

Title: Quantum foundations minus probability theory

Date: May 05, 2009 04:00 PM

URL: <http://pirsa.org/09050010>

Abstract: Researchers in quantum foundations claim (D'Ariano, Fuchs, ...):

Quantum = probability theory + x

and hence:

$x = \text{Quantum} - \text{probability theory}$

Guided by the metaphorical analogy:

probability theory / x = flesh / bones

we introduce a notion of quantum measurement within x, which, when flesing it with Hilbert spaces, provides orthodox quantum mechanical probability calculus.

**How much QM can we recover without a priori assuming instrumentalist concepts such as measurement and probability but just ‘compoundness’?**

Many say (D'Ariano, Fuchs, Hardy (?), Spekkens (?)...):

$$\text{quantum} = \text{probability theory} + x$$

and hence:

$$x = \text{quantum} - \text{probability theory}$$

Guided by the metaphorical analogy:

$$\frac{\text{flesh}}{\text{bones}} = \frac{\text{probability}}{x}$$

We start from  $x$ , (i) extract (the skeleton of) **classicality** from it, (ii) extract **probability** from it, (iii) extract **quantum measurements** from it, and (iv) describe **quantum-classical interaction** within it. Flesing it with FHilb yields QM probabilistic calculus.

Many say (D'Ariano, Fuchs, Hardy (?), Spekkens (?)...):

$$\text{quantum} = \text{probability theory} + x$$

and hence:

$$x = \text{quantum} - \text{probability theory}$$

Guided by the metaphorical analogy:

$$\frac{\text{flesh}}{\text{bones}} = \frac{\text{probability}}{x}$$

We start from  $x$ , (i) extract (the skeleton of) **classicality** from it, (ii) extract **probability** from it, (iii) extract **quantum measurements** from it, and (iv) describe **quantum-classical interaction** within it. Flesing it with FHilb yields QM probabilistic calculus.

**Is there a conceptual picture behind this game?**





## Classicizing quantumness

arXiv:0904.1997

*Somewhere out there, in ontic reality, is a world.*

*That world is called the ‘quantum universe’.*

*We would like to probe that world.*

*This requires ‘classical interfaces’.*

---

## Classicizing quantumness

arXiv:0904.1997

*Somewhere out there, in ontic reality, is a world.*

*That world is called the ‘quantum universe’.*

*We would like to probe that world.*

*This requires ‘classical interfaces’.*

---

We refer to ‘identifiable parts’ of it as **systems**,

and to their ‘identifiable changes’ as **processes**.

To joint parts and processes we refer by  $- \otimes -$ ,<sup>1</sup>

and to consecutive processes by  $- \circ -$ .<sup>1</sup>

---

<sup>1</sup>One could either think of this as structure of the interface or as structure of the system.

**Our language  $:=$  system, process,  $\otimes$ ,  $\circ$**





## Classicizing quantumness

arXiv:0904.1997

*Somewhere out there, in ontic reality, is a world.*

*That world is called the ‘quantum universe’.*

*We would like to probe that world.*

*This requires ‘classical interfaces’.*

---

We refer to ‘identifiable parts’ of it as **systems**,

and to their ‘identifiable changes’ as **processes**.

To joint parts and processes we refer by  $- \otimes -$ ,<sup>1</sup>

and to consecutive processes by  $- \circ -$ .<sup>1</sup>

---

<sup>1</sup>One could either think of this as structure of the interface or as structure of the system.

**Our language  $:=$  system, process,  $\otimes$ ,  $\circ$**

**Our language := system, process,  $\otimes$ ,  $\circ$**

**What does such a rigid stance buy us?**

Why does a tiger have stripes and a lion doesn't?



Why does a tiger have stripes and a lion doesn't?



prey ⊗ predator ⊗ environment  
|  
hunt  
↓  
dead prey ⊗ eating predator



Processes and their compositions:

$$f \equiv \begin{array}{c} | \\ \boxed{f} \\ | \end{array} \quad 1_A \equiv | \quad g \circ f \equiv \begin{array}{c} | \\ \boxed{g} \\ | \\ \boxed{f} \\ | \end{array} \quad f \otimes g \equiv \begin{array}{c} | \\ \boxed{f} \\ | \end{array} \begin{array}{c} | \\ \boxed{g} \\ | \end{array}$$

States and effects for systems, and resulting numbers:

$$I \xrightarrow{\psi} A \equiv \begin{array}{c} | \\ \nabla \psi \\ | \end{array} \quad A \xrightarrow{\pi} I \equiv \begin{array}{c} \triangle \pi \\ | \end{array} \quad I \xrightarrow{\pi} I \equiv \begin{array}{c} \diamond s \end{array}$$



Processes and their compositions:

$$f \equiv \begin{array}{c} | \\ \boxed{f} \\ | \end{array} \quad 1_A \equiv | \quad g \circ f \equiv \begin{array}{c} | \\ \boxed{g} \\ | \\ \boxed{f} \\ | \end{array} \quad f \otimes g \equiv \begin{array}{c} | \\ \boxed{f} \\ | \end{array} \begin{array}{c} | \\ \boxed{g} \\ | \end{array}$$

States and effects for systems, and resulting numbers:

$$I \xrightarrow{\psi} A \equiv \begin{array}{c} | \\ \nabla \psi \\ | \end{array} \quad A \xrightarrow{\pi} I \equiv \begin{array}{c} \triangle \pi \\ | \end{array} \quad I \xrightarrow{\pi} I \equiv \begin{array}{c} \diamond s \end{array}$$

Upside-down flipping i.e. adjoints:

<b>pics:</b>		
<b>cats:</b>	$A \xrightarrow{f} B$	$B \xrightarrow{f^\dagger} A$
<b>FHilb:</b>	linear map	its adjoint

# CLASSICAL INTERFACES



An classical interface is:

$$A \xrightarrow{\delta} A \otimes A = \text{cup}$$

$$A \xrightarrow{\varepsilon} I = \text{cap}$$

such that:

1.  $\varepsilon$  is a *unit* for  $\delta$ ;
2.  $\delta$  is *coassociative*;
3.  $\delta$  is *cocommutative*;
4.  $\delta$  is *isometry*;
5.  $\delta$  is *Frobenius*.

$$\text{cup with dot} \stackrel{(1)}{=} | \stackrel{(4)}{=} \text{cap with dot}$$

$$\text{cup with dot} \stackrel{(2)}{=} \text{cup with dot}$$

$$\text{cap with dot} \stackrel{(3)}{=} \text{cup with dot}$$

$$\text{cup with dot and cap} \stackrel{(5)}{=} \text{cup with dot and cap}$$

In **FHilb** we can encode all ONBs as

$$\delta : \mathcal{H} \rightarrow \mathcal{H} \otimes \mathcal{H} :: |i\rangle \mapsto |ii\rangle \quad \varepsilon : \mathcal{H} \rightarrow \mathbb{C} :: |i\rangle \mapsto 1.$$

**Thm.** Interfaces  $(\delta, \varepsilon)$  in **FHilb** exactly correspond with orthonormal bases on the underlying Hilbert space.



An *classical interface* is:

$$A \xrightarrow{\delta} A \otimes A = \text{cup}$$

$$A \xrightarrow{\varepsilon} I = \text{cap}$$

such that:

1.  $\varepsilon$  is a *unit* for  $\delta$ ;
2.  $\delta$  is *coassociative*;
3.  $\delta$  is *cocommutative*;
4.  $\delta$  is *isometry*;
5.  $\delta$  is *Frobenius*.

$$\text{cup with dot} \stackrel{(1)}{=} | \stackrel{(4)}{=} \text{cap with dot}$$

$$\text{cup with dot and cap} \stackrel{(2)}{=} \text{cap with dot and cup}$$

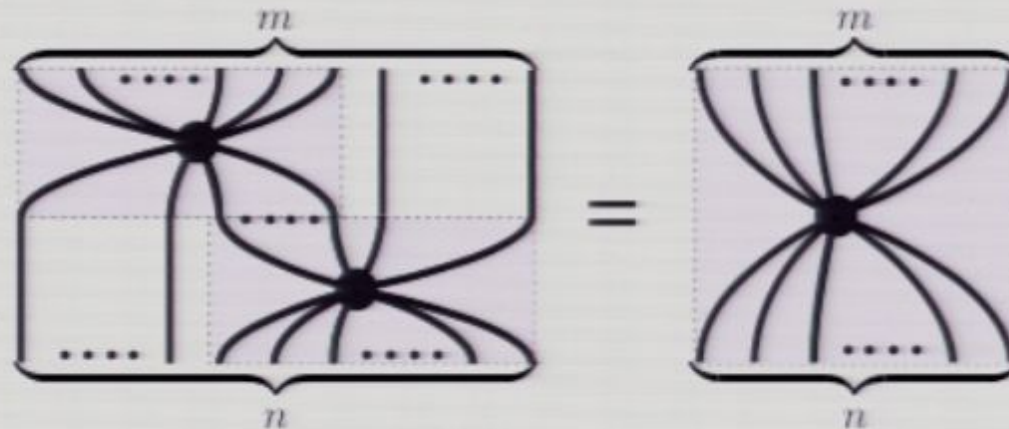
$$\text{cup with dot and cap with dot} \stackrel{(3)}{=} \text{cup with dot}$$

$$\text{cup with dot and cap with dot and cap} \stackrel{(5)}{=} \text{cup with dot and cap}$$

A classical interface is:

$$\left\{ \begin{array}{c} m \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ n \end{array} \right\} \mid n, m \in \mathbb{N}$$

invariant under flipping and swapping, and such that:



An *classical interface* is:

$$A \xrightarrow{\delta} A \otimes A = \text{cup}$$

$$A \xrightarrow{\varepsilon} I = \text{cap}$$

such that:

1.  $\varepsilon$  is a *unit* for  $\delta$ ;
2.  $\delta$  is *coassociative*;
3.  $\delta$  is *cocommutative*;
4.  $\delta$  is *isometry*;
5.  $\delta$  is *Frobenius*.

$$\text{cup with dot} \stackrel{(1)}{=} | \stackrel{(4)}{=} \text{cap with dot}$$

$$\text{cup with dot and cap} \stackrel{(2)}{=} \text{cap with dot and cup}$$

$$\text{cup with dot and cap with dot} \stackrel{(3)}{=} \text{cup with dot}$$

$$\text{cup with dot and cap with dot and cup} \stackrel{(5)}{=} \text{cup with dot and cap with dot}$$

In **FHilb** we can encode all ONBs as

$$\delta : \mathcal{H} \rightarrow \mathcal{H} \otimes \mathcal{H} :: |i\rangle \mapsto |ii\rangle \quad \varepsilon : \mathcal{H} \rightarrow \mathbb{C} :: |i\rangle \mapsto 1.$$

**Thm.** Interfaces  $(\delta, \varepsilon)$  in **FHilb** exactly correspond with orthonormal bases on the underlying Hilbert space.

An *classical interface* is:

$$A \xrightarrow{\delta} A \otimes A = \text{cup}$$

$$A \xrightarrow{\varepsilon} I = \text{cap}$$

such that:

1.  $\varepsilon$  is a *unit* for  $\delta$ ;
2.  $\delta$  is *coassociative*;
3.  $\delta$  is *cocommutative*;
4.  $\delta$  is *isometry*;
5.  $\delta$  is *Frobenius*.

$$\text{cup} \stackrel{(1)}{=} | \stackrel{(4)}{=} \text{cap}$$

$$\text{cup} \stackrel{(2)}{=} \text{cup}$$

$$\text{cap} \stackrel{(3)}{=} \text{cup}$$

$$\text{cup} \stackrel{(5)}{=} \text{cap}$$



In **FHilb** we can encode all ONBs as

$$\delta : \mathcal{H} \rightarrow \mathcal{H} \otimes \mathcal{H} :: |i\rangle \mapsto |ii\rangle \quad \varepsilon : \mathcal{H} \rightarrow \mathbb{C} :: |i\rangle \mapsto 1.$$

**Thm.** Interfaces  $(\delta, \varepsilon)$  in **FHilb** exactly correspond with orthonormal bases on the underlying Hilbert space.

An *classical interface* is:

$$A \xrightarrow{\delta} A \otimes A = \text{cup}$$

$$A \xrightarrow{\varepsilon} I = \text{cap}$$

such that:

1.  $\varepsilon$  is a *unit* for  $\delta$ ;
2.  $\delta$  is *coassociative*;
3.  $\delta$  is *cocommutative*;
4.  $\delta$  is *isometry*;
5.  $\delta$  is *Frobenius*.

$$\text{cup with dot} \stackrel{(1)}{=} | \stackrel{(4)}{=} \text{cap with dot}$$

$$\text{cup with dot and cap} \stackrel{(2)}{=} \text{cap with dot and cup}$$

$$\text{cap with dot and cup} \stackrel{(3)}{=} \text{cup with dot}$$

$$\text{cup with dot and cap with dot} \stackrel{(5)}{=} \text{cap with dot and cup with dot}$$

In **FHilb** we can encode all ONBs as

$$\delta : \mathcal{H} \rightarrow \mathcal{H} \otimes \mathcal{H} :: |i\rangle \mapsto |ii\rangle \quad \varepsilon : \mathcal{H} \rightarrow \mathbb{C} :: |i\rangle \mapsto 1.$$

**Thm.** Interfaces  $(\delta, \varepsilon)$  in **FHilb** exactly correspond with orthonormal bases on the underlying Hilbert space.

In **FHilb** we can encode all ONBs as

$$\delta : \mathcal{H} \rightarrow \mathcal{H} \otimes \mathcal{H} :: |i\rangle \mapsto |ii\rangle \quad \varepsilon : \mathcal{H} \rightarrow \mathbb{C} :: |i\rangle \mapsto 1.$$

**Thm.** Interfaces  $(\delta, \varepsilon)$  in **FHilb** exactly correspond with orthonormal bases on the underlying Hilbert space.

Processes and their compositions:

$$f \equiv \begin{array}{c} | \\ \boxed{f} \\ | \end{array} \quad 1_A \equiv | \quad g \circ f \equiv \begin{array}{c} | \\ \boxed{g} \\ | \\ \boxed{f} \\ | \end{array} \quad f \otimes g \equiv \begin{array}{c} | \\ \boxed{f} \\ | \end{array} \begin{array}{c} | \\ \boxed{g} \\ | \end{array}$$

States and effects for systems, and resulting numbers:

$$I \xrightarrow{\psi} A \equiv \begin{array}{c} | \\ \nabla \psi \\ | \end{array} \quad A \xrightarrow{\pi} I \equiv \begin{array}{c} \triangle \pi \\ | \end{array} \quad I \xrightarrow{\pi} I \equiv \begin{array}{c} \diamond s \end{array}$$



Why does a tiger have stripes and a lion doesn't?



prey ⊗ predator ⊗ environment

hunt

dead prey ⊗ eating predator



Processes and their compositions:


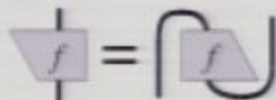
$$f \equiv \begin{array}{c} | \\ \boxed{f} \\ | \end{array} \quad 1_A \equiv | \quad g \circ f \equiv \begin{array}{c} | \\ \boxed{g} \\ | \\ \boxed{f} \\ | \end{array} \quad f \otimes g \equiv \begin{array}{c} | \\ \boxed{f} \\ | \end{array} \begin{array}{c} | \\ \boxed{g} \\ | \end{array}$$


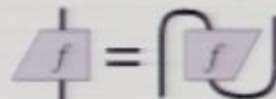
States and effects for systems, and resulting numbers:

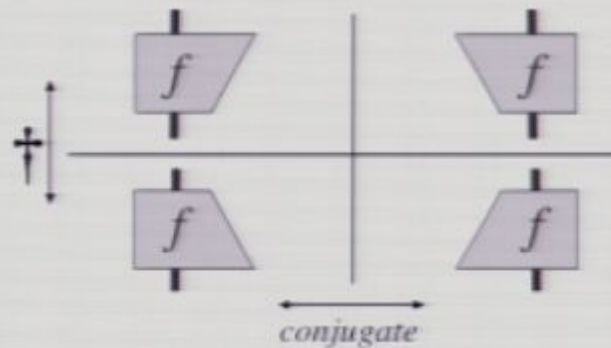
$$I \xrightarrow{\psi} A \equiv \begin{array}{c} | \\ \nabla \psi \\ | \end{array} \quad A \xrightarrow{\pi} I \equiv \begin{array}{c} | \\ \triangle \pi \\ | \end{array} \quad I \xrightarrow{\pi} I \equiv \begin{array}{c} | \\ \diamond s \\ | \end{array}$$

Upside-down flipping i.e. adjoints:

<b>pics:</b>		
<b>cats:</b>	$A \xrightarrow{f} B$	$B \xrightarrow{f^\dagger} A$
<b>FHilb:</b>	linear map	its adjoint

<b>pics:</b>		
<b>cats:</b>	$A \xrightarrow{f} B$	$f^* = A \xrightarrow{(\epsilon_A \otimes 1_B) \circ (1_A \otimes f^\dagger \otimes 1_B) \circ (1_A \otimes \eta_B)} B$
<b>FHilb:</b>	linear map	its conjugate

<b>pics:</b>		
<b>cats:</b>	$A \xrightarrow{f} B$	$f^* = B \xrightarrow{(\epsilon_B \otimes 1_A) \circ (1_B \otimes f \otimes 1_A) \circ (1_B \otimes \eta_A)} A$
<b>FHilb:</b>	linear map	its transposed



# MEASUREMENT $\leftrightarrow$ NO-SIGNALING

In **FHilb** we can encode all ONBs as

$$\delta : \mathcal{H} \rightarrow \mathcal{H} \otimes \mathcal{H} :: |i\rangle \mapsto |ii\rangle \quad \varepsilon : \mathcal{H} \rightarrow \mathbb{C} :: |i\rangle \mapsto 1.$$

**Thm.** Interfaces  $(\delta, \varepsilon)$  in **FHilb** exactly correspond with orthonormal bases on the underlying Hilbert space.



