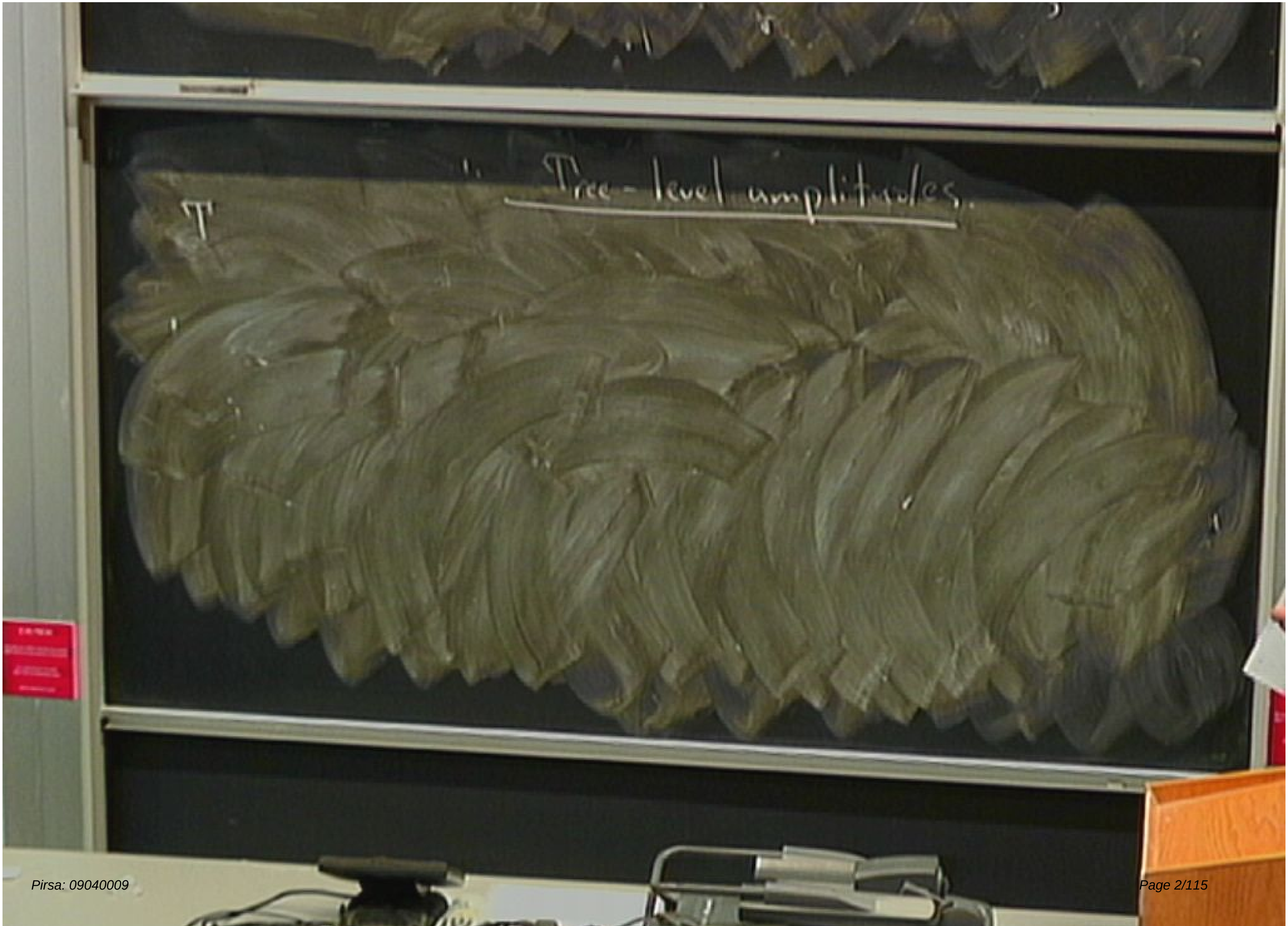
Title: Introduction to the Bosonic String Part B

Date: Apr 03, 2009 12:00 PM

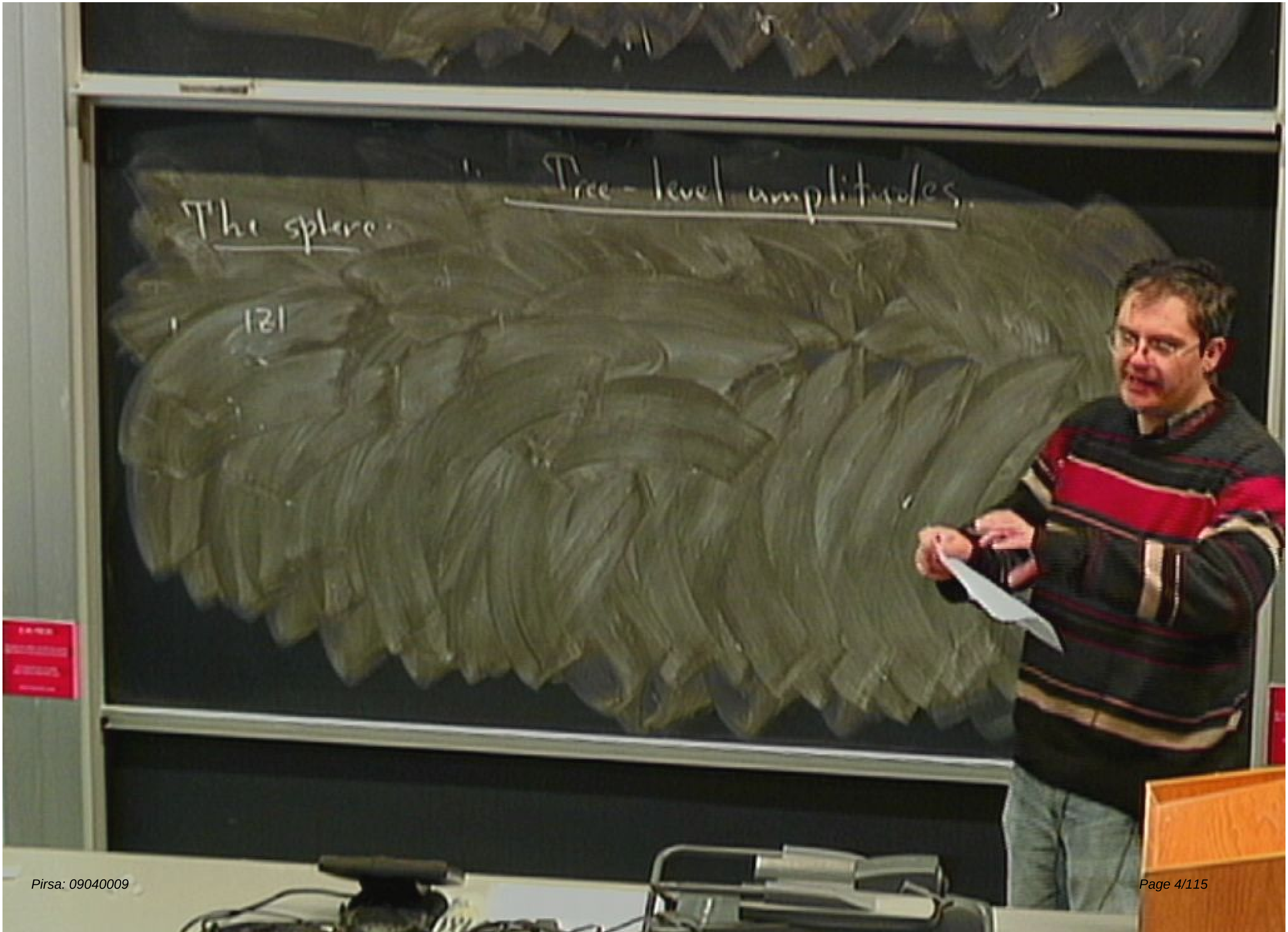
URL: http://pirsa.org/09040009

Abstract: This course provides a thorough introduction to the bosonic string based on the Polyakov path integral and conformal field theory. We introduce central ideas of string theory, the tools of conformal field theory, the Polyakov path integral, and the covariant quantization of the string. We discuss string interactions and cover the tree-level and one loop amplitudes. More advanced topics such as T-duality and D-branes will be taught as part of the course. The course is geared for M.Sc. and Ph.D. students enrolled in Collaborative Ph.D. Program in Theoretical Physics. Required previous course work: Quantum Field Theory (AM516 or equivalent). The course evaluation will be based on regular problem sets that will be handed in during the term. The primary text is the book: 'String theory. Vol. 1: An introduction to the bosonic string. J. Polchinski (Santa Barbara, KITP) . 1998. 402pp. Cambridge, UK: Univ. Pr. (1998) 402 p.' All interested students should contact Alex Buchel at abuchel@uwo.ca as soon as possible.


Tree-level amplitudes.

1. Tree-level amplitudes.

The sphere:


Tree-level amplitudes.
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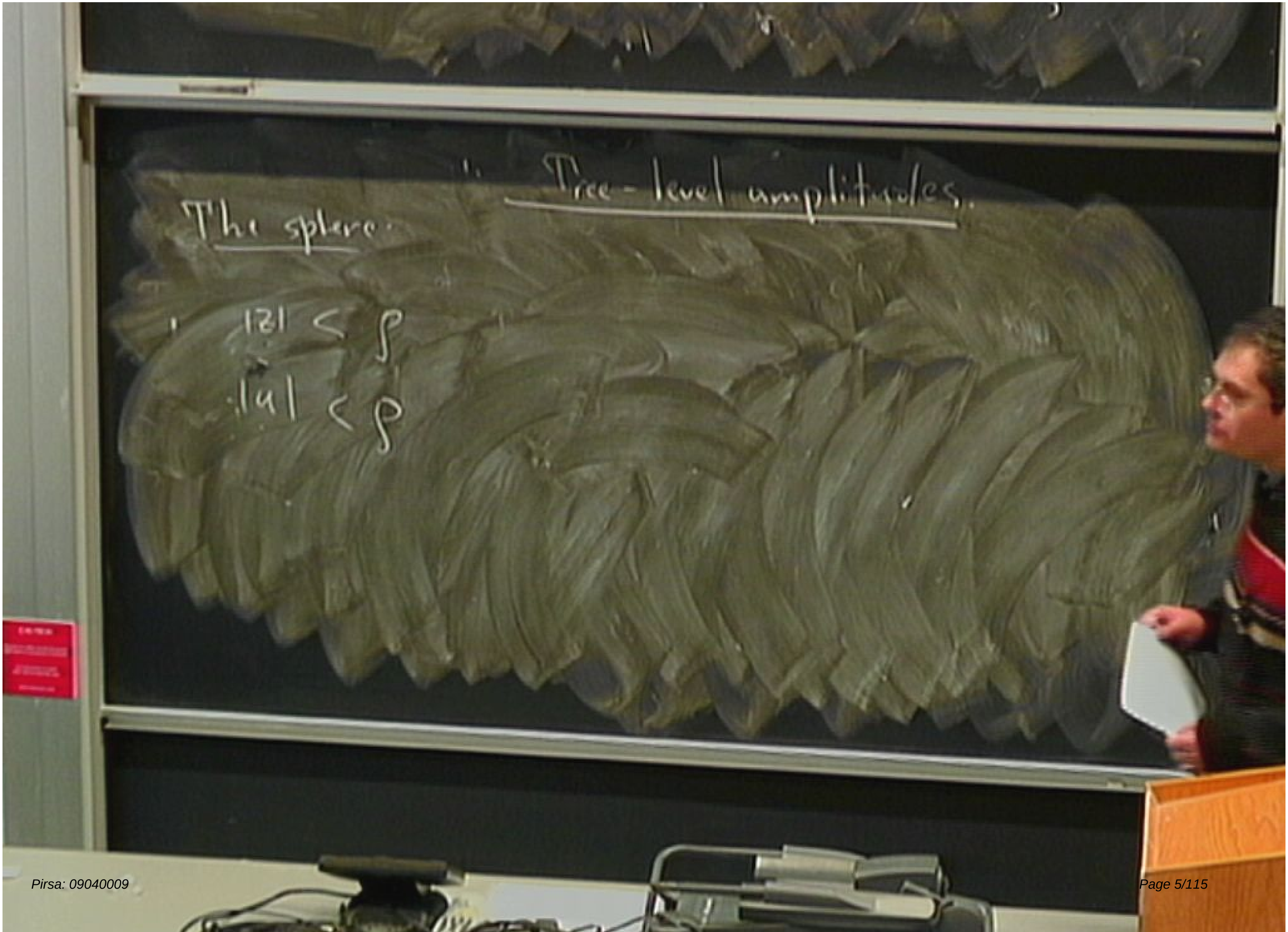
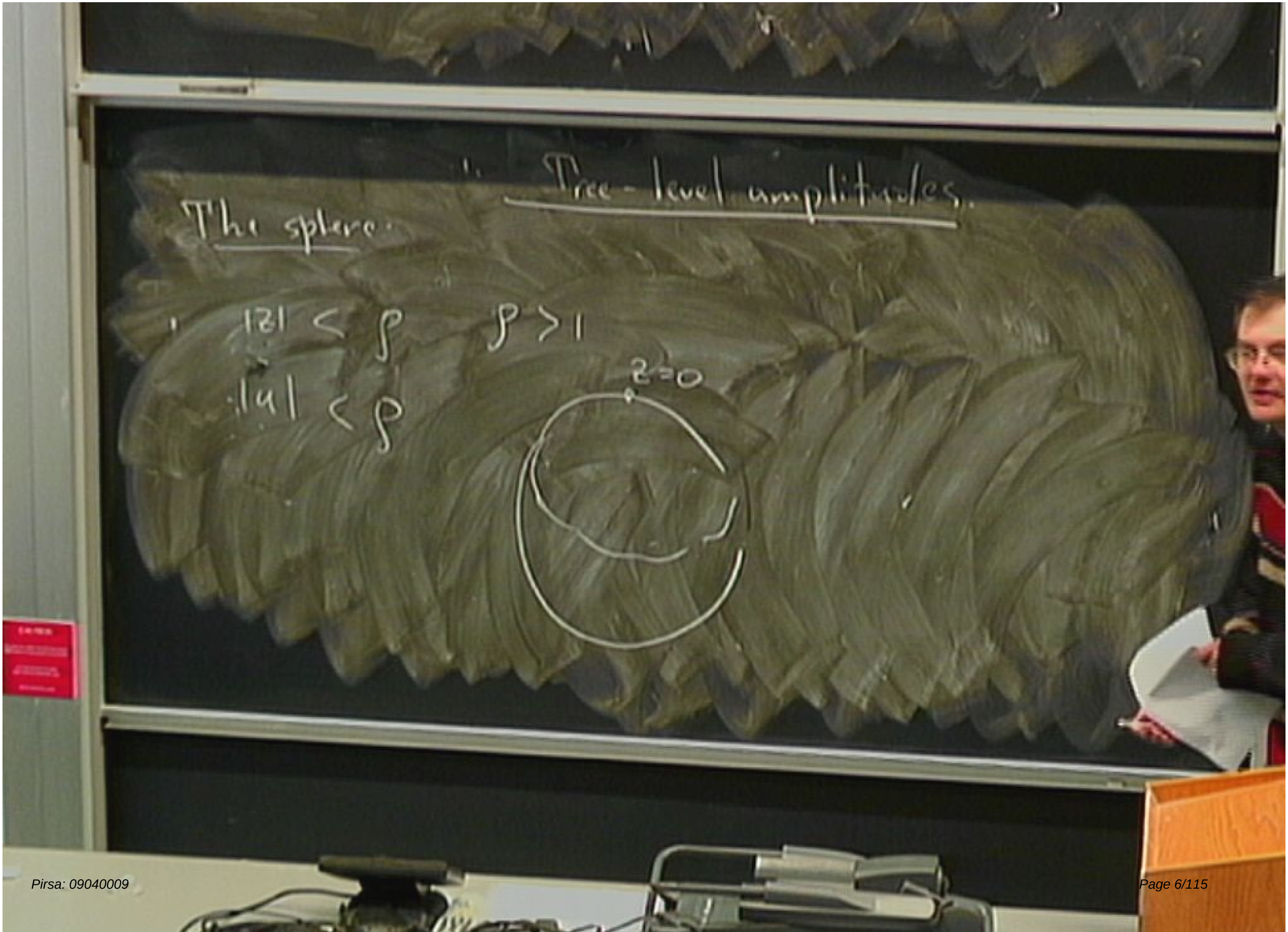

Tree-level amplitudes.
The sphere:
|z| < ρ
|u| < ρ

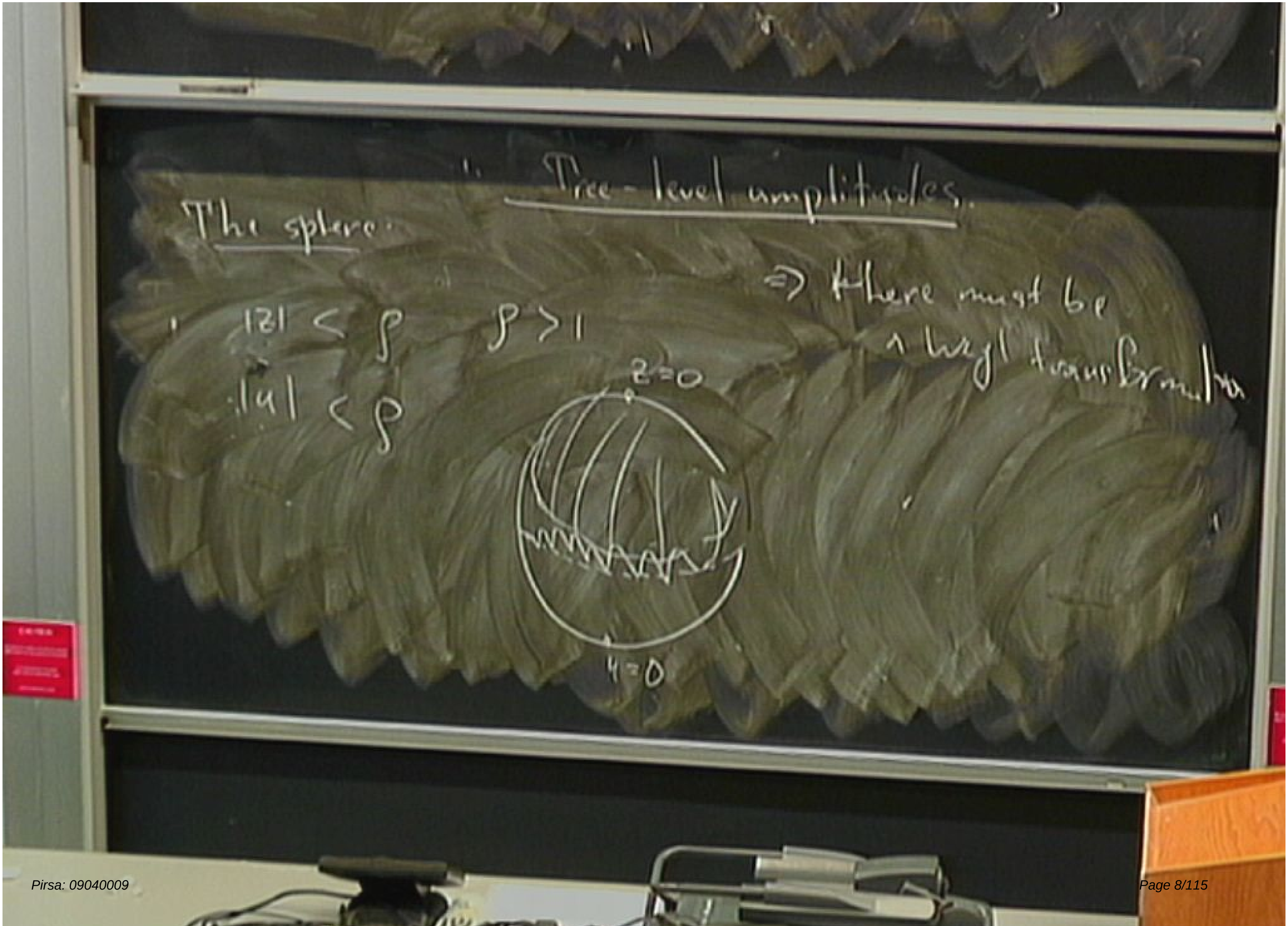

Tree-level amplitudes.
The sphere:
|z| < ρ
ρ > 1
|u| < ρ
z=0

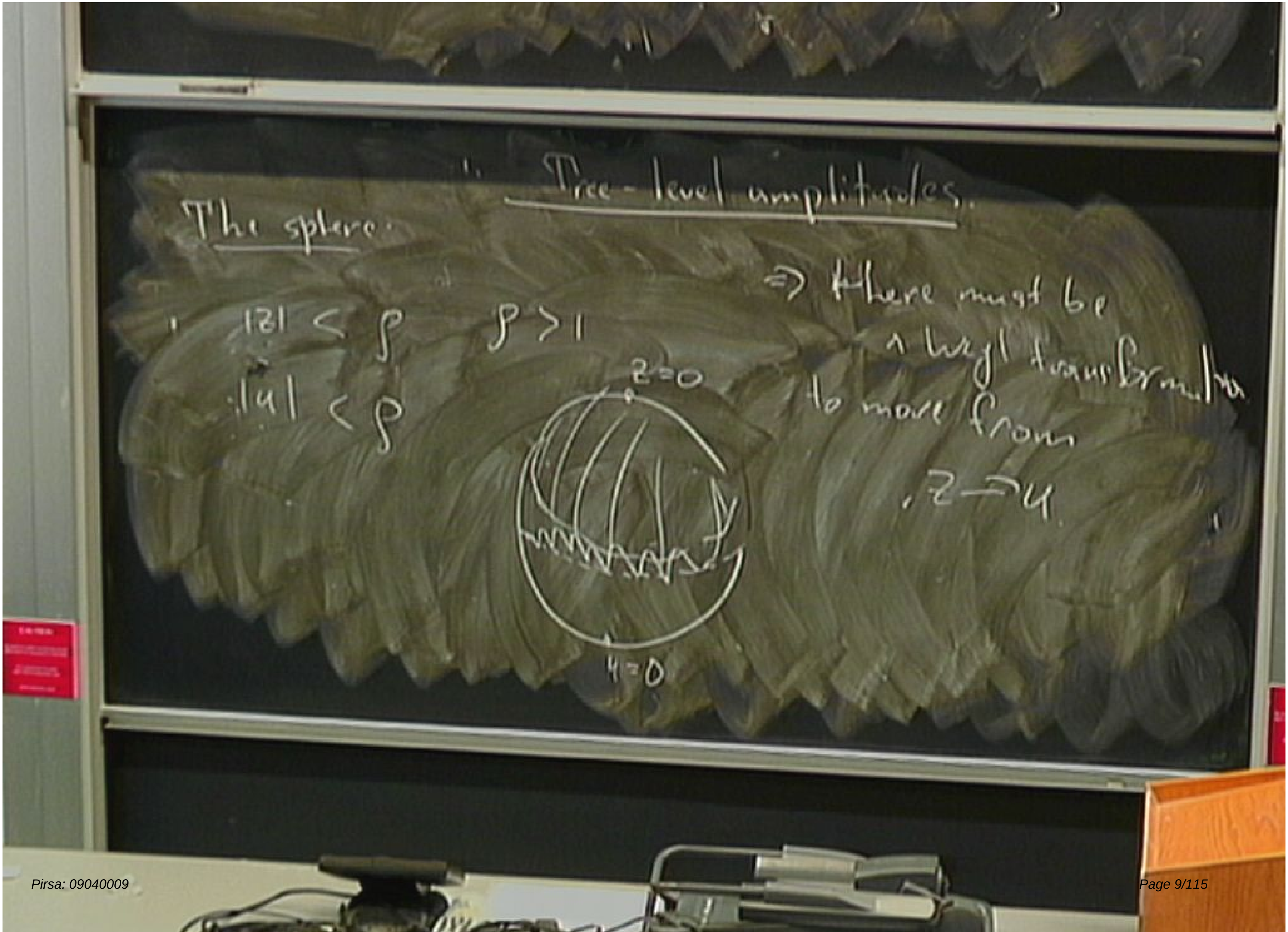
Tree-level amplitudes.

