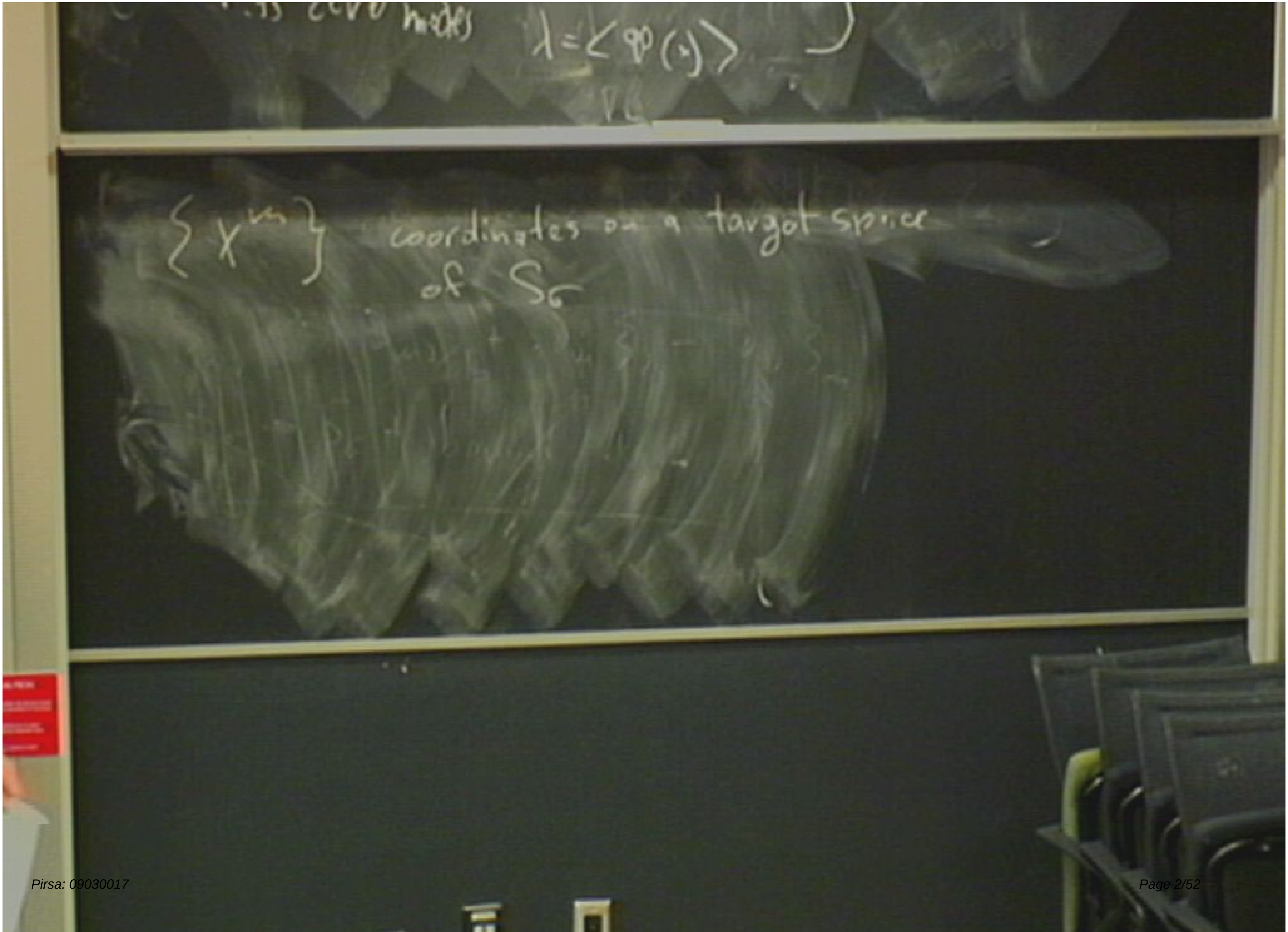
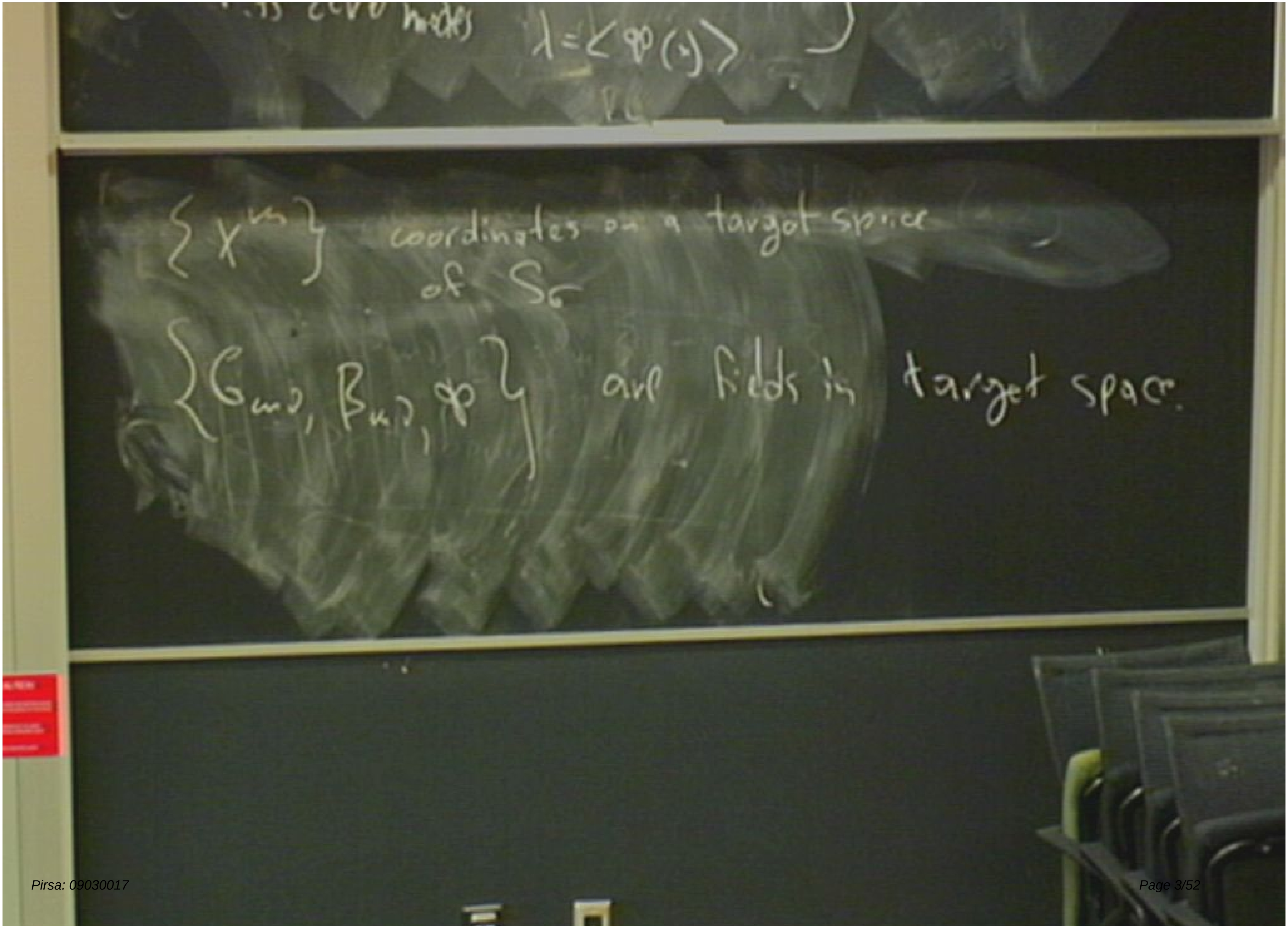
Title: Introduction to the Bosonic String Part B

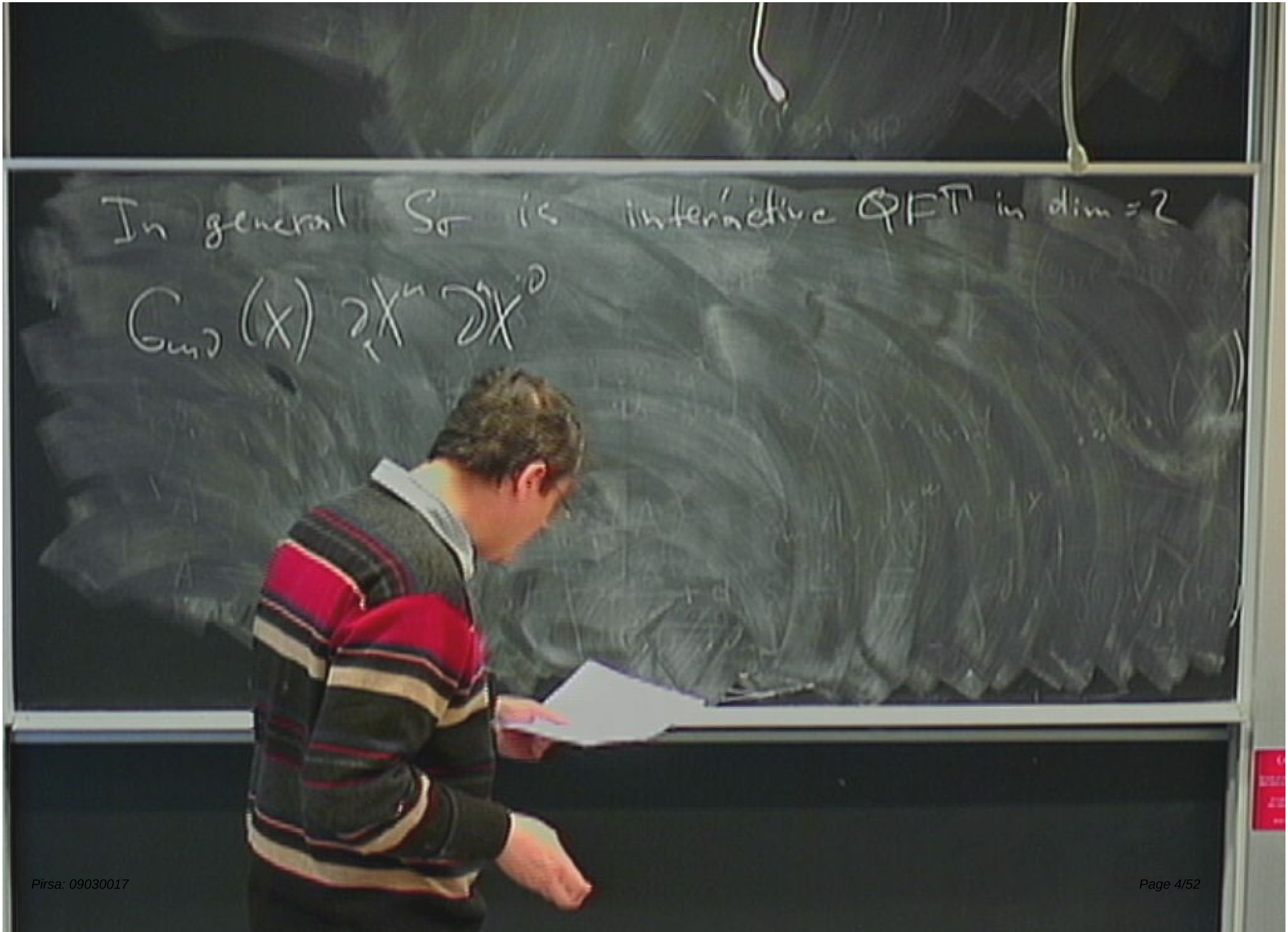
Date: Mar 27, 2009 12:00 PM

URL: http://pirsa.org/09030017

Abstract: This course provides a thorough introduction to the bosonic string based on the Polyakov path integral and conformal field theory. We introduce central ideas of string theory, the tools of conformal field theory, the Polyakov path integral, and the covariant quantization of the string. We discuss string interactions and cover the tree-level and one loop amplitudes. More advanced topics such as T-duality and D-branes will be taught as part of the course. The course is geared for M.Sc. and Ph.D. students enrolled in Collaborative Ph.D. Program in Theoretical Physics. Required previous course work: Quantum Field Theory (AM516 or equivalent). The course evaluation will be based on regular problem sets that will be handed in during the term. The primary text is the book: 'String theory. Vol. 1: An introduction to the bosonic string. J. Polchinski (Santa Barbara, KITP) . 1998. 402pp. Cambridge, UK: Univ. Pr. (1998) 402 p.' All interested students should contact Alex Buchel at abuchel@uwo.ca as soon as possible.


zero modes
λ = ⟨φ(x)⟩
{X^m} coordinates on a target space
of


zero modes
$\lambda = \langle \varphi(x) \rangle$
$\{X^m\}$ coordinates on a target space of Σ
$\{G_{mn}, B_{mn}, \varphi\}$ are fields in target space.


