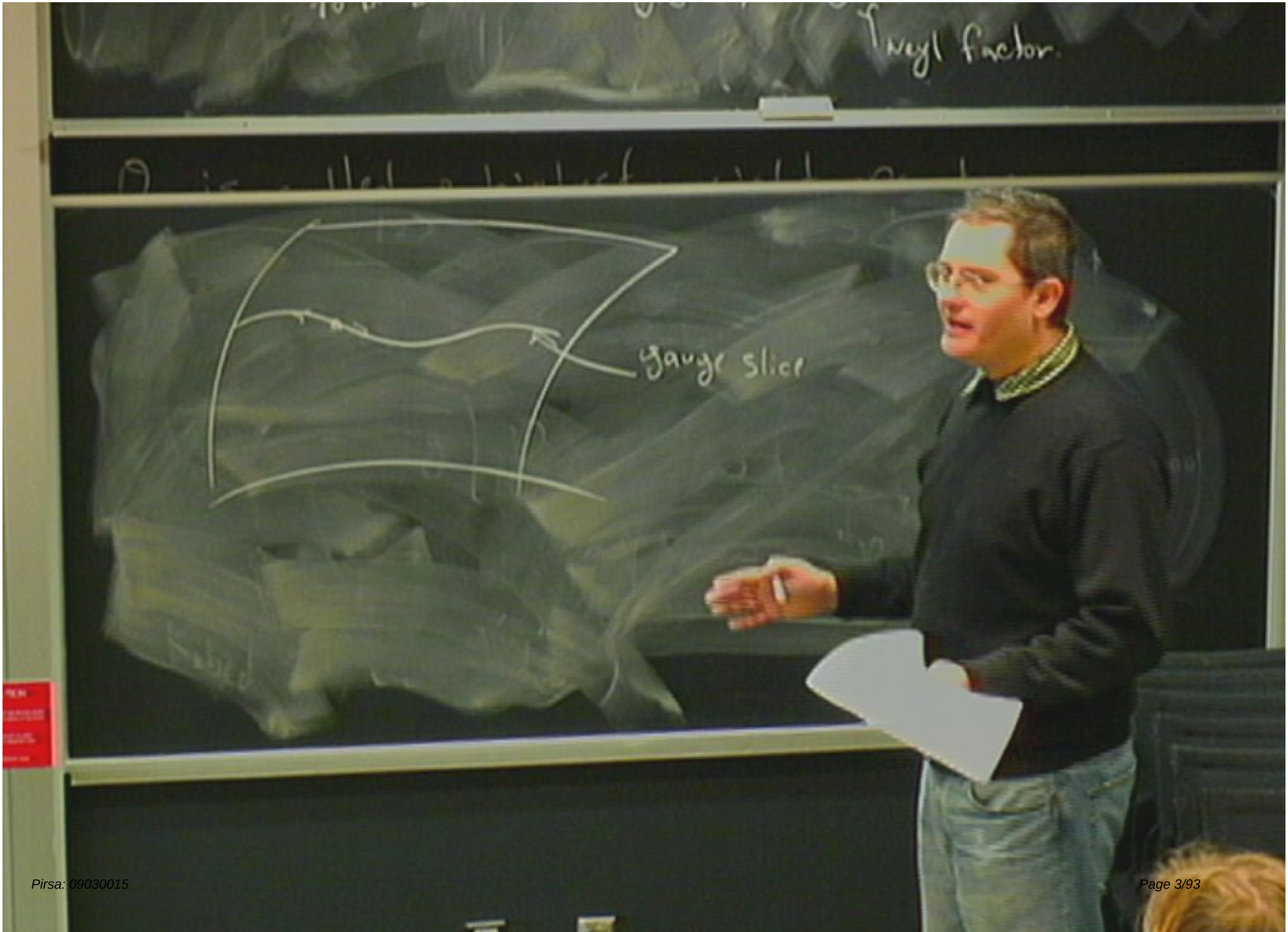
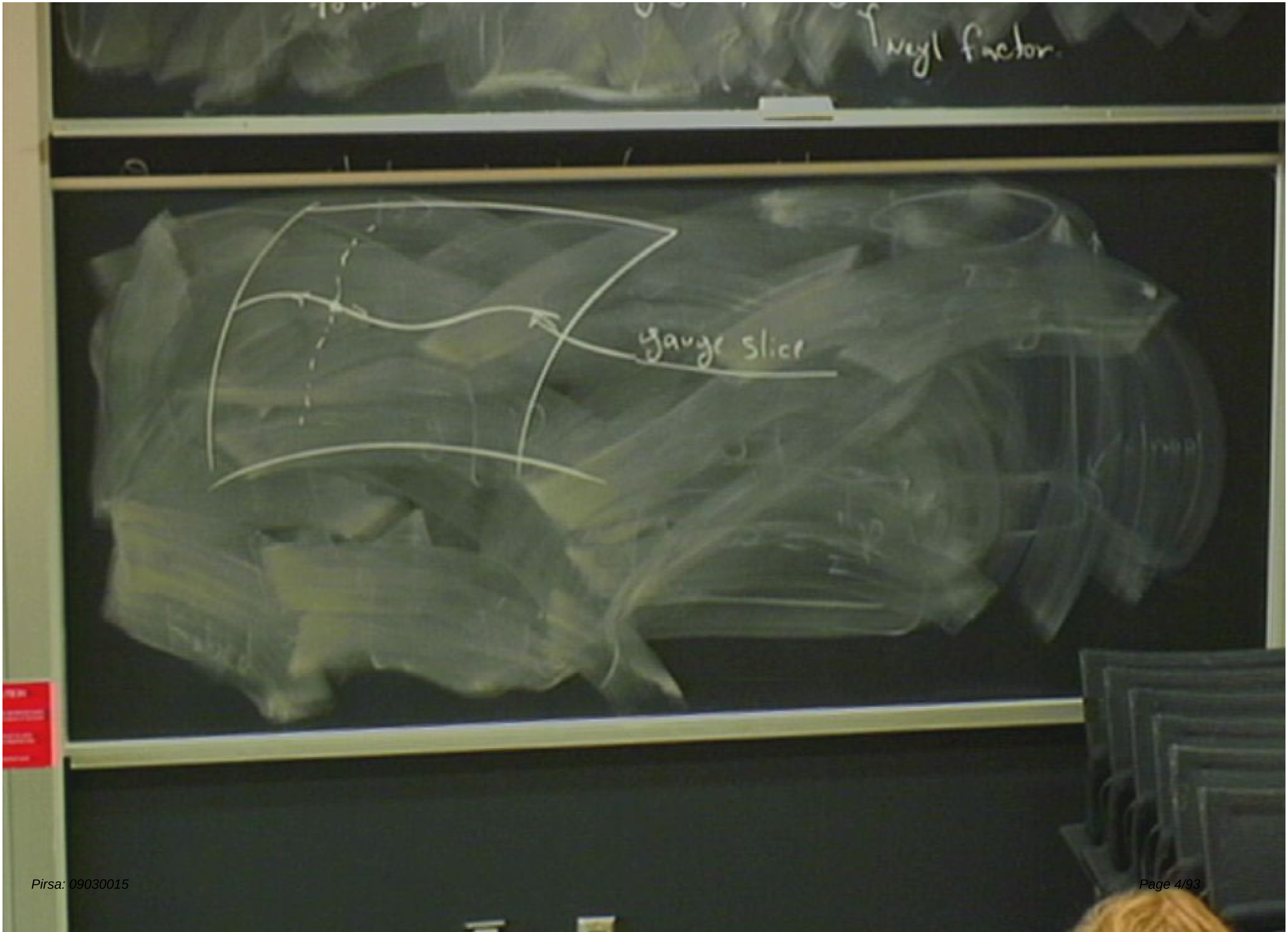
Title: Introduction to the Bosonic String Part B

Date: Mar 13, 2009 12:00 PM

URL: http://pirsa.org/09030015

Abstract: This course provides a thorough introduction to the bosonic string based on the Polyakov path integral and conformal field theory. We introduce central ideas of string theory, the tools of conformal field theory, the Polyakov path integral, and the covariant quantization of the string. We discuss string interactions and cover the tree-level and one loop amplitudes. More advanced topics such as T-duality and D-branes will be taught as part of the course. The course is geared for M.Sc. and Ph.D. students enrolled in Collaborative Ph.D. Program in Theoretical Physics. Required previous course work: Quantum Field Theory (AM516 or equivalent). The course evaluation will be based on regular problem sets that will be handed in during the term. The primary text is the book: 'String theory. Vol. 1: An introduction to the bosonic string. J. Polchinski (Santa Barbara, KITP) . 1998. 402pp. Cambridge, UK: Univ. Pr. (1998) 402 p.' All interested students should contact Alex Buchel at abuchel@uwo.ca as soon as possible.


