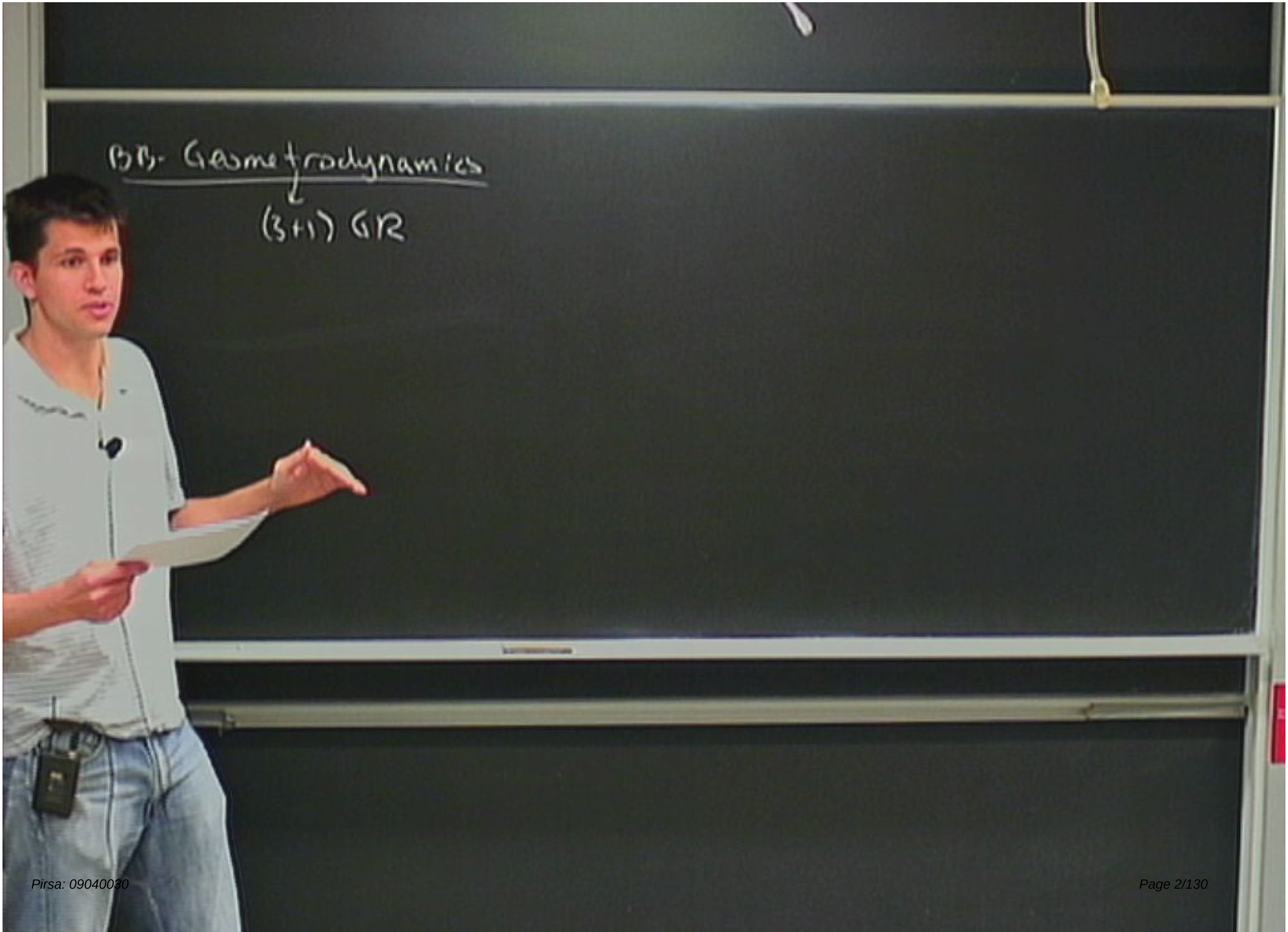
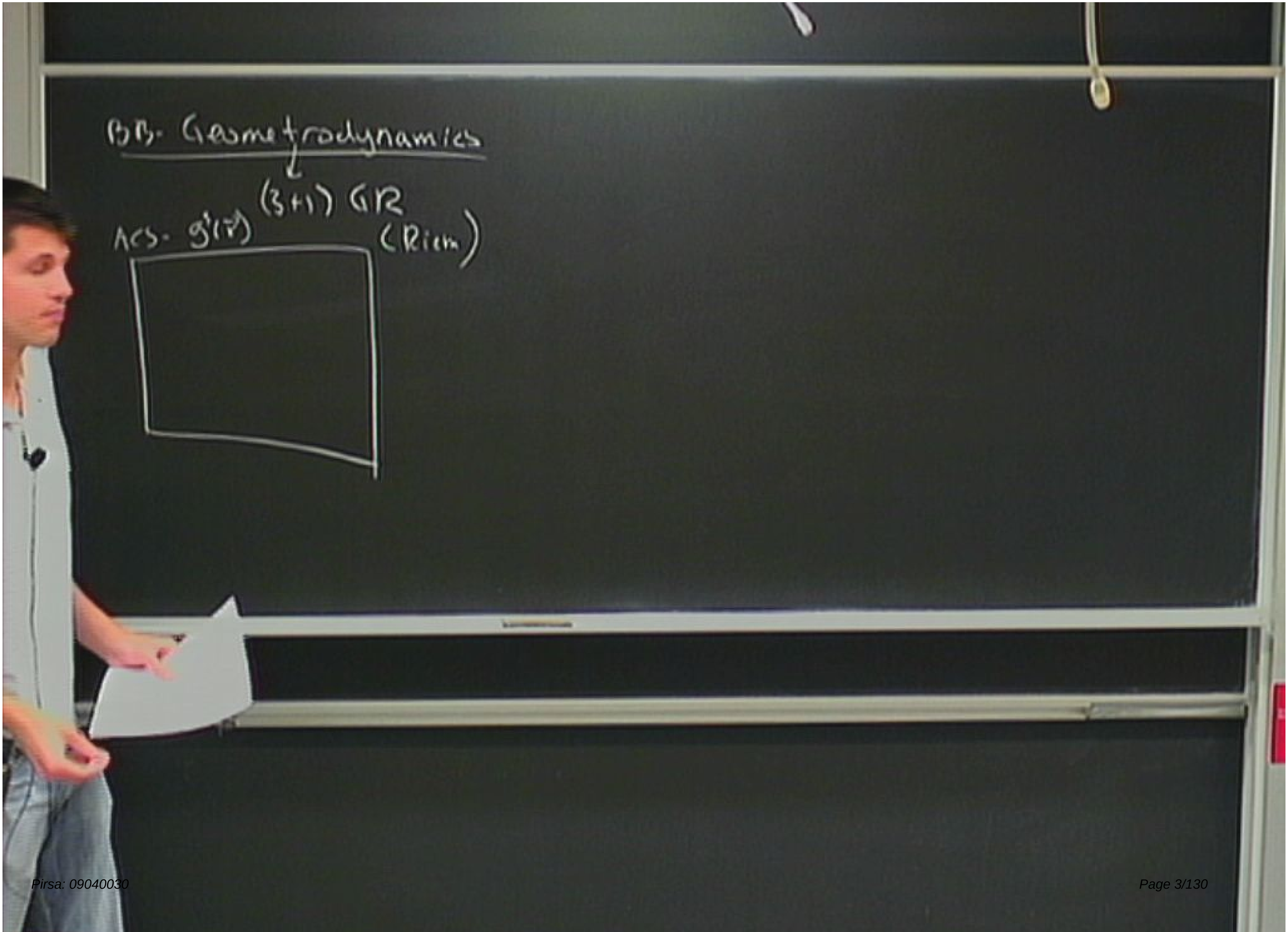
Title: Mini-Course on Mach's Principle - Lecture 9

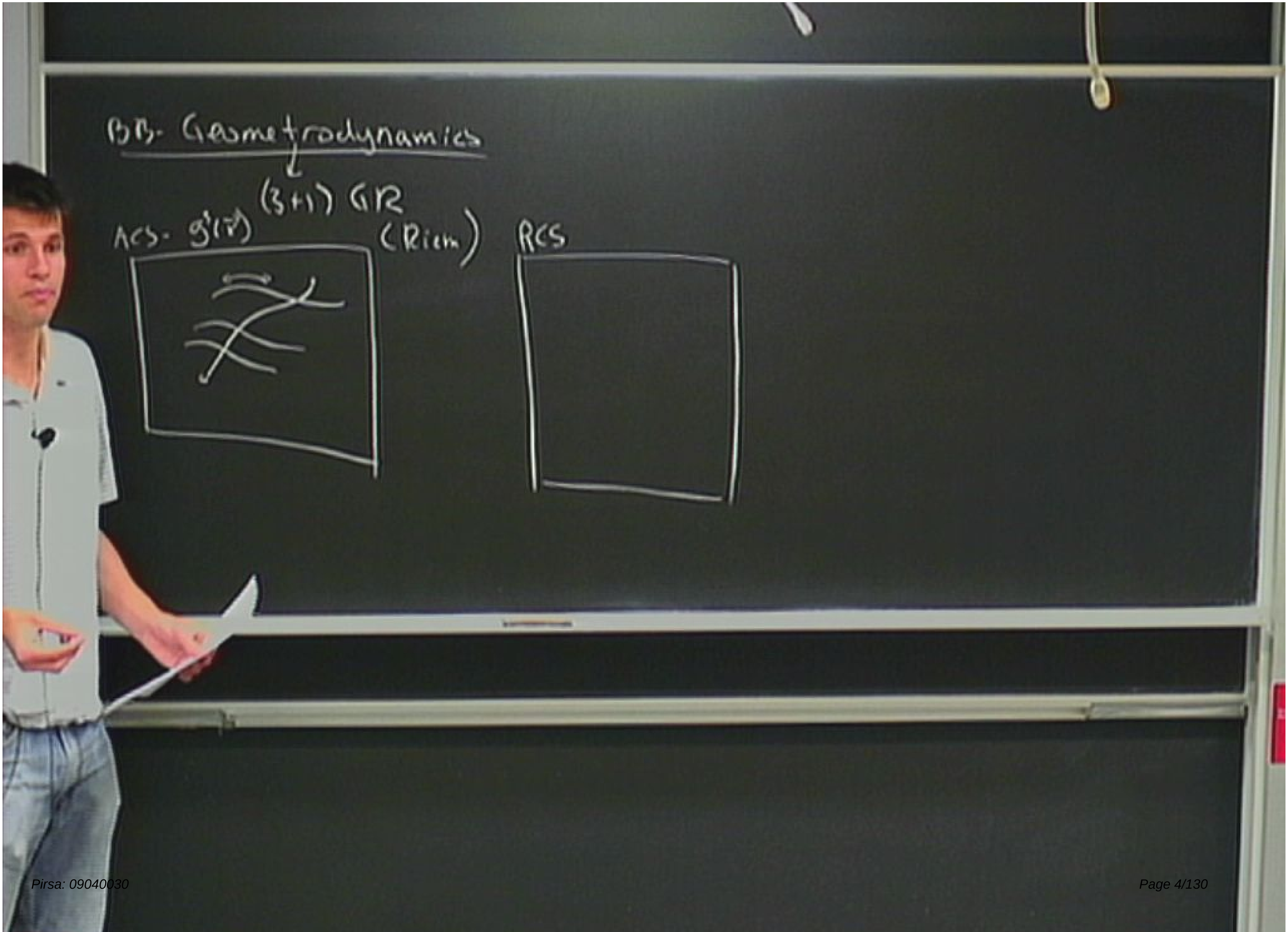
Date: Apr 22, 2009 10:30 AM

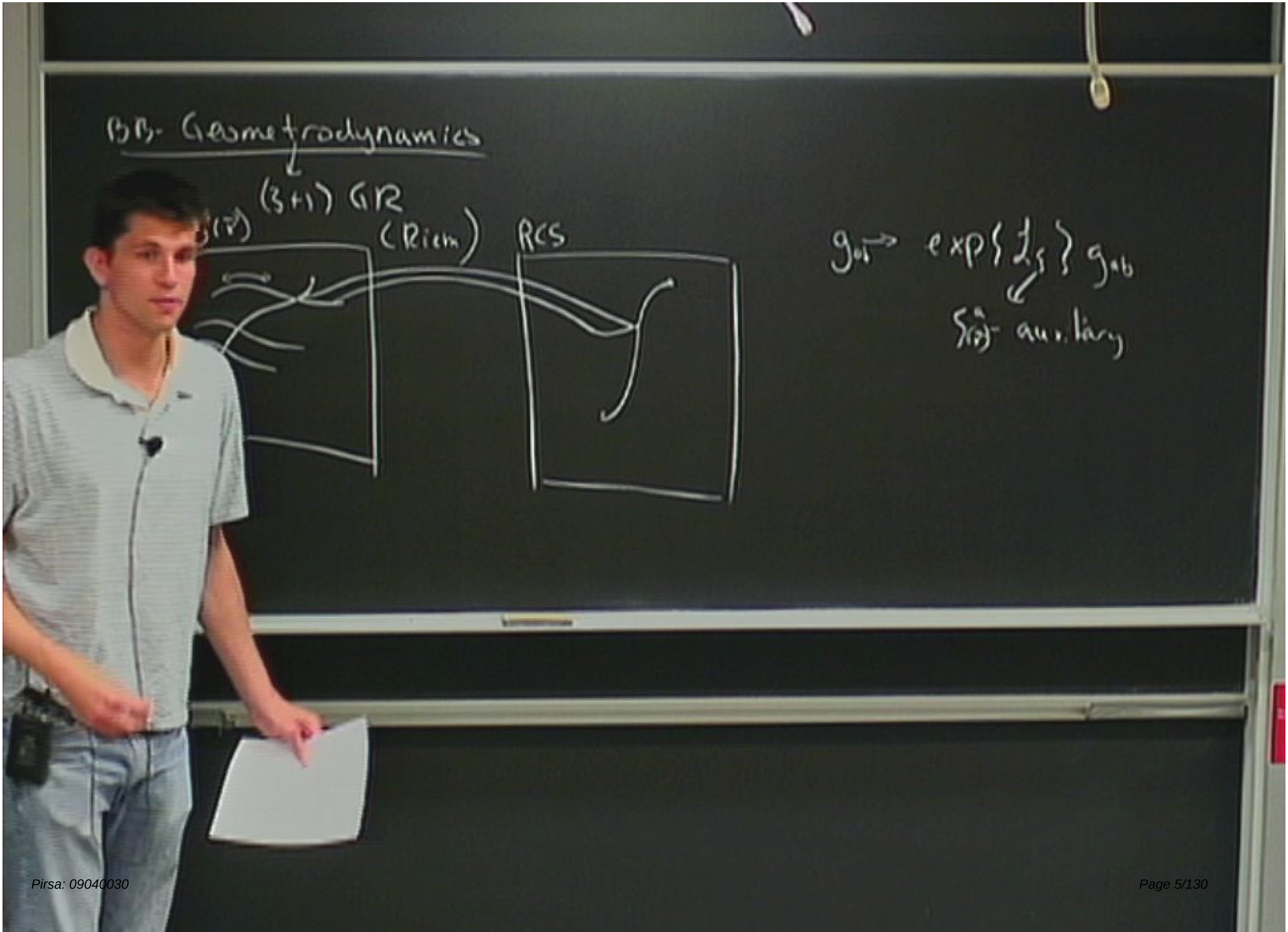
URL: http://pirsa.org/09040030

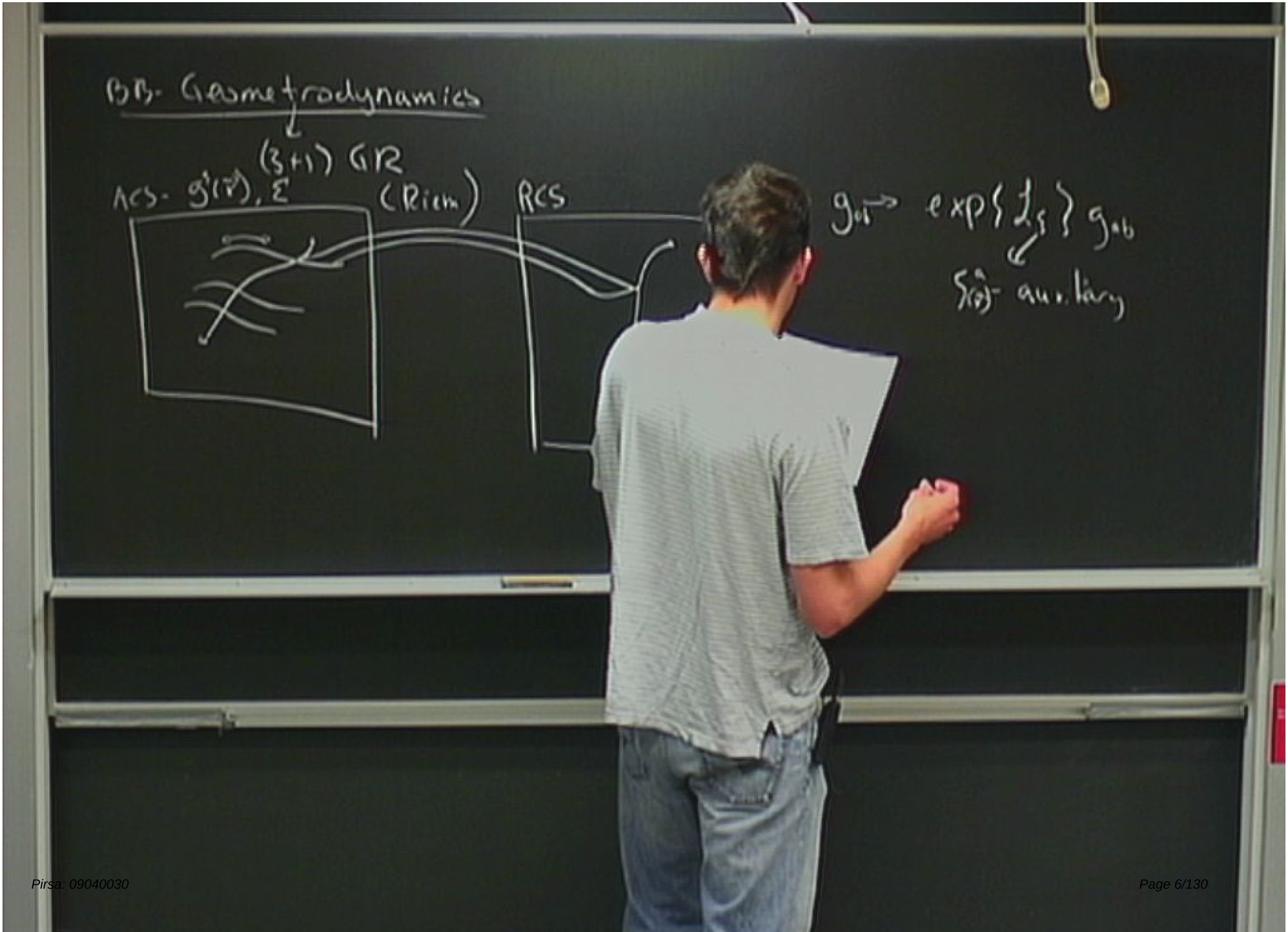
Abstract:


BB- Geometrodynamics
(3+1) GR


BB- Geometrodynamics
(3+1) GR
(Riem)


BB- Geometrodynamics
(3+1) GR
ACS-
(Riem)
RCS


BB- Geometrodynamics
(3+1) GR
(Riem)
RCS
auxiliary


BB- Geometrodynamics
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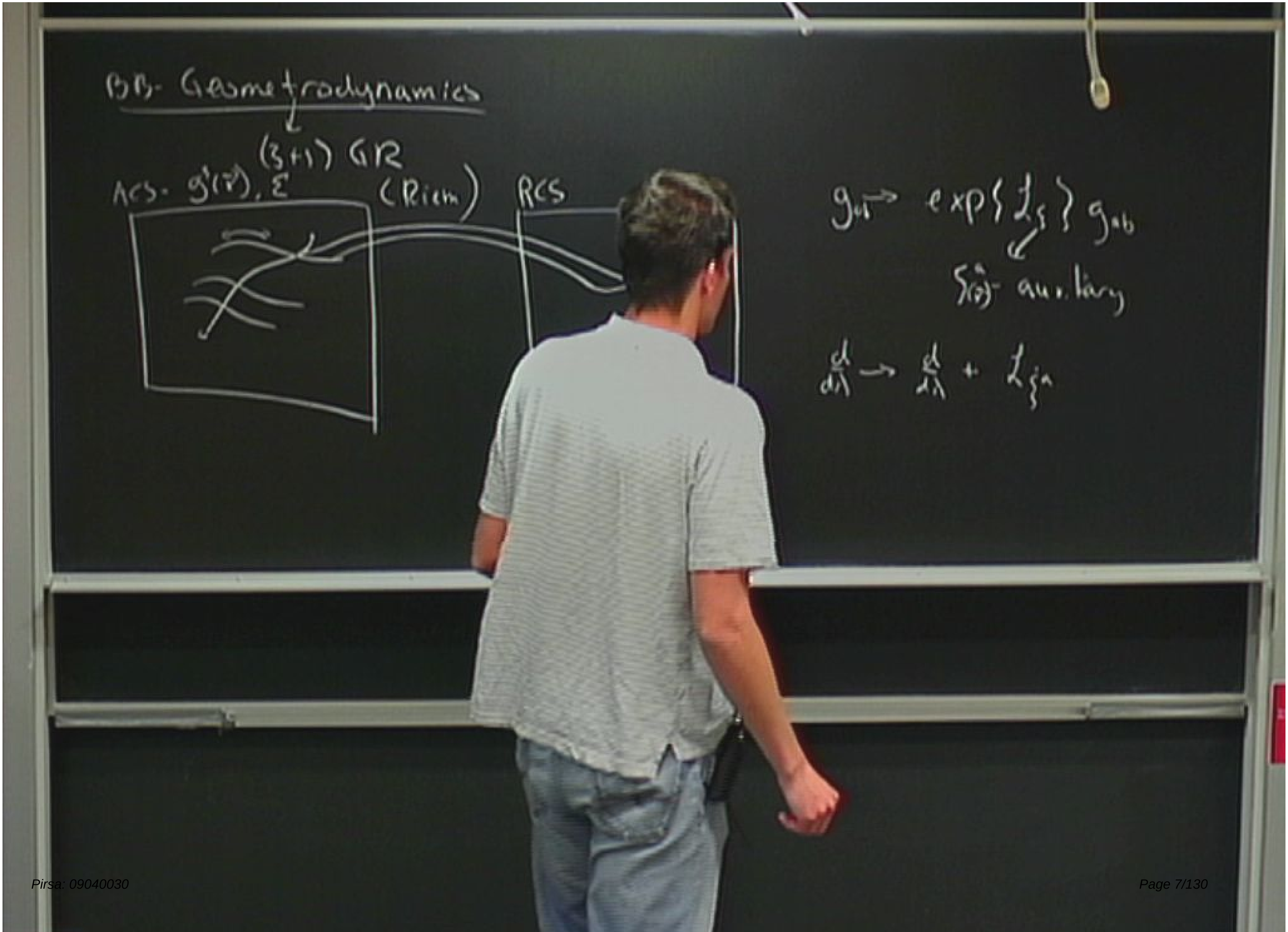
BB- Geometrodynamics
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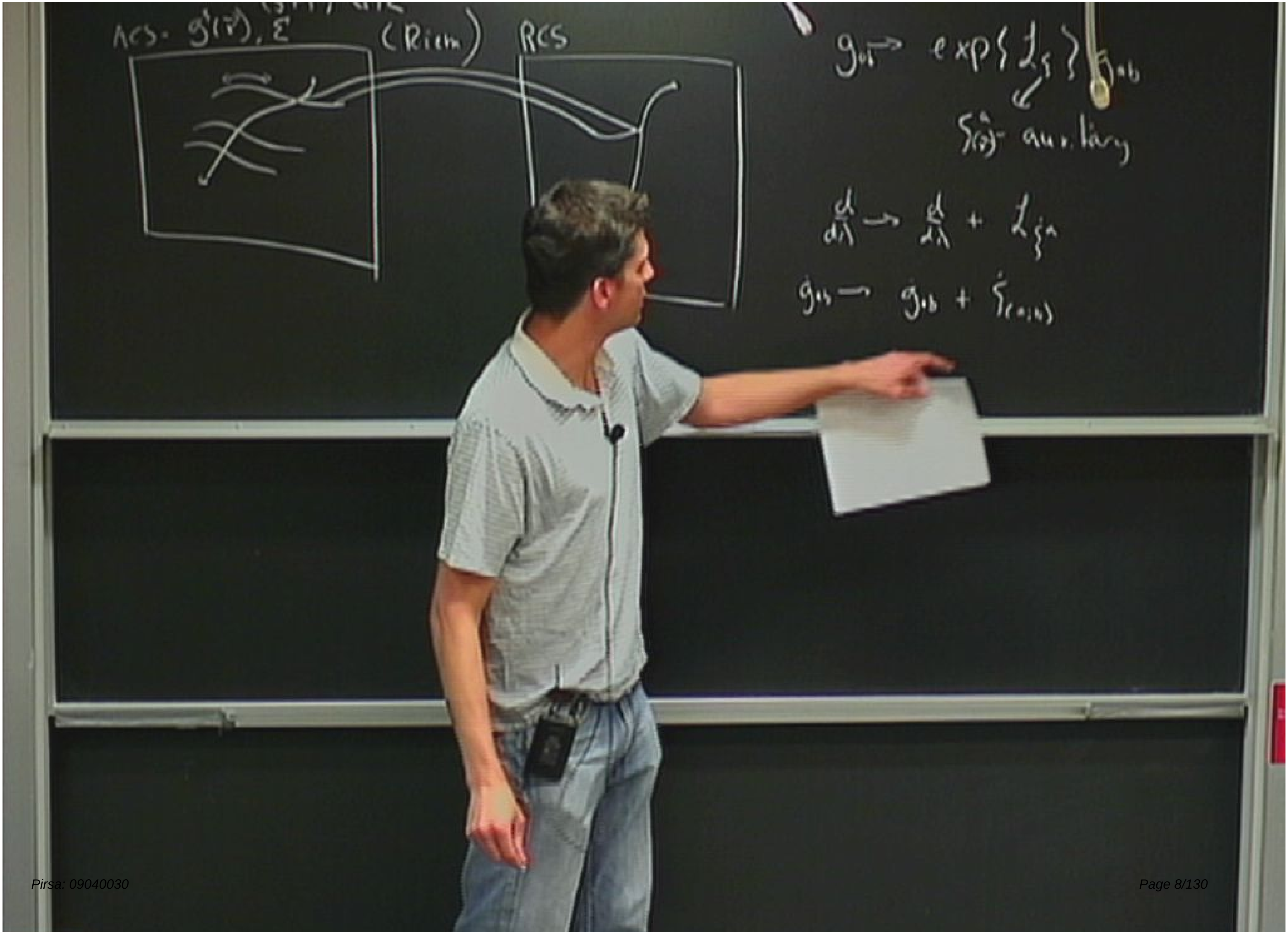
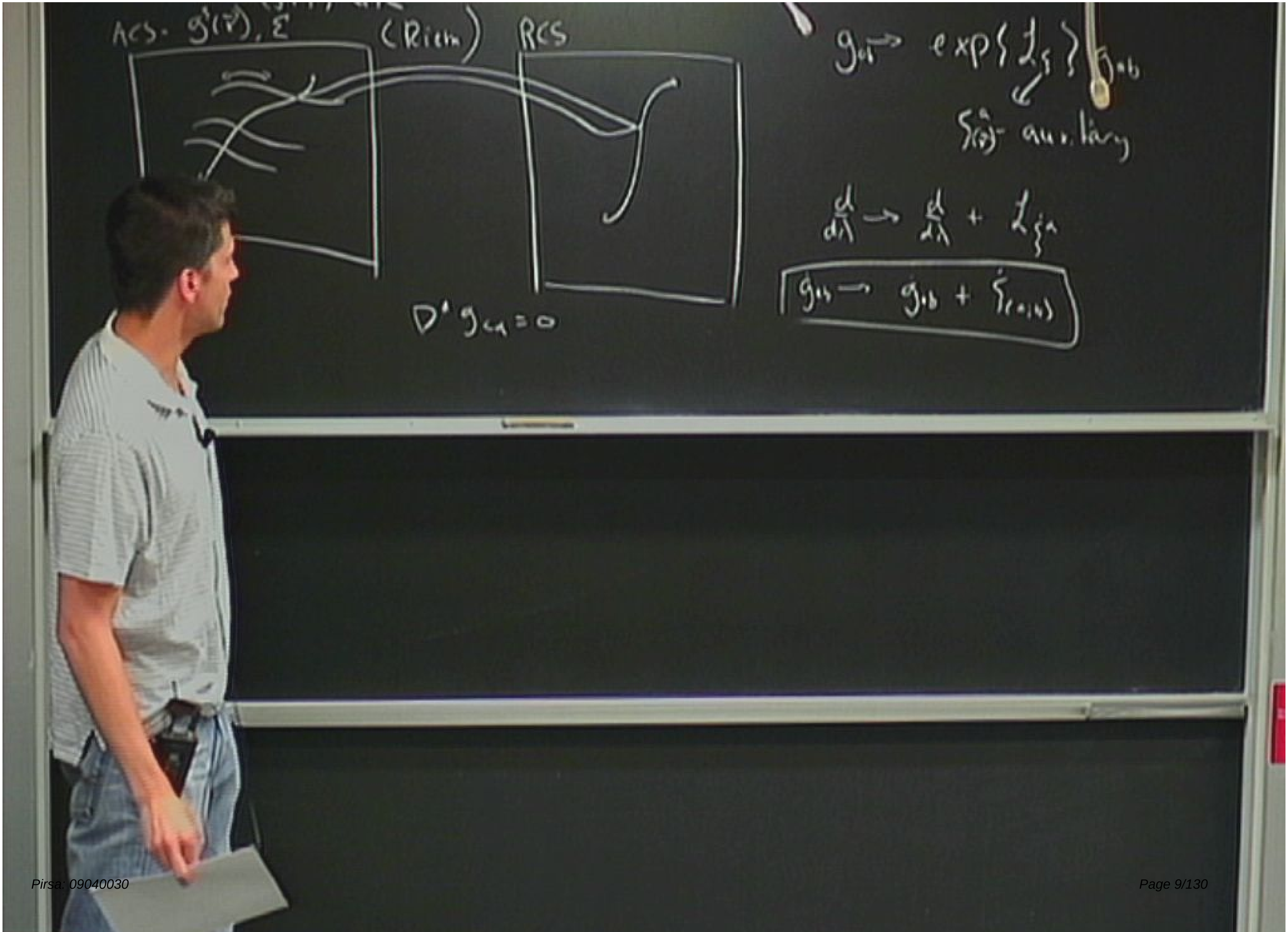
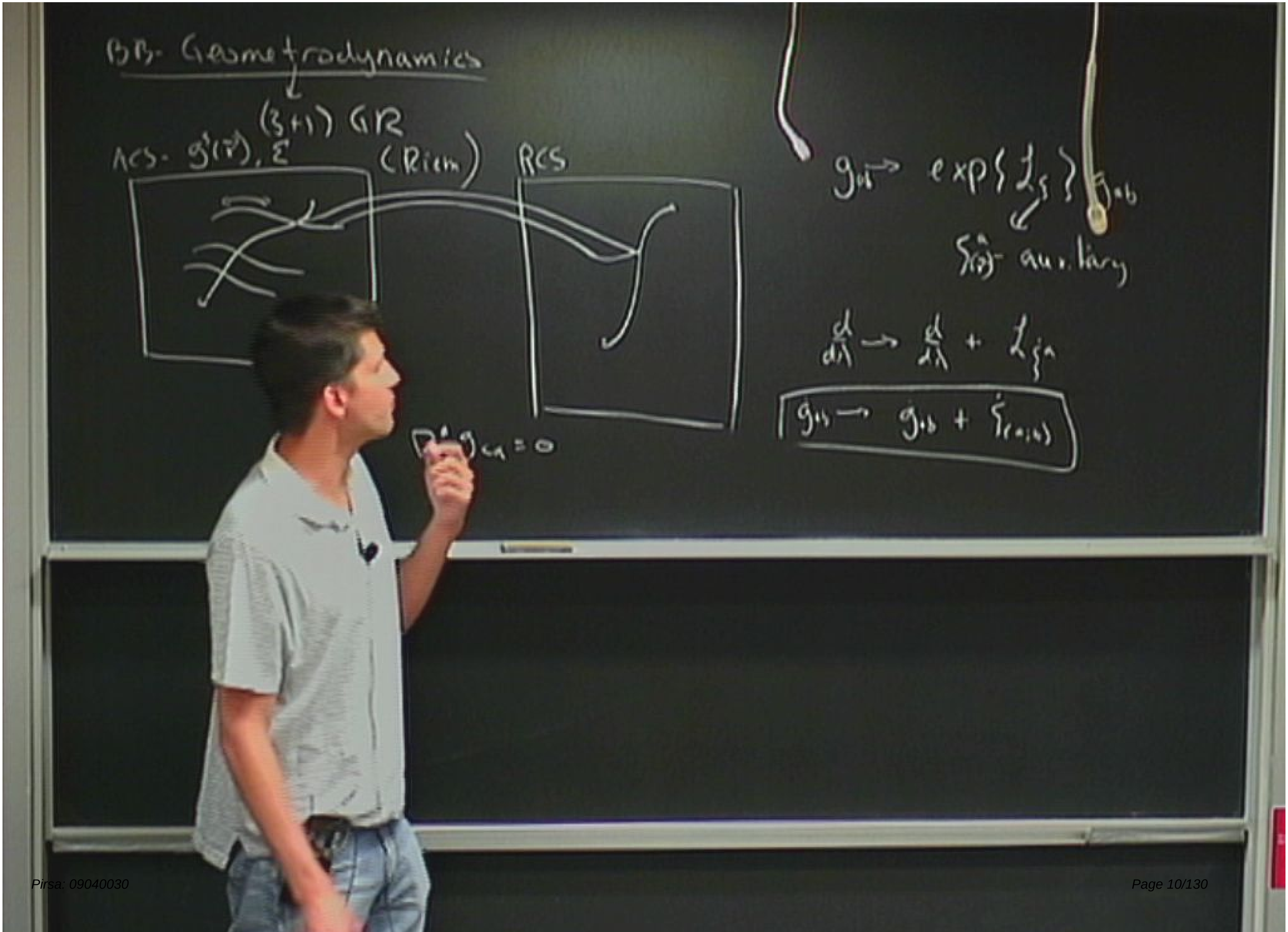
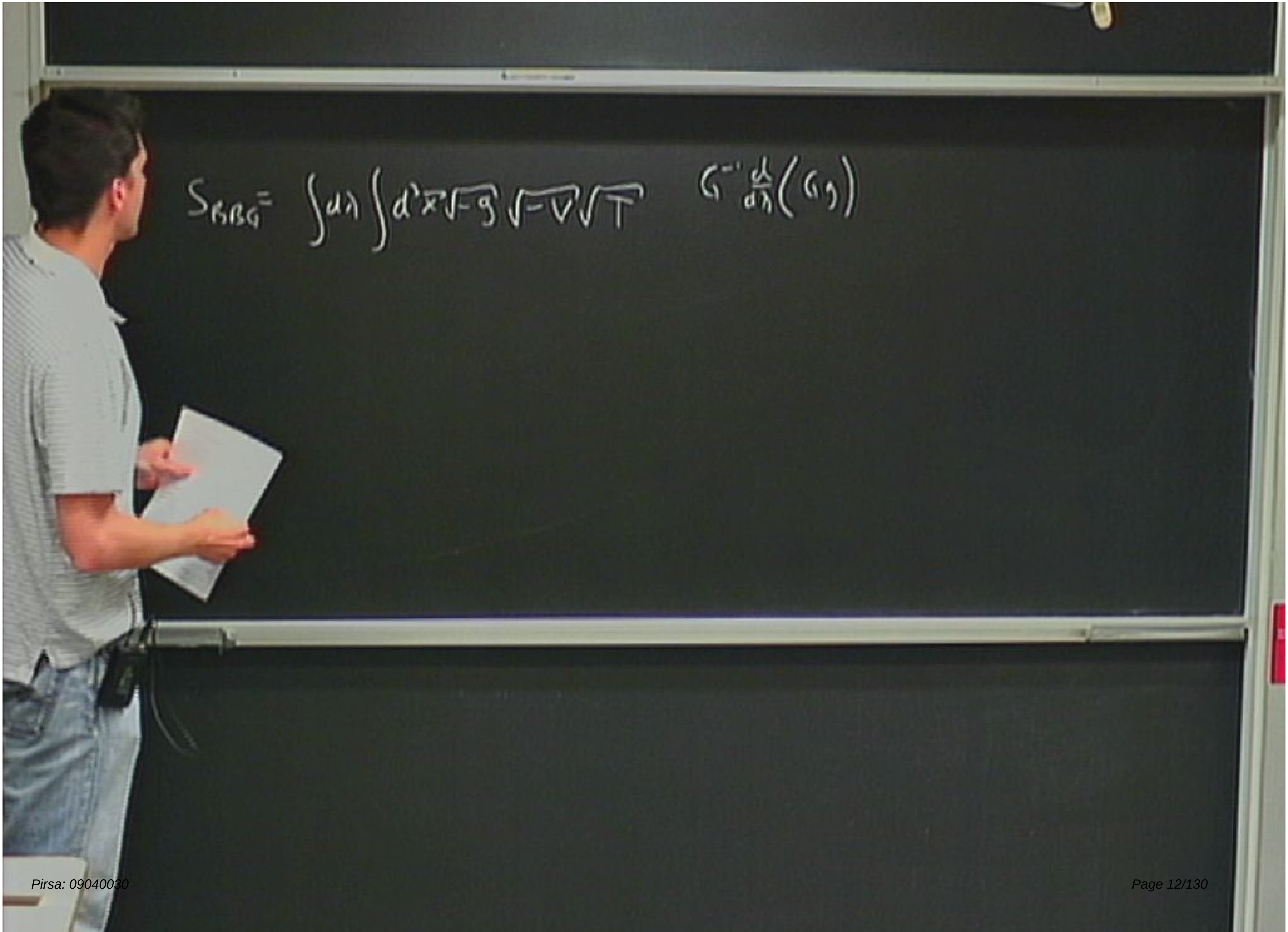
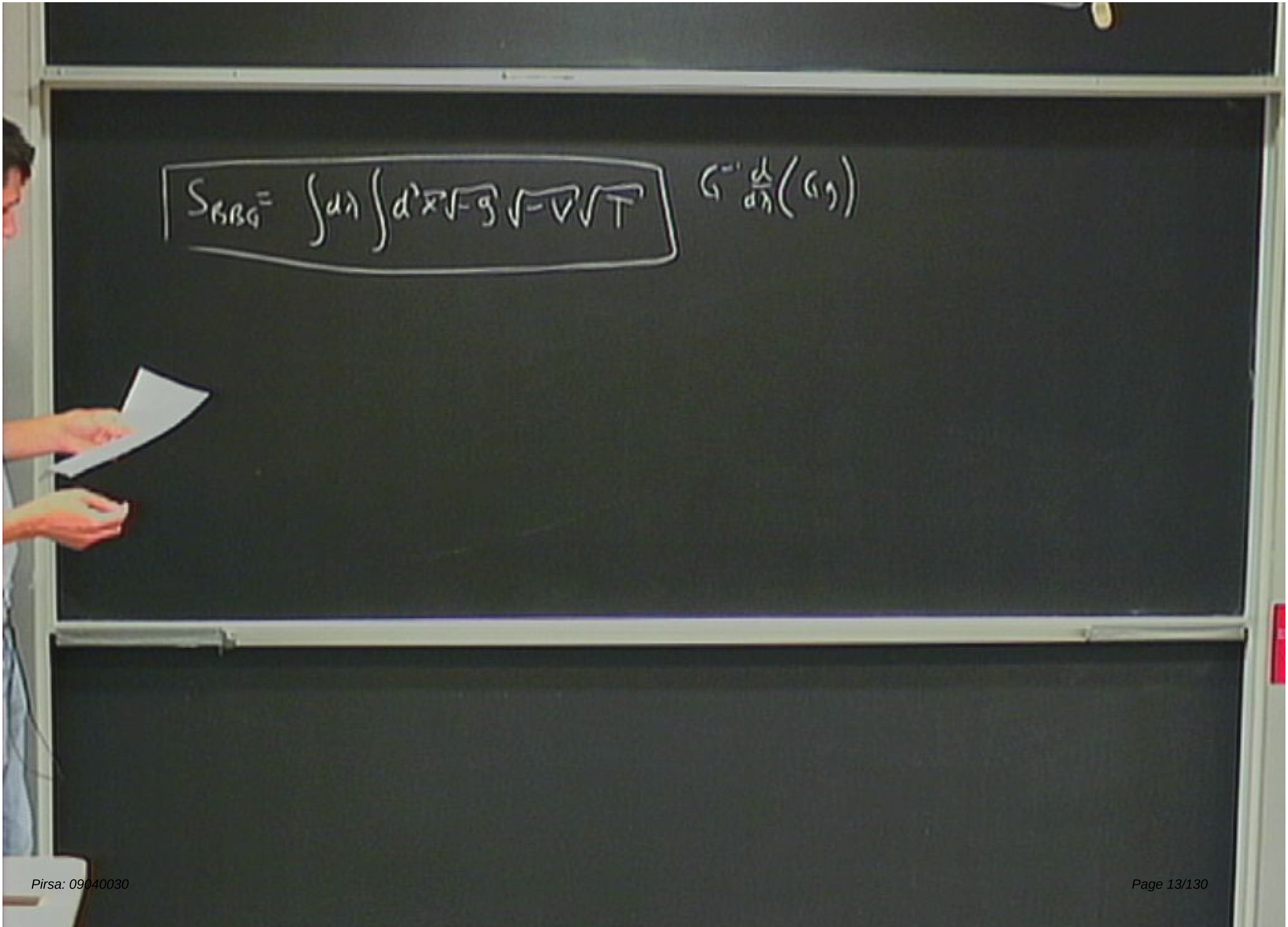