A classical interface is:

$$\left\{ \begin{array}{c} m \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ n \end{array} \right\} \mid n, m \in \mathbb{N}$$

invariant under flipping and swapping, and such that:

$$\begin{array}{c} m \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ n \end{array} = \begin{array}{c} m \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ n \end{array}$$


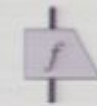
Processes and their compositions:

$$f \equiv \begin{array}{c} | \\ \boxed{f} \\ | \end{array} \quad 1_A \equiv | \quad g \circ f \equiv \begin{array}{c} | \\ \boxed{g} \\ | \\ \boxed{f} \\ | \end{array} \quad f \otimes g \equiv \begin{array}{c} | \\ \boxed{f} \\ | \end{array} \begin{array}{c} | \\ \boxed{g} \\ | \end{array}$$

States and effects for systems, and resulting numbers:

$$I \xrightarrow{\psi} A \equiv \begin{array}{c} | \\ \nabla \psi \\ | \end{array} \quad A \xrightarrow{\pi} I \equiv \begin{array}{c} \triangle \pi \\ | \end{array} \quad I \xrightarrow{\pi} I \equiv \begin{array}{c} \diamond s \\ | \end{array}$$

Upside-down flipping i.e. adjoints:

<b>pics:</b>		
<b>cats:</b>	$A \xrightarrow{f} B$	$B \xrightarrow{f^\dagger} A$
<b>FHilb:</b>	linear map	its adjoint

An classical interface is:

$$A \xrightarrow{\delta} A \otimes A = \text{cup}$$

$$A \xrightarrow{\varepsilon} I = \text{cap}$$

such that:

1.  $\varepsilon$  is a *unit* for  $\delta$ ;
2.  $\delta$  is *coassociative*;
3.  $\delta$  is *cocommutative*;
4.  $\delta$  is *isometry*;
5.  $\delta$  is *Frobenius*.

$$\text{cup with dot} \stackrel{(1)}{=} | \stackrel{(4)}{=} \text{cap with dot}$$

$$\text{cup with dot} \stackrel{(2)}{=} \text{cup with dot}$$

$$\text{cap with dot} \stackrel{(3)}{=} \text{cup with dot}$$

$$\text{cup with dot and cap} \stackrel{(5)}{=} \text{cup with dot and cap}$$

A classical interface is:

$$\left\{ \begin{array}{c} m \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ n \end{array} \right\} \mid n, m \in \mathbb{N}$$

invariant under flipping and swapping, and such that:

$$\begin{array}{c} m \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ n \end{array} = \begin{array}{c} m \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ n \end{array}$$



An classical interface is:

$$A \xrightarrow{\delta} A \otimes A = \text{cup}$$

$$A \xrightarrow{\varepsilon} I = \text{cap}$$

such that:

1.  $\varepsilon$  is a *unit* for  $\delta$ ;
2.  $\delta$  is *coassociative*;
3.  $\delta$  is *cocommutative*;
4.  $\delta$  is *isometry*;
5.  $\delta$  is *Frobenius*.

$$\text{cup} \stackrel{(1)}{=} | \stackrel{(4)}{=} \text{cap}$$

$$\text{cup} \stackrel{(2)}{=} \text{cup}$$

$$\text{cap} \stackrel{(3)}{=} \text{cup}$$

$$\text{cup} \stackrel{(5)}{=} \text{cap}$$




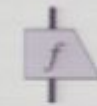
Processes and their compositions:

$$f \equiv \begin{array}{c} | \\ \boxed{f} \\ | \end{array} \quad 1_A \equiv | \quad g \circ f \equiv \begin{array}{c} | \\ \boxed{g} \\ | \\ \boxed{f} \\ | \end{array} \quad f \otimes g \equiv \begin{array}{c} | \\ \boxed{f} \\ | \end{array} \begin{array}{c} | \\ \boxed{g} \\ | \end{array}$$

States and effects for systems, and resulting numbers:

$$I \xrightarrow{\psi} A \equiv \begin{array}{c} | \\ \nabla \psi \\ | \end{array} \quad A \xrightarrow{\pi} I \equiv \begin{array}{c} \triangle \pi \\ | \end{array} \quad I \xrightarrow{\pi} I \equiv \begin{array}{c} \diamond s \end{array}$$

Upside-down flipping i.e. adjoints:

<b>pics:</b>		
<b>cats:</b>	$A \xrightarrow{f} B$	$B \xrightarrow{f^\dagger} A$
<b>FHilb:</b>	linear map	its adjoint

An *classical interface* is:

$$A \xrightarrow{\delta} A \otimes A = \text{cup}$$

$$A \xrightarrow{\varepsilon} I = \text{cap}$$

such that:

1.  $\varepsilon$  is a *unit* for  $\delta$ ;
2.  $\delta$  is *coassociative*;
3.  $\delta$  is *cocommutative*;
4.  $\delta$  is *isometry*;
5.  $\delta$  is *Frobenius*.

$$\text{cup with dot} \stackrel{(1)}{=} | \stackrel{(4)}{=} \text{cap with dot}$$

$$\text{cup with dot and cap} \stackrel{(2)}{=} \text{cap with dot and cup}$$

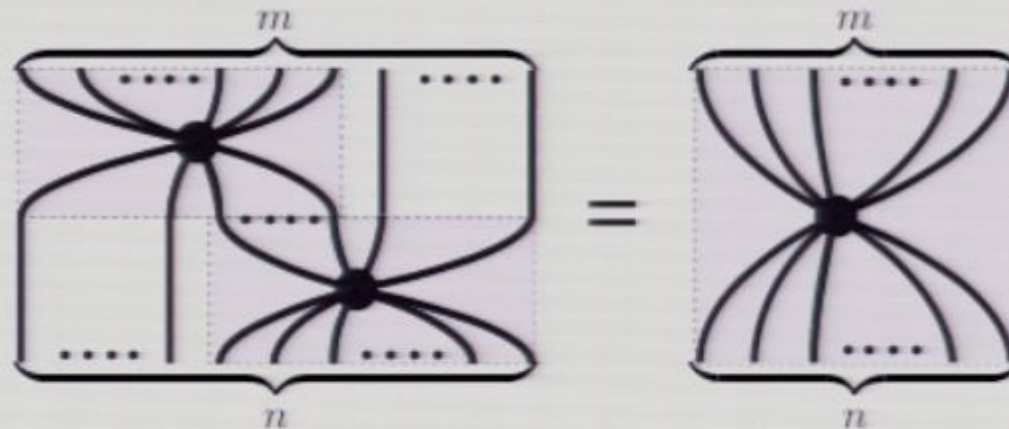
$$\text{cap with dot and cup} \stackrel{(3)}{=} \text{cup with dot}$$

$$\text{cup with dot and cap with dot} \stackrel{(5)}{=} \text{cap with dot and cup with dot}$$

A classical interface is:

$$\left\{ \begin{array}{c} m \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ n \end{array} \right\} \mid n, m \in \mathbb{N}$$

invariant under flipping and swapping, and such that:



In **FHilb** we can encode all ONBs as

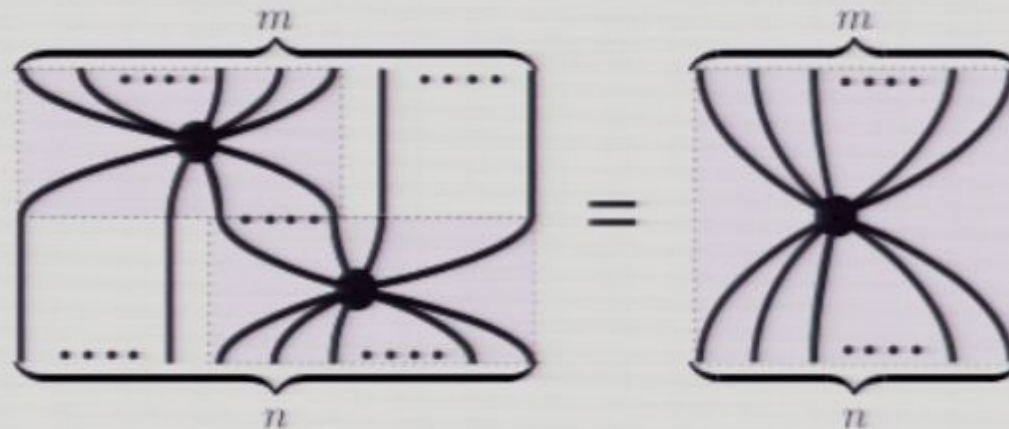
$$\delta : \mathcal{H} \rightarrow \mathcal{H} \otimes \mathcal{H} :: |i\rangle \mapsto |ii\rangle \quad \varepsilon : \mathcal{H} \rightarrow \mathbb{C} :: |i\rangle \mapsto 1.$$

**Thm.** Interfaces  $(\delta, \varepsilon)$  in **FHilb** exactly correspond with orthonormal bases on the underlying Hilbert space.

A classical interface is:

$$\left\{ \begin{array}{c} \text{Diagram of a classical interface with } m \text{ inputs and } n \text{ outputs} \\ \hline n, m \in \mathbb{N} \end{array} \right\}$$

invariant under flipping and swapping, and such that:

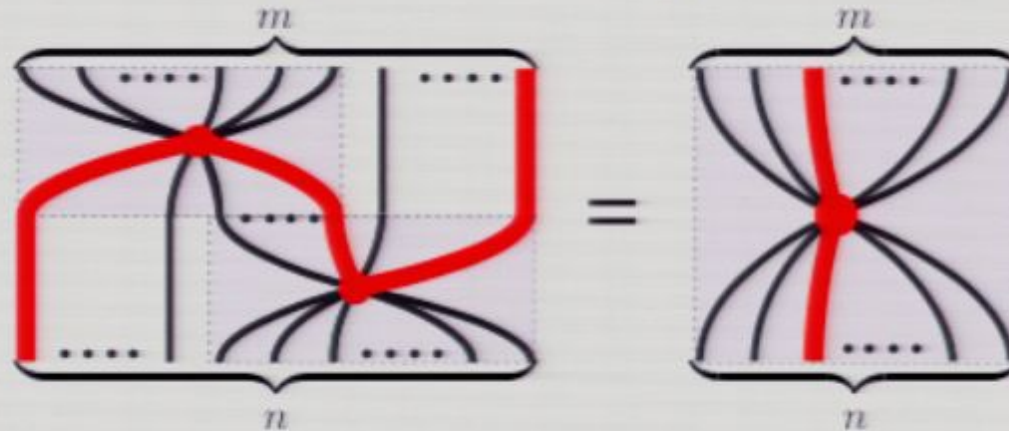




A classical interface is:

$$\left\{ \begin{array}{c} m \\ \text{diagram} \\ n \end{array} \mid n, m \in \mathbb{N} \right\}$$

invariant under flipping and swapping, and such that:



for instances  $\delta_0^2 =$   and  $\delta_2^0 =$  .

A classical interface is:

$$\left\{ \begin{array}{c} m \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ n \end{array} \right\} \mid n, m \in \mathbb{N}$$

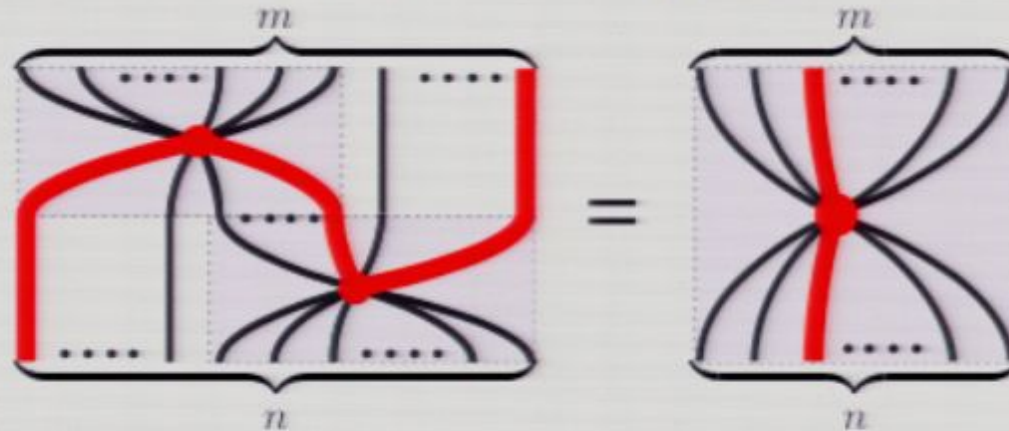
invariant under flipping and swapping, and such that:

$$\begin{array}{c} m \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ n \end{array} = \begin{array}{c} m \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ n \end{array}$$

A classical interface is:

$$\left\{ \begin{array}{c} m \\ \text{diagram} \\ n \end{array} \mid n, m \in \mathbb{N} \right\}$$

invariant under flipping and swapping, and such that:



for instances  $\delta_0^2 =$   and  $\delta_2^0 =$  .

An *classical interface* is:

$$A \xrightarrow{\delta} A \otimes A = \text{cup}$$

$$A \xrightarrow{\varepsilon} I = \text{cap}$$

such that:

1.  $\varepsilon$  is a *unit* for  $\delta$ ;
2.  $\delta$  is *coassociative*;
3.  $\delta$  is *cocommutative*;
4.  $\delta$  is *isometry*;
5.  $\delta$  is *Frobenius*.

$$\text{cup} \stackrel{(1)}{=} | \stackrel{(4)}{=} \text{cap}$$

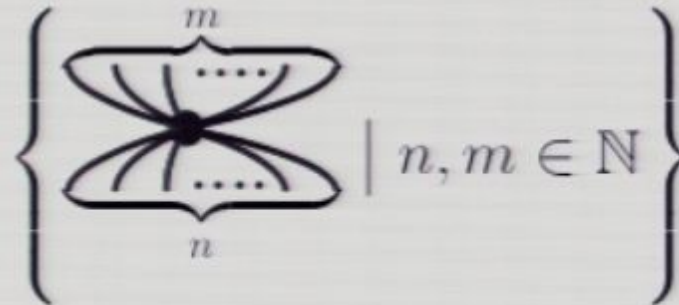
$$\text{cup} \circ \text{cup} \stackrel{(2)}{=} \text{cup} \circ \text{cup}$$

$$\text{cap} \circ \text{cap} \stackrel{(3)}{=} \text{cap} \circ \text{cap}$$

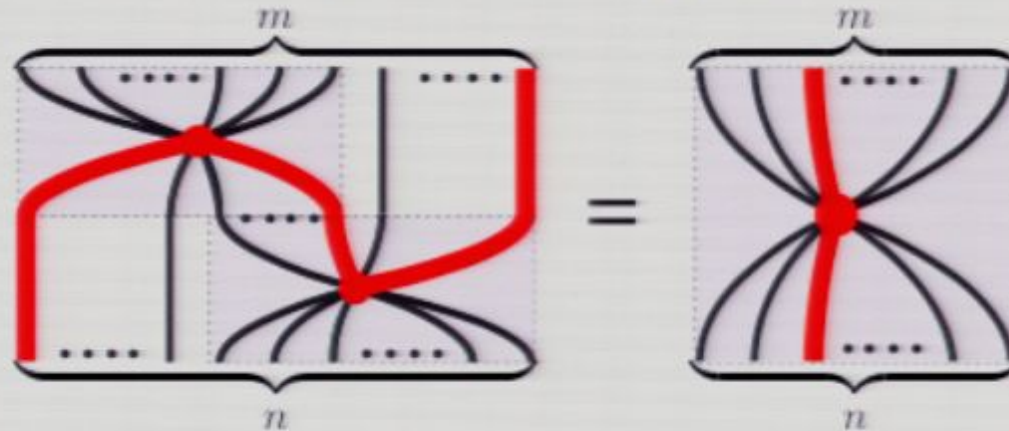
$$\text{cup} \circ \text{cap} \stackrel{(5)}{=} \text{cap} \circ \text{cup}$$



A classical interface is:

$$\left\{ \begin{array}{c} m \\ \text{...} \\ \text{...} \\ n \end{array} \right\} \mid n, m \in \mathbb{N}$$


invariant under flipping and swapping, and such that:



for instances  $\delta_0^2 =$   and  $\delta_2^0 =$  .



A classical interface is:

$$\left\{ \begin{array}{c} m \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ n \end{array} \right\} \mid n, m \in \mathbb{N}$$

invariant under flipping and swapping, and such that:

$$\begin{array}{c} m \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ n \end{array} = \begin{array}{c} m \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ n \end{array}$$

An classical interface is:

$$A \xrightarrow{\delta} A \otimes A = \text{cup}$$

$$A \xrightarrow{\varepsilon} I = \text{cap}$$

such that:

1.  $\varepsilon$  is a *unit* for  $\delta$ ;
2.  $\delta$  is *coassociative*;
3.  $\delta$  is *cocommutative*;
4.  $\delta$  is *isometry*;
5.  $\delta$  is *Frobenius*.

$$\text{cup with dot} \stackrel{(1)}{=} | \stackrel{(4)}{=} \text{cap with dot}$$

$$\text{cup over cup} \stackrel{(2)}{=} \text{cup over cup}$$

$$\text{cup over cap} \stackrel{(3)}{=} \text{cup}$$

$$\text{cup over cap with dot} \stackrel{(5)}{=} \text{cup over cap}$$

A classical interface is:

$$\left\{ \begin{array}{c} \text{Diagram of a classical interface with } m \text{ inputs and } n \text{ outputs} \\ \mid n, m \in \mathbb{N} \end{array} \right\}$$

invariant under flipping and swapping, and such that:

$$\begin{array}{c} \text{Diagram of two classical interfaces swapped} \\ = \\ \text{Diagram of a single classical interface} \end{array}$$

An *classical interface* is:

$$A \xrightarrow{\delta} A \otimes A = \text{cup}$$

$$A \xrightarrow{\varepsilon} I = \text{cap}$$

such that:

1.  $\varepsilon$  is a *unit* for  $\delta$ ;
2.  $\delta$  is *coassociative*;
3.  $\delta$  is *cocommutative*;
4.  $\delta$  is *isometry*;
5.  $\delta$  is *Frobenius*.

$$\text{cup} \stackrel{(1)}{=} | \stackrel{(4)}{=} \text{cap}$$

$$\text{cup} \circ \text{cup} \stackrel{(2)}{=} \text{cup} \circ \text{cup}$$

$$\text{cap} \circ \text{cap} \stackrel{(3)}{=} \text{cap} \circ \text{cap}$$

$$\text{cup} \circ \text{cap} \stackrel{(5)}{=} \text{cap} \circ \text{cup}$$



A classical interface is:

$$\left\{ \begin{array}{c} m \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ n \end{array} \right\} \mid n, m \in \mathbb{N}$$

invariant under flipping and swapping, and such that:

$$\begin{array}{c} m \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ n \end{array} = \begin{array}{c} m \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ n \end{array}$$



An *classical interface* is:

$$A \xrightarrow{\delta} A \otimes A = \text{cup}$$

$$A \xrightarrow{\varepsilon} I = \text{cap}$$

such that:

1.  $\varepsilon$  is a *unit* for  $\delta$ ;
2.  $\delta$  is *coassociative*;
3.  $\delta$  is *cocommutative*;
4.  $\delta$  is *isometry*;
5.  $\delta$  is *Frobenius*.

$$\text{cup with dot} \stackrel{(1)}{=} | \stackrel{(4)}{=} \text{cap with dot}$$

$$\text{cup with dot and cap} \stackrel{(2)}{=} \text{cap with dot and cup}$$

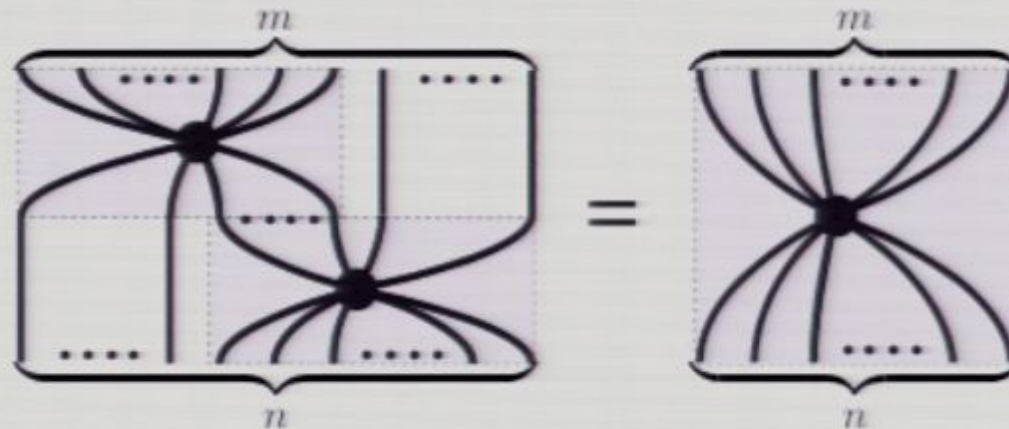
$$\text{cap with dot and cup} \stackrel{(3)}{=} \text{cup with dot}$$

$$\text{cup with dot and cap with dot} \stackrel{(5)}{=} \text{cap with dot and cup with dot}$$

A classical interface is:

$$\left\{ \begin{array}{c} m \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ n \end{array} \right\} \mid n, m \in \mathbb{N}$$

invariant under flipping and swapping, and such that:




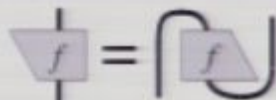
A classical interface is:


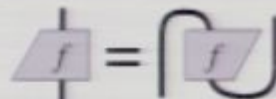
$$\left\{ \begin{array}{c} m \\ \text{diagram} \\ n \end{array} \mid n, m \in \mathbb{N} \right\}$$

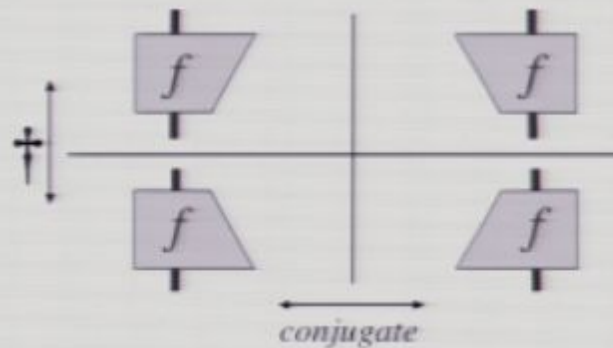
invariant under flipping and swapping, and such that:

$$\begin{array}{c} m \\ \text{diagram with red path} \\ n \end{array} = \begin{array}{c} m \\ \text{diagram with red path} \\ n \end{array}$$

for instances  $\delta_0^2 = \text{diagram}$  and  $\delta_2^0 = \text{diagram}$ .

