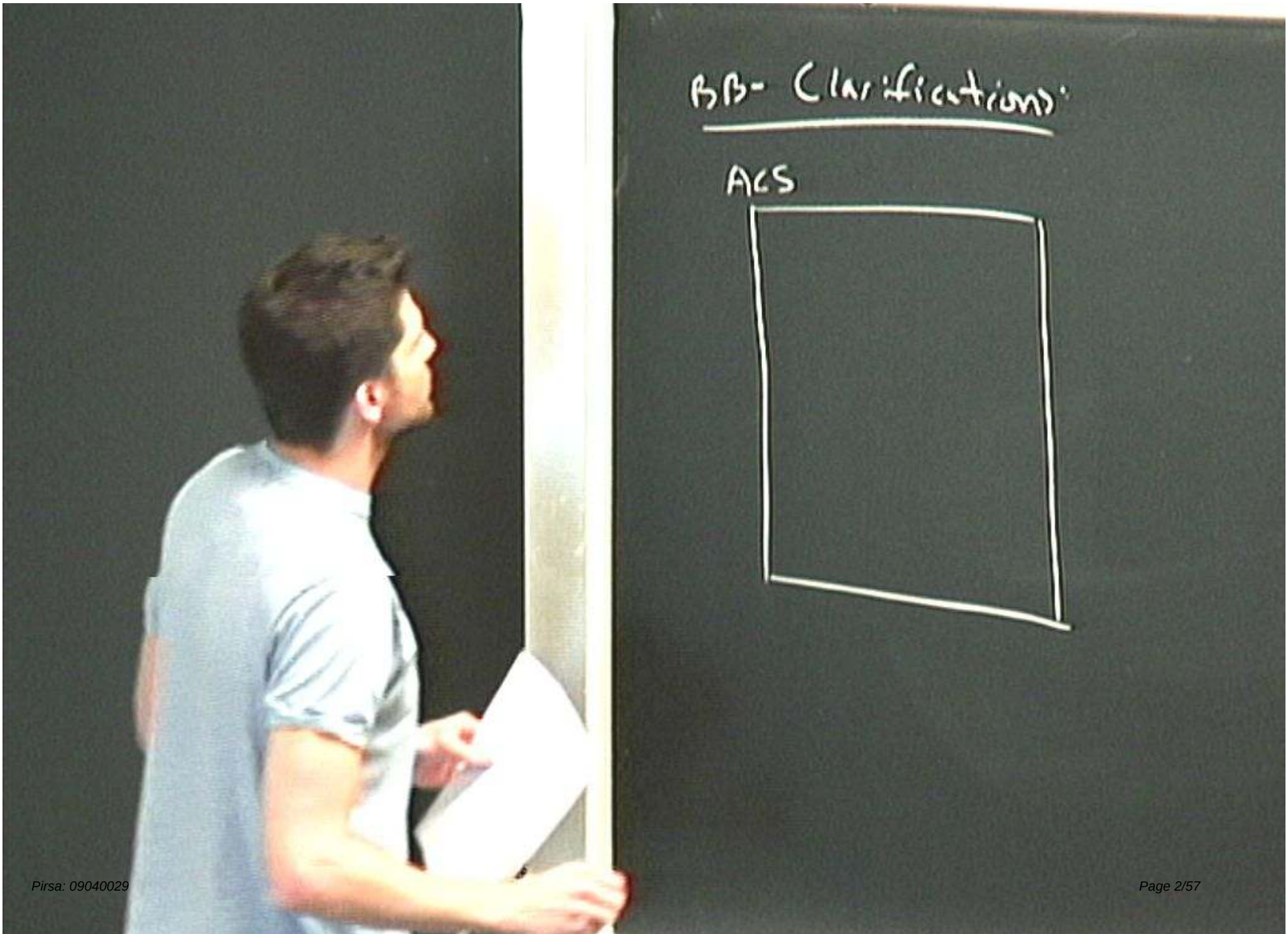
Title: Mini-Course on Mach's Principle - Lecture 8

Date: Apr 15, 2009 10:30 AM

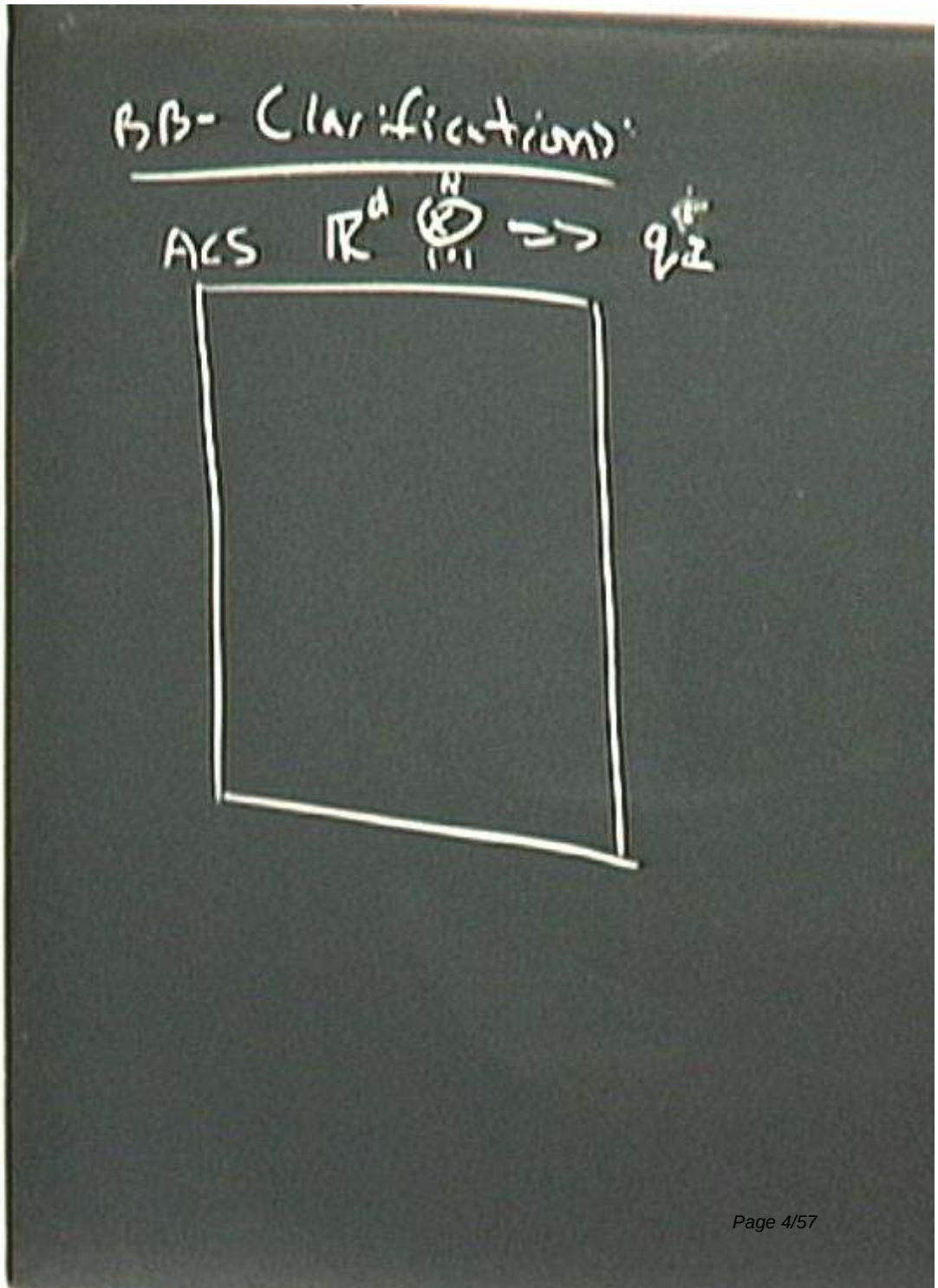
URL: http://pirsa.org/09040029

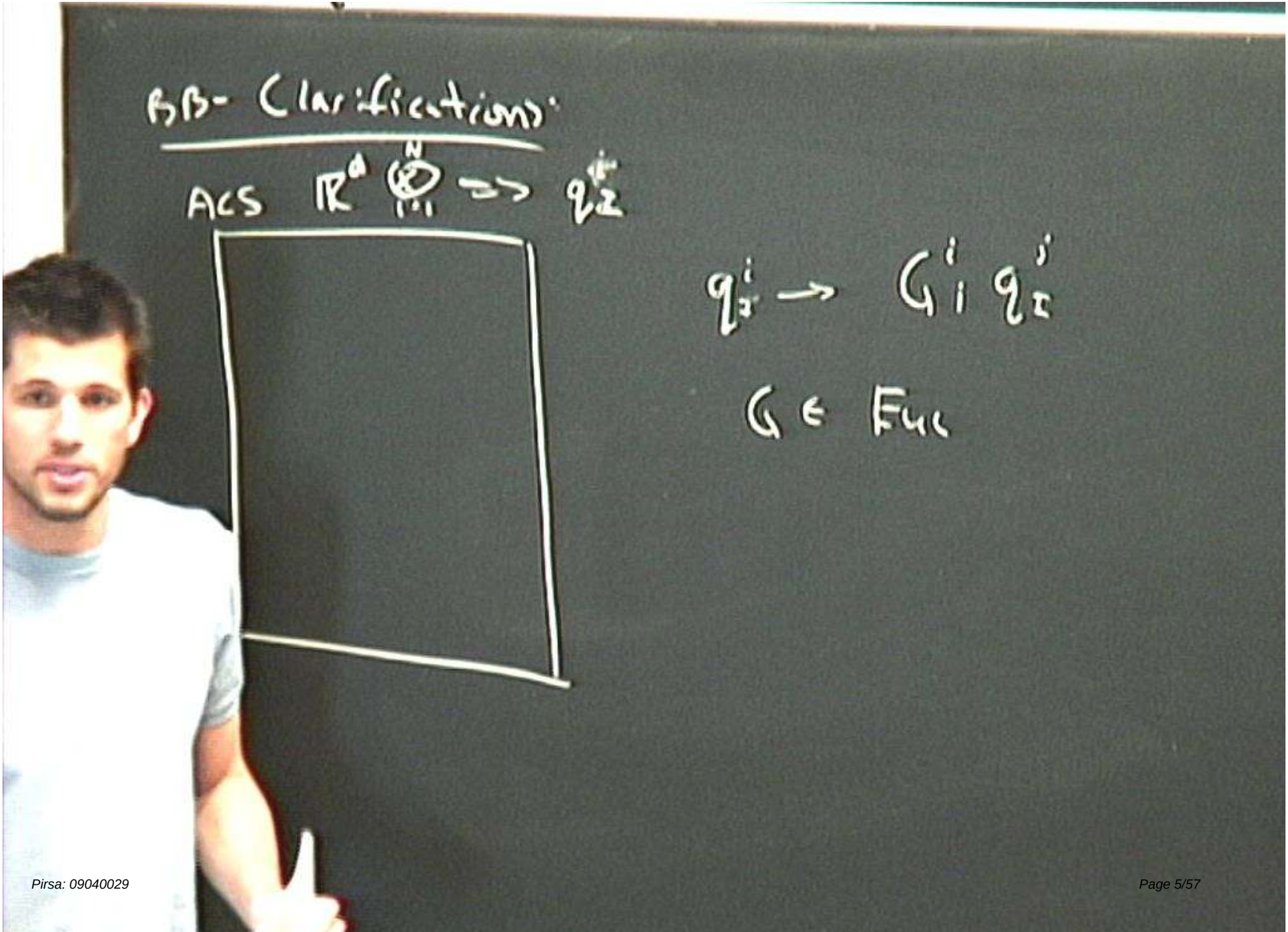
Abstract:


BB- Clarifications
ACS

BB- Clarifications:

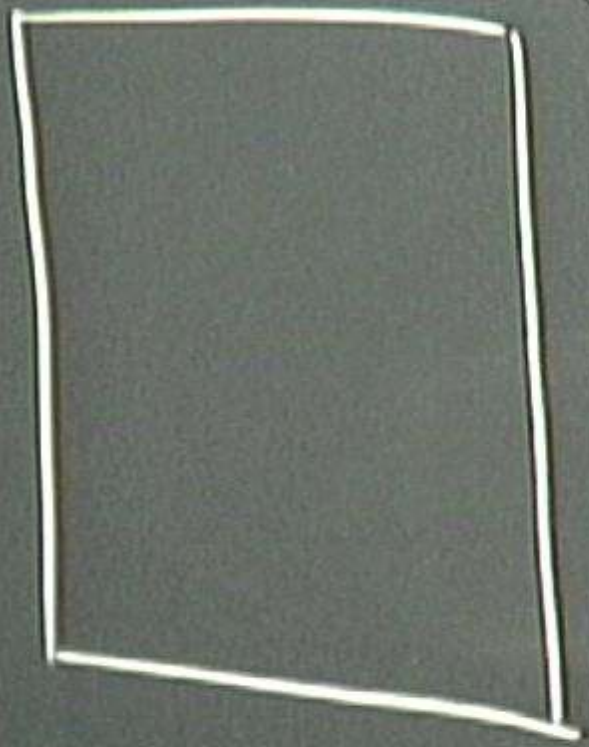
ACS $\mathbb{R}^d$ $\Rightarrow$ q^{th}


BB- Clarifications:
ACS $\mathbb{R}^4$ => q^{th}


BB- Clarifications:
ACS $\mathbb{R}^d \otimes_{i=1}^{N} \Rightarrow q^i_I$
$q^i_I \to G^i_j q^j_I$
$G \in Euc$

BB- Classifications:

ACS $\mathbb{R}^4 \otimes_{1,1} \Rightarrow q^{4n}_2$

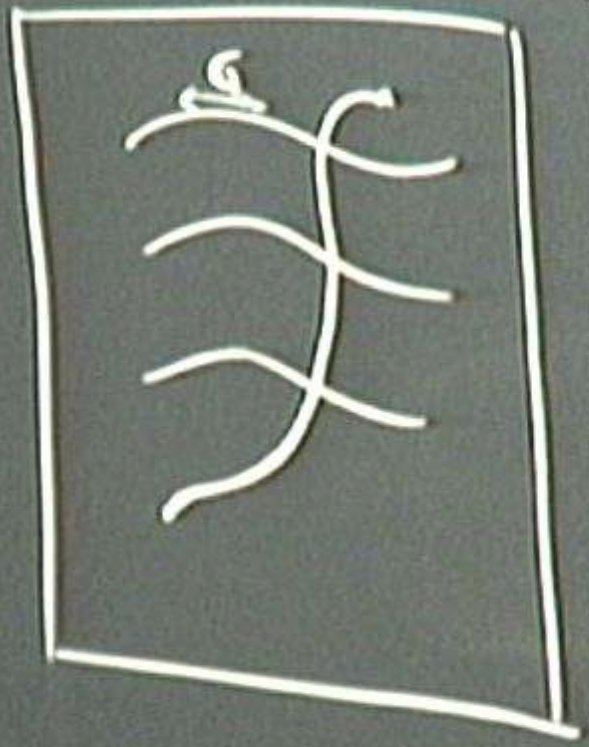


$q^i_x \rightarrow G^i_j q^j_x$

$G \in$ Euc / Sim / Lt...

BB- Clarifications:

ACS $\mathbb{R}^d \otimes_{1,1}^{N} \Rightarrow q_I^i$

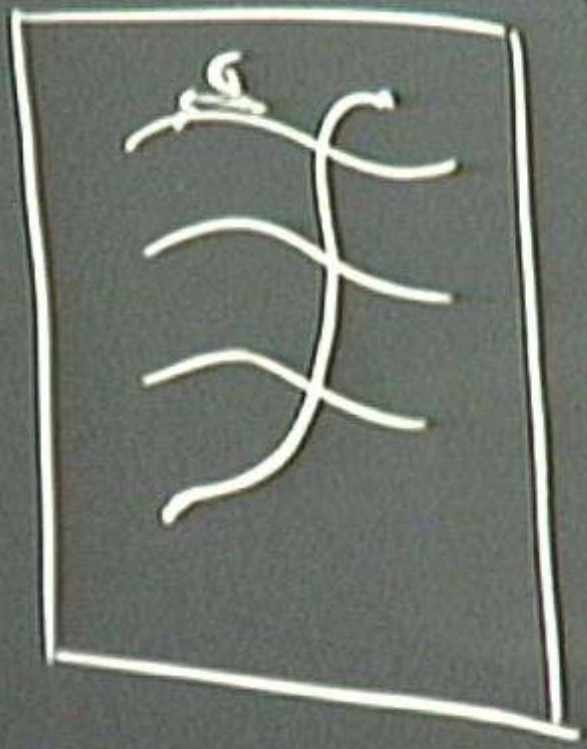


$q_I^i \to G^i{}_j \, q_I^j$

$G \in$ Euc/Sim/etc...

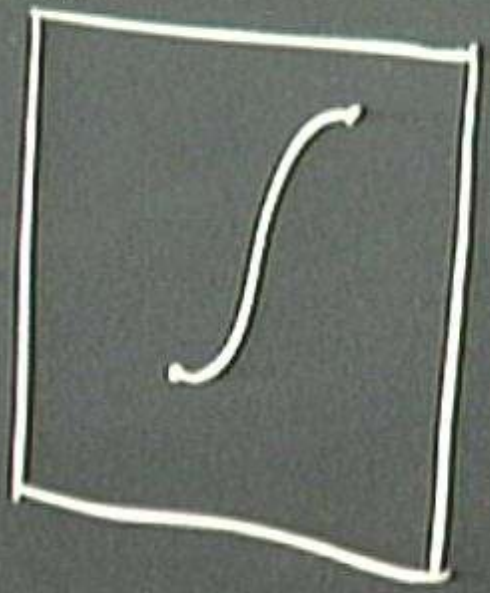
BB- Clarifications:

ACS $\mathbb{R}^d \otimes_{i=1}^{N} \Rightarrow q_I^i$



$q_I^i \to G^i{}_j\, q_I^j$

RCS $G \in$ Euc/Sim/etc...