In general S_σ is interactive QFT in dim = 2
$G_{\mu\nu}(X)\, \partial X^\mu \bar{\partial} X^\nu$

In general S_σ is interactive QFT in dim = 2

$$G_{\mu\nu}(X)\,\partial_a X^\mu \partial_b X^\nu = \left[G_{\mu\nu}(x_0) + G_{\mu\nu;\rho}\, Y^\rho + \dots \right] \partial_a Y^\mu \partial_b Y^\nu$$

$$X^\mu = x_0^\mu + Y^\mu(\sigma)$$

(x_0^μ: fixed classical)

In general S_σ is interactive QFT in dim = 2

$$G_{\mu\nu}(X)\,\partial X^\mu \partial X^\nu = \left[G_{\mu\nu}(x_0) + G_{\mu\nu;\rho} Y^\rho + \dots \right] \partial_a Y^\mu \partial_b Y^\nu$$

$$X^\mu = x_0^\mu + Y^\mu(\sigma)$$

↑ fixed classical

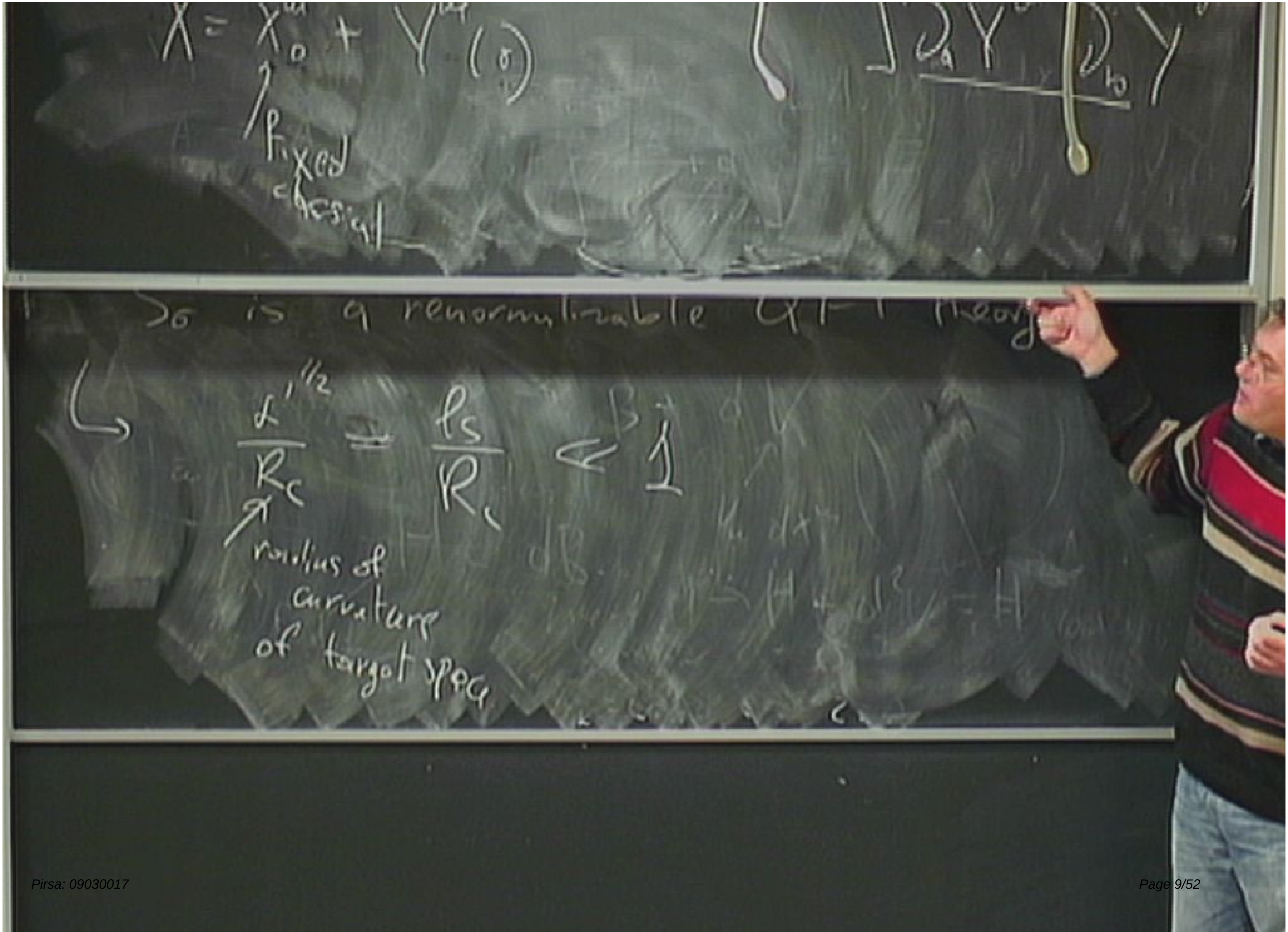
S_0 is a renormalizable QFT theory

fixed
classical

S_σ is a renormalizable QFT if...

$$\hookrightarrow \quad \frac{\alpha'^{1/2}}{R_c} \simeq \frac{\ell_s}{R_c}$$

↗ radius of curvature of target space



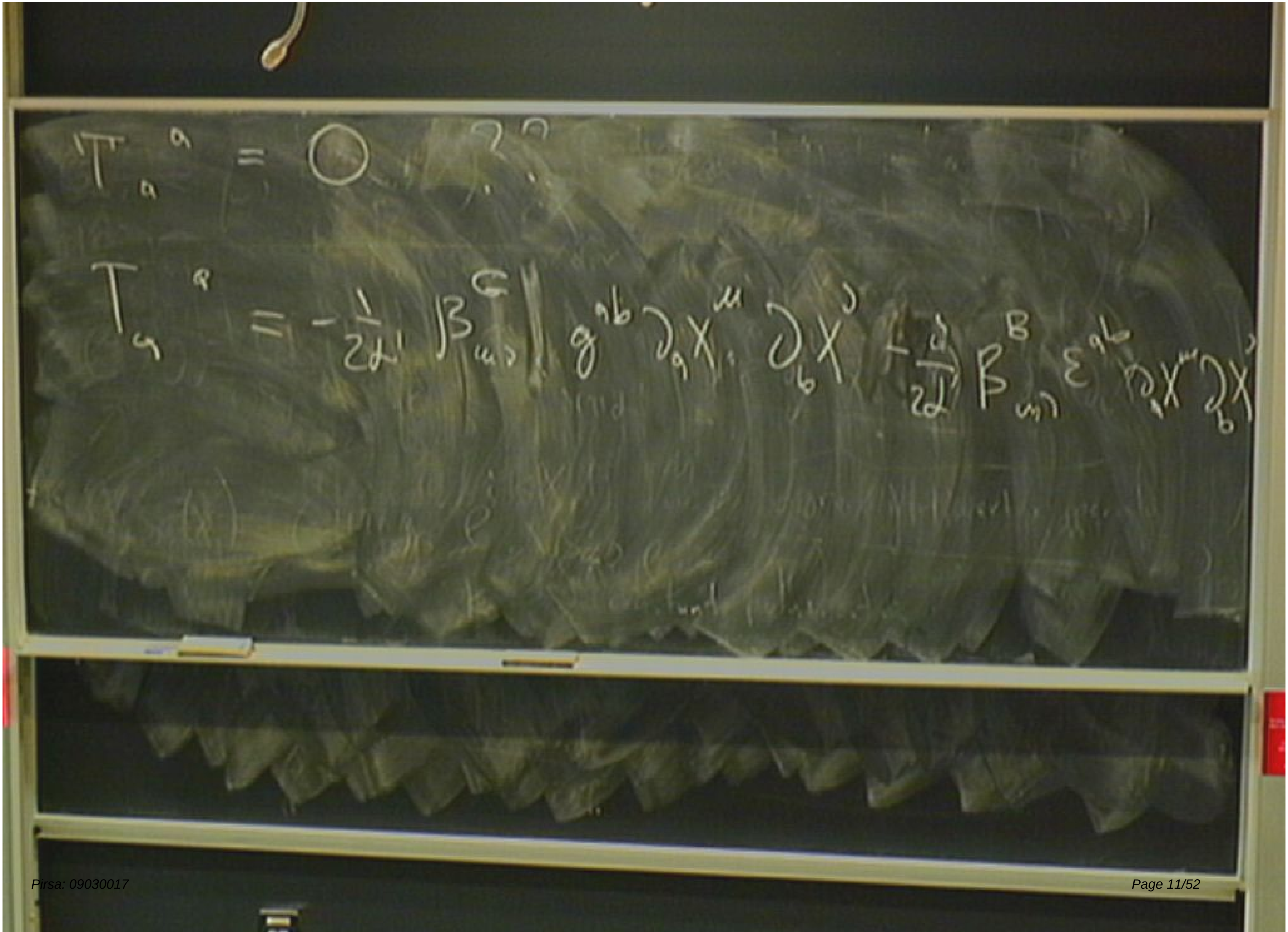
X = X_0 + Y(σ)
fixed classical
is a renormalizable QFT theory
α'^{1/2} / R_c ≃ ℓ_s / R_c << 1
radius of curvature of target space

fixed
classical

S_σ is a renormalizable QFT theory

↳ $$\frac{\alpha'^{1/2}}{R_c} = \frac{l_s}{R_c} \ll 1$$

↗ radius of curvature of target space


T_a^a = 0 ??
T_a^a = -1/(2α') β^G_{μν} g^{ab} ∂_a X^μ ∂_b X^ν + i/(2α') β^B_{μν} ε^{ab} ∂_a X^μ ∂_b X^ν

$$T_a{}^a = -\frac{1}{2\alpha'}\beta^G_{\mu\nu}\, g^{ab}\partial_a X^\mu \partial_b X^\nu + \frac{i}{2\alpha'}\beta^B_{\mu\nu}\,\varepsilon^{ab}\partial_a X^\mu \partial_b X^\nu - \frac{1}{2}\beta^\Phi R$$

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$$T_a{}^a = (\partial X)^2 + R$$

$$T_a{}^a = -\frac{1}{2\alpha'}\beta^G_{\mu\nu}\, g^{ab}\partial_a X^\mu \partial_b X^\nu - \frac{i}{2\alpha'}\beta^B_{\mu\nu}\,\epsilon^{ab}\partial_a X^\mu \partial_b X^\nu - \frac{1}{2}\beta^\Phi R$$

$$T_a{}^a = (\partial X)^2 + R$$

fixed
classical

S_σ is a renormalizable QFT theory

$$\hookrightarrow \quad \frac{\alpha'^{1/2}}{R_c} = \frac{l_s}{R_c} \ll 1$$

(R_c) radius of curvature of target space

$$- \frac{1}{2} \beta^{\Phi} R$$

$$\Pi'^{\,\varphi}_{q} = (\Phi)^2 + 4 \cdot R$$

$$\beta^{G}_{\mu\nu} = \alpha' \left[R_{\mu\nu} + 2 \nabla_\mu \nabla_\nu \Phi - \frac{1}{4} H_{\mu\lambda\rho} H_{\nu}{}^{\lambda\rho} \right] + O(\alpha'^2)$$

$$-\frac{1}{2}\beta^{\phi} R$$

$$T'^{\,q}_{\;q} = (\phi)^2 + \dots R$$

$$H = dB$$

$$\beta^{G}_{\mu\nu} = \alpha' \left[R_{\mu\nu} + 2\nabla_\mu \nabla_\nu \phi - \frac{1}{4} H_{\mu\lambda\rho} H_\nu{}^{\lambda\rho} \right] + O(\alpha'^2)$$

target space curvature

$$- \frac{1}{2} \beta^{\Phi} R$$

$$\Pi'{}_q{}^p = (\Phi)^2 + \cdot R$$

$H = dB$

$$\beta^G_{\mu\nu} = \alpha' \left[R_{\mu\nu} + 2 \nabla_\mu \nabla_\nu \Phi - \frac{1}{4} H_{\mu\lambda\rho} H_\nu{}^{\lambda\rho} \right] + O(\alpha'^2)$$

target space curvature

$$\beta^B_{\mu\nu} = \alpha' \left[- \frac{1}{2} \nabla^\rho H_{\rho\mu\nu} + \nabla^\rho \Phi \, H_{\rho\mu\nu} \right] + O(\alpha'^2)$$

$$\ldots + \alpha' \left[-\frac{1}{2}\nabla^2\varphi + \nabla_\mu\varphi\nabla^\mu\varphi - \frac{1}{24}H_{\mu\nu\rho}H^{\mu\nu\rho}\right] + O(\alpha'^2)$$

$$T_a{}^a = 0 \quad ??$$

$$T_a{}^a = -\frac{1}{2\alpha'}\beta^G_{\mu\nu}\, g^{ab}\partial_a X^\mu \partial_b X^\nu + \frac{i}{2\alpha'}\beta^B_{\mu\nu}\,\varepsilon^{ab}\partial_a X^\mu \partial_b X^\nu - \frac{1}{2}\beta^\Phi R$$

$$T'_a{}^a = \cancel{O(\alpha')^2} + \cdot R$$

$$\beta^{\Phi} = \frac{D-26}{6} + \alpha' \left[-\frac{1}{2}\nabla^2\Phi + \nabla_\mu \Phi \nabla^\mu \Phi - \frac{1}{24} H_{\mu\nu\rho} H^{\mu\nu\rho} \right] + O(\alpha'^2)$$

$$T_a{}^a = 0 \quad ??$$

$$T_a{}^a = -\frac{1}{2\alpha'} \beta^{G}_{\mu\nu}\, g^{ab} \partial_a X^\mu \partial_b X^\nu - \frac{i}{2\alpha'} \beta^{B}_{\mu\nu}\, \epsilon^{ab} \partial_a X^\mu \partial_b X^\nu - \frac{1}{2} \beta^{\Phi} R$$

$$T_a{}^a = \ldots{}^{c} + \ldots R$$

$$\beta_G = \beta_B = \beta_\varphi = 0 \Leftrightarrow V'(\langle\varphi\rangle)$$


