

Title: Exploring extra dimensions with cosmic acceleration

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Abstract: The standard cosmological model features two periods of accelerated expansion: an inflationary epoch at early times, and a dark energy dominated epoch at late times. These periods of accelerated expansion can lead to surprisingly strong constraints on models with extra dimensions. I will describe new mathematical results which enable one to reconstruct features of a higher-dimensional theory based on the behaviour of the accelerating four-dimensional cosmology. When applied to inflation, these results pose several interesting questions for the construction of concrete models. When applied to dark energy, they provide a new technique to combine measurements of dark energy parameters and constraints on variation of Newton's constant. This technique can transform near-future dark energy surveys into become powerful probes of extra dimensional physics.

Exploring extra dimensions with cosmic acceleration

Daniel Wesley
(Cambridge)

extra dimensions

(Kaluza-Klein, superstrings)

“cosmic acceleration”

$$\ddot{a} > 0$$

(inflation, dark energy)

- Review of classical energy conditions (null, strong)
- In compactified models, reduction of Einstein equations and energy condition(s) to

$$a \frac{d\zeta}{da} \geq \alpha_0 + \alpha_1 \zeta + \alpha_2 \zeta^2$$

$$\zeta^2 \leq F$$

resulting in e-folding bounds for accelerated expansion.

- Implications for inflation
- Implications for dark energy -- new technique to combine dark energy and G_N variation constraints to probe extra dimensions

Strong energy condition (SEC)

$$R_{MN} t^M t^N \geq 0 \quad \text{where } t^M \text{ is timelike or null}$$

This condition ensures that geodesics converge, and hence that “gravity is attractive.”

Null energy condition (NEC)

$$R_{MN} n^M n^N \geq 0 \quad \text{where } n^M \text{ is null}$$

This is the weakest of the energy conditions, and ensures lightlike observers always see a positive energy density.

Strong

$$R_{MN} t^M t^N \geq 0$$

Null

$$R_{MN} n^M n^N \geq 0$$

Satisfied by

- $\rho + P \geq 0$ and $\rho + 3P \geq 0$
- Scalars with $V(\varphi) \leq 0$
- ... classical I ID SUGRA + others
- anti-de Sitter $\Lambda < 0$
- dust and radiation

- $\rho + P \geq 0$
- Scalars with any $V(\varphi)$ and positive kinetic energy terms
- D-branes
- Implied by all other energy conditions

Violated by

- Scalars with $V(\varphi) > 0$ anywhere
- de Sitter $\Lambda > 0$
- D-branes

- Casimir energy (...unless averaged)
- Negative tension (orientifold planes)
- ghost condensates

Why do we need the NEC?

Causality violations

“warp drives,” traversable wormholes, time machines, CTCs

Morris, Thorne *Am. J. Phys.* **56** (1988) 395 ; Visser, Kar, Dadhich *Phys. Rev. Lett.* **90** (2003) 201102 ; Alcubierre *Class. Quant. Grav.* **11** (1994) L73 ; Krasnikov *Phys. Rev. D* **57** (1998) 4760 ; Morris, Thorne, Yurtsever *Phys. Rev. Lett.* **61** (1988) 1446 ; Hawking *Phys. Rev. D* **46** (1992) 603

Classical instabilities

Big Rips, Big Bounces, gradient instabilities ...

Cline, Jeon, Moore *Phys. Rev. D* **70** (2004) 043543 ; Hsu, Jenkins, Wise *Phys. Lett. B* **597** (2004) 270 ; Buniy, Hsu, Murray *Phys. Rev. D* **74** (2006) 063518 ; Caldwell *Phys. Lett. B* **545** (2002) 23 ; Caldwell, Kamionkowski, Weinberg *Phys. Rev. Lett.* **91** (2003) 071301 (many others...)

Quantum instabilities

non-unitarity, ghosts, vacuum decay

(many references...)

Breakdown of gravitational thermodynamics

perpetuum mobile, apparent second law violations

Dubovsky, Gregoire, Nicolis, Rattazzi *JHEP* **0603** (2006) 025 ; Arkani-Hamed, Dubovsky, Nicolis, Trincherini, Villadoro *JHEP* **05** (2007) 055 ; Dubovsky, Sibiryakov *Phys. Lett. B* **638** (2006) 509

(Gibbons '84, Maldacena/Nunez '01)

One cannot obtain a four-dimensional de Sitter universe from a higher-dimensional theory with static extra dimensions that satisfies the **strong** energy condition.*

Proof: $M_n = M_4 \times B$

$$ds^2 = g_{mn}(y)dy^m dy^n + W(y)^2 g_{\alpha\beta} dX^\alpha dX^\beta$$

$${}^{(n)}R_{\alpha\beta} = {}^{(4)}R_{\alpha\beta} - \frac{1}{4} \nabla_m \nabla^m (W^{-1/4}) g_{\alpha\beta}$$

$$\int_B W^{-1/4} {}^{(n)}R_{00} g^{00} = \int_B W^{-1/4} {}^{(4)}R_{00} g^{00} + \int_B (\partial W^{-1/4})^2$$

SEC requires $R_{00} \geq 0$ $\Lambda > 0 \implies {}^{(4)}R_{00} < 0$

areas for improvement:

- present-day universe is not de Sitter - what about equations of state differing from $w=-1$?
- extra dimensions could be dynamical - can this allow one to escape from the no-go?
- many higher-dimensional theories violate the SEC - can the energy condition be weakened?

(here w is the ratio of pressure to energy density, $w = P/\rho$)

New theorems

A family of results which pair an energy condition and a general feature of the extra-dimensional spacetime metric.*

Here is an example with one pair:

If the higher-dimensional spacetime is warped and Ricci-flat, and the higher-dimensional theory satisfies the Null Energy Condition, then *the four-dimensional w is different than -1 , is time-varying, or both*. In fact, *w cannot remain close to -1 for more than a few e-foldings*.

Furthermore, if the four-dimensional universe is accelerating, then *Newton's constant varies (in both instantaneous and secular senses)*.

4D universe with constant w embedded with k Ricci-flat extra dimensions, evolve by breathing mode only, with no other matter.

$$ds_4^2 = a(\eta)^2 (-d\eta^2 + dx_3^2)$$

$$a(\eta) \sim \eta^{2/(1+3w)}$$

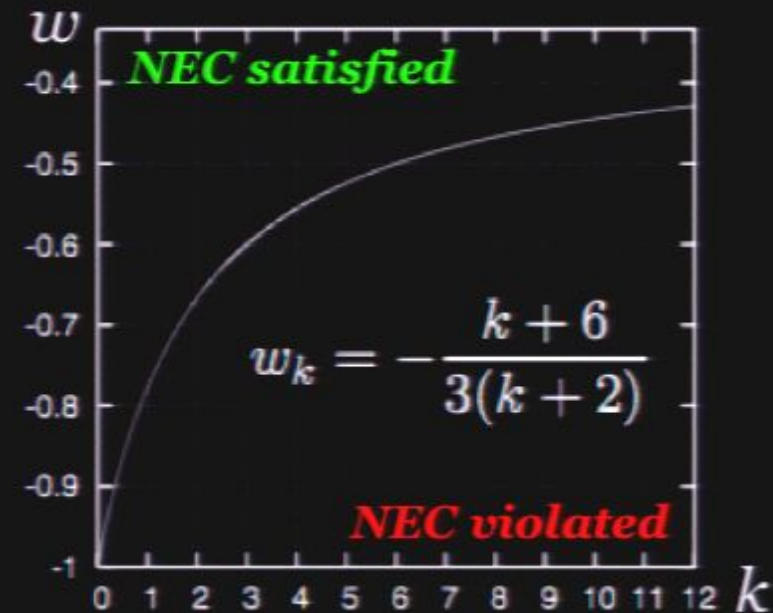
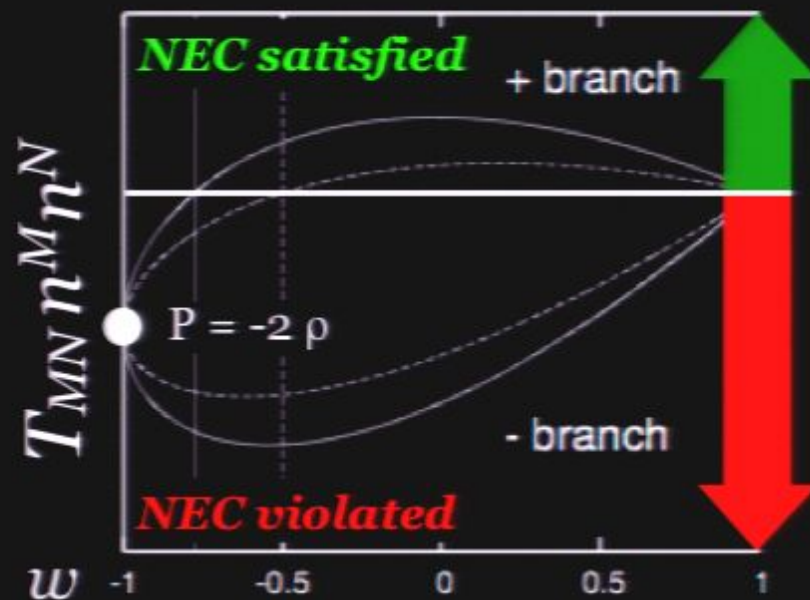
$$\rho + P = 3(1+w) \frac{(a')^2}{a^4} = \frac{(\psi')^2}{a^2}$$

$$V(\psi) = V_0 e^{\pm \sqrt{3(1+w)}\psi} \quad \psi(\eta) = \mp \left(\frac{1+w}{1+3w} \right) \ln \eta^6 + \psi_0$$

$$c = \sqrt{\frac{2k}{k+2}} \quad a(\eta) = e^{c\psi/2} A(\eta)$$

$$ds_{4+k}^2 = A(\eta)^2 (-d\eta^2 + dx_3^2) + \exp \left[\frac{2c}{k} \psi(\eta) \right] ds_k^2$$

Reconstruct (4+k)D metric and use Einstein equations to compute stress-energy tensor



$$V(\psi) = V_0 e^{\mp \sqrt{3(1+w)}\psi}$$

This unusual (even pathological) behavior is completely invisible from 4D

For a more general case...