Weyl factor.
gauge slice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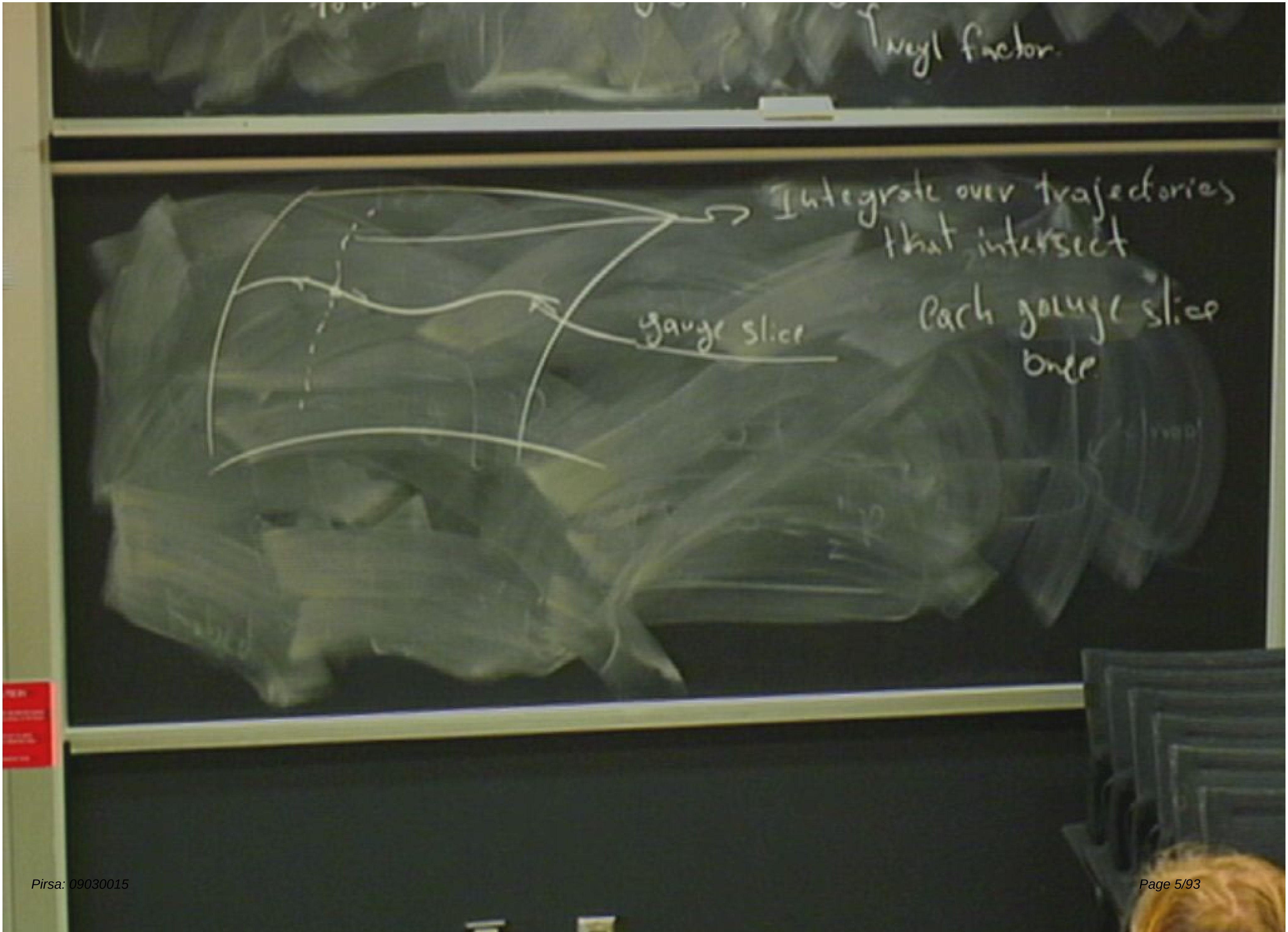
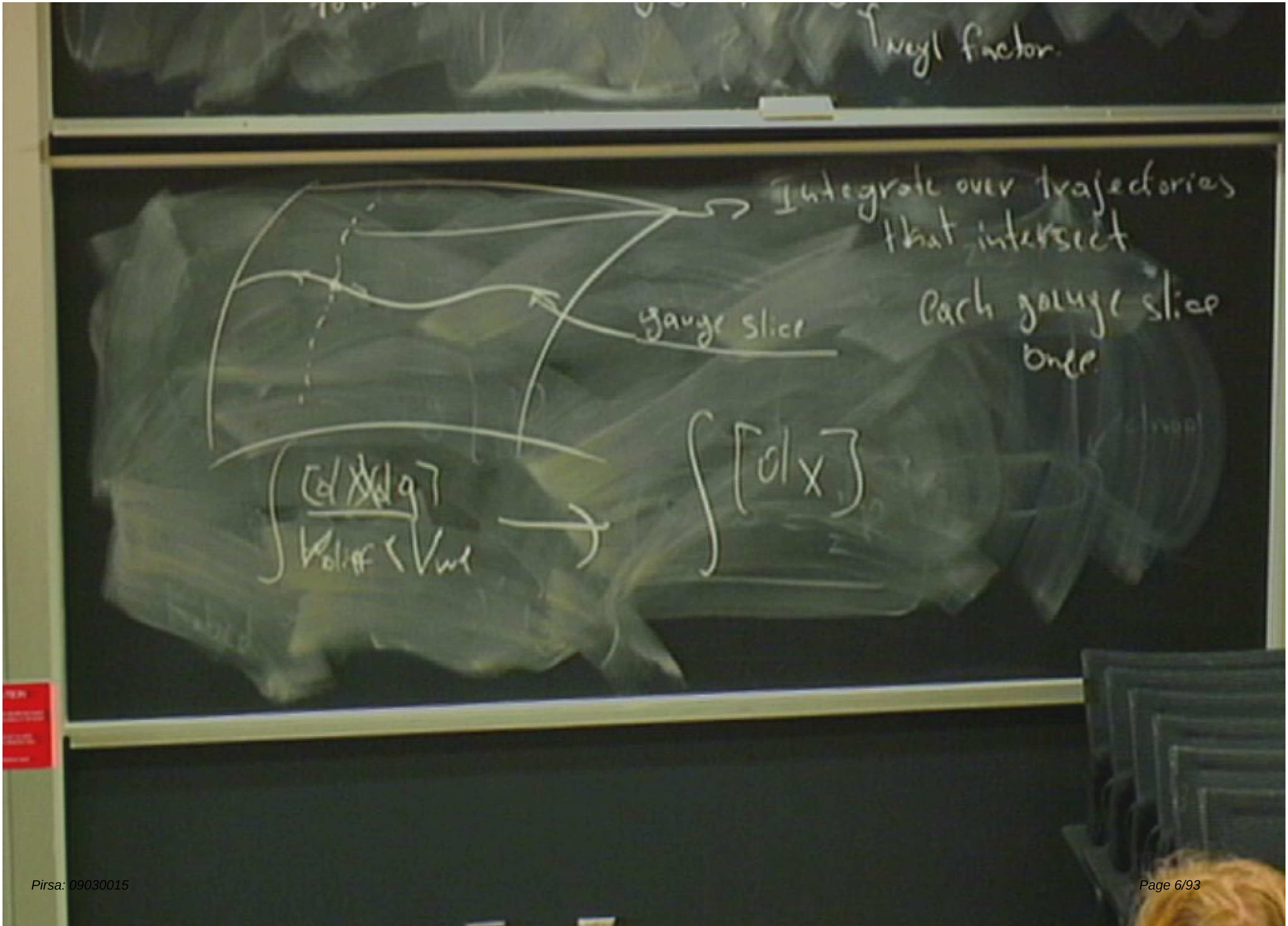