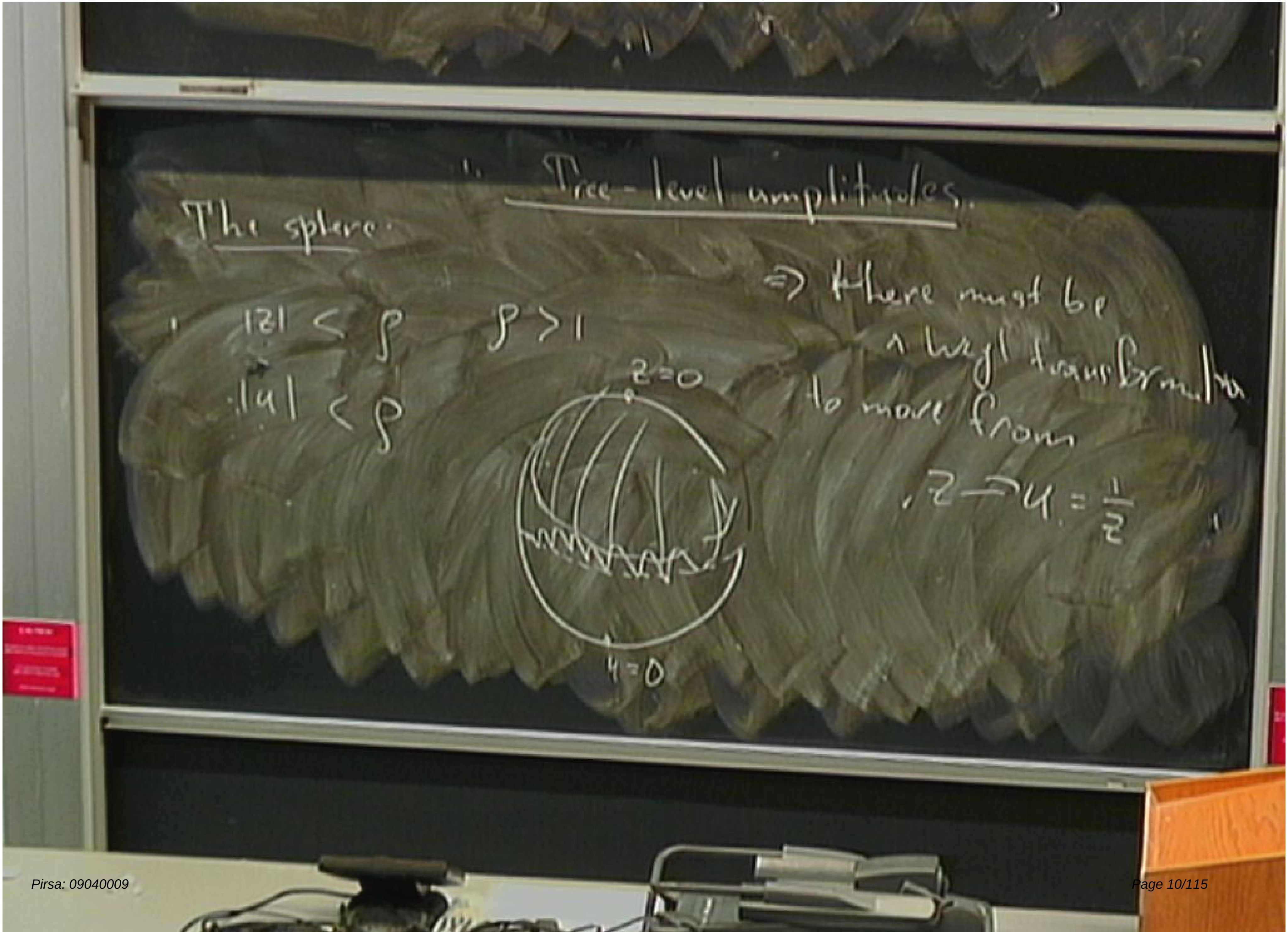
The sphere:

$|z| < \rho \qquad \rho > 1$

$|u| < \rho$


Tree-level amplitudes.
The sphere:
|z| < ρ
ρ > 1
|u| < ρ
⇒ there must be
a Weyl transformation
z=0
u=0


Tree-level amplitudes.
The sphere:
$|z| < \rho$
$\rho > 1$
$|u| < \rho$
$z=0$
$u=0$
$\Rightarrow$ there must be a Weyl transformation to move from
$z \to u$


Tree-level amplitudes.
The sphere.
$|z| < \rho$ $\rho > 1$
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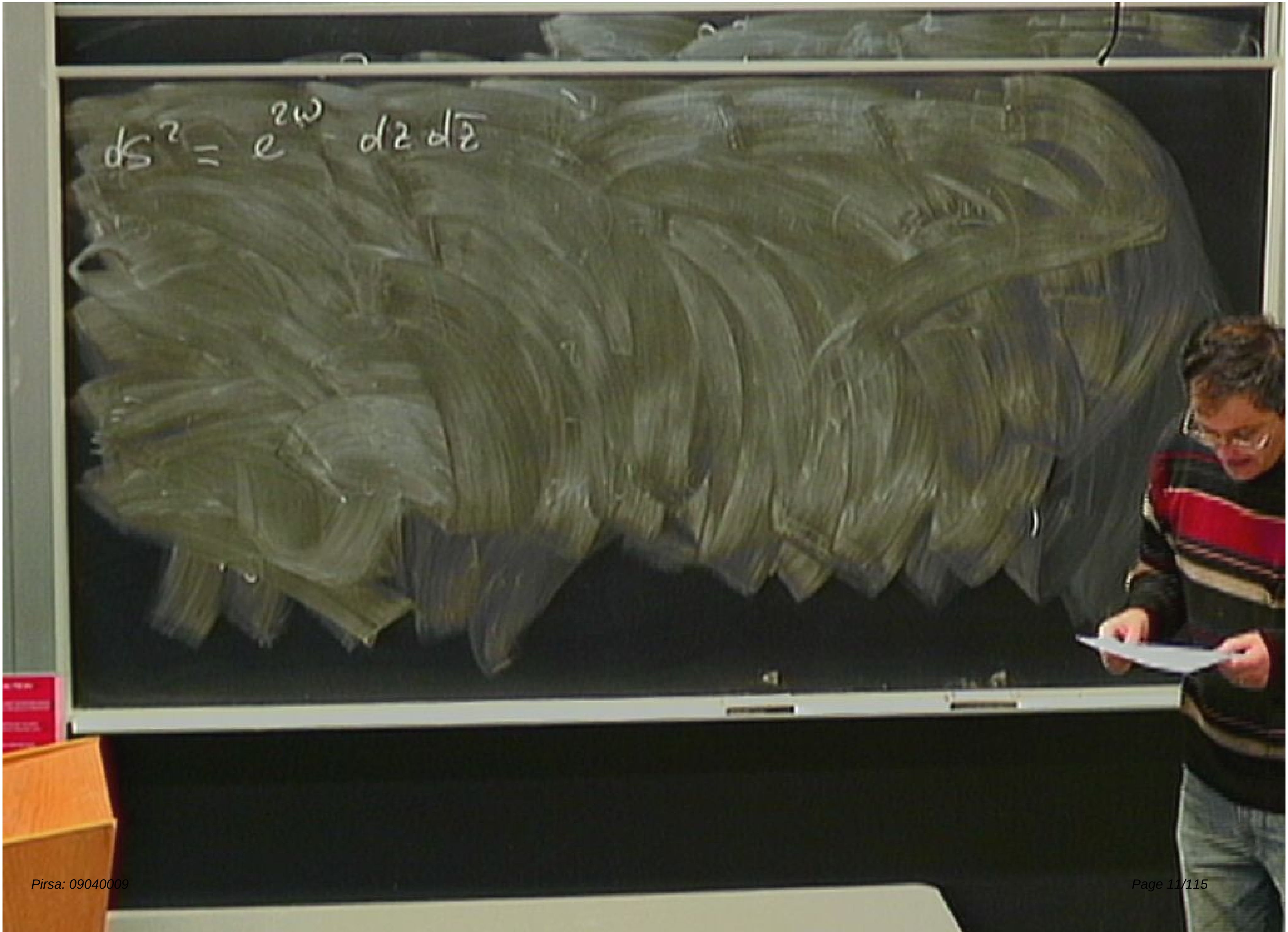
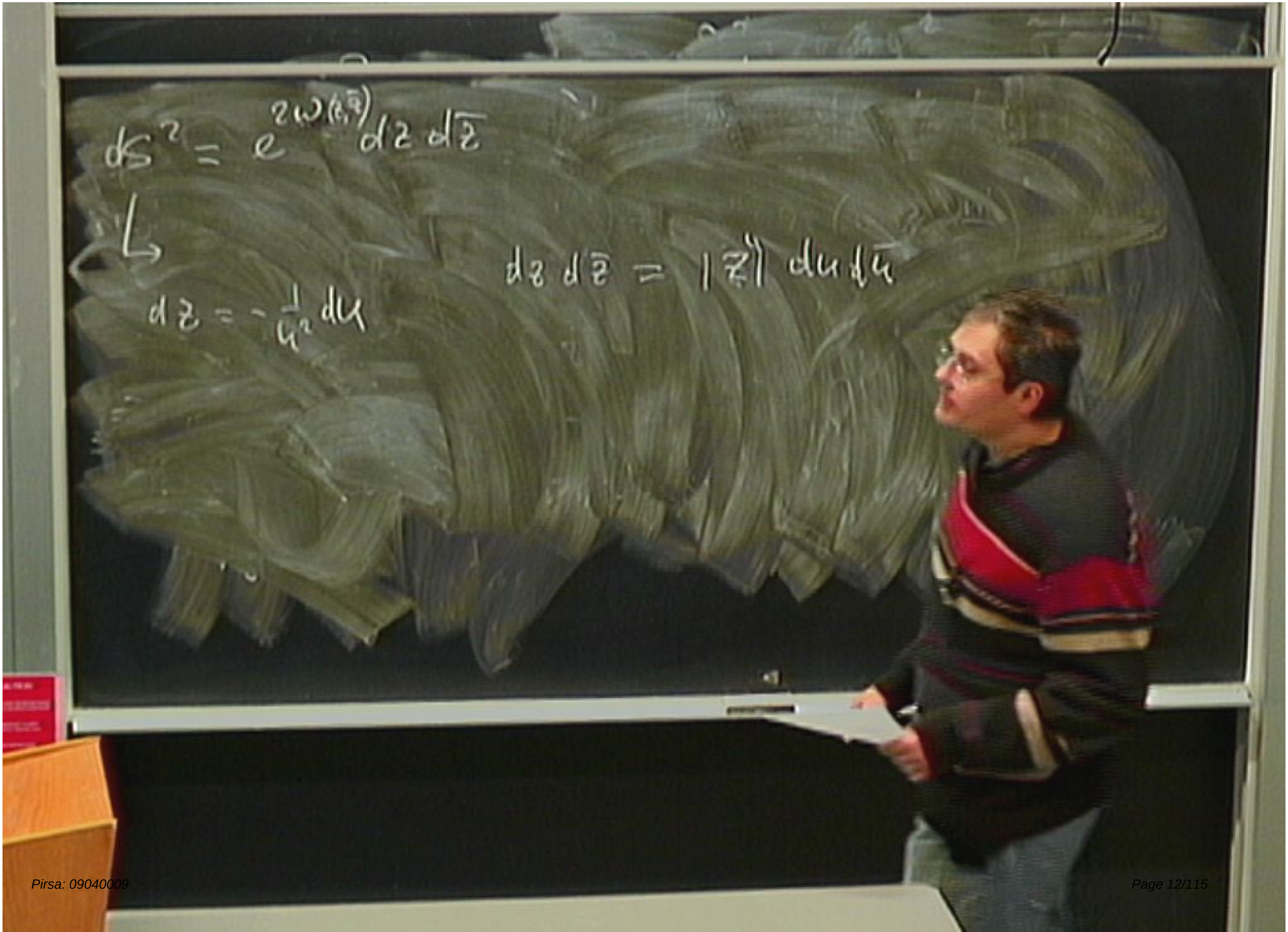
$\Rightarrow$ there must be a Weyl transformation to move from
$z \to u = \frac{1}{z}$


$ds^2 = e^{2\omega} dz\, d\bar{z}$


ds² = e^{2ω(z,z̄)} dz dz̄
dz = − 1/u² du
dz dz̄ = |z| du dū

1. Tree-level amplitudes.

The sphere:

$|z| < \rho \qquad \rho > 1$

$|u| < \rho$

$\Rightarrow$ there must be a Weyl transformation to move from

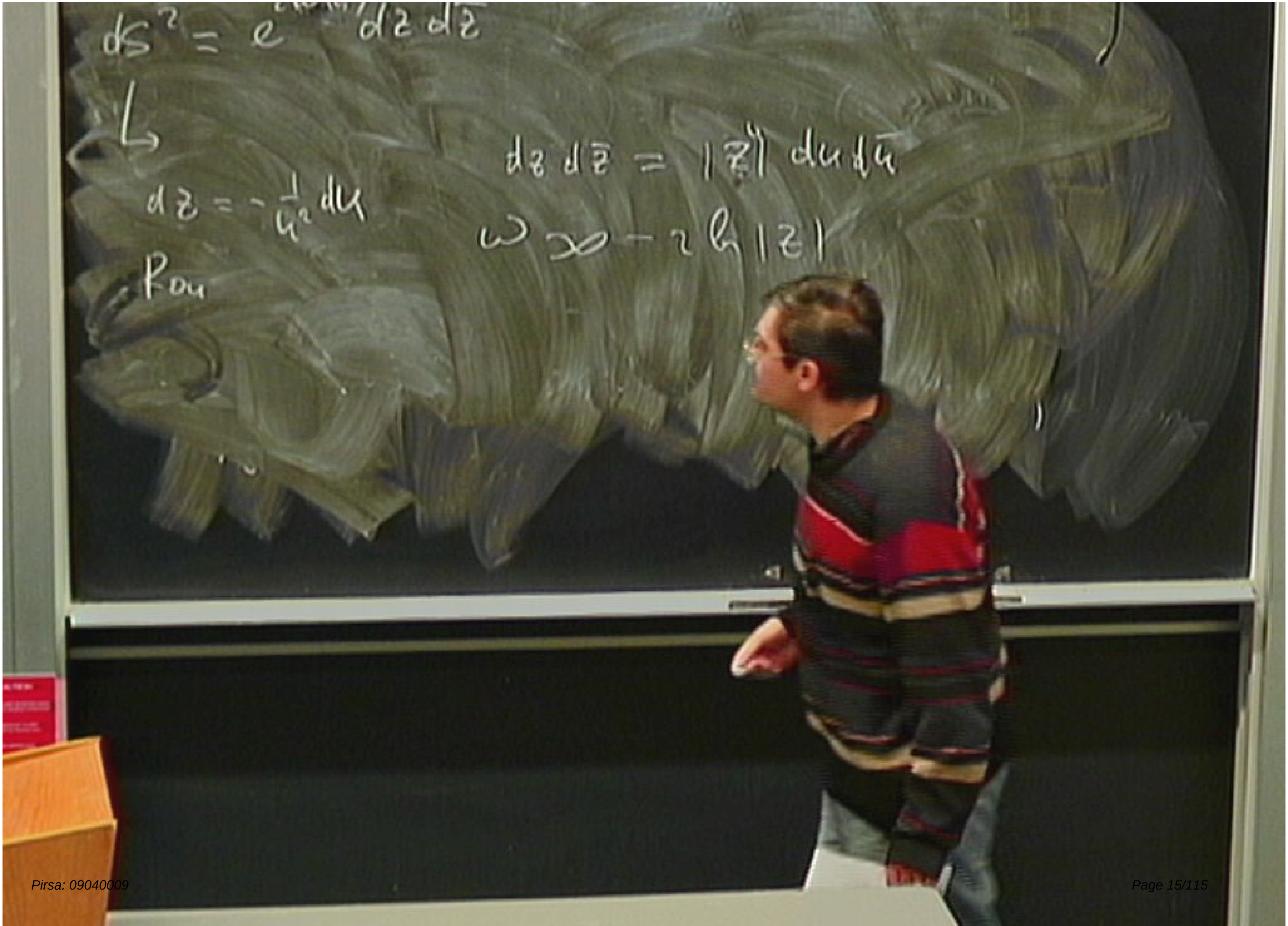
$z \to u = \frac{1}{z}$

$$ds^2 = e^{2\omega(z,\bar z)}\, dz\, d\bar z$$

$$dz = -\frac{1}{u^2}\, du$$

$$dz\, d\bar z = |z|^4\, du\, d\bar u$$

$$\omega \propto -2 \ln|z|$$


ds² = e dz dz̄
dz dz̄ = |z| du dū
dz = - 1/u² du
ω ∞ - 2 ln|z|
R₀ᵤ

$$ds^2 = e^{\cdots} dz\, d\bar{z}$$

$\hookrightarrow$

$$dz\, d\bar{z} = |z| \, du\, d\bar{u}$$

$$dz = -\frac{1}{u^2} du \qquad \omega \sim -2 \ln |z|$$

<u>Round metric:</u>

$$ds^2 = \frac{4r^2 \, dz\, d\bar{z}}{(1+z\bar{z})^2}$$

$$dz = -\frac{1}{u^2}\,du$$

$$\omega \propto -2\ln|z|$$

Round metric:

$$ds^2 = \frac{4r^2\,dz\,d\bar{z}}{(1+z\bar{z})^2}$$

$|z| < \rho \quad \rho > 1$

$dz = -\frac{1}{u^2} du$

$\omega \infty - 2\ln|z|$

Round metric :

$$ds^2 = \frac{4r^2\, dz\, d\bar{z}}{(1+z\bar{z})^2} = \frac{4r^2\, du\, d\bar{u}}{(1+u\bar{u})^2}$$

$|z| < \rho \quad \rho > 1$

$z - \frac{1}{z+5}$

$$dz = -\frac{1}{u^2}\,du$$

$$\omega \approx -2\ln|z|$$

Round metric :

radius^2 of a sphere.

$$ds^2 = \frac{4r^2\,dz\,d\bar{z}}{(1+z\bar{z})^2} = \frac{4r^2\,du\,d\bar{u}}{(1+u\bar{u})^2}$$

