

Title: Introduction to the Bosonic String Part B

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Abstract: This course provides a thorough introduction to the bosonic string based on the Polyakov path integral and conformal field theory. We introduce central ideas of string theory, the tools of conformal field theory, the Polyakov path integral, and the covariant quantization of the string. We discuss string interactions and cover the tree-level and one loop amplitudes. More advanced topics such as T-duality and D-branes will be taught as part of the course. The course is geared for M.Sc. and Ph.D. students enrolled in Collaborative Ph.D. Program in Theoretical Physics. Required previous course work: Quantum Field Theory (AM516 or equivalent). The course evaluation will be based on regular problem sets that will be handed in during the term. The primary text is the book: 'String theory. Vol. 1: An introduction to the bosonic string. J. Polchinski (Santa Barbara, KITP) . 1998. 402pp. Cambridge, UK: Univ. Pr. (1998) 402 p.' All interested students should contact Alex Buchel at abuchel@uwo.ca as soon as possible.

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1. Tree-level amplitudes.



The sphere.

1. Tree-level amplitudes.

The sphere.

121

Tree-level amplitudes.

The sphere.

$$|z| < \rho$$

$$|w| < \rho$$

Tree-level amplitudes.

The sphere.

Tree-level amplitudes.

$$|z| < \rho$$

$$\rho > 1$$

$$|y| < \rho$$

$$z=0$$



The sphere.

Tree-level amplitudes.

$$|z| < \rho$$

$$\rho > 1$$

$$|y| < \rho$$



The sphere.

$$|z| < \rho$$

$$\rho > 1$$

$$|y| < \rho$$

\Rightarrow there must be
a local transformation



Tree-level amplitudes.

The sphere.

$|z| < \rho$ $\rho > 1$
 $|y| < \rho$

\Rightarrow there must be
a Weyl transformation
to move from
 $z \rightarrow y$



The sphere.

Tree-level amplitudes.

$$|z| < \rho$$

$$\rho > 1$$

$$|y| < \rho$$

\Rightarrow There must be
a Weyl transformation
to move from

$$z \rightarrow y = \frac{1}{z}$$



$$ds^2 = e^{2\omega} dz d\bar{z}$$

$$ds^2 = e^{2\omega(z, \bar{z})} dz d\bar{z}$$

$$dz = -\frac{1}{u^2} du$$

$$dz d\bar{z} = |z'| du d\bar{u}$$

Tree-level amplitudes.

The sphere.

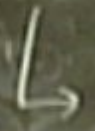
$|z| < \rho$
 $|w| < \rho$
 $\rho > 1$

\Rightarrow There must be
 a Weyl transformation
 to move from
 $z \rightarrow w = \frac{1}{z}$



$z \sim z \rightarrow w = \frac{1}{z}$

$$ds^2 = e^{2\omega(z, \bar{z})} dz d\bar{z}$$



$$dz = -\frac{1}{u^2} du$$

$$dz d\bar{z} = |z'|^2 du d\bar{u}$$

$$\omega \propto -2 \ln |z|$$

$$ds^2 = e^{2\omega} dz d\bar{z}$$

↳

$$dz = -\frac{1}{4r} du$$

Pou

$$dz d\bar{z} = |z'|^2 du d\bar{u}$$

$$\omega \propto -2 \ln |z|$$

$$ds^2 = e^{2\omega} dz d\bar{z}$$

↳

$$dz = -\frac{1}{4r} du$$

$$dz d\bar{z} = |z|^2 du d\bar{u}$$

$$\omega = -2 \ln |z|$$

Round metric:

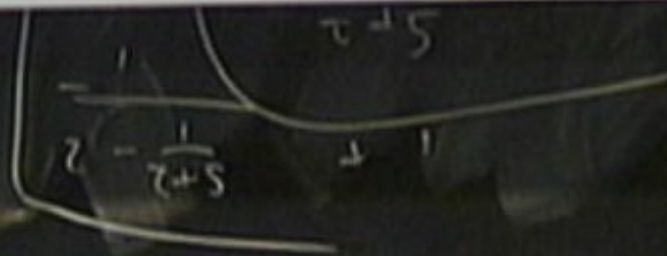
$$ds^2 = \frac{4r^2 dz d\bar{z}}{(1+z\bar{z})^2}$$

$$dz = -\frac{1}{4r^2} d\mu$$

Round metric:

$$ds^2 = \frac{4r^2 dz d\bar{z}}{(1+z\bar{z})^2}$$

$$|z| < \rho \quad \rho > 1$$

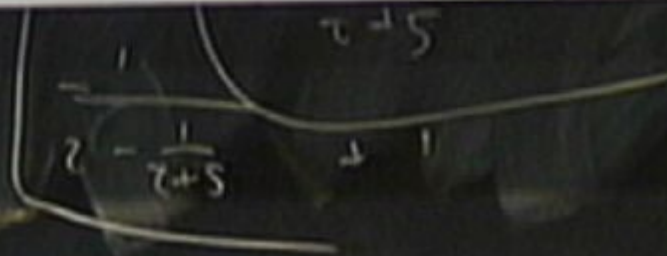


$$dz = -\frac{1}{4r} dr$$

Round metric:

$$ds^2 = \frac{4r^2 dz d\bar{z}}{(1+z\bar{z})^2} = \frac{4r^2 dr d\bar{r}}{(1+4r^2)^2}$$

