

Title: Introduction to the Bosonic String

Date: Apr 17, 2009 10:00 AM

URL: <http://pirsa.org/09040005>

Abstract: This course provides a thorough introduction to the bosonic string based on the Polyakov path integral and conformal field theory. We introduce central ideas of string theory, the tools of conformal field theory, the Polyakov path integral, and the covariant quantization of the string. We discuss string interactions and cover the tree-level and one loop amplitudes. More advanced topics such as T-duality and D-branes will be taught as part of the course. The course is geared for M.Sc. and Ph.D. students enrolled in Collaborative Ph.D. Program in Theoretical Physics. Required previous course work: Quantum Field Theory (AM516 or equivalent). The course evaluation will be based on regular problem sets that will be handed in during the term. The primary text is the book: 'String theory. Vol. 1: An introduction to the bosonic string. J. Polchinski (Santa Barbara, KITP) . 1998. 402pp. Cambridge, UK: Univ. Pr. (1998) 402 p.' All interested students should contact Alex Buchel at [abuchel@uwo.ca](mailto:abuchel@uwo.ca) as soon as possible.

keep-th paper:

0812.4408

prep-th paper:

0812.4408

M. Hedrick

$T$ -duality & Toroidal compactifications.

$T$ -duality & Toroidal compactifications.

$T^1$ -duality & Toroidal compactifications.

Bosonic:  $D=26$

W. Hodge

$T$ -duality & Toroidal compactifications

Bosonic :  $D=26$

SUSY :  $D=10$

string  
T-duality & Toroidal compactifications.

Bosonic :  $D=26$  } some must be compact. field.  
SUSY :  $D=10$  }



$$x^i \sim x^i + 2\pi R^i \quad i = 4 \dots 25$$

Characterist  
of

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$L \gg R^i$   
↑ characteristic  
size of a probe

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$L \gg R^i \rightarrow$  the probe dynamics is in 4d.  
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$\Rightarrow$  Toroidal comp in QFT  
 $\Rightarrow$  --- in ST

## Kaluza-Klein Compactification

$$ds^2 = G_{MN} dx^M dx^N$$

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$$ds^2 = \underbrace{G_{MN} dx^M dx^N}_{D\text{-dim}} =$$

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$$ds^2 = \underbrace{G_{MN} dx^M dx^N}_{D\text{-dim}} = \underbrace{G_{\mu\nu} dx^\mu dx^\nu}_{d\text{-dim}} + \underbrace{\dots}_{D=d+1}$$



## Kaluza-Klein Compactification

$$ds^2 = \underbrace{G_{MN}^{(D)} dx^M dx^N}_{D\text{-dim}} = \underbrace{G_{\mu\nu}^{(d)} dx^\mu dx^\nu}_{d\text{-dim}} + G_{dd} (dx^d + \dots)^2$$

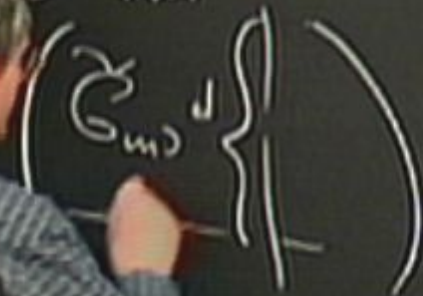
$D = d + 1$



Kaluza-Klein Compactification

$$ds^2 = \underbrace{G_{MN} dx^M dx^N}_{D\text{-dim}} = \underbrace{G_{\mu\nu} dx^\mu dx^\nu}_{d\text{-dim}} + G_{dd} \left( dx^d + A_\mu dx^\mu \right)^2$$

$d = \text{dim}$   
 $D = d + 1$



Kaluza-Klein compactification.

$$ds^2 = \underbrace{G_{MN}^D dx^M dx^N}_{D\text{-dim}} = \underbrace{G_{\mu\nu}^d dx^\mu dx^\nu}_{d\text{-dim}} + G_{dd} \left( dx^d + A_{\mu d} dx^\mu \right)^2$$

$D = d + 1$

$$G_{MN}^D = \begin{pmatrix} G_{\mu\nu}^d & A_{\mu d} \\ A_{\mu d} & G_{dd} \end{pmatrix} \leftarrow \text{most general decomposition.}$$

$$G_{UV}^D = \left( \begin{array}{c|c} \left. \begin{array}{c} J_{m \times d} \\ A_{m \times d} \end{array} \right\} & A_{m \times m} \\ \hline A_{m \times m} & G_{d \times d} \end{array} \right) \leftarrow \text{most general decomposition.}$$

$$X^d \sim X^d + 2\pi R$$

$$G_{MN}^D = \left( \begin{array}{c|c} \left. \begin{array}{c} J_{mn} \\ \hline A_{mn} \end{array} \right\} & A_{mn} \\ \hline A_{mn} & G_{MN} \end{array} \right) \leftarrow \text{most general decomposition.}$$

$$X^d \sim X^d + 2\pi R$$

Assumption of KK compactification.

$$\frac{\partial G_{mn}}{\partial X^d} = 0$$

$$\frac{\partial A_{mn}}{\partial X^d} = 0$$

$$\frac{\partial G_{MN}}{\partial X^d} = 0$$

$$G_{MN}^D = \left( \begin{array}{c|c} \left. \begin{array}{c} G_{mn} \\ A_{m1} \\ \vdots \\ A_{mD} \end{array} \right\} & A_{mD} \\ \hline A_{mD} & G_{DD} \end{array} \right) \leftarrow \text{most general decomposition.}$$

$$x^d \sim x^d + 2\pi R$$

Assumption of KK compactification.

$$\frac{\partial G_{mn}}{\partial x^d} = 0$$

$$\frac{\partial A_{mD}}{\partial x^d} = 0$$

$$\frac{\partial G_{DD}}{\partial x^d} = 0$$

In D-dim



$$G_{MN}^D = \left( \begin{array}{c|c} \left. \begin{array}{c} \text{Im} \\ \text{Re} \end{array} \right\} & A_{MN} \\ \hline A_{MN} & G_{MN} \end{array} \right) \leftarrow \text{most general decomposition.}$$

$$x^d \sim x^d + 2\pi R$$

Assumption of KK compactification.

$$\frac{\partial G_{MN}}{\partial x^d} = 0$$

$$\frac{\partial A_{MN}}{\partial x^d} = 0$$

$$\frac{\partial G_{dd}}{\partial x^d} = 0$$

In D-dim

$$S_D = \frac{1}{2\kappa_D^2} \int d^D x \sqrt{-G} R_D$$

$\Rightarrow \dim d = D-1$  in  $d$ -dim Diff.

$$X^u \rightarrow X^{u'}(x^d)$$

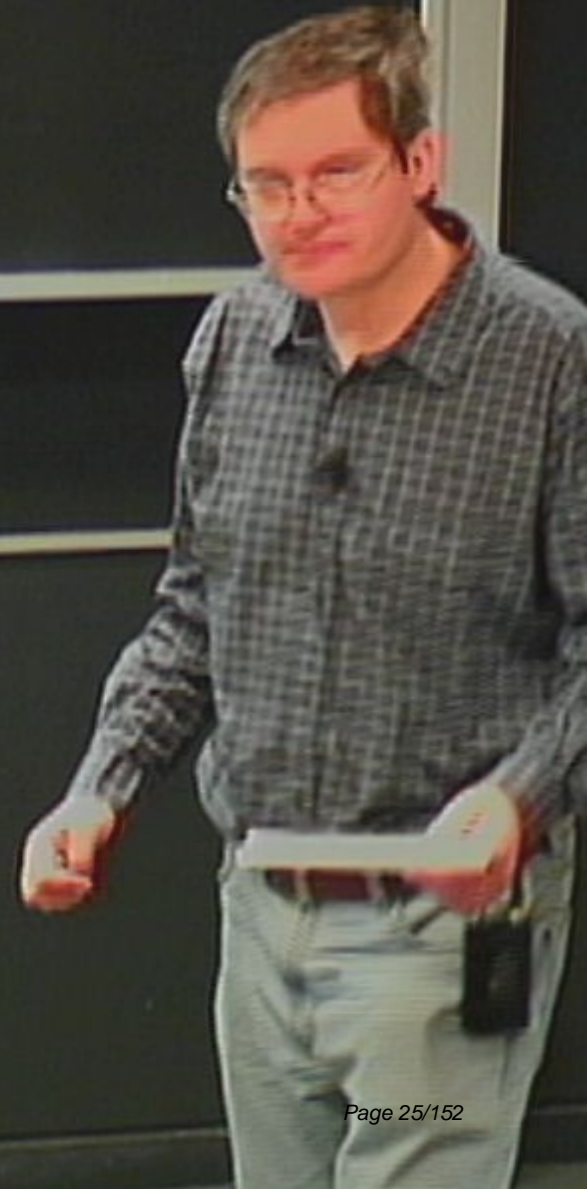
$$\Rightarrow X^d \rightarrow X^{d'} = X^d$$



$\Rightarrow \dim d = D-1$  in  $d$ -dim Diff.

$$X^u \rightarrow X^{u'}(x^d)$$

$$\Rightarrow X^d \rightarrow X^{d'} = X^d + \lambda(x^u)$$



$\Rightarrow \text{Ind} d = D - 1$  in  $d$ -dim Diff.

$$X^u \rightarrow X^{u'}(X^d)$$

$$\Rightarrow X^d \rightarrow X^{d'} = X^d + \lambda(X^u)$$



$\Rightarrow \text{Ind} d = D - 1$  in  $d$ -dim Diff.

$$X^u \rightarrow X^{u'}(X^d)$$

$$\Rightarrow X^d \rightarrow X^{d'} = X^d + \lambda(X^u)$$

→ transforms as a rank-2 tensor  
→ is a scalar

$G_{uv}$   $\rightarrow$  transforms as a rank-2 tensor.

