Title: Decoherence and Entanglement Dynamics of Coupled Qubits

Date: Mar 23, 2009 04:00 PM

URL: http://pirsa.org/09030047

Abstract: We study the entanglement dynamics and relaxation properties of a system of two interacting qubits in the two cases (I) two independent bosonic baths and (II) one common bath, at temperature T. The entanglement dynamics is studied in terms of the concurrence C(t) between the two spins and of the von Neumann entropy S(t) with respect to the bath, as a function of time. We prove that the system does thermalize. In the case (II) of a single bath, the existence of a decoherence-free (DFS) subspace makes entanglement dynamics very rich. We show that when the system is initially in a state with a component in the DFS the relaxation time is surprisingly long, showing the existence of semi-decoherence free subspaces. The equilibrium state in this case is not the Gibbs state. The entanglement dynamics for the single bath case is also studied as a function of temperature, coupling strength with the environment and strength of tunneling coupling. The case of the mixed state is finally shown and discussed.

DECOHERENCE AND ENTANGLEMENT DYNAMICS OF COUPLED QUBITS

G.Campagnano , A.H., and U.Weiss

arXiv:0807.1987v1

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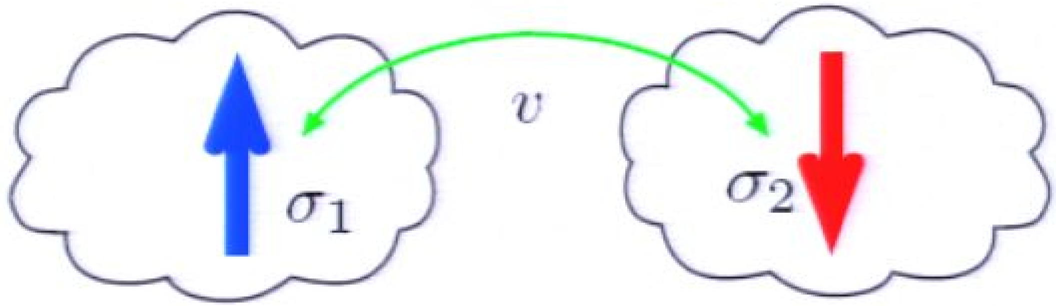
✤ OUTLINE:

- Model of coupled spins
- Dissipative environment
- Master equation approach
- Entanglement dynamics
- Relaxation

✤ Model:



$$H = H_S + H_B + H_I$$

$$H_S = -\frac{\Delta}{2}\sigma_1^x - \frac{\Delta}{2}\sigma_2^x - \frac{v}{2}\sigma_1^z\sigma_2^z$$

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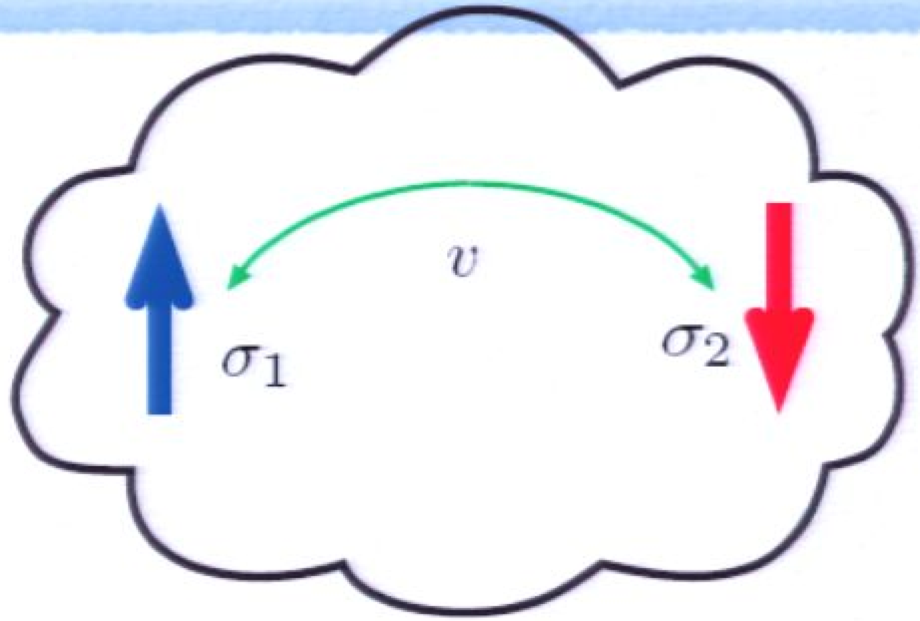

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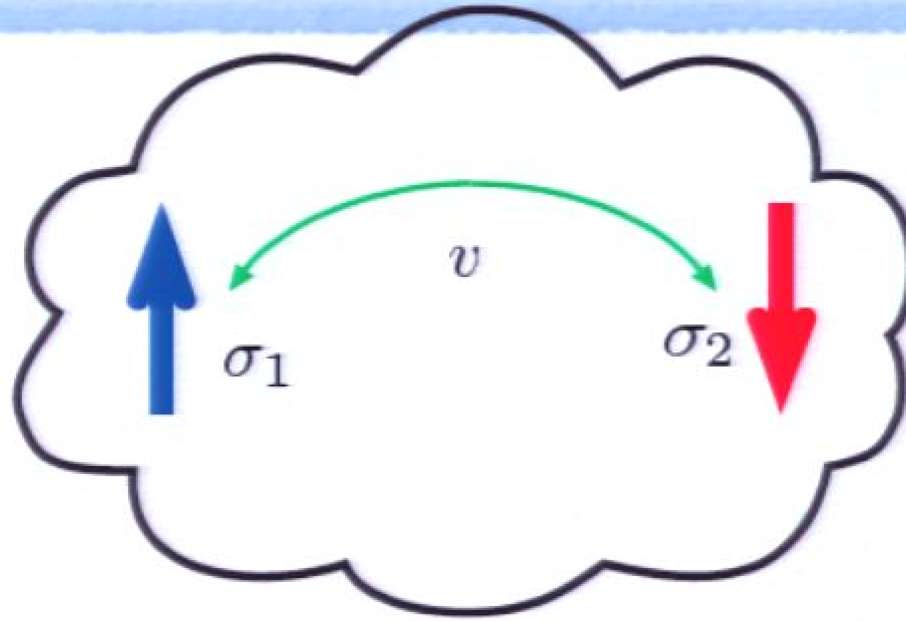