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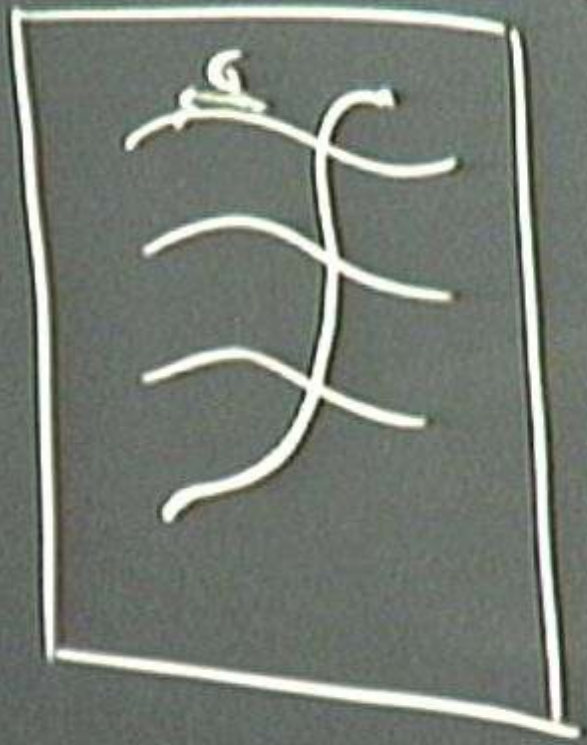
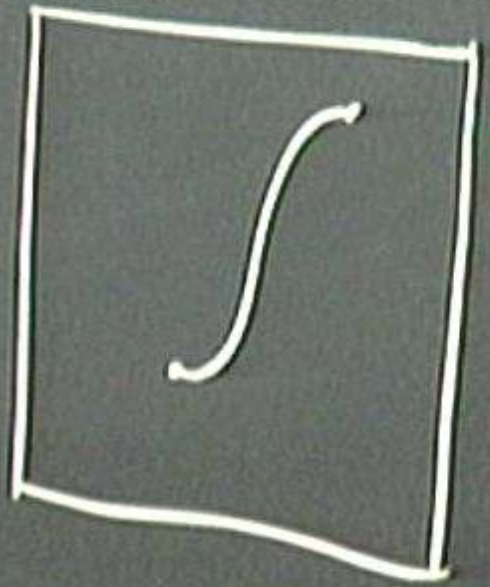


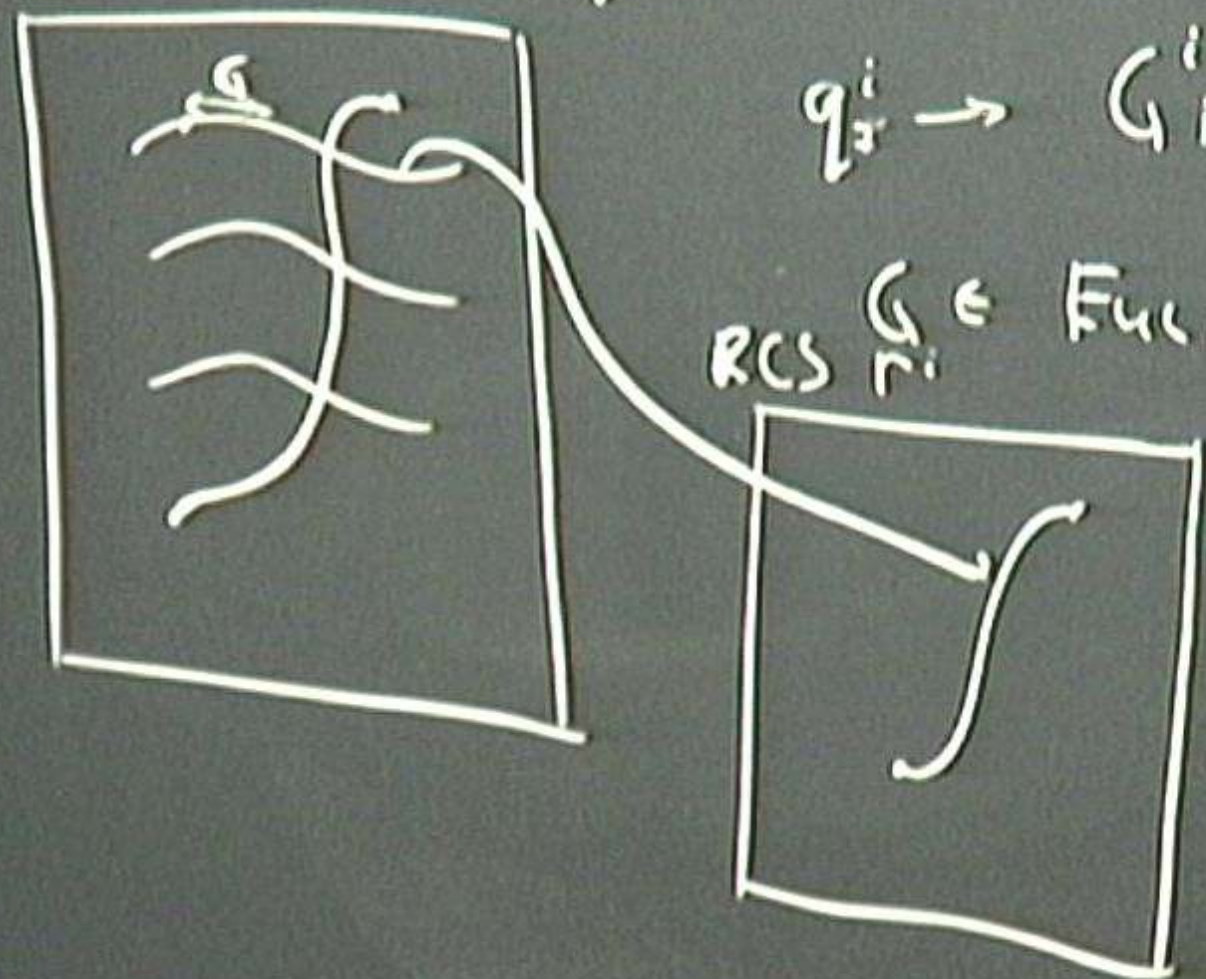
$q_I^i \to G^i_j \, q_I^j$

RCS r^i $G \in$ Euc/Sim/etc...



BB- Clarifications:

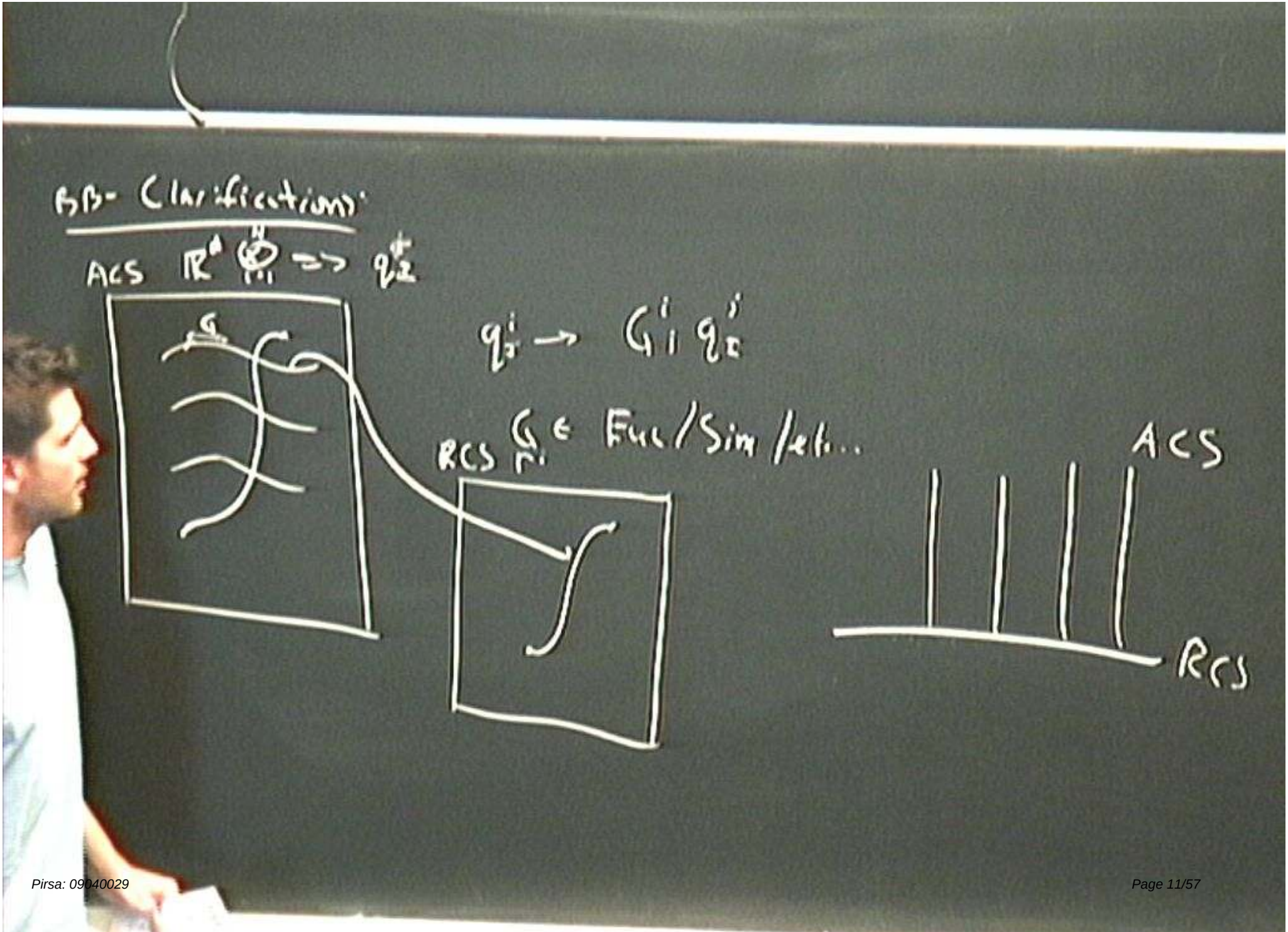
ACS $\mathbb{R}^d \otimes_{i=1}^{N} \Rightarrow q^i_I$



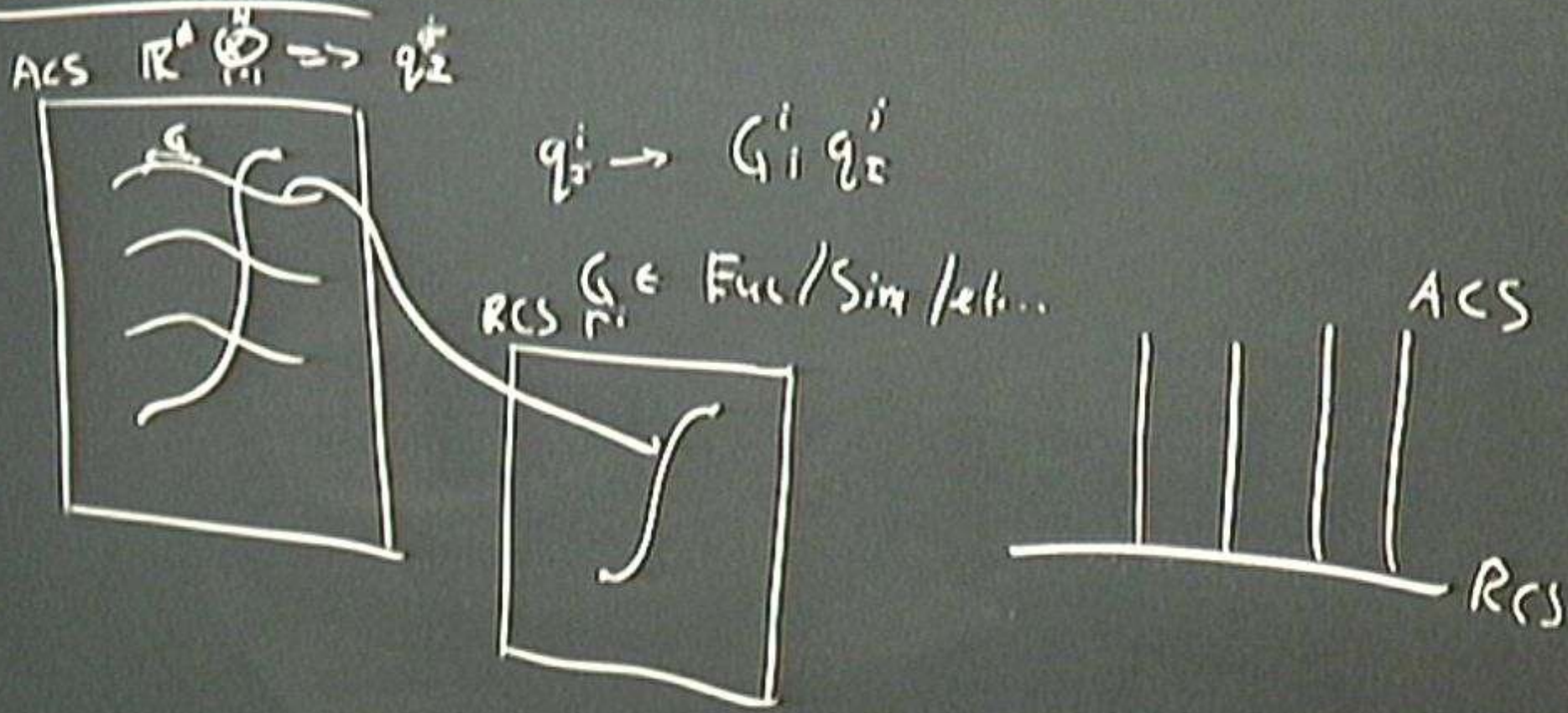
$q^i_I \to G^i_j \, q^j_I$

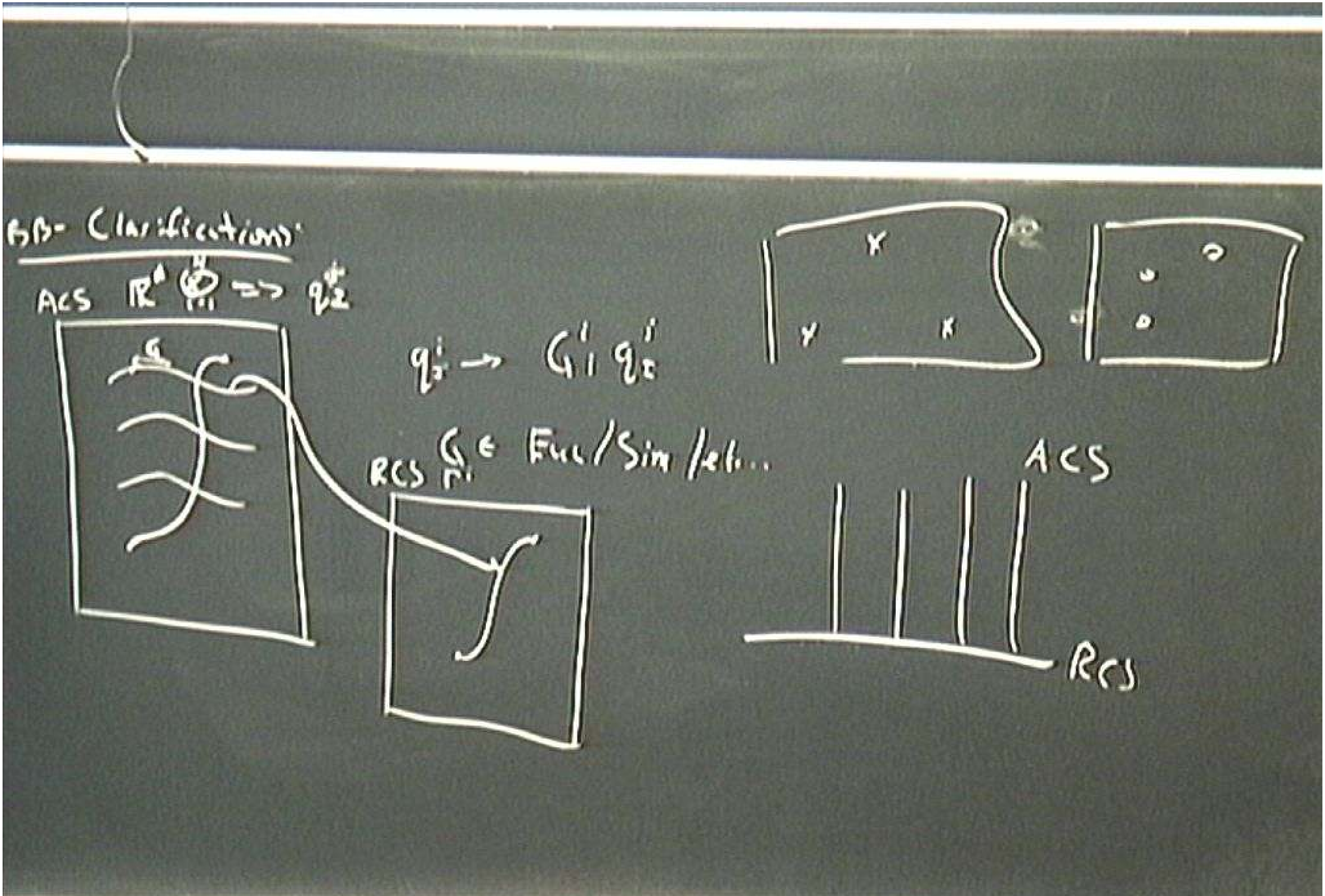
$G \in$ Euc/Sim/etc...

