Title: Integrability in gauge/string dualities

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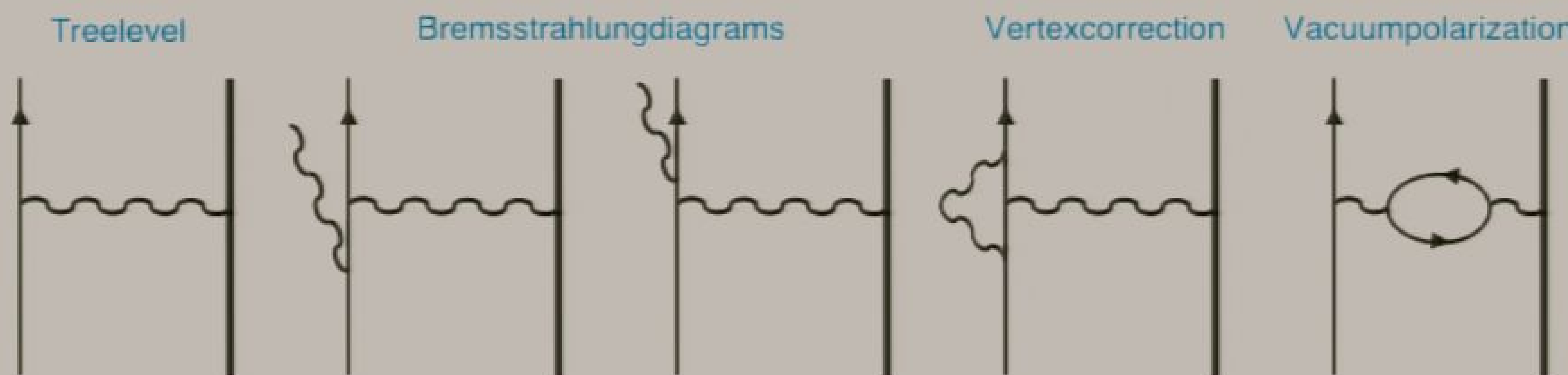
Abstract: Integrability in gauge/string dualities will be reviewed in a broad perspective with a particular emphasis on the recently proposed equations describing the full planar spectrum of anomalous dimensions in AdS/CFT [N.Gromov, V.Kazakov, PV]. These are a concise version of Thermodynamic Bethe equations, called Y-system, which generalize the asymptotic Bethe equations of Beisert and Staudacher (which yield the full spectrum of N=4 SYM for asymptotically long local operators) and incorporate the 4-loop results for the shortest twist two operators obtained by Bajnok and Janik from the dual string sigma model (thus reproducing perturbative gauge theory computations with thousands of diagrams). On the way, we will explain some of the interesting open problems in the field.

Electron and heavy proton

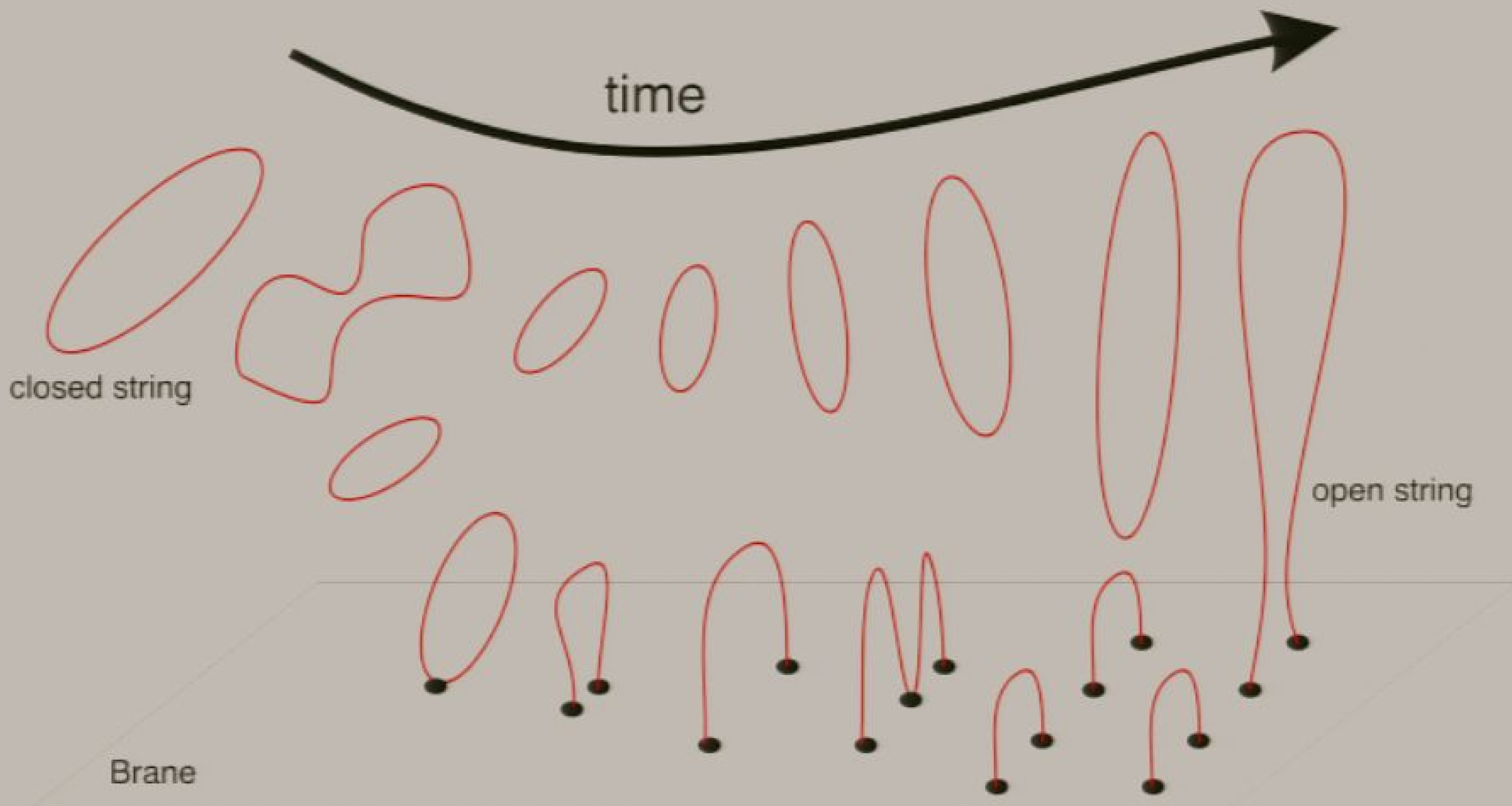
Either



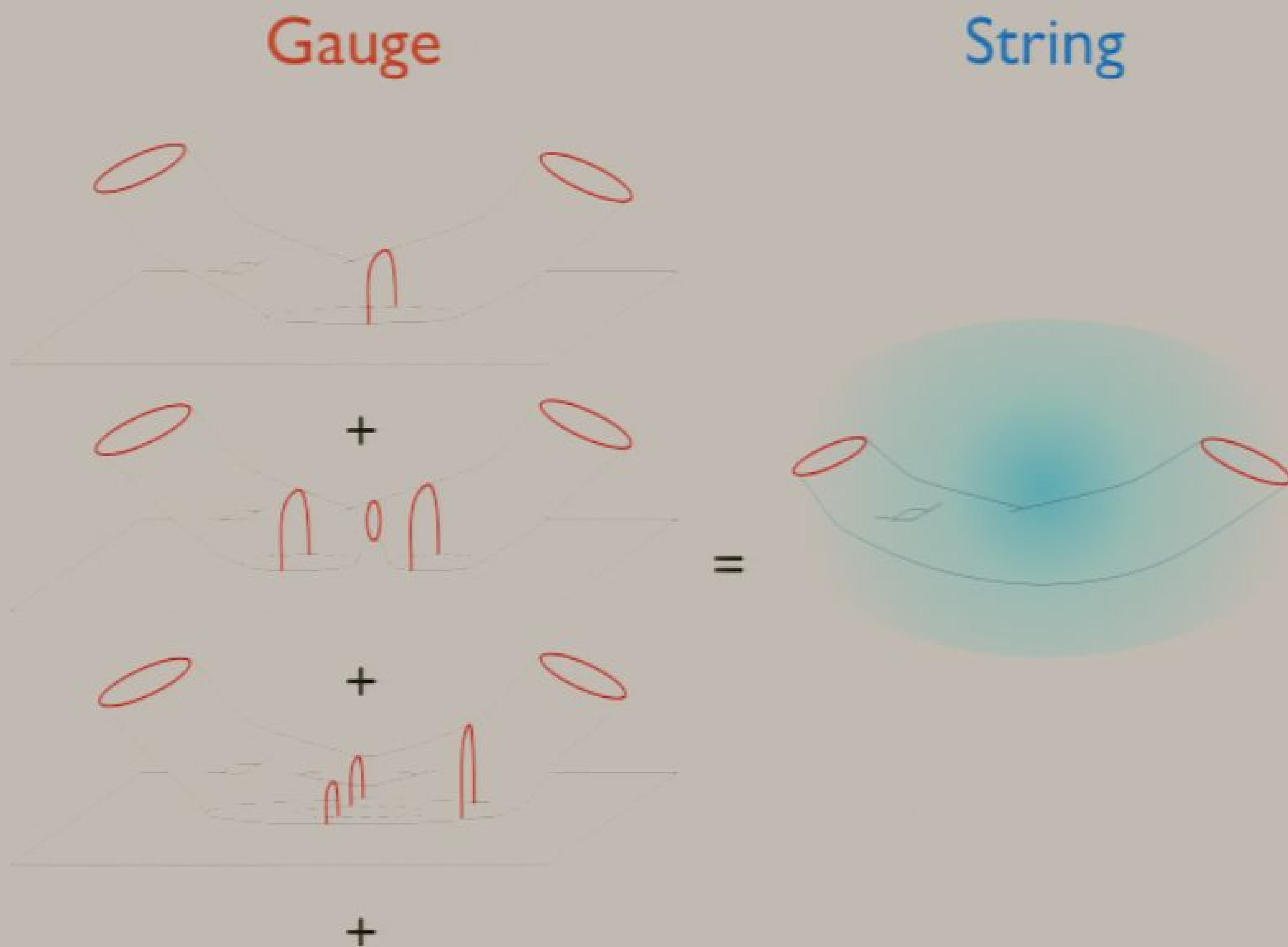
or

$$V(r) = -\frac{\alpha}{r}\left(1 + \frac{\alpha}{4\sqrt{\pi}}\frac{e^{-2m_e r}}{(m_e r)^{3/2}} + \dots\right)$$

Strings and Branes



Either Brane or Background

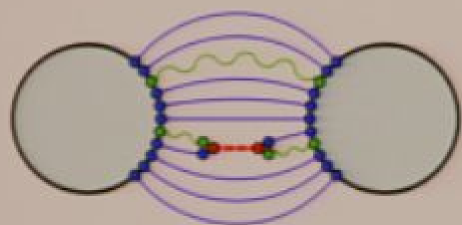


CFT

AdS

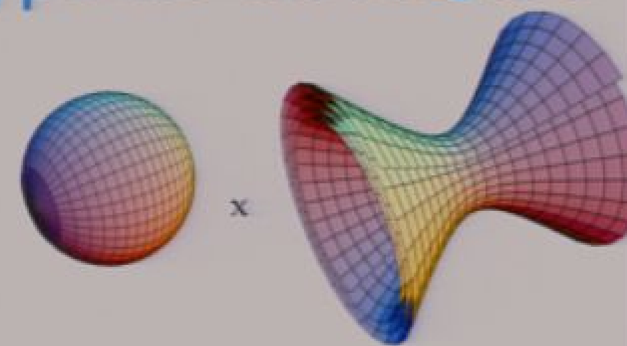
N=4 SYM in 4d

type IIB in $AdS_5 \times S^5$



dual to

[Maldacena]



$$\mathcal{L} = F^2 + (D\Phi)^2 + [\Phi, \Phi]^2 + \bar{\Psi}\mathcal{D}\Psi$$

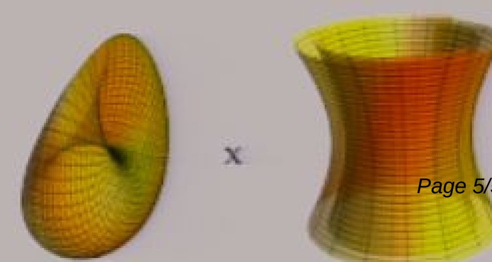
$$\mathcal{L} = (\partial x)^2 + \Lambda(x^2 - 1) + \dots$$
$$= \mathrm{str}\left(J^{(2)} \wedge *J^{(2)} + J^{(1)} \wedge J^{(3)}\right)$$

3D planar N=6
CS matter

dual to

[ABJM]

type IIA in $AdS_4 \times CP^3$



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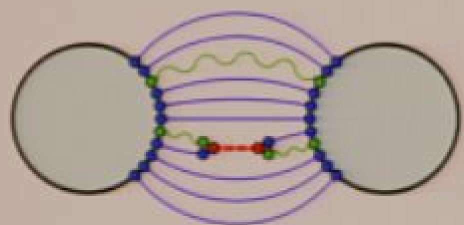

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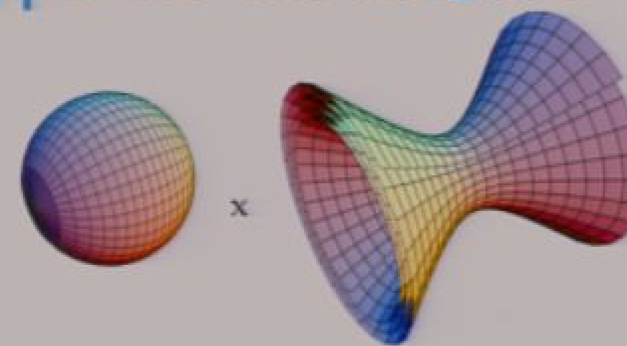

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$$\mathcal{L} = (\partial x)^2 + \Lambda(x^2 - 1) + \ldots$$

$$\lambda = g_{YM}^2 N = \frac{R^4}{\alpha'^2} = g_s N$$

(to set up these dualities we typically use stacks of N branes instead of a single brane)