(Riem)
RCS
auxiliary


ACS
(Riem)
RCS
auxiliary


BB- Geometrodynamics
(3+1) GR
ACS
(Riem)
RCS
auxiliary

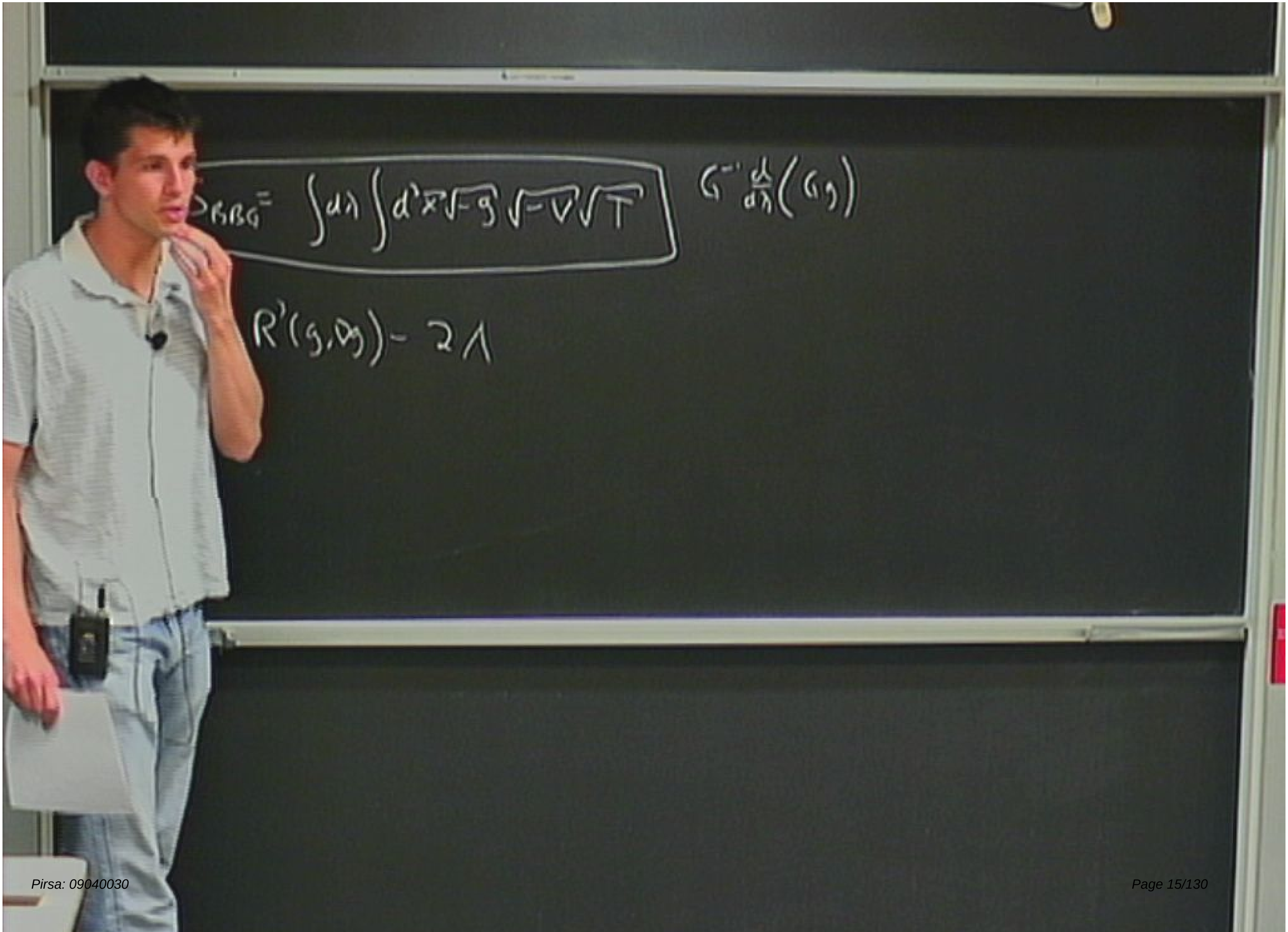

$\frac{d}{d\lambda} \to \frac{d}{d\lambda} + \mathcal{L}_{\xi^a}$
$D^a g_{ab} = 0$
$g_{ab} \to g_{ab} + \xi_{(a;b)}$

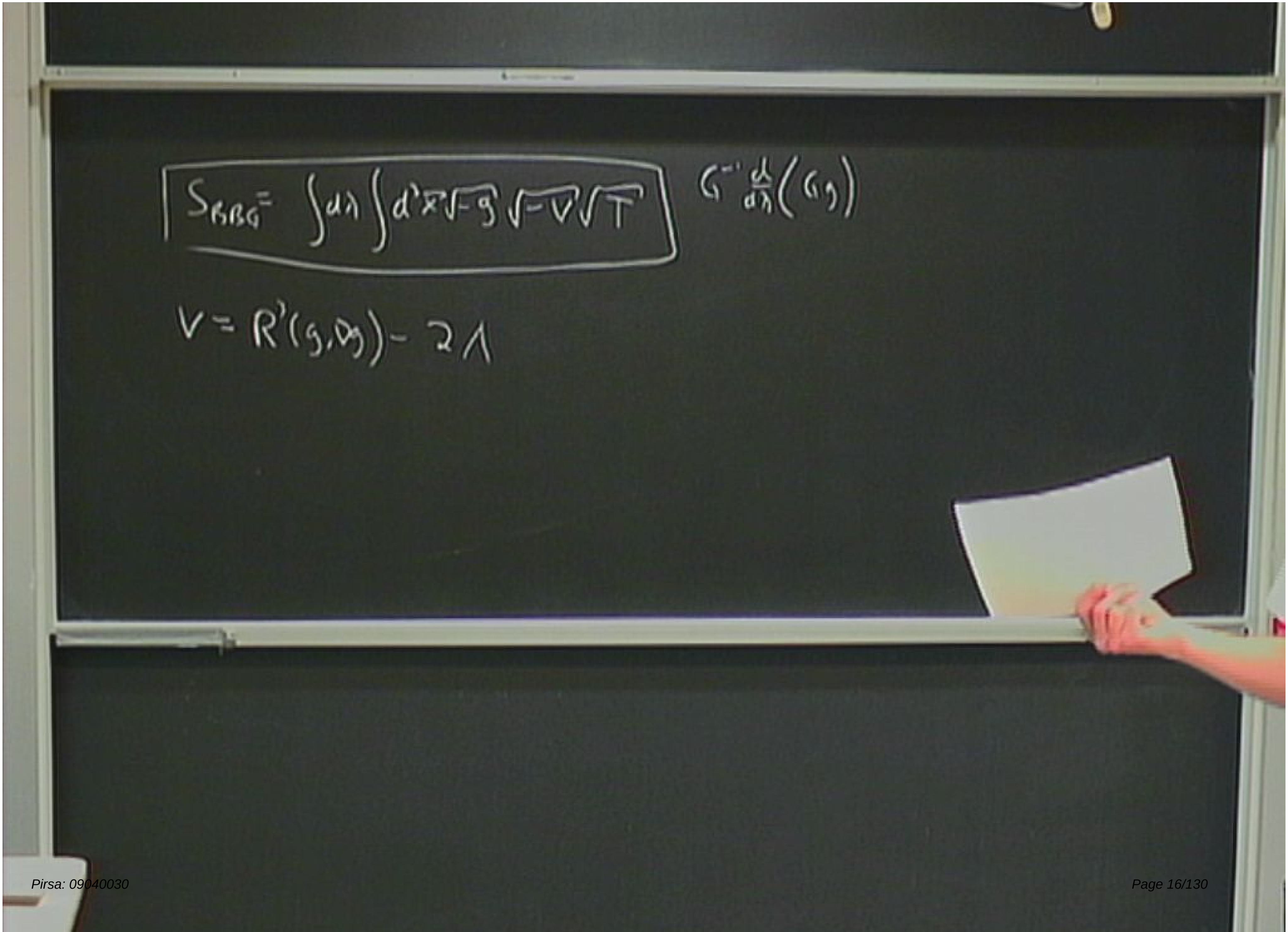

$S_{BBG} = \int d\lambda \int d^4x \sqrt{-g} \sqrt{-V} \sqrt{T}$
$G^{-1} \frac{d}{d\lambda}(Gg)$

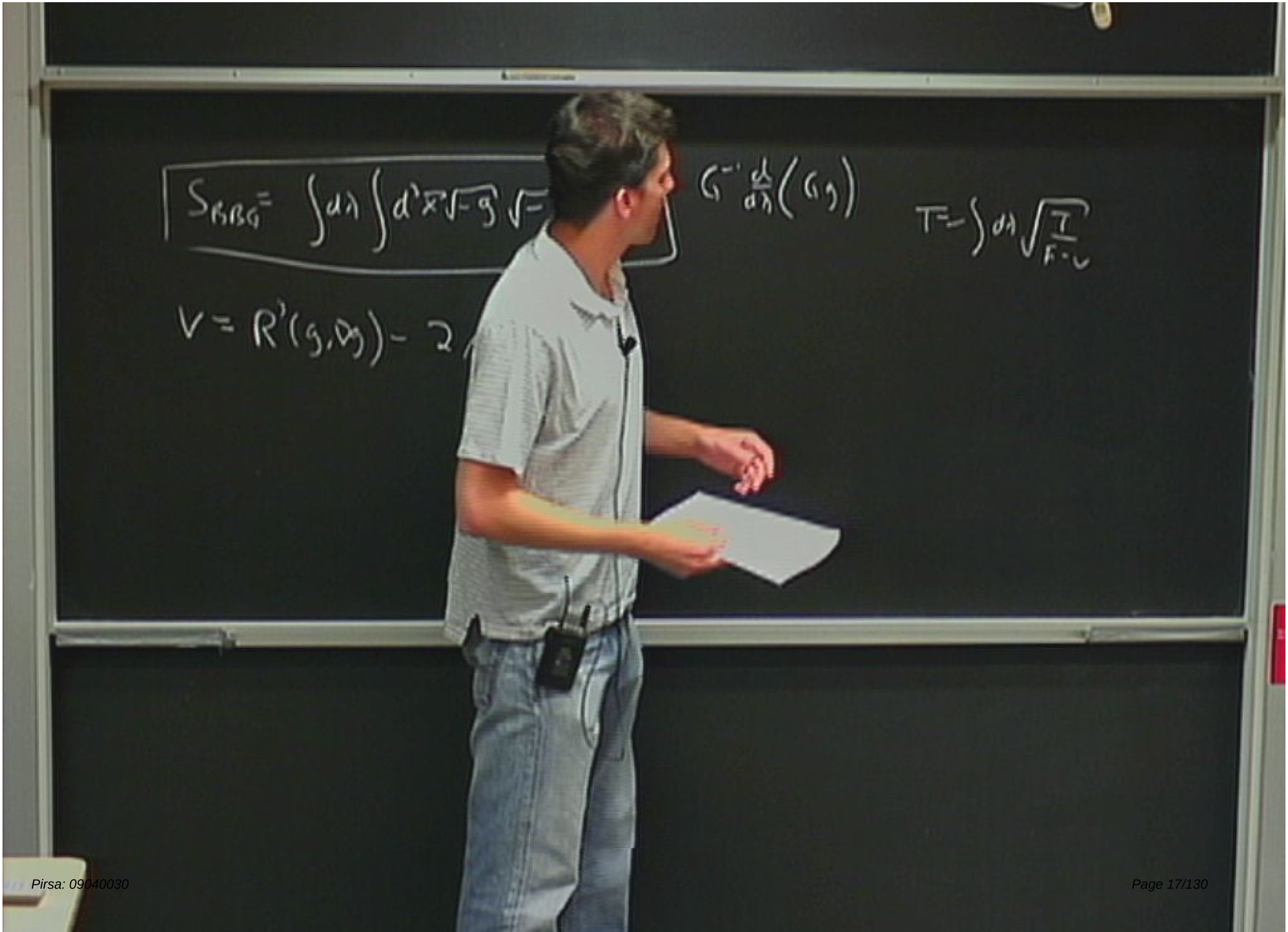

$S_{BBG} = \int d\lambda \int d^4x \sqrt{-g}\sqrt{-V}\sqrt{T}$
$G^{-1} \frac{d}{d\lambda}(G g)$

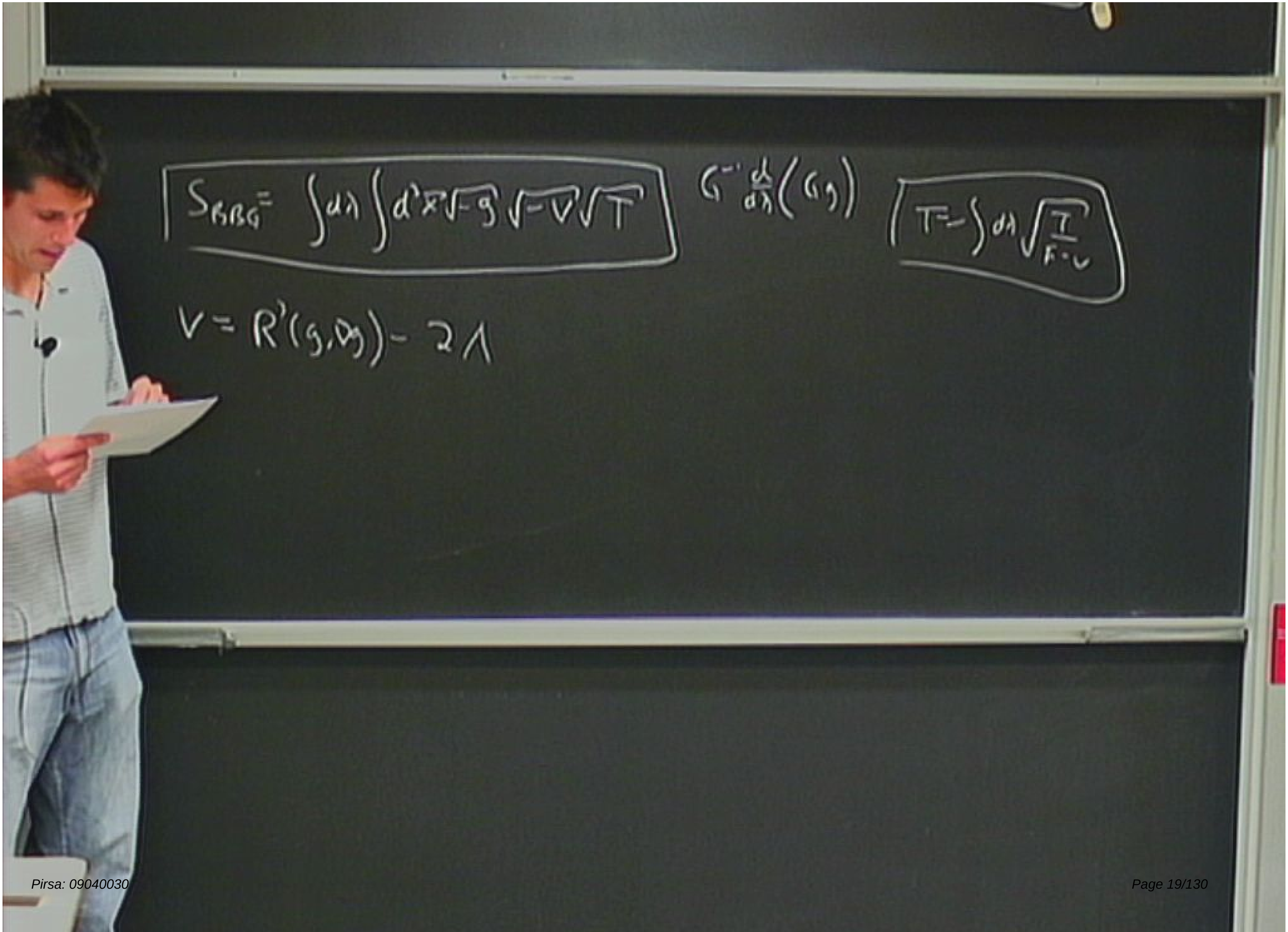
$$S_{BBG} = \int d\lambda \int d^4x \sqrt{-g}\,\sqrt{-V}\,\sqrt{T} \qquad G^{-1}\frac{d}{d\lambda}(G g)$$

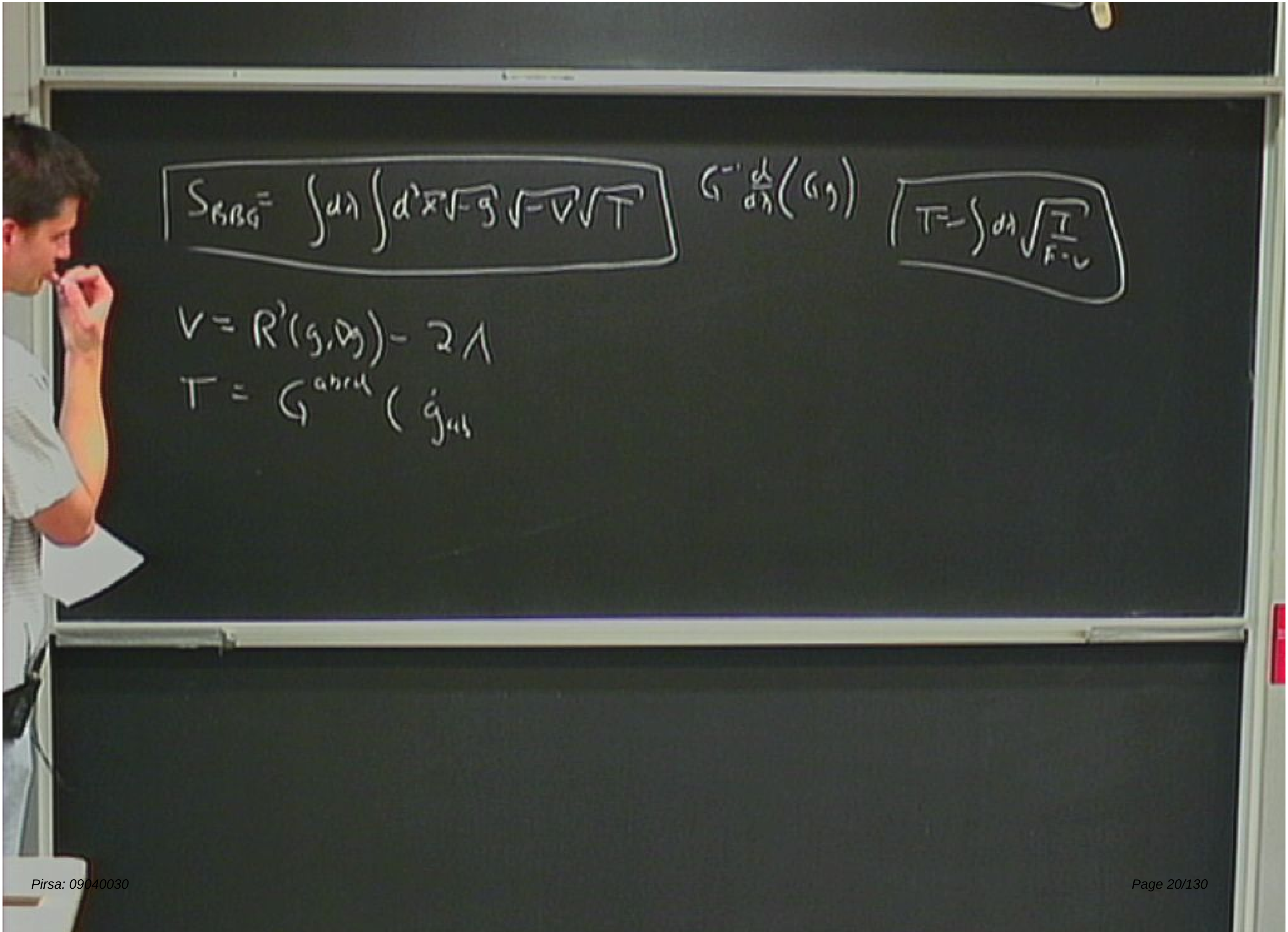
$$V = R^3(g, Dg) - 2\Lambda$$

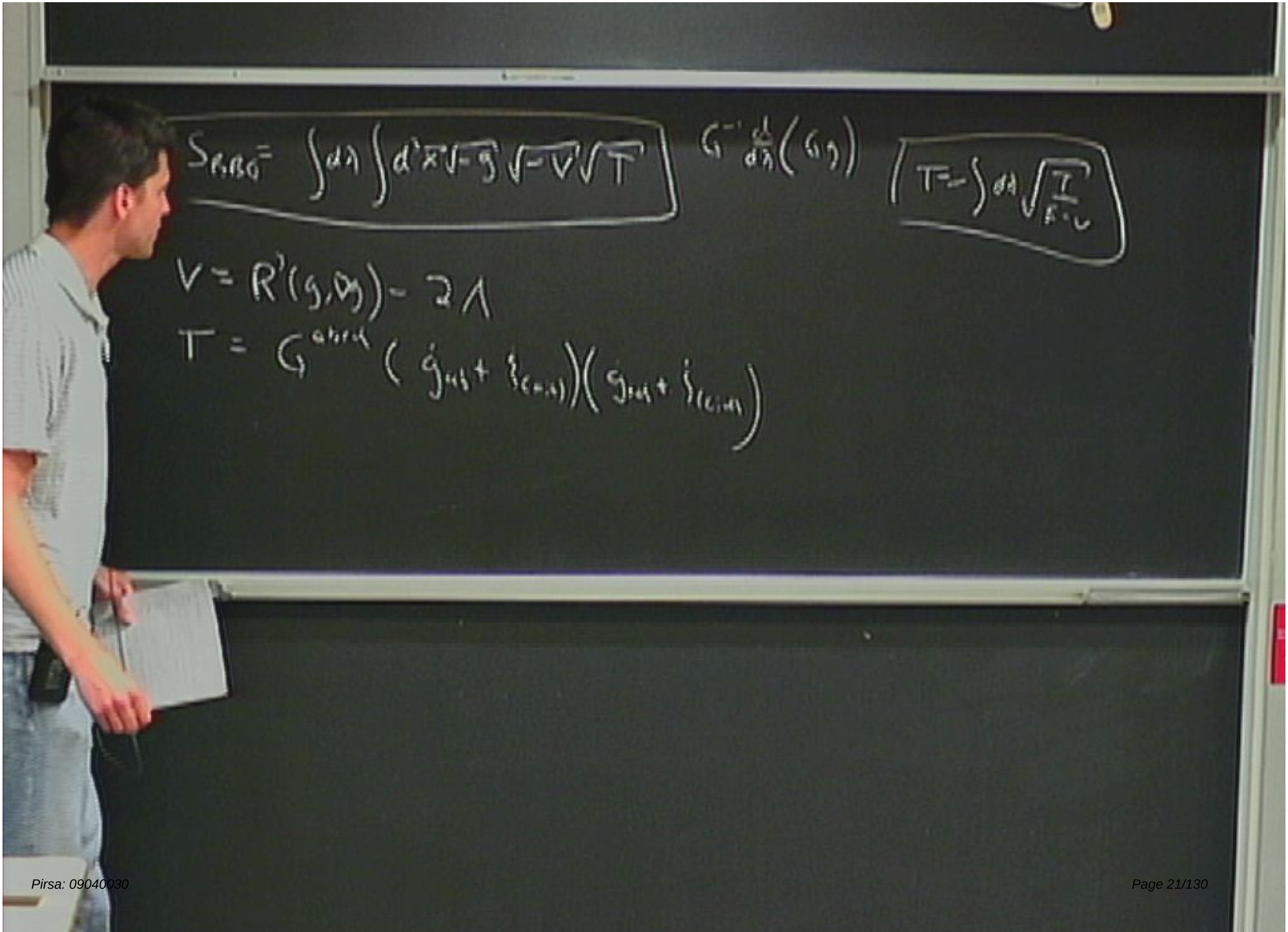

S_BBG = ∫dλ ∫d^4x √-g √-V √T
G^-1 d/dλ (G g)
R'(g,Dg) - 2Λ

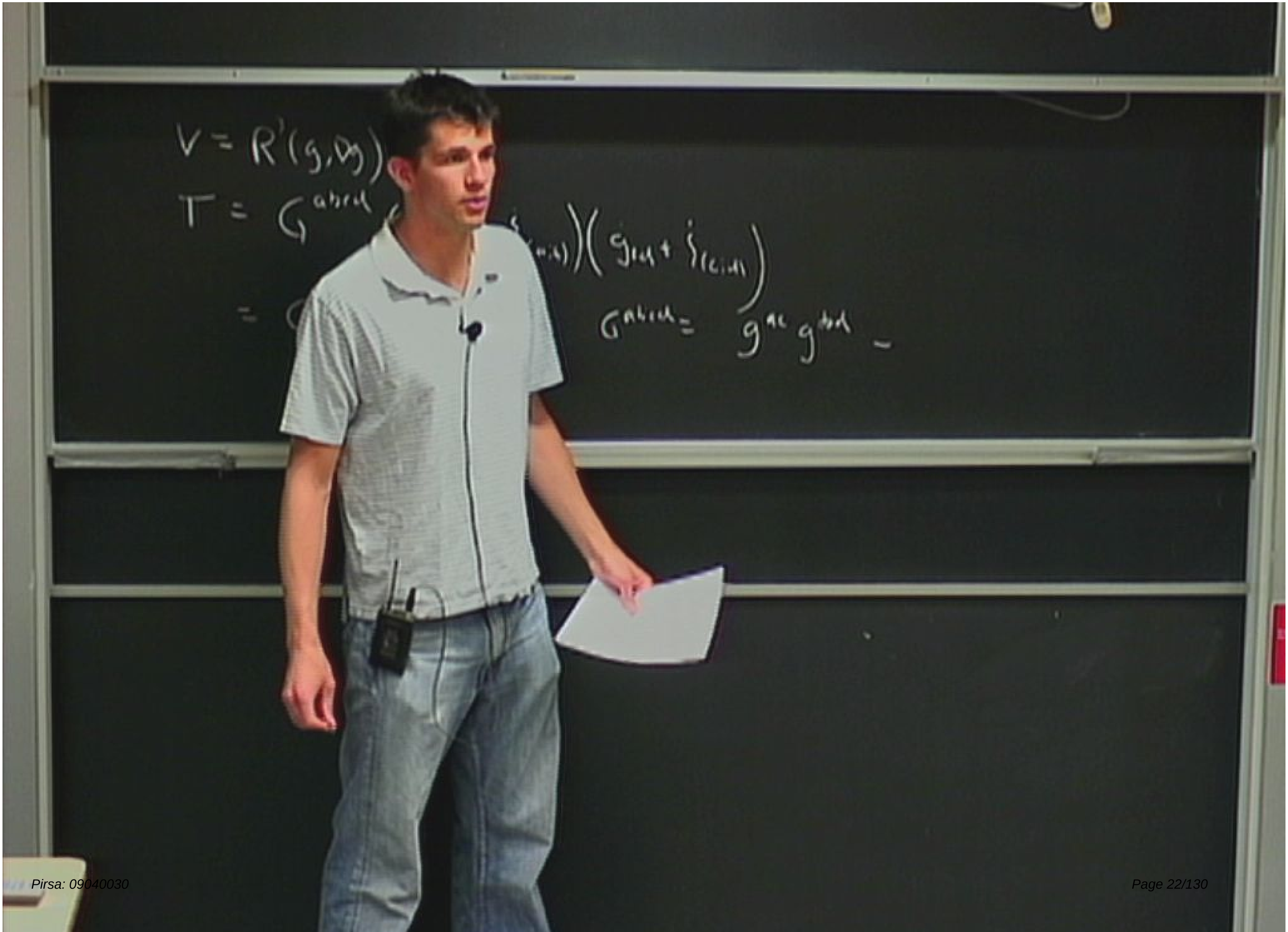

$S_{BBG} = \int d\lambda \int d^3x \sqrt{-g}\sqrt{-V}\sqrt{T}$
$G^{-1}\frac{d}{d\lambda}(G g)$
$V = R^3(g, Dg) - 2\Lambda$

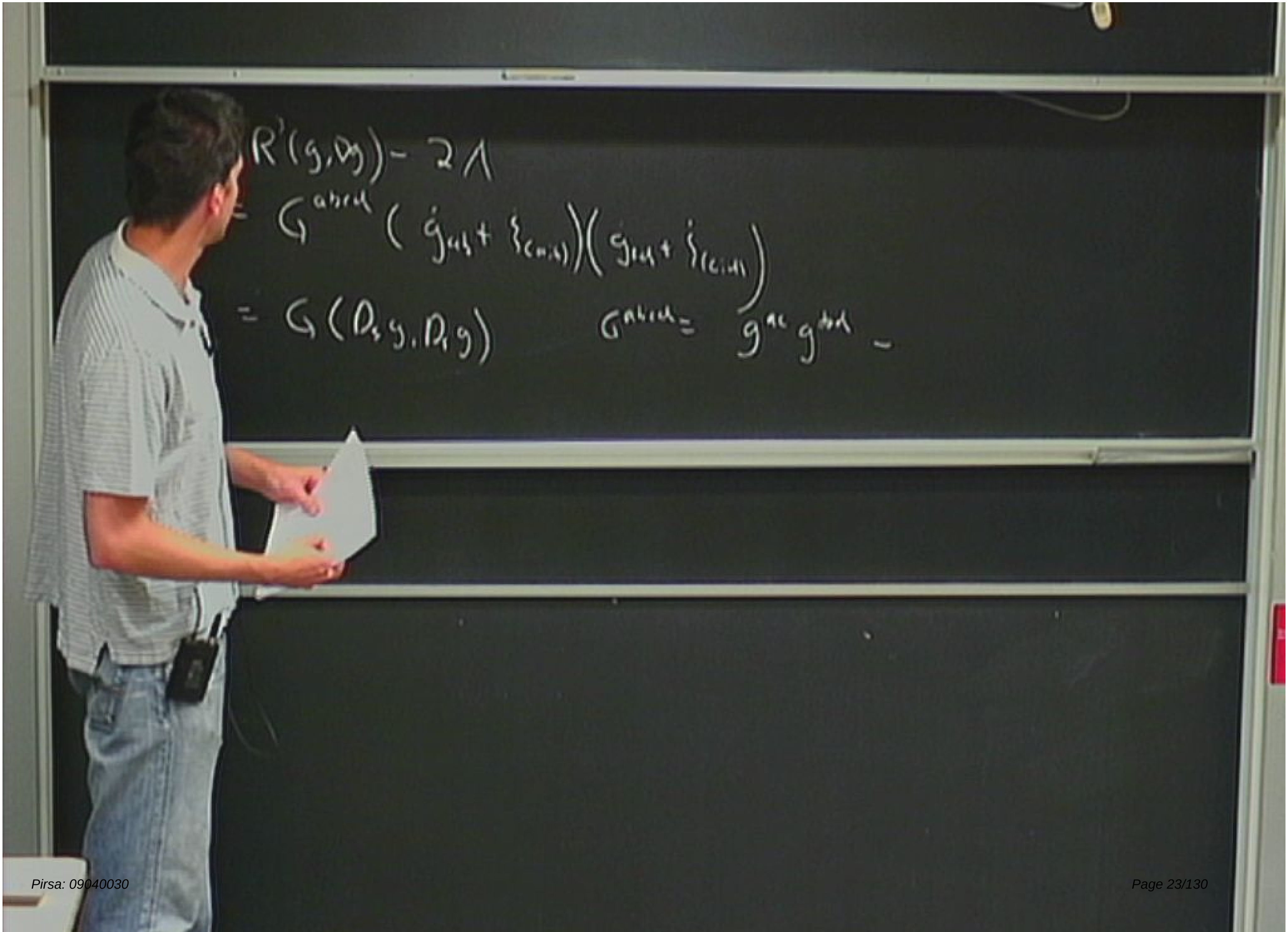



$S_{BBG} = \int d\lambda \int d^3x \sqrt{-g}\sqrt{-V}\sqrt{T}$
$G^{-1}\frac{d}{d\lambda}(Gg)$
$T = -\int d\lambda \sqrt{\frac{T}{F \cdot V}}$
$V = R^3(g, Dg) - 2\Lambda$


$S_{BBG} = \int d\lambda \int d^3x \sqrt{-g} \sqrt{-V} \sqrt{T}$
$G^{-1} \frac{d}{d\lambda} (G g)$
$T = \int d\lambda \sqrt{\frac{T}{R-V}}$
$V = R^3(g, Dg) - 2\Lambda$
$T = G^{abcd} (\dot{g}_{ab}$


$S_{BBG} = \int d\lambda \int d^3x \sqrt{-g} \sqrt{-V} \sqrt{T}$
$G^{-1} \frac{d}{d\lambda} (Gg)$
$V = R^3(g, Dg) - 2\Lambda$
$T = G^{abcd} (\dot{g}_{ab} + \xi_{(a;b)})(\dot{g}_{cd} + \xi_{(c;d)})$

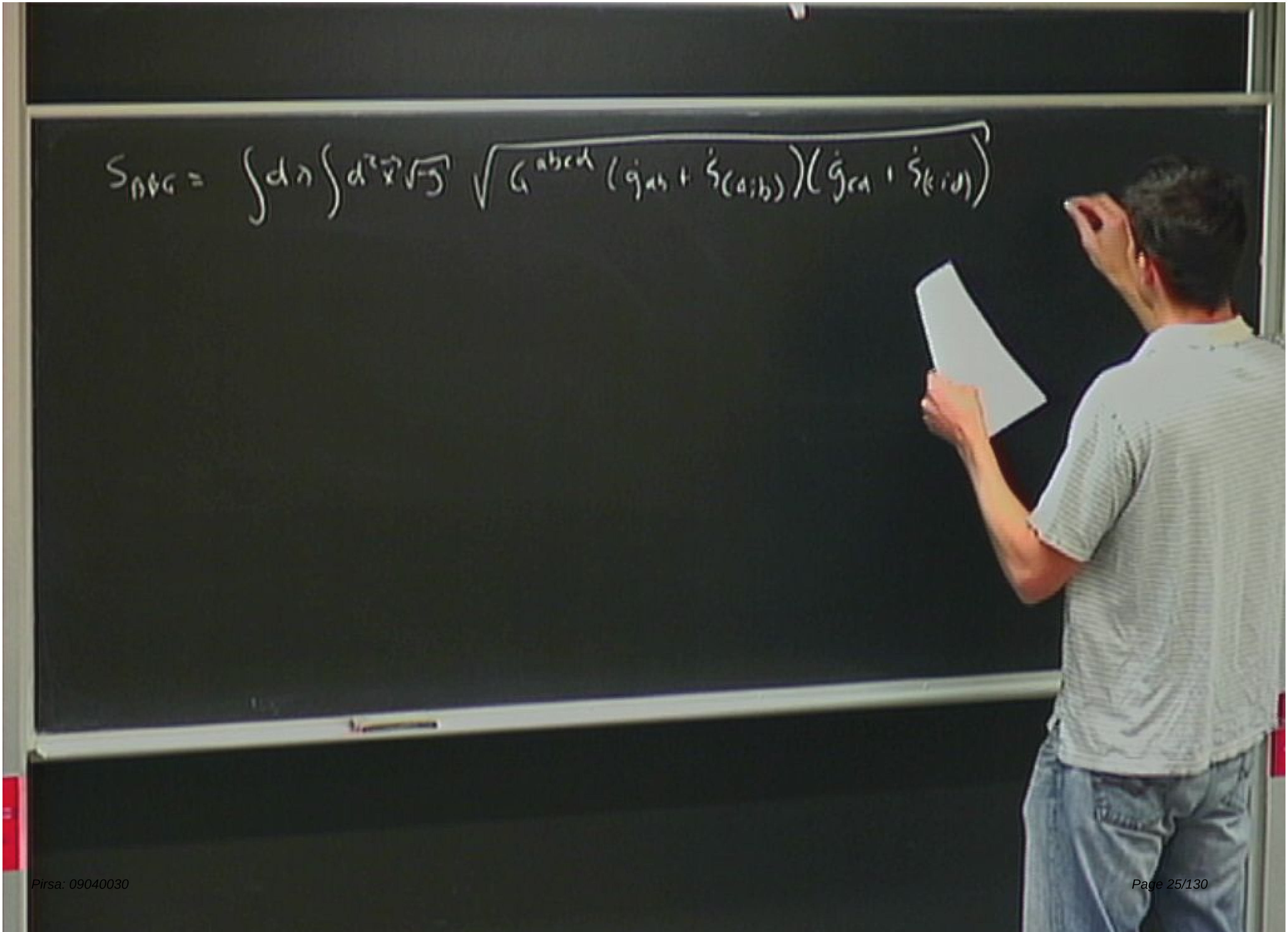


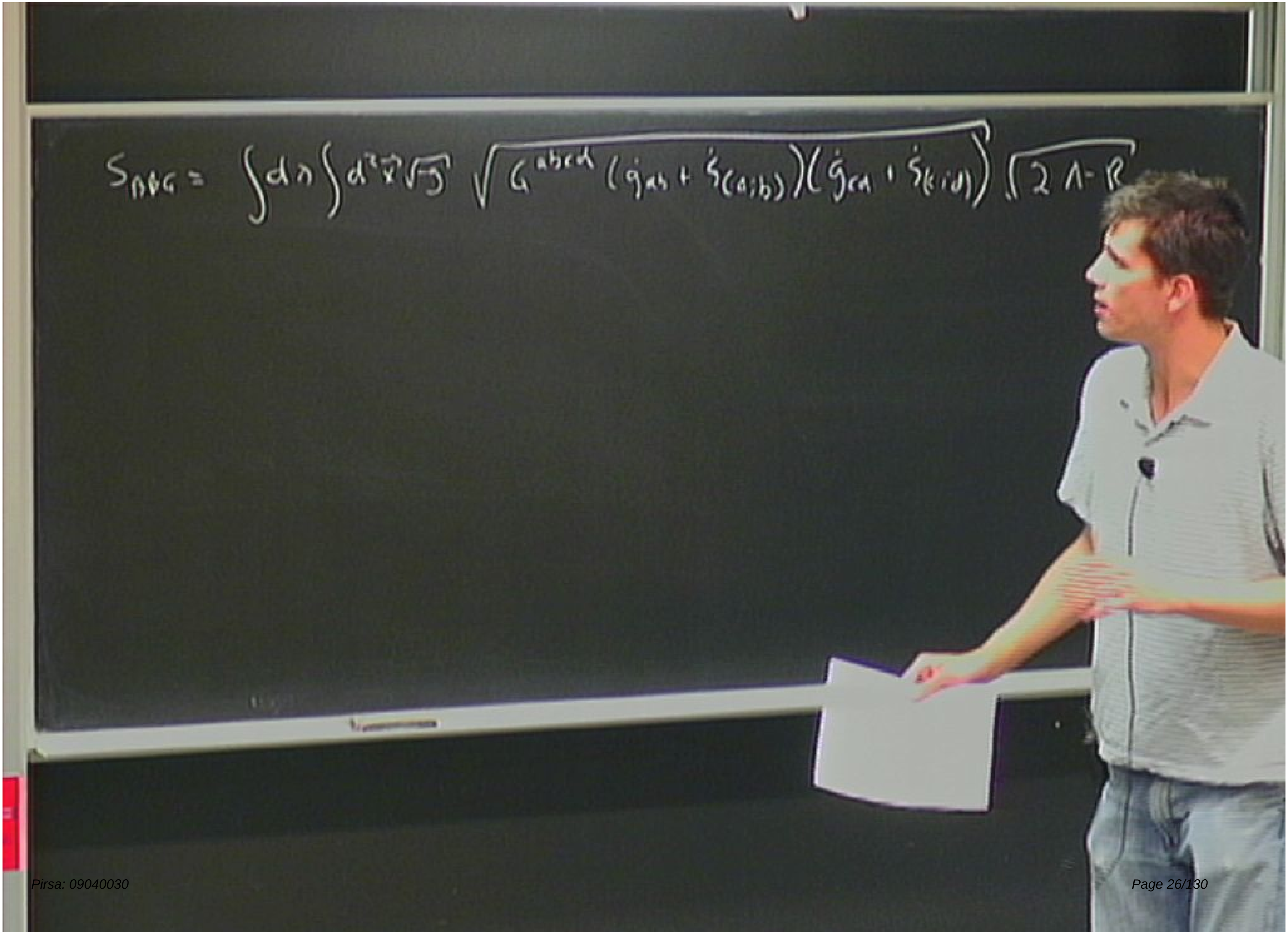

R'(g,Dg) − 2Λ
= G^{abcd} (ġ_{ab} + ξ_{(a;b)})(ġ_{cd} + ξ_{(c;d)})
= G(D_t g, D_t g)
G^{abcd} = g^{ac} g^{bd} −

$$V = R^3(g, Dg) - 2\Lambda$$

$$T = G^{abcd}\left(\dot{g}_{ab} + \xi_{(a;b)}\right)\left(\dot{g}_{cd} + \xi_{(c;d)}\right)$$

$$= G(D_t g, D_t g) \qquad G^{abcd} = g^{ac}g^{bd} - g^{ab}g^{cd}$$


$S_{BDG} = \int d\lambda \int d^3\vec{x}\sqrt{-g}\,\sqrt{G^{abcd}(\dot{g}_{ab} + \dot{\xi}_{(a;b)})(\dot{g}_{cd} + \dot{\xi}_{(c;d)})}$


$S_{BBG} = \int d\lambda \int d^3\vec{x} \sqrt{g} \sqrt{G^{abcd}(\dot{g}_{ab} + \dot{\xi}_{(a;b)})(\dot{g}_{cd} + \dot{\xi}_{(c;d)})} \sqrt{2\Lambda - R}$

$$S_{BSG} = \int d\lambda \int d^3x \sqrt{g} \sqrt{G^{abcd}(\dot{g}_{ab} + \dot{\xi}_{(a;b)})(\dot{g}_{cd} + \dot{\xi}_{(c;d)})} \sqrt{2\Lambda - R}$$

$$\dot{\xi} \downarrow N_a$$

$$S_{BBG} = \int d\lambda \int d^3\vec{x}\sqrt{g}\,\sqrt{G^{abcd}(\dot g_{ab} + \dot\xi_{(a;b)})(\dot g_{cd} + \dot\xi_{(c;d)})}\,\sqrt{2\Lambda - R}$$

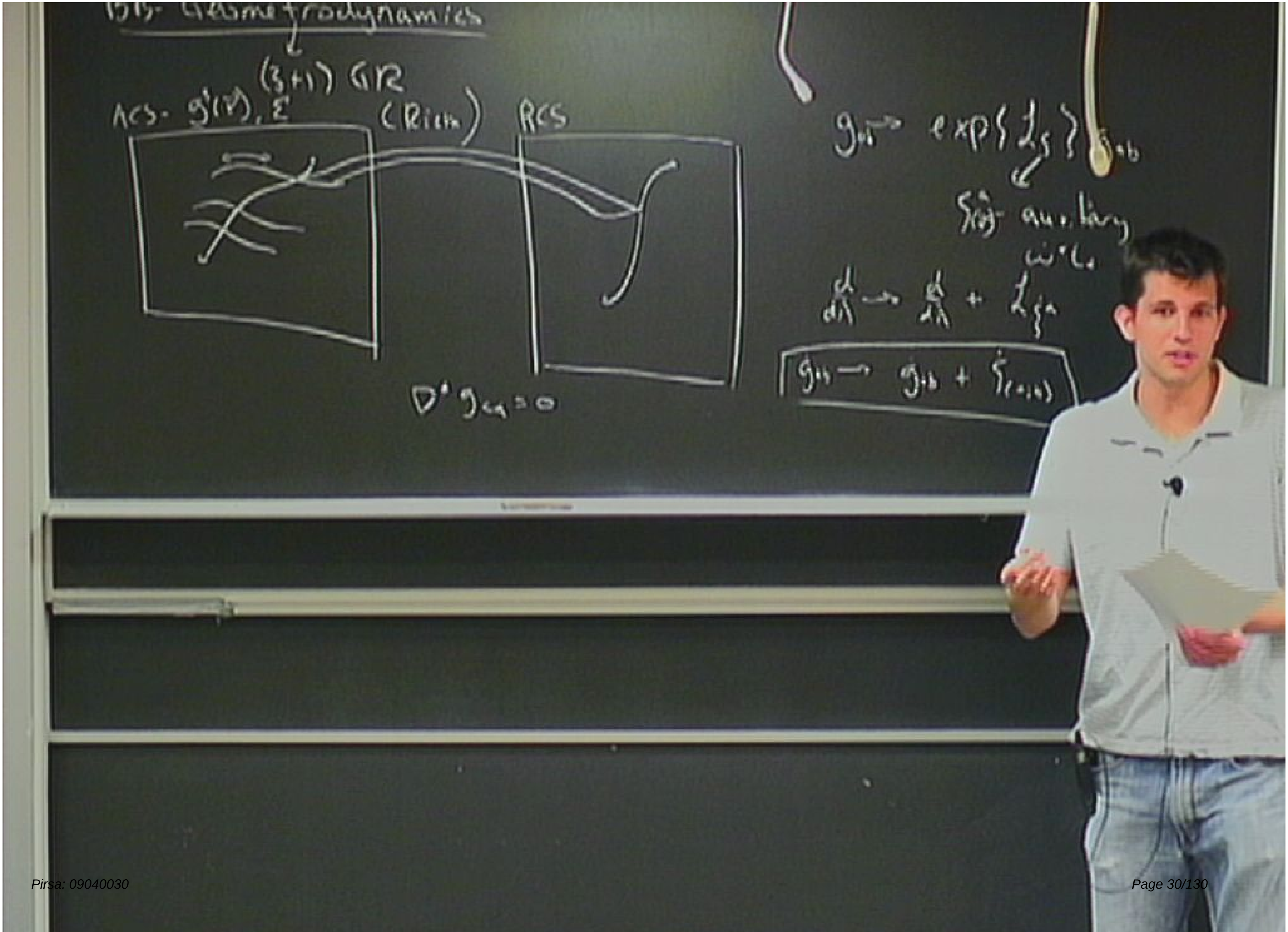
$\dot\xi \to N_a$

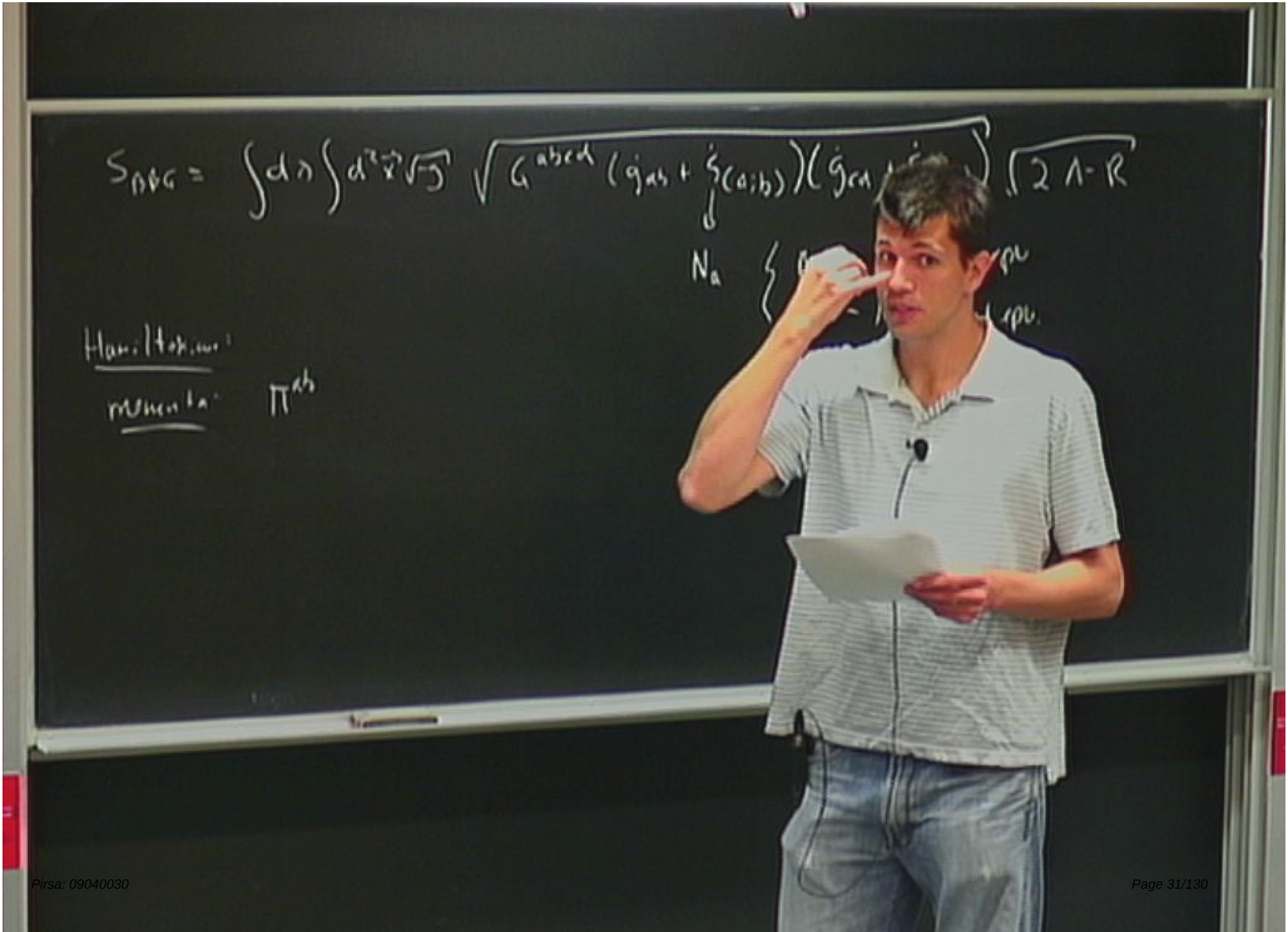
BB – $\dot\xi$ free eqn

BSW – N_a fixed eqn.

$$S_{BBG} = \int d\lambda \int d^3\vec{x}\,\sqrt{-g}\,\sqrt{G^{abcd}\left(\dot g_{ab} + \dot\xi_{(a;b)}\right)\left(\dot g_{cd} + \dot\xi_{(c;d)}\right)}\,\sqrt{2\Lambda - R}$$

N_a { BB – $\dot\xi$ free eqn / BSW – N_a fixed eqn.


