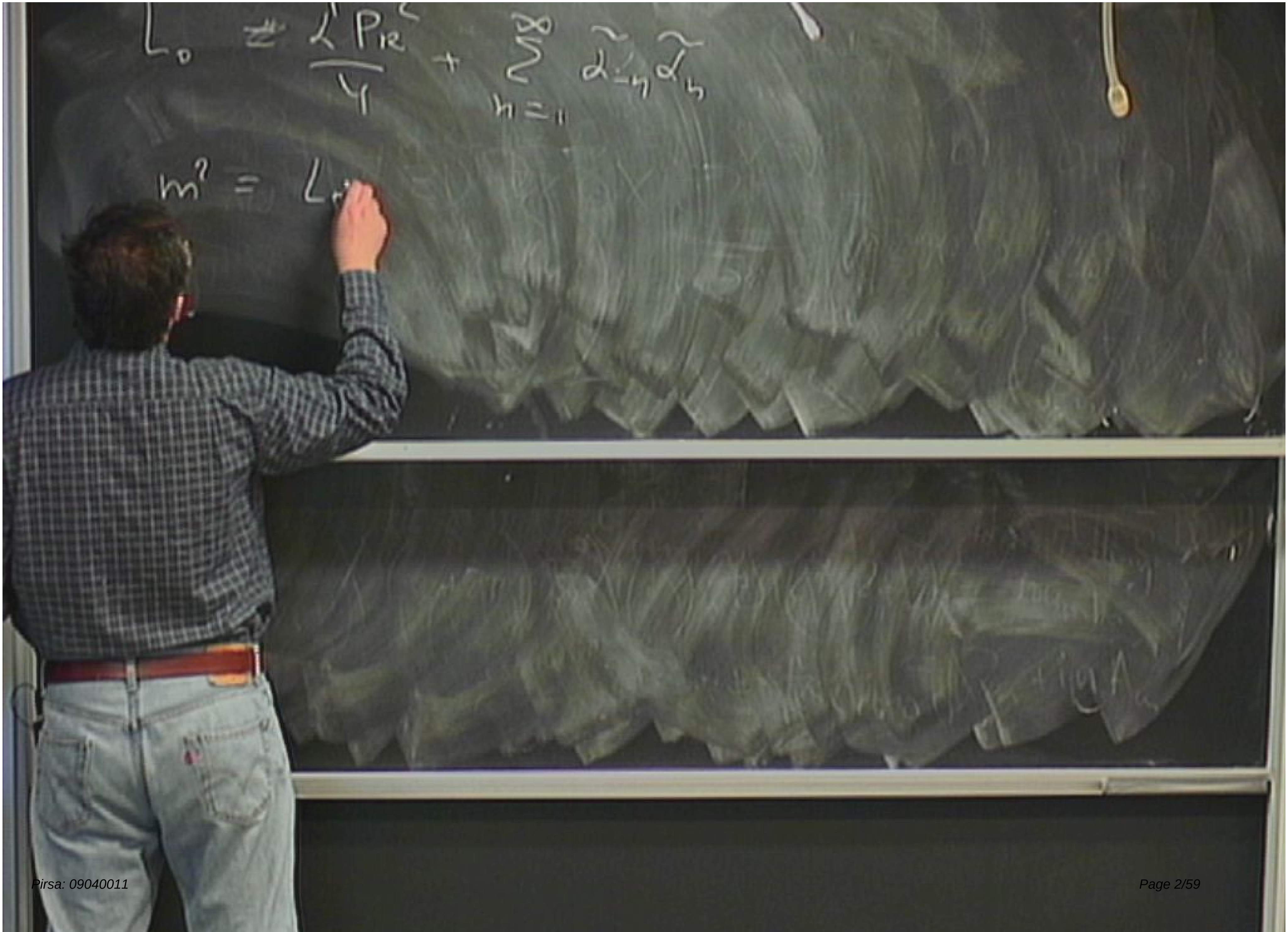
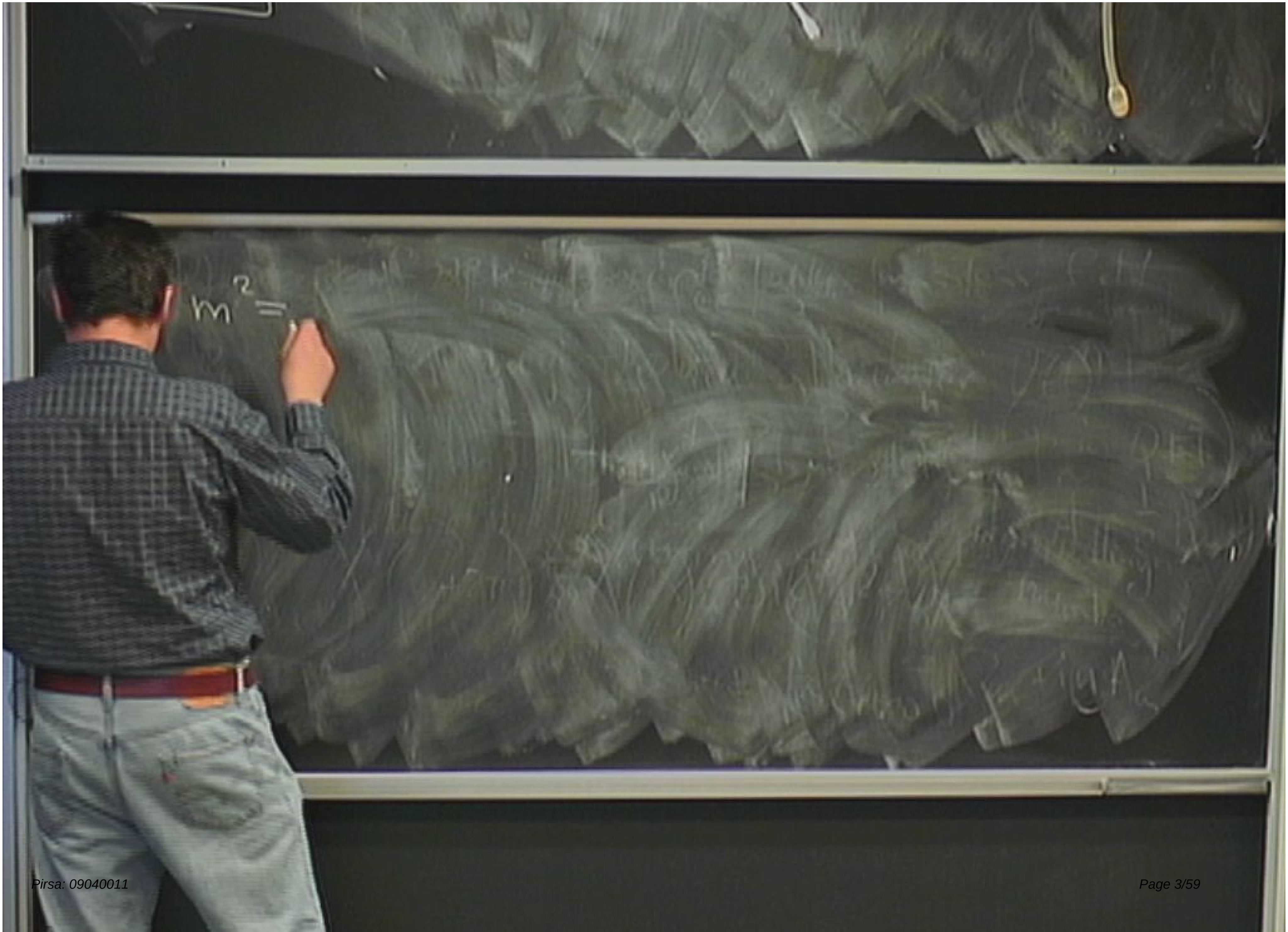
Title: Introduction to the Bosonic String Part B

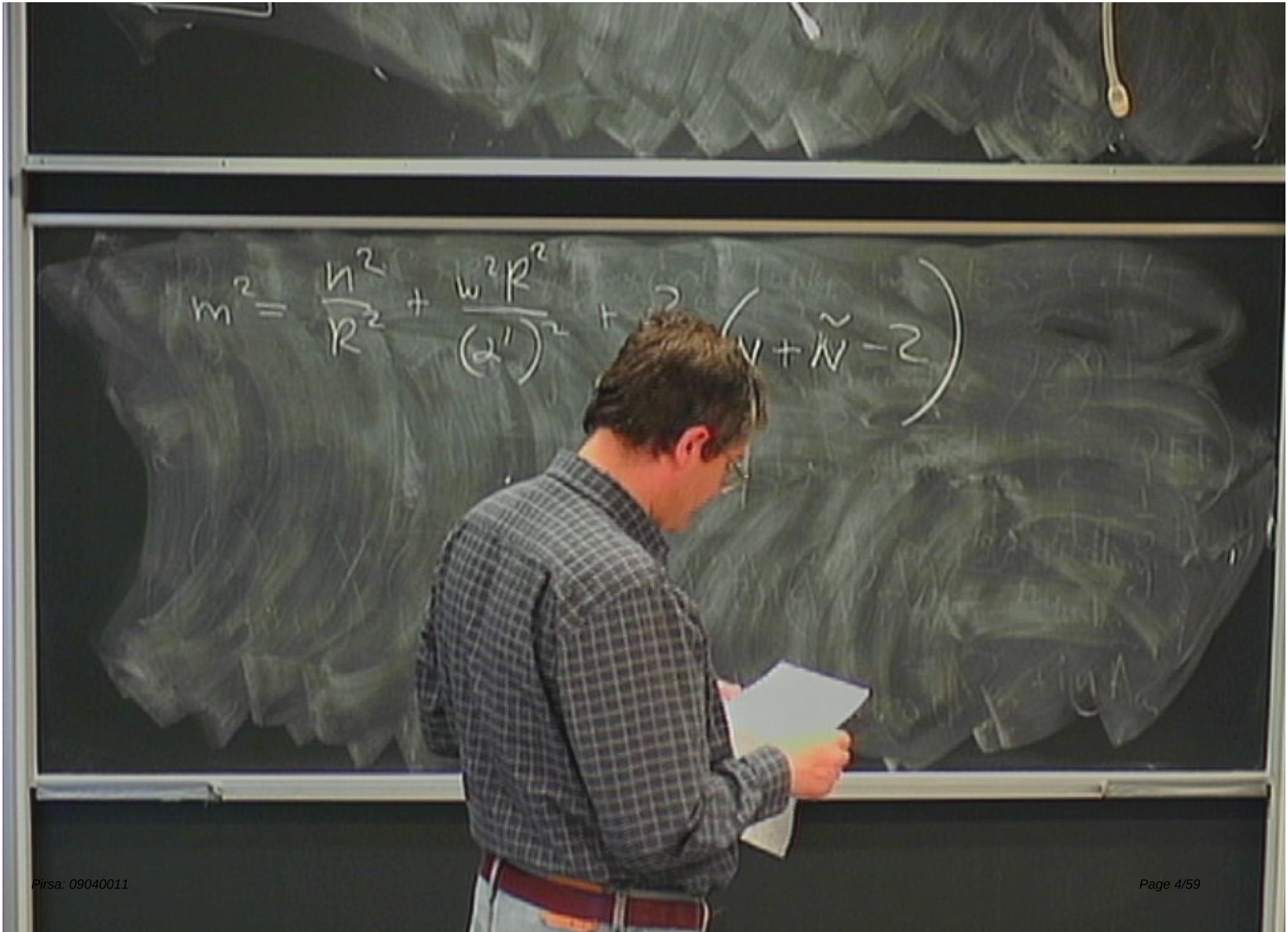
Date: Apr 17, 2009 12:00 PM

URL: http://pirsa.org/09040011

Abstract: This course provides a thorough introduction to the bosonic string based on the Polyakov path integral and conformal field theory. We introduce central ideas of string theory, the tools of conformal field theory, the Polyakov path integral, and the covariant quantization of the string. We discuss string interactions and cover the tree-level and one loop amplitudes. More advanced topics such as T-duality and D-branes will be taught as part of the course. The course is geared for M.Sc. and Ph.D. students enrolled in Collaborative Ph.D. Program in Theoretical Physics. Required previous course work: Quantum Field Theory (AM516 or equivalent). The course evaluation will be based on regular problem sets that will be handed in during the term. The primary text is the book: 'String theory. Vol. 1: An introduction to the bosonic string. J. Polchinski (Santa Barbara, KITP) . 1998. 402pp. Cambridge, UK: Univ. Pr. (1998) 402 p.' All interested students should contact Alex Buchel at abuchel@uwo.ca as soon as possible.


