Title: Deformed gauge theories and their string duals

Date: May 19, 2009 11:00 AM

URL: http://pirsa.org/09050009

Abstract: Gauge theories with deformed products of fields in the lagrangian constitute an interesting generalization of the gauge/string duality.
We review a systematic procedure to find the string duals of such theories, called the TsT transformation, and illustrate its properties by means of a few examples.

Perimeter Institute, May 19th, 2009

Deformed gauge theories and their string duals

Emiliano Imeroni

Université Libre de Bruxelles

based on `hep-th/0612032` (with *Asad Naqvi*), `arXiv:0808.1271` + work in progress

Outline

Introduction and motivations

The gauge/string duality and its *generalizations*

Gauge theory lagrangians with *deformed products*

String duals and the *TsT transformation*

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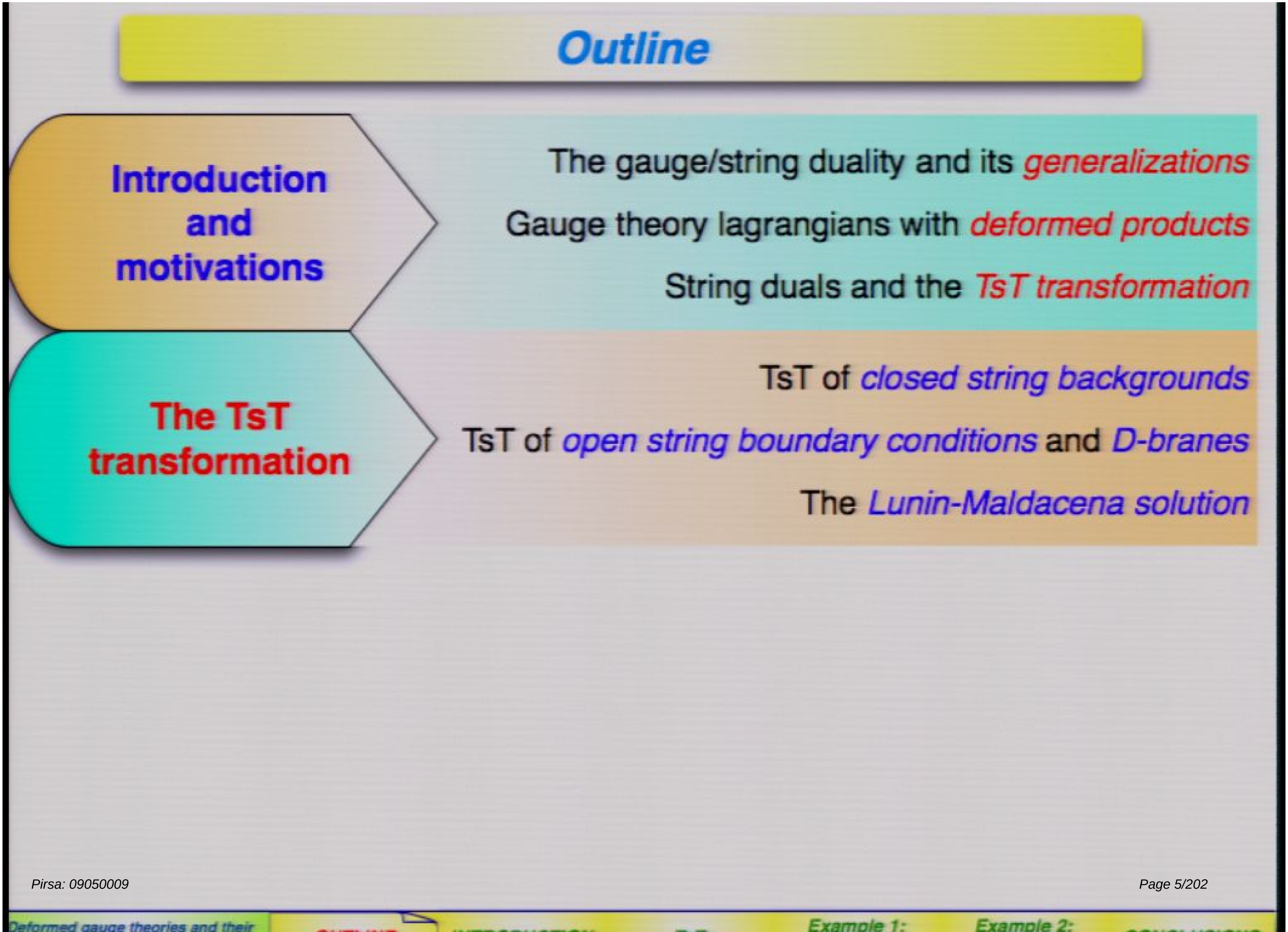
The TsT transformation

- TsT of *closed string backgrounds*
- TsT of *open string boundary conditions* and *D-branes*
- The *Lunin-Maldacena solution*



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Example 1:
β-deformed ABJM theory

ABJM theory and its string/M-theory dual

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Example 2:
Unquenched SQCD

- *Dipole deformation* of fivebrane theories and *SQCD*
- TsT of the *string dual of unquenched SQCD*

Deforming the gauge/string duality

AdS / CFT CORRESPONDENCE

Type IIB strings *on* $AdS_5 \times S^5$ $\xleftrightarrow{\text{EXACT DUALITY}}$ $\mathcal{N} = 4 \quad U(N)$ **Super Yang-Mills theory**

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Many possible GENERALIZATIONS ⇨

Study generalizations that arise when considering ***gauge theories*** with ***deformed products of fields*** in the lagrangian

$$\Phi_1(x)\Phi_2(x) \longrightarrow \Phi_1(x) \star \Phi_2(x)$$

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Non-commutative deformation

$$\Phi_1(x)\Phi_2(x) \to \Phi_1(x) \star \Phi_2(x) = e^{\frac{i}{2}\theta^{ij}\partial_{\xi^i}\partial_{\zeta^j}}\,\Phi_1(x+\xi)\Phi_2(x+\zeta)|_{\xi=\zeta=0}$$

$$= \Phi_1(x)\Phi_2(x) + \tfrac{i}{2}\theta^{ij}\partial_i\Phi_1\partial_j\Phi_2 + \ldots$$

$\Rightarrow$ $[x^i, x^j] = i\theta^{ij}$ Yang-Mills on the ***non-commutative torus***

Non-covariant, *non-causal*, *non-local*

Arises in string theory with a ***background B-field***

[Connes-Rieffel, Connes-Douglas-Schwarz, Seiberg-Witten, ...]

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$$\Phi_1(x)\Phi_2(x) \rightarrow \Phi_1(x) \star \Phi_2(x) = \Phi_1(x - \tfrac{L_2}{2})\Phi_2(x + \tfrac{L_1}{2})$$

$${L_a}^{\mu} = Q_a L^{\mu}$$ "DIPOLE VECTOR" OF A FIELD OF CHARGE Q_a

Non-local but *commutative*

[Bergman-Dasgupta-Ganor-Karczmarek-Rajesh]

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Dipole deformations along the *light-cone* give rise to *Schrödinger symmetric* theories whose DLCQ yields *(conformal) non-relativistic systems*

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Compactified dipole theories can give rise to *ordinary theories* with interesting properties

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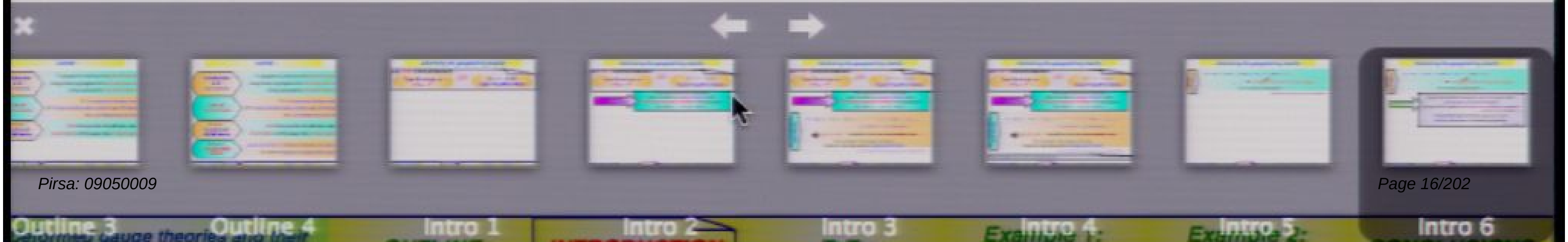

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Ordinary theory with *deformed interactions*

[Leigh-Strassler, ...]

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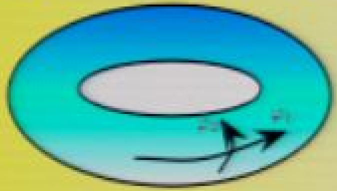
Ordinary theory with *deformed interactions*

All of these apparently unrelated theories can be seen in a unified framework as ***deformations*** *of ordinary Yang-Mills theory by* ***higher dimension gauge invariant operators***

Deforming the gauge/string duality

GRAVITY DUAL

From the point of view of the *gravity dual* the field theory deformation corresponds to a transformation on a ***torus*** [Lunin-Maldacena]



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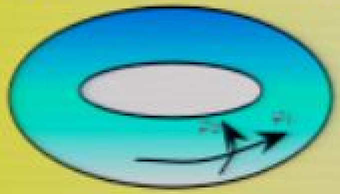

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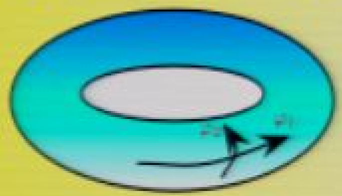

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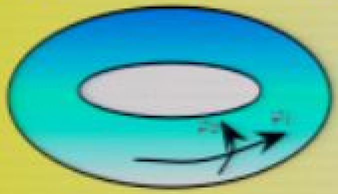

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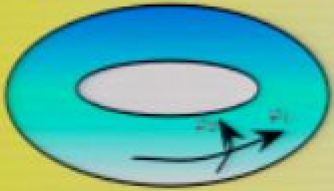

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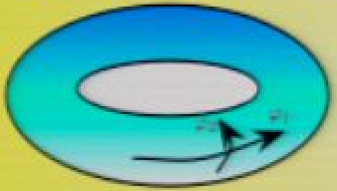

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Start with a gauge theory on a ***Dp-brane***: where is the ***TsT torus*** located?

	x^1	x^2	y^1	y^2
Dp	—	—	●	●

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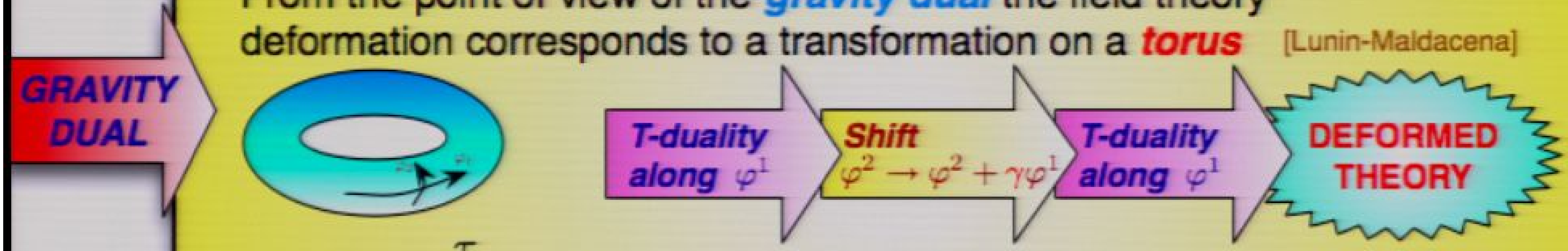

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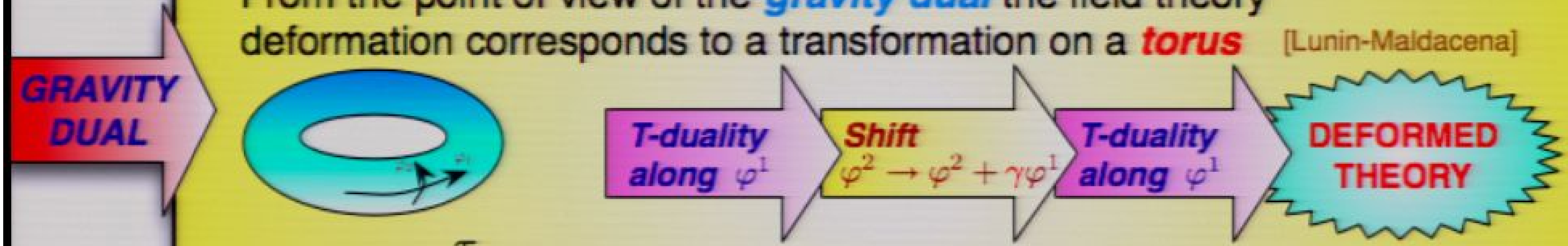

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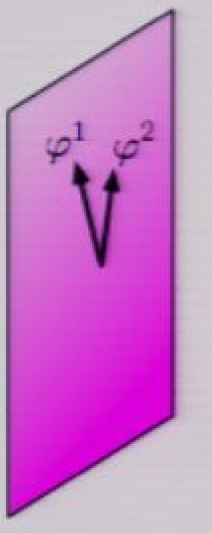

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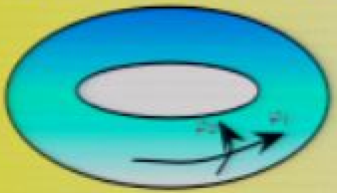

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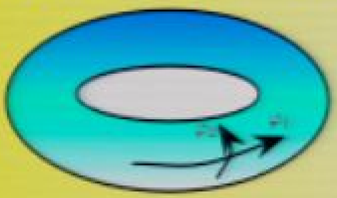

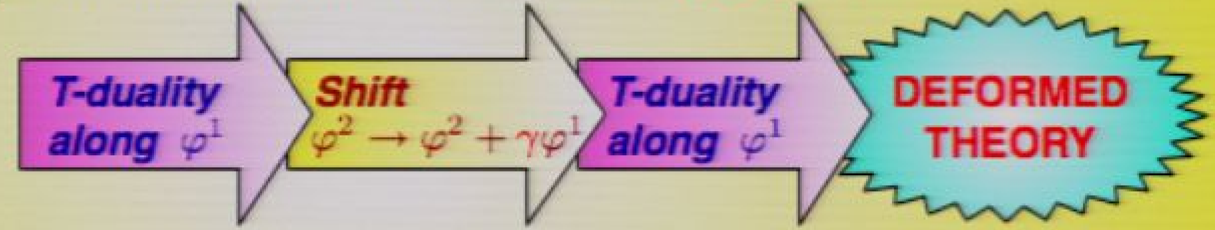

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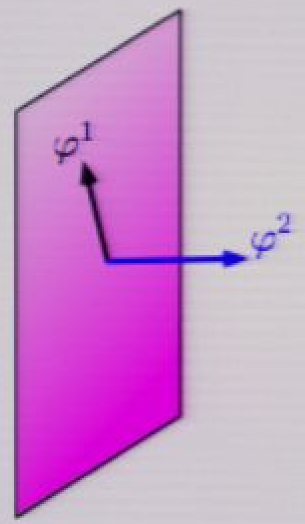

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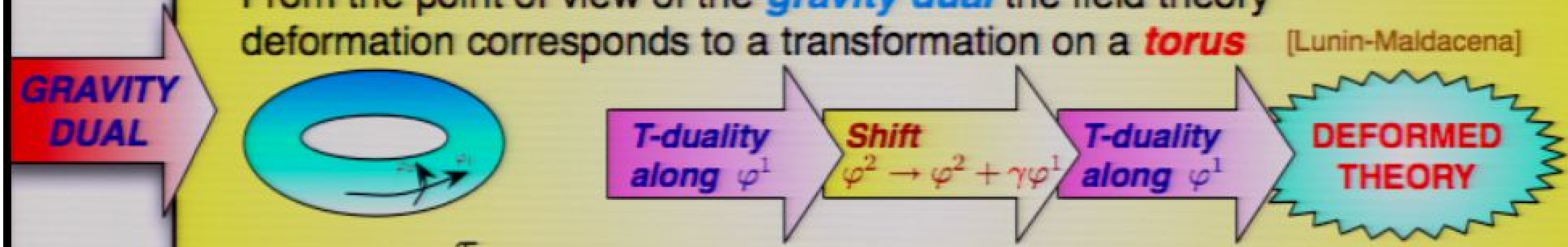

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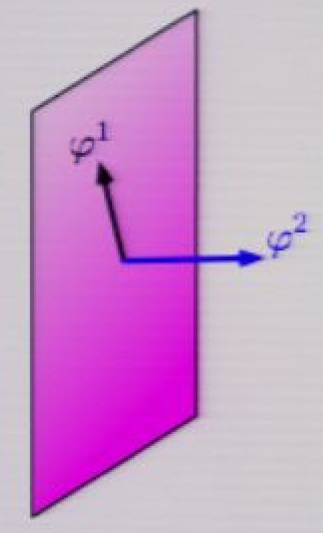

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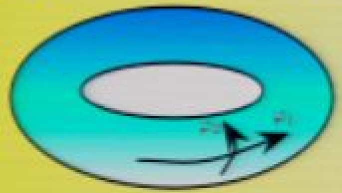

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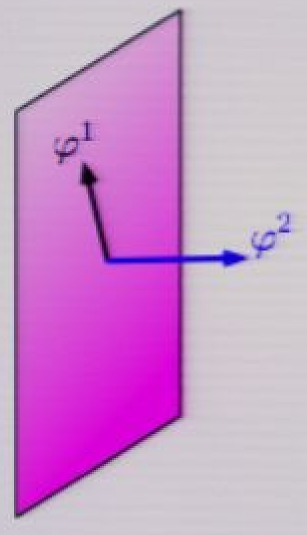

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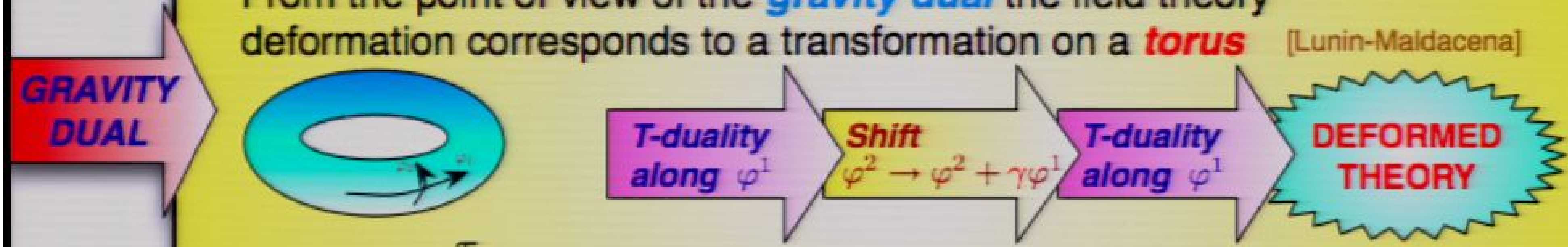
→ **DIPOLE THEORY**

Deforming the gauge/string duality

From the point of view of the *gravity dual* the field theory deformation corresponds to a transformation on a ***torus*** [Lunin-Maldacena]



GRAVITY DUAL

T-duality along φ^1 → ***Shift*** $\varphi^2 \to \varphi^2 + \gamma\varphi^1$ → ***T-duality along*** φ^1 → **DEFORMED THEORY**

$$\tau \to \tau' = \frac{\tau}{1+\gamma\tau}$$

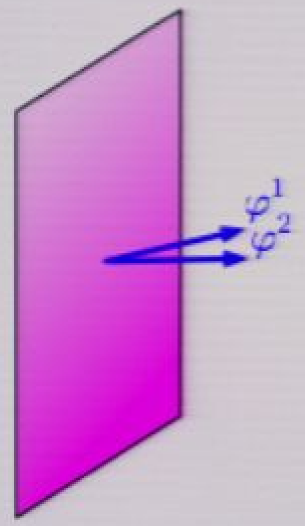
We can study these ***TsT deformations systematically!***

Start with a gauge theory on a ***Dp-brane***: where is the ***TsT torus*** located?