$G_{det}$   $\rightarrow$  is a scalar.

$$G_{uv}(x^m) \rightarrow G'_{uv}(x'^m) \equiv G_{uv}(x^m)$$

$G_{\mu\nu} \rightarrow$  transforms as a rank-2 tensor

$R_{\text{det}} \rightarrow$  is a scalar

$$G_{\mu\nu}(x^\mu) \rightarrow G'_{\mu\nu}(x'^\mu) \equiv G_{\mu\nu}(x^\mu)$$

$$A_{\mu\nu} \rightarrow (A'_{\mu\nu} = A_{\mu\nu} - \partial_\mu \lambda)$$

$G_{\mu\nu} \rightarrow$  transforms as a rank-2 tensor.

$G_{\text{det}} \rightarrow$  is a scalar.

$$G_{\text{det}}(x^\mu) \rightarrow G'_{\text{det}}(x'^{\mu'}) \equiv G_{\text{det}}(x^\mu)$$

$$A_\mu \rightarrow A'_\mu = A_\mu - \partial_\mu \lambda$$

← gauge transformations in d-dim.

⇒ Look at a massless scalar in D-dim  
 $G_{d-1} = 1$



⇒ Look at a massless scalar in  $D$ -dim

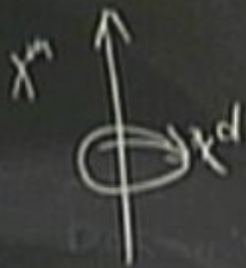
$$G_{\mu\nu} = \eta_{\mu\nu}$$

$$\mathcal{P}(x^\mu, x^\nu)$$

$\Rightarrow x^\mu$



⇒ Look at a massless scalar in  $\mathcal{D}$ -dim  
 $G_{d,d} = \mathbb{1}$



$$\Phi(x^m, x^d)$$

$$\square_D \Phi = 0 \Rightarrow \nabla_m \nabla^m \Phi = 0$$

$$\frac{1}{\sqrt{-g_D}} \partial_m \left[ \sqrt{-g_D} \partial^m \Phi \right] = 0$$

⇒ Toroidal comp in QFT

⇒ --- in ST

$$\int_{\mathcal{M}} \left[ \frac{1}{\sqrt{-g}} \left( \frac{\delta \mathcal{L}}{\delta g^{\mu\nu}} \right) G^{\mu\nu} \right] \delta g^{\mu\nu} = 0$$

$\Rightarrow$  moment of  $g$  along  $x^d$  is

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \left[ \frac{d}{dx} \left( \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x \psi^2 dx \right) \right] = 0$$

⇒ moment of  $\psi$  along  $x^d$  direction is quantized

$$P_d = \int p dx$$

$$P(x^d) = \sum_{n=-\infty}^{+\infty} p_n(x^d) e^{\frac{in x^d}{\hbar}}$$

$$\frac{1}{\sqrt{G^D}} \left[ \frac{\partial}{\partial x^D} G^{DN} \right] = 0$$

$\Rightarrow$  moment of  $\varphi$  along  $x^d$  direction is quantized

$$P_d = \frac{\hbar p}{2\pi}$$

$$\varphi(x^d) = \sum_{n=-\infty}^{+\infty} \varphi_n(x^d) e^{i \frac{n \hbar x^d}{2\pi}}$$

$\rightarrow e^{i P_d x^d}$

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \left[ \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \psi(x) e^{-ipx} dx \right] e^{ipx} dx = 0$$

⇒ Moment of  $\psi$  along  $x^d$  direction is quantized

$$P_d = \frac{d}{dx}$$

$$\psi(x^d) = \sum_{n=-\infty}^{+\infty} \psi(x^d_n) e^{i n x^d}$$

$$e^{i n x^d}$$

→  $e^{i p_d x^d}$

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \left[ \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \psi(x) e^{-ipx} dx \right] e^{ipx} dx = 0$$

⇒ moment of  $\psi$  along  $x^d$  direction is quantized

$$P_d = \frac{p}{\hbar}$$

$$\psi(x^d) = \sum_{p=-\infty}^{+\infty} \psi(x^d) e^{i p x^d / \hbar}$$

$$\psi(x^d) e^{i p x^d / \hbar}$$

$$\rightarrow e^{i p_d x^d}$$

$$\int_{-\infty}^{\infty} \left[ \int_{-\infty}^{\infty} \psi(x) \psi^*(x) dx \right] dx = 1$$

$\Rightarrow$  momentum of  $\psi$  along  $x^d$  direction is quantized

$$P_d = \frac{h}{R}$$

$$\rightarrow e^{iP_d x^d}$$

$$\psi(x^d) = \sum_{n=-\infty}^{+\infty} \psi_n(x^d) e^{i \frac{h n x^d}{R}}$$

$$\psi(x^d + 2\pi R) = \psi(x^d)$$



$$\int_{\mathcal{D}} \varphi = 0 \Rightarrow \underbrace{\nabla_M \nabla^M}_{d\text{-dim}} \varphi_{(n)} = \frac{\kappa^2}{R^2} \varphi_{(n)}(x)$$

$$\partial_M \left[ \sqrt{-g} G^{MN} \partial_N \varphi \right]$$

$$\int_{\Omega} \varphi = 0 \Rightarrow$$

$$\underbrace{\Delta_{\text{lin}} \Delta_{\text{lin}}^{\text{lin}} \varphi}_{d\text{-dim}} = \frac{\hbar^2}{2m} \varphi_n(x)$$

$$\left( \Delta_{\text{lin}} - \frac{\hbar^2}{2m} \right) \varphi_n = 0$$

$$\int_D \varphi = 0 \Rightarrow$$

$$\underbrace{\Delta_M \Delta^M}_{d\text{-dim}} \varphi_n(x) = \frac{\hbar^2}{k^2} \varphi_n(x)$$

$$\left( \Delta_d - \frac{\hbar^2}{k^2} \right) \varphi_n = 0$$

1 massless scalar in D-dim  $\equiv$

$$\square_D \varphi = 0 \Rightarrow$$

$$\underbrace{\nabla_M \nabla^M}_{d\text{-dim}} \varphi_n(x) = \frac{\eta^2}{R^2} \varphi_n(x)$$

$$\left( \square_D - \frac{\eta^2}{R^2} \right) \varphi_n = 0$$

1 massless scalar in  $D$ -dim  $\equiv$  Infinite set of massive scalars in  $d$ -dim  $m_n^2 = \frac{\eta^2}{R^2}$

$\Rightarrow$  In ST we derived EDM and Effective action  
for massless fields

⇒ In ST we derived FOM and FPR action  
for massless fields

$$m \sim \frac{1}{\Lambda} \sim \frac{1}{(\Lambda^2)^{1/2}}$$

$\Rightarrow$  In ST we derived EDM and Effective action  
for massless fields

$$m_s \sim \frac{1}{\ell_s} \sim \left(\frac{1}{\alpha'}\right)^{1/2}$$

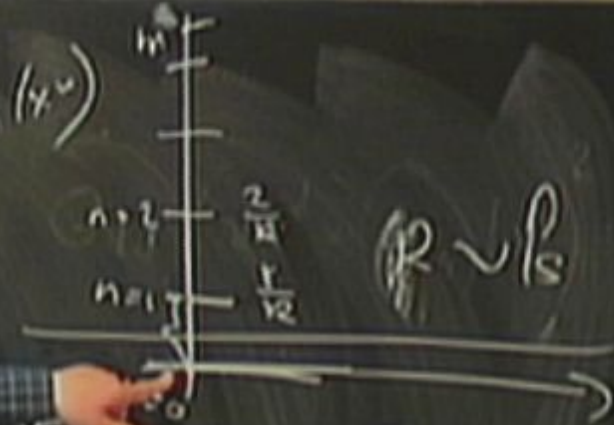


$$\square_D \varphi = 0 \Rightarrow$$

$$\underbrace{\Delta_{\text{in}} \Delta_{\text{in}}^{\text{in}}}_{\text{in}} \varphi_{n(\text{in})} = \frac{\hbar^2}{R^2} \varphi_n(x)$$

$$\left( \square_D - \frac{\hbar^2}{R^2} \right) \varphi_n = 0$$

1 massless scalar



massive scalars in  $D$ -dim

$$m_h^2 = \frac{\hbar^2}{R^2}$$



$\Rightarrow$  In ST we derived EDM and Effective action  
for massless fields

$$m_k \sim \frac{1}{\Lambda^2} \sim \frac{1}{(\Lambda^2)^{1/2}}$$

$\Rightarrow$  light modes is only  $n=0$

⇒ In ST we derived EDM and Effective action  
for massless fields

$$m_k \sim \frac{1}{\alpha} \sim \left(\frac{1}{\alpha}\right)^{1/2}$$

⇒ light mode is only  $n=0$

$$\varphi_0(x^M) = \varphi_0(x^\mu) \left( e^{\frac{i p_\nu}{\hbar} x^\nu} \right)$$

$\Rightarrow$  In ST we derived EOM and Effective action  
for massless fields

$$m_k \sim \frac{1}{\Lambda^2} \sim \left(\frac{1}{\Lambda}\right)^2$$

$\Rightarrow$  light modes is only  $n=0$

$$\varphi_0(x^M) = \varphi_0(x^\mu) \left( e^{\frac{i p_\nu}{k} x^\nu} \right)$$



$$x^d \rightarrow x^d + \lambda \nabla f(x^d)$$

$$A_{n+1} \rightarrow A_n + \lambda \nabla f(A_n)$$

$\rightarrow$  illegal

... of the function



$$x^d \rightarrow x^d + \lambda(x^m)$$

$$A_m \rightarrow A_m - \partial_m \lambda$$

$$P_n(x^m) e^{i \frac{n}{R} x^d}$$

wave  
function  
of  
"h"  
-mode



$$P_n(x^m) e^{i \frac{n}{R} x^d}$$

$$= e^{i \frac{n}{R} \lambda}$$

$$\Psi(x) \rightarrow \Psi(x) e^{i g \lambda}$$

g - is a charge.