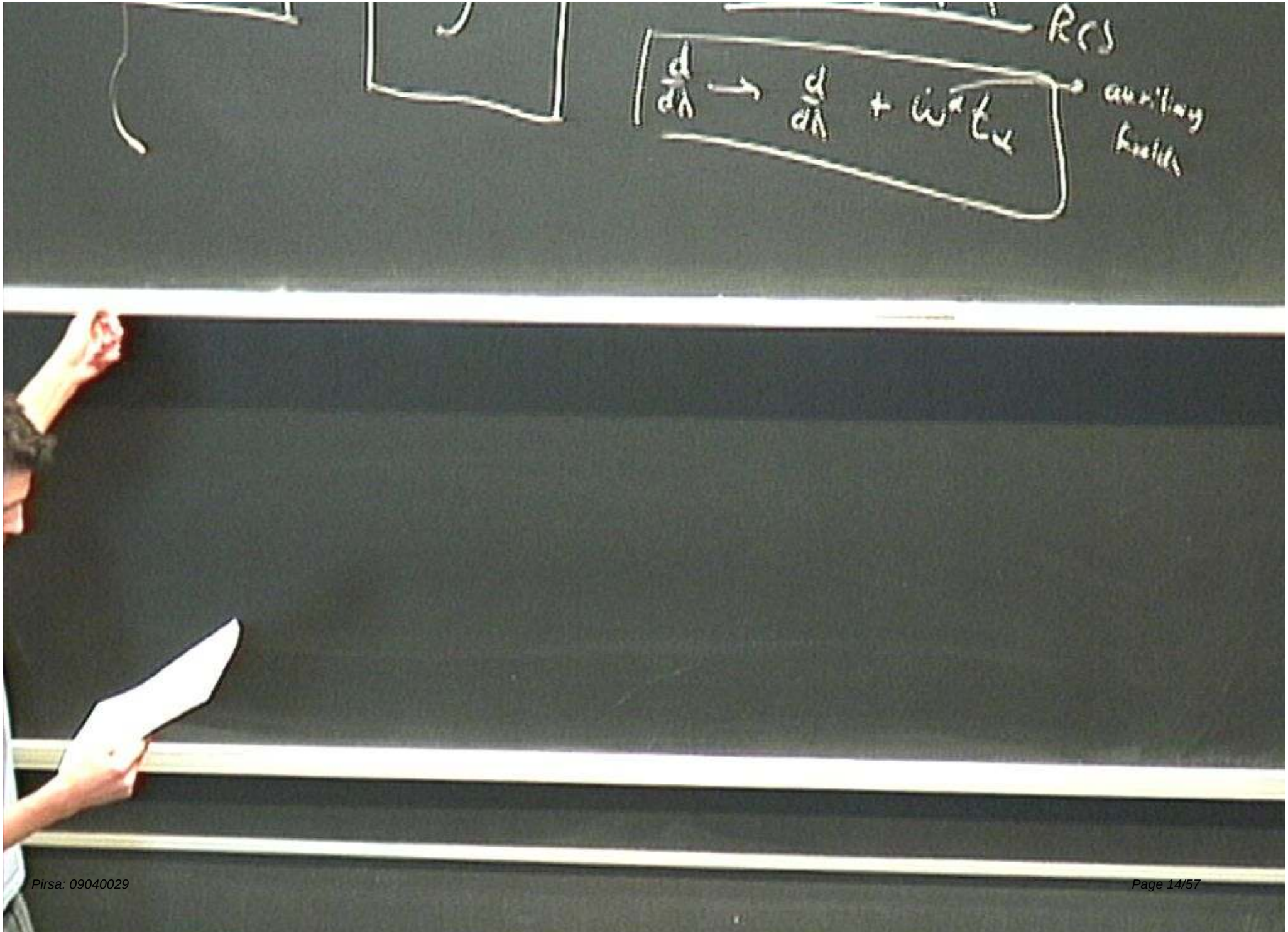
RCS r^i

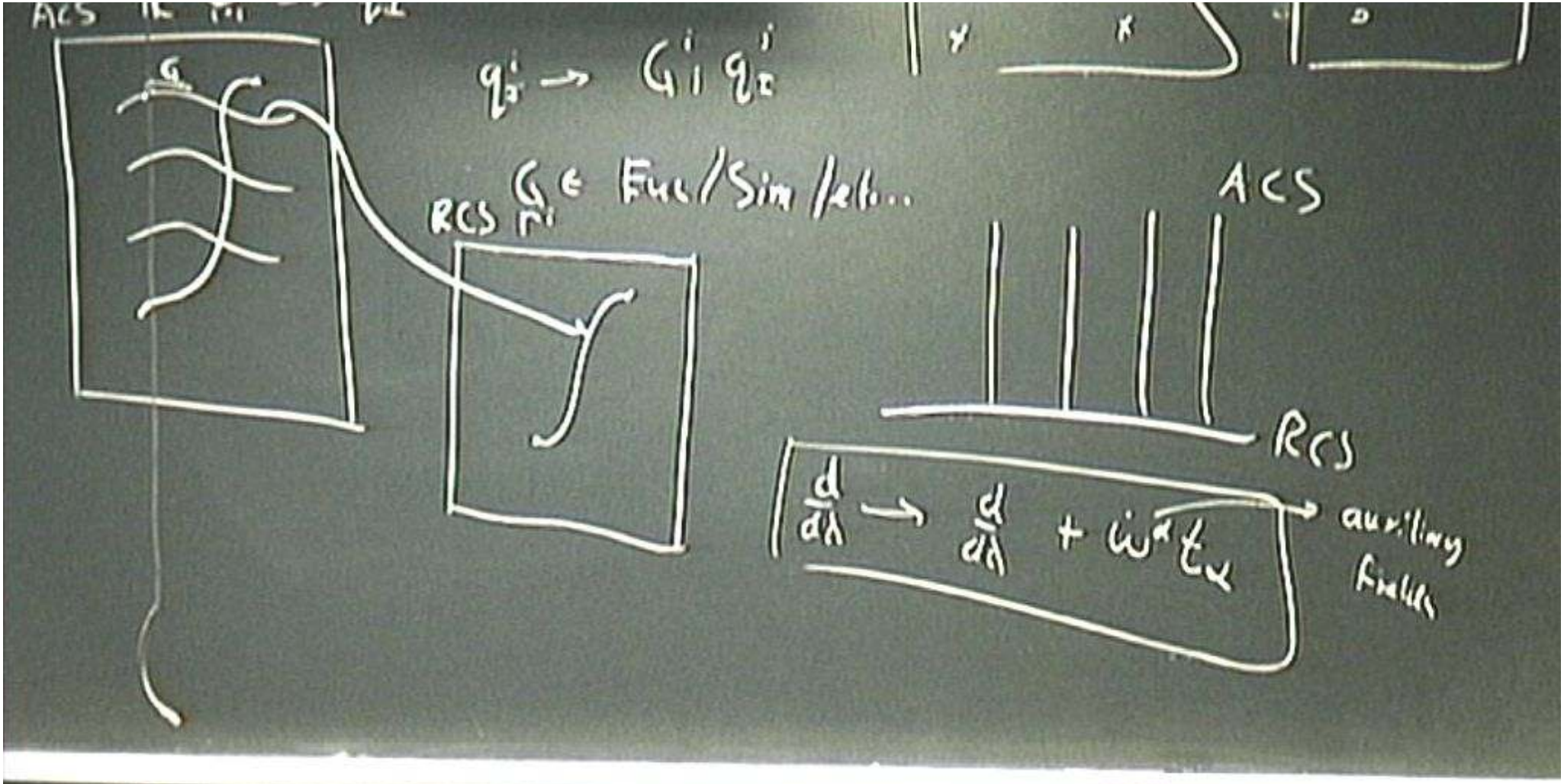

BB- Clarifications:
ACS
RCS
ACS
RCS

BB- Classifications:




BB- Classifications:
ACS
RCS
G ∈ Euc/Sim/etc...
ACS
RCS

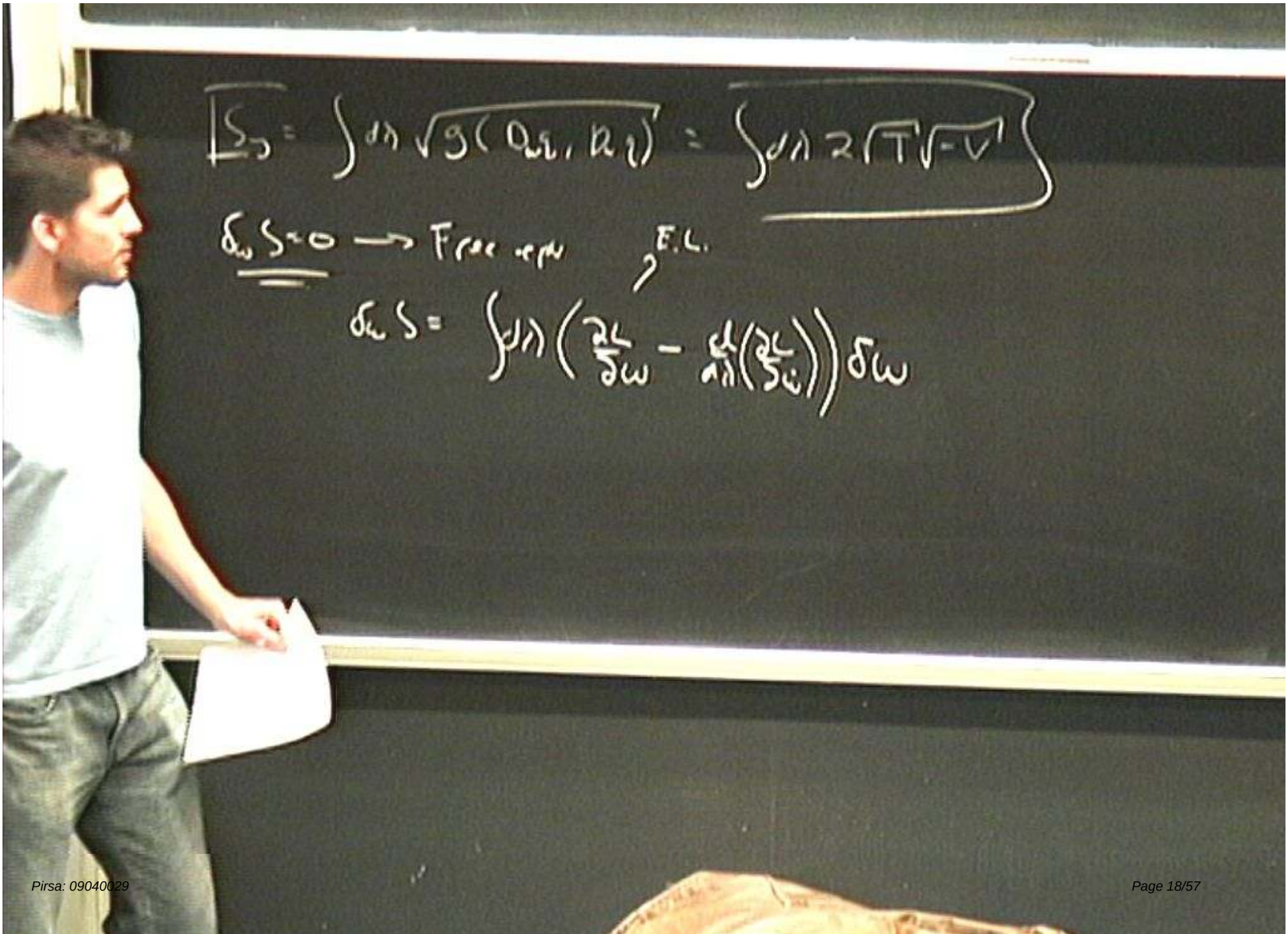

R(s)
$\frac{d}{dt} \to \frac{d}{dt} + \omega^a L_a$
auxiliary
fields

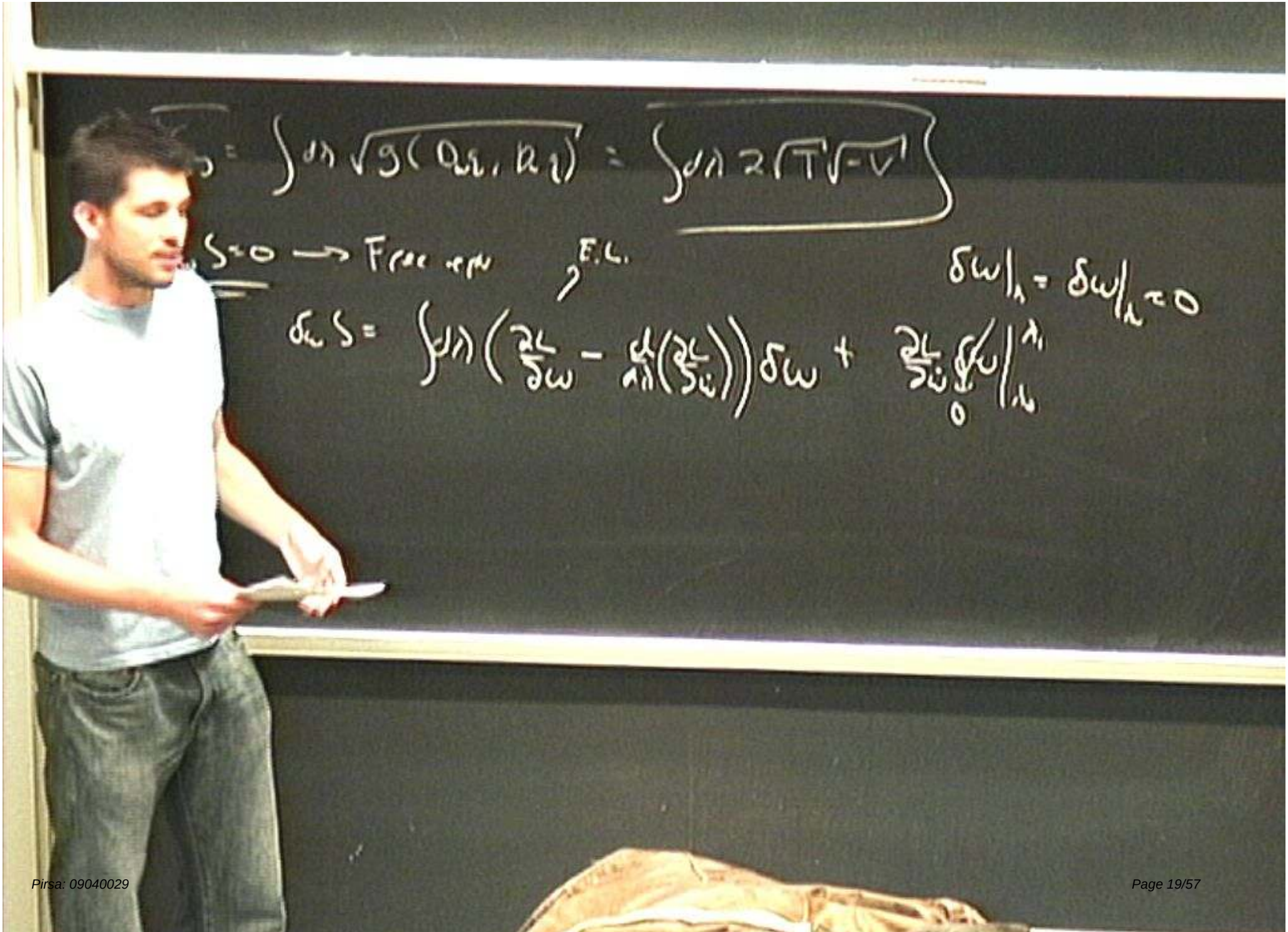

ACS
RCS
ACS
RCS
auxiliary

$$S_3 = \int d\lambda \sqrt{g(Q_{cl}, R_{cl})} = \int d\lambda \, 2\sqrt{T}\sqrt{-V}$$

$$\overline{S_J} = \int d\lambda \sqrt{g(Q_{cl}, R_{cl})} = \int d\lambda\, 2\sqrt{T}\sqrt{-V}$$

$\delta_{cl} S = 0 \rightarrow$ Free eqn


$S_g = \int d\lambda \sqrt{g(D_\omega q, D_\omega q)} = \int d\lambda\, 2\sqrt{T}\sqrt{-V}$
$\delta_\omega S = 0 \longrightarrow$ Free eqn
E.L.
$\delta_\omega S = \int d\lambda \left(\frac{\partial L}{\partial \omega} - \frac{d}{d\lambda}\left(\frac{\partial L}{\partial \dot\omega}\right)\right)\delta\omega$


$= \int d\lambda \sqrt{g(D_\lambda q, D_\lambda q)} = \int d\lambda\, 2\sqrt{T}\sqrt{-V}$
$S=0 \longrightarrow$ Free eqn
E.L.
$\delta\omega|_{\lambda_1} = \delta\omega|_{\lambda_2} = 0$
$\delta_\omega S = \int d\lambda \left(\frac{\partial L}{\partial \omega} - \frac{d}{d\lambda}\left(\frac{\partial L}{\partial \dot\omega}\right)\right)\delta\omega + \frac{\partial L}{\partial \dot\omega}\delta\omega\Big|_{\lambda_2}^{\lambda_1}$
0

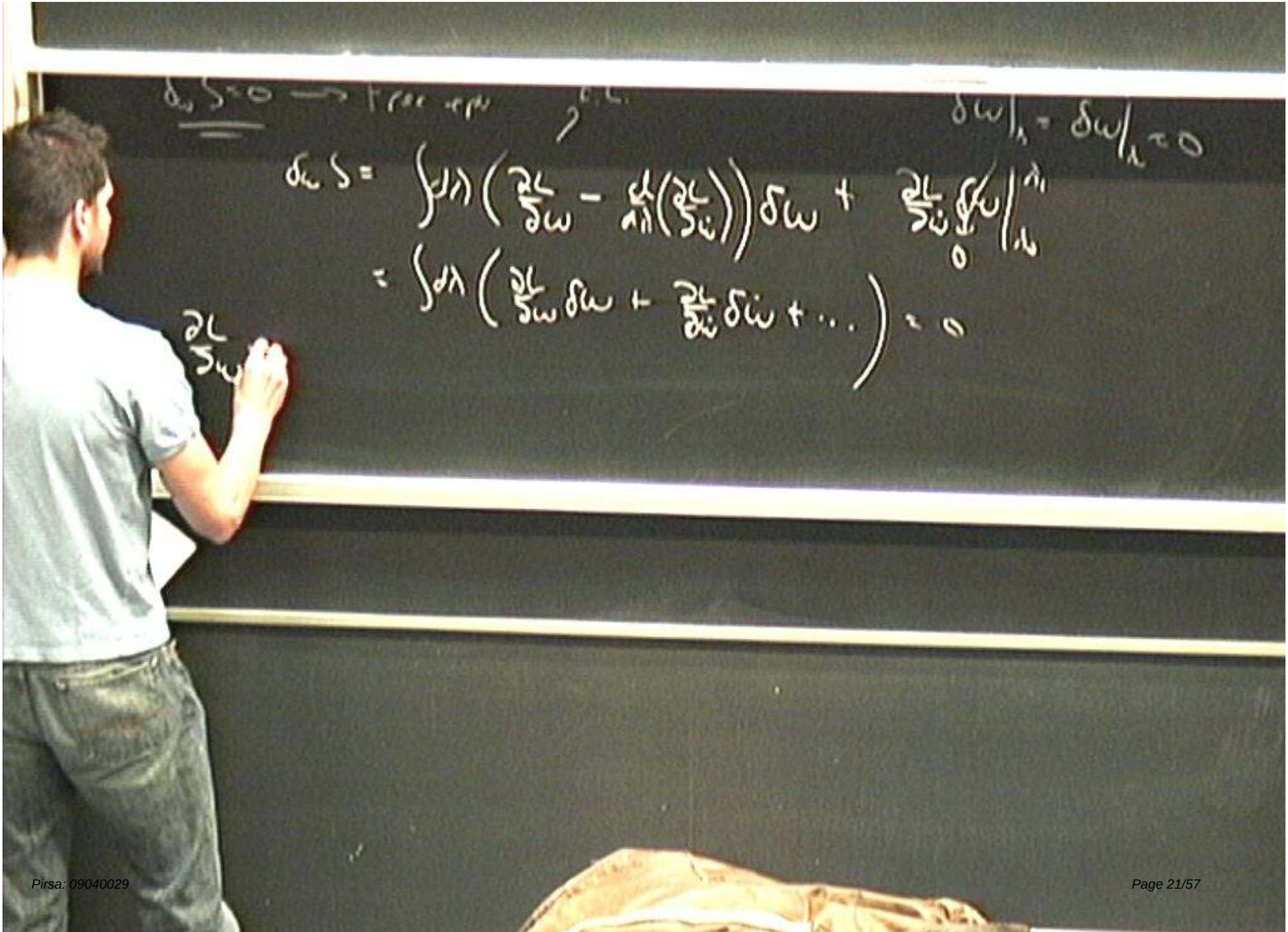
$$S_0 = \int d\lambda \sqrt{g(q_{cl}, \dot{q}_{cl})} = \int d\lambda\, 2\sqrt{T}\sqrt{-V}$$