$L_0 = \frac{\alpha' p_R^2}{4} + \sum_{n=1}^{\infty} \tilde{\alpha}_{-n} \tilde{\alpha}_n$
$m^2 = L$


$m^2 =$


$m^2 = \frac{n^2}{R^2} + \frac{w^2R^2}{(\alpha')^2} + 2 \left(N + \tilde{N} - 2 \right)$

$$m^2 = \frac{n^2}{R^2} + \frac{w^2 R^2}{(\alpha')^2} + \frac{2}{\alpha'}\left(N + \tilde{N} - 2\right)$$

$$N - \tilde{N} = 0$$

$$m^2 = \frac{n^2}{R^2} + \frac{w^2 R^2}{(\alpha')^2} + \frac{2}{\alpha'}\left(N + \tilde{N} - 2\right)$$

$$nw + N - \tilde{N} = 0$$

KK
massive
states

A_m' $\quad U(1)$

$$E = T \cdot 2\pi w R = \frac{1}{2\pi\alpha'}\, 2\pi w R = \frac{wR}{\alpha'}$$

$$m^2 = \frac{n^2}{R^2} + \frac{w^2 R^2}{(\alpha')^2} + \frac{2}{\alpha'}\left(N + \tilde{N} - 2\right)$$

from transverse oscillatory

$$nw + N - \tilde{N} = 0$$

KK massive states

$$\rightarrow E = T \, 2\pi w R = \frac{1}{2\pi\alpha'} 2\pi w R = \frac{wR}{\alpha'}$$

$A_\mu{}'$ $U(1)$

$$m^2 = \frac{n^2}{R^2} + \frac{w^2 R^2}{(\alpha')^2} + \frac{2}{\alpha'}\left(N + \tilde{N} - 2\right)$$

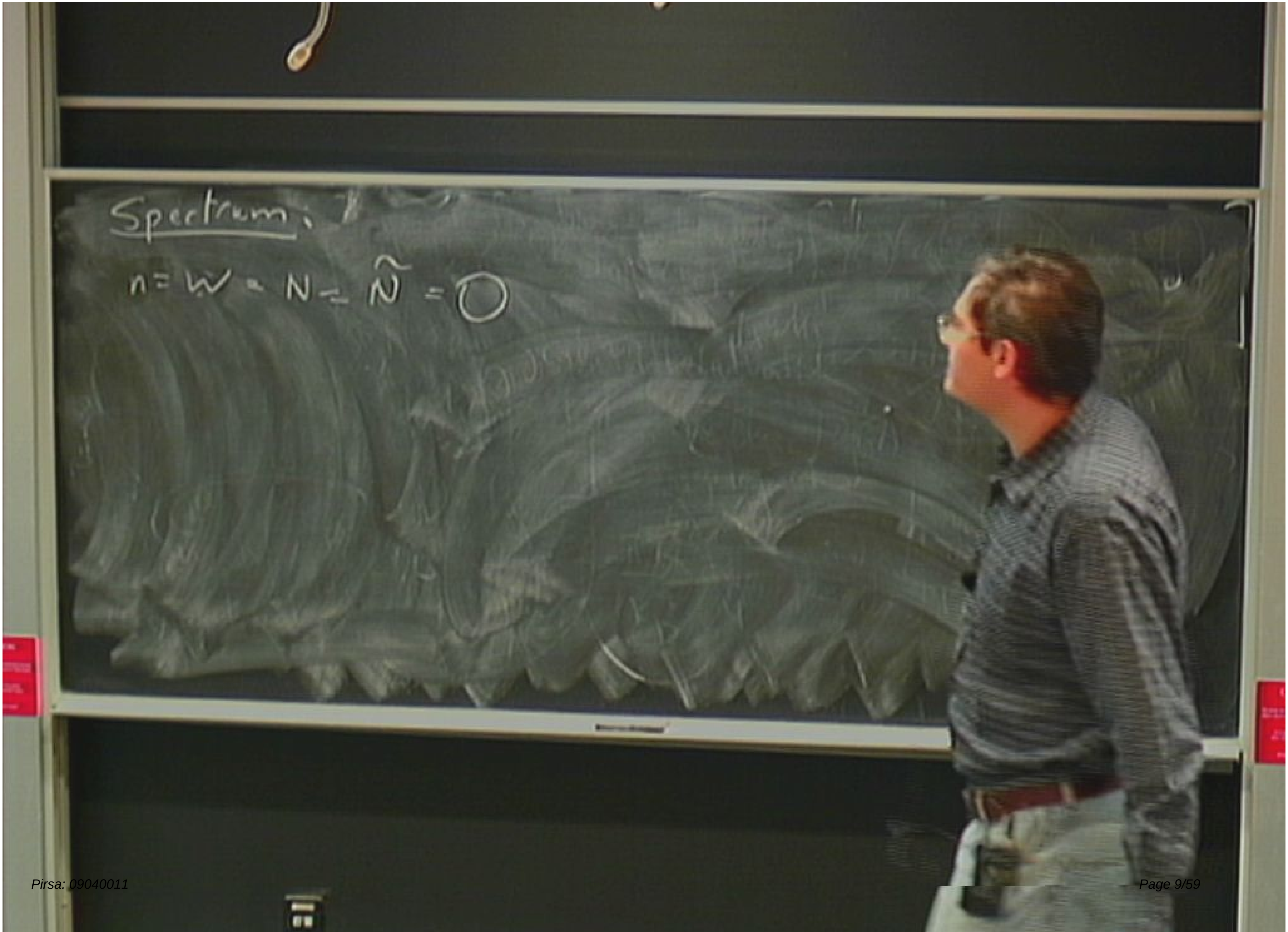
from transverse oscillatory

$$nw + N - \tilde{N} = 0$$

KK massive states

A_m $U(1)$

$$E = T\, 2\pi w R = \frac{1}{2\pi\alpha'} 2\pi w R = \frac{wR}{\alpha'}$$


Spectrum:
n = W = N = Ñ = 0

Spectrum:

$n = w = N = \tilde{N} = 0$ $\qquad m^2 = -\frac{4}{\alpha'}$ ✓ tachyon as before

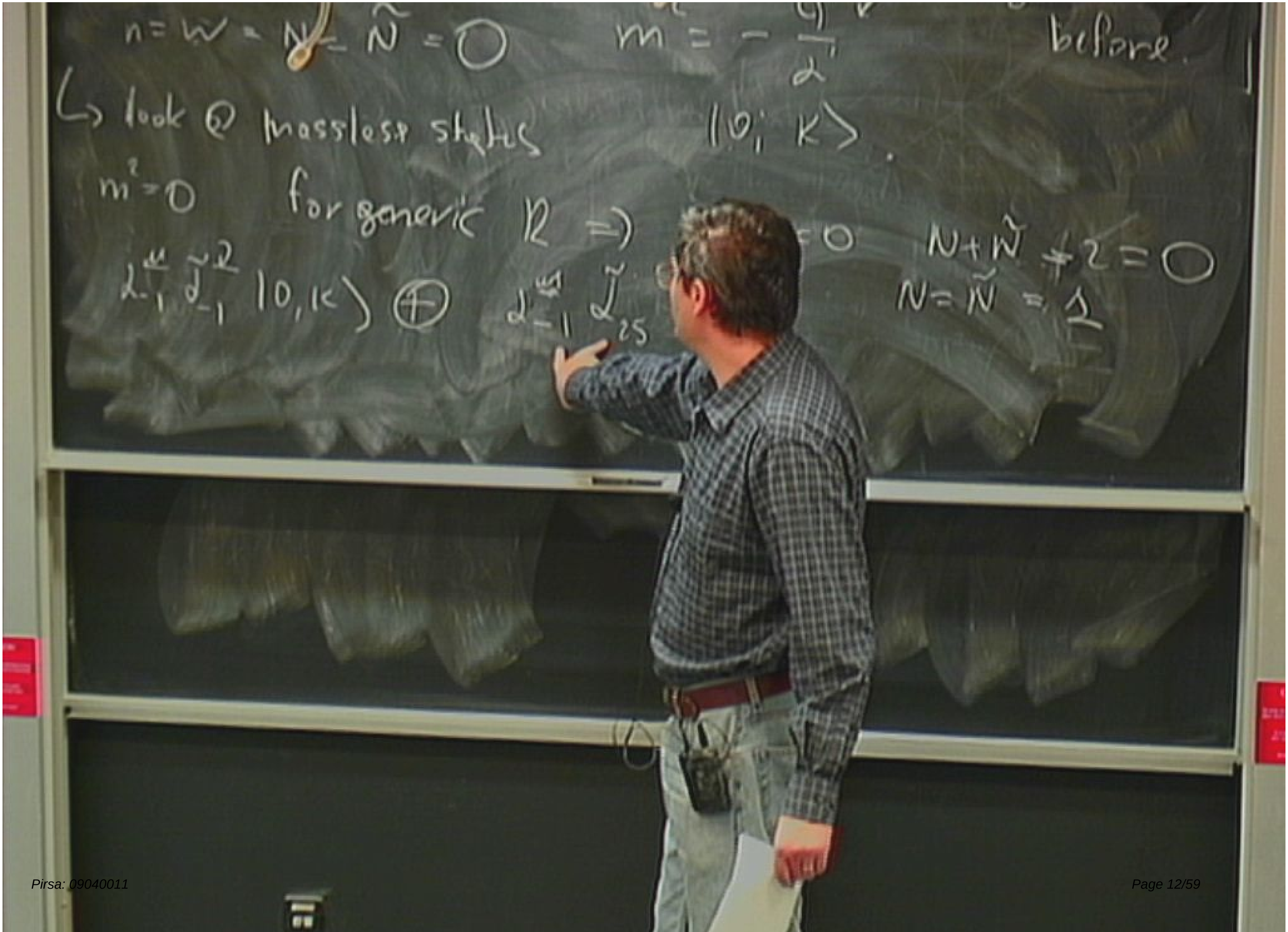
$|0; k\rangle$

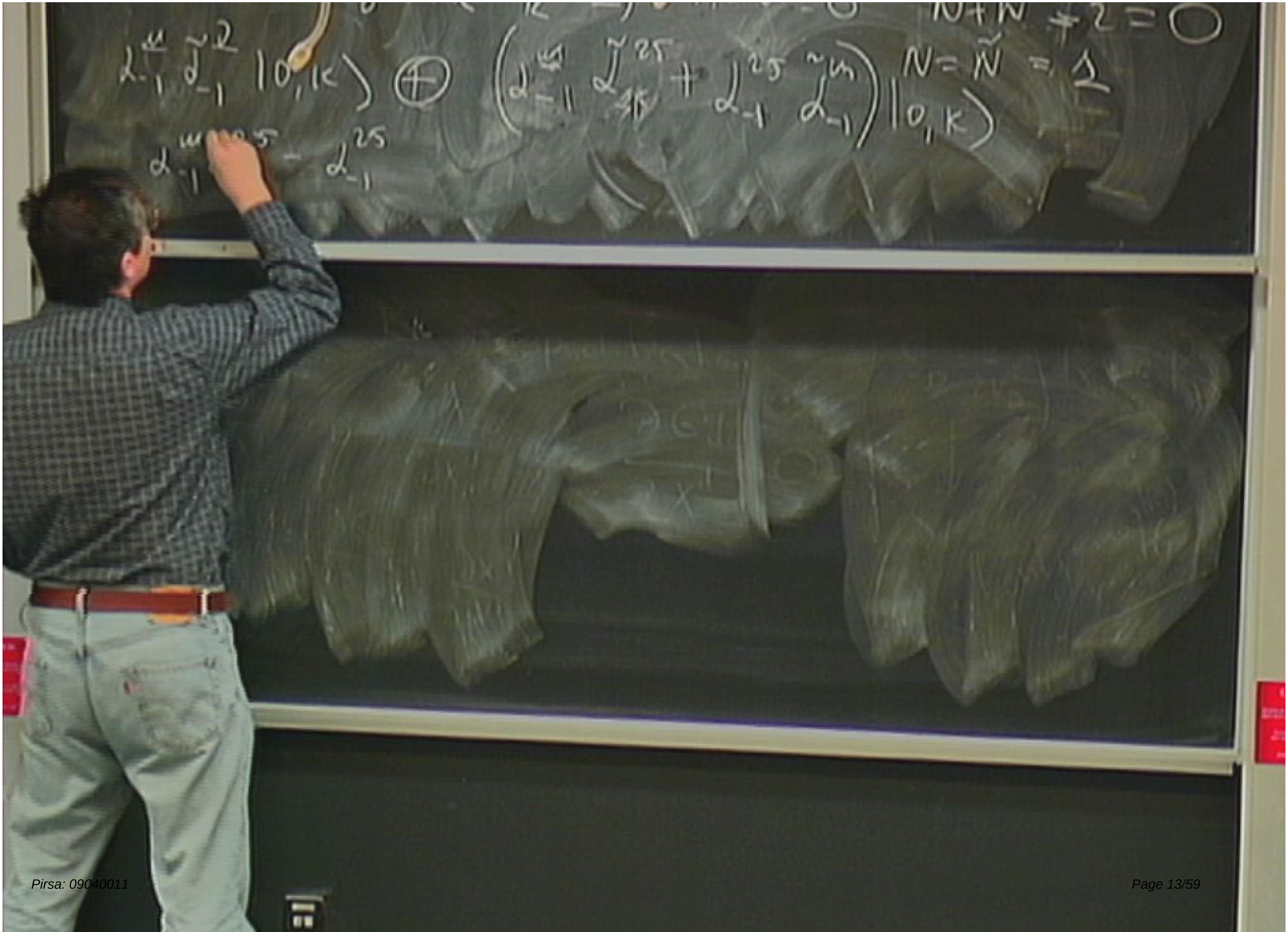
$n = w = N = \tilde{N} = 0 \qquad m^2 = -\frac{4}{\alpha'}$ tachyon as before.

