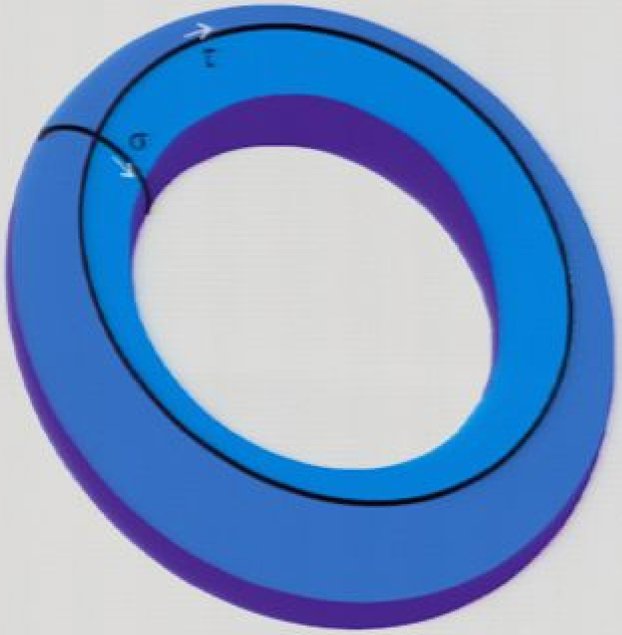

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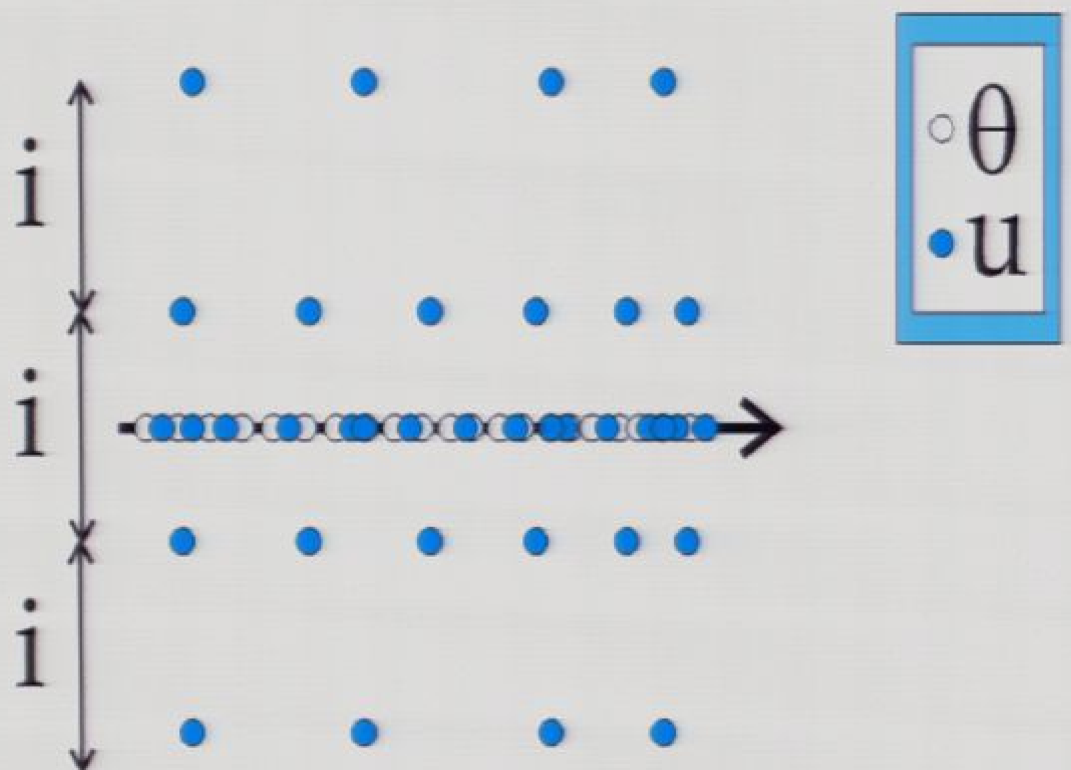

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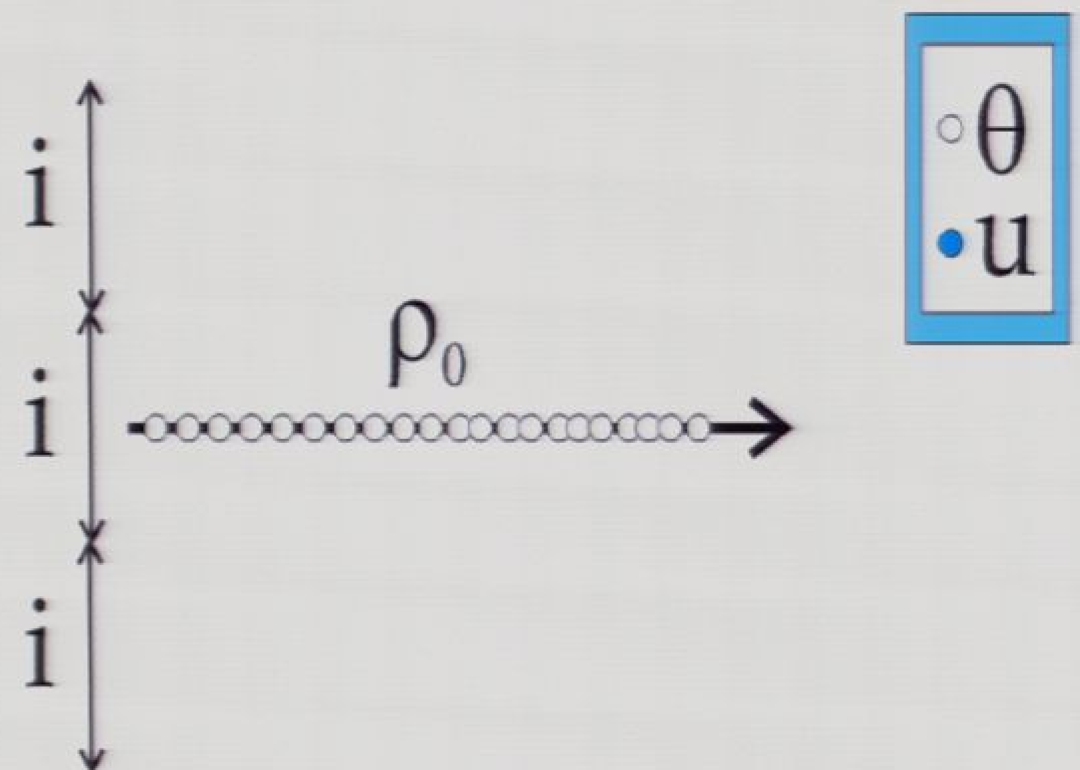

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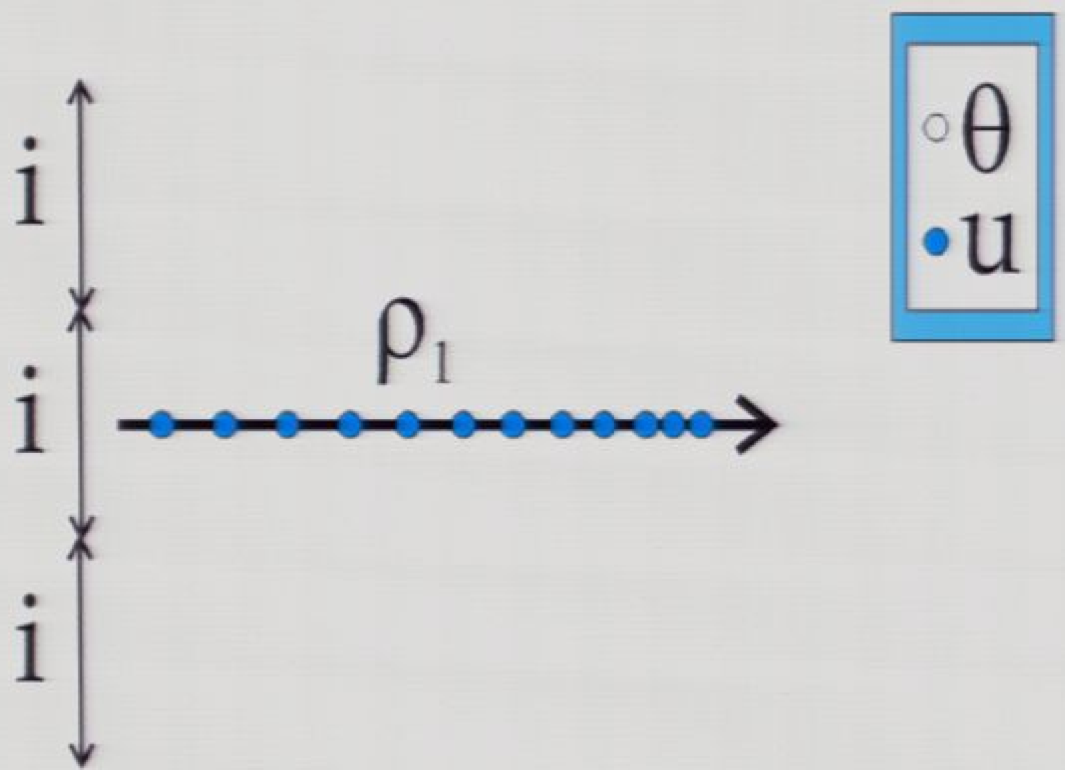

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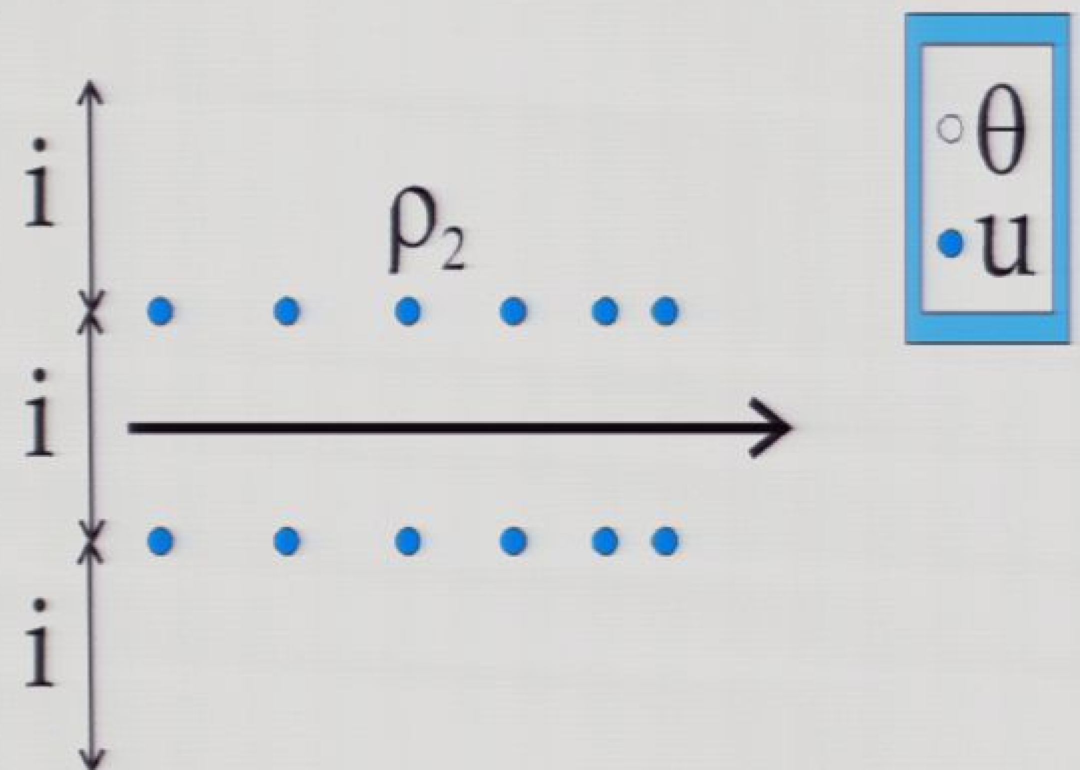
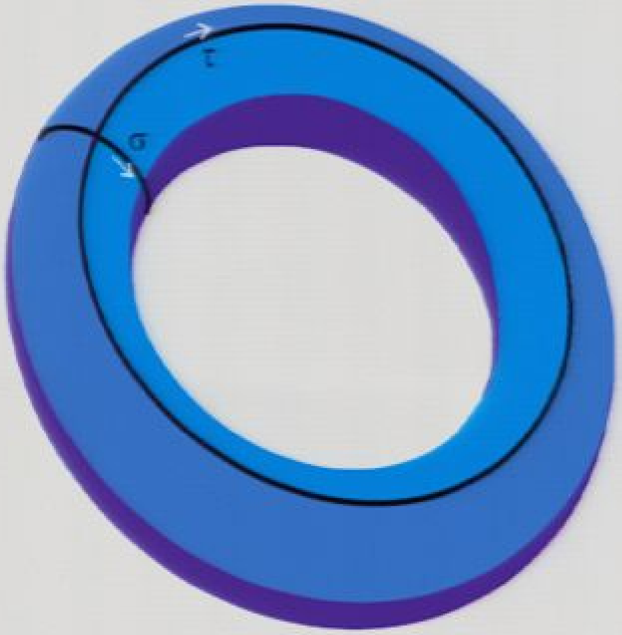


Vacuum



$$Z(\tau,\sigma) = Z(\sigma,\tau)$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$\sum e^{-E_n(L)R} \qquad \sum e^{-E_n(R)L}$$

$$\Downarrow$$

$$e^{-E_0(L)R}$$

I.e. from the asymptotical spectrum (infinite R) we can compute the Ground state energy for ANY finite volume!

$$E_0(L) = -\lim_{R\to\infty} \frac{\log \sum e^{-E_n(R)L}}{R}$$

S3 sigma model

Bethe ansatz:

$$1 = \prod_{\beta} \frac{u_j - \theta_\beta - i/2}{u_j - \theta_\beta + i/2} \prod_{i \neq j}^{J_u} \frac{u_j - u_i + i}{u_j - u_i - i}, \qquad su_R(2)$$

$$e^{-imL \sinh \pi\theta_\alpha} = \prod_{\beta \neq \alpha}^{L} S_0^2\left(\theta_\alpha - \theta_\beta\right) \prod_{j}^{J_u} \frac{\theta_\alpha - u_j + i/2}{\theta_\alpha - u_j - i/2} \prod_{k}^{J_v} \frac{\theta_\alpha - v_k + i/2}{\theta_\alpha - v_k - i/2},$$

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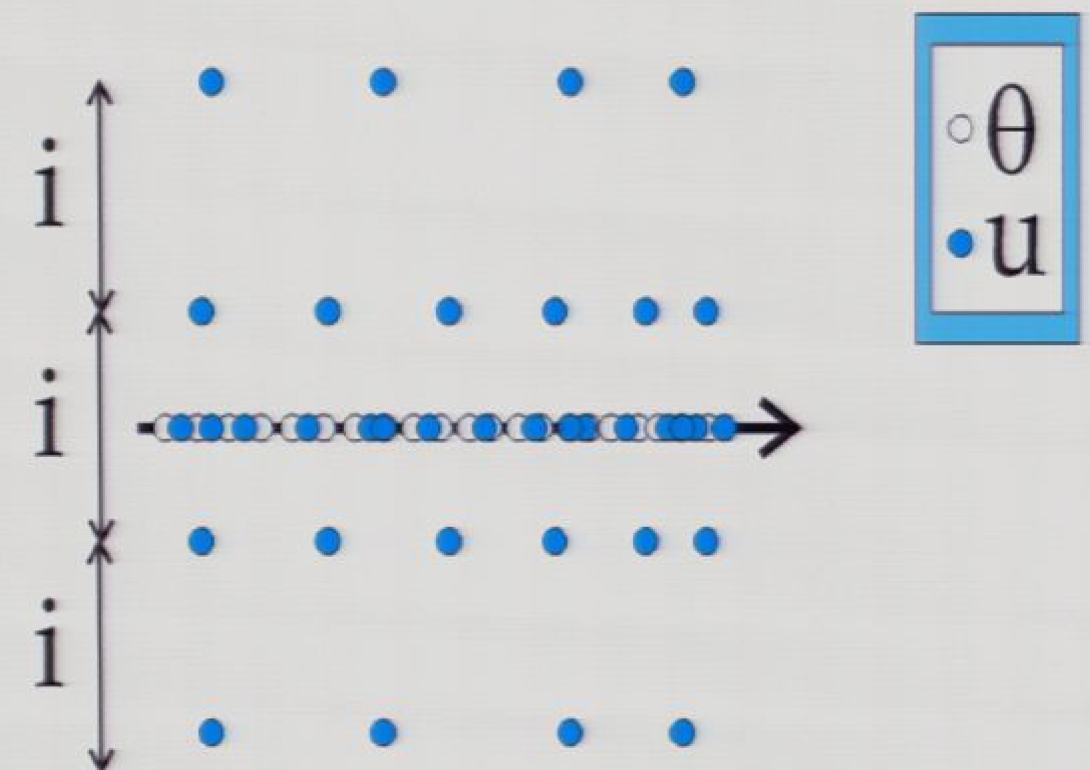

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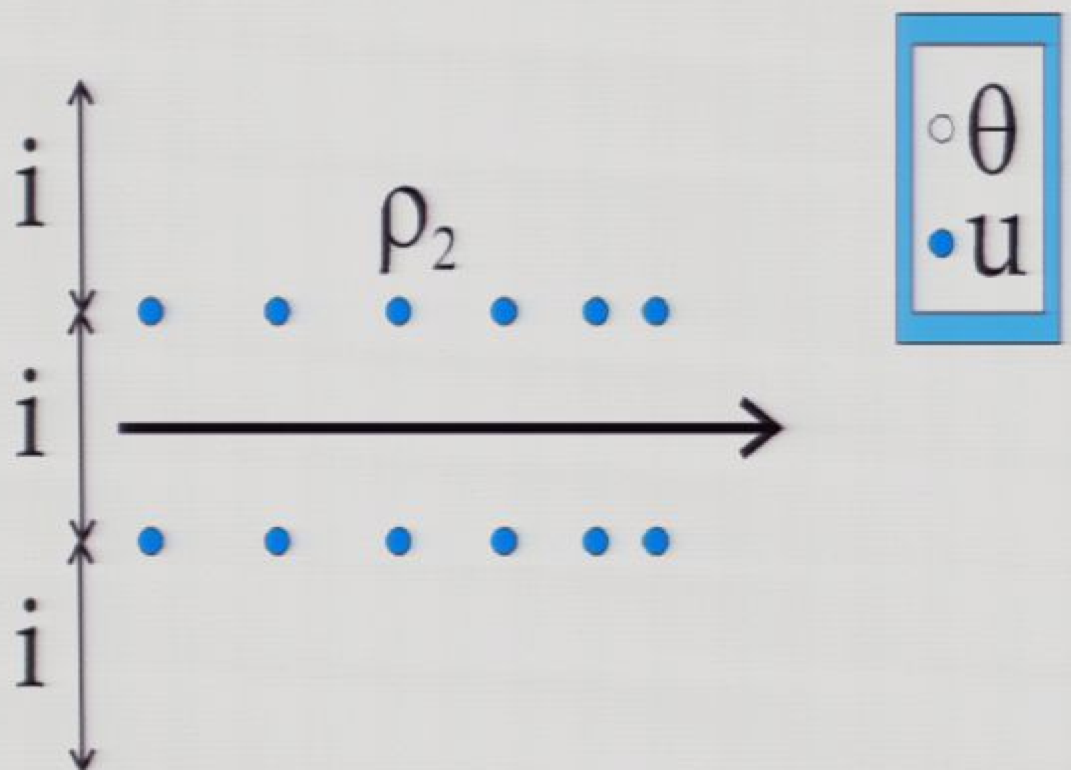

We define:

$$Y_k = \frac{\bar{\rho}_k}{\rho_k}$$

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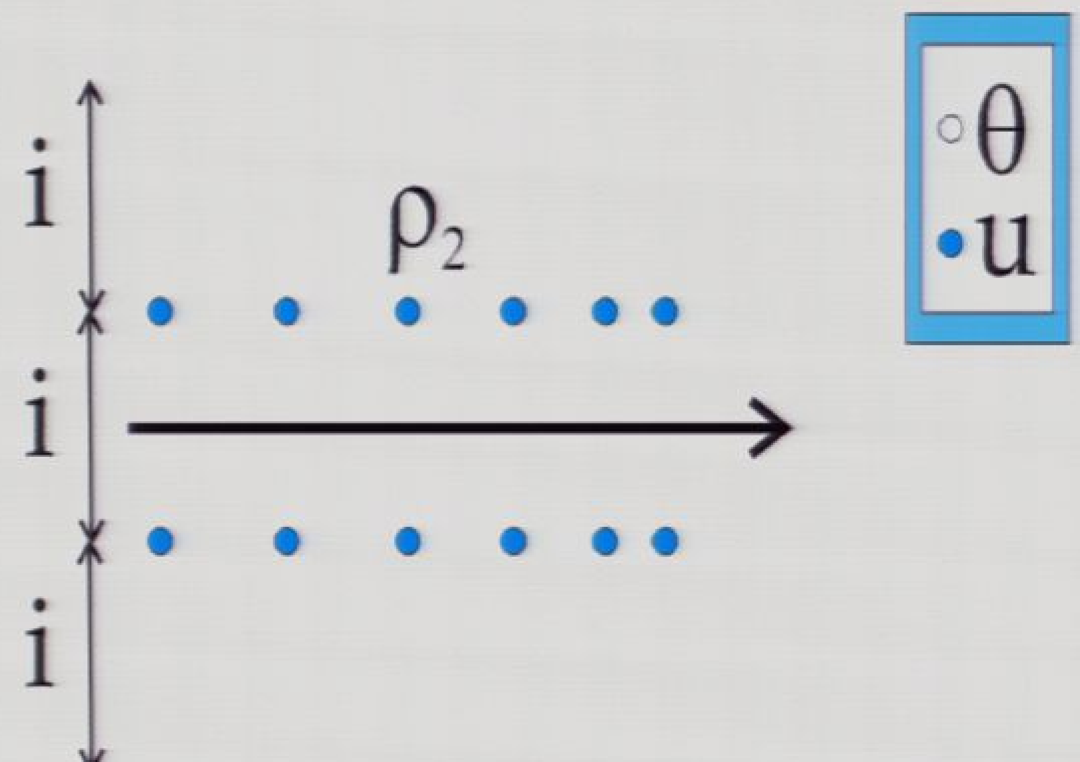

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Saddle point equation:

$$Y_n^+ Y_n^- = (1 + Y_{n+1})(1 + Y_{n-1})$$

$$E(L) = -\frac{1}{2}\int m \cosh(\pi x) \log(1 + Y_0)$$

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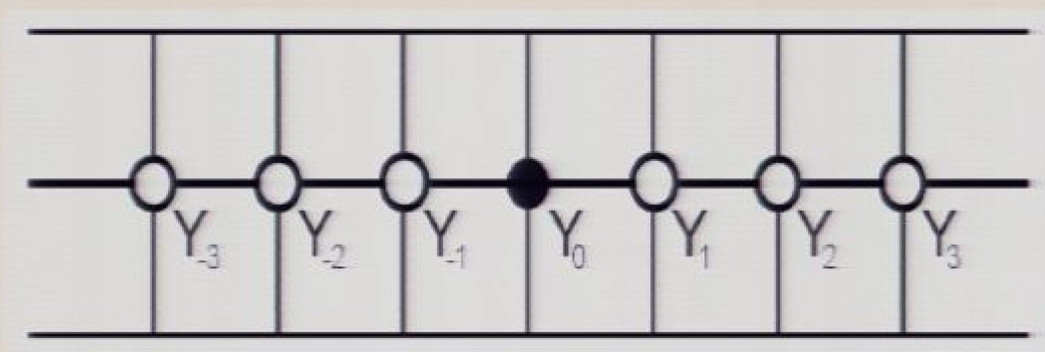

Above ground state

Dorey, Totteo,
Bazhanov

$$E(L) = -\frac{1}{2}\int m\cosh(\pi x)\log(1+Y_0)$$

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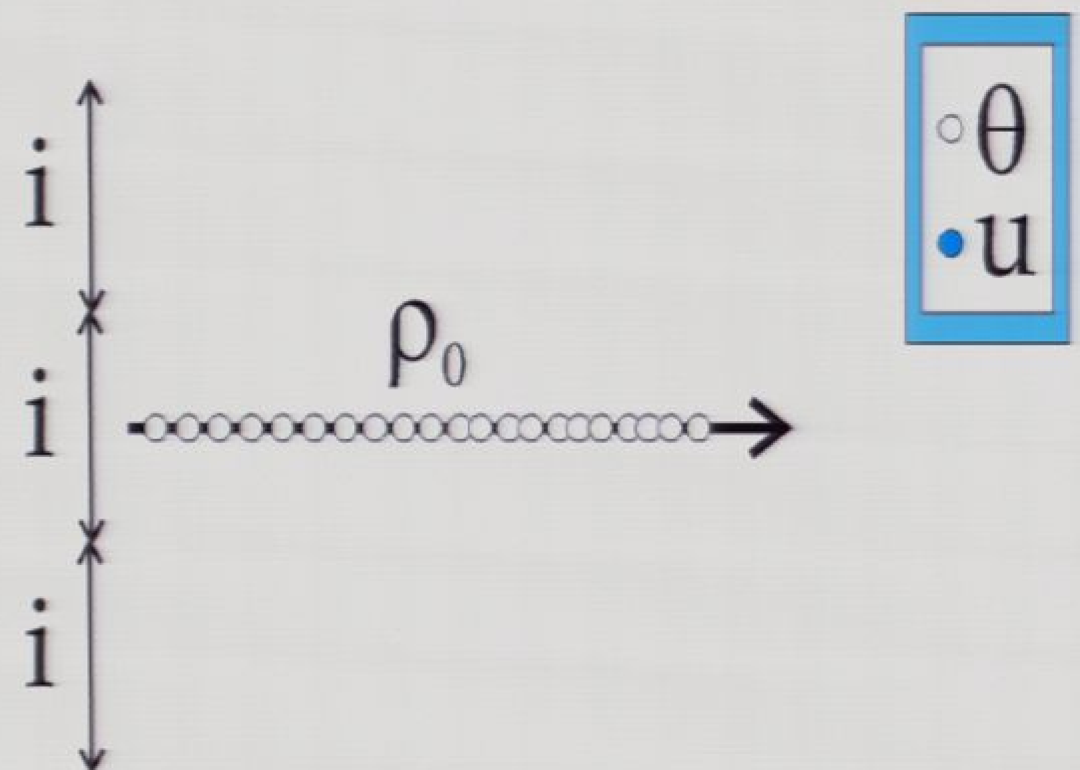

