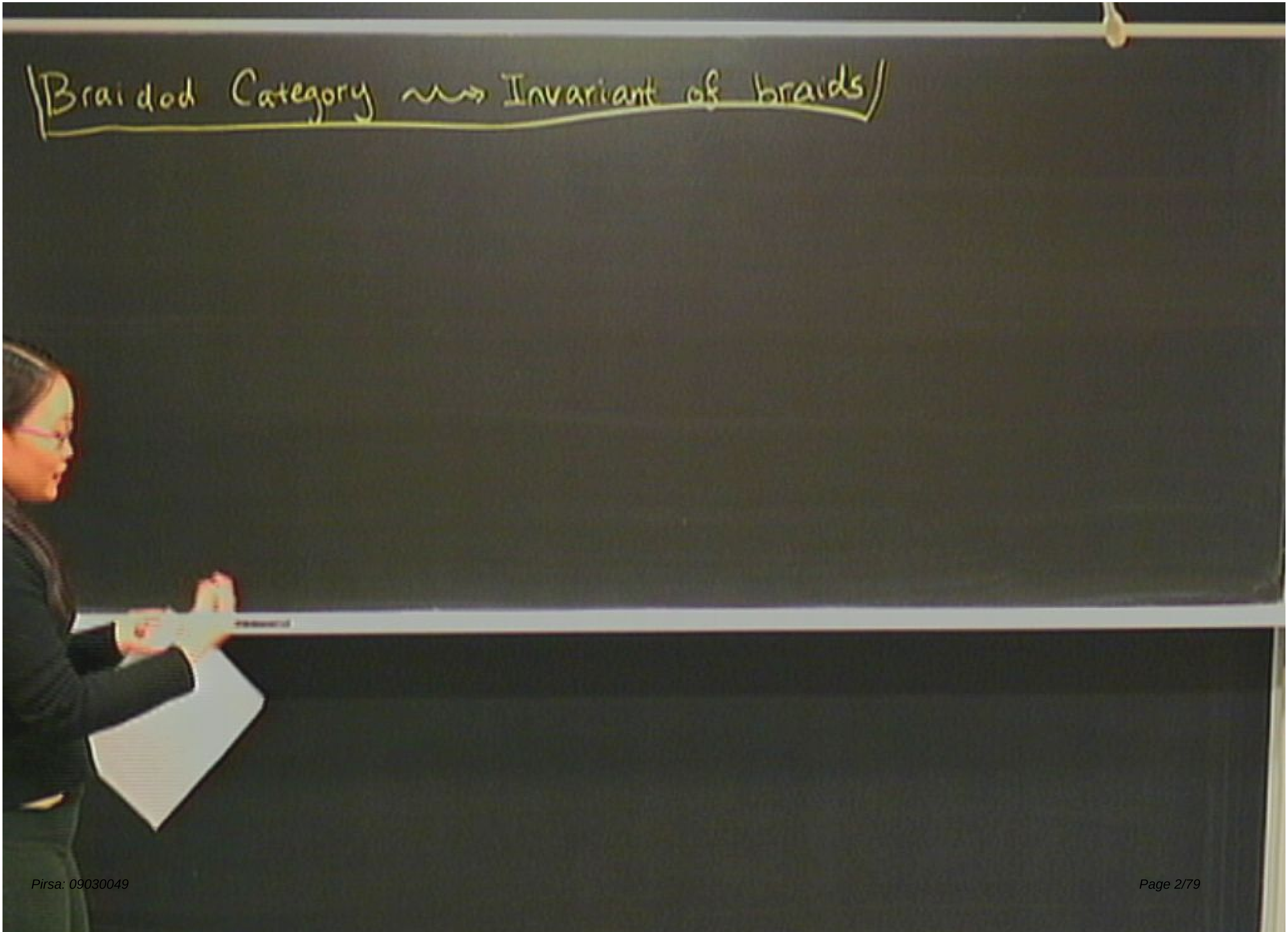
Title: Grad Talks 7

Date: Mar 30, 2009 11:00 AM

URL: http://pirsa.org/09030049

Abstract: Lecture on Quantum Groups by Lucy Zhang


Braided Category ⇝ Invariant of braids

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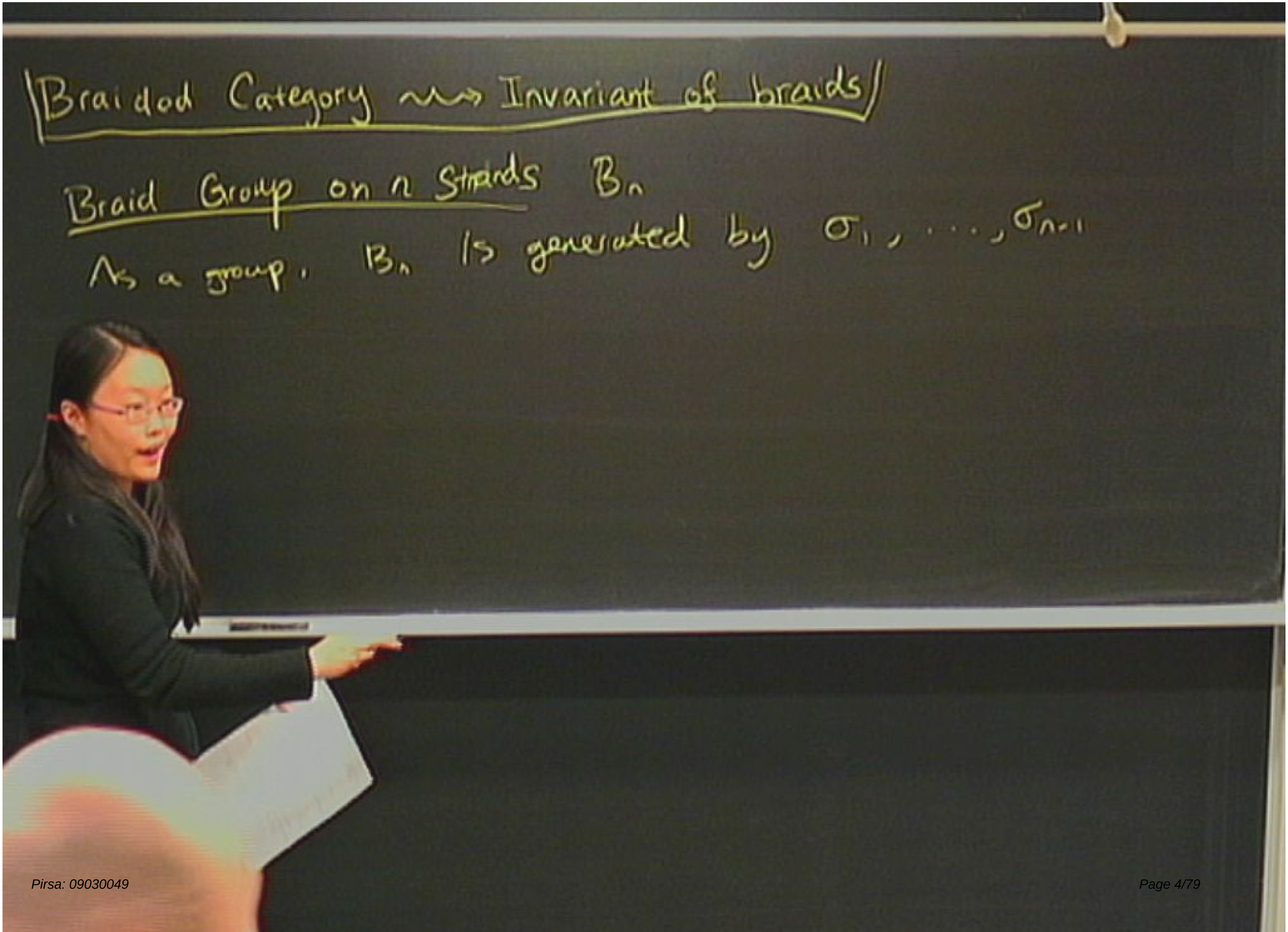
Braid Group on n Strands B_n



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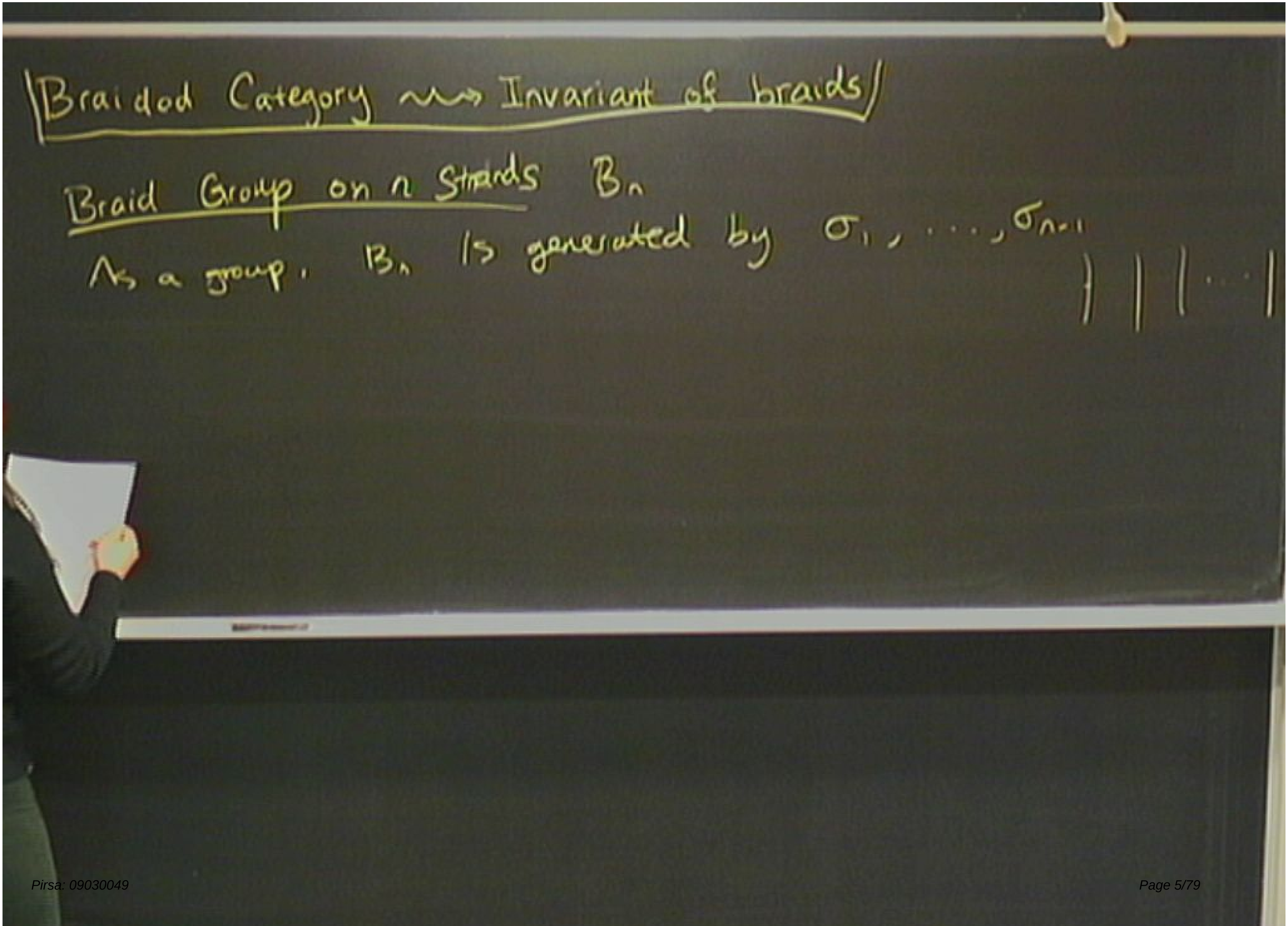
As a group, B_n is generated by $\sigma_1, \dots, \sigma_{n-1}$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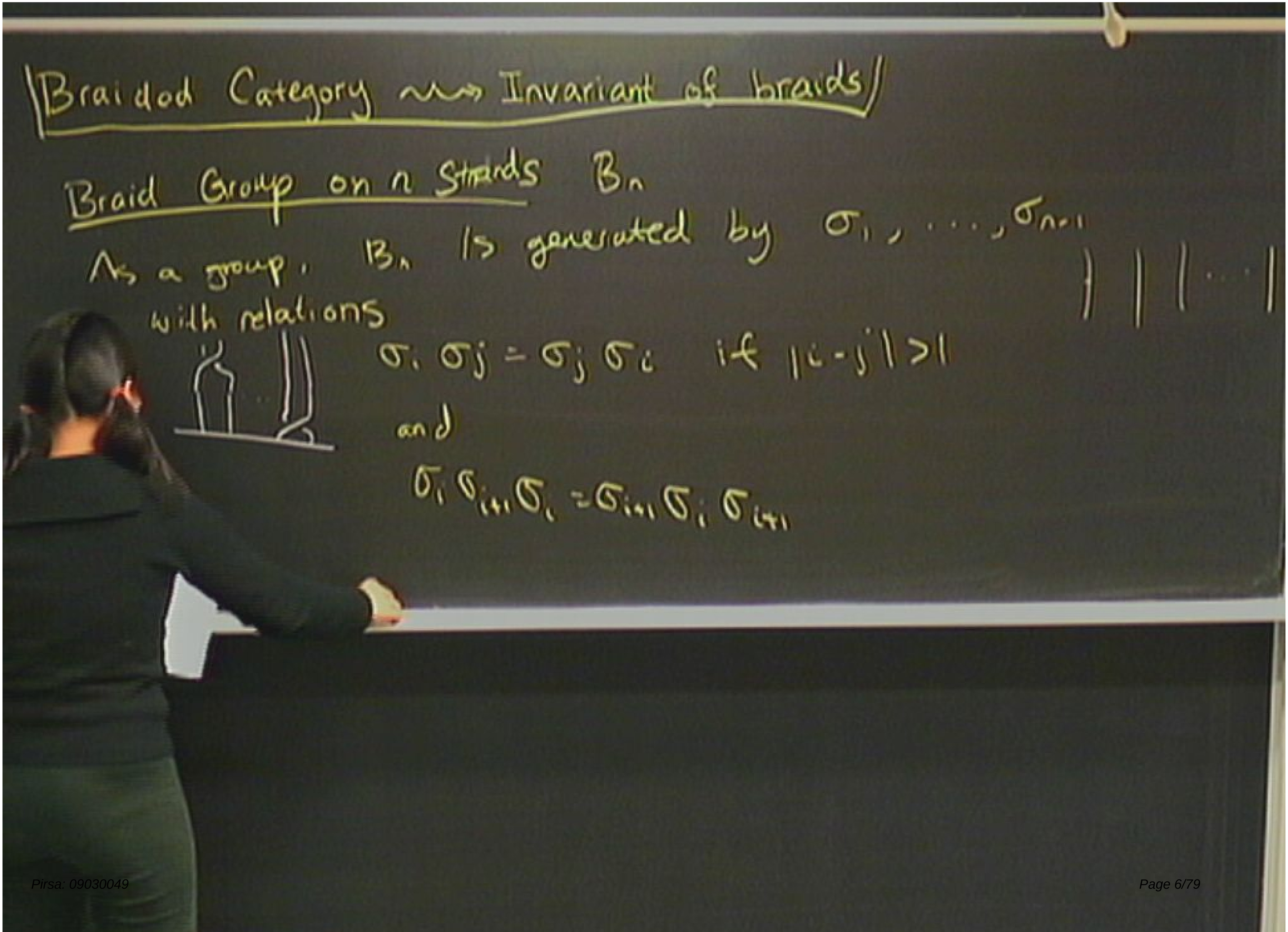
Braided Category ⇝ Invariant of braids
Braid Group on n Strands B_n
As a group, B_n is generated by $\sigma_1, \dots, \sigma_{n-1}$
with relations
$\sigma_i \sigma_j = \sigma_j \sigma_i$ if $|i-j| > 1$
and
$\sigma_i \sigma_{i+1} \sigma_i = \sigma_{i+1} \sigma_i \sigma_{i+1}$

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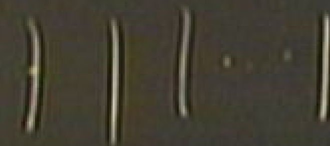
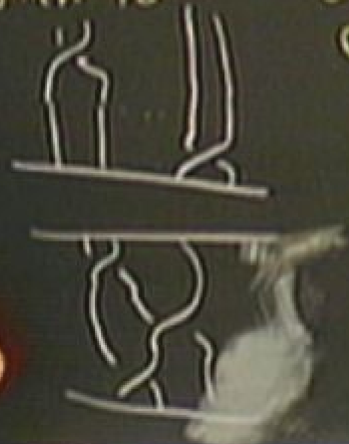


with relations



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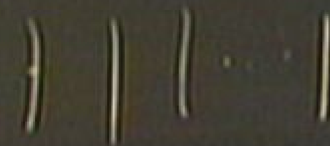

with relations

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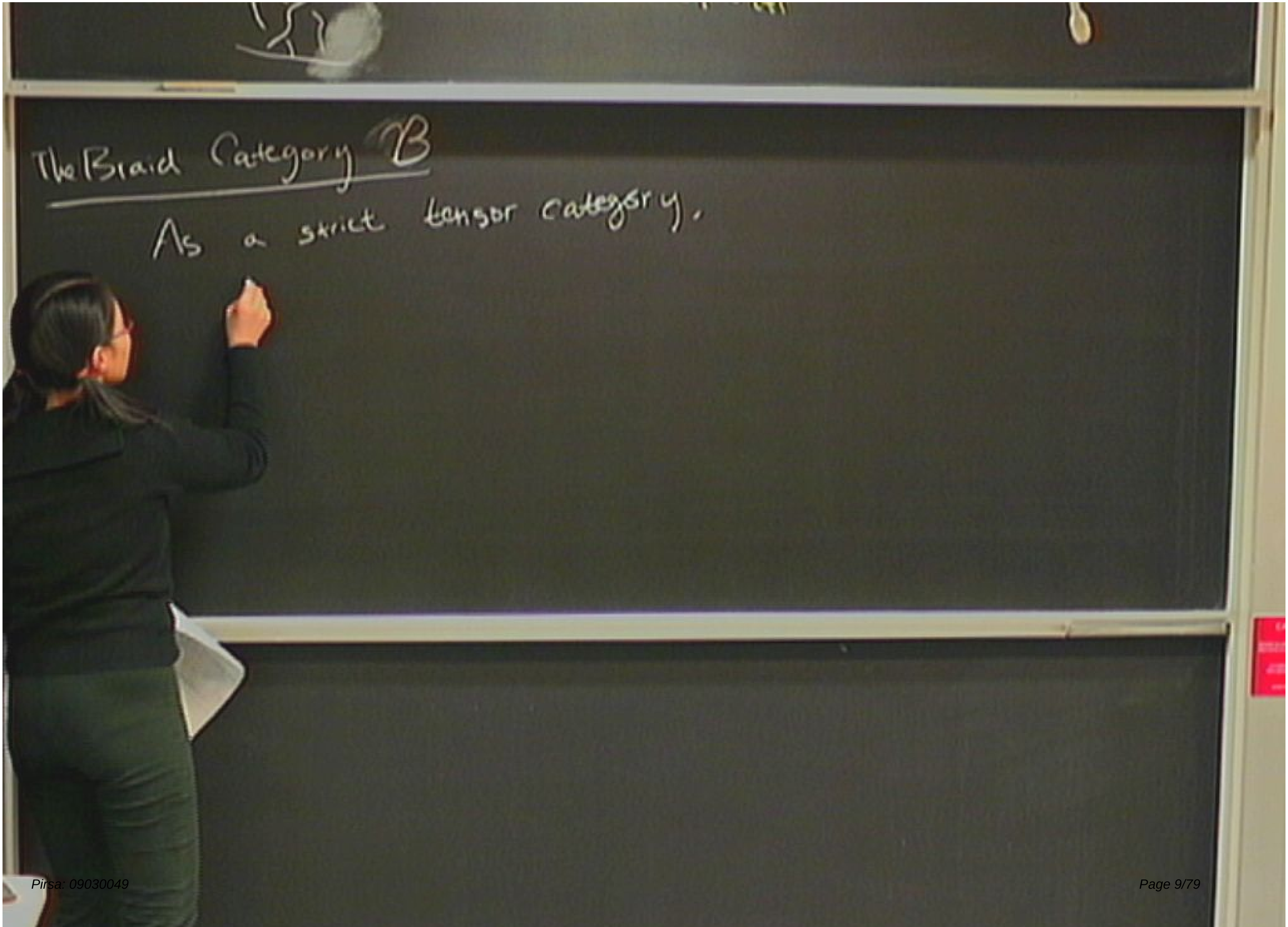
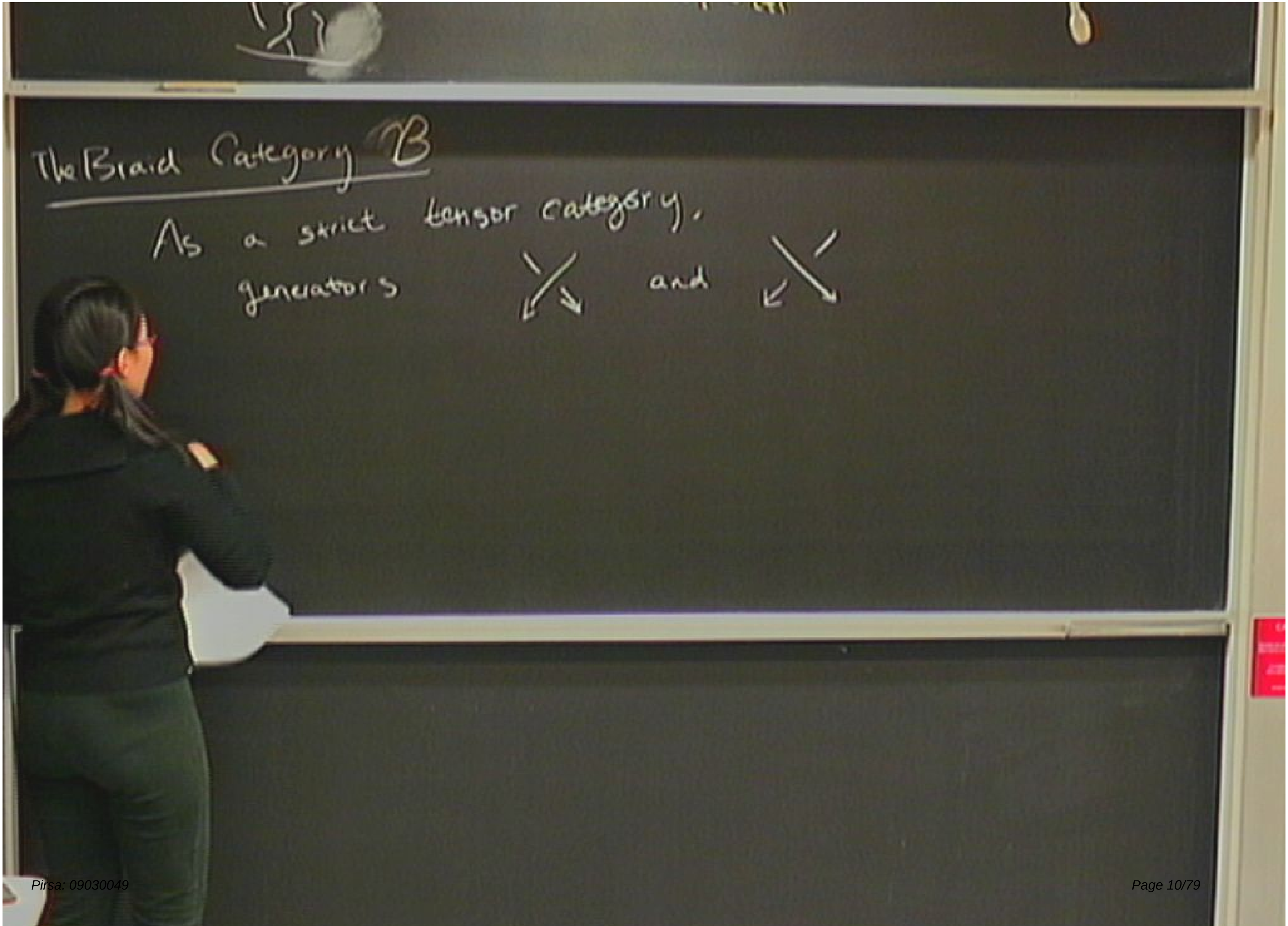

The Braid Category B
As a strict tensor category.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generators
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The Braid Category $\mathcal{B}$

As a strict tensor category.