$|z| < \rho \quad \rho > 1$



$$dz = -\frac{1}{4u} du$$

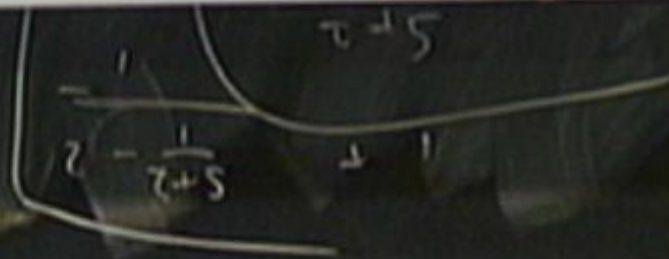
Rourol metric:

$$ds^2 = \frac{4r^2 dz d\bar{z}}{(1+z\bar{z})^2} = \frac{4r^2 du d\bar{u}}{(1+4u\bar{u})^2}$$

$\omega \propto -2 \ln |z|$
 \rightarrow radius² of a sphere.

$$R = \frac{2}{r^2}$$

$$|z| < \rho \quad \rho > 1$$



Recall

$$x > 0$$

$$k =$$

Recall

$$x > 0$$
$$= 2$$

$$K = 3x = 6 \quad m = 0$$

⇒ Riemann-Roch theorem

Recall $x > 0$ $k = 3x = 6$ $m = 0$
 $\left(\begin{array}{c} x \\ = \\ 2 \end{array} \right)$

\Rightarrow Riemann-Roch theorem

moduli = # of hol

Recall $g > 0$ $k = 3g = 6$ $m = 0$
($= 2$

⇒ Riemann-Roch theorem

moduli = # of holomorphic differentials $\delta y_{z,z}$

CKV = # of holomorphic vectors δz

Support no have & gaz

⇒ Suppose we have $f(z)$ is holomorphic near the
North pole (small z)

⇒ Suppose we have δg_{22} is holomorphic near the North pole (small z)

δg_{44}

⇒ Suppose we have g_{zz} is holomorphic near the North pole (small z)

$$\delta g_{44} = \left(\frac{\partial u}{\partial z}\right)^{-2} \delta g_{zz}$$

⇒ Suppose we have $f(z)$ is holomorphic near the North pole (small z)

$$\delta g_{44} = \left(\frac{\partial u}{\partial z}\right)^{-2} \delta g_{zz} = z^4 \delta g_{zz}$$

⇒ Suppose we have g_{zz} is holomorphic near the North pole (small z)

$$\delta g_{\mu\nu} = \left(\frac{\partial x}{\partial z}\right)^{-2} \delta g_{zz} = z^4 \delta g_{zz}$$

⇒

\Rightarrow Suppose we have $g_{z\bar{z}}$ is holomorphic near the North pole (small z)

$$\delta g_{44} = \left(\frac{\partial u}{\partial z}\right)^{-2} \delta g_{z\bar{z}} = z^4 \delta g_{z\bar{z}}$$

$$\Rightarrow \delta g_{z\bar{z}} \propto O\left(\frac{1}{z^4}\right)$$

⇒ Suppose we have g_{zz} is holomorphic near the North pole (small z)

$$\delta g_{\mu\nu} = \left(\frac{\partial x}{\partial z}\right)^{-2} \delta g_{zz} = z^4 \delta g_{zz}$$

⇒ $\delta g_{zz} \propto O\left(\frac{1}{z^4}\right)$ ⇒ There does not exist δg_{zz} which is simultaneously hol around $z=0$ and $\mu=0$

\Rightarrow Suppose we have g_{zz} is holomorphic near the North pole (small z)

$$\delta g_{\mu\nu} = \left(\frac{\partial x}{\partial z}\right)^{-2} \delta g_{zz} = z^4 \delta g_{zz}$$

$\Rightarrow \delta g_{zz} \propto O\left(\frac{1}{z^2}\right) \Rightarrow$ There does not exist δg_{zz} which is simultaneously hol around $z=0$ and $\mu=0$

$$\boxed{\mu=0}$$

The dual metric is
 $g_{\mu\nu}$

Handwritten scribbles and symbols at the top left of the chalkboard.