$\delta_\omega S = 0 \longrightarrow$ Free eqn $\overset{E.L.}{\longrightarrow}$

$\delta\omega|_{\lambda_1} = \delta\omega|_{\lambda_2} = 0$

$$\delta_\omega S = \int d\lambda \left(\frac{\partial L}{\partial \omega} - \frac{d}{d\lambda}\left(\frac{\partial L}{\partial \dot{\omega}}\right)\right)\delta\omega + \frac{\partial L}{\partial \dot{\omega}}\delta\omega\Big|_{\lambda_0}^{\lambda_1}$$

$$= \int d\lambda \left(\frac{\partial L}{\partial \omega}\delta\omega + \frac{\partial L}{\partial \dot{\omega}}\delta\dot{\omega} + \dots \right)$$


$\delta_\omega S = 0 \longrightarrow$
$\delta\omega|_{\lambda_1} = \delta\omega|_{\lambda_2} = 0$
$\delta_\omega S = \int d\lambda \left(\frac{\partial L}{\partial \omega} - \frac{d}{d\lambda}\left(\frac{\partial L}{\partial \dot{\omega}}\right)\right)\delta\omega + \frac{\partial L}{\partial \dot{\omega}}\delta\omega\Big|_{\lambda_1}^{\lambda_2}$
$= \int d\lambda \left(\frac{\partial L}{\partial \omega}\delta\omega + \frac{\partial L}{\partial \dot{\omega}}\delta\dot{\omega} + \dots \right) = 0$
$\frac{\partial L}{\partial \omega}$

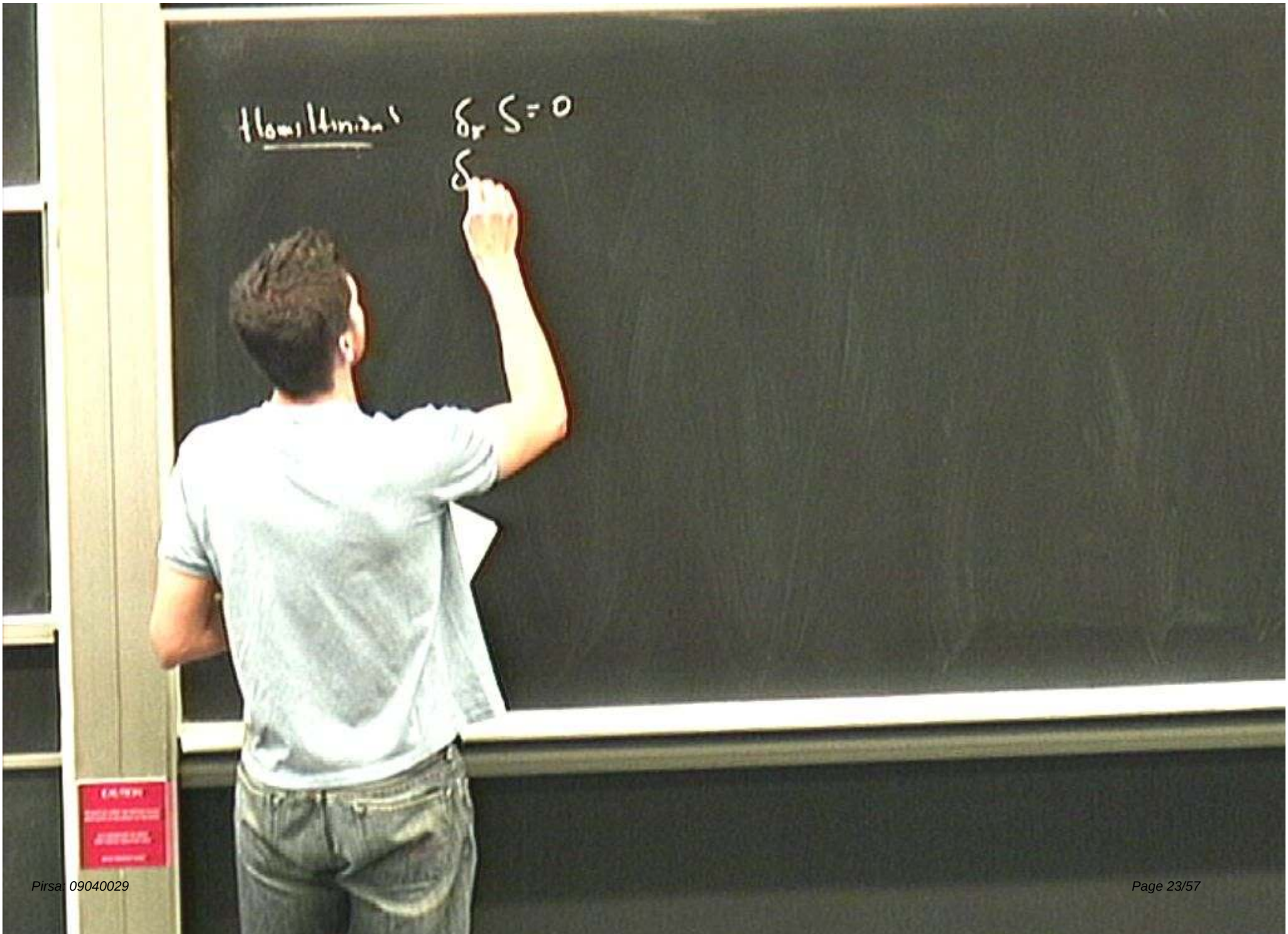
$\delta_\omega S = 0 \longrightarrow$ free eqn ?

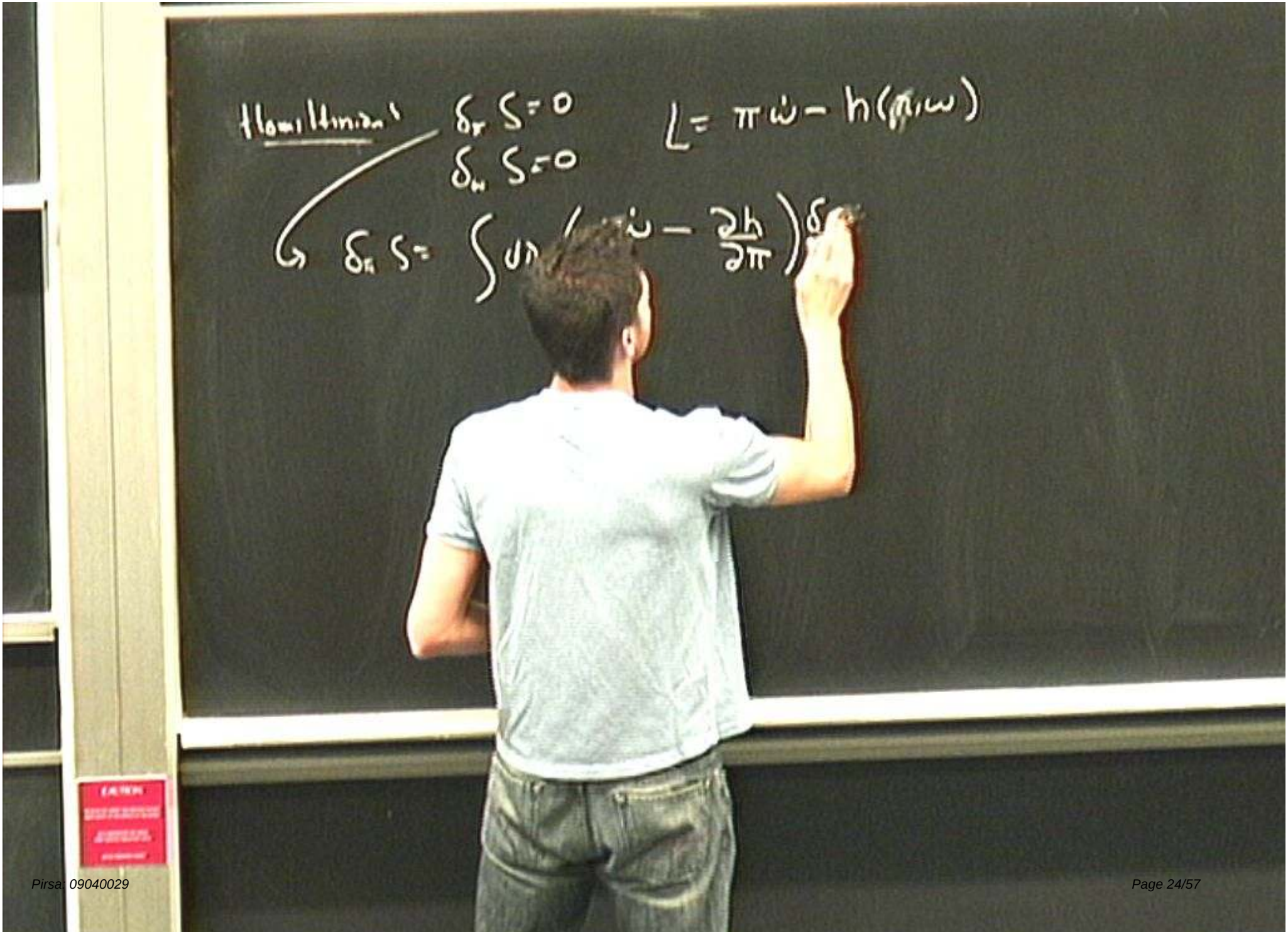
$\delta\omega|_{\lambda_1} = \delta\omega|_{\lambda_2} = 0$

$$\delta_\omega S = \int d\lambda \left(\frac{\partial L}{\partial \omega} - \frac{d}{d\lambda}\left(\frac{\partial L}{\partial \dot{\omega}}\right) \right) \delta\omega + \underbrace{\frac{\partial L}{\partial \dot{\omega}} \delta\omega \Big|_{\lambda_1}^{\lambda_2}}_{0}$$

$$= \int d\lambda \left(\frac{\partial L}{\partial \omega} \delta\omega + \frac{\partial L}{\partial \dot{\omega}} \delta\dot{\omega} + \dots \right) = 0$$

$\boxed{\frac{\partial L}{\partial \omega} = 0}$ and $\boxed{\frac{\partial L}{\partial \dot{\omega}} = 0} \Rightarrow$ Mach condition


Hamiltonian
$\delta_r S = 0$
δ


Hamiltonian
$\delta_\pi S = 0$
$\delta_\omega S = 0$
$L = \pi\dot{\omega} - h(\pi,\omega)$
$\delta_\pi S = \int dt \left(\dot{\omega} - \frac{\partial h}{\partial \pi}\right)\delta$

Hamiltonian $\delta_\pi S = 0$ $\quad \delta_\omega S = 0$ $\quad L = \pi\dot{\omega} - h(\pi,\omega)$

$$\delta_\pi S = \int d\lambda \left(\dot{\omega} - \frac{\partial h}{\partial \pi} \right) \delta\pi = 0$$

$$\boxed{\dot{\omega} = \frac{\partial h}{\partial \pi} = \{\omega, h\}} \quad (HI)$$

$$\delta_\omega S = \int d\lambda \left(-\frac{\partial h}{\partial \omega}\delta\omega + \pi\,\delta\dot{\omega} \right)$$

$$= \int d\lambda \left(-\frac{\partial h}{\partial \omega}\delta\omega - \dot{\pi}\,\delta\omega \right) + \cdots$$

$$\dot{\pi} = -\frac{\partial h}{\partial \omega} =$$

Hamiltonian: $\delta_\pi S = 0$ $\quad L = \pi\dot{\omega} - h(\pi,\omega)$

$\delta_\omega S = 0$

$$\delta_\pi S = \int d\lambda \left(\dot{\omega} - \frac{\partial h}{\partial \pi} \right) \delta\pi = 0$$

$$\boxed{\dot{\omega} = \frac{\partial h}{\partial \pi} = \{\omega, h\}} \quad (H1)$$

$$\delta_\omega S = \int d\lambda \left(-\frac{\partial h}{\partial \omega}\delta\omega + \pi\,\delta\dot{\omega} \right)$$

$$= \int d\lambda \left(-\frac{\partial h}{\partial \omega}\delta\omega - \dot{\pi}\,\delta\omega \right) + \oint$$