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✤ ENVIRONMENT:

The environment is made of a large number of harmonic oscillators

$$H_B = \sum_{\alpha;i=1,2} \hbar\omega_{\alpha,i}(b^{\dagger}_{\alpha,i}b_{\alpha,i} + \frac{1}{2})$$

Uncorrelated environments

$$H_I = \frac{1}{2}\left[\sigma_1^z \sum_{\alpha} c_{\alpha,1}(b^{\dagger}_{\alpha,1} + b_{\alpha,1}) + \sigma_2^z \sum_{\alpha} c_{\alpha,2}(b^{\dagger}_{\alpha,2} + b_{\alpha,2})\right]$$

For a single spin this is the "classic" spin boson problem:
Legget et al. RMP 1987

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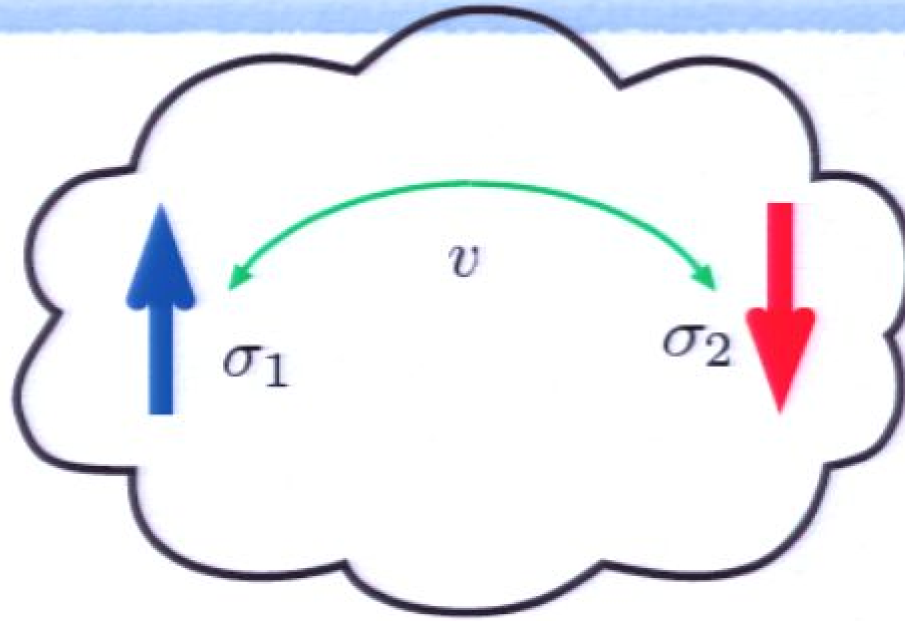

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Interaction Hamiltonian for a single bath

$$H_I = \frac{1}{2}(\sigma_1^z + \sigma_2^z)\sum_\alpha c_\alpha(b_\alpha^\dagger + b_\alpha)$$

Spectral Function for Ohmic baths

$$J_i(\omega) = K_i\, \omega^s \exp(-\omega/\omega_c)$$

Ohmic dissipation $s = 1$

We study the case $K_1 = K_2 \equiv \kappa/2\pi$, with the two baths at temperature T

We will consider the case of weak to moderate coupling → $K_i < 1$

✤ Reduced density matrix

$$\rho(t) = e^{-iHt}\rho(0)e^{iHt}$$

$$\rho(t) \quad \Rightarrow \quad \rho_s(t) = \text{Tr}_{env}\{\rho(t)\}$$

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How to describe the evolution of the reduced density matrix?

✤ Master Equation (Bloch-Redfield)

$$\dot{\rho}_{m',m}(t) = -i\omega_{m'm}\rho_{m'm}(t) + R_{m'mn'n}\rho_{n'n}(t)e^{i(\omega_{m'm}-\omega_{n'n})t}$$

where R is the Redfeld tensor defined as

$$\begin{aligned} R_{m'mn'n} := \quad & - \sum_k (\delta_{mn}\Gamma^{+}_{m'kkn'} + \delta_{n'm'}\Gamma^{-}_{nkkm}) \\ & + \Gamma^{+}_{nmm'n'} + \Gamma^{-}_{nmm'n'} \end{aligned}$$

✤ Entanglement for a bipartite system

Von Neumann Entropy $S = -\mathrm{Tr}(\rho \log_2 \rho)$

Concurrence $C = \mathrm{m}ax\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}$