$\hookrightarrow$ look @ massless states $|0; k\rangle$

$m^2 = 0$ for generic $R \Rightarrow n = w = 0 \quad N + \tilde{N} - 2 = 0$

$N = \tilde{N} = 1$


n = w = N − Ñ = 0
m = − q/2
before.
look @ massless states
|0; k⟩.
m² = 0
for generic R ⇒
N + Ñ
= 0
N = Ñ = 1
|0, k⟩ ⊕



$$\alpha_{-1}^{\mu}\tilde{\alpha}_{-1}^{\nu}|0,k\rangle \oplus \left(\alpha_{-1}^{\mu}\tilde{\alpha}_{-1}^{25} + \alpha_{-1}^{25}\tilde{\alpha}_{-1}^{\mu}\right)|0,k\rangle \qquad N = \tilde{N} = 1$$

$$\oplus\left(\alpha_{-1}^{\mu}\tilde{\alpha}_{-1}^{25} - \alpha_{-1}^{25}\tilde{\alpha}_{-1}^{\mu}\right)|0,k\rangle \oplus \alpha_{-1}^{25}\tilde{\alpha}_{-1}^{25}|0,k\rangle$$

$\hookrightarrow$ look @ massless states $|0;k\rangle$.

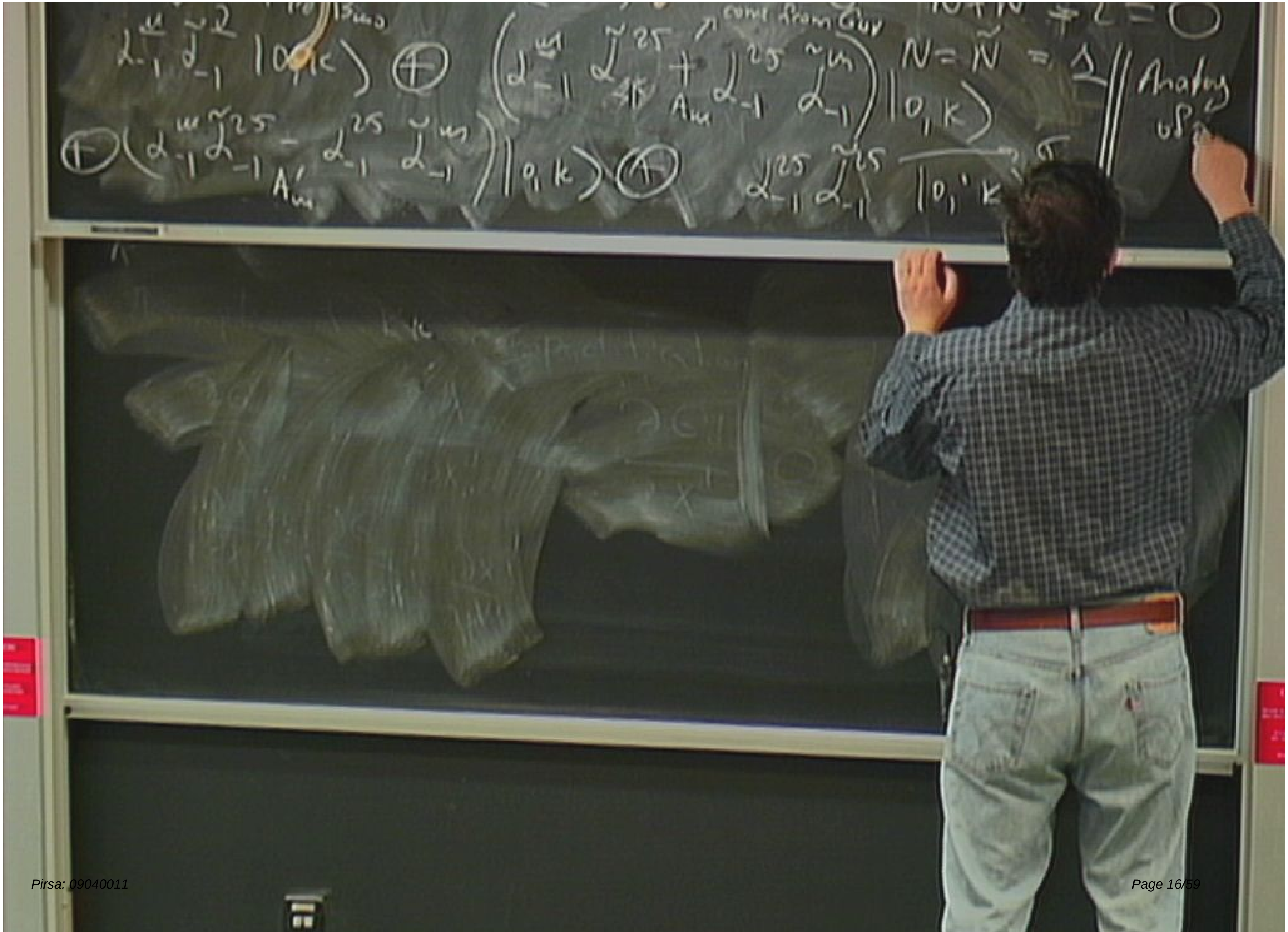
$m^2 = 0$ for generic $R \Rightarrow n = w = 0 \quad N + \tilde{N} - 2 = 0$

$N = \tilde{N} = 1$

$\checkmark\, G_{\mu\nu} + B_{\mu\nu}$

$$\alpha^{\mu}_{-1}\tilde{\alpha}^{\nu}_{-1}|0,k\rangle \oplus \left(\alpha^{\mu}_{-1}\tilde{\alpha}^{25}_{-1} + \alpha^{25}_{-1}\tilde{\alpha}^{\mu}_{-1}\right)|0,k\rangle$$

$$\oplus \left(\alpha^{\mu}_{-1}\tilde{\alpha}^{25}_{-1} - \alpha^{25}_{-1}\tilde{\alpha}^{\mu}_{-1}\right)|0,k\rangle \oplus \alpha^{25}_{-1}\tilde{\alpha}^{25}_{-1}|0,k\rangle$$


come from Guy
Analog

$$\oplus \left(\alpha_{-1}^{\mu}\tilde{\alpha}_{-1}^{25} - \alpha_{-1}^{25}\tilde{\alpha}_{-1}^{\mu}\right)|0,k\rangle \qquad A'_{\mu} \qquad \oplus \qquad \alpha_{-1}^{25}\tilde{\alpha}_{-1}^{25}|0,k\rangle \longrightarrow \sigma$$

$$\left(\alpha_{-1} \ldots A_{\mu}\,\alpha_{-1}\,\alpha_{-1}\right)|0,k\rangle$$

of QFT

"A" was a charge under A^{μ}

A'_{μ}

$$\ldots \; A_{m}\, \alpha_{-1}\, \alpha_{-1}\big)\,|0,k\rangle \qquad \ldots$$

$$\oplus \left(\alpha^{m}_{-1}\tilde{\alpha}^{25}_{-1} A'_{m}\; \alpha^{25}_{-1}\tilde{\alpha}^{m}_{-1}\right)|0,k\rangle \oplus \quad \alpha^{25}_{-1}\tilde{\alpha}^{25}_{-1}\,|0,'k\rangle \rightarrow \sigma$$

of QFT

"A" was a charge under A^{m}

"W" is a charge under A'_{m}

$A_w \, \alpha_{-1} \, \alpha_{-1} \,)\,|0,k\rangle$

of QFT

$\oplus \left(\alpha_{-1}^{w}\tilde{\alpha}_{-1}^{25} \, A'_w \, \alpha_{-1}^{25}\tilde{\alpha}_{-1}^{w}\right)|0,k\rangle \oplus \alpha_{-1}^{25}\tilde{\alpha}_{-1}^{25}\,|0,k\rangle$

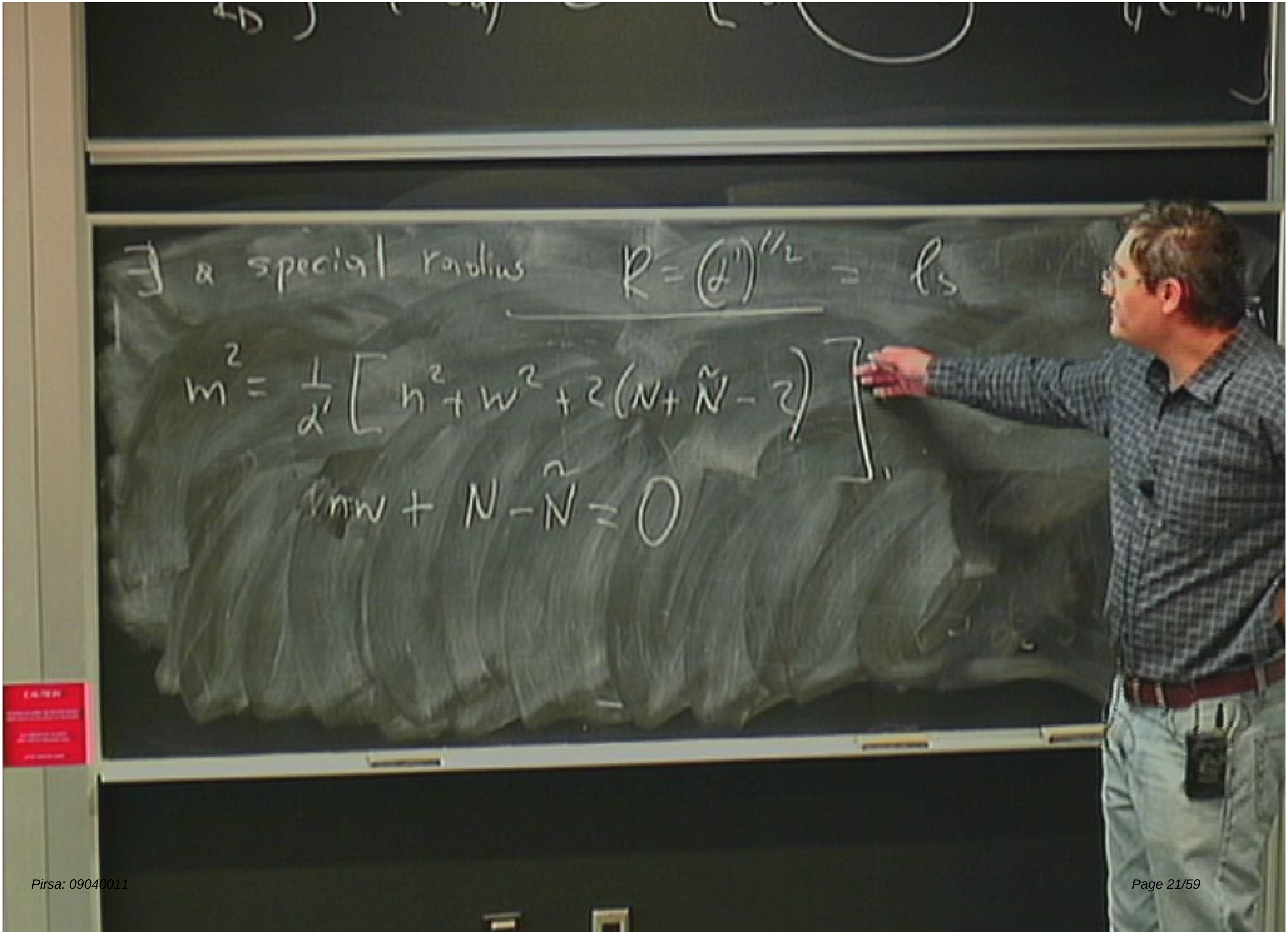
"n" was a charge under A^m

"w" is a charge under A'_m

↳ using operator-state correspondence

$$\left(\partial X^{w}\,\bar{\partial} X^{25} \pm \bar{\partial} X^{w}\,\partial X^{25}\right) e^{ikX}$$

$\exists$ a special radius $R = (\alpha')^{1/2}$


∃ a special radius R = (α')^1/2 = ℓs
m² = 1/α' [n² + w² + 2(N + Ñ − 2)],
nw + N − Ñ = 0

