Title: Physical Implementation of Quantum Random Walks

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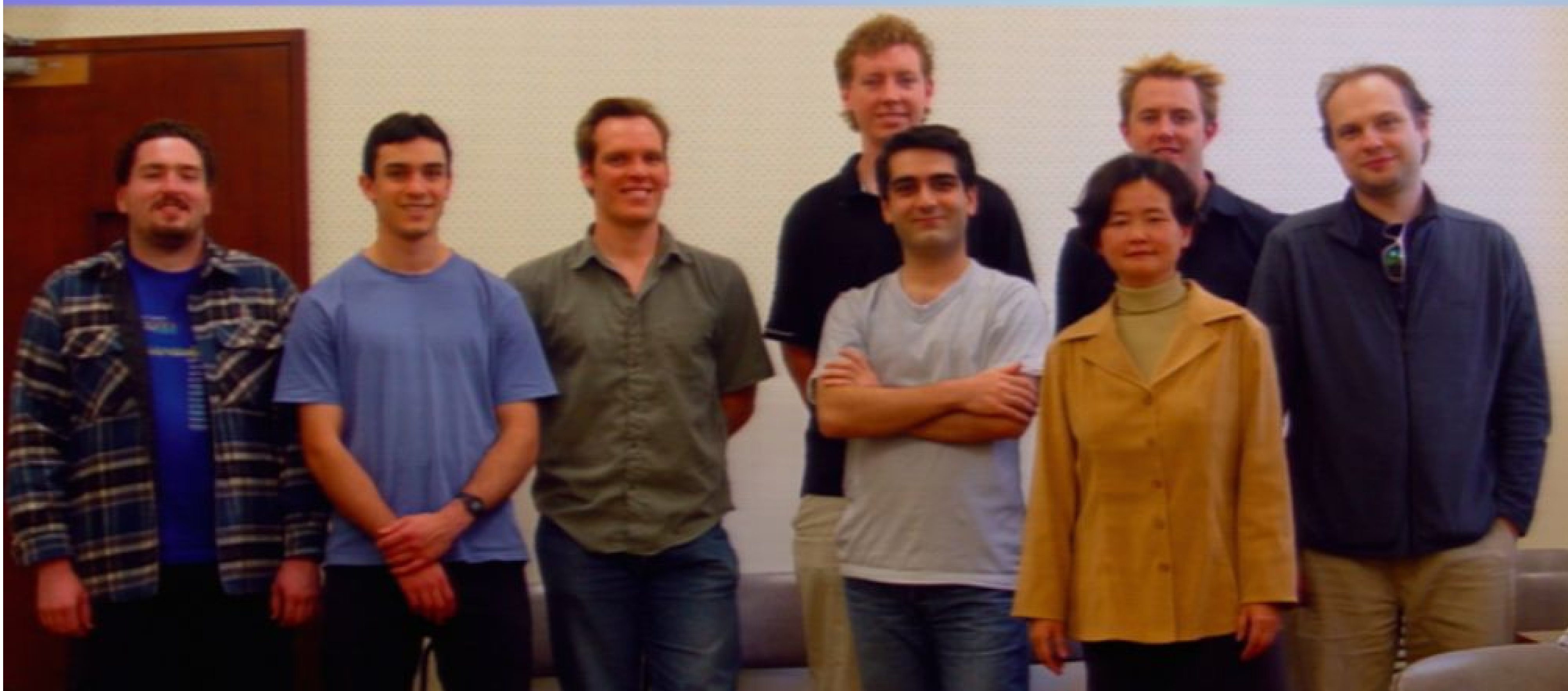
Abstract: Quantum random walks have received much interest due to their non-intuitive dynamics, which may hold a key to radically new quantum algorithms. What remains a major challenge is a physical realization that is experimentally viable, readily scalable, and not limited to specific connectivity criteria. In this seminar, I will present an implementation scheme for quantum walking on arbitrarily complex graphs. This scheme is particularly elegant since the walker is not required to physically step between the nodes; only flipping coins is suï¬ƒcient. In addition, by taking advantage of the inherent structure of the CS decomposition of unitary matrices, we are able to implement all coin operations necessary for each step of the walk simultaneously. This scheme can be physically realized using a variety of quantum systems, such as cold atoms trapped inside an optical lattice or electrons inside coupled quantum dots.

Physical implementation of quantum walks

Quantum Dynamics and Computation Group
The University of Western Australia

Physical implementation of quantum walks



$$i\hbar\frac{\partial}{\partial t}\psi\left(\vec{r}_1,\vec{r}_2,\ldots\vec{r}_N,t\right)=\left[\sum_{i=1}^{N}\left(-\frac{\hbar^2}{2m^*}\nabla^2_{r_i}+V(\vec{r}_i)\right)+\frac{e^2}{4\pi\varepsilon}\sum_{i>j=1}^{N}\frac{1}{r_{ij}}\right]\psi\left(\vec{r}_1,\vec{r}_2,\ldots\vec{r}_N,t\right)$$

Random walk applications

Random walk applications

Examples: DNA synopsis, animal foraging strategies, diffusion and mobility in materials, exchange rate forecast, stock market analysis, solving differential equations, quantum monte carlo, optimization, clustering and classification, graph connectivity, fractal theory, or even structure analysis of facebook, Google, MSN and Yahoo search engines.

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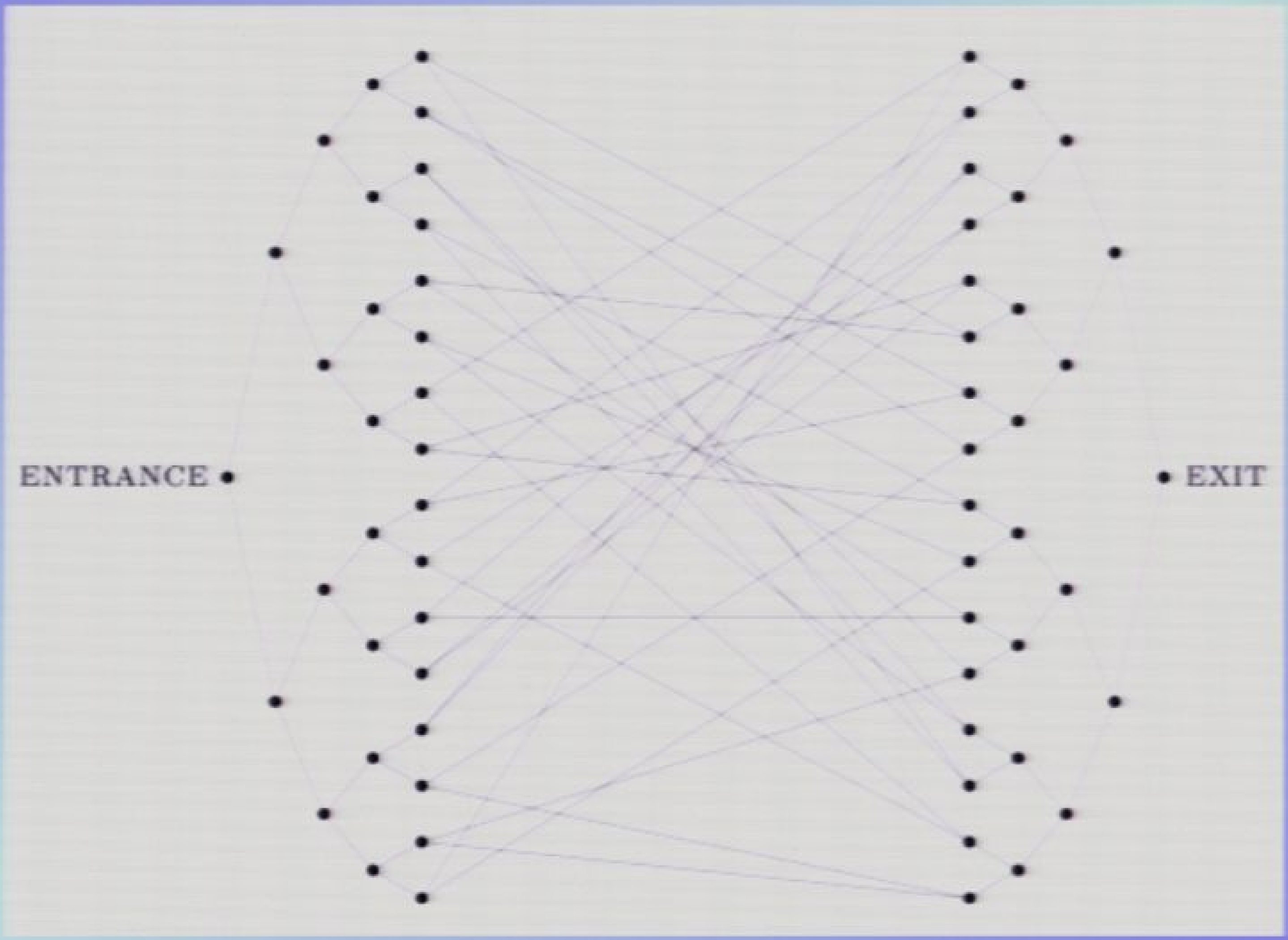
Quantum walk applications

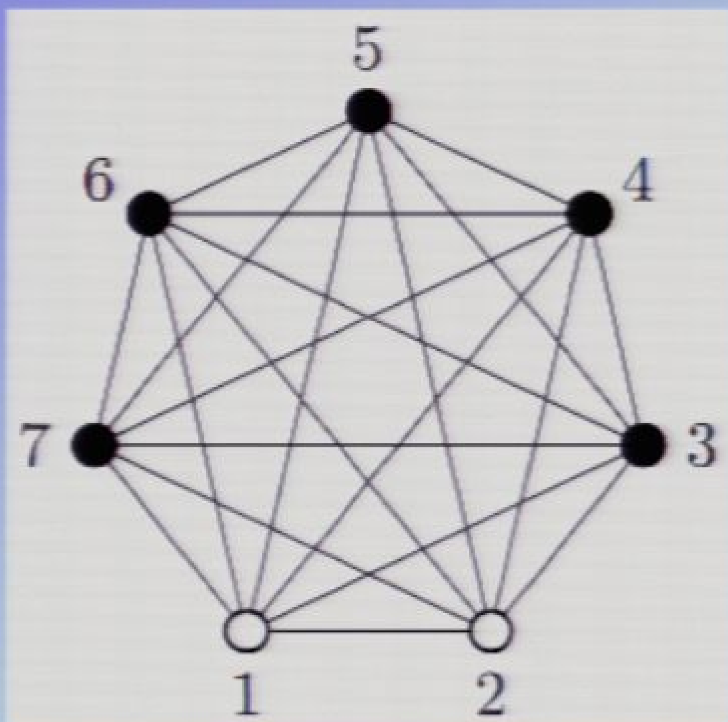
Examples: glued tree traversal, quantum query, quantum search, element distinctness, pattern recognition, hidden nonlinear structures, graph connectivity, group commutativity, and formula evaluation.

Quantum walk provides exponential speedup traversing a glued tree

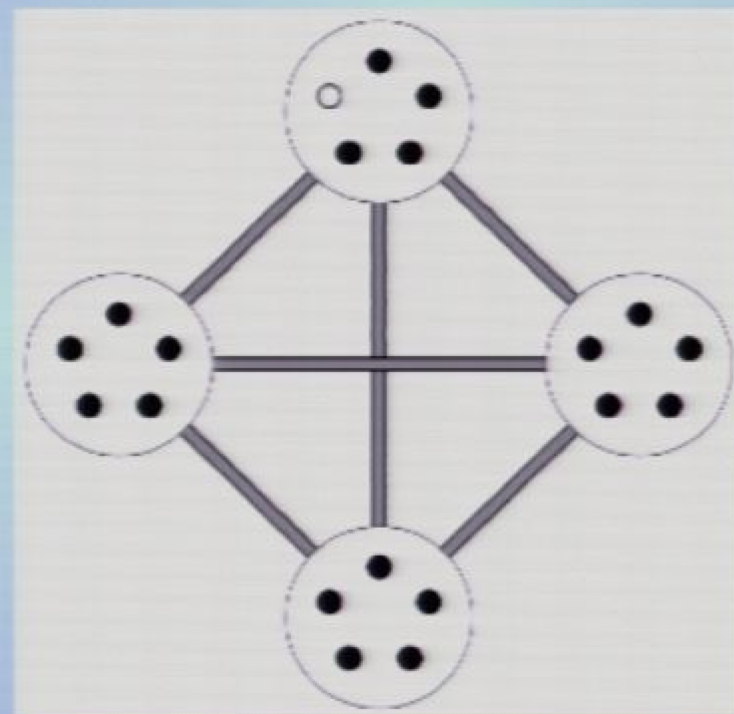


Childs et al 35th ACM Theory of Computing, 59 (2003)

Quantum walk provides polynomial speedup searching marked nodes in highly symmetrical graphs



Complete graph



M-partite complete graph

PRA: Shenvi et al 67, 052307 (2003); Childs and Goldstone 70, 022314 (2004)
Tulsi 78, 012310 (2008); Potoček et al 79, 012325 (2009); Reitzner et al 79, 012323 (2009)

Graph isomorphism
(long-standing open problem)

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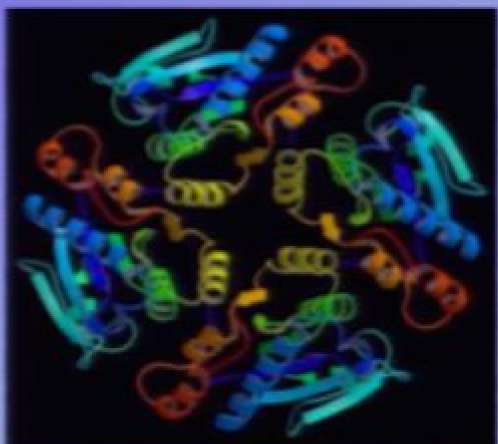

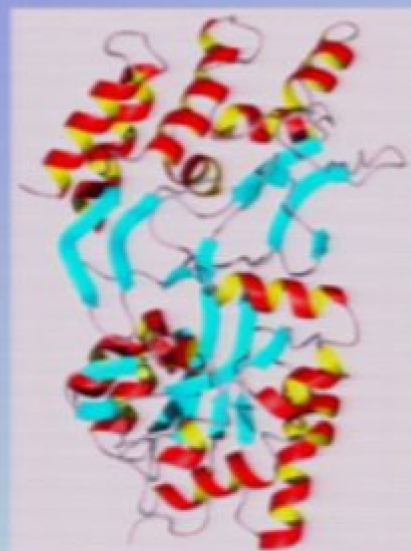

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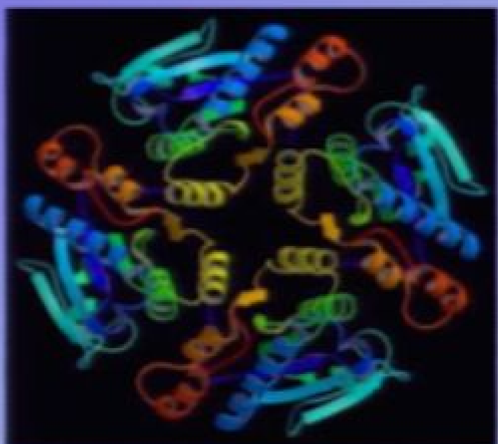

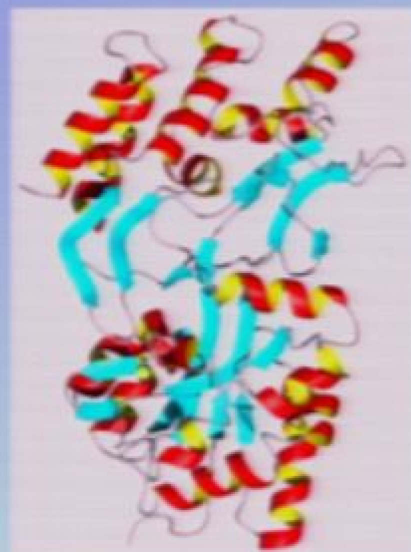

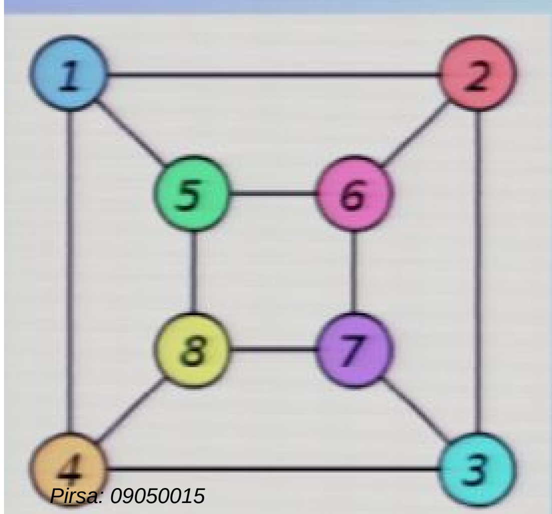

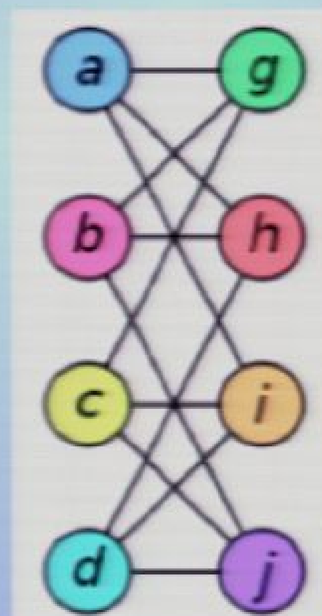

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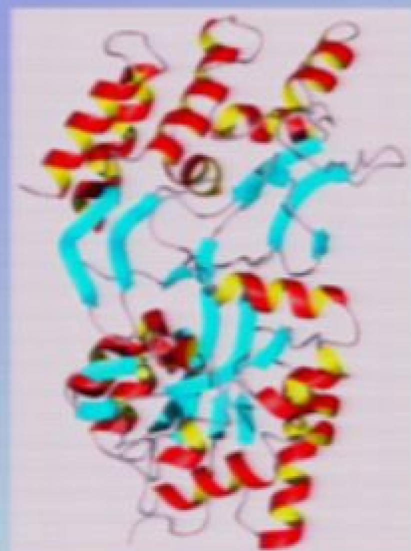

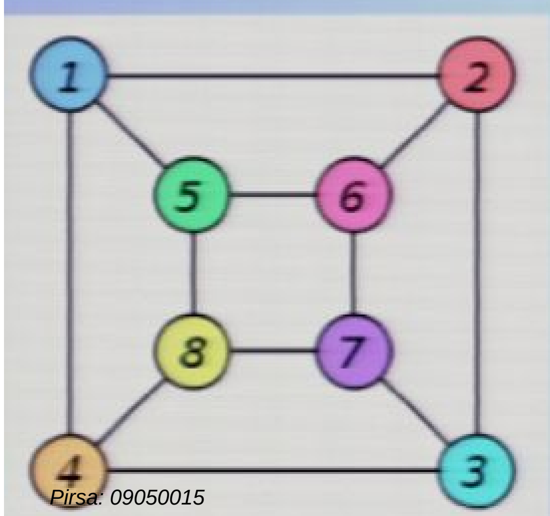

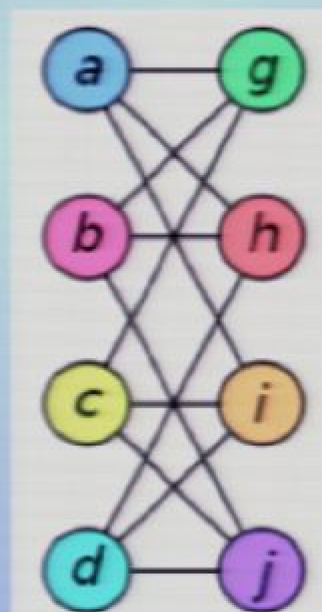

$f(a) = 1$
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$f(c) = 8$
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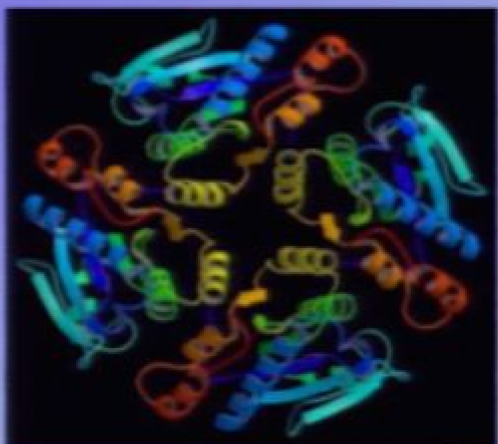
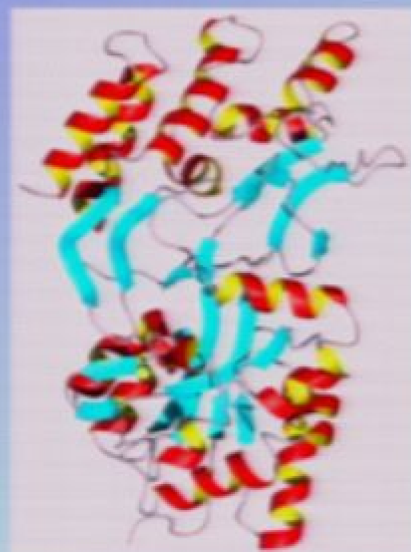
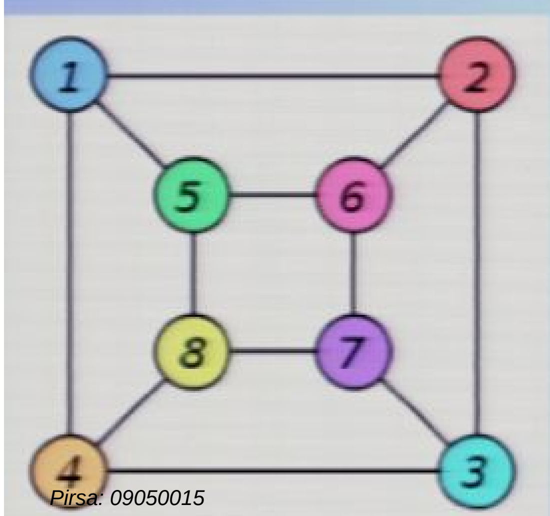


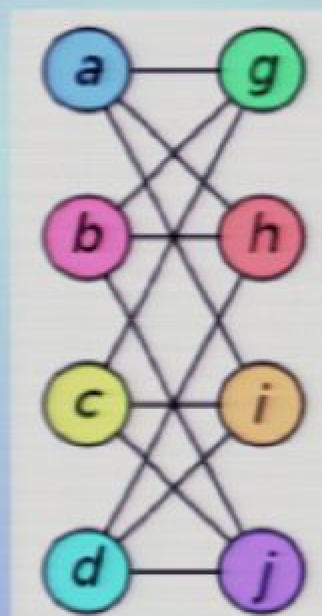


Best known classical algorithm

$$O(e^{\sqrt{n}+O(1)})$$

(Handbook of Graph Theory, 2004)





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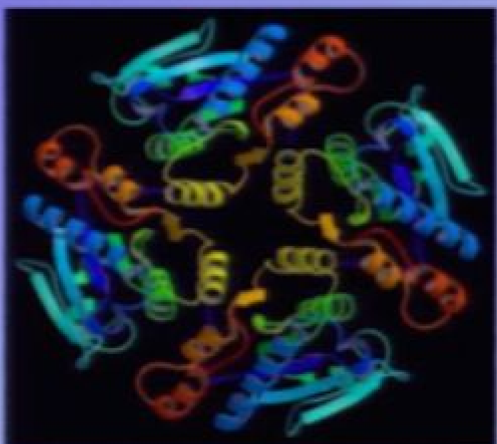

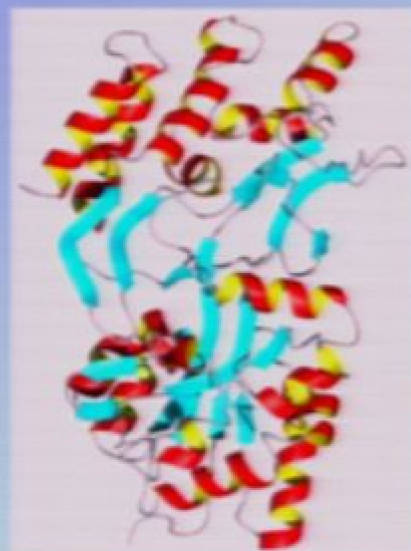

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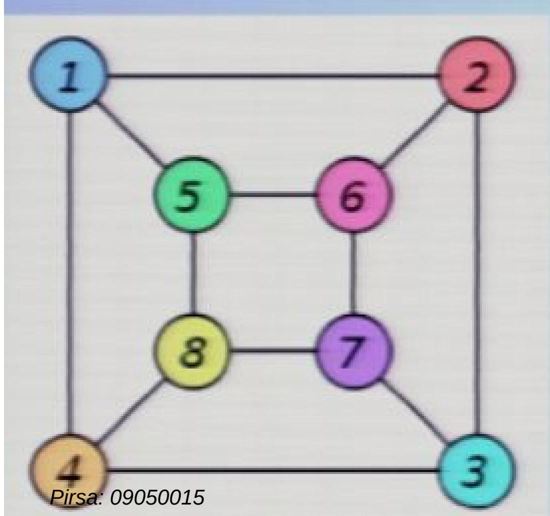

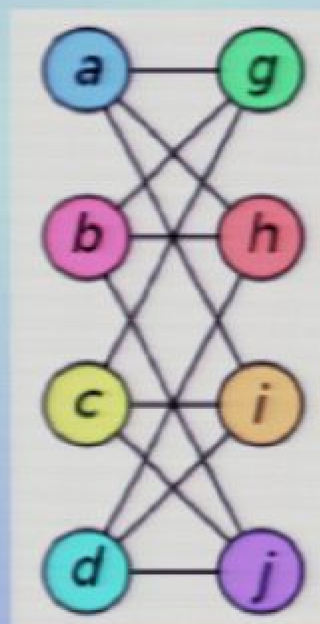

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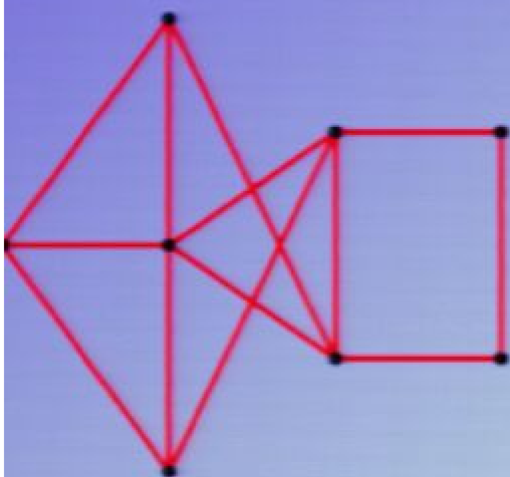
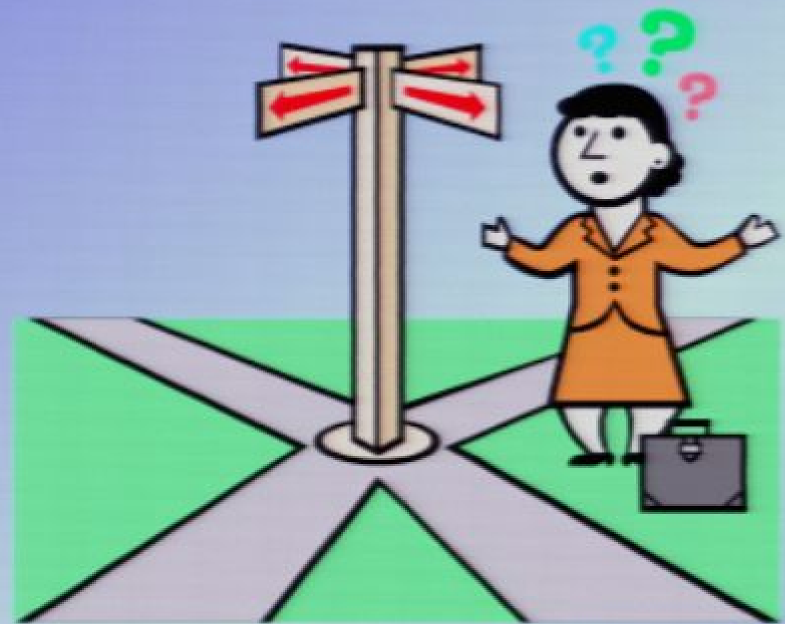
Alan Woods
Caiheng Li
(mathematicians
at UWA)

Random Walk on Graphs





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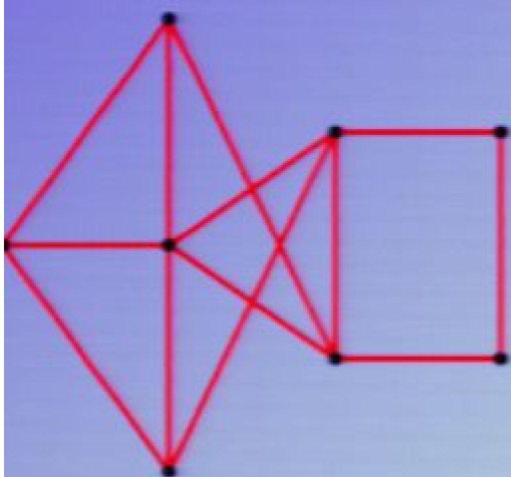

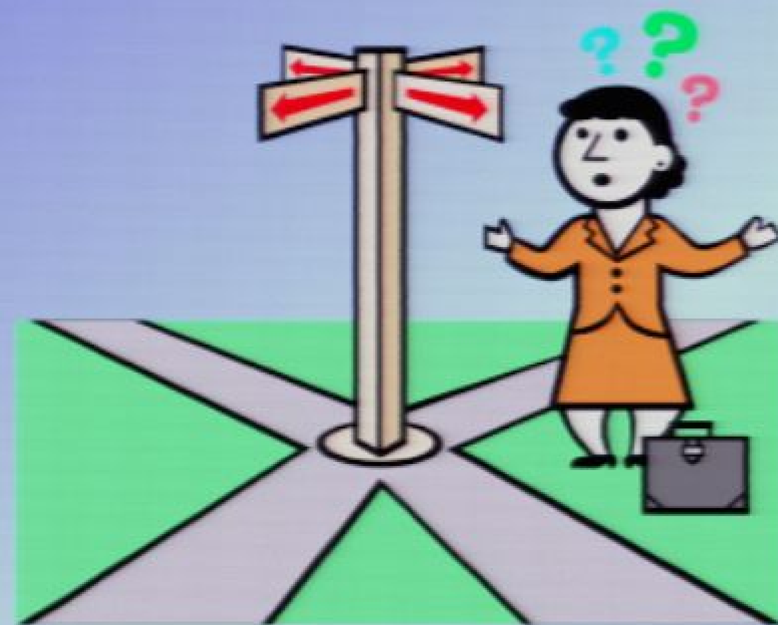

Classical : coin flip and move (stochastic)

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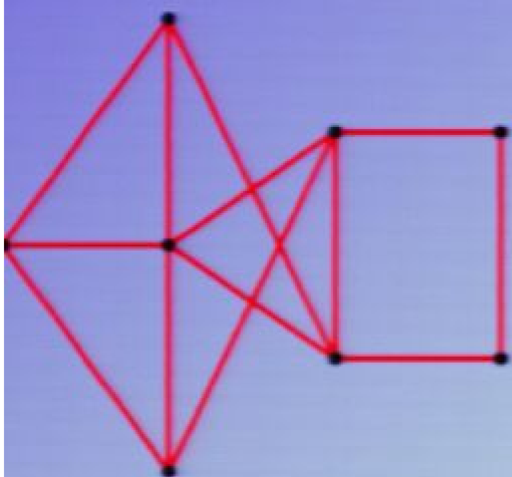
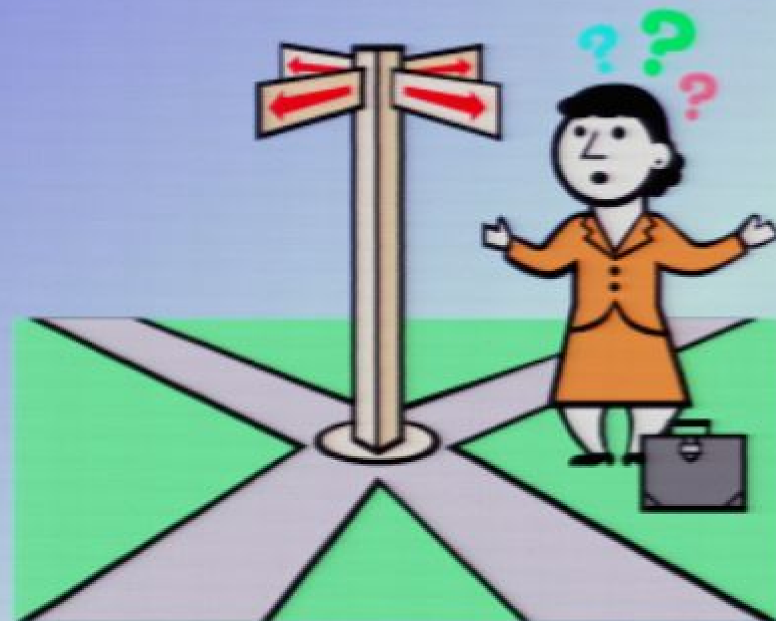


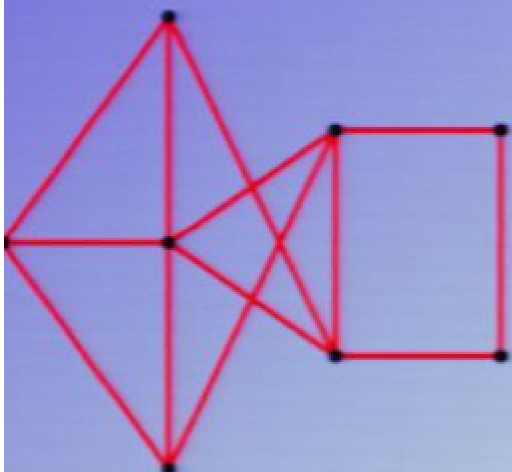


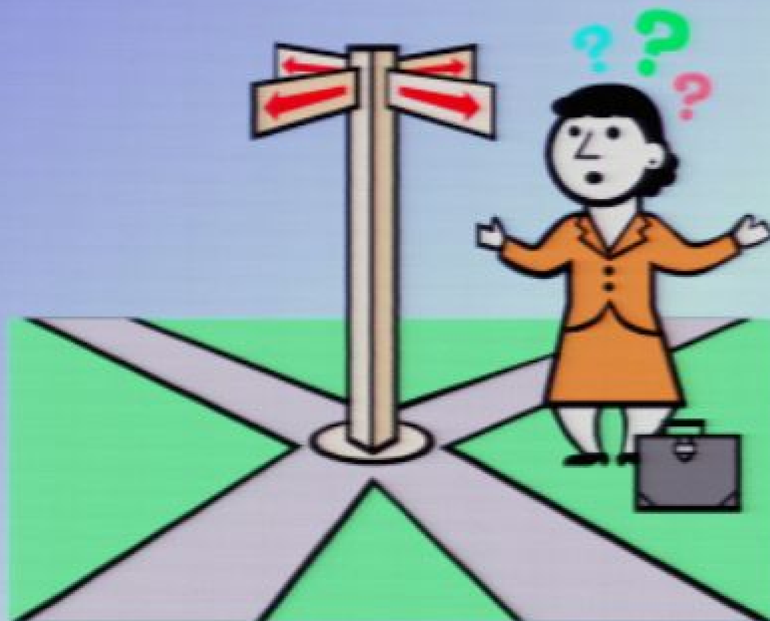
Classical : coin flip and move (stochastic)

$$\begin{pmatrix} p'_1 \\ \cdots \\ p'_n \end{pmatrix} = \begin{pmatrix} q_{11} & \cdots & q_{1n} \\ \cdots & \cdots & \cdots \\ q_{n1} & \cdots & q_{nn} \end{pmatrix} \begin{pmatrix} p_1 \\ \cdots \\ p_n \end{pmatrix}$$

probability

Random Walk on Graphs





Quantum : apply coin operator and evolve (unitary)

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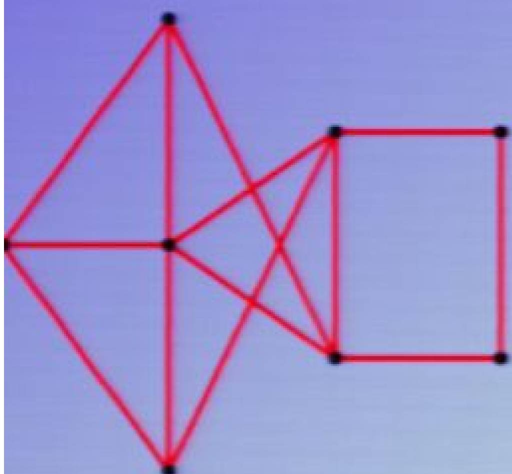

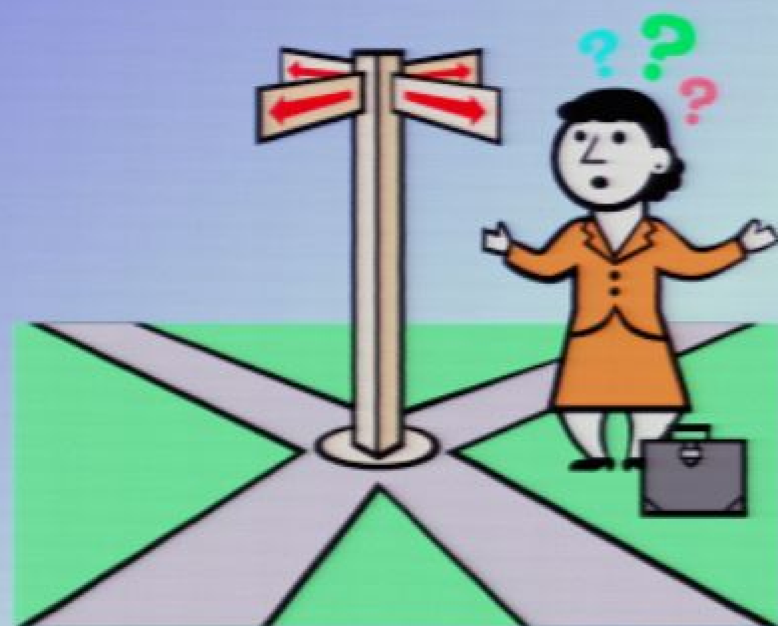

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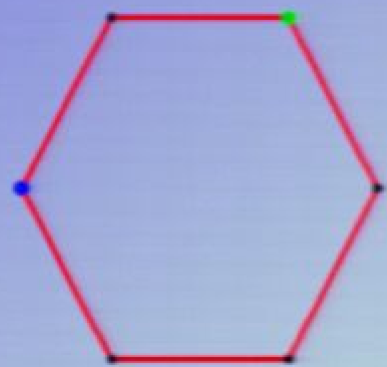
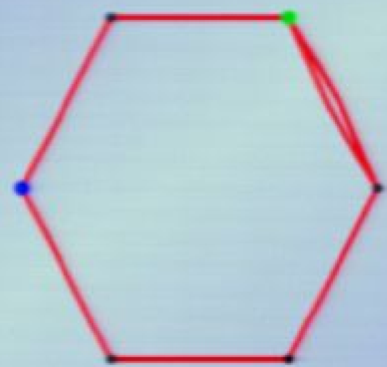
amplitude

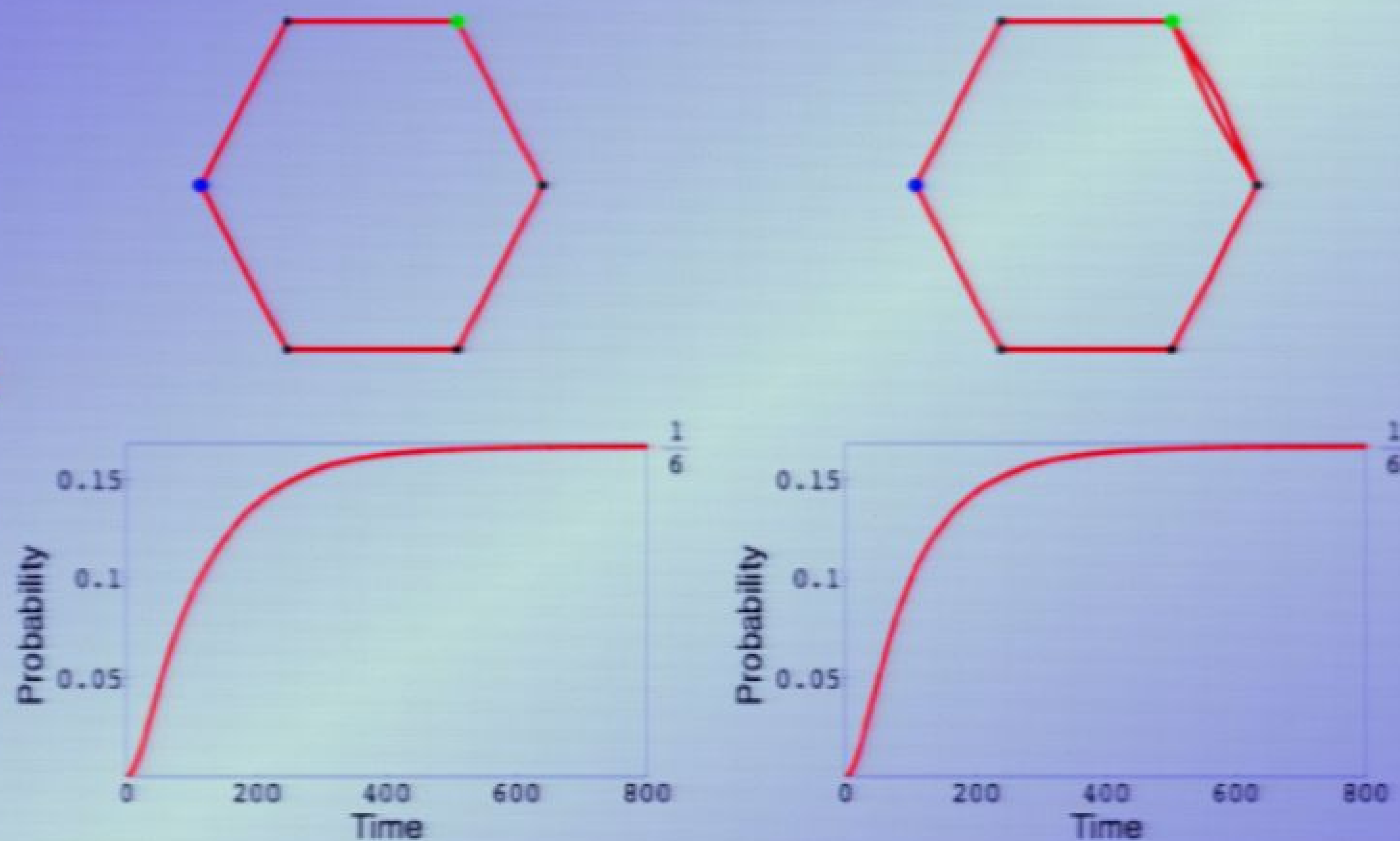
Random Walk on Graphs





Random Walk on Graphs

classical



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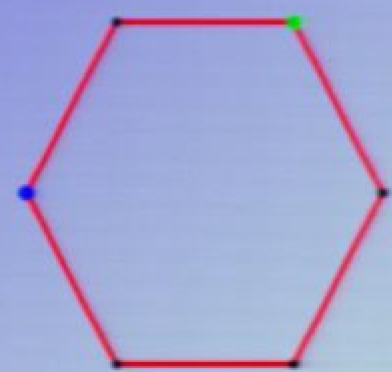

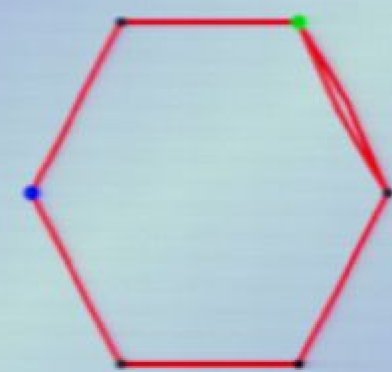

quantum

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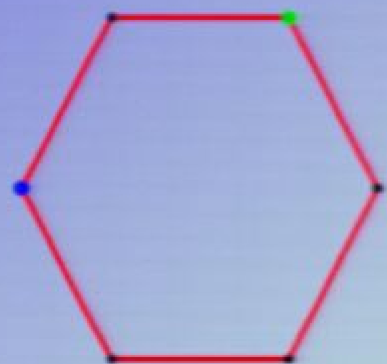
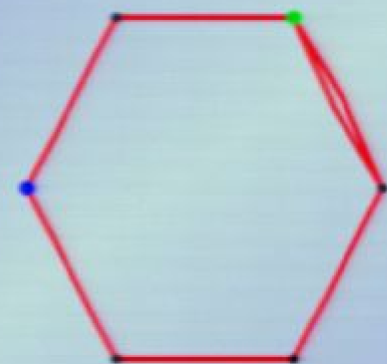
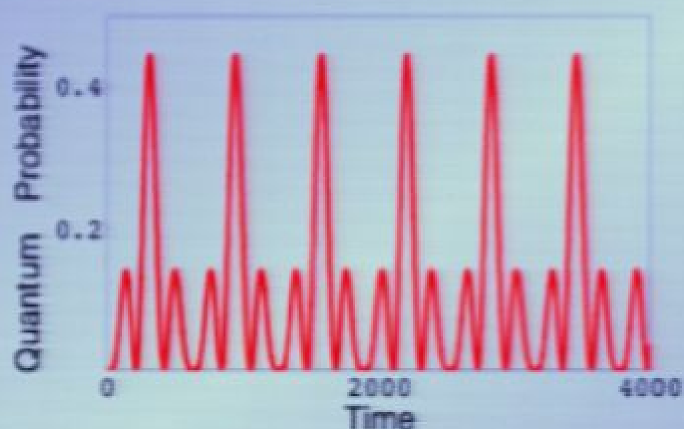


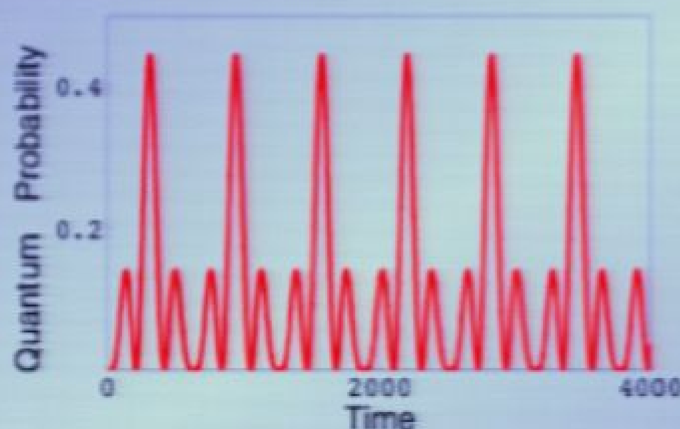


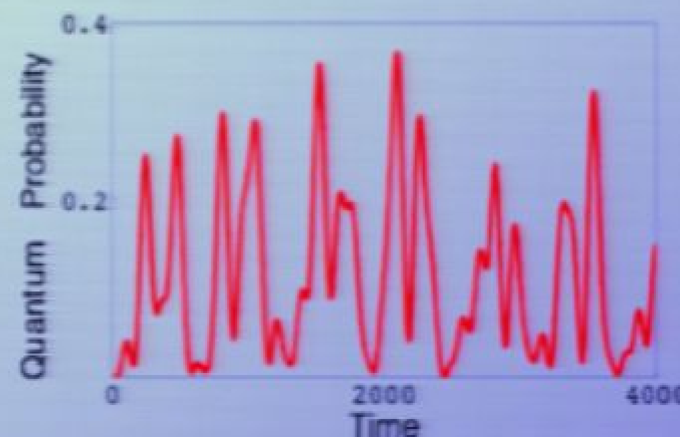
quantum



Random Walk on Graphs

quantum





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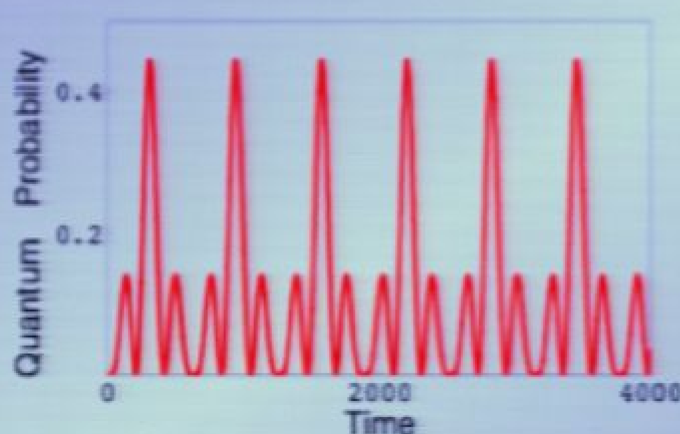

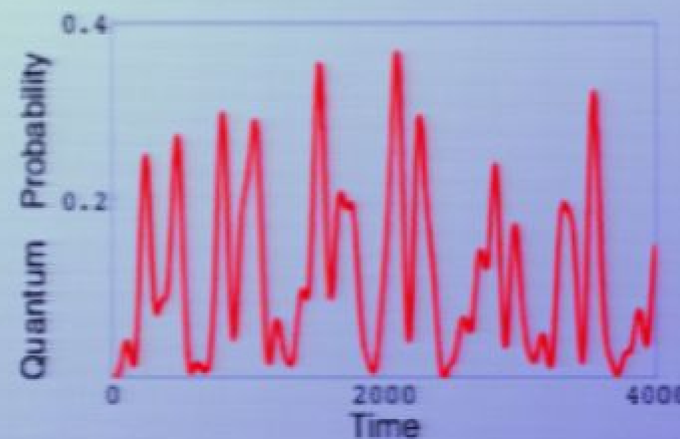

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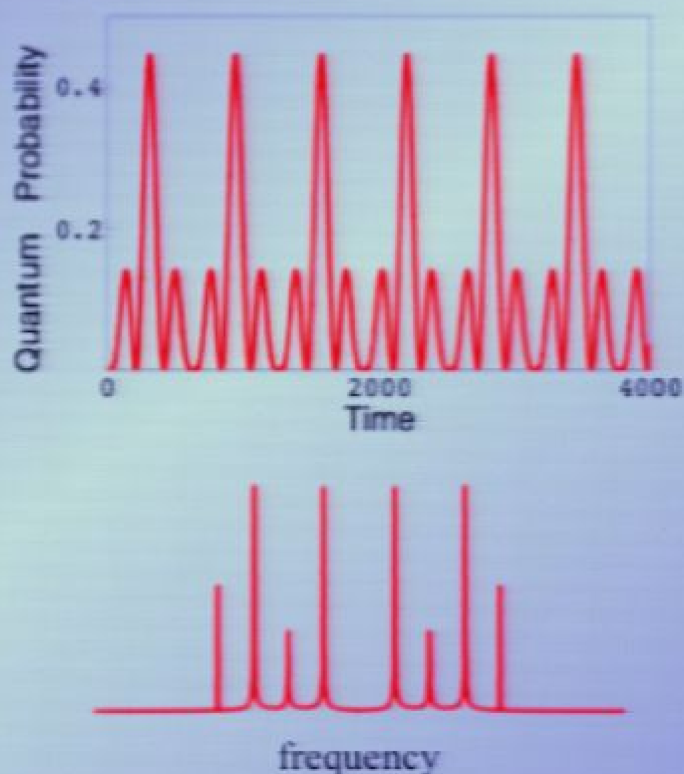
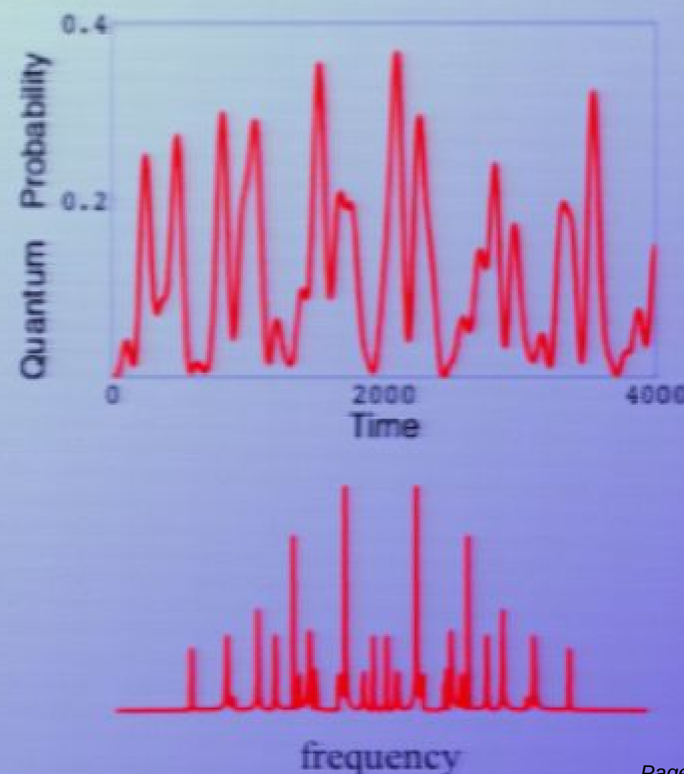
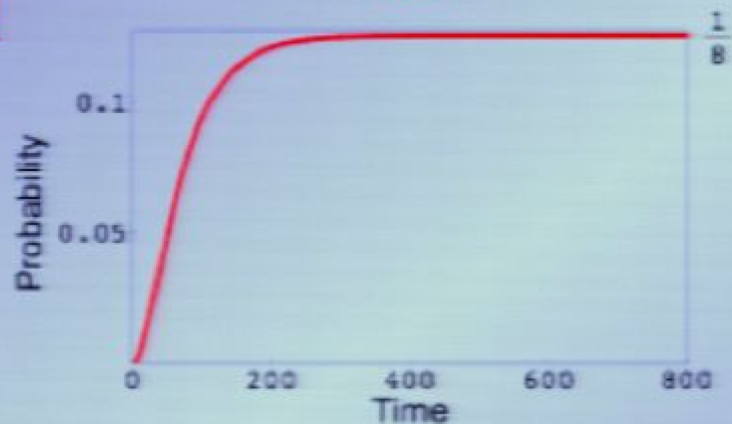




Random Walk on Graphs

Random Walk on Graphs

classical



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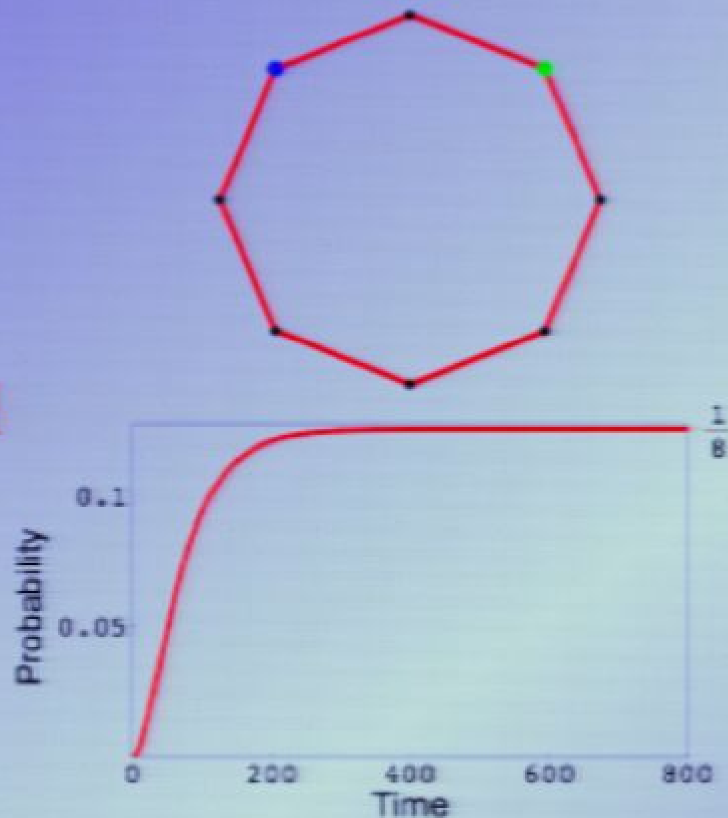