(3+1) GR
RCS


Hamiltonian:
momenta:

$$S_{BBG} = \int d\lambda \int d^3\vec{x}\,\sqrt{g}\,\sqrt{G^{abcd}\left(\dot{g}_{ab} + \dot{\xi}_{(a;b)}\right)\left(\dot{g}_{cd} + \dot{\xi}_{(c;d)}\right)}\,\sqrt{2\Lambda - R}$$

$\dot{\xi} \downarrow N_a$

$$N_a \begin{cases} BR - \dot{\xi} \text{ free eqn} \\ BSW - N_a \text{ fixed eqn.} \end{cases}$$

<u>Hamiltonian:</u>

<u>momenta:</u> $$\pi^{ab} = \frac{\partial L}{\partial \dot{g}_{ab}} \approx \sqrt{\frac{2\Lambda - R}{G(D_t g, D_t g)}}$$

$$S_{BBG} = \int d\lambda \int d^3\vec{x}\sqrt{-g}\,\sqrt{G^{abcd}(\dot g_{ab} + \dot\xi_{(a;b)})(\dot g_{cd} + \dot\xi_{(c;d)})}\,\sqrt{2\Lambda - R}$$

N_a { BR – $\dot\xi$ free eqn; BSW – N_a fixed eqn.

Hamiltonian:

momenta:

$$\pi^{ab} = \frac{\partial L}{\partial \dot g_{ab}} = \sqrt{\frac{2\Lambda - R}{G(\dot g, \dot g)}}\sqrt{-g}\; G^{ab}(\dot g, -)$$

$$S_{BBG} = \int d\lambda \int d^3\vec{x}\sqrt{-g}\,\sqrt{G^{abcd}(\dot g_{ab} + \dot\xi_{(a;b)})(\dot g_{cd} + \dot\xi_{(c;d)})}\,\sqrt{2\Lambda - R}$$

$\dot\xi_{(a;b)} \downarrow N_a$ { BB – $\dot\xi$ free var; BSW – N_a fixed var.

Hamiltonian:

momenta:

$$\pi^{ab} = \frac{\partial L}{\partial \dot g_{ab}} = \sqrt{\frac{2\Lambda - R}{G(\dot{g}, \dot{g})}}\sqrt{-g}\; G^{ab}(\dot{g}, \dots)$$

$$p^a = \frac{\partial L}{\partial \dot\xi_a} =$$

$$S_{BBG} = \int d\lambda \int d^3\vec{x}\sqrt{-g}\,\sqrt{G^{abcd}\,(\dot{g}_{ab} + \dot{\xi}_{(a;b)})(\dot{g}_{cd} + \dot{\xi}_{(c;d)})}\,\sqrt{2\Lambda - R}$$

N_a { BR – $\dot{\xi}$ free eqn. ; BSW – N_a fixed eqn.

Hamiltonian:

momenta:

$$\pi^{ab} = \frac{\partial L}{\partial \dot{g}_{ab}} = \sqrt{\frac{2\Lambda - R}{G(B_\xi g, B_\xi g)}}\sqrt{-g}\; G^{ab}(B_\xi g, -)$$

$$p^a = \frac{\partial L}{\partial \dot{\xi}_a} = \sqrt{\frac{2\Lambda - R}{G(B_\xi g, B_\xi g)}}\sqrt{-g}$$

$$S_{BBG} = \int d\lambda \int d^3\vec{x} \sqrt{-g} \sqrt{G^{abcd}(\dot{g}_{ab} + \dot{\xi}_{(a;b)})(\dot{g}_{cd} + \dot{\xi}_{(c;d)})} \sqrt{2\Lambda - R}$$

$\dot{\xi} \to N_a$ { BR – $\dot{\xi}$ free ep; BSW – N_a fixed ep. }

<u>Hamiltonian:</u>

<u>momenta:</u>

$$\pi^{ab} = \frac{\partial L}{\partial \dot{g}_{ab}} = \sqrt{\frac{2\Lambda - R}{G(\mathcal{B}g, \mathcal{B}g)}} \sqrt{-g}\, G^{ab}(\mathcal{B}g, -)$$

$$p^a = \frac{\partial L}{\partial \dot{\xi}_a} = \nabla_b \left\{ \sqrt{\frac{2\Lambda - R}{G(\mathcal{B}g, \mathcal{B}g)}} \sqrt{-g}\, G^{ab}(\mathcal{B}g, -) \right.$$

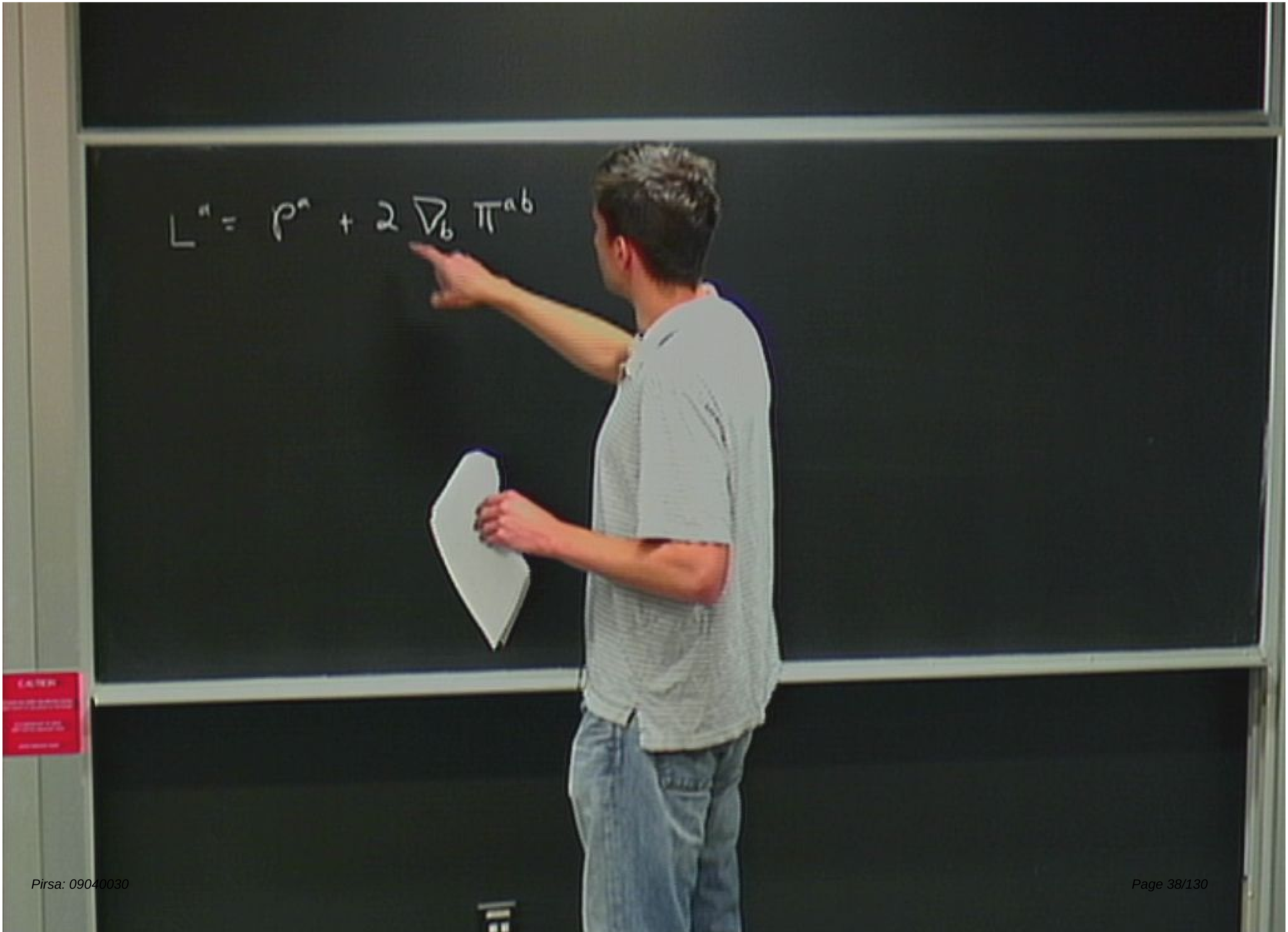
$$S_{BBG} = \int d\lambda \int d^3x \sqrt{-g} \sqrt{G^{abcd}(\dot{g}_{ab} + \dot{\xi}_{(a;b)})(\dot{g}_{cd} + \dot{\xi}_{(c;d)})} \sqrt{2\Lambda - R}$$

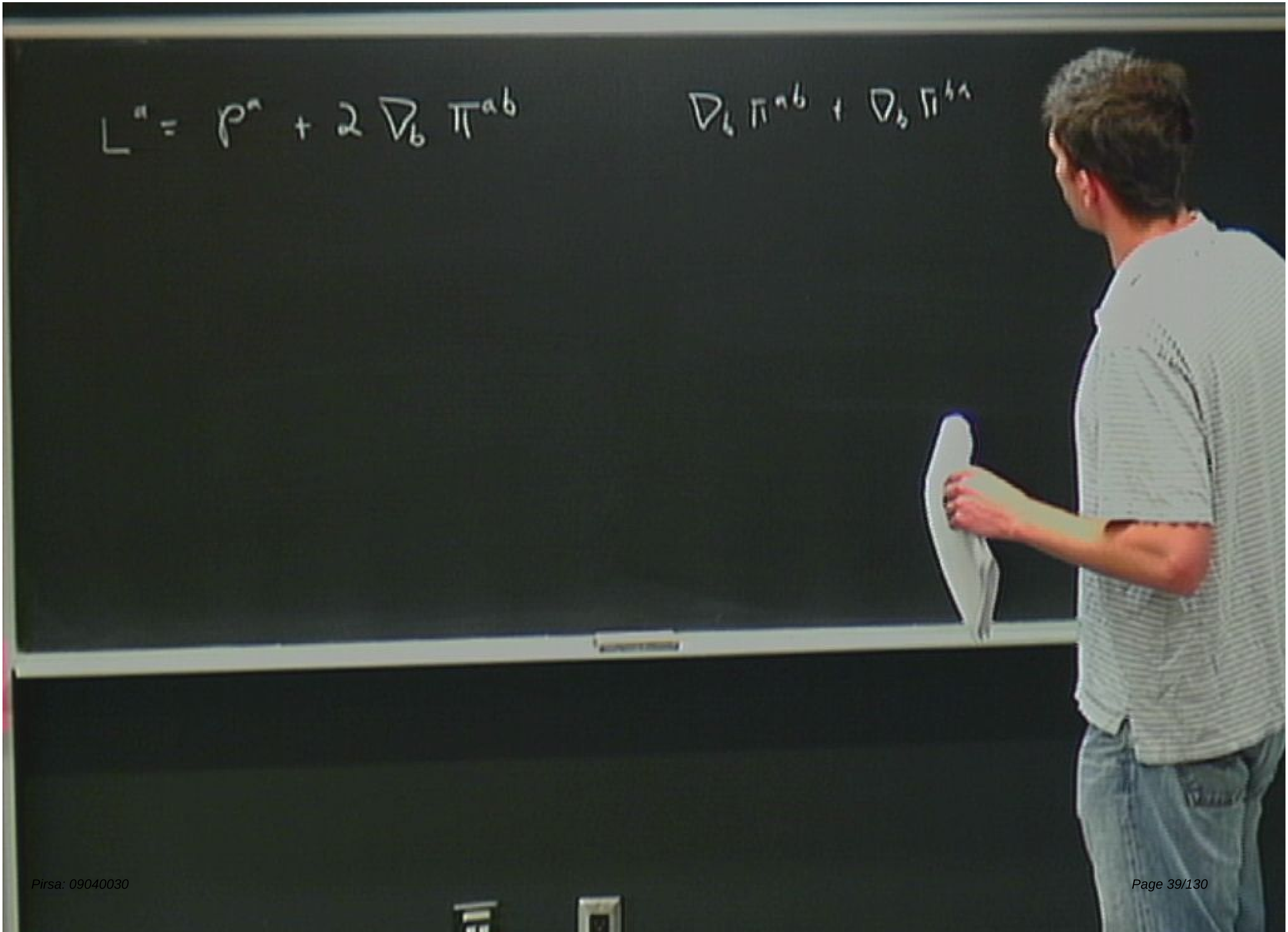
$\dot{\xi} \downarrow N_a$ $\begin{cases} BR - \dot{\xi} \text{ free eqn} \\ BSW - N_a \text{ fixed eqn.} \end{cases}$

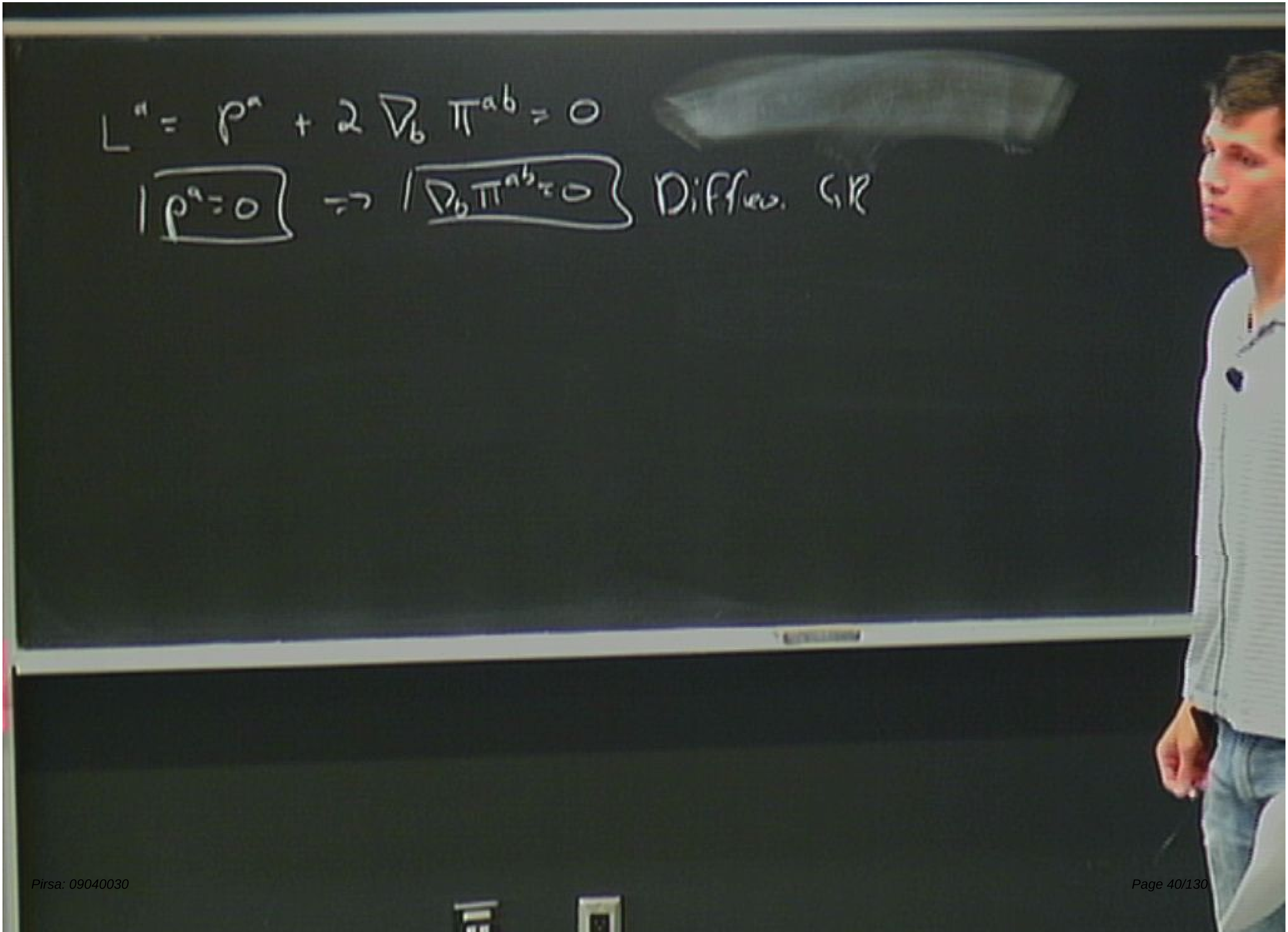
Hamiltonian momenta:

$$\pi^{ab} = \frac{\partial L}{\partial \dot{g}_{ab}} = \sqrt{\frac{2\Lambda - R}{G(B_g, B_g)}} \sqrt{-g}\; G^{ab}(B_g, -)$$

$$p^a = \frac{\partial L}{\partial \dot{\xi}_a} = -2\nabla_b \left\{ \sqrt{\frac{2\Lambda - R}{G(B_g, B_g)}} \sqrt{-g}\; G^{ab}(B_g, -) \right\}$$


$L^a = \rho^a + 2 \nabla_b \pi^{ab}$


$L^a = \rho^a + 2 \nabla_b \pi^{ab}$
$\nabla_b \pi^{ab} + \nabla_b \pi^{ba}$


$L^a = p^a + 2 \nabla_b \pi^{ab} = 0$
$p^a = 0 \Rightarrow \nabla_b \pi^{ab} = 0$ Diffeo. GR

$$L^a = p^a + 2\nabla_b \pi^{ab} = 0$$

$$\boxed{p^a = 0} \Rightarrow \boxed{\nabla_b \pi^{ab} = 0} \text{ Diffeo. GR}$$

<u>Ham Const</u>:

$$L^a = p^a + 2\nabla_b \pi^{ab} = 0$$

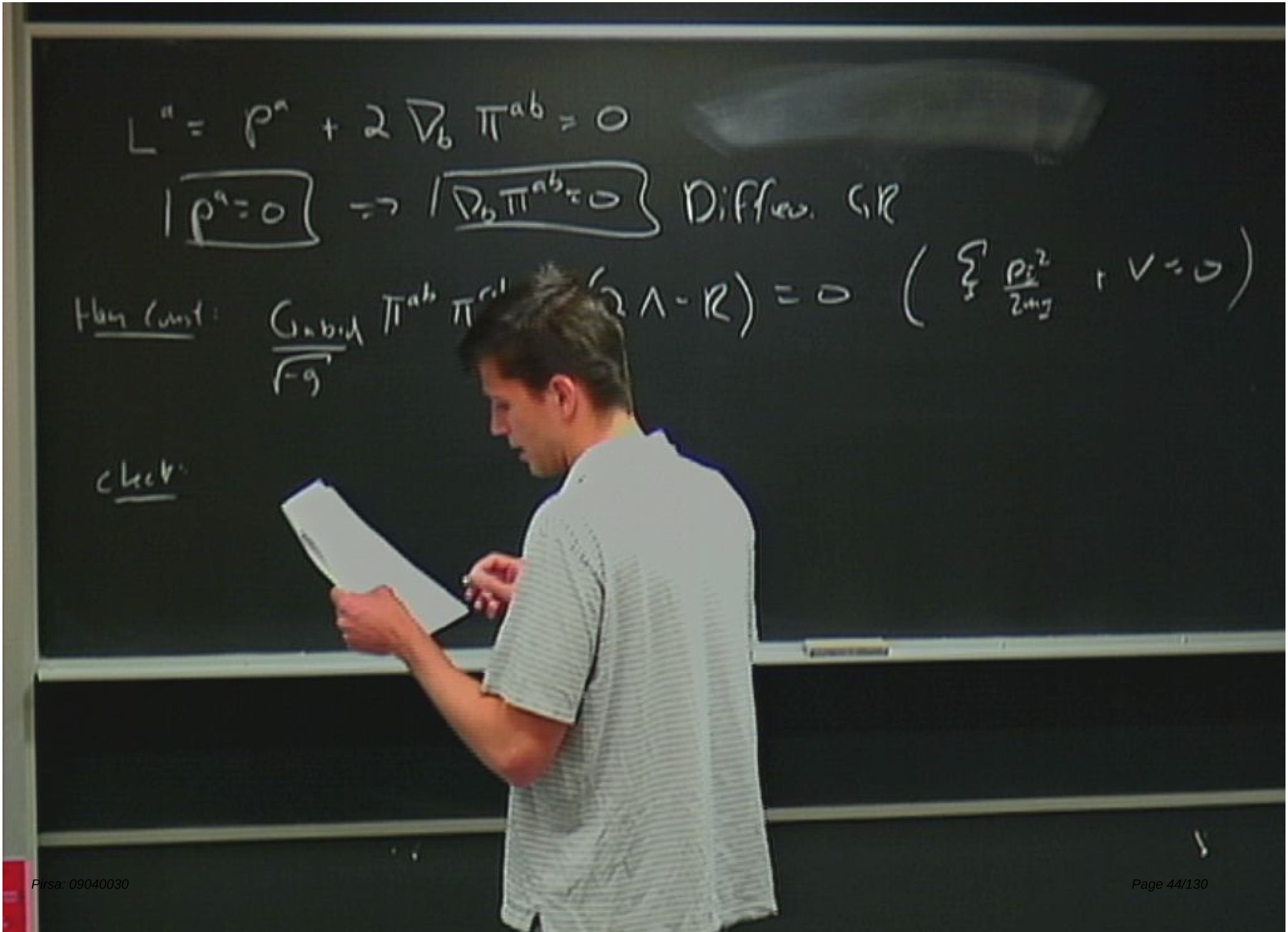
$$\boxed{p^a = 0} \Rightarrow \boxed{\nabla_b \pi^{ab} = 0} \quad \text{Diffeo. GR}$$

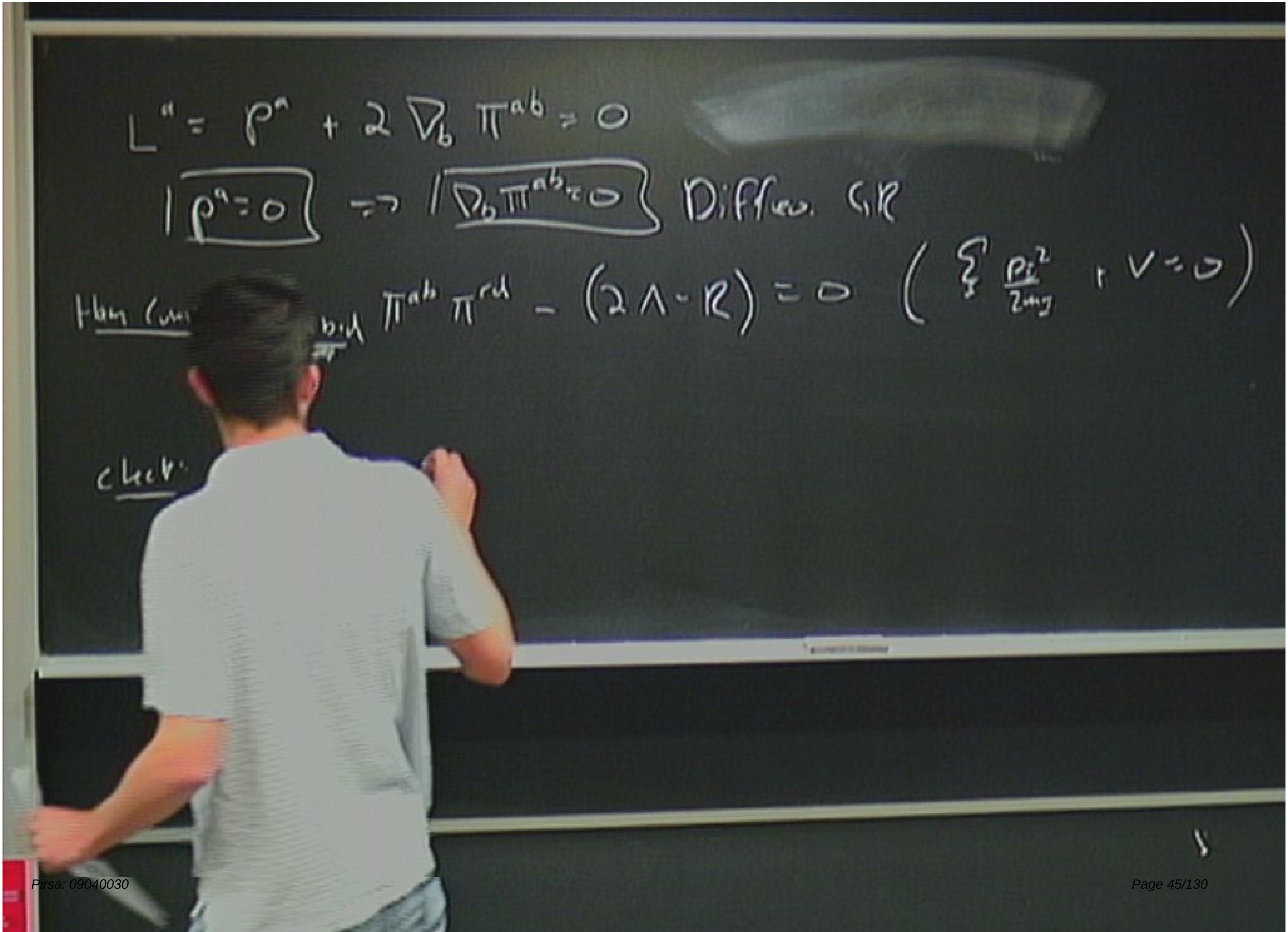
Ham Const: G_{abcd}

$$L^a = p^a + 2\nabla_b \pi^{ab} = 0$$

$$\boxed{p^a = 0} \Rightarrow \boxed{\nabla_b \pi^{ab} = 0} \text{ Diffeo. GR}$$

Ham Const: $G_{abcd}\pi^{ab}\pi^{cd} - (2\Lambda - R) = 0 \quad \left(\sum_i \frac{p_i^2}{2m_i} + V = 0 \right)$


$L^a = p^a + 2 \nabla_b \pi^{ab} = 0$
$p^a = 0 \Rightarrow \nabla_b \pi^{ab} = 0$ Diffeo. GR
Ham Const:
$\left(\sum_i \frac{p_i^2}{2m_i} + V = 0 \right)$
check:

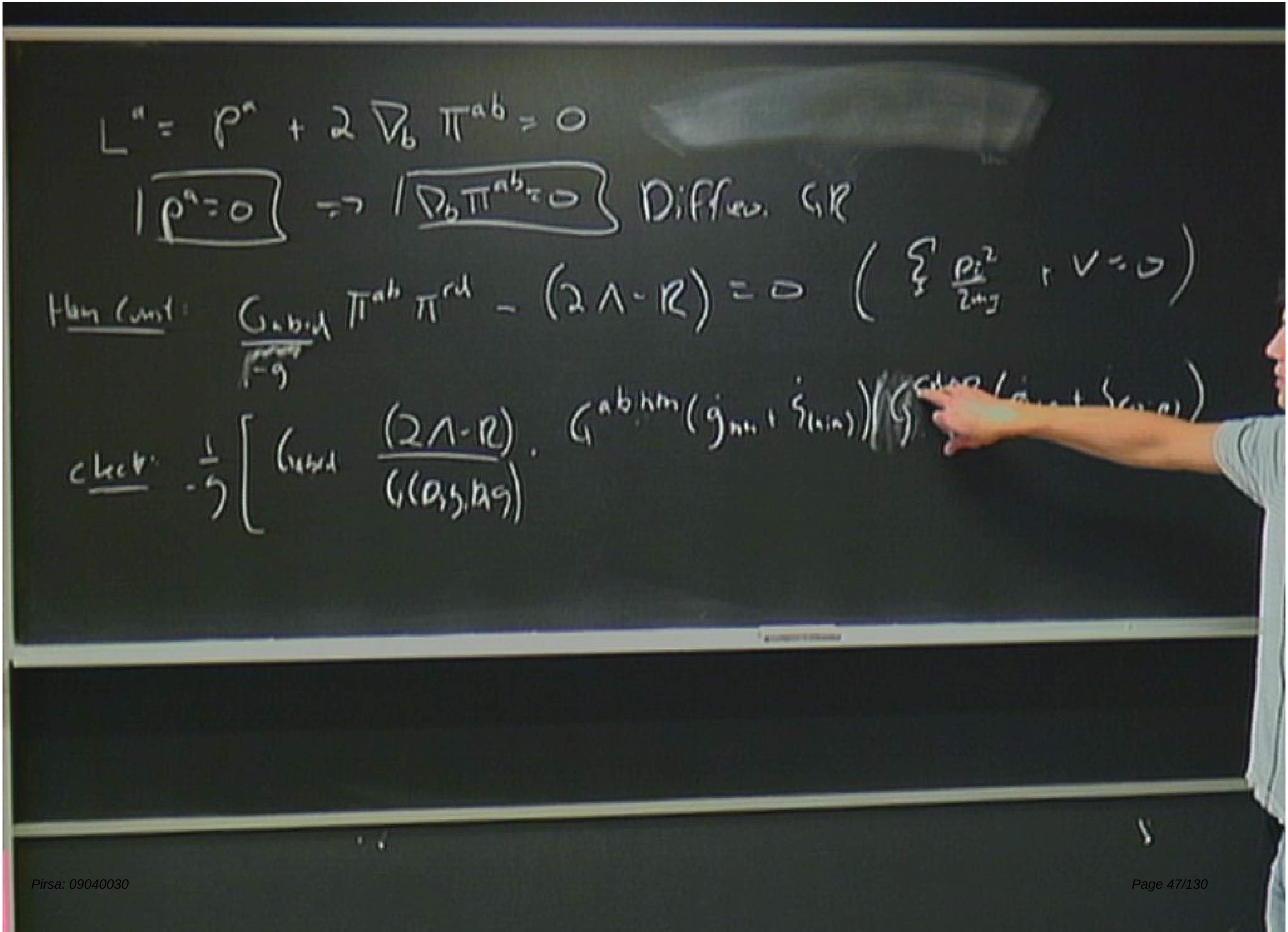

$L^a = p^a + 2 \nabla_b \pi^{ab} = 0$
$p^a = 0 \Rightarrow \nabla_b \pi^{ab} = 0$ Diffeo. GR
Ham Con
$\pi^{ab} \pi^{cd} - (2\Lambda - R) = 0$
$\left(\sum_i \frac{p_i^2}{2m_i} + V = 0 \right)$
check

$$L^a = p^a + 2\nabla_b \pi^{ab} = 0$$

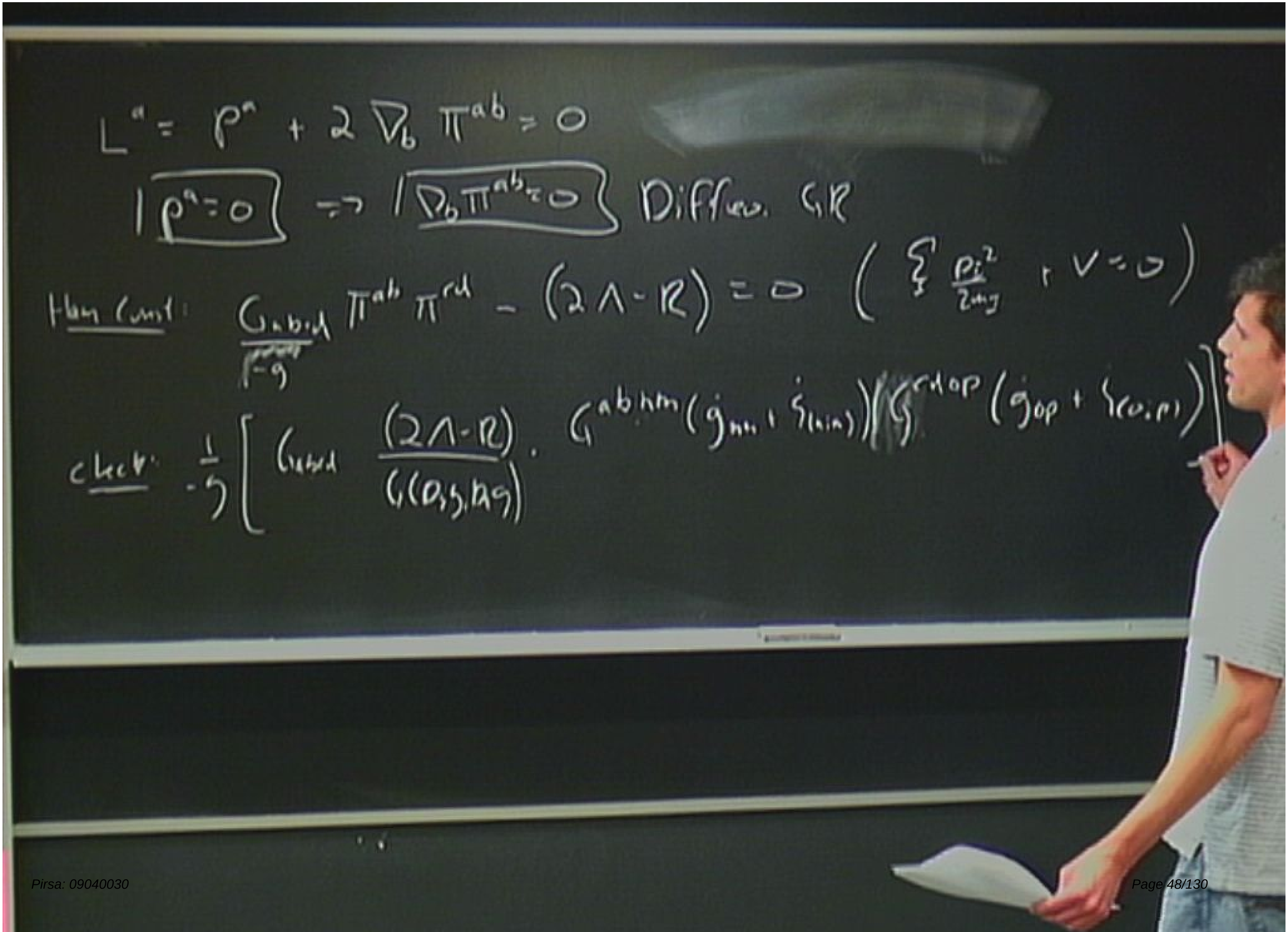
$$\boxed{p^a = 0} \Rightarrow \boxed{\nabla_b \pi^{ab} = 0} \quad \text{Diffeo. GR}$$

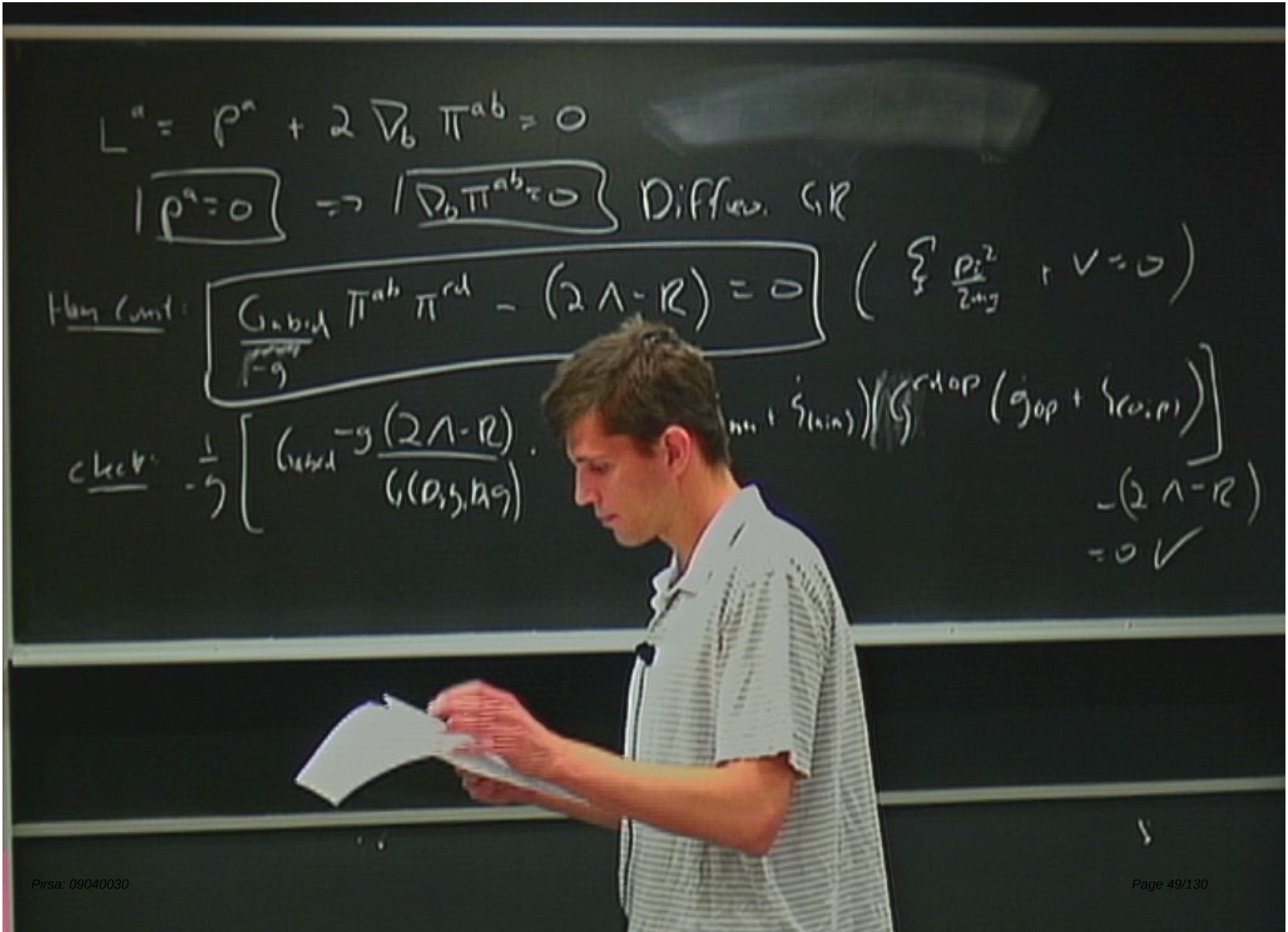
Ham Const: $$\frac{G_{abcd}}{\sqrt{-g}} \pi^{ab} \pi^{cd} - (2\Lambda - R) = 0 \qquad \left(\sum_i \frac{p_i^2}{2m_i} + V = 0 \right)$$

