Title: What is the objective face of a die?

Date: Mar 10, 2009 04:00 PM

URL: http://pirsa.org/09030041

Abstract: Quantum foundations in the light of gauge theories We will present the conjecture according to which the fact that q and p cannot be both ``observables" of the same quantum system indicates that there is a remnant universal symmetry acting on classical states. In order to unpack this claim we will generalize to unconstrained systems the gauge correspondence between properties defined by first-class constraints and gauge symmetries generated by these constraints. As we shall see, this means that the uncertainty principle might be encoded in the very definition of the canonical variables q and p. According to the ontology of quantum objects that stems from this analysis, the quantum-mechanical description of physical objects is complete.

What is the objective face of a die?

Quantum foundations in the light of gauge theories

Gabriel Catren

École Polytechnique/CNRS

Paris

Perimeter Institute for Theoretical Physics

March 10, 2009

“Gaugification” of quantum mechanics

- The aim of this talk is to apply to quantum mechanics a fundamental idea coming from gauge theories, namely...

“Gaugification” of quantum mechanics

- The aim of this talk is to apply to quantum mechanics a fundamental idea coming from gauge theories, namely...

- ... that there exists a “***gauge correspondence***” between ***properties*** of physical systems and ***symmetries*** generated by these properties,

“Gaugification” of quantum mechanics

- The aim of this talk is to apply to quantum mechanics a fundamental idea coming from gauge theories, namely...
- ... that there exists a “***gauge correspondence***” between ***properties*** of physical systems and ***symmetries*** generated by these properties,
- This approach relies strongly on an analysis of the geometric formulations of mechanics, namely...
  - ... the *symplectic* formulation of classical mechanics,
  - ... and *geometric quantization* (J.-M. Souriau & B. Kostant).

Constrained Hamiltonian Systems

The first-class constraints

$$\{G_a, G_b\} = C^c_{ab} G_c,$$

- define the constraint surface Σ

$$G_a\left(q^i, p_i\right) = 0,$$

(pre-symplectic submanifold of the phase space P)

Constrained Hamiltonian Systems

The first-class constraints

$$\{G_a, G_b\} = C^c_{ab} G_c,$$

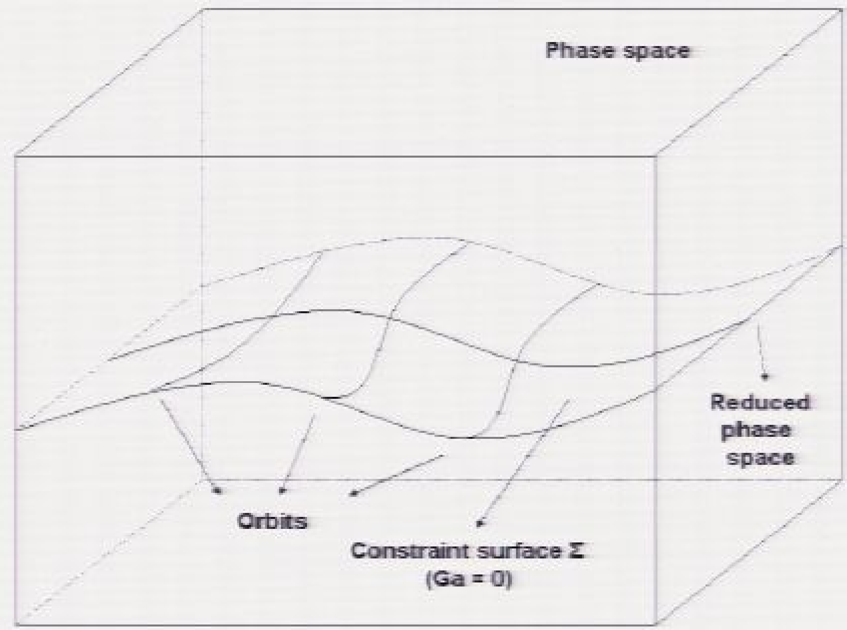
- define the constraint surface Σ

$$G_a\left(q^i, p_i\right) = 0,$$

(pre-symplectic submanifold of the phase space P)

- and generate gauge transf.

$$\delta_\epsilon F = \epsilon^a \{F, G_a\}$$



Constrained Hamiltonian Systems

The first-class constraints

$$\{G_a, G_b\} = C^c_{ab} G_c,$$

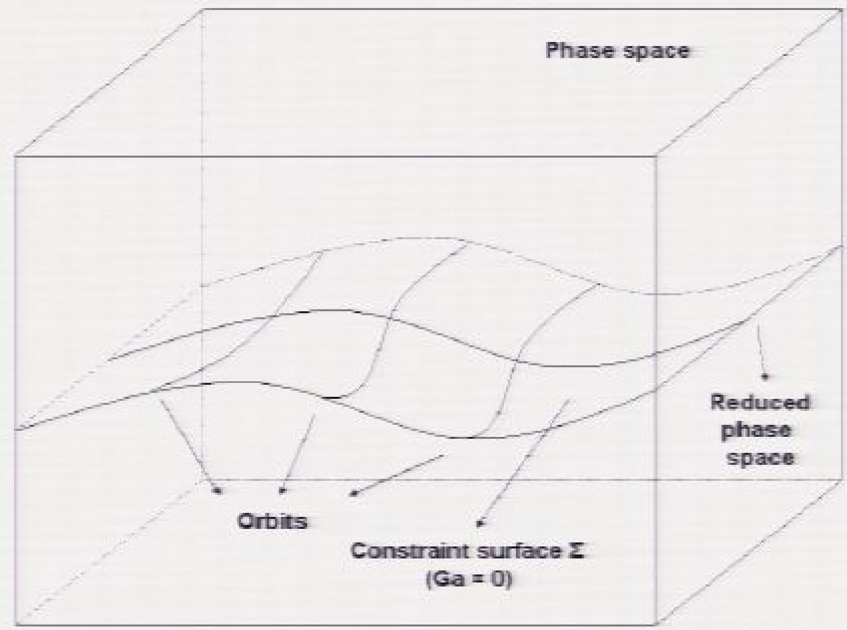
- define the constraint surface Σ

$$G_a\left(q^i, p_i\right) = 0,$$

(pre-symplectic submanifold of the phase space P)

- and generate gauge transf.

$$\delta_\epsilon F = \epsilon^a \{F, G_a\}$$



- The fact that the constraints are in involution means that the orbits generated by G_a remain in the constraint surface defined by G_b, which means that the gauge orbits define a null foliation of Σ (Frobenius' integrability condition).

Constrained Hamiltonian Systems

The first-class constraints

$$\{G_a, G_b\} = C^c_{ab} G_c,$$

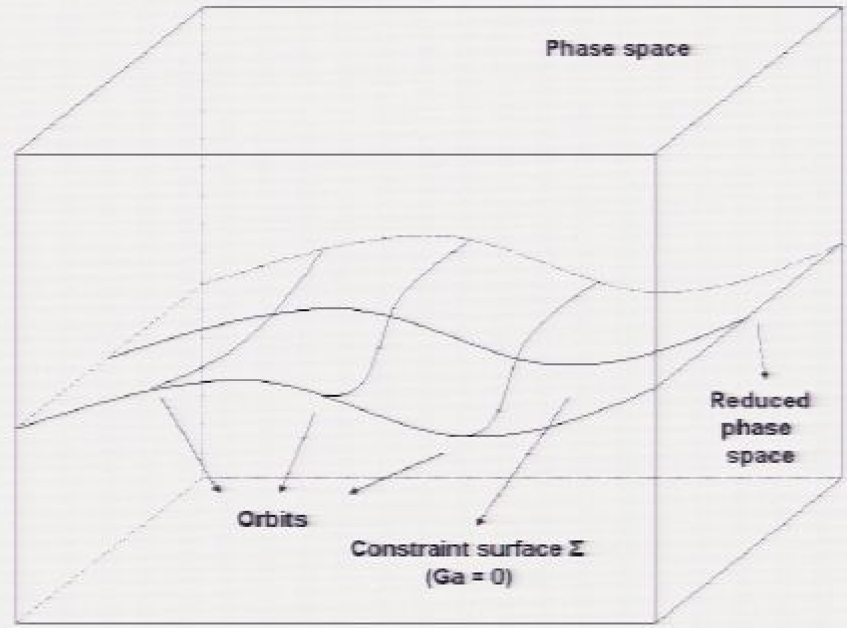
- define the constraint surface Σ

$$G_a\left(q^i, p_i\right) = 0,$$

(pre-symplectic submanifold of the phase space P)

- and generate gauge transf.

$$\delta_\epsilon F = \epsilon^a \{F, G_a\}$$



- The fact that the constraints are in involution means that the orbits generated by G_a remain in the constraint surface defined by G_b, which means that the gauge orbits define a null foliation of Σ (Frobenius' integrability condition).
- Hence, we can take the quotient of the constraint surface Σ by the gauge transformations. This quotient defines the reduced phase space P_{red}.

Observables

- The ***observables*** are functions on P_{red}, that is gauge invariant functions on Σ.

Observables

- The ***observables*** are functions on P_{red}, that is gauge invariant functions on Σ.
- Instead of working on P_{red}, it is generally more convenient to work in the original phase space...

1) by fixing the gauge, i.e. by selecting one representative in each gauge orbit.

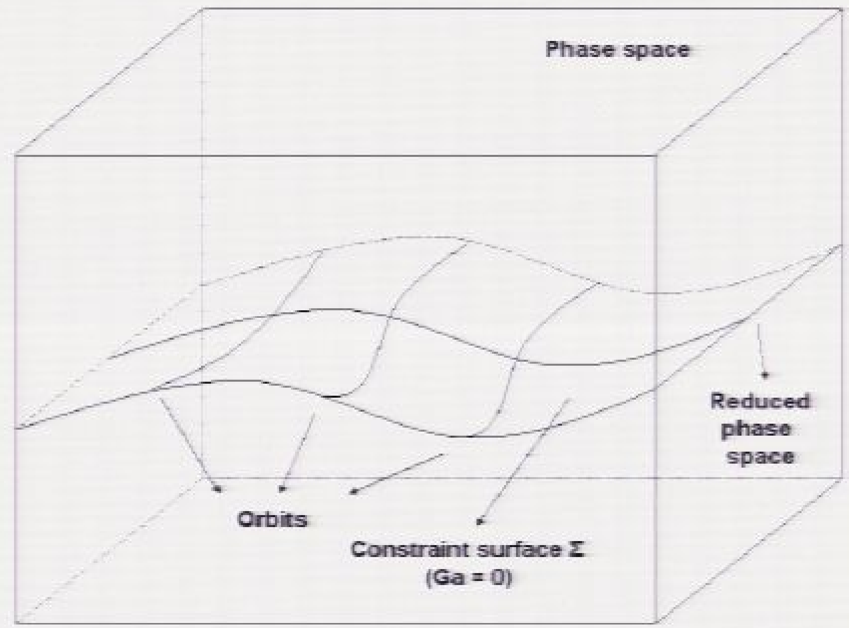
2) and by introducing a cohomological algorithm for identifying the observables, namely the BRST cohomology.