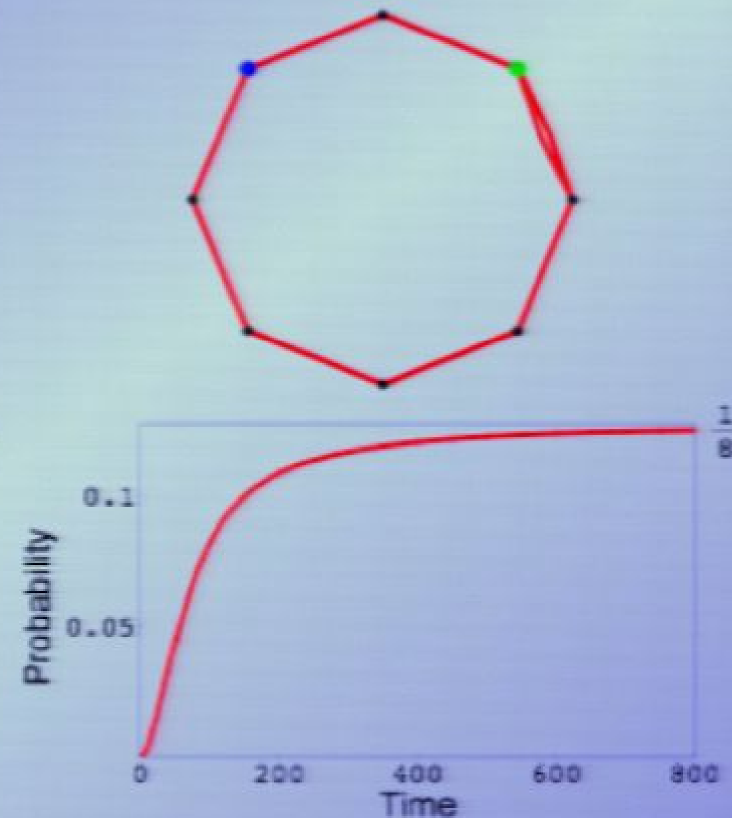

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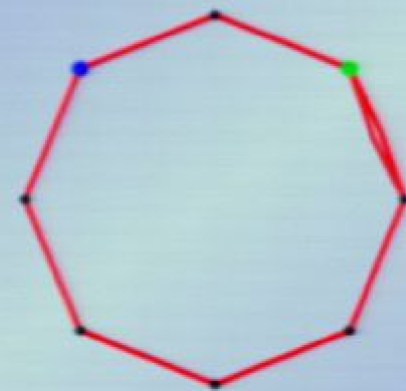

quantum

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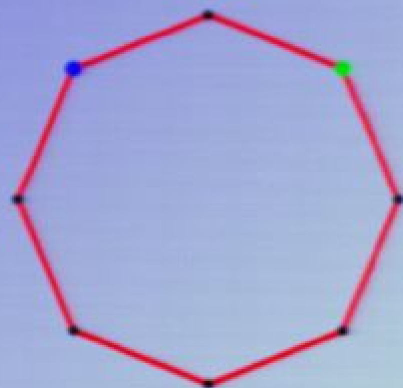

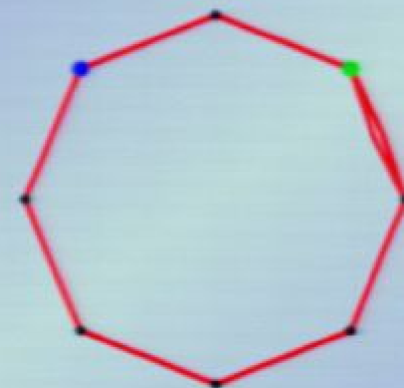

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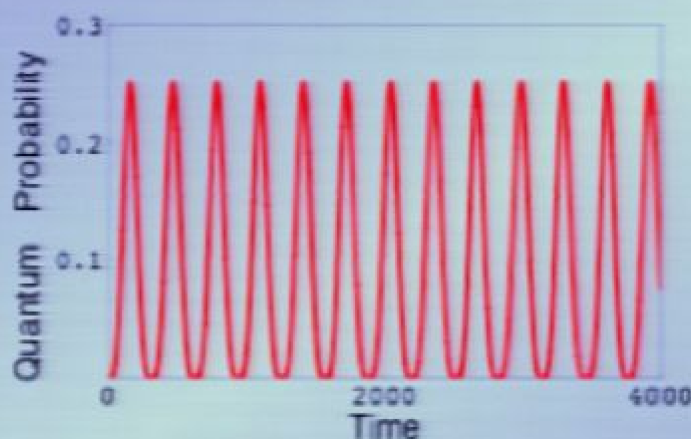

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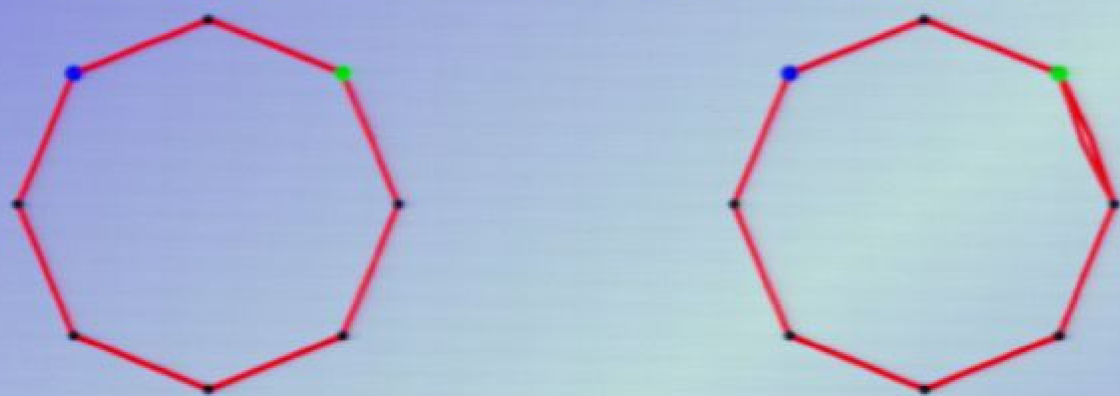

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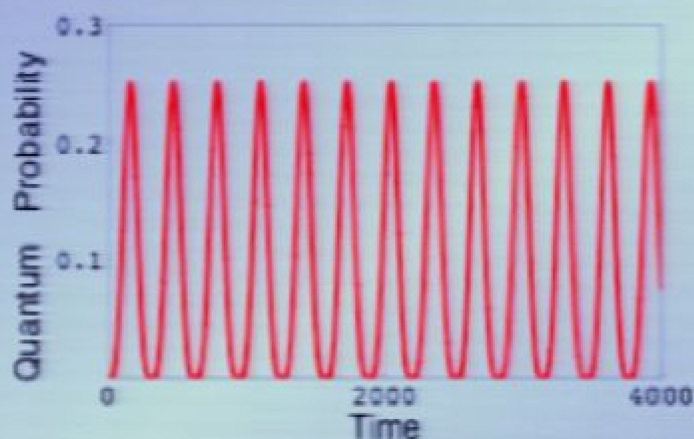
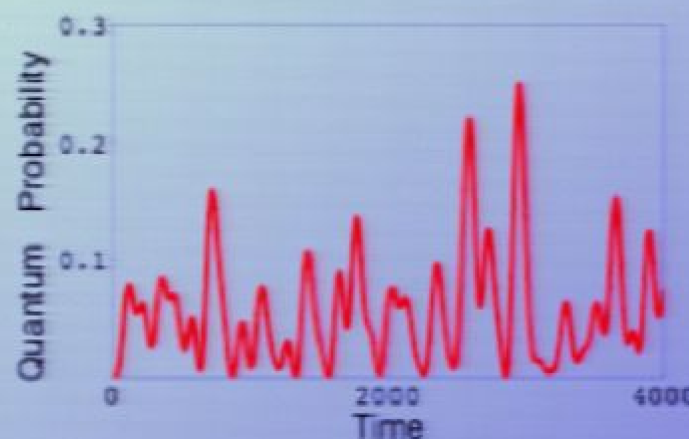
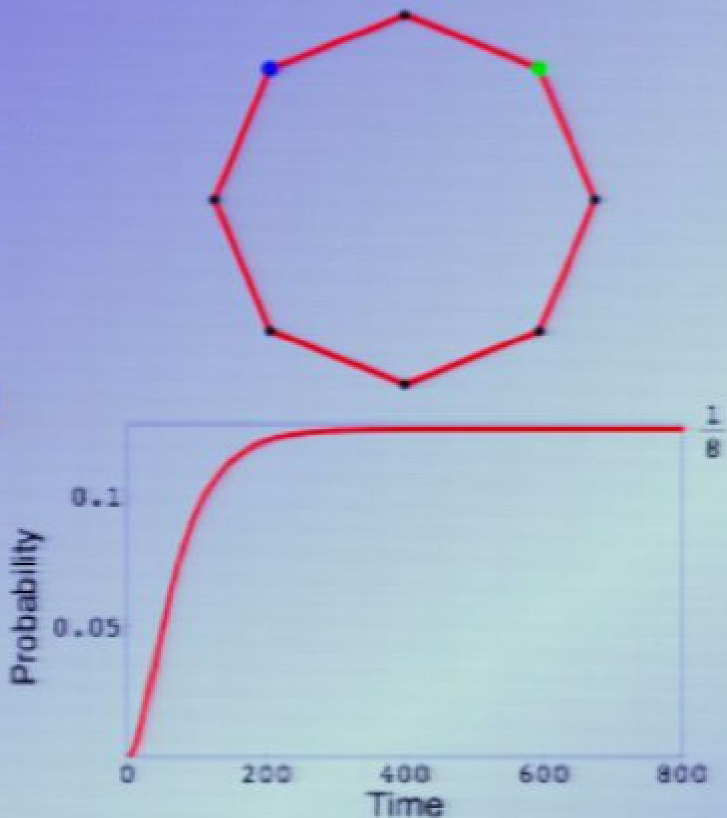
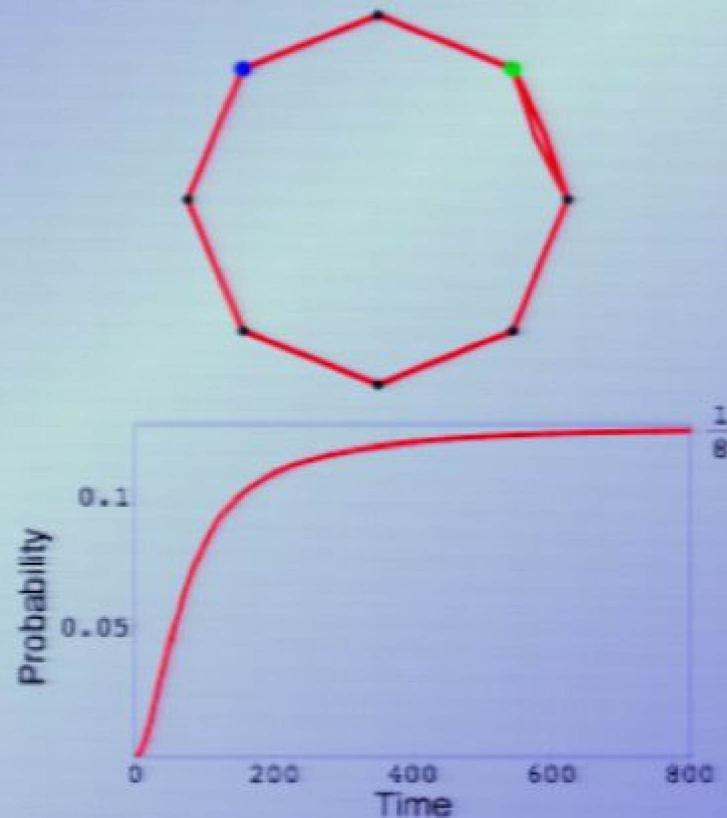




Random Walk on Graphs

classical





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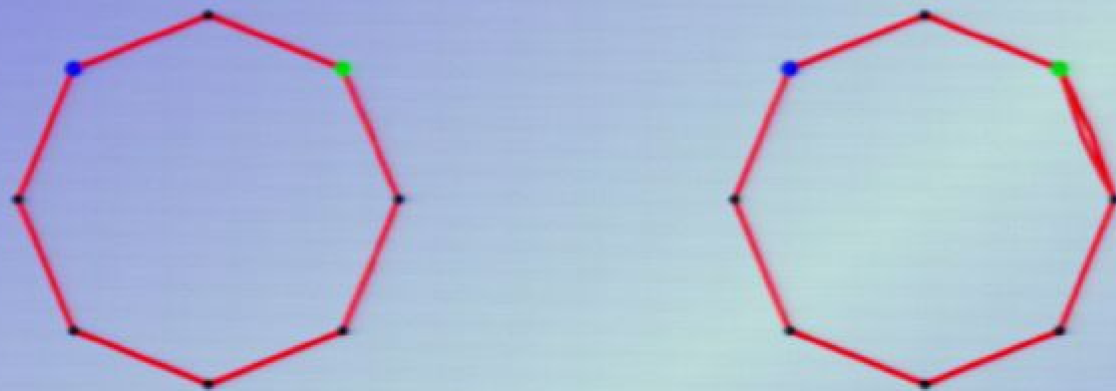

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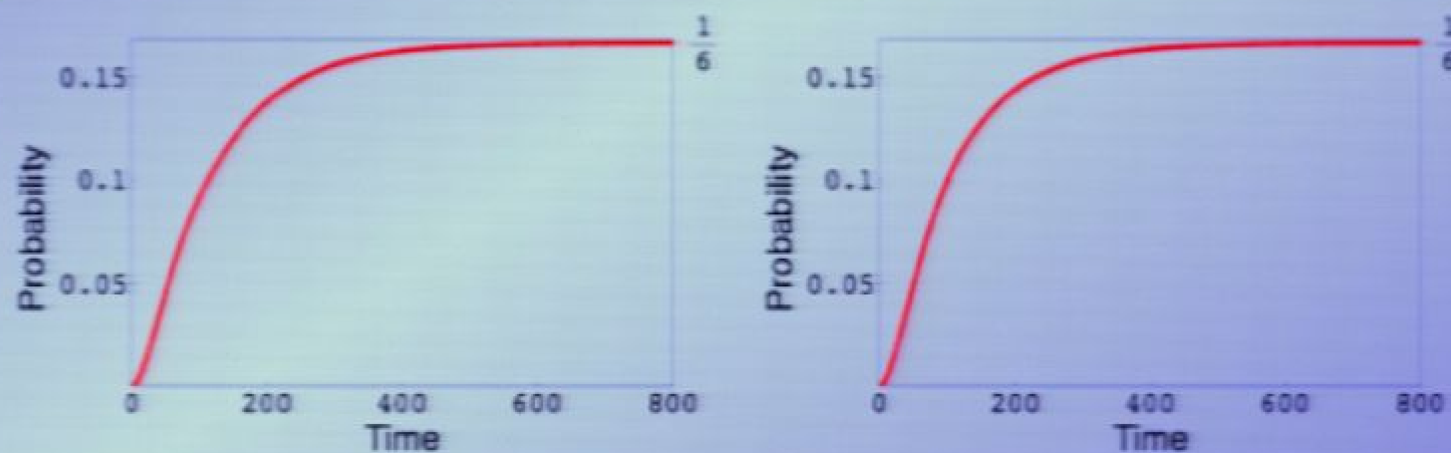

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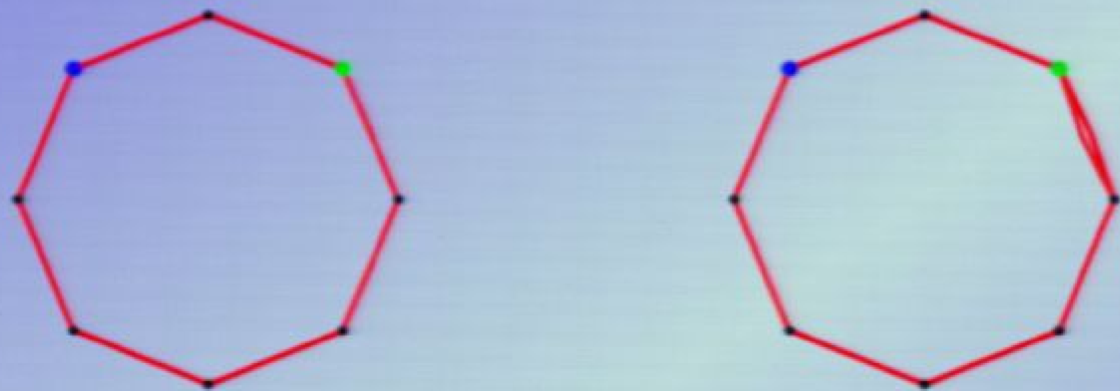

Random Walk on Graphs

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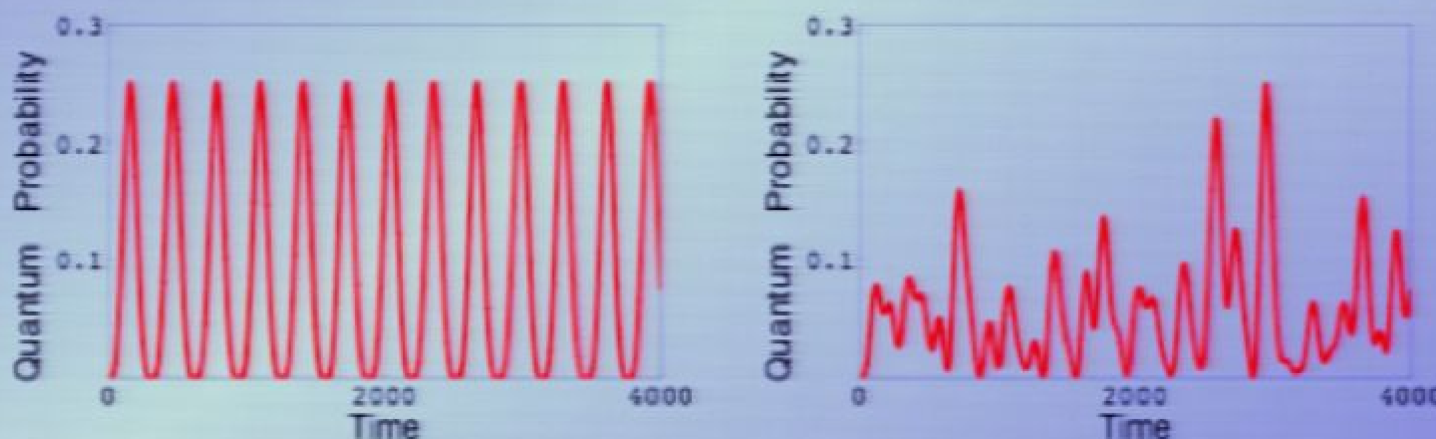
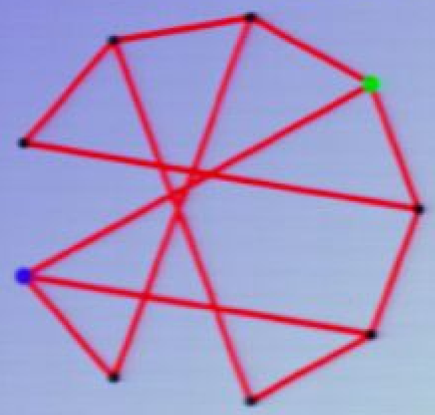
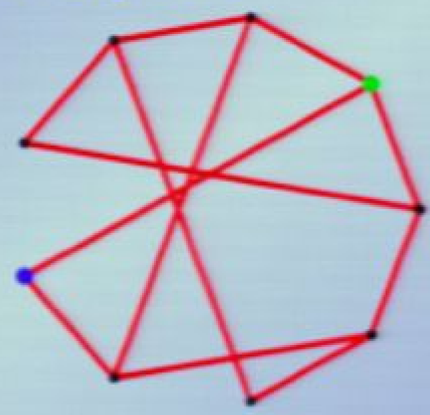


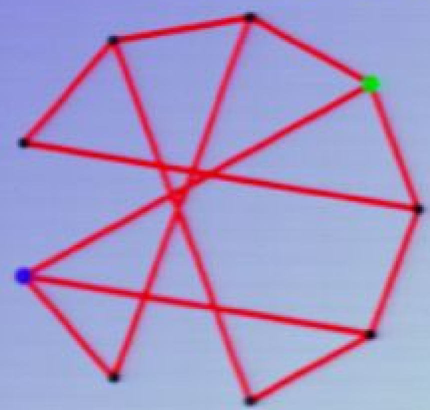
frequency

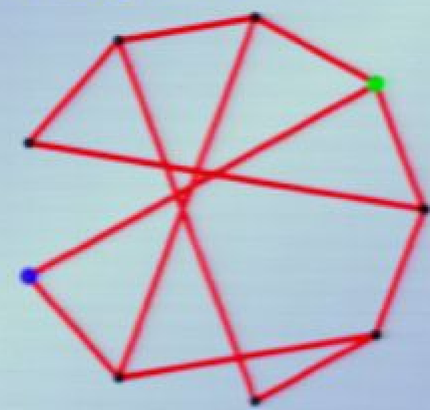
Random Walk on Graphs



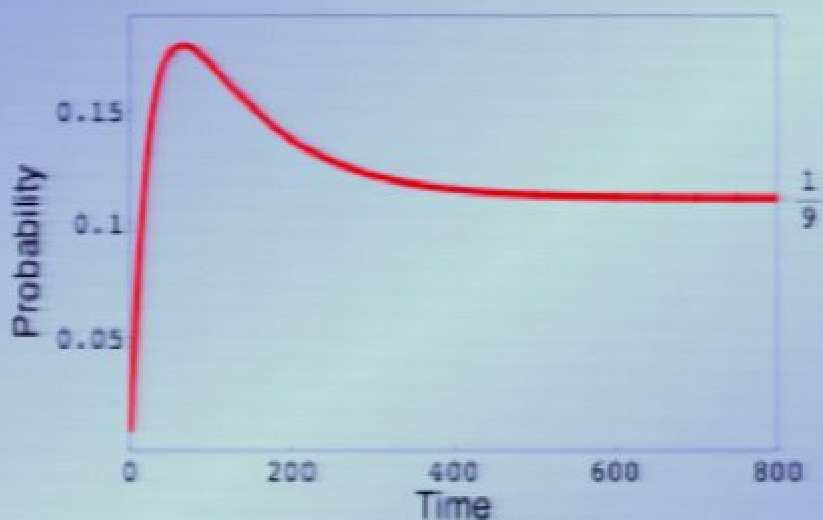


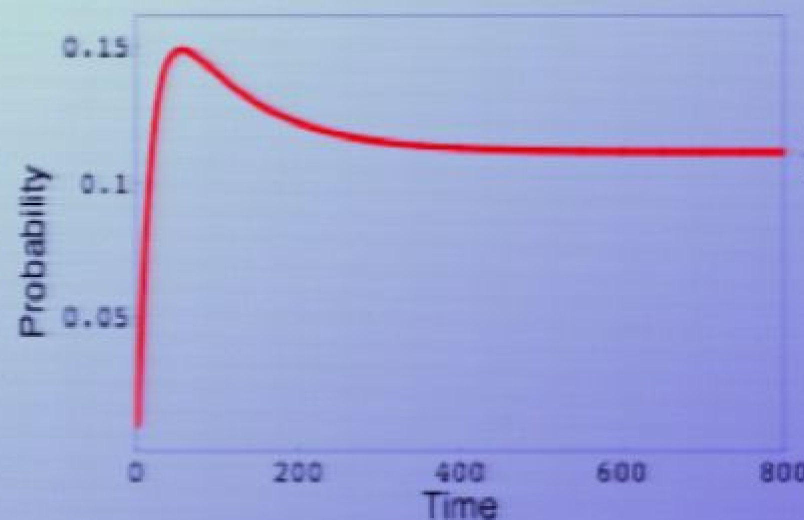
Random Walk on Graphs



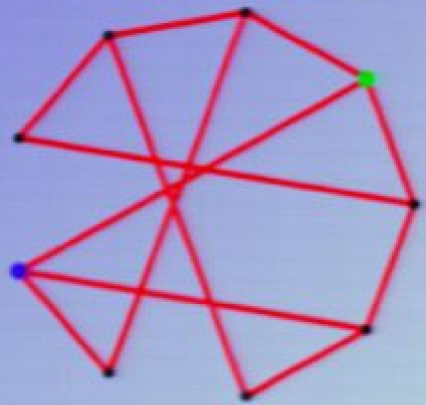


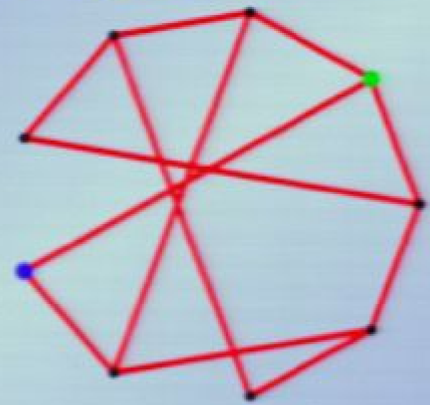
classical





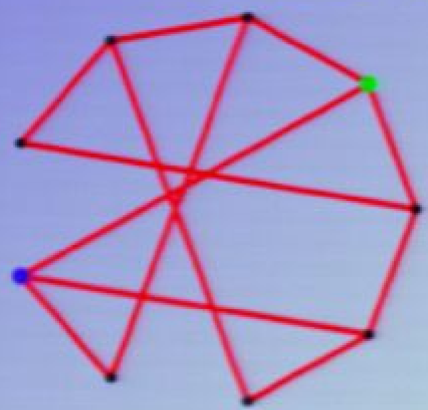
Random Walk on Graphs

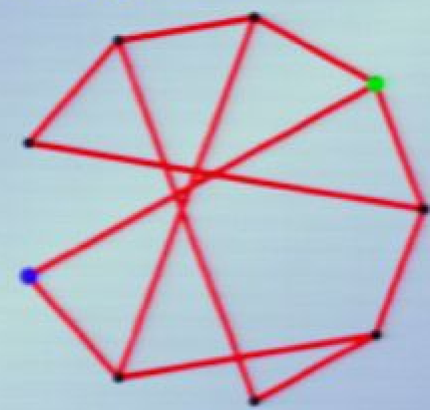




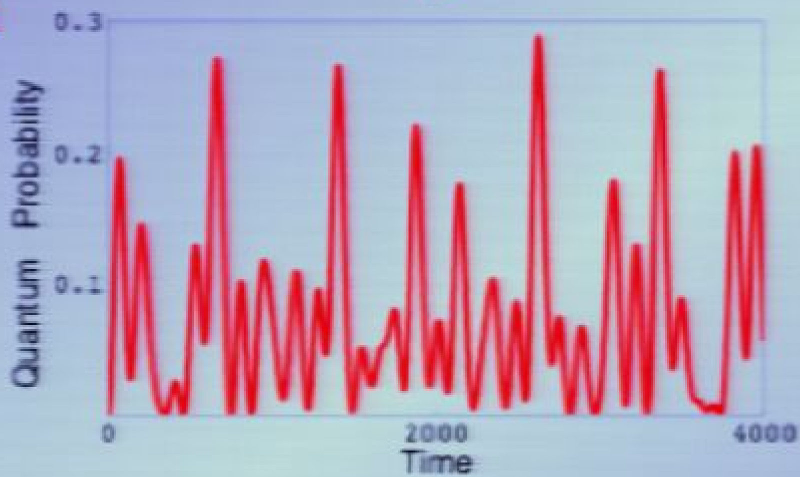
quantum

Random Walk on Graphs



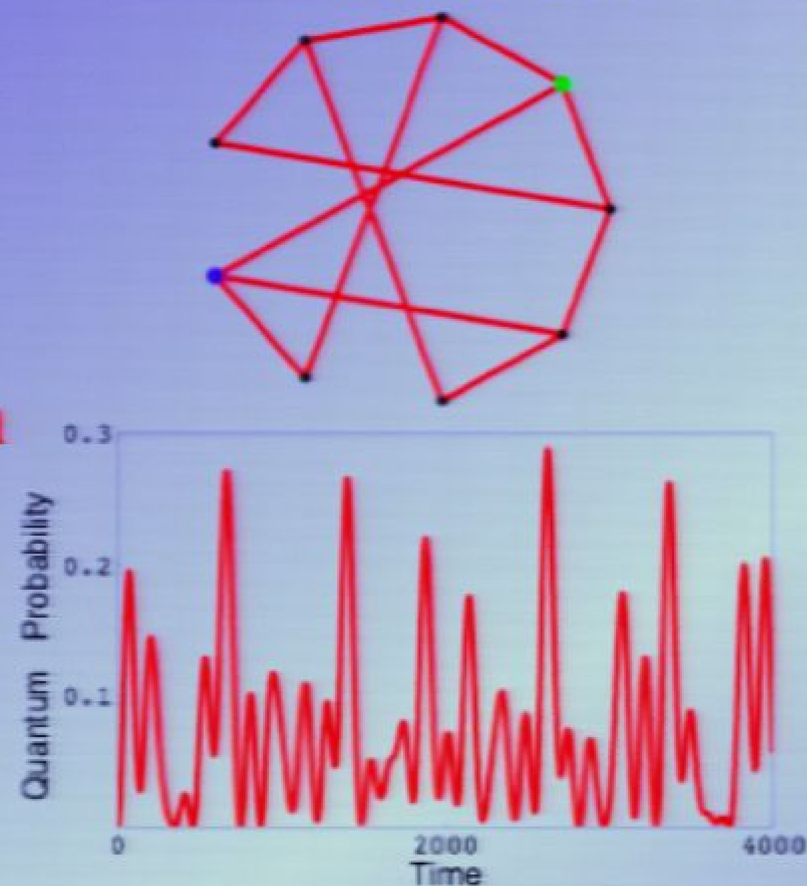


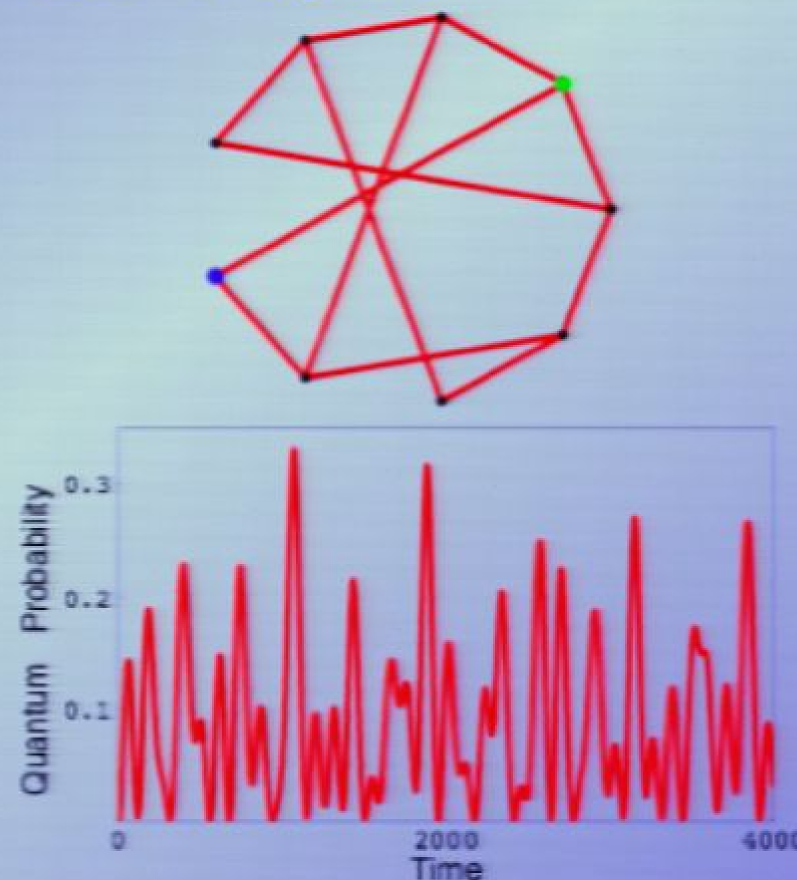
quantum



Random Walk on Graphs

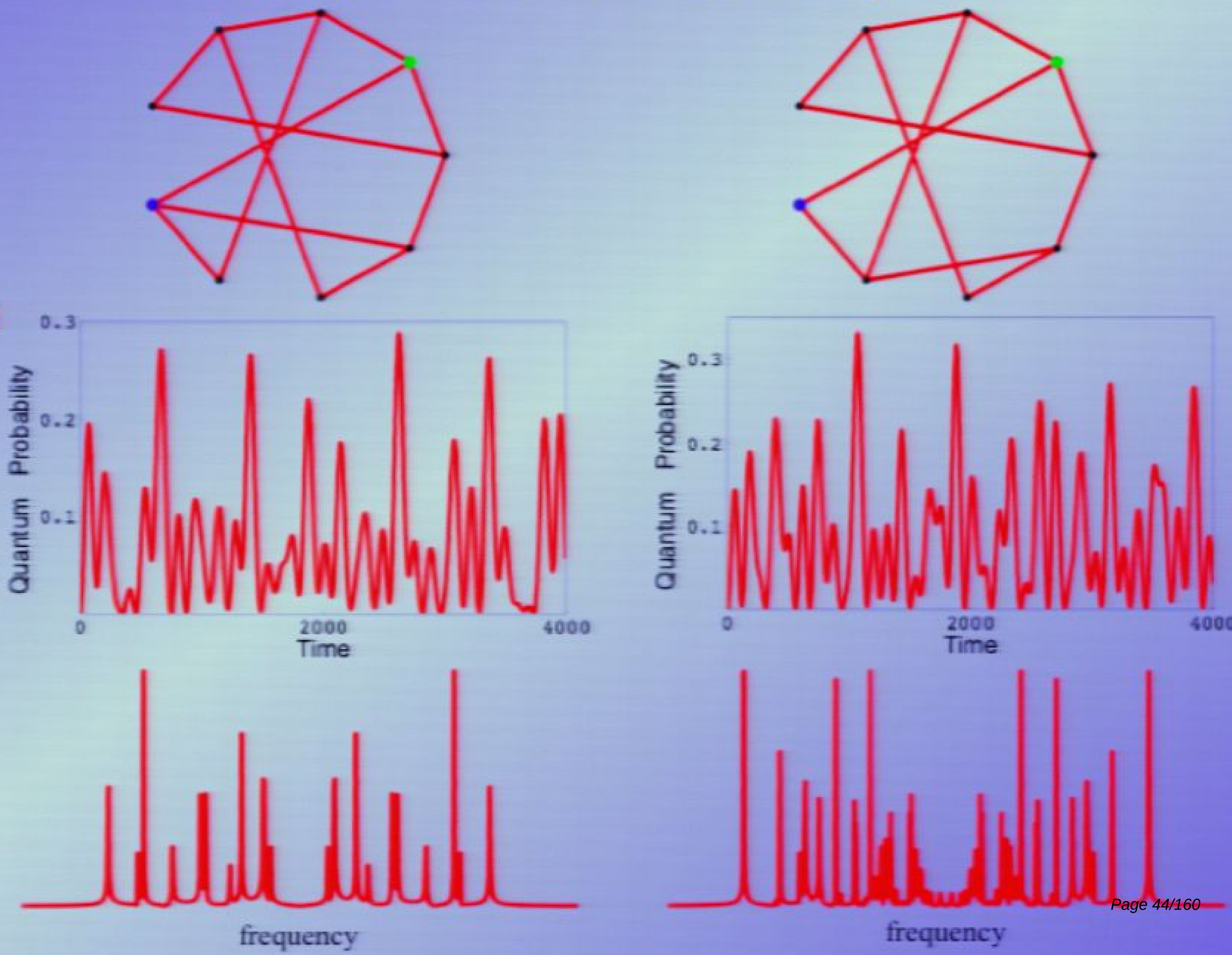
quantum





Random Walk on Graphs

quantum



Can we solve the GI problem by QW

Can we solve the GI problem by QW

Various attempts :

✧ Shiau, Joynt, Coppersmith
Quant. Info. Compu. 5, 492 (2005)

✧ Emms, Hancock, Severini, Wilson
J. Combinatorics 13, R34 (2006)

✧ Douglas and Wang,
J Phys A 41, 075303 (2008)

✧ Emms, Wilson, Hancock,
Pattern Recognition 42, 985 (2009)

Douglas and Wang, J Phys A 41, 075303 (2008)

A wide range of graphs tested		
Type of graph	Order of graph	Number in group
Eulerian	8	184
	9	1782
	10 (connected graphs)	30990
Cubic vertex-transitive	48	32
	60	26
	64	38
Planar	7	646
Simple	7	853
Tree	14	3159
Vertex-critical	10 (chi = 4)	2453
Edge-critical	12 (chi = 7)	395
Vertex-transitive	29 (connected graphs)	1181
Hypohamiltonian	26	2033
Hypohamiltonian, cubic (with girth at least 5)	28	34
Projective planes of order 16	273	13

Douglas and Wang, J Phys A 41, 075303 (2008)

Strongly regular graphs tested:		
Parameters	Number in group	Number tested (pairwise)
(16,6,2,2)	2	All
(25,12,5,6)	15	All
(26,10,3,4)	10	All
(28,12,6,4)	4	All
(29,14,6,7)	41	All
(35,18,9,9)	227	All
(36,14,4,6)	180	All
(36,15,6,6)	32548	All (529669878)
(37,18,8,9)	6760	All (22845420)
(40,12,2,4)	28	All
(45,12,3,3)	168	All
(64,18,2,6)	78	All

Physical implementation of quantum walks

GI-complete classes of graphs

- ✧ connected graphs
- ✧ graphs of diameter 2 and radius 1
- ✧ directed acyclic graphs
- ✧ regular graphs
- ✧ bipartite Eulerian graphs
- ✧ bipartite regular graphs
- ✧ line graphs
- ✧ chordal graphs
- ✧ regular self-complementary graphs

V. P. Zemlyachenko, P. M. Korneenko, and R. I. Tyshkevich, "The graph isomorphism problem", Zap. Nauch. Seminarov LOMI AN SSSR, 118, 83-158 (1982).

Physical implementation of quantum walks

Physical implementation of quantum walks

(1) Highly symmetrical graphs

Physical implementation of quantum walks

(1) Highly symmetrical graphs

Lines, circles, hypercubes, complete graphs, regular binary trees, toroidal lattice supergraph, etc

Physical implementation of quantum walks

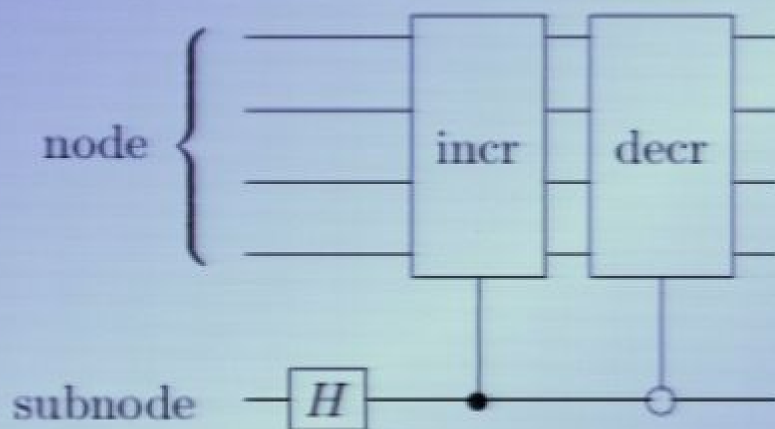
(1) Highly symmetrical graphs **Douglas and Wang, PRA (2009)**

Physical implementation of quantum walks

(1) Highly symmetrical graphs

Douglas and Wang, PRA (2009)

16-node cycle

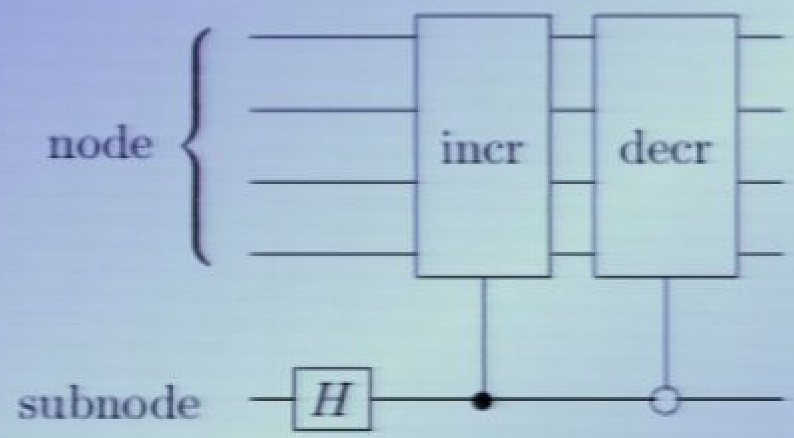


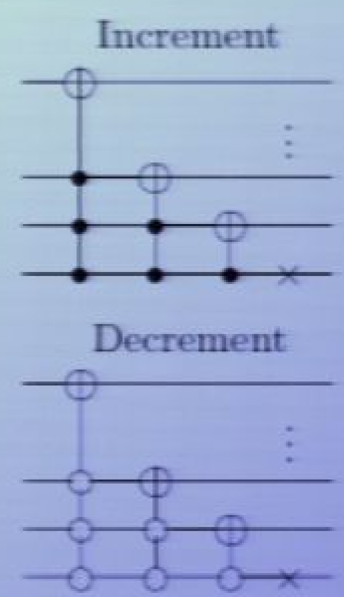
Physical implementation of quantum walks

(1) Highly symmetrical graphs

Douglas and Wang, PRA (2009)

16-node cycle



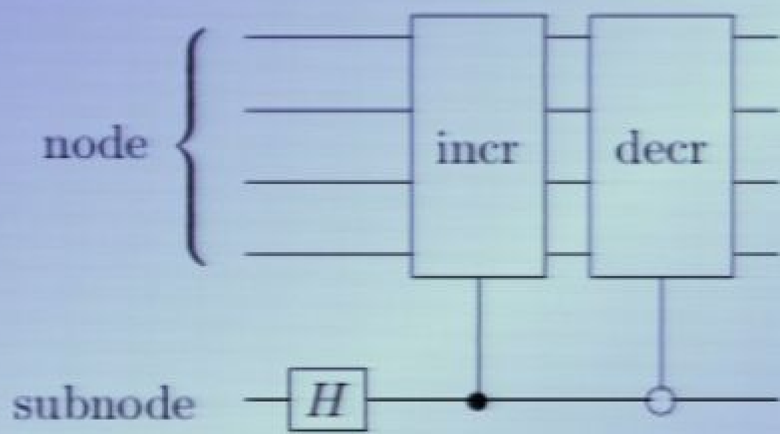


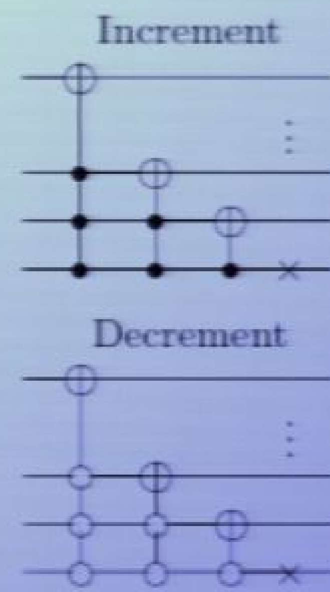
Physical implementation of quantum walks

(1) Highly symmetrical graphs

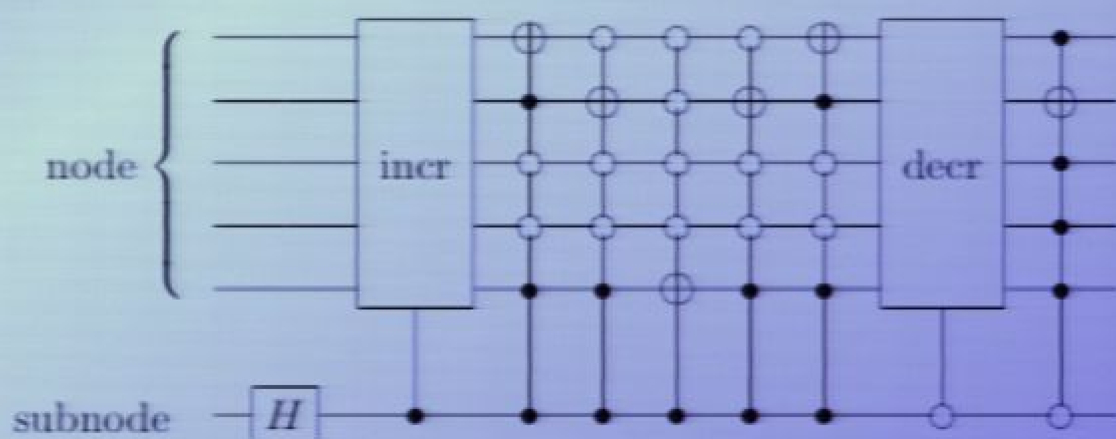
Douglas and Wang, PRA (2009)

16-node cycle





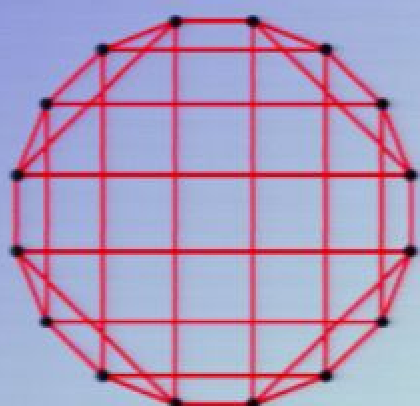
25-node cycle



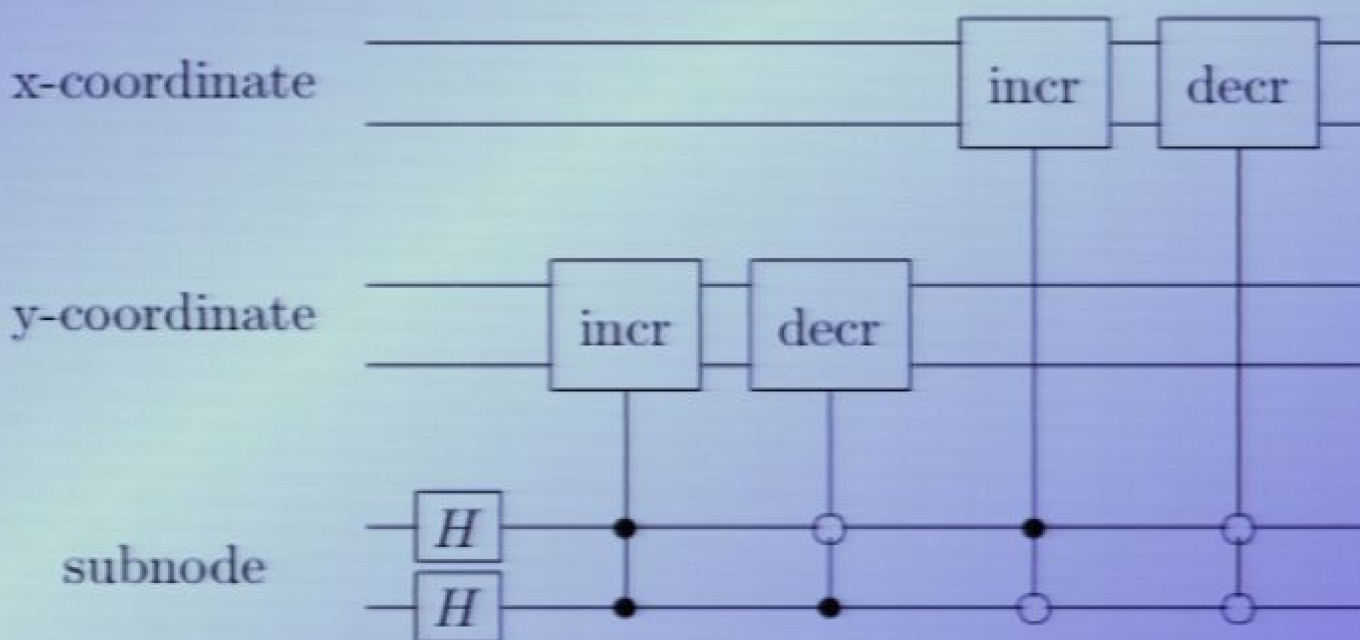
Physical implementation of quantum walks

(1) Highly symmetrical graphs

Douglas and Wang, PRA (2009)



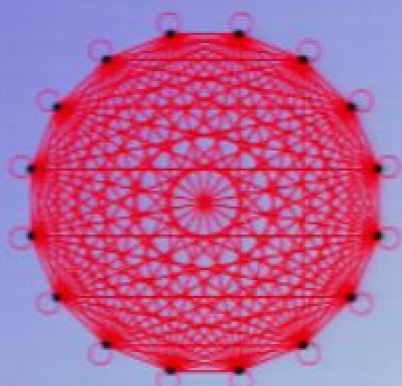
2D hypercycle



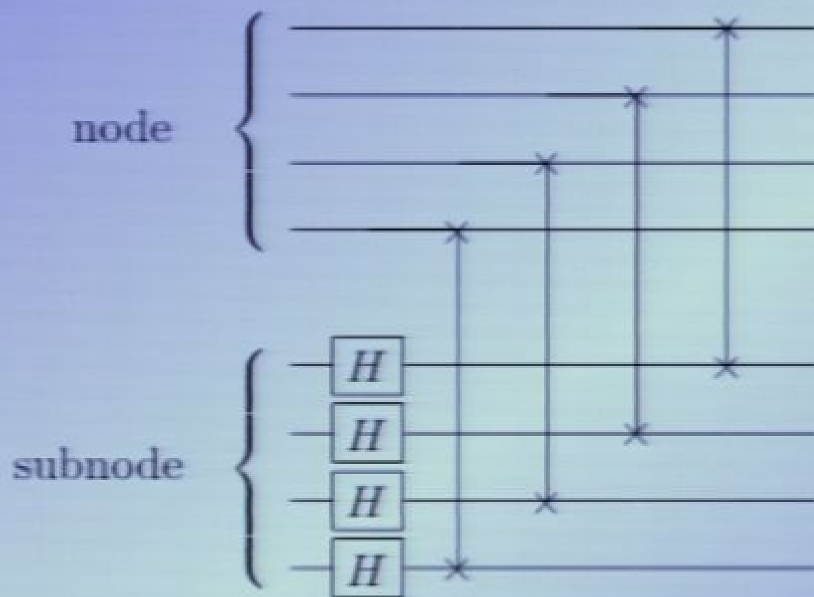
Physical implementation of quantum walks

(1) Highly symmetrical graphs

Douglas and Wang, PRA (2009)



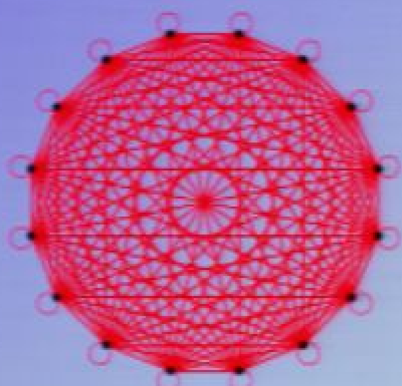
Complete 16-graph



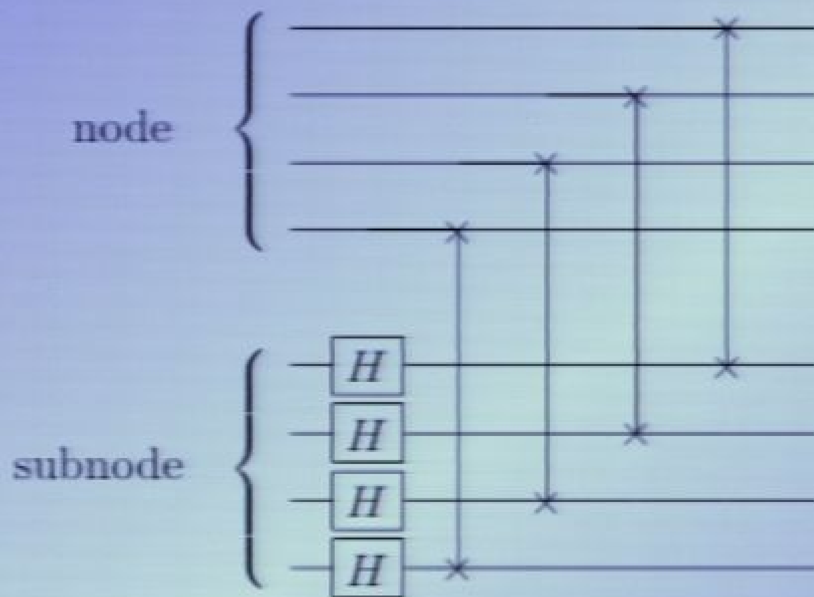
Physical implementation of quantum walks

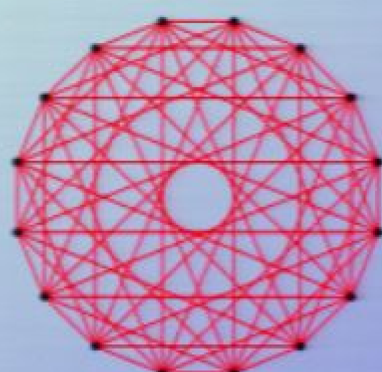
(1) Highly symmetrical graphs

Douglas and Wang, PRA (2009)

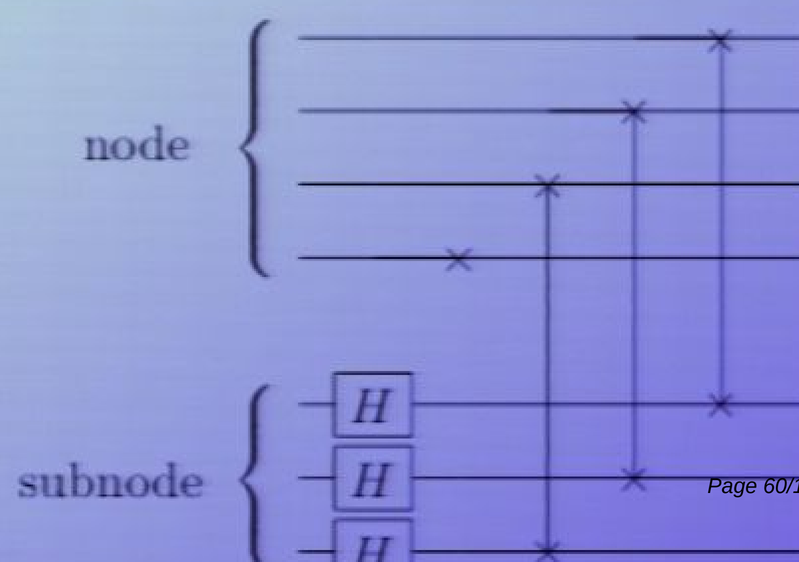


Complete 16-graph





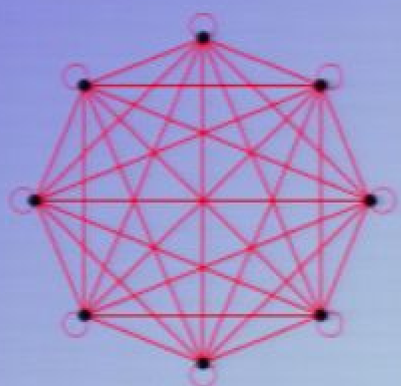
Modified 16-graph



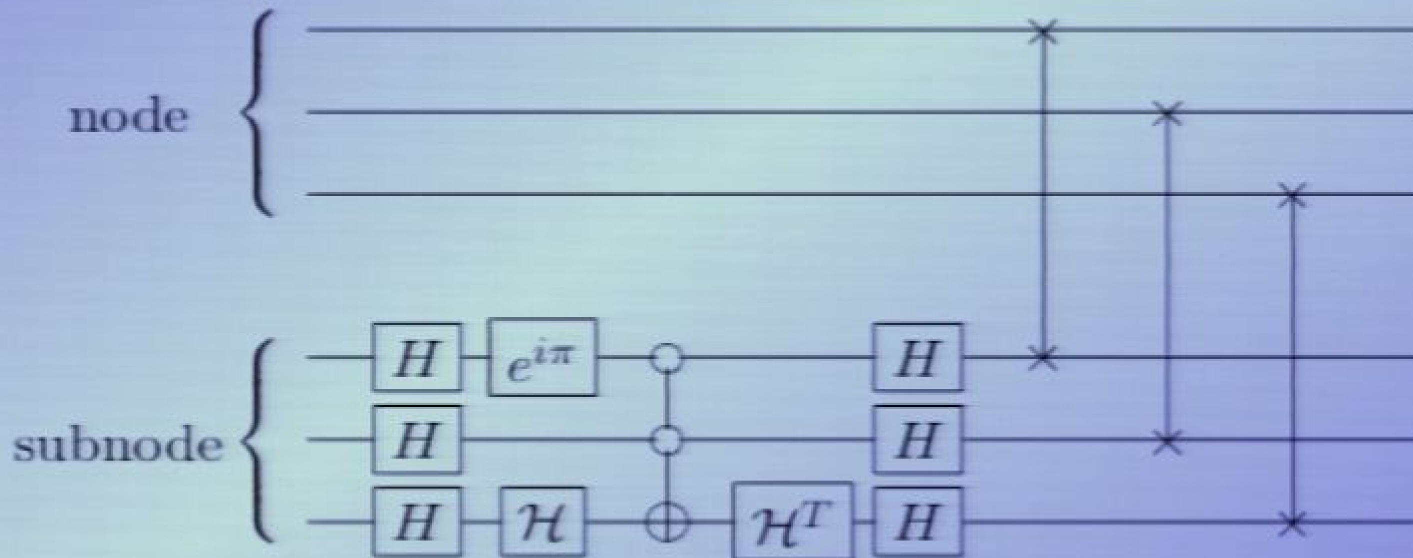
Physical implementation of quantum walks

(1) Highly symmetrical graphs

Douglas and Wang, PRA (2009)



