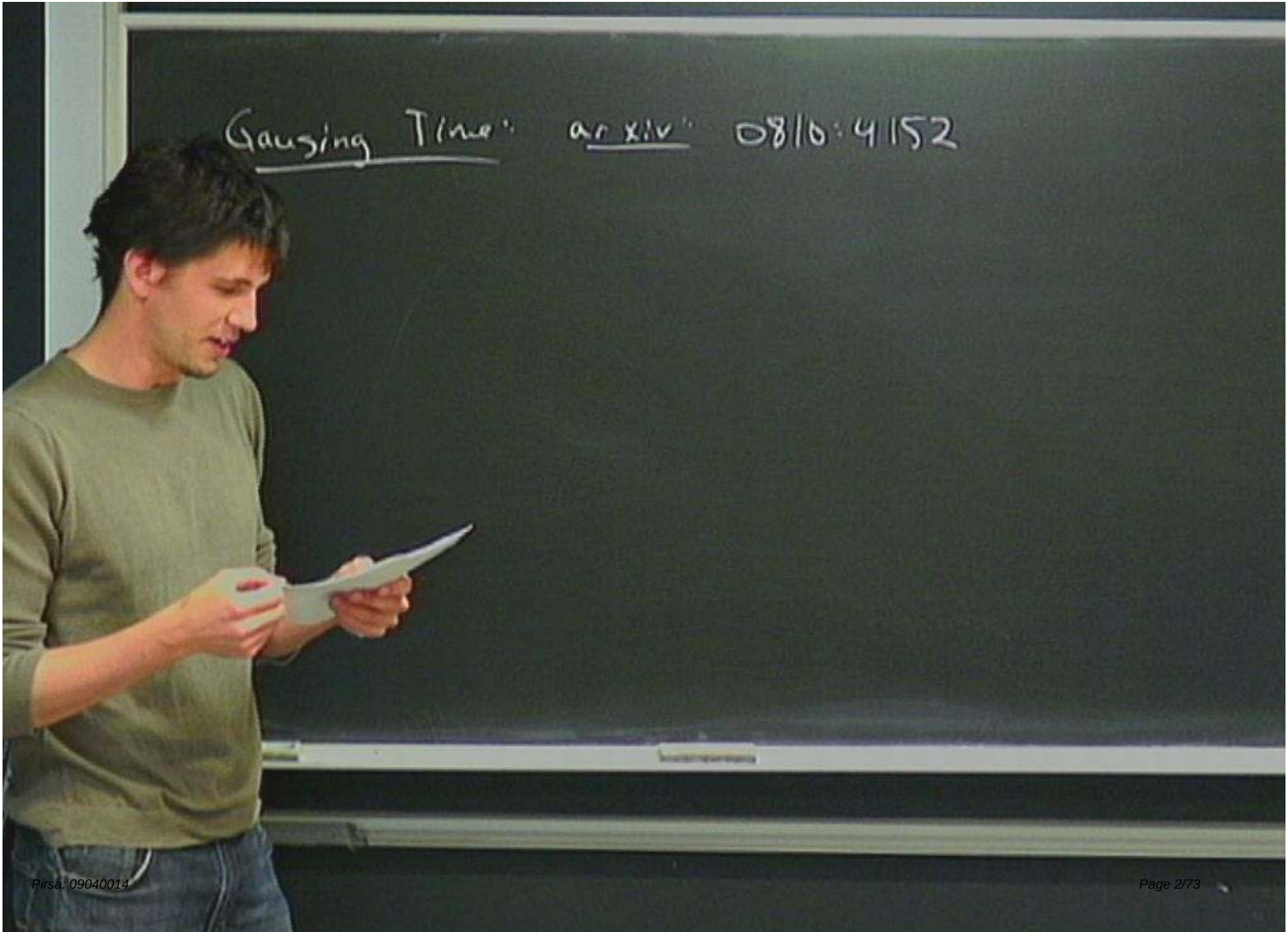
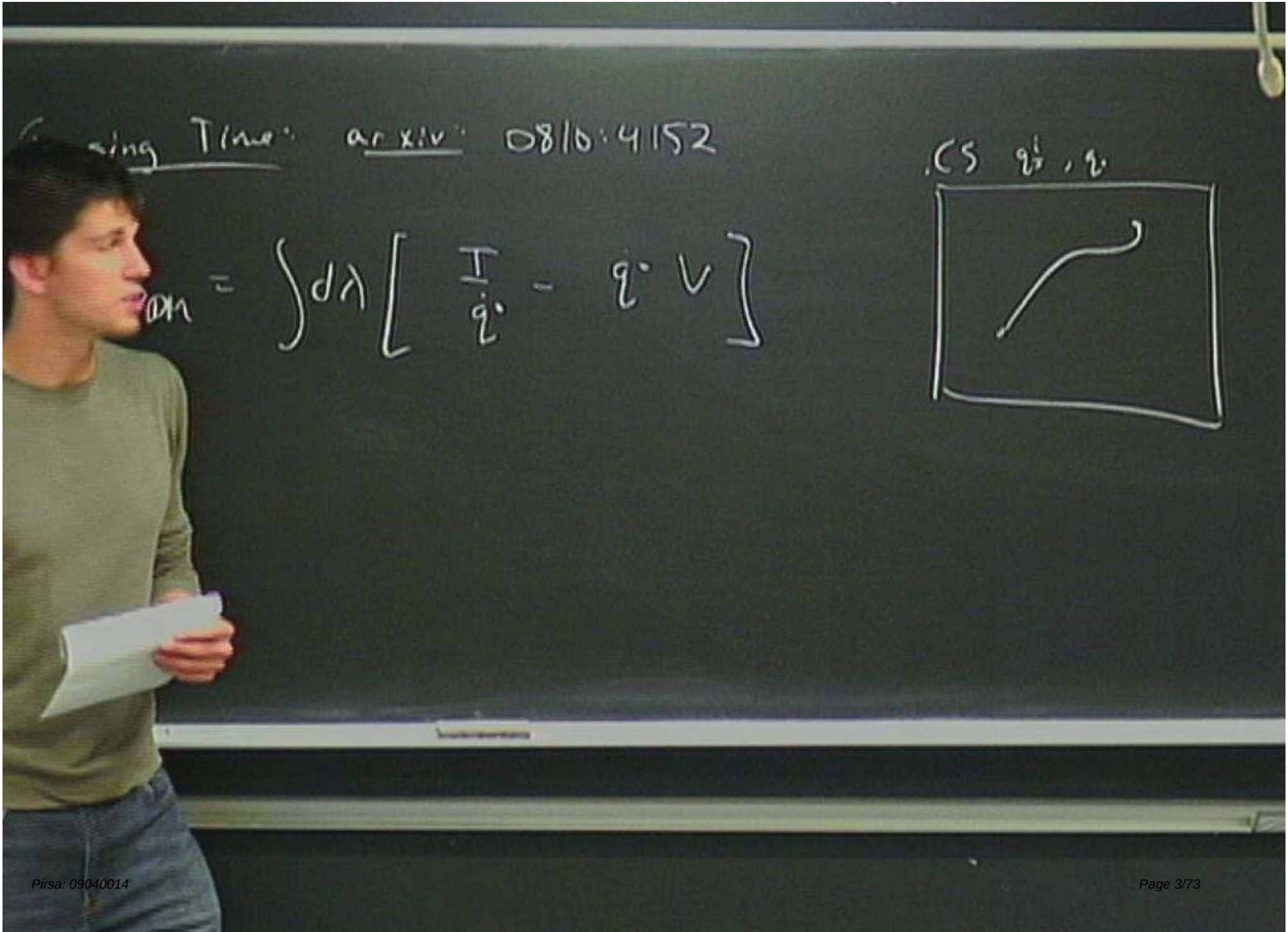
Title: Mini-Course on Mach's Principle - Lecture 7

Date: Apr 01, 2009 10:30 AM

URL: http://www.pirsa.org/09040014

Abstract:

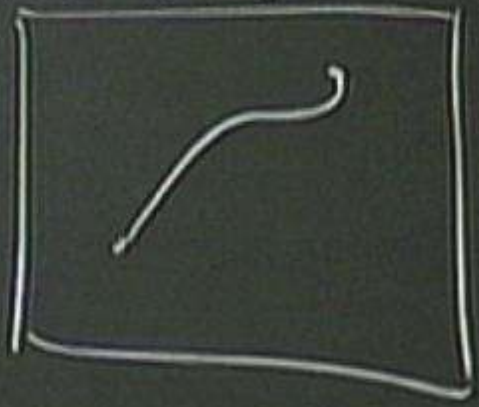

Gausing Time: arXiv: 0810.4152

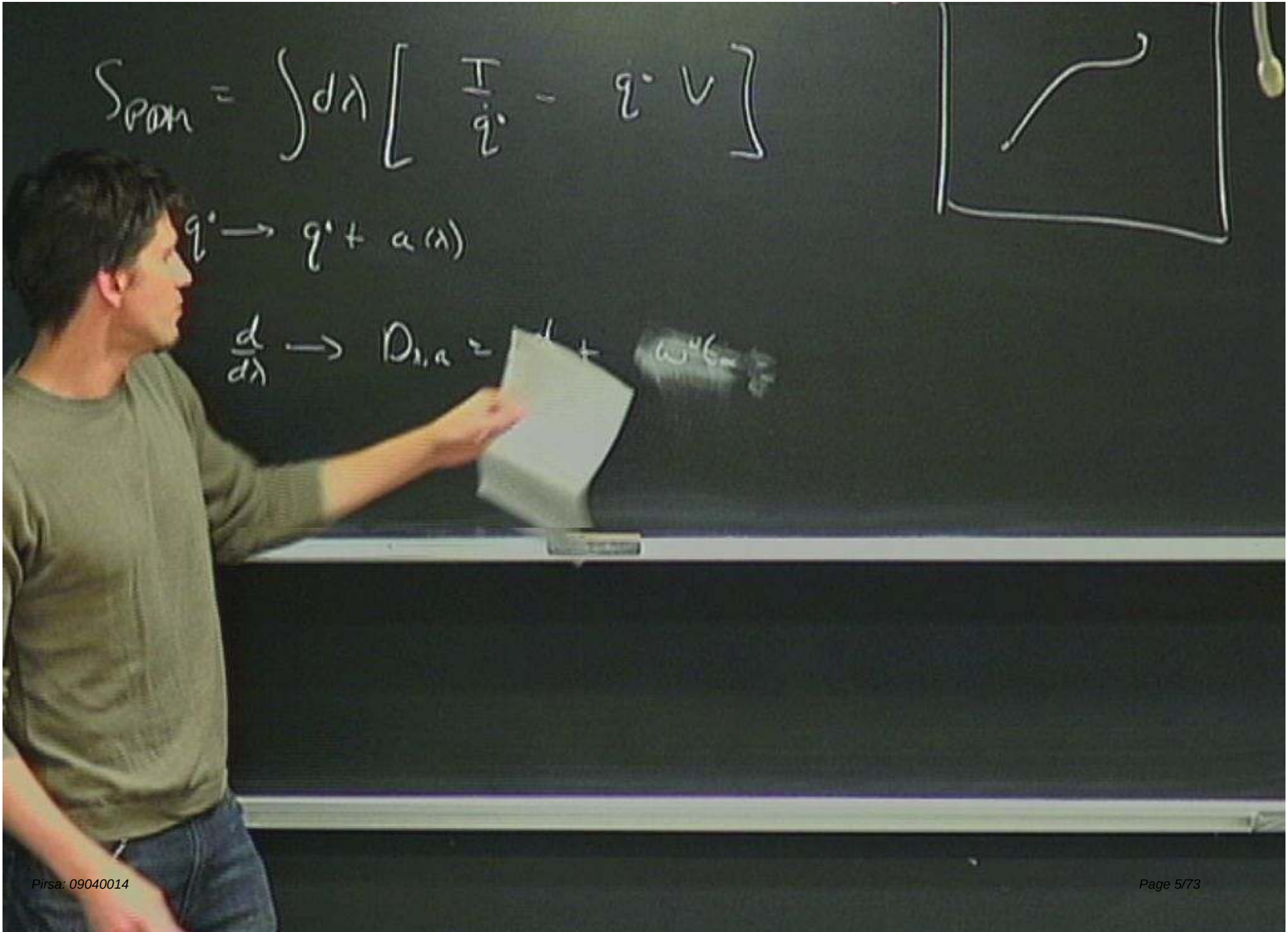

ing Time: arXiv: 0810.4152
$= \int d\lambda \left[\frac{I}{\dot{q}} - q \cdot V \right]$
CS q_i, q_i

Gauging Time: arXiv: 0810.4152

$$S_{POM} = \int d\lambda \left[\frac{I}{\dot{q}} - q \cdot V \right]$$

CS q^i, q^i



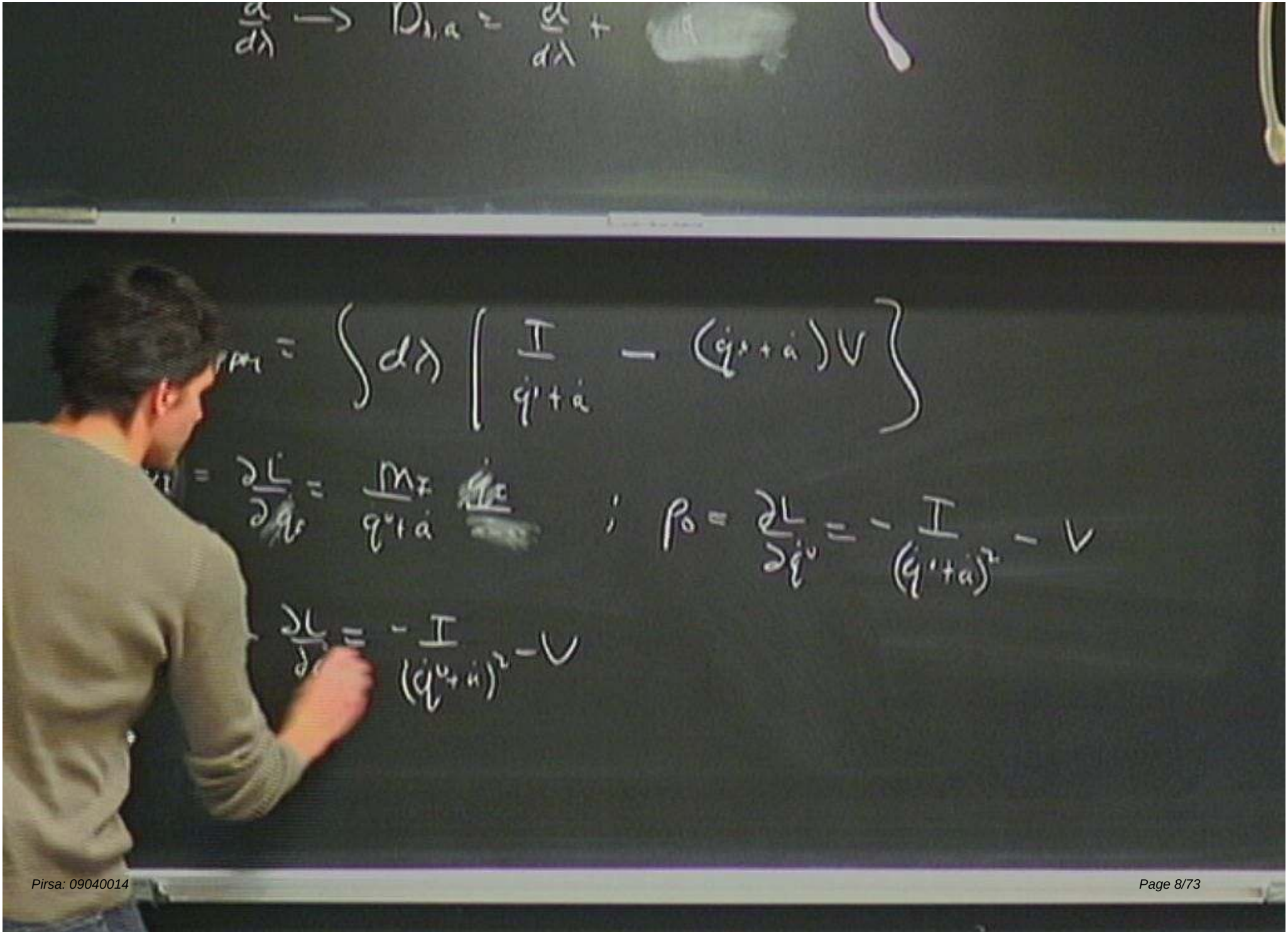

$S_{pom} = \int d\lambda \left[\frac{I}{\dot{q}} - q \cdot V \right]$
$q \rightarrow q + a(\lambda)$
$\frac{d}{d\lambda} \rightarrow D_{\lambda,a}$

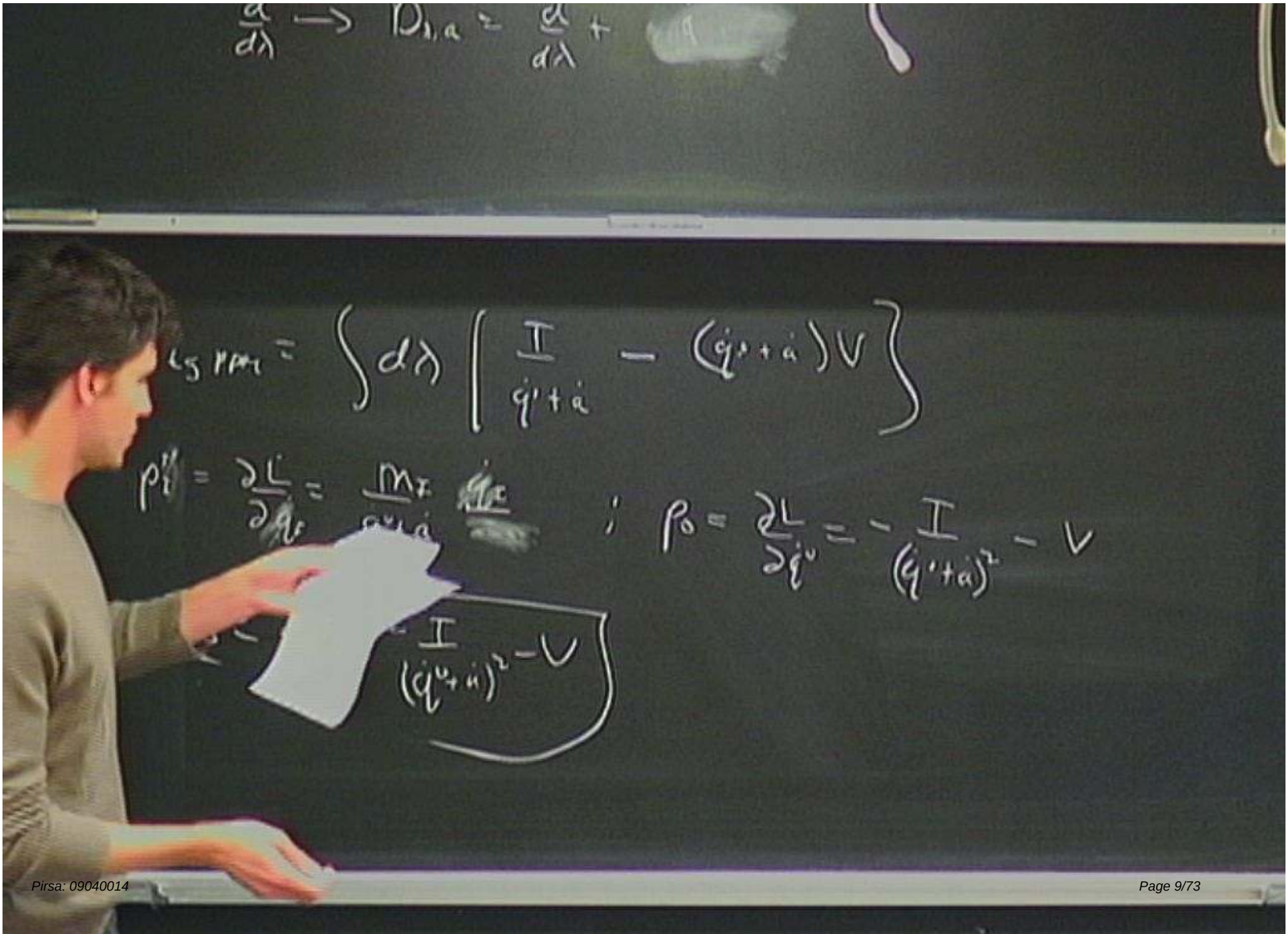
$\frac{d}{d\lambda} \longrightarrow D_{1,a} = \frac{d}{d\lambda} +$

$$S_{LSPM} = \int d\lambda \left\{ \frac{I}{\dot{q}^1 + \dot{a}} - (\dot{q}^1 + \dot{a}) V \right\}$$

$$\frac{d}{d\lambda} \longrightarrow D_{1,a} = \frac{d}{d\lambda} +$$

$$S_{LSPM} = \int d\lambda \left\{ \frac{I}{\dot{q}^1 + \dot{a}} - (\dot{q}^1 + \dot{a}) V \right\}$$


$\frac{d}{d\lambda} \longrightarrow D_{\lambda,a} = \frac{d}{d\lambda} +$
$= \int d\lambda \left\{ \frac{I}{\dot{q}' + a} - (\dot{q}' + a) V \right\}$
$= \frac{\partial L}{\partial \dot{q}} = \frac{m \dot{x}}{\dot{q}' + a}$; $p_0 = \frac{\partial L}{\partial \dot{q}^0} = - \frac{I}{(\dot{q}' + a)^2} - V$
$\frac{\partial L}{\partial \dot{x}} = - \frac{I}{(\dot{q}^0 + a)^2} - V$


$\frac{d}{d\lambda} \longrightarrow D_{\lambda,a} = \frac{d}{d\lambda} +$
$L_S = \int d\lambda \left\{ \frac{I}{\dot{q}' + \dot{a}} - (\dot{q}' + \dot{a}) V \right\}$
$p'' = \frac{\partial L}{\partial \dot{q}} = \frac{m \dot{q}}{\sqrt{\dot{q}^2}}$; $p_0 = \frac{\partial L}{\partial \dot{q}^0} = -\frac{I}{(\dot{q}' + \dot{a})^2} - V$
$\frac{I}{(\dot{q}^0 + \dot{a})^2} - V$

$$\frac{d}{d\lambda} \longrightarrow D_{\lambda,a} = \frac{d}{d\lambda} +$$

$$S_{Lagrange} = \int d\lambda \left\{ \frac{I}{q' + a} - (q' + a) V \right\}$$

$$p'_i = \frac{\partial L}{\partial q'_i} = \frac{m_i \dot{q}_i}{q' + a} \quad ; \quad p_0 = \frac{\partial L}{\partial \dot{q}^0} = -\frac{I}{(q' + a)^2} - V$$

$$\boxed{\pi_0 = \frac{\partial L}{\partial a} = -\frac{I}{(\dot{q}^0 + \dot{a})^2} - V}$$

$$\boxed{L_0 = \pi_0 - p_0 = 0} \quad \left(L_\alpha = \pi_\alpha - \sum_\tau p_\tau^\dagger t_\alpha q_\alpha \right)$$

$$\boxed{\pi_0 = 0} \quad (\text{Mach, Frepu.})$$

$L_0 = \Pi_0 - p_0 = 0$ $\quad (L_\alpha = \pi_\alpha - \sum_\tau \beta^\tau t_{\alpha\tau} q_\tau)$

