Title: Quantum Device Theory at the University of Waterloo

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Abstract:

Quantum device theory research at the University of Waterloo

F.K. Wilhelm

Institute for Quantum Computing (IQC) and Department of Physics and Astronomy, University of Waterloo, Canada

4-corners condmat meeting 2009, Perimeter Institute

The QDT group

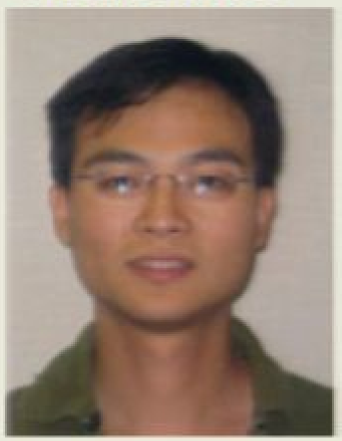
PI: Frank Wilhelm

PDF: Jay Gambetta

Brendan Osberg

Felix Motzoi

PDF: Bill Coish



Friend: Tzu-Chieh Wei

Peter Groszkowski

Farzad Qassemi

Research interests

Foundation: Condensed matter theory
Main interest: devices that can be used as qubits

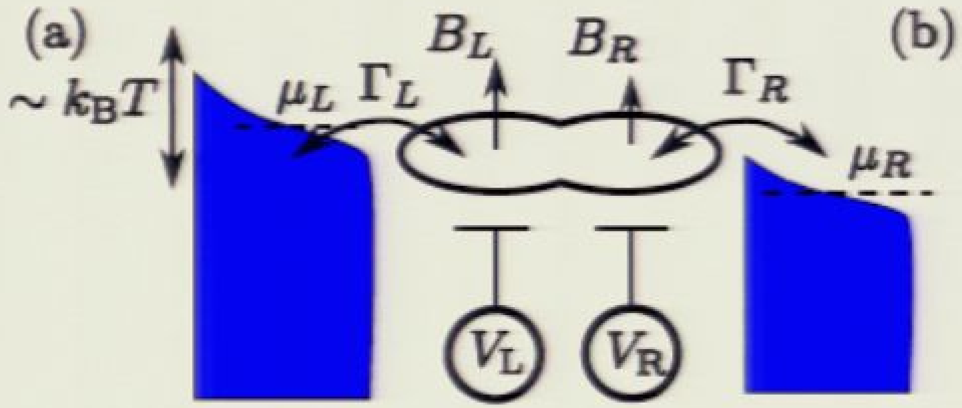
- Coherent spins in semiconductor quantum dots
- Quantum noise in nanostructures
- Decoherence theory
- Circuit quantum electrodynamics
- Quantum measurement theory
- Optimal control theory
- Quantum error correction

Few-body correlations far from equilibrium

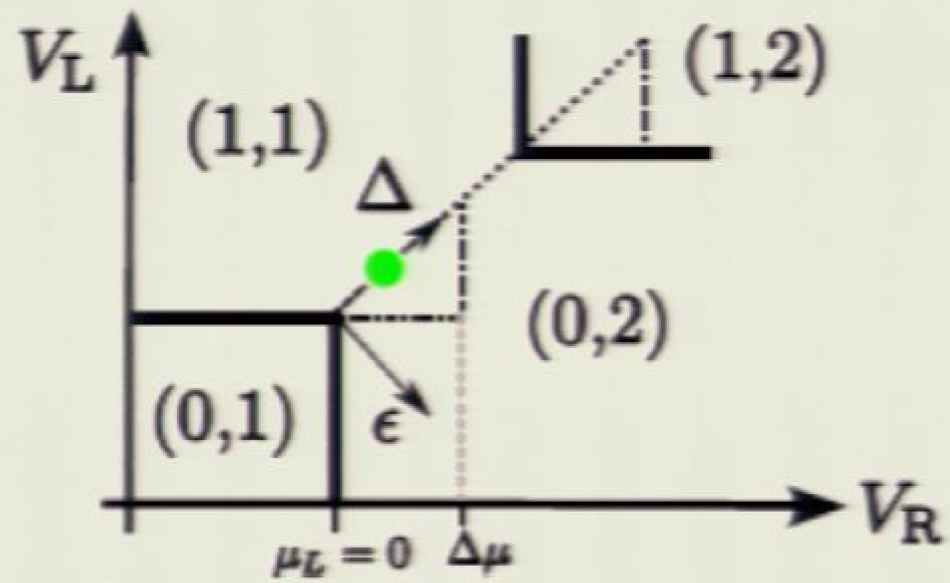
Contents

Stationary and transient leakage current in the Pauli spin blockade

Lateral double quantum dot



Charge stability diagram



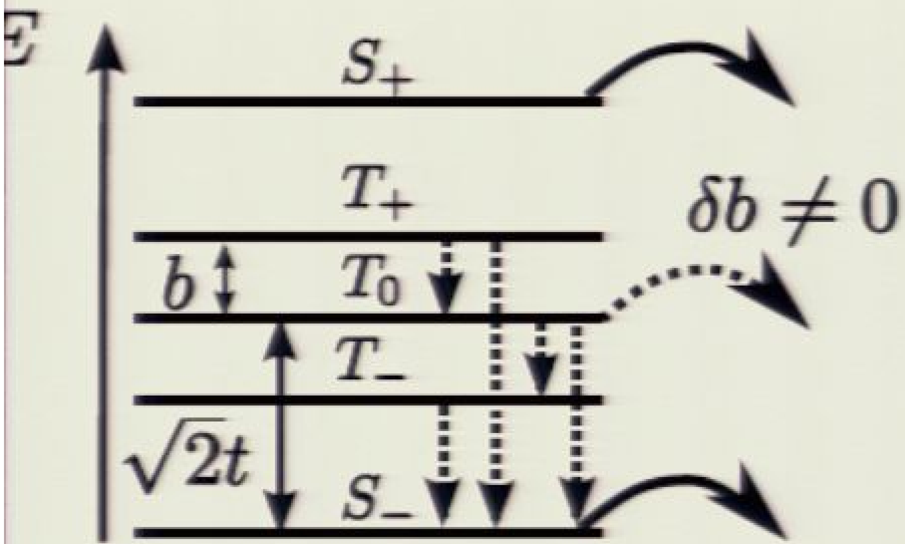
Hamiltonian

$H = H_{DD} + H_{Lead} + H_{D-L}$

$$H_{DD} = \sum_l \left(\frac{U}{2} n_l(n_l - 1) - V_l n_l\right) + U' n_L n_R - \sum_\sigma t\left(d^\dagger_{L\sigma} d_{R\sigma} + \text{h.c.}\right) + bS$$

IQC

Spin structure



- Tunnel contacts *not* spin-active!
- Transport cycle:

$$(n_L+1, n_R) \rightarrow (n_L, n_R+1 \\ \rightarrow (n_L, n_R) \rightarrow (n_L+1, n_R$$

- Can get stuck in triplet: Pauli spin blockade

- T: (1,1) Triplet states
- S: Hybridized (2,0) and (1,1) singlet
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$$H = \sum_j E_j |j\rangle\langle j| + \sum_{lk\sigma} \epsilon_{lk} c^\dagger_{lk\sigma} c_{lk\sigma} + \delta b |S\rangle\langle T_0| + \sum_{kl\sigma jj'} t_l A^{jj'}_{l\sigma} c_{lk\sigma} |j\rangle\langle j'| + \text{h.c.}$$

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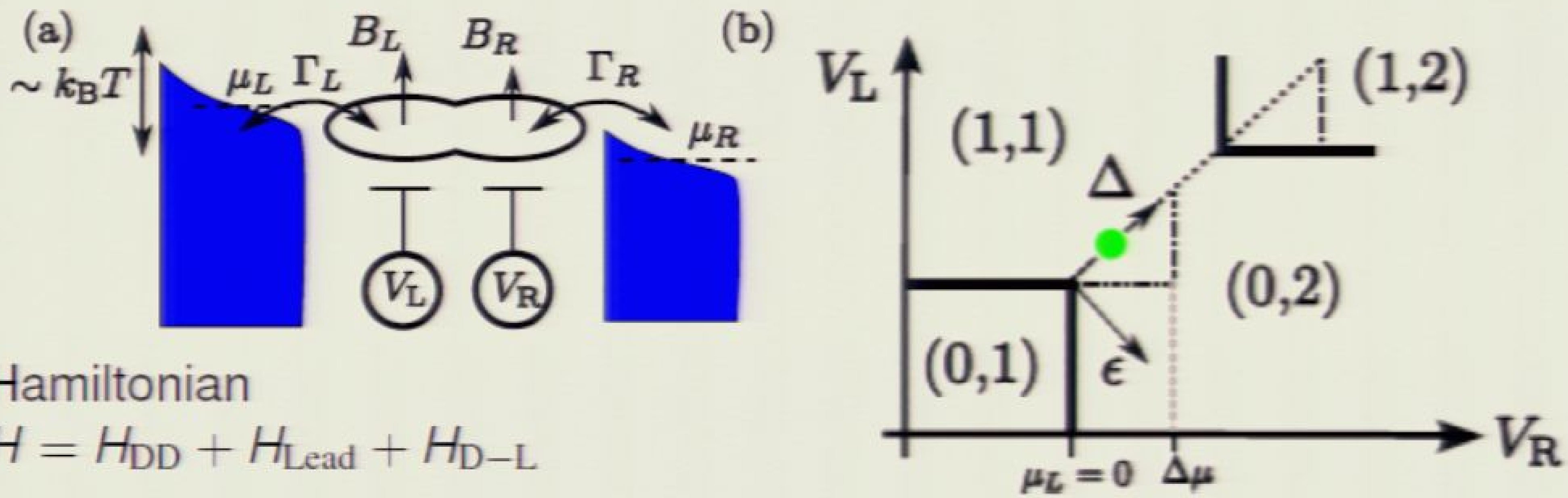

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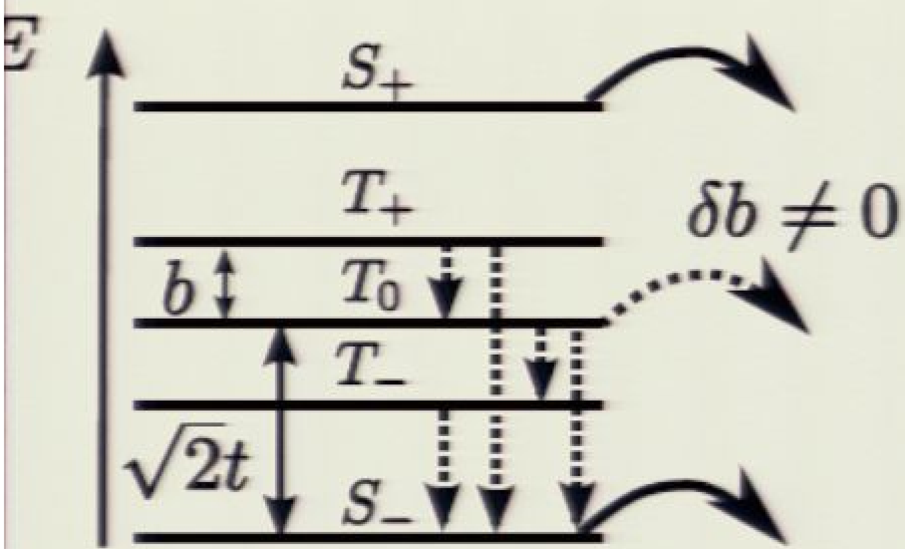