<b>pics:</b>		
<b>cats:</b>	$A \xrightarrow{f} B$	$f^\sharp = A \xrightarrow{(\epsilon_A \otimes 1_B) \circ (1_A \otimes f^\dagger \otimes 1_B) \circ (1_A \otimes \eta_B)} B$
<b>FHilb:</b>	linear map	its conjugate

<b>pics:</b>		
<b>cats:</b>	$A \xrightarrow{f} B$	$f^* = B \xrightarrow{(\epsilon_B \otimes 1_A) \circ (1_B \otimes f \otimes 1_A) \circ (1_B \otimes \eta_A)} A$
<b>FHilb:</b>	linear map	its transposed



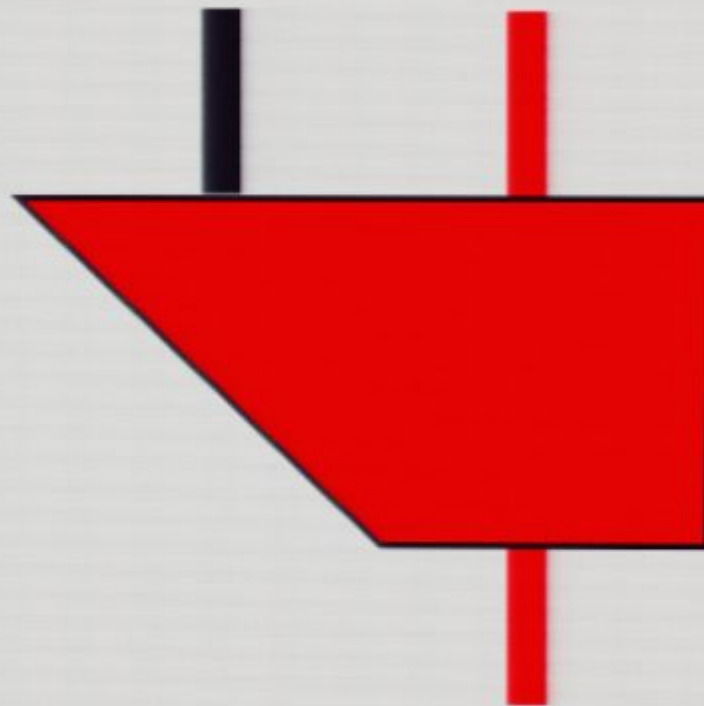
# MEASUREMENT $\leftrightarrow$ NO-SIGNALING





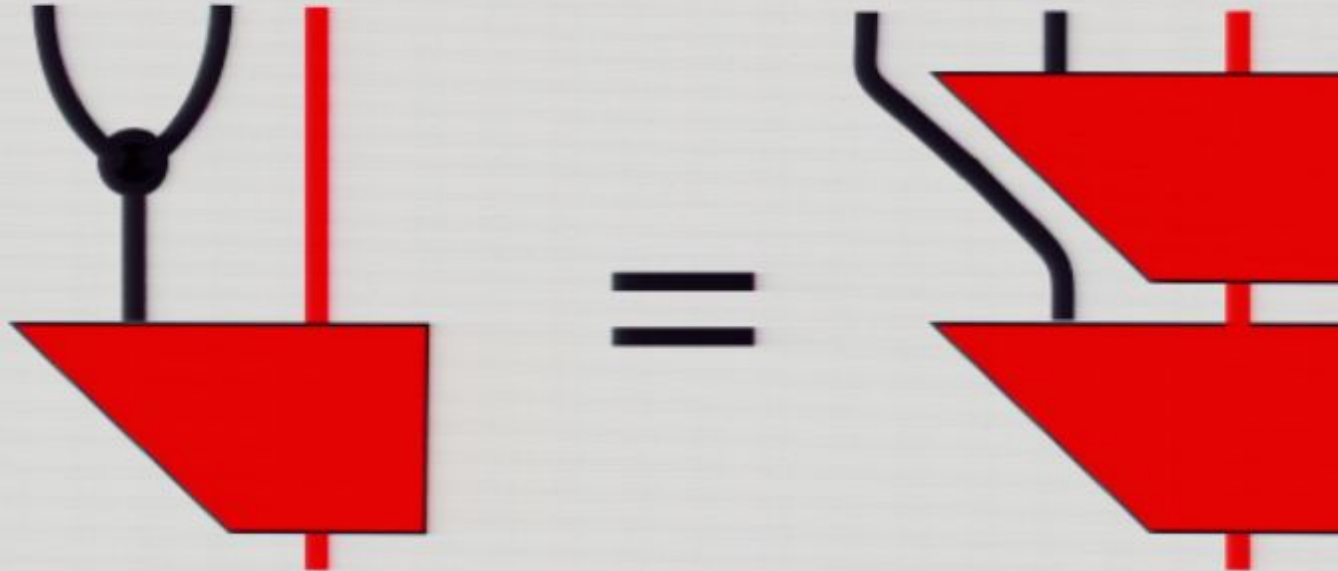
## Quantum measurement:

$$\mathcal{M} : A \rightarrow X \otimes A$$



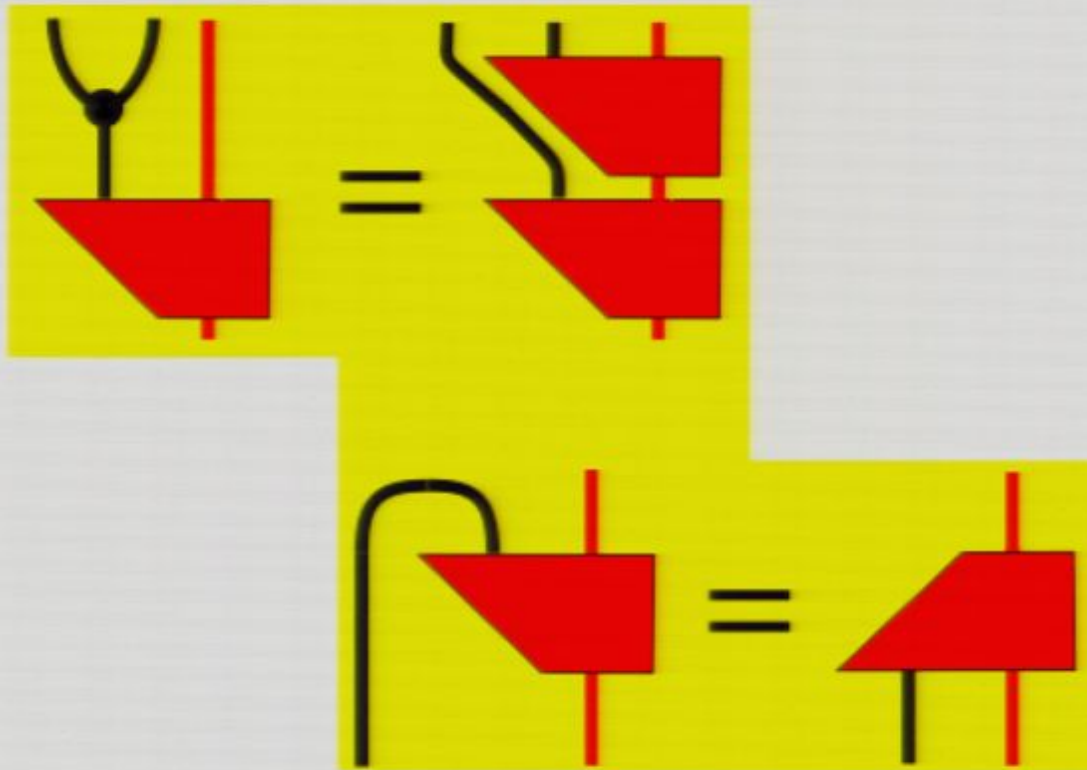
## Quantum measurement:

$$\mathcal{M} : A \rightarrow X \otimes A$$



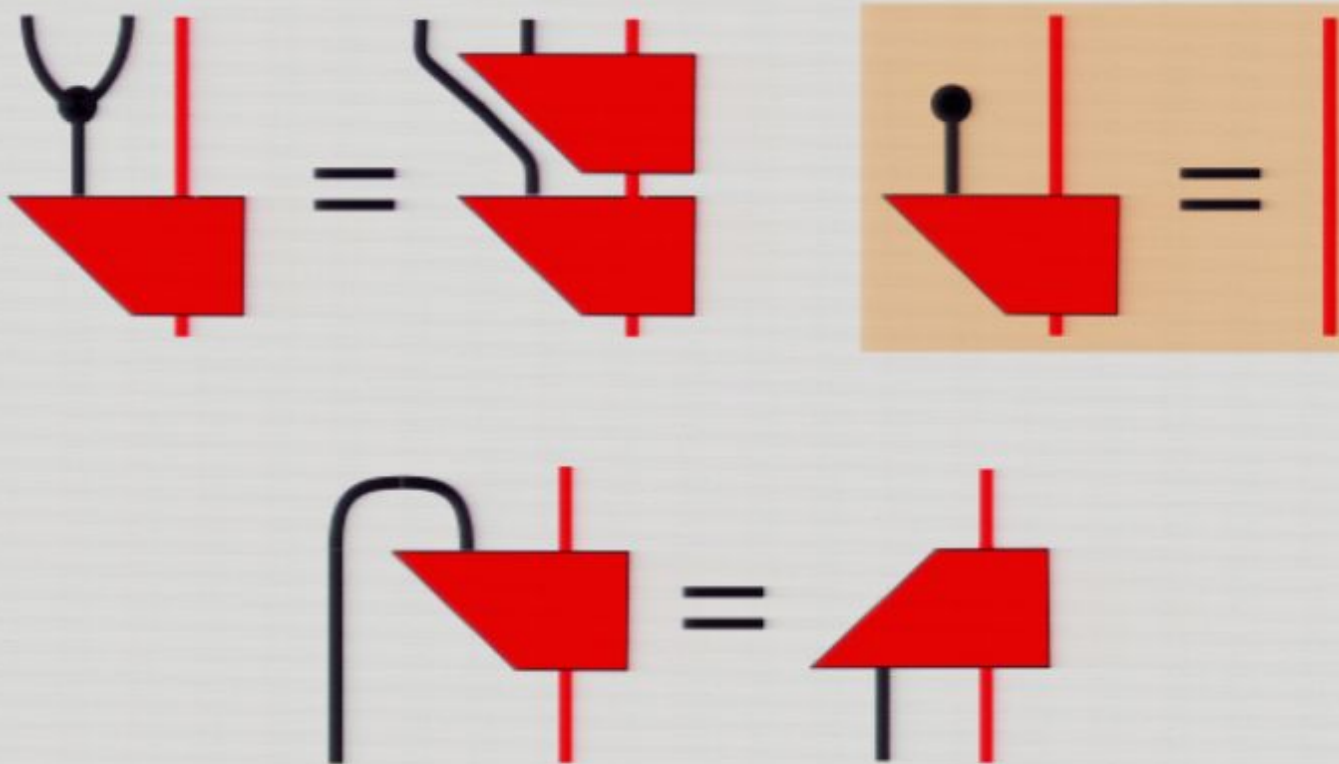
$\Rightarrow$  von Neumann projection postulate.

