Title: Membranes: living on the edge

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Interacting Membranes

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Outline

1. Introduction
2. M-theory
3. Branes in M-theory
4. Bagger-Lambert
5. ABJM and other generalisations
6. Membranes with a boundary?
7. What else can we learn?

M-theory is the strong coupling limit of the IIA string.
Its low energy effective action is given by eleven dimensional supergravity and its extended objects are the membrane and the fivebrane (these are both 1/2 BPS).
The relationship between M-theory and IIA is that M-theory on a circle of radius R is related the string coupling, g_s via:

$$g_s = \left(\frac{R}{l_p}\right)^{\frac{3}{2}} \tag{1}$$

The branes in M-theory relate to string theory as follows:

- A wrapped membrane is the fundamental string
- A nonwrapped membrane is the D2 brane
- A wrapped fivebrane is the D4 brane
- A nonwrapped fivebrane is the NS5 brane
- Momentum on the circle is the D0 brane

M-theory unfies string theories as can be seen by compactifying M-theory on different manifolds, for example:
M on T^2 is IIB string theory on S^1; the SL(2,Z) of IIB is now a geometric consequence of the toroidal compactification
M on K3 is the Heterotic string on T^3; the intersection form of two cycles on K3 is realted to the Cartan subalgebra of the Het string

We can work in eleven dimension directly and don't compactify.

We can describe the branes in two different ways.

There are solutions to eleven dimensional supergravity. This is typically a good description when N, the number of branes, is large.

We use this SUGRA description to analyse their properties.

There are world volume descriptions for single branes ie. Nambu Goto action for the membrane and some equations of motion describing a single fivebrane.

We are interested in interacting branes and their properties. We can use the supergravity description and take a low energy near horizon limit (valid at large N).
This gives

- $AdS_4 \times S^7$ for the membrane
- $AdS_7 \times S^4$ for the fivebrane

We can determine properties of the interacting brane theories from these supergravity duals.

In both cases the branes should be described by a conformal theory with 16 supercharges. This is a trivial consequence of the symmetries of the AdS spaces.
From looking at black holes in the AdS we can determine the thermodynamic properties.
This can also be correlated with scattering properties and in the case of the fivebrane, anomaly calculations.

The result is that the number of degrees of freedom for:

- the M2 scales as $N^{3/2}$
- the M5 scales as N^3

Both these indicate a curious interacting theory with new degrees of freedom. Explaining these degrees of freedom of the branes in M-theory is one of the major puzzles in field.