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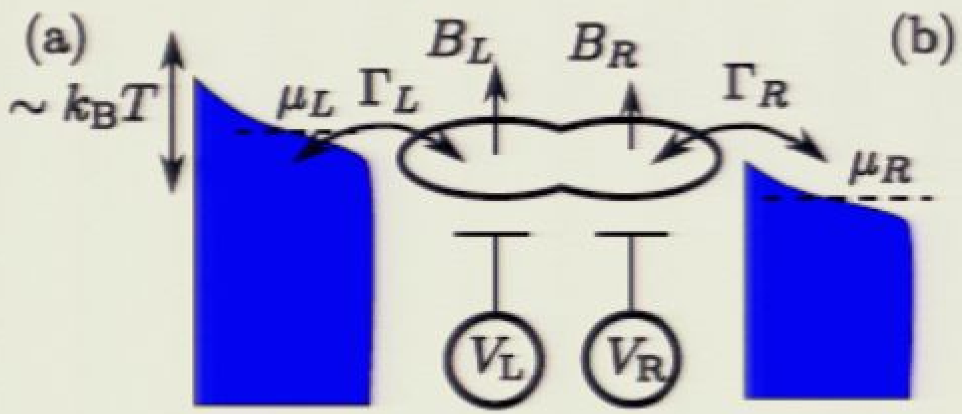

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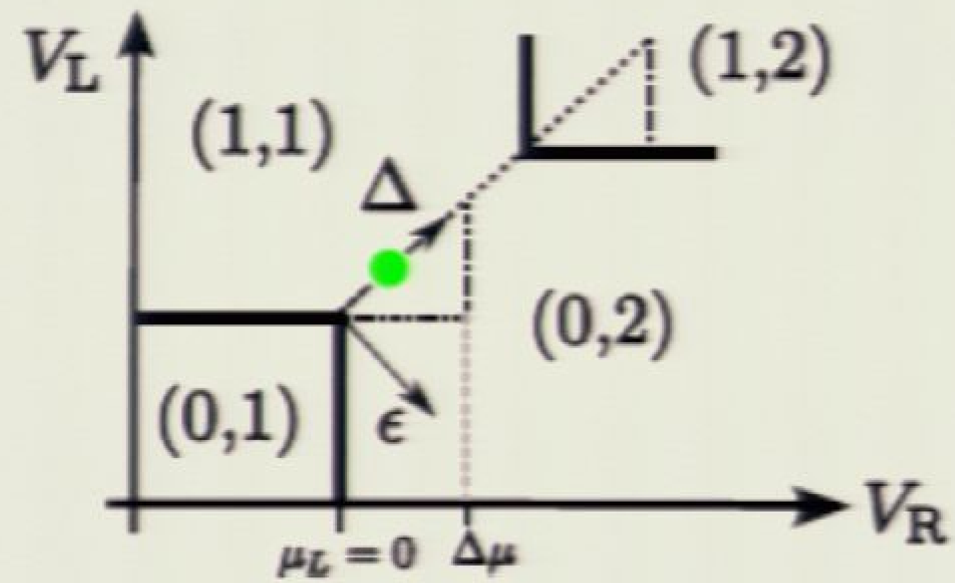

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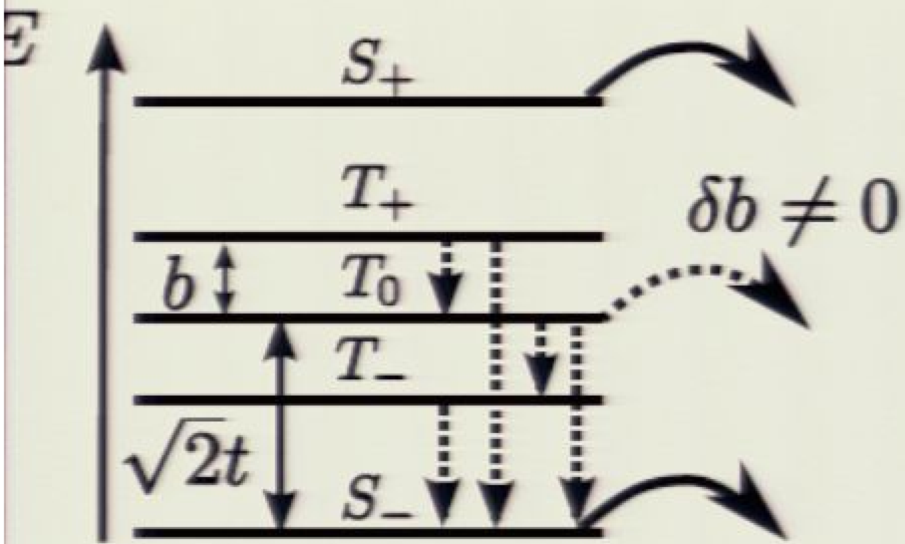

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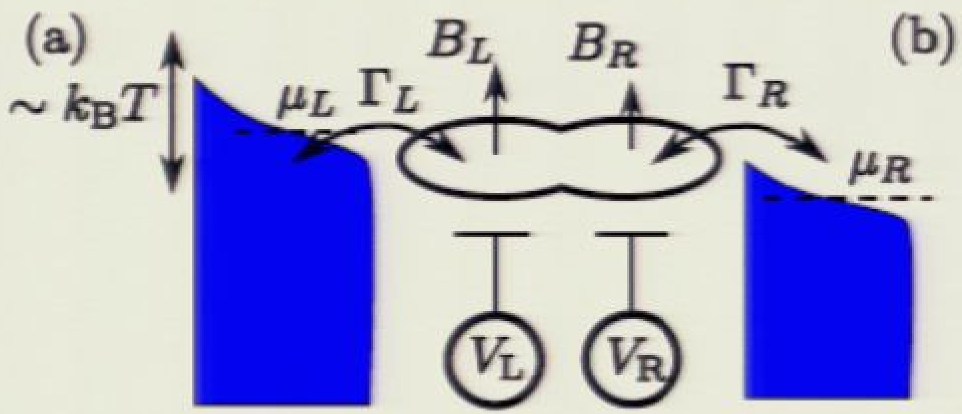

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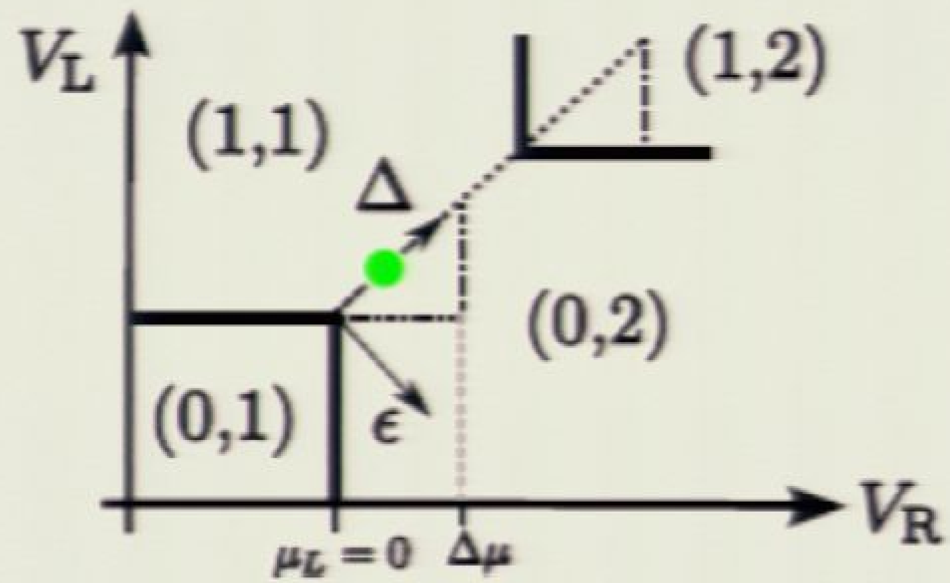

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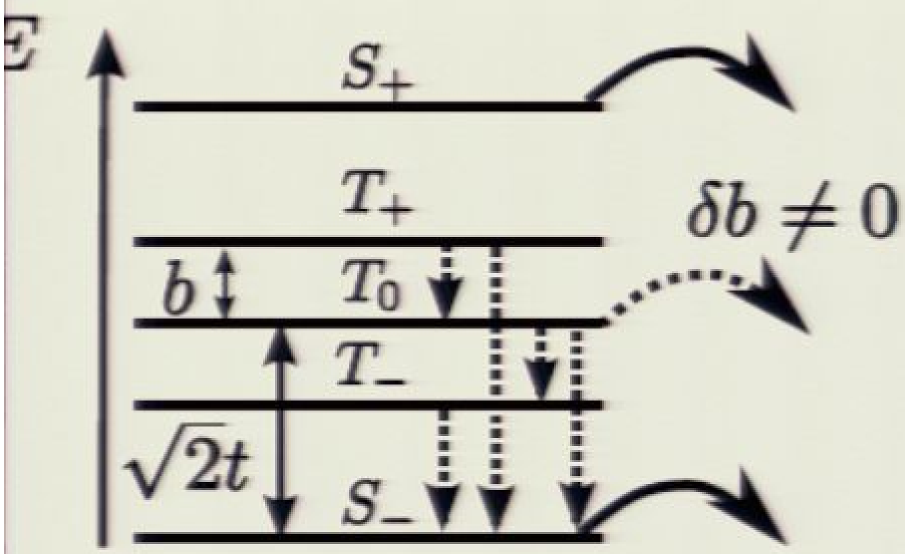

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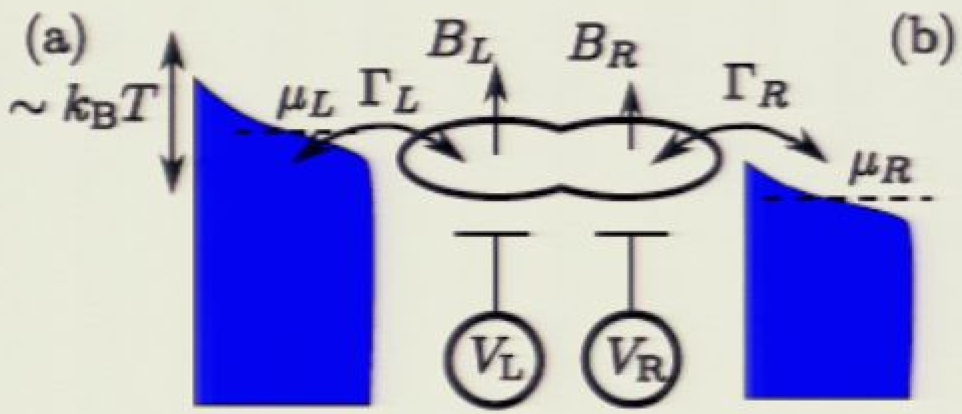