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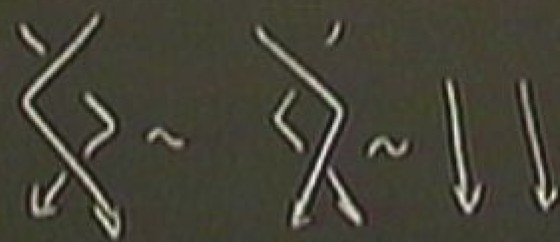
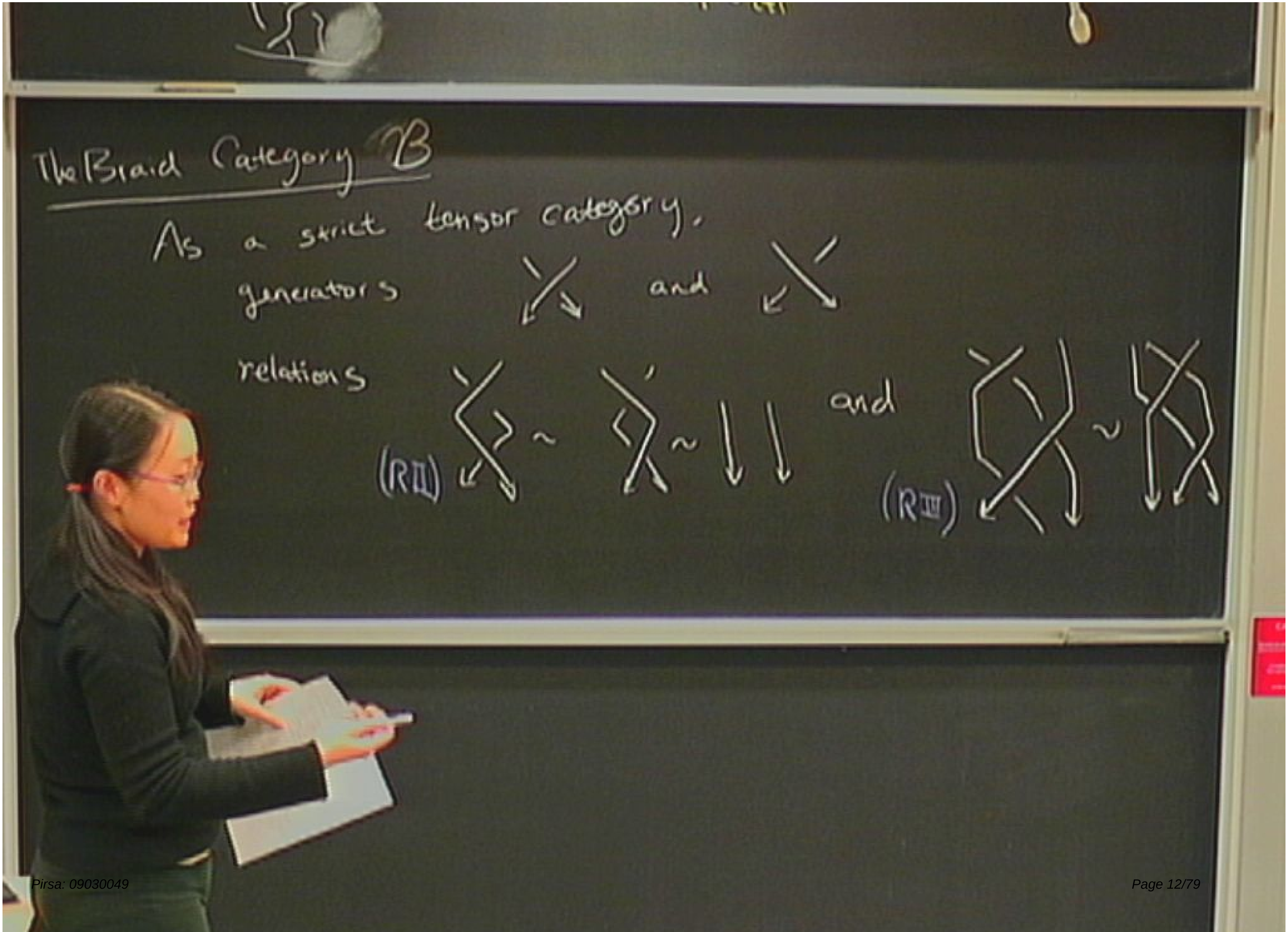
relations  and  $\sim$ 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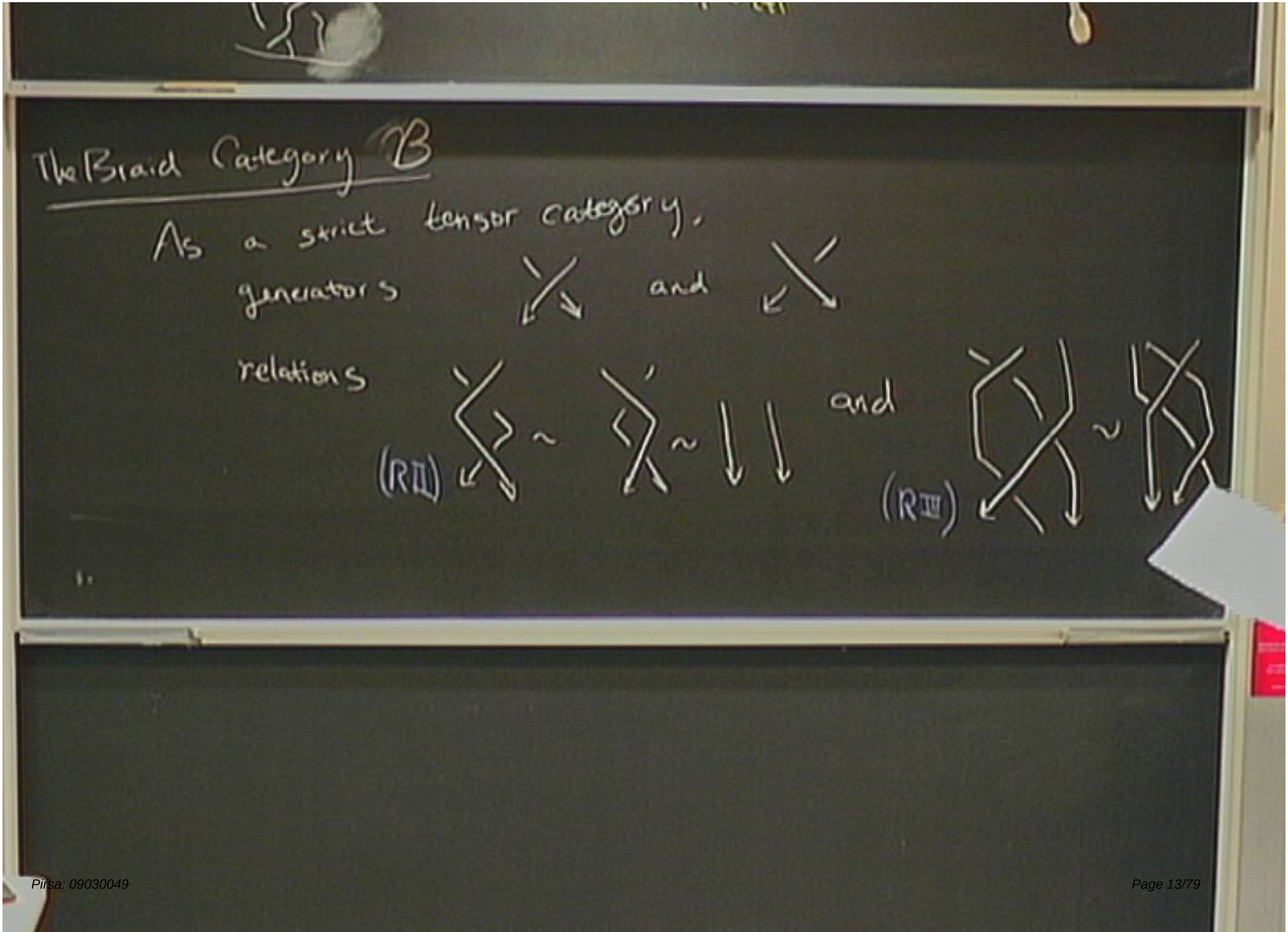
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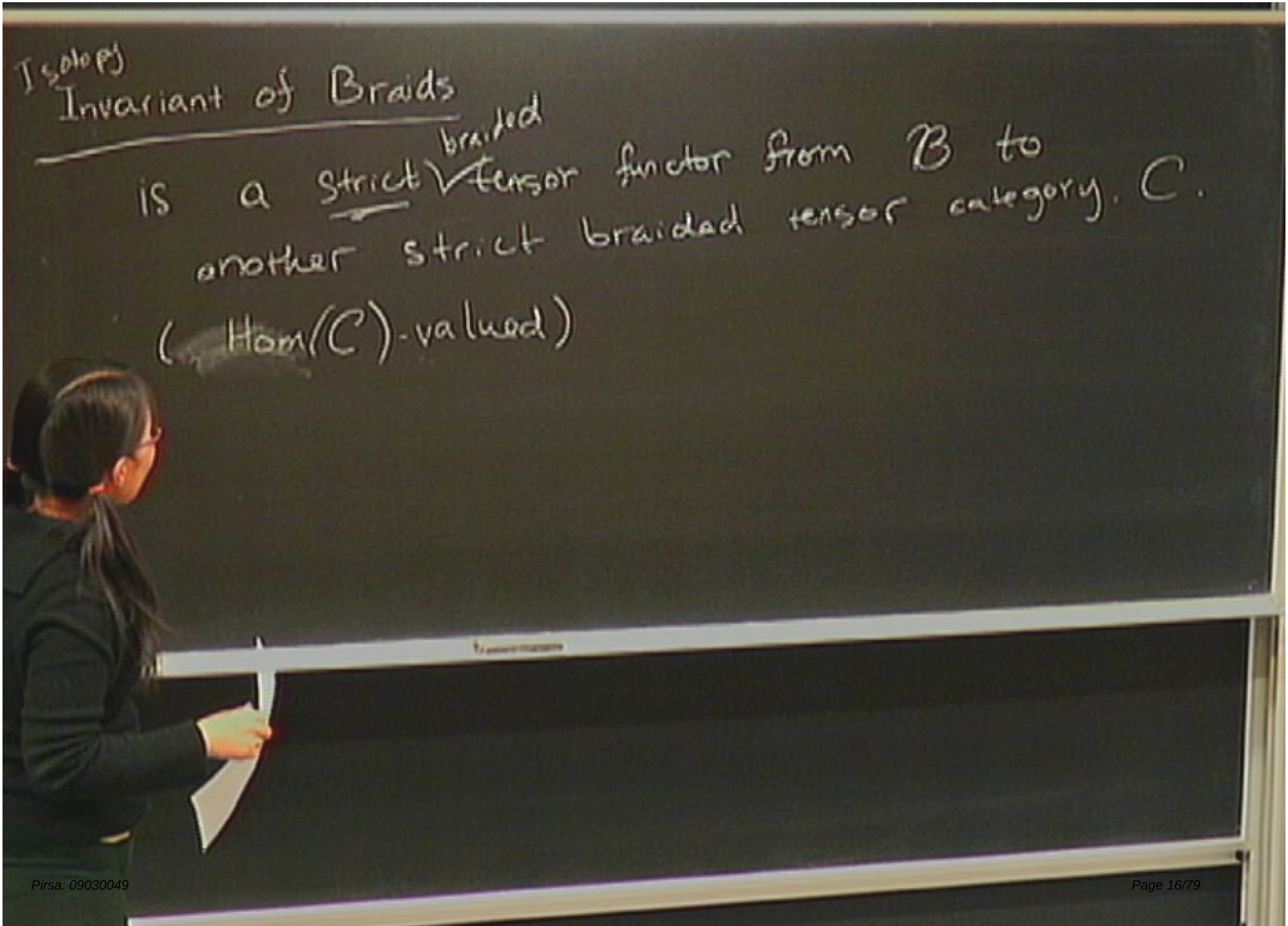
Invariant of Braids

is a strict tensor functor from $\mathcal{B}$ to another strict braided tensor category.

Isotopy
Invariant of Braids

is a strict tensor functor from $\mathcal{B}$ to another strict braided tensor category $\mathcal{C}$.

(Hom($\mathcal{C}$)-valued)


Isotopy
Invariant of Braids
braided
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[B]raided Tensor Functor $(F, \varphi_0, \varphi_2) : C \to D$

C

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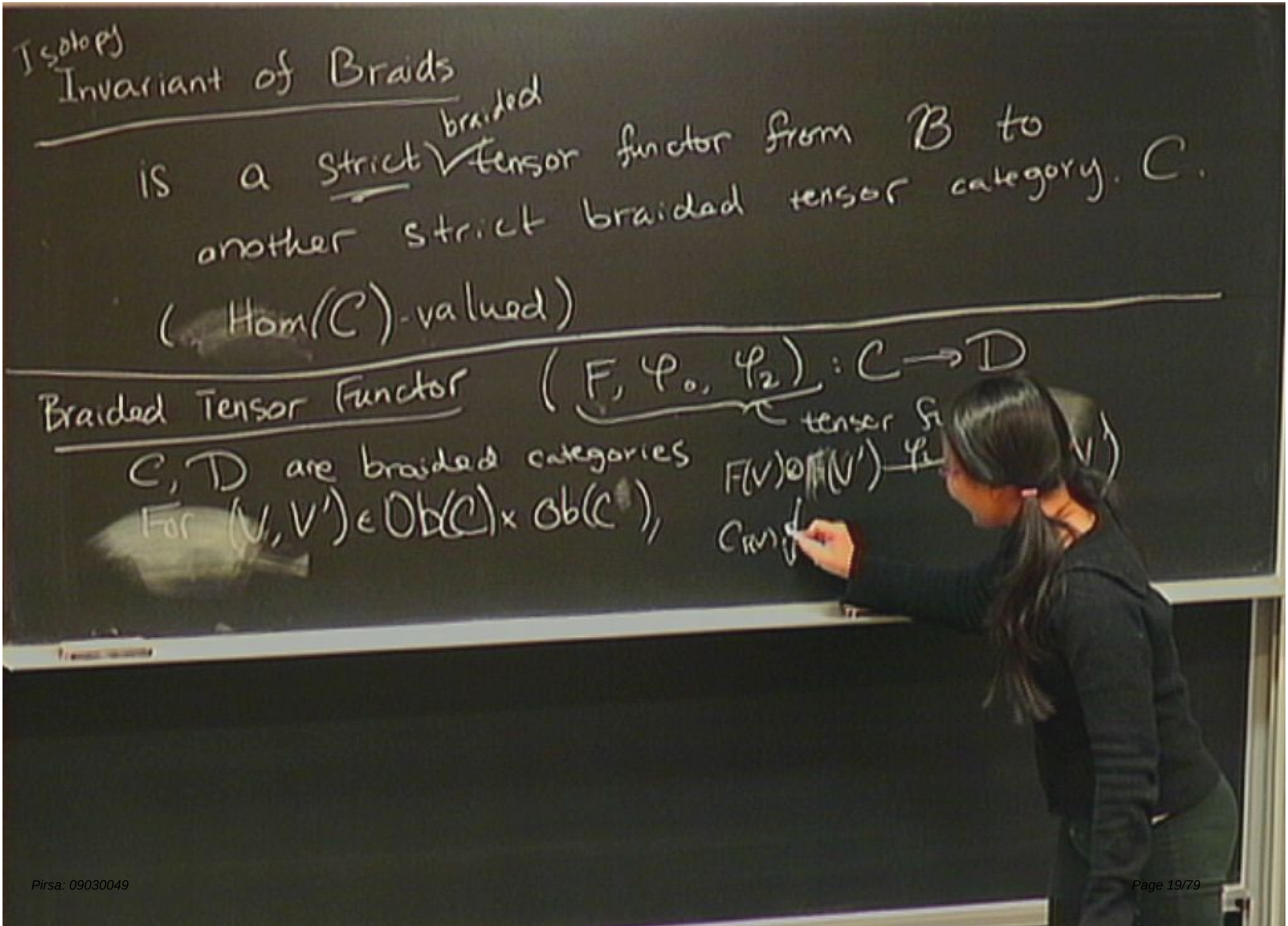
tensor functor

C, D are braided categories


Isotopy
Invariant of Braids
is a strict braided tensor functor from B to another strict braided tensor category. C.
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Braided Tensor Functor (F, φ0, φ2) : C → D
tensor fu
C, D are braided categories
For (V, V') ∈ Ob(C) × Ob(C'),
F(V)⊗F(V') φ2

Isotopy Invariant of Braids

is a strict braided tensor functor from $\mathcal{B}$ to another strict braided tensor category C.

(Hom(C)-valued)

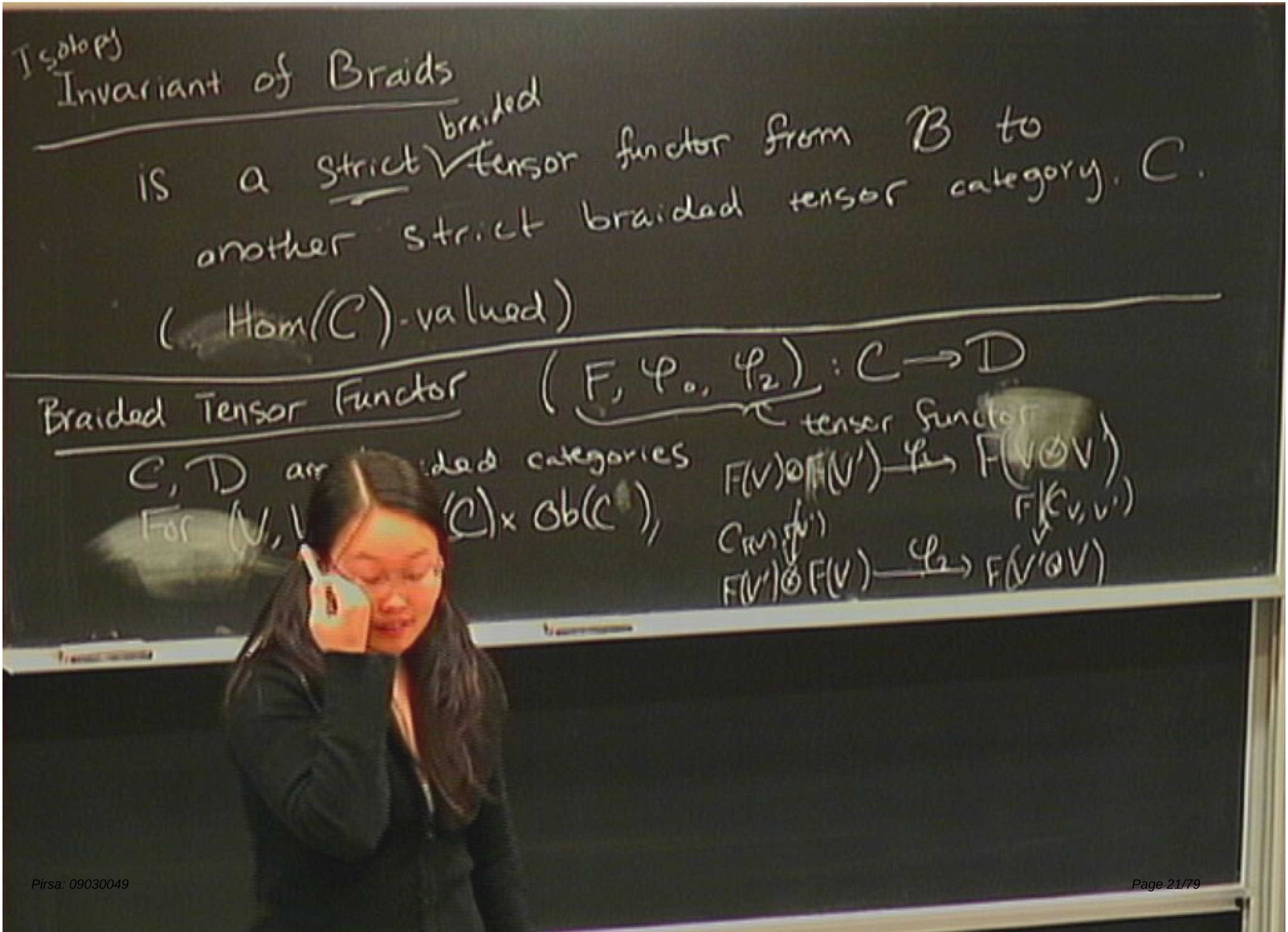
[illegible]d Tensor Functor $(F, \varphi_0, \varphi_2) : C \to D$

$(F, \varphi_0, \varphi_2)$ — tensor functor

[illegible] D are braided categories

[illegible]or $(V, V') \in Ob(C) \times Ob(C)$,

$$\begin{array}{ccc} F(V)\otimes F(V') & \xrightarrow{\varphi_2} & F(V\otimes V') \\ \downarrow c_{F(V),F(V')} & & \downarrow F(c_{V,V'}) \\ F(V')\otimes F(V) & \xrightarrow{\varphi_2} & F(V'\otimes V) \end{array}$$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C, D are braided categories
F(V)⊗F(V') —φ2→ F(V⊗V')
F(c_{V,V'})
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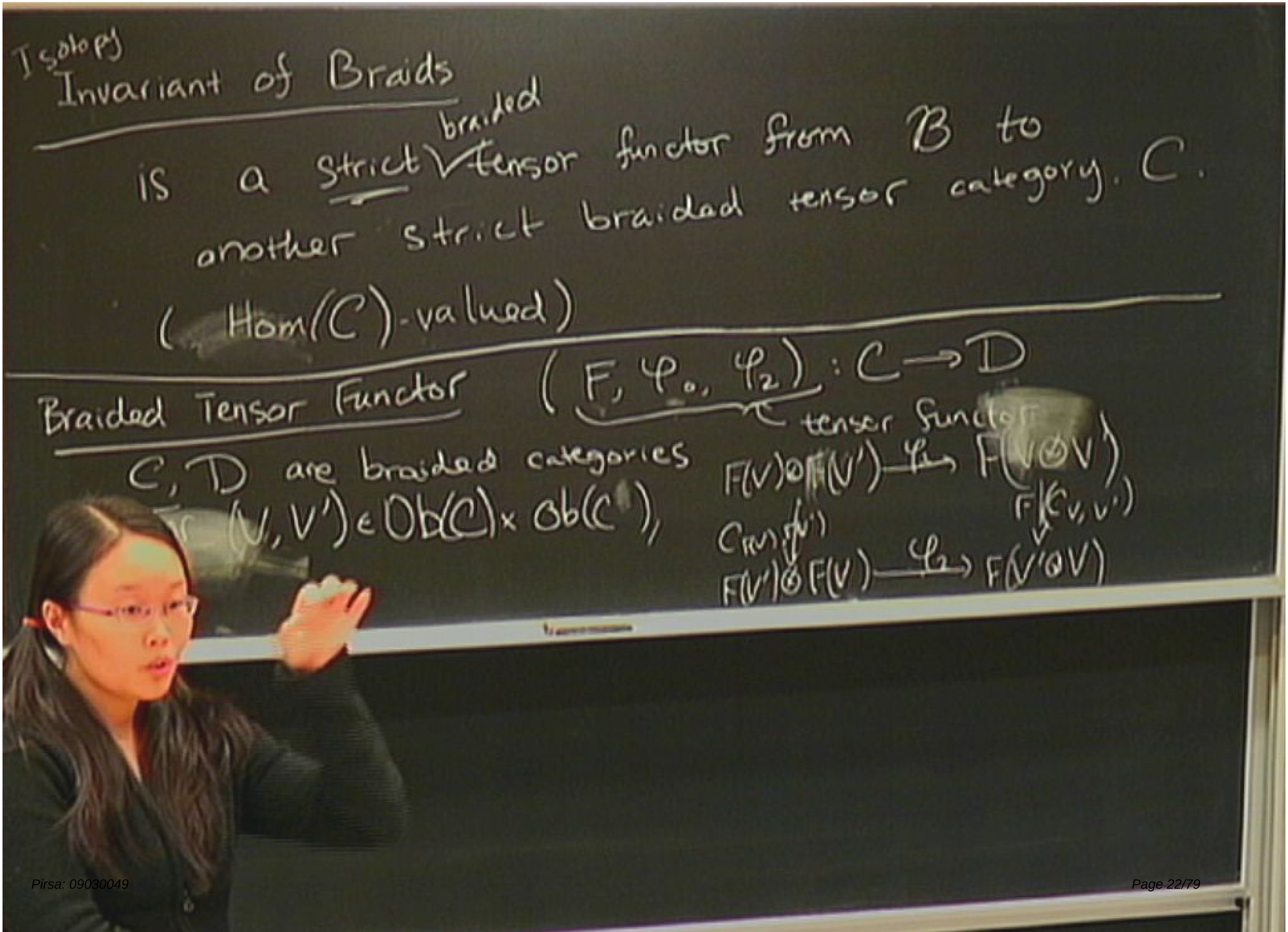
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Braided Tensor Functor
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tensor functor
C, D are braided categories
$(V, V') \in Ob(C) \times Ob(C)$,
$F(V) \otimes F(V') \xrightarrow{\varphi_2} F(V \otimes V')$
$c_{F(V),F(V')}$
$F(c_{V,V'})$
$F(V') \otimes F(V) \xrightarrow{\varphi_2} F(V' \otimes V)$

Isotopy
Invariant of Braids

is a $\underline{\text{strict}}$ braided tensor functor from $\mathcal{B}$ to another strict braided tensor category. C.

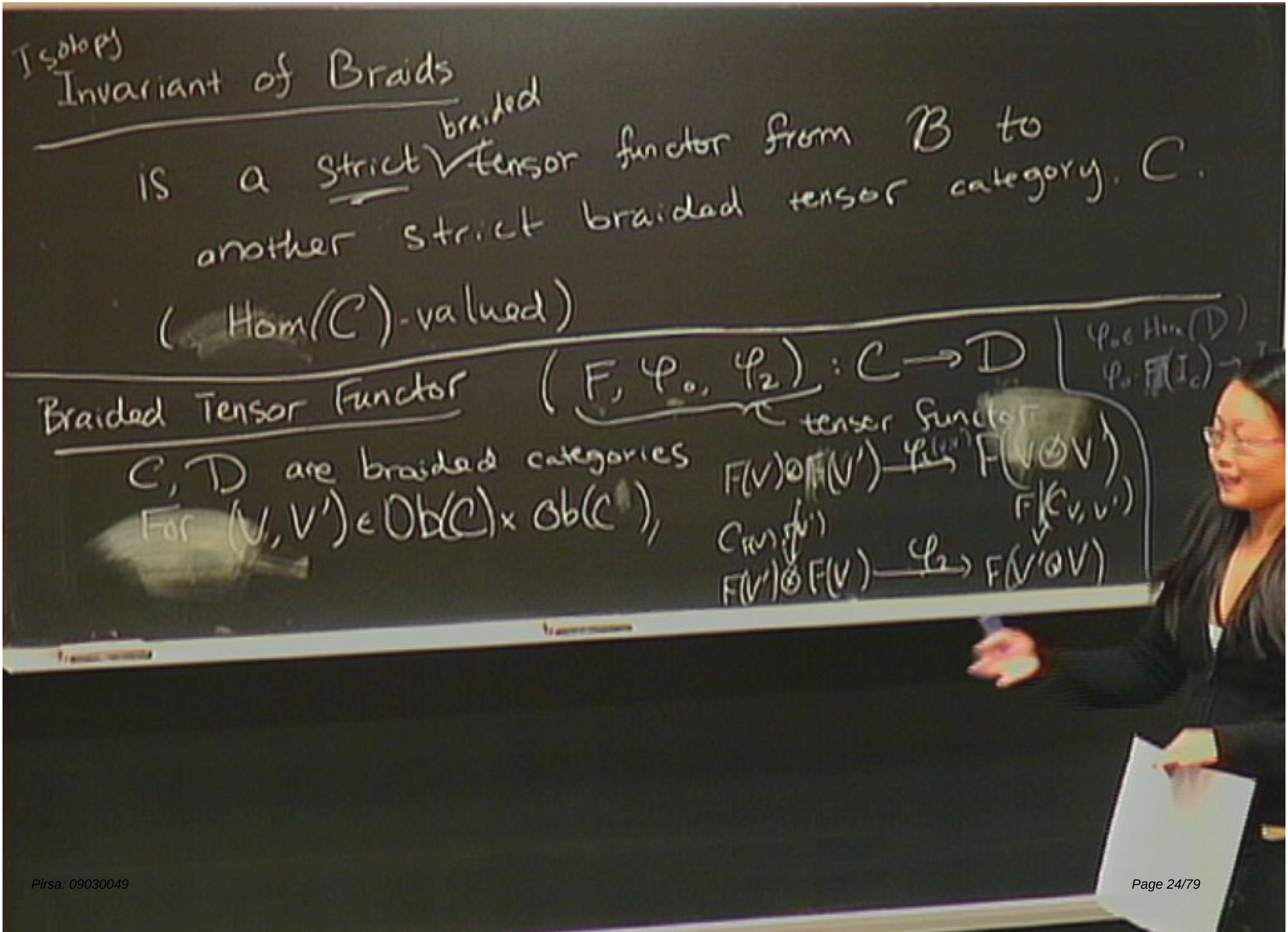
(Hom(C)-valued)

Braided Tensor Functor $(\underbrace{F, \varphi_0, \varphi_2}_{\text{tensor functor}}) : C \to D$

C, D are braided categories

For $(V, V') \in Ob(C) \times Ob(C)$,

$$\begin{array}{ccc} F(V) \otimes F(V') & \xrightarrow{\varphi_2} & F(V \otimes V') \\ \downarrow c_{F(V),F(V')} & & \downarrow F(c_{V,V'}) \\ F(V') \otimes F(V) & \xrightarrow{\varphi_2} & F(V' \otimes V) \end{array}$$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F(V)⊗F(V') → F(V⊗V')
F(V')⊗F(V) → F(V'⊗V)

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(Hom($\mathcal{C}$)-valued)