## Quantum measurement:



*Minimal requirements* for reasonable notion of measurement

## Quantum measurement:



Asserts *no-signaling*

**Thm.** In  $\mathbf{FHilb}$  the above rules yield exactly all projector spectra arising from self-adjoint operators.

**proof.**

Projection postulate  $\Rightarrow$

idempotence

$$P_i^2 = P_i$$

mutual orthogonality

$$P_i \circ P_{j \neq i} = 0$$

No signaling  $\Rightarrow$

Completeness of spectrum

$$\sum_i P_i = 1_{\mathcal{H}}$$

Minimal requirement II  $\Rightarrow$

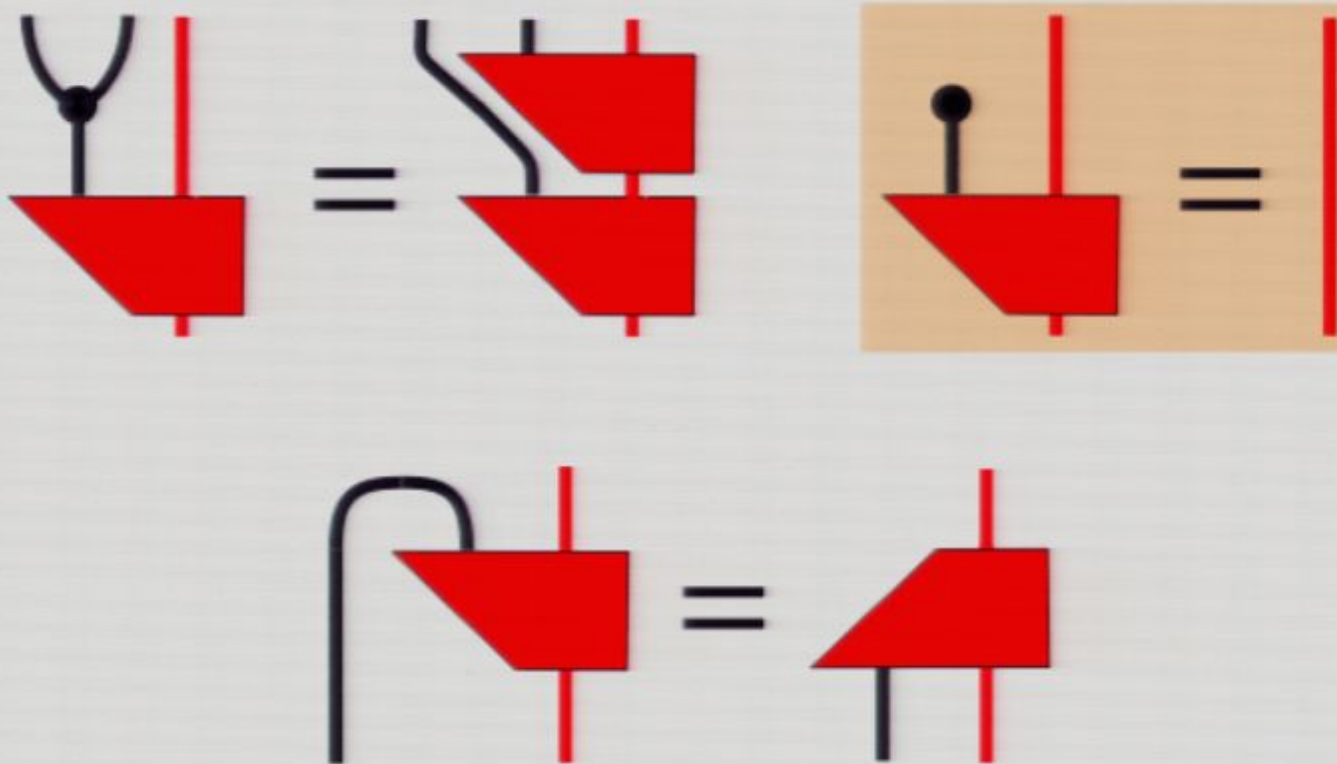
Orthogonality of projectors

$$P_i^\dagger = P_i$$

PROJECTOR  
 SPECTRUM



## Quantum measurement:



Asserts *no-signaling*

**Thm.** In FHilb the above rules yield exactly all projector spectra arising from self-adjoint operators.

**proof.**

Projection postulate  $\Rightarrow$

idempotence

$$P_i^2 = P_i$$

mutual orthogonality

$$P_i \circ P_{j \neq i} = 0$$

No signaling  $\Rightarrow$

Completeness of spectrum

$$\sum_i P_i = 1_{\mathcal{H}}$$

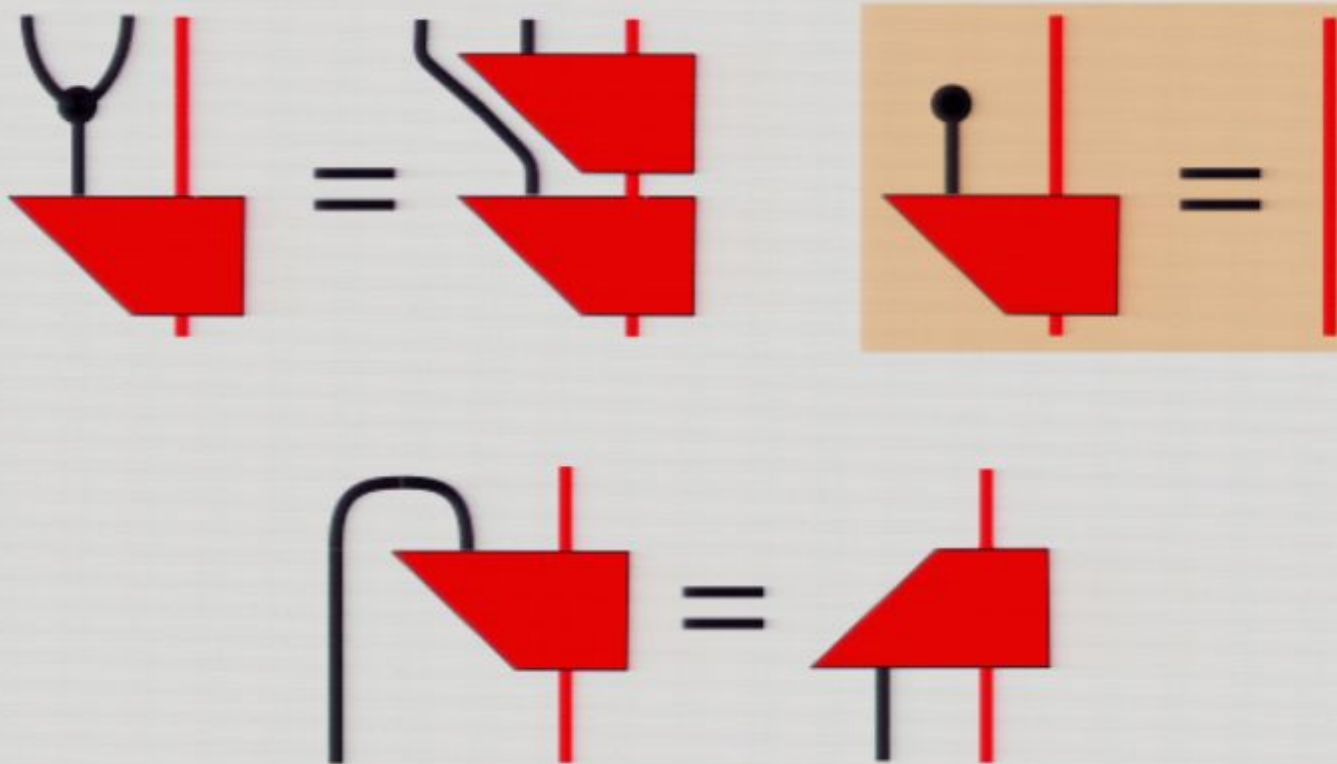
Minimal requirement II  $\Rightarrow$

Orthogonality of projectors

$$P_i^\dagger = P_i$$

PROJECTOR  
 SPECTRUM

## Quantum measurement:



Asserts *no-signaling*

**Thm.** In FHilb the above rules yield exactly all projector spectra arising from self-adjoint operators.

**proof.**

Projection postulate  $\Rightarrow$

idempotence

$$P_i^2 = P_i$$

mutual orthogonality

$$P_i \circ P_{j \neq i} = 0$$

No signaling  $\Rightarrow$

Completeness of spectrum

$$\sum_i P_i = 1_{\mathcal{H}}$$

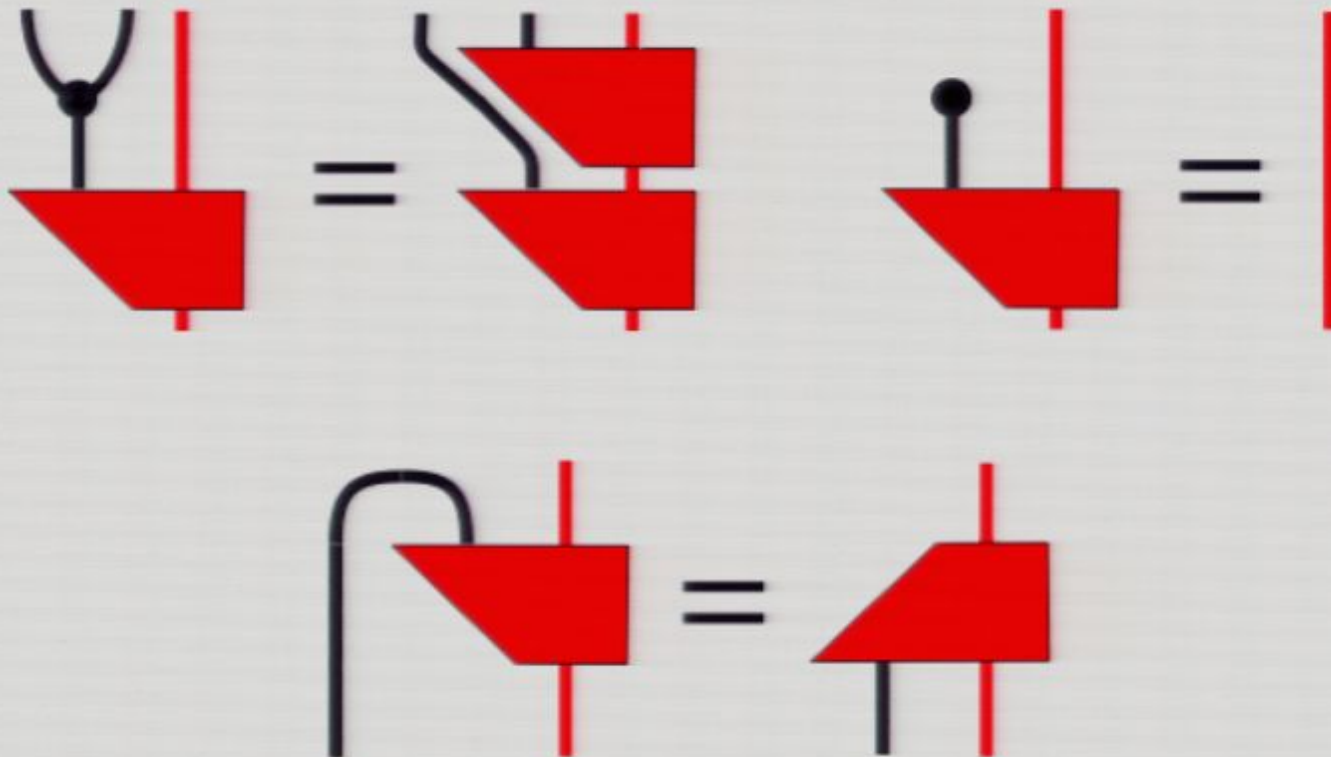
Minimal requirement II  $\Rightarrow$

Orthogonality of projectors

$$P_i^\dagger = P_i$$

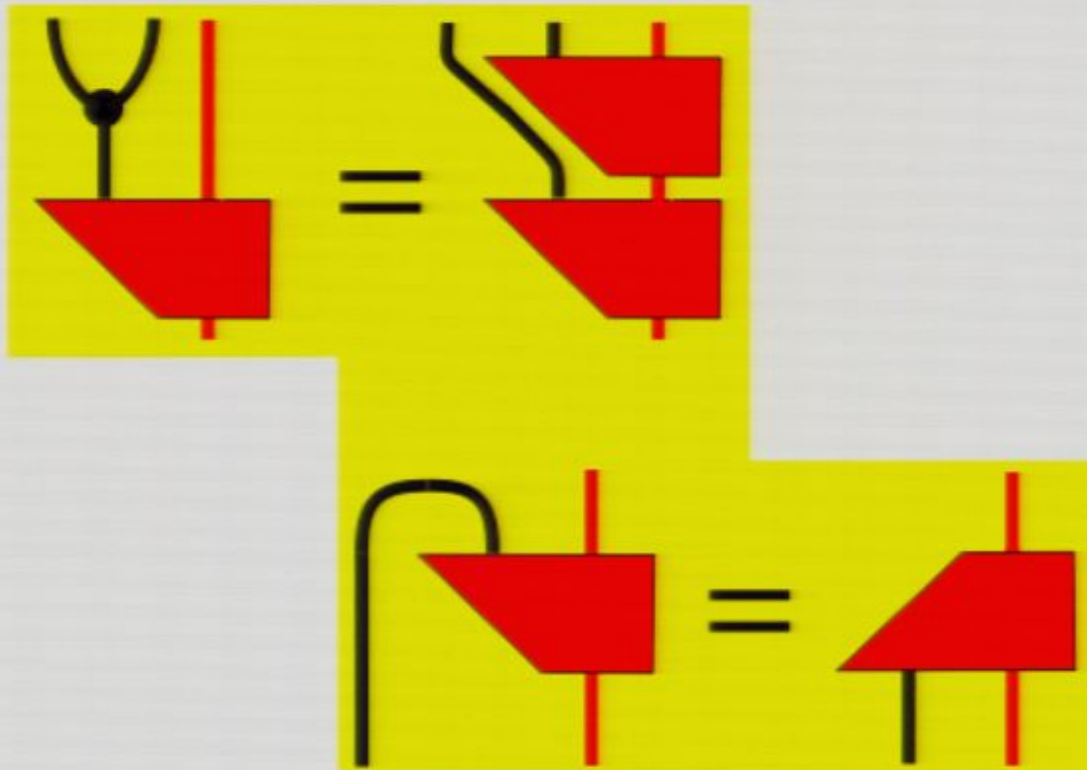
PROJECTOR  
 SPECTRUM

## Canonical observable :=



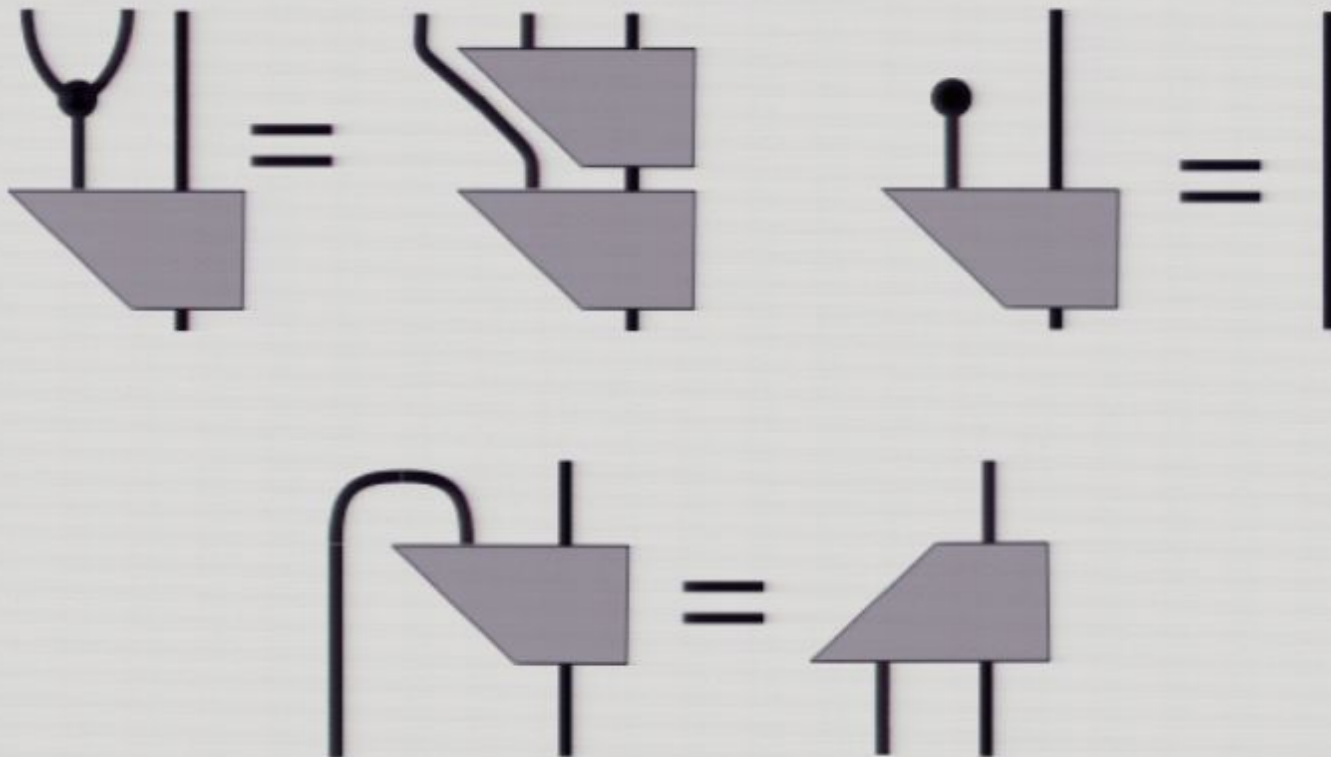


## Quantum measurement:

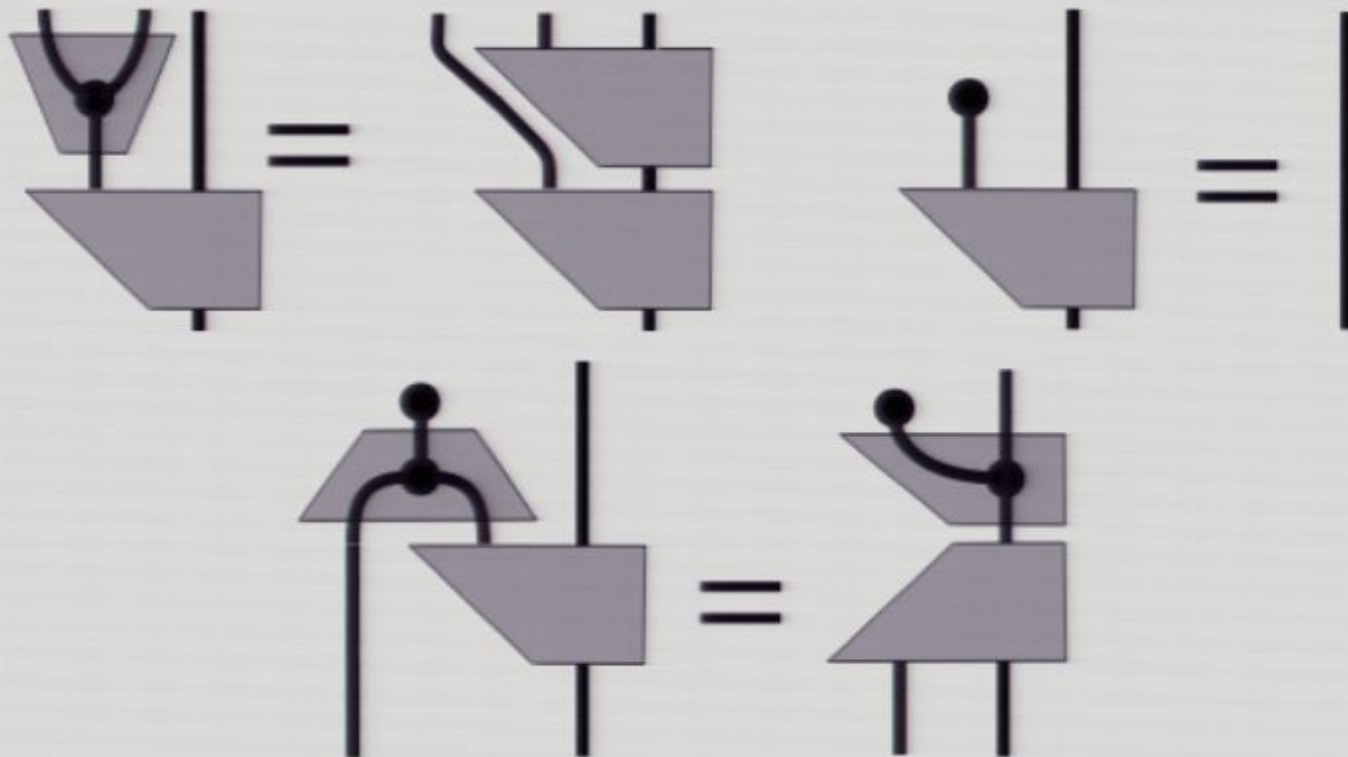


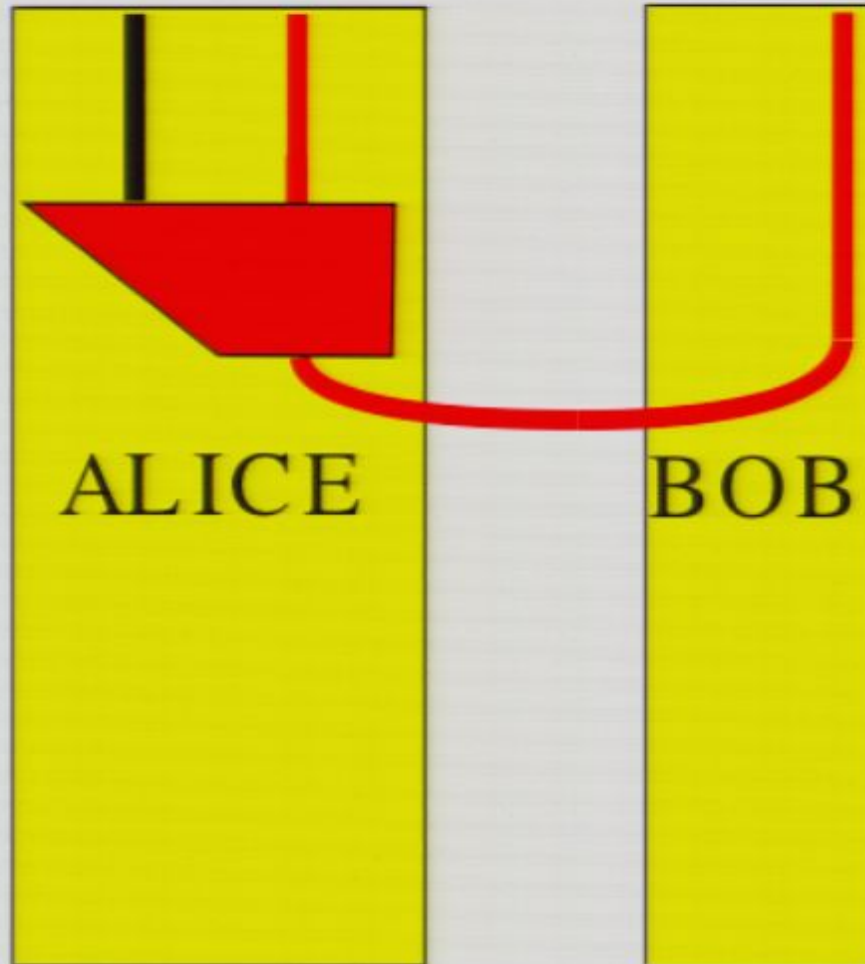
*Minimal requirements* for reasonable notion of measurement