Weyl factor.
Integrate over trajectories
that intersect
each gauge slice
once
gauge slice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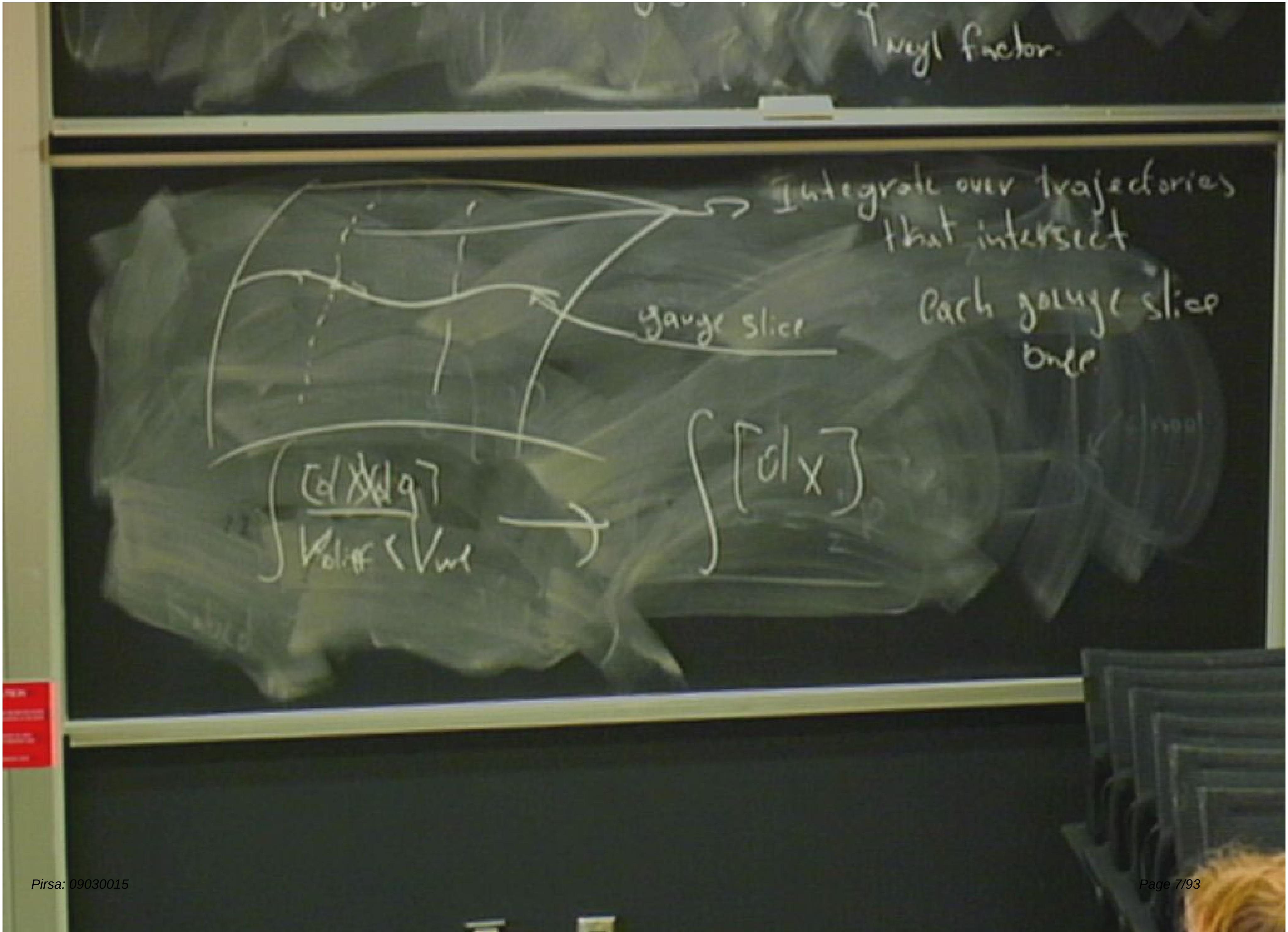
Weyl factor.
Integrate over trajectories
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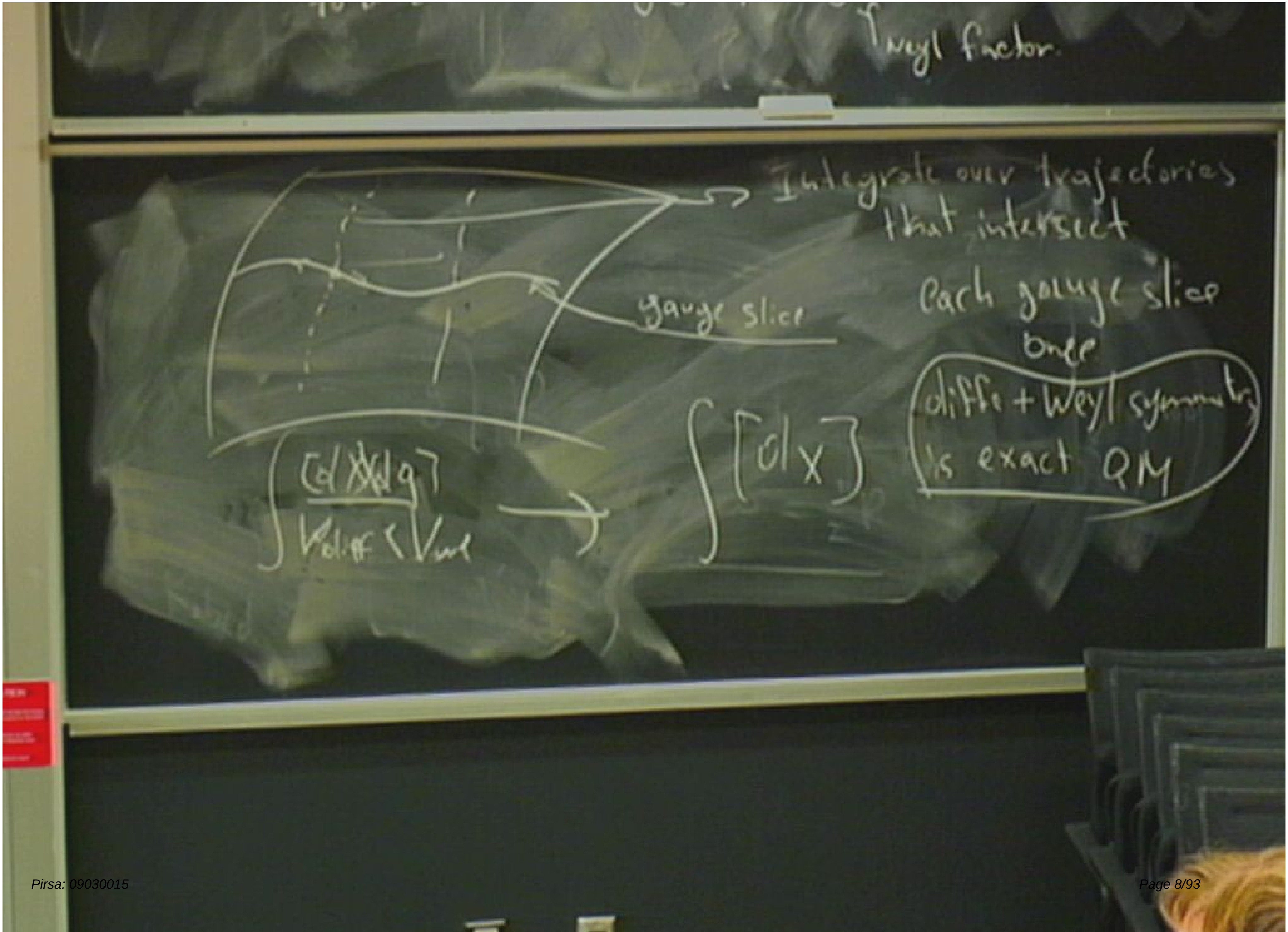
once
[dX][dg]
Vdiff × Vweyl
[dX]