$\beta_G = \beta_B = \beta_\varphi = 0 \iff V'(\langle\varphi\rangle) = 0$

$$\frac{D-26}{6} + \alpha'\left[-\frac{1}{2}\nabla^2\Phi + \nabla_\mu\Phi\nabla^\mu\Phi - \frac{1}{24}H_{\mu\nu\rho}H^{\mu\nu\rho}\right] + \mathcal{O}(\alpha'^2)$$

$$T_a{}^a = 0 \;\; ??$$

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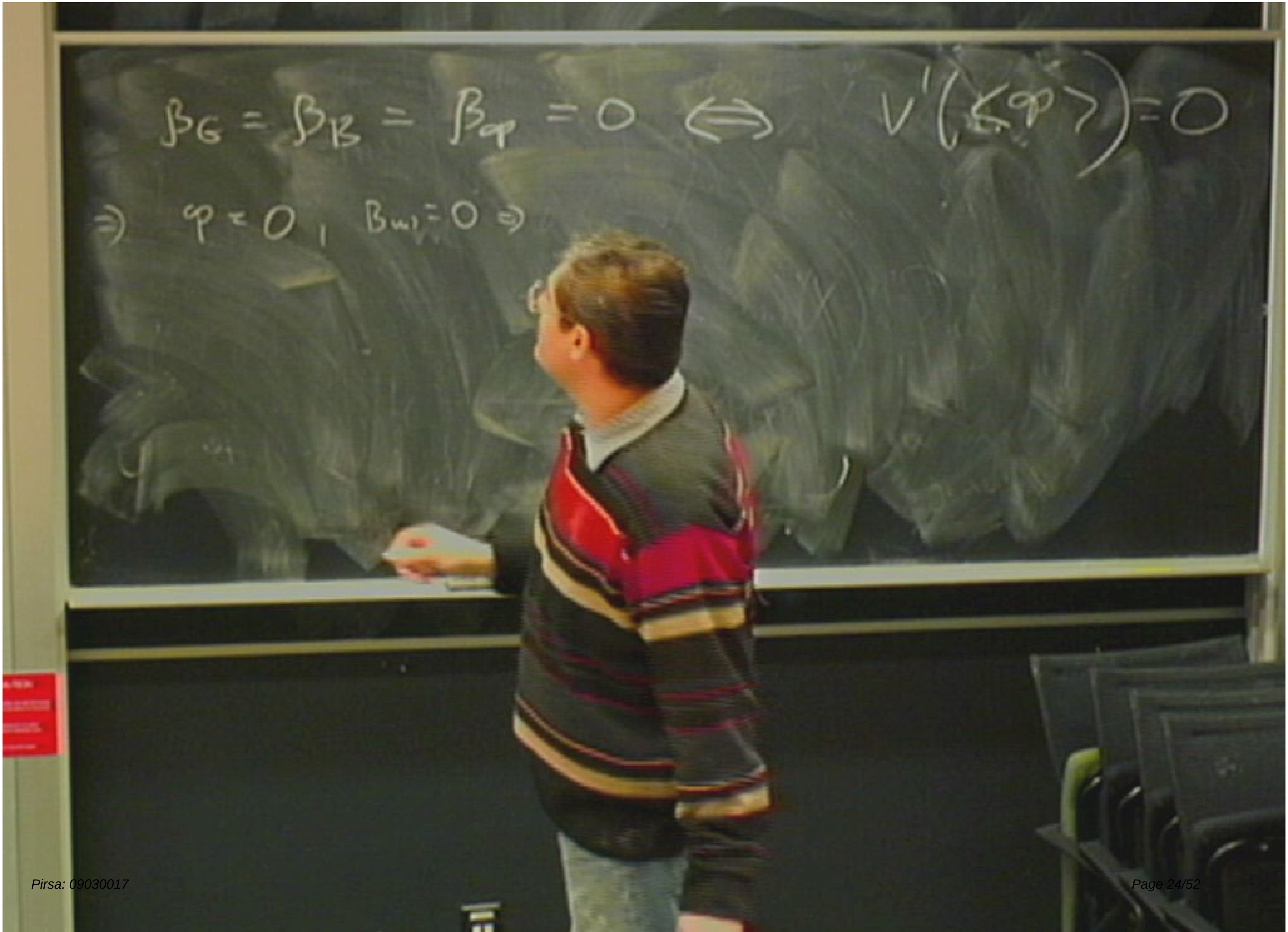
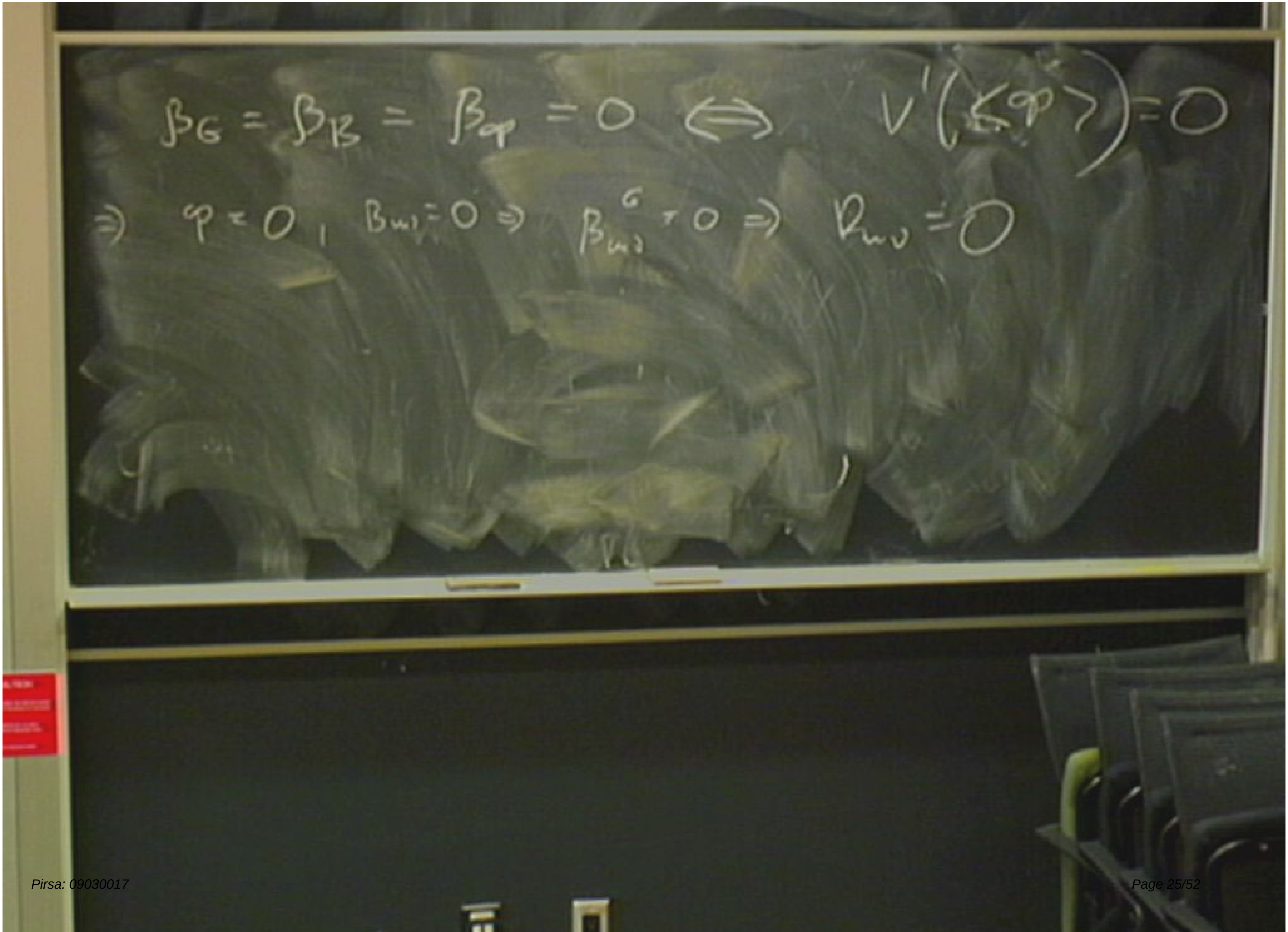
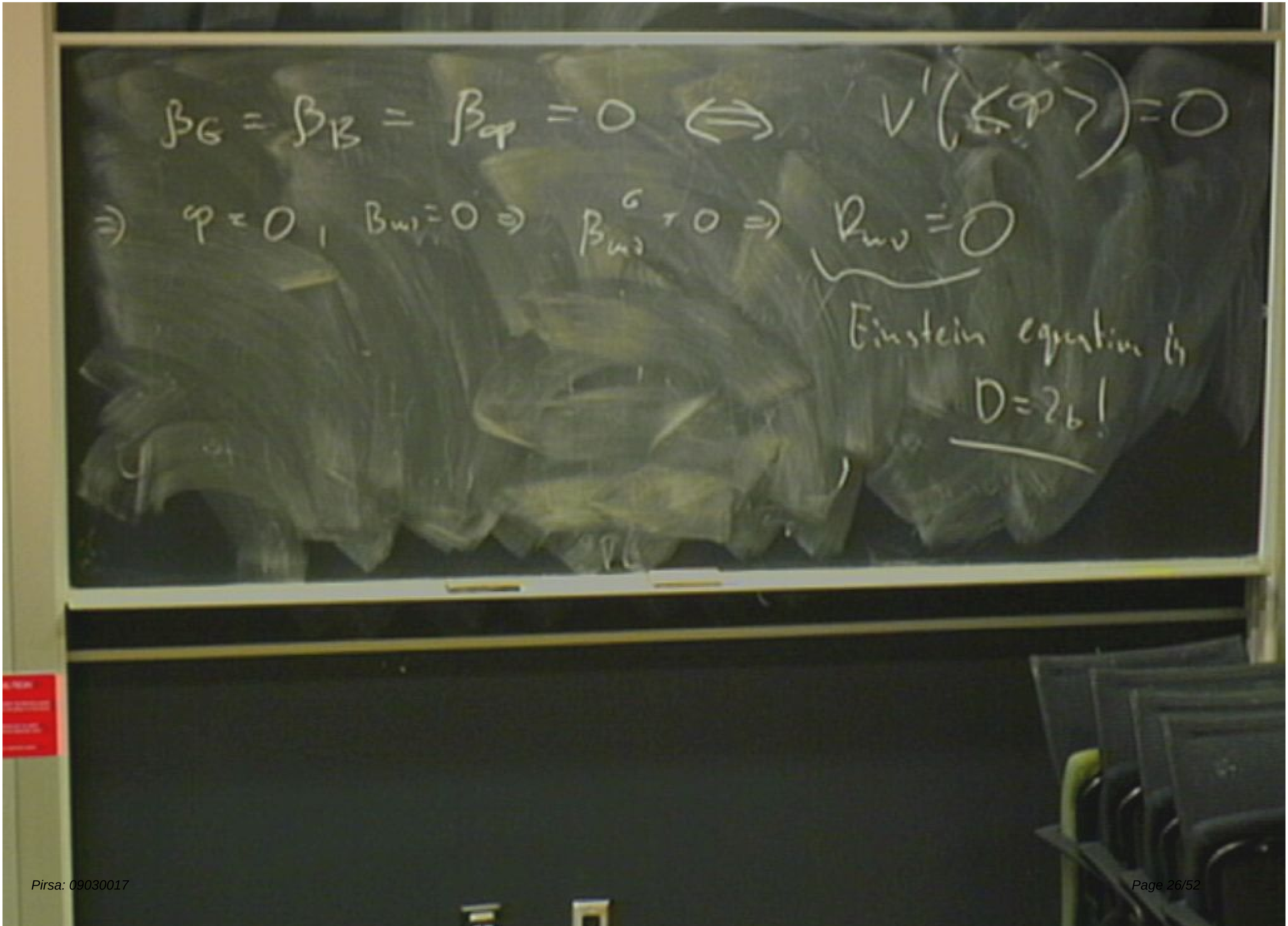
$$T_a{}^a = (\ldots)^2 + \ldots R$$


