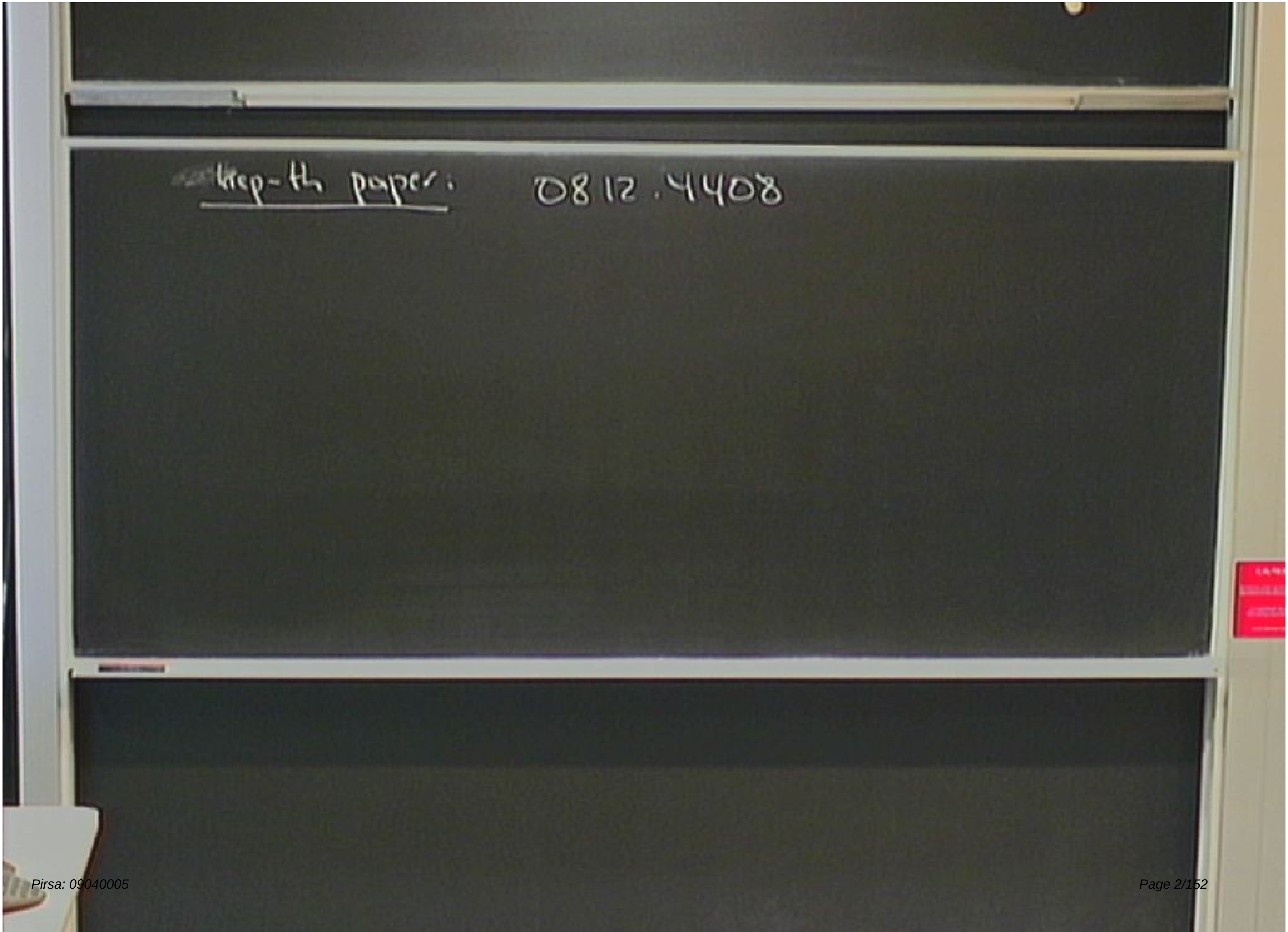
Title: Introduction to the Bosonic String

Date: Apr 17, 2009 10:00 AM

URL: http://pirsa.org/09040005

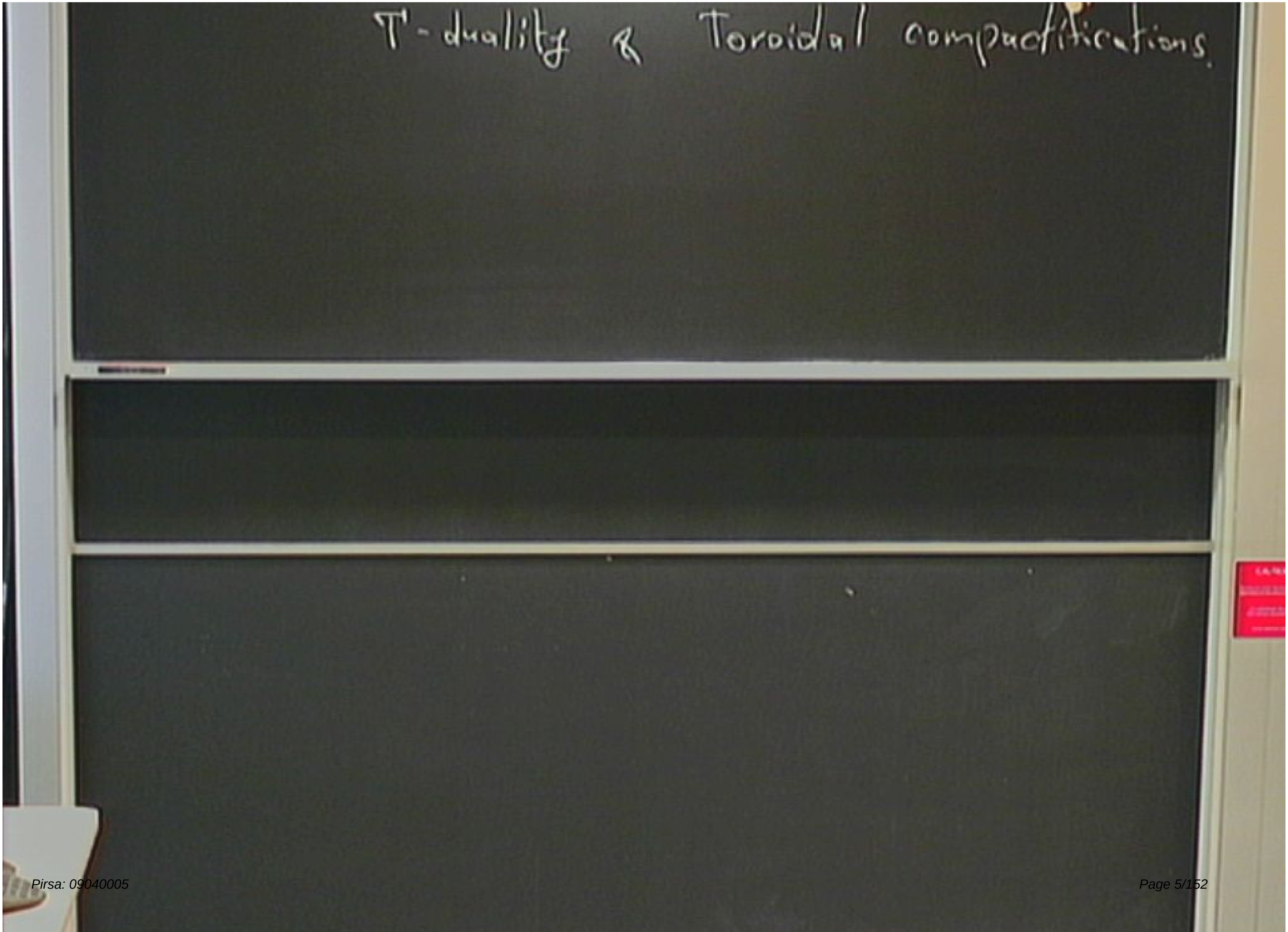
Abstract: This course provides a thorough introduction to the bosonic string based on the Polyakov path integral and conformal field theory. We introduce central ideas of string theory, the tools of conformal field theory, the Polyakov path integral, and the covariant quantization of the string. We discuss string interactions and cover the tree-level and one loop amplitudes. More advanced topics such as T-duality and D-branes will be taught as part of the course. The course is geared for M.Sc. and Ph.D. students enrolled in Collaborative Ph.D. Program in Theoretical Physics. Required previous course work: Quantum Field Theory (AM516 or equivalent). The course evaluation will be based on regular problem sets that will be handed in during the term. The primary text is the book: 'String theory. Vol. 1: An introduction to the bosonic string. J. Polchinski (Santa Barbara, KITP) . 1998. 402pp. Cambridge, UK: Univ. Pr. (1998) 402 p.' All interested students should contact Alex Buchel at abuchel@uwo.ca as soon as possible.


hep-th paper: 0812.4408

hep-th paper: 0812.4408

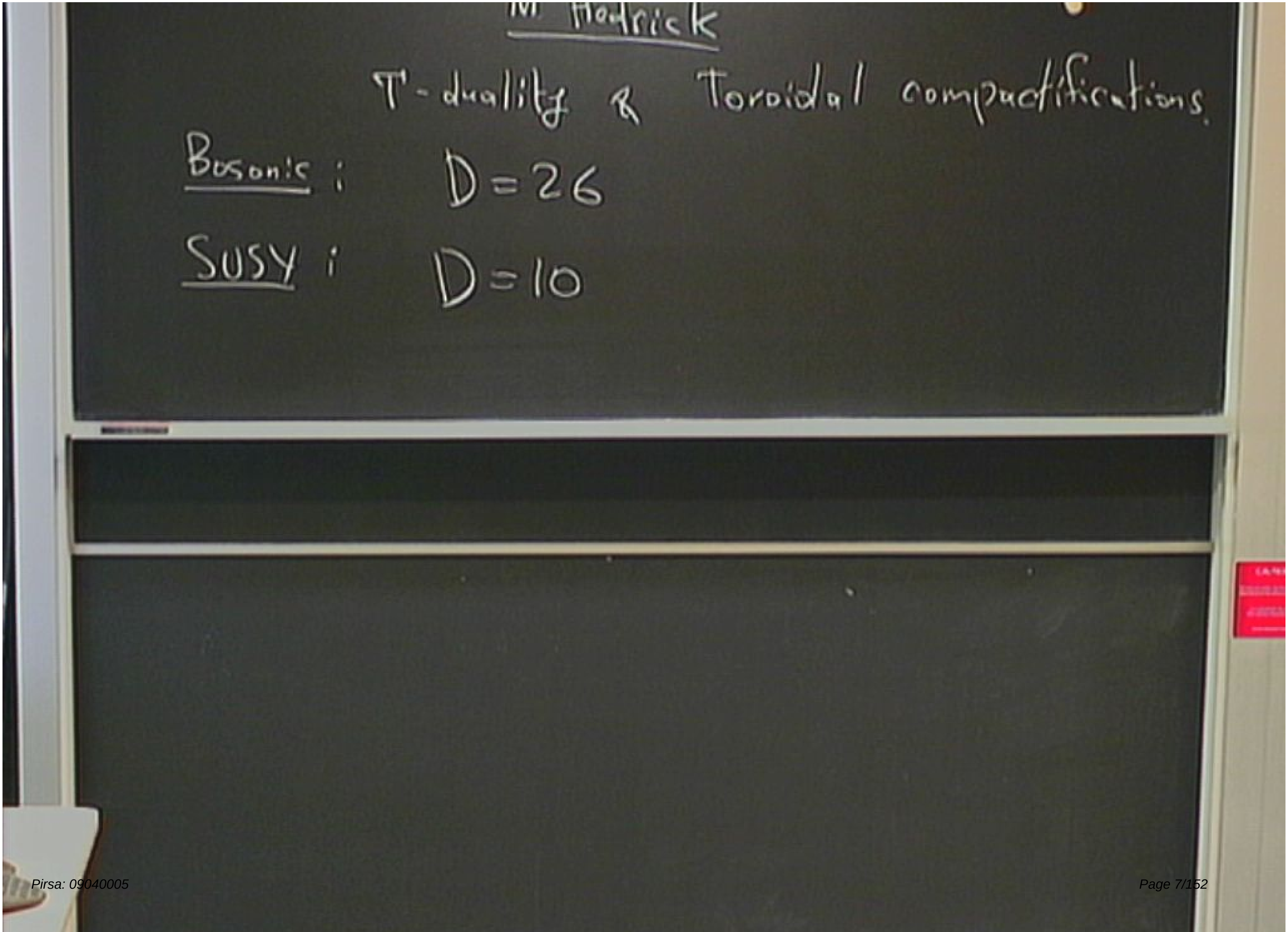
M Hedrick

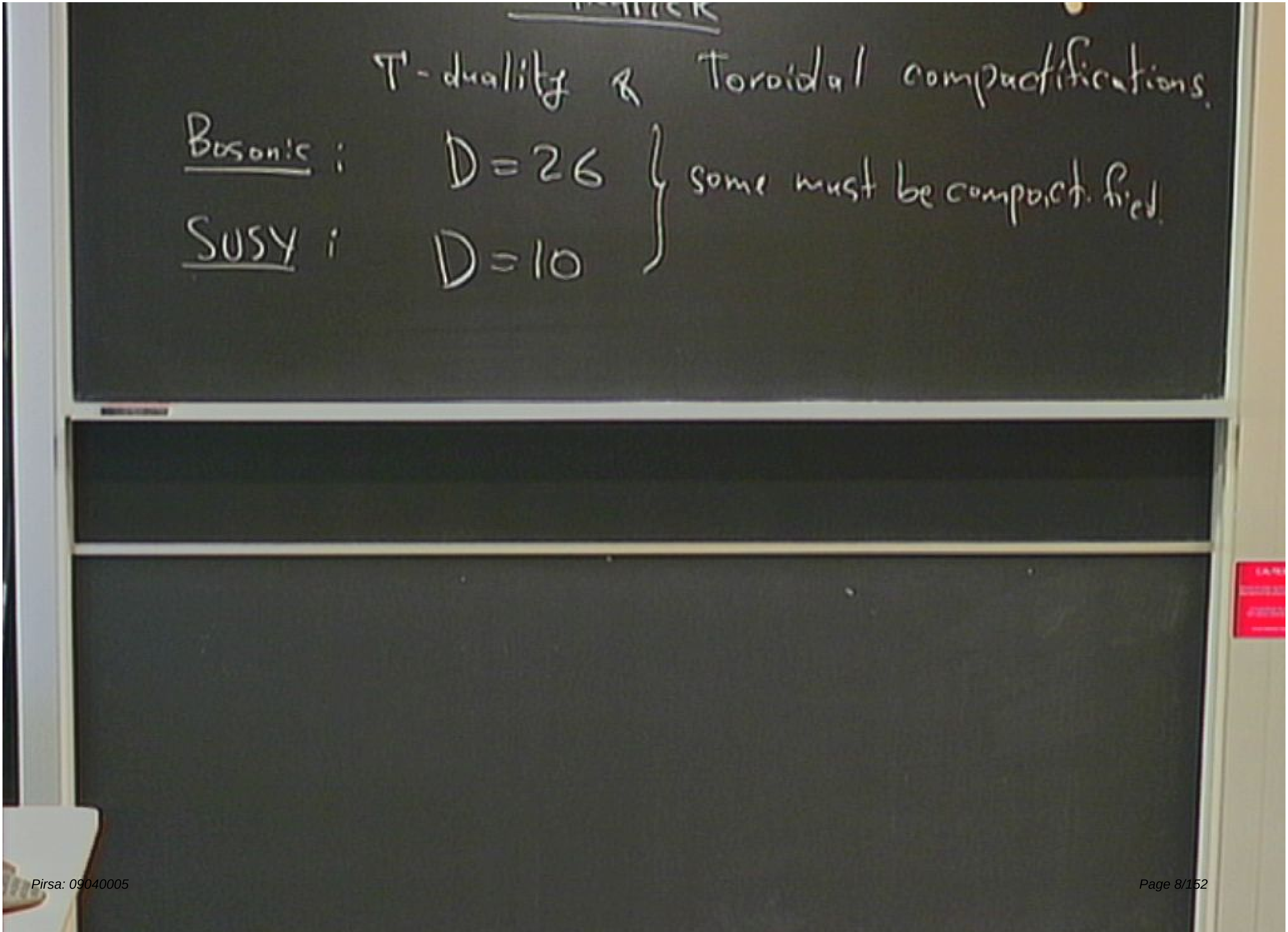
T-duality & Toroidal compactifications.

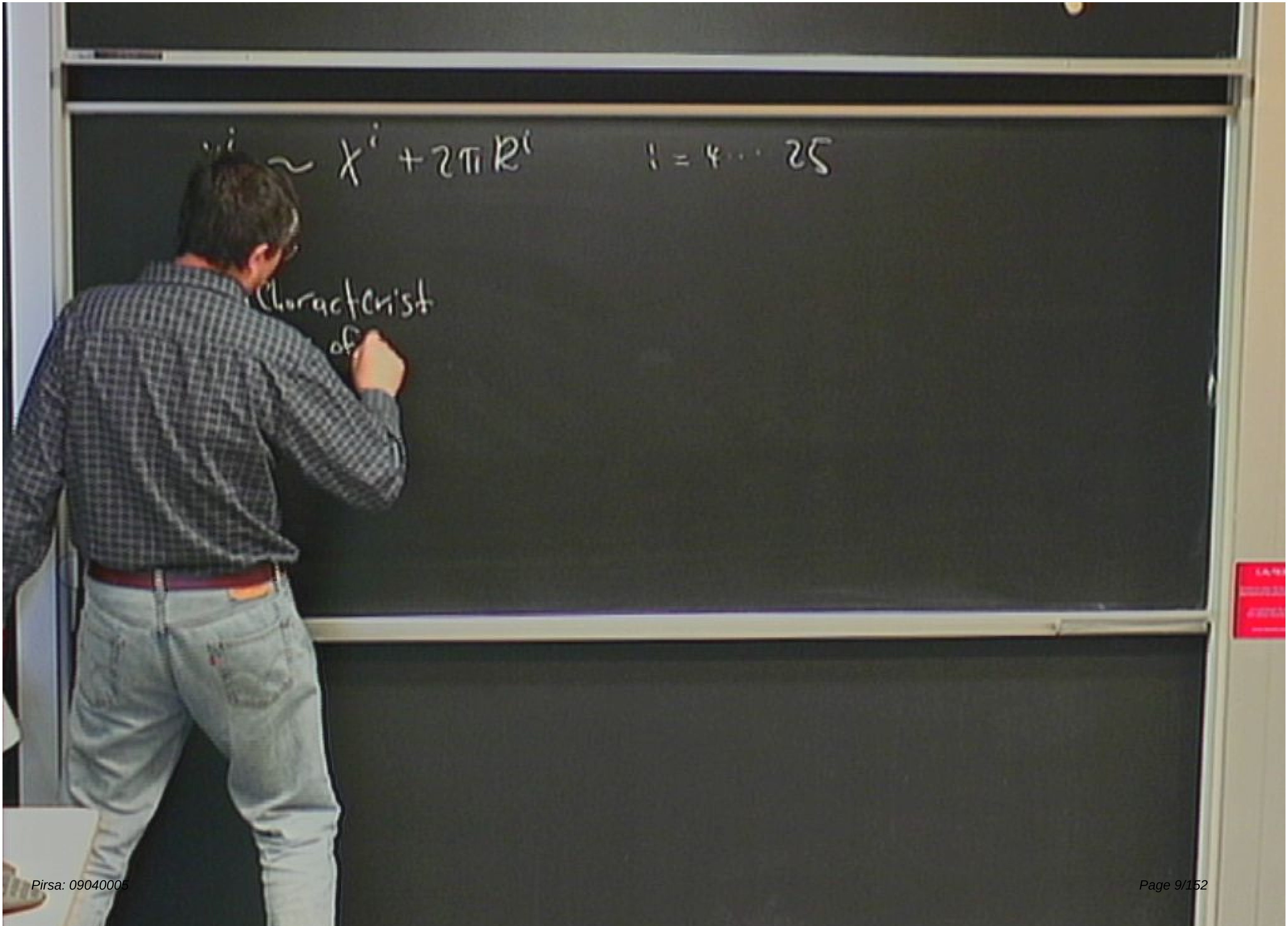

T-duality & Toroidal compactifications.

T-duality & Toroidal compactifications.

Bosonic : $D = 26$


T-duality & Toroidal compactifications
Bosonic : D=26
SUSY : D=10

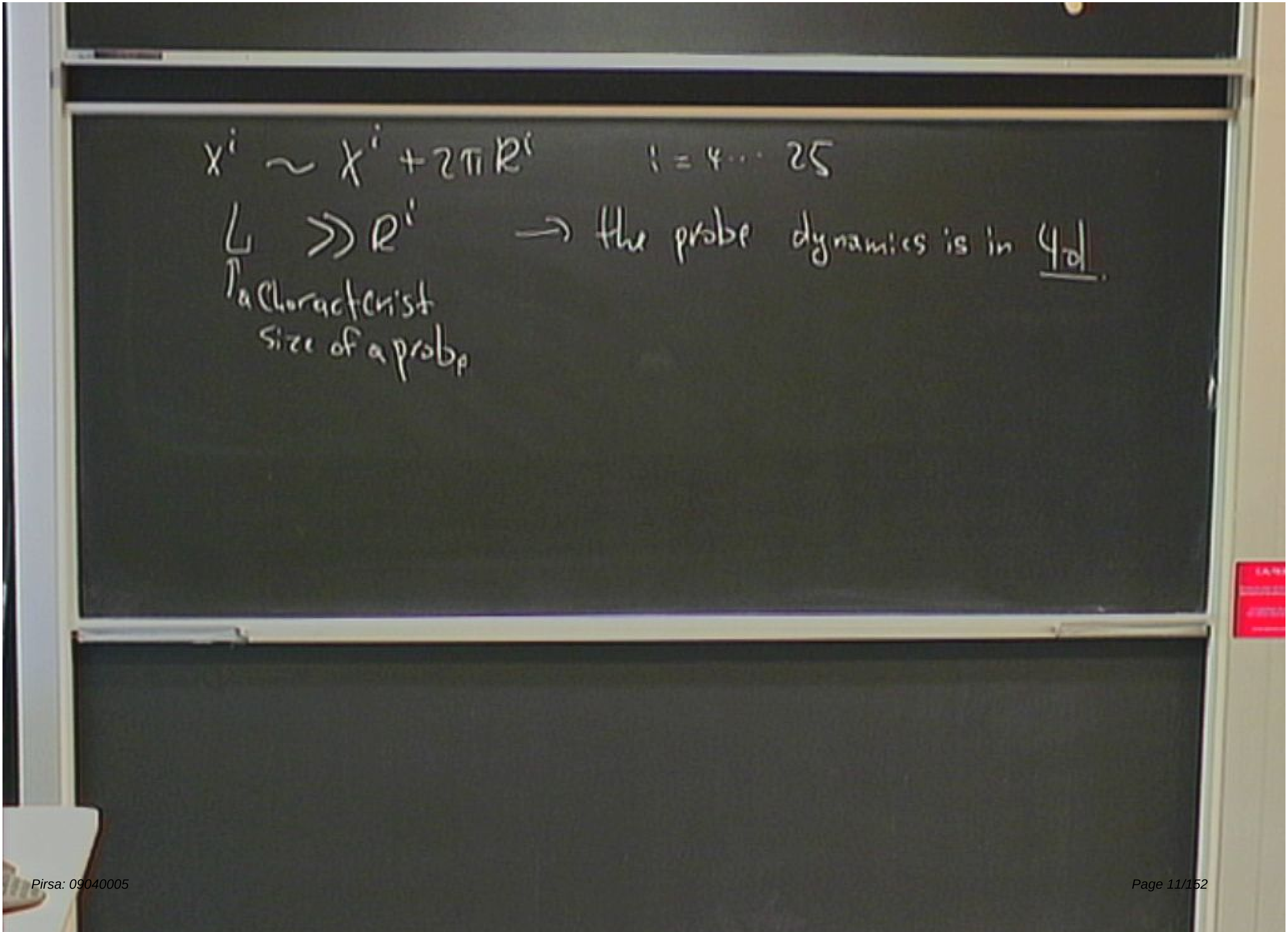

T-duality & Toroidal compactifications.
Bosonic : D=26
SUSY : D=10
some must be compactified


~ X^i + 2πR^i
i = 4 ··· 25
Characterist
of

$X^i \sim X^i + 2\pi R^i \qquad i = 4 \cdots 25$

$L \gg R^i$

↑ a characterist size of a prob$_p$


Xⁱ ~ Xⁱ + 2π Rⁱ   i = 4 ⋯ 25
L >> Rⁱ → the probe dynamics is in 4d.
↑ a characterist size of a probe

$$X^i \sim X^i + 2\pi R^i \qquad i = 4 \cdots 25$$

$L \gg R^i$ $\longrightarrow$ the probe dynamics is in <u>4d</u>.

↑ a characterist size of a probe

$X^i \sim X^i + 2\pi R^i \qquad i = 4 \cdots 25$

$L \gg R^i$ → the probe dynamics is in 4d.

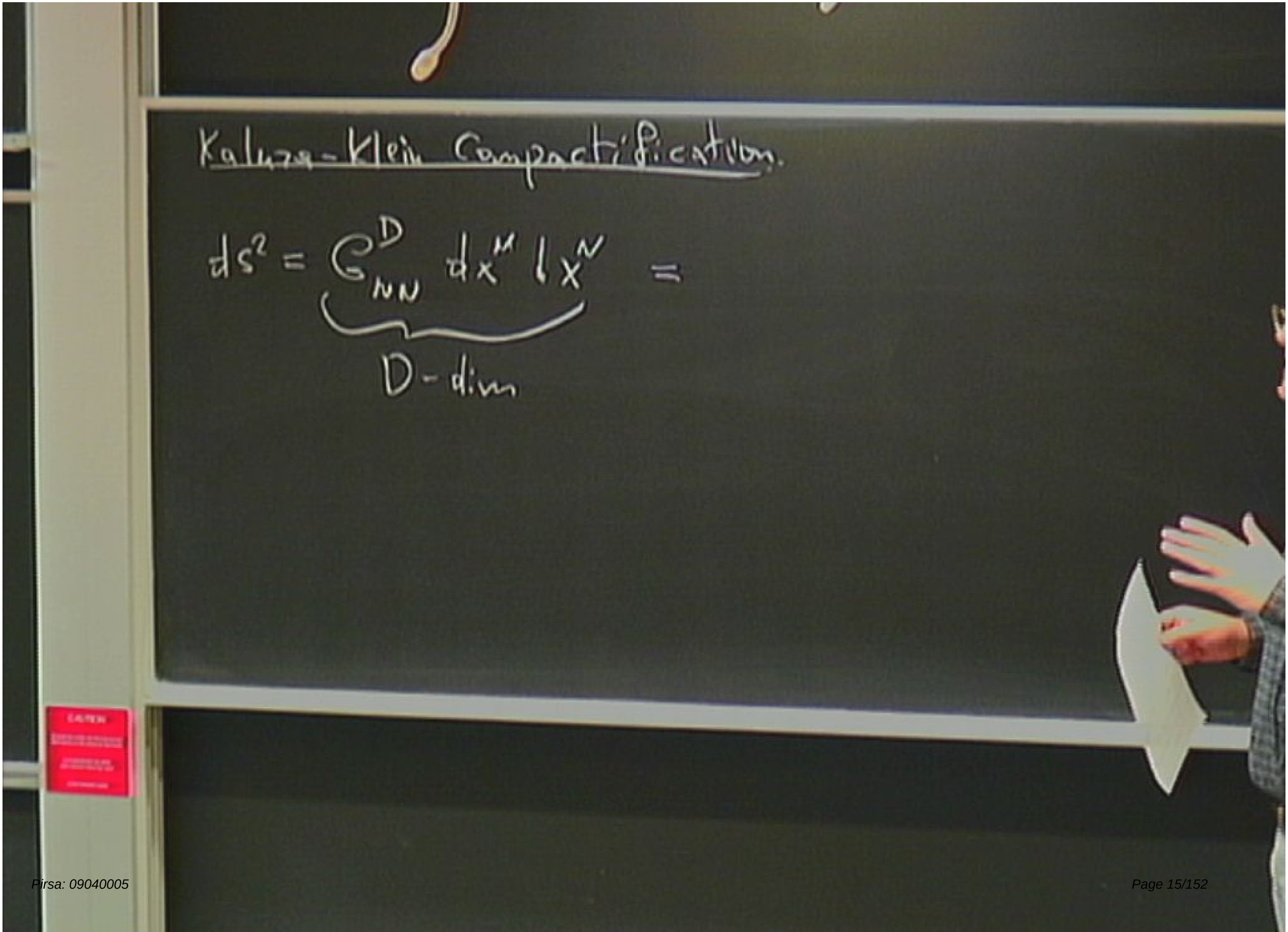
↑ a characterist size of a probe

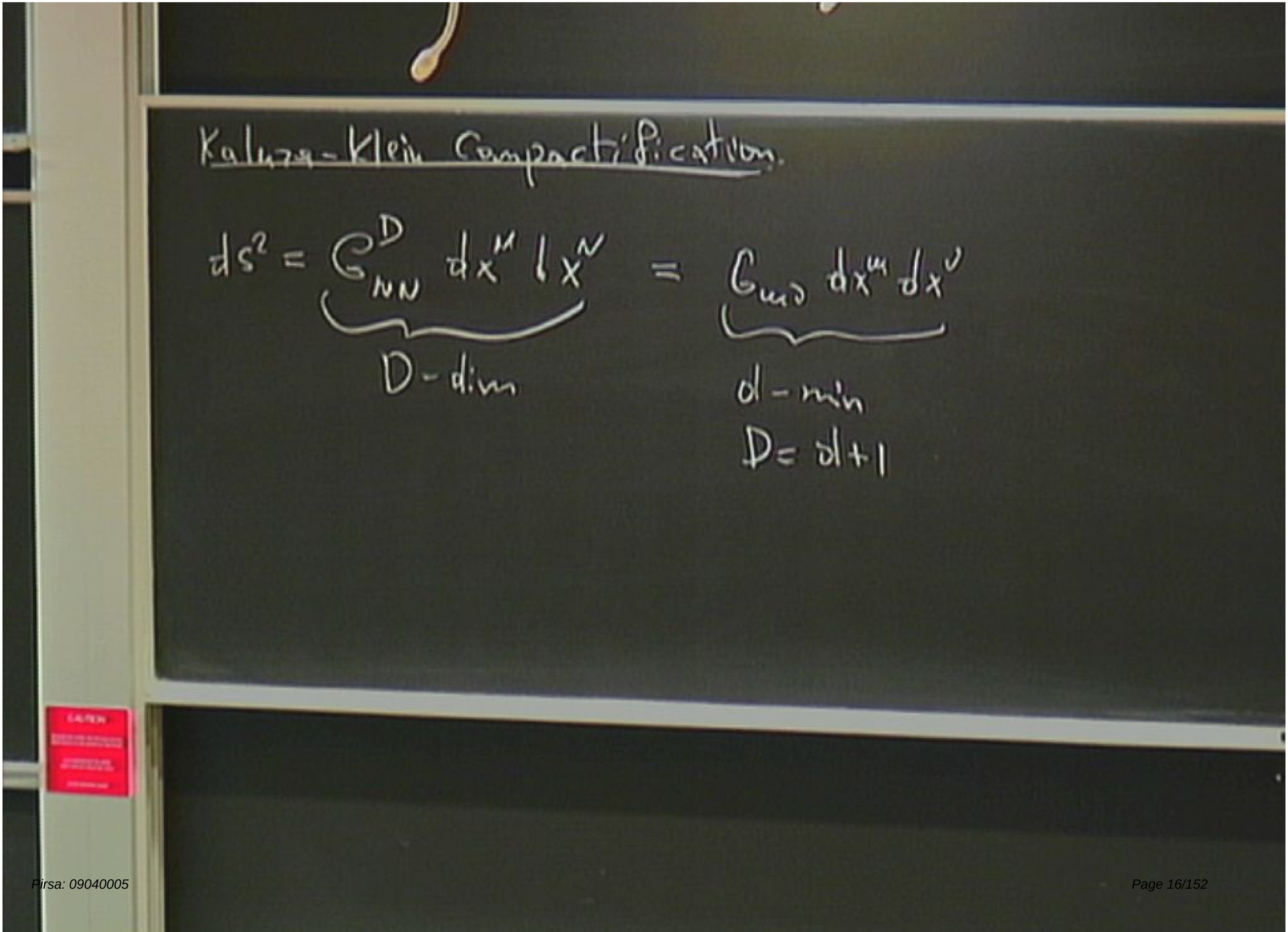
⇒ Toroidal comp in QFT

⇒ — — — in ST

Kaluza-Klein Compactification.

$$ds^2 = G^D_{MN}\, dx^M dx^N$$


Kaluza-Klein Compactification.
$ds^2 = G^D_{MN} dx^M dx^N =$
D-dim

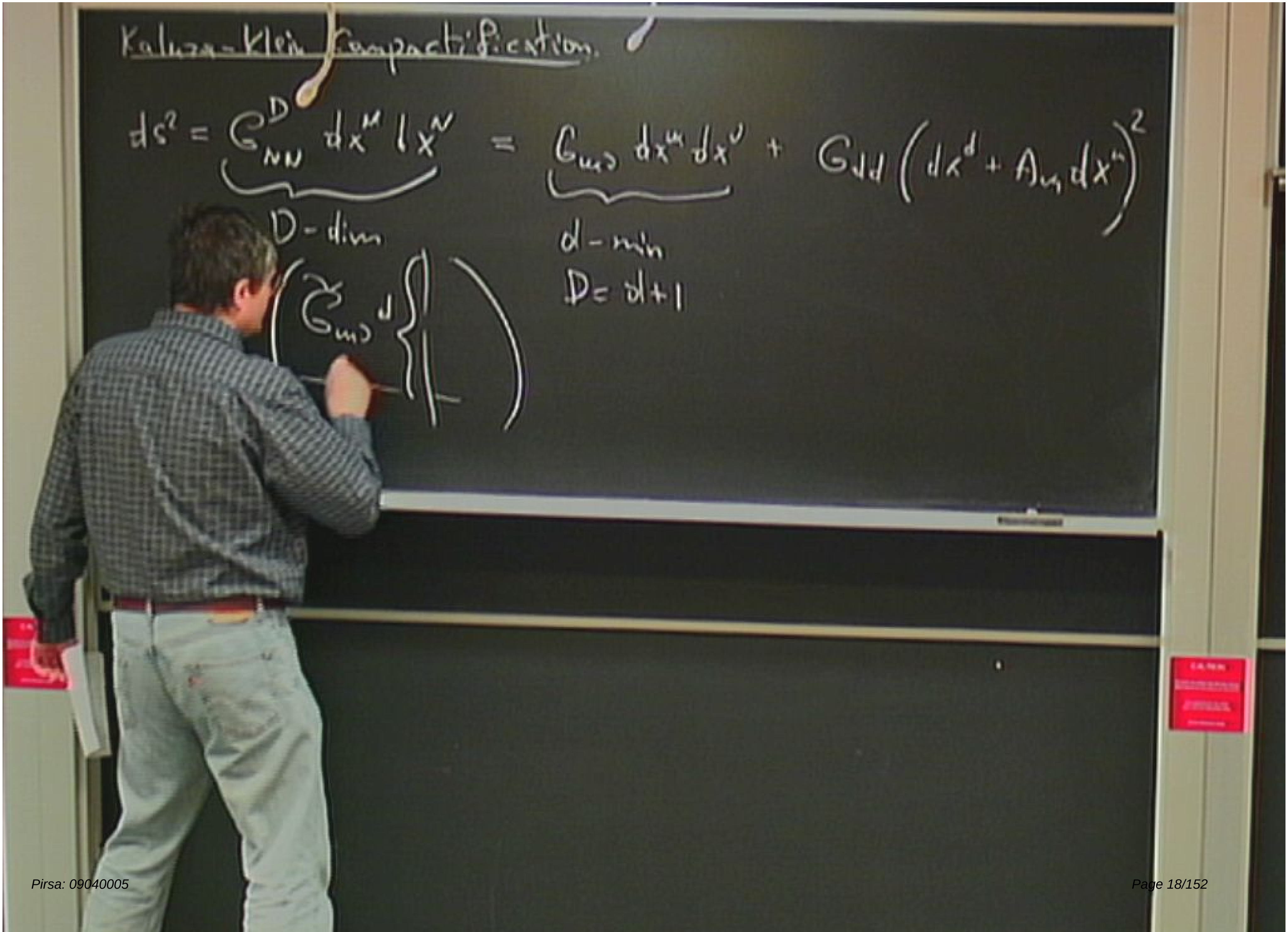


Kaluza-Klein Compactification
$ds^2 = G^D_{MN}\, dx^M dx^N = G_{\mu\nu}\, dx^\mu dx^\nu$
D-dim
d-dim
$D = d+1$

Kaluza-Klein Compactification.

$$ds^2 = \underbrace{G^D_{NN}\, dx^M\, dx^N}_{D\text{-dim}} = \underbrace{G_{\mu\nu}\, dx^\mu\, dx^\nu}_{d\text{-min}} + G_{dd}\left(dx^d + \right.$$

$D = d+1$


Kaluza-Klein Compactification.
$ds^2 = G^D_{MN}\, dx^M dx^N = G_{\mu\nu}\, dx^\mu dx^\nu + G_{dd}\left(dx^d + A_\mu dx^\mu\right)^2$
D-dim
d-dim
D = d+1
$\tilde{G}_{\mu\nu}$ d

Kaluza-Klein Compactification.

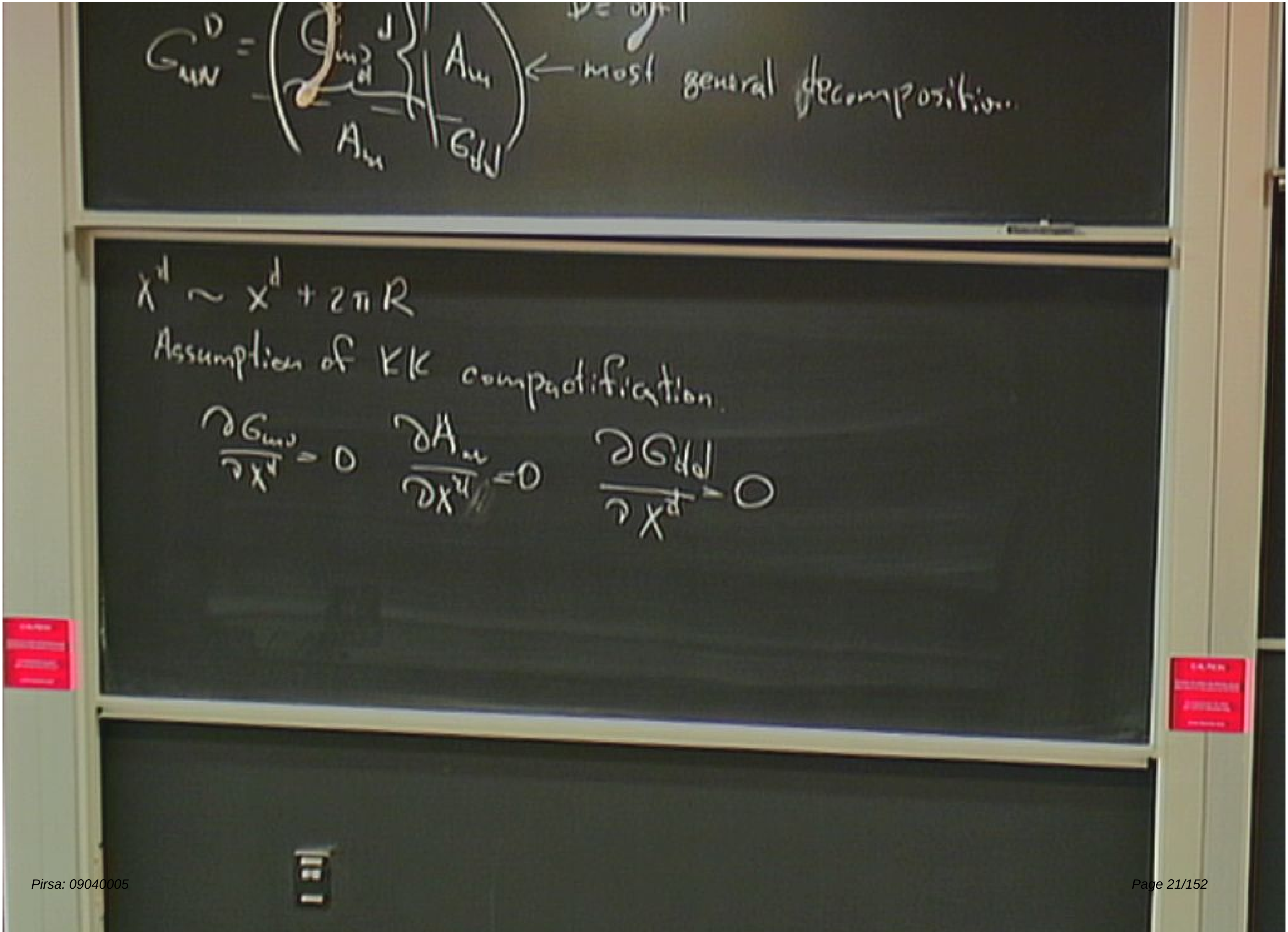
$$ds^2 = \underbrace{G^D_{MN}\, dx^M dx^N}_{D\text{-dim}} = \underbrace{G_{\mu\nu}\, dx^\mu dx^\nu}_{d\text{-dim}} + G_{dd}\left(dx^d + A_\mu dx^\mu\right)^2$$

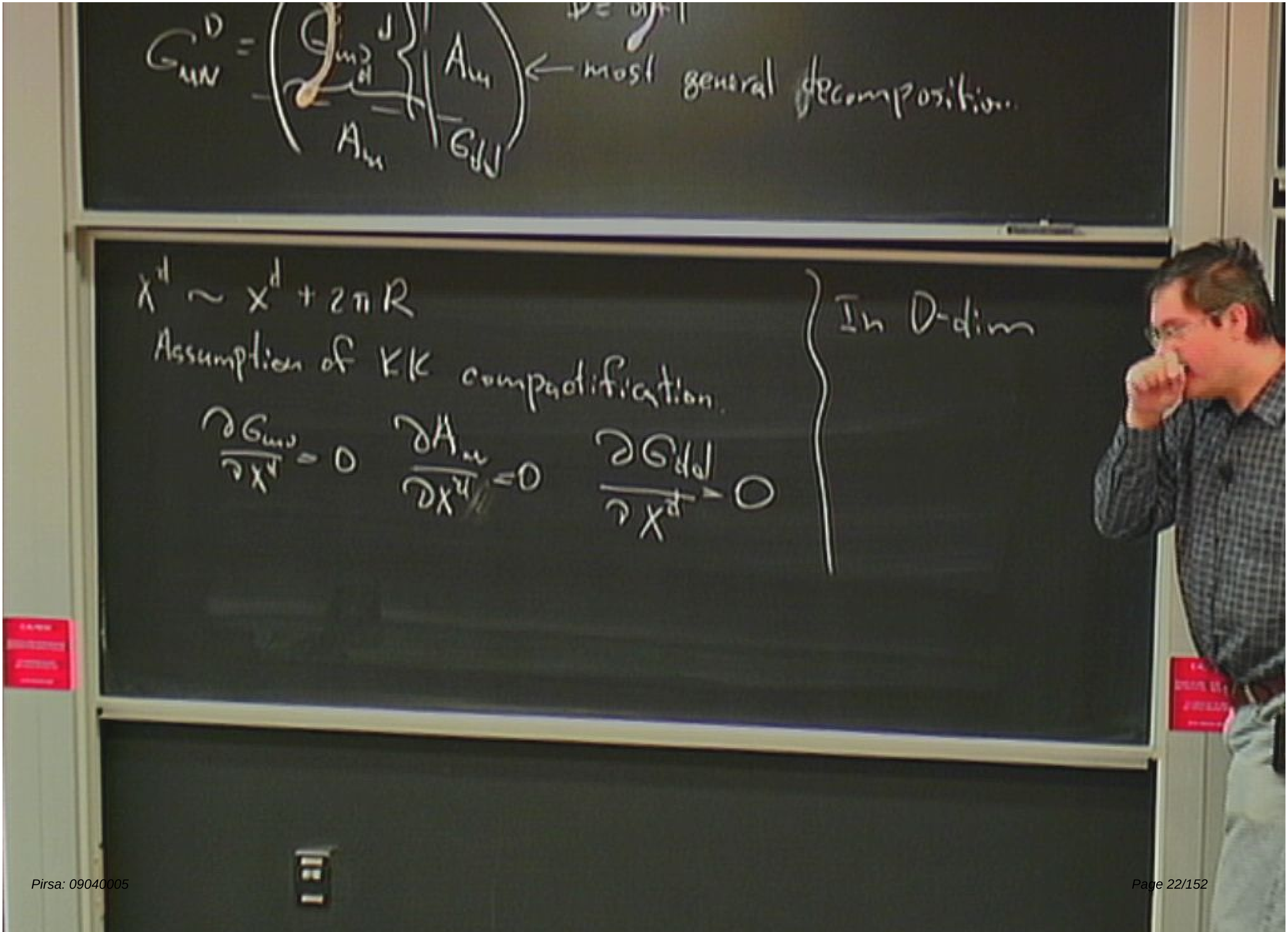
$D = d + 1$

$$G^D_{MN} = \begin{pmatrix} G_{\mu\nu} & A_\mu \\ A_\mu & G_{dd} \end{pmatrix} \leftarrow \text{most general decomposition}$$

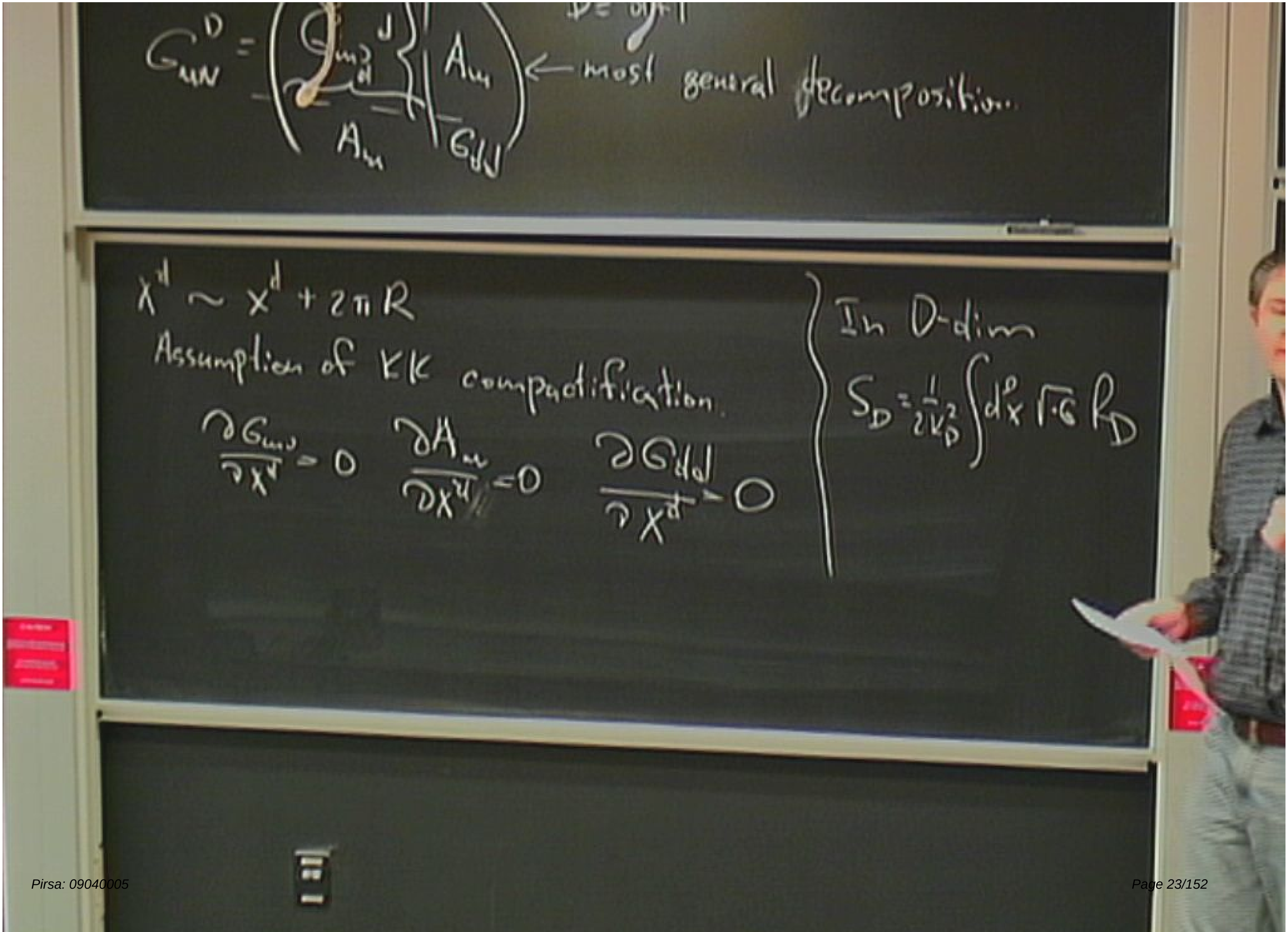
$$G^D_{MN} = \begin{pmatrix} g_{mn}^{\;d} & A_m \\ A_m & G_{dd} \end{pmatrix} \leftarrow \text{most general decomposition}$$