Handwritten mathematical expressions at the top right of the chalkboard, including $Q=4$ and $Q=20$.

Handwritten number '28' in the top left corner of the main chalkboard panel.

A large, dense area of the chalkboard completely obscured by heavy, overlapping horizontal and diagonal chalk strokes.

$Q = 4$ and $Q = 2$

$$2g^2 z^{-1} = 2g \frac{z^2}{z^2} = (g) n g \leftarrow (z) 2g$$

$z=0$ and $z=\infty$

$$z^2 \bar{z} = -z^2 \bar{z} \quad \frac{z^4}{z^2} = (z)^2 \bar{z} \quad \bar{z}(z) \leftarrow (z)^2 \bar{z}$$

z^2 to be holomorphic both about $z=0$ and $z=\infty$

$z=0$ and $z=\infty$

$$z^2 \frac{dz}{dz} = -z^2 \frac{dz}{dz} \rightarrow (z) dz \rightarrow (z) dz$$

z to be holomorphic both about $z=0$

and

$$z = \infty$$

$$z = 0$$

$z=0$ and $y=0$

$$\frac{\partial^2 z}{\partial z^2} = -z^2 \frac{\partial z}{\partial z} \rightarrow \frac{\partial^2 z}{\partial z^2} = -z^2 \frac{\partial z}{\partial z}$$

z to be holomorphic both about $z=0$ and

$$z = a_0 + a_1 z + a_2 z^2$$

$$z = 0 \Leftrightarrow z = \infty$$

$z=0$ and $z=\infty$

$$\frac{dz}{z^2} = -\frac{dz}{z^2}$$

$\frac{dz}{z^2}$ to be holomorphic both about $z=0$

and $z=\infty$

$$\frac{dz}{z^2} = a_0 + a_1 z + a_2 z^2$$

3 different complex parameters

Scalar expectation values.

(π^2)

Scalar expectation values.

$Z[J]$

Scalar expectation values.

$Z[J]$

generating functional

Scalar expectation values.

$$\underbrace{Z[J]}_{\substack{\uparrow \\ \text{generating} \\ \text{functional}}} = \langle e^{i \int d^4x T(x) X(x)} \rangle$$

Scalar expectation values.

$$\begin{aligned} Z[J] &= \langle e^{i \int d^2\sigma T(\sigma) X(\sigma)} \rangle \\ &= \int [dx] e^{-\frac{1}{4\pi\alpha'} \int d^2\sigma (\partial X)^2 + i \int d^2\sigma J(\sigma) X(\sigma)} \end{aligned}$$

generating functional

Expand $X^u(0) = \sum H$

Expand $X^{(n)}(s) = \sum_I X_I^{(n)} X_I(s)$

Expand $X^{(n)}(0) = \sum_I x_I^{(n)} X_I(0)$

$$\Delta^2 X_I = -\omega_I^2 X_I$$

Expand $X^{(n)}(0) = \sum_I x_I^{(n)} X_I(0)$

$$\Delta^2 X_I = -\omega_I^2 X_I$$

\int

Expand $X^{\mu}(\sigma) = \sum_I x_I^{\mu} X_I(\sigma)$

$$\Delta^2 X_I = -\omega_I^2 X_I$$

$$\int_M d^2\sigma \sqrt{g} X_I X_{I'} = \delta_{II'}$$

M
sphere

$$J_{\pm}^{\pm} = \int_{\sigma} \omega^2 \mathcal{J}^{\mu}(\sigma) X_{\pm}(\sigma)$$

$$J_{\pm}^{\epsilon} = \int_{\mathcal{O}(\mathbb{C})} J^{\mu}(\phi) X_{\pm}(\phi)$$

$$Z[J] = \prod_{I, \mu} \int d$$

$$J_{\pm}^{\mu} = \int d^4\sigma \, J^{\mu}(\sigma) X_{\pm}(\sigma)$$

$$Z[J] = \prod_{I, \mu} \int dX_{\pm}^{\mu}$$

$$J_{\pm}^{\omega} = \int d\Omega^2 \mathcal{J}^{\omega}(\Omega) X_{\pm}(\Omega)$$

$$Z[J] = \prod_{I, \omega} \int dY_{\pm}^{\omega} e^{-\frac{\omega^2 Y_{\pm}^{\omega} Y_{\pm}^{\omega}}{4\pi i \epsilon}}$$



$$J_I^u = \int d\sigma^2 \mathcal{J}^u(\sigma) X_I(\sigma)$$

$$Z[J] = \prod_{I, \mu} \int dX_I^\mu e^{-\frac{\omega_I^2 X_I^\mu X_{I, \mu}}{2} + i X_I^\mu J_{I, \mu}}$$