$\to$ Free $\to$ $\{\pi, h\} = 0$

$\pi = 0 \Rightarrow \dot{\pi} = 0$

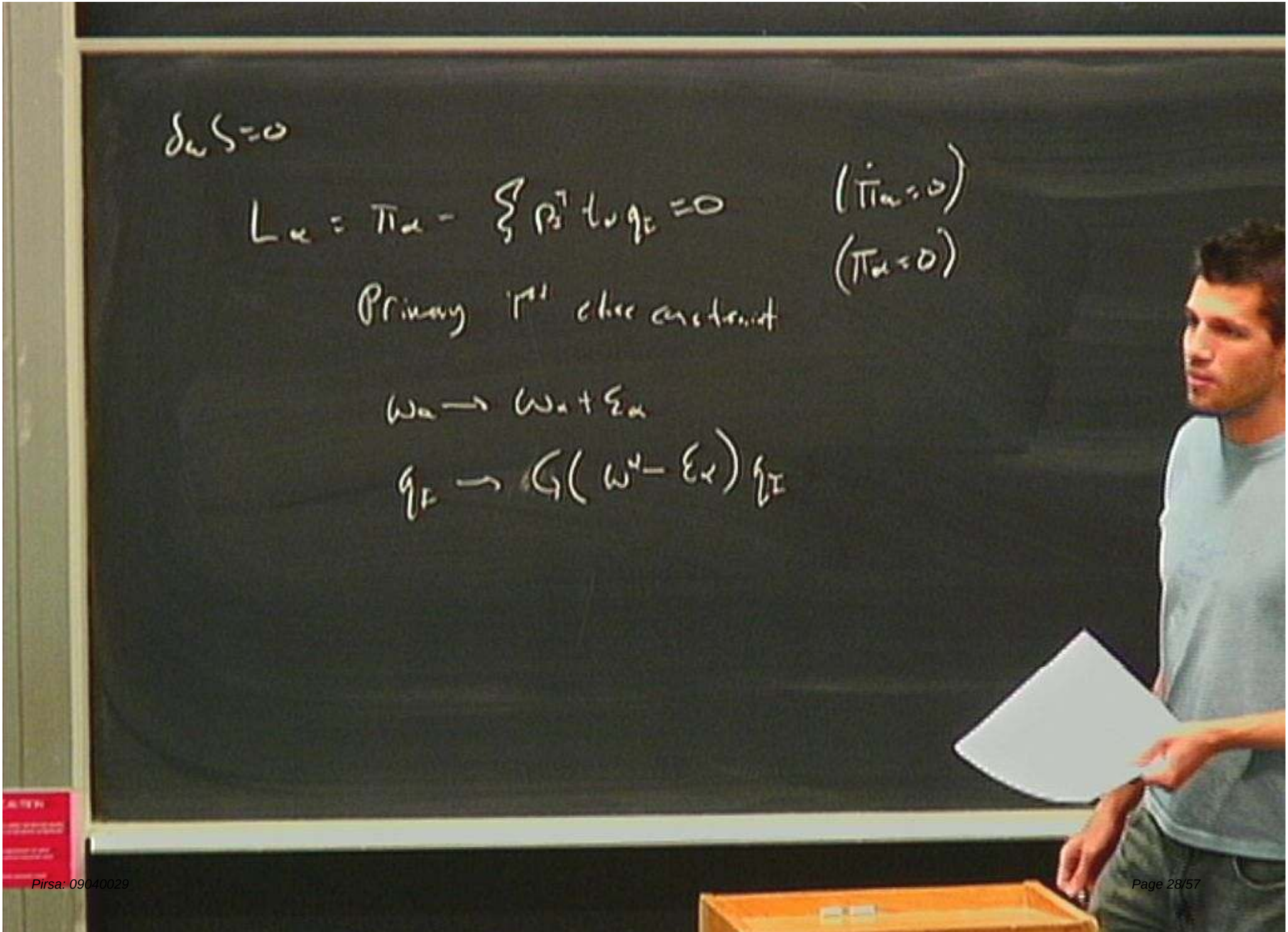
$\boxed{H2 \oplus \pi = 0}$

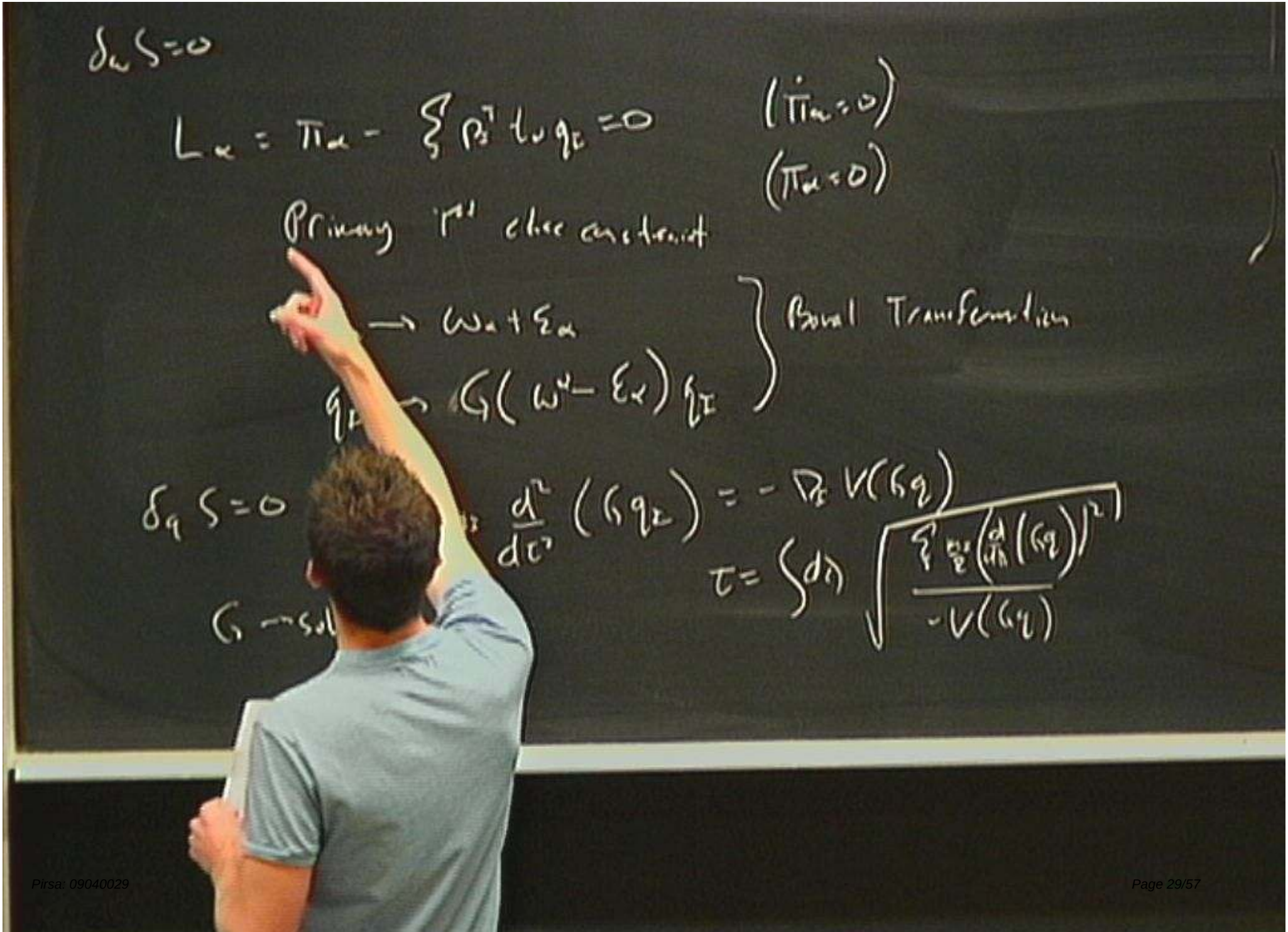
$$\boxed{\dot{\pi} = -\frac{\partial h}{\partial \omega} = \{\pi, h\}}$$

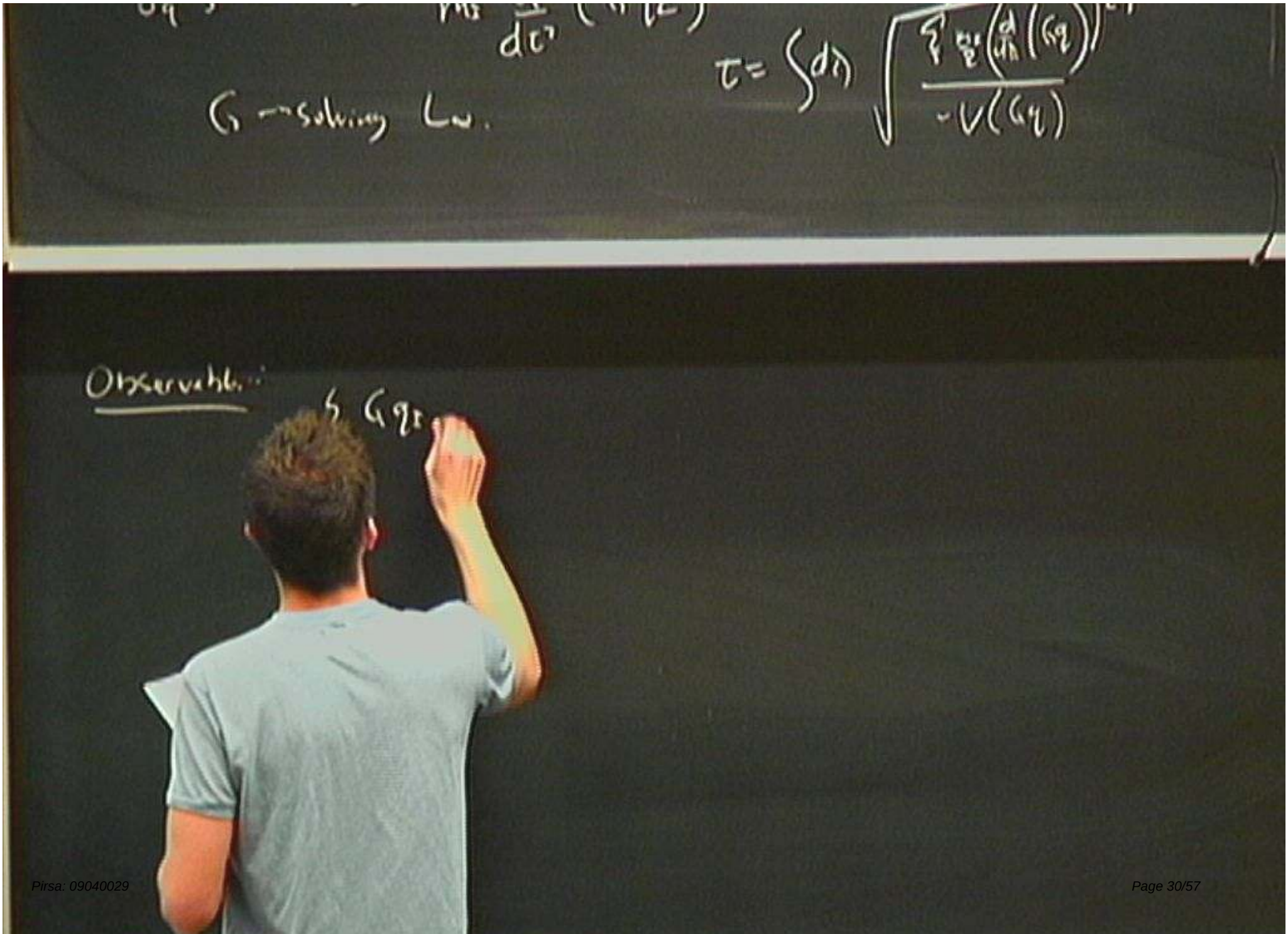
H2

$$\delta_\omega S = 0$$

$$L_\alpha = \pi_\alpha - \sum_i \beta^i t_\alpha q_i = 0$$


$\delta_\omega S = 0$
$L_\alpha = \pi_\alpha - \sum p_I^T t_\alpha q_I = 0$
$(\dot{\pi}_\alpha = 0)$
$(\pi_\alpha = 0)$
Primary 1st class constraint
$\omega_\alpha \to \omega_\alpha + \varepsilon_\alpha$
$q_I \to G(\omega^\alpha - \varepsilon_\alpha) q_I$


$\delta_\omega S = 0$
$L_\alpha = \pi_\alpha - \sum \beta_\alpha^I \dot{q}_I = 0$
$(\dot{\pi}_\alpha = 0)$
$(\pi_\alpha = 0)$
Primary 1st class constraint
$\omega_\alpha \to \omega_\alpha + \varepsilon_\alpha$
$q_I \to G(\omega^\alpha - \varepsilon_\alpha) q_I$
Gauge Transformation
$\delta_q S = 0$
$\frac{d^2}{d\tau^2}(G q_I) = -\partial_I V(G q)$
$\tau = \int d\eta \sqrt{\frac{\sum g_{IJ} \left(\frac{\partial}{\partial \eta}(G q)\right)^2}{-V(G q)}}$
$G \to$ sol


G - solving Law.
Observables:

$$\tau = \int d\tau \sqrt{\frac{\sum_i \frac{m_i}{2}\left(\frac{d}{d\tau}(Gq)\right)^2}{-V(Gq)}}$$

$G \to$ solving L_ω.

Observables:

$$\{ Gq_I, L_\omega \} = \{ \exp(i\omega_\alpha t_\alpha) q_I, \Pi_\alpha - \sum_i p_i^2 t_\beta q_i \}$$

$$=$$

$d\tau^2$

$$\tau = \int d\lambda \sqrt{\frac{\sum_i \frac{m_i}{2}\left(\frac{d}{d\lambda}(Gq)\right)}{-V(Gq)}}$$

$G \to$ solving L_α.

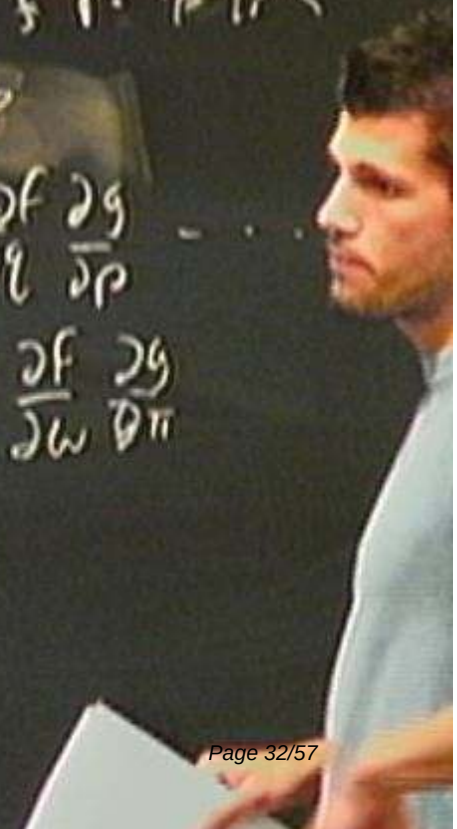
$(\omega_\alpha, \pi_\beta) = 1$

Observable:

$$\{Gq_I, L_\alpha\} = \{\exp(i\omega_\alpha t_\alpha) q_I, \pi_\beta - \sum_i p_i^2 t_\beta q_i\}$$

$$= t_\alpha G q_I - G t_\alpha q_I = 0$$

$$\{f(p,q,\pi,\omega), g, \ldots\} = \sum_{p,q,\pi,\omega} \frac{\partial f}{\partial q}\frac{\partial g}{\partial p} - \ldots + \frac{\partial f}{\partial \omega}\frac{\partial g}{\partial \pi}$$



G_t — solving L_ω.

$$\tau = \int d\eta \sqrt{\frac{\xi' g_{ij}(\phi^{-1}(q))}{-V(q)}}$$

$$(\omega_\alpha, \pi_\beta) = 1$$

Observables:

$$\{ G_t q_i, L_\alpha \} = \{ \exp(t\omega_\alpha L_\alpha) q_i, \pi_\beta - \{ p_i^{-1}(p) q_i \}$$

$$= \partial_\alpha G_t q_i - G_t \{ L_\alpha, q_i \} = 0$$

$$\{ F(p, q, \pi, \omega), \ldots \} = \sum \frac{\partial f}{\partial q} \frac{\partial g}{\partial p} - \ldots$$

$$+ \frac{\partial f}{\partial \omega} \frac{\partial g}{\partial \pi}$$

G_t — solving L_w.

$$\tau = \int d\eta \sqrt{\frac{\sum_i p_i^2\left(\frac{\partial}{\partial q_i}(Gq)\right)}{-V(Gq)}}$$

$$(\omega_\alpha, \pi_\beta) = \delta$$

Observables:

$$\{G q_i, L_\alpha\} = \{\exp(t\omega_\alpha t_\alpha) q_i, \pi_\beta - \sum p_i^2 f_\beta q_i\}$$

$$= t_\alpha G q_i - G t_\alpha q_i = 0$$

$G = \exp t_\alpha$

$\{t_\alpha, t_\beta\} = 0$

$$\{F(p,q,\pi,\omega), G(\dots)\} = \sum \frac{\partial F}{\partial q}\frac{\partial G}{\partial p} - \dots + \frac{\partial F}{\partial \omega}\frac{\partial G}{\partial \pi}$$

$$\tau = \int d\eta \sqrt{\frac{\sum_\alpha \frac{m_\alpha}{2}\left(\frac{d}{d\eta}(Gq)\right)}{-V(Gq)}}$$

$G \to$ solving L_α.

$(\omega_\alpha, \pi_\beta) = 1$

Observable:

$$\{Gq_I, L_\alpha\} = \{\exp(i\omega_\alpha t_\alpha) q_I, \pi_\beta - \sum_I p_I^i t_\beta q_I\}$$

$$= t_\alpha G q_I - G t_\alpha q_I = 0$$

$G = \exp t_\alpha$

$$\{F(p,q,\pi,\omega), g, \ldots\} = \sum_{p,q,\pi,\omega} \frac{\partial F}{\partial q}\frac{\partial g}{\partial p} - \ldots + \frac{\partial F}{\partial \omega}\frac{\partial g}{\partial \pi}$$

$\{t_\alpha, t_\alpha\} = 0$

$\{G^{-1}p', L_\alpha\} = 0$

$$\tau = \int d\tau \sqrt{\frac{\sum_i m_i \left(\frac{d}{d\tau}(Gq)\right)^2}{-V(Gq)}}$$

$G \to$ solving L_α.

$(\omega_\alpha, \pi_\beta) = 1$

Observables:

$$\{Gq_I, L_\alpha\} = \{\exp(i\omega_\alpha t_\alpha) q_I, \pi_\beta - \sum_I p_I^T t_\beta q_I\}$$

$$= t_\alpha G q_I - G t_\alpha q_I = 0$$

$G = \exp t_\alpha$

$$\{F(p,q,\pi,\omega), g, \ldots\} = \sum_{p,q,\pi,\omega} \frac{\partial F}{\partial q}\frac{\partial g}{\partial p} - \ldots + \frac{\partial F}{\partial \omega}\frac{\partial g}{\partial \pi}$$

$\{t_\alpha, t_\beta\}$

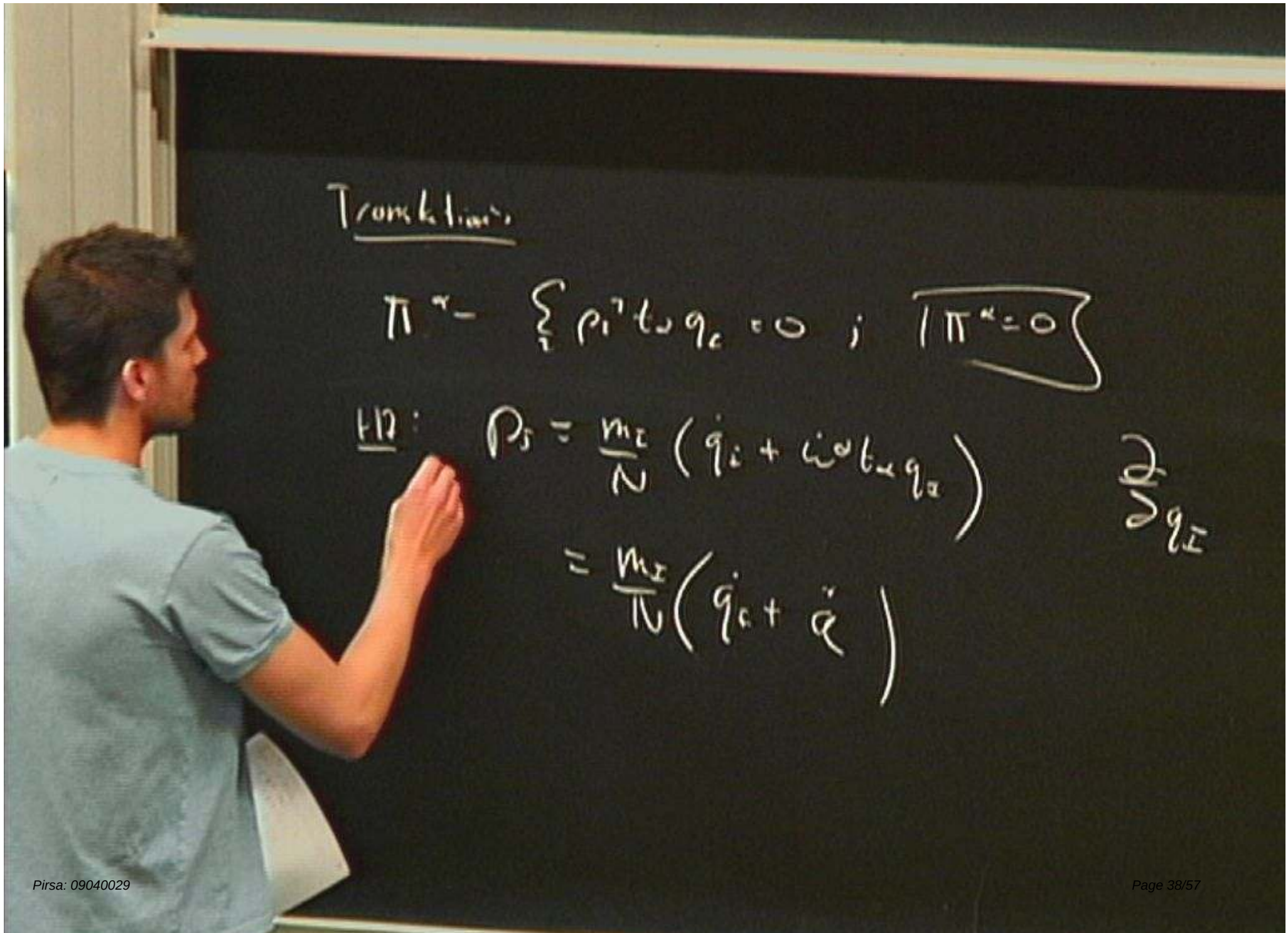
$$\{G^{-1}p', L_\alpha\} = 0$$

τ, q, p commute with L_α !!