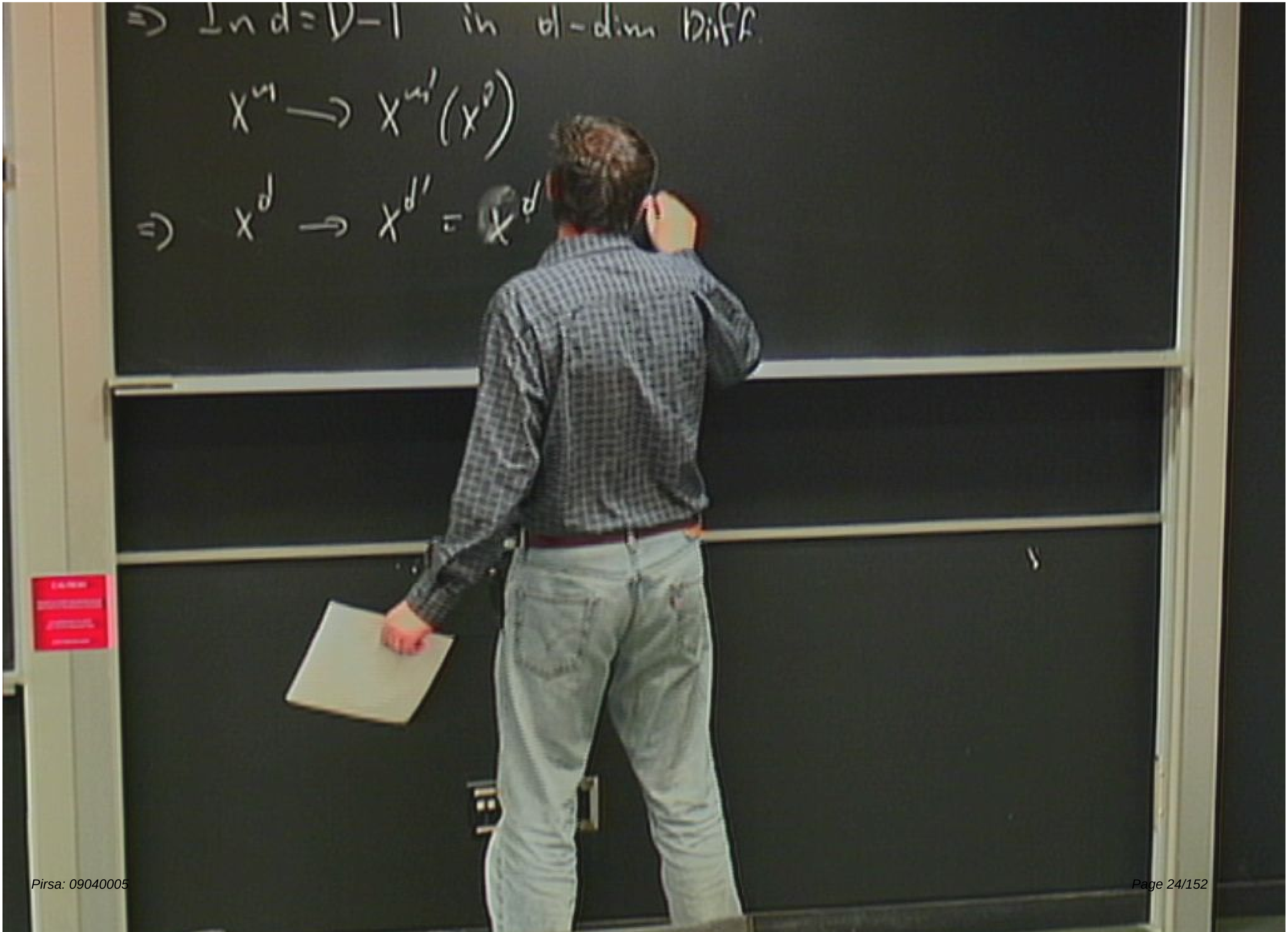
$$x^d \sim x^d + 2\pi R$$

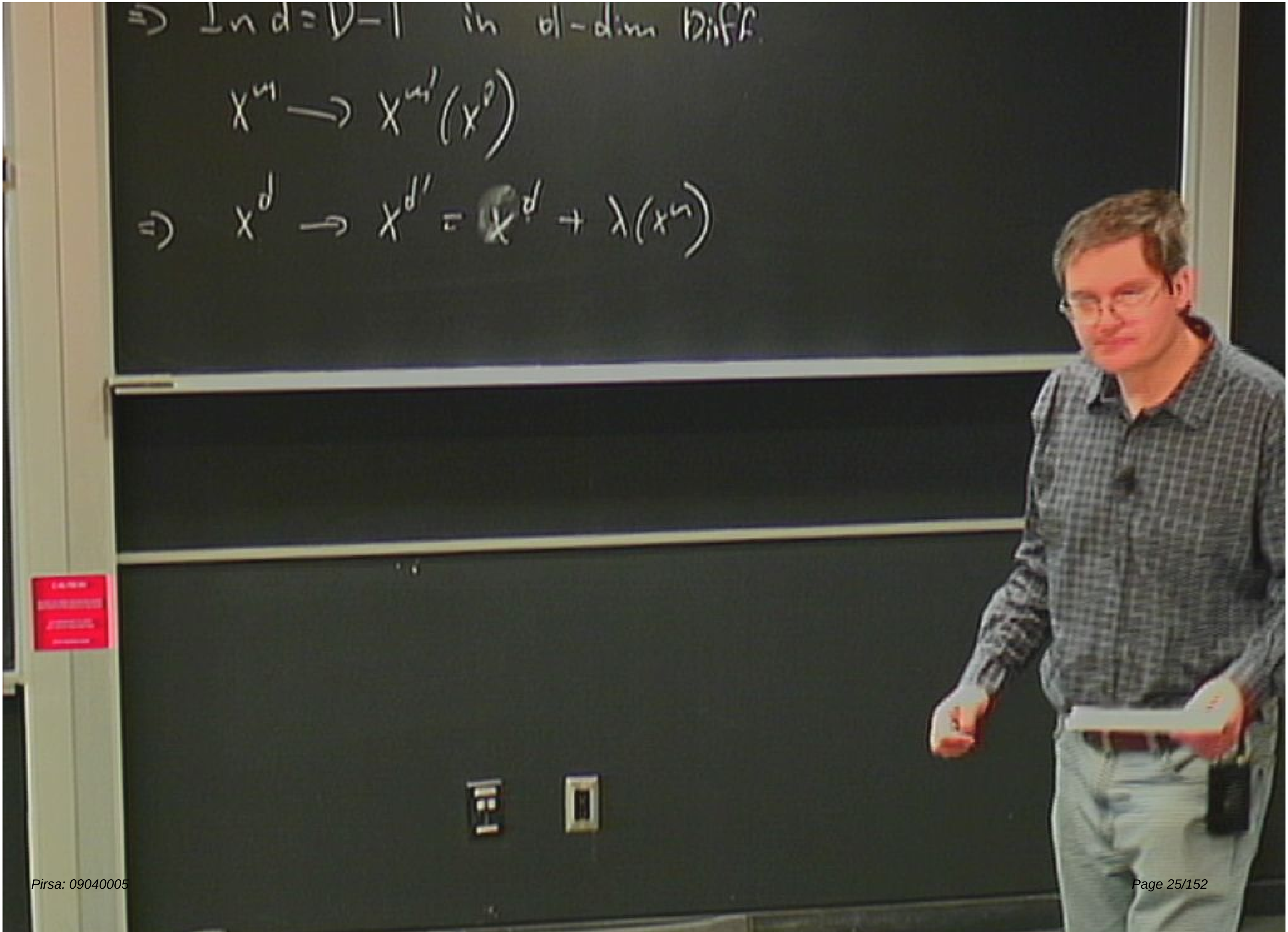

← most general decomposition
x^d ~ x^d + 2πR
Assumption of KK compactification
∂G_μν / ∂x^d = 0   ∂A_μ / ∂x^d = 0   ∂G_dd / ∂x^d = 0

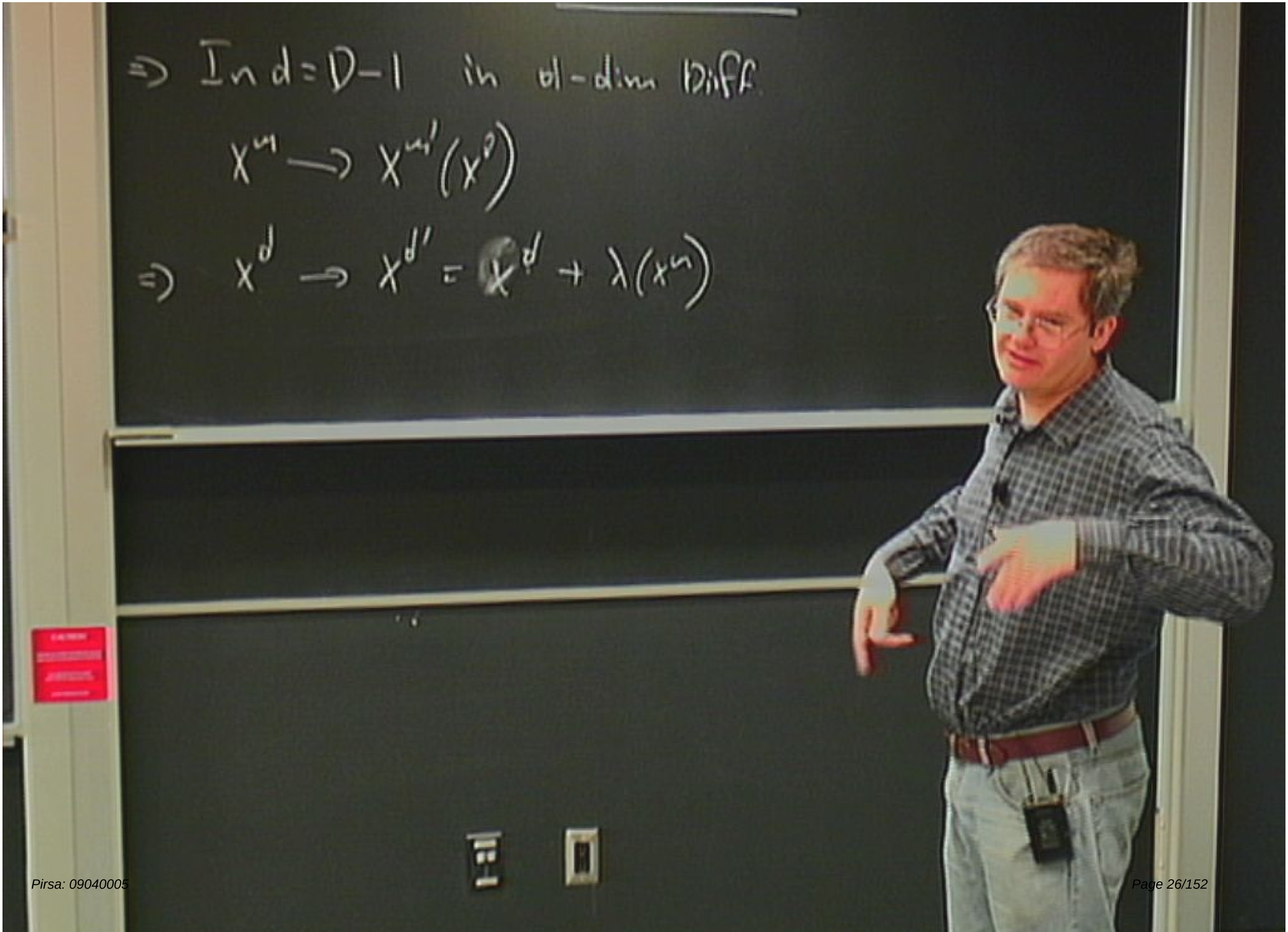


$G_{MN}^{D} = \begin{pmatrix} G_{\mu\nu} & A_\mu \\ A_\mu & G_{dd} \end{pmatrix}$ ← most general decomposition
$x^d \sim x^d + 2\pi R$
Assumption of KK compactification
$\frac{\partial G_{\mu\nu}}{\partial x^d} = 0 \quad \frac{\partial A_\mu}{\partial x^d} = 0 \quad \frac{\partial G_{dd}}{\partial x^d} = 0$
In D-dim


most general decomposition
x^d ~ x^d + 2πR
Assumption of KK compactification
In D-dim


⇒ Ind d = D−1 in d-dim Diff
x^μ → x^μ'(x^ν)
⇒ x^d → x^d' = x^d

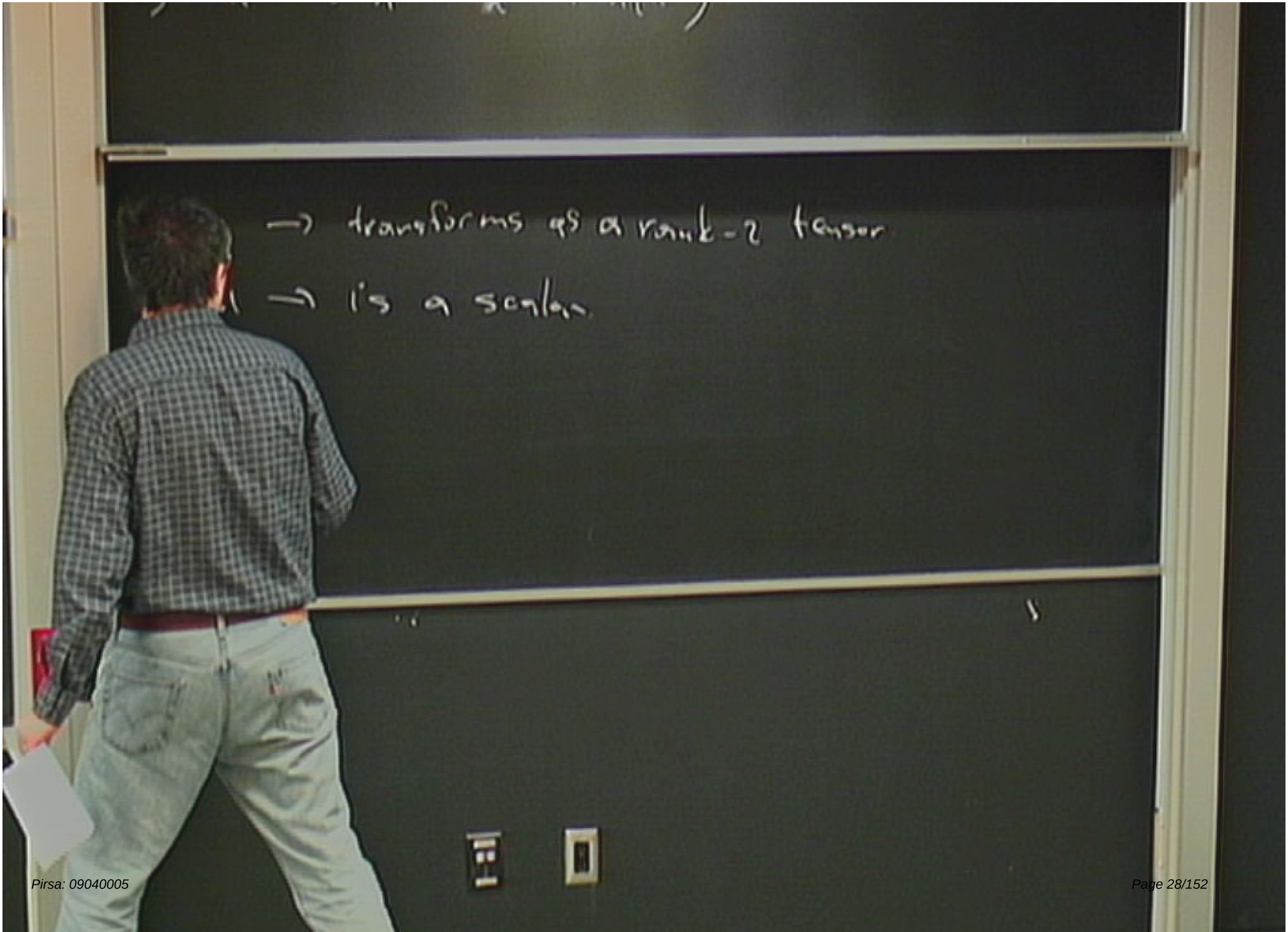

⇒ Ind d = D−1 in d-dim Diff.
x^μ → x^μ'(x^ν)
⇒ x^d → x^d' = x^d + λ(x^μ)

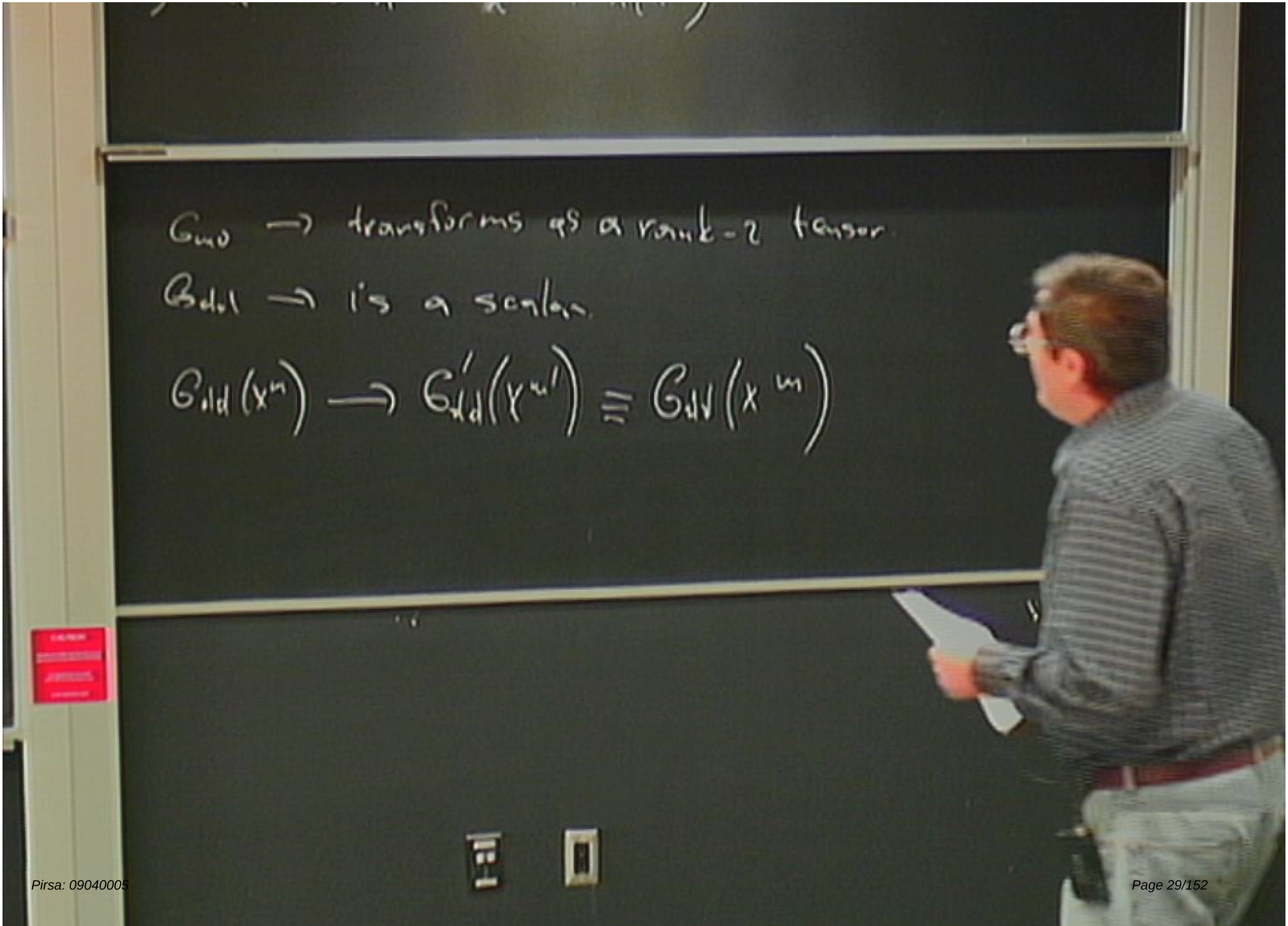

⇒ Ind = D−1 in d-dim Diff
$x^\mu \to x^{\mu'}(x^\nu)$
⇒ $x^d \to x^{d'} = x^d + \lambda(x^\mu)$

$\Rightarrow$ Ind $d = D-1$ in d-dim Diff

$$x^\mu \longrightarrow x^{\mu'}(x^\nu)$$

$$\Rightarrow \quad x^d \longrightarrow x^{d'} = x^d + \lambda(x^\mu)$$


→ transforms as a rank-2 tensor
→ is a scalar


$G_{\mu\nu} \rightarrow$ transforms as a rank-2 tensor
$G_{dd} \rightarrow$ is a scalar
$G_{dd}(x^{\mu}) \rightarrow G'_{dd}(x^{\mu'}) \equiv G_{dd}(x^{\mu})$

$G_{\mu\nu}$ → transforms as a rank-2 tensor

G_{dil} → it's a scalar

$$G_{dil}(x^{\mu}) \longrightarrow G'_{dil}(x^{\mu'}) \equiv G_{dil}(x^{\mu})$$

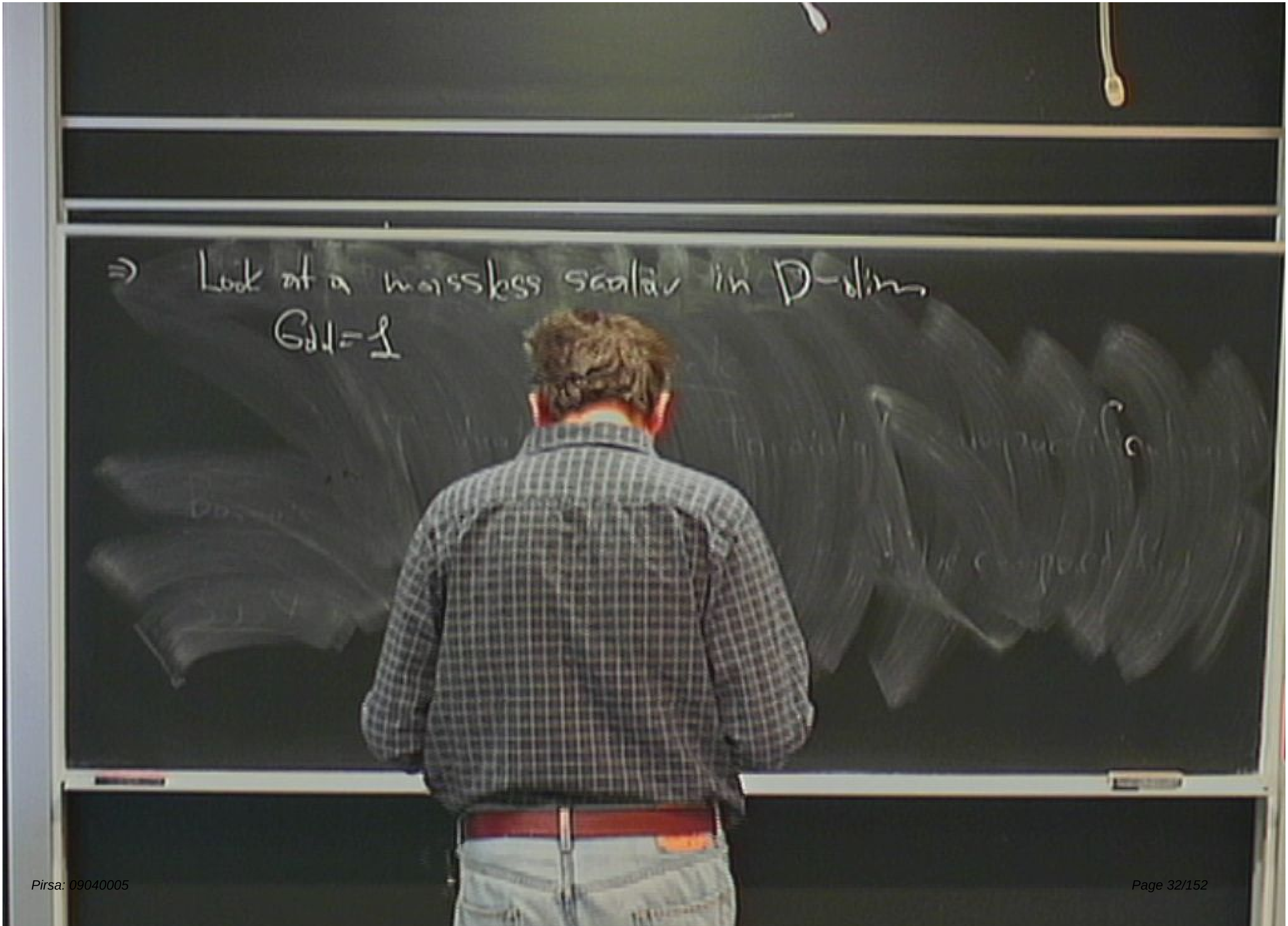
$$A_{\mu} \longrightarrow A'_{\mu} = A_{\mu} - \partial_{\mu}\lambda$$

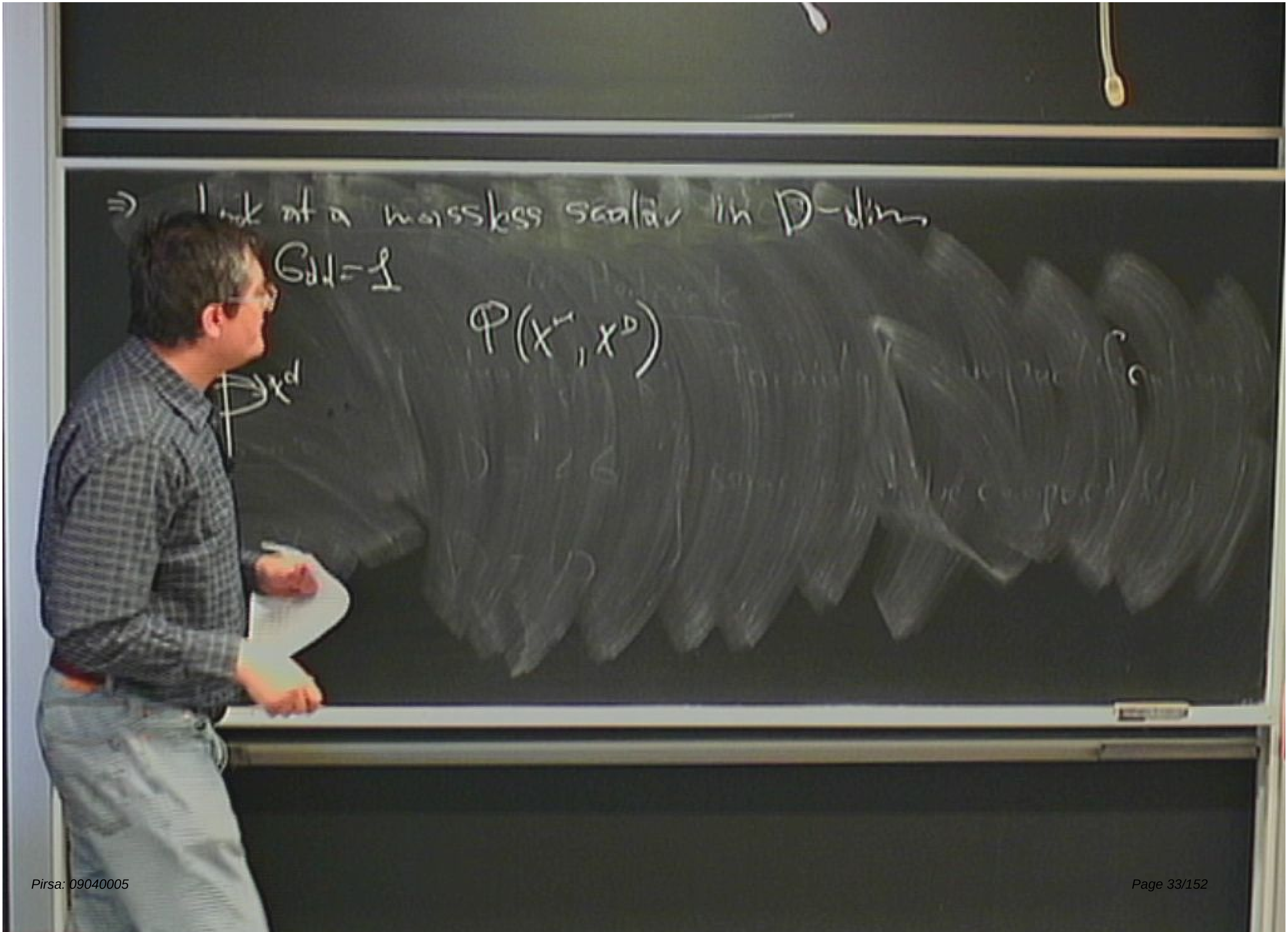
$G_{\mu\nu} \rightarrow$ transforms as a rank-2 tensor

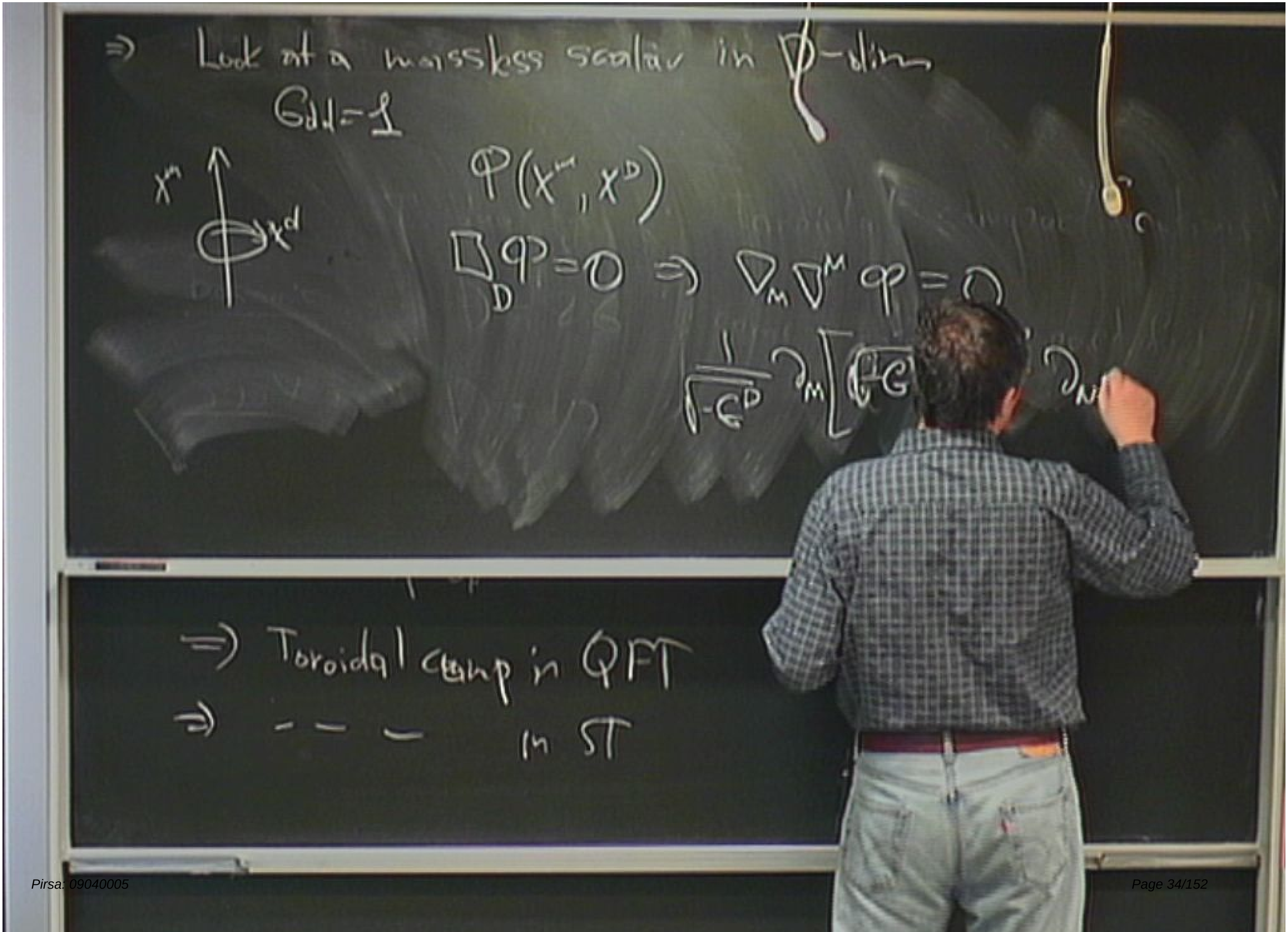
$G_{dd} \rightarrow$ is a scalar

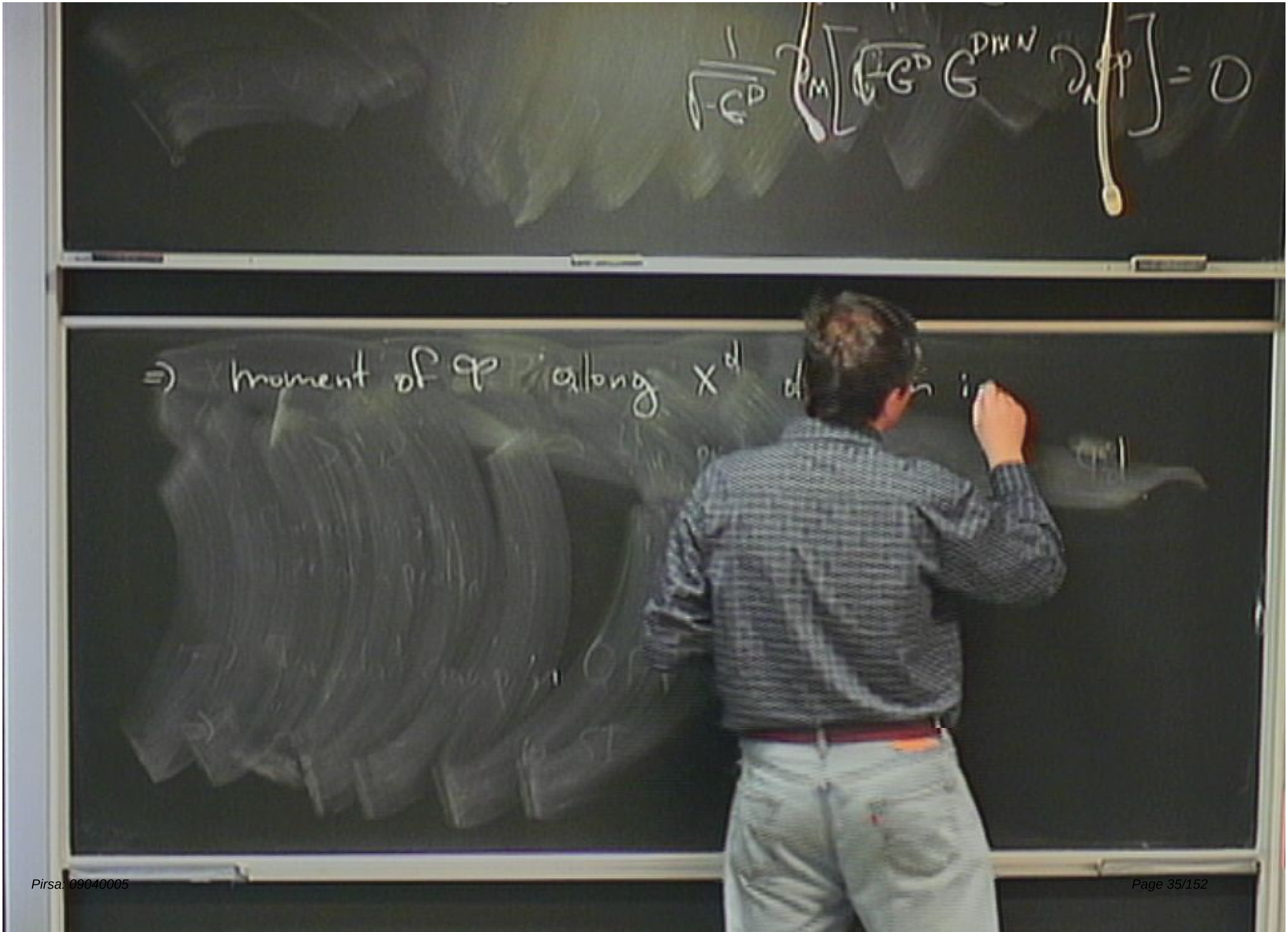
$$G_{dd}(x^{\mu}) \longrightarrow G'_{dd}(x^{\mu'}) \equiv G_{dd}(x^{\mu})$$

$$A_{\mu} \rightarrow A'_{\mu} = A_{\mu} - \partial_{\mu}\lambda$$ ← gauge transformations in d-dim.


⇒ Look at a massless scalar in D-dim


⇒ Look at a massless scalar in D-dim
$G_{dd} = 1$
$\varphi(x^\mu, x^D)$


⇒ Look at a massless scalar in D-dim
Gdd = 1
φ(x^m, x^D)
□_D φ = 0 ⇒ ∇_M ∇^M φ = 0
⇒ Toroidal comp in QFT
⇒ — — — in ST



$\frac{1}{\sqrt{-G^D}} \partial_M \left[\sqrt{-G^D}\, G^{DMN} \partial_N \varphi \right] = 0$
⇒ moment of φ along x^d

$$\frac{1}{\sqrt{-G^D}}\,\partial_M\left[\sqrt{-G^D}\,G^{DMN}\,\partial_N\varphi\right]=0$$

$\Rightarrow$ moment of φ along x^d direction is quantized

$$p_d = \frac{n}{R}$$

$$\varphi(x^\mu) = \sum_{n=-\infty}^{+\infty} \varphi_n(x^\mu)\, e^{\frac{inx^d}{R}}$$

$$\frac{1}{\sqrt{-G^D}} \partial_M \left[\sqrt{-G^D}\, G^{DMN} \partial_N \varphi \right] = 0$$

$\Rightarrow$ momentum of φ along x^d direction is quantized

$$p_d = \frac{n}{R} \quad \nearrow e^{i p_d x^d}$$

$$\varphi(x^\mu) = \sum_{n=-\infty}^{+\infty} \varphi_n(x^\mu)\, e^{\frac{i n x^d}{R}}$$

$$\frac{1}{\sqrt{-G^D}}\partial_M\left[\sqrt{-G^D}\, G^{DMN}\,\partial_N \varphi\right] = 0$$

$\Rightarrow$ moment of φ along x^d direction is quantized

$$P_d = \frac{n}{R} \qquad \nearrow e^{iP_d x^d}$$

$$\varphi(x^M) = \sum_{n=-\infty}^{+\infty} \varphi_n(x^\mu)\, e^{\frac{inx^d}{R}}$$

$$\frac{1}{\sqrt{-G^D}}\,\partial_M\left[\sqrt{-G^D}\,G^{D\,MN}\,\partial_N\varphi\right] = 0$$

$\Rightarrow$ moment of φ along x^d direction is quantized

$$p_d = \frac{n}{R} \nearrow e^{i p_d x^d}$$

$$\varphi(x^M) = \sum_{n=-\infty}^{+\infty} \varphi_n(x^\mu)\, e^{\frac{i n x^d}{R}}$$

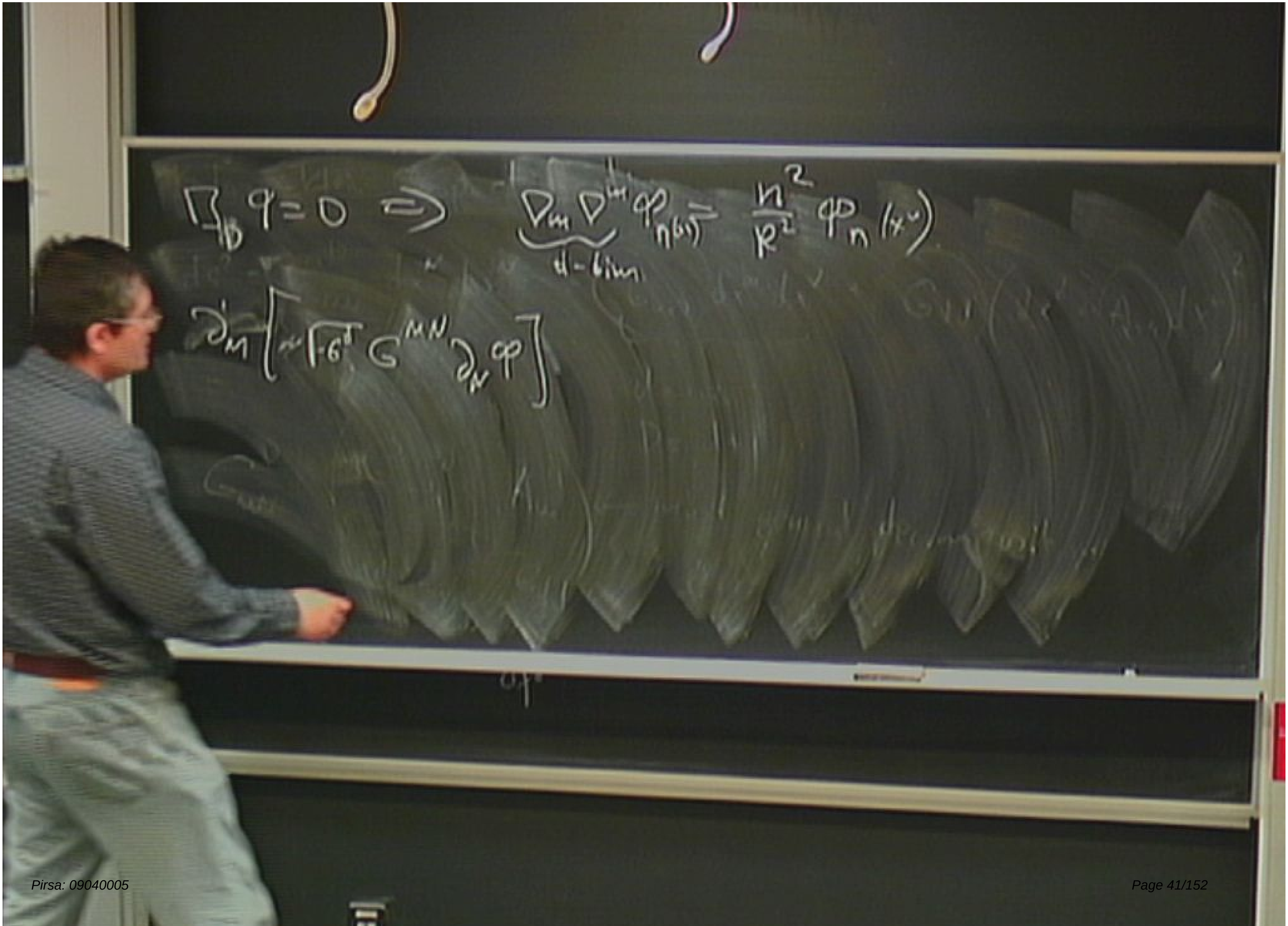
$$\frac{1}{\sqrt{-G^D}}\partial_M\left[\sqrt{-G^D}\, G^{DMN}\partial_N\varphi\right]=0$$

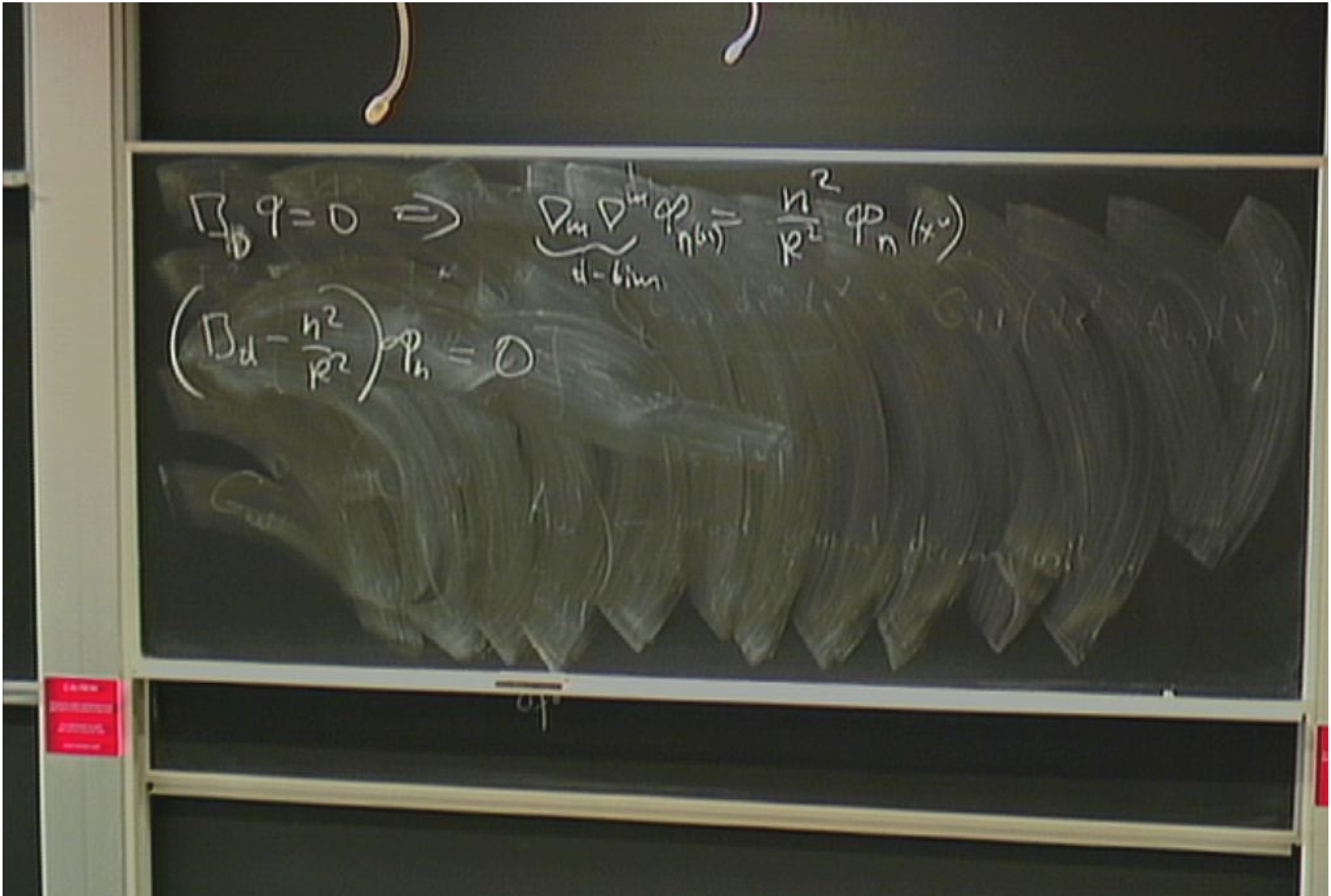
$\Rightarrow$ moment of φ along x^d direction is quantized

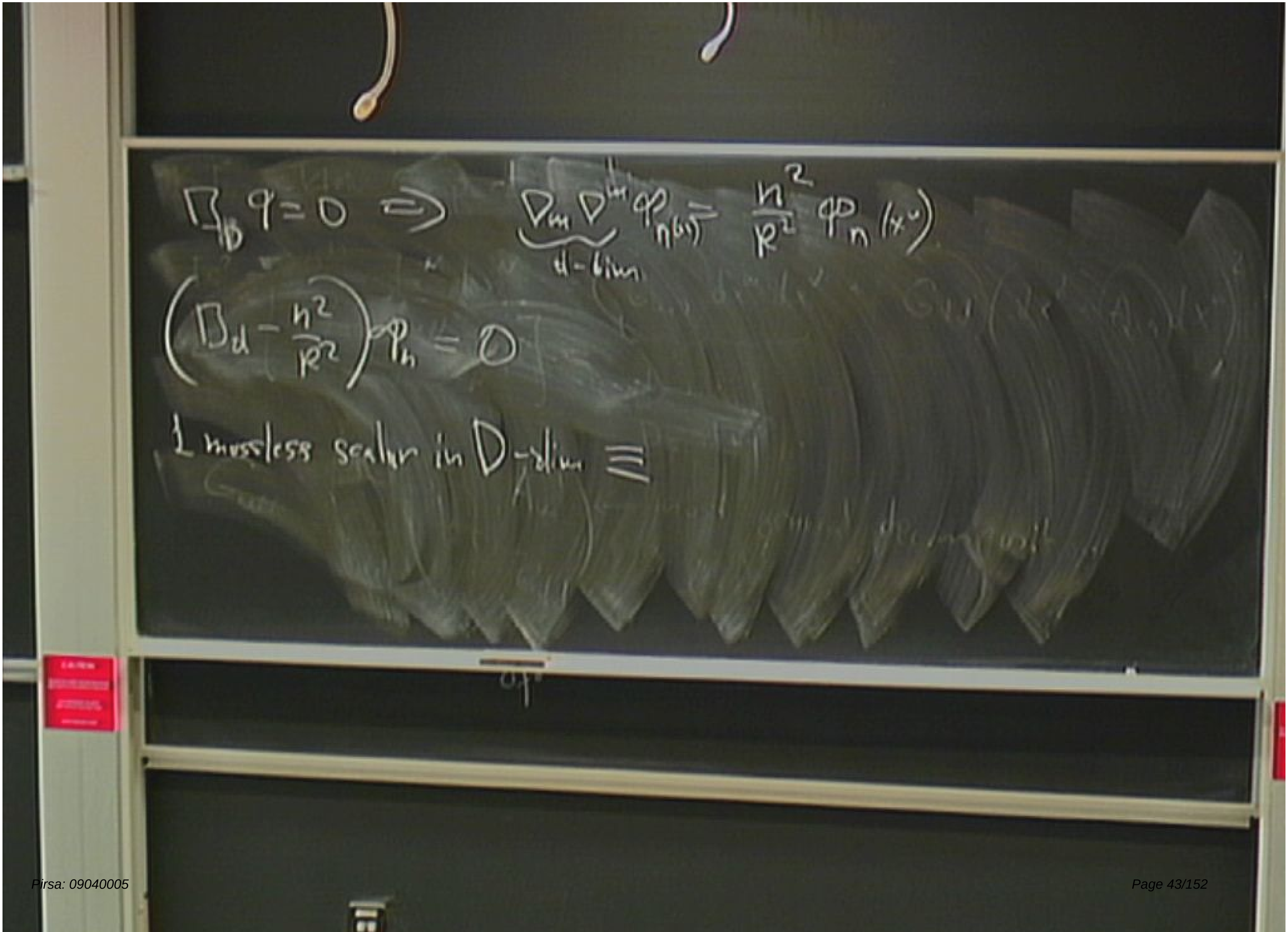
$$P_d = \frac{n}{R} \quad \nearrow e^{iP_d x^d}$$

$$\varphi(x^M) = \sum_{n=-\infty}^{+\infty} \varphi_n(x^\mu)\, e^{\frac{inx^d}{R}}$$

$$\varphi(x^d+2\pi R) = \varphi(x^d)$$


□_D φ = 0 ⇒ ∇_m ∇^m φ_n(x) = n²/R² φ_n(x^μ)
4-dim
∂_M [√-G G^MN ∂_N φ]