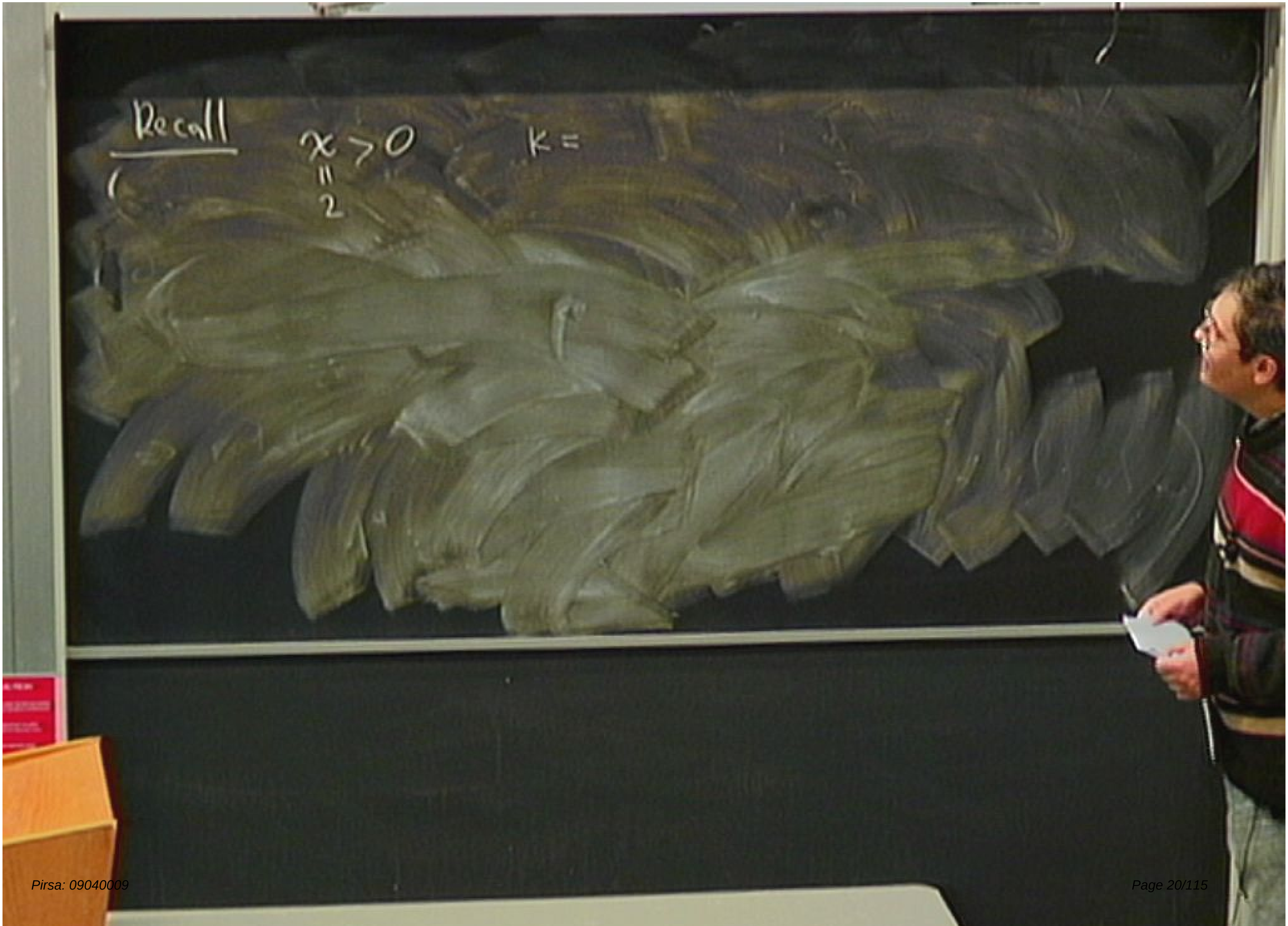
$$R = \frac{2}{r^2}$$

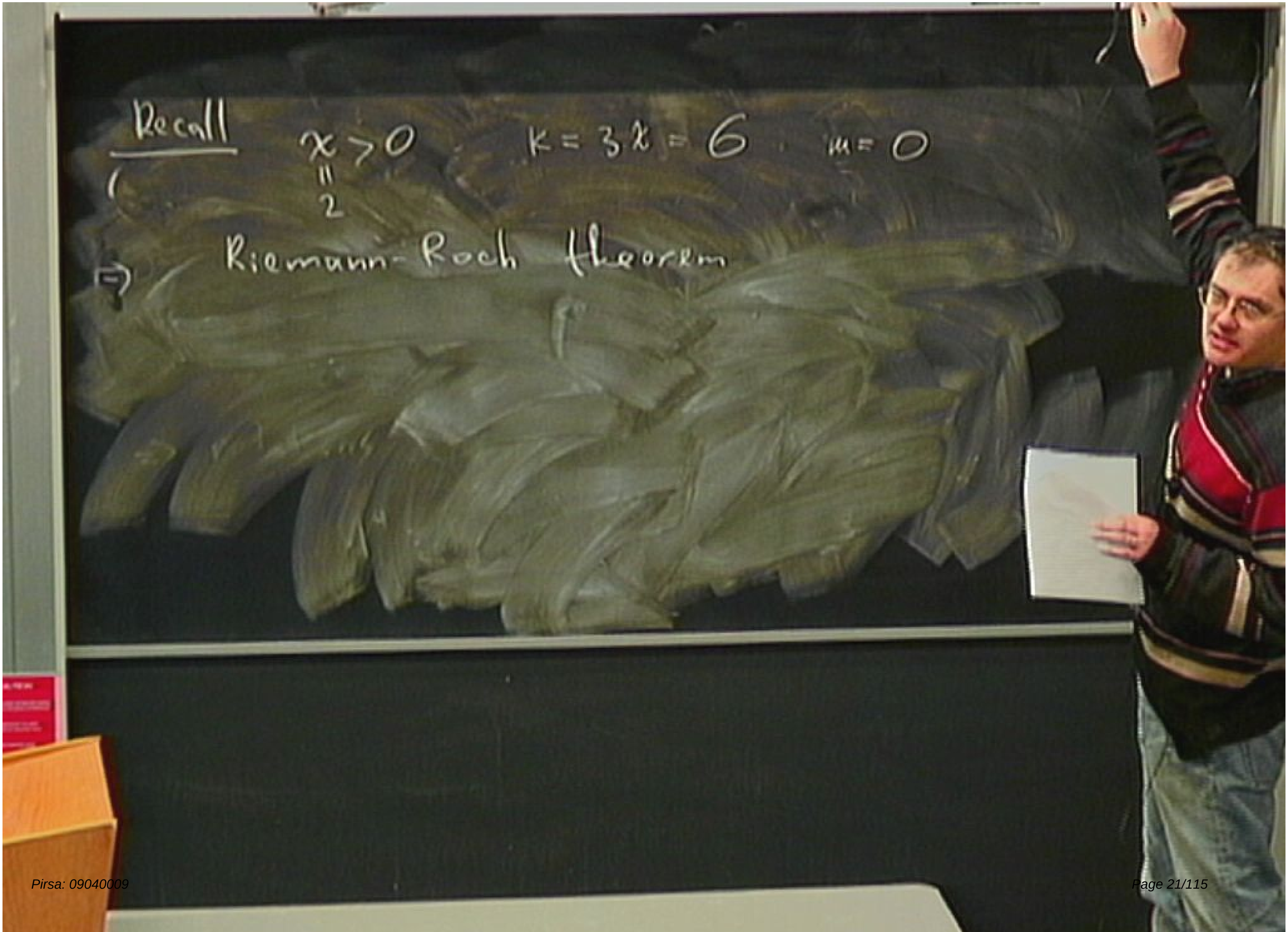
$|z| < \rho \quad \rho > 1$

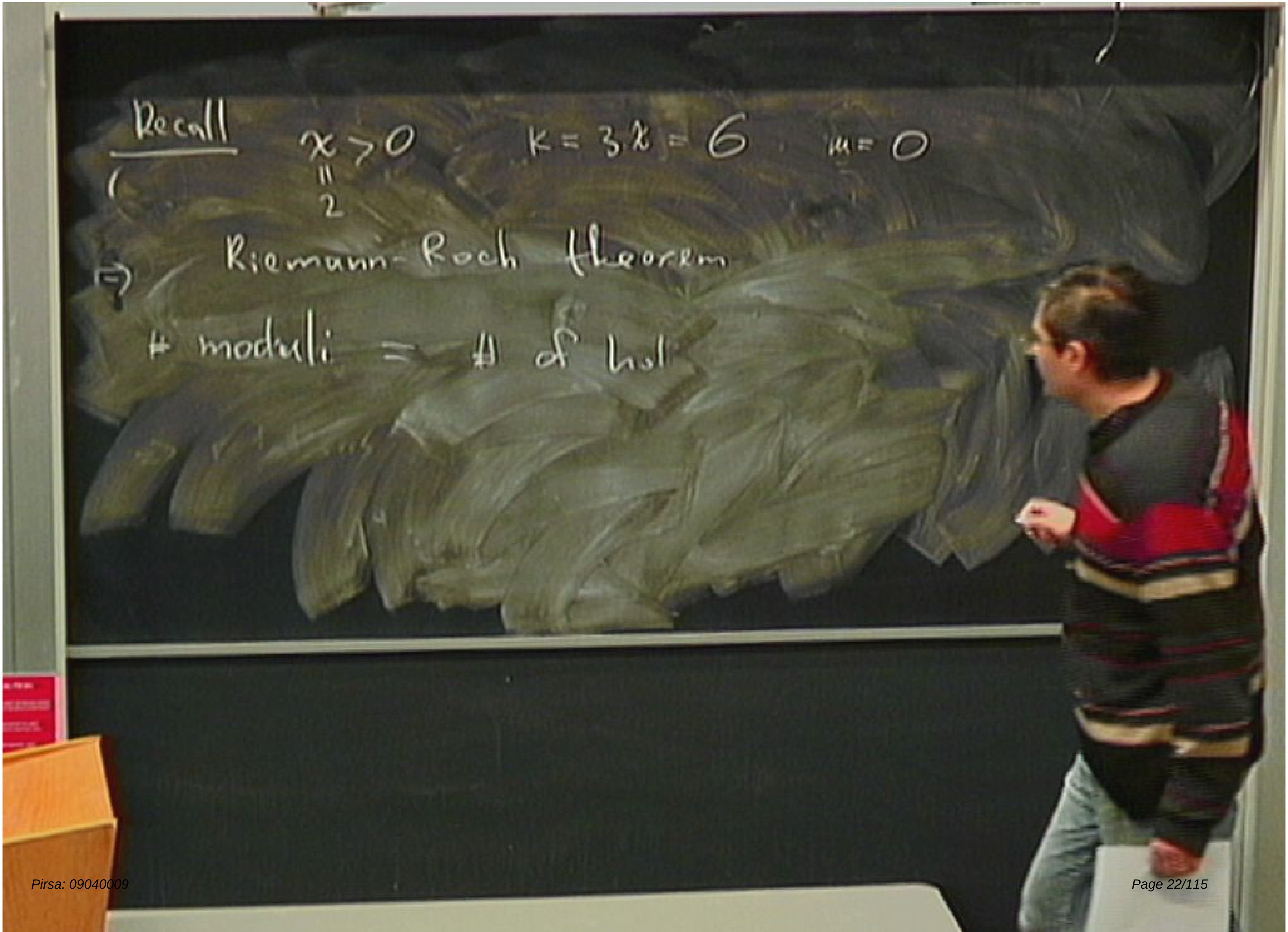
$\tau + 5$

$\frac{1}{}$

$\tau - \frac{1}{\tau+5}$


Recall
x > 0
2
K =


Recall
χ > 0
=
2
K = 3χ = 6
m = 0
Riemann-Roch theorem


Recall
χ > 0 = 2
K = 3χ = 6
m = 0
⇒ Riemann-Roch theorem
moduli = # of hol

Recall

$\left(\chi > 0 \right.$ $\qquad K = 3\chi = 6 \qquad m = 0$

$\chi = 2$

$\Rightarrow$ Riemann-Roch theorem

\# moduli = \# of holomorphic differentials δg_{zz}

Recall

$\underset{2}{\overset{\|}{\chi}} > 0 \qquad K = 3\chi = 6 \qquad m = 0$

$\Rightarrow$ Riemann-Roch theorem

moduli = # of holomorphic differentials δg_{zz}

CKV = # of holomorphic vectors δz

⇒ Suppose we have g_{a}

$\Rightarrow$ Suppose we have $g_{z\bar{z}}$ is holomorphic near the North pole (small z)

$\Rightarrow$ Suppose we have $\delta g_{\bar{z}\bar{z}}$ is holomorphic near the North pole (small z)

δg_{uu}

⇒ Suppose we have δg_{zz} is holomorphic near the North pole (small z)

$$\delta g_{uu} = \left(\frac{\partial u}{\partial z}\right)^{-2} \delta g_{zz}$$

⇒ Suppose we have δg_{zz} is holomorphic near the North pole (small z)

$$\delta g_{uu} = \left(\frac{\partial u}{\partial z}\right)^{-2} \delta g_{zz} = z^4 \, \delta g_{zz}$$

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⇒

⇒ Suppose we have δg_{zz} is holomorphic near the North pole (small z)

$$\delta g_{uu} = \left(\frac{\partial u}{\partial z}\right)^{-2} \delta g_{zz} = z^4 \, \delta g_{zz}$$

$$\Rightarrow \delta g_{zz} \propto \mathcal{O}\left(\frac{1}{z^4}\right)$$

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$$\Rightarrow \delta g_{zz} \propto O\left(\frac{1}{z^4}\right) \Rightarrow$$ There does not exist δg_{zz} which is simultaneously hol around $z=0$ and $u=0$

$\boxed{u=0}$

the dual work by

δT


z=0 and y=0

z=0 and u=0
δz(z) → δu(u) = ∂u/∂z δz = −z⁻² δz

$z = 0$ and $u = 0$

$$\delta z(z) \to \delta u(u) = \frac{\partial u}{\partial z}\,\delta z = -z^{-2}\,\delta z$$

δz to be holomorphic <u>both</u> about $z = 0$

and

$u = 0$ @ $z = \infty$

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$u = 0$

$$\delta z(z) \to \delta u(u) = \frac{\partial u}{\partial z}\,\delta z = -z^{-2}\,\delta z$$

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$$\delta z = a_0$$

$z=0$ and $u=0$

$$\delta z(z) \to \delta u(u) = \frac{\partial u}{\partial z}\,\delta z = -z^{-2}\,\delta z$$

δz to be holomorphic both about $z=0$ and $u=0$ @ $z=\infty$

$$\delta z = a_0 + a_1 z + a_2 z^2$$

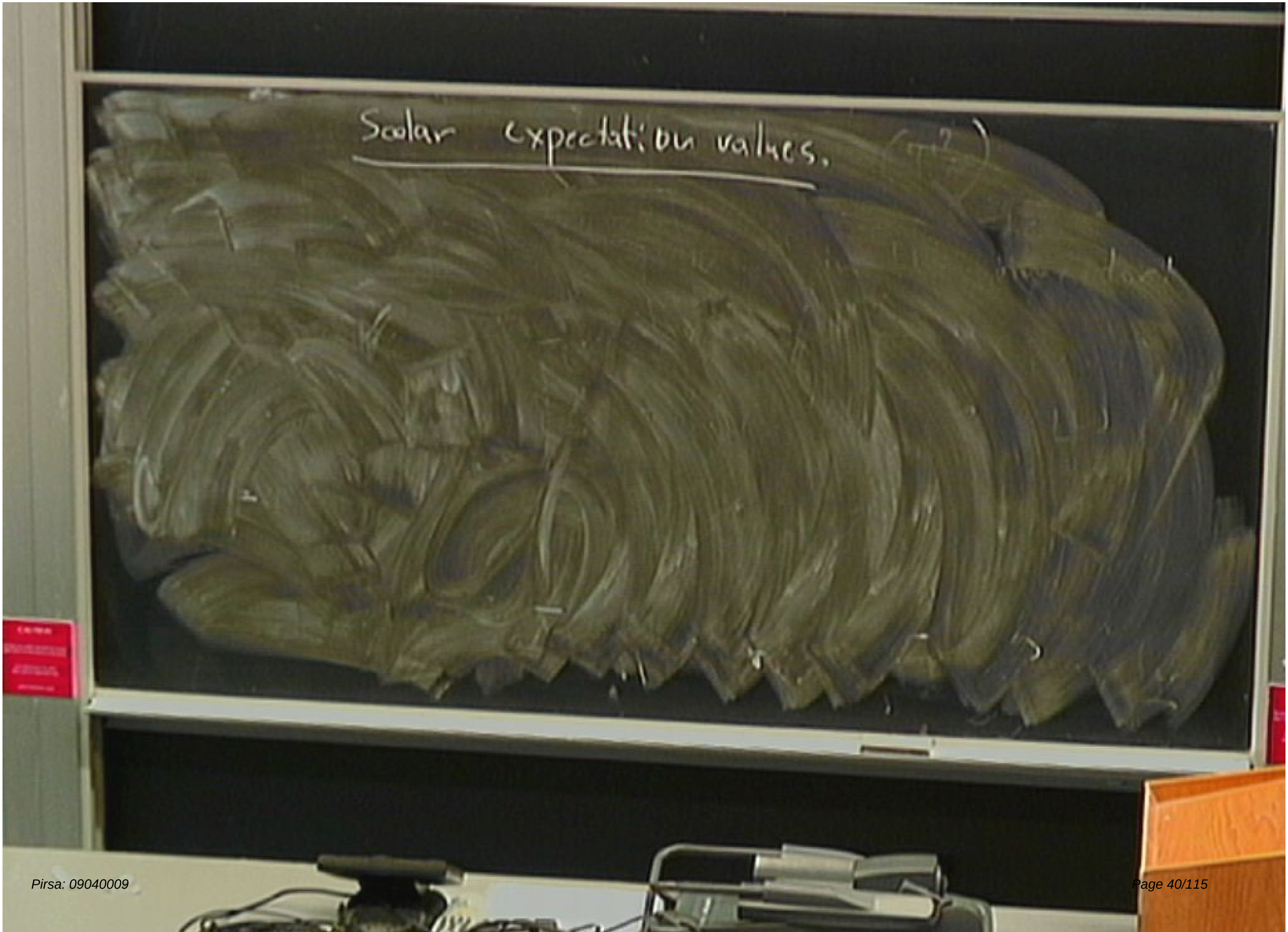
$z=0$ and $u=0$

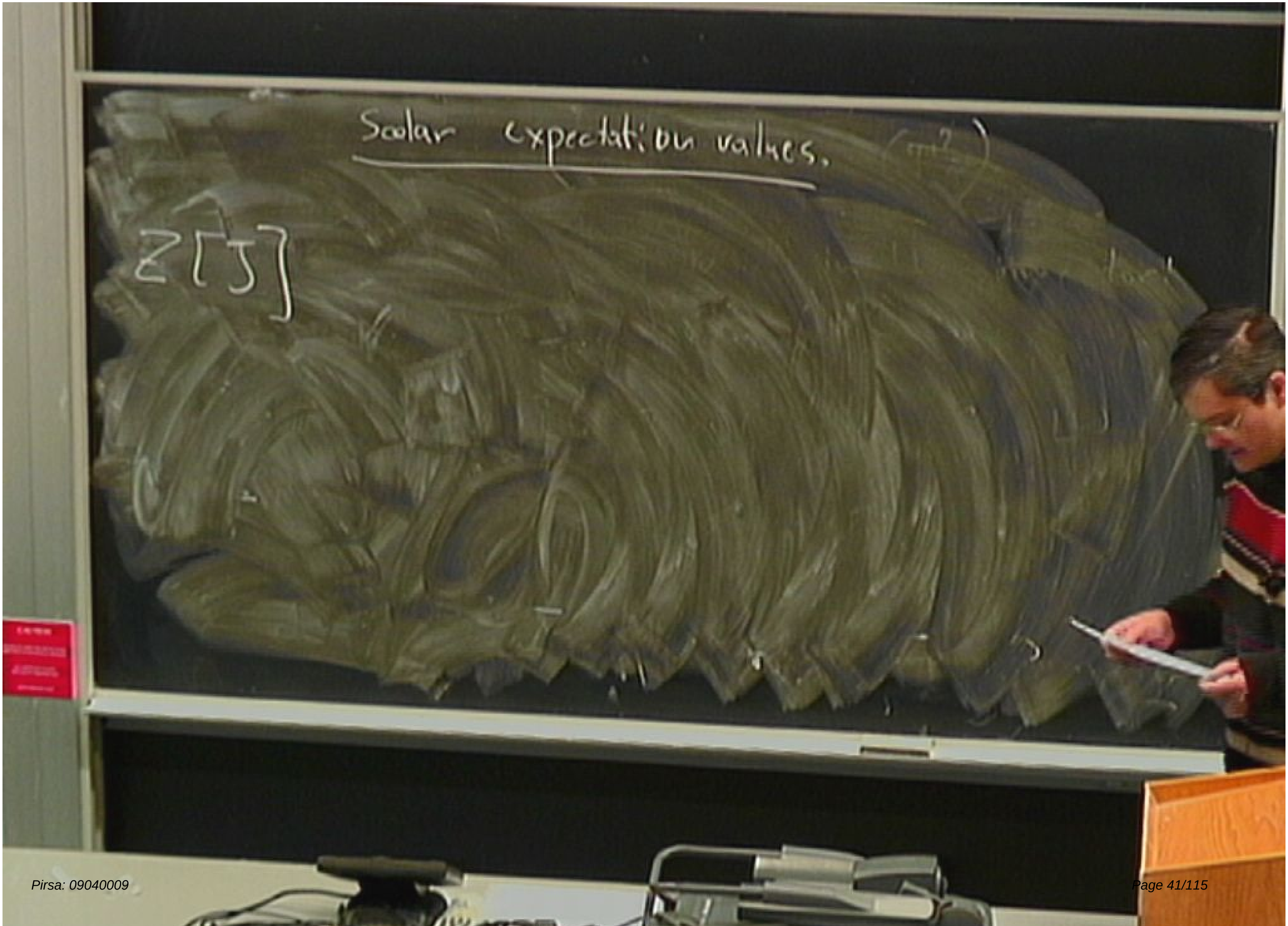
$$\delta z(z) \to \delta u(u) = \frac{\partial u}{\partial z}\,\delta z = -z^{-2}\,\delta z$$

δz to be holomorphic both about $z=0$
and
$u=0$ @ $z=\infty$

$$\delta z = a_0 + a_1 z + a_2 z^2$$

3 different complex parameters.


Scalar expectation values.


Scalar expectation values.
Z[J]

Scalar expectation values.