$$x^\mu \rightarrow x^\mu + \lambda(x^\mu)$$

$$A_\mu \rightarrow A_\mu - \partial_\mu \lambda$$

$\psi(x) \rightarrow \psi(x) e^{i\frac{n}{R}\lambda}$   
 $\psi$  is a charge  
 $\psi(x) = \underbrace{\varphi_n(x^\mu)}_{\text{wave function of } q_n \text{ "h"-mode}} e^{i\frac{n}{R}x^\mu}$

gauge field

$$\psi(x) \rightarrow \psi(x) e^{i\theta}$$

$\psi$  is a charge

$$\varphi_n(x^\mu) e^{i\frac{n}{R}x^\mu} \cdot e^{i\left(\frac{n}{R}\right)\lambda}$$

$P_j^{(n)} \equiv$  a KK charge!

$$A_\mu \rightarrow A'_\mu = A_\mu - \partial_\mu \lambda$$

gauge transformations in d-dim.



$$x^\mu \rightarrow x^\mu + \lambda(x^\mu)$$

$$A_\mu \rightarrow A_\mu - \partial_\mu \lambda$$

↑  
gauge field

$$\psi(x^\mu) e^{i \frac{n}{R} x^d}$$

wave function of  $q_n$   
"h"-mode

$$\psi(x^\mu) e^{i \frac{n}{R} x^d} \rightarrow \psi(x^\mu) e^{i \frac{n}{R} x^d} \cdot e^{i \left(\frac{n}{R}\right) \lambda}$$

$P_d^{(n)} \equiv$  a KK charge!

$h=0$  is neutral

$$A_\mu \rightarrow A'_\mu = A_\mu - \partial_\mu \lambda$$

← gauge transformations in d-dim.

$$I_{\text{rot}} = \frac{1}{2} \int dx \sqrt{-G^{\text{D}}} R^{\text{D}}$$

$$\frac{1}{2} \int dx \sqrt{-G^{\text{D}}}$$

L



$\frac{1}{2} \left( \frac{1}{2K_D} \right) dx \quad \sqrt{-G^D}$

$\frac{1}{16\pi G_N^D}$

$\int d^D x \sqrt{-G^D} \left( -\frac{1}{2} \partial_M \varphi \partial^M \varphi \right)$

$\int d^D x \sqrt{-G^D} \sqrt{G^D}$

$\left( -\frac{1}{2} \right)$

$\Delta^M \varphi = 0$

*depend on  $x^M$*

$\varphi(x^d + 2\pi R) = \varphi(x^d)$

$$\begin{aligned}
 & \frac{1}{16\pi G_N} \int d^d x \sqrt{-g} \left( -\frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi \right) \\
 & \xrightarrow{\text{depend only on } x^\mu} \frac{1}{16\pi G_N} \int d^d x \sqrt{-g} \left( -\frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi \right) \\
 & \xrightarrow{\text{d}} \frac{1}{16\pi G_N} \int d^d x \sqrt{-g} \left( -\frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi \right) \\
 & \xrightarrow{\text{d}} \frac{1}{16\pi G_N} \int d^d x \sqrt{-g} \left( -\frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi \right)
 \end{aligned}$$

$$\varphi(x^\mu)$$

expansion

To get action in  $d = D-1$  dim.  
with  $D$ -dim action and

$$\Delta_{\mu\nu} \partial^\mu \varphi = 0 \quad \left( -\frac{1}{2} \partial_\mu \partial^\mu \varphi \right)$$

Partialing  $x^0$

To get action in  $d = D - 1$  dim.  
start with  $D$ -dim action and  
complete it assuming nothing depends on  $x^0$

$\Delta_{\mu\nu} \Delta^{\mu\nu} \varphi = 0$        $(-\frac{1}{2} \eta_{\mu\nu} \partial^{\mu} \partial^{\nu} \varphi)$

To get action in  $d = D-1$  dim.  
start with  $D$ -dim action and  
complete it assuming nothing depends on  $x^D$

Limitations: truncated to massless modes

$$J_0 = \frac{1}{2k_0} \int d^D x \sqrt{-G^D} R =$$

$$G_{4+1} = e^{2\sigma(x^{\mu})}$$

$$J_0 = \frac{1}{2k_0} \int d^D x \sqrt{-G^D} R_D =$$

$$G_{d+1} = e^{2\sigma(x^{\mu})}$$

$$\sqrt{-G^D} = \sqrt{-G^d} e^{\sigma}$$

$$J_D = \frac{1}{2\kappa_D} \int d^D x \sqrt{-G^D} R_D =$$

$$G_{d+1} = e^{2\sigma(x^{\mu})}$$

$$\sqrt{-G^D} = \sqrt{-G^d} e^{\sigma}$$

$$R_D = R_d - 2e^{-\sigma} \nabla^2 \sigma - \frac{1}{4} e^{2\sigma} F_{\mu\nu} F^{\mu\nu}$$



$$J_0 = \frac{1}{2k_0} \int d^D x \sqrt{-G^D} R_D =$$

$$G_{d+1} = e^{2\sigma(x^{\mu})}$$

$$\sqrt{-G^D} = \sqrt{-G^d} e^{\sigma}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$R_D = R_d - 2e^{-\sigma} \nabla^2 e^{\sigma} - \frac{1}{4} e^{2\sigma} F_{\mu\nu} F^{\mu\nu}$$

$$J_0 = \frac{1}{2k_0} \int d^D x \sqrt{-G^D} R_D = \frac{1}{2k_0} \cdot 2\pi R \cdot \int d^d x$$

$$G_{d+1} = e^{2\sigma(x^{d+1})}$$

$$\sqrt{-G^D} = \sqrt{-G^d} e^{\sigma}$$

$$F_{uv} = \partial_u A_v - \partial_v A_u$$

$$R_D = R_d - 2e^{-\sigma} \nabla^2 e^{\sigma} = \frac{1}{4} e^{2\sigma} F_{uv} F^{uv}$$

$$J_0 = \frac{1}{2k_0} \int_{M^D} d^D x \sqrt{-G^D} R_D = \frac{1}{2k_0} \cdot 2\pi R \cdot \int_{M^d} d^d x \sqrt{-G^d} e^{\sigma}$$

$$G_{d+1} = e^{2\sigma(x^{d+1})}$$

$$\sqrt{-G^D} = \sqrt{-G^d} e^{\sigma}$$

$$F_{uv} = \partial_u A_v - \partial_v A_u$$

$$R_D = R_d - 2e^{-\sigma} \nabla^2 e^{\sigma} = \frac{1}{4} e^{2\sigma} F_{uv} F^{uv}$$

$$J_0 = \frac{1}{2k_0} \int_{\mathcal{M}^D} d^D x \sqrt{-G^D} R_D = \frac{1}{2k_0} \cdot 2\pi R \cdot \int_{\mathcal{M}^d} d^d x \sqrt{-G^d} e^{\sigma} \left[ R_d - \frac{1}{4} e^{2\sigma} F_{\mu\nu} F^{\mu\nu} \right]$$

$$G_{d+1} = e^{2\sigma(x^{\mu})}$$

$$\sqrt{-G^D} = \sqrt{-G^d} e^{\sigma}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$R_D = R_d - 2e^{-\sigma} \nabla^2 e^{\sigma} - \frac{1}{4} e^{2\sigma} F_{\mu\nu} F^{\mu\nu}$$

$$J_D = \frac{1}{2\kappa_0} \int_{M^D} d^D x \sqrt{-G^D} R_D = \frac{1}{2\kappa_0} \cdot 2\pi R \cdot \int_{M^d} d^d x \sqrt{-G^d} e^{\sigma} \left[ R_d - \frac{1}{4} e^{2\sigma} F_{\mu\nu} F^{\mu\nu} \right]$$

$$G_{d+1} = e^{2\sigma(x^{\mu})}$$

is a total derivative:

$$\sqrt{-G^D} = \sqrt{-G^d} e^{\sigma}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$R_D = R_d - \left( 2e^{-\sigma} \nabla^2 e^{\sigma} \right) - \frac{1}{4} e^{2\sigma} F_{\mu\nu} F^{\mu\nu}$$

$$J_0 = \frac{1}{2k_0} \int_{M^D} \sqrt{-G^D} R_D = \frac{1}{2k_0} \cdot 2\pi R \cdot \int_{M^d} \sqrt{-G^d} e^{\frac{\sigma}{2}} \left[ R_d - \frac{1}{4} e^{2\sigma} F_{\mu\nu} F^{\mu\nu} \right]$$

$$G_{d+1} = e^{2\sigma(x^{\mu})}$$

is a total derivative:

$$\sqrt{-G^D} = \sqrt{-G^d} e^{\sigma}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$R_D = R_d - \left( 2e^{-\sigma} \nabla^2 \sigma \right) - \frac{1}{4} e^{2\sigma} F_{\mu\nu} F^{\mu\nu}$$

$$\frac{\partial}{\partial x^\mu} \left( \frac{\partial \sigma}{\partial x^\mu} \right)$$

$$\frac{\partial}{\partial x^\mu} \left( \frac{\partial \sigma}{\partial x^\mu} \right)$$

$$L_0 = \int dx^d \sqrt{|G_{dd}|} = e^{\sigma} \cdot R$$

$\sigma$  is a modulus that determines the size of a  
 $d - 1$

$$L_0 = \int dx^d \sqrt{|G_{dd}|} = e^{\sigma} R$$

$\sigma$  is a modulus that determines the size of a d-circle



$$J_D = \frac{1}{2k_0} \int_{M^D} d^D x \sqrt{-G^D} R_D = \frac{1}{2k_0} \cdot 2\pi R \cdot \int d^d x \sqrt{-G} \left( e^{\frac{2\sigma}{4}} R_d - \frac{1}{4} e^{2\sigma} F_{\mu\nu} F^{\mu\nu} \right)$$

$$G_{d+1} = e^{2\sigma} G^d$$

$$\sqrt{-G^D} = \sqrt{-G^d} e^{\sigma}$$

$$R_D = R_d - \frac{2}{4} e^{-\sigma} \nabla^2 e^{\sigma} - \frac{1}{4} e^{2\sigma} F_{\mu\nu} F^{\mu\nu}$$

is a total derivative!