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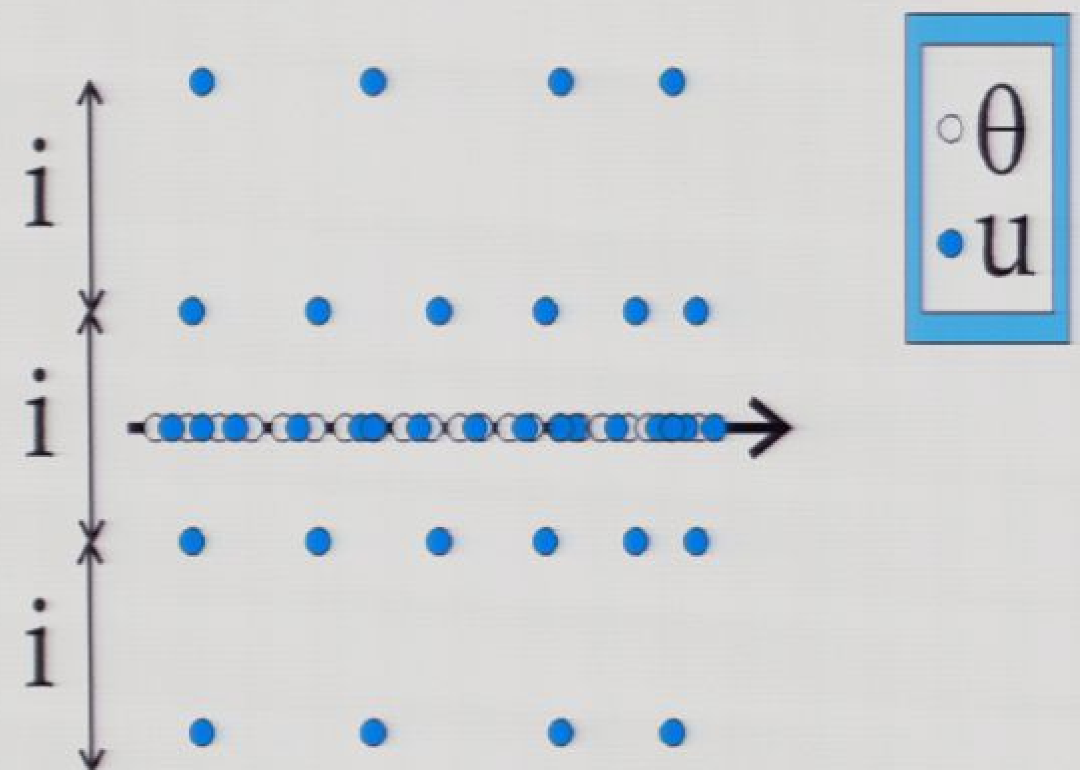

S3 sigma model

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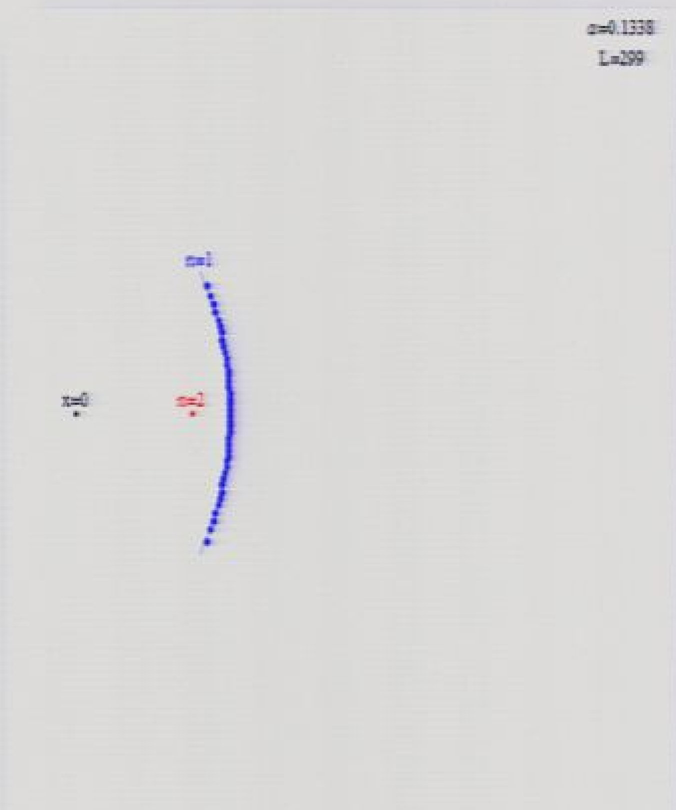
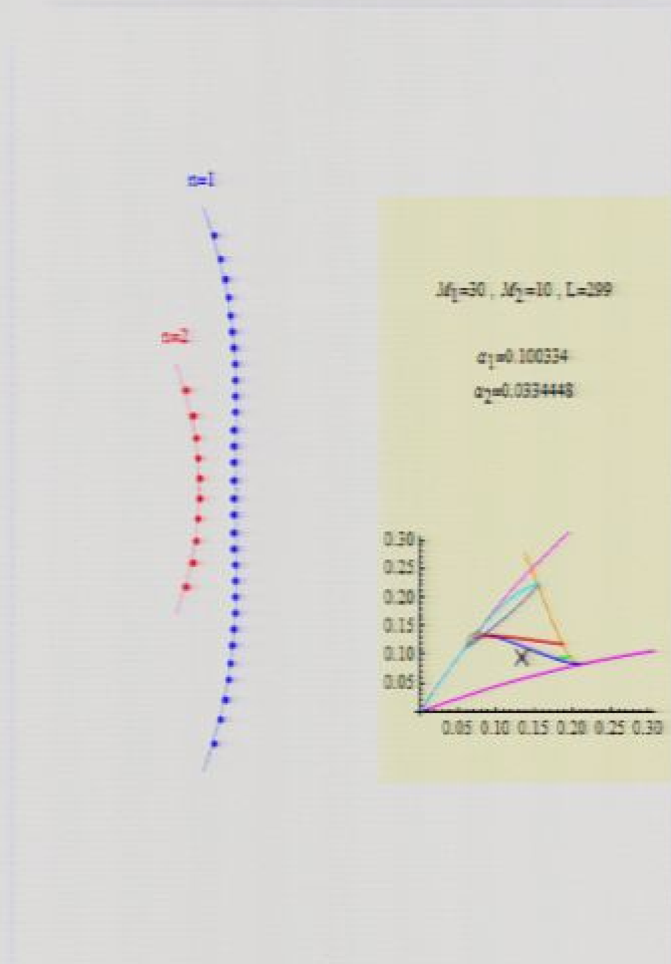
Numerical Solution





Numerical Solution

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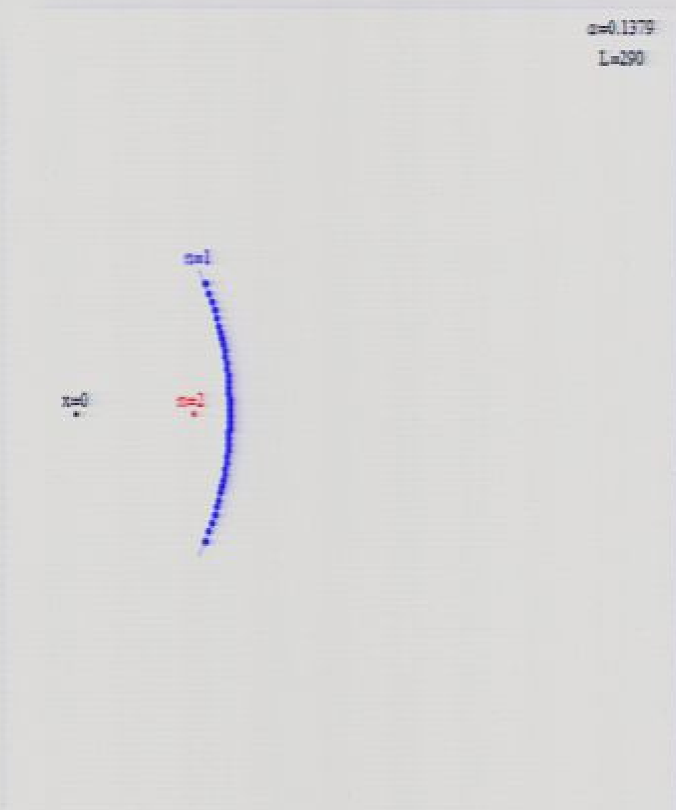

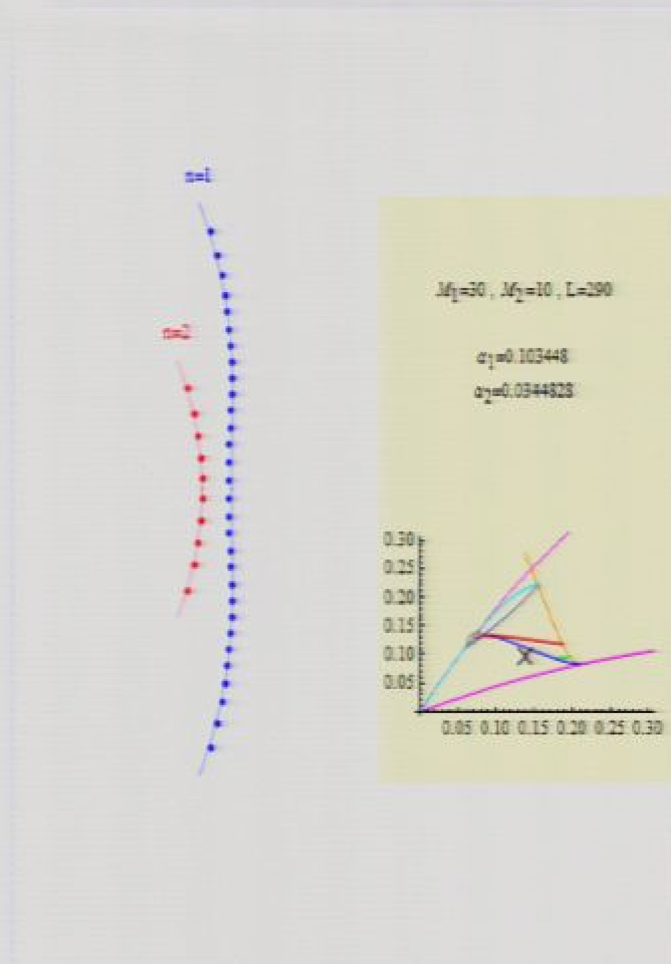

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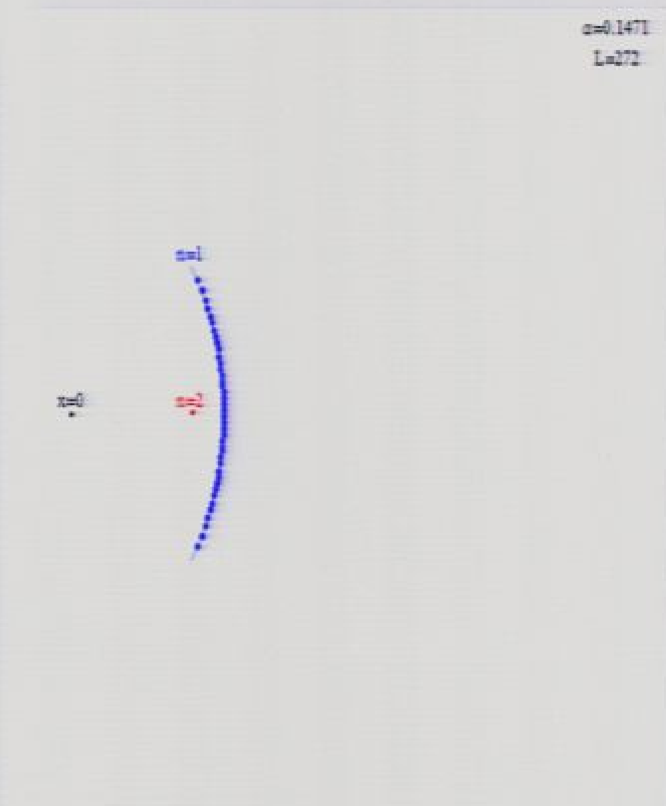

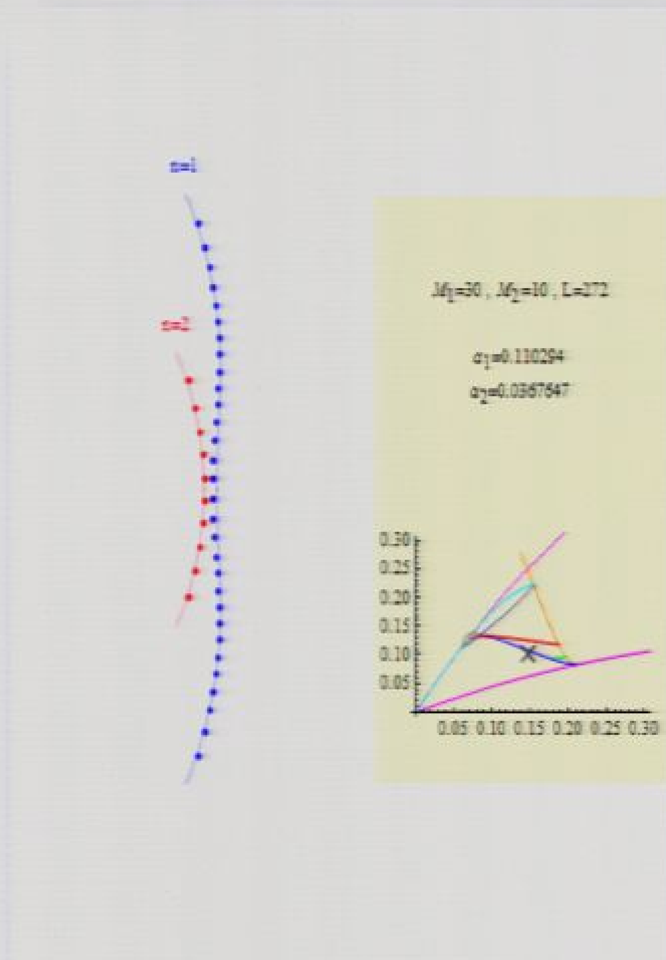

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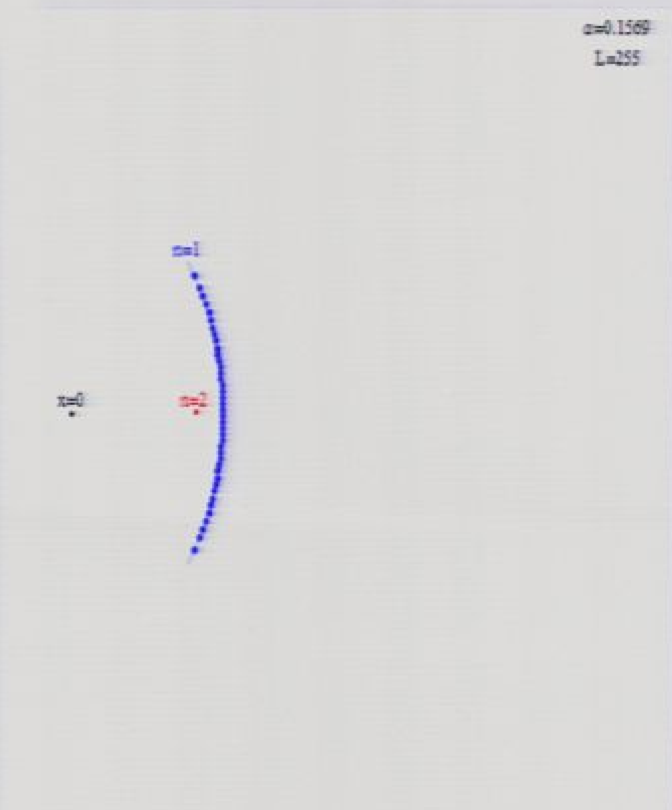

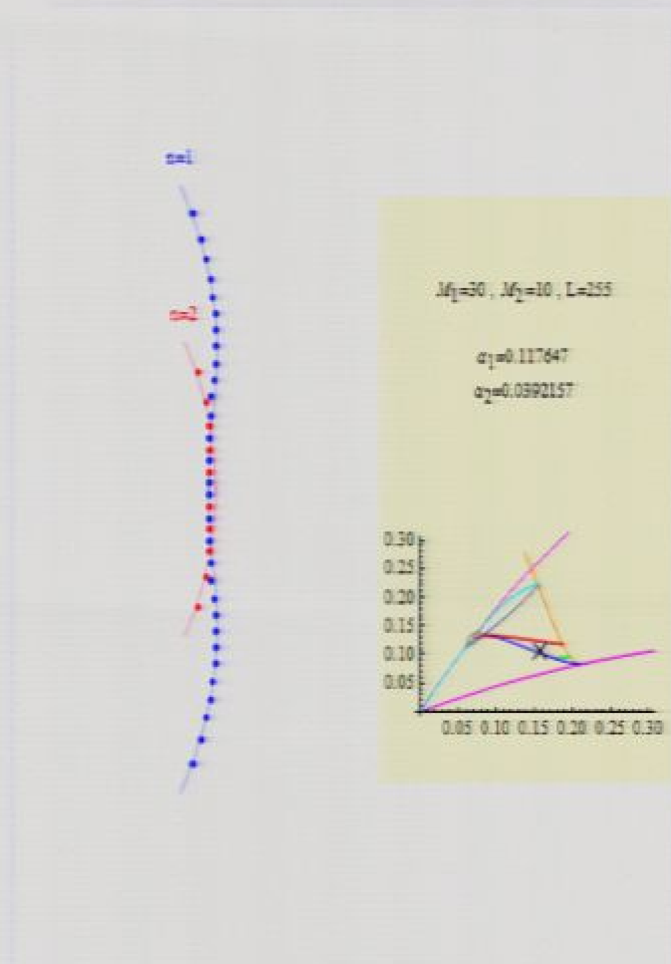

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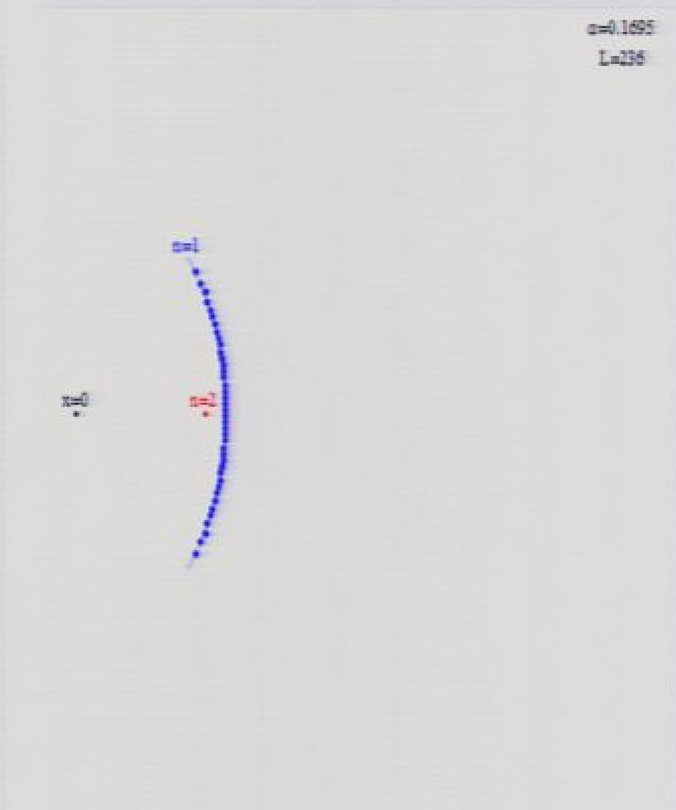

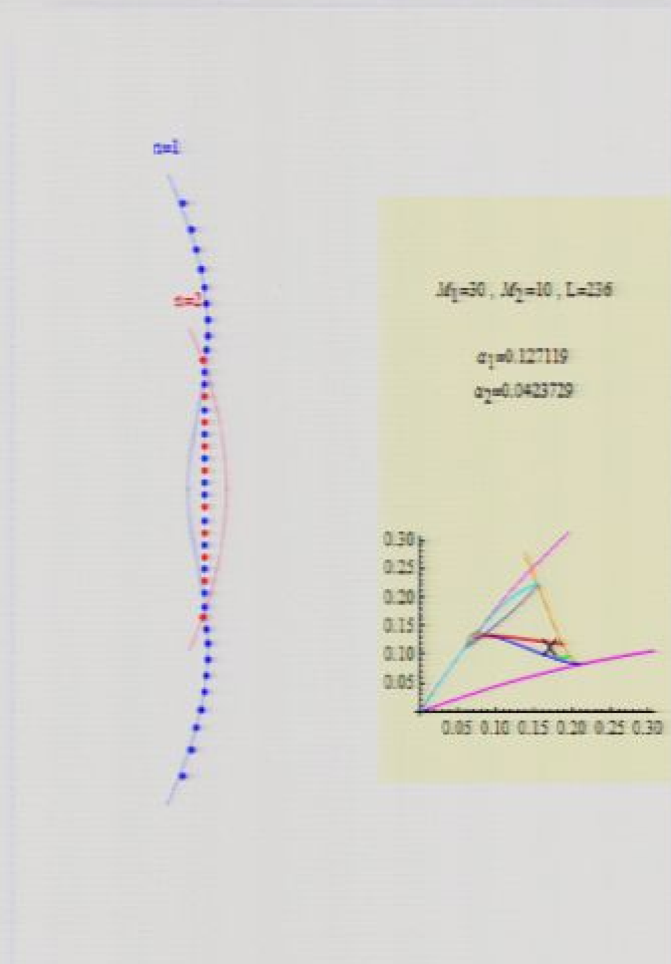

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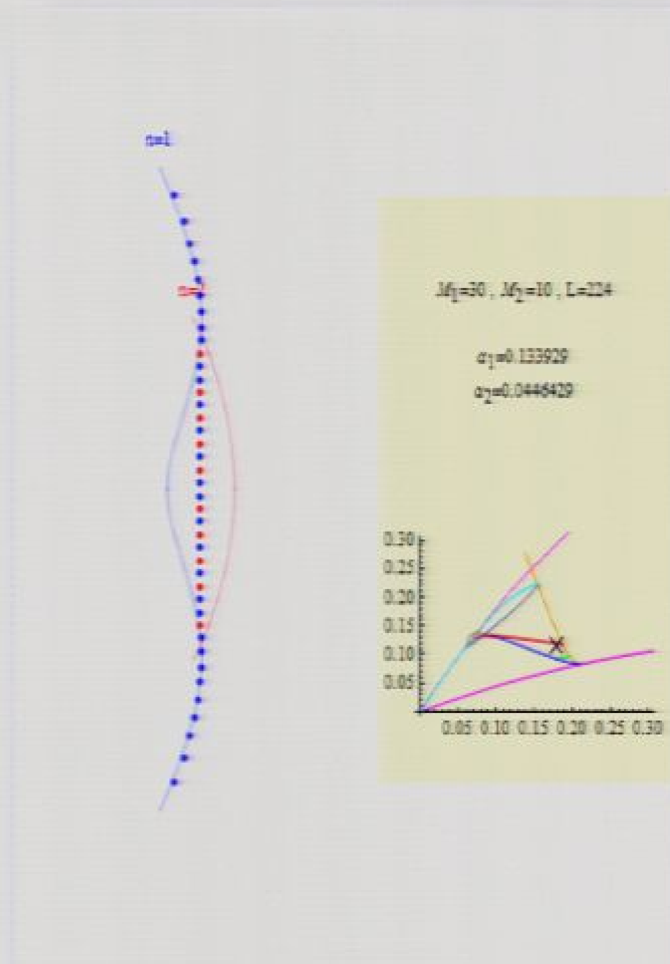

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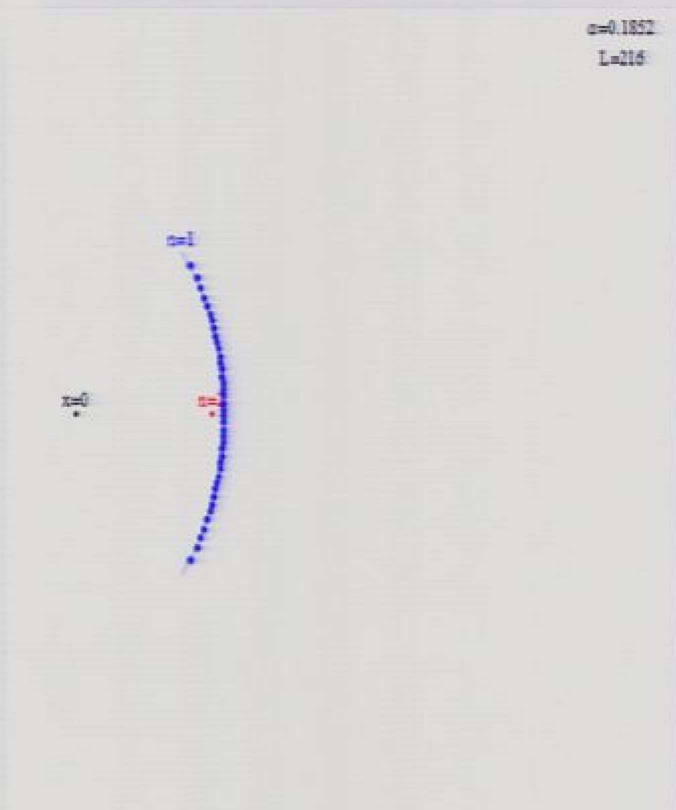