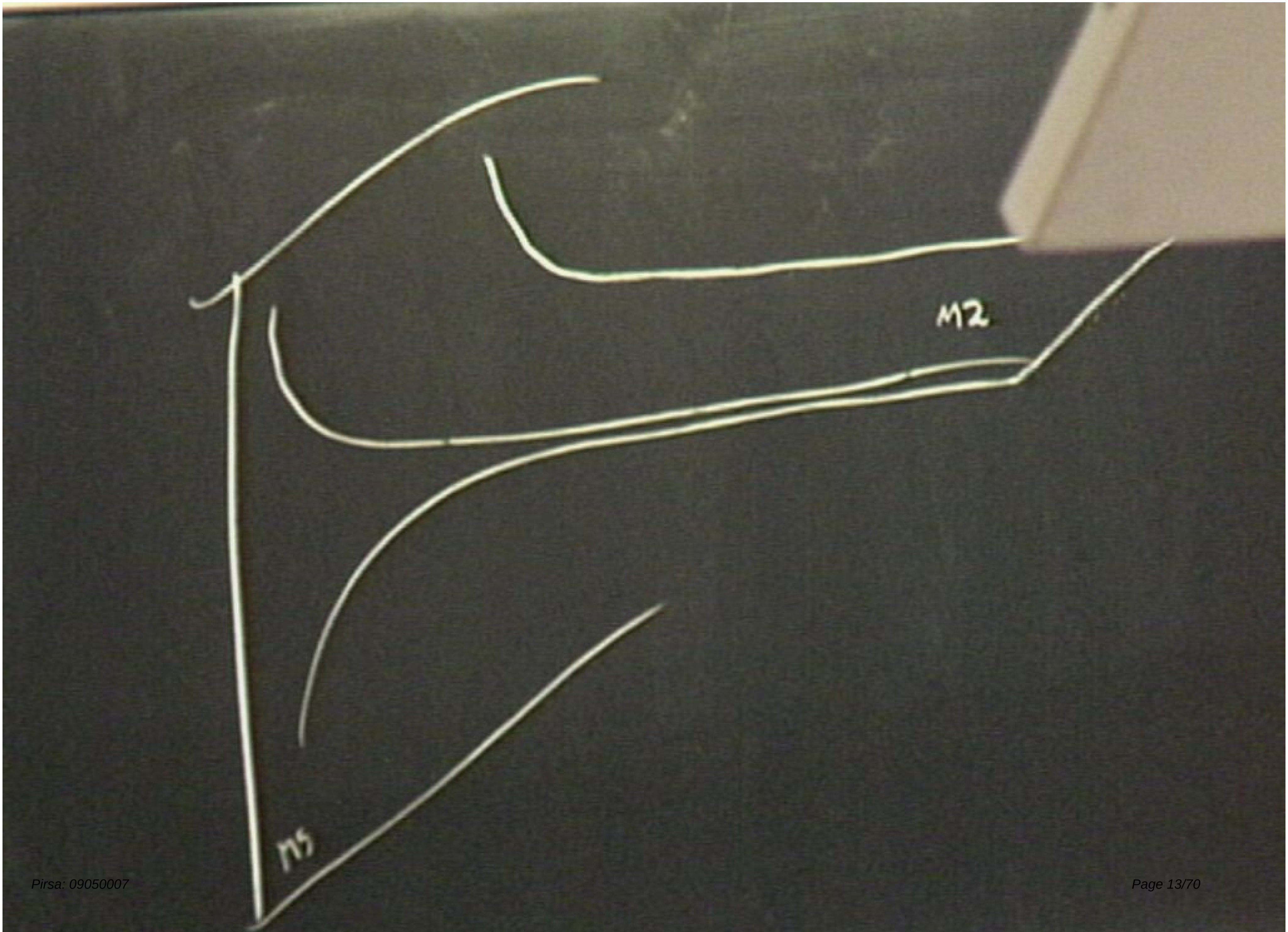
- What about M2-M5 interactions?
- there is one very interesting way in which M2s and M5s interact. A membrane can end on a fivebrane
- As such the fivebrane is sort of the D-brane in M-theory

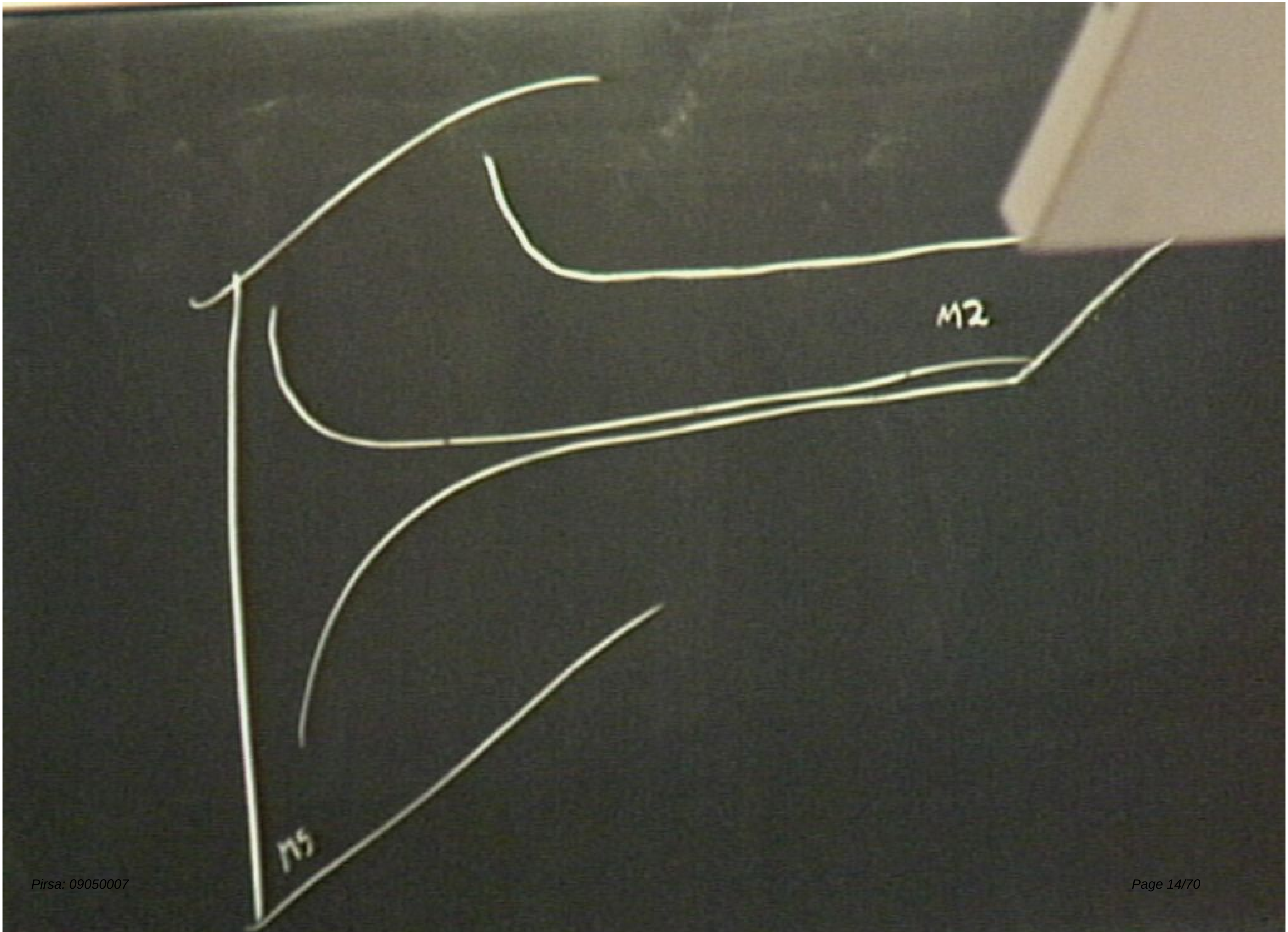
We can see the relation between the open membrane and the fivebrane in several ways.
One way is the self-dual string solution to the fivebrane equations of motion (Howe, Lambert and West).

