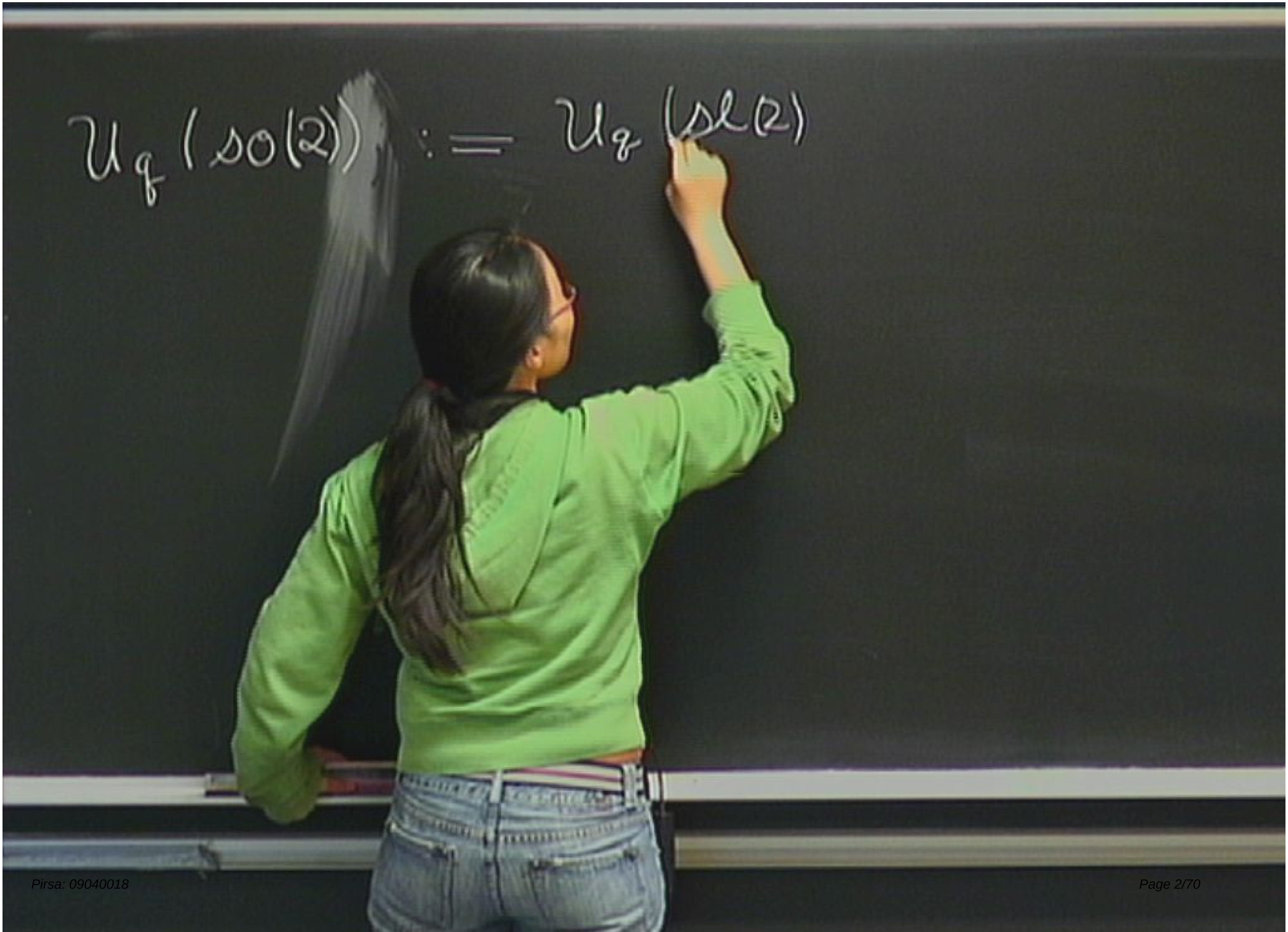
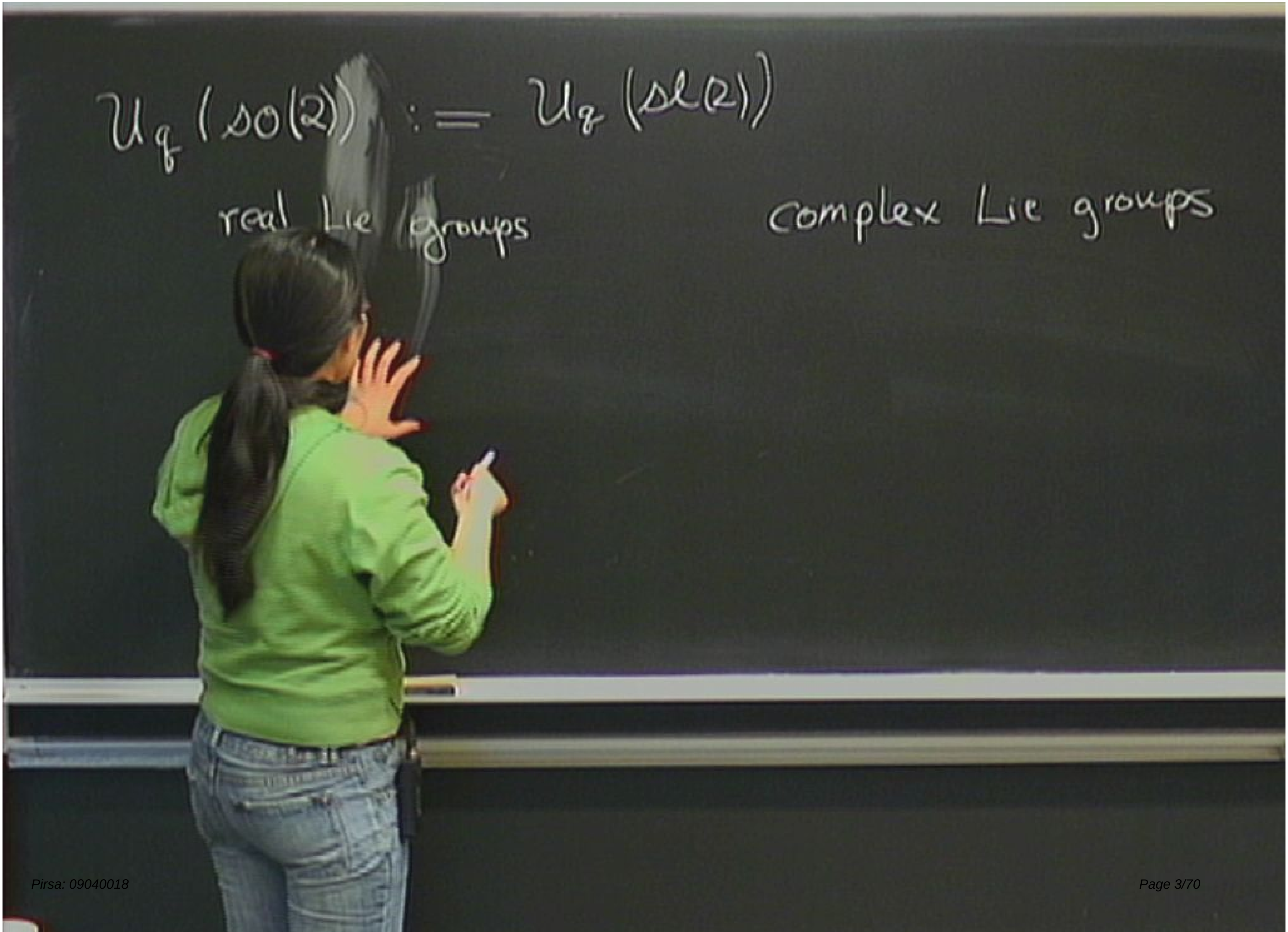
Title: Grad Talks 9

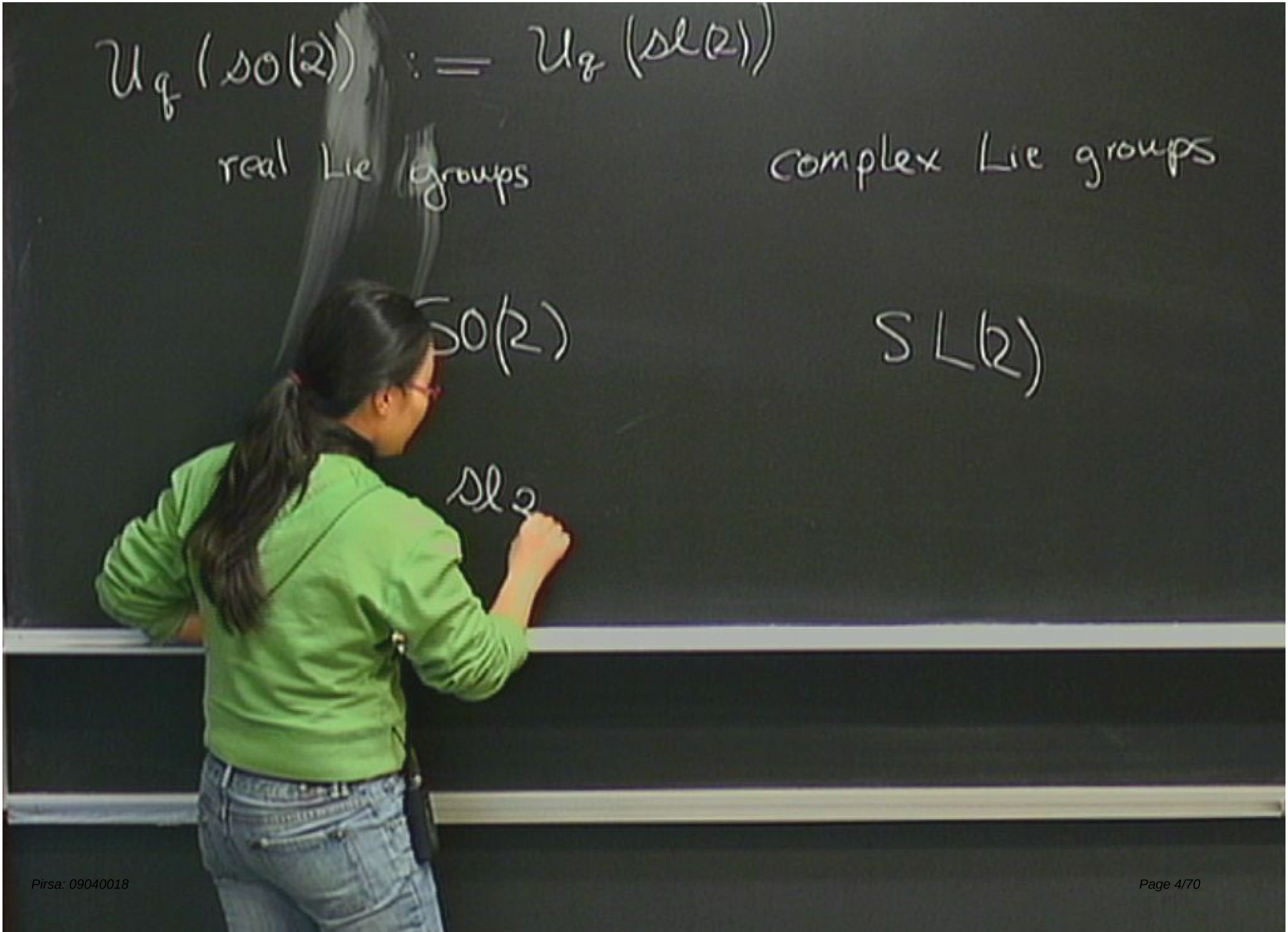
Date: Apr 13, 2009 11:00 AM

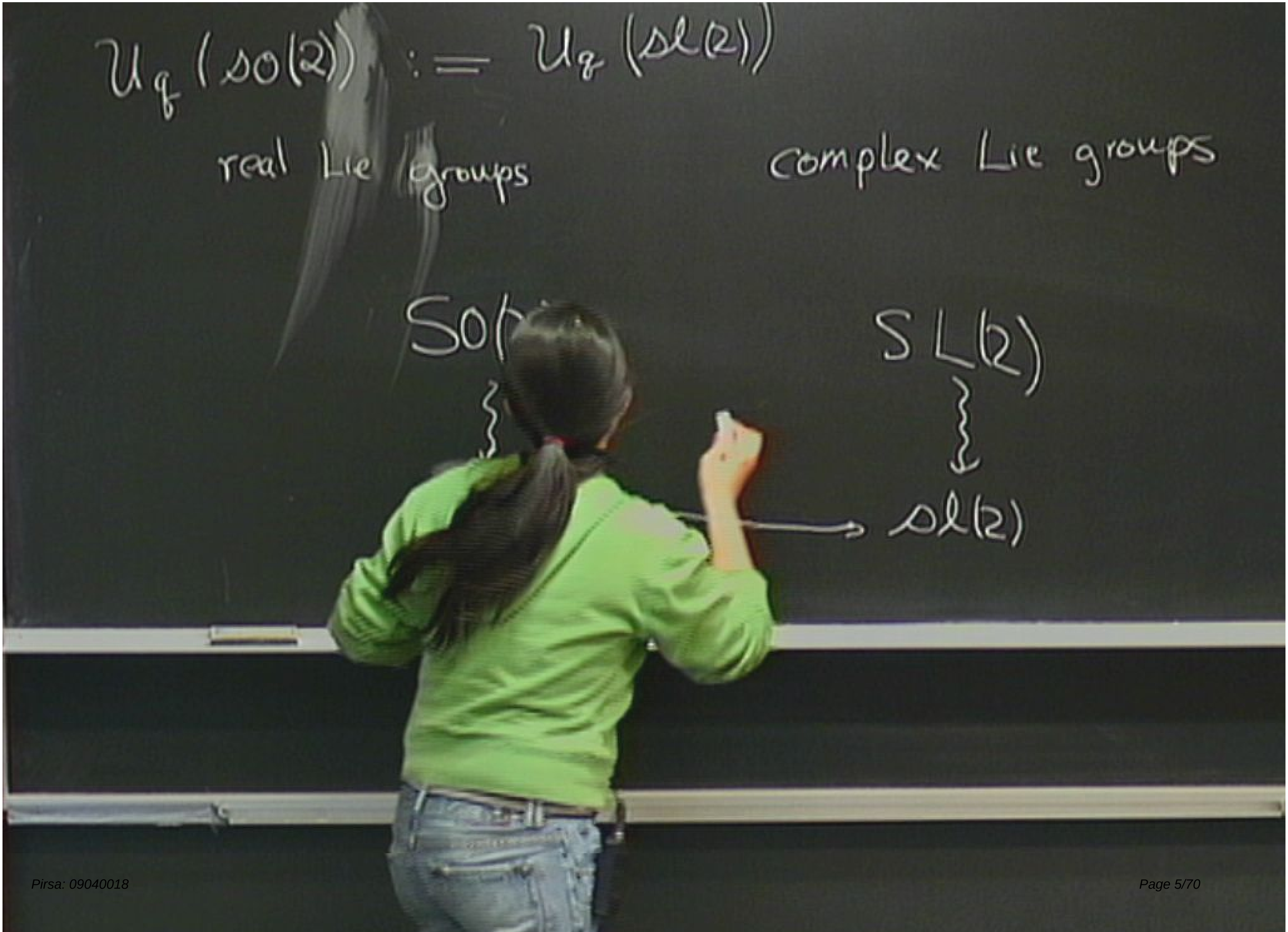
URL: http://pirsa.org/09040018

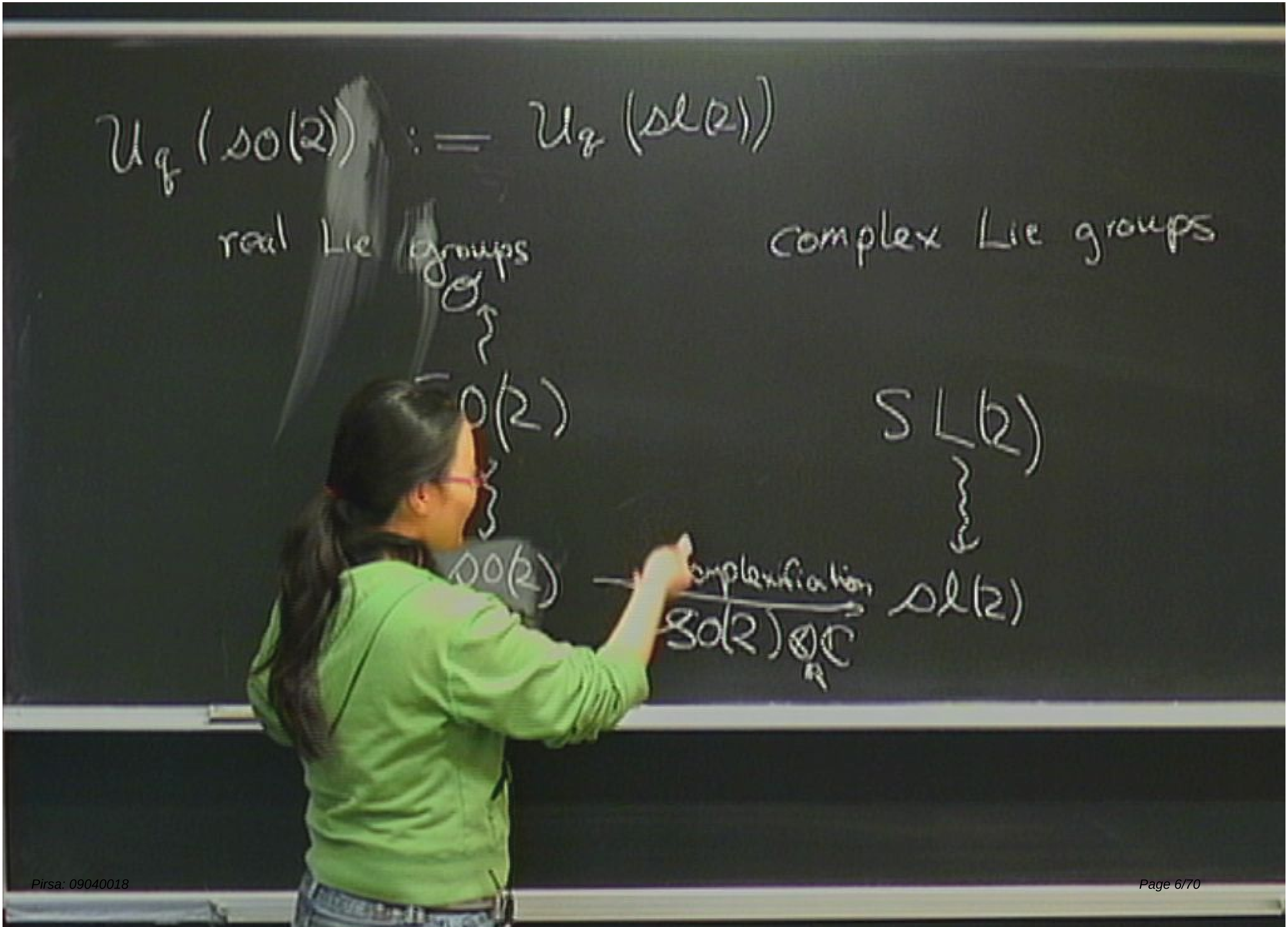
Abstract: Lecture on Quantum Groups by Lucy Zhang

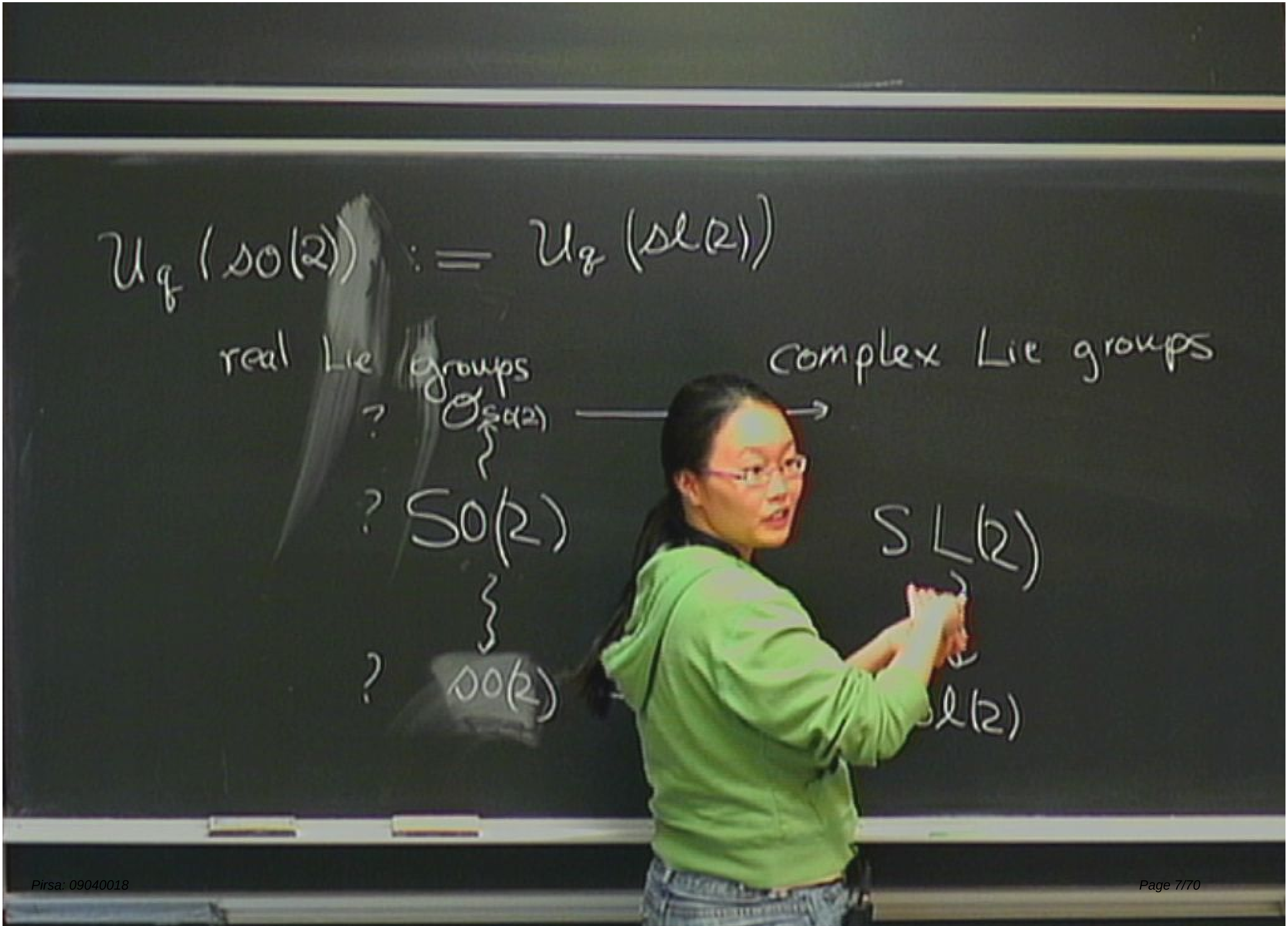

$\mathcal{U}_q(so(2)) := \mathcal{U}_q(sl(2))$


$U_q(so(2)) := U_q(sl(2))$
real Lie groups
complex Lie groups


$U_q(so(2)) := U_q(sl(2))$
real Lie groups
complex Lie groups
$SO(2)$
$SL(2)$
sl_2