- Extra dimensions could distort
 - Anisotropically
 - Inhomogeneously
- Warped extra dimensions
- Non-Ricci-flat extra dimensions
- Scalar field might not be breathing mode
- Metric moduli may not act like quintessence
- Completely different scalar could drive accel.
- ... etc

Assumptions

- Higher-dimensional action has Einstein-Hilbert form
...includes $g(\varphi)R$ and $F(R)$ models

$$S_{(4+k)} = \frac{1}{2\kappa^{2+k}} \int \left[R(g) + \mathcal{L}_m^{(4+k)} \right] \sqrt{-g} d^{4+k} X$$

- All four-dimensional statements refer to the Einstein frame metric and its associated cosmology.

$$S_{4D} = \frac{1}{2\ell_4^2} \int R(g) \sqrt{-g} d^4 x + \text{other terms}$$

- M closed and compact, or a quotient of such, as $M = M'/G$

$$g_{MN}^{(4+k)} dX^M dX^N = e^{2\Omega(t,y)} h_{\mu\nu}^{(4)}(t) dx^\mu dx^\nu + g_{\alpha\beta}^{(k)}(t,y) dy^\alpha dy^\beta$$

- Arbitrary other matter fields may be present in the action

Curved	Ricci-flat	CRF
$\mathcal{R} \neq 0$ on \mathcal{M}	$\mathcal{R} = 0$ on \mathcal{M}	$ds^2 = e^{2\Omega} ds_4^2 + e^{-2\Omega} ds_k^2$
<p><u>Non-Abelian continuous isometries</u>**</p> <p>... includes models which obtain 4D gauge symmetries by KK reduction</p> <p><u>Rugby-ball SLED</u></p> <p>** We only know these cannot be Ricci-flat, which is a stronger condition than "curvature-free."</p>	<p><u>Special holonomy</u></p> <p>$Sp(n)$ $Spin(7)$ $SU(n)$ (Calabi-Yau) G_2 (M theory)</p> <p><u>One-dimensional</u></p> <p>Original Kaluza-Klein Randall-Sundrum</p> <p><u>Tori</u></p> <p>R^n / Z^n with $R \geq 0$</p>	<p><u>Klebanov-Strassler warped throat</u></p> <p><u>Giddings-Kachru-Polchinski flux solutions</u></p>

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Curved

Ricci-flat

CRF

dS

Strong

(Gibbons et. al.)

Null

Null

$w > -1$

Strong

Null

Null

(transient)

(transient)

(transient)

Some tools

Decomposition of metric deformations

$$\frac{1}{2} \frac{d}{dt} g_{\alpha\beta}^{(k)} = \frac{1}{k} \xi g_{\alpha\beta}^{(k)} + \sigma_{\alpha\beta}$$

A-Averaging

$$\langle Q(t, y^\alpha) \rangle_A = \left(\int e^{A\Omega} Q \det(e_{\mathcal{M}}) d^k y \right) \left(\int e^{A\Omega} \det(e_{\mathcal{M}}) d^k y \right)^{-1}$$

Weighted integral of Q
over the compact space

Weighted volume of
the compact space

A useful lemma

For a perfect fluid $T_{\hat{\mu}\hat{\nu}} = \begin{pmatrix} \rho & \\ & P\delta_{\hat{m}\hat{n}} \end{pmatrix}$ if $\rho + P < 0$ then NEC is violated.

In a Kaluza-Klein spacetime, the most general stress-energy is

$$T_{\hat{A}\hat{B}} = \begin{pmatrix} \rho^D & 0_{\hat{0}\hat{n}} & J^{\hat{b}} \\ 0_{\hat{m}\hat{0}} & P_3^D \delta_{\hat{m}\hat{n}} & 0_{\hat{m}\hat{b}} \\ J^{\hat{a}} & 0_{\hat{a}\hat{n}} & \tau_{\hat{a}\hat{b}} \end{pmatrix} \quad \begin{aligned} \tau_{\hat{a}\hat{b}} &= P_k^D \delta_{\hat{a}\hat{b}} + \pi_{\hat{a}\hat{b}} \\ \delta^{\hat{a}\hat{b}} \pi_{\hat{a}\hat{b}} &= 0 \quad P_k^D = k^{-1} \delta^{\hat{a}\hat{b}} \tau_{\hat{a}\hat{b}} \end{aligned}$$

If either $\rho^D + P_3^D < 0$ or $\rho^D + P_k^D < 0$ then NEC violated

Dynamical metric hinders satisfying NEC

$$n^2 e^{-\phi} \langle e^{2\Omega} (\rho^D + P_3^D) \rangle_A = n^2 (\rho_T + P_T) - \frac{k+2}{2k} \xi_{0|A}^2 - \frac{k+2}{2k} \langle \xi_{\perp|A}^2 \rangle_A - \langle \sigma^2 \rangle_A$$

Must be
non-negative
to satisfy NEC

4D energy
density and
pressure

$$\begin{aligned} n^2 e^{-\phi} \langle e^{2\Omega} (\rho^D + P_k^D) \rangle_A &= \frac{n^2}{2} (\rho_T + 3P_T) + 2 \left(\frac{A}{4} - 1 \right) \frac{k+2}{2k} \langle \xi_{\perp|A}^2 \rangle_A \\ &\quad - \frac{k+2}{2k} \xi_{0|A}^2 - \langle \sigma^2 \rangle_A \\ &\quad + \left[\left(\frac{4}{k} - 1 \right) (A+2) + \left(4 - \frac{4}{k} \right) \right] \langle e^{2\Omega} (\partial\Omega)^2 \rangle_A \\ &\quad + \frac{k+2}{2k} \frac{n}{a^3} \frac{d}{dt} \left(\frac{a^3}{n} \xi_{0|A} \right) \end{aligned}$$

Dynamical metric helps
to satisfy NEC

$$\zeta = \frac{\dot{\xi}_{0|A}}{H(t)} = \text{fractional increase in volume per Hubble time}$$

Einstein equations and energy conditions cast as two simple inequalities

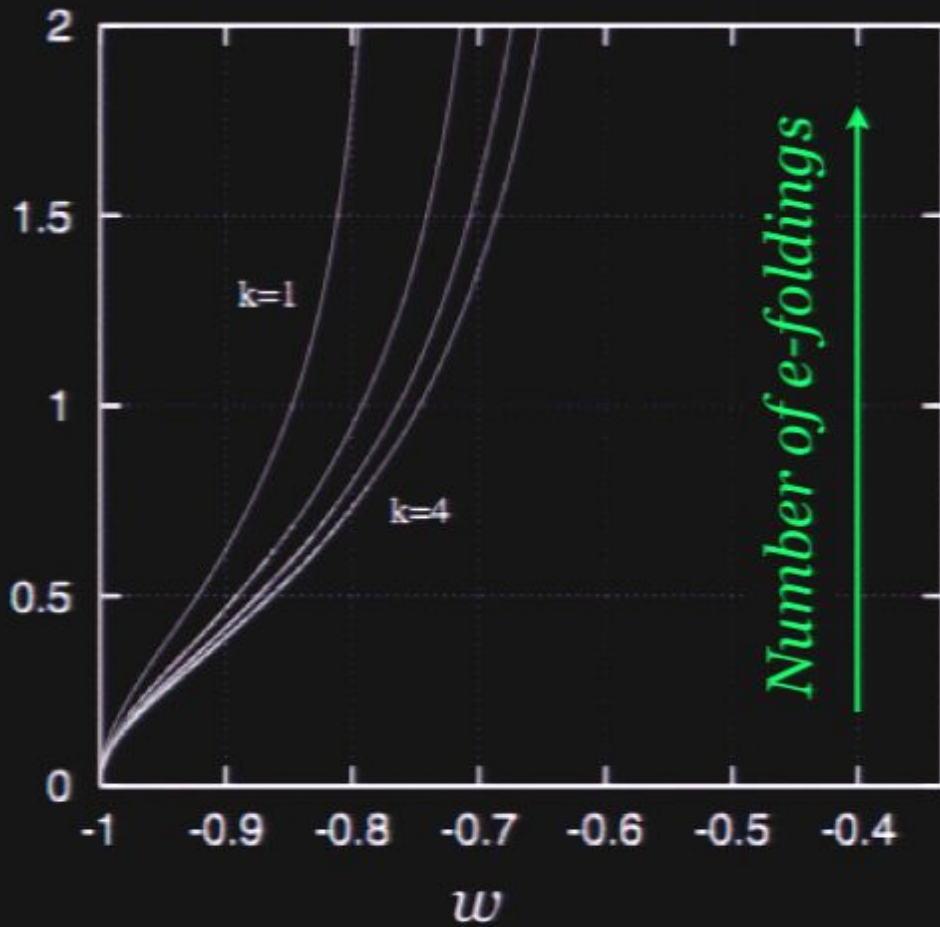
$$(1) \quad a \frac{d\zeta}{da} \geq \alpha_0 + \alpha_1 \zeta + \alpha_2 \zeta^2$$