$\varphi_0 \in \mathrm{Hom}(\mathcal{D})$

$\varphi_0 : F(I_c) \to I$

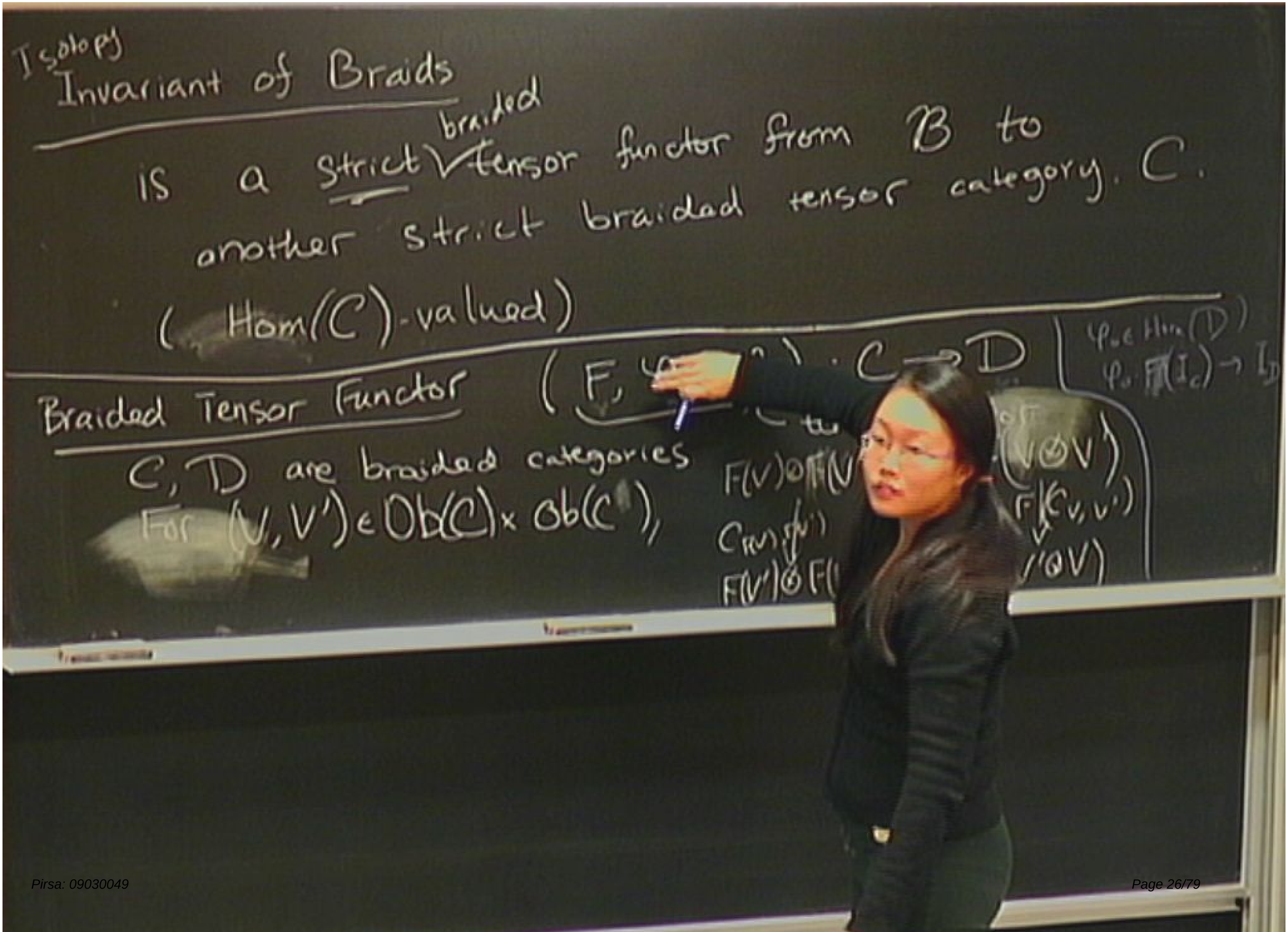
Braided Tensor Functor

$(F, \varphi_0, \varphi_2) : \mathcal{C} \to \mathcal{D}$ — tensor functor

$\mathcal{C}, \mathcal{D}$ are braided categories

For $(V, V') \in Ob(\mathcal{C}) \times Ob(\mathcal{C})$,

$$\begin{array}{ccc} F(V)\otimes F(V') & \xrightarrow{\varphi_2(V,V')} & F(V\otimes V') \\ \downarrow c_{F(V),F(V')} & & \downarrow F(c_{V,V'}) \\ F(V')\otimes F(V) & \xrightarrow{\varphi_2} & F(V'\otimes V) \end{array}$$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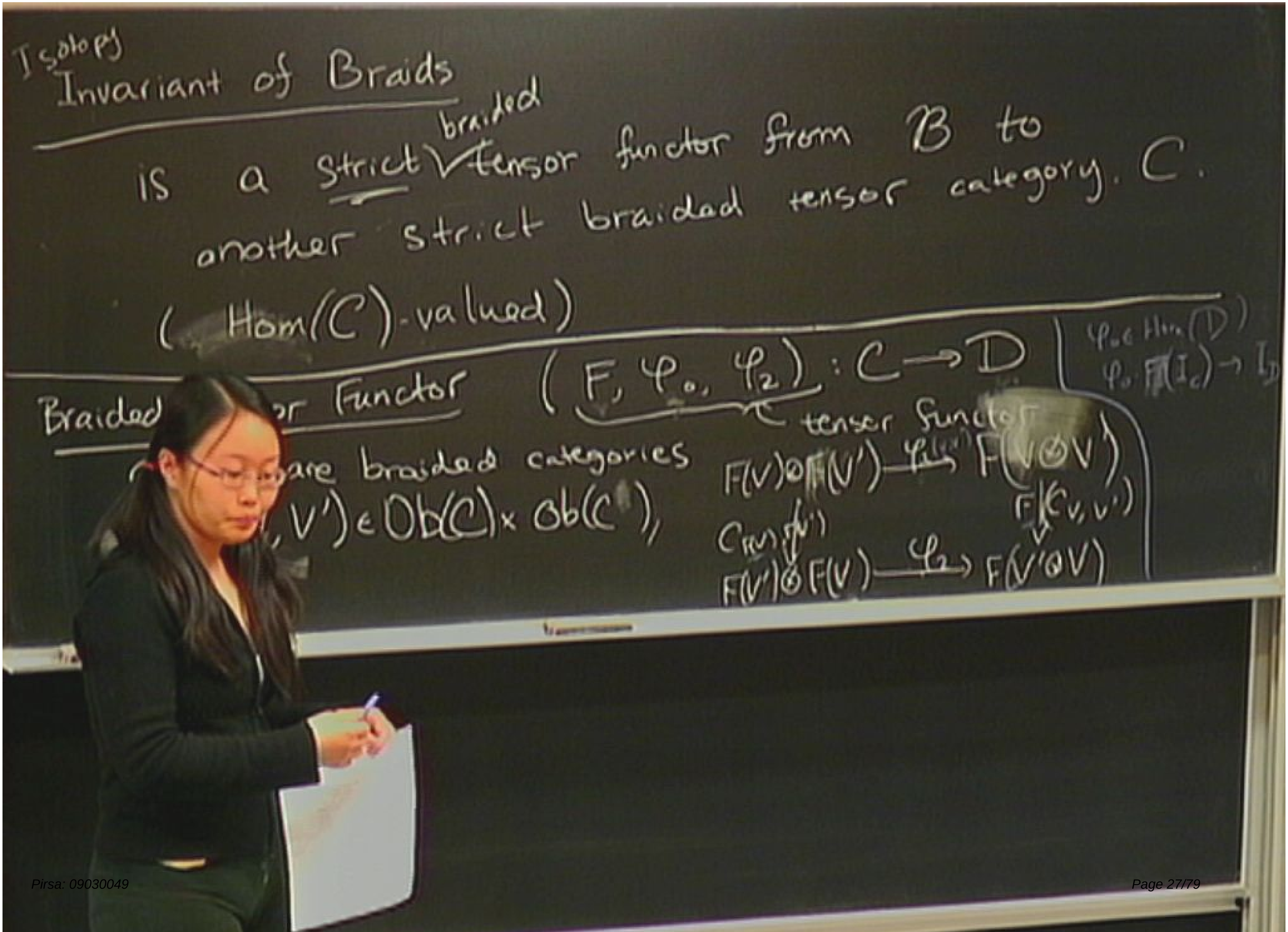
F(V)⊗F(V
F(V')⊗F(
φ₀ ∈ Hom(D)
φ₀: F(I_C) → I_D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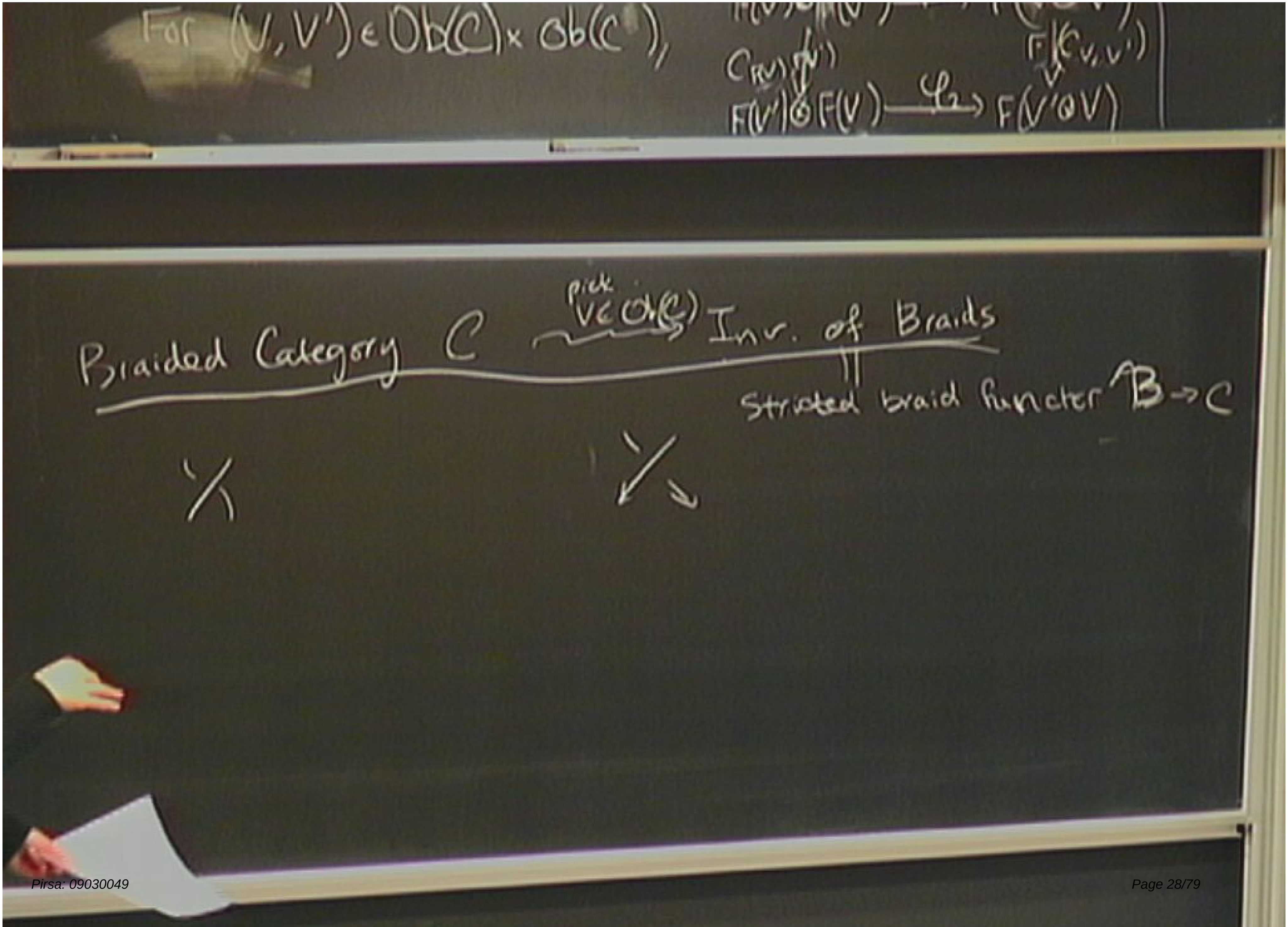
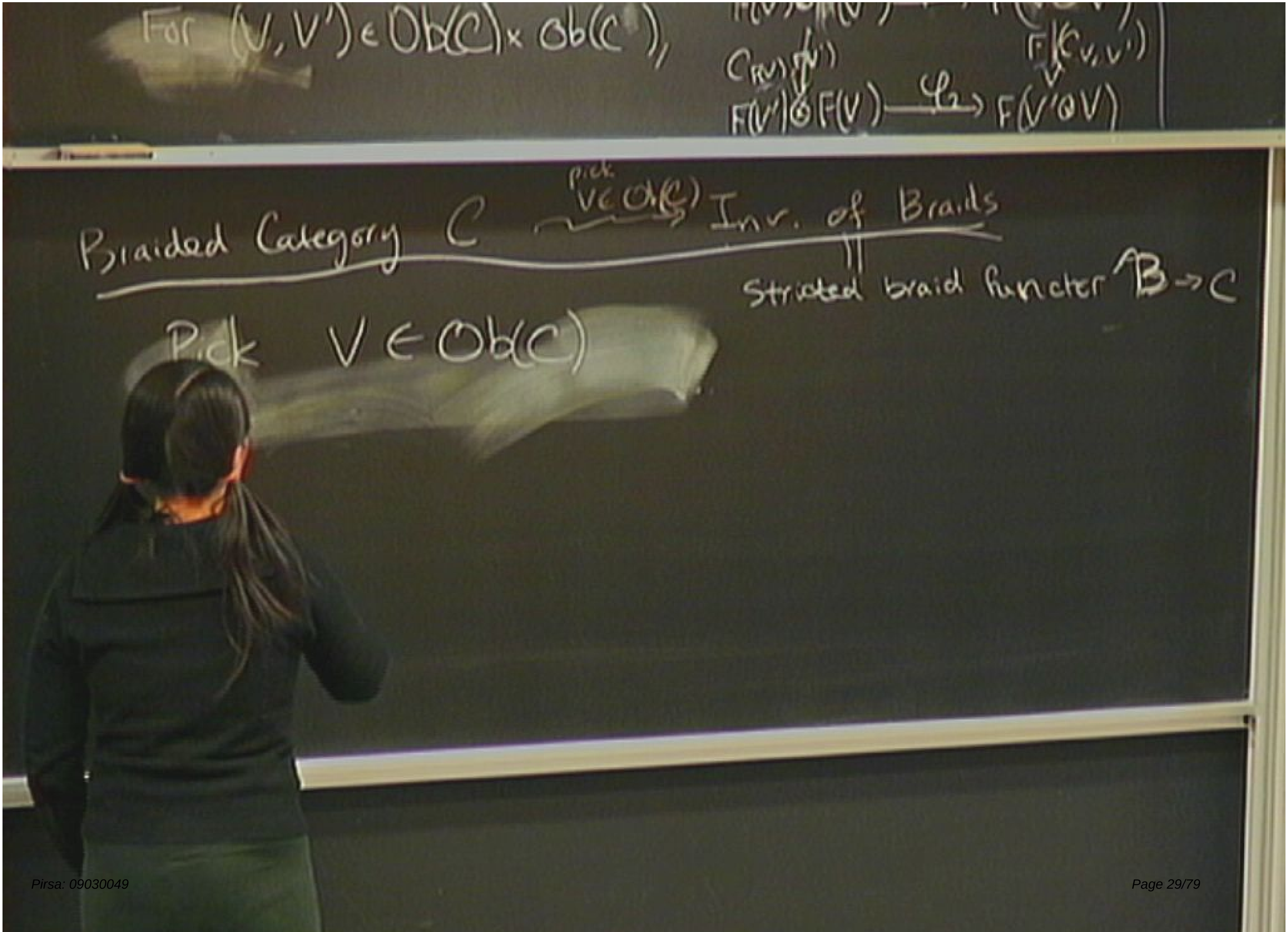

For (V, V') ∈ Ob(C) × Ob(C'),
F(V')⊗F(V) → F(V'⊗V)
Braided Category C
pick V ∈ Ob(C)
Inv. of Braids
Stricted braid functor B → C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Pick V ∈ Ob(C)

For $(V, V') \in Ob(C) \times Ob(C)$,

$$C_{F(V),F(V')}$$

$$F(V') \otimes F(V) \xrightarrow{\varphi_2} F(V' \otimes V)$$

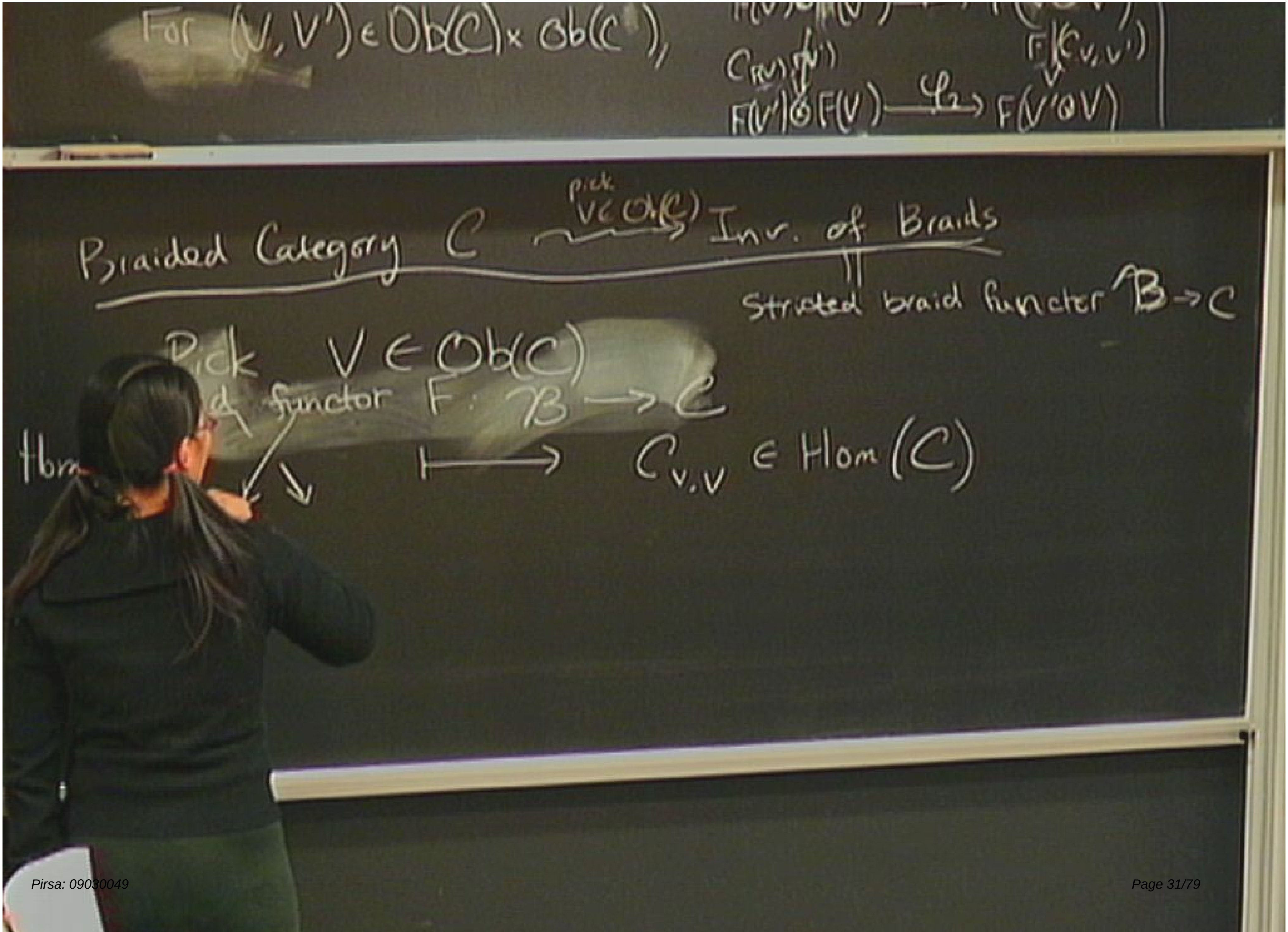
$F(c_{V,V'})$

Braided Category C $\overset{\text{pick } V \in Ob(C)}{\rightsquigarrow}$ Inv. of Braids