NON-COMMUTATIVE

	x^1	x^2	y^1	y^2
Dp	–	–	●	●

DIPOLE



$$f \star g = e^{i\pi\gamma(p_1^f p_2^g - p_2^f p_1^g)} f g$$

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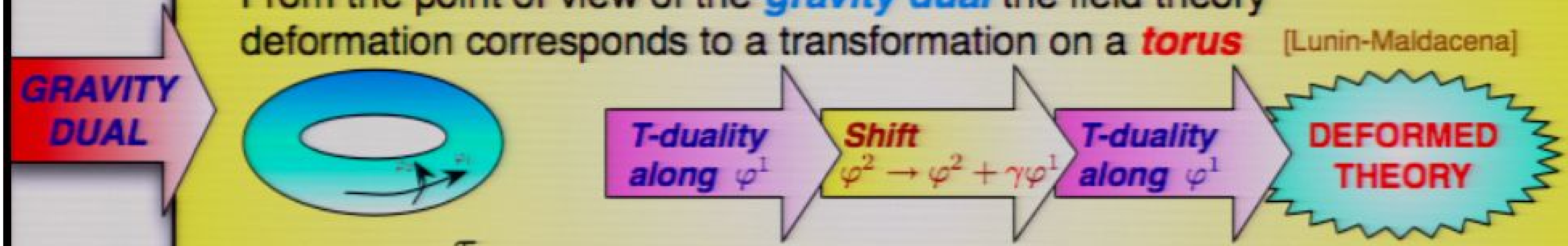

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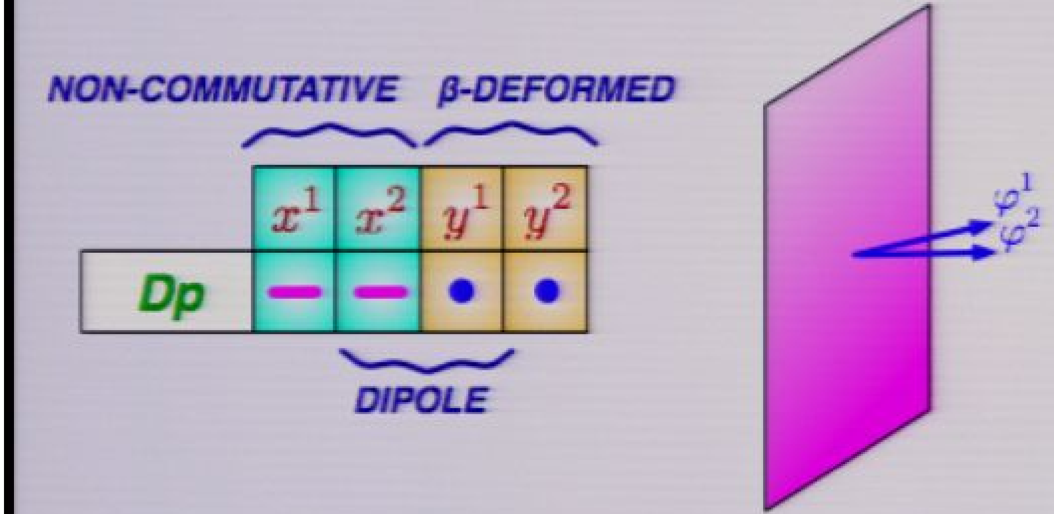

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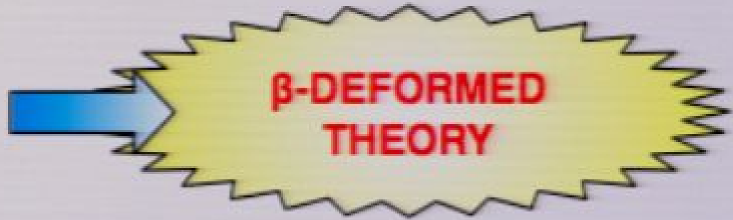
p_1 $U(1)$ ***CHARGE***

p_2 $U(1)$ ***CHARGE***



Deforming the gauge/string duality

GRAVITY DUAL

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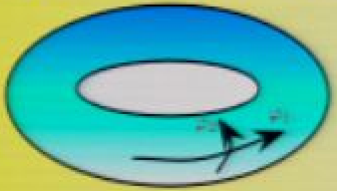

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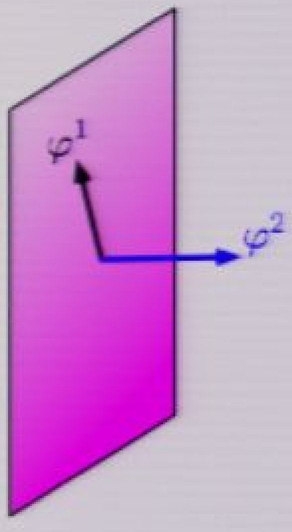
NON-COMMUTATIVE (x^1, x^2) β-DEFORMED (y^1, y^2)

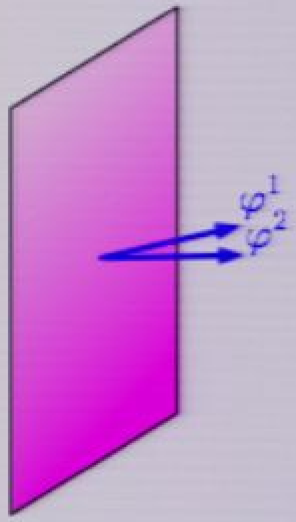
	x^1	x^2	y^1	y^2
Dp	—	—	●	●

DIPOLE (x^2, y^1)

NON-COMMUTATIVE



DIPOLE



β-DEFORMED

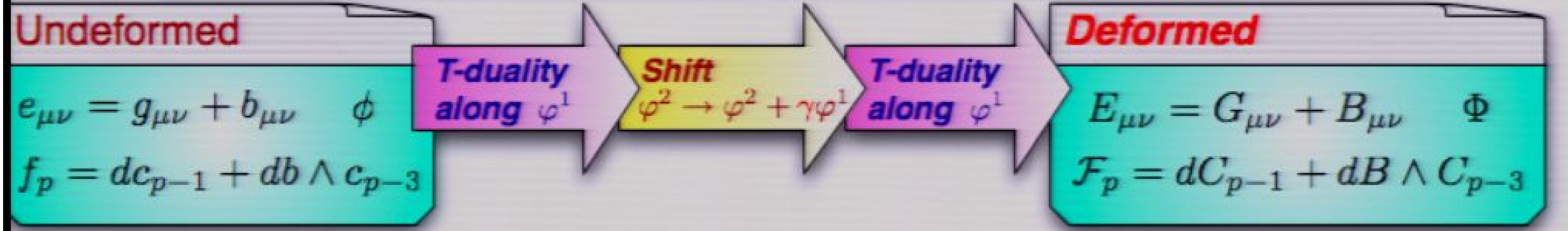
TsT for closed string backgrounds

Undeformed

$$e_{\mu\nu} = g_{\mu\nu} + b_{\mu\nu} \qquad \phi$$

$$f_p = dc_{p-1} + db \wedge c_{p-3}$$

TsT for closed string backgrounds



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T-duality along φ^1

Shift $\varphi^2 \to \varphi^2 + \gamma\varphi^1$

T-duality along φ^1

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NS-NS fields

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EXAMPLES $\quad G_{11} = \mathcal{M}\, g_{11} \qquad G_{12} = \mathcal{M}\, g_{12} \qquad G_{22} = \mathcal{M}\, g_{22}$

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EXAMPLES

$$G_{11} = \mathcal{M}\, g_{11} \qquad G_{12} = \mathcal{M}\, g_{12} \qquad G_{22} = \mathcal{M}\, g_{22}$$

If $b_{\mu\nu} = 0$: $\quad \mathcal{M} = \left\{1 + \gamma^2 \det\begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22}\end{pmatrix}\right\}^{-1}$

$$B_{12} = \gamma\mathcal{M} \det\begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22}\end{pmatrix} = \frac{\mathcal{M} - 1}{\gamma}$$

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$$\sum_q \mathcal{F}_q \wedge e^B = \sum_q f_q \wedge e^b + \gamma\left[\sum_q f_q \wedge e^b\right]_{[\varphi^1][\varphi^2]}$$

$$(\omega_{p[y]})_{\alpha_1\cdots\alpha_{p-1}} = (\omega_p)_{\alpha_1\cdots\alpha_{p-1}y}$$

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EXAMPLES

$$F_1 = f_1 + \gamma\,[f_3 + f_1 \wedge b]_{[\varphi^1][\varphi^2]}$$

$$\mathcal{F}_3 + F_1 \wedge B = f_3 + f_1 \wedge b + \gamma\left[f_5 + f_3 \wedge b + \frac{1}{2} f_1 \wedge b \wedge b\right]_{[\varphi^1][\varphi^2]}$$

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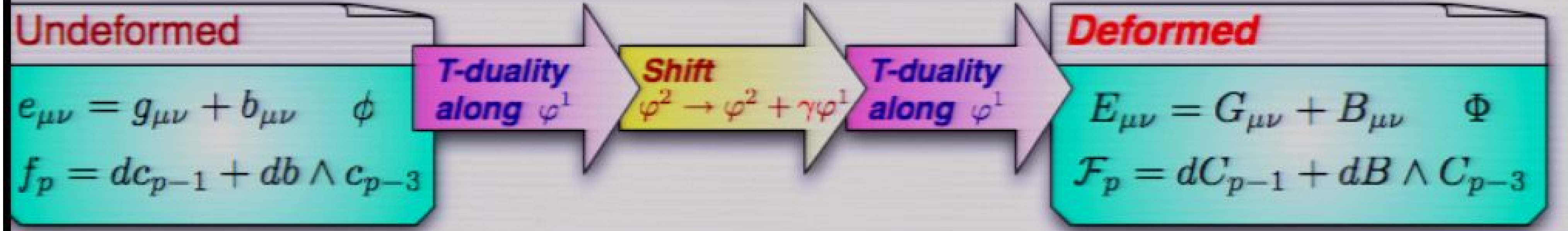
(gauge choice)

WESS-ZUMINO COUPLING IN D-BRANE WORLD-VOLUME ACTION

TsT for closed string backgrounds



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WESS-ZUMINO COUPLING IN D-BRANE WORLD-VOLUME ACTION

D*p* - D*(p+2)* coupling: ***Myers effect?***

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WESS-ZUMINO COUPLING IN D-BRANE WORLD-VOLUME ACTION

We need to study *open string boundary conditions* in detail

D-brane probes provide *connections* between gravity and field theory

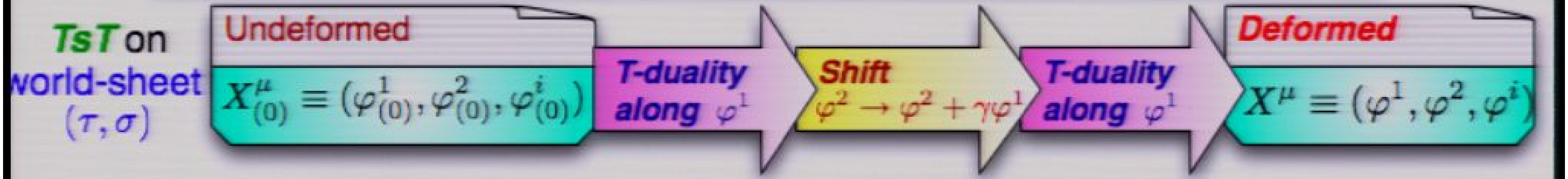
TsT for open strings and D-branes

TsT on world-sheet (τ, σ)

Undeformed

$$X^{\mu}_{(0)} \equiv (\varphi^1_{(0)}, \varphi^2_{(0)}, \varphi^i_{(0)})$$

TsT for open strings and D-branes



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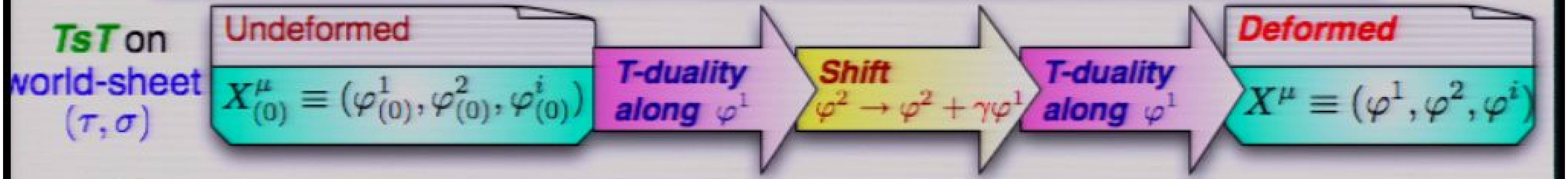

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$$\begin{cases} \partial_\alpha \varphi^1_{(0)} = \partial_\alpha \varphi^1 - \gamma \; B_{2\mu} \; \partial_\alpha \varphi^\mu - \gamma \; \eta_{\alpha\beta} \; \epsilon^{\beta\kappa} \; G_{2\mu} \; \partial_\kappa \varphi^\mu \\ \partial_\alpha \varphi^2_{(0)} = \partial_\alpha \varphi^2 + \gamma \; B_{1\mu} \; \partial_\alpha \varphi^\mu + \gamma \; \eta_{\alpha\beta} \; \epsilon^{\beta\kappa} \; G_{1\mu} \; \partial_\kappa \varphi^\mu \\ \partial_\alpha \varphi^i_{(0)} = \partial_\alpha \varphi^i \end{cases}$$

[Frolov, Alday-Arutyunov-Frolov, *E.I.*]

TsT for open strings and D-branes

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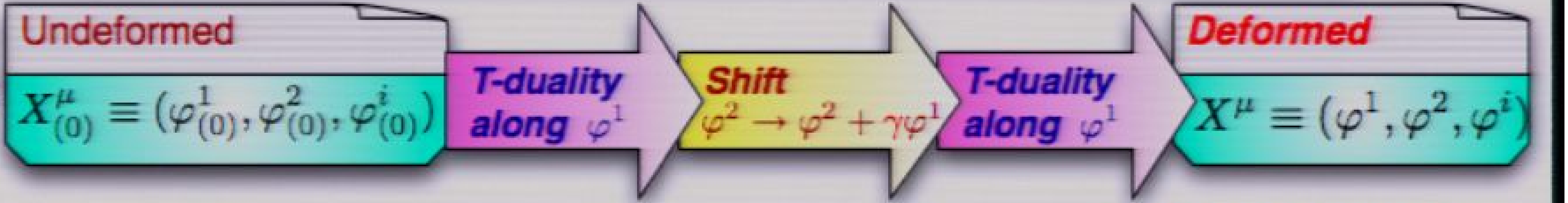

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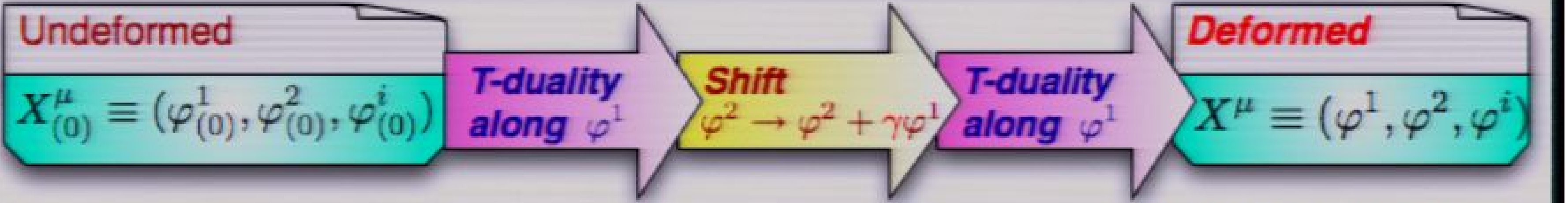
HOW DO BOUNDARY CONDITIONS TRANSFORM?

TsT for open strings and D-branes



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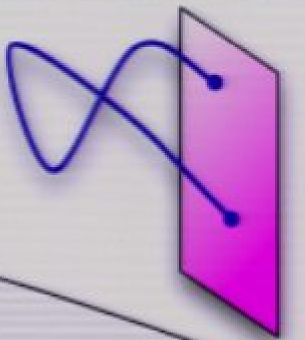


Open string boundary conditions

φ^μ | ▬ — *along* the D-brane (generalized Neumann)

$$g_{\mu\nu}\partial_\sigma \varphi^\nu_{(0)} - (b_{\mu\nu} + f_{\mu\nu})\,\partial_\tau \varphi^\nu_{(0)} = 0$$

WORLD-VOLUME GAUGE FIELD

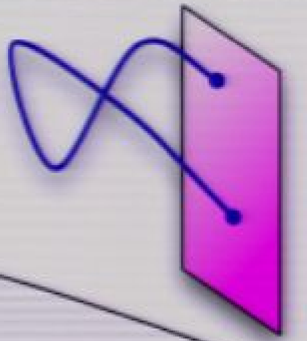
TsT for open strings and D-branes

TsT on world-sheet (τ, σ)

Undeformed $X^\mu_{(0)} \equiv (\varphi^1_{(0)}, \varphi^2_{(0)}, \varphi^i_{(0)})$ → **T-duality along** φ^1 → **Shift** $\varphi^2 \to \varphi^2 + \gamma\varphi^1$ → **T-duality along** φ^1 → **Deformed** $X^\mu \equiv (\varphi^1, \varphi^2, \varphi^i)$

$$\begin{cases} \partial_\alpha \varphi^1_{(0)} = \partial_\alpha \varphi^1 - \gamma \; B_{2\mu} \; \partial_\alpha \varphi^\mu - \gamma \; \eta_{\alpha\beta} \; \epsilon^{\beta\kappa} \; G_{2\mu} \; \partial_\kappa \varphi^\mu \\ \partial_\alpha \varphi^2_{(0)} = \partial_\alpha \varphi^2 + \gamma \; B_{1\mu} \; \partial_\alpha \varphi^\mu + \gamma \; \eta_{\alpha\beta} \; \epsilon^{\beta\kappa} \; G_{1\mu} \; \partial_\kappa \varphi^\mu \\ \partial_\alpha \varphi^i_{(0)} = \partial_\alpha \varphi^i \end{cases}$$

HOW DO BOUNDARY CONDITIONS TRANSFORM?