eigenvalues of: $\rho(\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$

Measure of entanglement for mixed states

INITIAL STATES

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\begin{aligned}
|\Psi_a\rangle &= 1/\sqrt{2}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle) \\
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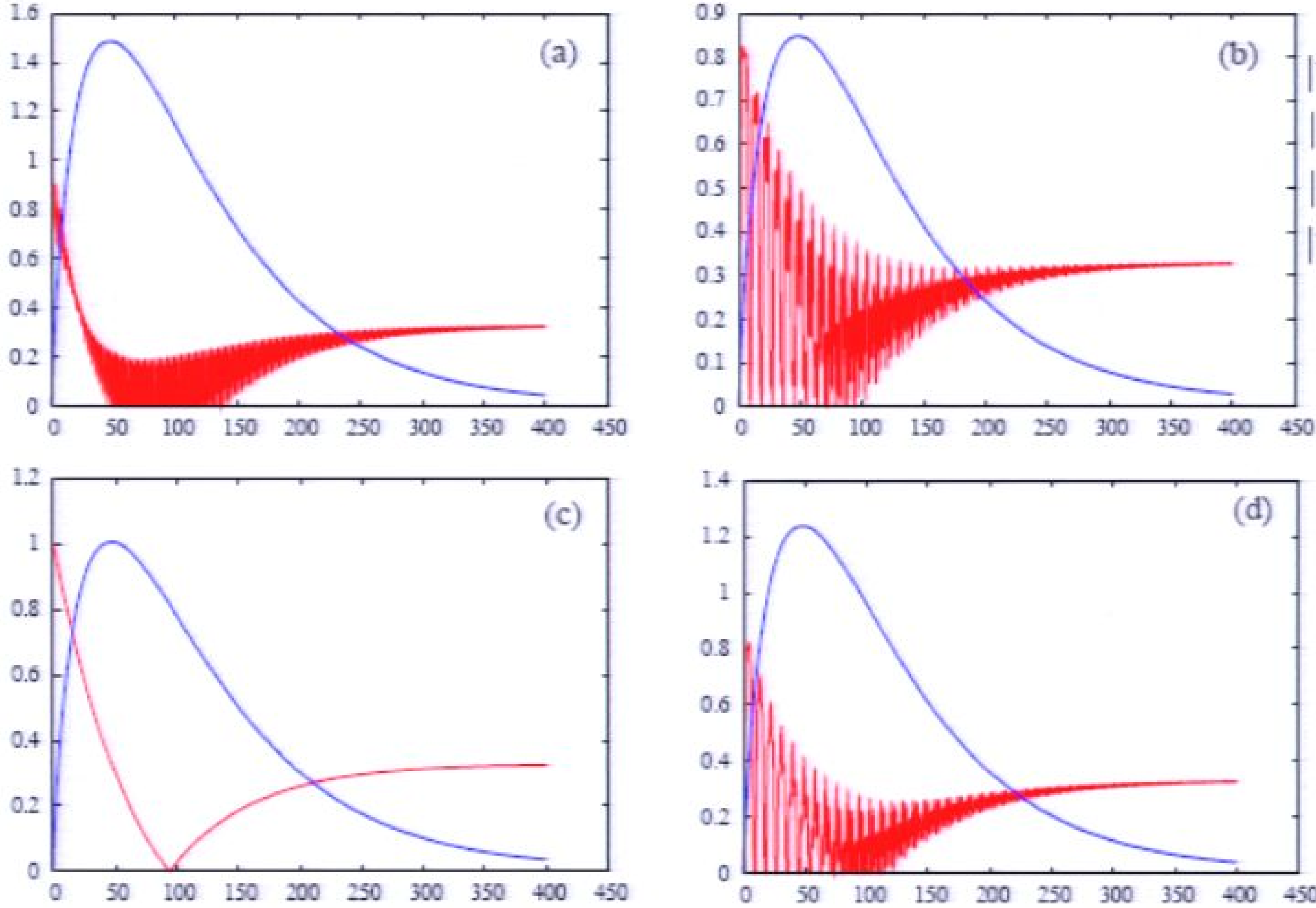
Two independent baths case



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$$\Delta = 1$$
$$\kappa = 0.01$$
$$\beta = 10$$

Single bath case

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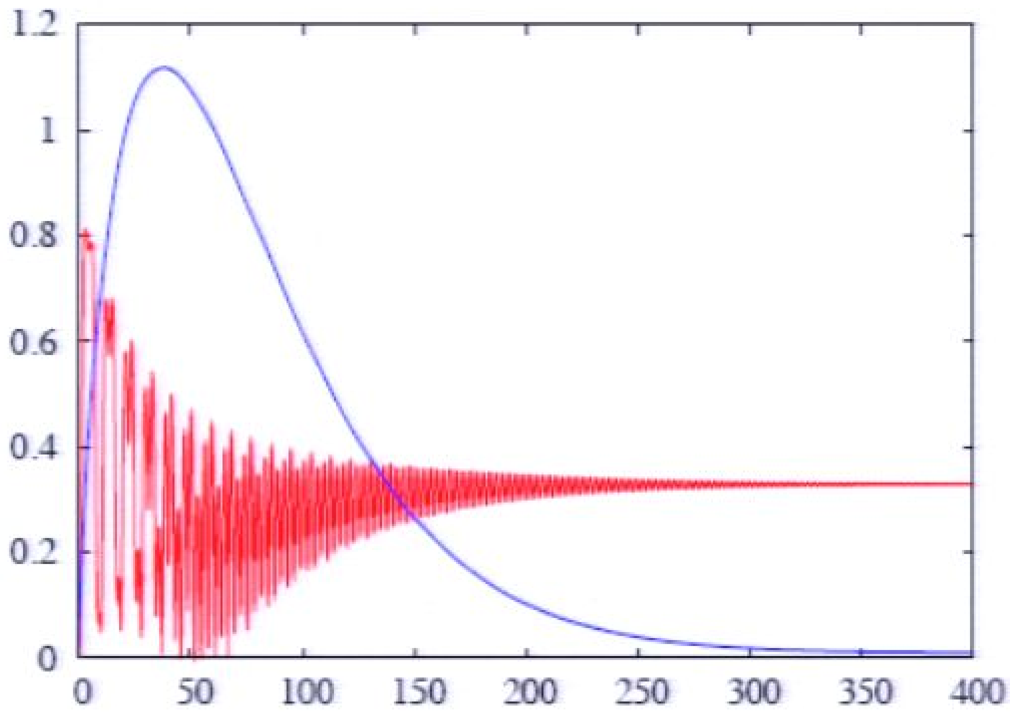