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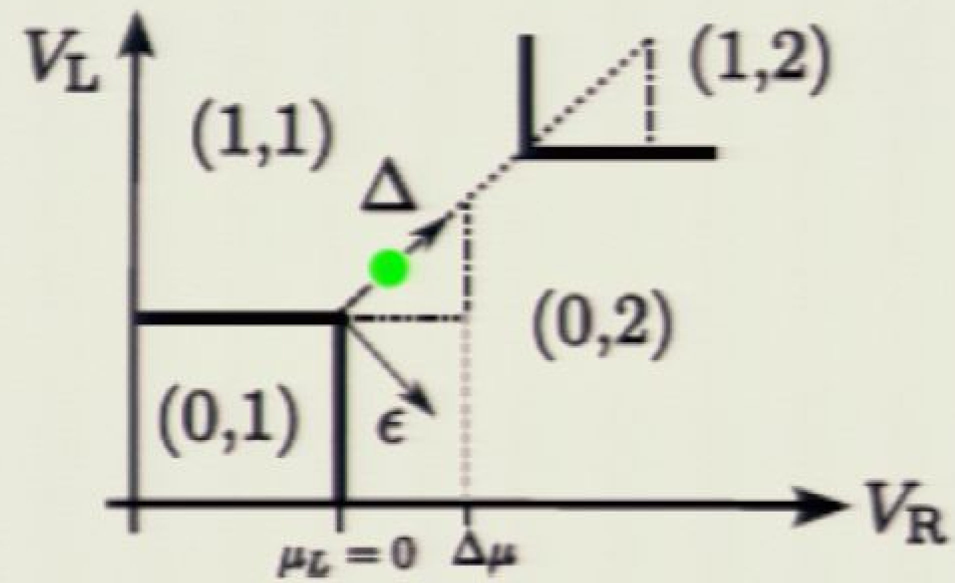

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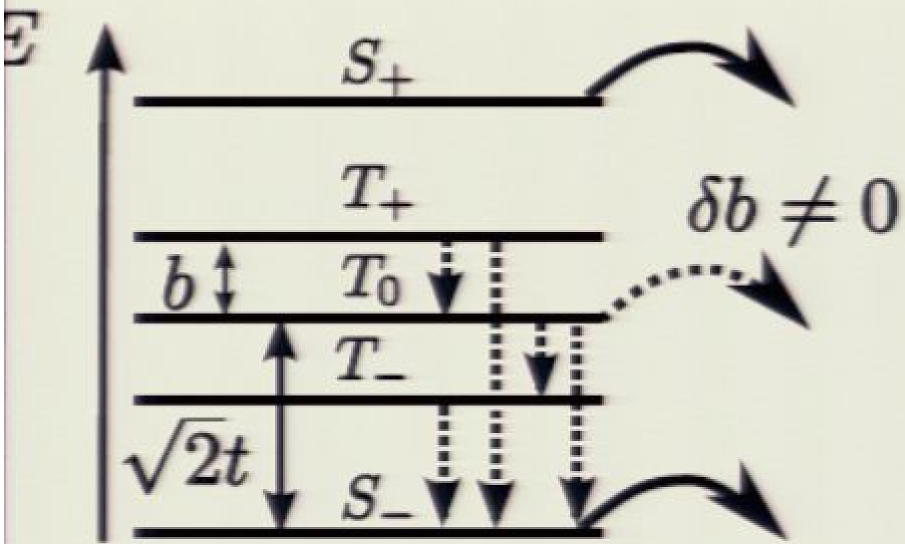

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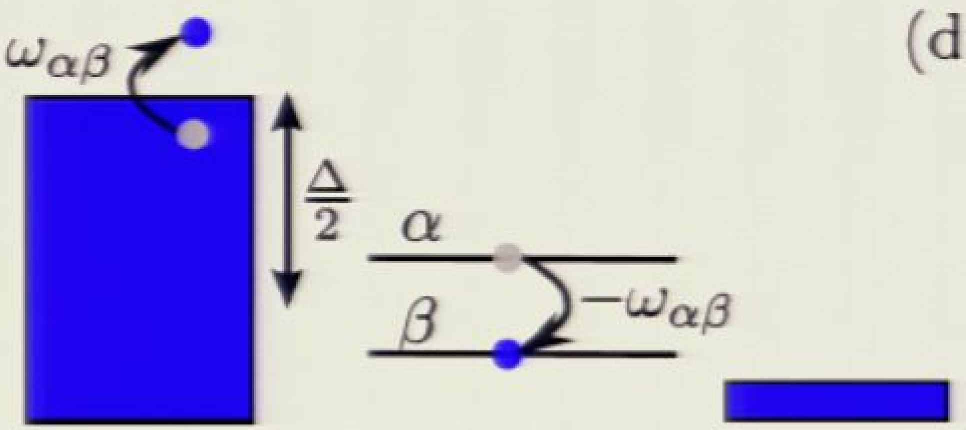
Spin-flip cotunneling



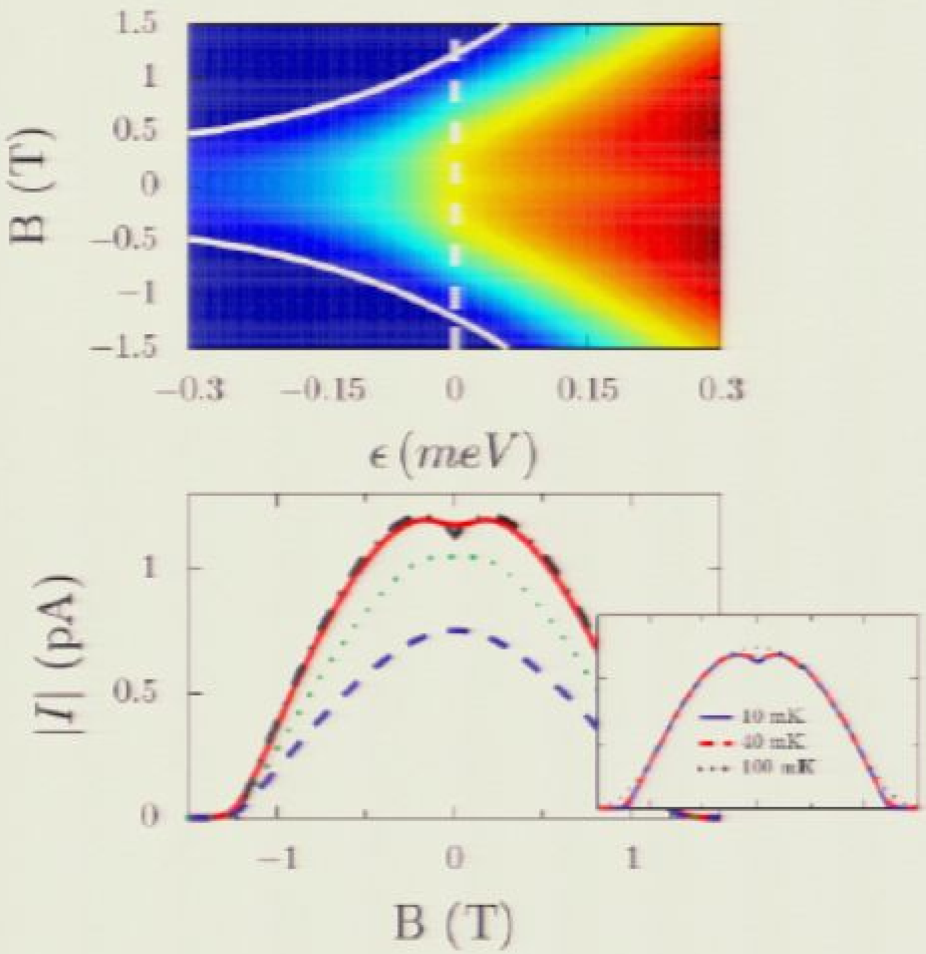
- second-order process
- second electron can have any spin
- High bias $\rightarrow$ no Kondo effect
- sf-cotuneling breaks Pauli blockade at long times

Mathematical procedure

- Schrieffer-Wolf transformation for higher-order processes
- Fermi's golden rule rates for processes
- Pauli master equation for probabilities, currents

Result: Fast spin initialization

Leakage current:



- cutoff at large B: $E_{T\_} < E_{S\_}$: spin blockade in presence at cotunneling at $T = 0$
- allows spin initialization — fast compared to inelastic spin relaxation
- Zero-field dip: At large δB, T_0 can directly escape via S_0

F. Qassemi, W.A. Coish, and FKW, PRL in press, arXiv:0812.2957

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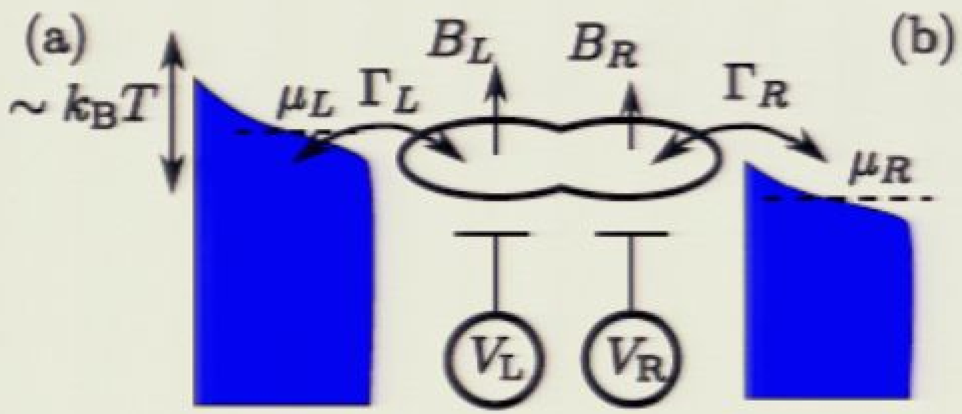