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$F_{\mu\nu} F^{\mu\nu}$$

$$J_0 = \frac{1}{2\kappa_0} \int_{M^D} d^D x \sqrt{-G^D} R_D = \frac{1}{2\kappa_0} \cdot 2\pi R \int_{M^d} d^d x \sqrt{-G} \left( e^{\frac{2\sigma}{\kappa_0}} R_d - \frac{1}{4} e^{2\sigma} F_{\mu\nu} F^{\mu\nu} \right)$$

$$G_{d+1} = e^{2\sigma} G^d$$

is a total derivative:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$\sqrt{-G^D} = \sqrt{-G^d} e^{2\sigma}$$

$$R_D = R_d - \frac{2}{\kappa_0} e^{-\frac{2\sigma}{\kappa_0}} \nabla^2 \sigma - \frac{1}{4} e^{\frac{2\sigma}{\kappa_0}} F_{\mu\nu} F^{\mu\nu}$$

$$J_D = \frac{1}{2k_D} \int_{M^D} d^D x \sqrt{-G^D} R_D = \frac{1}{2k_D} \cdot 2\pi R \cdot \int d^d x \sqrt{-G} \left( e^{\frac{2\sigma}{4}} R_d - \frac{1}{4} e^{2\sigma} F_{\mu\nu} F^{\mu\nu} \right)$$

$$G_{d+1} = e^{2\sigma} G^d$$

is a total derivative:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$\sqrt{-G^D} = \sqrt{-G^d} e^{2\sigma}$$

$$R_D = R_d - \frac{2}{4} e^{-\sigma} \nabla^2 e^{\frac{2\sigma}{4}} - \frac{1}{4} e^{2\sigma} F_{\mu\nu} F^{\mu\nu}$$

$$S_1 = \frac{1}{2\kappa_D^2} \int dx^D \sqrt{-G_D} \left[ R + \frac{1}{2} \nabla_m \phi \nabla^m \phi \right]$$

$$S_1 = \frac{1}{2\kappa_D^2} \int dx^D \sqrt{-G_D} \left[ R + \frac{1}{2} \nabla_m \varphi \nabla^m \varphi \right]$$

( $D=26$ ,  $\varphi$  is a dilaton)

For now Bms

$$S_{\text{D}} = \frac{1}{2\kappa_D^2} \int dx^D \sqrt{-G_D} \left[ R + \frac{1}{2} \nabla_m \varphi \nabla^m \varphi \right]$$

( $D=26$ ,  $\varphi$  is a dilaton)

For now **BMN**)

$$S_{\text{D}} = \frac{1}{2\kappa_D^2} \int dx^D \sqrt{-G_D} \left[ R + \frac{1}{2} \nabla_m \varphi \nabla^m \varphi \right] e^{-2\varphi}$$

( $D=26$ ,  $\varphi$  is a dilaton)

For now  $B_{MN}$ )

$$J_D = \frac{1}{2k_D} \int_{M^D} d^D x \sqrt{-G^D} R_D = \left( \frac{1}{2k_D} \cdot 2\pi R \right) \int d^d x \sqrt{-G} \left( \frac{1}{4} e^{2\sigma} F_{\mu\nu} F^{\mu\nu} \right)$$

$$G_{d+1} = e^{2\sigma} G^d$$

is a total derivative:

$$\sqrt{-G^D} = \sqrt{-G^d} e^{\sigma}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$R_D = R_d - \left( 2 e^{-\sigma} \nabla^2 e^{\sigma} \right) - \left( \frac{1}{4} e^{2\sigma} F_{\mu\nu} F^{\mu\nu} \right)$$

$\frac{1}{4} e^{2\sigma} F_{\mu\nu} F^{\mu\nu}$   
 $\frac{1}{4} e^{2\sigma} F_{\mu\nu} F^{\mu\nu}$   
 $\frac{1}{4} e^{2\sigma} F_{\mu\nu} F^{\mu\nu}$



$$S_{(1)} = \frac{1}{2\kappa_D^2} \int dx^D \sqrt{-G_D} \left[ R + \frac{1}{2} \nabla_m \varphi \nabla^m \varphi \right] e^{-2\varphi}$$

( $D=26$ ,  $\varphi$  is a dilaton)

For now  $B_{MN}$

$$= \frac{\pi R}{\kappa_D^2} \int dx^d (-G_d)^{\frac{1}{2}} e^{-2\varphi + 5}$$

$$S_{\text{D}} = \frac{1}{2\kappa_D^2} \int dx^D \sqrt{-G_D} \left[ R + \frac{1}{2} \nabla_m \varphi \nabla^m \varphi \right] e^{-2\varphi}$$

(D=26,  $\varphi$  is a dilaton)

For now  $B_{MN}$

$$= \frac{\pi R}{\kappa_D^2} \int dx^d (-G_d)^{1/2} e^{-2\varphi + 5\sigma} \left[ R_d - 4\nabla\varphi\nabla\sigma + 4\nabla\varphi\nabla\sigma - \frac{1}{4} e^{2\sigma} F_{\mu\nu}^2 \right]$$

$$S_{\text{D}} = \frac{1}{2\kappa_D^2} \int dx^D \sqrt{-G_D} \left[ R + \frac{1}{2} \nabla_m \varphi \nabla^m \varphi \right] e^{-2\varphi}$$

(D=26,  $\varphi$  is a dilaton)

For now  $B_{MN}$

$$= \frac{\pi R}{\kappa_D^2} \int dx^d (-G_d)^{\frac{1}{2}} e^{-2\varphi + 5\sigma} \left[ R_d \left( -4\nabla\varphi\nabla\varphi \right) + 4\nabla\varphi\nabla\varphi - \frac{1}{4} e^{2\sigma} F_{\mu\nu} F^{\mu\nu} \right]$$

$$S_{\text{D}} = \frac{1}{2\kappa_D^2} \int dx^D \sqrt{-G_D} \left[ R + \frac{1}{2} \nabla_m \varphi \nabla^m \varphi \right] e^{-2\varphi}$$

( $D=26$ ,  $\varphi$  is a dilaton)

For now  $B_{MN}$

$$= \frac{\pi R}{\kappa_D^2} \int dx^d (-G_d)^{\frac{1}{2}} e^{-2\varphi + 5\sigma} \left[ R_d - 4\nabla\varphi\nabla\sigma + 4\nabla\varphi\nabla\varphi - \frac{1}{4} e^{2\sigma} F_{\omega\omega}^2 \right]$$

$$= \frac{\pi R}{k_D^2} \int d^d x \sqrt{-G} e^{-2\varphi_d} \left[ R_d - (\nabla\phi)^2 + 4(\nabla\varphi_d)^2 - \frac{1}{4} e^{2\phi} F_{\mu\nu} F^{\mu\nu} \right]$$

$$\varphi_d \equiv \varphi - \frac{\phi}{\sqrt{2}}$$

$$= \frac{\pi R}{k_D^2} \int d^d x \sqrt{-G_{II}} e^{-2\varphi_d} \left[ R_{II} - (\nabla\phi)^2 + 4(\nabla\varphi_d)^2 - \right.$$

$$\varphi_d \equiv \varphi -$$

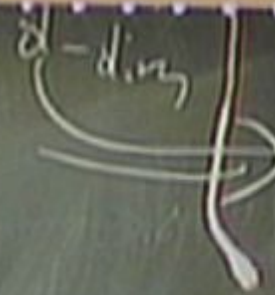
string  
same in  
d-dim

$$- \frac{1}{4} e^{2\phi} F_{\mu\nu} F^{\mu\nu}$$



sign for  $(\nabla\varphi_d)^2$   
is correct in Einstein frame.

$$p_d \equiv p - \frac{1}{2} \rho v^2$$

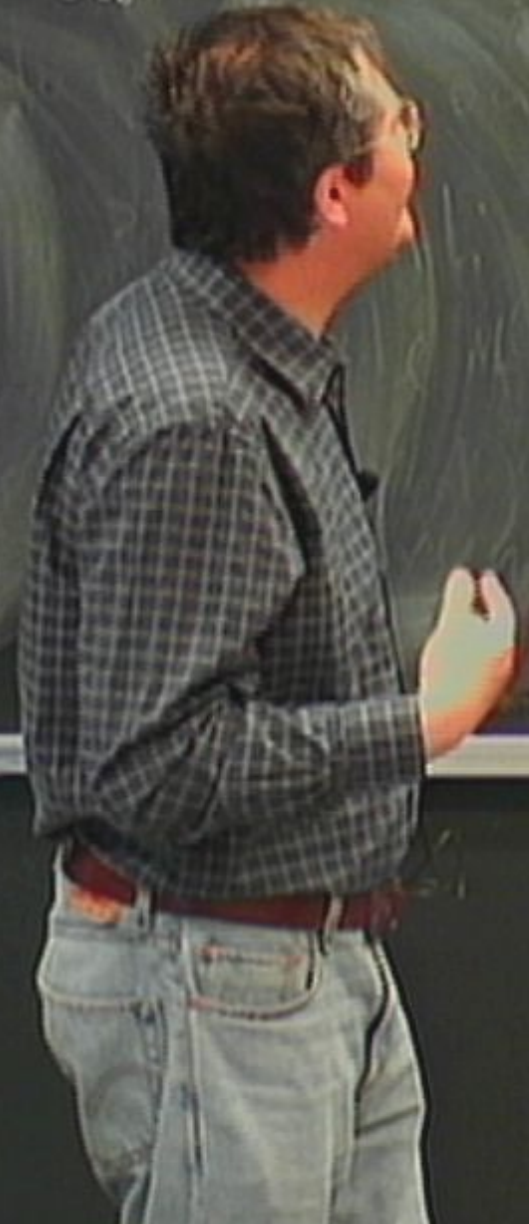


sign for  $(\nabla \cdot \mathbf{v})^2$   
is correct in Finsler frame.