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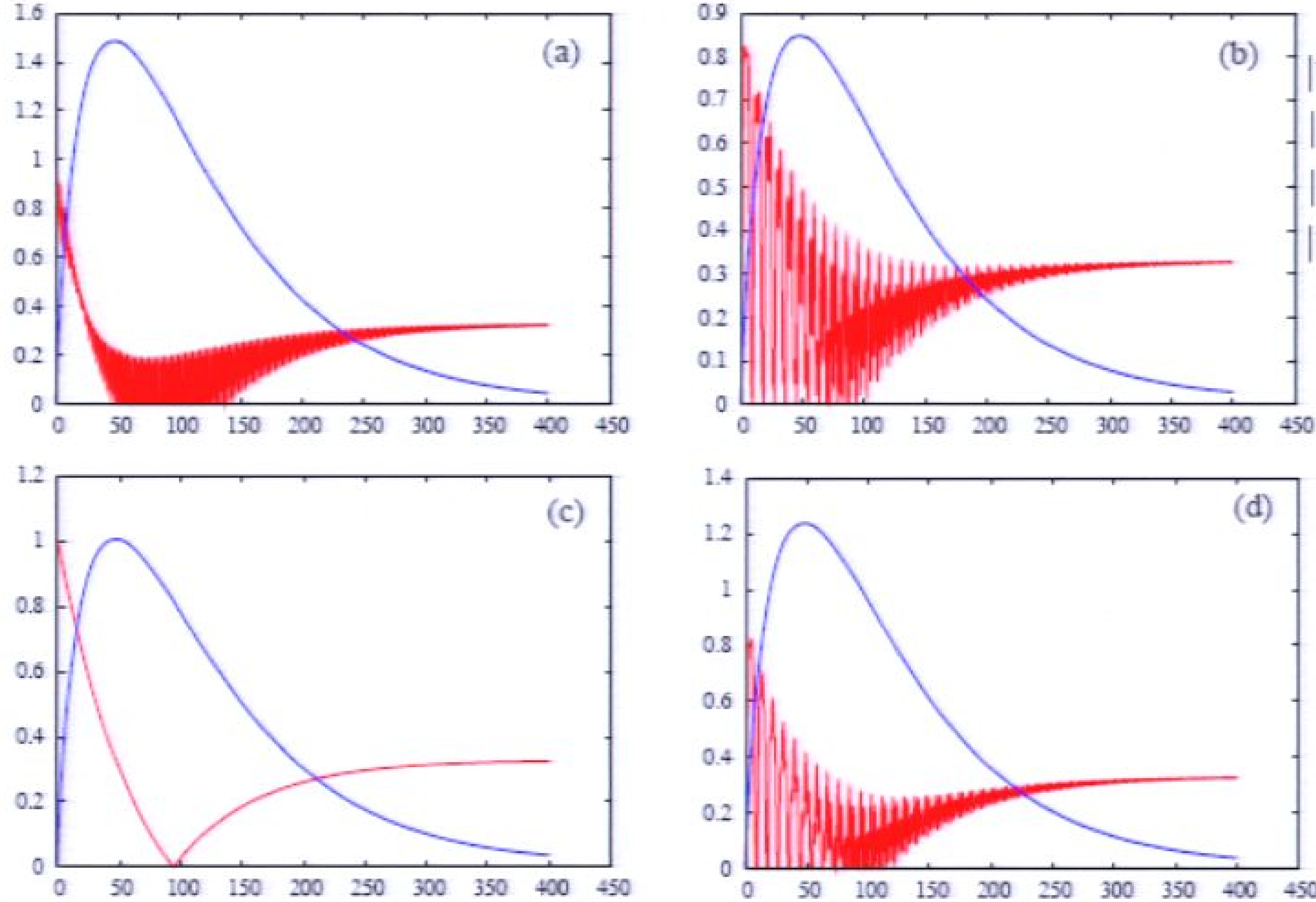

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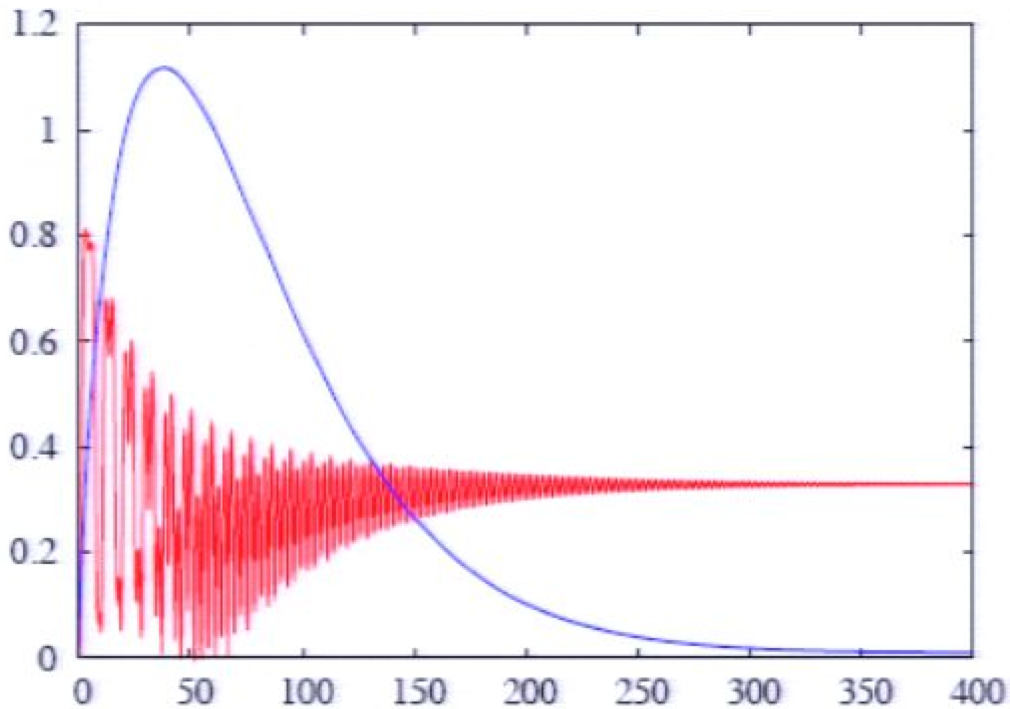

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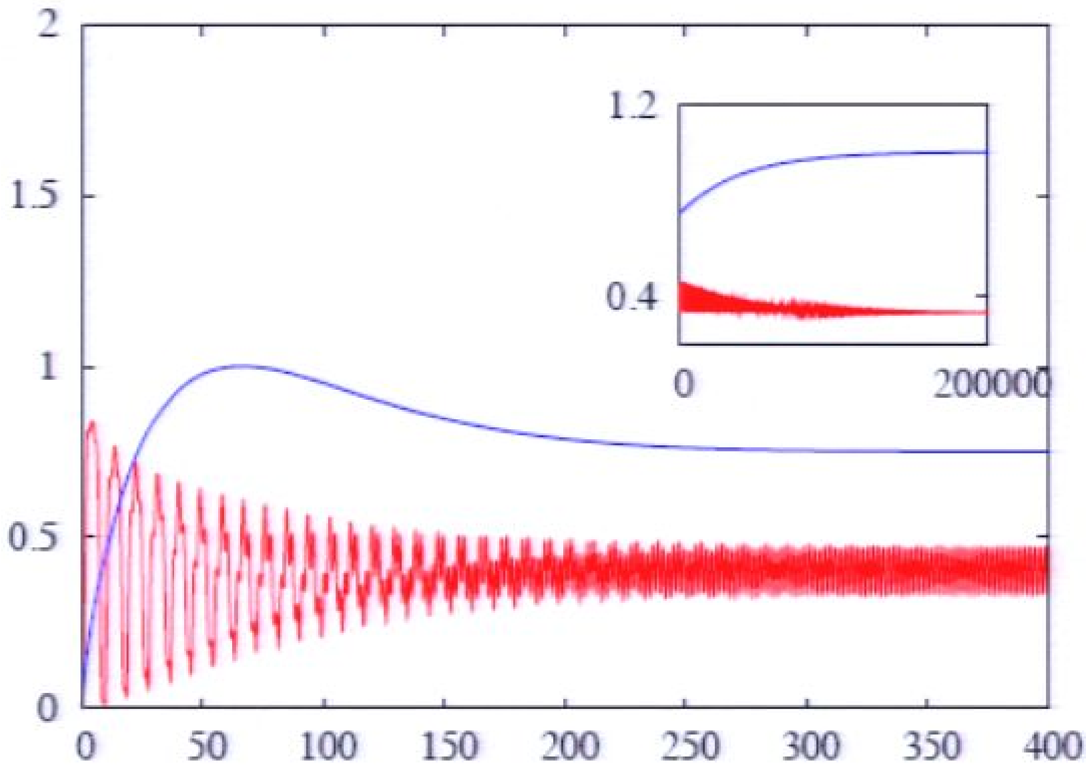
Initial state with a component in the DFS