$$(2) \quad \zeta^2 \leq F$$

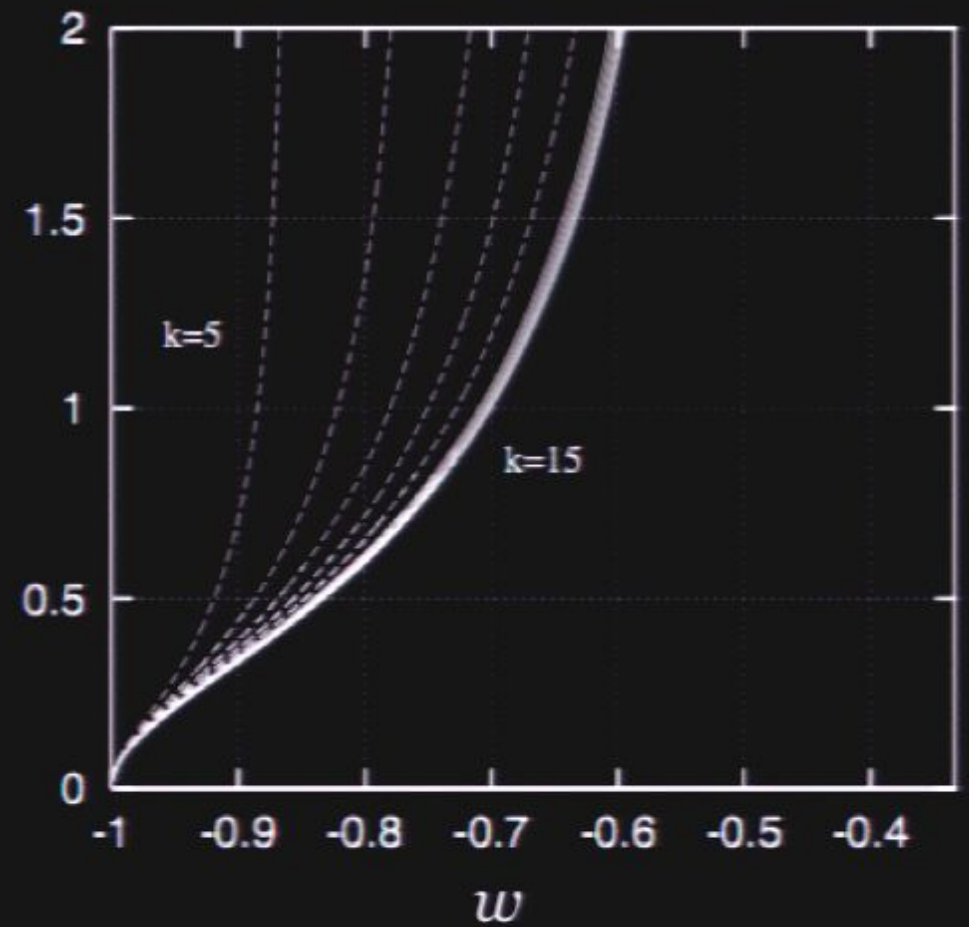
α_0 , α_1 , α_2 , and F are complicated functions of w , etc. If one cannot solve these equations, then a NEC-satisfying solution does not exist.

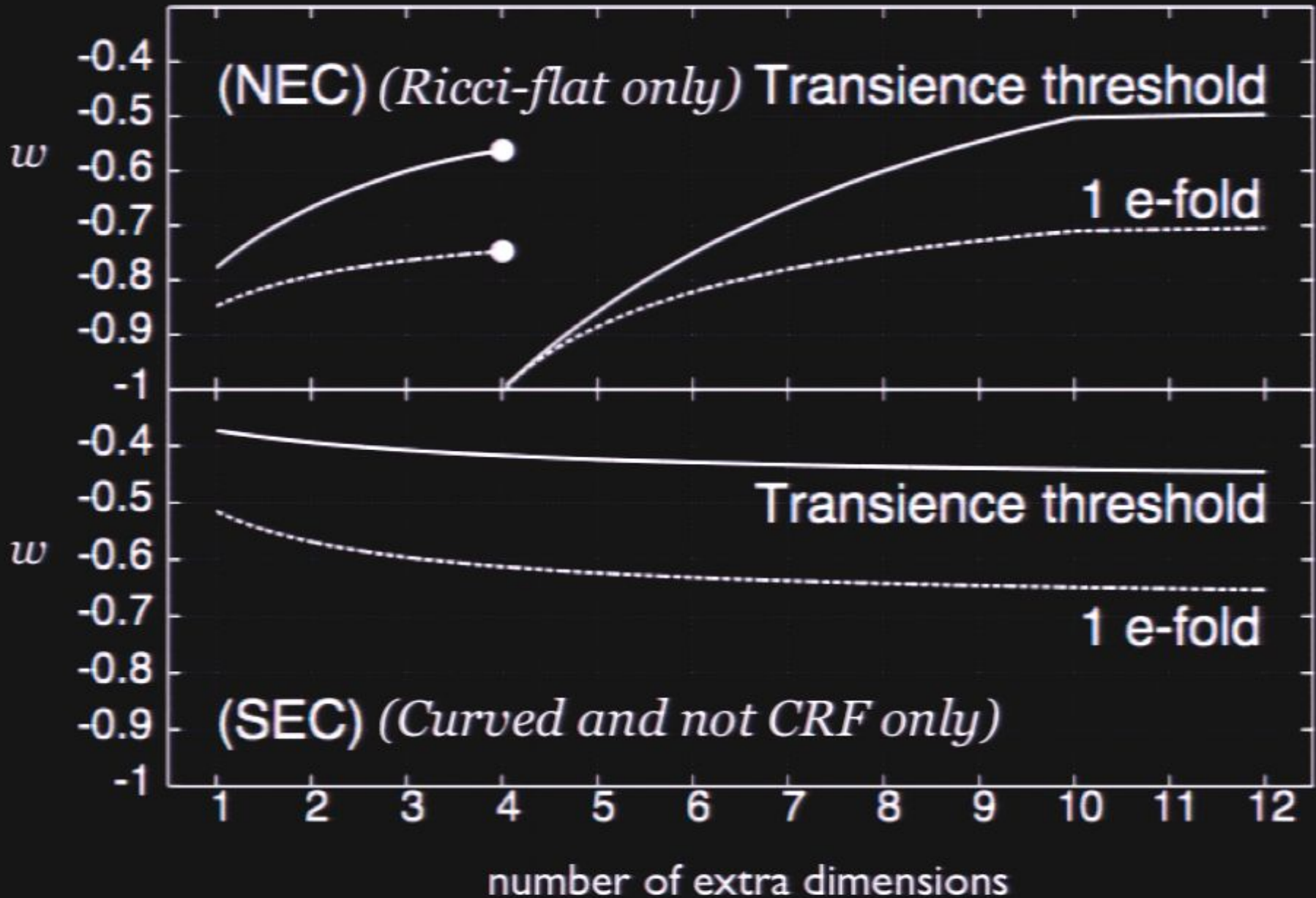
The solution with the maximal number of e-foldings consistent with NEC saturates (1) with boundary conditions set by (2).

$k=1-4$



$k=5-10$ and $k = 11 \dots$





“Loopholes”

1. Curvature of M

Could go either way, either helping or hindering NEC.
Important question: what metric should we use?

2. Quantum effects

If we are unable to impose the Einstein equations, how literally should we take the extra dimensions?

3. Higher-derivative corrections to GR

$$G_{MN} = T_{MN} + \Theta_{MN}[R, R_{AB}, R_{ABCD}]$$

if effectively NEC-violating, do we avoid usual problems?