Planar Limit $N \to \infty$

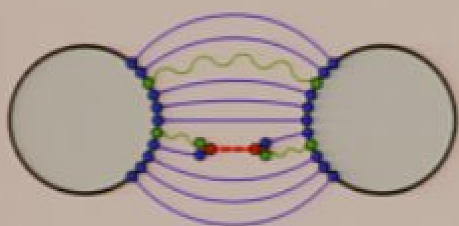
Planar CFT

Free Strings

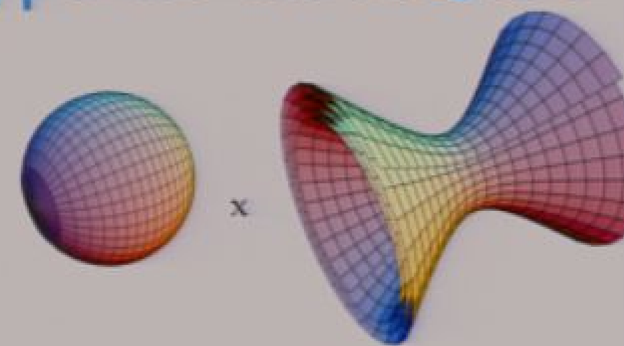
CFT

AdS

N=4 SYM in 4d



type IIB in $AdS_5 \times S^5$



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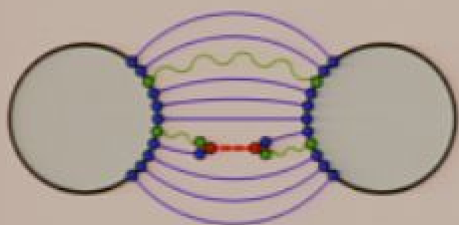
Strongly coupled gauge theory — Classical strings

Perturbative gauge theory — Highly quantum strings

CFT

N=4 SYM in 4d



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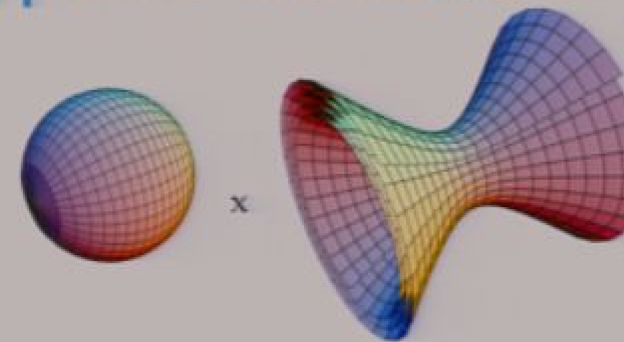


- Understand and define quantum gravity/string theory
- Powerful tool for studying strong coupling phenomena
- **Inspiration for solving for the first time non-trivial gauge theories**

Strongly coupled gauge theory

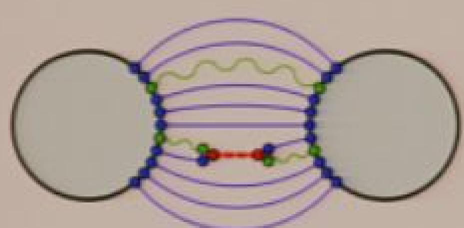
Classical strings

Perturbative gauge theory

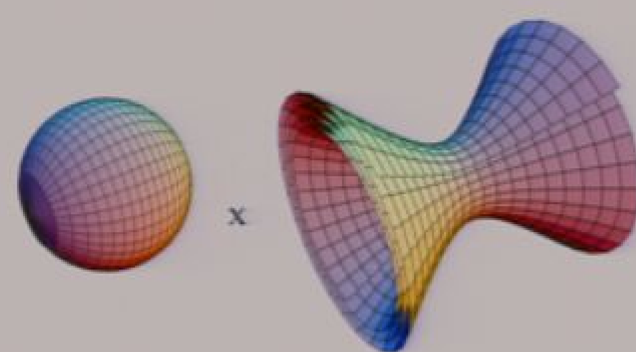
Highly quantum strings

Solve = ?

- Correlation functions:
  2 pt functions (**spectrum of anomalous dimensions**)
  3 pt functions (structure constants)
- S-matrix



anomalous dimensions



string energies

Integrability

Spin chains in N=4

$$\mathrm{tr}\left(\Phi_1\Phi_2(D_3)^2(\Phi_1)^5\ldots\right)$$

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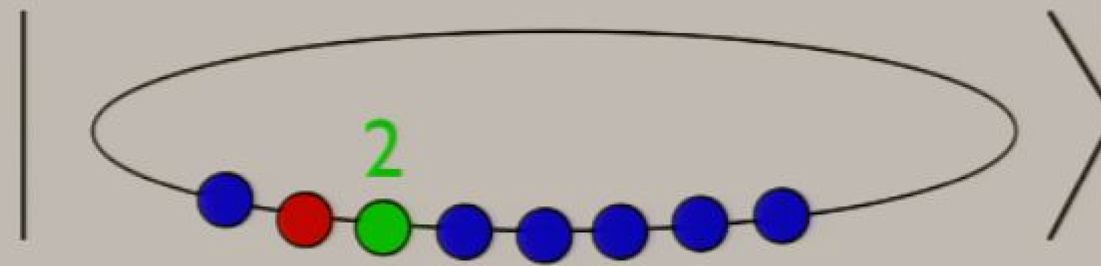

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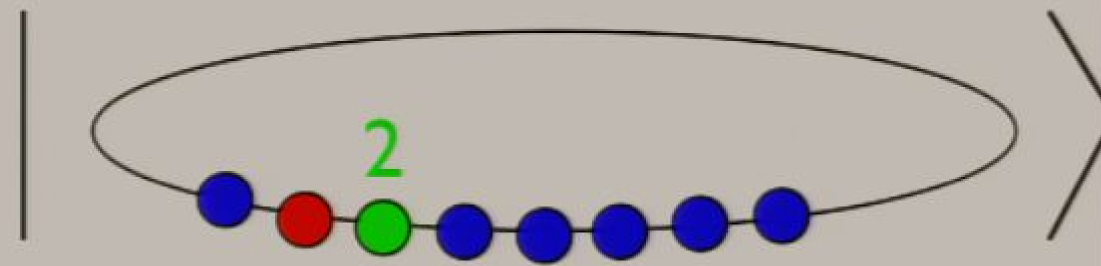

Mixing matrix or
Dilatation operator

$$\mathcal{O}_A^{ren}(x) = (e^{\,\hat{H}\log\Lambda})_{AB}\,\mathcal{O}_B(x)$$

$$\langle \mathcal{O}_A^{ren}(x)\mathcal{O}_B^{ren}(y)\rangle = \frac{\delta_{AB}}{|x-y|^{2\Delta}}$$

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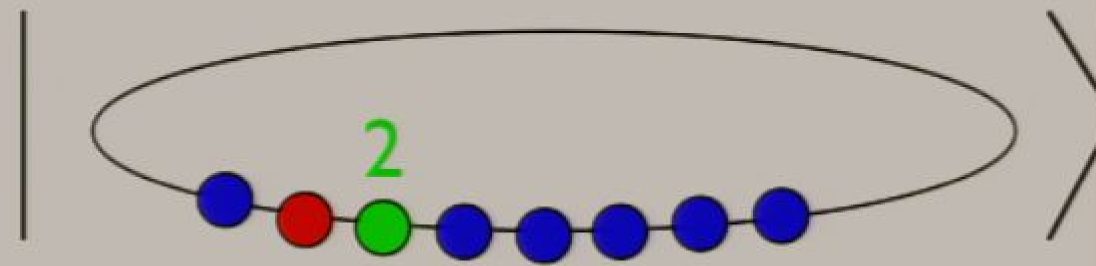

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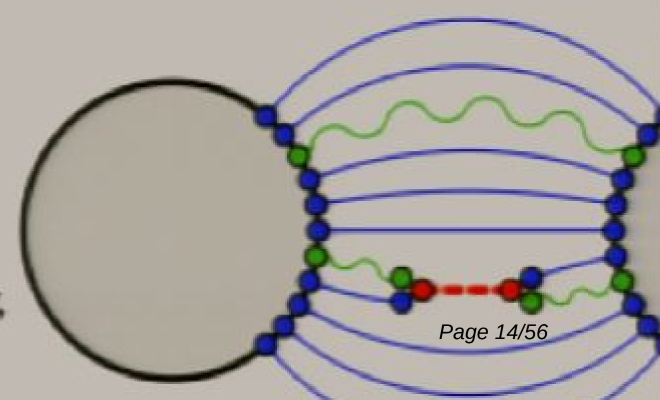
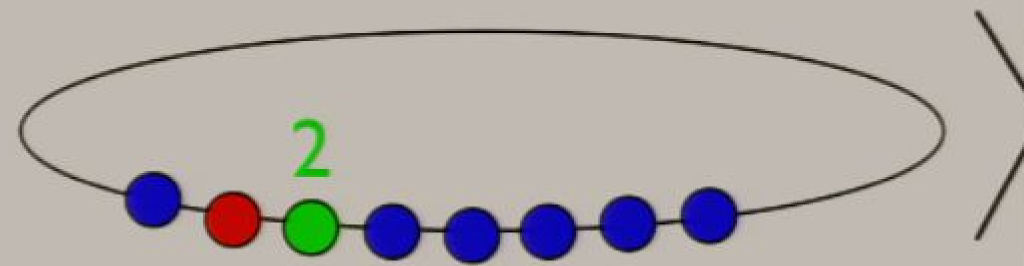


H is nearest neighbors to leading order in perturbation theory, next to nearest neighbors at next to leading order etc...

Spin chains in N=4

$\mathrm{tr}\left(\Phi_1\Phi_2(D_3)^2(\Phi_1)^5\ldots\right)$ | 

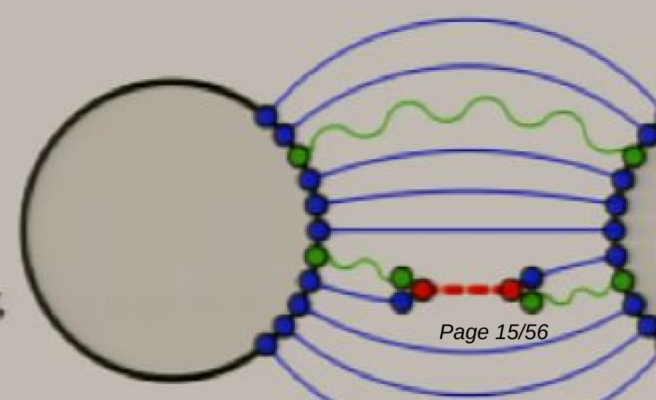
Mixing matrix or
Dilatation operator

Integrable spin chain
Hamiltonian

$$\mathcal{O}_A^{ren}(x) = (e^{\hat{H}\log\Lambda})_{AB}\,\mathcal{O}_B(x)$$

[Minahan, Zarembo; Beisert, Staudacher]

$$\langle \mathcal{O}_A^{ren}(x)\mathcal{O}_B^{ren}(y)\rangle = \frac{\delta_{AB}}{|x-y|^{2\Delta}}$$



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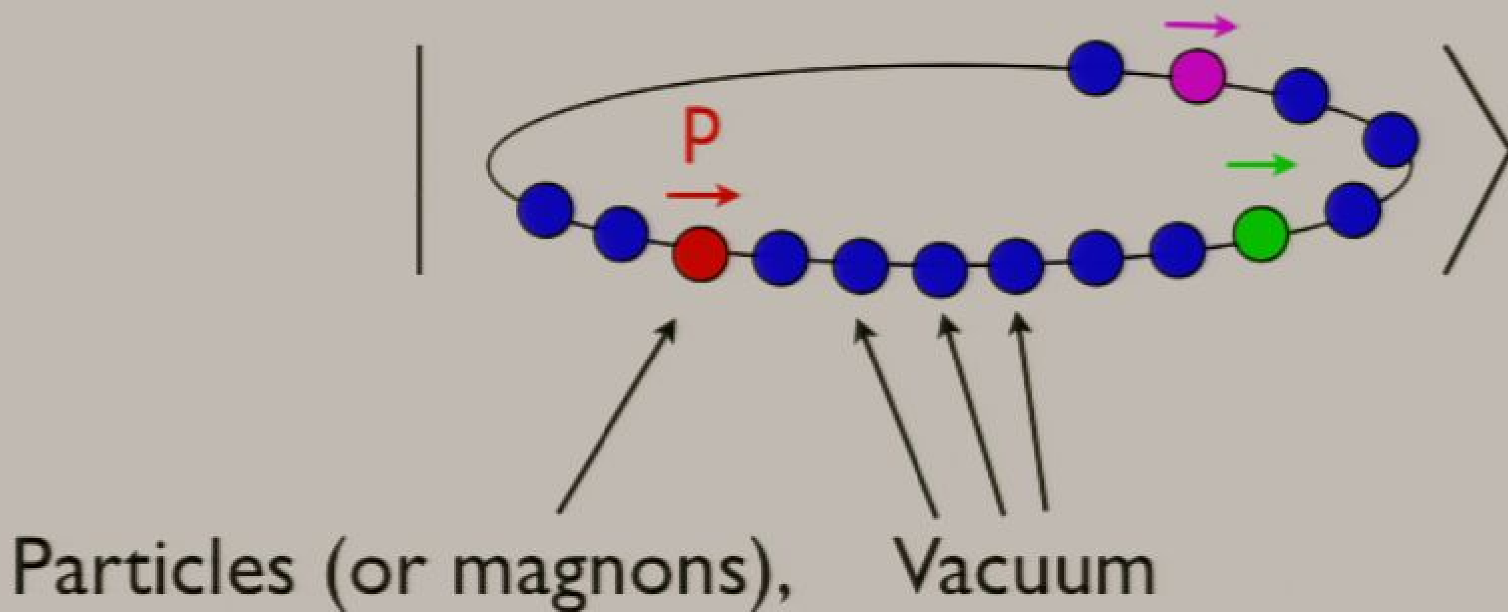
2d S-matrix in N=4