Restriction & projection

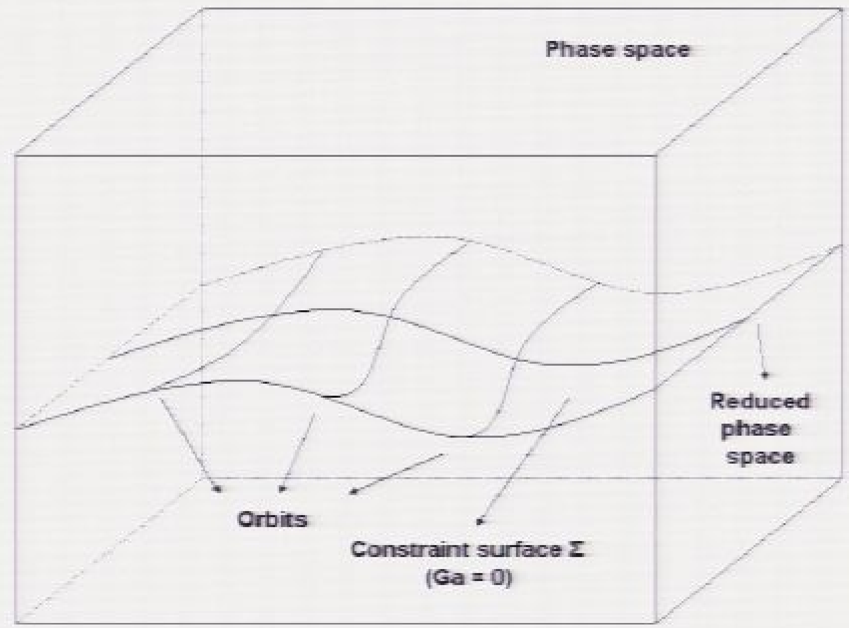
- If a state (q, p) satisfies the constraint

$$G_a(q, p) = 0,$$

we will say that G_a is a *property* of the state (q, p).



Restriction & projection



- If a state (q, p) satisfies the constraint

$$G_a(q, p) = 0,$$

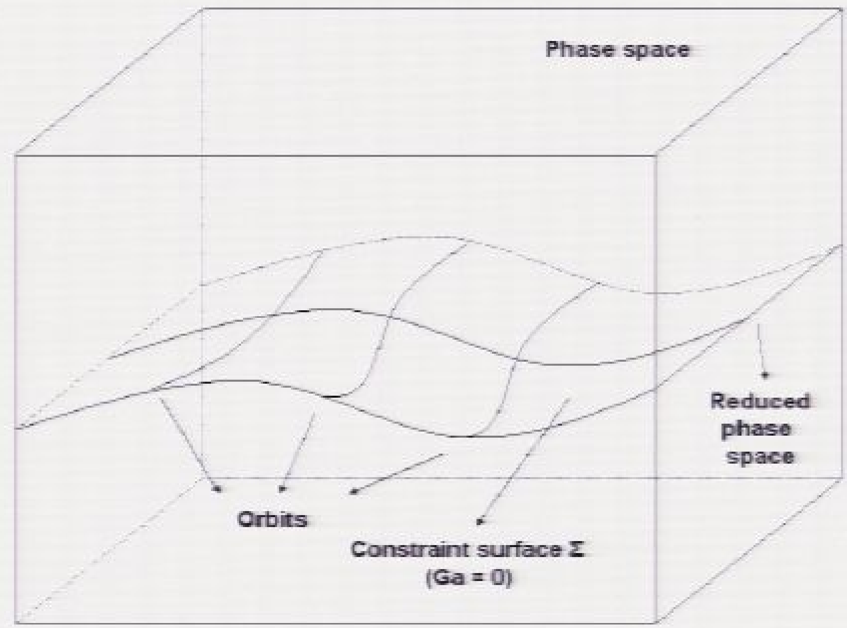
we will say that G_a is a *property* of the state (q, p).

- The attribution of properties strikes twice:

1) there is a ***restriction*** to the constraint surface,

2) and there is a ***projection*** to the orbit space defined by the gauge transf.

Restriction & projection



- If a state (q, p) satisfies the constraint

$$G_a(q, p) = 0,$$

we will say that G_a is a *property* of the state (q, p).

- The attribution of properties strikes twice:

  1) there is a ***restriction*** to the constraint surface,

  2) and there is a ***projection*** to the orbit space defined by the gauge transf.

- In other terms, "localization" to Σ entails the impossibility of localizing along the gauge orbits. Hence, there is a limit to the "complete" localization of states.

Gauge commutativity

- What is the obstruction to this “complete” localization or determination of states?

Gauge commutativity

- What is the obstruction to this "complete" localization or determination of states?
- Two constraints G_a and G_b can be both properties of the same states only if they commute (weakly):

$$\{G_a, G_b\} = C^c_{ab} G_c \approx 0.$$

Gauge commutativity

- What is the obstruction to this “complete” localization or determination of states?
- Two constraints G_a and G_b can be both properties of the same states only if they commute (weakly):

$$\{G_a, G_b\} = C^c_{ab} G_c \approx 0.$$

- Hence, non-commutativity is the obstruction to the attribution of different properties to the same state.

Gauge commutativity

- What is the obstruction to this “complete” localization or determination of states?
- Two constraints G_a and G_b can be both properties of the same states only if they commute (weakly):
$$\{G_a, G_b\} = C_{ab}^c G_c \approx 0.$$
- Hence, non-commutativity is the obstruction to the attribution of different properties to the same state.
- In other terms, if G_a is a property of a state (q, p), then (q, p) is defined modulo the gauge transformations generated by G_a.

Gauge commutativity

- What is the obstruction to this "complete" localization or determination of states?
- Two constraints G_a and G_b can be both properties of the same states only if they commute (weakly):
$$\{G_a, G_b\} = C^c_{ab} G_c \approx 0.$$
- Hence, non-commutativity is the obstruction to the attribution of different properties to the same state.
- In other terms, if G_a is a property of a state (q, p), then (q, p) is defined modulo the gauge transformations generated by G_a.
- Therefore, functions which are not invariant along the gauge orbits generated by G_a cannot also be properties of (q, p) (i.e. properties commute with the constraints).

Gauge commutativity

- What is the obstruction to this "complete" localization or determination of states?
- Two constraints G_a and G_b can be both properties of the same states only if they commute (weakly):
$$\{G_a, G_b\} = C^c_{ab} G_c \approx 0.$$
- Hence, non-commutativity is the obstruction to the attribution of different properties to the same state.
- In other terms, if G_a is a property of a state (q, p), then (q, p) is defined modulo the gauge transformations generated by G_a.
- Therefore, functions which are not invariant along the gauge orbits generated by G_a cannot also be properties of (q, p) (i.e. properties commute with the constraints).
- This imposes a sort of "non-commutative" limit to the process of determination of states: we can go on attributing properties to $(q, p) \Leftrightarrow$ the properties commute.

Gauge commutativity

- What is the obstruction to this "complete" localization or determination of states?
- Two constraints G_a and G_b can be both properties of the same states only if they commute (weakly):

$$\{G_a, G_b\} = C^c_{ab} G_c \approx 0.$$

- Hence, non-commutativity is the obstruction to the attribution of different properties to the same state.
- In other terms, if G_a is a property of a state (q, p), then (q, p) is defined modulo the gauge transformations generated by G_a.
- Therefore, functions which are not invariant along the gauge orbits generated by G_a cannot also be properties of (q, p) (i.e. properties commute with the constraints).
- This imposes a sort of "non-commutative" limit to the process of determination of states: we can go on attributing properties to $(q, p) \Leftrightarrow$ the properties commute.
- The fact that the observables do not permit to distinguish states belonging to the same gauge orbit does not mean that the theory is incomplete, i.e. that it could exist hypothetical hidden variables that would permit to distinguish gauge equivalent states.

Relevance for quantum mechanics

- According to the theory of constrained Hamiltonian systems, there exists a "***gauge correspondence***" between...

  properties (constraints) $\leftrightsquigarrow$ ***symmetries*** (gauge transformations),

  being non-commutativity the obstruction to the attribution of different properties to the same state.

Relevance for quantum mechanics

- According to the theory of constrained Hamiltonian systems, there exists a "***gauge correspondence***" between...

  properties (constraints) $\leftrightsquigarrow$ ***symmetries*** (gauge transformations),

  being non-commutativity the obstruction to the attribution of different properties to the same state.

- Symmetries reduce the amount of observable information that is necessary for describing a physical system.

Relevance for quantum mechanics

- According to the theory of constrained Hamiltonian systems, there exists a "***gauge correspondence***" between...

  $$\textit{\textbf{properties}} \text{ (constraints)} \leftrightsquigarrow \textit{\textbf{symmetries}} \text{ (gauge transformations)},$$

  being non-commutativity the obstruction to the attribution of different properties to the same state.

- Symmetries reduce the amount of observable information that is necessary for describing a physical system.

- On the other hand, q and p cannot be both "observables" of the same quantum system.

Relevance for quantum mechanics

- According to the theory of constrained Hamiltonian systems, there exists a "***gauge correspondence***" between...

  $$\textit{\textbf{properties}} \text{ (constraints)} \leftrightsquigarrow \textit{\textbf{symmetries}} \text{ (gauge transformations)},$$

  being non-commutativity the obstruction to the attribution of different properties to the same state.
- Symmetries reduce the amount of observable information that is necessary for describing a physical system.
- On the other hand, q and p cannot be both "observables" of the same quantum system.
- Just an idea... maybe the reason for this quantum "uncertainty" is that we forgot to take the quotient by a remanent "quantum" symmetry.

Relevance for quantum mechanics

- According to the theory of constrained Hamiltonian systems, there exists a "***gauge correspondence***" between...

  properties (constraints) $\leftrightsquigarrow$ ***symmetries*** (gauge transformations),

  being non-commutativity the obstruction to the attribution of different properties to the same state.

- Symmetries reduce the amount of observable information that is necessary for describing a physical system.

- On the other hand, q and p cannot be both "observables" of the same quantum system.

- Just an idea... maybe the reason for this quantum "uncertainty" is that we forgot to take the quotient by a remanent "quantum" symmetry.

- Roughly speaking... it might be the case that...

  Classical mechanics / "quantum" symmetry $=$ ***Quantum mechanics***.

Constraints vs. observable properties

- We will then try to generalize to unconstrained systems the *gauge correspondence* between *properties* defined by G_a and *symmetry transf.* generated by G_a.

Constraints vs. observable properties

- We will then try to generalize to unconstrained systems the *gauge correspondence* between *properties* defined by G_a and *symmetry transf.* generated by G_a.
- To do so, we will distinguish between...

  • ... the ***properties given by the constraints*** $G_a \in \mathcal{C}^\infty(P)$, which define the reduced subspace of possible physical states of the theory.

  • ... and the ***properties given by the observables*** $f(q,p) \in \mathcal{C}^\infty(P_{red})$, which permit us to select one possible solution among all possible physical solutions.