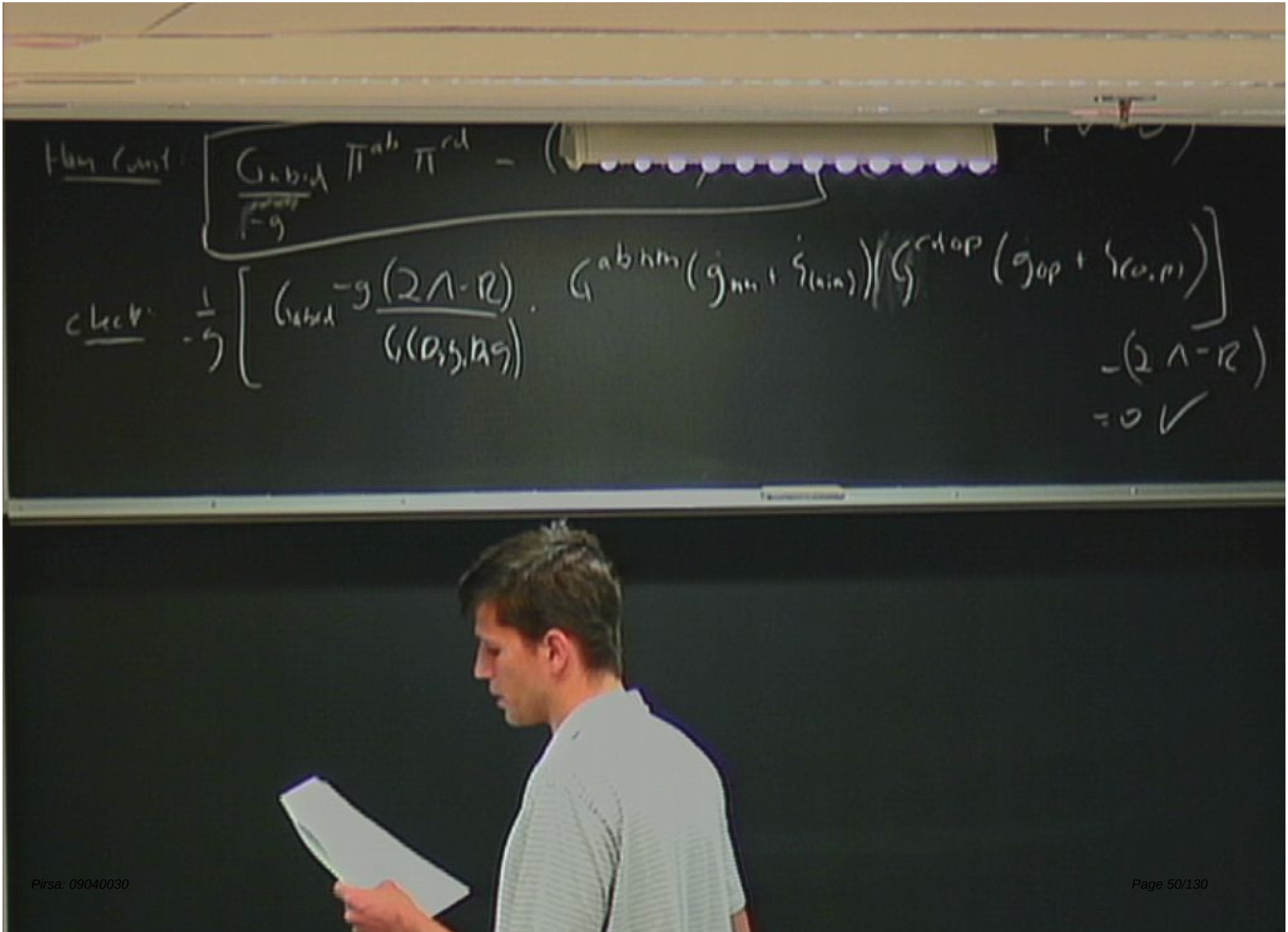
check: $$\frac{1}{\sqrt{-g}} \Big[G_{abcd} \, \frac{(2\Lambda - R)}{G(D_3 g, D_3 g)} \cdot G^{abhm} (\dot{g}_{hm} + \xi_{(h;m)}) \, G^{cdop} (\dot{g}_{op} + \xi_{(o;p)})$$



$L^a = p^a + 2 \nabla_b \pi^{ab} = 0$
$p^a = 0 \Rightarrow \nabla_b \pi^{ab} = 0$ Diffeo. GR
Ham Const: $\frac{G_{abcd}}{\sqrt{-g}} \pi^{ab} \pi^{cd} - (2\Lambda - R) = 0$ $\left(\sum \frac{p_i^2}{2m_i} + V = 0 \right)$
check:

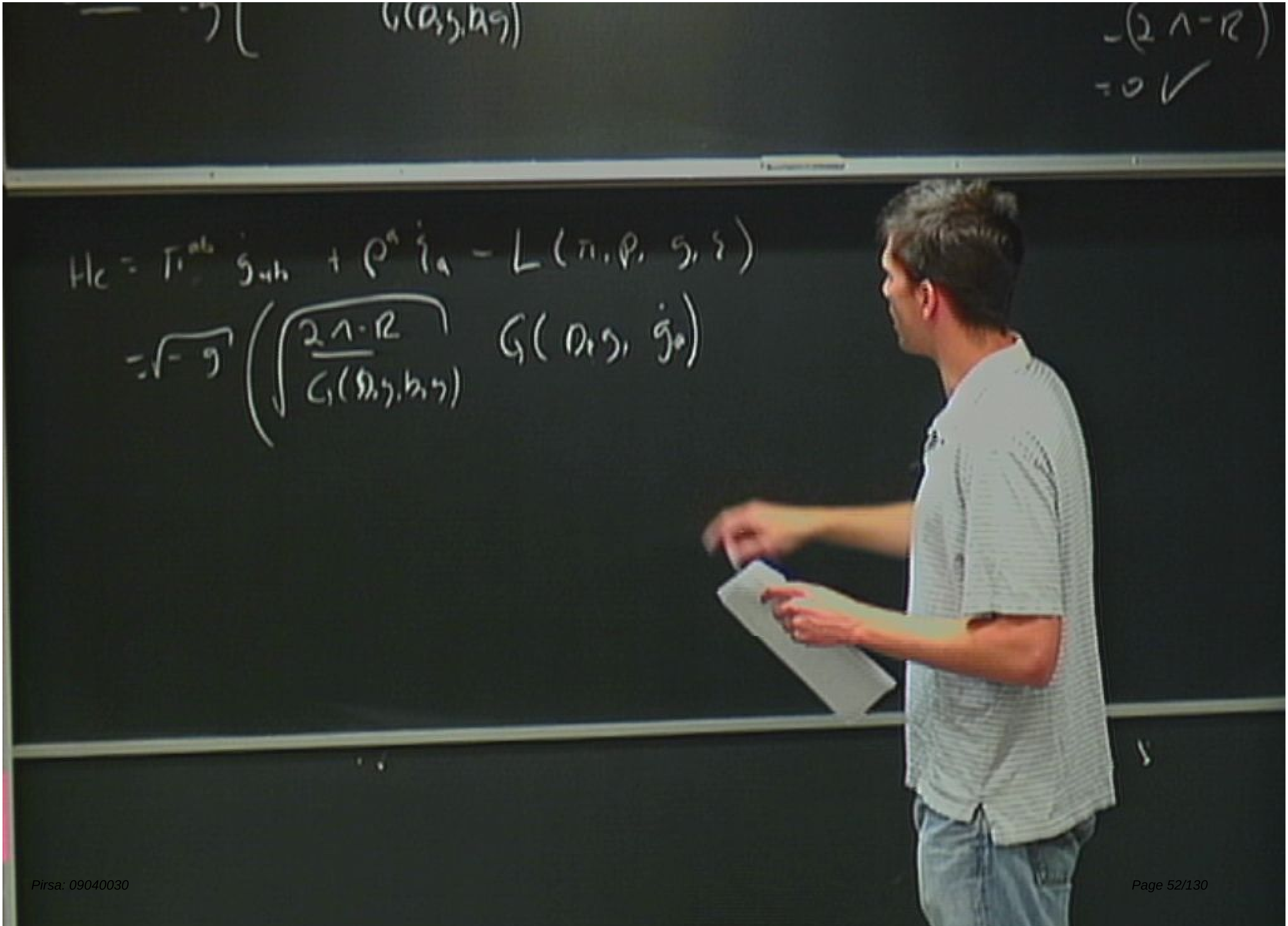

$L^a = p^a + 2 \nabla_b \pi^{ab} = 0$
$p^a = 0 \Rightarrow \nabla_b \pi^{ab} = 0$ Diffeo. GR
Ham Const: $\frac{G_{abcd}}{\sqrt{-g}} \pi^{ab} \pi^{cd} - (2\Lambda - R) = 0$
$\left(\sum \frac{p_i^2}{2m_i} + V = 0 \right)$
check: $\frac{1}{-g} \left[G_{abcd} \frac{(2\Lambda - R)}{G(D_g g, D_g g)} \cdot G^{abmn}(\dot g_{mn} + \xi_{(m;n)}) G^{cdop}(\dot g_{op} + \xi_{(o;p)}) \right]$


L^a = p^a + 2 ∇_b π^ab = 0
p^a = 0 ⇒ ∇_b π^ab = 0 Diffeo. GR
Ham Const:
G_abcd π^ab π^cd − (2Λ − R) = 0
√-g
check:
−(2Λ − R)
= 0 ✓



Ham Const:
check:
= 0 ✓

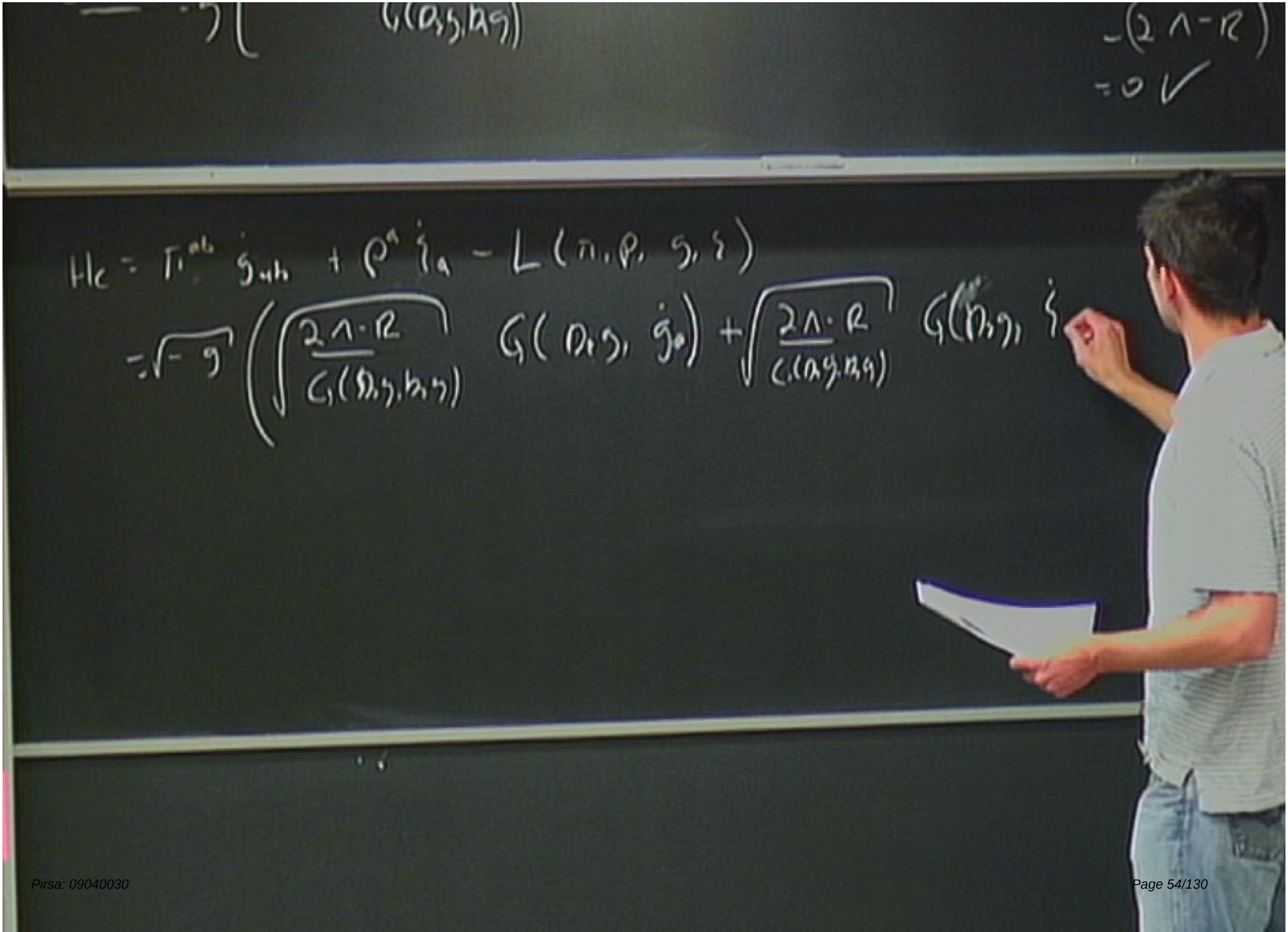

Hc = π^ab ġ_ab + ρ^a ξ̇_a − L(π, ρ, g, ξ)

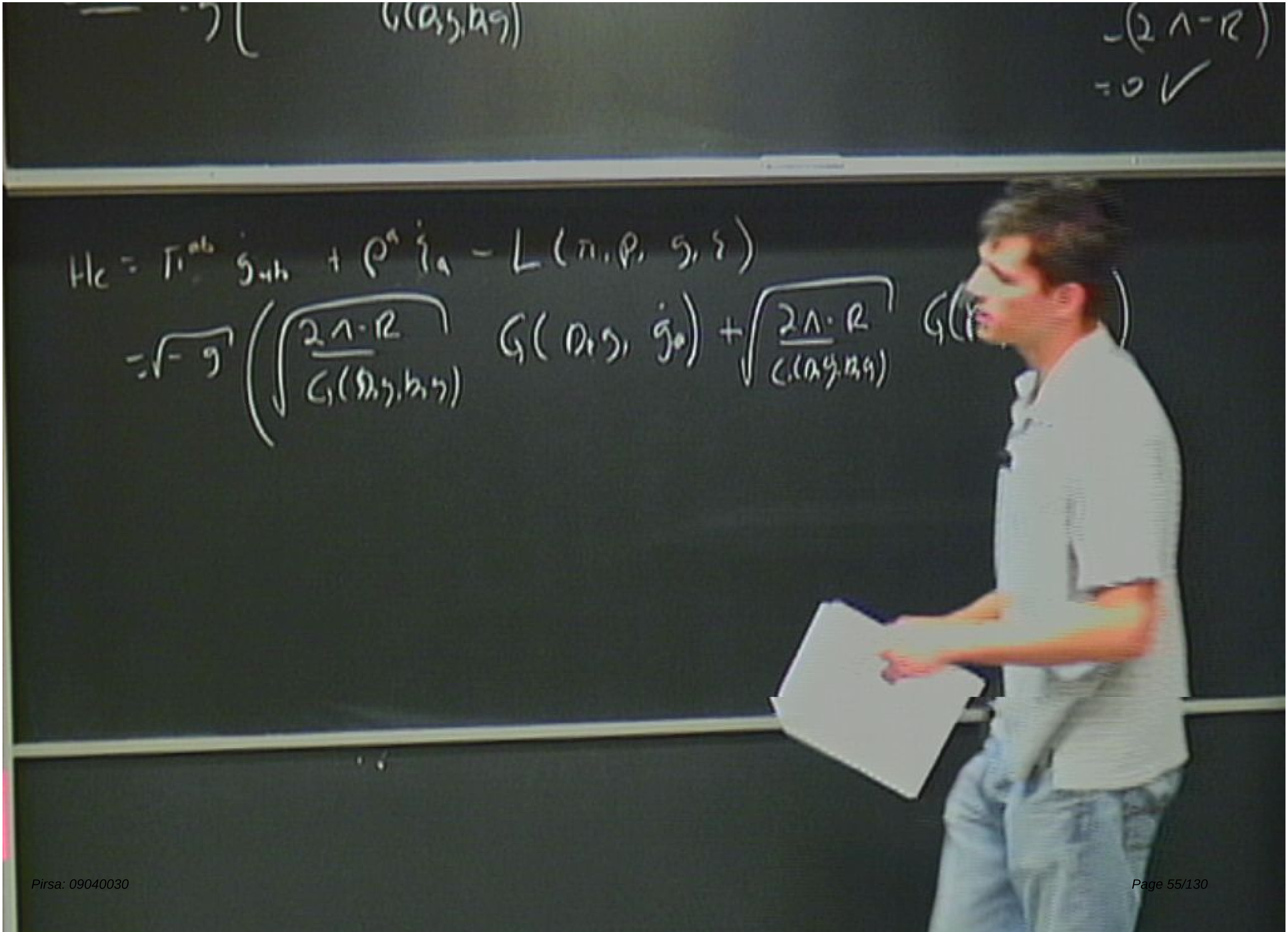


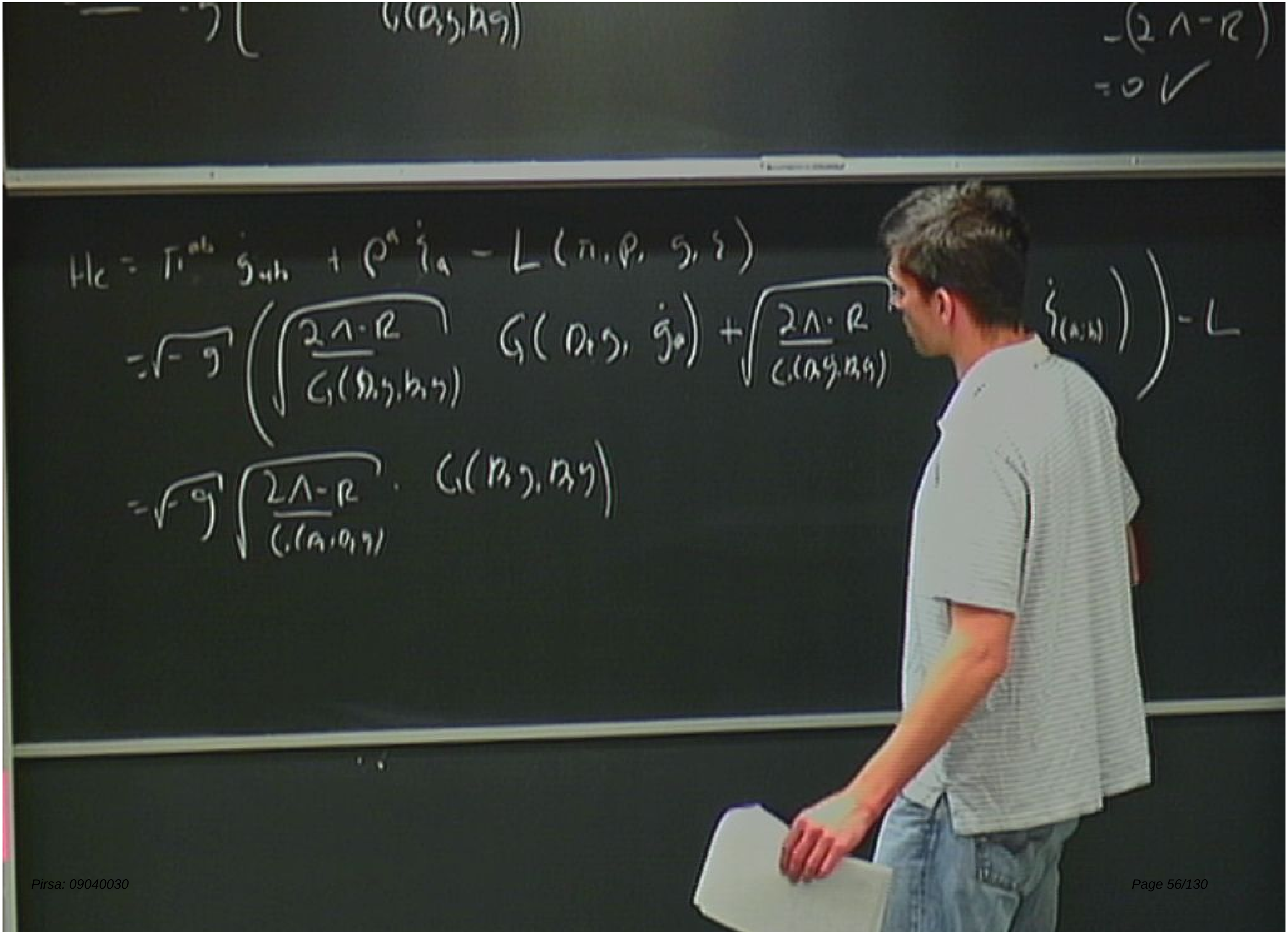
$$\ldots \; G(D_3 g, D_3 g)) \qquad -(2\Lambda - R) = 0 \checkmark$$

$$H_c = \pi^{ab}\dot{g}_{ab} + p^a \dot{\xi}_a - L(\pi, p, g, \xi)$$

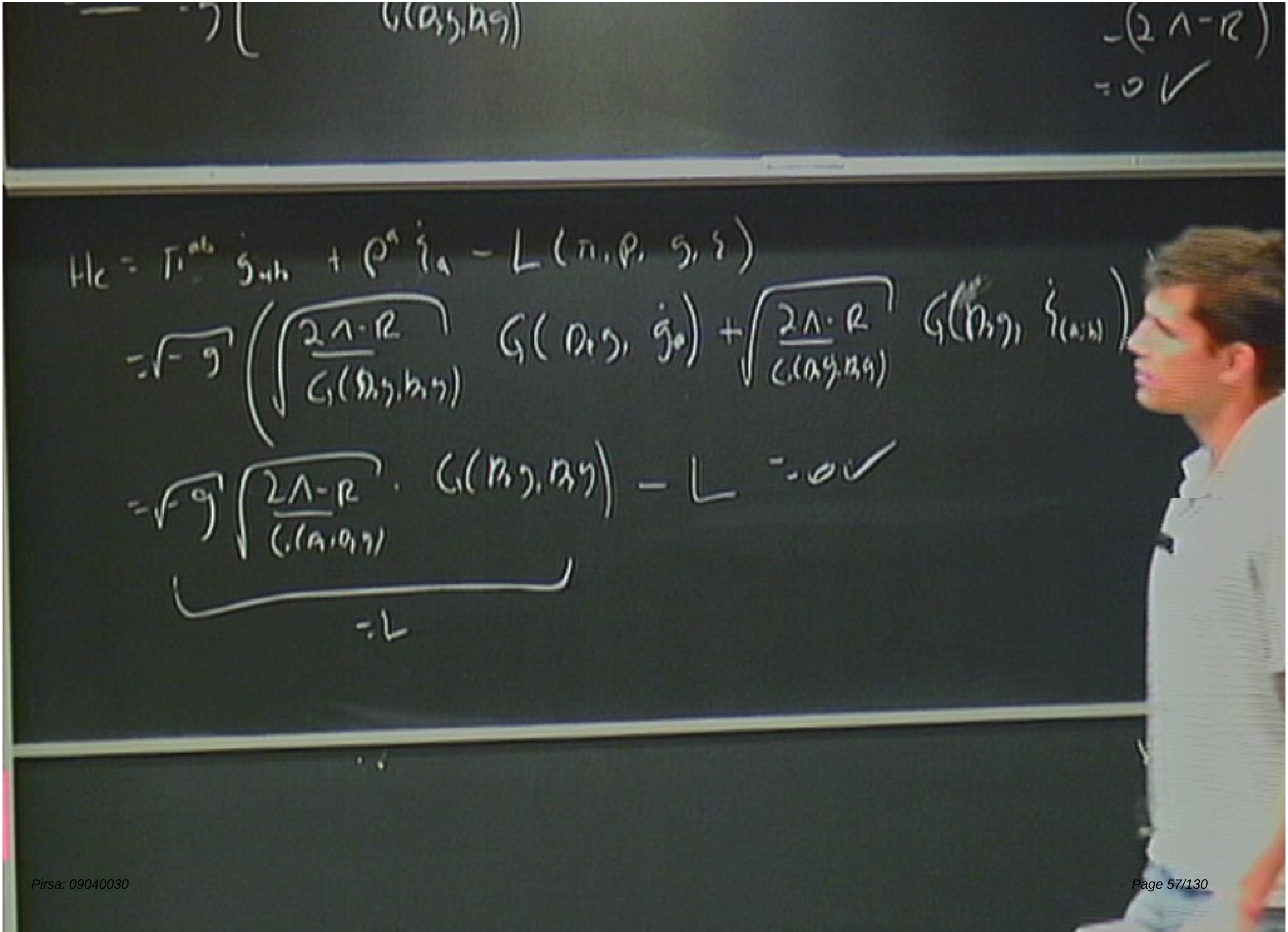
$$= \sqrt{-g}\left(\sqrt{\frac{2\Lambda - R}{G(D_3 g, D_3 g)}}\; G(D_3 g, \dot{g}_{ab}) + \right.$$


Hc = π^ab ġ_ab + p^a q̇_a − L(π, p, g, q)


$-(2\Lambda - R)$
$= 0$ ✓
$H_c = \pi^{ab}\dot{g}_{ab} + p^a \dot{q}_a - L(\pi, p, g, q)$



G(D₃g, D₃g)
−(2Λ − R)
= 0 ✓
Hc = π^ab ġ_ab + ρ^a ξ̇_a − L(π, ρ, g, ξ)
= √−g (√(2Λ · R / G(D₃g, D₃g)) G(D₃g, ġ_a) + √(2Λ · R / G(D₃g, D₃g)) ξ̇(a;b))) − L
= √−g √(2Λ − R / G(D₃g, D₃g)) · G(D₃g, D₃g)

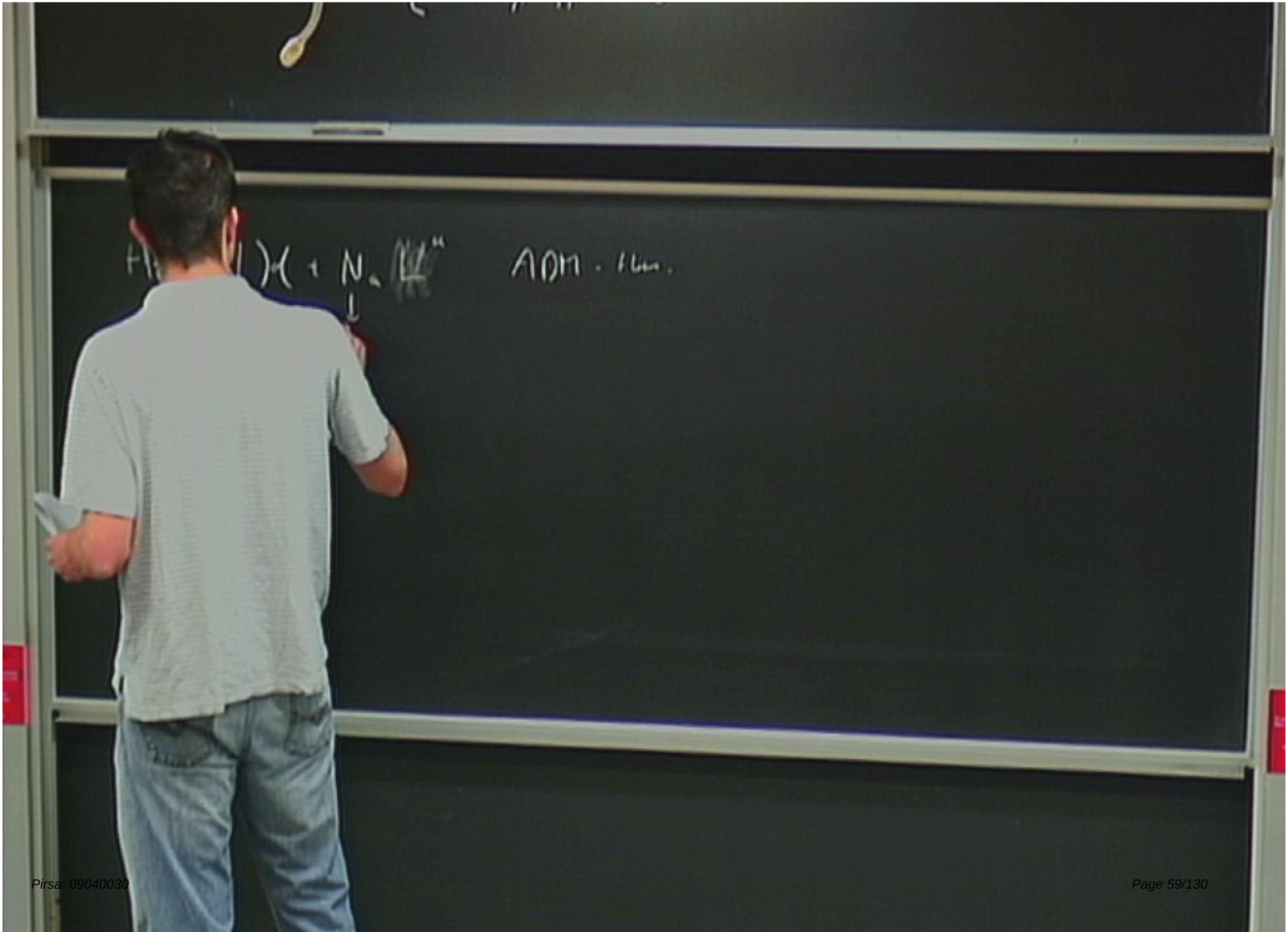

-(2Λ-R)
=0 ✓
Hc = π^ab ġ_ab + p^a ξ̇_a - L(π, p, g, ξ)
= √-g (√(2Λ-R / G(Dg, Dg)) G(Dg, ġ_ab) + √(2Λ-R / G(Dg, Dg)) G(Dg, ξ̇_(a;b)))
= √-g √(2Λ-R / G(Dg, Dg)) · G(Dg, Dg) - L = 0 ✓
= L

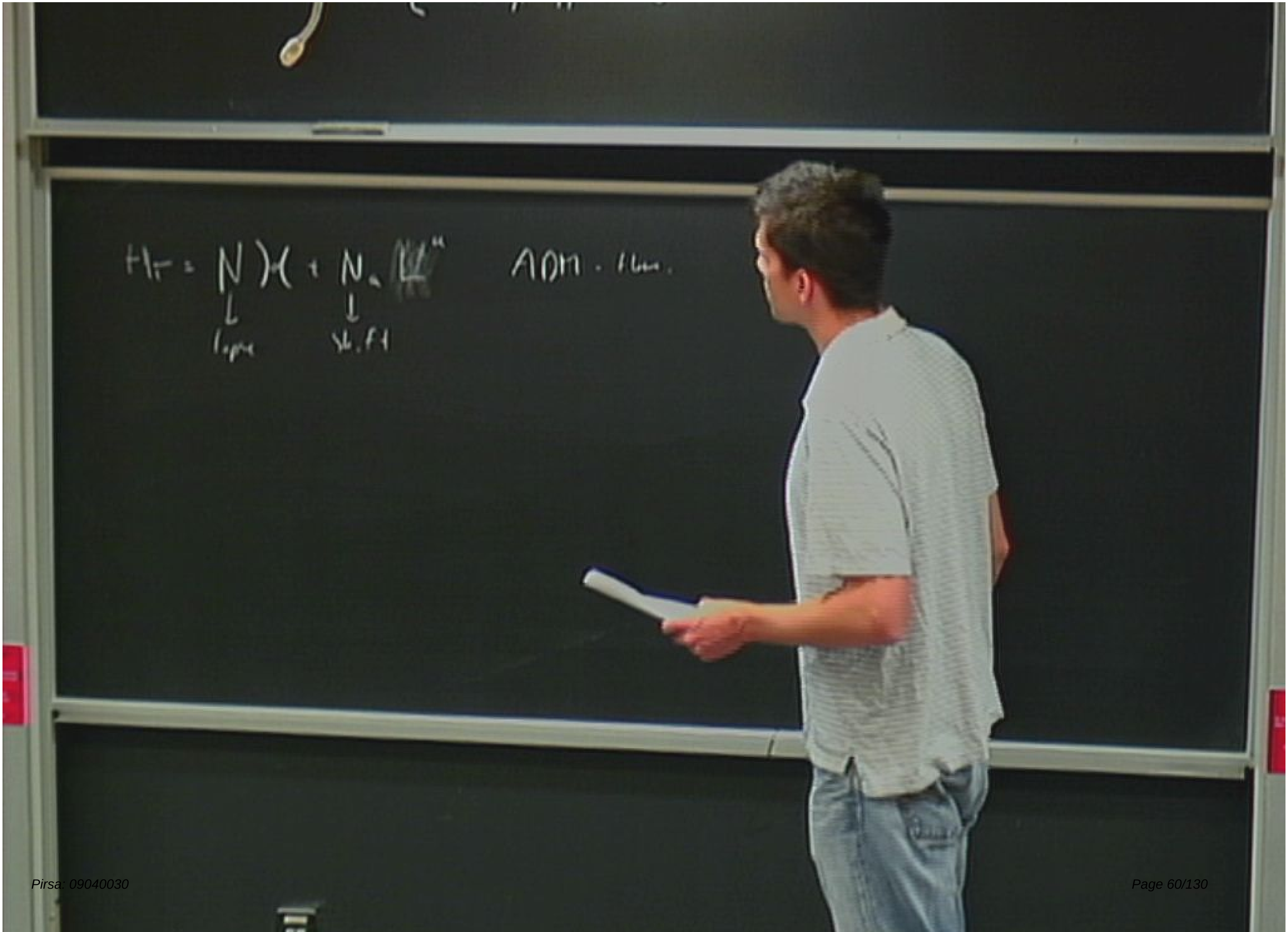
$$\ldots \; \sqrt{-g}\left[\quad \frac{\ldots}{G(\partial_a g, \partial_a g)} \right] \qquad -(2\Lambda - R) = 0 \checkmark$$

$$H_c = \pi^{ab}\dot{g}_{ab} + \rho^a \dot{\xi}_a - L(\pi, \rho, g, \xi)$$

$$= \sqrt{-g}\left(\sqrt{\frac{2\Lambda - R}{G(\partial_a g, \partial_a g)}}\; G(\partial_a g, \dot{g}_a) + \sqrt{\frac{2\Lambda - R}{G(\partial_a g, \partial_a g)}}\; G(\partial_a g, \dot{\xi}_{(a;b)})\right) - L$$

$$= \underbrace{\sqrt{-g}\sqrt{\frac{2\Lambda - R}{G(\partial_a g, \partial_a g)}} \cdot G(\partial_a g, \partial_a g)}_{=L} - L = 0 \checkmark$$




ADM

$$H_T = N\,\mathcal{H} + N_a\,\mathcal{H}^a \qquad \text{ADM - form.}$$

$N \downarrow$ lapse, $N_a \downarrow$ shift

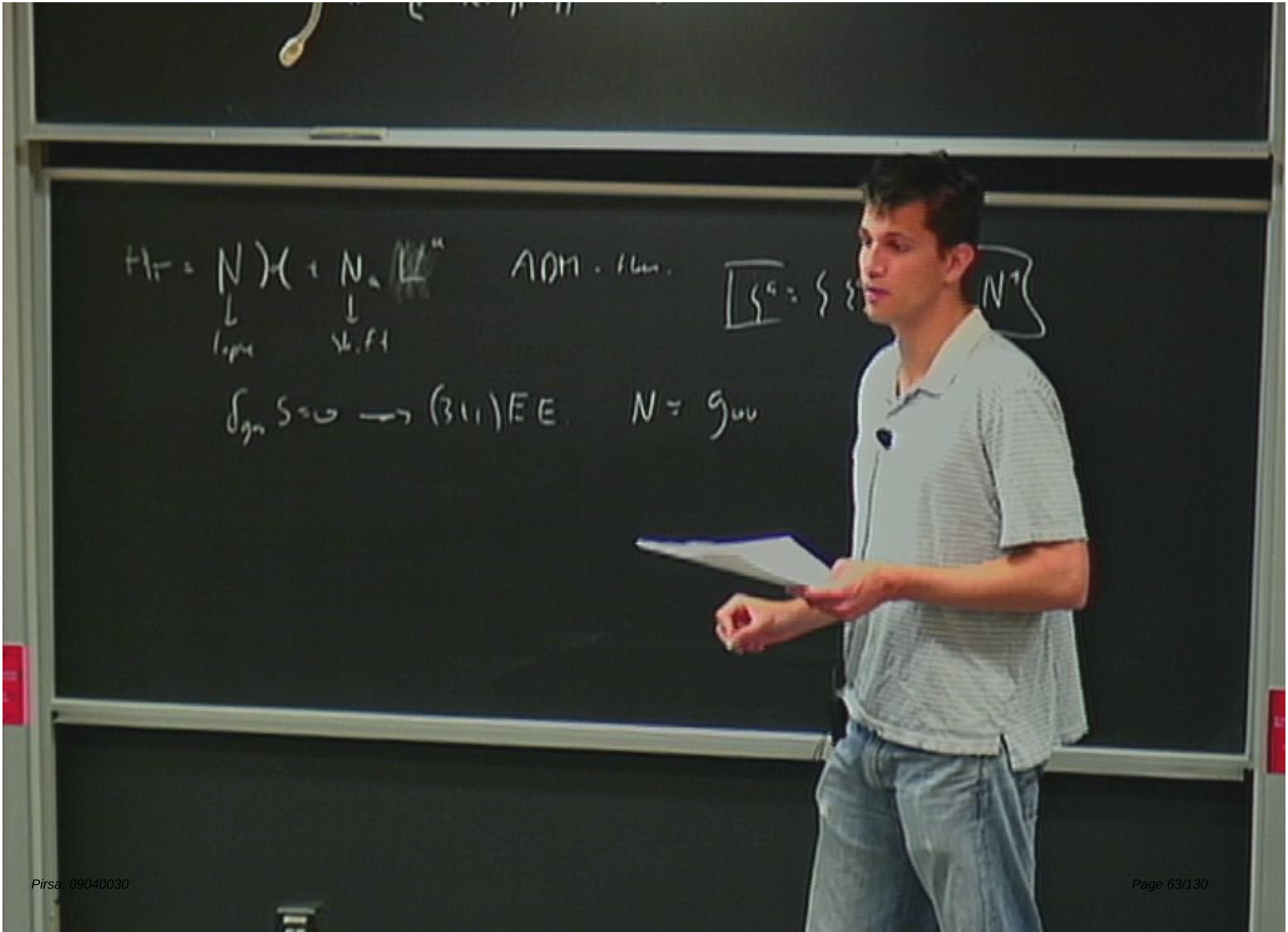
$$\dot{\xi}^a = \{\xi^a, H_T\} =$$

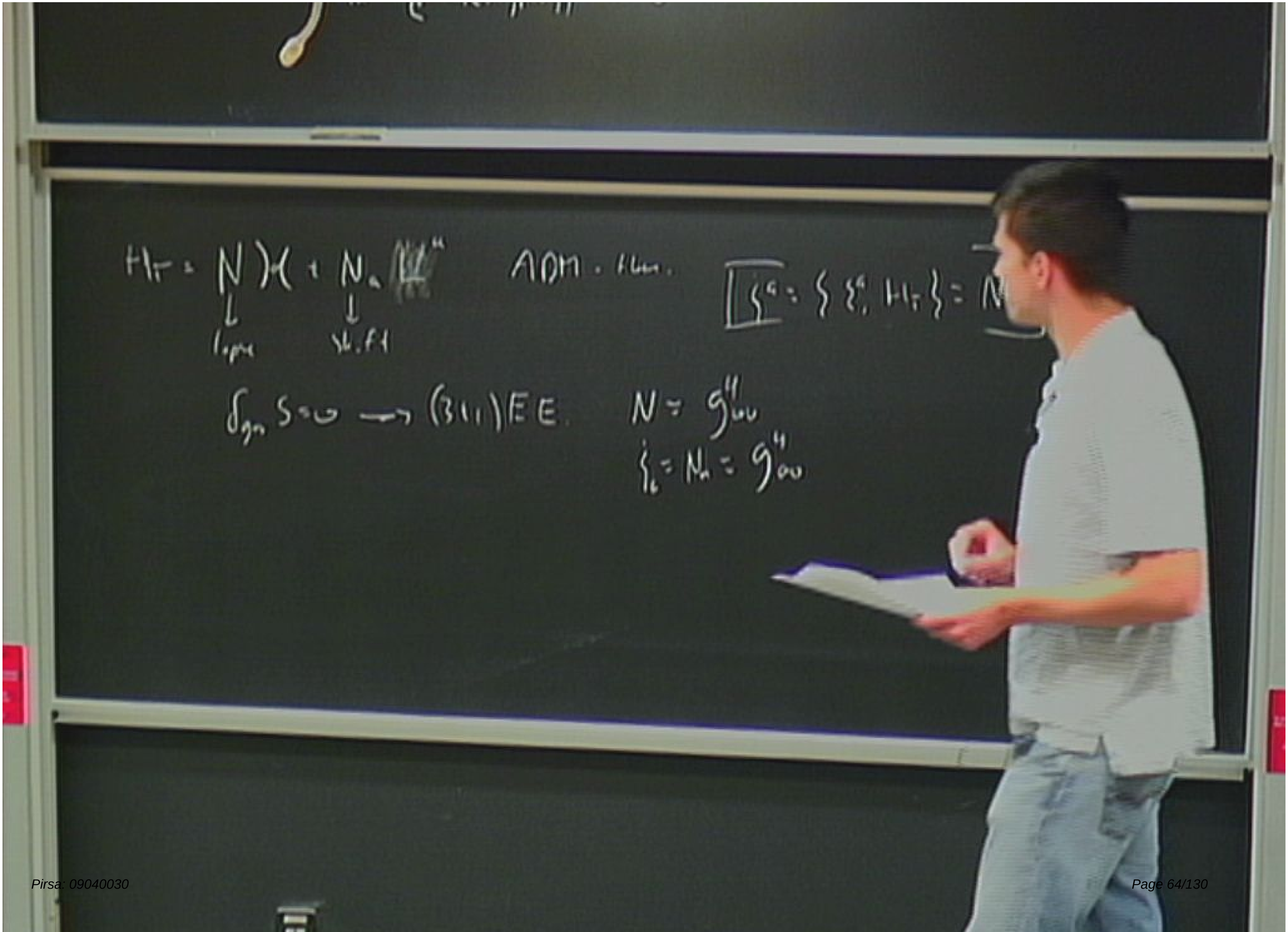
$$H_T = N\mathcal{H} + N_a \mathcal{H}^a$$

ADM. Ham.