Open string boundary conditions

φ^μ ▬ *along* the D-brane (generalized Neumann)

$$g_{\mu\nu}\partial_\sigma \varphi^\nu_{(0)} - (b_{\mu\nu} + f_{\mu\nu})\, \partial_\tau \varphi^\nu_{(0)} = 0$$

WORLD-VOLUME GAUGE FIELD

⇨

$$[G_{\mu\nu} + \gamma\,(f_{1\mu}G_{2\nu} - f_{2\mu}G_{1\nu})]\, \partial_\sigma \varphi^\nu$$
$$-[B_{\mu\nu} + f_{\mu\nu} + \gamma\,(f_{1\mu}B_{2\nu} - f_{2\mu}B_{1\nu})]\, \partial_\tau \varphi^\nu = 0$$

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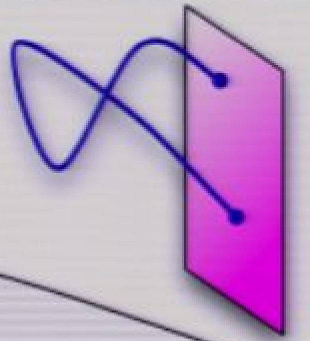

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WORLD-VOLUME GAUGE FIELD

⇒

$$[G_{\mu\nu} + \gamma\,(f_{1\mu}G_{2\nu} - f_{2\mu}G_{1\nu})]\, \partial_\sigma \varphi^\nu$$
$$- [B_{\mu\nu} + f_{\mu\nu} + \gamma\,(f_{1\mu}B_{2\nu} - f_{2\mu}B_{1\nu})]\, \partial_\tau \varphi^\nu = 0$$

If $f_{\mu\nu} = 0$: $\quad G_{\mu\nu}\ \partial_\sigma \varphi^\nu - B_{\mu\nu}\ \partial_\tau \varphi^\nu = 0 \quad$ Neumann with $F = 0$

TsT for open strings and D-branes

TsT on world-sheet (τ,σ)

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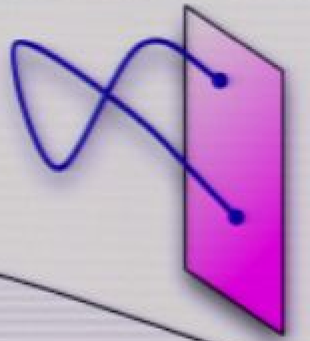
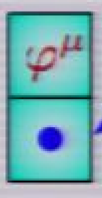


Open string boundary conditions



transverse to the D-brane (Dirichlet)

$$\partial_\tau \varphi^\mu_{(0)} = 0$$

TsT for open strings and D-branes

TsT on world-sheet (τ, σ)

Undeformed

$X^\mu_{(0)} \equiv (\varphi^1_{(0)}, \varphi^2_{(0)}, \varphi^i_{(0)})$

T-duality along φ^1

Shift $\varphi^2 \to \varphi^2 + \gamma\varphi^1$

T-duality along φ^1

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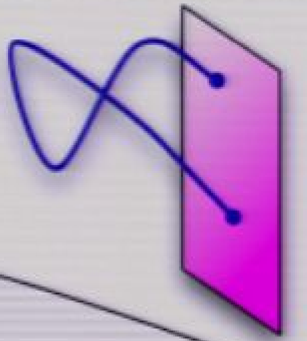
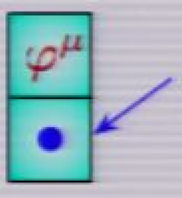


Open string boundary conditions



transverse to the D-brane (Dirichlet)

$\partial_\tau \varphi^\mu_{(0)} = 0 \Rightarrow \partial_\tau \varphi^\mu = 0$ Dirichlet

TsT for open strings and D-branes

TsT on
world-sheet
(τ,σ)

Undeformed
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T-duality along φ^1

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HOW DO BOUNDARY CONDITIONS TRANSFORM?

Open string boundary conditions

φ^μ
●

transverse to the D-brane
(Dirichlet)

$\partial_\tau\varphi^\mu_{(0)} = 0 \Rightarrow \partial_\tau\varphi^\mu = 0$ Dirichlet

UNLESS

φ^1	φ^2
●	●

$$\Rightarrow \begin{cases} G_{2\nu}\ \partial_\sigma\varphi^\nu - B_{2\nu}\ \partial_\tau\varphi^\nu + \frac{1}{\gamma}\ \partial_\tau\varphi^1 = 0 \\ G_{1\nu}\ \partial_\sigma\varphi^\nu - B_{1\nu}\ \partial_\tau\varphi^\nu - \frac{1}{\gamma}\ \partial_\tau\varphi^2 = 0 \end{cases}$$

TsT for open strings and D-branes

TsT on world-sheet (τ,σ)

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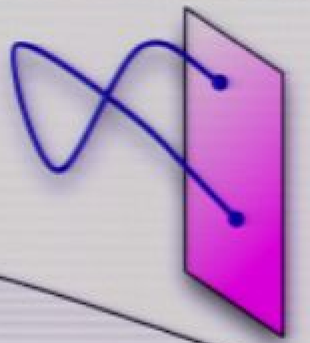
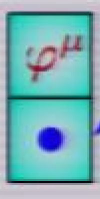
HOW DO BOUNDARY CONDITIONS TRANSFORM?



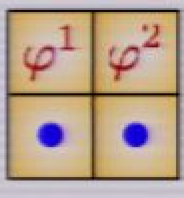
Open string boundary conditions



transverse to the D-brane (Dirichlet)

$\partial_\tau \varphi^\mu_{(0)} = 0$ ➡ $\partial_\tau \varphi^\mu = 0$ Dirichlet

UNLESS

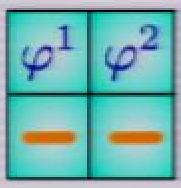


➡

$$\begin{cases} G_{2\nu}\ \partial_\sigma \varphi^\nu - B_{2\nu}\ \partial_\tau \varphi^\nu + \frac{1}{\gamma}\ \partial_\tau \varphi^1 = 0 \\ G_{1\nu}\ \partial_\sigma \varphi^\nu - B_{1\nu}\ \partial_\tau \varphi^\nu - \frac{1}{\gamma}\ \partial_\tau \varphi^2 = 0 \end{cases}$$

Neumann with

$$F_{12} = \frac{1}{\gamma}$$



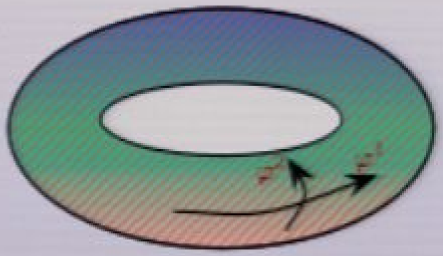
TsT for open strings and D-branes

Summary of D-brane boundary conditions along the (φ_1, φ_2) *torus*

TsT for open strings and D-branes

Summary of D-brane boundary conditions along the (φ_1, φ_2) *torus*

Undeformed



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Summary of D-brane boundary conditions along the (φ_1, φ_2) *torus*

Undeformed	***Deformed***

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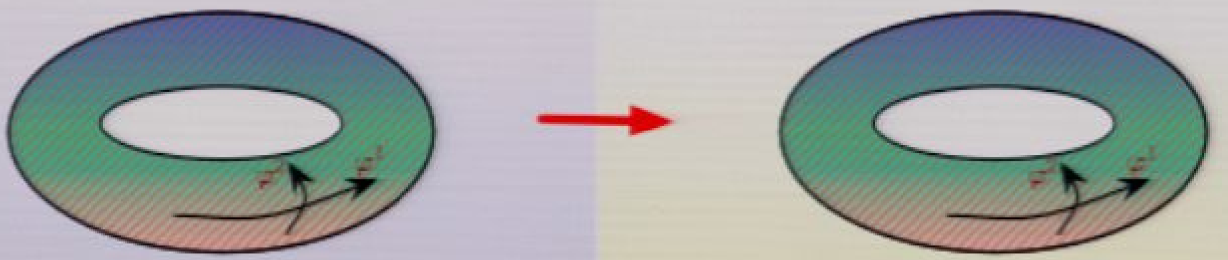

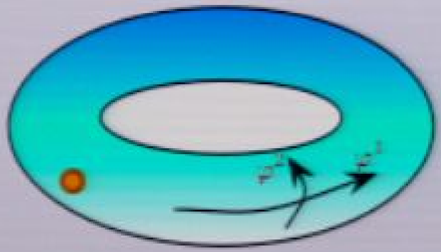

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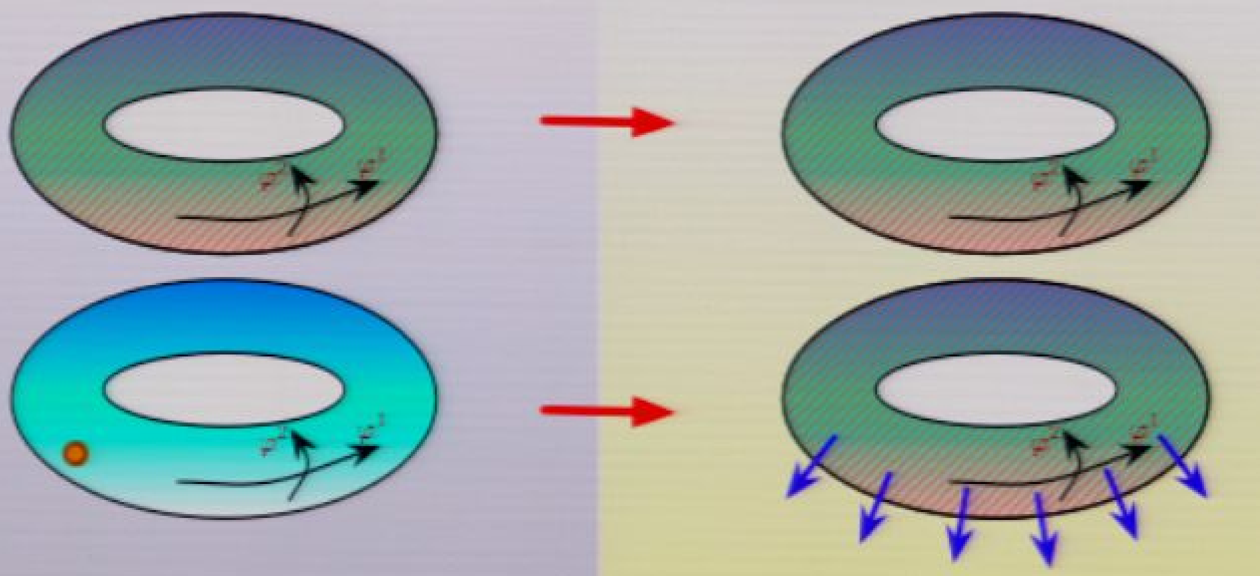
Undeformed | ***Deformed***



$F_{12} = \frac{1}{\gamma}$

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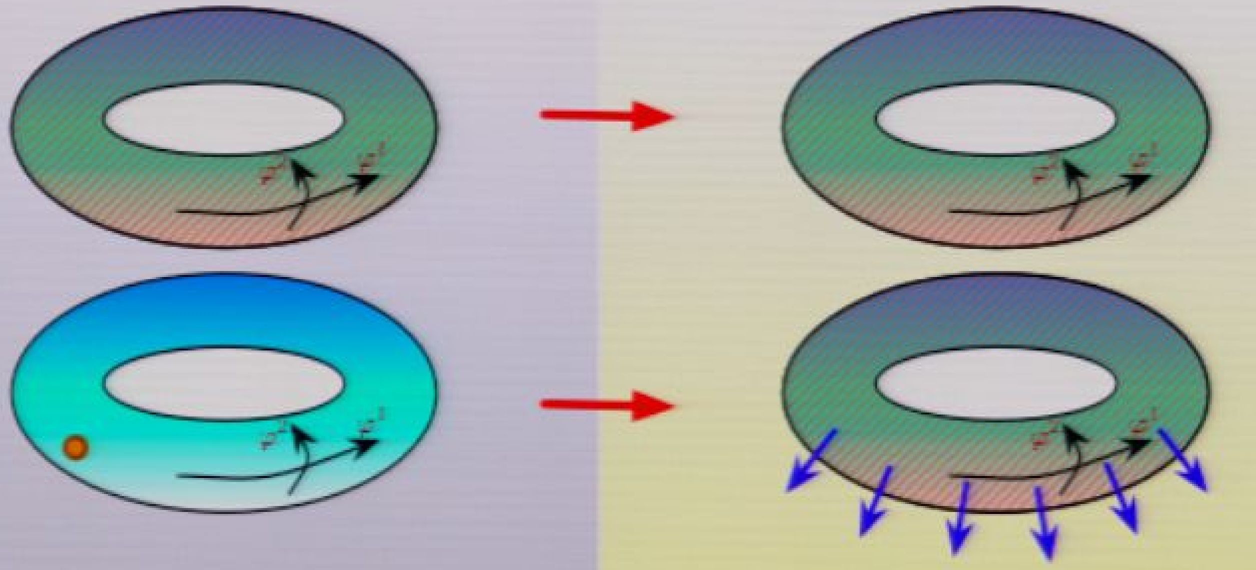


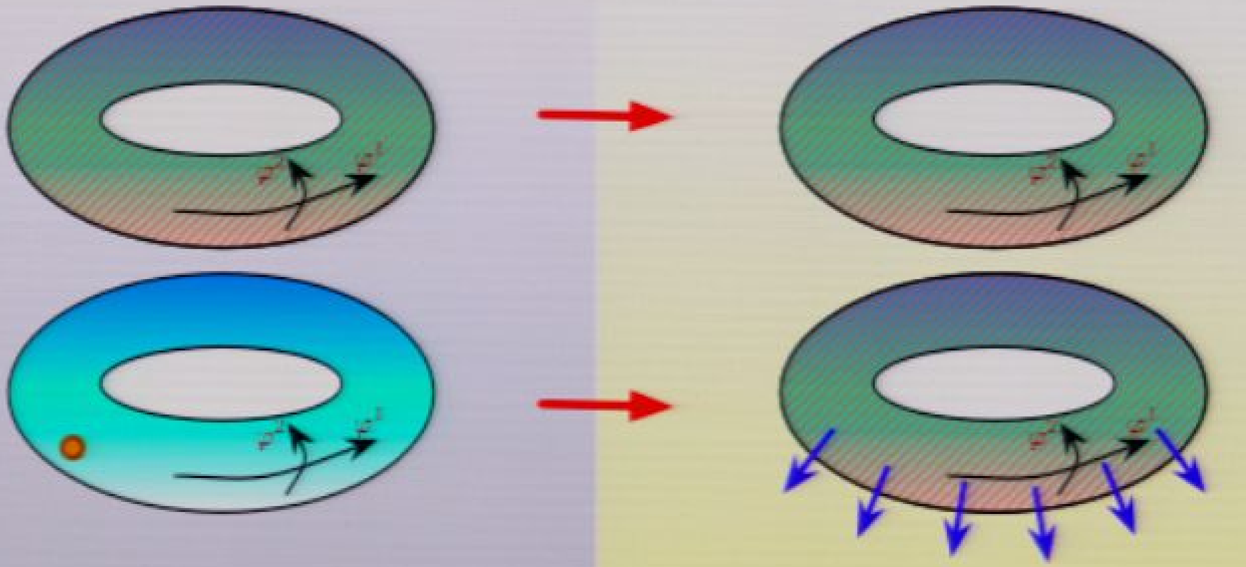
$$F_{12} = \frac{1}{\gamma}$$

➡ A *D-brane transverse to the two-torus* turns into a *D-brane wrapping the torus* with ***magnetic world-volume flux***

TsT for open strings and D-branes

Summary of D-brane boundary conditions along the (φ_1, φ_2) *torus*

Undeformed	***Deformed***



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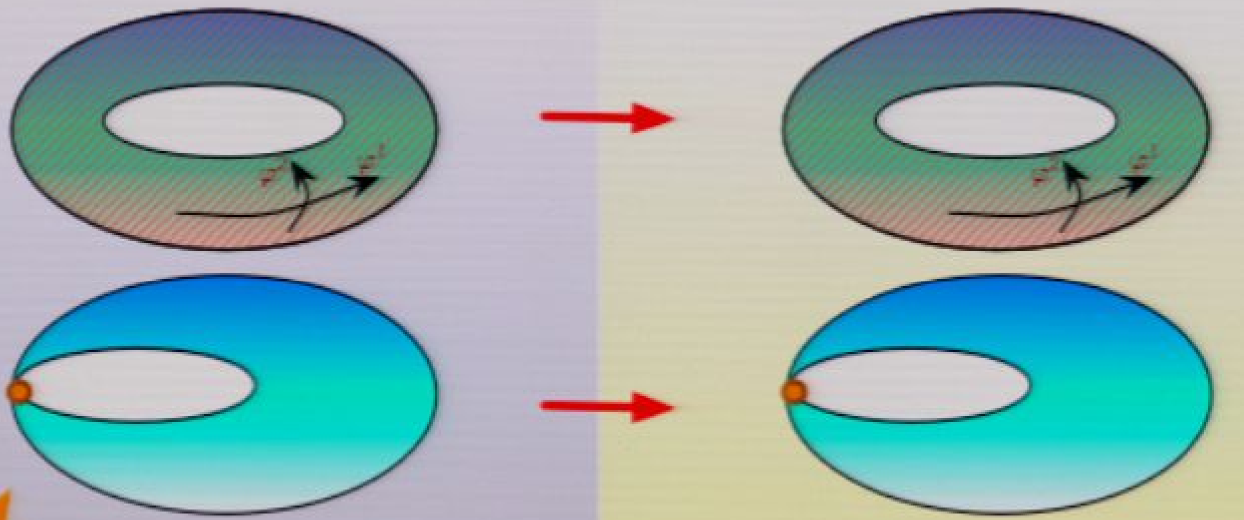
Undeformed	***Deformed***



➡ A *D-brane transverse to the two-torus* turns into a ***D-brane wrapping the torus*** with ***magnetic world-volume flux***