\Rightarrow all integrals are gaussian except for the

$$X_0 =$$

$\# CKV = \#$ of holomorphic vectors

$I_{,m}$
 \Rightarrow all integrals are gaussian except for the

$$X_0 = \left[\int \rho^2 r_g \right]^{1/2}$$

CKV = # of holomorphic

$$z[f] = i(2\pi)^d$$



$$z[j] = i(2\pi)^j$$

$$Z[J] = \prod_{I, \mu} \int dX_I^\mu e^{-\frac{\omega_I^2 X_I^\mu X_I^\mu}{2\pi d} + iX_I^\mu J_{I\mu}}$$

⇒ all integrals are gaussian except for t

$$X_0 = \left[\int d^2\sigma \gamma \right]^{-1/2}$$



$$Z(J) = \prod_{I, \mu} \int dX_I^{\mu} e^{-\frac{\omega_I^2 X_I^{\mu} Y_{I, \mu} + i X_I^{\mu} J_{I, \mu}}{\omega_I^2}} \int d\phi e^{i\phi \omega_I^2 \sigma_I(\phi)}$$

⇒ all integrals are gaussian except for the

$$X_0 = \left[\int \rho_0 \rho \right]^{-1/2}$$



$\Lambda_0 = (\dots)$

$$z[\mathcal{J}] = i (2\pi)^D \delta^D[\mathcal{J}_0] \prod_{\mathcal{J} \neq 0}$$

$$z[\tau] = i (2\pi)^D \delta^D[\tau_0] \prod_{\tau \neq 0} \left(\frac{m^2 \tau}{\omega^2 \tau} \right)^{D/2}$$

$$Z[J] = i (2\pi)^D \delta^D[J_0] \prod_{J \neq 0} \left(\frac{m^2 \delta^D}{\omega_J^2} \right)^{D/2} e$$

$$z(J) = i (2\pi)^D \delta^D[J_0] \prod_{J \neq 0} \left(\frac{m^2 \omega^2}{\omega_I^2} \right)^{D/2} e^{-\frac{\pi i}{\omega_I} J_0 J_I}$$

$$z(J) = i (2\pi)^D \delta^D(J_0) \prod_{J \neq 0} \left(\frac{m^2 J^2}{\omega_J^2} \right)^{D/2} e^{-\frac{\pi i}{\omega_J} J_0 J_I}$$

$$= i (2\pi)^D \delta^D(J_0)$$



Expand $X^{\mu}(\sigma) = \sum_I x_I^{\mu} X_I(\sigma)$

$$\Delta^2 X_I = -\omega_I^2 X_I$$

$$\int_M d^2\sigma g X_I X_{I'} = \delta_{II'}$$

M
Sphere

$$Z[J] = i (2\pi)^D \delta^D[J_0] \prod_{J \neq 0} \left(\frac{m^2}{\omega_J^2} \right)^{D/2} e^{-\frac{\pi i}{\omega_J} J_J J_I}$$

$$= i (2\pi)^D \delta^D(J_0) \det' \left(\frac{-\Delta^2}{4\pi^2} \right)$$

$$Z[J] = i (2\pi)^D \delta^D[J_0] \prod_{J \neq 0} \left(\frac{m^2 d}{\omega_J^2} \right)^{D/2} e^{-\frac{\pi i}{\omega_J} J_J J_I}$$

$$= i (2\pi)^D \delta^D(J_0) \left[\det' \left(\frac{-\Delta^2}{4\pi^2 d'} \right) \right]^{-D/2}$$

$$Z[J] = i (2\pi)^D \delta^D[J_0] \prod_{J \neq 0} \left(\frac{m^2 \omega^2}{\omega_I^2} \right)^{D/2} e^{-\frac{\pi i}{\omega_I} \frac{J_0 J_I}{\omega_I}}$$

$$= i (2\pi)^D \delta^D(J_0) \left[\det' \left(\frac{-\Delta^2}{4\pi^2 \epsilon'} \right) \right]^{-D/2}$$

$$Z(J) = \prod_{I, \omega} \int dX_I^\omega e^{-\frac{\omega^2 X_I^\omega X_I^\omega}{\omega \Gamma \omega} + i X_I^\omega J_{I, \omega}}$$

⇒ all integrals are gaussian except for the

$$X_0 = \left[\int d^2 \sigma \right]^{-1/2}$$



\Rightarrow all integrals are gaussian except for the

$$X_0 = \left[\int d^2\sigma \mathcal{J}_0 \right]^{-1/2}$$

$$Z(\mathcal{J}) = i (2\pi)^D \delta(\mathcal{J}_0) \prod_{\mathcal{J} \neq 0} \left(\frac{\pi}{\omega_{\mathcal{J}}^2} \right) e^{-\frac{\mathcal{J}^2}{\omega_{\mathcal{J}}^2}}$$

$$= i (2\pi)^D \delta^D(\mathcal{J}_0) \left[\det' \left(\frac{-\nabla^2}{4\pi^2 \alpha'} \right) \right]^{-D/2} \cdot e^{-\frac{1}{2} \int d^2\sigma d^2\sigma' \mathcal{J}(\sigma) \mathcal{J}(\sigma')}$$