$\mathcal{U}_q(so(2)) := \mathcal{U}_q(sl(2))$
real Lie groups
complex Lie groups
SO(
SL(2)
sl(2)


$U_q(so(2)) := U_q(sl(2))$
real Lie groups
complex Lie groups
$SO(2)$
$SL(2)$
$so(2) \xrightarrow{\text{complexification}} sl(2)$
$so(2) \otimes_{\mathbb{R}} \mathbb{C}$


$U_q(so(2)) := U_q(sl(2))$
real Lie groups
complex Lie groups
? $\mathcal{O}_{SO(2)}$
? $SO(2)$
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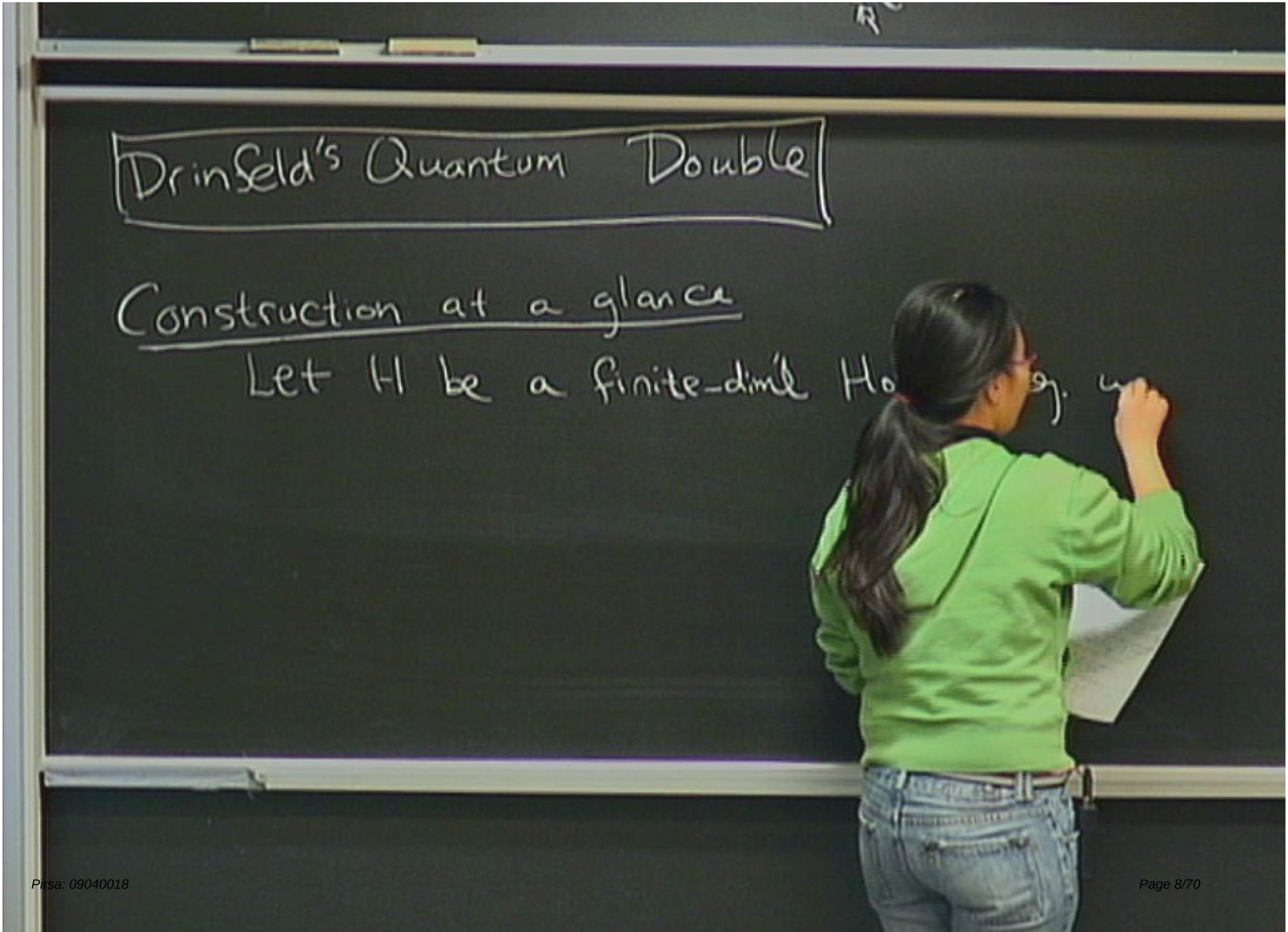
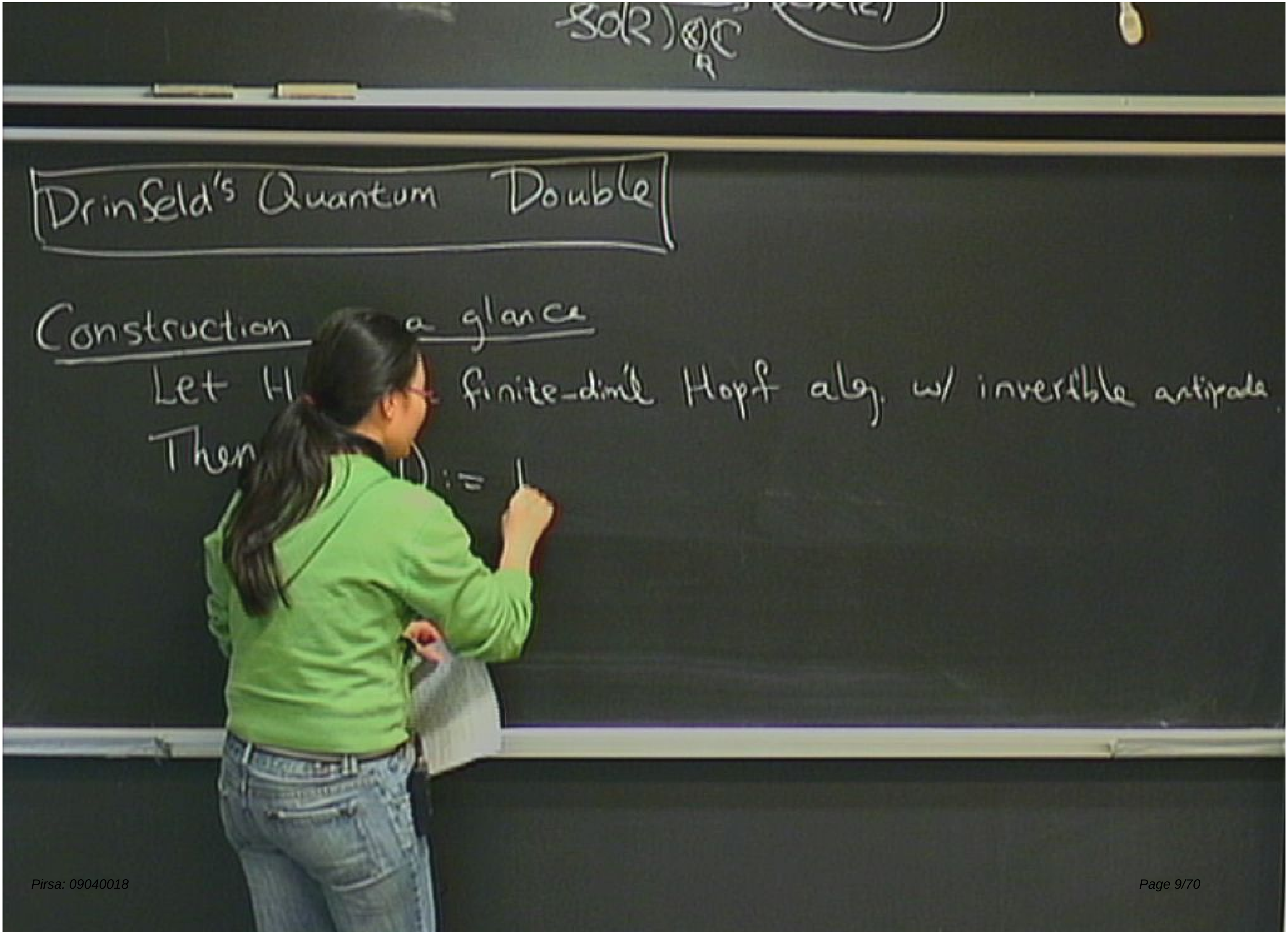
? $so(2)$


Drinfeld's Quantum Double
Construction at a glance
Let H be a finite-dim'l Ho



Drinfeld's Quantum Double
Construction a glance
Let H finite-dim'l Hopf alg. w/ invertible antipode
Then

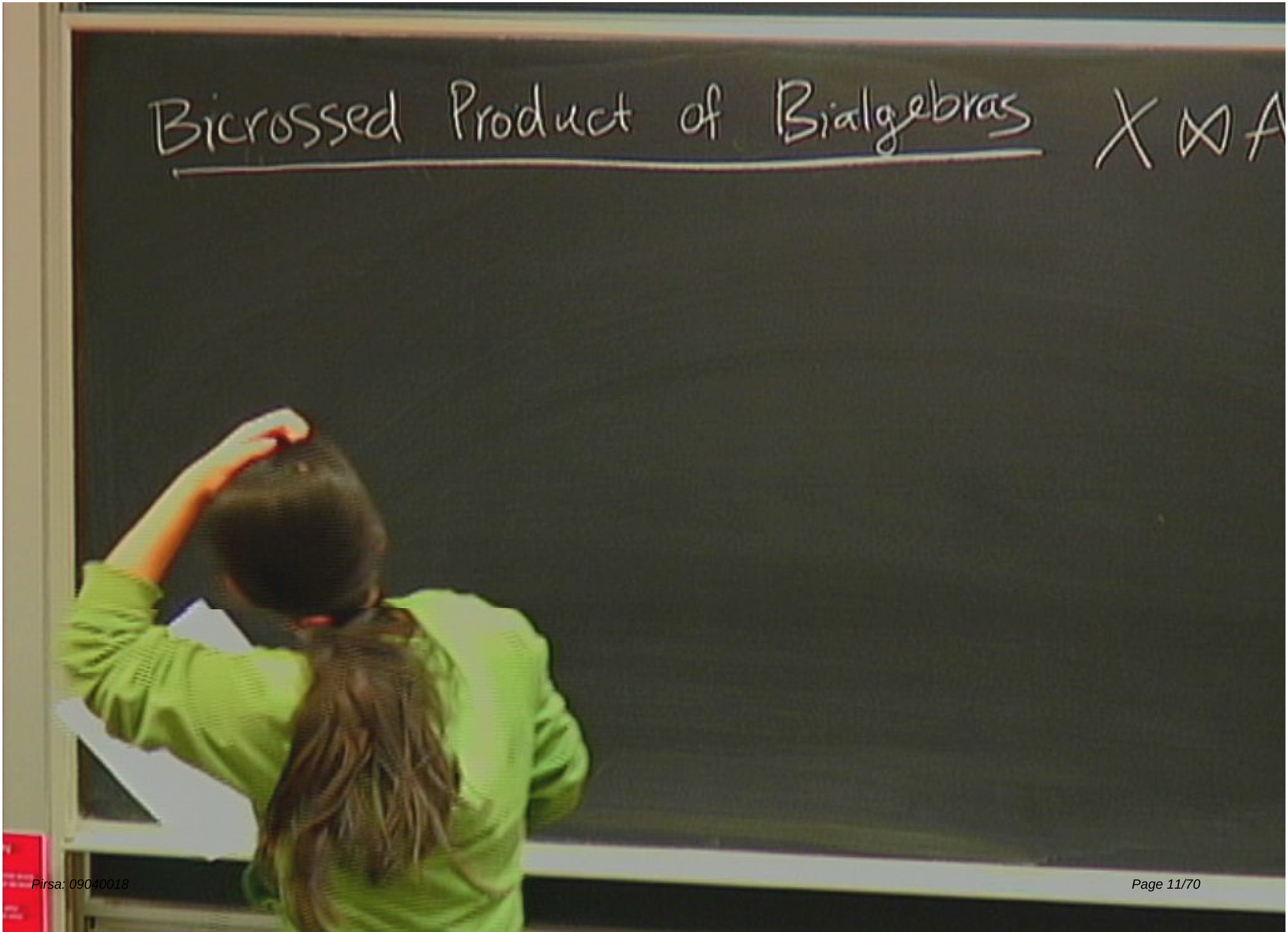
$SO(2)\otimes_{\mathbb{R}}\mathbb{C}$

Drinfeld's Quantum Double

Construction at a glance

Let H be a finite-dim'l Hopf alg. w/ invertible antipode.

Then $D(H) := (H^{op})^* \bowtie H$ based on coadj. rep'n


Bicrossed Product of Bialgebras
X ⋈ A


Bicrossed Product of Bialgebras
X ⋈ A
Semi-direct Product of Groups
H ⋊φ K

Bicrossed Product of Bialgebras $X \bowtie A$

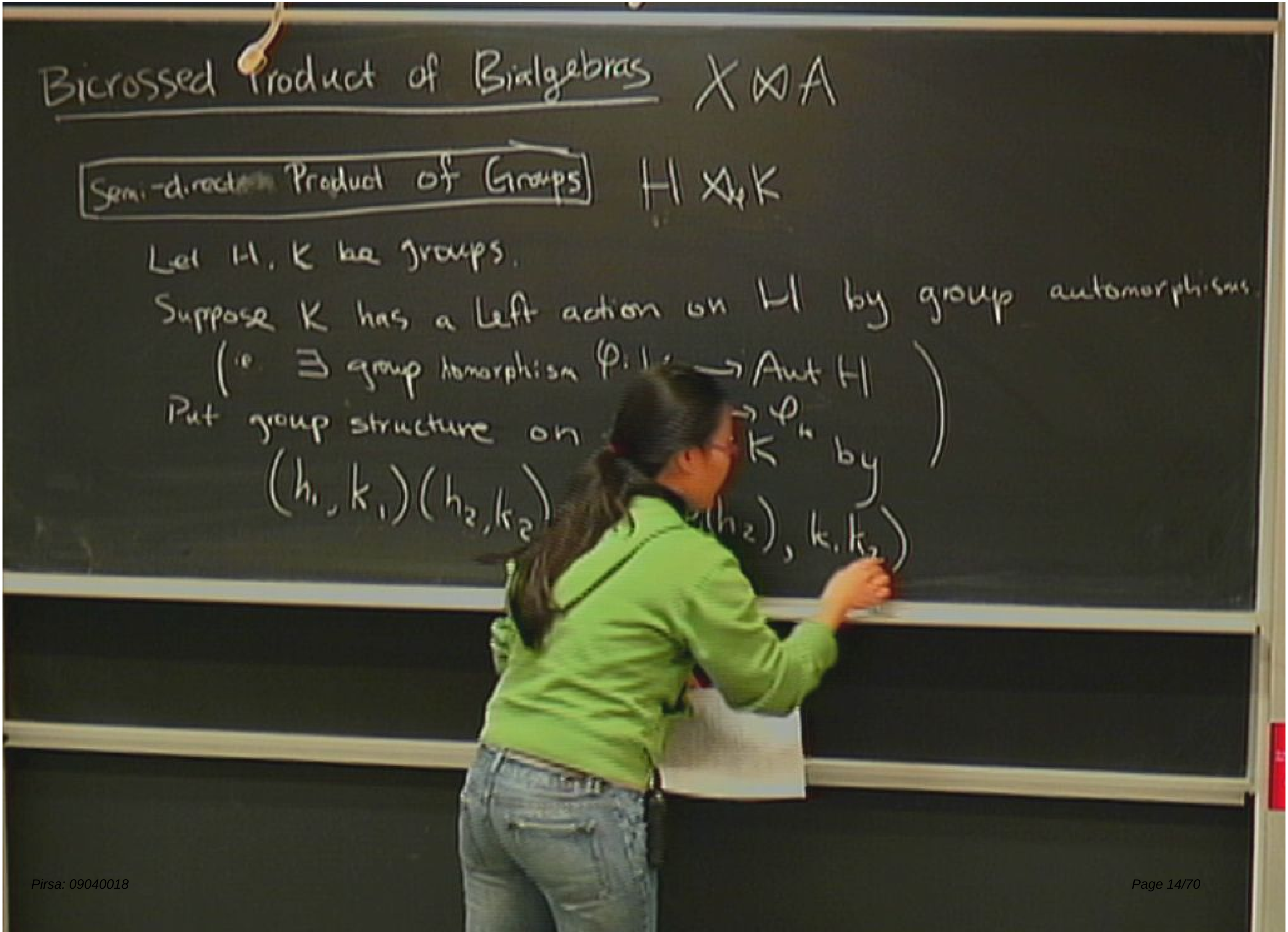
Semi-direct Product of Groups $H \rtimes_{\varphi} K$

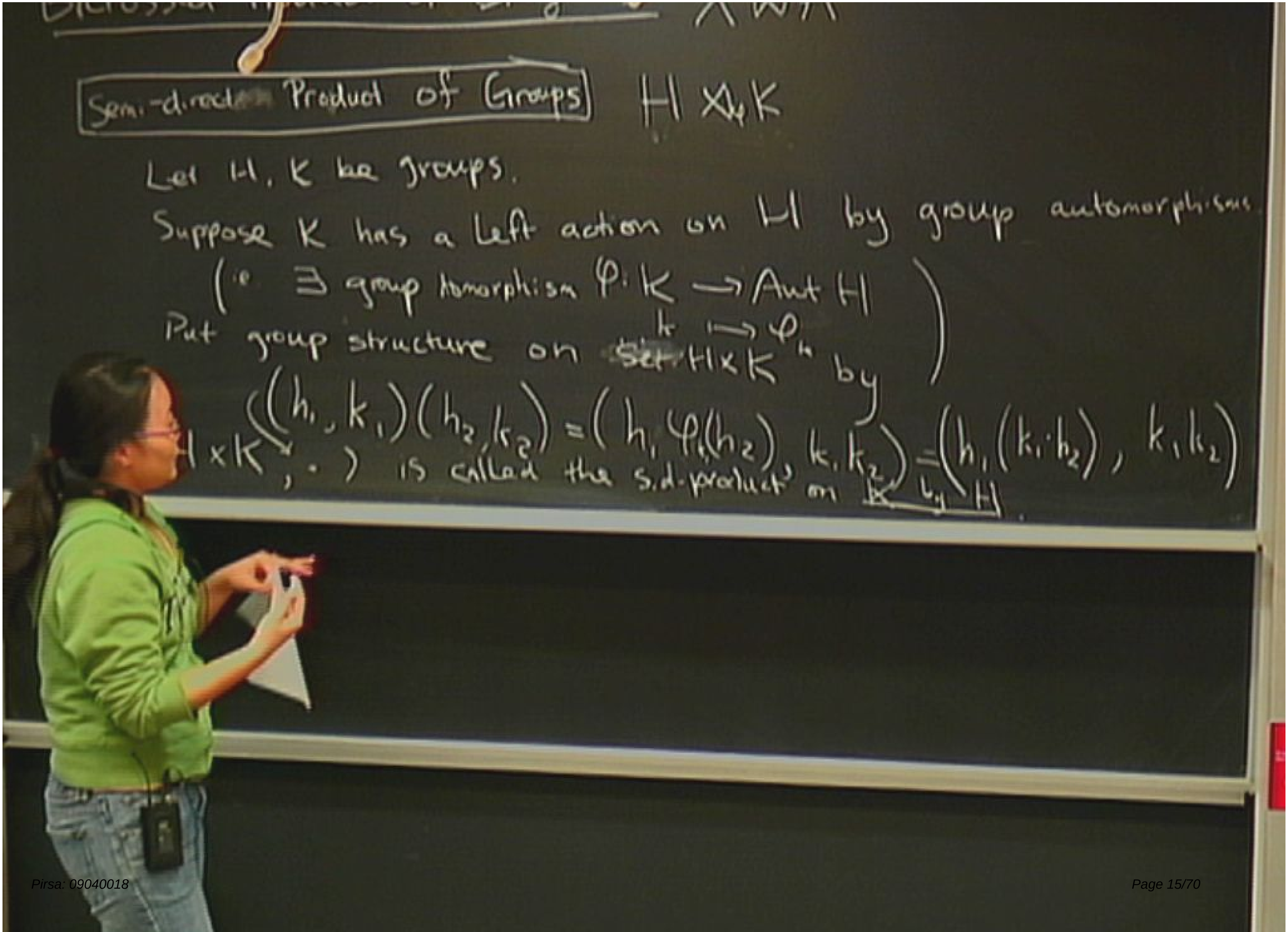
Let H, K be groups.

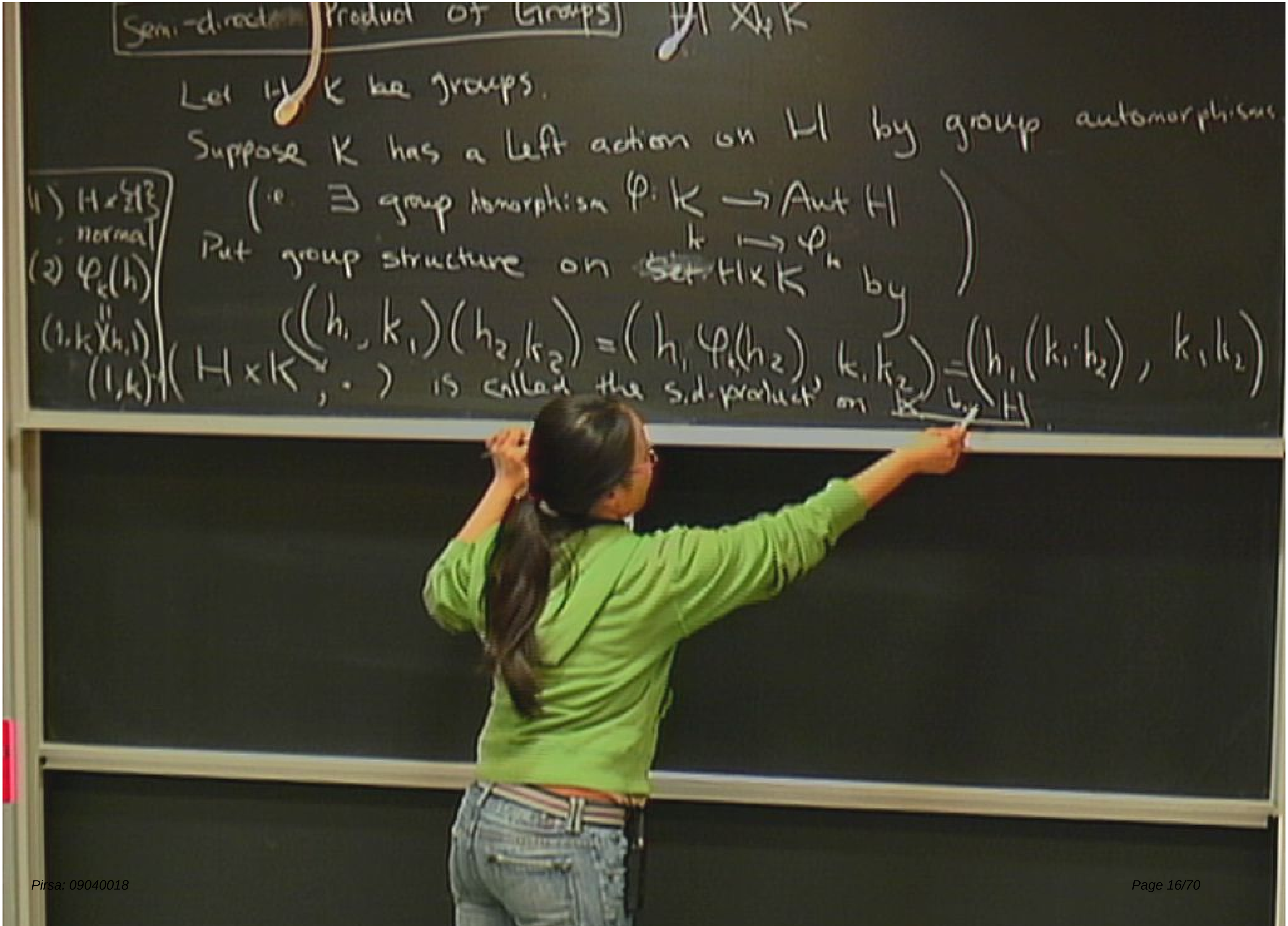
Suppose K has a left action on H by group automorphisms.