$$|\Psi_d\rangle = |\uparrow\downarrow\rangle$$



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$$\rho_{ij}(t) = \rho_{ij}(0)\exp[-(\gamma_{ij} + i\omega_{ij})t] \quad i \neq j$$

$$\begin{aligned} \gamma_{13} &\propto \coth(\beta\omega_{21}/2) - 1 \\ \gamma_{12} &\propto \coth(\beta\omega_{21}/2) + 1 \\ \frac{\gamma_{12}}{\gamma_{13}} &\sim 10^3 \end{aligned}$$

$$\omega_{21} = \frac{1}{2}\left(-v + \sqrt{v^2 + 4\Delta^2}\right)$$

Relaxation

Double Bath
(no DFS)

$$\rho_{eq} = \frac{1}{Z} e^{-\beta H_s}$$

Single Bath
(DFS)

$$\rho_{eq}^{(mono)} = \mathrm{diag}\left(\frac{|A|^2}{Z_3} e^{-\beta E_1}, \frac{|A|^2}{Z_3} e^{-\beta E_2}, |B|^2, \frac{|A|^2}{Z_3} e^{-\beta E_4} \right)$$

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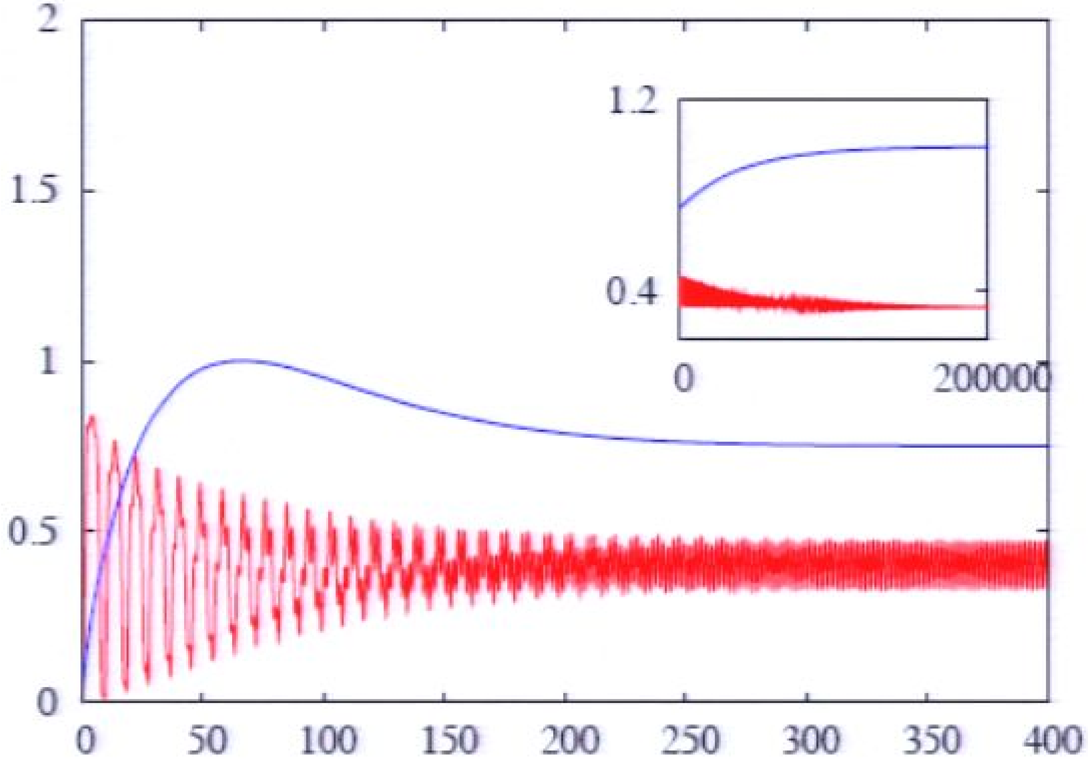

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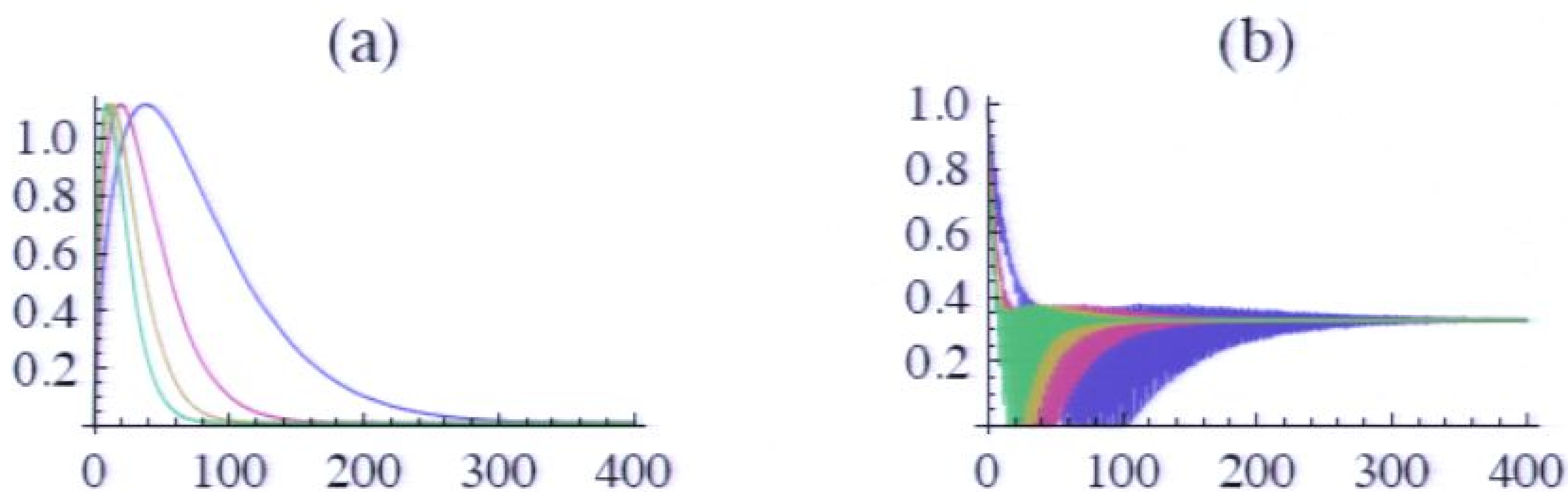
STUDY IN K