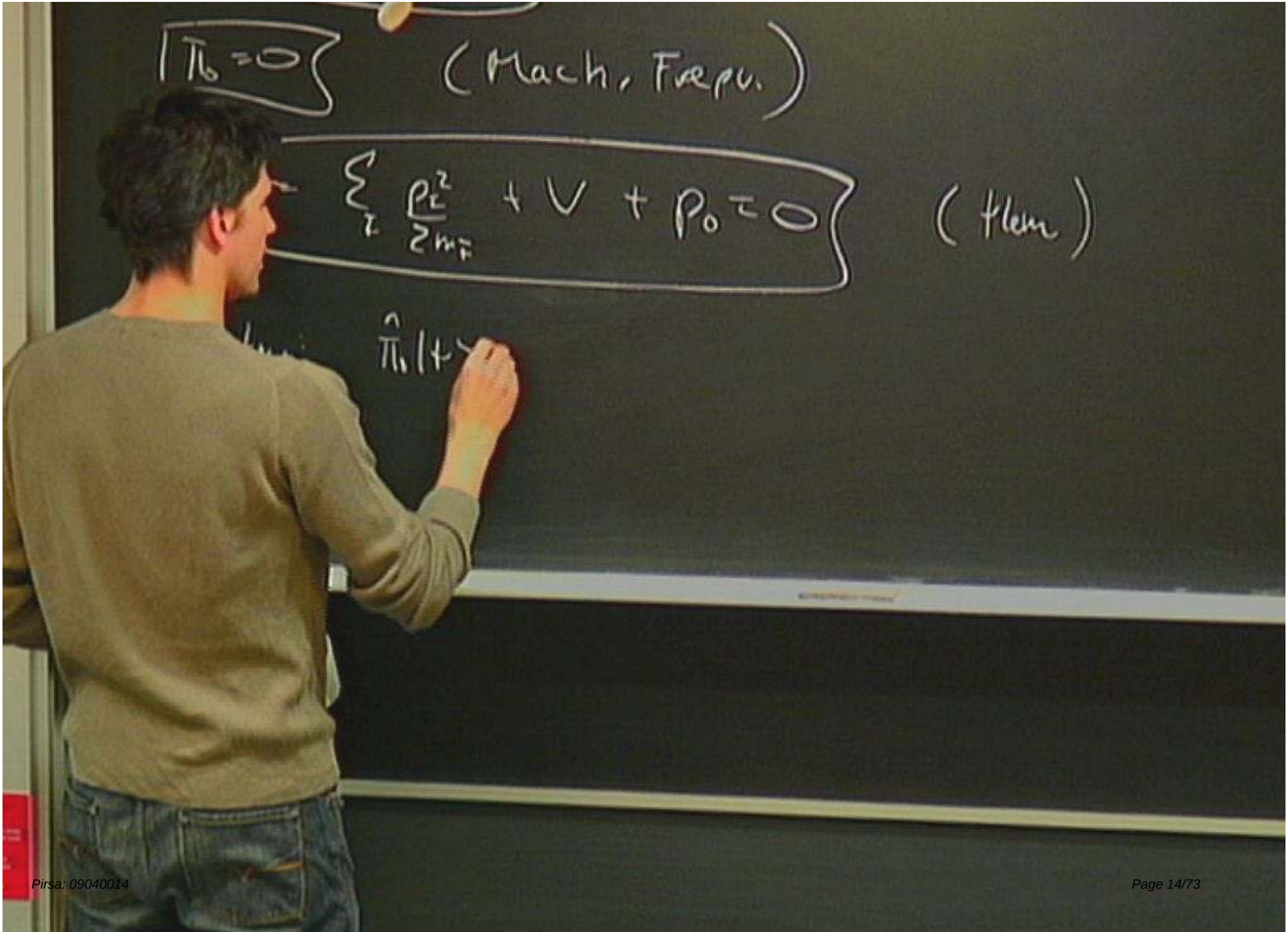
$\Pi_0 = 0$ $\quad$ (Mach, Freq.)

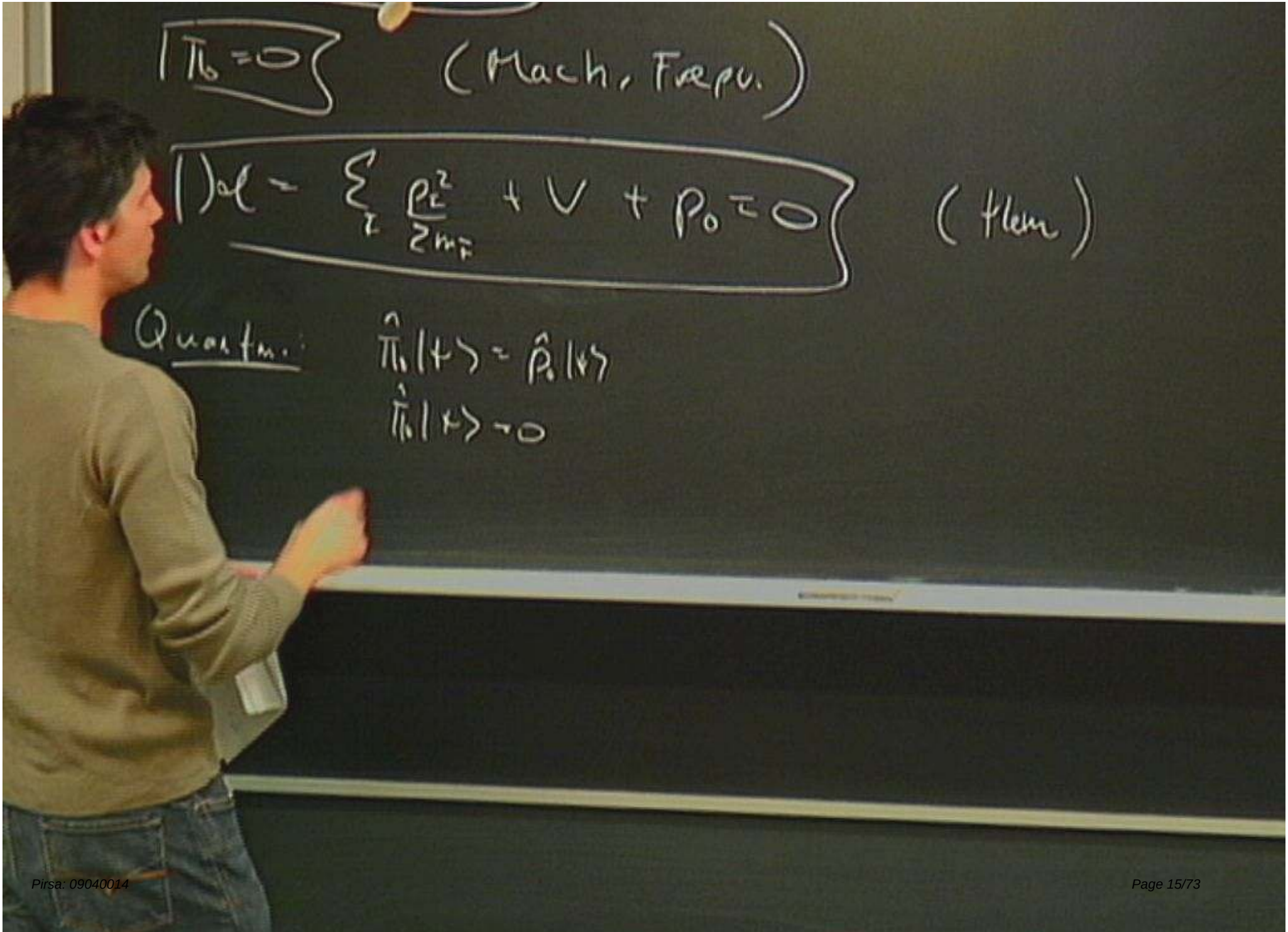
$\alpha = \sum_i \frac{p_i^2}{2m_i}$

$L_0 = \pi_0 - p_0 = 0$ $\left(L_\alpha = \pi_\alpha - \sum_\tau \beta^\tau t_{\alpha\tau} q_\tau \right)$

$\pi_0 = 0$ (Mach, Freq.)

$\mathcal{H} - \sum_i \frac{p_i^2}{2m_i} + V + p_0 = 0$ (Ham)


$\Pi_0 = 0$
(Mach, Freq.)
$\sum_i \frac{p_i^2}{2m_i} + V + p_0 = 0$
(Ham)


$\pi_0 = 0$ (Mach, Fraps.)
$H = \sum_i \frac{p_i^2}{2m_i} + V + p_0 = 0$ (Klein)
Quantum: $\hat{\pi}_0 |\psi\rangle = \hat{p}_0 |\psi\rangle$
$\hat{\pi}_0 |\psi\rangle = 0$

$\pi_0 = 0$ (Mach. Fæpu.)

$H = \sum_i \frac{p_i^2}{2m_i} + V + p_0 = 0$ (Hem)

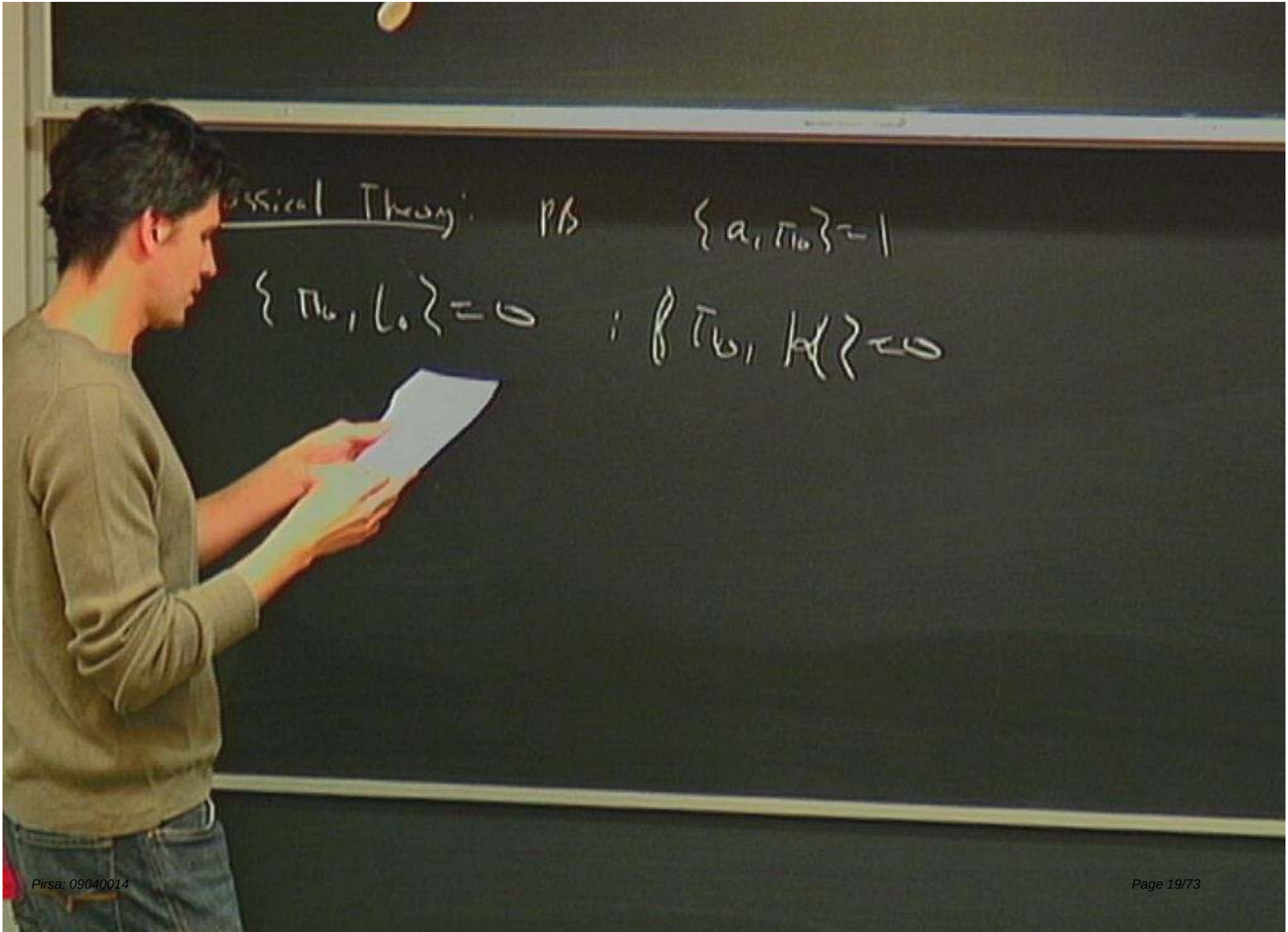
Quantm.: $\hat{\pi}_0 |+\rangle = \hat{p}_0 |+\rangle$

$\hat{\pi}_0 |+\rangle = 0$

Classical Theory: PB $\{a, \Pi_b\} = 1$

Classical Theory: PB $\{a, \Pi_a\} = 1$

$\{\Pi_a, L_a\} = 0$


ssical Theory: PB
{a, π_b} = 1
{π_b, L_b} = 0
: β {π_b, H} = 0

Classical Theory: PB $\{a, \Pi_0\} = 1$

$\{\Pi_0, L_0\} = 0$: β $\{\Pi_0, H\} = 0$ $\{\phi_0, H\} = 0$

Classical Theory: PB $\{a, \pi_0\} = 1$

$\{\pi_0, L_0\} = 0$; $\{\pi_0, H_c\} = 0$ $\{\phi_0, H_c\} = 0$

$H_c = \sum_i P_i^T \dot{q}_i + a\pi_0 + \dot{q}_0 P_0 - L$

Classical Theory: PB $\{a, \pi_b\} = 1$

$\{\pi_b, L_b\} = 0$; $\{\pi_b, H_c\} = 0$ $\{\pi_b, H\} = 0$

$H_c = \sum_c P_i^T \dot{q}_i + a\pi_b + \dot{q}_0 P_0 - L = (\dots) = 0$

Classical Theory: PB $\{a, \pi_0\} = 1$

$\{\pi_0, L_0\} = 0$; $\{\pi_0, H\} = 0$ $\{p_0, H\} = 0$

$H_c = \sum_i p_i^T \dot{q}_i + \dot{a}\pi_0 + \dot{q}_0 p_0 - L = (\ldots) = 0$

q_i, p_i

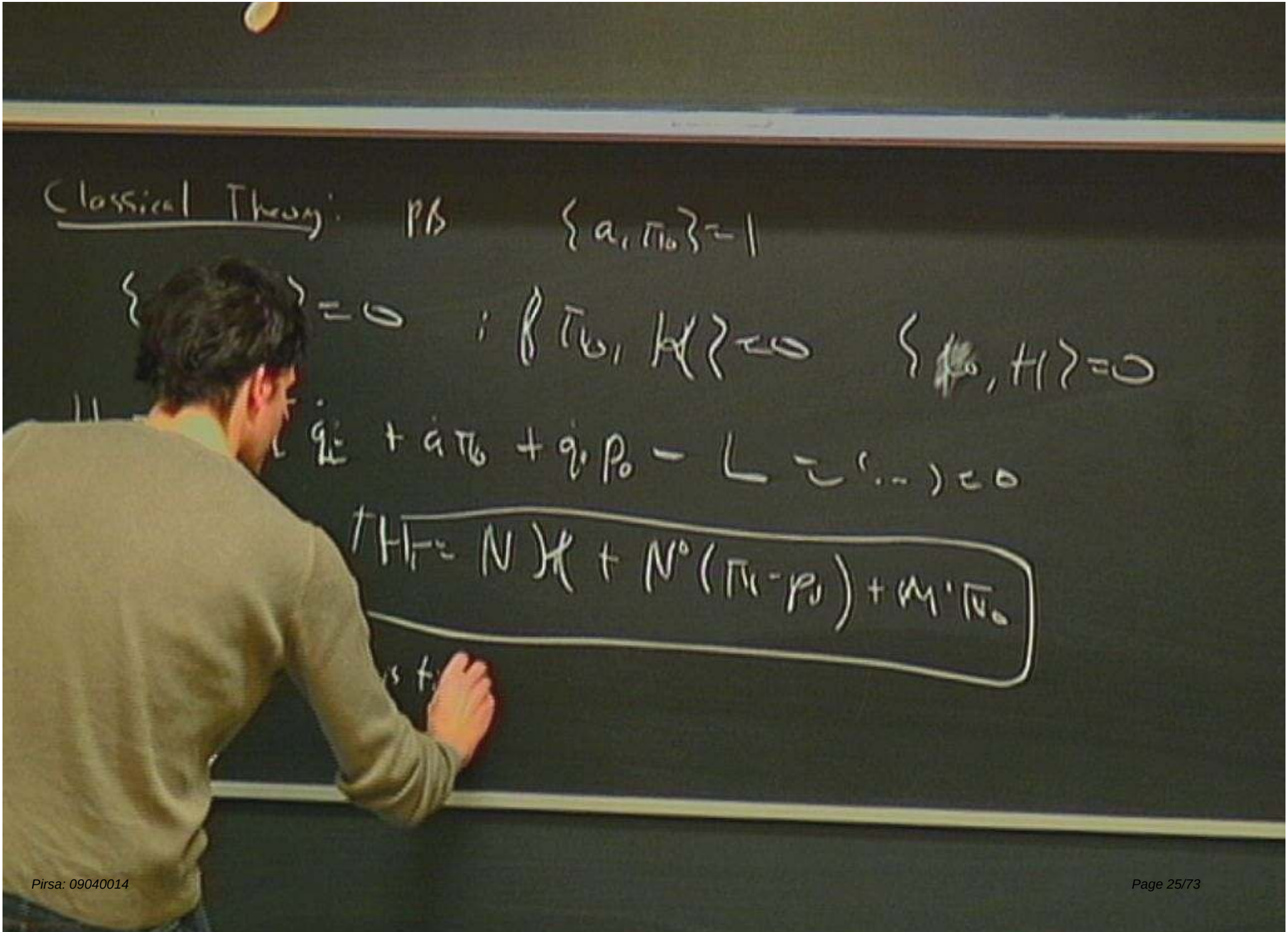
Classical Theory: PB $\{a, \pi_0\} = 1$

$\{\pi_0, L_0\} = 0$; $\{\pi_0, \mathcal{H}\} = 0$ $\{L_0, H\} = 0$

$H_c = \sum_i p_i^T \dot{q}_i + \dot{a}\pi_0 + \dot{q}_0 p_0 - L = (\ldots) = 0$

$\dot{q}_i \quad \dot{p}_i$

$$\boxed{H_T = N\mathcal{H} + N^0(\pi_i - p_0) + M^i \pi_0}$$


Classical Theory: PB
{a, π0} = 1
= 0 ; {π0, H} = 0
{p0, H} = 0
+ ȧ π0 + q̇0 p0 − L = (...) = 0
H = N H + N^0 (π1 − p0) + M^1 π0

Classical Theory: PB $\{a, \pi_0\} = 1$

$\{\pi_0, L_0\} = 0$; $\{\pi_0, \mathcal{H}\} = 0$ $\{L_0, H\} = 0$

$H_c = \sum_i p_i^T \dot{q}_i + \dot{a}\pi_0 + \dot{q}_0 p_0 - L = (\ldots) = 0$

$\dot{q}_i, \dot{p}_i$ $\quad \boxed{H_T = N\mathcal{H} + N^0(\pi_i - p_i) + M^i \pi_0}$

$\Downarrow$

Newton's Laws $N \to$ ephemeris time

Classical Theory: PB $\{a, \pi_0\} = 1$

$\{\pi_0, L_0\} = 0$; $\{\pi_0, \mathcal{H}\} = 0$ $\{\mathcal{H}_0, H\} = 0$

$H_c = \sum_i p_i^T \dot{q}_i + \dot{a}\pi_0 + \dot{q}_0 p_0 - L = (\ldots) = 0$

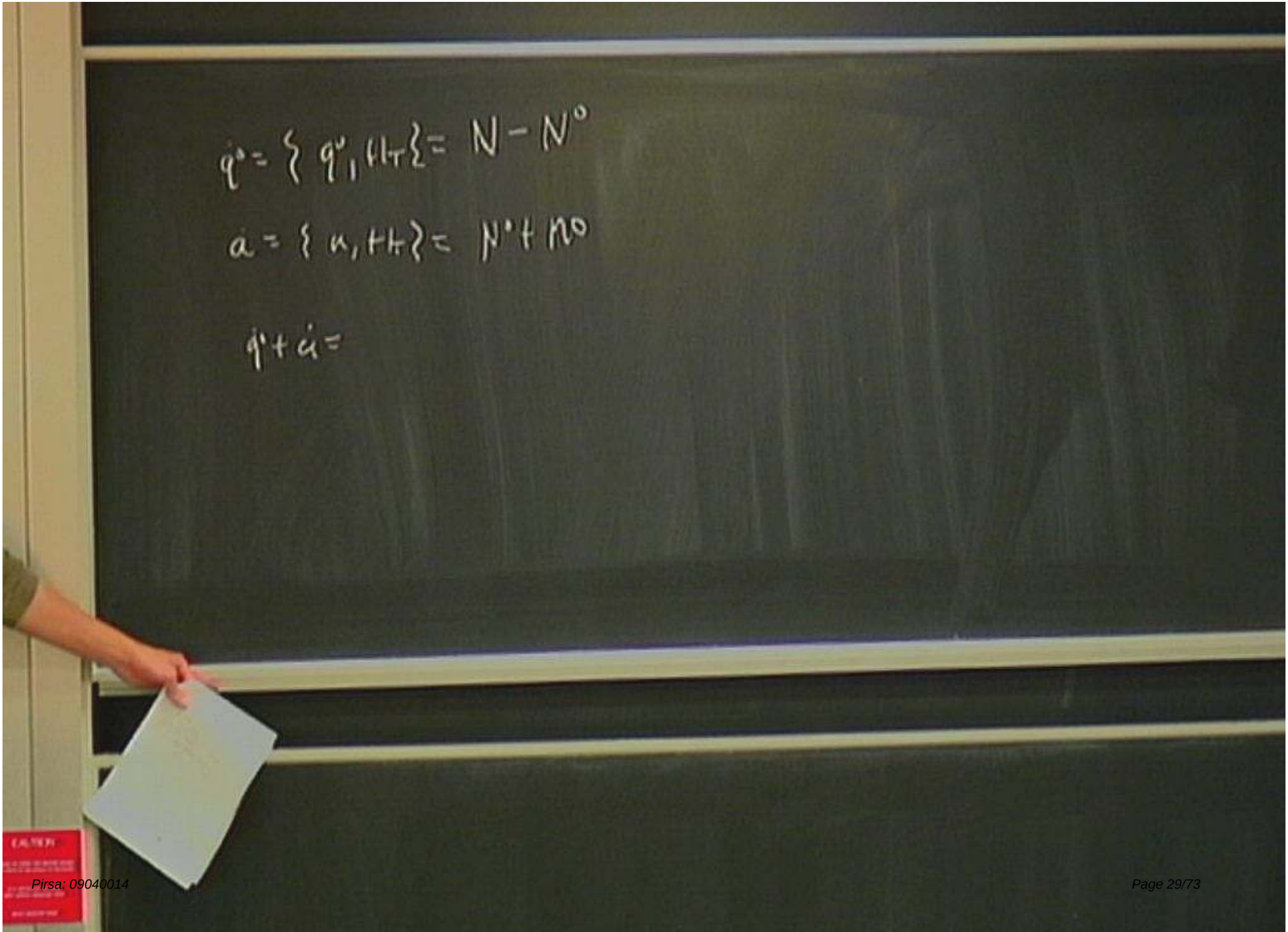
q^i_I, p^i_I $\boxed{H_T = N\mathcal{H} + N^0(\pi_0 - p_0) + M^i \pi_0}$