(i.e. $\exists$ group homomorphism $\varphi : K \longrightarrow \mathrm{Aut}\, H$, $k \longmapsto \varphi_k$)

P


Bicrossed Product of Bialgebras X⋈A
Semi-direct Product of Groups H ⋊ K
Let H, K be groups.
Suppose K has a left action on H by group automorphisms.
(i.e. ∃ group homomorphism φ: → Aut H
→ φ
Put group structure on K by
(h1, k1)(h2, k2) (h2), k1k2)


Semi-direct Product of Groups
$H \rtimes_{\varphi} K$
Let H, K be groups.
Suppose K has a left action on H by group automorphisms
(i.e. ∃ group homomorphism $\varphi: K \to \mathrm{Aut}\, H$, $k \mapsto \varphi_k$)
Put group structure on ~~set~~ $H \times K$ by
$(h_1, k_1)(h_2, k_2) = (h_1 \varphi_{k_1}(h_2), k_1 k_2) = (h_1 (k_1 \cdot h_2), k_1 k_2)$
$(H \times K, \cdot)$ is called the s.d. product of K on H.


Semi-direct Product of Groups
H ⋊_φ K
Let H, K be groups.
Suppose K has a left action on H by group automorphisms
(i.e. ∃ group homomorphism φ: K → Aut H
k ↦ φ_k)
Put group structure on set H×K by
((h₁, k₁)(h₂, k₂)) = (h₁ φ_k₁(h₂), k₁k₂) = (h₁ (k₁·h₂), k₁k₂)
(H×K, ·) is called the s.d. product of K by H.
(1) H ◁ {1}×… normal
(2) φ_k(h) = (1,k)(h,1)(1,k)⁻¹

(2) $\varphi_k(h)$ Put group structure on set $H \times K$ by

$(1,k)(h,1)$ $\quad (h_1, k_1)(h_2, k_2) = (h_1\varphi_{k_1}(h_2), k_1k_2) = (h_1(k_1 \cdot h_2), k_1k_2)$

$(1,k)$ $(H \times K, \cdot)$ is called the s.d. product on K by H.

Bicrossed product of Group $\quad H\,{}_{\beta}\!\bowtie_{\alpha} K$

Setup

A pair (H, K) of groups is saidsd to be matched

(2) $\varphi_k(h)$ Put group structure on set "$H \times K$" by

$(1,k)(h,1)$ $(h_1,k_1)(h_2,k_2) = (h_1\varphi_{k_1}(h_2), k_1k_2) = (h_1(k_1 \cdot h_2), k_1k_2)$

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Bicrossed product of Group $H \,{}_\beta\!\bowtie_\alpha K$

Setup

A pair (H,K) of groups is said to be matched

if $\exists$ $\alpha : K \times H \to H$ left action of group K on set H

$\beta : K \times H \to K$ right action of group H on set K

$(2)\ \varphi_k(h) = (1,k)(h,1)(1,k)^{-1}$

Put group structure on set "$H \times K$" by

$$(h_1,k_1)(h_2,k_2) = (h_1\varphi_{k_1}(h_2),\ k_1k_2) = (h_1(k_1\cdot h_2),\ k_1k_2)$$

$(H\times K, \cdot)$ is called the s.d. product of $\underline{K \text{ by } H}$.

Setup

A pair (H,K) of groups is said to be <u>matched</u>

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$\beta: K\times H \to K$ right action of group H on set K

s.t.

$(zz')^y = z^{z'\cdot y}\, z'^{\,y}$

$z\cdot(\ldots y) = z^y \cdot \ldots$

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$(1,k)(h,1)$ $(h_1, k_1)(h_2, k_2) = (h_1 \varphi_{k_1}(h_2), k_1 k_2) = (h_1 (k_1 \cdot h_2), k_1 k_2)$
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A pair (H, K) of groups is said to be matched
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$\beta : K \times H \to K$ right " " H " " K
s.t.
$(z z')^y = z^{z' \cdot y} z'^y$
$z \cdot (y y') = (z \cdot y)(z^y \cdot y')$

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s.t.

$$(zz')^y = z^{z'\cdot y} z'^y$$

$$z\cdot(yy') = (z\cdot y)(z^y \cdot y')$$

$$z \cdot 1 = 1$$

$$1^y = 1$$

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$\shortparallel$

$(1,k)(h,1)$ $\quad (h_1, k_1)(h_2, k_2) = (h_1 \varphi_{k_1}(h_2), k_1 k_2) = (h_1 (k_1 \cdot h_2), k_1 k_2)$

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A pair (H, K) of groups is said to be <u>matched</u>

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$$\alpha(k, h) =: k \cdot h$$

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(2) $\varphi_k(h)$ Put group structure on "set $H \times K$" by

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$(1,k)$ $(H \times K, \cdot)$ is called the s.d. product on K by H.

Biсrossed Product of Groups $\quad H \,{}_\beta\!\bowtie_\alpha K$

Setup

A pair (H, K) of groups is said to be matched

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$(z z')^y = z^{z' \cdot y} z'^{\, y}$

$z \cdot (y y') = (z \cdot y)(z^y \cdot y')$

$z \cdot 1 = 1$

$1^y = 1$

$\forall\, y, y' \in H$, $z, z' \in K$

$\alpha(k,h) =: k \cdot h$

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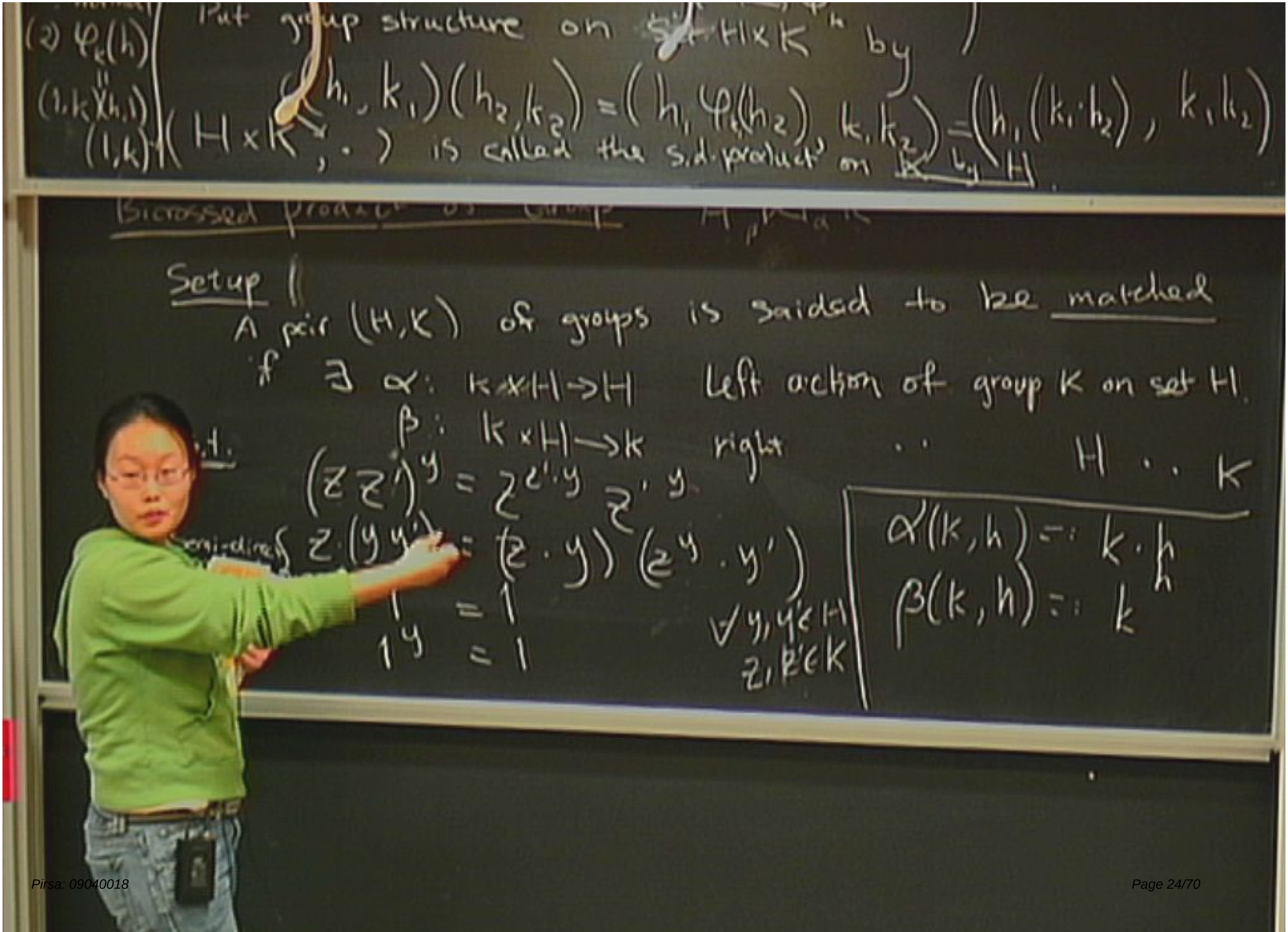
(2) $\varphi_k(h)$
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$(1,k)^{-1}$
Put group structure on set $H \times K$ by
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Bicrossed product of groups
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s.t.
$(z z')^y = z^{z' \cdot y} z'^y$
$z \cdot (y y') = (z \cdot y)(z^y \cdot y')$
$1 = 1$
$1^y = 1$
$\forall y, y' \in H$
$z, z' \in K$
$\alpha(k, h) =: k \cdot h$
$\beta(k, h) =: k^h$

(2) $\varphi_k(h)$ Put group structure on set $H \times K$ by

$(1,k)(h,1)$

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Bicrossed product of groups $H \,{}_\beta\!\bowtie_\alpha K$

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semi-direct product

$$z \cdot (y y') = (z \cdot y)(z^y \cdot y')$$

$$z \cdot 1 = 1$$

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$\forall y, y' \in H$, $z, z' \in K$

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(2) $\varphi_k(h)$ = $(1,k)(h,1)(1,k)^{-1}$

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$(H \times K, \cdot)$ is called the s.d. product of K by H.

Bicrossed Product of Groups $H \,{}_\beta\!\bowtie_\alpha K$

Setup

A pair (H, K) of groups is said to be matched

if $\exists$ $\alpha : K \times H \to H$ left action of group K on set H

$\beta : K \times H \to K$ right " " H " " K

s.t.

$$(z z')^y = z^{z' \cdot y} z'^{y}$$

semi-direct product $\left\{ \begin{array}{l} z \cdot (y y') = (z \cdot y)(z^y \cdot y') \\ z \cdot 1 = 1 \end{array} \right.$

$$1^y = 1$$

$\forall\, y, y' \in H$, $z, z' \in K$

$\alpha(k, h) =: k \cdot h$

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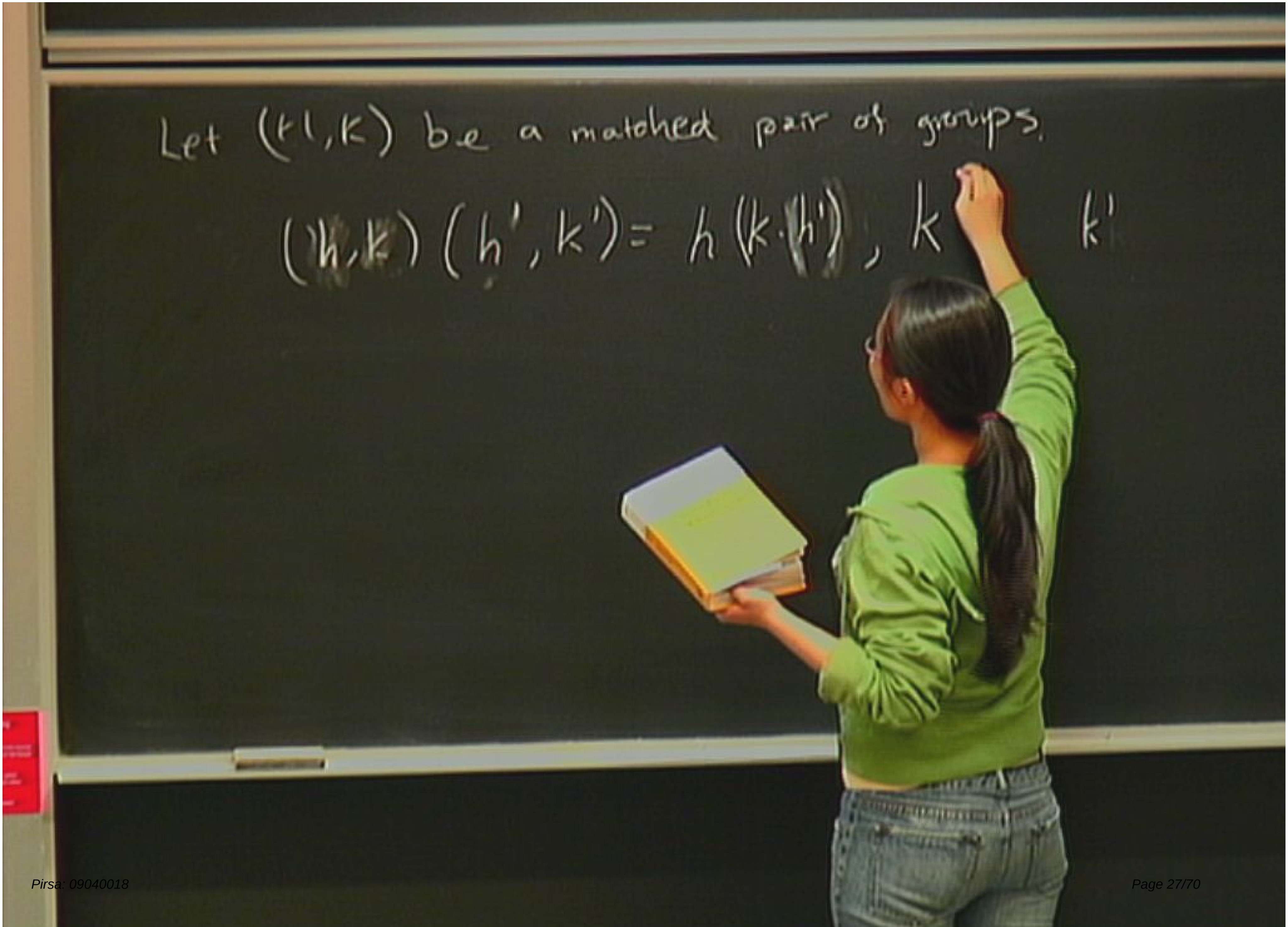
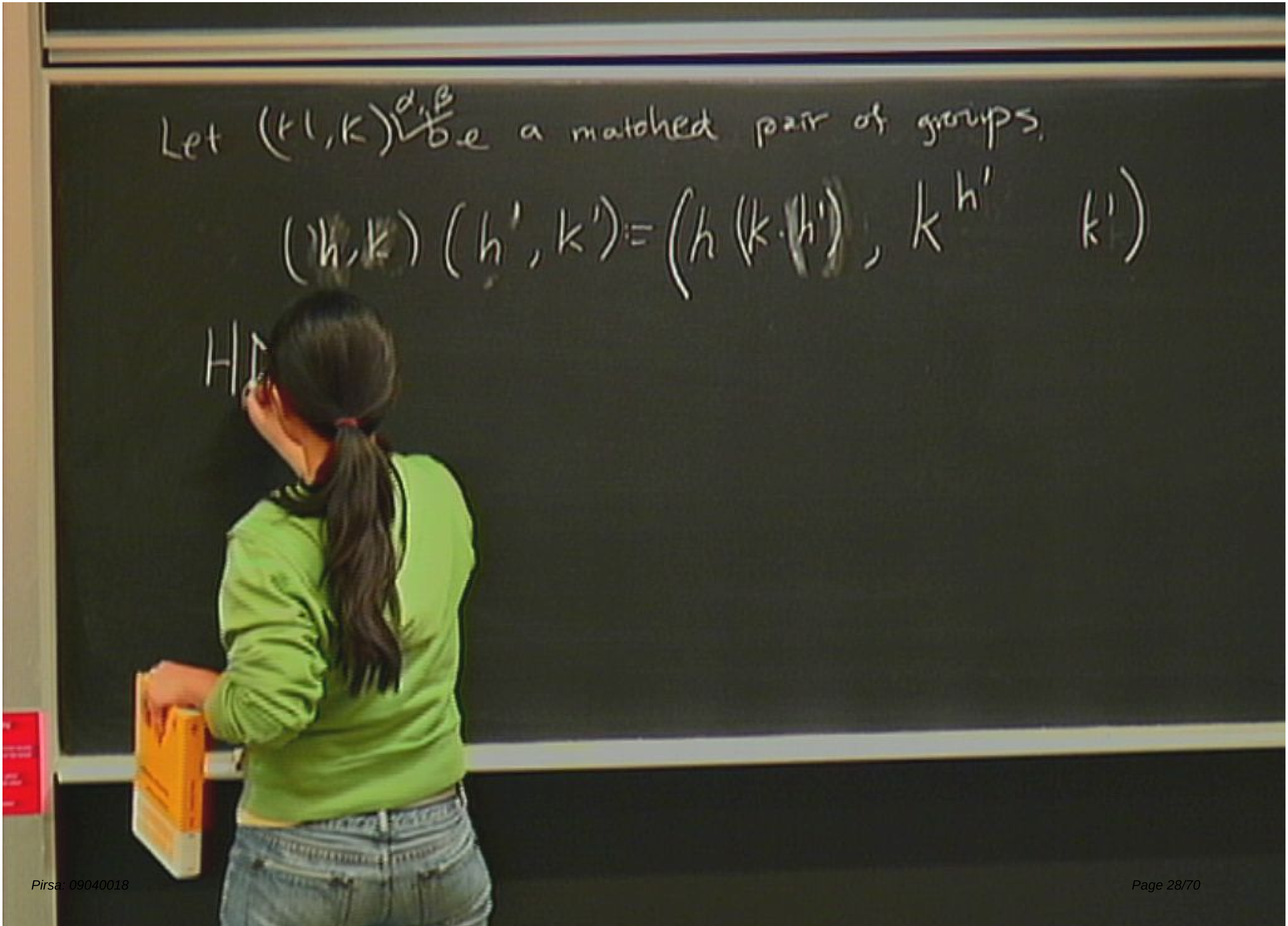
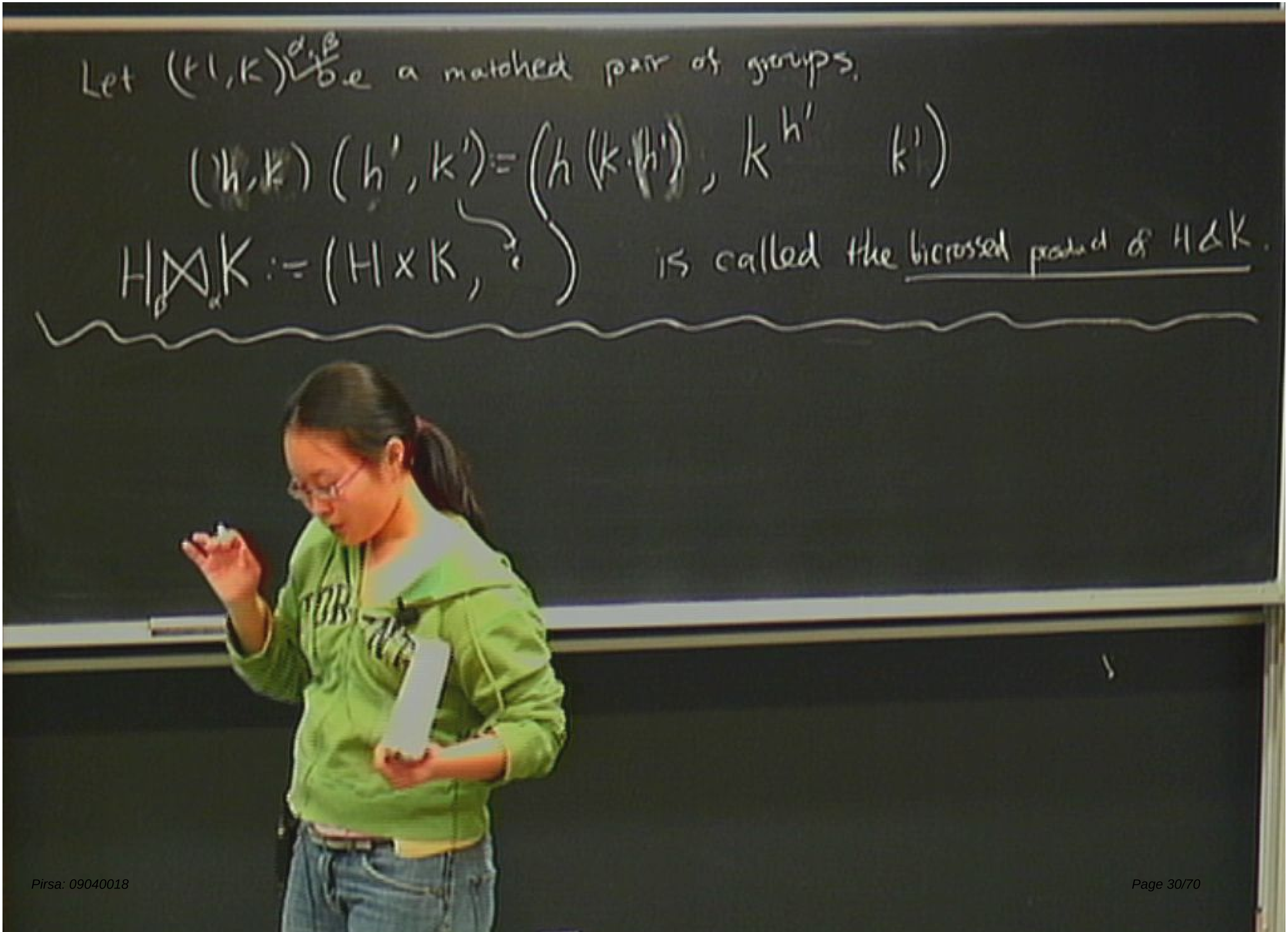