Constraints vs. observable properties

- We will then try to generalize to unconstrained systems the *gauge correspondence* between *properties* defined by G_a and *symmetry transf.* generated by G_a.

- To do so, we will distinguish between...

  • ... the ***properties given by the constraints*** $G_a \in \mathcal{C}^\infty(P)$, which define the reduced subspace of possible physical states of the theory.

  • ... and the ***properties given by the observables*** $f(q,p) \in \mathcal{C}^\infty(P_{red})$, which permit us to select one possible solution among all possible physical solutions.

- We will then claim that the gauge correspondence should be valid for both kinds of properties, i.e. not only for properties given by G_a, but also for the observable properties.

Constraints vs. observable properties

- We will then try to generalize to unconstrained systems the *gauge correspondence* between *properties* defined by G_a and *symmetry transf.* generated by G_a.
- To do so, we will distinguish between...

  • ... the ***properties given by the constraints*** $G_a \in \mathcal{C}^\infty(P)$, which define the reduced subspace of possible physical states of the theory.

  • ... and the ***properties given by the observables*** $f(q,p) \in \mathcal{C}^\infty(P_{red})$, which permit us to select one possible solution among all possible physical solutions.
- We will then claim that the gauge correspondence should be valid for both kinds of properties, i.e. not only for properties given by G_a, but also for the observable properties.
- It is worth noting that this is not the case in classical mechanics since observable properties do not generate symmetry transformations.

Constraints vs. observable properties

- We will then try to generalize to unconstrained systems the *gauge correspondence* between *properties* defined by G_a and *symmetry transf.* generated by G_a.
- To do so, we will distinguish between...

  • ... the ***properties given by the constraints*** $G_a \in \mathcal{C}^\infty(P)$, which define the reduced subspace of possible physical states of the theory.

  • ... and the ***properties given by the observables*** $f(q, p) \in \mathcal{C}^\infty(P_{red})$, which permit us to select one possible solution among all possible physical solutions.
- We will then claim that the gauge correspondence should be valid for both kinds of properties, i.e. not only for properties given by G_a, but also for the observable properties.
- It is worth noting that this is not the case in classical mechanics since observable properties do not generate symmetry transformations.
- In other terms, since any two distinct points in P_{red} are supposed to describe states which are physically different, there can be no symmetry acting on P_{red}.

Quantum symmetry

- The proposed generalization of the gauge correspondence to unconstrained systems will forbid localization in phase space even if there are no constraints.

Constraints vs. observable properties

- We will then try to generalize to unconstrained systems the *gauge correspondence* between *properties* defined by G_a and *symmetry transf.* generated by G_a.
- To do so, we will distinguish between...
  - ... the ***properties given by the constraints*** $G_a \in \mathcal{C}^\infty(P)$, which define the reduced subspace of possible physical states of the theory.
  - ... and the ***properties given by the observables*** $f(q,p) \in \mathcal{C}^\infty(P_{red})$, which permit us to select one possible solution among all possible physical solutions.
- We will then claim that the gauge correspondence should be valid for both kinds of properties, i.e. not only for properties given by G_a, but also for the observable properties.
- It is worth noting that this is not the case in classical mechanics since observable properties do not generate symmetry transformations.
- In other terms, since any two distinct points in P_{red} are supposed to describe states which are physically different, there can be no symmetry acting on P_{red}.

Quantum symmetry

- The proposed generalization of the gauge correspondence to unconstrained systems will forbid localization in phase space even if there are no constraints.

Quantum symmetry

- The proposed generalization of the gauge correspondence to unconstrained systems will forbid localization in phase space even if there are no constraints.

- The remanent “quantum symmetry” that forbids localization in an unconstrained (reduced) phase space P_{red} will be generated by the very (reduced) canonical variables q and p of P_{red}.

Quantum symmetry

- The proposed generalization of the gauge correspondence to unconstrained systems will forbid localization in phase space even if there are no constraints.
- The remanent "quantum symmetry" that forbids localization in an unconstrained (reduced) phase space P_{red} will be generated by the very (reduced) canonical variables q and p of P_{red}.
- In other terms, the main idea of this approach is that the conceptual justification of the uncertainty principle is somehow encoded in the very definitions of q and p.

Quantum symmetry

- The proposed generalization of the gauge correspondence to unconstrained systems will forbid localization in phase space even if there are no constraints.
- The remanent "quantum symmetry" that forbids localization in an unconstrained (reduced) phase space P_{red} will be generated by the very (reduced) canonical variables q and p of P_{red}.
- In other terms, the main idea of this approach is that the conceptual justification of the uncertainty principle is somehow encoded in the very definitions of q and p.
- So this is our minimal prescription for trying to understand quantum mechanics... let's simply try to understand what a momentum p is !!!

Quantum symmetry

- The proposed generalization of the gauge correspondence to unconstrained systems will forbid localization in phase space even if there are no constraints.
- The remanent "quantum symmetry" that forbids localization in an unconstrained (reduced) phase space P_{red} will be generated by the very (reduced) canonical variables q and p of P_{red}.
- In other terms, the main idea of this approach is that the conceptual justification of the uncertainty principle is somehow encoded in the very definitions of q and p.
- So this is our minimal prescription for trying to understand quantum mechanics... let's simply try to understand what a momentum p is !!!

What is mechanics?

- If the “quantum” symmetry does not stem from the existence of constraints... what could be the conceptual motivation for formulating this new symmetry principle?

What is mechanics?

- If the "quantum" symmetry does not stem from the existence of constraints... what could be the conceptual motivation for formulating this new symmetry principle?

What is mechanics?

- If the “quantum” symmetry does not stem from the existence of constraints... what could be the conceptual motivation for formulating this new symmetry principle?
- In order to answer, it might be necessary to go back to very basic questions, such as
  - What is mechanics?
  - What is an observable like the momentum p?

What is mechanics?

- If the "quantum" symmetry does not stem from the existence of constraints... what could be the conceptual motivation for formulating this new symmetry principle?

- In order to answer, it might be necessary to go back to very basic questions, such as

  - What is mechanics?
  - What is an observable like the momentum p?

- Usual "pre-relativistic" answers: mechanics is the study of the evolution of physical systems in time and p is related to $\dot{q}(t)$, i.e. to an effective physical motion in q.

What is mechanics?

- If the “quantum” symmetry does not stem from the existence of constraints... what could be the conceptual motivation for formulating this new symmetry principle?
- In order to answer, it might be necessary to go back to very basic questions, such as
  - What is mechanics?
  - What is an observable like the momentum p?
- Usual “pre-relativistic” answers: mechanics is the study of the evolution of physical systems in time and p is related to $\dot{q}(t)$, i.e. to an effective physical motion in q.
- These answers depend on the existence of a temporal structure, i.e. of an external parameter t (or an internal clock) and a conjugated Hamiltonian $h(q, p)$.

Temporal structures

- Nevertheless, a temporal structure is an additional layer of structure which is not encoded in the very definition of a symplectic manifold and its canonical variables q and p.

Temporal structures

- Nevertheless, a temporal structure is an additional layer of structure which is not encoded in the very definition of a symplectic manifold and its canonical variables q and p.

- Indeed, a temporal structure can be defined...

  - ... either by introducing an external parameter t and a Hamiltonian function $h(q, p)$ on phase space (for subsystems of the universe).

  - ... or by selecting an internal physical clock in a parameterized system with a Hamiltonian constraint of the form $H = p_0^2 - h^2(q, p) = 0$ (for cosmological systems).

Temporal structures

- Nevertheless, a temporal structure is an additional layer of structure which is not encoded in the very definition of a symplectic manifold and its canonical variables q and p.

- Indeed, a temporal structure can be defined...

  - ... either by introducing an external parameter t and a Hamiltonian function $h(q, p)$ on phase space (for subsystems of the universe).

  - ... or by selecting an internal physical clock in a parameterized system with a Hamiltonian constraint of the form $H = p_0^2 - h^2(q, p) = 0$ (for cosmological systems).

- Moreover, there are systems (e.g. the gravitational field) which do not have a (canonical) temporal structure.

Temporal structures

- Nevertheless, a temporal structure is an additional layer of structure which is not encoded in the very definition of a symplectic manifold and its canonical variables q and p.

- Indeed, a temporal structure can be defined...

  - ... either by introducing an external parameter t and a Hamiltonian function $h(q, p)$ on phase space (for subsystems of the universe).

  - ... or by selecting an internal physical clock in a parameterized system with a Hamiltonian constraint of the form $H = p_0^2 - h^2(q, p) = 0$ (for cosmological systems).

- Moreover, there are systems (e.g. the gravitational field) which do not have a (canonical) temporal structure.

- Hence, the answers to the questions *What is mechanics?* and *What is an observable like* p*?* cannot depend on the presupposition of a temporal structure.

Symplectic diffeomorphisms

- In order to answer these questions, we can begin by noting that the very definition of physical observables $f \in \mathcal{C}^\infty(M)$ is related to the action of a group (of symplectic diffeomorphisms) on phase space.

Temporal structures

- Nevertheless, a temporal structure is an additional layer of structure which is not encoded in the very definition of a symplectic manifold and its canonical variables q and p.

- Indeed, a temporal structure can be defined...

  - ... either by introducing an external parameter t and a Hamiltonian function $h(q,p)$ on phase space (for subsystems of the universe).

  - ... or by selecting an internal physical clock in a parameterized system with a Hamiltonian constraint of the form $H = p_0^2 - h^2(q,p) = 0$ (for cosmological systems).

- Moreover, there are systems (e.g. the gravitational field) which do not have a (canonical) temporal structure.

- Hence, the answers to the questions *What is mechanics?* and *What is an observable like p?* cannot depend on the presupposition of a temporal structure.

Symplectic diffeomorphisms

- In order to answer these questions, we can begin by noting that the very definition of physical observables $f \in \mathcal{C}^\infty(M)$ is related to the action of a group (of symplectic diffeomorphisms) on phase space.