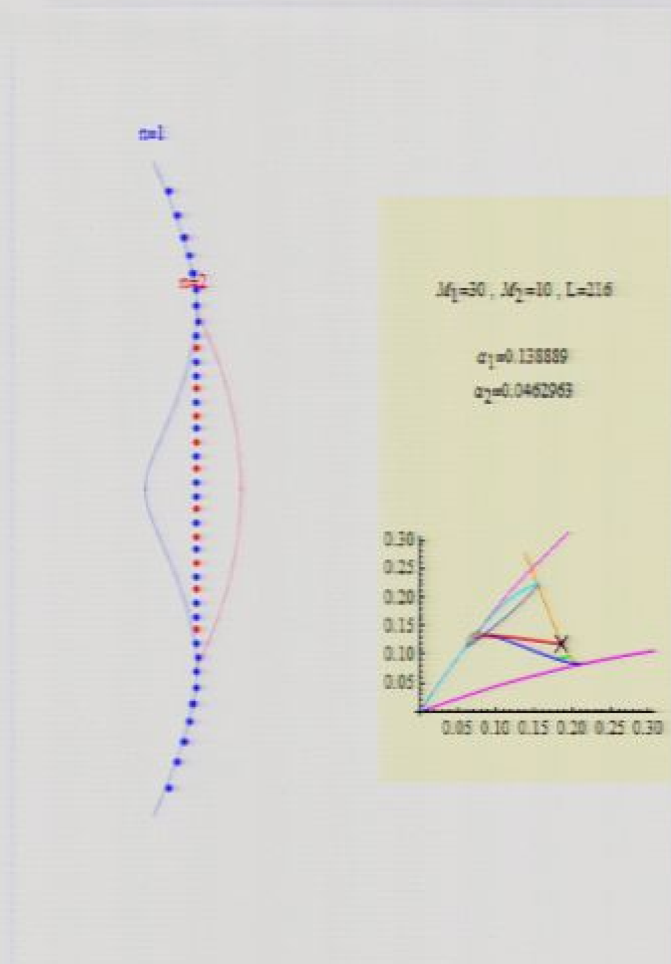

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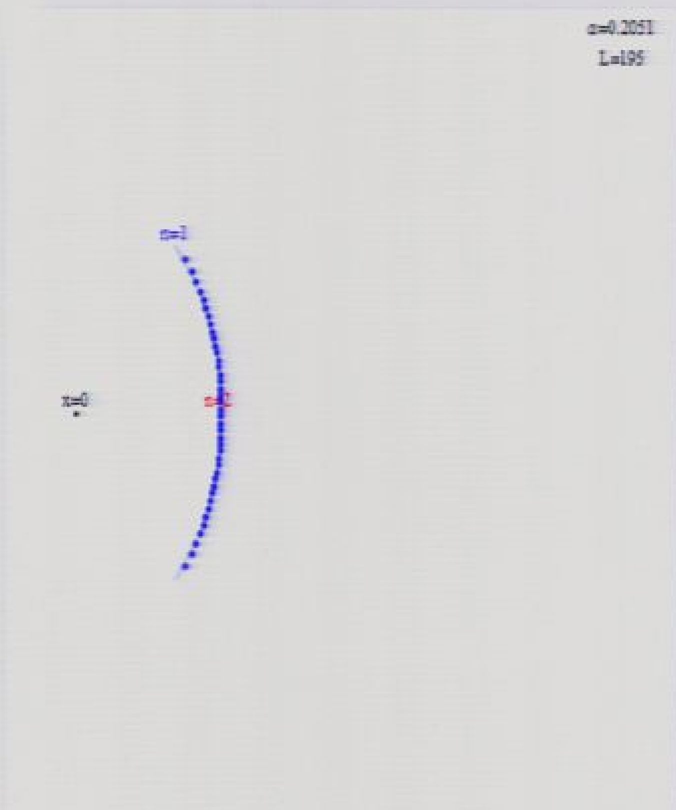

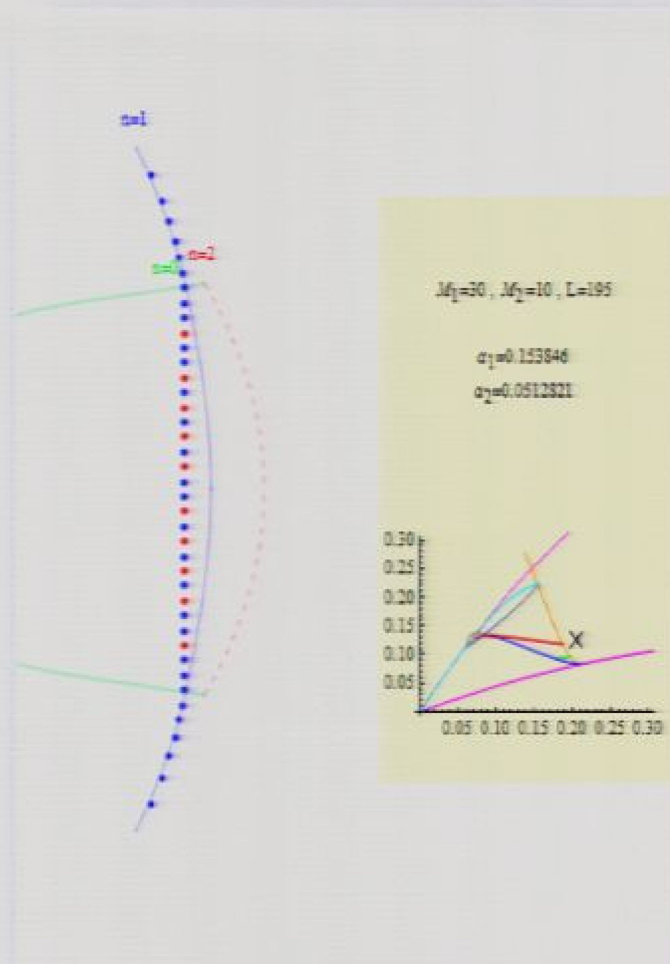

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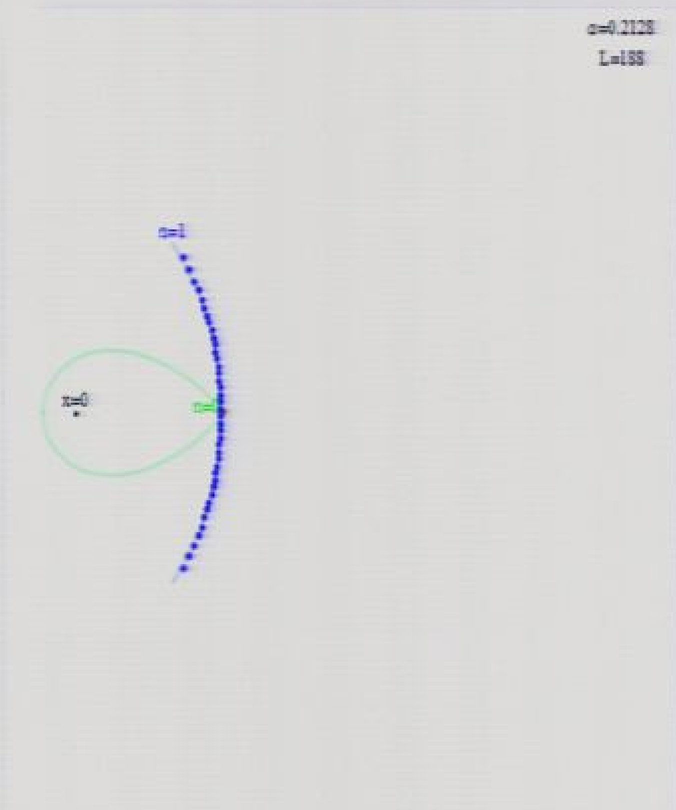

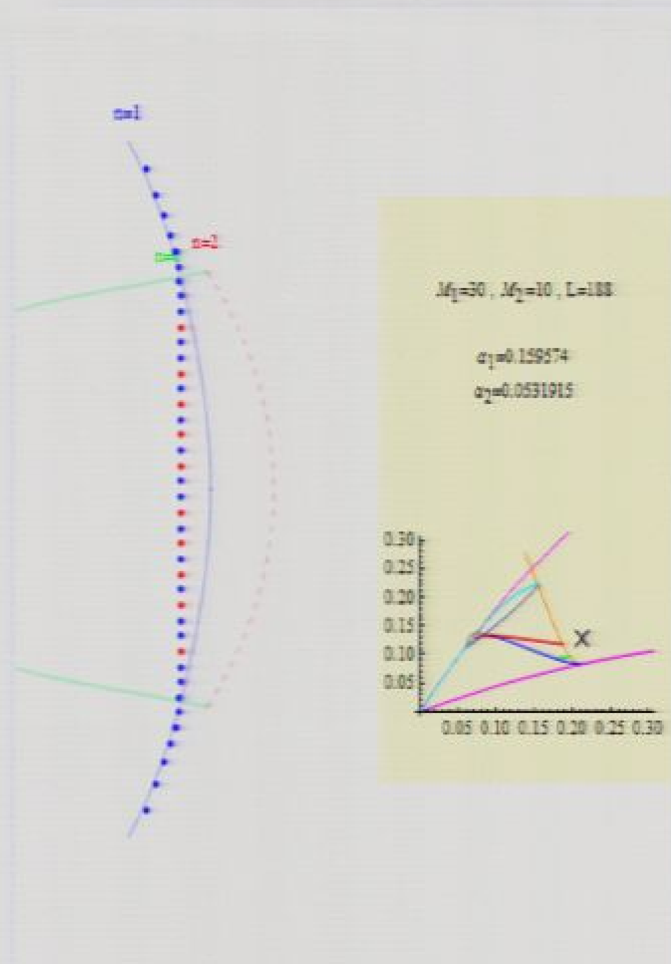

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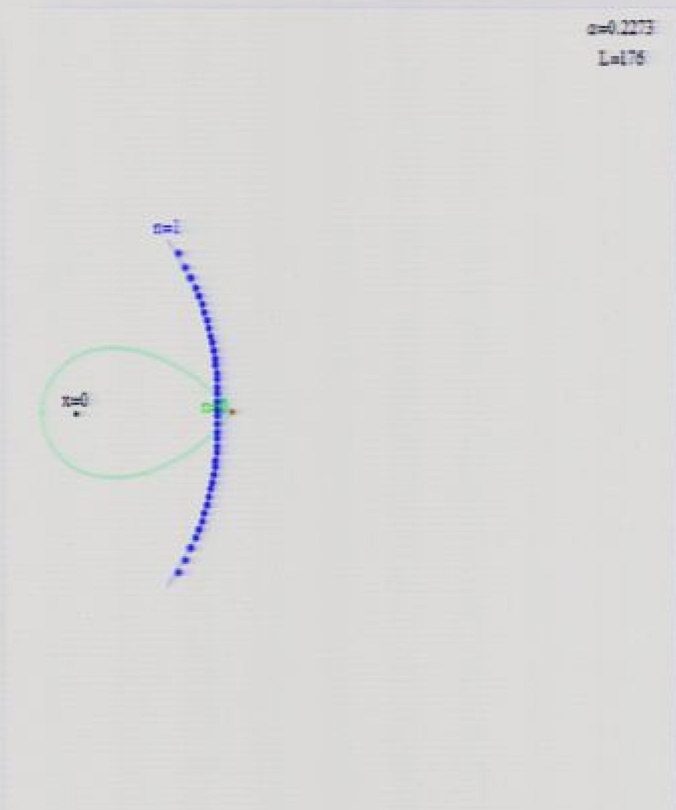

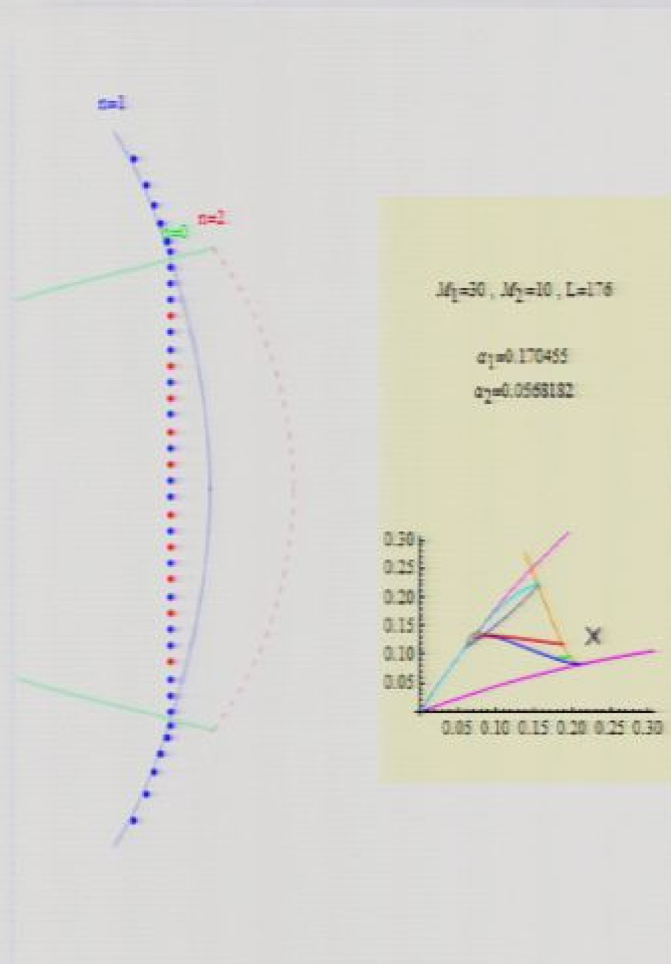

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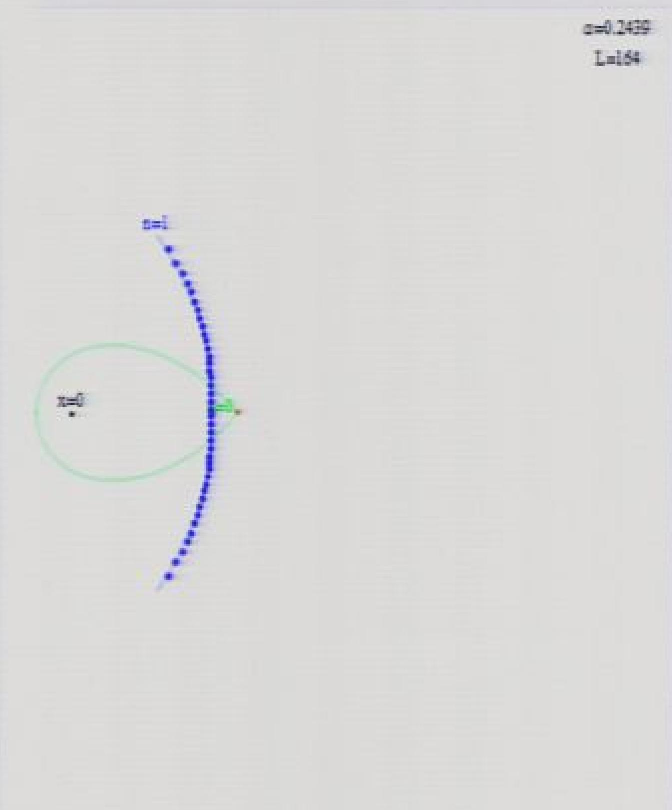

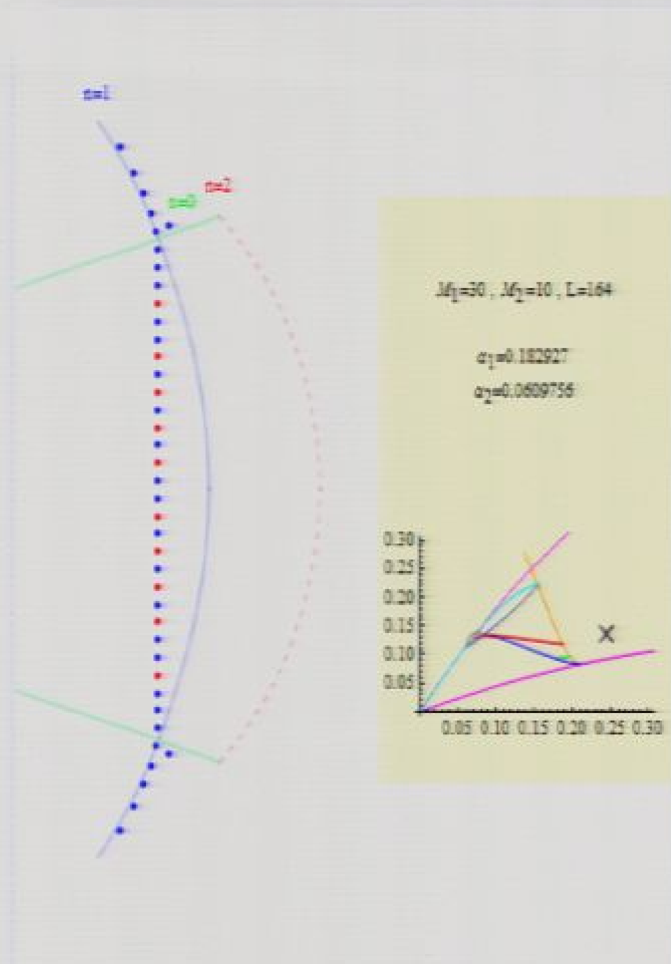

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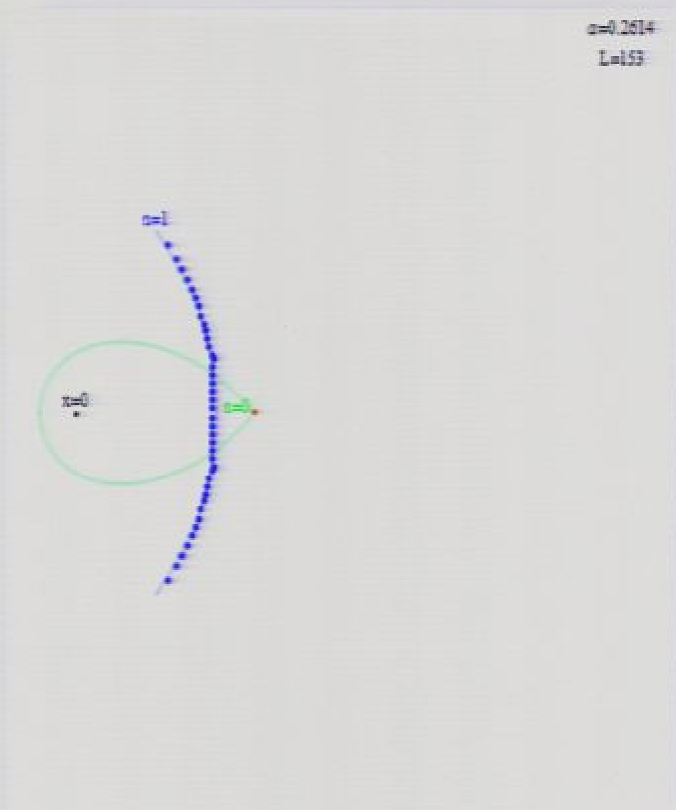

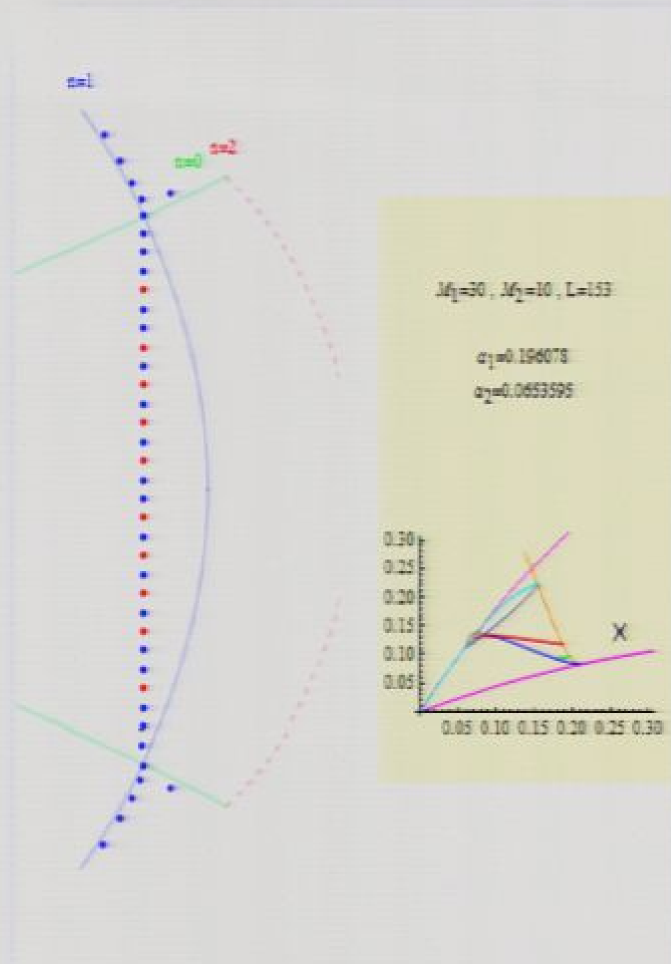

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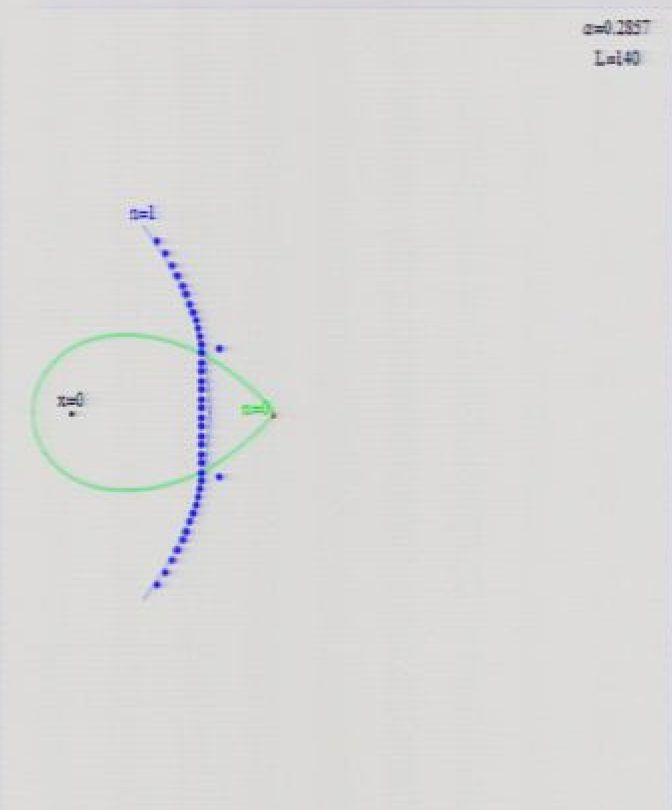

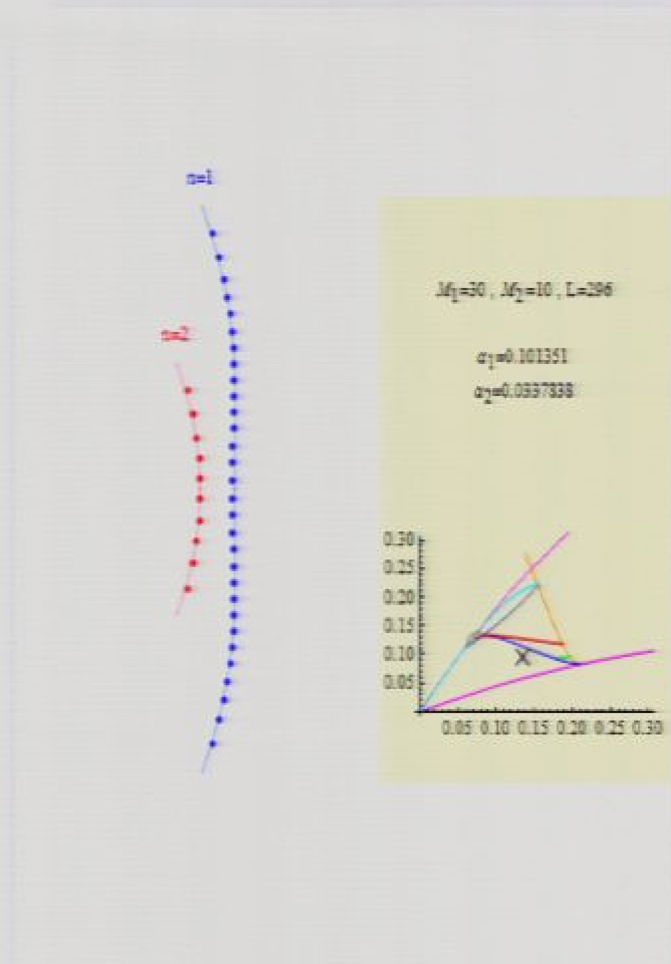

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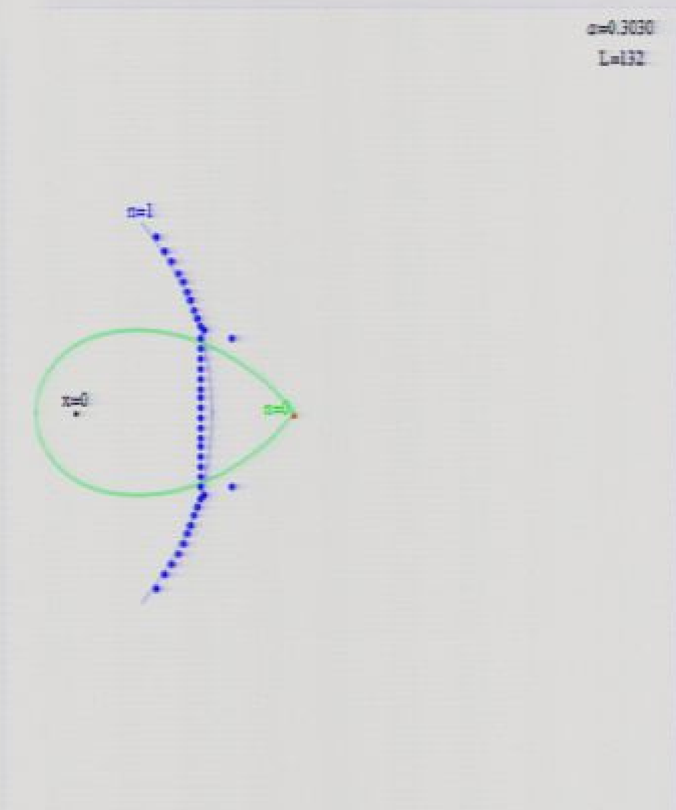

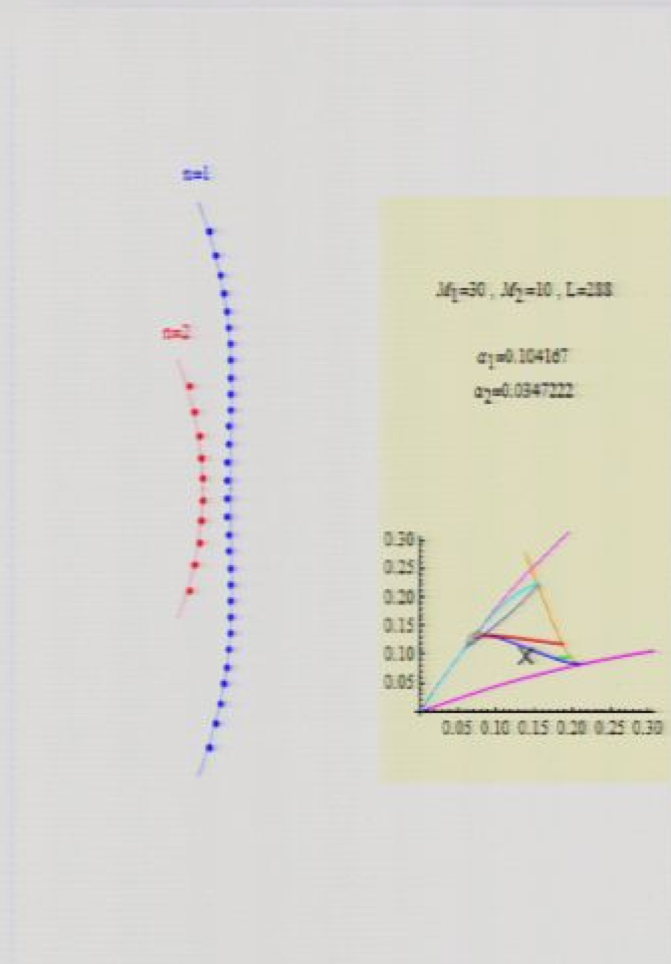

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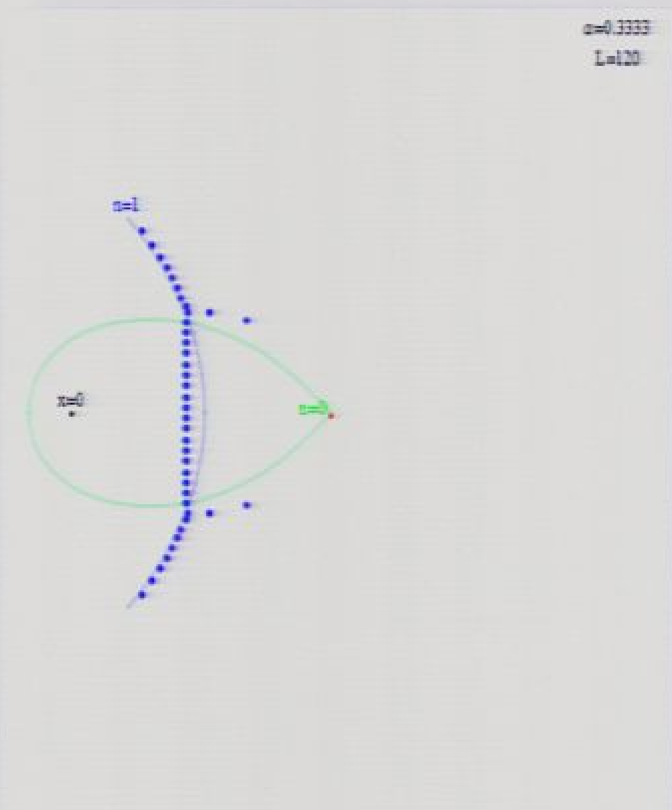