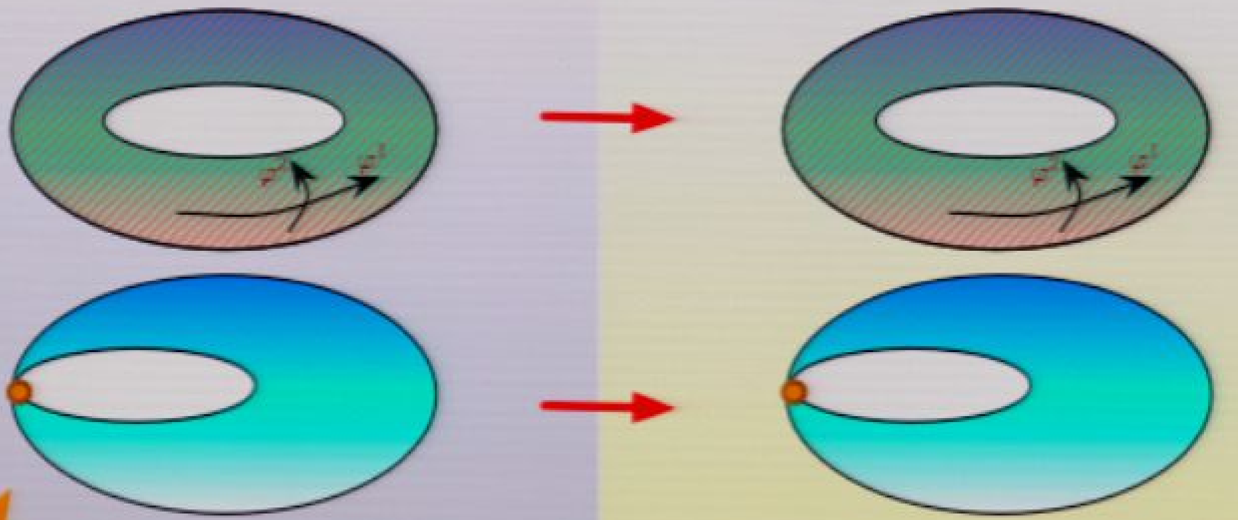
It is a consistent configuration only if γ is ***quantized***

For generic γ the brane has to sit at *special points* where cycles shrink

TsT for open strings and D-branes

Summary of D-brane boundary conditions along the (φ_1, φ_2) *torus*

Undeformed	***Deformed***



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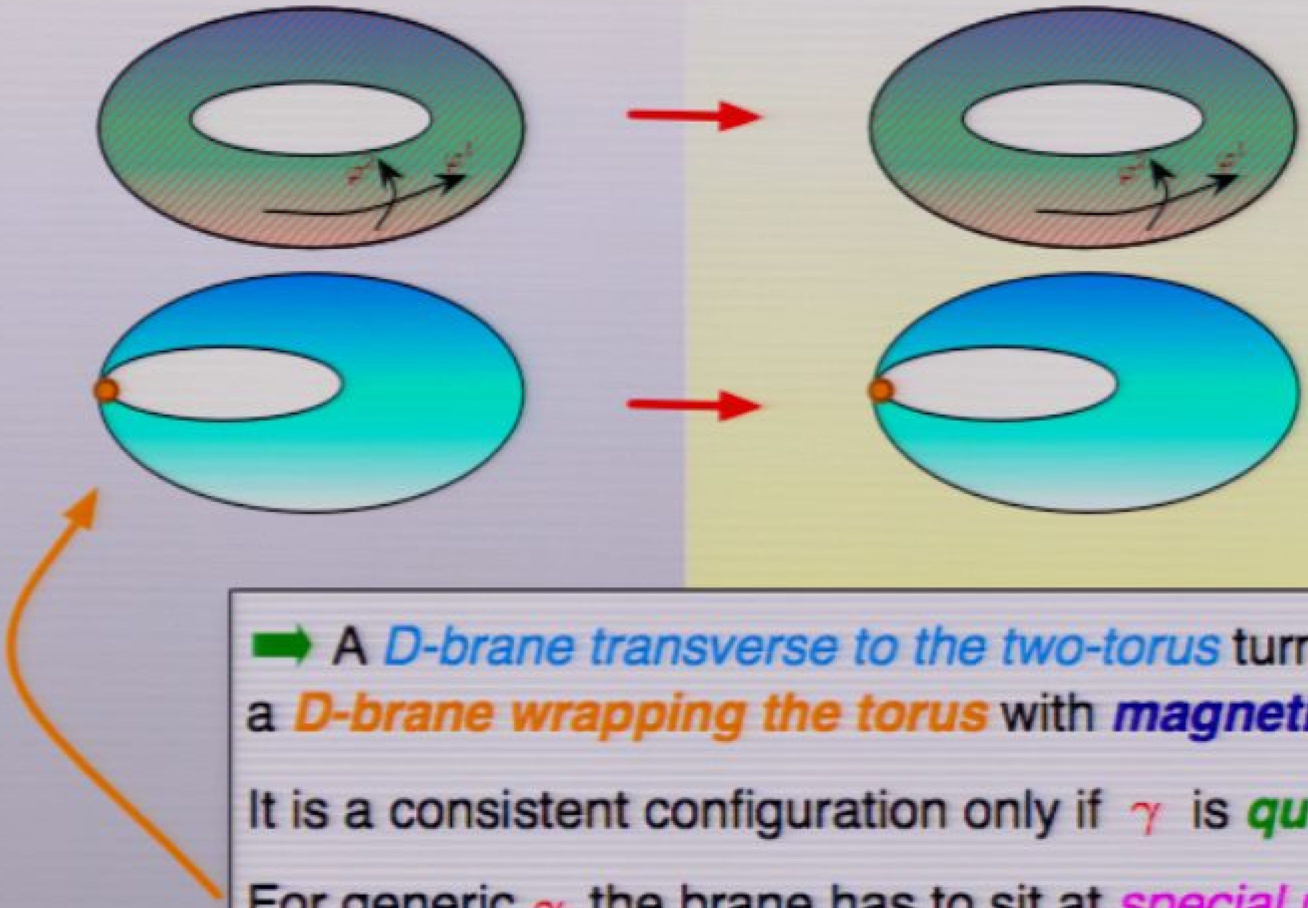

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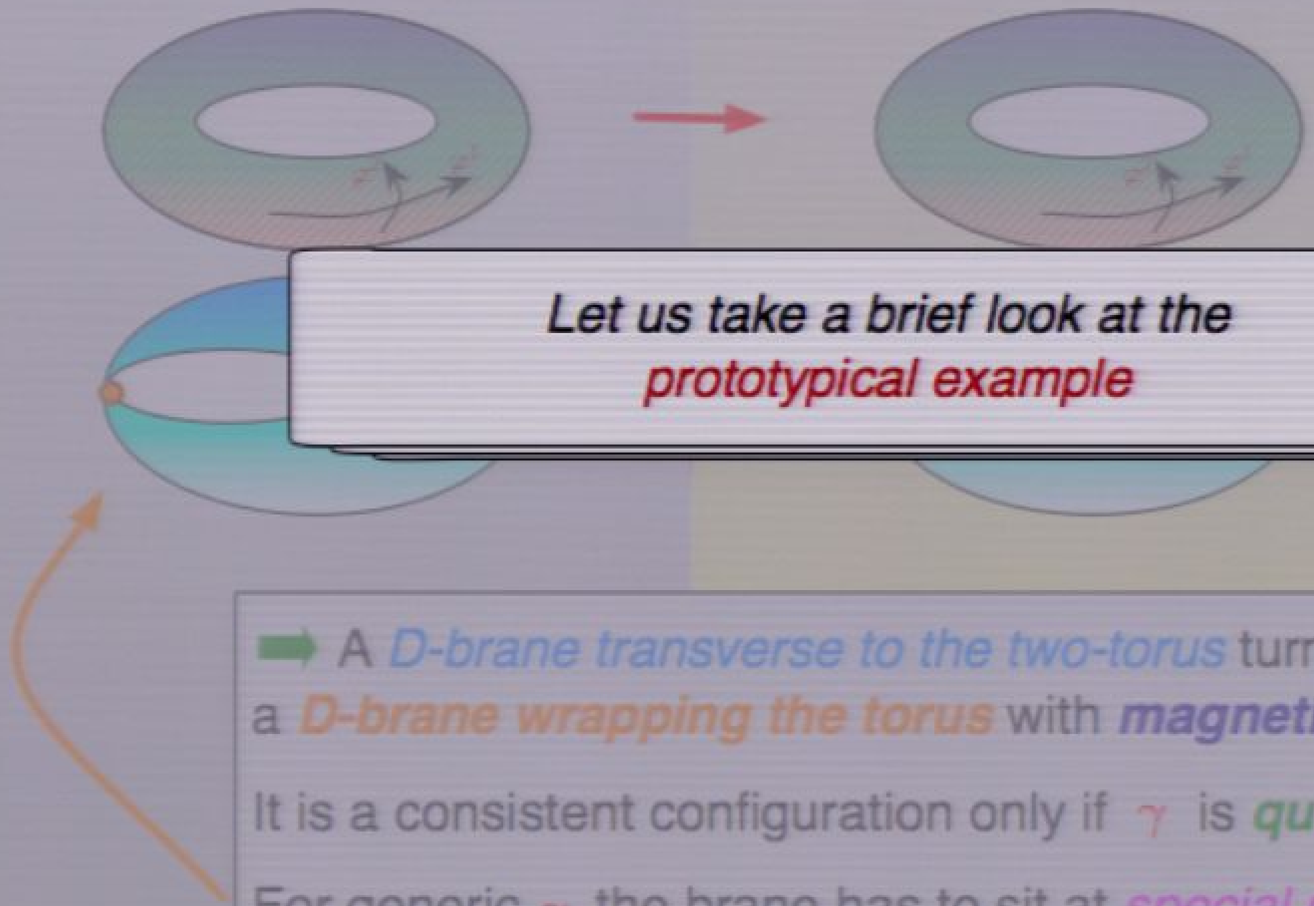


Let us take a brief look at the
prototypical example

A *D-brane transverse to the two-torus* turns into a ***D-brane wrapping the torus*** with ***magnetic world-volume flux***

It is a consistent configuration only if γ is ***quantized***

For generic γ the brane has to sit at *special points* where cycles shrink

The β-deformed theory and its gravity dual

$\mathcal{N}=4$ Super Yang-Mills

SUPERPOTENTIAL

$$W \;= \mathrm{Tr}\,(\quad \Phi_1\Phi_2\Phi_3 - \qquad \Phi_1\Phi_3\Phi_2)$$

The β-deformed theory and its gravity dual

$\mathcal{N}=4$ Super Yang-Mills

SUPERPOTENTIAL

$$W \;= \mathrm{Tr}\,(\quad \Phi_1\Phi_2\Phi_3 - \quad \Phi_1\Phi_3\Phi_2)$$

Gravity dual: $AdS_5 \times S^5$

$$ds^2 = ds^2_{AdS_5} + R^2\left[\sum_i \left(d\mu_i^2 + \quad \mu_i^2 d\phi_i^2\right)\right]$$

$$e^{2\Phi} = 1$$

$$C_4 = \omega_4 + 4R^4 \quad \omega_1 \wedge d\phi_1 \wedge d\phi_2 \wedge d\phi_3$$

$$R^4 = 4\pi g_s N$$

$$\sum_i \mu_i^2 = 1 \qquad \mu_1 = \cos\alpha \quad \mu_2 = \sin\alpha\cos\theta \quad \mu_3 = \sin\alpha\sin\theta$$

$$d\omega_1 = -\cos\alpha\sin^3\alpha\sin\theta\cos\theta d\alpha\wedge d\theta \quad d\omega_4 = \omega_{AdS_5}$$

The β-deformed theory and its gravity dual

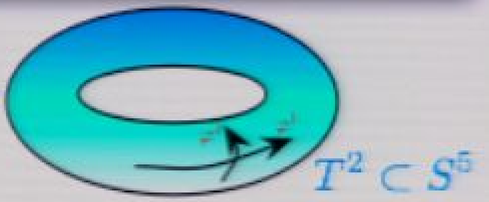
$\mathcal{N}=4$ **Super Yang-Mills** → **Exactly marginal deformation** → **β-deformed theory**

[Leigh-Strassler]

SUPERPOTENTIAL

$$W_\gamma = \mathrm{Tr}\left(e^{i\pi\gamma}\Phi_1\Phi_2\Phi_3 - e^{-i\pi\gamma}\Phi_1\Phi_3\Phi_2\right)$$

$$\begin{cases} \phi_1 = \varphi^3 - \varphi^2 \\ \phi_2 = \varphi^3 + \varphi^1 + \varphi^2 \\ \phi_3 = \varphi^3 - \varphi^1 \end{cases}$$



Gravity dual: the LM solution

$$ds^2 = ds^2_{AdS_5} + R^2\left[\sum_i\left(d\mu_i^2 + \mathcal{M}\mu_i^2 d\phi_i^2\right) + \hat{\gamma}^2\mathcal{M}\mu_1^2\mu_2^2\mu_3^2\Big(\sum_i d\phi_i\Big)^2\right]$$

$$e^{2\Phi} = \mathcal{M}$$

$$B = -\hat{\gamma}R^2\mathcal{M}\left(\mu_1^2\mu_2^2 d\phi_1\wedge d\phi_2 + \mu_2^2\mu_3^2 d\phi_2\wedge d\phi_3 + \mu_3^2\mu_1^2 d\phi_3\wedge d\phi_1\right)$$

$$C_2 = -4\hat{\gamma}R^2\,\omega_1\wedge\left(d\phi_1 + d\phi_2 + d\phi_3\right)$$

$$C_4 = \omega_4 + 4R^4\mathcal{M}\,\omega_1\wedge d\phi_1\wedge d\phi_2\wedge d\phi_3$$

$$R^4 = 4\pi g_s N \qquad \hat{\gamma} = R^2\,\gamma \qquad \mathcal{M}^{-1} = 1 + \hat{\gamma}^2(\mu_1^2\mu_2^2 + \mu_2^2\mu_3^2 + \mu_3^2\mu_1^2)$$

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[Lunin-Maldacena]

The β-deformed theory and its gravity dual

$\mathcal{N}=4$ Super Yang-Mills

Exactly marginal deformation

[Leigh-Strassler]

β-deformed theory

- ➡ $\mathcal{N}=1$ supersymmetry
- ➡ superconformal
- ➡ lagrangian description

SUPERPOTENTIAL

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[Lunin-Maldacena]

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$\mathcal{N}=4$ Super Yang-Mills → **Exactly marginal deformation** → **β-deformed theory**

[Leigh-Strassler]

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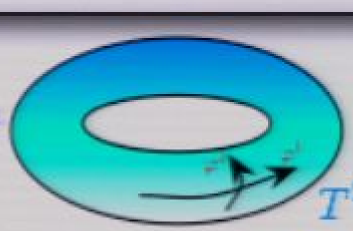

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$$e^{2\Phi} = \mathcal{M}$$

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$$C_2 = -4\hat{\gamma}R^2\,\omega_1\wedge\left(d\phi_1 + d\phi_2 + d\phi_3\right)$$

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[Lunin-Maldacena]

The β-deformed theory and its gravity dual

$\mathcal{N}=4$ Super Yang-Mills

SUPERPOTENTIAL

$$W \; = \mathrm{Tr}\,(\qquad \Phi_1\Phi_2\Phi_3 - \qquad \Phi_1\Phi_3\Phi_2)$$

Gravity dual: $AdS_5\times S^5$

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$$R^4 = 4\pi g_s N$$

$$\sum_i \mu_i^2 = 1 \qquad \mu_1 = \cos\alpha \quad \mu_2 = \sin\alpha\cos\theta \quad \mu_3 = \sin\alpha\sin\theta$$

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The β-deformed theory and its gravity dual

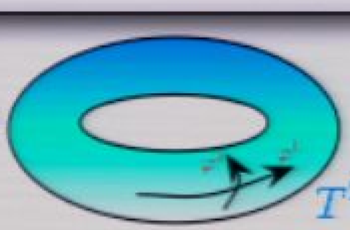
$\mathcal{N}=4$ **Super Yang-Mills** → **Exactly marginal deformation** → **β-deformed theory**

[Leigh-Strassler]

SUPERPOTENTIAL

$$W_\gamma = \mathrm{Tr}\,\left(e^{i\pi\gamma}\Phi_1\Phi_2\Phi_3 - e^{-i\pi\gamma}\Phi_1\Phi_3\Phi_2\right)$$

$$\begin{cases}\phi_1 = \varphi^3 - \varphi^2\\ \phi_2 = \varphi^3 + \varphi^1 + \varphi^2\\ \phi_3 = \varphi^3 - \varphi^1\end{cases}$$



$T^2 \subset S^5$

Gravity dual: the LM solution

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- superconformal
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The β-deformed theory and its gravity dual

$\mathcal{N} = 4$ **Super Yang-Mills**

Exactly marginal deformation

β-deformed theory

symmetry

cription

SUPERPOTEN

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The TsT transformation of D-branes can be used to study extended objects in the gravity dual

For instance, the study of giant gravitons yields information on the moduli space and "mesonic" BPS spectrum of the β-deformed theory

[Pirrone, *E.I.*-Naqvi, Butti-Forcella-Martucci-Minasian-Petrini-Zaffaroni]

Gravity dual: the LM solution

$d\phi_i)^2$

$\wedge d\phi_1)$

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No Signal

VGA-1

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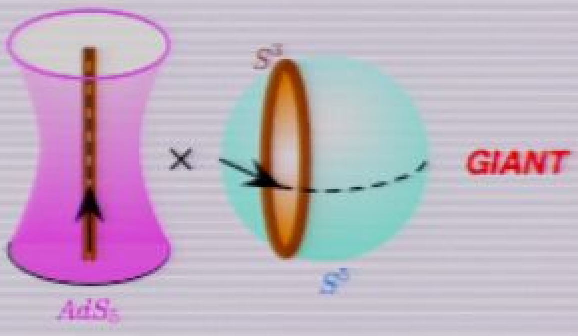

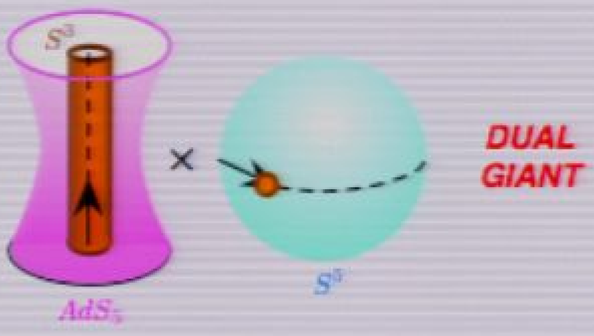

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ABJM Chern-Simons-matter theory

Recently there has been a lot of progress on **superconformal Chern-Simons-matter theories** living on **multiple M2-branes**

[Bagger-Lambert, Gustavsson, ...]