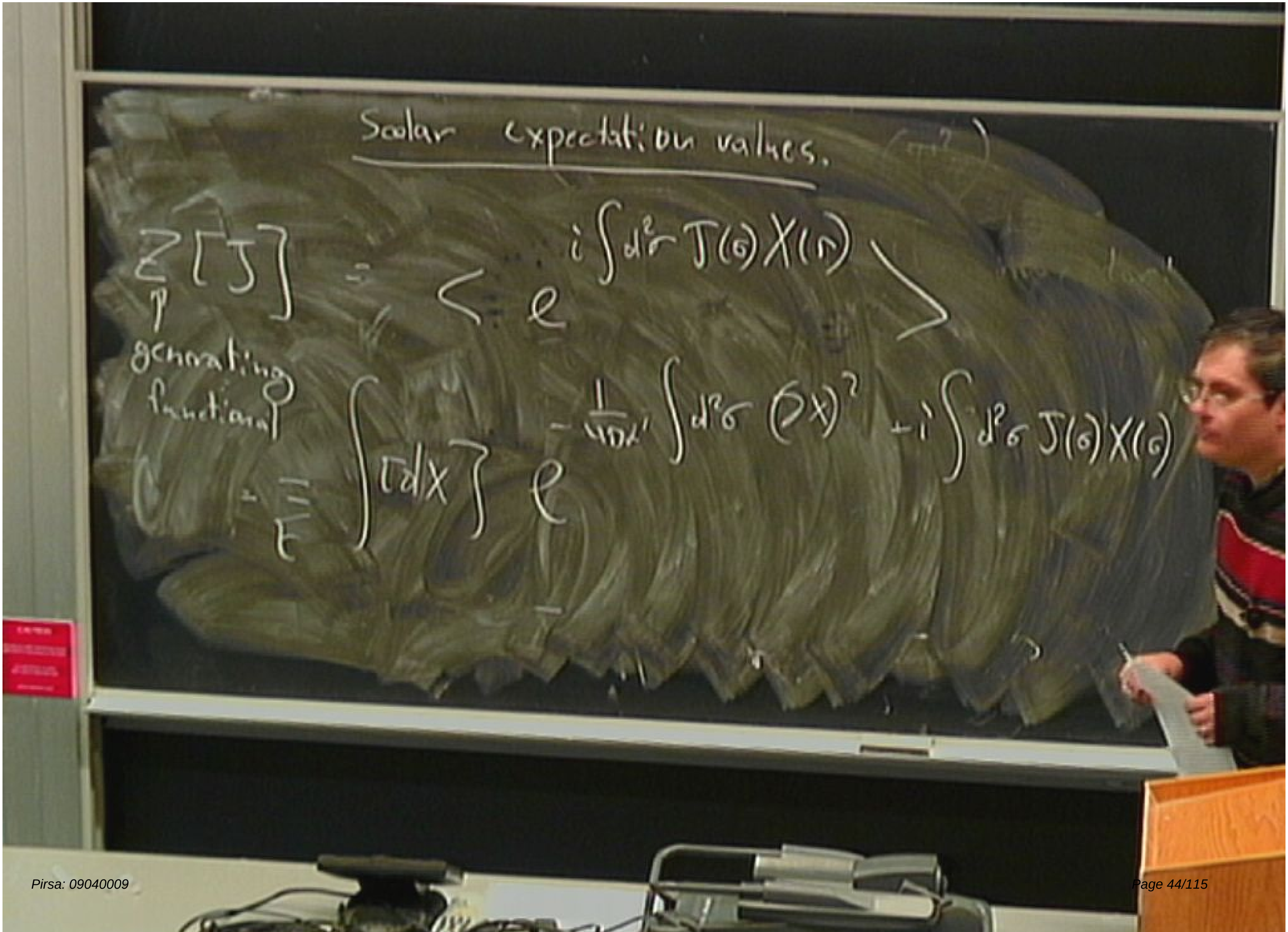
$Z[J]$

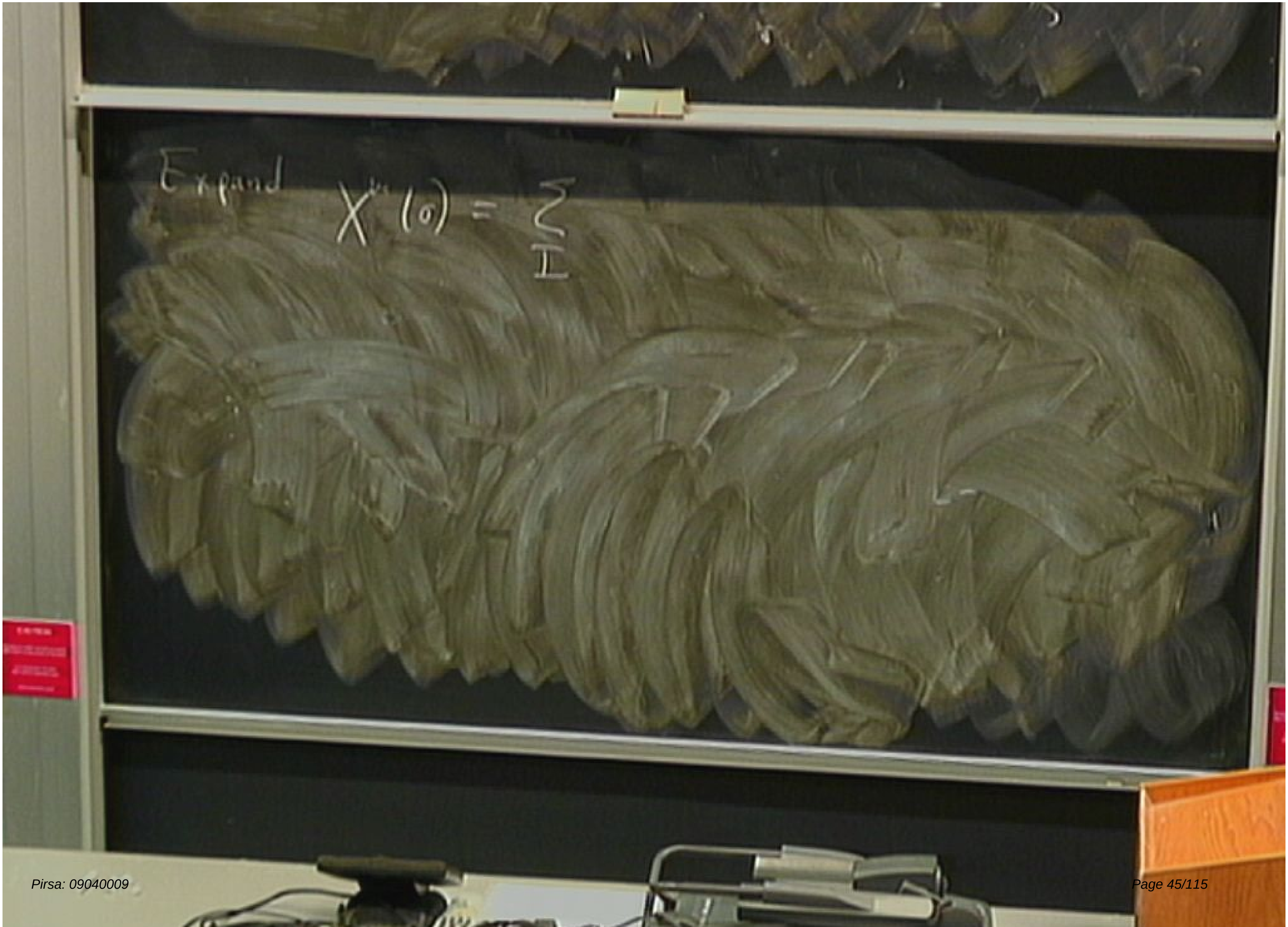
generating functional

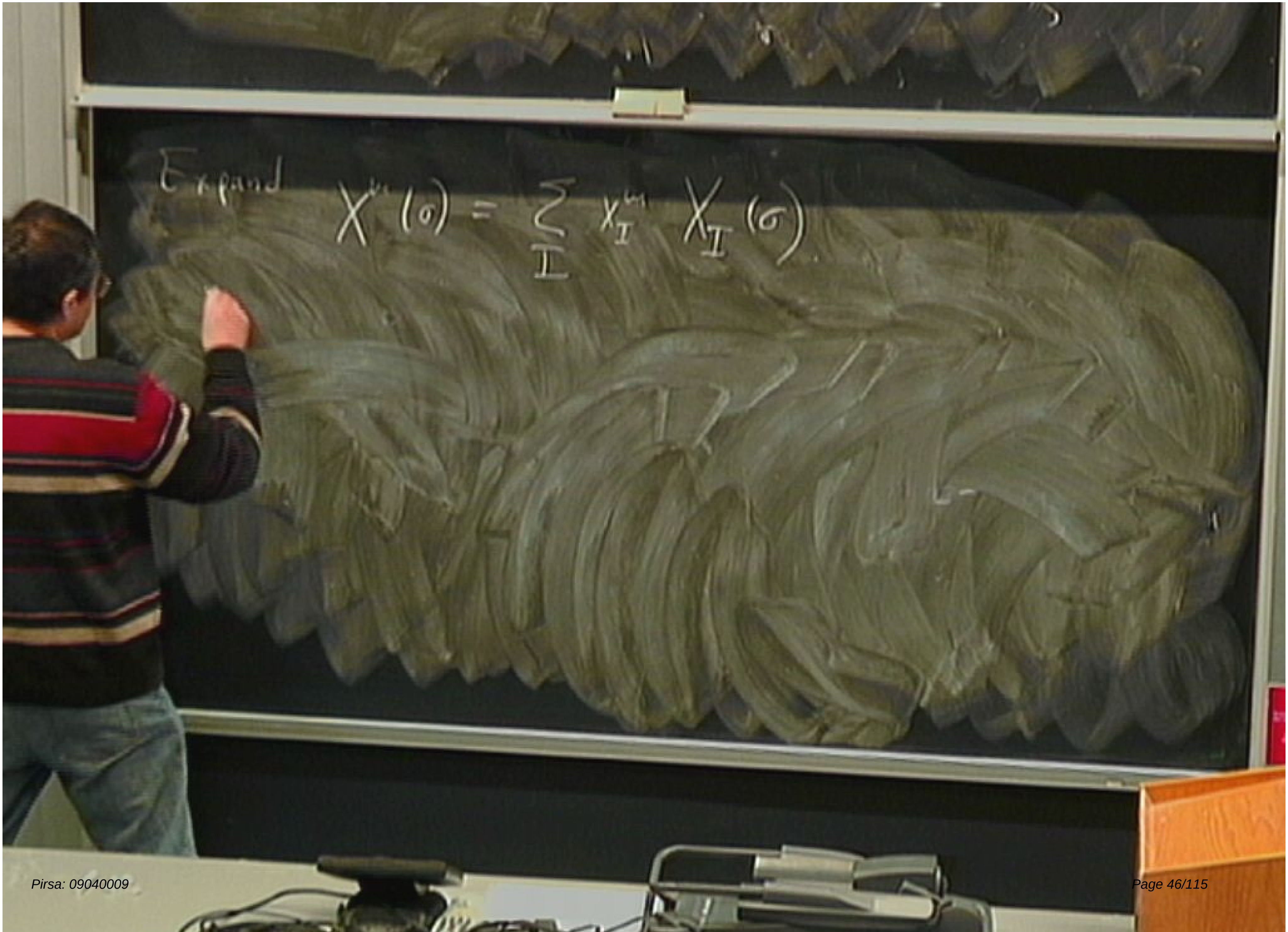
Scalar expectation values.

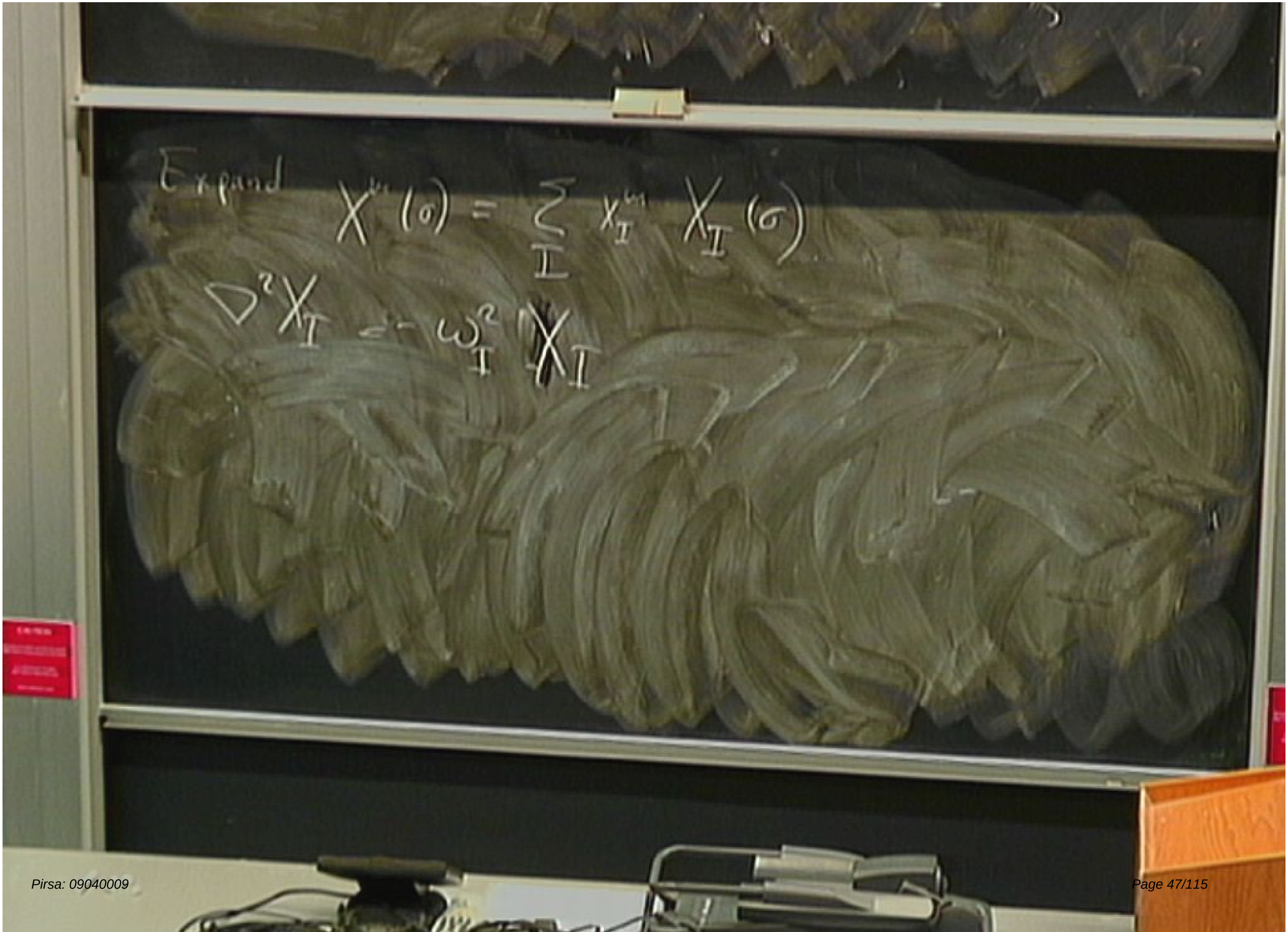
$$Z[J] = \left\langle e^{i\int d^2\sigma\, J(\sigma) X(\sigma)} \right\rangle$$

generating functional


Scalar expectation values.
$Z_P[J] = \langle e^{i\int d^2\sigma\, J(\sigma) X(\sigma)} \rangle$
generating functional
$= \int [dX]\, e^{-\frac{1}{4\pi\alpha'}\int d^2\sigma\, (\partial X)^2 + i\int d^2\sigma\, J(\sigma) X(\sigma)}$


Expand $X^{\mu}(\sigma) = \sum_{I}$


Expand $X^\mu(\sigma) = \sum_I x_I^\mu X_I(\sigma)$


Expand $X^\mu(\sigma) = \sum_I x_I^\mu X_I(\sigma)$
$\nabla^2 X_I = -\omega_I^2 X_I$

Expand $X^\mu(\sigma) = \sum_I x_I^\mu \, X_I(\sigma)$

$\nabla^2 X_I = -\omega_I^2 X_I$

$\int$

Expand $X^\mu(\sigma) = \sum_I x_I^\mu X_I(\sigma)$

$$\nabla^2 X_I = -\omega_I^2 X_I$$

$$\int_{M} d^2\sigma \sqrt{g}\, X_I X_{I'} = \delta_{II'}$$

sphere

$$J^{\mu}_{I} = \int d^2\sigma \; J^{\mu}(\sigma)\, X_I(\sigma)$$

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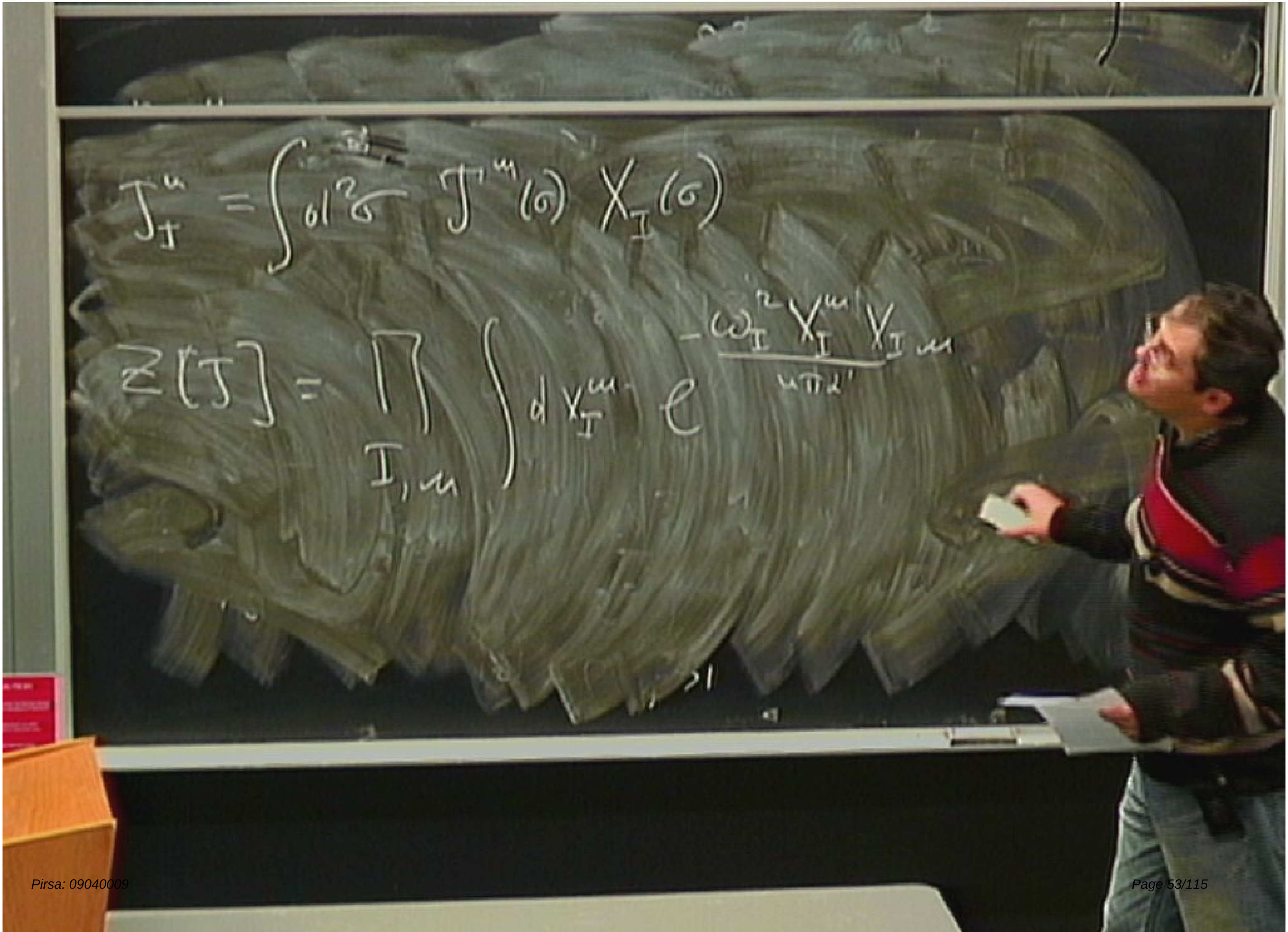
$$Z[J] = \prod_{I,\mu} \int d$$

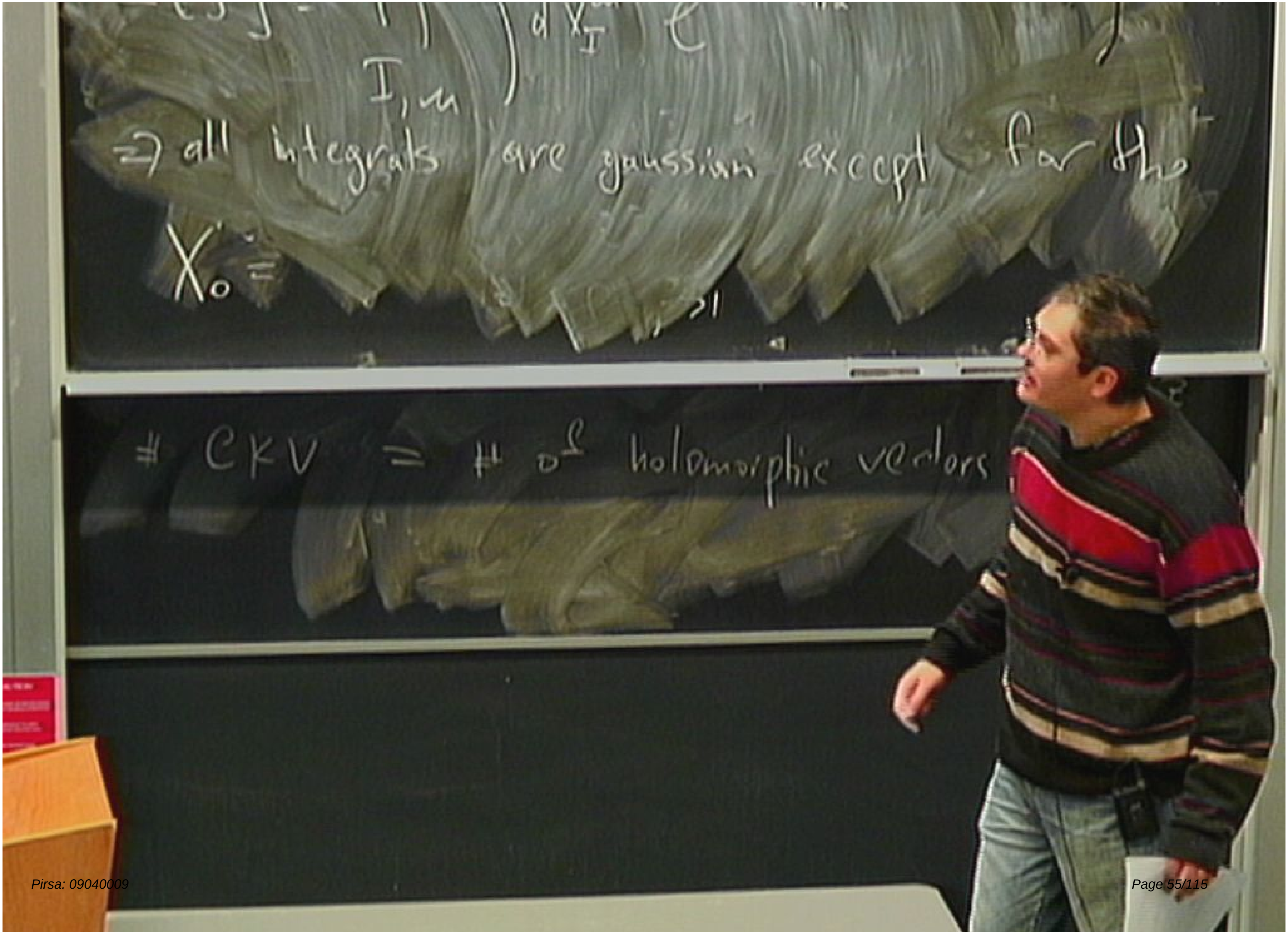
$$J^\mu_I = \int d^2\sigma \, J^\mu(\sigma)\, X_I(\sigma)$$

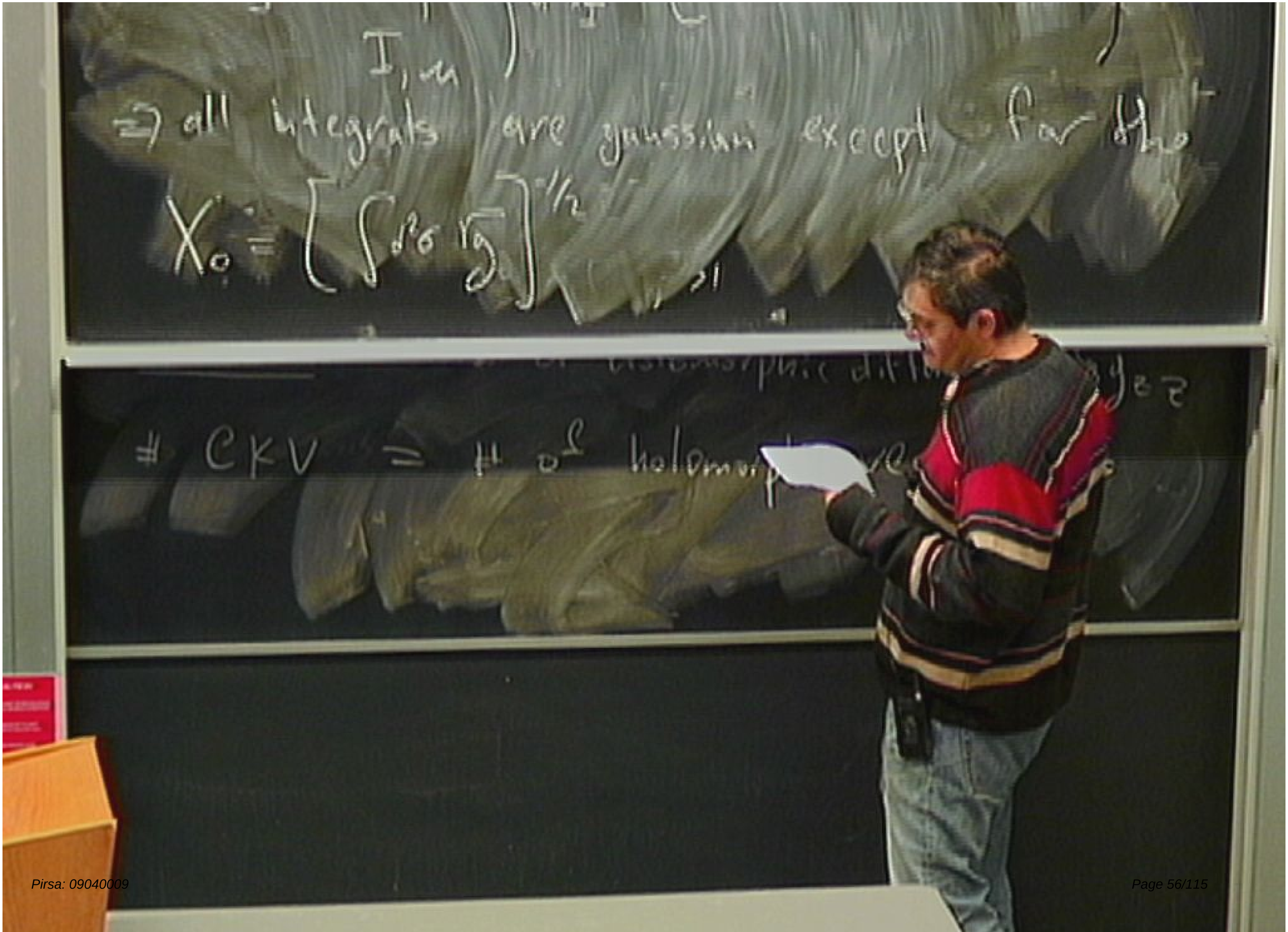
$$Z[J] = \prod_{I,\mu} \int d X^\mu_I \ldots$$