Special relativity



⇒ Dirac eff. action includes only massless fields





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⇒ they are neutral under  $U(1)$

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⇒ massive fields are charged

$\mathcal{P} \rightarrow$

- ⇒ Our eff. action includes only massless fields
- ⇒ they are neutral under  $U(1)$
- ⇒ massive fields are charged

$$D_\mu \psi \rightarrow$$

$$i \bar{\psi} \gamma^\mu A_\mu \psi$$

⇒ Dirac eff. action includes only massless fields

⇒ they are neutral under  $U(1)$

⇒ fields are charged

$$\begin{aligned} \rightarrow D_\mu &= \partial_\mu + i q A_\mu \\ &= \partial_\mu + i \frac{q}{\hbar c} A_\mu \end{aligned}$$

⇒ Dirac eff. action includes only massless fields

⇒ they are neutral under  $U(1)$

⇒ massive fields are charged

$\vec{P}_m \rightarrow D_m = \nabla_m + i(\vec{P}_m \cdot \vec{A}_m)$

$\tilde{A}_m = \frac{1}{g} A_m$

$= \nabla_m + i \frac{g}{r} A_m = \nabla_m + i \tilde{A}_m$

$$= \frac{\pi R}{\kappa_D^2} \int d^d X \sqrt{-G_{11}} e^{-2\varphi_d} \left[ R_{11} - (\nabla\phi)^2 + 4(\nabla\varphi_d)^2 - \right.$$

$\varphi_d \equiv$

string frame in d-dim

$$- \frac{R^2}{4} e^{2\phi} \sim F_{\mu\nu} F^{\mu\nu}$$



sign for  $(\nabla\varphi_d)^2$

is correct in Einstein frame.

$$= \frac{\pi R}{k_D^2} \int d^d X \sqrt{-G_{11}} e^{-2\varphi_d} \left[ R_{11} - (\nabla\phi)^2 + 4(\nabla\varphi_d)^2 - \right.$$

$$\varphi_d \equiv \varphi - \frac{\phi}{2}$$

string frame in  $d$ -dim

$$- R^2 e^{2\phi} \sim F_{\mu\nu} F^{\mu\nu}$$

sign for  $(\nabla\varphi_d)^2$

is correct in Einstein frame

$$F_{uv} \rightarrow R \cdot \tilde{F}_{uv}$$

⇒ relation between gravitational and gauge couplings  
↳ upon KK reduction



$$F_{uv} \rightarrow R \cdot \tilde{F}_{uv}$$

$\Rightarrow$  relation between gravitational and gauge couplings  
in d-dim KK reduction

$$\frac{1}{2\kappa_d^2} = \frac{\pi R}{16\pi^2 \kappa_D^2}$$

$$F_{uv} \rightarrow R \cdot \tilde{F}_{uv}$$

⇒ relation between gravitational and gauge coupling  
in d-dim KK reduction

$$\frac{1}{2k_D^2} = \frac{\pi R}{4k_D^2}$$

$$\frac{1}{4g_4^2}$$

$$F_{uv} \rightarrow R \cdot \tilde{F}_{uv}$$

⇒ relation between gravitational and gauge couplings  
 (upon dimensional reduction)

$$\frac{1}{2\kappa_D^2} = \frac{\pi R}{4\kappa_4^2}$$

$$\frac{1}{4g_4^2} \tilde{F}_{uv} \tilde{F}_{uv}$$

$$F_{uv} \rightarrow R \cdot \tilde{F}_{uv}$$

⇒ relation between gravitational and gauge couplings  
 (upon KK reduction)

$$\frac{1}{2k_D^2} = \frac{\pi R}{4k_D^2}$$

$$\frac{1}{4g_4^2} \tilde{F}_{uv} \tilde{F}_{uv}$$

$$\frac{1}{4g_4^2} = \frac{\pi R}{k_D^2} \frac{1}{4}$$

$$F_{uv} \rightarrow R \cdot \tilde{F}_{uv}$$

⇒ relation between gravitational and gauge couplings  
 (in 4D on KK reduction)

$$\frac{1}{2k_D^2} = \frac{\pi R}{k_D^2}$$

$$\frac{1}{4g_4^2} \tilde{F}_{uv} \tilde{F}_{uv}$$

$$\frac{1}{4g_D^2} = \frac{\pi R}{k_D^2} \frac{1}{4} \left[ \dots \right]^2$$

$$F_{uv} \rightarrow R \cdot \tilde{F}_{uv}$$

⇒ relation between gravitational and gauge couplings  
 (upon KK reduction)

$$\frac{1}{2k_D^2} = \frac{\pi R}{k_D^2}$$

$$\frac{1}{4g_D^2} \tilde{F}_{uv} \tilde{F}_{uv}$$

$$\frac{1}{4g_D^2} = \left( \frac{\pi R}{k_D^2} \right) \frac{1}{4} \cdot [2]$$

$$g_D^2 = \frac{2k_D^2}{L^2}$$

$$S_2 = -\frac{1}{24k_D^2} \int d^D x \sqrt{-G^{DD}} e^{-2\varphi} H_{MNP} H^{MNP}.$$

$$S_2 = -\frac{1}{2^{11} \kappa_D^2} \int d^D x \sqrt{-G} e^{-2\varphi} H_{MNP} H^{MNP}$$

$H = dB$        $B_{MN} \rightarrow \begin{cases} B_{\mu\nu} \rightarrow \text{antisymmetric in } \mu, \nu \text{ dim} \end{cases}$



$$S_2 = -\frac{1}{2^{4k_D}} \int d^D x \sqrt{-G} e^{-2\varphi} H_{MNP} H^{MNP}$$

$$H = dB \quad B_{MN} \rightarrow \begin{cases} B_{\mu\nu} \rightarrow \text{antisymmetric in } d\text{-dim} \\ B_{\mu d} \end{cases}$$

$$S_2 = -\frac{1}{2^{11} k_D^2} \int d^D x \sqrt{-G} e^{-2\varphi} H_{MNP} H^{MNP}$$

$$H = dB \quad B_{MN} \rightarrow \begin{cases} B_{\mu\nu} \rightarrow \text{antisymmetric in } \mu, \nu \\ B_{\mu d} \rightarrow A_\mu \end{cases}$$

$$S_2 = -\frac{1}{2^{11} \kappa_D^2} \int d^D x \sqrt{-G} e^{-2\varphi} H_{MNP} H^{MNP}$$

$H = dB$

$B_{MN} \rightarrow \begin{cases} B_{\mu\nu} \rightarrow \text{antisymmetric in } \mu, \nu \\ \underbrace{B_{\mu d}}_{\sim} \rightarrow A_\mu \rightarrow \text{independent gauge field} \end{cases}$

$$-\frac{1}{4g_d^2} F_{\mu\nu}^2$$

$$-\frac{1}{4g_d^2} = \left( \frac{1}{2} \right) \left( \frac{1}{2} \right)$$

$$S_2 = -\frac{1}{2^{11} k_D^2} \int d^D x \sqrt{-G^{DD}} e^{-2\phi} H_{MNP} H^{MNP}$$

$$H = dB$$

$B_{MN}$

$\left\{ \begin{array}{l} B_{\mu\nu} \rightarrow \text{antisymmetric in } \mu, \nu \\ B_{\mu d} \rightarrow A_{\mu} \rightarrow \text{included in gauge field} \end{array} \right.$

$$S_2 = -\frac{\pi R}{12 k_D^2} \int d^4 x \sqrt{-G_4} e^{-2\phi}$$

$$\frac{1}{4g^2} F_{\mu\nu} F^{\mu\nu} \quad \frac{1}{4g^2} \text{tr} \left( \frac{1}{2} F_{\mu\nu} F^{\mu\nu} \right)$$

$$S_2 = -\frac{1}{24k_D^2} \int d^D x \sqrt{-G} e^{-2\varphi} H_{MNP} H^{MNP}$$

$$H = dB$$

$B_{MN} \rightarrow \begin{cases} B_{\mu\nu} \rightarrow \text{antisymmetric} \\ B_{\mu d} \rightarrow \dots \\ B_{dd} \rightarrow \dots \end{cases}$

$$S_2 = -\frac{\pi R}{12k_D^2} \int d^4 x \sqrt{-G_4} e^{-2\varphi} \left[ \tilde{H}_{\mu\nu\rho} \tilde{H}^{\mu\nu\rho} \right]$$