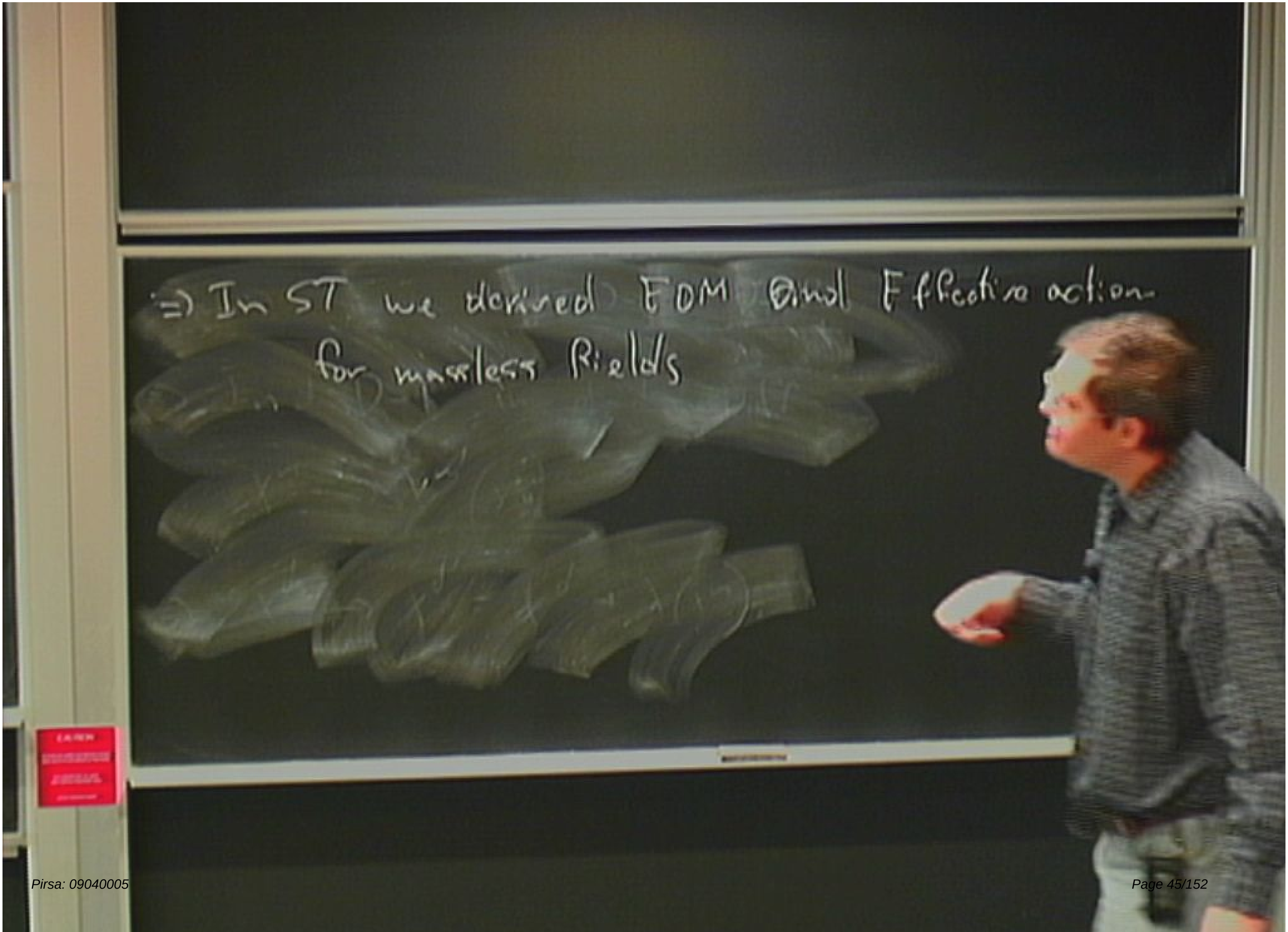

$\Box_B \varphi = 0 \Rightarrow \nabla_m \nabla^m \varphi_{n(m)} = \frac{n^2}{R^2} \varphi_n(x^\mu)$
d-dim
$\left(\Box_d - \frac{n^2}{R^2}\right)\varphi_n = 0$

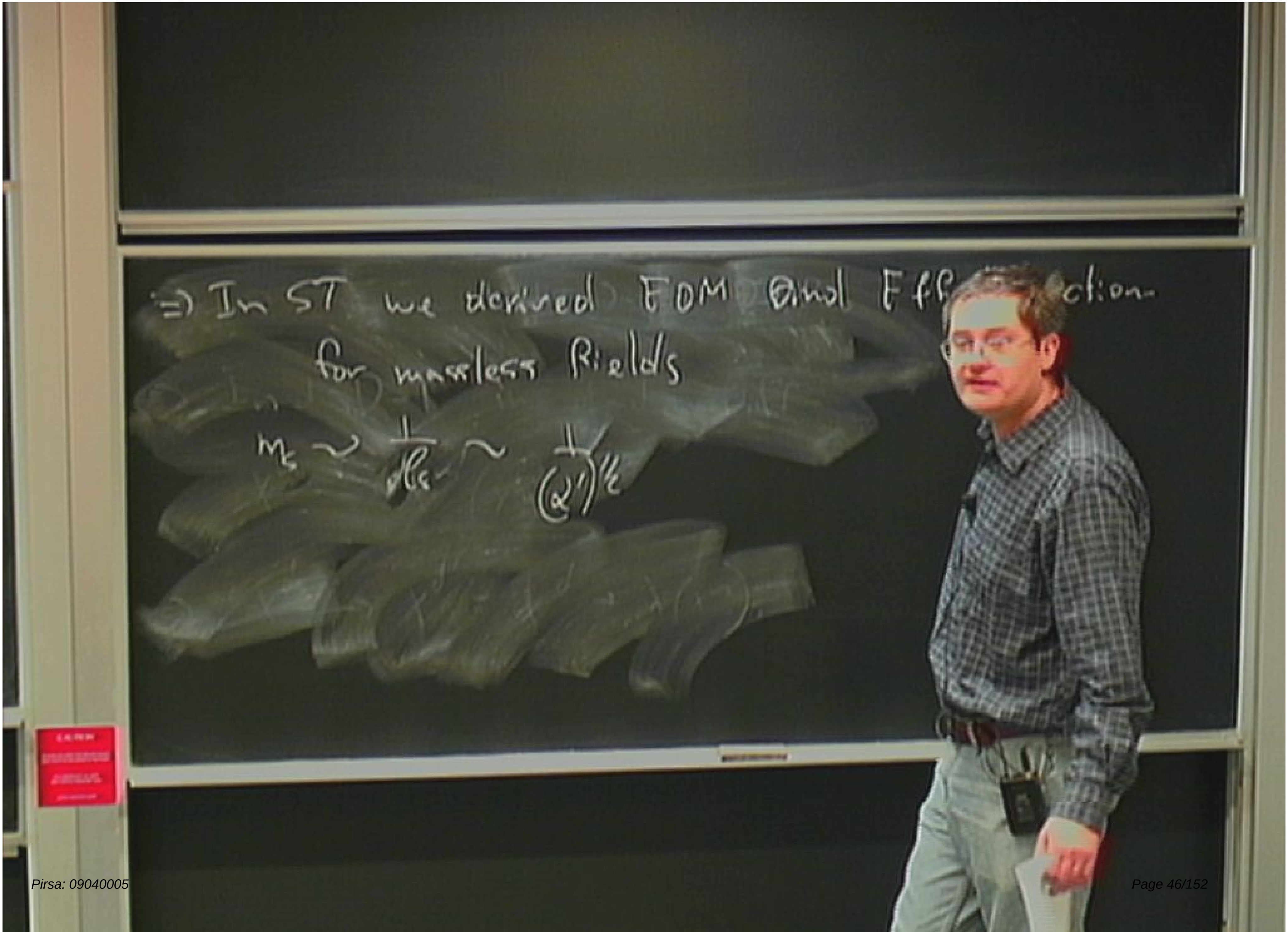

□_D φ = 0 ⇒ D_μ D^μ φ_n(x) = n²/R² φ_n(x^μ)
d-dim
(□_d − n²/R²) φ_n = 0
1 massless scalar in D-dim ≡

$$\Box_D \varphi = 0 \Rightarrow \underbrace{\nabla_\mu \nabla^\mu}_{d\text{-dim}} \varphi_{n(x)} = \frac{n^2}{R^2}\varphi_n(x^\mu)$$

$$\left(\Box_d - \frac{n^2}{R^2}\right)\varphi_n = 0$$

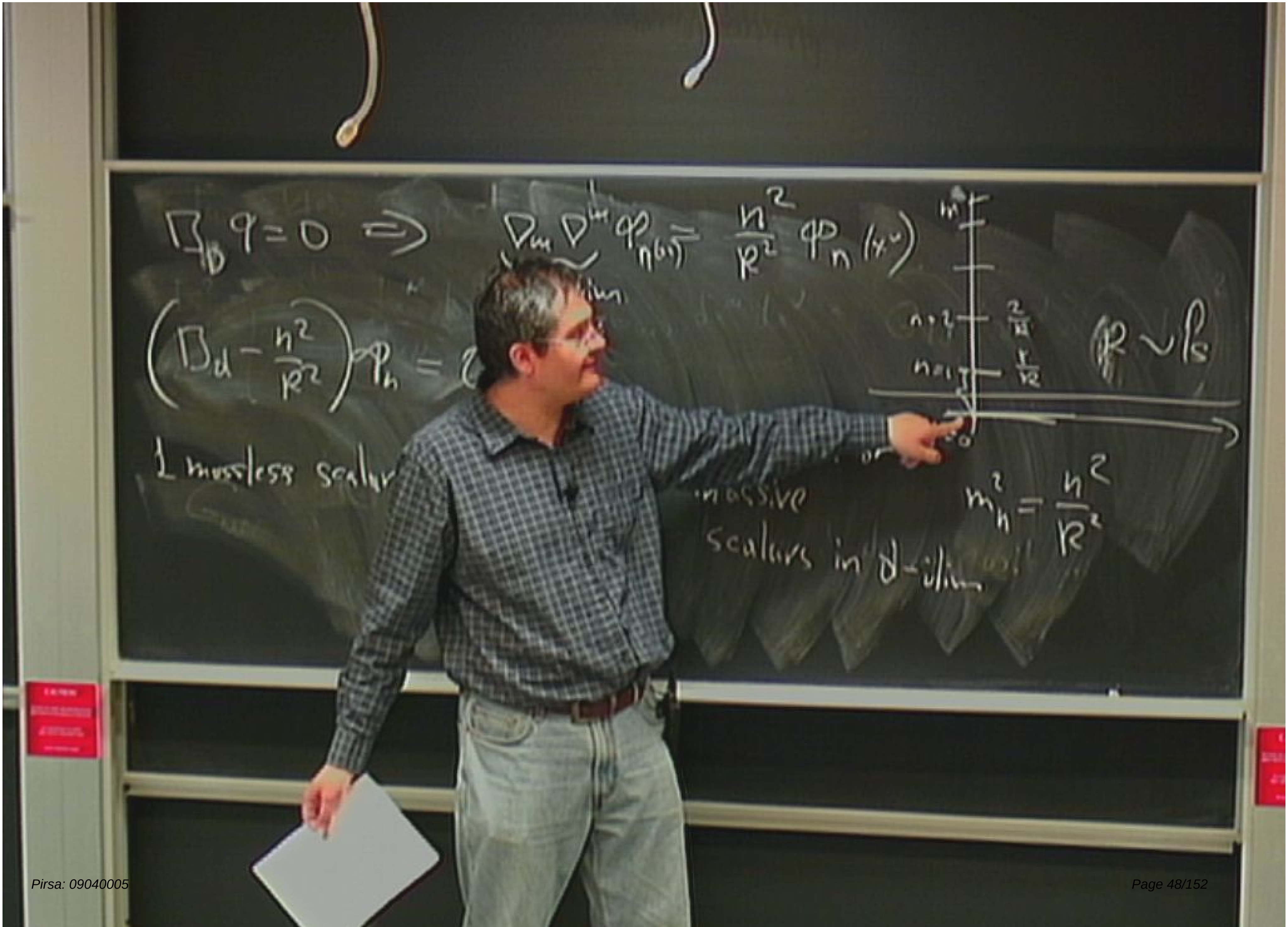
1 massless scalar in D-dim $\equiv$ Infinite set of massive scalars in d-dim $\quad m_n^2 = \frac{n^2}{R^2}$

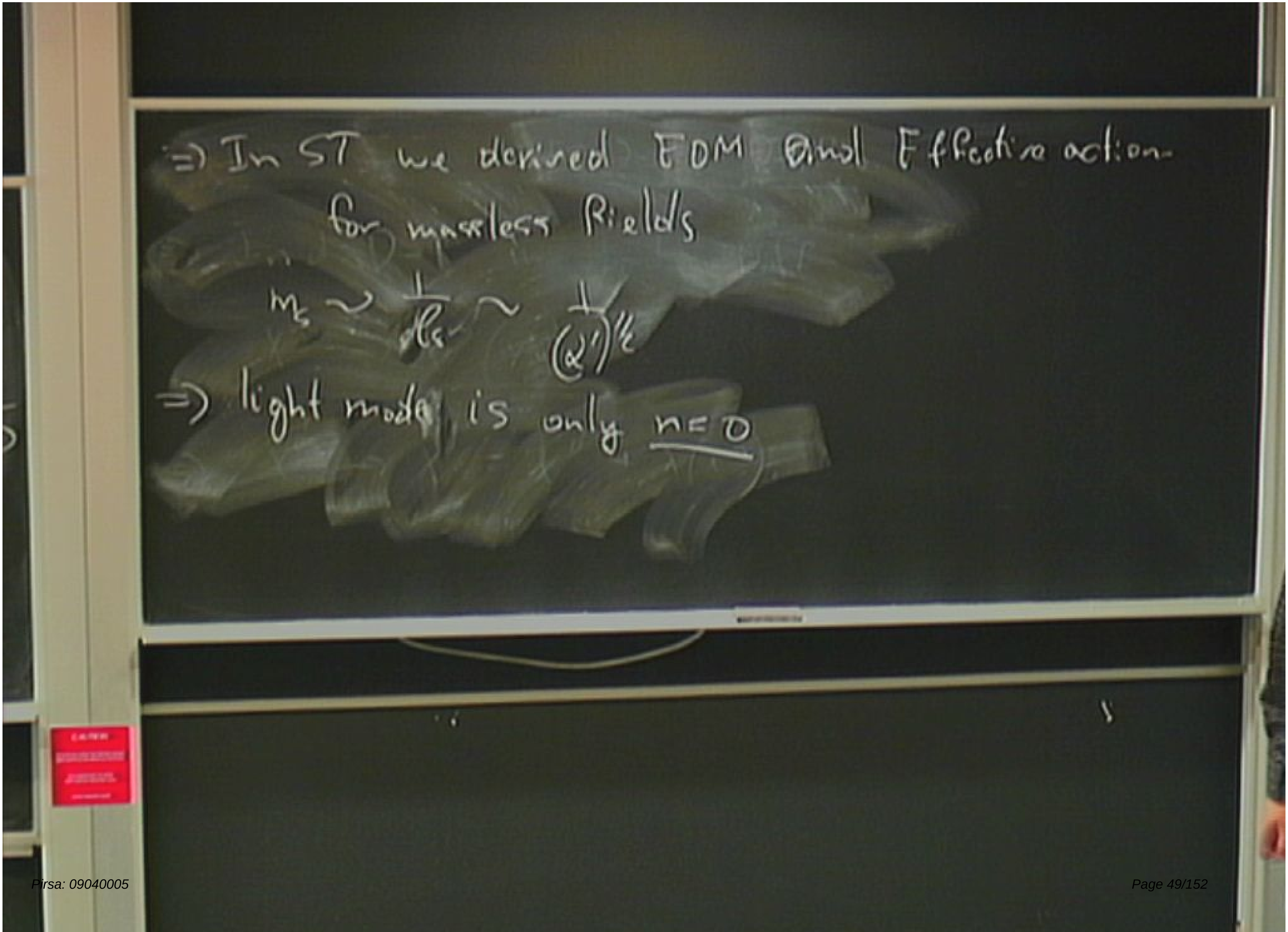

⇒ In ST we derived EOM and Effective action
for massless fields

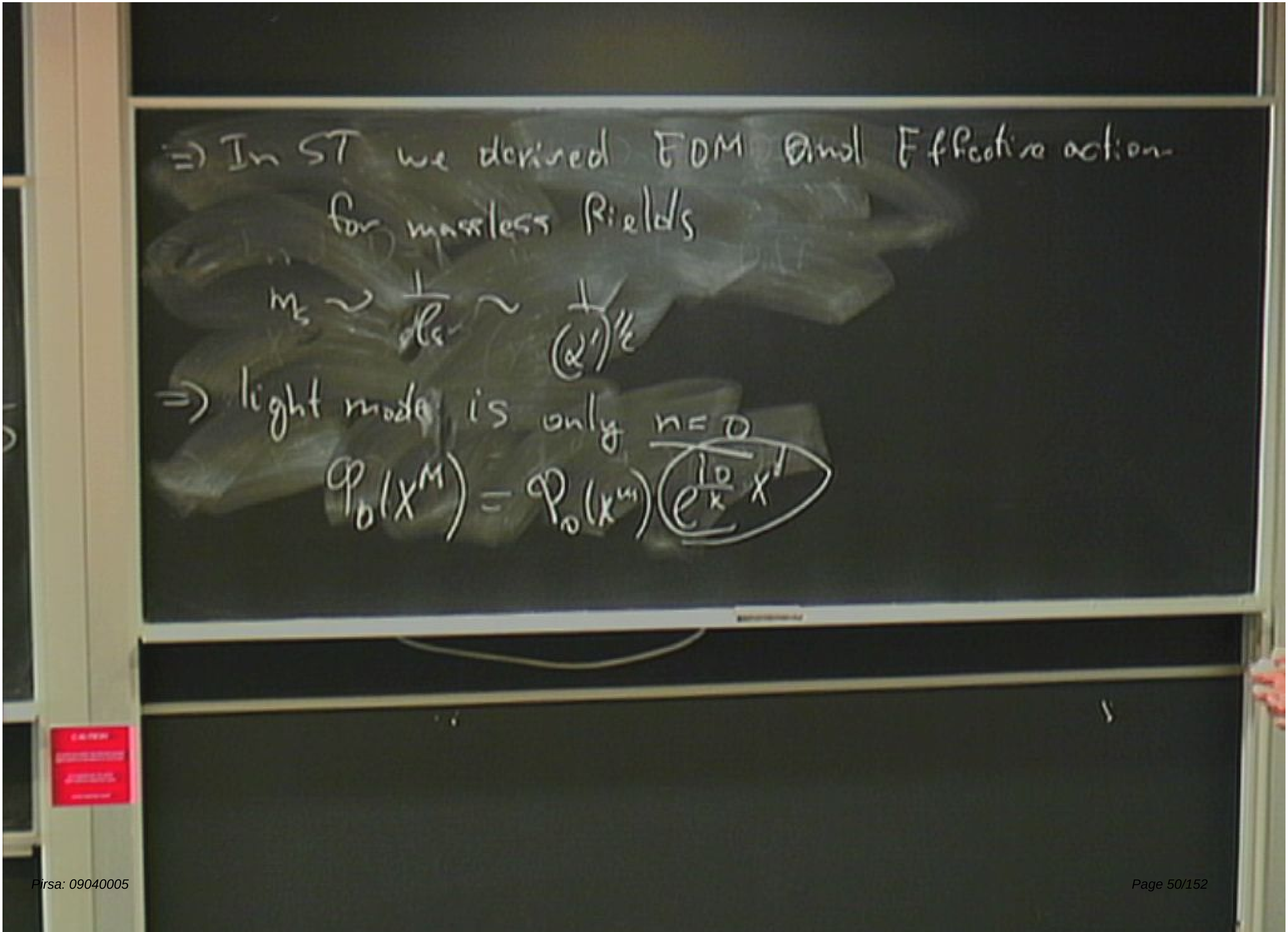

⇒ In ST we derived EOM and Eff
ction
for massless fields
$m_s \sim \frac{1}{\ell_s} \sim \frac{1}{(\alpha')^{1/2}}$

⇒ In ST we derived EOM and Effective action
for massless fields

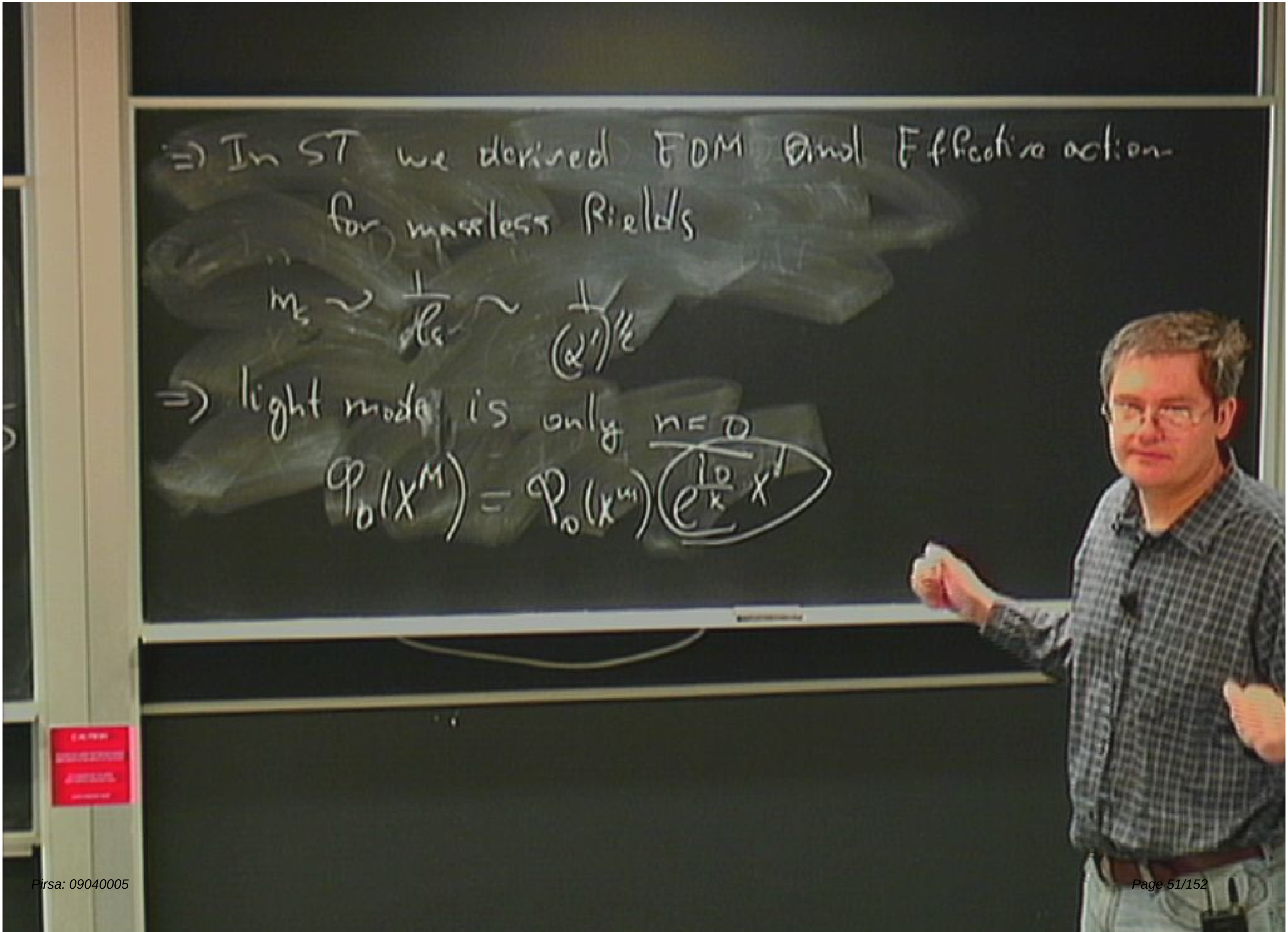
$$m_s \sim \frac{1}{l_s} \sim \frac{1}{(\alpha')^{1/2}}$$

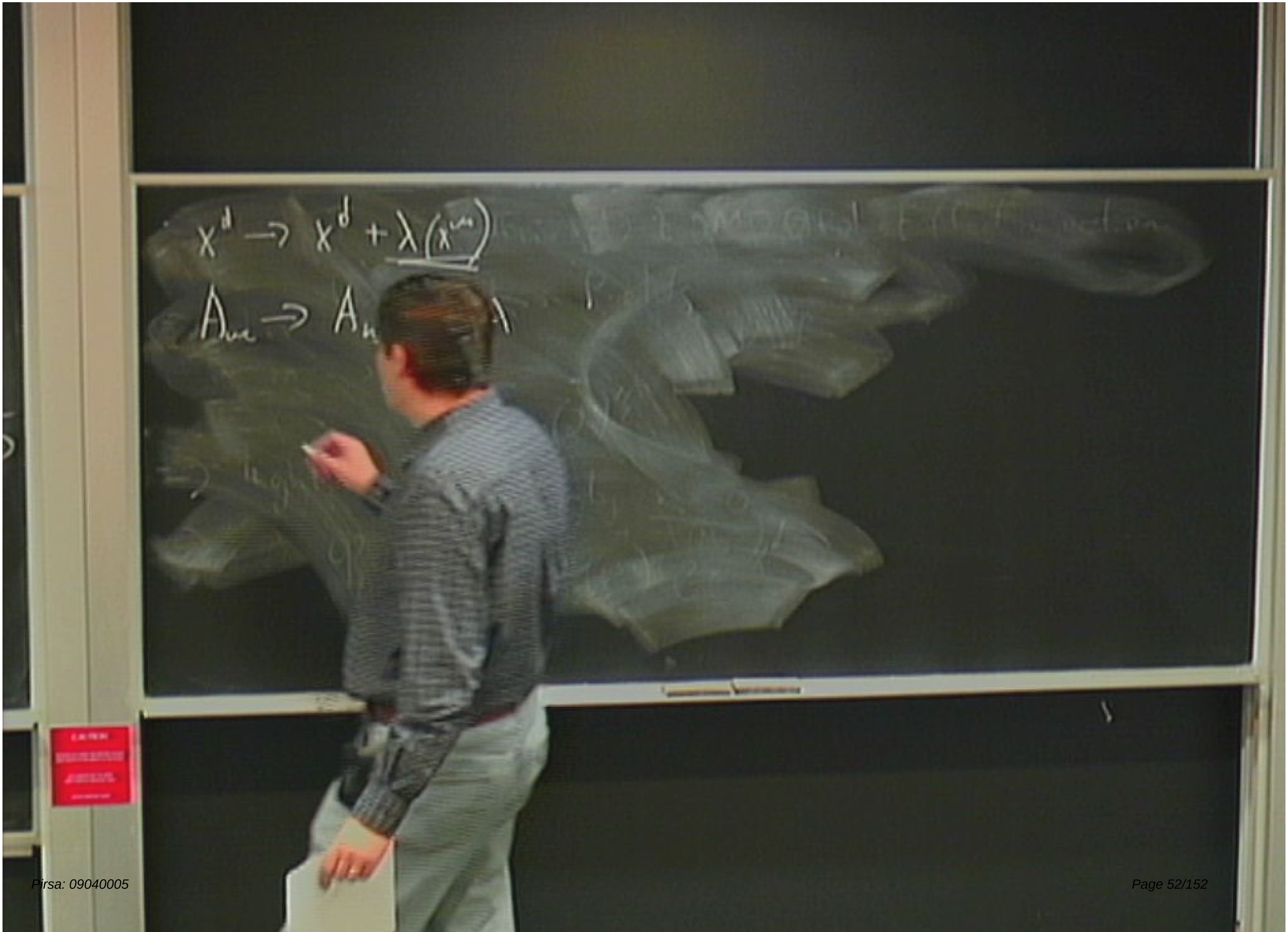

1 massless scalar
massive
scalars in d-dim

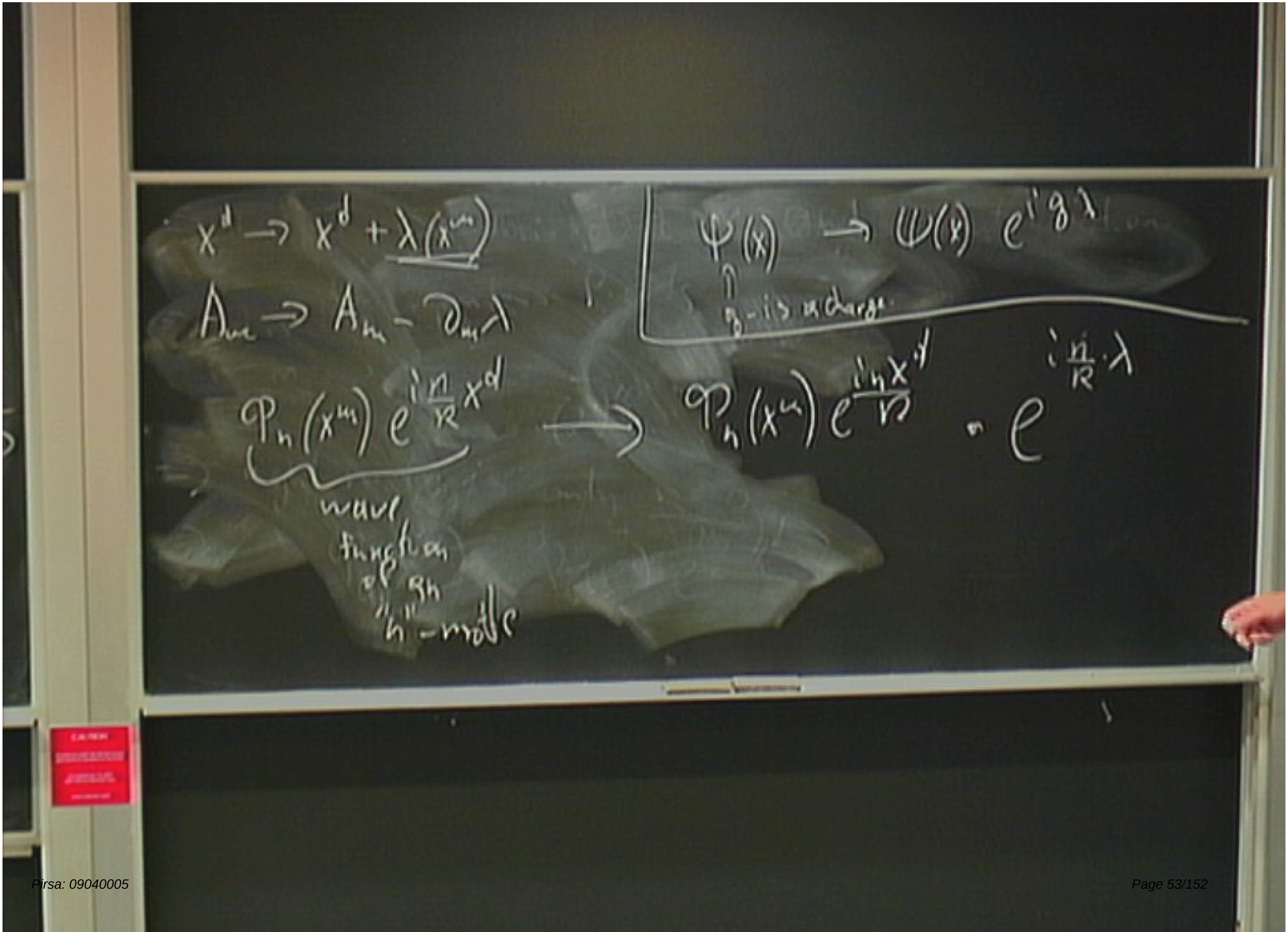

⇒ In ST we derived EOM and Effective action for massless fields
$m_s \sim \frac{1}{l_s} \sim \frac{1}{(\alpha')^{1/2}}$
⇒ light mode is only n=0

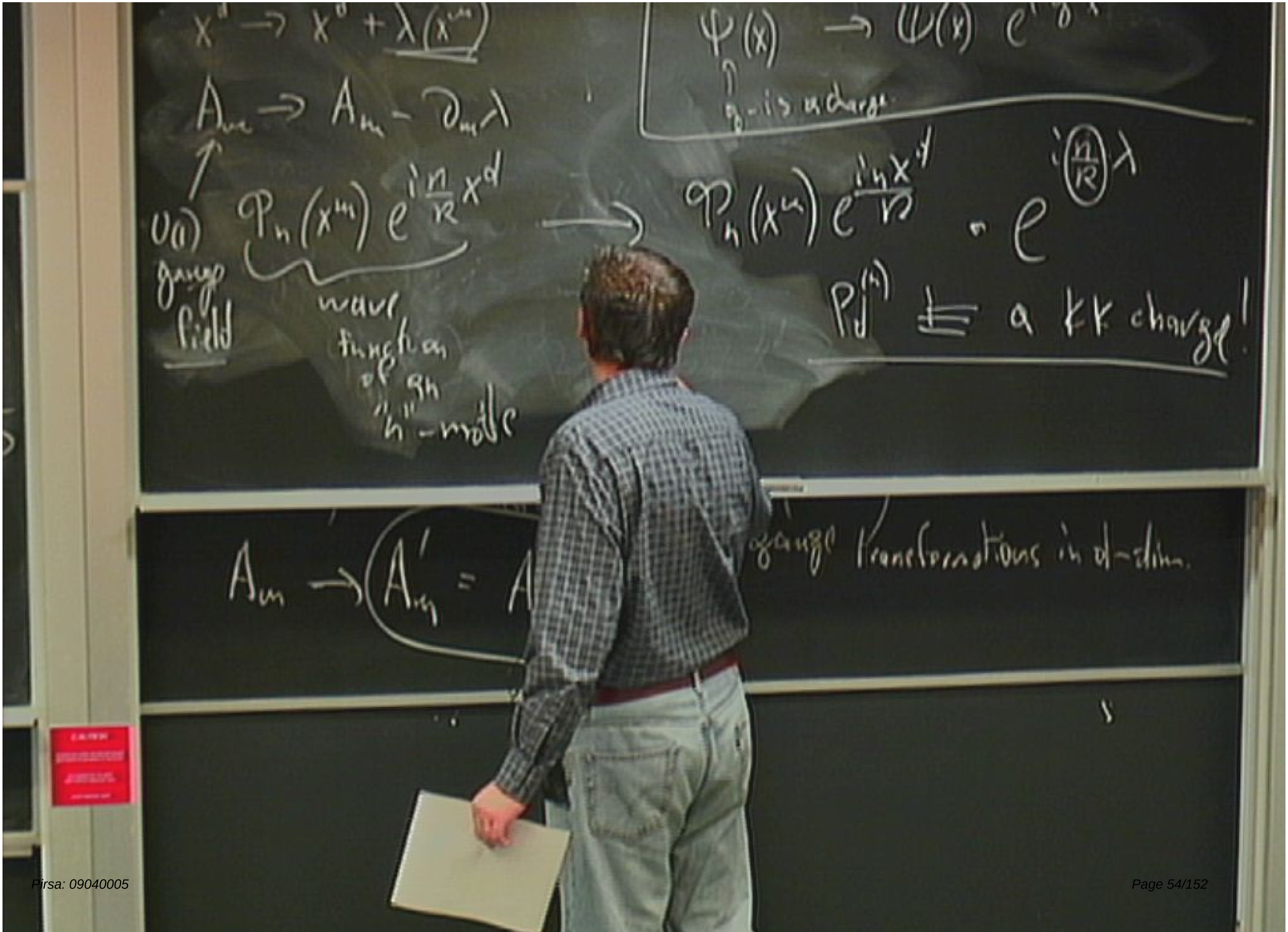


⇒ In ST we derived EOM and Effective action for massless fields
$m_s \sim \frac{1}{l_s} \sim \frac{1}{(\alpha')^{1/2}}$
⇒ light mode is only n=0
$\varphi_0(X^M) = \varphi_0(x^\mu)\left(e^{\frac{i0}{R}x^4}\right)$


⇒ In ST we derived EOM and Effective action
for massless fields
⇒ light mode is only n=0




x^d → x^d + λ(x^μ)
A_μ → A_μ − ∂_μ λ
φ_n(x^μ) e^{i n/R x^d} → φ_n(x^μ) e^{i n x^d / R} · e^{i n/R · λ}
wave function of an "n"-mode
ψ(x) → ψ(x) e^{i g λ}
g - is a charge


U(1) gauge field
wave function of an "n"-mode
a KK charge!
gauge transformations in d-dim.

$$x^d \to x^d + \lambda(x^\mu)$$

$$\varphi(x) \to \varphi(x)\, e^{iq\lambda}$$

q - is a charge

$$A_\mu \to A_\mu - \partial_\mu \lambda$$

U(1) gauge field

$$\varphi_n(x^\mu)\, e^{i\frac{n}{R}x^d} \to \varphi_n(x^\mu)\, e^{i\frac{n}{R}x^d} \cdot e^{i\left(\frac{n}{R}\right)\lambda}$$

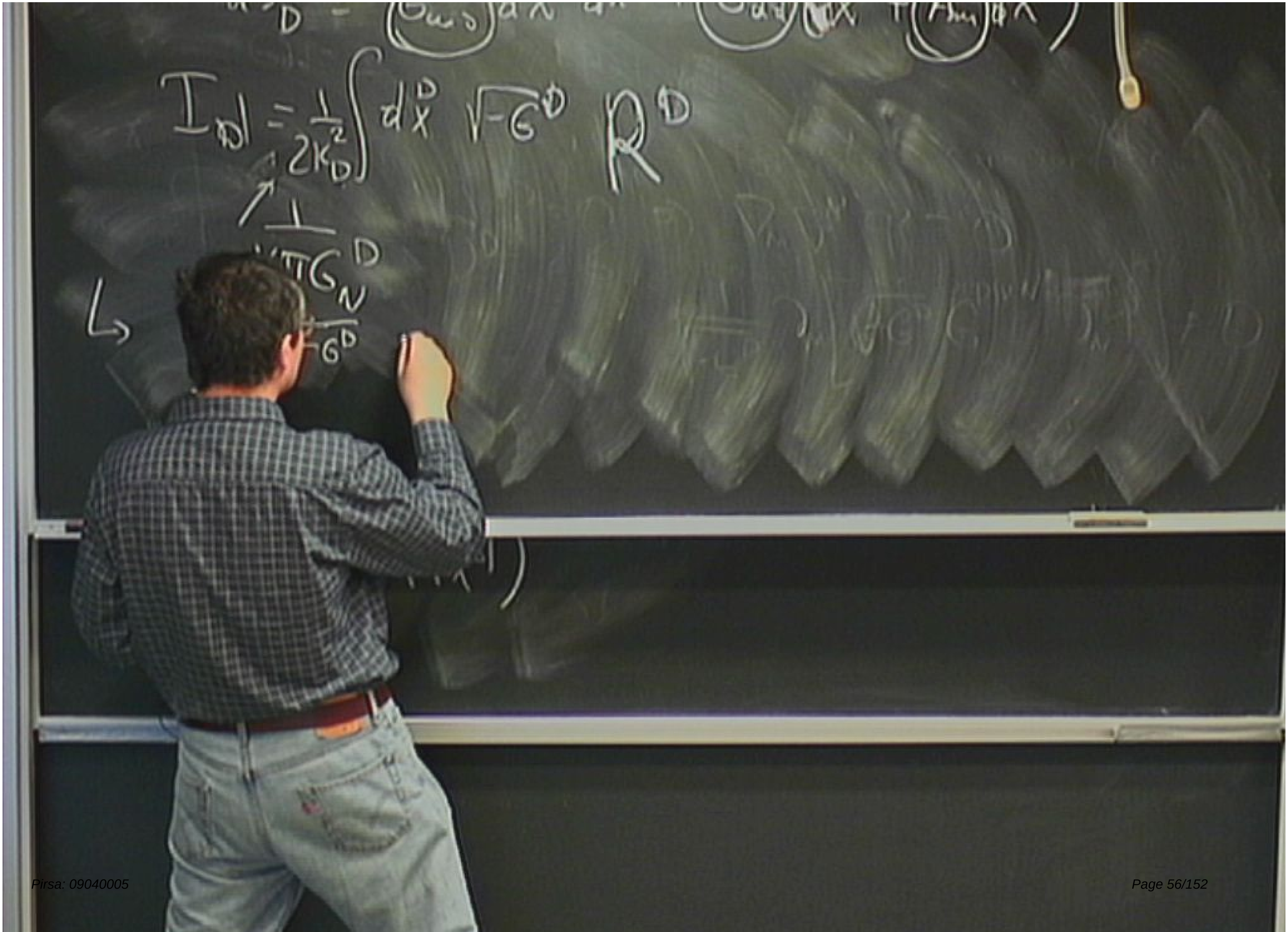
wave function of an "n"-mode

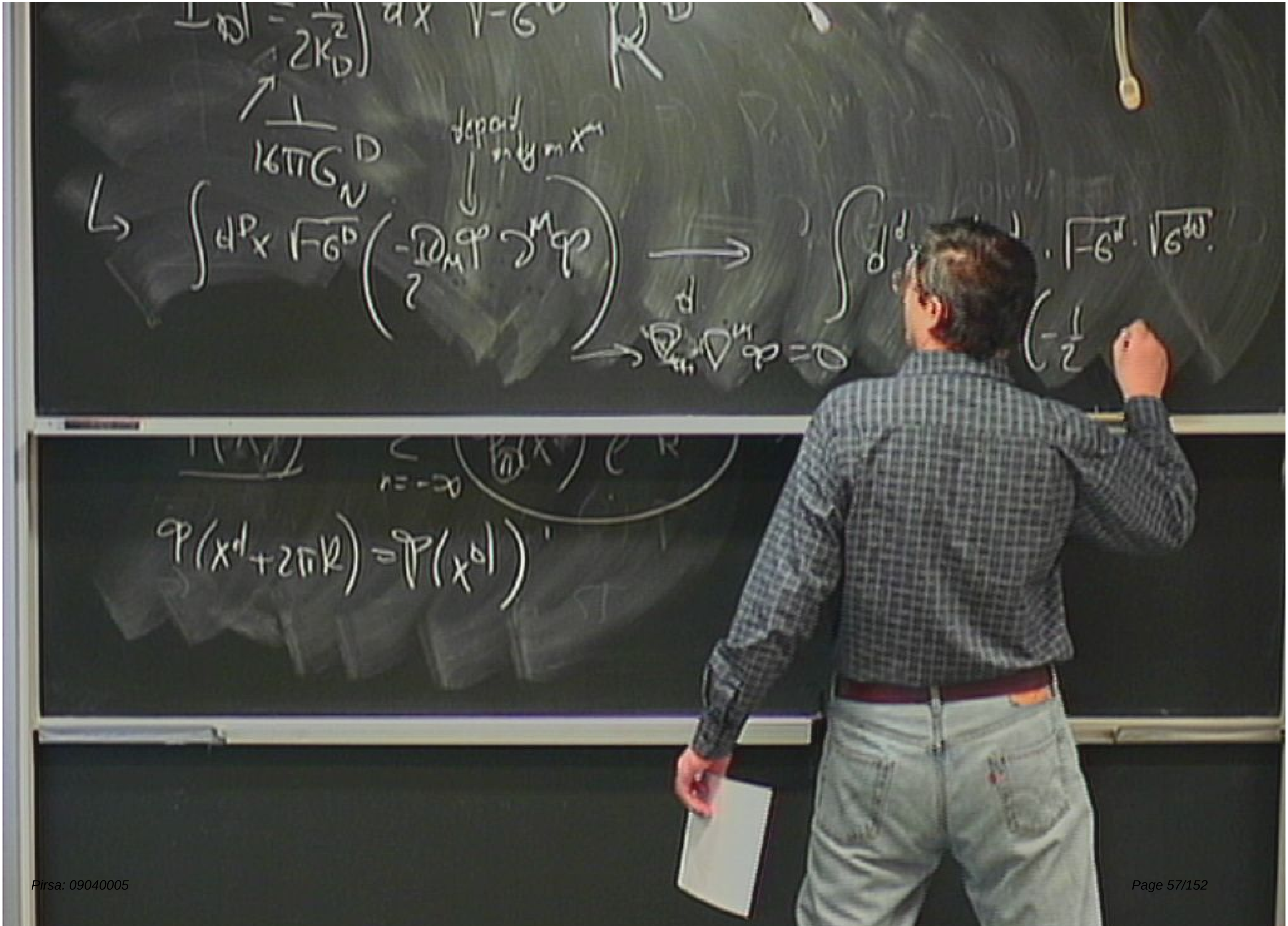
$P_d^{(n)} \equiv$ a KK charge!

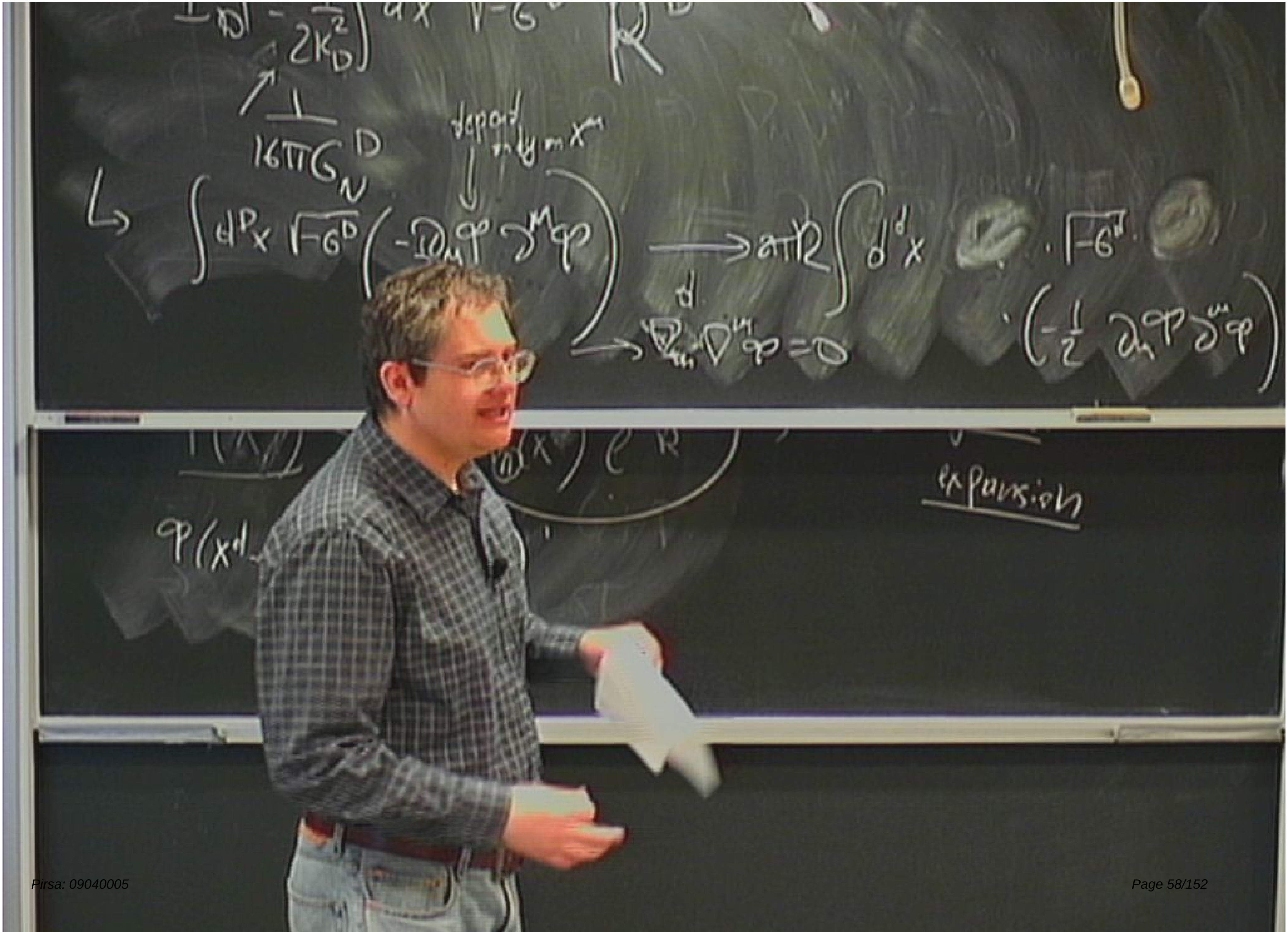
n=0 is neutral!