Translations:

$$\Pi^{\alpha} - \sum_i p_i^{\alpha} t_{i\alpha} q_{i\alpha} = 0 \; ; \quad \boxed{\Pi^{\alpha} = 0}$$

HO: $p_j = \frac{m_i}{N} \left(\dot{q}_i + i\omega t_{i\alpha} q_{i\alpha} \right)$


Translations
$\Pi^\alpha - \sum_i p_i^\alpha t_\alpha q_i = 0$; $\boxed{\Pi^\alpha = 0}$
HD: $p_I = \frac{m_I}{N}(\dot{q}_I + \dot{\omega}^\alpha t_\alpha q_I)$
$= \frac{m_I}{N}(\dot{q}_I + \dot{\alpha})$
$\frac{\partial}{\partial q_I}$

$$\Pi^{\alpha} = \sum_i p_i^{\alpha} \dot{q}_i = 0 \; ; \quad \boxed{\Pi^{\alpha} = 0}$$

HW: $\boxed{P_j = \frac{m_i}{N}\left(\dot{q}_i + \omega b_{i\alpha} q_\alpha\right)}$ $\frac{\partial}{\partial q_i}$

$$= \frac{m_i}{N}\left(\dot{q}_i + \ddot{\alpha}\right)$$

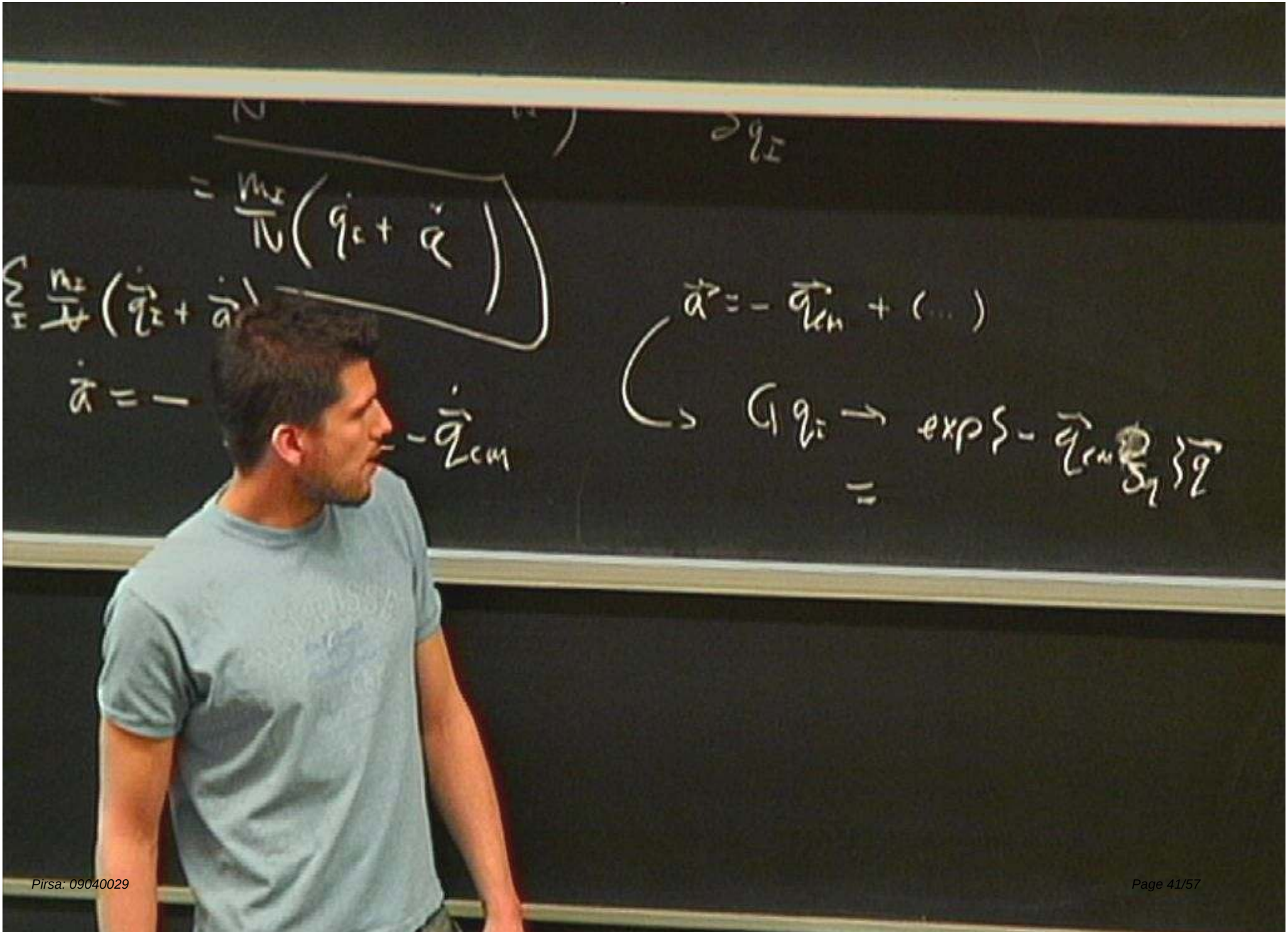
$$\sum_i \frac{m_i}{N}\left(\dot{q}_i + \ddot{\alpha}\right) = 0$$

HW: $\boxed{p_I = \frac{m_I}{N}\left(\dot{q}_I + \dot{\omega}^\alpha t_\alpha q_I\right)}$ $\qquad \frac{\partial}{\partial q_I}$

$$= \frac{m_I}{N}\left(\dot{q}_I + \dot{\vec{a}}\right)$$

$$\sum_I \frac{m_I}{N}\left(\dot{\vec{q}}_I + \dot{\vec{a}}\right) = 0 \qquad \vec{a} = -\vec{q}_{cm} + (\ldots)$$

$$\dot{\vec{a}} = -\frac{\sum_I m_I \vec{q}_I}{M_{tot}} = -\dot{\vec{q}}_{cm}$$


$= \frac{m_I}{N}(\dot{q}_I + \dot{\vec{a}})$
$\sum_I \frac{m_I}{N}(\dot{\vec{q}}_I + \dot{\vec{a}})$
$\dot{\vec{a}} = -$
$-\dot{\vec{q}}_{cm}$
$\vec{a} = -\vec{q}_{cm} + (\ldots)$
$G q_I \to \exp\{-\vec{q}_{cm} \frac{\partial}{\partial q}\} \vec{q}$
$=$

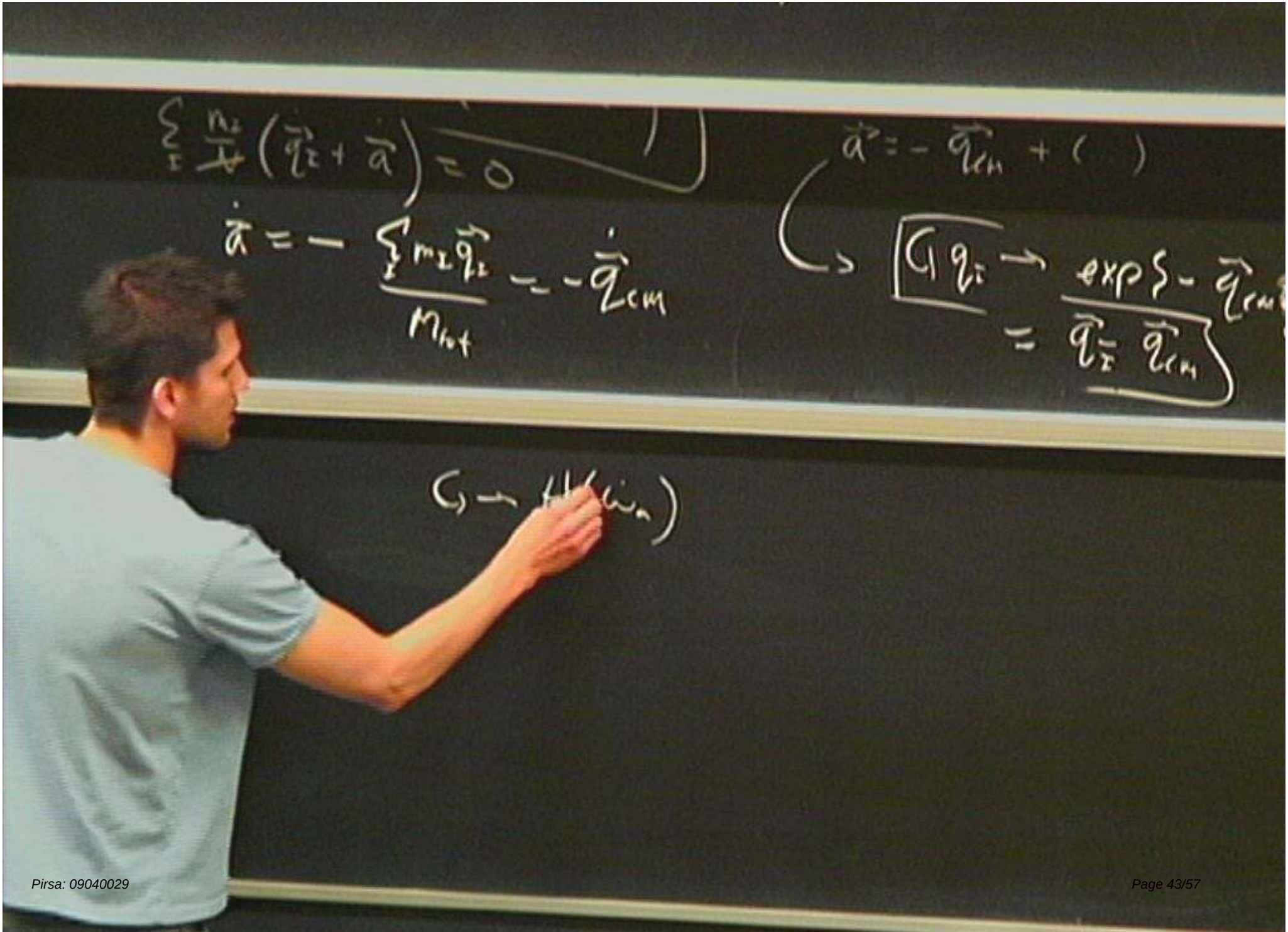
$= \frac{m_i}{M}(\dot{q}_i + \dot{a})$

$\sum_i \frac{m_i}{M}(\dot{q}_i + \dot{a}) = 0$

$\dot{a} = -\frac{\sum_i m_i \dot{q}_i}{M_{tot}} = -\dot{q}_{cm}$

$\vec{a} = -\vec{q}_{cm} + (\dots)$

$G\,q_i \to \exp\{-\vec{q}_{cm}\frac{\partial}{\partial \vec{q}}\}\,\vec{q}_i = \vec{q}_i - \vec{q}_{cm}$


$\sum_i \frac{m_i}{M_{tot}} (\vec{q}_i + \vec{a}) = 0$
$\vec{a} = -\frac{\sum_i m_i \vec{q}_i}{M_{tot}} = -\vec{q}_{cm}$
$\vec{a}' = -\vec{q}_{cm} + (\quad)$
$\vec{q}_i \rightarrow \exp\{-\vec{q}_{cm}$
$= \vec{q}_i - \vec{q}_{cm}$

$$\sum_i \frac{m_i}{M_{tot}} (\vec{q}_i + \vec{a}) = 0$$

$$\vec{a} = -\frac{\sum_i m_i \vec{q}_i}{M_{tot}} = -\vec{q}_{cm}$$

$$\vec{a} = -\vec{q}_{cm} + (\quad)$$

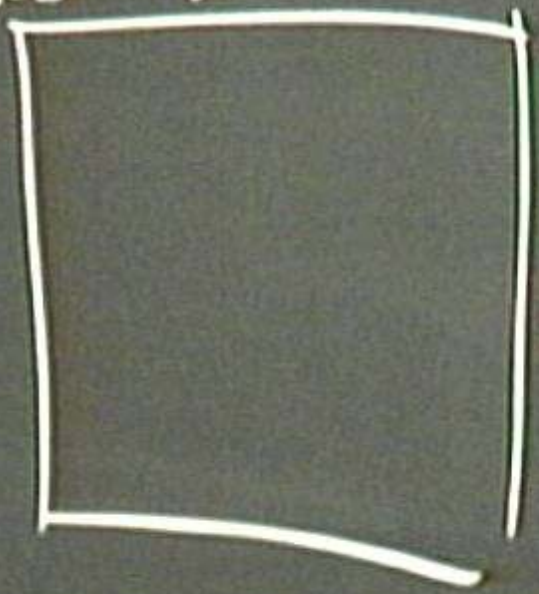
$$G\,\vec{q}_i \to \exp\{-\vec{q}_{cm}\} = \vec{q}_i - \vec{q}_{cm}$$

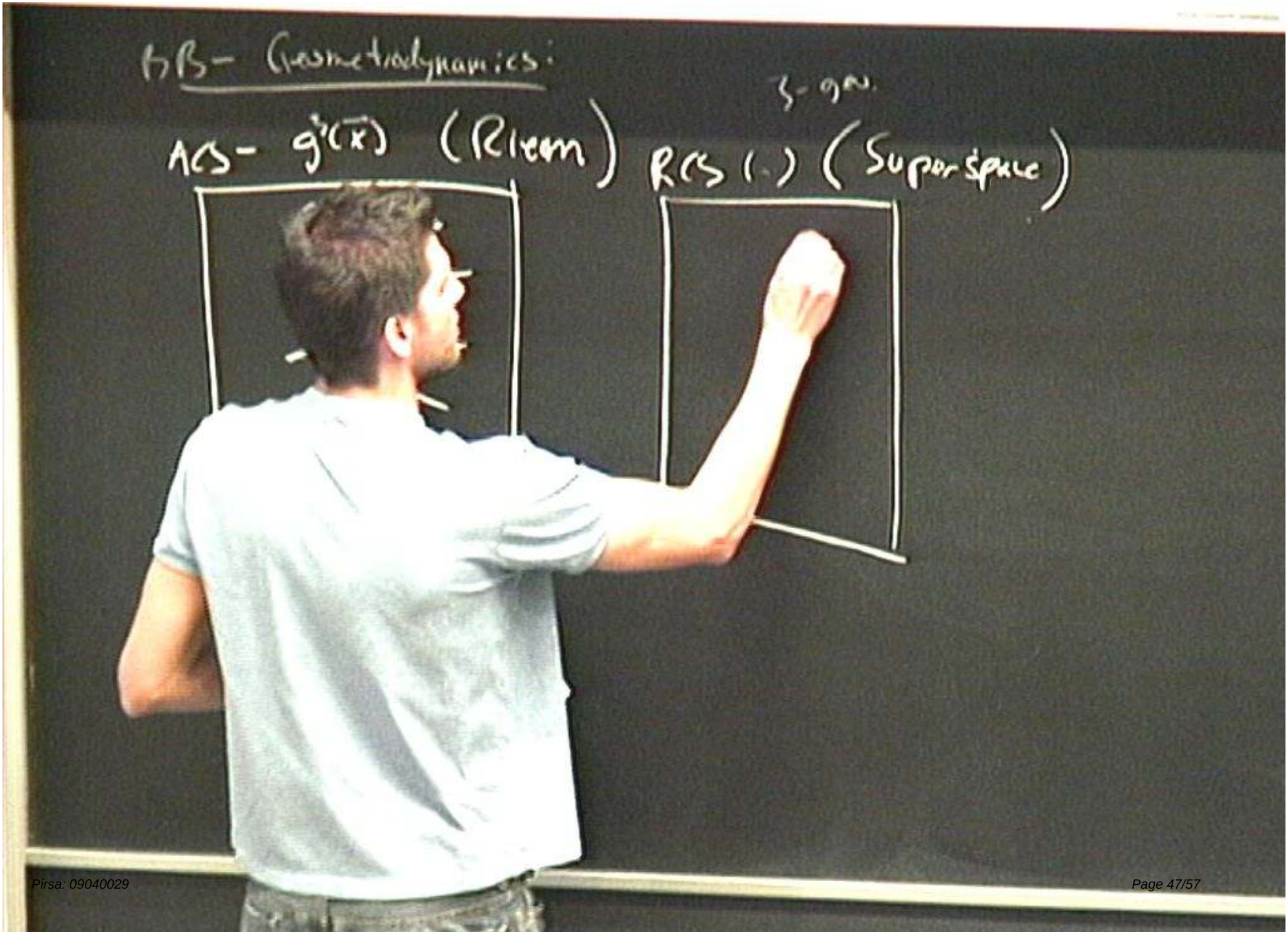
$$G \to H(\cdot\omega_n)$$

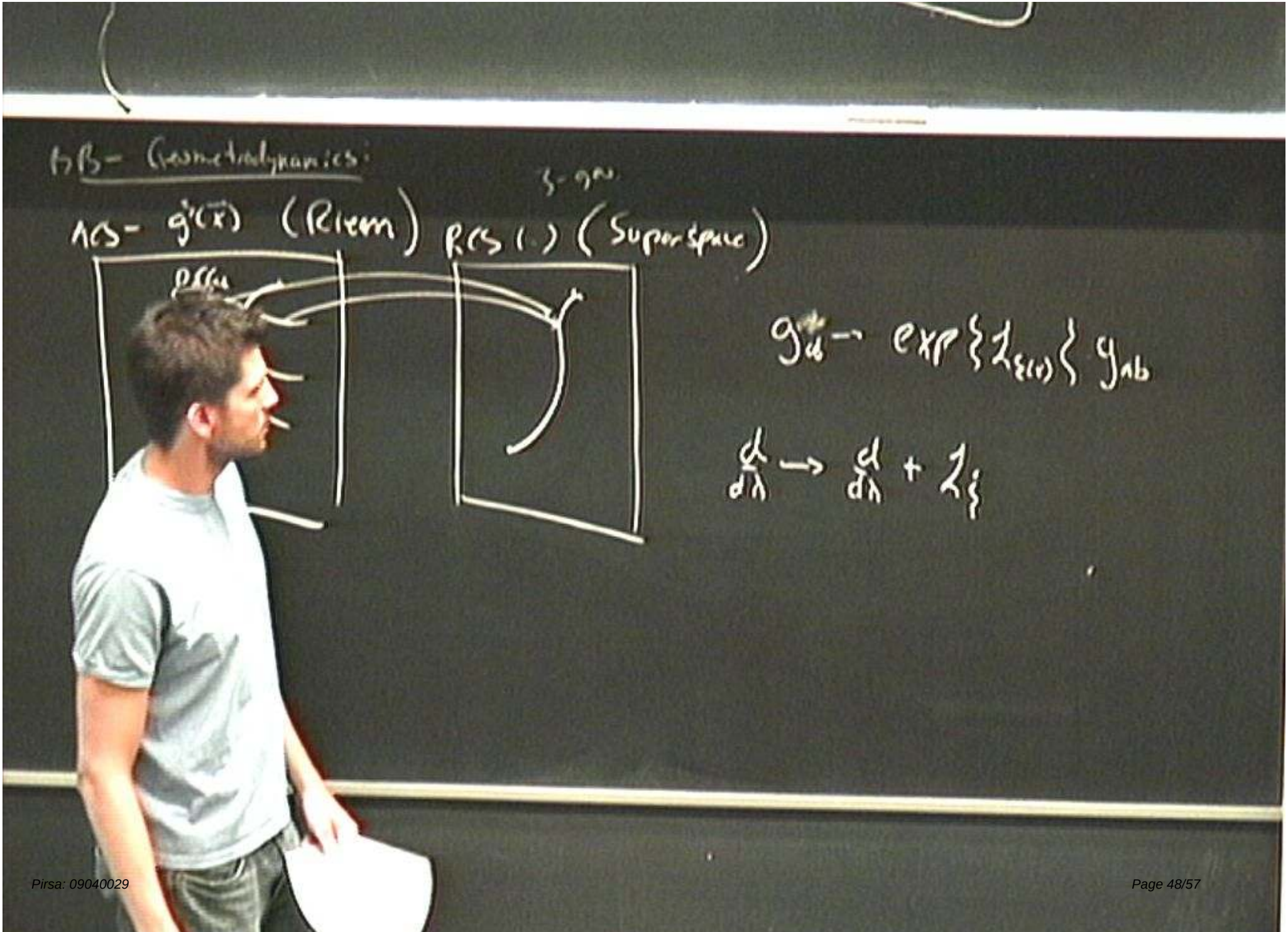
6.3 – Geometrodynamics:

5B- Geometrodynamics:

1CS- $g^3(x)$




HB- Geometrodynamics:
ACS- g^ij(x) (Riem)
RCS(.) (Superspace)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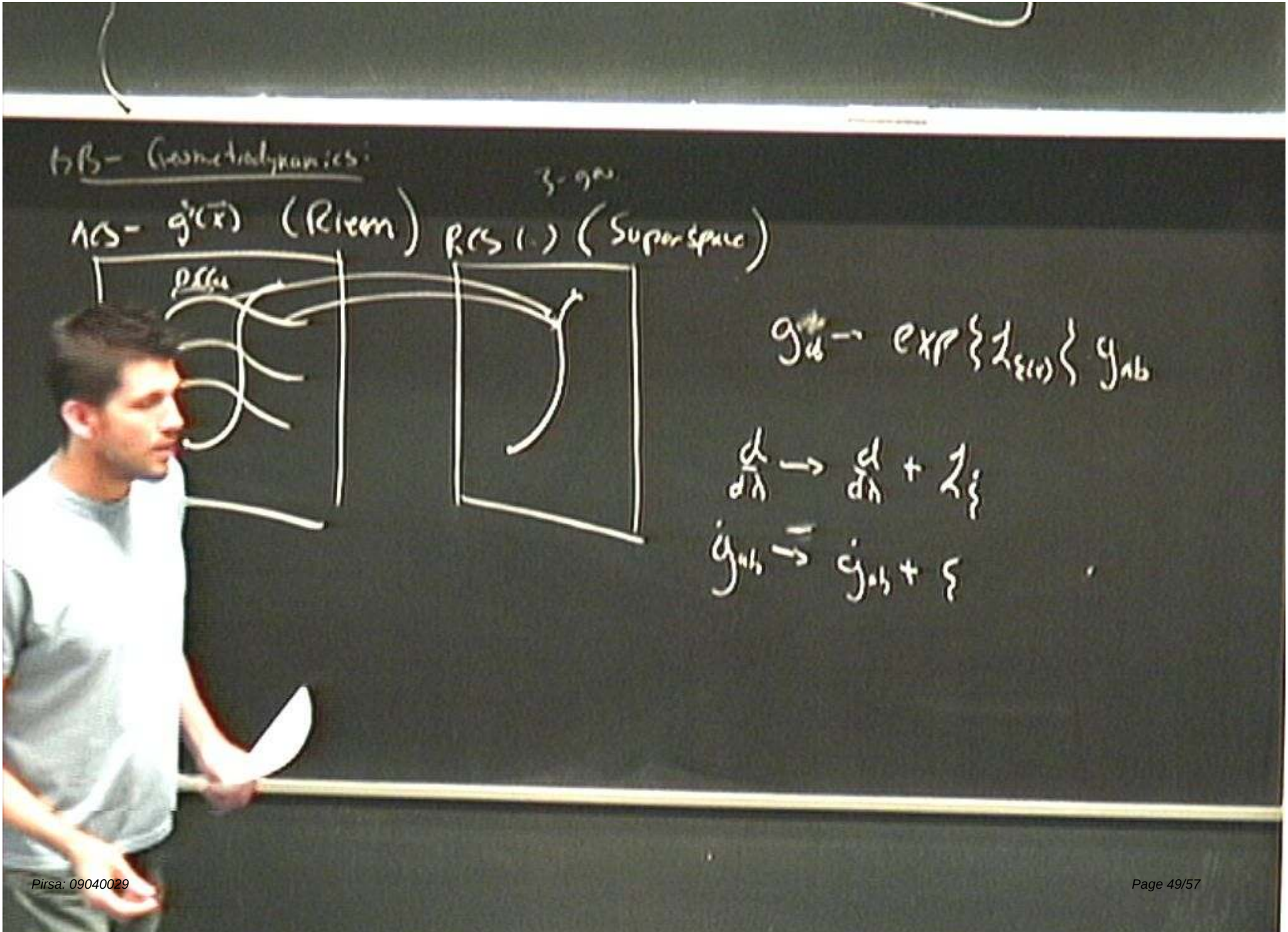
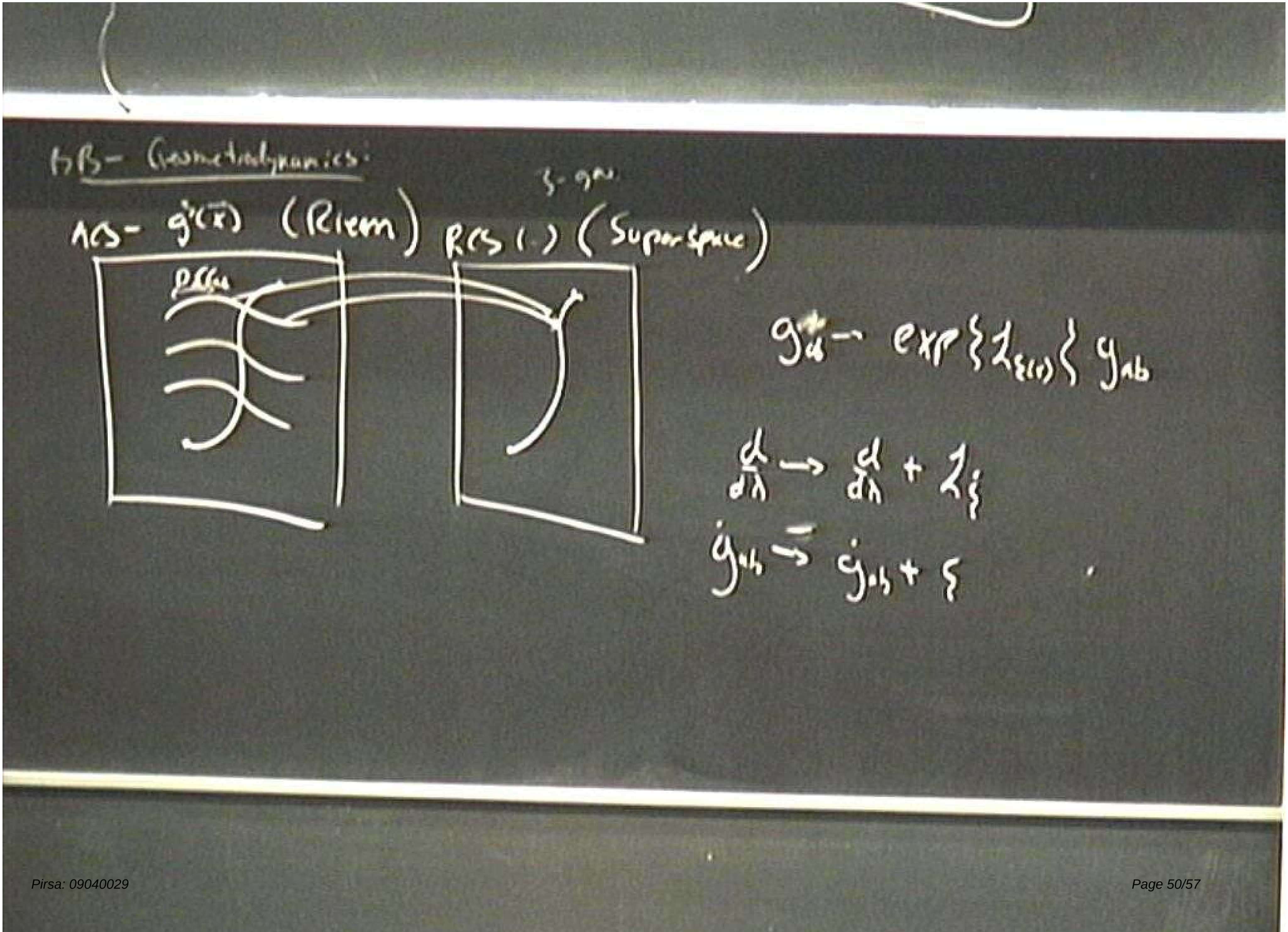
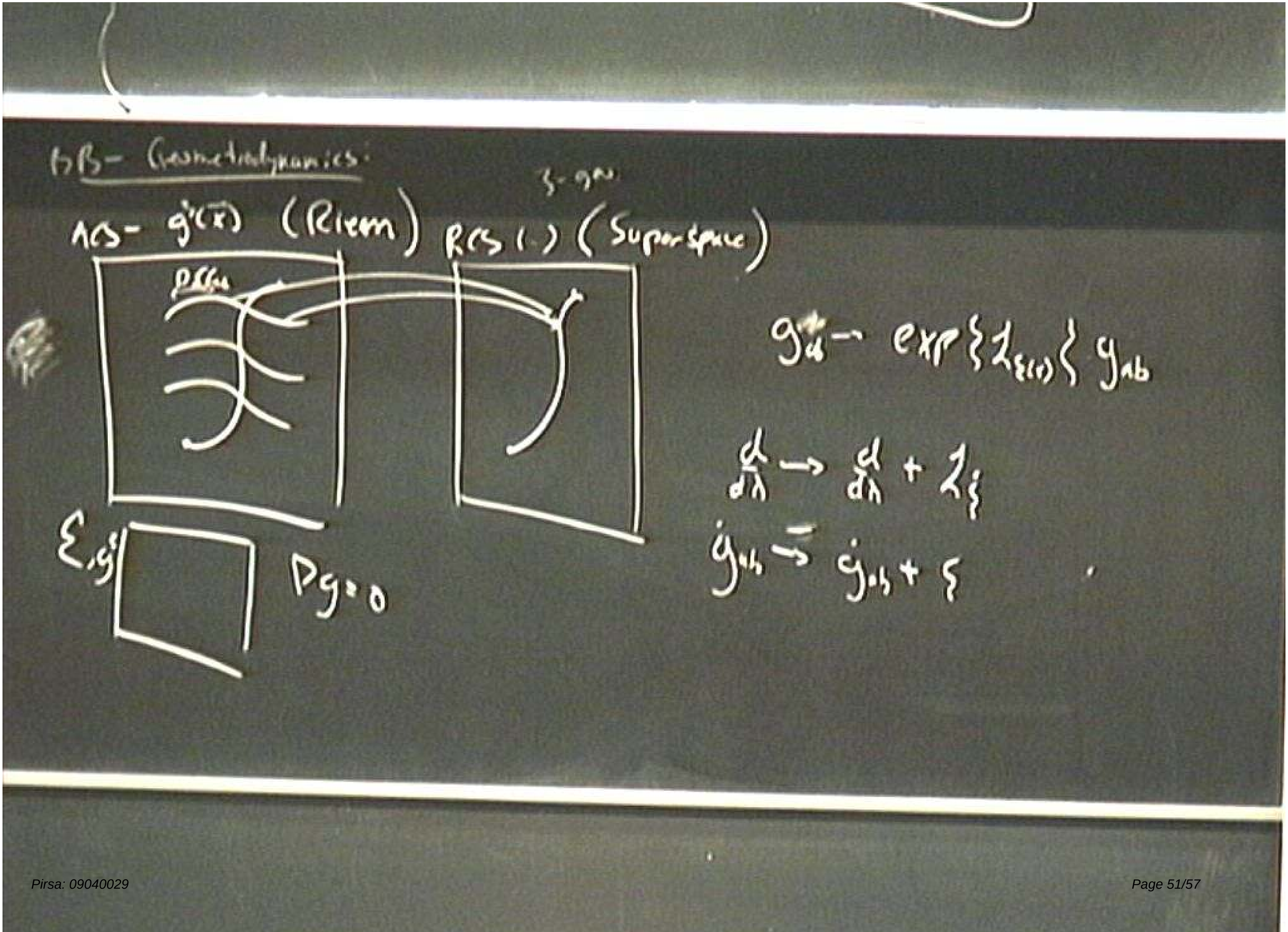
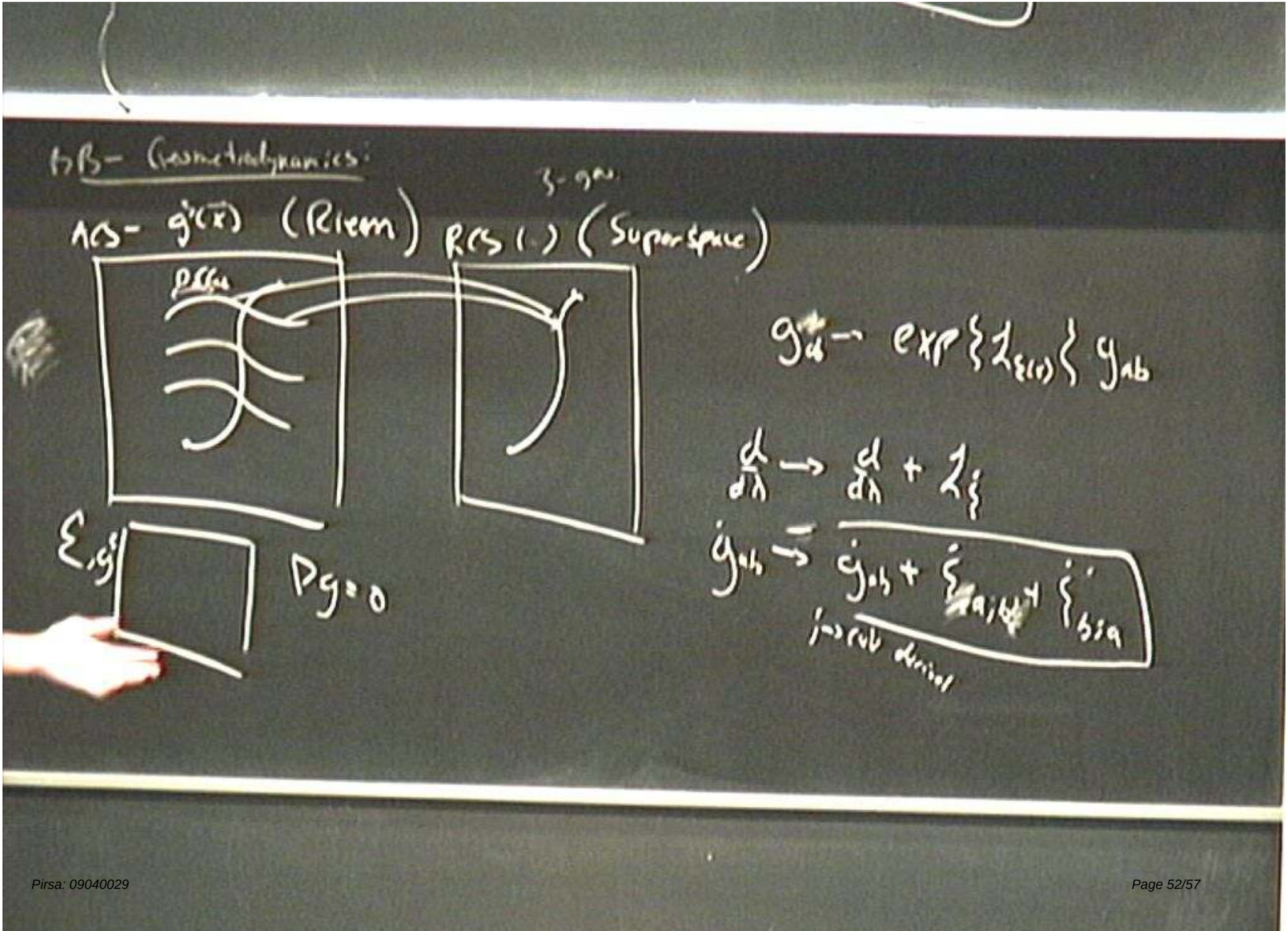
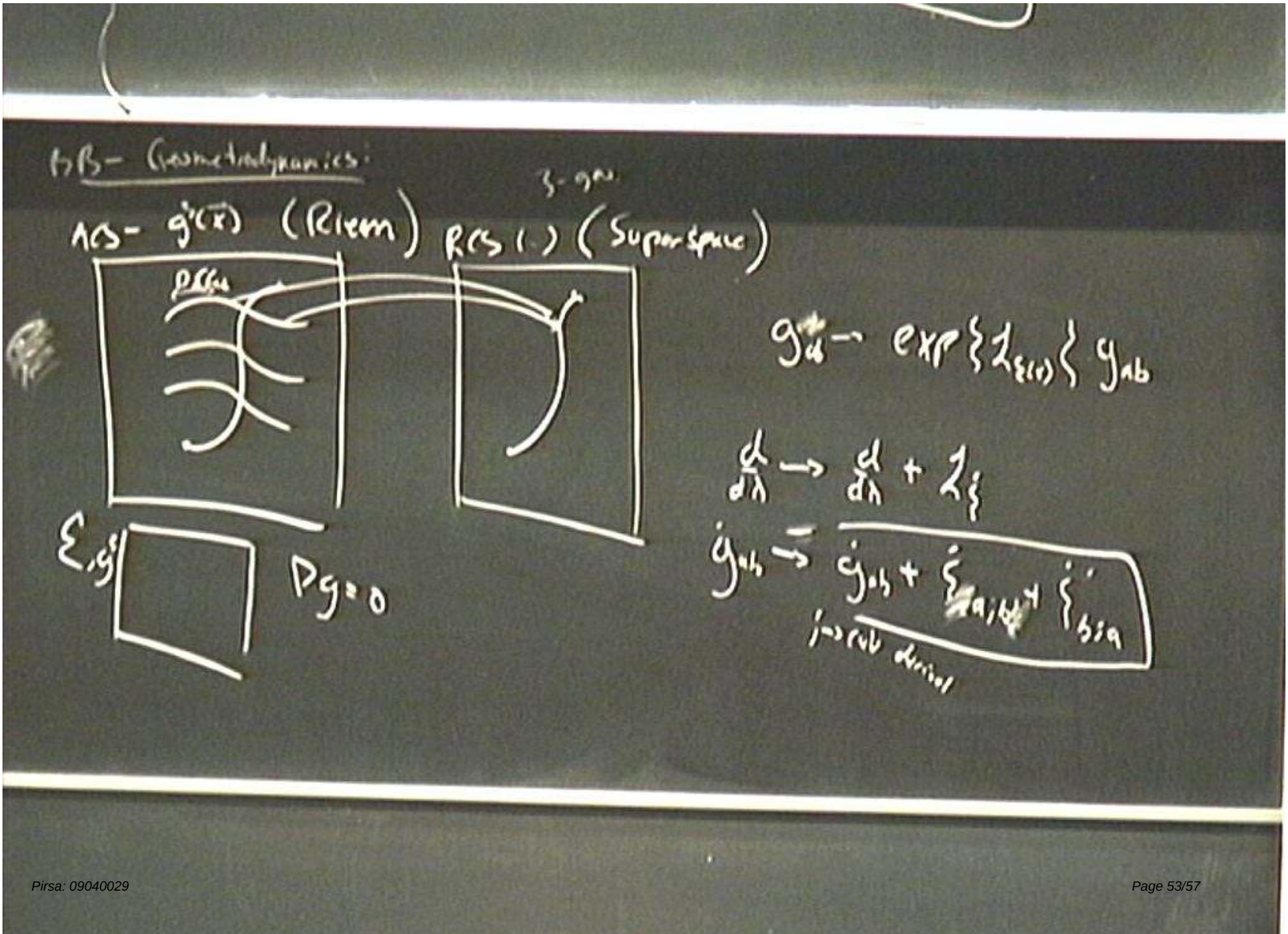
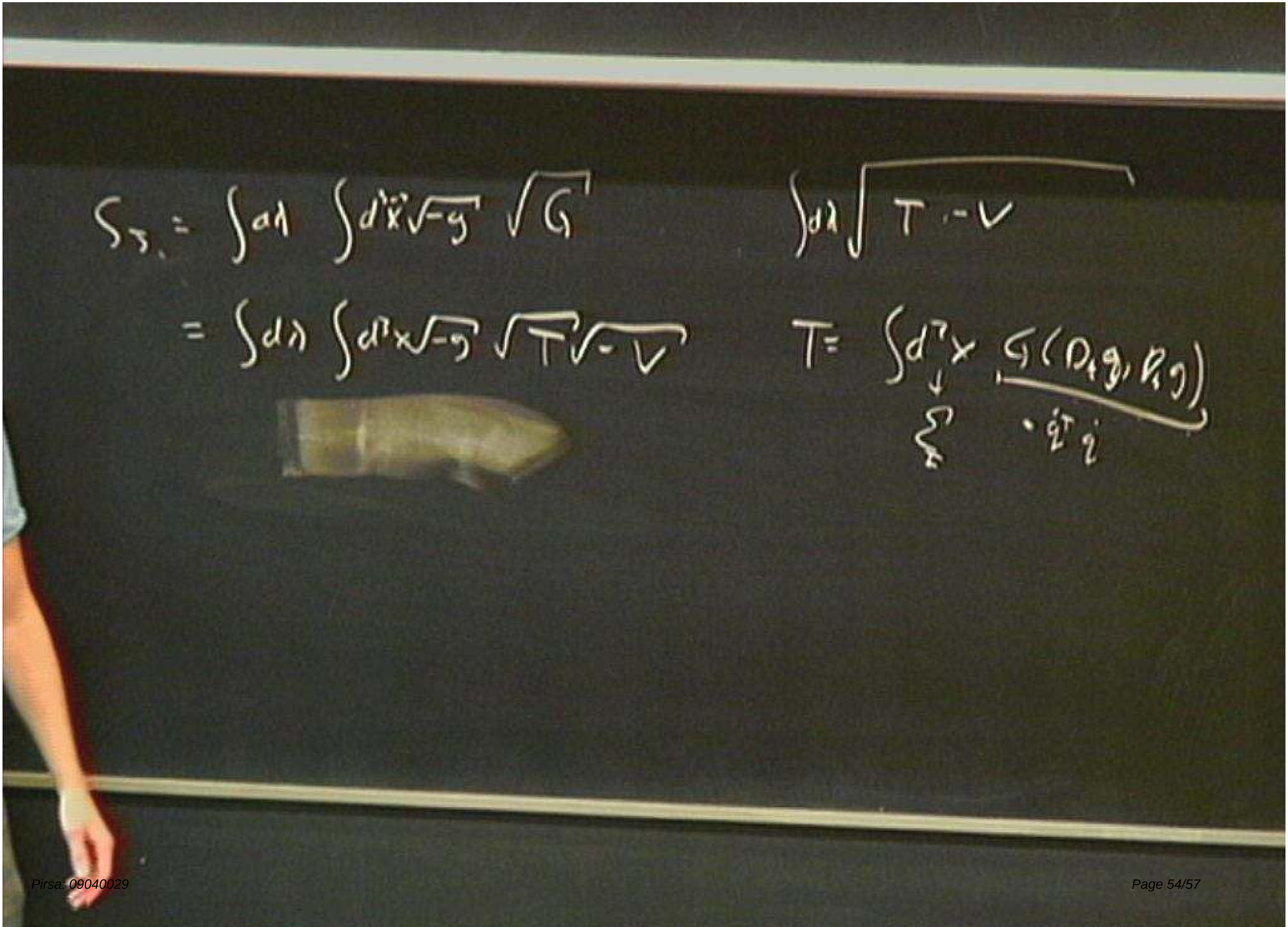