4. Negative-tension objects (not really a loophole, these violate NEC)

braneworld boundaries, O-planes

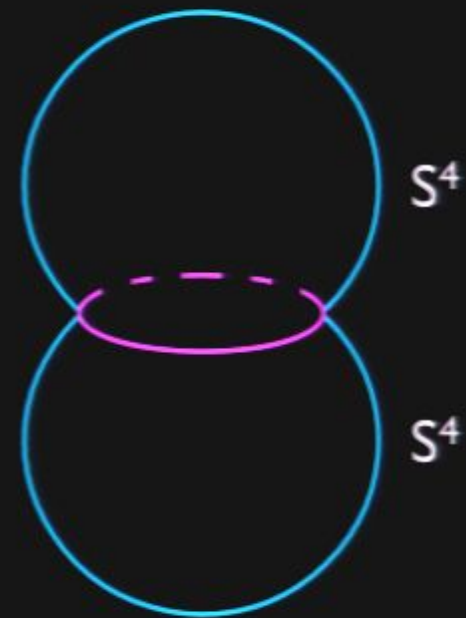
Bubbles of nothing

$$S = S_{\text{matter}} + S_{\text{gravity}} + \sum_i T_i \int_{\partial \mathcal{M}_i} \sqrt{h} d^{d-1}x$$



Negative-tension “Coleman-de Luccia instantons”

$$S = \int P(X) - V(\phi) \\ X = -\frac{1}{2}(\partial\phi)^2$$



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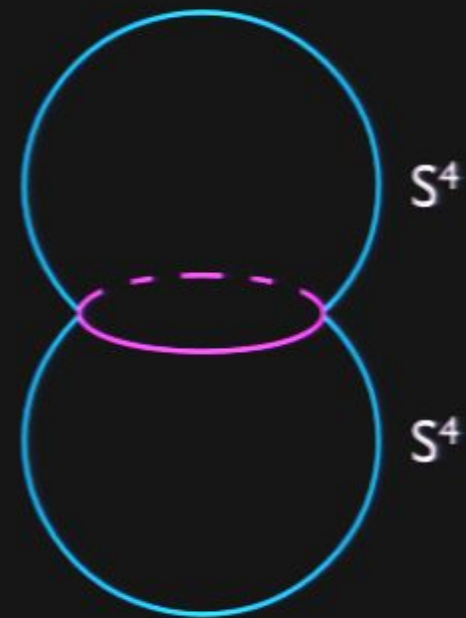
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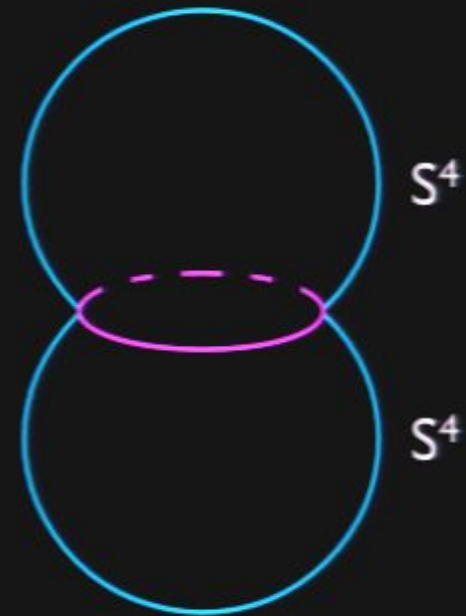
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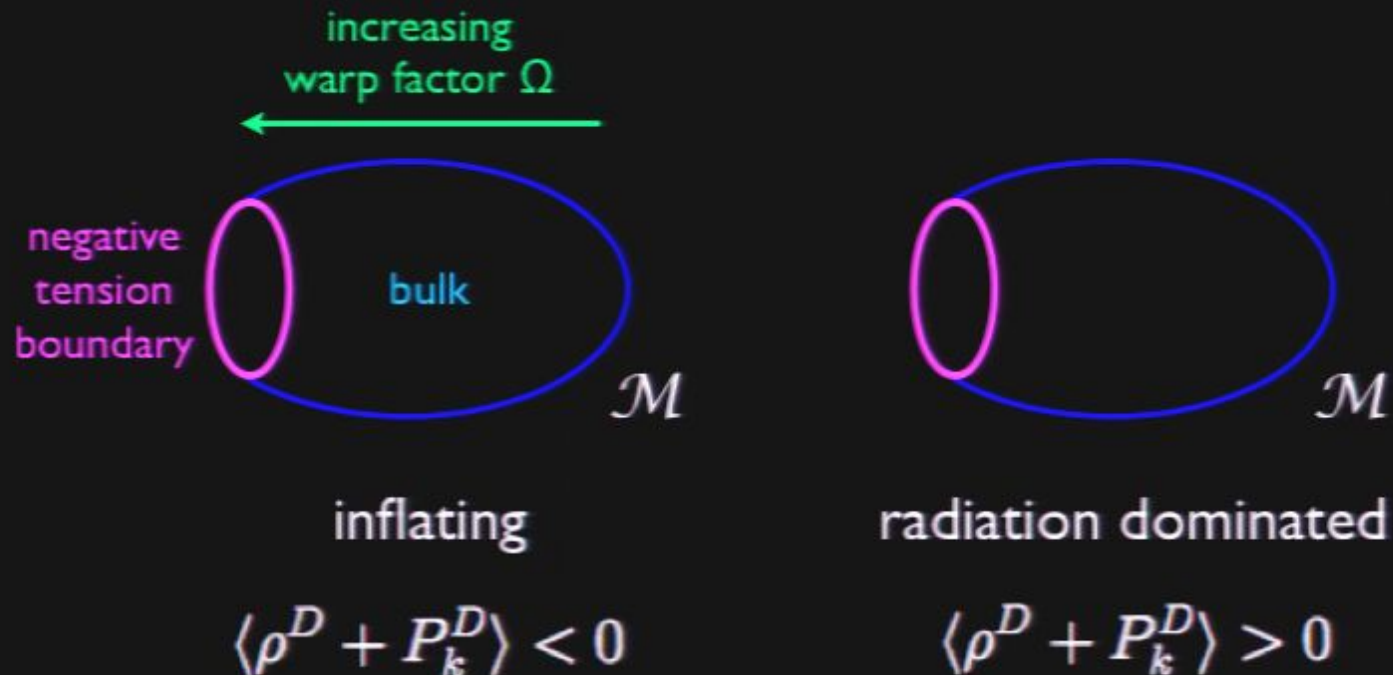


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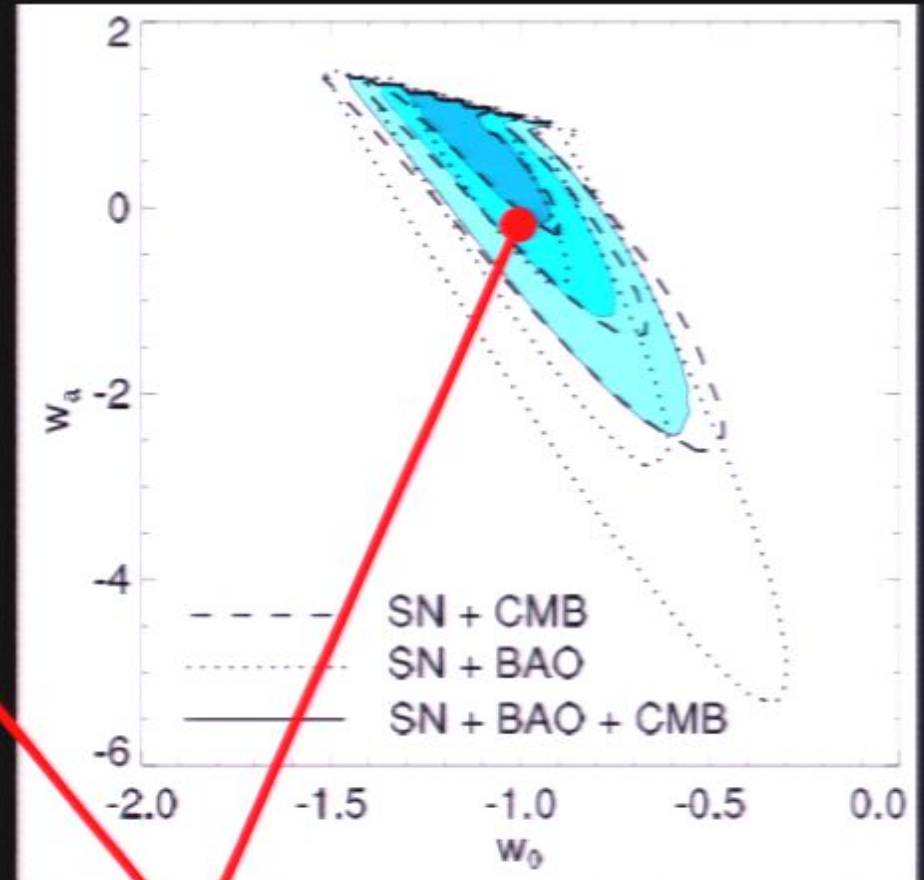
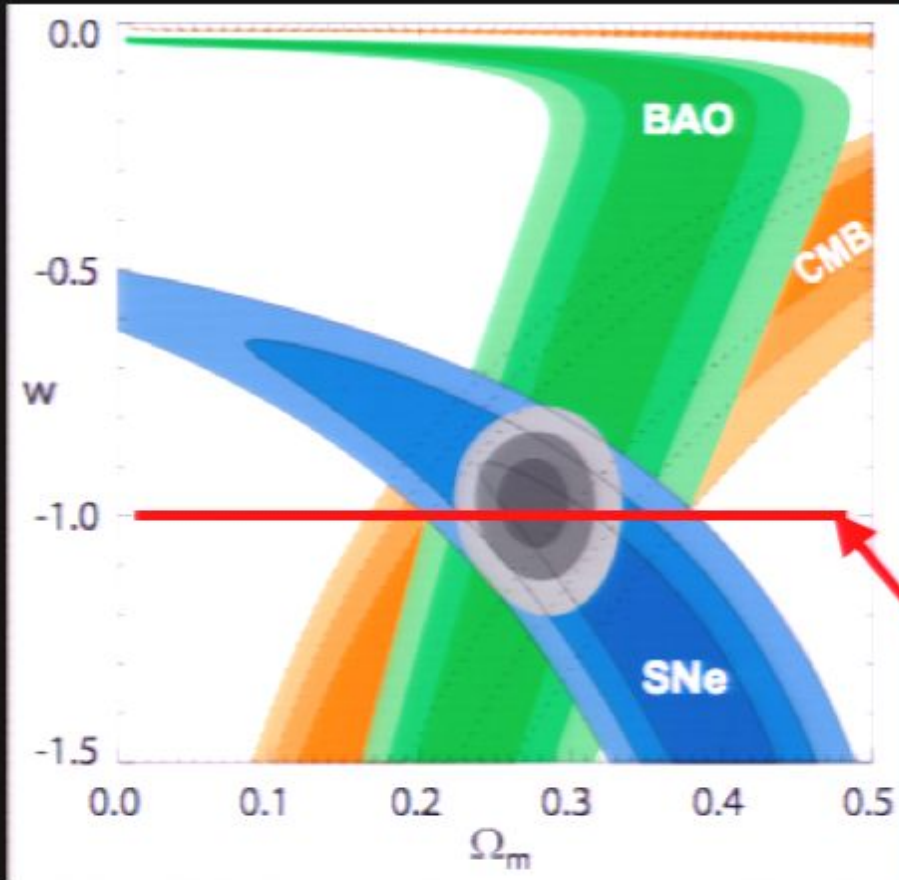