βG = βB = βφ = 0 ⟺ V'(⟨φ⟩) = 0
⇒ φ = 0 , B_wl = 0 ⇒


$\beta_G = \beta_B = \beta_\varphi = 0 \Leftrightarrow V'(\langle\varphi\rangle) = 0$
$\Rightarrow \varphi = 0, \; B_{\mu\nu} = 0 \Rightarrow \beta^G_{\mu\nu} = 0 \Rightarrow R_{\mu\nu} = 0$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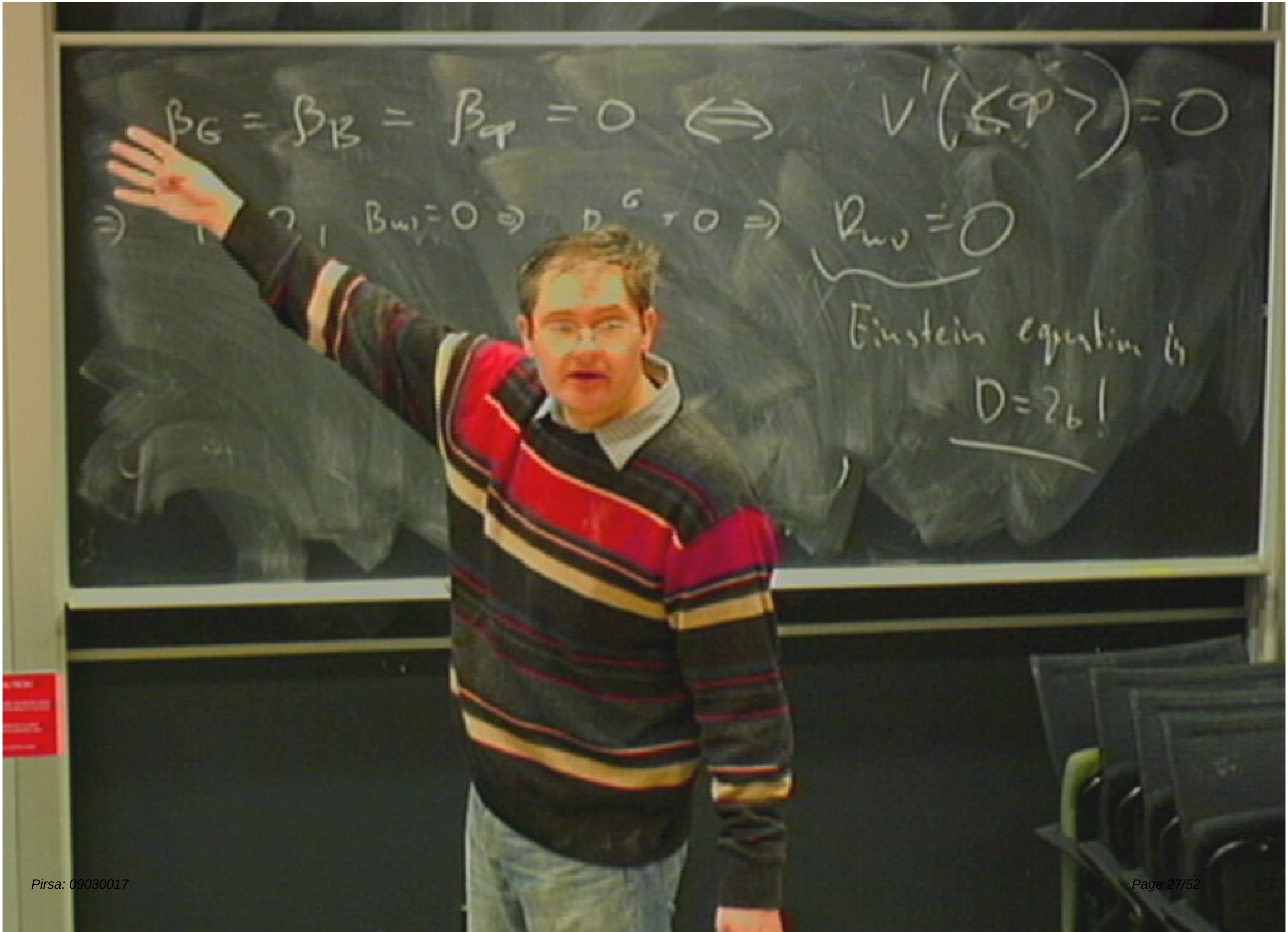
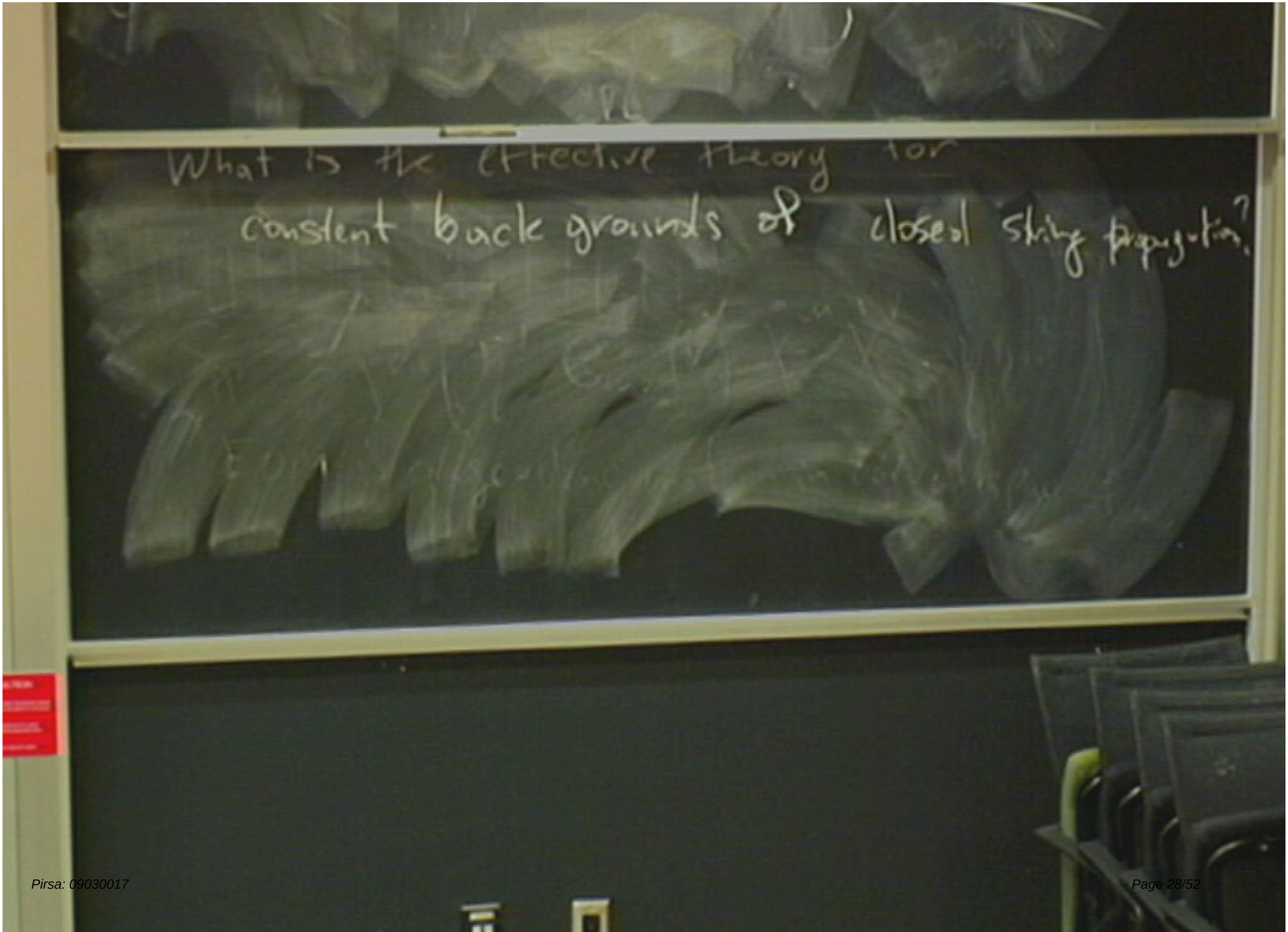
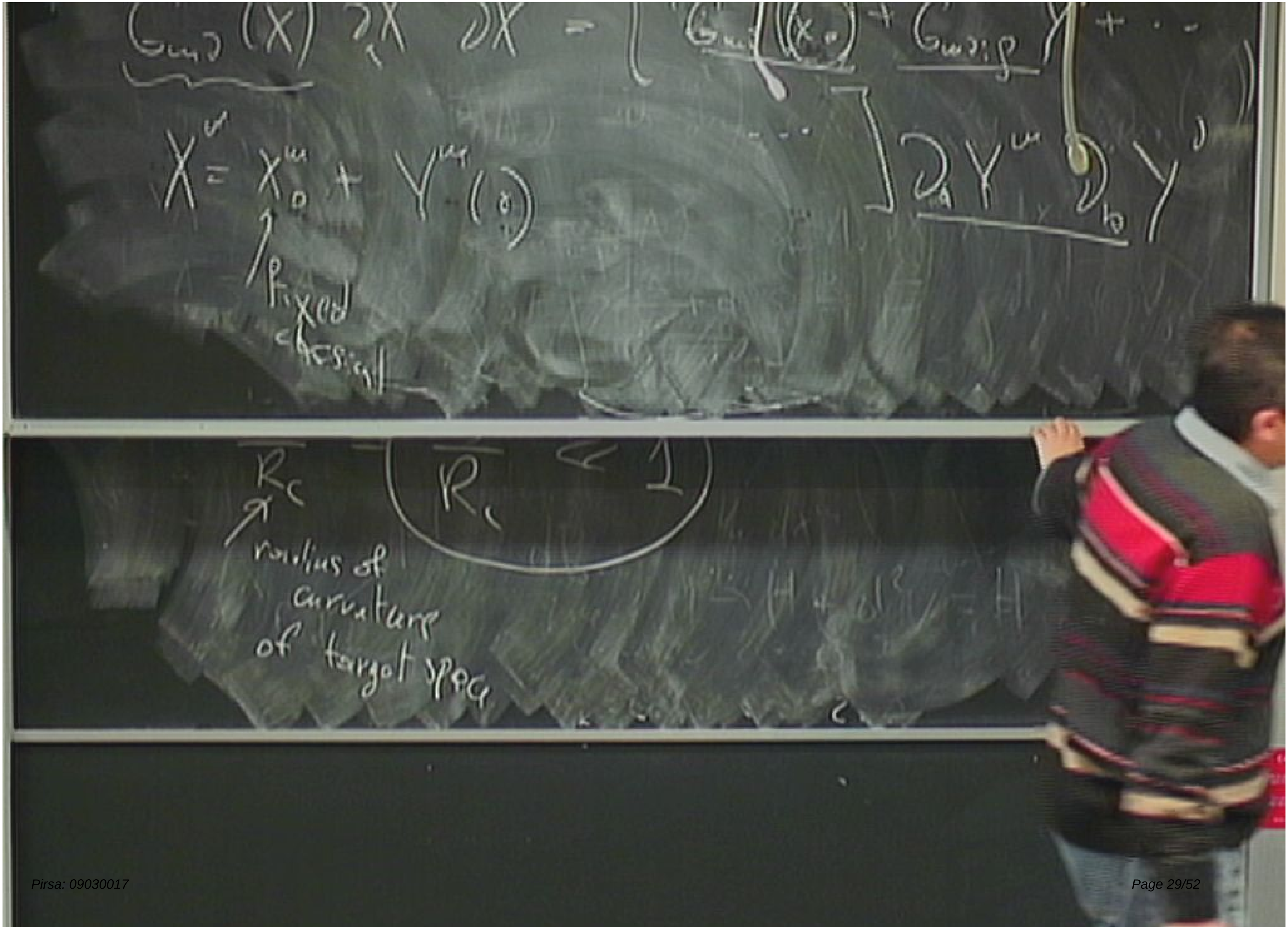
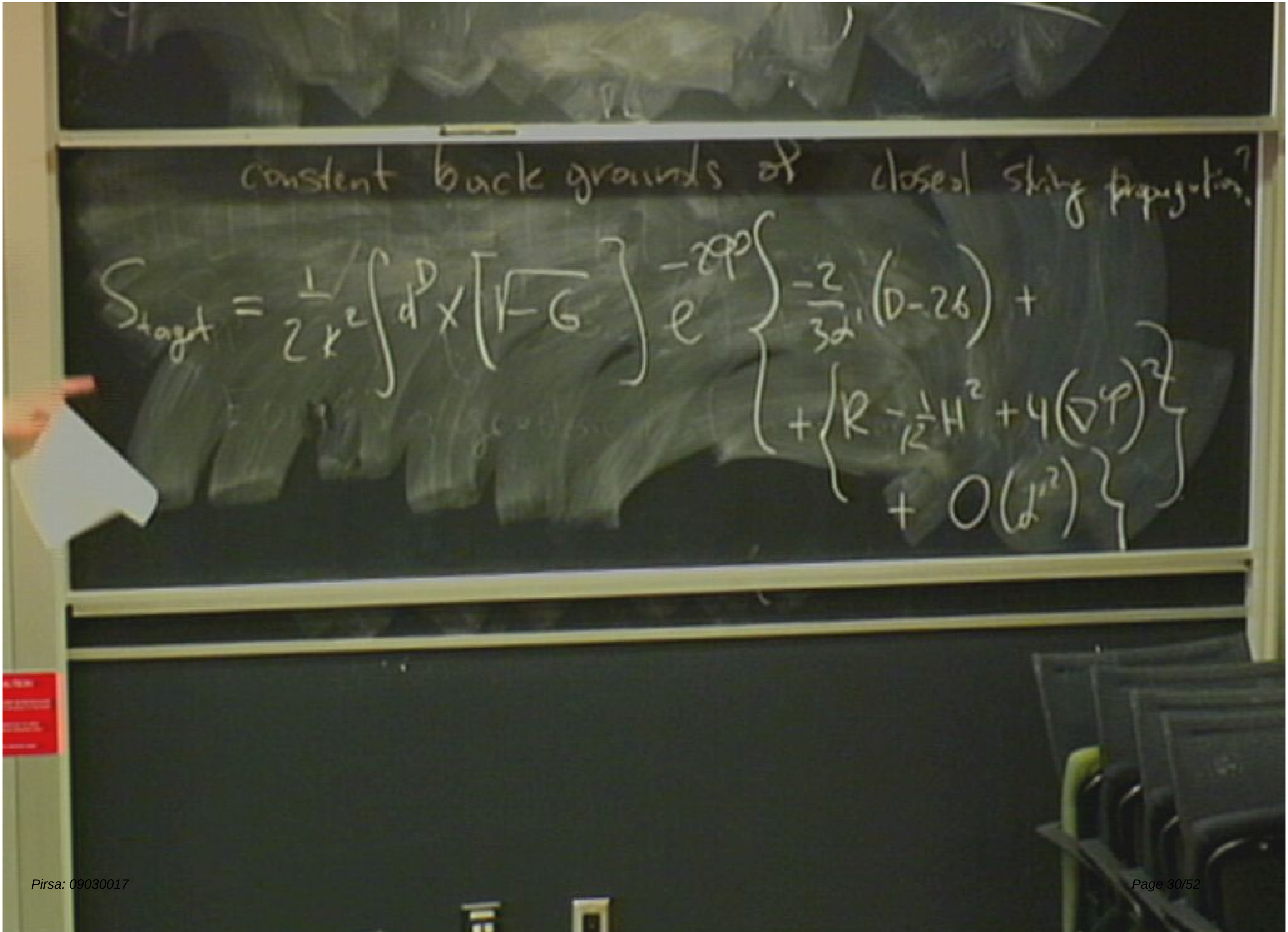
Einstein equation in
$D = 26$!


βG = βB = βφ = 0 ⟺ V'(⟨φ⟩) = 0
Bμν = 0 ⇒
Rμν = 0
Einstein equation in
D = 26!


What is the effective theory for
constant backgrounds of closed string propagation?


fixed
classical
radius of
curvature
of target space

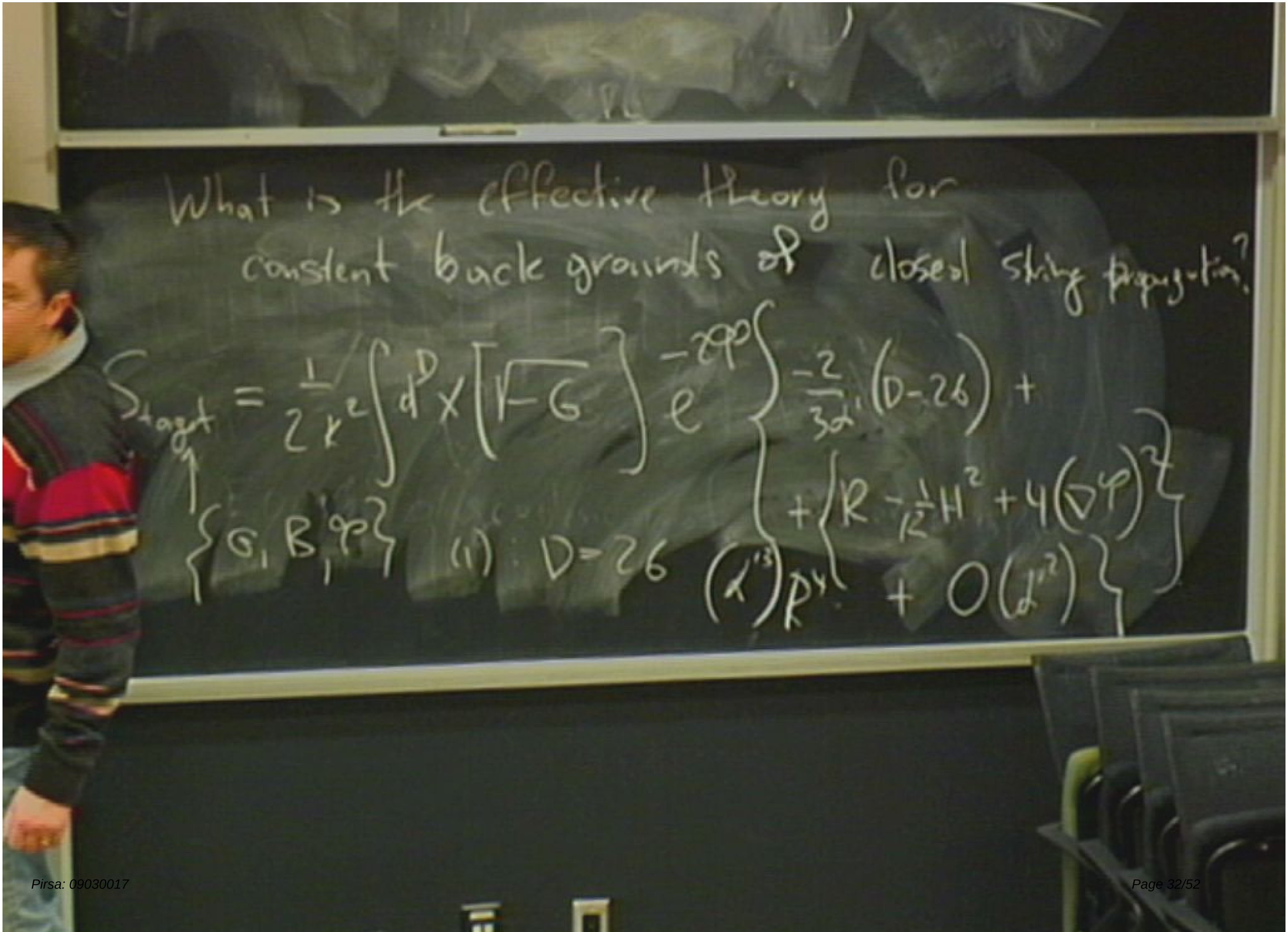

constant back grounds of closed string propagation?
$S_{target} = \frac{1}{2\kappa^2}\int d^D x \left[\sqrt{-G}\right] e^{-2\varphi}\left\{ \frac{-2}{3\alpha'}(D-26) + \left\{R - \frac{1}{12}H^2 + 4(\nabla\varphi)^2\right\} + O(\alpha'^2)\right\}$