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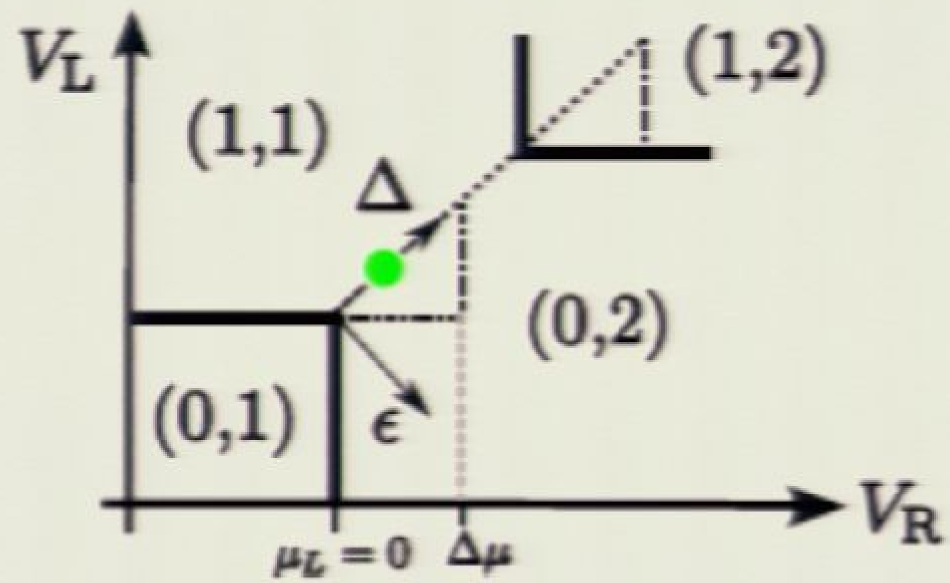

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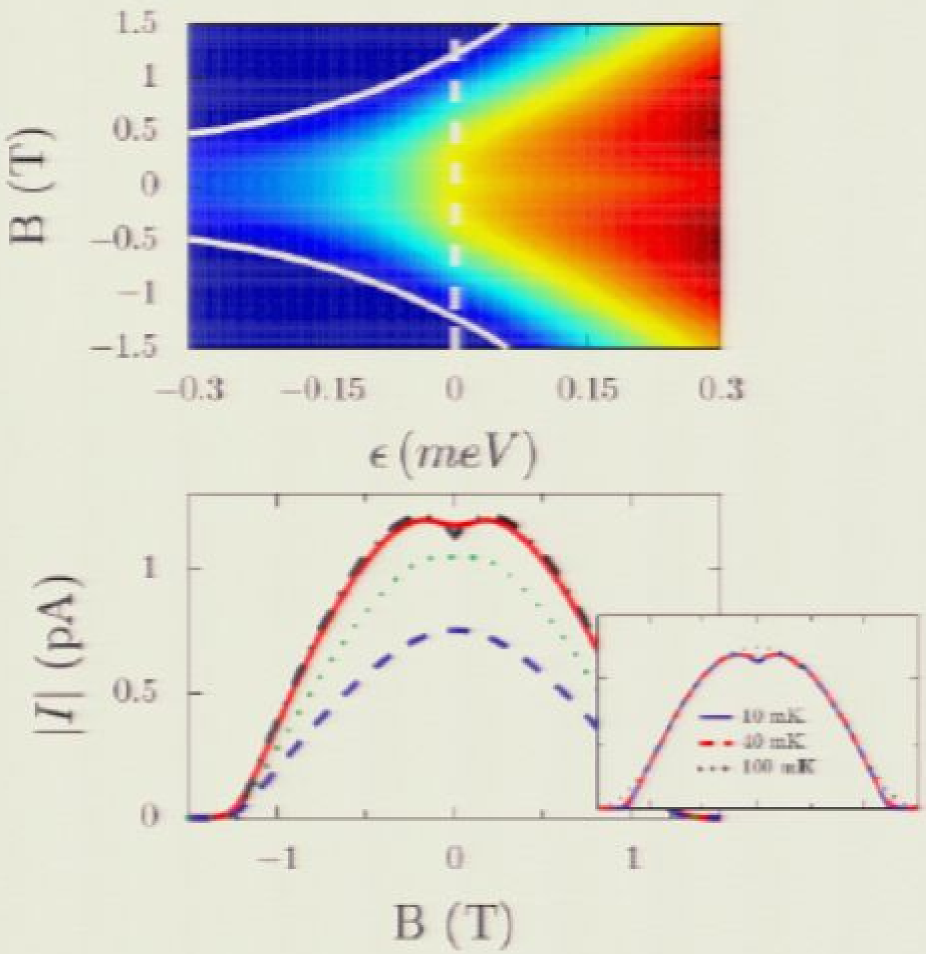

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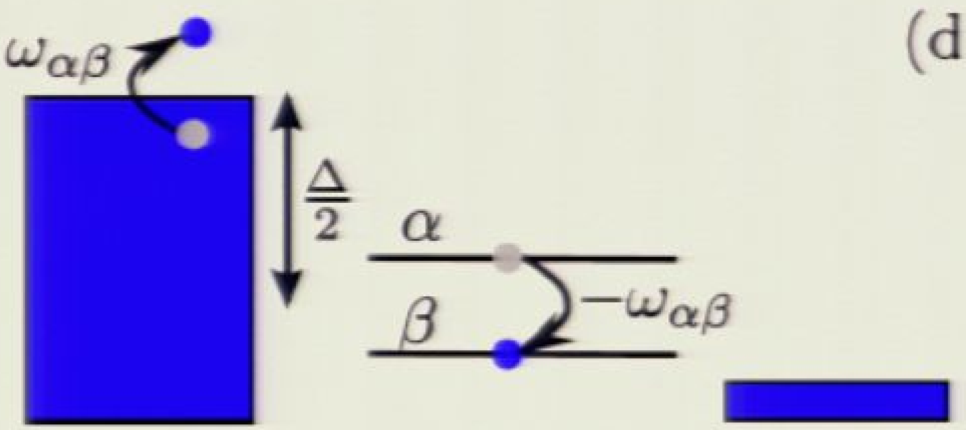

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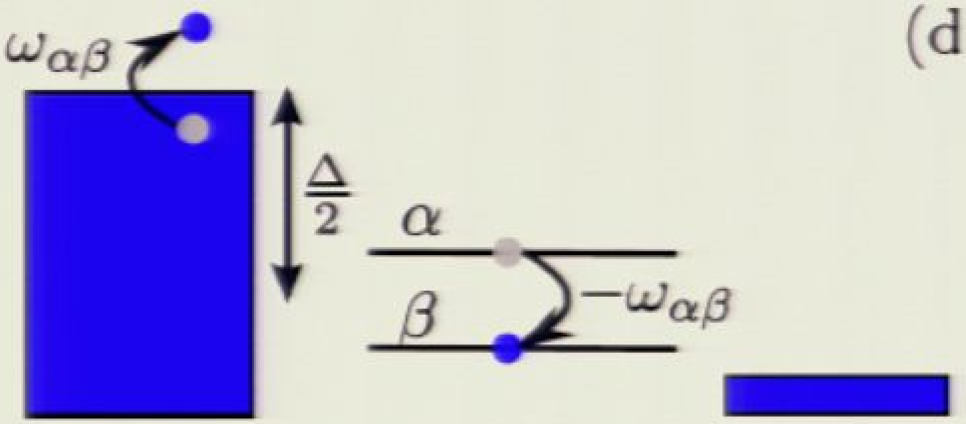

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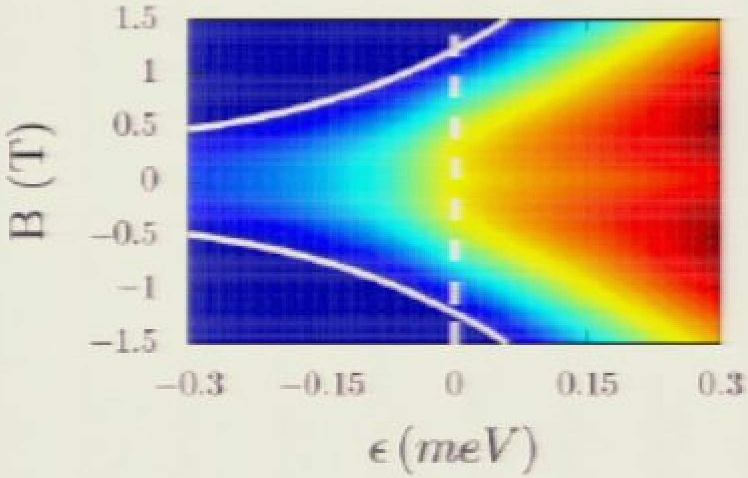