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MOTIVATIONS

- Not many examples of *3d superconformal theories*
- *M2-brane theory* and definition of M-theory
- Interacting conformal points of *condensed matter systems*

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ABJM theory

- *lives on* ***N*** *M2-branes probing a* $\mathbb{C}^4/\mathbb{Z}_k$ *orbifold*
- $U(N)_k \times U(N)_{-k}$ *Chern-Simons gauge theory*
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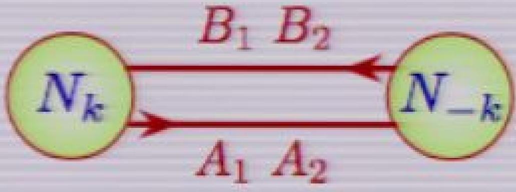

[Aharony-Bergman-Jafferis-Maldacena]

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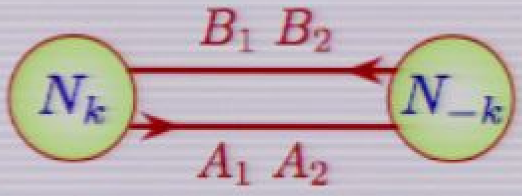

GENERALIZATIONS

Less supersymmetric Chern-Simons-Matter theories

[Benna-Klebanov-Klose-Smedbäck, Hosomichi-Lee-Lee-Lee-Park, Hanany *et al.*, Martelli-Sparks, Gaiotto-Tomasiello, ...]

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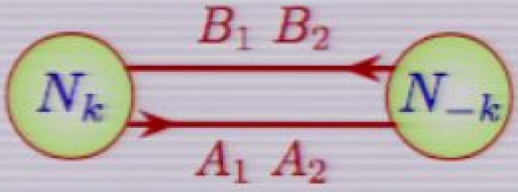

GENERALIZATIONS

Less supersymmetric Chern-Simons-Matter theories

A marginal deformation yields the ***β-deformed ABJM theory***, an $\mathcal{N}=2$ ***superconformal theory*** that admits an ***exact lagrangian description***

$$W \to W_\gamma = \frac{4\pi}{k}\,\mathrm{Tr}\left(e^{-i\pi\gamma/2}A_1B_1A_2B_2 - e^{i\pi\gamma/2}A_1B_2A_2B_1\right)$$

Gravity dual of ABJM theory

$AdS_4 \times \mathbb{CP}_3$

$$ds^2 = \frac{R^3}{k}\Big[\tfrac{1}{4}ds^2_{AdS_4} + d\xi^2 + \quad \cos^2\xi \sin^2\xi \left(d\psi + \tfrac{1}{2}\cos\theta_1 d\varphi_1 - \tfrac{1}{2}\cos\theta_2 d\varphi_2\right)^2$$
$$+\tfrac{1}{4}\cos^2\xi(d\theta_1^2 + \quad \sin^2\theta_1 d\varphi_1^2) + \tfrac{1}{4}\sin^2\xi(d\theta_2^2 + \quad \sin^2\theta_2 d\varphi_2^2)\Big]$$

$$e^{2\Phi} = \frac{R^3}{k^3}$$

$$R = 32\pi^2 kN$$

$$F_2 = k\Big(-\cos\xi\sin\xi d\xi \wedge (2d\psi + \cos\theta_1 d\varphi_1 - \cos\theta_2 d\varphi_2)$$
$$-\tfrac{1}{2}\cos^2\xi\sin\theta_1 d\theta_1 \wedge d\varphi_1 - \tfrac{1}{2}\sin^2\xi\sin\theta_2 d\theta_2 \wedge d\varphi_2\Big)$$

$$F_4 = -\frac{3R^3}{8}\ \omega_{AdS_4}$$

[Nilsson-Pope, Watamura, Sorokin-Tkach-Volkov, Aharony-Bergman-Jafferis-Maldacena]

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The type IIA gravity dual
is valid when $N, k \to \infty$, $\lambda = N/k$ fixed and $k \ll N \ll k^5$

When $N \gg k^5$ the appropriate description is
the $AdS_4 \times S^7/\mathbb{Z}_k$ solution of 11-dimensional supergravity

[Nilsson-Pope, Watamura, Sorokin-Tkach-Volkov, Aharony-Bergman-Jafferis-Maldacena]

Gravity dual of β-deformed ABJM theory

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$U(1)_{\varphi_1} \times U(1)_{\varphi_2}$

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$$+\tfrac{1}{4}\cos^2\xi(d\theta_1^2 + \mathcal{M}\sin^2\theta_1 d\varphi_1^2) + \tfrac{1}{4}\sin^2\xi(d\theta_2^2 + \mathcal{M}\sin^2\theta_2 d\varphi_2^2)$$

$$+\hat{\gamma}^2\mathcal{M}\cos^4\xi\sin^4\xi\sin^2\theta_1\sin^2\theta_2 d\psi^2\Big]$$

$$\hat{\gamma} = \frac{R^3}{4k}\gamma$$

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$$-\tfrac{1}{2}\cos^2\xi\sin\theta_1 d\theta_1\wedge d\varphi_1 - \tfrac{1}{2}\sin^2\xi\sin\theta_2 d\theta_2\wedge d\varphi_2\Big)$$

$$F_4 = -\frac{3R^3}{8}\left(\omega_{AdS_4} + 4\hat{\gamma}\cos^3\xi\sin^3\xi\sin\theta_1\sin\theta_2 d\xi\wedge d\psi\wedge d\theta_1\wedge d\theta_2\right)$$

$$-\frac{R^3}{8}d(\hat{\gamma}\mathcal{M}\cos^2\xi\sin^2\xi(\cos^2\xi\sin^2\theta_1 - \sin^2\xi\sin^2\theta_2))\wedge d\psi\wedge d\varphi_1\wedge d\varphi_2$$

$$B = -\frac{\hat{\gamma}\mathcal{M}R^3}{k}\cos^2\xi\sin^2\xi\,\Big(\tfrac{1}{2}\cos^2\xi\sin^2\theta_1\cos\theta_2 d\psi\wedge d\varphi_1 + \tfrac{1}{2}\sin^2\xi\sin^2\theta_2\cos\theta_1 d\psi\wedge d\varphi_2$$

$$+\tfrac{1}{4}\left(\sin^2\theta_1\sin^2\theta_2 + \cos^2\xi\sin^2\theta_1\cos^2\theta_2 + \sin^2\xi\sin^2\theta_2\cos^2\theta_1\right)d\varphi_1\wedge d\varphi_2\Big)$$

$$\mathcal{M}^{-1} = 1 + \hat{\gamma}^2\cos^2\xi\sin^2\xi\left(\sin^2\theta_1\sin^2\theta_2 + \cos^2\xi\sin^2\theta_1\cos^2\theta_2 + \sin^2\xi\sin^2\theta_2\cos^2\theta_1\right)$$

TsT of D-brane probes

The ***moduli space*** of the ***undeformed ABJM theory*** can be read on the action of a ***probe D2-brane*** in $AdS_4 \times \mathbb{CP}_3$ (or ***M2-brane*** in $AdS_4 \times S^7/\mathbb{Z}_k$)

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D*p*-brane action

$$S_{\mathrm{D}p} = -\tau_p \int d^{p+1}\sigma \, e^{-\Phi} \sqrt{-\det\left(\hat{G}_{ab} + \hat{B}_{ab} + F_{ab}\right)} + \tau_p \int_{\mathcal{M}_{p+1}} \sum_q \hat{C}_q \wedge e^{\hat{B}+F}$$

TsT of D-brane probes

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D2 probe

	AdS_4				$\mathbb{CP}_3$					
	x^0	x^1	x^2	r	ξ	ψ	θ_1	θ_2	φ_1	φ_2
D2	—	—	—	●	●	●	●	●	●	●

TsT of D-brane probes

The *moduli space* of the *undeformed ABJM theory* can be read on the action of a *probe D2-brane* in $AdS_4 \times \mathbb{CP}_3$ (or *M2-brane* in $AdS_4 \times S^7/\mathbb{Z}_k$)

Dp-brane

$AdS_4 \times \mathbb{CP}_3$

$$ds^2 = \frac{R^3}{k}\Big[\tfrac{1}{4}ds^2_{AdS_4} + d\xi^2 + \cos^2\xi\sin^2\xi\left(d\psi + \tfrac{1}{2}\cos\theta_1 d\varphi_1 - \tfrac{1}{2}\cos\theta_2 d\varphi_2\right)^2$$
$$+\tfrac{1}{4}\cos^2\xi(d\theta_1^2 + \sin^2\theta_1 d\varphi_1^2) + \tfrac{1}{4}\sin^2\xi(d\theta_2^2 + \sin^2\theta_2 d\varphi_2^2)\Big]$$

$$e^{2\Phi} = \frac{R^3}{k^3}$$

$$F_2 = k\Big(-\cos\xi\sin\xi d\xi\wedge(2d\psi + \cos\theta_1 d\varphi_1 - \cos\theta_2 d\varphi_2)$$
$$-\tfrac{1}{2}\cos^2\xi\sin\theta_1 d\theta_1\wedge d\varphi_1 - \tfrac{1}{2}\sin^2\xi\sin\theta_2 d\theta_2\wedge d\varphi_2\Big)$$

$$F_4 = -\frac{3R^3}{8}\ \omega_{AdS_4}$$

TsT of D-brane probes

The *moduli space* of the *undeformed ABJM theory* can be read on the action of a *probe D2-brane* in $AdS_4 \times \mathbb{CP}_3$ (or *M2-brane* in $AdS_4 \times S^7/\mathbb{Z}_k$)

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$$S_{\mathrm{D}p} = -\tau_p \int d^{p+1}\sigma \, e^{-\Phi} \sqrt{-\det\left(\hat{G}_{ab} + \hat{B}_{ab} + F_{ab}\right)} + \tau_p \int_{\mathcal{M}_{p+1}} \sum_q \hat{C}_q \wedge e^{\hat{B}+F}$$

D2 probe

	AdS_4				$\mathbb{CP}_3$					
	x^0	x^1	x^2	r	ξ	ψ	θ_1	θ_2	φ_1	φ_2
D2	—	—	—	●	●	●	●	●	●	●

SUBSTITUTE AND DUALIZE 3d GAUGE FIELD

$\mathbb{C}^4/\mathbb{Z}_k$

MODULI SPACE

TsT of D-brane probes

The *moduli space* of the *undeformed ABJM theory* can be read on the action of a *probe D2-brane* in $AdS_4 \times \mathbb{CP}_3$ (or *M2-brane* in $AdS_4 \times S^7/\mathbb{Z}_k$)

D*p*-brane action

$$S_{\mathrm{D}p} = -\tau_p \int d^{p+1}\sigma \, e^{-\Phi} \sqrt{-\det\left(\hat{G}_{ab} + \hat{B}_{ab} + F_{ab}\right)} + \tau_p \int_{\mathcal{M}_{p+1}} \sum_q \hat{C}_q \wedge e^{\hat{B}+F}$$

D2 probe

	AdS_4				$\mathbb{CP}_3$					
	x^0	x^1	x^2	r	ξ	ψ	θ_1	θ_2	φ_1	φ_2
D2	—	—	—	●	●	●	●	●	●	●

SUBSTITUTE AND DUALIZE 3d GAUGE FIELD

$\mathbb{C}^4/\mathbb{Z}_k$

MODULI SPACE

What about the *moduli space* of the *β-deformed ABJM theory* ?

TsT of D-brane probes

The *moduli space* of the *undeformed ABJM theory* can be read on the action of a *probe D2-brane* in $AdS_4 \times \mathbb{CP}_3$ (or *M2-brane* in $AdS_4 \times S^7/\mathbb{Z}_k$)

Dp-brane action

$$S_{\mathrm{D}p} = -\tau_p \int d^{p+1}\sigma \, e^{-\Phi} \sqrt{-\det\left(\hat{G}_{ab} + \hat{B}_{ab} + F_{ab}\right)} + \tau_p \int_{\mathcal{M}_{p+1}} \sum_q \hat{C}_q \wedge e^{\hat{B}+F}$$

D2 probe

	AdS_4				$\mathbb{CP}_3$					
	x^0	x^1	x^2	r	ξ	ψ	θ_1	θ_2	φ_1	φ_2
D2	—	—	—	●	●	●	●	●	●	●

SUBSTITUTE AND DUALIZE 3d GAUGE FIELD

$\mathbb{C}^4/\mathbb{Z}_k$

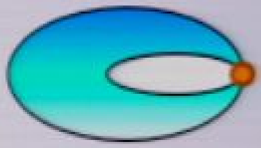
MODULI SPACE

What about the *moduli space* of the *β-deformed ABJM theory* ?

TsT

GENERIC γ *D2-brane* sitting at special points where *the torus shrinks*

	x^0	x^1	x^2	r	ξ	ψ	θ_1	θ_2	φ_1	φ_2
D2	—	—	—	●	●	●	●	●	●	●



$\mathcal{M} = 1$

(SIX POSSIBILITIES)

TsT of D-brane probes

The *moduli space* of the *undeformed ABJM theory* can be read on the action of a *probe D2-brane* in $AdS_4 \times \mathbb{CP}_3$ (or *M2-brane* in $AdS_4 \times S^7/\mathbb{Z}_k$)

D*p*-brane action

$$S_{\mathrm{D}p} = -\tau_p \int d^{p+1}\sigma \, e^{-\Phi} \sqrt{-\det\left(\hat{G}_{ab} + \hat{B}_{ab} + F_{ab}\right)} + \tau_p \int_{\mathcal{M}_{p+1}} \sum_q \hat{C}_q \wedge e^{\hat{B}+F}$$

D2 probe

$$\overbrace{\qquad\qquad}^{AdS_4} \quad \overbrace{\qquad\qquad\qquad}^{\mathbb{CP}_3}$$

	x^0	x^1	x^2	r	ξ	ψ	θ_1	θ_2	φ_1	φ_2
D2	—	—	—	●	●	●	●	●	●	●

SUBSTITUTE AND DUALIZE 3d GAUGE FIELD

$\mathbb{C}^4/\mathbb{Z}_k$

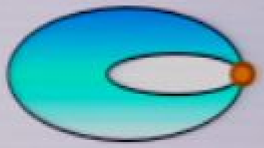
MODULI SPACE

What about the *moduli space* of the *β-deformed ABJM theory* ?

TsT

GENERIC γ *D2-brane* sitting at special points where *the torus shrinks*

	x^0	x^1	x^2	r	ξ	ψ	θ_1	θ_2	φ_1	φ_2
D2	—	—	—	●	●	●	●	●	●	●

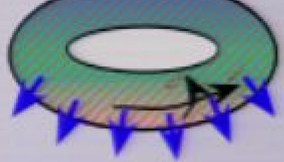


$\mathcal{M} = 1$

(SIX POSSIBILITIES)

RATIONAL γ *D4-brane* with *electric* and *magnetic flux*

	x^0	x^1	x^2	r	ξ	ψ	θ_1	θ_2	φ_1	φ_2
D4	—	—	—	●	●	●	●	●	—	—

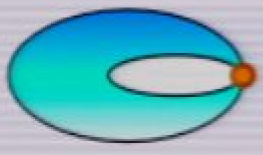


$F_{\varphi_1\varphi_2} = \frac{1}{\gamma}$

$F_{a\varphi_1} \quad F_{a\varphi_2}$

D2-brane probe

Embedding	x^0	x^1	x^2	r	ξ	ψ	θ_1	θ_2	φ_1	φ_2
D2	—	—	—	●	●	●	●	●	●	●



SIX POSSIBILITIES:
CHOOSE $\xi = 0$

$$S_{\rm D2} = -\tau_2 \int d^3\sigma e^{-\Phi} \sqrt{-\det(\hat{G}_{ab} + \hat{B}_{ab} + F_{ab})} + \tau_2 \int_{\mathcal{M}_3} (\hat{C}_3 + \hat{C}_1 \wedge (\hat{B} + F))$$

D2-brane probe

TsT transformation of $AdS_4 \times \mathbb{CP}_3$

$$\begin{aligned} ds^2 = \frac{R^3}{k}\Big[&\tfrac{1}{4}ds^2_{AdS_4} + d\xi^2 + \mathcal{M}\cos^2\xi\sin^2\xi\left(d\psi + \tfrac{1}{2}\cos\theta_1 d\varphi_1 - \tfrac{1}{2}\cos\theta_2 d\varphi_2\right)^2 \\ &+\tfrac{1}{4}\cos^2\xi(d\theta_1^2 + \mathcal{M}\sin^2\theta_1 d\varphi_1^2) + \tfrac{1}{4}\sin^2\xi(d\theta_2^2 + \mathcal{M}\sin^2\theta_2 d\varphi_2^2) \\ &+\hat{\gamma}^2\mathcal{M}\cos^4\xi\sin^4\xi\sin^2\theta_1\sin^2\theta_2 d\psi^2\Big] \end{aligned}$$

$$e^{2\Phi} = \frac{R^3}{k^3}\mathcal{M}$$

$$\begin{aligned} F_2 = k\Big(&-\cos\xi\sin\xi d\xi \wedge (2d\psi + \cos\theta_1 d\varphi_1 - \cos\theta_2 d\varphi_2) \\ &-\tfrac{1}{2}\cos^2\xi\sin\theta_1 d\theta_1 \wedge d\varphi_1 - \tfrac{1}{2}\sin^2\xi\sin\theta_2 d\theta_2 \wedge d\varphi_2\Big) \end{aligned}$$

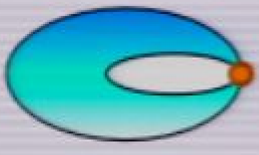
$$\begin{aligned} F_4 = -\frac{3R^3}{8}&\left(\omega_{AdS_4} + 4\hat{\gamma}\cos^3\xi\sin^3\xi\sin\theta_1\sin\theta_2 d\xi \wedge d\psi \wedge d\theta_1 \wedge d\theta_2\right) \\ &-\frac{R^3}{8}d(\hat{\gamma}\mathcal{M}\cos^2\xi\sin^2\xi(\cos^2\xi\sin^2\theta_1 - \sin^2\xi\sin^2\theta_2)) \wedge d\psi \wedge d\varphi_1 \wedge d\varphi_2 \end{aligned}$$

$$\begin{aligned} B = -\frac{\hat{\gamma}\mathcal{M}R^3}{k}\cos^2\xi\sin^2\xi\,\Big(&\tfrac{1}{2}\cos^2\xi\sin^2\theta_1\cos\theta_2 d\psi \wedge d\varphi_1 + \tfrac{1}{2}\sin^2\xi\sin^2\theta_2\cos\theta_1 d\psi \wedge d\varphi_2 \\ &+\tfrac{1}{4}\left(\sin^2\theta_1\sin^2\theta_2 + \cos^2\xi\sin^2\theta_1\cos^2\theta_2 + \sin^2\xi\sin^2\theta_2\cos^2\theta_1\right) d\varphi_1 \wedge d\varphi_2\Big) \end{aligned}$$

$$\mathcal{M}^{-1} = 1 + \hat{\gamma}^2\cos^2\xi\sin^2\xi\left(\sin^2\theta_1\sin^2\theta_2 + \cos^2\xi\sin^2\theta_1\cos^2\theta_2 + \sin^2\xi\sin^2\theta_2\cos^2\theta_1\right)$$