Let (H,K) be a matched pair of groups.
(h,k)(h',k') = h(k·h'), k k'

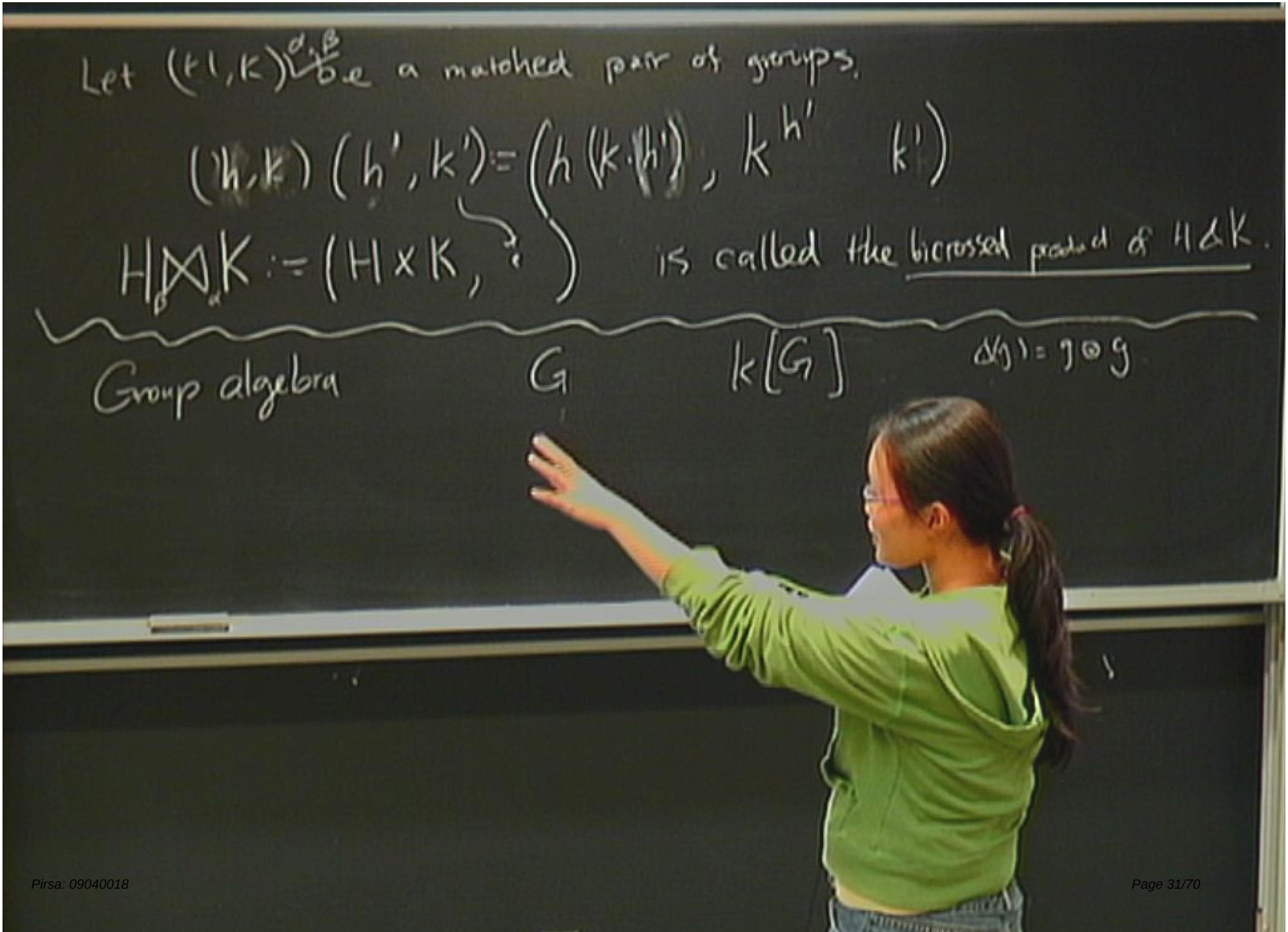

Let $(H,K)^{\alpha,\beta}$ be a matched pair of groups.
$(h,k)(h',k') = (h(k\cdot h'), k^{h'}\ k')$
H

Let $(H,K)^{\alpha,\beta}$ be a matched pair of groups.

$$(h,k)(h',k') := \left(h(k\cdot h'),\ k^{h'}\ \ k'\right)$$

$$H {}_{\beta}\!\bowtie_{\alpha} K := (H \times K,\ \cdot\ )$$ is called the bicrossed product of


Let $(H,K)^{\alpha,\beta}$ be a matched pair of groups.
$(h,k)(h',k') := (h(k\cdot h'), k^{h'} k')$
$H {}_{\beta}\!\bowtie_{\alpha} K := (H \times K, \cdot)$ is called the bicrossed product of H & K.


Let (H,K)^{α,β} be a matched pair of groups.
(h,k)(h',k') = (h(k·h'), k^{h'} k')
H ⋈ K := (H×K, ·) is called the bicrossed product of H & K.
Group algebra
G
k[G]
Δ(g) = g⊗g

Let $(H,K)^{\alpha,\beta}$ be a matched pair of groups.

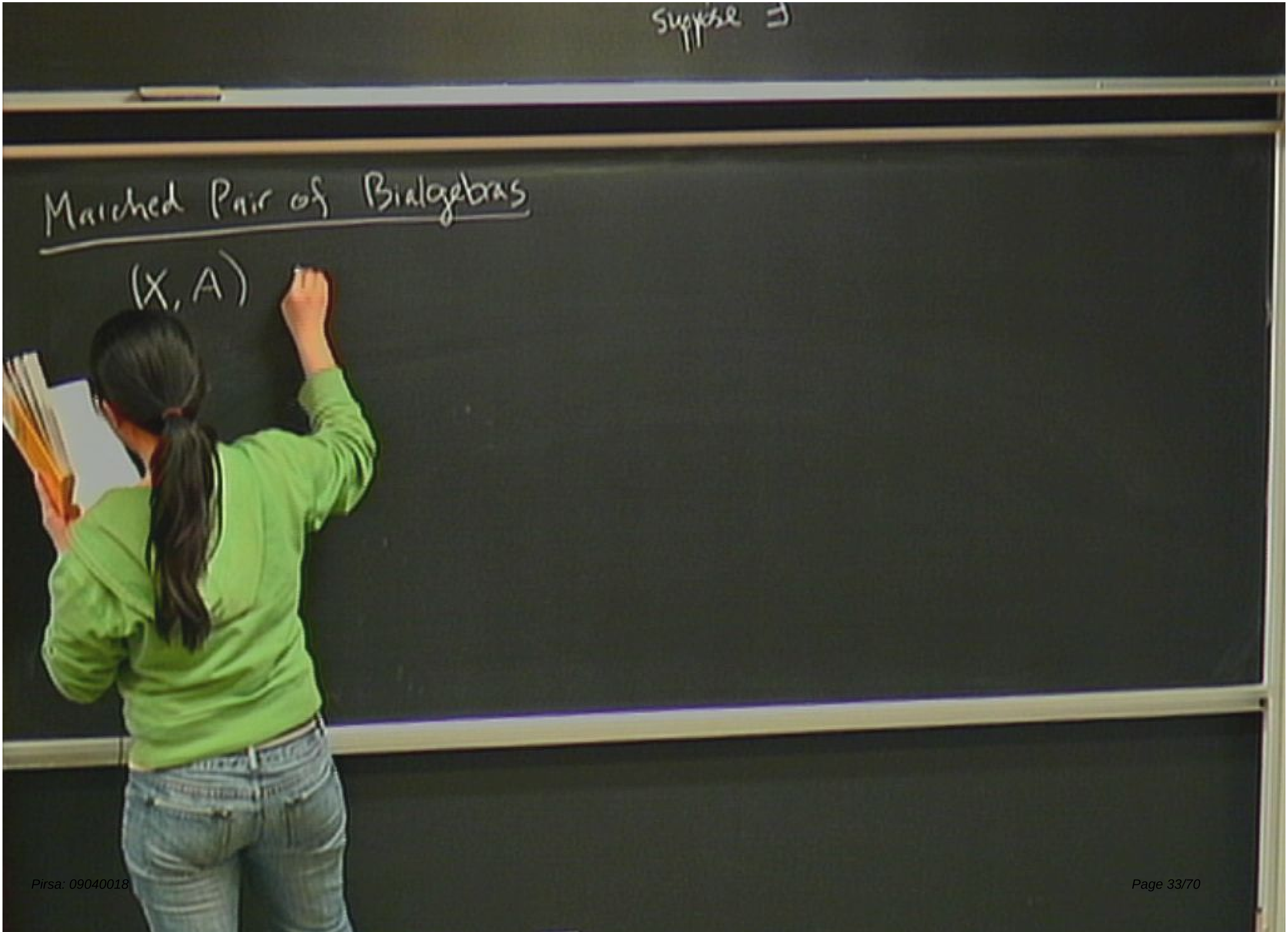
$$(h,k)(h',k') = \left(h(k\cdot h'),\ k^{h'}\ k'\right)$$

$H {}_{\beta}\!\bowtie_{\alpha} K := (H\times K, \cdot)$ is called the bicrossed product of H & K.

Group algebra $\qquad G \qquad k[G] \leftarrow$ bialg. $\quad \Delta(g) = g\otimes g$

~~matched pair~~ groups (H,K): $\qquad k[H]\bowtie k[K] = k[H\bowtie K]$

suppose $\exists$


suppose ∃
Matched Pair of Bialgebras
(X, A)

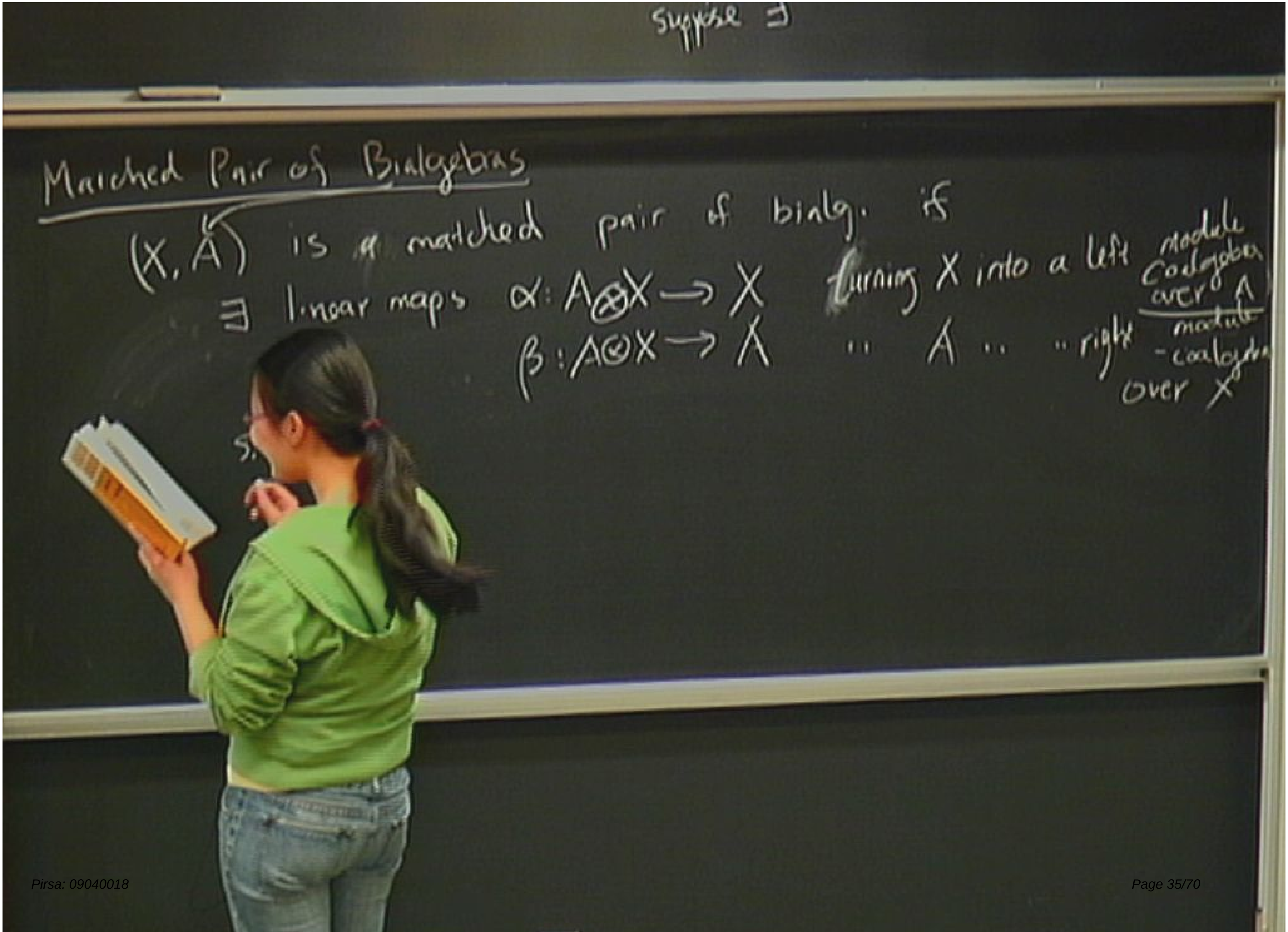
suppose $\exists$

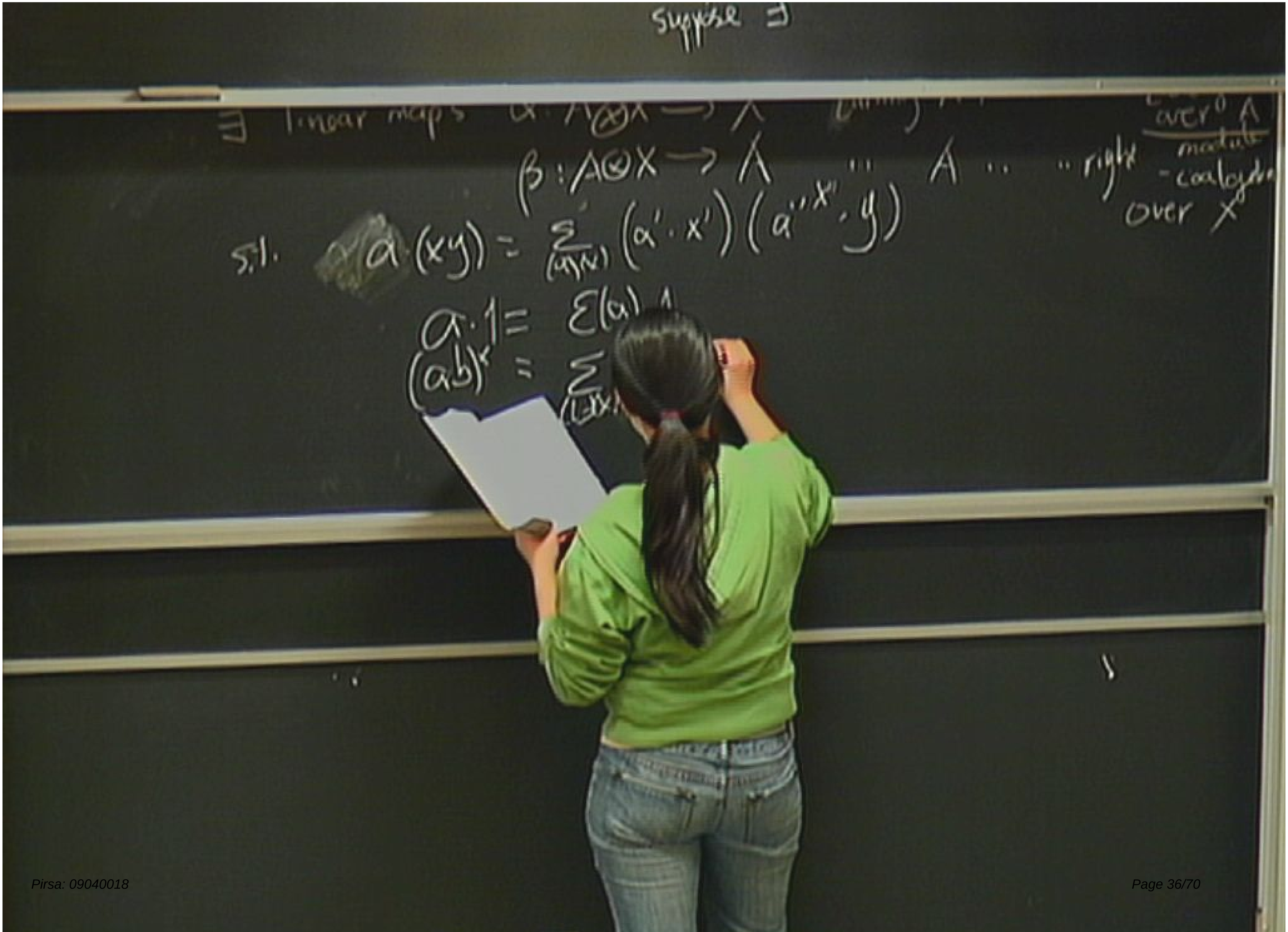
Matched Pair of Bialgebras

(X, A) is a matched pair of bialg. if

$\exists$ linear maps $\alpha : A \otimes X \to X$ turning X into a left module coalgebra over

$\beta : A \otimes X \to A$


suppose ∃
Matched Pair of Bialgebras
(X, A) is a matched pair of bialg. if
∃ linear maps α: A⊗X ⟶ X turning X into a left module coalgebra over A
β: A⊗X ⟶ A " A " " right module - coalgebra over X


suppose ∃
∃ linear maps α: A⊗X → A
β: A⊗X → A
right module-coalgebra over X
s.t. a·(xy) = Σ (a'·x')(a''x'·y)
a·1 = ε(a)1
(ab)x = Σ

suppose $\exists$

Matched Pair of Bialgebras

(X, A) is a matched pair of bialg. if

$\exists$ linear maps $\alpha : A \otimes X \to X$ turning X into a left module coalgebra over A

$\beta : A \otimes X \to A$ " " A " " right module-coalgebra over X

s.t. $a \cdot (xy) = \sum_{(a)(x)} (a' \cdot x')(a''^{x''} \cdot y)$

$a \cdot 1 = \varepsilon(a) 1$

$(ab)^x = \sum_{(b)(x)} a^{b' \cdot x'} b''^{x''}$ $\quad \sum a''$

$1^x = \varepsilon(x) 1$

suppose $\exists$

(X, A) is a matched pair of bialg. if

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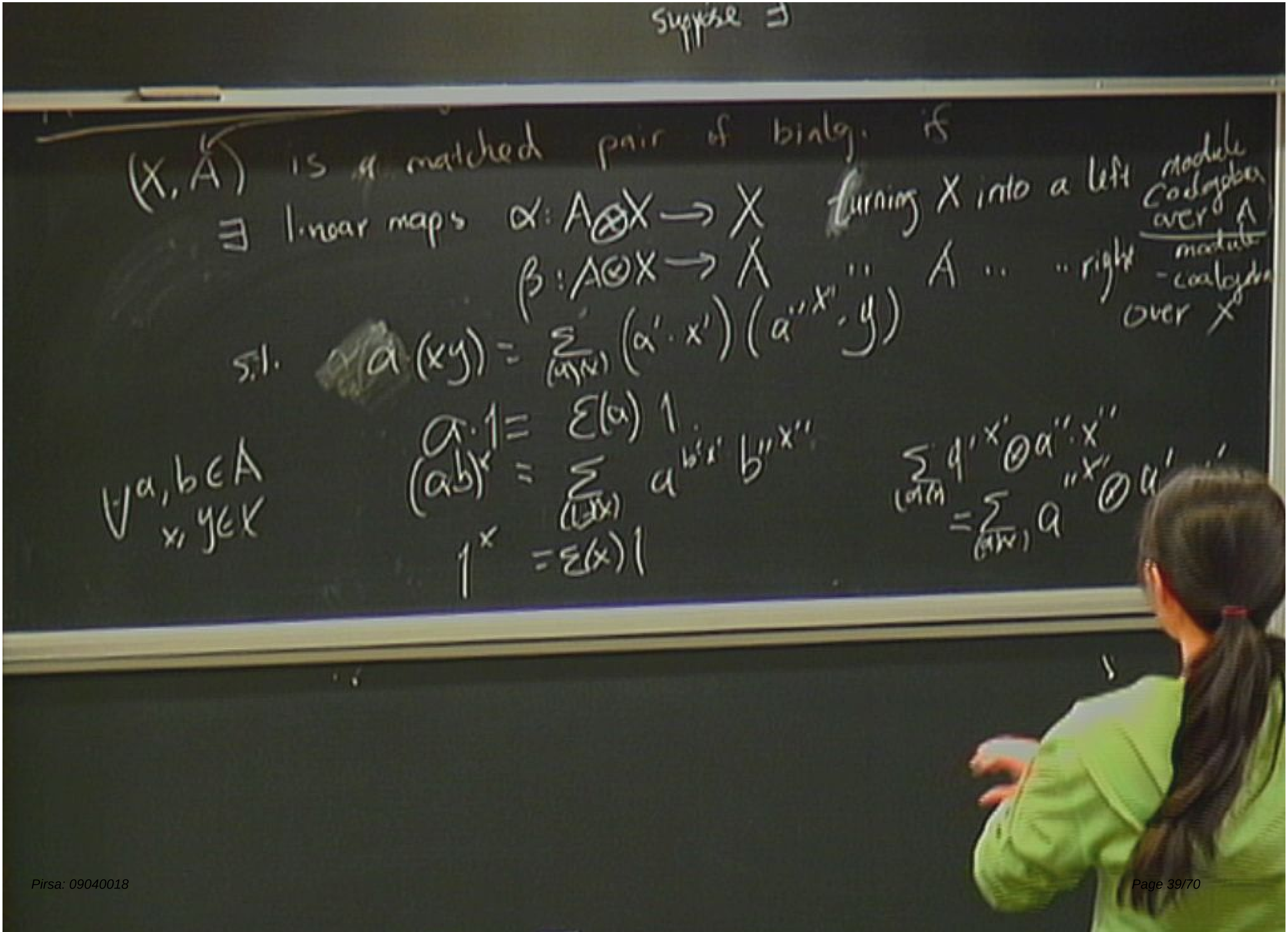
$1^x = \varepsilon(x) 1$

$\sum_{(a)(x)} a'^{x'} \otimes a'' \cdot x''$

$= \sum_{(a)(x)} a''^{x''} \otimes a' \cdot x'$


suppose ∃
(X, A) is a matched pair of bialg. if
∃ linear maps α: A⊗X → X turning X into a left module coalgebra over A
β: A⊗X → A " A " right module -coalgebra over X
s.t. a·(xy) = Σ(a)(x) (a'·x')(a''^x'·y)
a·1 = ε(a) 1
(ab)^x = Σ(b)(x) a^(b'·x') b''^x''
1^x = ε(x) 1
∀ a,b∈A x,y∈X
Σ(a)(x) a'^x' ⊗ a''·x''
= Σ(a)(x) a''^x'' ⊗ a'·x'

suppose $\exists$

(X, A) is a matched pair of bialg. if

$\exists$ linear maps $\alpha : A \otimes X \to X$ turning X into a left module coalgebra over A