|| Stricted braid functor $B \to C$

Pick $V \in Ob(C)$

$\longmapsto$ $c_{V,V} \in Hom(C)$



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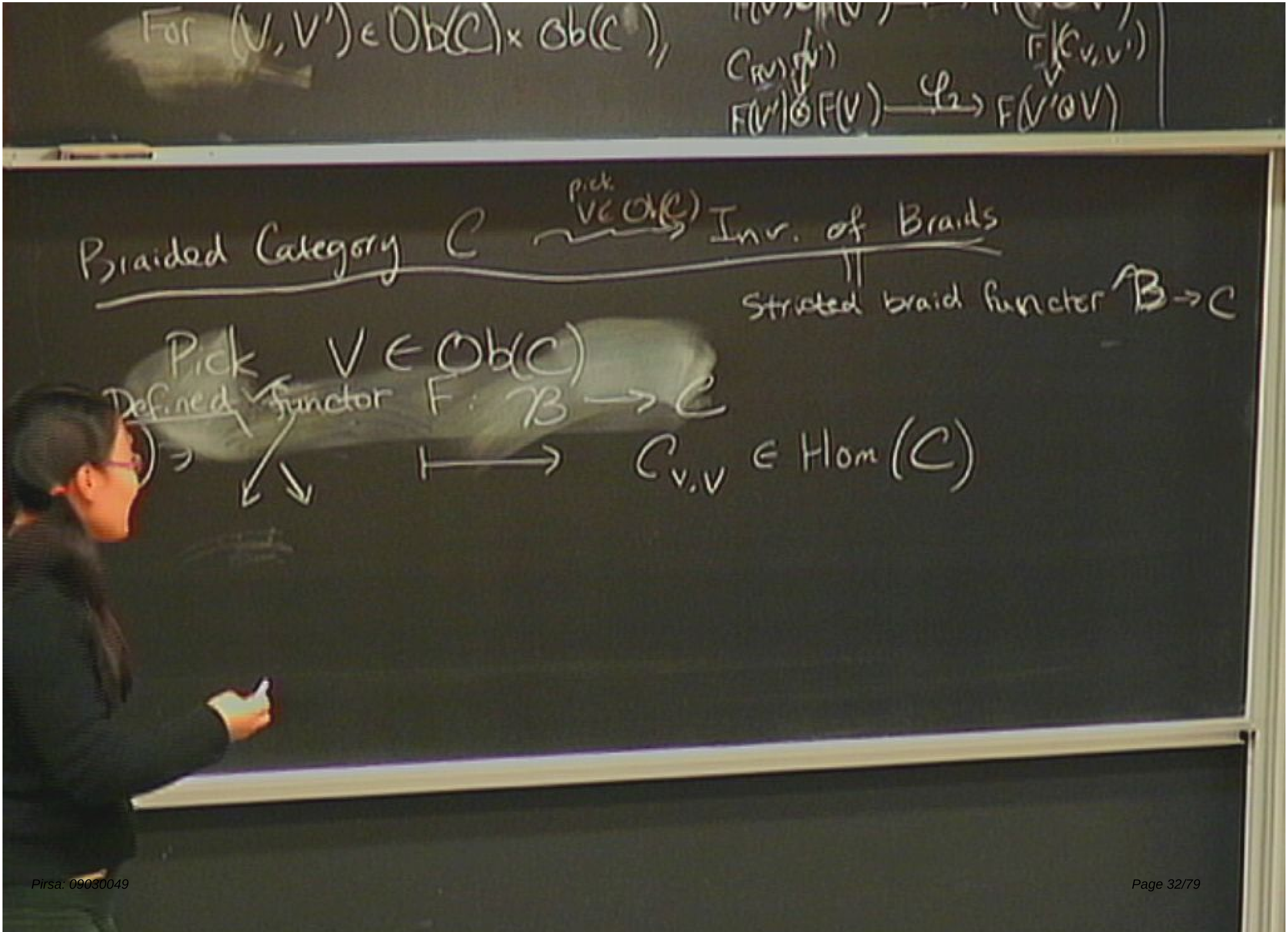
Inv. of Braids
Stricted braid functor B → C
Pick V ∈ Ob(C)
functor F: B → C
C v,V ∈ Hom(C)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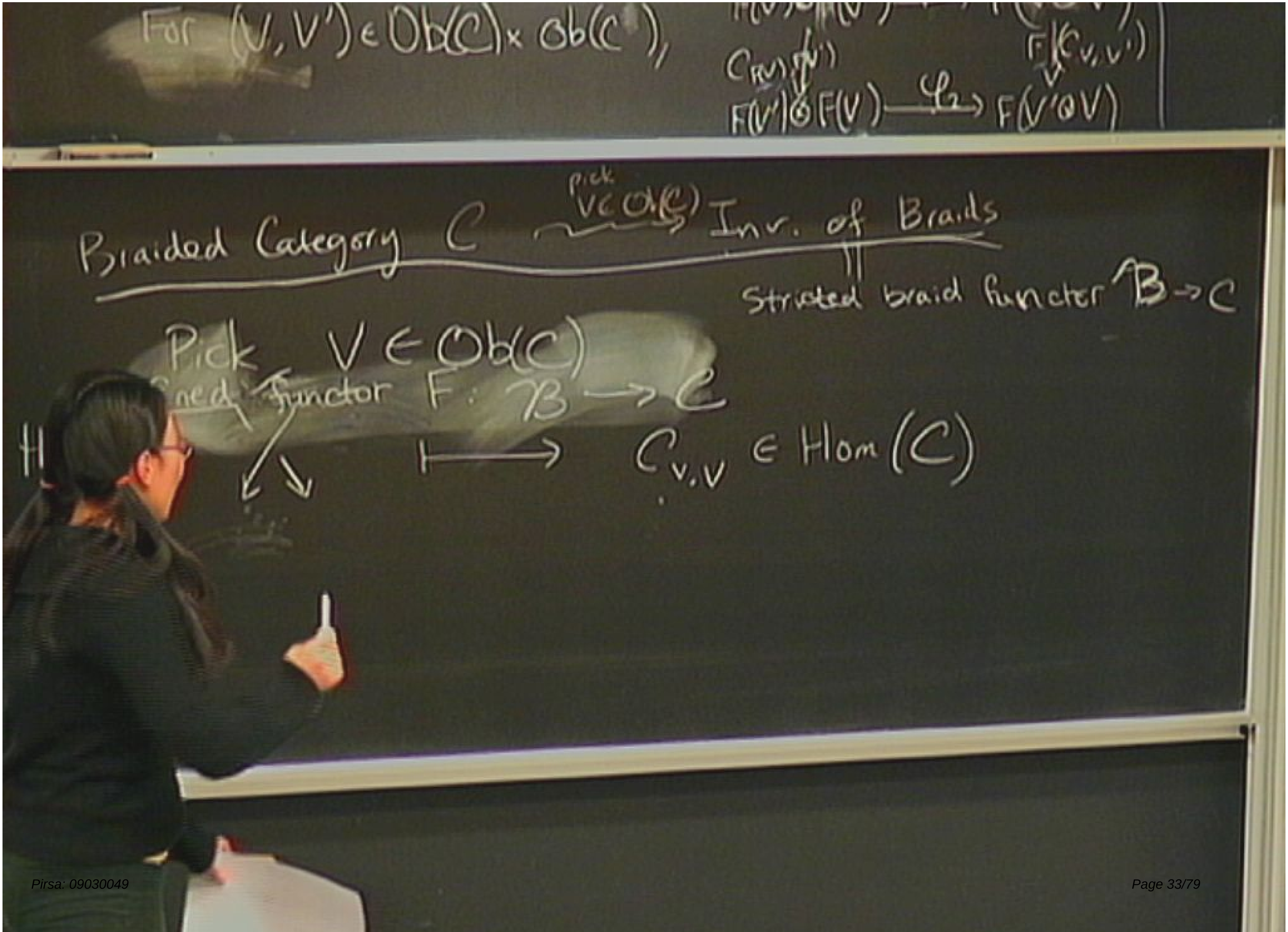
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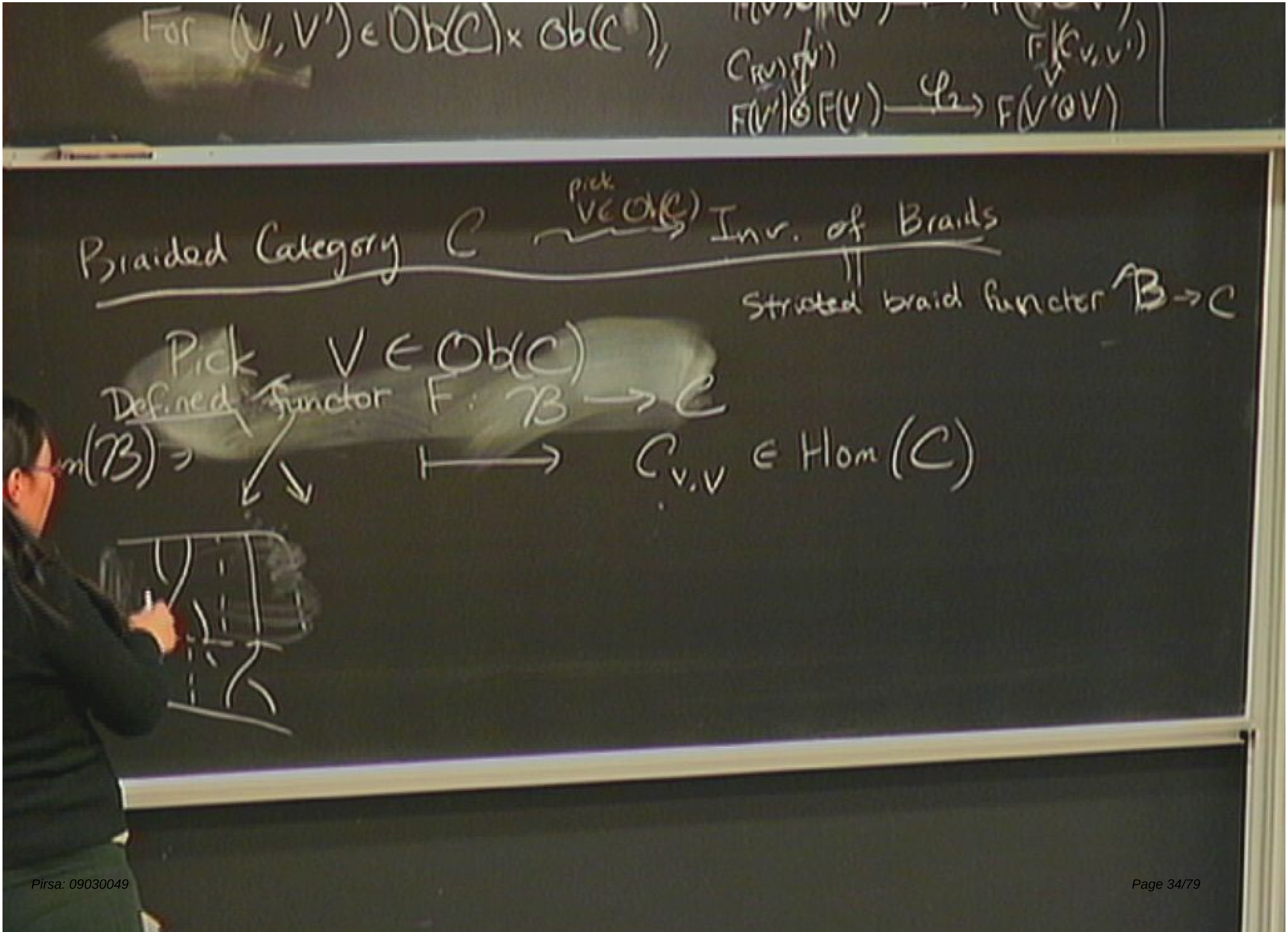
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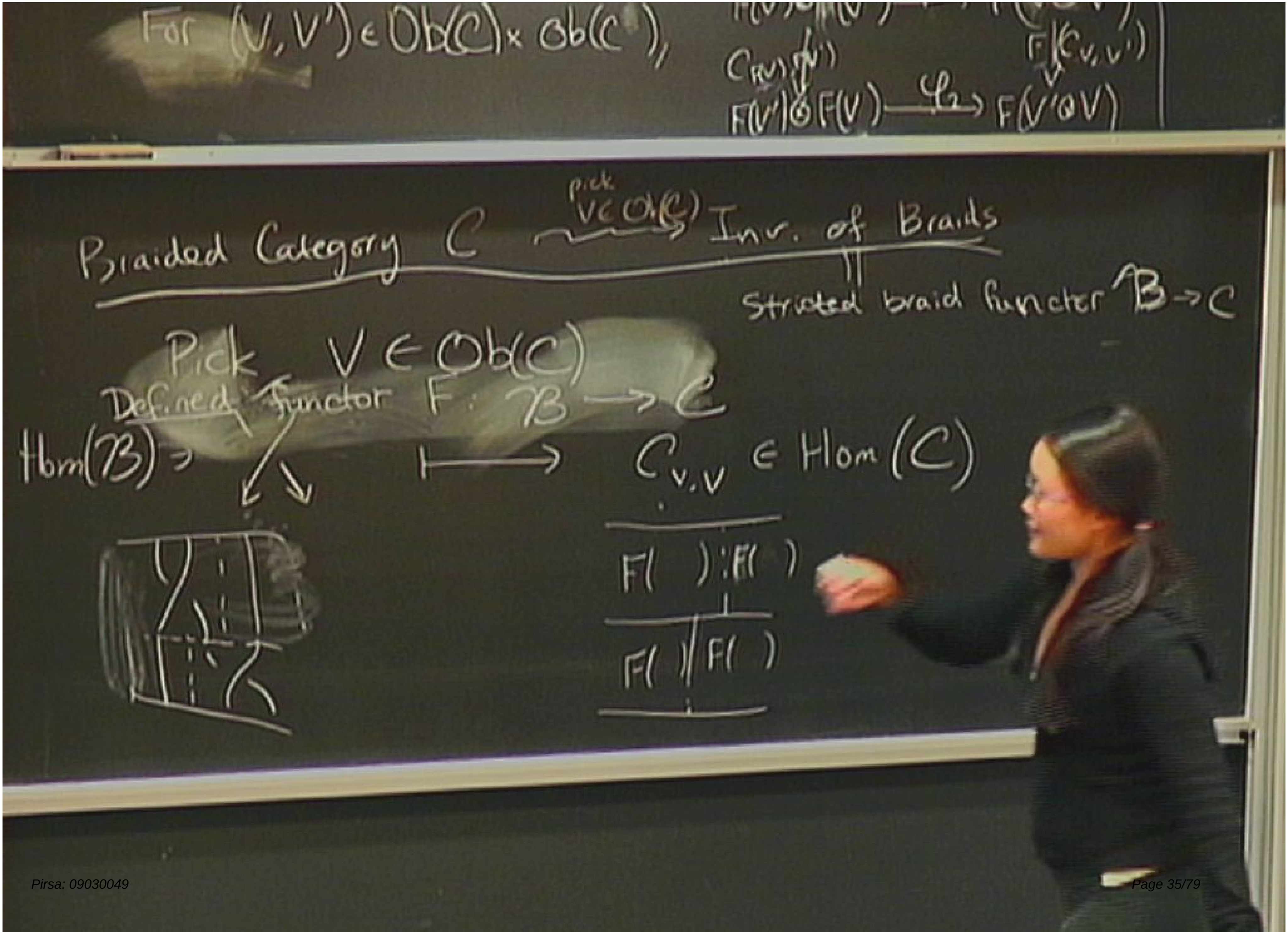
Stricted braid functor B → C
Pick V ∈ Ob(C)
Defined functor F: B → C
C_{V,V} ∈ Hom(C)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F( ) F( )
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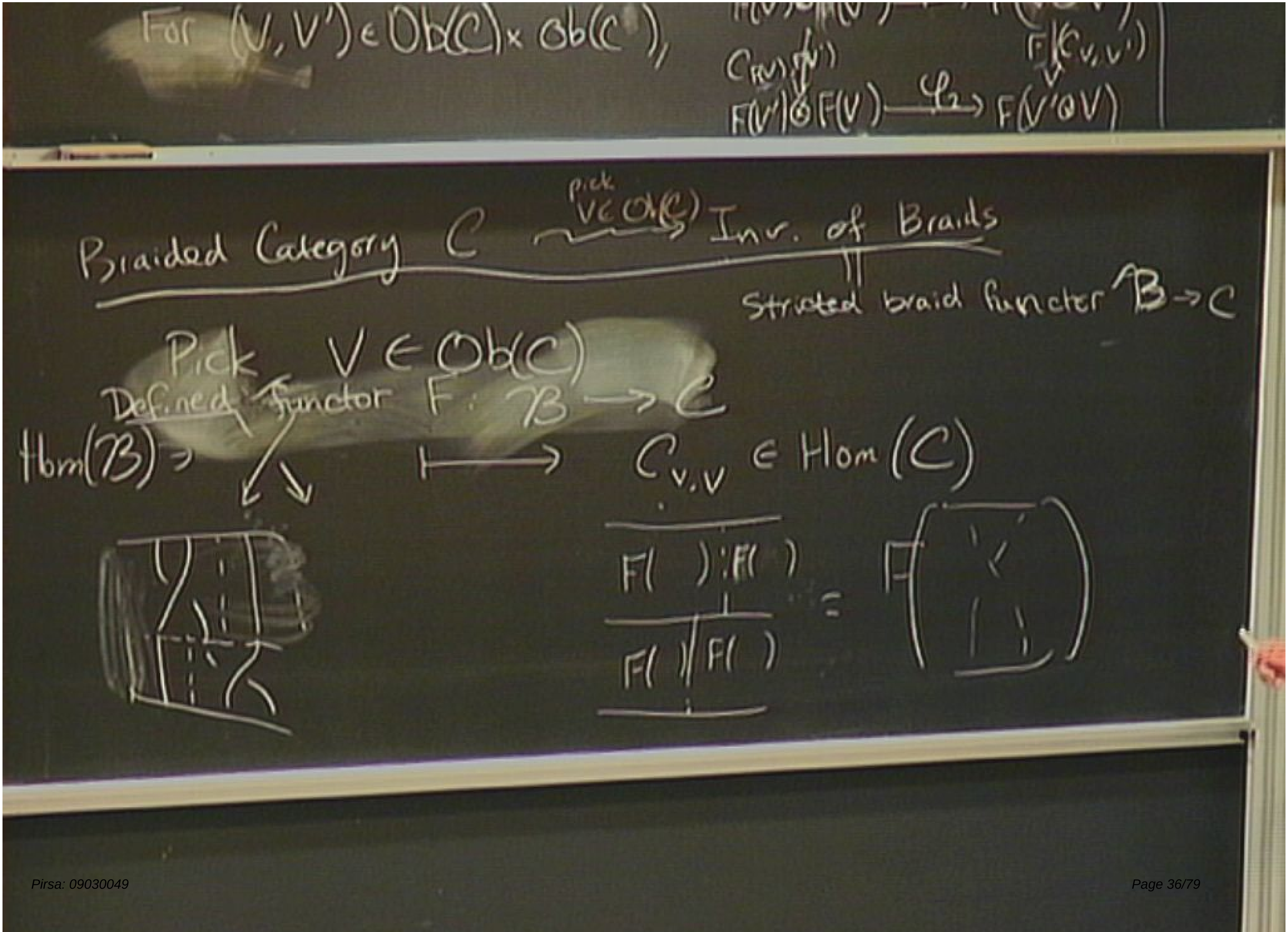
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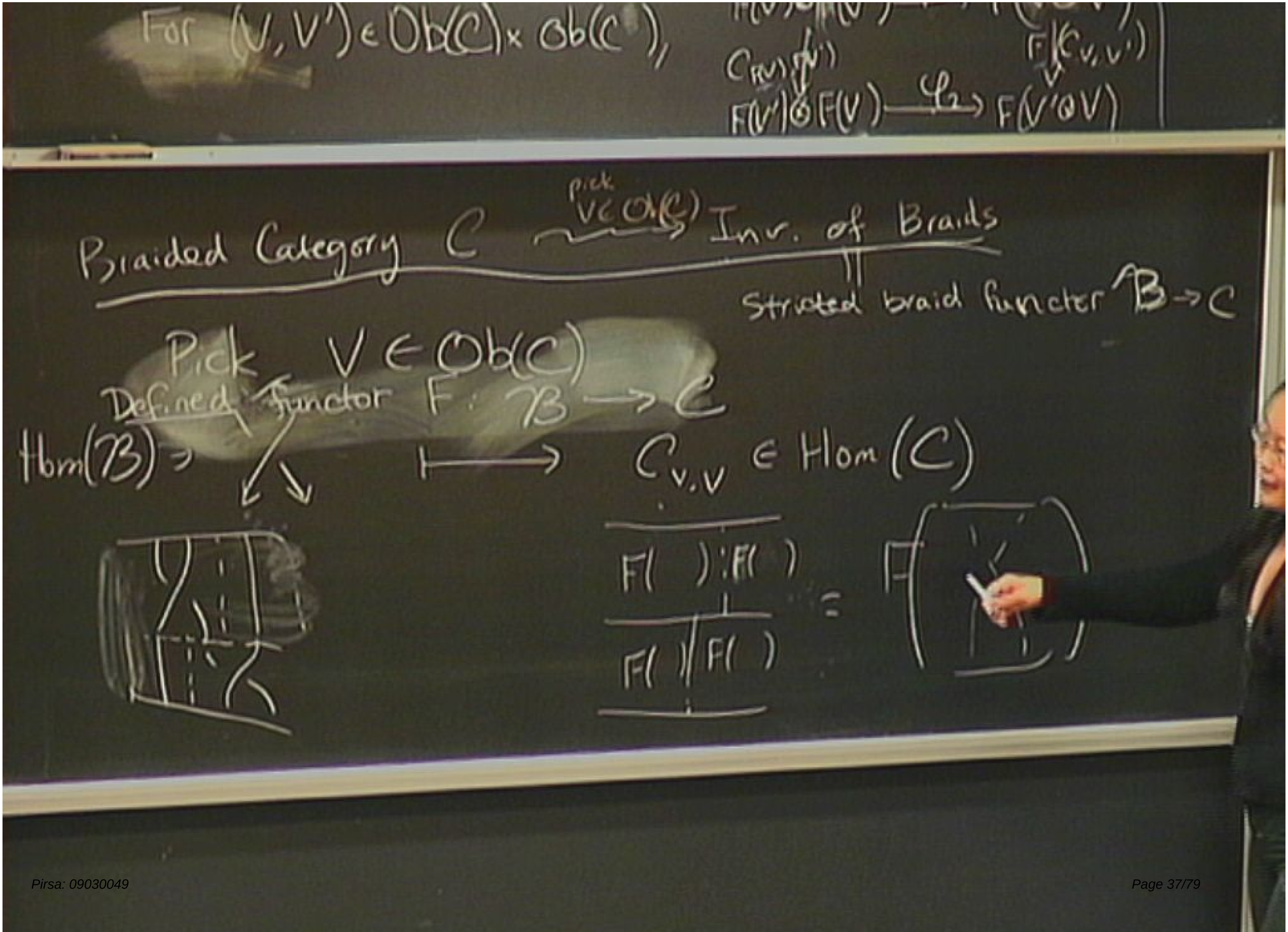
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F( ) F( )
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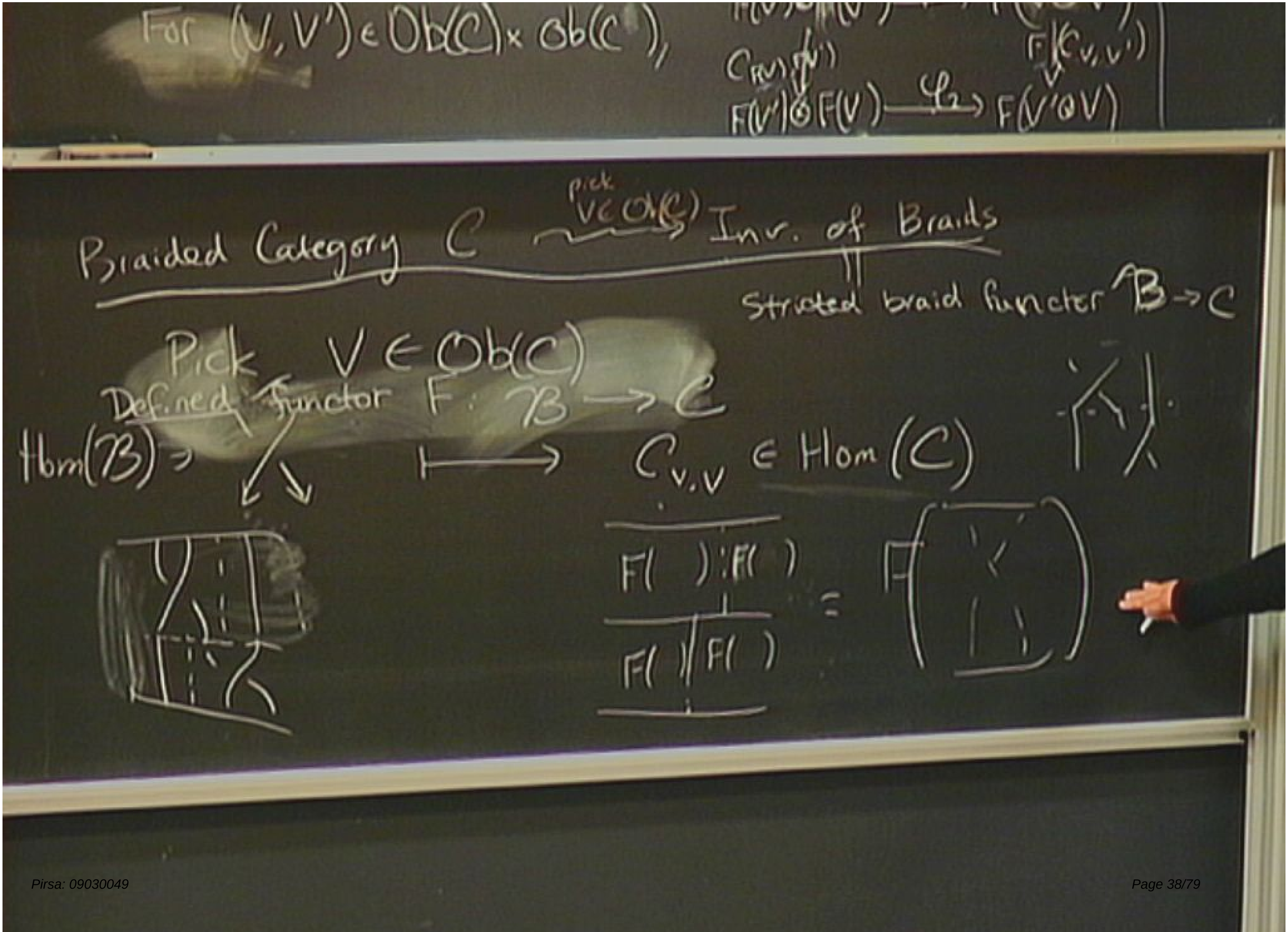
=
F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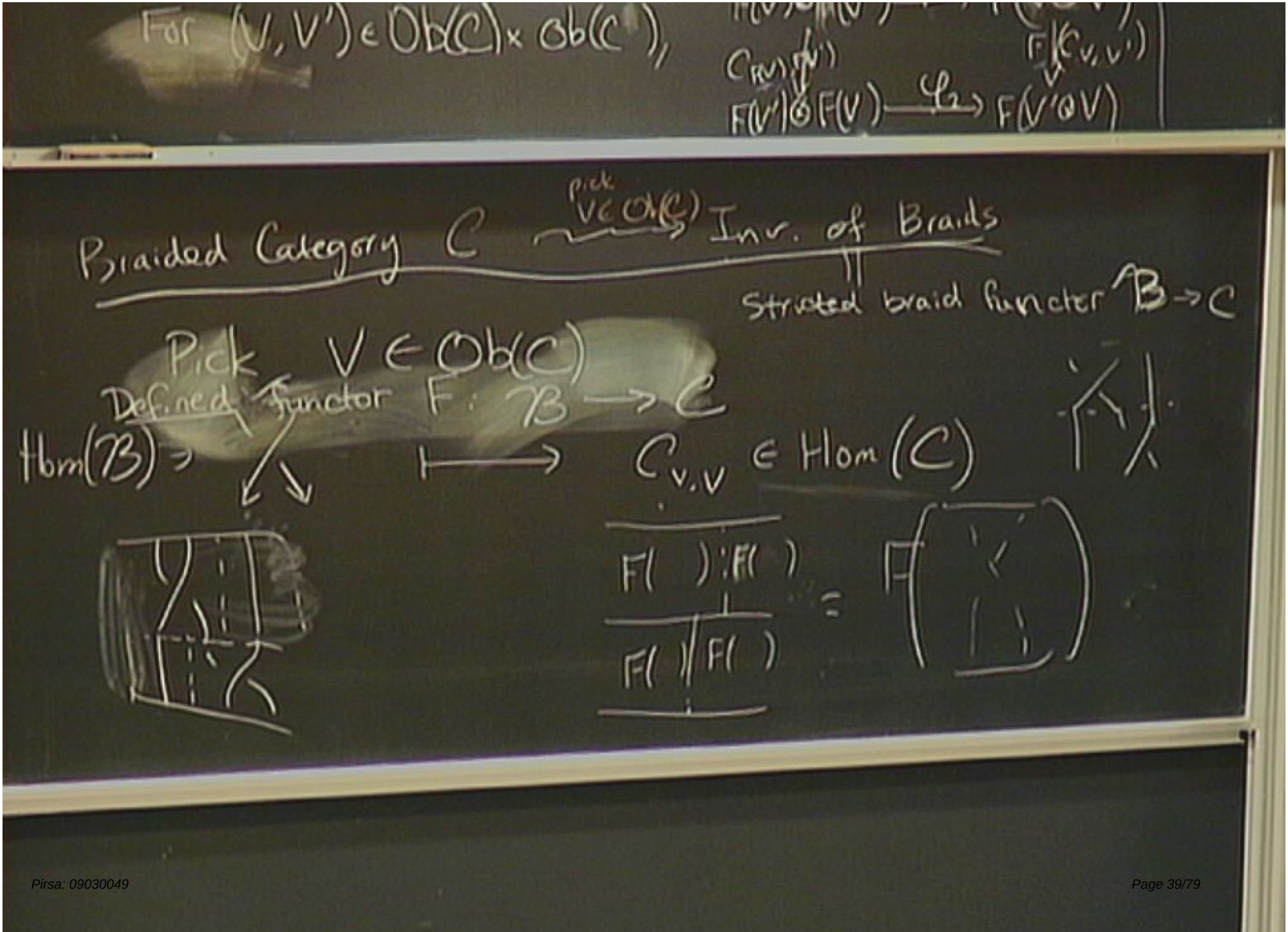
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Pick V ∈ Ob(C)
Defined functor F: B → C
Hom(B) ∋ ⟼ C_{V,V} ∈ Hom(C)
F( ) : F( )
F( ) | F( )
= F( )

For $(V, V') \in Ob(C) \times Ob(C)$,

$C_{F(V),F(V')}$ $\qquad$ $F(C_{V,V'})$

$F(V') \otimes F(V) \xrightarrow{\varphi_2} F(V' \otimes V)$

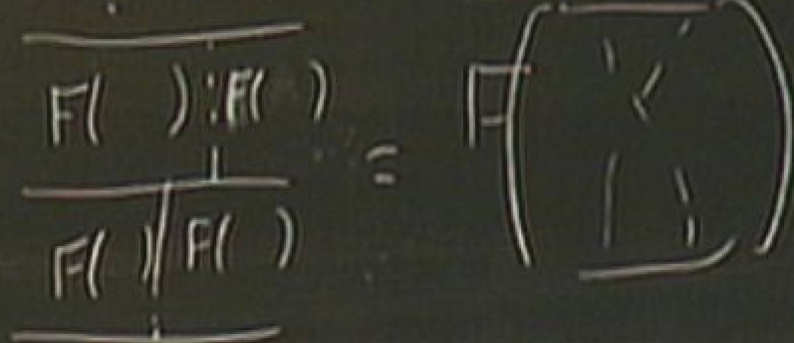
Braided Category C $\xrightarrow{\text{pick } V \in Ob(C)}$ Inv. of Braids

|| Stricted braid functor $\mathcal{B} \to C$

Pick $V \in Ob(C)$

Defined functor $F : \mathcal{B} \to C$

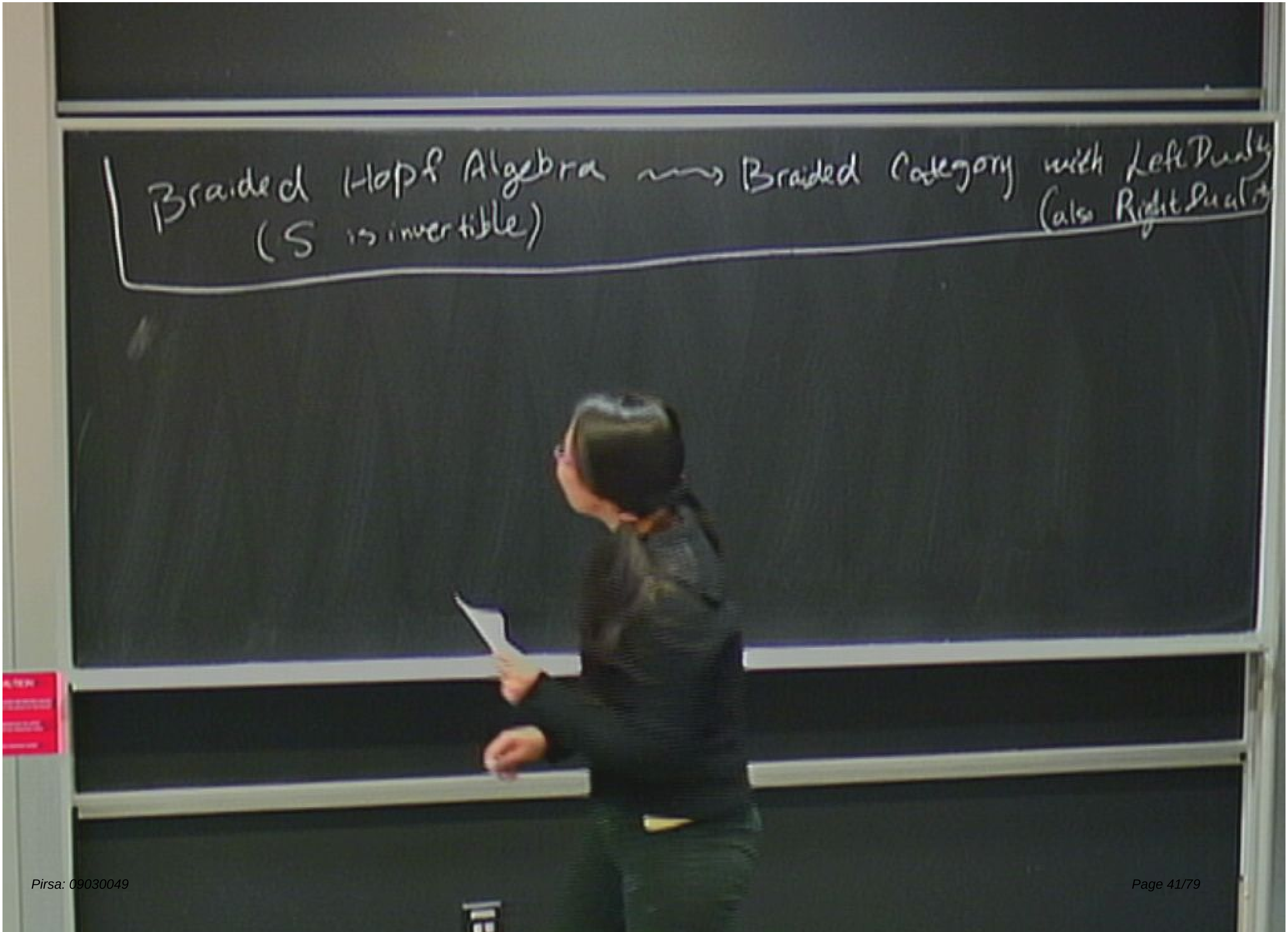
$Hom(\mathcal{B}) \ni$ $\longmapsto$ $C_{V,V} \in Hom(C)$

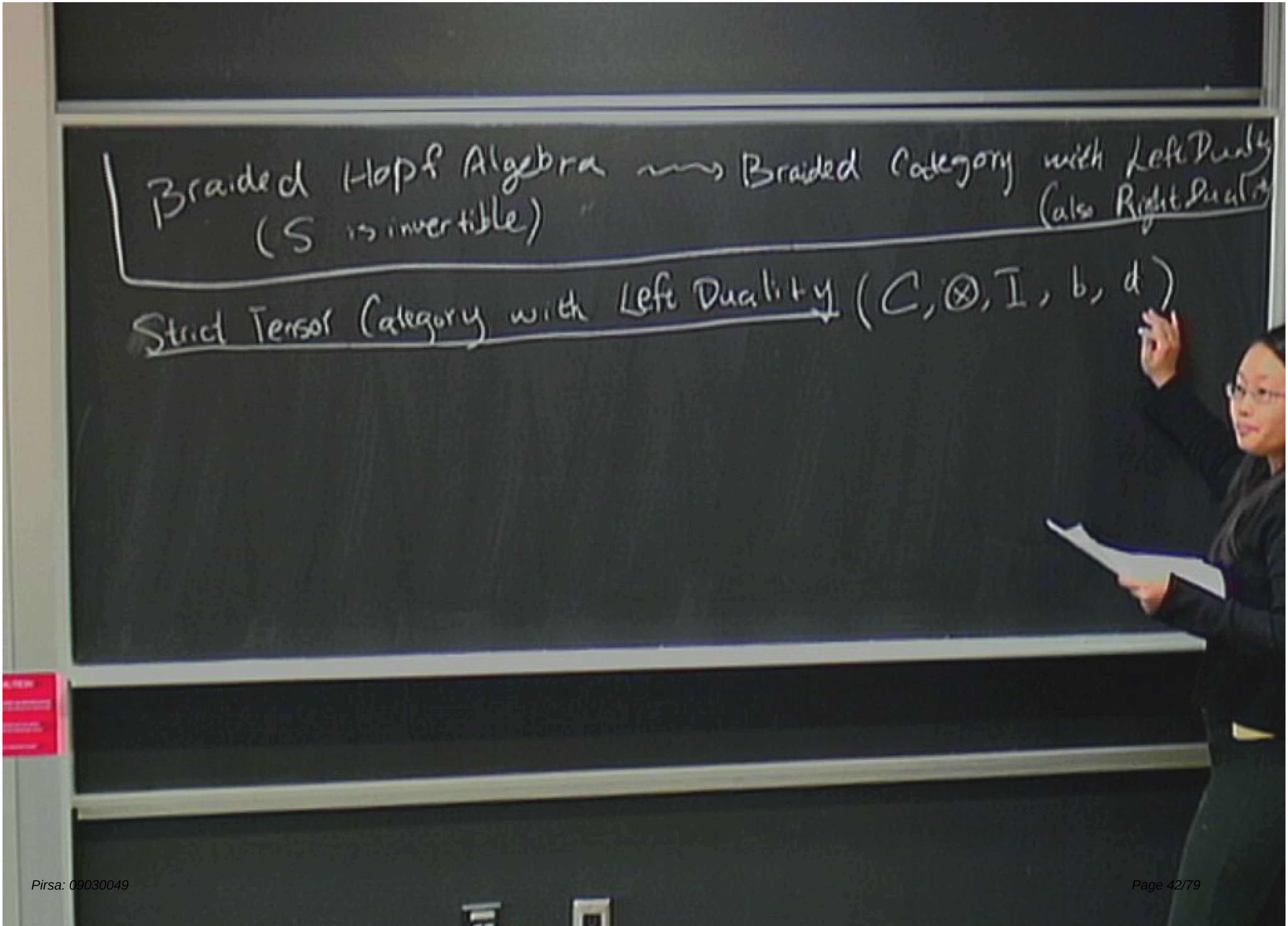


$F(\ ) \vdots F(\ )$

$F(\ ) | F(\ )$

$= F(\ \ )$


Braided Hopf Algebra ⇝ Braided Category with Left Duality
(S is invertible)
(also Right Duality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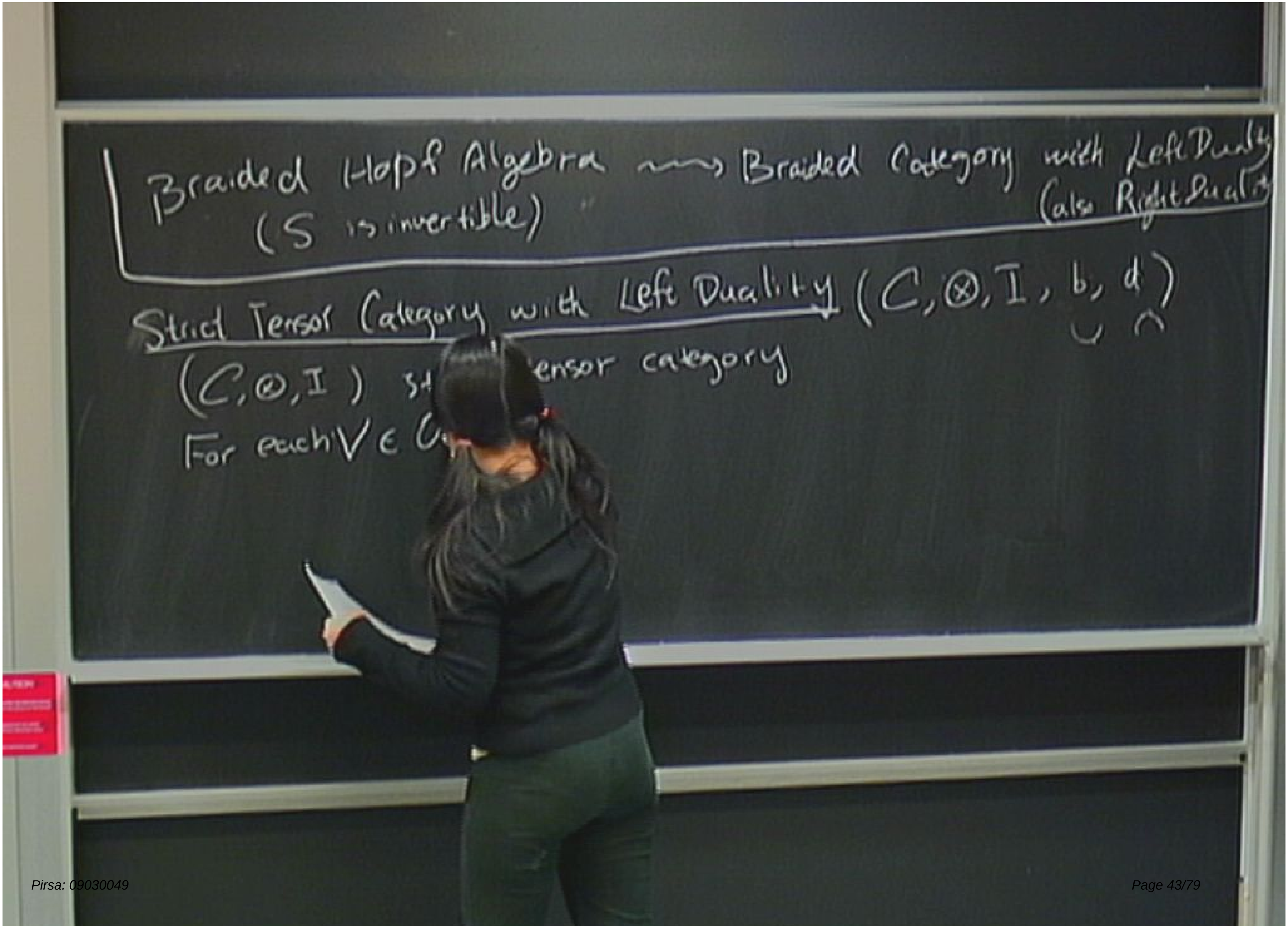
Strict Tensor Category with Left Duality (C, ⊗, I, b, d)