$\exists$ a special radius $R = (\alpha')^{1/2} = \ell_s$

$$m^2 = \frac{1}{\alpha'}\left[n^2 + w^2 + 2(N + \tilde{N} - 2) \right],$$

$$nw + N - \tilde{N} = 0$$

$$\tilde{N} = N + nw$$

$m^2 = 0$

$n^2 + w^2 + 2N +$

$+$

$\exists$ a special radius $R = (\alpha')^{1/2} = l_s$

$$m^2 = \frac{1}{\alpha'}\left[n^2 + w^2 + 2(N + \tilde{N} - 2) \right]$$

$$nw + N - \tilde{N} = 0$$

$$\tilde{N} = N + nw$$

$m^2 = 0$

$$n^2 + w^2 + 2N + 2N + 2nw = 4$$

$$\boxed{(n+w)^2 + 4N = 4}$$

New massless states:

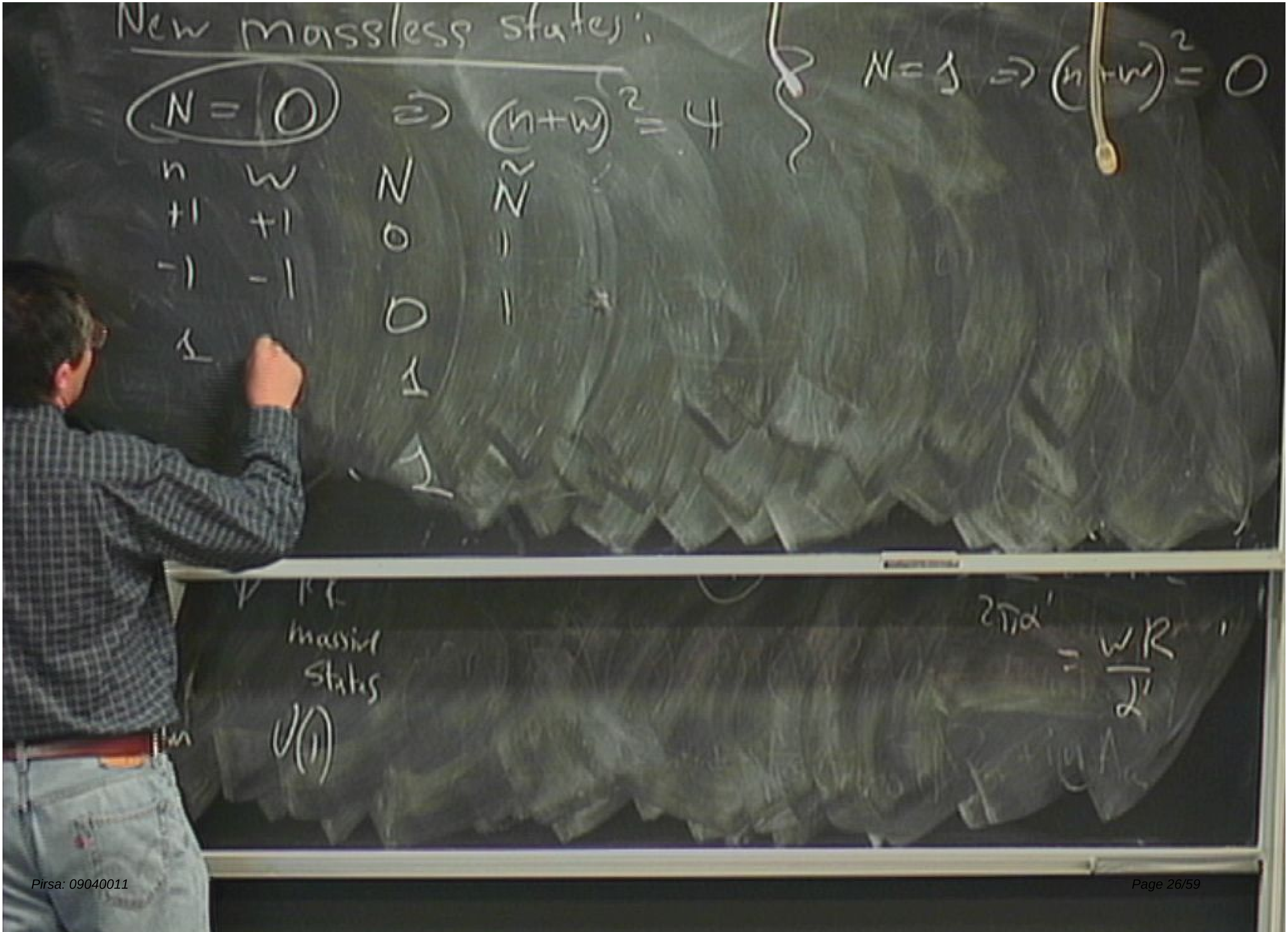
$$N = 0 \Rightarrow (n+w)^2 = 4$$

n	w
$+1$	$+1$
-1	-1

New massless states:

$$N = 0 \Rightarrow (n+w)^2 = 4$$

n	w	N	$\tilde{N}$
+1	+1	0	1
−1	−1	0	1


New massless states:
N = 0 ⇒ (n+w)^2 = 4
N = 1 ⇒ (n+w)^2 = 0
n w N Ñ
+1 +1 0 1
-1 -1 0 1
massive
states
= wR/α'

New massless states:

$N = 0 \Rightarrow (n+w)^2 = 4$

$N = 1 \Rightarrow (n+w)^2 = 0$

n	w	N	$\tilde{N}$
+1	+1	0	1
−1	−1	0	1
1	−1	1	0
−1	+1	1	0

$\left.\begin{array}{l} n = +2 \\ w = -2 \end{array}\right\} ?$

↓ KK

massive states

A_m $\quad$ $U(1)$

$2\pi\alpha'$

$= \frac{wR}{\alpha'}$

$$N = 0 \Rightarrow (n+w)^2 = 4 \qquad N = 1 \Rightarrow (n+w)^2 = 0$$

n	w	N	$\tilde{N}$
+1	+1	0	1
−1	−1	0	1
1	−1	1	0
−1	+1	1	0

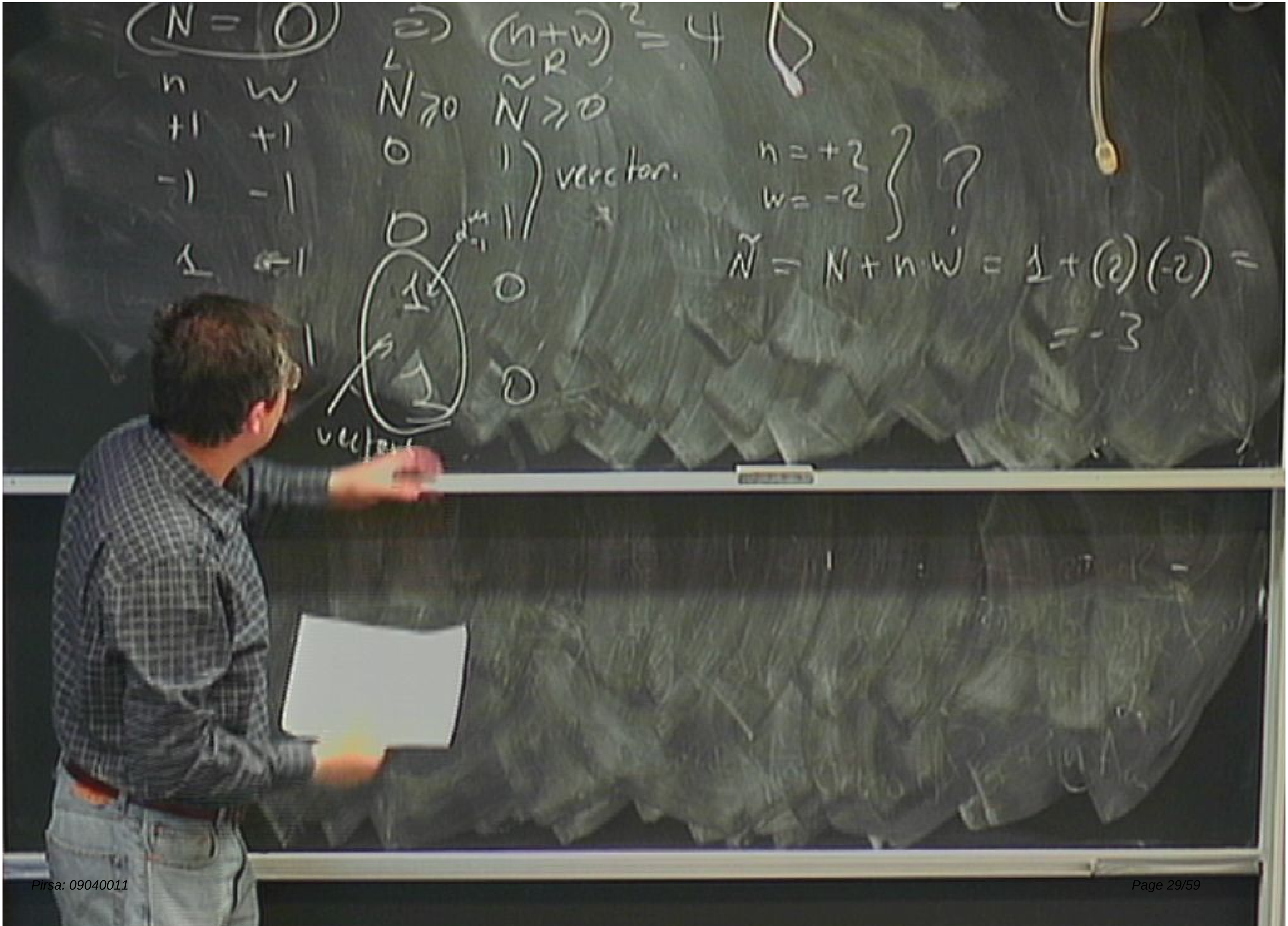
$$\left.\begin{matrix} n = +2 \\ w = -2 \end{matrix}\right\} ?$$

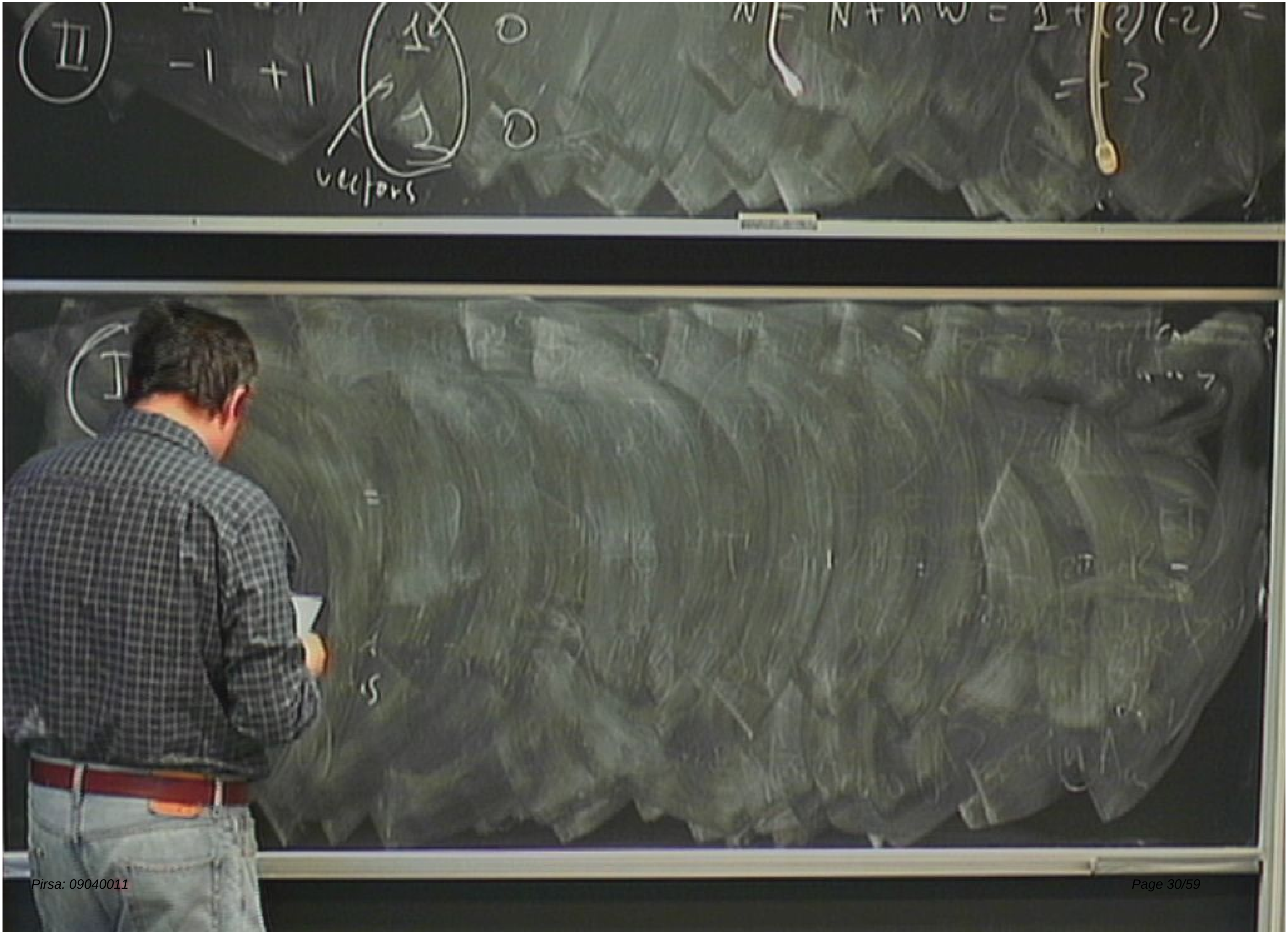
$$\tilde{N} = N + nw = 1 + (2)(-2) = -3$$