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s.t. $a \cdot (xy) = \sum_{(a),(x)} (a' \cdot x')(a''^{x''} \cdot y)$

$$a \cdot 1 = \varepsilon(a)\, 1$$

$$(ab)^x = \sum_{(b),(x)} a^{b' \cdot x'}\, b''^{x''}$$

$$1^x = \varepsilon(x)\, 1$$

$\forall a, b \in A$, $x, y \in X$

$$\sum_{(a),(x)} a'^{x'} \otimes a'' \cdot x' \quad = \sum_{(a),(x)} a$$

suppose $\exists$

(X, A) is a matched pair of bialg. if

$\exists$ linear maps $\alpha: A \otimes X \to X$ turning X into a left module coalgebra over A

$\beta: A \otimes X \to A$ " A .. .. right module-coalgebra over X

s.t. $a \cdot (xy) = \sum_{(a),(x)} (a' \cdot x')(a''^{x''} \cdot y)$

$A \otimes A \ni \Delta(a) = \sum_{(a)} a' \otimes a''$

$a \cdot 1 = \varepsilon(a) 1$

$\forall a, b \in A$, $x, y \in X$

$(ab)^x = \sum_{(b),(x)} a^{b' \cdot x'} b''^{x''}$

$\sum_{(a),(x)} a'^{x'} \otimes a'' \cdot x'' = \sum_{(a),(x)} a''^{x''} \otimes a' \cdot x'$

$1^x = \varepsilon(x) 1$

Suppose $\exists$

(X, A) is a matched pair of bialg. if

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$A \otimes A \ni \Delta(a) = \sum_{(a)} a' \otimes a''$

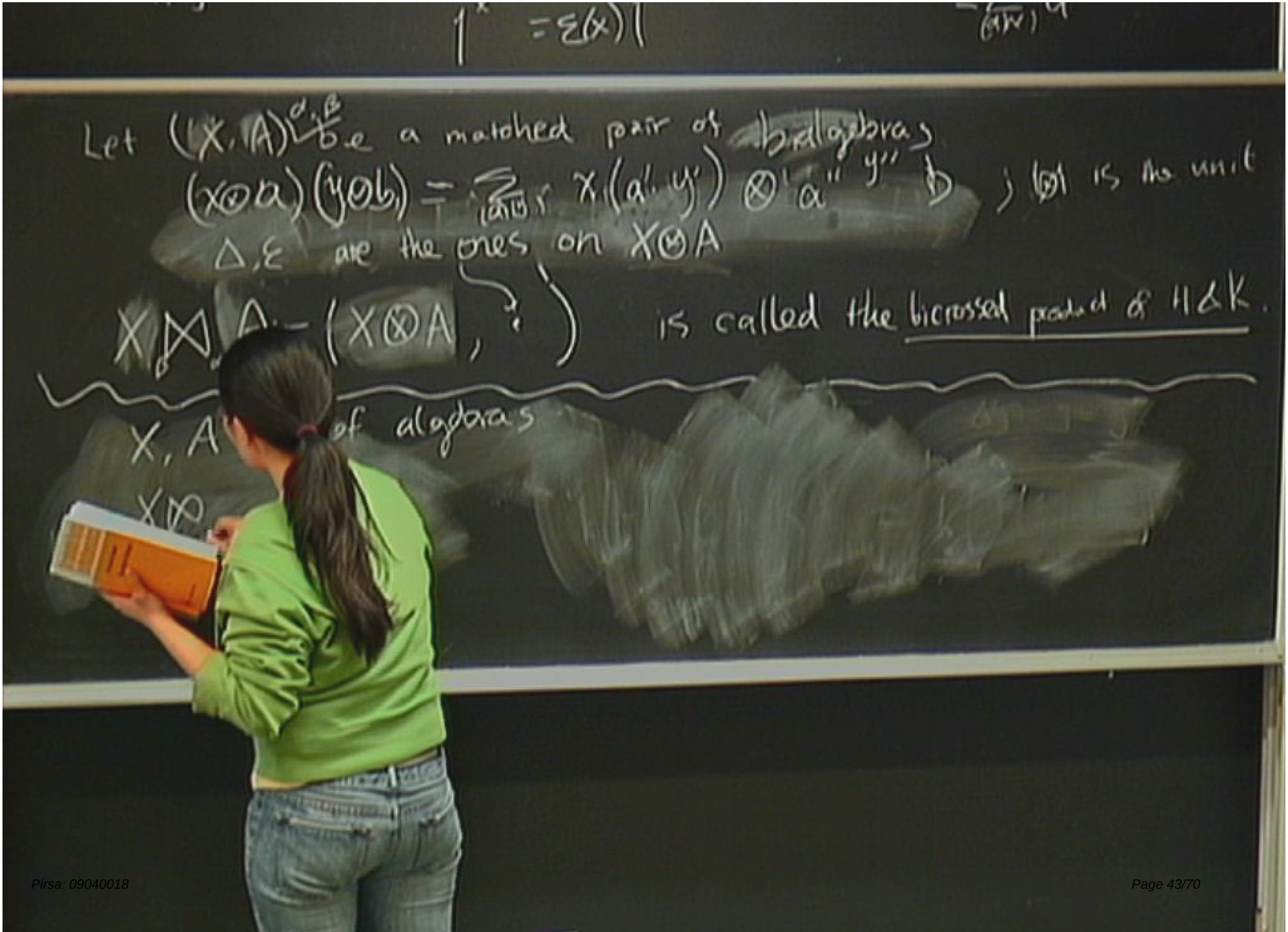
$a \cdot 1 = \varepsilon(a) 1$

$(ab)^x = \sum_{(b),(x)} a^{b' \cdot x'} b''^{x''}$

$1^x = \varepsilon(x) 1$

$\forall a, b \in A, \; x, y \in X$

$\sum_{(a),(x)} a'^{x'} \otimes a'' \cdot x'' = \sum_{(a),(x)} a''^{x''} \otimes a' \cdot x'$



= ε(x) 1
Let (X, A) be a matched pair of bialgebras
(x⊗a)(y⊗b) = Σ x(a'·y') ⊗ a''·y'' b ; 1⊗1 is the unit
Δ, ε are the ones on X⊗A
X ⋈ A = (X⊗A, ·) is called the bicrossed product of H & K.
X, A of algebras

Let (X, A) be a matched pair of bialgebras

$$(x \otimes a)(y \otimes b) = \sum_{(a)(y)} x (a' \cdot y') \otimes a'' \cdot y'' b \quad ; \quad 1 \otimes 1 \text{ is the unit}$$

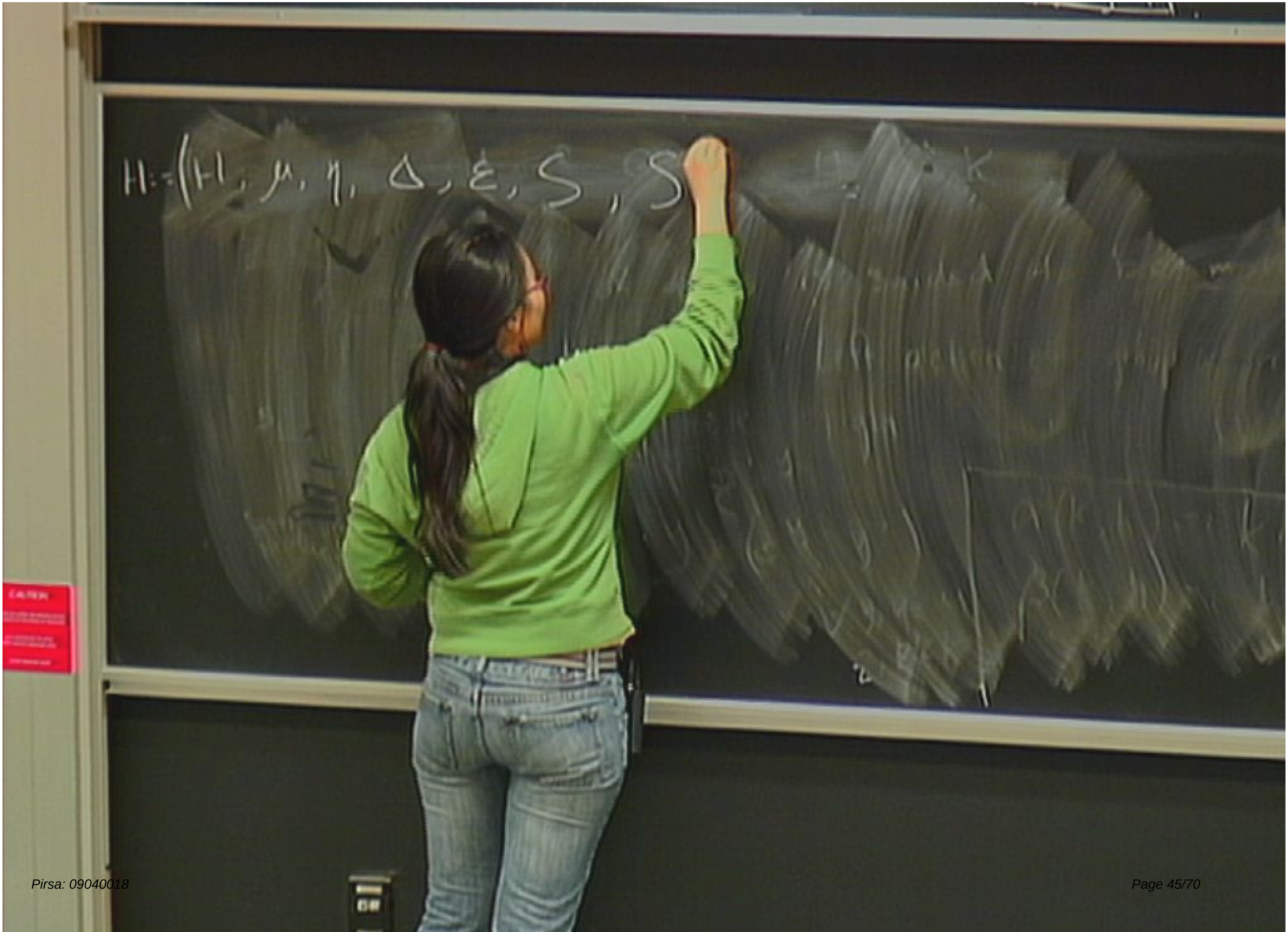
Δ, ε are the ones on $X \otimes A$

$X {}_{\beta}\!\bowtie_{\alpha} A = (X \otimes A, \cdot)$ is called the bicrossed product of H & K.

X, A Hopf algebras (matched)

$X \bowtie A$ has antipode

$$S(x \otimes a) = \sum_{(x)(a)} S_A(a'') \cdot S_X(x'') \otimes S_A(a') \cdot S_X(x')$$

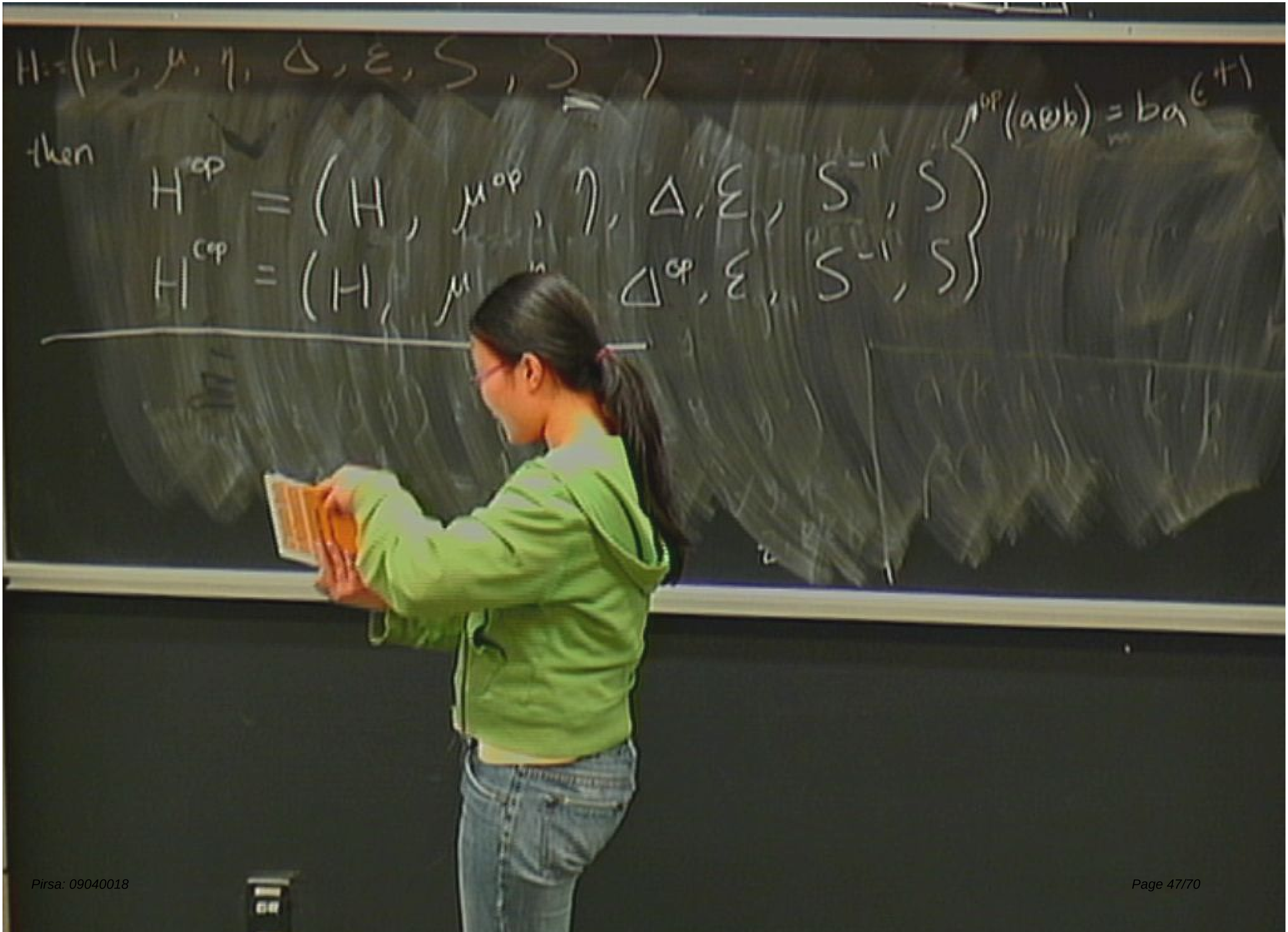

H:=(H, μ, η, Δ, ε, S, S

$H := (H, \mu, \eta, \Delta, \varepsilon, S, S^{-1})$

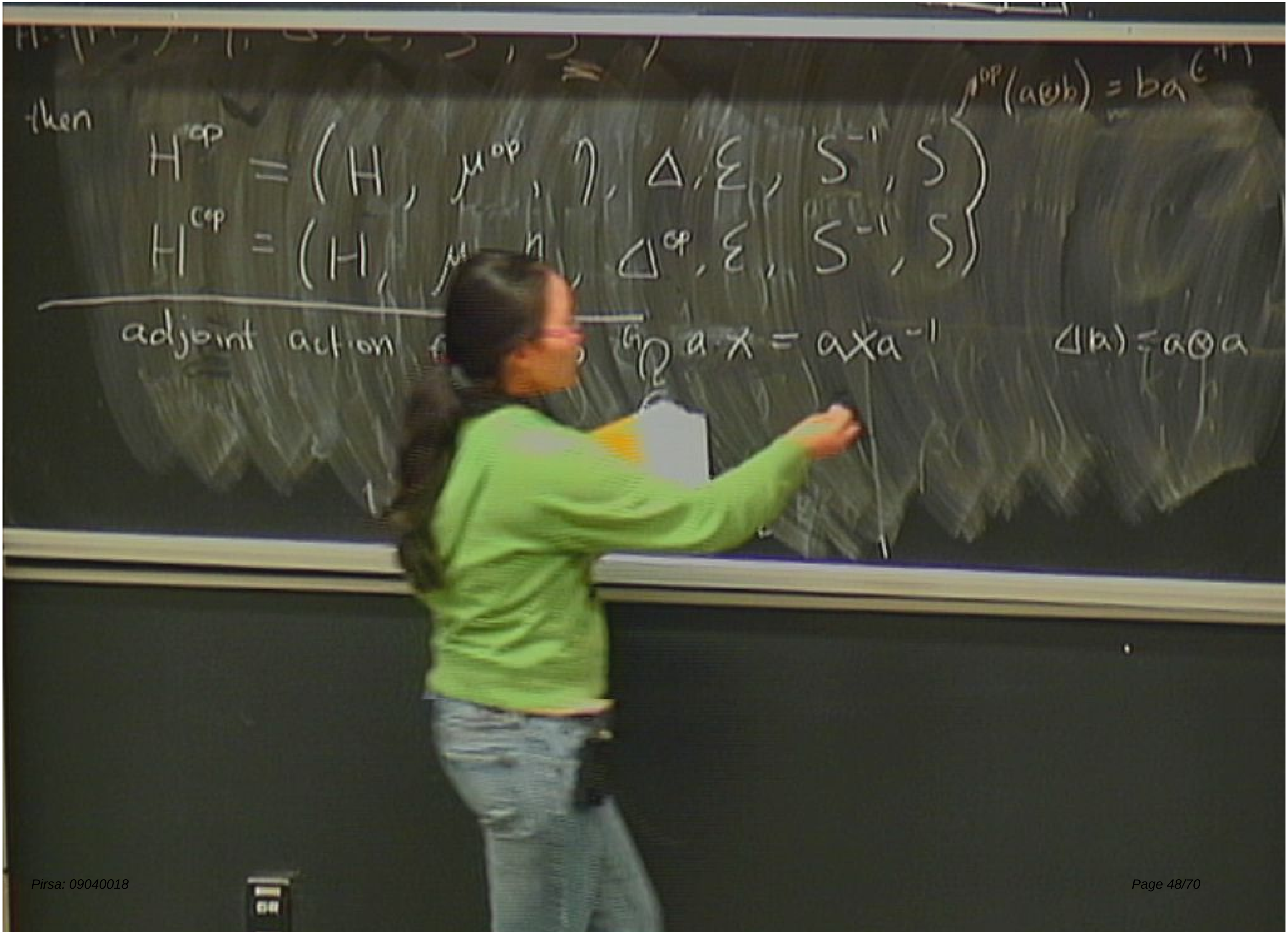
then

$$H^{op} = (H, \mu^{op}, \eta, \Delta, \varepsilon, S^{-1}, S) \qquad \mu^{op}(a \otimes b) = ba \ (*)$$

$$H^{cop} = (H, \mu, \eta, \Delta^{op}, \varepsilon, S^{-1}, S)$$



$H := (H, \mu, \eta, \Delta, \varepsilon, S, S$
$\mu^{op}(a \otimes b) = ba$
then
$H^{op} = (H, \mu^{op}, \eta, \Delta, \varepsilon, S^{-1}, S)$
$H^{cop} = (H, \mu$
$\Delta^{op}, \varepsilon, S^{-1}, S)$


then
$\mu^{op}(a\otimes b) = ba$
$H^{op} = (H, \mu^{op}, \eta, \Delta, \varepsilon, S^{-1}, S)$
$H^{cop} = (H, \mu, \eta, \Delta^{op}, \varepsilon, S^{-1}, S)$
adjoint action
$a \cdot x = axa^{-1}$
$\Delta(a) = a\otimes a$

then

$$H^{op} = (H, \mu^{op}, \eta, \Delta, \varepsilon, S^{-1}, S)$$

$$H^{cop} = (H, \mu, \eta, \Delta^{op}, \varepsilon, S^{-1}, S)$$

$\mu^{op}(a \otimes b) = ba$

adjoint action for groups $G \curvearrowright G$ $a \cdot x = axa^{-1}$

$\Delta(a) = a \otimes a$

$S(a) = a^{-1}$

General

$H^{op} = (H, \mu^{op}, \eta, \Delta, \varepsilon, S^{-1}, S)$

$H^{cop} = (H, \mu, \eta, \Delta^{op}, \varepsilon, S^{-1}, S)$

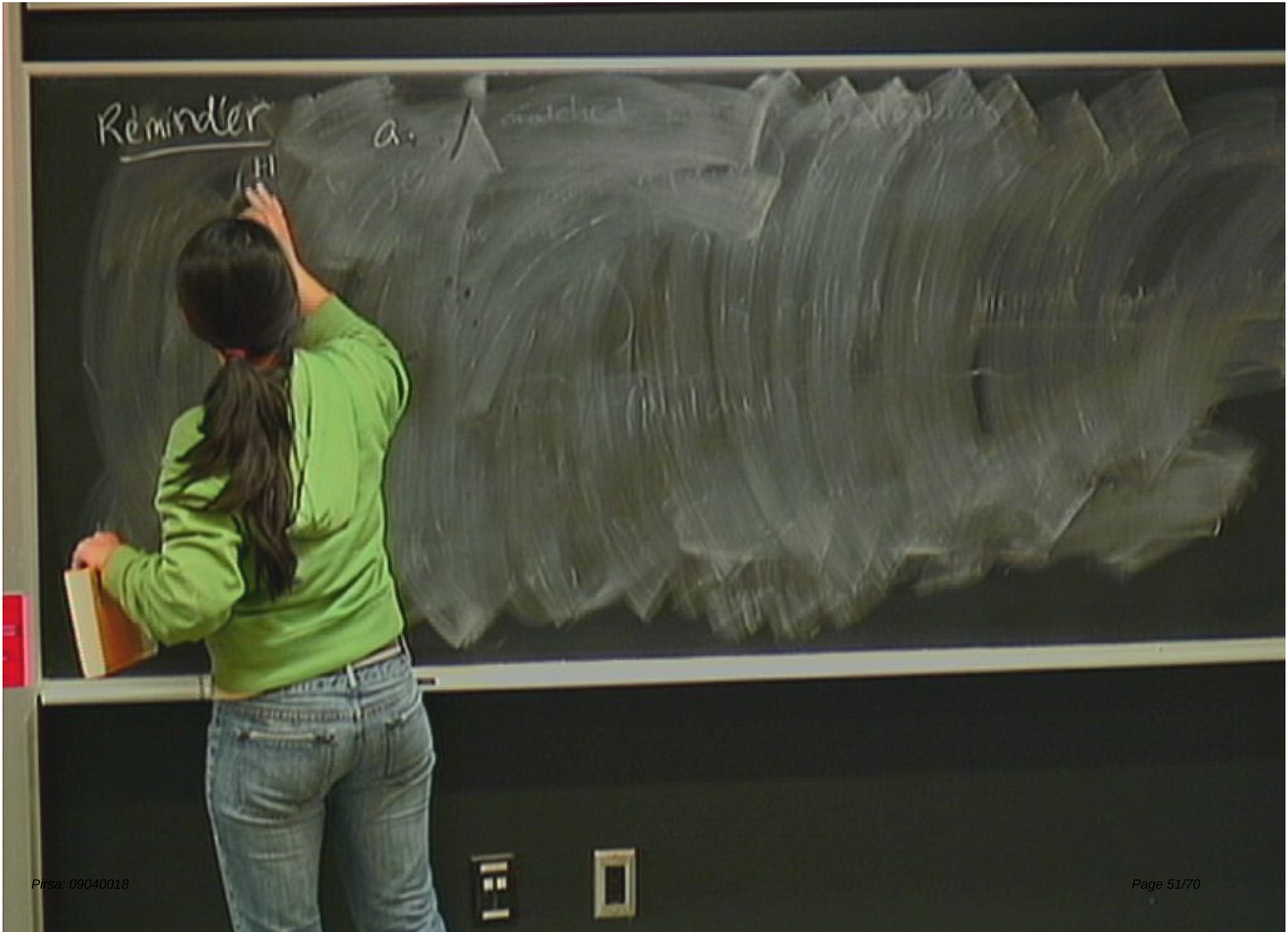
adjoint action for groups $G \curvearrowright G$ $a \cdot x = axa^{-1}$