Accelerating to decelerating transition after inflation



Additional sources of NEC violation may be required,
in addition to the negative tension boundaries.

Applications to future dark energy surveys



figures from Kowalski et al. 0804.4142

data is consistent with a cosmological constant

Problem 1: easy to construct models which give w very close to -1

Slow-roll scalar field with potential

$$3H\dot{\varphi} = -\frac{dV(\varphi)}{d\varphi}$$

$$w = \frac{\frac{1}{2}\dot{\varphi}^2 - V(\varphi)}{\frac{1}{2}\dot{\varphi}^2 + V(\varphi)} \sim -1 + \frac{M_{\text{Pl}}^2}{3} \left(\frac{V'}{V} \right)^2$$

Extremely ambitious space missions may get $w_{\text{dark}} \lesssim -0.955$ at 3σ , which requires $\left| \frac{V'}{V} \right| \lesssim \frac{0.37}{M_{\text{Pl}}}$

\therefore Easy to choose parameters such that we can never distinguish this simple model from Λ .

Problem 2: What will confirming de Sitter tell us?

The “inverse problem”

How can we use the observed properties of dark energy to learn about its origin?

Example: suppose a reliable source informs us that $w_{dark} = -1$ and constant.

Possibility I:

A simple cosmological constant Λ .

Possibility II:

Metastable de Sitter string vacuum

... one of 10^{300} choices given choice of Calabi-Yau (or other manifold) configuration of D/anti-D branes, O-planes, flux winding numbers, etc.

Observational prospects

(work-in-progress with Paul Steinhardt)

More precise measurements of $w(a)$, combined with bounds on variation of fundamental constants, can provide surprisingly powerful probes of extra-dimensional physics.

As dark energy surveys provide ever-more stringent constraints on the dark energy equation-of-state, we can progressively rule out families of extra-dimensional models - even if the measurements are consistent with Λ

Einstein equations and energy conditions
cast as two simple inequalities

$$a \frac{d\zeta}{da} \geq \alpha_0 + \alpha_1 \zeta + \alpha_2 \zeta^2$$

$$\zeta^2 \leq F$$

Where ζ measures the rate-of-change of extra-dimensional volume,
and $\alpha_0, \alpha_1, \alpha_2$, and F are complicated functions of w , etc.

4D Newton constant $G_N^{(4)} = \frac{G_N^{(4+k)}}{\text{Vol}(M)}$

$$\frac{\dot{G}_N}{G_N} = -H(t)\zeta \qquad \frac{G_N(t_1)}{G_N(t_0)} = \exp \int_{a(t_1)}^{a(t_0)} \zeta(a) \frac{da}{a}$$

Dark energy parameterization

$$w_{dark} = w_0 + w_a(1 - a) + w_b(1 - a)^2$$

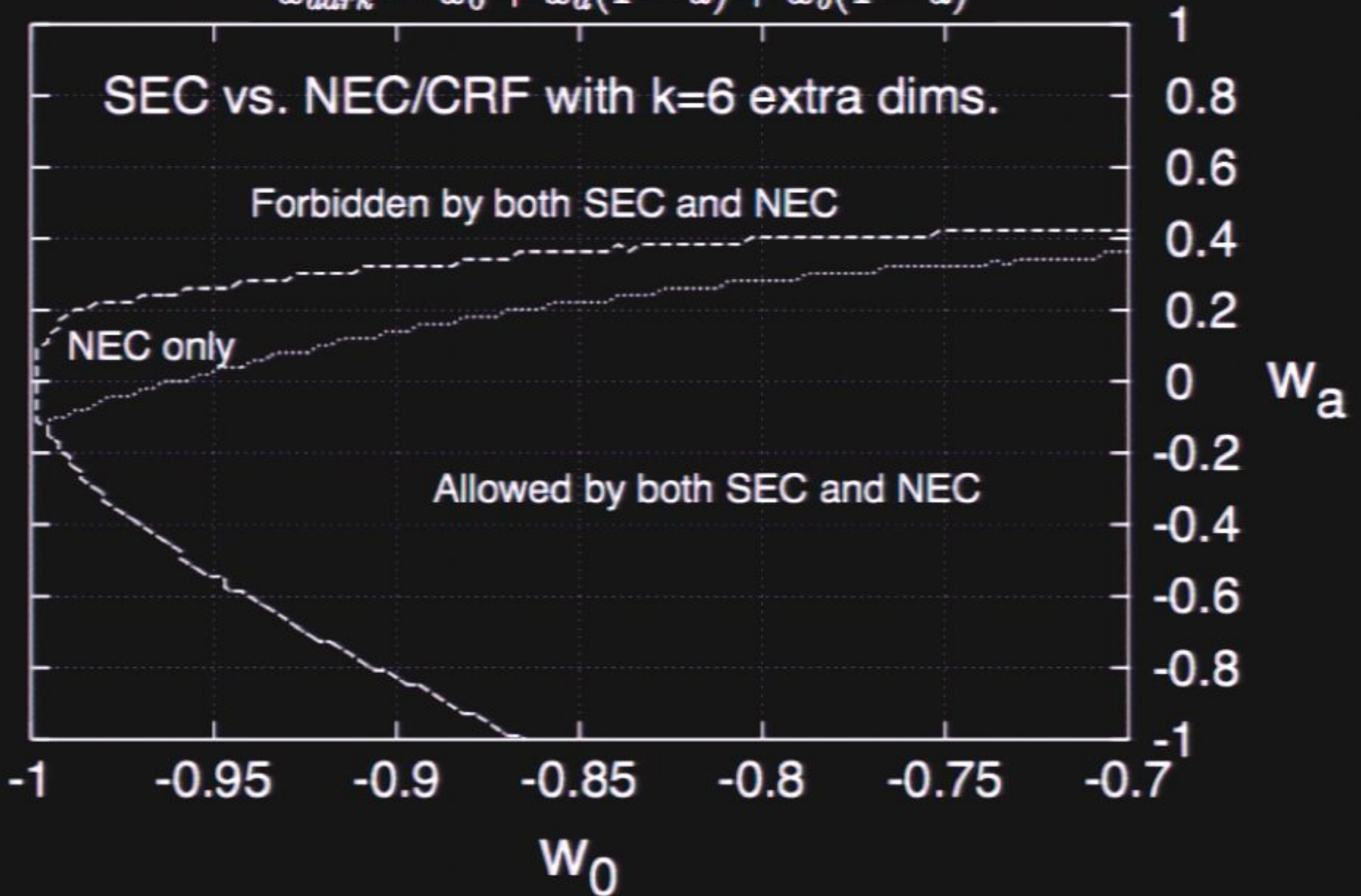
DETF linear
parameterization

This term allows us to ensure $w_{dark} > -1$ even when the scale factor is significantly different from unity