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\begin{aligned}
v &= 0.7 \\
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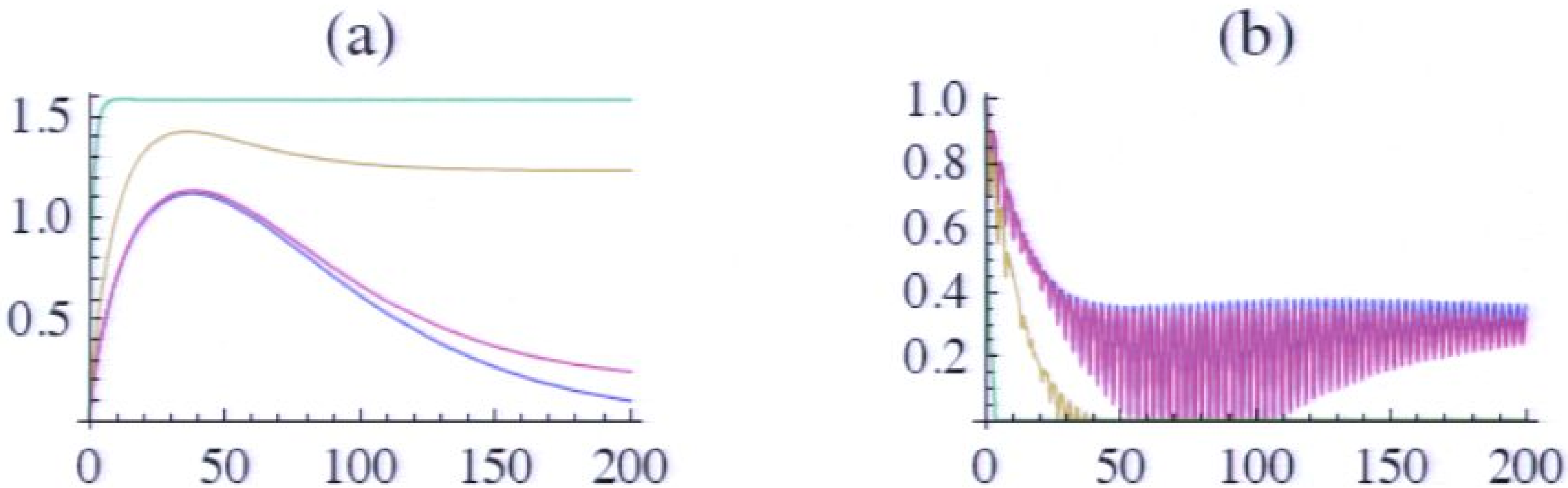
\kappa &= 0.1, 0.2, 0.3, 0.4 \\
\beta &= 10
\end{aligned}
$$

Study in Temperature



$$
\begin{aligned}
v &= 0.7 \\
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\beta &= 20, 5, 1, 0.1
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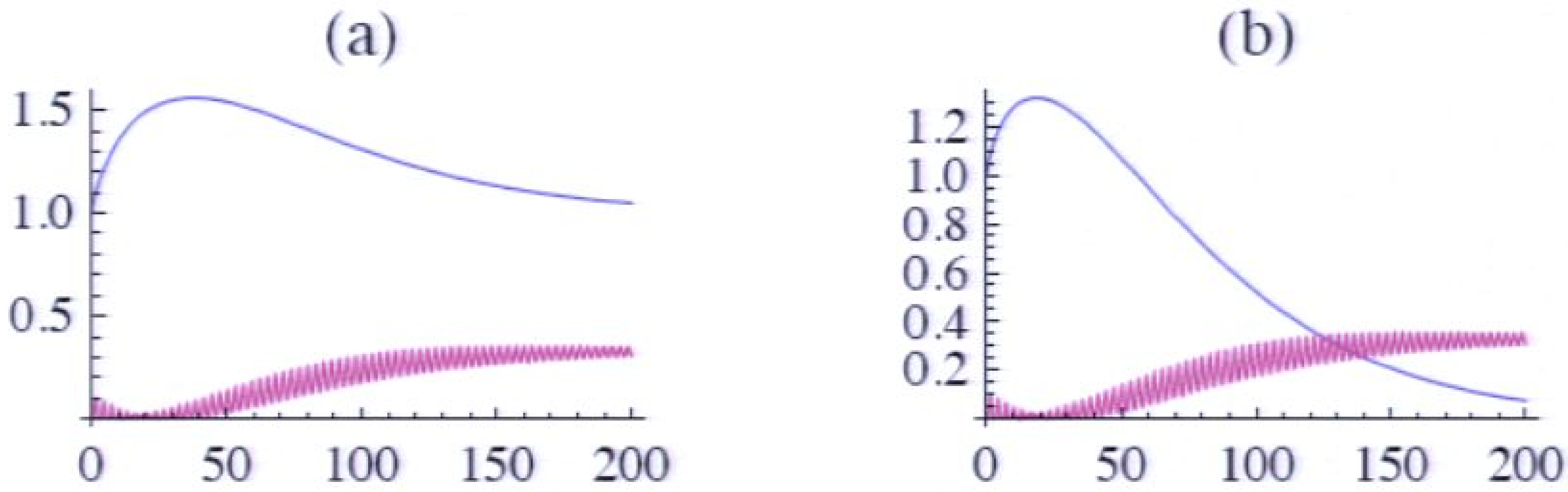
THE MIXED STATE



$$\begin{aligned} v &= 0.7 \\ \Delta &= 1 \\ \kappa &= 0.01 \\ \beta &= 20 \end{aligned}$$