What is the effective theory for constant backgrounds of closed string propagation?

$$S_{target} = \frac{1}{2\kappa^2}\int d^D x \sqrt{-G}\, e^{-2\varphi}\left\{ -\frac{2}{3\alpha'}(D-26) + \left\{ R - \frac{1}{12}H^2 + 4(\nabla\varphi)^2 \right\} + O(\alpha'^2) \right\}$$

$\uparrow$ $\{G, B, \varphi\}$

(1) $D = 26$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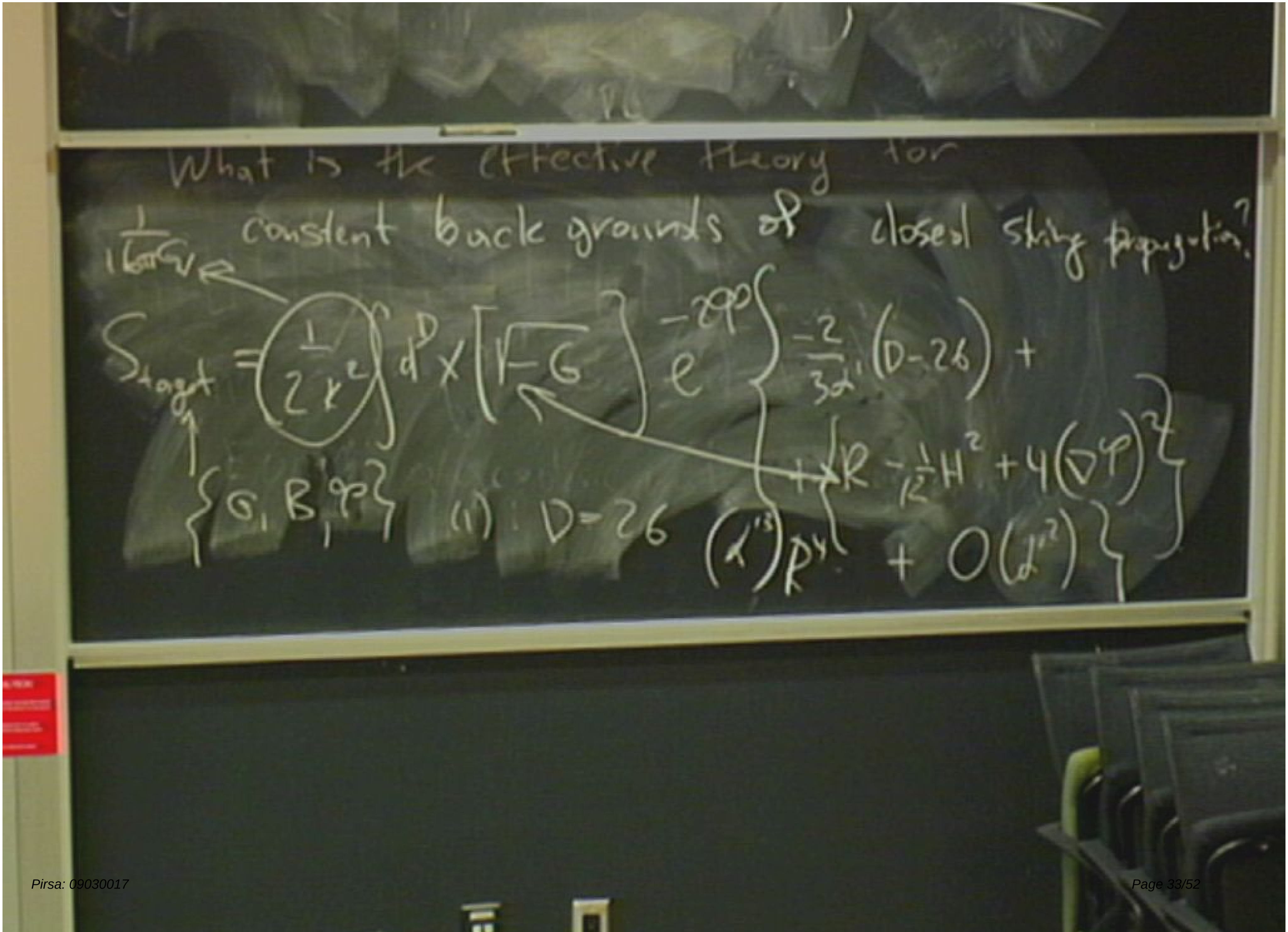
$\{G, B, \varphi\}$
(1) $D=26$
$(\alpha'^3)R^4$


What is the effective theory for
constant backgrounds of closed string propagation?
1/16πG_N
S_target = 1/2κ² ∫ d^D x √-G e^-2φ { -2/3α' (D-26) + R - 1/12 H² + 4(∇φ)² + O(α'²) }
{G, B, φ}
(i) D=26

$$G_{\mu\nu} \rightarrow G^{E}_{\mu\nu} = e^{-4\varphi/(D-2)}\, G_{\mu\nu}$$

↑ Einstein frame metric

↑ Sigma-model (or string frame metric)

$S^{E} =$

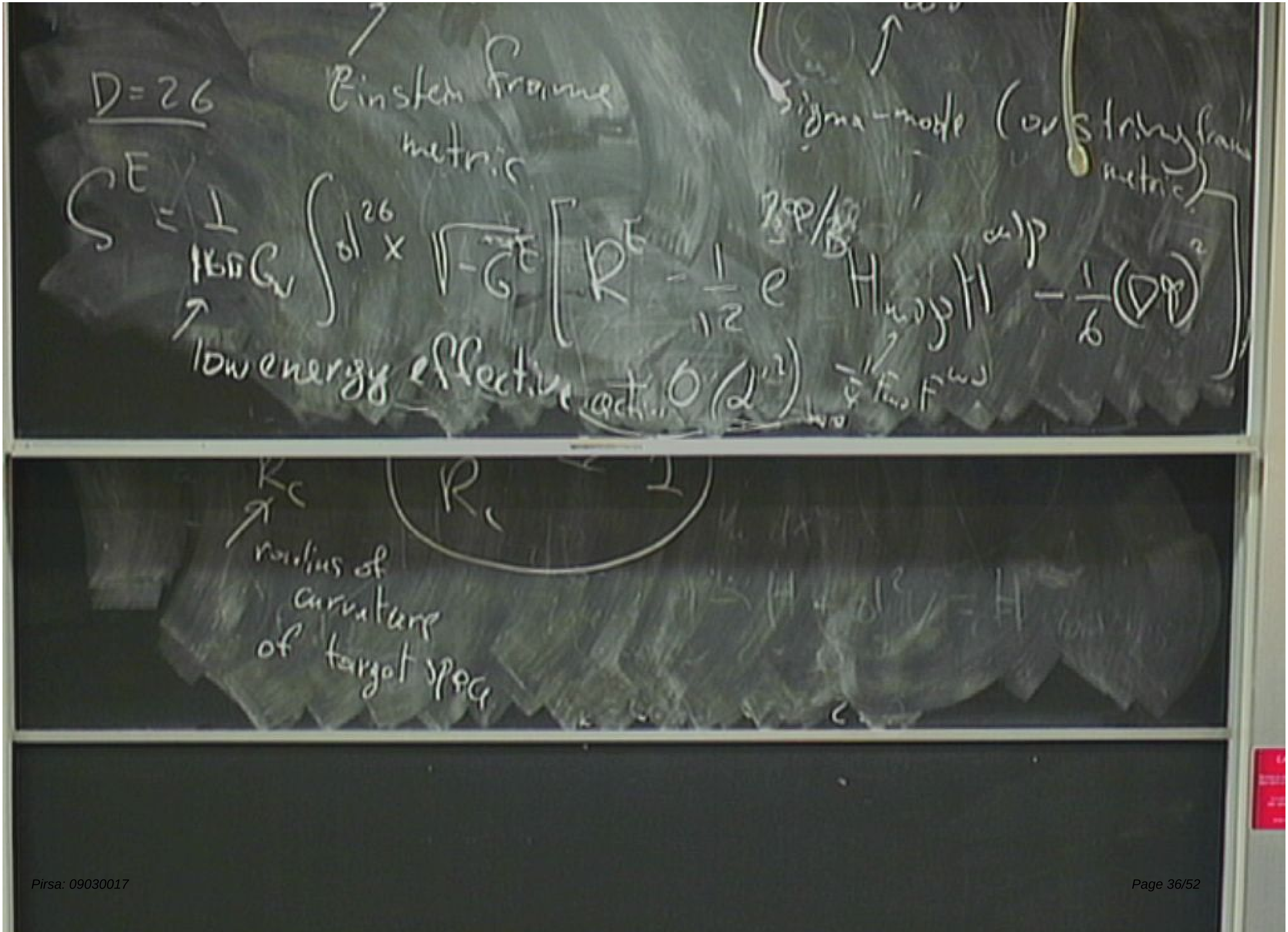
$$G_{\mu\nu} \to G^E_{\mu\nu} = e^{-4\varphi/(D-2)} G_{\mu\nu}$$

Einstein frame metric

Sigma-model (or string frame metric)

$\underline{D=26}$

$S^E =$


D=26
Einstein frame metric
sigma-model (or string frame metric)
low energy effective action O(α'²)
radius of curvature of target space

metric

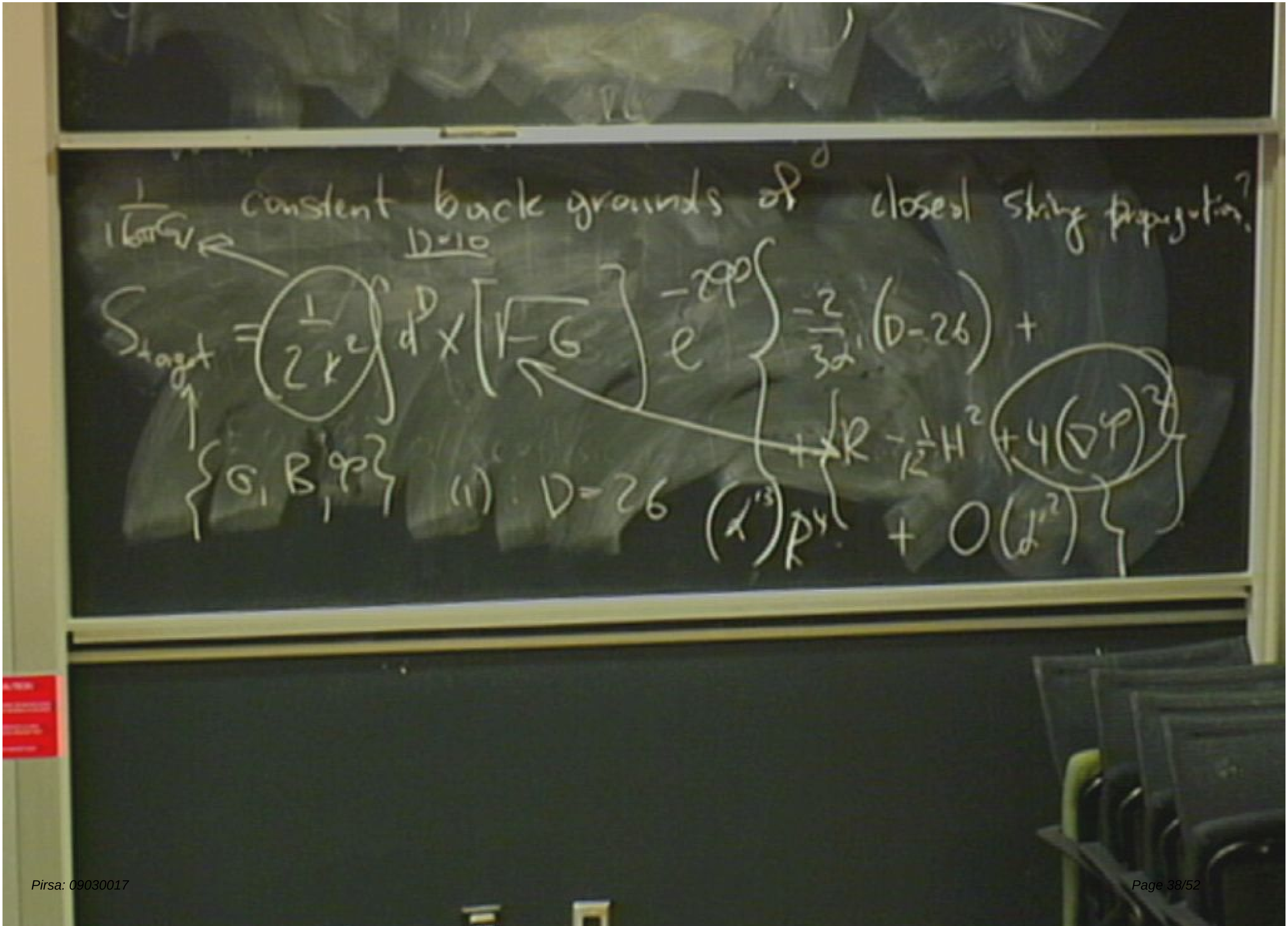
$$S^E = \frac{1}{16\pi G_N}\int d^{26}x\,\sqrt{-G^E}\left[R^E - \frac{1}{12}e^{-8\varphi/3}H_{\mu\nu\rho}H^{\mu\nu\rho} - \frac{1}{6}(\nabla\varphi)^2\right]$$