Braided Hopf Algebra ⇝ Braided Category with Left Duality
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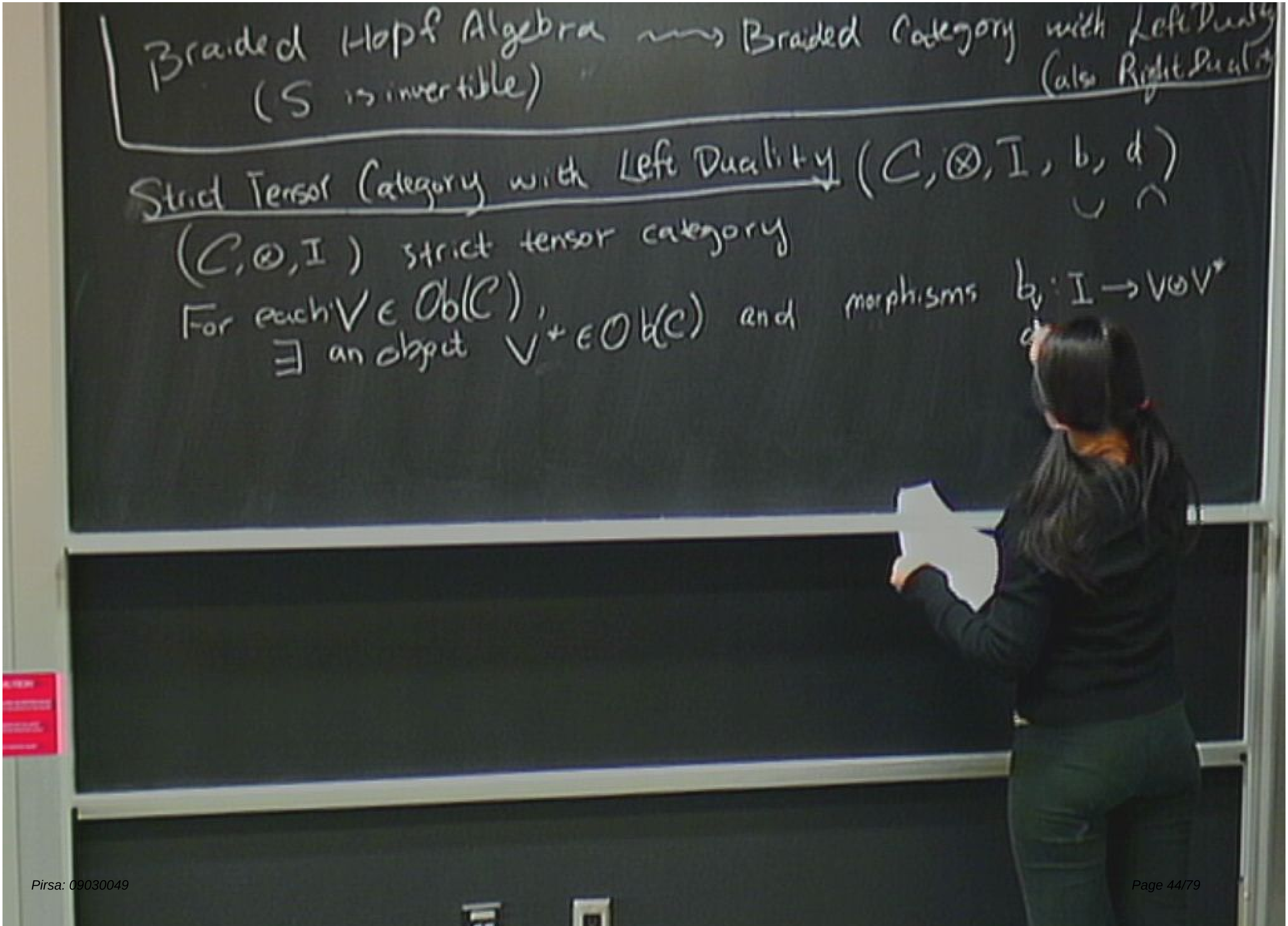
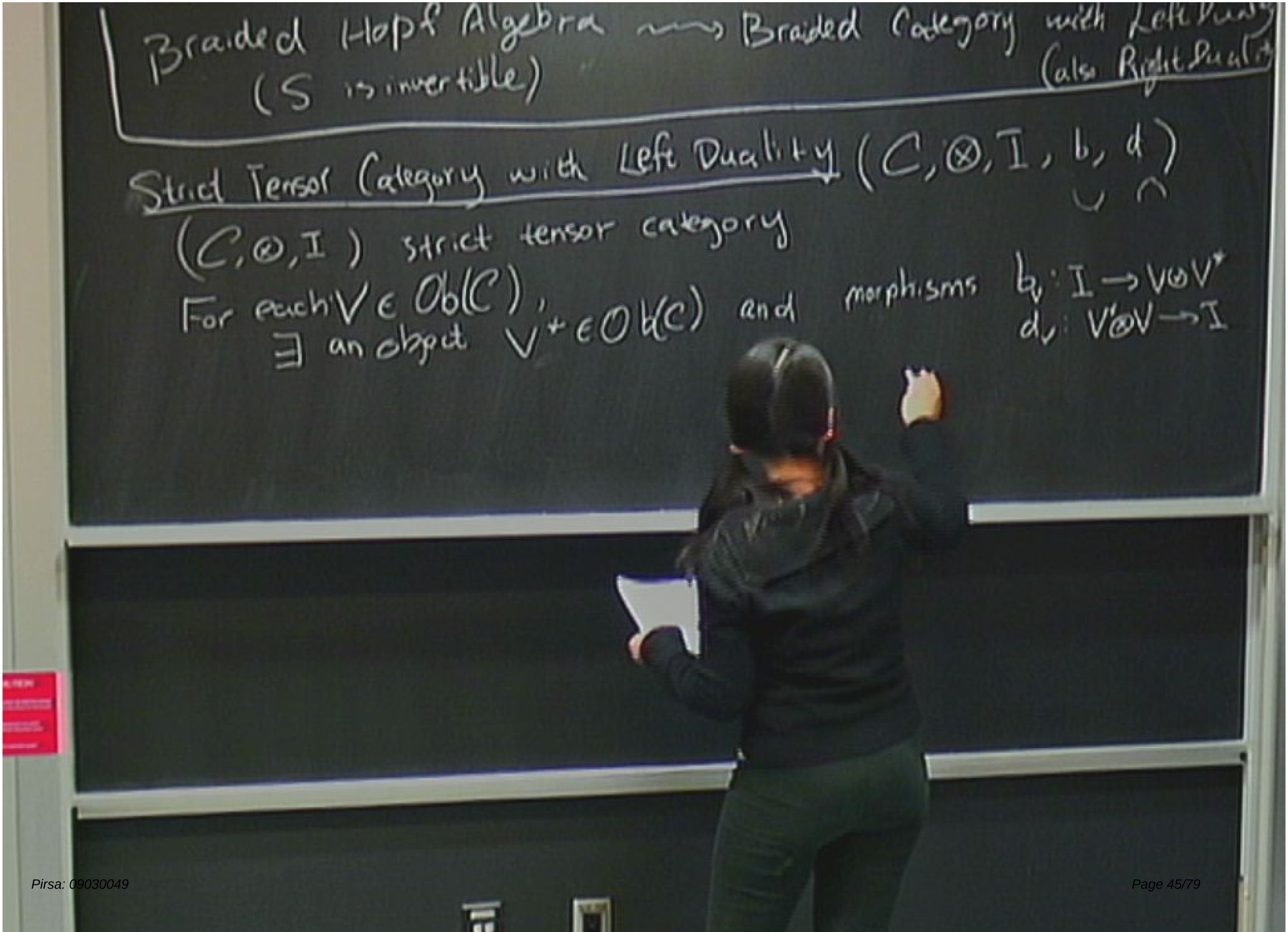
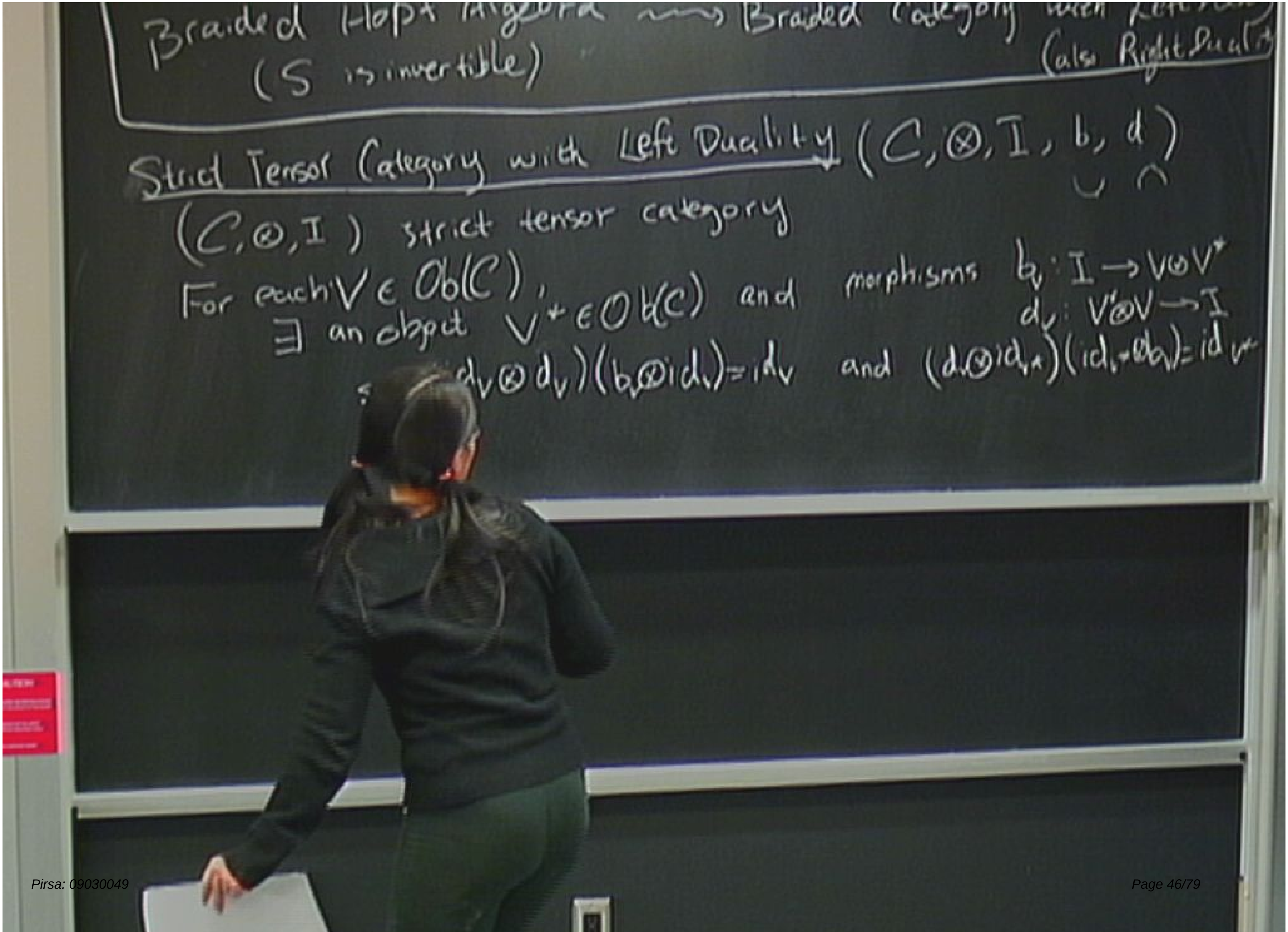
Strict Tensor Category with Left Duality (C, ⊗, I, b, d)
(C, ⊗, I) st ensor category
For each V ∈ C


Braided Hopf Algebra ⇝ Braided Category with Left Duality
(S is invertible)
(also Right Duality)
Strict Tensor Category with Left Duality (C, ⊗, I, b, d)
(C, ⊗, I) strict tensor category
For each V ∈ Ob(C), ∃ an object V* ∈ Ob(C) and morphisms b_V : I → V⊗V*


Braided Hopf Algebra ⇝ Braided Category with Left Duality
(S is invertible) (also Right Duality)
Strict Tensor Category with Left Duality (C, ⊗, I, b, d)
(C, ⊗, I) strict tensor category
For each V ∈ Ob(C),
∃ an object V* ∈ Ob(C) and morphisms b_V : I → V⊗V*
d_V : V*⊗V → I


Braided Hopf Algebra ⇝ Braided Category with
(S is invertible)
(also Right Duality)
Strict Tensor Category with Left Duality $(\mathcal{C}, \otimes, I, b, d)$
$(\mathcal{C}, \otimes, I)$ strict tensor category
For each $V \in Ob(\mathcal{C})$, $\exists$ an object $V^* \in Ob(\mathcal{C})$ and morphisms $b_V : I \to V \otimes V^*$
$d_V : V^* \otimes V \to I$
s.t. $(id_V \otimes d_V)(b_V \otimes id_V) = id_V$ and $(d_V \otimes id_{V^*})(id_{V^*} \otimes b_V) = id_{V^*}$

Braided Hopf Algebra $\rightsquigarrow$ Braided Category with Left Duality
(S is invertible) (also Right Duality)

Strict Tensor Category with Left Duality $(\mathcal{C}, \otimes, I, b, d)$
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad b = \cup, \; d = \cap$

$(\mathcal{C}, \otimes, I)$ strict tensor category

For each $V \in Ob(\mathcal{C})$,
$\exists$ an object $V^* \in Ob(\mathcal{C})$ and morphisms $b_V : I \to V \otimes V^*$, $d_V : V^* \otimes V \to I$

s.t. $(id_V \otimes d_V)(b_V \otimes id_V) = id_V$ and $(d_V \otimes id_{V^*})(id_{V^*} \otimes b_V) = id_{V^*}$

Braided Hopf Algebra $\rightsquigarrow$ Braided Category with Left Duality
(S is invertible) (also Right Duality)

Strict Tensor Category with Left Duality $(\mathcal{C}, \otimes, I, b, d)$

$(\mathcal{C}, \otimes, I)$ strict tensor category

For each $V \in Ob(\mathcal{C})$, $\downarrow_V (id_V)$ $\uparrow_{V^*} (id_{V^*})$ and morphisms b_V [illegible] $\otimes V^*$

$\exists$ an object $V^* \in Ob(\mathcal{C})$ d_V [illegible] I

s.t. $(id_V \otimes d_V)(b_V \otimes id_V) = id_V$ and $(d_V \otimes id_{V^*}$ [illegible] id_{V^*}

Braided Hopf Algebra $\rightsquigarrow$ Braided Category with Left Duality
(S is invertible) (also Right Duality)

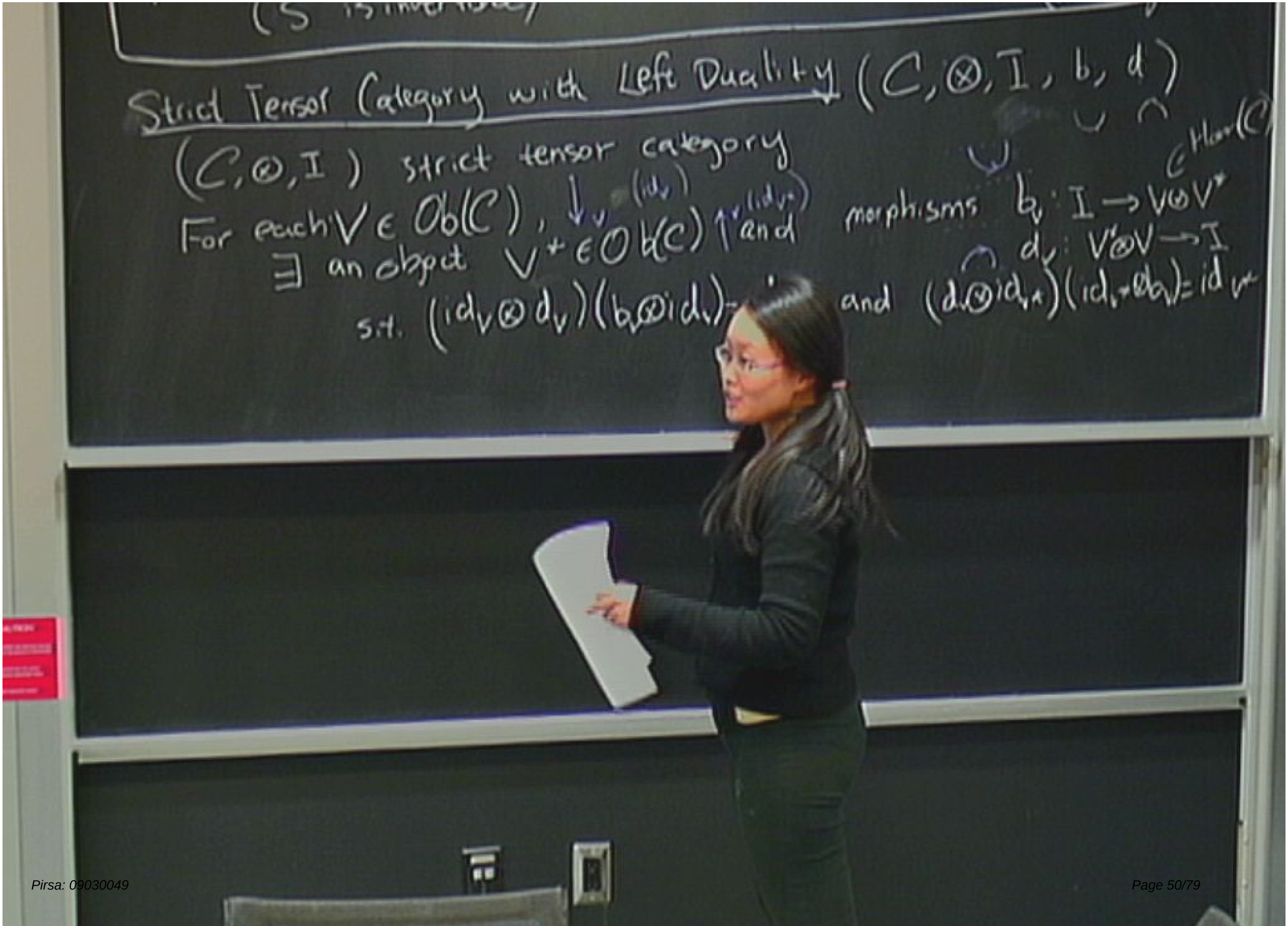
Strict Tensor Category with Left Duality $(\mathcal{C}, \otimes, I, b, d)$

$(\mathcal{C}, \otimes, I)$ strict tensor category

For each $V \in Ob(\mathcal{C})$, $\downarrow_V$ (id_V) $\exists$ an object $V^* \in Ob(\mathcal{C})$ $\uparrow_{V^*}$ (id_{V^*}) and morphisms $b_V : I \to V$...

$d_V : V^* \otimes V$...

s.t. $(id_V \otimes d_V)(b_V \otimes id_V) = id_V$ and $(d_V \otimes id_{V^*})(id_{V^*}$...


Strict Tensor Category with Left Duality (C, ⊗, I, b, d)
(C, ⊗, I) strict tensor category
For each V ∈ Ob(C), ∃ an object V* ∈ Ob(C) and morphisms b_V : I → V⊗V*
d_V : V*⊗V → I
s.t. (id_V ⊗ d_V)(b_V ⊗ id_V) = id_V and (d_V ⊗ id_V*)(id_V* ⊗ b_V) = id_V*

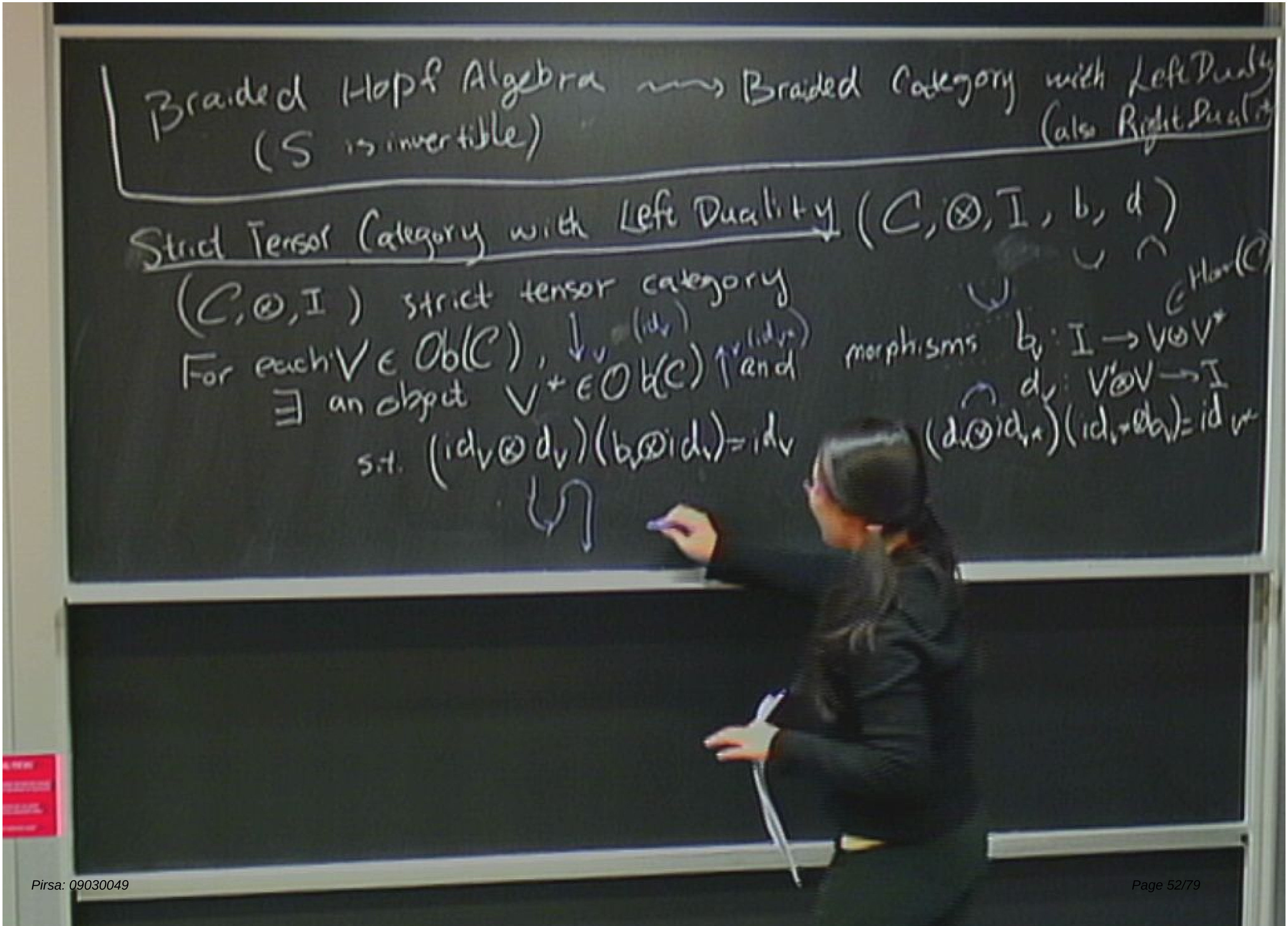
Braided Hopf Algebra $\rightsquigarrow$ Braided Category with Left Duality
(S is invertible) (also Right Duality)

Strict Tensor Category with Left Duality $(\mathcal{C}, \otimes, I, b, d)$

$(\mathcal{C}, \otimes, I)$ strict tensor category

For each $V \in Ob(\mathcal{C})$, $\downarrow_V (id_V)$ $\exists$ an object $V^* \in Ob(\mathcal{C})$ $\uparrow_V (id_{V^*})$ and morphisms $b_V : I \to V \otimes V^*$ $\in Hom(\mathcal{C})$, $d_V : V^* \otimes V \to I$

s.t. $(id_V \otimes d_V)(\ldots \, b_V) = id_V$ and $(d_V \otimes id_{V^*})(id_{V^*} \otimes b_V) = id_{V^*}$


Braided Hopf Algebra ⇝ Braided Category with Left Duality
(S is invertible)
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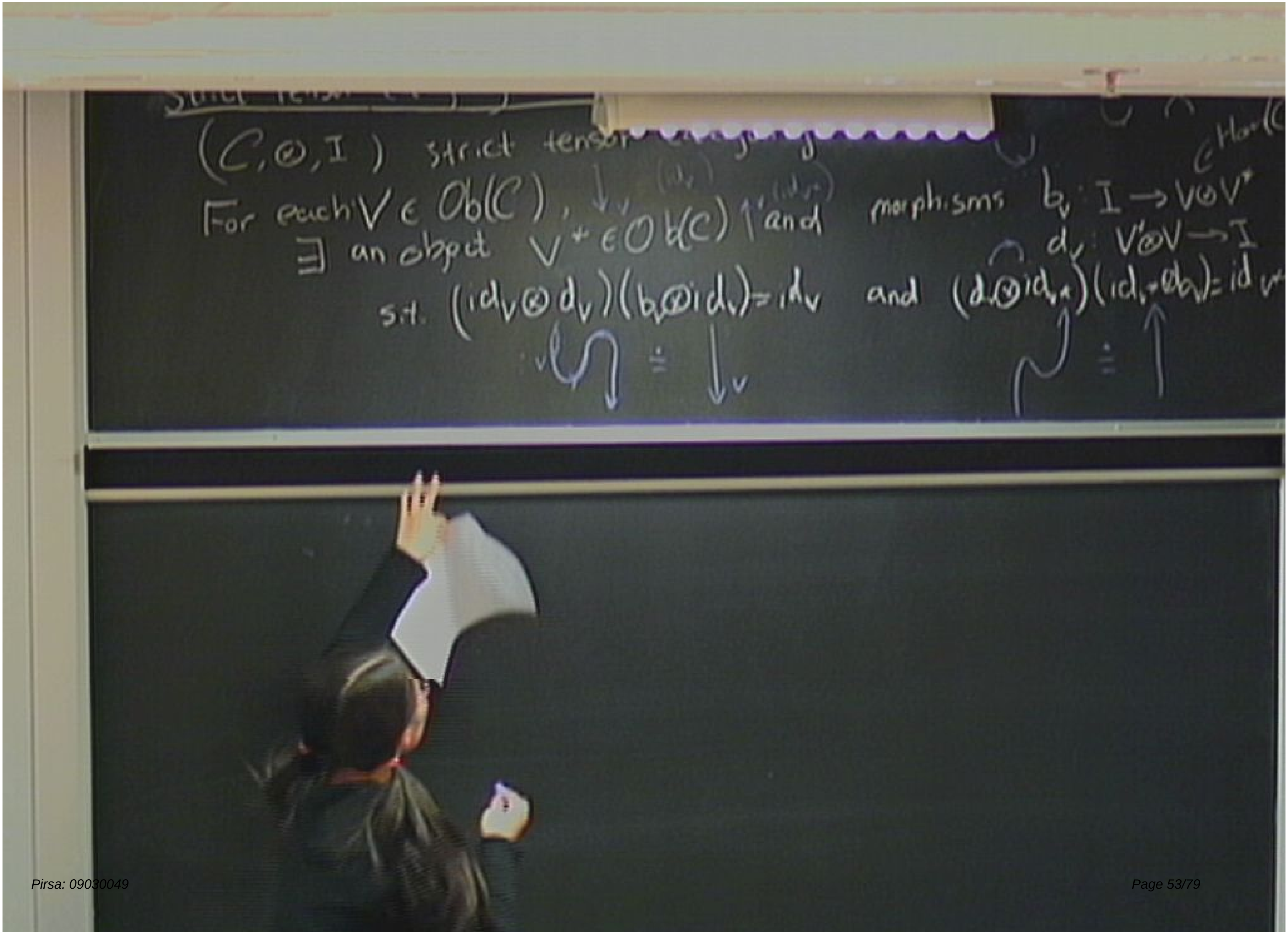
Strict Tensor Category with Left Duality (C, ⊗, I, b, d)
(C, ⊗, I) strict tensor category
For each V ∈ Ob(C), ↓V (idV) ↑V (idV*)
∃ an object V* ∈ Ob(C) and morphisms bV : I → V⊗V*
dV : V*⊗V → I
s.t. (idV ⊗ dV)(bV ⊗ idV) = idV
(dV ⊗ idV*)(idV* ⊗ bV) = idV*