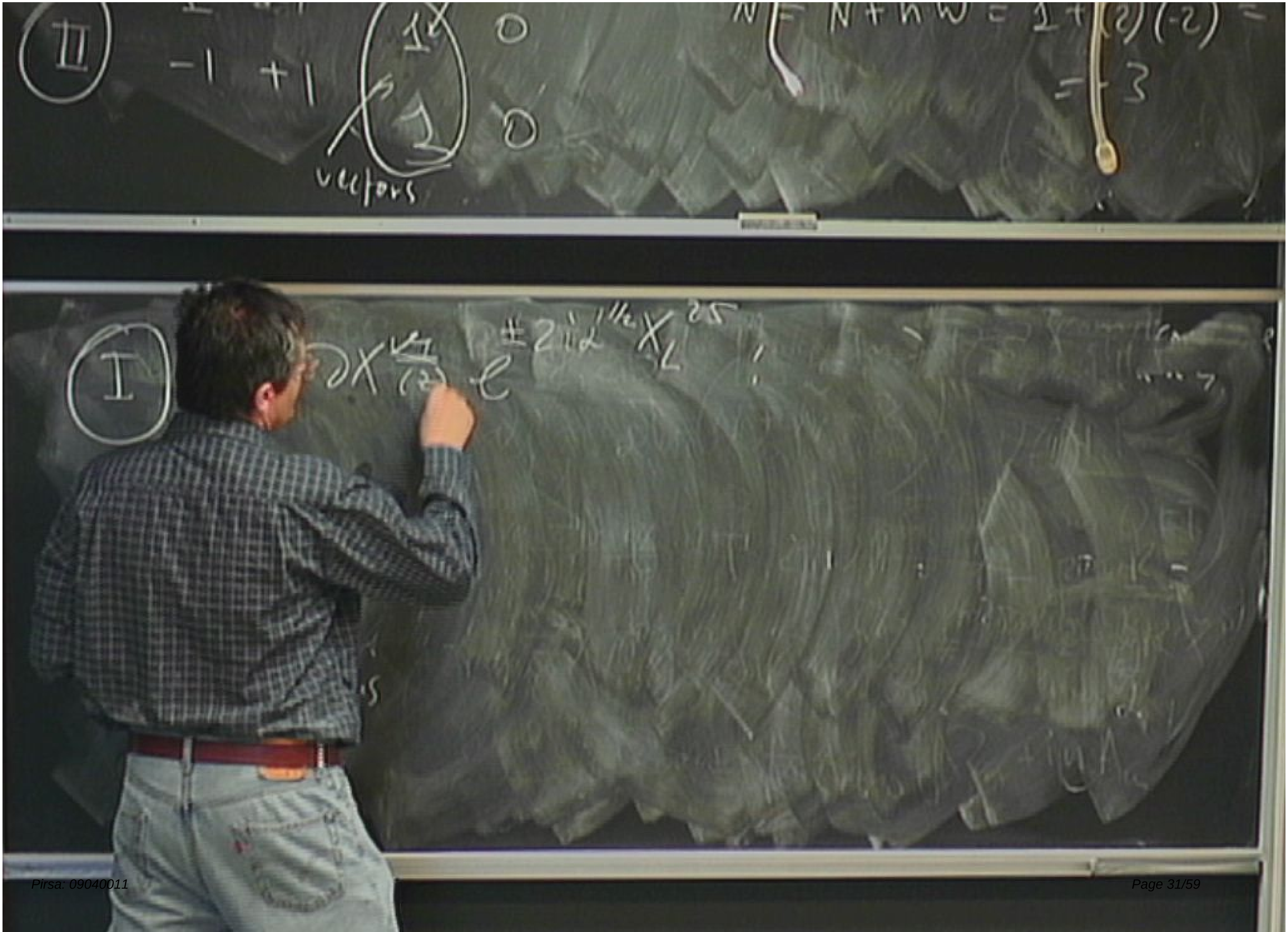
↓ KK massive states

A_m $U(1)$

$$\frac{1}{2\pi\alpha'} 2\pi w R = \frac{wR}{\alpha'}$$


N = 0 ⇒ (n+w)^2 = 4
n w N ≥ 0 Ñ ≥ 0
+1 +1 0 1
-1 -1
vector.
n = +2
w = -2
?
Ñ = N + nw = 1 + (2)(-2) =
= -3
vectors


vectors


vectors

II −1 +1 1 0
1 0
vectors

$N = N + nw = 1 + (2)(-2) =$
$= -3$

I : $\partial X^{\mu}(\bar{z})\, e^{\pm 2i\alpha'^{1/2} X_R^{25}(\bar{z})}\, e^{ikX}$; ...

II : $\partial X^{\mu}(\bar{z})\, e^{\pm 2i\alpha'^{1/2} X_L^{25}(z)}\, e^{ikX}$

$$\mathrm{II} \quad -1 \quad +1 \quad 1 \quad 0$$

$$1 \quad 0$$

vectors

$$N = N + nw = 1 + (2)(-2) = -3$$

$$\mathrm{I} : \partial X^{\mu}(z)\, e^{\pm 2i\alpha'^{1/2} X_R^{25}(\bar z)}\, e^{ikX} ;$$

$$\mathrm{II} : \bar\partial X^{\mu}(\bar z)\, e^{\pm 2i\alpha'^{1/2} X_L^{25}(z)}\, e^{ikX}$$

$$\Rightarrow R \neq (\alpha')^{1/2} \Longrightarrow 1 + 1$$ gauge bosons

$$U(1) \times U(1)$$

$\textcircled{II}$ $\quad -1 \quad +1 \quad 1 \quad 0$

$\quad\quad\quad\quad\quad\quad\quad 1 \quad 0$

vectors

$N = N + nw = 1 + (2)(-2) =$

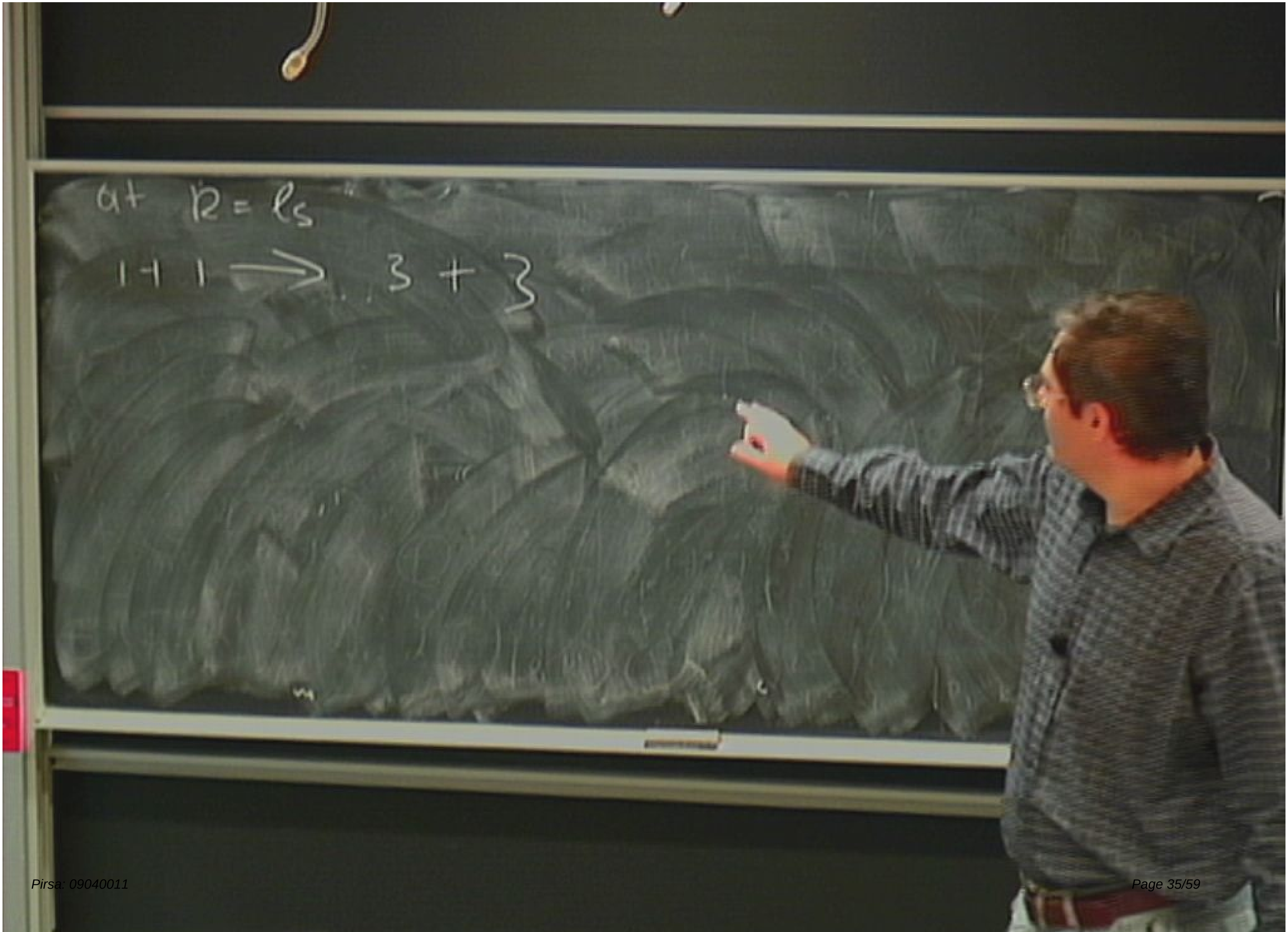
$= -3$

$\textcircled{I}$: $\partial X^{\mu}(\bar{z})\, e^{\pm 2i\alpha'^{1/2} X_R^{25}(\bar{z})}\, e^{ikX}$; ...

$\textcircled{II}$: $\partial X^{\mu}(\bar{z})\, e^{\pm 2i\alpha'^{1/2} X_L^{25}(z)}\, e^{ikX}$