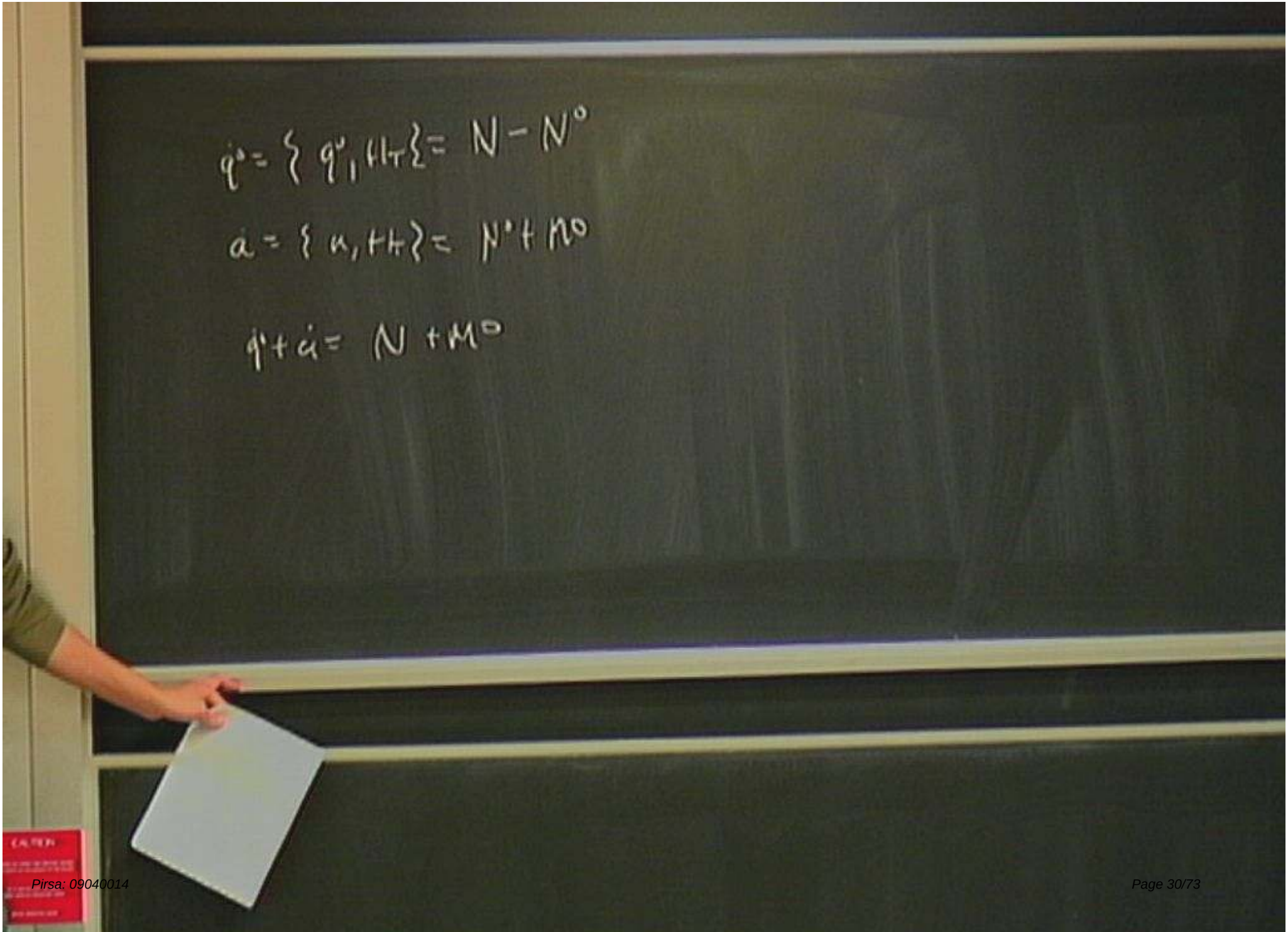
$\Downarrow$

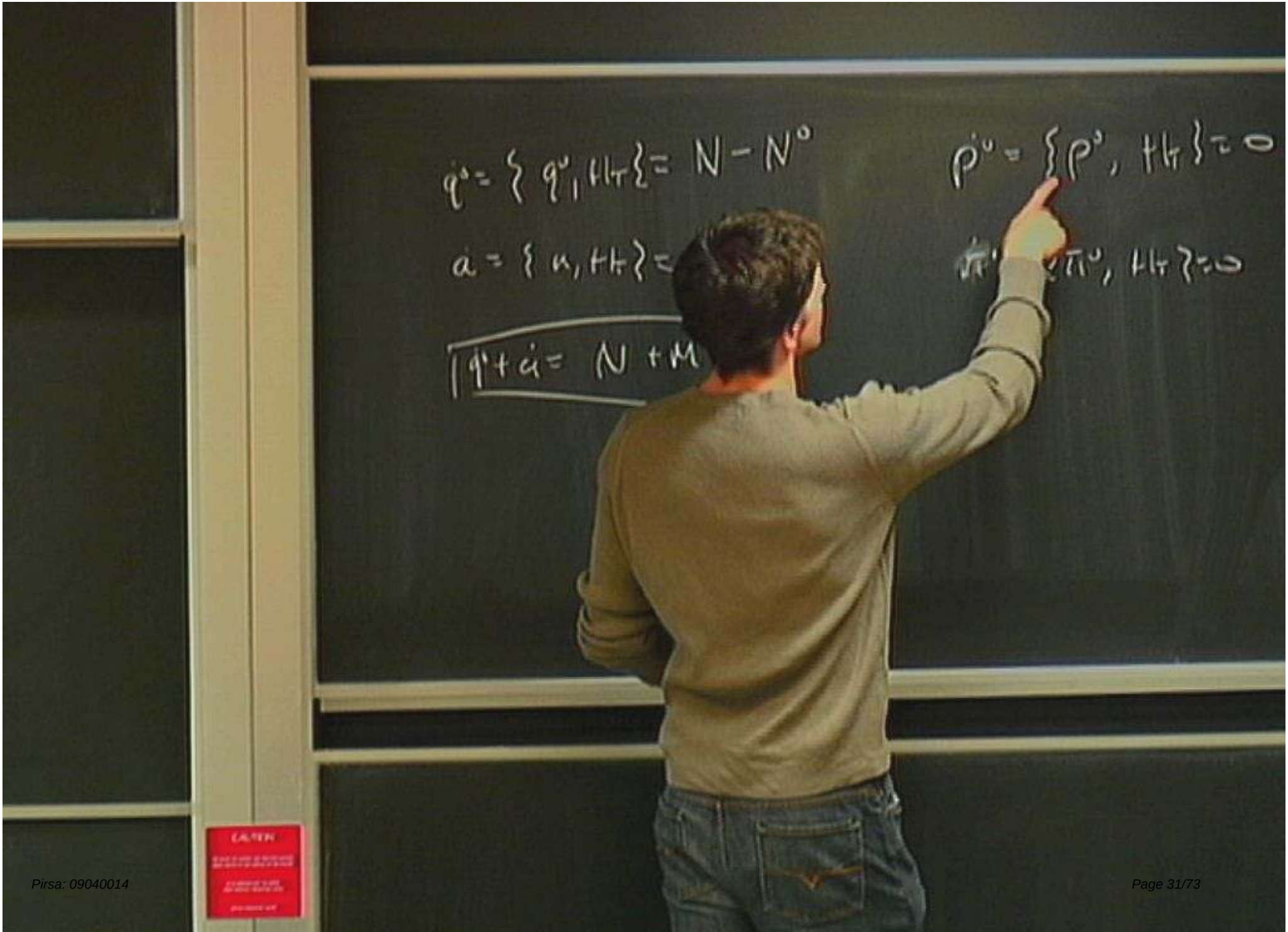
Newton's Laws $N \to$ ephemeris time $N d\lambda = dt_N$

$q^\alpha = \{ q^\alpha, H_T \} = N - N^\alpha$

$\dot{a} = \{ a, H_T \} =$


$q^0 = \{ q^0, H_T \} = N - N^0$
$a = \{ \kappa, H_T \} = N^0 + \kappa_0$
$q^0 + a =$


$q^0 = \{ q^0, H_T \} = N - N^0$
$a = \{ a, H_T \} = N^+ + M^0$
$q^+ + a = N + M^0$


$q^0 = \{ q^0, H_T \} = N - N^0$
$\dot{q} = \{ q, H_T \} =$
$q^0 + \dot{q} = N + M$
$\dot{p}^0 = \{ p^0, H_T \} = 0$
$\pi^0, H_T \} = 0$

$\dot{q}^0 = \{ q^0, H_T \} = N - N^0$

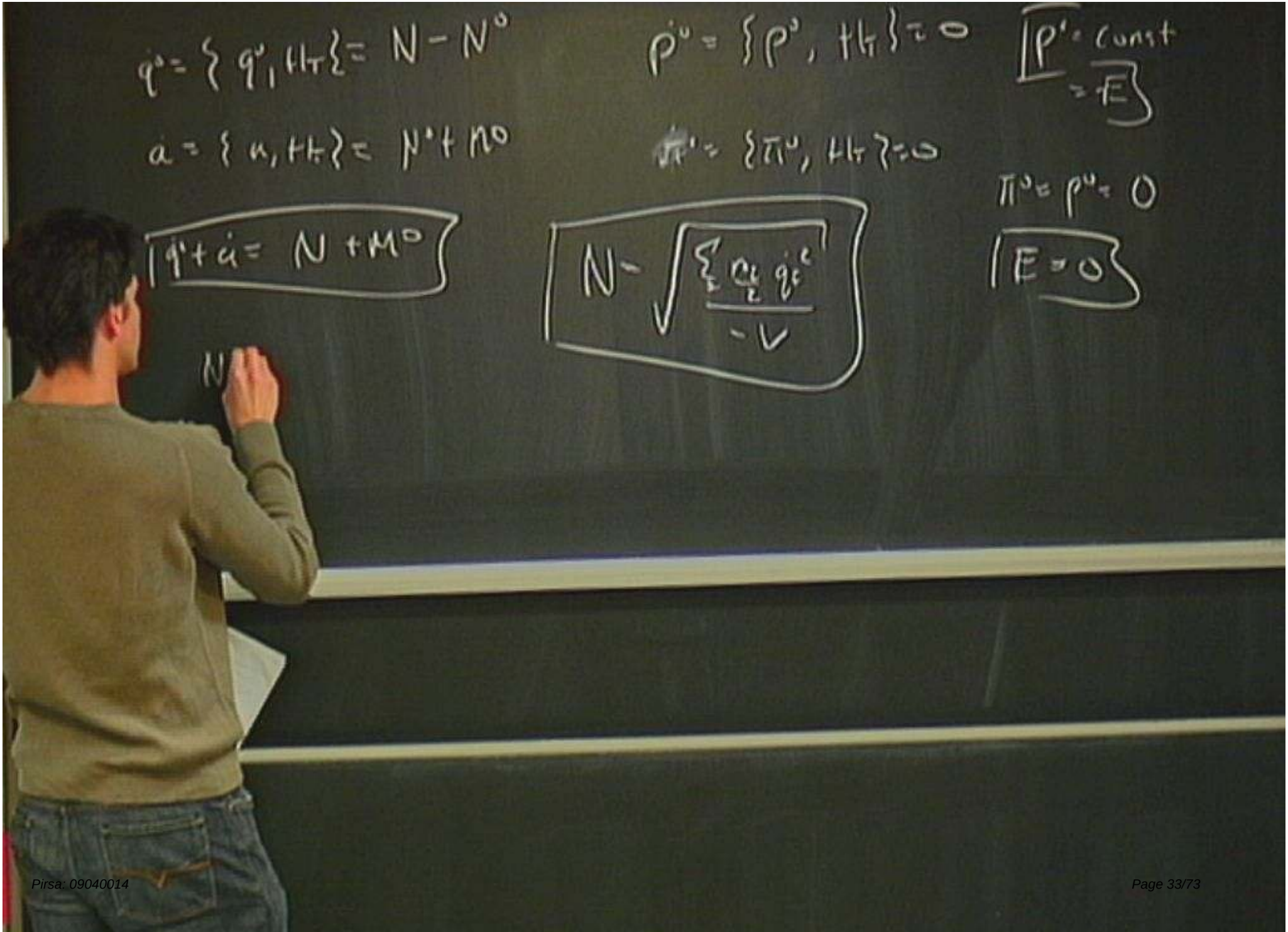
$\dot{a} = \{ a, H_T \} = N' + M^0$

$\boxed{\dot{q}^1 + \dot{a} = N + M^0}$

$\dot{p}^0 = \{ p^0, H_T \} = 0$

$\dot{\pi}^0 = \{ \pi^0, H_T \} = 0$

$\boxed{p^1 = \text{const} = E}$


$q^0 = \{ q^0, H_+ \} = N - N^0$
$\dot{P}^0 = \{ P^0, H_+ \} = 0$
$P^i = \text{const} = E$
$a = \{ a, H_+ \} = N^+ + M^0$
$\dot{\pi}^0 = \{ \pi^0, H_+ \} = 0$
$\pi^0 = P^0 = 0$
$q^+ + a = N + M^0$
$N = \sqrt{\frac{\sum_i a_i q_i^2}{V}}$
$E = 0$