(C, ⊗, I) strict tensor
For each V ∈ Ob(C), ∃ an object V* ∈ Ob(C) and morphisms b_V : I → V⊗V*
d_V : V*⊗V → I
s.t. (id_V ⊗ d_V)(b_V ⊗ id_V) = id_V and (d_V ⊗ id_V*)(id_V* ⊗ b_V) = id_V*

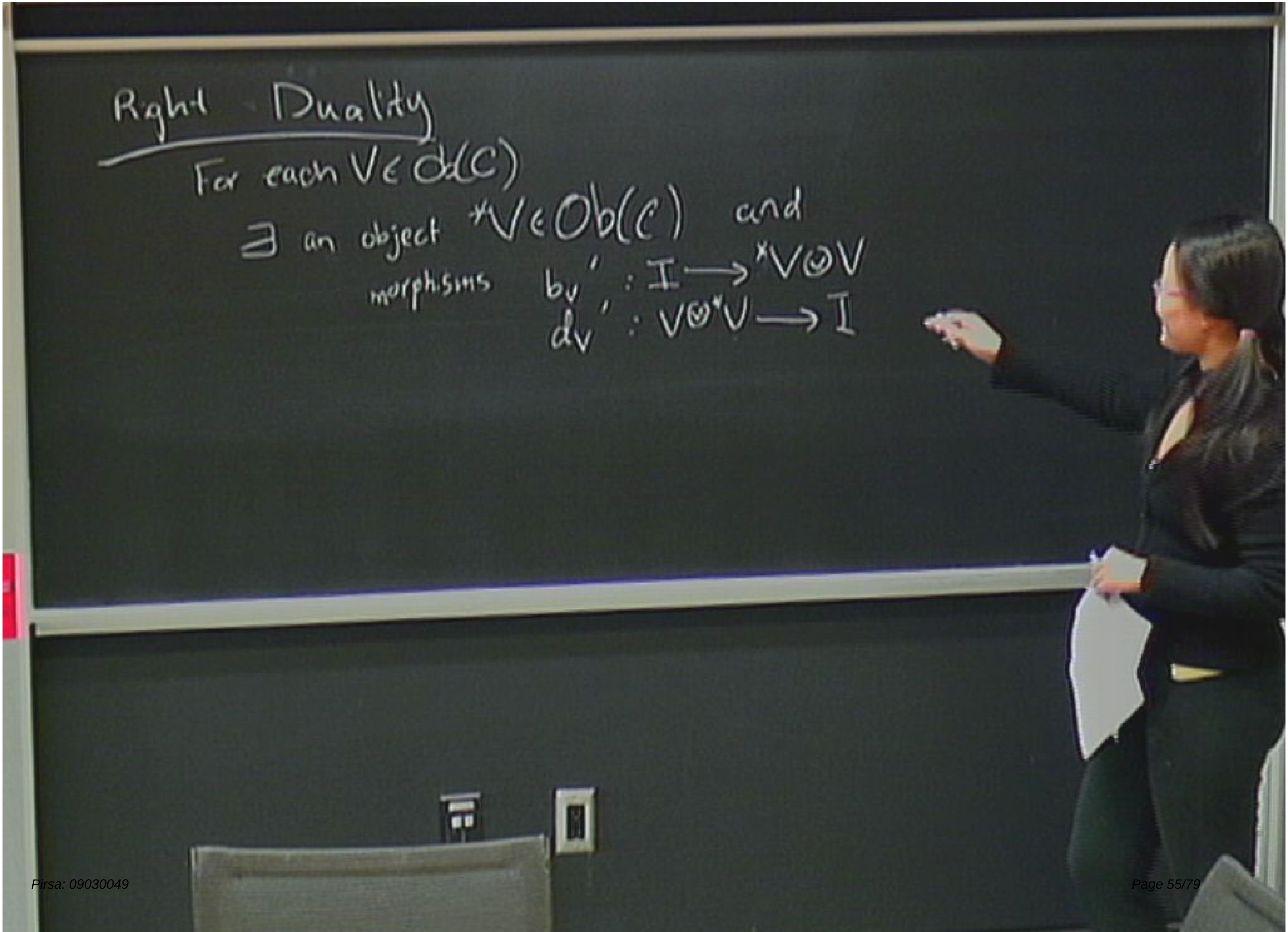
Strict Tensor Category

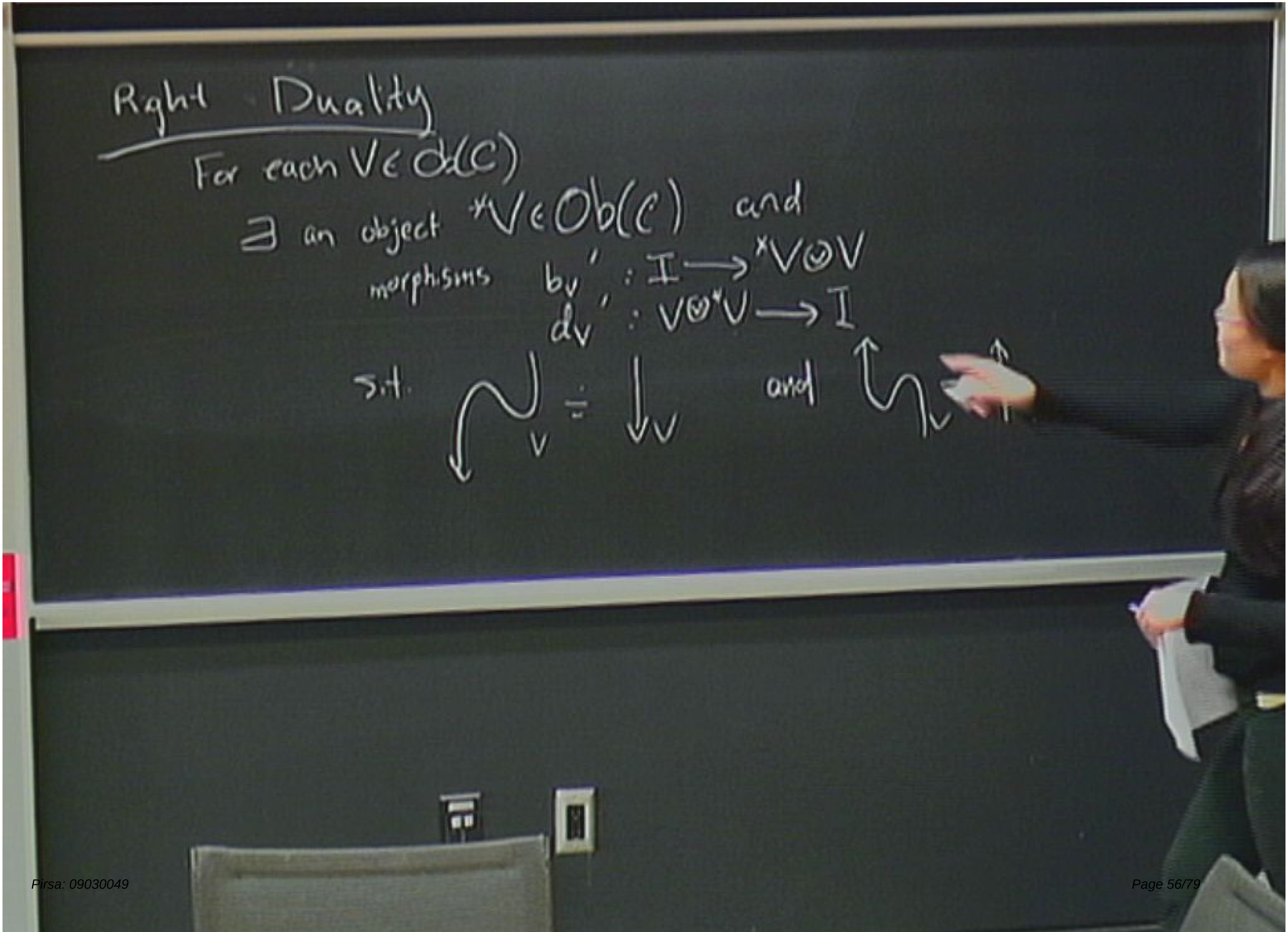
$(\mathcal{C}, \otimes, I)$ strict tensor category

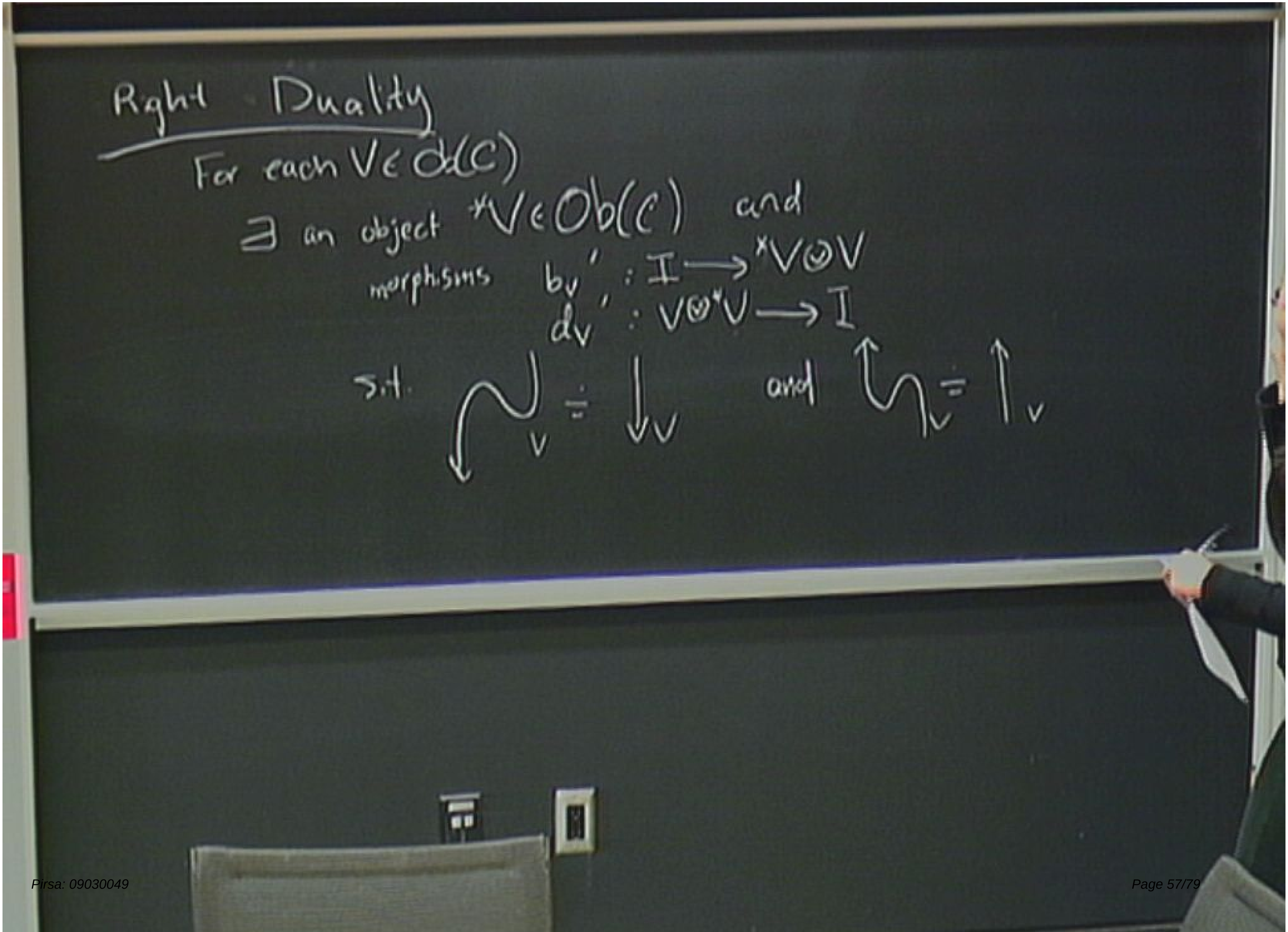
For each $V \in Ob(\mathcal{C})$, $\exists$ an object $V^* \in Ob(\mathcal{C})$ and morphisms $b_V : I \to V \otimes V^*$

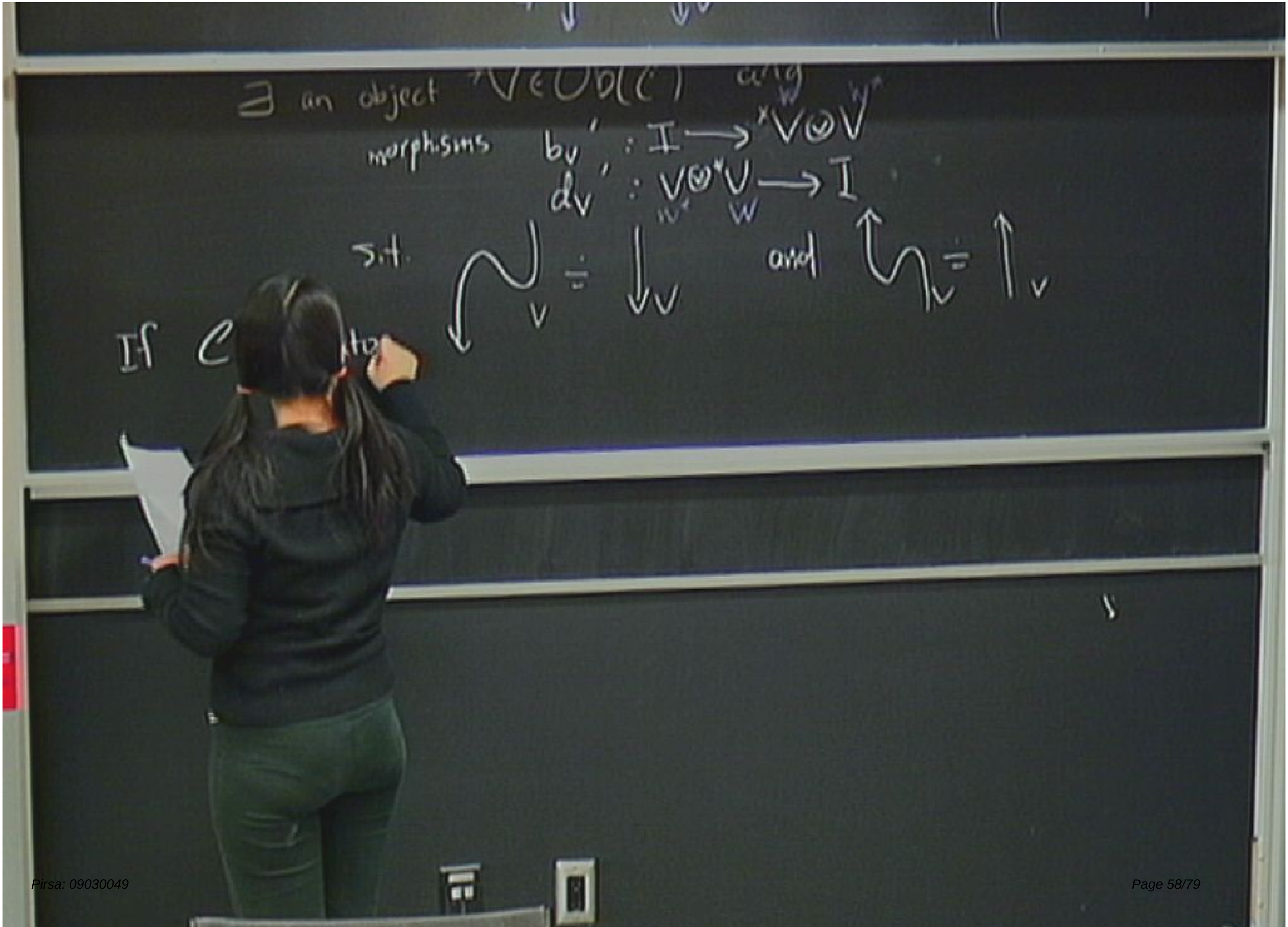
$d_V : V^* \otimes V \to I$

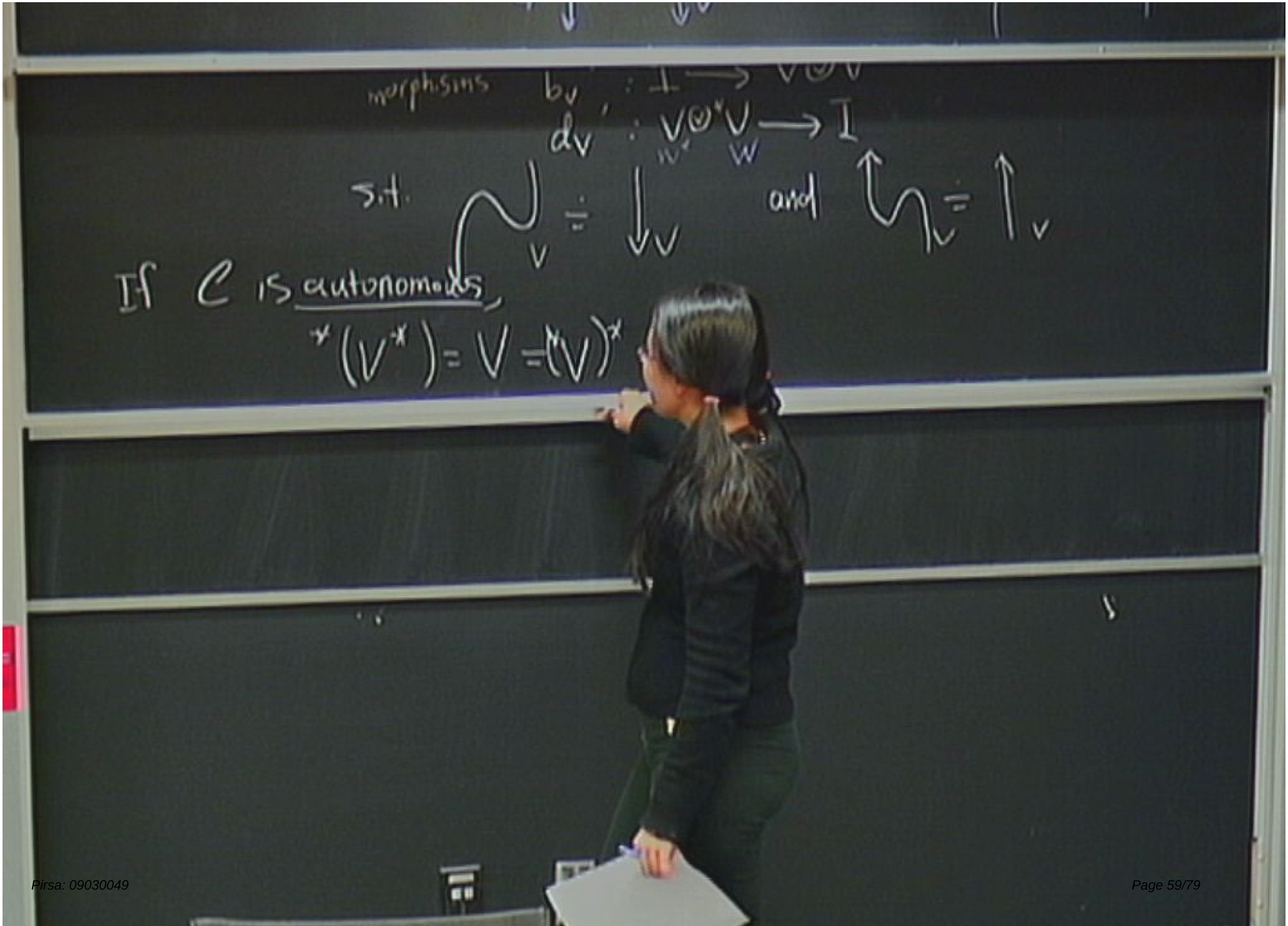
s.t. $(id_V \otimes d_V)(b_V \otimes id_V) = id_V$ and $(d_V \otimes id_{V^*})(id_{V^*} \otimes b_V) = id_{V^*}$


Right Duality
For each V ∈ Ob(C)
∃ an object *V ∈ Ob(C) and
morphisms b_V' : I ⟶ *V ⊗ V
d_V' : V ⊗ *V ⟶ I


Right Duality
For each V ∈ Ob(C)
∃ an object *V ∈ Ob(C) and
morphisms b_V' : I → *V⊗V
d_V' : V⊗*V → I
s.t.
V = V
and


Right Duality
For each V ∈ Ob(C)
∃ an object *V ∈ Ob(C) and
morphisms b_V' : I → *V⊗V
d_V' : V⊗*V → I
s.t.
= ↓V
and
= ↑V


∃ an object *V ∈ Ob(C) and
morphisms b_V' : I ⟶ *V ⊗ V
d_V' : V ⊗ *V ⟶ I
s.t.
and


morphisms $b_V : I \longrightarrow V \otimes {}^*V$
$d_V' : V \otimes {}^*V \longrightarrow I$
s.t.
and
If $\mathcal{C}$ is autonomous,
${}^*(V^*) = V = ({}^*V)^*$

Right Duality

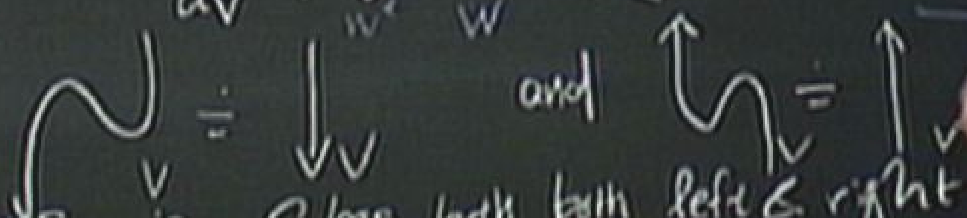
For each $V \in Ob(\mathcal{C})$

$\exists$ an object ${}^*V \in Ob(\mathcal{C})$ and

morphisms $b_V' : I \to {}^*V \otimes V$

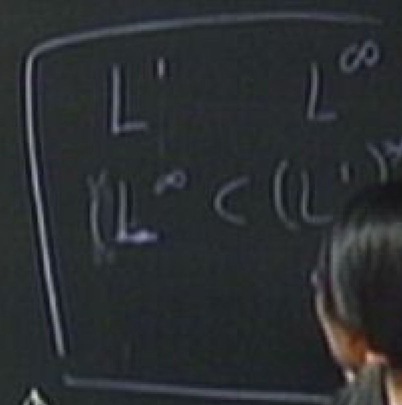
$d_V' : V \otimes {}^*V \to I$

s.t.



If $\mathcal{C}$ is autonomous, i.e. $\mathcal{C}$ has both left & right

then ${}^*(V^*) = V = ({}^*V)^*$



Right Duality

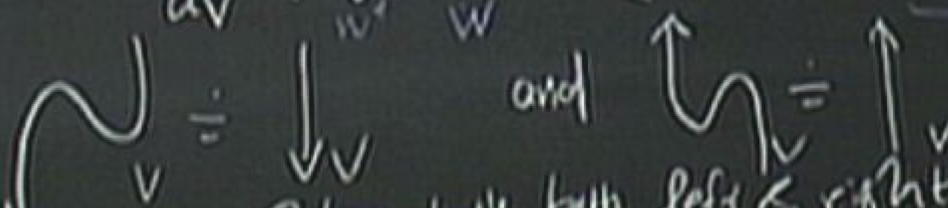
For each $V \in Ob(\mathcal{C})$

$\exists$ an object ${}^*V \in Ob(\mathcal{C})$ and

morphisms $b_V' : I \longrightarrow {}^*V \otimes V$

$d_V' : V \otimes {}^*V \longrightarrow I$

s.t.



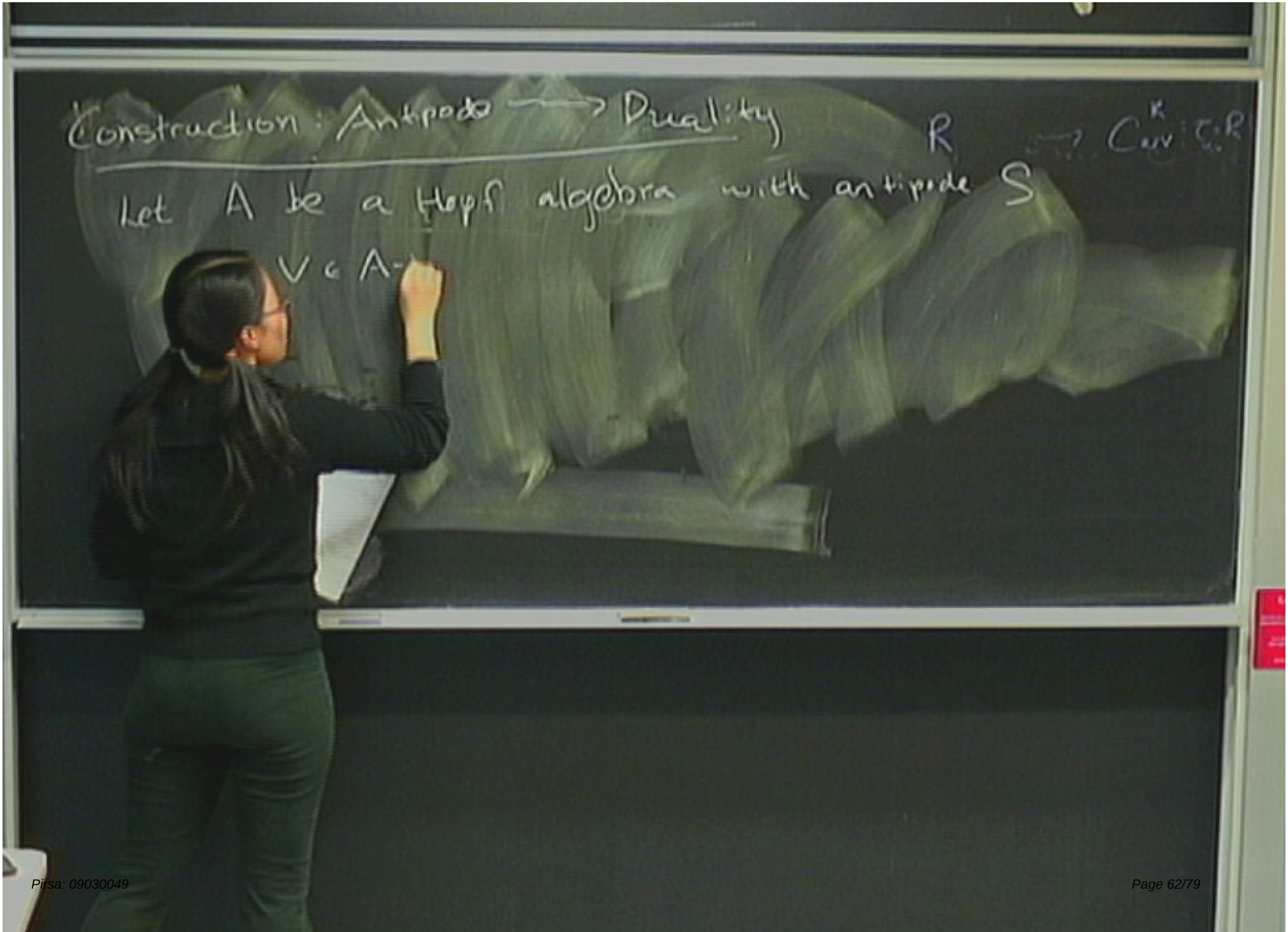
If $\mathcal{C}$ is autonomous, i.e. $\mathcal{C}$ has both left & right duality,