Complete 8-graph with Grover Coin

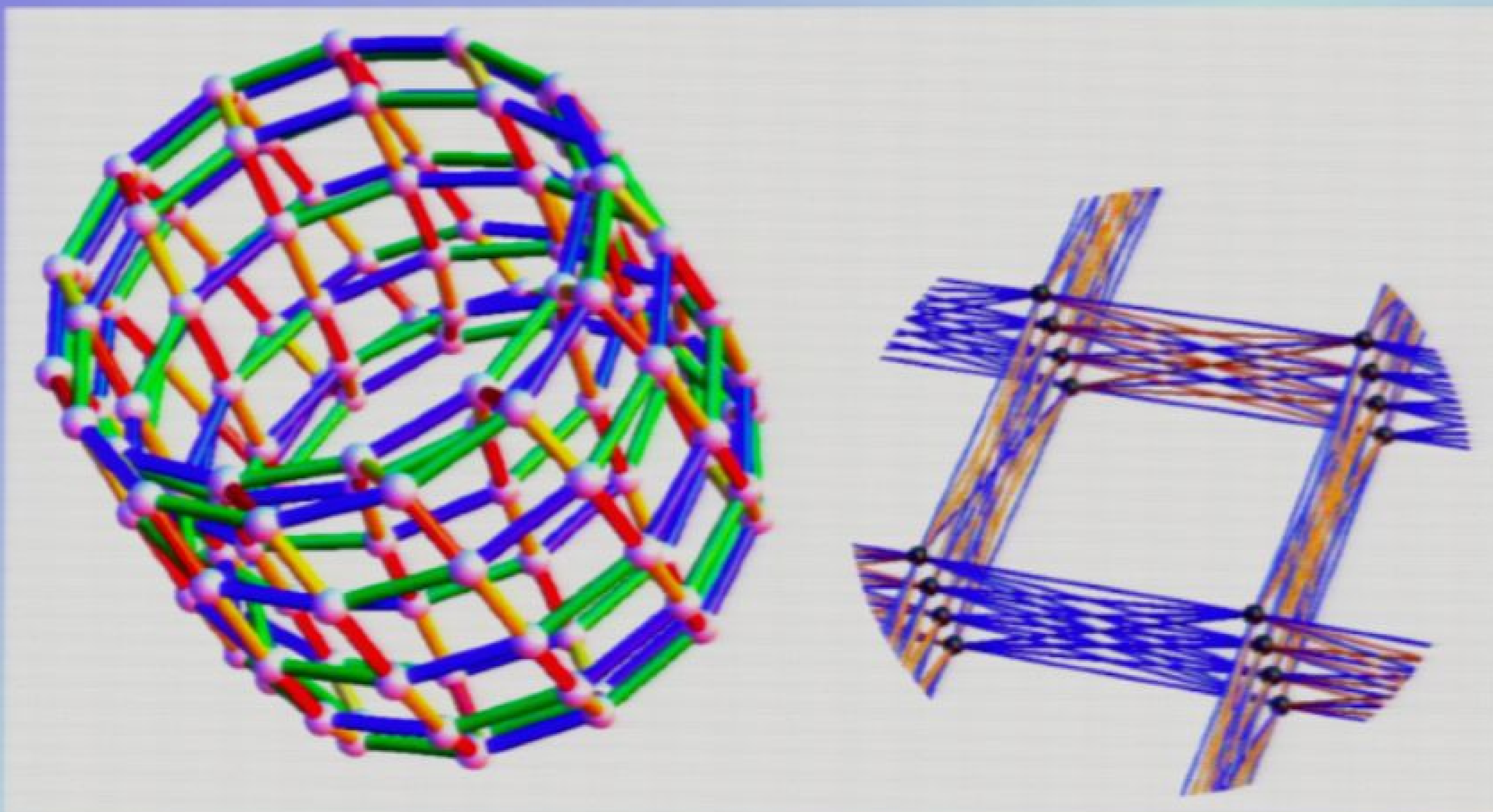


$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}; \quad \mathcal{H} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$$

Physical implementation of quantum walks

(1) Highly symmetrical graphs

Douglas and Wang, PRA (2009)

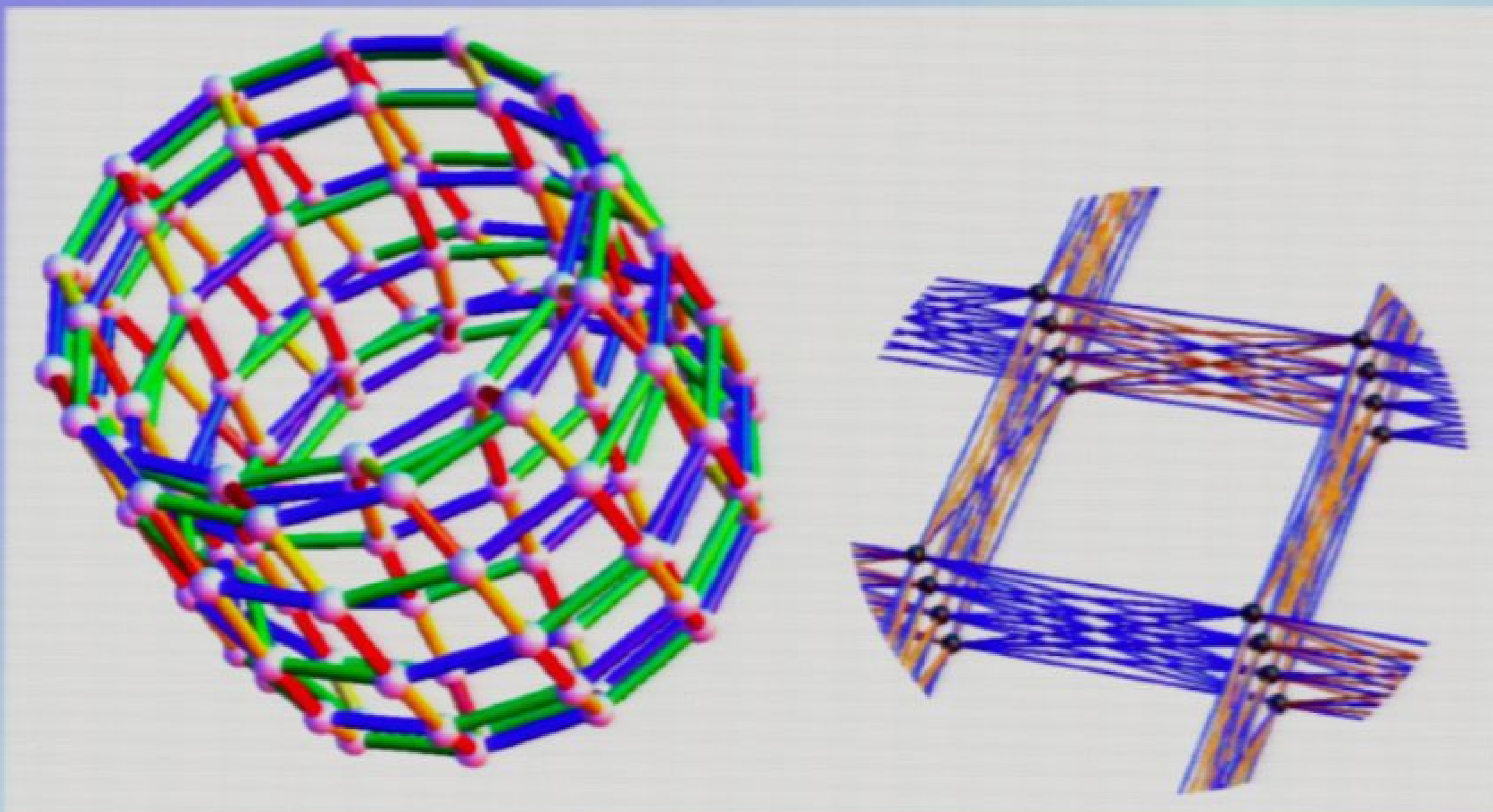


(8 x 8 x 4) toroidal lattice supergraph

Physical implementation of quantum walks

(1) Highly symmetrical graphs

Douglas and Wang, PRA (2009)



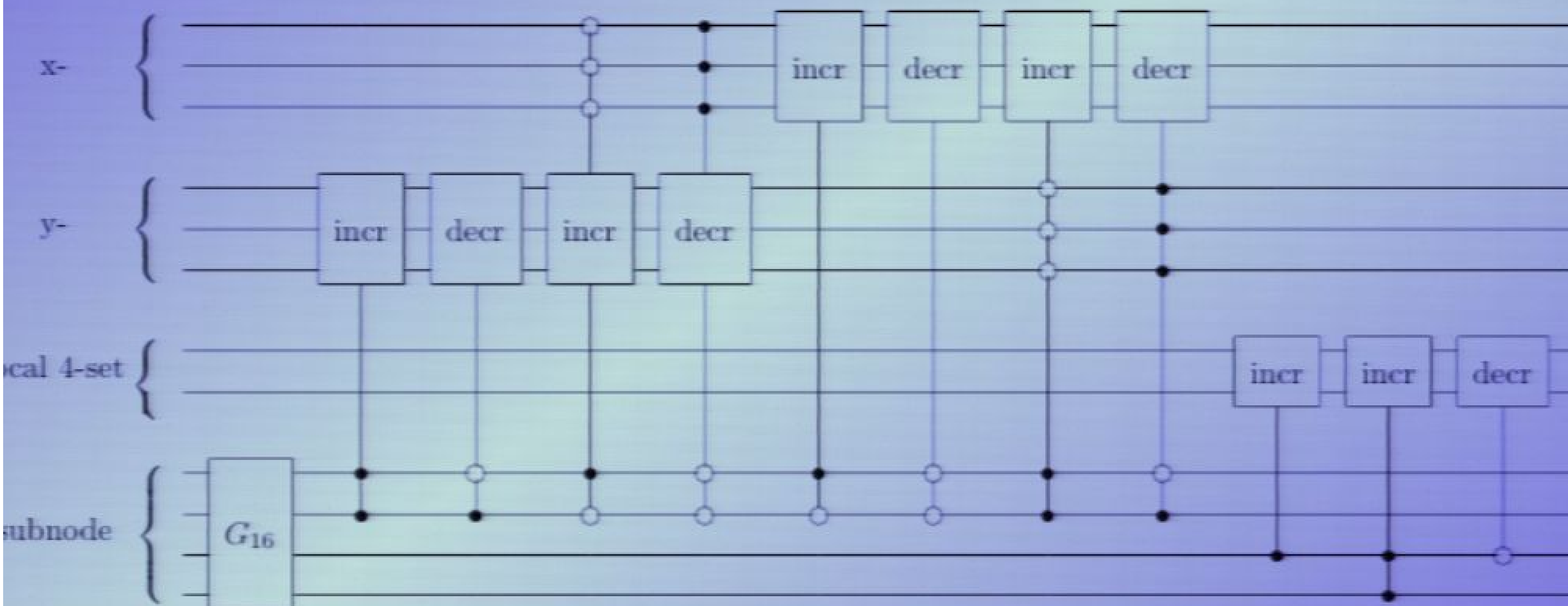
(8 x 8 x 4) toroidal lattice supergraph

Menicucci et al, PRL 101, 130501 (2008)

Physical implementation of quantum walks

(1) Highly symmetrical graphs

Douglas and Wang, PRA (2009)

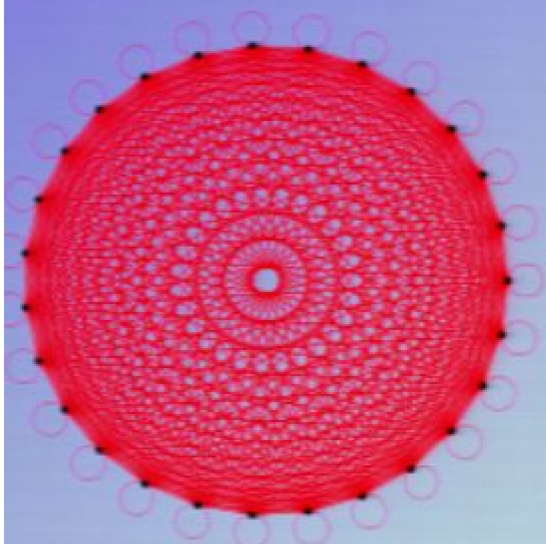


(8 x 8 x 4) toroidal lattice supergraph

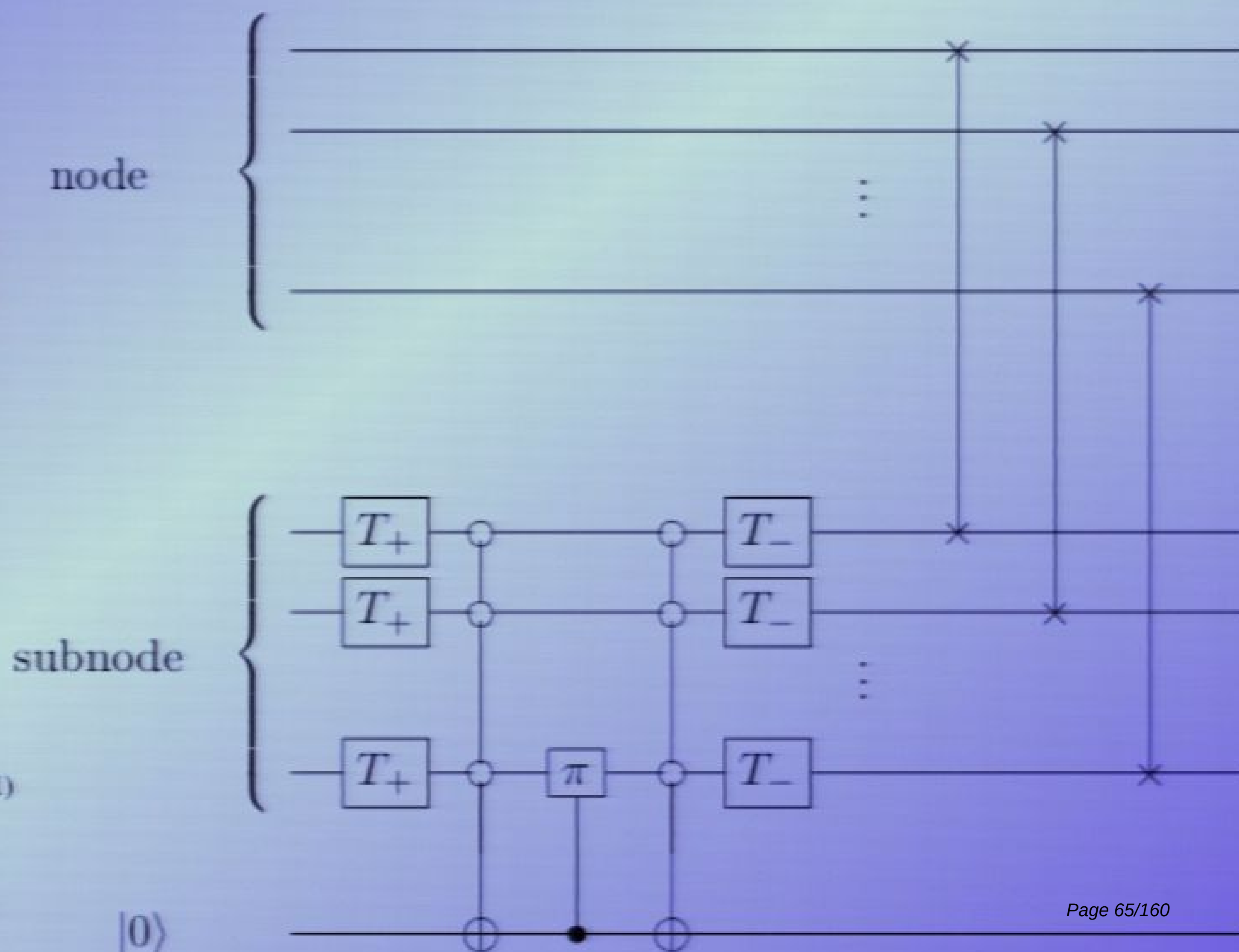
Physical implementation of quantum walks

(1) Highly symmetrical graphs

Douglas and Wang, PRA (2009)



Complete 3^n-graph



$$(T_\pm)_{j,k} = \frac{1}{\sqrt{3}} e^{\pm i \frac{2\pi}{3}(j-1)(k-1)}$$

qutrit equivalent of Hadamard

Physical implementation of quantum walks

(1) Highly symmetrical graphs

Physical implementation of quantum walks

(1) Highly symmetrical graphs

(2) Sparse and other special graphs

Aharonov, Tashma, Proc. ACM STOC 20–29, (2003)

Berry, Ahokas, Cleve, Sanders, Commum. Math. Phys. 270, 359 (2007)

Osborne, PRL 101, 140503 (2008)

Childs, arXiv:0810.0312 (2009)

Physical implementation of quantum walks

(1) Highly symmetrical graphs

(2) Sparse and other special graphs

Aharonov, Tashma, Proc. ACM STOC 20–29, (2003)

Berry, Ahokas, Cleve, Sanders, Commum. Math. Phys. 270, 359 (2007)

Osborne, PRL 101, 140503 (2008)

Childs, arXiv:0810.0312 (2009)

(3) Graphs with arbitrary complexity

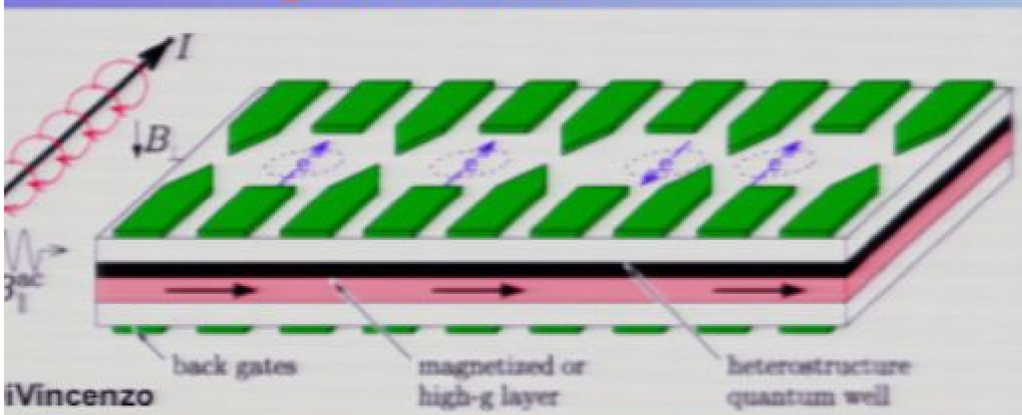
Manouchehri and J.B. Wang, J. Phys. A 41, 065304 (2008)

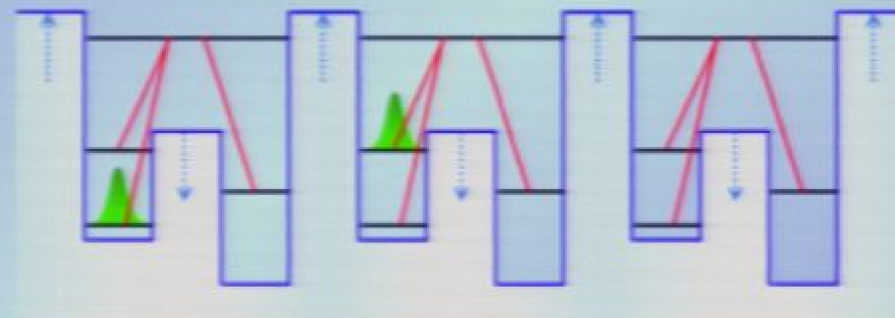
Manouchehri and Wang, PRL (submitted, 2009)

Physical implementation of quantum walks
(graphs with arbitrary complexity)

Physical implementation of quantum walks
(graphs with arbitrary complexity)

Quantum Dots

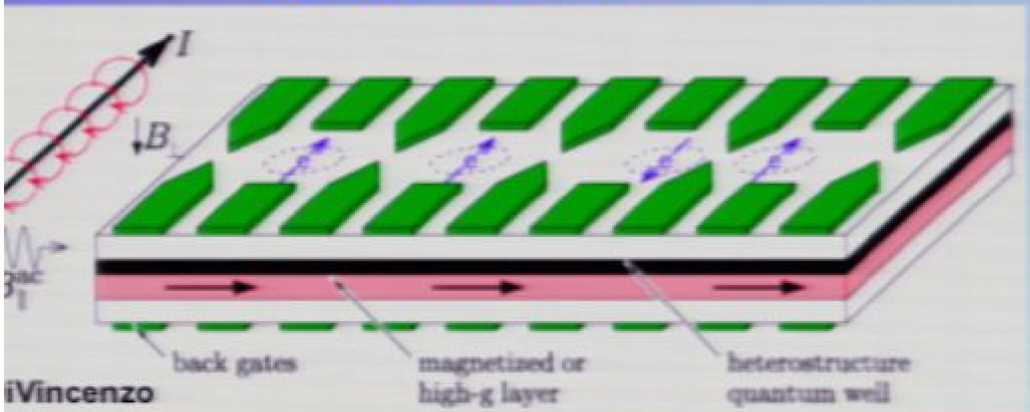


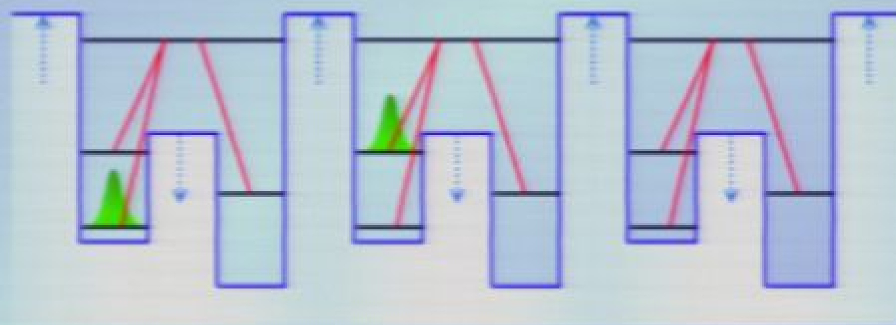


K. Manouchehri and J.B. Wang
Quantum walks in an array of quantum dots
J. Phys. A 41, 065304 (2008)

Physical implementation of quantum walks
(graphs with arbitrary complexity)

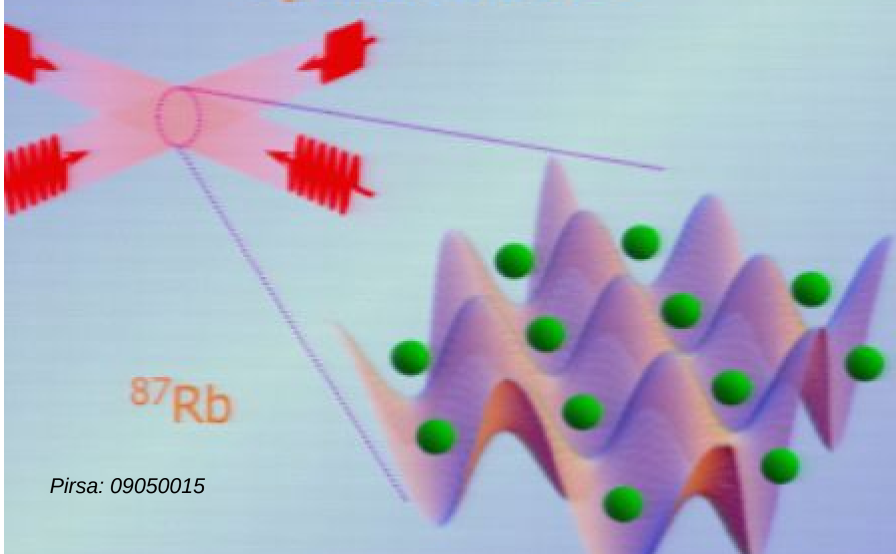
Quantum Dots

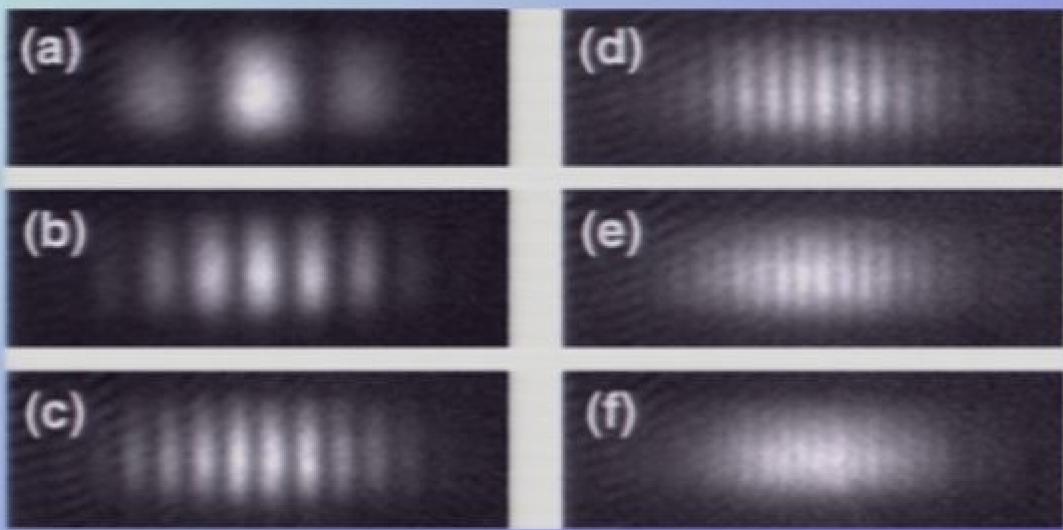




K. Manouchehri and J.B. Wang
Quantum walks in an array of quantum dots
J. Phys. A 41, 065304 (2008)

Optical Lattice





K. Manouchehri and J.B. Wang
(PRL submitted 2009)

Optical Lattice: Principle

Optical Lattice: Principle

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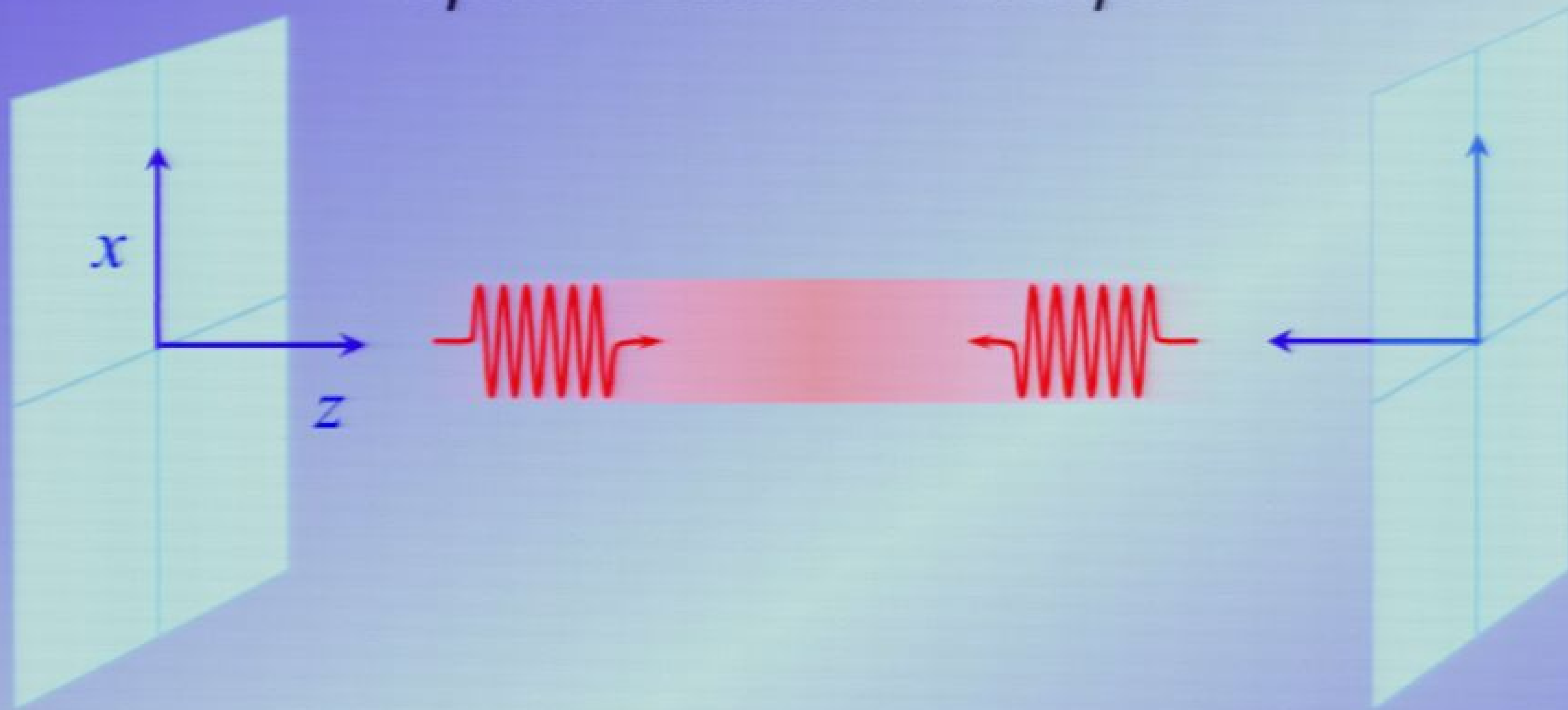
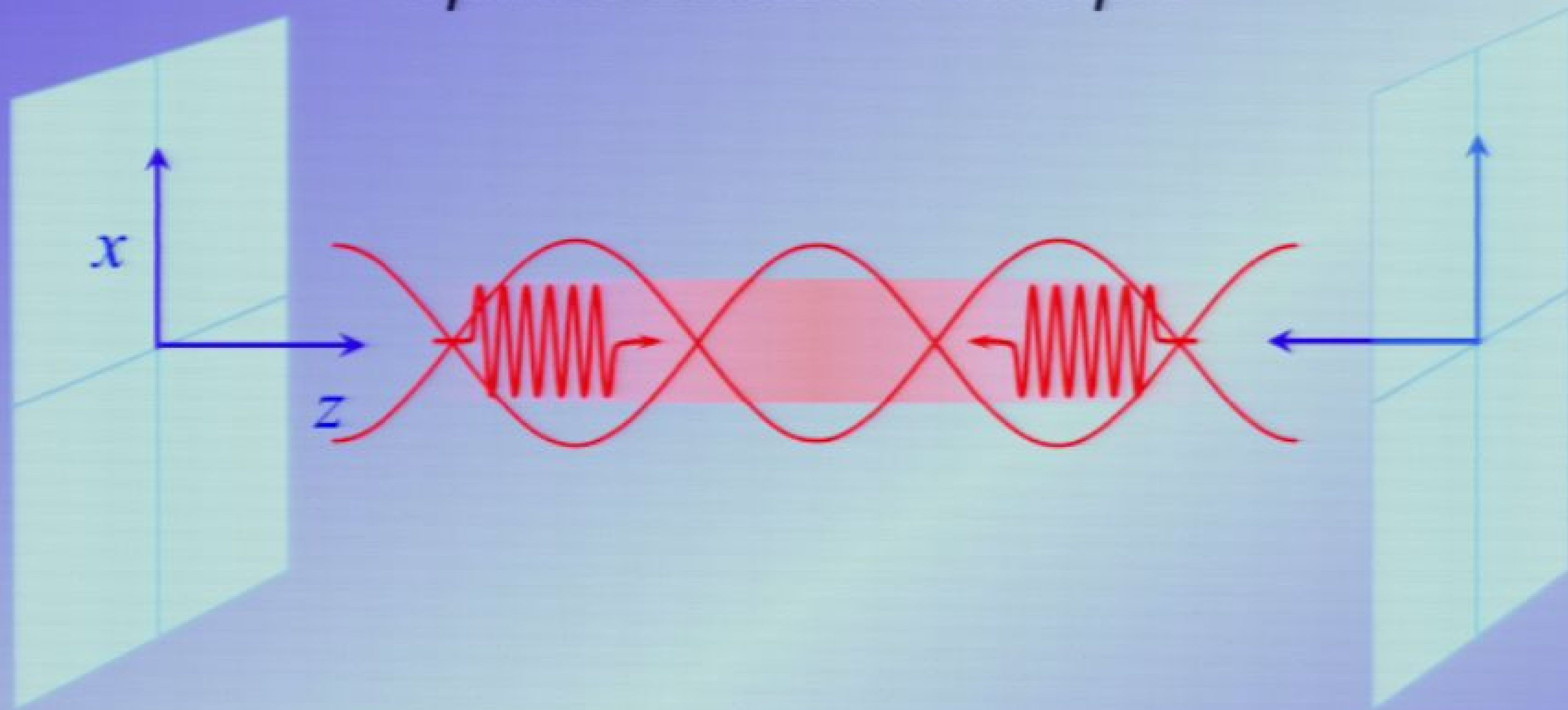
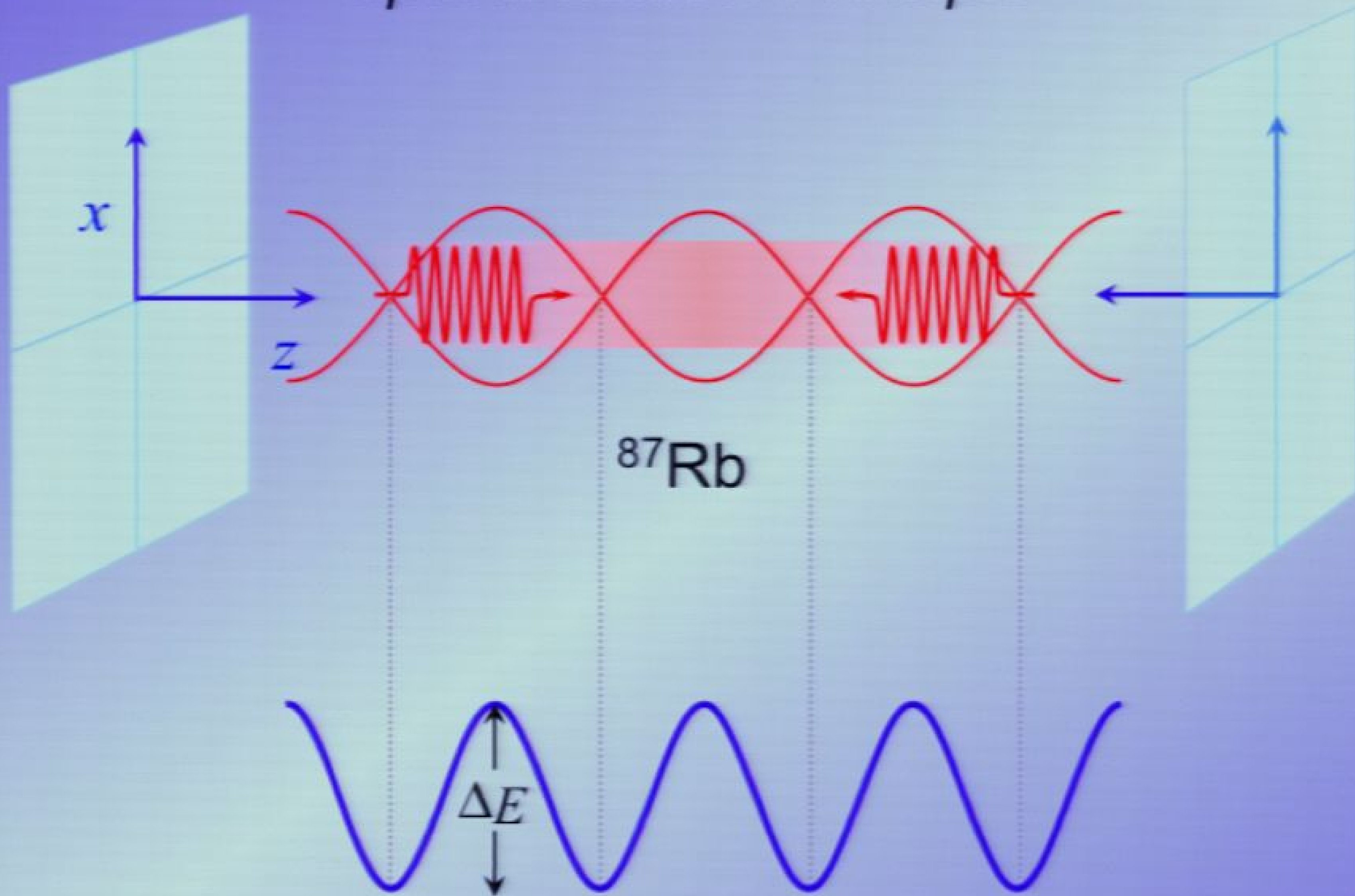