$$H_{\mu\nu\rho} = \epsilon_{\mu\nu\rho}{}^{\sigma}\partial_{\sigma}\phi \tag{2}$$

where

$$\phi = \frac{Q_2}{r^2} \tag{3}$$


M2
MS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It describes the membrane emerging from the fivebrane (from the fivebrane perspective).
We can analyse properties of the self-dual string from this solution and use tools such as:

- anomaly cancellation- we can determine the dofs as a function of Q_2 and Q_5
- goldstone mode analysis
- low energy scattering
- decoupling limits

In string theory, we can also describe a D1 string ending on a D3 brane in a similar way. It appears as a monopole on the D3 brane.
In string theory though, we can describe the same system from the D1 brane perspective (look at its 1/2 BPS equation).

- That equation is the Nahm equation.
- The soltution produces a fuzzy two sphere whose radius blow up to infintiy generating an extra two dimensions that give the three brane at this point.
- Can we do the same for the membrane and describe the self-dual string from its pespective?

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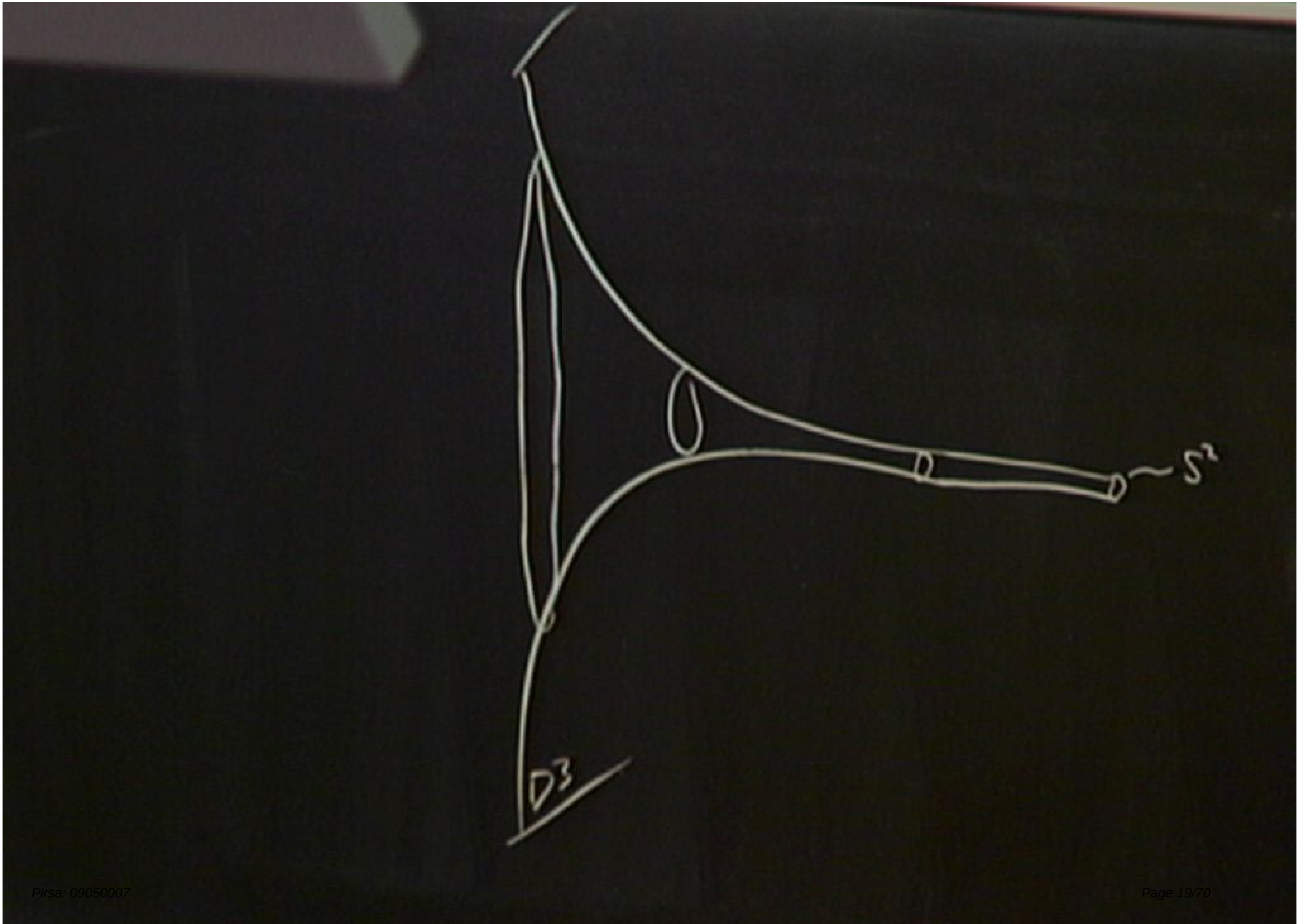
~ S²
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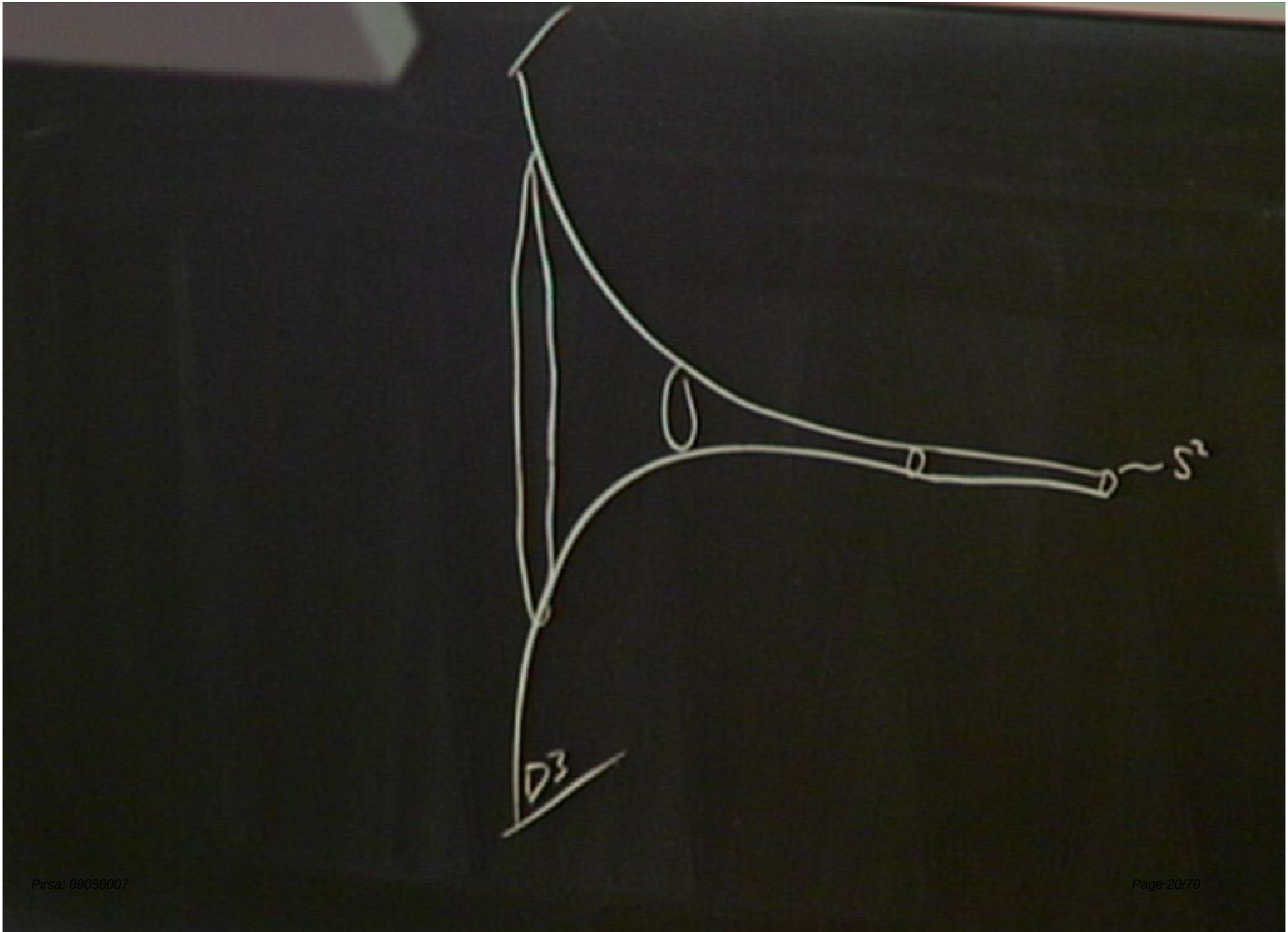