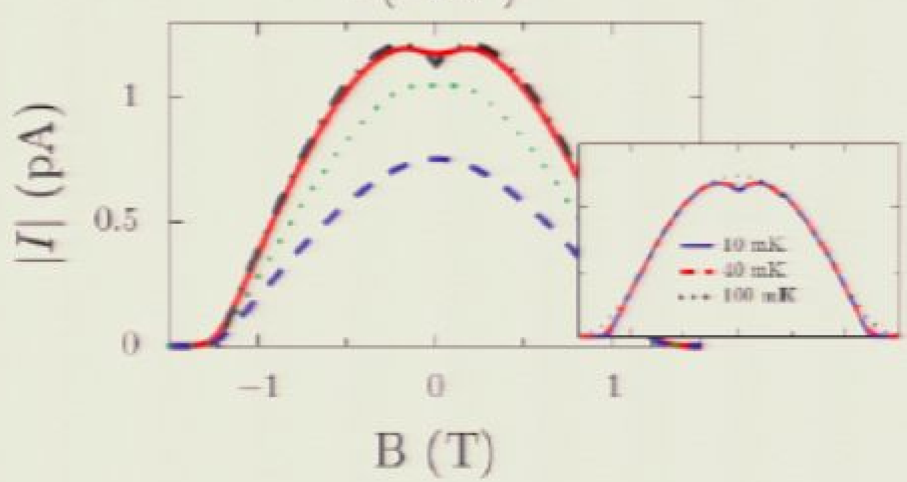

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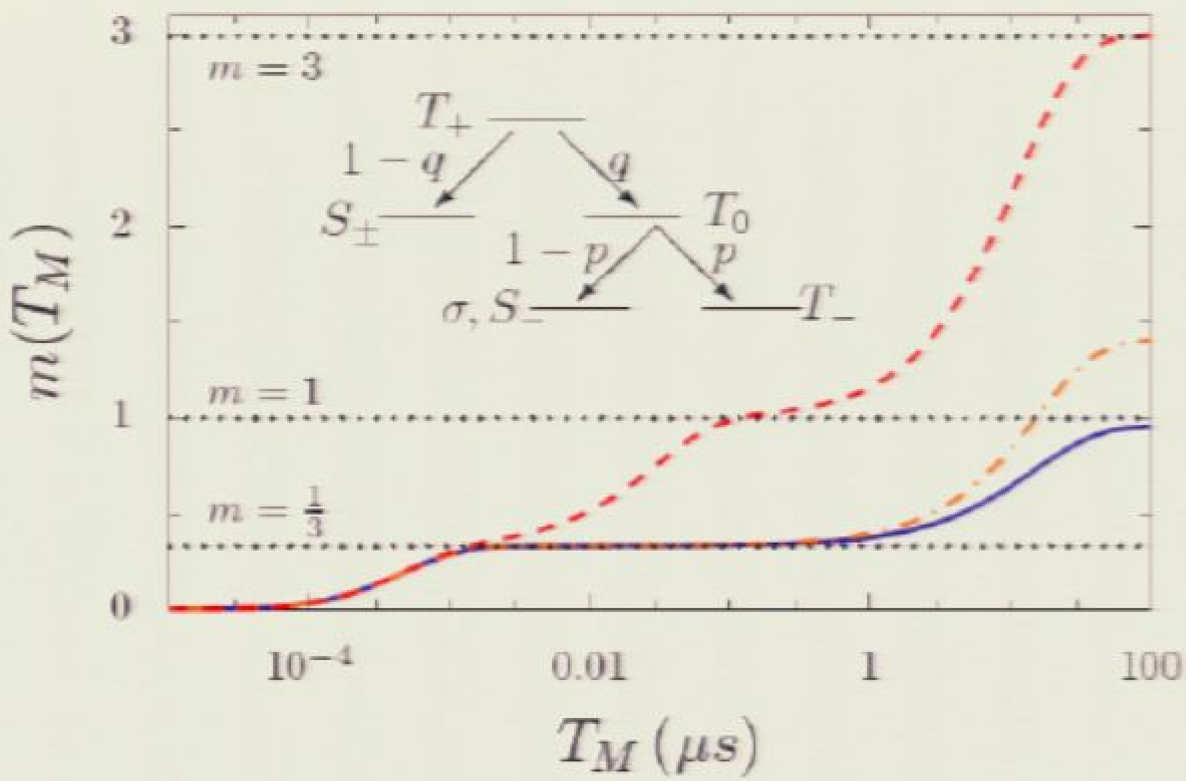
Itinerant current statistics



- Transferred electron number (measurement time)
- Fractional values from rate branching ratio

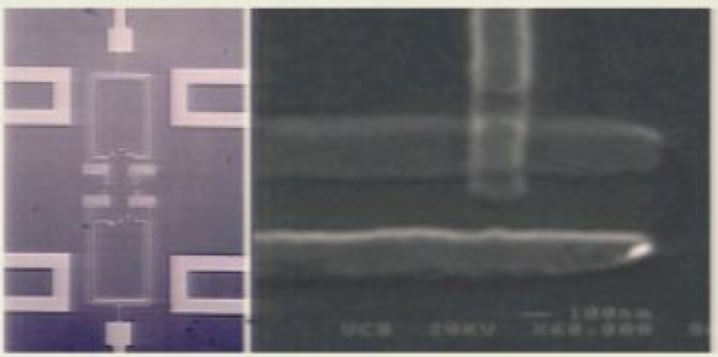
- $m = 1/3$: 3 of 4 levels blocking
- $m = 1$: 2 of 4 levels blocking
- $m = 3$: 1 of 4 levels blocking

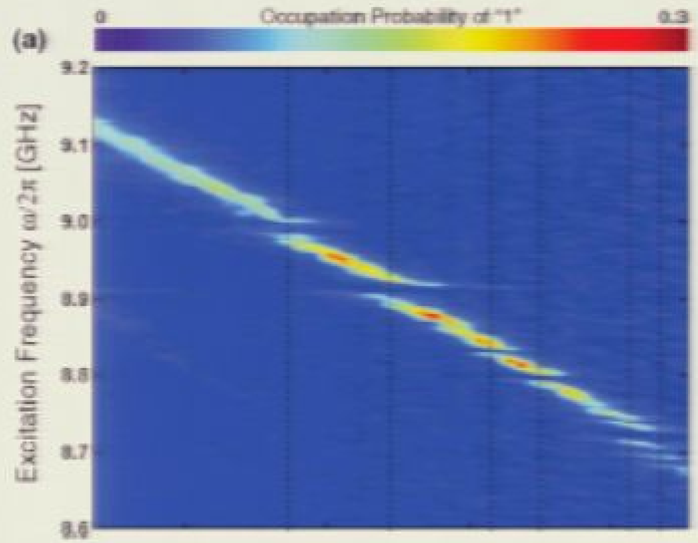
Outline

1. Spin in quantum dots
2. Noise in tunnel junctions

Superconducting tunnel junctions

Tool for applications: Bolometers, SQUIDs, qubits





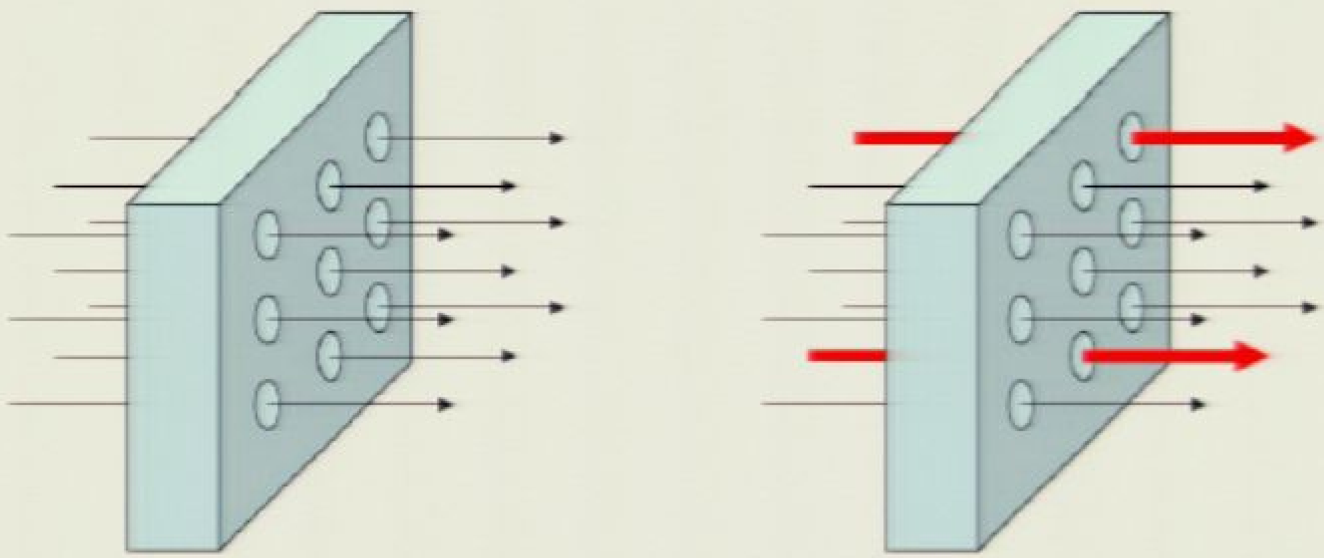
Excess noise in junctions limits sensitive experiments:

Qubit: R.W. Simmonds *et al.*, PRL 2004.

Bolometer / Mixer: P. Dieleman *et al.*, PRL 1997.

SQUIDs: F. Wellstood *et al.*, APL 2004.

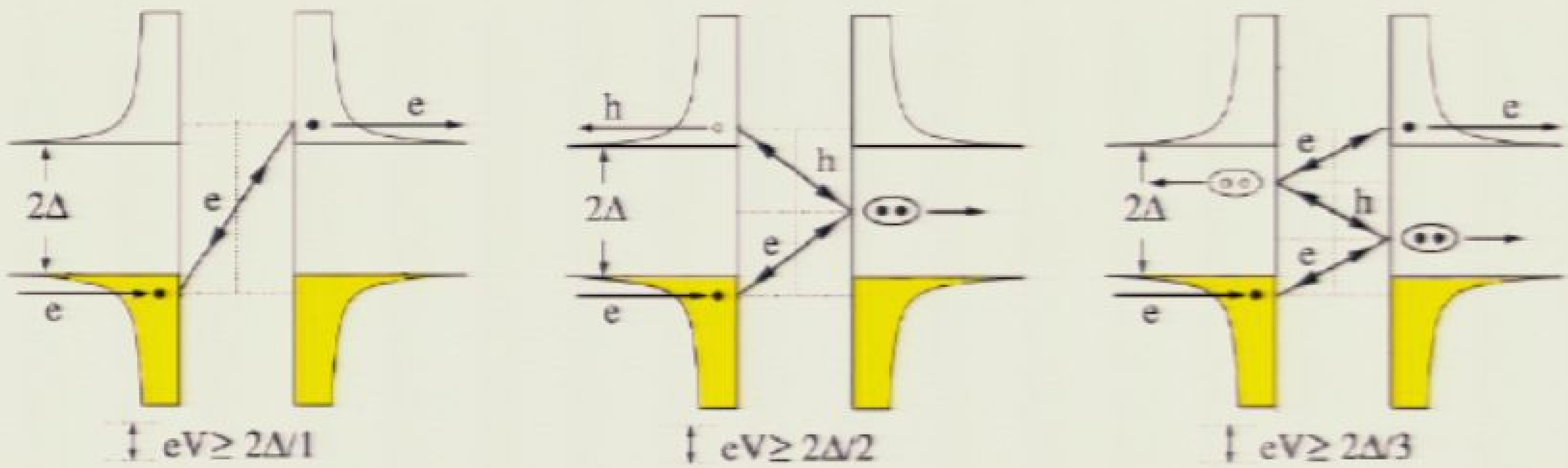
Rough junctions



Homogenous tunnel junction:
Transmission $T \ll 1$, M modes
Phase qubit: $M \simeq 10^6$, $T \simeq 0.003$

Rough junction: $T \propto e^{-d\sqrt{2m(U-E)}}$
Pinhole(s), $T \simeq 1$.