D2-brane probe

Embedding	x^0	x^1	x^2	r	ξ	ψ	θ_1	θ_2	φ_1	φ_2
D2	—	—	—	●	●	●	●	●	●	●



SIX POSSIBILITIES:
CHOOSE $\xi = 0$

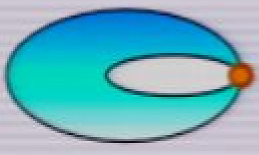
$$S_{\mathrm{D2}} = -\tau_2 \int d^3\sigma e^{-\Phi}\sqrt{-\det(\hat{G}_{ab}+\hat{B}_{ab}+F_{ab})} + \tau_2 \int_{\mathcal{M}_3} (\hat{C}_3 + \hat{C}_1 \wedge (\hat{B}+F))$$

$\mathcal{M} = 1$ when the torus shrinks

SAME LAGRANGIAN AS IN THE UNDEFORMED CASE FOR $\xi = 0$

D2-brane probe

Embedding	x^0	x^1	x^2	r	ξ	ψ	θ_1	θ_2	φ_1	φ_2
D2	—	—	—	●	●	●	●	●	●	●



SIX POSSIBILITIES: CHOOSE $\xi = 0$

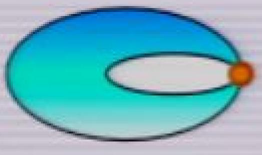
$$S_{\rm D2} = -\tau_2 \int d^3\sigma e^{-\Phi}\sqrt{-\det(\hat{G}_{ab}+\hat{B}_{ab}+F_{ab})} + \tau_2 \int_{\mathcal{M}_3} (\underbrace{\hat{C}_3 + \hat{C}_1 \wedge (\hat{B}}_{\text{UNDEFORMED } C_3} + F))$$

$\mathcal{M} = 1$ when the torus shrinks

SAME LAGRANGIAN AS IN THE UNDEFORMED CASE FOR $\xi = 0$

D2-brane probe

Embedding	x^0	x^1	x^2	r	ξ	ψ	θ_1	θ_2	φ_1	φ_2
D2	—	—	—	●	●	●	●	●	●	●



SIX POSSIBILITIES:

CHOOSE $\xi = 0$

$$S_{\rm D2} = -\tau_2 \int d^3\sigma e^{-\Phi}\sqrt{-\det(\hat{G}_{ab}+\hat{B}_{ab}+F_{ab})} + \tau_2 \int_{\mathcal{M}_3} (\underbrace{\hat{C}_3 + \hat{C}_1 \wedge (\hat{B}}_{\text{UNDEFORMED } C_3} + \boxed{F}))$$

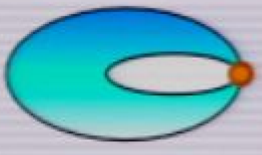
$\mathcal{M} = 1$ when the torus shrinks

SAME LAGRANGIAN AS IN THE UNDEFORMED CASE FOR $\xi = 0$

DUALIZE THE 3d GAUGE FIELD VIA LAGRANGE MULTIPLIER $\frac{\tau_2 k}{2}\int \alpha \, dF$

D2-brane probe

Embedding	x^0	x^1	x^2	r	ξ	ψ	θ_1	θ_2	φ_1	φ_2
D2	—	—	—	●	●	●	●	●	●	●



SIX POSSIBILITIES: CHOOSE $\xi = 0$

$$S_{\rm D2} = -\tau_2 \int d^3\sigma e^{-\Phi}\sqrt{-\det(\hat{G}_{ab}+\hat{B}_{ab}+F_{ab})} + \tau_2 \int_{\mathcal{M}_3} (\underbrace{\hat{C}_3 + \hat{C}_1 \wedge (\hat{B}}_{\text{UNDEFORMED } C_3} + \bigcirc\!\!\!\!F\,))$$

$\mathcal{M} = 1$ when the torus shrinks

SAME LAGRANGIAN AS IN THE UNDEFORMED CASE FOR $\xi = 0$

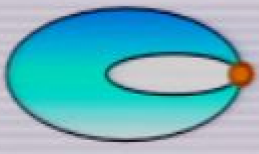
DUALIZE THE 3d GAUGE FIELD VIA LAGRANGE MULTIPLIER $\frac{\tau_2 k}{2}\int \alpha\, dF$

$$S_{\rm D2}^{\rm TsT} = -\frac{\tau_2 R^3}{4}\int d^3\sigma \left[(\partial_a\rho)^2 + \frac{\rho^2}{4}\left((\partial_a\theta_1)^2 + (\partial_a\varphi_1)^2 + (\partial_a\alpha)^2 + 2\cos\theta_1\partial_a\varphi_1\partial_a\alpha\right)\right]$$

$r = \rho^2$

D2-brane probe

Embedding	x^0	x^1	x^2	r	ξ	ψ	θ_1	θ_2	φ_1	φ_2
D2	—	—	—	●	●	●	●	●	●	●



SIX POSSIBILITIES:
CHOOSE $\xi = 0$

$$S_{\rm D2} = -\tau_2 \int d^3\sigma e^{-\Phi}\sqrt{-\det(\hat{G}_{ab}+\hat{B}_{ab}+F_{ab})} + \tau_2 \int_{\mathcal{M}_3} (\underbrace{\hat{C}_3 + \hat{C}_1 \wedge (\hat{B}}_{\text{UNDEFORMED } C_3} + \boxed{F}))$$

$\mathcal{M} = 1$ when the torus shrinks

SAME LAGRANGIAN AS IN THE UNDEFORMED CASE FOR $\xi = 0$

DUALIZE THE 3d GAUGE FIELD VIA LAGRANGE MULTIPLIER $\frac{\tau_2 k}{2}\int \alpha \, dF$

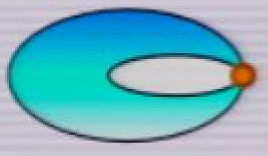
$$\boxed{S^{\rm TsT}_{\rm D2} = -\frac{\tau_2 R^3}{4}\int d^3\sigma \left[(\partial_a\rho)^2 + \frac{\rho^2}{4}\underbrace{((\partial_a\theta_1)^2 + (\partial_a\varphi_1)^2 + (\partial_a\alpha)^2 + 2\cos\theta_1\partial_a\varphi_1\partial_a\alpha)}_{\text{THREE-SPHERE}}\right]}$$

$r = \rho^2$

D2-brane probe

Embedding

	x^0	x^1	x^2	r	ξ	ψ	θ_1	θ_2	φ_1	φ_2
D2	—	—	—	●	●	●	●	●	●	●



SIX POSSIBILITIES: CHOOSE $\xi = 0$

$$S_{\rm D2} = -\tau_2 \int d^3\sigma e^{-\Phi}\sqrt{-\det(\hat{G}_{ab} + \hat{B}_{ab} + F_{ab})} + \tau_2 \int_{\mathcal{M}_3} (\underbrace{\hat{C}_3 + \hat{C}_1 \wedge (\hat{B}}_{} + \boxed{F}))$$

$\mathcal{M} = 1$ when the torus shrinks

UNDEFORMED C_3

SAME LAGRANGIAN AS IN THE UNDEFORMED CASE FOR $\xi = 0$

DUALIZE THE 3d GAUGE FIELD VIA LAGRANGE MULTIPLIER $\frac{\tau_2 k}{2}\int \alpha\, dF$

$$\boxed{S^{\rm TsT}_{\rm D2} = -\frac{\tau_2 R^3}{4}\int d^3\sigma \left[(\partial_a\rho)^2 + \frac{\rho^2}{4}\underbrace{((\partial_a\theta_1)^2 + (\partial_a\varphi_1)^2 + (\partial_a\alpha)^2 + 2\cos\theta_1 \partial_a\varphi_1\partial_a\alpha)}_{}\right]}$$

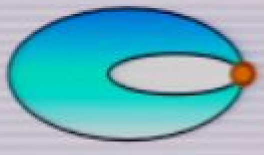
$r = \rho^2$

THREE-SPHERE

α HAS PERIODICITY $4\pi/k$

D2-brane probe

Embedding	x^0	x^1	x^2	r	ξ	ψ	θ_1	θ_2	φ_1	φ_2
D2	—	—	—	●	●	●	●	●	●	●



SIX POSSIBILITIES: CHOOSE $\xi = 0$

$$S_{\rm D2} = -\tau_2 \int d^3\sigma e^{-\Phi}\sqrt{-\det(\hat{G}_{ab}+\hat{B}_{ab}+F_{ab})} + \tau_2 \int_{\mathcal{M}_3} (\underbrace{\hat{C}_3 + \hat{C}_1 \wedge (\hat{B}}_{} + F))$$

$\mathcal{M} = 1$ when the torus shrinks

UNDEFORMED C_3

SAME LAGRANGIAN AS IN THE UNDEFORMED CASE FOR $\xi = 0$

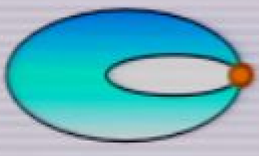
DUALIZE THE 3d GAUGE FIELD VIA LAGRANGE MULTIPLIER $\frac{\tau_2 k}{2}\int \alpha \, dF$

$$S^{\rm TsT}_{\rm D2} = -\frac{\tau_2 R^3}{4}\int d^3\sigma \left[(\partial_a\rho)^2 + \frac{\rho^2}{4}\left((\partial_a\theta_1)^2 + (\partial_a\varphi_1)^2 + (\partial_a\alpha)^2 + 2\cos\theta_1\partial_a\varphi_1\partial_a\alpha\right)\right]$$

The abelian *moduli space* is made up of *six copies* of $\mathbb{C}^2/\mathbb{Z}_k$

D2-brane probe

Embedding	x^0	x^1	x^2	r	ξ	ψ	θ_1	θ_2	φ_1	φ_2
D2	—	—	—	●	●	●	●	●	●	●



SIX POSSIBILITIES: CHOOSE $\xi = 0$

$$S_{\mathrm{D2}} = -\tau_2 \int d^3\sigma e^{-\Phi}\sqrt{-\det(\hat{G}_{ab}+\hat{B}_{ab}+F_{ab})} + \tau_2 \int_{\mathcal{M}_3} (\underbrace{\hat{C}_3 + \hat{C}_1 \wedge (\hat{B}}_{\text{UNDEFORMED } C_3} + F))$$

$\mathcal{M} = 1$ when the torus shrinks

SAME LAGRANGIAN AS IN THE UNDEFORMED CASE FOR $\xi = 0$

DUALIZE THE 3d GAUGE FIELD VIA LAGRANGE MULTIPLIER $\frac{\tau_2 k}{2}\int \alpha \, dF$

$$S^{\mathrm{TsT}}_{\mathrm{D2}} = -\frac{\tau_2 R^3}{4}\int d^3\sigma \left[(\partial_a\rho)^2 + \frac{\rho^2}{4}\left((\partial_a\theta_1)^2 + (\partial_a\varphi_1)^2 + (\partial_a\alpha)^2 + 2\cos\theta_1\partial_a\varphi_1\partial_a\alpha\right)\right]$$

The abelian *moduli space* is made up of *six copies* of $\mathbb{C}^2/\mathbb{Z}_k$

Gauge theory

Superpotential $W_\gamma = \frac{4\pi}{k}\,\mathrm{Tr}\left(e^{-i\pi\gamma/2}A_1B_1A_2B_2 - e^{i\pi\gamma/2}A_1B_2A_2B_1\right)$

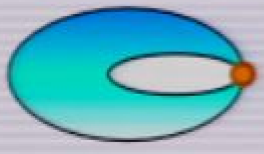
F-term equations $B_1A_2B_2 - e^{i\pi\gamma}B_2A_2B_1 = 0 \quad B_2A_1B_1 - e^{i\pi\gamma}B_1A_1B_2 = 0$

$A_2B_2A_1 - e^{i\pi\gamma}A_1B_2A_2 = 0 \quad A_1B_1A_2 - e^{i\pi\gamma}A_2B_1A_1 = 0$

D2-brane probe

Embedding

	x^0	x^1	x^2	r	ξ	ψ	θ_1	θ_2	φ_1	φ_2
D2	—	—	—	●	●	●	●	●	●	●



SIX POSSIBILITIES: CHOOSE $\xi = 0$

$$S_{\rm D2} = -\tau_2 \int d^3\sigma e^{-\Phi}\sqrt{-\det(\hat G_{ab} + \hat B_{ab} + F_{ab})} + \tau_2 \int_{\mathcal{M}_3} (\underbrace{\hat C_3 + \hat C_1 \wedge (\hat B}_{\text{UNDEFORMED } C_3} + \boxed{F}))$$

$\mathcal{M} = 1$ when the torus shrinks

SAME LAGRANGIAN AS IN THE UNDEFORMED CASE FOR $\xi = 0$

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$$A_2B_2A_1 - e^{i\pi\gamma}A_1B_2A_2 = 0 \qquad A_1B_1A_2 - e^{i\pi\gamma}A_2B_1A_1 = 0$$

➡ *Six possibilities* to set two out of the four fields to zero: each one spans $\mathbb{C}^2/\mathbb{Z}_k$

D4-brane probe and new branches of vacua

For ***rational*** $\gamma = m/n$ we expect ***new branches*** to arise ⟺ ***D4-branes***

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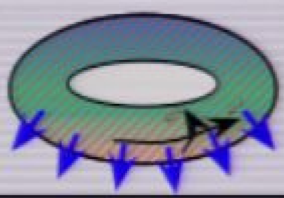
D4	—	—	—	●	●	●	●	●	—	—



$F_{\varphi_1\varphi_2} = \frac{1}{\gamma}$

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D4-brane probe and new branches of vacua

TsT transformation of $AdS_4 \times \mathbb{CP}_3$

$$ds^2 = \frac{R^3}{k}\Big[\tfrac{1}{4}ds^2_{AdS_4} + d\xi^2 + \mathcal{M}\cos^2\xi\sin^2\xi\left(d\psi + \tfrac{1}{2}\cos\theta_1 d\varphi_1 - \tfrac{1}{2}\cos\theta_2 d\varphi_2\right)^2$$
$$+\tfrac{1}{4}\cos^2\xi(d\theta_1^2 + \mathcal{M}\sin^2\theta_1 d\varphi_1^2) + \tfrac{1}{4}\sin^2\xi(d\theta_2^2 + \mathcal{M}\sin^2\theta_2 d\varphi_2^2)$$
$$+\hat{\gamma}^2\mathcal{M}\cos^4\xi\sin^4\xi\sin^2\theta_1\sin^2\theta_2 d\psi^2\Big]$$

$$e^{2\Phi} = \frac{R^3}{k^3}\mathcal{M}$$

$$F_2 = k\Big(-\cos\xi\sin\xi d\xi\wedge(2d\psi + \cos\theta_1 d\varphi_1 - \cos\theta_2 d\varphi_2)$$
$$-\tfrac{1}{2}\cos^2\xi\sin\theta_1 d\theta_1\wedge d\varphi_1 - \tfrac{1}{2}\sin^2\xi\sin\theta_2 d\theta_2\wedge d\varphi_2\Big)$$

$$F_4 = -\frac{3R^3}{8}\left(\omega_{AdS_4} + 4\hat{\gamma}\cos^3\xi\sin^3\xi\sin\theta_1\sin\theta_2 d\xi\wedge d\psi\wedge d\theta_1\wedge d\theta_2\right)$$
$$-\frac{R^3}{8}d(\hat{\gamma}\mathcal{M}\cos^2\xi\sin^2\xi(\cos^2\xi\sin^2\theta_1 - \sin^2\xi\sin^2\theta_2))\wedge d\psi\wedge d\varphi_1\wedge d\varphi_2$$

$$B = -\frac{\hat{\gamma}\mathcal{M}R^3}{k}\cos^2\xi\sin^2\xi\ \Big(\tfrac{1}{2}\cos^2\xi\sin^2\theta_1\cos\theta_2 d\psi\wedge d\varphi_1 + \tfrac{1}{2}\sin^2\xi\sin^2\theta_2\cos\theta_1 d\psi\wedge d\varphi_2$$
$$+\tfrac{1}{4}\left(\sin^2\theta_1\sin^2\theta_2 + \cos^2\xi\sin^2\theta_1\cos^2\theta_2 + \sin^2\xi\sin^2\theta_2\cos^2\theta_1\right)d\varphi_1\wedge d\varphi_2\Big)$$

$$\mathcal{M}^{-1} = 1 + \hat{\gamma}^2\cos^2\xi\sin^2\xi\left(\sin^2\theta_1\sin^2\theta_2 + \cos^2\xi\sin^2\theta_1\cos^2\theta_2 + \sin^2\xi\sin^2\theta_2\cos^2\theta_1\right)$$

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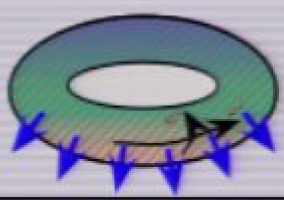
Embedding

	x^0	x^1	x^2	r	ξ	ψ	θ_1	θ_2	φ_1	φ_2
D4	—	—	—	●	●	●	●	●	—	—



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Trade the *Wilson lines* on the torus with *periodic scalars* $A_{\varphi_1} = \frac{\phi_2}{\gamma}$ $A_{\varphi_2} = -\frac{\phi_1}{\gamma}$, *integrate* over the two torus and and *dualize* the 3d gauge field

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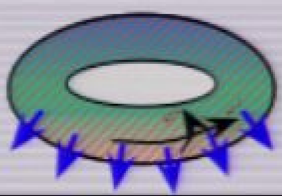

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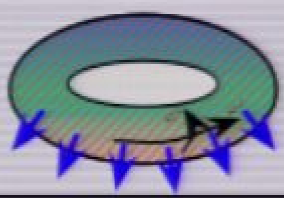
D4	—	—	—	●	●	●	●	●	—	—



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D2-BRANE PROBE IN UNDEFORMED BACKGROUND

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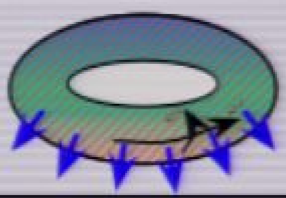
D4	—	—	—	●	●	●	●	●	—	—



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D2-BRANE PROBE IN UNDEFORMED BACKGROUND

HOWEVER THE PERIODICITY OF ϕ_1 AND ϕ_2 IS $2\pi/n$

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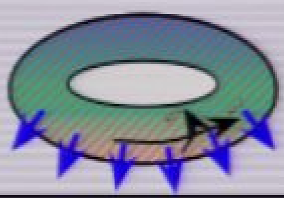
D4	—	—	—	●	●	●	●	●	—	—



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The abelian ***moduli space*** is a $\mathbb{Z}_n \times \mathbb{Z}_n$ ***orbifold*** of $\mathbb{C}^4/\mathbb{Z}_k$

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Lesson

β-deformed theories show *new features*
for *quantized values* of the deformation parameter