S²
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We need an action, but we don't have one. The answer is to backwards engineer.

- Construct the equation which has fuzzy three funnel solutions, this was done by Basu and Harvey
- Find an action that has N=8 supersymmetry and the Basu Harvey equation as a BPS equation, done by Bagger and Lambert

This action is defines the interacting membrane. (Note, it is not given from some fundamental principle but sort of phenomenologically constructed.)

Bagger and Lambert proposed a theory with $\mathcal{N} = 8$ supersymmetry to describe multiple coincident membranes. The novel insight allowing this construction is that the fields take values in a non-associative algebra, denoted here by $\mathcal{A}$. This non-associative algebra, also called a three algebra, is endowed with a totally antisymmetric three-bracket instead of the standard commutator found in Lie algebras. The three-bracket or triple product is given by the antisymmetrised associator. For example the associator of three transverse scalars is

$$\langle X^I, X^J, X^K \rangle = (X^I \cdot X^J) \cdot X^K - X^I \cdot (X^J \cdot X^K) \tag{4}$$

and the three bracket is then

$$[X^I, X^J, X^K] = \frac{1}{12} \langle X^{[I}, X^J, X^{K]} \rangle . \tag{5}$$

One can introduce a basis $\{T^a\}$ of $\mathcal{A}$ satisfying

$$[T^a, T^b, T^c] = f^{abc}{}_d T^d \,, \qquad (6)$$

where the totally antisymmetric structure constants (We raise and lower algebraic indices with a positive definite trace form metric which, in this paper, we take to be simply δ_{ab}.) f^{abcd} obey the fundamental identity, akin to the Jacobi identity of Lie algebras, given by

$$f^{efg}{}_{d} f^{abc}{}_{g} = f^{efa}{}_{g} f^{bcg}{}_{d} + f^{efb}{}_{g} f^{cag}{}_{d} + f^{efc}{}_{g} f^{abg}{}_{d} \,. \qquad (7)$$

We remark that at this stage we have not specified the dimension of the algebra which we shall denote by n.
To make the supersymmetry algebra close it is necessary to introduce non-propagating fields $\tilde{A}_{\mu}{}^{b}{}_{a}$, which gauge the transformation:

$$\delta X^I_a = \Lambda_{cd} f^{cdb}{}_a X^I_b \equiv \tilde{\Lambda}^b{}_a X^I_b \,. \tag{8}$$

The gauge field is antisymmetric as a consequence of the antisymmetry of $f^{cda}{}_b$ so the gauge group $G \subseteq SO(n)$.