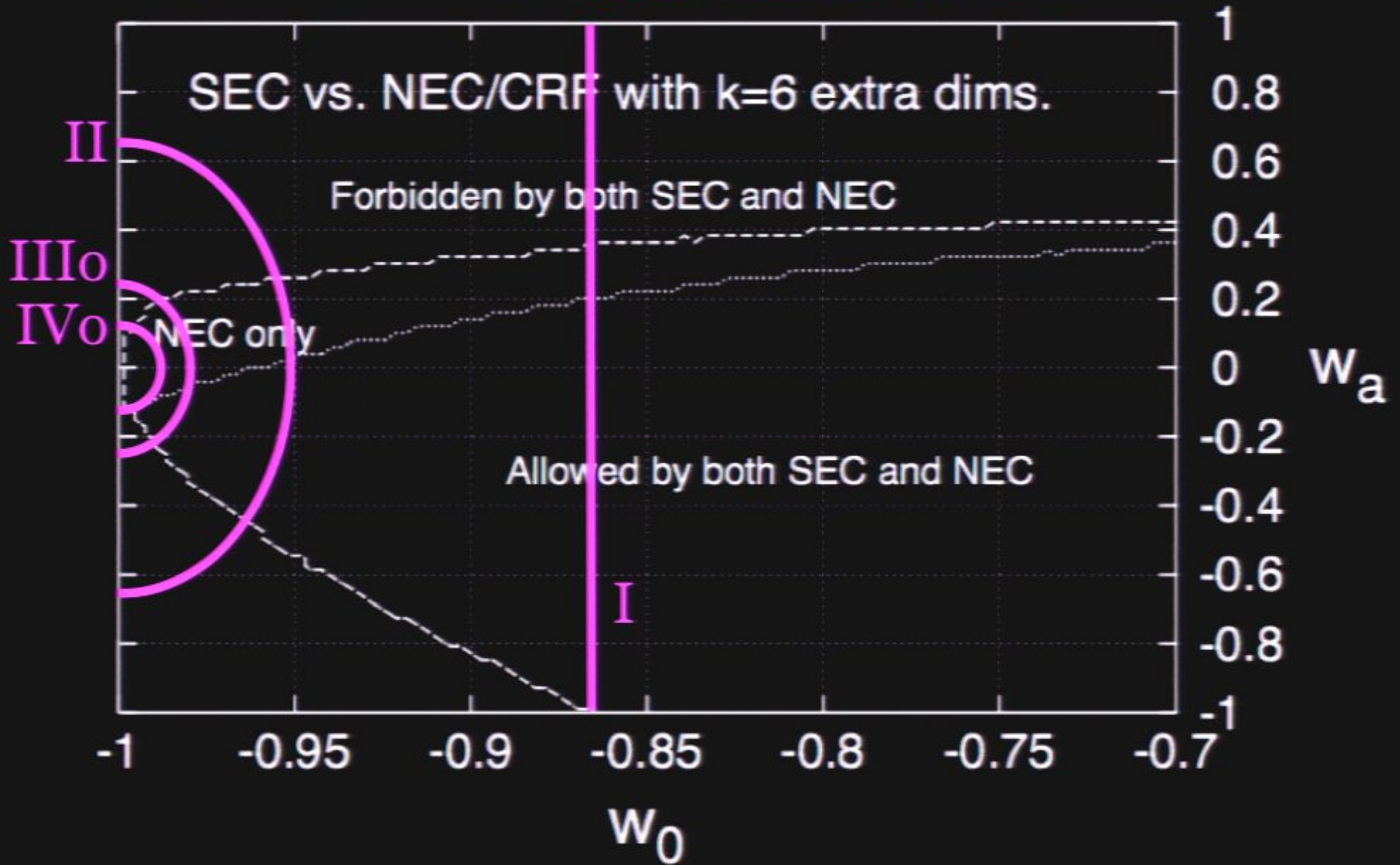
We explored model space by running many different realizations of dark energy models and ζ evolution on Cosmos, the UK national cosmology supercomputer



$$w_{dark} = w_0 + w_a(1 - a) + w_b(1 - a)^2$$



DETF constraints



G_N variation constraints

BBN constraints using WMAP Ω_b and HEP N_ν
Copi, Davis, Krauss, PRL 92 171301 (2004)

$$\frac{G_{BBN}}{G_0} = 1.01^{+0.20}_{-0.16}$$

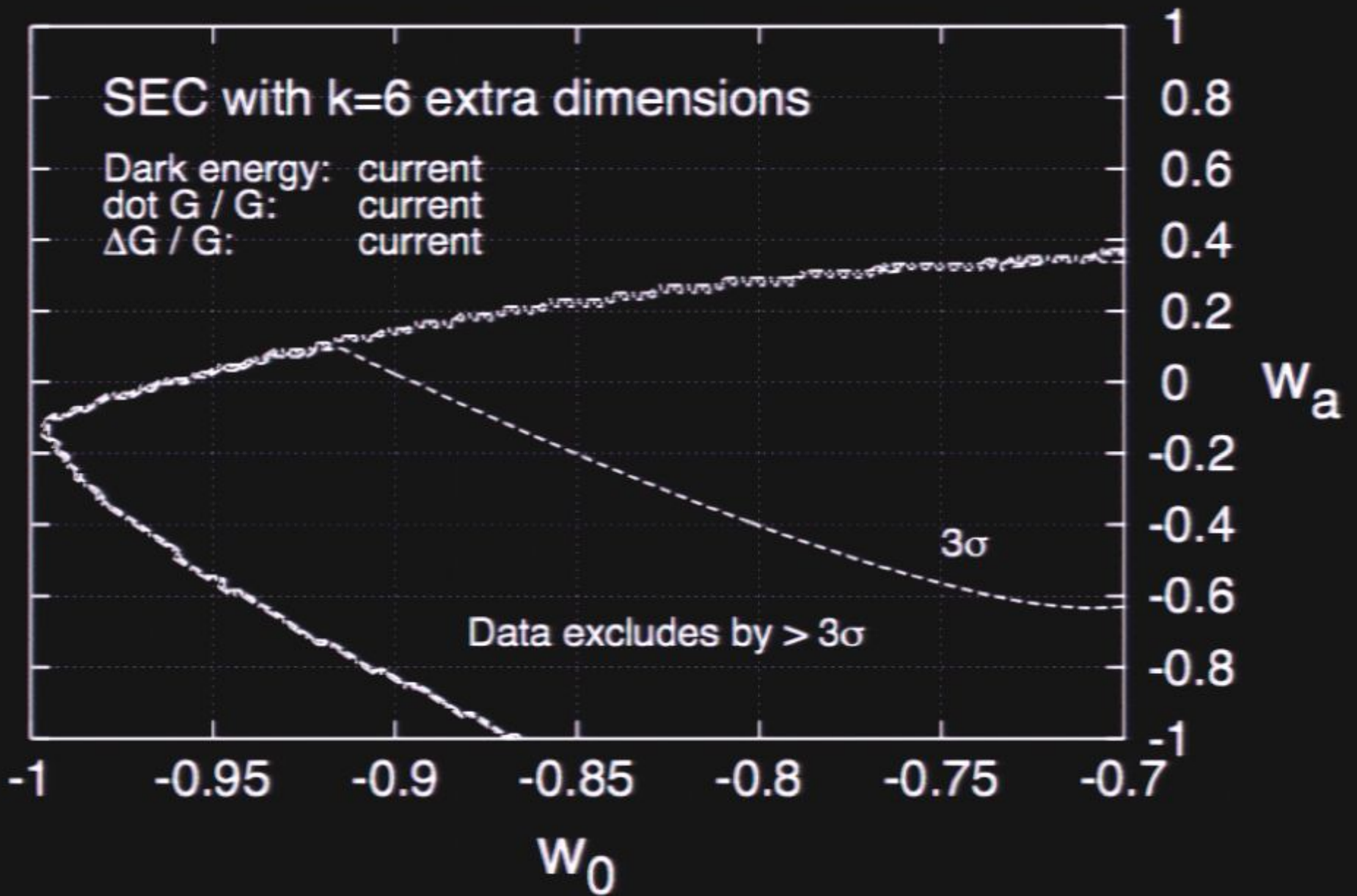
Radar ranging data from
Mariner 10, Mercury and Venus