$J_I^\mu = \int d^2\sigma \, J^\mu(\sigma) \, X_I(\sigma)$
$Z[J] = \prod_{I,\mu} \int dY_I^\mu \, e^{-\frac{\omega_I^2 Y_I^\mu Y_{I\mu}}{4\pi\alpha'}}$

$$J^{\mu}_{I} = \int d^2\sigma \; J^{\mu}(\sigma)\, X_I(\sigma)$$

$$Z[J] = \prod_{I,\mu} \int dX^{\mu}_{I}\; e^{-\frac{\omega_I^2 X_I^{\mu} X_{I\mu}}{4\pi\alpha'} + i X_I^{\mu} J_{I\mu}}$$


⇒ all integrals are gaussian except for the
X₀ =
CKV = # of holomorphic vectors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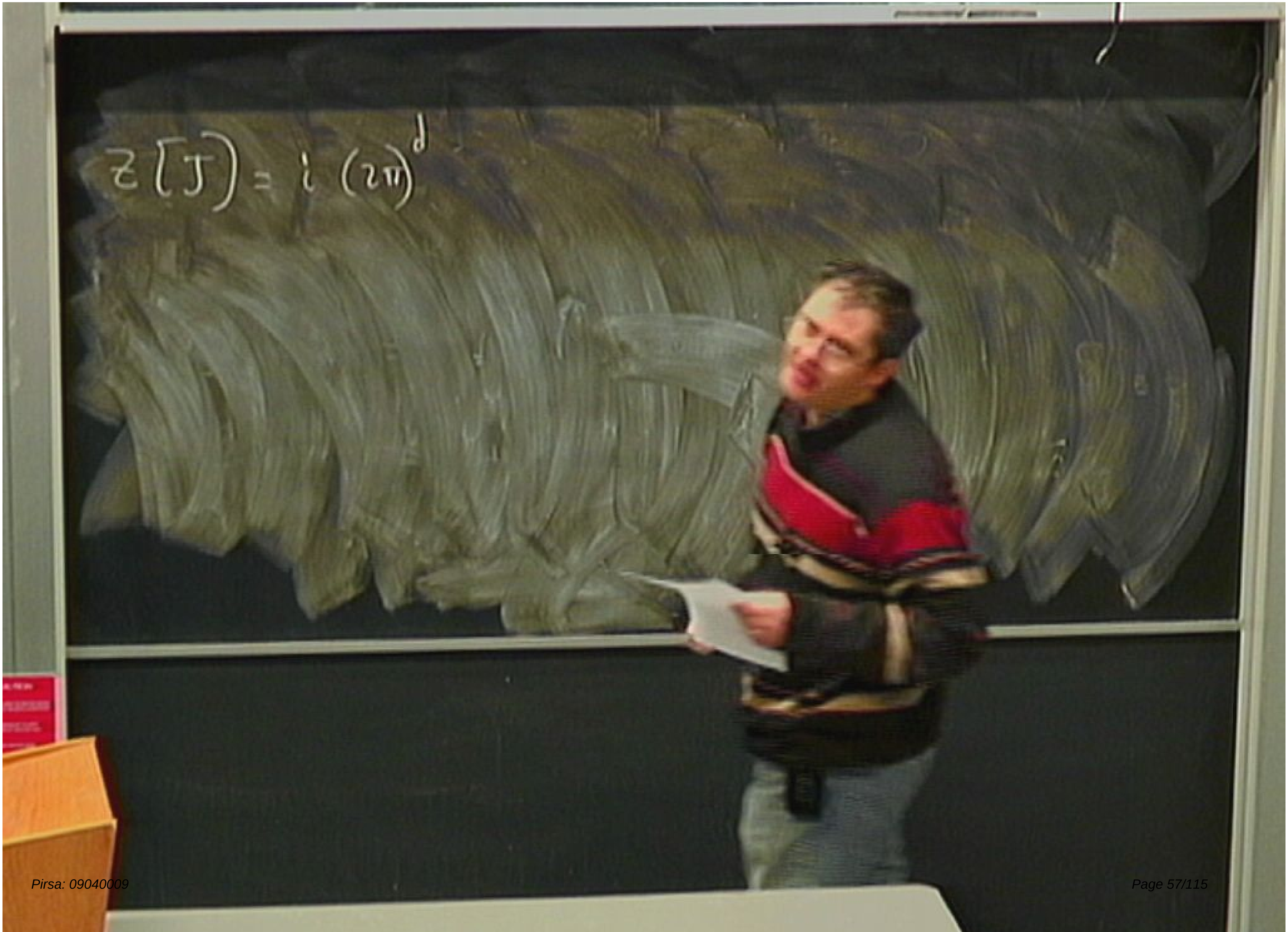
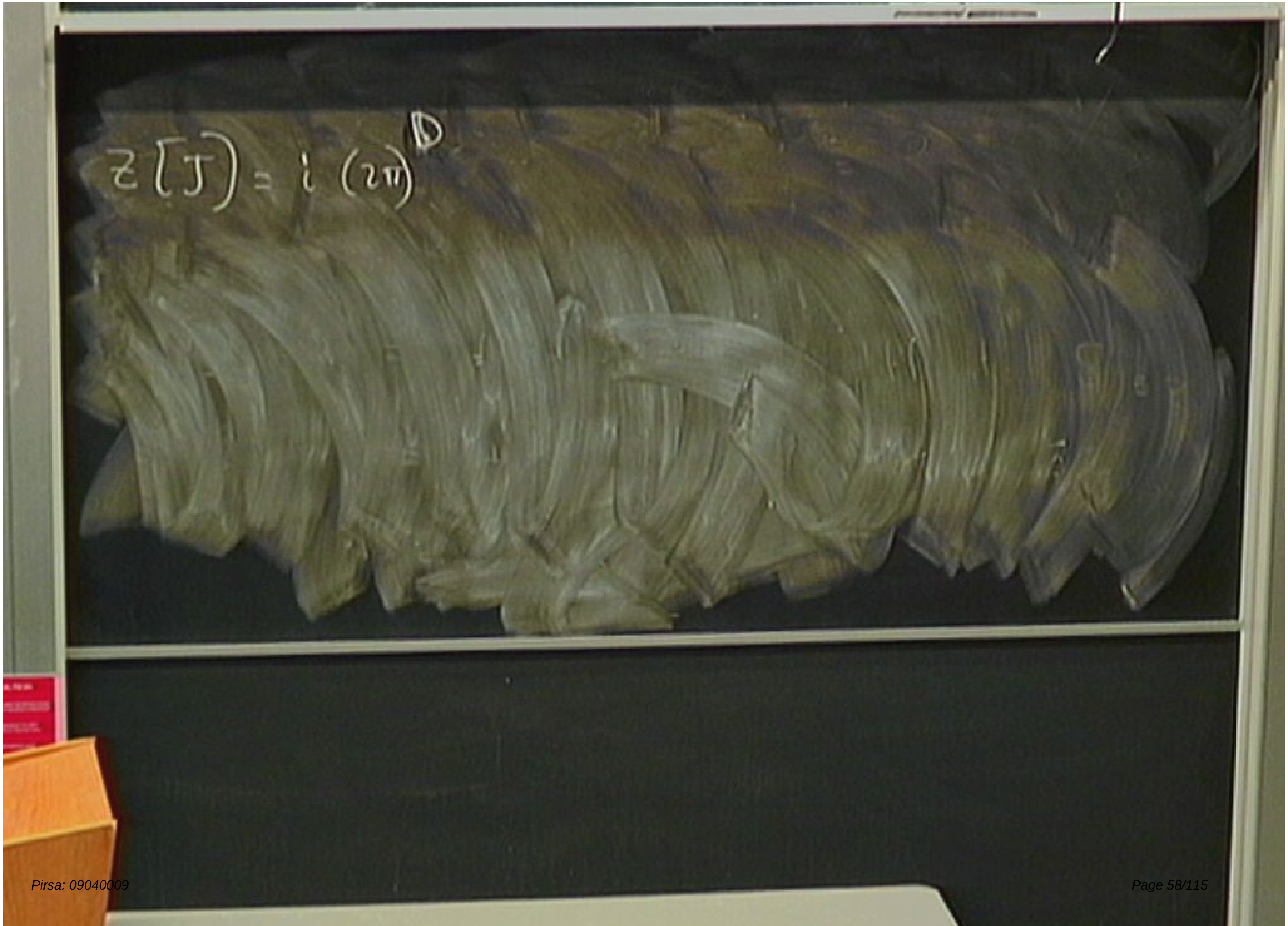
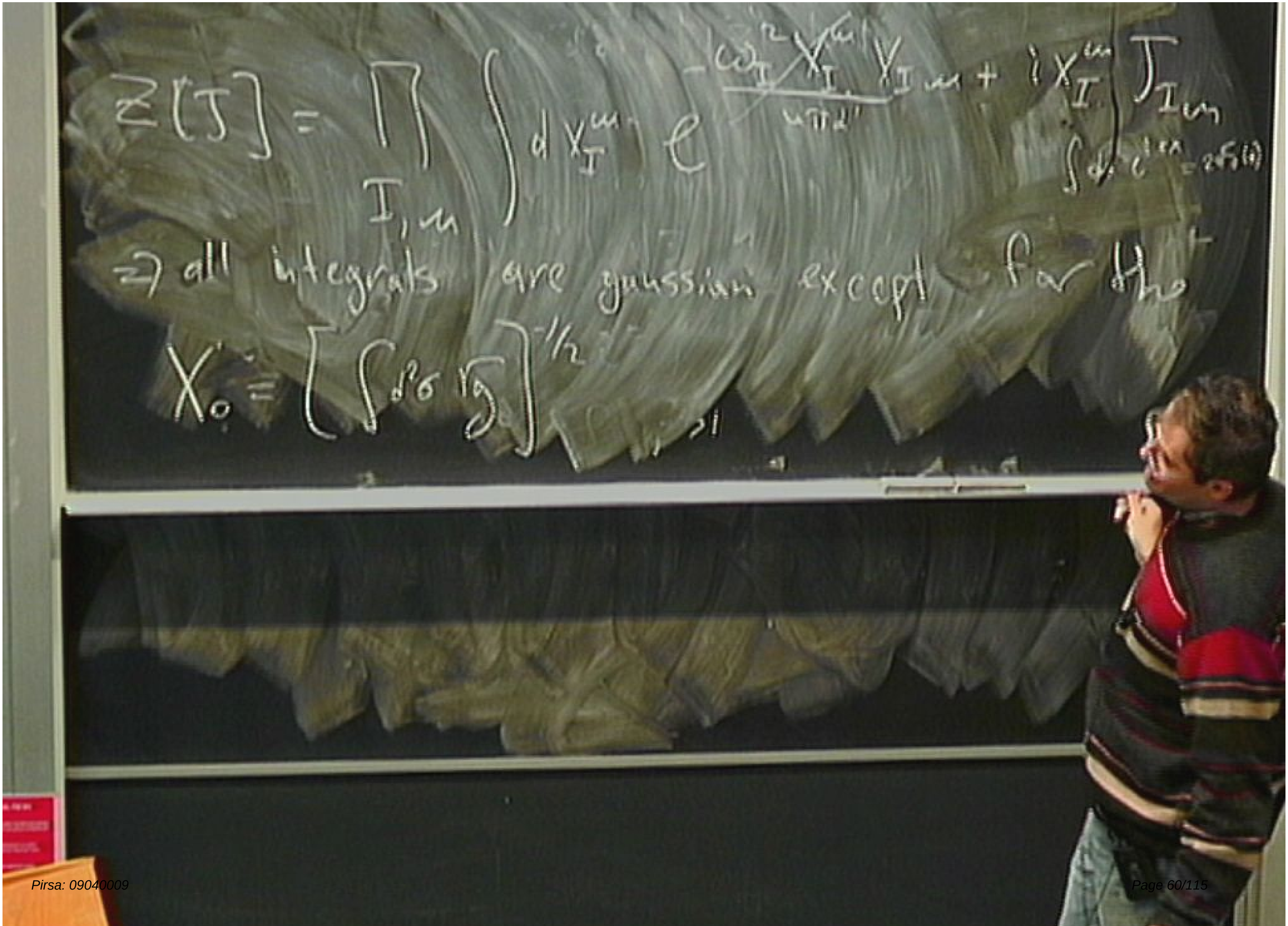

Z[J] = i (2π)^d


Z[J] = i (2π)^D

$$Z[J] = \prod_{I,m} \int dX_I^m \, e^{-\frac{\omega_I^2 X_I^m X_{Im}}{4\pi\alpha'} + i X_I^m J_{Im}}$$

$\Rightarrow$ all integrals are gaussian except for t

$$X_0 = \left[\int d^2\sigma \sqrt{g}\right]^{-1/2}$$

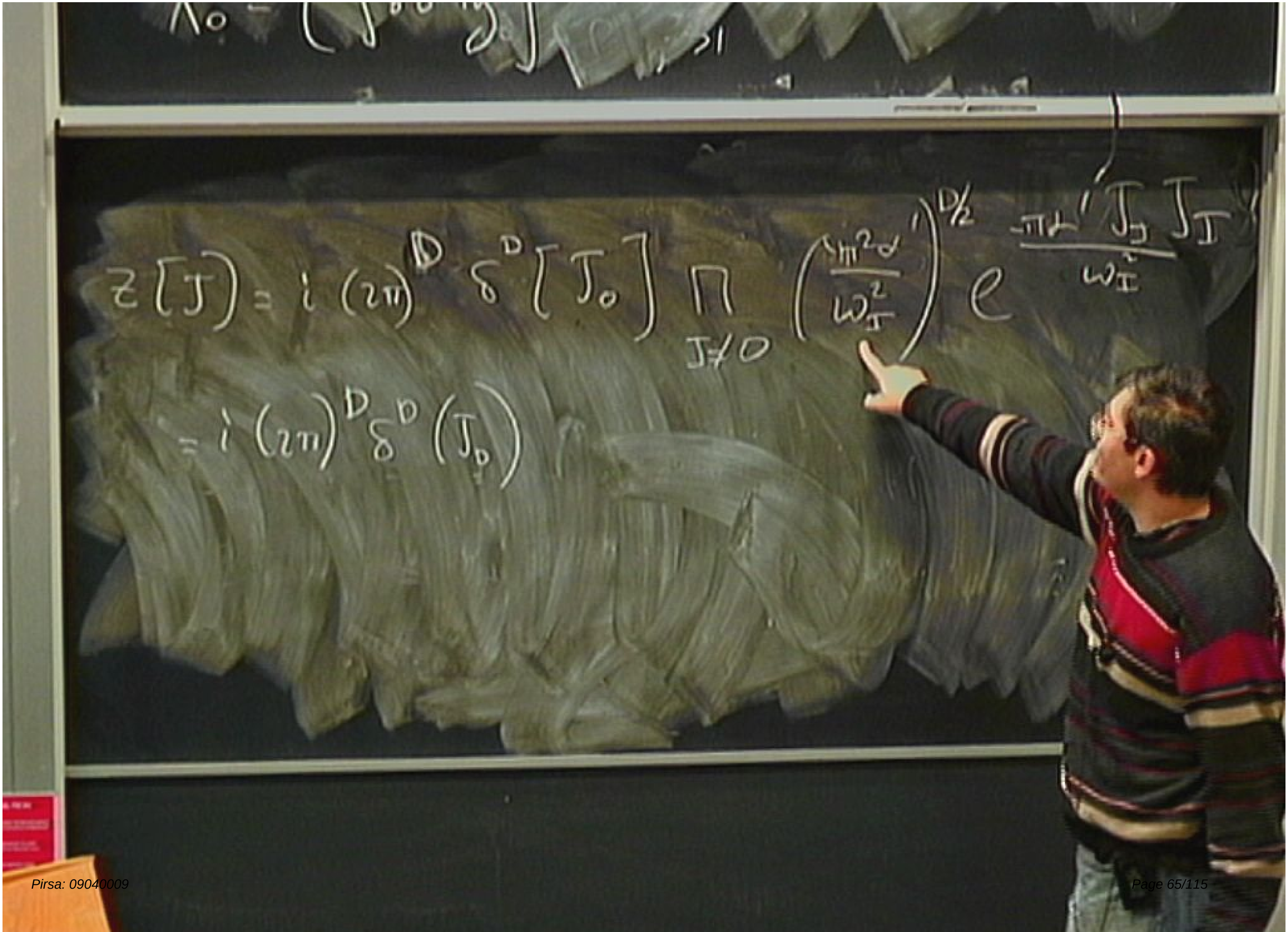

$Z[J] = \prod_{I,m} \int dX_I^{lm} \, e^{-\frac{\omega_I^2 X_I^{lm} X_{I\,lm}}{4\pi\lambda'} + i X_I^{lm} J_{I\,lm}}$
$\int dx \, e^{ikx} = 2\pi\delta(k)$
⇒ all integrals are gaussian except for the
$X_0 = \left[\int d^2\sigma \sqrt{g}\right]^{-1/2}$

$$Z[J] = i\,(2\pi)^{D}\,\delta^{D}[J_0]\prod_{J\neq 0}$$

$$Z[J] = i\,(2\pi)^{D}\,\delta^{D}[J_0]\prod_{J\neq 0}\left(\frac{\pi^2\alpha'}{\omega_I^2}\right)^{D/2}$$

$$Z[J] = i\,(2\pi)^{D}\,\delta^{D}[J_0]\prod_{J\neq 0}\left(\frac{4\pi^2\alpha'}{\omega_J^2}\right)^{D/2} e$$

$$Z[J] = i\,(2\pi)^{D}\,\delta^{D}[J_0]\prod_{I\neq 0}\left(\frac{\pi^2\alpha'}{\omega_I^2}\right)^{D/2} e^{\frac{-\pi\alpha' J_I J_I}{\omega_I^2}}$$

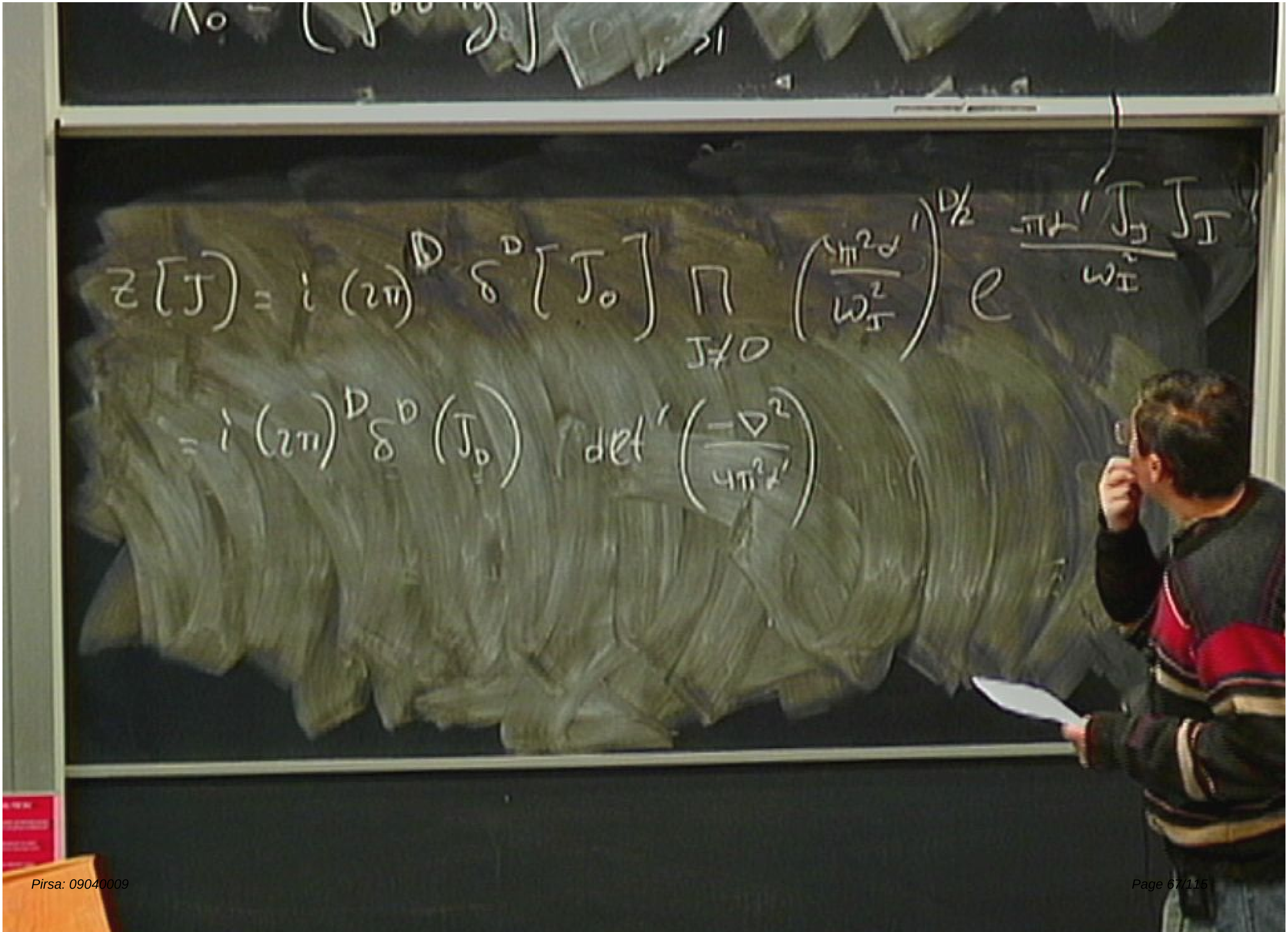

$Z[J] = i\,(2\pi)^D \delta^D[J_0] \prod_{I\neq 0} \left(\frac{4\pi^2\alpha'}{\omega_I^2}\right)^{D/2} e^{\frac{-\pi\alpha' J_I J_I}{\omega_I^2}}$
$= i\,(2\pi)^D \delta^D(J_0)$