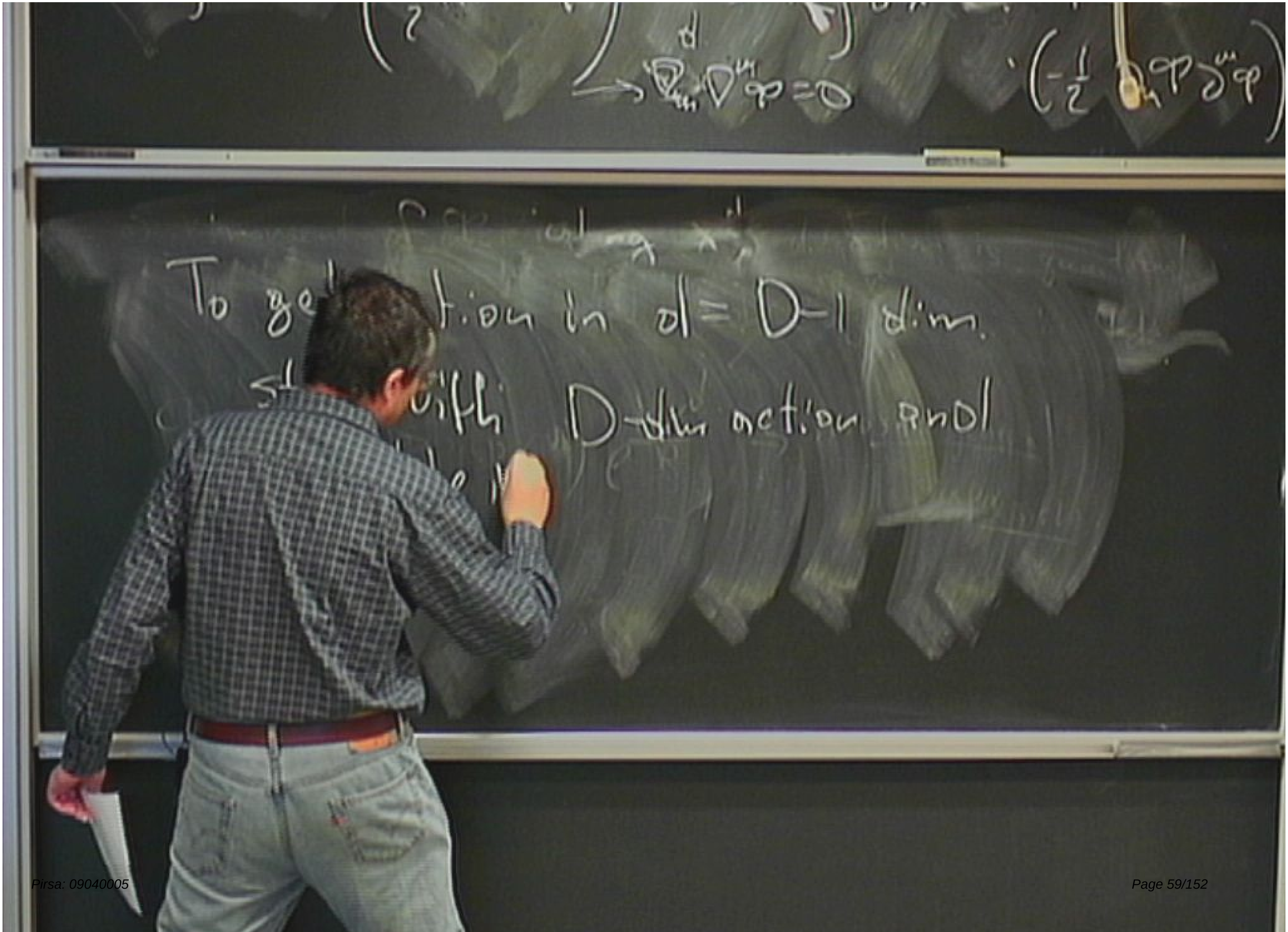
$$A_\mu \to A'_\mu = A_\mu - \partial_\mu \lambda$$

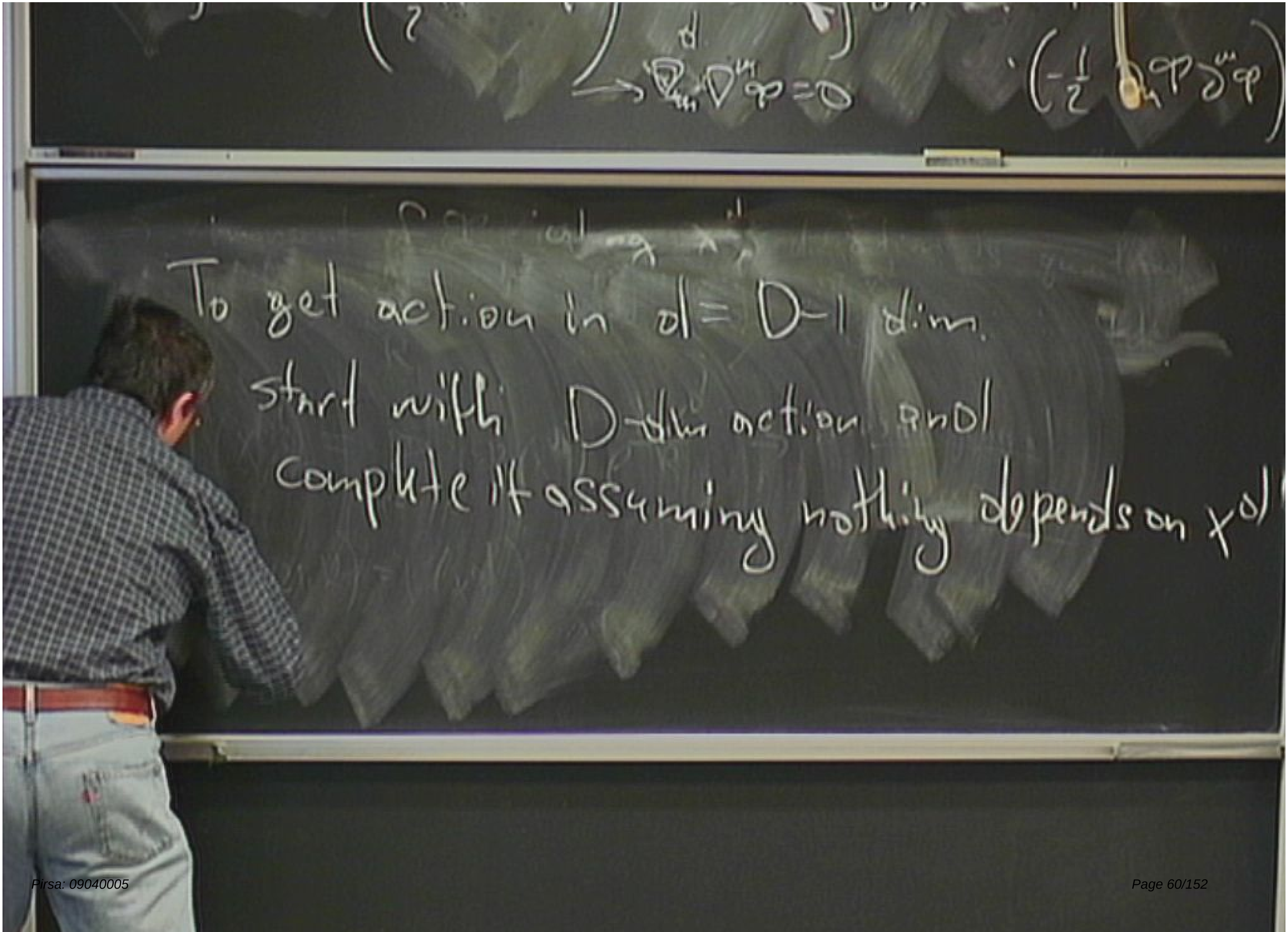
← gauge transformations in d-dim.


I = 1/2K_D^2 ∫ d^D x √-G^D R^D


1/16πG_N^D
depend only on x^m
φ(x^d + 2πR) = φ(x^d)


expansion


$\nabla_\mu \nabla^\mu \varphi = 0$
$\left(-\frac{1}{2}\,\partial_\mu\varphi\,\partial^\mu\varphi\right)$
in d = D−1 dim.
D-dim action and

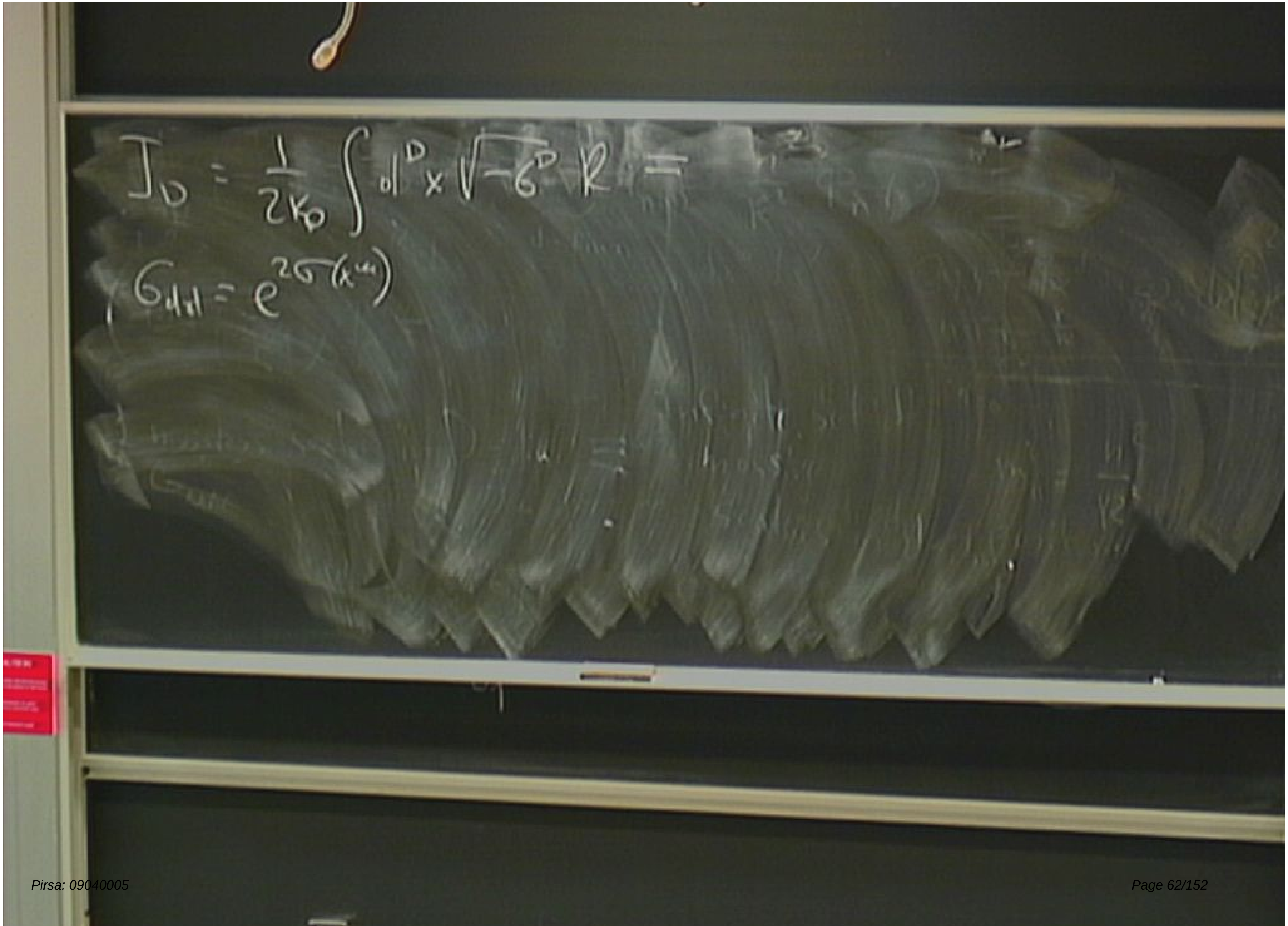

To get action in d = D-1 dim.
start with D-dim action and
complete it assuming nothing depends on x^d

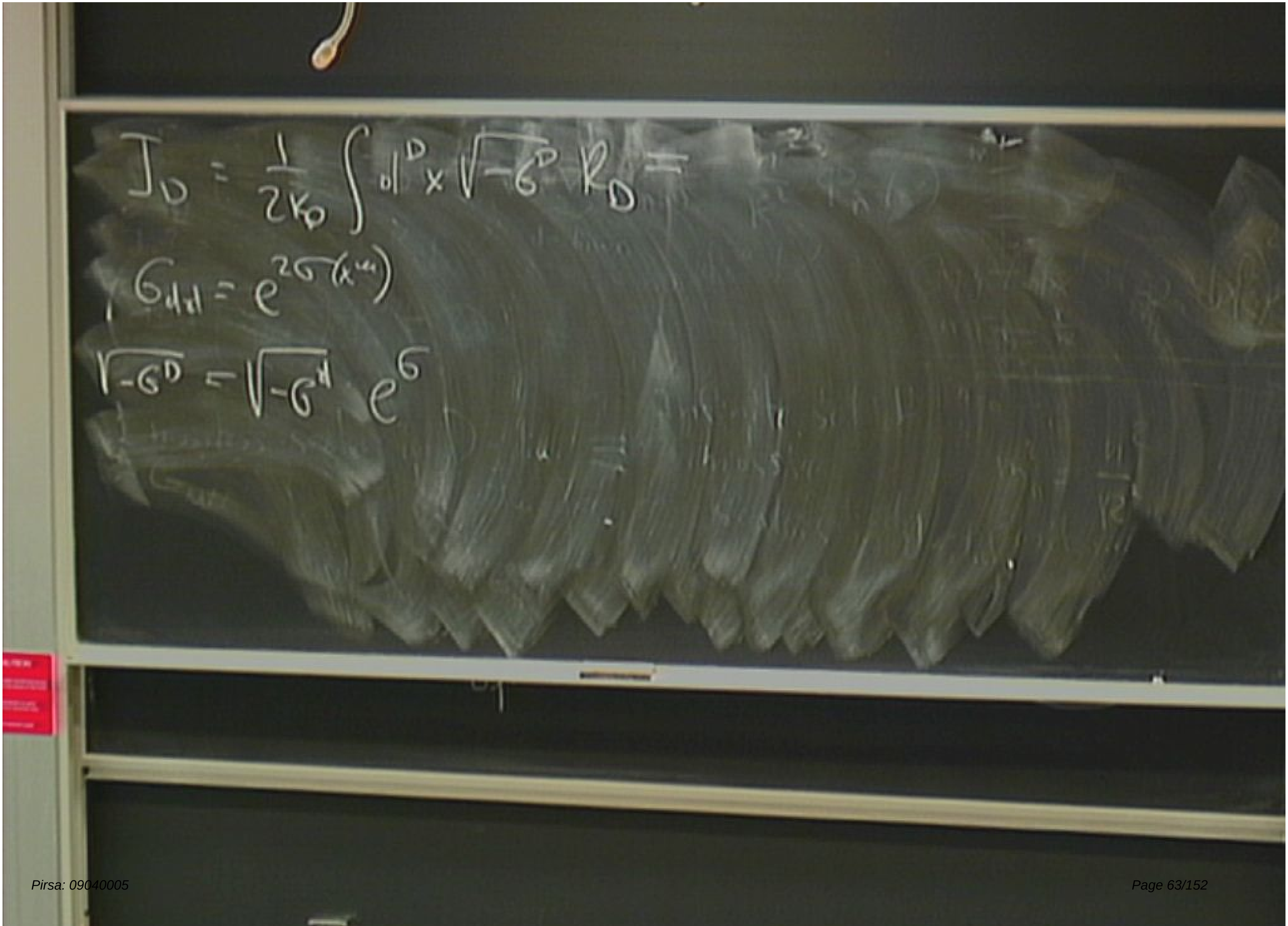
$\partial_\mu \partial^\mu \varphi = 0$

$\left(-\frac{1}{2}\partial_\mu \varphi \partial^\mu \varphi\right)$

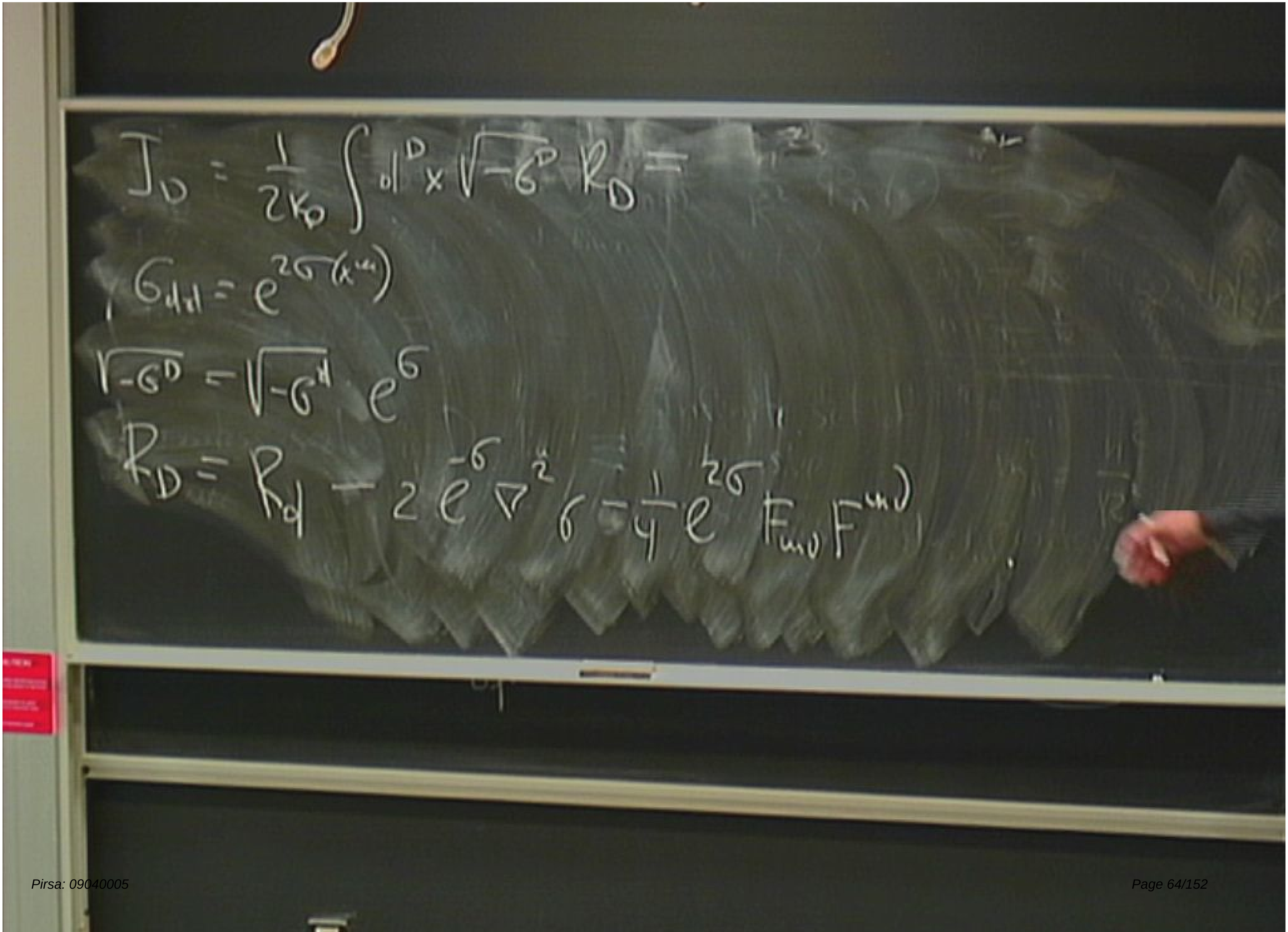
To get action in $d = D-1$ dim.
start with D-dim action and
complete it assuming nothing depends on x^d

Limitations: truncates to massless modes


$I_D = \frac{1}{2\kappa_D}\int d^Dx \sqrt{-G^D}\, R =$
$G_{d+1} = e^{2\sigma(x^{\mu})}$



$I_D = \frac{1}{2\kappa_D}\int d^Dx \sqrt{-G^D}\, R_D =$
$G_{d+1} = e^{2\sigma(x^{\mu})}$
$\sqrt{-G^D} = \sqrt{-G^d}\; e^{\sigma}$



$I_D = \frac{1}{2\kappa_D}\int d^D x \sqrt{-G^D}\, R_D =$
$G_{d+1} = e^{2\sigma(x^\mu)}$
$\sqrt{-G^D} = \sqrt{-G^d}\, e^{\sigma}$
$R_D = R_d - 2e^{-\sigma}\nabla^2\sigma - \frac{1}{4}e^{2\sigma}F_{\mu\nu}F^{\mu\nu}$

$$I_D = \frac{1}{2\kappa_D}\int d^Dx \sqrt{-G^D}\, R_D =$$

$$G_{d+1} = e^{2\sigma(x^{\mu})}$$

$$\sqrt{-G^D} = \sqrt{-G^d}\; e^{\sigma}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$R_D = R_d - 2e^{-\sigma}\nabla^2 e^{\sigma} - \frac{1}{4}e^{2\sigma} F_{\mu\nu}F^{\mu\nu}$$

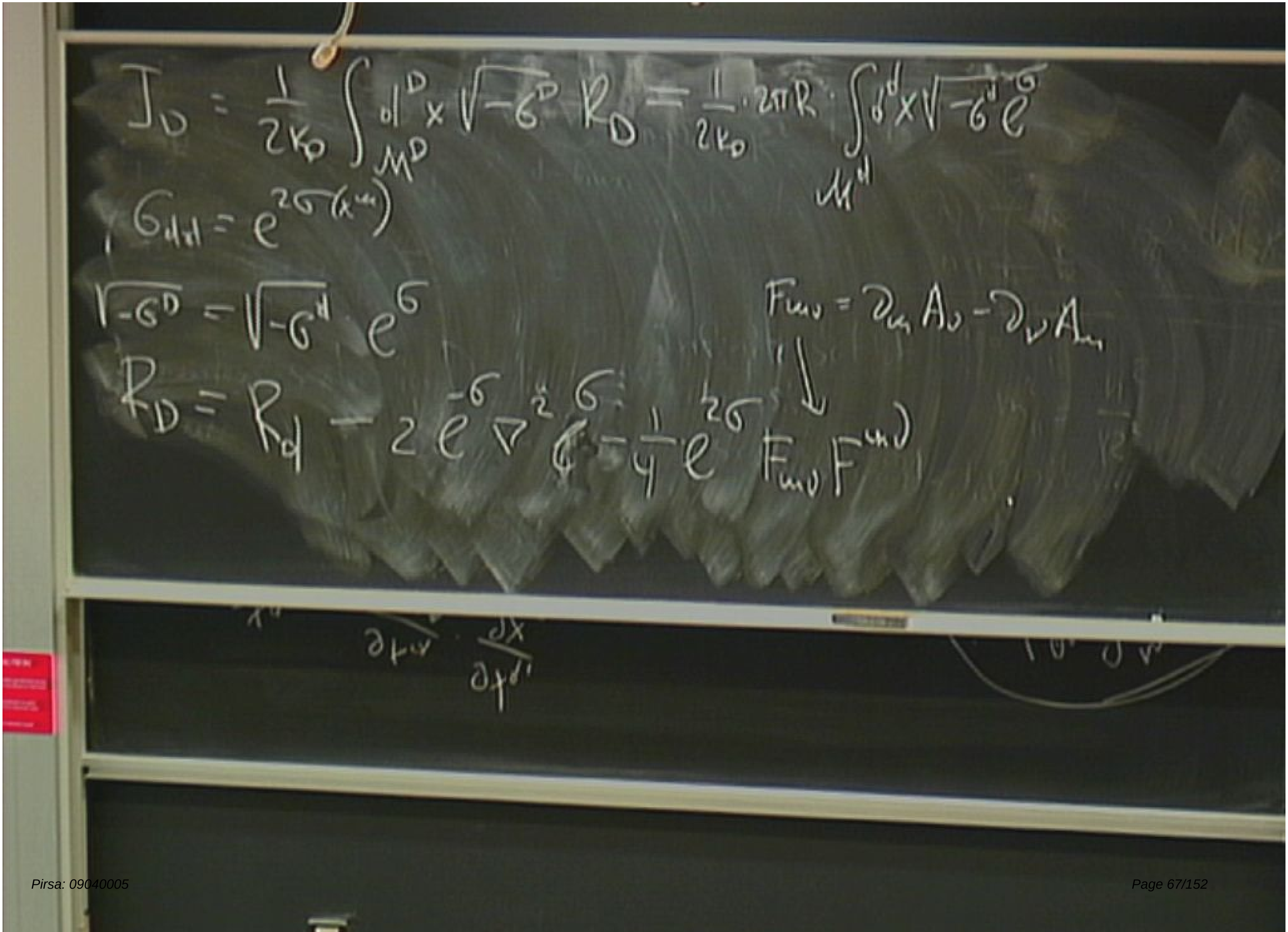
$$I_D = \frac{1}{2\kappa_D}\int d^D x \sqrt{-G^D}\, R_D = \frac{1}{2\kappa_D}\cdot 2\pi R \cdot \int d^d x$$

$$G_{d+1} = e^{2\sigma(x^\mu)}$$

$$\sqrt{-G^D} = \sqrt{-G^d}\, e^{\sigma}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$R_D = R_d - 2e^{-\sigma}\nabla^2 e^{\sigma} - \frac{1}{4}e^{2\sigma}F_{\mu\nu}F^{\mu\nu}$$


$I_D = \frac{1}{2\kappa_D} \int_{M^D} d^D x \sqrt{-G^D}\, R_D = \frac{1}{2\kappa_D} \cdot 2\pi R \cdot \int_{M^d} d^d x \sqrt{-G^d}\, e^{\sigma}$
$G_{dd} = e^{2\sigma(x^\mu)}$
$\sqrt{-G^D} = \sqrt{-G^d}\, e^{\sigma}$
$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$
$R_D = R_d - 2 e^{-\sigma} \nabla^2 e^{\sigma} - \frac{1}{4} e^{2\sigma} F_{\mu\nu} F^{\mu\nu}$

$$I_D = \frac{1}{2\kappa_D}\int_{M^D} d^D x \sqrt{-G^D}\, R_D = \frac{1}{2\kappa_D}\cdot 2\pi R \cdot \int_{M^d} d^d x \sqrt{-G^d}\, e^{\sigma}\left[R_d - \frac{1}{4} e^{2\sigma} F_{\mu\nu}F^{\mu\nu}\right]$$

$$G_{dd} = e^{2\sigma(x^{\mu})}$$

$$\sqrt{-G^D} = \sqrt{-G^d}\; e^{\sigma}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$R_D = R_d - 2 e^{-\sigma} \nabla^2 e^{\sigma} - \frac{1}{4} e^{2\sigma} F_{\mu\nu}F^{\mu\nu}$$

$$J_D = \frac{1}{2\kappa_D}\int_{M^D} d^D x \sqrt{-G^D}\, R_D = \frac{1}{2\kappa_D}\cdot 2\pi R \cdot \int_{M^d} d^d x \sqrt{-G^d}\, e^{\sigma}\left[R_d - \frac{1}{4} e^{2\sigma} F_{\mu\nu}F^{\mu\nu}\right]$$

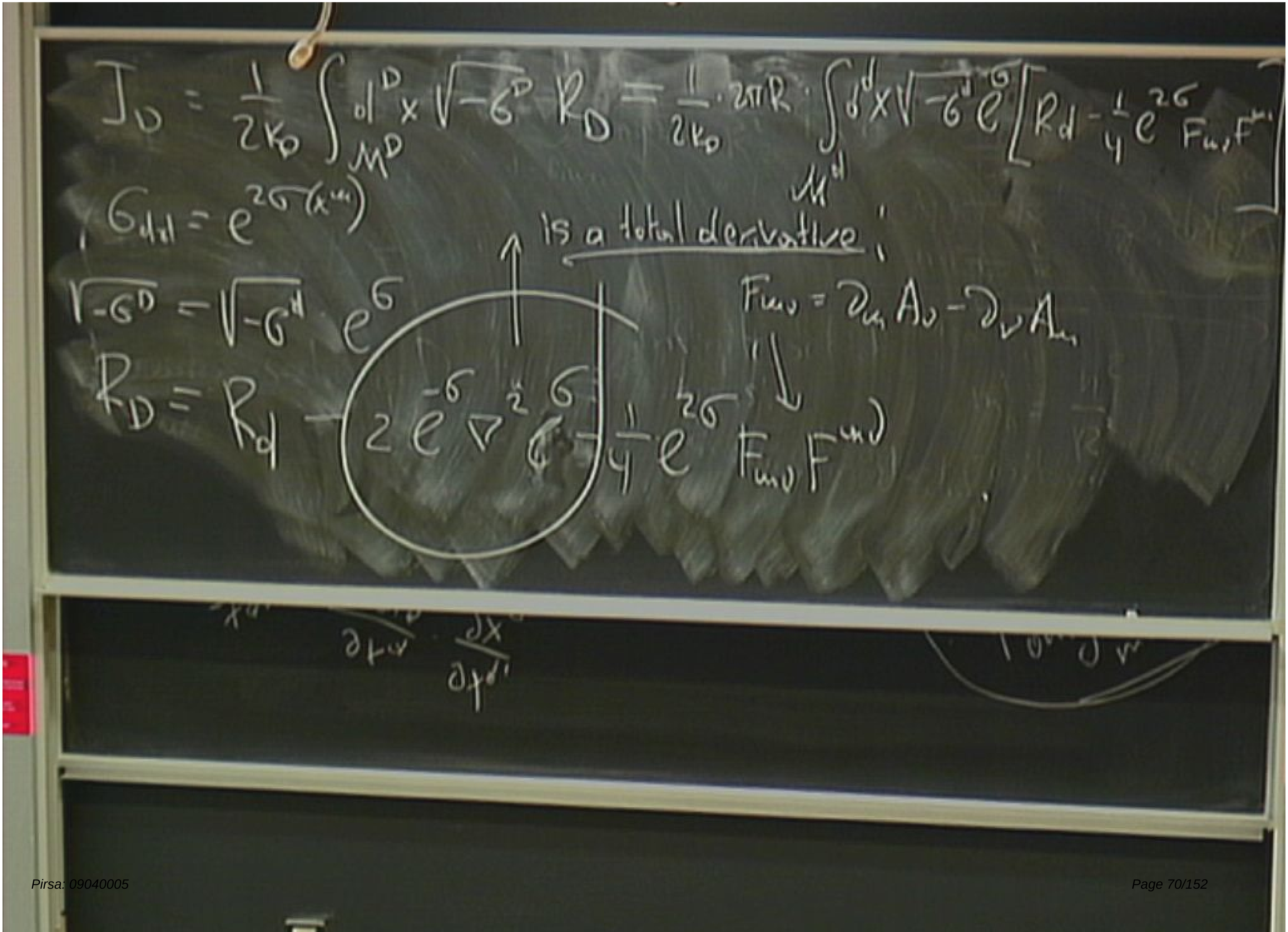
$$G_{dd} = e^{2\sigma(x^\mu)}$$

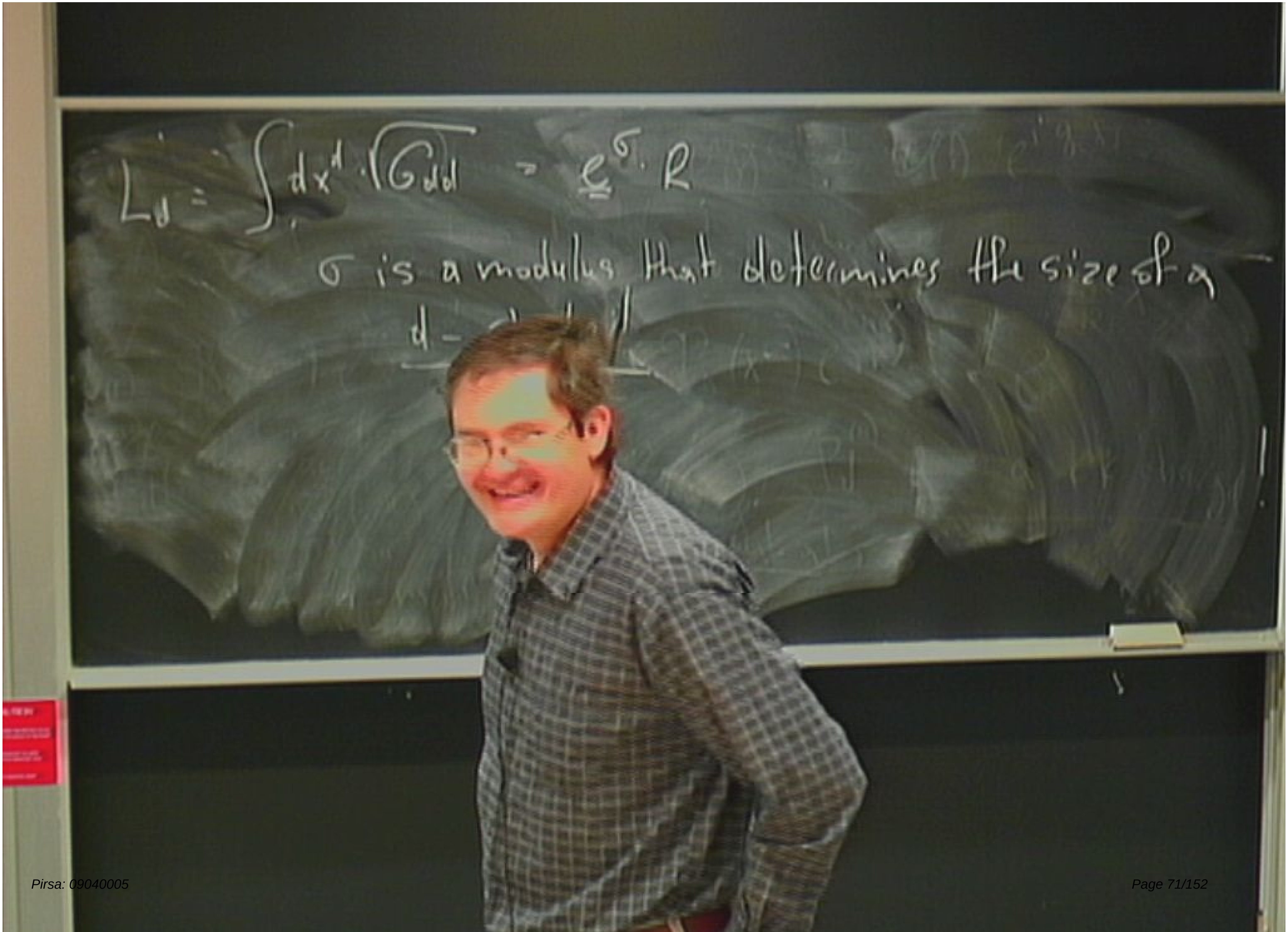
$$\sqrt{-G^D} = \sqrt{-G^d}\, e^{\sigma}$$

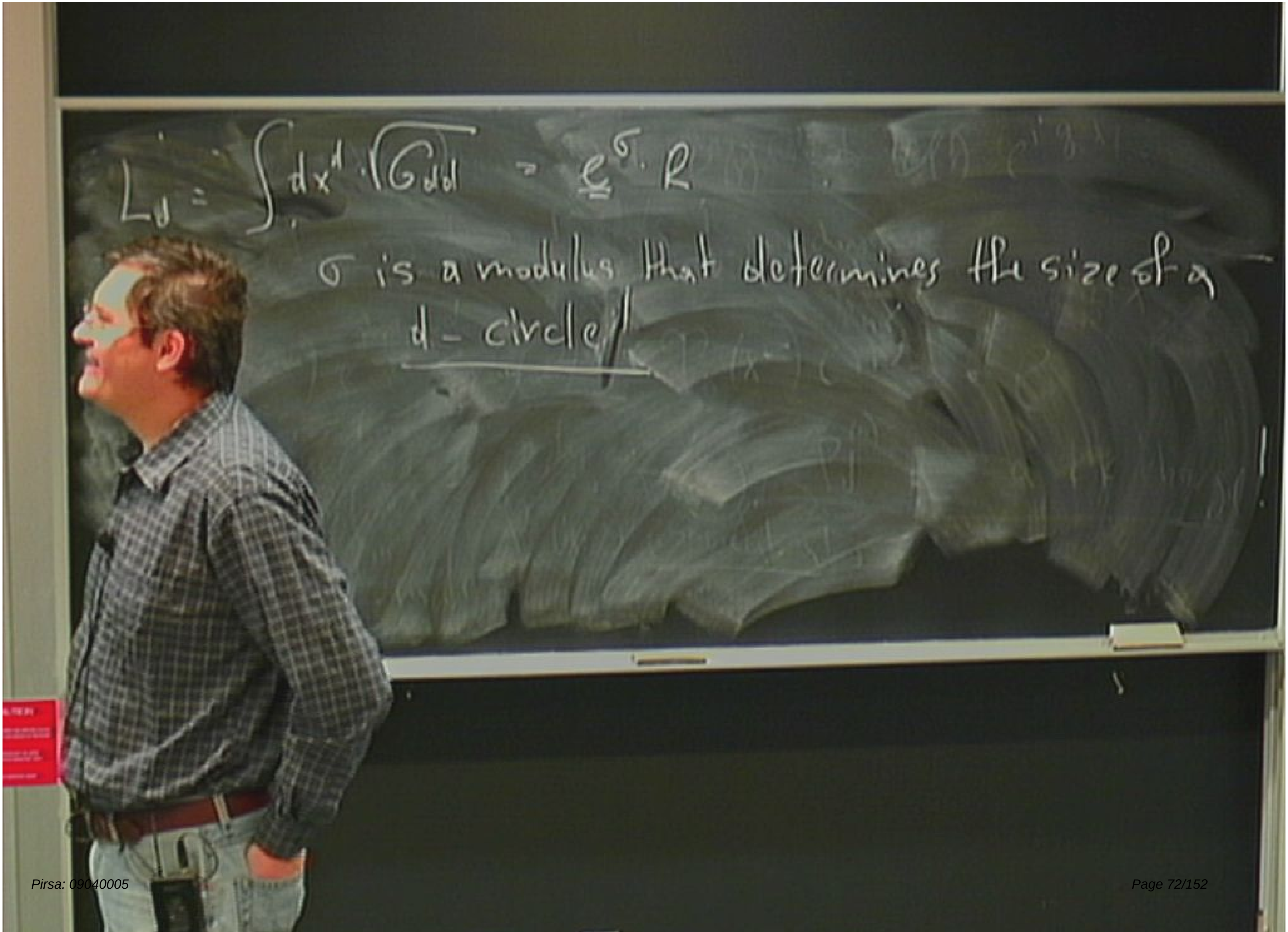
$$R_D = R_d - \left(2 e^{-\sigma}\nabla^2 e^{\sigma} - \frac{1}{4} e^{2\sigma} F_{\mu\nu}F^{\mu\nu}\right)$$

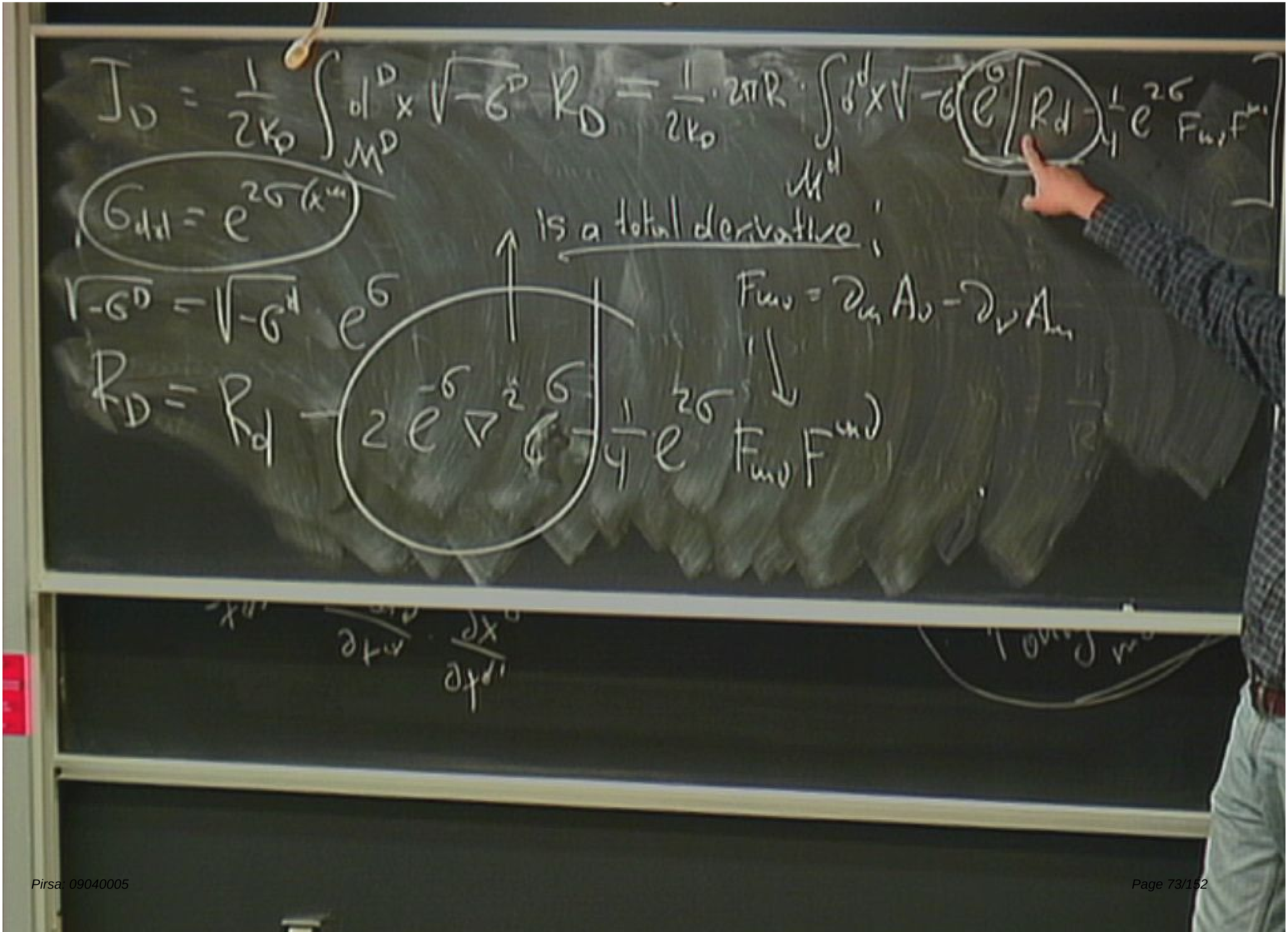
is a total derivative;

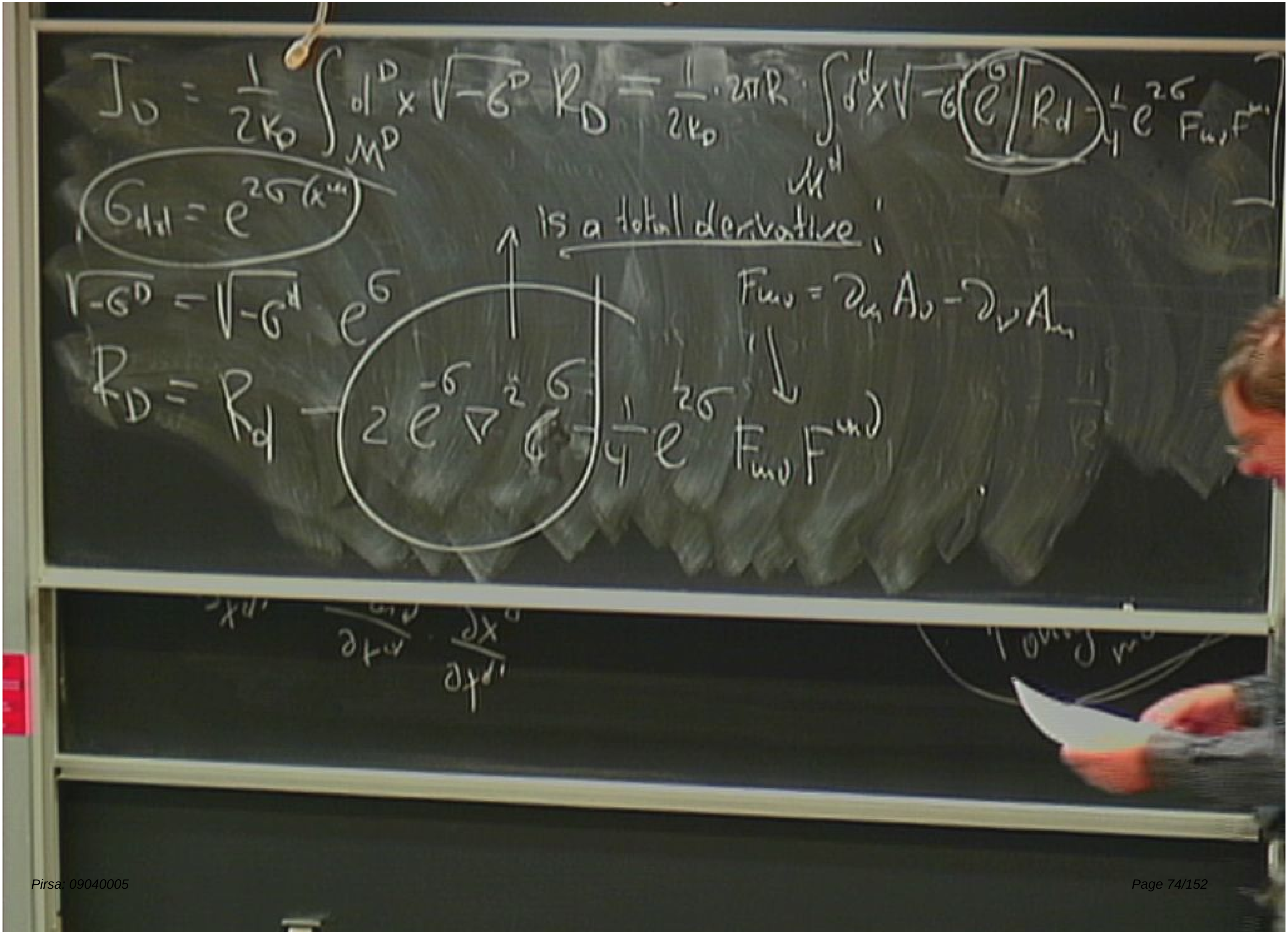
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$


I_D = 1/2κ_D ∫_{M^D} d^D x √(-G^D) R_D = 1/2κ_D · 2πR · ∫_{M^d} d^d x √(-G^d) e^σ [R_d - 1/4 e^{2σ} F_{μν}F^{μν}]
G_{dd} = e^{2σ(x^μ)}
is a total derivative
√(-G^D) = √(-G^d) e^σ
F_{μν} = ∂_μ A_ν - ∂_ν A_μ
R_D = R_d - (2 e^{-σ} ∇² e^σ - 1/4 e^{2σ} F_{μν} F^{μν})


L_g = ∫dx^d √G_dd = e^σ · R
σ is a modulus that determines the size of a


L_d = ∫dx^d √G_dd = e^σ · R
σ is a modulus that determines the size of a
d-circle


$I_D = \frac{1}{2\kappa_D} \int_{M^D} d^D x \sqrt{-G^D}\, R_D = \frac{1}{2\kappa_D} \cdot 2\pi R \cdot \int_{M^d} d^d x \sqrt{-G}\, e^{\sigma} \left[R_d - \frac{1}{4} e^{2\sigma} F_{\mu\nu} F^{\mu\nu} \right]$
$G_{d+1} = e^{2\sigma(x^{\mu})}$
$\sqrt{-G^D} = \sqrt{-G^d}\, e^{\sigma}$
$R_D = R_d - \left(2 e^{-\sigma} \nabla^2 e^{\sigma} - \frac{1}{4} e^{2\sigma} F_{\mu\nu} F^{\mu\nu} \right)$
is a total derivative;
$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$



is a total derivative

$$S_D = \frac{1}{2\kappa_D}\int_{M^D} d^Dx \sqrt{-G^D}\, R_D = \frac{1}{2\kappa_D}\cdot 2\pi R \cdot \int_{M^d} d^dx \sqrt{-G}\, e^{\sigma}\left[R_d - \frac{1}{4} e^{2\sigma} F_{\mu\nu}F^{\mu\nu}\right]$$