GB - Geometrodynamics:
ACS - $g^{ij}(\vec{x})$ (Riem)
RCS (.) (Superspace)
$g_{ab} \to \exp\{\mathcal{L}_{\xi(x)}\}\, g_{ab}$
$\frac{d}{d\lambda} \to \frac{d}{d\lambda} + \mathcal{L}_{\dot{\xi}}$
$\dot{g}_{ab} \to \dot{g}_{ab} + \xi$


BB– Geometrodynamics:
(Riem)
(Superspace)
exp


Geometrodynamics:
(Riem)
(Superspace)
Dg=0
exp


GB - Geometrodynamics:
3-gen
ACS - $g^3(\vec{x})$ (Riem)
RCS ( ) (Superspace)
$g_{ab} \to \exp\{\mathcal{L}_{\xi(x)}\} g_{ab}$
$\frac{d}{d\lambda} \to \frac{d}{d\lambda} + \mathcal{L}_{\dot{\xi}}$
$\dot{g}_{ab} \to \dot{g}_{ab} + \dot{\xi}_{a;b} + \dot{\xi}_{b;a}$
$\mathcal{E}, g^3$
$\nabla g = 0$


GB - Geometrodynamics:
3-gen.
ACS - $g^3(\vec{x})$ (Riem)
RCS ( ) (Superspace)
$g_{ab} \to \exp\{\mathcal{L}_{\xi(x)}\}\, g_{ab}$
$\frac{d}{d\lambda} \to \frac{d}{d\lambda} + \mathcal{L}_{\dot\xi}$
$\dot g_{ab} \to \dot g_{ab} + \dot\xi_{a;b} + \dot\xi_{b;a}$
$\mathcal{E}, g^3$
$\nabla g = 0$


$S = \int d\lambda \int d^3x \sqrt{-g} \sqrt{G}$
$= \int d\lambda \int d^3x \sqrt{-g} \sqrt{T} \sqrt{-V}$
$\int d\lambda \sqrt{T - V}$
$T = \int d^3x \, G(D_t g, D_t g)$

$$S_3 = \int d\lambda \int d^3x \sqrt{-g}\,\sqrt{G} \qquad \int d\lambda \sqrt{T - V}$$

$$= \int d\lambda \int d^3x \sqrt{-g}\,\sqrt{T}\sqrt{-V} \qquad T = \int d^3x\, \underbrace{G(D_t g, D_t g)}_{\dot{q}^T \dot{q}}$$

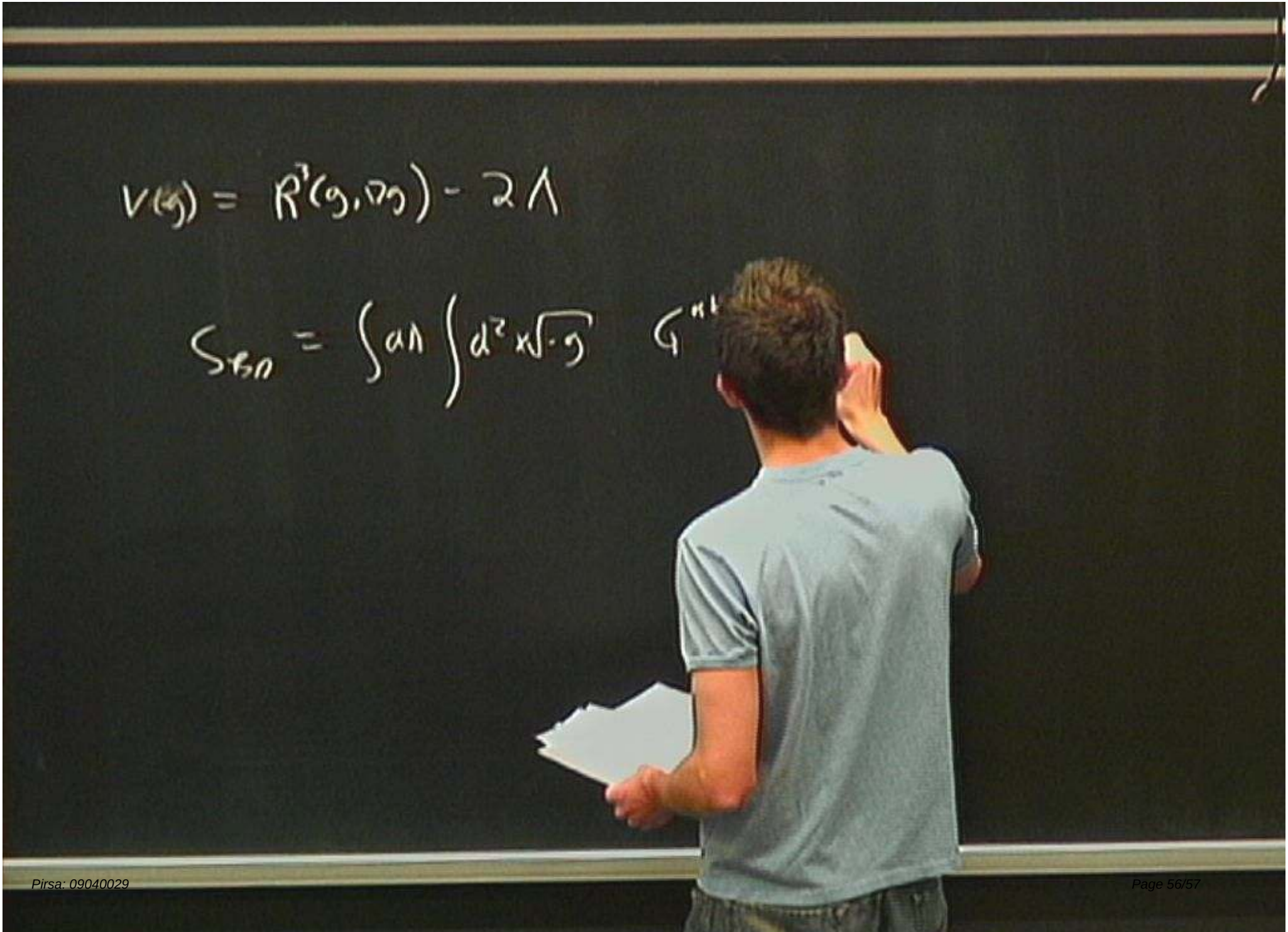
$$\sum$$

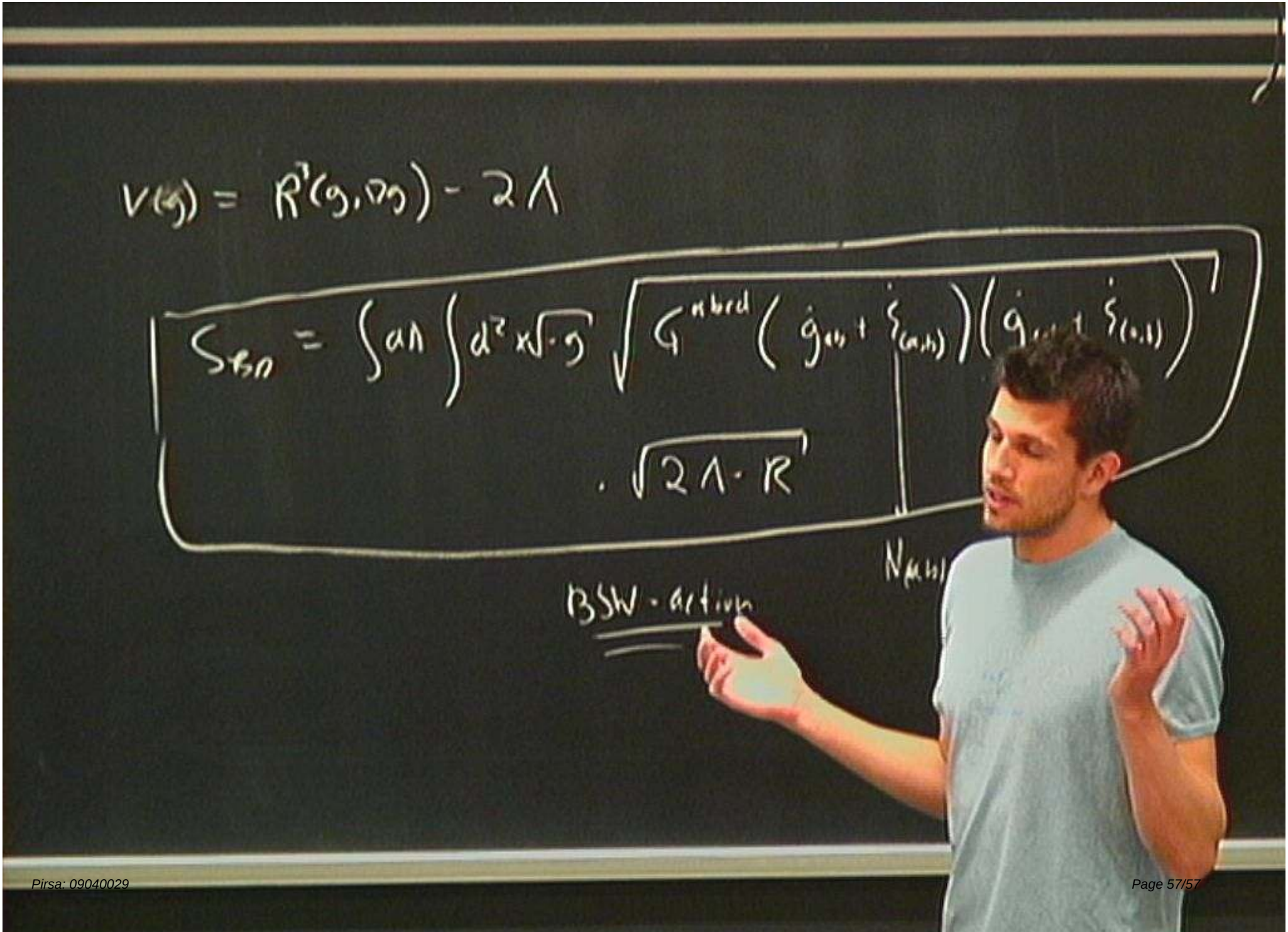
G^{abcd} $\quad \dot{g}_{ab}\,\dot{g}_{cd}$

$$G^{abcd} = C_1 g^{ab} g^{cd} + C_2 g^{ac} g^{bd} + C_3 g^{ad} g^{bc}$$

$$= C_1 g^{ab} g^{cd} + C_2 g^{ac} g^{bd}$$

$$= g^{ac} g^{bd} + \alpha\, g^{ab} g^{cd}$$


$V(g) = R^3(g, \partial g) - 2\Lambda$
$S_{EH} = \int d\Lambda \int d^4x \sqrt{-g}$


$V(g) = R^2(g, \partial g) - 2\Lambda$
$S_{BSW} = \int d\Lambda \int d^2x \sqrt{-g} \sqrt{G^{abcd}(\bar{g}_{ab} + \xi_{(a,b)})(\bar{g}_{cd} + \xi_{(c,d)})}$
$\cdot \sqrt{2\Lambda \cdot R}$
BSW - action