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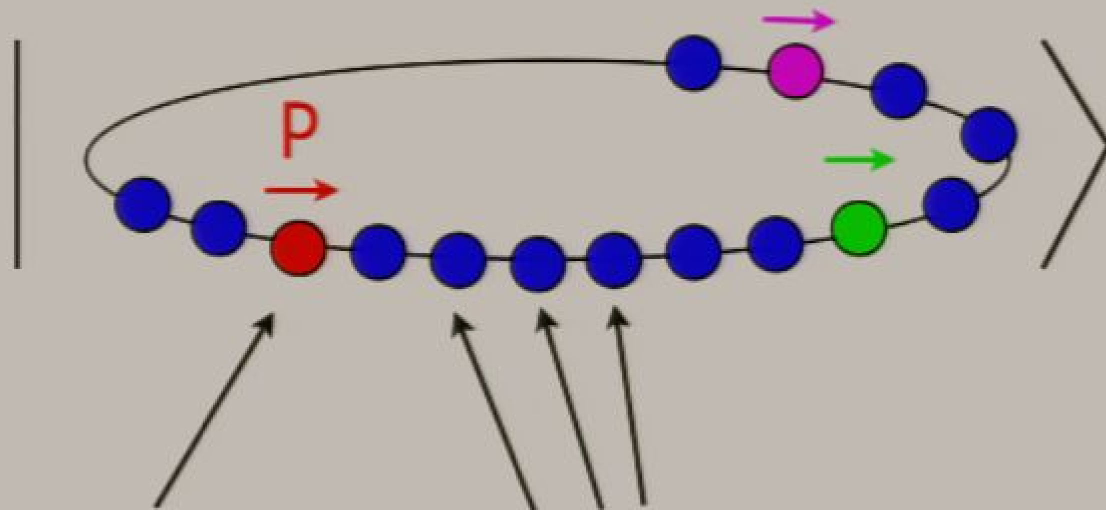
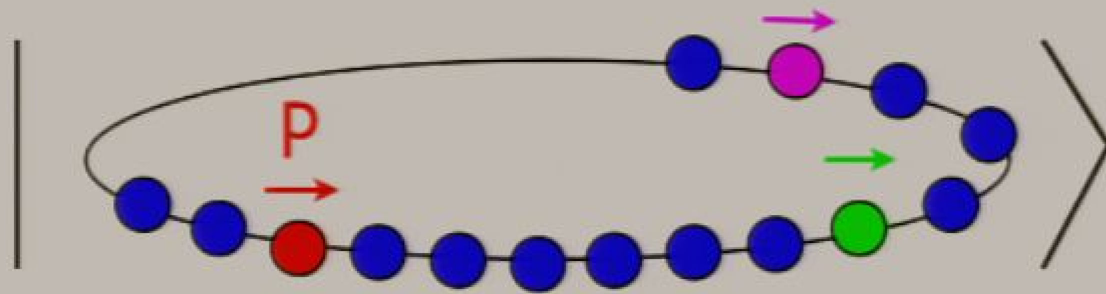


Particles (or magnons), Vacuum

Particles can scatter, e.g. $S(p,k)$ [Staudacher]

Particles transform in $PSU(2|2)^2$ extended [Beisert]

2d S-matrix in N=4



$$H \longrightarrow S(p,k)$$

$$PSU(2,2|4) \longrightarrow PSU(2|2)^2 \text{ extended}$$

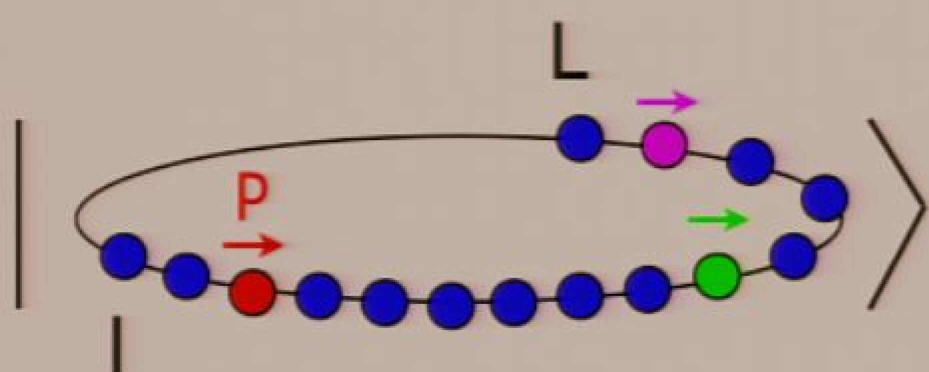
S-matrix (up to a scalar factor) and magnon dispersion relation almost fixed by symmetry

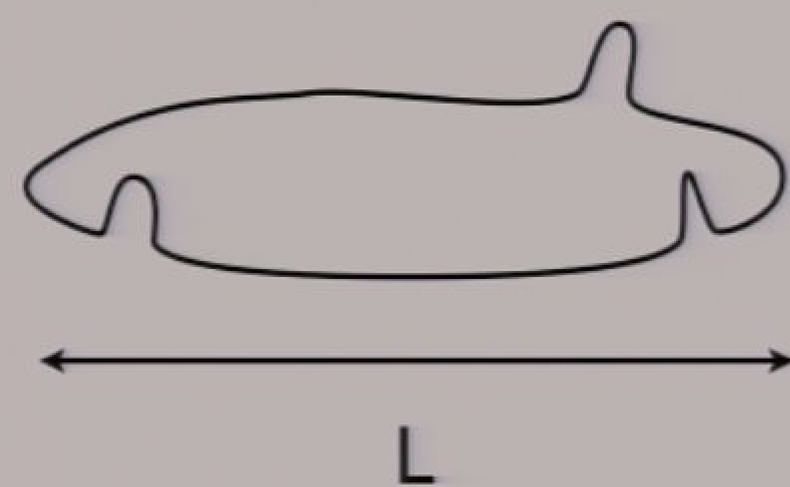
[Beisert; Aryutunov, Frolov, Zamaklar]

2d S-matrix in AdS/CFT

Spin chain magnons in an operator with L fields

Worldsheet excitations in light-cone gauged string theory. 2D QFT in a circle of size L





Previous arguments hold for both string and gauge theory

Asymptotic Bethe equations and integrable 2D QFT

In 1+1D $\quad Q_1 = \sum p_j \ , \ Q_2 = \sum p_j^2 \ , \ \Rightarrow \ \{p_1, p_2\} = \{p'_1, p'_2\}$

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If (integrability!) $\quad \exists Q_3 = \sum p_j^3 \; \Rightarrow \; \{p_1, p_2, p_3\} = \{p'_1, p'_2, p'_3\}$

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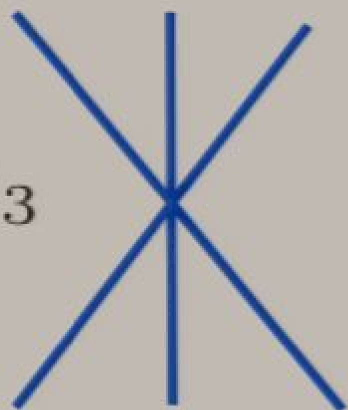
$e^{i\alpha Q_3}$



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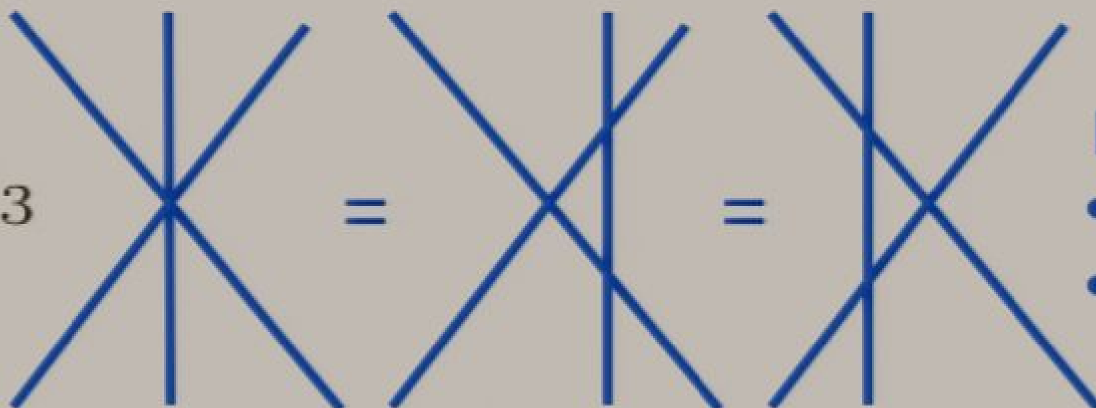

Integrable theories:

- Factorized scattering.
- S-matrices obey YB.

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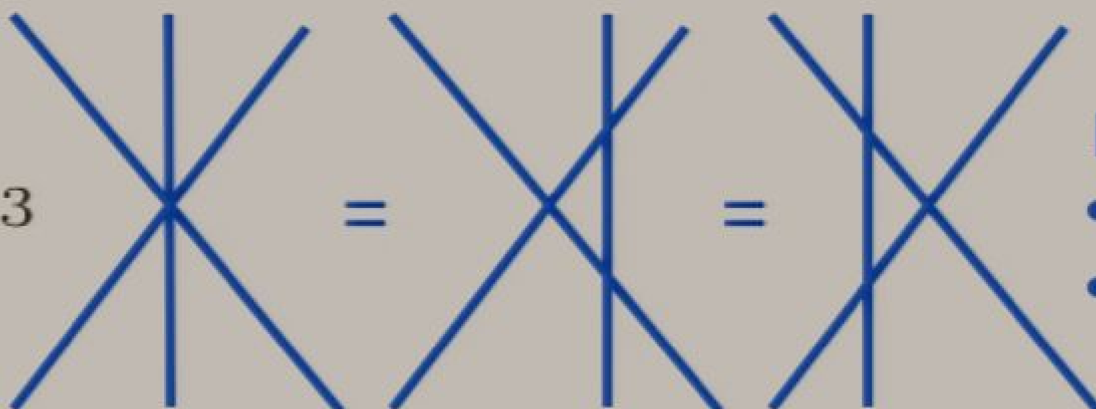

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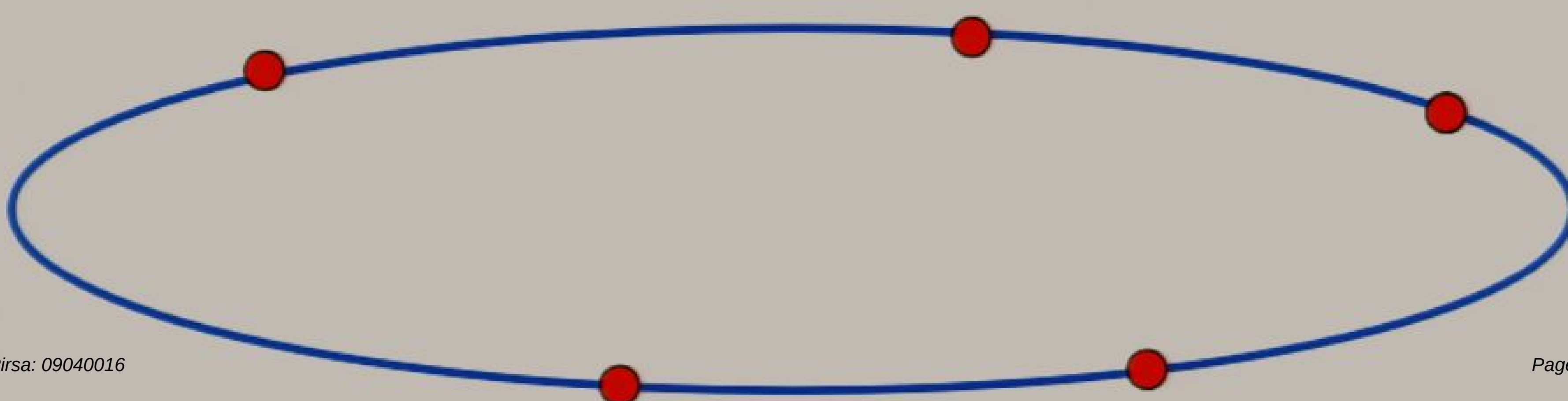

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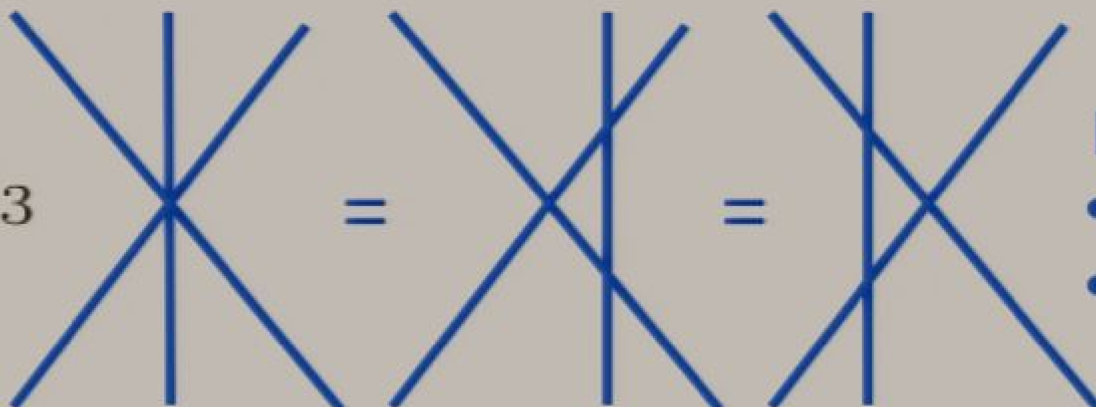

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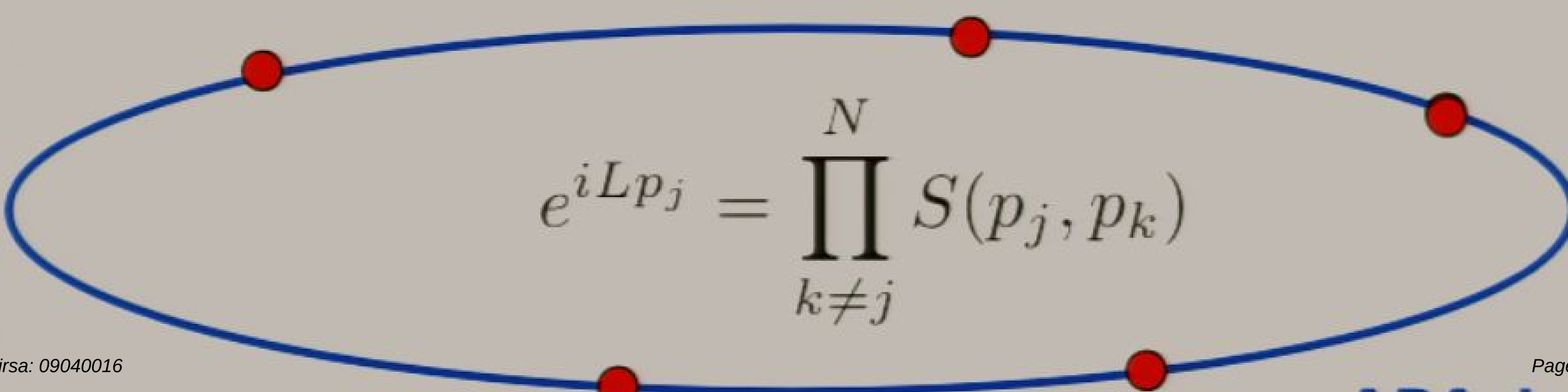
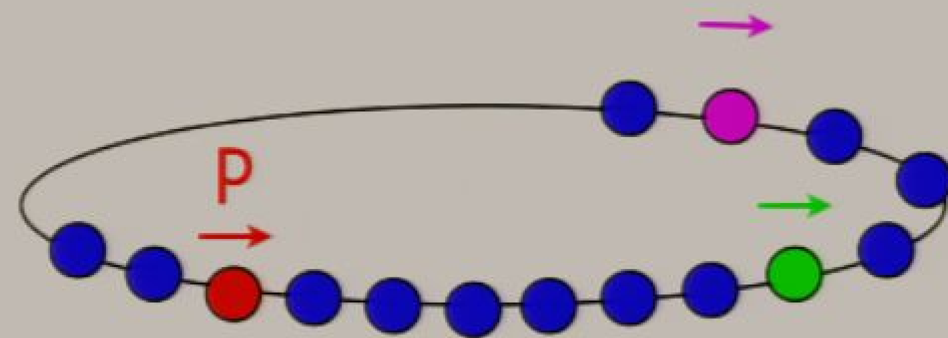


$$e^{iLp_j} = \prod_{k \neq j}^{N} S(p_j, p_k)$$

ABA: large L!