As a consequence of the transformation law (8) the group G is restricted by insisting that one may write:

$$\tilde{A}_{\mu}{}^{b}{}_{a} = f^{cdb}{}_{a} A_{\mu\, cd} \tag{9}$$

for some $n \times n$ matrix valued $A_{\mu\, cd}$ with $f^{cdb}{}_{a}$ satisfying the fundamental identity which implies f^{abcd} must be an invariant four form of the group.

The Lagrangian for the full $\mathcal{N} = 8$ theory including these gauge fields is given by

$$\mathcal{L} = -\frac{1}{2}D^{\mu}X^{aI}D_{\mu}X^{I}_{a} + \frac{i}{2}\bar{\Psi}^{a}\Gamma^{\mu}D_{\mu}\Psi_{a} + \frac{i}{4}\bar{\Psi}_{b}\Gamma_{IJ}\Psi_{a}X^{I}_{c}X^{J}_{d}f^{abcd}$$
$$-V(X) + \frac{1}{2}\epsilon^{\mu\nu\lambda}\left(f^{abcd}A_{\mu ab}\partial_{\nu}A_{\lambda cd} + \frac{2}{3}f^{cda}{}_{g}f^{efgb}A_{\mu ab}A_{\nu cd}A_{\lambda ef}\right.$$

with bosonic potential

$$V(X) = \frac{1}{12}Tr\left([X^{I}, X^{J}, X^{K}]^{2}\right), \qquad (10)$$

the supersymmetry transformations are

$$\delta X^I_a = i\bar{\epsilon}\Gamma^I\Psi_a, \quad (11)$$

$$\delta\Psi_a = D_\mu X^I_a\Gamma^\mu\Gamma^I\epsilon - \frac{1}{6}X^I_b X^J_c X^K_d f^{bcd}{}_a\Gamma^{IJK}\epsilon, \quad (12)$$

$$\delta\tilde{A}_\mu{}^b{}_a = i\bar{\epsilon}\Gamma_\mu\Gamma^I X^I_c\Psi_d f^{cdb}{}_a, \quad (13)$$

where the covariant derivative acts as $D_\mu X_a = \partial_\mu X_a - \tilde{A}_\mu{}^b{}_a X_b$.

Now, we must solve the fundamental identity and specify the algebra. The remarkable fact is that there is only one solution! So after all the general discussion on general algebras. In fact only

$$f^{abcd} = \epsilon^{abcd} \tag{14}$$

is allowed.

For this case, the twisted Chern-Simons action becomes simply:

$$\begin{aligned}\mathcal{L}_{TCS} &= Tr\left(A^+ dA^+ + \frac{2}{3}A^+ \wedge A^+ \wedge A^+\right) \\ &- Tr\left(A^- dA^- + \frac{2}{3}A^- \wedge A^- \wedge A^-\right) \\ &= \mathcal{L}_{CS}[A^+] - \mathcal{L}_{CS}[A^-]\,.\end{aligned}$$

ie. has decomposed into two $SU(2)$ Chern-Simons theories with a relative minus sign.

What about the coupling?
In fact, one can put in a constant k, the level of the Chern-Simons theory. This then via susy is also the coupling constant of the gauge field to matter.
It is quantised and thus not renormaised. Thus we have a true quantum conformal theory.
We must interpret this coupling k. Recall that M-theory contains no dimensionaless coupling.

Obvious problem.
How many interacting membranes does it describe??
We don't have an arbitrary algebra.

Need to do something, possibilities:

- Do not use δ^{ab} to raise indices but allow negative directions, in particular allow Lorentzian signiture
- Drop overall antisymmetry condition on the structure constants
- Forget about 3-algebras and just generalise the $SU(2) \times SU(2)$ case to $G \times G$.

First case, was done by Benvenuti, Gomez, Toni and Verlinde.
The key problem with this is that there is now a ghost like mode. The conjecture due to Schwarz is that one must introdice a shift symmetry to allow it to be gauged away. One can then show the theory reduces to Yang-Mills theory.
The coupling though arises through spontaneous symmetry breaking of the conformal theory.

The second and third options turn out to be equivalent. The approach of generalising the product guage group is due to Aharony, Bergmann, Jafferis and Maldacena and the lifting of antisymmetry on the structure constants is due to Bagger and Lambert.
These choices no longer leave $\mathcal{N} = 8$ susy. The straight generalisation to product gauge groups gives only $\mathcal{N} = 6$.

Interpreting ABJM.
The key is to work out the moduli space of vacua for the $U(N) \times U(N)$ theory. This gives:

$$\frac{\mathbf{R}^{8N}}{\mathbf{Z}_k} / S_N \tag{15}$$

This is the same as the moduli space of N membranes on a $\mathbf{Z}_k$ orbifold.