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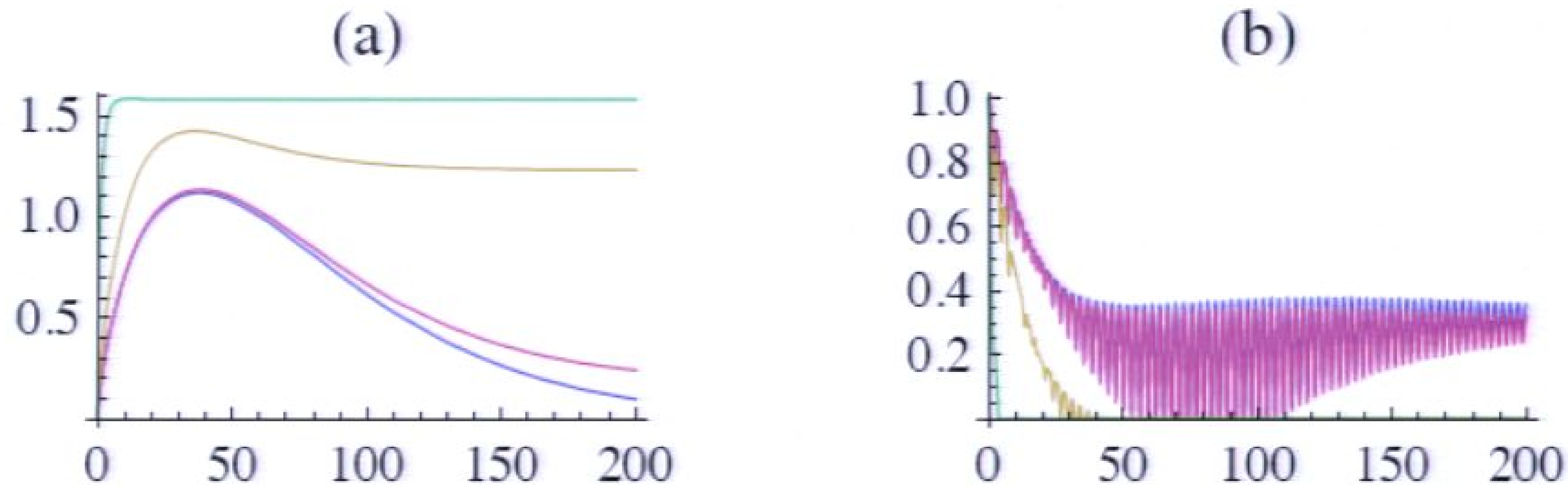
$$\begin{aligned} \rho_{mix}^{(1)} &= (\rho_a + \rho_c)/2 \\ \rho_{mix}^{(2)} &= (\rho_a + \rho_b)/2 \end{aligned}$$

STUDY IN TEMPERATURE



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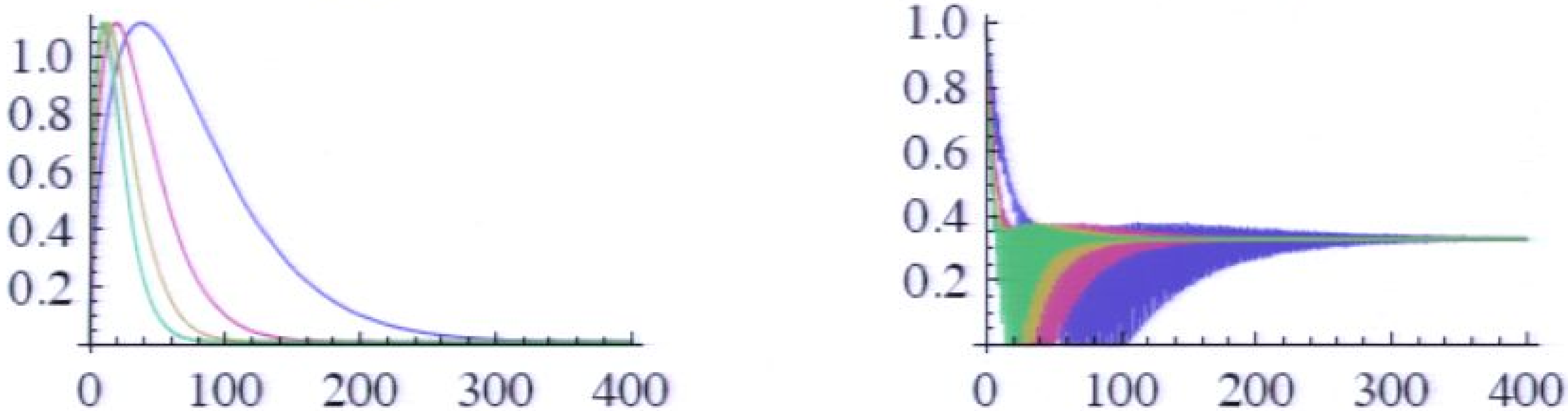
STUDY IN K

(a) (b)



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STUDY IN TEMPERATURE

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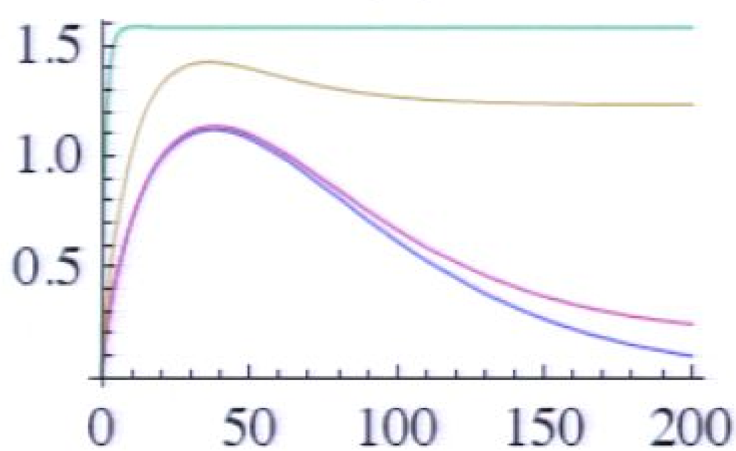
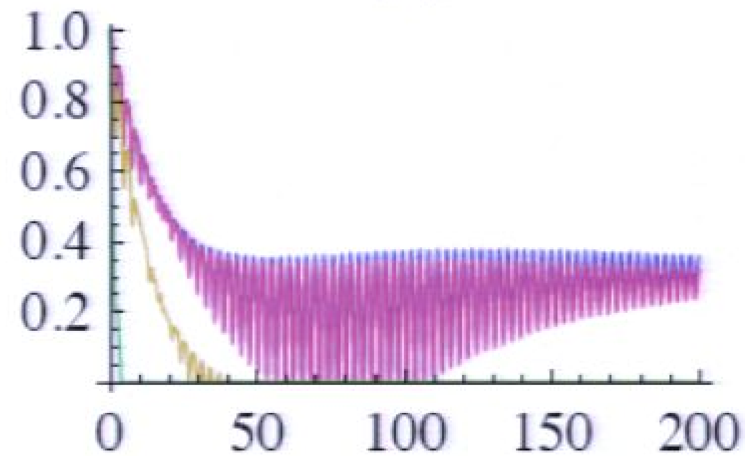