Bethe Equations

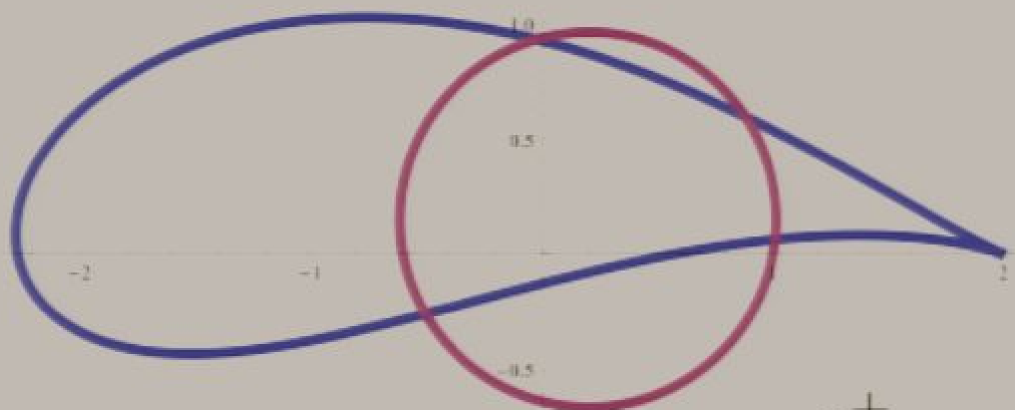
$$0 = \left(e^{iLp_j} \prod_{k \neq j}^{M} \hat{S}(p_j, p_k) - 1 \right) \Bigg| \quad \Bigg\rangle$$



$$\Delta = J + \sum_{j=1}^{M} \sqrt{1 + \lambda \sin^2 \frac{p_j}{2}} + \ldots$$

[Staudacher; Beisert, Staudacher; Janik; Beisert, Eden, Staudacher; Beisert, Hernandez, Lopez; Arutyunov, Frolov, Zamaklar]

Explicitly

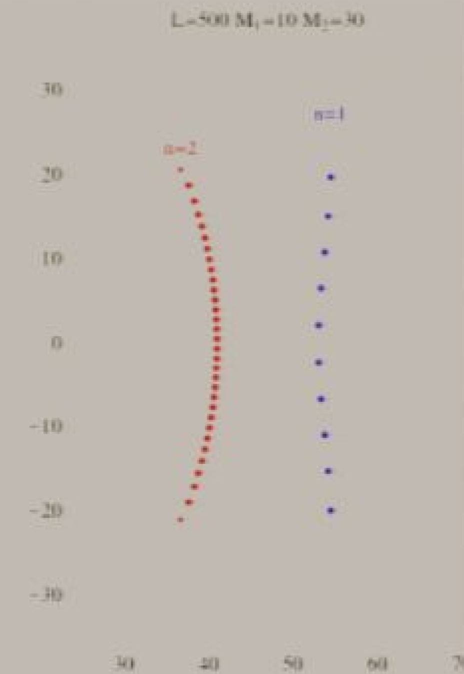


$$\frac{x^+}{x^-} = e^{ip}$$

$$x^+ + \frac{1}{x^+} - x^- - \frac{1}{x^-} = \frac{i}{g}$$

$$x^+ + \frac{1}{x^+} + x^- + \frac{1}{x^-} = \frac{2u}{g} = 2\left(x + \frac{1}{x}\right)$$

[Beisert, Staudacher; Beisert, Eden, Staudacher; Beisert, Hernandez, Lopez]



$$1 = \frac{\mathbf{u_1} - u_2 + \frac{i}{2}}{\mathbf{u_1} - u_2 - \frac{i}{2}} \frac{\mathbf{x_1} - \frac{1}{x_4^+}}{\mathbf{x_1} - \frac{1}{x_4^-}},$$

$$1 = \frac{\mathbf{u_2} - u_2 - i}{\mathbf{u_2} - u_2 + i} \frac{\mathbf{u_2} - u_1 + \frac{i}{2}}{\mathbf{u_2} - u_1 - \frac{i}{2}} \frac{\mathbf{u_2} - u_3 + \frac{i}{2}}{\mathbf{u_2} - u_3 - \frac{i}{2}},$$

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$$1 = \left(\frac{\mathbf{x_4^-}}{\mathbf{x_4^+}}\right)^L \frac{\mathbf{u_4} - u_4 + i}{\mathbf{u_4} - u_4 - i} \frac{x_1 - \frac{1}{\mathbf{x_4^-}}}{x_1 - \frac{1}{\mathbf{x_4^+}}} \frac{\mathbf{x_4^-} - x_3}{\mathbf{x_4^+} - x_3} \frac{x_7 - \frac{1}{\mathbf{x_4^-}}}{x_7 - \frac{1}{\mathbf{x_4^+}}} \frac{\mathbf{x_4^-} - x_5}{\mathbf{x_4^+} - x_5} \sigma_{\mathrm{BES}}(\mathbf{u_4}, u_4)$$

$$1 = \frac{\mathbf{u_5} - u_6 + \frac{i}{2}}{\mathbf{u_5} - u_6 - \frac{i}{2}} \frac{\mathbf{x_5} - x_4^+}{\mathbf{x_5} - x_4^-}$$

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$$\prod_{k=1}^{K_5} \frac{u_{6,j} - u_{5,k} + \frac{i}{2}}{u_{6,j} - u_{5,k} - \frac{i}{2}}$$

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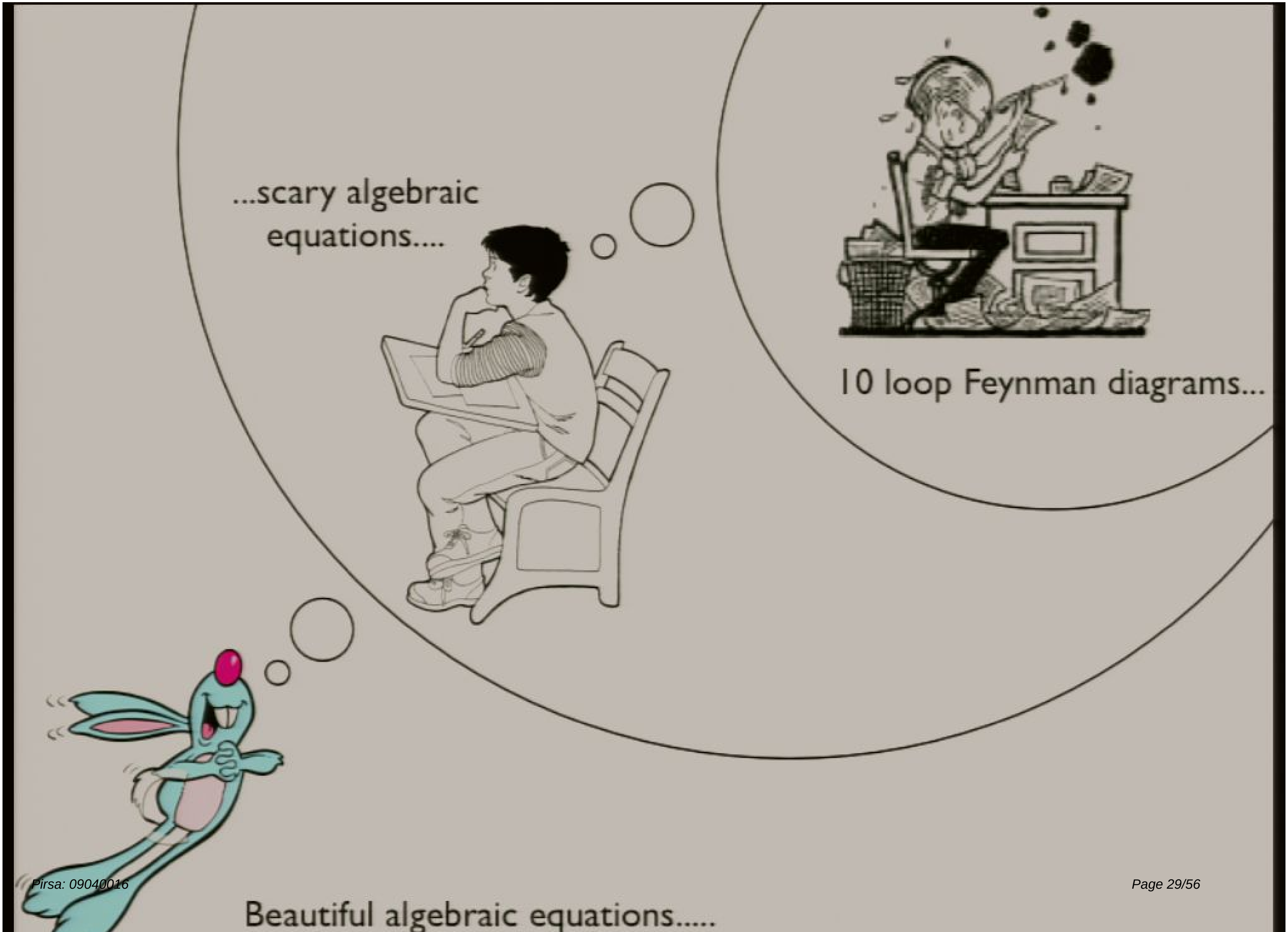
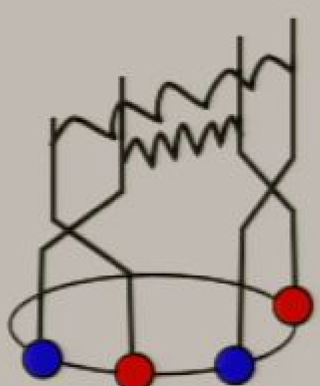

...scary algebraic
equations....
10 loop Feynman diagrams...
Beautiful algebraic equations.....

What about small operators?



we postpone discussing some interesting points concerning the origin of the higher charges Q_3 , integrability and generalizations

[...,Matsubara, Zamolodchikov
Dorey and Tateo,...]

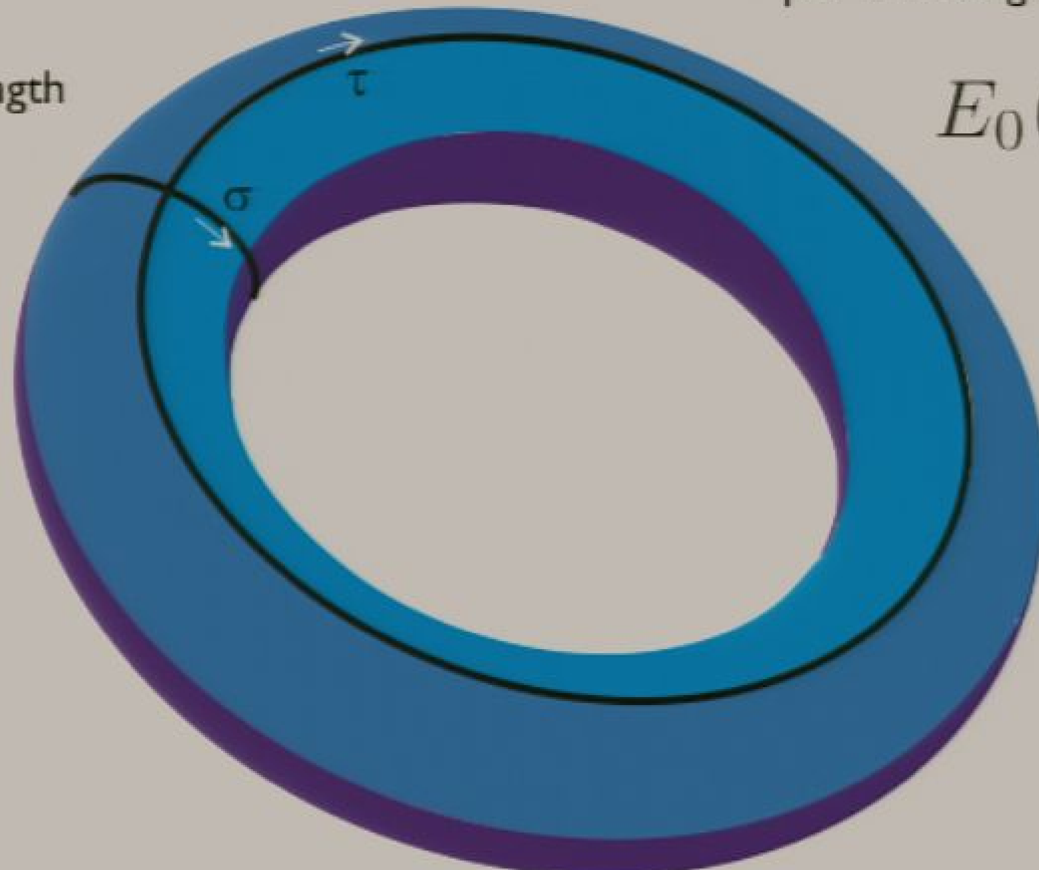
TBA Wick rotation

Ground state energy at size L = free energy
per unit length at temperature 1/L

$$E_0(L) = f(L)$$

Asymptotic Bethe Ansatz
equations in a circle of length
L at zero temperature

$$Z = e^{-RE_0(L)}$$



$$Z = e^{-Rf(L}$$

Exact BAE at
temperature 1/L

A priori this is valid for the ground state energy only.