\Rightarrow all integrals are gaussian except for the

$$X_0 = \left[\int d^2\sigma \mathcal{J} \right]^{-1/2}$$

$$Z(\mathcal{J}) = i (2\pi)^D \delta(\mathcal{J}_0) \prod_{\mathcal{J} \neq 0} \left(\frac{\pi}{\omega_{\mathcal{J}}^2} \right) e^{-\frac{\mathcal{J}^2}{2\omega_{\mathcal{J}}^2}}$$

$$= i (2\pi)^D \delta^D(\mathcal{J}_0) \left[\det' \left(\frac{-\nabla^2}{4\pi^2 \alpha'} \right) \right]^{-D/2} \cdot e^{-\frac{1}{2} \int d^2\sigma d^2\sigma' \mathcal{J}(\sigma) \mathcal{J}(\sigma') \mathcal{G}'(\sigma, \sigma')}$$

$$G'(G, G)$$

$$G'(s_1, s_2) = \sum_{I \neq 0} \frac{2\pi i^I}{\omega_I^2}$$

$$G'(\sigma_1, \sigma_2) = \sum_{T \neq 0} \frac{2\pi\omega_T}{\omega_T^2} X_T(\sigma_1) X_T(\sigma_2)$$

$$G(\sigma_1, \sigma_2) = \sum_{I \neq 0} \frac{2\pi\alpha'}{\omega_I^2} \underbrace{X_I(\sigma_1)}_{\text{Eigenfunctions of } -D^2} X_I(\sigma_2)$$

Eigenfunctions of $-D^2$

$$G'(s_1, s_2) = \sum_{I \neq 0} \frac{2\pi\alpha'}{\omega_I^2} \underbrace{X_I(s_1)}_{\text{Eigenfunctions of } -D^2} X_I(s_2)$$

Eigenfunctions of $-D^2$

$$-\frac{1}{2\pi\alpha'}$$

$$G'(r_1, r_2) = \sum_{l \neq 0} \frac{2\pi\alpha'}{\omega_l^2} \underbrace{X_l(r_1)}_{\text{Eigenfunctions of } -\nabla^2} X_l(r_2)$$

Eigenfunctions of $-\nabla^2$

$$-\frac{1}{2\pi\alpha'} \nabla^2 G'(r_1, r_2) = \sum_{l \neq 0} X_l(r_1) X_l(r_2)$$

$$G'(r_1, r_2) = \sum_{l \neq 0} \frac{2\pi\alpha'}{\omega_l^2} \underbrace{X_l(r_1)}_{\text{Eigenfunctions of } -\Delta^2} X_l(r_2)$$

Eigenfunctions of $-\Delta^2$

$$-\frac{1}{2\pi\alpha'} \Delta^2 G'(r_1, r_2) = \sum_{l \neq 0} X_l(r_1) X_l(r_2)$$

$$G'(r_1, r_2) = \sum_{l \neq 0} \frac{2\pi\alpha'}{\omega_l^2} \underbrace{X_l(r_1)}_{\text{Eigenfunctions of } -\nabla^2} X_l(r_2)$$

Eigenfunctions of $-\nabla^2$

$$-\frac{1}{2\pi\alpha'} \nabla_1^2 G'(r_1, r_2) = \sum_{l \neq 0} X_l(r_1) X_l(r_2) = \frac{\delta^2(r_1 - r_2)}{\mathcal{V}}$$

$$G'(r_1, r_2) = \sum_{l \neq 0} \frac{2\pi\alpha'}{\omega_l^2} \underbrace{X_l(r_1)}_{\text{Eigenfunctions of } -\nabla^2} X_l(r_2)$$

Eigenfunctions of $-\nabla^2$

$$-\frac{1}{2\pi\alpha'} \nabla_1^2 G'(r_1, r_2) = \sum_{l \neq 0} X_l(r_1) X_l(r_2) = \frac{\delta^2(r_1 - r_2)}{\mathcal{G}} - X_0^2$$

$$G'(r_1, r_2) = \sum_{I \neq 0} \frac{2\pi\alpha'}{\omega_I^2} \underbrace{X_I(r_1)}_{\text{Eigenfunctions of } -\Delta^2} X_I(r_2)$$