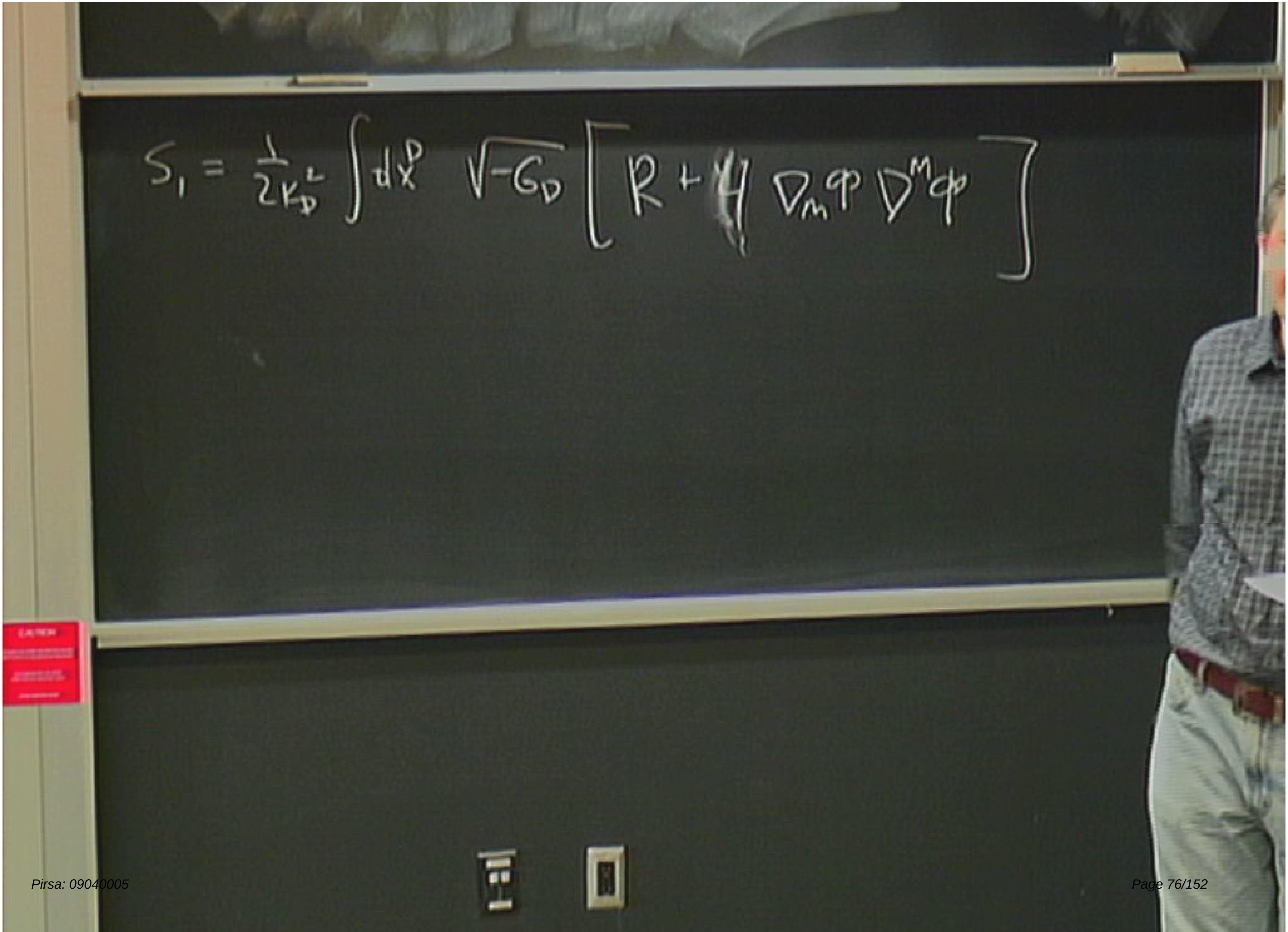
$$G_{d+1\,d+1} = e^{2\sigma(x^\mu)}$$

$$\sqrt{-G^D} = \sqrt{-G^d}\; e^{\sigma}$$

$$R_D = R_d - \left(2 e^{-\sigma} \nabla^2 e^{\sigma} - \frac{1}{4} e^{2\sigma} F_{\mu\nu}F^{\mu\nu}\right)$$

is a total derivative;

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$


$S_1 = \frac{1}{2\kappa_D^2} \int d^D x \sqrt{-G_D} \left[R + 4 \nabla_M \varphi \nabla^M \varphi \right]$

$$S_1 = \frac{1}{2\kappa_D^2}\int d x^D \sqrt{-G_D}\left[R + 4\, \nabla_\mu \varphi \nabla^\mu \varphi \right]$$

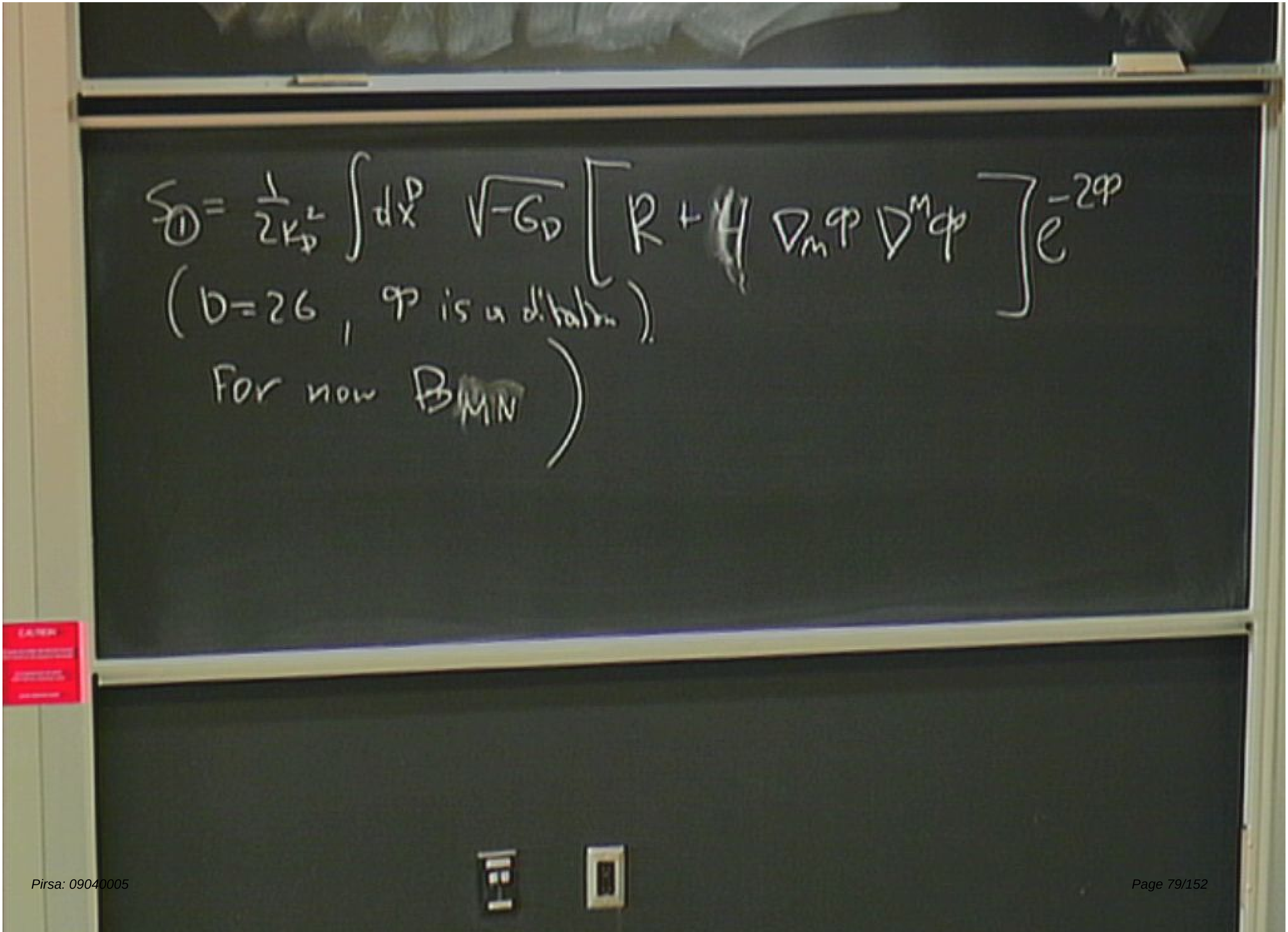
($D = 26$, φ is the dilaton)

For now $B_{\mu\nu} = 0$

$$S_{(1)} = \frac{1}{2\kappa_D^2}\int d^Dx \, \sqrt{-G_D}\left[R + 4 D_M\varphi D^M\varphi \right]$$

($D=26$, φ is a dilaton).

For now B_{MN})


S_① = 1/(2κ_D²) ∫ dx^D √(−G_D) [R + 4 D_M φ D^M φ] e^{−2φ}
(D = 26 , φ is a dilaton)
For now B_MN)

$$I_D = \frac{1}{2\kappa_D}\int_{M^D} d^D x\,\sqrt{-G^D}\,R_D = \frac{1}{2\kappa_D}\cdot 2\pi R \int_{M^d} d^d x \sqrt{-G}\, e^{\sigma}\left[R_d - \frac{1}{4} e^{2\sigma} F_{\mu\nu}F^{\mu\nu}\right]$$

$$G_{dd} = e^{2\sigma(x^\mu)}$$

$$\sqrt{-G^D} = \sqrt{-G^d}\; e^{\sigma}$$

$$R_D = R_d - \left(2 e^{-\sigma}\nabla^2 e^{\sigma} - \frac{1}{4} e^{2\sigma} F_{\mu\nu}F^{\mu\nu}\right)$$

is a total derivative;

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$S_{(1)} = \frac{1}{2\kappa_D^2}\int dx^D \sqrt{-G_D}\left[R + 4\, \nabla_M \varphi \nabla^M \varphi \right] e^{-2\varphi}$$

(D = 26 , φ is a dilaton).

For now B_{MN})

$$= \frac{\pi R}{\kappa_D^2}\int dx^d \left(-G_d\right)^{1/2} e^{-2\varphi + \sigma}$$

$$S_{①} = \frac{1}{2\kappa_D^2}\int dx^D \sqrt{-G_D}\left[R + 4\,\nabla_M\varphi\,\nabla^M\varphi\right]e^{-2\varphi}$$

($D = 26$, φ is a dilaton)

For now B_{MN}

$$= \frac{\pi R}{\kappa_D^2}\int dx^d\,(-G_d)^{1/2}\,e^{-2\varphi+\sigma}\left[R_d - 4\nabla\varphi\nabla\sigma + 4\nabla\varphi\nabla\varphi - \frac{1}{4}e^{2\sigma}F_{\mu\nu}F^{\mu\nu}\right]$$

$$S_{①} = \frac{1}{2\kappa_D^2}\int dx^D \sqrt{-G_D}\left[R + 4\nabla_M\varphi\nabla^M\varphi\right]e^{-2\varphi}$$

($D = 26$, φ is a dilaton)

For now B_{MN}

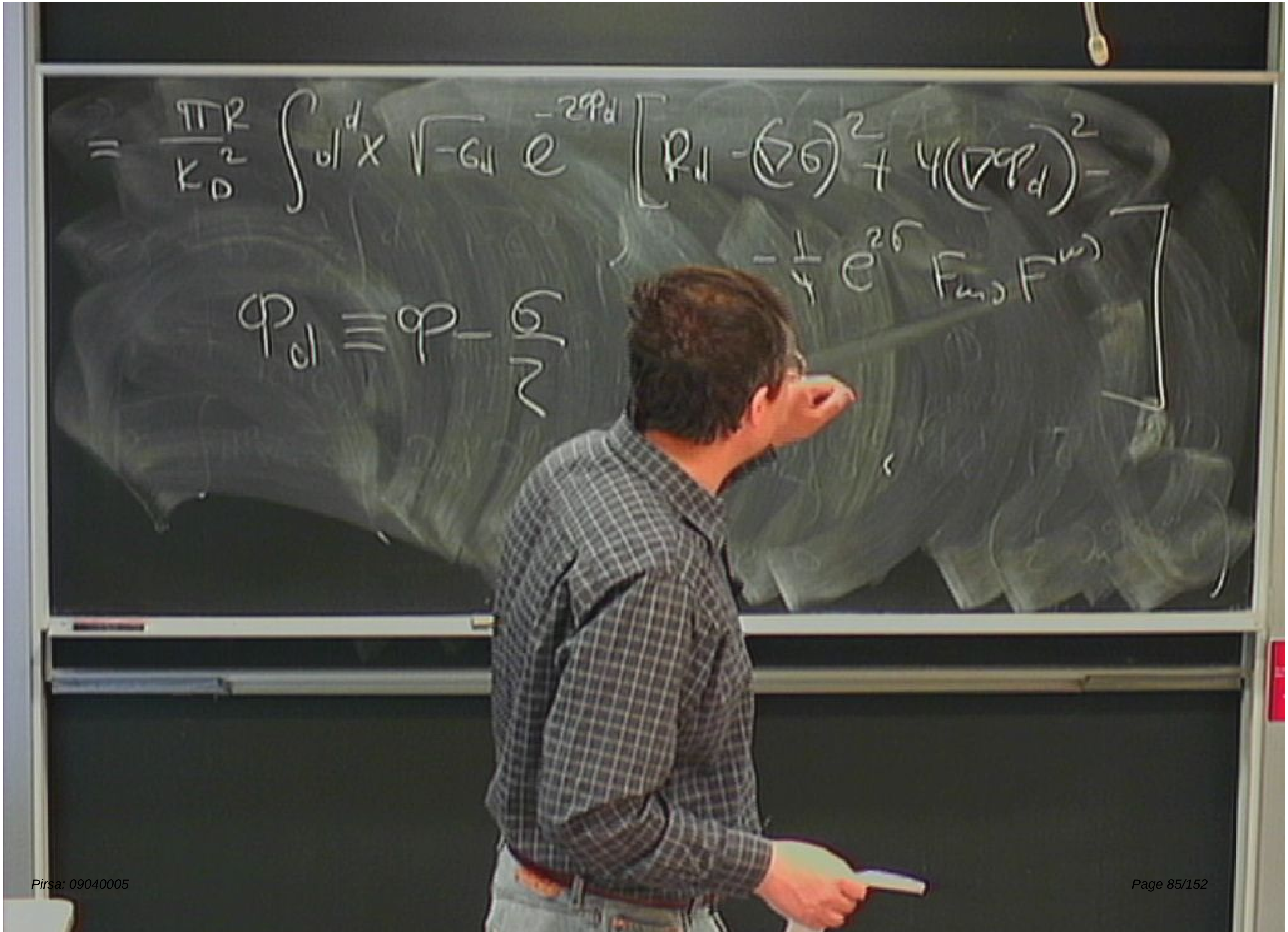
$$= \frac{\pi R}{\kappa_D^2}\int dx^d\left(-G_d\right)^{1/2} e^{-2\varphi+\sigma}\left[R_d - 4\nabla\varphi\nabla\sigma + 4\nabla\varphi\nabla\varphi - \frac{1}{4}e^{2\sigma}F_{\mu\nu}F^{\mu\nu}\right]$$

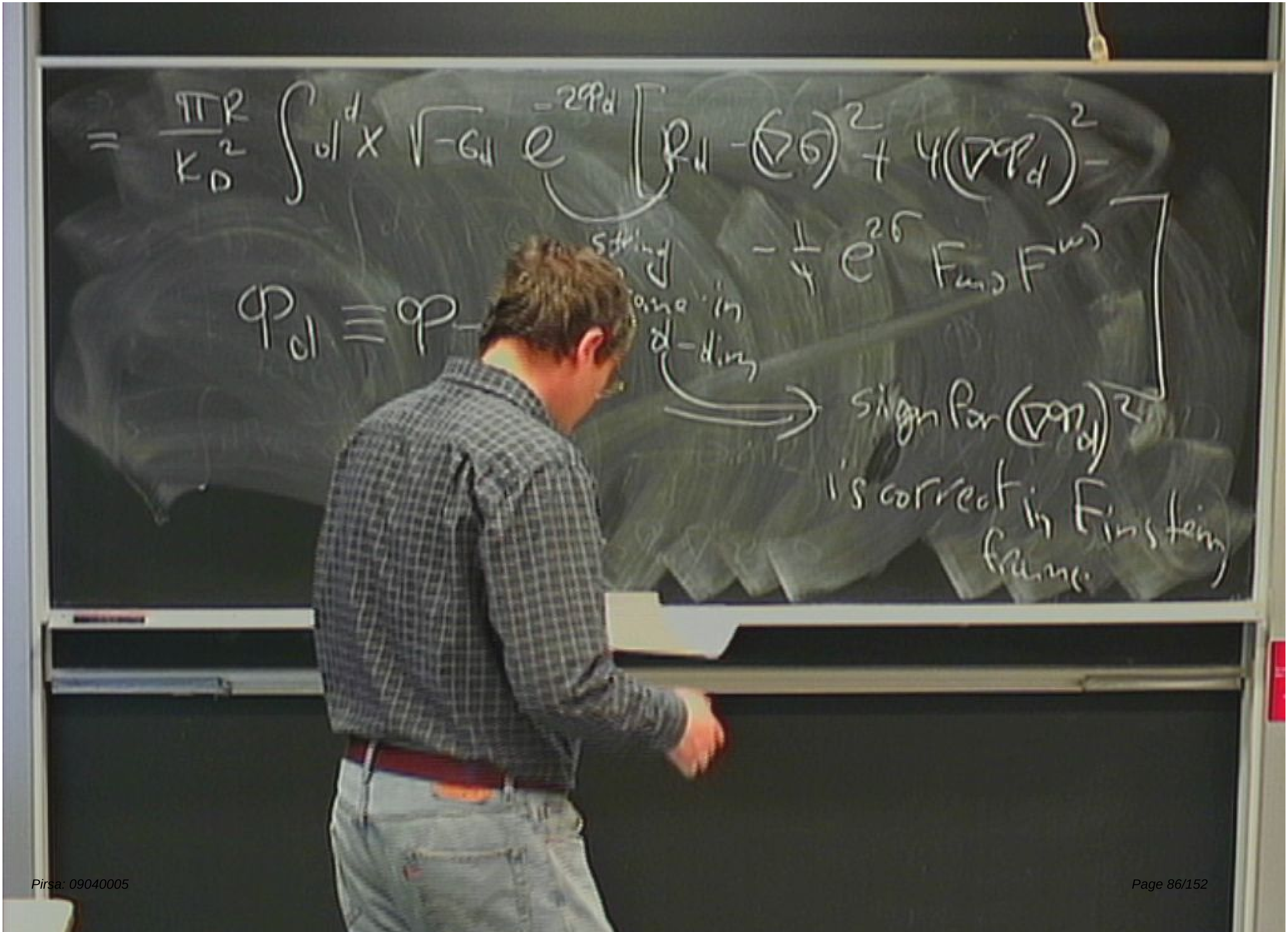
$$S_{①} = \frac{1}{2\kappa_D^2} \int d x^D \sqrt{-G_D} \left[R + 4 \nabla_M \varphi \nabla^M \varphi \right] e^{-2\varphi}$$

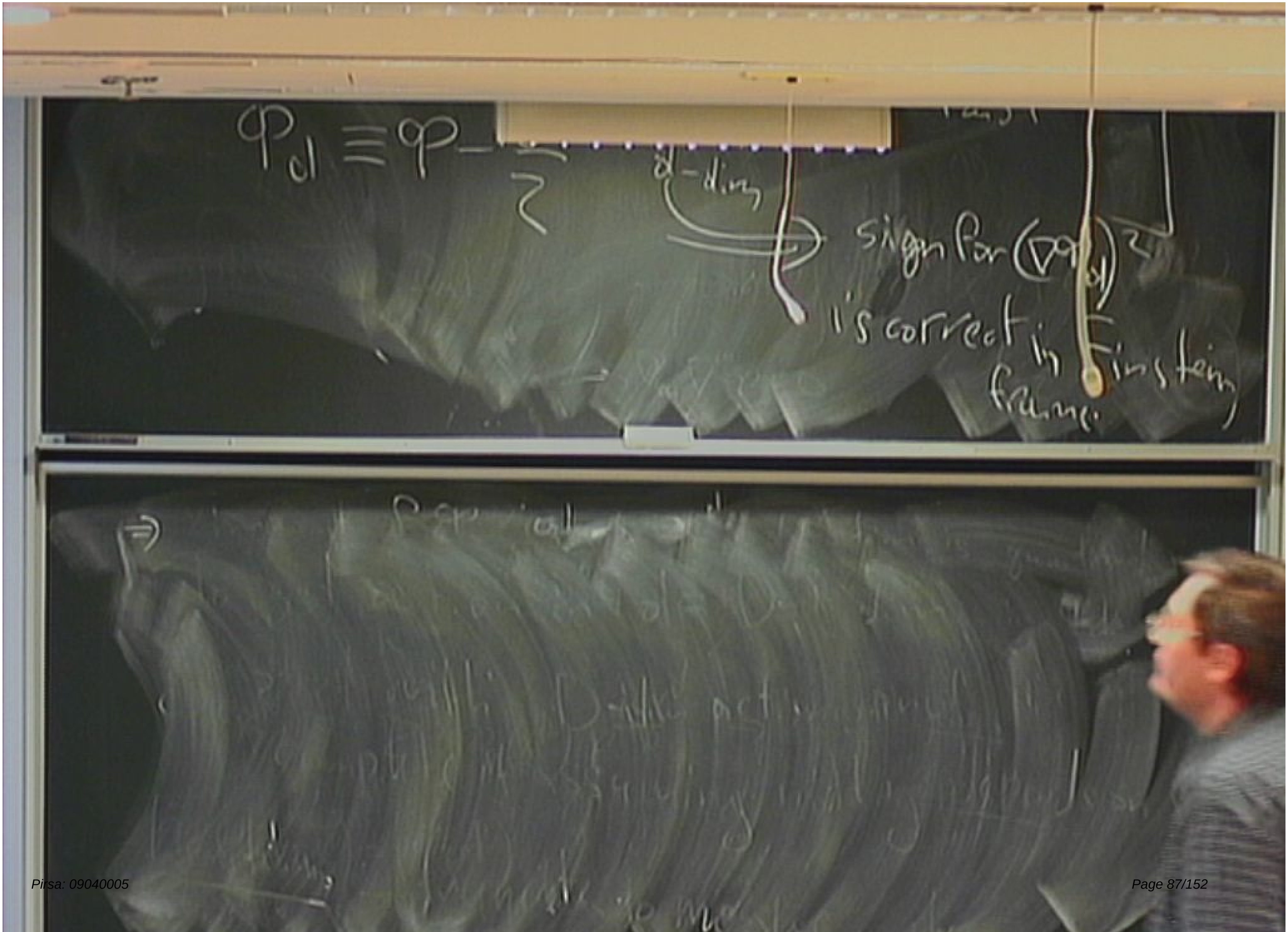
(D = 26 , φ is a dilaton).

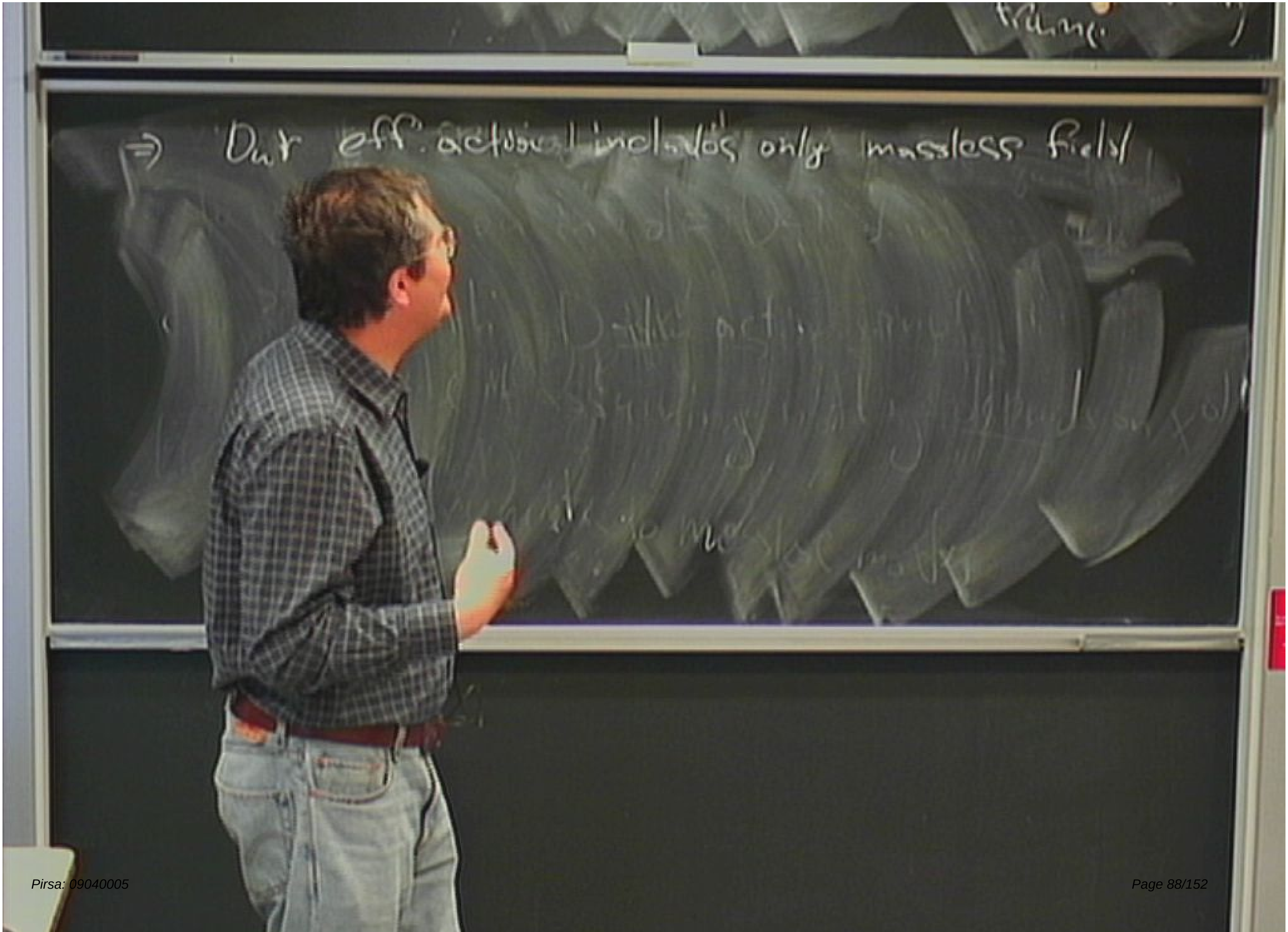
For now B_{MN})

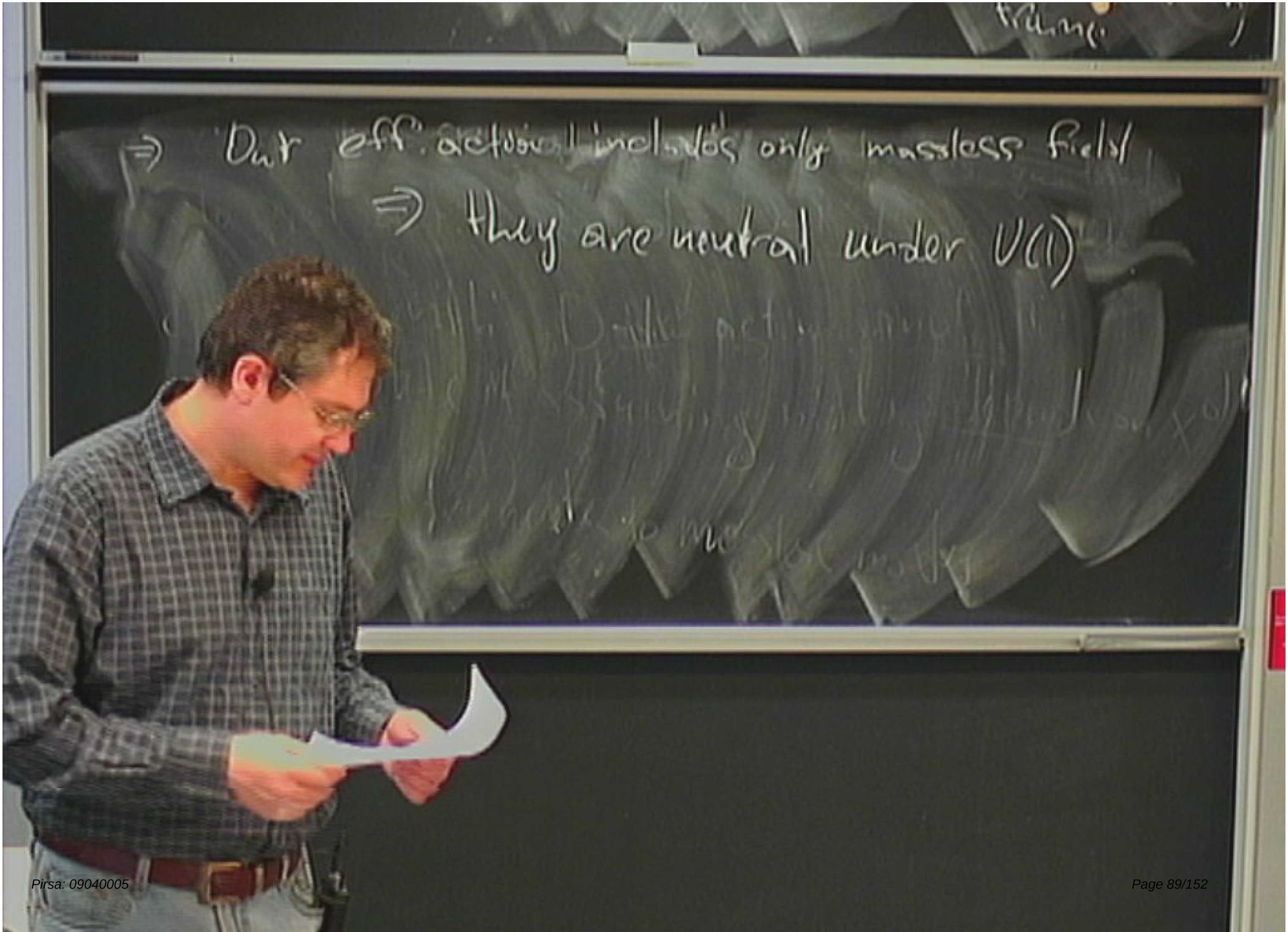
$$= \frac{\pi R}{\kappa_D^2} \int dx^d (-G_d)^{1/2} e^{-2\varphi+\sigma} \left[R_d - 4 \nabla\varphi \nabla\sigma + 4 \nabla\varphi\nabla\varphi - \frac{1}{4} e^{2\sigma} F_{\mu\nu} F^{\mu\nu} \right]$$

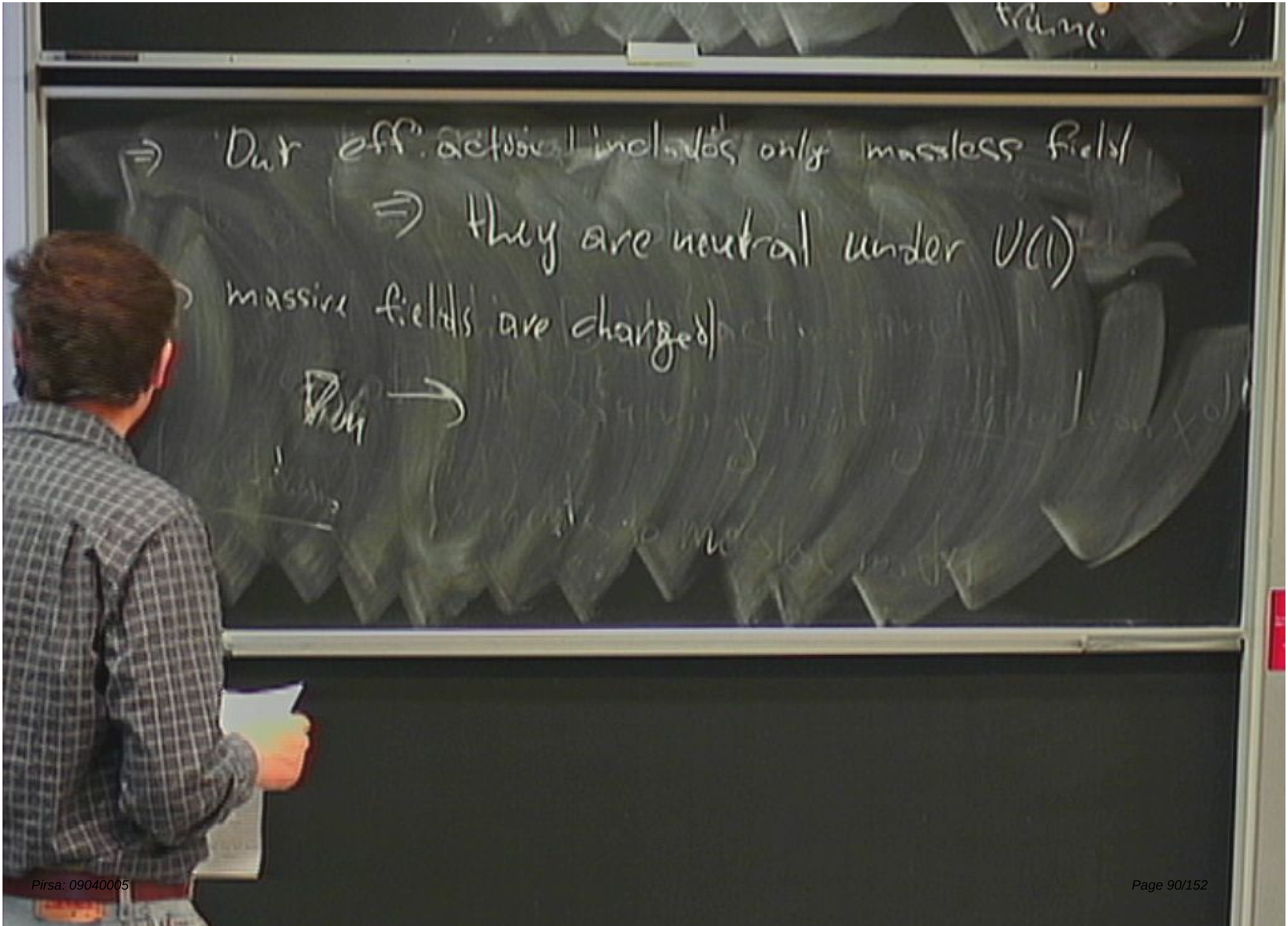

$= \frac{\pi R}{\kappa_D^2} \int d^d x \sqrt{-G_d}\, e^{-2\varphi_d} \left[R_d - (\nabla\sigma)^2 + 4(\nabla\varphi_d)^2 - \frac{1}{4} e^{2\sigma} F_{\mu\nu}F^{\mu\nu} \right]$
$\varphi_d \equiv \varphi - \frac{\sigma}{2}$

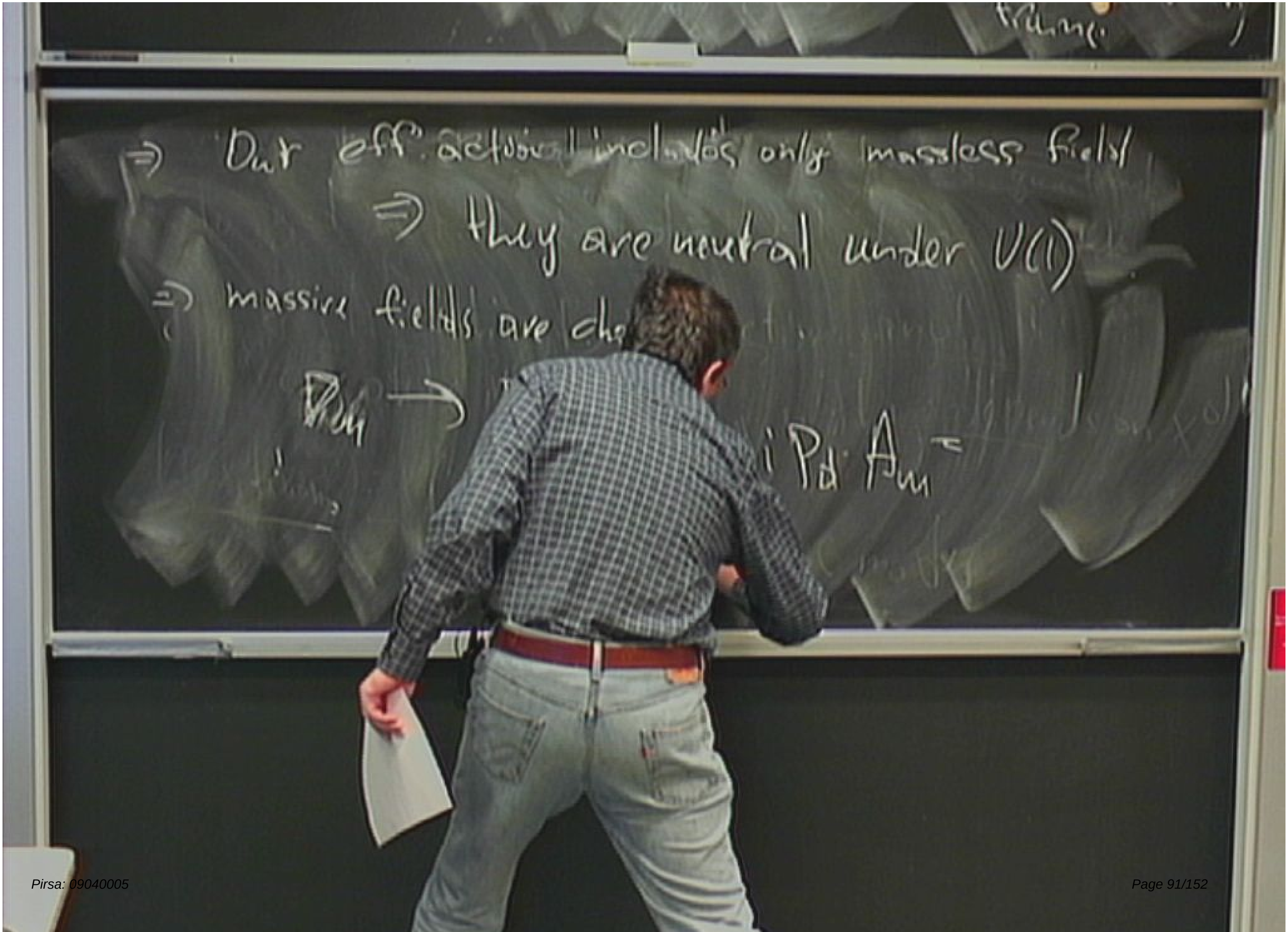

string
frame in
d-dim
sign for
is correct in Einstein
frame.

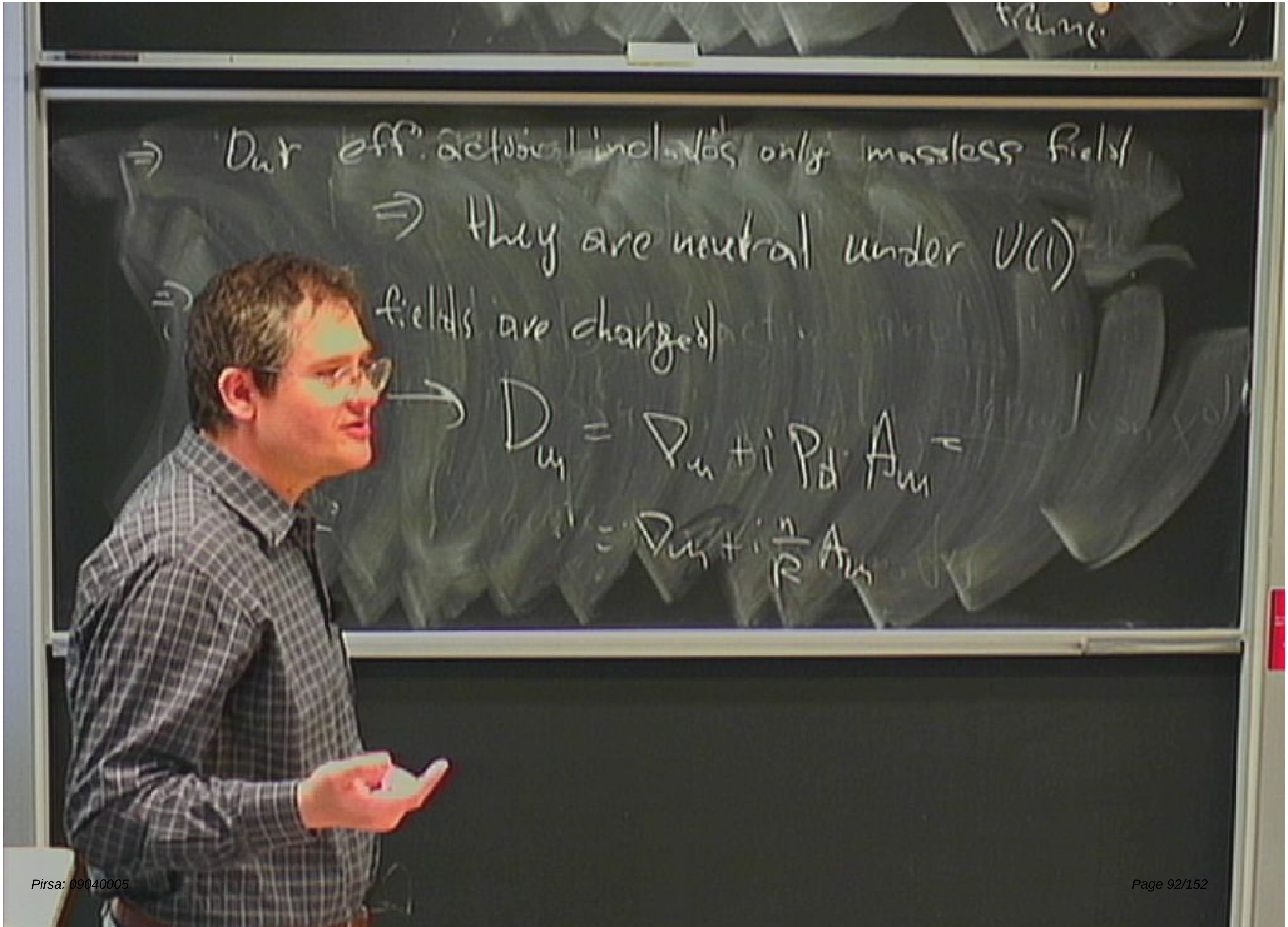

d-dim
sign for $(\nabla\varphi_{cl})^2$
is correct in Einstein frame.


⇒ Our eff. action includes only massless field


⇒ Our eff. action includes only massless field
⇒ they are neutral under U(1)


⇒ Our eff. action includes only massless field
⇒ they are neutral under U(1)
massive fields are charged


⇒ Our eff. action includes only massless field
⇒ they are neutral under U(1)
⇒ massive fields are


eff. action includes only massless field
they are neutral under U(1)
fields are charged
D_μ = ∇_μ + i P_d A_μ
= ∇_μ + i n/R A_μ

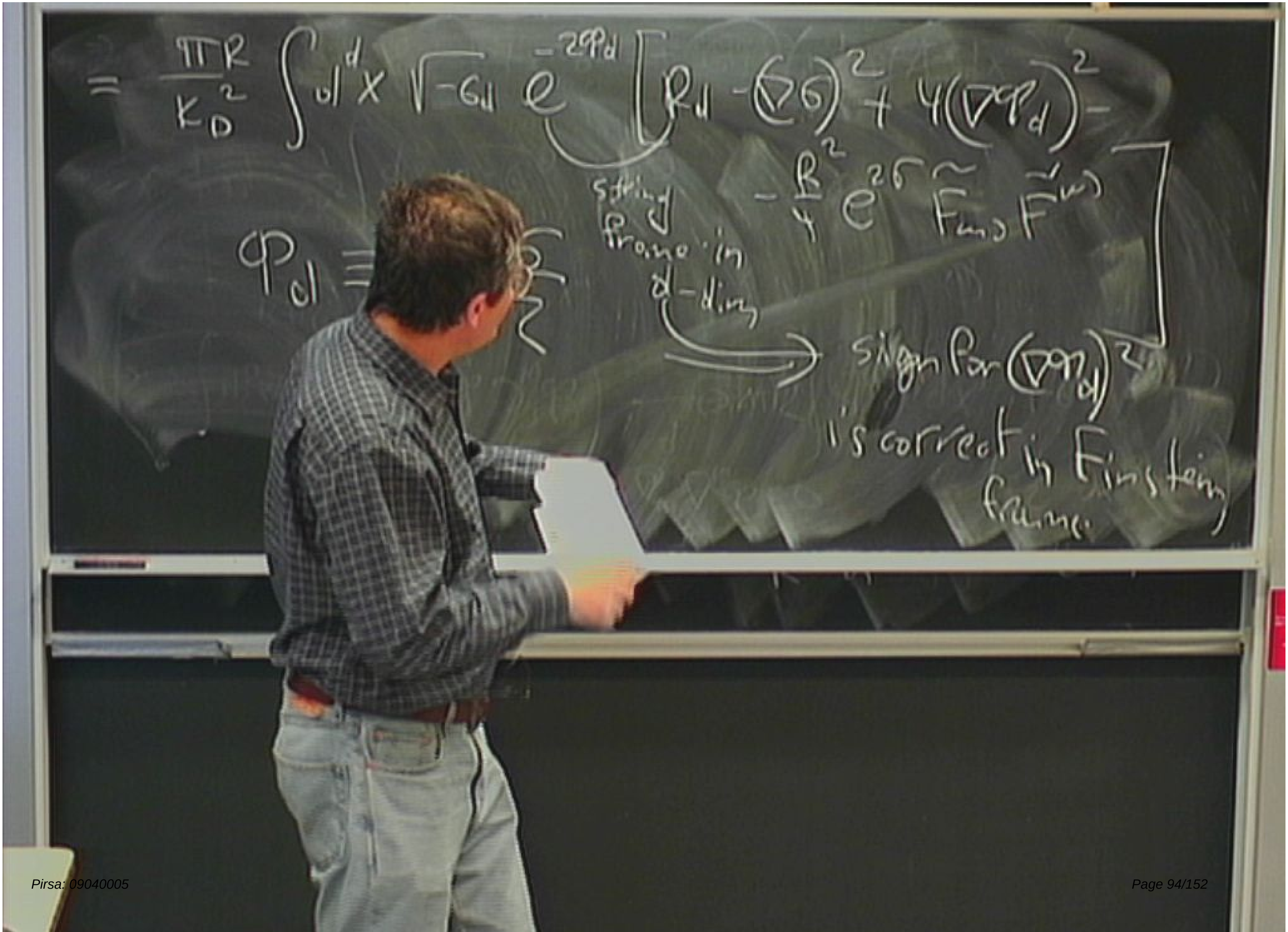
⇒ Our eff. action includes only massless field

⇒ they are neutral under $U(1)$

⇒ massive fields are charged

$$\tilde{A}_\mu = \frac{1}{R} A_\mu$$

$$\partial_\mu \to D_\mu = \partial_\mu + i\,(p_d)\,A_\mu = \partial_\mu + i\frac{n}{R} A_\mu = \partial_\mu + i\,(n)\,\tilde{A}_\mu$$


string frame in d-dim
sign for $(\nabla\varphi_d)^2$
is correct in Einstein frame

$$= \frac{\pi R}{\kappa_D^2}\int d^dx \sqrt{-G_d}\, e^{-2\varphi_d}\left[R_d - (\nabla\sigma)^2 + 4(\nabla\varphi_d)^2 - \frac{R^2}{4} e^{2\sigma}\tilde{F}_{(\mu\nu)}\tilde{F}^{(\mu\nu)}\right]$$

string frame in d-dim

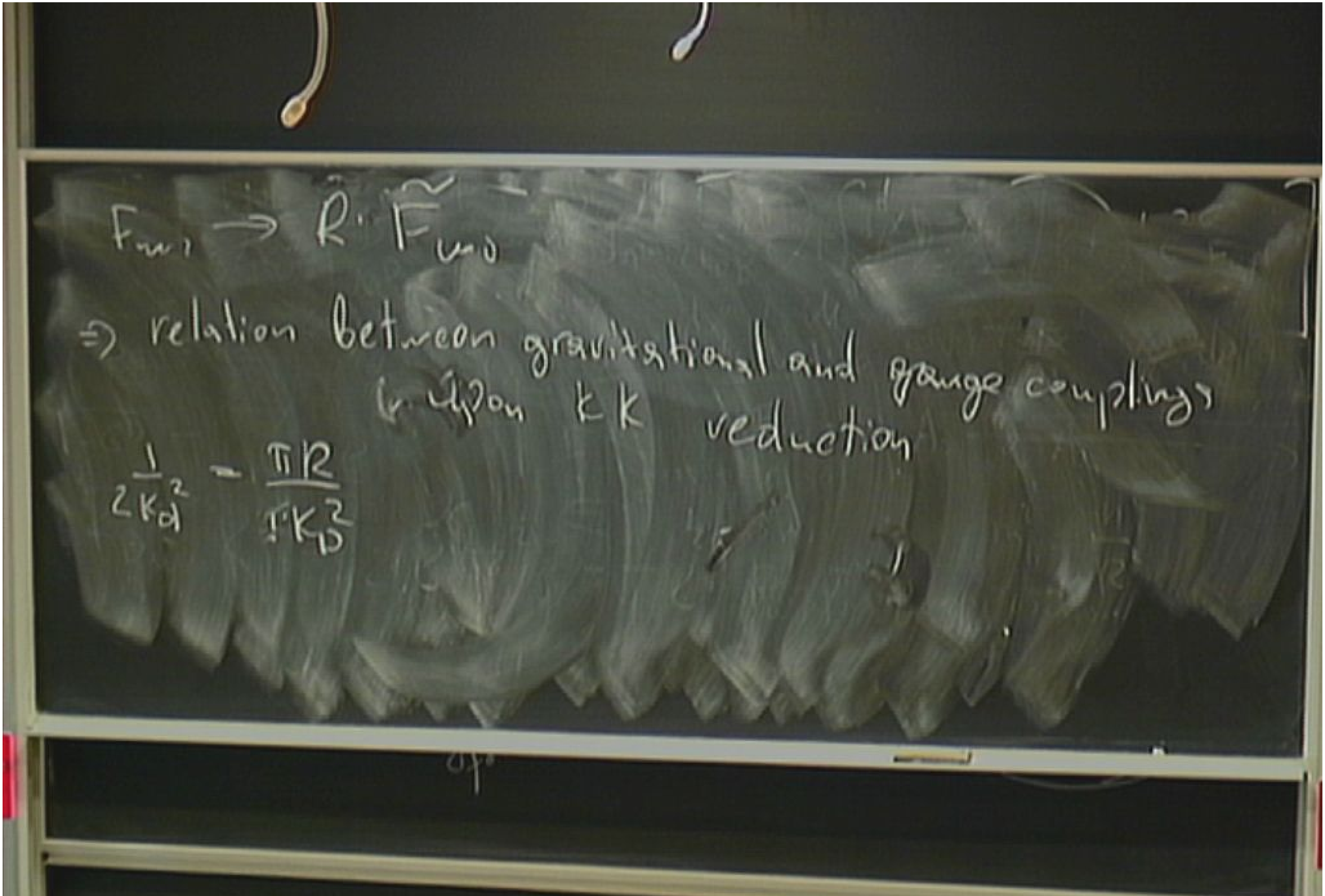
$$\varphi_d \equiv \varphi - \frac{\sigma}{2}$$

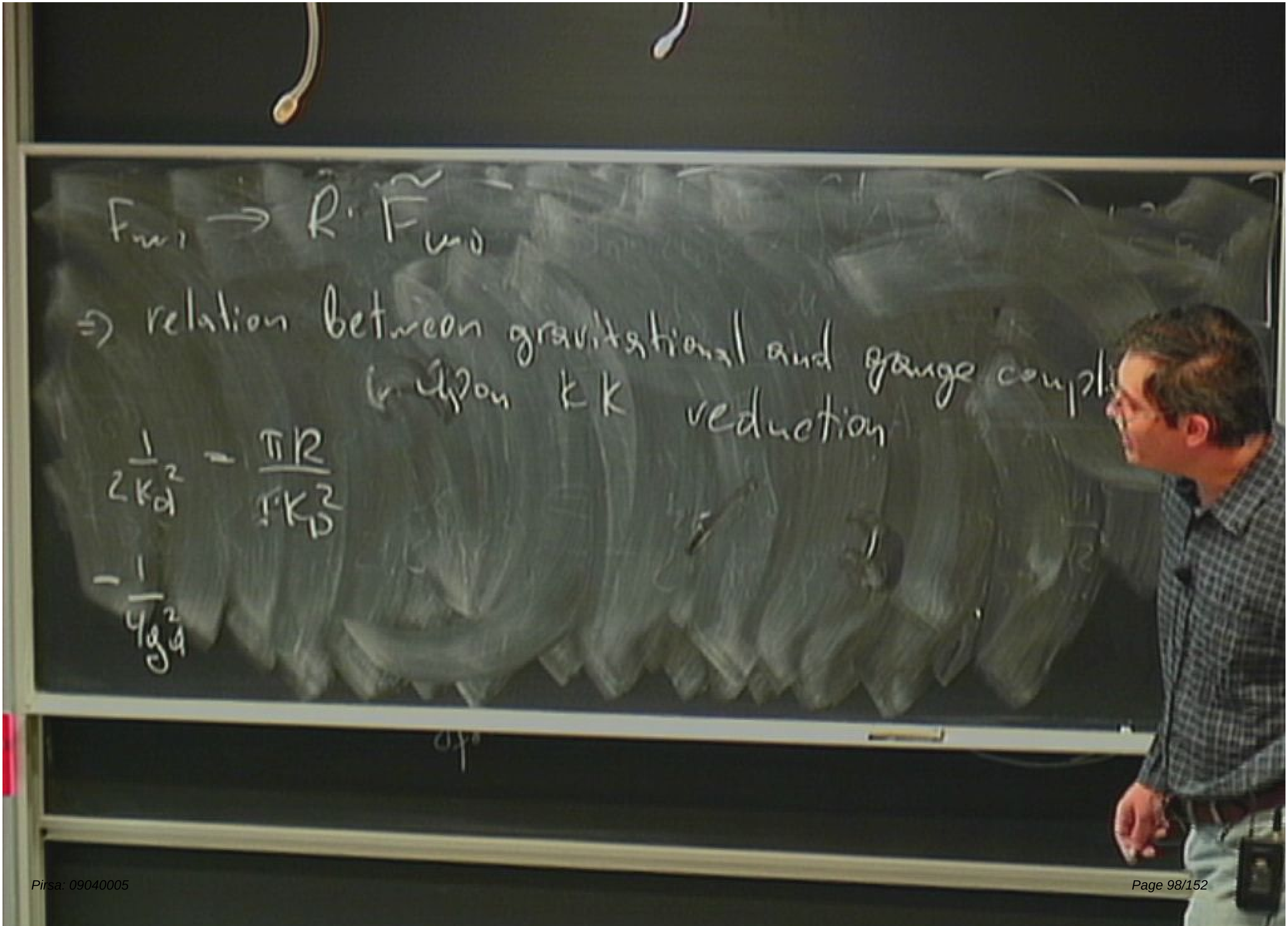
→ sign for $(\nabla\varphi_d)^2$ is correct in Einstein frame.