## Canonical observable := Copying

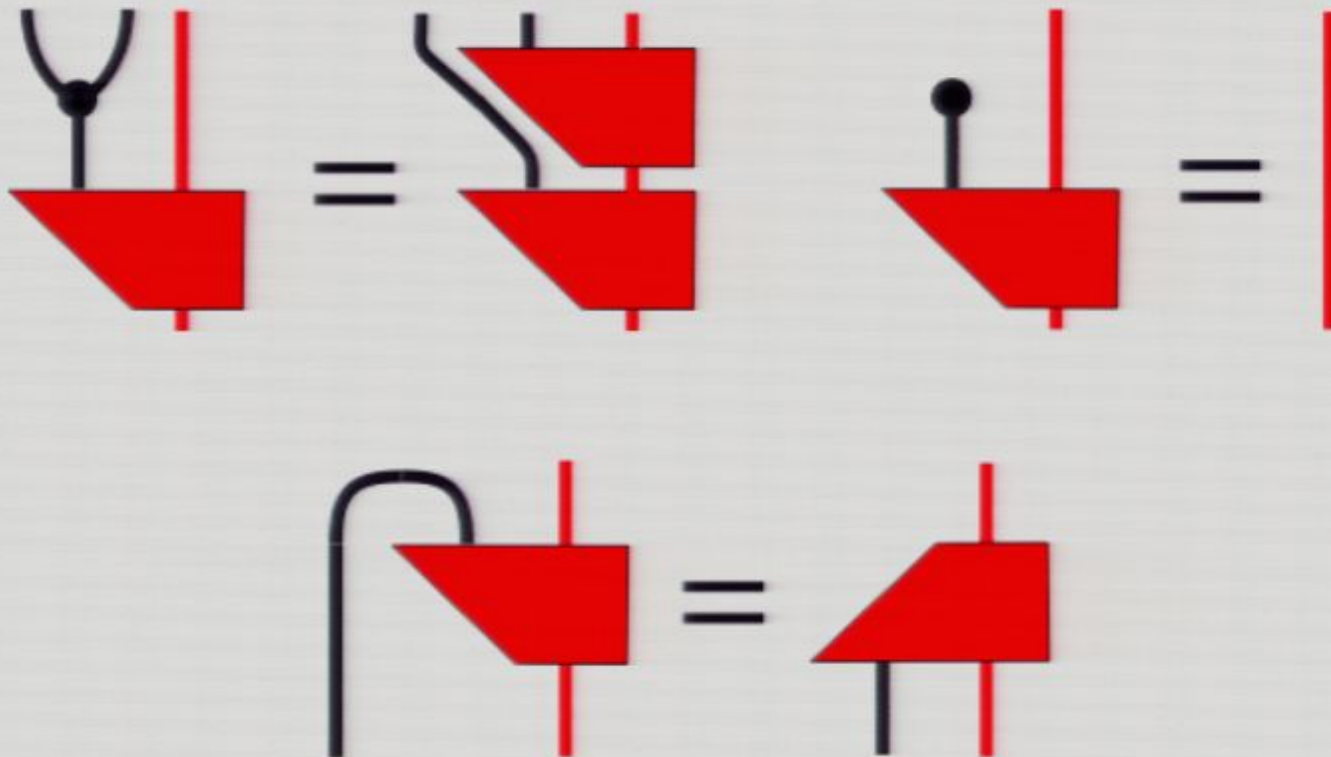


## Canonical observable := Copying

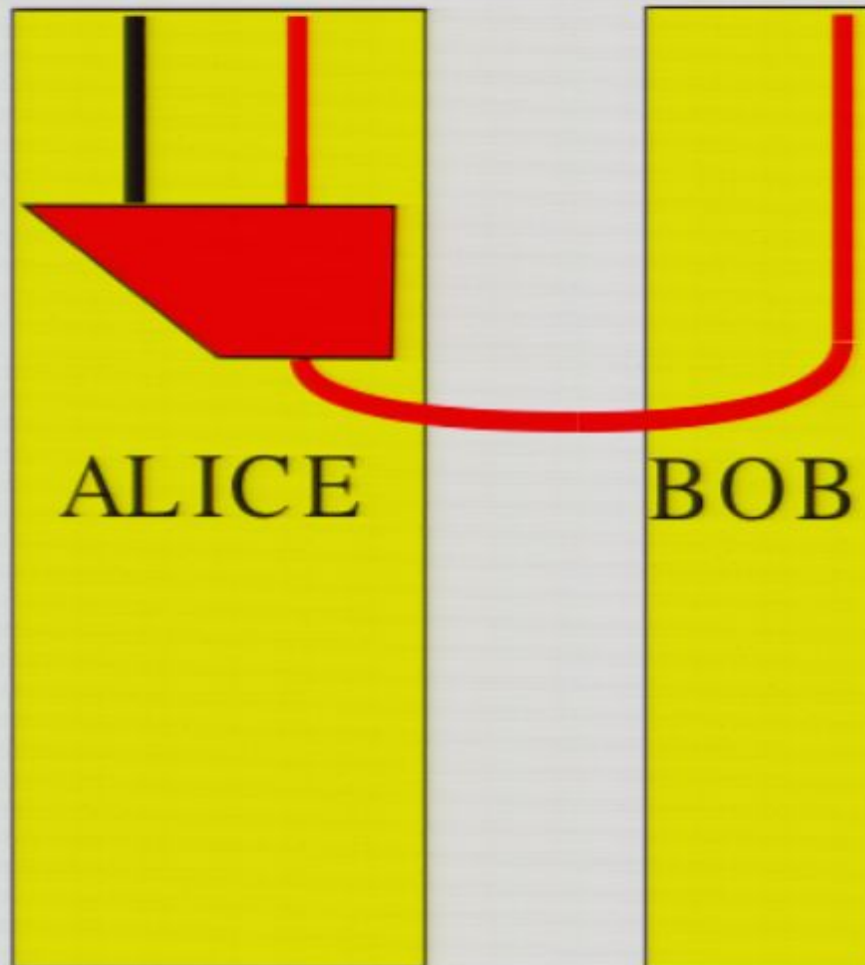




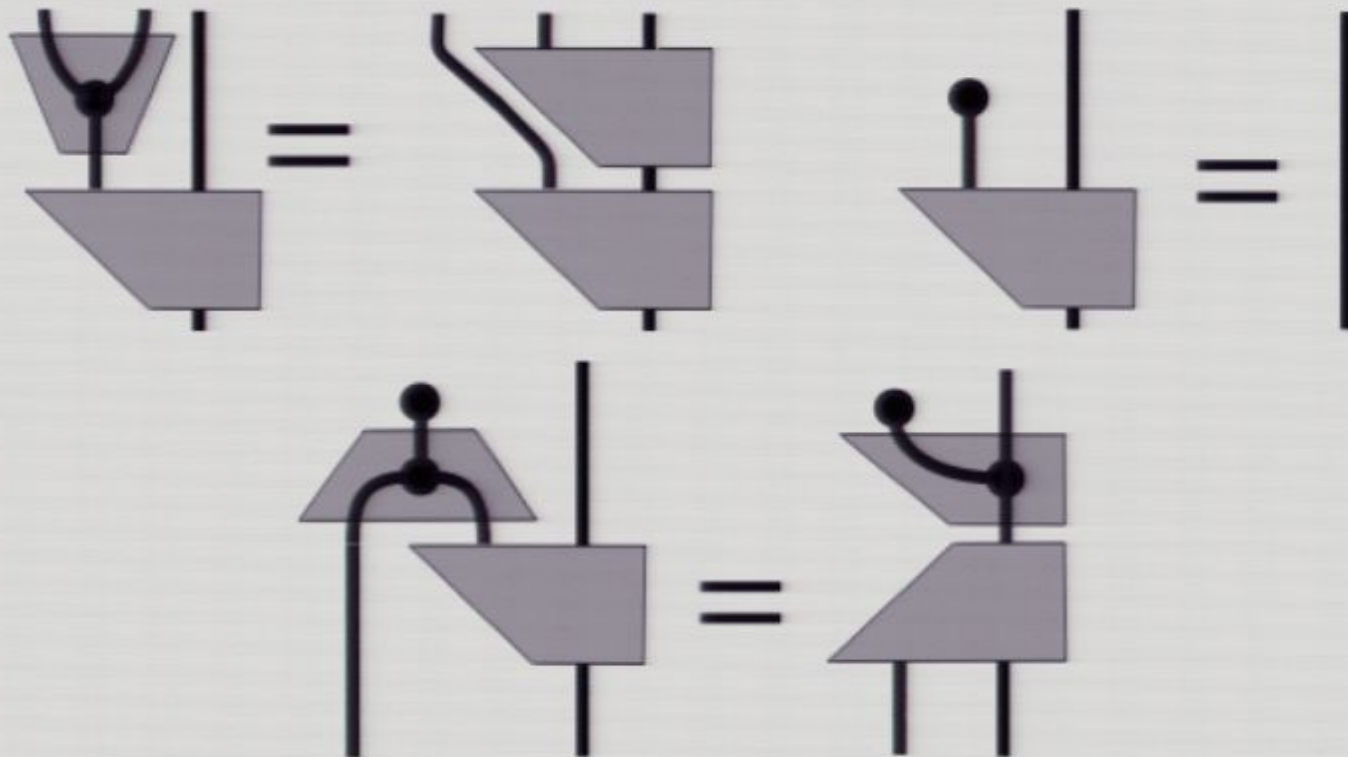
## Canonical observable :=



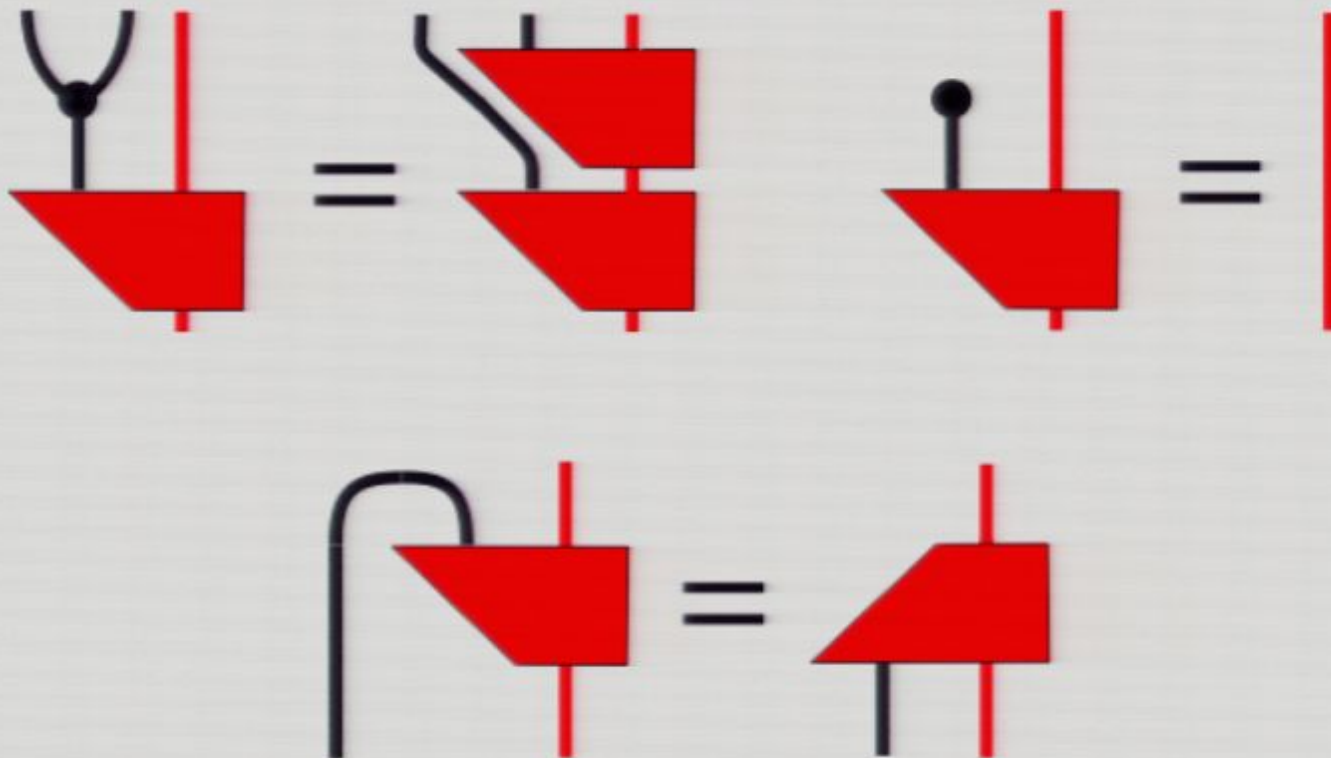


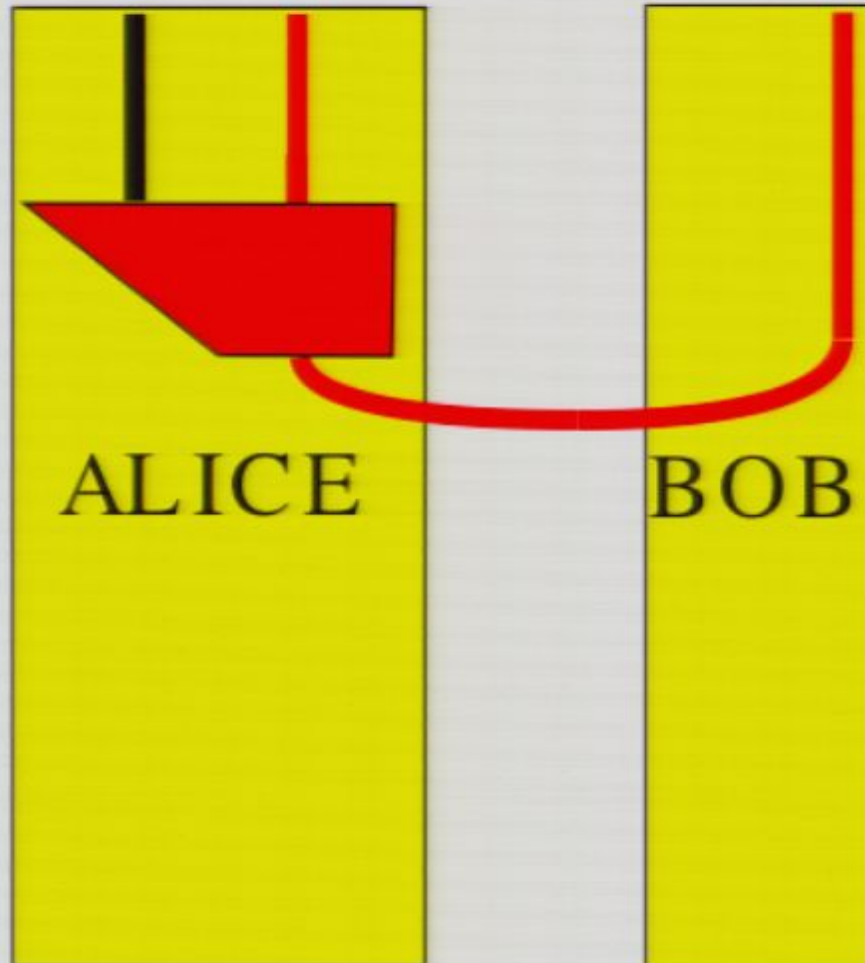


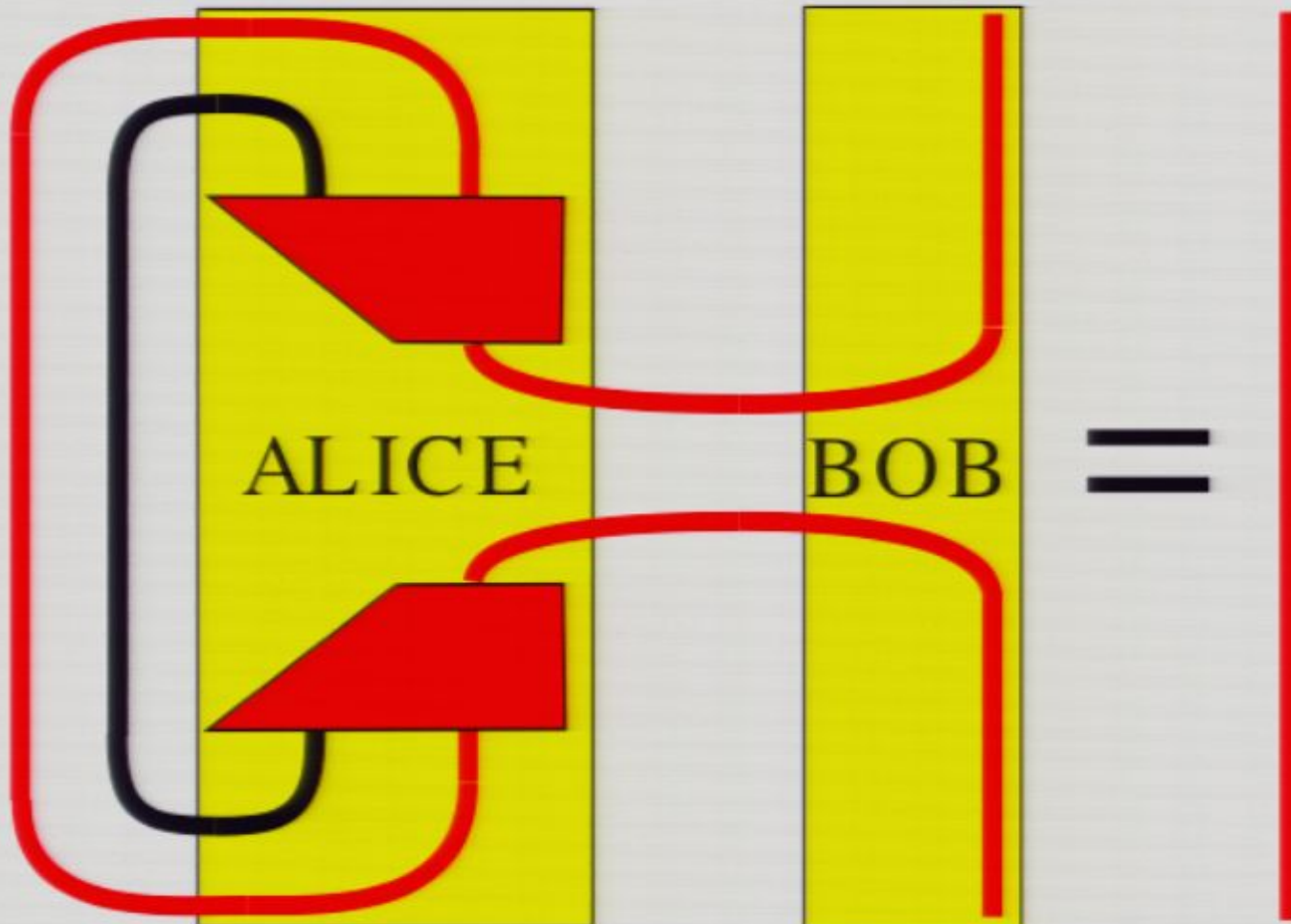
## Canonical observable := Copying



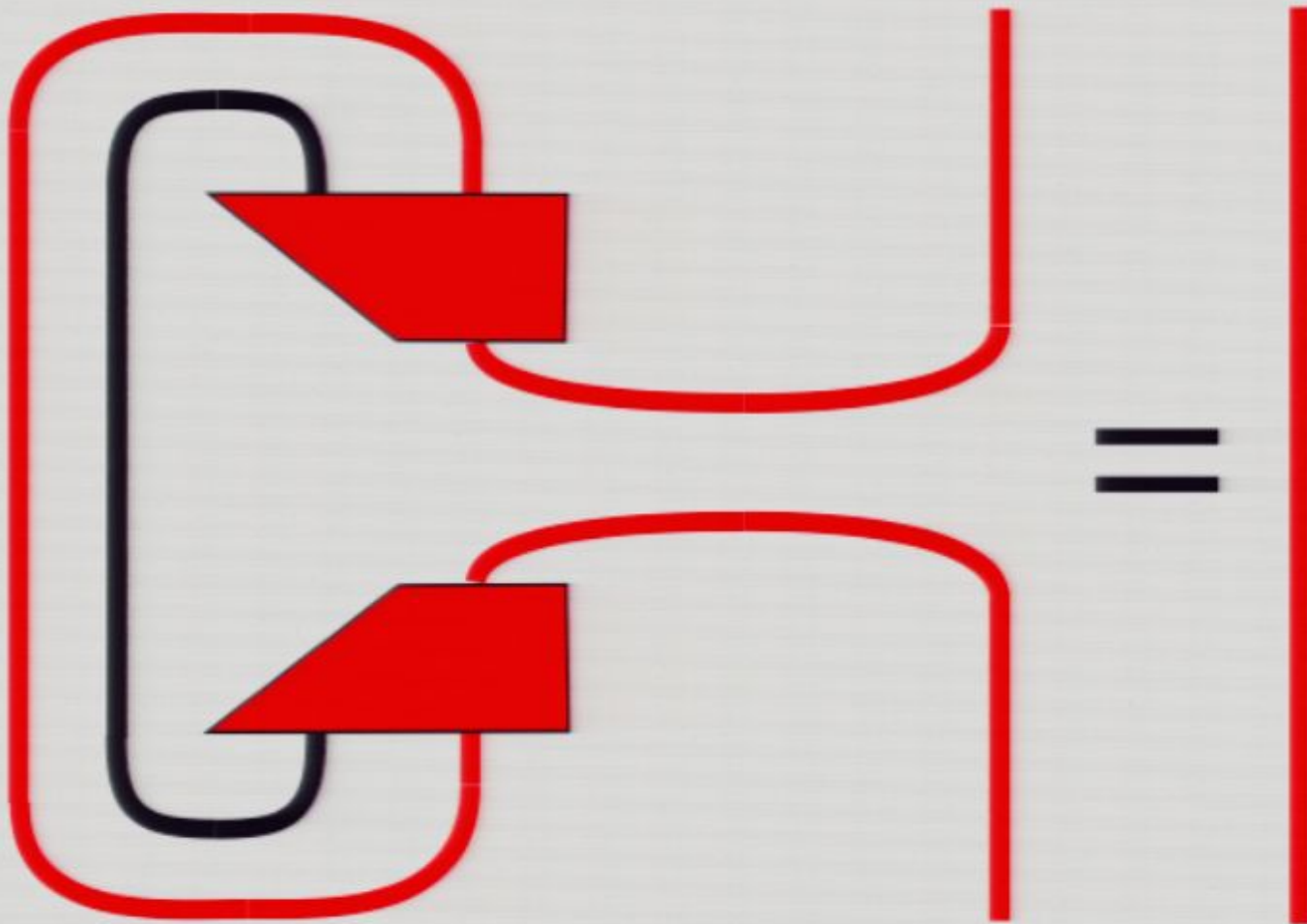
**Canonical observable :=**

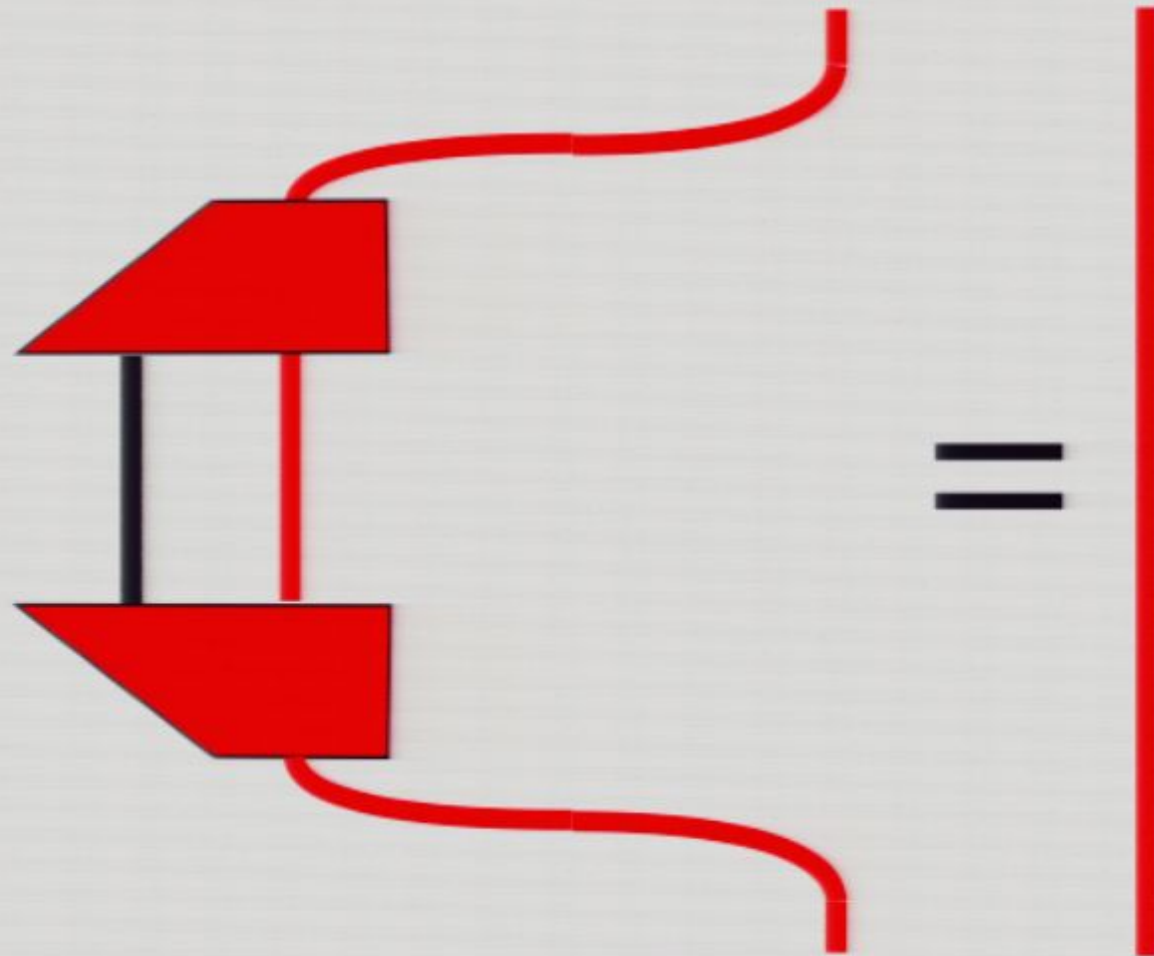


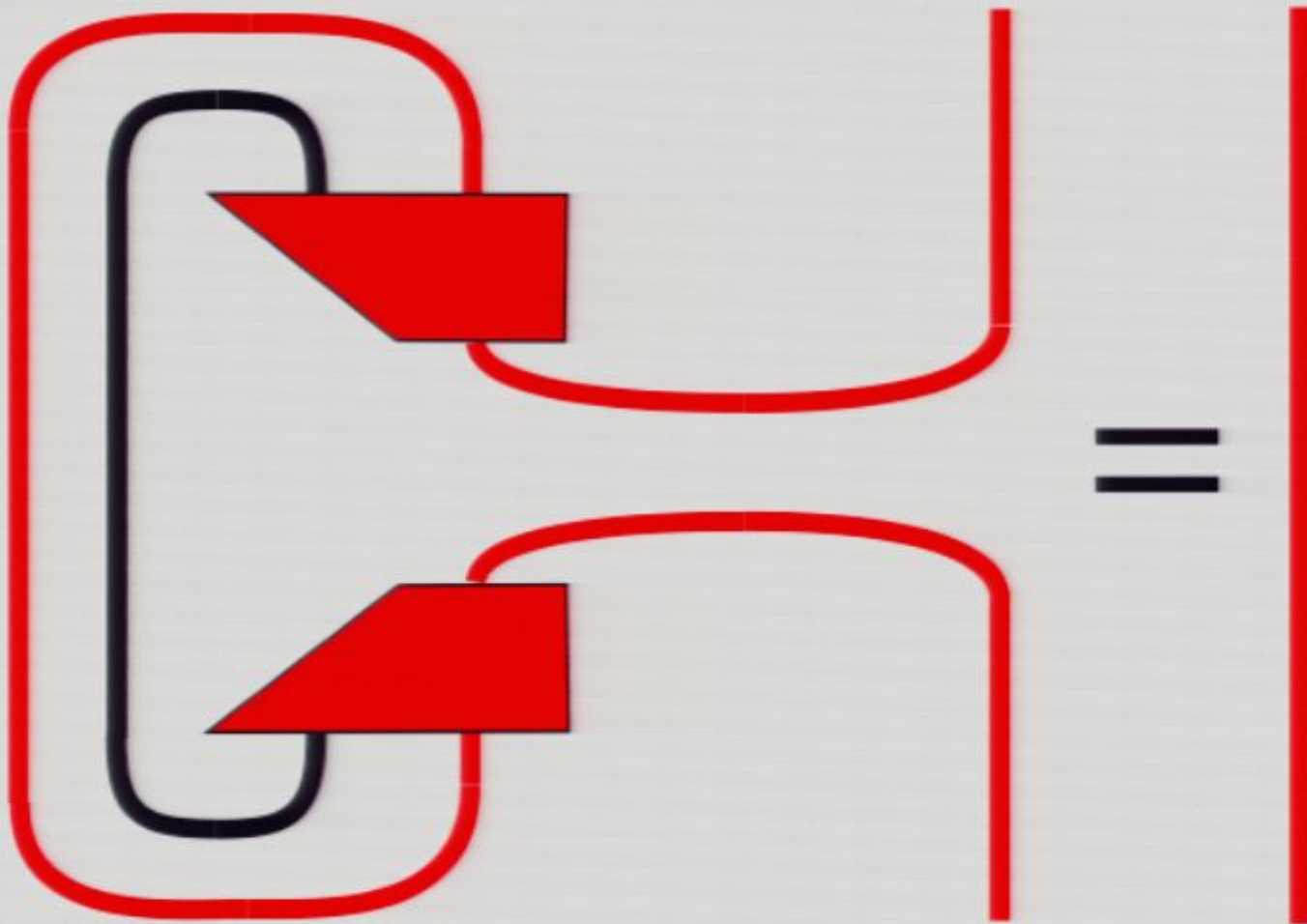


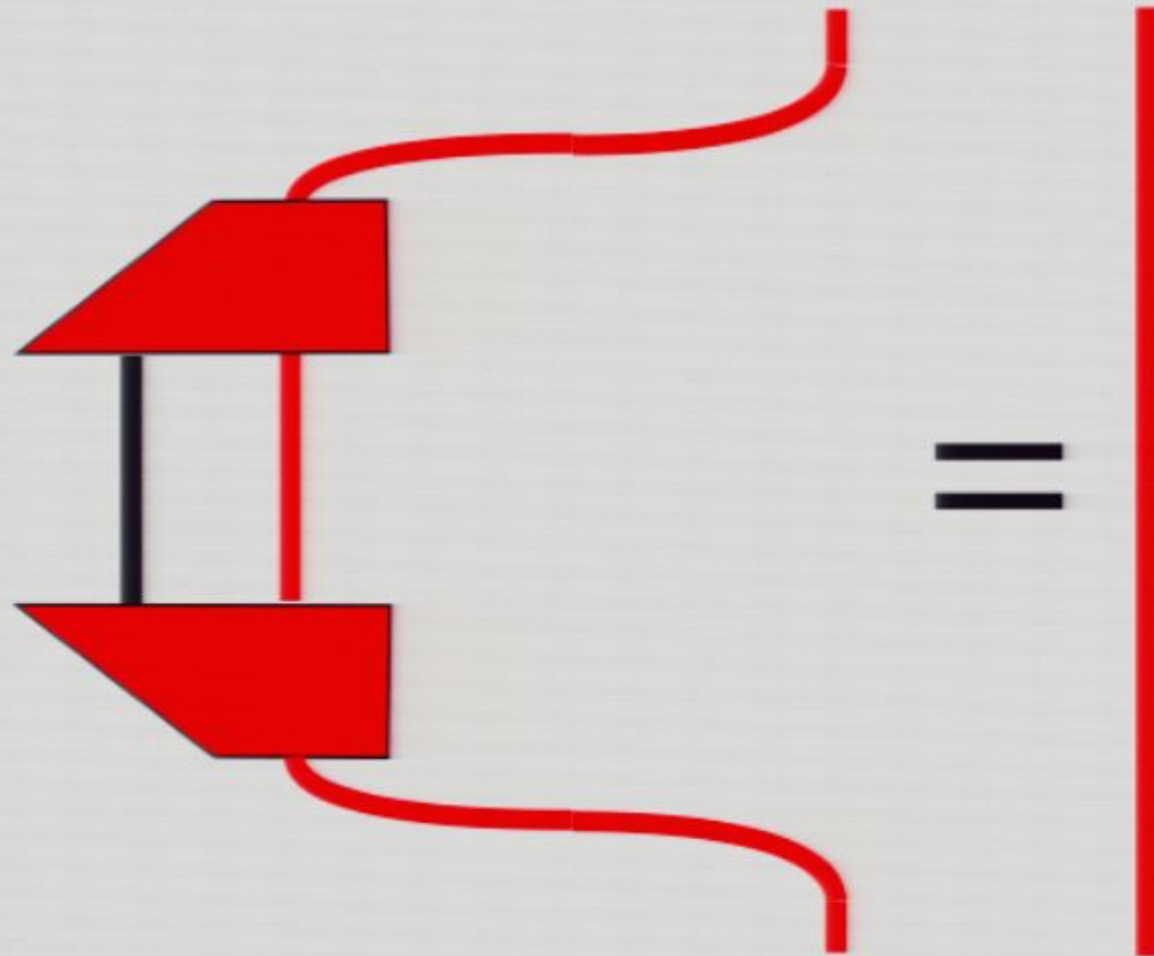


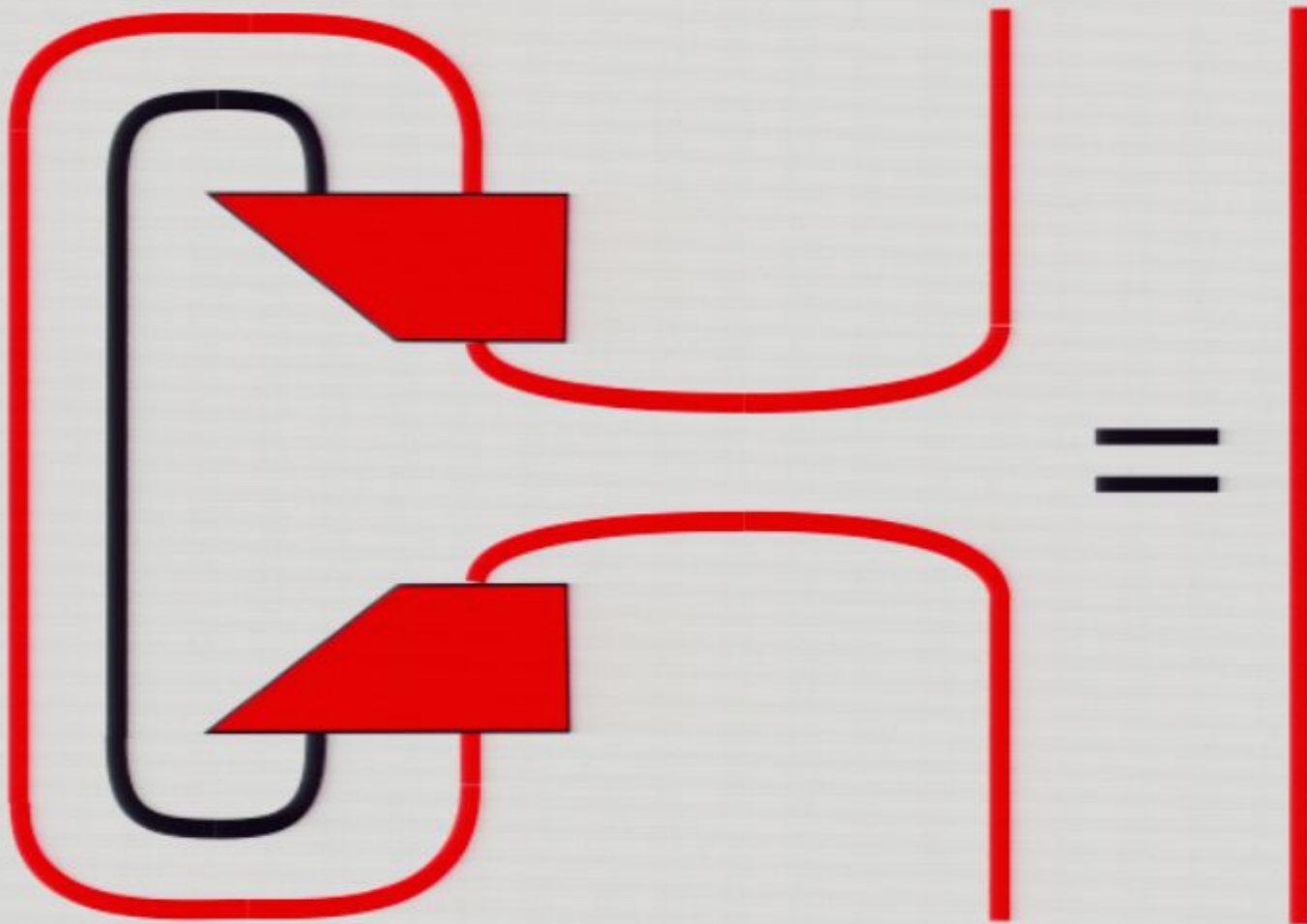




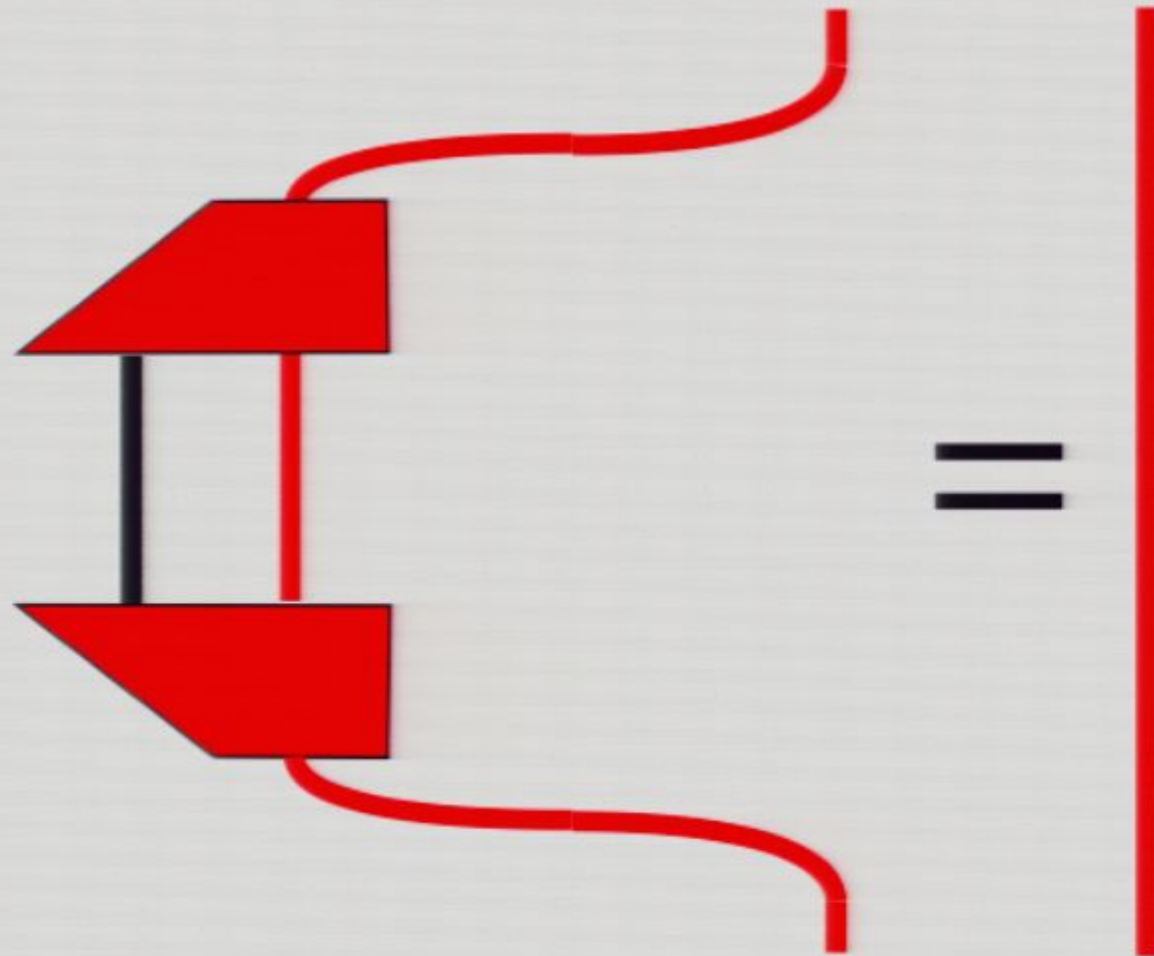


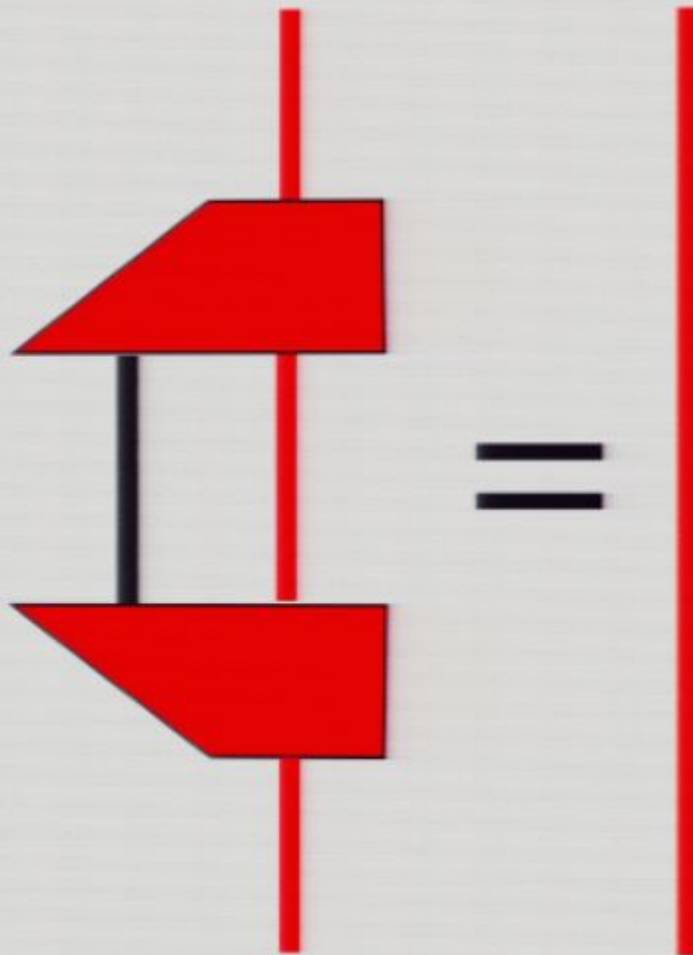


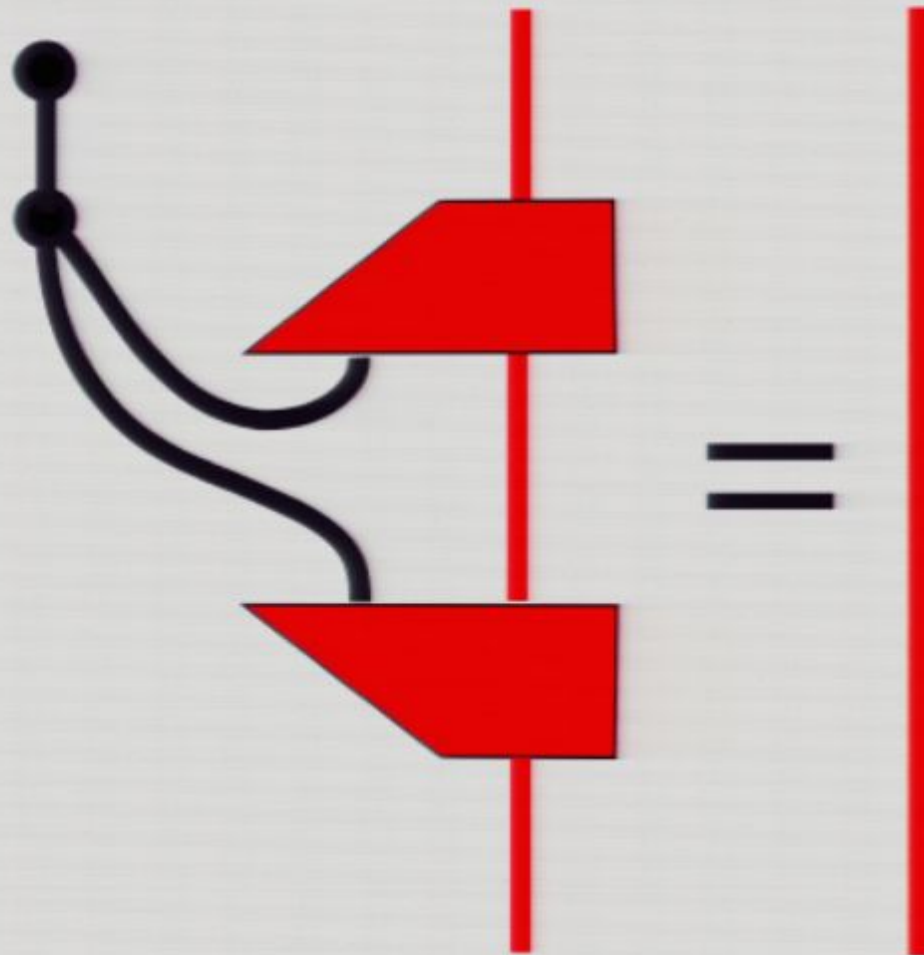


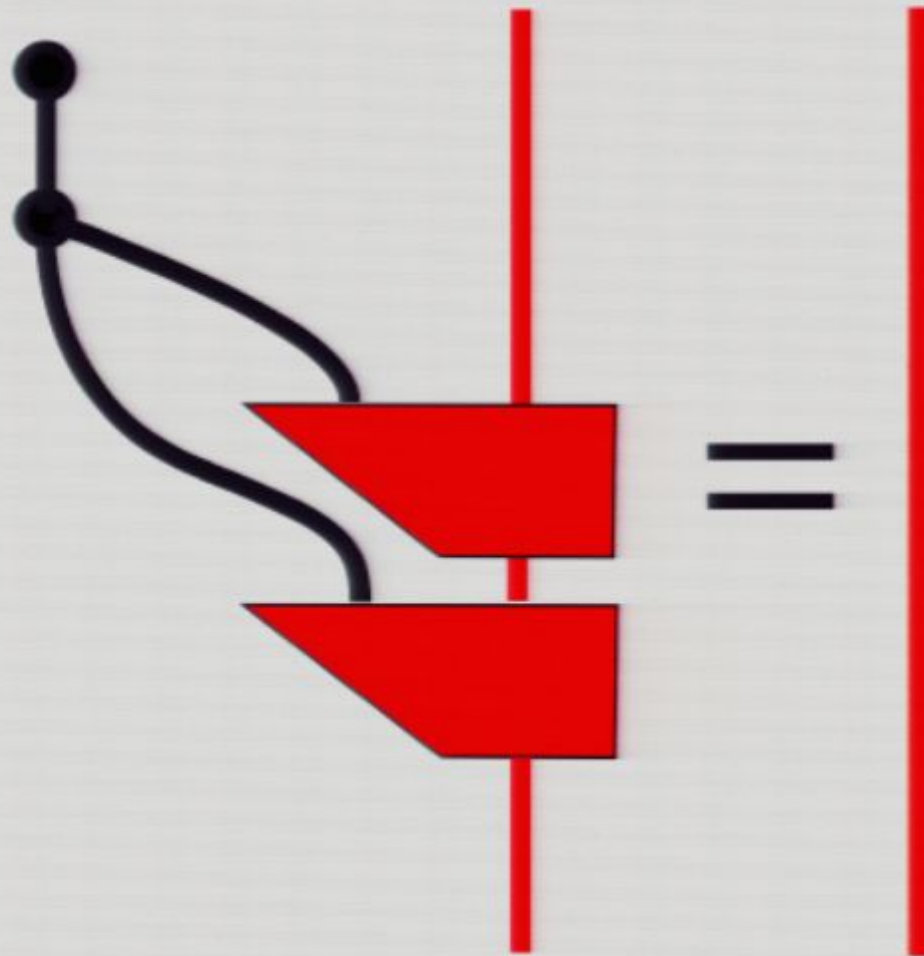


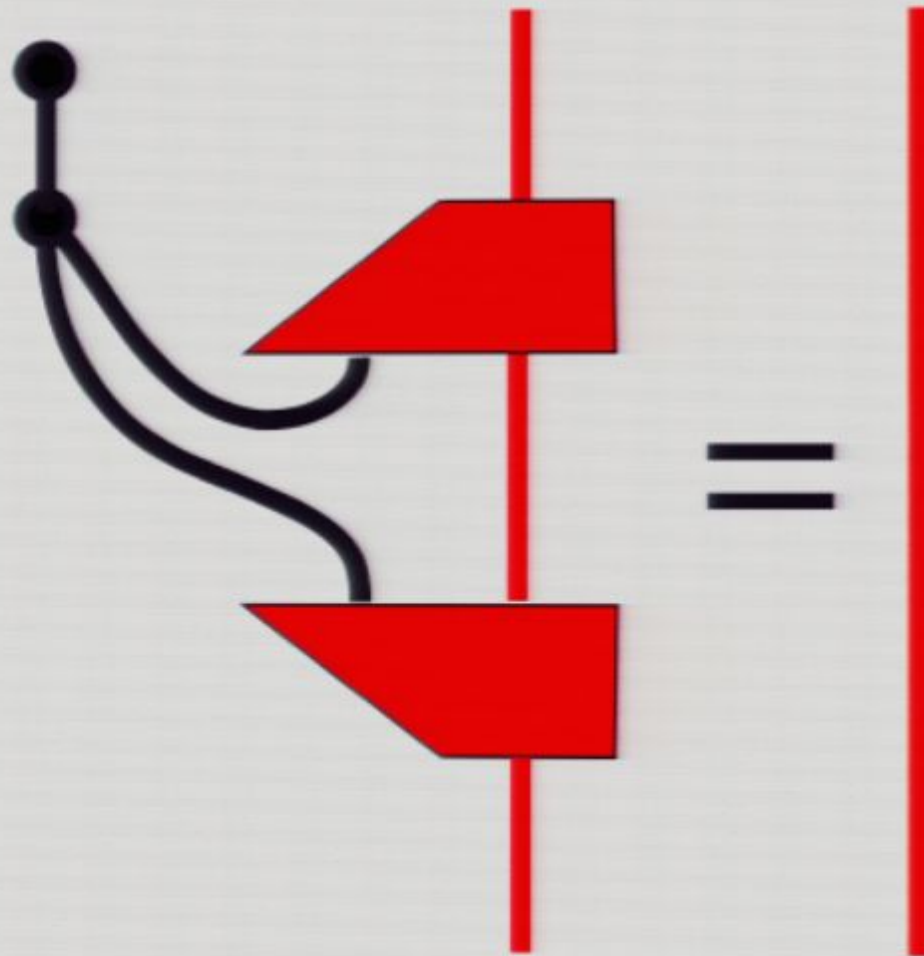




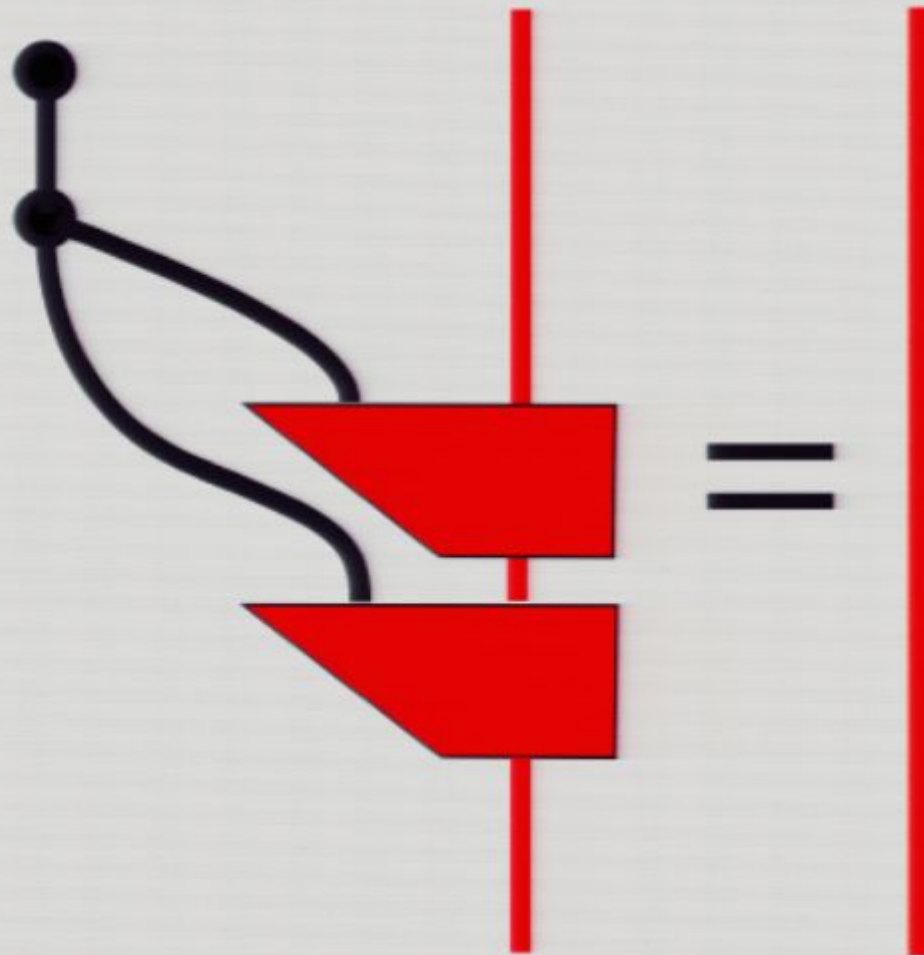


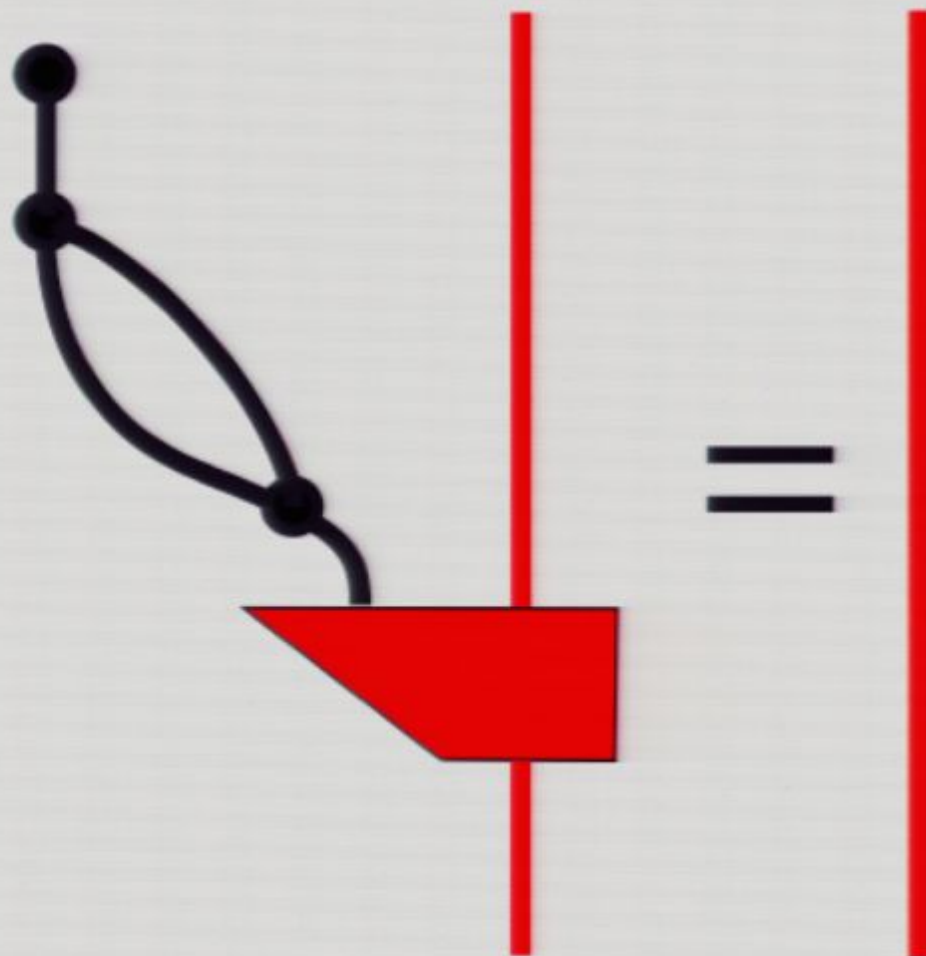


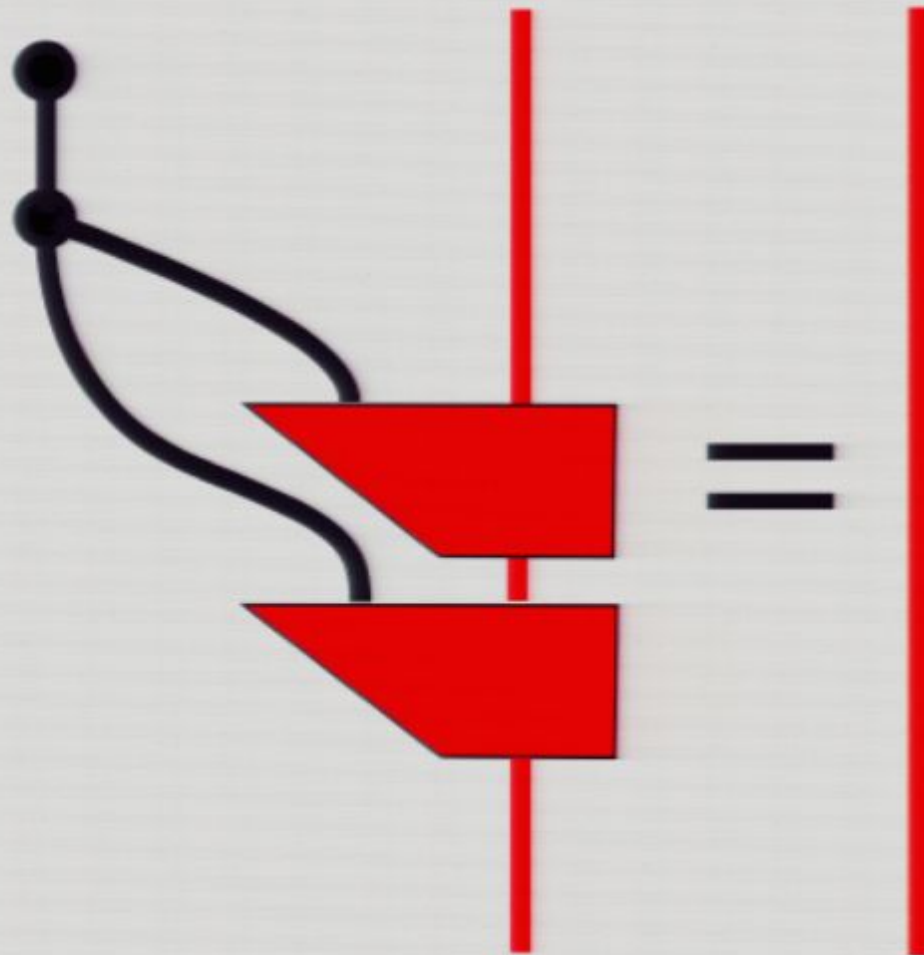


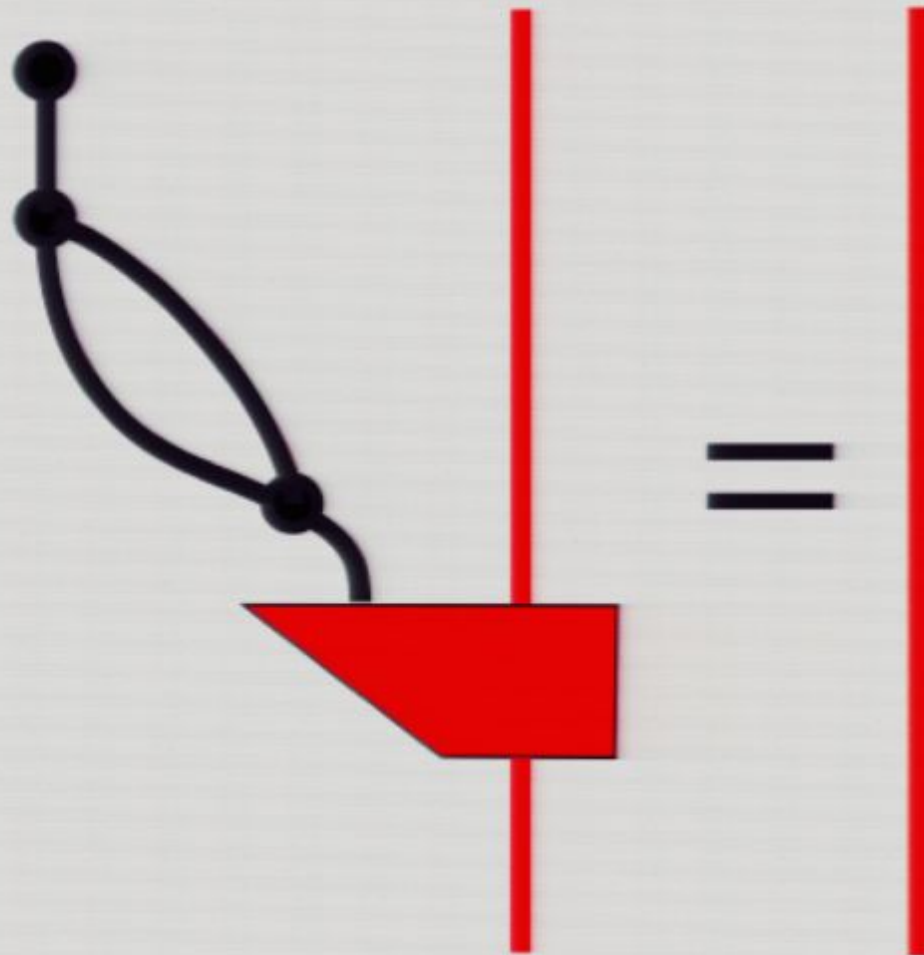


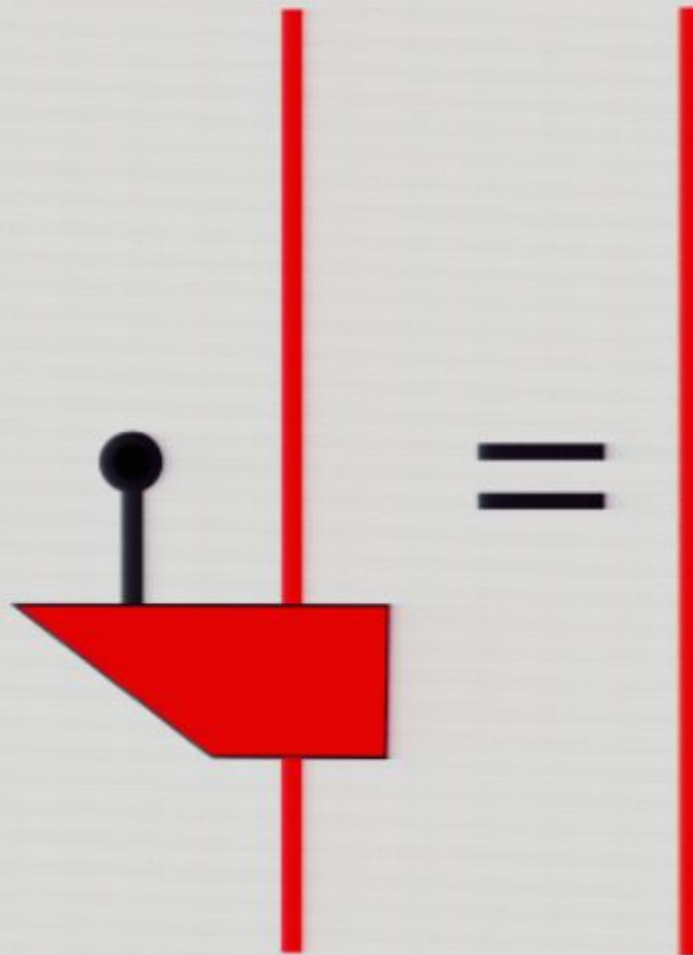






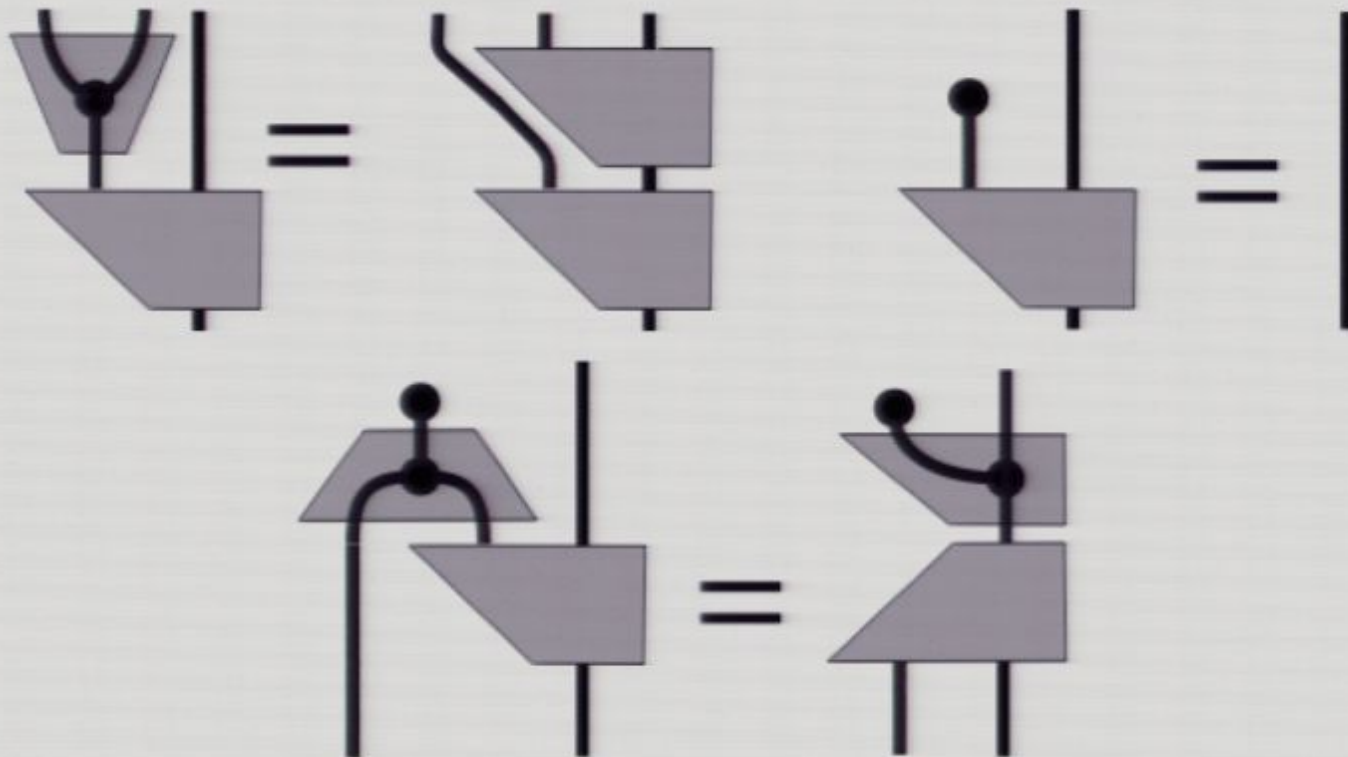




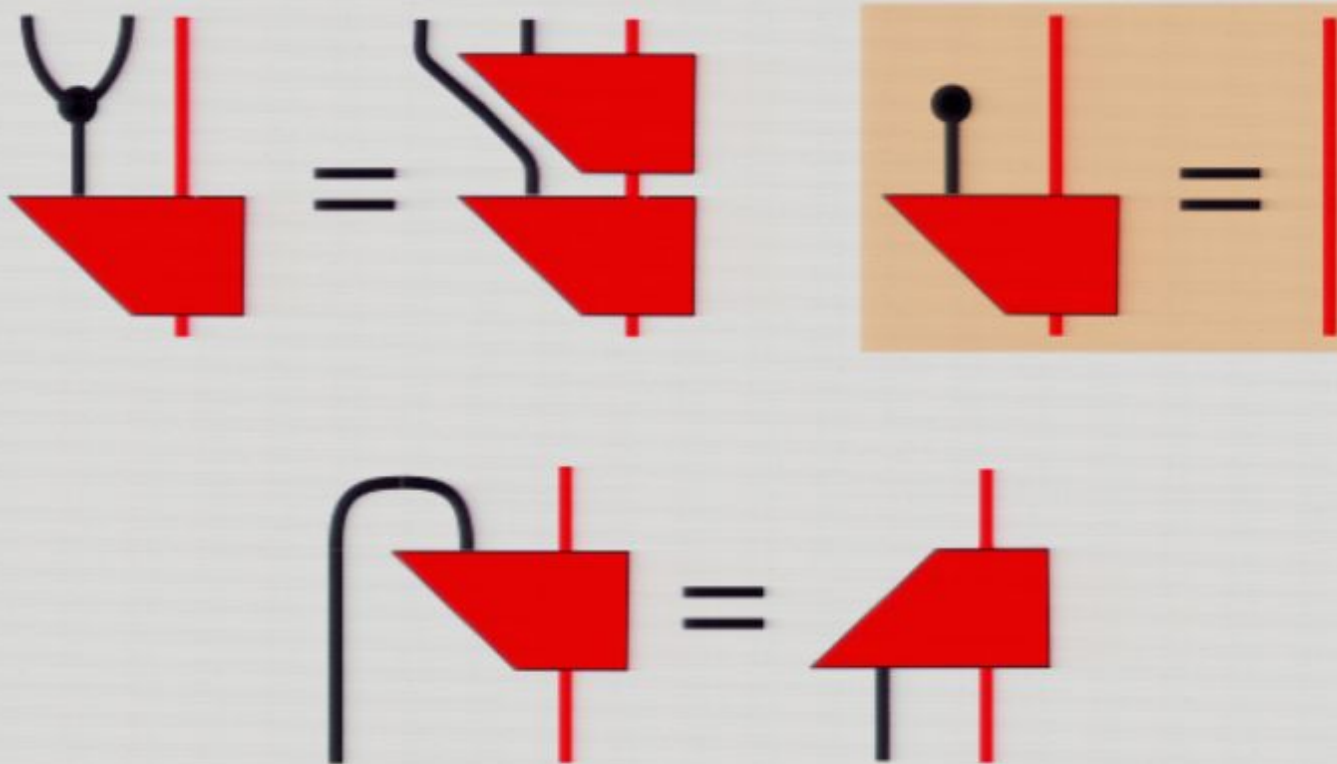