Symplectic diffeomorphisms

- In order to answer these questions, we can begin by noting that the very definition of physical observables $f \in \mathcal{C}^\infty(M)$ is related to the action of a group (of symplectic diffeomorphisms) on phase space.
- To clarify this point we have to take into account that phase space is a symplectic manifold (M, ω), i.e. a manifold endowed with 2-form ω which is closed

$$d\omega = 0$$

and non-degenerate

$$\omega(X, \cdot) = 0 \Leftrightarrow X = 0.$$

Symplectic diffeomorphisms

- In order to answer these questions, we can begin by noting that the very definition of physical observables $f \in \mathcal{C}^\infty(M)$ is related to the action of a group (of symplectic diffeomorphisms) on phase space.
- To clarify this point we have to take into account that phase space is a symplectic manifold (M, ω), i.e. a manifold endowed with 2-form ω which is closed

$$d\omega = 0$$

and non-degenerate

$$\omega(X, \cdot) = 0 \Leftrightarrow X = 0.$$

- The symplectic structure ω induces a correspondence:

Observables $f \in \mathcal{C}^\infty(M) \mapsto$ *classical operators* $v_f \in TM.$

Symplectic diffeomorphisms

- In order to answer these questions, we can begin by noting that the very definition of physical observables $f \in \mathcal{C}^\infty(M)$ is related to the action of a group (of symplectic diffeomorphisms) on phase space.
- To clarify this point we have to take into account that phase space is a symplectic manifold (M, ω), i.e. a manifold endowed with 2-form ω which is closed

$$d\omega = 0$$

and non-degenerate

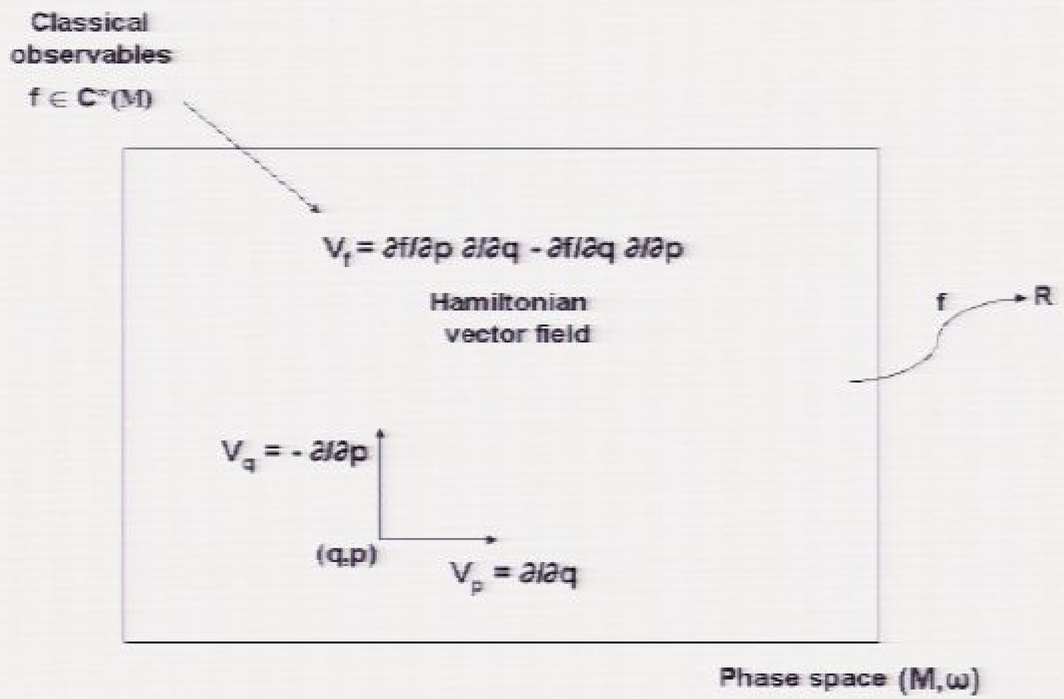
$$\omega(X, \cdot) = 0 \Leftrightarrow X = 0.$$

- The symplectic structure ω induces a correspondence:

Observables $f \in \mathcal{C}^\infty(M) \mapsto$ *classical operators* $v_f \in TM$.

- Classical operators $v_f \in TM$ generate *symplectic diffeomorphisms* of phase space (M, ω), i.e. diffeomorphisms of M that preserve the symplectic structure ω.

Symplectic mechanics



- Correspondence between *observables* and *classical operators* (Hamiltonian vector fields):

$$\begin{aligned} \omega^\sharp : T^*M \quad &\rightarrow \quad TM \\ df \quad &\mapsto \quad v_f = \frac{\partial f}{\partial p}\frac{\partial}{\partial q} - \frac{\partial f}{\partial q}\frac{\partial}{\partial p} \qquad \text{such that} \qquad \omega(v_f.\cdot) = df \end{aligned}$$

$$\mathcal{L}_{v_f}\omega = (di_{v_f} + i_{v_f}d)\omega = d^2 f = 0$$

Symplectic diffeomorphisms

- In order to answer these questions, we can begin by noting that the very definition of physical observables $f \in \mathcal{C}^\infty(M)$ is related to the action of a group (of symplectic diffeomorphisms) on phase space.
- To clarify this point we have to take into account that phase space is a symplectic manifold (M, ω), i.e. a manifold endowed with 2-form ω which is closed

$$d\omega = 0$$

and non-degenerate

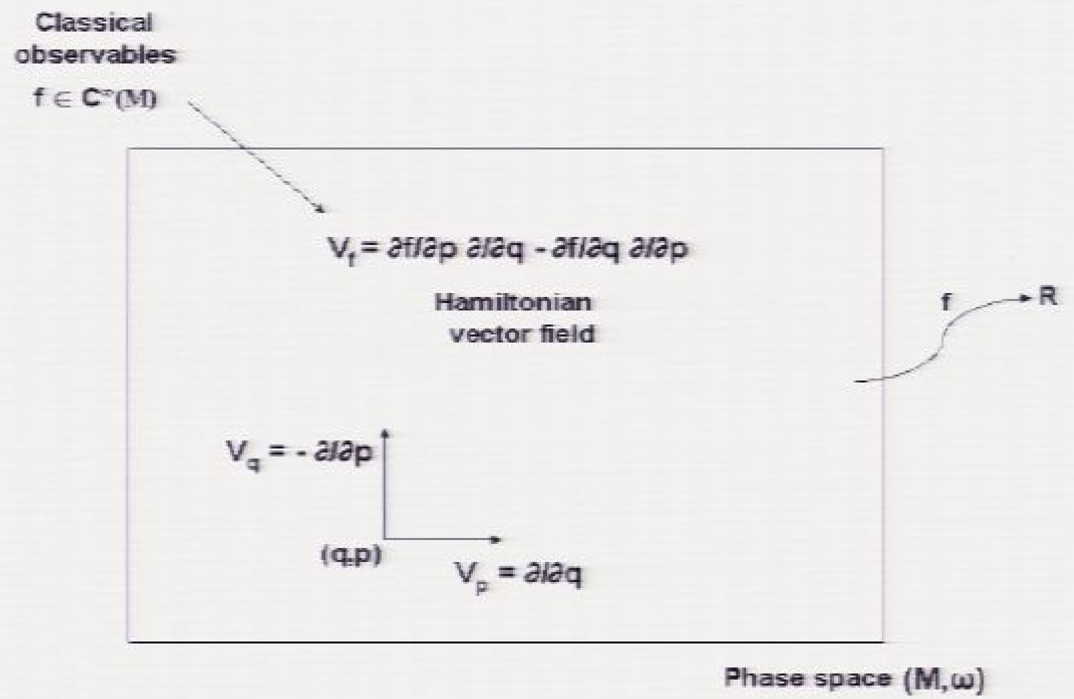
$$\omega(X, \cdot) = 0 \Leftrightarrow X = 0.$$

- The symplectic structure ω induces a correspondence:

Observables $f \in \mathcal{C}^\infty(M) \mapsto$ *classical operators* $v_f \in TM$.

- Classical operators $v_f \in TM$ generate *symplectic diffeomorphisms* of phase space (M, ω), i.e. diffeomorphisms of M that preserve the symplectic structure ω.

Symplectic mechanics



- Correspondence between *observables* and *classical operators* (Hamiltonian vector fields):

$$\begin{aligned} \omega^\sharp : T^*M \quad &\rightarrow \quad TM \\ df \quad &\mapsto \quad v_f = \frac{\partial f}{\partial p}\frac{\partial}{\partial q} - \frac{\partial f}{\partial q}\frac{\partial}{\partial p} \end{aligned} \quad \text{such that} \quad \omega(v_f, \cdot) = df$$

$$\mathcal{L}_{v_f}\omega = (di_{v_f} + i_{v_f}d)\omega = d^2 f = 0$$

Momenta vs. velocities

- In particular, p generates canonical transformations of q through the action of the classical operator $v_p = \frac{\partial}{\partial q} \in TM$:

$$\mathcal{L}ie_{v_p} q = \{q, p\} = 1.$$

Momenta vs. velocities

- In particular, p generates canonical transformations of q through the action of the classical operator $v_p = \frac{\partial}{\partial q} \in TM$:

$$\mathcal{L}ie_{v_p} q = \{q, p\} = 1.$$

- If one can define a Hamiltonian $h(q, p)$ and a physical clock t, then the momentum p can be related to the velocity $\dot{q}(t)$ through Hamilton's equation $\dot{q}(t) = -\frac{\partial h(q,p)}{\partial p}$.

Momenta vs. velocities

- In particular, p generates canonical transformations of q through the action of the classical operator $v_p = \frac{\partial}{\partial q} \in TM$:

$$\mathcal{L}ie_{v_p} q = \{q, p\} = 1.$$

- If one can define a Hamiltonian $h(q, p)$ and a physical clock t, then the momentum p can be related to the velocity $\dot{q}(t)$ through Hamilton's equation $\dot{q}(t) = -\frac{\partial h(q,p)}{\partial p}$.
- However, it is only through the mediation of the Hamiltonian $h(q, p)$ that the canonical transf. of q generated by p becomes a transf. of q *in time*, i.e. a physical motion.

Momenta vs. velocities

- In particular, p generates canonical transformations of q through the action of the classical operator $v_p = \frac{\partial}{\partial q} \in TM$:

$$\mathcal{L}ie_{v_p} q = \{q, p\} = 1.$$

- If one can define a Hamiltonian $h(q,p)$ and a physical clock t, then the momentum p can be related to the velocity $\dot{q}(t)$ through Hamilton's equation $\dot{q}(t) = -\frac{\partial h(q,p)}{\partial p}$.
- However, it is only through the mediation of the Hamiltonian $h(q,p)$ that the canonical transf. of q generated by p becomes a transf. of q *in time*, i.e. a physical motion.
- If we abstract the existence of $h(q,p)$, we cannot pass from canonical transformations to effective physical motions.

Momenta vs. velocities

- In particular, p generates canonical transformations of q through the action of the classical operator $v_p = \frac{\partial}{\partial q} \in TM$:

$$\mathcal{L}ie_{v_p} q = \{q, p\} = 1.$$

- If one can define a Hamiltonian $h(q,p)$ and a physical clock t, then the momentum p can be related to the velocity $\dot{q}(t)$ through Hamilton's equation $\dot{q}(t) = -\frac{\partial h(q,p)}{\partial p}$.
- However, it is only through the mediation of the Hamiltonian $h(q,p)$ that the canonical transf. of q generated by p becomes a transf. of q *in time*, i.e. a physical motion.
- If we abstract the existence of $h(q,p)$, we cannot pass from canonical transformations to effective physical motions.
- In other terms, the intrinsic definition of p as a generator of canonical transf. of q does not make any reference to temporal processes.

Momenta vs. velocities

- In particular, p generates canonical transformations of q through the action of the classical operator $v_p = \frac{\partial}{\partial q} \in TM$:

$$\mathcal{L}ie_{v_p} q = \{q, p\} = 1.$$

- If one can define a Hamiltonian $h(q,p)$ and a physical clock t, then the momentum p can be related to the velocity $\dot{q}(t)$ through Hamilton's equation $\dot{q}(t) = -\frac{\partial h(q,p)}{\partial p}$.
- However, it is only through the mediation of the Hamiltonian $h(q,p)$ that the canonical transf. of q generated by p becomes a transf. of q *in time*, i.e. a physical motion.
- If we abstract the existence of $h(q,p)$, we cannot pass from canonical transformations to effective physical motions.
- In other terms, the intrinsic definition of p as a generator of canonical transf. of q does not make any reference to temporal processes.
- However, what information about the state (q,p) is conveyed by p if it is not related to the velocity of an effective physical motion?