Eigenfunctions of $-\Delta^2$

$$\frac{-\frac{1}{2\pi\alpha'} \Delta^2 G'(r_1, r_2)}{\frac{1}{\alpha'}} = \sum_{I \neq 0} X_I(r_1) X_I(r_2) = \frac{\delta^2(r_1 - r_2)}{\alpha'} - X_0^2$$

different complex parameters

$$G'(\sigma_1, \sigma_2) = \sum_{I \neq 0} \frac{2\pi\alpha'}{\omega_I^2} \underbrace{X_I(\sigma_1)}_{\text{Eigenfunctions of } -\Delta^2} X_I(\sigma_2)$$

Eigenfunctions of $-\Delta^2$

$$\frac{-\frac{1}{2\pi\alpha'} \Delta^2 G'(\sigma_1, \sigma_2)}{\int \mathcal{D}\sigma X_0} = \sum_{I \neq 0} X_I(\sigma_1) X_I(\sigma_2) = \frac{\delta^2(\sigma_1 - \sigma_2)}{\int \mathcal{D}\sigma} - X_0^2$$

$$\int \mathcal{D}\sigma X_0 G'(\sigma_1, \sigma_2) d^2\sigma = 0$$

different complex parameters

⇒ Restrict to a sphere.

- dx

(π^2)

⇒ Restrict to a sphere.

$$-d\Omega^2 = c^2 \omega^2 d\alpha d\bar{\alpha}$$



⇒ Restrict to a sphere.

$$d\Omega = \frac{2\omega}{c} d\alpha d\bar{z}$$

→ Restrict to a sphere.

$$d\bar{z}^2 = \int_C \omega \, dz \, d\bar{z}$$

$$G'(5, 6)$$

→ Restrict to a sphere.

$$d\bar{z}^2 = c^{2w} dz d\bar{z}$$

$$G'(b_1, b_2) = -\frac{c'}{2} \ln |z_1 - z_2|^2$$

→ Restrict to a sphere.

$$d\bar{z}^2 = c^{2w} dz d\bar{z}$$

$$G'(b_1, b_2) = -\frac{c'}{2} \ln |z_1 - z_2|^2 + f(z_1, \bar{z}_1) + f(z_2, \bar{z}_2)$$

$$G'(\sigma_1, \sigma_2) = \sum_{I \neq 0} \frac{2\pi\alpha'}{\omega_I^2} \underbrace{X_I(\sigma_1)}_{\text{Eigenfunctions of } -D^2} X_I(\sigma_2)$$

Eigenfunctions of $-D^2$

$$\frac{-\frac{1}{2\pi\alpha'} \nabla_{1,2}^2 G'(\sigma_1, \sigma_2)}{\int \mathcal{G} X_0} = \sum_{I \neq 0} X_I(\sigma_1) X_I(\sigma_2) = \frac{\delta^2(\sigma_1 - \sigma_2)}{\int \mathcal{G}} - X_0^2$$

$$\int \mathcal{G} X_0 G'(\sigma_1, \sigma_2) d^2\sigma = 0$$

$$d\mathcal{G} = e^{\omega} dz d\bar{z}$$

$$G'(z, \bar{z}) = -\frac{\alpha'}{2} \ln |z_1 - z_2|^2 + \frac{f(z_1, \bar{z}_1)}{4} + \frac{f(z_2, \bar{z}_2)}{4}$$

$$f(z, \bar{z}) = \frac{\alpha' X_0^2}{4} \int_{\sigma(z')} e^{2\omega(z', \bar{z}')} \ln |z - z'|^2$$

↑ splic

$$G'(\sigma_1, \sigma_2) = \sum_{I \neq 0} \frac{2\pi\alpha'}{\omega_I^2} \underbrace{X_I(\sigma_1)}_{\text{Eigenfunctions of } -D^2} X_I(\sigma_2)$$

Eigenfunctions of $-D^2$

$$\frac{-\frac{1}{2\pi\alpha'} \nabla_{1,2}^2 G'(\sigma_1, \sigma_2)}{\int \mathcal{D}\sigma X_0} = \sum_{I \neq 0} X_I(\sigma_1) X_I(\sigma_2) = \frac{\delta^2(\sigma_1 - \sigma_2)}{\int \mathcal{D}\sigma} - X_0^2$$

$$\int \mathcal{D}\sigma X_0 G'(\sigma_1, \sigma_2) d^2\sigma = 0 \quad *$$

$$d\Phi = e^{\omega} dz d\bar{z}$$

$$G'(b, \bar{b}) = -\frac{\alpha'}{2} \ln |z_1 - z_2|^2 + \frac{f(z_1, \bar{z}_1)}{4} + \frac{f(z_2, \bar{z}_2)}{4}$$

$$f(z, \bar{z}) = \frac{\alpha' X_0^2}{4} \int_{\sigma(z')}^z e^{2\omega(z', \bar{z}')} \ln |z - z'|^2 + \textcircled{K}$$