These features can be captured by
TsT-transformed D-brane probes in the gravity duals

Quantities captured by D-brane probes that
do not transform under TsT
will be *invariant* under the deformation

$S_{D4} = -\tau_4 \int$ … $) \wedge (\hat{B} + F))$

Trade the … $= -\frac{\phi_1}{\gamma},$
integrate

D2-BRANE PROBE IN UNDEFORMED BACKGROUND

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Gravity dual of unquenched SQCD

We now turn to a more "realistic" theory: $\mathcal{N}=1$ ***Supersymmetric QCD***

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QUARKS GAUGE VECTOR

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QUARKS

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QUARTIC SUPERPOTENTIAL $\mathcal{W}$

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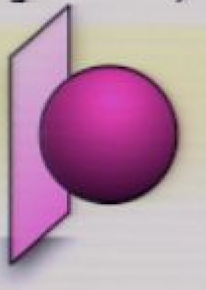
The ***gravity dual*** is given by the backreaction of N_c ***"color" D5-branes*** and N_f ***"flavor" D5-branes*** wrapped on submanifolds of a non-compact Calabi-Yau space



[Maldacena-Núñez, Casero-Núñez-Paredes]

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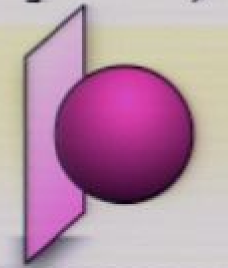

In the *quenched approximation* $N_f \ll N_c$ the ***flavor branes*** can be treated as ***probes***

[Karch-Katz]

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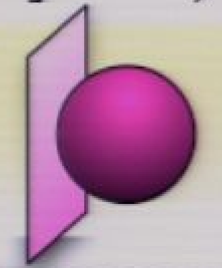

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COLOR ***FLAVOR***

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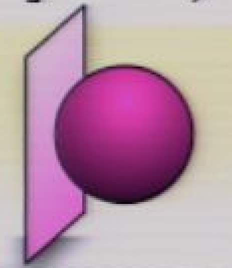

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COLOR

FLAVOR

Localization of the branes is difficult

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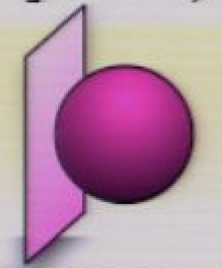

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COLOR ***FLAVOR***

Localization of the branes is difficult

SMEARING

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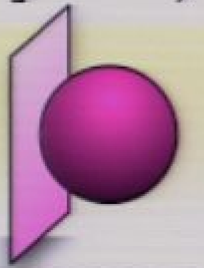

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[Casero-Núñez-Paredes]

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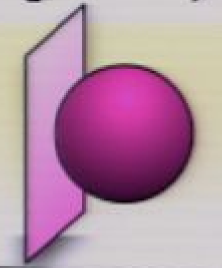

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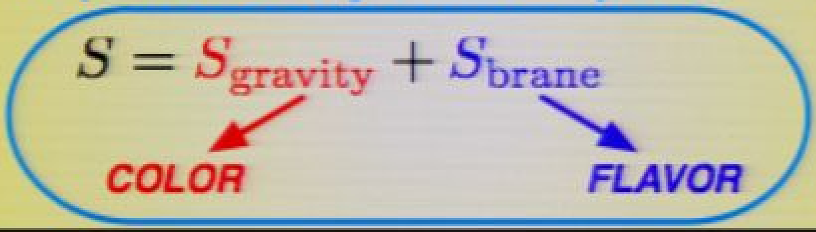

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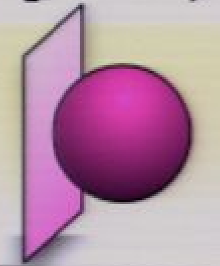

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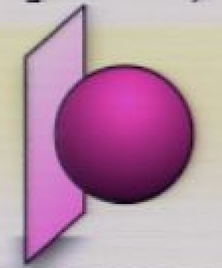

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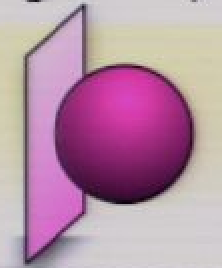

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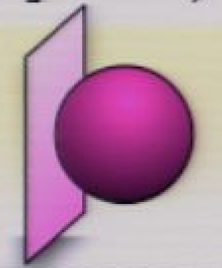

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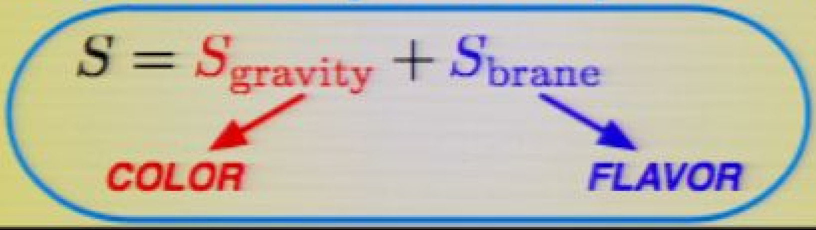

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due to source term

$$S_{D5}^{(\mathrm{WZ})} = \frac{\tau_5 N_f}{4\pi^2}\int \mathrm{Vol}(\mathcal{Y}_4)\wedge C_6$$

SMEARING

[Casero-Núñez-Paredes]

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$ds^2 =$ … $\sin^2\tilde\theta d\tilde\varphi^2\Big)$

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WHY?

It was shown in the case of the
dual of $\mathcal{N}=1$ Super Yang-Mills that **TsT** helps
in *disentangling the gauge theory dynamics* from
the unwanted *Kaluza-Klein modes*
[Gürsoy-Núñez]

It is interesting to study
the effects of the transformation
on a solution of *supergravity plus branes*

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$4\pi^2 \int$

[Casero-Núñez-Paredes]

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TsT of the gravity dual of SQCD

TsT of the CNP solution along $(\varphi, \tilde{\varphi})$

$$ds^2 = e^{\phi}\Big[dx_{1,3}^2 + 4Y\,d\rho^2 + H\left(d\theta^2 + \mathcal{M}\sin^2\theta d\varphi^2\right) + G\left(d\tilde{\theta}^2 + \mathcal{M}\sin^2\tilde{\theta}d\tilde{\varphi}^2\right)$$
$$+\mathcal{M}Y\left(d\psi + \cos\theta d\varphi + \cos\tilde{\theta}d\tilde{\varphi}\right)^2 + \gamma^2 e^{2\phi}\mathcal{M}GHY\sin^2\theta\sin^2\tilde{\theta}d\psi^2\Big]$$

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$$B = \gamma e^{2\phi}\mathcal{M}\Big[HY\sin^2\theta\cos\tilde{\theta}d\psi\wedge d\varphi - GY\sin^2\tilde{\theta}\cos\theta d\psi\wedge d\tilde{\varphi}$$
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$$\mathcal{M}^{-1} = 1 + \gamma^2 e^{2\phi}\left(GH\sin^2\theta\sin^2\tilde{\theta} + HY\sin^2\theta\cos^2\tilde{\theta} + GY\cos^2\theta\sin^2\tilde{\theta}\right)$$

Deformed solution and brane sources

What are the *D-brane sources* of the deformed solution?

CNP flavor branes $S = S_{\rm IIB} + S_{\rm D5_f}$

	x^0	x^1	x^2	x^3	ρ	ψ	θ	$\tilde\theta$	φ	$\tilde\varphi$
$\rm D5_f$	—	—	—	—	—	—	—	—	—	—

SMEARING

TsT of the gravity dual of SQCD

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$\underbrace{\theta \quad \tilde{\theta} \quad \varphi \quad \tilde{\varphi}}_{\text{SMEARING}}$

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$D5_{\rm f}$	—	—	—	—	—	—	—	—	—	—

SMEARING

TsT

Transformed flavor branes $F_{\varphi\tilde\varphi} = \frac{1}{\gamma}$

	x^0	x^1	x^2	x^3	ρ	ψ	θ	$\tilde\theta$	φ	$\tilde\varphi$
$D7_{\rm f}$	—	—	—	—	—	—	—	—	—	—

SMEARING ?

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TsT

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SMEARING ?

D7-brane action

$$S^{(\mathrm{WZ})}_{\mathrm{D7}} = \frac{N_7 \tau_7}{4} \int \mathrm{Vol}(\mathcal{Y}_2) \wedge \left[C_8 + C_6 \wedge (B+F) + \frac{1}{2} C_4 \wedge (B+F) \wedge (B+F) \right]$$

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TsT

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SMEARING ?

N_7 *FLAVOR BRANES*

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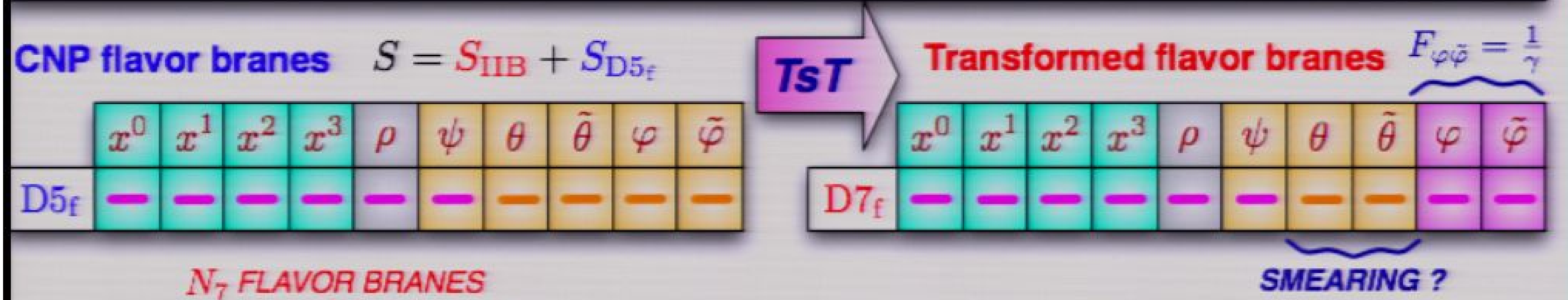

CNP flavor branes $S = S_{\rm IIB} + S_{\rm D5_f}$

	x^0	x^1	x^2	x^3	ρ	ψ	θ	$\tilde\theta$	φ	$\tilde\varphi$
$\rm D5_f$	—	—	—	—	—	—	—	—	—	—

TsT

Transformed flavor branes $\underbrace{F_{\varphi\tilde\varphi} = \frac{1}{\gamma}}$

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SMEARING ?

N_7 FLAVOR BRANES

D7-brane action

$$S_{\rm D7}^{\rm (WZ)} = \frac{N_7 \tau_7}{4} \int {\rm Vol}(\mathcal{Y}_2) \wedge \left[C_8 + C_6 \wedge (B+F) + \frac{1}{2} C_4 \wedge (B+F) \wedge (B+F) \right]$$

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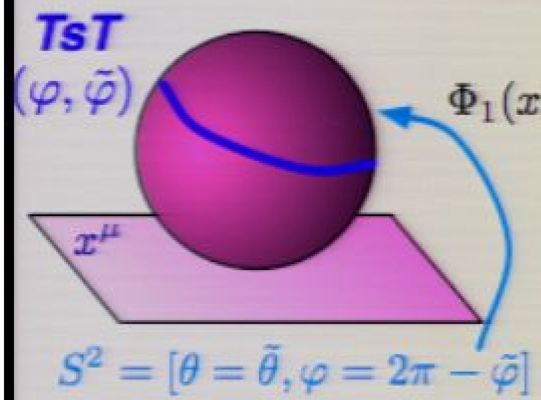
Does it imply the *quantization of* γ ?

Features of the deformed solution

The gauge theory counterpart of the *TsT transformation* is a *dipole deformation* of the theory on the five-branes



6D DIPOLE THEORY

$$\Phi_1(x) \star \Phi_2(x) = \Phi_1(x - L_2/2)\Phi_2(x + L_1/2)$$

[Gürsoy-Núñez]

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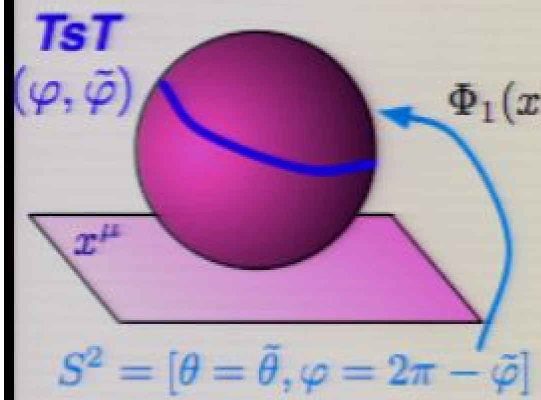

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KALUZA-KLEIN REDUCTION

4D ORDINARY THEORY

MODIFIED INTERACTIONS OF KALUZA-KLEIN MODES

[Gürsoy-Núñez]

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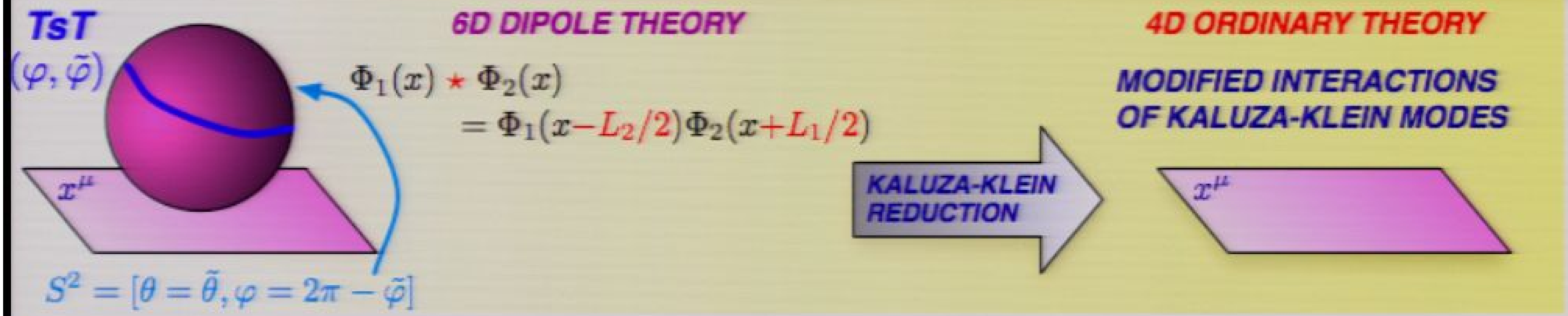

TsT

$(\varphi, \tilde{\varphi})$

x^μ

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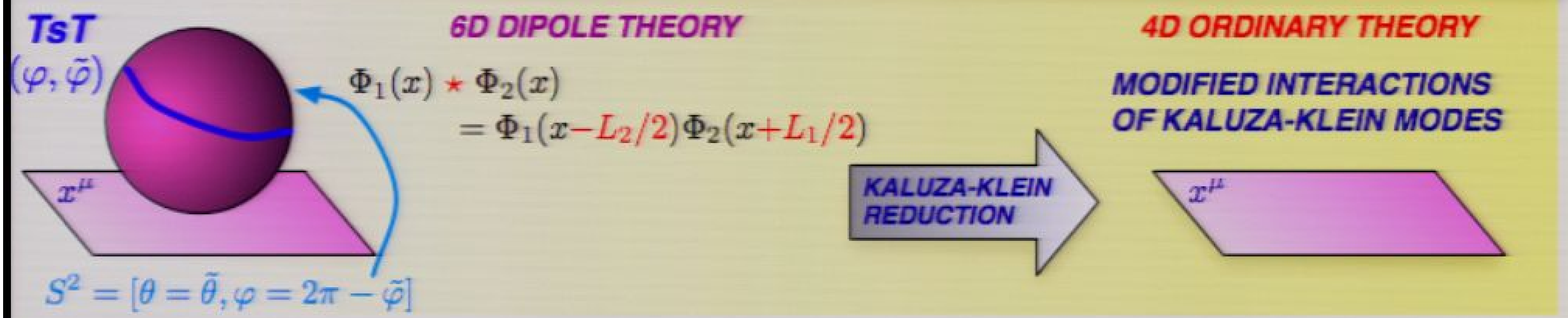

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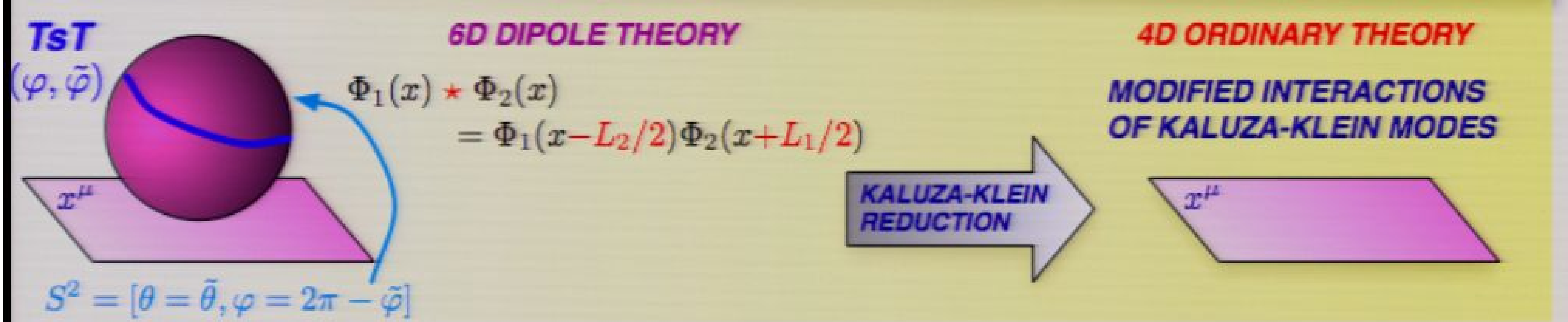

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At least *two types* of interesting features to study:

- ***Universal*** features, independent of the Kaluza-Klein dynamics and of the deformation of quark interactions
- ***New*** features due to the deformation of quark interactions

Features of the deformed solution

Universal features

- ***Gauge coupling*** and ***theta angle*** are computed via a "color" D5-brane probe on $S^2 = [\theta = \tilde{\theta}, \varphi = 2\pi - \tilde{\varphi}]$, which is ***invariant under TsT***

$$\frac{8\pi^2}{g^2_{\mathrm{YM}}} = 2(H(\rho) + G(\rho)) \qquad \theta_{\mathrm{YM}} = \frac{2N_f - N_c}{2}(\psi - \psi_0)$$