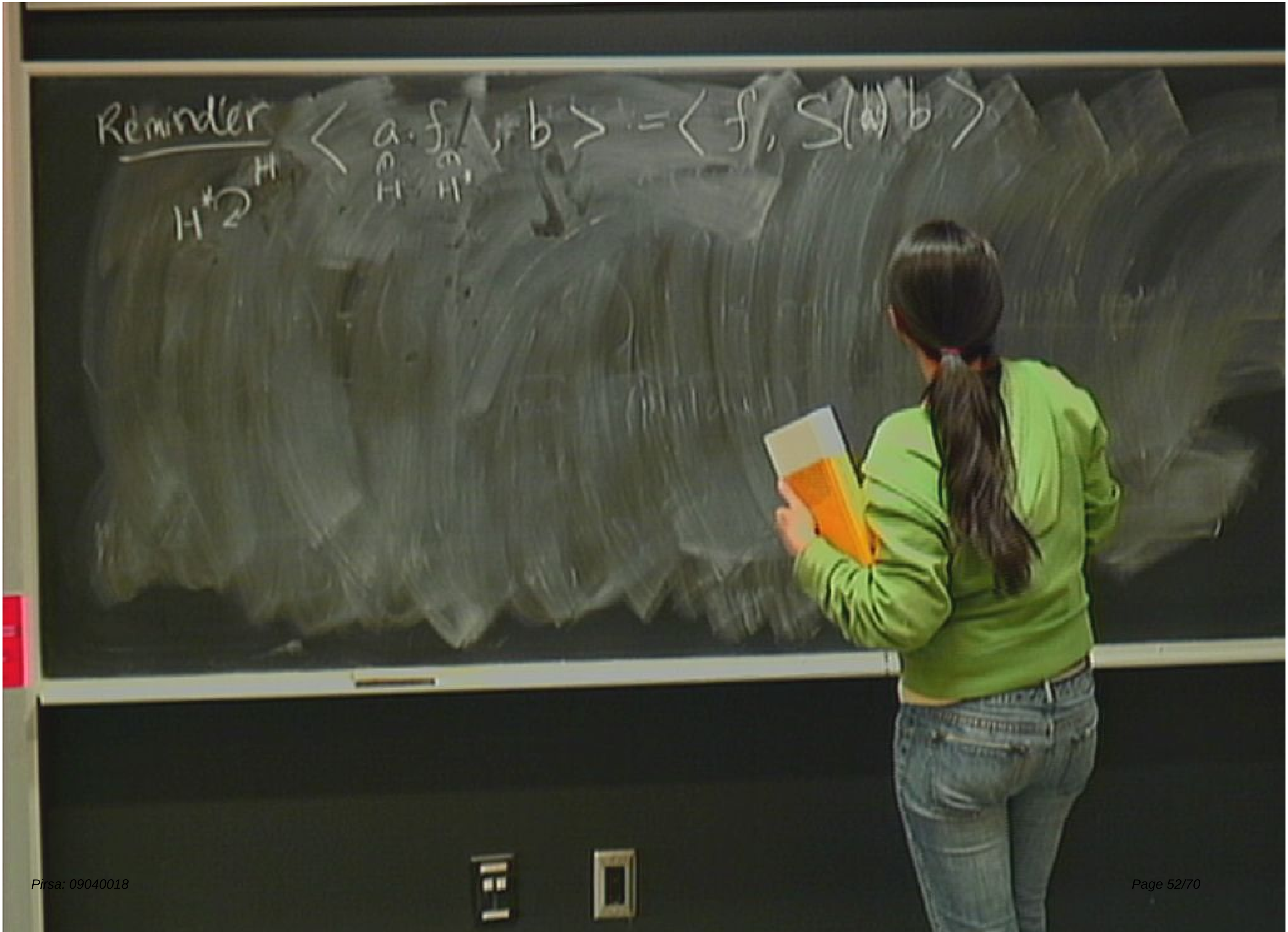
$\Delta(a) = a \otimes a$

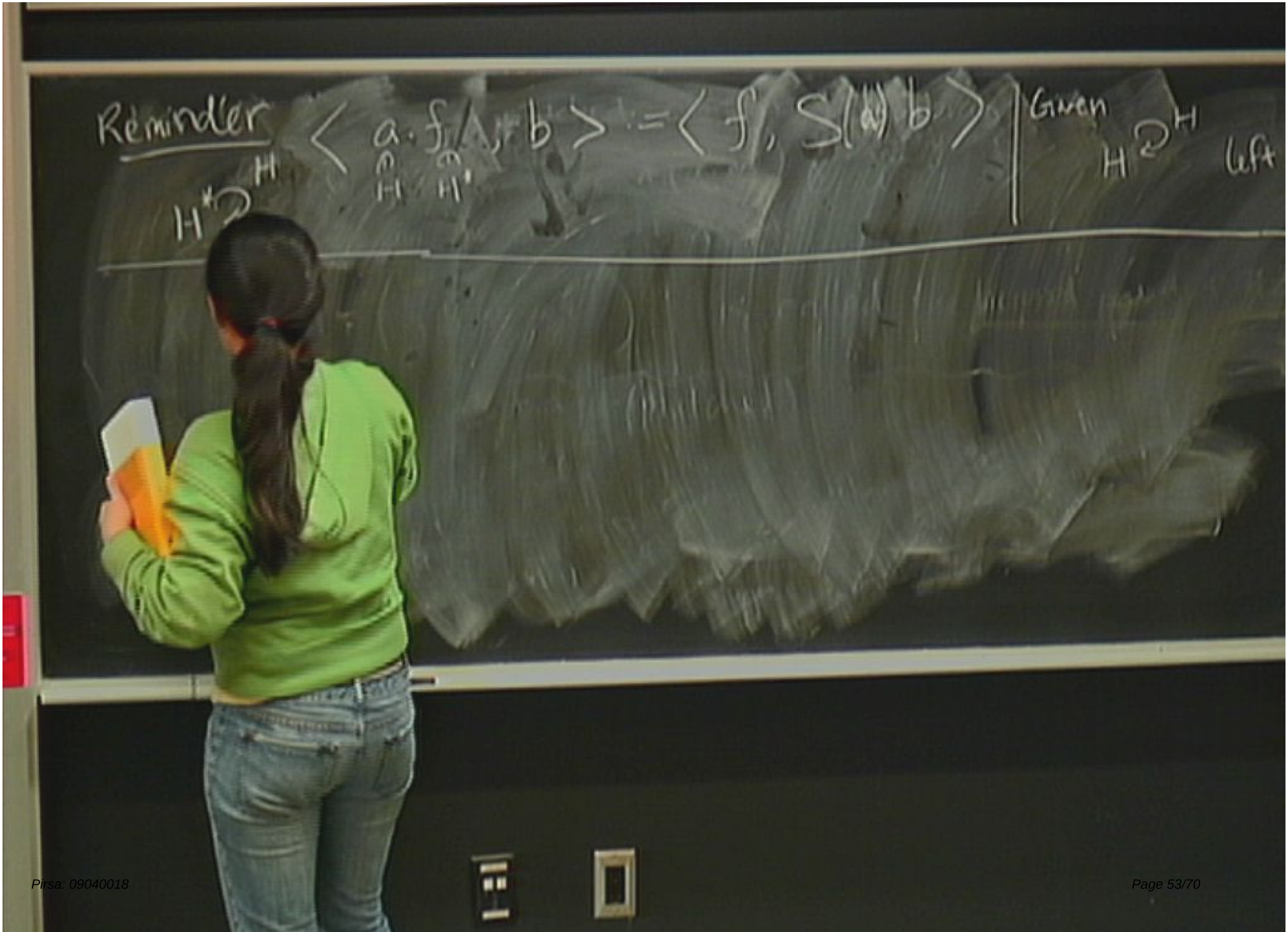
$S(a) = a^{-1}$

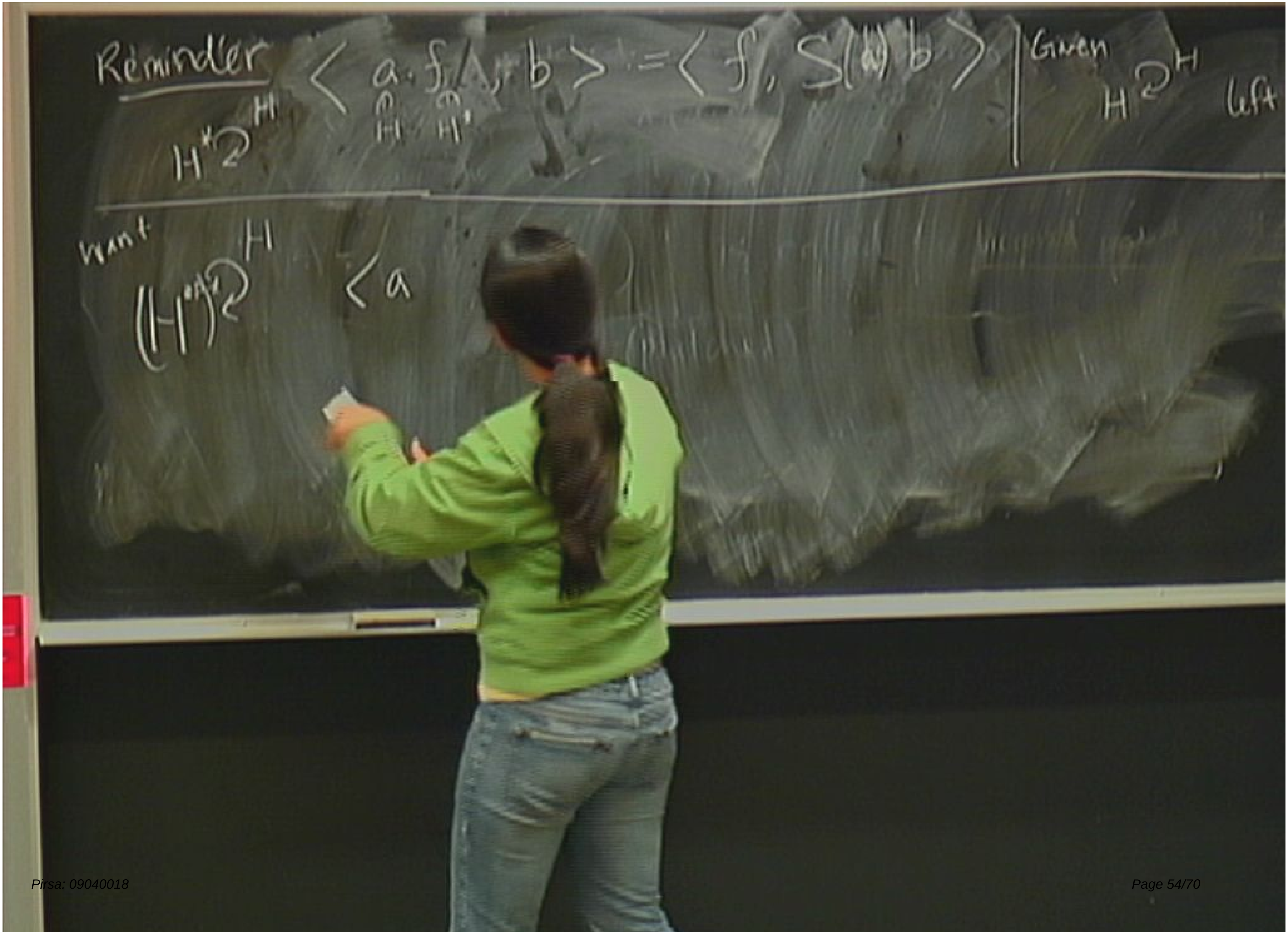
Generally, ~~ad~~ adj action for a Hopf alg. $H \curvearrowright H$

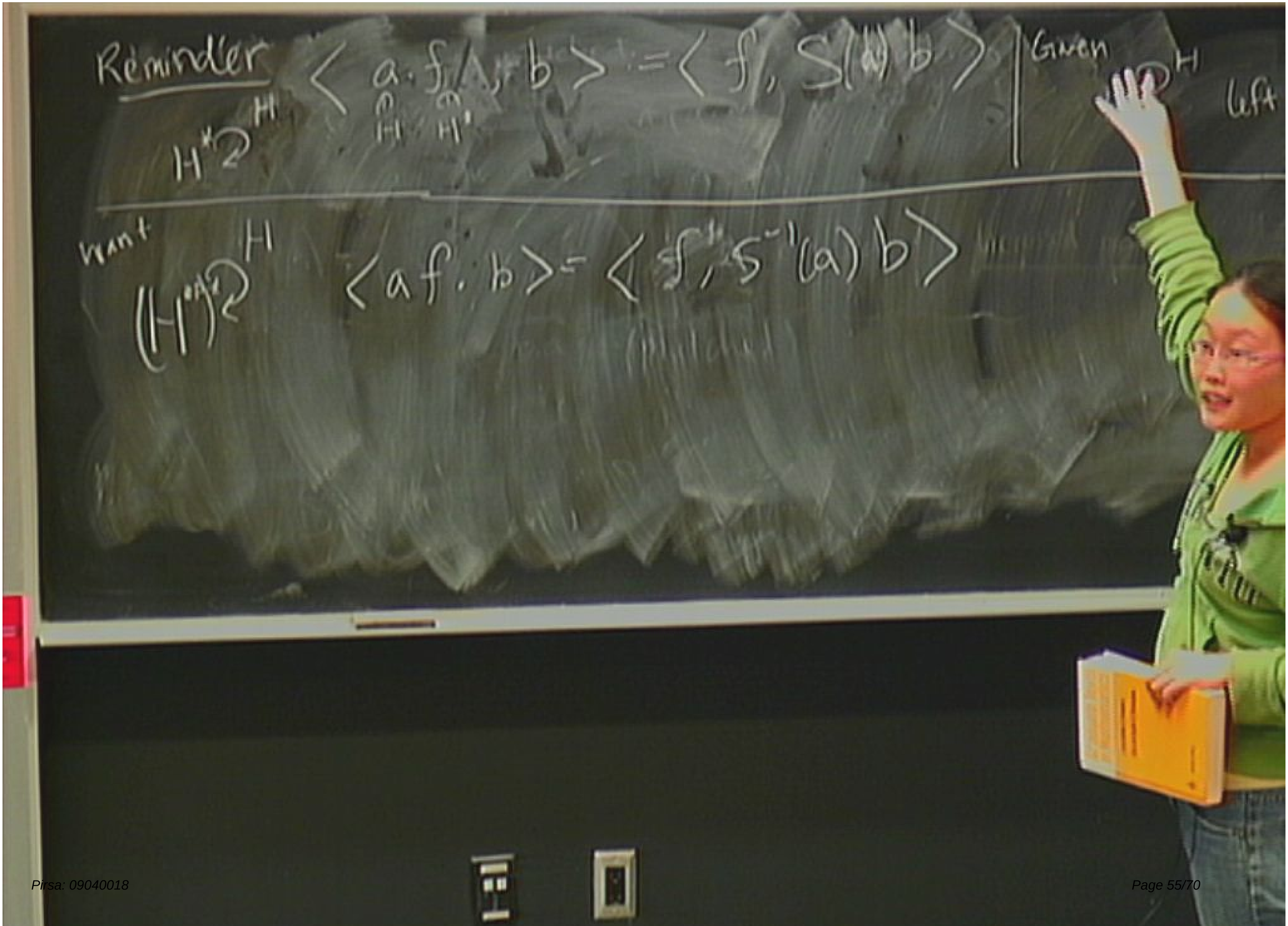
$a \cdot x = \sum_{(a)} a' x S(a'')$

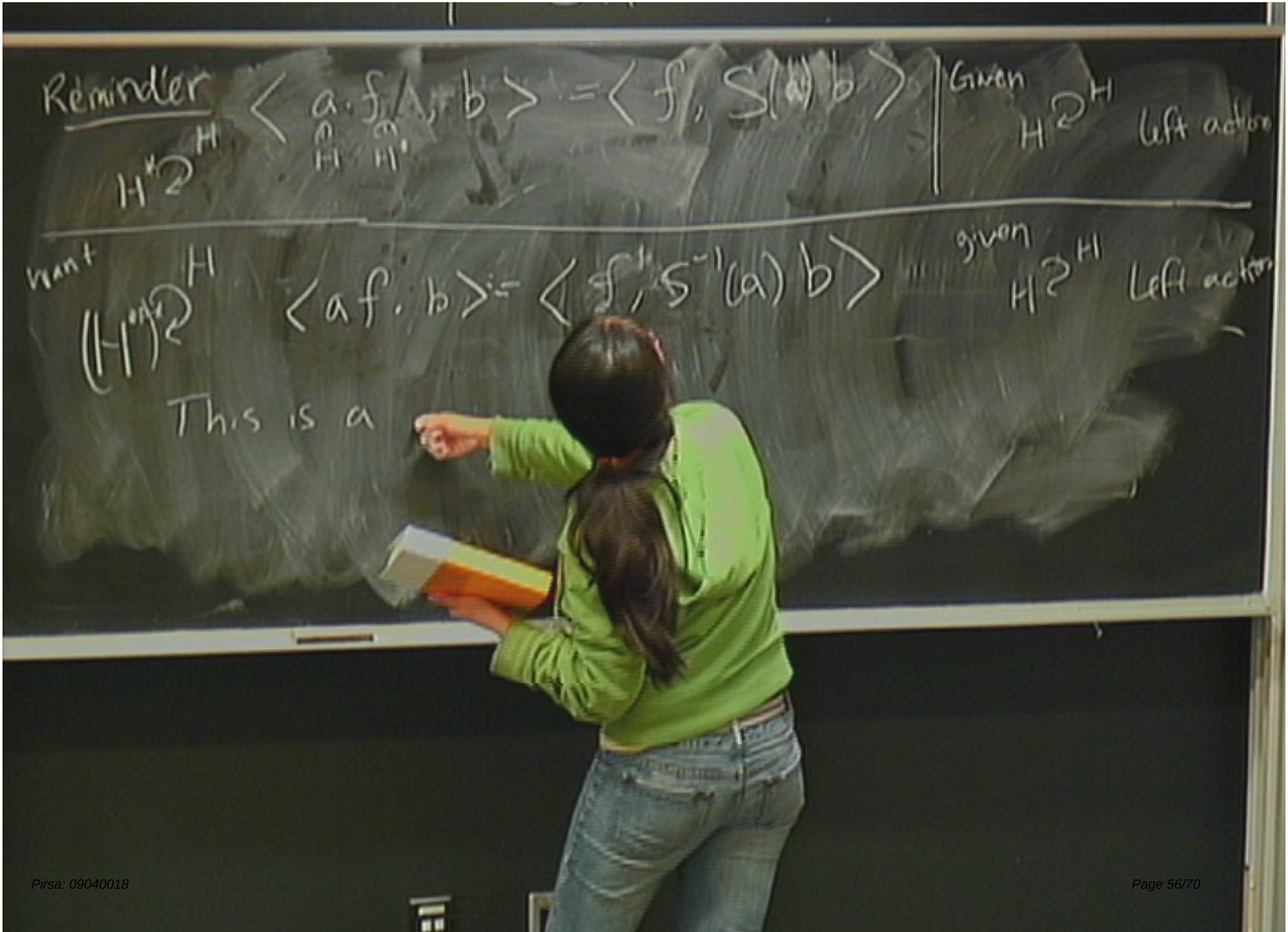

Reminder


Reminder


Reminder
Given
left


Reminder
Given
left
want


Reminder
< a·f, b > := < f, S(a) b >
Given
left
Want
< af, b > = < f, S^{-1}(a) b >



Reminder
$\langle a.f, b \rangle := \langle f, S(a) b \rangle$
$H^* \supseteq H$
Given
$H \supseteq H$
left action
want
$(H^{op})^* \supseteq H$
$\langle af, b \rangle := \langle f, S^{-1}(a) b \rangle$
given
$H \supseteq H$
Left action
This is a

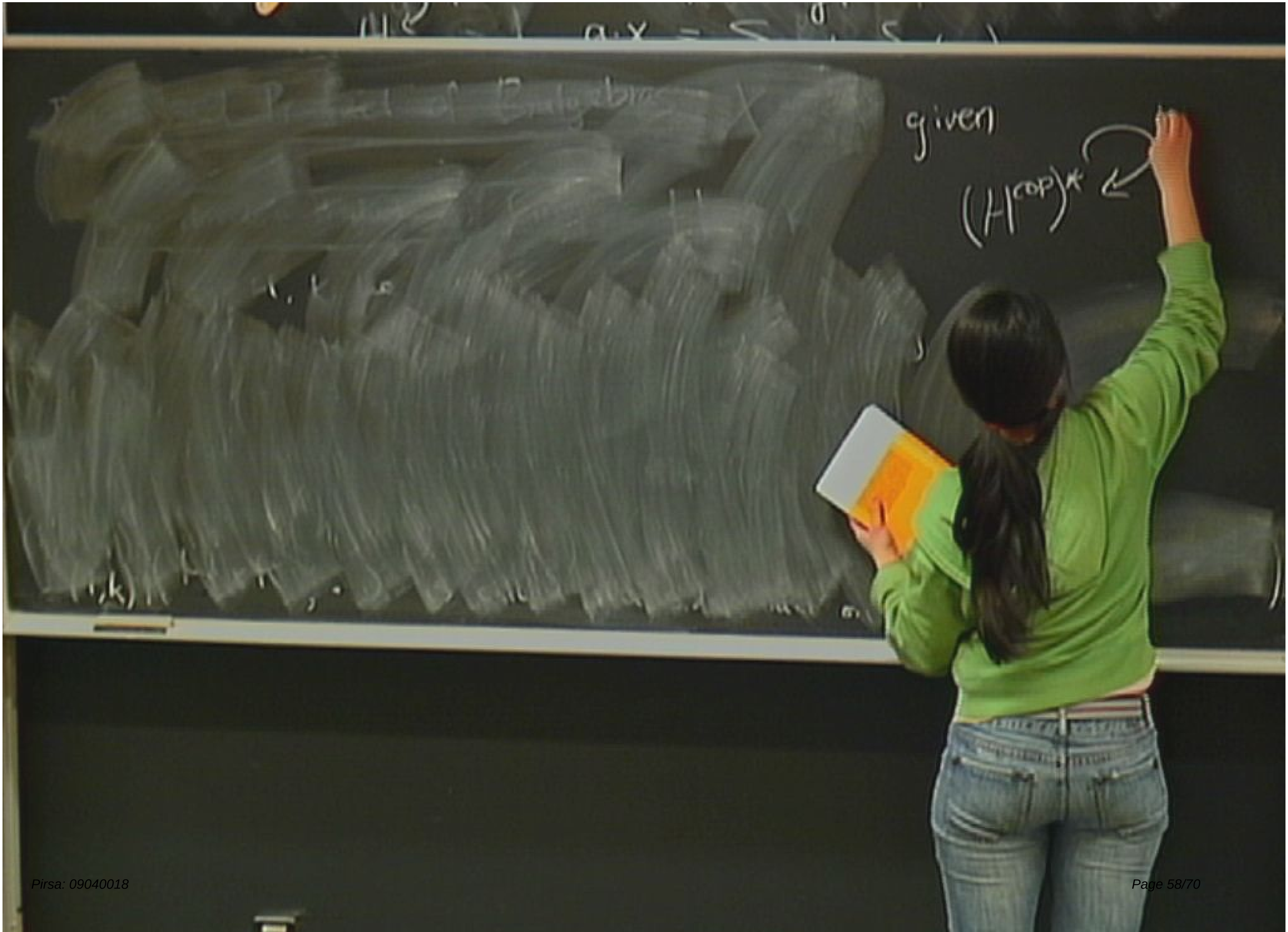
Reminder $H^* \curvearrowleft^{H}$ $\langle \underset{H}{a} \cdot \underset{H^*}{f}, b \rangle := \langle f, S(a) b \rangle$