↑ arbitrary constant

↑ sphere

→ Scattering of tachyons

A_{S^2}

→ Scattering of tachyons

$$A_S^z = \langle e^{i k_1 X(\sigma_1)} \dots e^{i k_n} \rangle$$

→ Scattering of tachyons

$$A_{S^2} = \left\langle e^{i k_i X(\sigma_i)} \dots e^{i k_n X(\sigma_n)} \right\rangle_{S^2}$$

→ Scattering of tachyons

$$A_{S^2} = \left\langle e^{i k_1 X(\sigma_1)} \dots e^{i k_n X(\sigma_n)} \right\rangle_{S^2}$$
$$T(\sigma) = \sum_{i=1}^n k_i \delta^2(\sigma - \sigma_i)$$

→ Scattering of tachyons

$$A_{S^2} = \left\langle e^{i k_1 X(\sigma_1)} \dots e^{i k_n X(\sigma_n)} \right\rangle_{S^2}$$

$$T(\sigma) = \sum_{i=1}^n k_i \delta^2(\sigma - \sigma_i)$$

$$\left\langle e^{i \int T(\sigma) X(\sigma)} \right\rangle$$

Arbitrary
constant

→ Scattering of tachyons

$$A_{S^2}^{(n)} = \left\langle e^{i k_1 X(\sigma_1)} \dots e^{i k_n X(\sigma_n)} \right\rangle_{S^2}$$

$$J(\sigma) = \sum_{i=1}^n k_i \delta^2(\sigma - \sigma_i)$$
$$\left\langle e^{i \int J(\sigma) X(\sigma)} \right\rangle$$

$$A_{S_2}^{(s)} = i C_{S_2}^X \delta^D$$

$$A_{S_2}^{(s)} = i C_{S_2}^x \delta^D \left(\begin{matrix} \mathcal{N}_3 \\ \mathcal{K}_3 \end{matrix} \right)$$

$$\delta^D(S_0)$$

$$X_0^{-D} \left(\det \frac{-\Delta^2}{\sqrt{|\Delta|}} \right)^{D/2} S_2$$

$$A_{S_2}^{(s)} = i C_{S_2}^X \delta^D \left(\frac{\sqrt{2}}{2} k_0 \right) e^{\frac{2\pi i k_0 x}{\lambda}}$$

$$\delta^D(S_0)$$

$$X_0^{-D} \left(\det \frac{-\Delta^2}{4\pi^2} \right)^{D/2} S_2$$

$$A_{S_2}^{(s)}$$

$$= i C_{S_2}^X$$

$$\delta^D \left(\sum_{i=1}^D k_i \right)$$

$$e^{-\sum_{k \in S_2} k_i k_j G'(0)}$$

$$\delta^D(\mathcal{J}_0)$$

$$X_0^{-D} \left(\det \frac{-\Delta^2}{\text{mid}^2} \right)^{D/2}_{S_2}$$



$$A_{S_2}^{(s)} = i C_{S_2}^X \delta^D \left(\sum_{i=1}^D k_i \right) e^{-i \sum_{k=1}^D k_i k_j G'(0, S)}$$

$$\delta^D(S_0)$$

$$X_0^{-D} \left(\det \frac{-\delta^2}{\omega^2} \right)_{S_2}^{D/2}$$

$$A_{S_2}^{(n)} = i C_{S_2}^X \delta^D \left(\prod_{i=1}^n k_i \right) e^{-i \sum_{i=1}^n k_i k_j G'(\sigma_i, \sigma_j)} - \frac{1}{2} \sum_{l=1}^n k_l^L G'(\sigma_i, \sigma_l)$$

$$\delta^D(\vec{J}_0)$$

$$X_0^{-D} \left(\det \frac{\delta^2 S_2}{\delta \sigma^2} \right)^{D/2}$$

$$A_{S_2}^{(n)} = i C_{S_2}^X \delta^D \left(\prod_{i=1}^n k_i \right) e^{i \sum_{i=1}^n k_i k_j G'(\sigma_i, \sigma_j)} - \frac{1}{2} \sum_{i=1}^n k_i^L G'(\sigma_i, \sigma_i)$$

$$X_0^{-D} \left(\det' \frac{-\Delta^2}{4\pi^2 d} \right)_{S_2}^{D/2}$$

$$\int d\sigma^L d\sigma'^L J(\sigma) J(\sigma') G'(\sigma, \sigma')$$

$$A_{S_2}^{(n)} = i \int_{S_2} \delta^D \left(\frac{\mathbf{r}_i}{r_0} \right) \left[\sum_{i,j} \kappa_i \kappa_j G(\sigma_i, \sigma_j) - \frac{1}{2} \sum_{i=1}^n \kappa_i^2 G'(\sigma_i) \right]$$