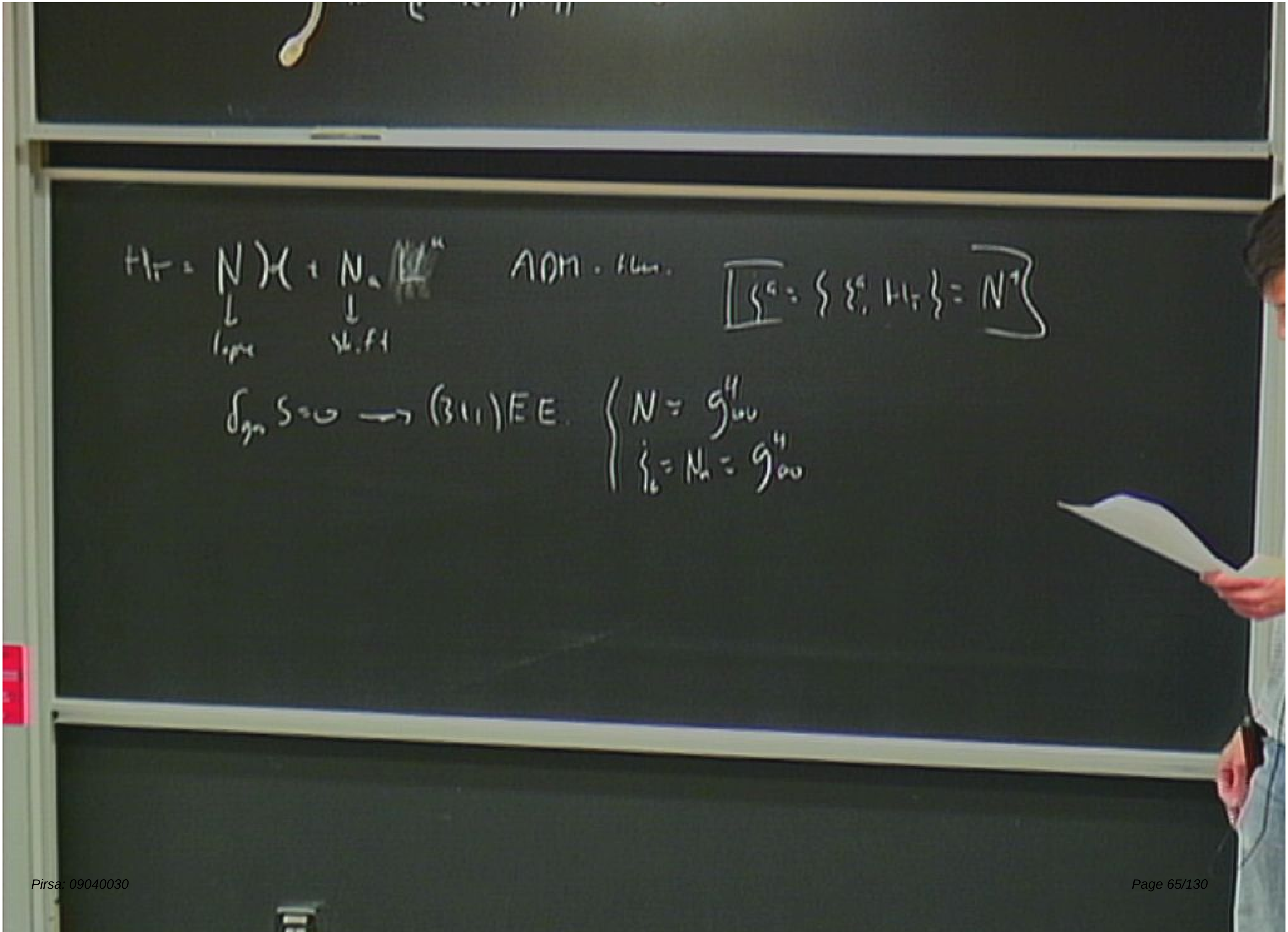
N → lapse, N_a → shift

$$\dot{\xi}^a = \{\xi^a, H_T\} = N^a$$


ADM . Ham.
(3+1) E E

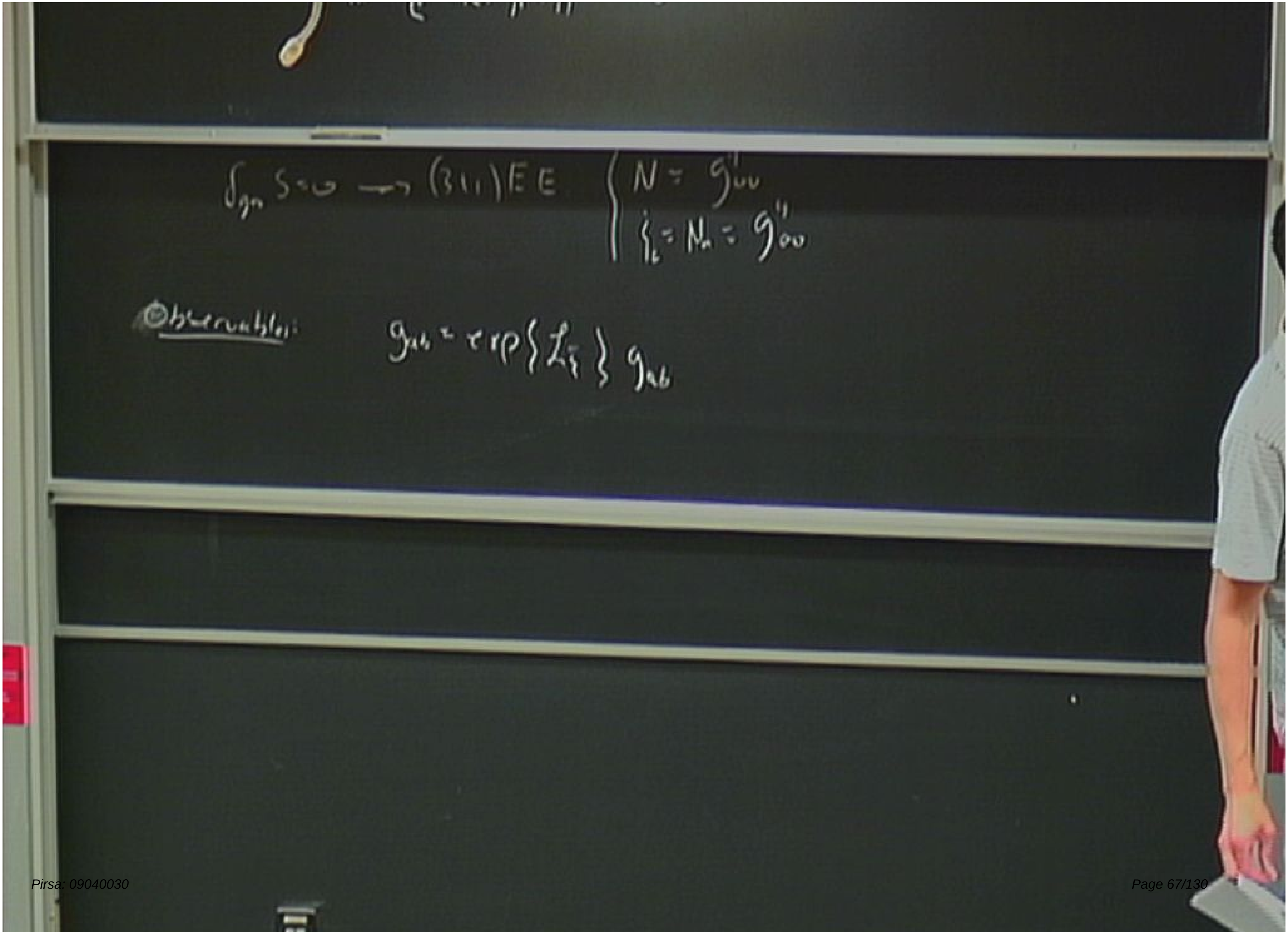


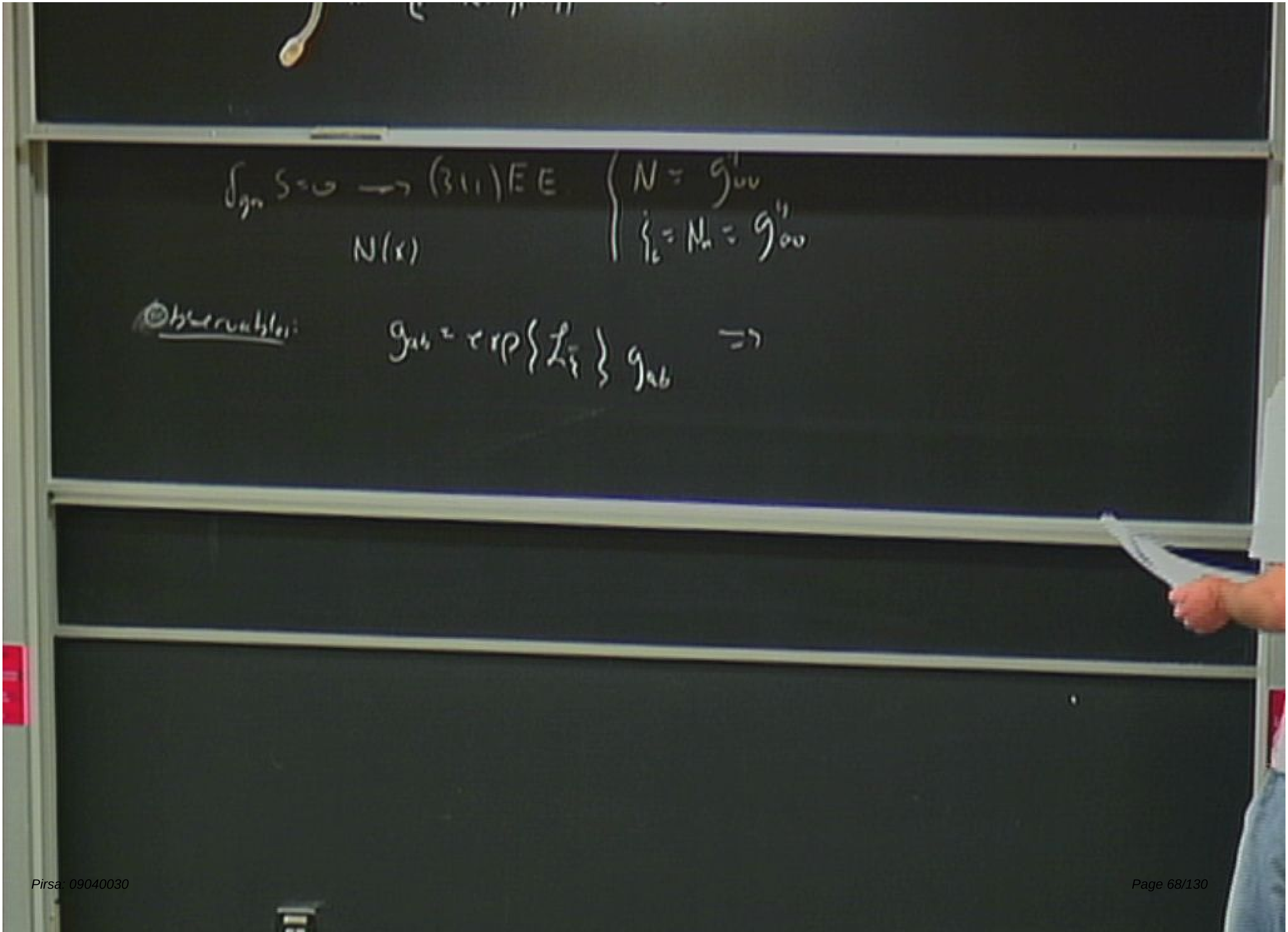
H_T = N ℋ + N_a ℋ^a
ADM . Ham.
lapse
sh. ft
δ_gμν S = 0 → (3+1) E E
N = g^00
ξ_i = N_a = g^0a


H_T = N H + N_a H^a
ADM. Ham.
lapse
shift
ξ^a = {ξ^a, H_T} = N^a
δ_g S = 0 → (3+1) E E
N = g^4_00
ξ_i = N_a = g^4_0a

$\delta_{g_{\mu\nu}} S = 0 \longrightarrow (3,1)\,EE \quad \begin{cases} N = g^{-1}_{00} \\ \dot{\xi}_0 = N_a = g^{-1}_{a0} \end{cases}$

Observables:

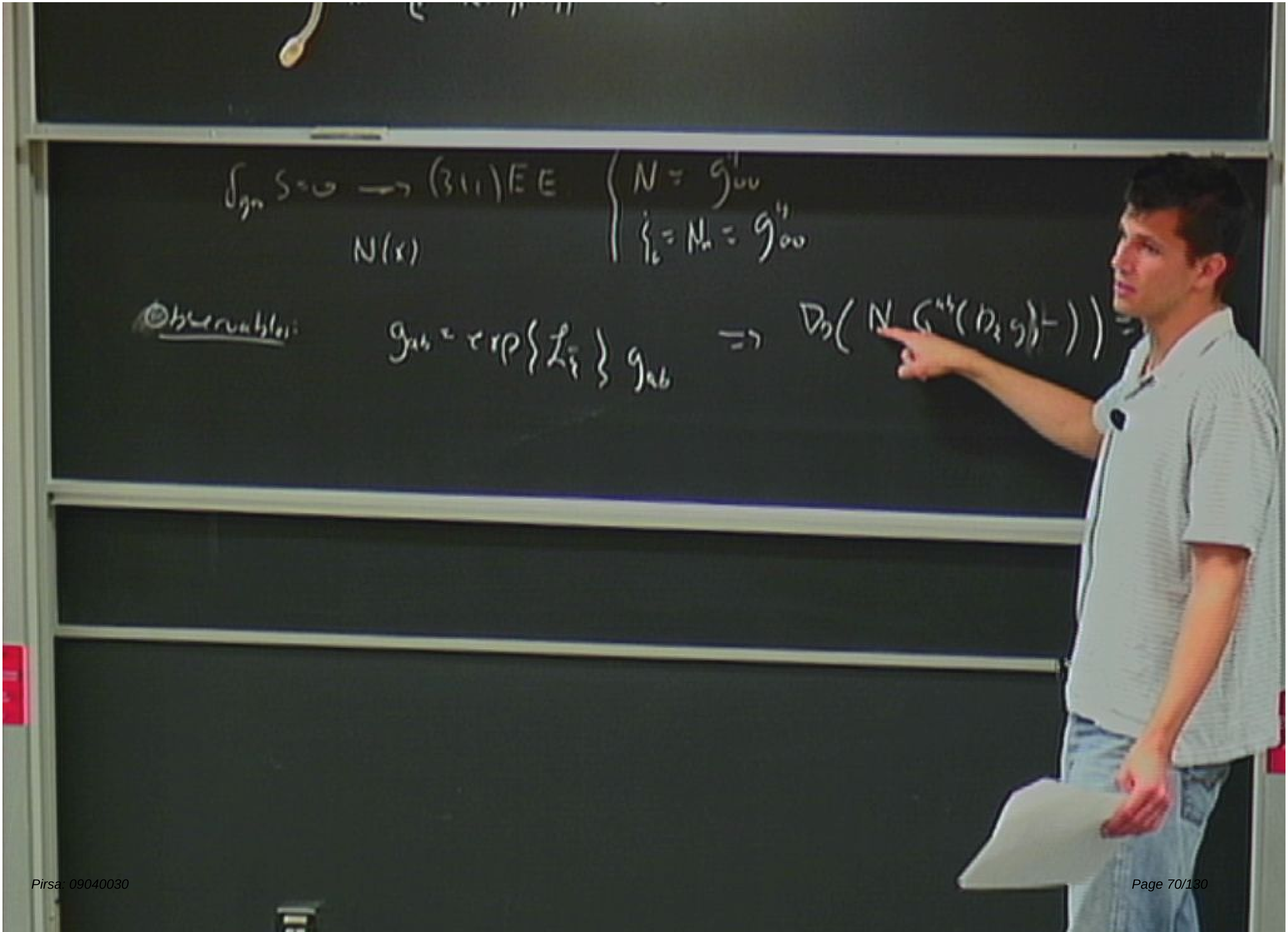

δ_gm S = 0 ⟶ (3+1) EE
N = g'_00
ξ_t = N_n = g''_00
Observables:
g_ab = exp{ L_ξ } g_ab


δ_g S = 0 → (3+1) EE
N = g'_00
N(x)
Observables:
g_ab = exp{L_ξ} g_ab ⇒

$$\delta_{g_{ab}} S = 0 \longrightarrow (3+1) EE \quad \begin{cases} N = g^{tt}_{00} \\ \xi_b = N_a = g^{ti}_{00} \end{cases}$$

$N(x)$

Observables: $g_{ab} = \exp\{\mathcal{L}_\xi\}\, g_{ab} \Rightarrow D_b\left(N\, G^{ab}(D_i g)\ldots\right) = 0$


N(x)
Observables:

$$\delta_{g_{ab}} S = 0 \longrightarrow (3+1)\,EE \qquad \begin{cases} N = g^{\prime}_{00} \\ \dot{\xi}_b = N_a = g^{\prime}_{00} \end{cases}$$

$N(x)$

Observables: $\quad g_{ab} = \exp\{\mathcal{L}_{\xi}\}\, g_{ab} \implies D_b\left(N\, G^{ab}(D_a g) \ldots\right) = 0$

$\delta_{g_{ab}} S = 0 \longrightarrow (3+1)\,EE$

$N(\vec{x})$

$$\begin{cases} N = g^{4}_{00} \\ \xi_b = N_b = g^{4}_{0b} \end{cases}$$

reflection inv.

$\Uparrow$

Observables: $g_{ab} = \exp\{\mathcal{L}_{\xi}\}\, g_{ab} \implies D_b\left(N\, G^{ab}(D_c g)\ldots)\right) = 0$

$\delta_{g_{ab}} S = 0 \longrightarrow (3+1)\,EE$ $\begin{cases} N = g^{44}_{00} \\ \xi_b = N_a = g^{4}_{a0} \end{cases}$

$N(\vec{x})$

reflection inv.

$\Uparrow$

Observables: $g_{ab} = \exp\{\mathcal{L}_{\vec{\xi}}\}\, g_{ab} \Rightarrow D_b\left(N\, G^{ab}(D_k g)\ldots\right) = 0$

lapse shift

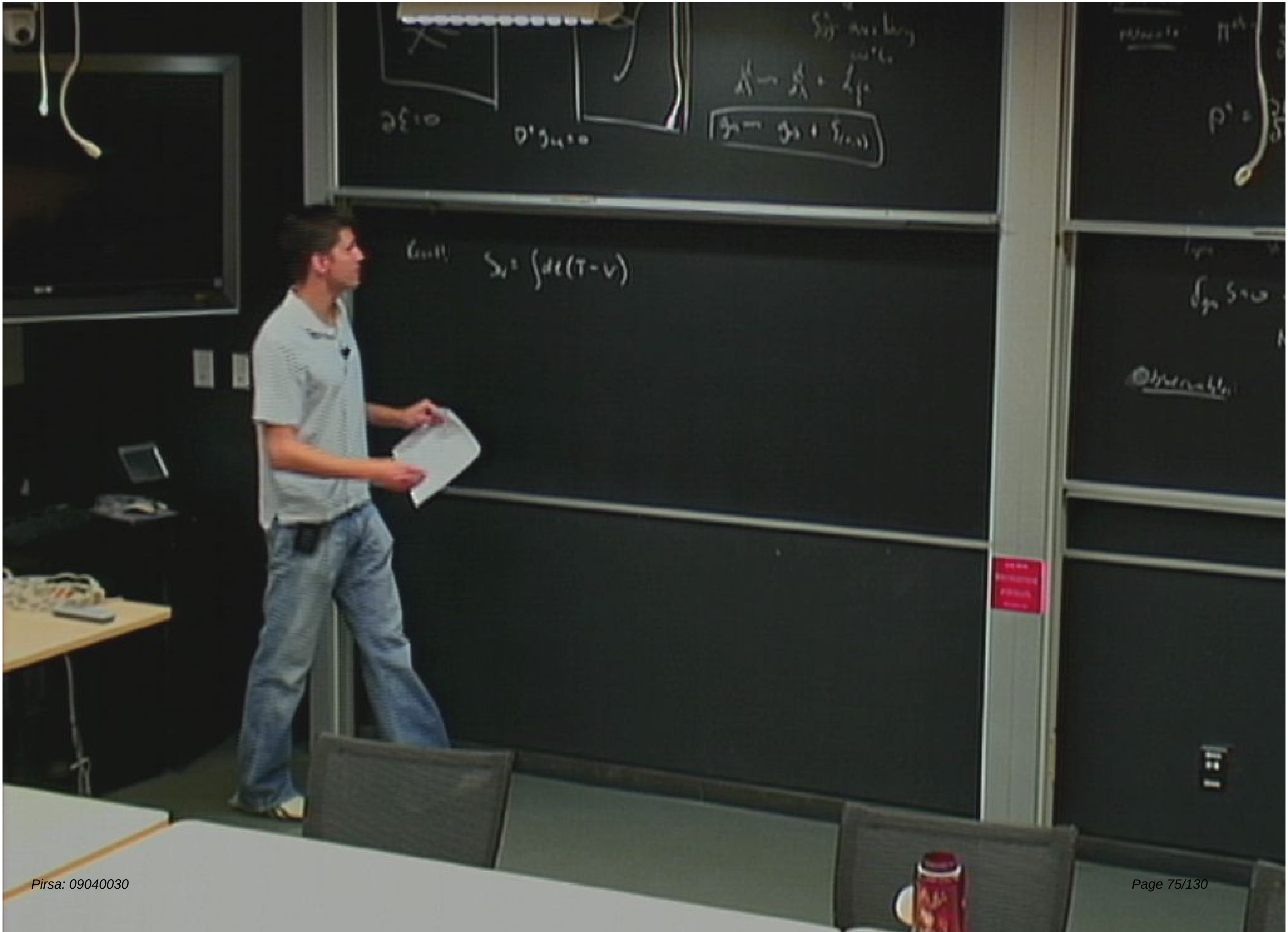
$$\delta_{g_{\mu\nu}} S = 0 \longrightarrow (3+1)\,EE \qquad \begin{cases} N = g^{4}_{00} \\ \xi_b = N_a = g^{4}_{0a} \end{cases}$$

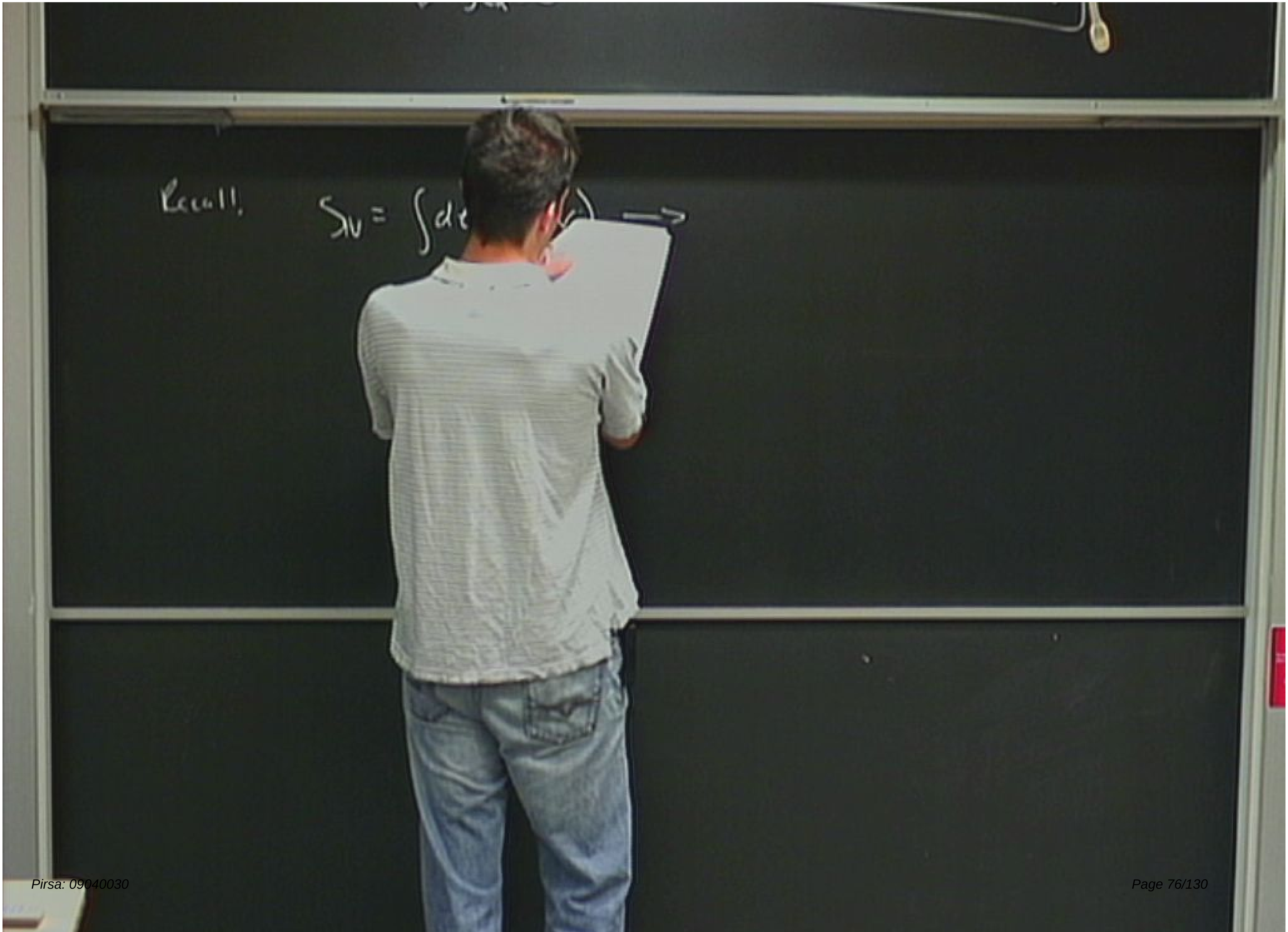
$N(\vec{x})$

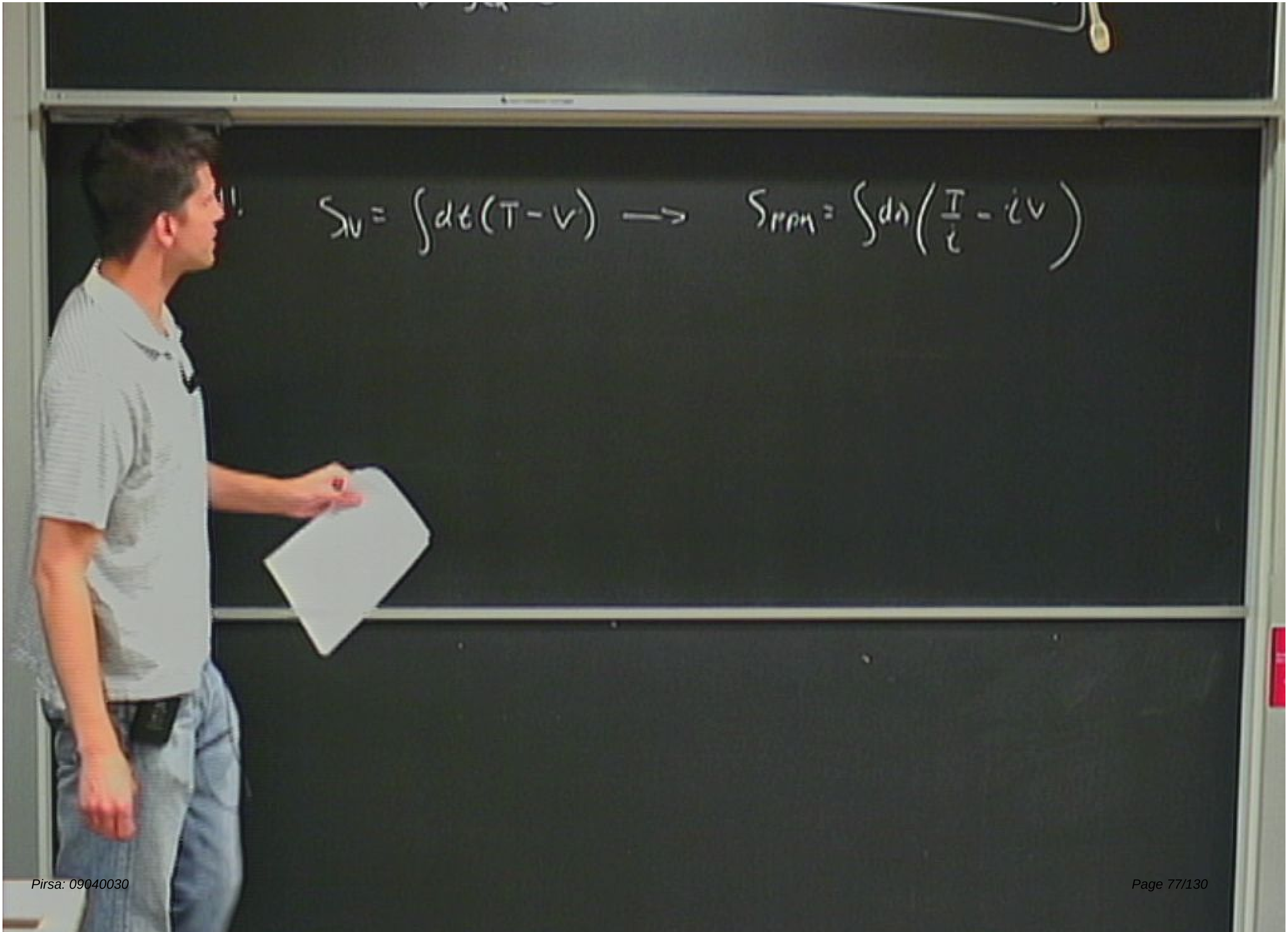
refoliation inv.

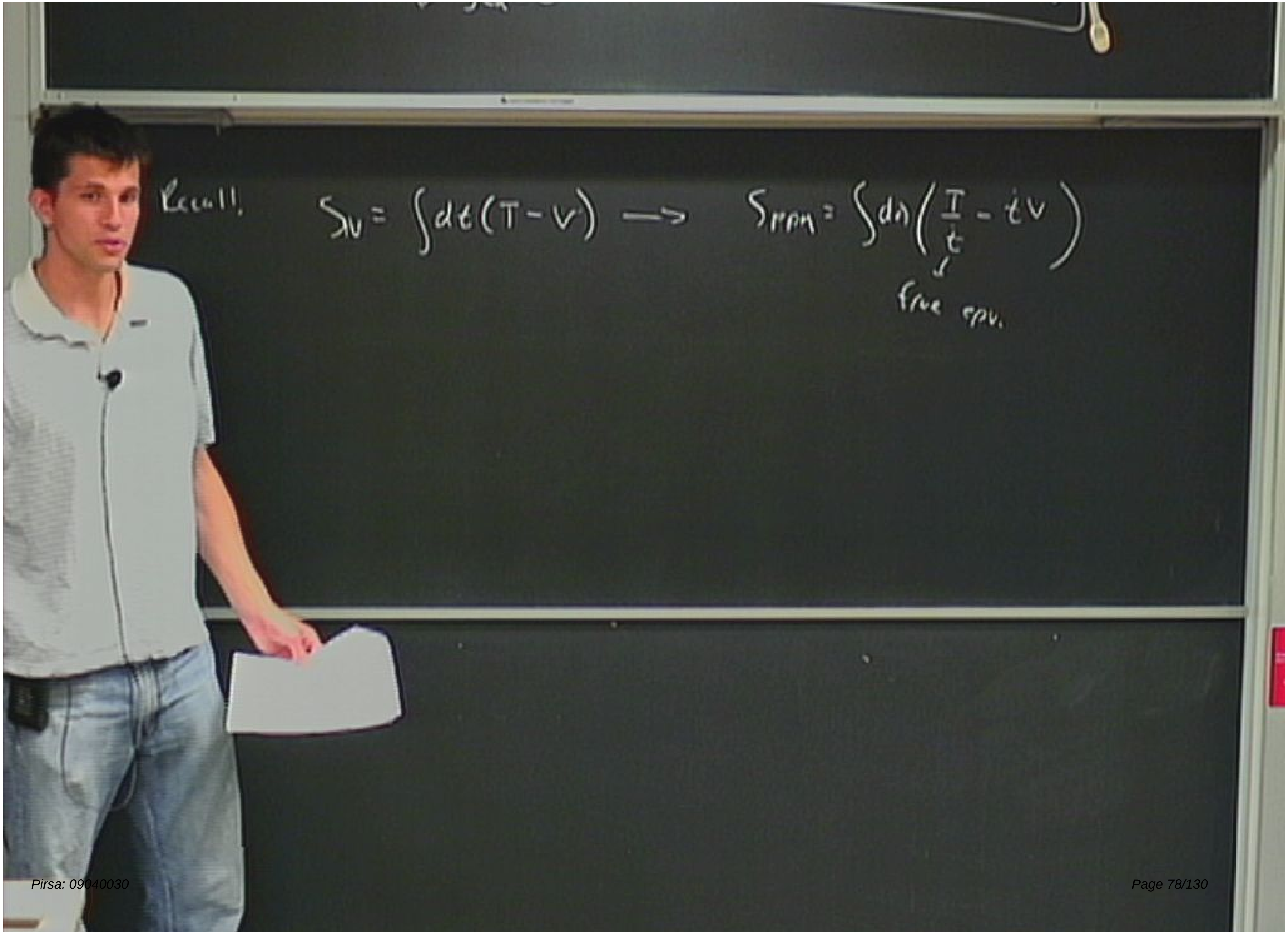
$\Uparrow$

Observables: $\quad g_{ab} = \exp\{\mathcal{L}_{\vec{\xi}}\}\, g_{ab} \implies D_b\left(N\, G^{ab}(D_c g)^{-}\right) = 0$

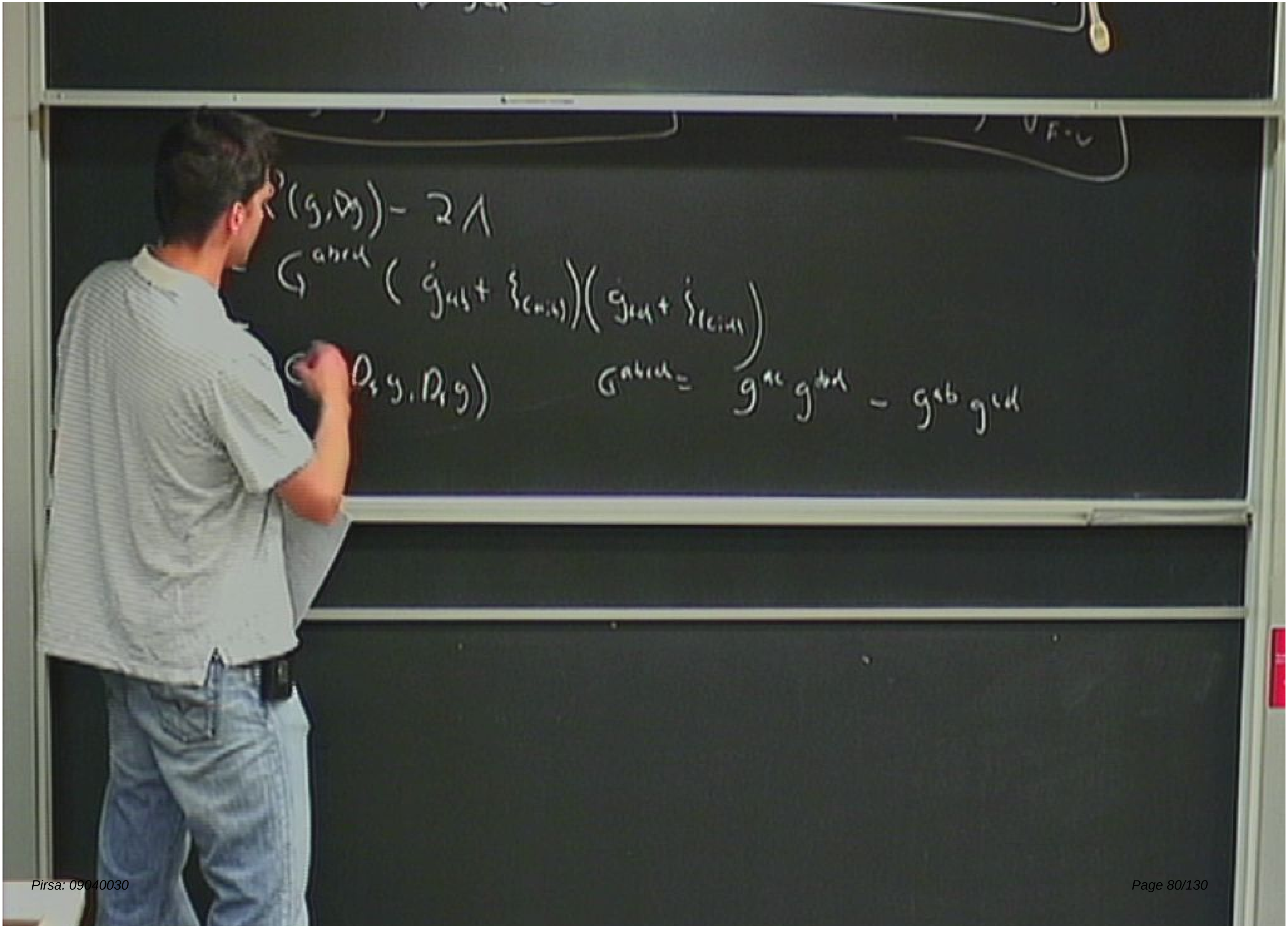


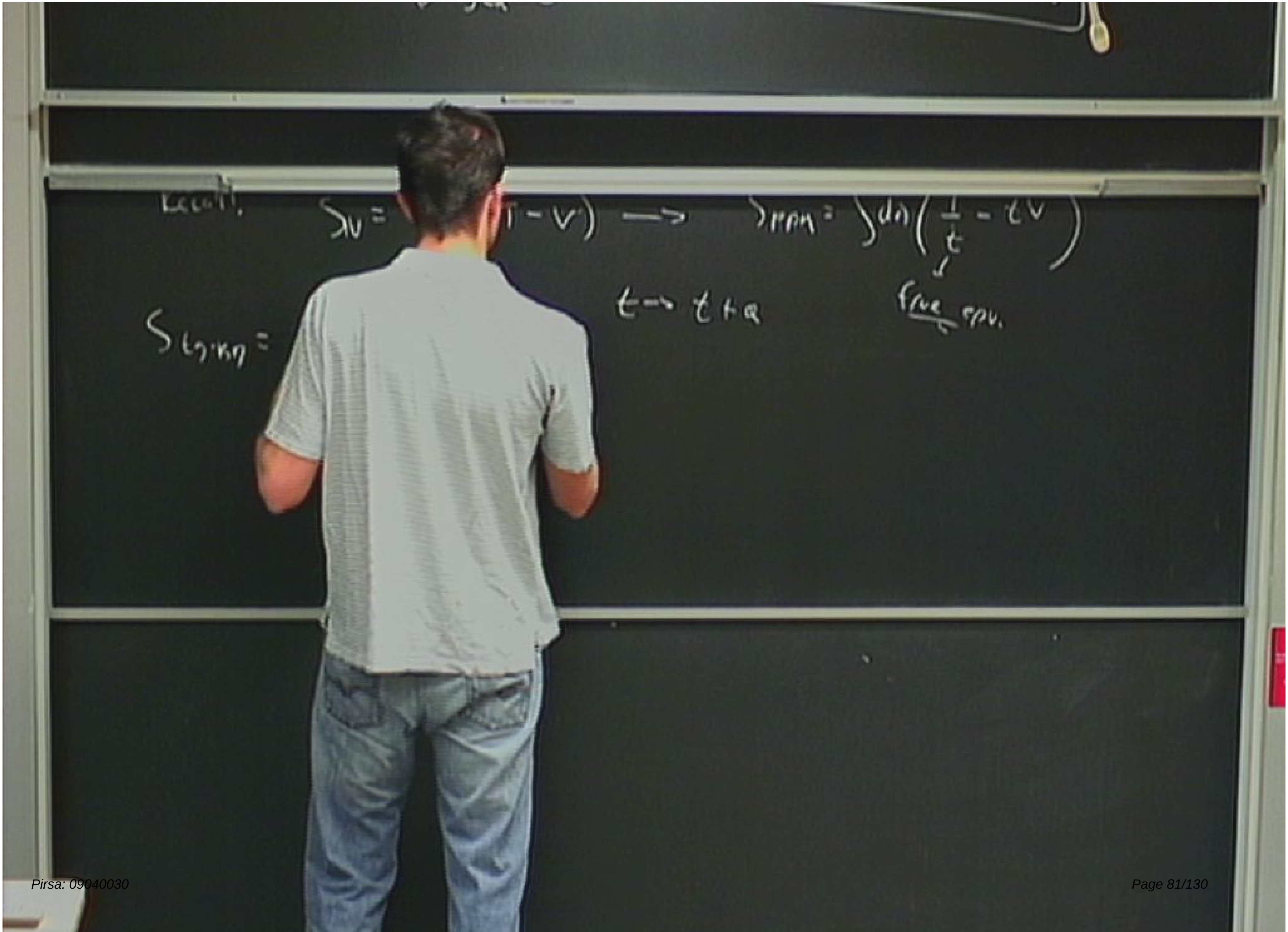

Recall!