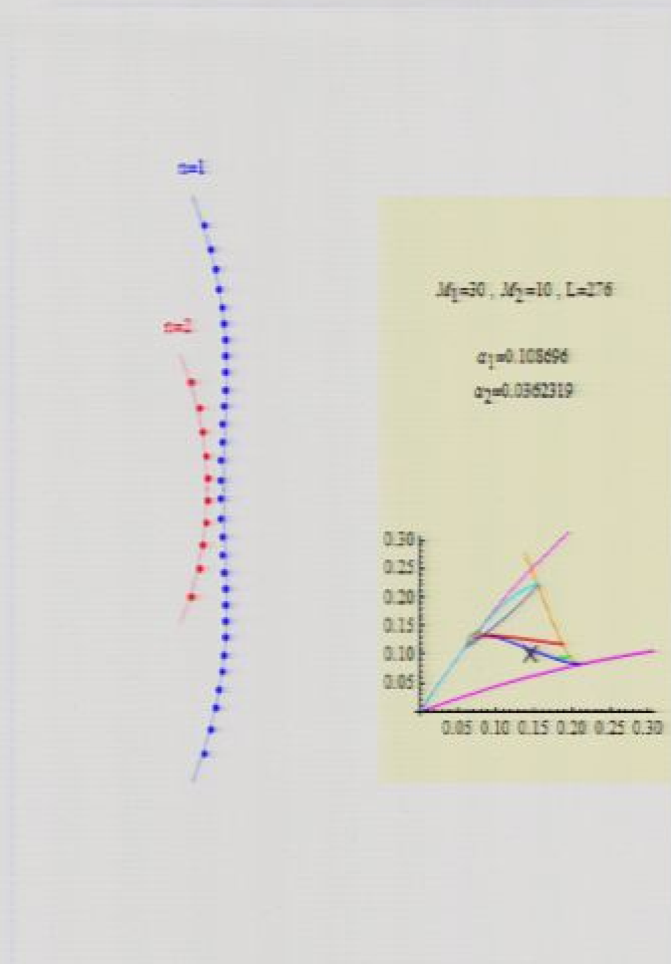

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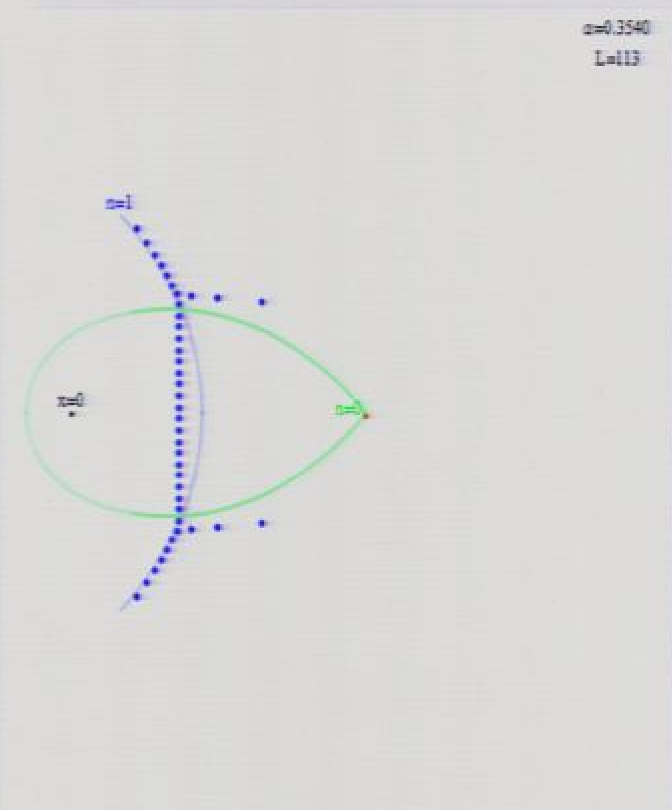

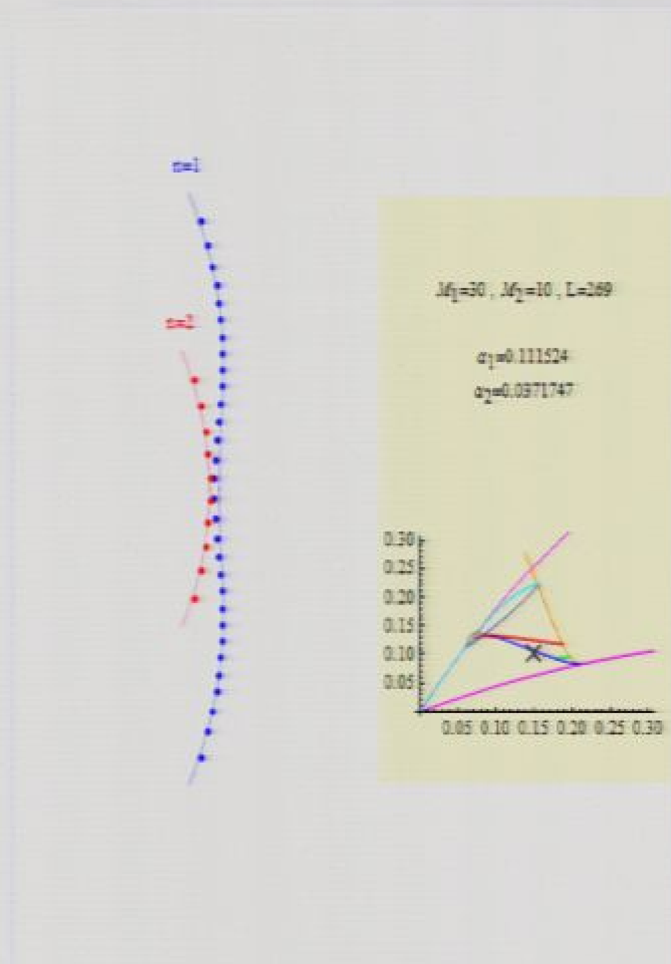

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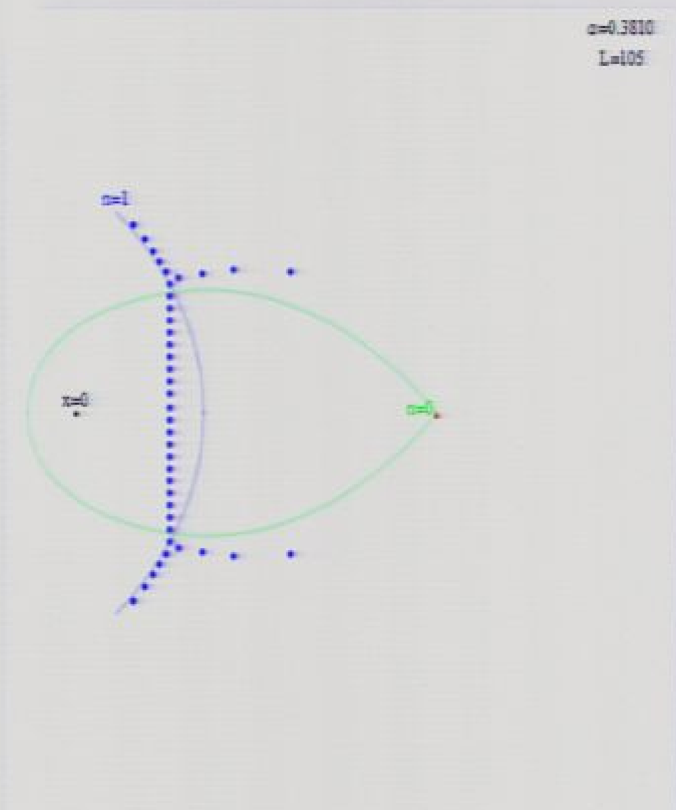

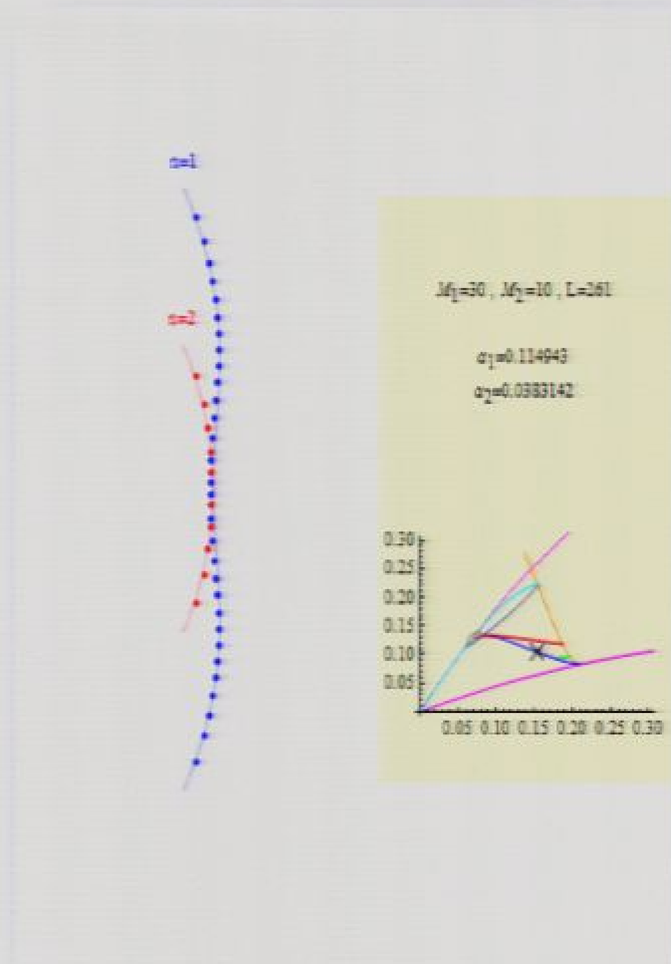

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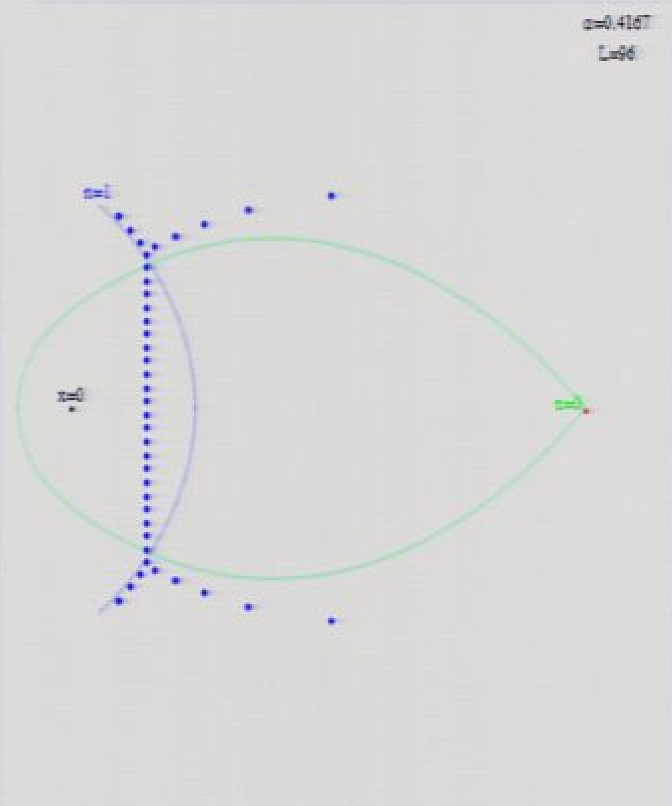

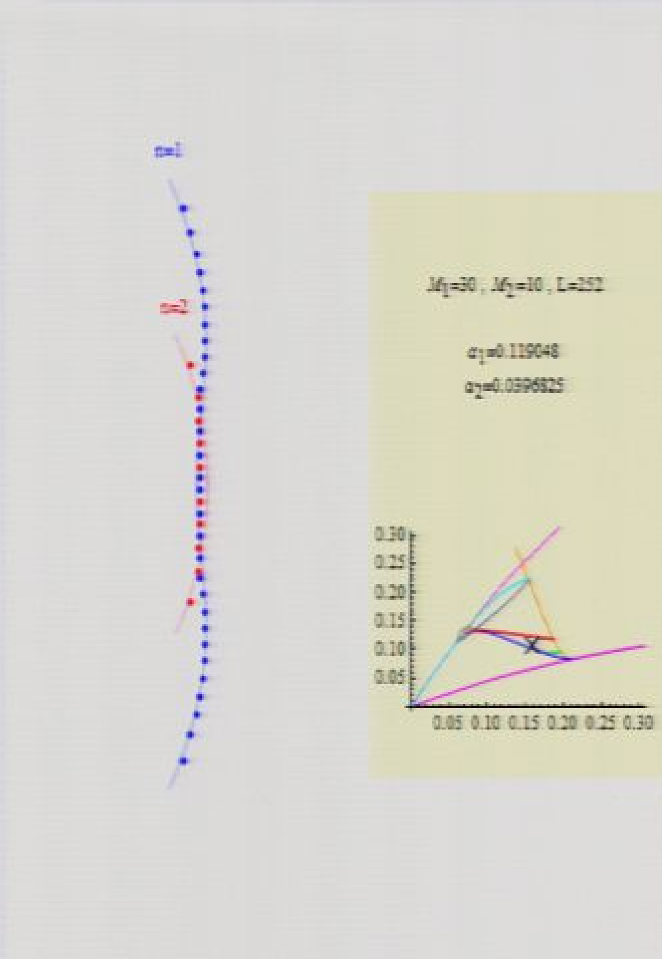

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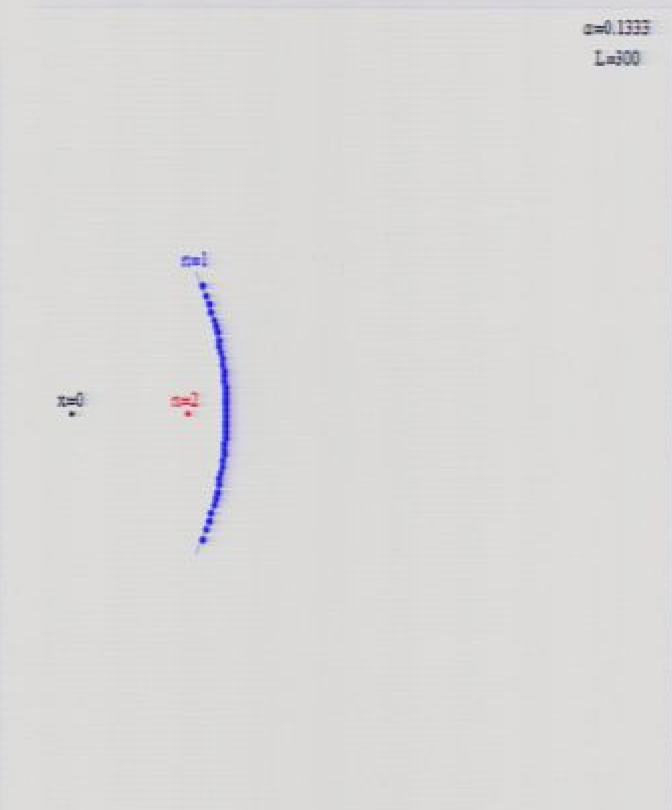

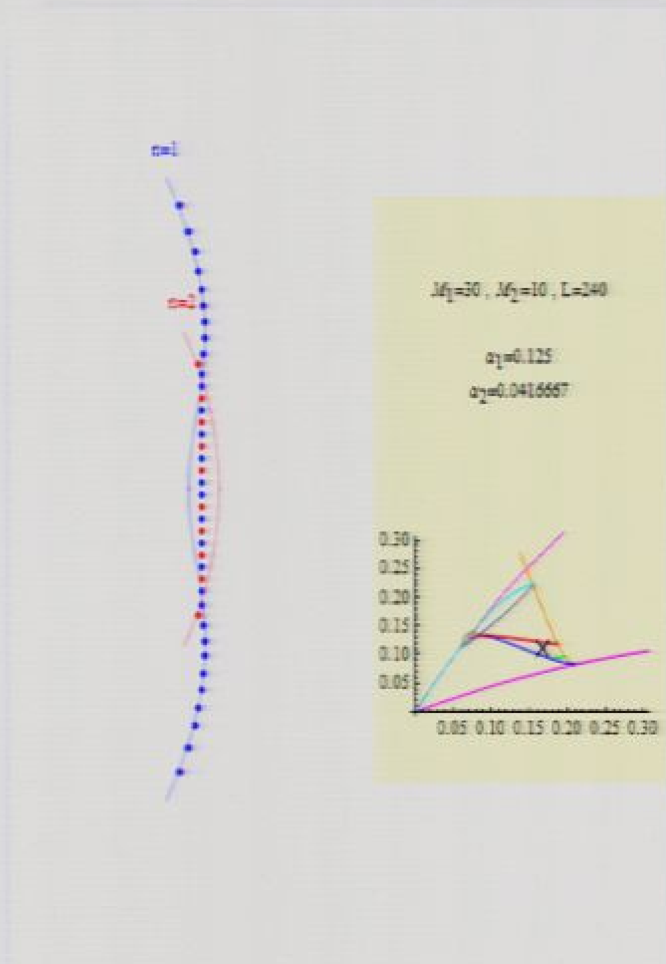

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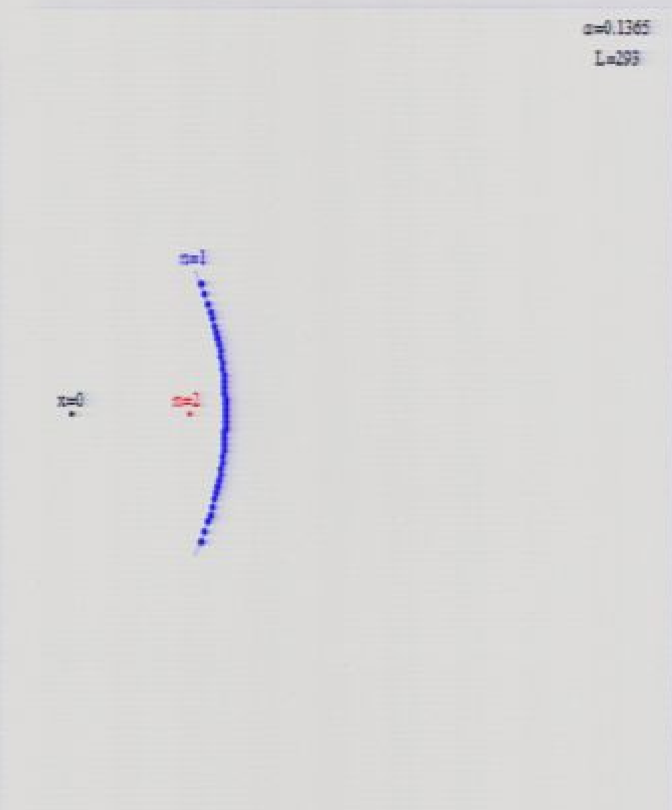

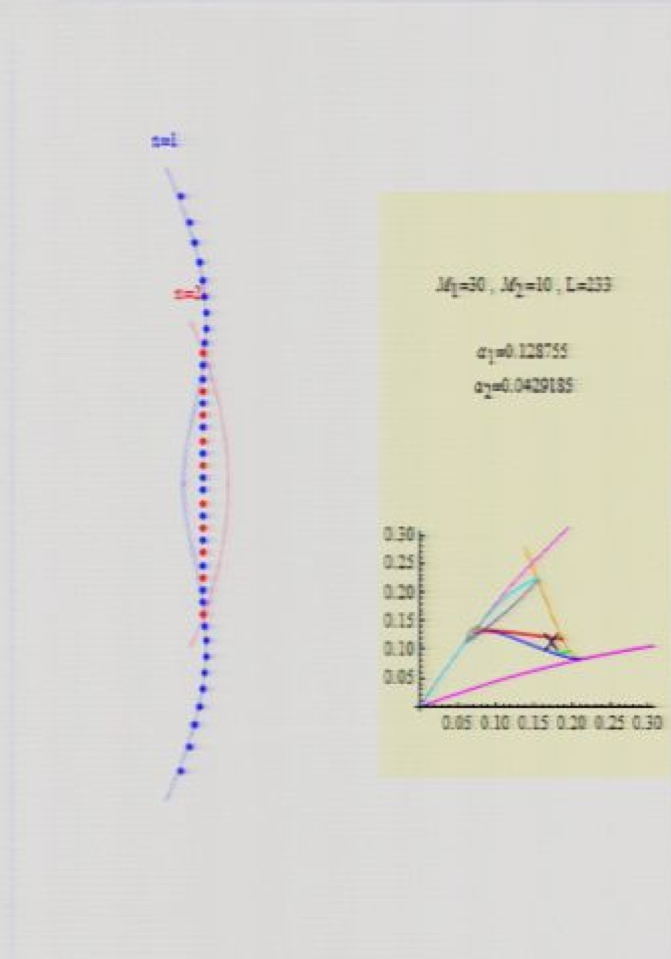

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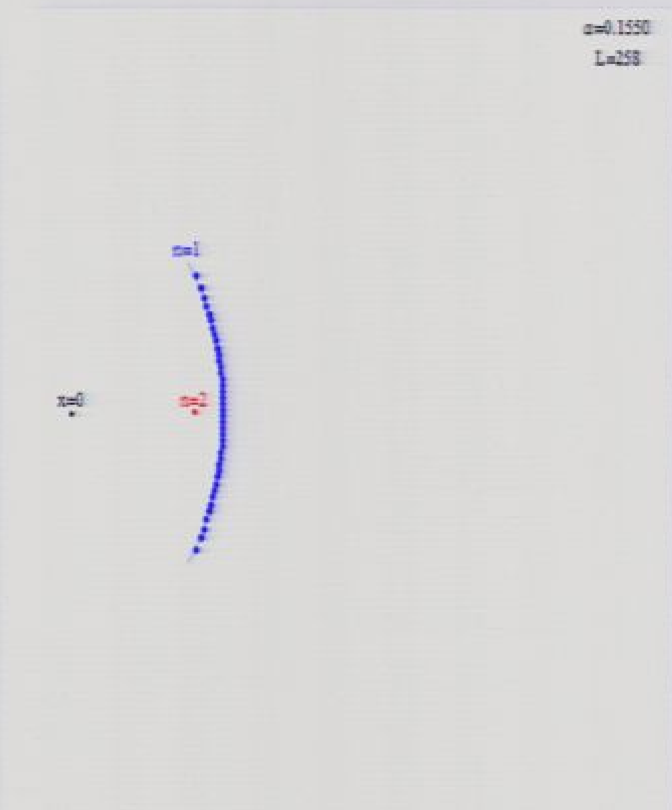