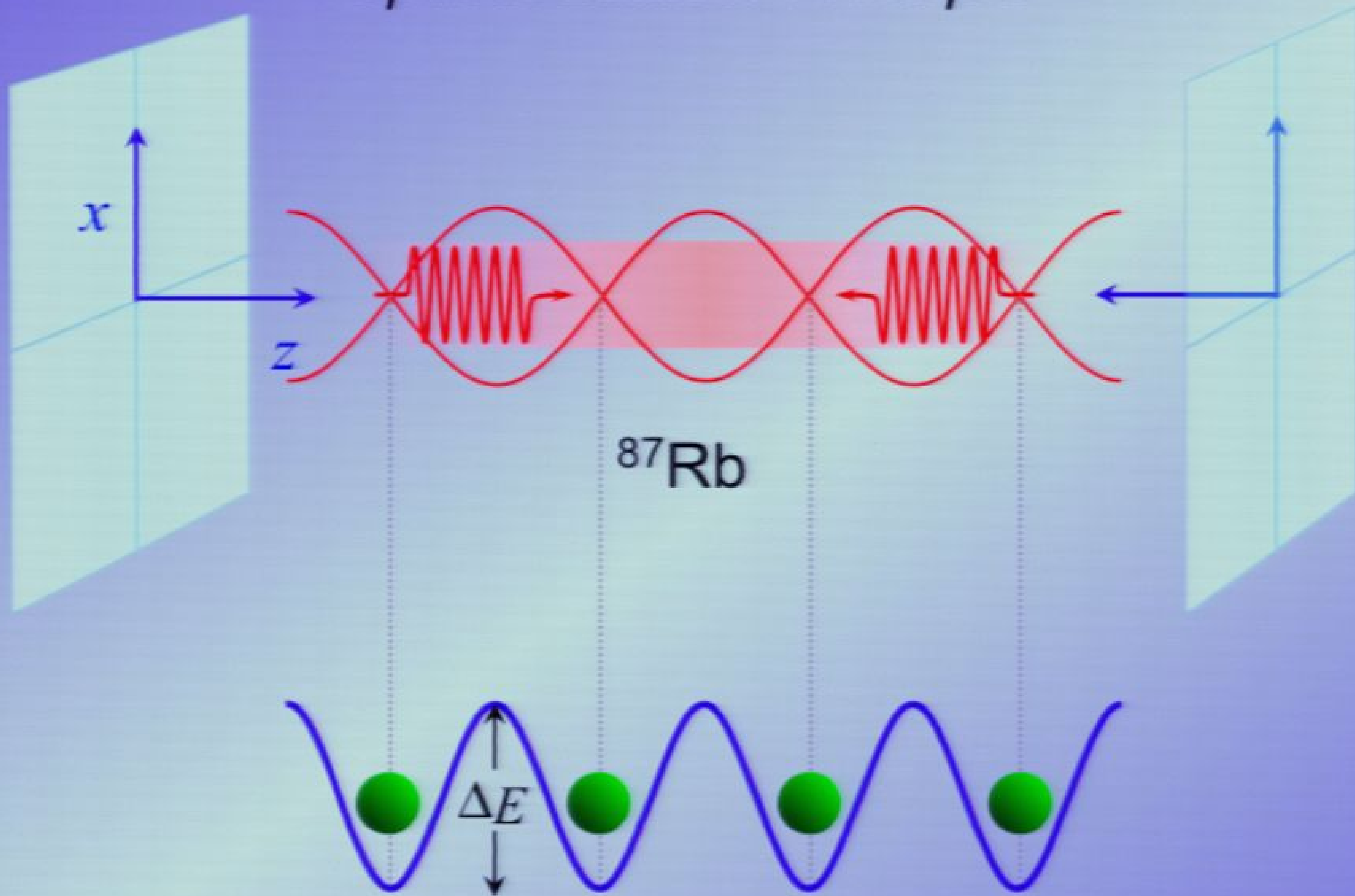
Optical Lattice: Principle



Optical Lattice: Principle

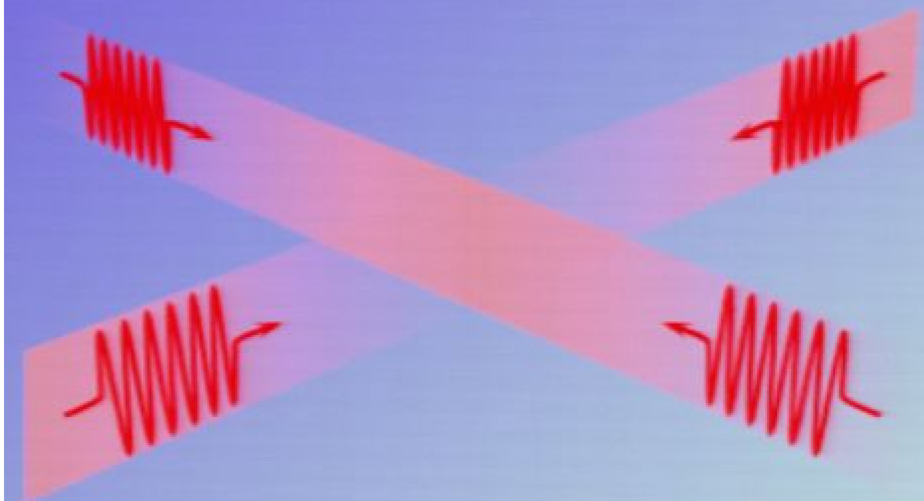


Optical Lattice: Principle

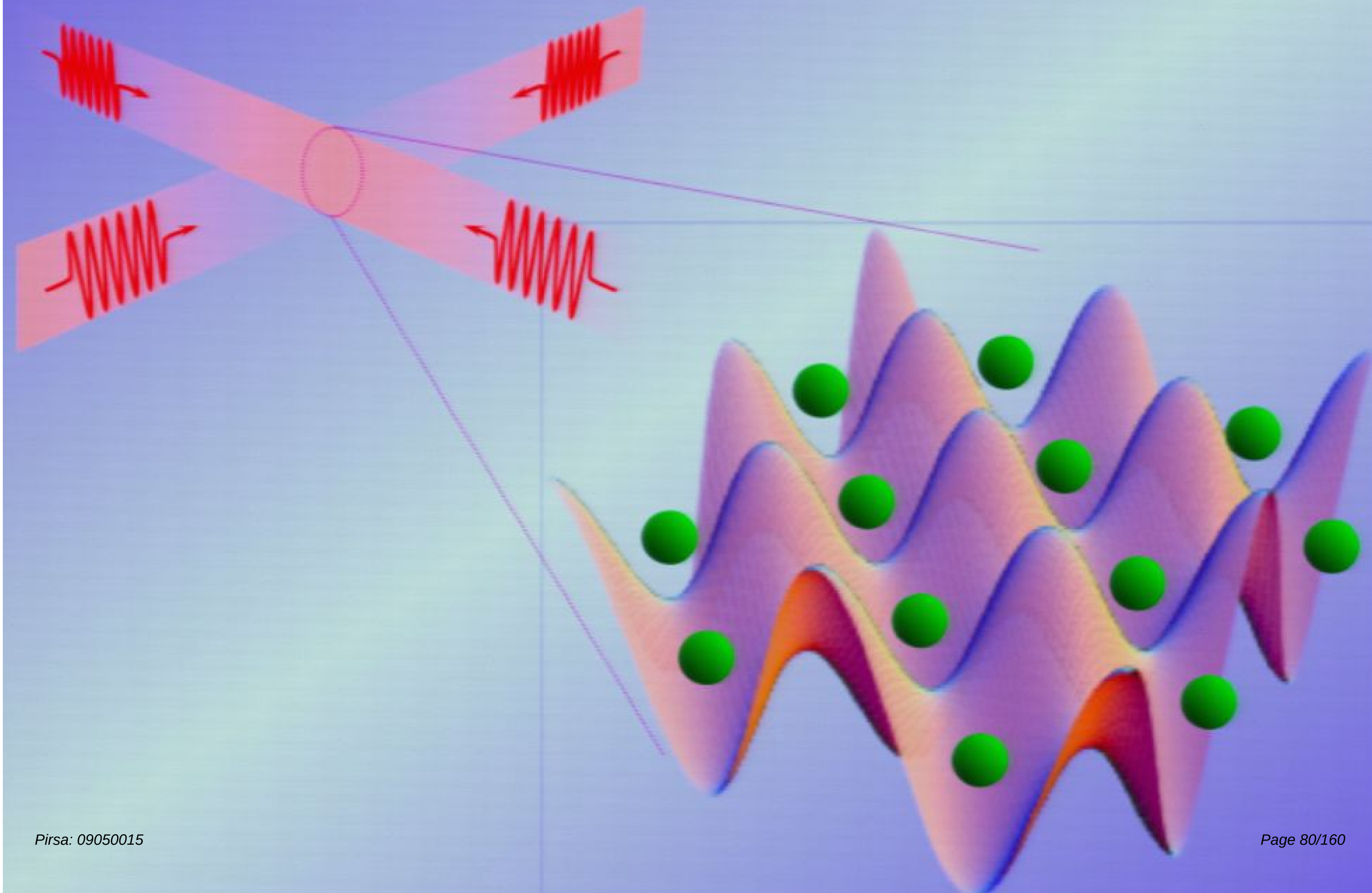


2D Optical Lattice

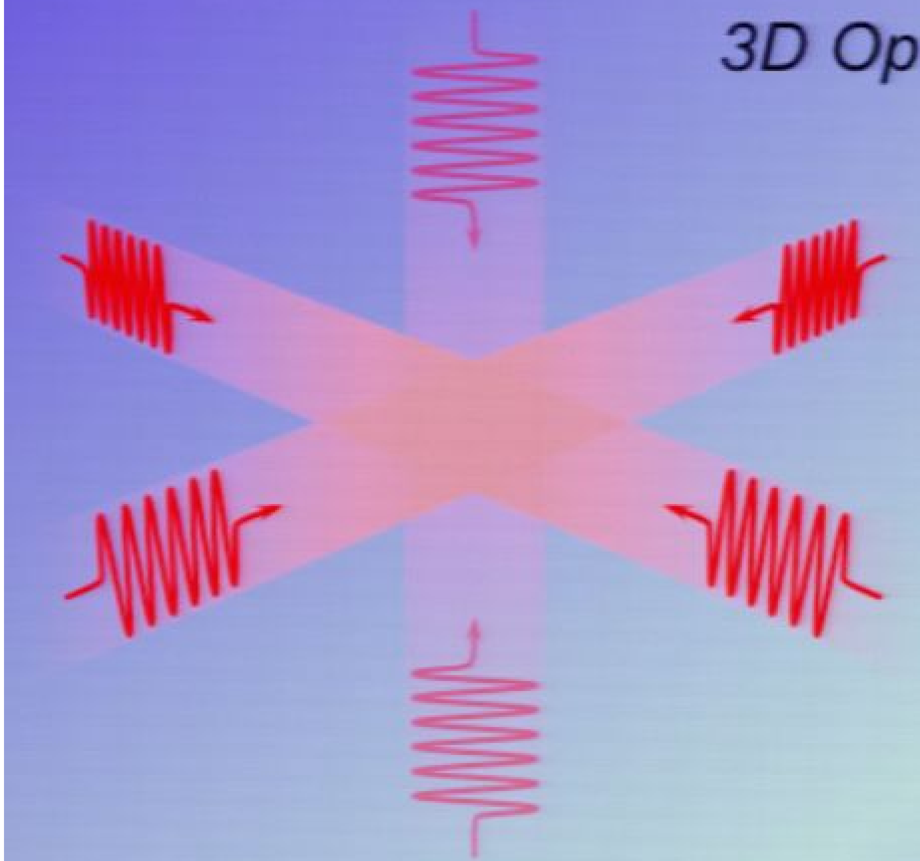
2D Optical Lattice



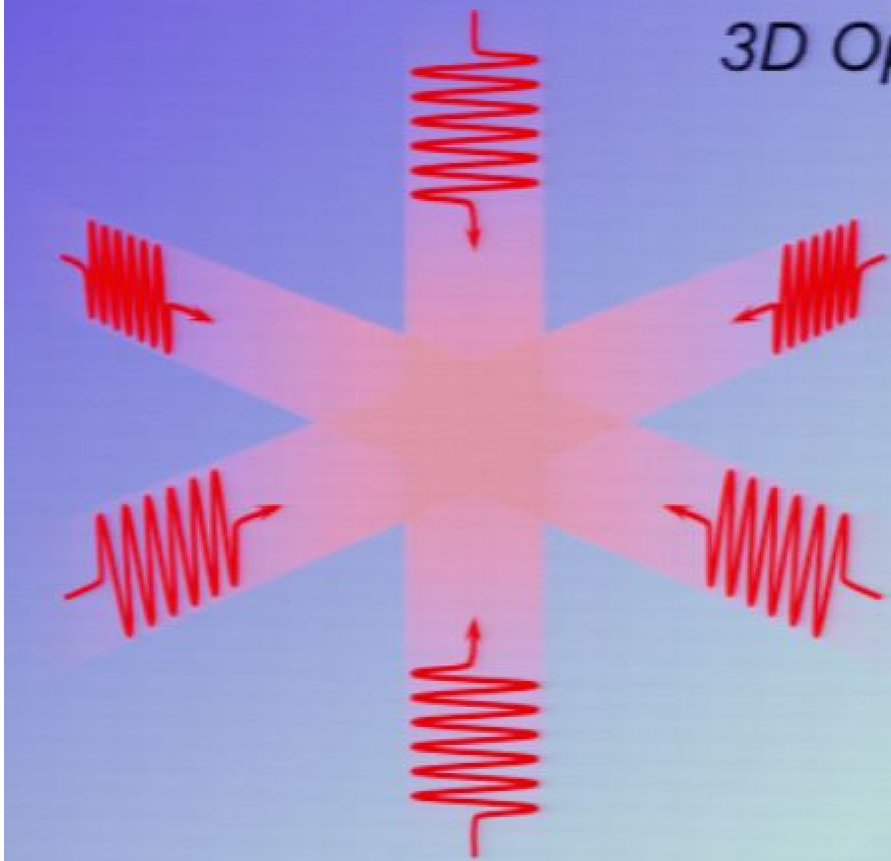
2D Optical Lattice



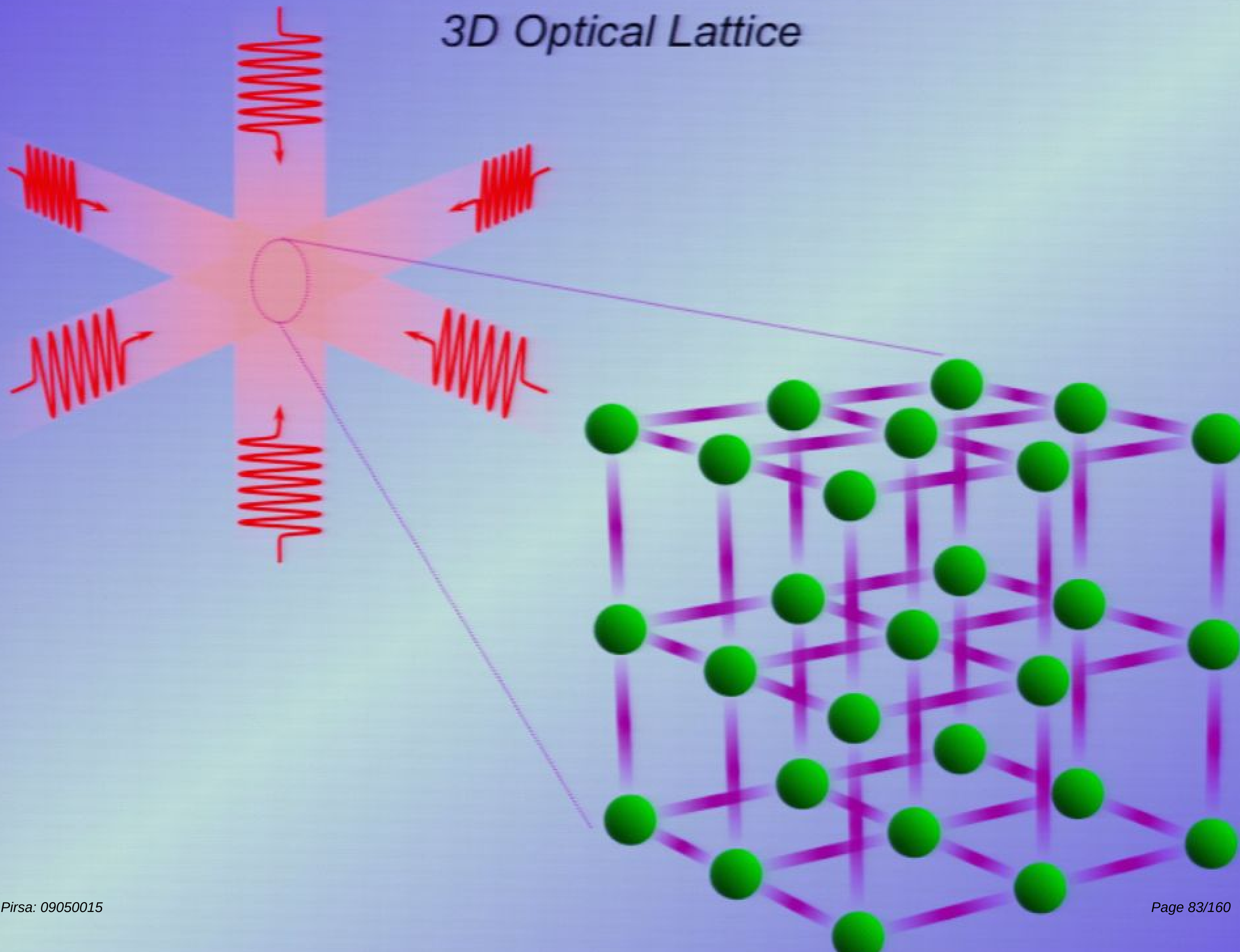
3D Optical Lattice

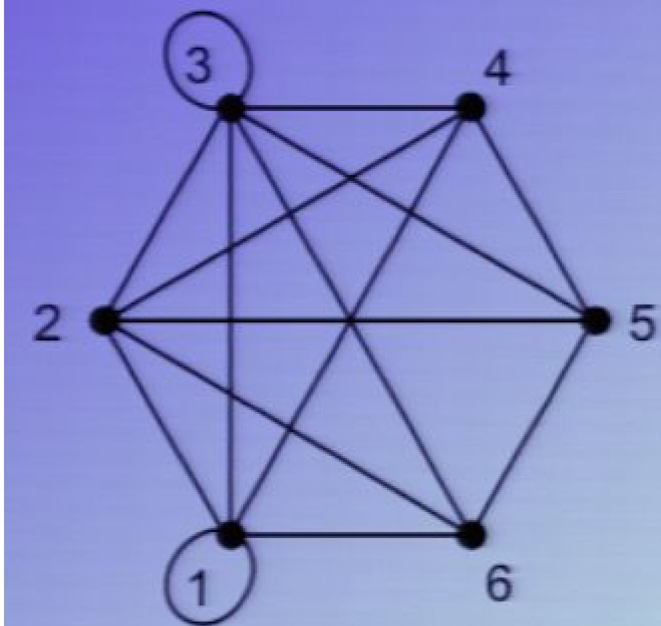


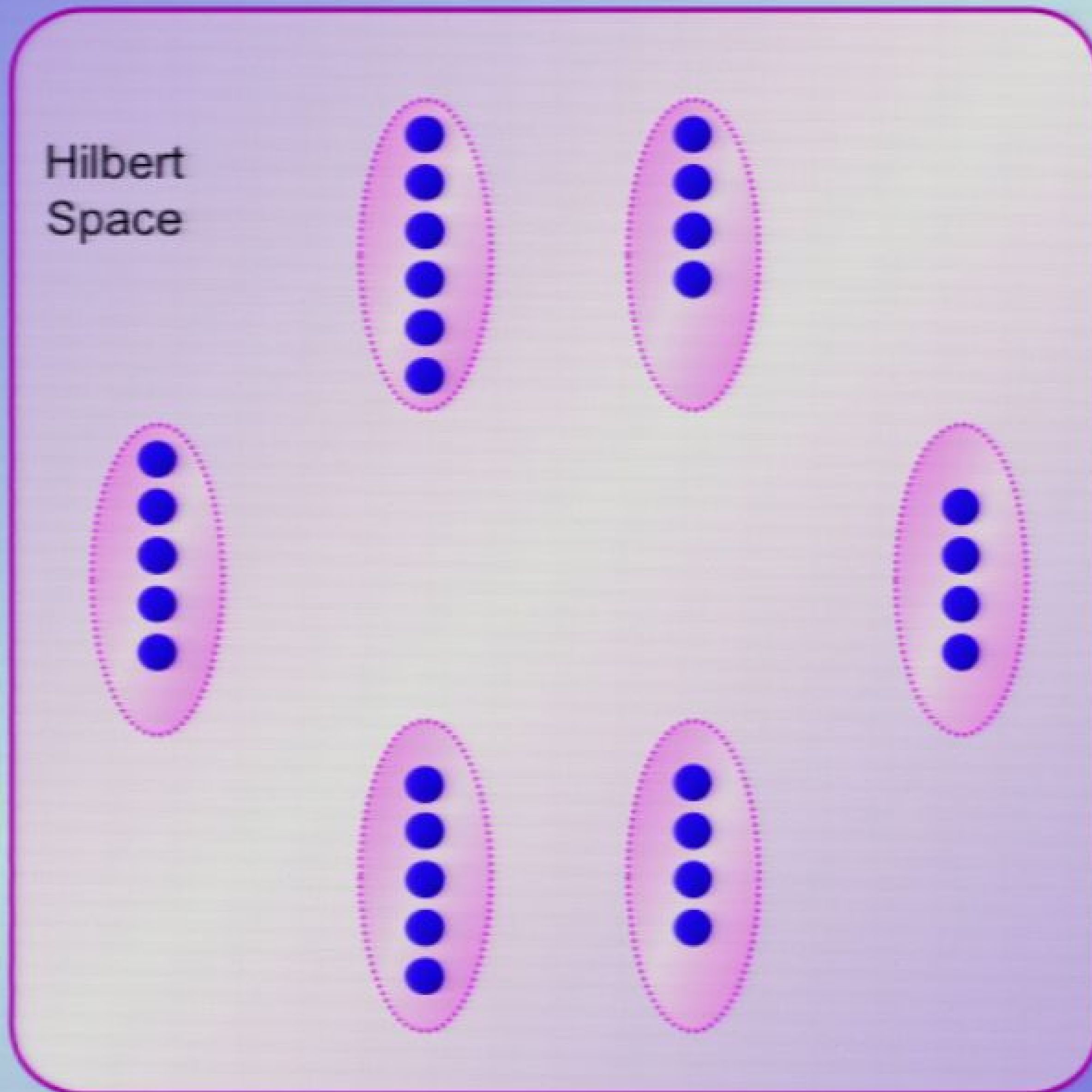
3D Optical Lattice

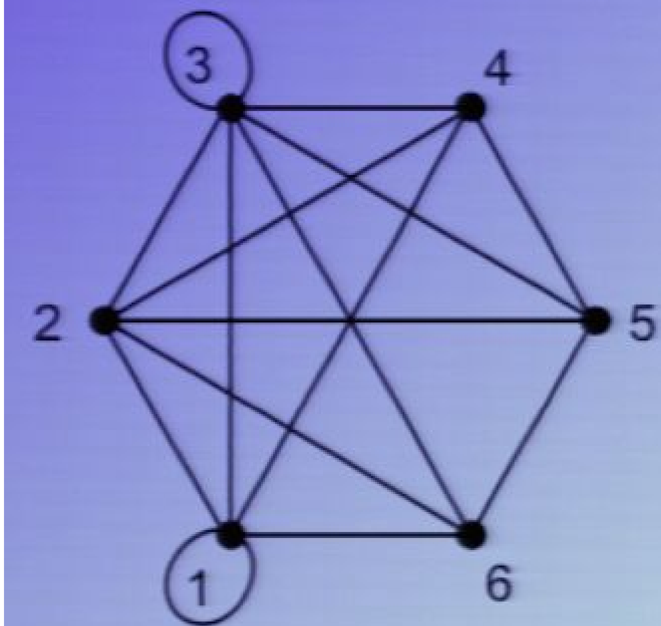


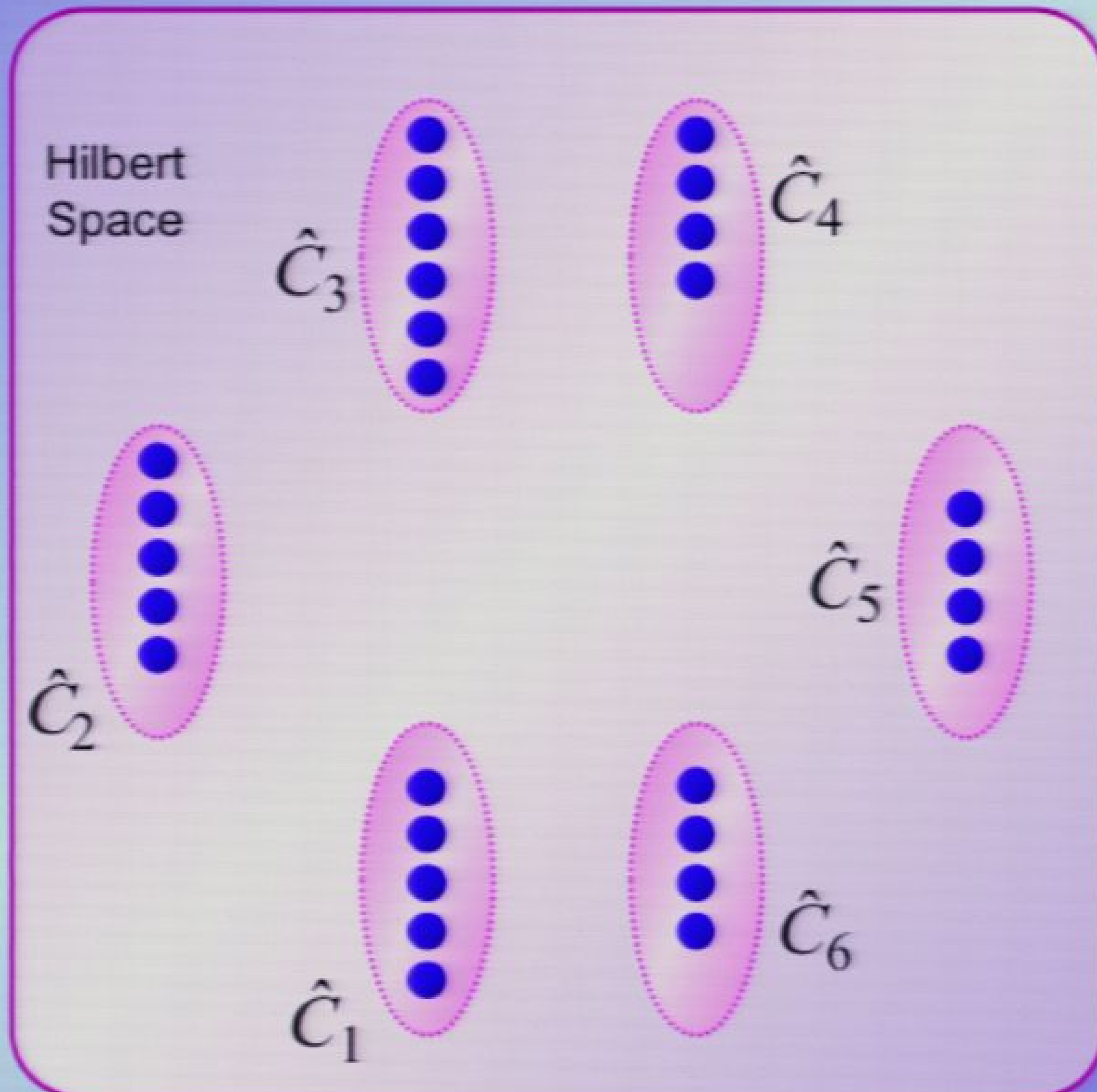
3D Optical Lattice



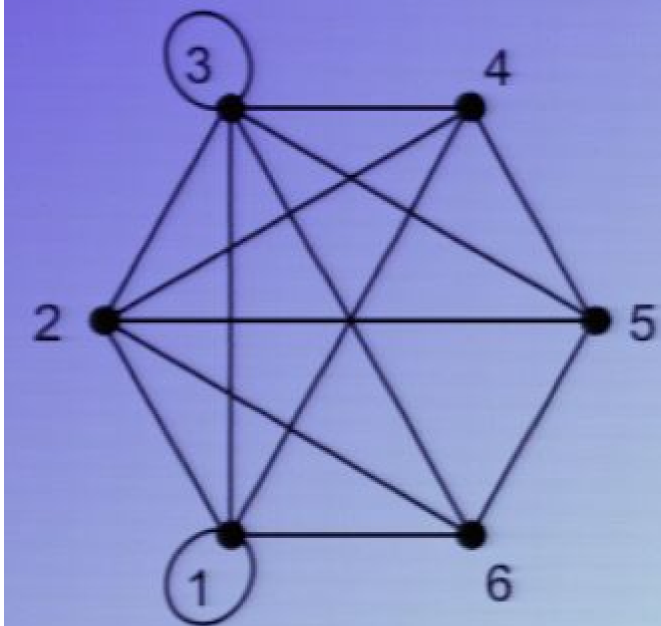

3
4
2
5
1
6

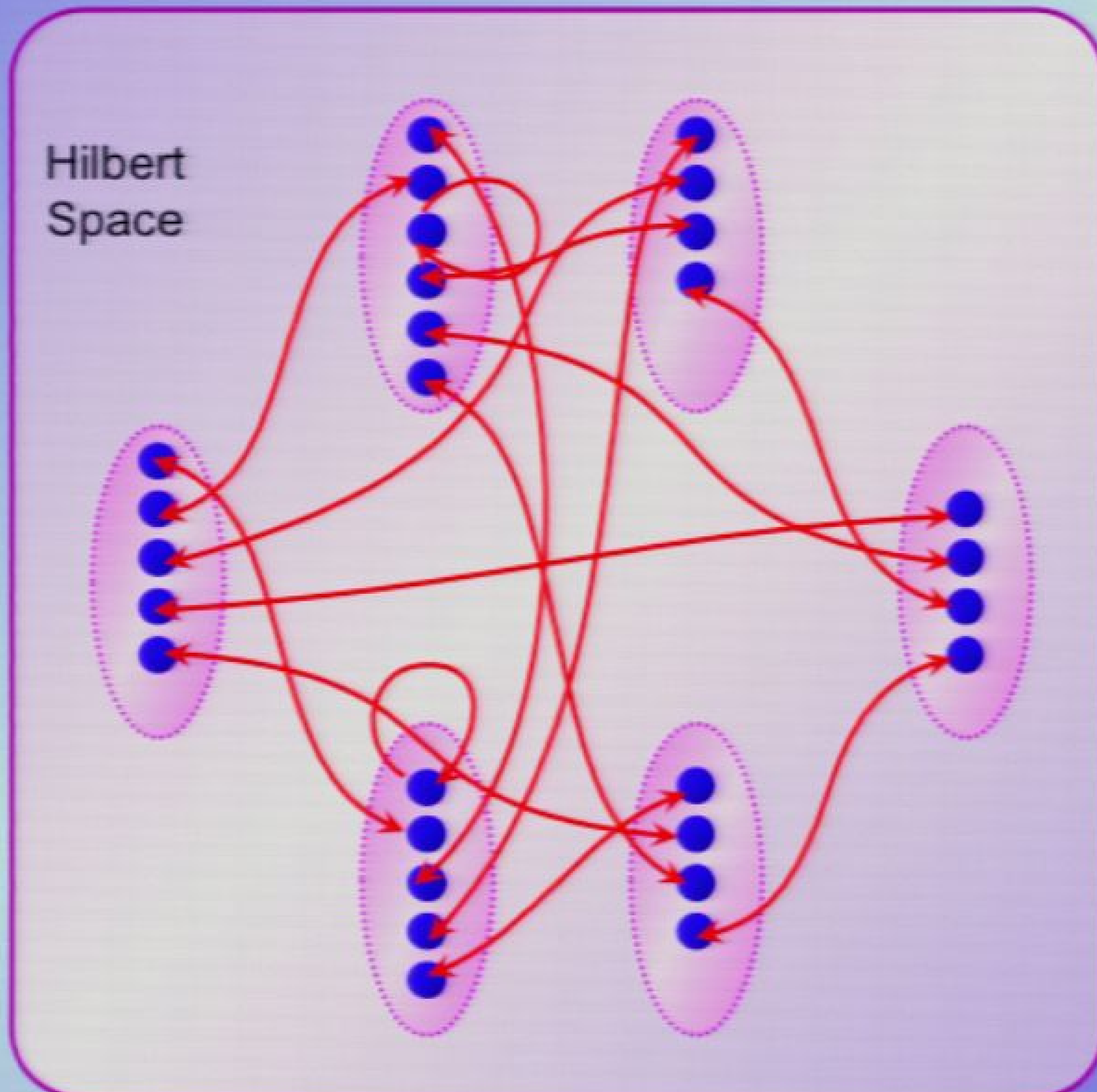

Hilbert
Space



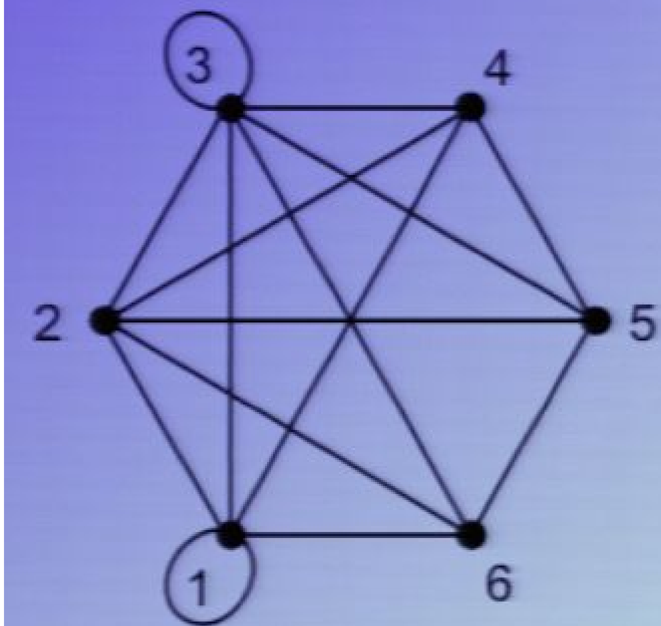


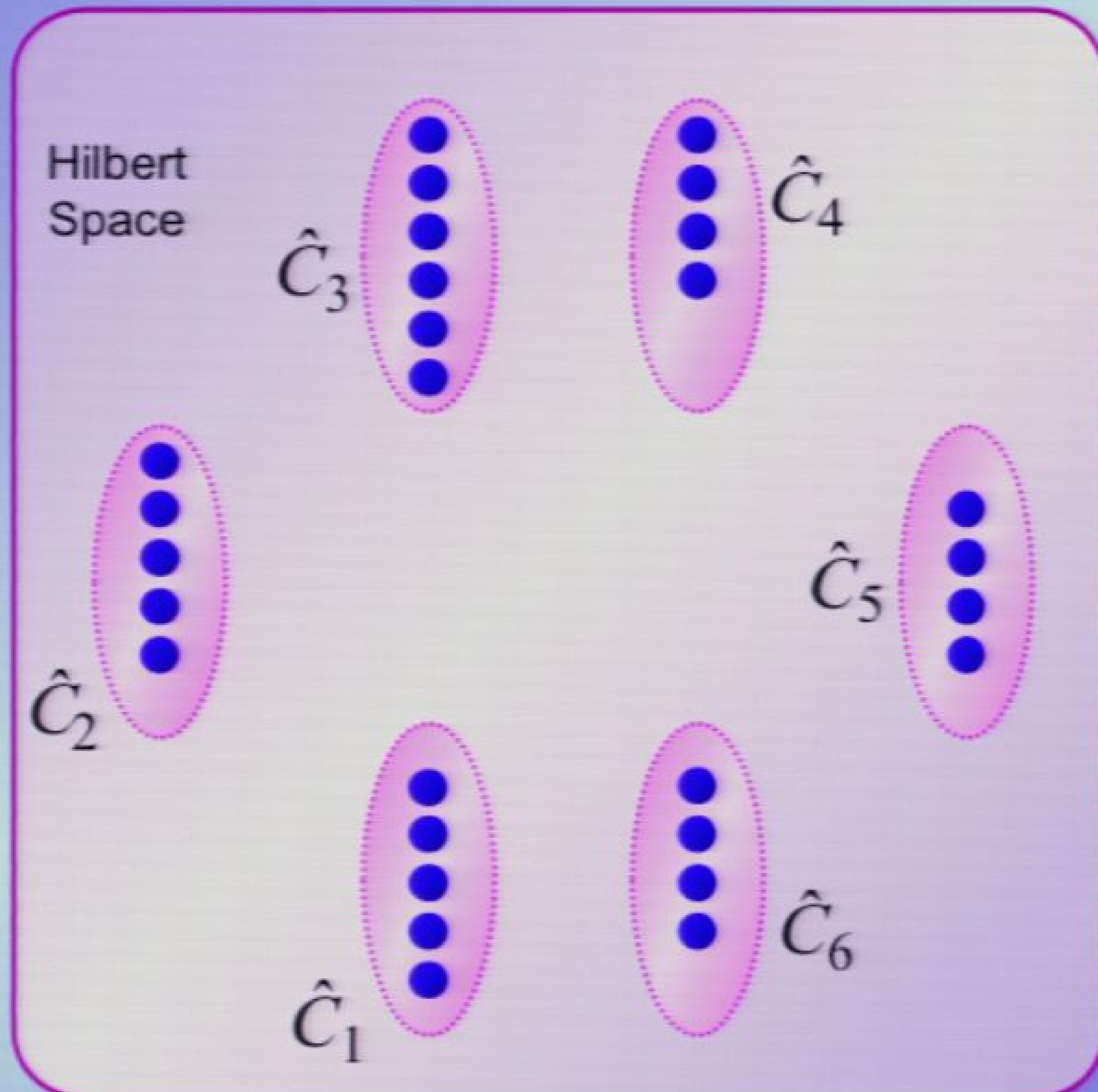
$$\left|\psi_f\right\rangle = \ldots \qquad \hat{C}\left|\psi_0\right\rangle$$





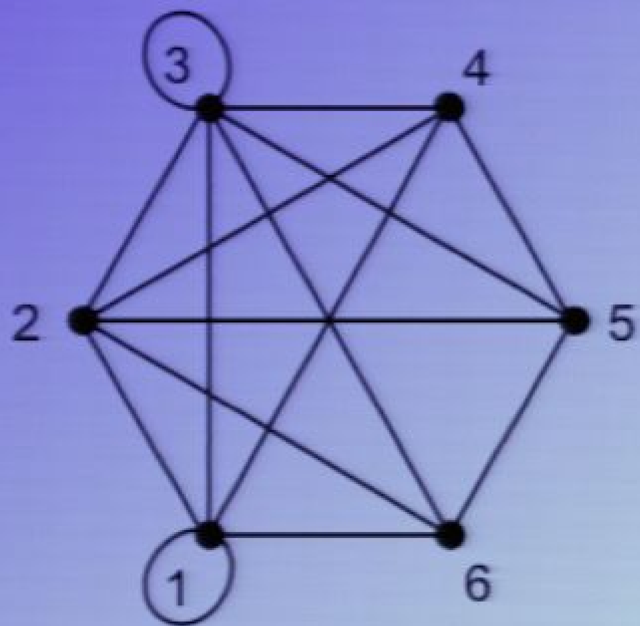
$$\left|\psi_f\right\rangle = \ldots \qquad \hat{T}\,\hat{C}\left|\psi_0\right\rangle$$

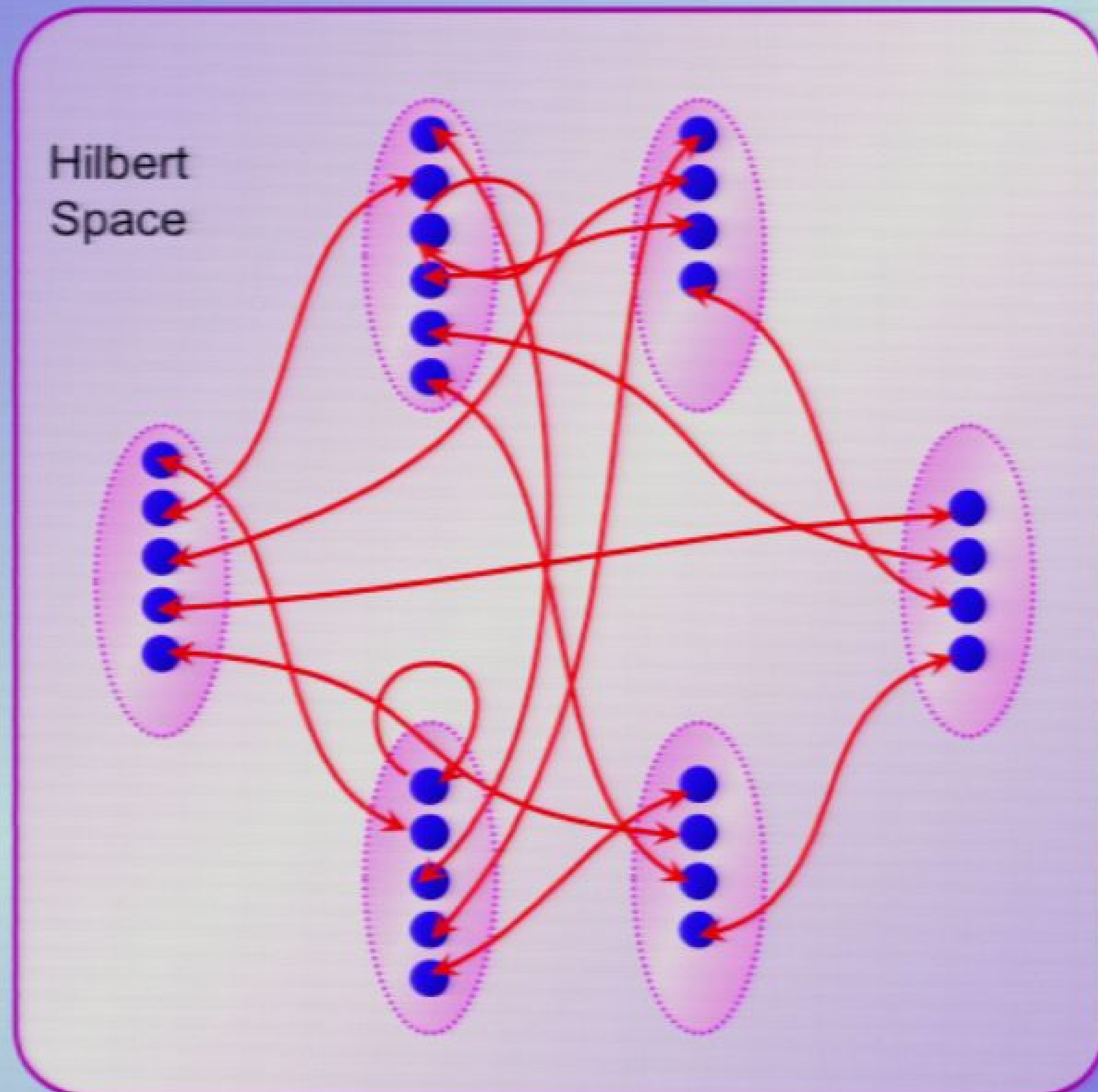




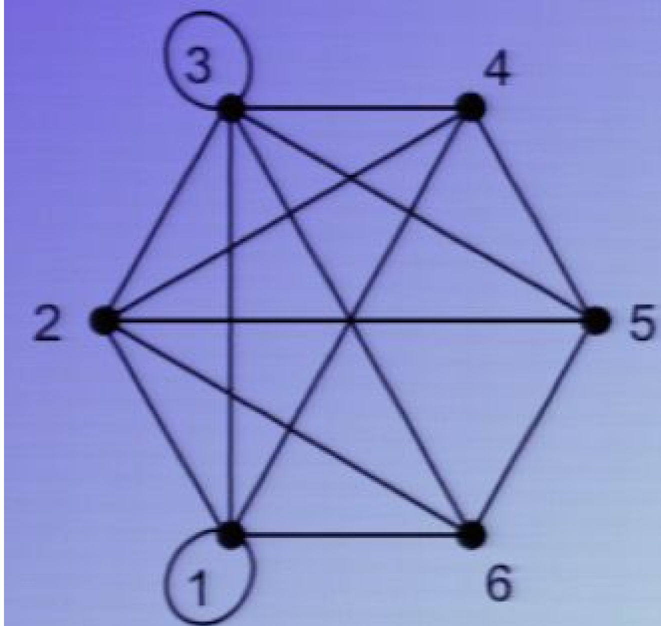
$$\left|\psi_f\right\rangle = \ldots$$

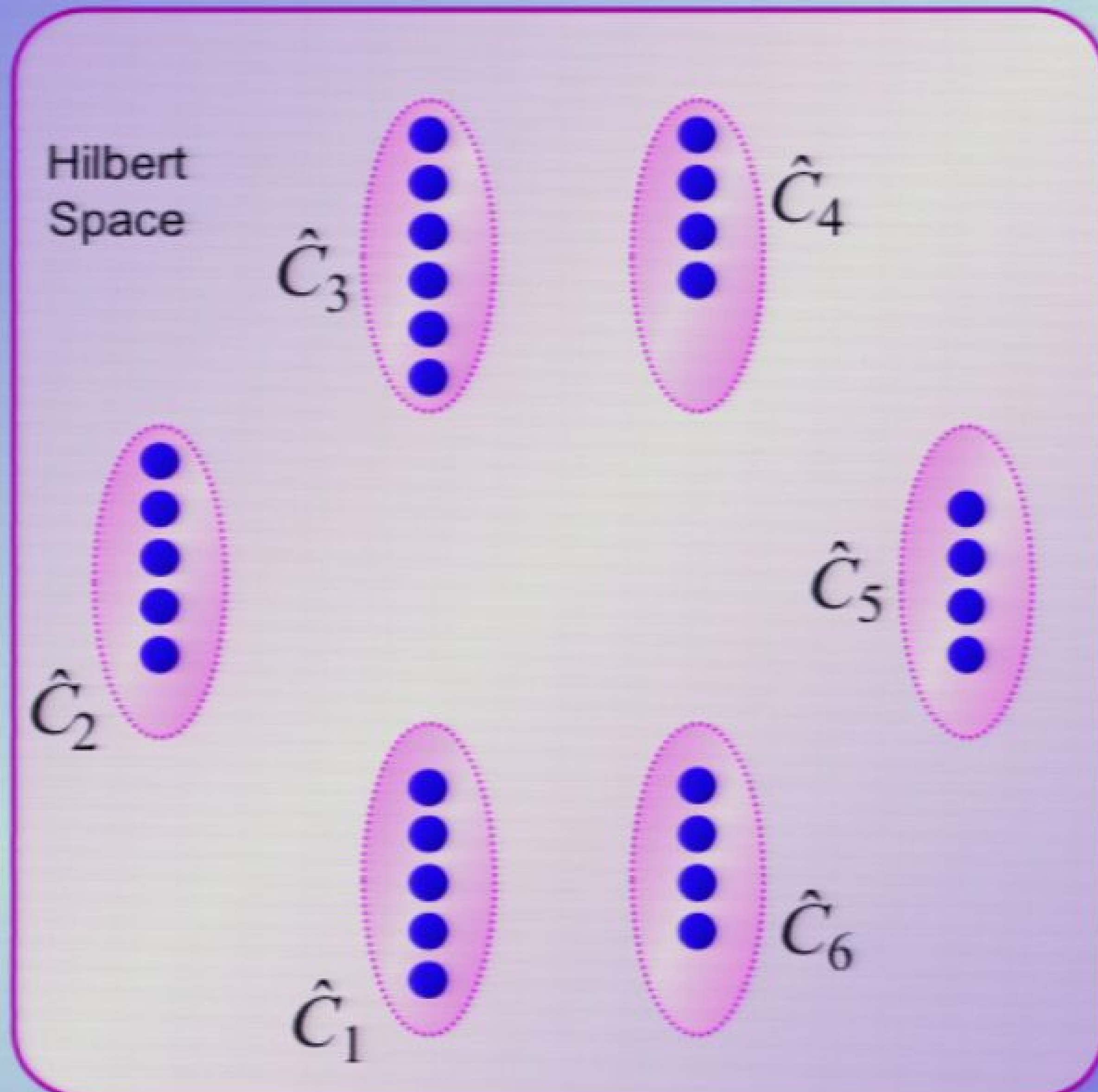
$$\hat{C}\left|\psi_0\right\rangle$$



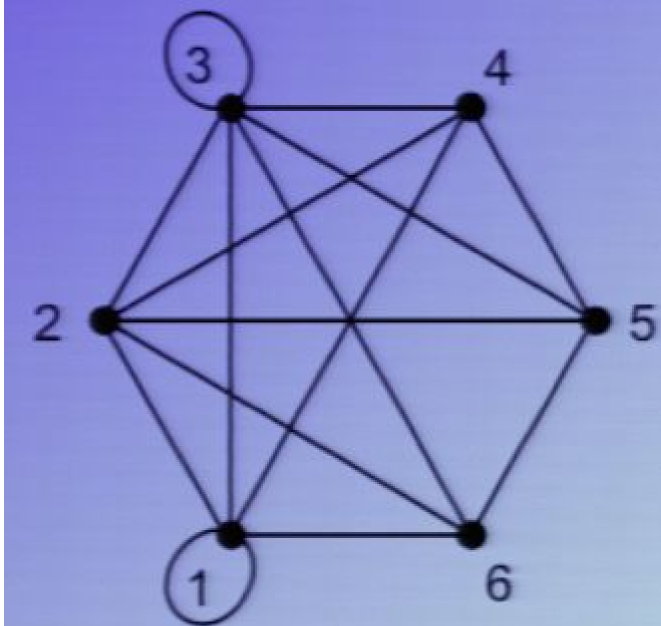


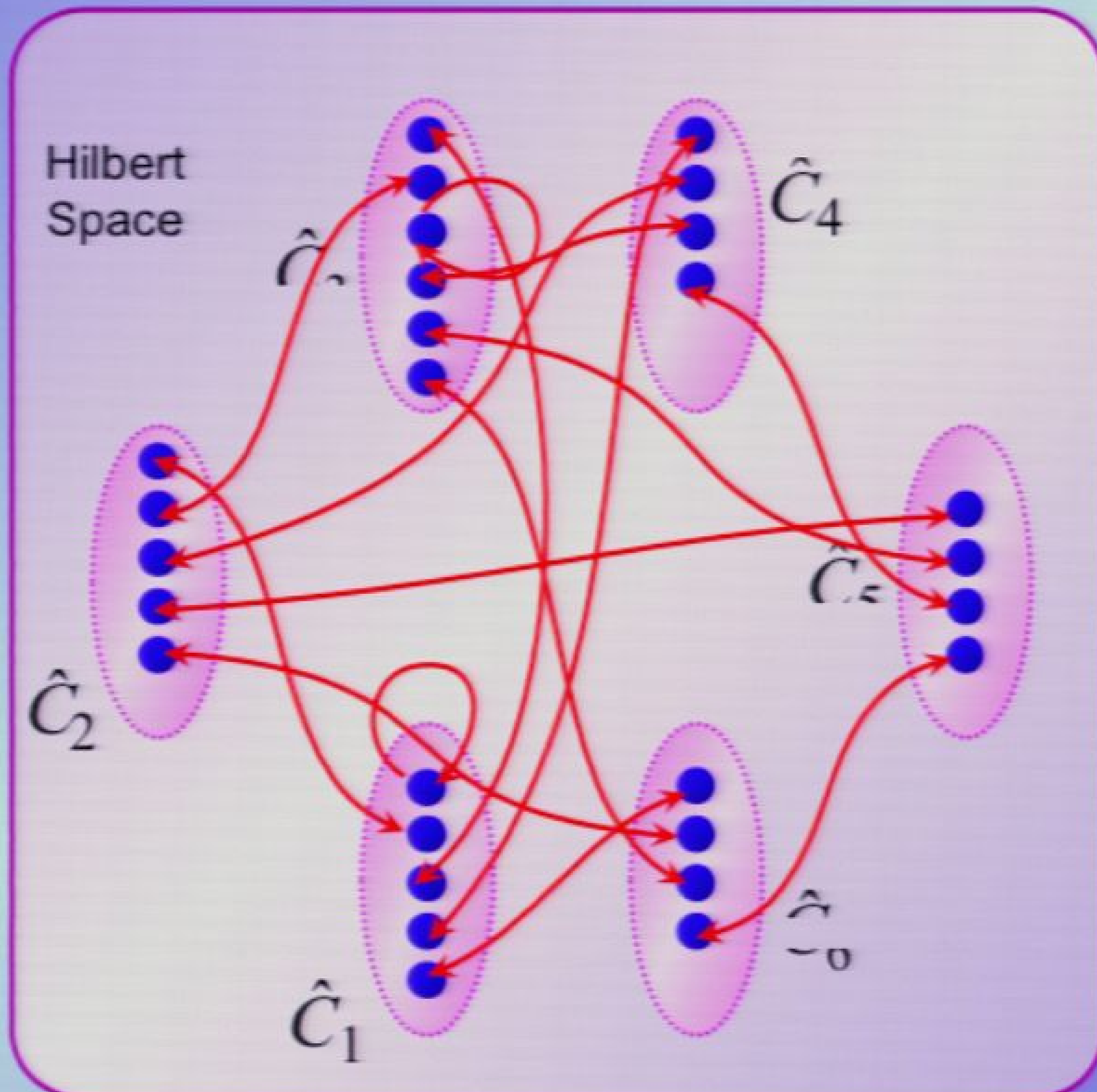
$$\left|\psi_f\right\rangle = \ldots \qquad \hat{T}\,\hat{C}\left|\psi_0\right\rangle$$



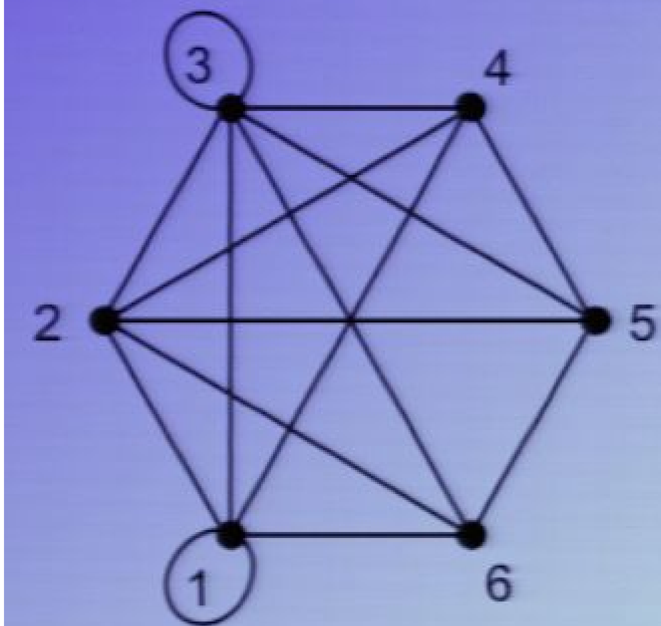


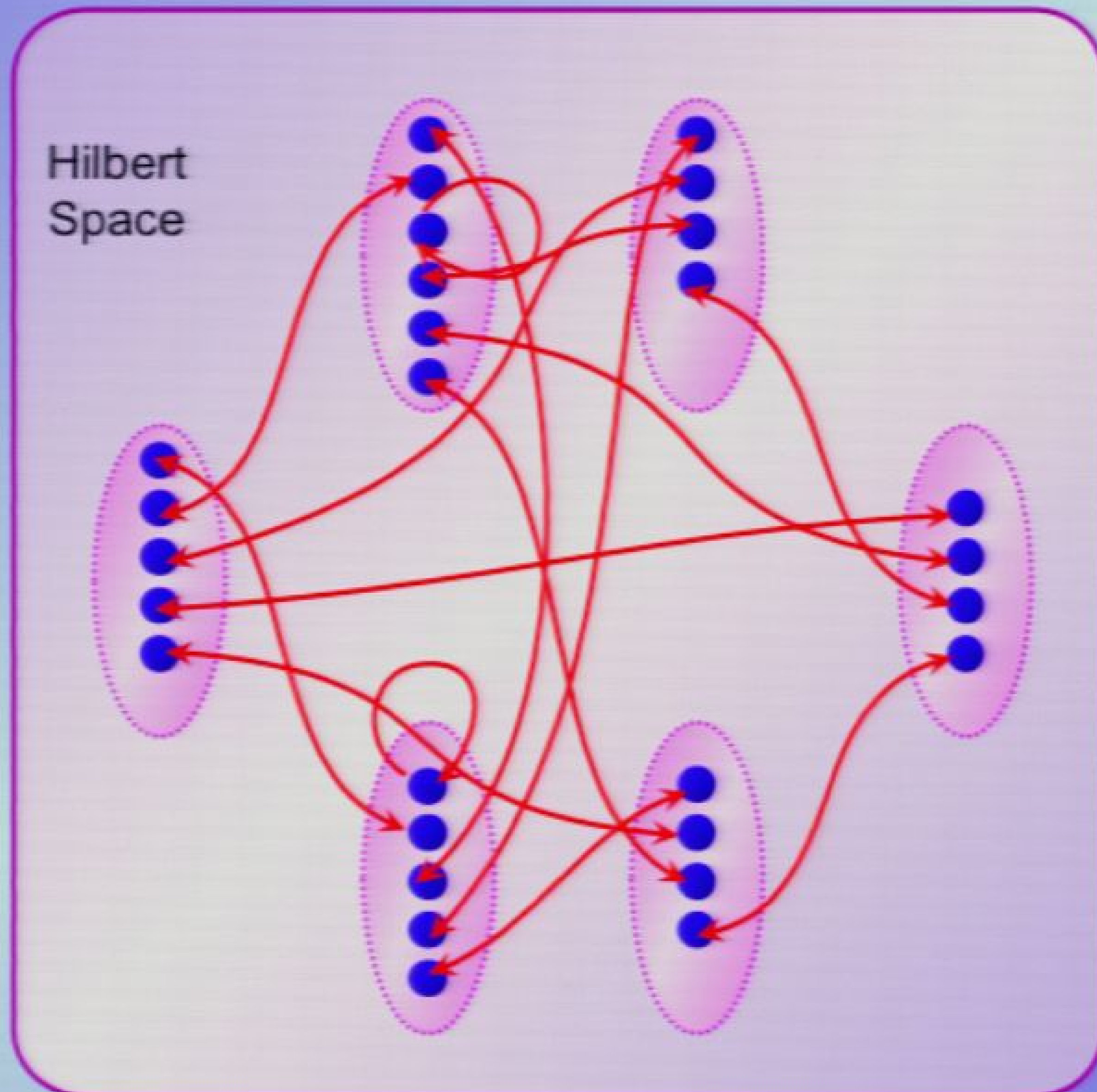
$$\left|\psi_f\right\rangle = \ldots \qquad \hat{C}\,\hat{T}\,\hat{C}\left|\psi_0\right\rangle$$



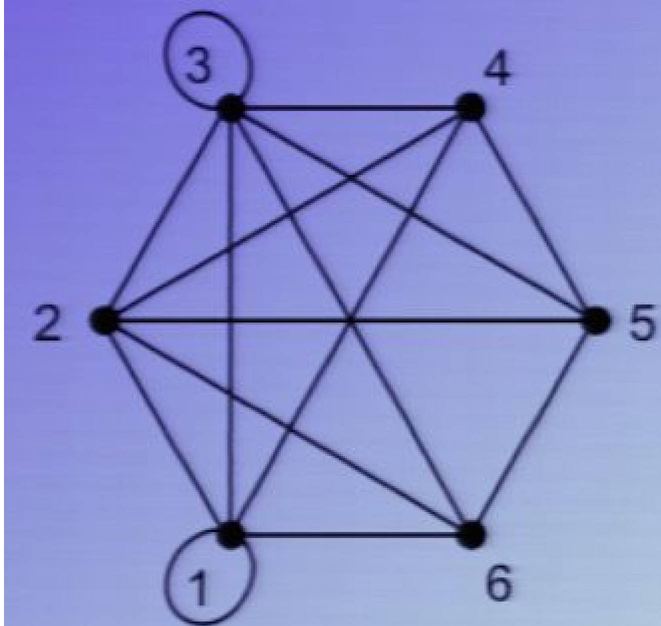


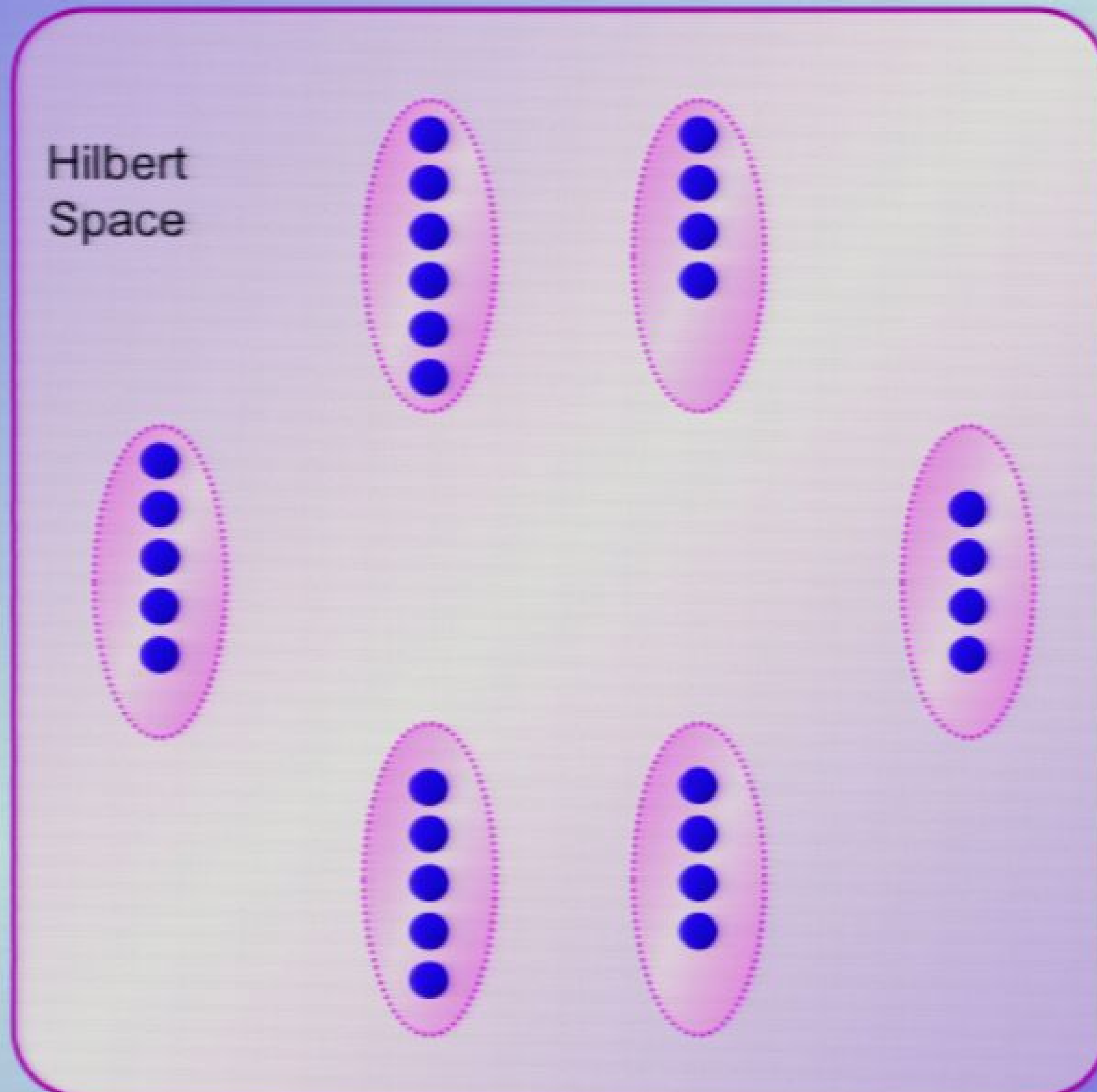
$$\left|\psi_f\right\rangle = \ldots \qquad \hat{T}\,\hat{C}\,\hat{T}\,\hat{C}\,\hat{T}\,\hat{C}\left|\psi_0\right\rangle$$





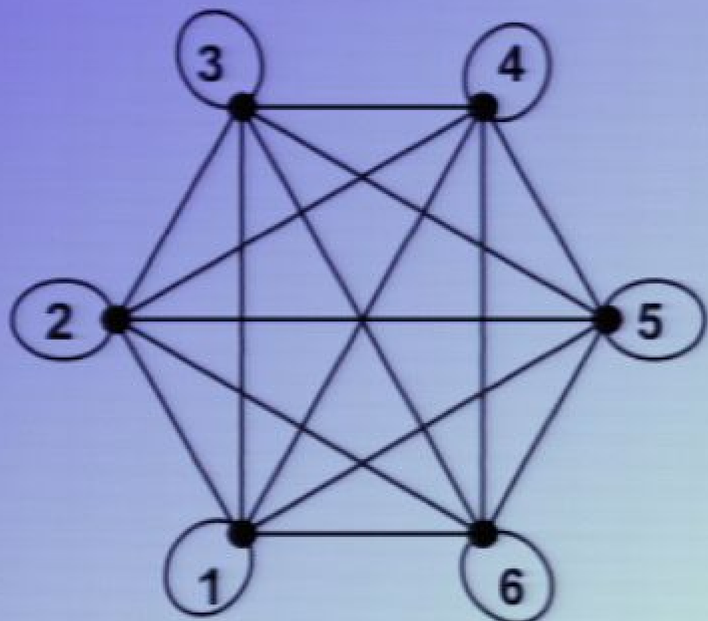
$$|\psi_f\rangle = \ldots \qquad \hat{T}\,\hat{C}\,\hat{T}\,\hat{C}\,\hat{T}\,\hat{C}\,|\psi_0\rangle$$



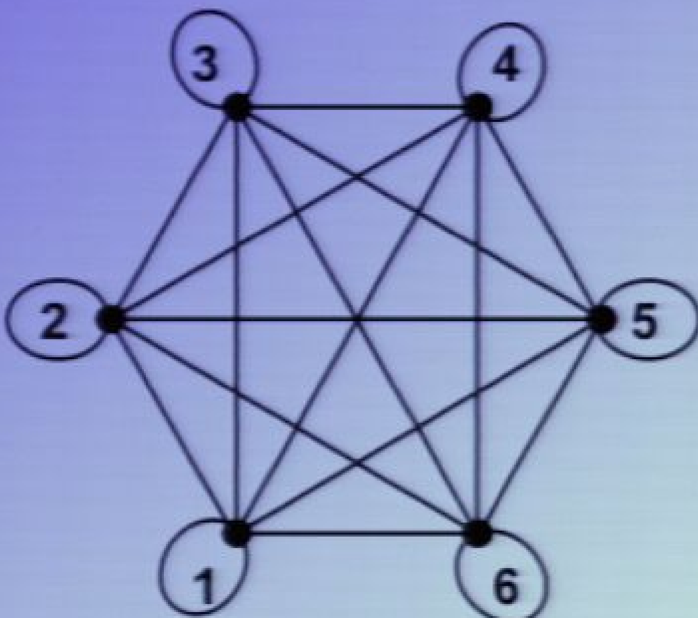


$$|\psi_f\rangle = \ldots \qquad \hat{T}\hat{C}\hat{T}\hat{C}\hat{T}\hat{C}\hat{T}\hat{C}|\psi_0\rangle$$

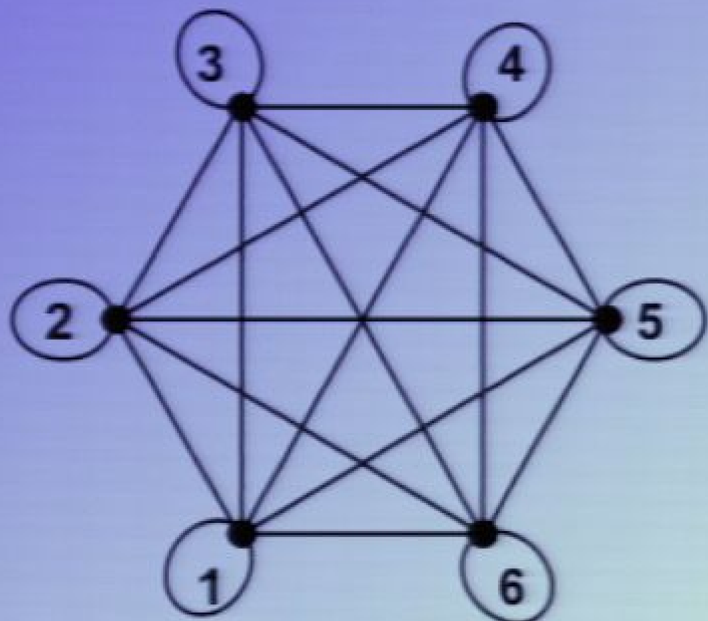
Getting Rid of Translation Operator

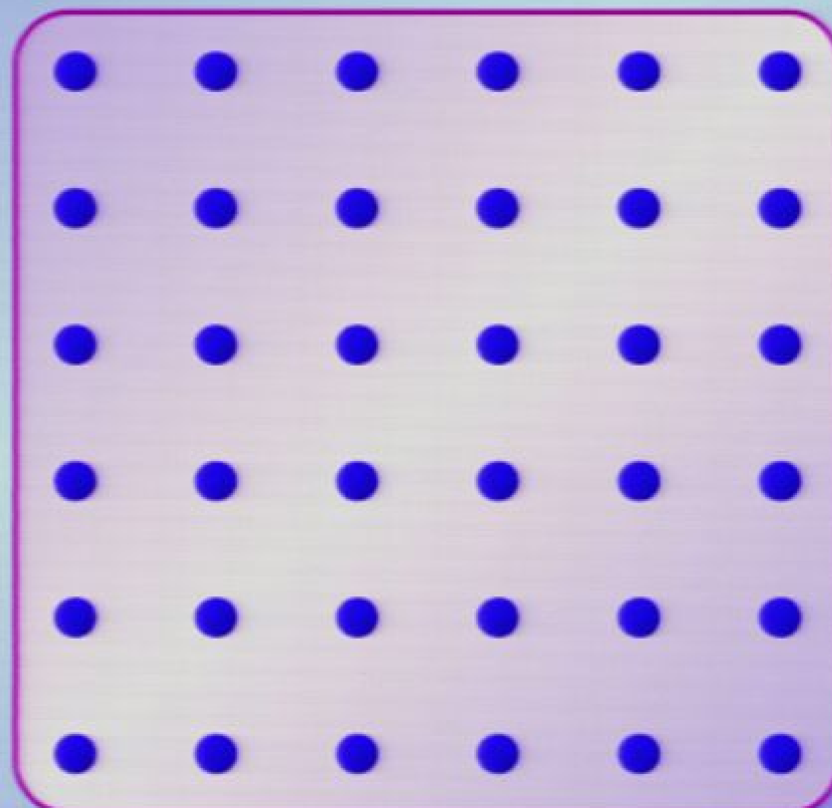


Getting Rid of Translation Operator

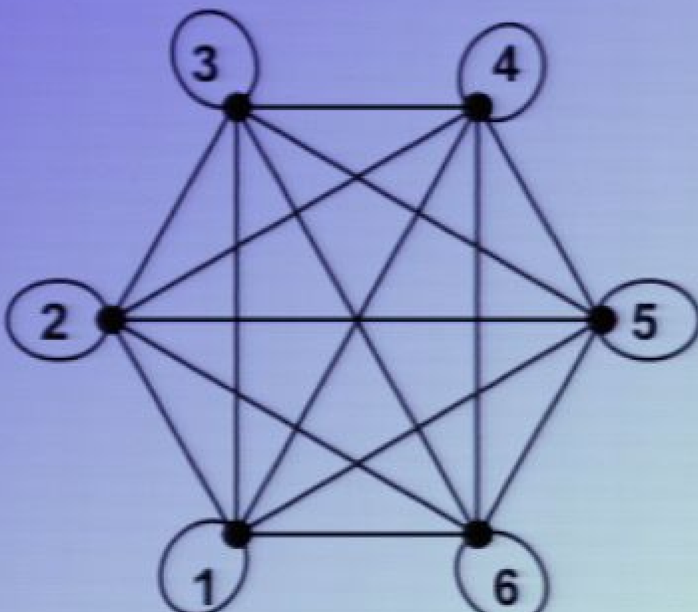


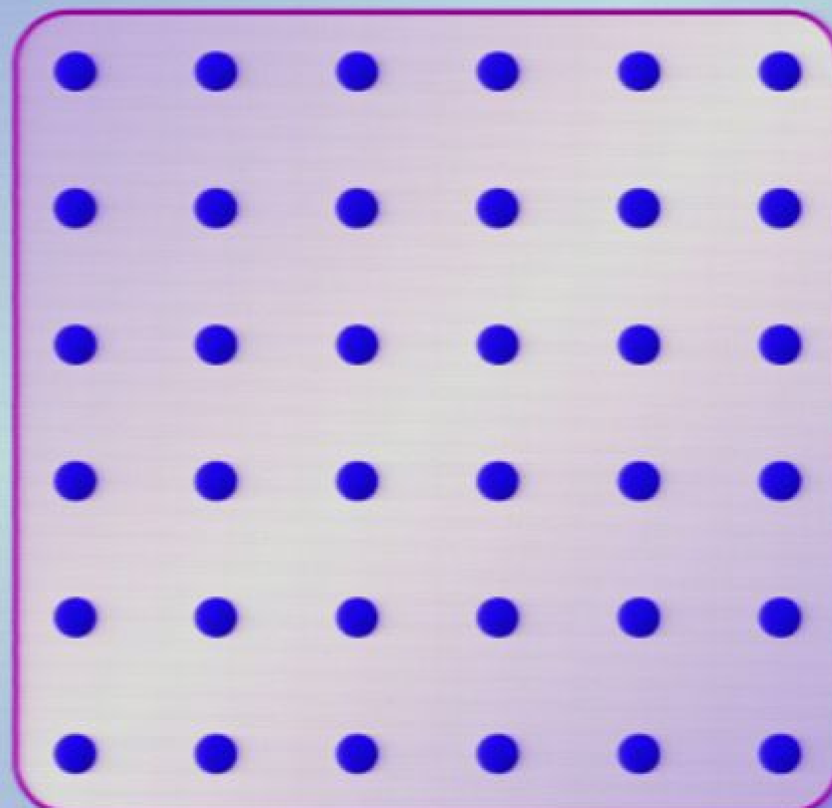
Getting Rid of Translation Operator





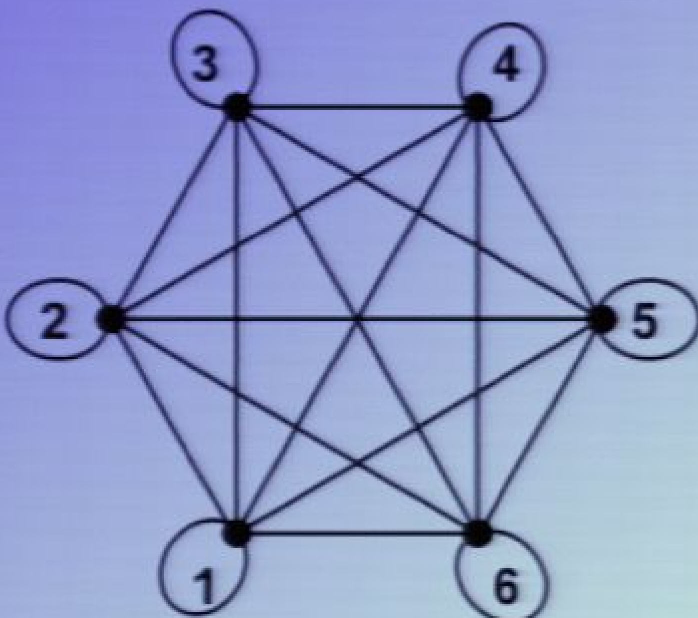
Getting Rid of Translation Operator

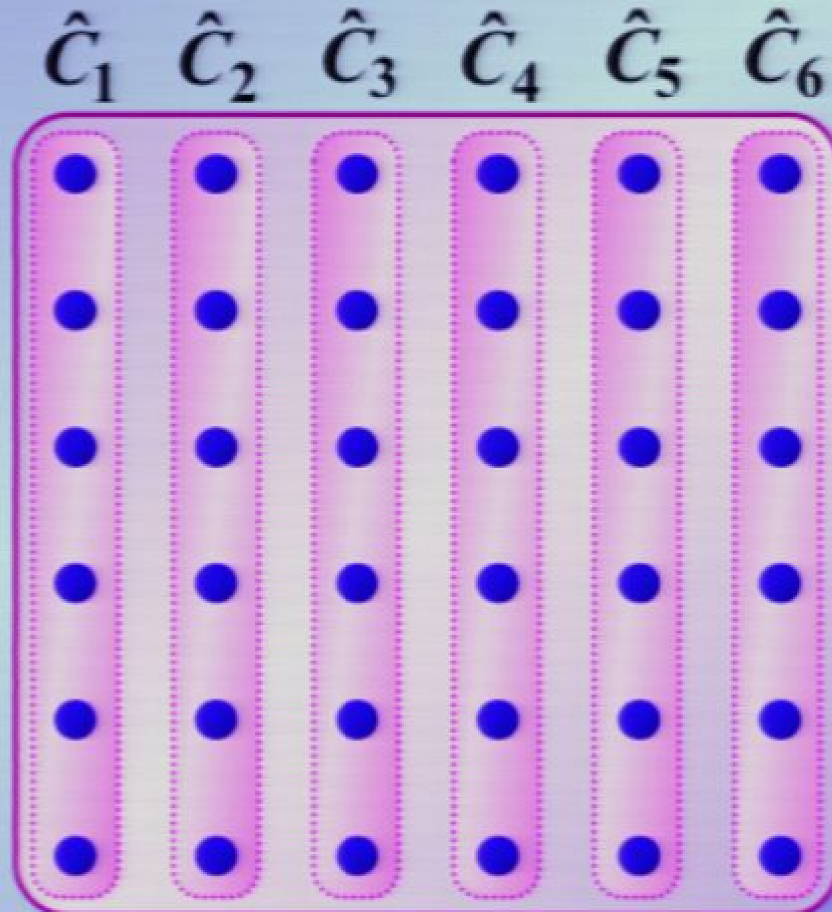




$$\left|\psi_f\right\rangle = \ldots \qquad \hat{C}\left|\psi_0\right\rangle$$

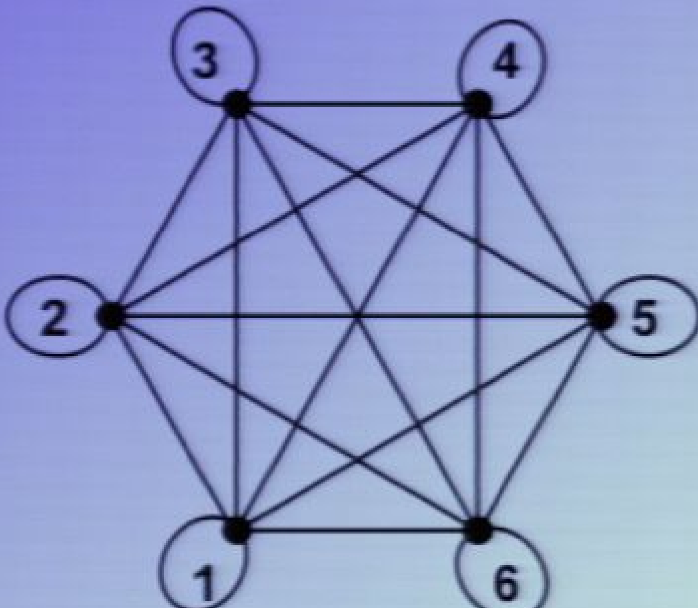
Getting Rid of Translation Operator

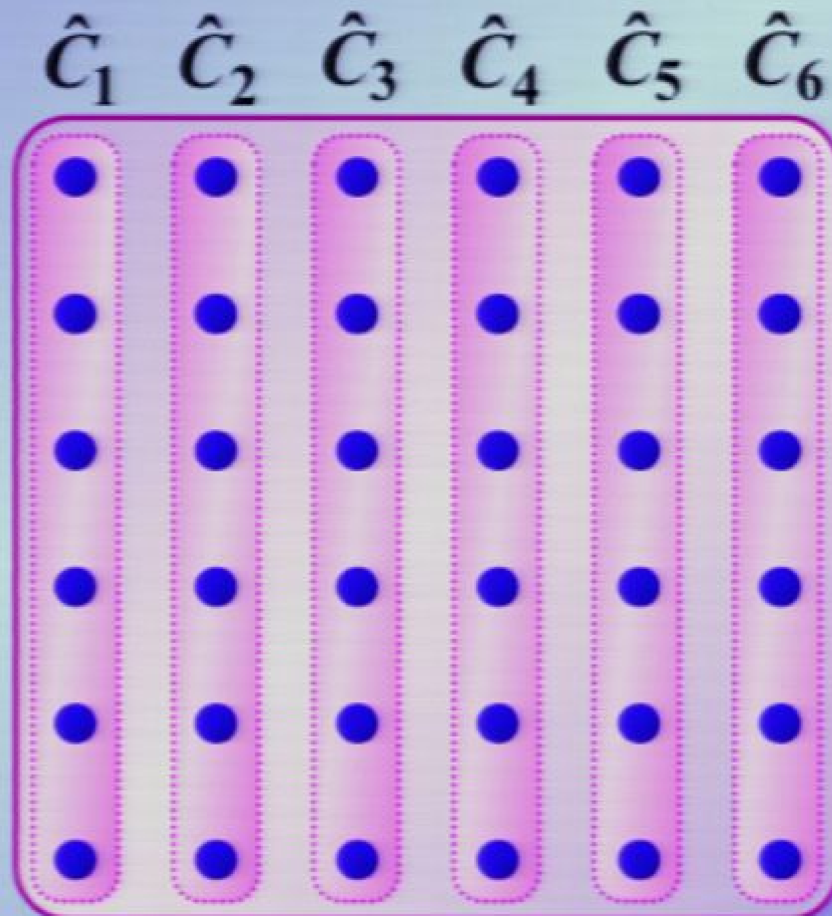




$$\left|\psi_f\right\rangle = \ldots \qquad \hat{C}\left|\psi_0\right\rangle$$

Getting Rid of Translation Operator

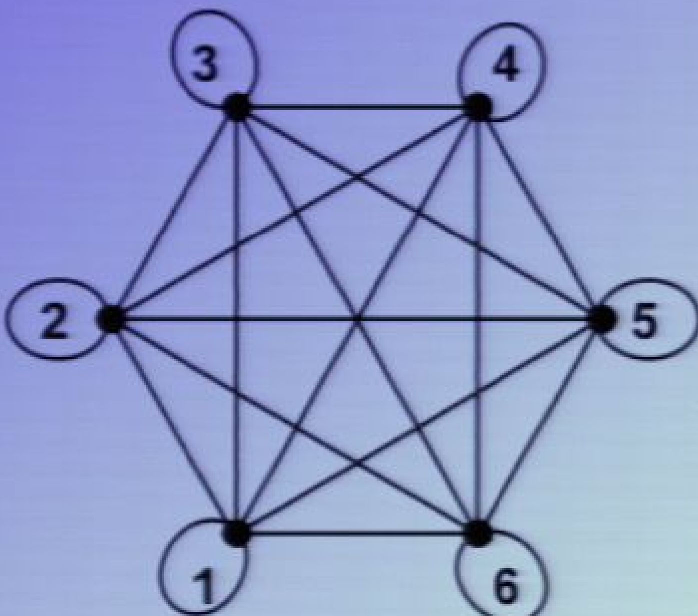




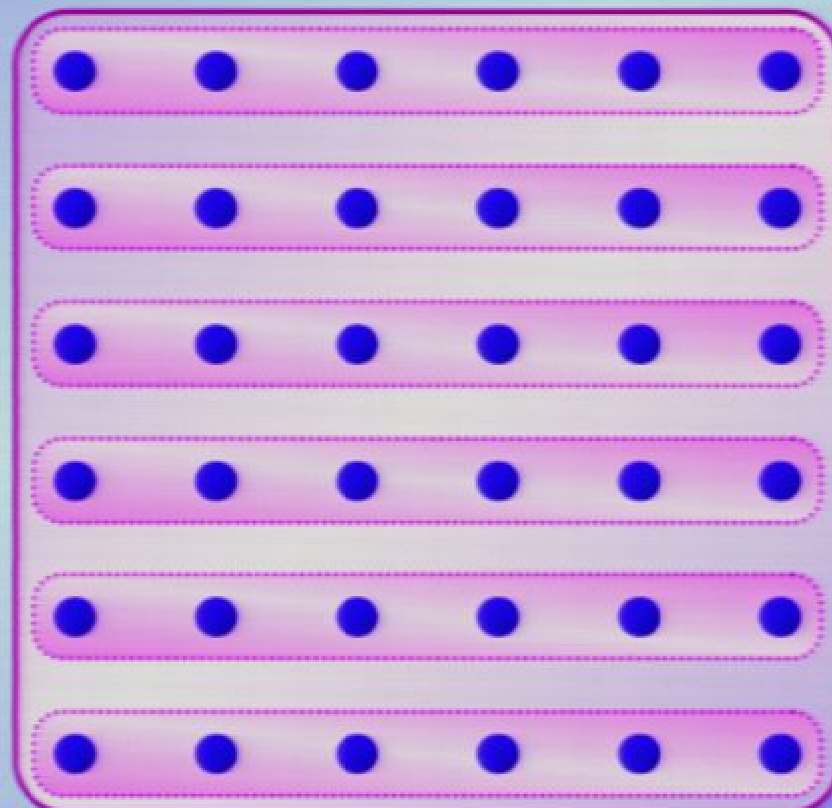
$$|k,l\rangle \xrightarrow{\hat{T}} |l,k\rangle$$