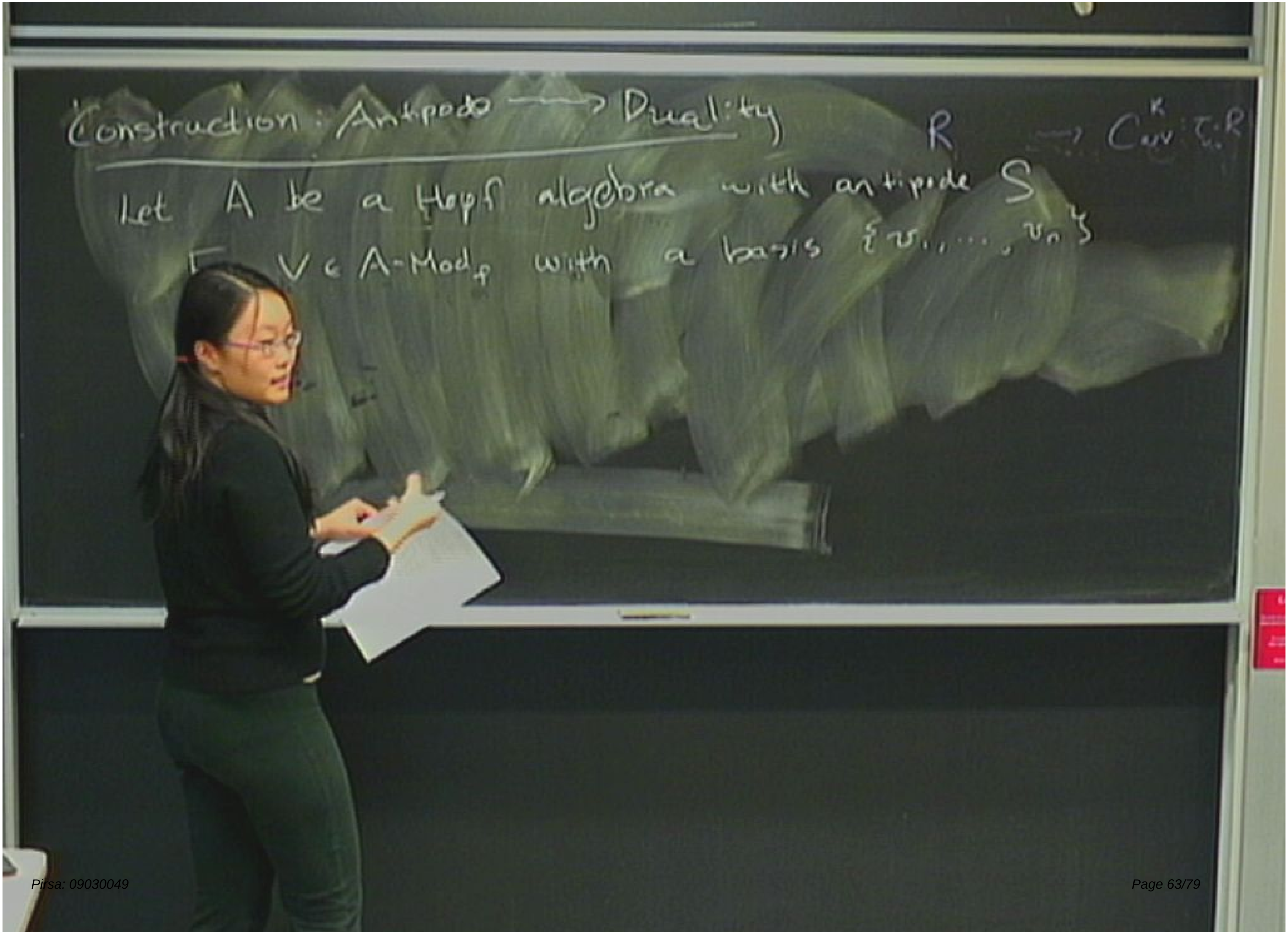
then ${}^*(V^*) = V = ({}^*V)^*$

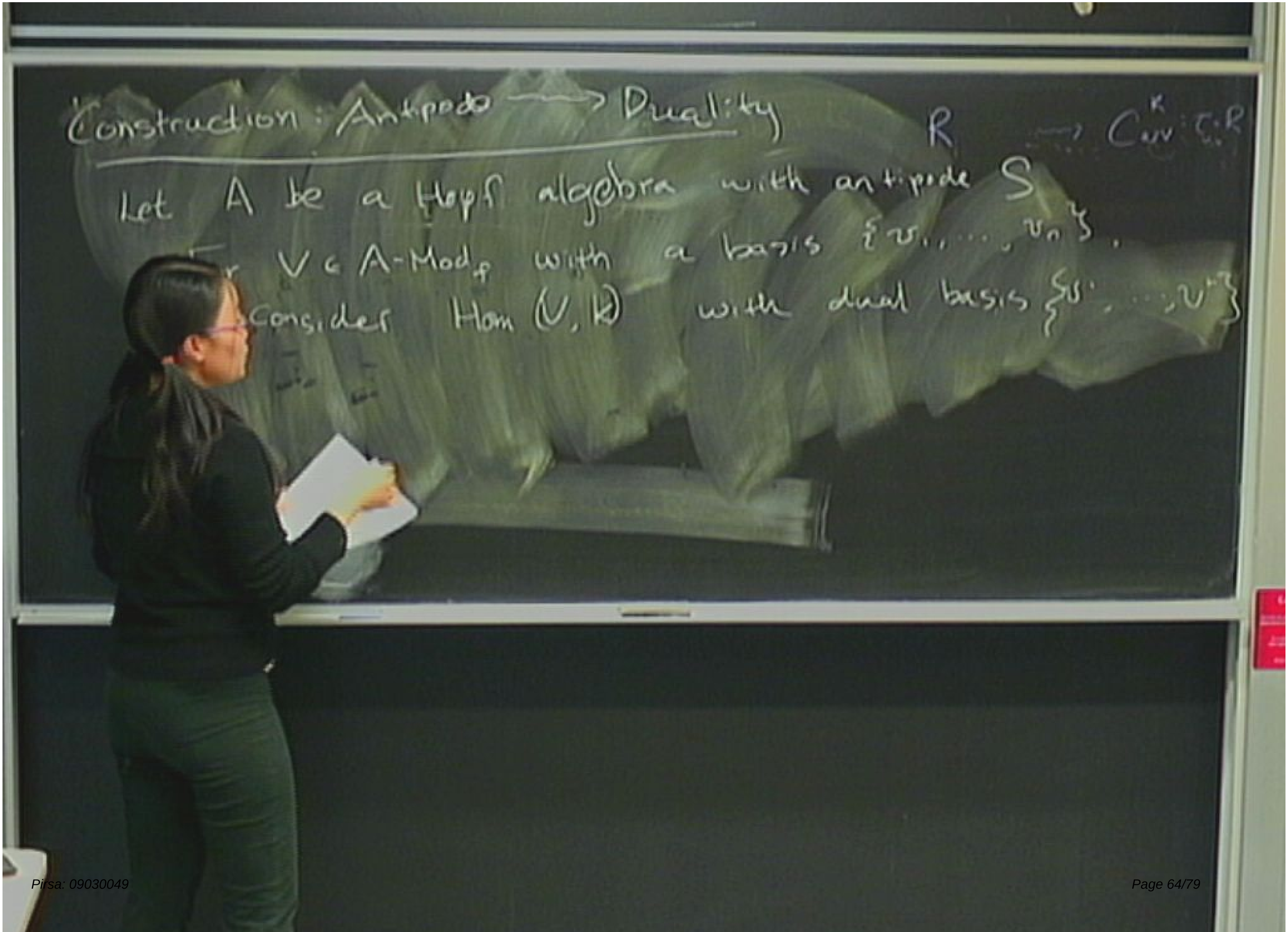
$L^1 \quad L^\infty$

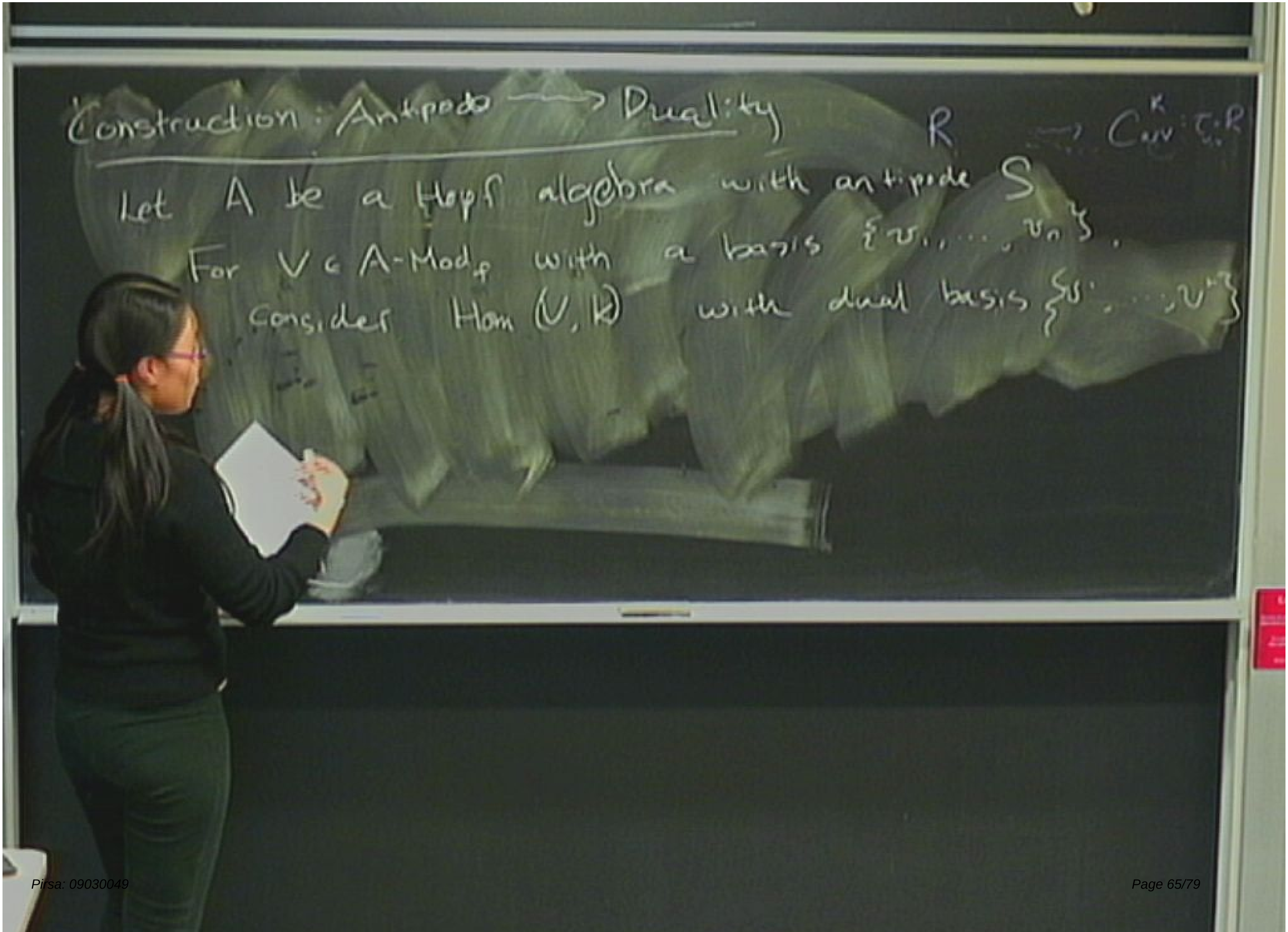
$L^\infty \subset (L^1)^*$

$L^1 \subset (L^\infty)^*$


Construction: Antipode ⟶ Duality
R
Let A be a Hopf algebra with antipode S
V ∈ A-


Construction: Antipode ⟶ Duality
Let A be a Hopf algebra with antipode S
V ∈ A-Mod_f with a basis {v_1, ..., v_n}


Construction: Antipode ⟶ Duality
Let A be a Hopf algebra with antipode S
V ∈ A-Mod_f with a basis {v_1, …, v_n}.
Consider Hom (V, k) with dual basis {v^1, …, v^n}


Construction: Antipode ⟶ Duality
R
Let A be a Hopf algebra with antipode S
For V ∈ A-Mod_f with a basis {v_1, ..., v_n}.
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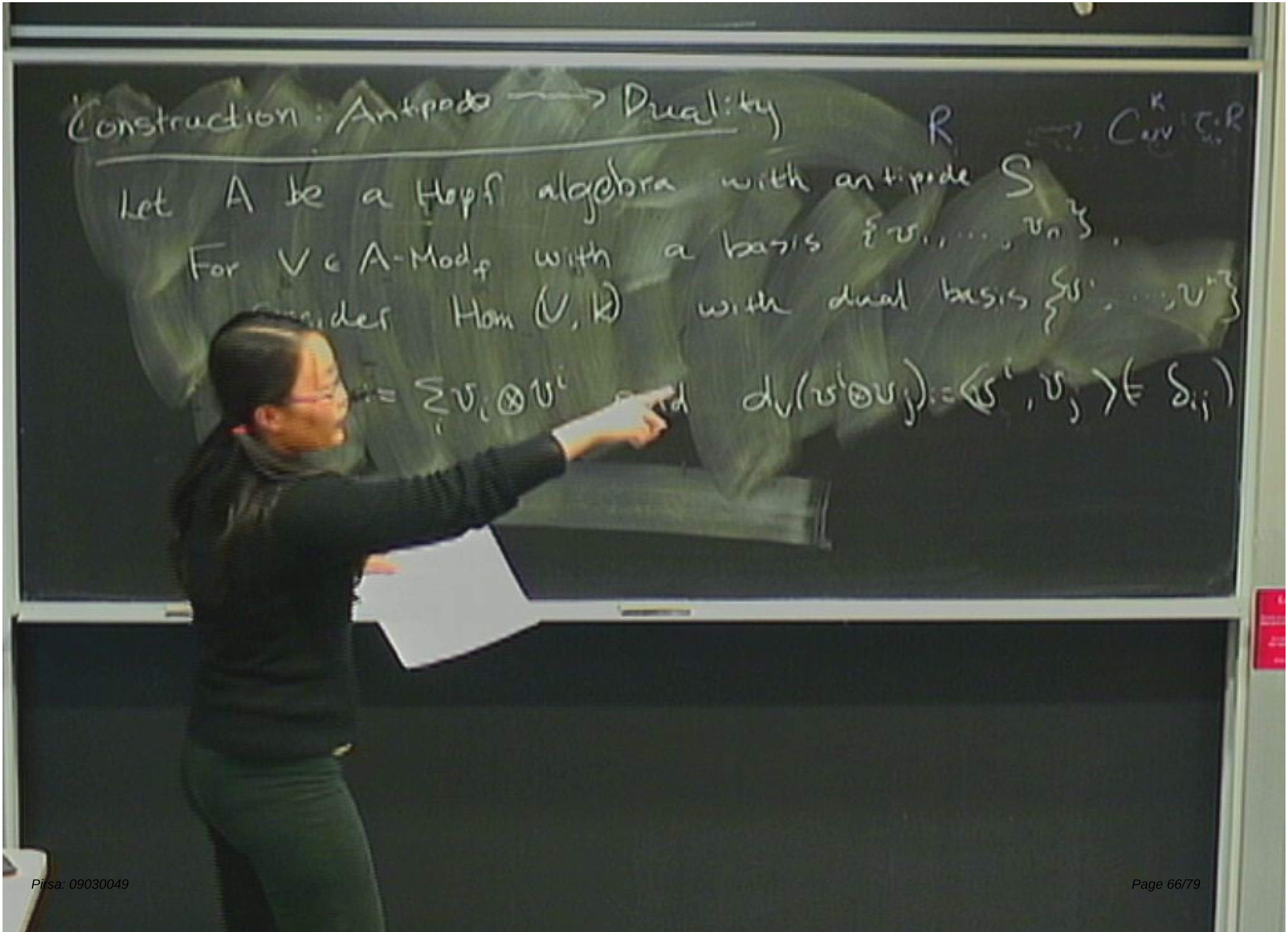
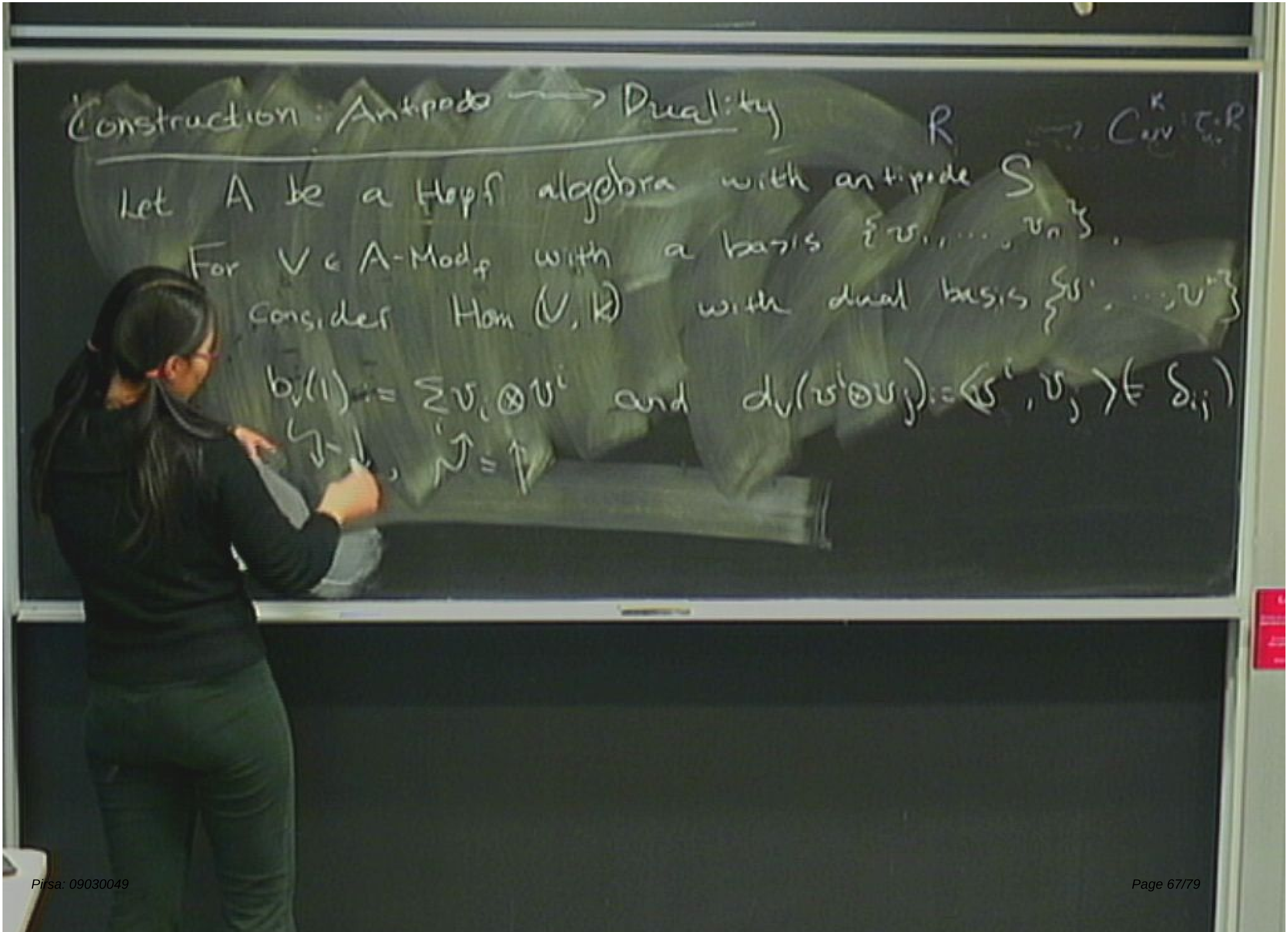
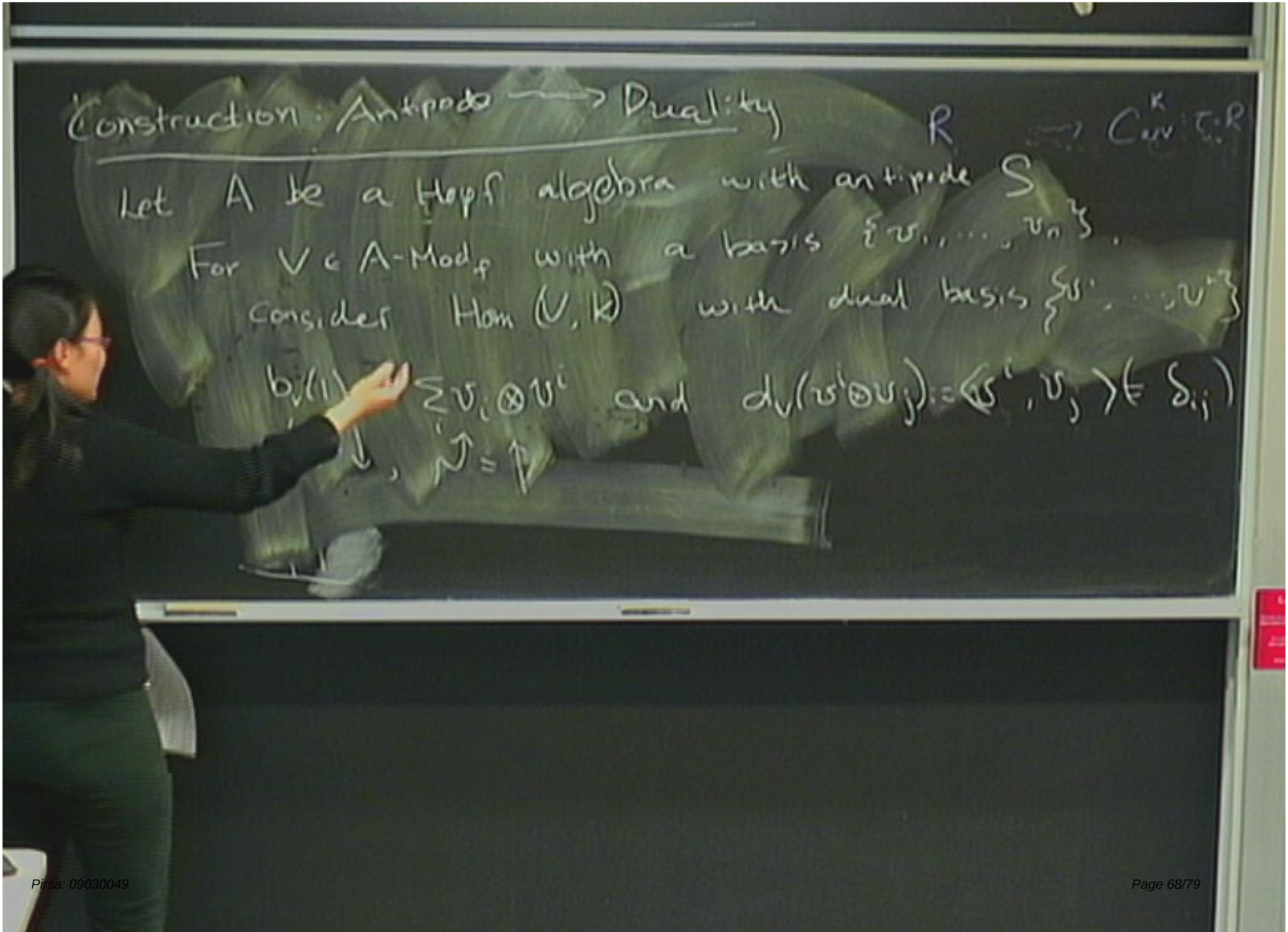
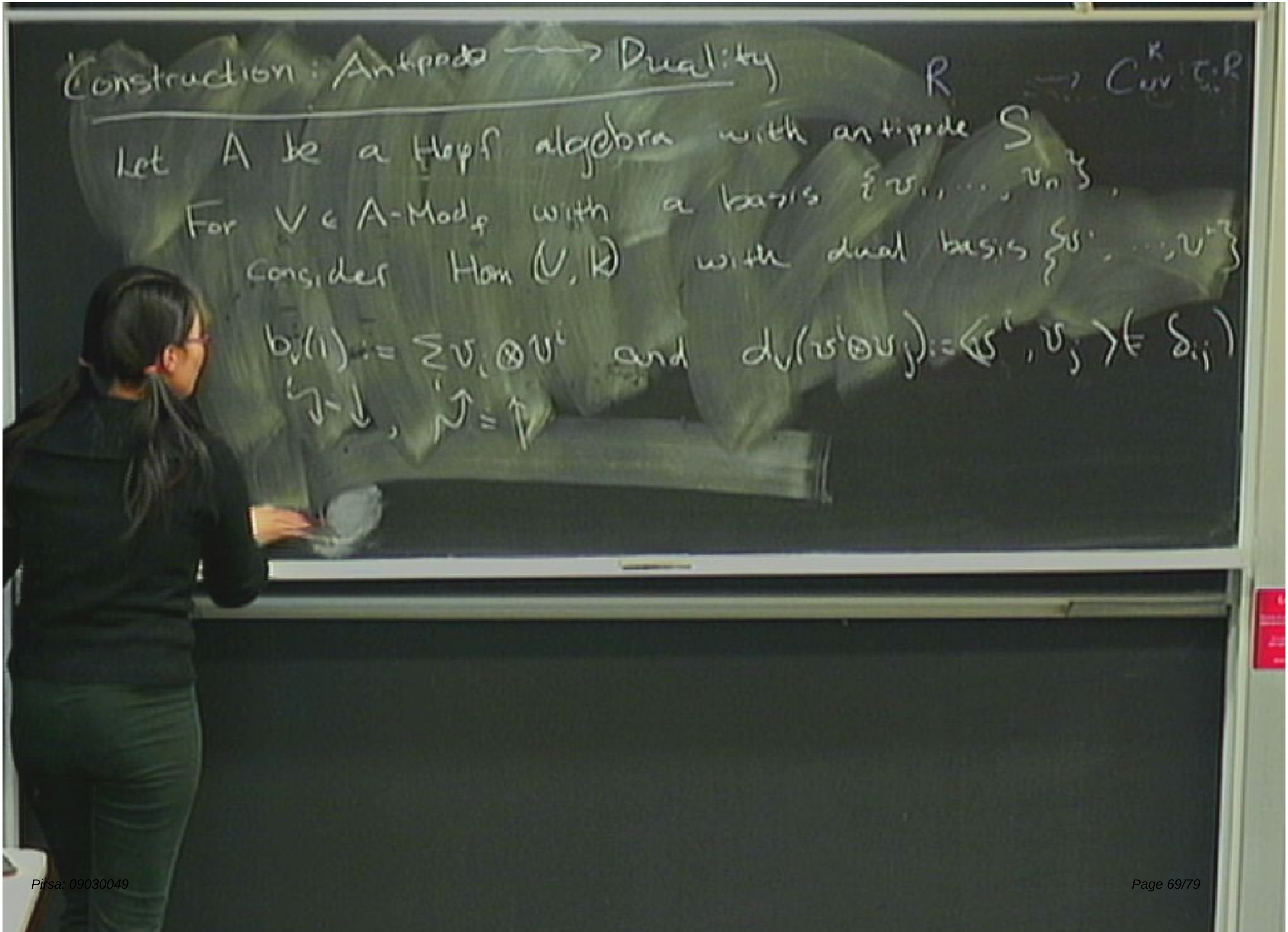
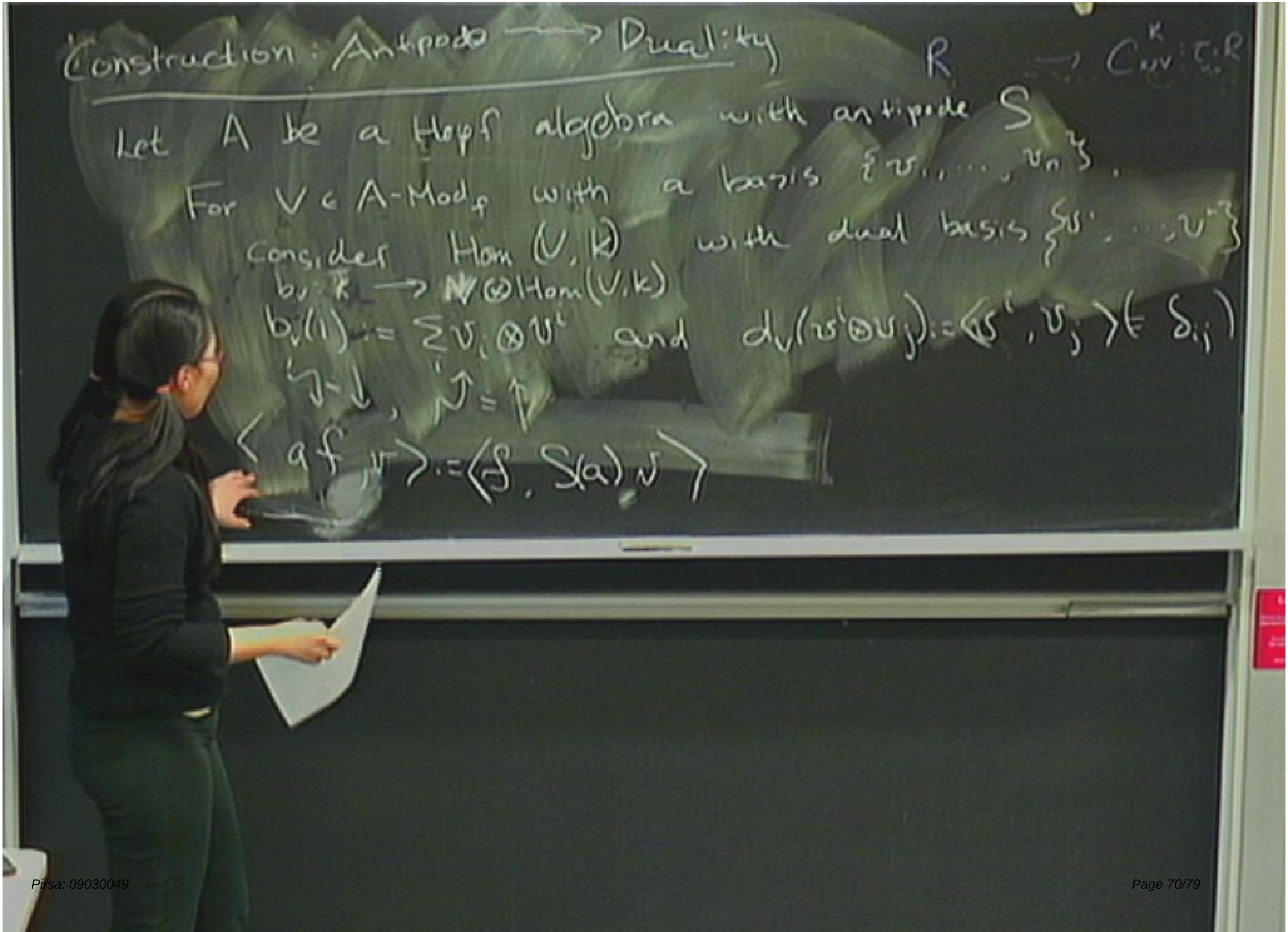

Construction: Antipode → Duality
Let A be a Hopf algebra with antipode S
For V ∈ A-Mod_f with a basis {v_1, ..., v_n}.
Hom (V, k) with dual basis {v^1, ..., v^n}
= Σ_i v_i ⊗ v^i
d_V(v^i ⊗ v_j) := ⟨v^i, v_j⟩ (= δ_ij)


Construction: Antipode ⟶ Duality
R ⟶ C^K_{V,V} τ∘R
Let A be a Hopf algebra with antipode S
For V ∈ A-Mod_f with a basis {v_1, ..., v_n}.
Consider Hom(V, k) with dual basis {v^1, ..., v^n}
b_V(1) := Σ v_i ⊗ v^i and d_V(v^i ⊗ v_j) := ⟨v^i, v_j⟩ (= δ_ij)


Construction: Antipode ⇝ Duality
Let A be a Hopf algebra with antipode S.
For V ∈ A-Mod$_f$ with a basis $\{v_1, \dots, v_n\}$.
consider Hom(V, k) with dual basis $\{v^1, \dots, v^n\}$
$b_V(1) = \sum_i v_i \otimes v^i$ and $d_V(v^i \otimes v_j) := \langle v^i, v_j \rangle (= \delta_{ij})$
↓, V* = ↑


Construction: Antipode ⟶ Duality
Let A be a Hopf algebra with antipode S
For V ∈ A-Mod_f with a basis {v_1, ..., v_n},
Consider Hom(V, k) with dual basis {v^1, ..., v^n}
b_V(1) := Σ_i v_i ⊗ v^i and d_V(v^i ⊗ v_j) := ⟨v^i, v_j⟩ (= δ_ij)


Construction: Antipode ⟶ Duality
R ⟶ C^{K}_{UV}
Let A be a Hopf algebra with antipode S
For V ∈ A-Mod$_f$ with a basis $\{v_1, \ldots, v_n\}$.
Consider Hom(V, k) with dual basis $\{v^1, \ldots, v^n\}$
$b_V: k \to V \otimes \mathrm{Hom}(V,k)$
$b_V(1) := \sum_i v_i \otimes v^i$ and $d_V(v^i \otimes v_j) := \langle v^i, v_j \rangle = \delta_{ij}$
$\langle af, v \rangle := \langle f, S(a) v \rangle$

Construction: Antipode $\longrightarrow$ Duality

$R \rightsquigarrow C^{K}_{UV}$ [illegible]

Let A be a Hopf algebra with antipode S.