## Canonical observable := Copying



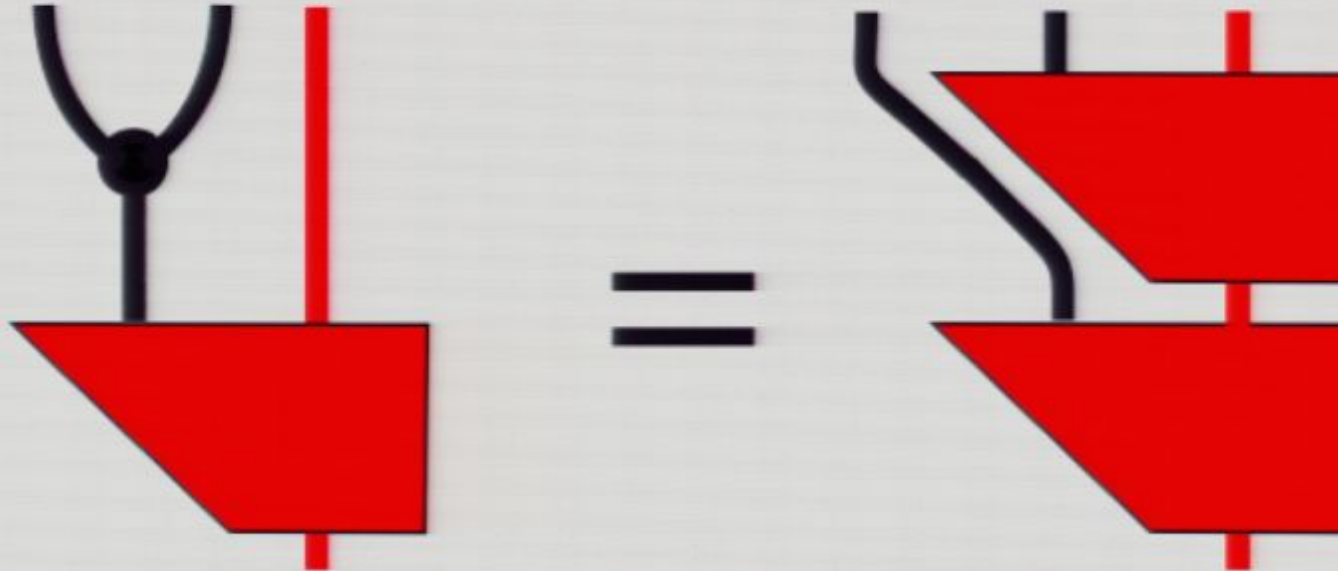
## Quantum measurement:



Asserts *no-signaling*

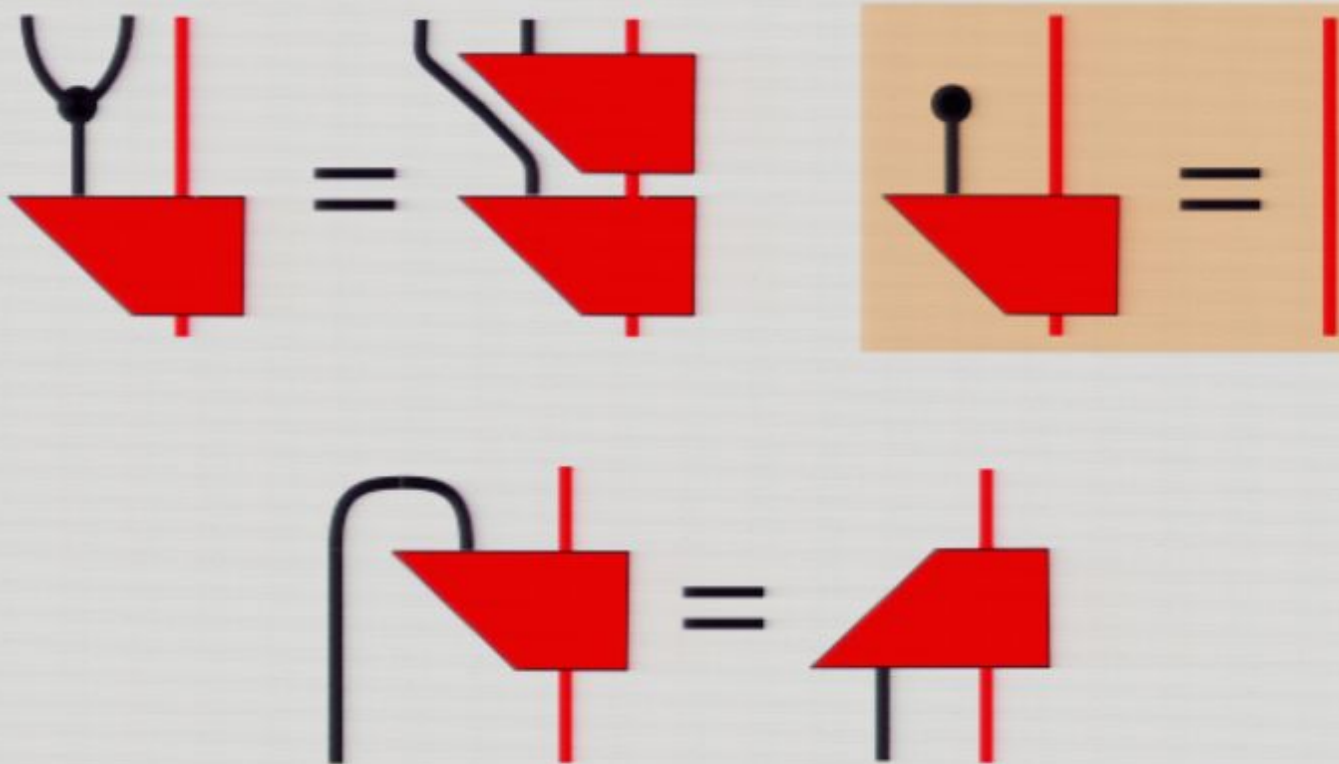
## Quantum measurement:

$$\mathcal{M} : A \rightarrow X \otimes A$$

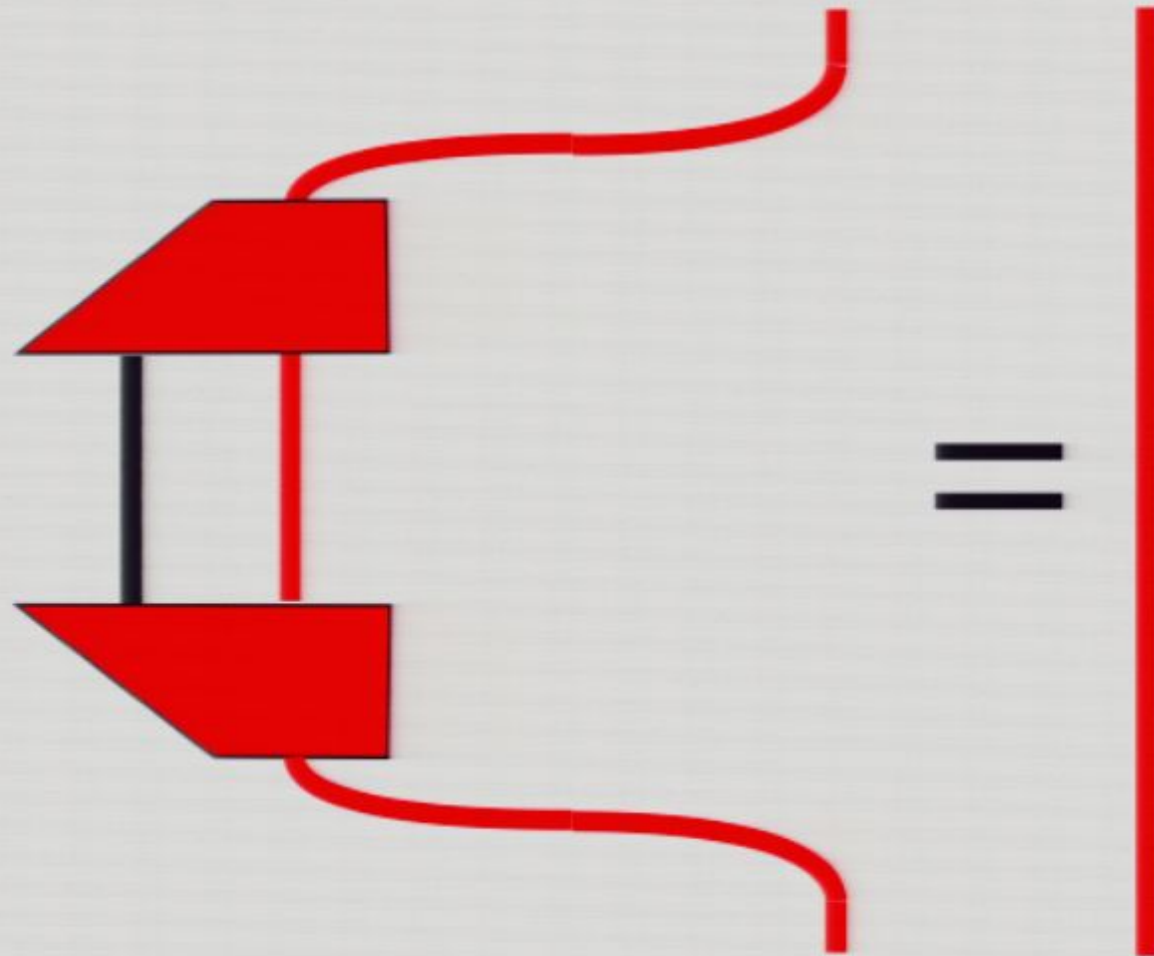


$\Rightarrow$  von Neumann projection postulate.

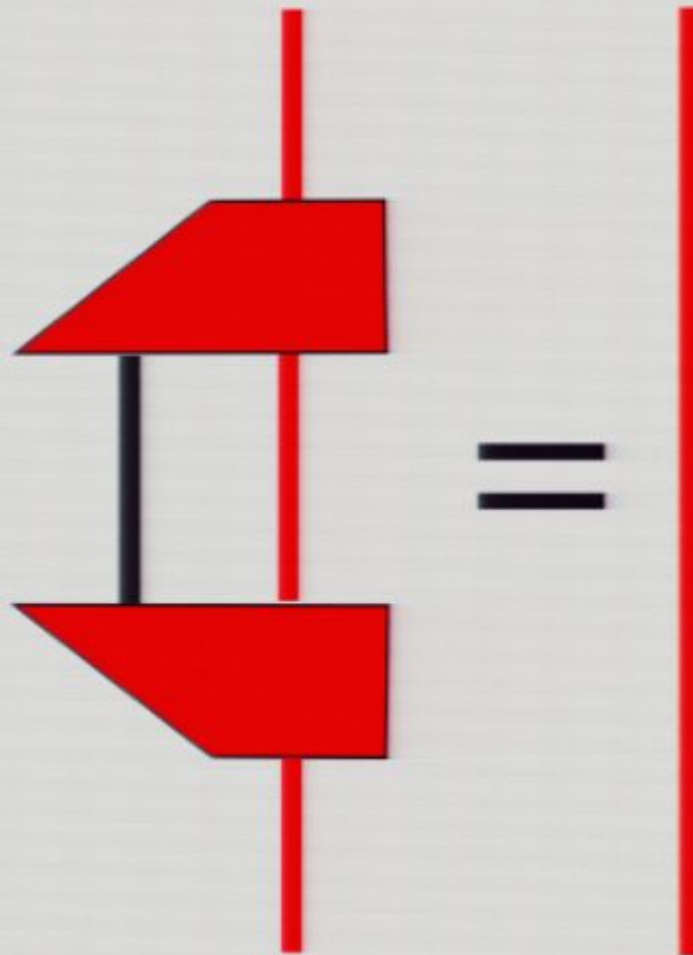
## Quantum measurement:



Asserts *no-signaling*



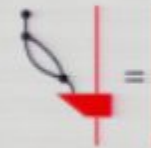




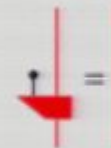
# EXPOSING CLASSICAL VS QUANTUM

How much QM can we recover without a priori  
suming instrumentalist concepts such as meas-  
ment and probability but just 'compoundness'

Search



Page 36



Page 37

EXPERIMENTAL CLASSICAL IN (QUANTUM)

Page 38

An *classical interface* is:

$$A \xrightarrow{\delta} A \otimes A = \text{cup}$$

$$A \xrightarrow{\varepsilon} I = \text{cap}$$

such that:

1.  $\varepsilon$  is a *unit* for  $\delta$ ;
2.  $\delta$  is *coassociative*;
3.  $\delta$  is *cocommutative*;
4.  $\delta$  is *isometry*;
5.  $\delta$  is *Frobenius*.

$$\text{cup with top dot} \stackrel{(1)}{=} | \stackrel{(4)}{=} \text{cap with bottom dot}$$