Weyl factor.
Integrate over trajectories
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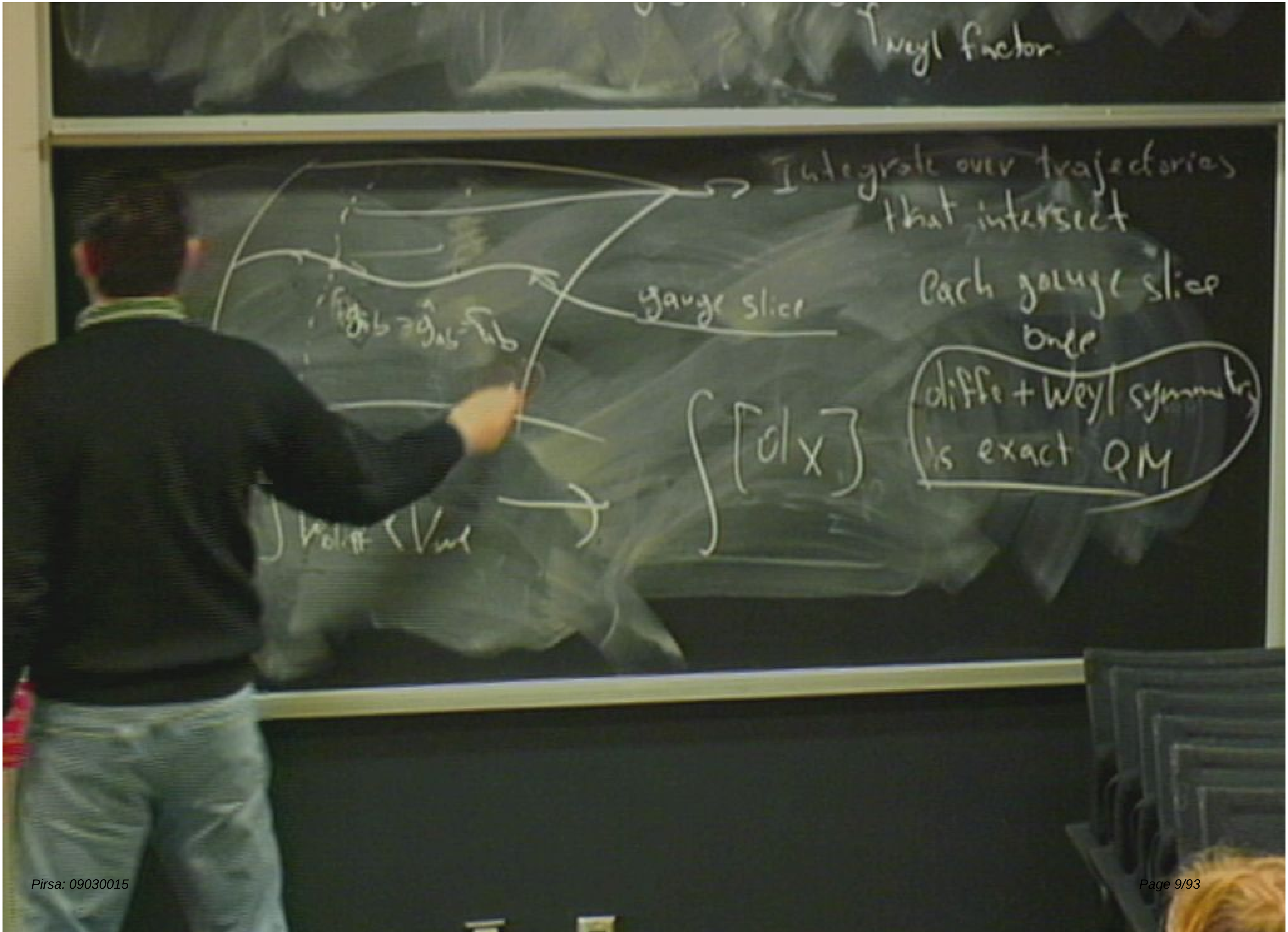
each gauge slice
once
gauge slice
diffs + Weyl symmetry
is exact QM
[dX dg]
Vdiff × VWeyl