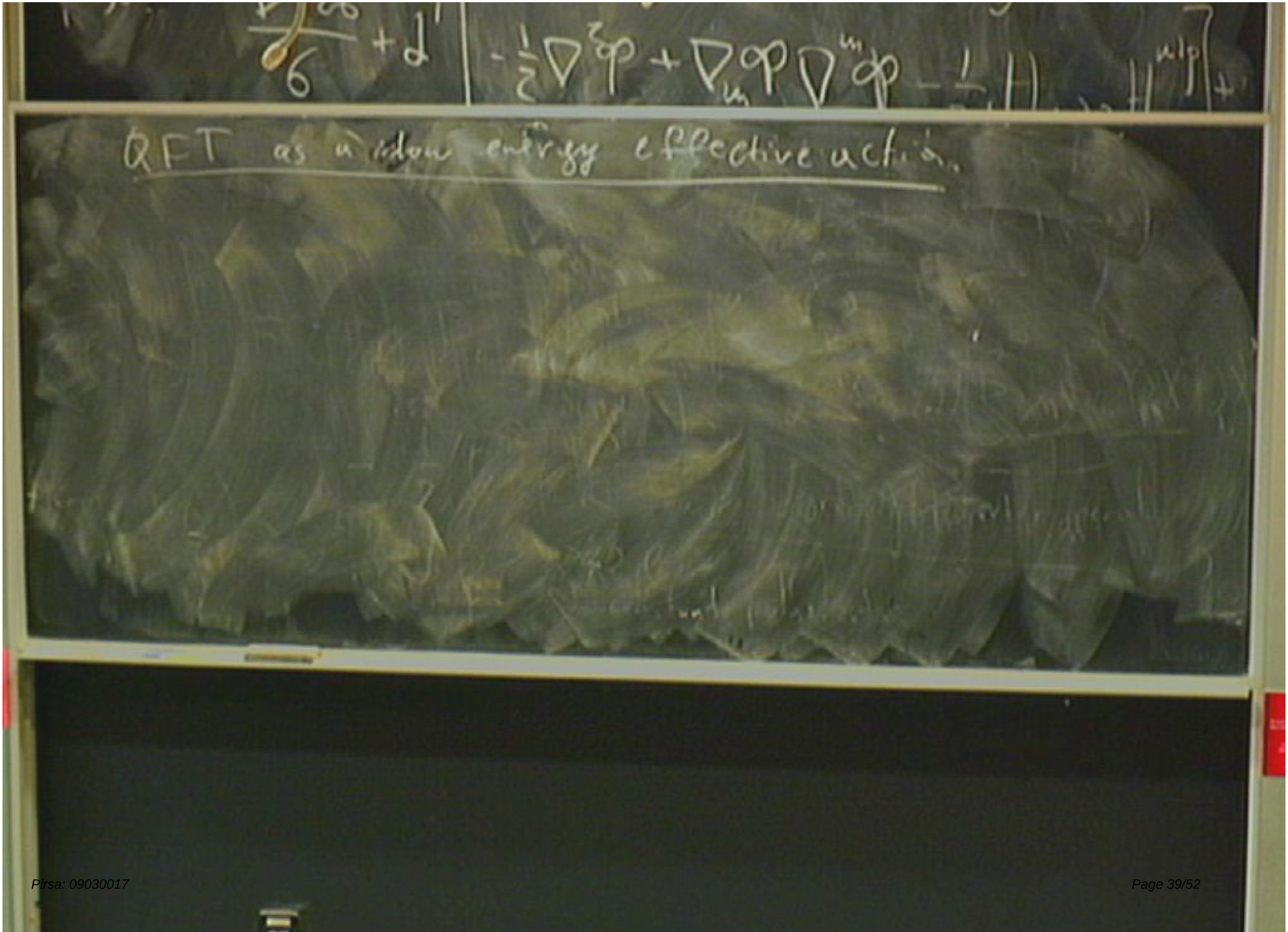
(string frame metric)

low energy effective action $O(\alpha'^2)$ — $-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$ (bosonic string)

$$\hookrightarrow \frac{\alpha'^{1/2}}{R_c} = \frac{l_s}{R_c} \ll 1$$

R_c: radius of curvature of target space


constant backgrounds of closed string propagation?
1/16πG_N
D=10
S_target = 1/2κ² ∫ d^D x √-G e^-2φ { -2/3α' (D-26) + R - 1/12 H² + 4(∇φ)² + O(α'²) }
{G, B, φ}
(1) : D=26
(α'³) R⁴


QFT as a low energy effective action.

QFT as a low energy effective action.

$$\mathcal{L} = -(\partial\varphi)^2 - (\partial\chi)^2 - V(\varphi, \chi)$$

$V =$

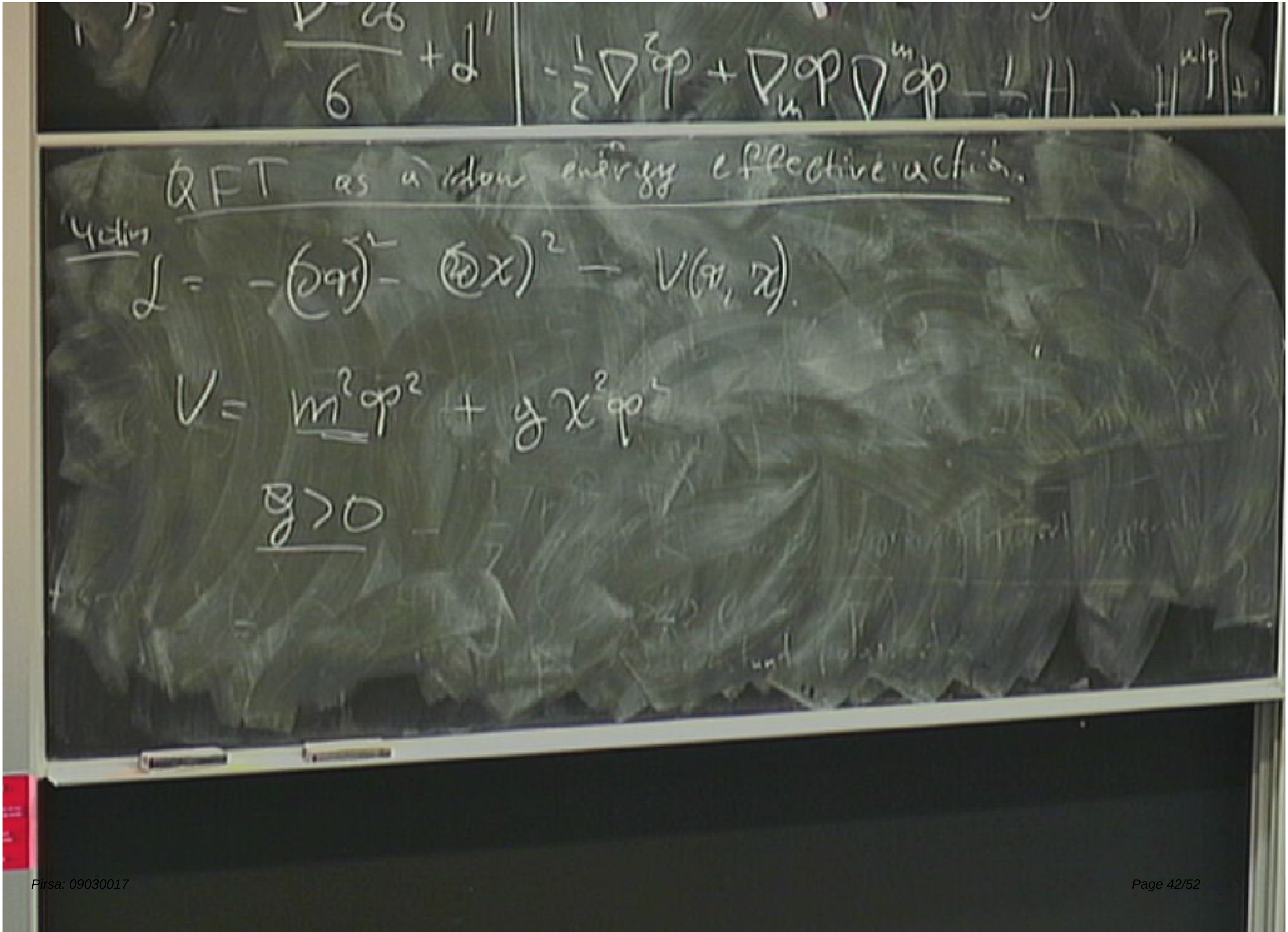
$$\frac{\nabla^2 \ldots}{6} + \mathcal{L}' \quad \Big| \quad -\frac{1}{2}\nabla^2\varphi + \nabla_m\varphi\nabla^m\varphi - \frac{1}{\ldots}H \ldots H^{\mu\nu\rho}$$

QFT as a low energy effective action.

Yukawa

$$\mathcal{L} = -(\partial\varphi)^2 - (\partial\chi)^2 - V(\varphi, \chi).$$

$$V = m^2\varphi^2 + g\chi^2\varphi^2$$


QFT as a low energy effective action.
Yukawa
$\mathcal{L} = -(\partial\varphi)^2 - (\partial\chi)^2 - V(\varphi, \chi)$
$V = m^2\varphi^2 + g\chi^2\varphi^2$
$g > 0$

$$\frac{\nabla \ldots}{6} + \mathcal{L}' \qquad -\frac{1}{2}\nabla^2\varphi + \nabla_\mu\varphi\nabla^\mu\varphi - \ldots$$

QFT as a low energy effective action.