$q^\mu = \{ q^\mu, H_T \} = N - N^0$

$p^\mu = \{ p^\mu, H_T \} = 0$

$p^\mu = \text{const} = E$

$a = \{ \pi, H_T \} = N' + N^0$

$\pi^\mu = \{ \pi^\mu, H_T \} = 0$

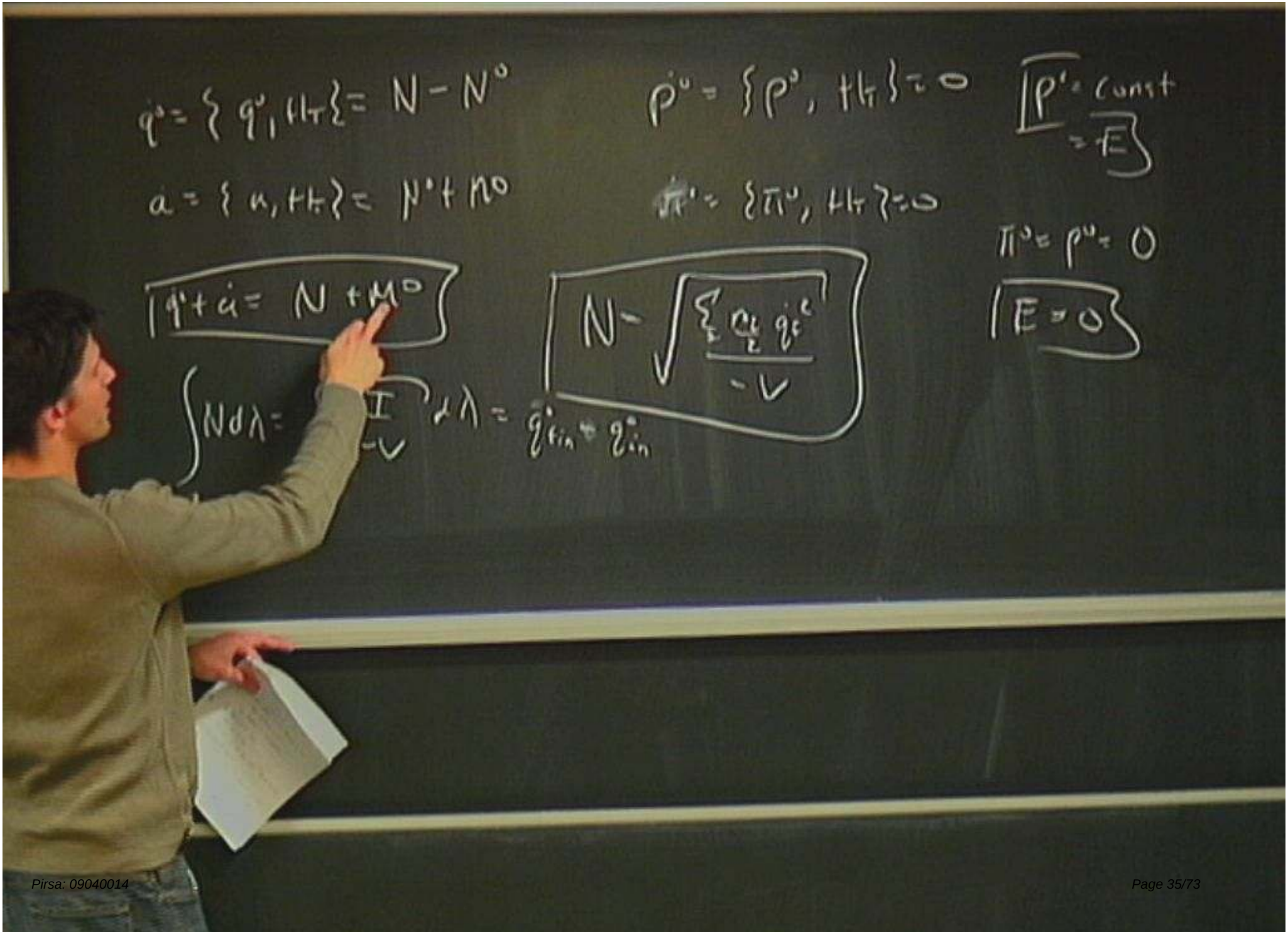
$\pi^\mu = p^\mu = 0$

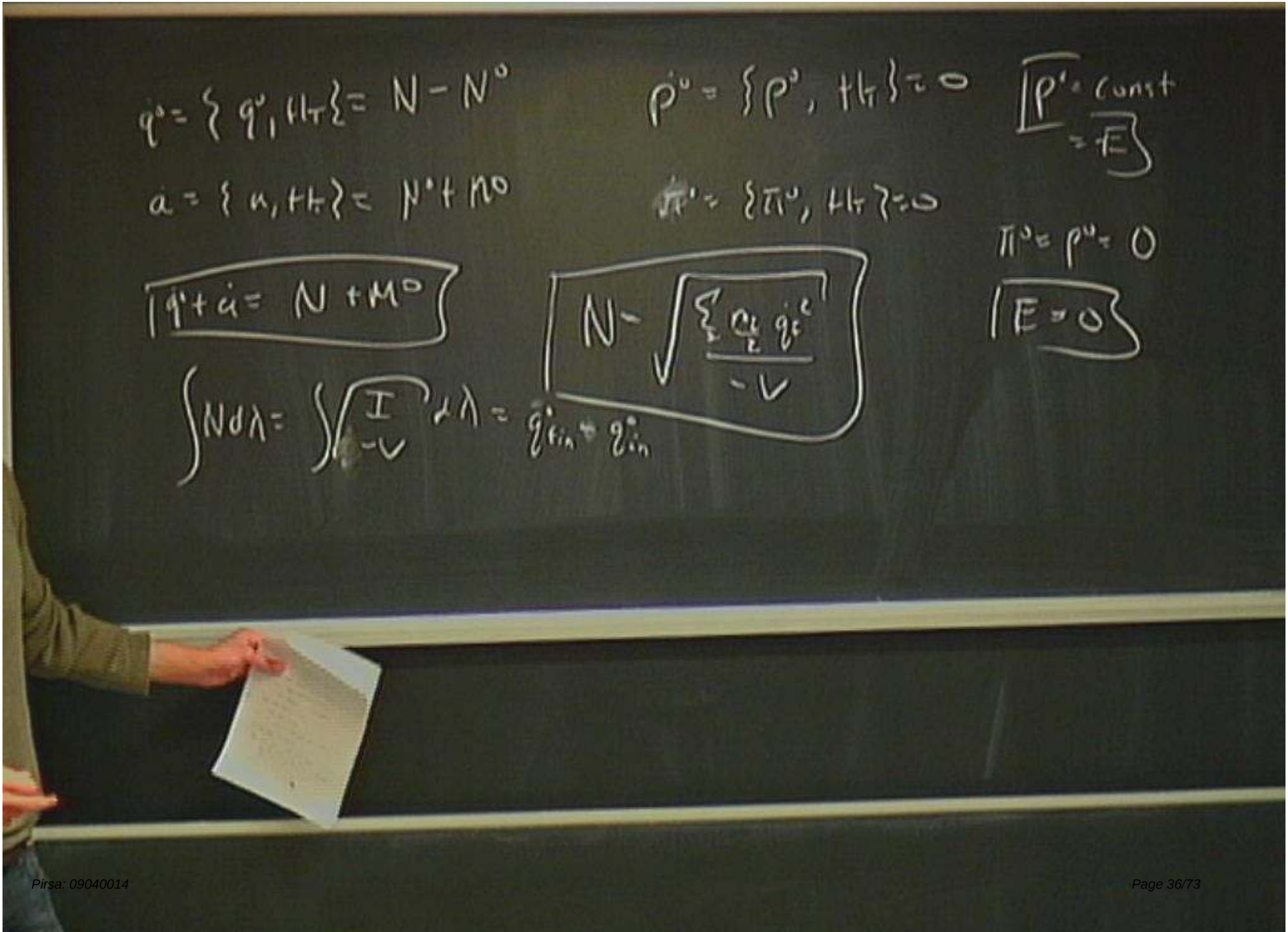
$q' + a = N + N^0$

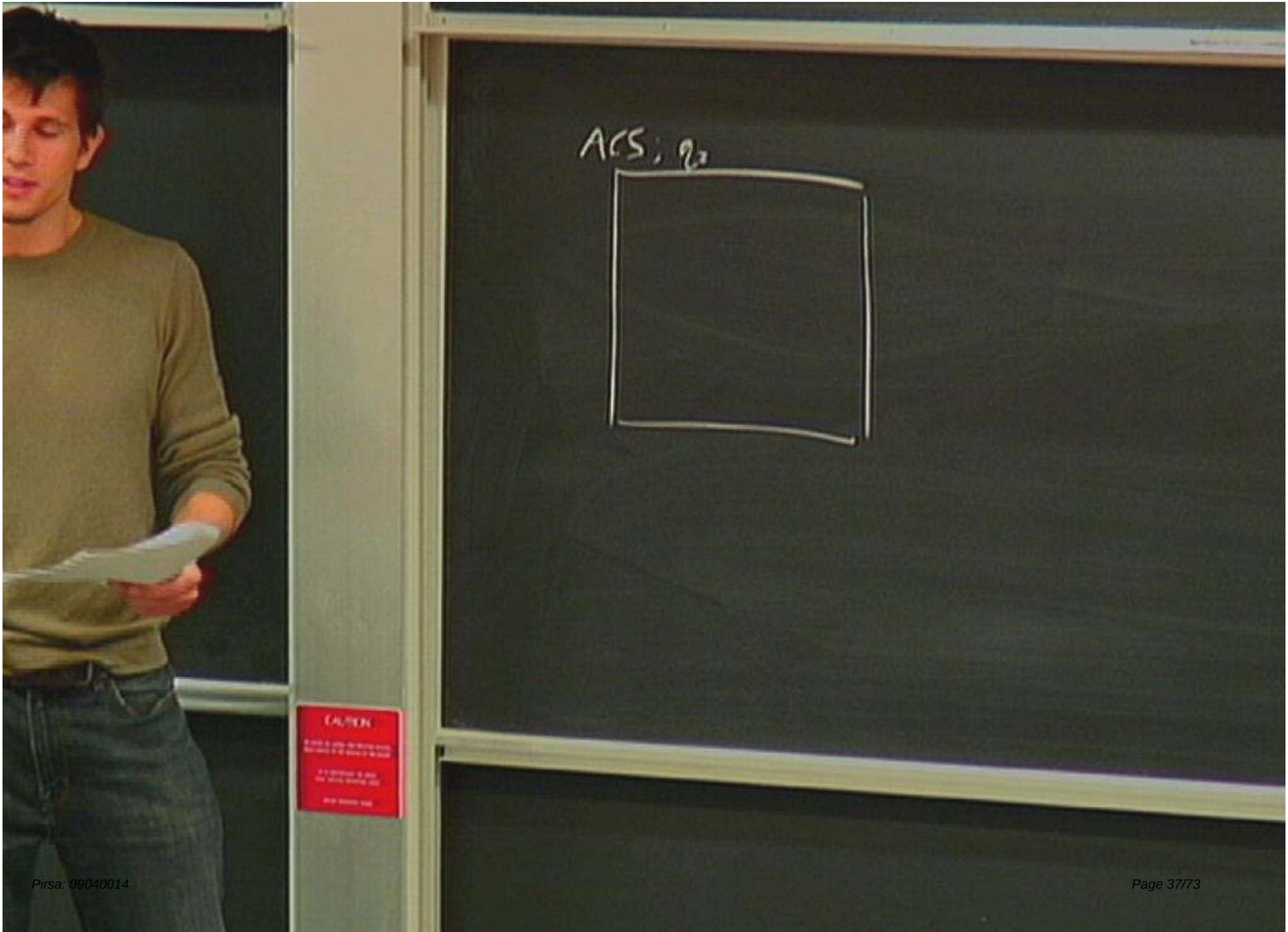
$N = \sqrt{\frac{\{ q, q' \}}{-V}}$

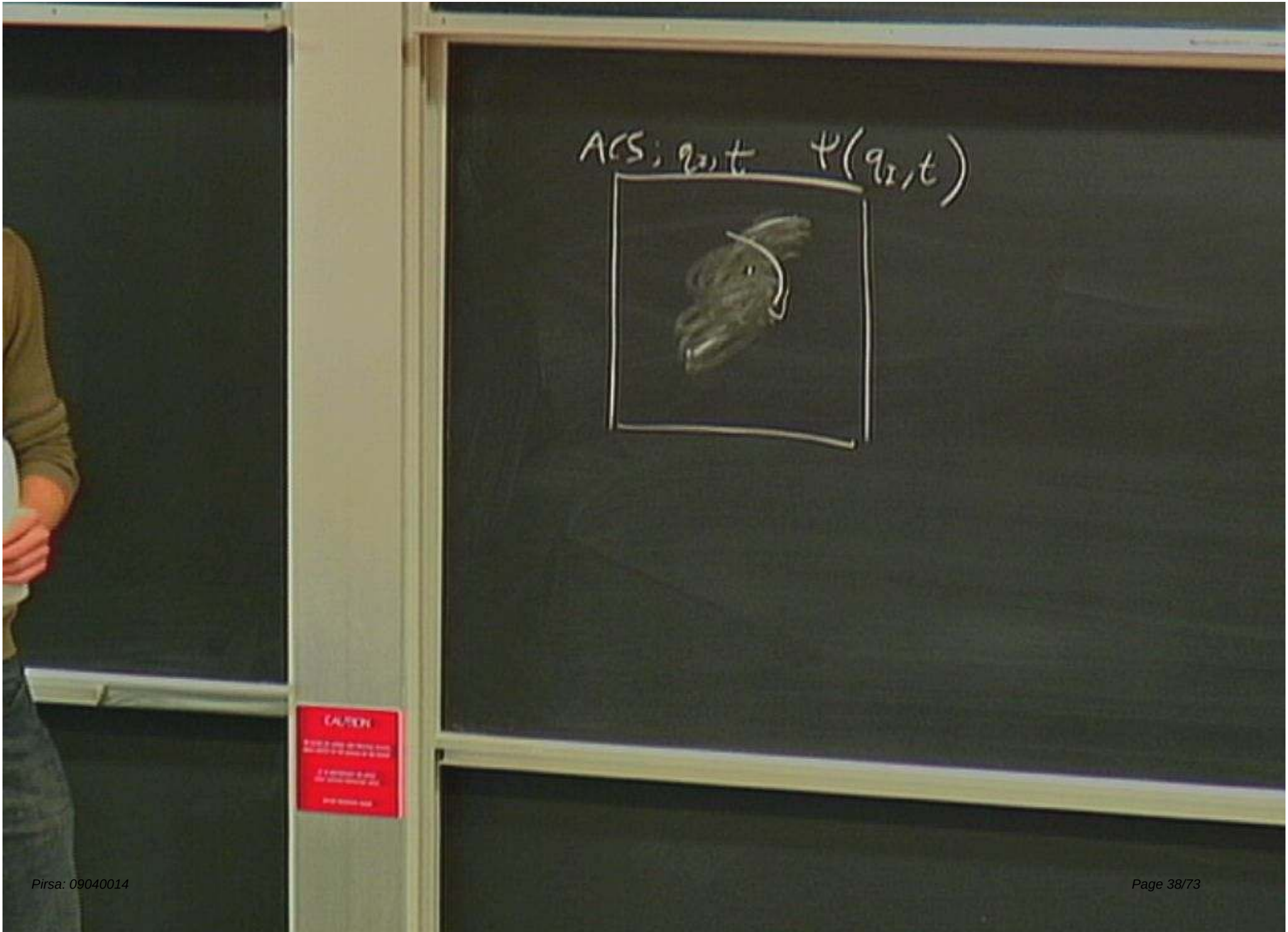
$E = 0$

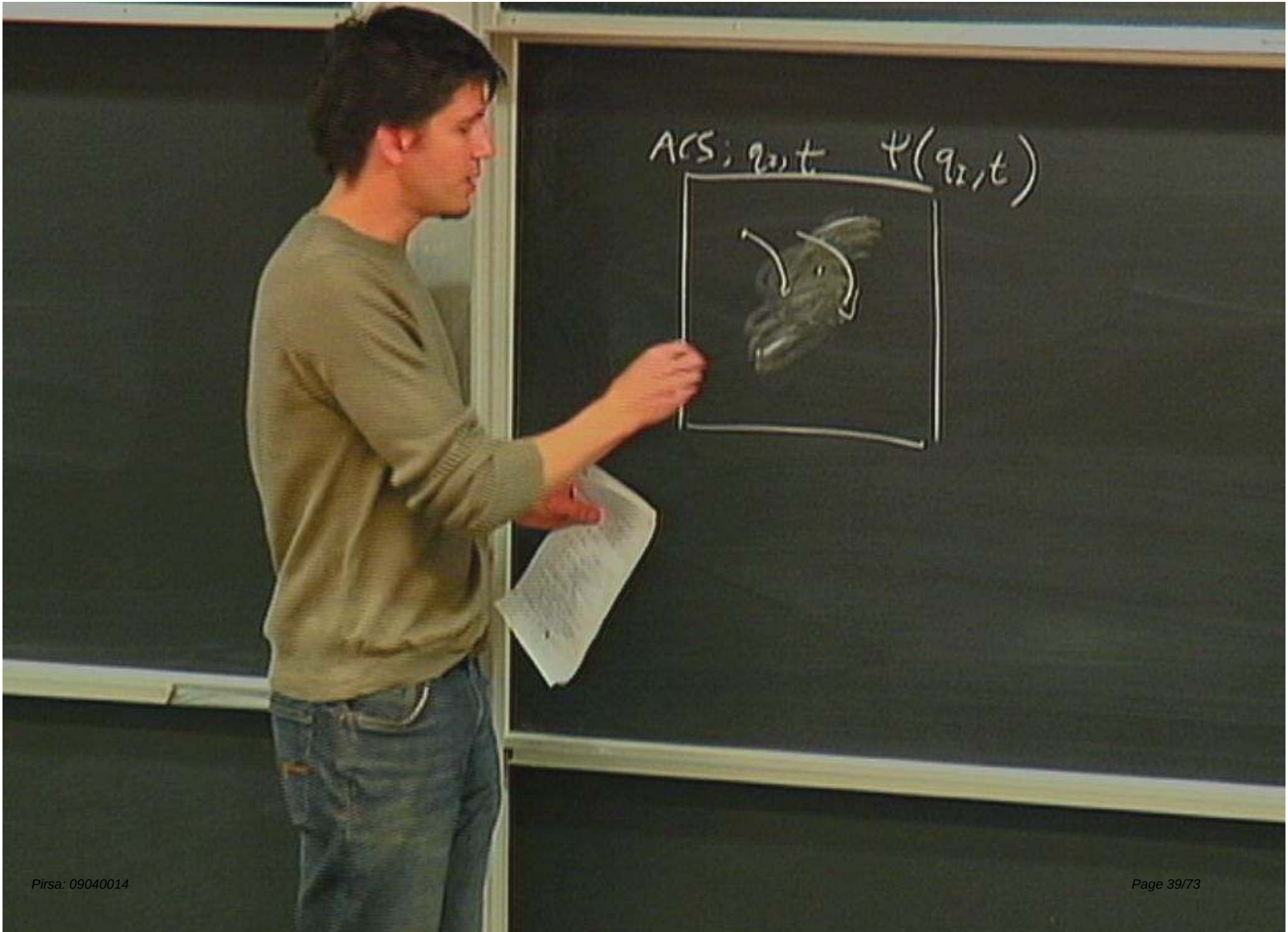
$\int N d\lambda = \int \sqrt{\frac{I}{-V}} d\lambda$

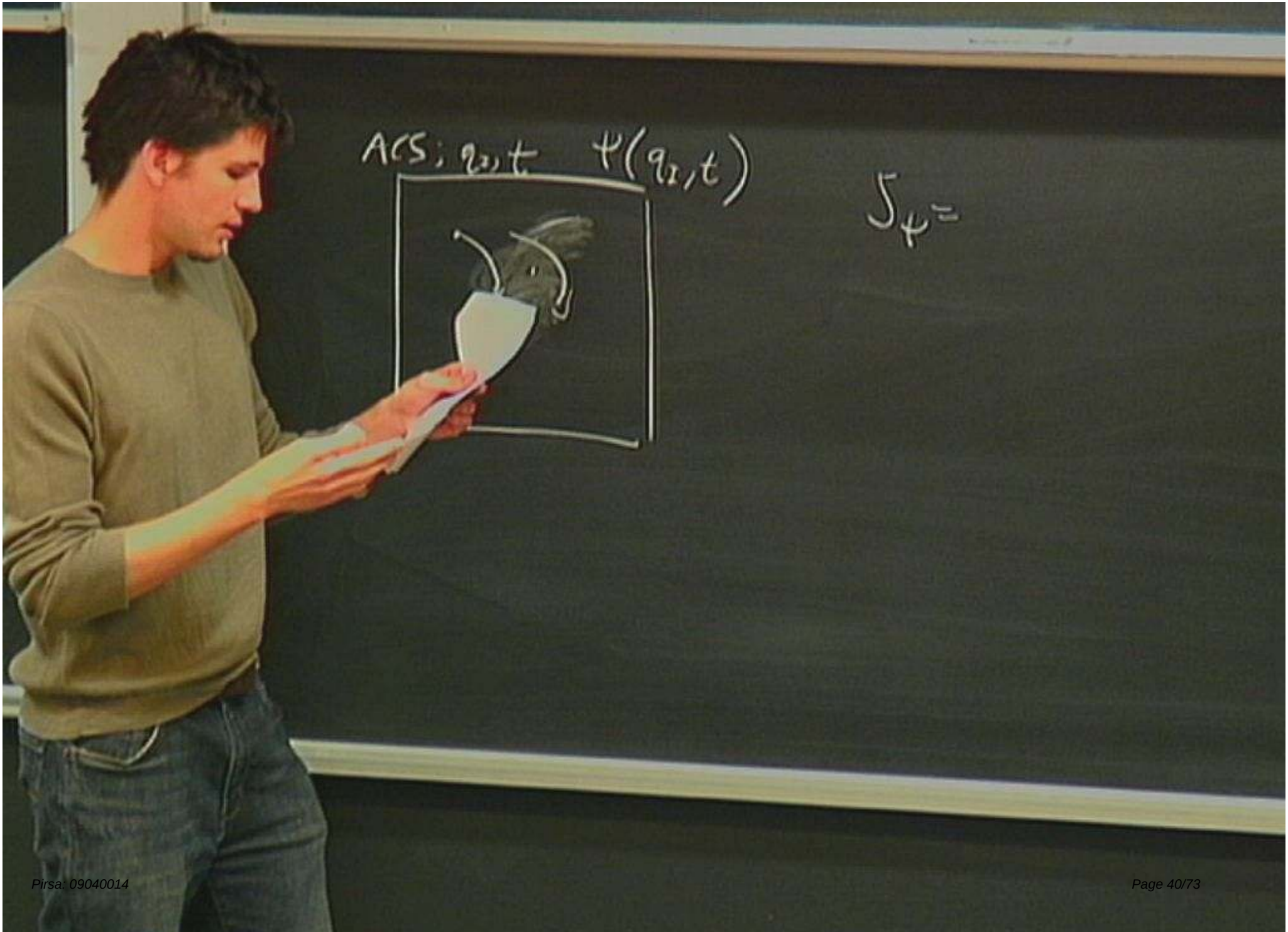

$q^\mu = \{ q^\mu, H_T \} = N - N^\mu$
$p^\mu = \{ p^\mu, H_T \} = 0$
$p^\mu = \text{const} = E$
$a = \{ \kappa, H_T \} = \lambda' + \kappa\lambda$
$\pi^\mu = \{ \pi^\mu, H_T \} = 0$
$\pi^\mu = p^\mu = 0$
$E = 0$
$\dot{q}^\mu + \dot{a} = N + M^\mu$
$N = \sqrt{\frac{\sum_i a_i \dot{q}_i^2}{-V}}$
$\int N d\lambda = \frac{I}{-V} d\lambda = \dot{q}^\mu_{in} - \dot{q}^\mu_{in}$


$q^\mu = \{ q^\mu, H_T \} = N - N^\mu$
$\dot{p}^\mu = \{ p^\mu, H_T \} = 0$
$p^\mu = \text{const} = E$
$a = \{ a, H_T \} = N' + M^0$
$\dot{\pi}^\mu = \{ \pi^\mu, H_T \} = 0$
$\pi^\mu = p^\mu = 0$
$q' + a = N + M^0$
$N = \sqrt{\frac{\sum_i a_i \dot{q}^i}{-V}}$
$E = 0$
$\int N d\lambda = \int \sqrt{\frac{I}{-V}} d\lambda = q^i_{fin} - q^i_{in}$


AdS; Q


ACS; q_2, t
$\Psi(q_1, t)$


ACS; q, t
Ψ(q, t)


ACS; q, t
ψ(q, t)
∫ψ =

ACS: q_i, t $\Psi(q_i, t)$

$$S_\Psi = \int dt\, dq_i \left\{ \Psi^+ \hat{H} \Psi \right\}$$

$$\hat{H} = \sum_i \frac{\hat{p}_i^2}{2m_i} + V(\hat{q}_i) + \hat{p}_i$$

ACS: q_i, t $\quad \psi(q_i, t)$

$$S_\psi = \int dt\, dq_i \left\{ \psi^+ \hat{H} \psi \right\}$$

$$\hat{H} = \sum_i \frac{\hat{p}_i^2}{2m_i} + V(\hat{q}_i) + \hat{p}_0$$

$$p_i = +i\hbar \frac{\partial}{\partial q_i} \quad \hat{q}_i \cdot q_i \quad ; \hat{p}_0 = -i\hbar \frac{\partial}{\partial t}$$

ACS: q_i, t $\Psi(q_i, t)$

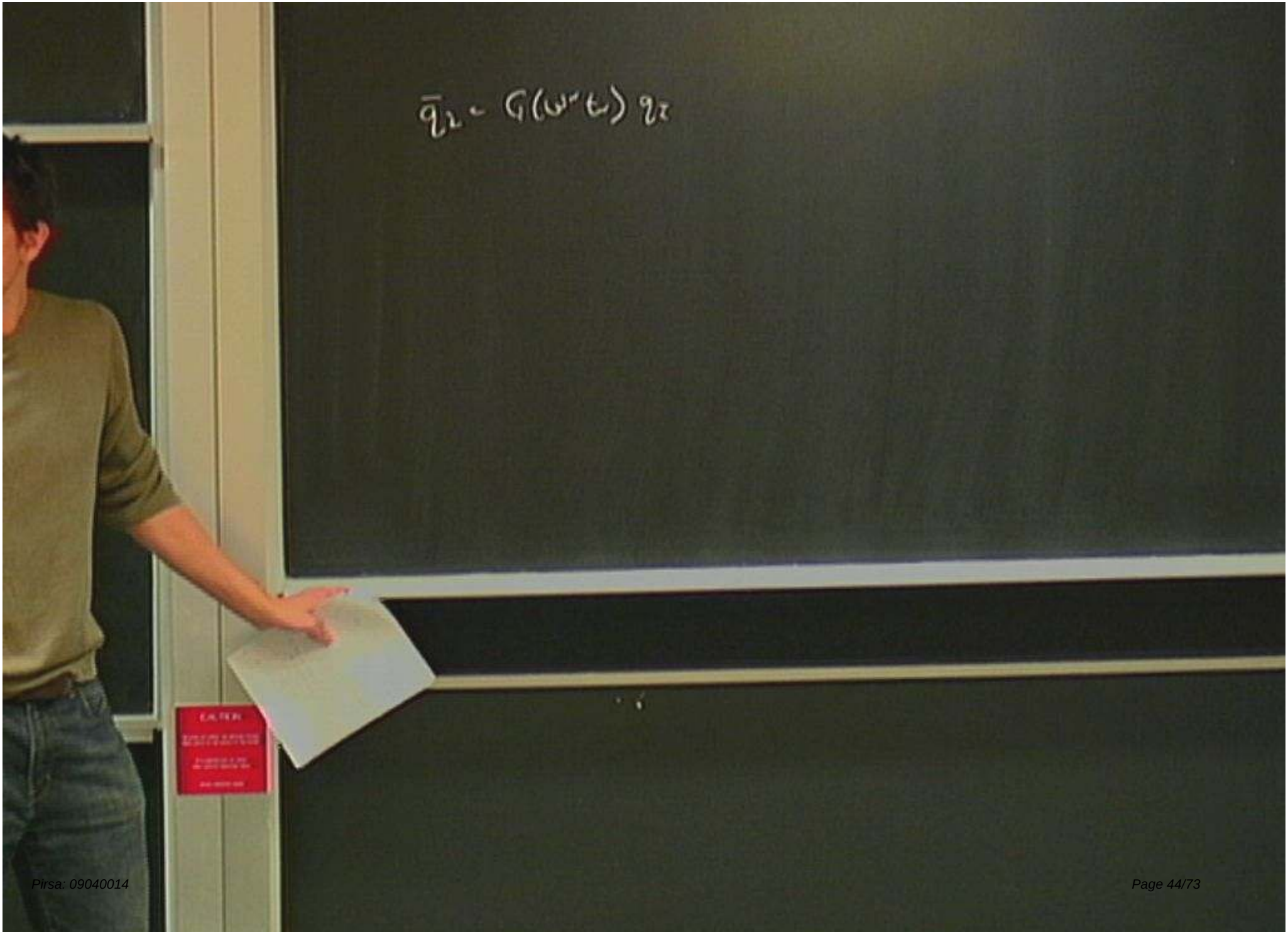
$$S_\Psi = \int dt\, dq_i \left\{ \Psi^+ \hat{H} \Psi \right\}$$

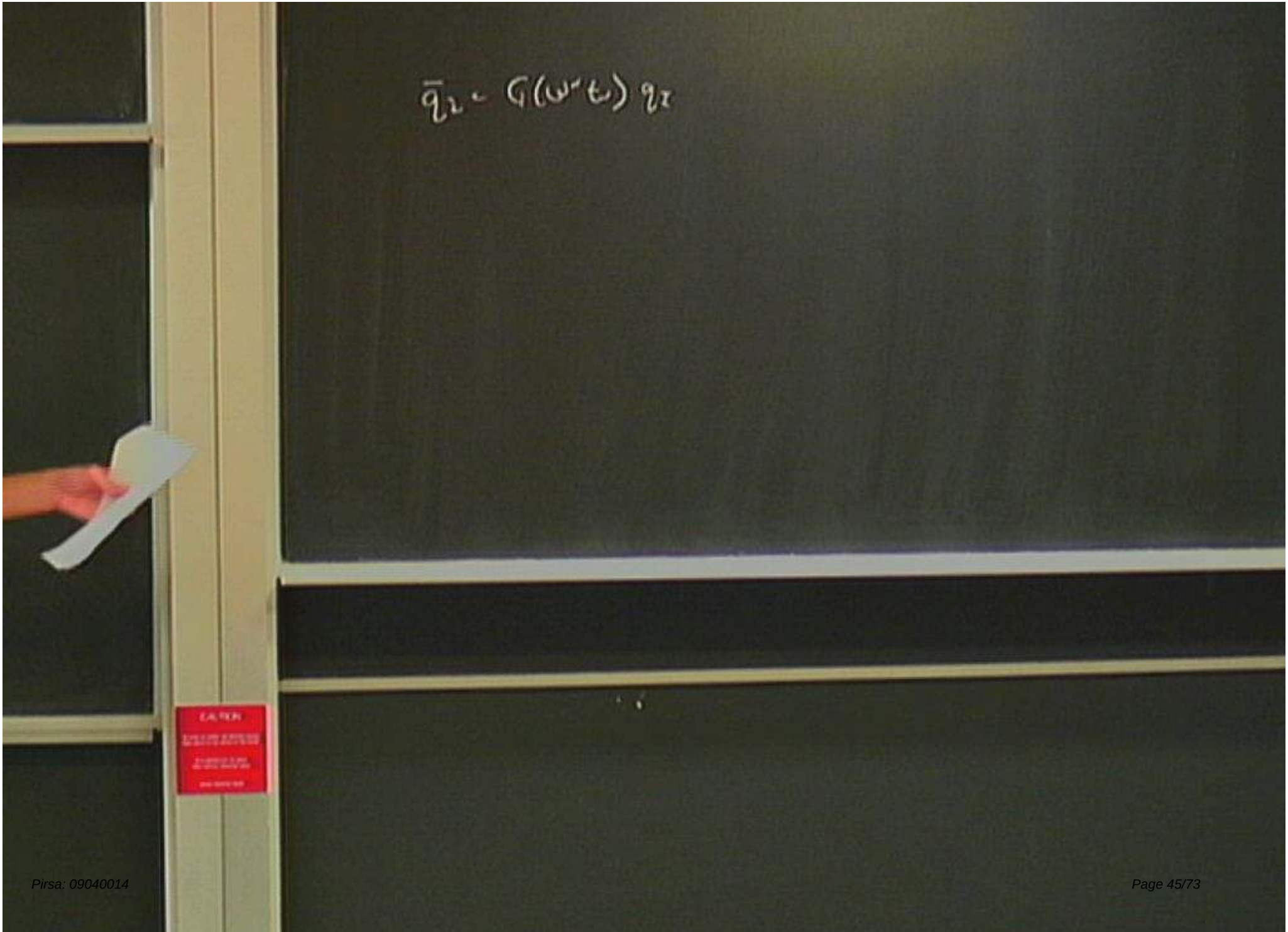
$$\hat{H} = \sum_i \frac{\hat{p}_i^2}{2m_i} + V(q_i) = i\hbar \frac{\partial}{\partial t}$$