$$|\psi_f\rangle = \ldots \qquad \hat{C}|\psi_0\rangle$$

Getting Rid of Translation Operator



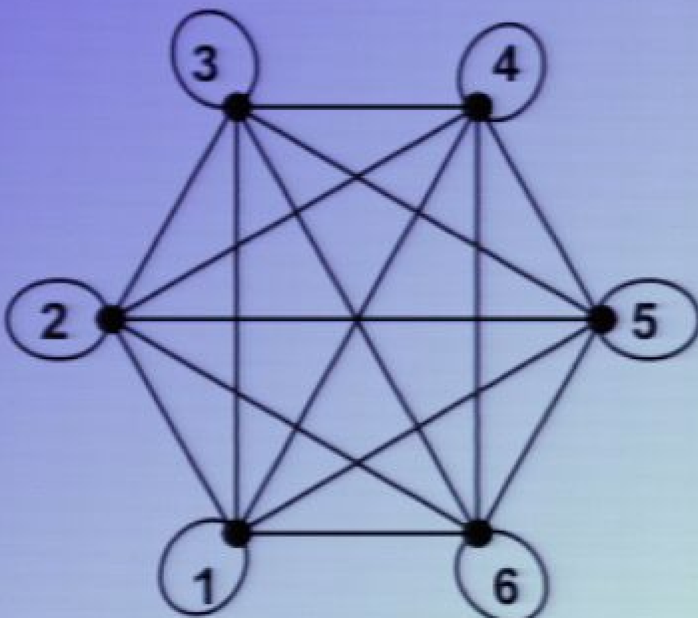
$$|k,l\rangle \xrightarrow{\hat{T}} |l,k\rangle$$

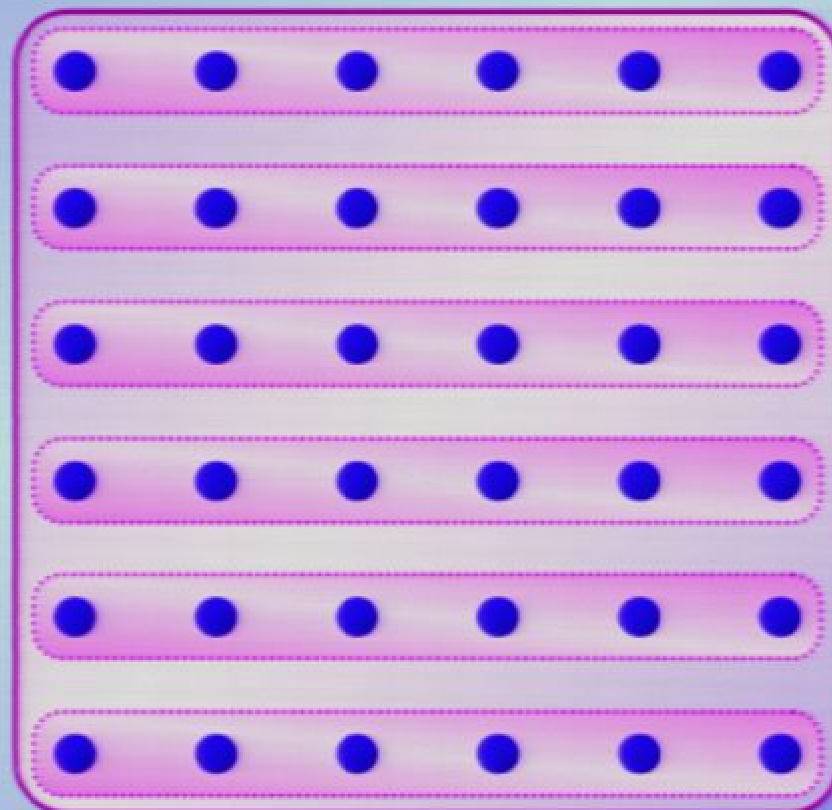


$$|\psi_f\rangle = \ldots$$

$$\hat{T}\,\hat{C}\,|\psi_0\rangle$$

Getting Rid of Translation Operator

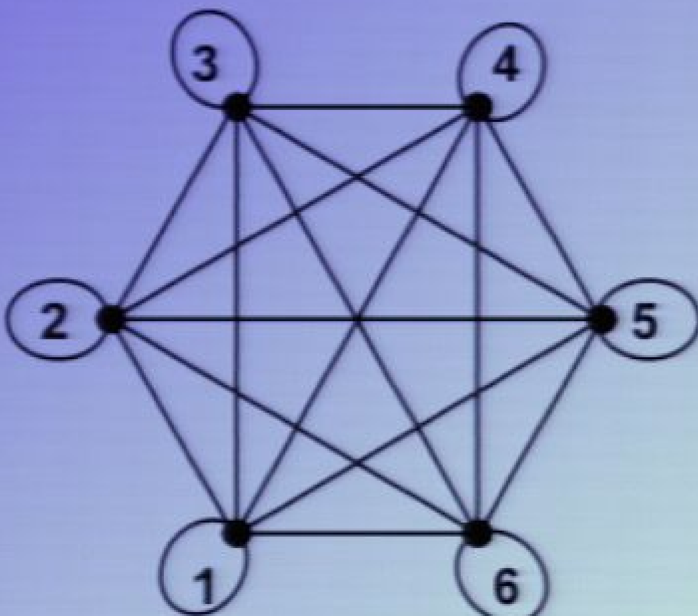


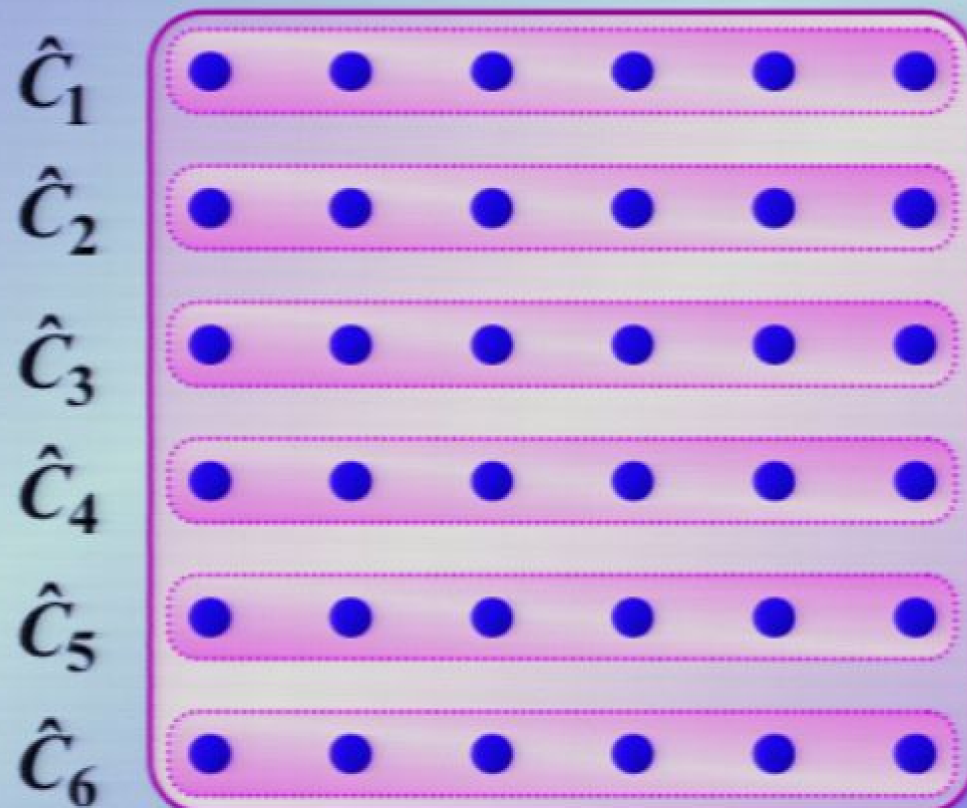


$$|k,l\rangle \xrightarrow{\hat{T}} |l,k\rangle$$

$$|\psi_f\rangle = \ldots \qquad \hat{C}\,\hat{T}\,\hat{C}\,|\psi_0\rangle$$

Getting Rid of Translation Operator



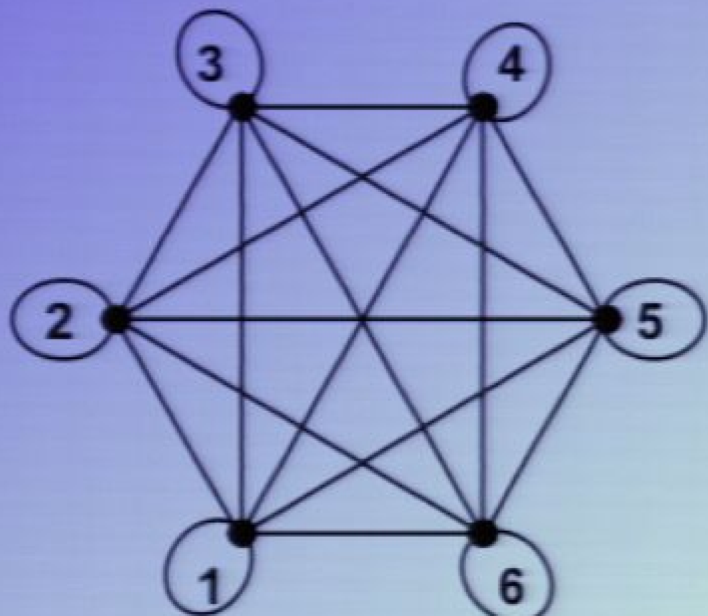


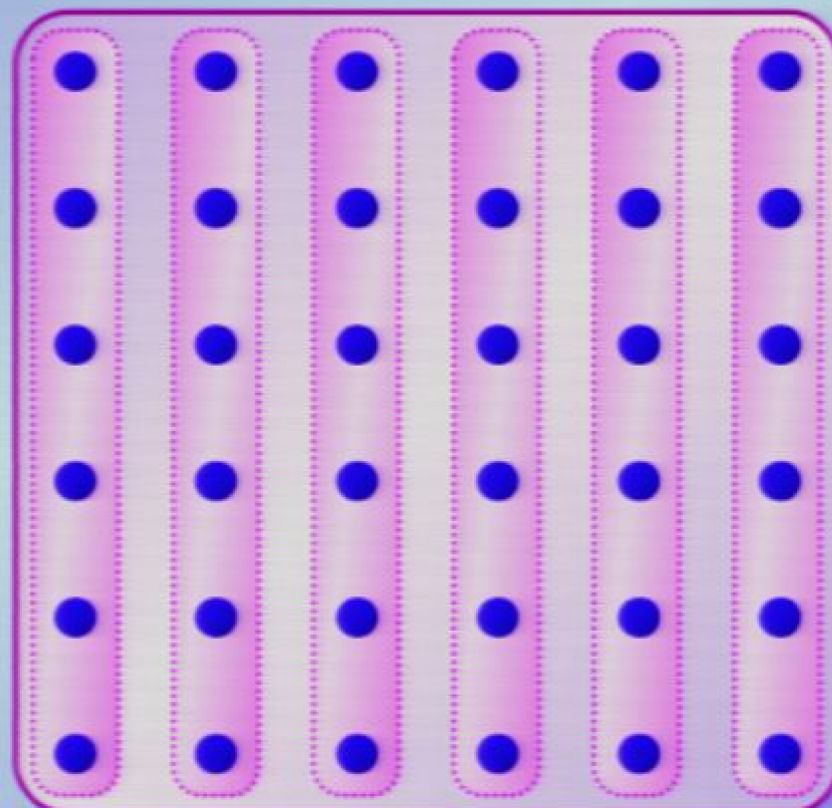
$$|k,l\rangle \xrightarrow{\hat{T}} |l,k\rangle$$

$$|\psi_f\rangle = \ldots \qquad \underbrace{\hat{C}\,\hat{T}}\,\hat{C}\,|\psi_0\rangle$$

C_H C_V

Getting Rid of Translation Operator



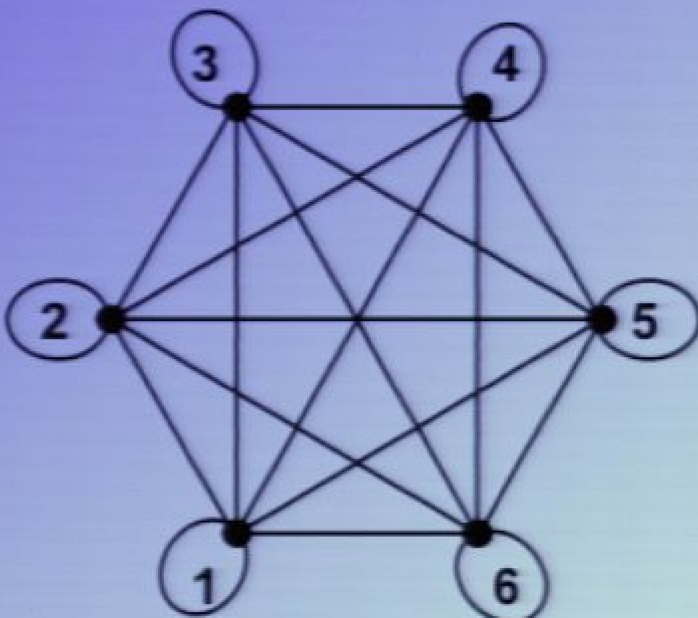


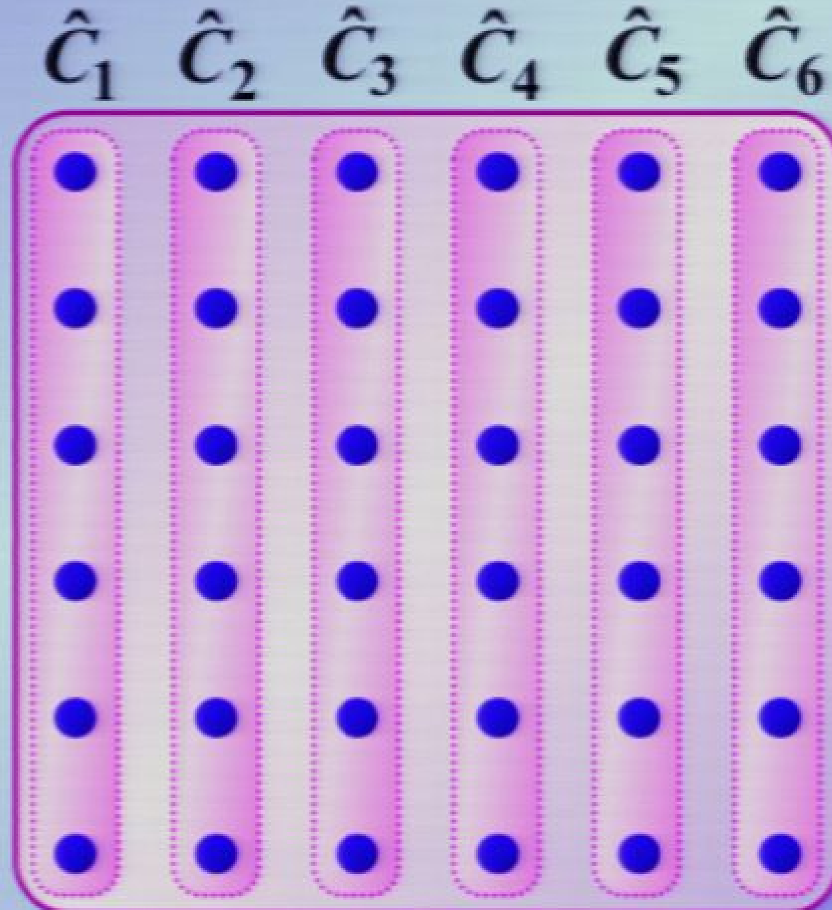
$$|k,l\rangle \xrightarrow{\hat{T}} |l,k\rangle$$

$$|\psi_f\rangle = \ldots \qquad \hat{C}\,\hat{T}\,\underbrace{\hat{C}\,\hat{T}}\,\hat{C}\,|\psi_0\rangle$$

$C_H \; C_V$

Getting Rid of Translation Operator

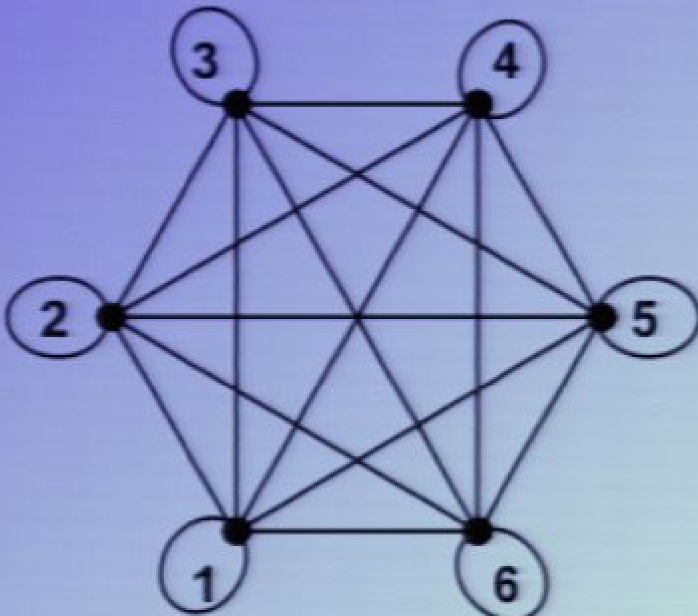


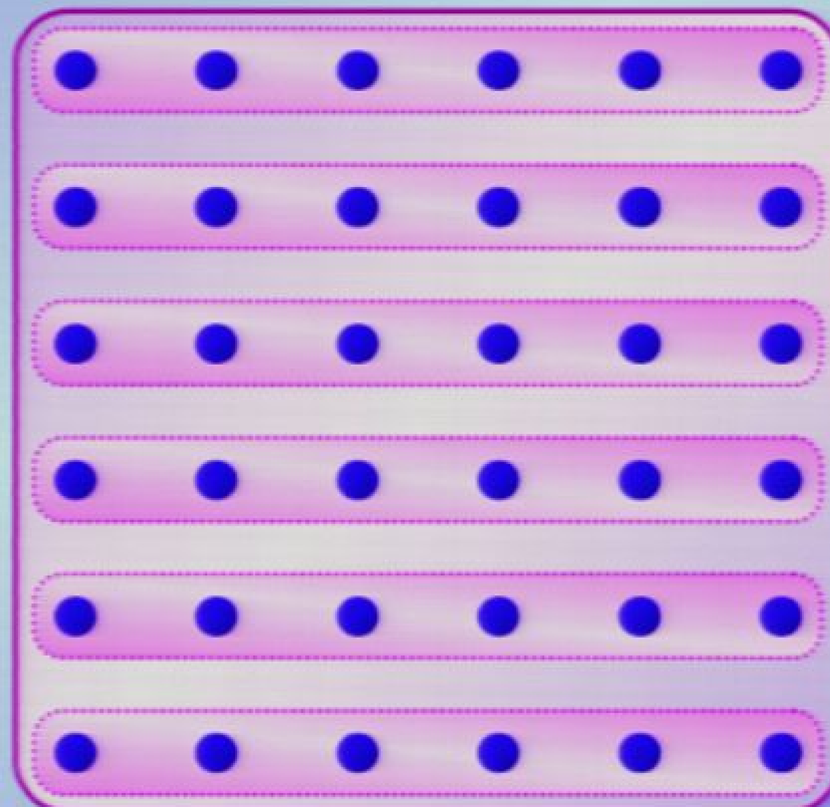


$$\left|k,l\right\rangle \xrightarrow{\hat{T}} \left|l,k\right\rangle$$

$$\left|\psi_f\right\rangle = \ldots \qquad \underbrace{\hat{C}\,\hat{T}}_{C_V}\,\underbrace{\hat{C}\,\hat{T}}_{C_H}\,\underbrace{\hat{C}}_{C_V}\left|\psi_0\right\rangle$$

Getting Rid of Translation Operator

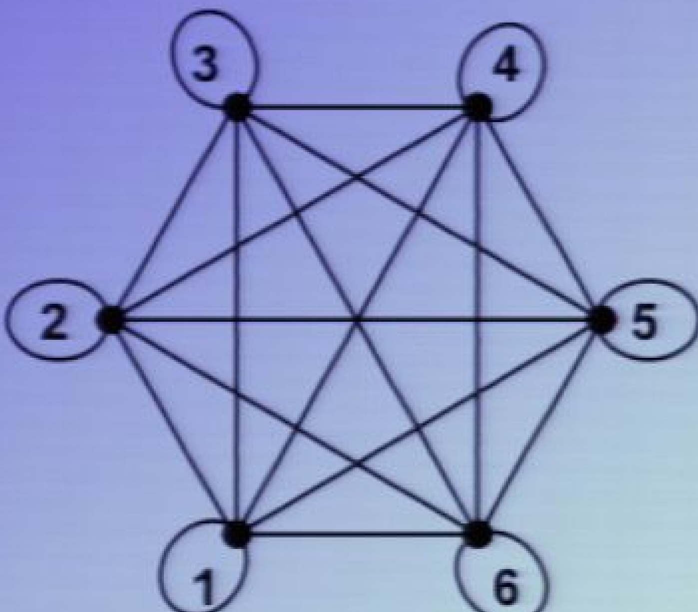


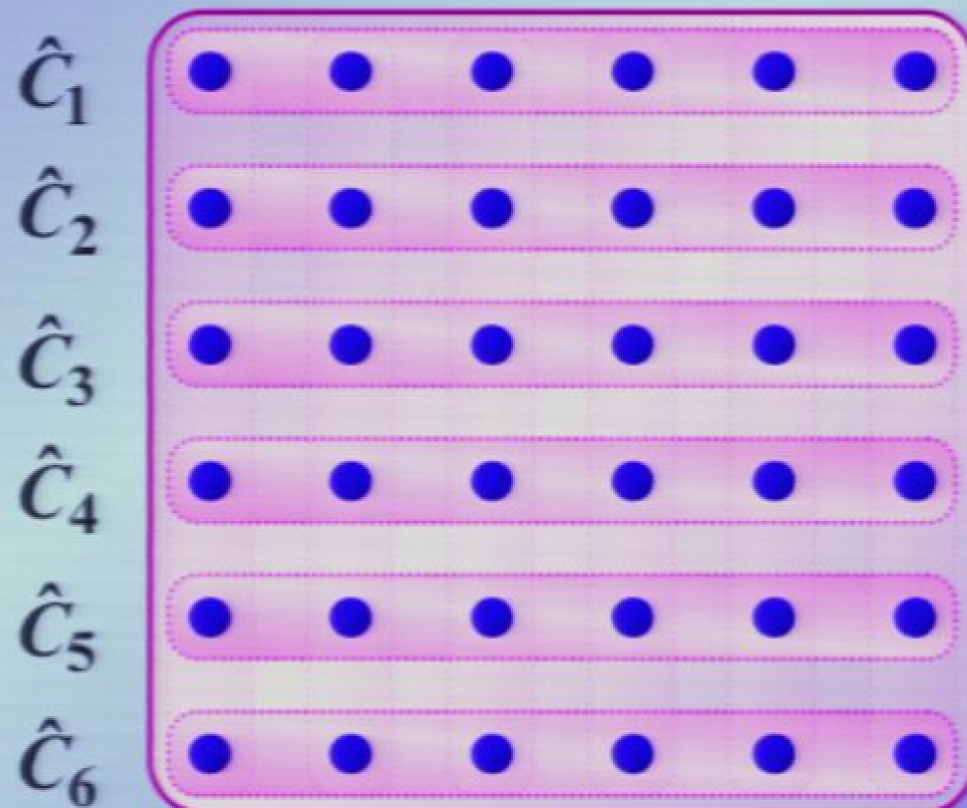


$$|k,l\rangle \xrightarrow{\hat{T}} |l,k\rangle$$

$$|\psi_f\rangle = \ldots \qquad \hat{C}\,\hat{T}\,\underbrace{\hat{C}\,\hat{T}}_{C_V}\,\underbrace{\hat{C}\,\hat{T}}_{C_H}\,\underbrace{\hat{C}}_{C_V}|\psi_0\rangle$$

Getting Rid of Translation Operator

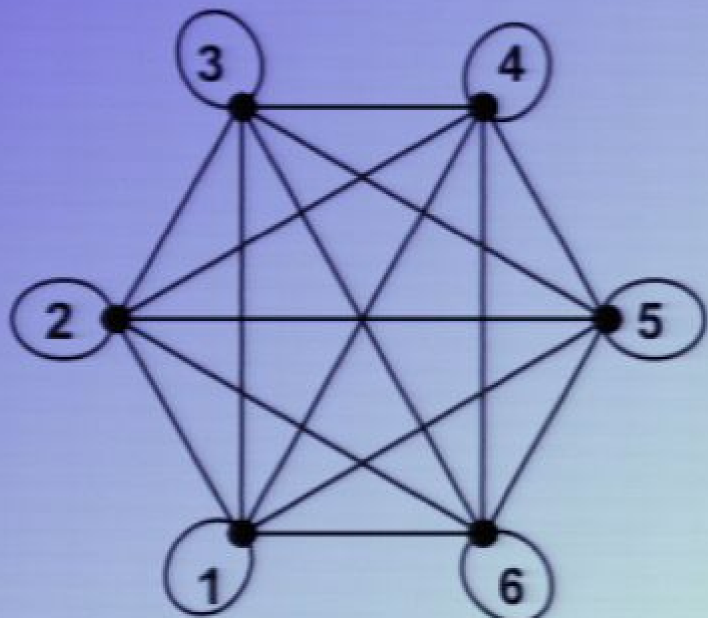


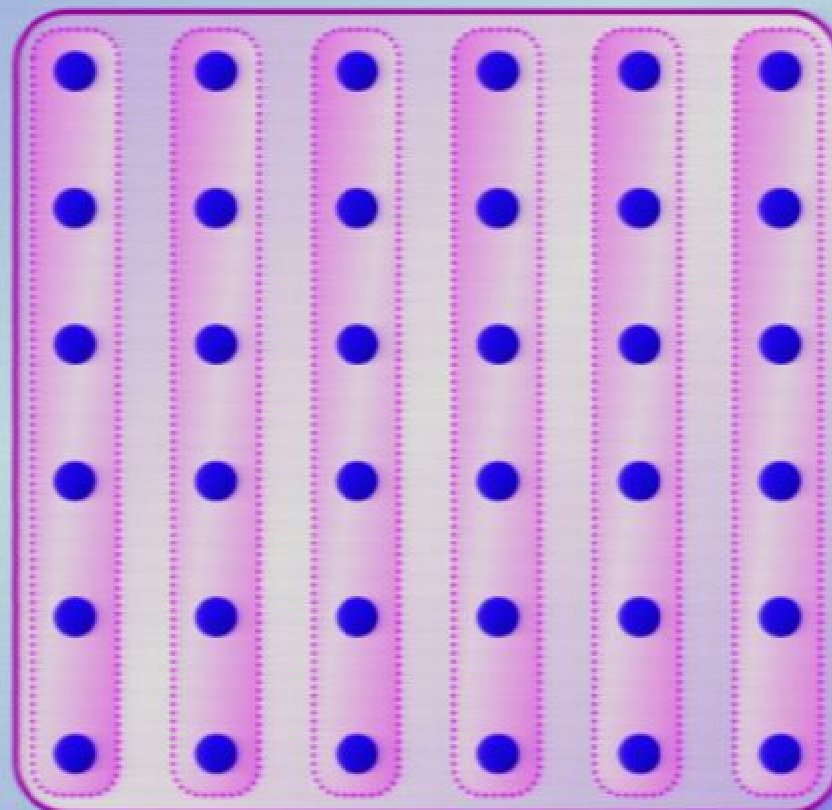


$$|k,l\rangle \xrightarrow{\hat{T}} |l,k\rangle$$

$$|\psi_f\rangle = \ldots \quad \hat{T}\,\underbrace{\hat{C}\,\hat{T}}_{C_H}\,\underbrace{\hat{C}\,\hat{T}}_{C_V}\,\underbrace{\hat{C}\,\hat{T}}_{C_H}\,\underbrace{\hat{C}}_{C_V}|\psi_0\rangle$$

Getting Rid of Translation Operator



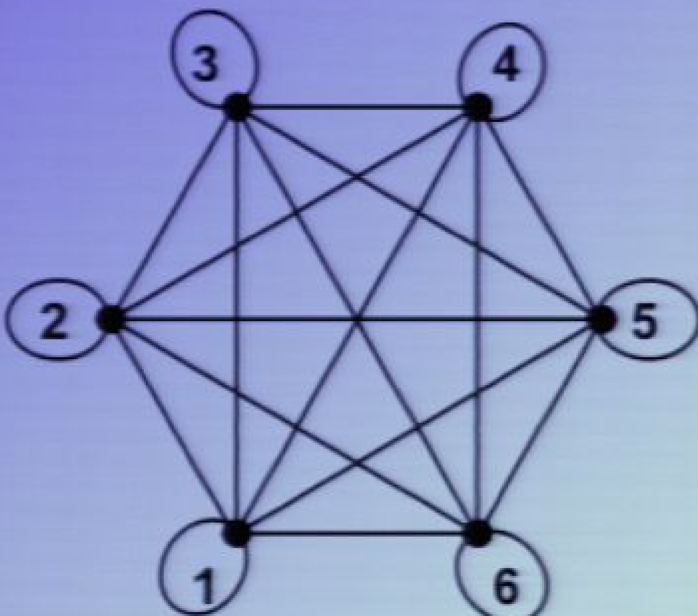


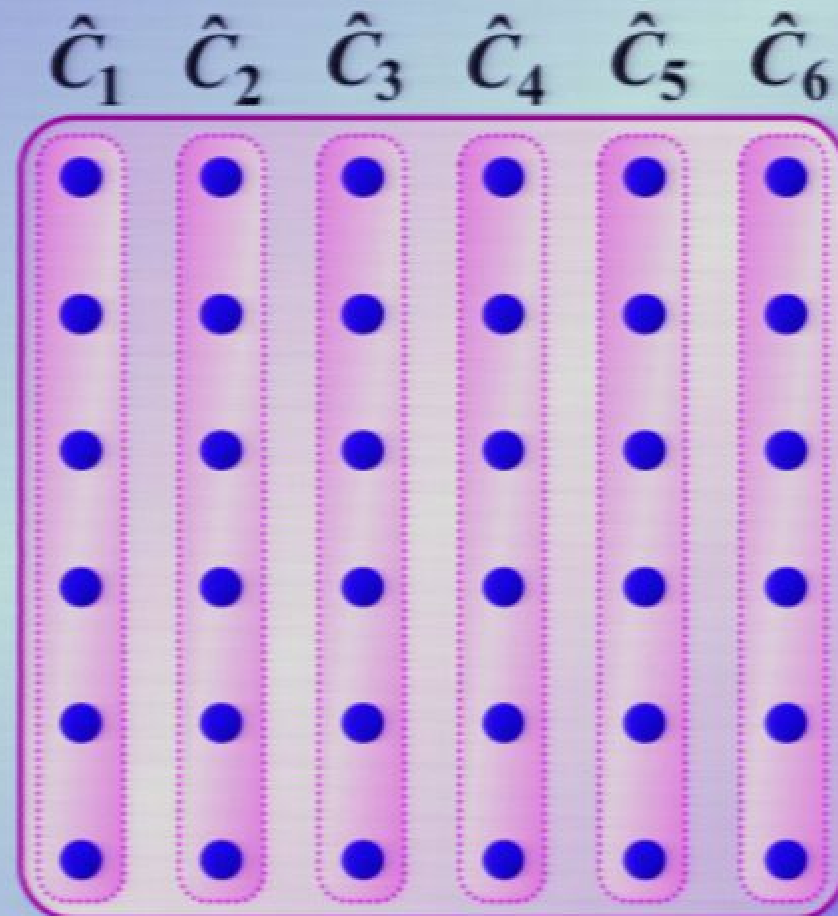
$$|k,l\rangle \xrightarrow{\hat{T}} |l,k\rangle$$

$$|\psi_f\rangle = \ldots \quad \hat{C}\,\hat{T}\,\underbrace{\hat{C}\,\hat{T}}_{C_H}\,\underbrace{\hat{C}\,\hat{T}}_{C_V}\,\underbrace{\hat{C}\,\hat{T}}_{C_H}\,\hat{C}\,|\psi_0\rangle$$

C_V

Getting Rid of Translation Operator

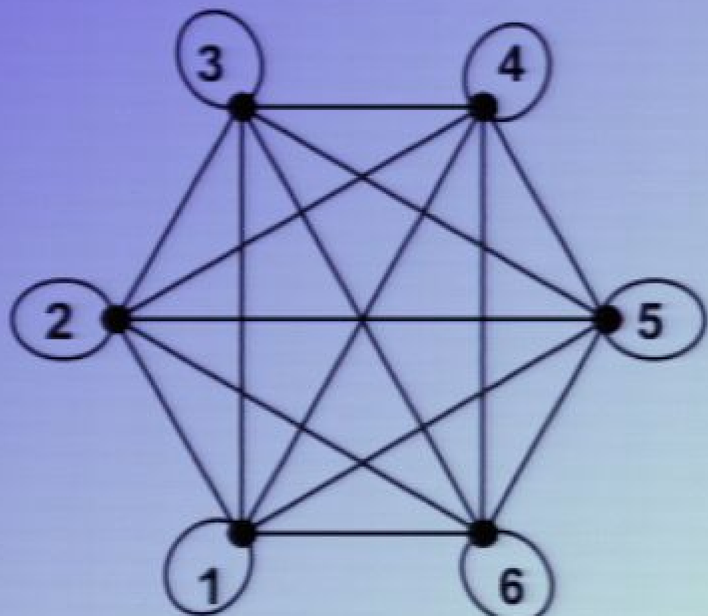


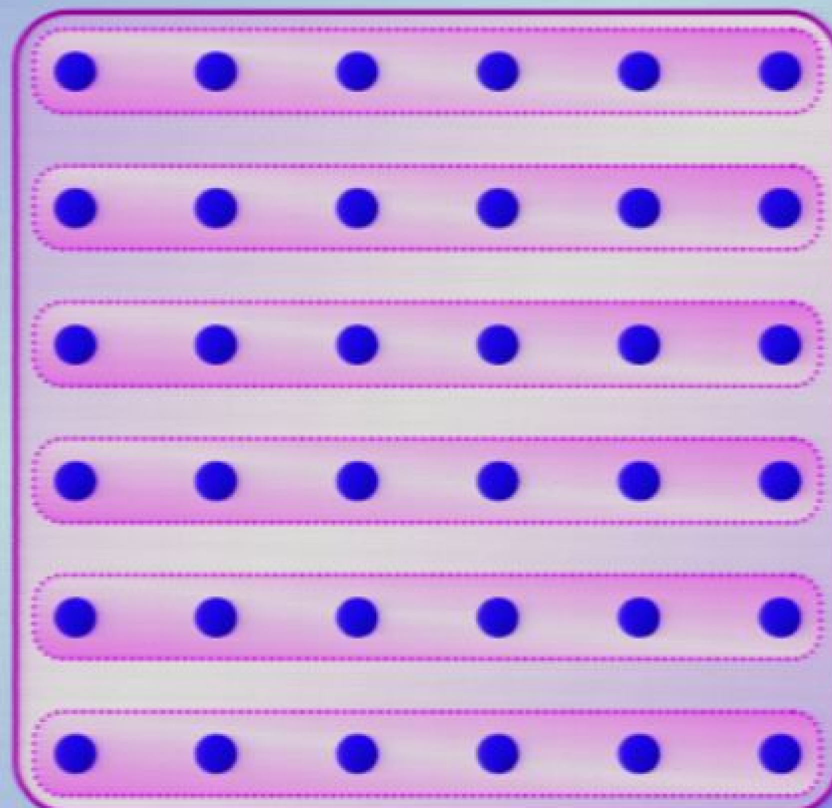


$$|k,l\rangle \xrightarrow{\hat{T}} |l,k\rangle$$

$$|\psi_f\rangle = \ldots \underbrace{\hat{C}\hat{T}}_{C_V}\underbrace{\hat{C}\hat{T}}_{C_H}\underbrace{\hat{C}\hat{T}}_{C_V}\underbrace{\hat{C}\hat{T}}_{C_H}\underbrace{\hat{C}}_{C_V}|\psi_0\rangle$$

Getting Rid of Translation Operator



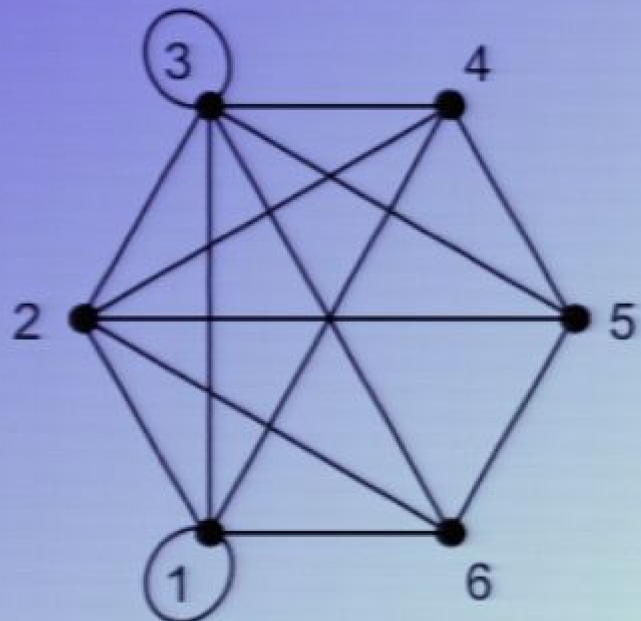


$$|k,l\rangle \xrightarrow{\hat{T}} |l,k\rangle$$

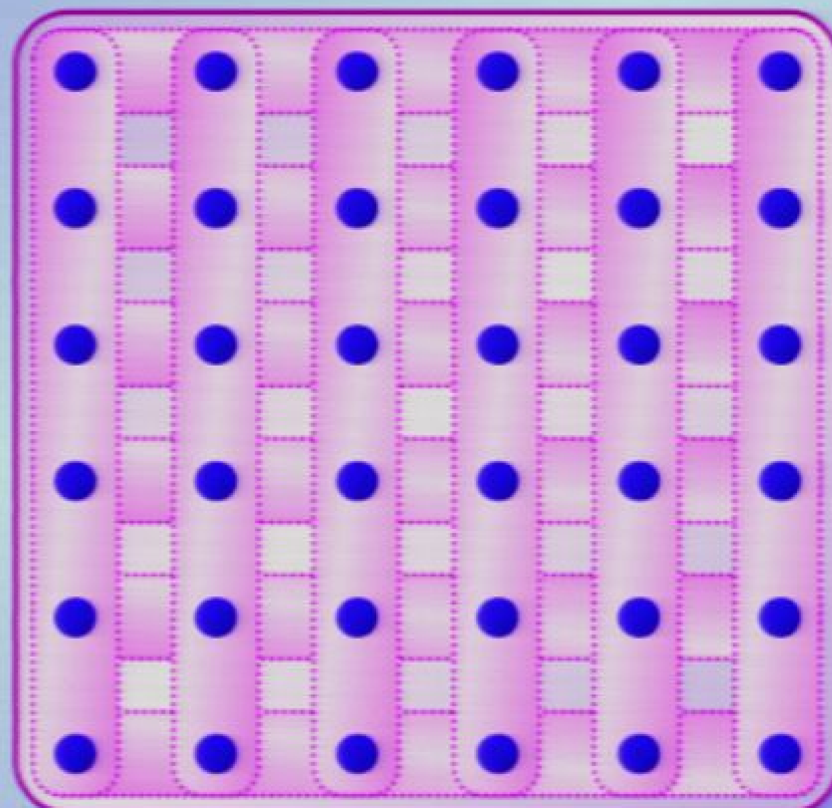
$$|\psi_f\rangle = \ldots \hat{T}\hat{C}\hat{T}\hat{C}\hat{T}\hat{C}\hat{T}\hat{C}\hat{T}\hat{C}|\psi_0\rangle$$

C_V C_H C_V C_H C_V

Getting Rid of Translation Operator

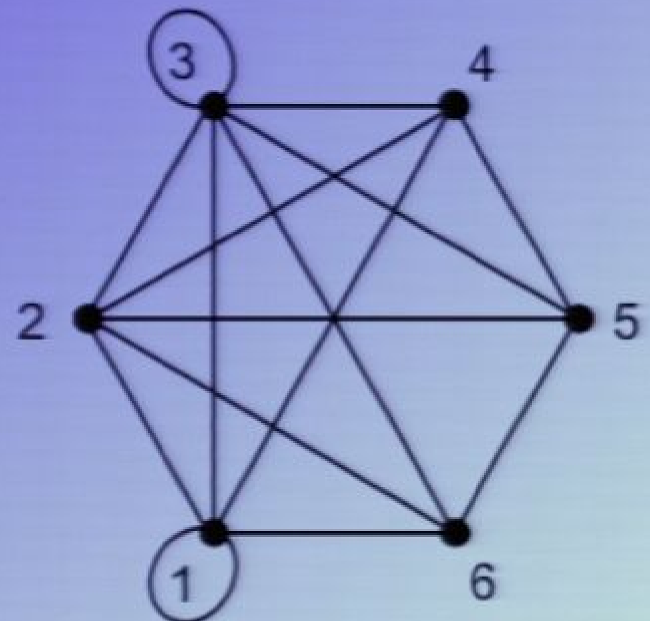


$$|k,l\rangle \xrightarrow{\hat{T}} |l,k\rangle$$

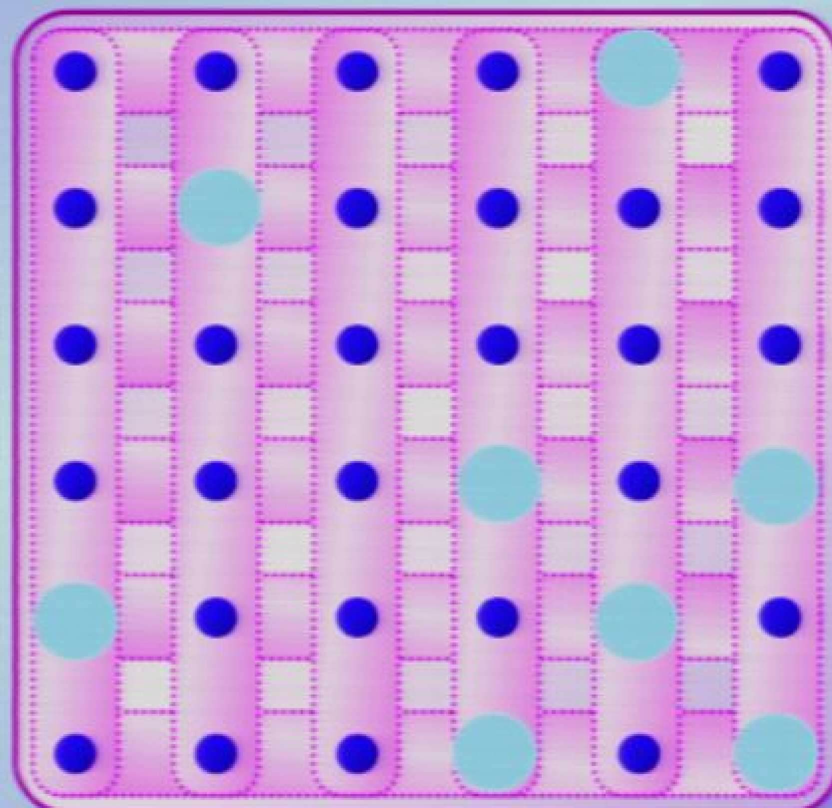


$$|\psi_f\rangle = \ldots \hat{T}\,\underbrace{\hat{C}\,\hat{T}}_{C_V}\,\underbrace{\hat{C}\,\hat{T}}_{C_H}\,\underbrace{\hat{C}\,\hat{T}}_{C_V}\,\underbrace{\hat{C}\,\hat{T}}_{C_H}\,\underbrace{\hat{C}}_{C_V}|\psi_0\rangle$$

Getting Rid of Translation Operator



$$\left|k,l\right\rangle \xrightarrow{\hat{T}} \left|l,k\right\rangle$$



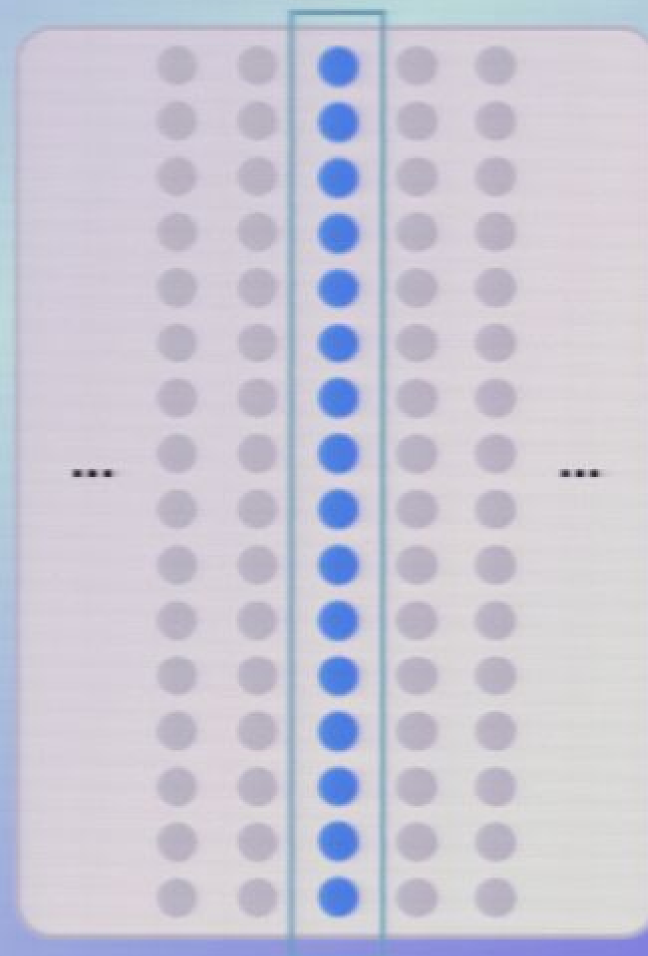
$$\left|\psi_f\right\rangle = \ldots \hat{T}\,\underbrace{\hat{C}\,\hat{T}}_{C_V}\,\underbrace{\hat{C}\,\hat{T}}_{C_H}\,\underbrace{\hat{C}\,\hat{T}}_{C_V}\,\underbrace{\hat{C}\,\hat{T}}_{C_H}\,\underbrace{\hat{C}}_{C_V}\left|\psi_0\right\rangle$$

Implementing the Coin Operator

$\hat{c}_j^V$

In general, all elements are non-zero.