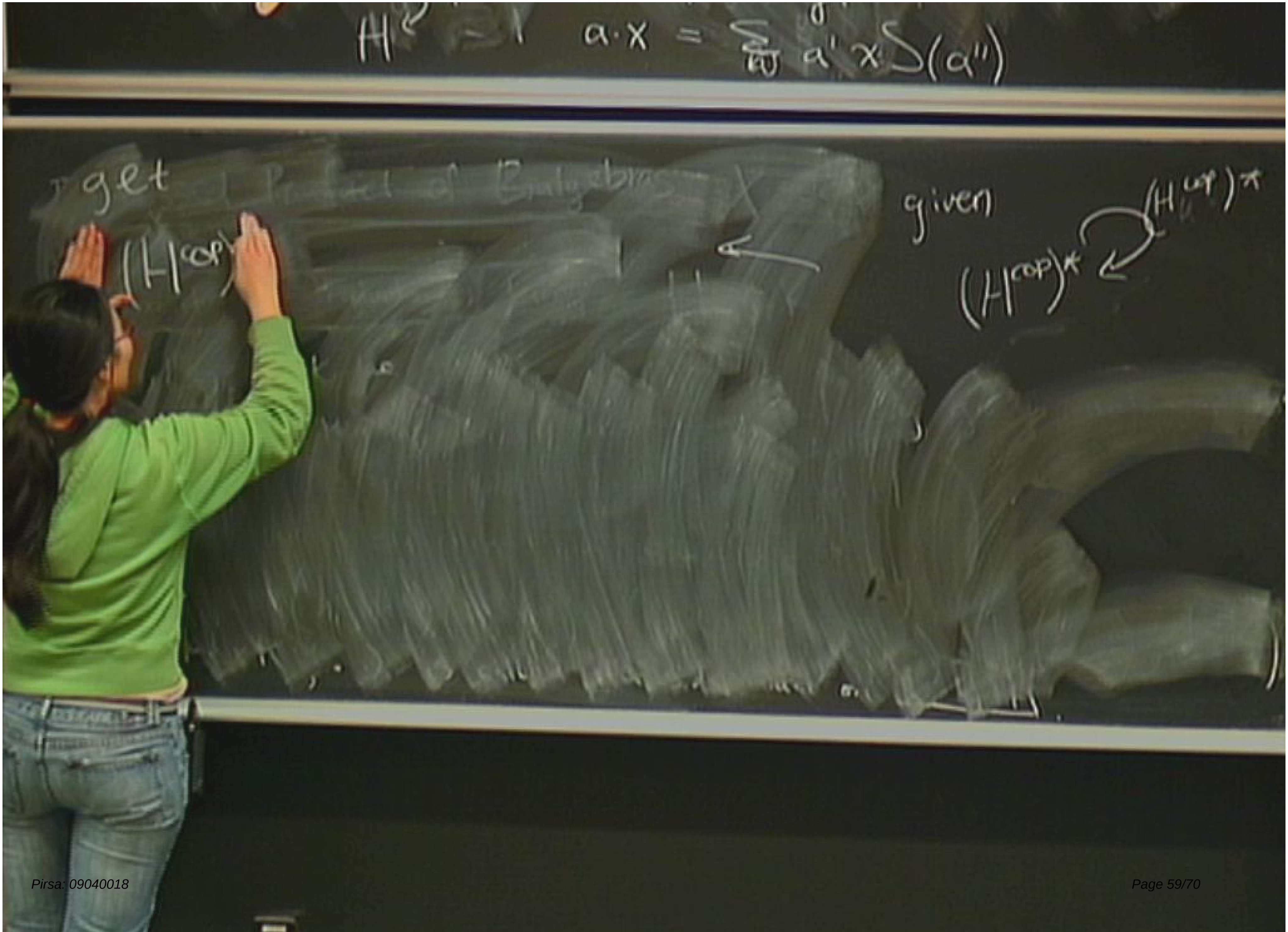
Given $H \curvearrowleft^{H}$ left action

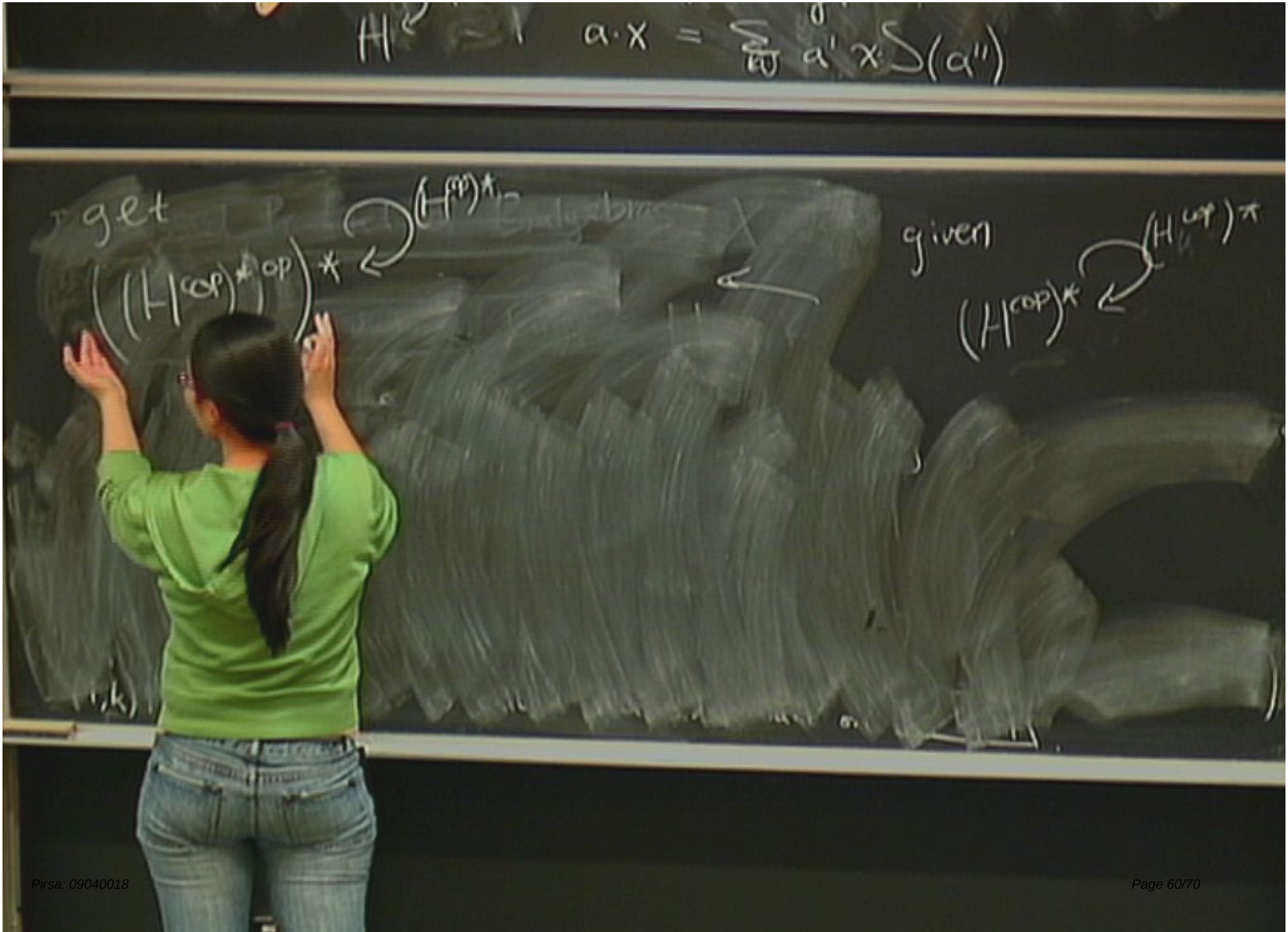
want $(H^{op})^* \curvearrowleft^{H}$ $\langle a f, b \rangle := \langle f, S^{-1}(a) b \rangle$

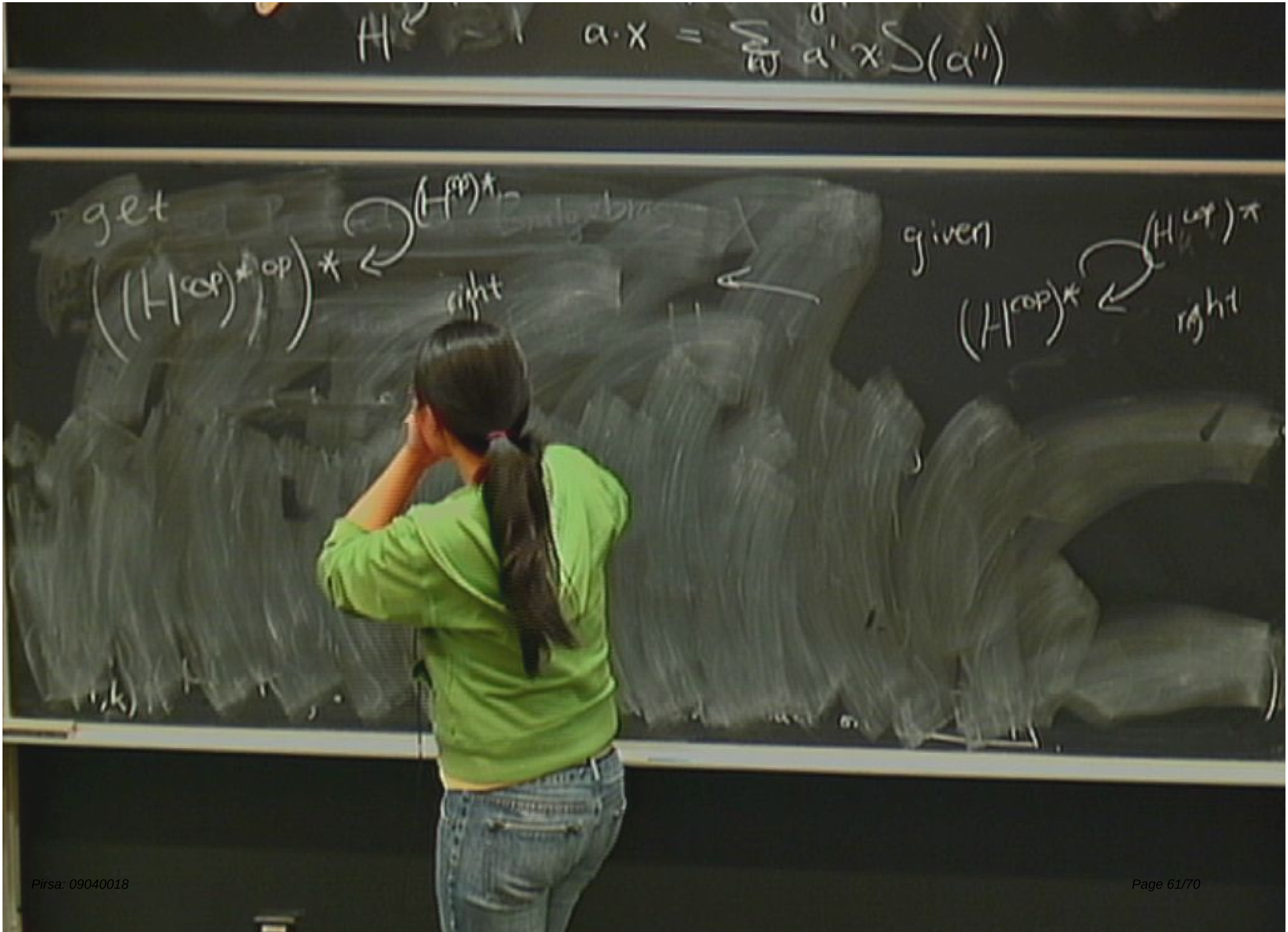
given $H \curvearrowleft^{H}$ Left action

This turns $(H^{op})^*$ into a left module-coalg. over H.

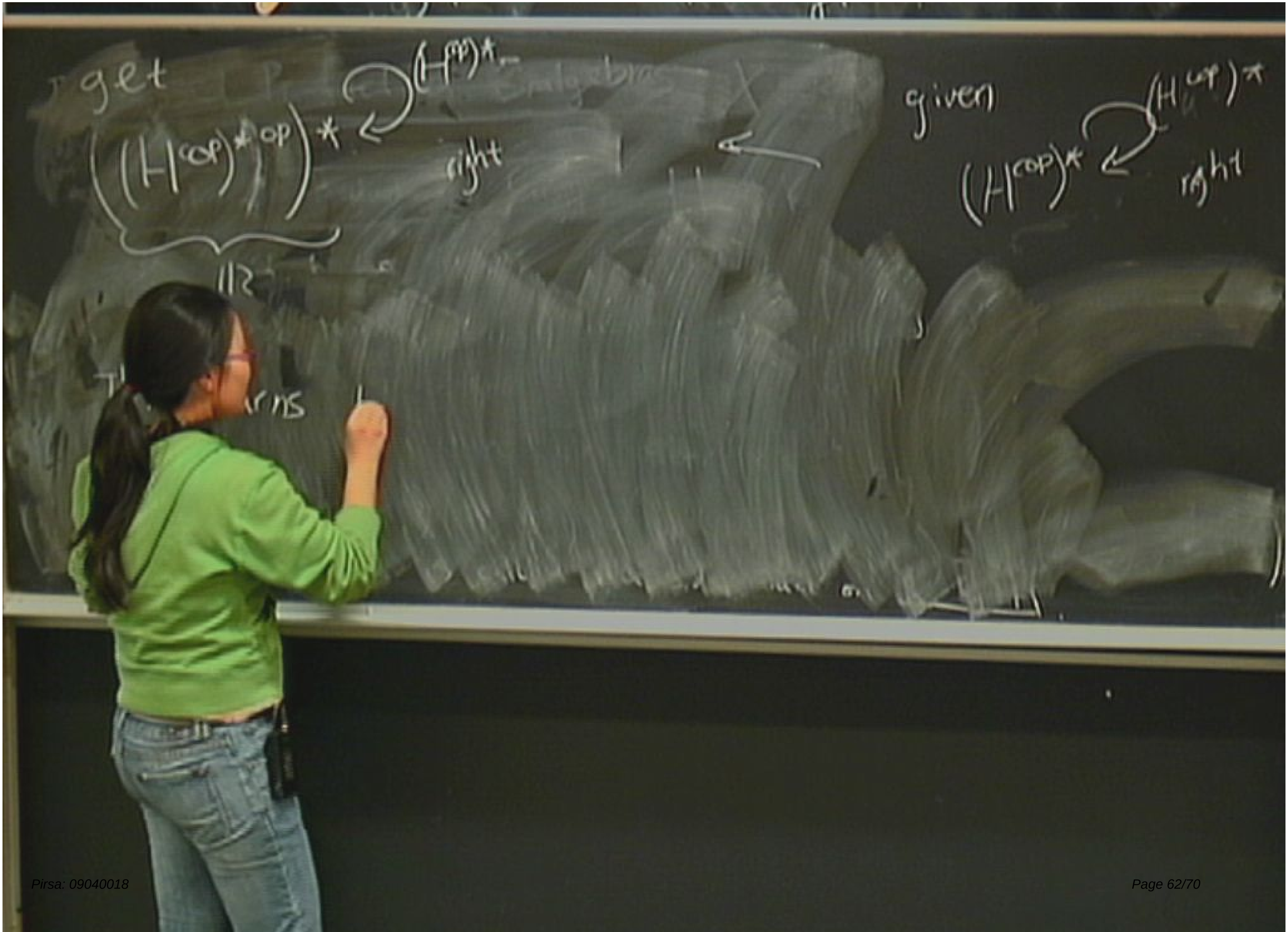

given
$(H^{cop})^*$


a·x = Σ a' x S(a'')
get
(H^cop)
given
(H^cop)*
(H^cop)*


a·x = Σ a' x S(a'')
get
((H^cop)*^op)*
given
(H^cop)*



a·x = Σ a' x S(a'')
get
((H^cop)*op)*
(H^op)*
right
given
(H^op)*
(H^cop)*
right


get
((H^cop)*op)*
right
given
(H^cop)*
(H^cop)*
right

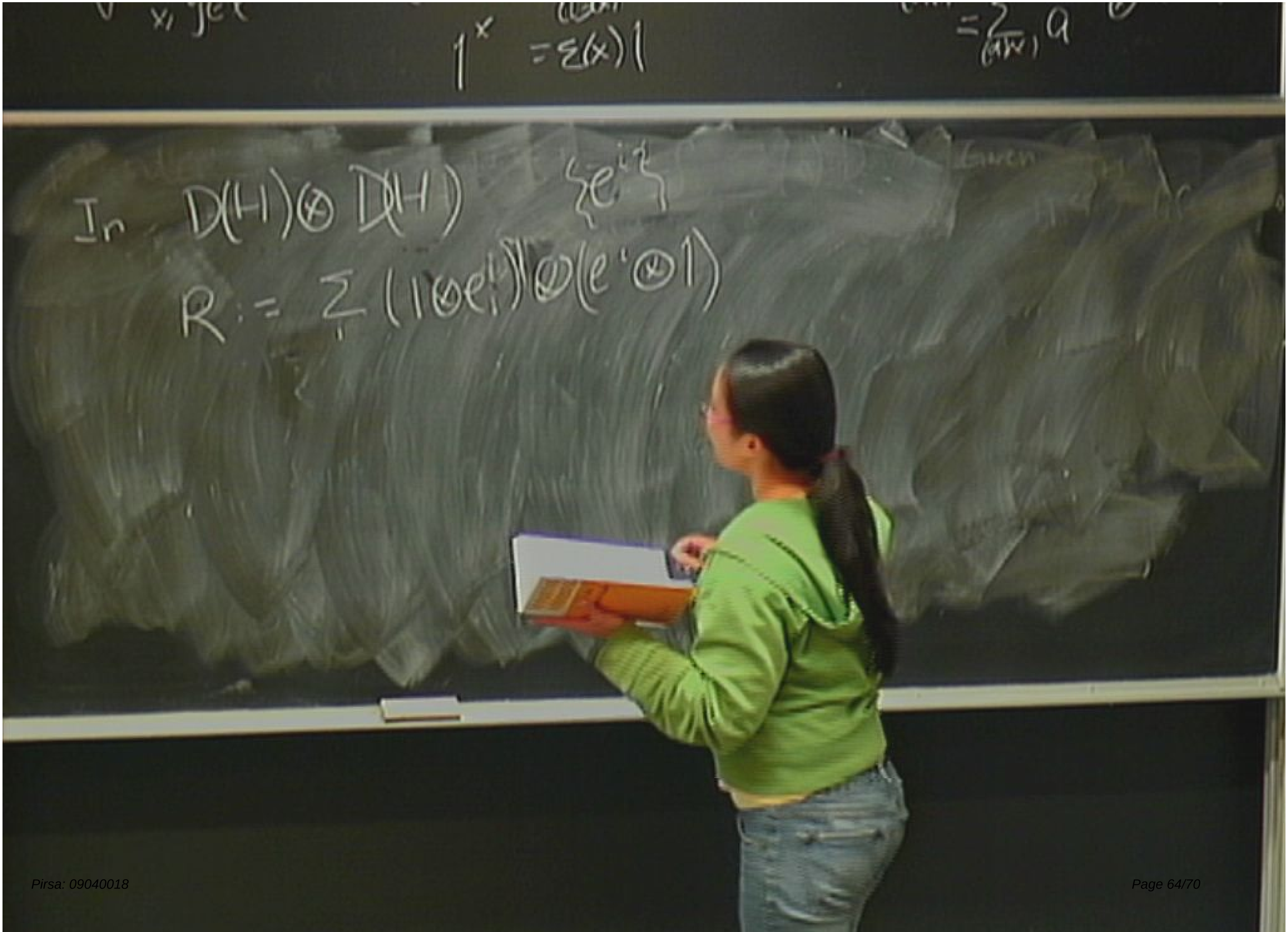
H^{ξ} $a \cdot x = \sum_{(a)} a' x S(a'')$