$F_{\mu\nu} \to R \cdot \tilde{F}_{\mu\nu}$

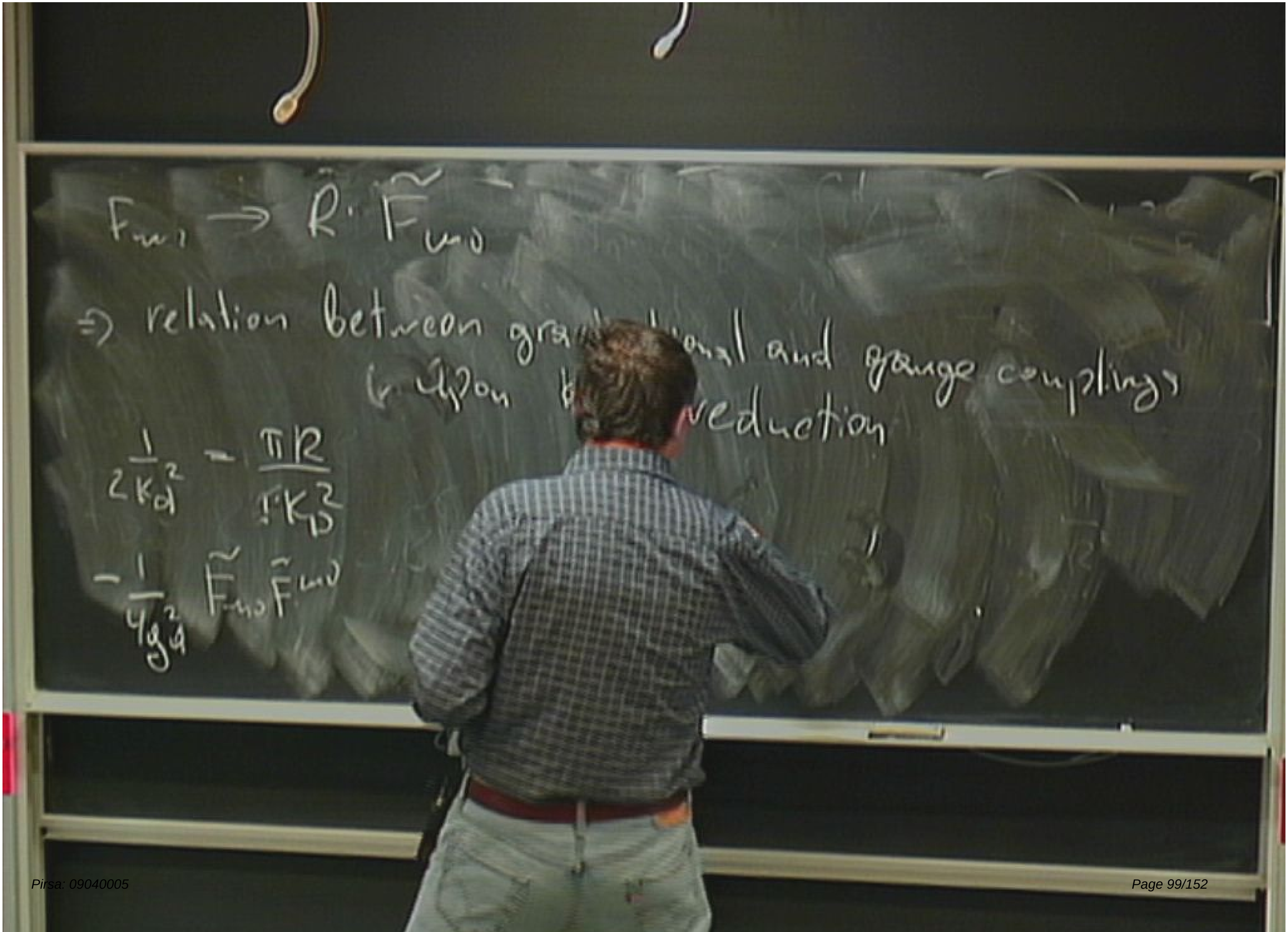
$\Rightarrow$ relation between gravitational and gauge couplings

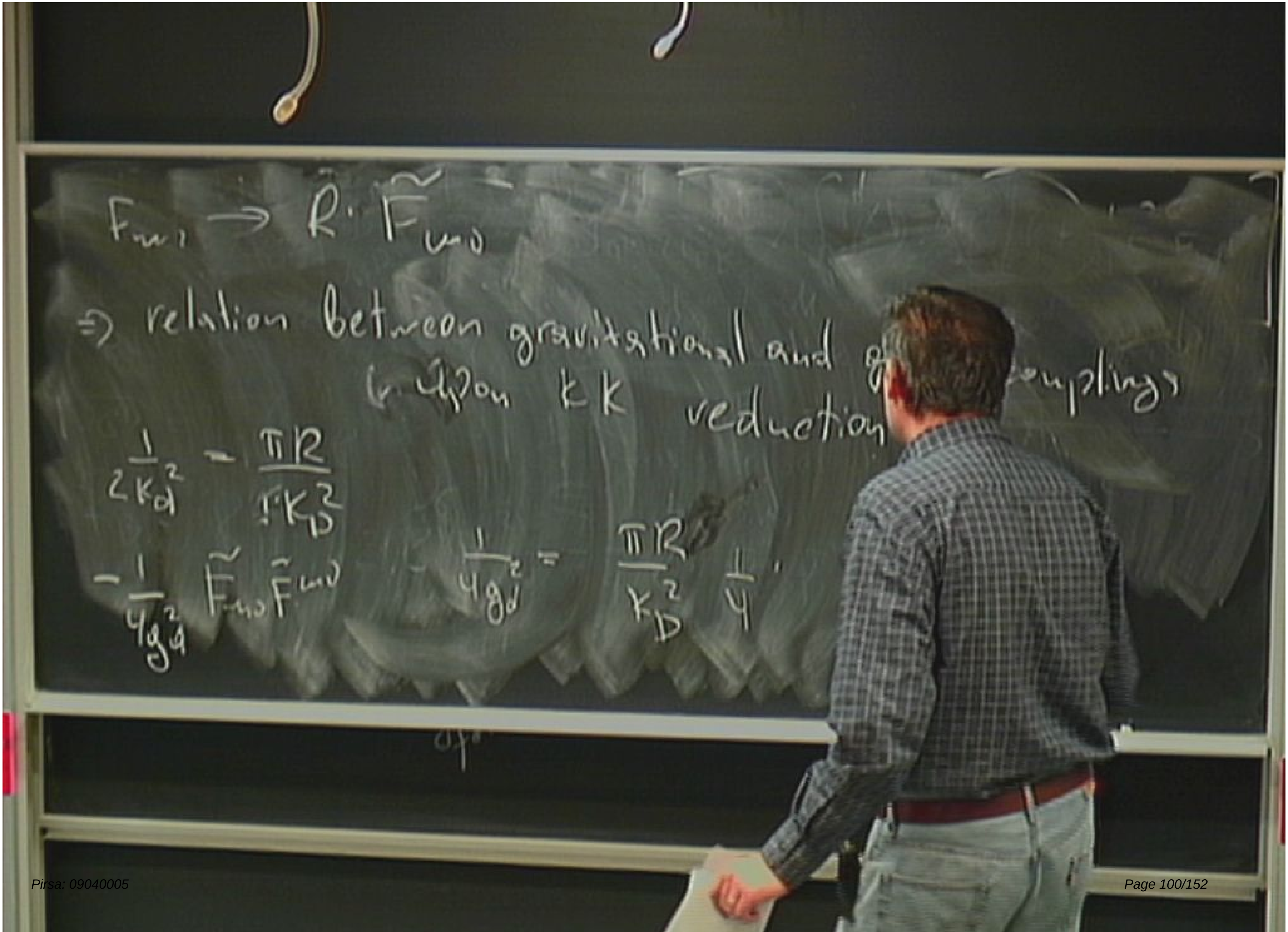
upon KK reduction


$F_{\mu\nu} \to R \cdot \tilde{F}_{\mu\nu}$
⇒ relation between gravitational and gauge couplings
upon KK reduction
$\frac{1}{2\kappa_d^2} = \frac{\pi R}{2\kappa_D^2}$



relation between gravitational and gauge coupl
upon KK reduction


$F_{\mu\nu} \rightarrow \tilde{R} \cdot \tilde{F}_{\mu\nu}$
⇒ relation between
and gauge couplings
reduction
$\frac{1}{2\kappa_d^2} = \frac{\pi R}{\kappa_D^2}$
$-\frac{1}{4g_4^2} \tilde{F}_{\mu\nu} \tilde{F}^{\mu\nu}$

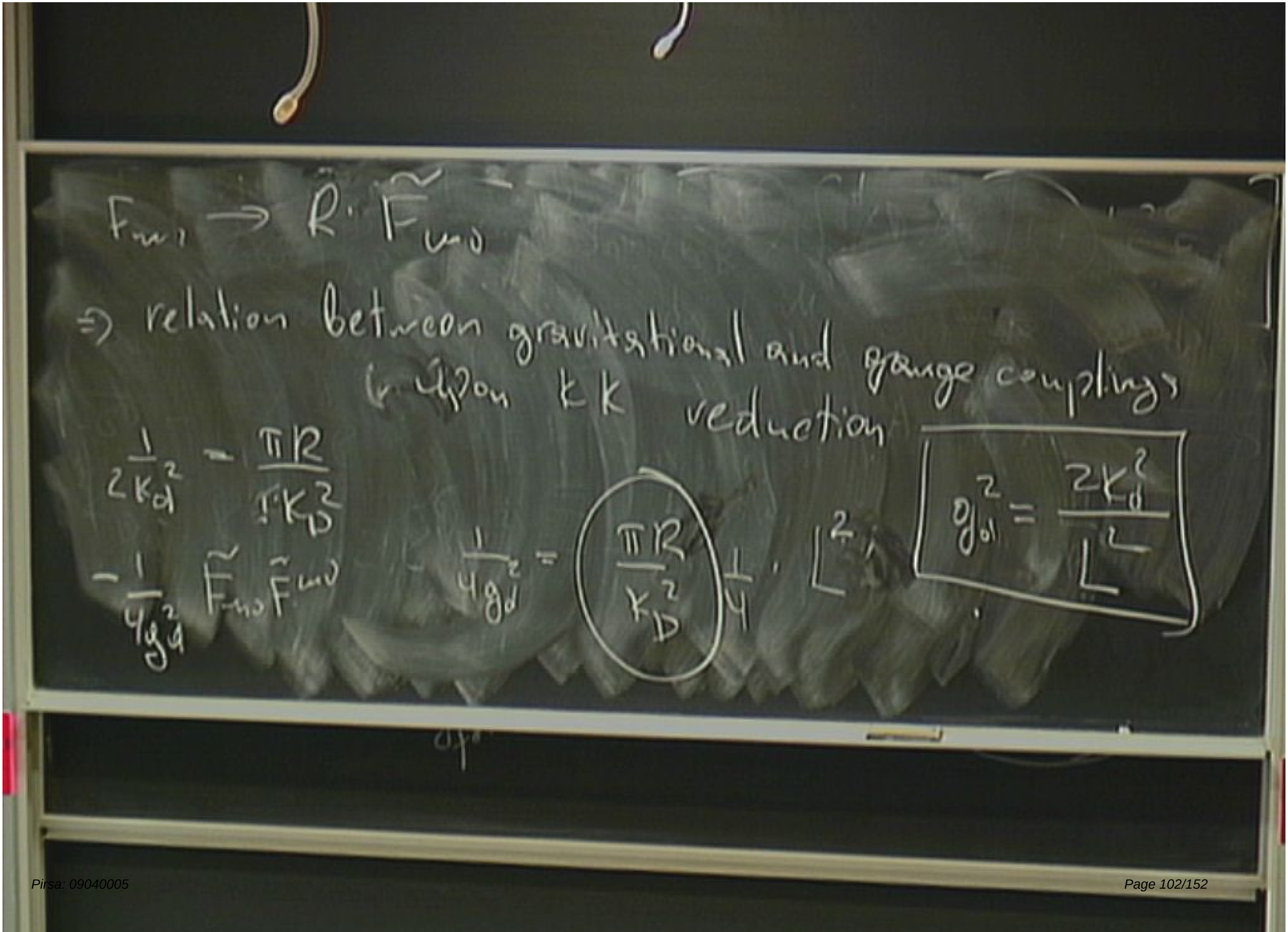

F_μν → R · F̃_μν
⇒ relation between gravitational and g couplings
(upon KK reduction)
1/(2κ_d²) = πR/(κ_D²)
−1/(4g_4²) F̃_μν F̃^μν
1/(4g_d²) = πR/(κ_D²) · 1/4

$$F_{\mu\nu} \rightarrow R \cdot \tilde{F}_{\mu\nu}$$

$\Rightarrow$ relation between gravitational and gauge couplings (i.e. upon KK reduction)

$$\frac{1}{2\kappa_d^2} = \frac{\pi R}{\kappa_D^2}$$

$$-\frac{1}{4g_d^2} \tilde{F}_{\mu\nu} \tilde{F}^{\mu\nu} \qquad \frac{1}{4g_d^2} = \frac{\pi R}{\kappa_D^2} \cdot \frac{1}{4} \cdot L^2 .$$


$F_{\mu\nu} \to R \cdot \tilde{F}_{\mu\nu}$
⇒ relation between gravitational and gauge couplings
(i.e. upon KK reduction)
$\frac{1}{2\kappa_d^2} = \frac{\pi R}{[illegible]\,\kappa_D^2}$
$-\frac{1}{4g_d^2} \tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu}$
$\frac{1}{4g_d^2} = \frac{\pi R}{\kappa_D^2} \frac{1}{4} \cdot L^2$
$g_d^2 = \frac{2\kappa_d^2}{L^2}$

$$-\frac{1}{4g_d^2}\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu} \qquad \frac{1}{4g_d^2} = \frac{\Pi R}{\kappa_D^2}\,\frac{1}{4}, \quad L^2$$

$$S_2 = -\frac{1}{2\cdot 4\kappa_D^2}\int d^Dx\,\sqrt{-G^D}\,e^{-2\varphi}\,H_{MNP}H^{MNP}.$$

$$-\frac{1}{4g_4^2}\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu} \qquad \frac{1}{4g_d^2} = \frac{\pi R}{\kappa_D^2}\,\frac{1}{4}\,, L^2 \qquad \boxed{\frac{}{g_d^2} \quad L^2}$$

$$S_2 = -\frac{1}{2\cdot 4\kappa_D^2}\int d^D x\,\sqrt{-G^D}\,e^{-2\varphi}\,H_{MNP}H^{MNP}.$$

$$H = dB$$

$B_{MN} \rightarrow \Big\{\; B_{\mu\nu} \rightarrow$ antisymmetric in d-dim

$$-\frac{1}{4g_d^2}\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu} \qquad \frac{1}{4g_d^2} = \frac{\pi R}{\kappa_D^2}\frac{1}{4},\ L^2,\ \boxed{g_d \ldots L^2}$$

$$S_2 = -\frac{1}{2\cdot 4\kappa_D^2}\int d^Dx \sqrt{-G^D}\, e^{-2\varphi}\, H_{MNP} H^{MNP}.$$

$$H = dB \qquad B_{MN} \rightarrow \begin{cases} B_{\mu\nu} \rightarrow \text{antisymmetric in } d\text{-dim} \\ B_{\mu d} \end{cases}$$

$$-\frac{1}{4g_d^2}\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu} \qquad \frac{1}{4g_d^2} = \frac{\pi R}{\kappa_D^2}\,\frac{1}{4}\,L^2$$

$$S_2 = -\frac{1}{2\cdot 4\kappa_D^2}\int d^Dx\,\sqrt{-G^D}\,e^{-2\varphi}\,H_{MNP}H^{MNP}.$$

$$H = dB$$

$B_{MN} \rightarrow$
- $B_{\mu\nu} \rightarrow$ antisymmetric in d-dim
- $B_{\mu d} \rightarrow A'_\mu$

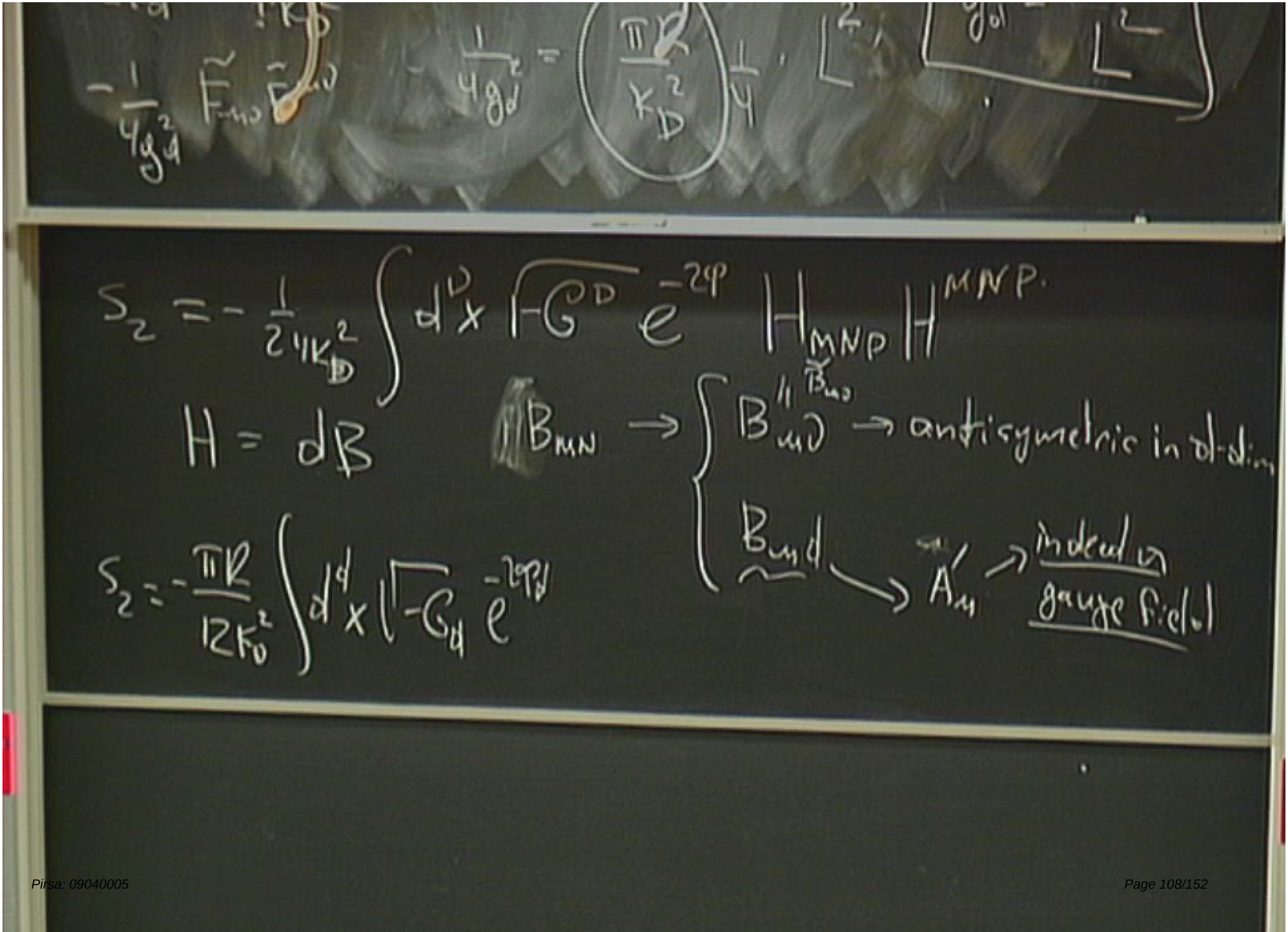
$$-\frac{1}{4g_4^2}\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu} \qquad \frac{1}{4g_d^2} = \frac{\pi R}{\kappa_D^2}\frac{1}{4}, \quad L^2 \qquad \boxed{\; g_d \;\; L^2 \;}$$

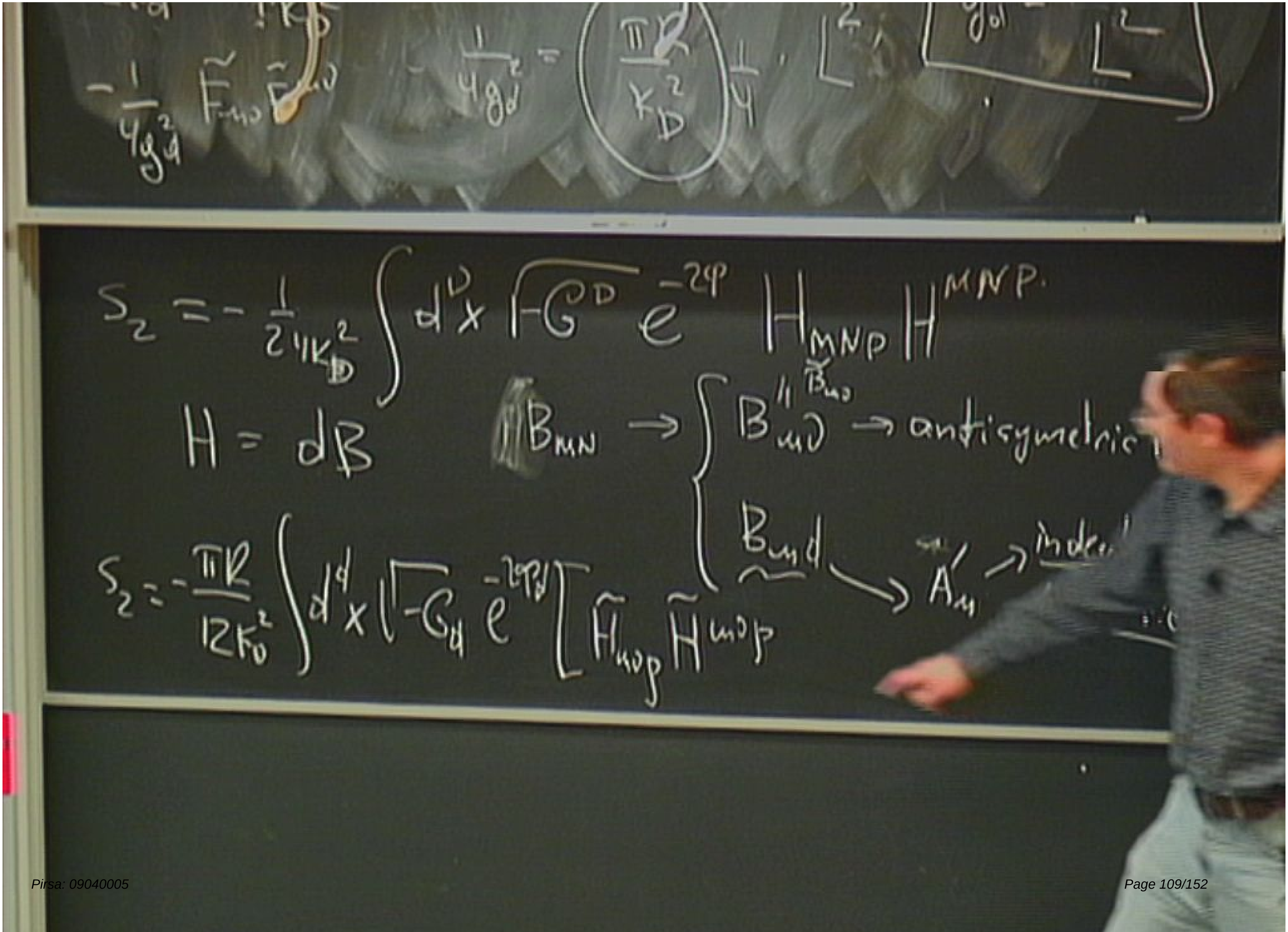
$$S_2 = -\frac{1}{2\cdot 4\kappa_D^2}\int d^Dx\,\sqrt{-G^D}\,e^{-2\varphi}\,H_{MNP}H^{MNP}.$$

$$H = dB$$

$B_{MN} \rightarrow$

- $B_{\mu\nu} \rightarrow$ antisymmetric in d-dim
- $B_{\mu d} \rightarrow A'_\mu \rightarrow$ <u>indeed a gauge field</u>


S_2 = - 1/(2 4K_D^2) ∫ d^D x √-G^D e^{-2φ} H_{MNP} H^{MNP}
H = dB
B_{MN} → B_{μν} → antisymmetric in d-dim
B_{μd} → A_μ → induced gauge field
S_2 = - πR/(12K_D^2) ∫ d^d x √-G_d e^{-2φ_d}


$S_2 = -\frac{1}{2\,4\kappa_D^2} \int d^D x \sqrt{-G^D}\, e^{-2\varphi} H_{MNP} H^{MNP}$
$H = dB$
$B_{MN} \rightarrow \{ B_{\mu\nu} \rightarrow$ antisymmetric
$B_{\mu d} \rightarrow A_\mu \rightarrow$ indep
$S_2 = -\frac{\pi R}{12\kappa_D^2} \int d^d x \sqrt{-G_d}\, e^{-2\varphi_d} [\tilde{H}_{\mu\nu\rho} \tilde{H}^{\mu\nu\rho}$

$$-\frac{1}{4g_4^2}\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu} \quad -\frac{1}{4g_d^2} = \frac{\pi R}{\kappa_D^2}\frac{1}{4}, \; L^2, \quad g_d \; L^2$$

$$S_2 = -\frac{1}{2\,4\kappa_D^2}\int d^Dx\,\sqrt{-G^D}\,e^{-2\varphi}\,H_{MNP}H^{MNP}$$

$$H = dB \qquad B_{MN} \rightarrow \begin{cases} B_{\mu\nu} \, (= \tilde{B}_{\mu\nu}) \rightarrow \text{antisymmetric in d-dim} \\ \tilde{B}_{\mu d} \longrightarrow A'_\mu \rightarrow \underline{\text{indeed a gauge field}} \end{cases}$$

$$S_2 = -\frac{\pi R}{12\kappa_D^2}\int d^dx\,\sqrt{-G_d}\,e^{-2\varphi_d}\left[\tilde{H}_{\mu\nu\rho}\tilde{H}^{\mu\nu\rho} + 3e^{-2\sigma}F'_{\mu\nu}F'^{\mu\nu}\right]$$

$$-\frac{1}{4g_4^2}\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu} \quad -\frac{1}{4g_d^2} = \frac{\pi R}{\kappa_D^2}\frac{1}{4}, \quad L^2, \quad g_d \quad L^2$$

$$S_2 = -\frac{1}{2\,4\kappa_D^2}\int d^Dx\,\sqrt{-G^D}\,e^{-2\varphi}\,H_{MNP}H^{MNP}.$$

$$H = dB \qquad B_{MN} \rightarrow \begin{cases} B_{\mu\nu} \;(\tilde{B}_{\mu\nu}) \rightarrow \text{antisymmetric in d-dim} \\ \tilde{B}_{\mu d} \rightarrow A'_\mu \rightarrow \underline{\text{indeed a gauge field}} \end{cases}$$

$$S_2 = -\frac{\pi R}{12\kappa_D^2}\int d^dx\,\sqrt{-G_d}\,e^{-2\varphi_d}\left[\tilde{H}_{\mu\nu\rho}\tilde{H}^{\mu\nu\rho} + 3e^{-2\sigma}F'_{\mu\nu}F'^{\mu\nu}\right]$$

$$S_2 = -\frac{1}{24\kappa_D^2}\int d^D x \sqrt{-G^D}\, e^{-2\varphi} H_{MNP} H^{MNP}$$

$$H = dB$$

$$B_{MN} \to \begin{cases} B_{\mu\nu} \ (= \tilde{B}_{\mu\nu}) \to \text{antisymmetric in } d\text{-dim} \\ \underset{\sim}{B_{\mu d}} \to A'_\mu \to \text{induced gauge field} \end{cases}$$

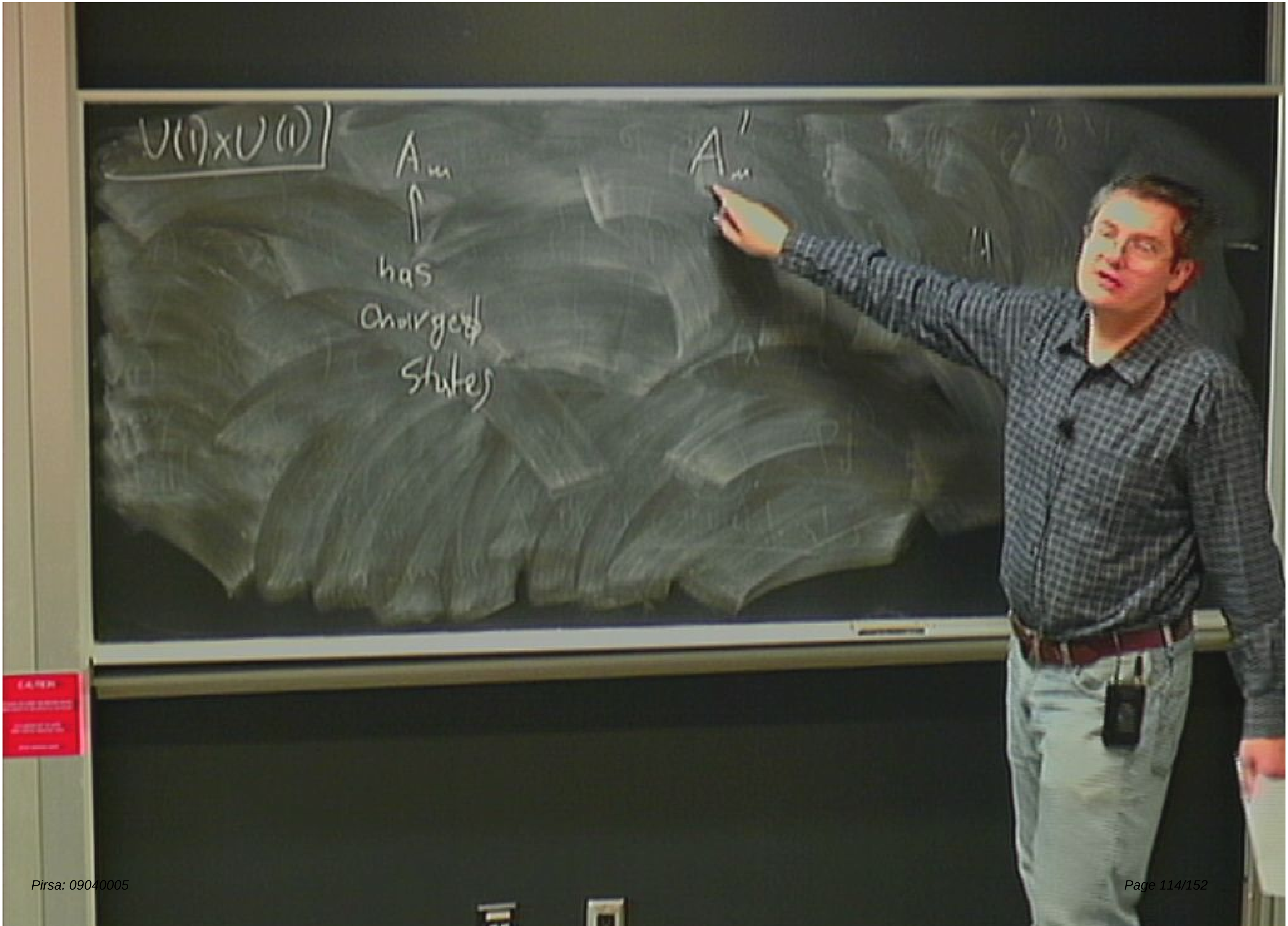
$$S_2 = -\frac{\pi R}{12\kappa_D^2}\int d^d x \sqrt{-G_d}\, e^{-2\varphi_d}\left[\tilde{H}_{\mu\nu\rho}\tilde{H}^{\mu\nu\rho} + 3e^{-2\sigma} F'_{\mu\nu} F'^{\mu\nu}\right]$$

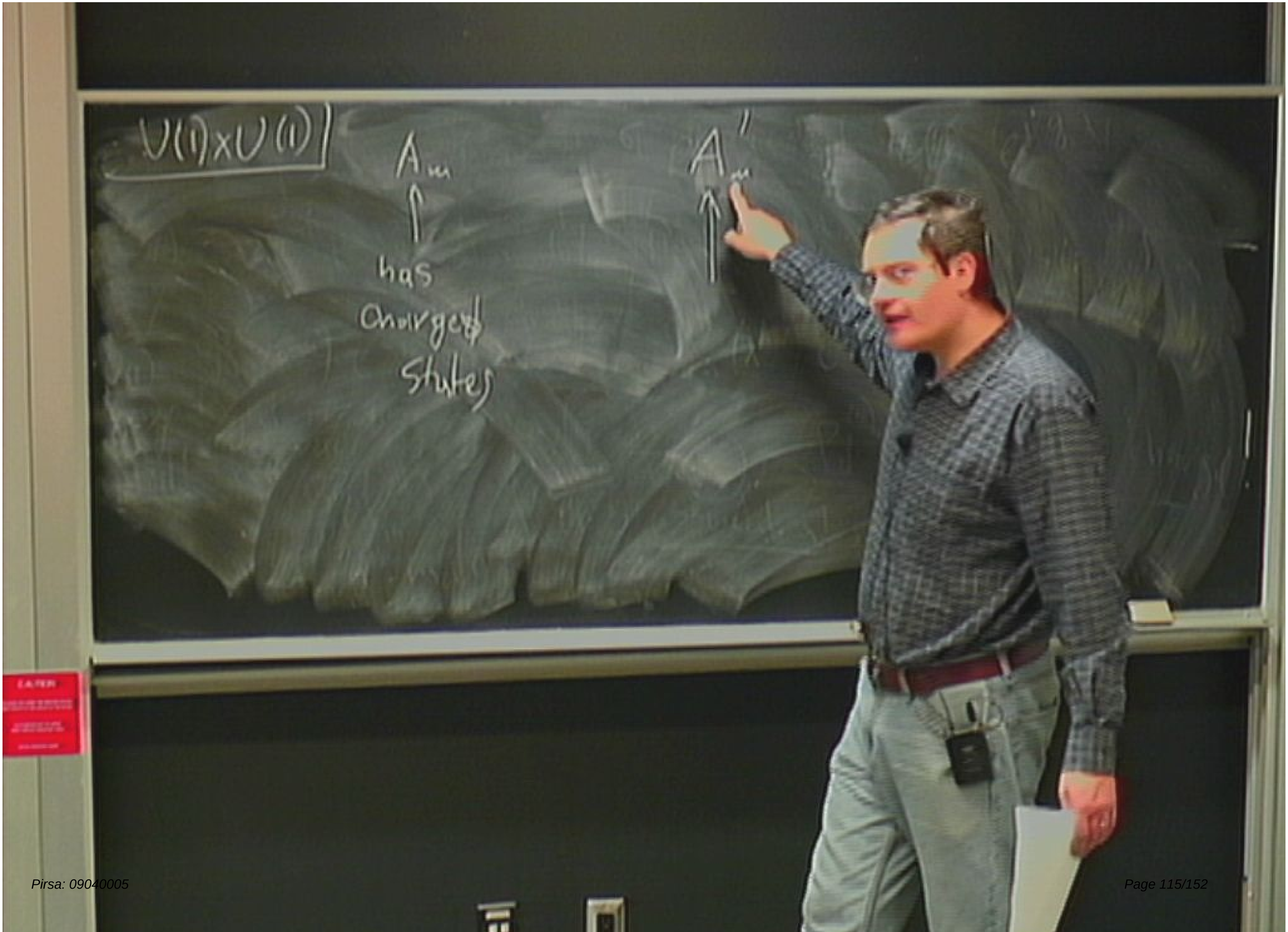
$$S_2 = -\frac{1}{24\kappa_D^2}\int d^Dx\,\sqrt{-G^D}\,e^{-2\varphi}\,H_{MNP}H^{MNP}.$$

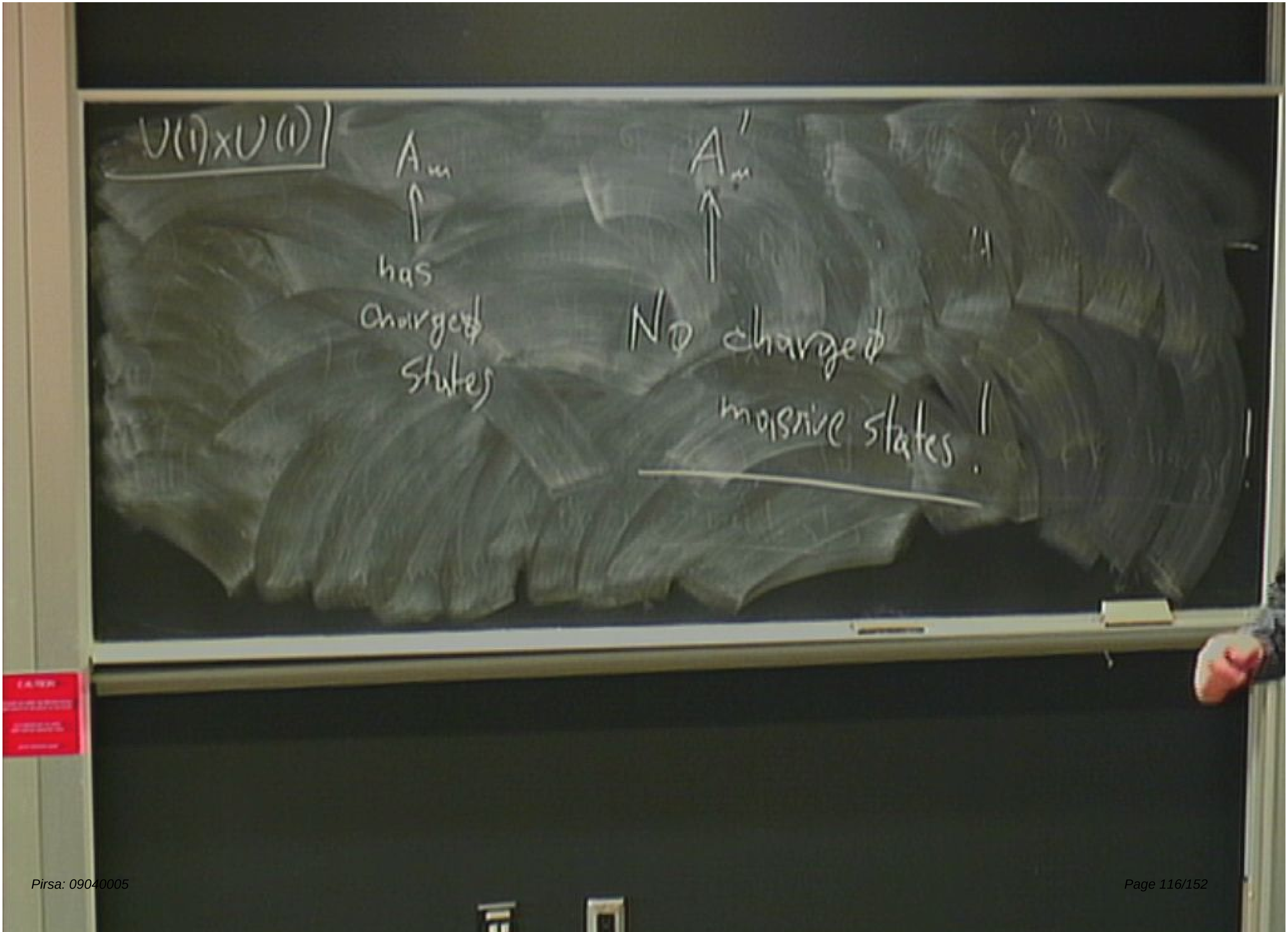
$$H = dB$$

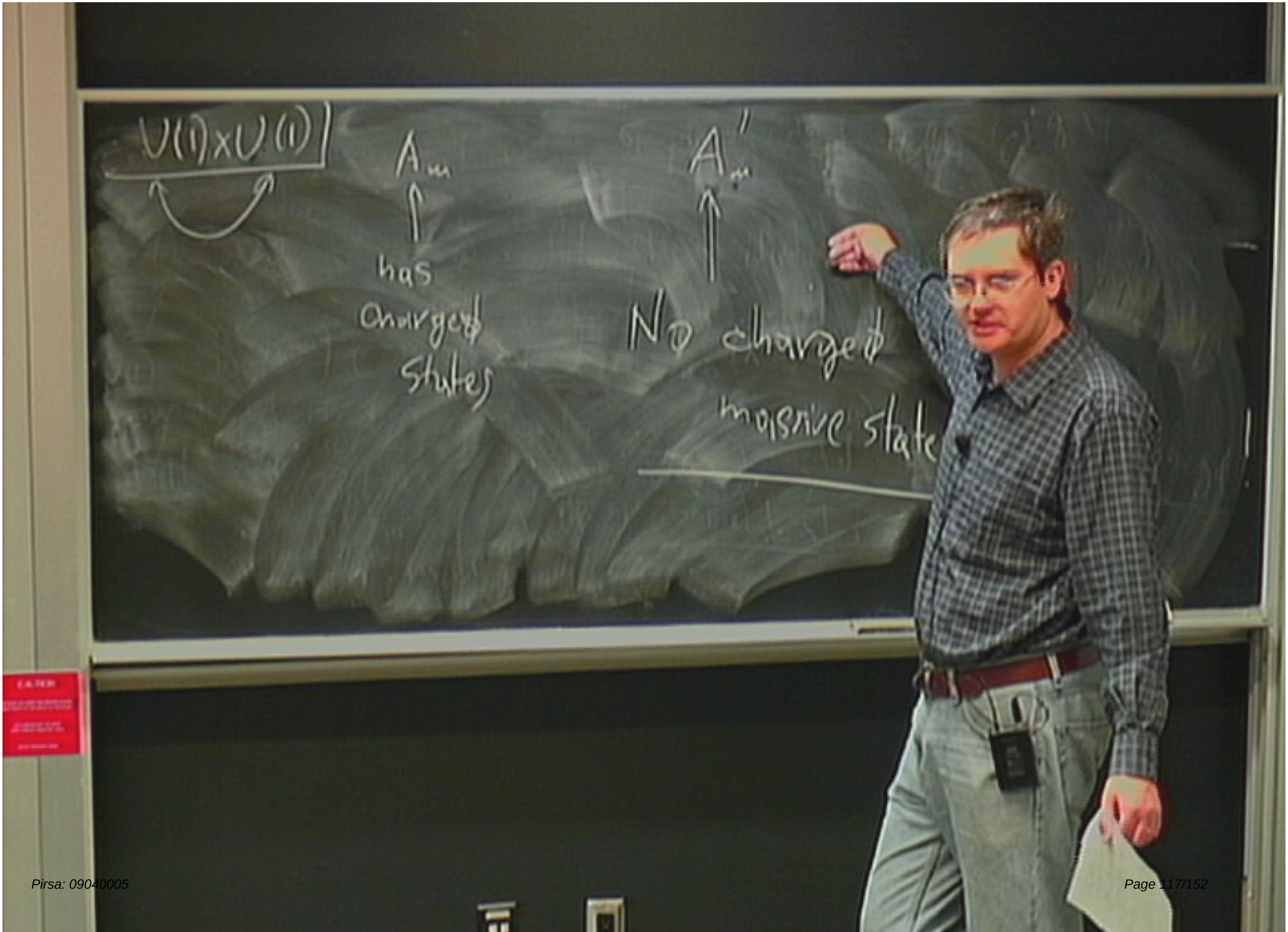
$$B_{MN} \to \begin{cases} B_{\mu\nu} \,(\tilde{B}_{\mu\nu}) \to \text{antisymmetric in d-dim} \\ B_{\mu d} \to A'_\mu \to \text{induced gauge field} \end{cases}$$

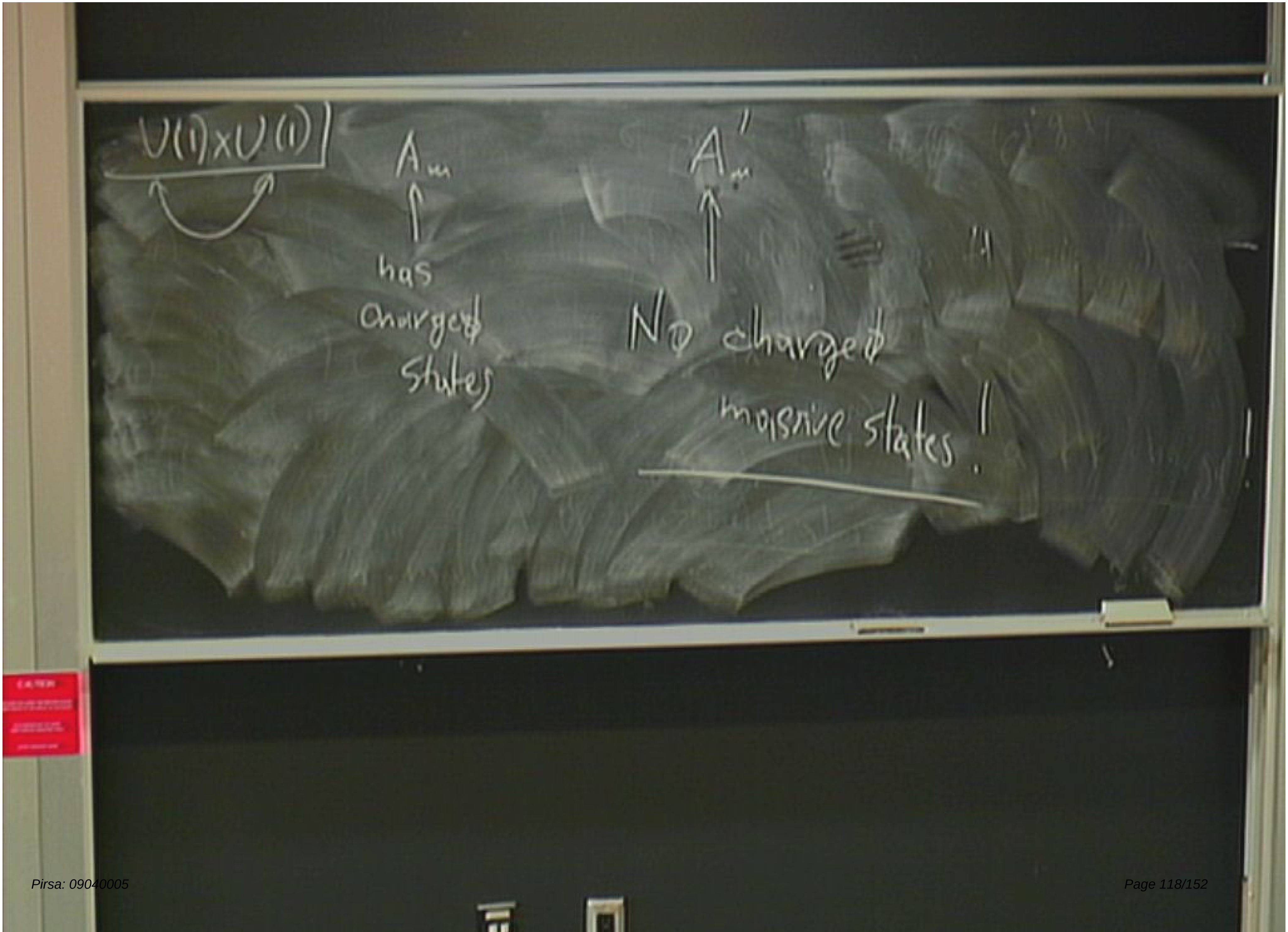
$$S_2 = -\frac{\pi R}{12\kappa_0^2}\int d^dx\,\sqrt{-G_d}\,e^{-2\Phi_d}\left[\tilde{H}_{\mu\nu\rho}\tilde{H}^{\mu\nu\rho} + 3e^{-2\sigma}F'_{\mu\nu}F'^{\mu\nu}\right]$$


U(1)xU(1)
Aμ
has
charged
states
A'μ


U(1)×U(1)
A_μ
has
charged
states
A'_μ


U(1)xU(1)
A_μ
has charged states
A'_μ
No charged massive states!