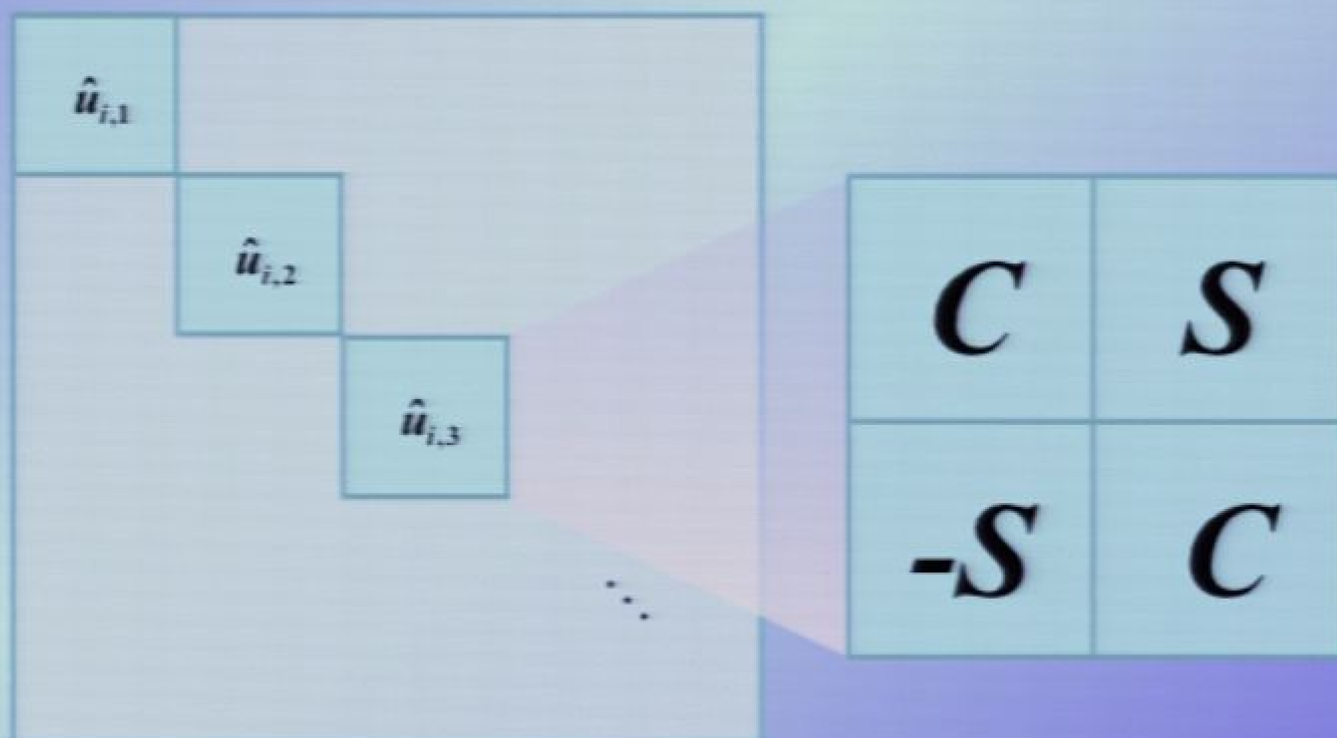
$\mathbf{A}_j^V$



CS Decomposition

$$\hat{c}_j^V = \prod_{i=1}^{N-1} \hat{U}_i(d_i)$$

$$\hat{U}_i(d_i) =$$

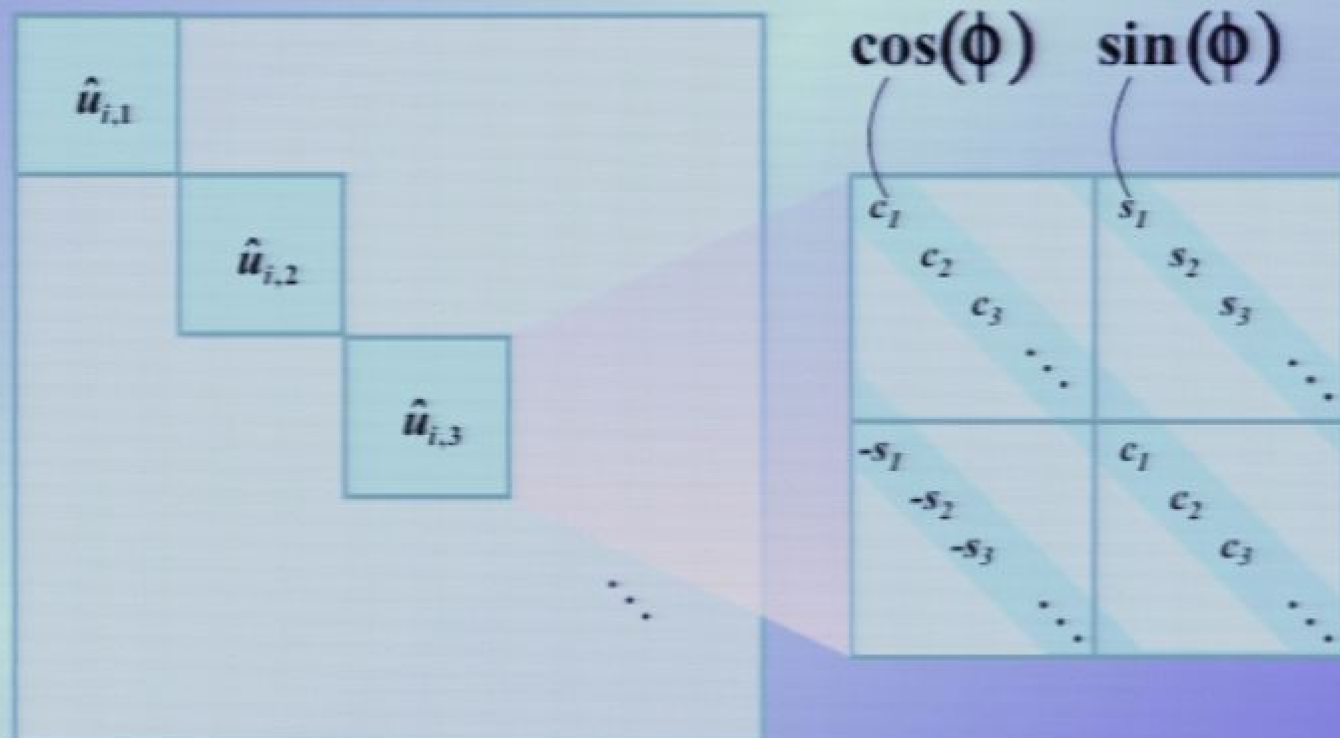


$$d_i = 2,4,8\ldots N/2$$

CS Decomposition

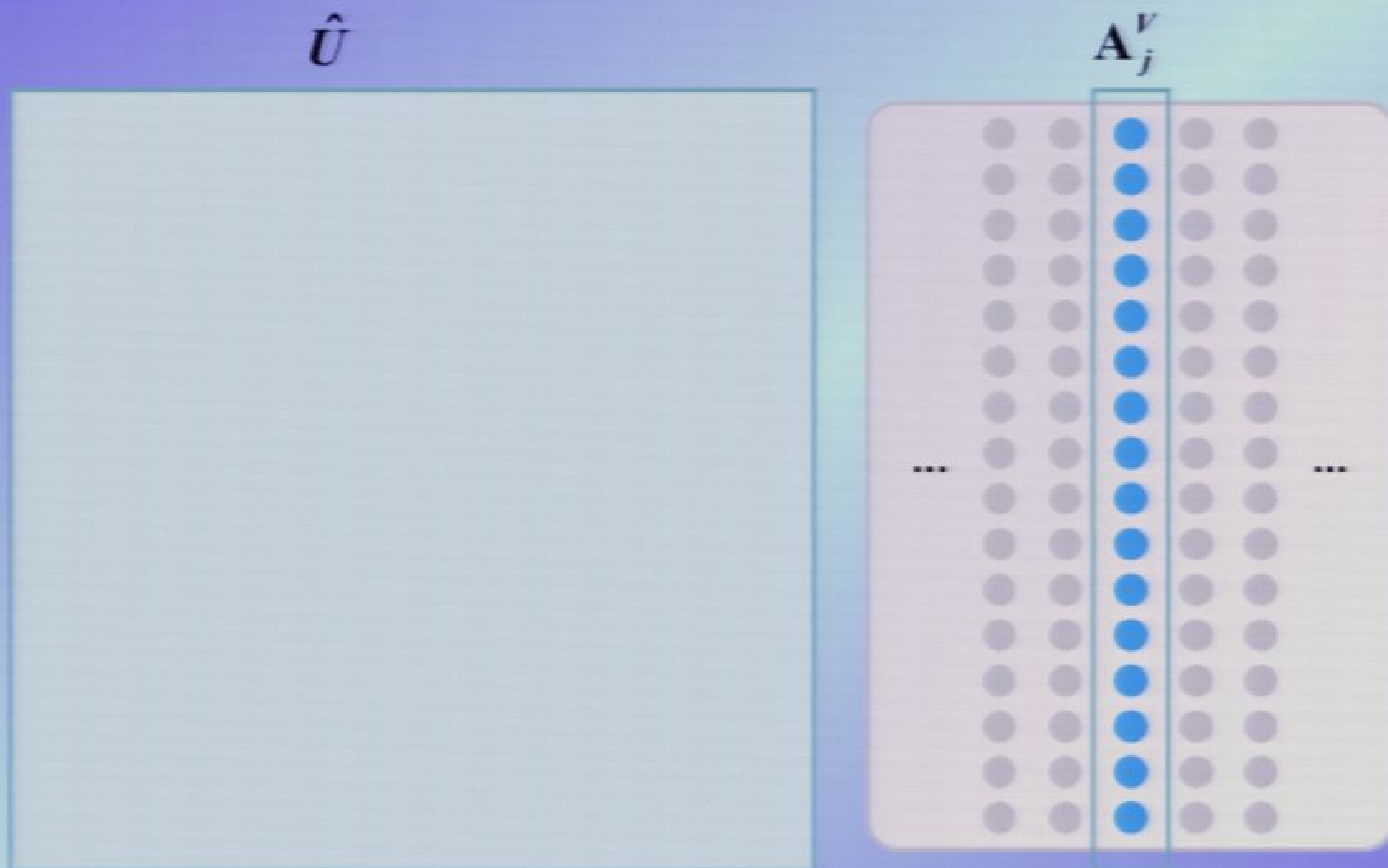
$$\hat{c}_j^V = \prod_{i=1}^{N-1} \hat{U}_i(d_i)$$

$$\hat{U}_i(d_i) =$$



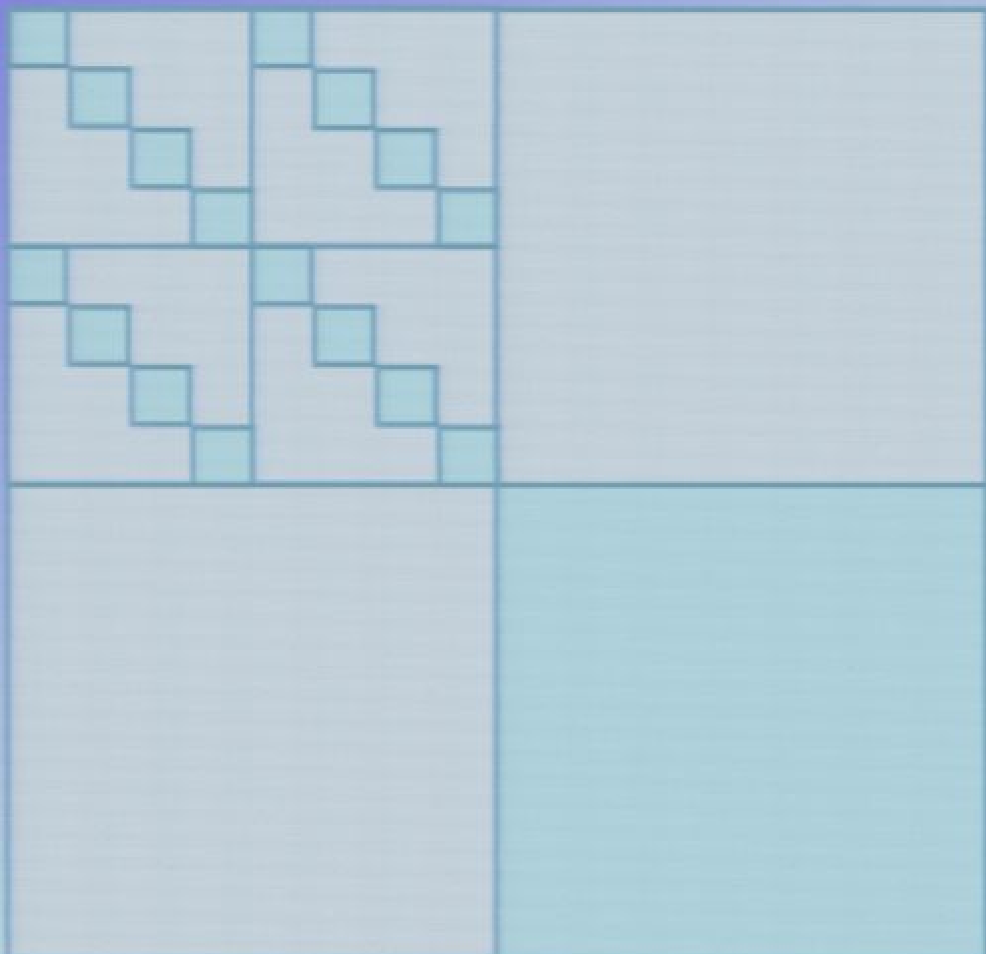
$$d_i = 2,4,8\ldots N/2$$

Implementing the Coin Operator

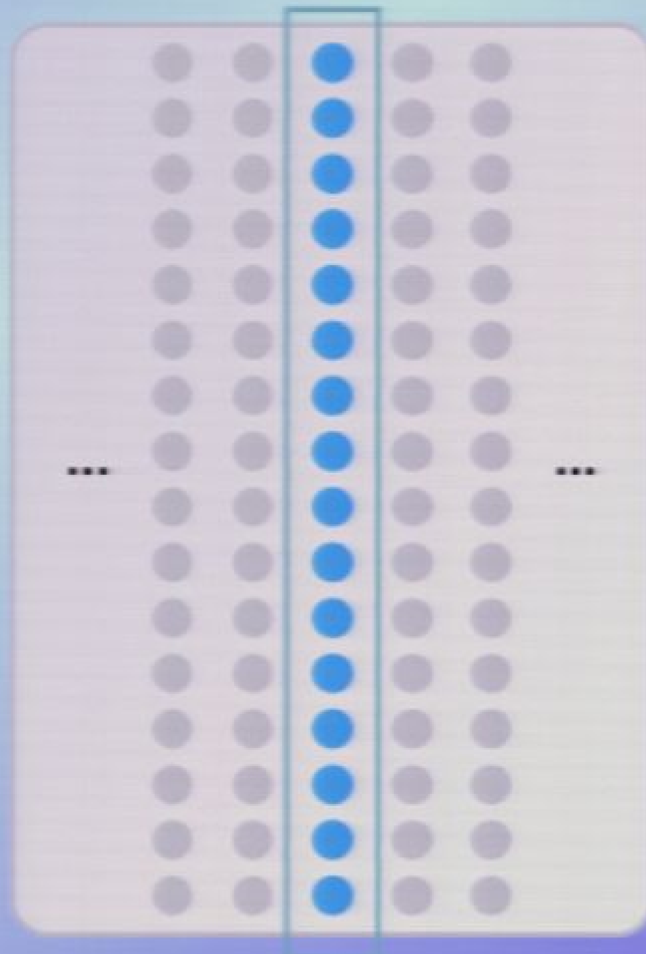


Implementing the Coin Operator

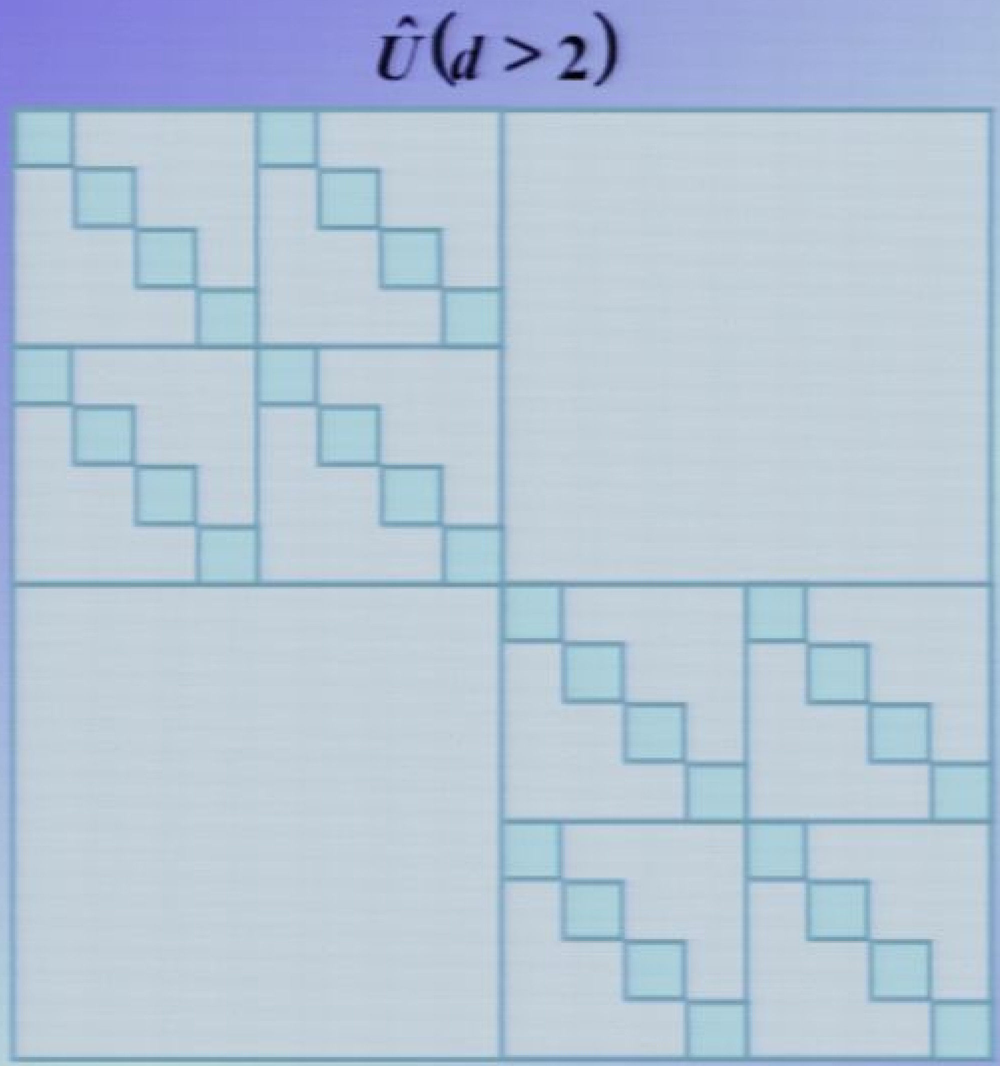
$\hat{U}(d>2)$

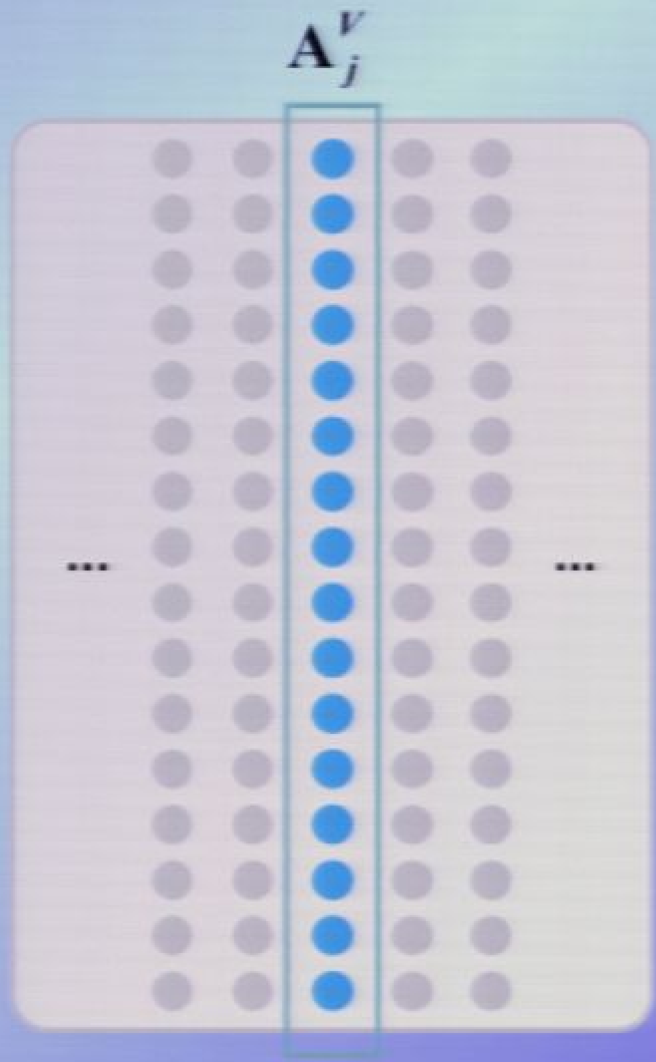


$\mathbf{A}_j^V$



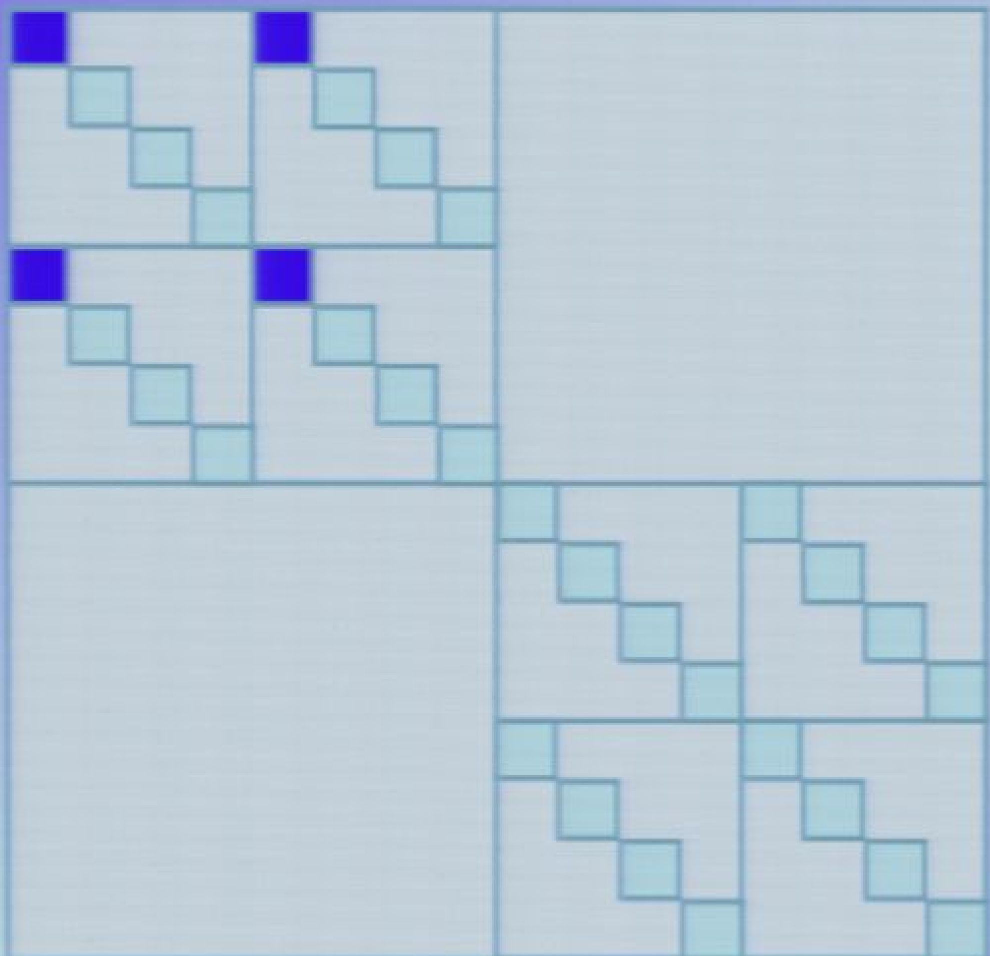
Implementing the Coin Operator



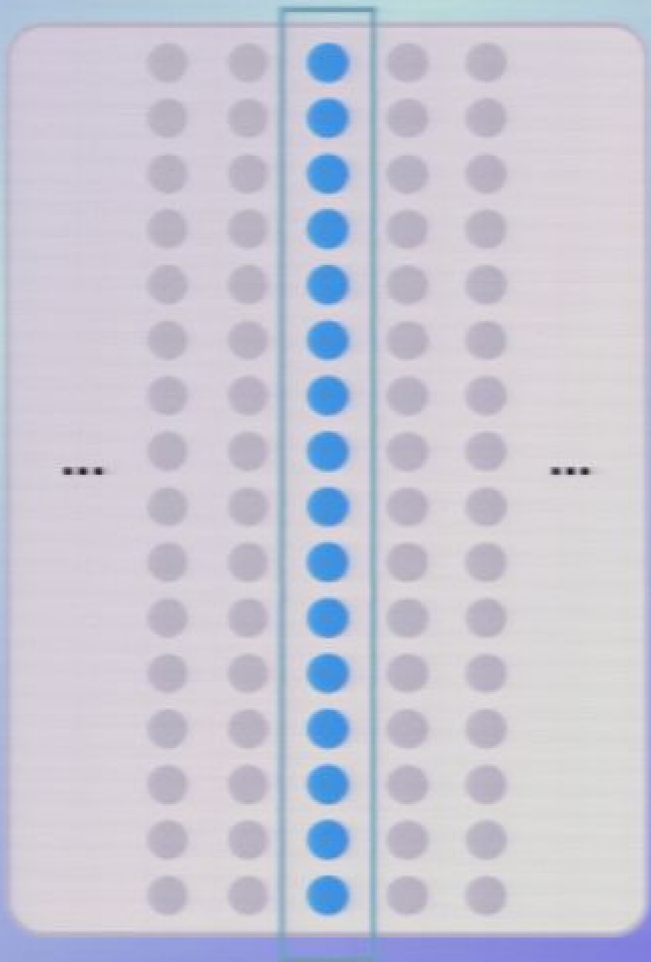


Implementing the Coin Operator

$\hat{U}(d > 2)$

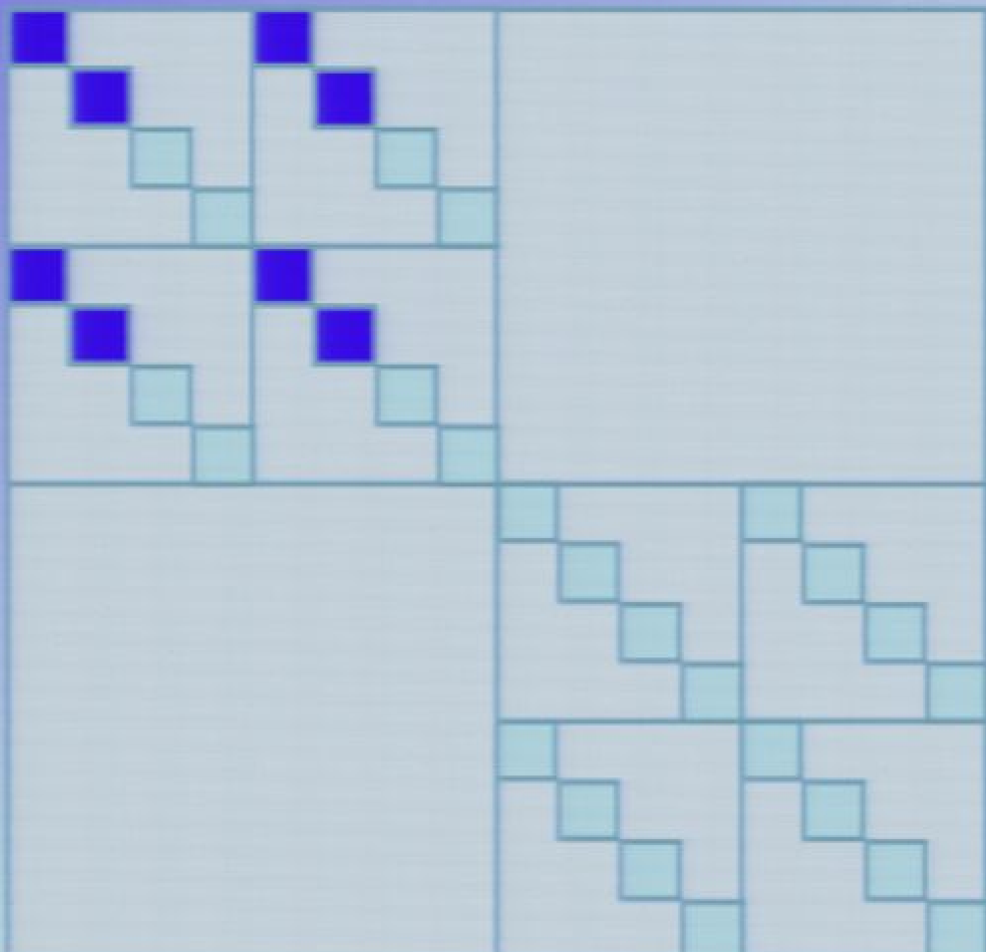


$\mathbf{A}_j^V$

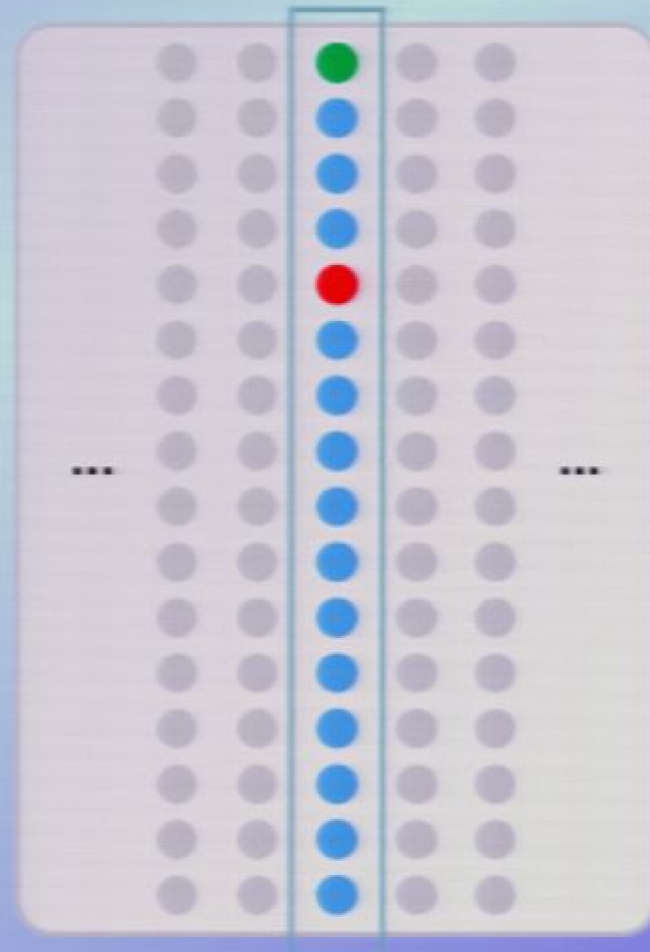


Implementing the Coin Operator

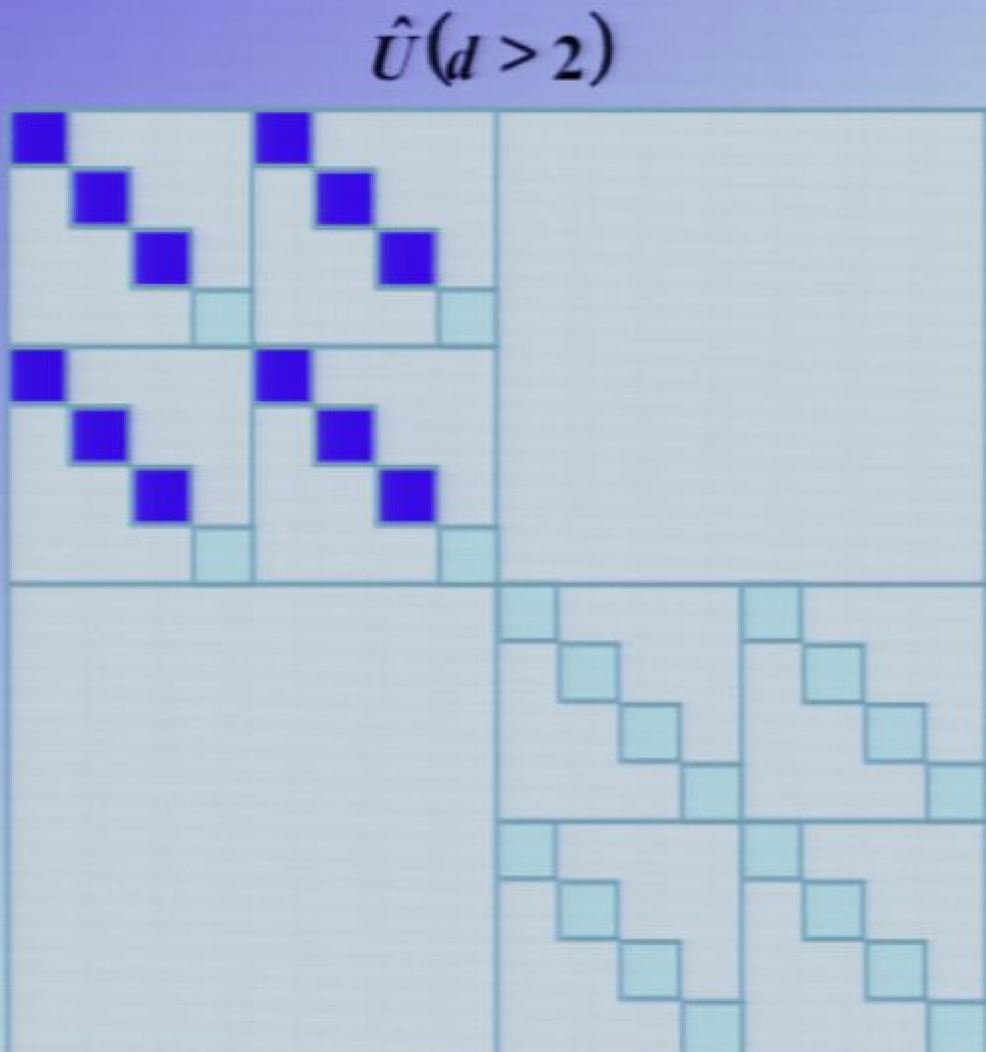
$\hat{U}(d > 2)$

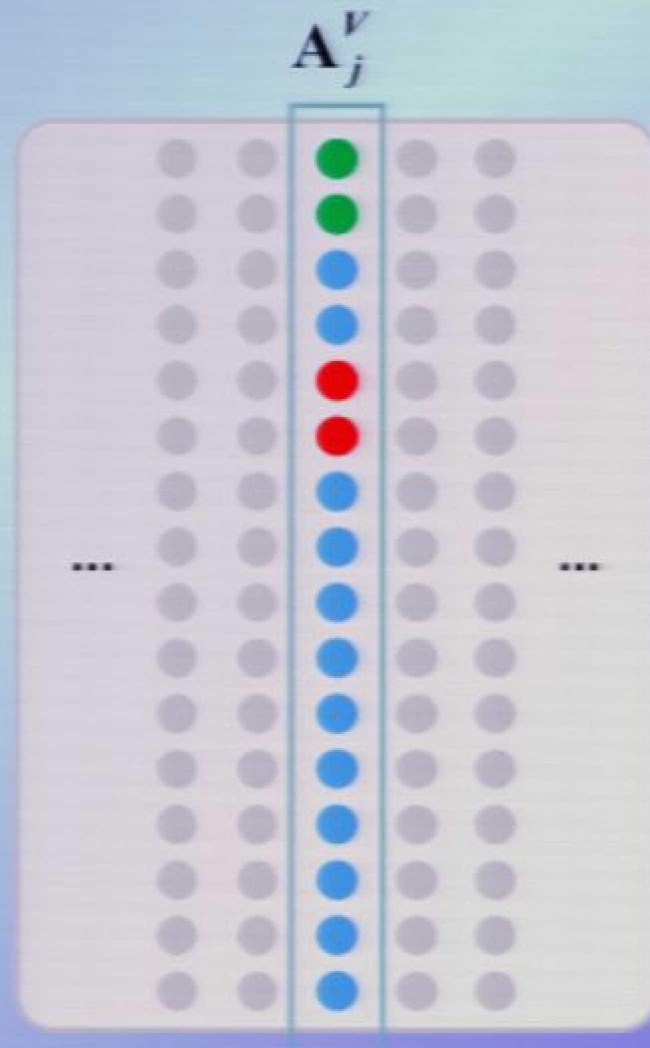


$\mathbf{A}_j^V$

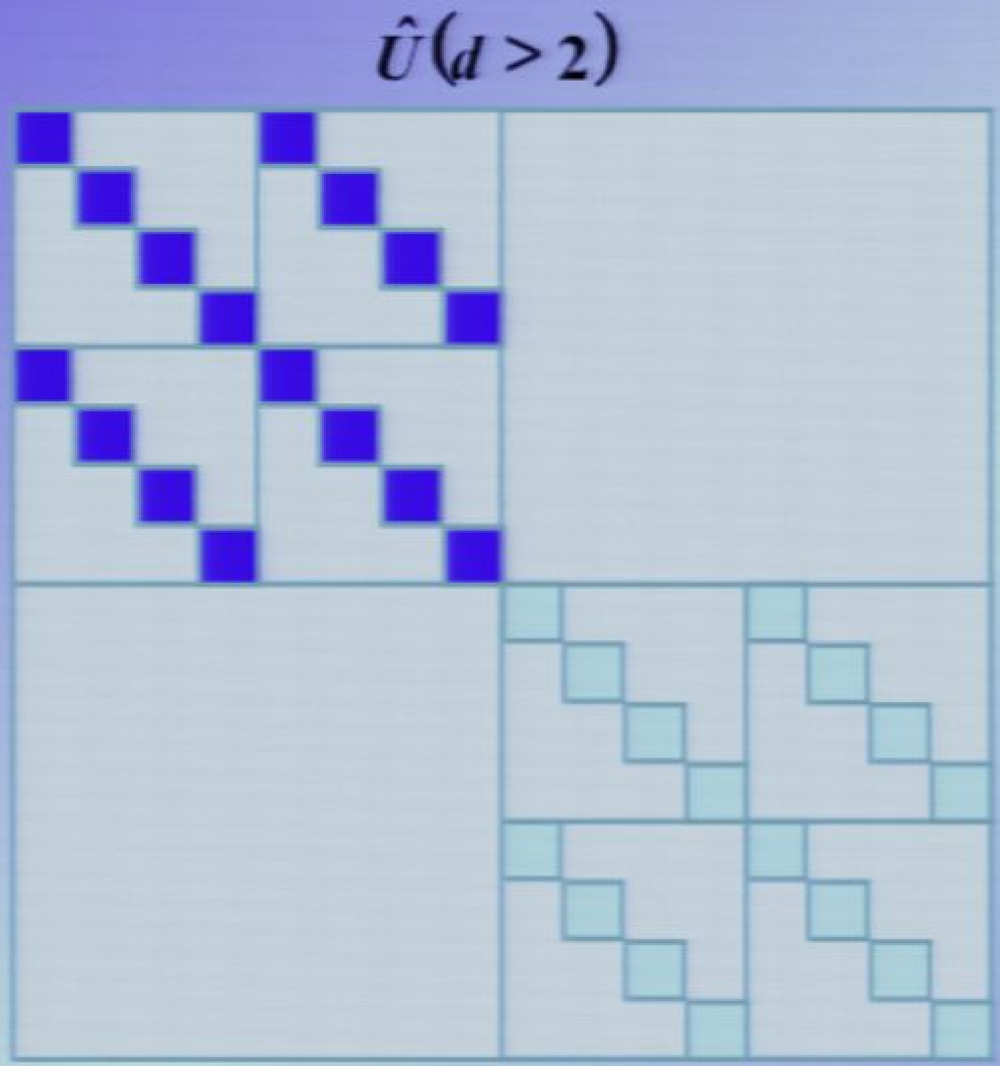


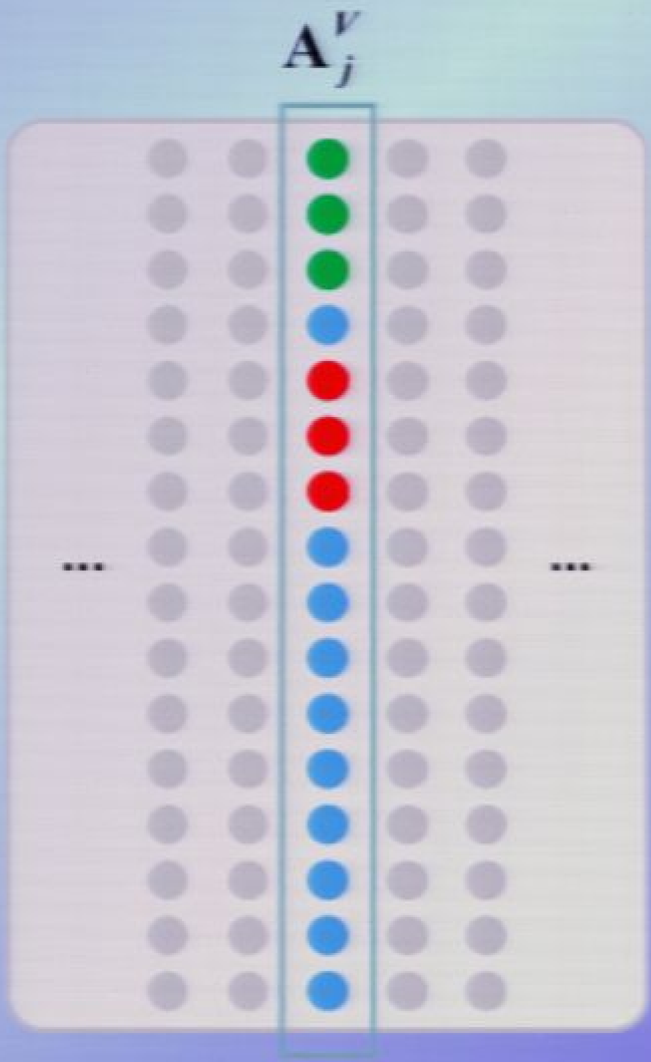
Implementing the Coin Operator



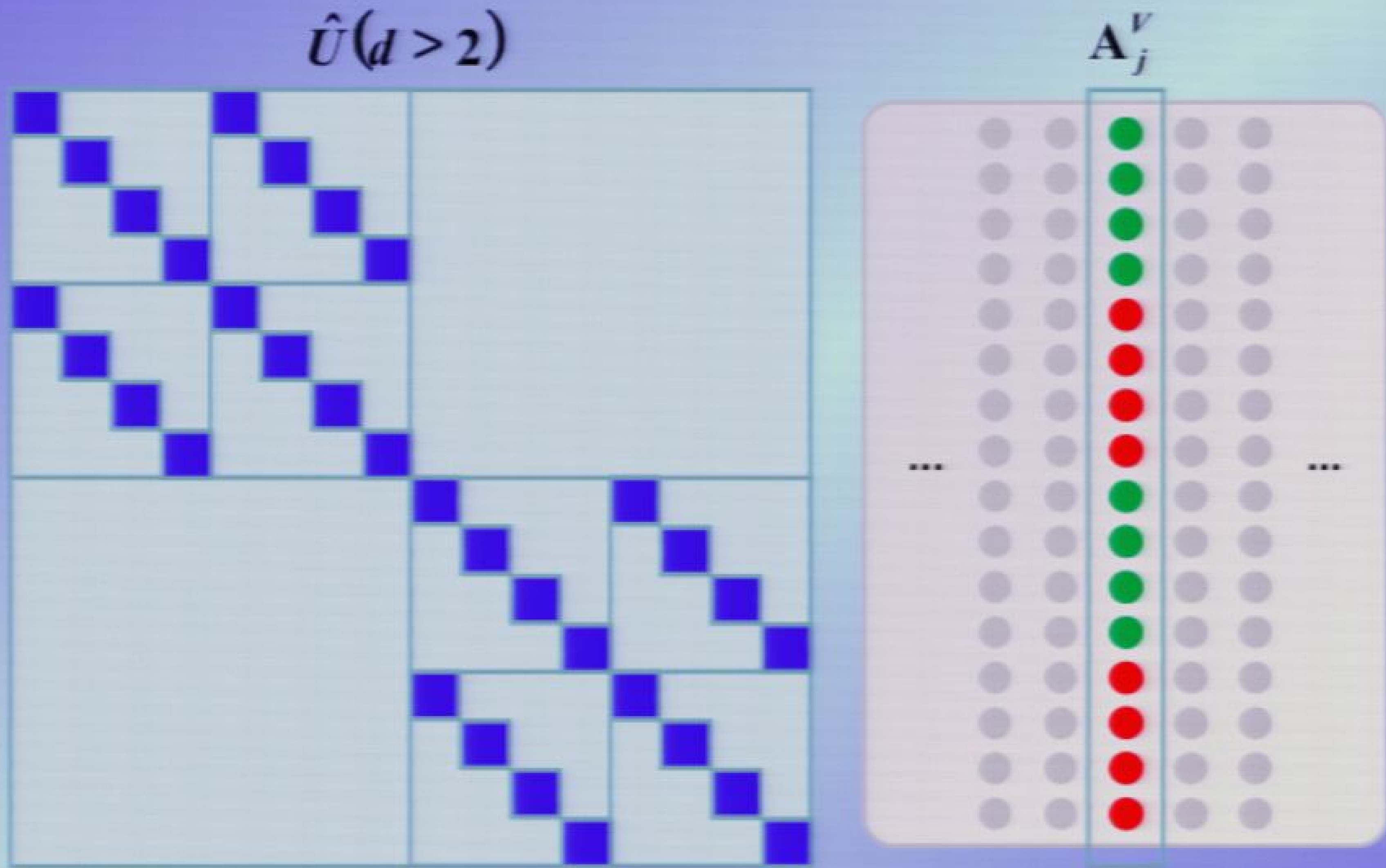


Implementing the Coin Operator



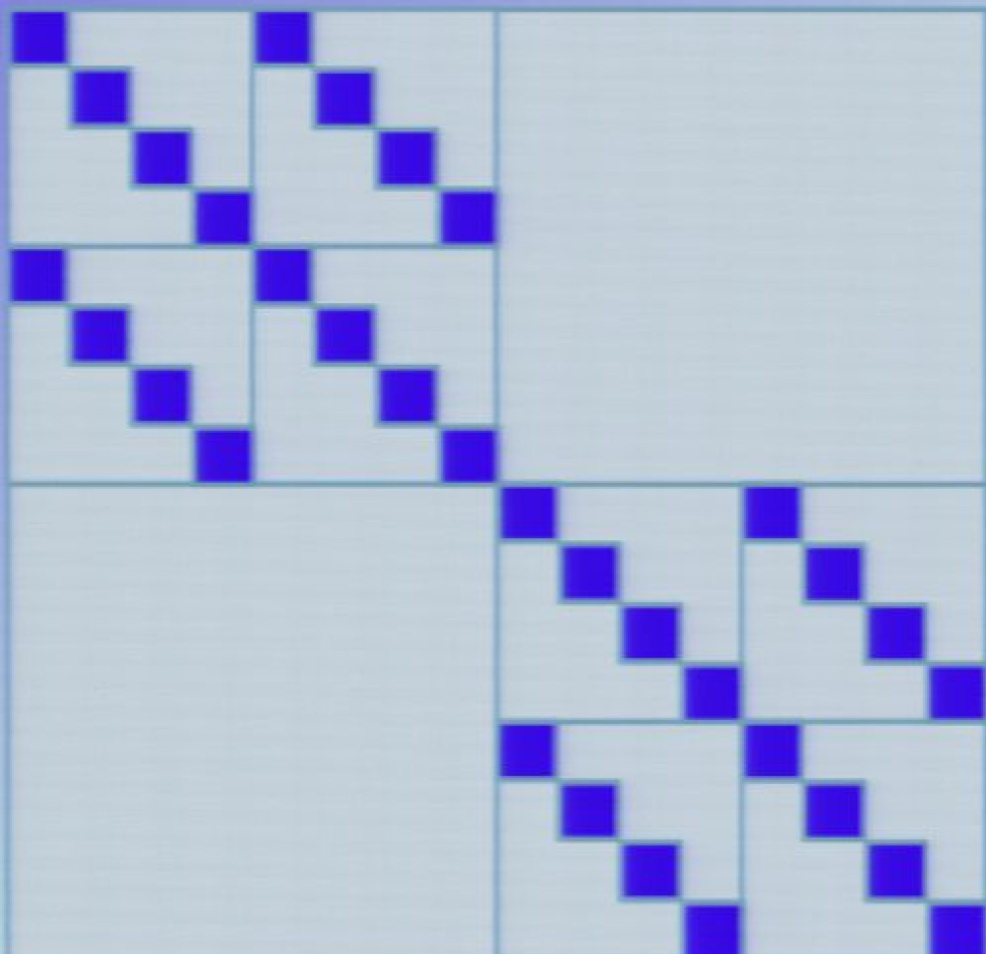


Implementing the Coin Operator

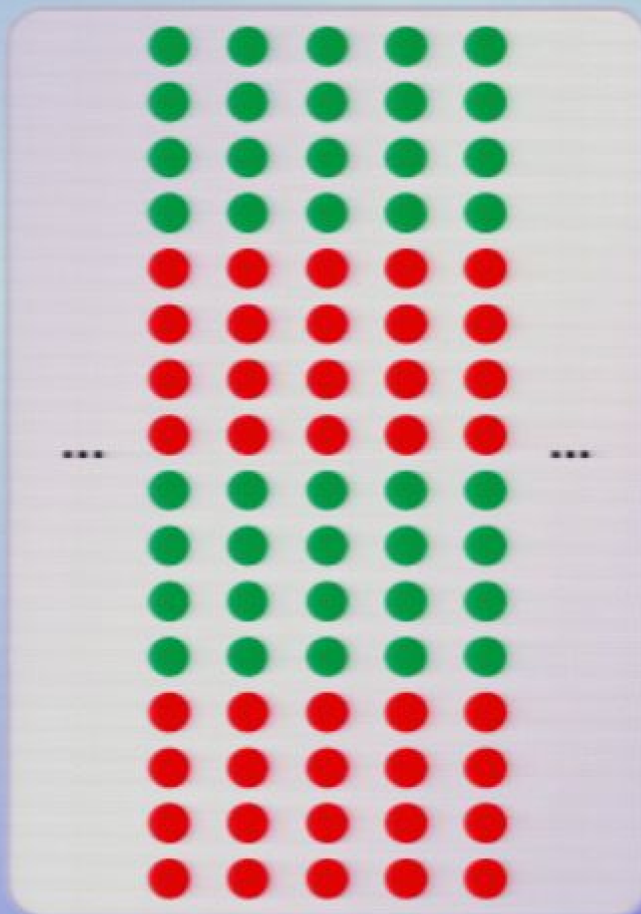


Implementing the Coin Operator

$\hat{U}(d > 2)$



$\mathbf{A}_j^V$



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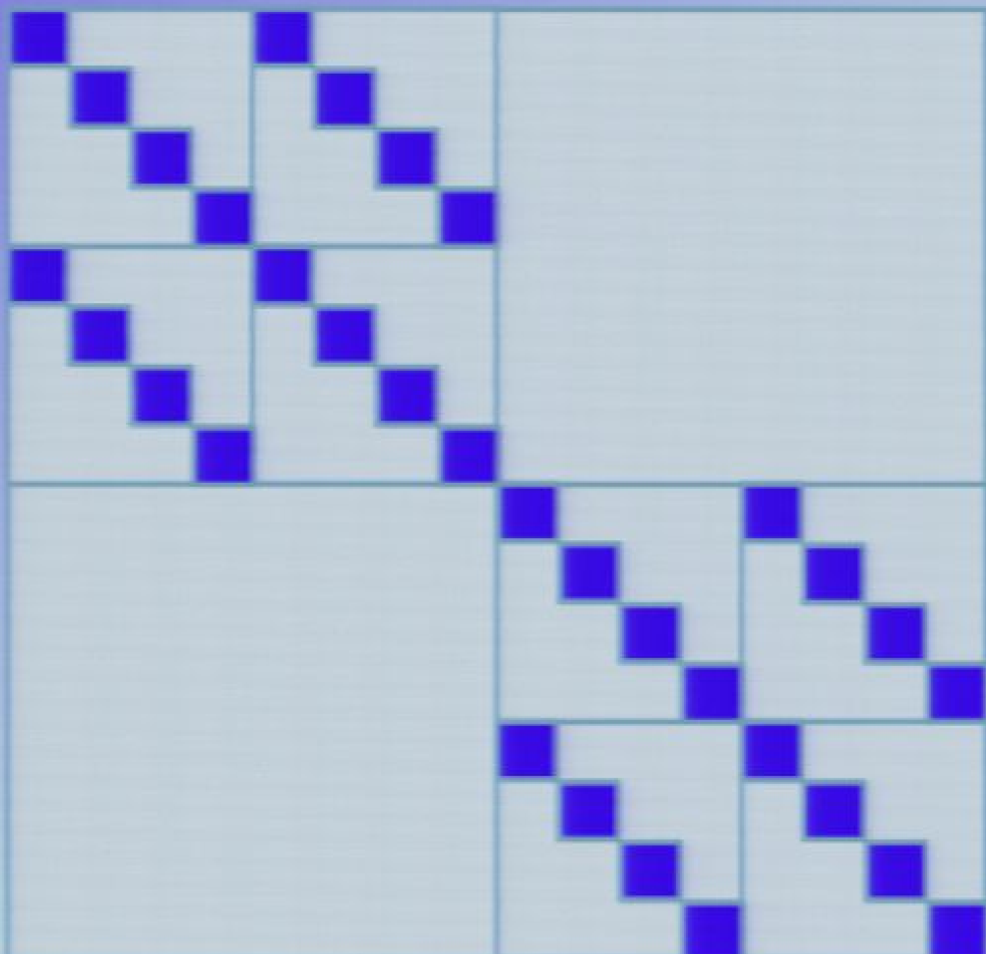

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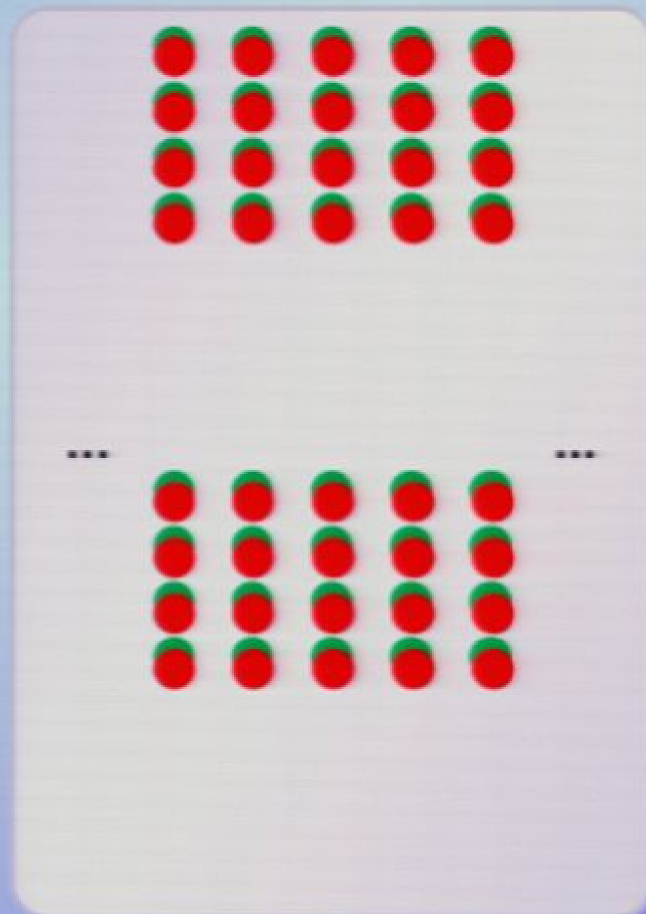

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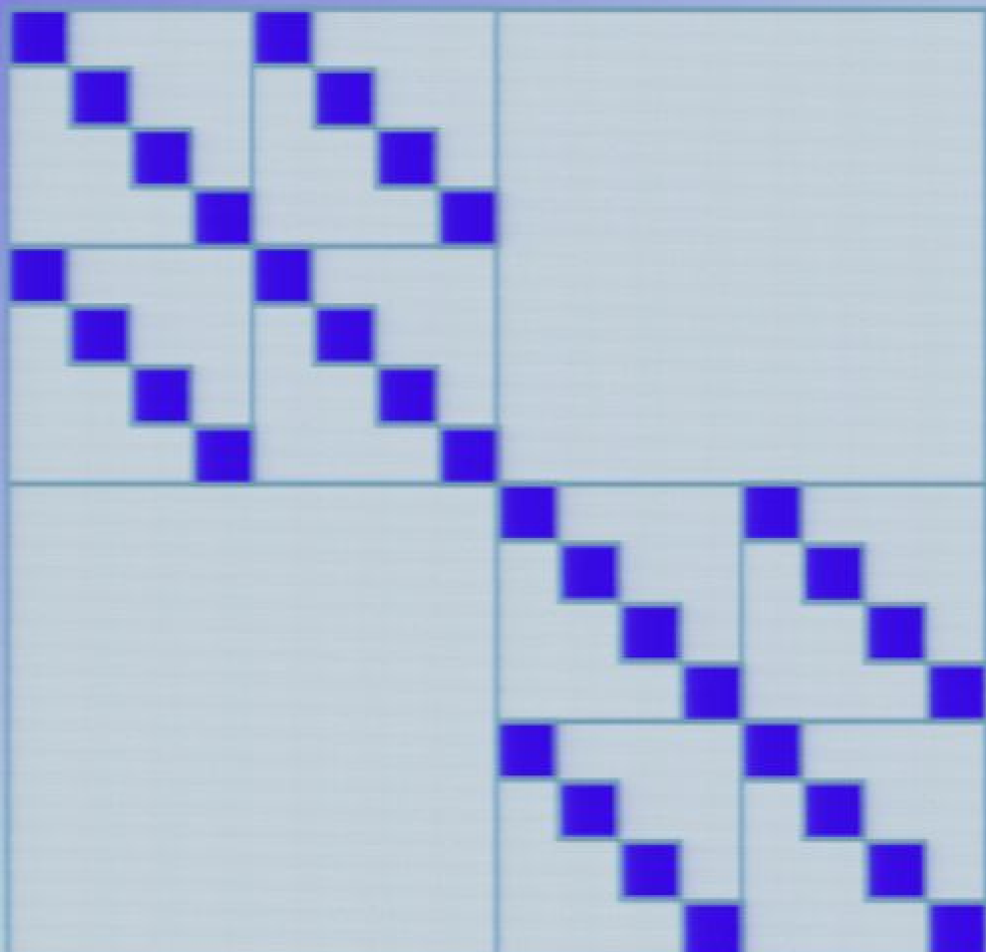