$\Rightarrow R \neq (\alpha')^{1/2} \Longrightarrow 1 + 1$ gauge bosons

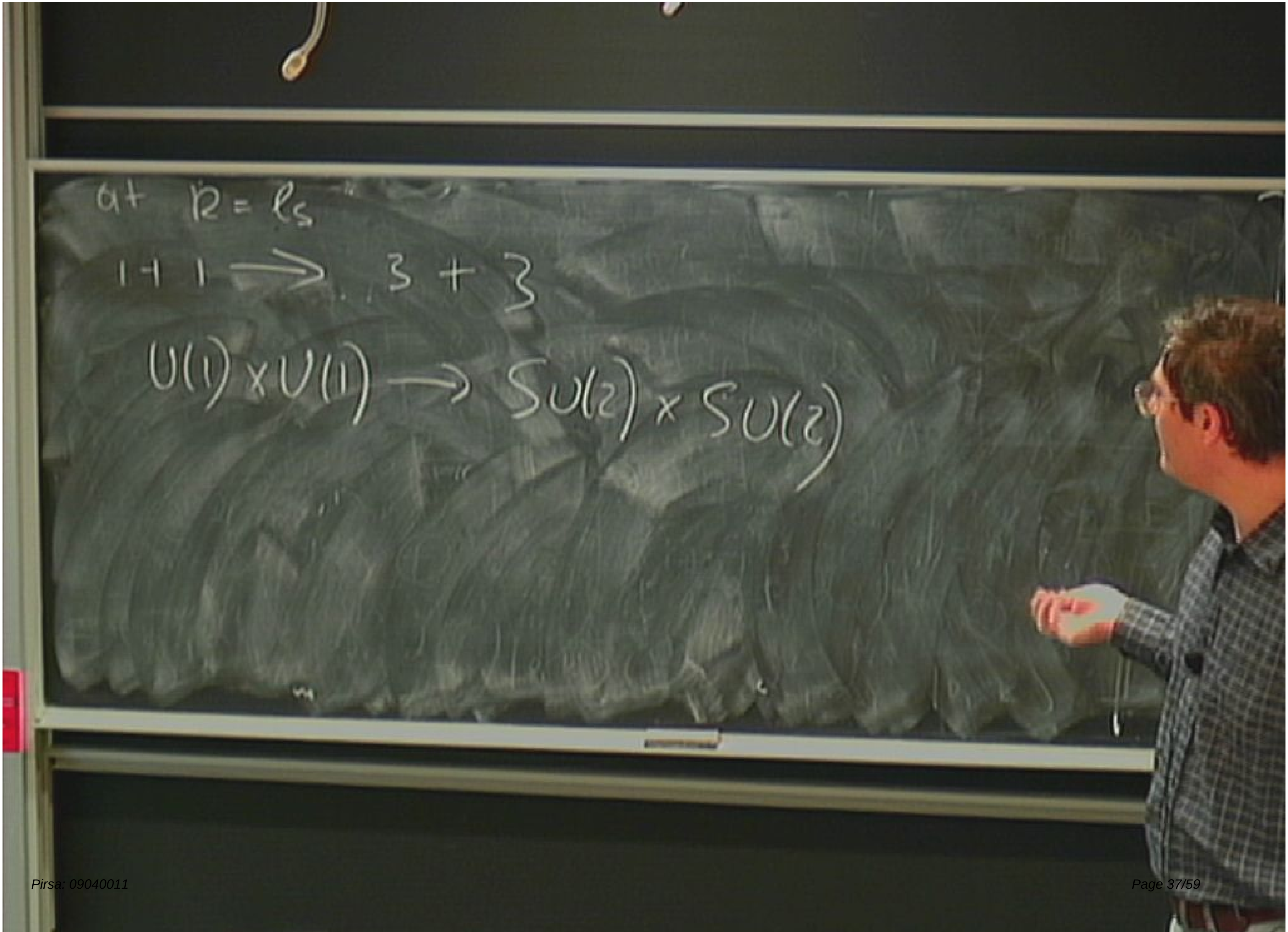
$U(1) \times U(1)$

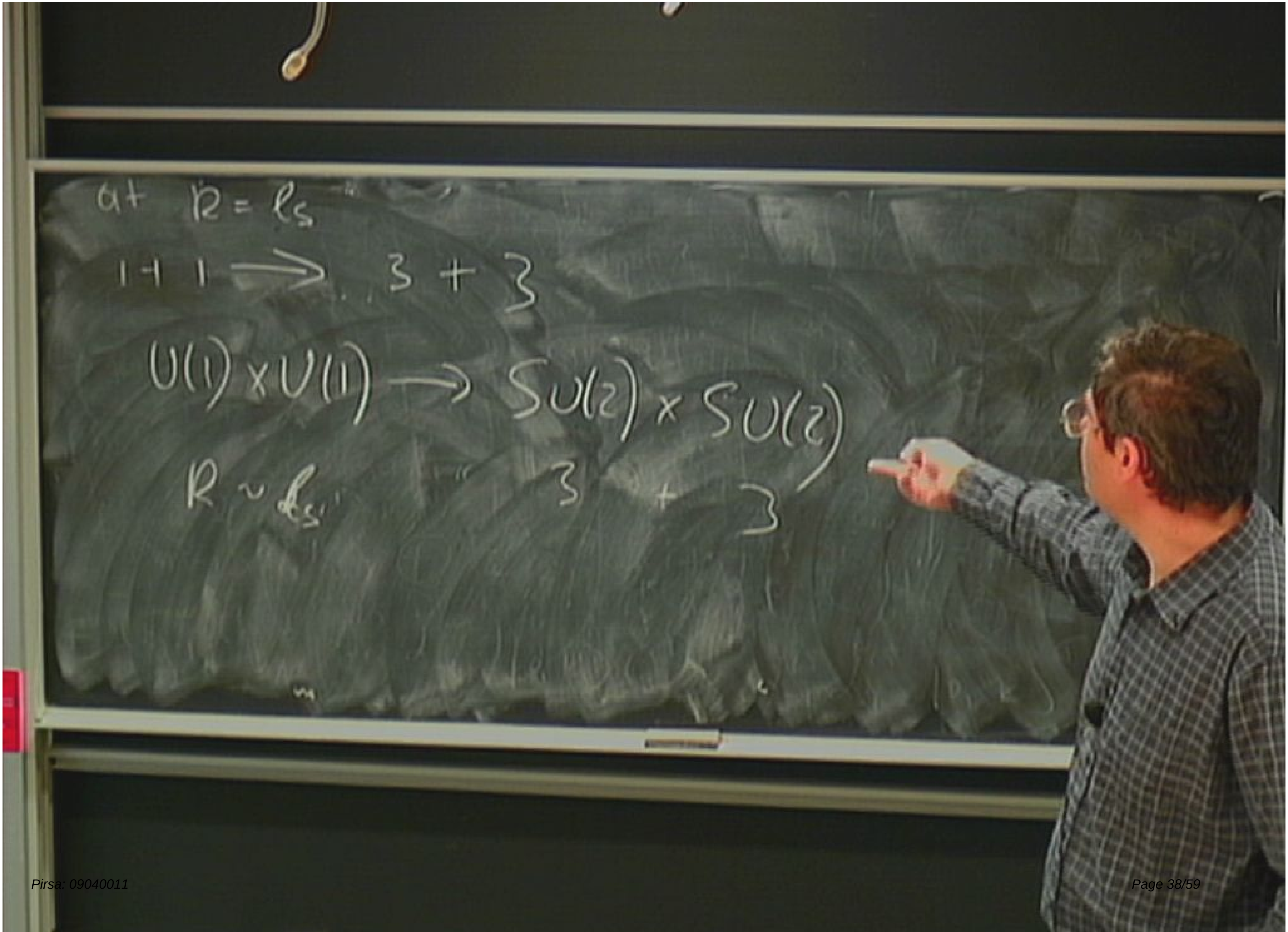

at $R = \ell_s$
$1+1 \Rightarrow 3+3$

at $R = \ell_s$

$$1 + 1 \longrightarrow 3 + 3$$

$$U(1) \times U(1) \longrightarrow SU(2) \times SU(2)$$


at $R = l_s$
$1+1 \to 3+3$
$U(1) \times U(1) \to SU(2) \times SU(2)$


at R = l_s
1+1 → 3 + 3
U(1) x U(1) → SU(2) x SU(2)
R ~ l_s 3 + 3

at $R = \ell_s$

$$1+1 \longrightarrow 3+3$$

$$U(1)\times U(1) \longrightarrow SU(2)\times SU(2)$$

$R \sim \ell_s$ $\quad 3 + 3$

4 gauge bosons are massive $\quad m^2 \propto |R - \alpha'^{1/2}|^2$

T-duality in string theory

φ a massless scalar field in D $\equiv$ infinite # of $m_n = \frac{n}{R}$

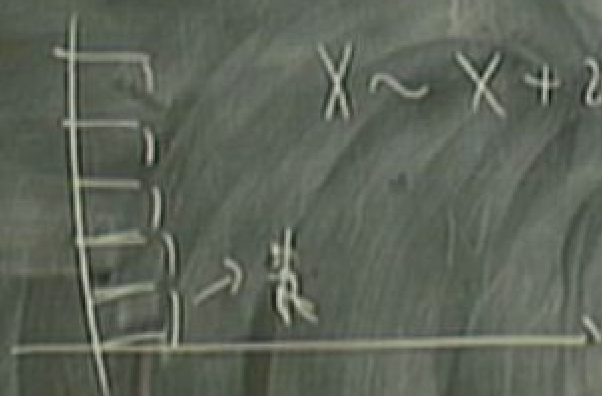
$D = d + 1$

T-duality in string theory

φ a massless scalar field in D $\equiv$ infinite # of $m_n = \frac{n}{R}$

$D = d+1$

$X \sim X + 2\pi R$

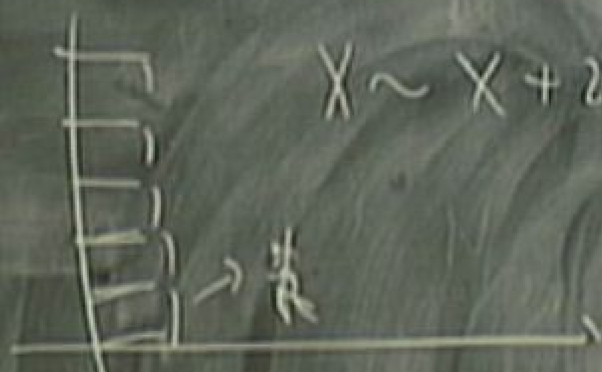


T-duality in string theory

φ a massless scalar field in D $\equiv$ infinite # of $m_n = \frac{n}{R}$

$D = d+1$

$X \sim X + 2\pi R$



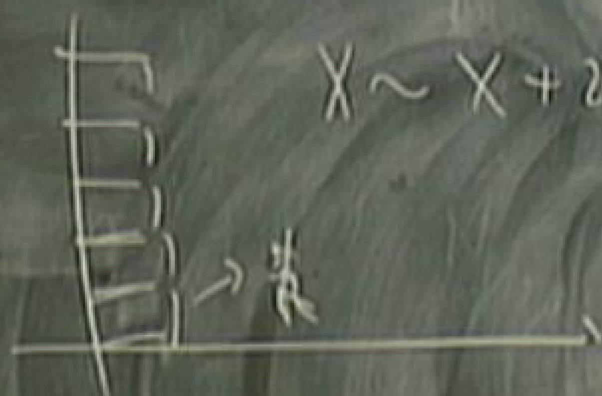
$R \to \infty$

T-duality in string theory

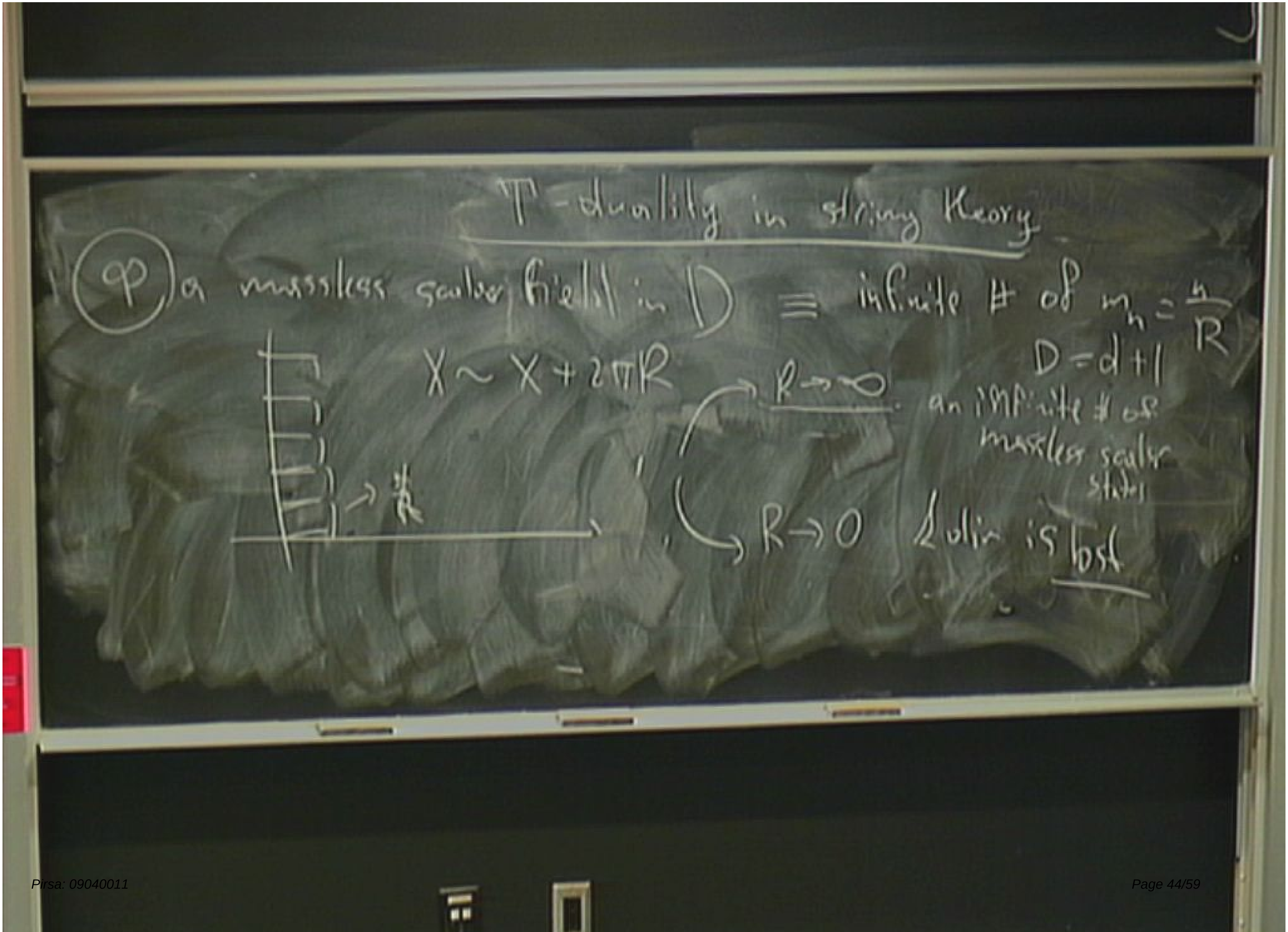
φ a massless scalar field in D $\equiv$ infinite # of $m_n = \frac{n}{R}$