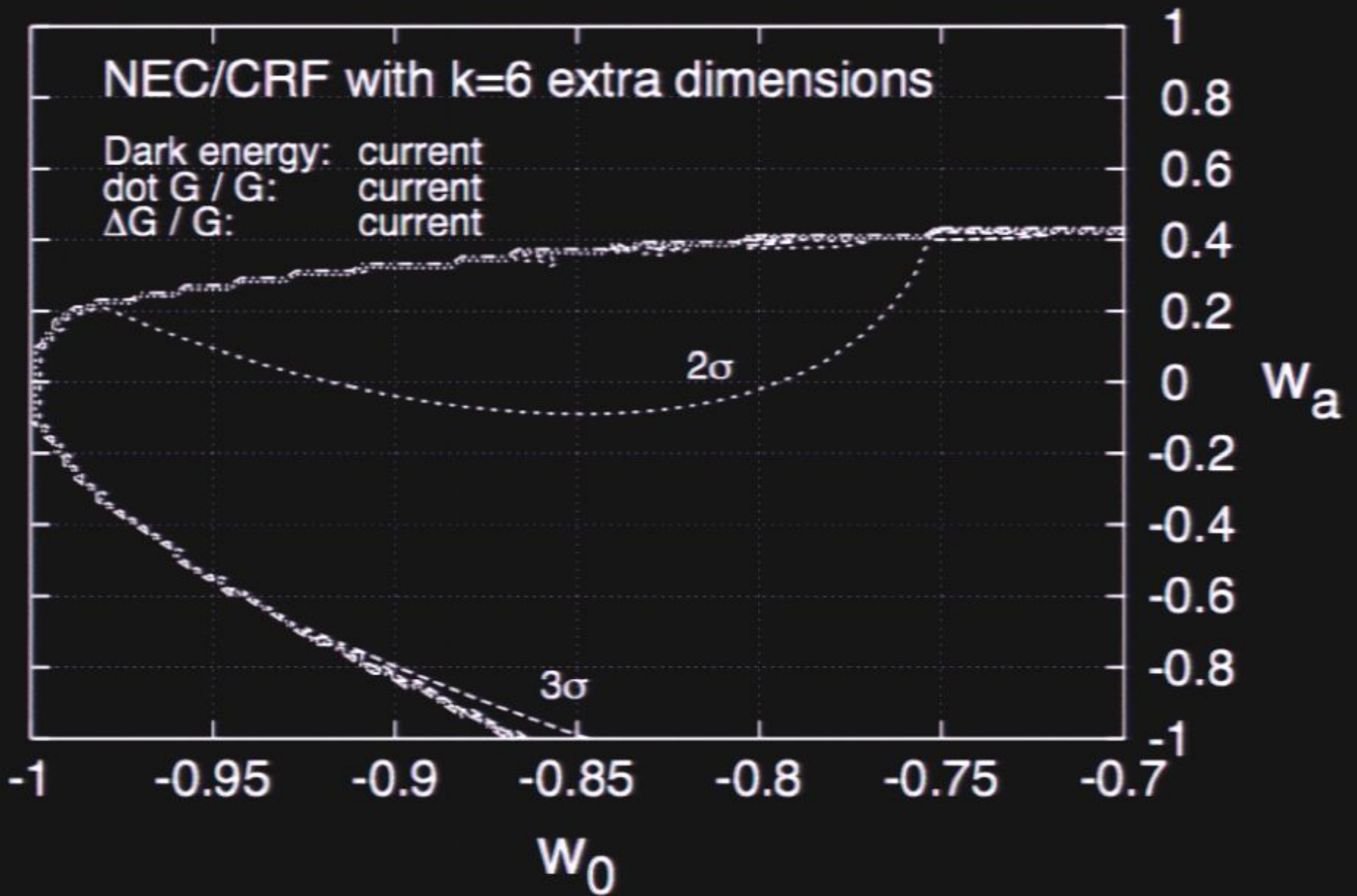
$$\dot{G}/G = (0 \pm 2) \times 10^{-12} \text{ yr}^{-1}$$