$B_{\mu d} \rightarrow A_{\mu} \rightarrow \text{index}$

$\frac{1}{4g^2} F_{\mu\nu} F^{\mu\nu}$ 
 $\frac{1}{4g^2} = \left( \frac{1}{2} \right)^2$

$$S_2 = -\frac{1}{2^{11} k_D^2} \int d^D x \sqrt{-G^{DD}} e^{-2\phi} H_{MNP} H^{MNP}$$

$H = dB$ 
 $B_{MN} \rightarrow \begin{cases} B_{\mu\nu} \rightarrow \text{antisymmetric in } \mu, \nu \text{ dim} \\ B_{\mu d} \rightarrow \text{indeed a gauge field} \end{cases}$

$$S_2 = -\frac{\pi R}{12 k_D^2} \int d^4 x \sqrt{-G_4} e^{-2\phi} \left[ \tilde{H}_{\mu\nu\rho} \tilde{H}^{\mu\nu\rho} + 3e^{-2\phi} F'_{\mu\nu} F'^{\mu\nu} \right]$$

$$-\frac{1}{4g^2} F_{\mu\nu} F^{\mu\nu} \quad -\frac{1}{4g^2} \text{tr} \left( \frac{1}{2} F_{\mu\nu} F^{\mu\nu} \right)$$

$$S_2 = -\frac{1}{24k_D^2} \int d^D x \sqrt{-G} e^{-2\phi} H_{MNP} H^{MNP}$$

$$H = dB$$

$B_{MN}$

$\rightarrow \left\{ \begin{array}{l} B_{\mu\nu} \rightarrow \text{antisymmetric in } d\text{-dim} \\ B_{\mu d} \rightarrow \text{indeed a gauge field} \end{array} \right.$

$$S_2 = -\frac{\pi R}{12k_D^2} \int d^4 x \sqrt{-G_4} e^{-2\phi} \left[ \tilde{H}_{\mu\nu\rho} \tilde{H}^{\mu\nu\rho} + 3e^{-2\phi} F'_{\mu\nu} F'^{\mu\nu} \right]$$

$$S_2 = -\frac{1}{24\kappa_D^2} \int d^D x \sqrt{-G^{DD}} e^{-2\varphi} H_{MNP} H^{MNP}$$

$$H = dB$$

$B_{MN}$

$B_{MN} \rightarrow \begin{cases} B_{MN}^{\parallel} \rightarrow \text{antisymmetric in d-dim} \\ B_{MN}^{\perp} \end{cases}$

$B_{MN}^{\perp} \rightarrow A_M \rightarrow \text{indeed a gauge field}$

$$S_2 = -\frac{\pi R}{12\kappa_0^2} \int d^4 x \sqrt{-G_4} e^{-2\varphi} \left[ \tilde{H}_{MNP} \tilde{H}^{MNP} + 3e^{-2\sigma} F_{MN} F^{MN} \right]$$



$$\textcircled{S_2} = -\frac{1}{24\kappa_D^2} \int d^D x \sqrt{-G^{DD}} e^{-2\varphi} H_{MNP} H^{MNP}$$

$$H = dB$$

$B_{MN}$

$\rightarrow \left\{ \begin{array}{l} B_{\mu\nu} \rightarrow \text{antisymmetric in d-dim} \\ B_{\mu d} \end{array} \right.$

$B_{\mu d} \rightarrow A_\mu \rightarrow \text{indeed a gauge field}$

$$S_2 = -\frac{\pi R}{12\kappa_0^2} \int d^4 x \sqrt{-G_4} e^{-2\varphi} \left[ \tilde{H}_{\mu\nu\rho} \tilde{H}^{\mu\nu\rho} + 3e^{-2\sigma} F_{\mu\nu} F^{\mu\nu} \right]$$

$V(I) \times V(I)$

$A_m$



has  
changed  
states

$A'_m$



EATEN

$V(\mathbb{R}) \times V(\mathbb{R})$

$A_m$



has  
an infinite  
states

$A'$



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MICHIGAN  
LIBRARY

$V(I) \times V(I)$

$A_m$

$A'_m$

↑  
has  
charged  
states

↑  
No charged  
massive states!

$$U(1) \times U(1)$$



$A_m$

↑  
has  
charged  
states

$A'_m$

↑  
No charged  
massive state



CAUTION  
DO NOT TOUCH  
ELECTRICAL  
EQUIPMENT  
OR YOU MAY  
BE SHOCKED  
OR BURNED

$$V(I) \times U(I)$$



$A_m$

has  
charged  
states

$A'_m$

No charged  
massive states



Toroidal compactification in CFT

$$X \sim X + 2\pi R$$

Toroidal compactification in CFT

$$X \sim X + 2\pi R, \quad (G_{\theta\theta} = 1)$$

$$\frac{1}{2\pi\alpha'} \int_{M_2} d^2z \partial X \bar{\partial} X$$



# Toroidal compactification in CFT

$$X \sim \underline{X + 2\pi R}, \quad (G_{\text{orb}} = 1)$$

$$\frac{1}{2\pi\alpha'} \int_{M_2} d^2z \partial X \bar{\partial} X$$

Toroidal compactification in CFT

$$X \sim \underline{X + 2\pi R} \quad (G_{\text{orb}} = 1)$$

$$\frac{1}{2\pi\alpha'} \int_{M_2} d^2z \partial X \bar{\partial} X \rightarrow \text{some FOM, OPE's}$$

Tub

$$V_{\text{un}} + \frac{1}{R} A_{\text{un}} \quad V_{\text{un}} + \frac{1}{R} B_{\text{un}}$$

$M_2$

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Periodicity has 2 effects:

(1)

Periodicity has 2 effects:

(a) RP

Periodicity has effects:

(4)  $e^{2\pi i P} = 1 \Rightarrow P = \frac{1}{2015}$

Periodicity has 2 effects:

$$(4) \quad e^{2\pi i R P} = 1 \quad \Rightarrow \quad P = \frac{1}{R/5} \quad (\text{45 in QFT})$$

Periodicity has 2 effects:

(a)  $R_p = 1 \rightarrow P = \frac{1}{R^{1/5}} \text{ (45 in QFT)}$

$(\sigma + 2\pi) = X(i)$

Periodicity has 2 effects:

$$e^{iR\omega} = 1 \rightarrow P = \frac{1}{R} \quad (\text{as in QFT})$$

$$X(\sigma + 2\pi) = X(\sigma) +$$



- Periodicity has 2 effects:

$$(a) \quad e^{2\pi i R P} = 1 \quad \rightarrow \quad P = \frac{h}{R} \quad (\text{as in QFT})$$

$$(b) \quad \chi(\sigma + 2\pi) = \chi(\sigma) + 2\pi R w \quad \leftarrow \text{is a winding number.}$$

- Periodicity has 2 effects:

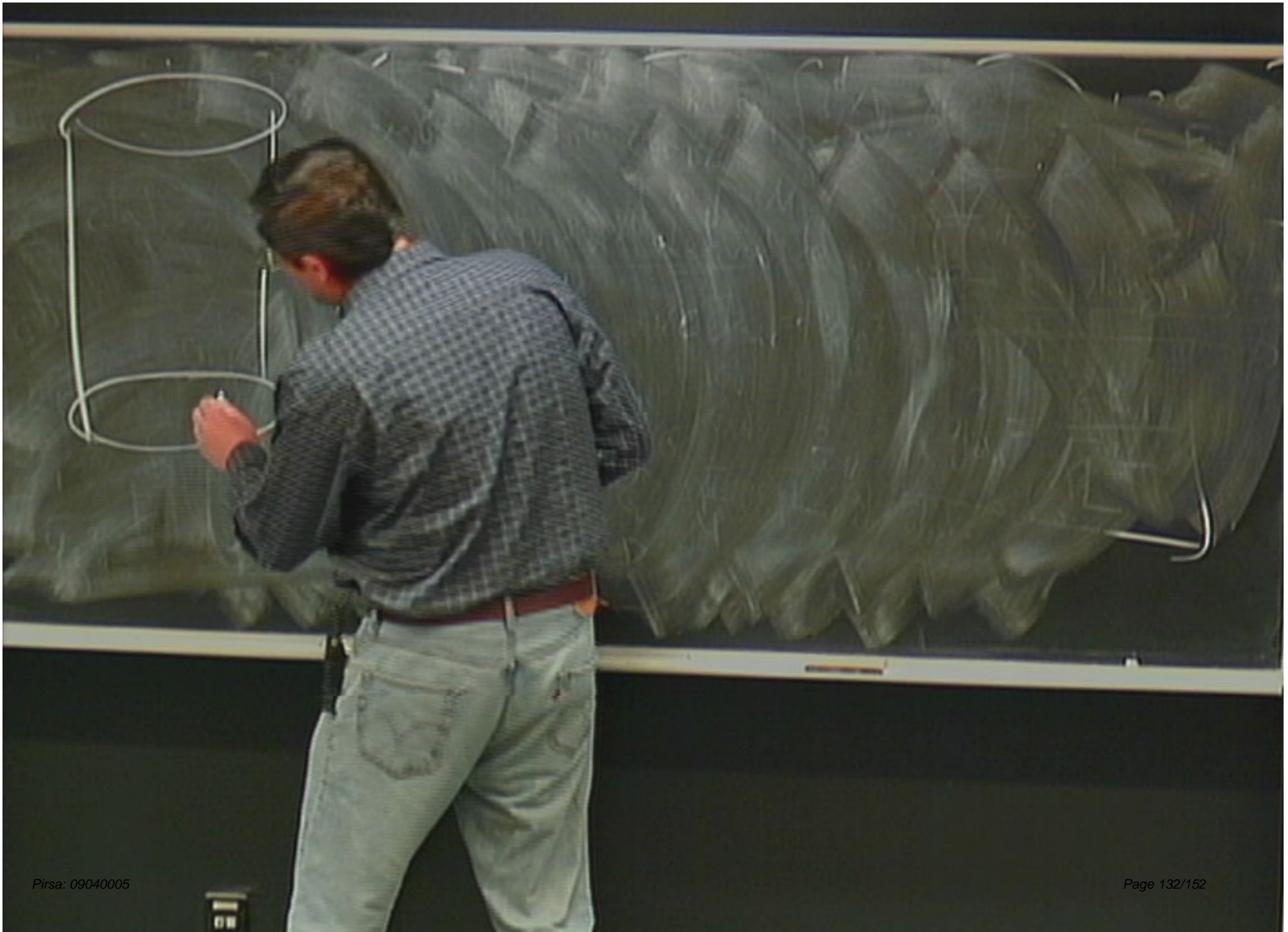
(a)  $e^{2\pi i R P} = 1 \rightarrow P = \frac{h}{R}$  (as in QFT)