What does p measure?

- Indeed, if we want to *define* p as a quantity that generates canonical transf. of q, there should exist a relationship between the particular value of p and the transf. that it generates.

What does p measure?

- Indeed, if we want to *define* p as a quantity that generates canonical transf. of q, there should exist a relationship between the particular value of p and the transf. that it generates.

- In other terms, what information about the canonical transf. of q is conveyed by the particular value of p?

What does p measure?

- Indeed, if we want to *define* p as a quantity that generates canonical transf. of q, there should exist a relationship between the particular value of p and the transf. that it generates.

- In other terms, what information about the canonical transf. of q is conveyed by the particular value of p?

- If the particular value of a velocity $\dot{q}$ conveys information about the transformation of q in time...

What does p measure?

- Indeed, if we want to *define* p as a quantity that generates canonical transf. of q, there should exist a relationship between the particular value of p and the transf. that it generates.

- In other terms, what information about the canonical transf. of q is conveyed by the particular value of p?

- If the particular value of a velocity $\dot{q}$ conveys information about the transformation of q in time...

- ... we could in principle expect that the particular value of p conveys information about some kind of "virtual velocity" associated to the canonical transformations of q.

Momenta as “virtual velocities”

- However, this is not the case in classical mechanics.

Momenta as “virtual velocities”

- However, this is not the case in classical mechanics.
- In other terms, the canonical transf. of q generated by p does not depend on the value of p. Indeed, p and $p' = p + k$ define the same transf. of q:

$$\mathcal{L}ie_{v_{p+k}} q = \{q, p + k\} = \{q, p\} = 1.$$

Momenta as “virtual velocities”

- However, this is not the case in classical mechanics.

- In other terms, the canonical transf. of q generated by p does not depend on the value of p. Indeed, p and $p' = p + k$ define the same transf. of q:

$$\mathcal{L}ie_{v_{p+k}} q = \{q, p+k\} = \{q, p\} = 1.$$

- This is a consequence of the fact that the classical correspondence between observables and classical operators is not injective (or faithful)

$$f(q,p) = k \in \mathbb{R} \quad \mapsto \quad v_k = \frac{\partial k}{\partial p}\frac{\partial}{\partial q} - \frac{\partial k}{\partial q}\frac{\partial}{\partial p} = 0.$$

Momenta as "virtual velocities"

- However, this is not the case in classical mechanics.
- In other terms, the canonical transf. of q generated by p does not depend on the value of p. Indeed, p and $p' = p + k$ define the same transf. of q:

$$\mathcal{L}ie_{v_{p+k}} q = \{q, p + k\} = \{q, p\} = 1.$$

- This is a consequence of the fact that the classical correspondence between observables and classical operators is not injective (or faithful)

$$f(q,p) = k \in \mathbb{R} \quad \mapsto \quad v_k = \frac{\partial k}{\partial p}\frac{\partial}{\partial q} - \frac{\partial k}{\partial q}\frac{\partial}{\partial p} = 0.$$

- In other terms, the correspondence between the Poisson algebra of observables and the Lie algebra of classical operators is not a Lie algebra isomorphism.

Classical operators commute...

- It is easy to see that the fact that the canonical transf. generated by v_p does not depend on the particular value of p is a consequence of the commutativity of v_p and v_q.
  - Indeed, since the transition from p to $p' = p + k$ is generated by $v_q = -\frac{\partial}{\partial p}$...
  - ... the variation of v_p under a variation of p (generated by v_q) is given by

$$\mathcal{L}ie_{v_q} v_p = [v_p, v_q] = v_{\{q,p\}} = v_1 = 0.$$

Classical operators commute...

- It is easy to see that the fact that the canonical transf. generated by v_p does not depend on the particular value of p is a consequence of the commutativity of v_p and v_q.
  - Indeed, since the transition from p to $p' = p + k$ is generated by $v_q = -\frac{\partial}{\partial p}$...
  - ... the variation of v_p under a variation of p (generated by v_q) is given by

$$\mathcal{L}ie_{v_q} v_p = [v_p, v_q] = v_{\{q,p\}} = v_1 = 0.$$

- The commutativity of v_p and v_q is then the obstruction that prevents us from defining observables as properties that quantify *faithfully* virtual transf.

Classical operators commute...

- It is easy to see that the fact that the canonical transf. generated by v_p does not depend on the particular value of p is a consequence of the commutativity of v_p and v_q.
  - Indeed, since the transition from p to $p' = p + k$ is generated by $v_q = -\frac{\partial}{\partial p}$...
  - ... the variation of v_p under a variation of p (generated by v_q) is given by

$$\mathcal{L}ie_{v_q} v_p = [v_p, v_q] = v_{\{q,p\}} = v_1 = 0.$$

- The commutativity of v_p and v_q is then the obstruction that prevents us from defining observables as properties that quantify *faithfully* virtual transf.
- If we still want to interpret p as a "virtual velocity" that quantifies *faithfully* the virtual (timeless) transformations of q...
  - ... we have to force the non-commutativity of the operators associated to q and p,
  - and we have to provide a physical interpretation of theses virtual transf.

Momenta as "virtual velocities"

- However, this is not the case in classical mechanics.
- In other terms, the canonical transf. of q generated by p does not depend on the value of p. Indeed, p and $p' = p + k$ define the same transf. of q:

$$\mathcal{L}ie_{v_{p+k}} q = \{q, p + k\} = \{q, p\} = 1.$$

- This is a consequence of the fact that the classical correspondence between observables and classical operators is not injective (or faithful)

$$f(q,p) = k \in \mathbb{R} \quad \mapsto \quad v_k = \frac{\partial k}{\partial p}\frac{\partial}{\partial q} - \frac{\partial k}{\partial q}\frac{\partial}{\partial p} = 0.$$

- In other terms, the correspondence between the Poisson algebra of observables and the Lie algebra of classical operators is not a Lie algebra isomorphism.

Classical operators commute...

- It is easy to see that the fact that the canonical transf. generated by v_p does not depend on the particular value of p is a consequence of the commutativity of v_p and v_q.
  - Indeed, since the transition from p to $p' = p + k$ is generated by $v_q = -\frac{\partial}{\partial p}$...
  - ... the variation of v_p under a variation of p (generated by v_q) is given by

$$\mathcal{L}ie_{v_q} v_p = [v_p, v_q] = v_{\{q,p\}} = v_1 = 0.$$

Classical operators commute...

- It is easy to see that the fact that the canonical transf. generated by v_p does not depend on the particular value of p is a consequence of the commutativity of v_p and v_q.
  - Indeed, since the transition from p to $p' = p + k$ is generated by $v_q = -\frac{\partial}{\partial p}$...
  - ... the variation of v_p under a variation of p (generated by v_q) is given by

$$\mathcal{L}ie_{v_q} v_p = [v_p, v_q] = v_{\{q,p\}} = v_1 = 0.$$

- The commutativity of v_p and v_q is then the obstruction that prevents us from defining observables as properties that quantify *faithfully* virtual transf.

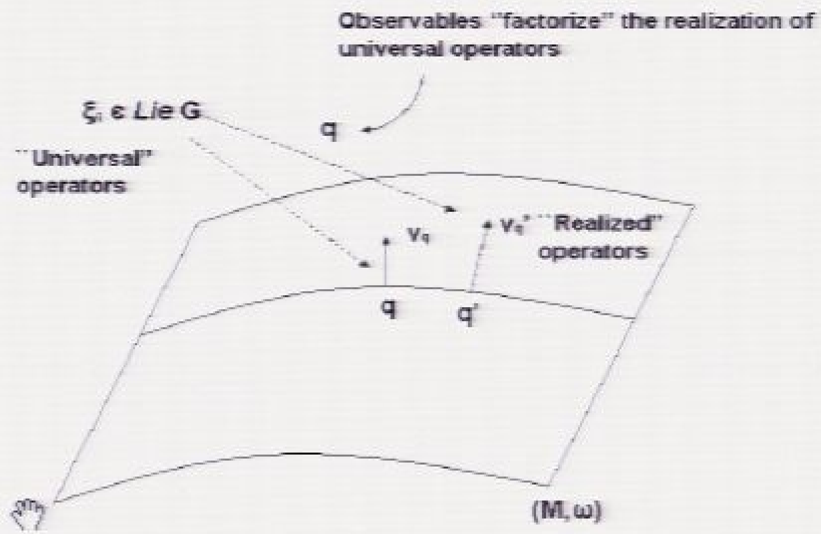
Classical operators commute...

- It is easy to see that the fact that the canonical transf. generated by v_p does not depend on the particular value of p is a consequence of the commutativity of v_p and v_q.
  - Indeed, since the transition from p to $p' = p + k$ is generated by $v_q = -\frac{\partial}{\partial p}$...
  - ... the variation of v_p under a variation of p (generated by v_q) is given by

$$\mathcal{L}ie_{v_q} v_p = [v_p, v_q] = v_{\{q,p\}} = v_1 = 0.$$

- The commutativity of v_p and v_q is then the obstruction that prevents us from defining observables as properties that quantify *faithfully* virtual transf.

Classical realization

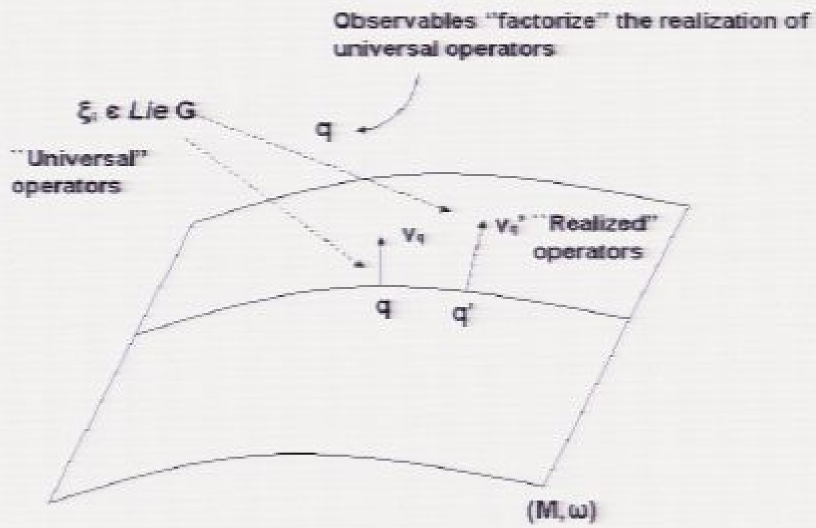


- In classical mechanics, a non-trivial transf. of p does not affect the classical operator v_p:

$$\mathcal{L}ie_{v_q} p = \{p, q\} = -1$$

$$\mathcal{L}ie_{v_q} v_p = [v_p, v_q] = v_{\{p,q\}} = v_{-1} = 0$$

Classical realization



- In classical mechanics, a non-trivial transf. of p does not affect the classical operator v_p:

$$\mathcal{L}ie_{v_q} p = \{p, q\} = -1$$

$$\mathcal{L}ie_{v_q} v_p = [v_p, v_q] = v_{\{p,q\}} = v_{-1} = 0$$

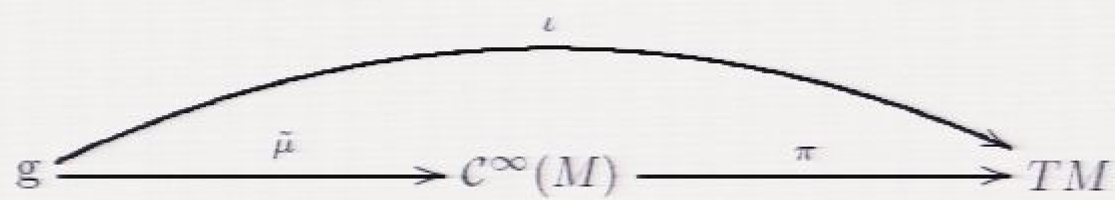
- Hence, states with different p do not necessarily realize differently the universal operator ξ_p.