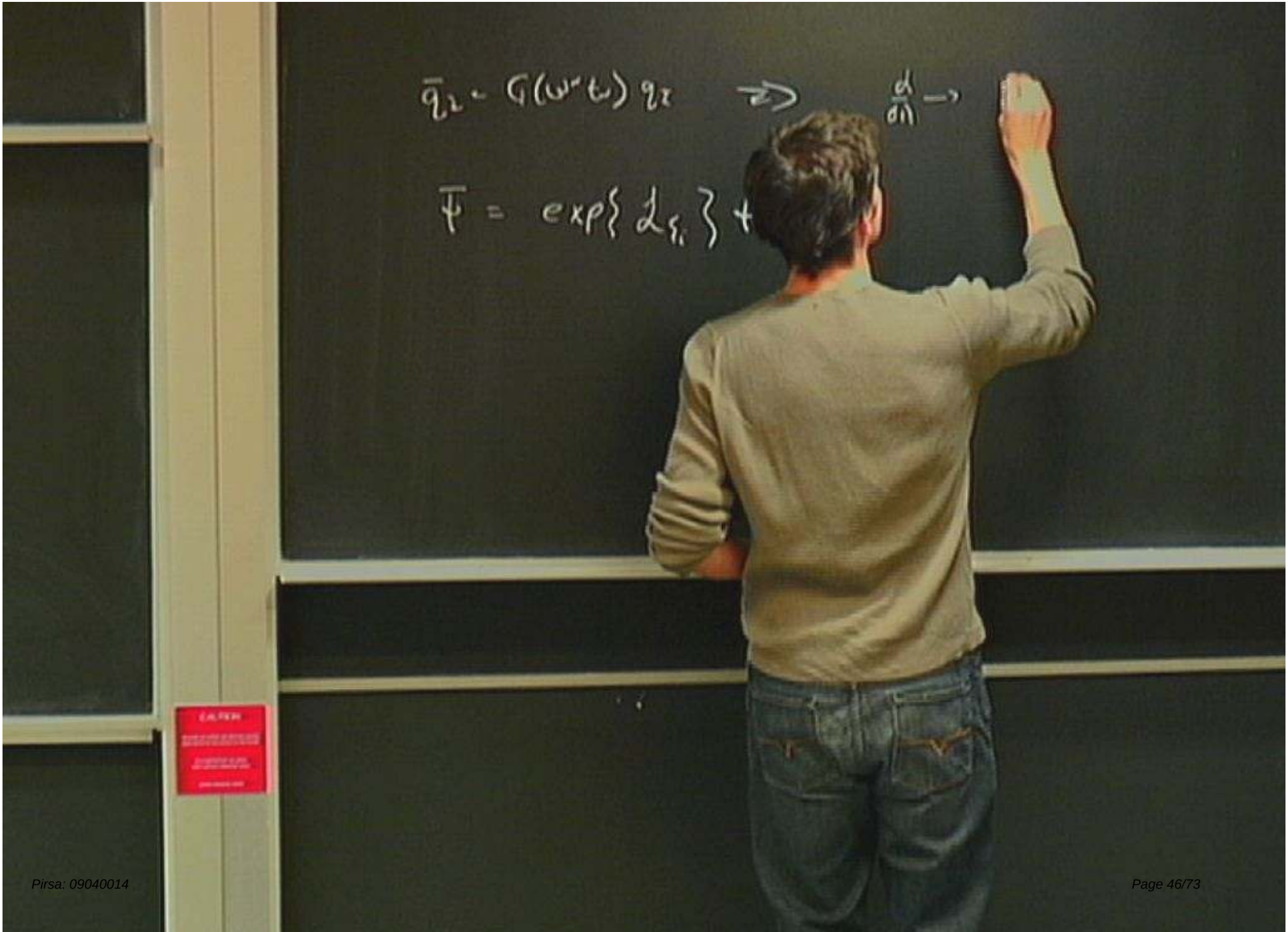
$$p_i = +i\hbar \frac{\partial}{\partial q_i} \quad \hat{q}_i = q_i \; ; \; \hat{p}_0 = -i\hbar \frac{\partial}{\partial t}$$

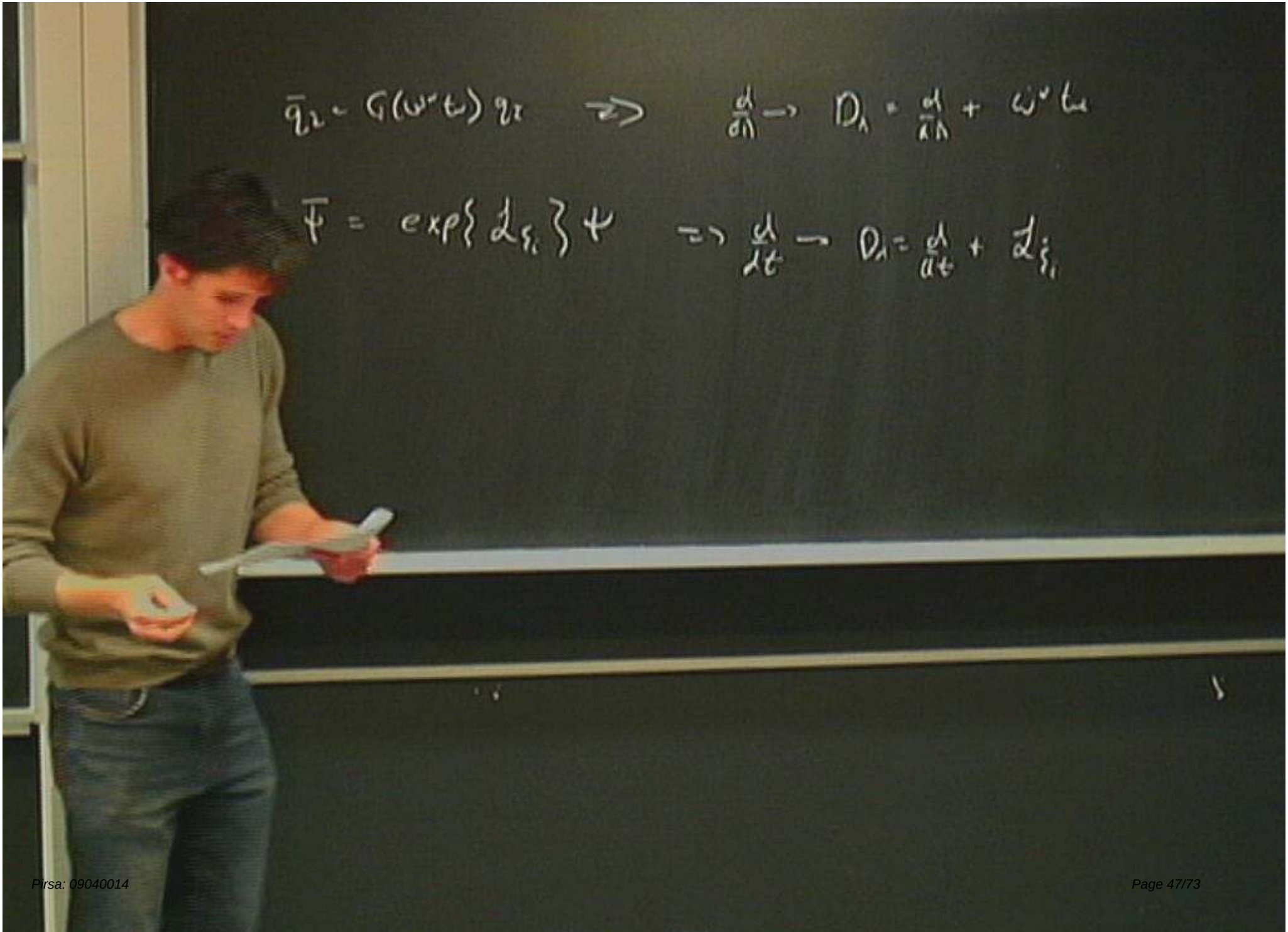
$$\delta_{\Psi^+} \delta = 0 \Rightarrow \hat{H} \Psi = 0$$

$$\delta_\Psi S = 0 \Rightarrow \hat{H} \Psi^+ = 0$$


$\bar{q}_i = G(\omega - \omega_i)\, q_i$

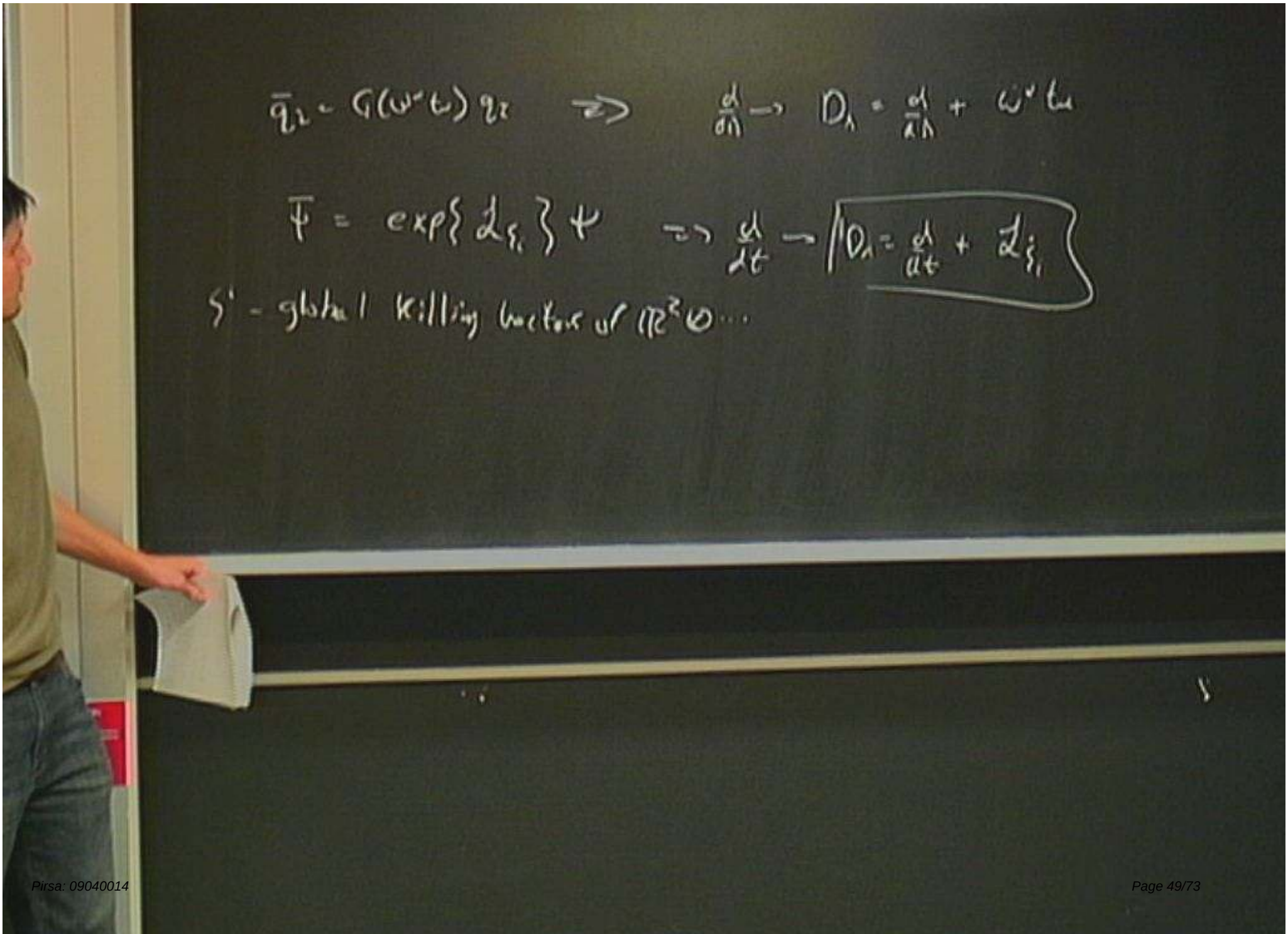

$\bar{q}_i = G(\omega t)\, q_i$

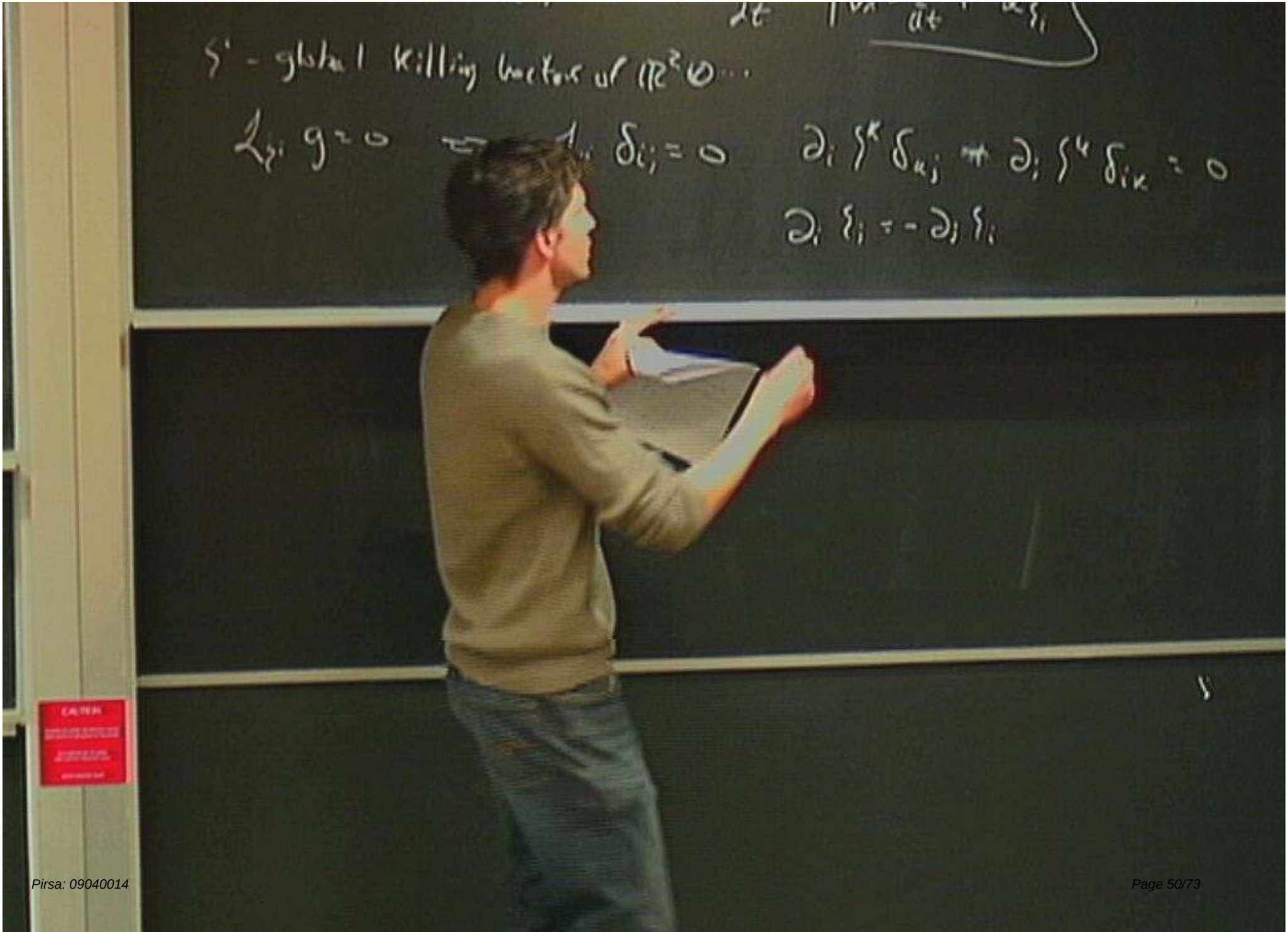

$\bar{q}_i = G(\omega t)\, q_i \Rightarrow \frac{d}{d\lambda} \rightarrow$
$\bar{\Psi} = \exp\{ \lambda_{\xi_i} \} +$

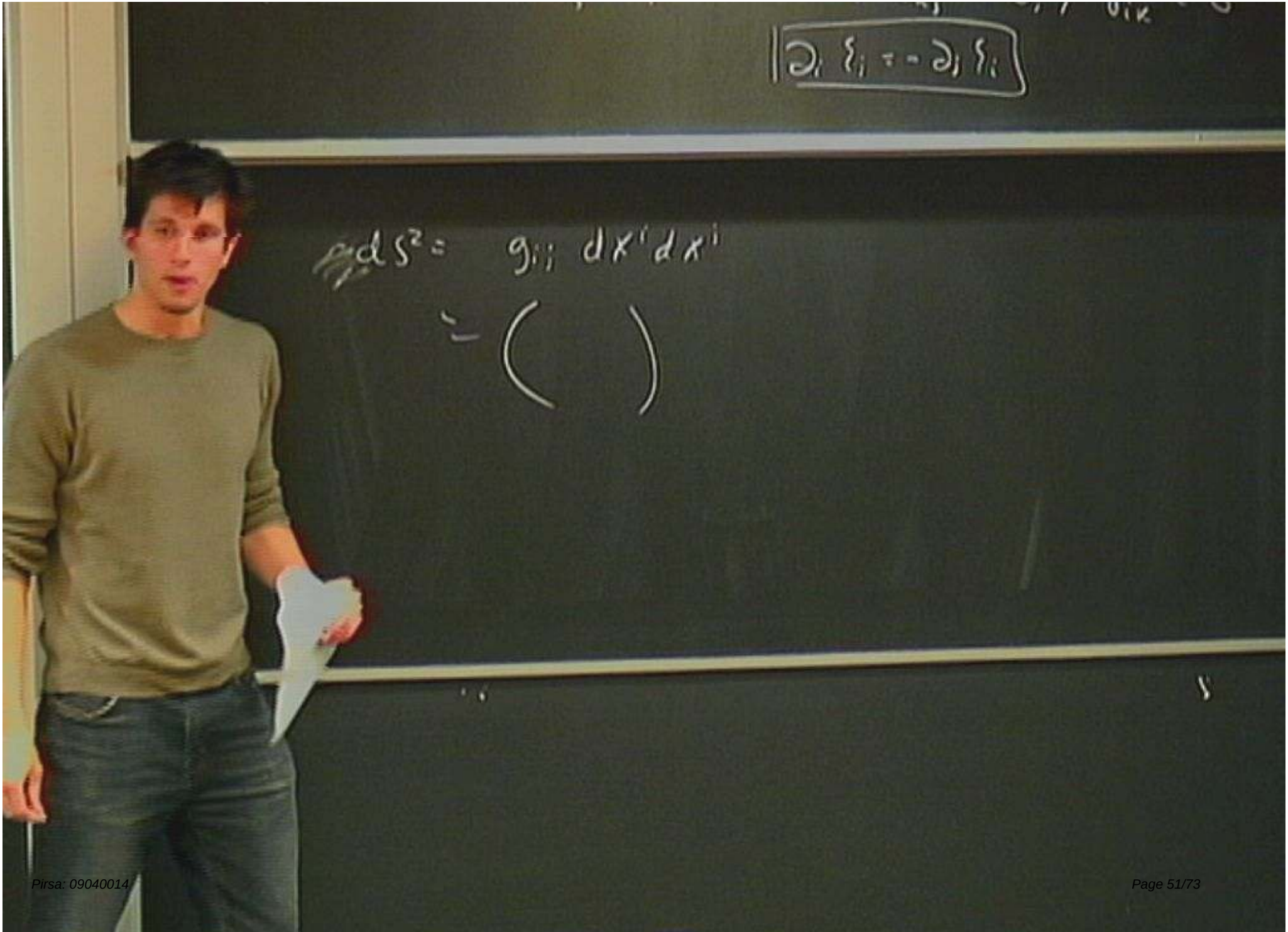

$\bar{q}_i = G(\omega \cdot t)\, q_i \Rightarrow \frac{d}{dt} \to D_t = \frac{d}{dt} + \omega \cdot L$
$F = \exp\{\alpha_{\xi_i}\}\psi \Rightarrow \frac{d}{dt} \to D_t = \frac{d}{dt} + \alpha_{\xi_i}$

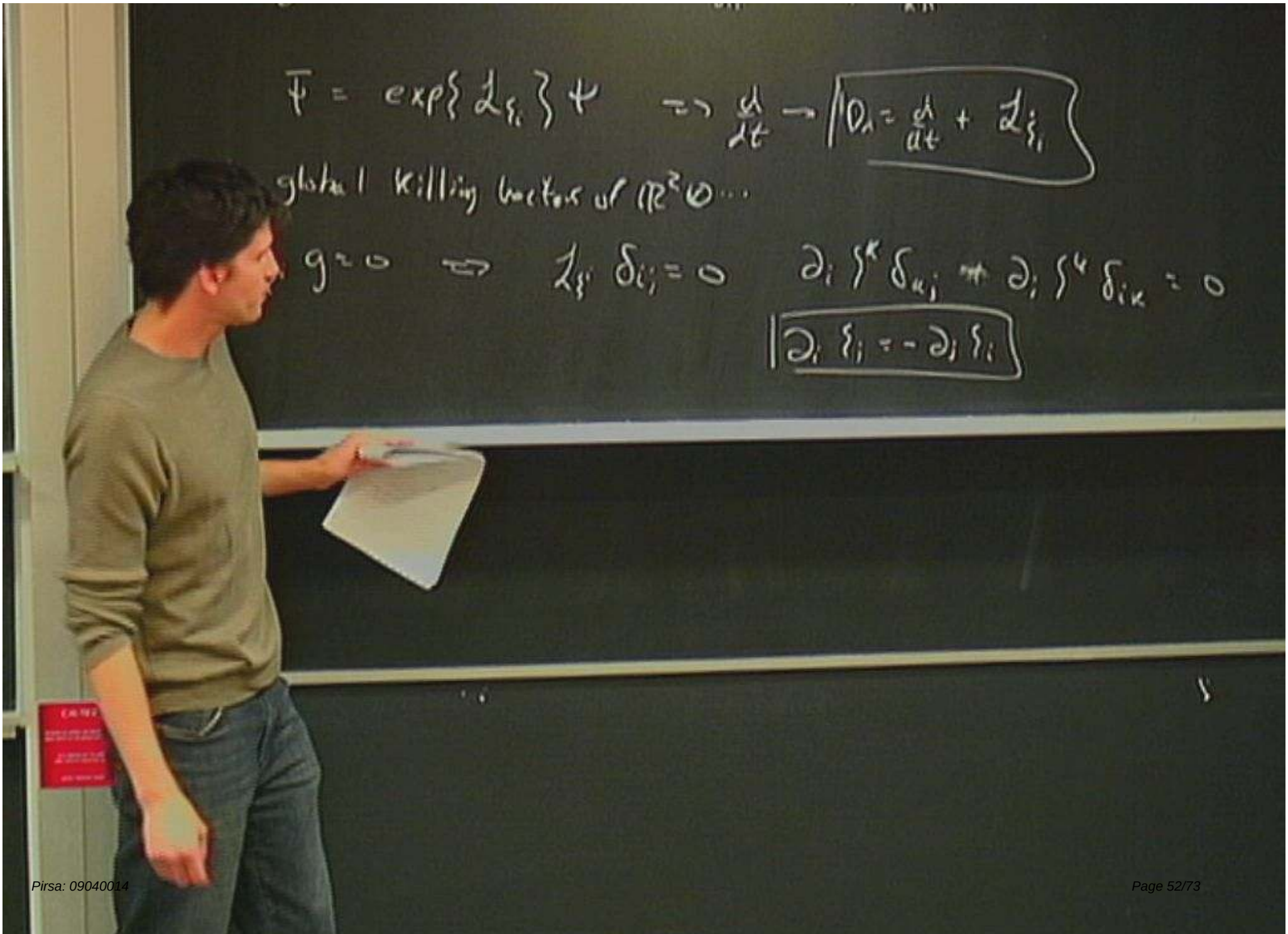
$$\bar{q}_i = G(\omega \cdot t)\, q_i \quad \Rightarrow \quad \frac{d}{dh} \to D_\lambda = \frac{d}{d\lambda} + \omega \cdot t_\omega$$

$$\Psi = \exp\{\alpha_{\xi_i}\}\,\psi \quad \Rightarrow \quad \frac{d}{dt} \to D_\lambda = \frac{d}{dt} + \alpha_{\xi_i}$$