2D QFT argument - contrary to everything so far this is very stringy!

[...,Matsubara, Zamolodchikov
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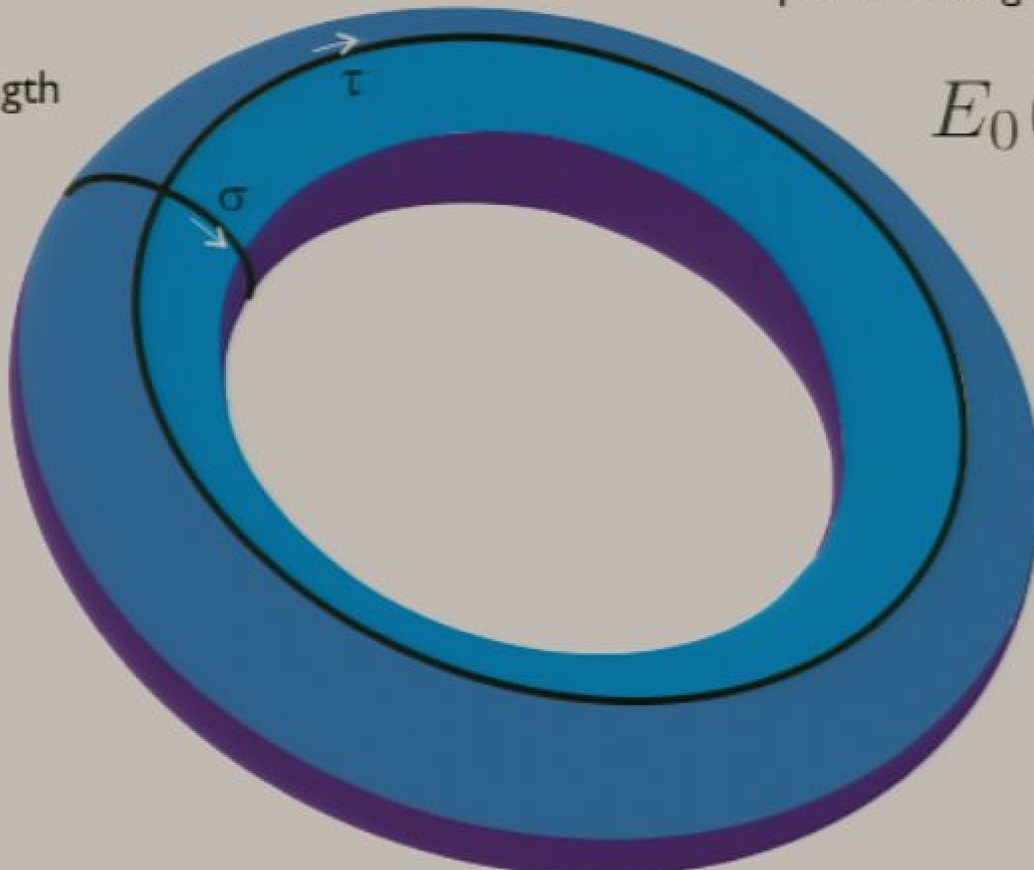

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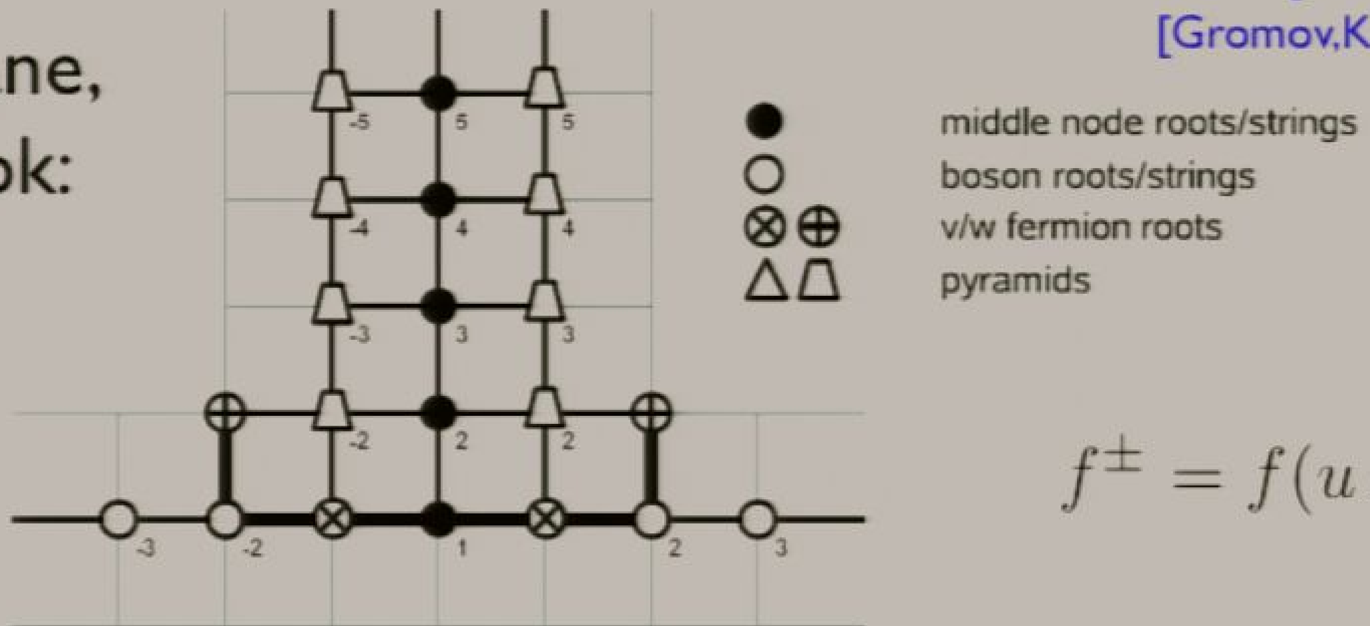
The AdS/CFT Y-system, the full Planar Spectrum

[Gromov,Kazakov,PV]
[Gromov,Kazakov,Kozak, PV]

a,s plane,
T-hook:



$$f^{\pm} = f(u \pm i/2)$$

$$\frac{Y_{a,s}^{+} Y_{a,s}^{-}}{Y_{a+1,s} Y_{a-1,s}} = \frac{(1 + Y_{a,s+1})(1 + Y_{a,s-1})}{(1 + Y_{a+1,s})(1 + Y_{a-1,s})}$$

The energy depends only on the magnon dispersion relation and on the Y-functions found from solving the Y-system:

$$E = \sum_j \epsilon_1(u_{4,j}) + \sum_{a=1}^{\infty} \int_{-\infty}^{\infty} \frac{du}{2\pi i} \frac{\partial \epsilon_a^*}{\partial u} \log\left(1 + Y_{a,0}^*(u)\right)$$

$$Y_{1,0}(u_{4,j}) = -1$$

+ boundary conditions

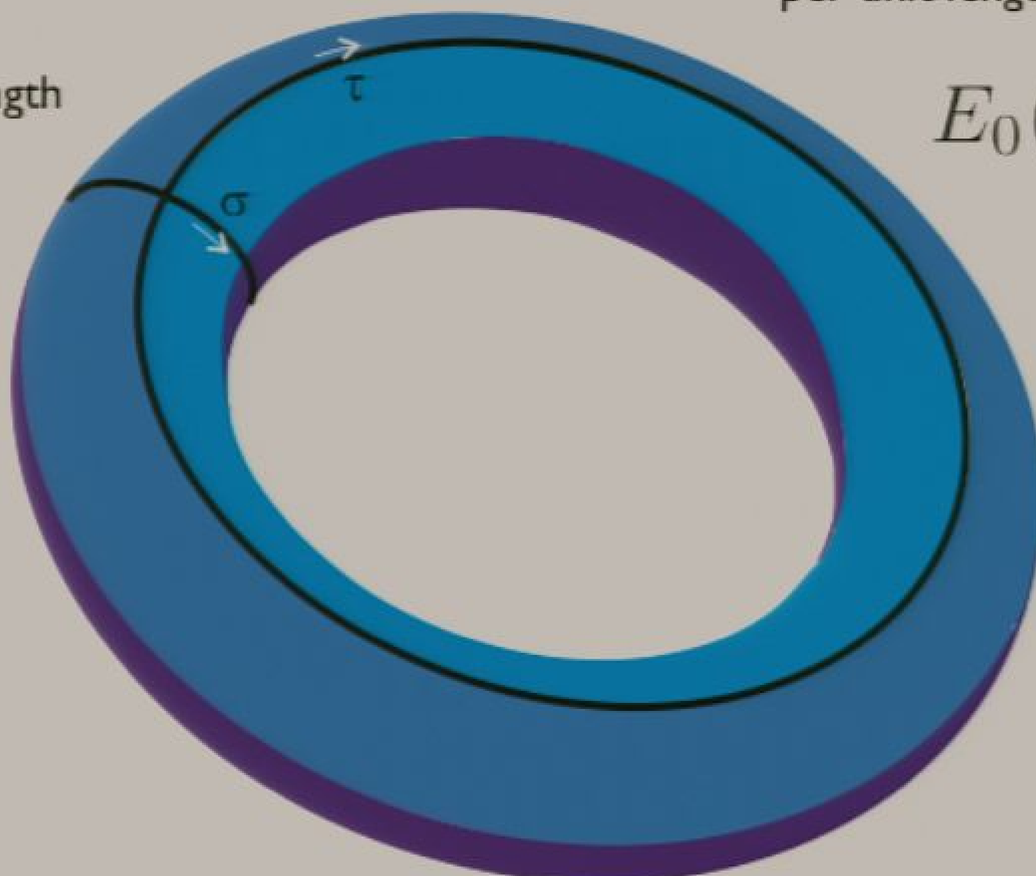
[...,Matsubara, Zamolodchikov
Dorey and Tateo,...]

TBA Wick rotation

Ground state energy at size L = free energy
per unit length at temperature 1/L

Asymptotic Bethe Ansatz equations in a circle of length L at zero temperature

$$Z = e^{-RE_0(L)}$$



$$E_0(L) = f(L)$$

$$Z = e^{-Rf(L}$$

Exact BAE at
temperature 1/L

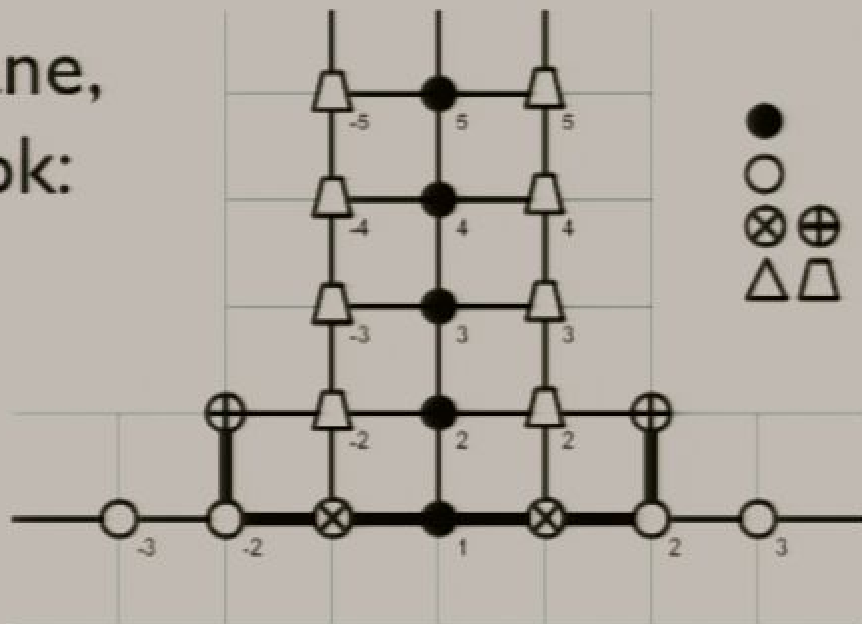
A priori this is valid for the ground state energy only.

2D QFT argument - contrary to everything so far this is very stringy!

The AdS/CFT Y-system, the full Planar Spectrum

[Gromov,Kazakov,PV]
[Gromov,Kazakov,Kozak, PV]

a,s plane, T-hook:



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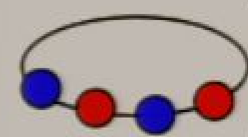
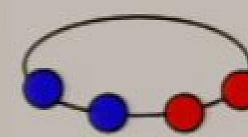
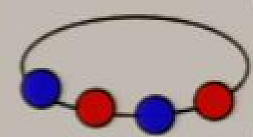
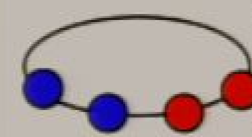
Quantum strings and the four loop Konishi operator

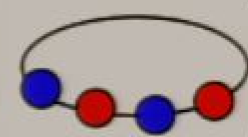
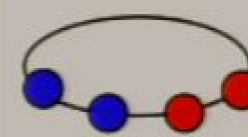
$$Konishi = \mathrm{tr}[Z,X]^2 = \Big|$$  $$\Big\rangle - \Big|$$  $$\Big\rangle$$

Quantum strings and the four loop Konishi operator

$Konishi = \mathrm{tr}[Z,X]^2 = \Big|$  $\Big\rangle$ - $\Big|$  $\Big\rangle$

$$\Delta_{Konishi} = 12g^2 - 48g^4 + 336g^6 + (-2584 + 384\zeta_3 - 1440\zeta_5)g^8$$

Quantum strings and the four loop Konishi operator

$Konishi = \mathrm{tr}[Z,X]^2 = \Big|$  $\Big\rangle$ - $\Big|$  $\Big\rangle$

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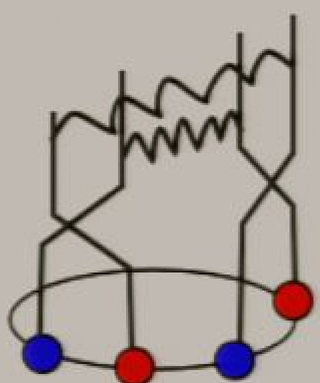


+ ....

Reproduces the YM 4 loop computation

involving 130000 Feynman diagrams!

(done in components)

[Bajnok,Janik]
[Gromov,Kazakov,PV]

[Fiamberti, Stambrogio, Sieg, Zanon]
[Velizhanin]

The Y-system incorporates both the Asymptotic Bethe equations and the leading Luscher corrections and generalizes the latter for any operator.