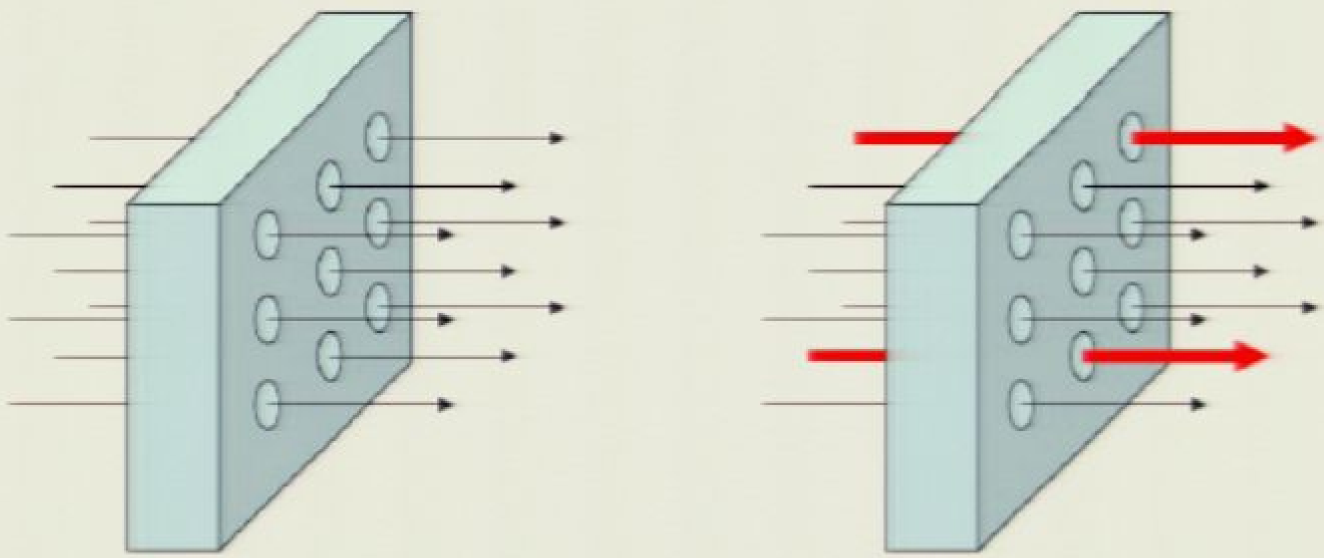
Multiple Andreev reflection



transferred charge 1e   transferred charge 2e   transferred charge 3e

Transferred charge $n \geq e\frac{2\Delta}{eV}$
Reflection at off-diagonal (electron-hole potential)
$\Rightarrow$ can take place above gap.
$V \rightarrow 0$: Supercurrent-carrying bound state

Rough junctions



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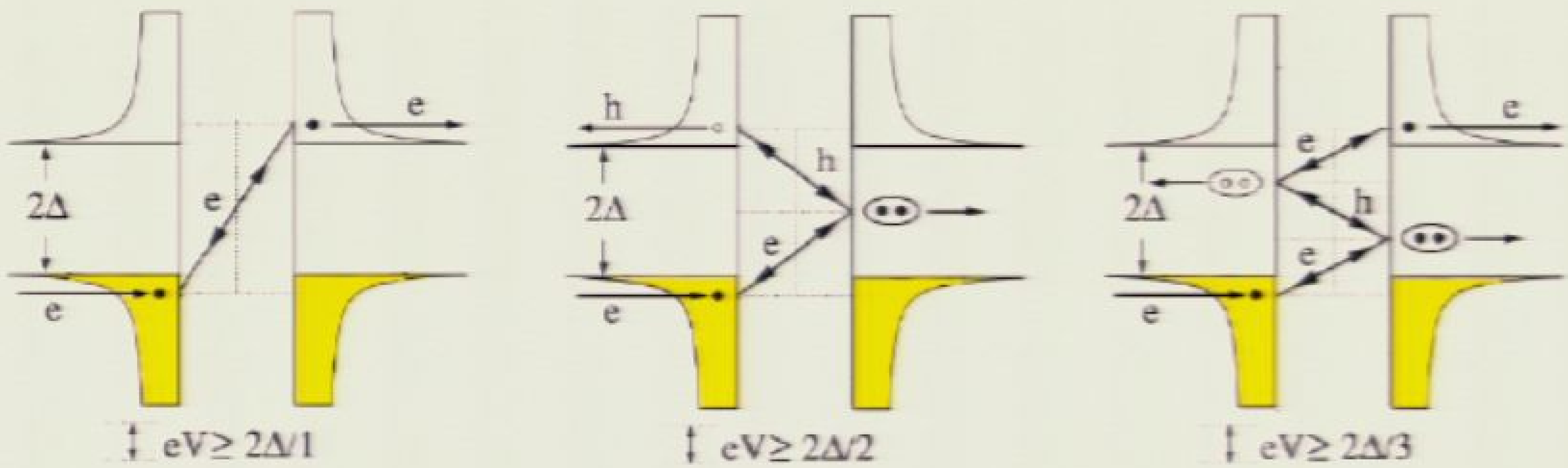

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Transport and shot noise

$T = 0$ quantum transport, no thermal noise.

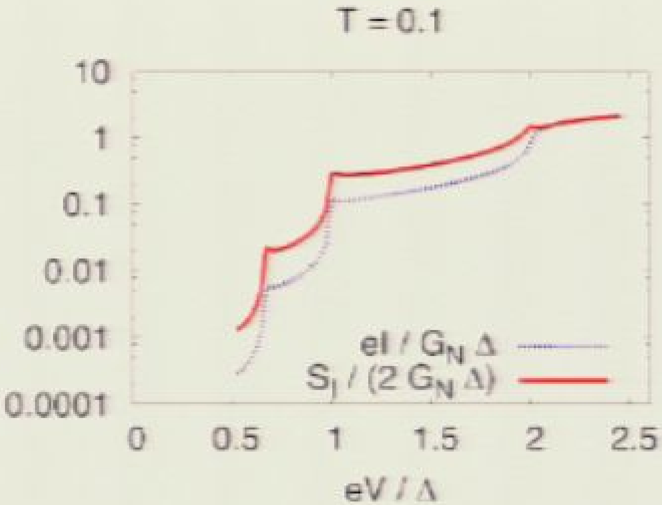
Conductance:
$G_N = G_Q \sum_n T_n$
Shot noise:
$S_I = 2eVG_Q \sum_n T_n(1 - T_n)$
Fano factor: $F = S_I/2eI$,
Tunnel junction: $F = 1$

Technique: Full counting statistics of charge transfer
Cumulant generating function $S(\chi)$:

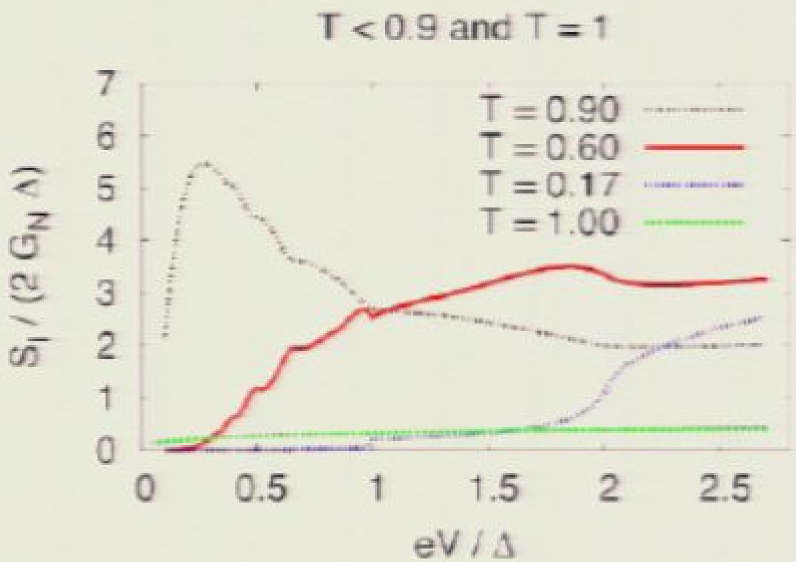
$$e^{-S(\chi)} = \sum_N P(N) e^{iN\chi}$$

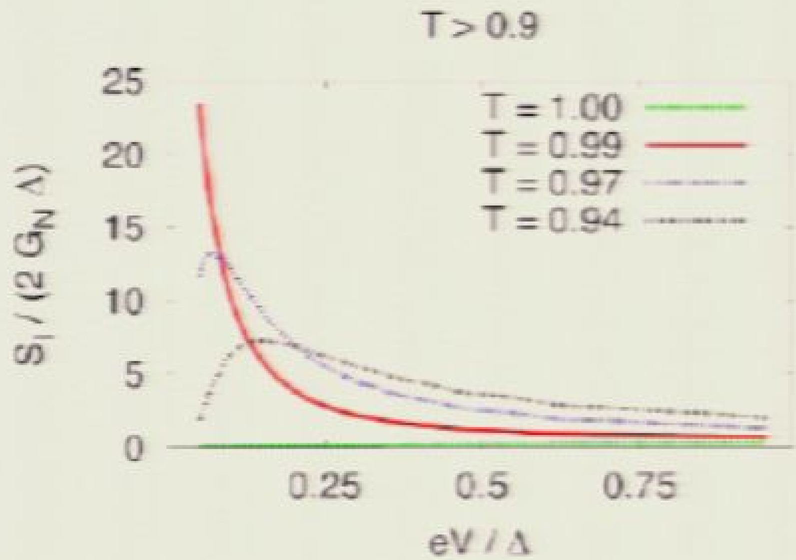
Calculated from Keldyh Green's function including counting field (Levitov, Lesovik, Nazarov, Belzig, 1996 —)

Single mode contact



Subharmonic gap structure
Crossover to $S \propto 2eN \propto 1/V$
Residual noise at $T = 1$: Certain about transport happening, uncertain about number of cycles (supergap AR!)





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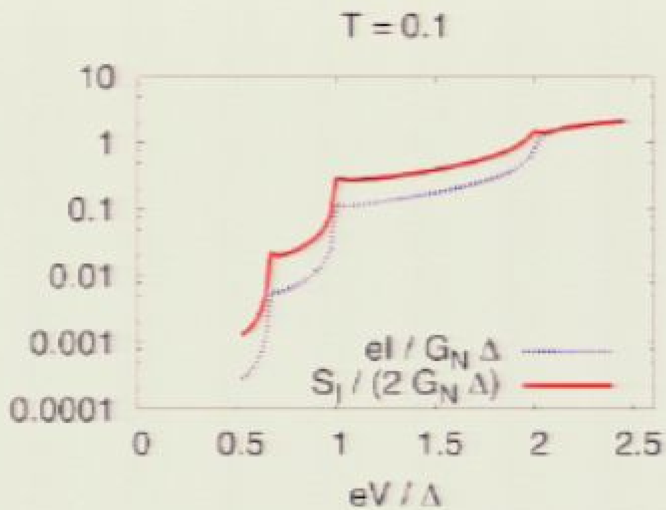

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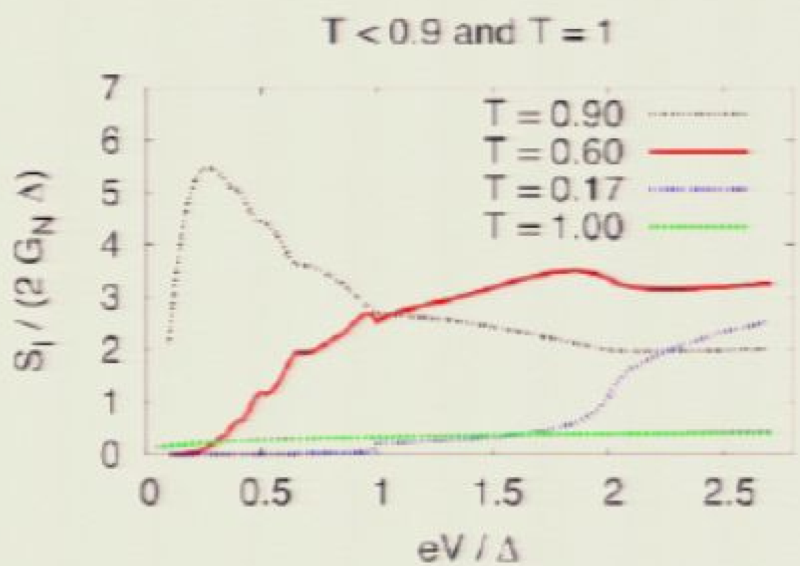