Co-Momentum Map

- The symplectic action is *Hamiltonian* if there exists a map

$$\begin{aligned} \tilde{\mu} : \mathrm{g} &\rightarrow \mathcal{C}^\infty(M) \\ \xi &\mapsto f_\xi \end{aligned}$$

(called *co-momentum map*) such that the following diagram commutes



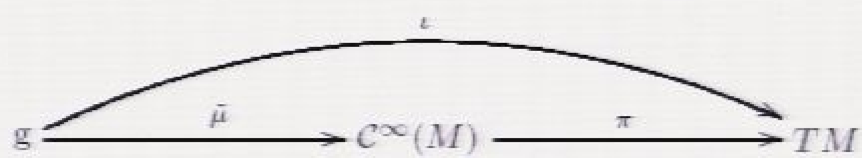
Universal operator $\xi \longrightarrow$ Observable $f_\xi \longrightarrow$ Realized operator v_{f_ξ}

Co-Momentum Map

- The symplectic action is *Hamiltonian* if there exists a map

$$\begin{aligned} \tilde{\mu} : \mathrm{g} &\rightarrow \mathcal{C}^{\infty}(M) \\ \xi &\mapsto f_{\xi} \end{aligned}$$

(called *co-momentum map*) such that the following diagram commutes



Universal operator ξ ⟶ Observable f_{ξ} ⟶ Realized operator $v_{f_{\xi}}$

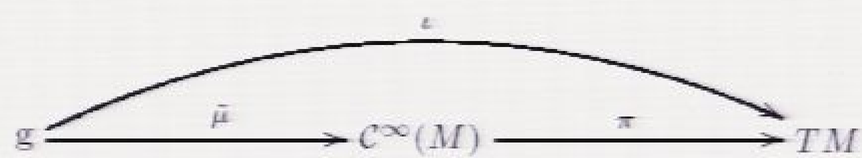
Adobe Acrobat Professional - [Quantum%20foundations%20-%20Perimeter%20Institut%202009%20%28IV%29[1].pdf]

Co-Momentum Map

- The symplectic action is *Hamiltonian* if there exists a map

$$\tilde{\mu} : \mathfrak{g} \rightarrow \mathcal{C}^\infty(M)$$
$$\xi \mapsto f_\xi$$

(called *co-momentum map*) such that the following diagram commutes



Universal operator ξ ⟶ Observable f_ξ ⟶ Realized operator v_{f_ξ}

– p. 78/

Classical operators commute...

- It is easy to see that the fact that the canonical transf. generated by v_p does not depend on the particular value of p is a consequence of the commutativity of v_p and v_q.
  - Indeed, since the transition from p to $p' = p + k$ is generated by $v_q = -\frac{\partial}{\partial p}$...
  - ... the variation of v_p under a variation of p (generated by v_q) is given by

$$\mathcal{L}ie_{v_q} v_p = [v_p, v_q] = v_{\{q,p\}} = v_1 = 0.$$

- The commutativity of v_p and v_q is then the obstruction that prevents us from defining observables as properties that quantify *faithfully* virtual transf.
- If we still want to interpret p as a "virtual velocity" that quantifies *faithfully* the virtual (timeless) transformations of q...
  - ... we have to force the non-commutativity of the operators associated to q and p,
  - and we have to provide a physical interpretation of theses virtual transf.

Let's summarize...

- To define the observables of a gauge theory, one has to take the quotient by gauge symmetries.

Let's summarize...

- To define the observables of a gauge theory, one has to take the quotient by gauge symmetries.
- Since q and p cannot be both "observables" of the same quantum system...

Let's summarize...

- To define the observables of a gauge theory, one has to take the quotient by gauge symmetries.
- Since q and p cannot be both "observables" of the same quantum system...
- ... we have proposed the hypothesis according to which maybe we have forgotten to take the quotient by a remanent "quantum" symmetry (which can only rely on the very definitions of q and p).

Let's summarize...

- To define the observables of a gauge theory, one has to take the quotient by gauge symmetries.
- Since q and p cannot be both "observables" of the same quantum system...
- ... we have proposed the hypothesis according to which maybe we have forgotten to take the quotient by a remanent "quantum" symmetry (which can only rely on the very definitions of q and p).
- Hence, we want to generalize the gauge correspondence between constraints and symmetries to the physical observables of unconstrained theories.

Let's summarize...

- To define the observables of a gauge theory, one has to take the quotient by gauge symmetries.
- Since q and p cannot be both "observables" of the same quantum system...
- ... we have proposed the hypothesis according to which maybe we have forgotten to take the quotient by a remanent "quantum" symmetry (which can only rely on the very definitions of q and p).
- Hence, we want to generalize the gauge correspondence between constraints and symmetries to the physical observables of unconstrained theories.
- And in fact physical observables $f \in C^\infty(M)$ do generate symplectic diffeomorphisms of (M, ω). In particular, p is the generator of canonical transformations of q.

Let's summarize...

- To define the observables of a gauge theory, one has to take the quotient by gauge symmetries.
- Since q and p cannot be both "observables" of the same quantum system...
- ... we have proposed the hypothesis according to which maybe we have forgotten to take the quotient by a remanent "quantum" symmetry (which can only rely on the very definitions of q and p).
- Hence, we want to generalize the gauge correspondence between constraints and symmetries to the physical observables of unconstrained theories.
- And in fact physical observables $f \in C^\infty(M)$ do generate symplectic diffeomorphisms of (M, ω). In particular, p is the generator of canonical transformations of q.
- However, the presupposition of a temporal structure (mis)leads us naturally to interpret these transf. generated by p in terms of velocities $\dot{q}$ and effective physical motions.

Let's summarize...

- To define the observables of a gauge theory, one has to take the quotient by gauge symmetries.
- Since q and p cannot be both "observables" of the same quantum system...
- ... we have proposed the hypothesis according to which maybe we have forgotten to take the quotient by a remanent "quantum" symmetry (which can only rely on the very definitions of q and p).
- Hence, we want to generalize the gauge correspondence between constraints and symmetries to the physical observables of unconstrained theories.
- And in fact physical observables $f \in C^\infty(M)$ do generate symplectic diffeomorphisms of (M, ω). In particular, p is the generator of canonical transformations of q.
- However, the presupposition of a temporal structure (mis)leads us naturally to interpret these transf. generated by p in terms of velocities $\dot{q}$ and effective physical motions.
- If we abstract this temporal structure, the fact remains that p generates "virtual" (non-temporal) transf. of q that could be interpreted as symmetry transf.

Problems

- Nevertheless, we have encountered a certain number of problems, namely...

  1) The fact that v_q and v_p commute prevents us from interpreting p as a kind of "virtual" (or timeless) velocity that quantifies *faithfully* the virtual transf. of q.

  In other terms, the particular value of p does not convey any information about the canonical transf. generated by p.

  2) We do not know what the physical meaning of these virtual transf. could be.

Problems

- Nevertheless, we have encountered a certain number of problems, namely...

  1) The fact that v_q and v_p commute prevents us from interpreting p as a kind of "virtual" (or timeless) velocity that quantifies *faithfully* the virtual transf. of q.

  In other terms, the particular value of p does not convey any information about the canonical transf. generated by p.

  2) We do not know what the physical meaning of these virtual transf. could be.

- In order to solve these problems, we will propose two postulates:

  • **Quantum Postulate**, which will permit us to provide a timeless definition of p as a quantity that quantifies *faithfully* the virtual transformations of q.

  • **Gauge Postulate**, which will provide a physical interpretation of these virtual transformations.

Problems

- Nevertheless, we have encountered a certain number of problems, namely...

  1) The fact that v_q and v_p commute prevents us from interpreting p as a kind of "virtual" (or timeless) velocity that quantifies *faithfully* the virtual transf. of q.

  In other terms, the particular value of p does not convey any information about the canonical transf. generated by p.

  2) We do not know what the physical meaning of these virtual transf. could be.

- In order to solve these problems, we will propose two postulates:

  • **Quantum Postulate**, which will permit us to provide a timeless definition of p as a quantity that quantifies *faithfully* the virtual transformations of q.

  • **Gauge Postulate**, which will provide a physical interpretation of these virtual transformations.

- In order to introduce them we will first present in rough terms the ontology of physical objects that stems from these postulates. It is worth stressing that this ontology matches to a certain extent an intuitive "phenomenological" conception about objects.

Problems

- Nevertheless, we have encountered a certain number of problems, namely...

  1) The fact that v_q and v_p commute prevents us from interpreting p as a kind of "virtual" (or timeless) velocity that quantifies *faithfully* the virtual transf. of q.

  In other terms, the particular value of p does not convey any information about the canonical transf. generated by p.

  2) We do not know what the physical meaning of these virtual transf. could be.

- In order to solve these problems, we will propose two postulates:

  • **Quantum Postulate**, which will permit us to provide a timeless definition of p as a quantity that quantifies *faithfully* the virtual transformations of q.

  • **Gauge Postulate**, which will provide a physical interpretation of these virtual transformations.

- In order to introduce them we will first present in rough terms the ontology of physical objects that stems from these postulates. It is worth stressing that this ontology matches to a certain extent an intuitive "phenomenological" conception about objects.

What is an object?

- An ***object*** is a physical configuration

♠ that has different ***phases***, ***aspects*** or ***profiles***,

♠ such that the different phases are related by means of certain ***phase transformations***,

♠ and which is defined by a set of ***objective properties*** that permit us to identify and recognize the object in spite of the variations of its phases.

What is an object?

- An ***object*** is a physical configuration

  ♠ that has different ***phases***, ***aspects*** or ***profiles***,

  ♠ such that the different phases are related by means of certain ***phase transformations***,

  ♠ and which is defined by a set of ***objective properties*** that permit us to identify and recognize the object in spite of the variations of its phases.