(b)



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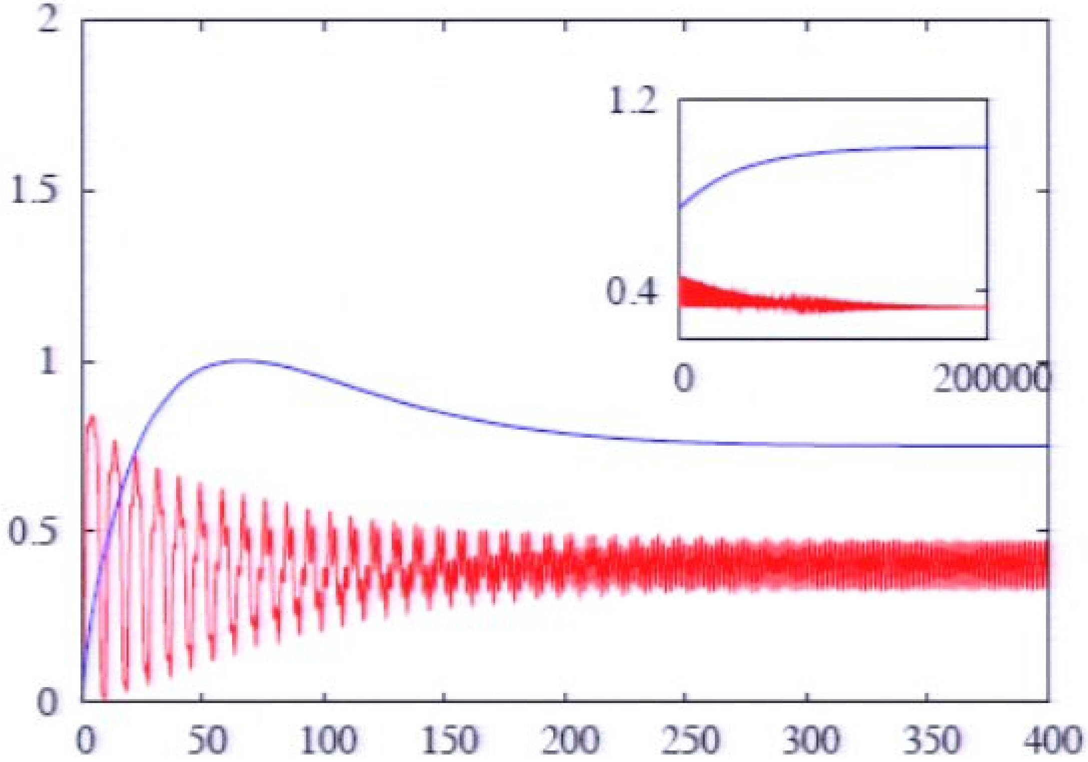

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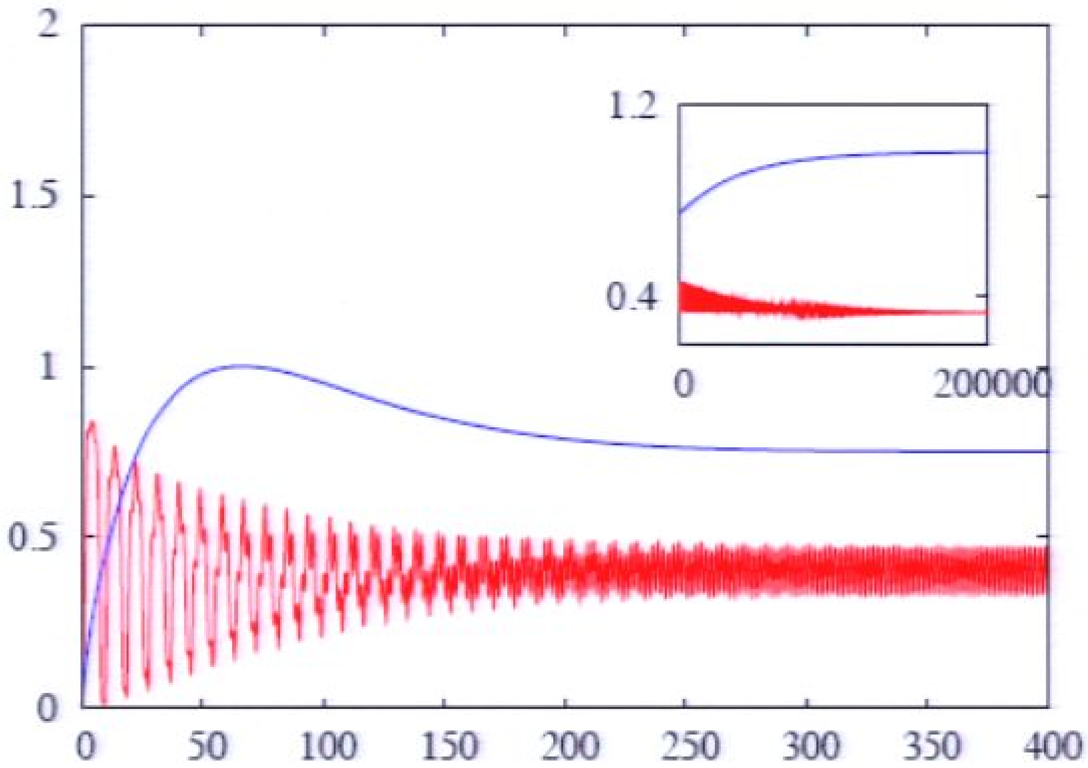

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