$$\int d\sigma d\sigma' \underline{I}(\sigma) \underline{I}(\sigma') G'(\sigma, \sigma')$$

$$\left(\det \frac{\delta^2 \mathcal{L}}{\delta \sigma^2} \right)_{S_2}^{D/2}$$

$$A_{S_2}^{(n)} = i C_{S_2} \delta \left(\sum_{i=1}^n k_i \right) e^{i \sum_{i=1}^n \phi_i} - \frac{1}{2} \sum_{i=1}^n k_i^2 G(\phi_i, \phi_i)$$

$$\delta^D(\vec{S}_0)$$

$$\int d\phi^L d\phi'^L \underline{J}(\phi) \underline{I}(\phi') G(\phi, \phi')$$

$$X_0^{-D} \left(\det \frac{-\nabla^2}{m^2} \right)_{S_2}^{D/2}$$



$$G'(z, \bar{z}) = -\frac{\alpha'}{2} \ln |z_1 - z_2|^2 + \frac{f(z_1, \bar{z}_1)}{2} + \frac{f(z_2, \bar{z}_2)}{2}$$

$$f(z, \bar{z}) = \frac{\alpha' X_0^2}{4} \int_{\mathcal{D}} d^2 z' e^{2\omega(z, \bar{z}')} \ln |z - z'|^2 + K$$

Arbitrary constant

$G'_T =$

$$G = \sum_{i=1}^n \kappa_i \delta^2 (z - z_i)$$

$$J(\sigma) X_0(\sigma)$$

$$G'(z, \bar{z}) = -\frac{\alpha'}{2} \ln |z_1 - z_2|^2 + \frac{f(z_1, \bar{z}_1)}{2} + \frac{f(z_2, \bar{z}_2)}{2}$$

$$f(z, \bar{z}) = \frac{\alpha' X_0^2}{4} \int d\sigma^2 z' e^{2\omega(z, \bar{z})} \ln |z - z'|^2 + k$$

$$-\frac{\alpha'}{2} \ln d_{12}^2$$

$$G'_T = -\alpha' \omega(\tau, \sigma) + \frac{\alpha'}{2} \ln z^2$$

↑ arbitrary constant

$$J(\sigma) = \sum_{i=1}^n k_i \delta^2(\sigma - \sigma_i)$$

$$\int J(\sigma) X_+(\sigma)$$

$$G'(z_1, z_2) = -\frac{\alpha}{2} \ln |z_1 - z_2|^2 + \frac{f(z_1, \bar{z}_1)}{2} + \frac{f(z_2, \bar{z}_2)}{2}$$

$$f(z, \bar{z}) = \frac{\alpha' X_0^2}{4} \int d^2 z' e^{2\omega(z, \bar{z})} \ln |z - z'|^2 + k$$

$$-\frac{\alpha'}{2} \ln d^2 z$$

$$G'_r = -\alpha' \omega(r, \theta) + 2f(z, \bar{z})$$

Arbitrary constant

$$J(\sigma) = \sum_{i=1}^n k_i \delta^2(\sigma - \sigma_i)$$

$$: \int J(\sigma) X_4(\sigma)$$

$$A_{S_2}^{(S)} = i C_{L_2} \delta_0^D \left(\sum k_i \right) e^{-\frac{\alpha'}{2} \sum_{i=1}^n k_i^2 \omega(k_i)} \prod_{\substack{i,j=1 \\ i < j}}^n |z_{ij}|^{\alpha' k_i k_j}$$

$$A_{S_2}^{(S)} = i C_{lm} \sum_0^D (\sum k_i) e^{-\frac{\alpha^2}{2} \sum_{i=1}^n k_i^2 \omega(k_i)}$$

$\left(\prod_{\substack{i,j=1 \\ i < j}}^n |z_{ij}| \right)^{\alpha^2 k_i k_j}$

$$A_{S_2}^{(5)} = i C_{L_n} \delta^D(\sum k_i) e^{-\frac{\alpha'}{2} \sum k_i^2 \omega(k_i)} \prod_{\substack{i,j=1 \\ i < j}}^n |z_{ij}|^{\alpha' k_i k_j}$$

$$A^{(n)} \int \prod_{i=1}^{n-3} |z_{ij}|^{\alpha' k_i k_j} \rightarrow$$



$$A_{S_2}^{(g)} = i C_{\text{un}} \int_0^D (\sum k_i) e^{-\frac{\alpha'}{2} \sum k_i^2 \omega(k_i)} \prod_{i < j}^{i, j=1}^n |z_{ij}|^{\alpha' k_i \cdot k_j}$$

$A^{(n)}$ $\int_{\text{mod } \mathbb{Z}^2}$ $\prod_{i=1}^{n-3}$ $|z_{ij}|^{\alpha' k_i \cdot k_j}$ \rightarrow Virasoro amplitudes