- This means that the objective properties must be by definition invariant under phase transformations

What is an object?

- An ***object*** is a physical configuration

  ♠ that has different ***phases***, ***aspects*** or ***profiles***,

  ♠ such that the different phases are related by means of certain ***phase transformations***,

  ♠ and which is defined by a set of ***objective properties*** that permit us to identify and recognize the object in spite of the variations of its phases.

- This means that the objective properties must be by definition invariant under phase transformations

- To summarize... there is no objective entity without an invariant identity (Quine), nor without different “phenomenological” phases (Husserl).

Phase transformations

- The notion of phase transformations...
  - ... is not directly related to temporal processes,
  - ... and it defines a *universal* notion of invariance which stems from the very notion of a generic object, independent of the particular symmetries that it may have.

Phase transformations

- The notion of phase transformations...
  - ... is not directly related to temporal processes,
  - ... and it defines a *universal* notion of invariance which stems from the very notion of a generic object, independent of the particular symmetries that it may have.
- Indeed, phase transformations arise from the fact that *any* physical system is, independent of its particular symmetries, a *multifaceted object*.

Phase transformations

- The notion of phase transformations...
  - ... is not directly related to temporal processes,
  - ... and it defines a *universal* notion of invariance which stems from the very notion of a generic object, independent of the particular symmetries that it may have.
- Indeed, phase transformations arise from the fact that *any* physical system is, independent of its particular symmetries, a *multifaceted object*.
- It is worth stressing that the fact that a phase is not an objective property does not mean that different phases cannot be distinguished.

Phase transformations

- The notion of phase transformations...
  - ... is not directly related to temporal processes,
  - ... and it defines a *universal* notion of invariance which stems from the very notion of a generic object, independent of the particular symmetries that it may have.
- Indeed, phase transformations arise from the fact that *any* physical system is, independent of its particular symmetries, a *multifaceted object*.
- It is worth stressing that the fact that a phase is not an objective property does not mean that different phases cannot be distinguished.
- For instance, the different numbers of a die permit us to distinguish its different non-objective phases. This is why the toss of a die can produce different results.

Quantum postulate

- **Quantum postulate**: the correspondence between observables and operators has to be an isomorphism of Lie algebras...

Phase transformations

- The notion of phase transformations...
  - ... is not directly related to temporal processes,
  - ... and it defines a *universal* notion of invariance which stems from the very notion of a generic object, independent of the particular symmetries that it may have.
- Indeed, phase transformations arise from the fact that *any* physical system is, independent of its particular symmetries, a *multifaceted object*.
- It is worth stressing that the fact that a phase is not an objective property does not mean that different phases cannot be distinguished.
- For instance, the different numbers of a die permit us to distinguish its different non-objective phases. This is why the toss of a die can produce different results.

Quantum postulate

- **Quantum postulate**: the correspondence between observables and operators has to be an isomorphism of Lie algebras...

Quantum postulate

- **Quantum postulate**: the correspondence between observables and operators has to be an isomorphism of Lie algebras...
- ... which implies in particular that the operators associated to q and p no longer commute.

Quantum postulate

- **Quantum postulate**: the correspondence between observables and operators has to be an isomorphism of Lie algebras...
- ... which implies in particular that the operators associated to q and p no longer commute.
- This postulate guarantees that the particular value of an objective property p quantifies *faithfully*, as a kind of "virtual velocity", the corresponding timeless transf. of q.

Quantum postulate

- **Quantum postulate**: the correspondence between observables and operators has to be an isomorphism of Lie algebras...
- ... which implies in particular that the operators associated to q and p no longer commute.
- This postulate guarantees that the particular value of an objective property p quantifies *faithfully*, as a kind of "virtual velocity", the corresponding timeless transf. of q.
- For instance, the values of the vertical angular momentum of the quantum objects $|L_z = \frac{1}{2}\rangle$ and $|L_z = 1\rangle$ specify faithfully how the objects transform *differently* under the same *universal* rotation $\theta = 2\pi$ around the vertical axe:

$$e^{i\hat{L}_z 2\pi/\hbar}|\frac{1}{2}\rangle = e^{i\pi}|\frac{1}{2}\rangle = -|\frac{1}{2}\rangle \quad \text{“slower” virtual velocity}$$

$$e^{i\hat{L}_z 2\pi/\hbar}|1\rangle = e^{i2\pi}|1\rangle = |1\rangle \quad \text{“faster” virtual velocity.}$$

Quantum postulate

- **Quantum postulate**: the correspondence between observables and operators has to be an isomorphism of Lie algebras...
- ... which implies in particular that the operators associated to q and p no longer commute.
- This postulate guarantees that the particular value of an objective property p quantifies *faithfully*, as a kind of "virtual velocity", the corresponding timeless transf. of q.
- For instance, the values of the vertical angular momentum of the quantum objects $|L_z = \frac{1}{2}\rangle$ and $|L_z = 1\rangle$ specify faithfully how the objects transform *differently* under the same *universal* rotation $\theta = 2\pi$ around the vertical axe:

$$e^{i\hat{L}_z 2\pi/\hbar}|\frac{1}{2}\rangle = e^{i\pi}|\frac{1}{2}\rangle = -|\frac{1}{2}\rangle \quad \text{“slower” virtual velocity}$$
$$e^{i\hat{L}_z 2\pi/\hbar}|1\rangle = e^{i2\pi}|1\rangle = |1\rangle \quad \text{“faster” virtual velocity.}$$

- However, how can we interpret *physically* these virtual transformations if they cannot be understood as effective physical motions in time?

Gauge postulate

- **Gauge postulate**: the transf. generated by an objective property are phase transf. of the system.

Gauge postulate

- **Gauge postulate**: the transf. generated by an objective property are phase transf. of the system.
- This postulate extends to the very definition of the canonical variables q and p of unconstrained systems the gauge correspondence between constraints and symmetries.

Gauge postulate

- **Gauge postulate**: the transf. generated by an objective property are phase transf. of the system.
- This postulate extends to the very definition of the canonical variables q and p of unconstrained systems the gauge correspondence between constraints and symmetries.
- This generalization depends on an *universal* concept of symmetry valid for *any* object, namely symmetries associated to phase transformations.

Gauge postulate

- **Gauge postulate**: the transf. generated by an objective property are phase transf. of the system.
- This postulate extends to the very definition of the canonical variables q and p of unconstrained systems the gauge correspondence between constraints and symmetries.
- This generalization depends on an *universal* concept of symmetry valid for *any* object, namely symmetries associated to phase transformations.
- This postulate clarifies the fact that p is not related to a transf. of q in time...

Gauge postulate

- **Gauge postulate**: the transf. generated by an objective property are phase transf. of the system.
- This postulate extends to the very definition of the canonical variables q and p of unconstrained systems the gauge correspondence between constraints and symmetries.
- This generalization depends on an *universal* concept of symmetry valid for *any* object, namely symmetries associated to phase transformations.
- This postulate clarifies the fact that p is not related to a transf. of q in time...
- ... but rather to the "timeless" intrinsic structure of objects, i.e. to the fact that any object is a superposition of phases.

Gauge postulate

- **Gauge postulate**: the transf. generated by an objective property are phase transf. of the system.
- This postulate extends to the very definition of the canonical variables q and p of unconstrained systems the gauge correspondence between constraints and symmetries.
- This generalization depends on an *universal* concept of symmetry valid for *any* object, namely symmetries associated to phase transformations.
- This postulate clarifies the fact that p is not related to a transf. of q in time...
- ... but rather to the "timeless" intrinsic structure of objects, i.e. to the fact that any object is a superposition of phases.
- If p_1 is an objective property of the object $|p_1\rangle$, then $|p_1\rangle$ is defined modulo the phase transf. generated by $\hat{p}$:

$$e^{i\hat{p}\lambda/\hbar}|p_1\rangle = e^{ip_1\lambda}|p_1\rangle \simeq |p_1\rangle.$$

Gauge postulate

- **Gauge postulate**: the transf. generated by an objective property are phase transf. of the system.
- This postulate extends to the very definition of the canonical variables q and p of unconstrained systems the gauge correspondence between constraints and symmetries.
- This generalization depends on an *universal* concept of symmetry valid for *any* object, namely symmetries associated to phase transformations.
- This postulate clarifies the fact that p is not related to a transf. of q in time...
- ... but rather to the "timeless" intrinsic structure of objects, i.e. to the fact that any object is a superposition of phases.

Gauge postulate

- **Gauge postulate**: the transf. generated by an objective property are phase transf. of the system.
- This postulate extends to the very definition of the canonical variables q and p of unconstrained systems the gauge correspondence between constraints and symmetries.
- This generalization depends on an *universal* concept of symmetry valid for *any* object, namely symmetries associated to phase transformations.
- This postulate clarifies the fact that p is not related to a transf. of q in time...
- ... but rather to the “timeless” intrinsic structure of objects, i.e. to the fact that any object is a superposition of phases.
- If p_1 is an objective property of the object $|p_1\rangle$, then $|p_1\rangle$ is defined modulo the phase transf. generated by $\hat{p}$:

$$e^{i\hat{p}\lambda/\hbar}|p_1\rangle = e^{ip_1\lambda}|p_1\rangle \simeq |p_1\rangle.$$

The Uncertainty Principle Revisited

- Heisenberg uncertainty principle is a direct consequence of the circular imbrication established by the gauge postulate between phases and objective properties.

The Uncertainty Principle Revisited

- Heisenberg uncertainty principle is a direct consequence of the circular imbrication established by the gauge postulate between phases and objective properties.
- Objective properties of an object $\mathcal{O}$...
  - are invariant under phase transformations,
  - and generate these very phase transformations (gauge postulate).

The Uncertainty Principle Revisited

- Heisenberg uncertainty principle is a direct consequence of the circular imbrication established by the gauge postulate between phases and objective properties.
- Objective properties of an object $\mathcal{O}$...
    - are invariant under phase transformations,
    - and generate these very phase transformations (gauge postulate).
- Therefore, an objective property of an object $\mathcal{O}$ has to be invariant under the phase transformations *generated by all the other objective properties* of the same $\mathcal{O}$.

The Uncertainty Principle Revisited

- Heisenberg uncertainty principle is a direct consequence of the circular imbrication established by the gauge postulate between phases and objective properties.
- Objective properties of an object $\mathcal{O}$...
  - are invariant under phase transformations,
  - and generate these very phase transformations (gauge postulate).
- Therefore, an objective property of an object $\mathcal{O}$ has to be invariant under the phase transformations *generated by all the other objective properties* of the same $\mathcal{O}$.
- If f, g are objective properties of $\mathcal{O}$, then they have to satisfy $\mathcal{L}ie_{v_g} f = \{f, g\} = 0$.