[dX]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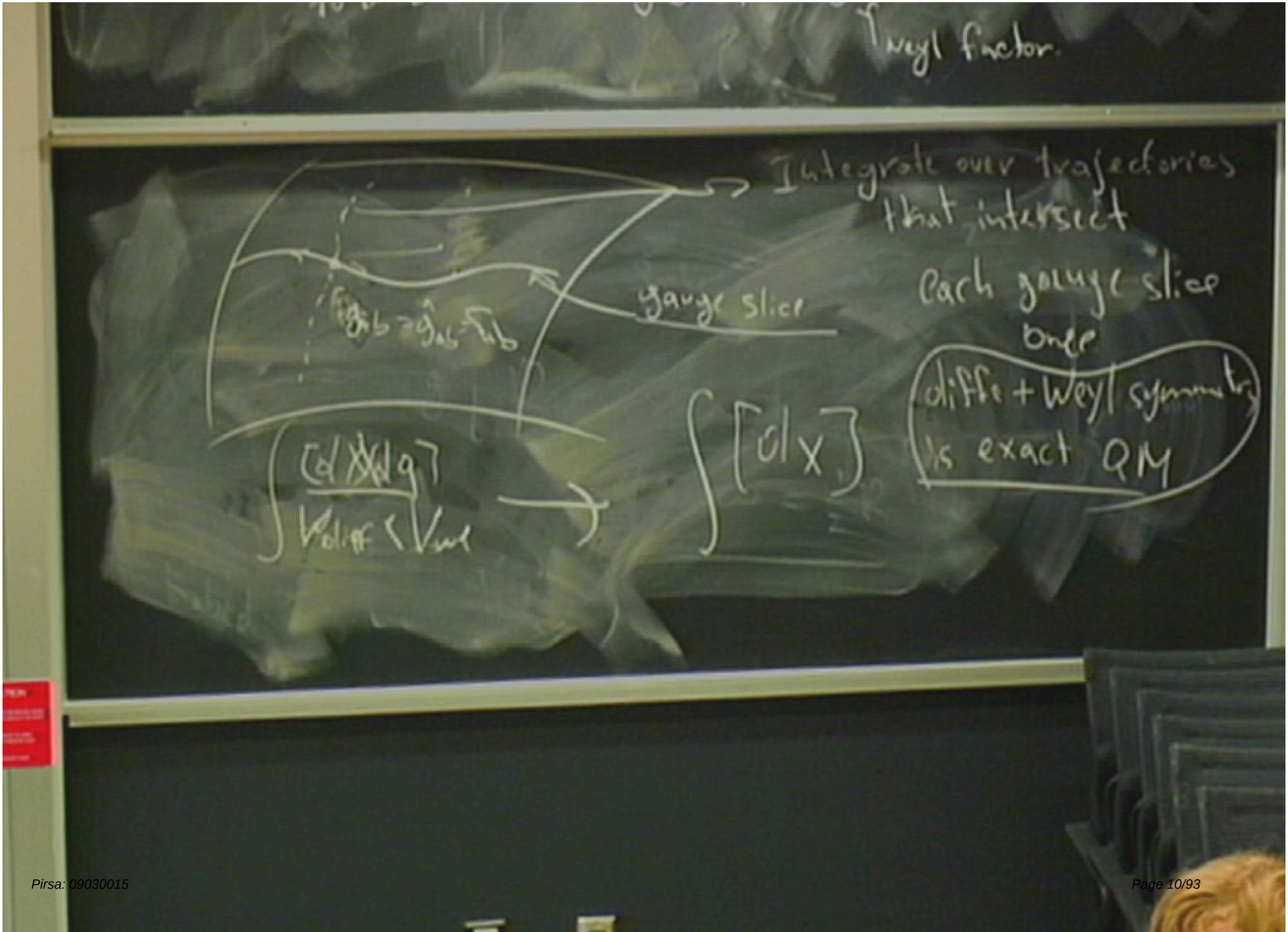
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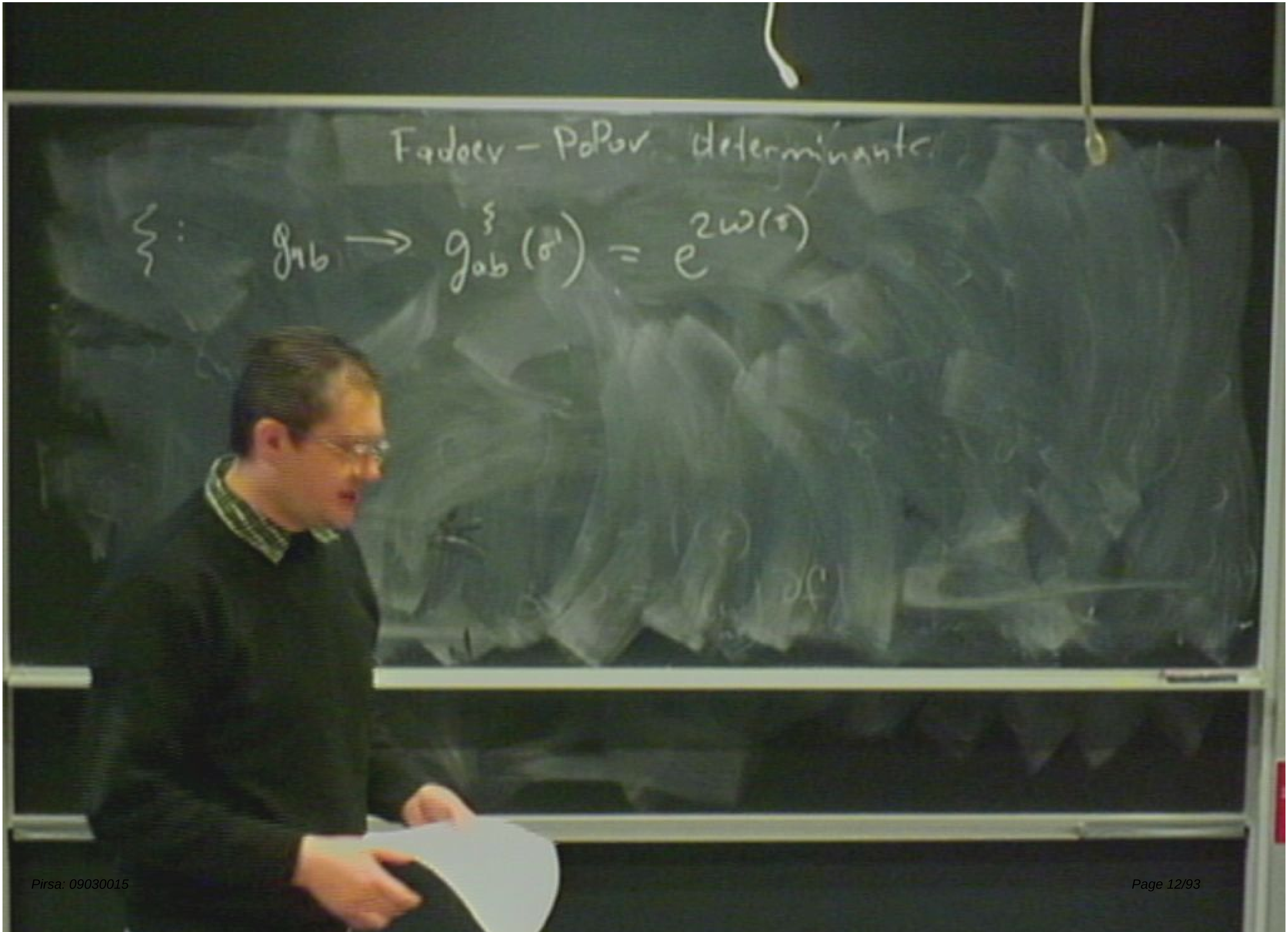
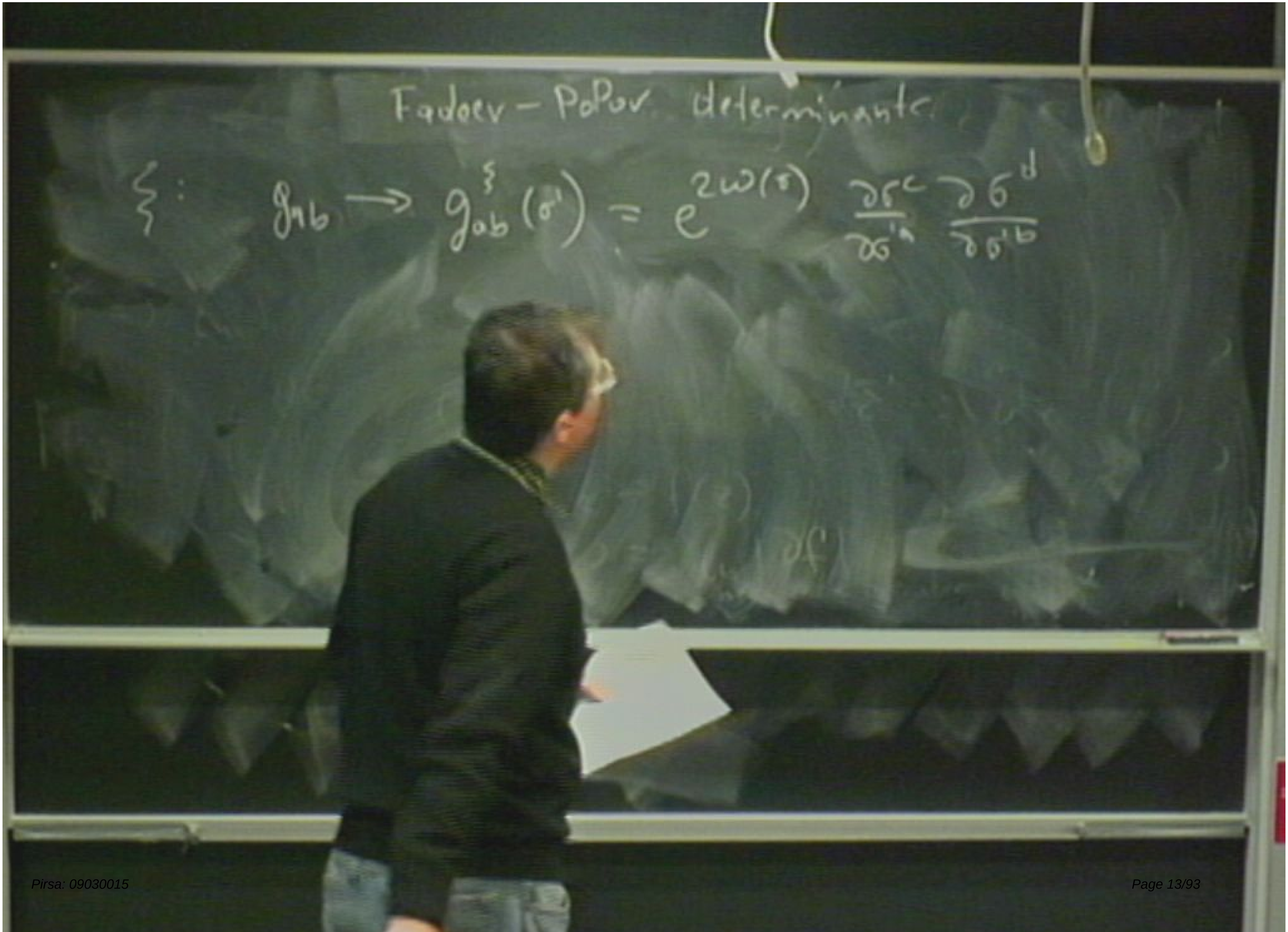
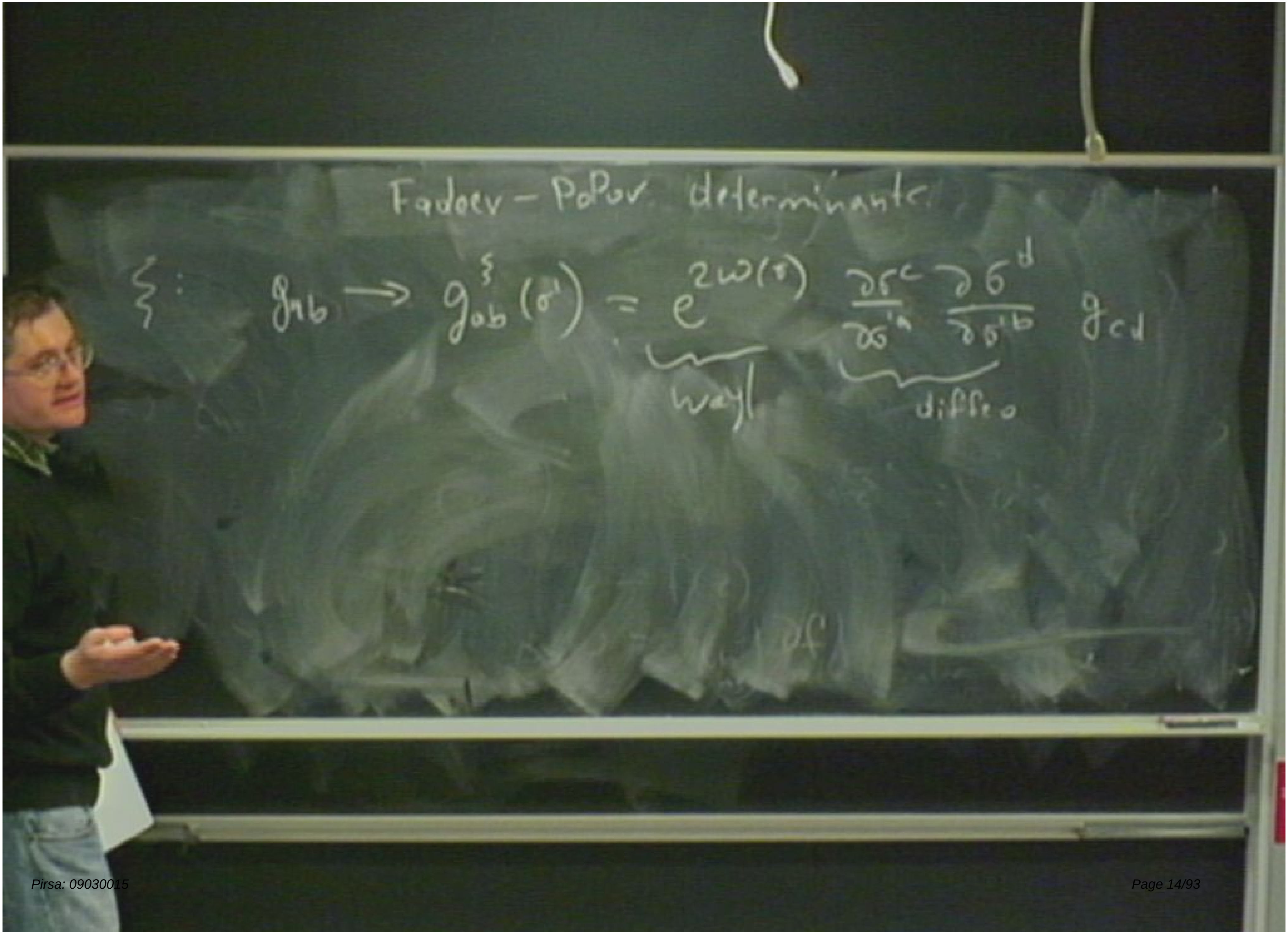
Fadeev - Popov determinants