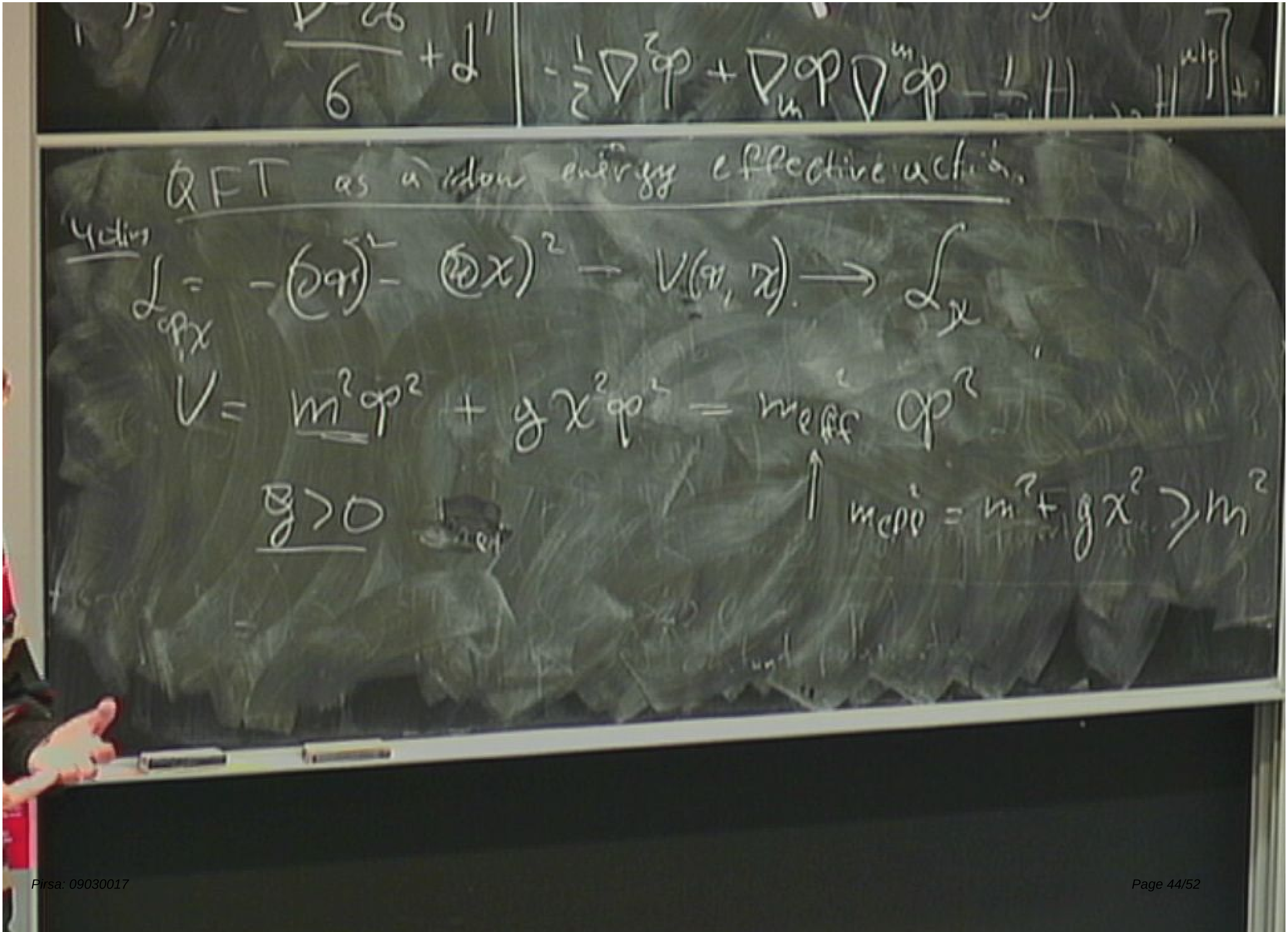
Yukawa

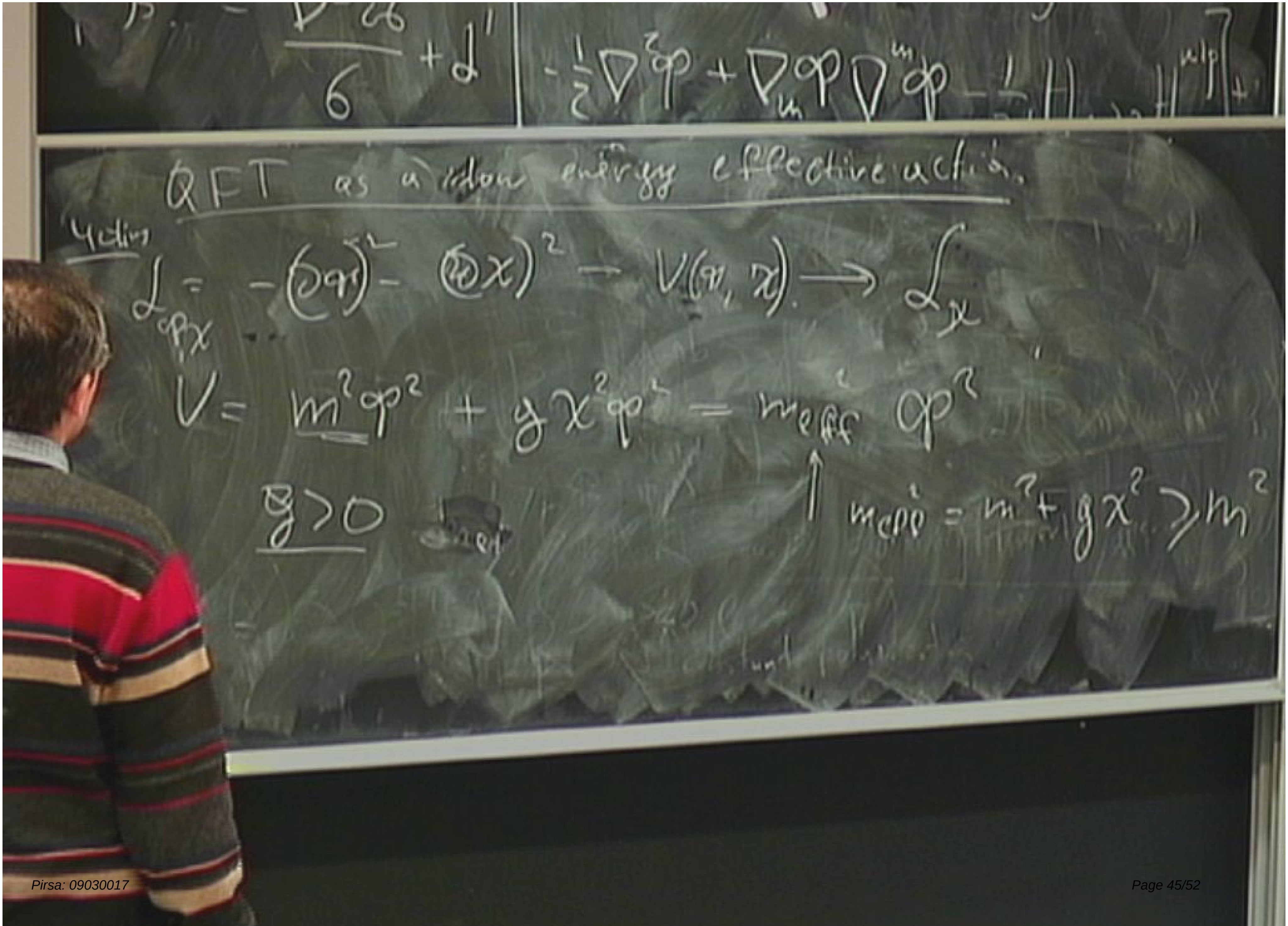
$$\mathcal{L} = -(\partial\varphi)^2 - (\partial\chi)^2 - V(\varphi, \chi).$$

$$V = m^2\varphi^2 + g\chi^2\varphi^2 = m_{eff}^2\,\varphi^2$$

$g > 0$

$$m_{eff}^2 = m^2 + g\chi^2 > m^2$$


QFT as a low energy effective action.
Yukawa
$\mathcal{L}_{\varphi\chi} = -(\partial\varphi)^2 - (\partial\chi)^2 - V(\varphi, \chi) \longrightarrow \mathcal{L}_\chi$
$V = m^2\varphi^2 + g\chi^2\varphi^2 = m_{eff}^2\,\varphi^2$
$g > 0$
$m_{eff}^2 = m^2 + g\chi^2 > m^2$

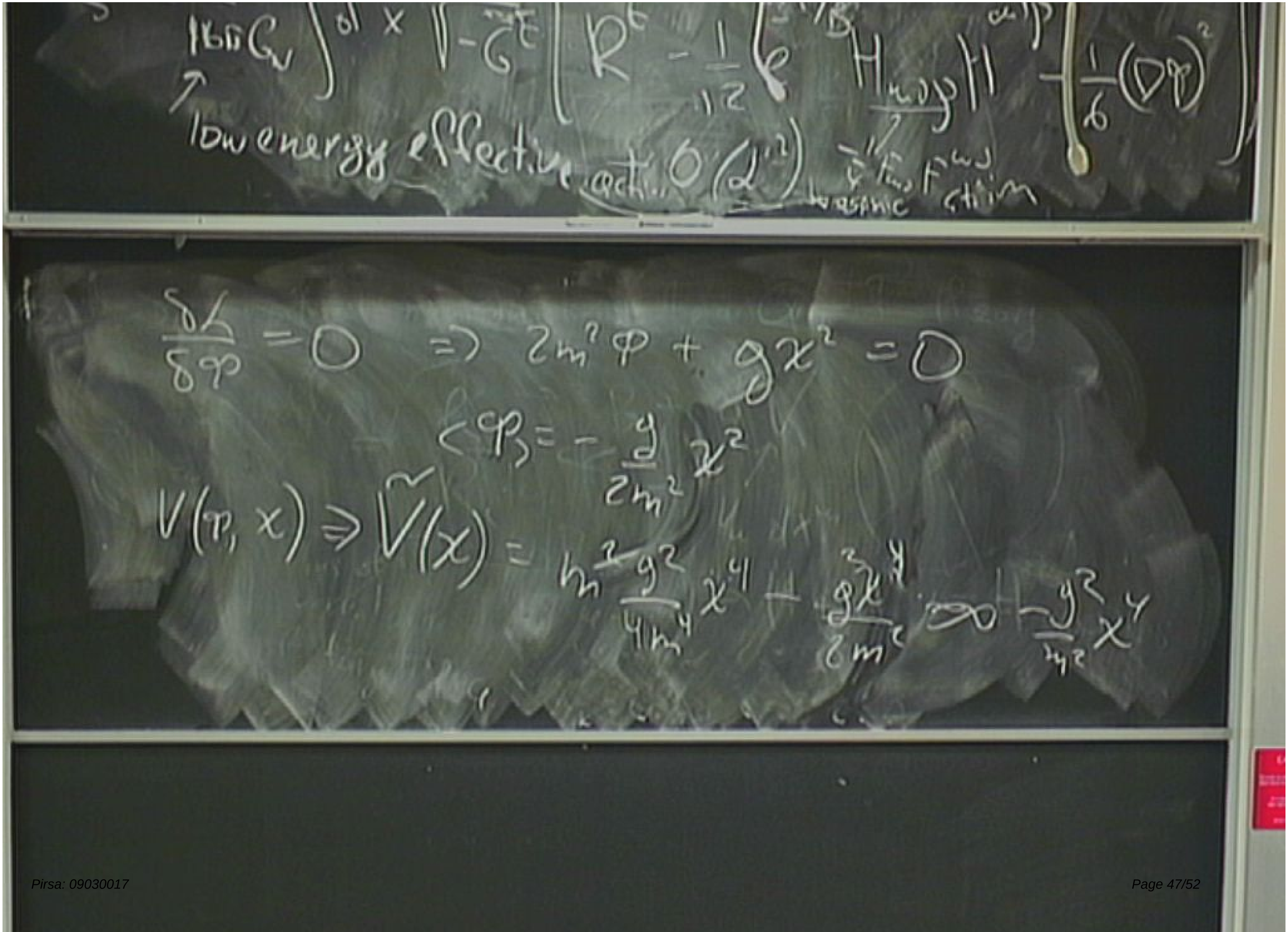

QFT as a low energy effective action
$\mathcal{L}_{\varphi\chi} = -(\partial\varphi)^2 - (\partial\chi)^2 - V(\varphi, \chi) \longrightarrow \mathcal{L}_\chi$
$V = m^2\varphi^2 + g\chi^2\varphi^2 = m_{eff}^2\,\varphi^2$
$g > 0$
$m_{eff}^2 = m^2 + g\chi^2 > m^2$

$16\pi G_N \int d^{10}x \sqrt{-G^E} \left[R^E - \frac{1}{12} e^{-\phi} H_{\mu\nu\rho} H^{\mu\nu\rho} - \frac{1}{6} (\nabla\phi)^2 \right]$

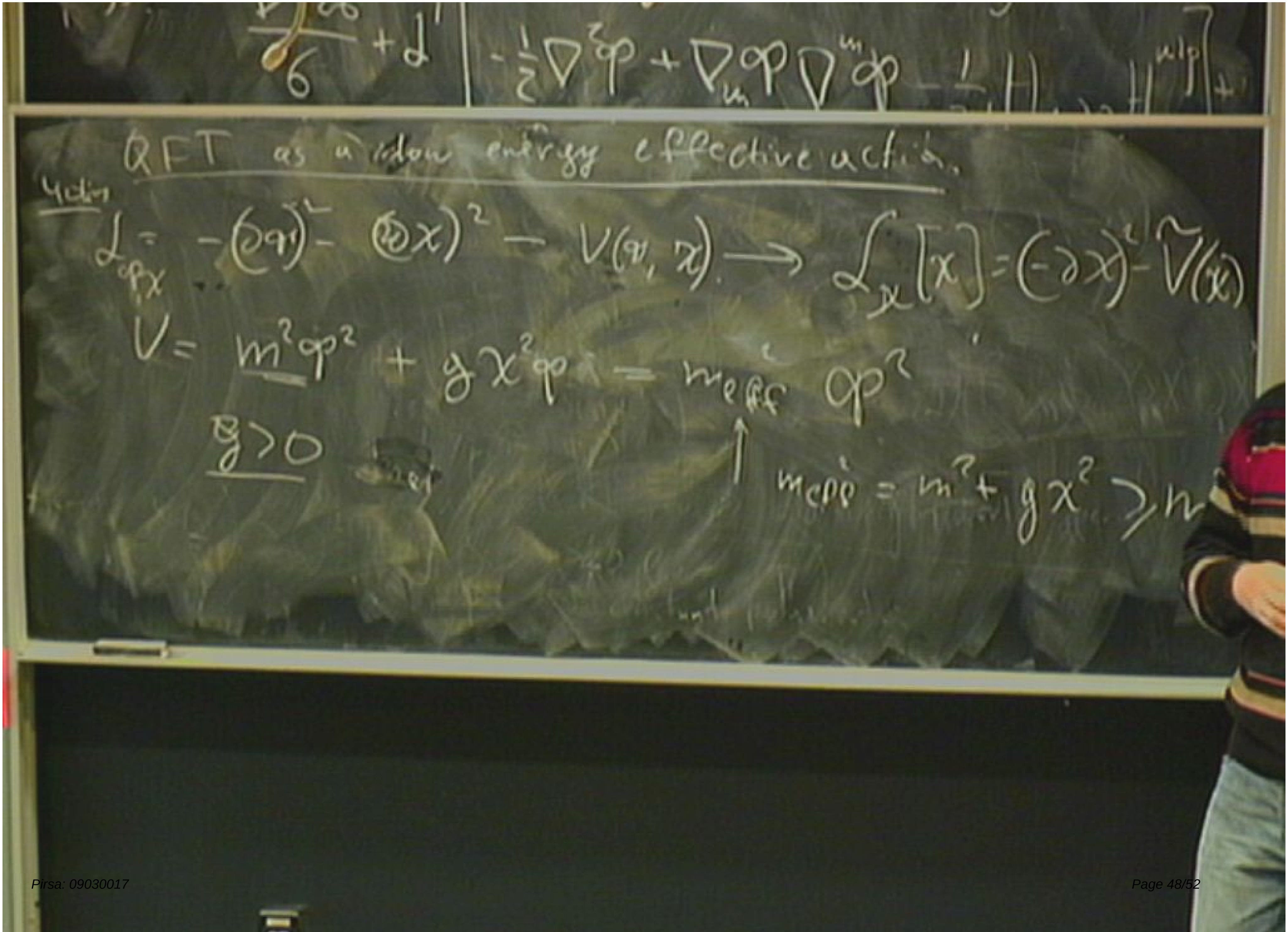
low energy effective action $O(\alpha'^2)$ $-\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$ bosonic action

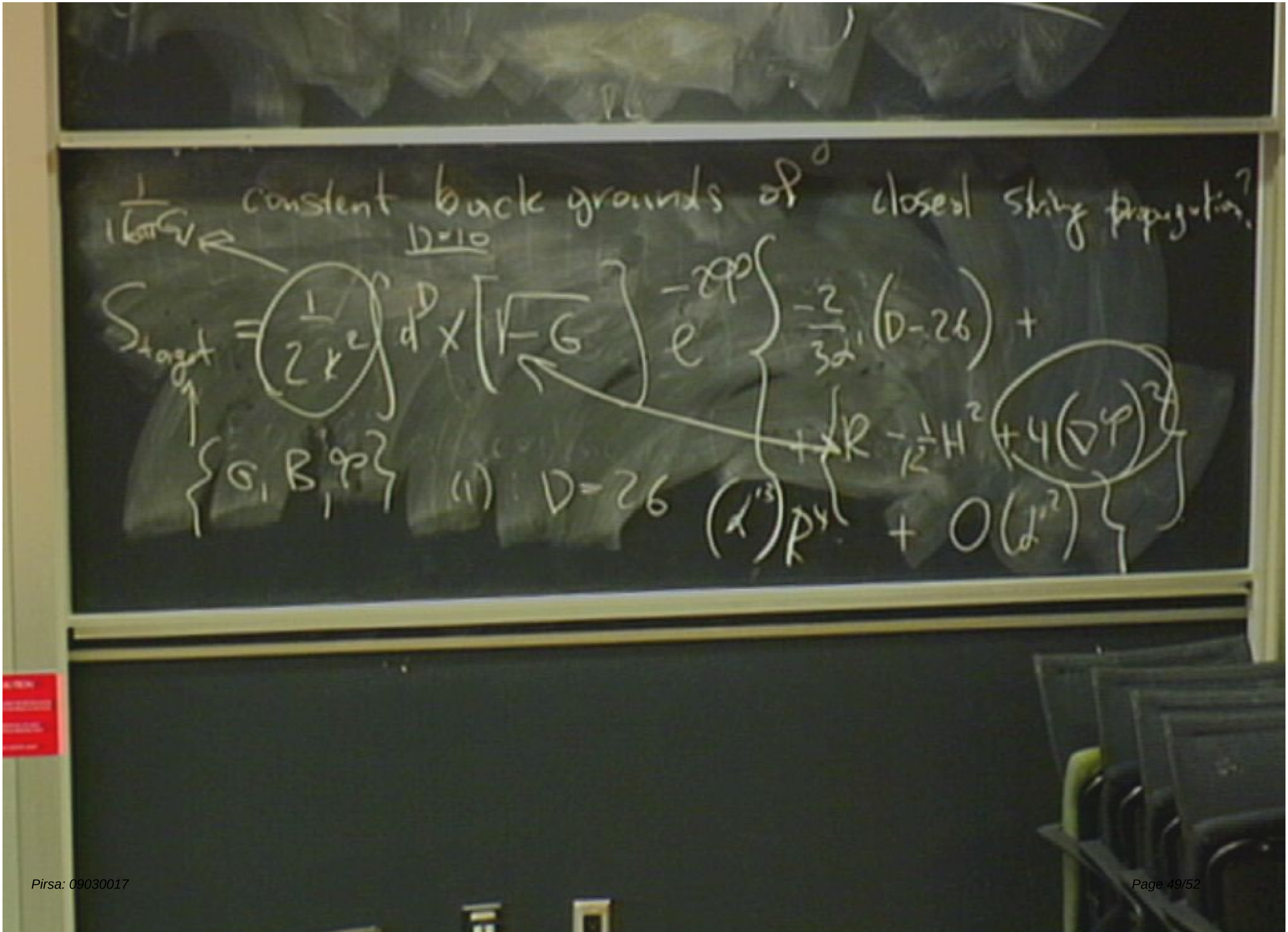
$$\frac{\delta \mathcal{L}}{\delta \varphi} = 0 \Rightarrow 2m^2 \varphi + g x^2 = 0$$