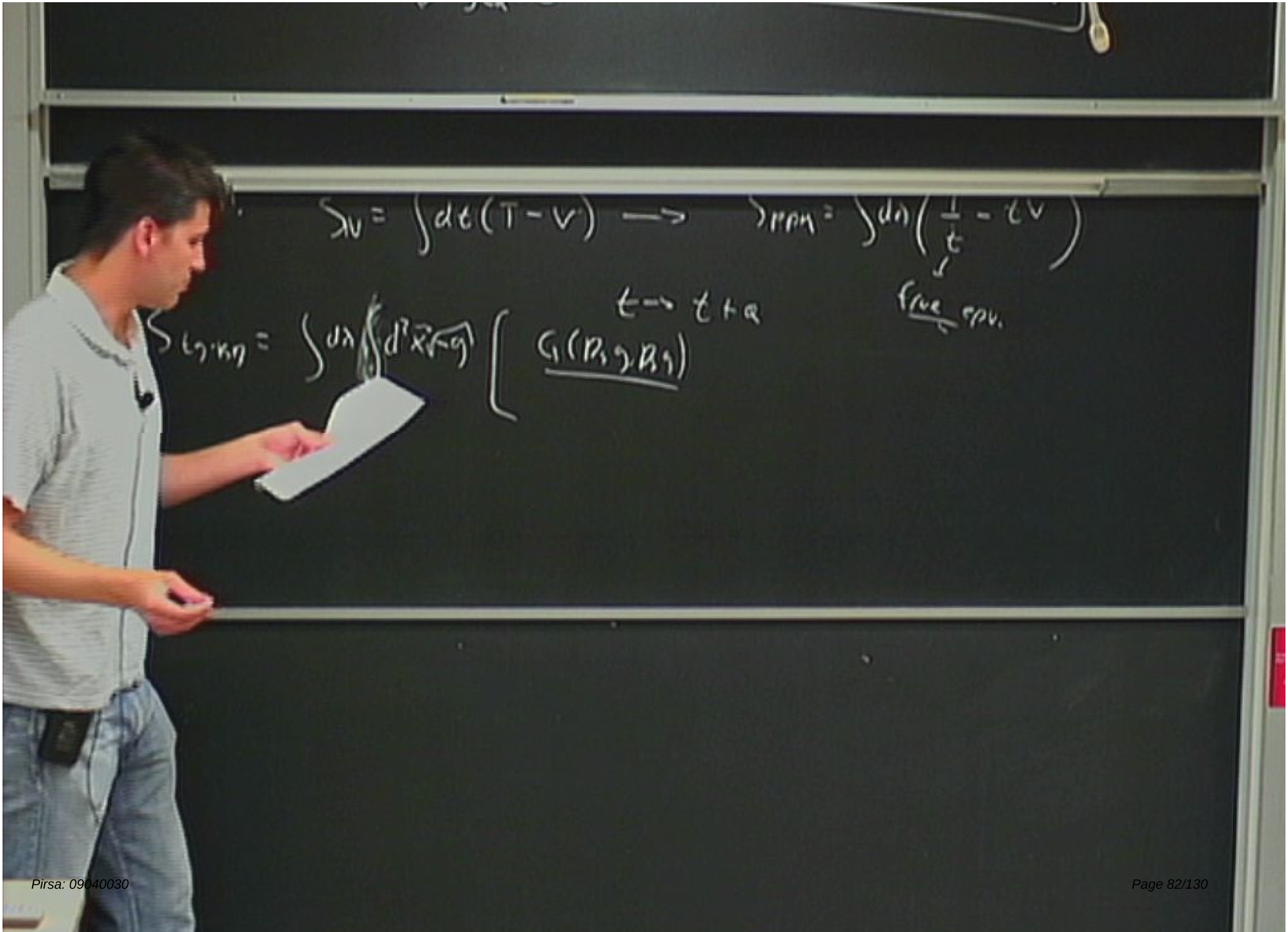

S_W = ∫dt (T − V) ⟶ S_rpm = ∫dn (T/i − iV)

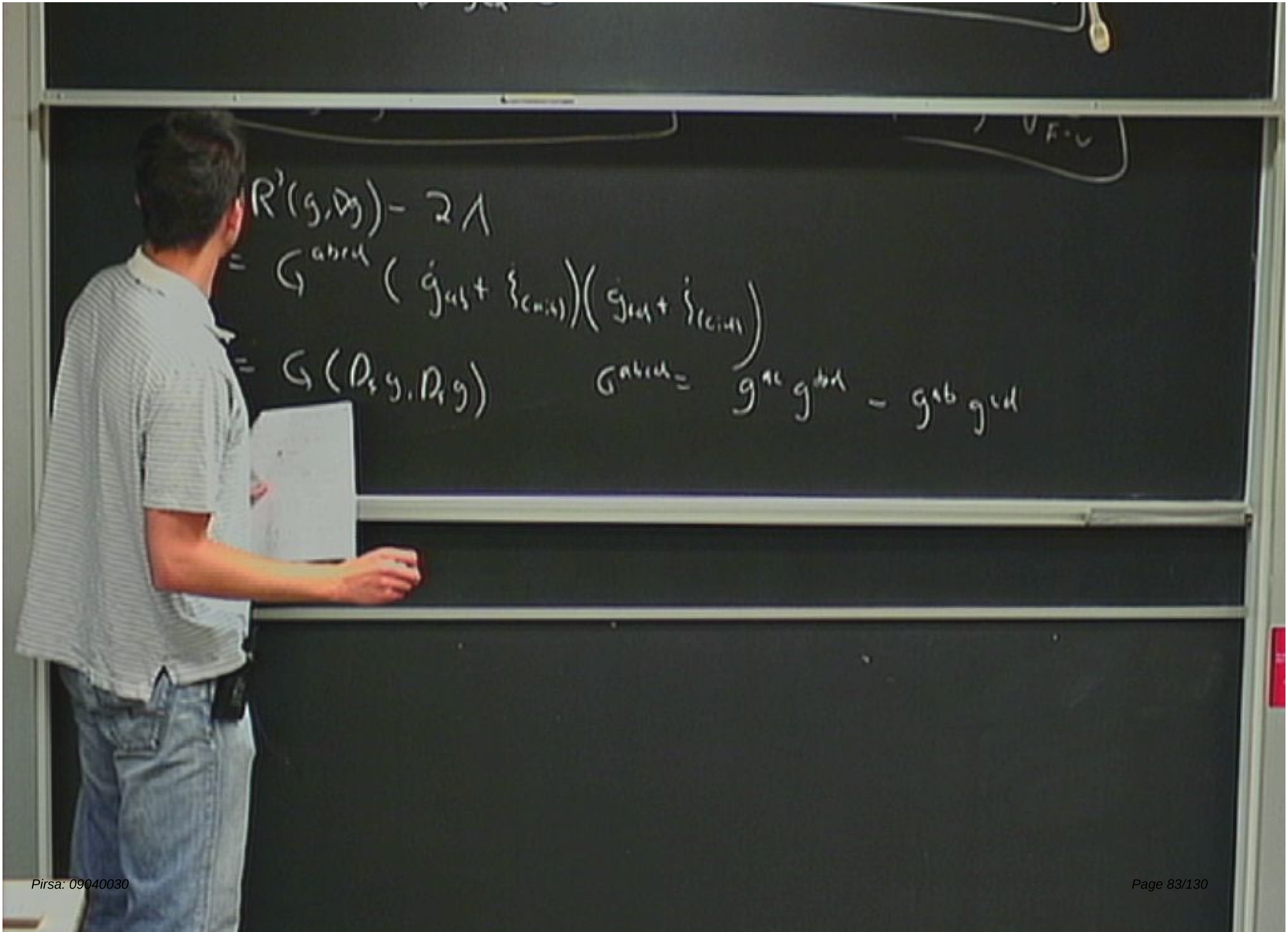

Recall!
$S_{NV} = \int dt \, (T - V) \longrightarrow S_{rpm} = \int d\lambda \left(\frac{T}{\dot{t}} - \dot{t} V \right)$
free eqv.

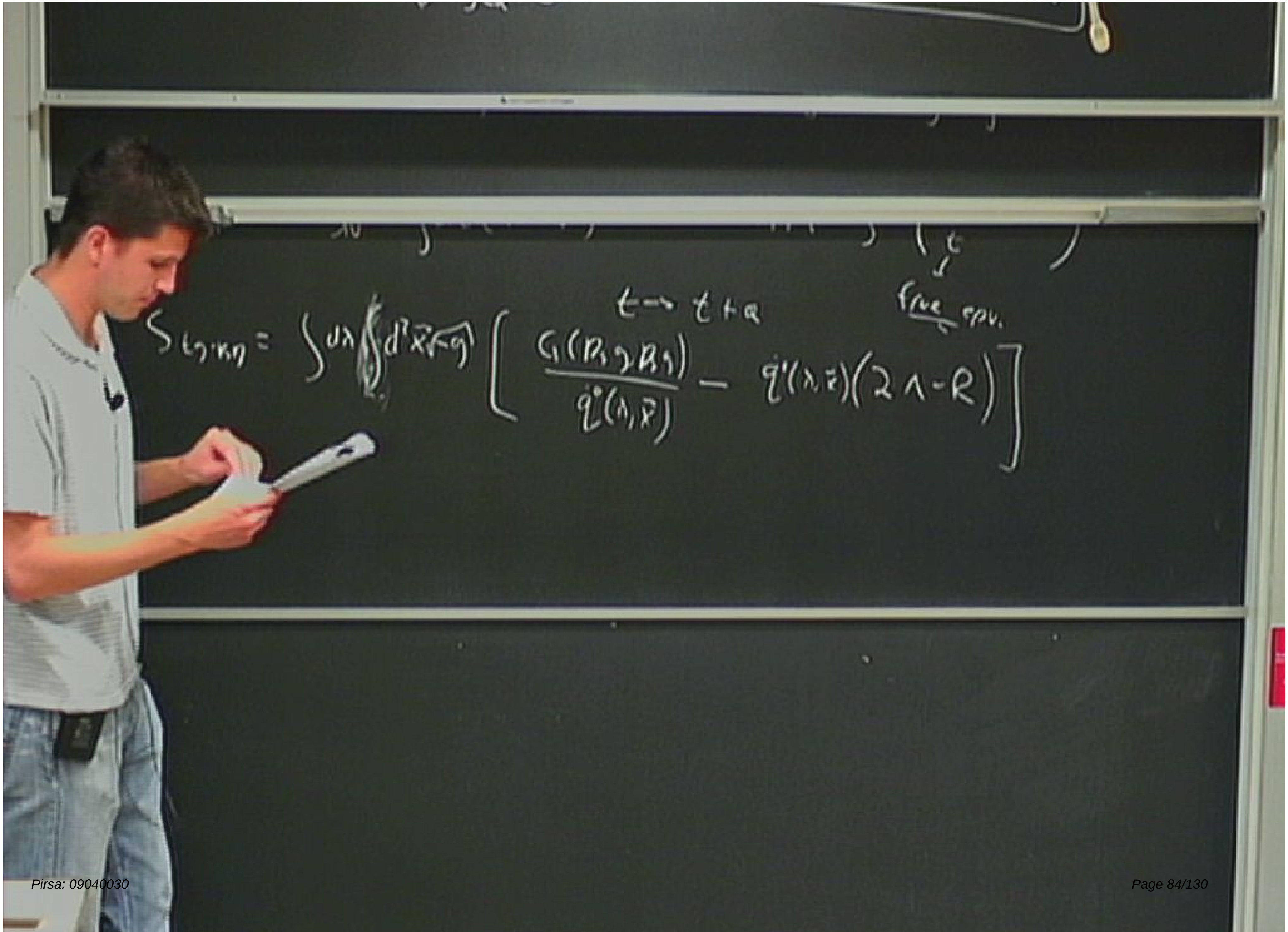
$S_{NV} = \int dt (T - V) \longrightarrow S_{ppm} = \int d\lambda \left(\frac{T}{\dot{t}} - \dot{t} V \right)$
$t \longrightarrow t + a$
free eqn.

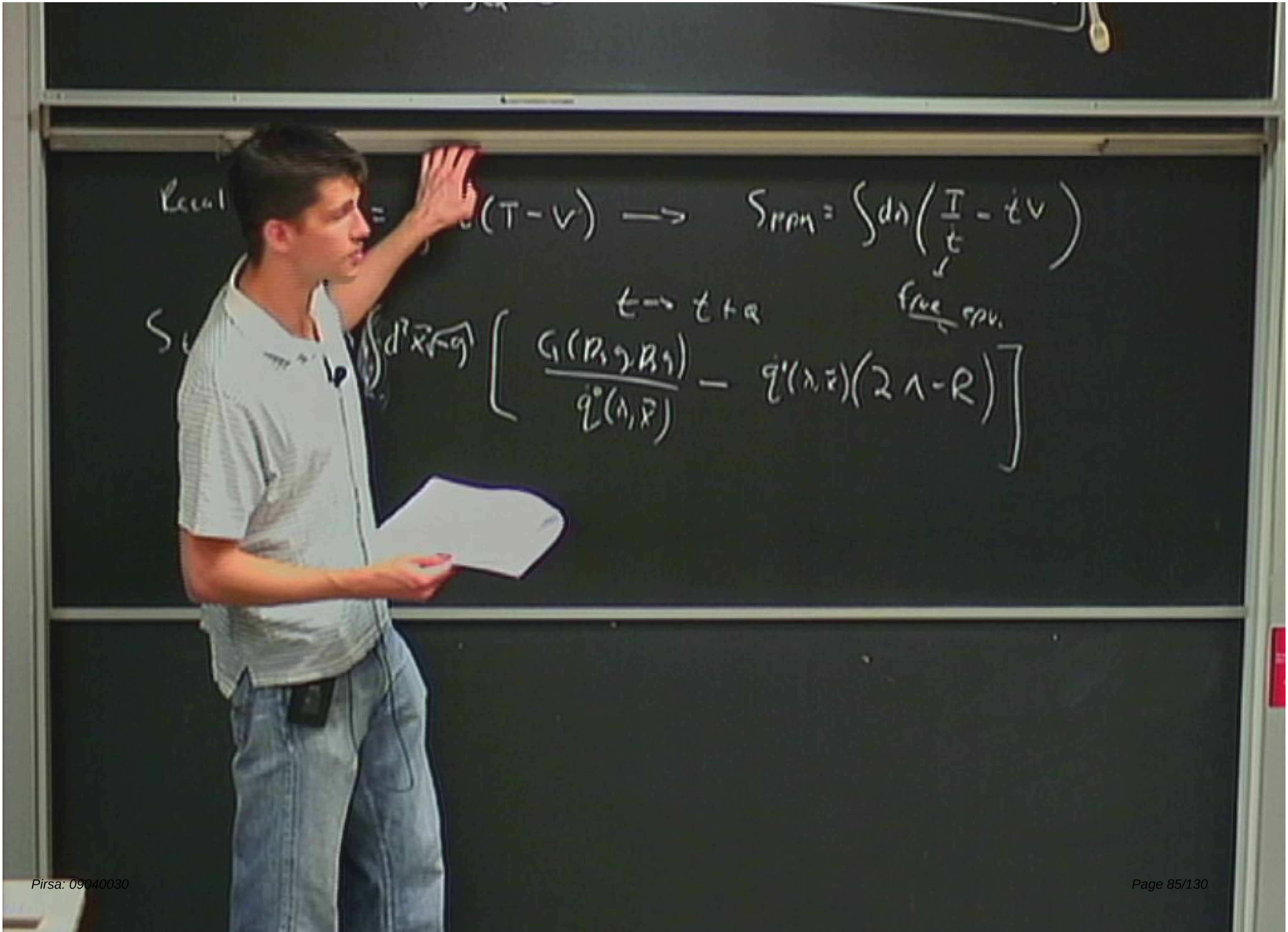

(g, Dg) − 2Λ
G^abcd = g^ac g^bd − g^ab g^cd

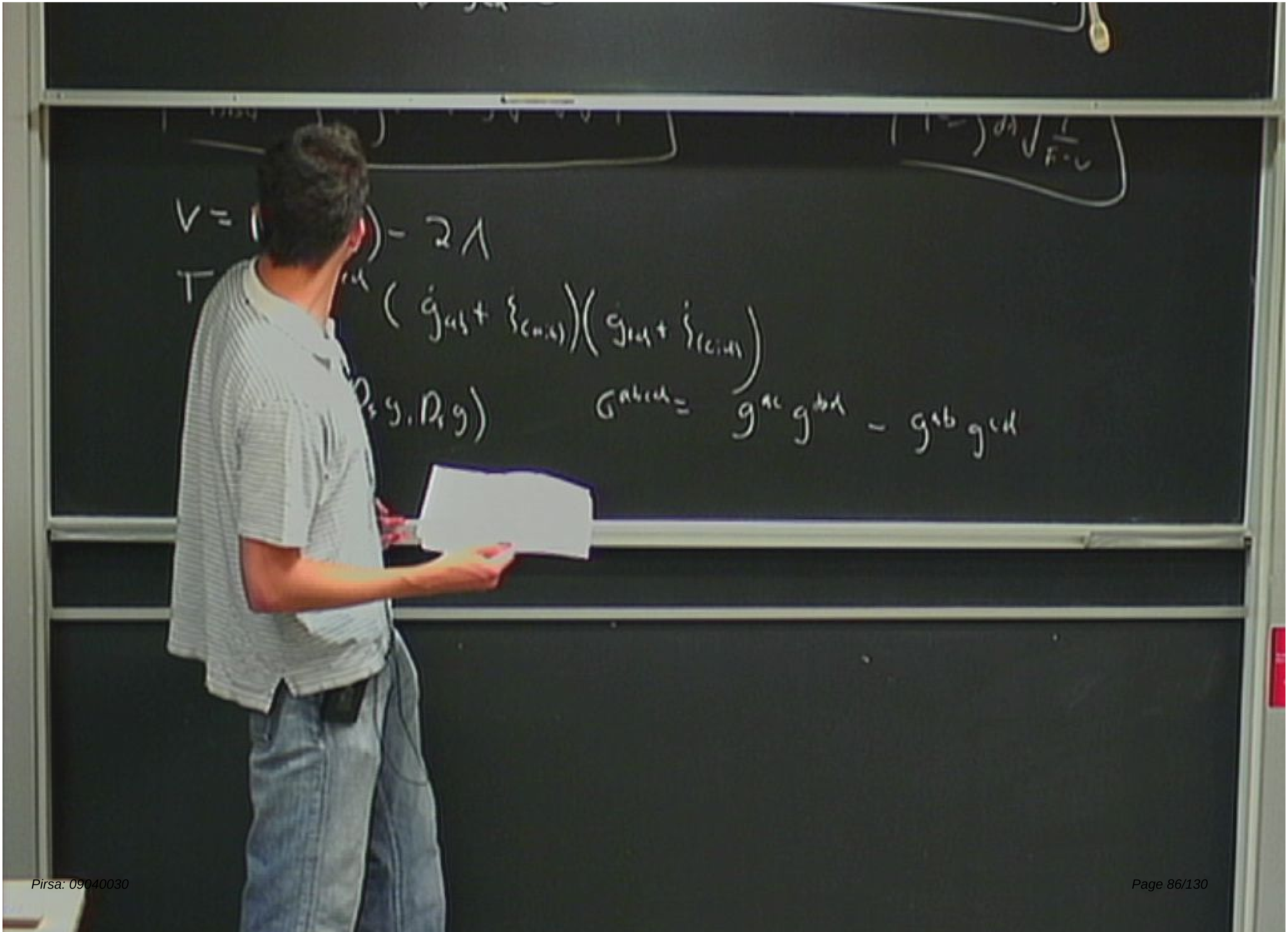

t → t + a
free eqn.

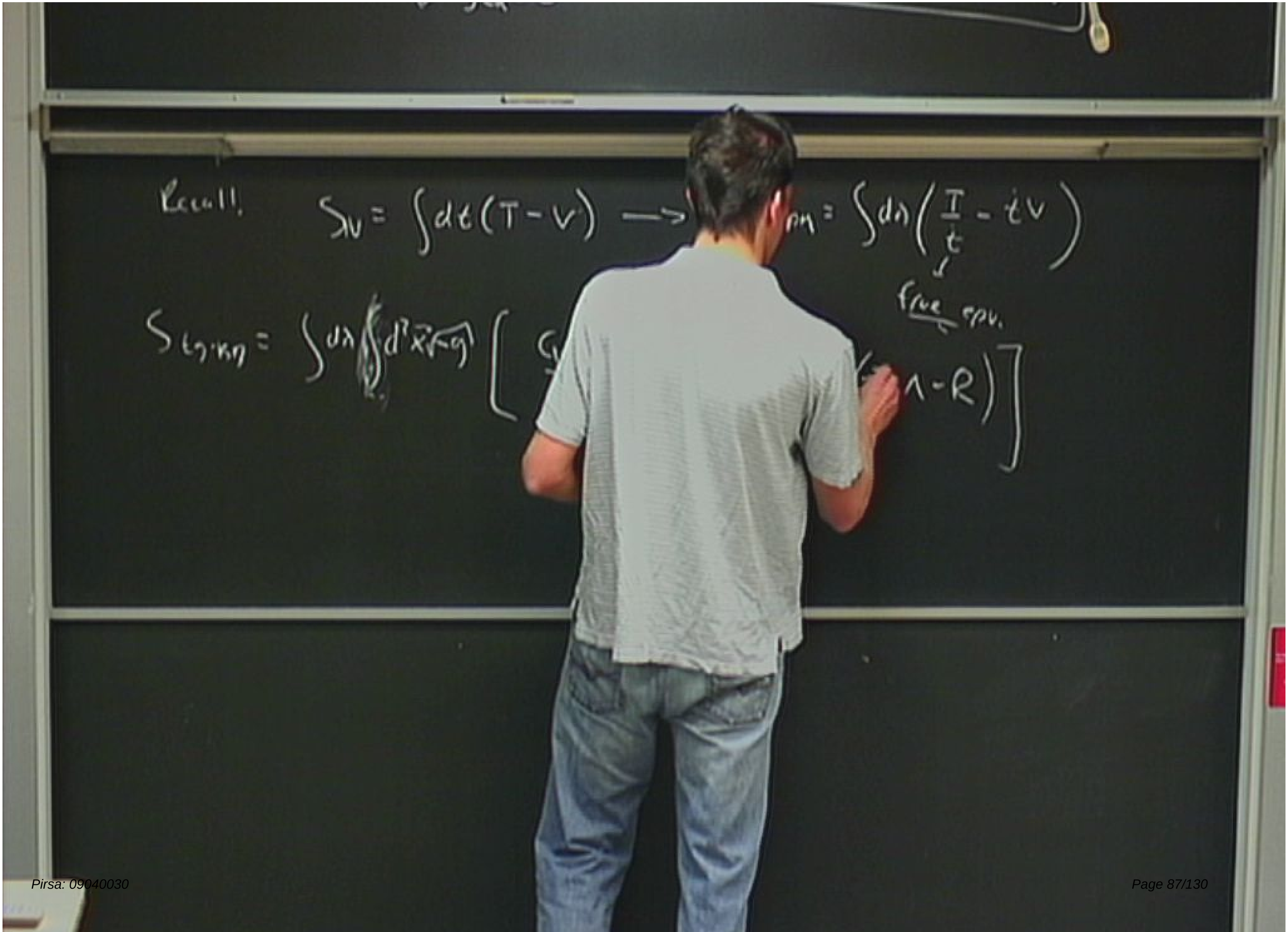

S = ∫dt (T − V)
t → t + a
free

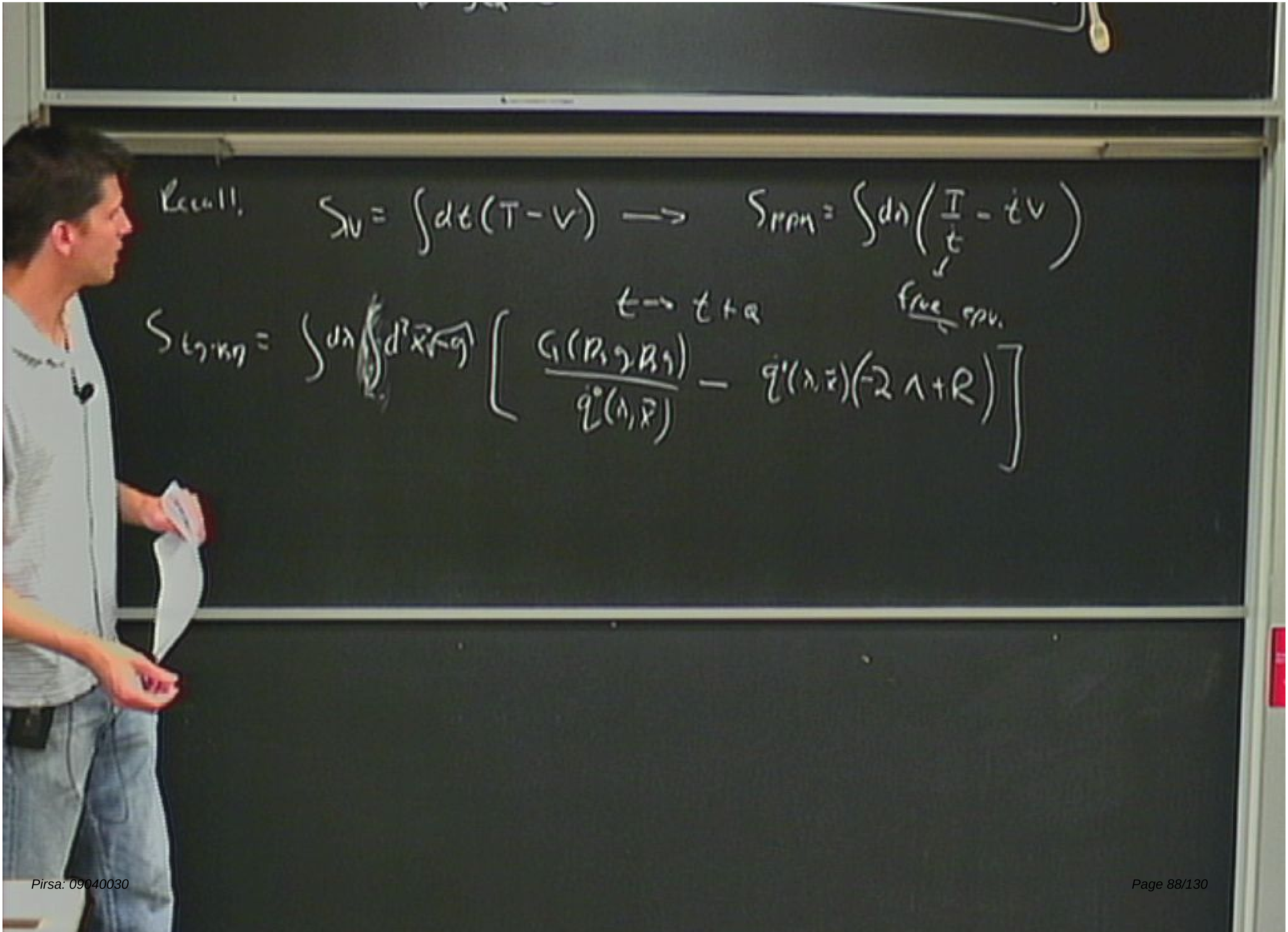

R'(g, Dg) - 2Λ
= G^abcd (ġab + ξ(a;b))(ġcd + ξ(c;d))
= G(Dt g, Dt g)
G^abcd = g^ac g^bd - g^ab g^cd

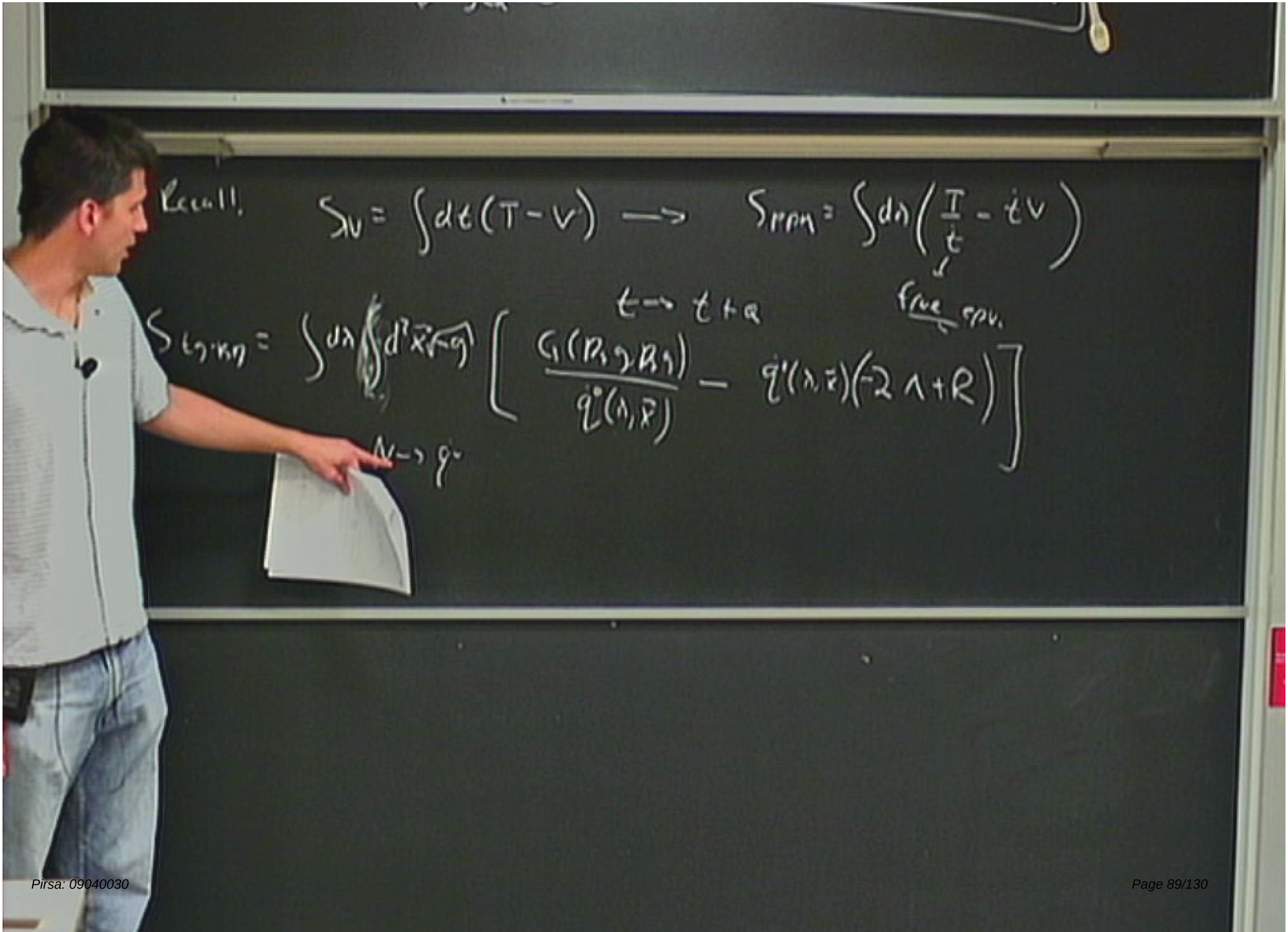


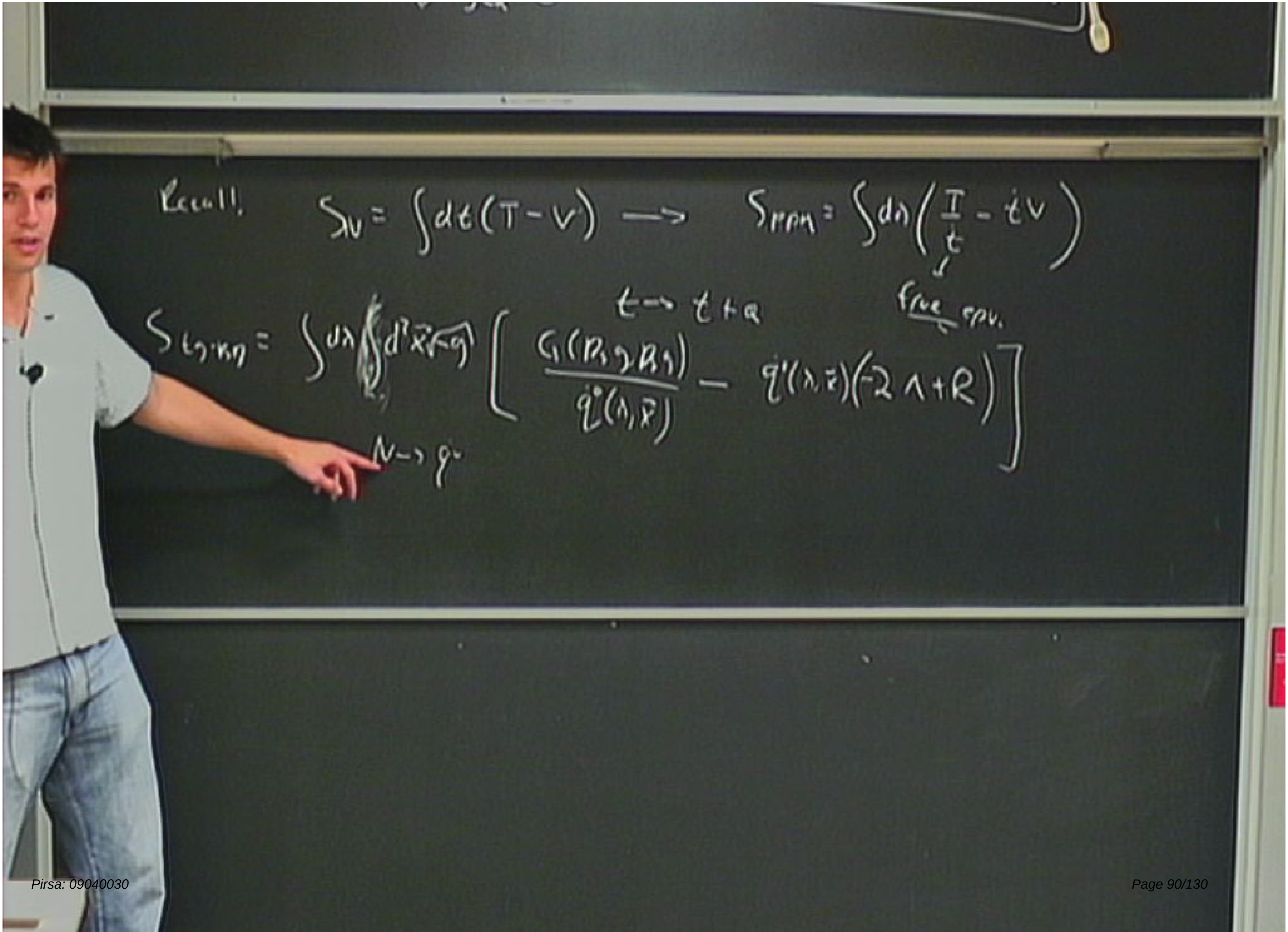

Recall

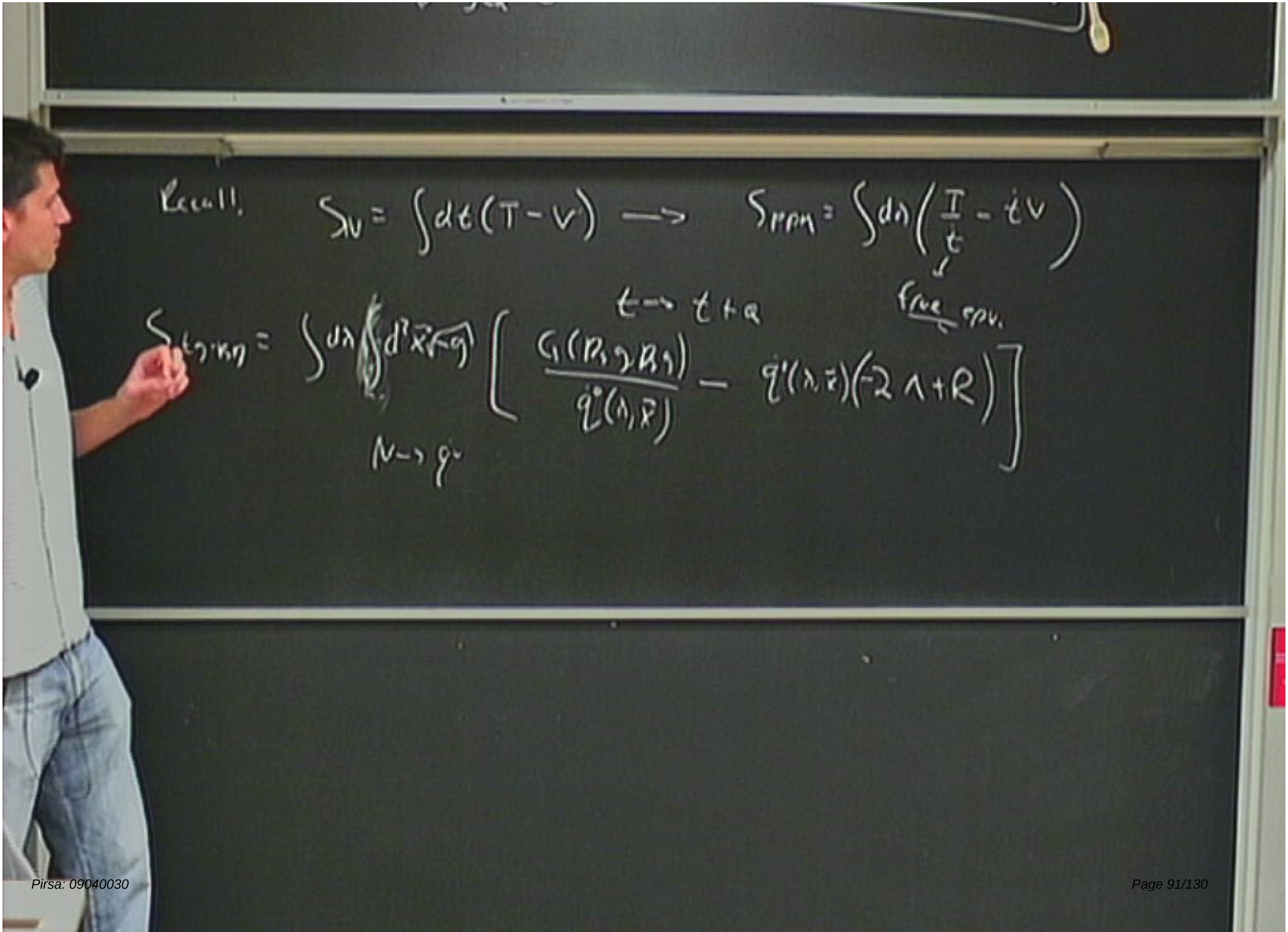

$G^{abcd} = g^{ac} g^{bd} - g^{ab} g^{cd}$


Recall!
free equ.


Recall!
S_NV = ∫dt (T − V) ⟶ S_repa = ∫dλ (T/ṫ − ṫV)
t ⟶ t + a
free eqn.
S_typ.rep = ∫dλ ∫d³x √g [G(p,g,p,g)/ġ(λ,x) − ġ(λ,x)(−2Λ + R)]


Recall!
t → t + a
free eqn.


Recall!
t → t + a
True eqv.
N → q̇


Recall!
S = ∫dt (T − V) ⟶ S = ∫dλ (T/ṫ − ṫV)
t ⟶ t + a
free eqn.
N ⟶ q

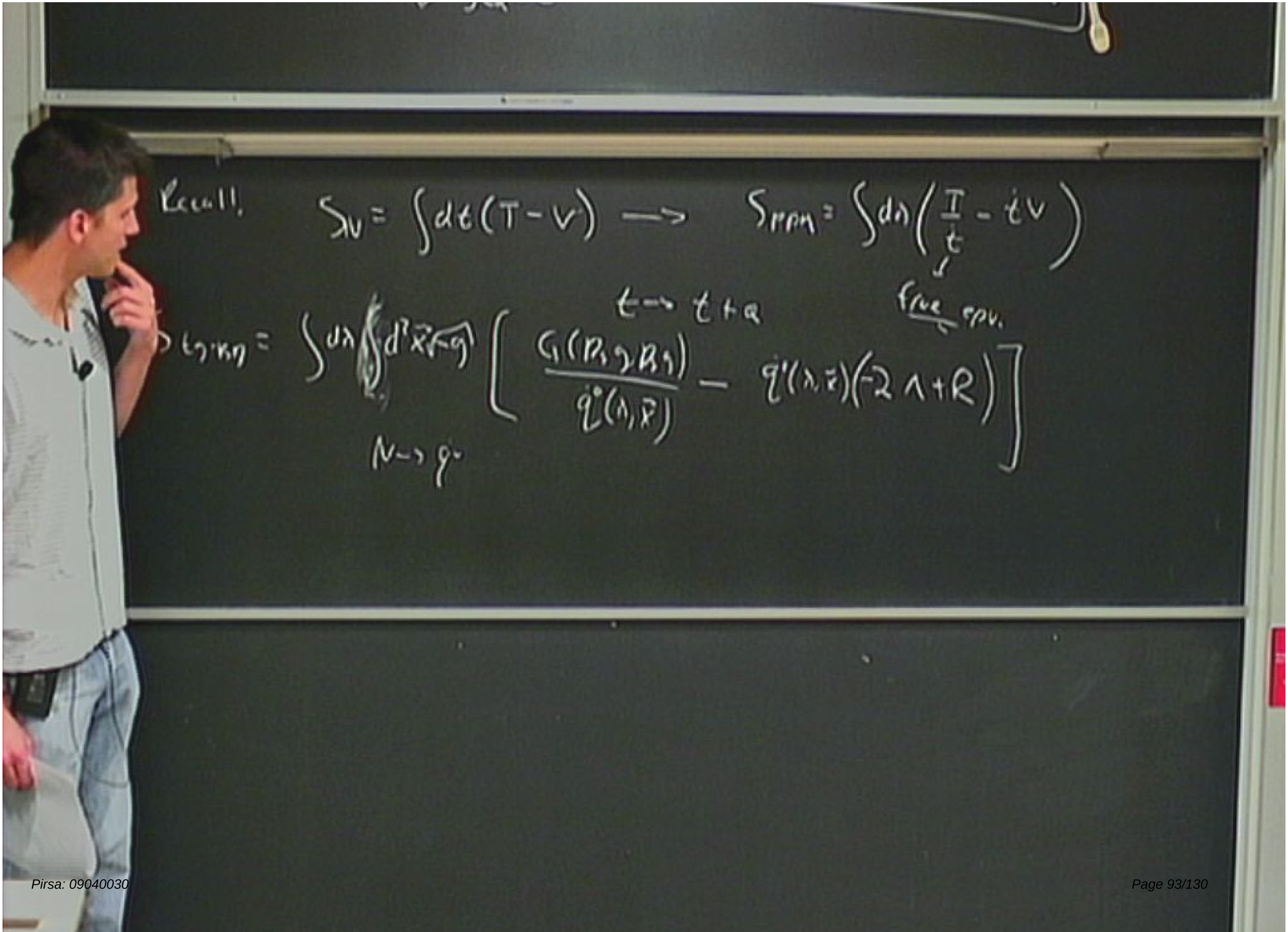
Recall!