$\frac{1}{k}$ provides the coupling and may be taken to be small ie. k large.
In this perturbative limit, taking the same limit on the S^7 of the $AdS_4 \times S^7$ near horizon geometry of the membrane gives a new geometry of $AdS_4 \times CP^3$.
This is a new perturbative compactification of M-theory with a new AdS/CFT correspondence. Much work using integrable structures has been applied to this system.

Back to the self-dual string.
Take the membrane to now have a boundary and calculate the boundary dynamics using BL or ABJM.
We will need to understad how susy works when there is a boundary.

General Approach to supersymmetry with a boundary
Supersymmetry is typically broken by the presence of a boundary. Obvious since one breaks translation invariance.
One can restore a fraction of susy though through the addition of appropriate boundary terms in the action.
This boundary terms then encodes the boundary dynamics.

A $\mathcal{N} = 1$ scalar superfield is given by

$$\Phi = a + \theta\psi - \theta^2 f \,, \tag{16}$$

and can be integrated over superspace to form an action

$$S_0 = \int d^3x \int d^2\theta \, \Phi = \int d^3x \, f \,. \tag{17}$$

The supersymmetry transformations are,

$$\delta\Phi = \epsilon Q \Phi \Rightarrow \begin{cases} \delta a = \epsilon\psi \\ \delta\psi_\alpha = -\epsilon_\alpha f + (\gamma^\mu \epsilon)_\alpha \partial_\mu a \\ \delta f = -\epsilon\gamma^\mu \partial_\mu \psi \,, \end{cases} \tag{18}$$

In the presence of a boundary the supersymmetry transformation gives:

$$\delta S_0 = -\partial_\mu(\epsilon\gamma^\mu\psi). \tag{19}$$

Consider the following boundary action

$$S_1 = -\int d^3x\partial_3\Phi|_{=0} = -\int d^3x\partial_3 a, \tag{20}$$

with supersymmetry variation

$$\delta S_1 = -\int d^3x\, \partial_3(\epsilon\psi)\,. \tag{21}$$

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Then the combination $S_0 \pm S_1$ has variation

$$\delta[S_0 \pm S_1] = \mp \int d^3x \partial_3[\epsilon(1 \pm \gamma^3)\psi] = \mp \int d^3x \partial_3[2\epsilon_\mp \psi_\pm] \quad (22)$$

where we have defined projected spinors
$\psi_\pm \equiv P_\pm \psi \equiv \frac{1}{2}(1 \pm \gamma^3)\psi$. Then

$$[S_0 \pm S_1] = 0 \Leftrightarrow \epsilon_\mp = 0 \,. \quad (23)$$

Thus the modified action preserves half $\mathcal{N} = (1,0)$ or $(0,1)$ supersymmetry.

We are also free to suppliment this with any theory that has susy on the boundary. In particular with codimension one reductions of the original superfields. This is useful since it allows us to add in a term that allows the nonpropagating auxilliary fields to be easily integrated out.

We carry this out for ABJM in $= 2$ superfield formulation.
There are two choices, to preserve: $\mathcal{N} = (2,0)$ or $\mathcal{N} = (1,1)$
For $\mathcal{N} = (2,0)$ this proceedure gives:

$$\mathcal{L}_{(2,0)\,bound} = 2\kappa\,[v_n v^n + \chi_- \gamma^m \partial_m \chi_- + \psi_- \gamma^m \partial_m \psi_- + a \partial_m \partial^m a] + \xi^*_{A-} \xi^A_+ + \frac{1}{16\kappa} Z^*_A Z^A Z^*_B Z^B \quad (24)$$

The equation of motion is then:

$$\partial_3 Z^A - \frac{1}{8\kappa} Z^A Z^*_B Z^B = 0. \tag{25}$$

How should we interpret this?
Let us consider searching for 1/2 supersymmetric bosonic vacuum solutions of the closed membrane theory and employing the Bogomolny trick. (Assuming only x^3 dependence). Then the Hamiltonian is given by

$$H = \partial_3 Z^A \partial_3 Z^*_A + \frac{1}{64\kappa^2}\left(Z^*_A Z^A\right)^3 \tag{26}$$

$$= |\partial_3 Z^A - \frac{1}{8\kappa} Z_B Z^*_B Z^A|^2 + \frac{1}{16\kappa}\partial_3(Z^*_A Z^A Z^*_B Z^B). \tag{27}$$

Then the minimum energy configuration satisfies the BPS bound

$$\partial_3 Z^A - \frac{1}{8\kappa} Z^A Z^*_B Z^B = 0. \tag{28}$$

So we see that our 'natural' boundary condition obtained from the generalized theory corresponds exactly to the BPS equation.