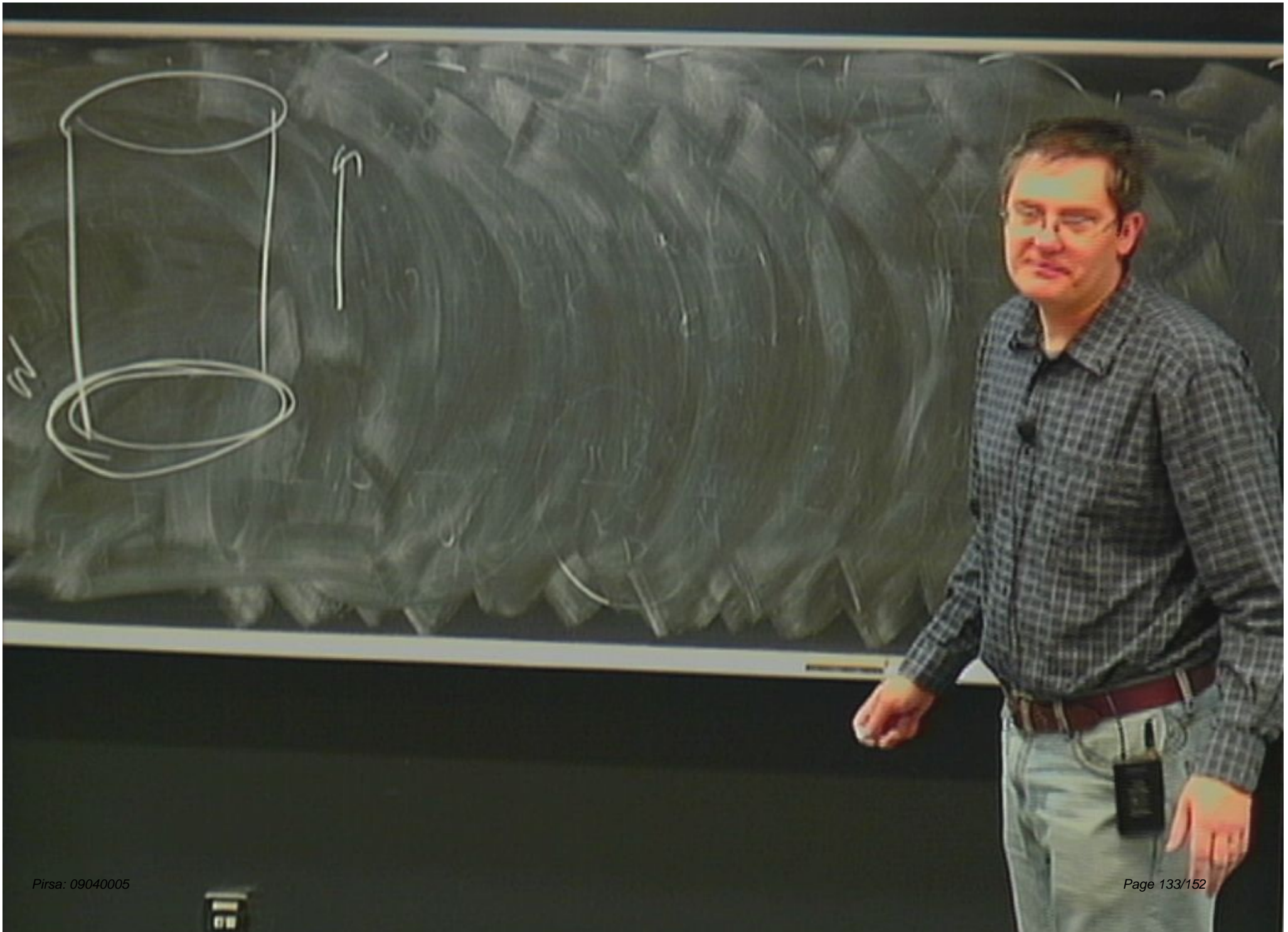
(b)  $\chi(\sigma + 2\pi) = \chi(\sigma) + 2\pi R W$  is a winding number.

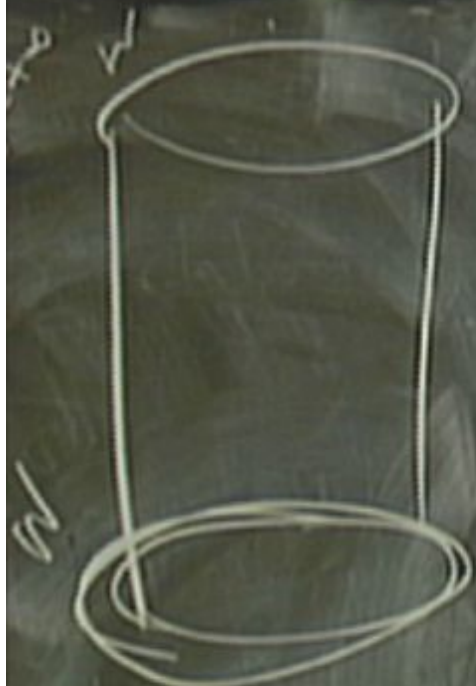
- Periodicity has 2 effects:

(a)  $e^{2\pi i R P} = 1 \rightarrow P = \frac{1}{R}$  (as in QFT)

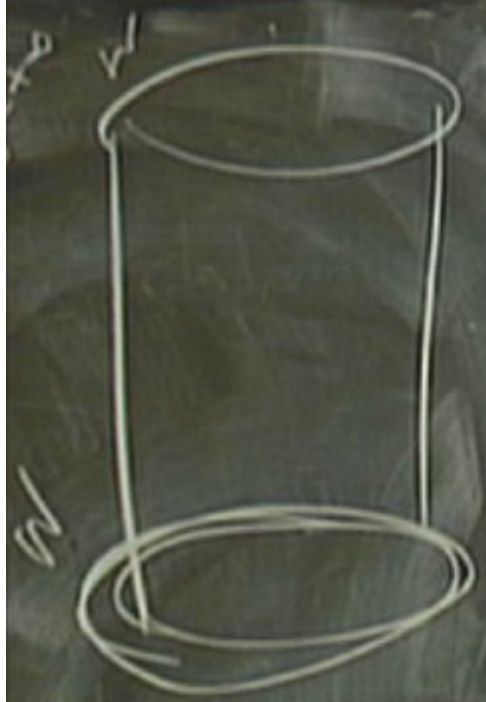
(b)  $\chi(\sigma + 2\pi) = \chi(\sigma) + 2\pi R w$  is a winding number.





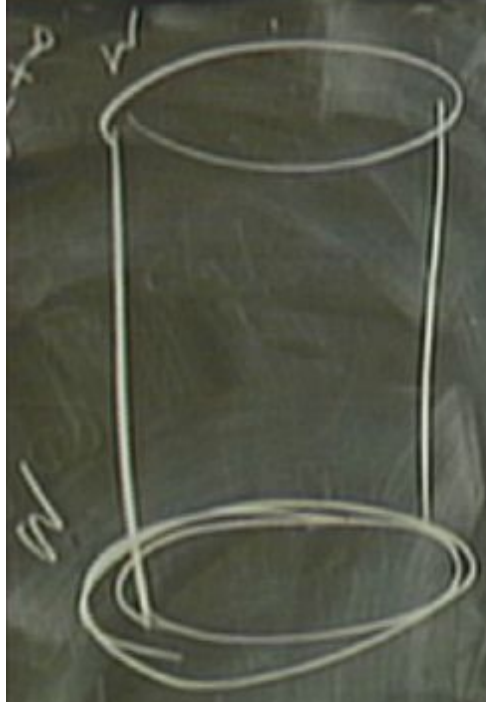


$w$  is a topological (conserved quantity)

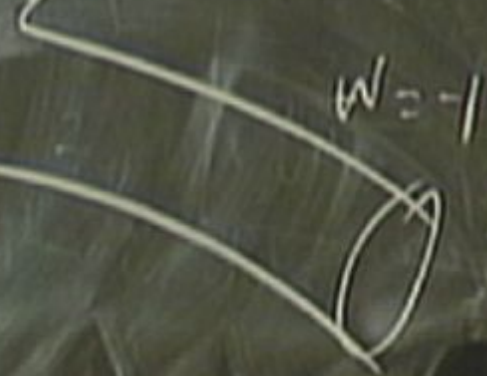
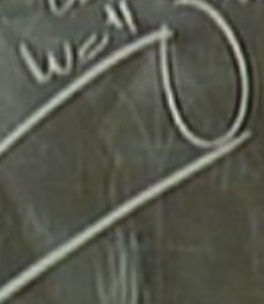