$$\text{cup with top dot, then cup} \stackrel{(2)}{=} \text{cup, then cup with top dot}$$

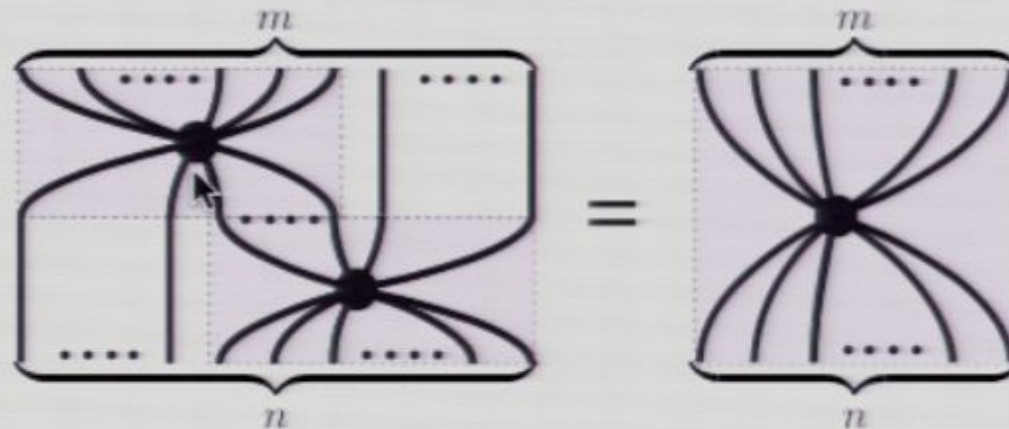
$$\text{cap with bottom dot, then cap} \stackrel{(3)}{=} \text{cap, then cap with bottom dot}$$

$$\text{cup with top dot, then cap with bottom dot} \stackrel{(5)}{=} \text{cap with bottom dot, then cup with top dot}$$

*A classical interface is:*

$$\left\{ \begin{array}{c} m \\ \text{---} \text{---} \text{---} \text{---} \\ \text{---} \text{---} \text{---} \text{---} \\ n \end{array} \right\} \mid n, m \in \mathbb{N}$$

invariant under flipping and swapping, and such that:





An classical interface is:

$$A \xrightarrow{\delta} A \otimes A = \text{cup}$$

$$A \xrightarrow{\varepsilon} I = \text{cap}$$

such that:

1.  $\varepsilon$  is a *unit* for  $\delta$ ;
2.  $\delta$  is *coassociative*;
3.  $\delta$  is *cocommutative*;
4.  $\delta$  is *isometry*;
5.  $\delta$  is *Frobenius*.

$$\text{cup with dot} \stackrel{(1)}{=} | \stackrel{(4)}{=} \text{cap with dot}$$

$$\text{cup with dot and cap} \stackrel{(2)}{=} \text{cap with dot and cup}$$

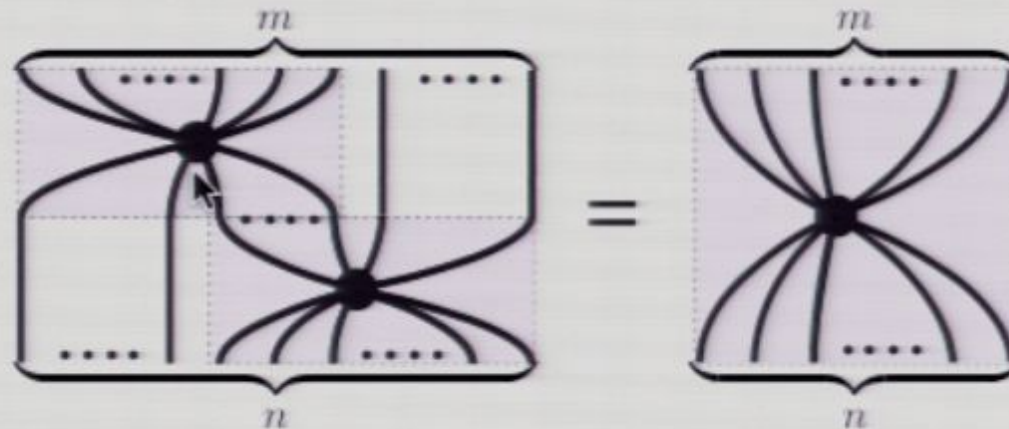