$\bar{q}_2 = G(\omega^\alpha t_\alpha) q_2 \Rightarrow \frac{d}{d\lambda} \to D_\lambda = \frac{d}{d\lambda} + \omega^\alpha t_\alpha$
$\bar{\Psi} = \exp\{\mathcal{L}_{\xi_i}\}\Psi \Rightarrow \frac{d}{dt} \to D_\lambda = \frac{d}{dt} + \mathcal{L}_{\dot{\xi}_i}$
ξ^i - global Killing vectors of $\mathbb{R}^2 \otimes \cdots$


ξ^i - global Killing vectors of $\mathbb{R}^2 \otimes \cdots$
$\mathcal{L}_{\xi^i} g = 0 \Leftrightarrow \mathcal{L}_{\xi^i} \delta_{ij} = 0$
$\partial_i \xi^k \delta_{kj} + \partial_j \xi^k \delta_{ik} = 0$
$\partial_i \xi_j = -\partial_j \xi_i$

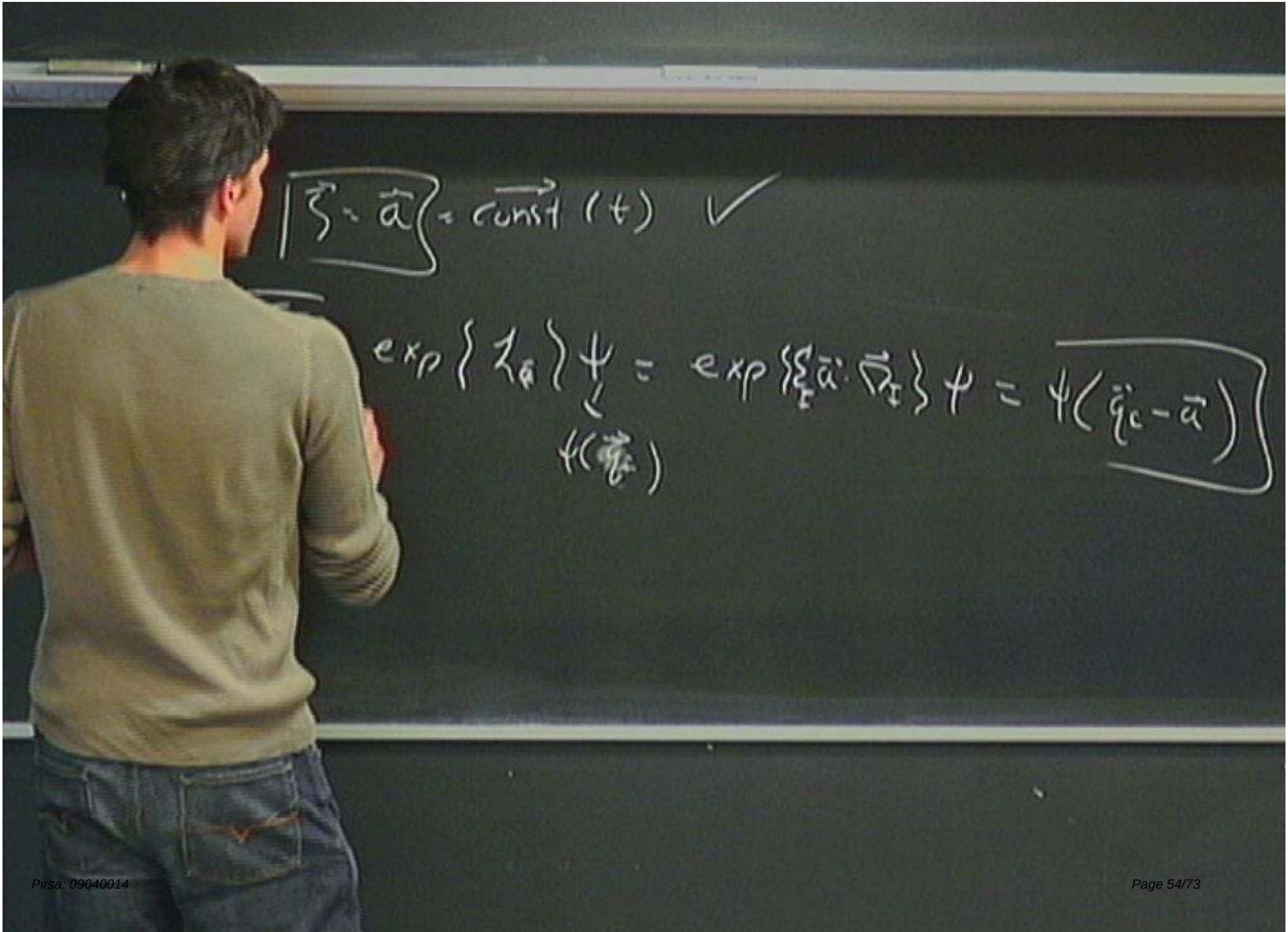

$\partial_i \xi_i = -\partial_i \xi_i$
$ds^2 = g_{ij}\, dx^i dx^j$


$\mathcal{F} = \exp\{\mathcal{L}_{\xi_i}\}\psi \Rightarrow \frac{d}{dt} \rightarrow D_\lambda = \frac{d}{dt} + \mathcal{L}_{\xi_i}$
global Killing vectors of $\mathbb{R}^2 \otimes \cdots$
$g = 0 \Rightarrow \mathcal{L}_{\xi_i} \delta_{ij} = 0 \quad \partial_i \xi^k \delta_{kj} + \partial_j \xi^k \delta_{ik} = 0$
$\partial_i \xi_j = -\partial_j \xi_i$

$\partial_i \xi_j = -\partial_j \xi_i$

$ds^2 = g_{ij}\, dx^i dx^j$

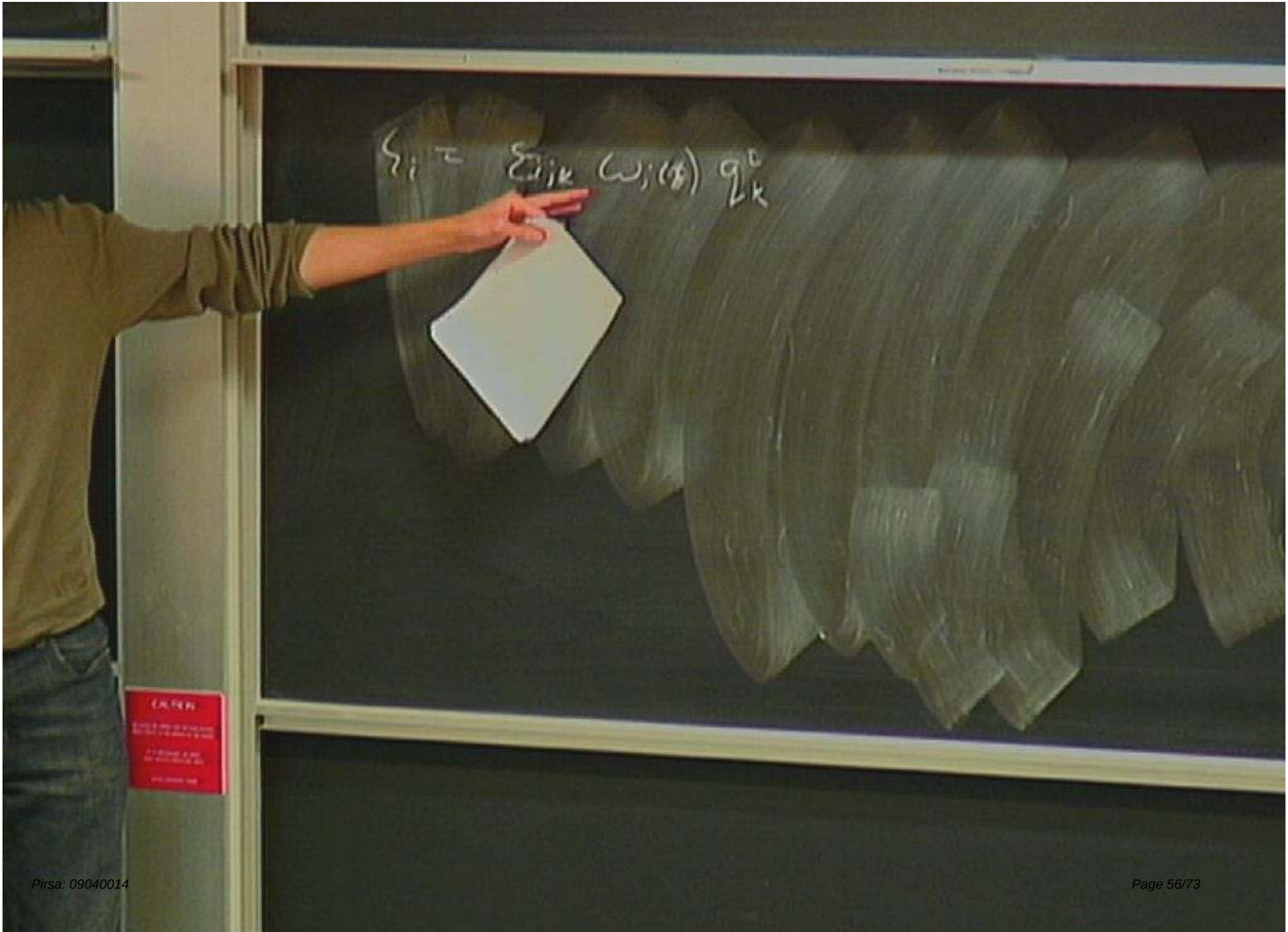
$= \begin{pmatrix} & \\ & \end{pmatrix} = dx^1 + dy^2 + \ldots$

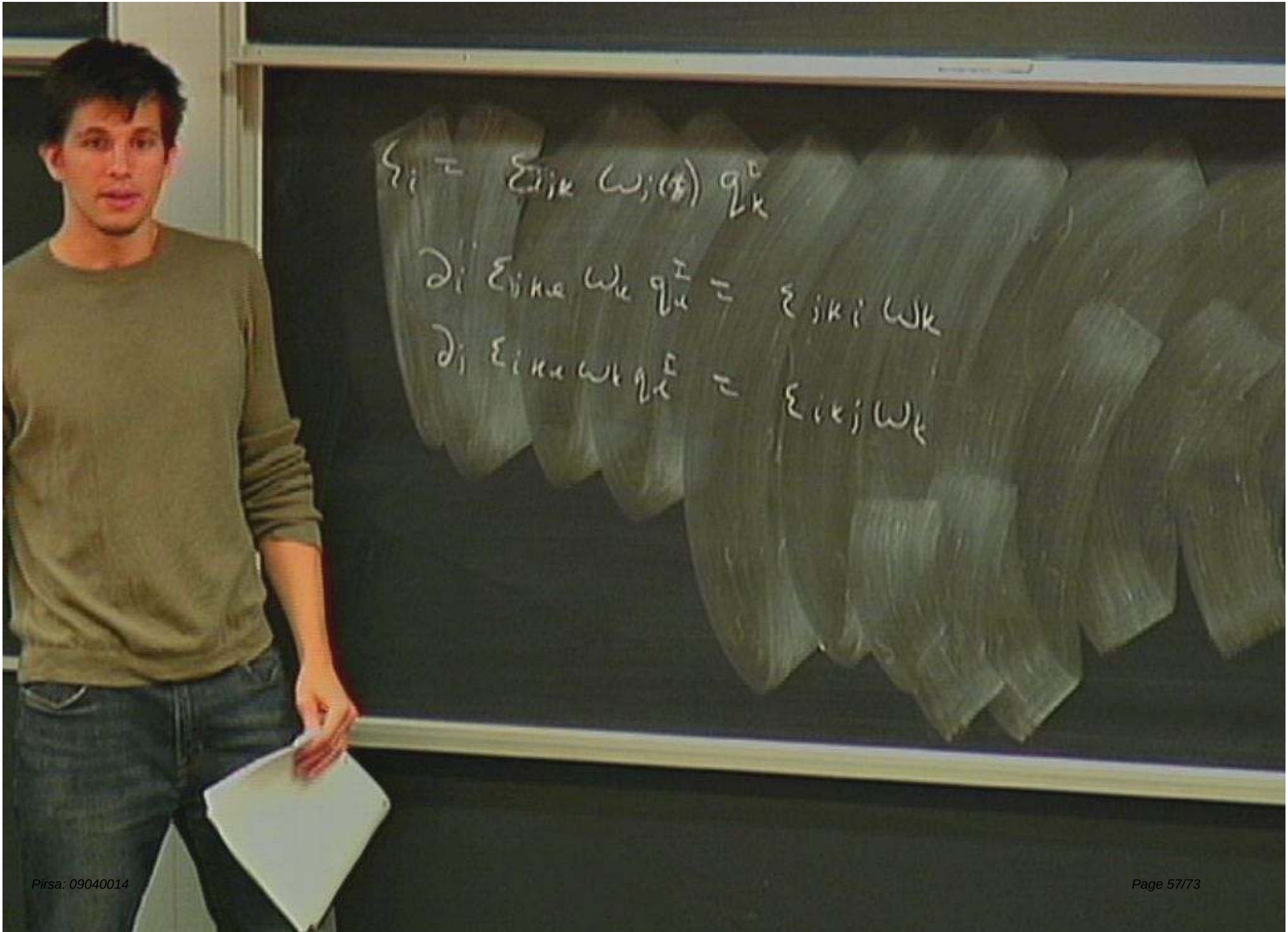

$\vec{\xi} \cdot \vec{a} = \text{const}\,(t)$ ✓
$\exp\{\hat{x}_a\}\psi(\vec{\xi}) = \exp\{\xi_a \vec{a} \cdot \vec{\nabla}_x\}\psi = \psi(\vec{\xi} - \vec{a})$

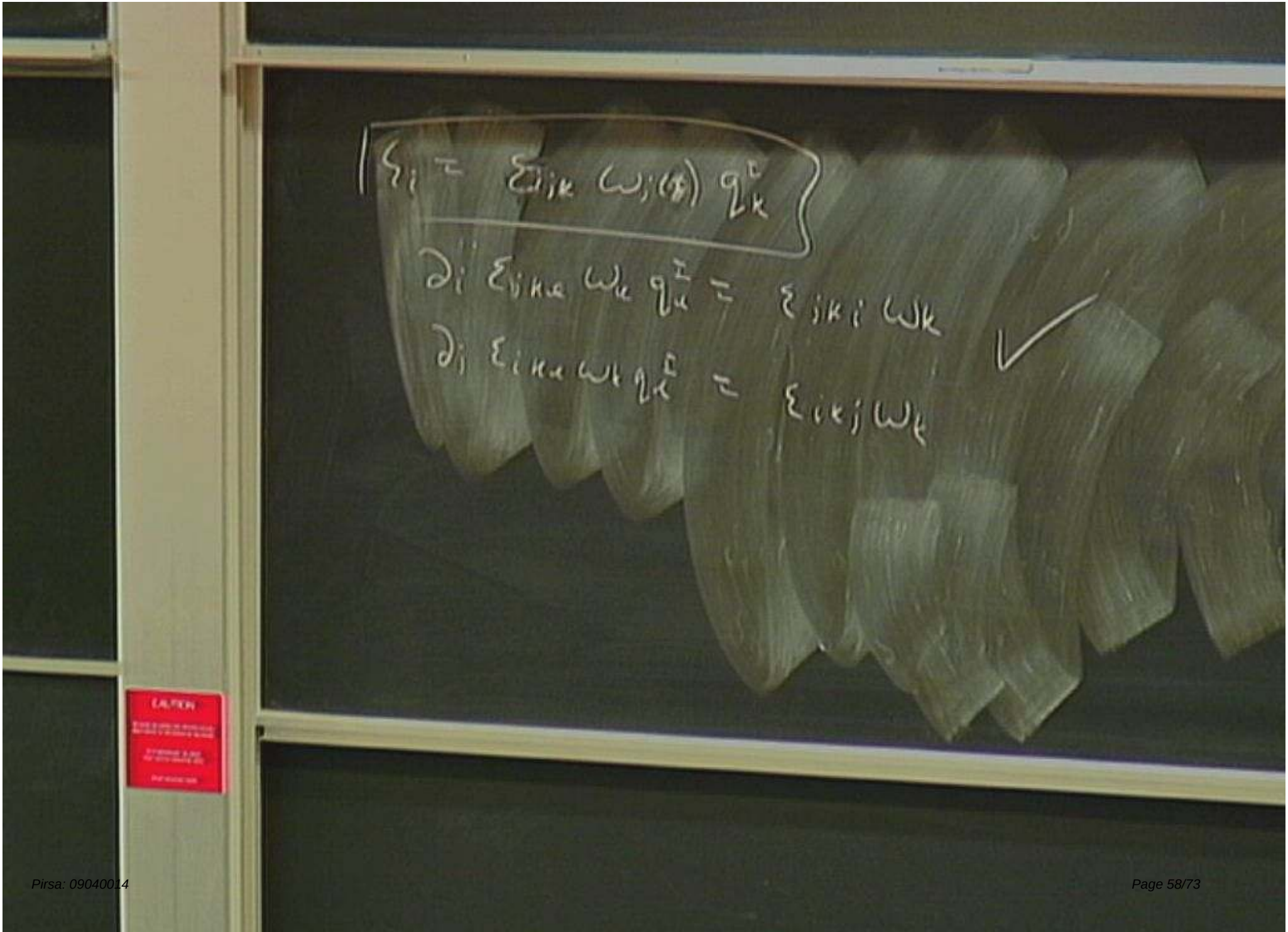
Sol'ns. $\boxed{\vec{\zeta} = \vec{a}} = \overrightarrow{\text{const}}\,(t)$ ✓

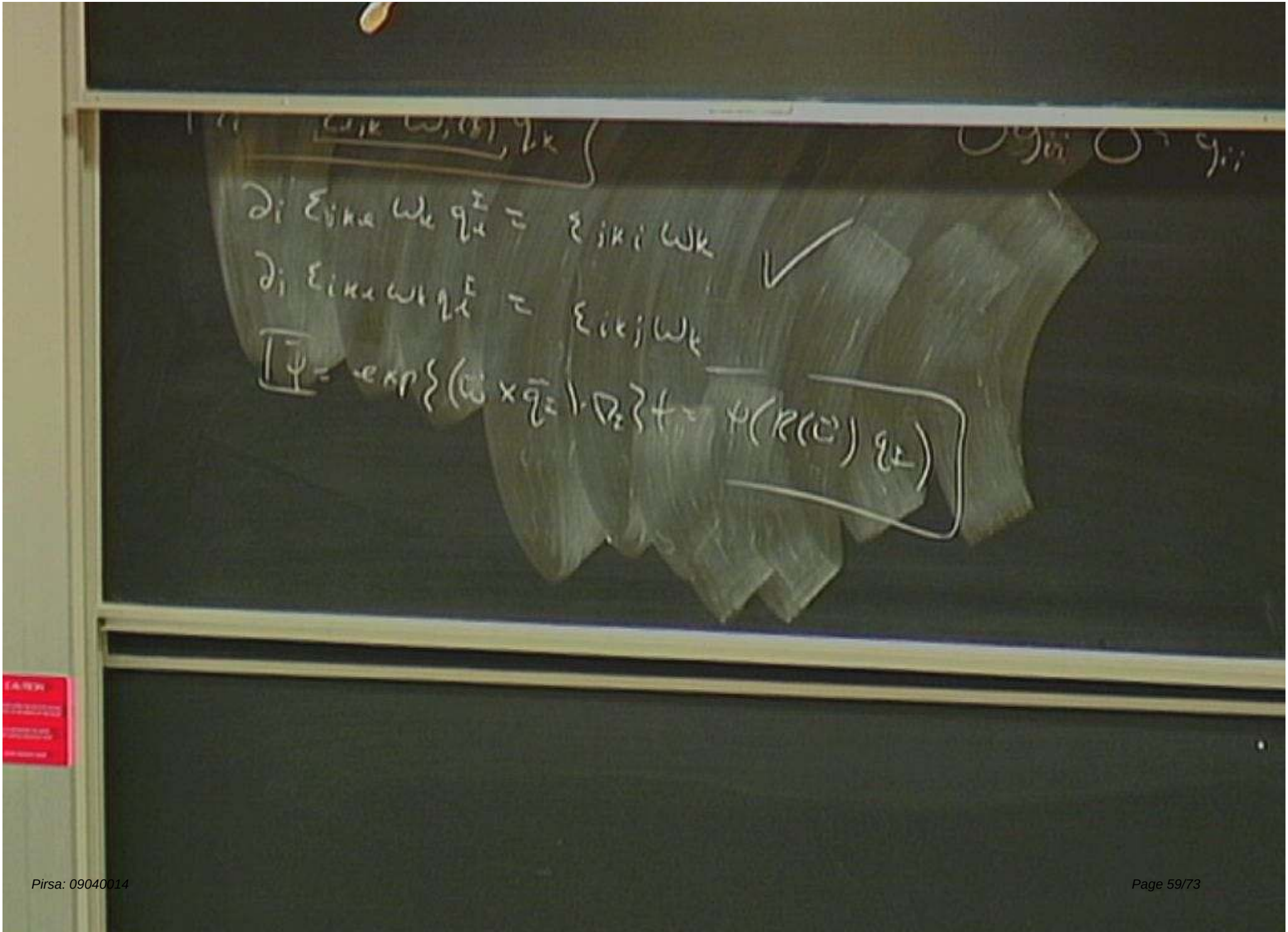
$$\boxed{\bar{\psi} \to \exp\{\mathcal{L}_{\vec{a}}\}\psi = \exp\{\sum_I \vec{a}\cdot\vec{\nabla}_I\}\psi = \psi(\vec{q}_c - \vec{a})}$$