Expand $X^\mu(\sigma) = \sum_I x_I^\mu X_I(\sigma)$

$\nabla^2 X_I = -\omega_I^2 X_I$

$\int_{M} d^2\sigma \sqrt{g}\, X_I X_{I'} = \delta_{II'}$

M sphere

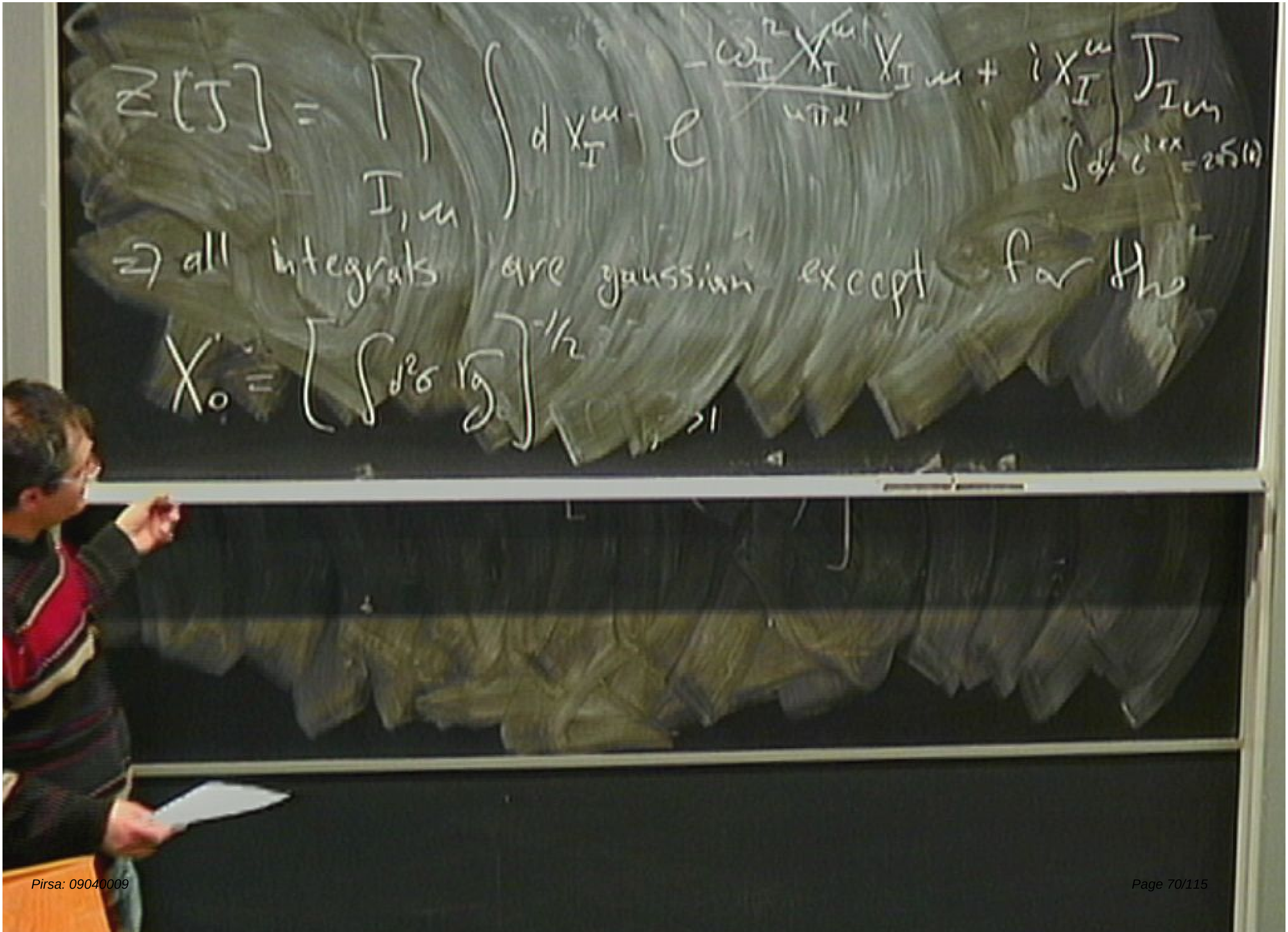

$Z[J] = i\,(2\pi)^D \delta^D[J_0] \prod_{I\neq 0} \left(\frac{4\pi^2\alpha'}{\omega_I^2}\right)^{D/2} e^{\frac{-\pi\alpha' J_I J_I}{\omega_I^2}}$
$= i\,(2\pi)^D \delta^D(J_0)\ \det'\left(\frac{-\nabla^2}{4\pi^2\alpha'}\right)$

$$Z[J] = i\,(2\pi)^D\,\delta^D[J_0]\prod_{J\neq 0}\left(\frac{4\pi^2\alpha'}{\omega_I^2}\right)^{D/2} e^{\frac{-\pi\alpha' J_J J_I}{\omega_I^2}}$$

$$= i\,(2\pi)^D\,\delta^D(J_0)\left[\det{}'\left(\frac{-\nabla^2}{4\pi^2\alpha'}\right)\right]^{-D/2}$$

$$Z[J] = i\,(2\pi)^D\,\delta^D[J_0] \prod_{I\neq 0} \left(\frac{\pi^2\alpha'}{\omega_I^2}\right)^{D/2} e^{-\frac{\pi\alpha' J_I J_I}{\omega_I^2}}$$

$$= i\,(2\pi)^D\,\delta^D(J_0)\left[\det{}'\left(\frac{-\nabla^2}{4\pi^2\alpha'}\right)\right]^{-D/2}\cdot$$


$Z[J] = \prod_{I,\mu} \int dX_I^\mu \, e^{-\frac{\omega_I^2 X_I^\mu X_{I\mu}}{4\pi\alpha'} + i X_I^\mu J_{I\mu}}$
$\int dx\, e^{ikx} = 2\pi\delta(k)$
$\Rightarrow$ all integrals are gaussian except for the
$X_0 = \left[\int d^2\sigma \sqrt{g}\right]^{-1/2}$

⇒ all integrals are gaussian except for the

$X_0 = \left[\int d^2\sigma \sqrt{g}\right]^{-1/2}$

$Z(J) = i\,(2\pi)^D\,\delta(J_0)\prod_{I\neq 0}\left(\frac{\pi\,\alpha'}{\omega_I^2}\right) e^{\,\cdots\,\omega_I^2}$

$= i\,(2\pi)^D\delta^D(J_0)\left[\det{}'\left(\frac{-\nabla^2}{4\pi^2\alpha'}\right)\right]^{-D/2} \cdot e^{-\frac{1}{2}\int d^2\sigma\, d^2\sigma'\, J(\sigma)J(\sigma')}$

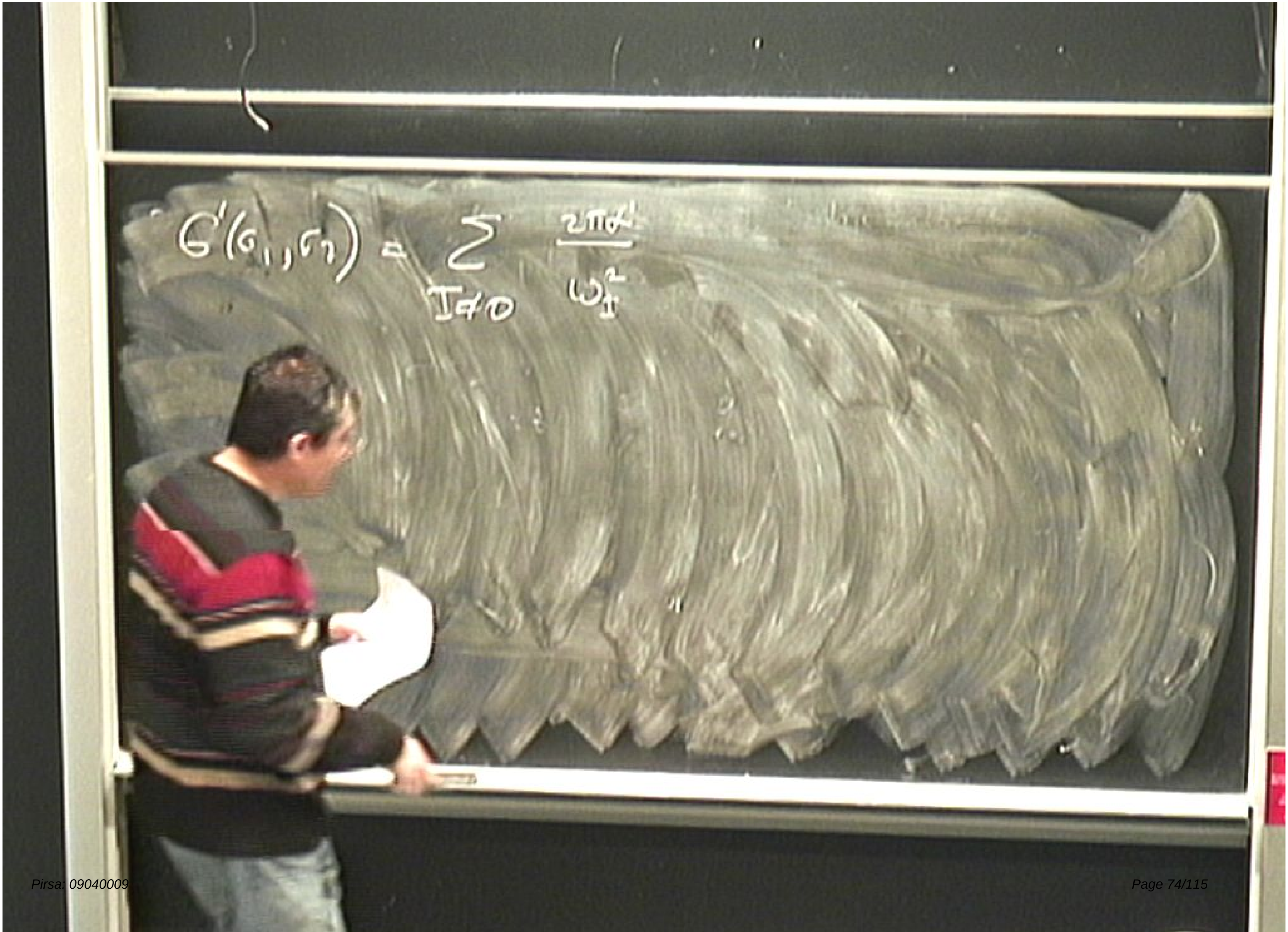
$\Rightarrow$ all integrals are gaussian except for the

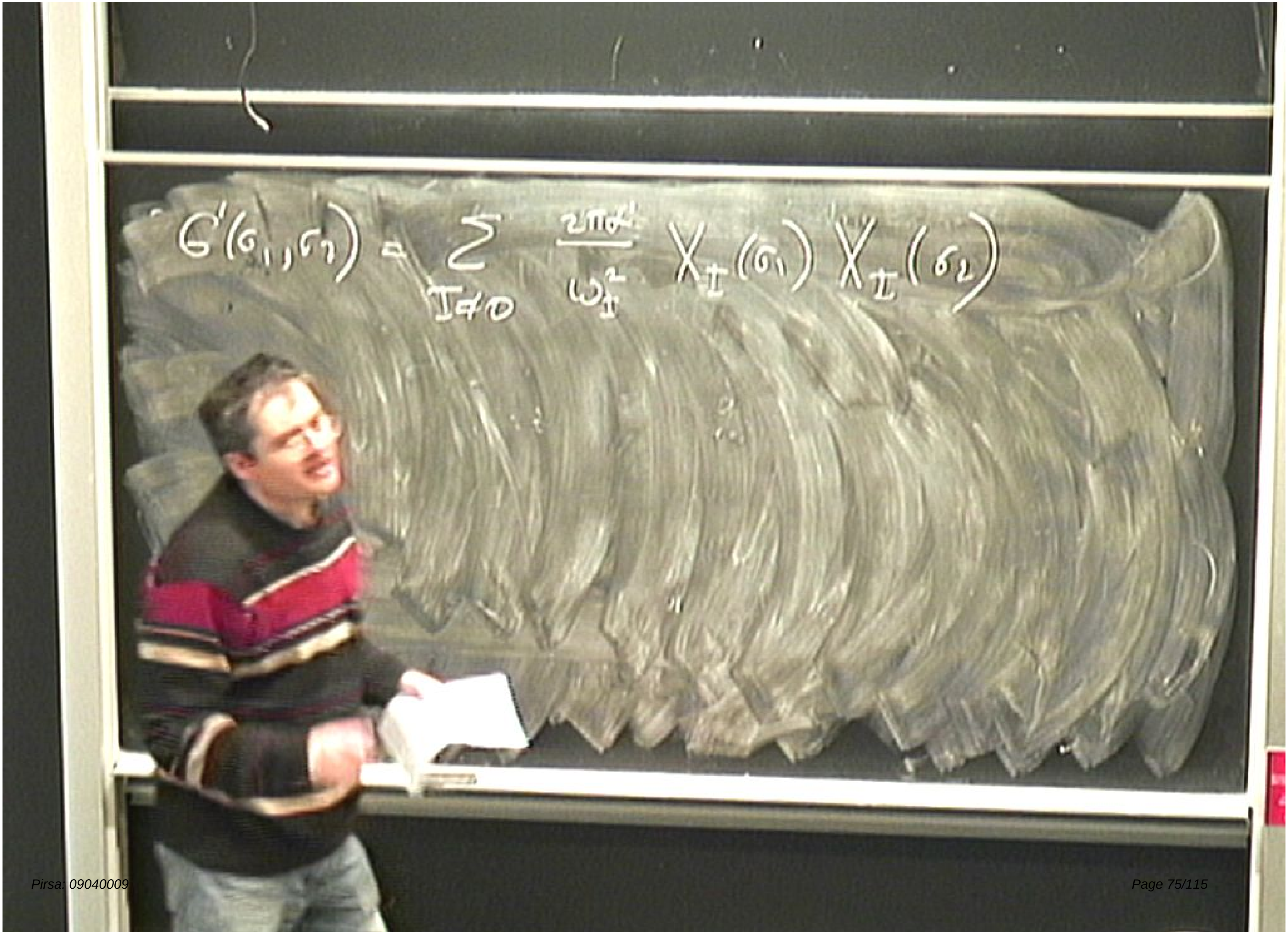
$$X_0 = \left[\int d^2\sigma \sqrt{g_0}\right]^{-1/2}$$

$$Z(J) = i\,(2\pi)^D\,\delta(J_0)\prod_{I\neq 0}\left(\frac{\pi\,\sigma}{\omega_I^2}\right) e^{\omega_I}$$

$$= i\,(2\pi)^D\,\delta^D(J_0)\left[\det{}'\left(\frac{-\nabla^2}{4\pi^2\alpha'}\right)\right]^{-D/2} \cdot e^{-\frac{1}{2}\int d^2\sigma\, d^2\sigma'\, J(\sigma) J(\sigma')\, G'(\sigma,\sigma')}$$


$G'(\sigma_1, \sigma_2)$


$G'(\sigma_1,\sigma_2) = \sum_{I\neq 0} \frac{2\pi\alpha'}{\omega_I^2}$


$G'(\sigma_1, \sigma_2) = \sum_{I \neq 0} \frac{2\pi\alpha'}{\omega_I^2} X_I(\sigma_1) X_I(\sigma_2)$

$$G'(\sigma_1,\sigma_2) = \sum_{I\neq 0} \frac{2\pi\alpha'}{\omega_I^2} \underbrace{X_I(\sigma_1)}_{\text{Eigenfunctions of } -\nabla^2} X_I(\sigma_2)$$

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different complex parameters.

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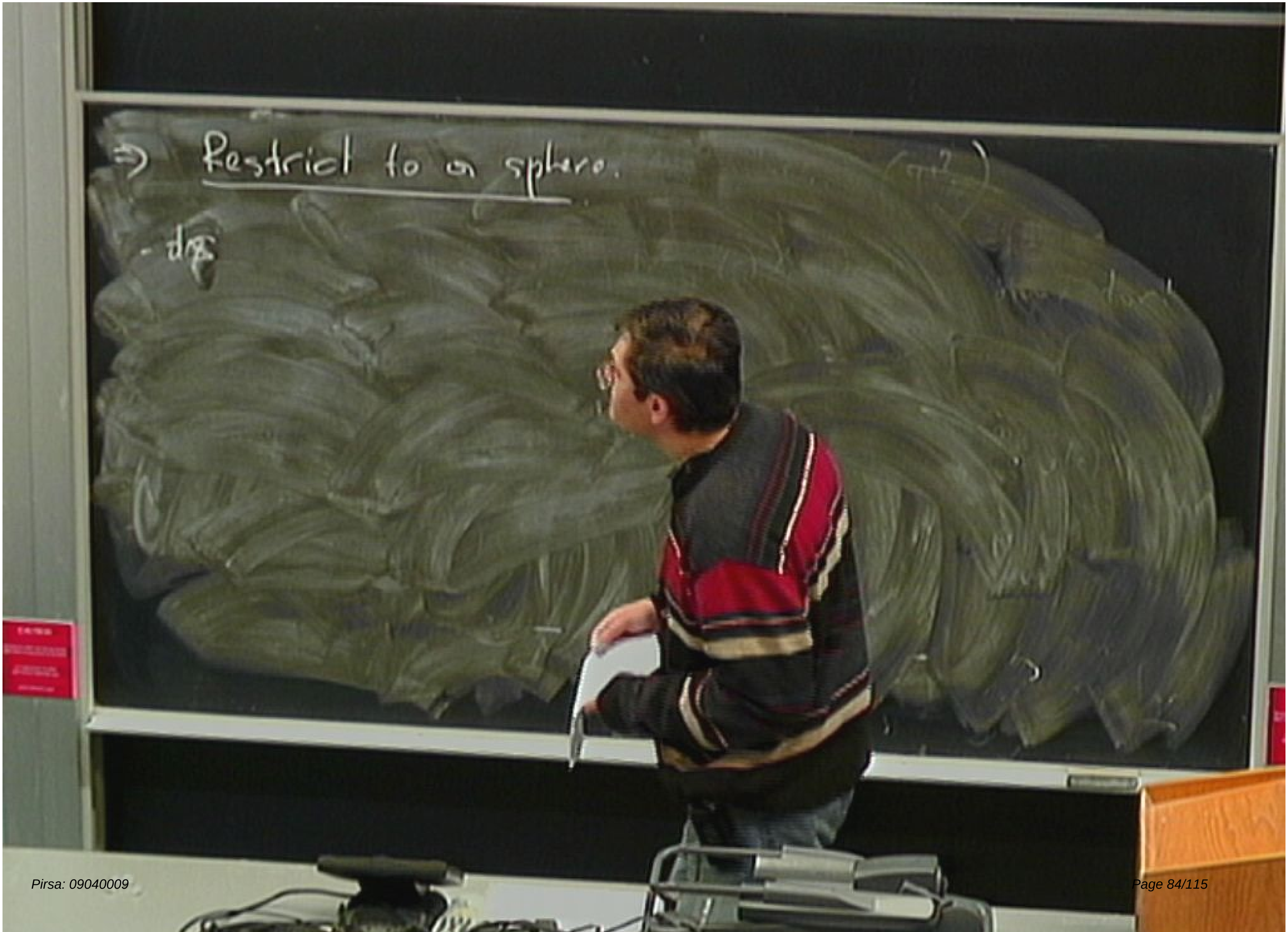
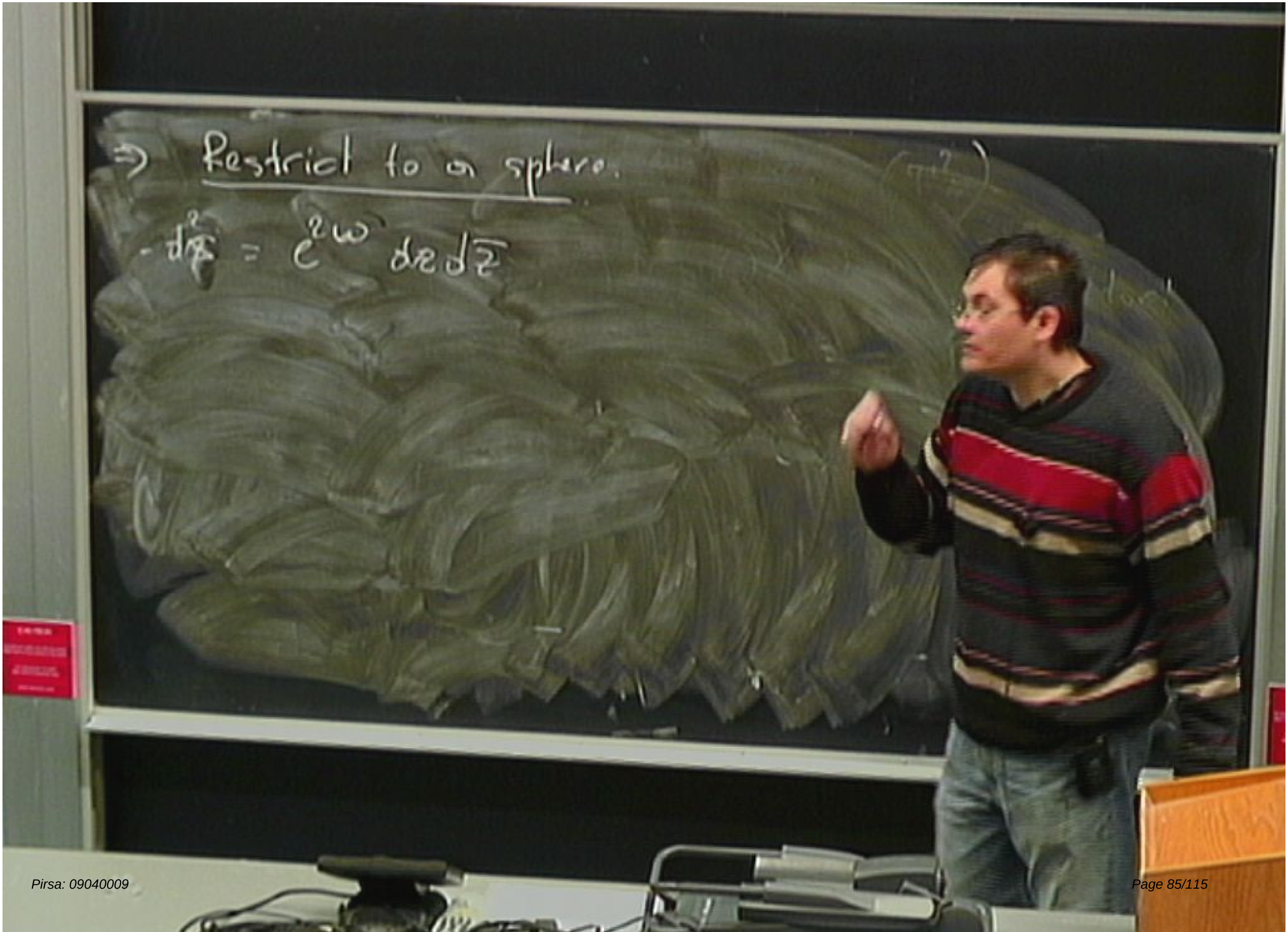
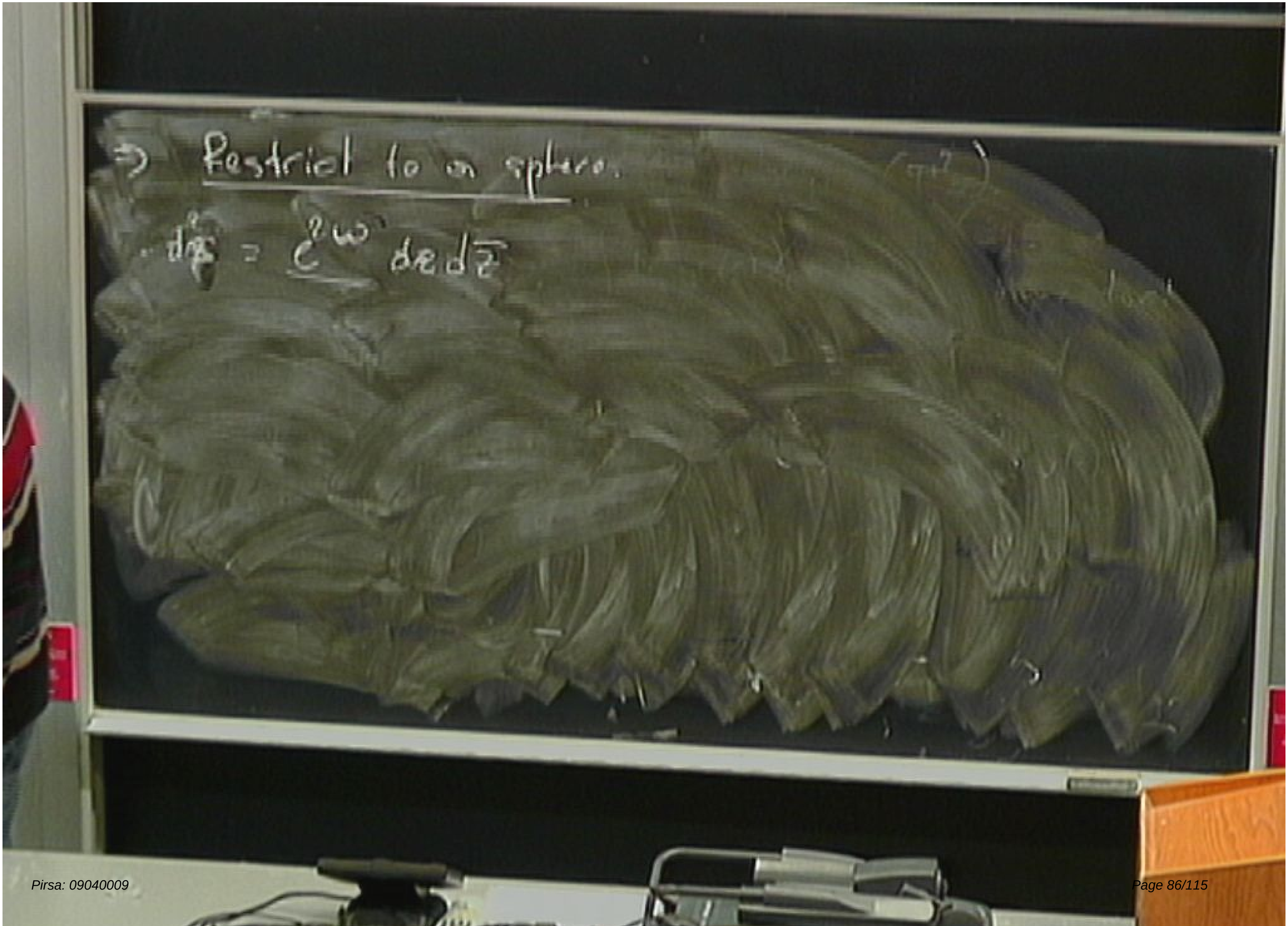
$$\int \sqrt{g}\, X_0\, G'(\sigma_1,\sigma_2)\, d^2\sigma = 0$$