Fadeev – PoPov determinants
$\xi : \quad g_{ab} \longrightarrow g^{\xi}_{ab}(\sigma') = e^{2\omega(\sigma)}$


Fadeev - PoPov determinants
$\xi: \quad g_{ab} \longrightarrow g^{\xi}_{ab}(\sigma') = e^{2\omega(\sigma)} \frac{\partial\sigma^c}{\partial\sigma'^a} \frac{\partial\sigma^d}{\partial\sigma'^b}$

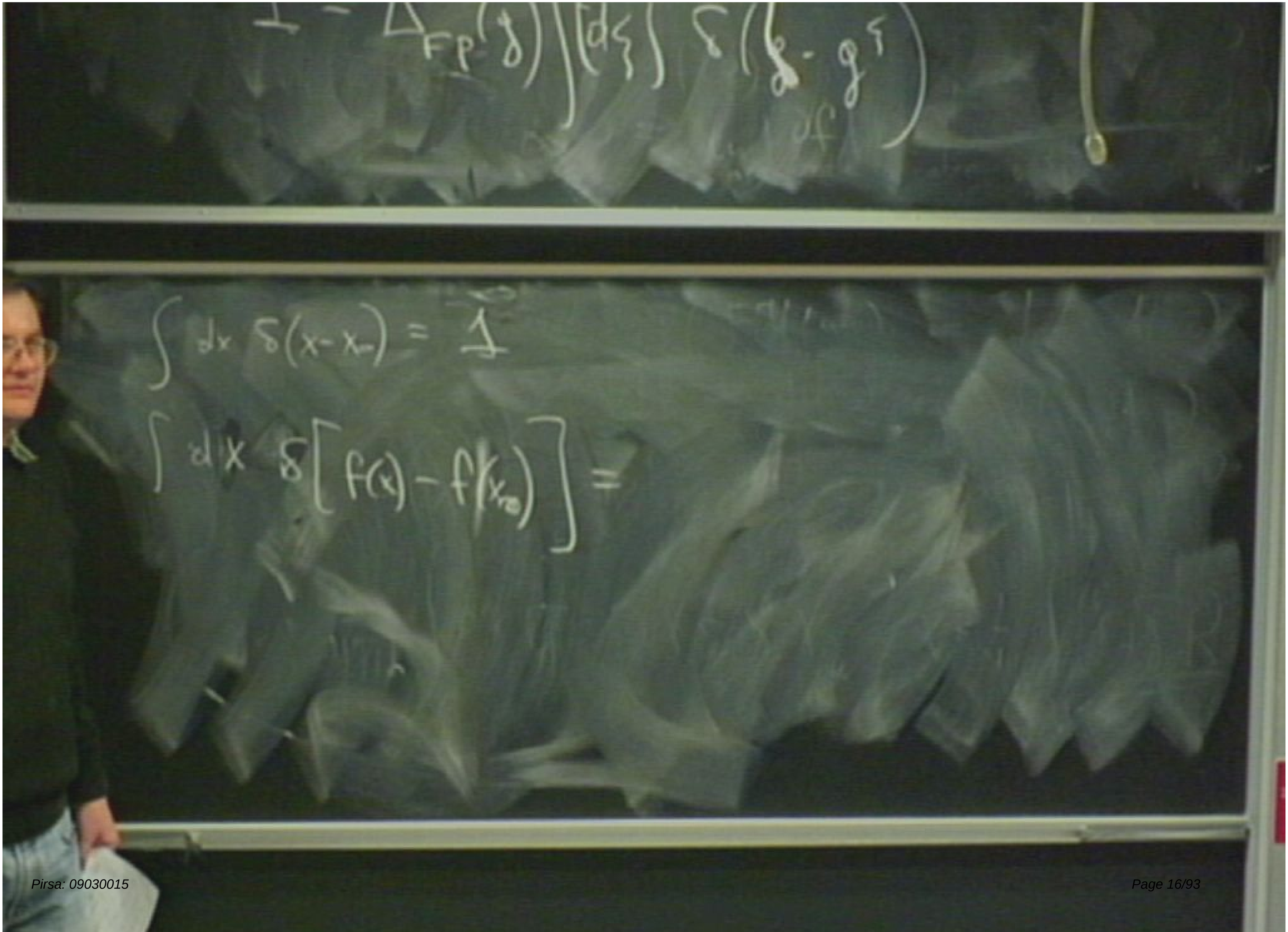

Fadeev – PoPov determinante
$\xi : \quad g_{ab} \longrightarrow g^{\xi}_{ab}(\sigma') = \underbrace{e^{2\omega(\sigma)}}_{\text{weyl}} \underbrace{\frac{\partial \sigma^c}{\partial \sigma'^a} \frac{\partial \sigma^d}{\partial \sigma'^b}}_{\text{diffeo}} g_{cd}$

Fadeev – Popov determinants

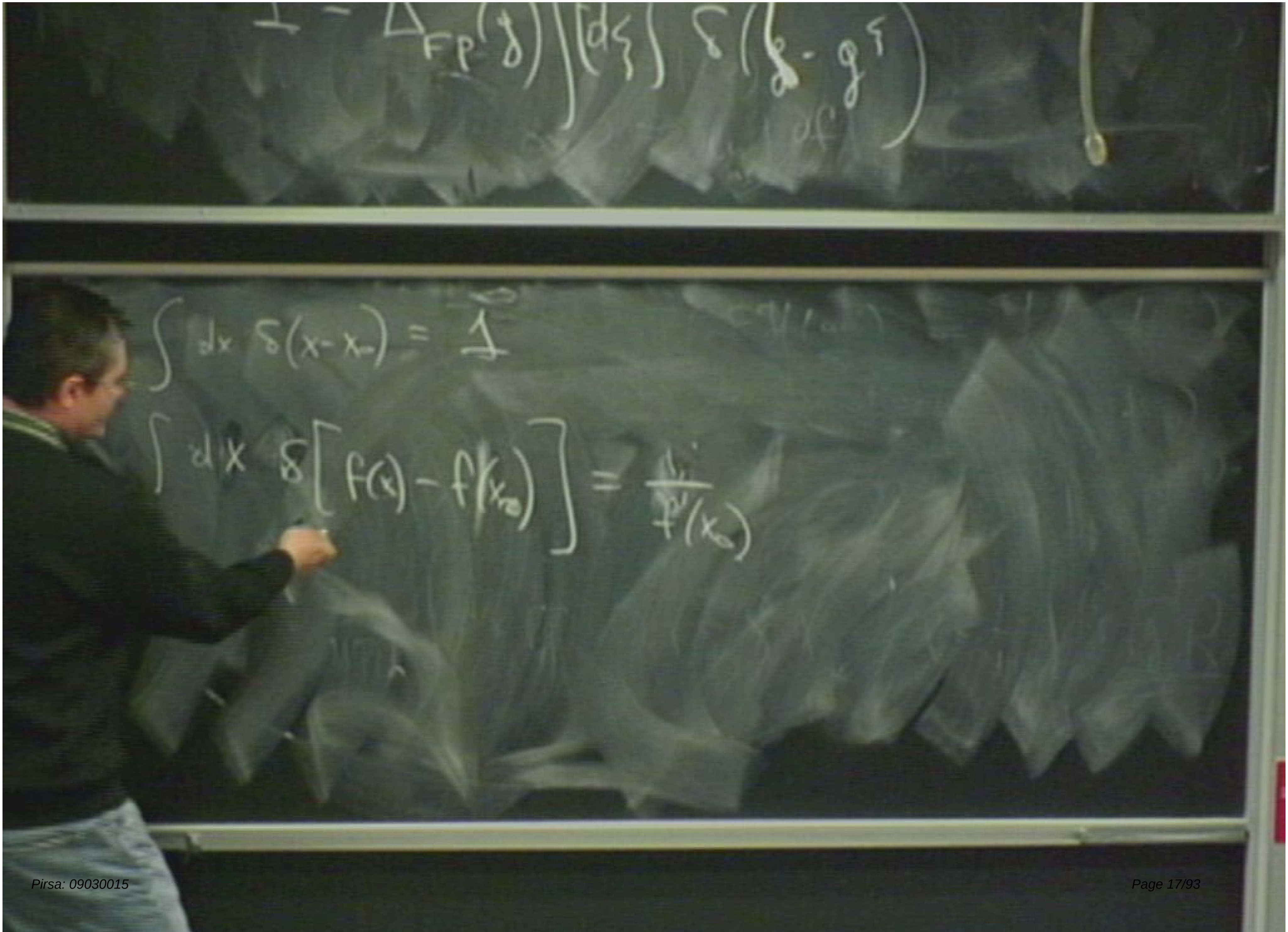
$$\xi: \quad g_{ab} \longrightarrow g^{\xi}_{ab}(\sigma') = \underbrace{e^{2\omega(\sigma)}}_{\text{weyl}} \underbrace{\frac{\partial \sigma^c}{\partial \sigma'^a} \frac{\partial \sigma^d}{\partial \sigma'^b}}_{\text{diffeo}} g_{cd}$$

Introduce FP measure

$$1 = \Delta_{FP}(g) \int [d\xi] \, \delta\left(g - g^{\xi}\right)$$



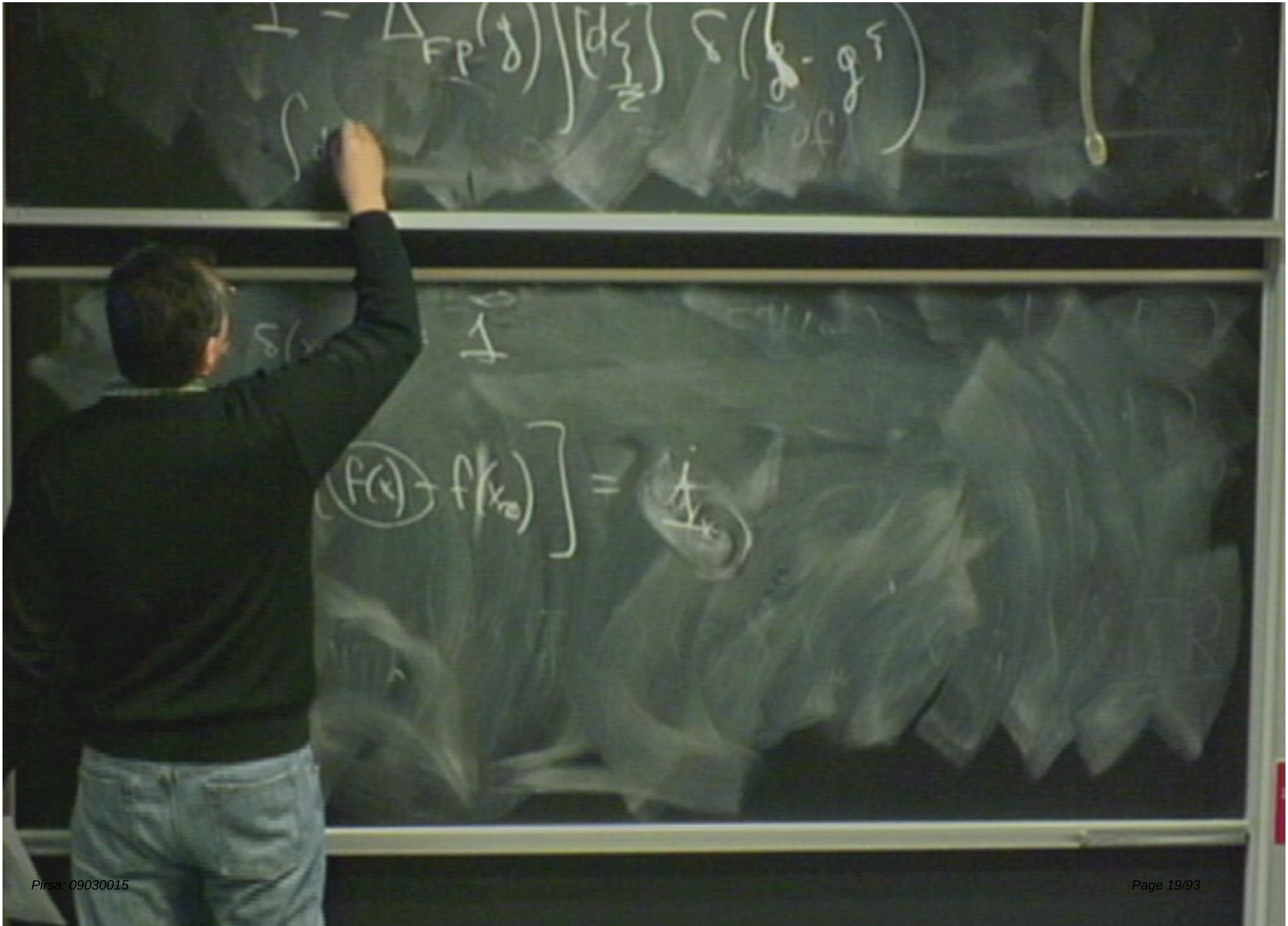
$1 = \Delta_{FP}(g) \int [d\xi] \, \delta(g - g^{\xi})$
$\int dx \, \delta(x - x_0) = 1$
$\int dx \, \delta[f(x) - f(x_0)] =$