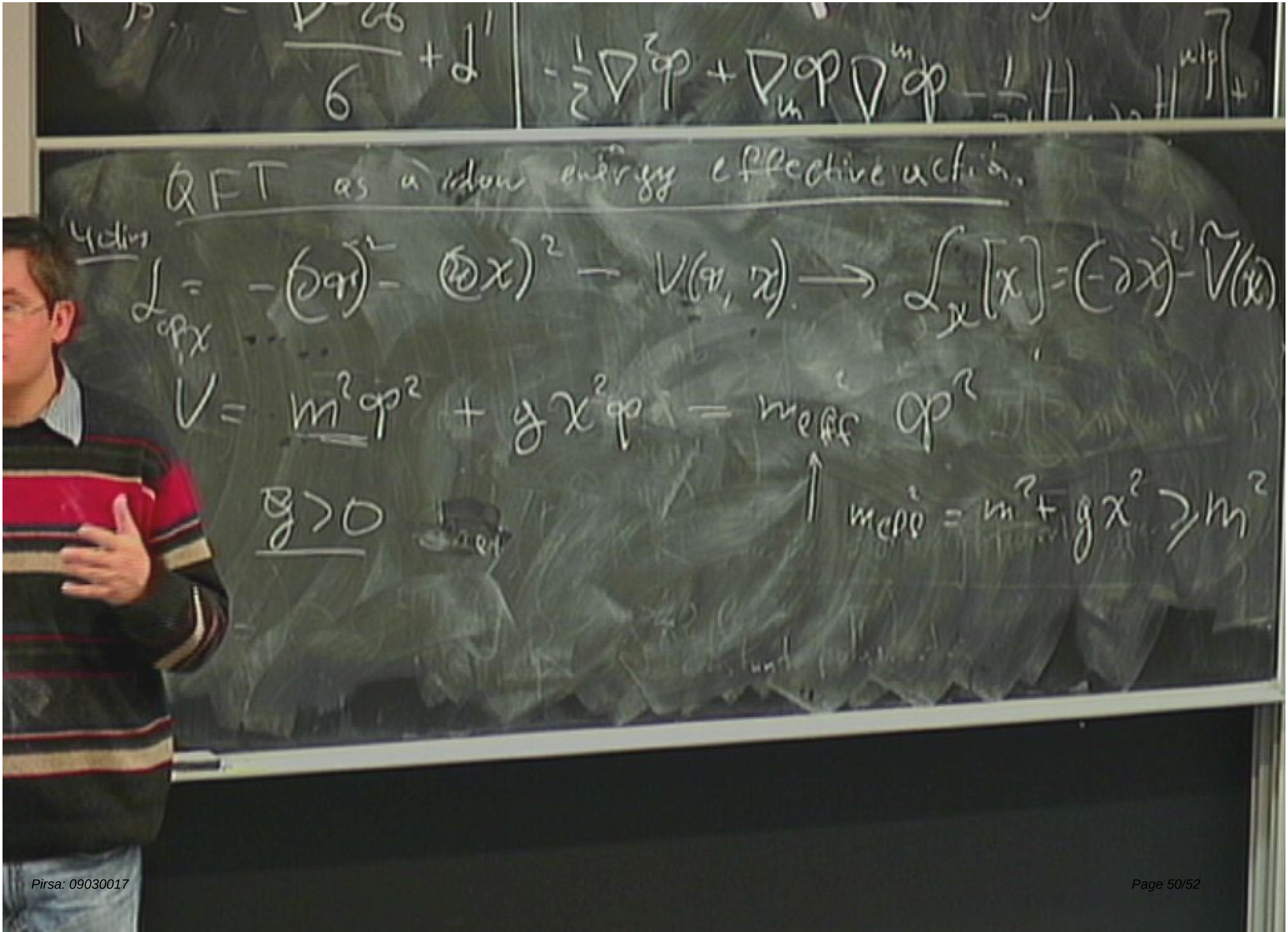
$$\varphi = -\frac{g}{2m^2} x^2$$

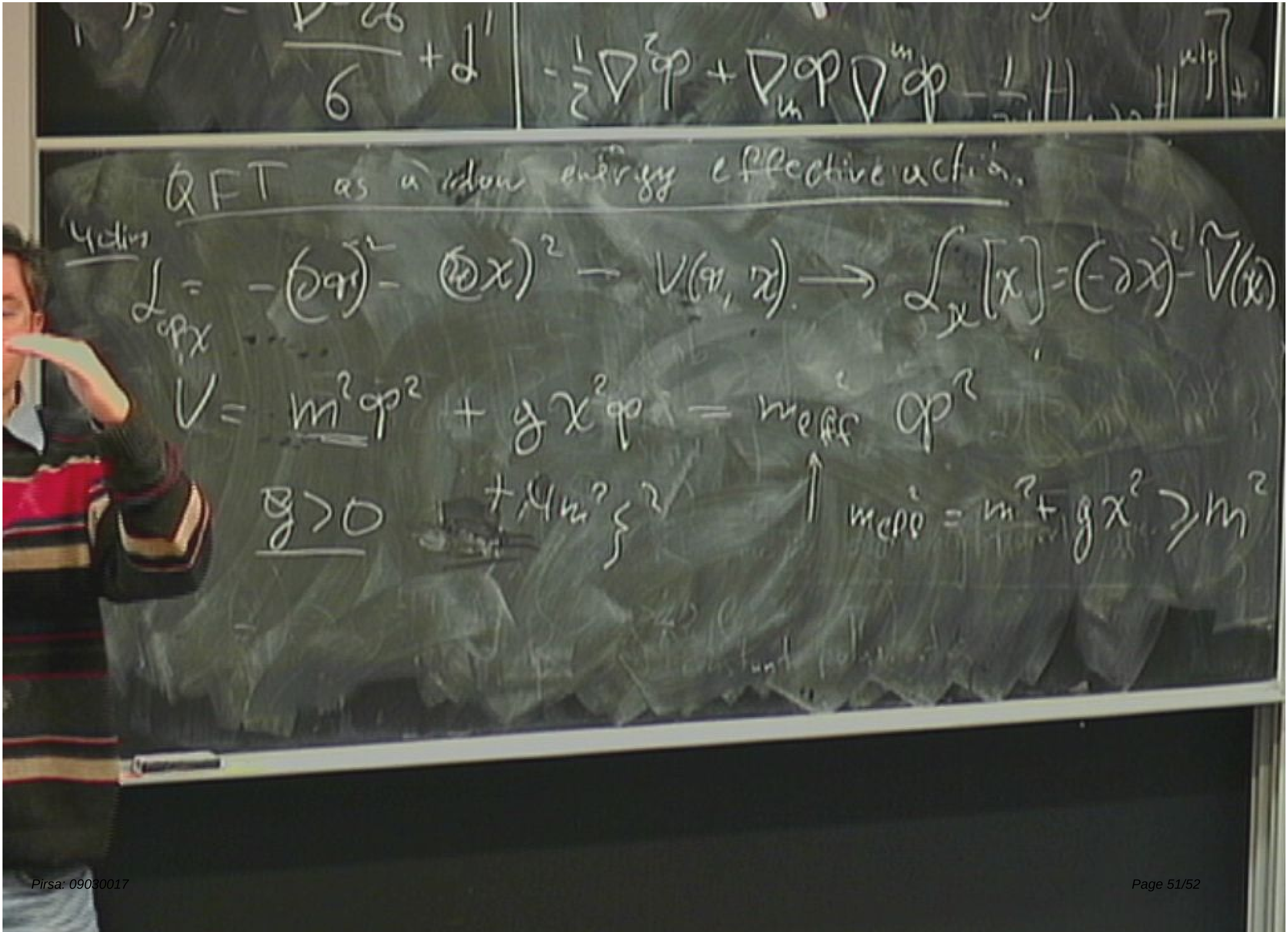


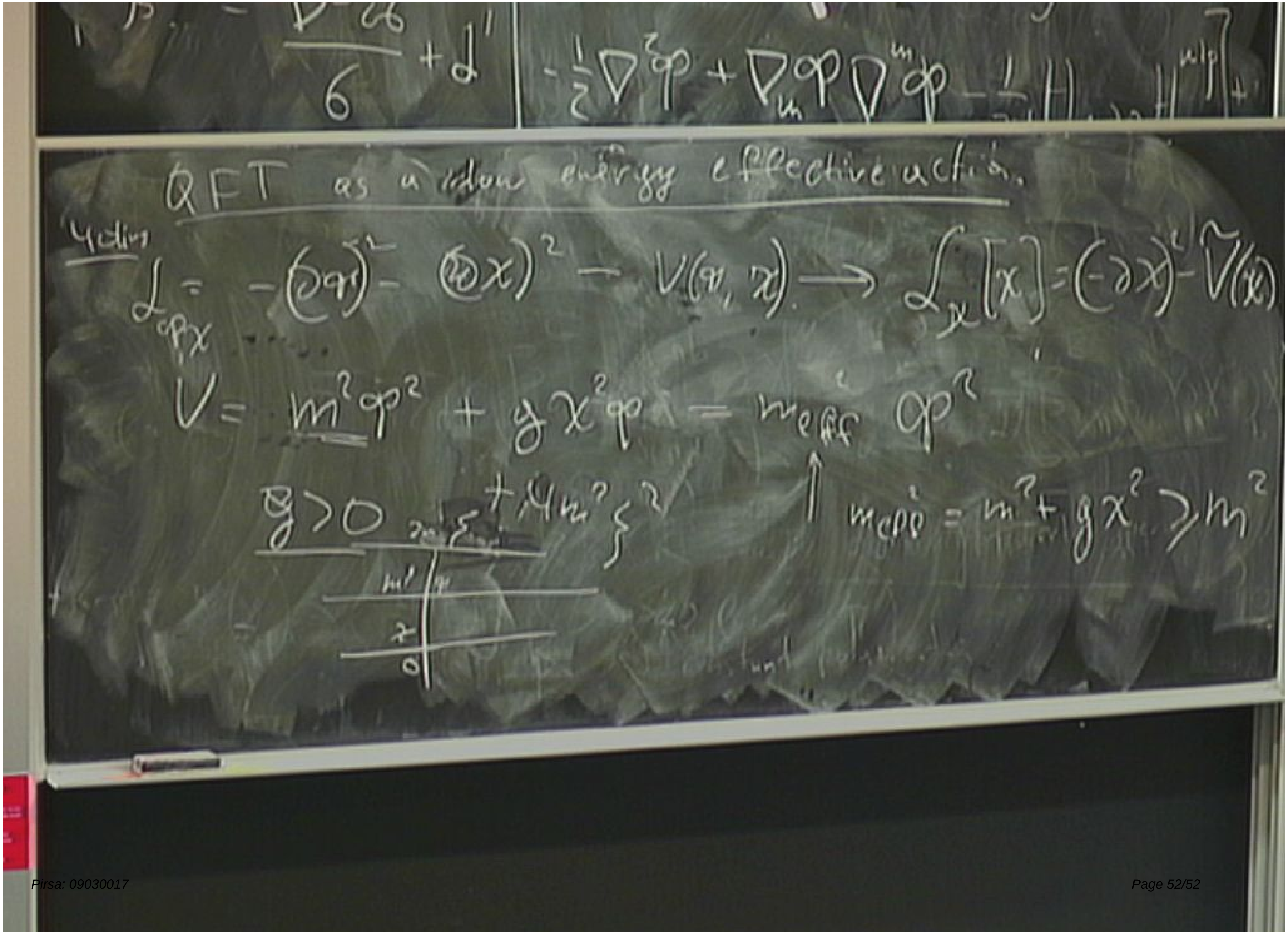
16πG_N
low energy effective action O(α'^2)
δL/δφ = 0 ⇒ 2m²φ + gx² = 0
<φ> = − g/2m² x²
V(φ, x) ⇒ Ṽ(x) = m² g²/4m⁴ x⁴ − g x⁴/2m² = − g²/4m² x⁴


QFT as a low energy effective action.
V = m²φ² + gχ²φ² = m²eff φ²
g > 0
meff² = m² + gχ²


constant backgrounds of closed string propagation?
1/16πG_N
D=10
S_target = 1/2κ² ∫ d^D x √-G e^{-2φ} { -2/3α' (D-26) + α'R - 1/12 H² + 4(∇φ)² + O(α'²) }
{G, B, φ}
(1) D=26
(α'³) R⁴


QFT as a low energy effective action.
$\mathcal{L}_{\varphi\chi} = -(\partial\varphi)^2 - (\partial\chi)^2 - V(\varphi,\chi) \longrightarrow \mathcal{L}_\chi[\chi] = (-\partial\chi)^2 - \tilde{V}(\chi)$
$V = m^2\varphi^2 + g\chi^2\varphi^2 = m_{eff}^2\,\varphi^2$
$g > 0$
$m_{eff}^2 = m^2 + g\chi^2 > m^2$


QFT as a low energy effective action.


QFT as a low energy effective action.
L = -(∂φ)² - (∂χ)² - V(φ, χ) → L_χ[χ] = (-∂χ)² - Ṽ(χ)
V = m²φ² + gχ²φ² = m_eff² φ²
g > 0
m_eff² = m² + gχ² > m²