U(1)xU(1)
A_μ
has
charged
states
A'_μ
No charged
massive states


U(1)xU(1)
Aμ
has
charged
states
A'μ
No charged
massive states!

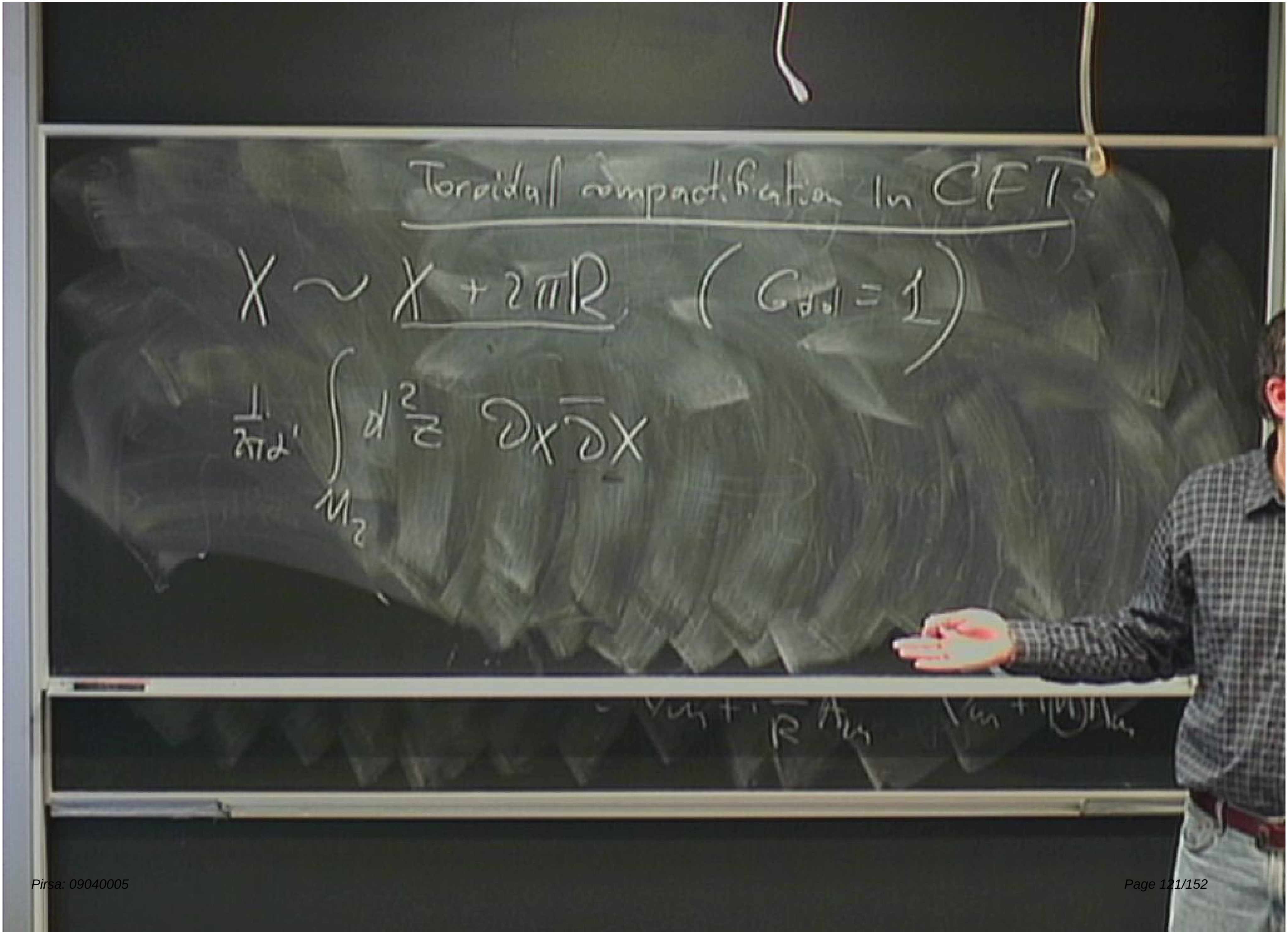
Toroidal compactification in CFT

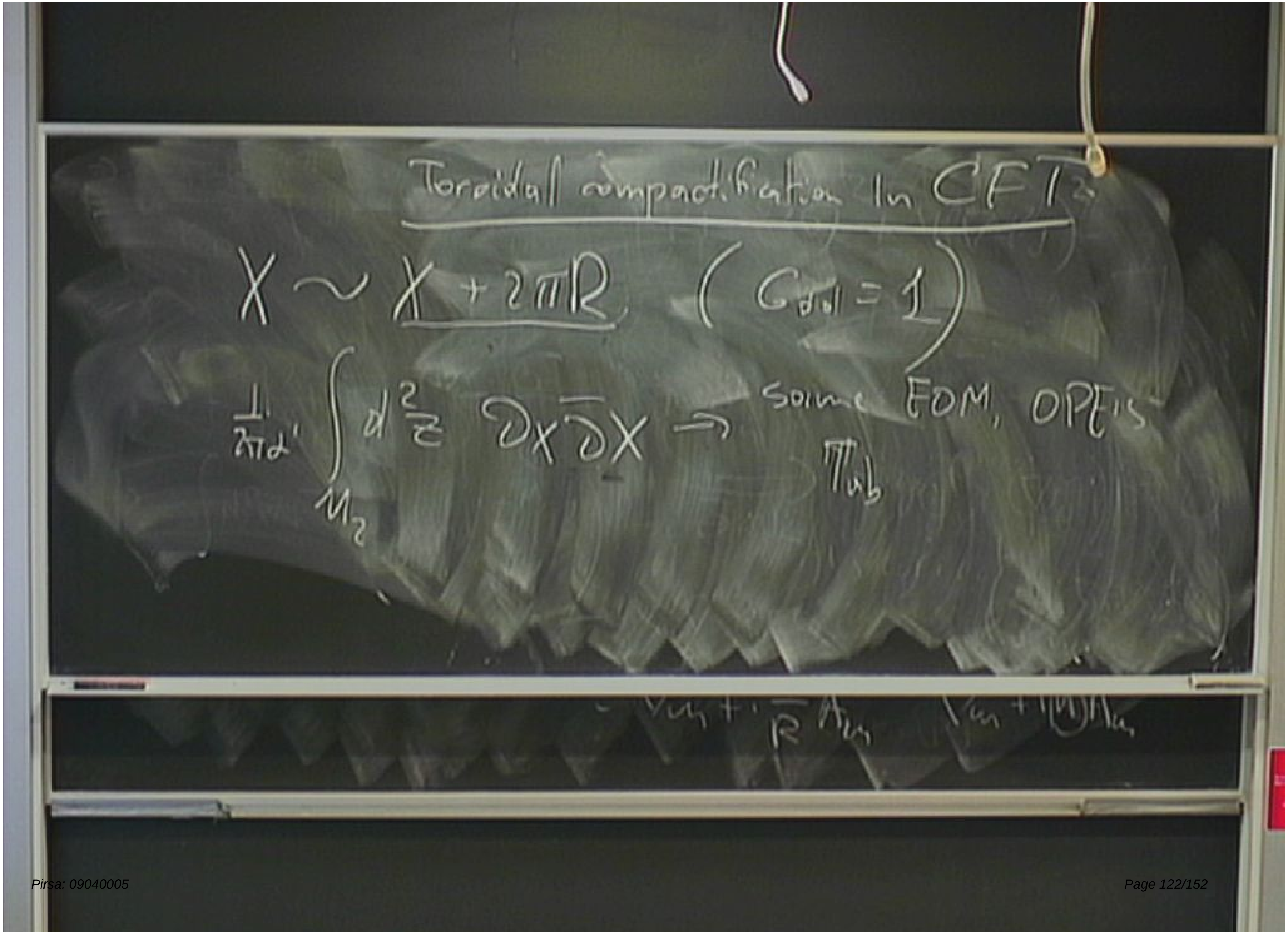
$$X \sim X + 2\pi R$$

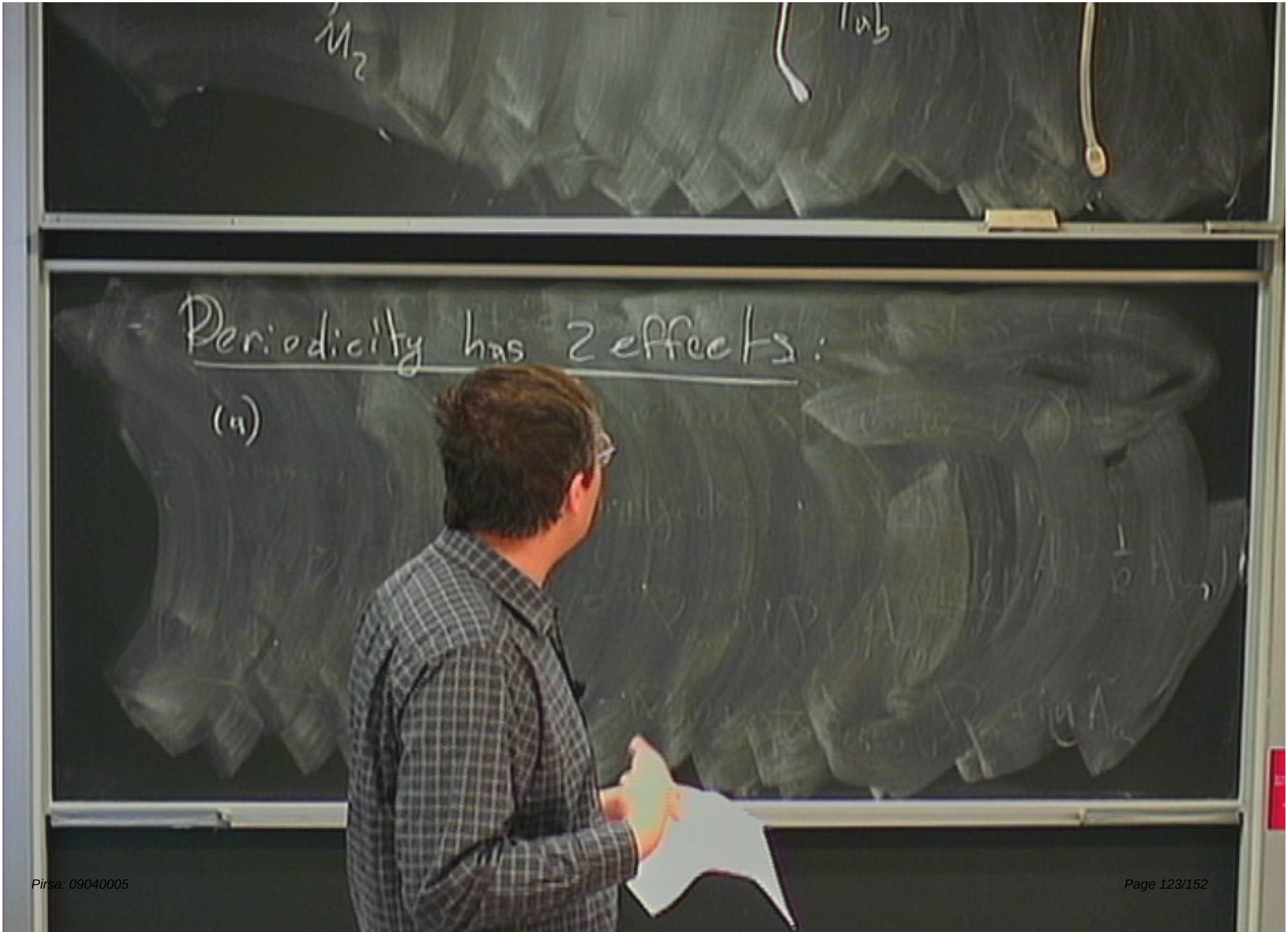
Toroidal compactification in CFT_2

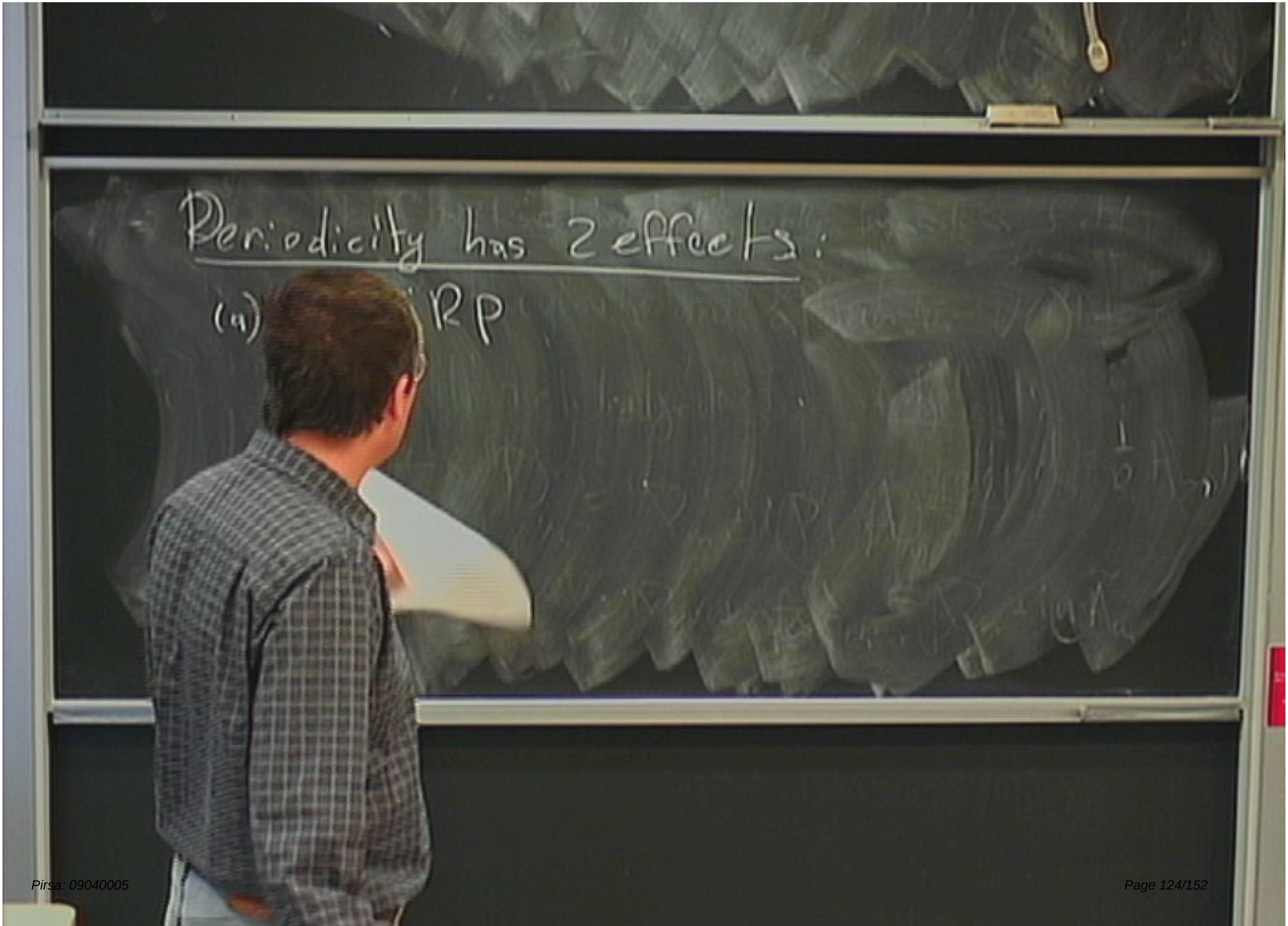
$$X \sim X + 2\pi R \quad (G_{dd} = 1)$$

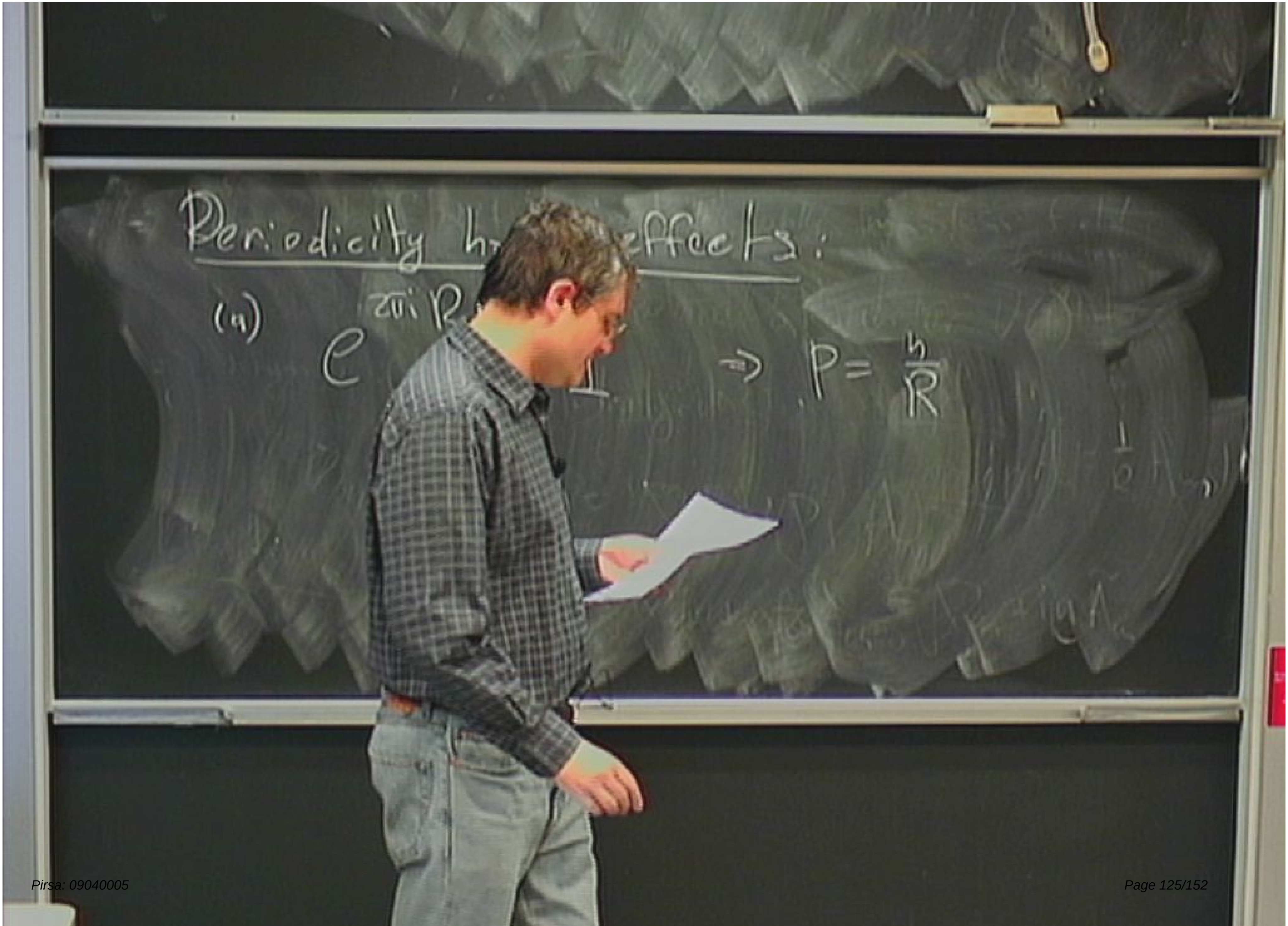
$$\frac{1}{2\pi\alpha'}\int_{M_2} d^2z \, \partial X \bar{\partial} X$$


Toroidal compactification in CFT2
X ~ X + 2πR (c_tot = 1)
1/(2πα') ∫_{M_2} d²z ∂X ∂̄X


Toroidal compactification in CFT2
X ~ X + 2πR (c_tot = 1)
1/2πα' ∫ d²z ∂X ∂̄X → some EOM, OPE's
M2
T_ab

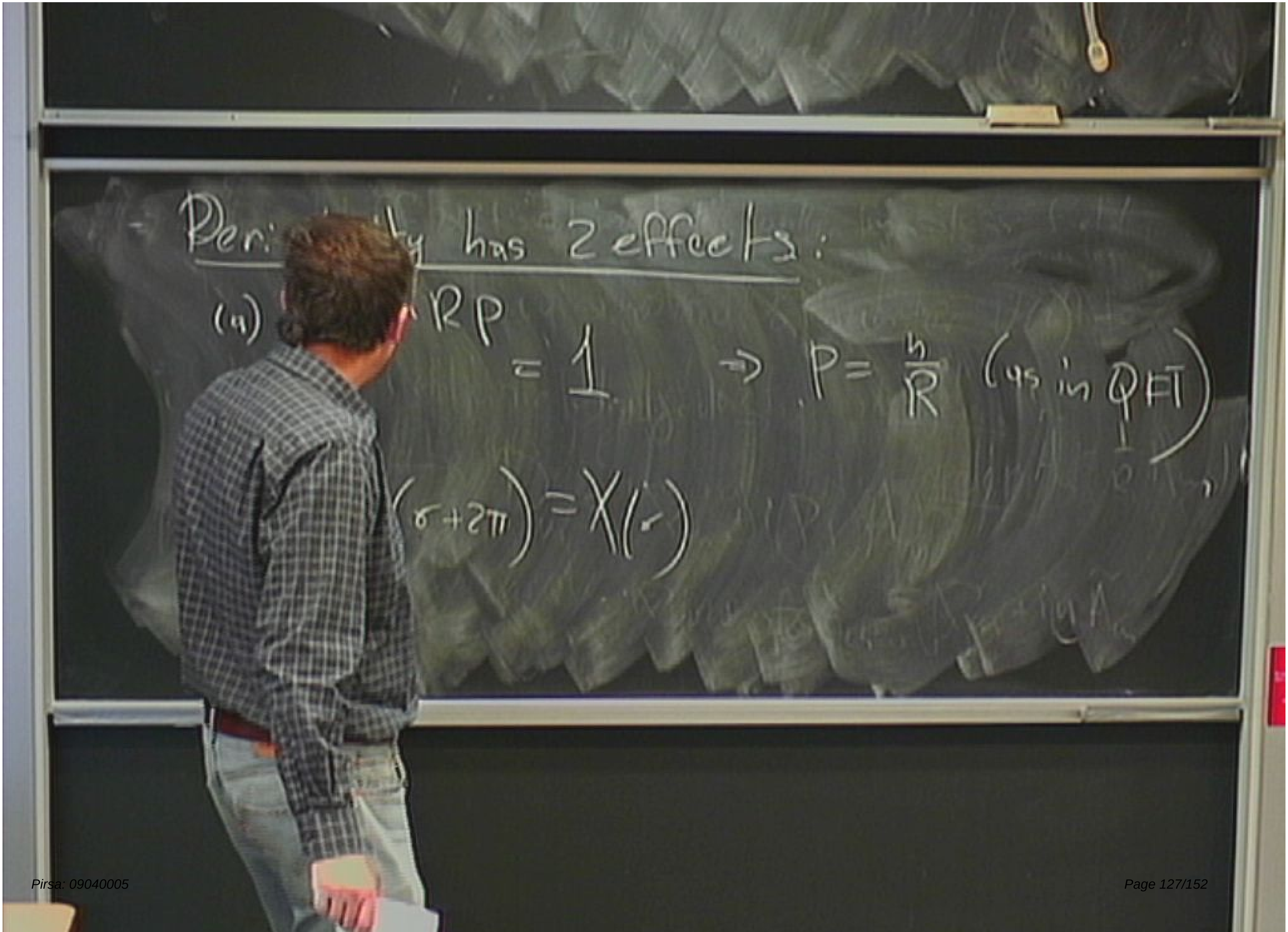

Periodicity has 2 effects:
(a)

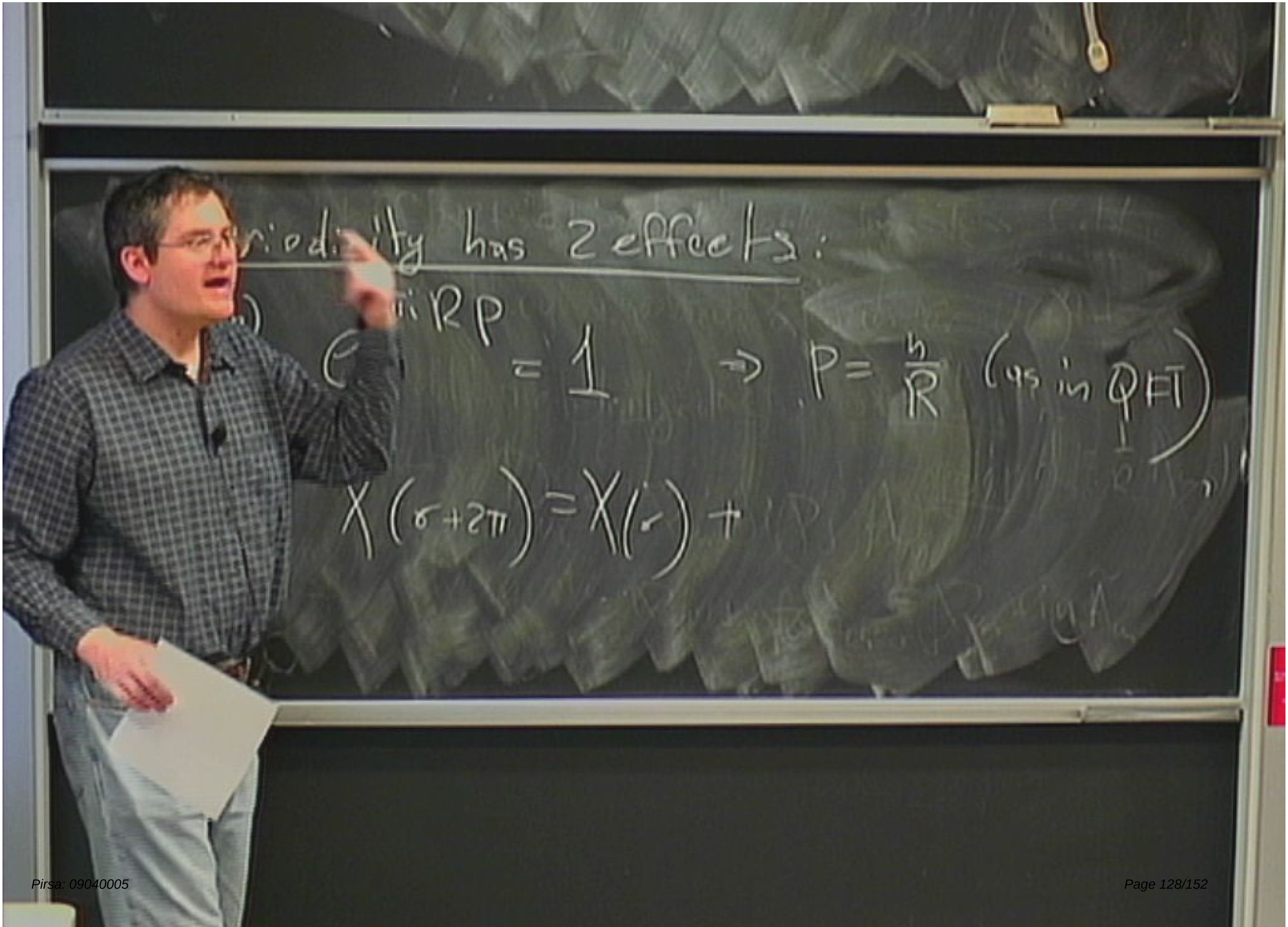

Periodicity has 2 effects:
(a)
RP


Periodicity
effects:
(a)
P = h/R

Periodicity has 2 effects:

(a) $e^{2\pi i R P} = 1 \Rightarrow P = \frac{n}{R}$ (as in QFT)


has 2 effects:
(a)
RP
= 1
⇒ P = n/R (as in QFT)
(σ+2π) = X(


has 2 effects:
RP
= 1
⇒ P = n/R (as in QFT)
X(σ+2π) = X(σ) +

Periodicity has 2 effects:

(a) $e^{2\pi i R p} = 1 \Rightarrow p = \frac{n}{R}$ (as in QFT)

(b) $X(\sigma + 2\pi) = X(\sigma) + 2\pi R w$ — w is a winding number.

Periodicity has 2 effects:

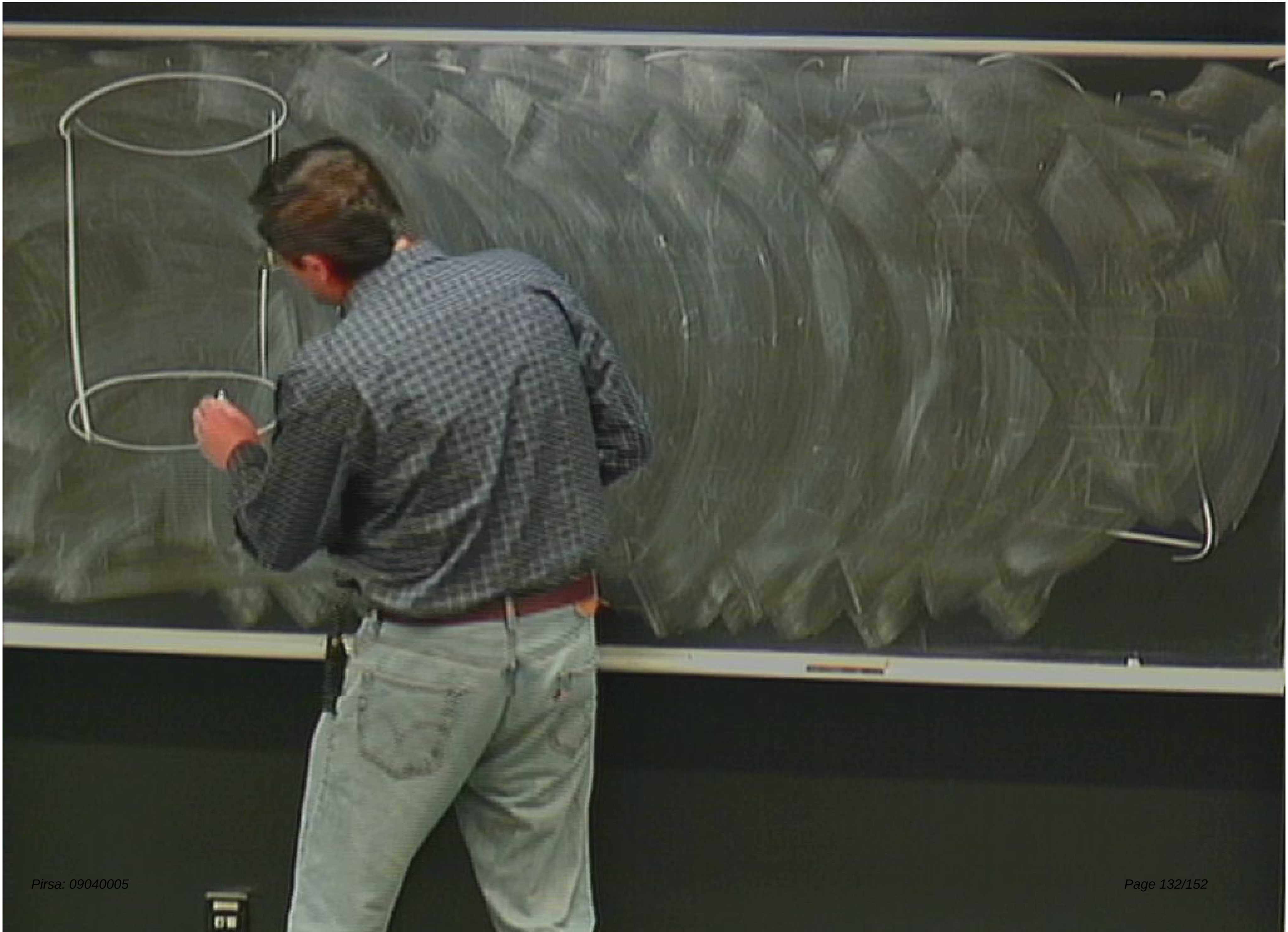
(a) $e^{2\pi i R P} = 1 \Rightarrow P = \frac{n}{R}$ (as in QFT)

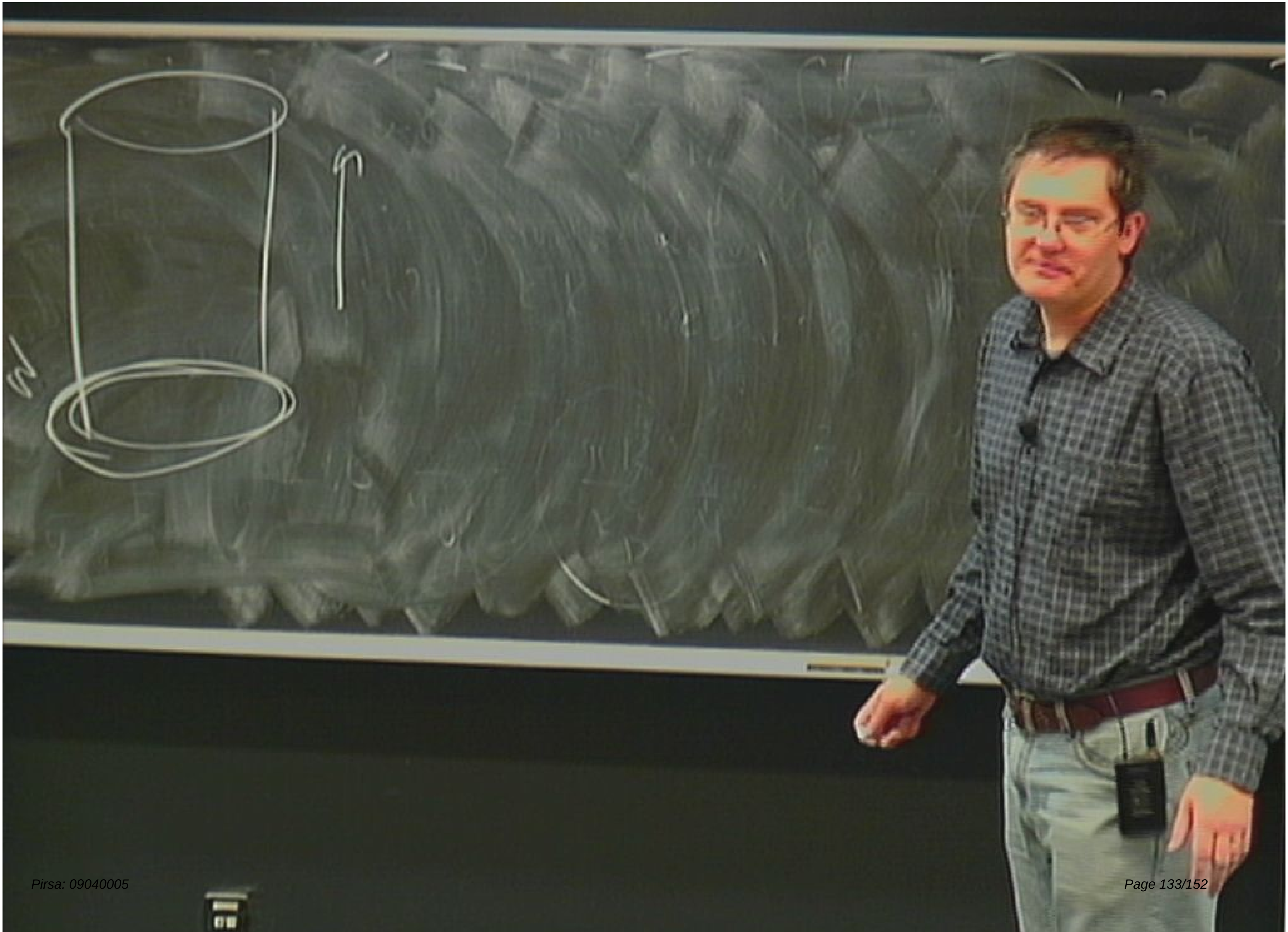
(b) $X(\sigma + 2\pi) = X(\sigma) + 2\pi R w$ — w is a winding number.

Periodicity has 2 effects:

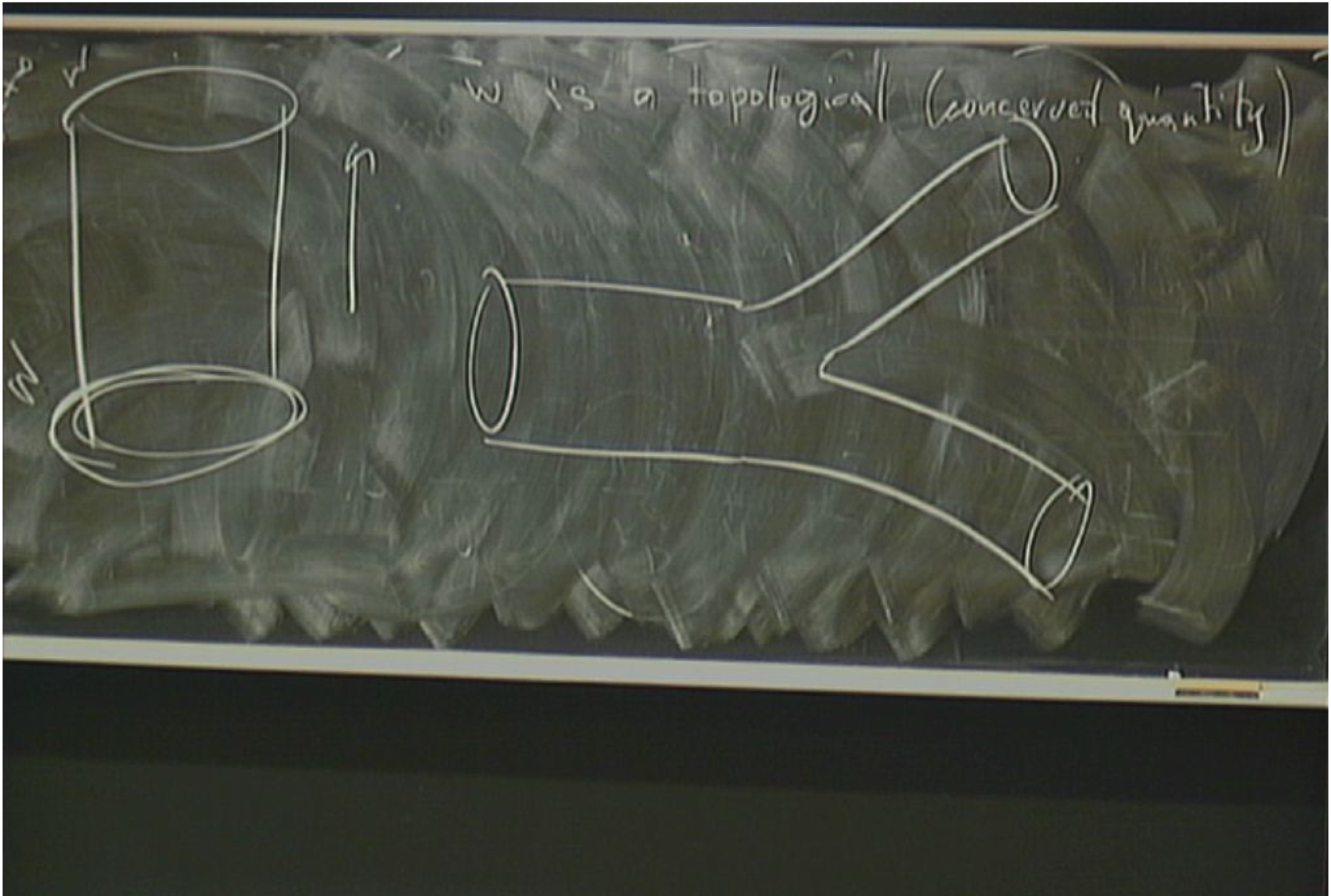
(a) $e^{2\pi i R p} = 1 \Rightarrow p = \frac{n}{R}$ (as in QFT)

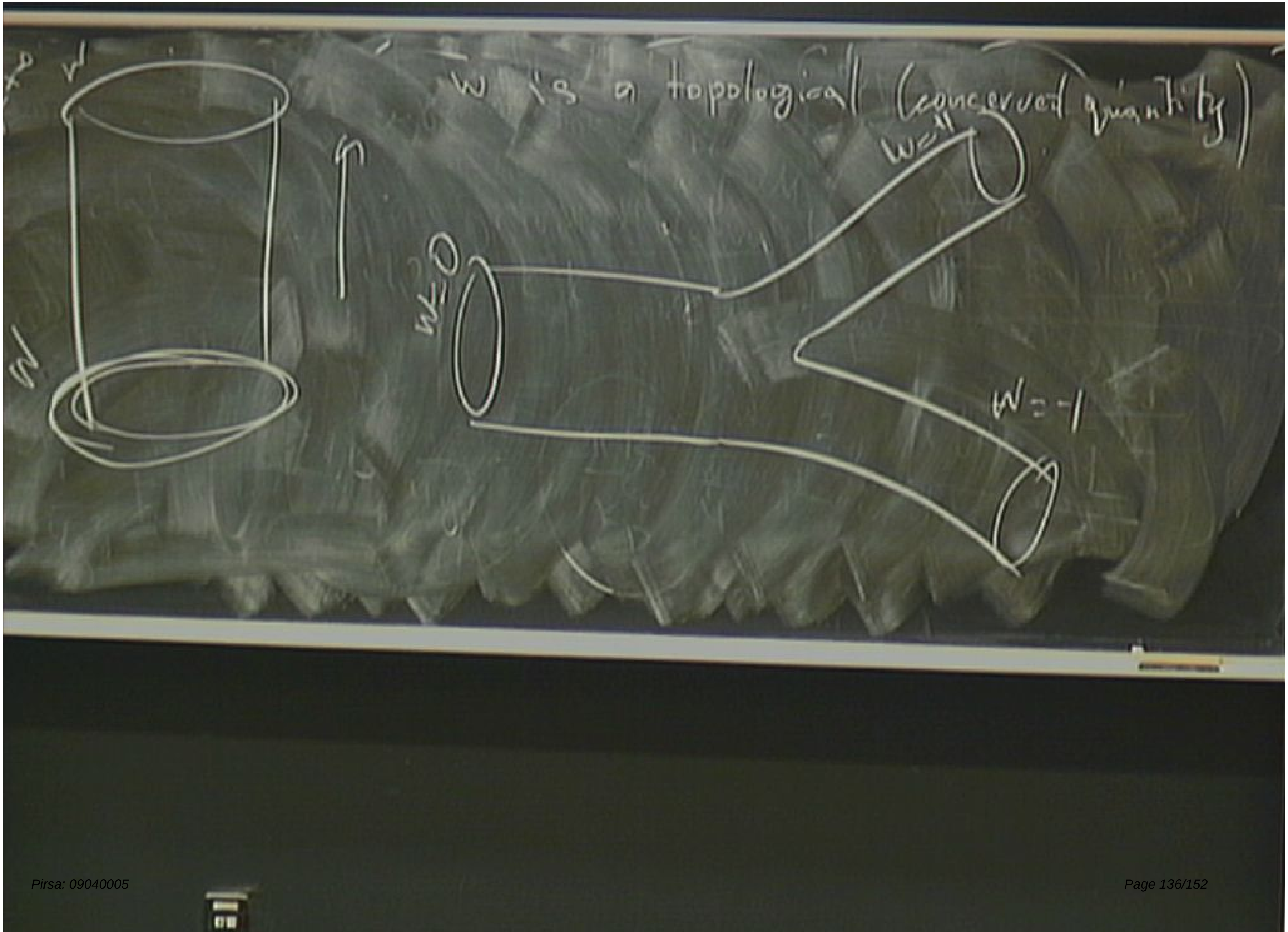
(b) $X(\sigma + 2\pi) = X(\sigma) + 2\pi R w$ — w is a winding number.