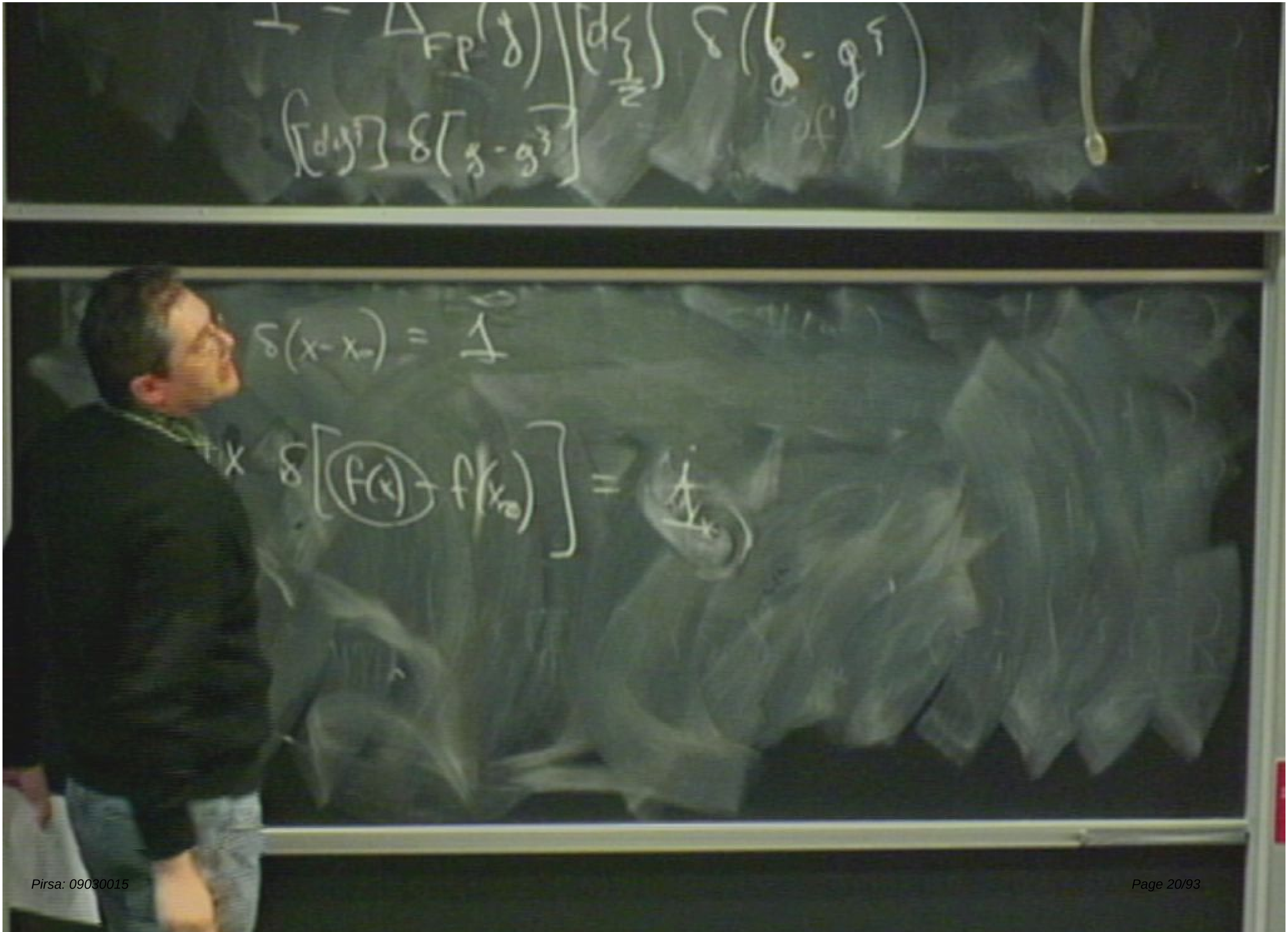

$1 = \Delta_{FP}(g) \int d\xi \int \delta(g - g^{\xi})$
$\int dx\, \delta(x - x_0) = 1$
$\int dx\, \delta[f(x) - f(x_0)] = \frac{1}{f'(x_0)}$

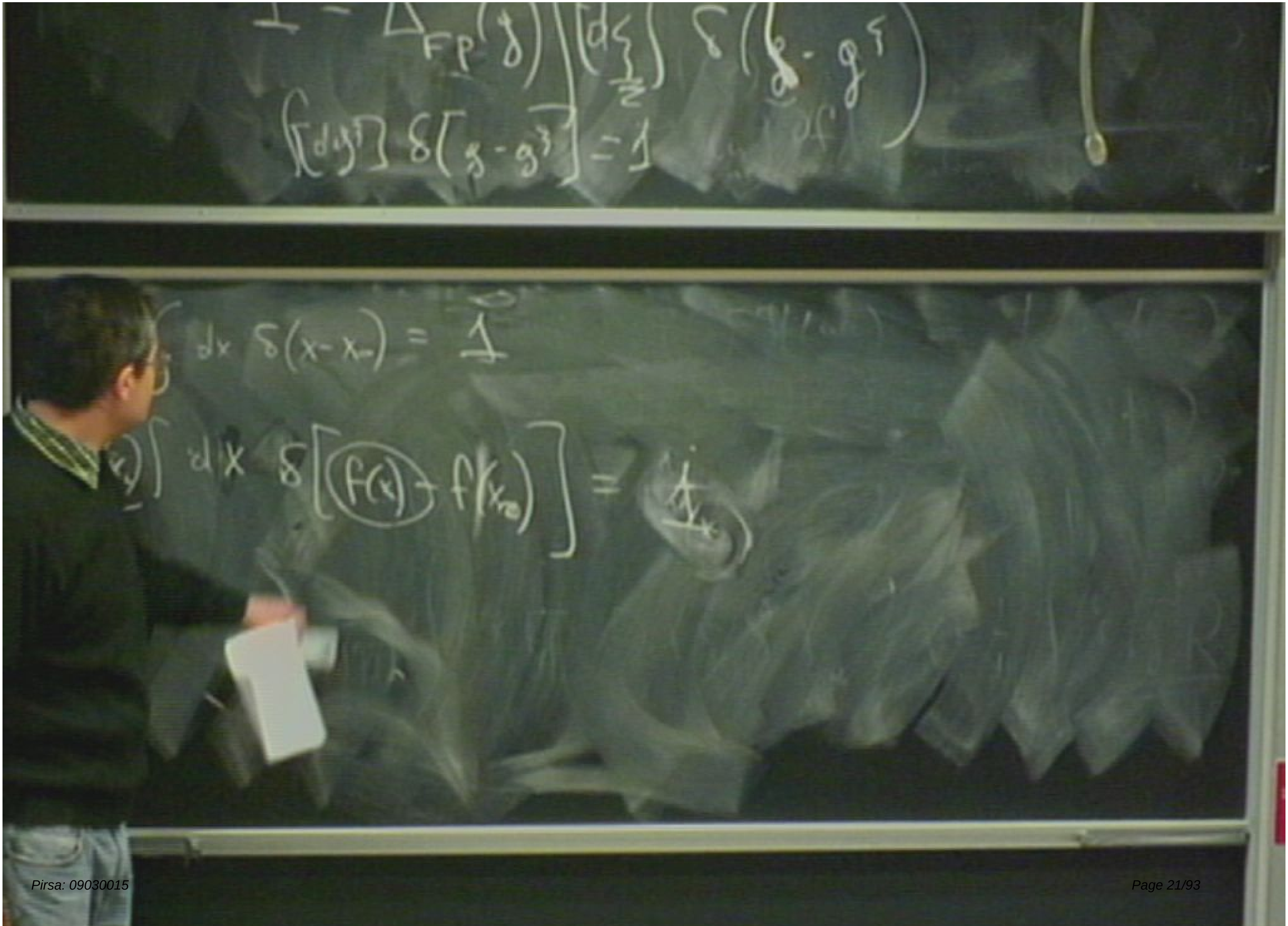
$$1 = \Delta_{FP}(g) \int [d\xi] \int \delta\left(g - g^{\xi}\right)$$

$$\int dx\, \delta(x - x_0) = 1$$

$$\underbrace{f'(x_0)}_{\Delta_{FP}} \int dx\, \delta\left[f(x) - f(x_0)\right] = 1$$







1 = Δ_FP(g) ∫[dξ] δ(g - g^ξ)
∫[dg^ξ] δ[g - g^ξ] = 1
∫ dx δ(x - x_0) = 1
∫ dx δ[f(x) - f(x_0)] =

$$1 = \Delta_{FP}(g) \int [d\xi] \, \delta\left(g - g^{\xi}\right)$$

$$\int [dg^{\xi}] \, \delta\left[g - g^{\xi}\right] = 1$$

$$\int dx \, \delta(x - x_0) = 1$$

$$\underbrace{f'(x_0)}_{\Delta_{FP}} \int dx \, \delta\left[f(x) - f(x_0)\right] = 1$$

$$\int d f(x) \left[\delta\left(F - f(x)\right)\right] = 1$$

(i) we assume $[d\xi]$

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(ii) we assume that δ is gauge invariant.