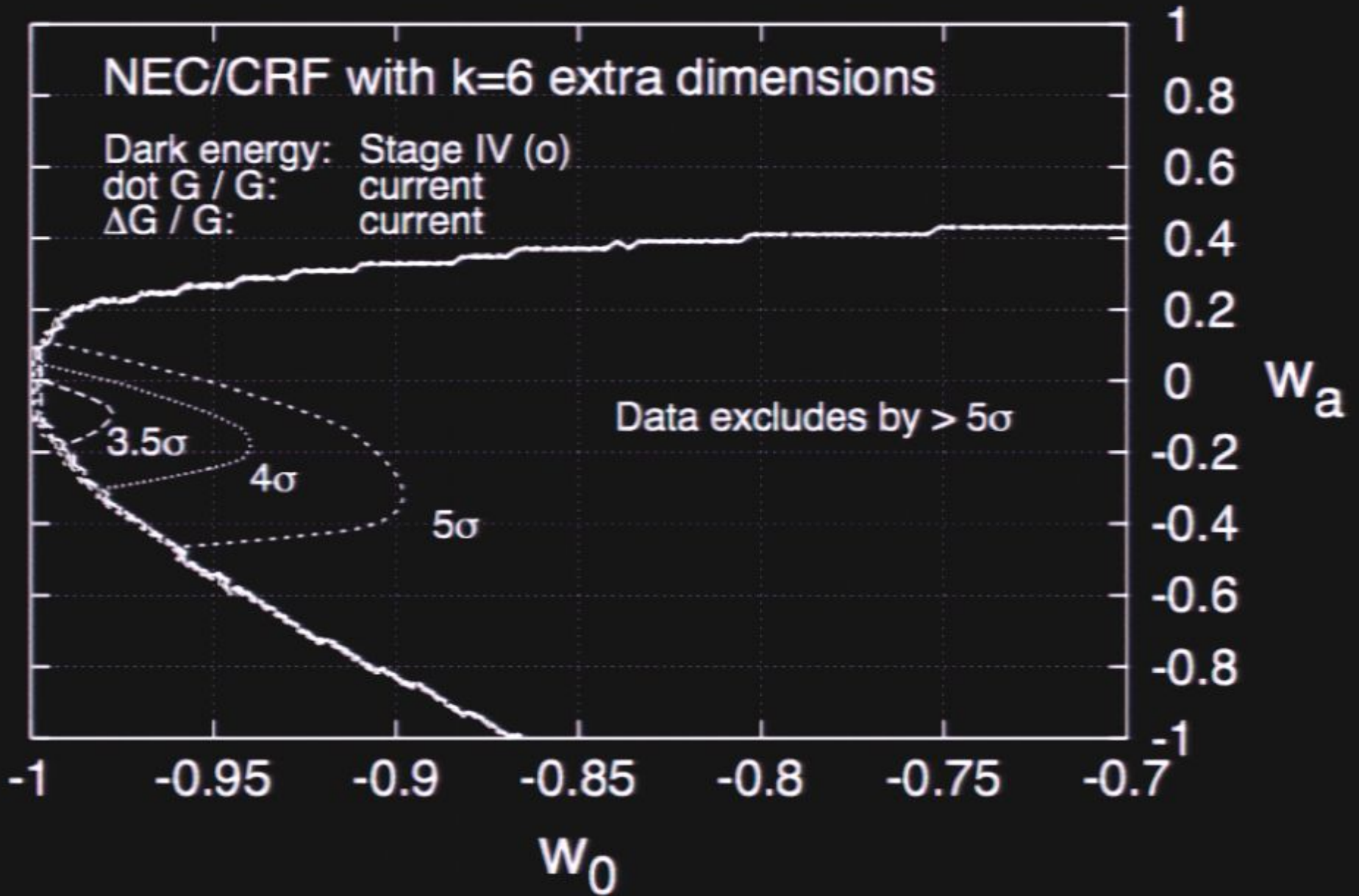
PSR 1913+16

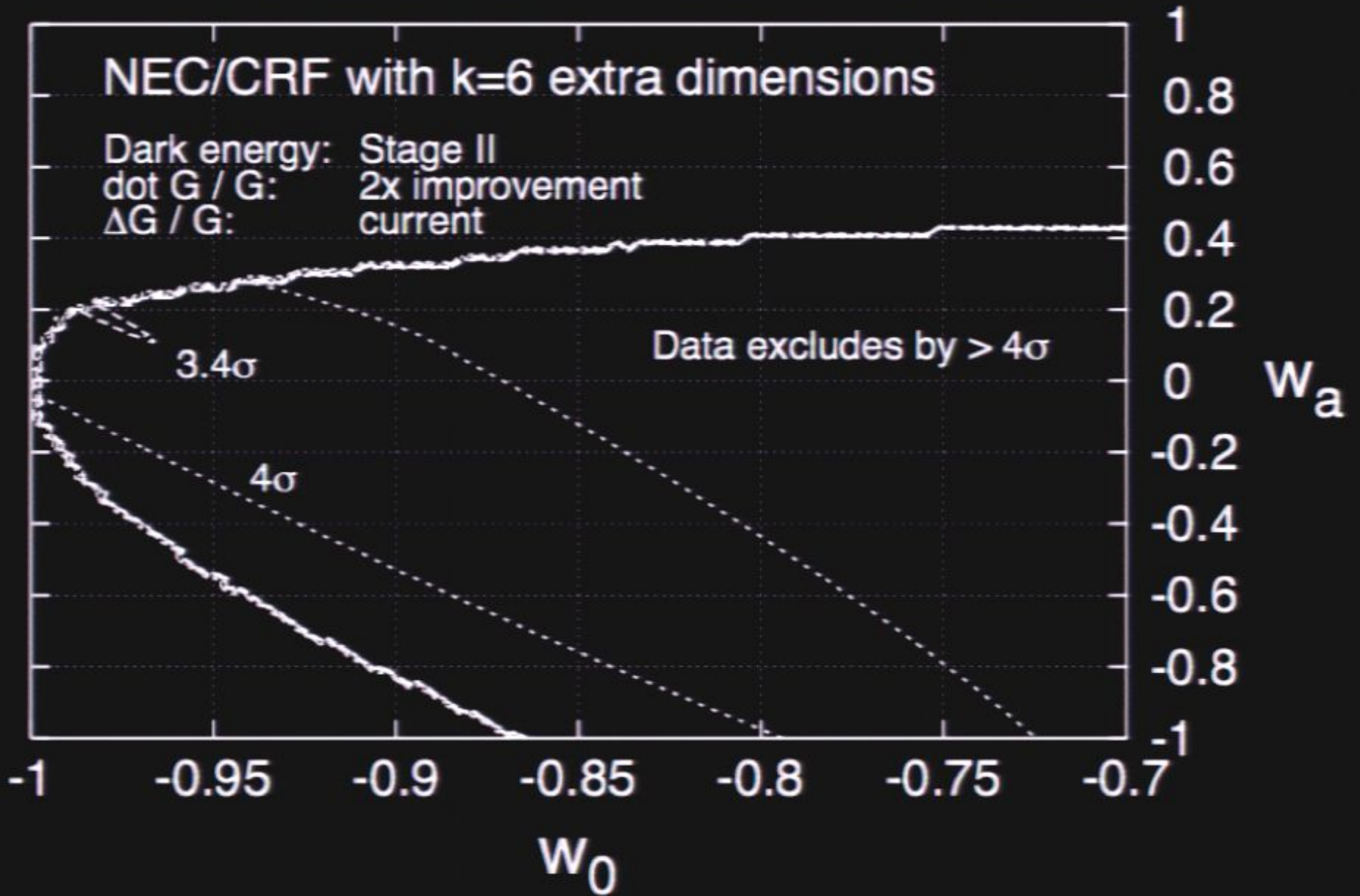
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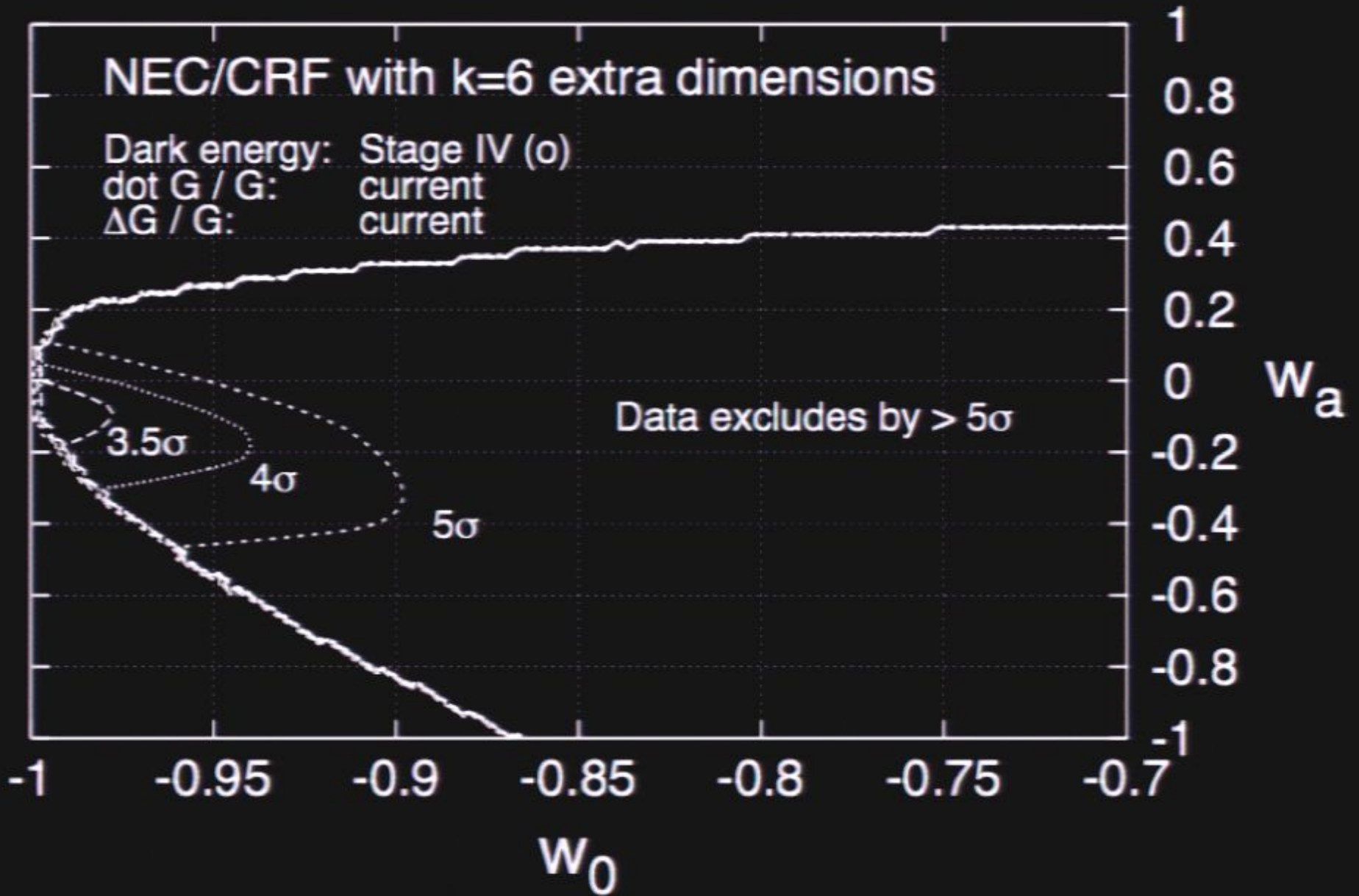
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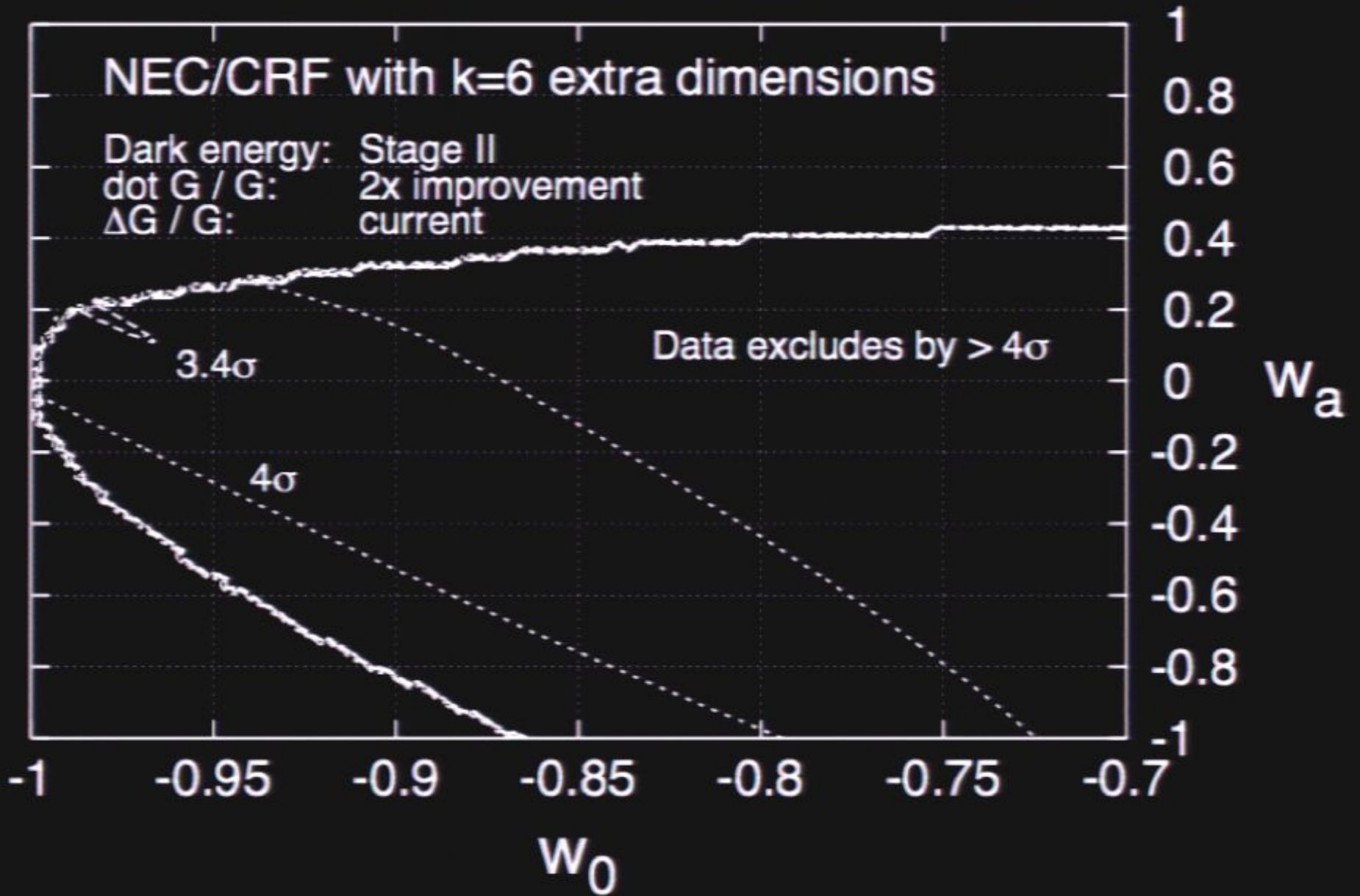












Conclusions

- Obtaining an accelerating 4D universe puts surprisingly strong constraints on theories with extra dimensions
- Suggests a series of interesting questions about inflationary models, their sources for NEC violation, and the nature of the accelerating/decelerating transition
- A novel technique for leveraging future dark energy constraints with G_N variation constraints, which can provide a powerful probe of extra-dimensional physics.