$X \sim X + 2\pi R$

$D = d + 1$



$R \to \infty$: an infinite # of massless scalar states

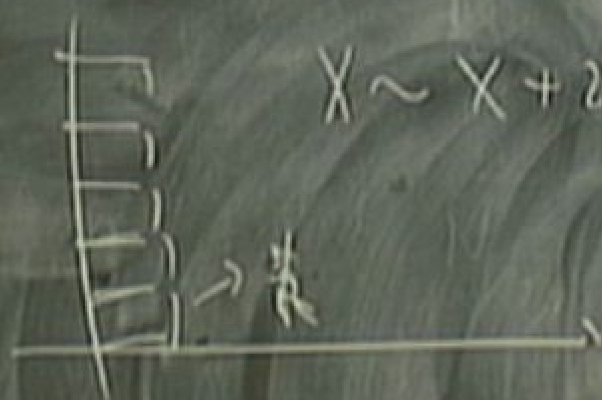

T-duality in string theory
φ a massless scalar field in D ≡ infinite # of m_n = n/R
X ~ X + 2πR
D = d+1
R→∞ an infinite # of massless scalar states
R→0 d-dim is lost

T-duality in string theory

φ a massless scalar field in D $\equiv$ infinite # of $m_n = \frac{n}{R}$

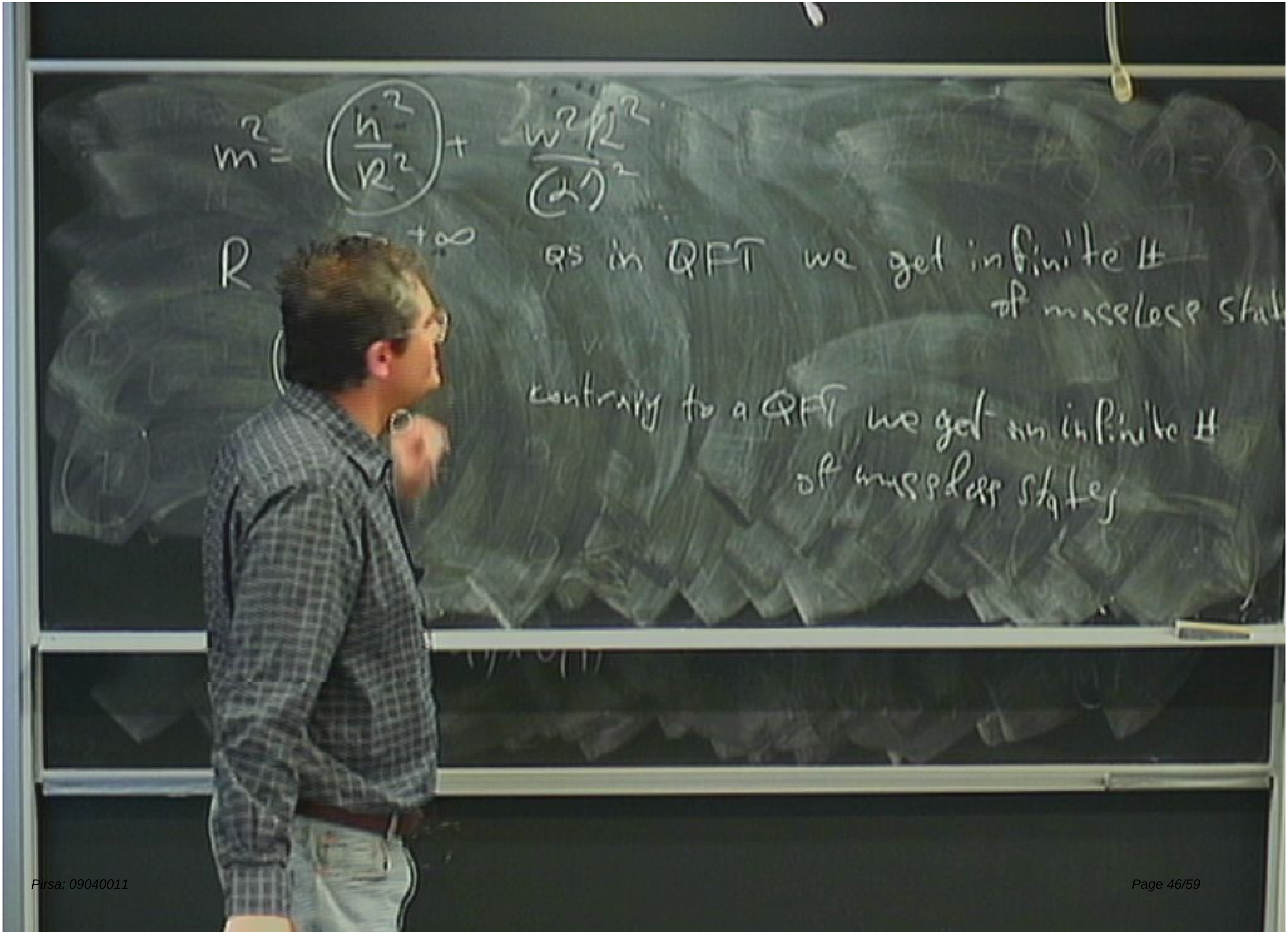
$D = d+1$

$X \sim X + 2\pi R$



$R \to \infty$. an infinite # of massless scalar states

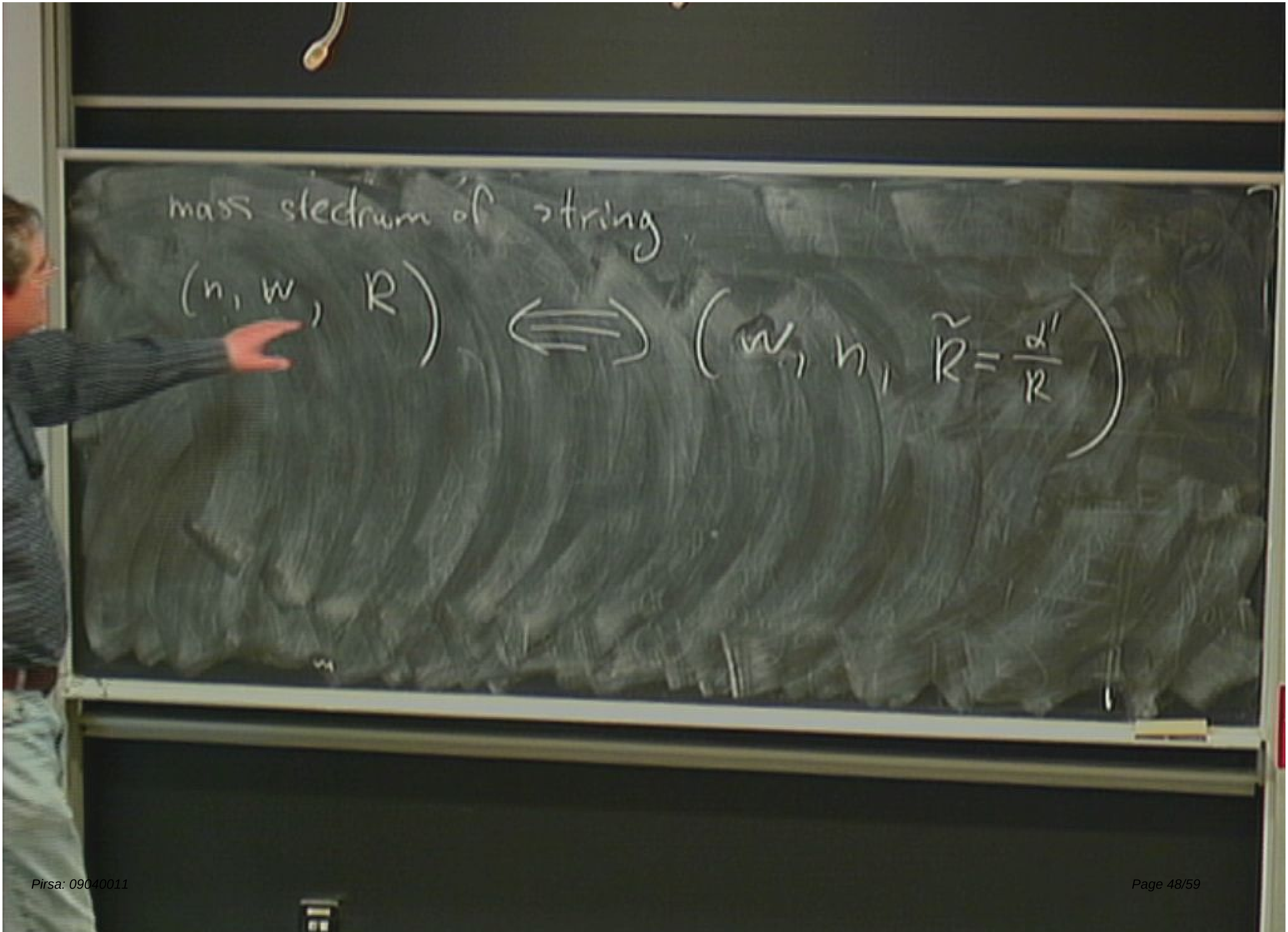
$R \to 0$ ℓulin is lost


m² = n²/R² + w²R²/(α')²
R → +∞
as in QFT we get infinite # of massless stat
contrary to a QFT we get an infinite # of massless states

$$m^2 = \frac{n^2}{R^2} + \frac{w^2 R^2}{(\alpha')^2} = \frac{w^2}{(R^*)^2} \qquad R^* = \frac{\alpha'}{R}$$

$R \to +\infty$ as in QFT we get infinite # of massless states

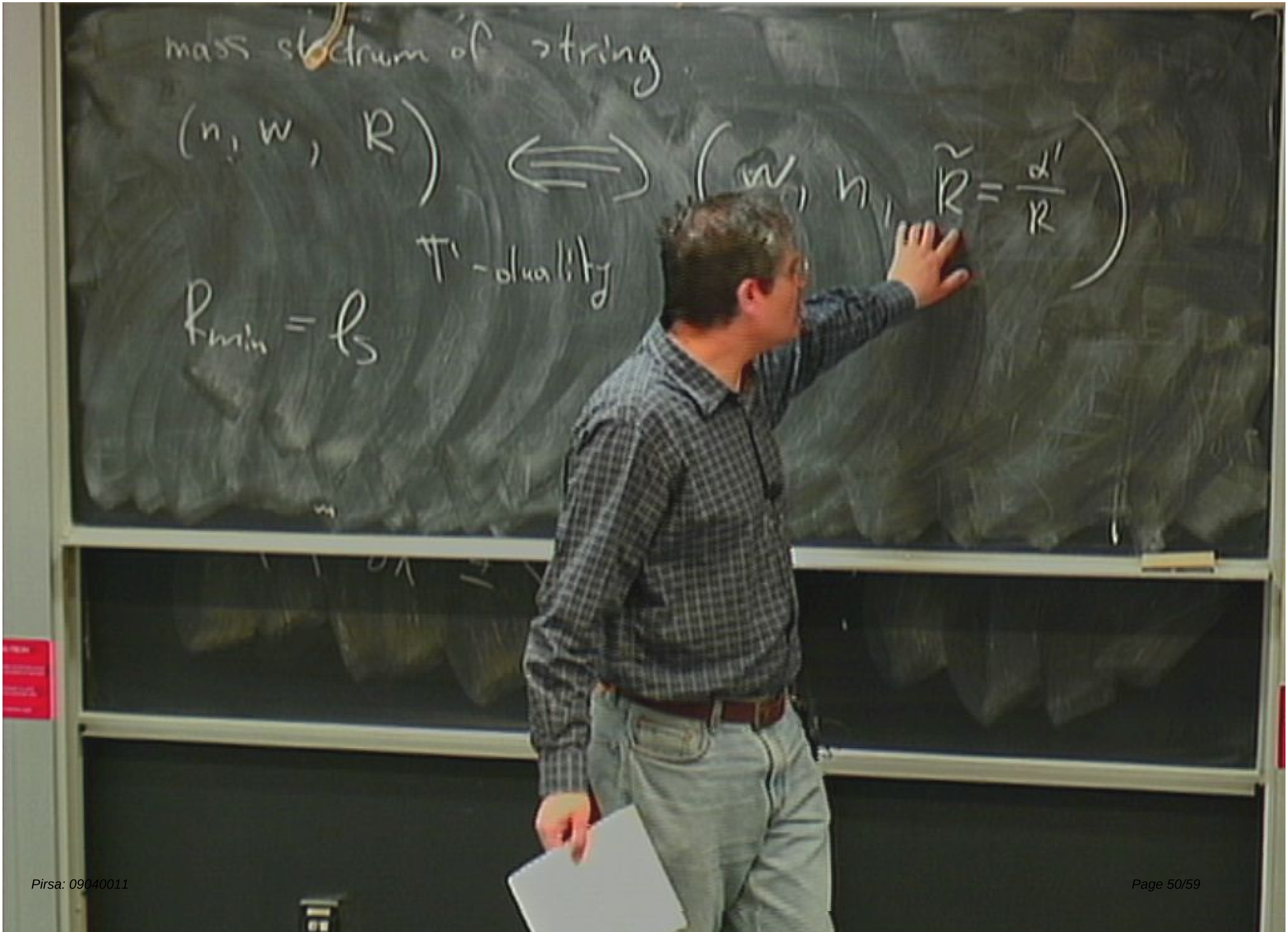
$R \to 0$ contrary to a QFT, we get an infinite # of massless states