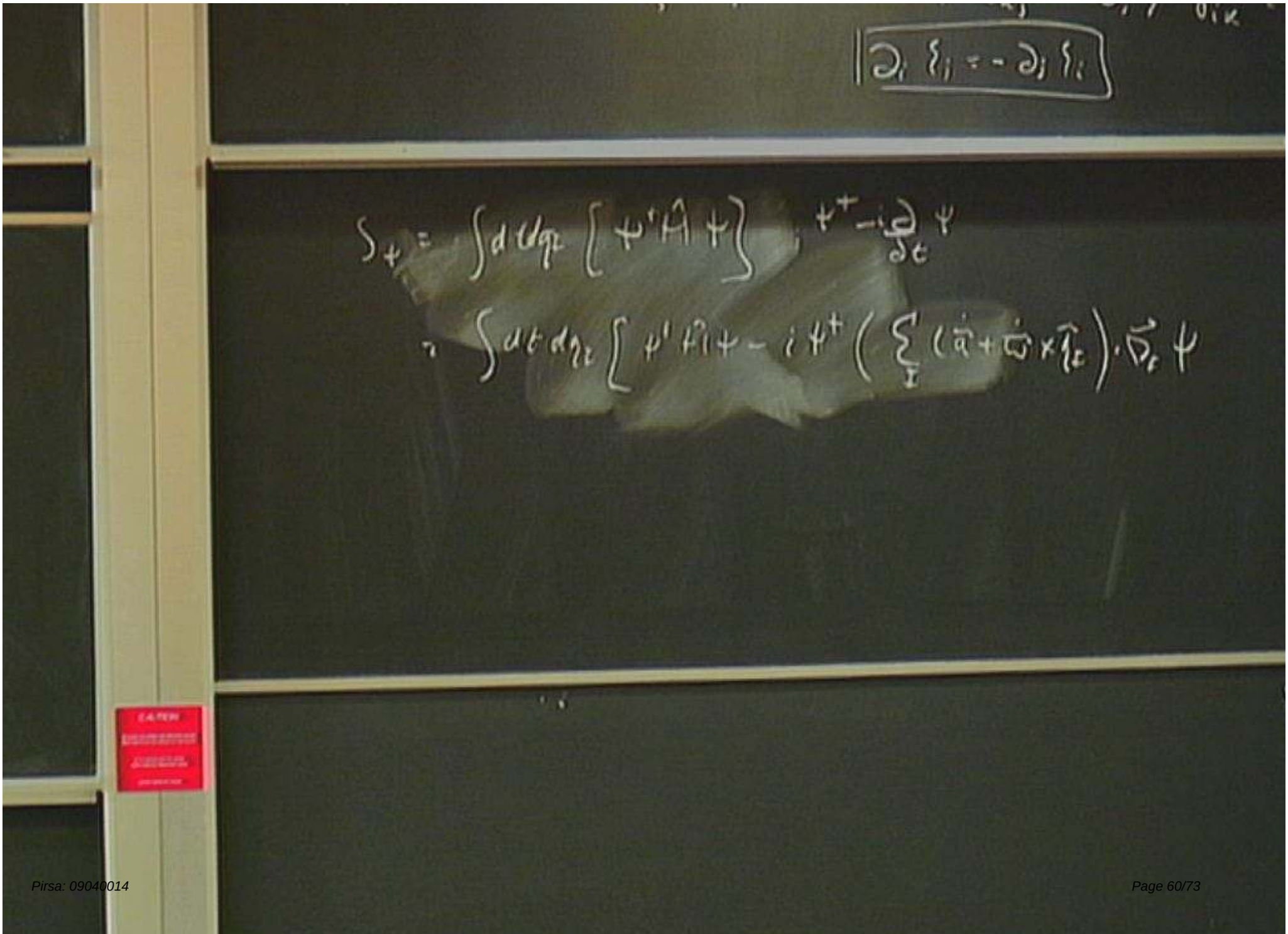
$\psi(\vec{q}_c)$

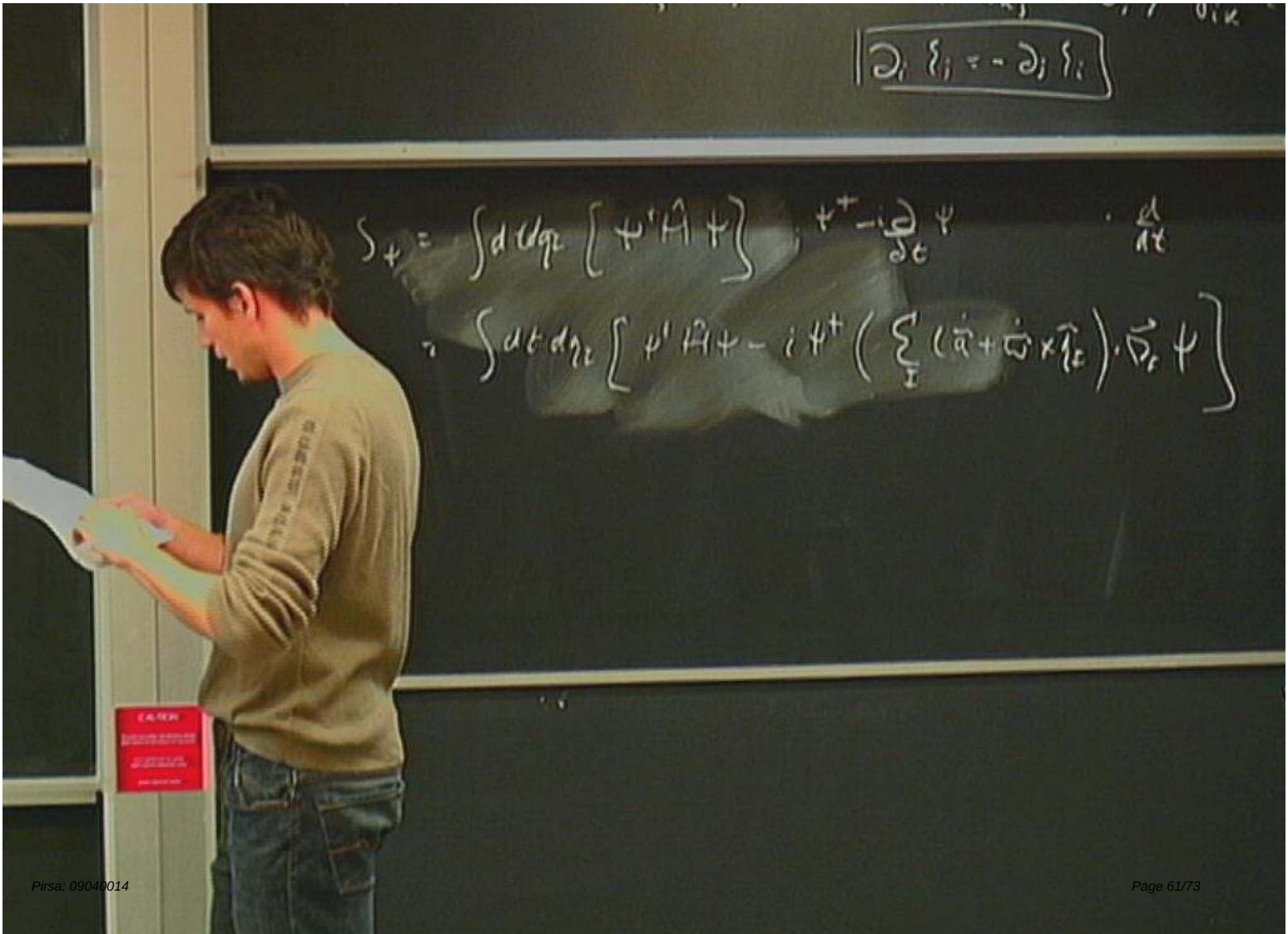

$\dot{\xi}_i = \varepsilon_{ijk}\,\omega_j(t)\,q^0_k$


$\xi_i = \varepsilon_{ijk}\,\omega_j(t)\,q^0_k$
$\partial_i\,\varepsilon_{ikl}\,\omega_k\,q^1_l = \varepsilon_{jki}\,\omega_k$
$\partial_i\,\varepsilon_{ikl}\,\omega_k\,q^2_l = \varepsilon_{ikj}\,\omega_k$


$\xi_i = \varepsilon_{ijk}\,\omega_j(t)\,q^2_k$
$\partial_i\,\varepsilon_{jka}\,\omega_k\,q^2_a = \varepsilon_{jki}\,\omega_k$
$\partial_i\,\varepsilon_{ika}\,\omega_k\,q^2_a = \varepsilon_{ikj}\,\omega_k$


$\partial_i \varepsilon_{ikl} \omega_k q_l^{\Sigma} = \varepsilon_{ikl} \omega_k$
$\partial_i \varepsilon_{ikl} \omega_k q_l^{E} = \varepsilon_{ikl} \omega_k$
$\Psi = \exp\{(\vec{\omega} \times \vec{q}_\Sigma) \cdot \nabla_\Sigma\} \psi = \psi(R(\vec{\omega}) q_\Sigma)$



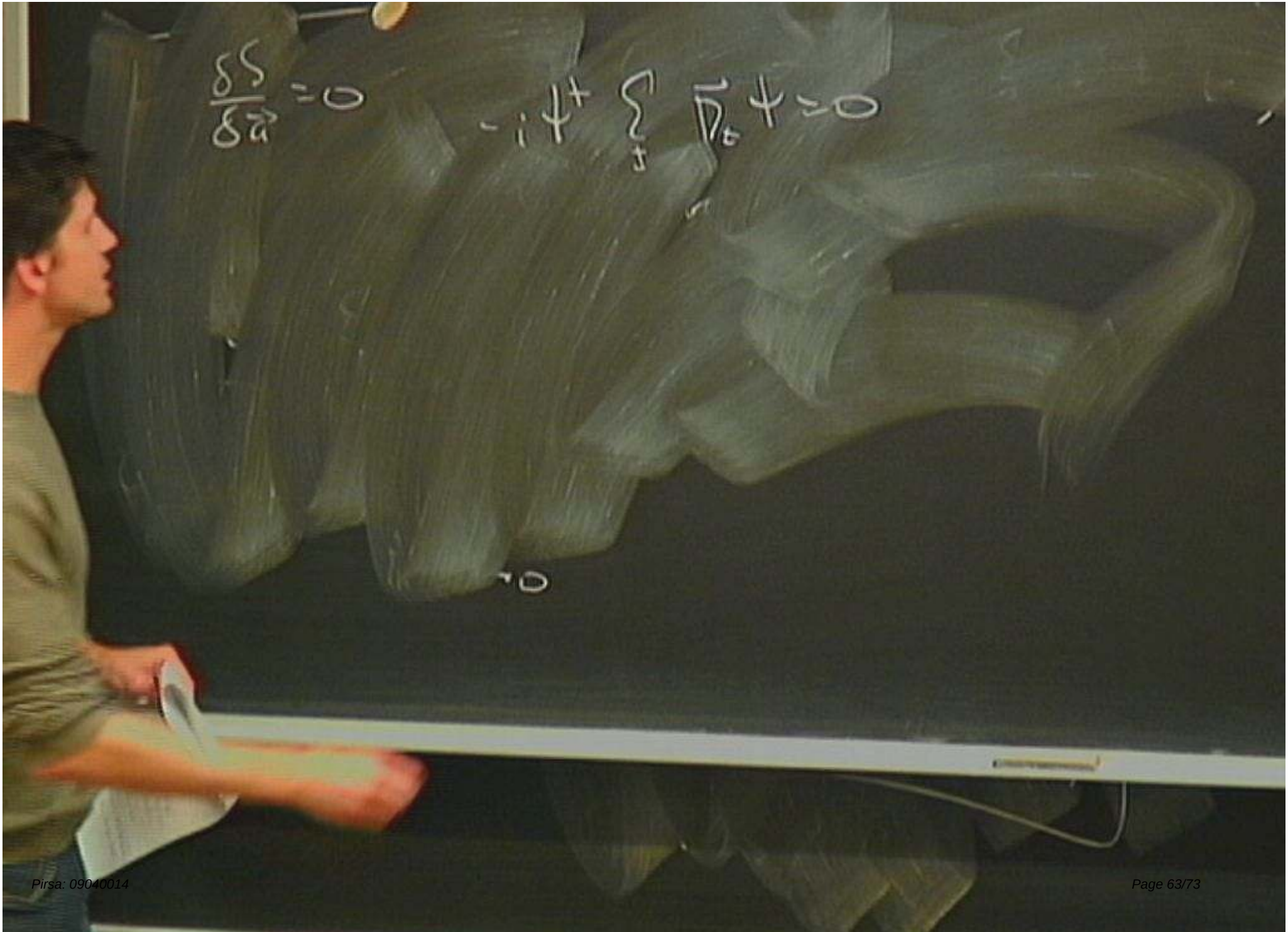

$\boxed{\partial_i \xi_j = -\partial_j \xi_i}$
$S_\psi = \int dt\, dq_I \left[\psi^\dagger \hat{H} \psi \right]$
$\psi^\dagger - i \frac{\partial}{\partial t} \psi$
$\frac{d}{dt}$
$= \int dt\, dq_I \left[\psi^\dagger \hat{H} \psi - i \psi^\dagger \left(\sum_I (\dot{\vec{a}} + \dot{\vec{\omega}} \times \vec{q}_I) \cdot \vec{\nabla}_I \psi \right] \right.$

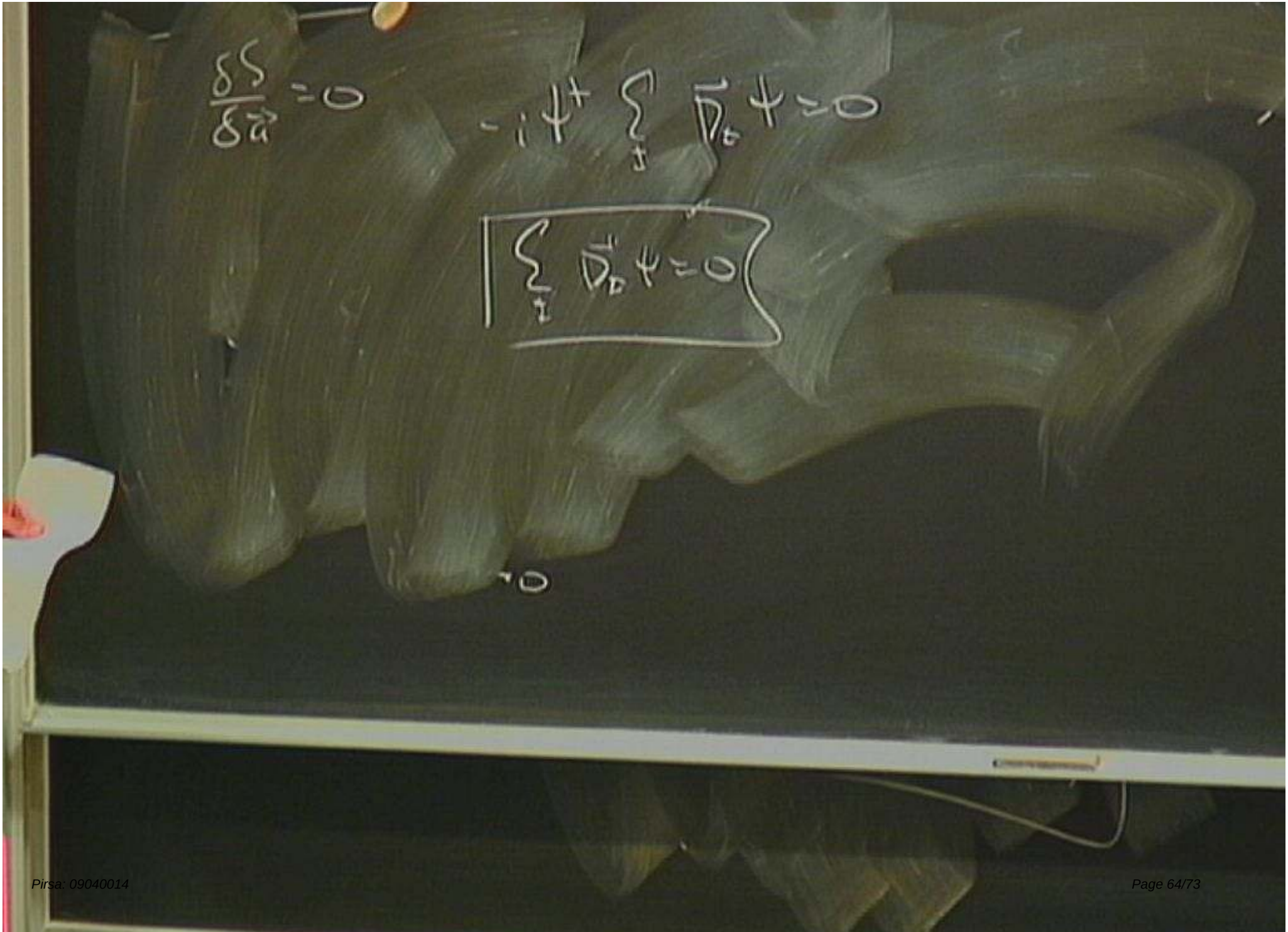
$$\boxed{\partial_i \xi_j = -\partial_j \xi_i}$$

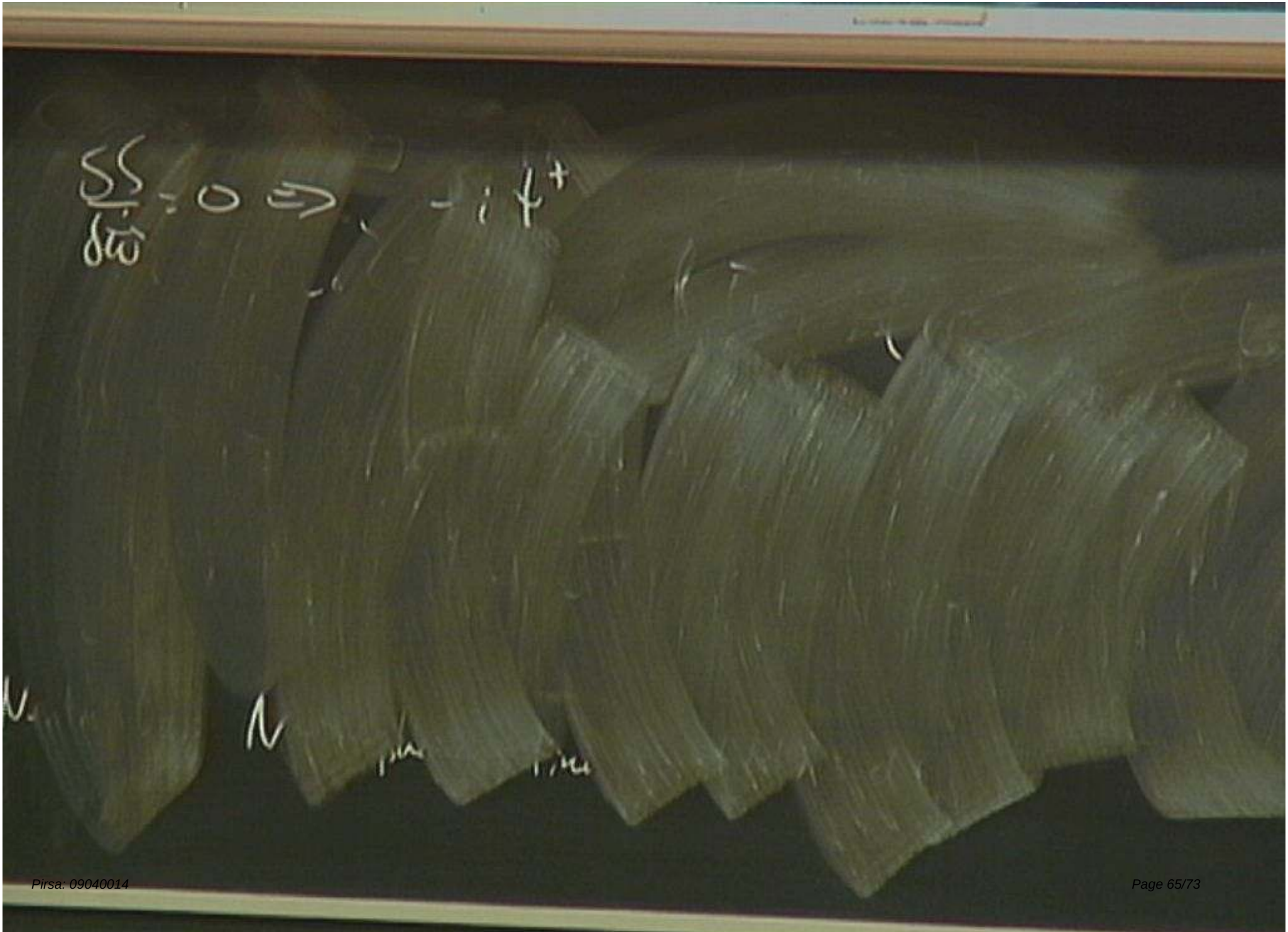
$$S_\psi = \int dt\, dq_I \left[\psi^\dagger \hat{H} \psi \right] \quad ; \quad \psi^\dagger - i\frac{\partial}{\partial t}\psi \qquad \cdot \frac{d}{dt}$$