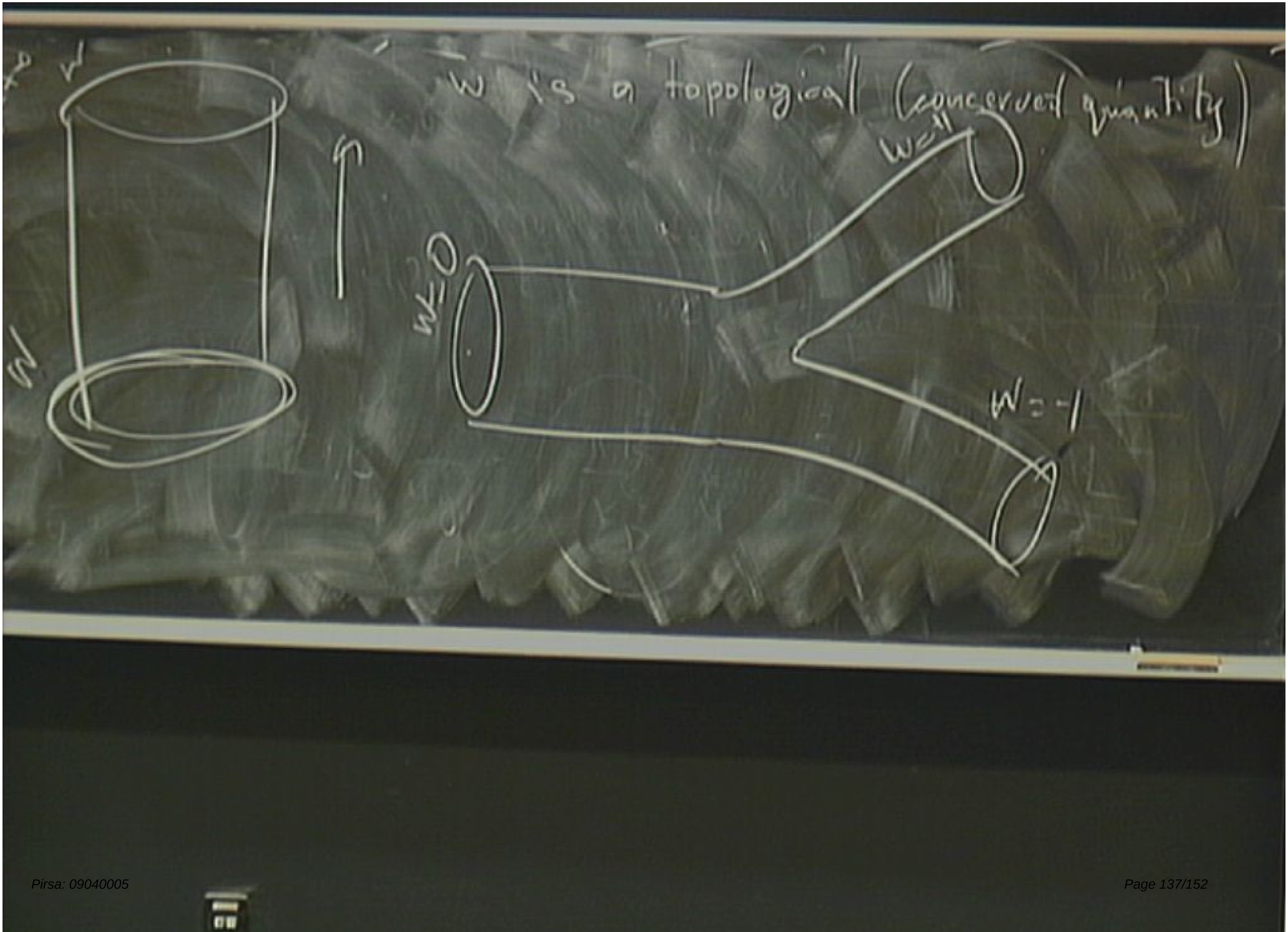


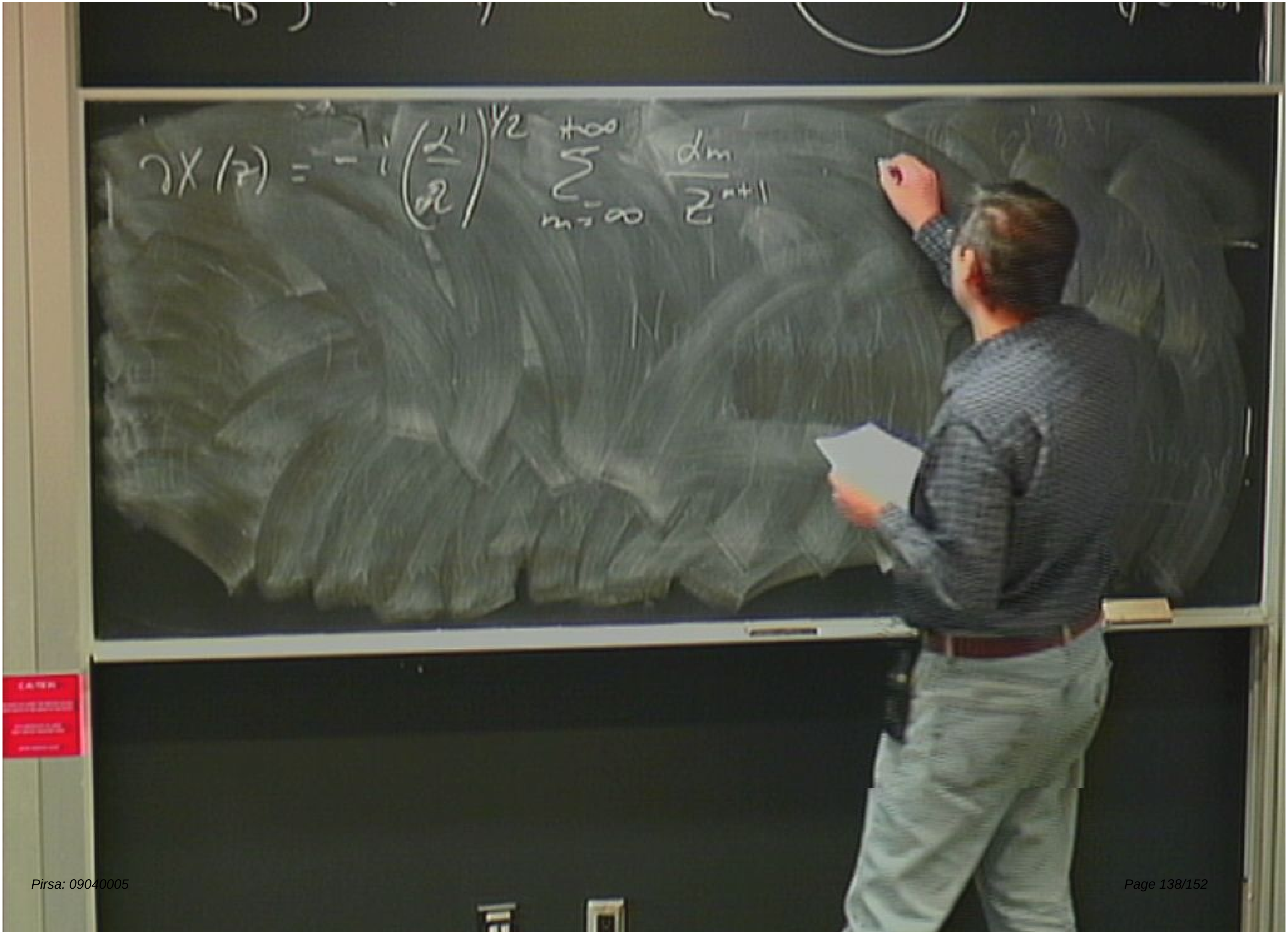


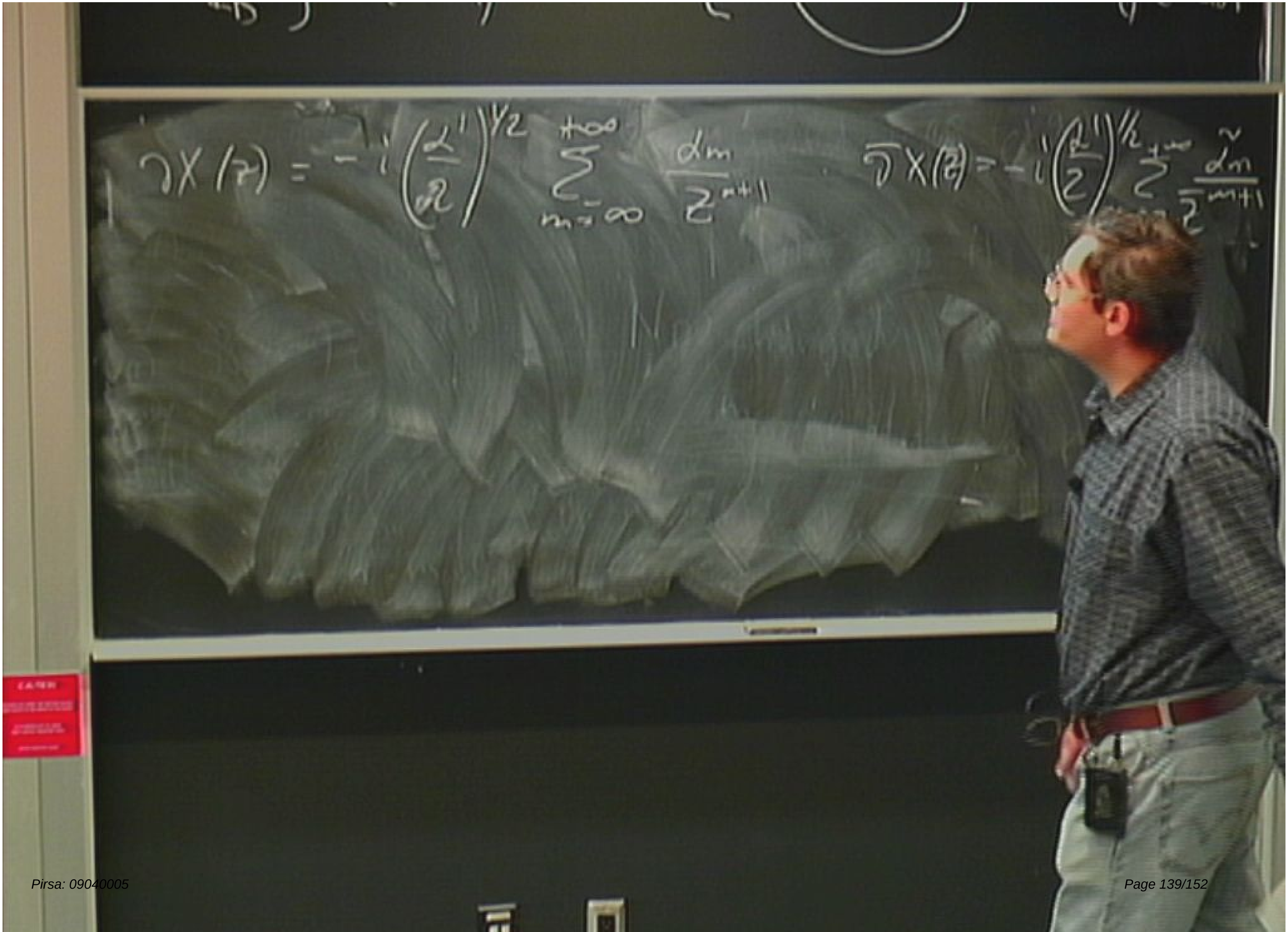

w is a topological (conserved quantity)

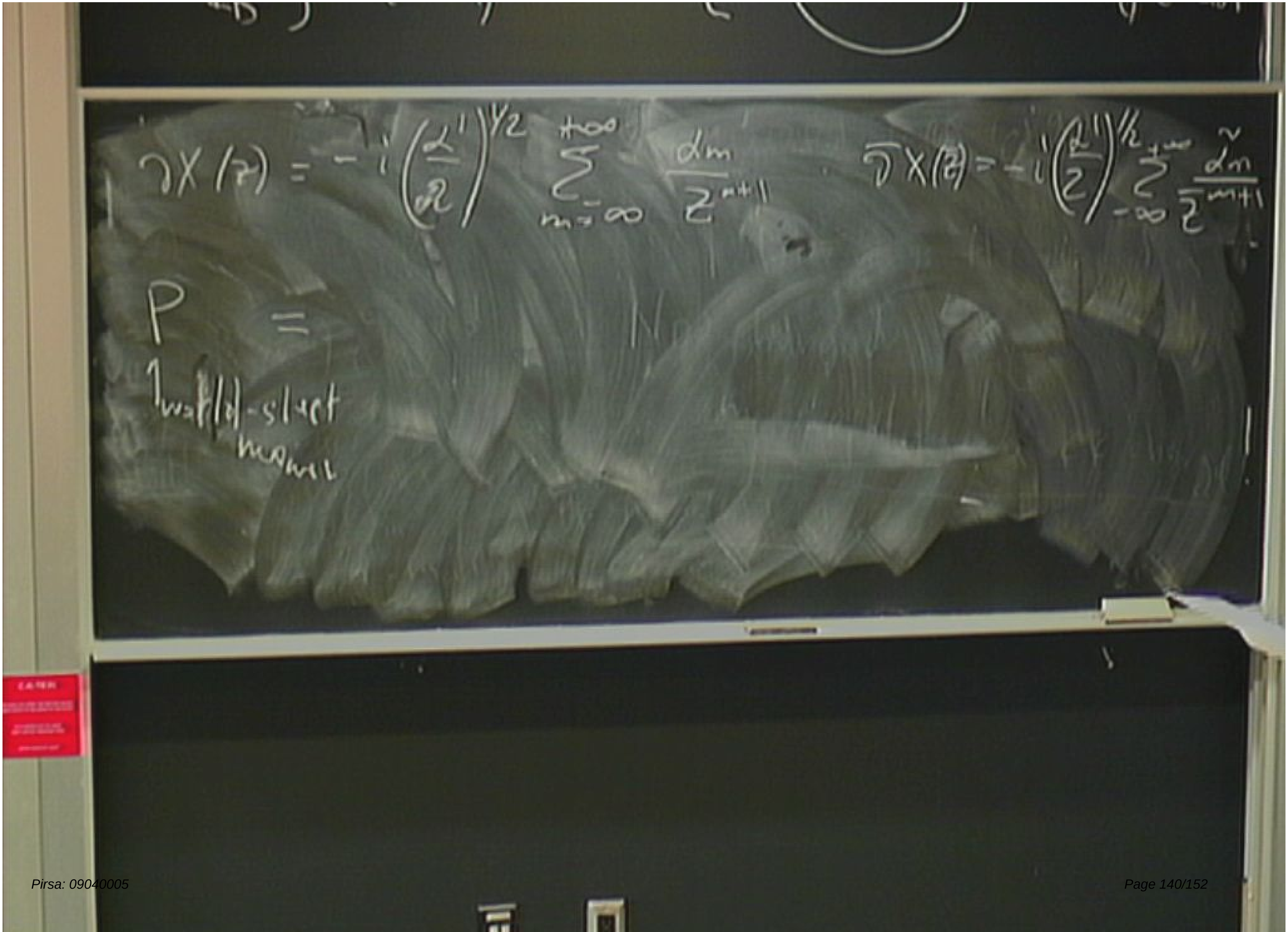

W is a topological (conserved quantity)


W is a topological (conserved quantity)
W=+1
W=0
W=-1
W


W is a topological (conserved quantity)
W=0
W=+1
W=-1




∂X(z) = -i (α'/2)^{1/2} Σ_{m=-∞}^{+∞} α_m / z^{m+1}
∂̄X(z̄) = -i (α'/2)^{1/2} Σ^{+∞} α̃_m / z̄^{m+1}

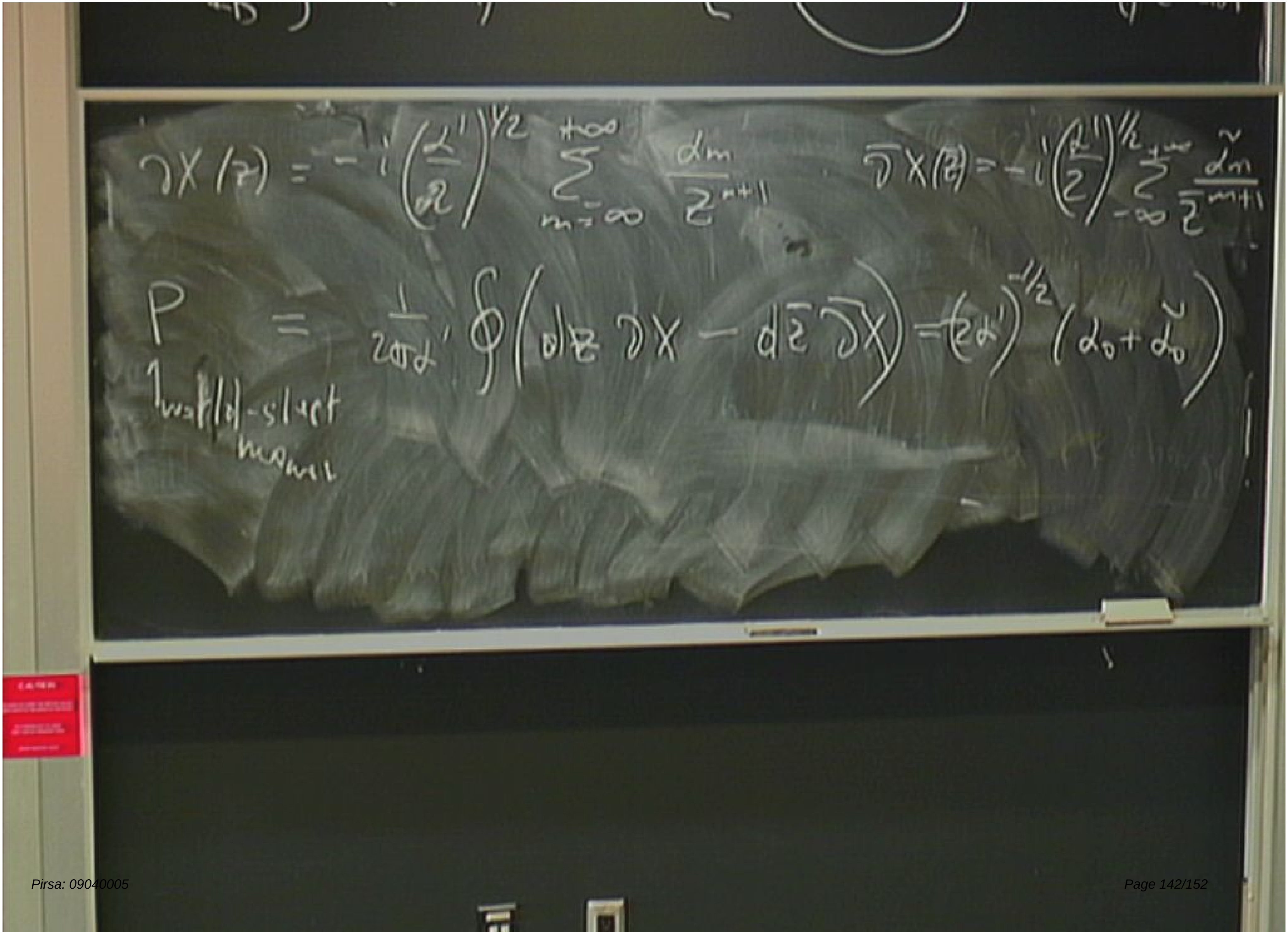

∂X(z) = −i (α'/2)^{1/2} Σ_{m=−∞}^{+∞} α_m / z^{m+1}
∂̄X(z̄) = −i (α'/2)^{1/2} Σ_{−∞}^{+∞} α̃_m / z̄^{m+1}
P =
1 world-sheet
momentum

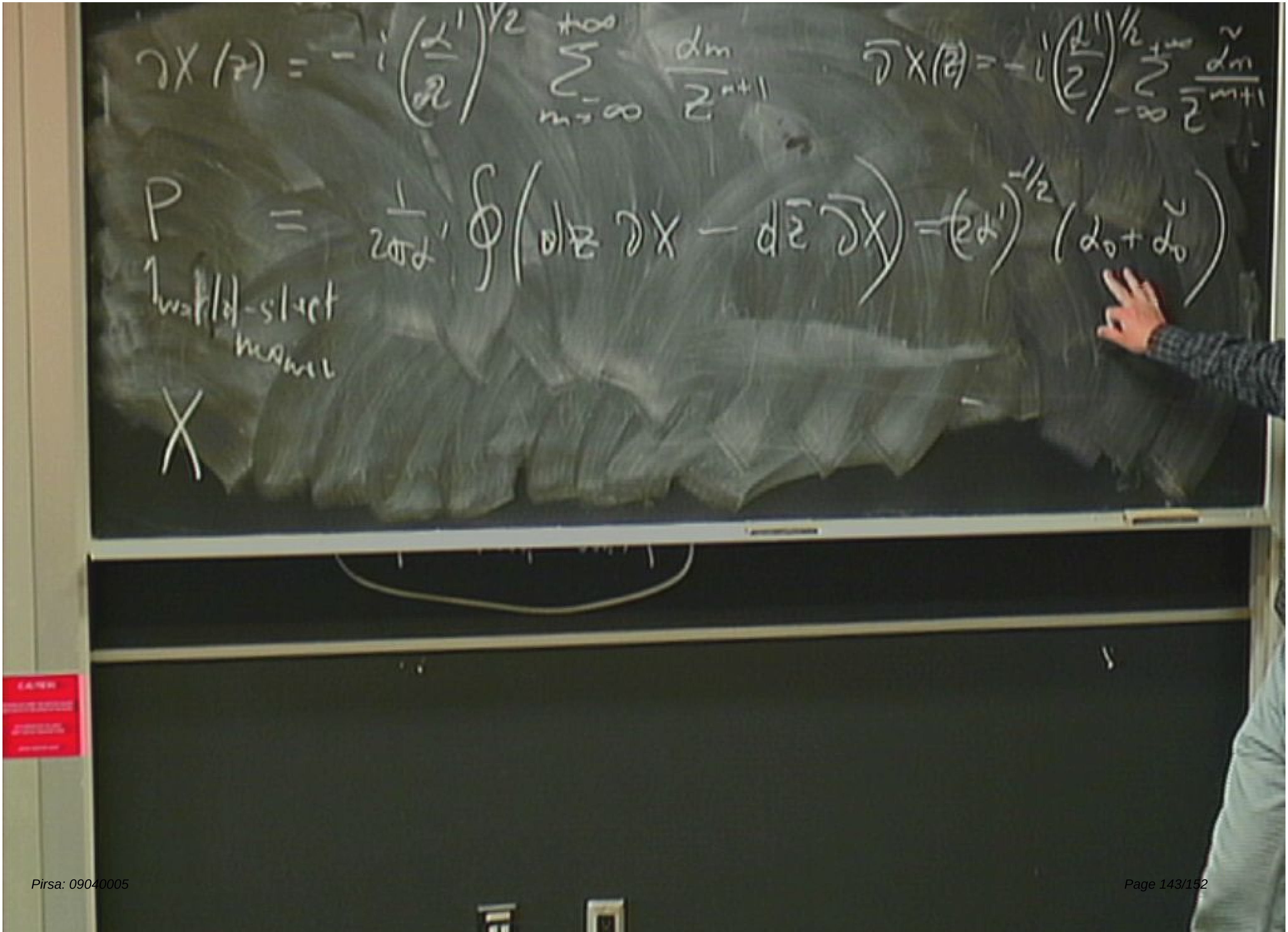
$$\partial X(z) = -i\left(\frac{\alpha'}{2}\right)^{1/2}\sum_{m=-\infty}^{+\infty}\frac{\alpha_m}{z^{m+1}}$$

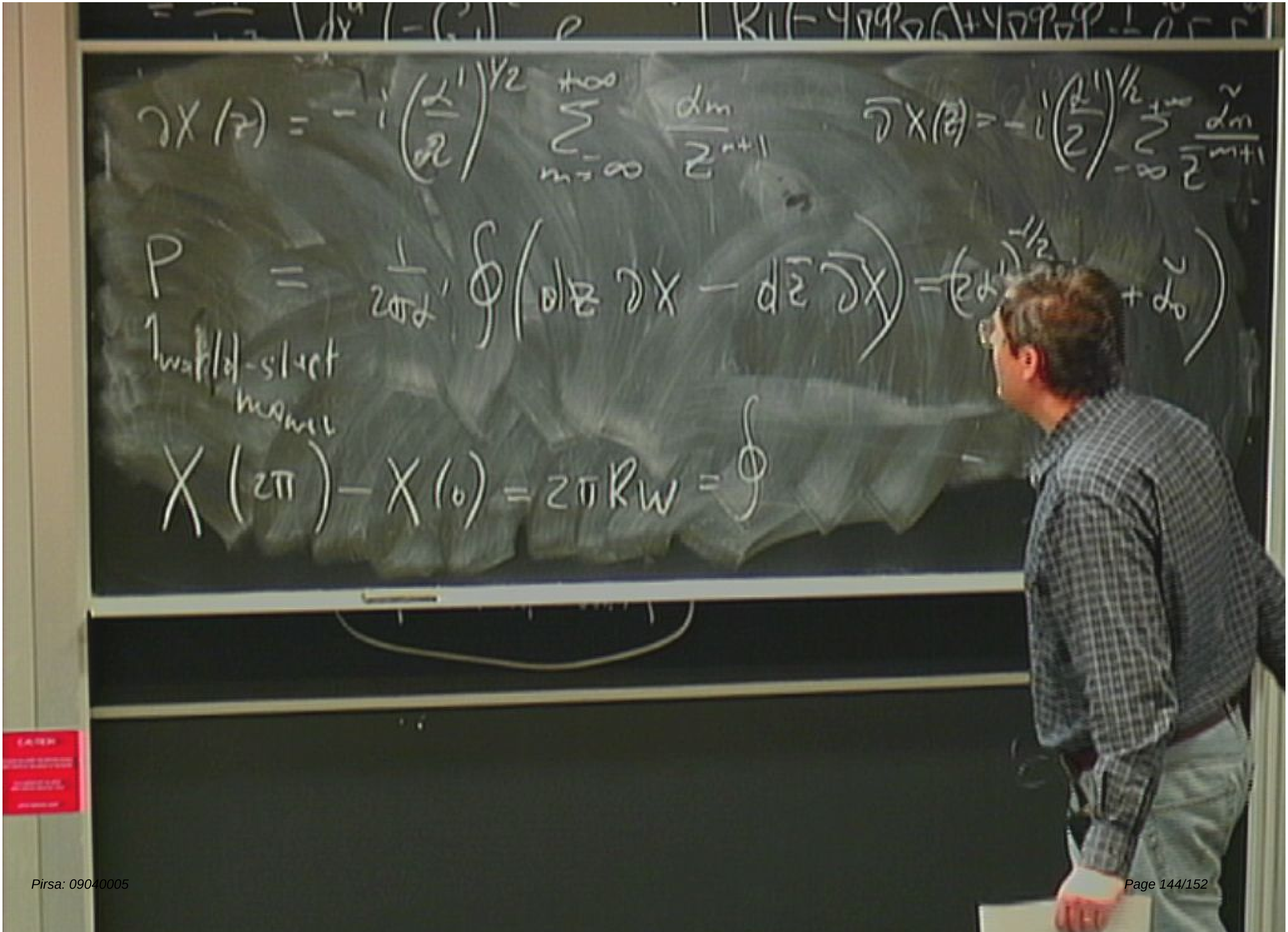
$$\bar\partial X(\bar z) = -i\left(\frac{\alpha'}{2}\right)^{1/2}\sum_{-\infty}^{+\infty}\frac{\tilde\alpha_m}{\bar z^{m+1}}$$

$$P = \frac{1}{2\pi\alpha'}\oint\left(dz\,\partial X - d\bar z\,\bar\partial X\right)$$

world-sheet momentum


$\partial X(z) = -i\left(\frac{\alpha'}{2}\right)^{1/2} \sum_{m=-\infty}^{+\infty} \frac{\alpha_m}{z^{m+1}}$
$\bar\partial X(\bar z) = -i\left(\frac{\alpha'}{2}\right)^{1/2} \sum_{-\infty}^{+\infty} \frac{\tilde\alpha_m}{\bar z^{m+1}}$
$P = \frac{1}{2\pi\alpha'} \oint \left(dz\, \partial X - d\bar z\, \bar\partial X\right) = (2\alpha')^{-1/2}\left(\alpha_0 + \tilde\alpha_0\right)$


$\partial X(z) = -i\left(\frac{\alpha'}{2}\right)^{1/2}\sum_{m=-\infty}^{+\infty}\frac{\alpha_m}{z^{m+1}}$
$\bar\partial X(\bar z) = -i\left(\frac{\alpha'}{2}\right)^{1/2}\sum_{-\infty}^{+\infty}\frac{\tilde\alpha_m}{\bar z^{m+1}}$
$P = \frac{1}{2\pi\alpha'}\oint\left(dz\,\partial X - d\bar z\,\bar\partial X\right) = (2\alpha')^{-1/2}(\alpha_0 + \tilde\alpha_0)$
world-sheet
momentum
X


∂X(z) = −i (α'/2)^1/2 Σ_{m=−∞}^{+∞} α_m / z^{m+1}
∂̄X(z̄) = −i (α'/2)^1/2 Σ_{−∞}^{+∞} α̃_m / z̄^{m+1}
P = 1/(2πα') ∮ (dz ∂X − dz̄ ∂̄X)
X(2π) − X(0) = 2πRw = ∮

$$\partial X(z) = -i\left(\frac{\alpha'}{2}\right)^{1/2}\sum_{m=-\infty}^{+\infty}\frac{\alpha_m}{z^{m+1}} \qquad \bar\partial X(\bar z) = -i\left(\frac{\alpha'}{2}\right)^{1/2}\sum_{-\infty}^{+\infty}\frac{\tilde\alpha_m}{\bar z^{m+1}}$$

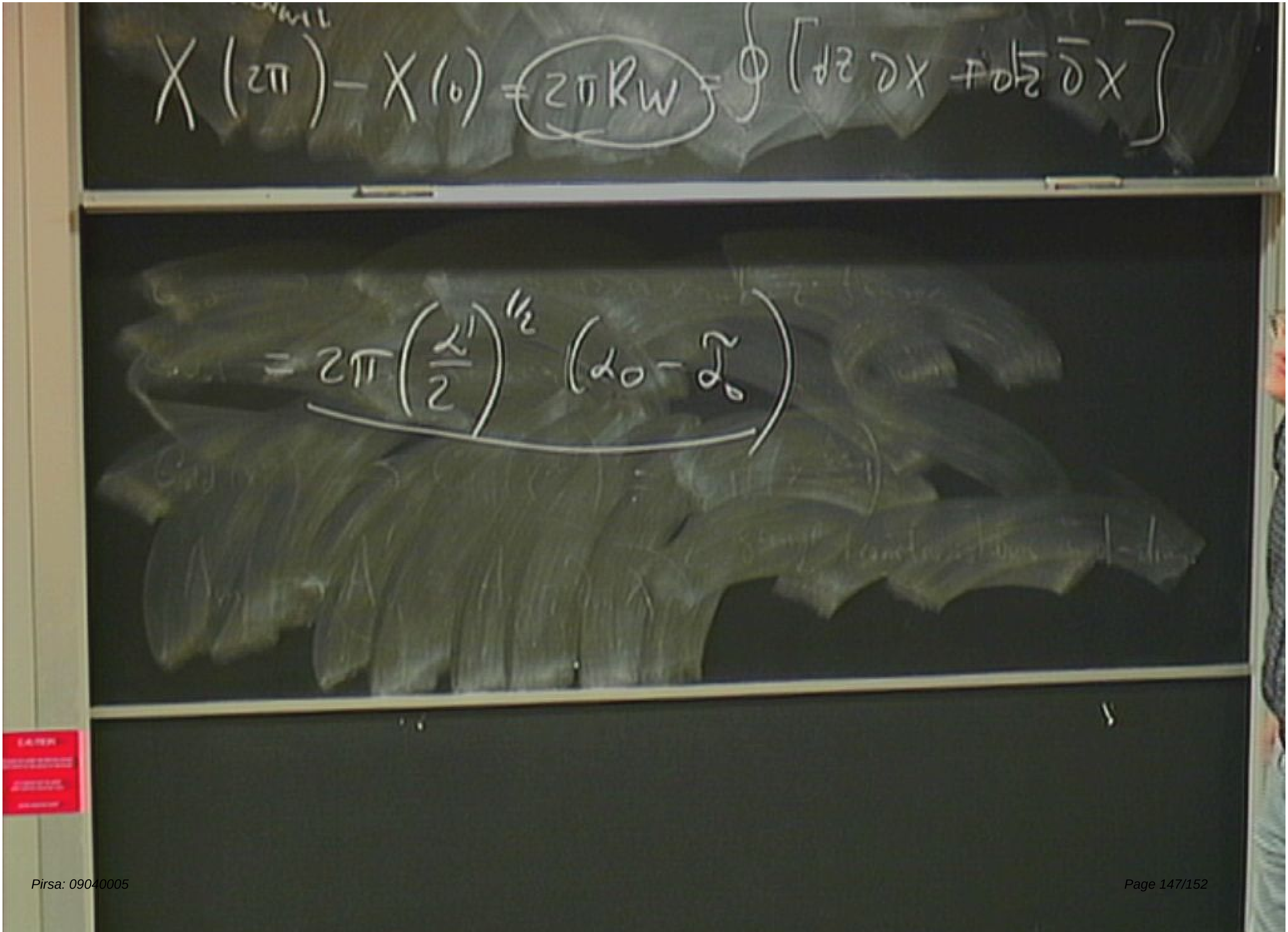
$$P = \frac{1}{2\pi\alpha'}\oint\left(dz\,\partial X - d\bar z\,\bar\partial X\right) = (2\alpha')^{-1/2}\left(\alpha_0+\tilde\alpha_0\right)$$

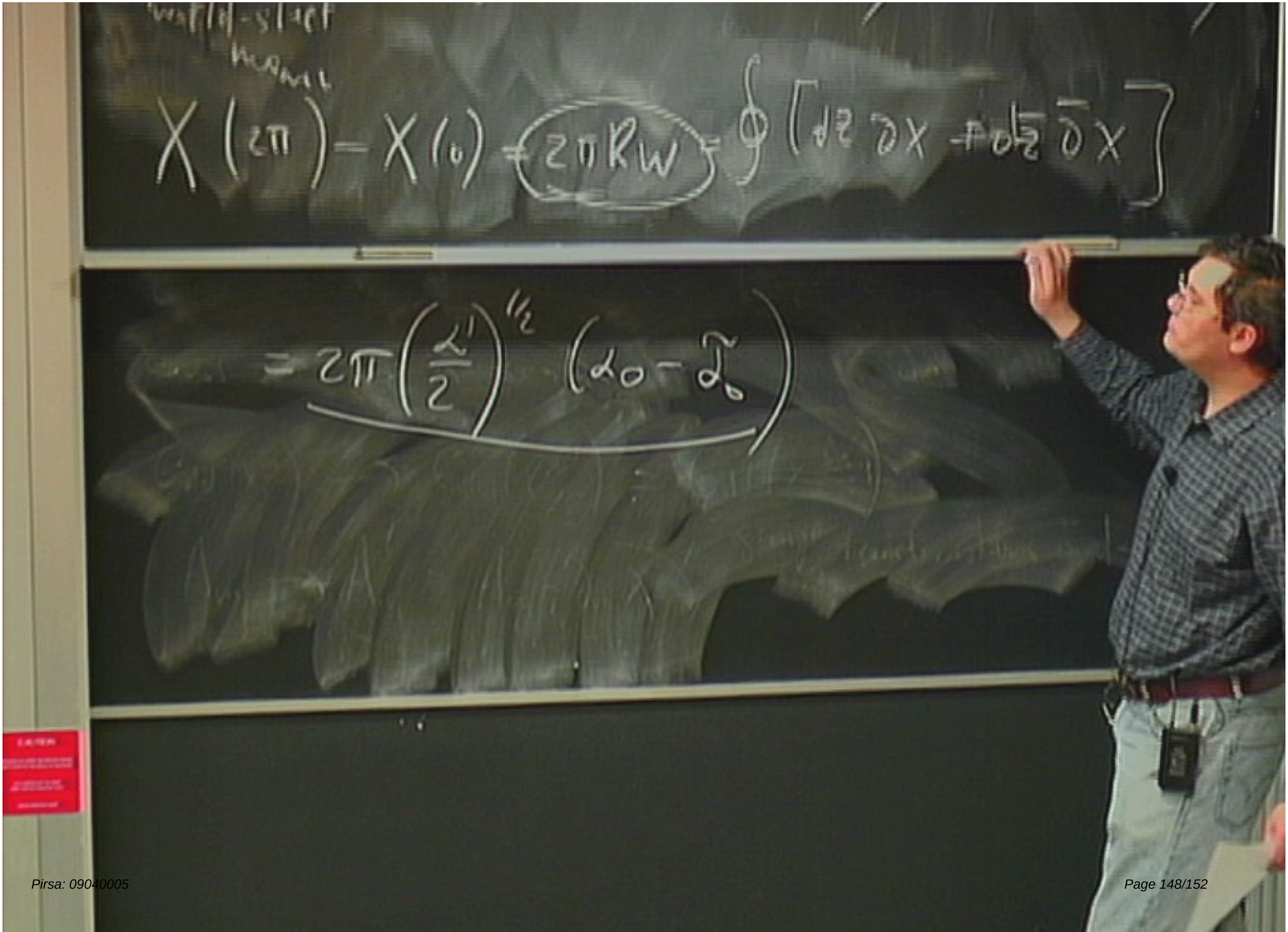
↑ world-sheet momentum

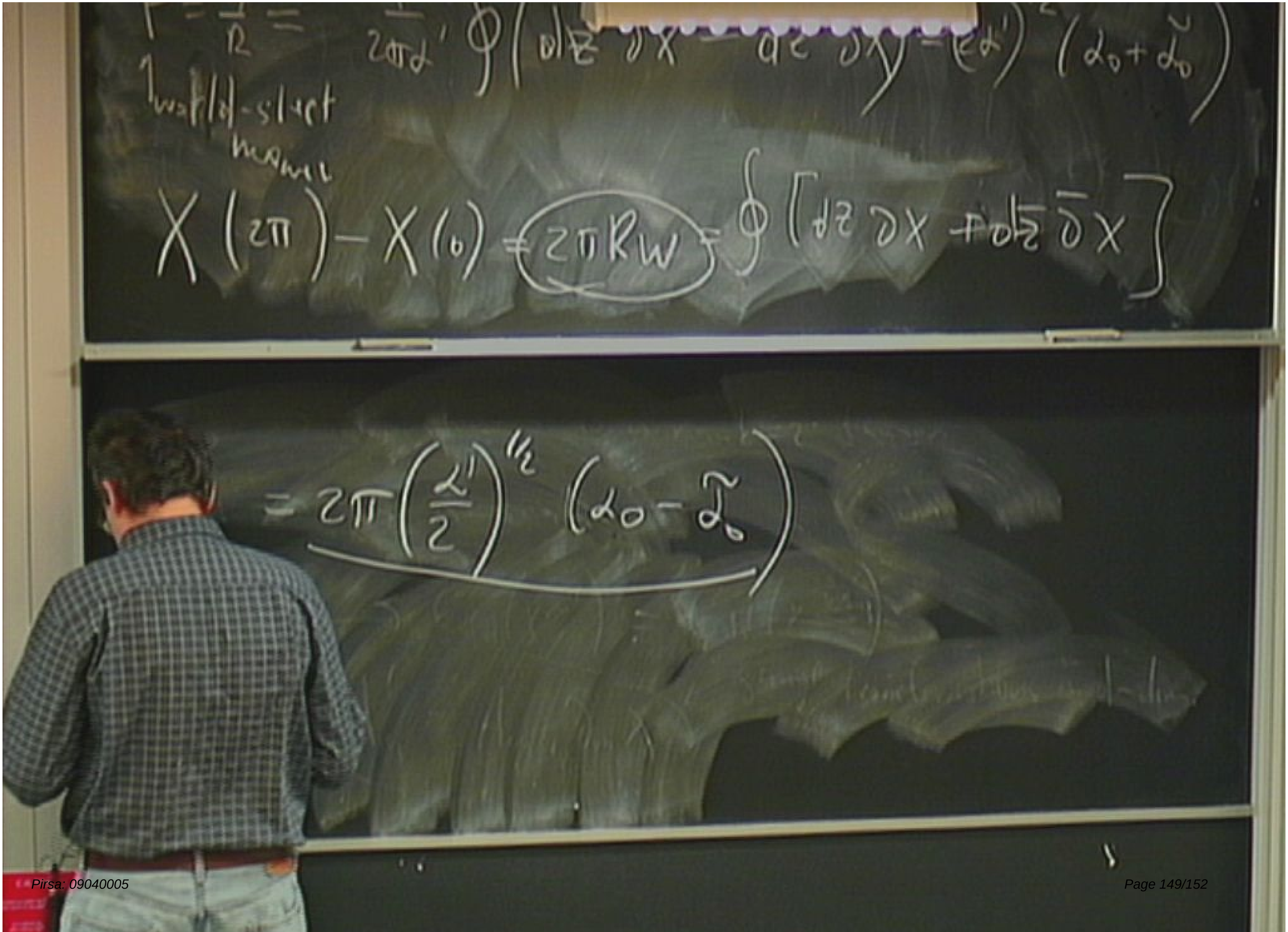
$$X(2\pi) - X(0) = 2\pi R w = \oint\left[dz\,\partial X + d\bar z\,\bar\partial X\right]$$

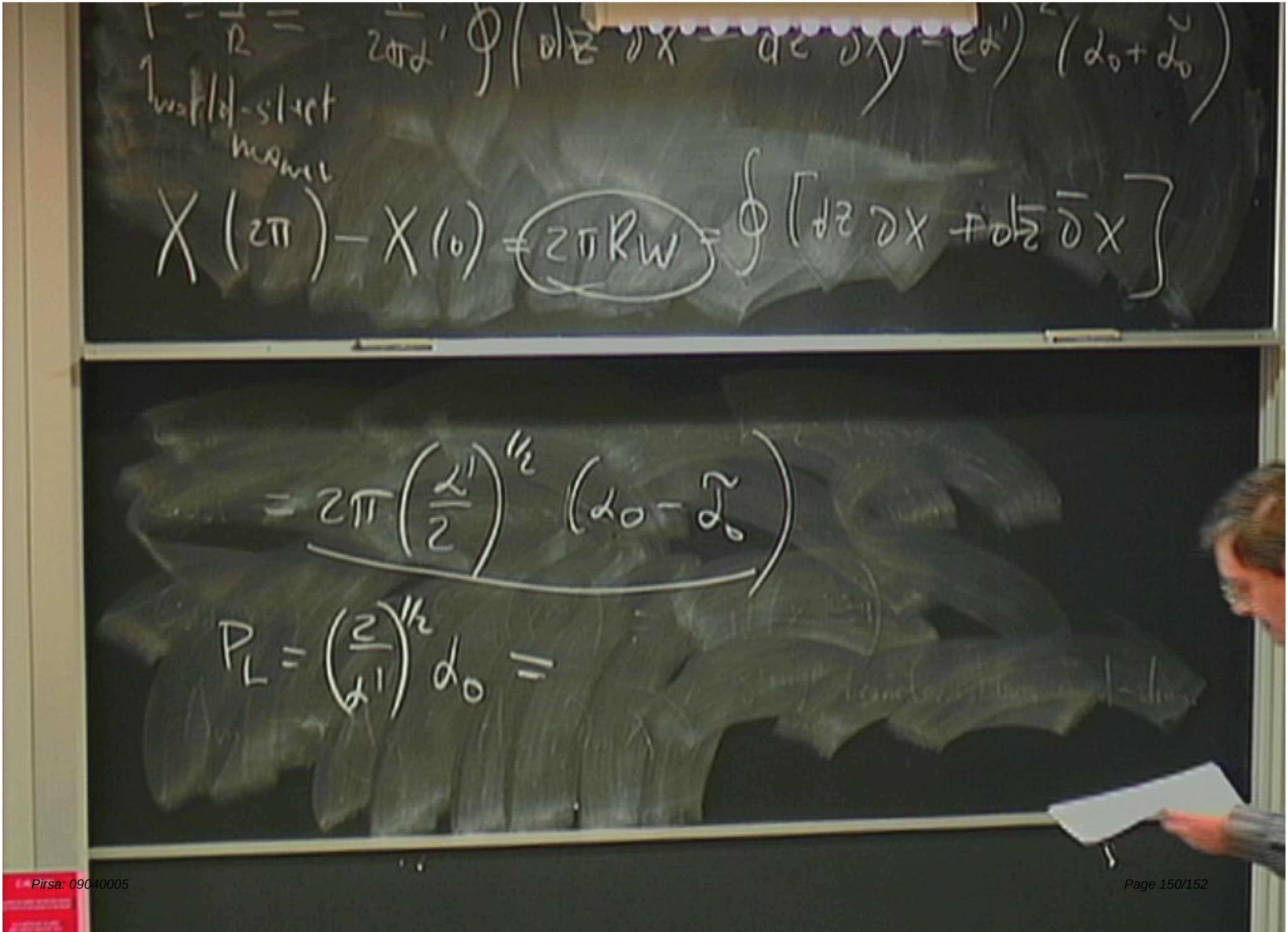
$$X(2\pi) - X(0) = 2\pi R W = \oint \left[dz\, \partial X + d\bar{z}\, \bar{\partial} X \right]$$

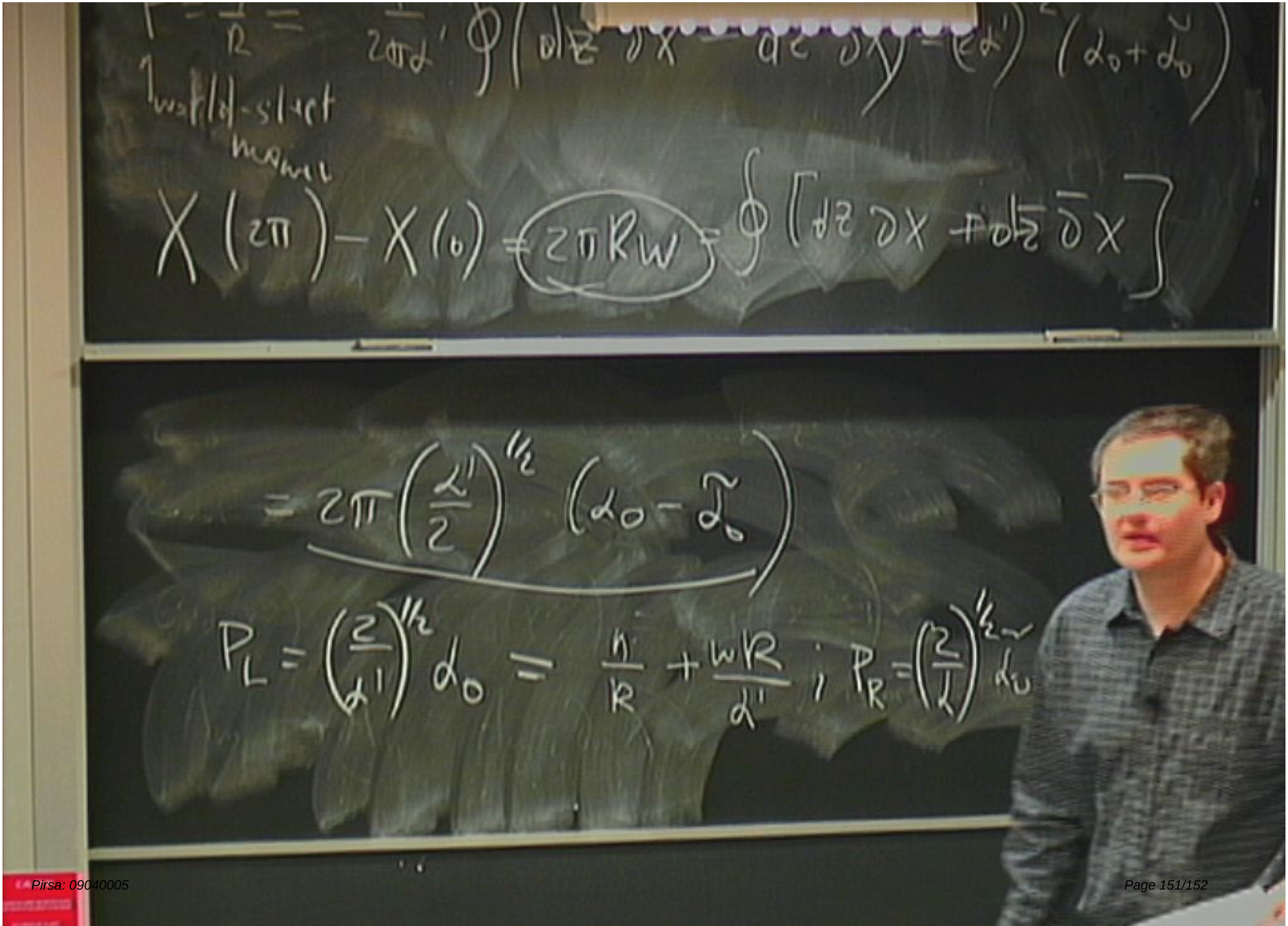
$$= 2\pi \left(\frac{\alpha'}{2}\right)^{1/2} \left(\alpha_0 - \tilde{\alpha}_0\right)$$


X(2π) − X(0) = 2πRW = ∮ [dz ∂X + dz̄ ∂̄X]
= 2π (α'/2)^1/2 (α0 − α̃0)


X(2π) − X(0) = 2πRW = ∮[dz ∂X + dz̄ ∂̄X]
= 2π(α'/2)^{1/2} (α₀ − α̃₀)


world-sheet
X(2π) − X(0) = 2πRW = ∮[dz ∂X + dz̄ ∂̄X]
= 2π(α'/2)^{1/2} (α₀ − α̃₀)


world-sheet
X(2π) − X(0) = 2πRW = ∮[dz ∂X + dz̄ ∂̄X]
= 2π(α'/2)^1/2 (α0 − α̃0)
PL = (2/α')^1/2 α0 =


X(2π) − X(0) = 2πRw = ∮[dz ∂X + dz̄ ∂̄X]
= 2π(α'/2)^1/2 (α0 − α̃0)
PL = (2/α')^1/2 α0 = n/R + wR/α' ; PR = (2/α')^1/2 α̃0

$$p = \frac{n}{R} = \frac{1}{2\pi\alpha'}\oint\left(dz\,\partial X - d\bar{z}\,\bar{\partial} X\right) = \left(\frac{2}{\alpha'}\right)^{1/2}\left(\alpha_0 + \tilde{\alpha}_0\right)$$

world-sheet momentum

$$X(2\pi) - X(0) = 2\pi R w = \oint\left[dz\,\partial X + d\bar{z}\,\bar{\partial} X\right]$$

$$= 2\pi\left(\frac{\alpha'}{2}\right)^{1/2}\left(\alpha_0 - \tilde{\alpha}_0\right)$$

$$P_L = \left(\frac{2}{\alpha'}\right)^{1/2}\alpha_0 = \frac{n}{R} + \frac{wR}{\alpha'}\ ;\quad P_R = \left(\frac{2}{\alpha'}\right)^{1/2}\tilde{\alpha}_0 = \frac{n}{R} - \frac{wR}{\alpha'}$$