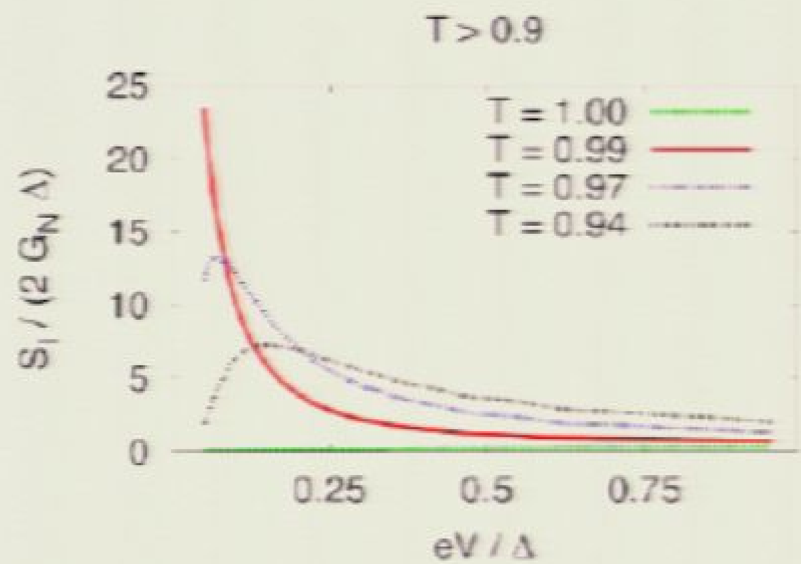

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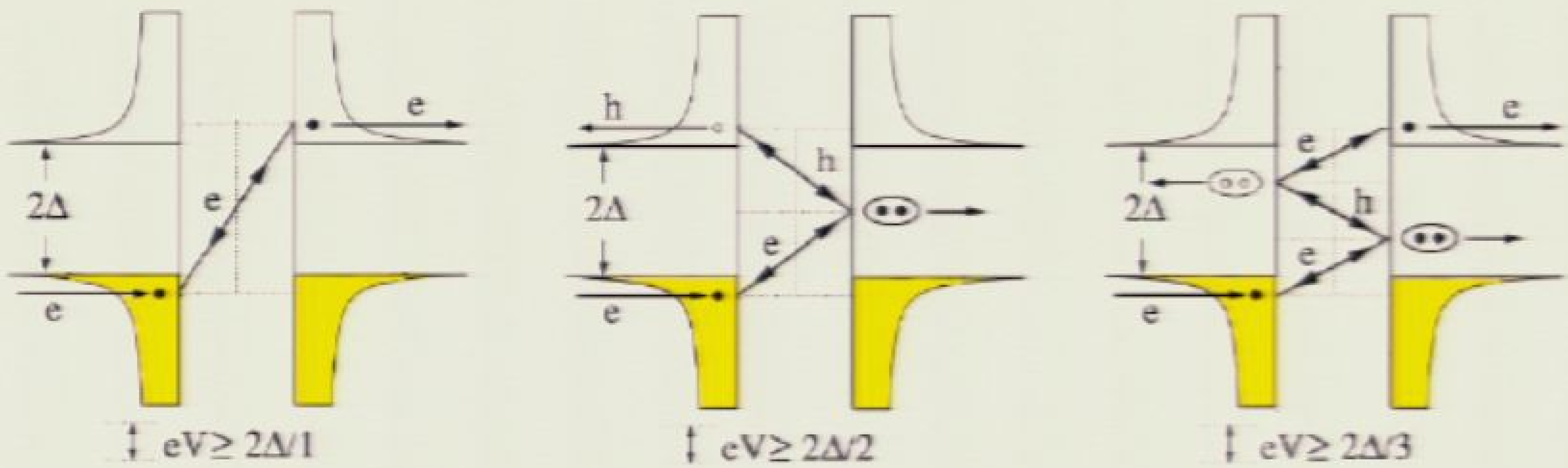

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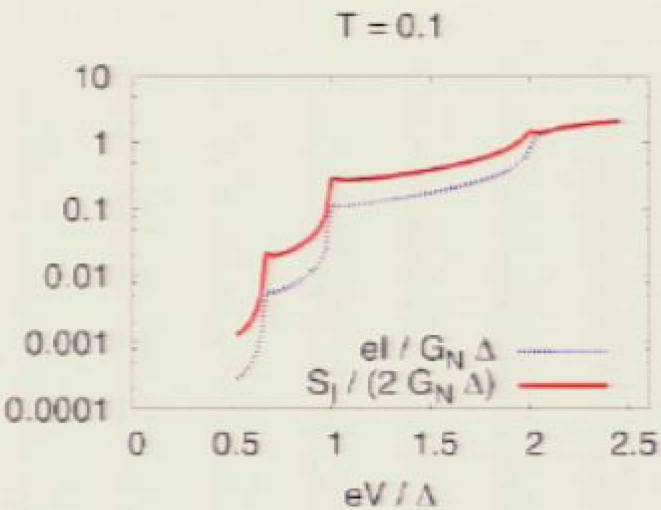
Conductance:
$G_N = G_Q \sum_n T_n$
Shot noise:
$S_I = 2eVG_Q \sum_n T_n(1 - T_n)$
Fano factor: $F = S_I/2eI$,
Tunnel junction: $F = 1$

Technique: Full counting statistics of charge transfer
Cumulant generating function $S(\chi)$:

$$e^{-S(\chi)} = \sum_N P(N) e^{iN\chi}$$

Calculated from Keldyh Green's function including counting field (Levitov, Lesovik, Nazarov, Belzig, 1996 —)

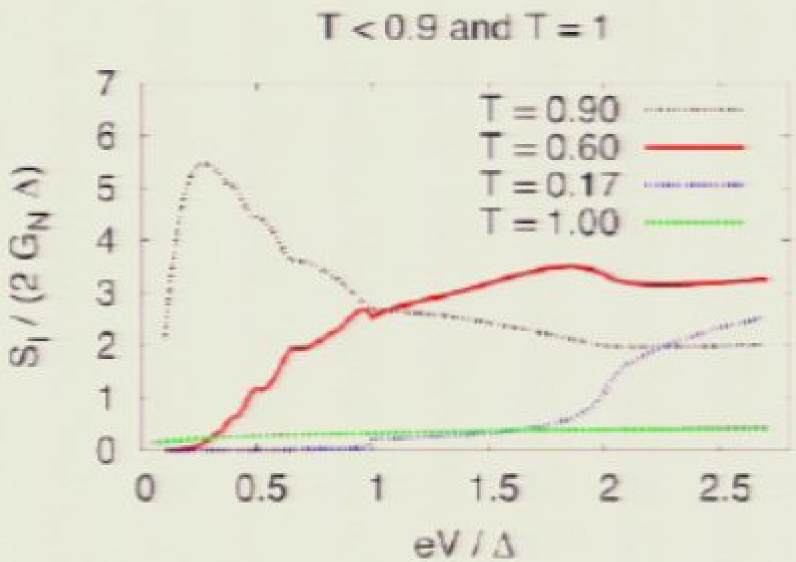
Single mode contact

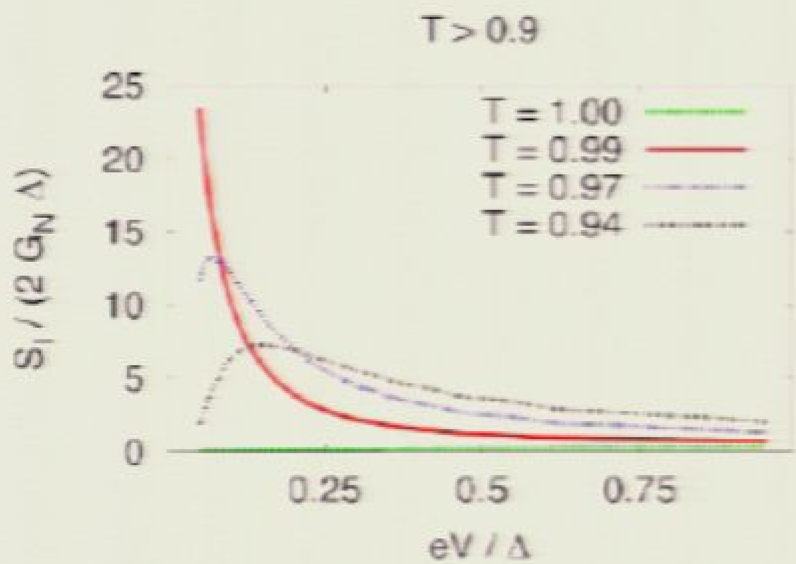


Subharmonic gap structure

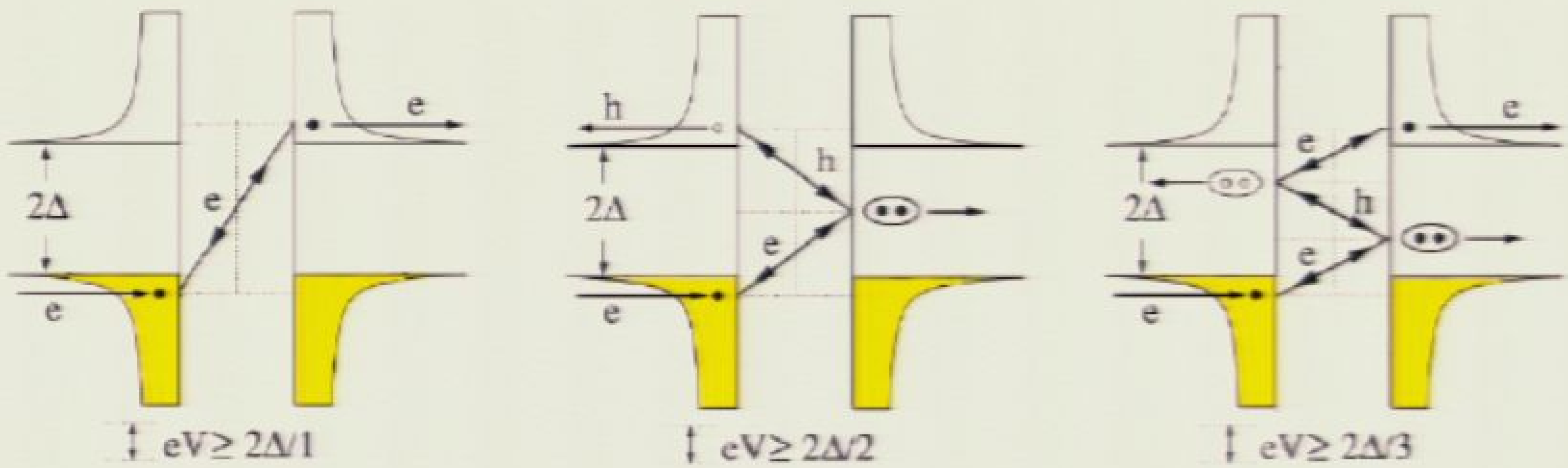
Crossover to $S \propto 2eN \propto 1/V$

Residual noise at $T = 1$: Certain about transport happening, uncertain about number of cycles (supergap AR!)