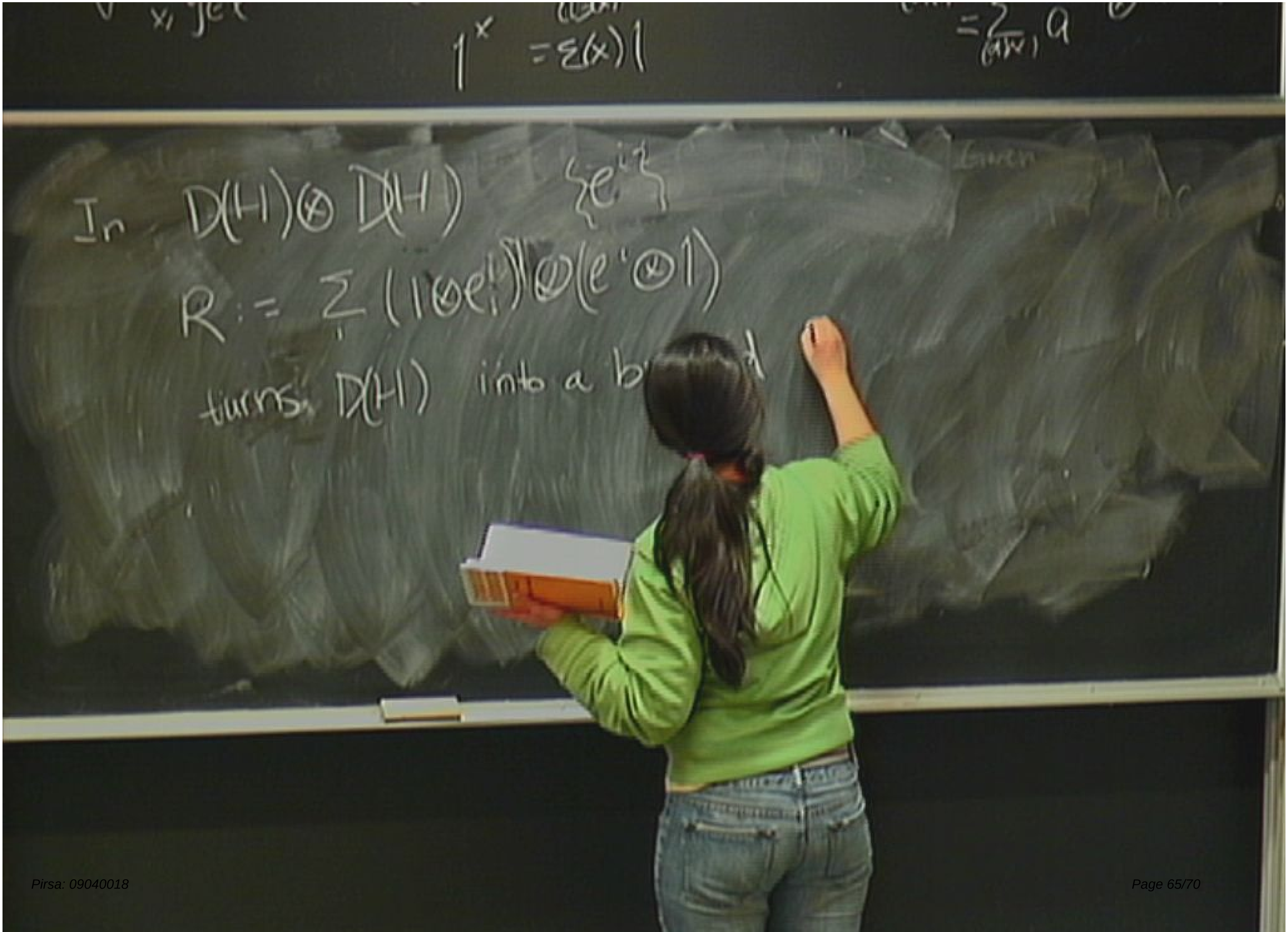
get

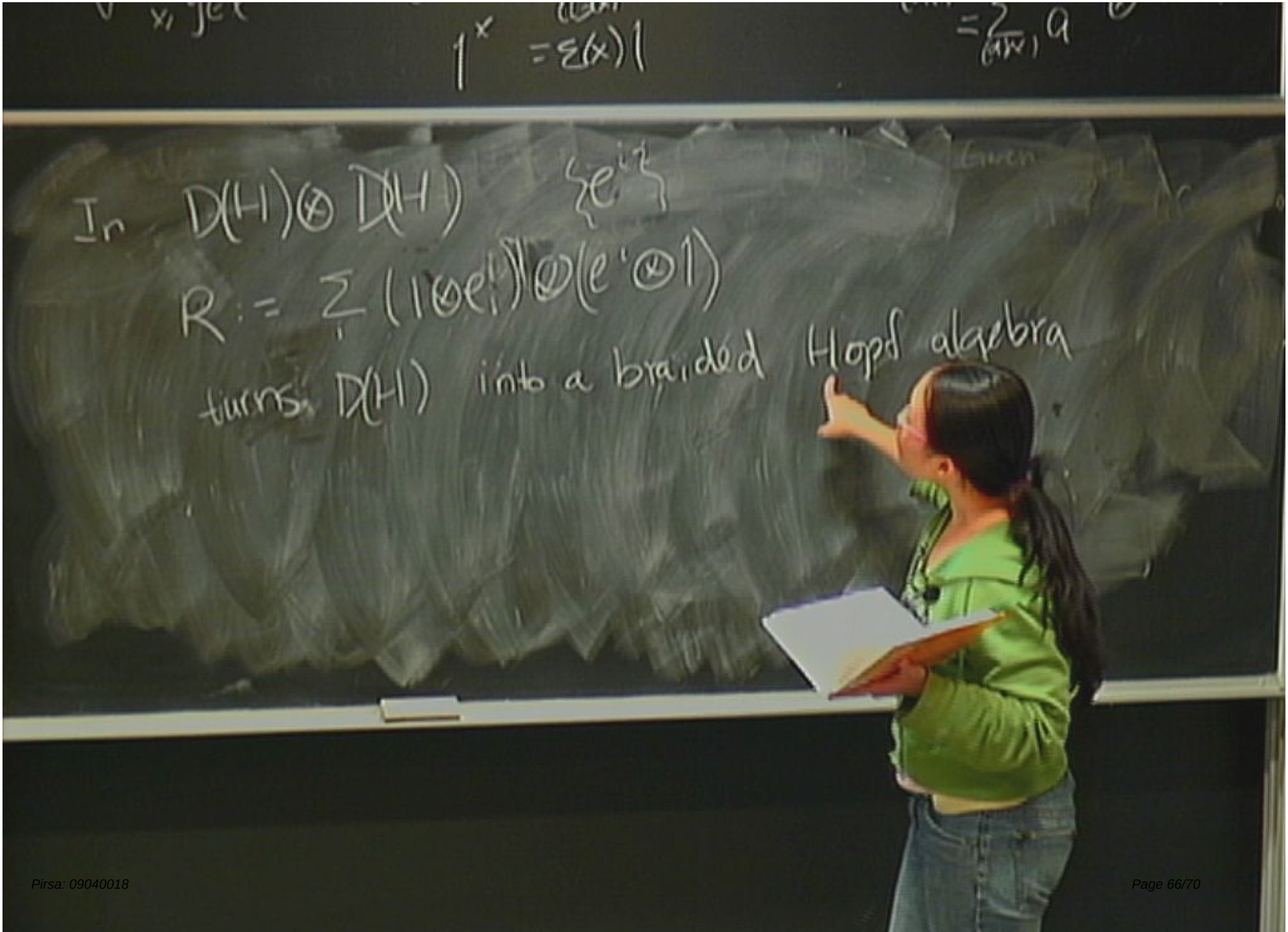
$$\underbrace{\left(\left(H^{cop}\right)^{*\,op}\right)^{*}}_{\substack{\cong \\ H}} \xleftarrow{\text{right}} \left(H^{op}\right)^{*} \cong \left(H^{op}\right)^{*}$$

given $\left(H^{cop}\right)^{*} \xleftarrow{\text{right}} \left(H^{cop}\right)^{*}$

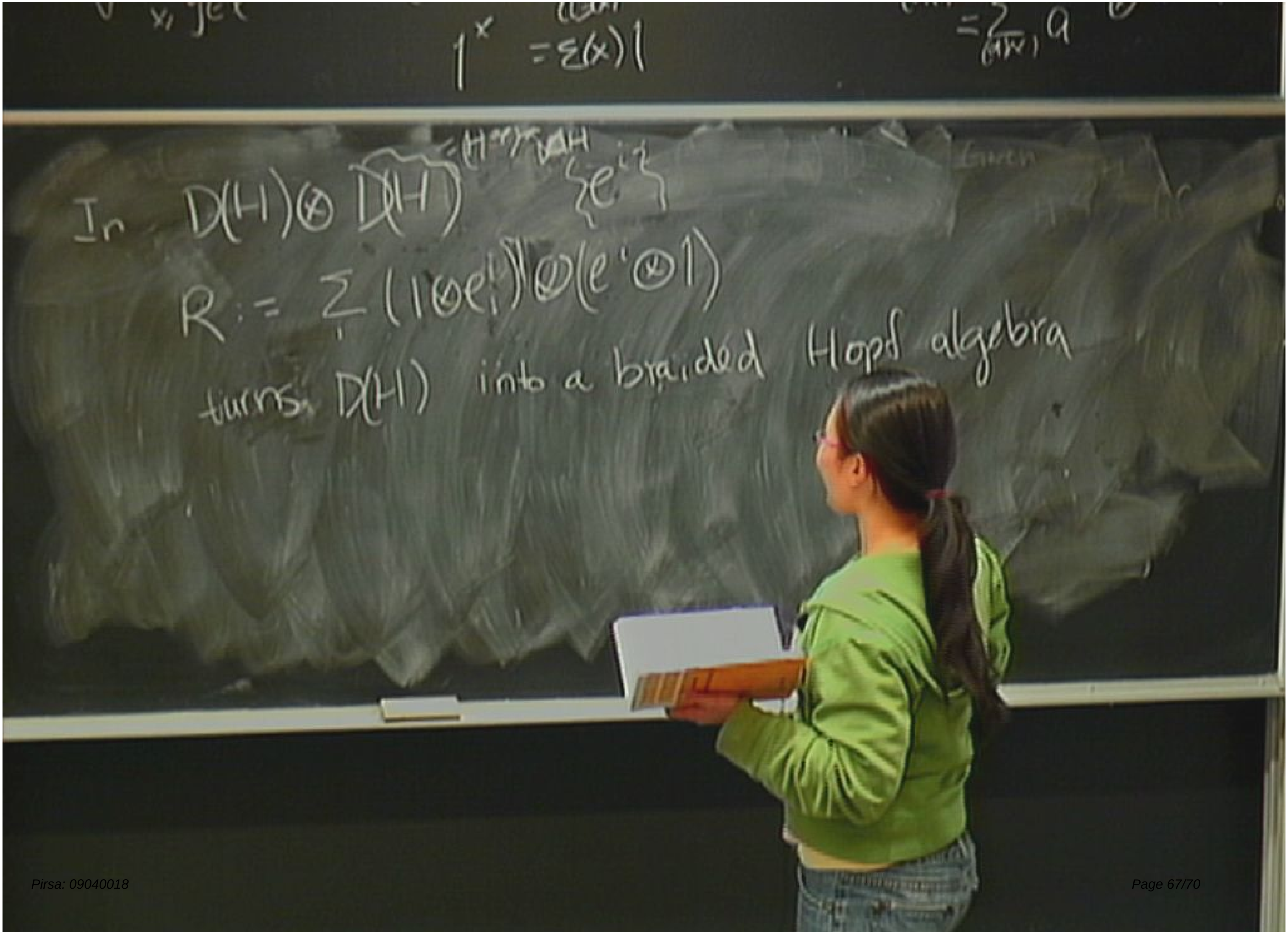
This turns H into a right module-coalg. over $(H^{o}$


1^x = ε(x)1
In D(H)⊗D(H)
{e^i}
R := Σ_i (1⊗e_i)⊗(e^i⊗1)

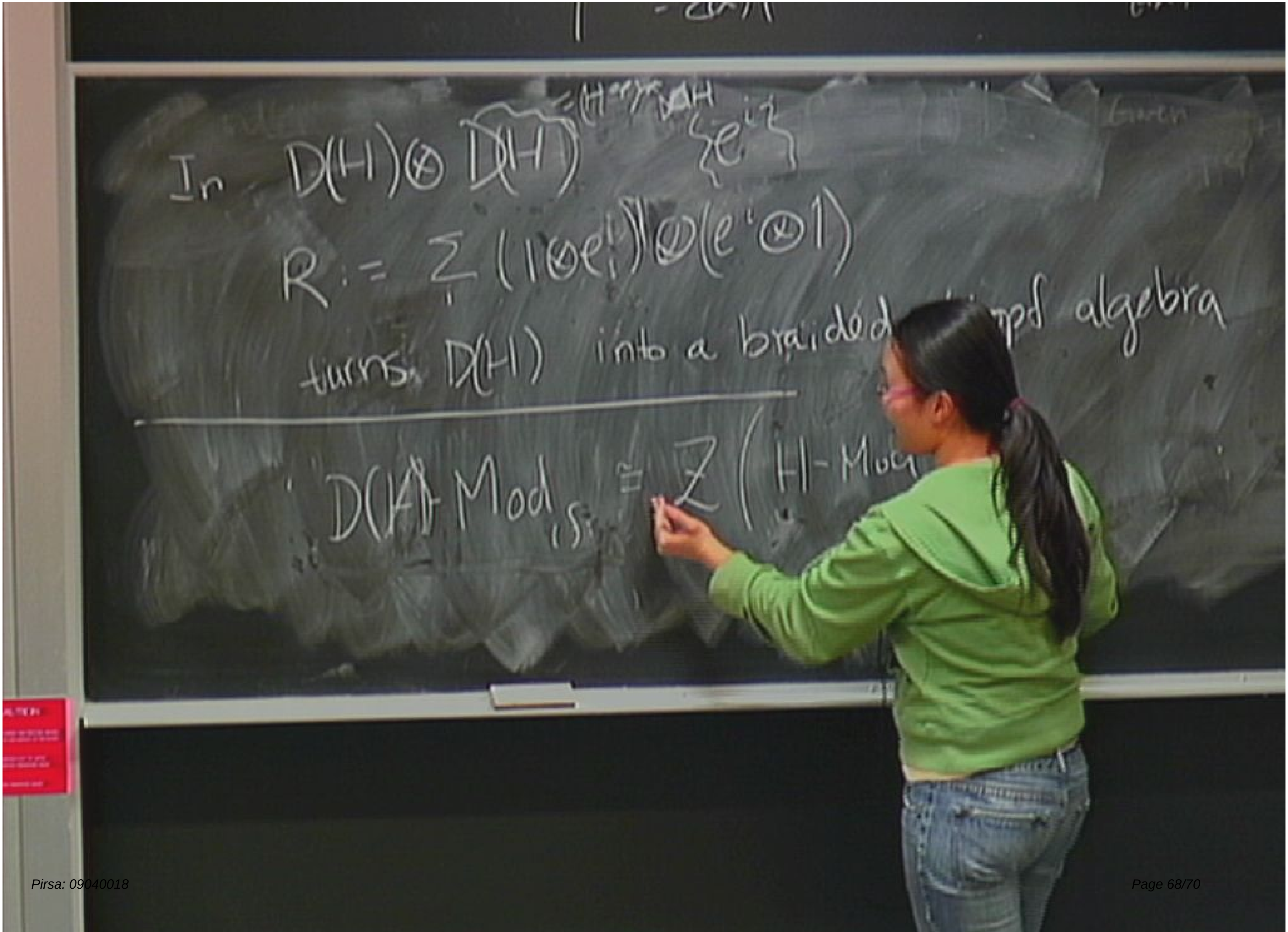

In D(H) ⊗ D(H)
R := Σ (1 ⊗ e_i) ⊗ (e^i ⊗ 1)
turns D(H) into a

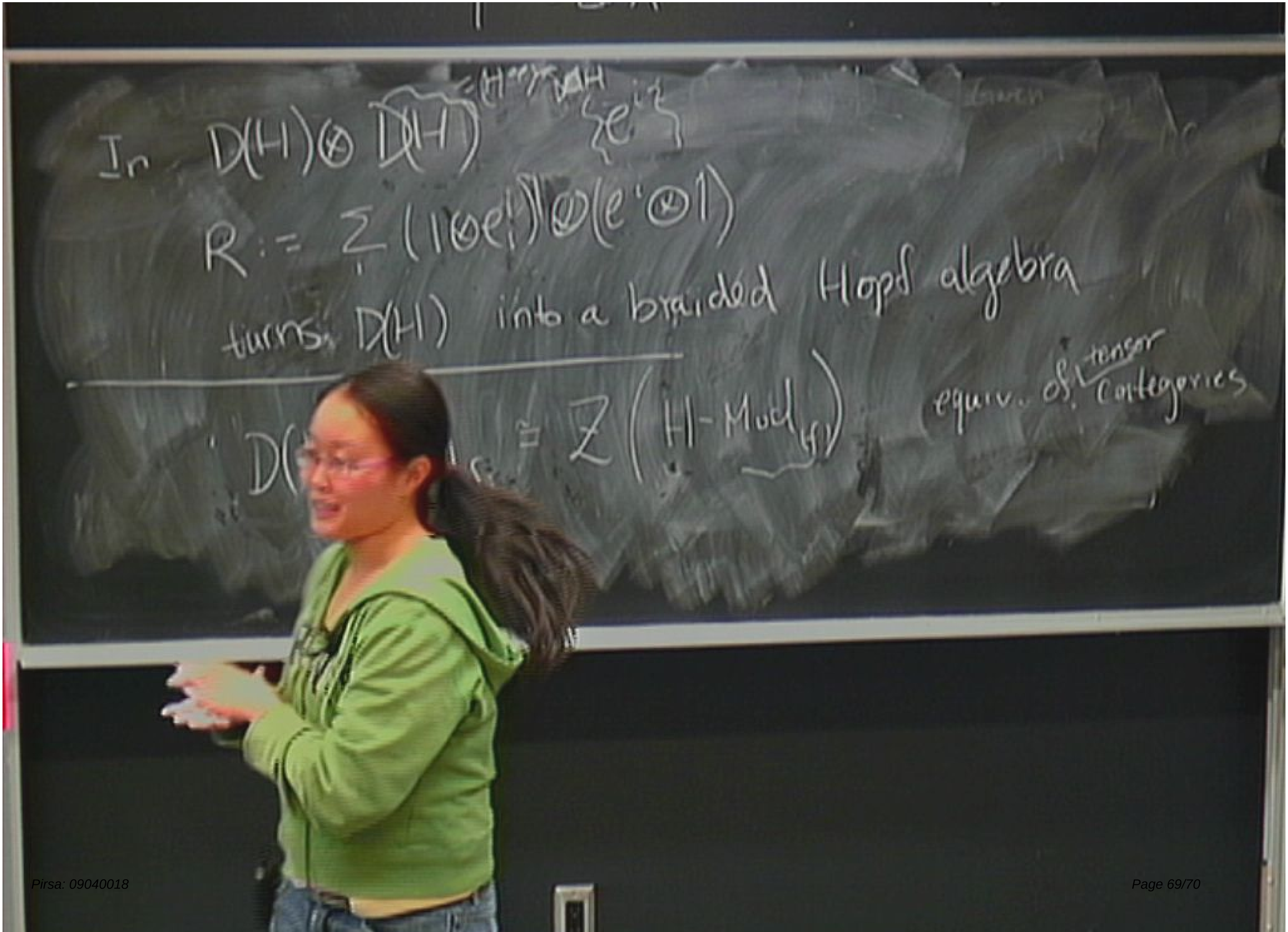


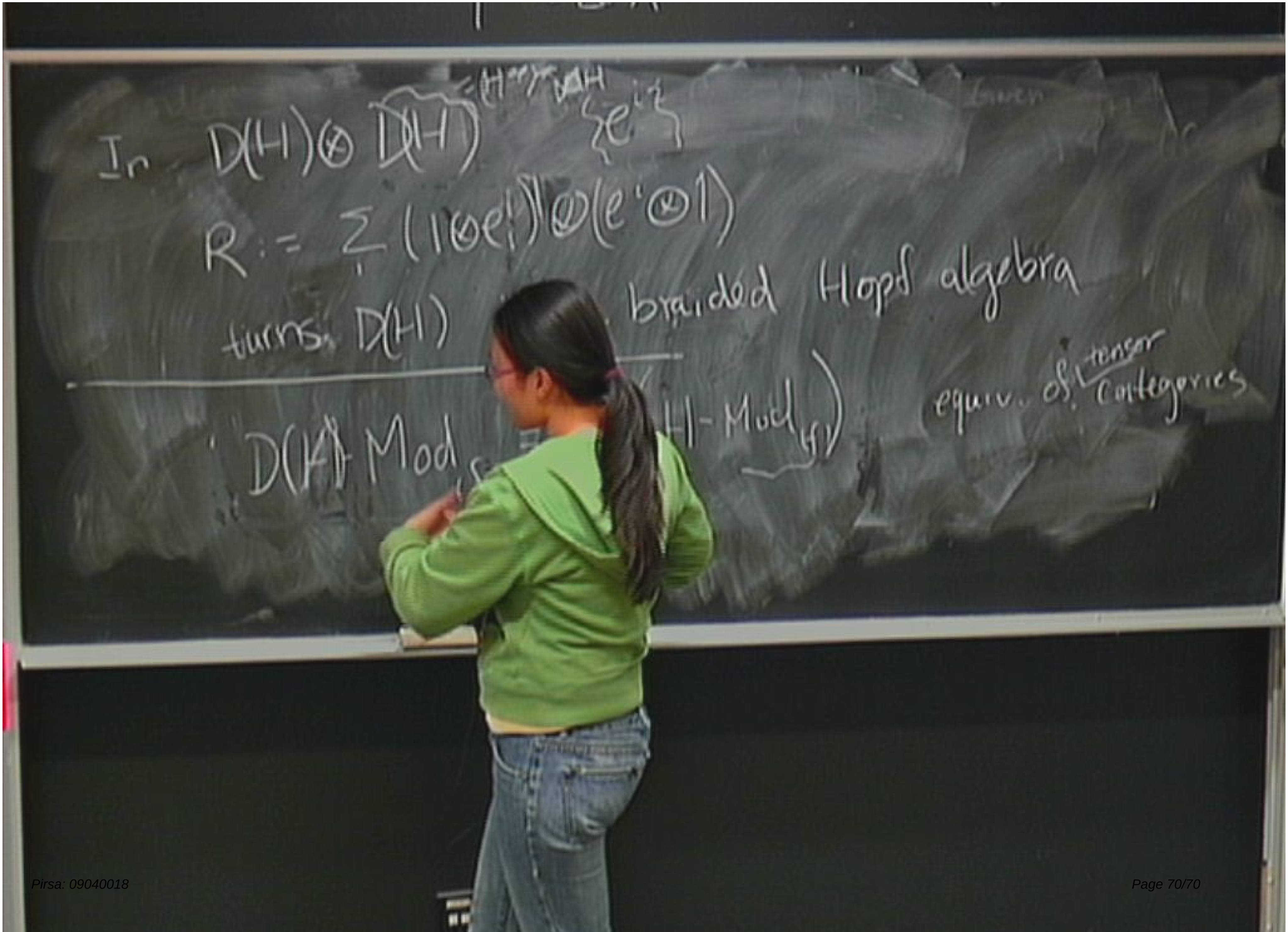
$1^x = \varepsilon(x) 1$
In $D(H) \otimes D(H)$ $\{e^i\}$
$R := \sum_i (1 \otimes e_i) \otimes (e^i \otimes 1)$
turns $D(H)$ into a braided Hopf algebra



1^x = ε(x)1
In D(H) ⊗ D(H)
R := Σ_i (1⊗e_i)⊗(e^i⊗1)
turns D(H) into a braided Hopf algebra


In D(H)⊗D(H)
R := Σ (1⊗e^i)⊗(e^i⊗1)
turns D(H) into a braided
pf algebra
D(H)-Mod
Z(H-Mo


In D(H) ⊗ D(H)
R := Σ (1⊗e_i) ⊗ (e^i ⊗ 1)
turns D(H) into a braided Hopf algebra
Z(H-Mod)
equiv. of tensor categories


In D(H) ⊗ D(H)
{e^i}
R := Σ (1⊗e_i) ⊗ (e^i ⊗ 1)
turns D(H)
braided Hopf algebra
D(H)-Mod
H-Mod
equiv. of tensor categories