different complex parameters.


⇒ Restrict to a sphere.


⇒ Restrict to a sphere.
$-ds^2 = e^{2\omega} dz\, d\bar{z}$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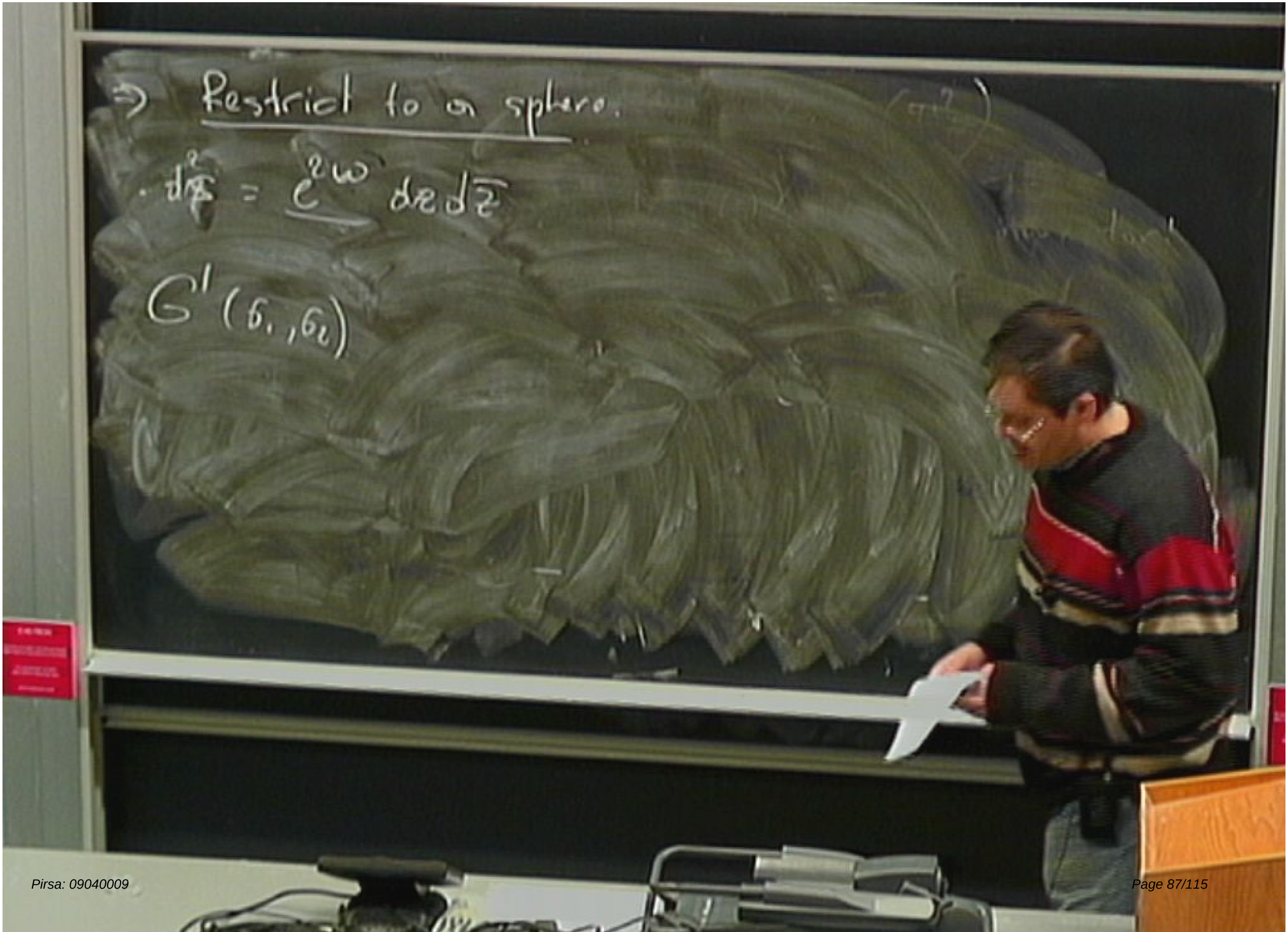
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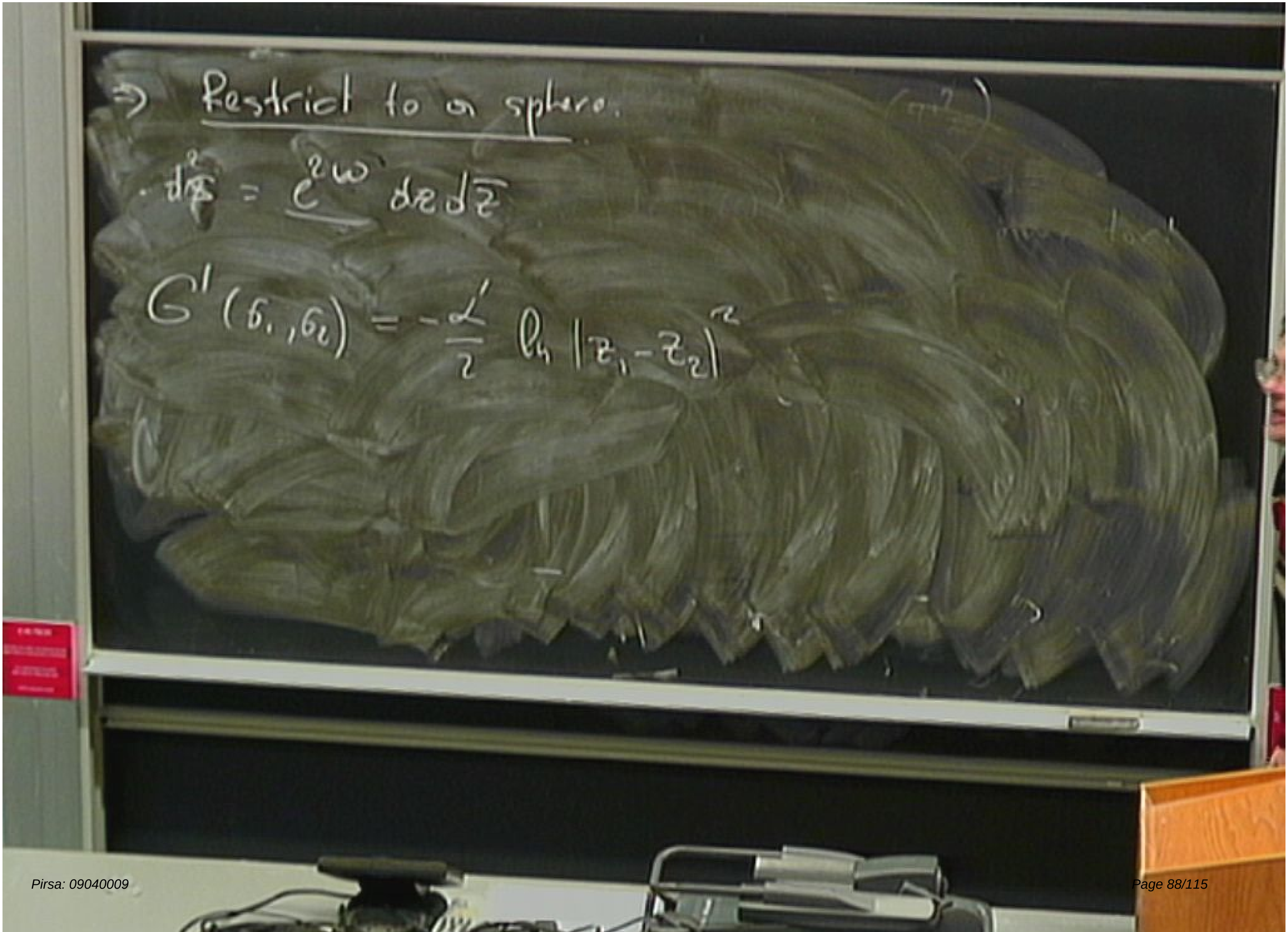
G'(σ₁, σ₂)


⇒ Restrict to a sphere.
$ds^2 = e^{2\omega} dz\, d\bar{z}$
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⇒ Restrict to a sphere.

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$$\int \sqrt{g}\, X_0\, G'(\sigma_1,\sigma_2)\, d^2\sigma = 0$$

$$d^2\sigma = e^{2\omega}\, dz\, d\bar{z}$$

$$G'(\sigma_1, \sigma_2) = -\frac{\alpha'}{2} \ln |z_1 - z_2|^2 + f(z_1, \bar{z}_1) + f(z_2, \bar{z}_2)$$

$$f(z, \bar{z}) = \frac{\alpha' X_0^2}{4} \int d^2 z'\, e^{2\omega(z', \bar{z}')} \ln |z - z'|^2$$

T_{sphere}

$$G'(\sigma_1,\sigma_2) = \sum_{I\neq 0} \frac{2\pi\alpha'}{\omega_I^2} \underbrace{X_I(\sigma_1)}_{\text{Eigenfunctions of } -\nabla^2} X_I(\sigma_2)$$

$$-\frac{1}{2\pi\alpha'} \nabla_1^2 G'(\sigma_1,\sigma_2) = \sum_{I\neq 0} X_I(\sigma_1) X_I(\sigma_2) = \frac{\delta^2(\sigma_1-\sigma_2)}{\sqrt{g}} - X_0^2$$

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$$d\sigma = e^{2\omega}\, dz\, d\bar{z}$$

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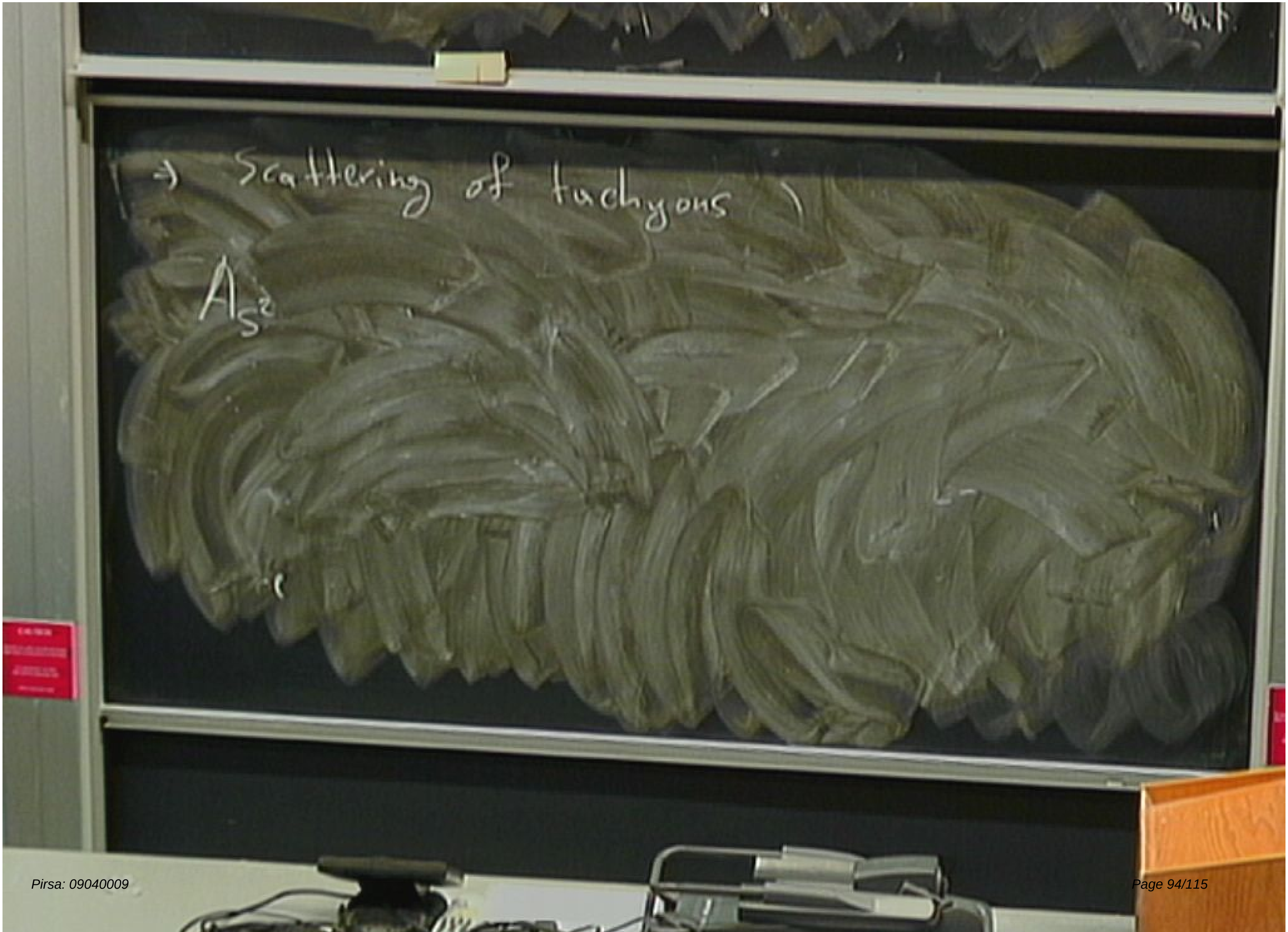
$$f(z,\bar{z}) = \frac{\alpha' X_0^2}{4}\int d^2z'\, e^{2\omega(z',\bar{z}')}\ln|z-z'|^2 + K$$

arbitrary constant

sphere


→ Scattering of tachyons)
A_{S^2}

→ Scattering of tachyons

$$A_{S^2} = \langle e^{ik_1 X(\sigma_1)} \cdots e^{ik_n}$$

→ Scattering of tachyons

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$$J(\sigma) = \sum_{i=1}^{n} k_i \, \delta^2(\sigma - \sigma_i)$$

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arbitrary constant

→ Scattering of tachyons

$$A^{(n)}_{S^2} = \left\langle e^{ik_1 X(\sigma_1)} \cdots e^{ik_n X(\sigma_n)} \right\rangle_{S^2}$$