Multiple Andreev reflection



transferred charge 1e transferred charge 2e transferred charge 3e

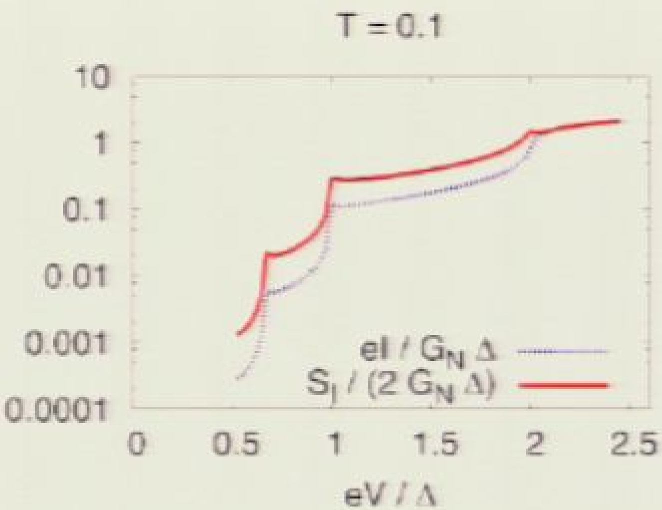
Transferred charge $n \geq e\frac{2\Delta}{eV}$

Reflection at off-diagonal (electron-hole potential)

$\Rightarrow$ can take place above gap.

$V \to 0$: Supercurrent-carrying bound state

Single mode contact



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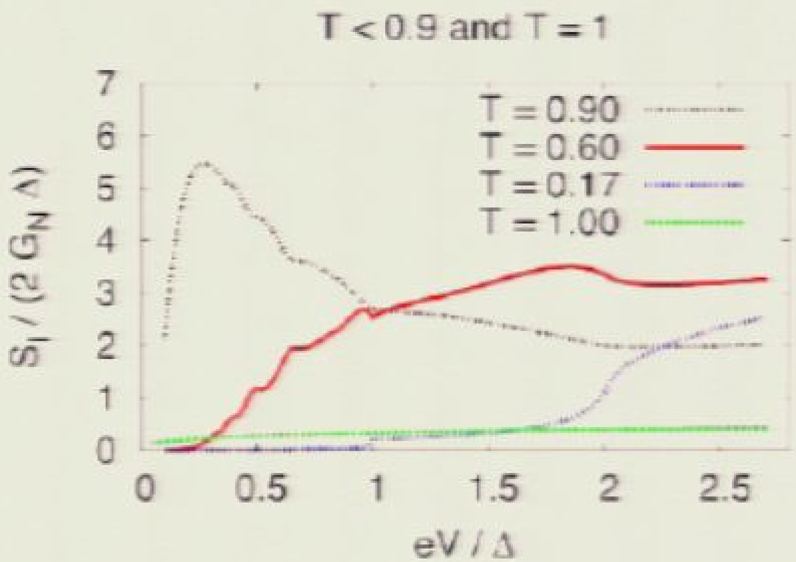
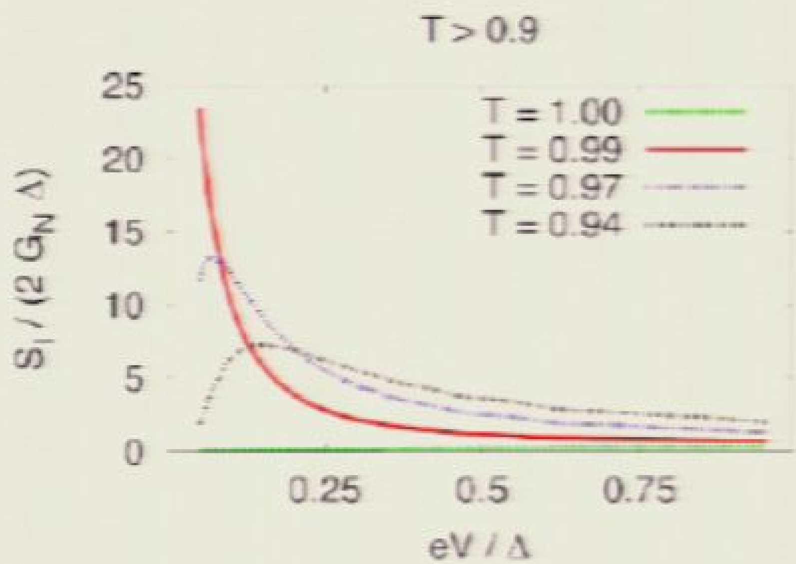
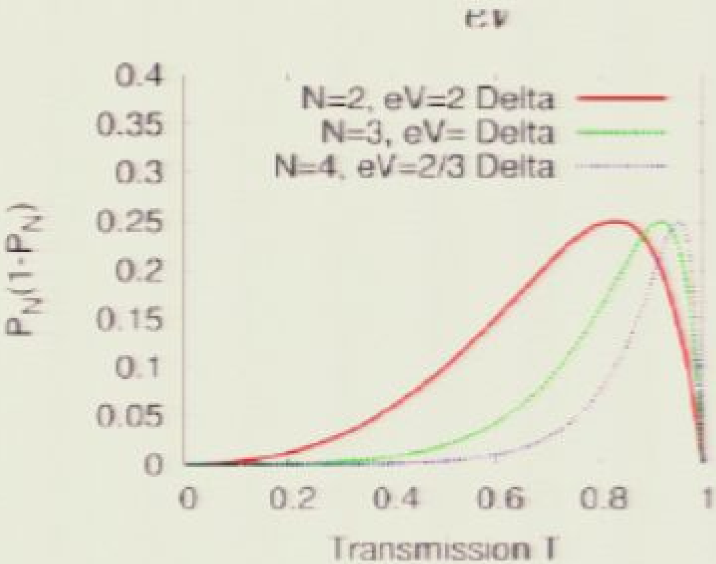
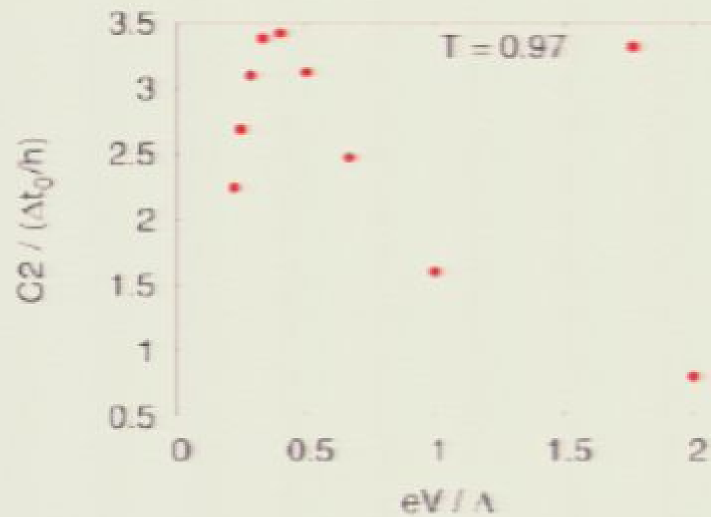




Counting Andreev cycles





Qualitative prediction $S \propto 2enT^n(1 - T^n)$ for n-th order Andreev

Divergence as $1/V^\alpha$.

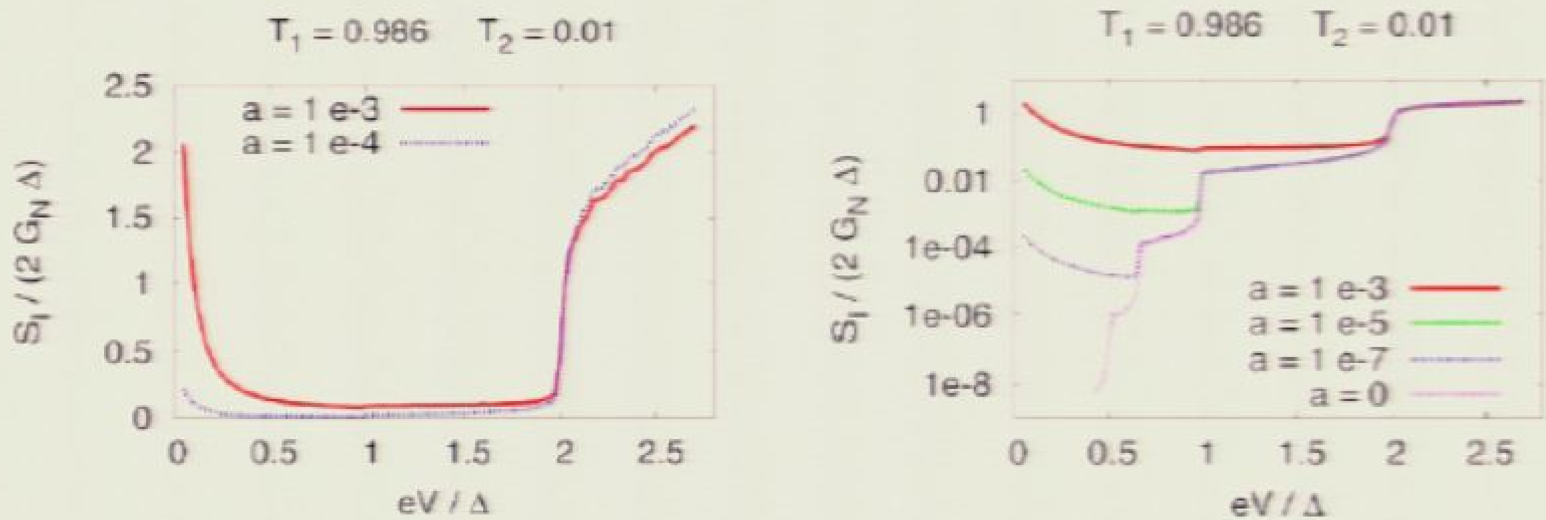
Qualitative: $\alpha = 1$, full model $\alpha \simeq 0.8$.

G. Heinrich and FKW, arXiv:0808.3705

Full tunnel junctions

Tunnel contact with fraction *a* pinholes

$$\rho(T) = a\delta(T - T_1) + (1 - a)\delta(T - T_2)$$



Pinhole can be hidden in current but dominate low-V-noise

Summary

- Spin-flip cotunneling
- Noise in superconducting tunnel junctions