$$\text{cup with dot and cap with dot} \stackrel{(3)}{=} \text{cup with dot}$$

$$\text{cup with dot and cap with dot and cap} \stackrel{(5)}{=} \text{cup with dot and cap with dot}$$

A classical interface is:

$$\left\{ \begin{array}{c} m \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ n \end{array} \right\} \mid n, m \in \mathbb{N}$$

invariant under flipping and swapping, and such that:



An classical interface is:

$$A \xrightarrow{\delta} A \otimes A = \text{cup}$$

$$A \xrightarrow{\varepsilon} I = \text{cap}$$

such that:

1.  $\varepsilon$  is a *unit* for  $\delta$ ;
2.  $\delta$  is *coassociative*;
3.  $\delta$  is *cocommutative*;
4.  $\delta$  is *isometry*;
5.  $\delta$  is *Frobenius*.

$$\text{cup} \stackrel{(1)}{=} | \stackrel{(4)}{=} \text{cap}$$

$$\text{cup} \circ \text{cup} \stackrel{(2)}{=} \text{cup} \circ \text{cup}$$

$$\text{cap} \circ \text{cap} \stackrel{(3)}{=} \text{cap} \circ \text{cap}$$

$$\text{cup} \circ \text{cap} \stackrel{(5)}{=} \text{cap} \circ \text{cup}$$

**How much QM can we recover without a priori assuming instrumentalist concepts such as measurement and probability but just ‘compoundness’?**