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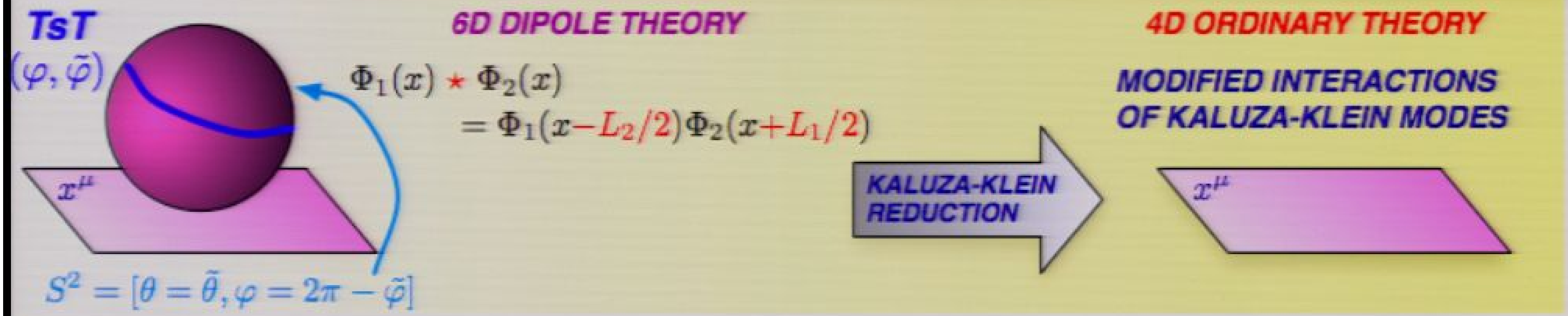

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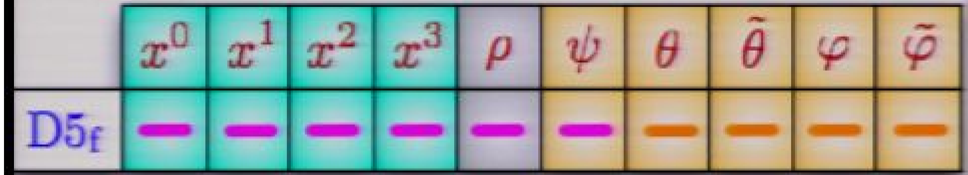

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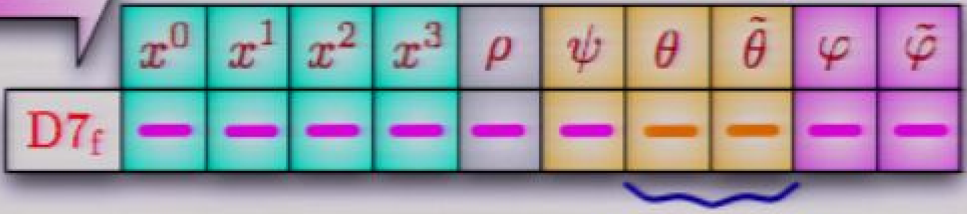

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TsT of the gravity dual of SQCD

The "type A" CNP solution

$$ds^2 = e^{\phi}\Big[dx_{1,3}^2 + 4Y\,d\rho^2 + H\left(d\theta^2 + \sin^2\theta d\varphi^2\right) + G\left(d\tilde{\theta}^2 + \sin^2\tilde{\theta}d\tilde{\varphi}^2\right)$$
$$+ \; Y\left(d\psi + \cos\theta d\varphi + \cos\tilde{\theta}d\tilde{\varphi}\right)^2\Big]$$

$U(1)_{\varphi} \times U(1)_{\tilde{\varphi}}$

$$e^{2\Phi} = e^{2\phi}$$

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TsT of the gravity dual of SQCD

TsT of the CNP solution along $(\varphi, \tilde{\varphi})$

$$ds^2 = e^{\phi}\Big[dx_{1,3}^2 + 4Y\,d\rho^2 + H\left(d\theta^2 + \mathcal{M}\sin^2\theta d\varphi^2\right) + G\left(d\tilde{\theta}^2 + \mathcal{M}\sin^2\tilde{\theta}d\tilde{\varphi}^2\right)$$
$$+\mathcal{M}Y\left(d\psi + \cos\theta d\varphi + \cos\tilde{\theta}d\tilde{\varphi}\right)^2 + \gamma^2 e^{2\phi}\mathcal{M}GHY\sin^2\theta\sin^2\tilde{\theta}d\psi^2\Big]$$

$$e^{2\Phi} = e^{2\phi}\mathcal{M}$$

$$\mathcal{F}_3 = -\frac{N_c}{4}\sin\tilde{\theta}d\tilde{\theta}\wedge d\tilde{\varphi}\wedge(d\psi + \cos\theta d\varphi)$$
$$-\frac{N_f - N_c}{4}\sin\theta d\theta\wedge d\varphi\wedge(d\psi + \cos\tilde{\theta}d\tilde{\varphi}) - F_1\wedge B$$

$$B = \gamma e^{2\phi}\mathcal{M}\Big[HY\sin^2\theta\cos\tilde{\theta}d\psi\wedge d\varphi - GY\sin^2\tilde{\theta}\cos\theta d\psi\wedge d\tilde{\varphi}$$
$$-\left(GH\sin^2\theta\sin^2\tilde{\theta} + HY\sin^2\theta\cos^2\tilde{\theta} + GY\cos^2\theta\sin^2\tilde{\theta}\right)d\varphi\wedge d\tilde{\varphi}\Big]$$

$$F_1 = -\gamma\left[\frac{N_c}{4}\sin\tilde{\theta}\cos\theta d\tilde{\theta} - \frac{N_f - N_c}{4}\sin\theta\cos\tilde{\theta}d\theta\right]$$

$$\mathcal{M}^{-1} = 1 + \gamma^2 e^{2\phi}\left(GH\sin^2\theta\sin^2\tilde{\theta} + HY\sin^2\theta\cos^2\tilde{\theta} + GY\cos^2\theta\sin^2\tilde{\theta}\right)$$

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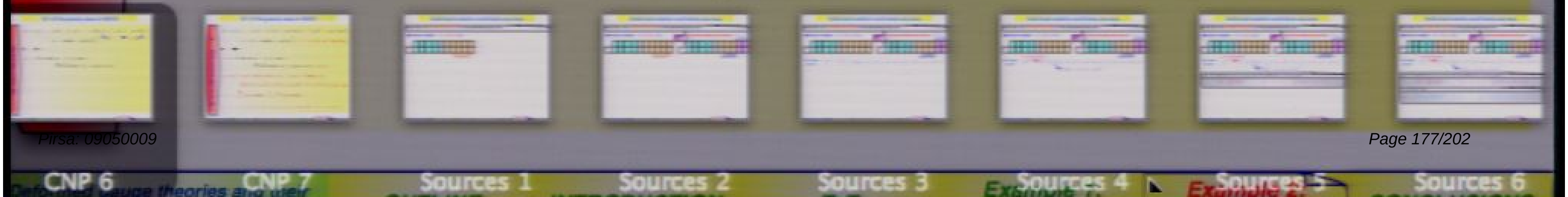
$\longleftarrow U(1)_{\varphi} \times U(1)_{\tilde{\varphi}} \longrightarrow$

$$e^{2\Phi} = e^{2\phi}$$

$$F_3 = -\frac{N_c}{4}\sin\tilde{\theta}d\tilde{\theta}\wedge d\tilde{\varphi}\wedge\left(d\psi + \cos\theta d\varphi\right) - \frac{N_f - N_c}{4}\sin\theta d\theta\wedge d\varphi\wedge\left(d\psi + \cos\tilde{\theta}d\tilde{\varphi}\right)$$



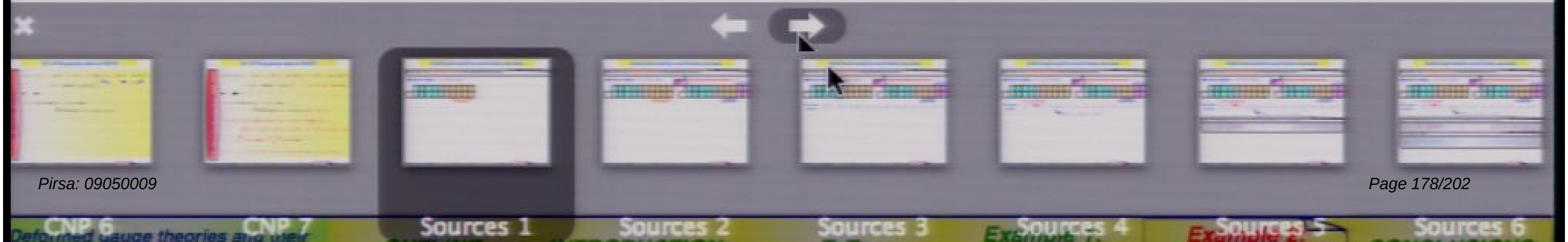
Deformed solution and brane sources

What are the *D-brane sources* of the deformed solution?

CNP flavor branes $S = S_{\text{IIB}} + S_{\text{D5}_\text{f}}$

	x^0	x^1	x^2	x^3	ρ	ψ	θ	$\tilde{\theta}$	φ	$\tilde{\varphi}$
D5_f	—	—	—	—	—	—	—	—	—	—

$\underbrace{\theta \;\; \tilde{\theta} \;\; \varphi \;\; \tilde{\varphi}}_{\text{SMEARING}}$



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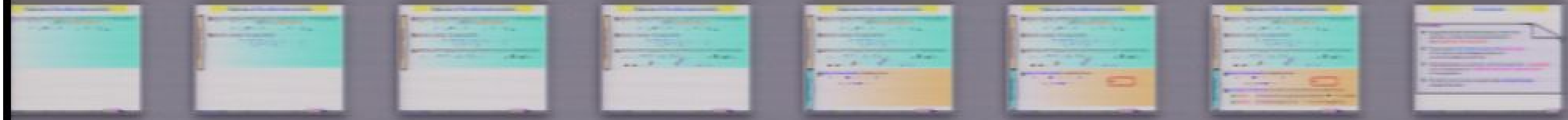

eatures 1 Features 2 Features 3 Features 4 Features 5 Features 6 Features 7 Conclusions

Features of the deformed solution

Universal features

- ***Gauge coupling*** and ***theta angle*** are computed via a "color" D5-brane probe on $S^2 = [\theta = \tilde{\theta}, \varphi = 2\pi - \tilde{\varphi}]$, which is *invariant under TsT*

$$\frac{8\pi^2}{g^2_{\rm YM}} = 2(H(\rho) + G(\rho)) \qquad \theta_{\rm YM} = \frac{2N_f - N_c}{2}(\psi - \psi_0)$$

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- ***Exact Seiberg duality*** is implemented as

$$H(\rho) \leftrightarrow G(\rho) \qquad N_c \leftrightarrow N_f - N_c$$

(or $(\theta, \varphi) \leftrightarrow (\tilde{\theta}, \tilde{\varphi})$)

$\boxed{\text{and } \ \gamma \leftrightarrow -\gamma}$

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- ***Two types of mesons*** as excitations on the world-volume of flavor branes
  - *Generic* γ ***D5-branes*** sitting at special points where the $(\varphi, \tilde{\varphi})$ torus shrinks
  - *Rational* γ ***D7-branes*** wrapped on the $(\varphi, \tilde{\varphi})$ torus with magnetic flux

Conclusions

In summary...

- Gauge theories with ***deformed product of fields*** in the lagrangian constitute interesting generalizations of the ***gauge/string correspondence***
- They are dual to ***TsT transformations*** of the ***string duals*** of the original undeformed gauge theories and can thus be studied systematically
- ***New phenomena*** occur when the deformation parameter γ is ***quantized*** such as the appearance of ***additional branches of mesonic vacua*** of the gauge theory
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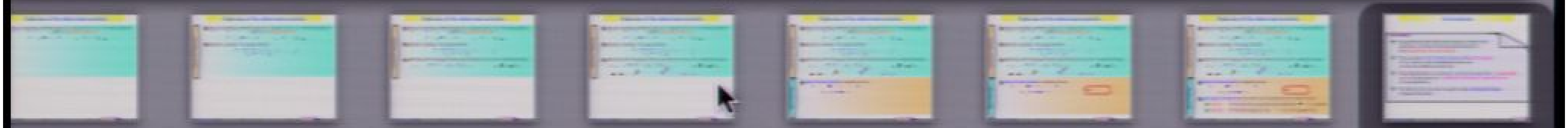

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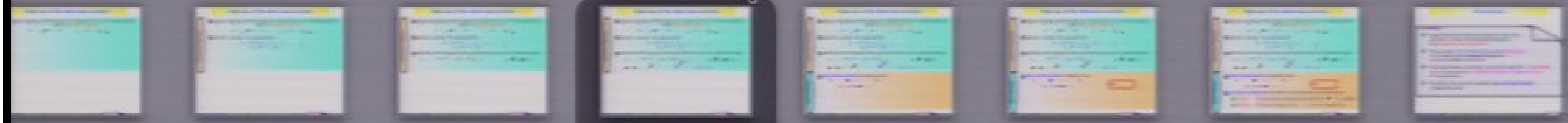

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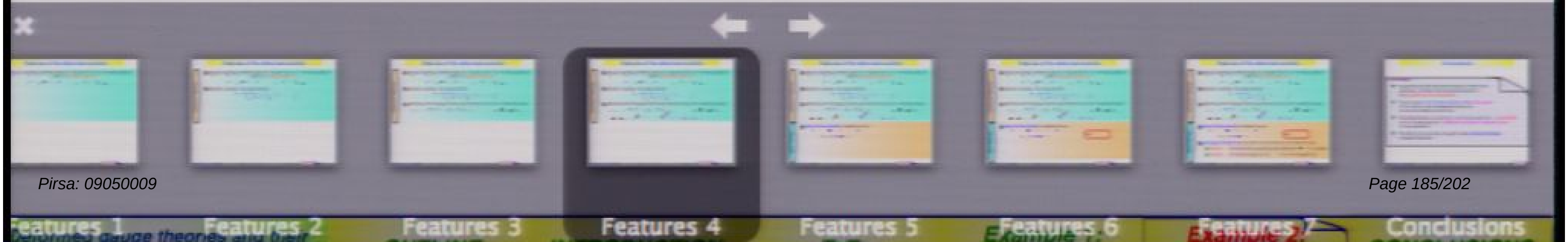

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No Signal

VGA-1

No Signal
VGA-1

No Signal
VGA-1

No Signal

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TsT

Transformed flavor branes $\underbrace{F_{\varphi\tilde\varphi} = \frac{1}{\gamma}}$

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$\rm D7_f$	—	—	—	—	—	—	—	—	—	—

SMEARING ?

N_7 FLAVOR BRANES

D7-brane action

$$S^{\rm (WZ)}_{\rm D7} = \frac{N_7\tau_7}{4}\int {\rm Vol}(\mathcal{Y}_2)\wedge\left[C_8 + C_6\wedge(B+F) + \frac{1}{2}C_4\wedge(B+F)\wedge(B+F)\right]$$

$${\rm Vol}(\mathcal{Y}_2) = \sin\theta\; d\theta\wedge\sin\tilde\theta\; d\tilde\theta$$

Type IIB equations are modified by the presence of the sources

$$dF_1 = \frac{N_7}{4}{\rm Vol}(\mathcal{Y}_2) \qquad d\mathcal{F}_3 + H_3\wedge F_1 = -\frac{N_7}{4}{\rm Vol}(\mathcal{Y}_2)\wedge(B+2\pi^2F)$$

Our solution has

$$dF_1 = \frac{\gamma N_f}{4}\sin\theta\sin\tilde\theta d\theta\wedge d\tilde\theta \qquad d\mathcal{F}_3 + H_3\wedge F_1 = \frac{N_f}{4}\sin\theta\sin\tilde\theta d\theta\wedge d\varphi\wedge d\tilde\theta\wedge d\tilde\varphi - dF_1\wedge B$$

MATCHING

$N_7 = \gamma N_f$ ***flavor D7-branes***

Does it imply the *quantization of γ* ?

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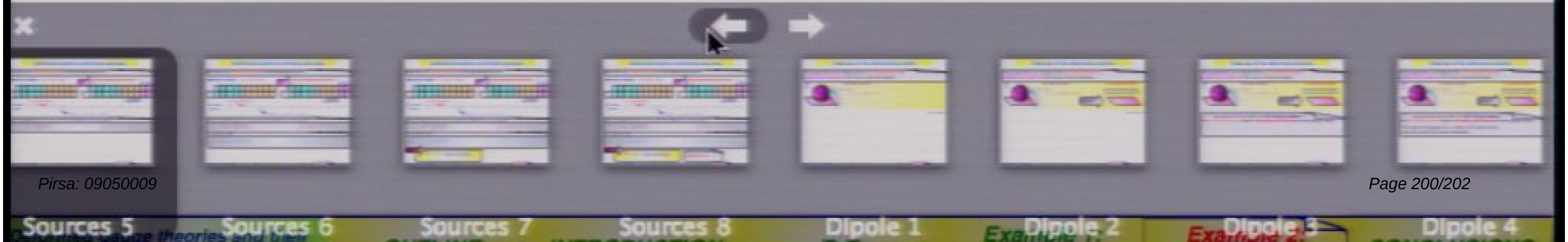


The gravity dual of SQCD

The "type A" CNP solution

$ds^2 =$ … $\sin^2 \tilde\theta d\tilde\varphi^2 \Big)$

We want to study the
TsT transformation of the **CNP solution**

$e^{2\Phi} = e^{2\phi}$

… $N_c)$

WHY?

It was shown in the case of the
dual of $\mathcal{N}=1$ *Super Yang-Mills* that *TsT* helps
in *disentangling the gauge theory dynamics* from
the unwanted *Kaluza-Klein modes*

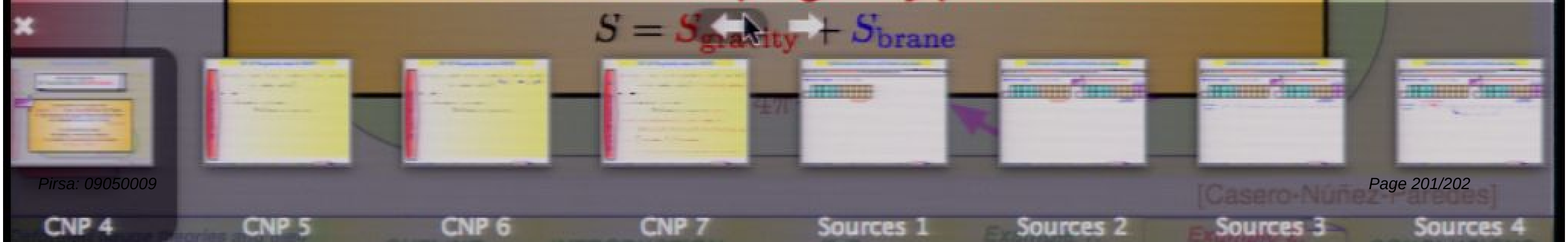
[Gürsoy-Núñez]

It is interesting to study
the effects of the transformation
on a solution of *supergravity plus branes*

$$S = S_{\text{gravity}} + S_{\text{brane}}$$

4π

[Casero-Núñez-Paredes]



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