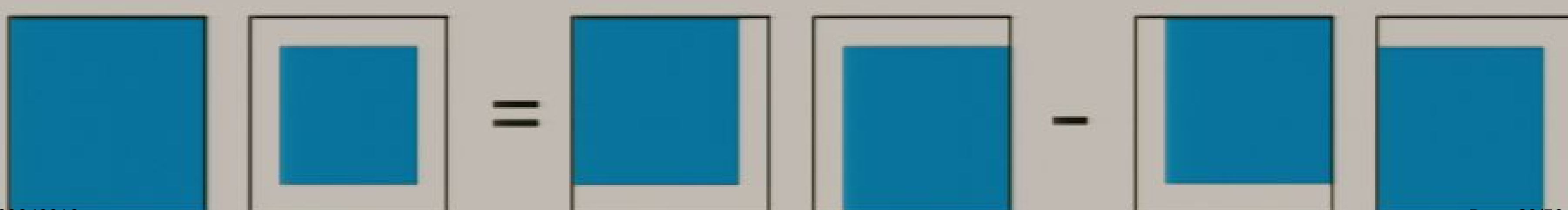
Hirota Dynamics

The Y-system is equivalent to the integrable Hirota equations: $T^+_{a,s}T^-_{a,s} = T_{a,s+1}T_{a+1,s} + T_{a-1,s}$

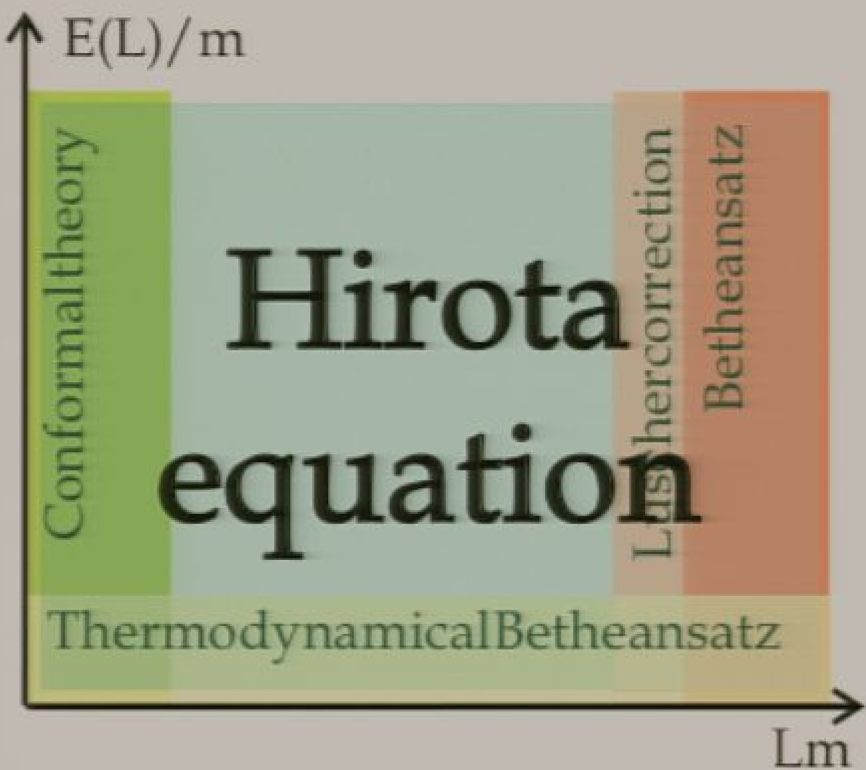
- Hirota can be written as the compatibility equation for a linear Lax auxiliary problem

$$L \cdot T = 0\ ,\ \ M \cdot T = 0$$

- Beautifully solved in terms of determinants exploiting the Jacobi relation:

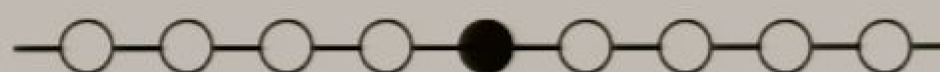


SU(2) PCF, an example



- On the example of the SU(2) PCF we were able to explore this ideas to reduce the Y-system equations

$$Y_n^+ Y_n^- = (1 + Y_{n+1})(1 + Y_{n-1})$$



to a **single** DdV like integral equation of trivial numerical iterative solution.

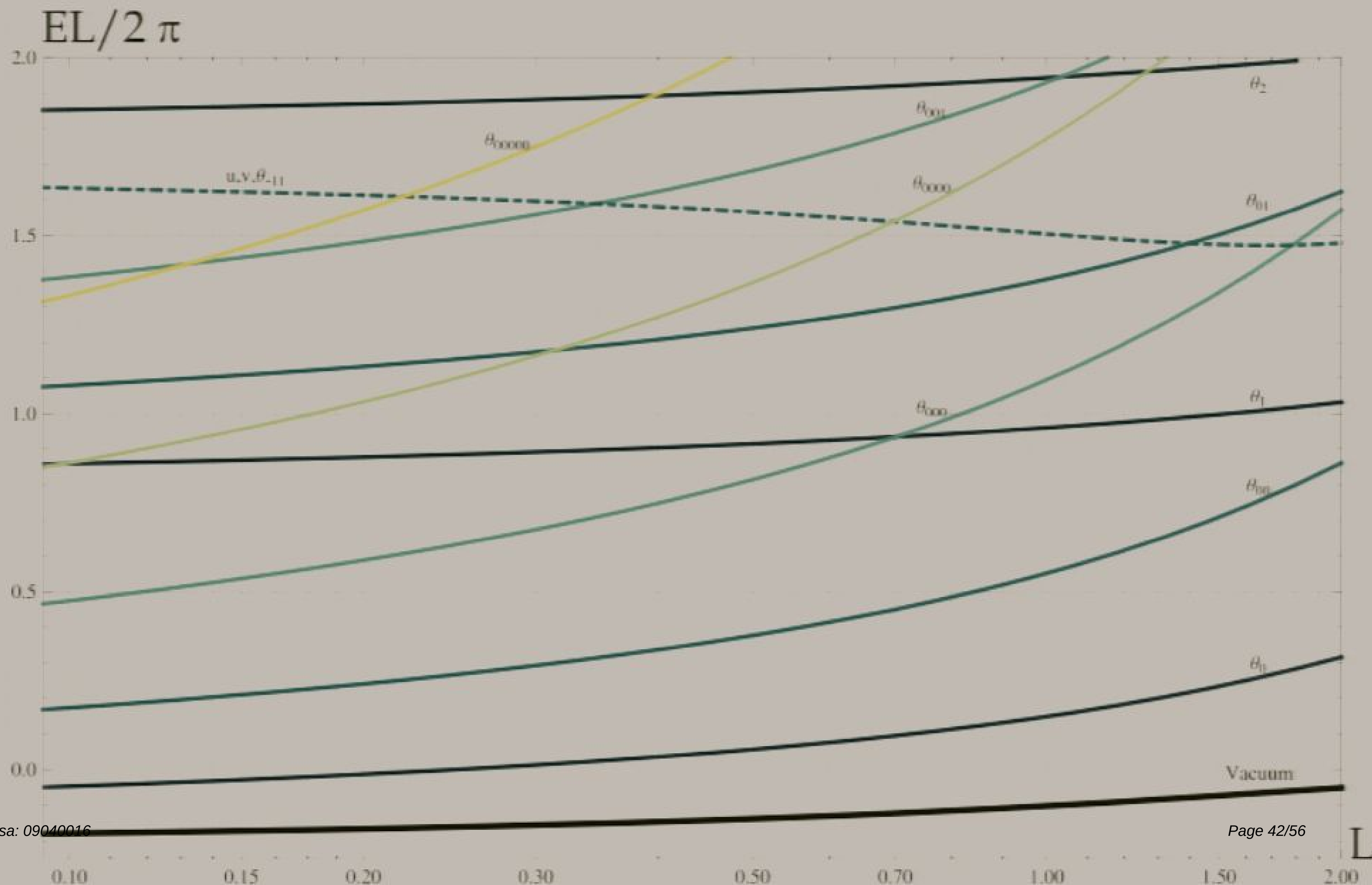
Simple numeric implementation

athematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help (70%) mar. 19:24

Numerics U(1) sector

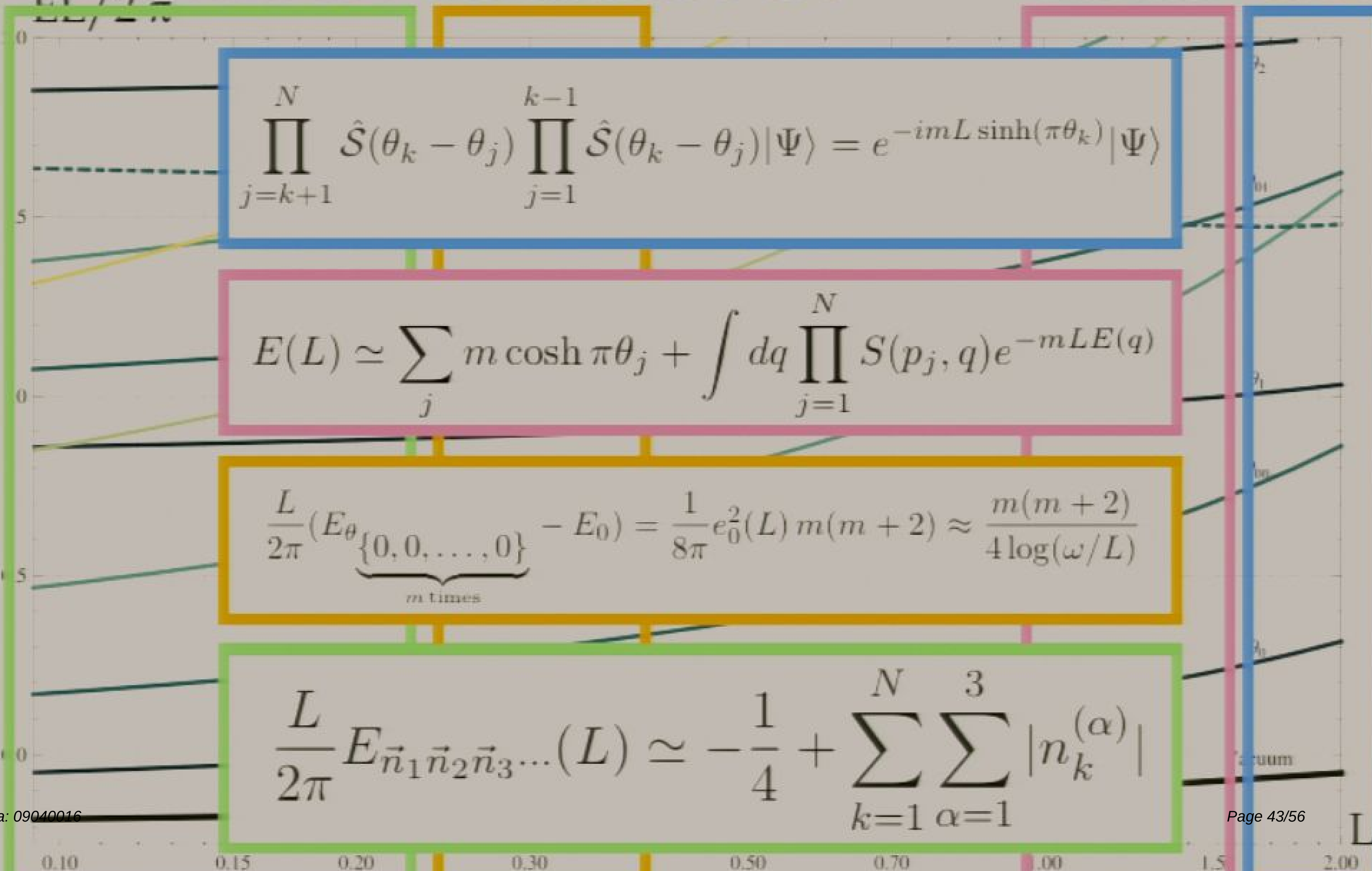
```
S0[x_] = I * Gamma[-(x / (2 I))] Gamma[1 / 2 + x / (2 I)] / (Gamma[x / (2 I)] Gamma[1 / 2 - x / (2 I)]);
K0[x_] = D[Log[S0[x] ^ 2], x] / (2 * Pi * I);

n = {0, 0, 0}; M = Length[n];
L = 1 / 2;

X = ArcCosh[8 Log[10] / L] / π;

eq[i_, v_] := L * Sinh[Pi * x[i]] + Sum[If[i == j, 0, Log[S0[x[i] - x[j]] ^ 2] / I], {j, M}] - 2 n[[i]] Pi + v[[i]];
BAE[v_] := Table[x[i], {i, M}] /. FindRoot[Table[Re[eq[j, v]], {j, M}], Table[{x[i], 2 i / M - 1 / 2}, {i, M}]];

F[S_] := FunctionInterpolation[S, {x, -X, X}, InterpolationPoints → 30];

r[k_, y_] := Log[(A[k][y] - 1) / (Abs[A[k][y]] - 1)];
rc[k_, y_] := Conjugate[r[k, y]];

θ[0] = BAE[Table[0, {j, M}]]

{-0.707446, -1.55606 × 10^-16, 0.707446}

A[0] = F[-Exp[-L * Cosh[Pi * x]] * Product[S0[x - θ[0][[j]] + I / 2] ^ 2, {j, M}]];

A[k_] := A[k] = F[(-Exp[(-L) * Cosh[Pi * x]]) * Product[S0[x - θ[k - 1][[j]] + I / 2] ^ 2, {j, M}] *
  Exp[NIntegrate[K0[x - y] * r[k - 1, y] - K0[x - y + I] * rc[k - 1, y] + 1, {y, -X, x, X},
      Method -> PrincipalValue] - 2 * X - rc[k - 1, x]]]
phase[k_][x_] := NIntegrate[2 * Im[K0[x - y - I / 2] * r[k - 1, y]] + 1, {y, -X, X}] - 2 * X;
θ[k_] := θ[k] = BAE[Table[phase[k][θ[k - 1][[j]]], {j, M}]];

En[k_] := Sum[Cosh[Pi * θ[k][[j]]], {j, M}] - NIntegrate[Re[r[k, y]] Cosh[Pi * y], {y, -X, X}];

Table[En[j], {j, 1, 8}]

{10.2414, 10.2425, 10.2425, 10.2424, 10.2424, 10.2424, 10.2424, 10.2424}
```