The Uncertainty Principle Revisited

- Heisenberg uncertainty principle is a direct consequence of the circular imbrication established by the gauge postulate between phases and objective properties.
- Objective properties of an object $\mathcal{O}$...
  - are invariant under phase transformations,
  - and generate these very phase transformations (gauge postulate).
- Therefore, an objective property of an object $\mathcal{O}$ has to be invariant under the phase transformations *generated by all the other objective properties* of the same $\mathcal{O}$.
- If f, g are objective properties of $\mathcal{O}$, then they have to satisfy $\mathcal{L}ie_{v_g} f = \{f, g\} = 0$.
- Hence, the set of objective properties of $\mathcal{O}$ has to define a commutative Poisson algebra.

The Uncertainty Principle Revisited

- Heisenberg uncertainty principle is a direct consequence of the circular imbrication established by the gauge postulate between phases and objective properties.
- Objective properties of an object $\mathcal{O}$...
  - are invariant under phase transformations,
  - and generate these very phase transformations (gauge postulate).
- Therefore, an objective property of an object $\mathcal{O}$ has to be invariant under the phase transformations *generated by all the other objective properties* of the same $\mathcal{O}$.
- If f, g are objective properties of $\mathcal{O}$, then they have to satisfy $\mathcal{L}ie_{v_g} f = \{f, g\} = 0$.
- Hence, the set of objective properties of $\mathcal{O}$ has to define a commutative Poisson algebra.
- In particular, if p is an objective property of $\mathcal{O}$, then q is necessarily, like the face of a die, a phase with no objective value.

Reconstructing quantum mechanics

- In the framework of geometric quantization, the quantum postulate and the gauge postulate are implemented in two different steps, namely...

  - ... the ***prequantization*** of the symplectic manifold (which implements the quantum postulate),

  - ... and the ***polarization*** of the quantum states (which implements the gauge postulate).

Reconstructing quantum mechanics

- In the framework of geometric quantization, the quantum postulate and the gauge postulate are implemented in two different steps, namely...

  - ... the ***prequantization*** of the symplectic manifold (which implements the quantum postulate),

  - ... and the ***polarization*** of the quantum states (which implements the gauge postulate).

- The prequantization formalism forces the non-commutativity of the operators associated to q and p by defining a $U(1)$-fiber bundle over (M, ω) endowed with a hermitian connection of curvature given by ω.

Reconstructing quantum mechanics

- In the framework of geometric quantization, the quantum postulate and the gauge postulate are implemented in two different steps, namely...

  - ... the ***prequantization*** of the symplectic manifold (which implements the quantum postulate),

  - ... and the ***polarization*** of the quantum states (which implements the gauge postulate).

Reconstructing quantum mechanics

- In the framework of geometric quantization, the quantum postulate and the gauge postulate are implemented in two different steps, namely...

  - ... the ***prequantization*** of the symplectic manifold (which implements the quantum postulate),

  - ... and the ***polarization*** of the quantum states (which implements the gauge postulate).

- The prequantization formalism forces the non-commutativity of the operators associated to q and p by defining a $U(1)$-fiber bundle over (M, ω) endowed with a hermitian connection of curvature given by ω.

Reconstructing quantum mechanics

- In the framework of geometric quantization, the quantum postulate and the gauge postulate are implemented in two different steps, namely...

  - ... the ***prequantization*** of the symplectic manifold (which implements the quantum postulate),

  - ... and the ***polarization*** of the quantum states (which implements the gauge postulate).

- The prequantization formalism forces the non-commutativity of the operators associated to q and p by defining a $U(1)$-fiber bundle over (M, ω) endowed with a hermitian connection of curvature given by ω.

- The physical states are the *polarized sections* of the quantum bundle, that is the sections ψ that satisfy

$$\nabla_{\mathcal{D}}\psi = 0,$$

where $\mathcal{D}$ is a Lagrangian foliation of (M, ω), i.e. a maximal (dim$\mathcal{D} = n$) isotropic ($\omega(X, Y) = 0$, $\forall X, Y \in \mathcal{D}$) and integrable distribution.

Insights of geometric quantization

Some characteristic features of quantum mechanics acquire a geometric explanation:

- Quantum non-commutativity acquires the same geometric meaning that non-commutativity of parallel transports in general relativity and Yang-Mills theory.

Insights of geometric quantization

Some characteristic features of quantum mechanics acquire a geometric explanation:

- Quantum non-commutativity acquires the same geometric meaning that non-commutativity of parallel transports in general relativity and Yang-Mills theory.
- The discrete spectra of some physical quantities is a consequence of the topology of the fiber bundle (like Dirac's quantization of charge).

On quantum objects & dice

- The notion of phase permits us to understand how it can be the case that the result of a measurement can be a non-objective *phase* of the object.

- Indeed, the superposition principle acquires an intuitive meaning: an object is a *superposition* of phases (which are in turn quantum objects).

On quantum objects & dice

- The notion of phase permits us to understand how it can be the case that the result of a measurement can be a non-objective *phase* of the object.

On quantum objects & dice

- The notion of phase permits us to understand how it can be the case that the result of a measurement can be a non-objective *phase* of the object.

- Indeed, the superposition principle acquires an intuitive meaning: an object is a *superposition* of phases (which are in turn quantum objects).

On quantum objects & dice

- The notion of phase permits us to understand how it can be the case that the result of a measurement can be a non-objective *phase* of the object.
- Indeed, the superposition principle acquires an intuitive meaning: an object is a *superposition* of phases (which are in turn quantum objects).
- Einstein was literally right... quantum objects are like dice, i.e. configurations with different phases that can be tossed.

On quantum objects & dice

- The notion of phase permits us to understand how it can be the case that the result of a measurement can be a non-objective *phase* of the object.
- Indeed, the superposition principle acquires an intuitive meaning: an object is a *superposition* of phases (which are in turn quantum objects).
- Einstein was literally right... quantum objects are like dice, i.e. configurations with different phases that can be tossed.
- In particular, it is meaningless to ask what the objective position q of an object $\mathcal{O}$ with a well-defined momentum p is.

On quantum objects & dice

- The notion of phase permits us to understand how it can be the case that the result of a measurement can be a non-objective *phase* of the object.
- Indeed, the superposition principle acquires an intuitive meaning: an object is a *superposition* of phases (which are in turn quantum objects).
- Einstein was literally right... quantum objects are like dice, i.e. configurations with different phases that can be tossed.
- In particular, it is meaningless to ask what the objective position q of an object $\mathcal{O}$ with a well-defined momentum p is.
- Even if such an object has no objective position, it will appear in a particular position q_1 if a position measurement is performed.

Quantum relationalism

- It is worth noting that this quantum ontology entails a commitment to what we could call *intrinsic relationalism*.

Quantum relationalism

- It is worth noting that this quantum ontology entails a commitment to what we could call *intrinsic relationalism*.
- Indeed, the very fact that an object is a superposition of non-objective phases...

Quantum relationalism

- It is worth noting that this quantum ontology entails a commitment to what we could call *intrinsic relationalism*.
- Indeed, the very fact that an object is a superposition of non-objective phases...
- ... permits us to define its invariant objective properties as the "relational differences" between its phases.

Quantum relationalism

- It is worth noting that this quantum ontology entails a commitment to what we could call *intrinsic relationalism*.
- Indeed, the very fact that an object is a superposition of non-objective phases...
- ... permits us to define its invariant objective properties as the “relational differences” between its phases.
- This relational character of quantum mechanics does not stem from the extrinsic relations between different objects...

Quantum relationalism

- It is worth noting that this quantum ontology entails a commitment to what we could call *intrinsic relationalism*.
- Indeed, the very fact that an object is a superposition of non-objective phases...
- ... permits us to define its invariant objective properties as the "relational differences" between its phases.
- This relational character of quantum mechanics does not stem from the extrinsic relations between different objects...
- ... but rather from the intrinsic relations between the different phases of a single object.

Quantum relationalism

- It is worth noting that this quantum ontology entails a commitment to what we could call *intrinsic relationalism*.
- Indeed, the very fact that an object is a superposition of non-objective phases...
- ... permits us to define its invariant objective properties as the “relational differences” between its phases.
- This relational character of quantum mechanics does not stem from the extrinsic relations between different objects...
- ... but rather from the intrinsic relations between the different phases of a single object.
- In this way, we can now provide an answer to the quantum koan *What is the objective face of a die?*, namely...

Quantum relationalism

- It is worth noting that this quantum ontology entails a commitment to what we could call *intrinsic relationalism*.
- Indeed, the very fact that an object is a superposition of non-objective phases...
- ... permits us to define its invariant objective properties as the “relational differences” between its phases.
- This relational character of quantum mechanics does not stem from the extrinsic relations between different objects...
- ... but rather from the intrinsic relations between the different phases of a single object.
- In this way, we can now provide an answer to the quantum koan *What is the objective face of a die?*, namely...
- ... that the objective properties of a die are the relational (and faithful) generators of the transformations between its non-objective phases.

Objective quantum probabilities

- This interpretation suggests a particular comprehension of (quantum) probabilities.

Objective quantum probabilities

- This interpretation suggests a particular comprehension of (quantum) probabilities.
- We could say that the distribution probability that specifies the probability of obtaining *tail* or *head* when throwing a coin might be understood...

  - ... neither as a description of the *instantaneous state* or the *temporal evolution* of the coin
  - ... nor as a consequence of our subjective lack of knowledge of its deterministic trajectory,
  - ... nor as a property of an *ensemble* of coins,
  - ... but rather as a faithful description of the intrinsic structure of a single coin, i.e. of the fact that a coin is a *single multifaceted object* with two similar phases.

Objective quantum probabilities

- This interpretation suggests a particular comprehension of (quantum) probabilities.
- We could say that the distribution probability that specifies the probability of obtaining *tail* or *head* when throwing a coin might be understood...
  - ... neither as a description of the *instantaneous state* or the *temporal evolution* of the coin
  - ... nor as a consequence of our subjective lack of knowledge of its deterministic trajectory,
  - ... nor as a property of an *ensemble* of coins,
  - ... but rather as a faithful description of the intrinsic structure of a single coin, i.e. of the fact that a coin is a *single multifaceted object* with two similar phases.
- In the same way, quantum probabilities could be considered objective in the sense that they convey information about the intrinsic multifaceted structure of *single* “dice-like” objects that can be tossed in different “tables”.

Conclusions...

- Since in quantum mechanics q and p cannot be both “observables” of the same physical system...

Objective quantum probabilities

- This interpretation suggests a particular comprehension of (quantum) probabilities.
- We could say that the distribution probability that specifies the probability of obtaining *tail* or *head* when throwing a coin might be understood...
  - ... neither as a description of the *instantaneous state* or the *temporal evolution* of the coin
  - ... nor as a consequence of our subjective lack of knowledge of its deterministic trajectory,
  - ... nor as a property of an *ensemble* of coins,
  - ... but rather as a faithful description of the intrinsic structure of a single coin, i.e. of the fact that a coin is a *single multifaceted object* with two similar phases.
- In the same way, quantum probabilities could be considered objective in the sense that they convey information about the intrinsic multifaceted structure of *single* "dice-like" objects that can be tossed in different "tables".

Conclusions...

- Since in quantum mechanics q and p cannot be both “observables” of the same physical system...

Conclusions...

- Since in quantum mechanics q and p cannot be both “observables” of the same physical system...