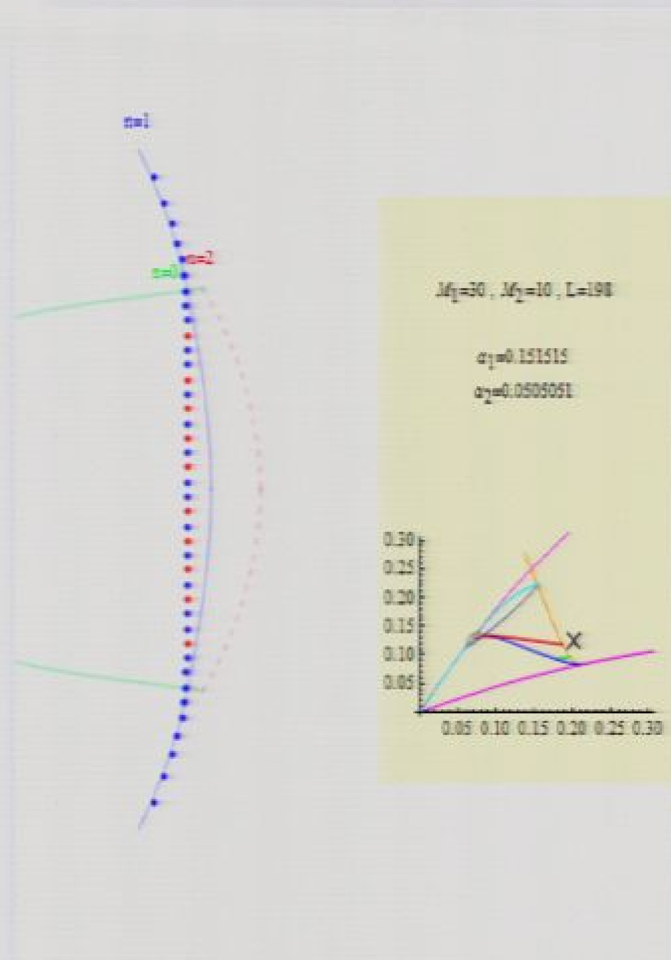

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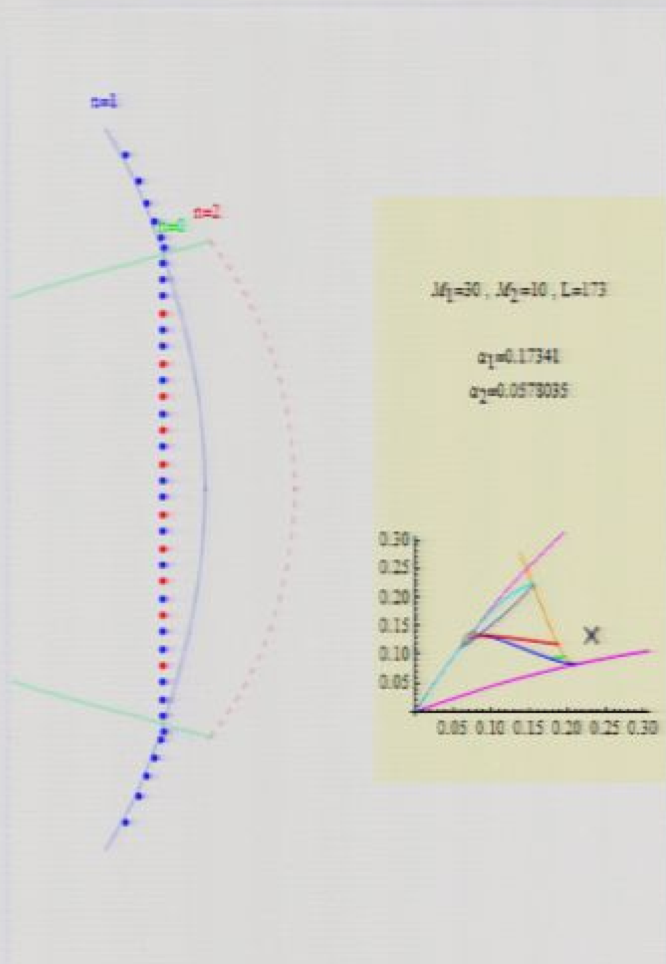
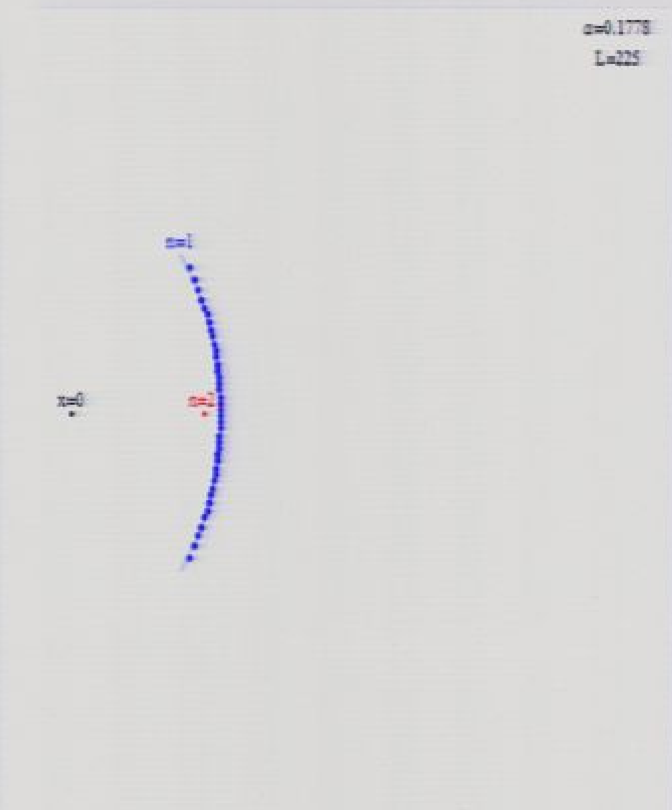
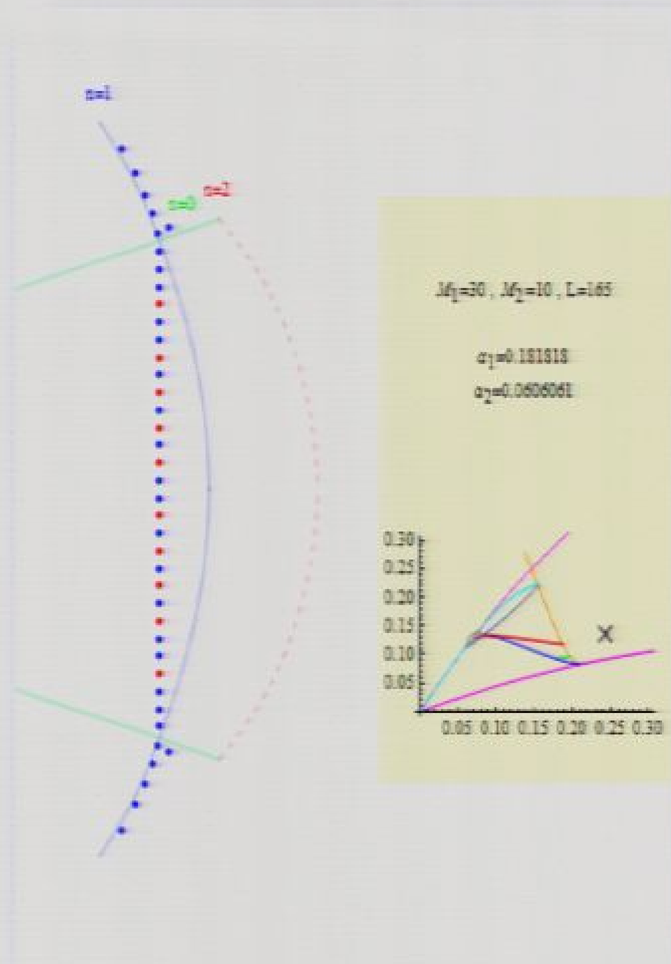


Numerical Solution

$$\left(\frac{u_j + i/2}{u_j - i/2}\right)^L = \prod_{k=1(k\neq j)}^{J} \frac{u_j - u_k + i}{u_j - u_k - i}$$





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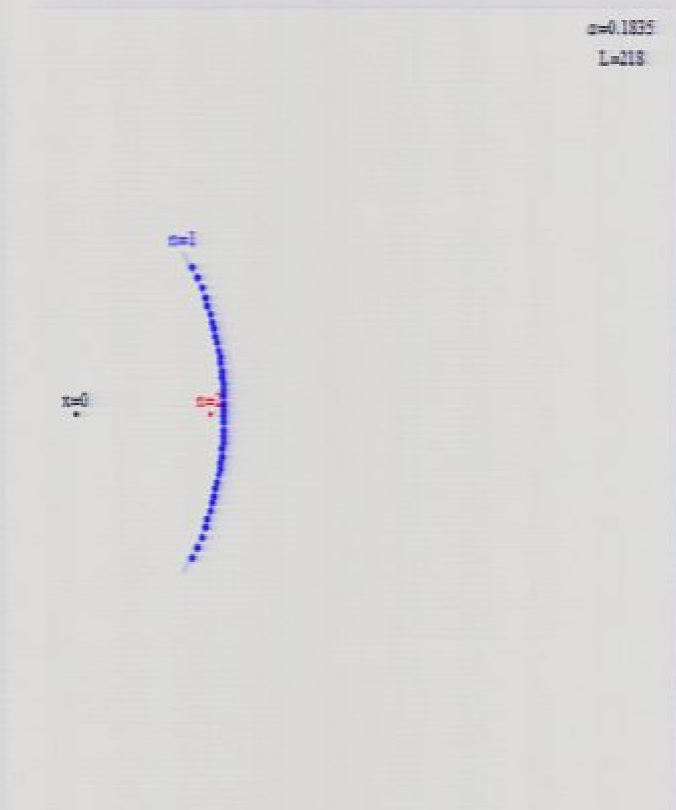

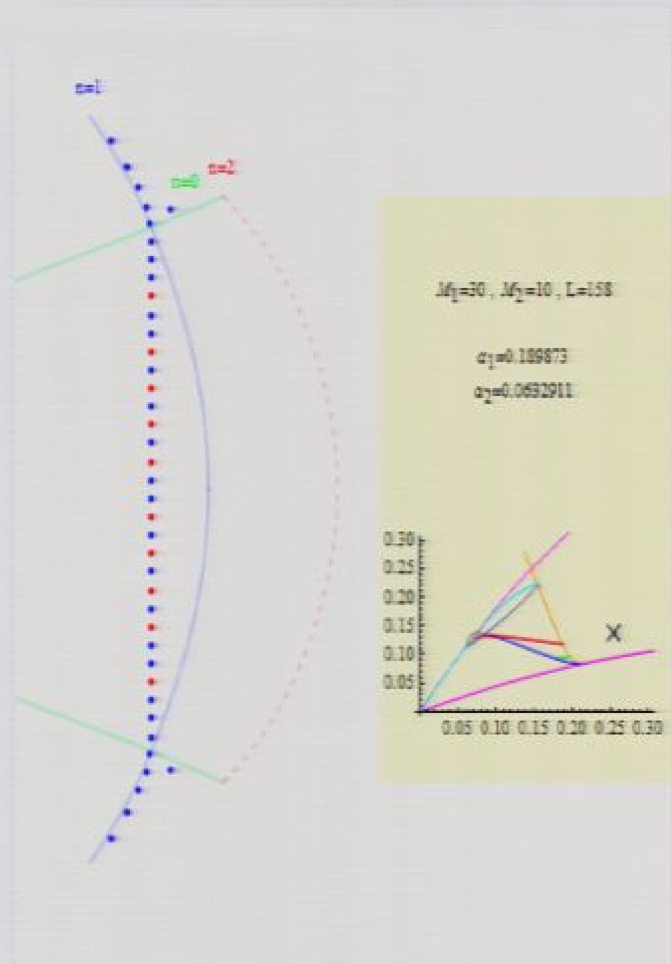

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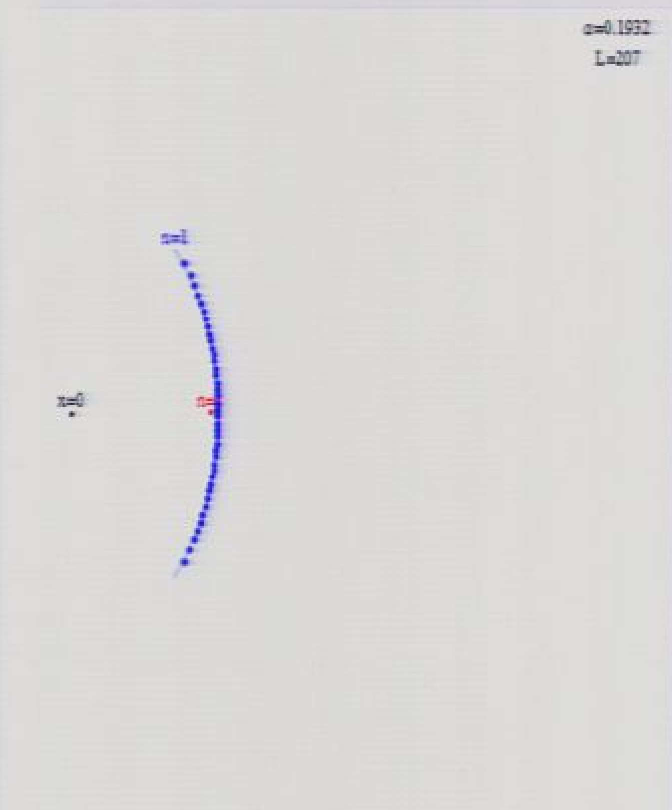

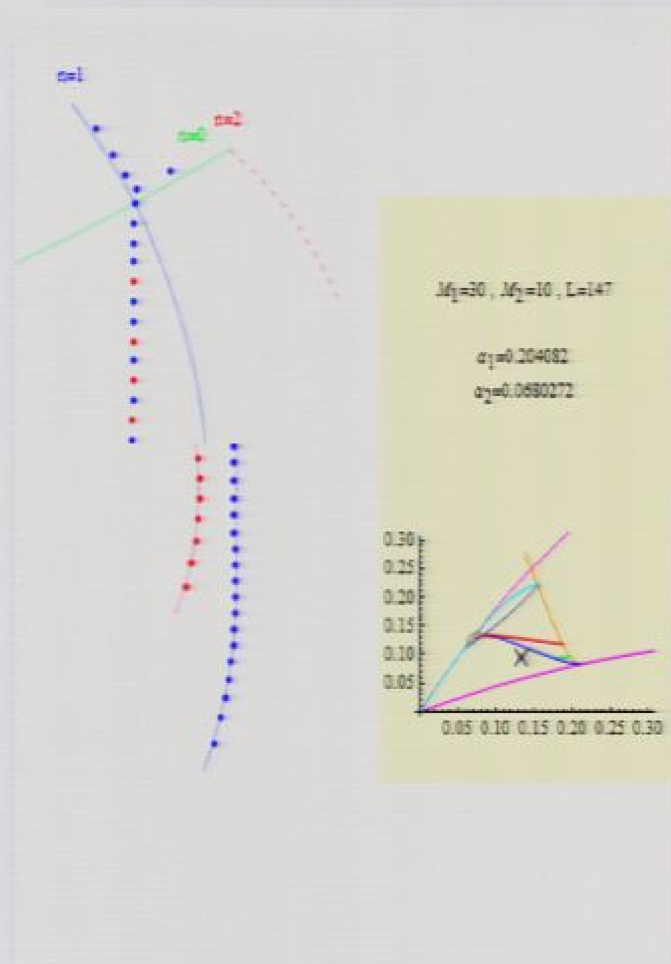

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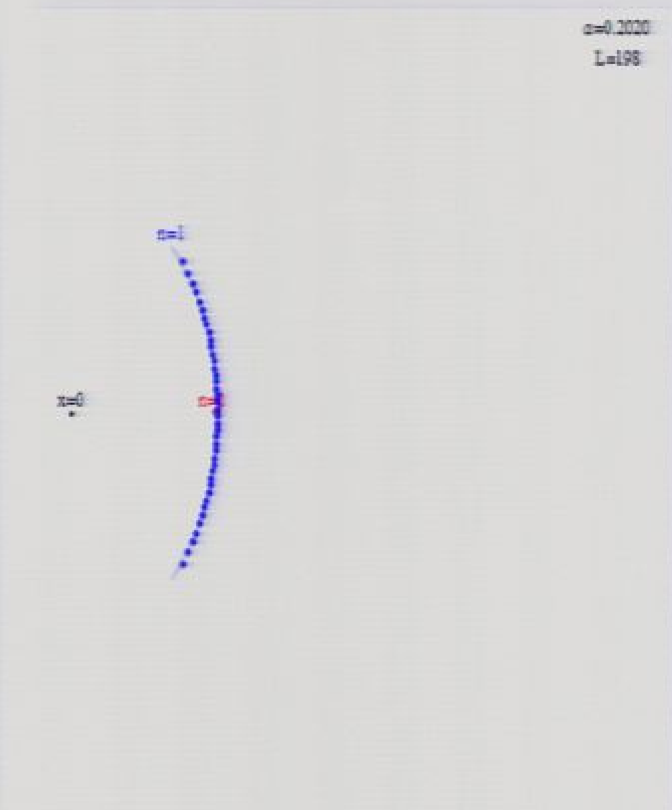

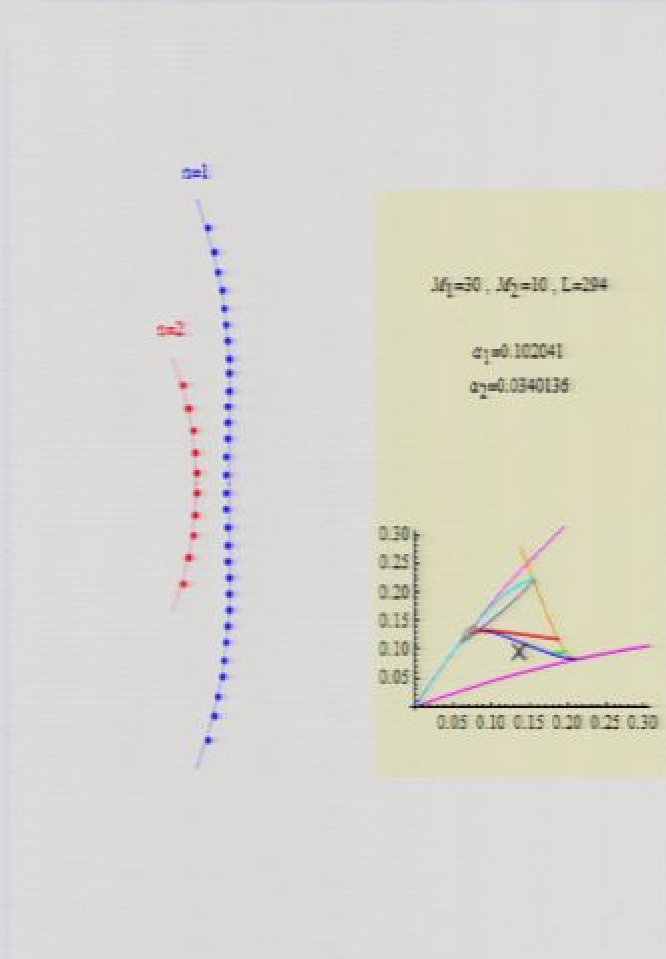

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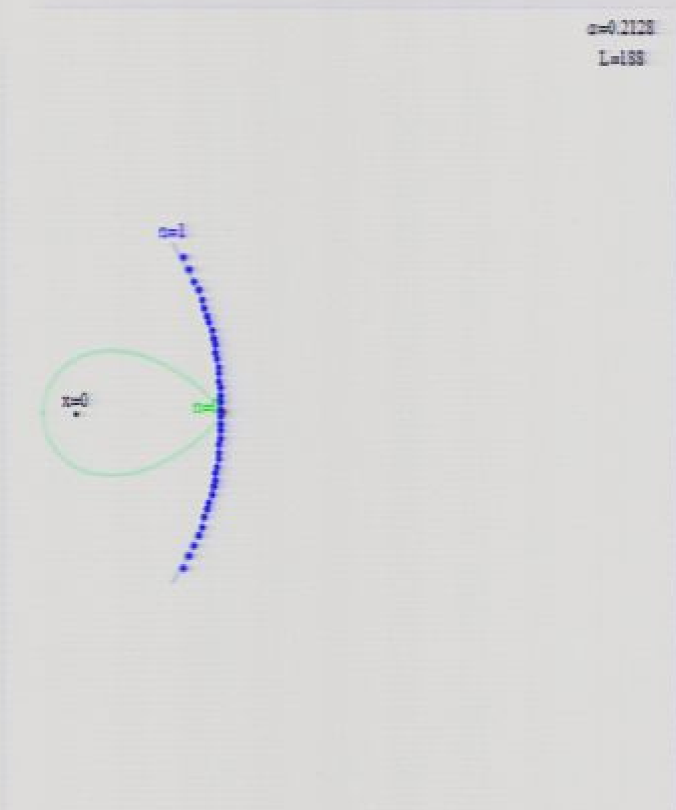

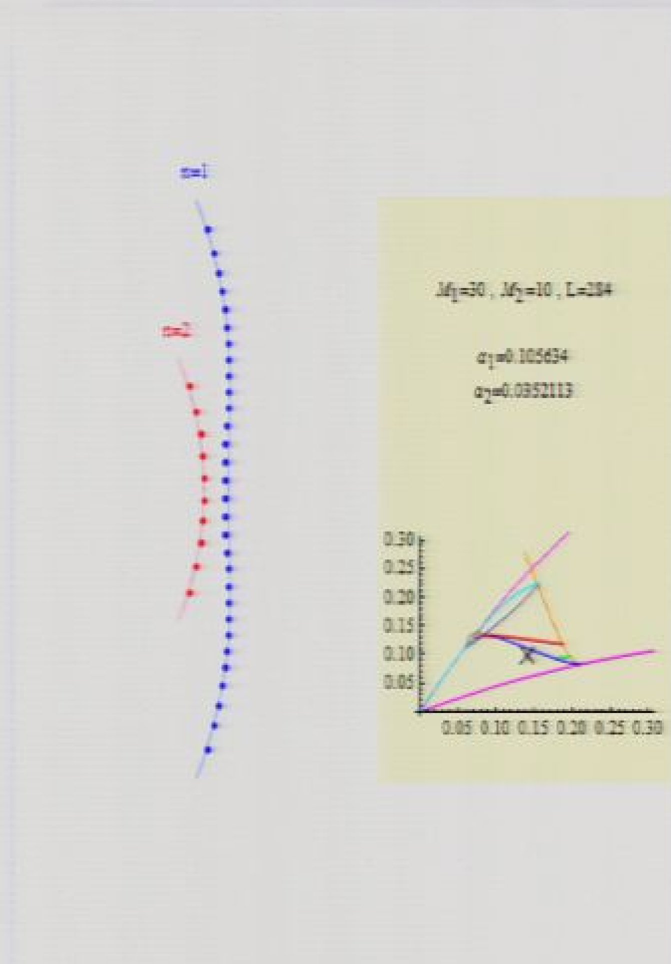

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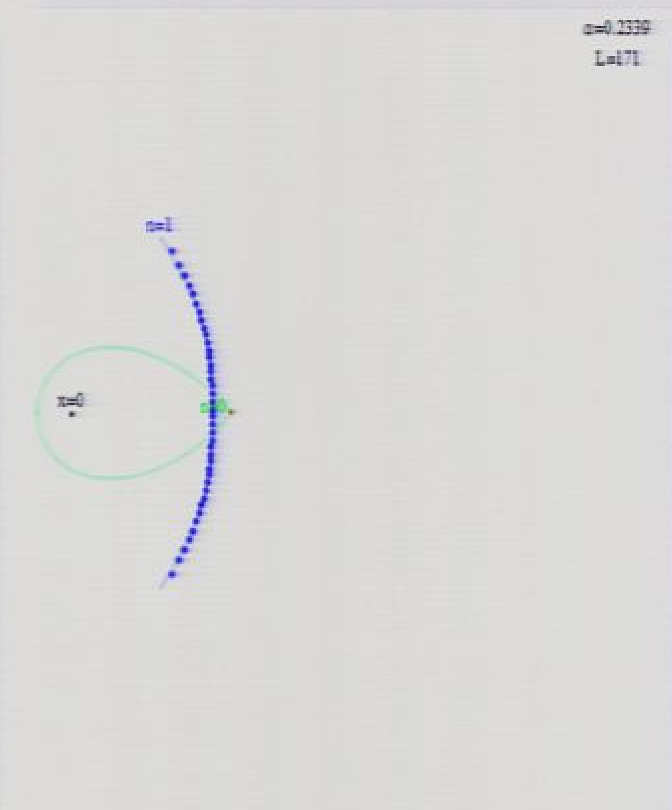

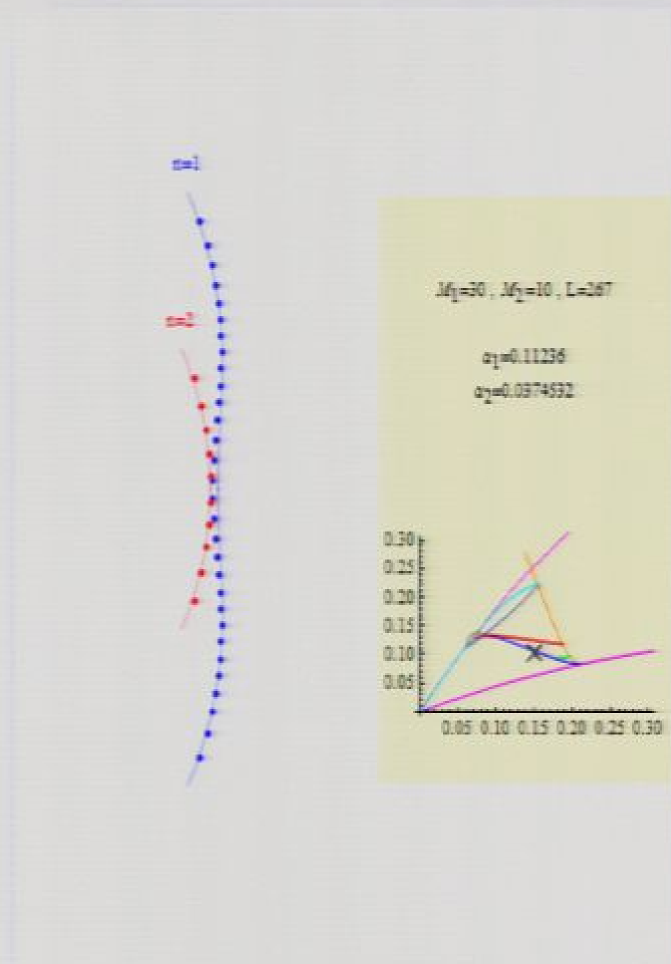

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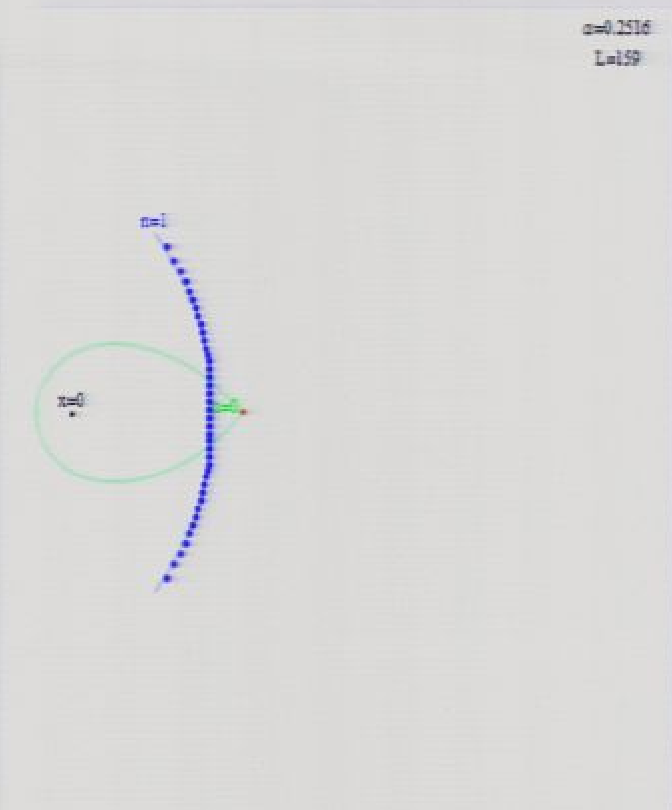