The exact spectrum of the SU(2) PCF



The exact spectrum of the SU(2) PCF

Free CFT **Perturbative CFT** **Luscher** **BAE**



$$\prod_{j=k+1}^{N} \hat{S}(\theta_k - \theta_j) \prod_{j=1}^{k-1} \hat{\mathcal{S}}(\theta_k - \theta_j) |\Psi\rangle = e^{-imL\sinh(\pi\theta_k)} |\Psi\rangle$$

$$E(L) \simeq \sum_j m\cosh\pi\theta_j + \int dq \prod_{j=1}^{N} S(p_j, q) e^{-mLE(q)}$$

$$\frac{L}{2\pi}(E_{\theta_{\underbrace{\{0,0,\ldots,0\}}_{m\text{ times}}}} - E_0) = \frac{1}{8\pi} e_0^2(L)\, m(m+2) \approx \frac{m(m+2)}{4\log(\omega/L)}$$

$$\frac{L}{2\pi} E_{\vec{n}_1\vec{n}_2\vec{n}_3\cdots}(L) \simeq -\frac{1}{4} + \sum_{k=1}^{N} \sum_{\alpha=1}^{3} |n_k^{(\alpha)}|$$

Next

- Do the same for AdS/CFT and reduce the Y-system to a single DdV integral equation. Not just a technical detail, it might contain important physics!
- Konishi: The **plot**, 5 loops, correction to the $\lambda^{1/4}$ behavior at strong coupling
- BKFL, the exact intercept, connection with DIS...
- Semi-classical string limit

Two miracles so far

- Higher charges exists and hence the quantum theory is integrable
- The Hirota equations governing their exact spectrum are integrable

Q_3 in string theory, Algebraic Curves

$\exists A(x)$ (8x8 matrix) which is flat on the e.o.m

[Bena, Polchinski, Roiban]

$$= \Omega(x) = P \exp \int A(g; x)$$

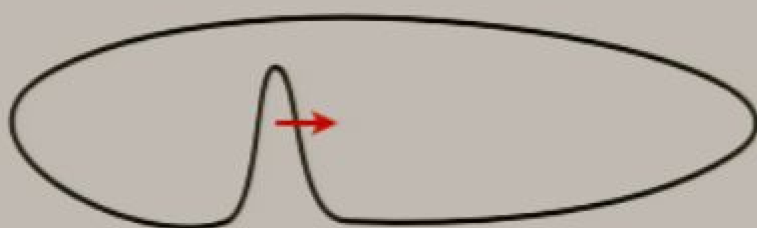
Eigenvalues are conserved... for any x!!

Classical Strings ⟷ Algebraic Curves

[Kazakov, Marshakov, Minahan, Zarembo; Beisert, Kazakov, Sakai, Zarembo]

Semi-classics

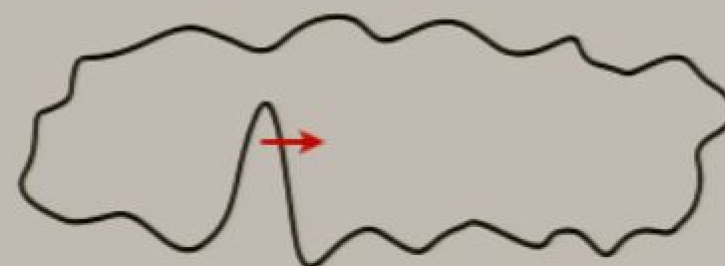
Classical motion



[Kazakov, Marshakov, Minahan, Zarembo]
[Beisert, Kazakov, Sakai, Zarembo]

Riemann Surface

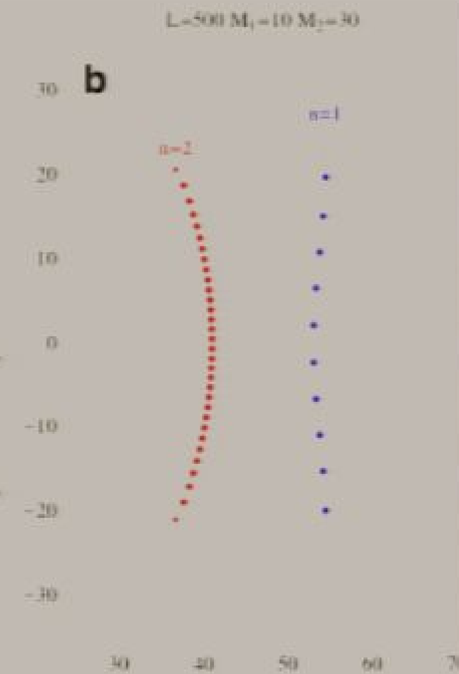
Perturbation

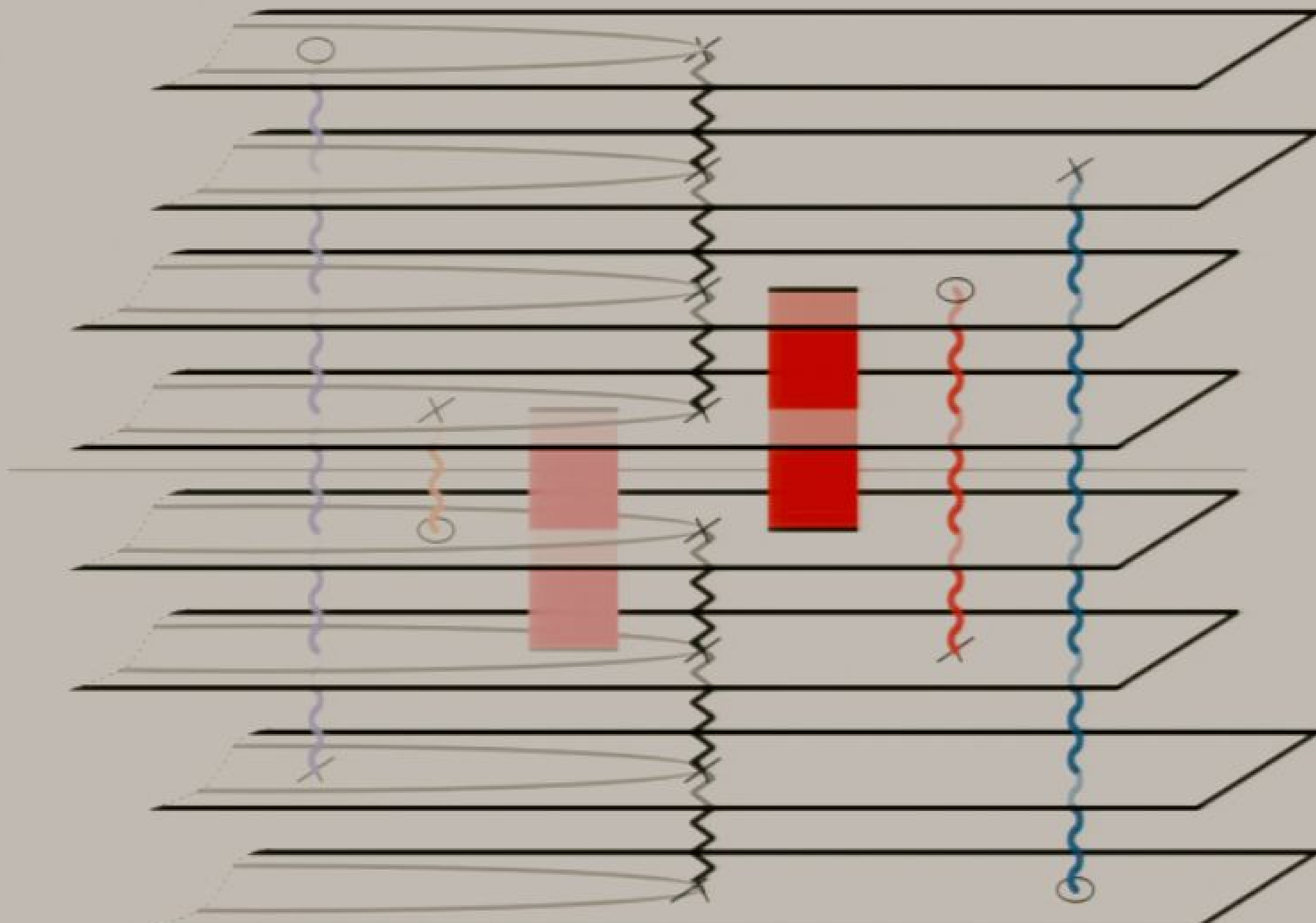


[Beisert, Freyhult] (YM side)
[Gromov, PV]

Extra poles in the Riemann Surface

Semiclassics





Cuts = classical condensation of Bethe roots. [KMMZ]

Quasi-classical Bethe equations can be derived!

[AFS]
[Gromov, PV]

Two weapons

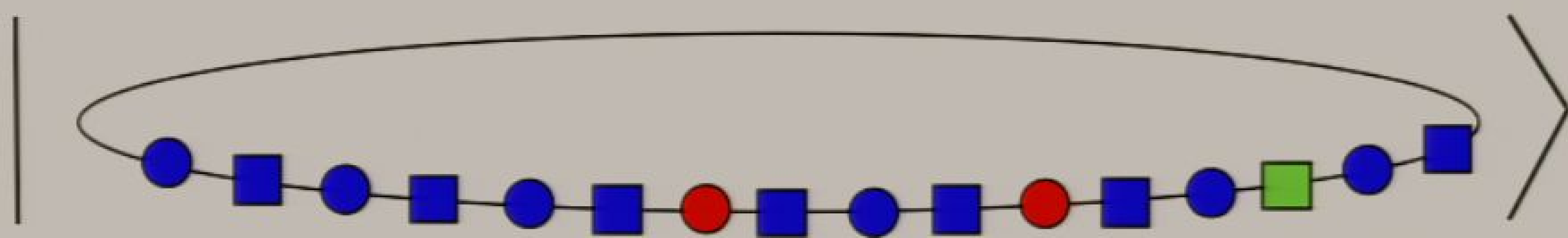
Thus, when finding a new integrable gauge/string duality we should pay special attention to

- Symmetry, in particular SU(2|2) extended is highly constraining
- Discretization of the algebraic semi-classical curves

indeed...

Spin chains in the ABJM theory

$$tr\left(Y^1 Y_1^\dagger Y^1 Y_1^\dagger Y^1 Y_1^\dagger Y^2 Y_1^\dagger Y^1 Y_1^\dagger Y^2 Y_1^\dagger Y^1 \bar{\Psi}_1 Y^1 Y_1^\dagger\right)$$



Integrable 2-loop spin chain Hamiltonian [Minahan, Zarembo]

Integrable string coset [Aryutunov, Frolov]

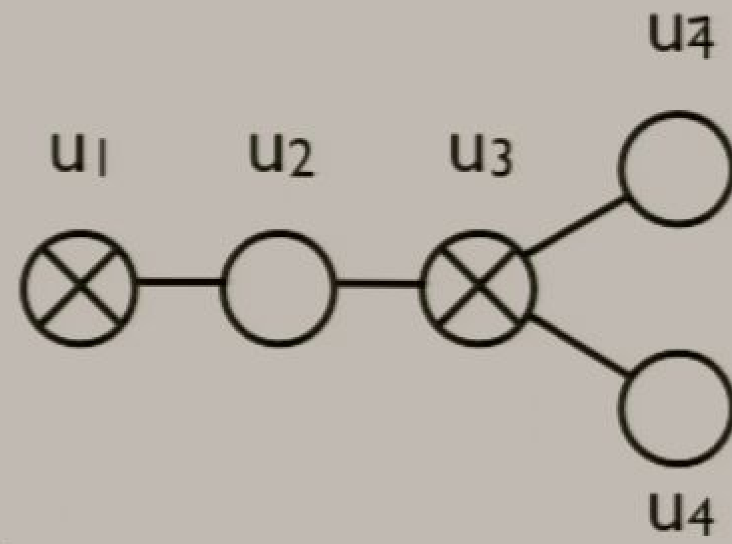
H → S(p,k) : OSp(2,2|6) → SU(2|2) extended [Gaiotto, Giombi, Yin]

String algebraic curves and semiclassics [Gromov,PV]

All loop asymptotic Bethe equations [Gromov,PV]

[Gromov,PV]

$$\prod_{k=1}^{K_2}\frac{u_{1,j}-u_{2,k}+\frac{i}{2}}{u_{1,j}-u_{2,k}-\frac{i}{2}}$$



$$1=\frac{\mathbf{u_1}-u_2+\frac{i}{2}}{\mathbf{u_1}-u_2-\frac{i}{2}}\,\frac{\mathbf{x_1}-\frac{1}{x_4^+}}{\mathbf{x_1}-\frac{1}{x_4^-}}\,\frac{\mathbf{x_1}-\frac{1}{x_{\bar 4}^+}}{\mathbf{x_1}-\frac{1}{x_{\bar 4}^-}}\,,$$

$$1=\frac{\mathbf{u_2}-u_2-i}{\mathbf{u_2}-u_2+i}\,\frac{\mathbf{u_2}-u_1+\frac{i}{2}}{\mathbf{u_2}-u_1-\frac{i}{2}}\,\frac{\mathbf{u_2}-u_3+\frac{i}{2}}{\mathbf{u_2}-u_3-\frac{i}{2}}\,,$$