$w$  is a topological (conserved quantity)

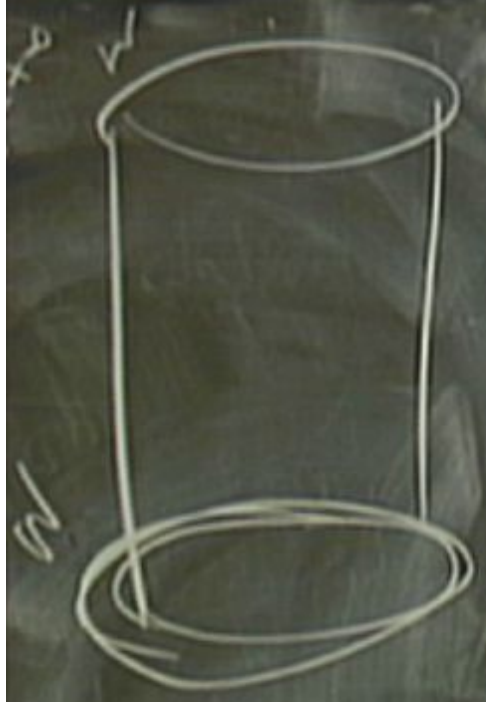




$w$  is a topological (conserved quantity)







$w$  is a topological (conserved quantity)



$$\gamma_X(z) = -i \left( \frac{z-i}{z+i} \right)^{1/2} \sum_{m=-\infty}^{+\infty} \frac{d_m}{z^{n+1}}$$



$$X(z) = -i \left(\frac{z^{-1}}{z}\right)^{1/2} \sum_{m=-\infty}^{+\infty} \frac{dm}{z^{m+1}}$$

$$\overline{X}(z) = -i \left(\frac{z^{-1}}{z}\right)^{1/2} \sum_{m=-\infty}^{+\infty} \frac{dm}{z^{m+1}}$$



$$\chi(z) = -i \left(\frac{z'}{z}\right)^{1/2} \sum_{m=-\infty}^{+\infty} \frac{dm}{z^{m+1}}$$

$$\bar{\chi}(z) = -i \left(\frac{z'}{z}\right)^{1/2} \sum_{m=-\infty}^{+\infty} \frac{dm}{z^{m+1}}$$

P =  
 world-sheet  
 moment

$$\chi(z) = -i \left(\frac{z'}{z}\right)^{1/2} \sum_{m=-\infty}^{+\infty} \frac{dm}{z^{m+1}}$$

$$\bar{\chi}(z) = -i \left(\frac{z'}{z}\right)^{1/2} \sum_{m=-\infty}^{+\infty} \frac{dm}{z^{m+1}}$$

$$P = \frac{1}{2\pi i} \oint \left( dz \chi - d\bar{z} \bar{\chi} \right)$$

world-sheet  
moment

$$\mathcal{X}(z) = -i \left(\frac{\alpha'}{2}\right)^{1/2} \sum_{m=-\infty}^{+\infty} \frac{\alpha_m}{z^{m+1}}$$

$$\bar{\mathcal{X}}(z) = -i \left(\frac{\alpha'}{2}\right)^{1/2} \sum_{m=-\infty}^{+\infty} \frac{\bar{\alpha}_m}{z^{m+1}}$$

$$P = \frac{1}{2\pi\alpha'} \oint \left( dz \mathcal{X} - d\bar{z} \bar{\mathcal{X}} \right) = \left(\frac{\alpha'}{2}\right)^{-1/2} (\alpha_0 + \bar{\alpha}_0)$$

world-sheet momentum

$$\partial X(z) = -i \left(\frac{\alpha'}{2}\right)^{1/2} \sum_{m=-\infty}^{+\infty} \frac{\alpha_m}{z^{m+1}}$$

$$\bar{\partial} X(\bar{z}) = -i \left(\frac{\alpha'}{2}\right)^{1/2} \sum_{m=-\infty}^{+\infty} \frac{\bar{\alpha}_m}{\bar{z}^{m+1}}$$

$$P = \frac{1}{2\pi\alpha'} \oint \left( dz \partial X - d\bar{z} \bar{\partial} X \right) = \left(\frac{\alpha'}{2}\right)^{-1/2} (\alpha_0 + \bar{\alpha}_0)$$

↑ world-sheet momentum

X



$$\partial X(z) = -i \left(\frac{z'}{z}\right)^{1/2} \sum_{m=-\infty}^{+\infty} \frac{d_m}{z^{m+1}}$$

$$\bar{\partial} X(z) = -i \left(\frac{z'}{z}\right)^{1/2} \sum_{m=-\infty}^{+\infty} \frac{d_m}{z^{m+1}}$$

$$P = \frac{1}{2\pi i} \oint_{\text{wald-streit}} \left( d\bar{z} \partial X - dz \bar{\partial} X \right) = \left( d\bar{z}^{1/2} + dz^{1/2} \right)$$

wald-streit  
wandel

$$X(z\pi) - X(0) = 2\pi R_W = \oint$$



$$\partial X(z) = -i \left(\frac{\alpha'}{2}\right)^{1/2} \sum_{m=-\infty}^{+\infty} \frac{\alpha_m}{z^{m+1}}$$

$$\bar{\partial} X(\bar{z}) = -i \left(\frac{\alpha'}{2}\right)^{1/2} \sum_{m=-\infty}^{+\infty} \frac{\bar{\alpha}_m}{\bar{z}^{m+1}}$$

$$P = \frac{1}{2\pi\alpha'} \oint_{\text{unit circle}} (dz \partial X - d\bar{z} \bar{\partial} X) = \left(\frac{\alpha'}{2}\right)^{1/2} (\alpha_0 + \bar{\alpha}_0)$$

↑  
Wahlst-Start  
Wahlst

$$X(z_2) - X(z_1) = 2\pi\alpha' W = \oint [dz \partial X + d\bar{z} \bar{\partial} X]$$

$$X(z\pi) - X(0) = 2\pi R_W = \oint \left[ \frac{1}{z} \partial X + \frac{1}{z} \bar{\partial} X \right]$$

$$= 2\pi \left( \frac{L''}{2} \right)^{1/2} (\alpha_0 - \bar{\alpha}_0)$$

$$X(2\pi) - X(0) = 2\pi R W = \oint [R \partial X + \rho \sqrt{\sigma} \bar{\partial} X]$$

$$= 2\pi \left( \frac{L''}{2} \right)^{1/2} (\alpha_0 - \bar{\alpha}_0)$$

wahl-sicht  
normal

$$X(z\pi) = X(0) + \underbrace{z\pi R_w}_{\text{Wahl-sicht}} \left\{ \int_{\alpha_0}^{\alpha_1} \partial x + \int_{\alpha_1}^{\alpha_2} \bar{\partial} x \right\}$$

$$= z\pi \left( \frac{\alpha_1}{2} \right)^{1/2} (\alpha_0 - \alpha_2)$$



CAUTION  
DO NOT TOUCH  
EQUIPMENT  
UNLESS  
AUTHORIZED

$$\frac{1}{R} = \frac{1}{2\pi d} \oint \left( \frac{\partial \phi}{\partial z} \partial x - \frac{\partial \phi}{\partial x} \partial z \right) = \frac{1}{2\pi d} (\alpha_0 + \alpha_0')$$

↑ world-sheet  
manifold

$$X(z, \bar{z}) - X(0) = 2\pi R W = \oint \left[ \frac{1}{2} \partial z \partial x + \frac{1}{2} \bar{\partial} z \bar{\partial} x \right]$$

$$= 2\pi \left( \frac{L'}{2} \right)^{1/2} (\alpha_0 - \alpha_0')$$

$$\frac{1}{2} = \frac{1}{2\pi d} \oint (dZ \partial X - d\bar{Z} \partial \bar{X}) = (d\alpha')^2 (d_0 + \tilde{d}_0)$$

world-sheet  
manifold

$$X(2\pi) - X(0) = 2\pi R W = \oint [dZ \partial X + d\bar{Z} \bar{\partial} X]$$

$$= 2\pi \left( \frac{L'}{2} \right)^{1/2} (d_0 - \tilde{d}_0)$$

$$P_L = \left( \frac{2}{\alpha'} \right)^{1/2} d_0 =$$

$$\frac{1}{R} = \frac{1}{200d} \oint (d_1 \partial_x - d_2 \partial_x) = (d_1' - (d_0 + d_0'))$$

world-sheet  
manifold

$$X(z\pi) - X(0) = 2\pi R W \oint [d_1 \partial_x + d_2 \bar{\partial}_x]$$

$$= 2\pi \left(\frac{L'}{2}\right)^{1/2} (d_0 - d_0')$$

$$P_L = \left(\frac{2}{\alpha'}\right)^{1/2} d_0 = \frac{1}{R} + \frac{wR}{\alpha'} ; P_R = \left(\frac{2}{\alpha'}\right)^{1/2} d_0'$$



$$\frac{1}{R} = \frac{1}{2\pi d'} \oint \left( \frac{\partial^2 \phi}{\partial x^2} - \frac{\partial^2 \phi}{\partial y^2} \right) (d_0 + d_0')$$

↑  
 work/d-slat  
 mass

$$X(z\pi) - X(0) = \underbrace{2\pi RW}_{\text{MKU2}} \oint \left[ \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} \right]$$

$$= 2\pi \left( \frac{L''}{2} \right)^{1/2} (d_0 - d_0')$$

$$P_L = \left( \frac{2}{d_1'} \right)^{1/2} d_0 = \frac{n}{R} + \frac{WR}{d_1'} ; P_R = \left( \frac{2}{L} \right)^{1/2} d_0 = \frac{n}{R} - \frac{WR}{d_1'}$$