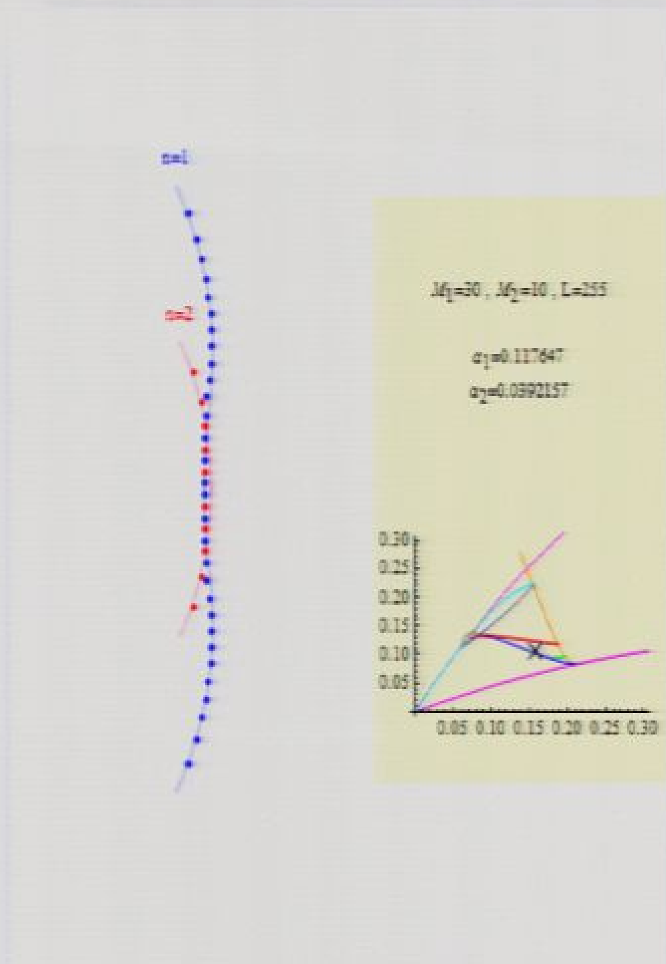

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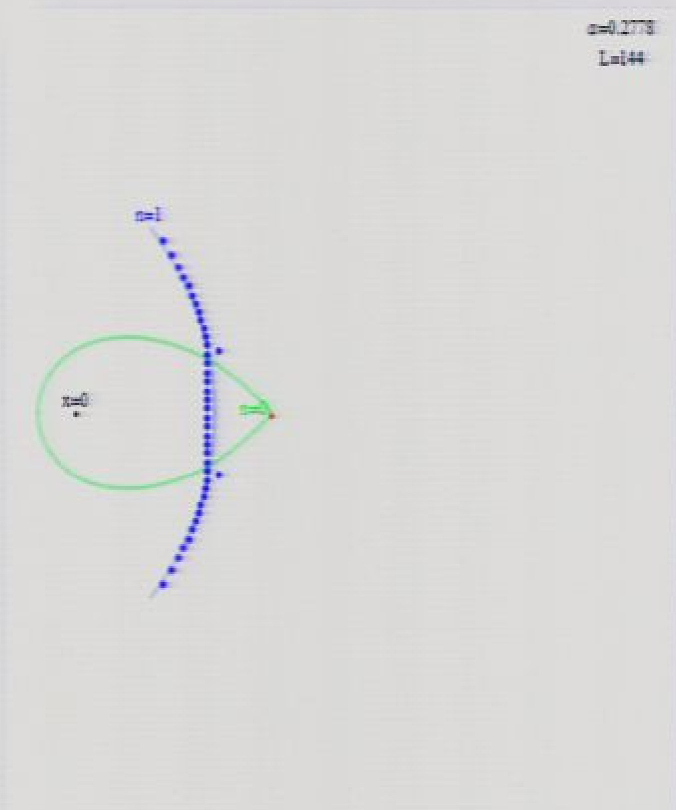

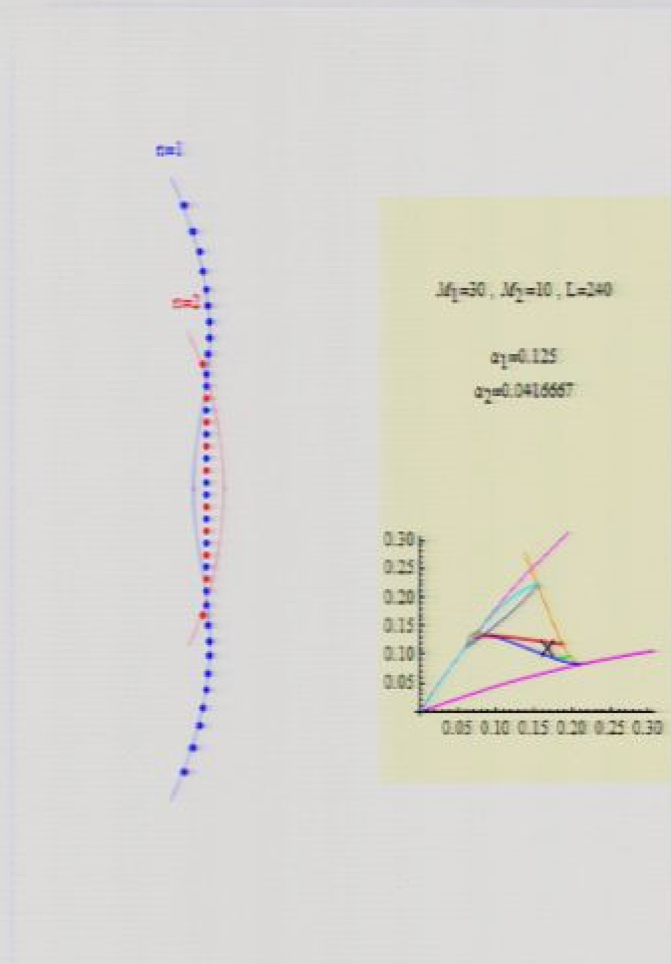

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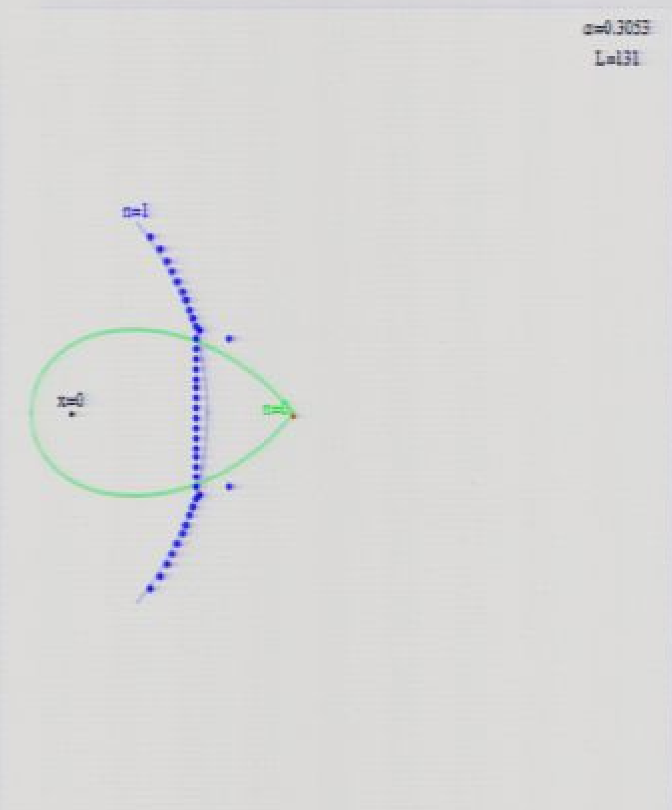

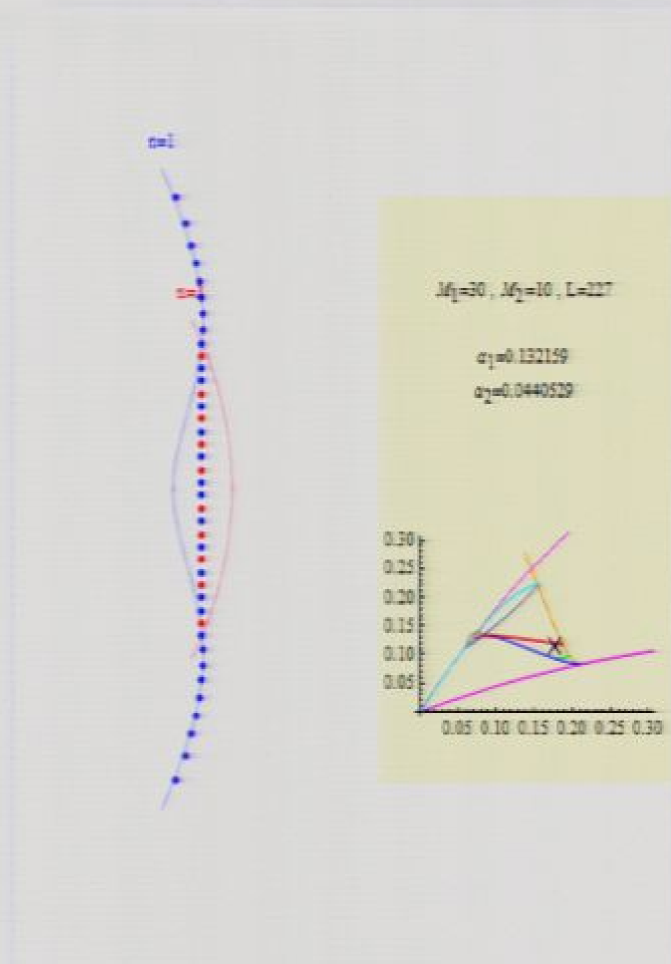

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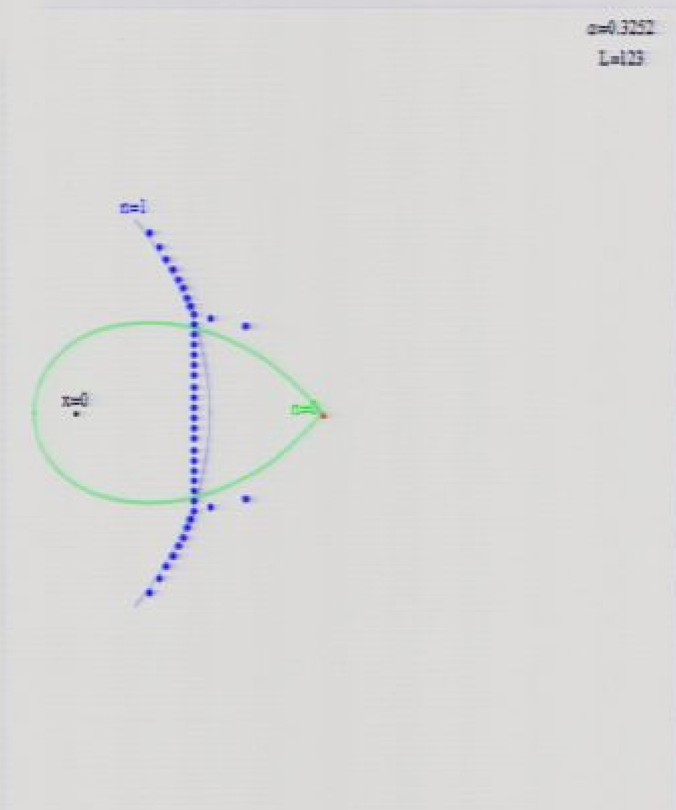

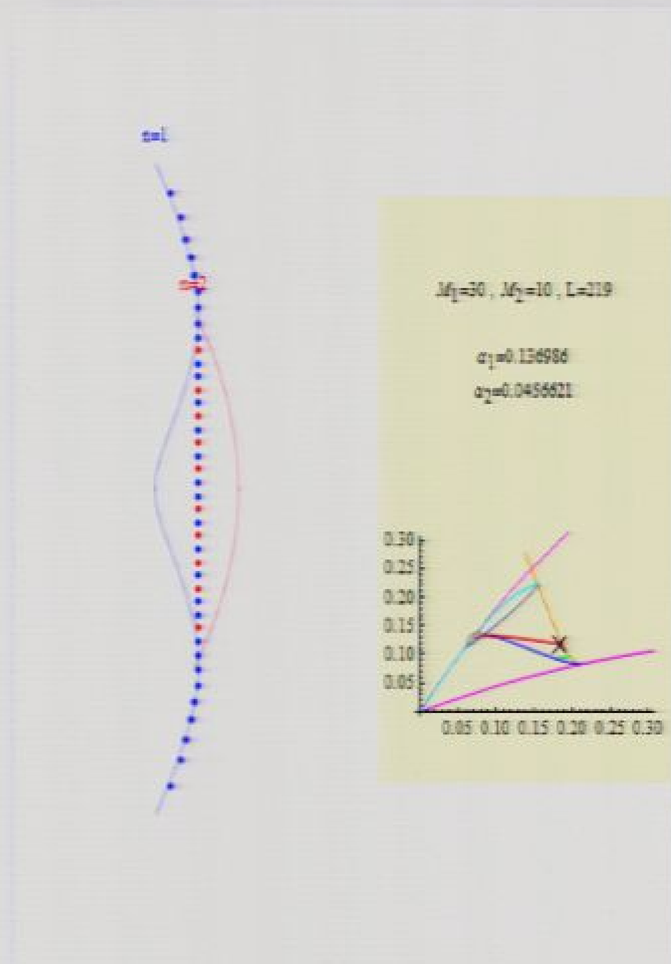

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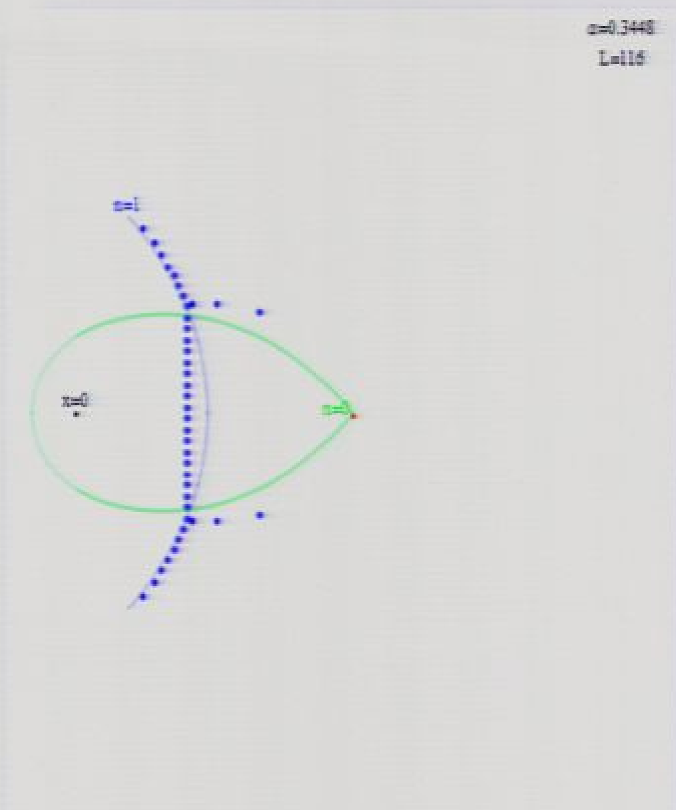

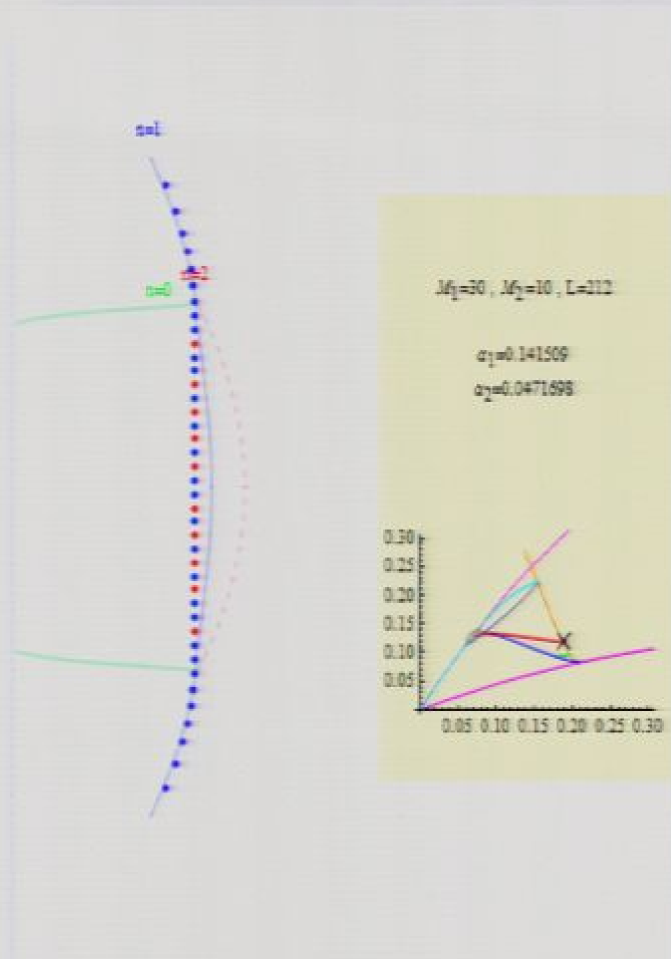

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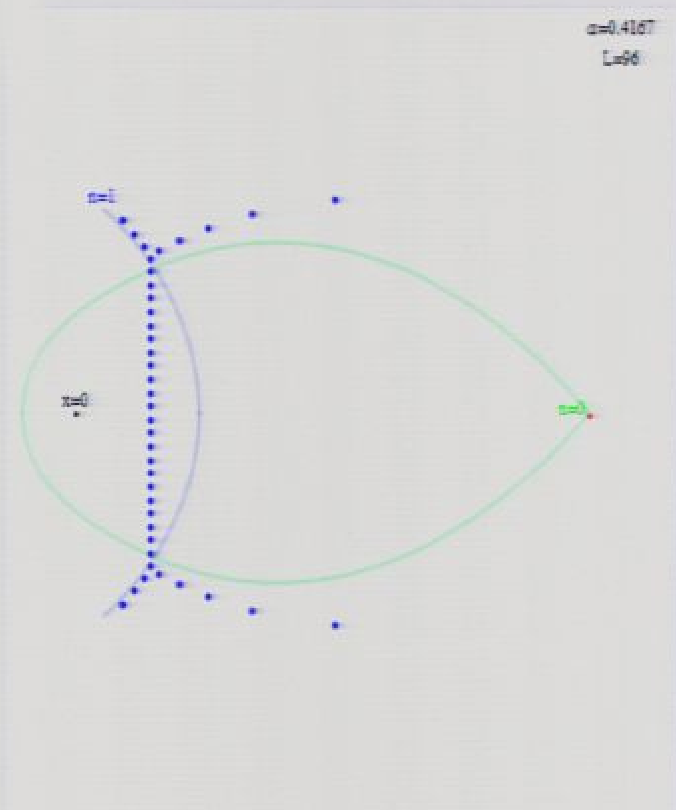

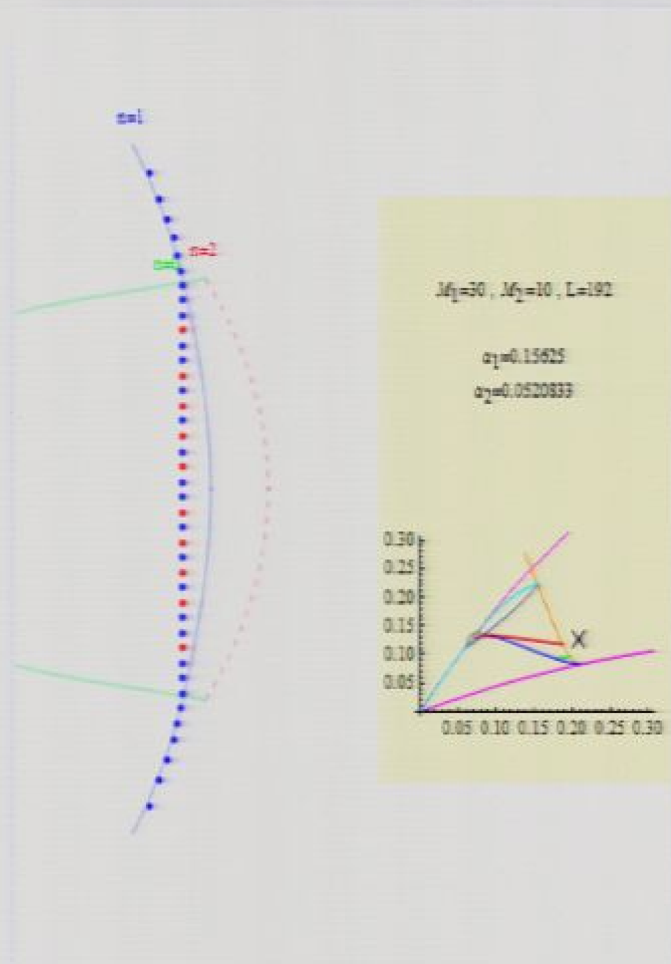

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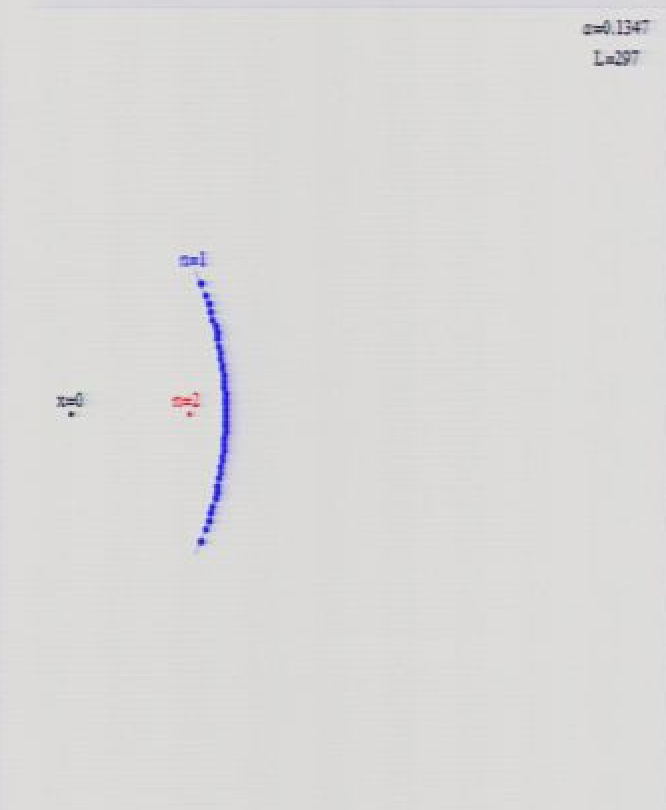

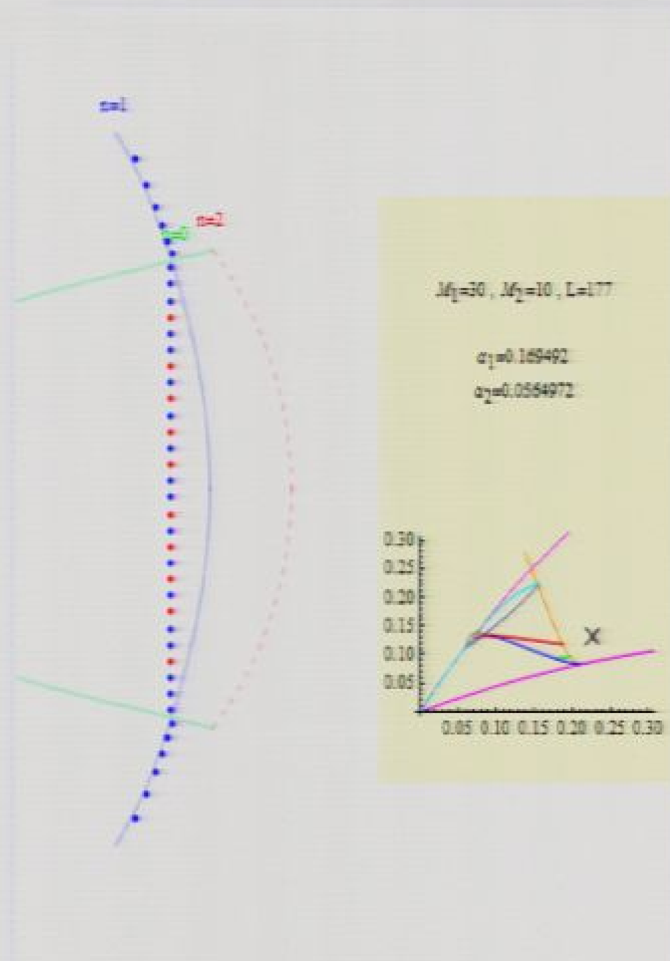

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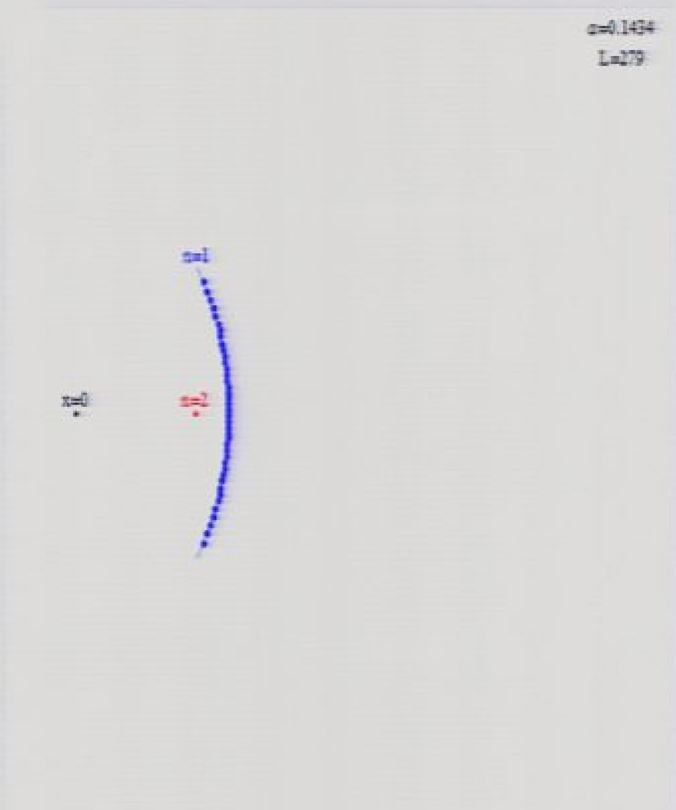