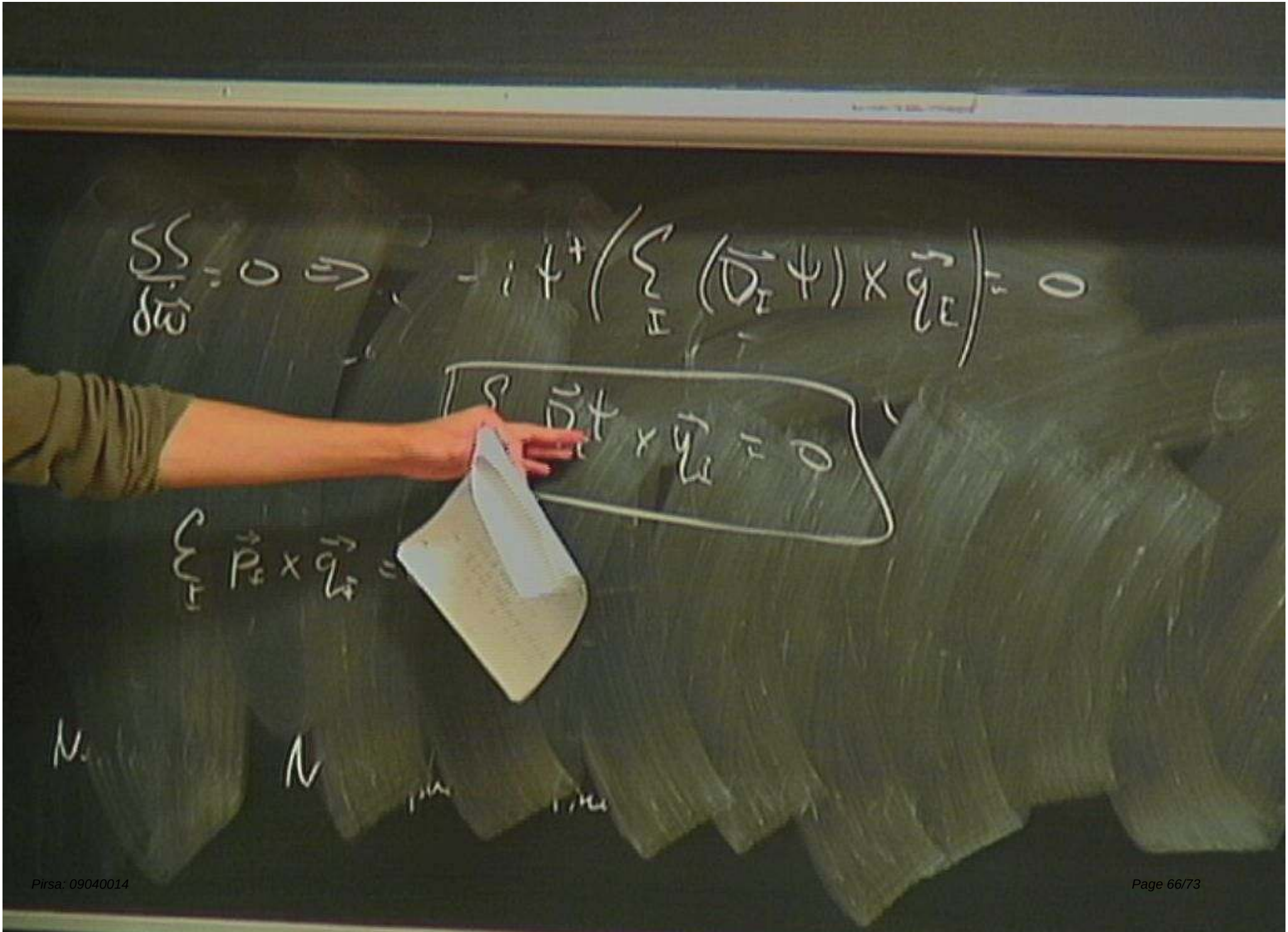
$$= \int dt\, dq_I \left[\psi^\dagger \hat{H} \psi - i\psi^\dagger \left(\sum_I (\dot{\vec{a}} + \dot{\vec{\omega}} \times \vec{q}_I) \cdot \vec{\nabla}_I \psi \right] \right.$$

Fepv. $\vec{a}, \vec{\omega}$


$\frac{\delta S}{\delta \vec{a}} = 0$
$-i\psi^{\dagger} \sum_{j} \vec{\nabla}_{b} \psi = 0$
$= 0$


$\frac{\delta S}{\delta \vec{a}} = 0$
$-i\psi^\dagger \sum_j \vec{D}_b \psi = 0$
$\sum_i \vec{D}_b \psi = 0$
$= 0$


$\frac{\delta S}{\delta \dot{\omega}} = 0 \Rightarrow$
$-i f^+$


$\frac{\delta S}{\delta \omega} = 0 \Rightarrow$
$-i\hbar^{\dagger}\left(\sum_I (\vec{\nabla}_I \Psi) \times \vec{p}_I\right) = 0$
$\sum \vec{p}_I \times \vec{q}_I = 0$
$\sum_I \vec{P}_I \times \vec{q}_I =$
N
N

$$\frac{\delta S}{\delta \vec{\omega}} = 0 \Rightarrow -i\hbar^{\dagger}\left(\sum_I (\vec{\nabla}_I \psi) \times \vec{q}_I\right) = 0$$

$$\boxed{\sum_i \vec{\nabla}_i t \times \vec{q}_i = 0}$$

$$\sum_i \vec{p}_i \times \vec{q}_i = 0$$

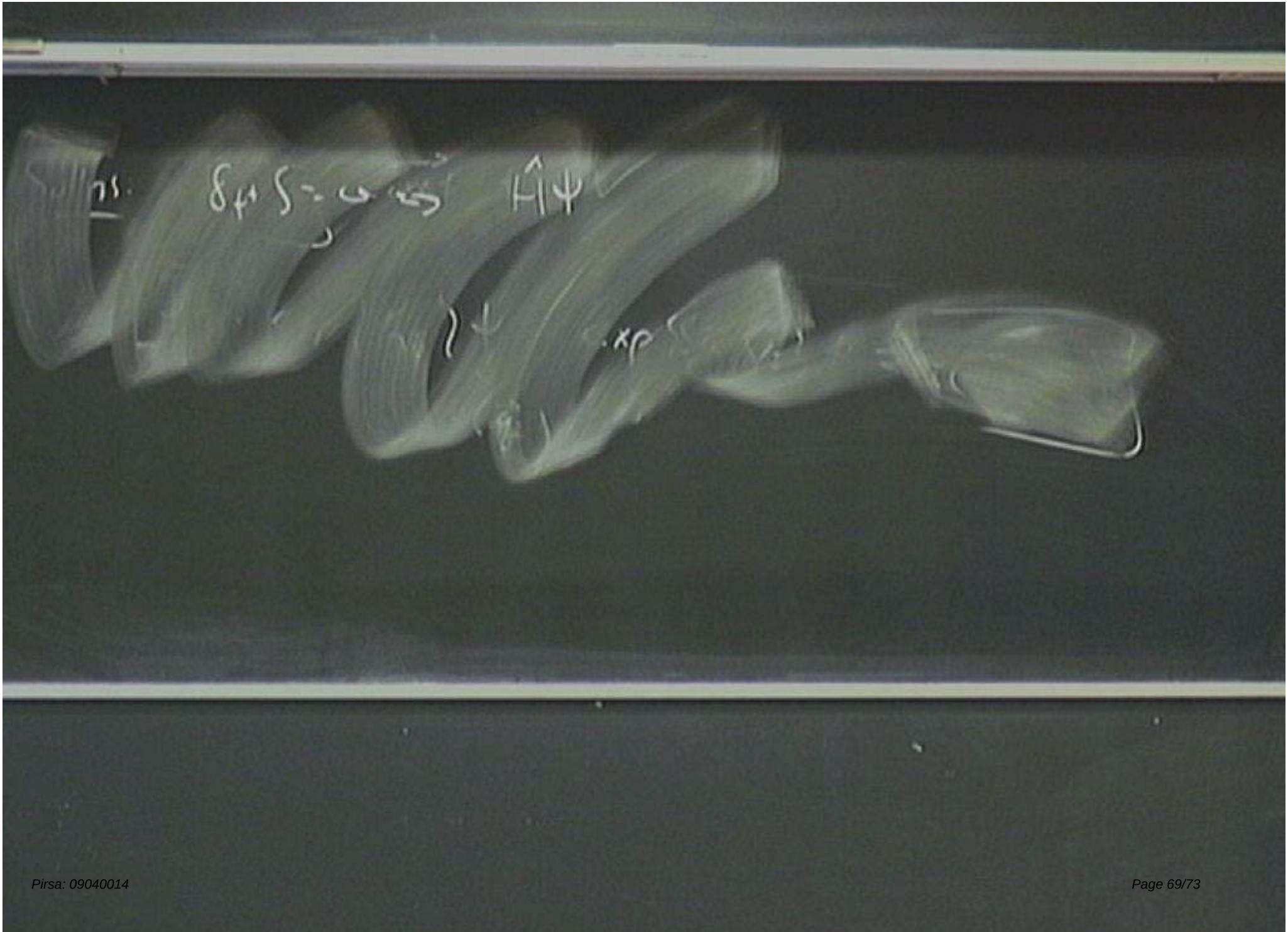
N. N

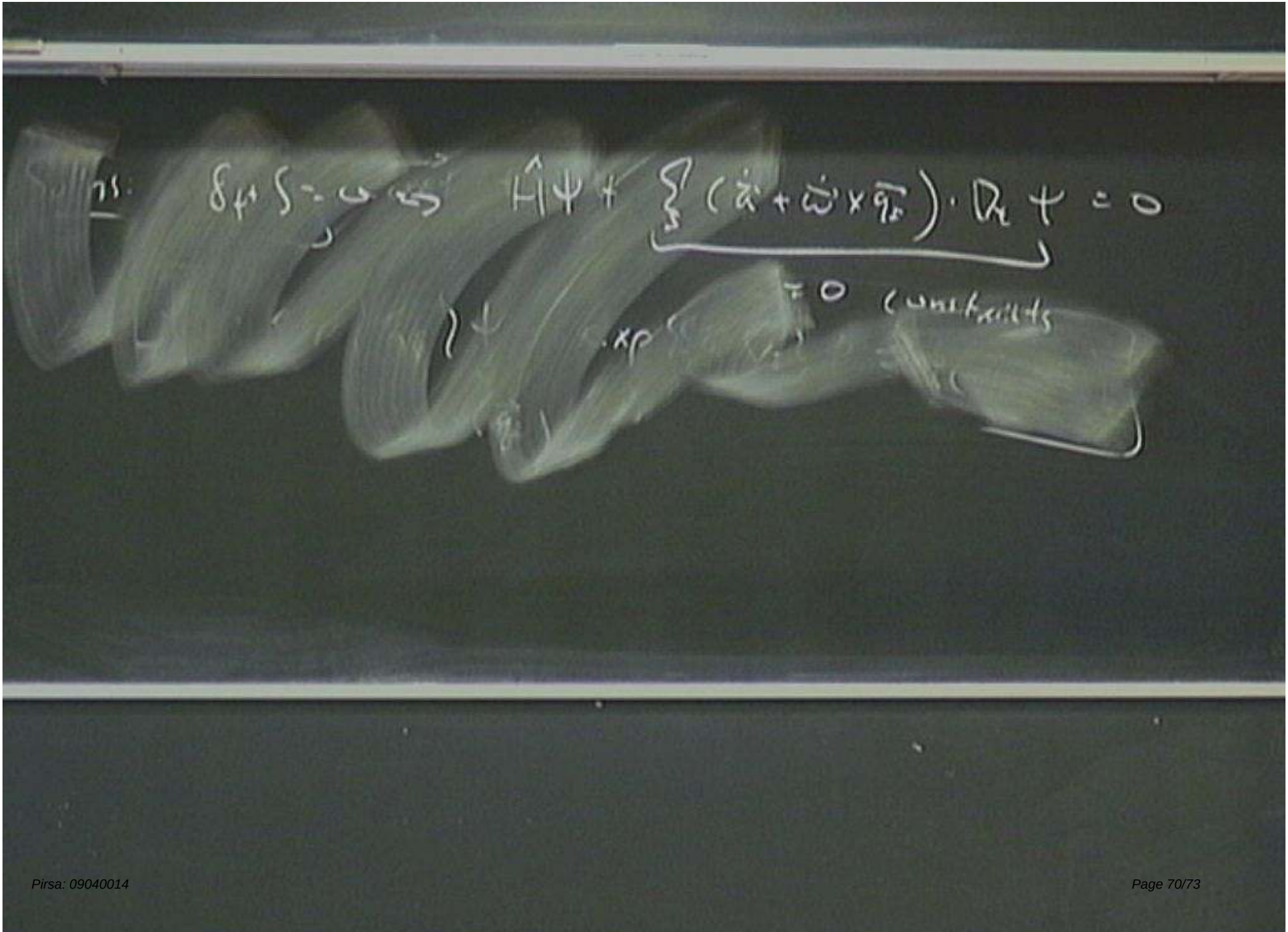
$$\frac{\delta S}{\delta \vec{\omega}} = 0 \Rightarrow -i\psi^{+}\left(\sum_I (\vec{\nabla}_I \psi) \times \vec{q}_I\right) = 0$$

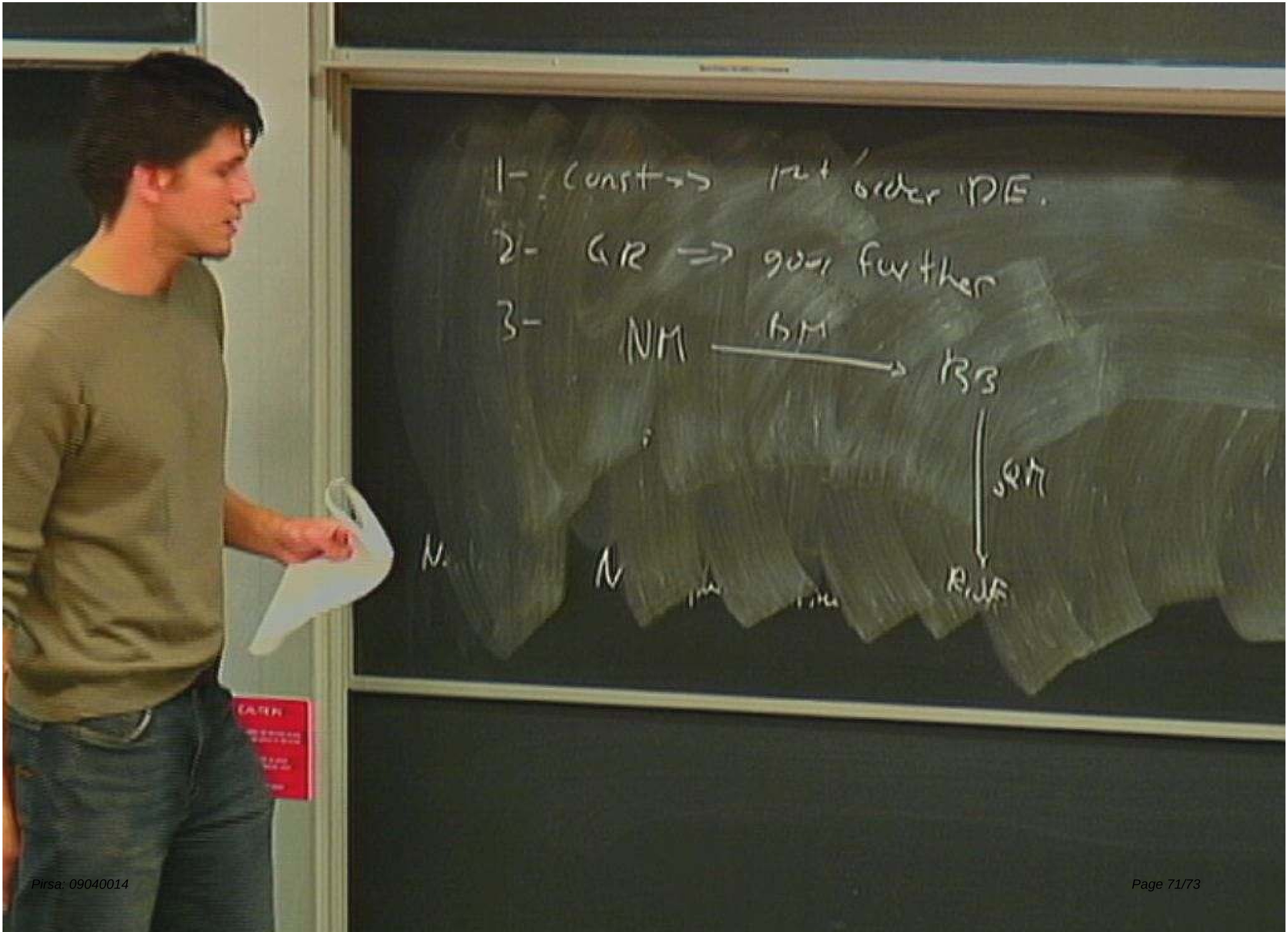
$$\boxed{\sum_I \vec{\nabla}_I \psi \times \vec{q}_I = 0} = -\sum_I \vec{q}_I \times \vec{\nabla}\psi_I = 0$$

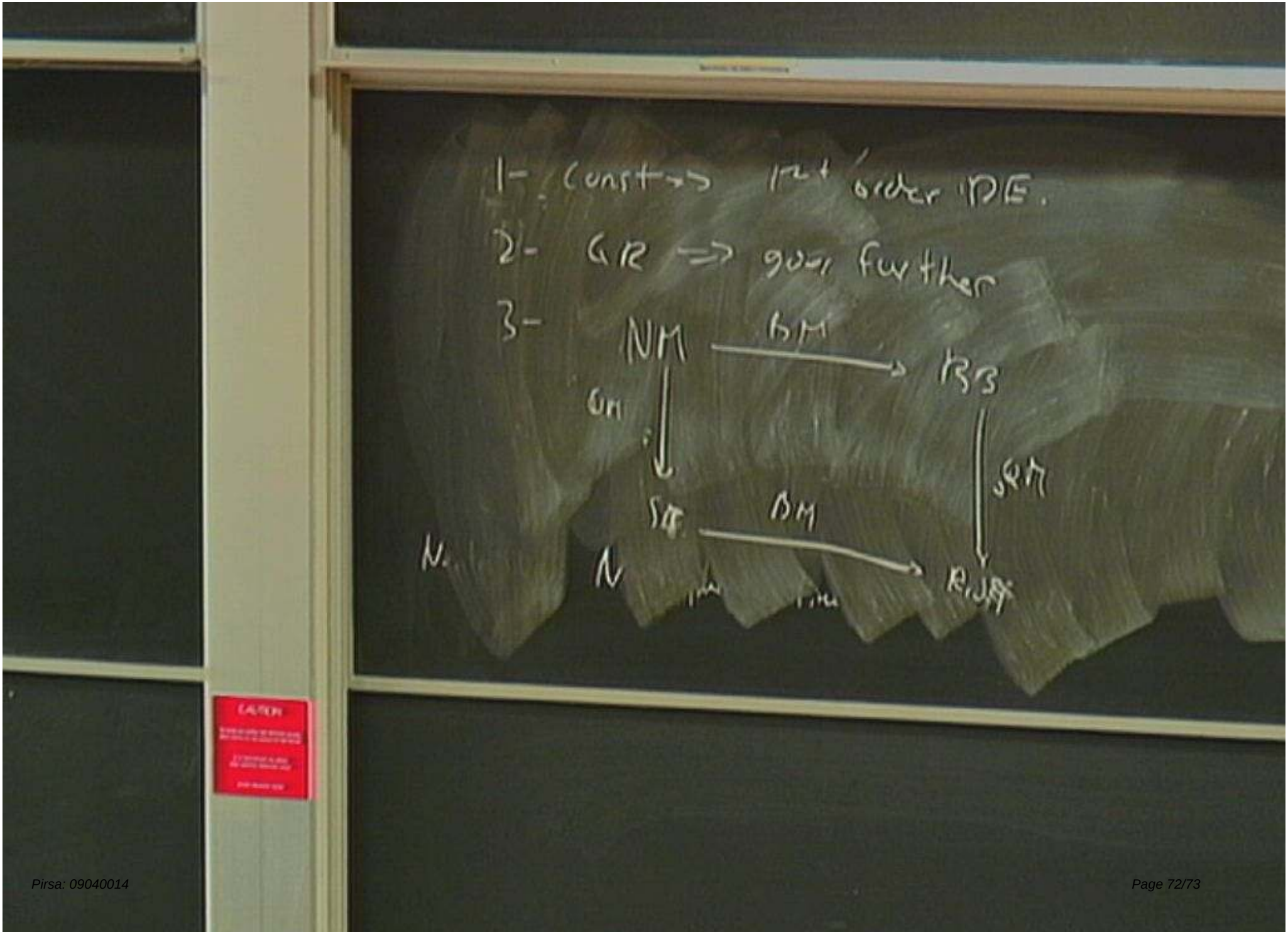
$$\sum_I \vec{p}_I \times \vec{q}_I = 0 \qquad -\sum \vec{q}_I \times \vec{p}_I = 0$$

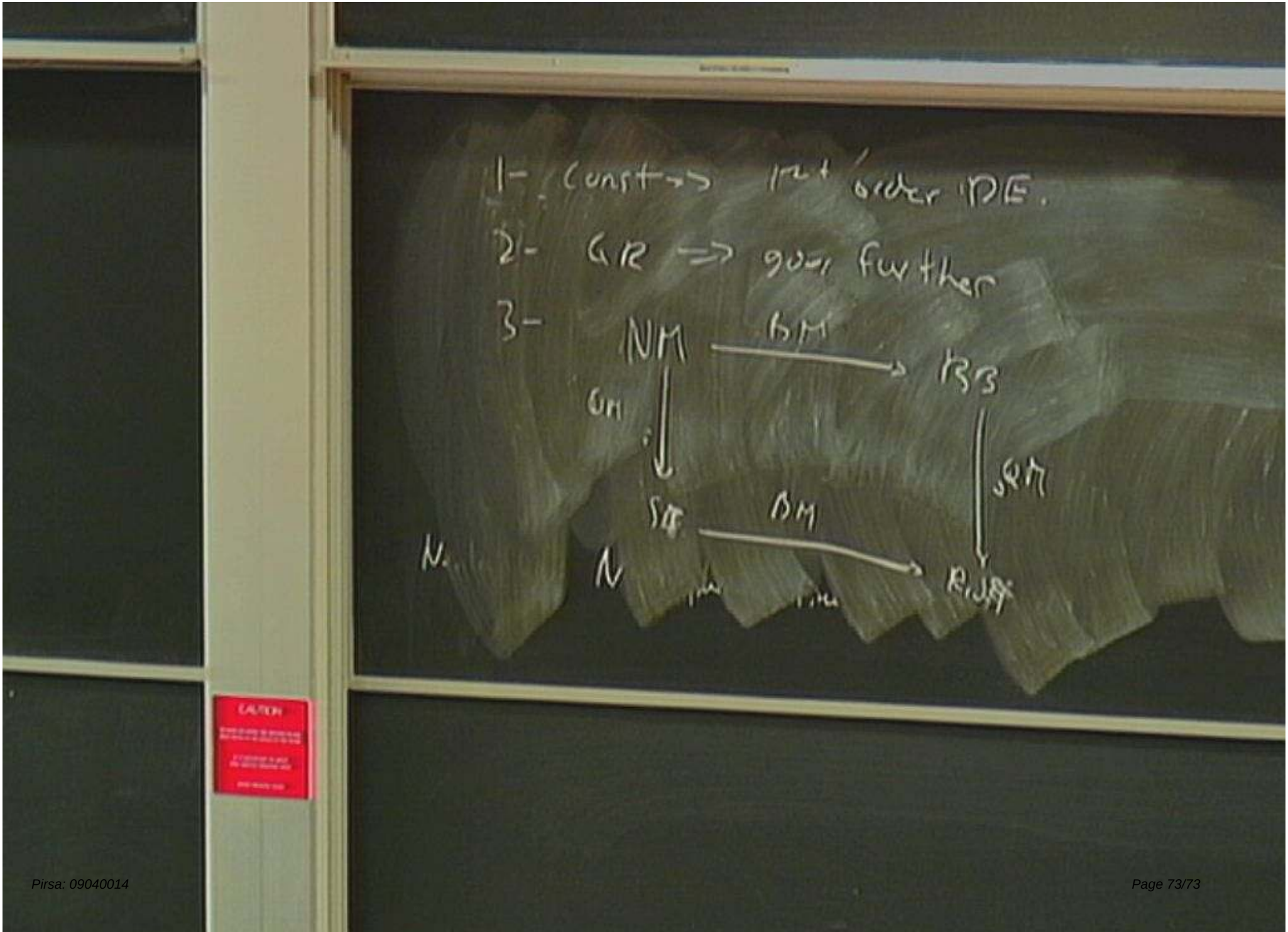
N. N






1- Const → 1st order DE.
2- GR ⇒ go further
3- NM
BM
BS
CAUTION


1- Const => 1st order DE.
2- GR => goes further
3-
NM
BM
BB
GM
SM
BM
N
N


1- Const => 1st order DE.
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NM
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N