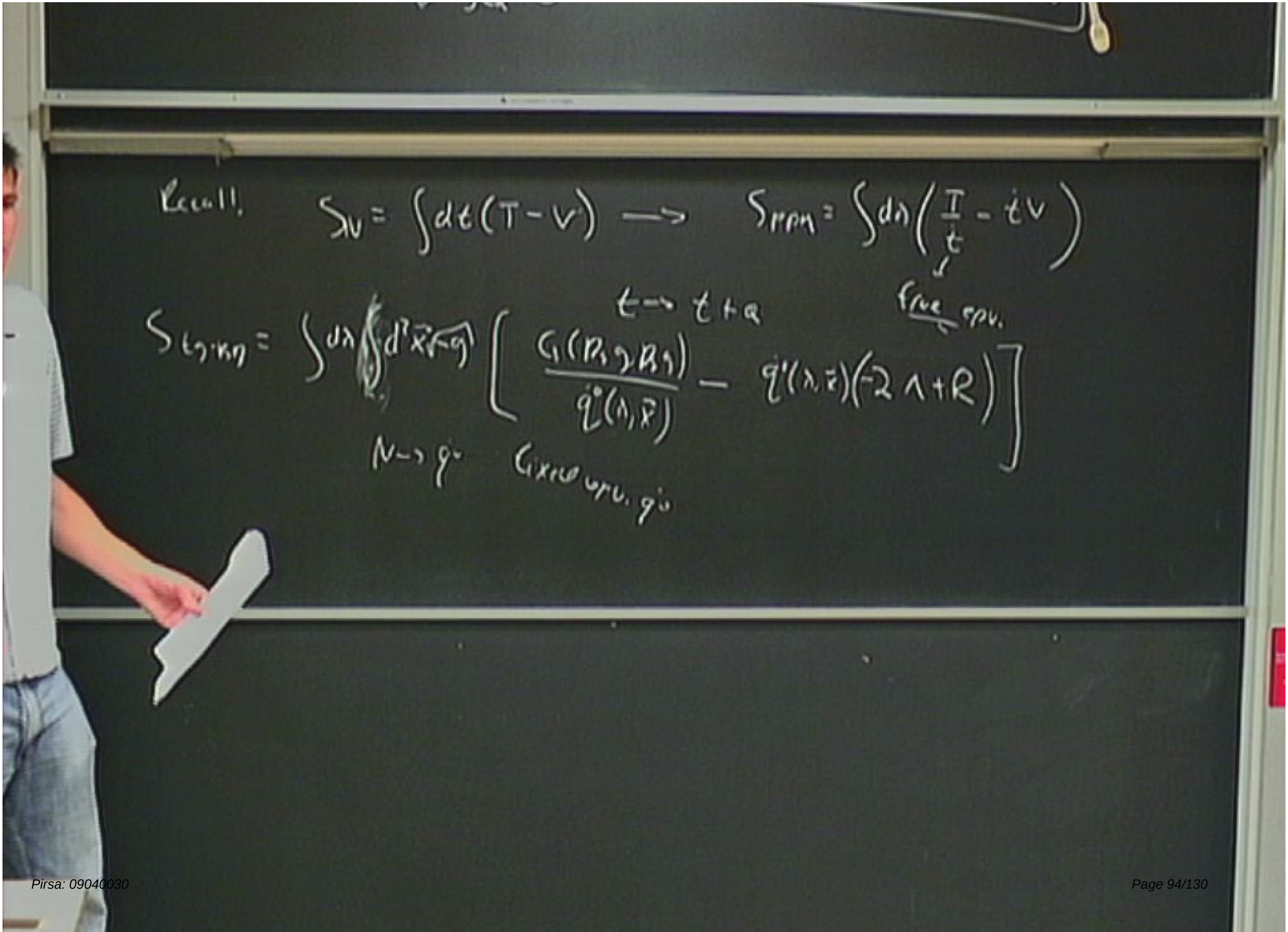
$$S_{NV} = \int dt\,(T - V) \longrightarrow S_{RPM} = \int d\lambda \left(\frac{T}{\dot{t}} - \dot{t} V \right)$$

$t \longrightarrow t + a$ Free eqn.

$$S_{RPM} = \int d\lambda \int d^3\vec{x} \sqrt{-g} \left[\frac{G(P, g, P, g)}{\dot{q}(\lambda, \vec{x})} - \dot{q}(\lambda, \vec{x}) \left(-2\Lambda + R \right) \right]$$

$N \rightarrow \dot{q}$


Recall!
S_NV = ∫dt(T − V) ⟶ S_PPN = ∫dλ(T/ṫ − ṫV)
free eqn.
t ⟶ t + a
S_GR = ∫dλ ∫d³x √−g [G(Pg, Pg)/ġ(λ, x) − ġ(λ, x)(−2Λ + R)]
N → g


Recall!
$S_{NV} = \int dt\,(T - V) \longrightarrow S_{\text{BM}} = \int d\lambda \left(\frac{T}{\dot{t}} - \dot{t} V \right)$
$t \to t + a$
free fn.
$S = \int d\lambda \int d^3\vec{x}\,\sqrt{-g} \left[\frac{G(P, g, P, g)}{\dot{q}(\lambda, \vec{x})} - \dot{q}(\lambda, \vec{x})(-2\Lambda + R) \right]$
$N \to \dot{q}$

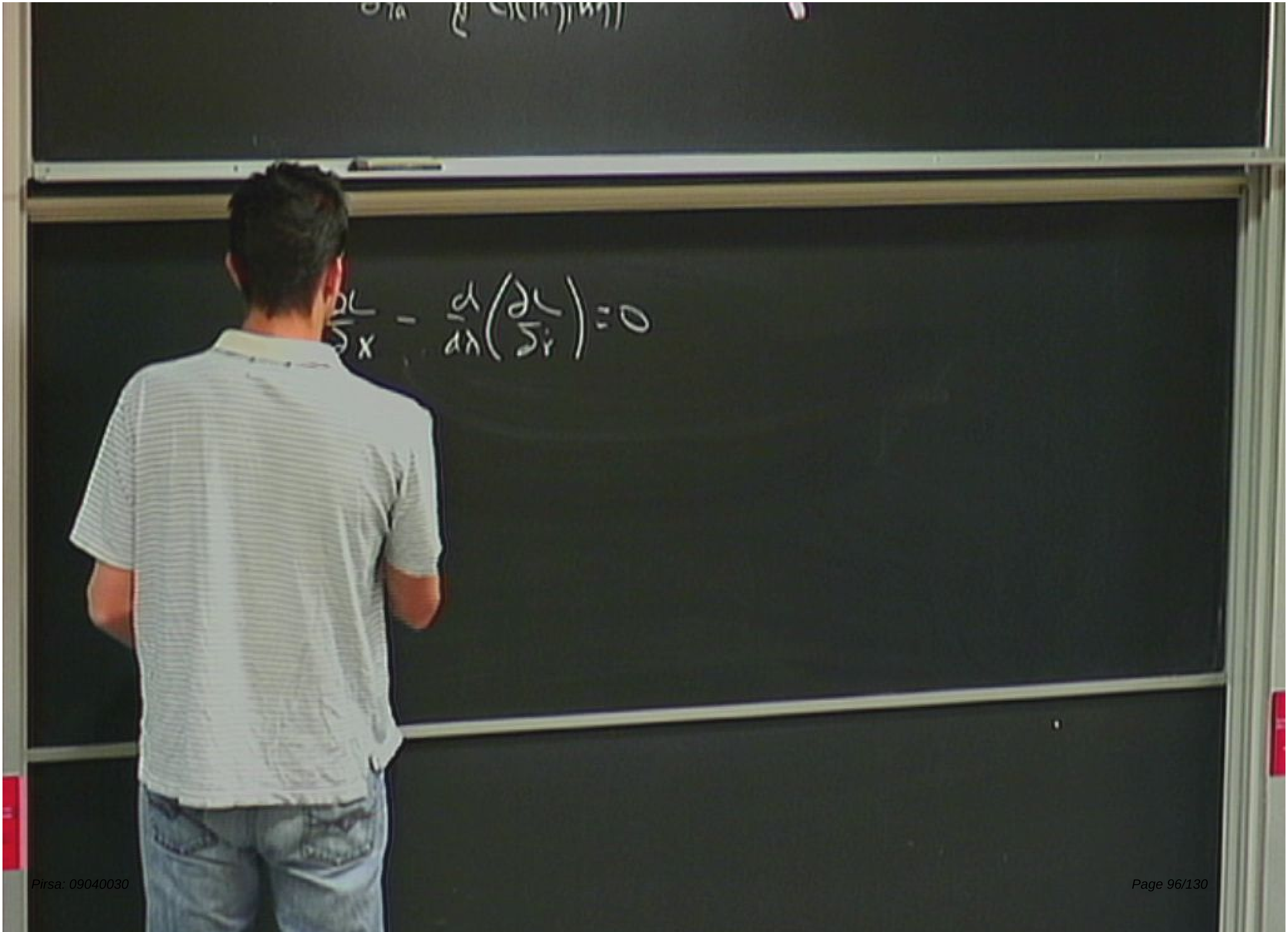
Recall!

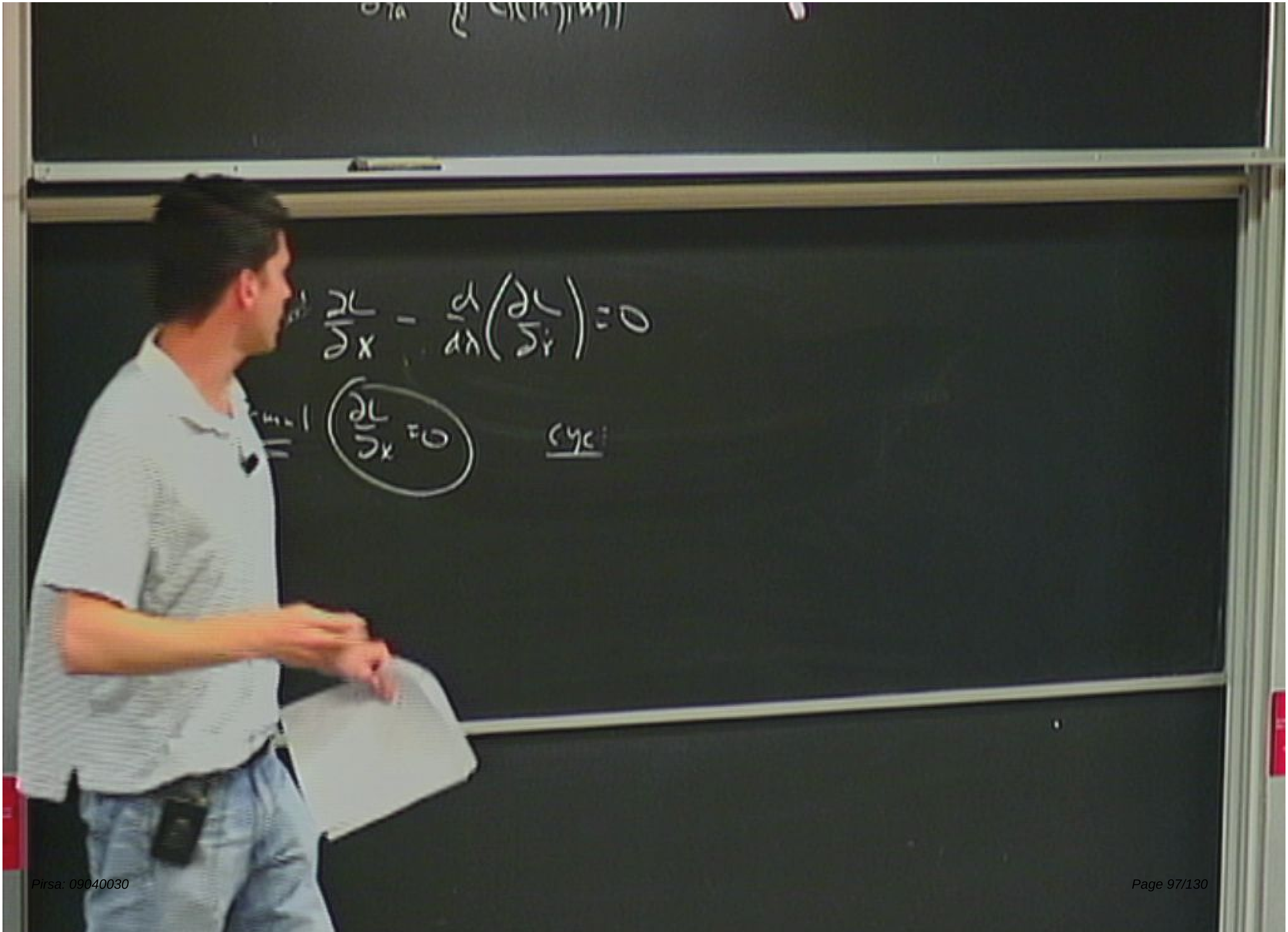
$$S_{NV} = \int dt\,(T - V) \longrightarrow S_{repa} = \int d\lambda \left(\frac{T}{\dot t} - \dot t V \right)$$

$t \to t + a$ free eqn.

$$S_{tgrav} = \int d\lambda \int d^3\vec{x}\sqrt{-q} \left[\frac{G(P_{ij}, P_{kl})}{\dot q(\lambda, \vec{x})} - \dot q(\lambda, \vec{x})\left(-2\Lambda + R\right) \right]$$

$N \to \dot q$ Gauss eqn. $\dot q$


∂L/∂x − d/dλ (∂L/∂ẋ) = 0

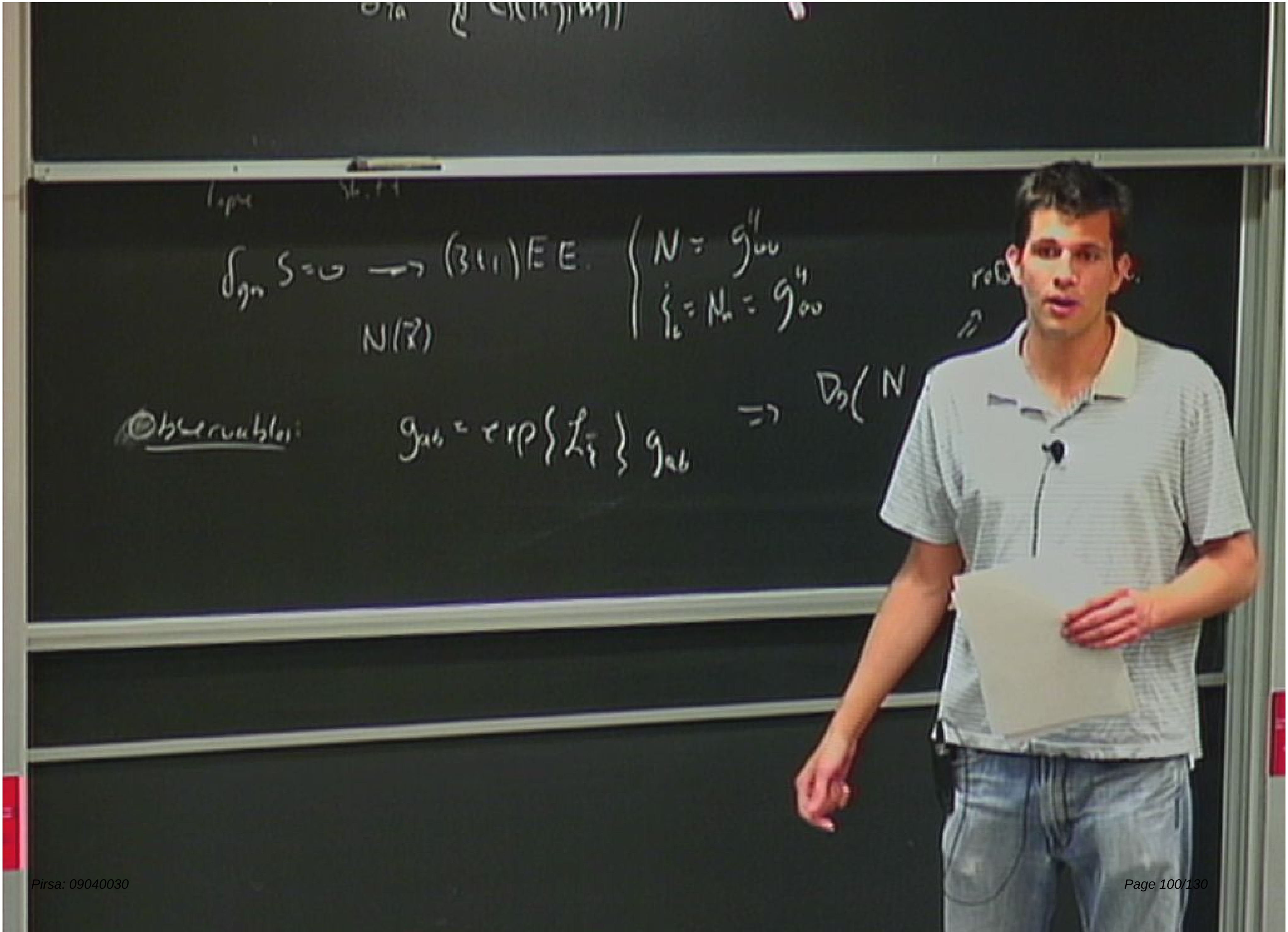

$\frac{\partial L}{\partial x} - \frac{d}{d\lambda}\left(\frac{\partial L}{\partial \dot{x}}\right) = 0$
$\frac{\partial L}{\partial x} = 0$
cyc

$$\frac{\partial L}{\partial x} - \frac{d}{d\lambda}\left(\frac{\partial L}{\partial \dot{x}}\right) = 0$$

λ - mal $\frac{\partial L}{\partial x} = 0$ cycl $\frac{\partial L}{\partial \dot{x}} = \text{const.}$

$$\frac{\partial L}{\partial x} - \frac{d}{d\lambda}\left(\frac{\partial L}{\partial \dot{x}}\right) = 0$$

$\lambda = m$! $\frac{\partial L}{\partial x} = 0$ cyc! $\frac{\partial L}{\partial \dot{x}} = \text{const.}$ (Newton) $= 0$


(3+1) E E.
N(ξ)
Observables:

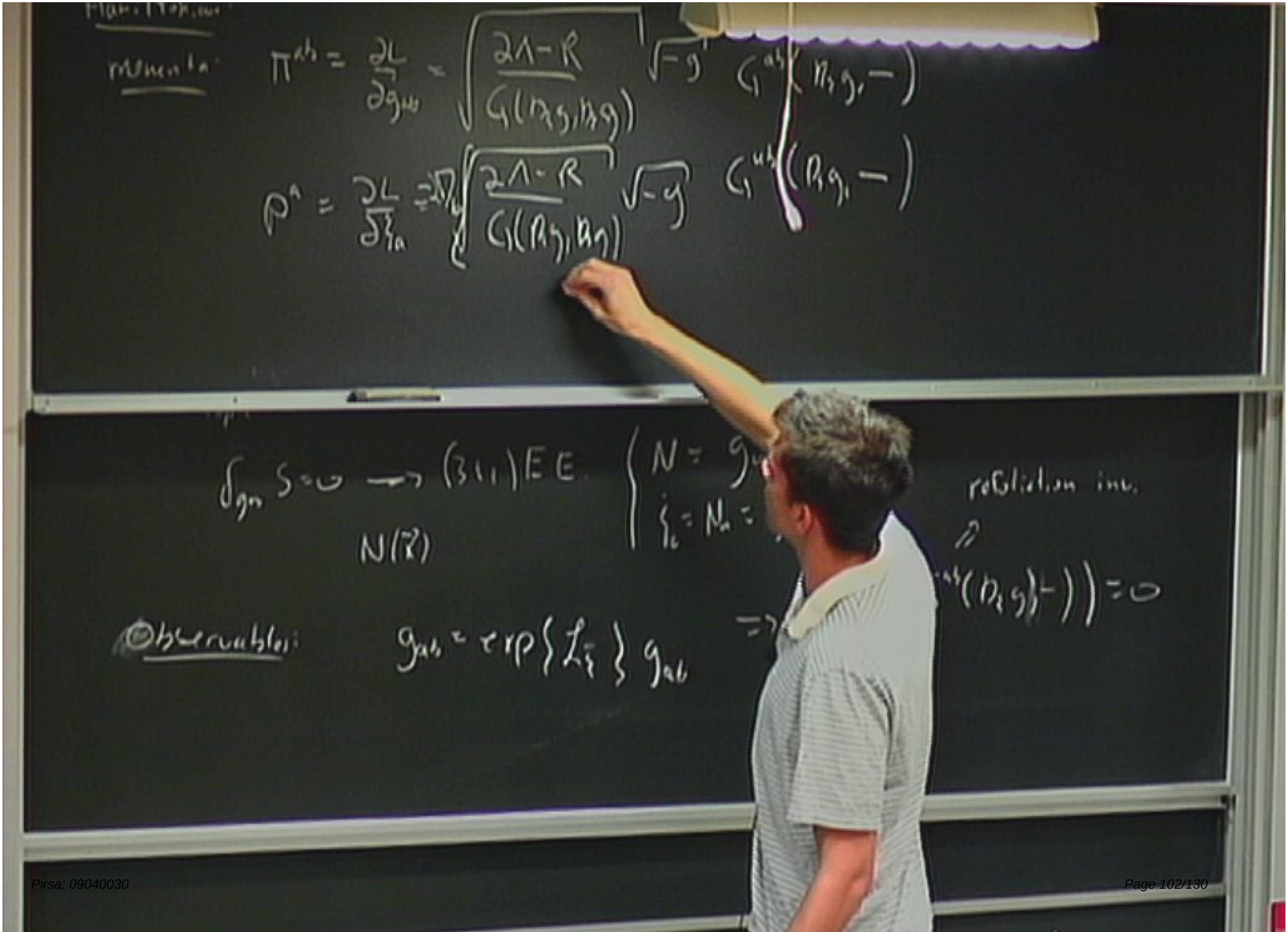
$$\delta_{g_{\mu\nu}} S = 0 \longrightarrow (3+1)\,EE \qquad \begin{cases} N = g^{4}_{00} \\ \xi_a = N_a = g^{4}_{0a} \end{cases}$$

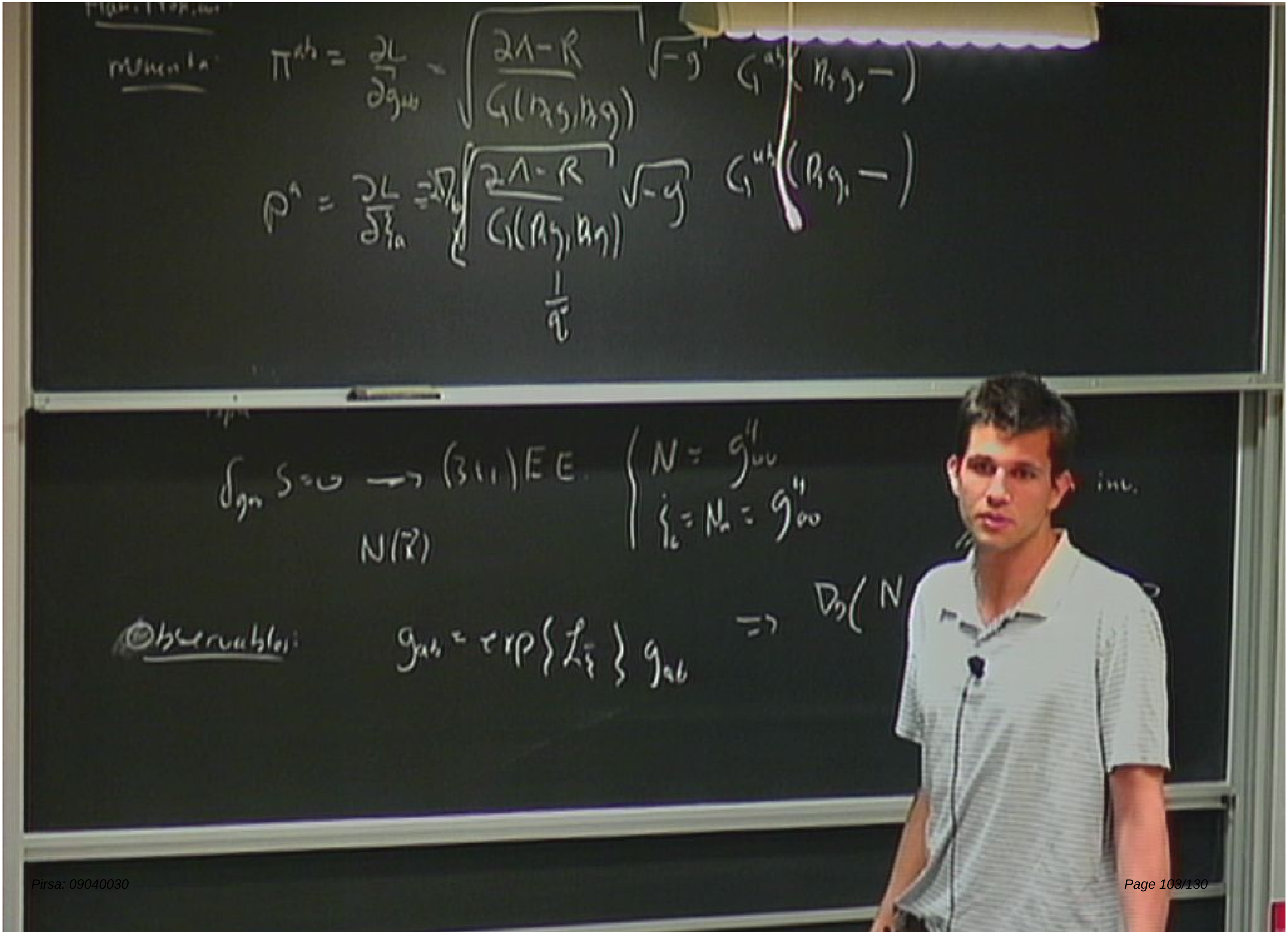
$N(\xi)$

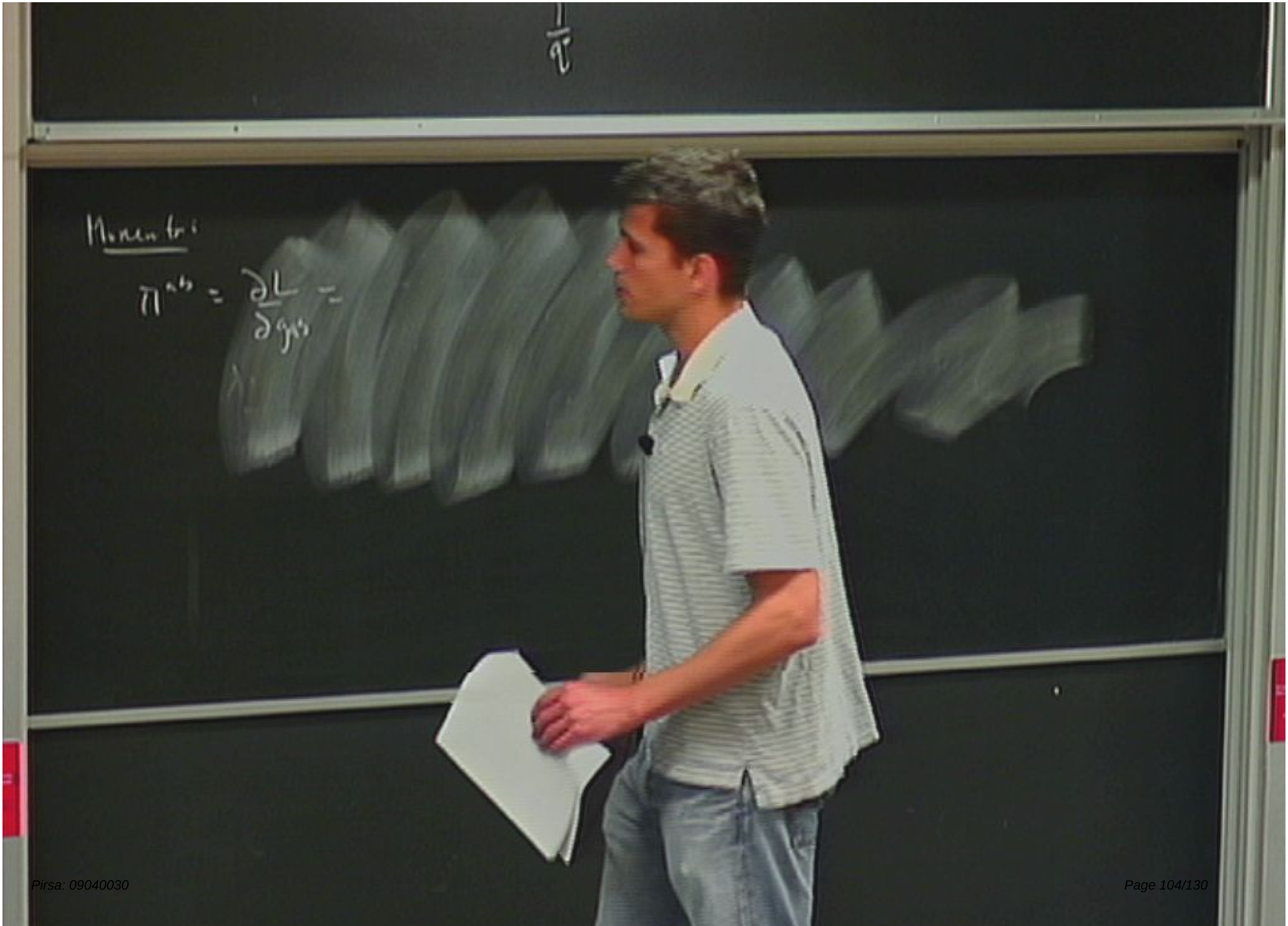
refoliation inv.

$\Uparrow$

Observables: $g_{ab} = \exp\{\mathcal{L}_{\xi}\}\, g_{ab} \Rightarrow D_b\left(N\, G^{ab}(D_c g)\ldots\right) = 0$


momenta
Observables:
(3+1) E E
rotation inv.


momenta:
Observables:

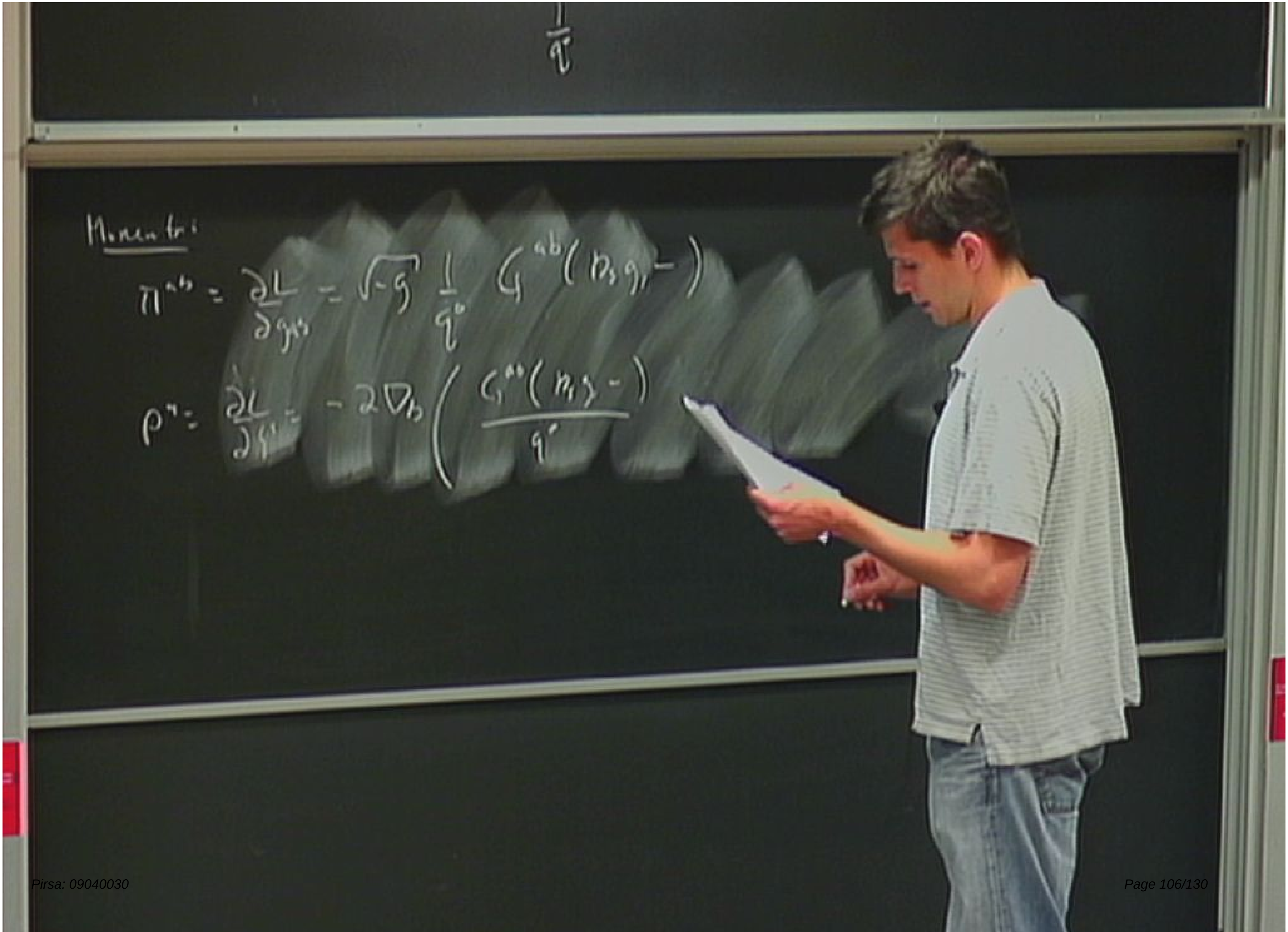

Momenta:
$\pi^{ab} = \frac{\partial L}{\partial \dot{g}_{ab}} =$

$\frac{1}{q^2}$

Momenta:

$\pi^{ab} = \frac{\partial L}{\partial \dot{g}_{ab}} = \sqrt{-g}\,\frac{1}{q^2}\,G^{ab}(n, g, \ldots)$

$p^a = \frac{\partial L}{\partial \dot{q}^a} =$


Momenta:

$$\frac{1}{q^0}$$

Momenta:

$$\pi^{ab} = \frac{\partial L}{\partial \dot g_{ab}} = \sqrt{-g}\,\frac{1}{q^0}\,G^{ab}(n, g, \ldots)$$

$$p^a = \frac{\partial L}{\partial \dot q^a} = -2\nabla_b\left(\frac{G^{ab}(n, g, \ldots)\sqrt{-g}}{q^0}\right) \Rightarrow \boxed{L^a = p^a - \nabla_b \pi^{ab} = 0}$$

$$\frac{1}{q^0}$$

Momenta:

$$\pi^{ab} = \frac{\partial L}{\partial \dot{g}_{ab}} = \sqrt{-g}\,\frac{1}{q^0}\,G^{ab}(n_3 g, -)$$

$$p^a = \frac{\partial L}{\partial \dot{q}^a} = -2\nabla_b\left(\frac{G^{ab}(n_3 g -)\sqrt{-g}}{q^0}\right) \Rightarrow \boxed{L^a = p^a - \nabla_b \pi^{ab} = 0}$$

$$p^0 = \frac{\partial L}{\partial \dot{q}^0} =$$

$$\frac{1}{q^0}$$

Momenta:

$$\pi^{ab} = \frac{\partial L}{\partial \dot{g}_{ab}} = \sqrt{-g}\,\frac{1}{q^0}\,G^{ab}(n, g, -)$$

$$p^a = \frac{\partial L}{\partial \dot{q}^a} = -2\nabla_b\left(\frac{G^{ab}(n, g, -)\sqrt{-g}}{q^0}\right) \Rightarrow \boxed{L^a = p^a - \nabla_b \pi^{ab} = 0}$$

$$p^0 = \frac{\partial L}{\partial \dot{q}^0} = -\sqrt{-g}\left(\frac{G(n, g, n, g)}{q^{0\,2}} + (R - 2\Lambda)\right)$$

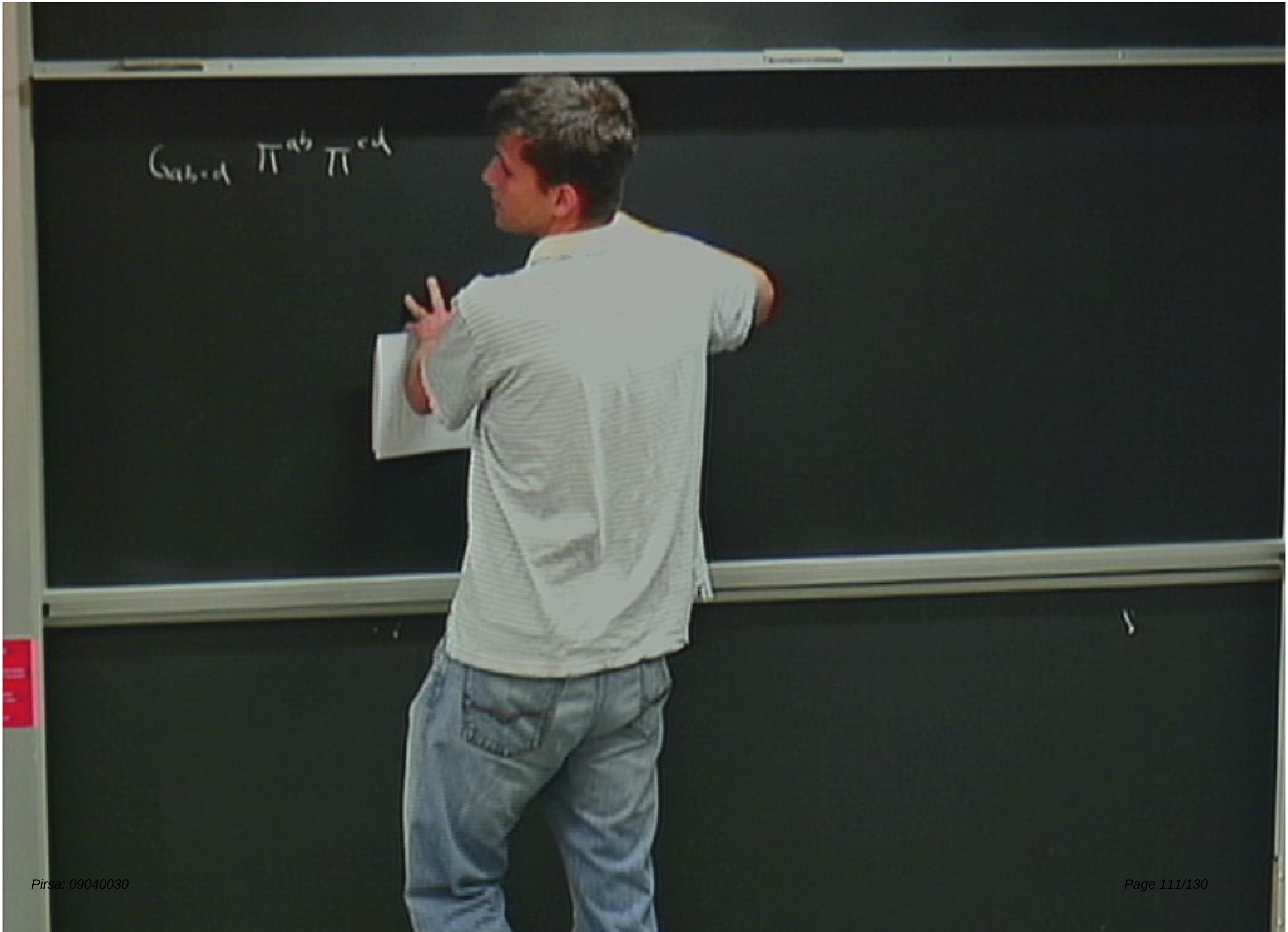
$$\frac{1}{q^0}$$

Momenta:

$$\pi^{ab} = \frac{\partial L}{\partial \dot{g}_{ab}} = \sqrt{-g}\,\frac{1}{q^0}\,G^{ab}(n_3 g_1 -)$$

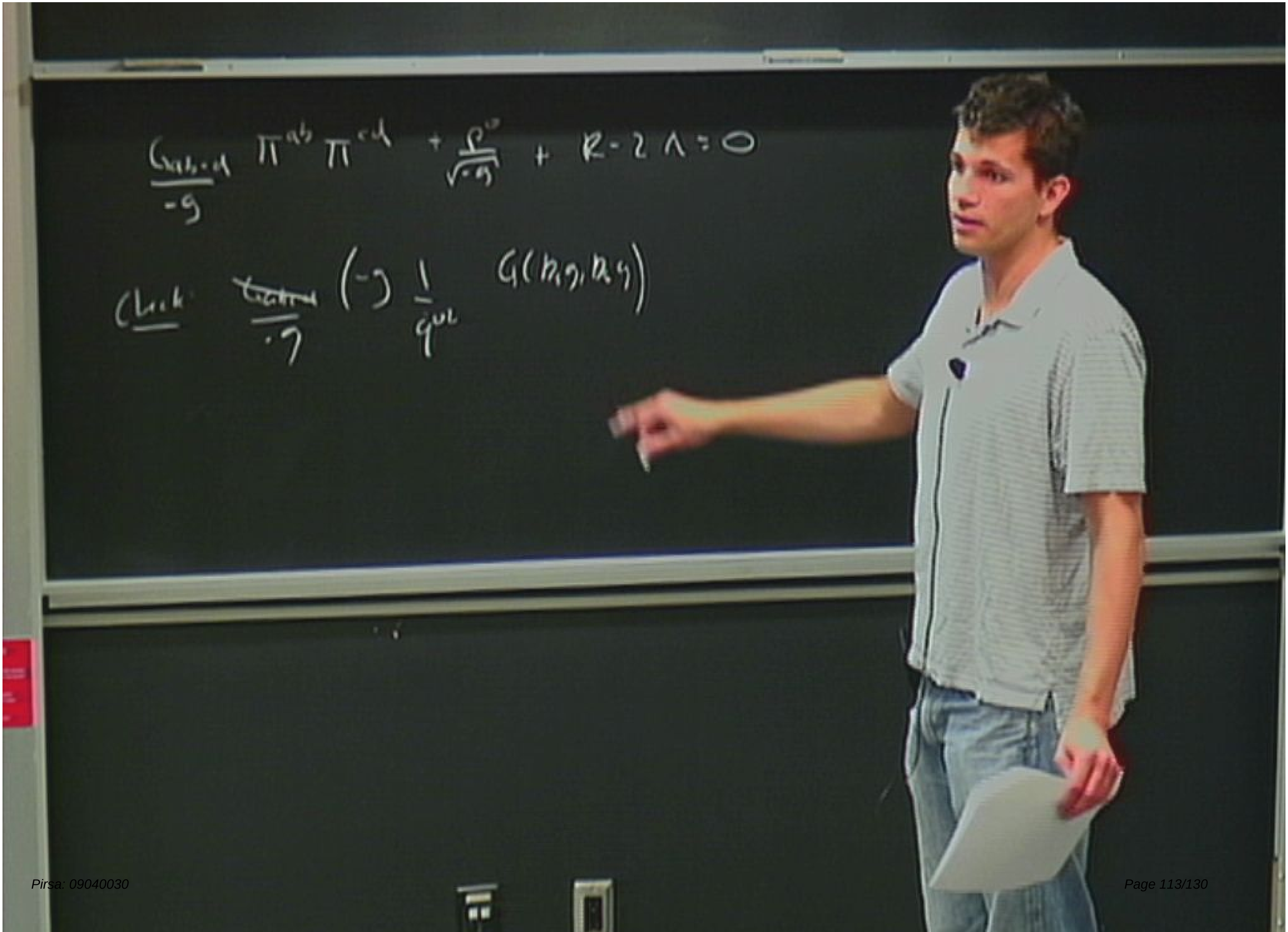
$$p^a = \frac{\partial L}{\partial \dot{q}^a} = -2\nabla_b\left(\frac{G^{ab}(n_3 g -)\sqrt{-g}}{q^0}\right) \Rightarrow \boxed{L^a = p^a - \nabla_b \pi^{ab} = 0}$$

$$p^0 = \frac{\partial L}{\partial \dot{q}^0} = -\sqrt{-g}\left(\frac{G(n_3 g n_3 g)}{q^{02}} + (R - 2\Lambda)\right)$$


$G_{ab,cd} \pi^{ab} \pi^{cd}$

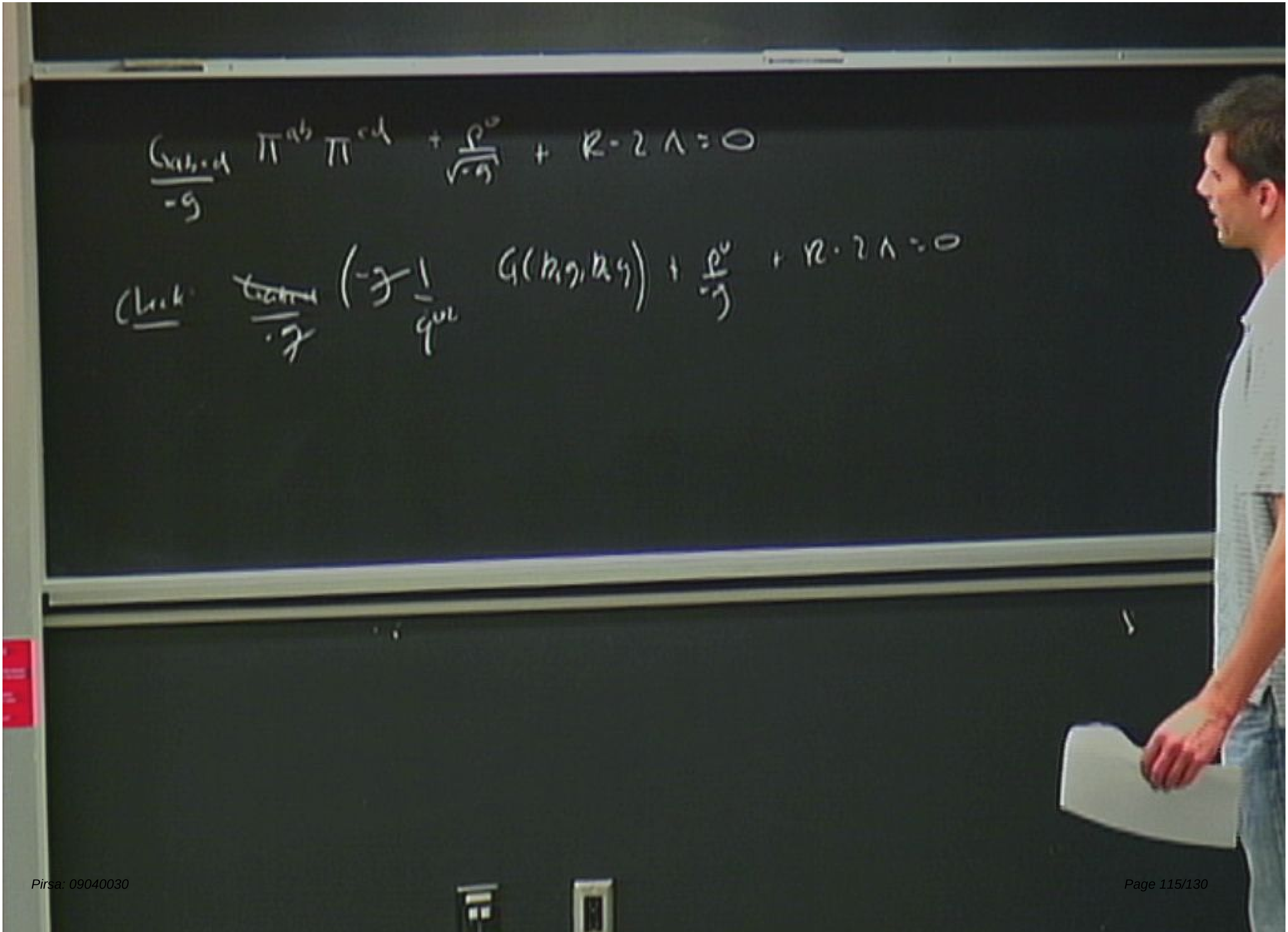
$$\frac{G_{abcd}}{-g}\,\pi^{ab}\,\pi^{cd} + \frac{\rho^{\omega}}{\sqrt{-g}} + R - 2\Lambda = 0$$