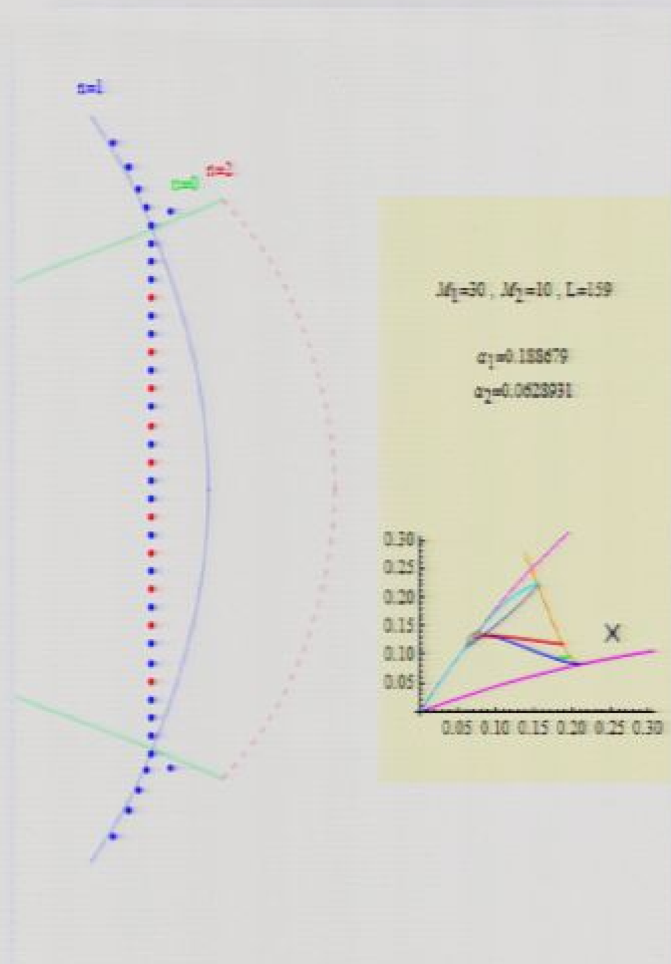

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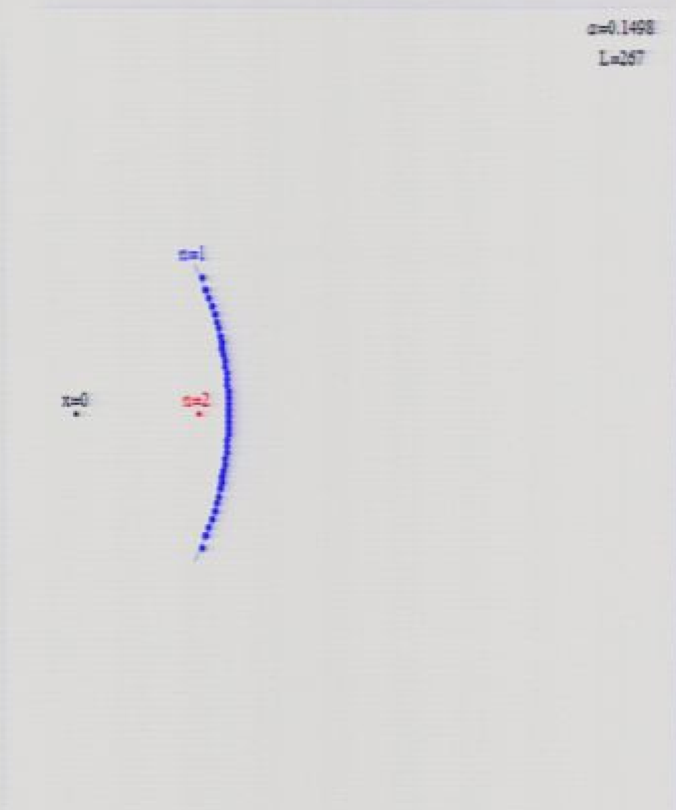

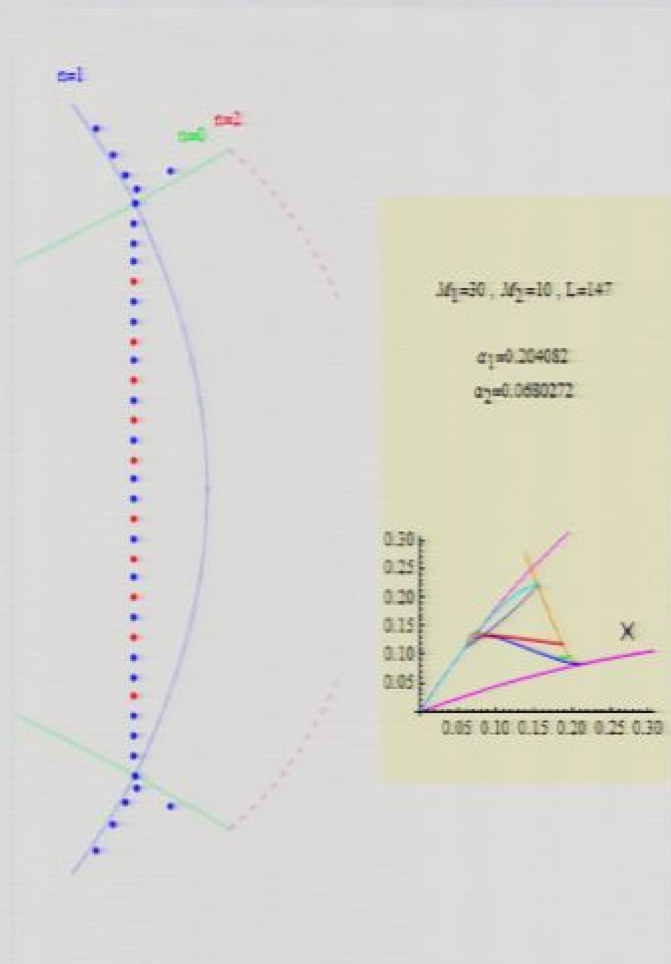

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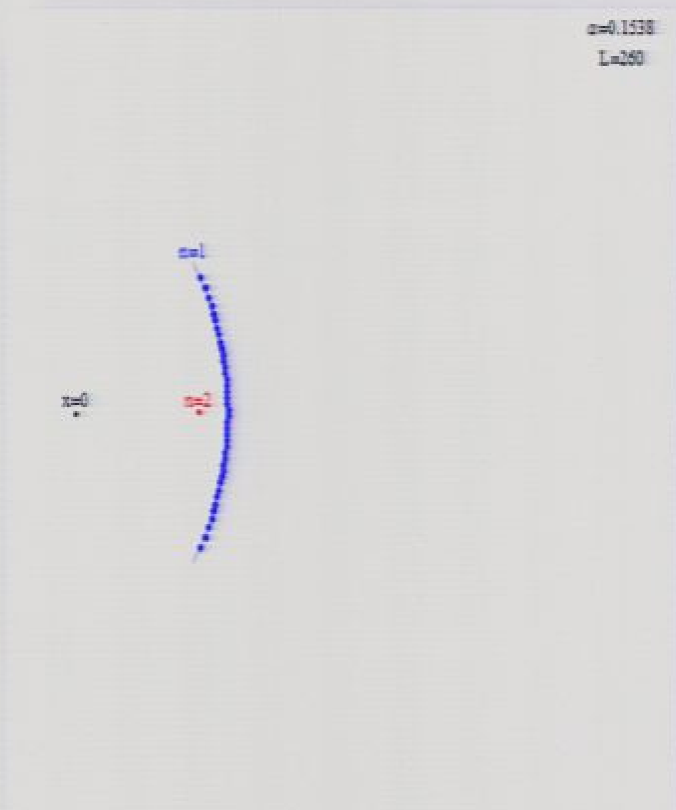

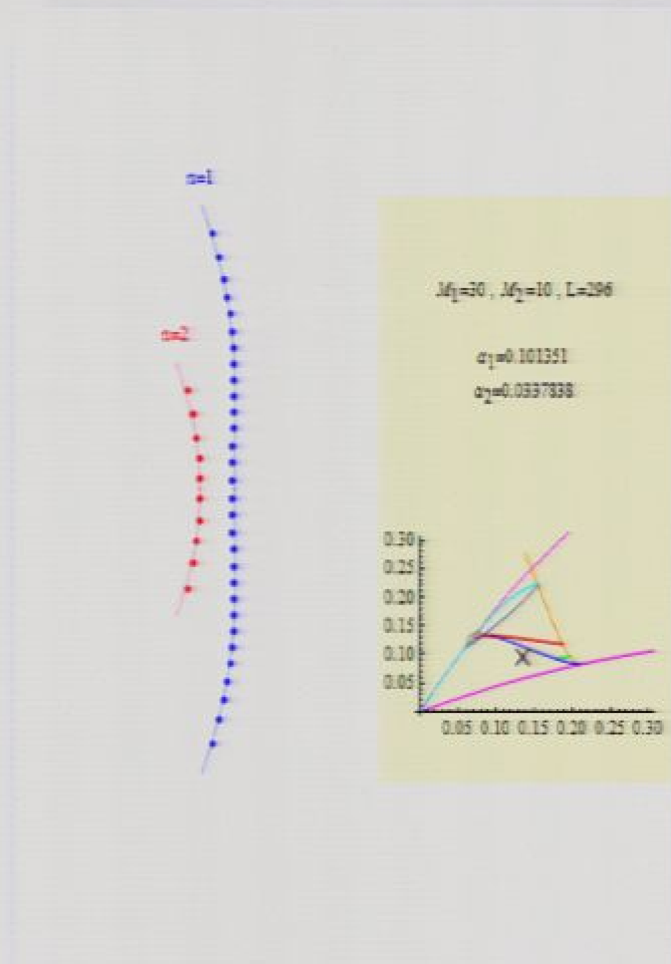

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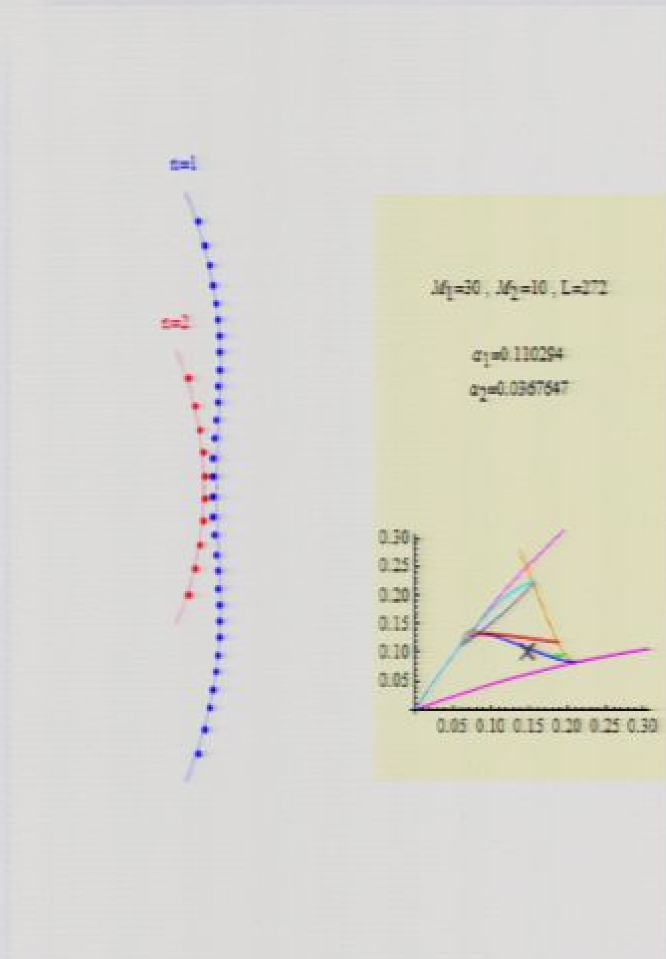

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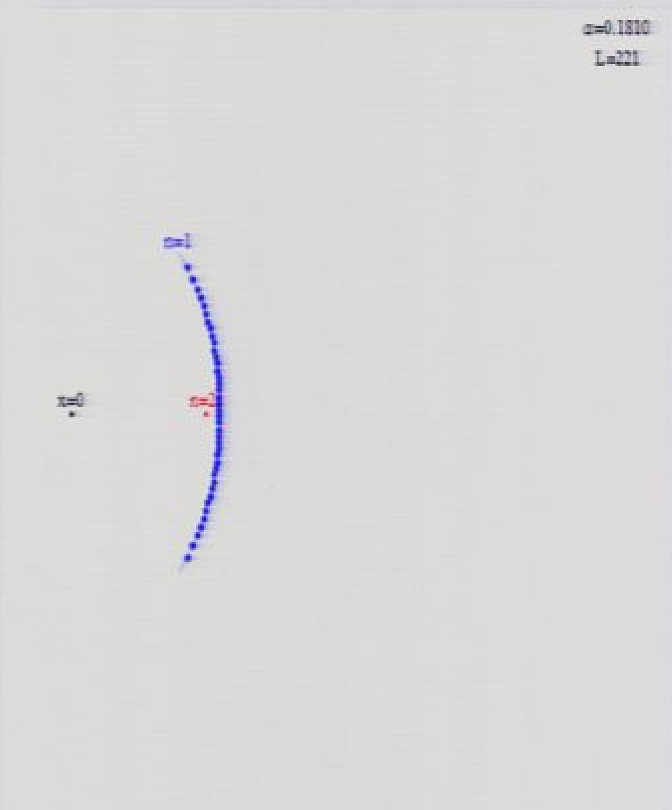

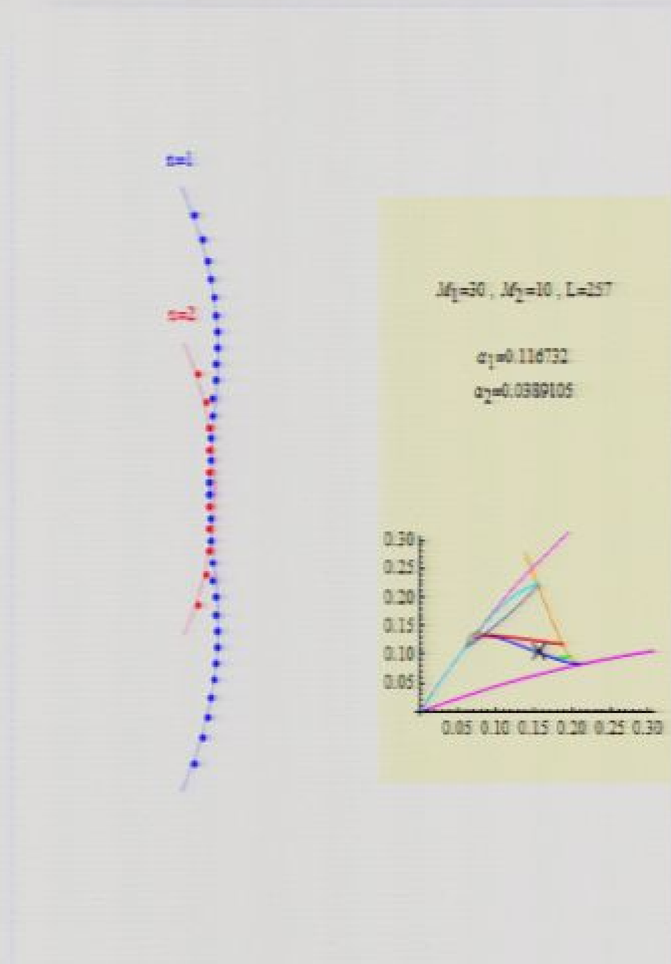

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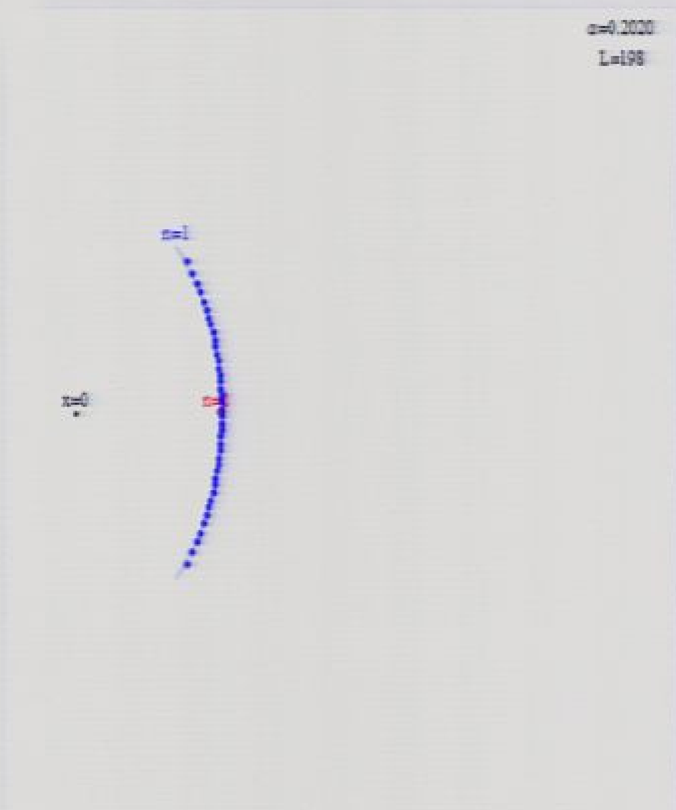
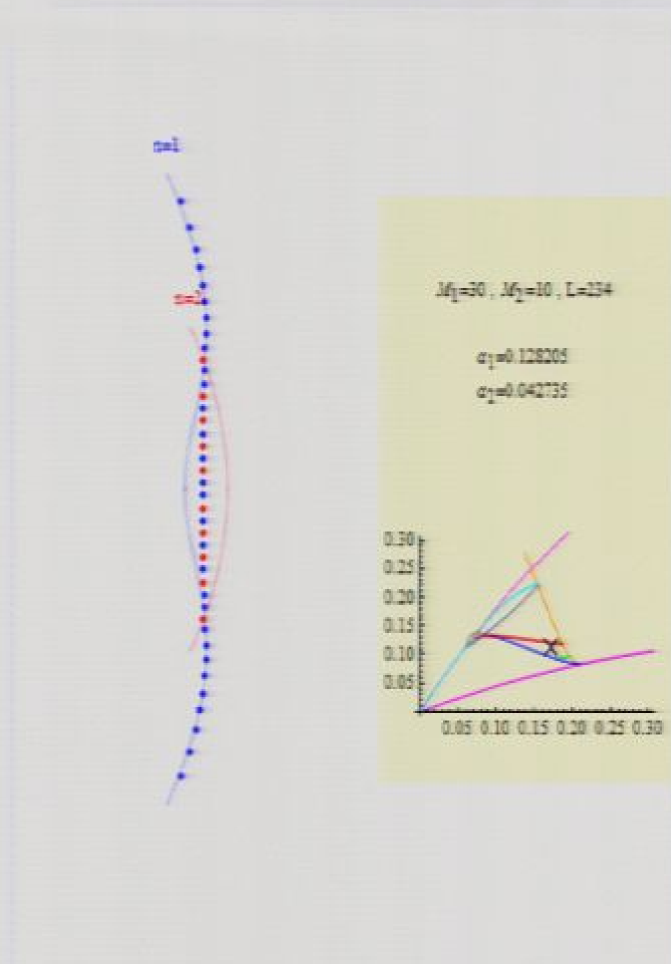
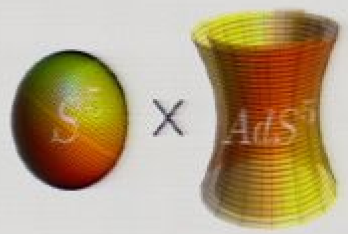




AdS/CFT correspondence

AdS/CFT duality: $$S = \frac{T}{2}\int \partial_\mu \vec{u} \cdot \partial^\mu \vec{u} \;\; d\sigma d\tau$$



String tension $T = \dfrac{\sqrt{\lambda}}{2\pi}$ **`t Hooft coupling** $\lambda = g_{YM}^2 N$

String coupling $g_s = \dfrac{\lambda}{4\pi N}$ **Number of colors** N

Anomalous dimensions = spectrum of 2D integrable field theories

S3 sigma model

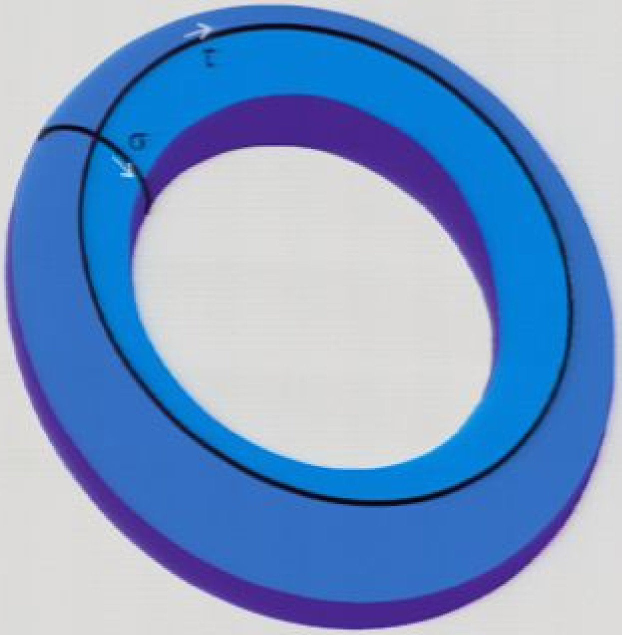
Bethe ansatz:

$$1 = \prod_{\beta}^{J_u} \frac{u_j - \theta_\beta - i/2}{u_j - \theta_\beta + i/2} \prod_{i \neq j}^{J_u} \frac{u_j - u_i + i}{u_j - u_i - i}, \qquad su_R(2)$$

$$e^{-imL \sinh \pi\theta_\alpha} = \prod_{\beta \neq \alpha}^{L} S_0^2\left(\theta_\alpha - \theta_\beta\right) \prod_{j}^{J_u} \frac{\theta_\alpha - u_j + i/2}{\theta_\alpha - u_j - i/2} \prod_{k}^{J_v} \frac{\theta_\alpha - v_k + i/2}{\theta_\alpha - v_k - i/2},$$

$$1 = \prod_{\beta}^{J_v} \frac{v_k - \theta_\beta - i/2}{v_k - \theta_\beta + i/2} \prod_{l \neq k}^{J_v} \frac{v_k - v_l + i}{v_k - v_l - i}, \qquad su_L(2)$$

Vacuum



$$Z(\tau,\sigma) = Z(\sigma,\tau)$$

S3 sigma model

Bethe ansatz:

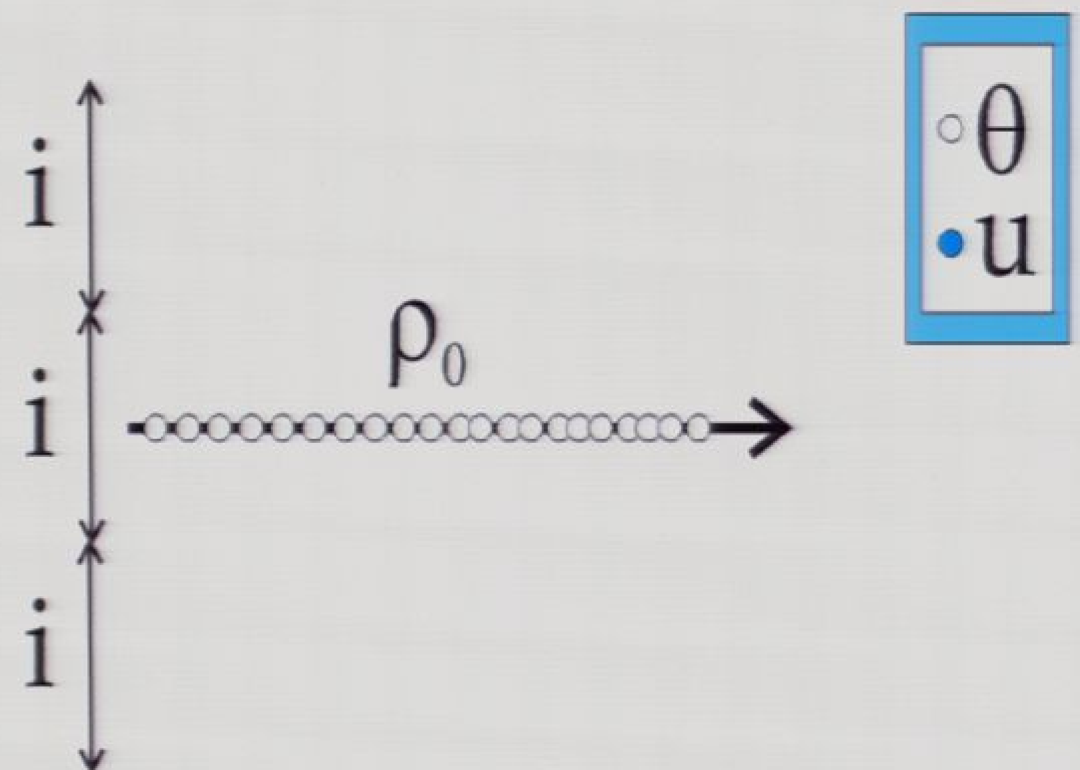
$$1 = \prod_{\beta}^{J_u} \frac{u_j - \theta_\beta - i/2}{u_j - \theta_\beta + i/2} \prod_{i \neq j}^{J_u} \frac{u_j - u_i + i}{u_j - u_i - i}, \qquad su_R(2)$$

$$e^{-imL \sinh \pi\theta_\alpha} = \prod_{\beta \neq \alpha}^{L} S_0^2\left(\theta_\alpha - \theta_\beta\right) \prod_{j}^{J_u} \frac{\theta_\alpha - u_j + i/2}{\theta_\alpha - u_j - i/2} \prod_{k}^{J_v} \frac{\theta_\alpha - v_k + i/2}{\theta_\alpha - v_k - i/2},$$

$$1 = \prod_{\beta}^{J_v} \frac{v_k - \theta_\beta - i/2}{v_k - \theta_\beta + i/2} \prod_{l \neq k}^{J_v} \frac{v_k - v_l + i}{v_k - v_l - i}, \qquad su_L(2)$$