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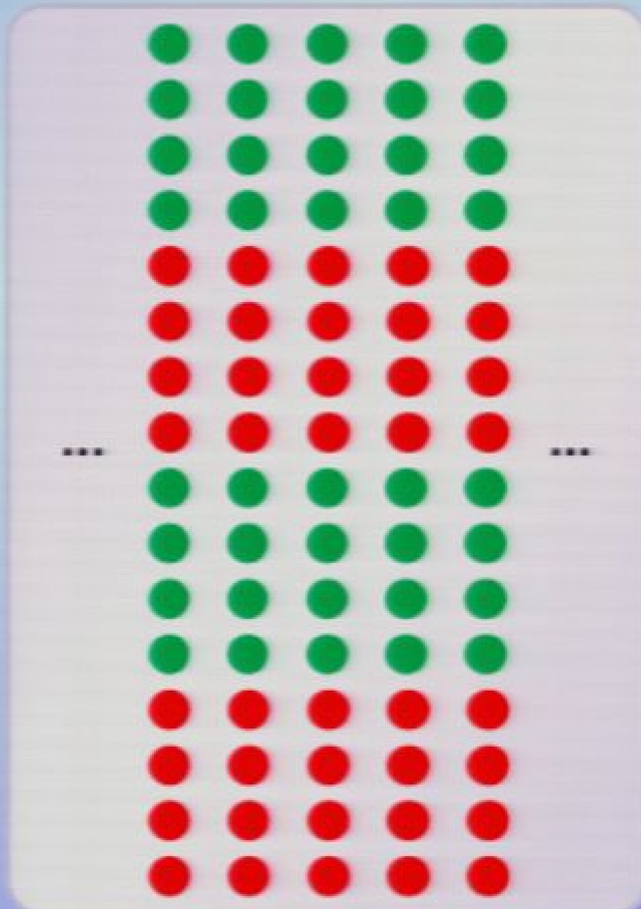
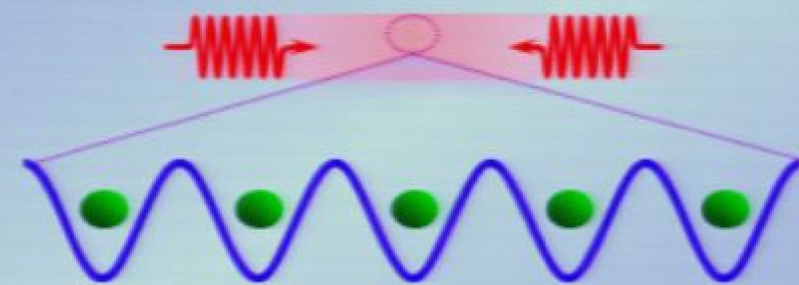
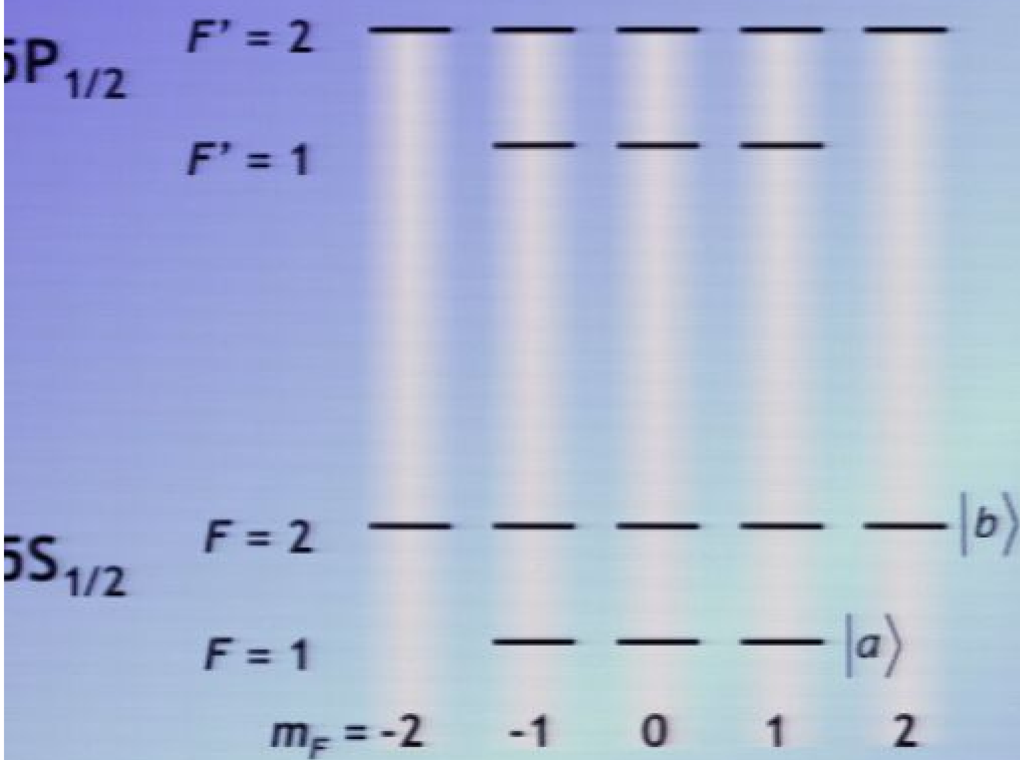


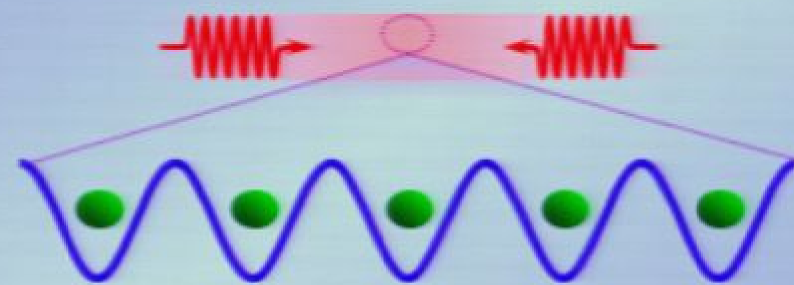
Atomic structure



Atomic structure

^{7}Rb





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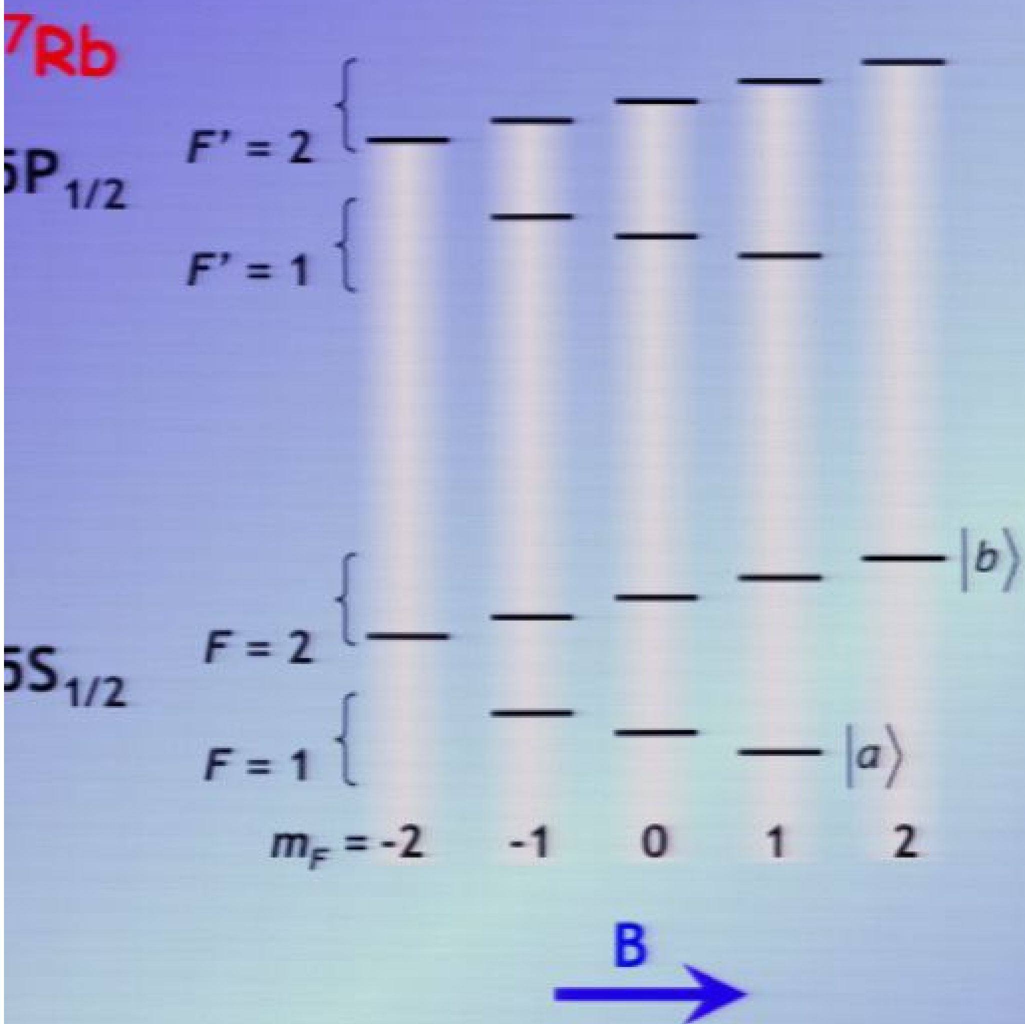
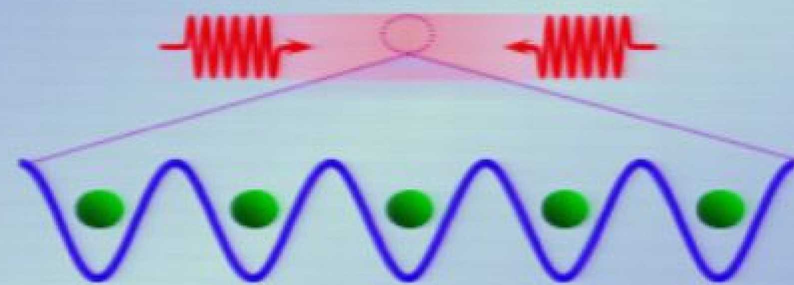
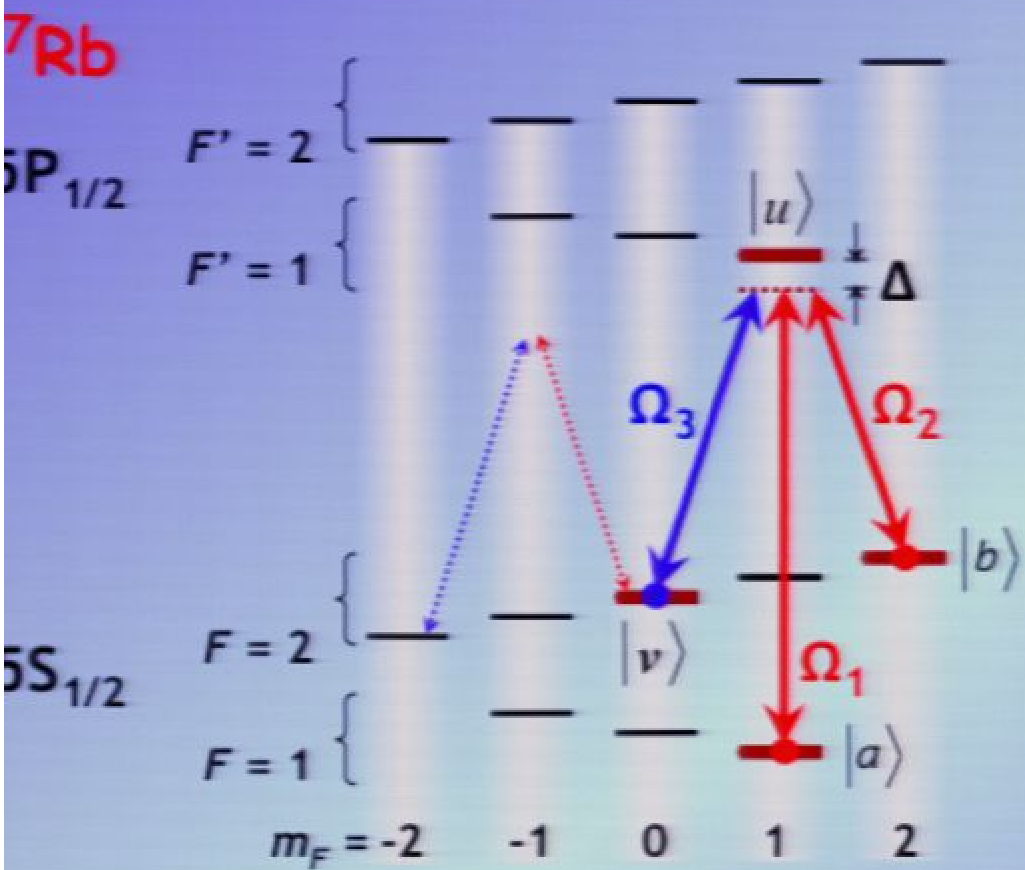
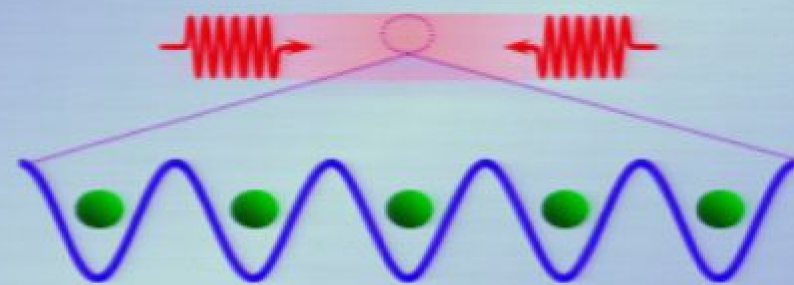




Unitary rotation





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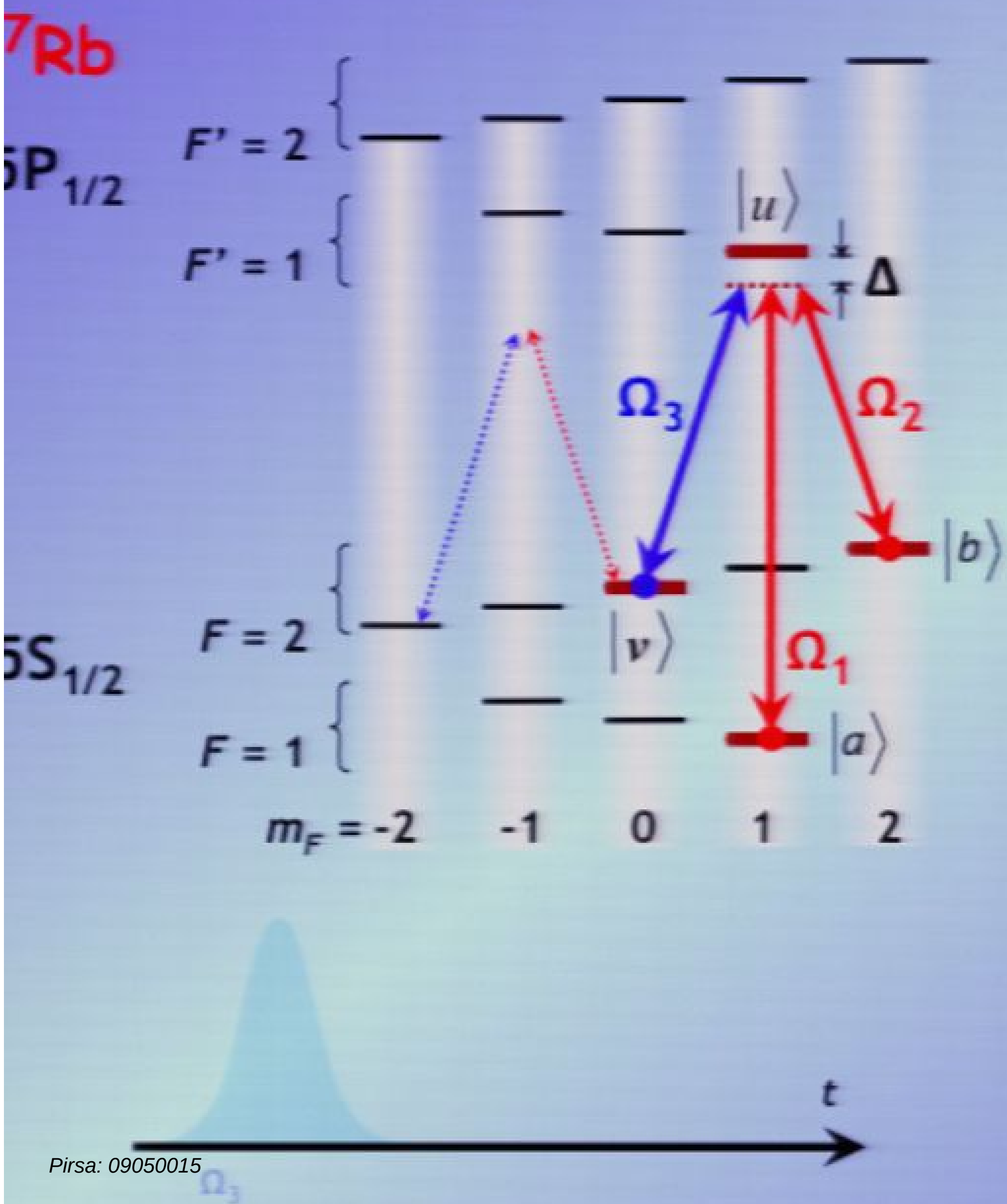

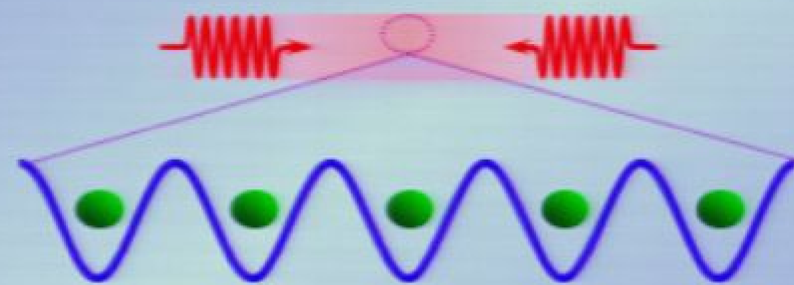

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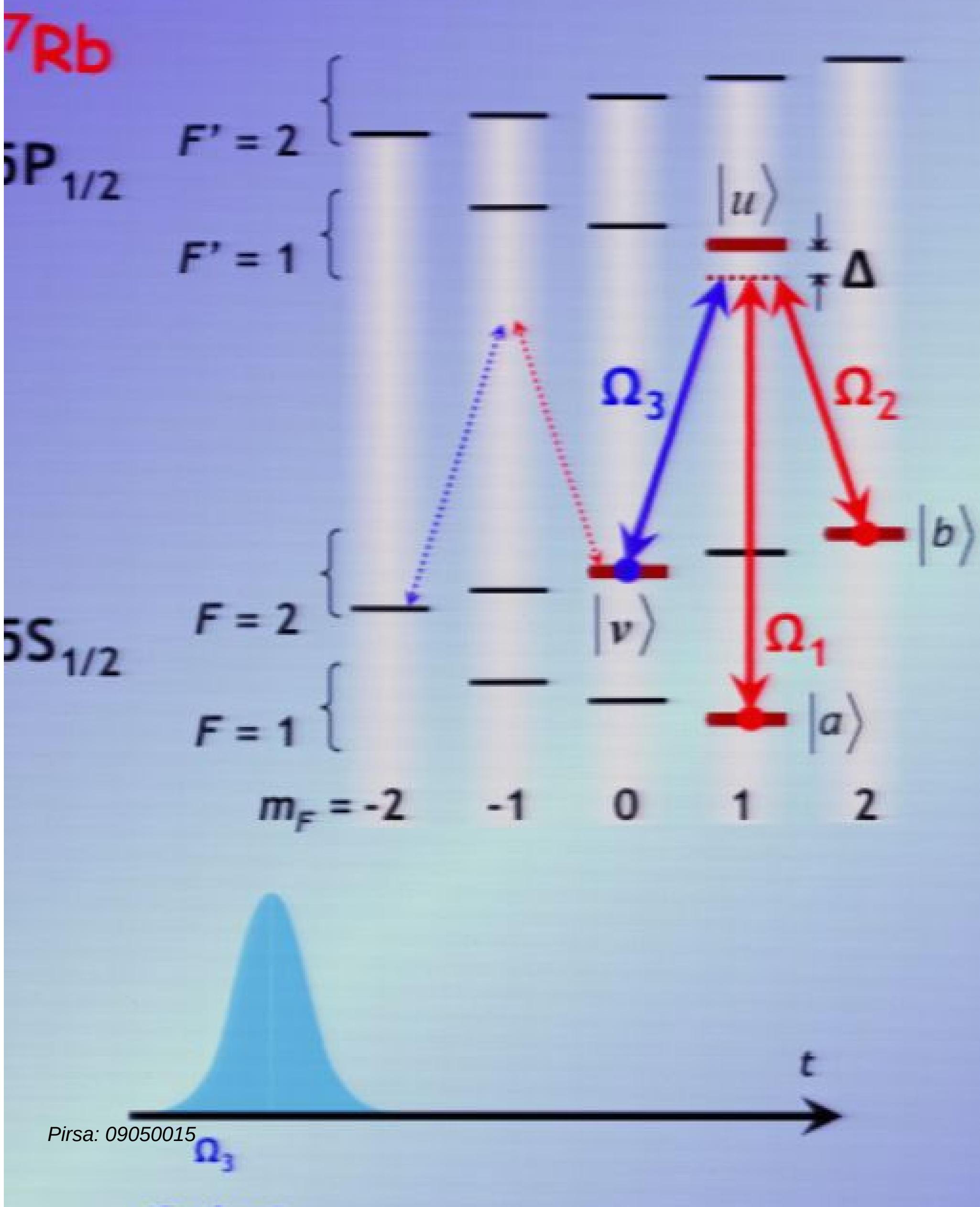

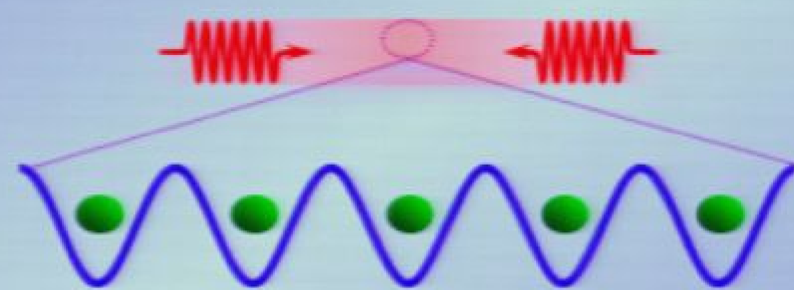

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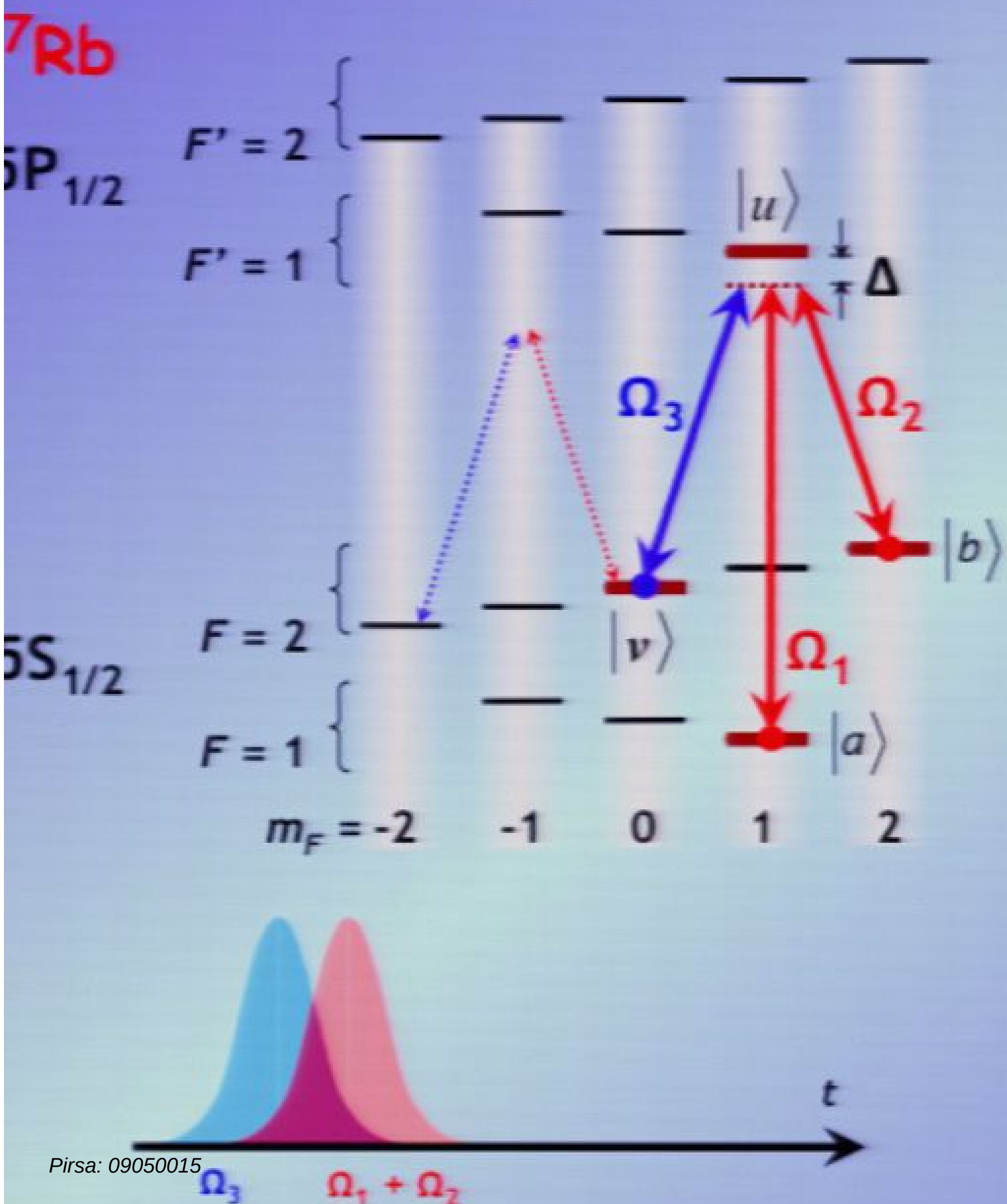

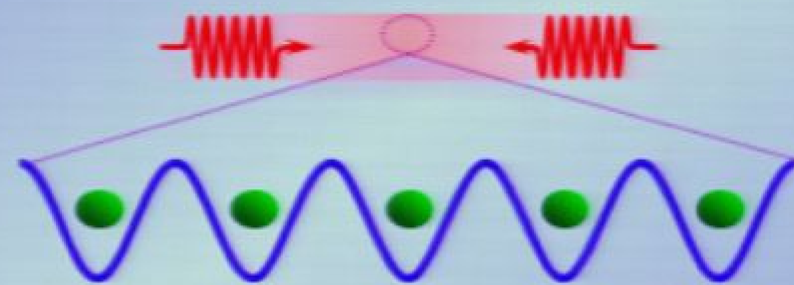

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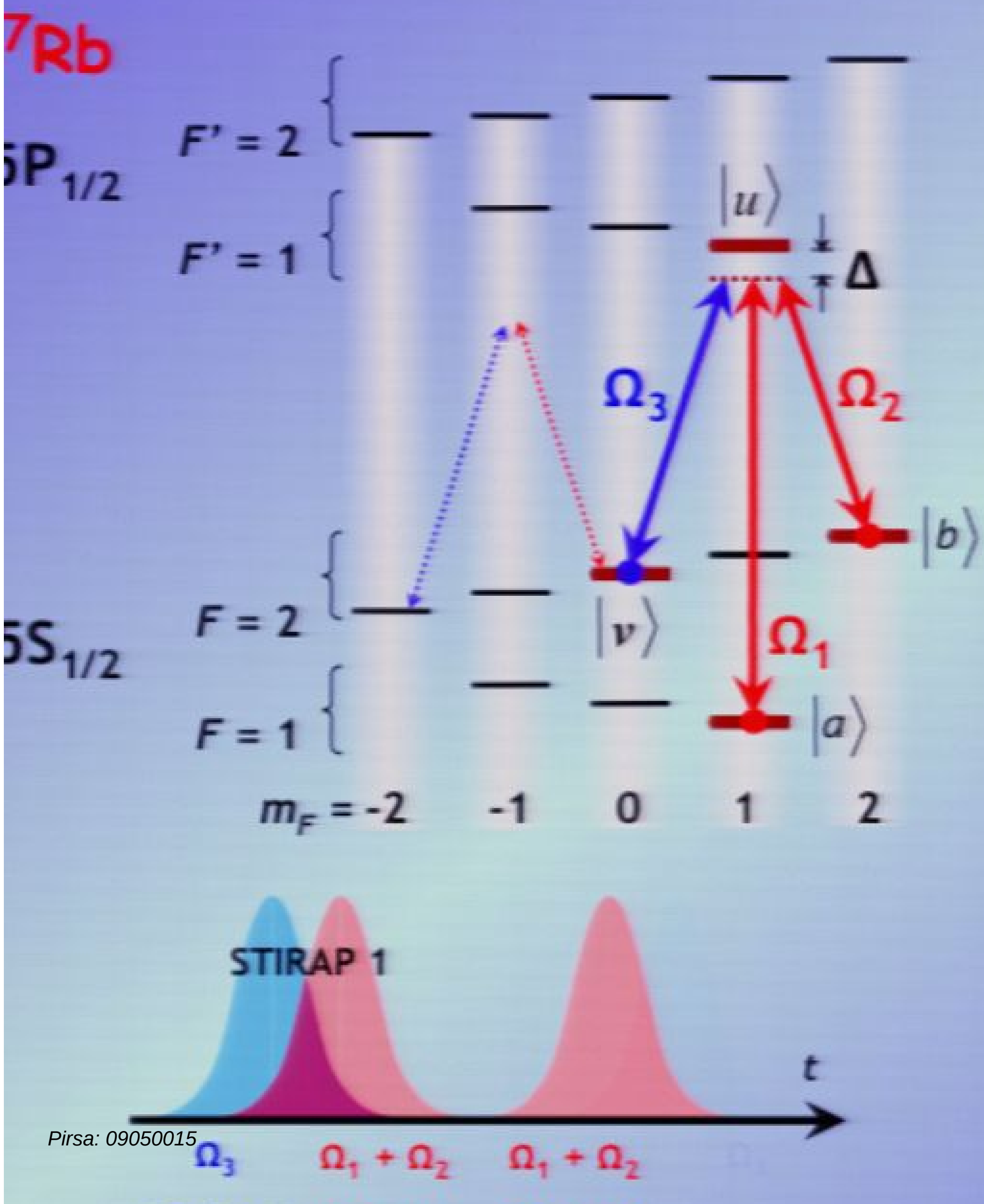

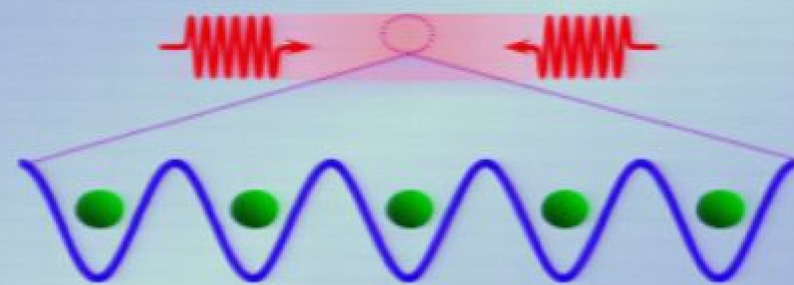

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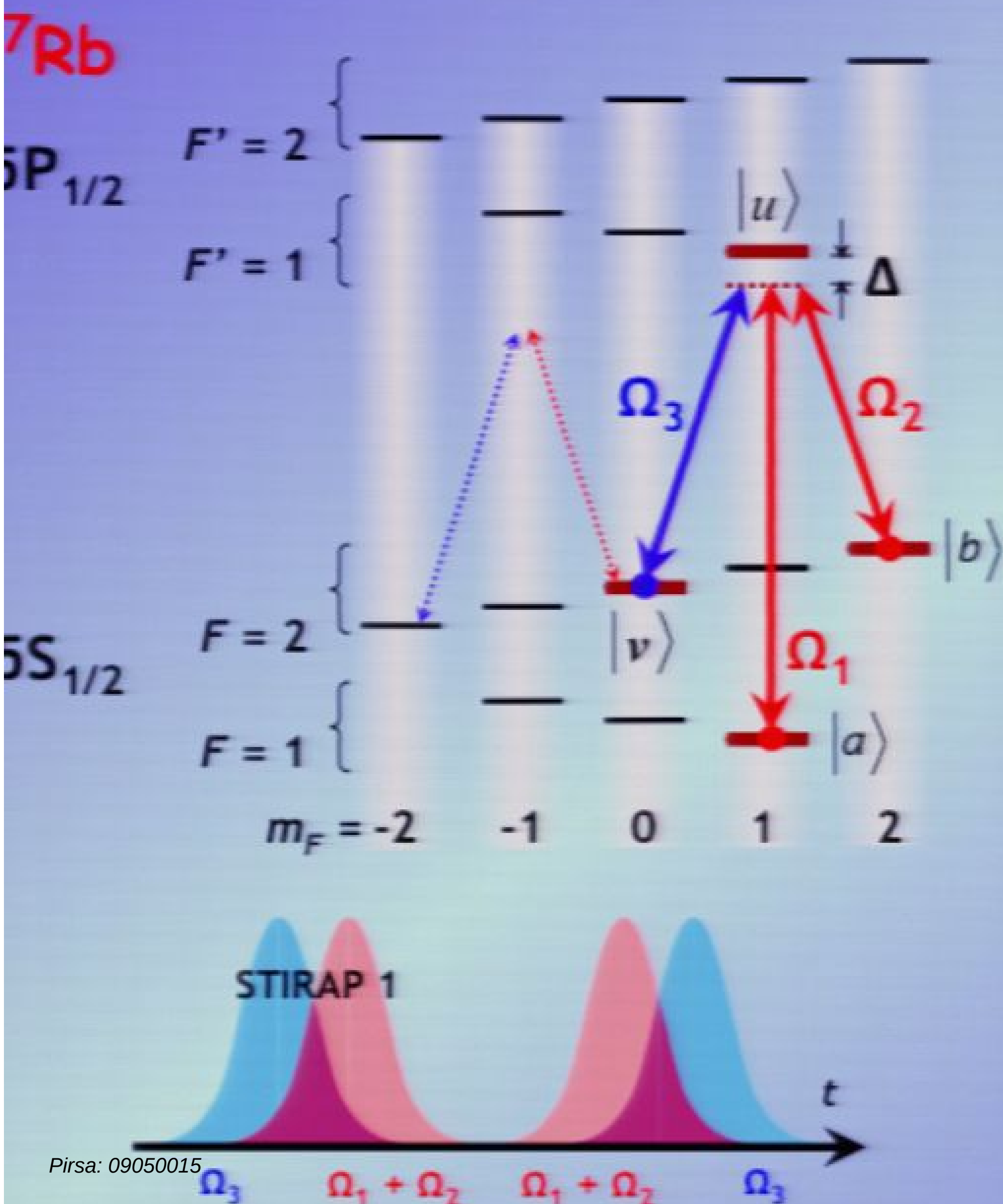

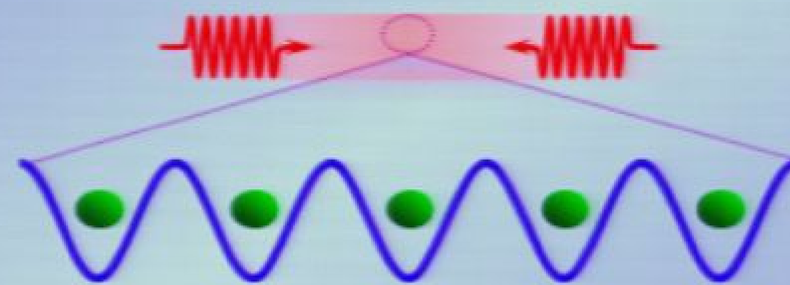

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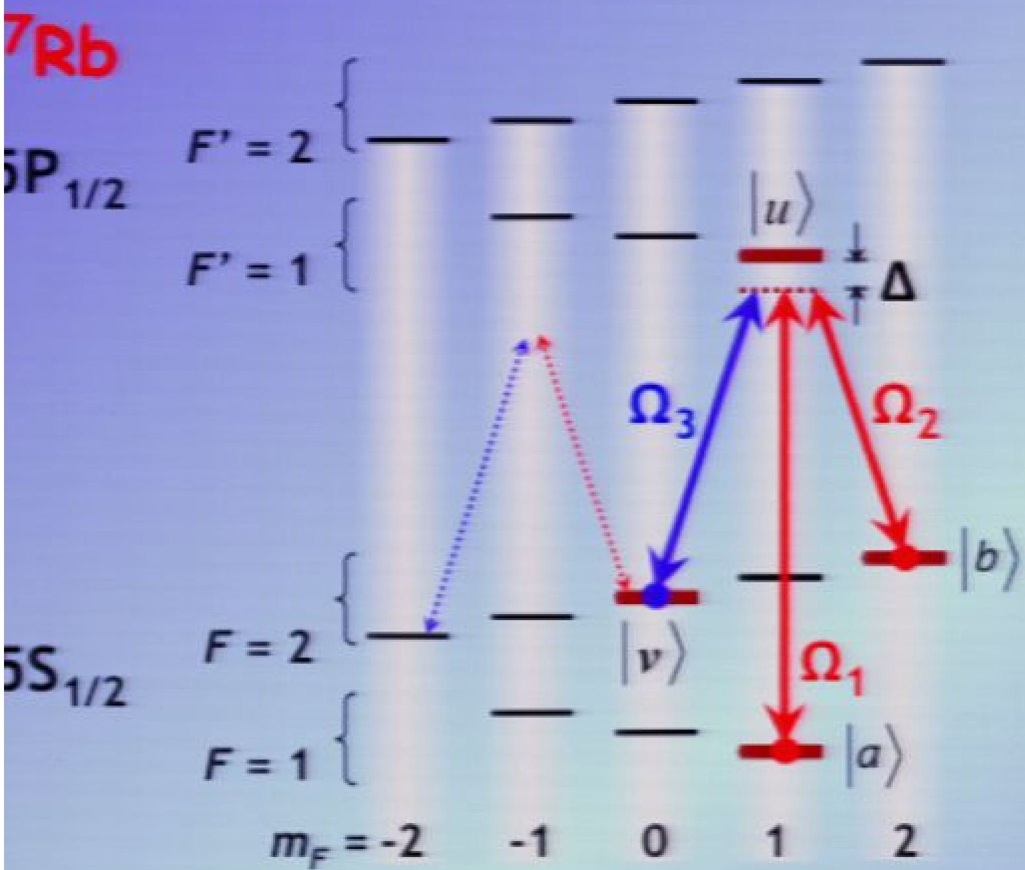
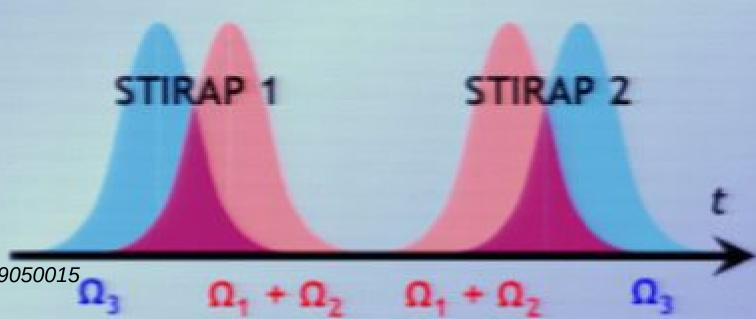
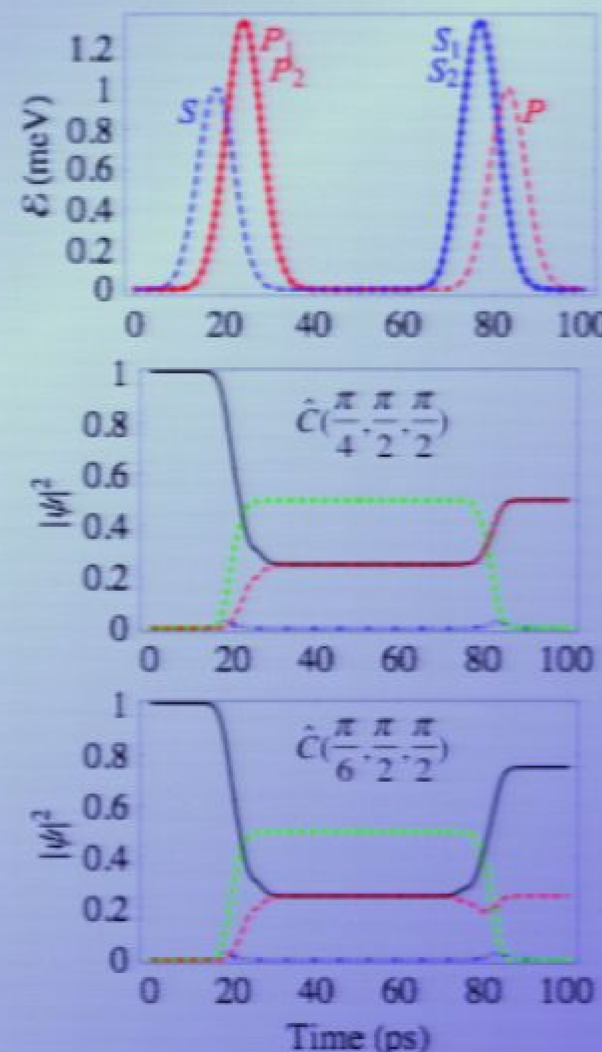




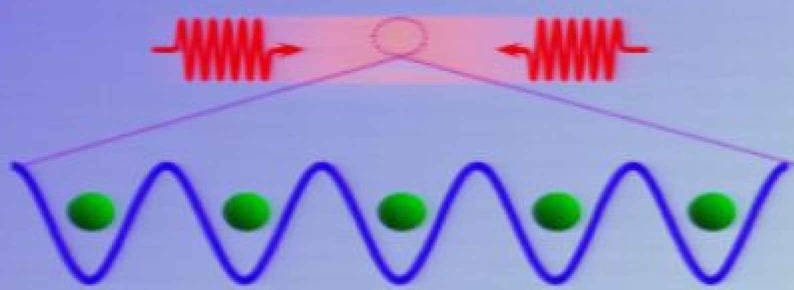


$$\hat{C}(\theta,\phi_1,\phi_2) = \begin{pmatrix} \cos(\theta) & \sin(\theta)\, e^{i\phi_1} \\ \sin(\theta)\, e^{i\phi_2} & -\cos(\theta)\, e^{i(\phi_1+\phi_2)} \end{pmatrix}$$

State-dependent transport

PRL 82, 1975 (1999)
Contemporary Phys 45, 367 (2004)
Phys. Rev. A 65, 032318 (2002)
PRA 77, 041601 (2008)

^{7}Rb



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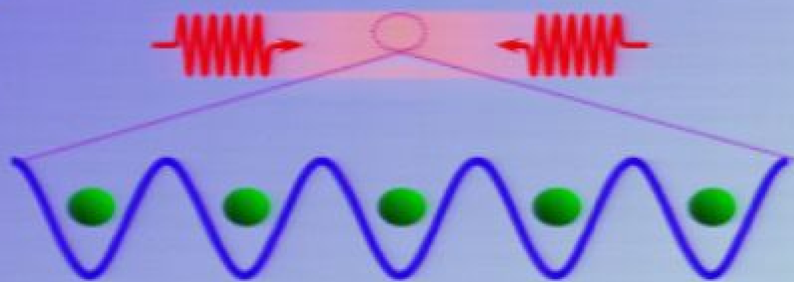

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Sum[ClebschGordan[{i, mi}, {s, mf − mi}, {F, mf}]
[i, mi, s, mf − mi], {mi, −i, i}]

$$\left(-\frac{1}{2}\right)\left[\frac{3}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right] + 0\left[\frac{3}{2}, -\frac{3}{2}, \frac{1}{2}, \frac{5}{2}\right] + 0\left[\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}\right] + \frac{\sqrt{3}}{2}\left[\frac{3}{2}, \frac{3}{2}, \frac{1}{2}, -\frac{1}{2}\right]$$

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$$0\left[\frac{3}{2}, -\frac{3}{2}, \frac{1}{2}, \frac{7}{2}\right] + 0\left[\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{5}{2}\right] + 0\left[\frac{3}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}\right] + 1\left[\frac{3}{2}, \frac{3}{2}, \frac{1}{2}, \frac{1}{2}\right]$$

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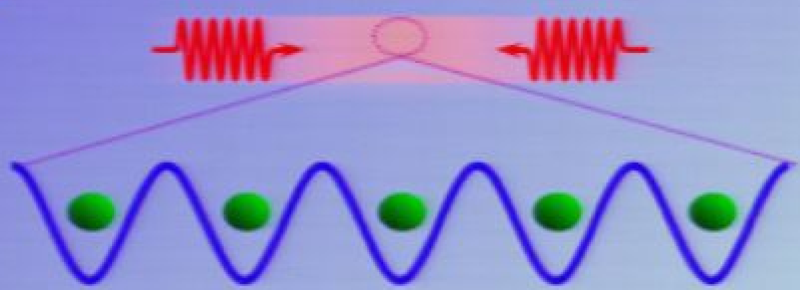

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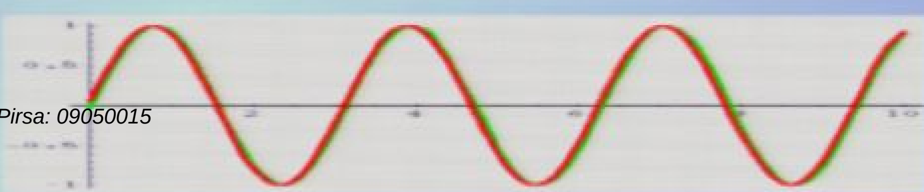

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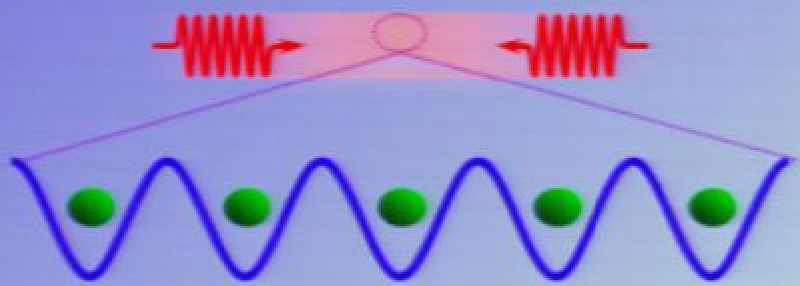

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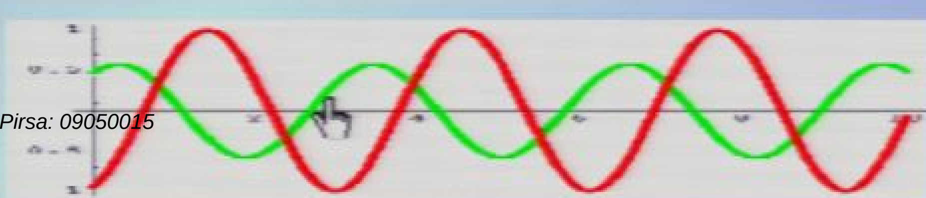

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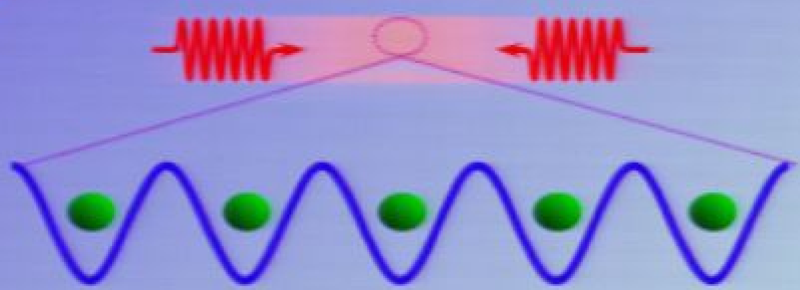

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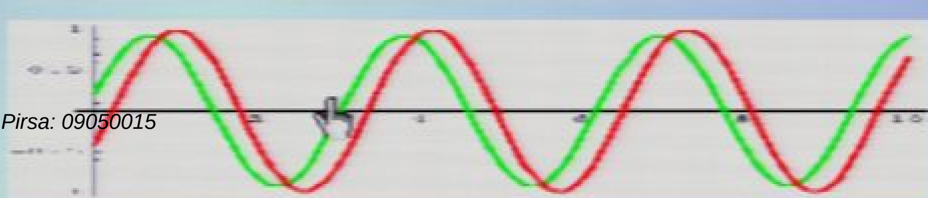

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Artist Impression – implement quantum walk

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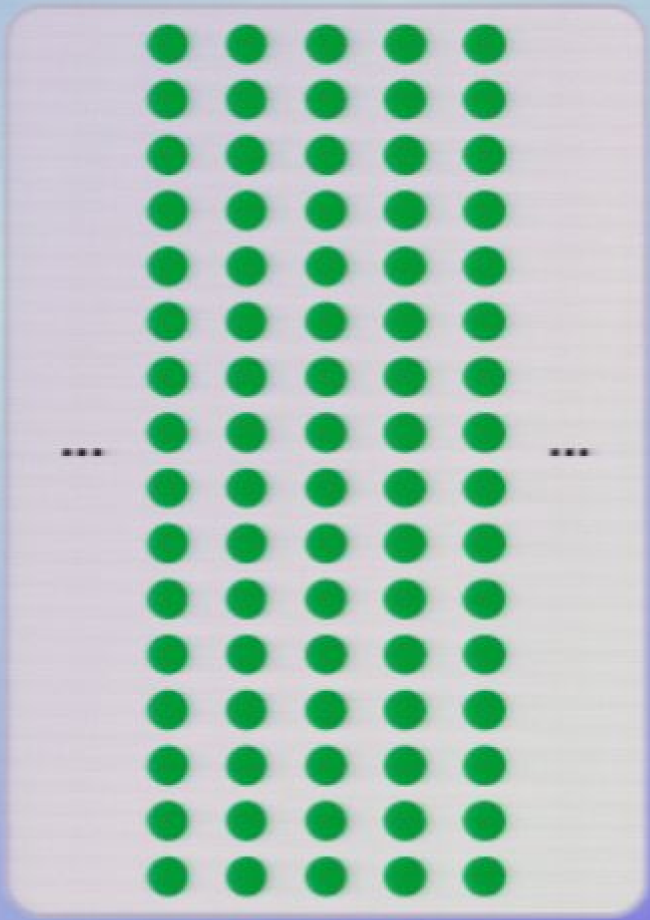

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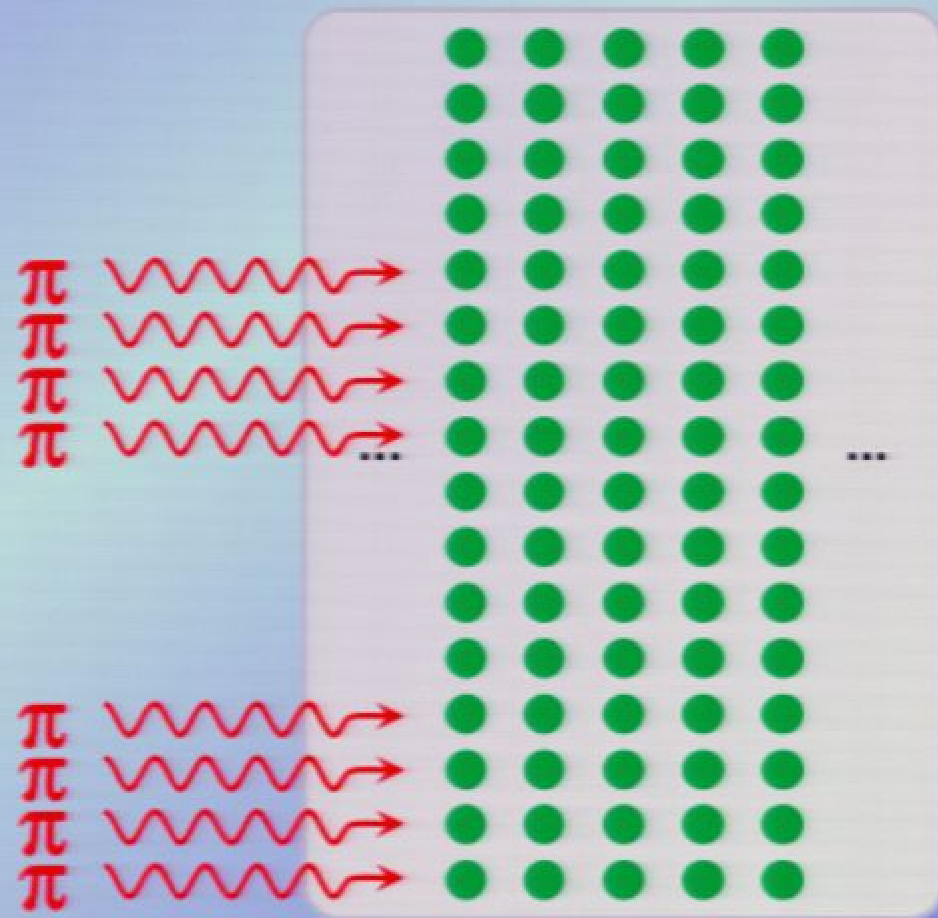

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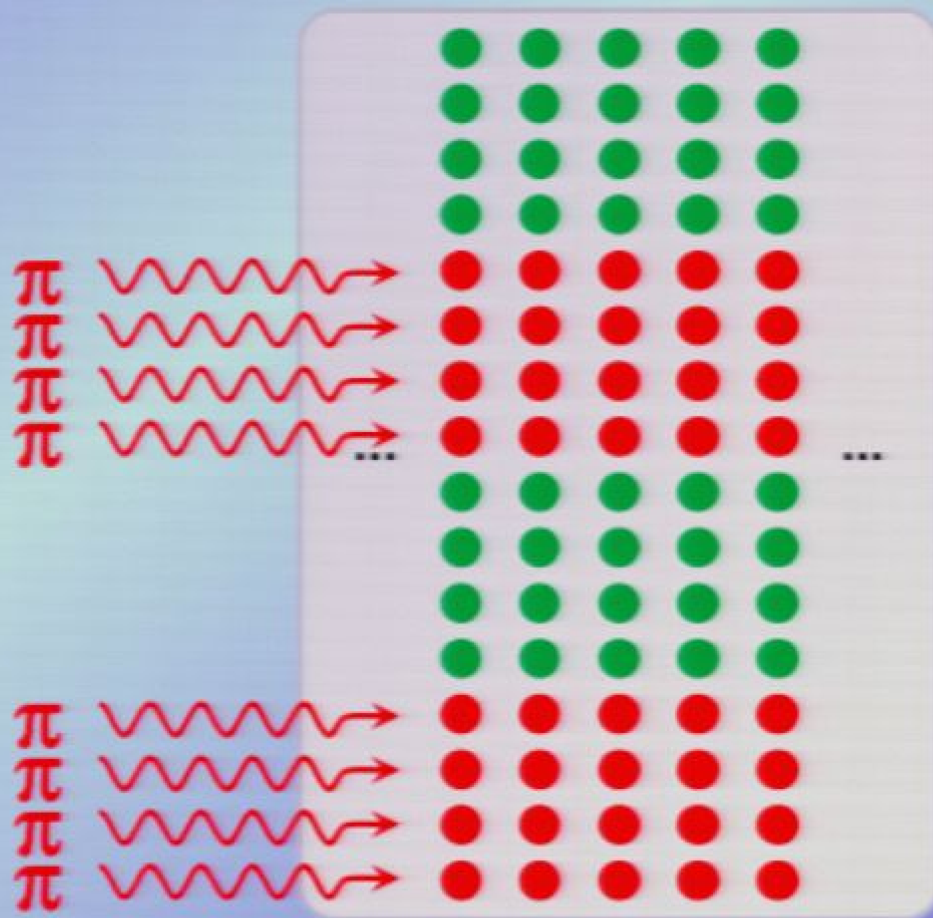