The $\mathcal{N} = (1,1)$ leads to:

$$\begin{aligned}\mathcal{L}_{(1,1)\,bound} \;=\; & 2\kappa v_n v^n + 2\kappa\chi_+\gamma^m\partial_m\chi_+ + 4\kappa\chi_+\lambda_- + 2\kappa\psi_-\gamma^m\partial_m\psi_- \\ & -2\kappa(f+\partial_3 a)^2 + \kappa bb \\ & \frac{1}{2}\xi^*\gamma^3\xi + \frac{1}{2}F^*Z + \frac{1}{2}Z^*F + \frac{1}{2}\partial_3(Z^*Z) \qquad (29)\end{aligned}$$

The equations of motion with this boundary action are:

$$\partial_3 Z^A - \frac{1}{4\kappa}\epsilon^{AC}\epsilon_{BD} W^{\dagger B} Z^{\dagger}_C W^{\dagger D} = 0\,, \qquad (30)$$

$$\partial_3 W_A + \frac{1}{4\kappa}\epsilon_{AC}\epsilon^{BD} Z^{\dagger}_B W^{\dagger C} Z^{\dagger}_D = 0 \qquad (31)$$

with

$$\mathrm{Tr}((ZZ^{\dagger} - W^{\dagger}W)) = 0\,, \quad \mathrm{Tr}((Z^{\dagger}Z - WW^{\dagger})) = 0 \qquad (32)$$

The same equations have also been observed as Bogmolnyi equations.
So far they have been solved and their brane interpretation not known....

- This is the beginning of the self-dual string.
- What about including all SUSY. We expect (4,4) susy multiplet.
- Idea: use the superspace formalism developed by Cederwall et al. and the techniques of van Neiwenhuizen to work out the supermultiplet on the boundary.
- Interpret in terms of the fivebrane?????

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Some more possibilities

- A host of situtions can arise now the constriants on f are relaxed. The theory will have lower susy but there can now be exotic gauge groups. What do these mean for the membrane??
- Perturbative membrane calculation are also now possible, conformal invariance directly checked at two loops
- Adding higher order corrections to get the membrane outside of the low energy limit ie. l_p corrections
- Relation to global limits of gauged supergravity

Some comments about what we can learn from the perturbation theory.

- k can't be renormalised, not even by finite shifts since that won't have a sensible space time interpretation.
- However, it is known that Chern-Simons theories can have a finite shift renormalisation of k.
- One has to be careful to work this out with BL theory since there are also Fermions.
- Result: BL (and massive deformations) produce a shift of $k \to k+2$.
- ABJM has no shift is one generates the scalar potential from integrating out the gauginos in the Chern-Simons multiplet.

Big questions still remain:

- $N^{3/2}$ degrees of freedom??
- Why is susy only manifest for the specific N=2 case??

A comment on $N^{3/2}$ dofs.
Consider a fuzzy three sphere whose radius, R goes like $N^{1/2}$
This was the case for the Basu-Harvey fuzzy funnel.
A fuzzy three sphere has a finite number of degrees of freedom because it has both an ultraviolet and infrared cut-off.

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The number of degrees of freedom of fields on such a space would then go like $N^{3/2}$, in the large N limit. This has been made more precise using the details of fuzzy spheres.
The picture is then of membranes with internal 3 spheres.
For the case where the cutoff is removed or the radius goes to infinity then one produces the fivebrane. Making this work requires many details to work out- Matsuo et al, Bandos and Townsend.
Still no description of this at finite N.

What is the relation to Yang-Mills ie the D2 brane theory?
It is a bit unusual. Giving a scalar field a vev causes the gauge field to become dynamical (Mukhi and Papageorakis). Lets see this for the original BL theory.

$$X^{(8)} = \phi^4 = g \tag{33}$$

$$A_\mu^{i4} = A_\mu^i \qquad \frac{1}{2}A^{ij}\epsilon^{ijk} = B_\mu^k \tag{34}$$

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Recently, this has been shown to be true for the higher derivative corrected D2 brane theory and the membrane with the Lorentzian metric

Can we now answer any M-theory questions using this new membrane description???

Still much to do!!!