For $V \in A\text{-Mod}_f$ with a basis $\{v_1, \ldots, v_n\}$.

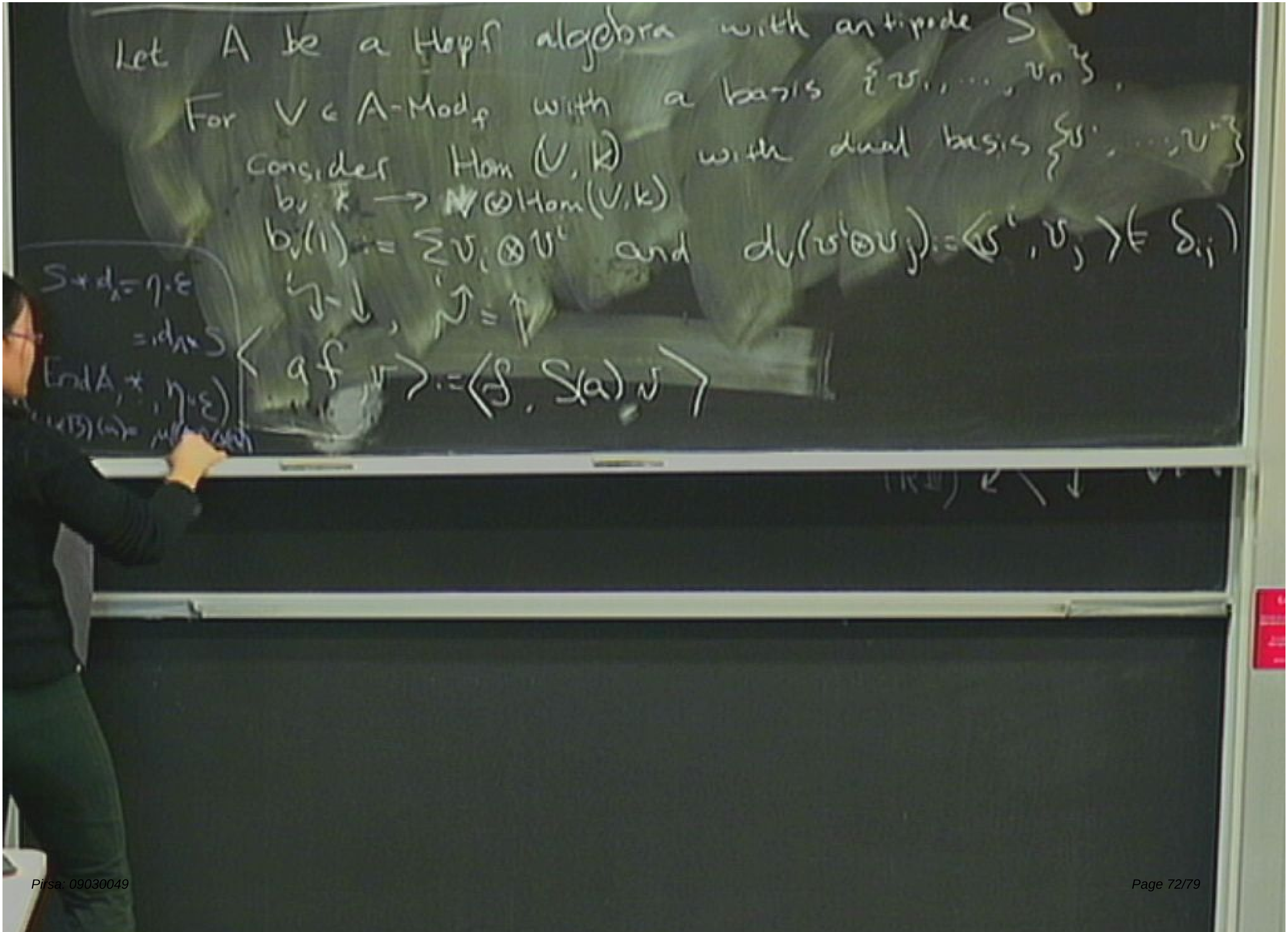
Consider $\mathrm{Hom}(V,k)$ with dual basis $\{v^1, \ldots, v^n\}$

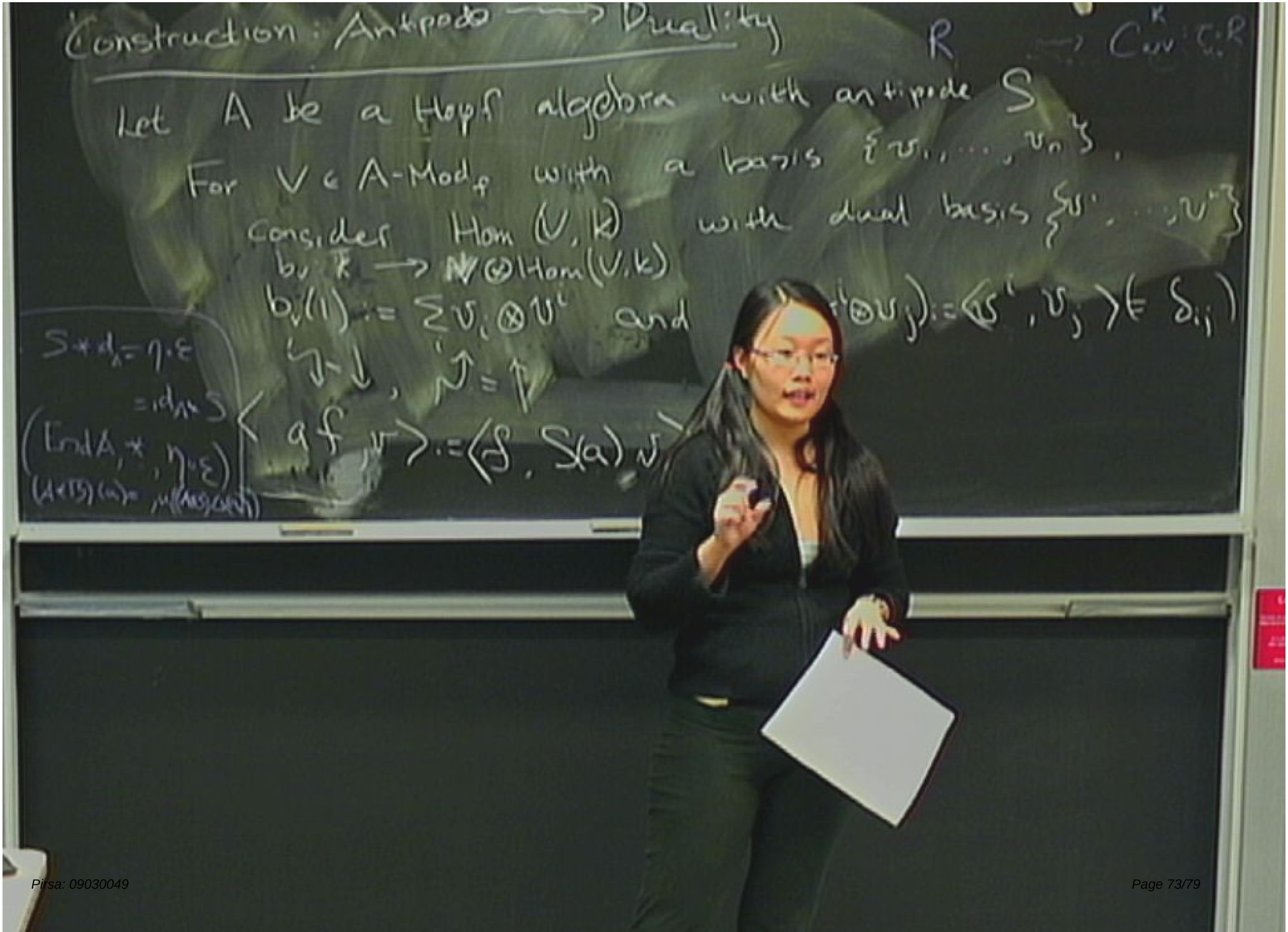
$b_V : k \longrightarrow V \otimes \mathrm{Hom}(V,k)$

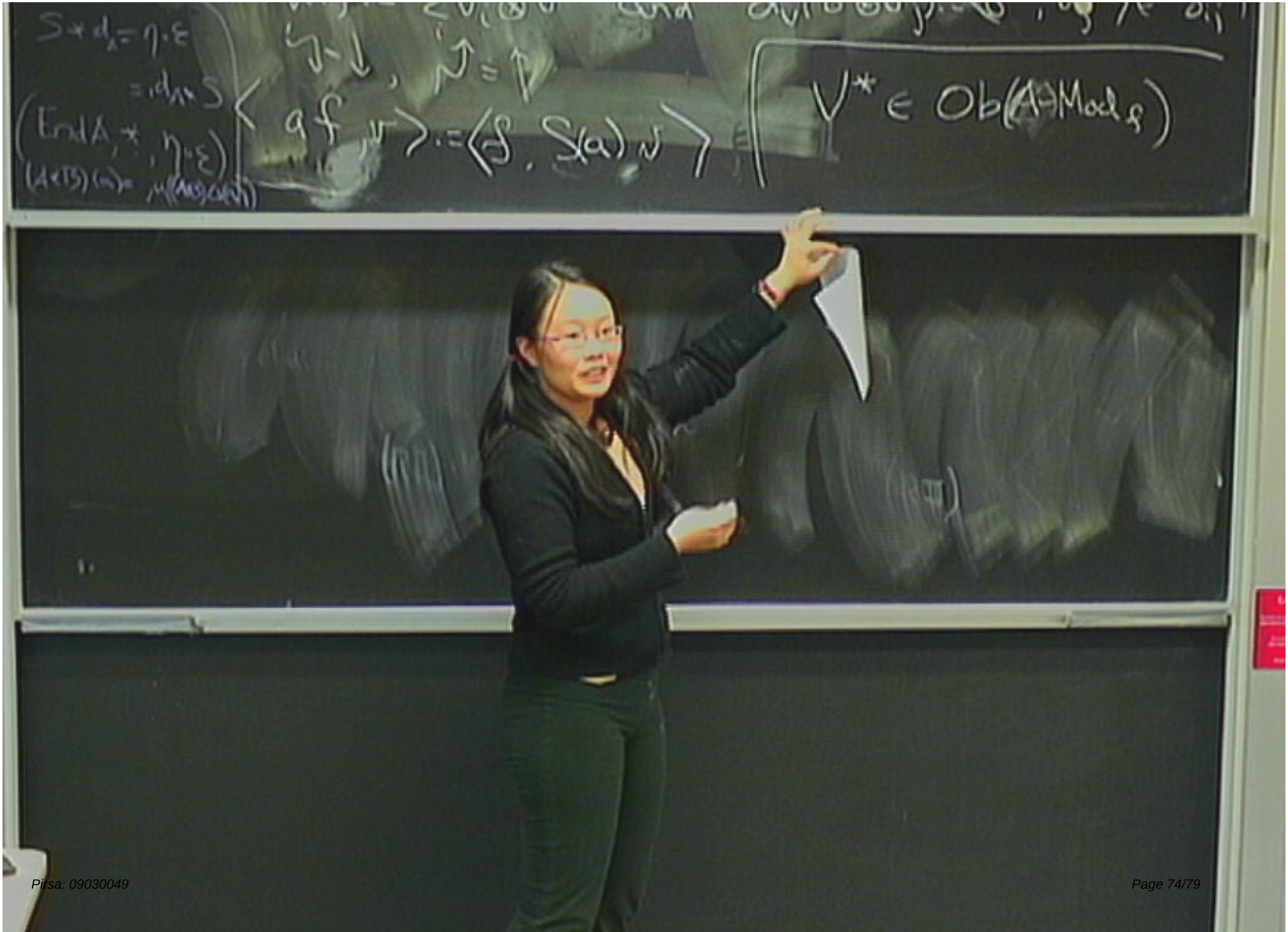
$b_V(1) := \sum_i v_i \otimes v^i$ and $d_V(v^i \otimes v_j) := \langle v^i, v_j \rangle (= \delta_{ij})$

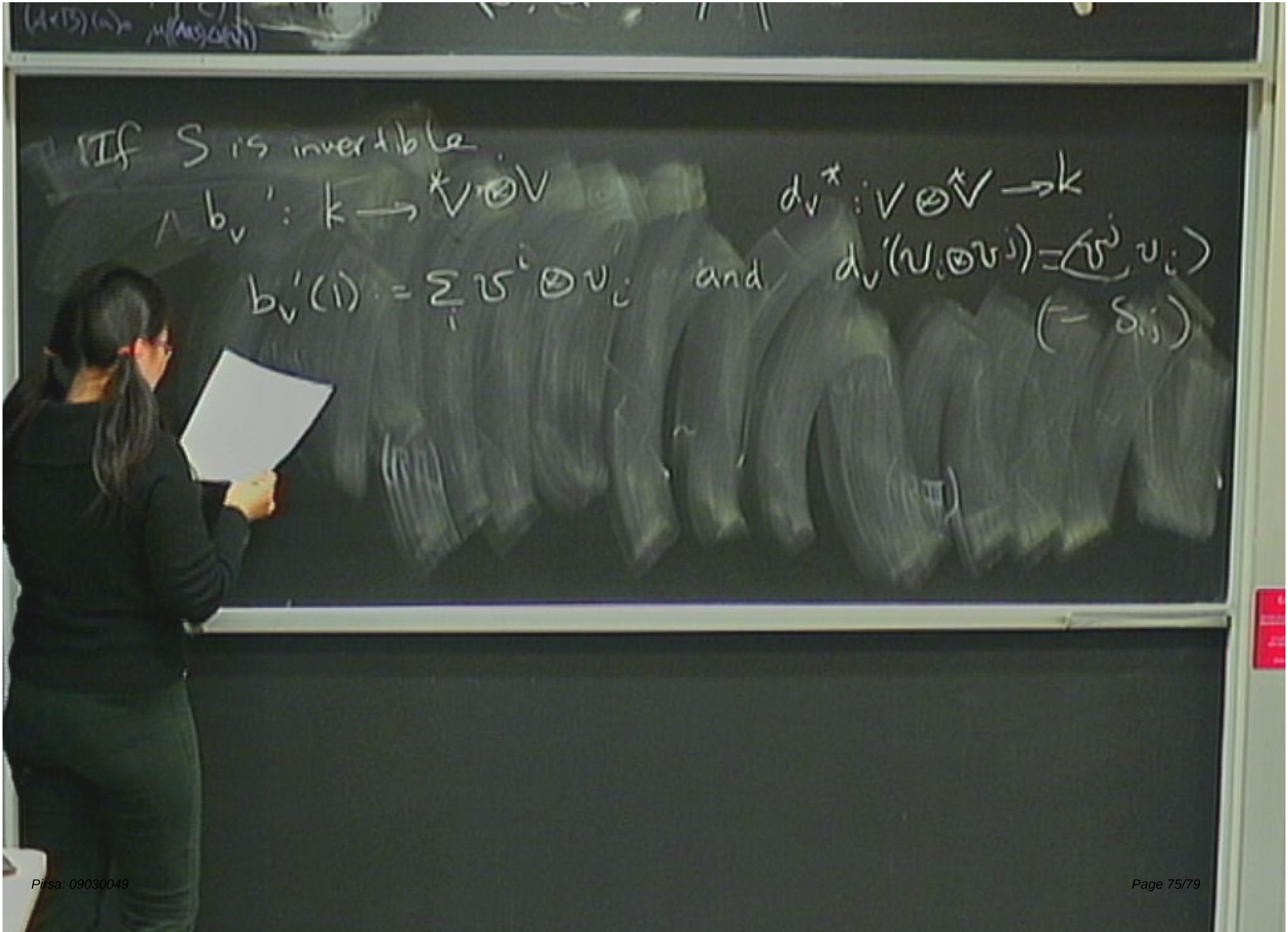
$v = \downarrow$, $v^* = \uparrow$

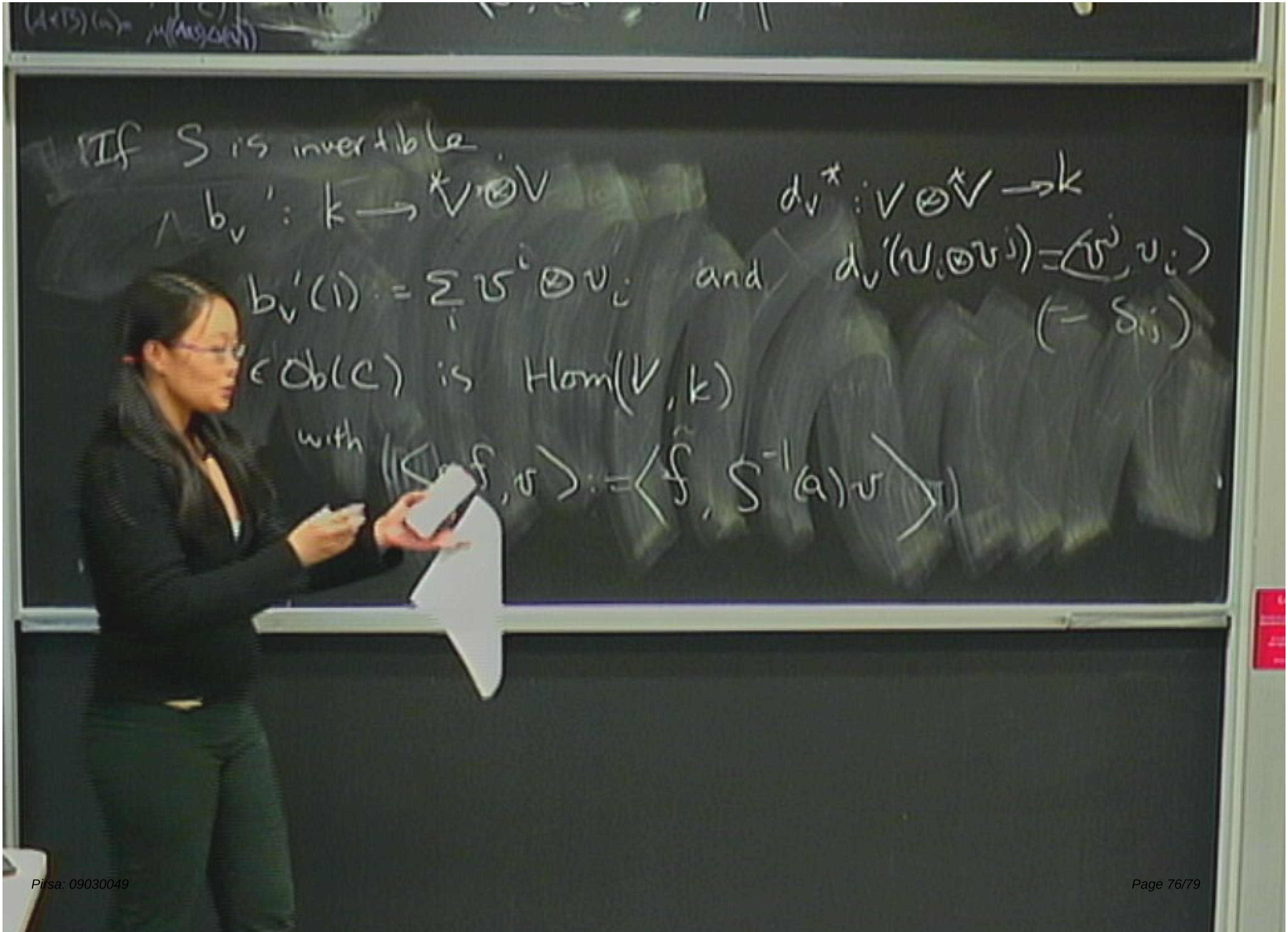
$\langle af, v \rangle := \langle f, S(a)v \rangle$

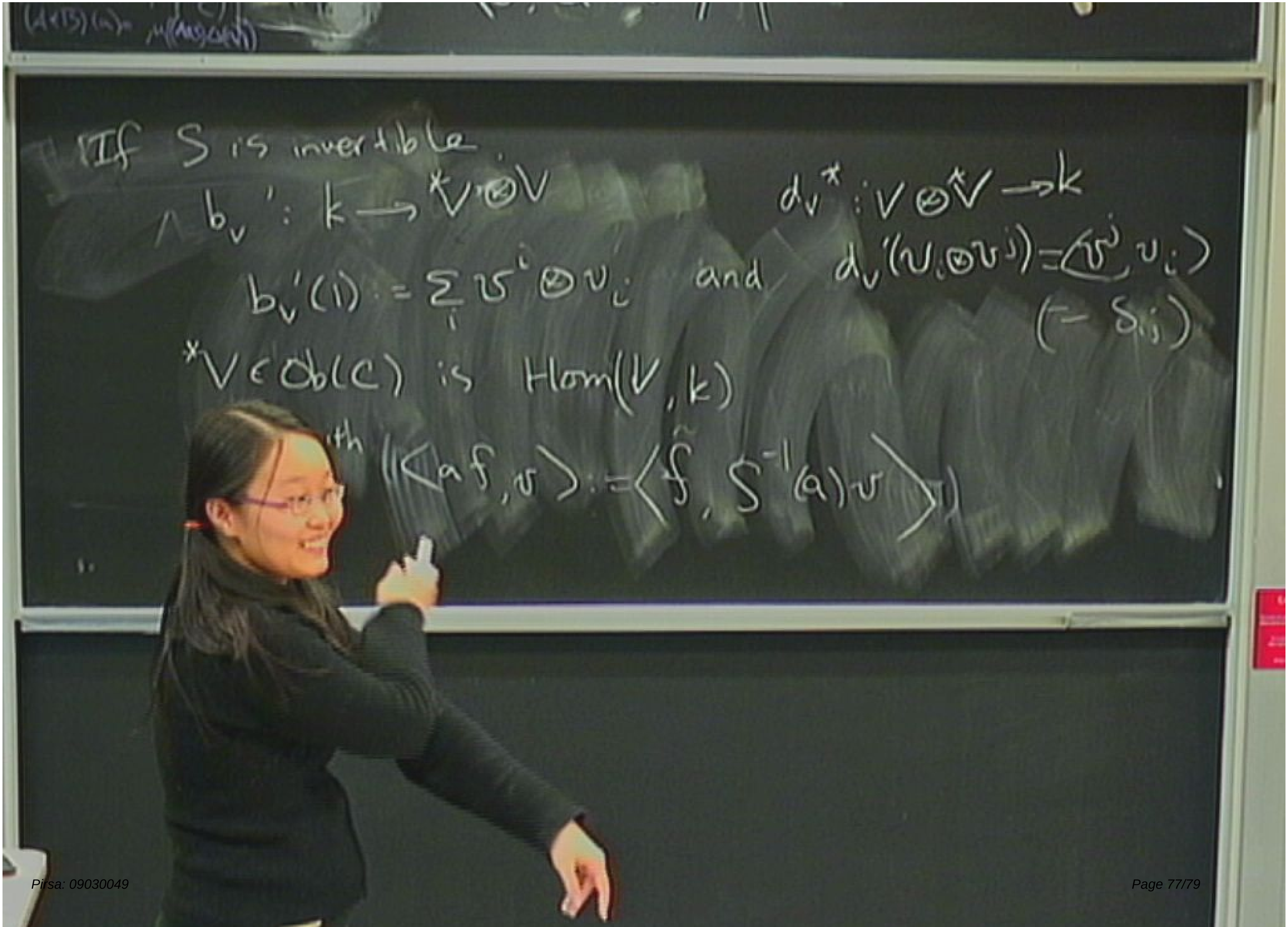

Let A be a Hopf algebra with antipode S
For V ∈ A-Mod_f with a basis {v_1, ..., v_n}.
Consider Hom(V, k) with dual basis {v^1, ..., v^n}
b_V: k → V ⊗ Hom(V, k)
b_V(1) := Σ v_i ⊗ v^i and d_V(v^i ⊗ v_j) := ⟨v^i, v_j⟩ (= δ_ij)
⟨a f, v⟩ := ⟨f, S(a) v⟩
S * id_A = η ∘ ε
= id_A * S
End A, *, η ∘ ε)


Construction: Antipode ⟶ Duality
Let A be a Hopf algebra with antipode S
For V ∈ A-Mod with a basis {v1, ..., vn}.
Consider Hom(V, k) with dual basis
b_V(1) := Σ vi ⊗ v^i and
⟨af, v⟩ := ⟨f, S(a) v⟩




II If S is invertible
b_v' : k → *V ⊗ V
d_v* : V ⊗ *V → k
b_v'(1) = Σ_i v^i ⊗ v_i and d_v'(v_i ⊗ v^j) = ⟨v^j, v_i⟩
(= δ_ij)


If S is invertible
b_v' : k → *V ⊗ V
d_v* : V ⊗ *V → k
b_v'(1) := Σ_i v^i ⊗ v_i
and
d_v'(v_i ⊗ v^j) = ⟨v^j, v_i⟩
(= δ_ij)
∈ Ob(C) is Hom(V, k)
with
⟨f, S^{-1}(a)v⟩


If S is invertible
b_V' : k → *V ⊗ V
d_V* : V ⊗ *V → k
b_V'(1) := Σ_i v^i ⊗ v_i
and
d_V'(v_i ⊗ v^j) = ⟨v^j, v_i⟩
(= δ_ij)
*V ∈ Ob(C) is Hom(V, k)
with (⟨af, v⟩ := ⟨f, S^{-1}(a)v⟩)

II If S is invertible

$b_V' : k \to {}^*V \otimes V \qquad d_V^* : V \otimes {}^*V \to k$

$b_V'(1) := \sum_i v^i \otimes v_i$ and $d_V'(v_i \otimes v^j) = \langle v^j, v_i \rangle$ $(= \delta_{ij})$

${}^*V \in Ob(C)$ is $Hom(V, k)$

with $(\langle af, v \rangle := \langle f, S^{-1}(a) v \rangle)$

II If S is invertible

$\wedge\ b_V' : k \to {}^*V \otimes V$ $\quad d_V^* : V \otimes {}^*V \to k$

$b_V'(1) := \sum_i v^i \otimes v_i$ and $d_V'(v_i \otimes v^j) = \langle v^j, v_i \rangle$ $(= \delta_{ij})$

${}^*V \in \mathrm{Ob}\, A\text{-Mod}$ is $\mathrm{Hom}(V, k)$

with $(\langle af, v \rangle := \langle \tilde{f}, S^{-1}(a) v \rangle)$