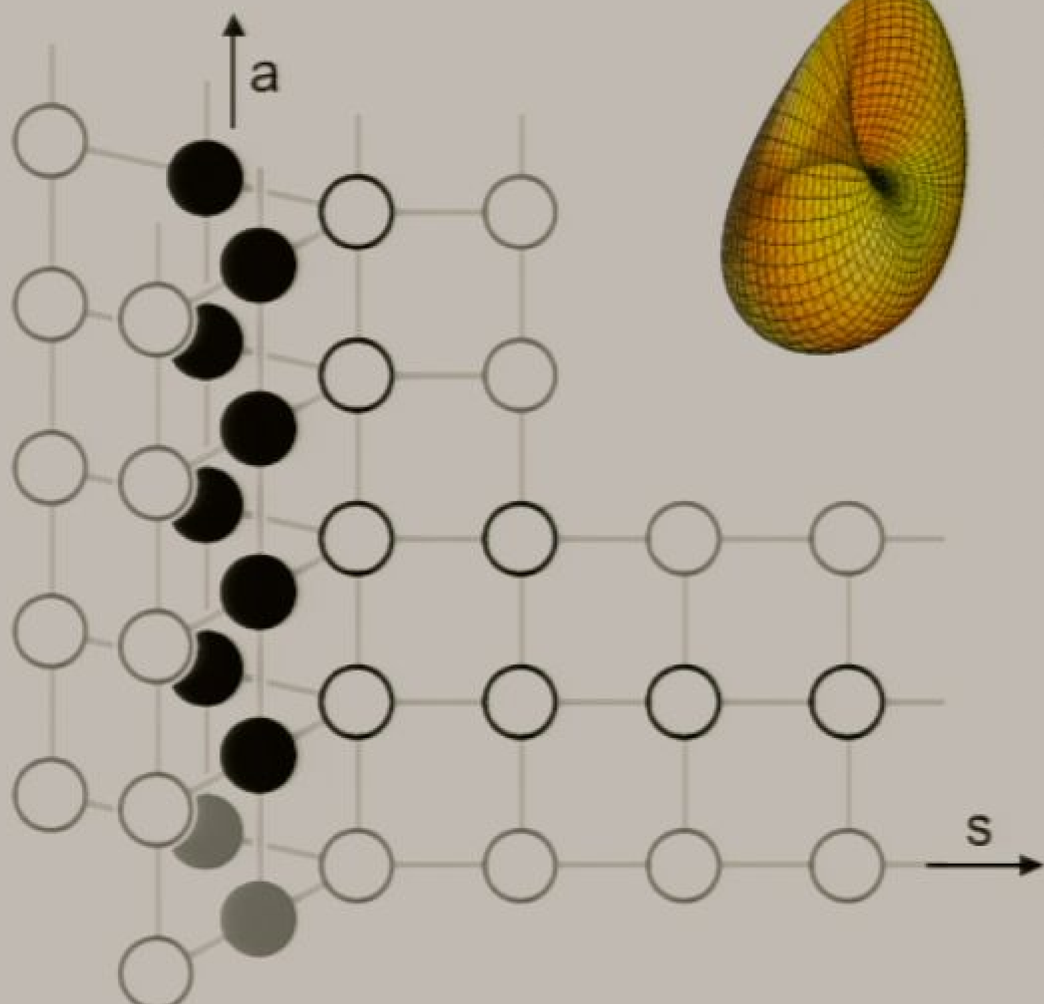
$$1=\frac{\mathbf{u_3}-u_2+\frac{i}{2}}{\mathbf{u_3}-u_2-\frac{i}{2}}\,\frac{\mathbf{x_3}-x_4^+}{\mathbf{x_3}-x_4^-}\,\frac{\mathbf{x_3}-x_{\bar 4}^+}{\mathbf{x_3}-x_{\bar 4}^-}$$

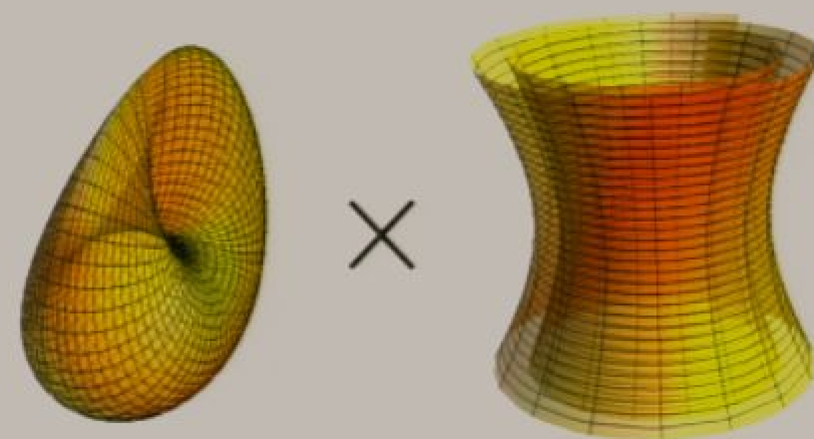
$$1=\left(\frac{\mathbf{x_4^-}}{\mathbf{x_4^+}}\right)^L\frac{\mathbf{u_4}-u_4+i}{\mathbf{u_4}-u_4-i}\,\frac{x_1-\frac{1}{\mathbf{x_4^-}}}{x_1-\frac{1}{\mathbf{x_4^+}}}\,\frac{\mathbf{x_4^-}-x_3}{\mathbf{x_4^+}-x_3}\,\sigma_{\rm BES}(\mathbf{u_4},u_4)\,\sigma_{\rm BES}(\mathbf{u_4},u_{\bar 4})\,,$$

$$1=\left(\frac{\mathbf{x_{\bar 4}^-}}{\mathbf{x_{\bar 4}^+}}\right)^L\frac{\mathbf{u_{\bar 4}}-u_{\bar 4}+i}{\mathbf{u_{\bar 4}}-u_{\bar 4}-i}\,\frac{x_1-\frac{1}{\mathbf{x_{\bar 4}^-}}}{x_1-\frac{1}{\mathbf{x_{\bar 4}^+}}}\,\frac{\mathbf{x_{\bar 4}^-}-x_3}{\mathbf{x_{\bar 4}^+}-x_3}\,\sigma_{\rm BES}(\mathbf{u_{\bar 4}},u_{\bar 4})\,\sigma_{\rm BES}(\mathbf{u_{\bar 4}},u_4)\,,$$

[Gromov, Kazakov, PV]

Full spectrum of planar ABJM



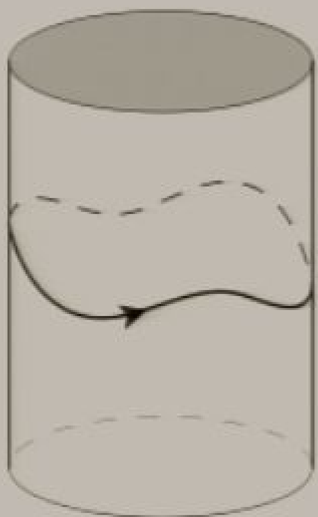


- The known examples of integrable gauge theories are

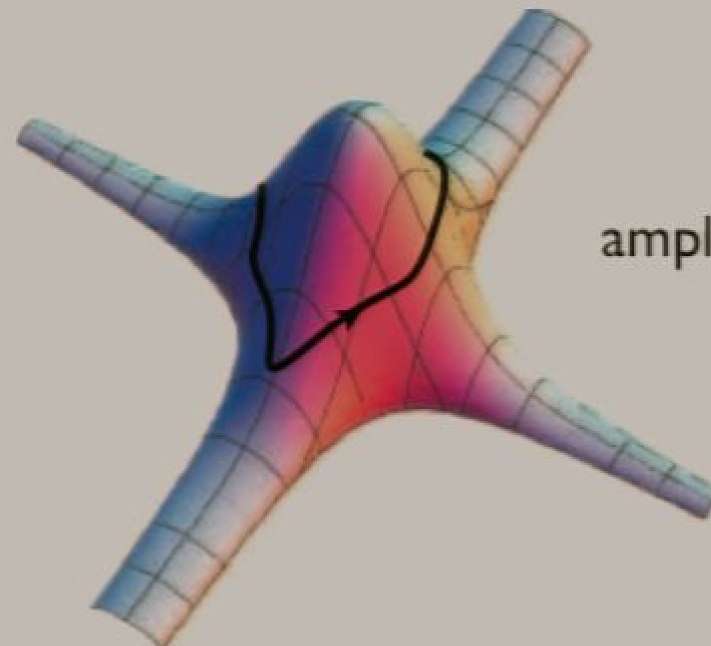
super-conformal gauge theories with gravity duals

- What is the landscape of integrable theories?
- Can we find new gauge/gravity dualities based on the integrable structures?
- How close to QCD can we get?
- What about going beyond the spectrum?

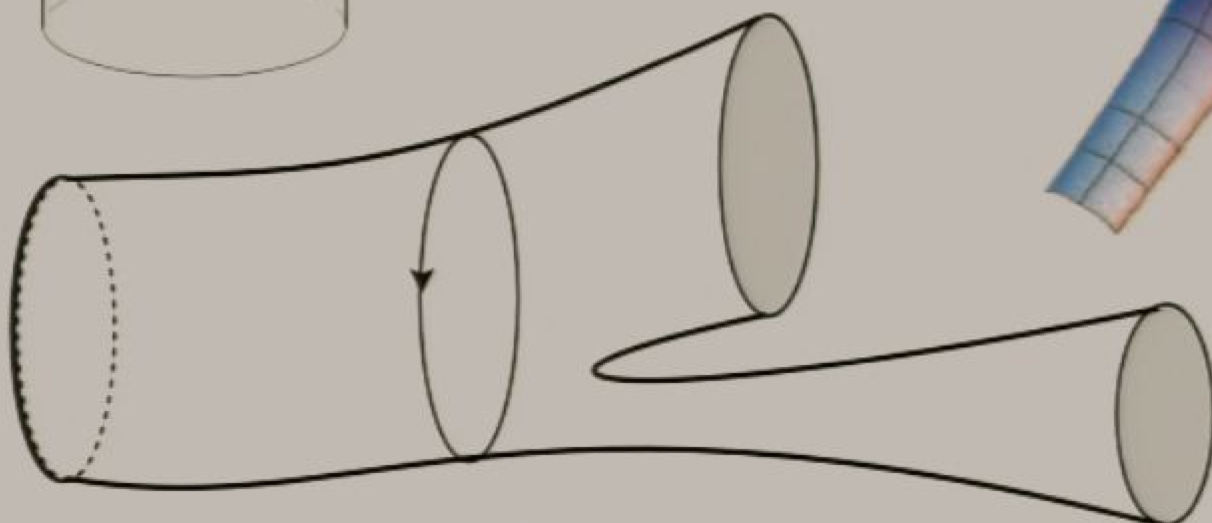
From the string 2D QFT point of view it seems reasonable...



spectrum



amplitudes



3 pt functions

Different topologies, same integrable structure?

see e.g. [Beisert, Ricci, Tseytlin, Wolf], [Berkovits, Maldacena] [Drummond, Henn, Plefka] for very interesting first steps

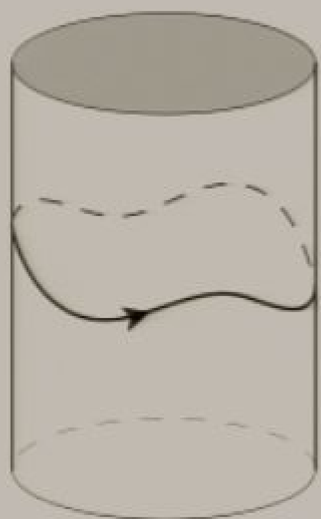
Conclusions and future

- Integrability can provide us with a very elegant description of the full planar spectrum of some gauge theories and their corresponding gravity dual.
- The study of the full planar spectrums needs to be further simplified and a nice plot of $\Delta(\lambda)$ for the Konishi operator would be most welcome

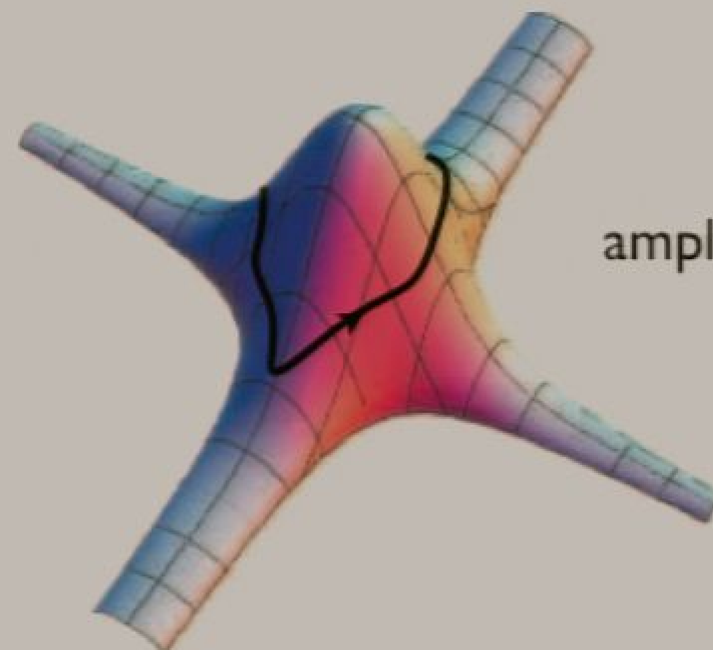
[Gromov, Kazakov, PV] work in progress

- A thorough classification of integrable theories is still missing and might teach us a lot about realistic gauge theories and the corresponding gravity duals
- Integrability techniques ought to be applied beyond the spectrum.

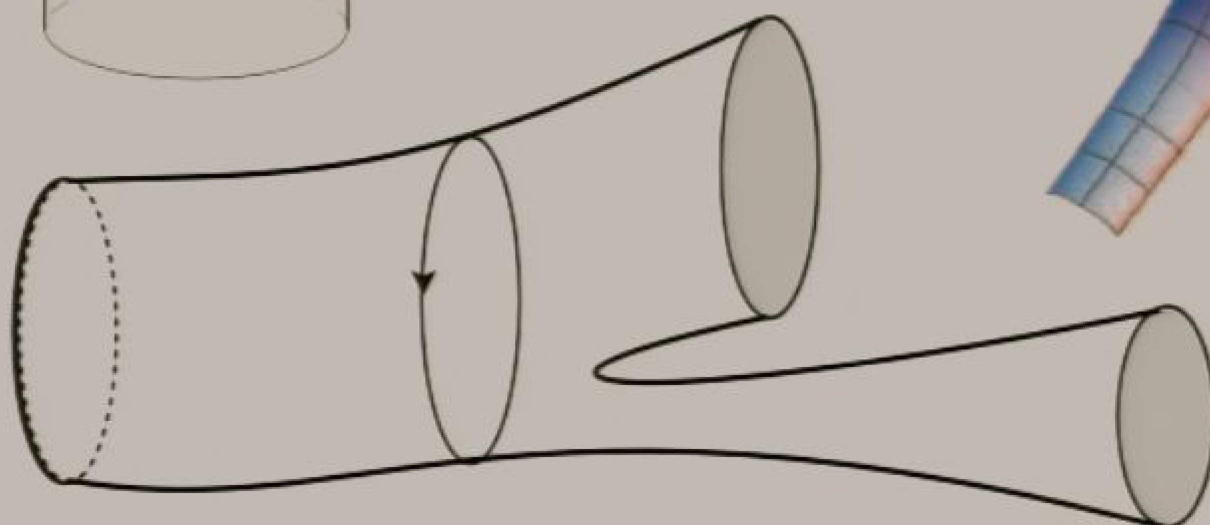
From the string 2D QFT point of view it seems reasonable...



spectrum



amplitudes



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