Check: $\frac{G_{abcd}}{-g}\,(-g)\,\frac{1}{g^{\omega 2}} \ldots$


Check

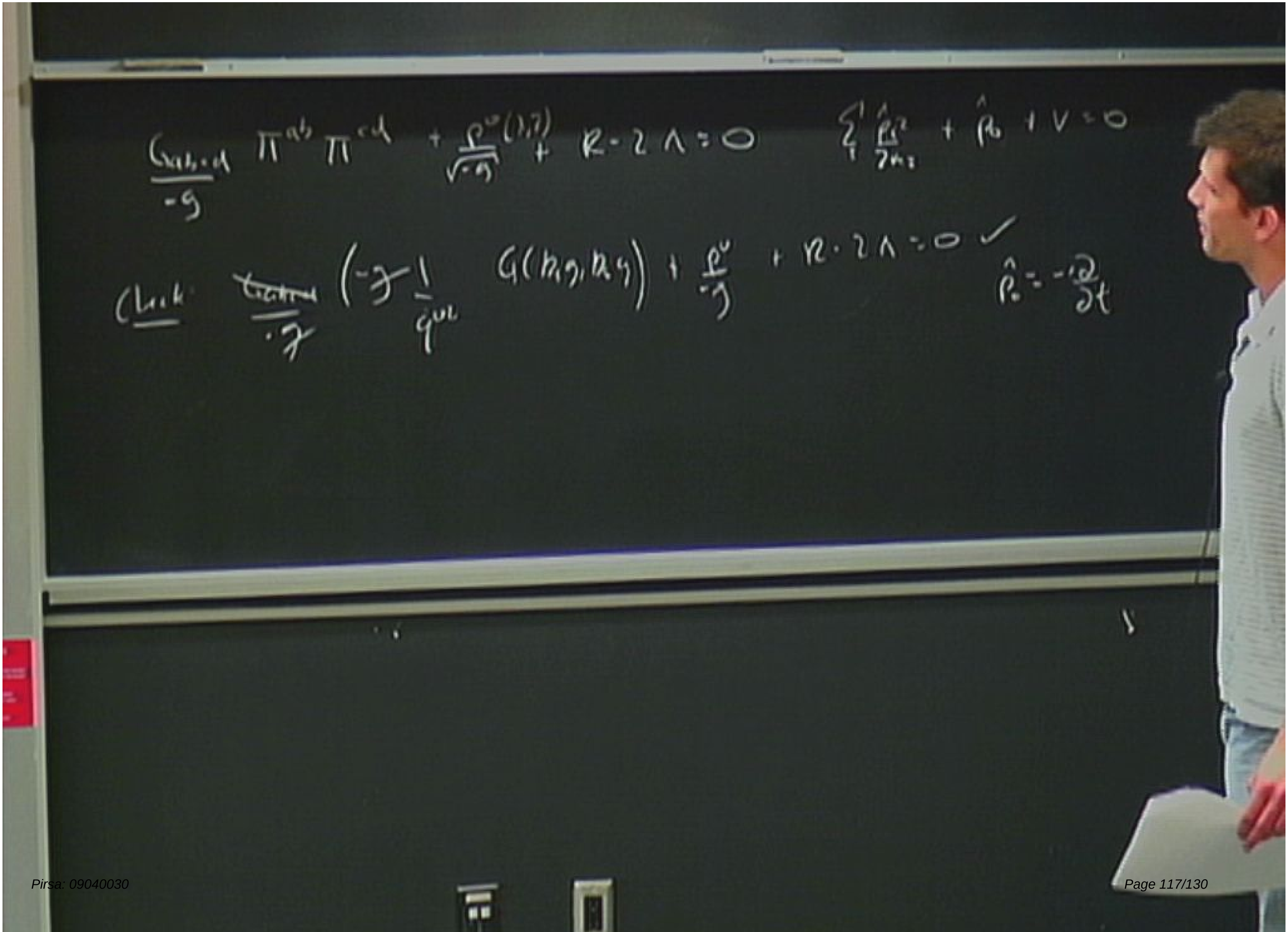
$$\frac{G_{abcd}}{-g}\,\pi^{ab}\pi^{cd} + \frac{\rho^{u}}{\sqrt{-g}} + R - 2\Lambda = 0$$

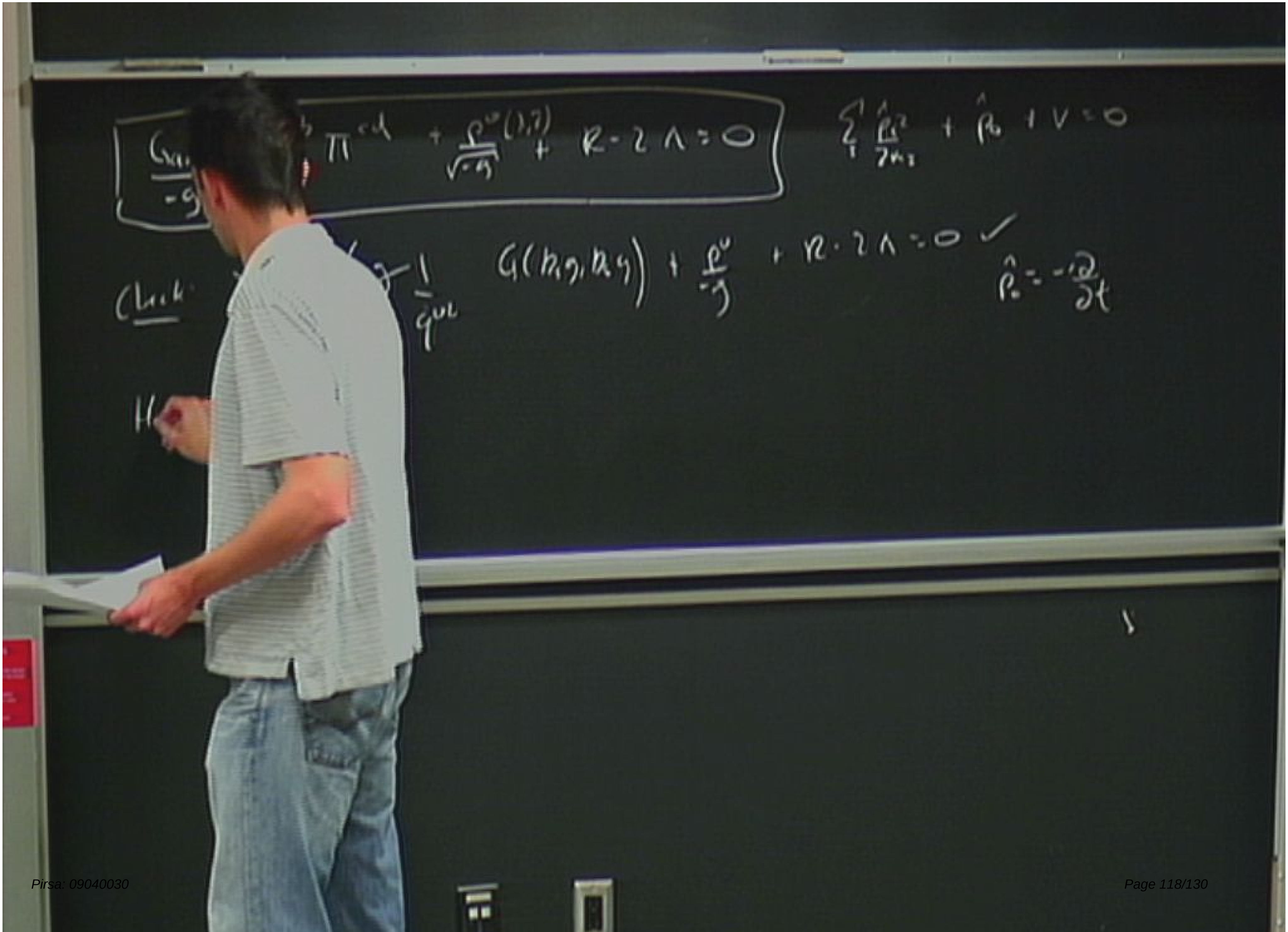
Check: $\frac{\cdots}{\cdot \mathcal{J}}\left(-\mathcal{J}\,\frac{1}{q^{u2}} \quad G(\partial_a q, \partial_b q)\right)$

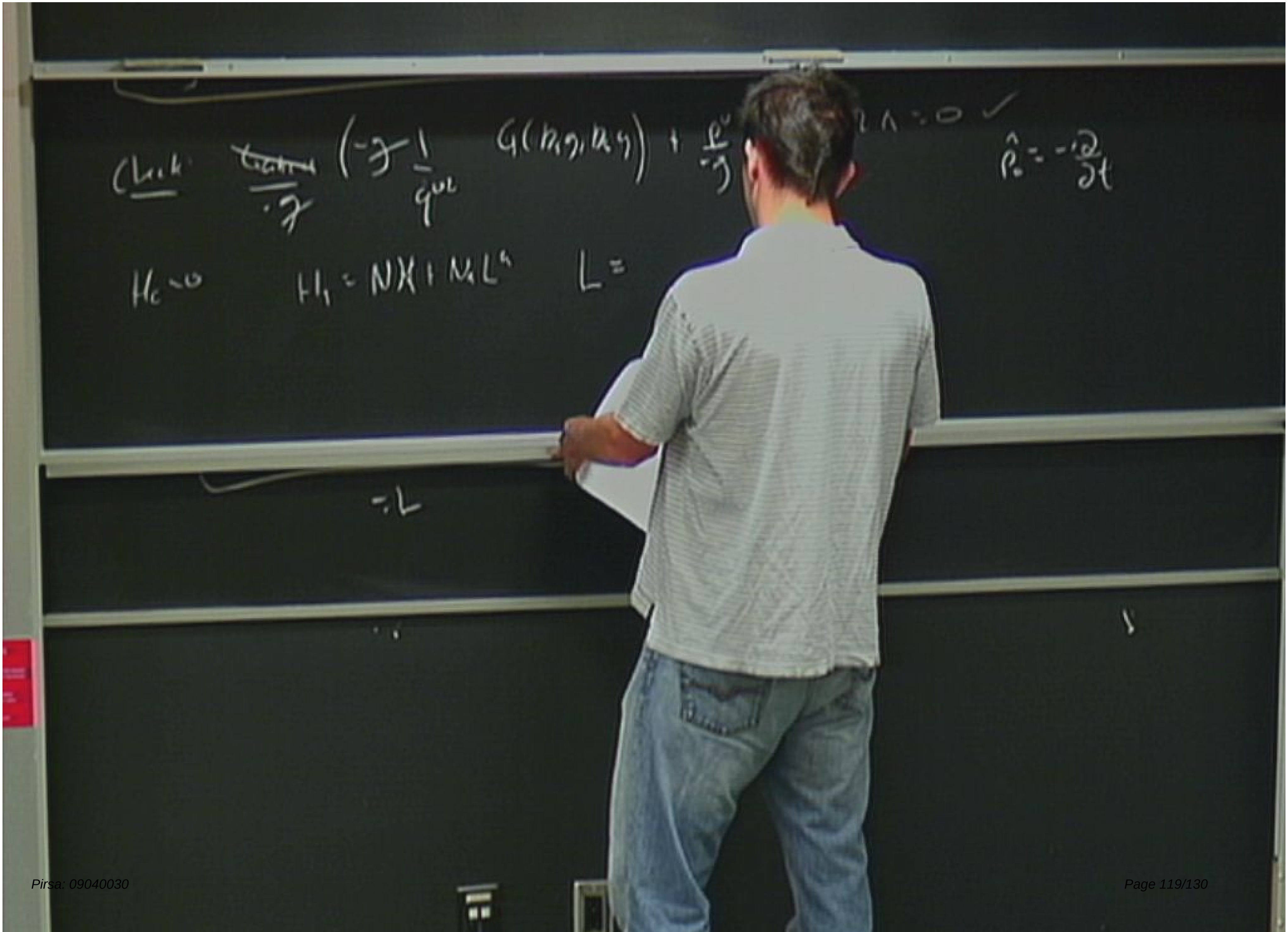

$\frac{G_{abcd}}{-g} \pi^{ab} \pi^{cd} + \frac{\rho^0}{\sqrt{-g}} + R - 2\Lambda = 0$
Check:
$\frac{1}{-g}\left(-g \frac{1}{q^{02}} G(\pi_g, \pi_g)\right) + \frac{\rho^0}{-g} + R - 2\Lambda = 0$

$$\frac{G_{abcd}}{-g}\,\pi^{ab}\pi^{cd} + \frac{\rho^0}{\sqrt{-g}} + R - 2\Lambda = 0 \qquad \sum_i \frac{p_i^2}{2m_i} + p_0 + V = 0$$

Check: $\frac{1}{-g}\left(-g\,\frac{1}{g^{02}}\,G(p_g, p_g)\right) + \frac{\rho^0}{-g} + R - 2\Lambda = 0$ ✓


Check
R - 2 Λ = 0


R - 2 Λ = 0
Check
G(h, g, h, g) + R - 2 Λ = 0

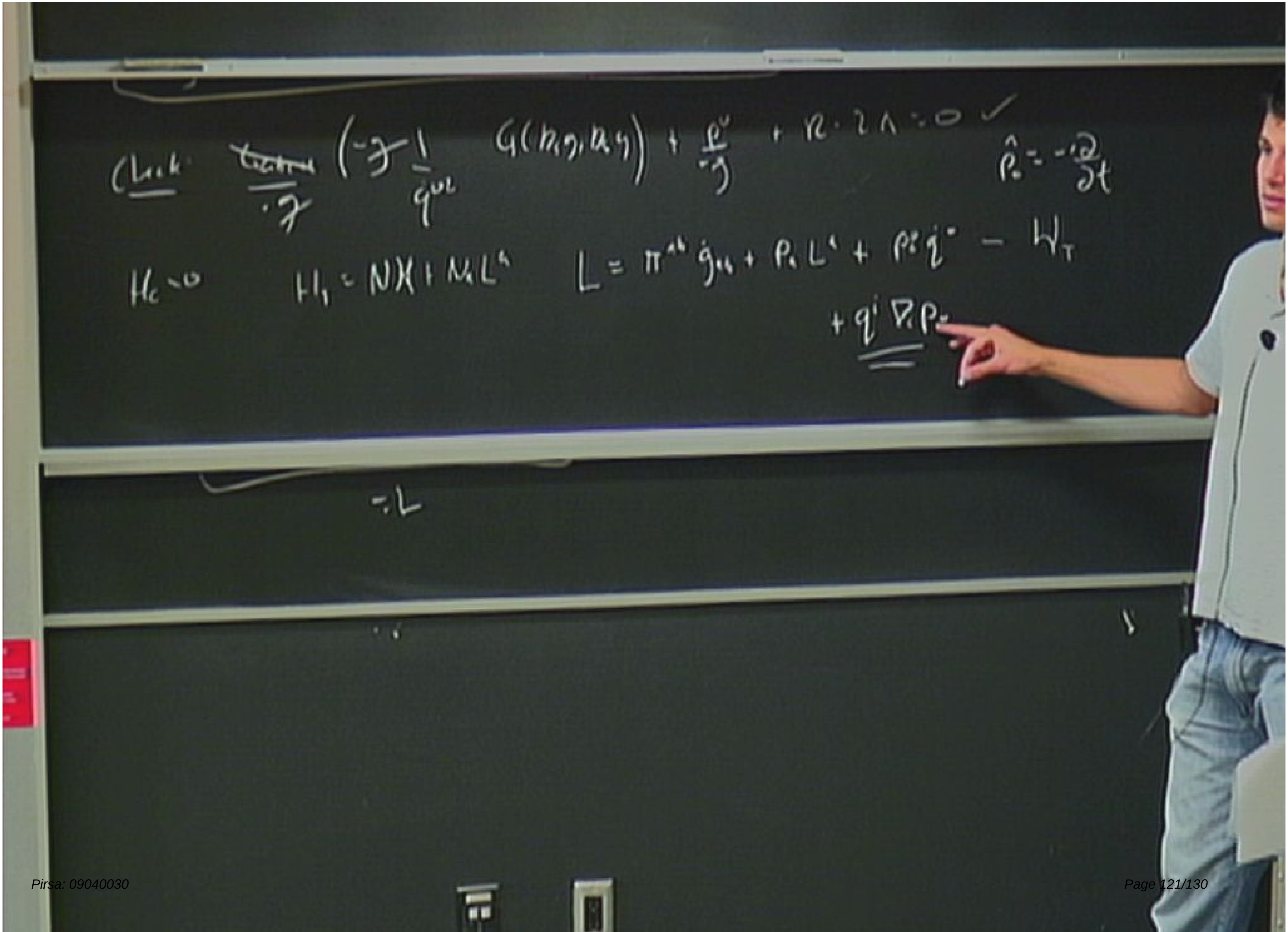

Check
L =

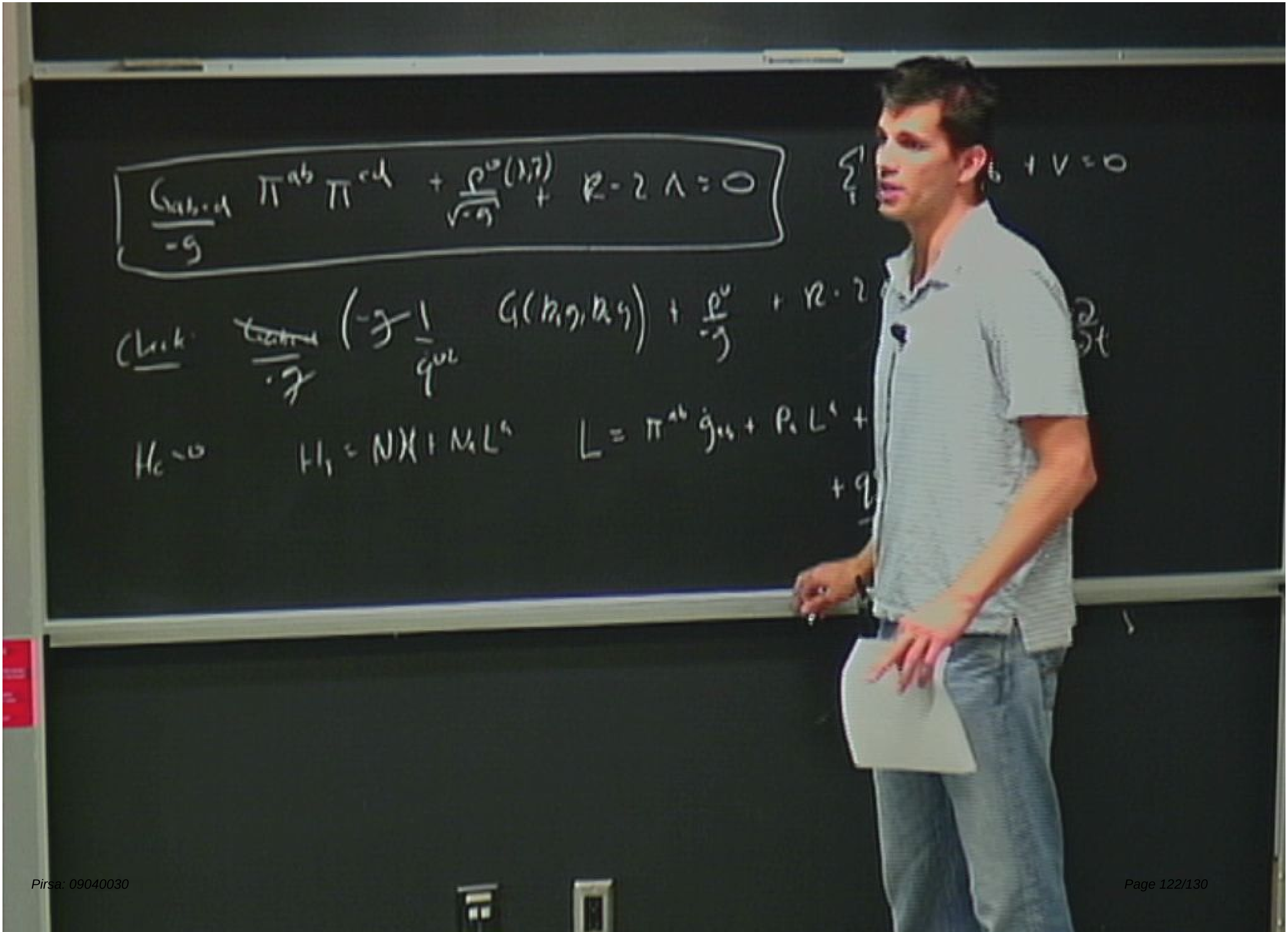
Check: $\frac{\delta}{\delta \mathcal{F}}\left(-\mathcal{F}\frac{1}{q^{0\,2}}\, G(\pi_{ij}, \pi_{kl})\right) + \frac{p^0}{-g} + R - 2\Lambda = 0$ ✓

$\hat{p}_0 = -i\frac{\partial}{\partial t}$

$H_c = 0 \qquad H_T = N\mathcal{H} + N_a L^a \qquad L = \pi^{ab}\dot{g}_{ab} + P_a L^a + p^0\dot{q}^0 - H_T$

$= L$


Check
H_c = 0
H_T = NH + N_a L^a
L = π^ab ġ_ab + P_a L^a + P^0 q̇^0 − H_T


Check

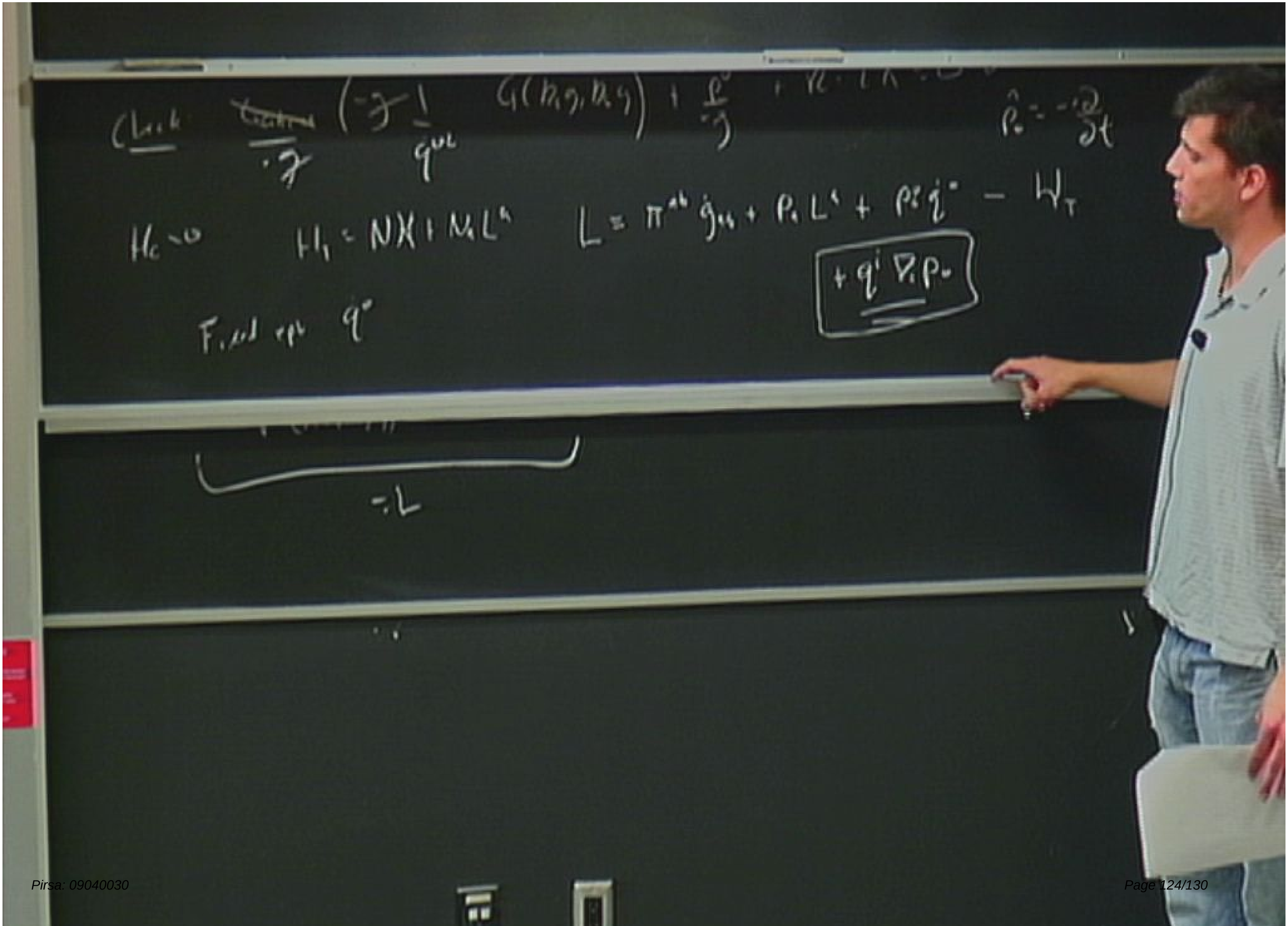
Check: $\frac{}{\cdot \chi} \left(-\chi \frac{1}{q^{0 2}} \quad G(\dots) \right) + \frac{p^0}{\cdot \chi} + R \cdot 2\Lambda = 0$ ✓

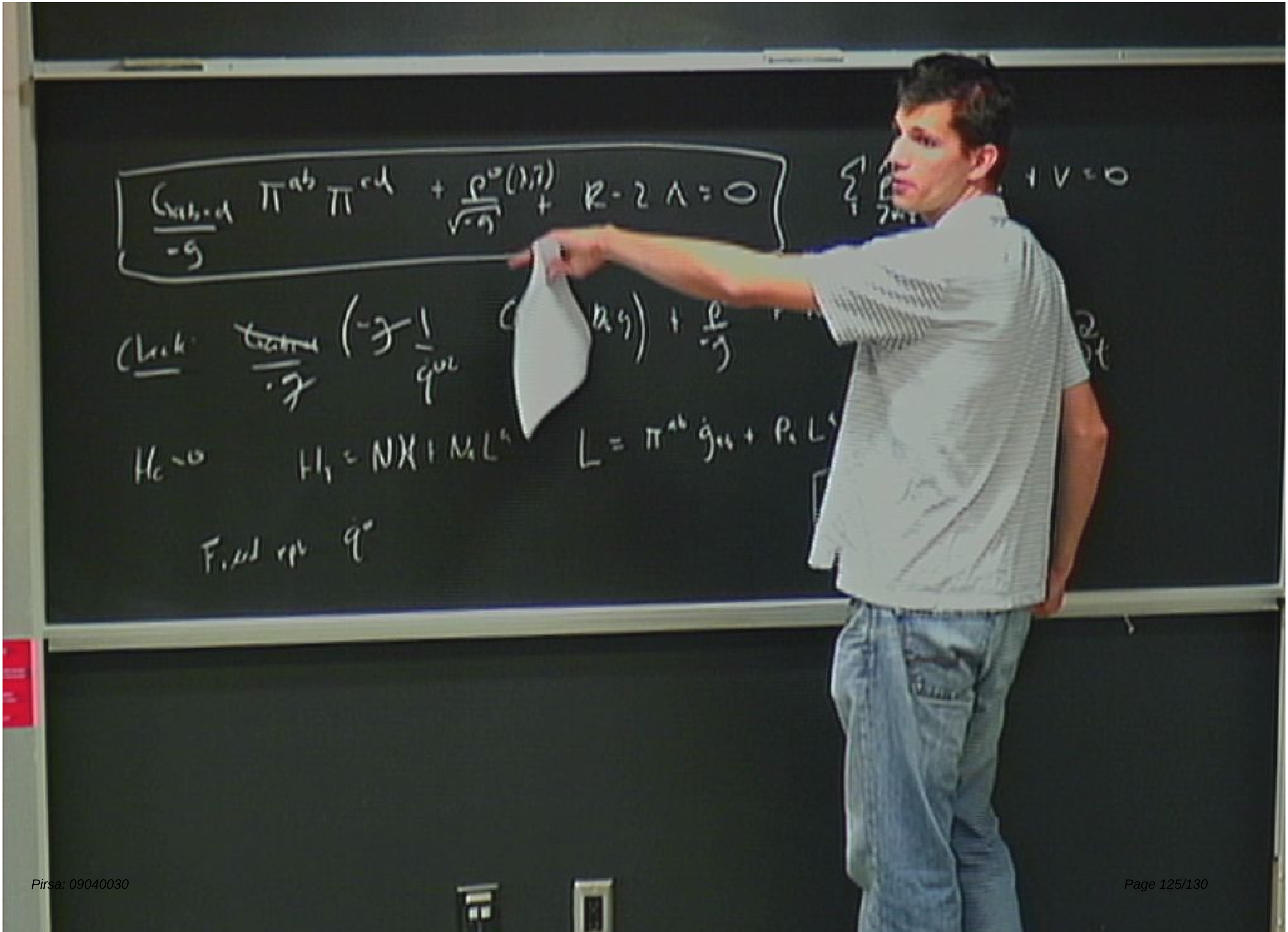
$\hat{p}_0 = -i\frac{\partial}{\partial t}$

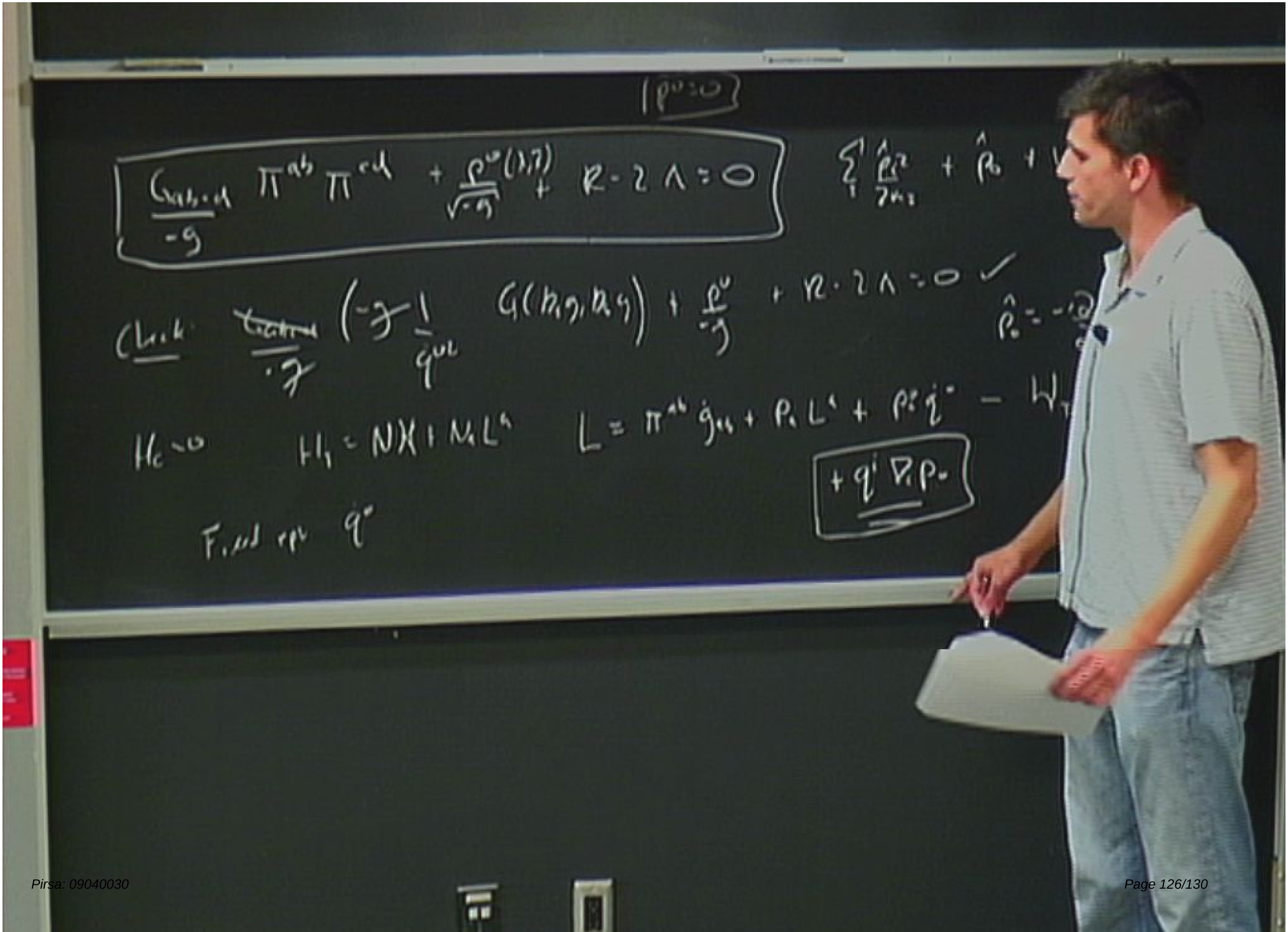
$H_c \sim 0 \qquad H_1 = N\mathcal{H} + N_a L^a \qquad L = \pi^{ab}\dot{g}_{ab} + P_a L^a + p^0 \dot{q}^0 - H_T$

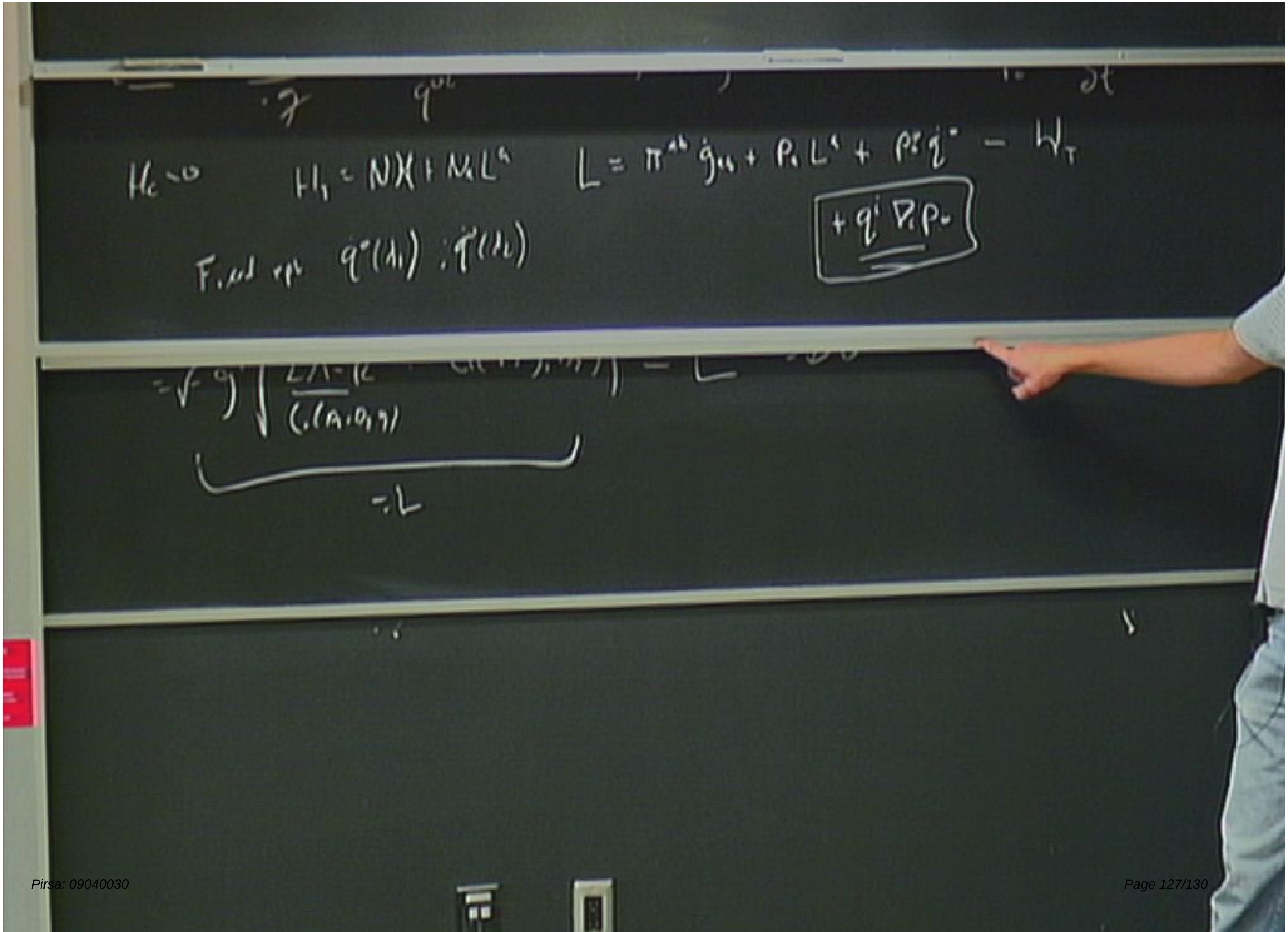
$+ q^i \nabla_i p_0$

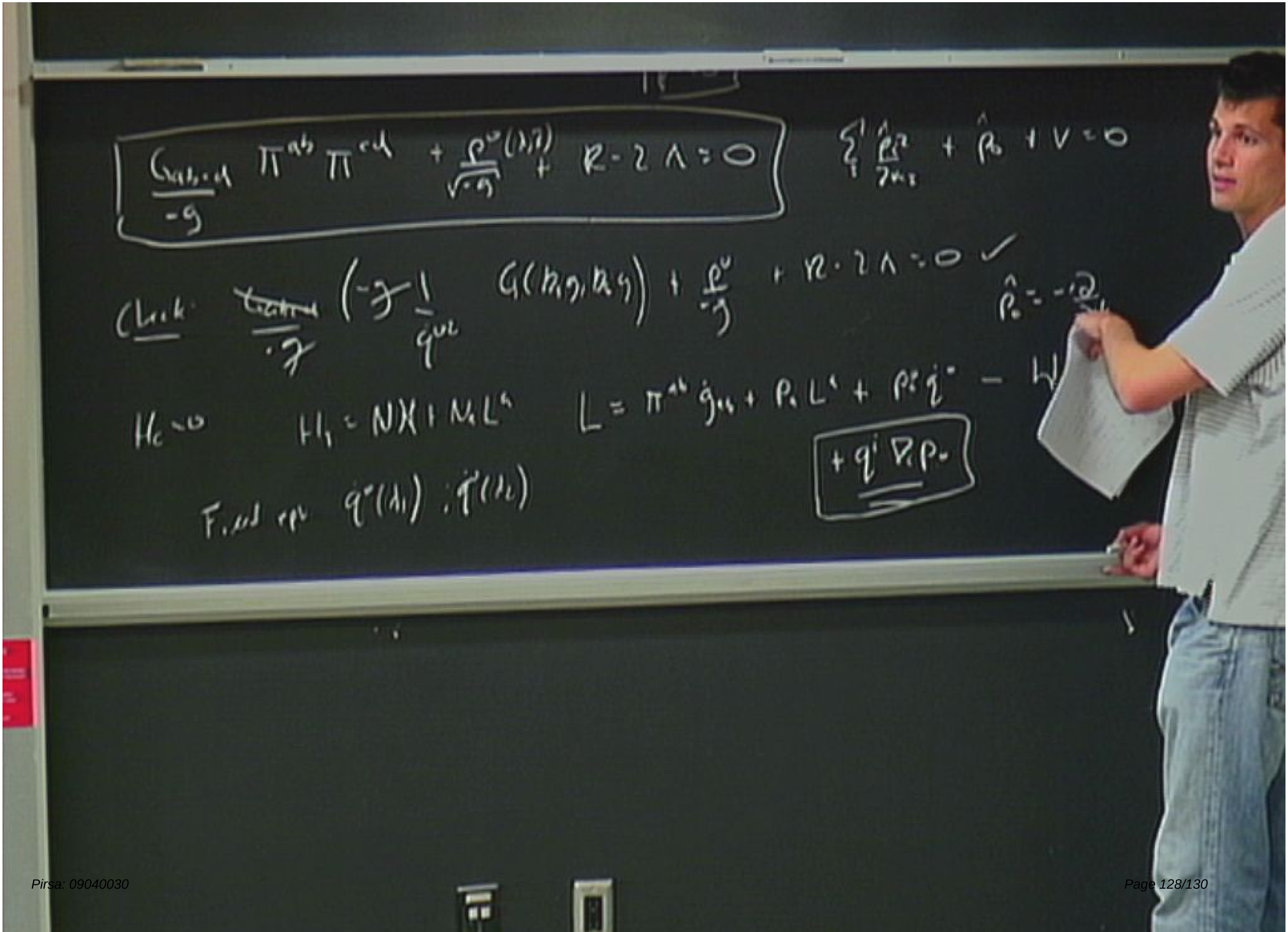
$= L$


Check
Find eqn q^0









$$\frac{G_{abcd}}{-g}\pi^{ab}\pi^{cd} + \frac{\rho^{0}(\lambda,?)}{\sqrt{-g}} + R - 2\Lambda = 0$$

$$\sum_i \frac{\hat{p}_i^2}{2m_i} + \hat{p}_0 + V = 0$$

Check: $\frac{G_{abcd}}{-g}\left(-g\frac{1}{\dot{q}^{0\,2}}\, G(\dot{g}, \dot{g})\right) + \frac{\rho^{0}}{\sqrt{-g}} + R - 2\Lambda = 0$ ✓

$$\hat{p}_0 = -i\frac{\partial}{\partial t}$$

$H_c = 0 \qquad H_T = N\mathcal{H} + N_a L^a \qquad L = \pi^{ab}\dot{g}_{ab} + P_a \dot{L}^a + p_i^0 \dot{q}^0 - H_T$

$$+ q^i \nabla_i p_0$$

Find eqs $q^0(\lambda_1)$; $q^0(\lambda_2)$

Ham Const: $\frac{G_{abcd}}{\sqrt{-g}} \pi^{ab} \pi^{cd} - ($ ... $)$

check: $\frac{1}{-g}\left[G_{abcd} \frac{-g(2\Lambda - R)}{G(\partial_t g, \partial_t g)} \cdot G^{abnm}(\dot{g}_{nm} + \nabla_{(n} \xi_{m)}) G^{cdop}(\dot{g}_{op} + \nabla_{(o}\xi_{p)}) \right]$

$-(2\Lambda - R)$

$= 0$ ✓

$$\frac{G_{abcd}}{-g} \pi^{ab} \pi^{cd} + \frac{\rho^0(\lambda, \gamma)}{\sqrt{-g}} + R - 2\Lambda = 0$$

$\sum_i \frac{\hat{p}_i^2}{2m_i} + \hat{p}_0 + V = 0$

check: $\frac{1}{-g}\left(-g \frac{1}{\dot{q}^{0\,2}} G(\partial_t g, \partial_t g)\right) + \frac{\rho^0}{-g} + R - 2\Lambda = 0$ ✓

$\hat{p}_0 = -i\frac{\partial}{\partial t}$

$H_c = 0 \qquad H_T = N\mathcal{H} + N_a L^a \qquad L = \pi^{ab}\dot{g}_{ab} + P_a L^a + p_0 \dot{q}^0 - H_T$

$+ q^i \nabla_i p_0$

Fixed eps $q^0(\lambda_1) ; q^0(\lambda_2)$