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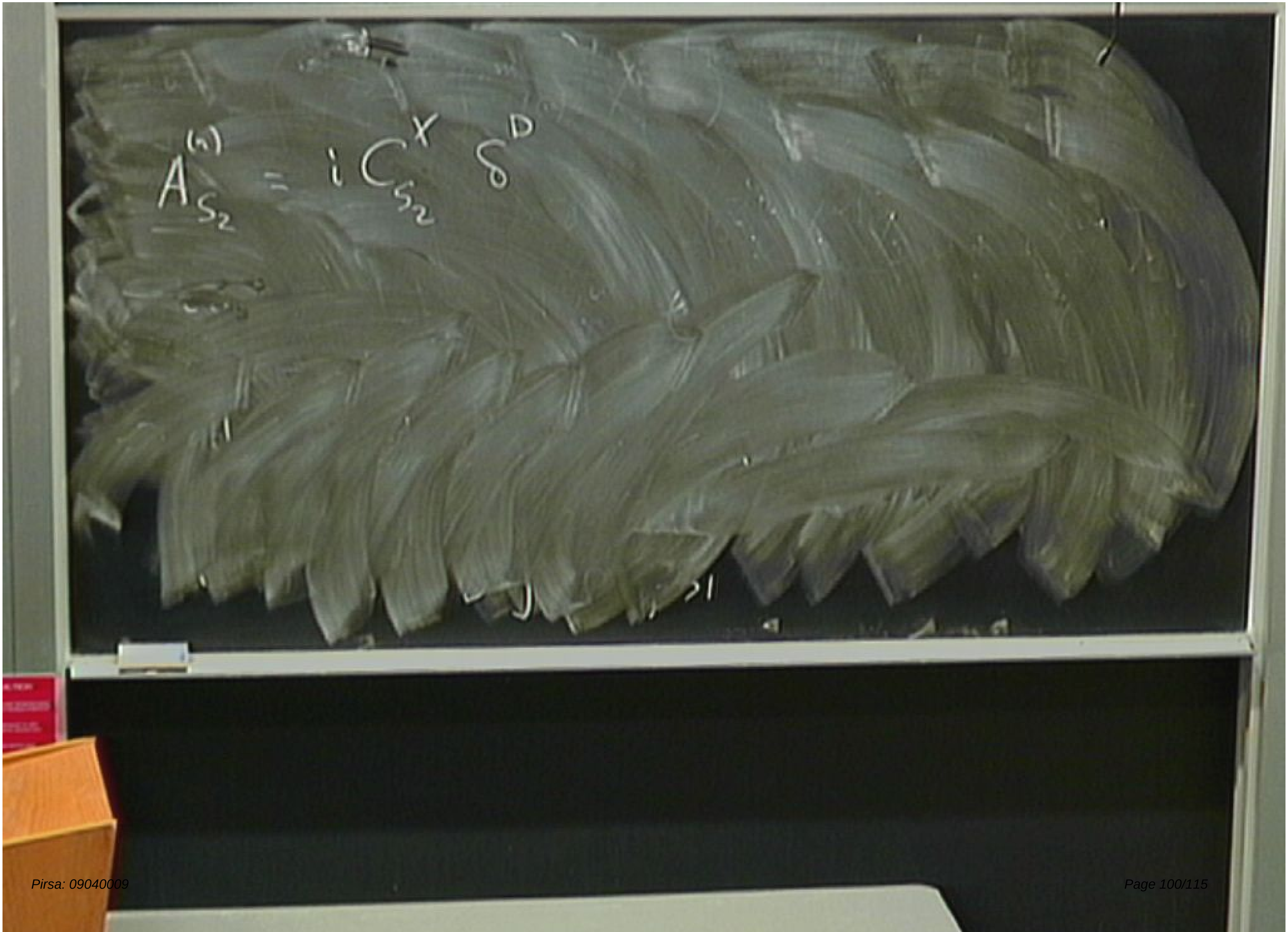
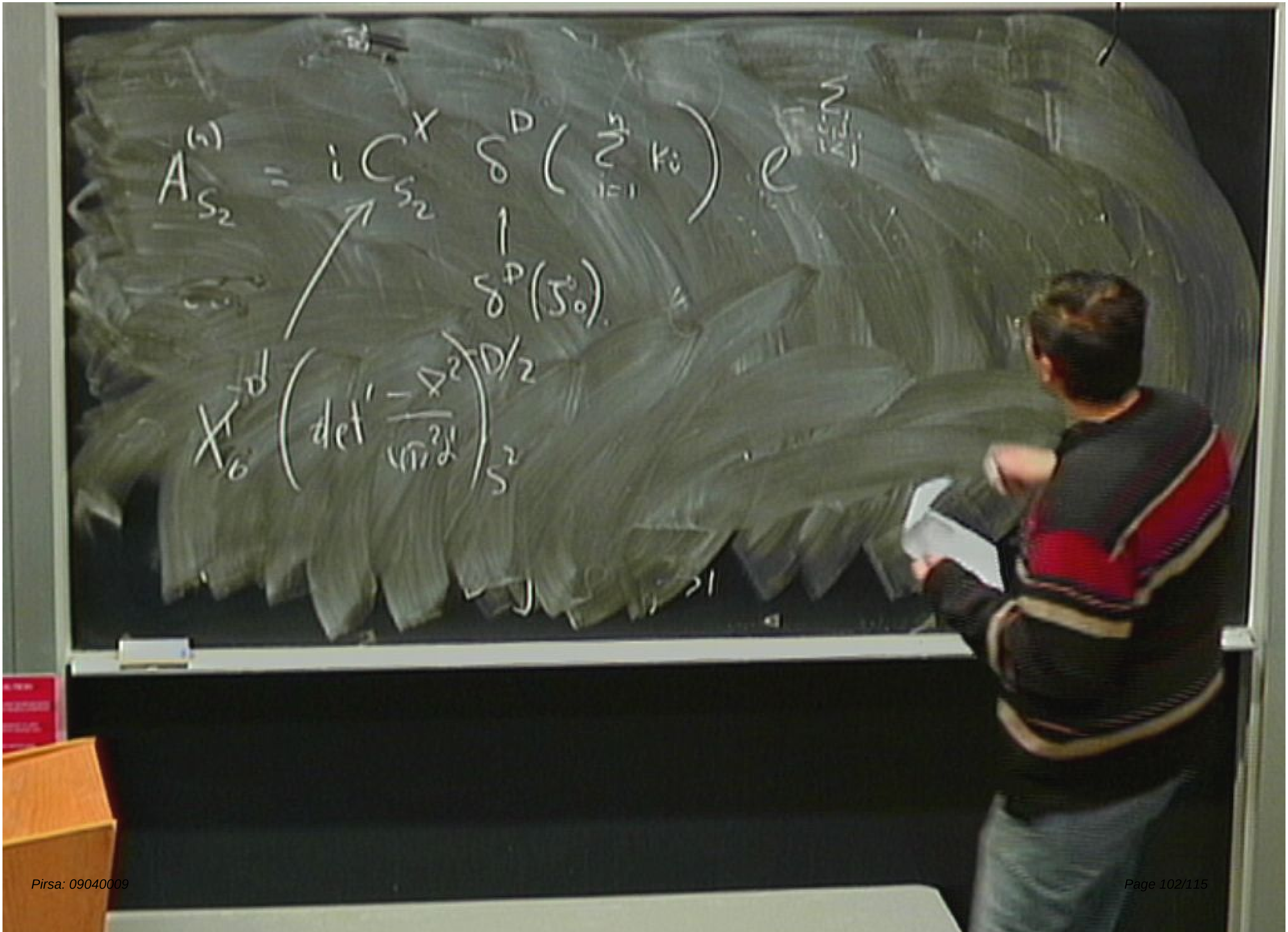
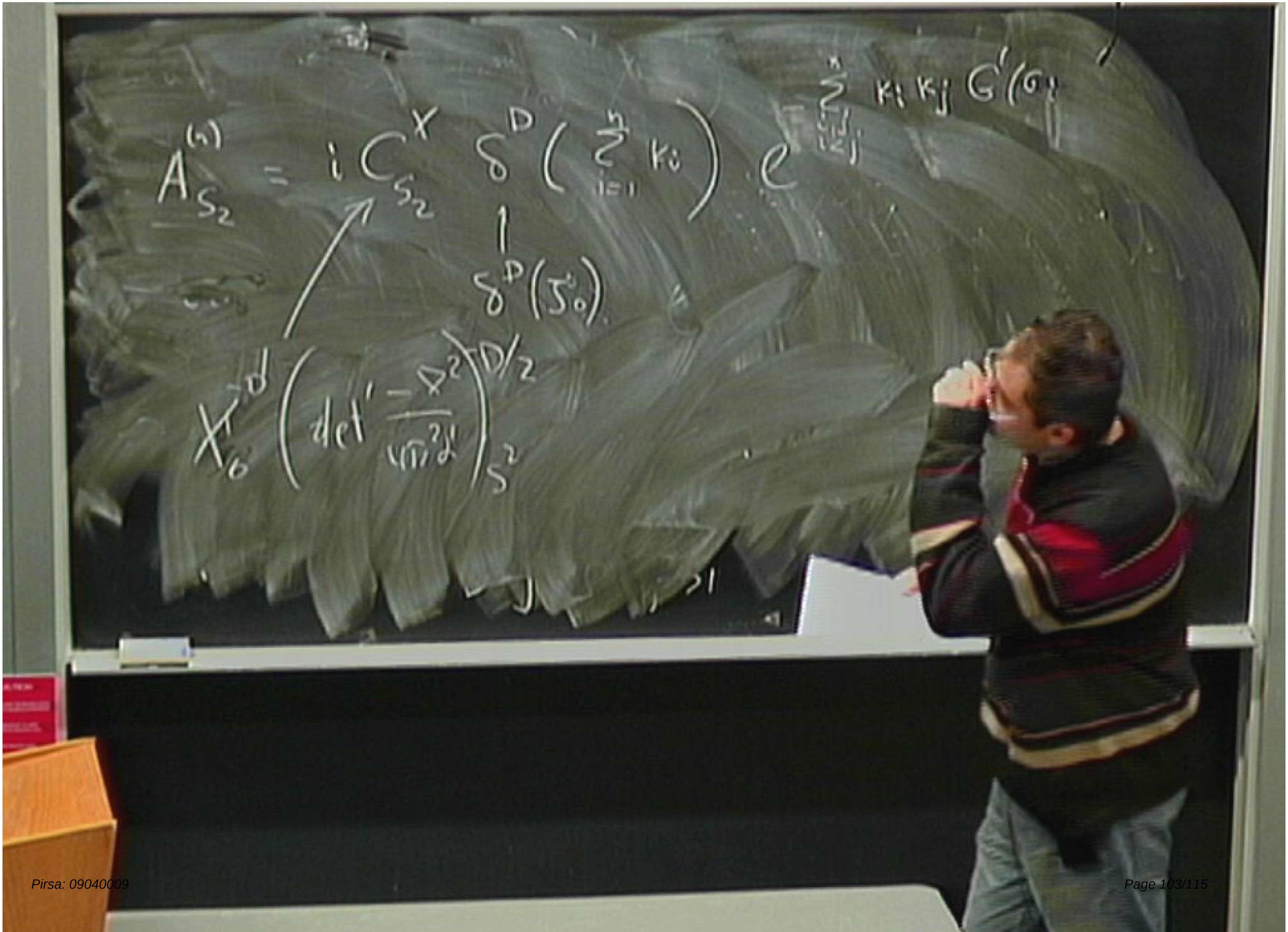

$A^{(n)}_{S_2} = i\, C^{X}_{S_2}\, \delta^{D}$

$$A^{(n)}_{S_2} = i\, C^{X}_{S_2}\, \delta^D\left(\sum_{i=1}^{n} k_i\right)$$

$$\delta^D(\zeta_0)$$

$$X_0^{-d}\left(\det{}' \frac{-\Delta^2}{4\pi^2\alpha'}\right)^{-D/2}_{S^2}$$


$A^{(n)}_{S_2} = i C^{X}_{S_2} \, \delta^D\left(\sum_{i=1}^{n} k_i\right) e^{\sum_{i<j}}$
$\delta^D(\zeta_0)$
$X_0^{-d} \left(\det' \frac{-\nabla^2}{(\pi)^2 \alpha'}\right)^{-D/2}_{S^2}$


$A^{(n)}_{S_2} = i C^{\chi}_{S_2} \delta^D\left(\sum_{i=1}^{n} k_i\right) e^{\sum_{i<j}^{n} k_i k_j G'(\sigma_i}$
$\delta^D(\zeta_0)$
$\chi_0^{-d} \left(\det' \frac{-\nabla^2}{4\pi^2\alpha'}\right)^{-D/2}_{S^2}$

$$A^{(n)}_{S_2} = i\, C^{X}_{S_2}\, \delta^D\left(\sum_{i=1}^{n} k_i\right) e^{\sum_{\substack{i,j=1 \\ i<j}}^{n} k_i k_j\, G'(\sigma_i, \sigma_j)}$$

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$$A^{(n)}_{S_2} = i\, C^X_{S_2}\, \delta^D\left(\sum_{i=1}^{n} k_i\right) e^{-\sum_{i<j}^{n} k_i k_j G'(\sigma_i,\sigma_j) - \frac{1}{2}\sum_{i=1}^{n} k_i^2 G'(\sigma_i,\sigma_i)}$$

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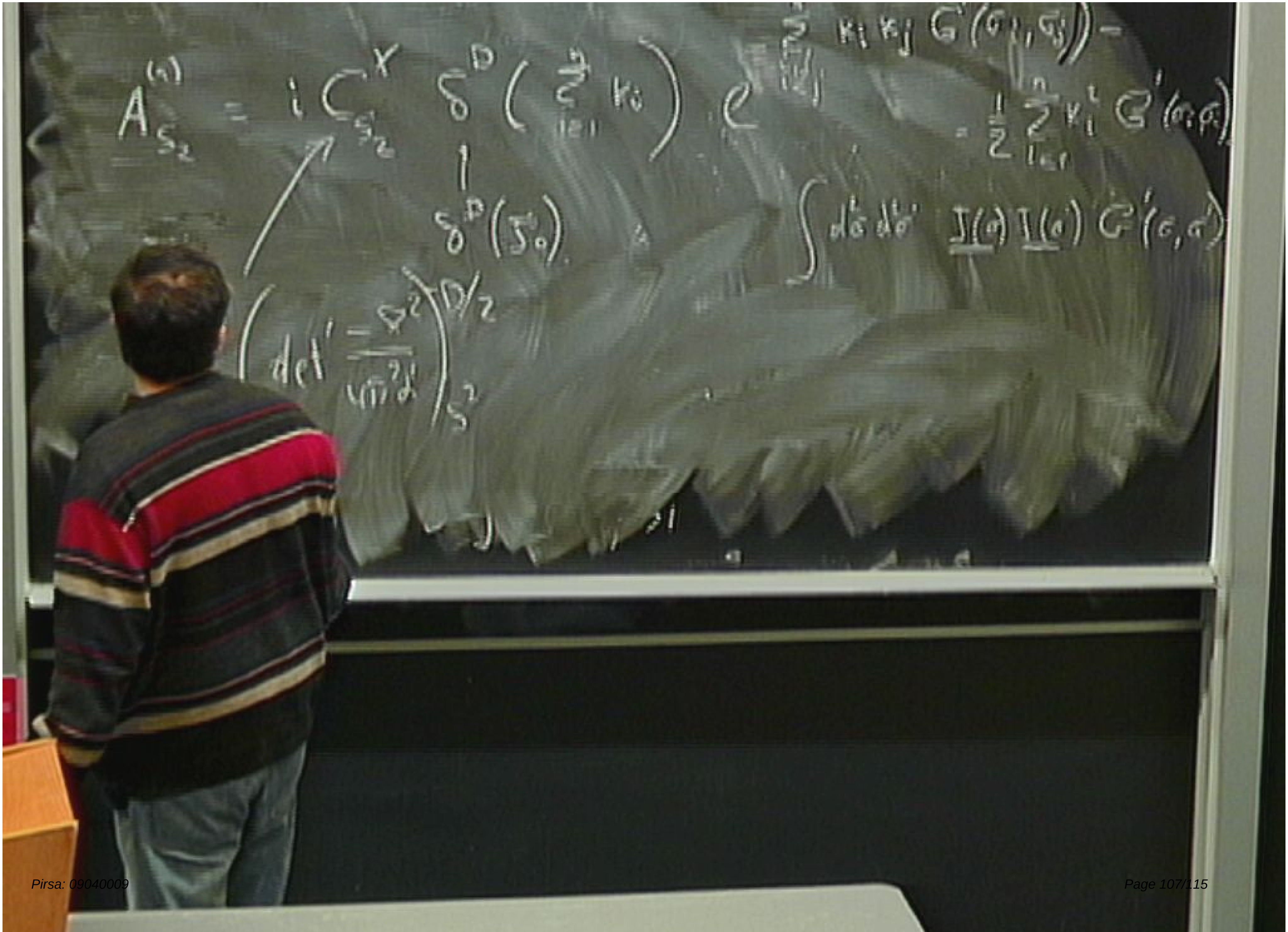
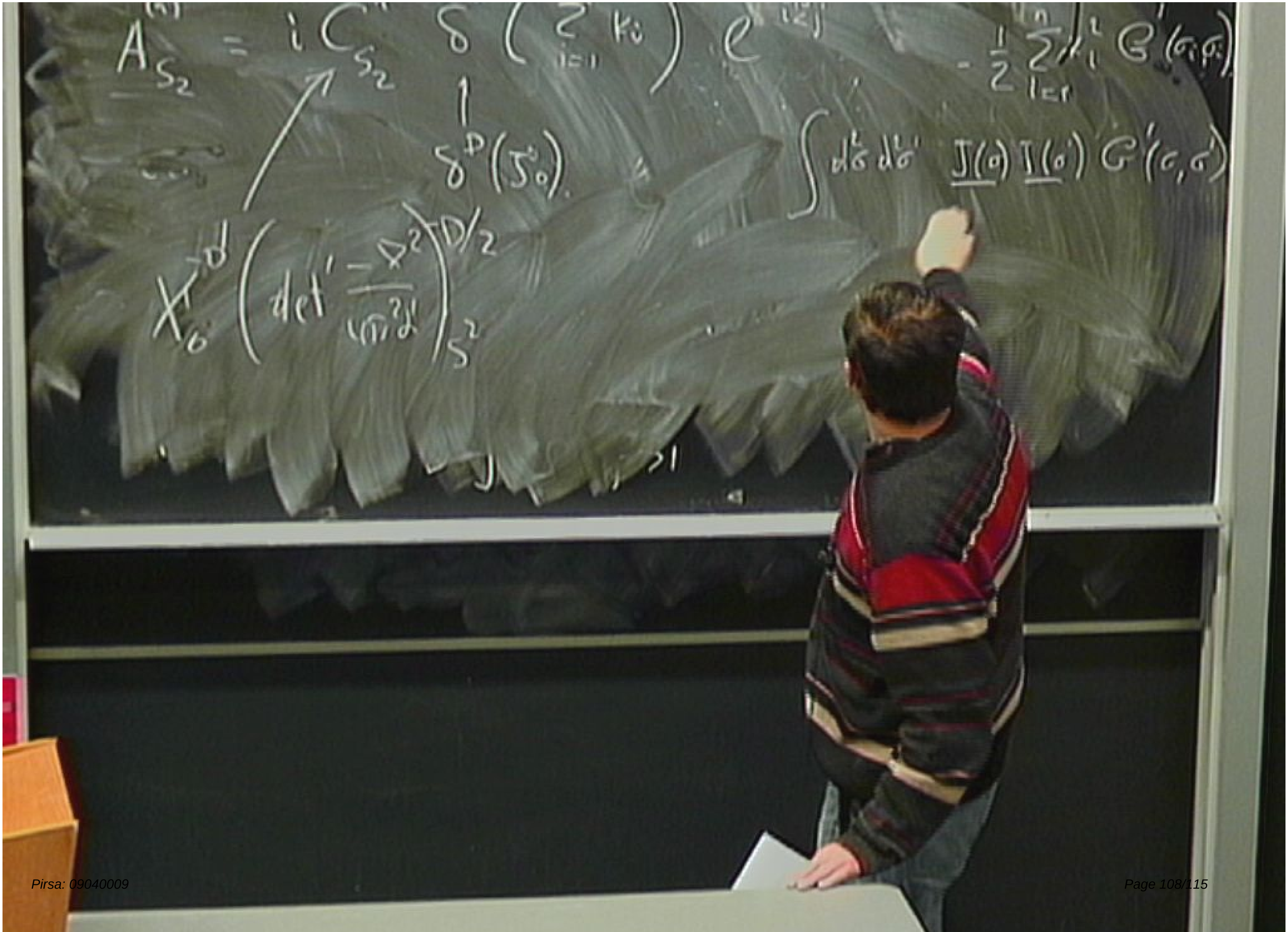
$$\delta^{D}(\zeta_0)$$

$$\int d^2\sigma\, d^2\sigma'\; J(\sigma) J(\sigma')\, G'(\sigma,\sigma')$$

$$X_0^{-d}\left(\det{}'\,\frac{-\nabla^2}{4\pi^2\alpha'}\right)_{S^2}^{-D/2}$$





$$G'(\sigma_1,\sigma_2) = -\frac{\alpha'}{2}\ln|z_1-z_2|^2 + f(z_1,\bar z_1) + f(z_2,\bar z_2)$$

$$f(z,\bar z) = \frac{\alpha' X_0^2}{4}\int d^2z'\, e^{2\omega(z',\bar z')}\ln|z-z'|^2 + K$$

arbitrary constant

$G'_r =$

$$\langle e \;\ldots\; e \rangle_{S^2}$$

$$\ldots) = \sum_{i=1}^{n} k_i\,\delta^2(\sigma-\sigma_i)$$

$$i\int J(\sigma)X_\mu(\sigma)$$

$$\langle e$$

$$G'(\sigma_1,\sigma_2) = -\frac{\alpha'}{2}\ln|z_1-z_2|^2 + f(z_1,\bar z_1) + f(z_2,\bar z_2)$$

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arbitrary constant

$$-\frac{\alpha'}{2}\ln d_{12}^2 \qquad G'_r = -\alpha'\omega(\sigma,\sigma) + \frac{\alpha'}{2}\ln z^2$$

$$J(\sigma) = \sum_{i=1}^{n} k_i\,\delta^2(\sigma-\sigma_i) \qquad i\int J(\sigma)X(\sigma)$$

$$\langle e$$

$$G'(\sigma_1,\sigma_2) = -\frac{\alpha'}{2}\ln|z_1-z_2|^2 + f(z_1,\bar z_1) + f(z_2,\bar z_2)$$

$$f(z,\bar z) = \frac{\alpha' X_0^2}{4}\int d'^2z'\, e^{2\omega(z',\bar z')}\ln|z-z'|^2 + k$$

arbitrary constant

$$-\frac{\alpha'}{2}\ln d_{12}^2 \qquad \boxed{G'_r = -\alpha'\,\omega(\sigma,\sigma) + 2f(z,\bar z)},$$

$$J(\sigma) = \sum_{i=1}^{n} k_i\,\delta^2(\sigma-\sigma_i) \qquad i\int J(\sigma)X_\mu(\sigma)$$

$$\langle e$$

$$A^{(n)}_{S_2} = i C_{S_2} \, \delta^D\left(\sum k_i\right) e^{-\frac{\alpha'}{2}\sum_i k_i^2 \omega(r_i)} \prod_{\substack{i,j=1 \\ i<j}}^{n} |z_{ij}|^{\alpha' k_i \cdot k_j}$$

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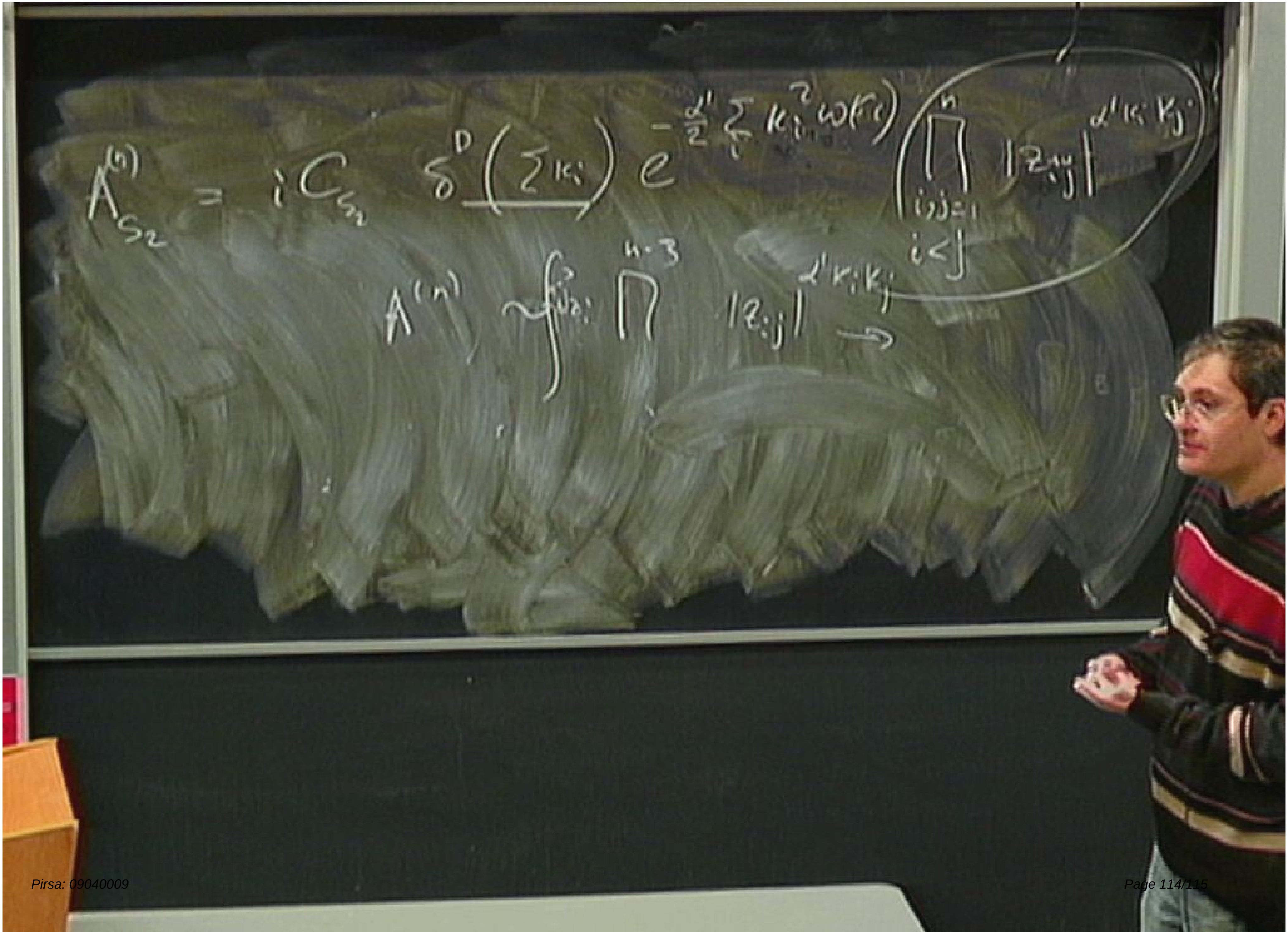
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$A^{(n)} \sim \int d^2 z_i \prod^{n-3} |z_{ij}|^{\alpha' k_i k_j} \rightarrow$

$$A^{(n)}_{S_2} = i C_{S_2} \, \delta^D\left(\sum k_i\right) e^{-\frac{\alpha'}{2}\sum_i k_i^2 \omega(\xi_i)} \prod_{\substack{i,j=1\\ i<j}}^{n} |z_{ij}|^{\alpha' k_i \cdot k_j}$$

$$A^{(n)} \sim \int d^2 z_i \prod^{n-3} |z_{ij}|^{\alpha' k_i \cdot k_j} \longrightarrow \text{Virasoro amplitudes}$$