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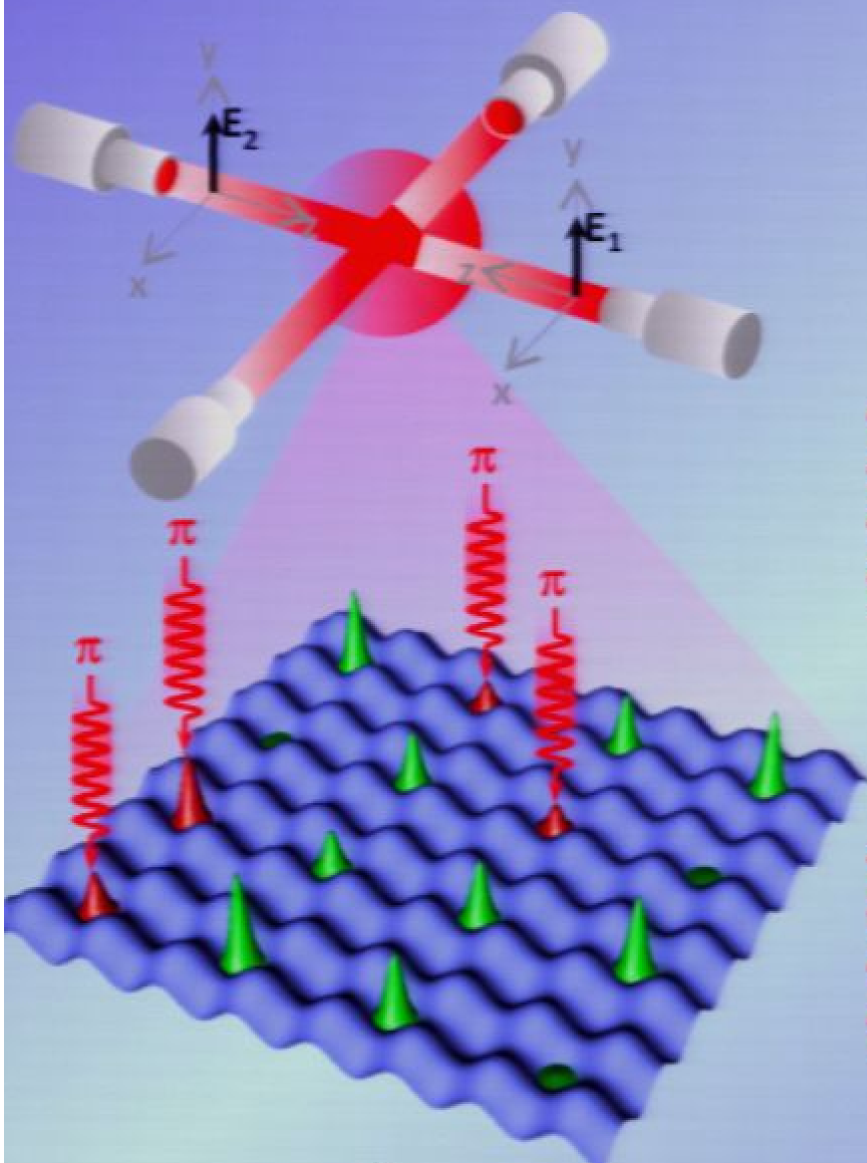

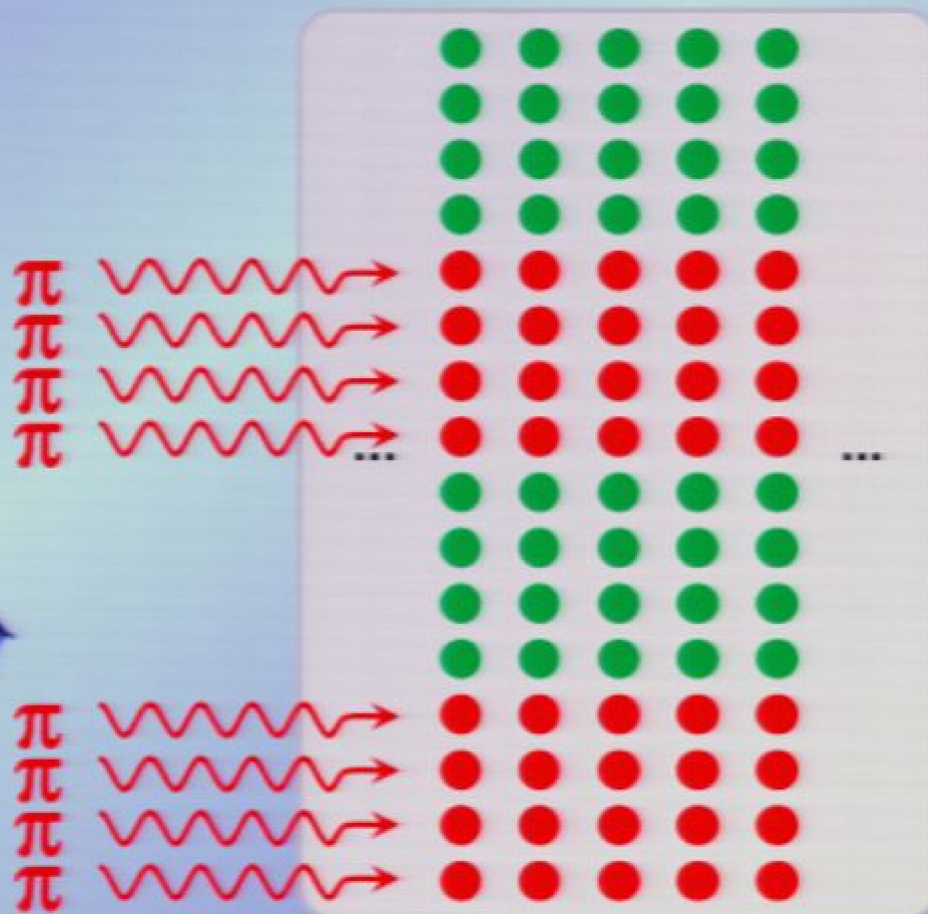

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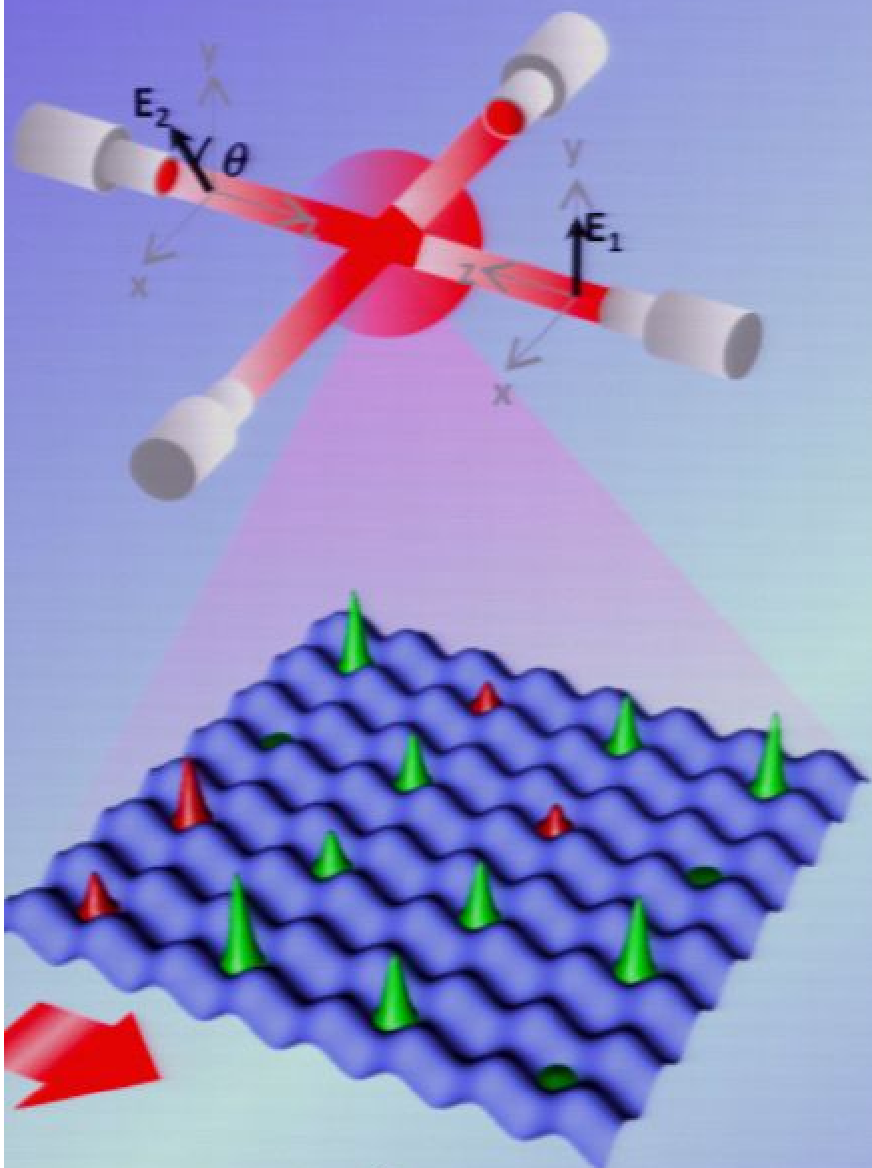

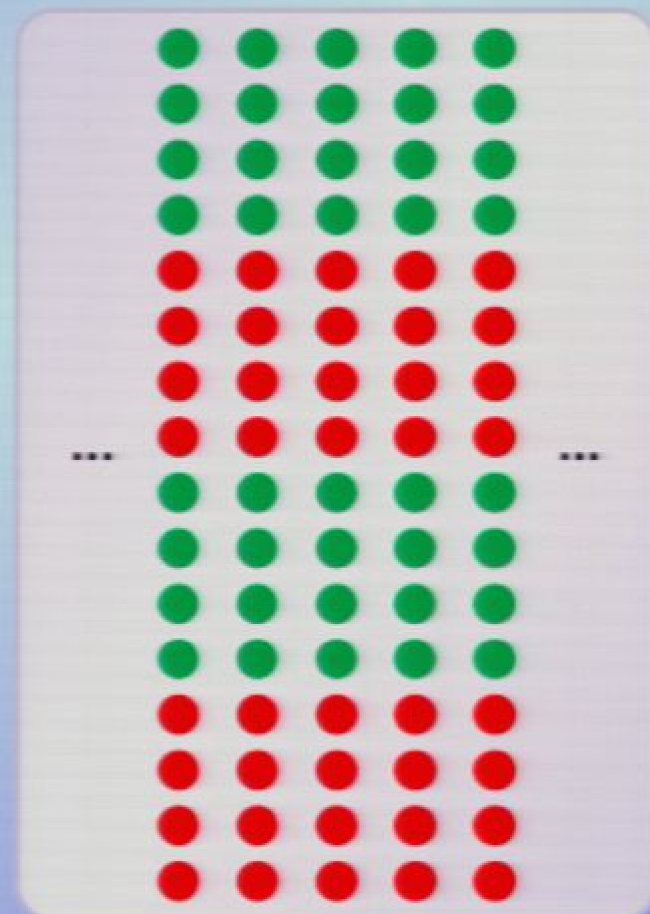

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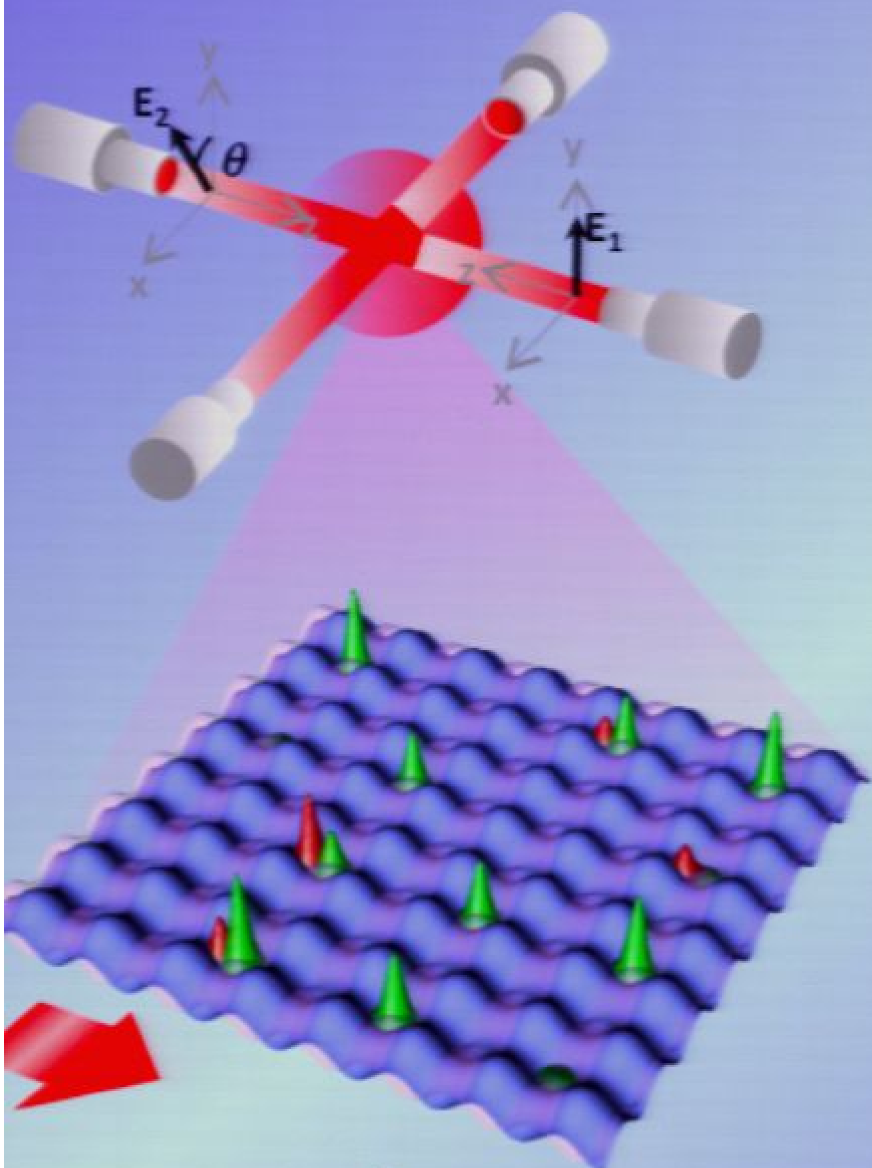

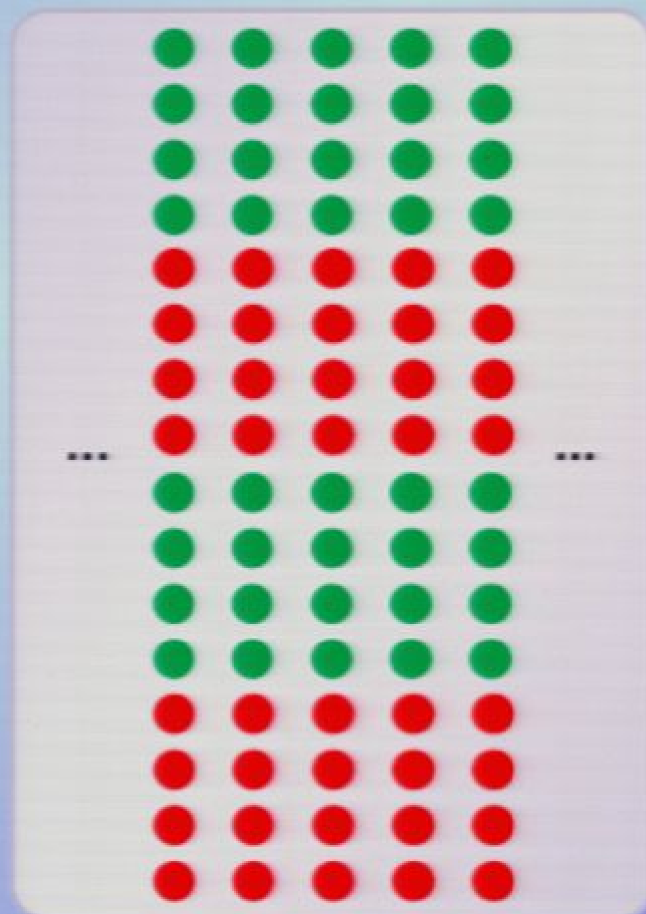

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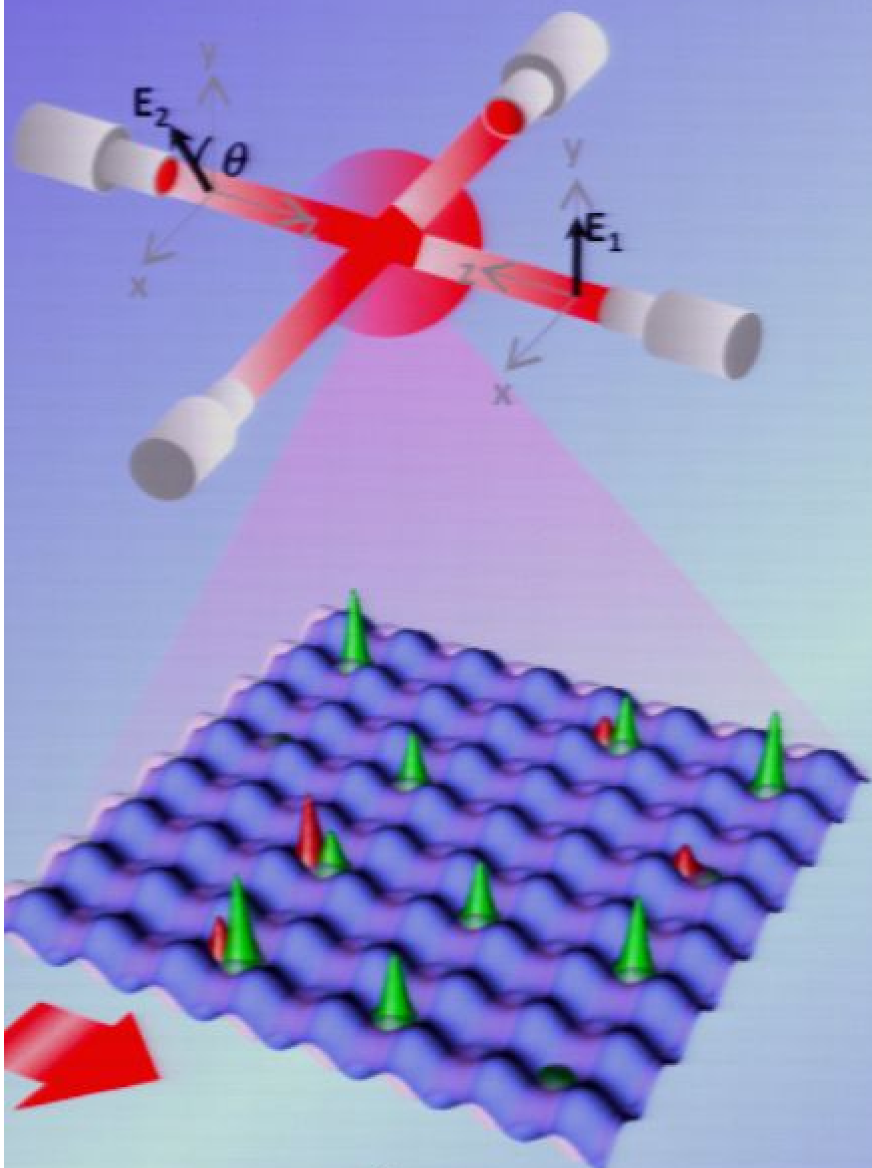

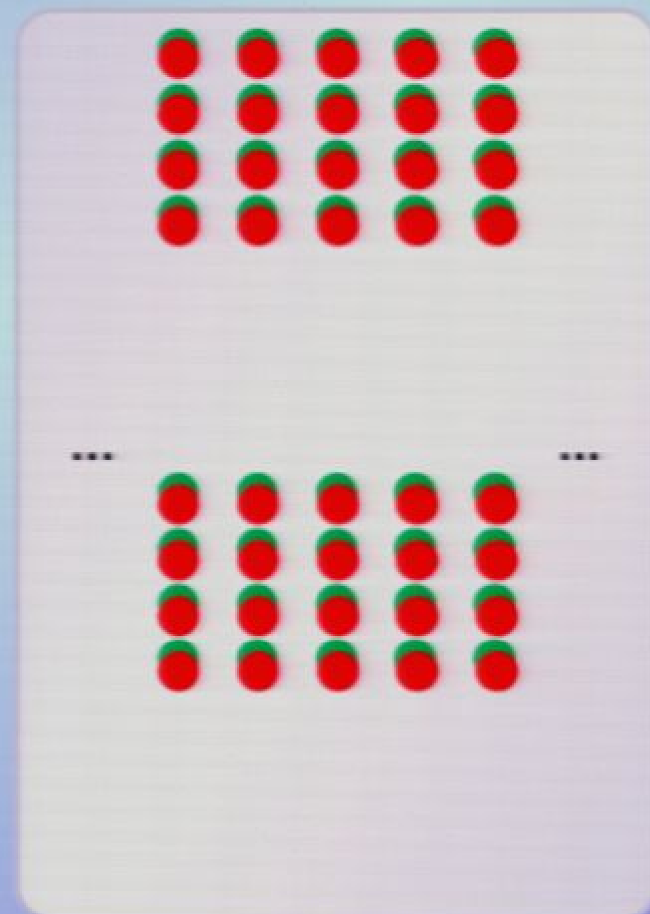

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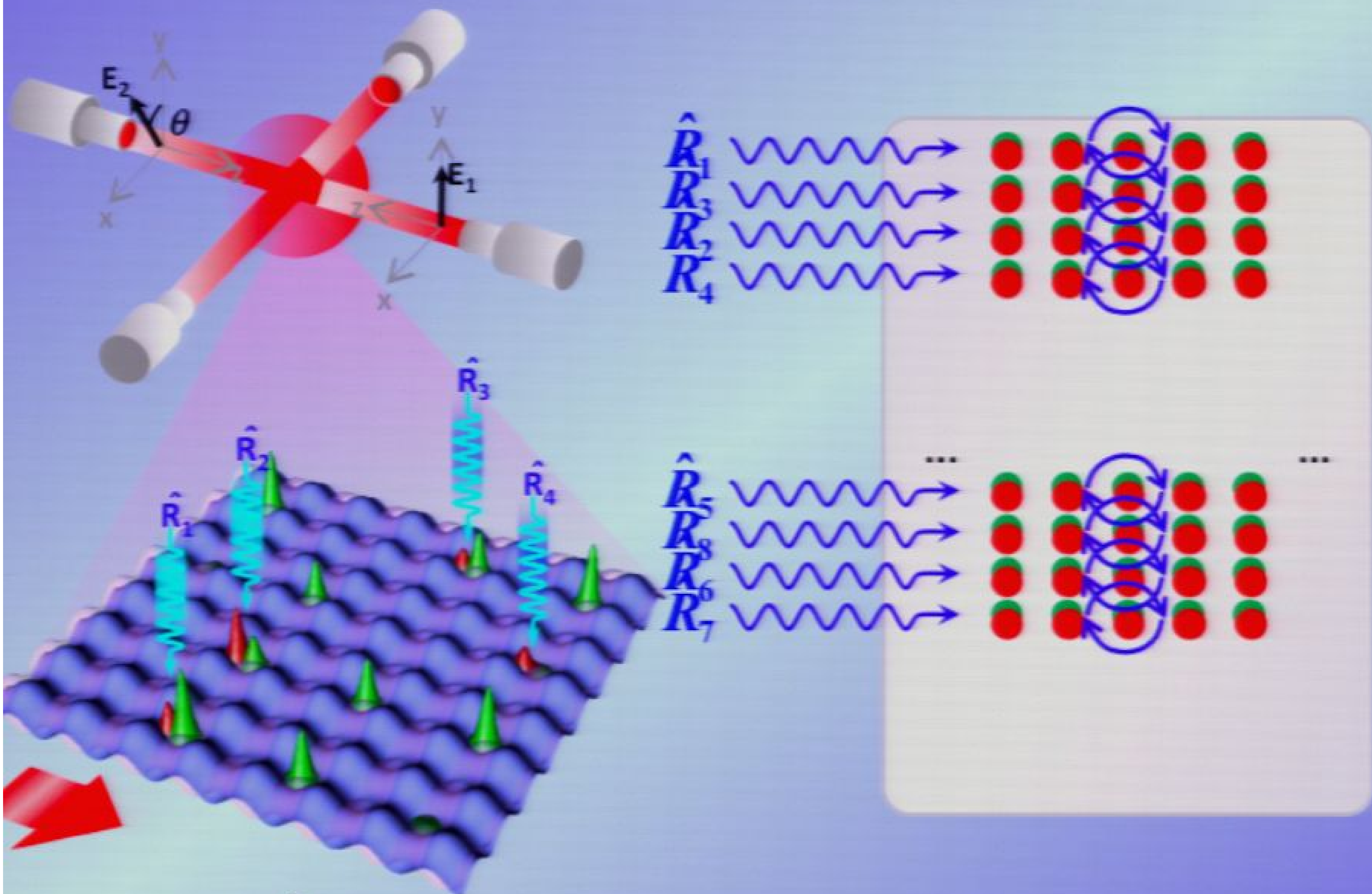

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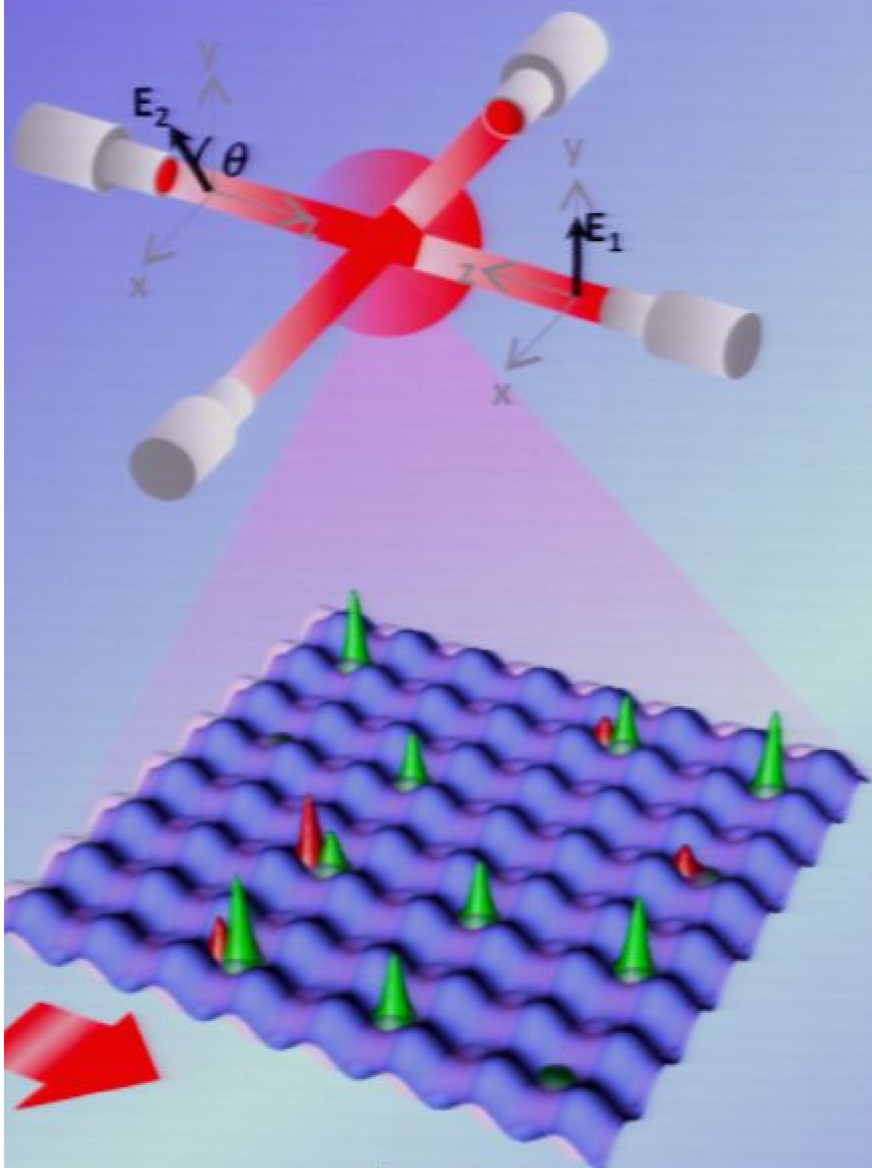

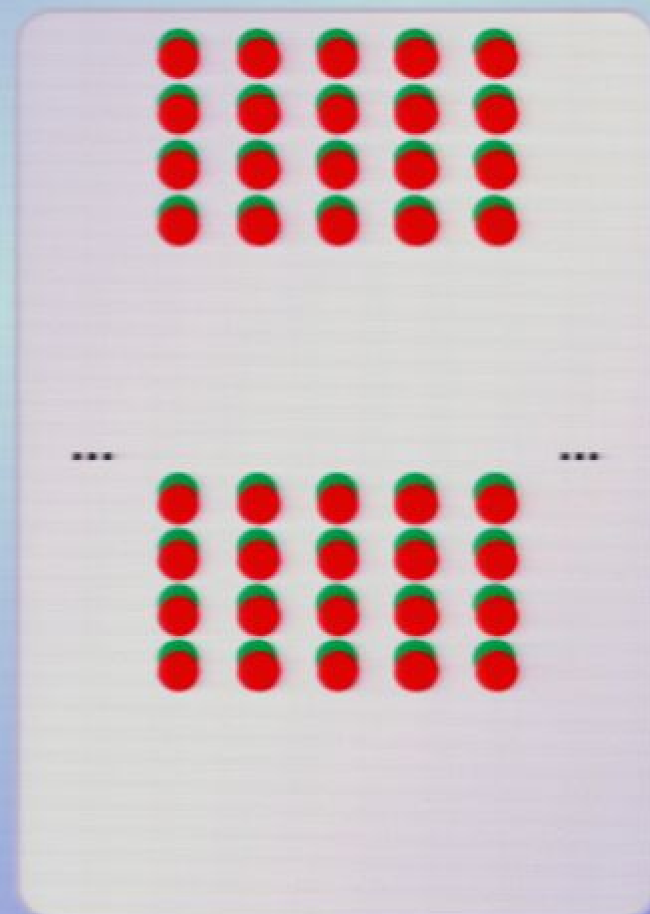

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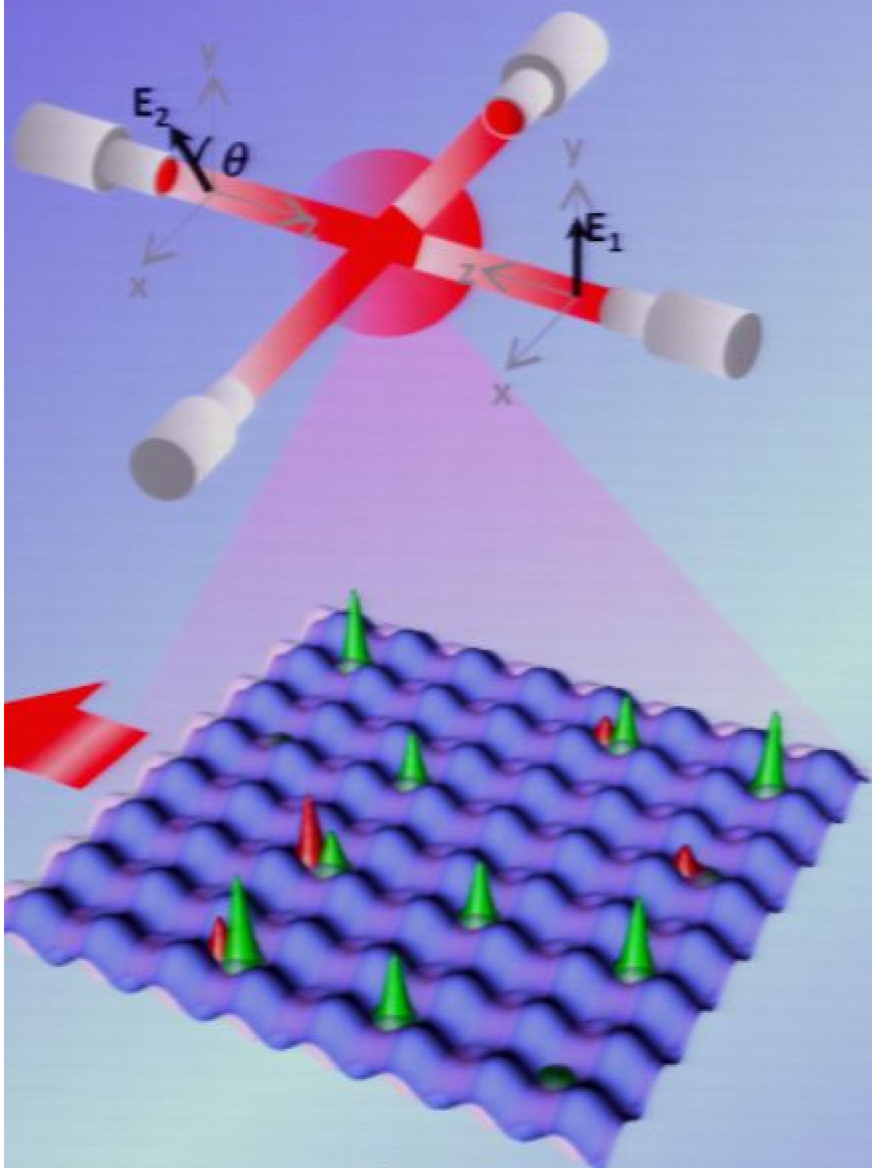

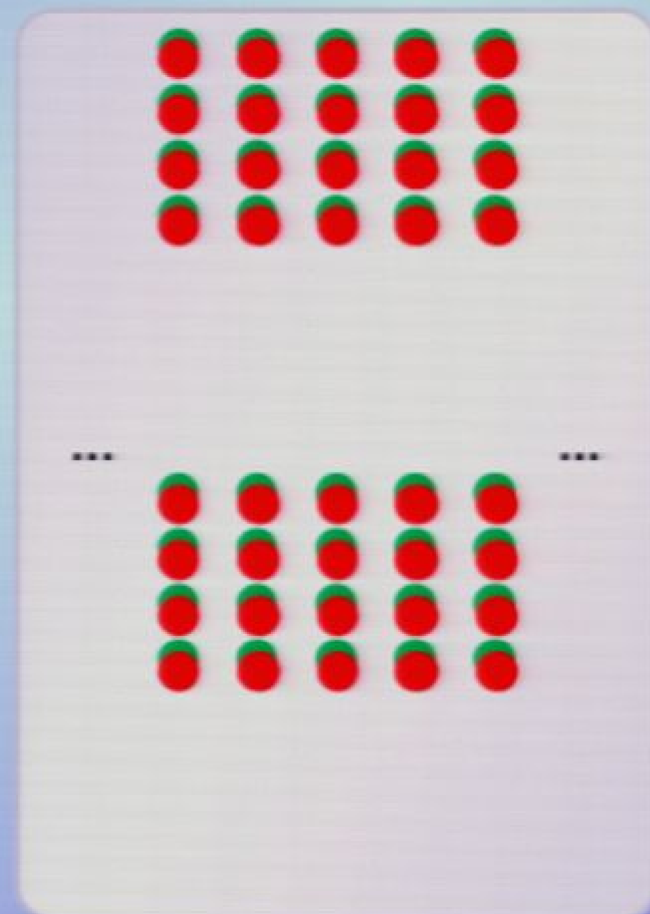

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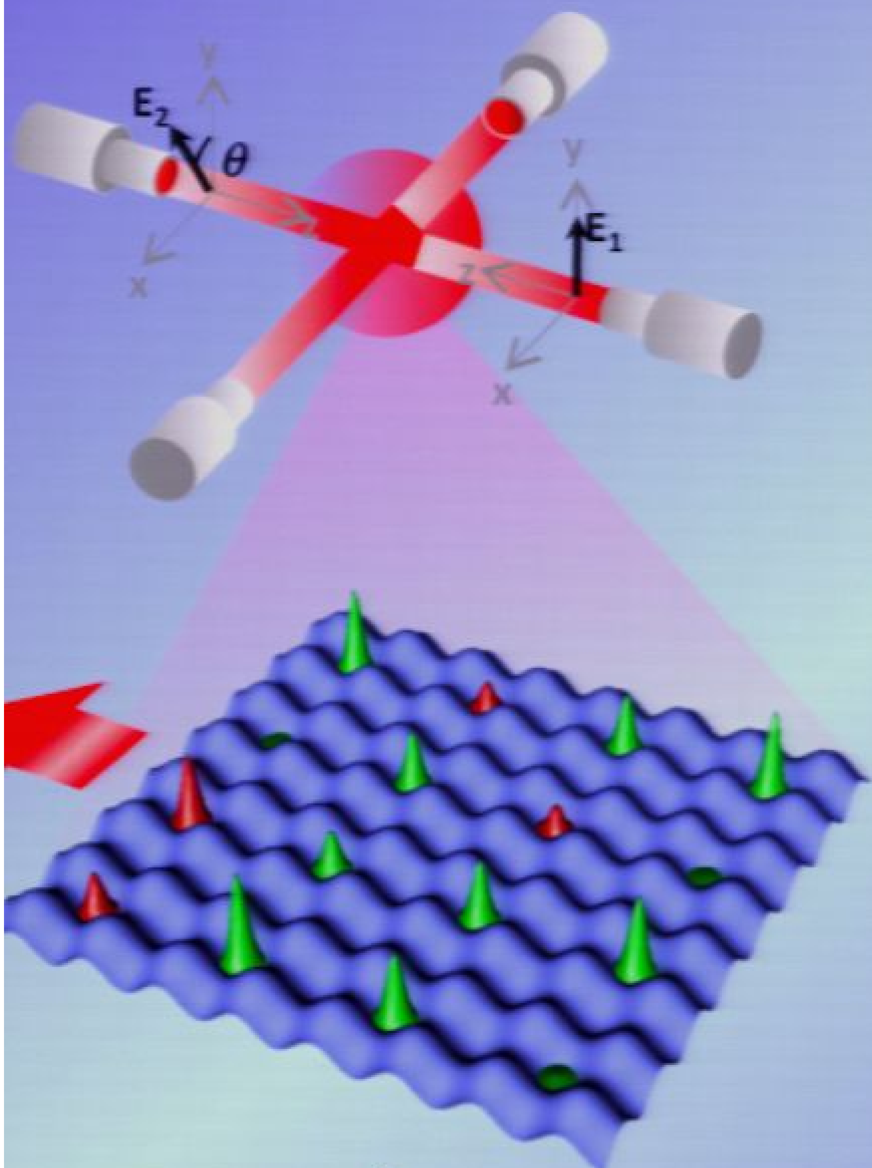

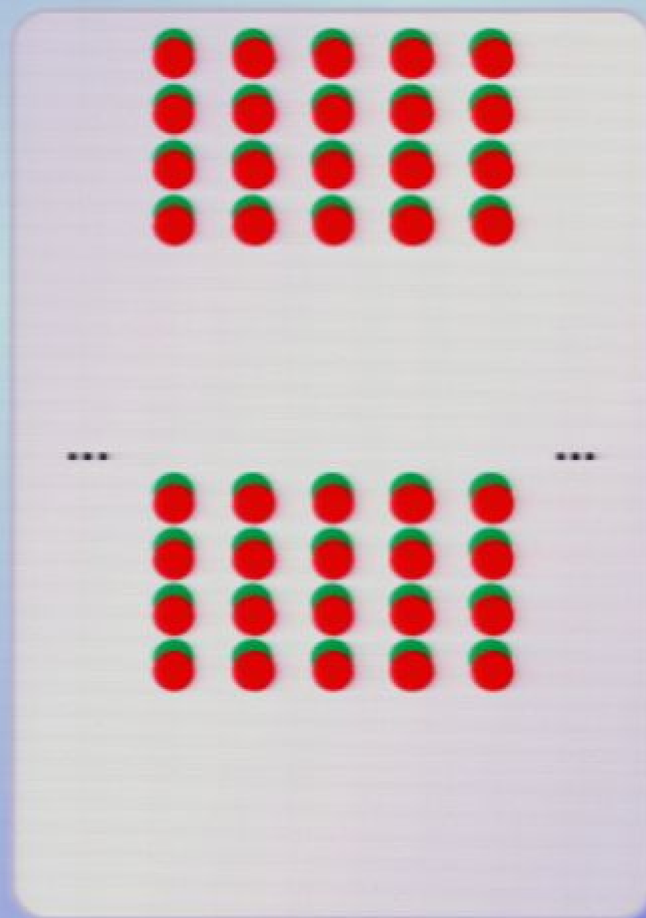

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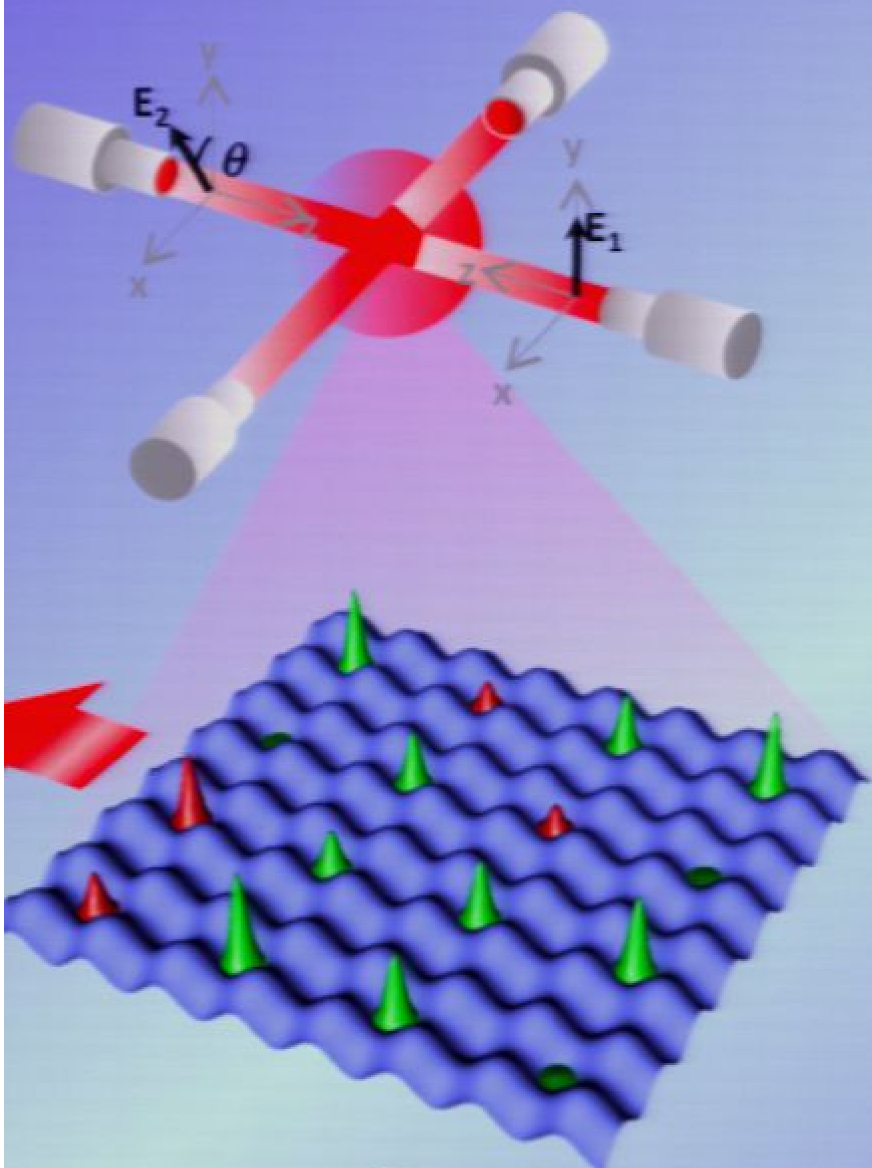

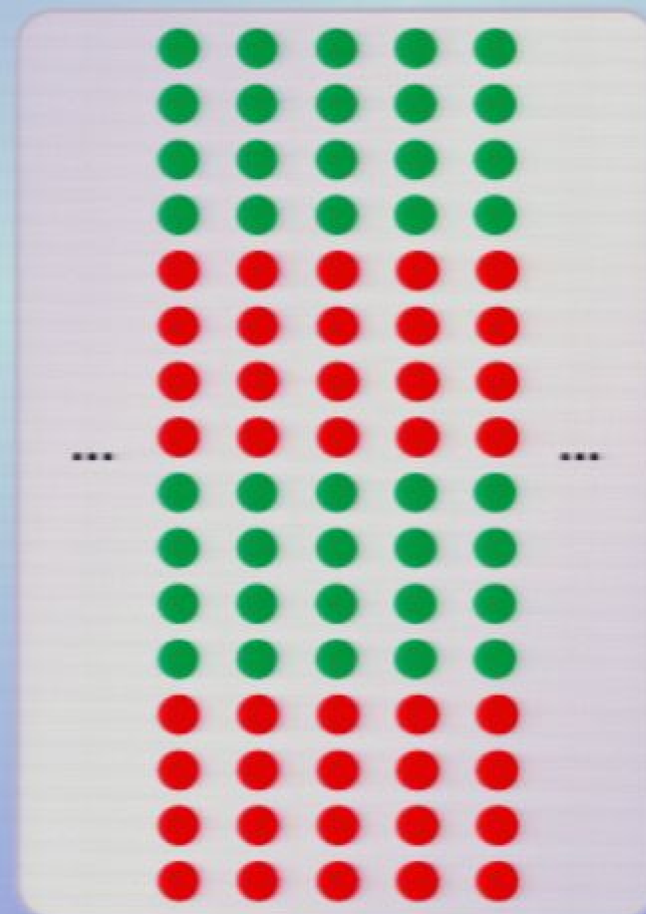

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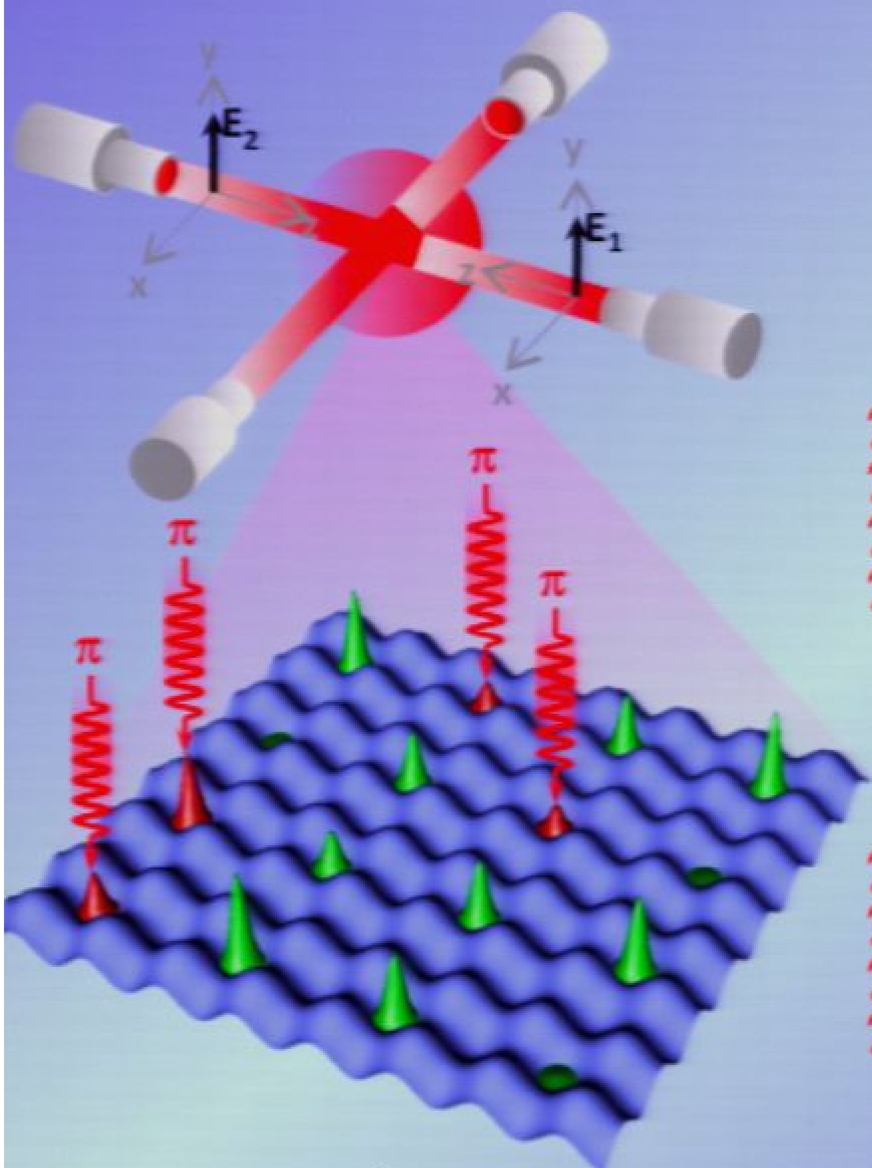
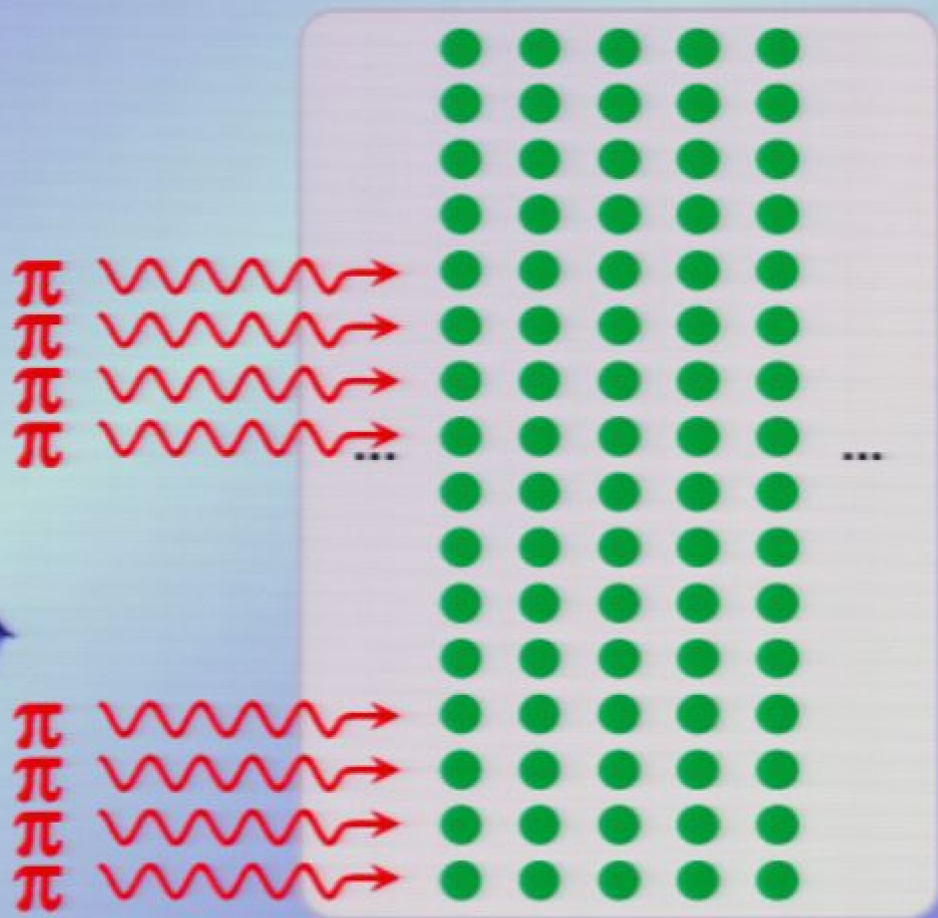
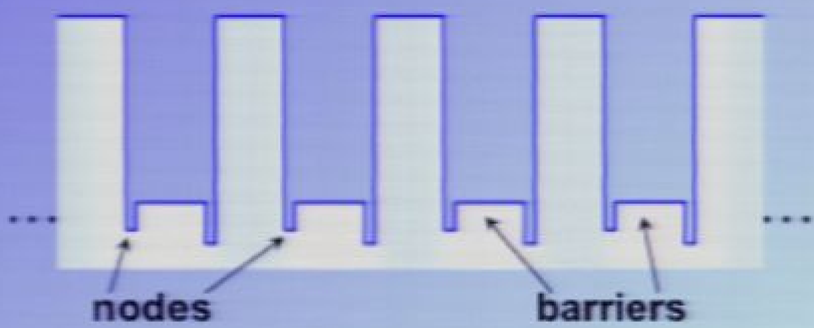
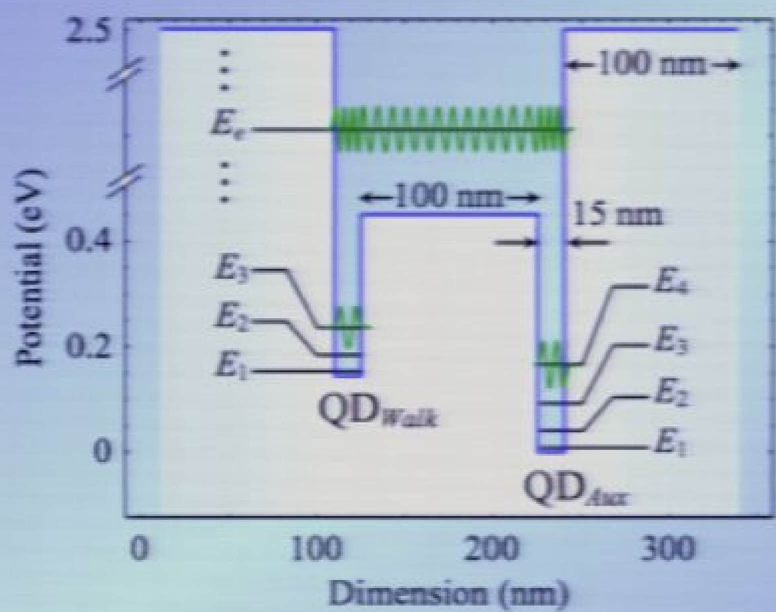


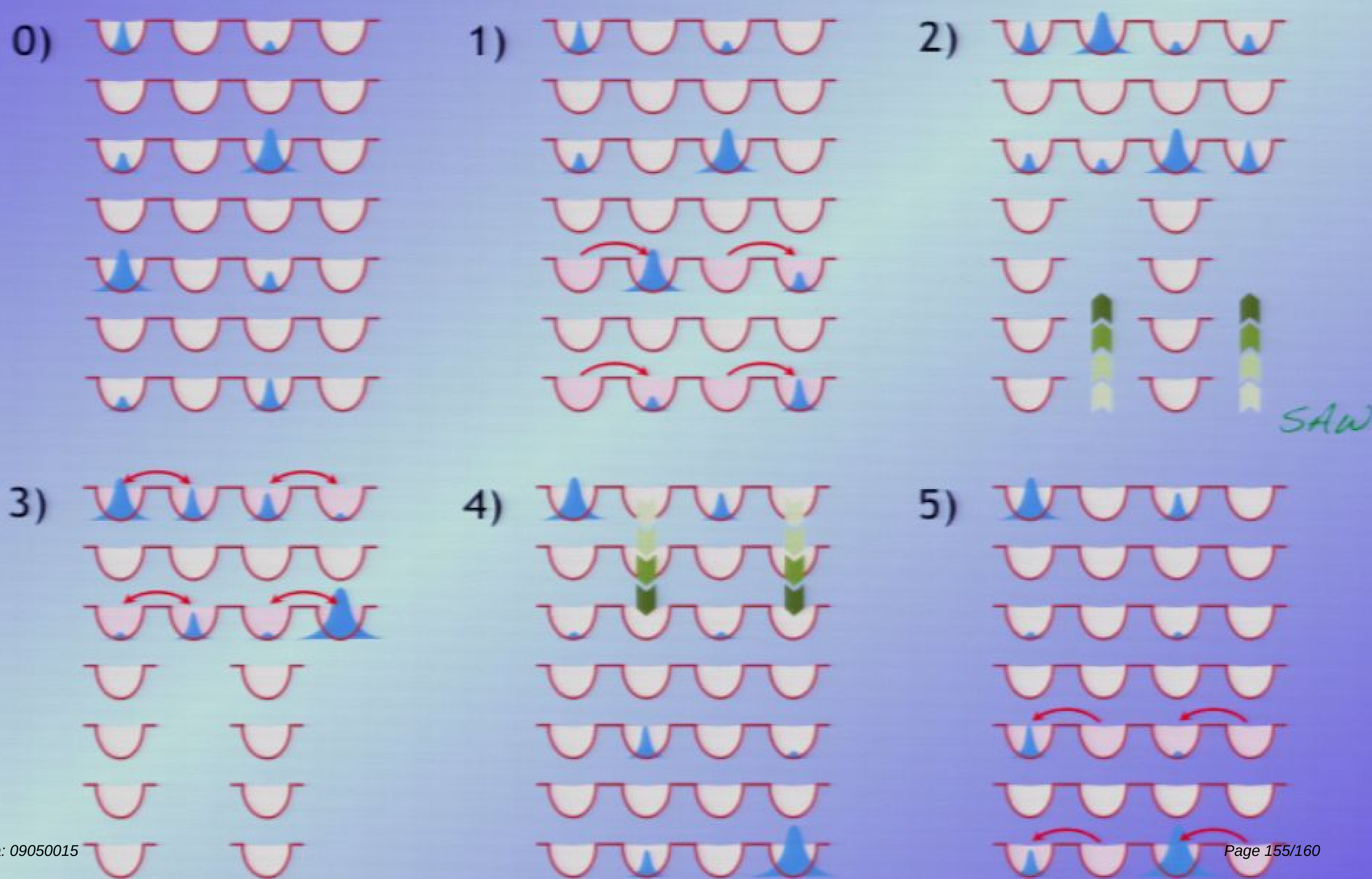


Quantum dot arrays





Quantum Dot Conveyor Belt



Conclusions

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➢ Actual walking is not required; only flipping coins is sufficient.

➢ Due to inherent structure of CS decomposition, all coin operations at each walk step can be done simultaneously for entire graph.

➢ This scheme can be physically realized using a variety of quantum systems.

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