Ground state from ABA

- The typical configuration of roots is



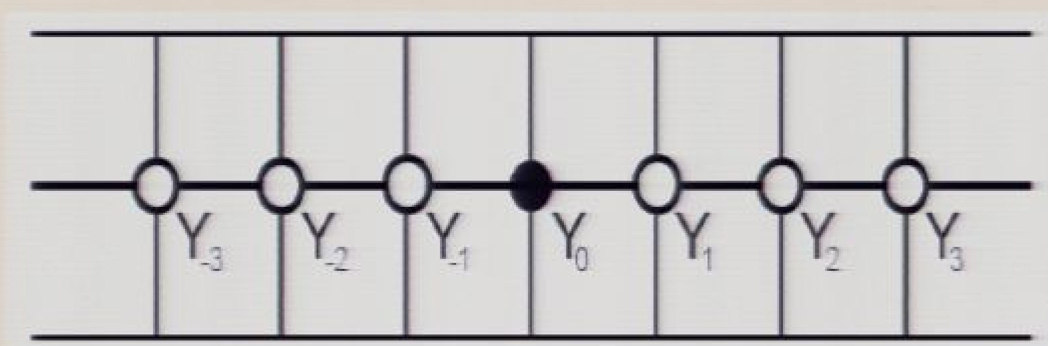
We define:

$$Y_k = \frac{\bar{\rho}_k}{\rho_k}$$

Saddle point equation:

$$Y_n^+ Y_n^- = (1 + Y_{n+1})(1 + Y_{n-1})$$

$$E(L) = -\frac{1}{2}\int m \cosh(\pi x) \log(1 + Y_0)$$



Above ground state

Dorey, Totteo, Bazhanov

$$E(L) = -\frac{1}{2}\int m\cosh(\pi x)\log(1+Y_0)$$

Above ground state

Dorey, Totteo, Bazhanov

$$E(L) = -\frac{1}{2}\int m\cosh(\pi x)\log(1+Y_0)$$

$$\Downarrow$$

$$E(L) = -\frac{1}{2}\int m\cosh(\pi x)\log(1+Y_0) + \sum_{j=1}^{N} m\cosh(\pi\theta_j)\,,$$

$$Y_0(\theta_j + i/2) = -1$$

Above ground state

Dorey, Totteo, Bazhanov

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$$E(L) = -\frac{1}{2}\int m\cosh(\pi x)\log(1+Y_0)+\sum_{j=1}^{N} m\cosh(\pi\theta_j)\ ,$$

$$Y_0(\theta_j + i/2) = -1$$

$$Y_n^+Y_n^- = (1+Y_{n+1})(1+Y_{n-1})$$

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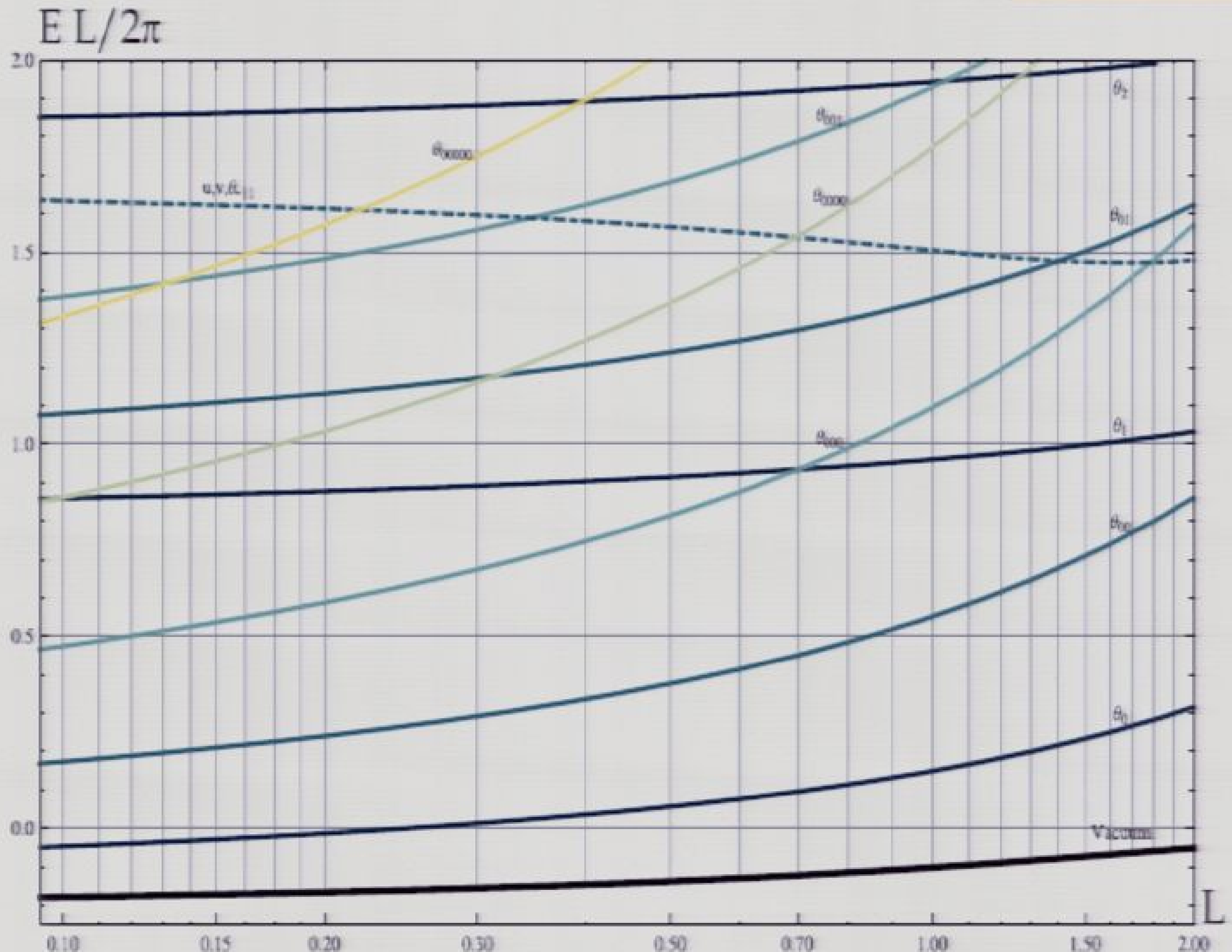
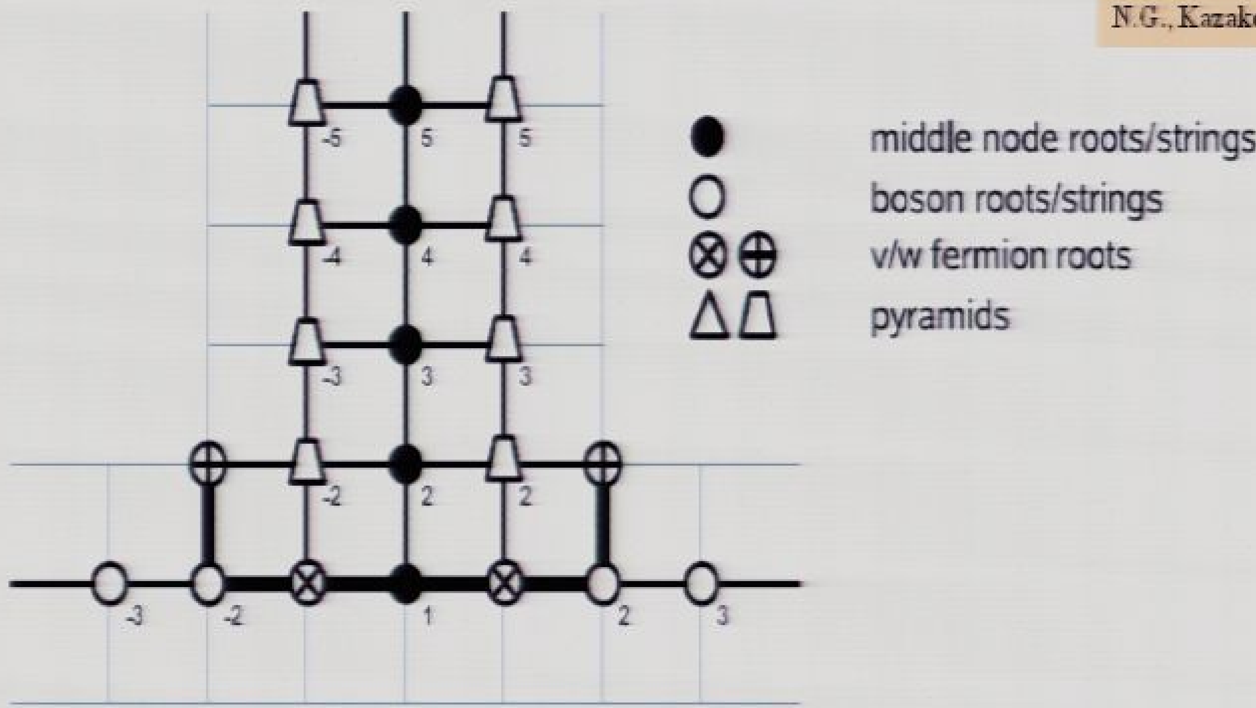
Spectrum of SO(4)

NG, V.Kazakov, P.Vieira



AdS/CFT Generalization

N.G., Kazakov, Vieira



AdS/CFT Generalization

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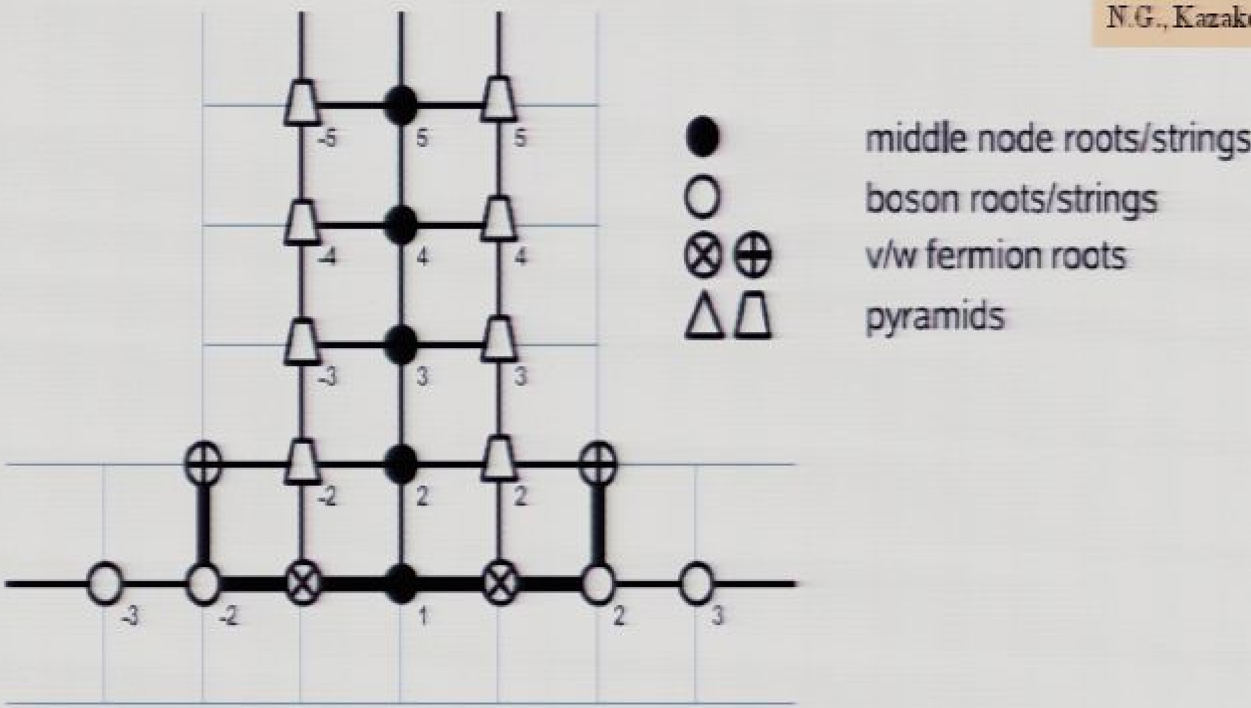
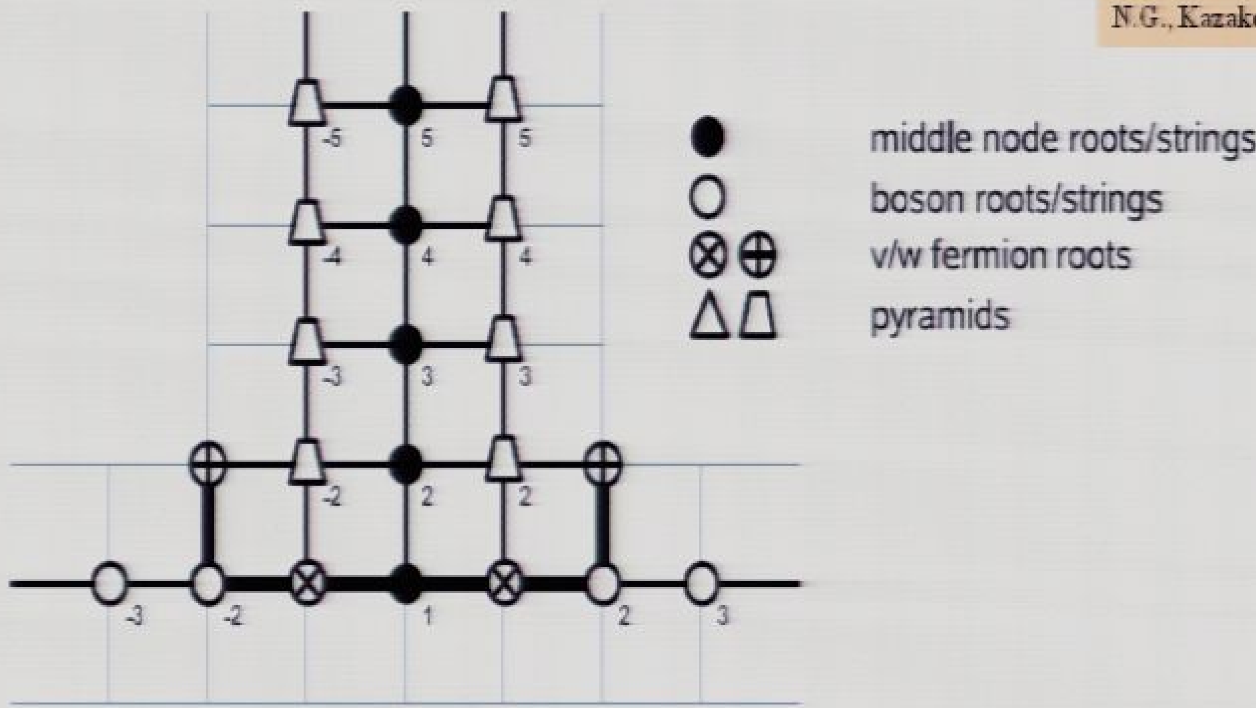


$$\frac{Y_{a,s}^{+}Y_{a,s}^{-}}{Y_{a+1,s}Y_{a-1,s}} = \frac{(1+Y_{a,s+1})(1+Y_{a,s-1})}{(1+Y_{a+1,s})(1+Y_{a-1,s})}$$

AdS/CFT Generalization

N.G., Kazakov, Vieira



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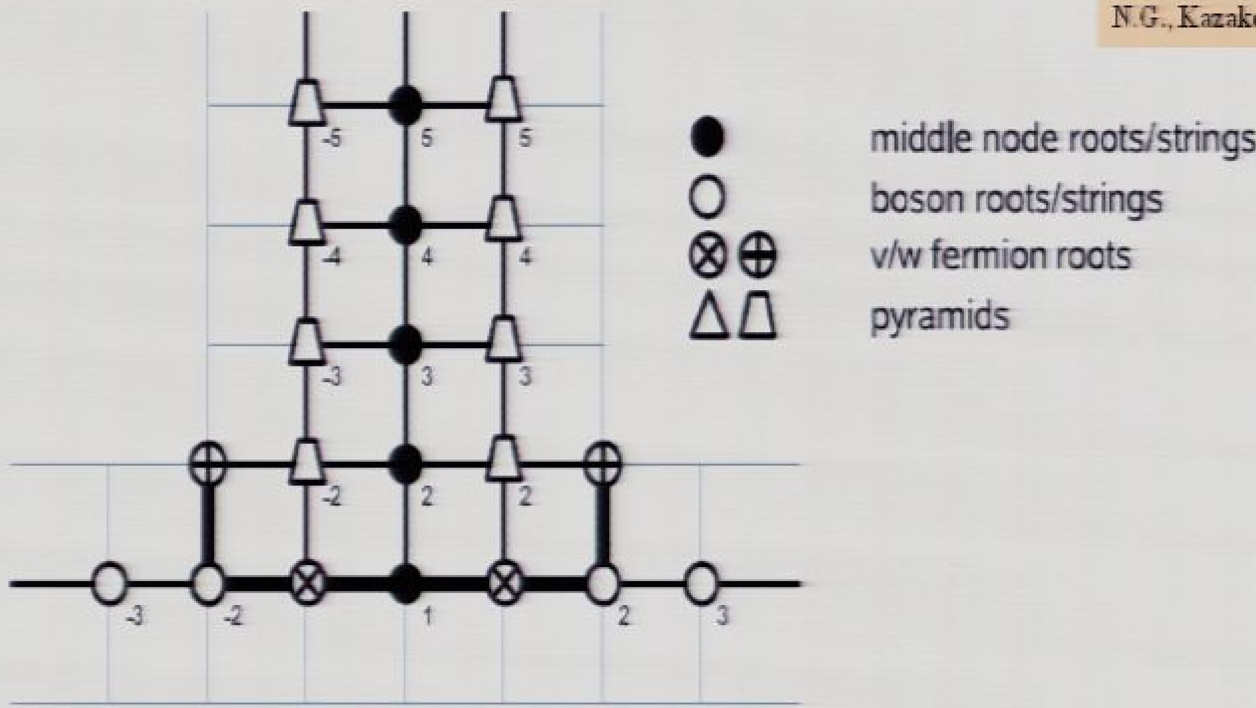
$$E = \sum \epsilon_1(u_{4,j}) + \sum^{\infty} \int_{-\infty}^{\infty} \frac{du}{2\pi i} \frac{\partial \epsilon_a^*}{\partial u} \log\left(1 + Y_{a,0}^*(u)\right)$$

AdS/CFT Generalization

N.G., Kazakov, Vieira



$$\frac{Y^+_{a,s}Y^-_{a,s}}{Y_{a+1,s}Y_{a-1,s}} = \frac{(1+Y_{a,s+1})(1+Y_{a,s-1})}{(1+Y_{a+1,s})(1+Y_{a-1,s})}$$

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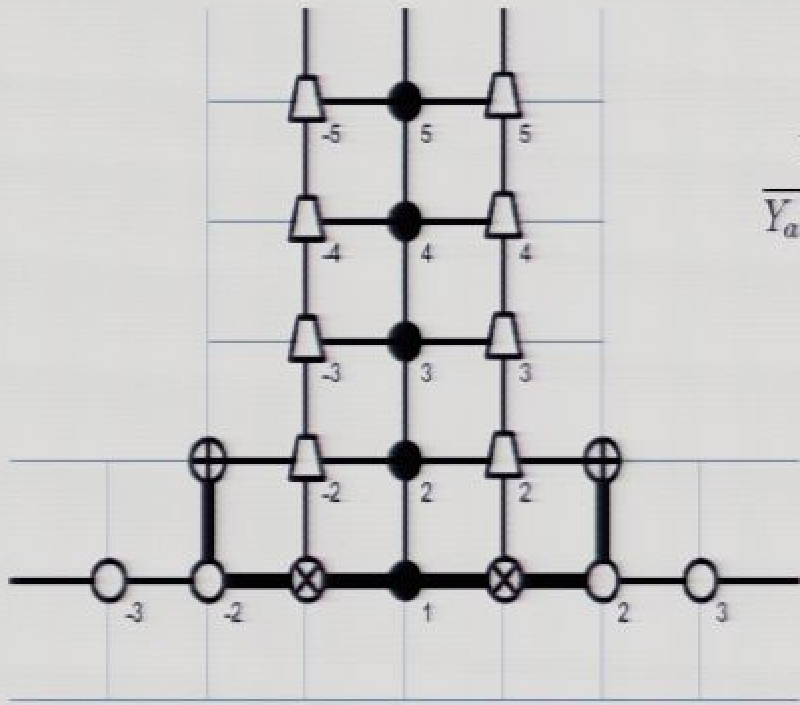
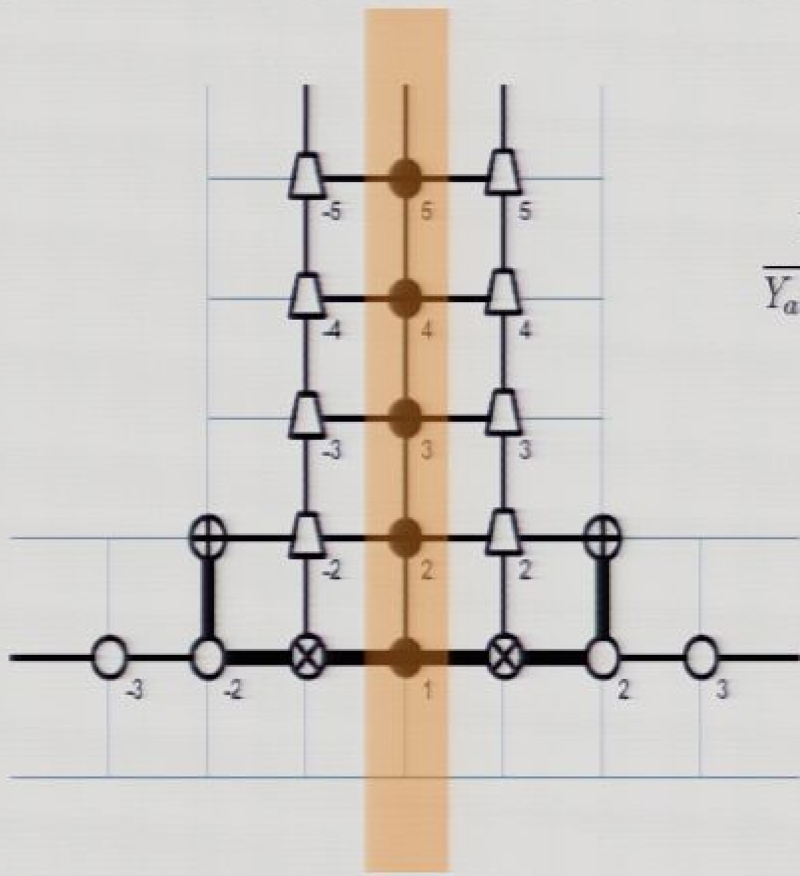
Large L limit or weak coupling



$$\frac{Y_{a,s}^{+}Y_{a,s}^{-}}{Y_{a+1,s}Y_{a-1,s}} = \frac{(1+Y_{a,s+1})(1+Y_{a,s-1})}{(1+Y_{a+1,s})(1+Y_{a-1,s})}$$

Large L limit or weak coupling



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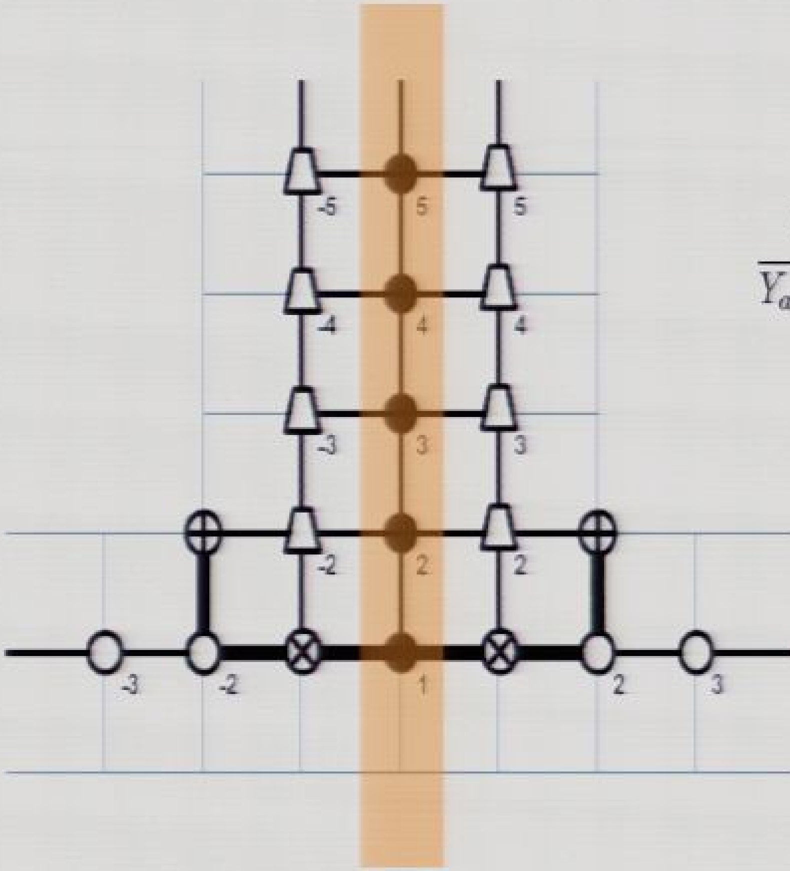

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Use Hirota equation:

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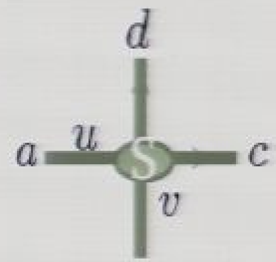
$$\text{where } Y_{a,s} = \frac{T_{a,s+1}T_{a,s-1}}{T_{a+1,s}T_{a-1,s}}\ .$$

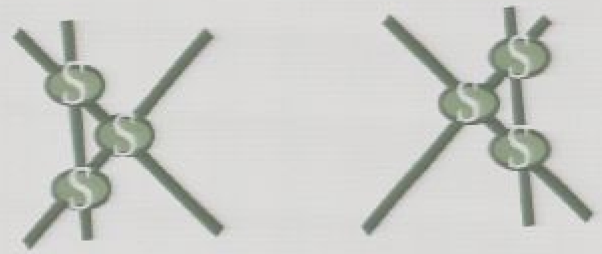
S-matrix

SU(2|2) invariant tensor with 4 fundamental indeses

$$S_{ab}^{cd}(u,v)$$



Obeys Yang-Baxter:



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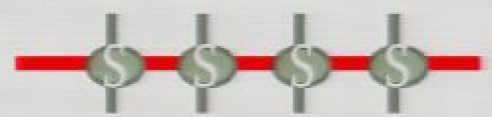
Then:



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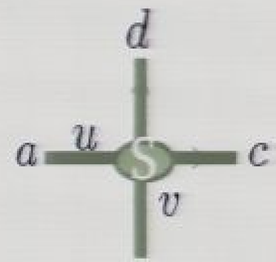

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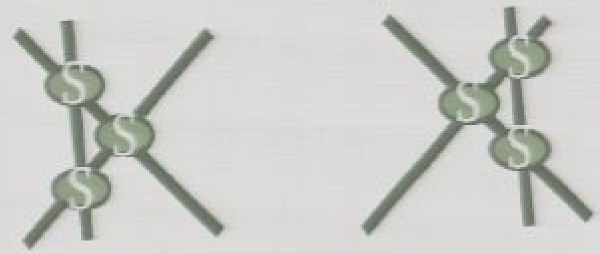

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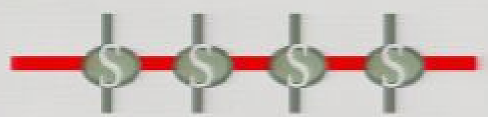

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The eigevalues solve Hirota

for rep = a

Konishi operator

The simplest operator

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In agreement with perturbation theory!!

4-loops!

Kotikov, Lipatov, Rej, Staudacher and Velizhanin
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Conclutions

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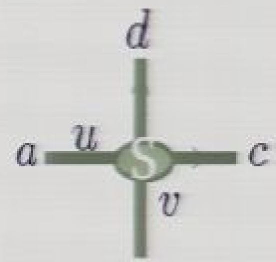

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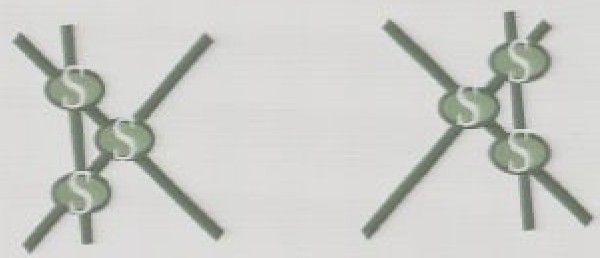

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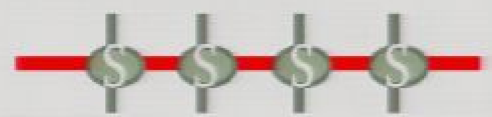

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