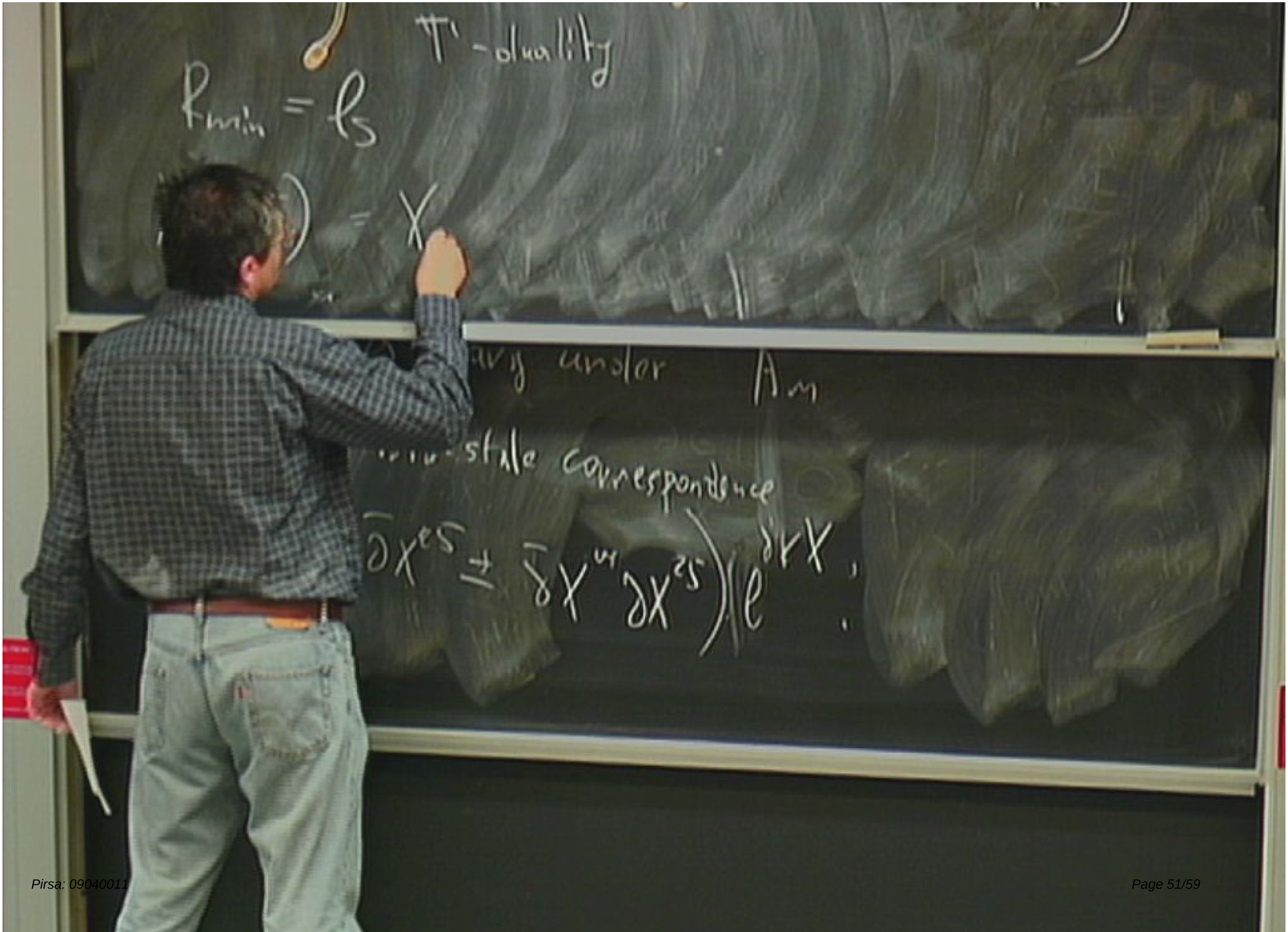

mass spectrum of string
$(n, w, R) \Longleftrightarrow (w, n, \tilde{R} = \frac{\alpha'}{R})$

mass spectrum of string.

$$(n, w, R) \Longleftrightarrow \left(w, n, \tilde{R} = \frac{\alpha'}{R}\right)$$

T-duality


mass spectrum of string
$(n, w, R) \Longleftrightarrow (w, n, \tilde{R} = \frac{\alpha'}{R})$
T-duality
$R_{min} = l_s$


T-duality
$R_{min} = \ell_s$
under A_m
state correspondence

T-duality

$R_{min} = \ell_s$

$$X(z,\bar z) = X_L(z) + X_R(\bar z)$$

... is a charg under A_μ

↳ using operator-state correspondence

$$(\partial X^\mu \bar\partial X^{25} \pm \bar\partial X^\mu \partial X^{25})e^{ikX}$$

T-duality

$R_{min} = \ell_s \qquad \partial X = \cdots \qquad \bar{\partial} X = \cdots$

$$X(z,\bar z) = X_L(z) + X_R(\bar z)$$

... is a charg under A_m

using operator-state correspondence

$$\left(\partial X^{\mu}\,\bar{\partial} X^{25} \pm \bar{\partial} X^{\mu}\,\partial X^{25}\right) e^{ik\cdot X}$$

$$X(z,\bar z) = X_L(z) + X_R(\bar z)$$

$$T: \quad X_L(z) \to X_L(z)$$

$$X_R(\bar z) \to -X_R(\bar z)$$

$$R \to \frac{\alpha'}{R}$$

$$X(z,\bar z) = X_L(z) + X_R(\bar z)$$

$$T: \quad \left. \begin{array}{l} X_L(z) \to X_L(z) \\ X_R(\bar z) \to -X_R(\bar z) \\ R \to \frac{\alpha'}{R} \end{array} \right\}$$

leads to the same OPE's ...

is preserve by string interactions

$$X(z,\bar{z}) = X_L(z) + X_R(\bar{z})$$

T: $X_L(z) \to X_L(z)$, $X_R(\bar{z}) \to -X_R(\bar{z})$, $R \to \frac{\alpha'}{R}$

leads to the same OPE's

is preserve by string interactions

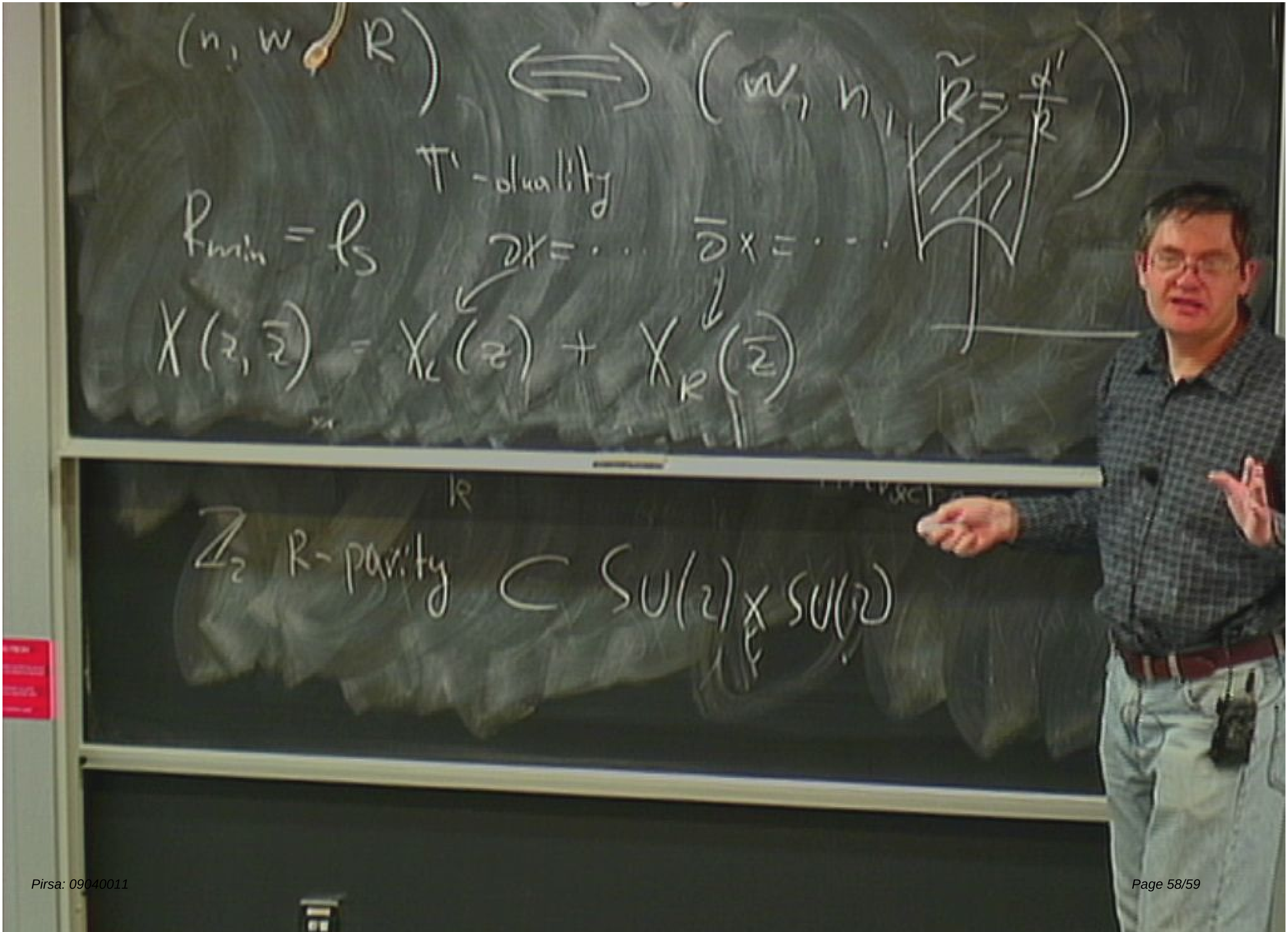
$\mathbb{Z}_2$ R-parity $\subset SU(2) \times SU(2)$

$$X(z,\bar{z}) = X_L(z) + X_R(\bar{z})$$

T : $X_L(z) \to X_L(z)$
$X_R(\bar{z}) \to - X_R(\bar{z})$
$R \to \frac{\alpha'}{R}$

Leads to the same OPE's . . .
is preserve by string interactions

$\mathbb{Z}_2$ R-parity $\subset SU(2) \times_F SU(2)$


(n, w, R) ⟺ (w, n, R̃ = α'/R)
T-duality
R_min = l_s
∂X = . . .
∂̄X = . . .
X(z, z̄) = X_L(z) + X_R(z̄)
Z_2 R-parity ⊂ SU(2) x SU(2)

$r_{min} = \ell_s \qquad \partial X = \ldots \qquad \bar\partial X = \ldots$

$$X(z,\bar z) = X_L(z) + X_R(\bar z)$$

$$T: \quad \begin{array}{l} X_L(z) \to X_L(z) \\ X_R(\bar z) \to -X_R(\bar z) \\ R \to \frac{\alpha'}{R} \end{array} \Bigg\}$$

leads to the same OPE's . . .

is preserve by string interactions

$\mathbb{Z}_2$ R-parity $\subset SU(2) \times SU(2)$