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$$\int [d\xi]\, \delta(\xi - \xi^{0}) = 1$$

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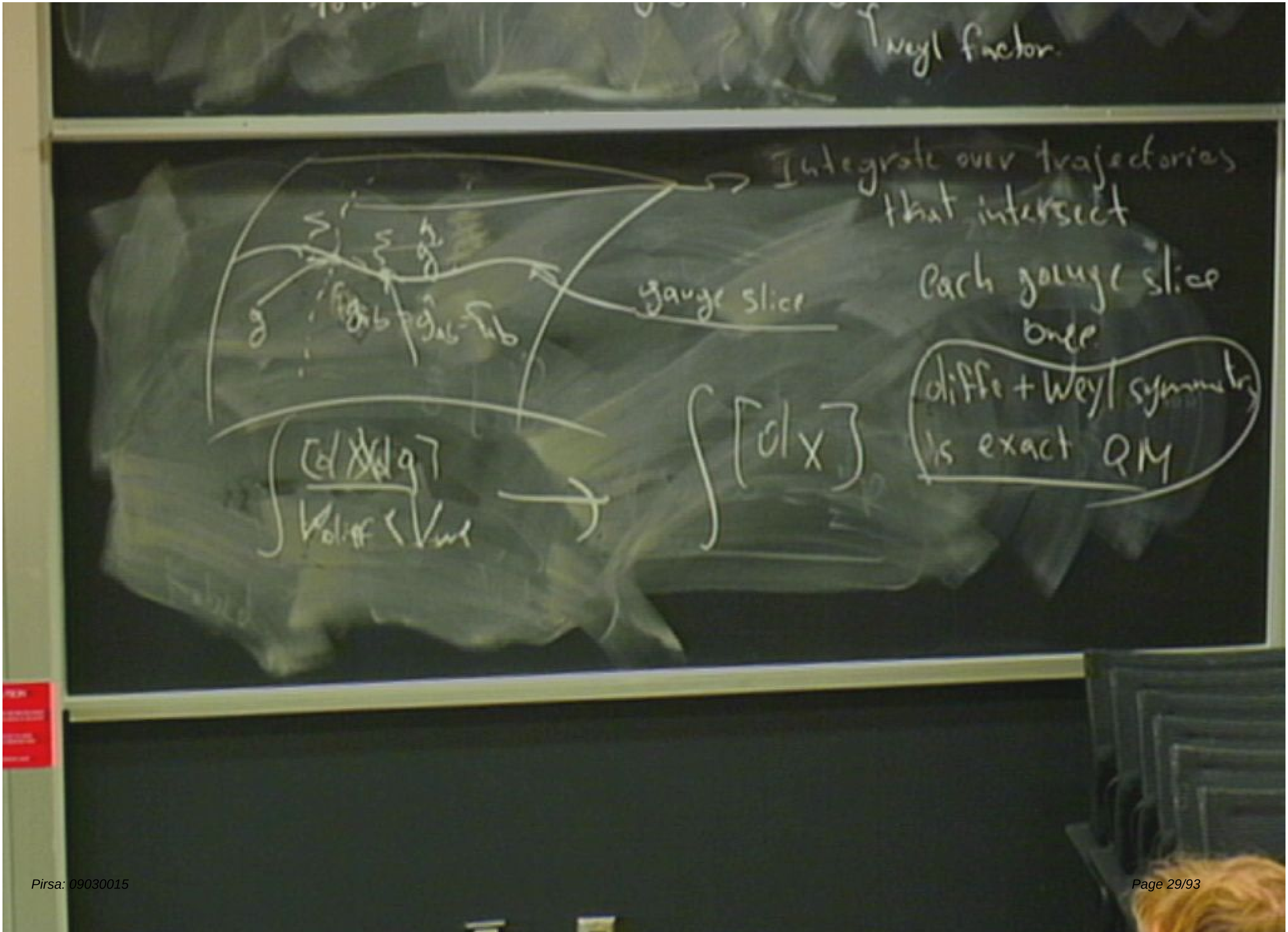
$$\int [d\xi]\, \delta(\xi - \xi^0) = 1$$

$$\Delta_{FP}(g) =$$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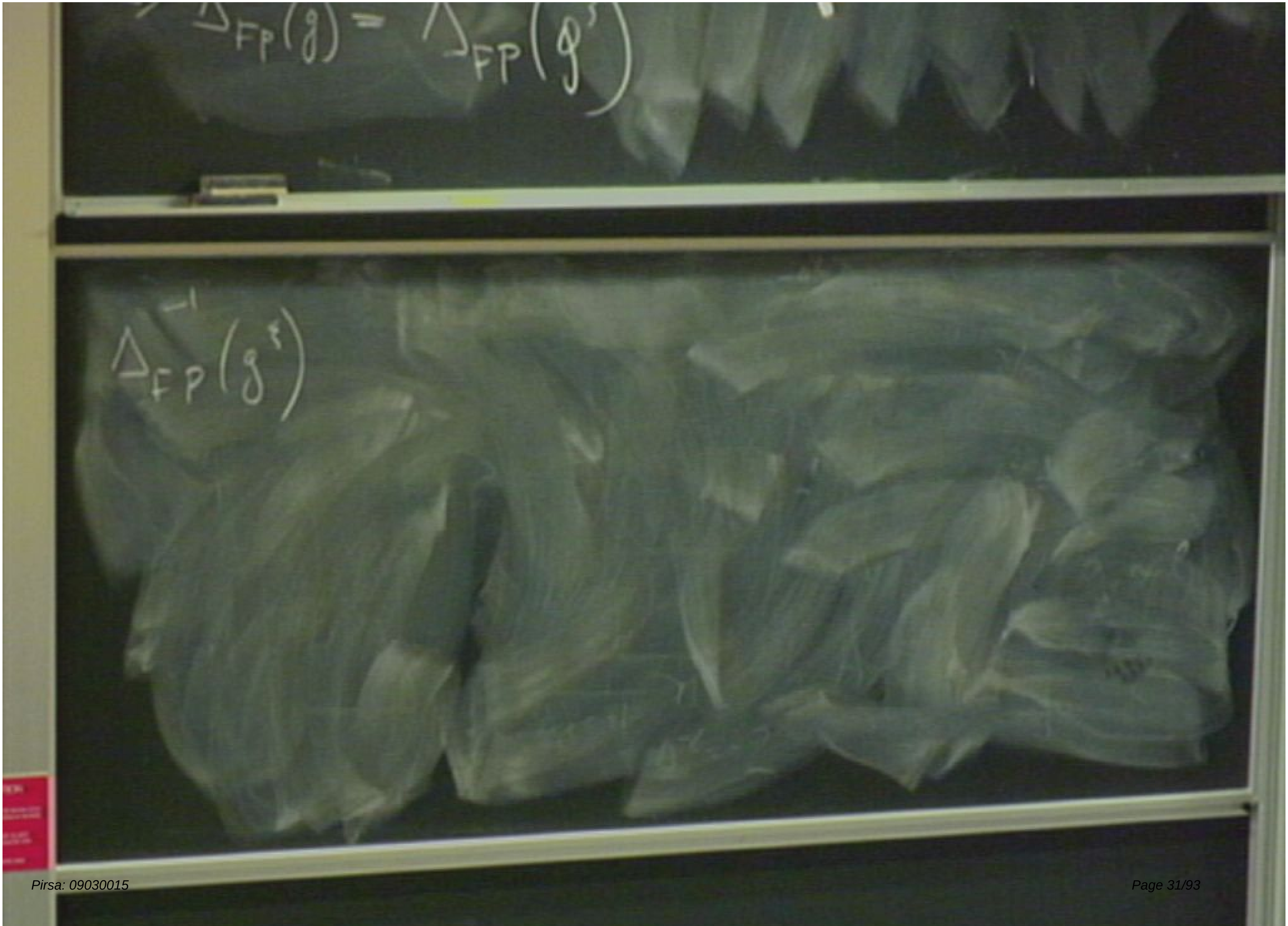
(i) we assume $[d\xi]$ is gauge invariant

(ii) we assume that δ is gauge invariant.

$$\int [d\xi]\, \delta(\xi - \xi^0) = 1$$

$$\hookrightarrow \Delta_{FP}(g) = \Delta_{FP}(g^{\xi})$$


$\Delta_{FP}(g) = \Delta_{FP}(g^{\xi})$
$\Delta^{-1}_{FP}(g^{\xi})$

$$\Delta_{FP}(g) = \Delta_{FP}(g^{\xi})$$

$$\Delta_{FP}^{-1}(g^{\xi}) = \int [d\xi'] \, \delta\left(g^{\xi} - g^{\xi'}\right)$$

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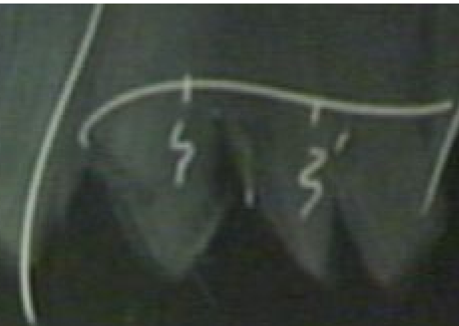

$$\Delta_{FP}^{-1}(g^{\xi}) = \int [d\xi'] \, \delta(g^{\xi} - g^{\xi'}) = \int$$

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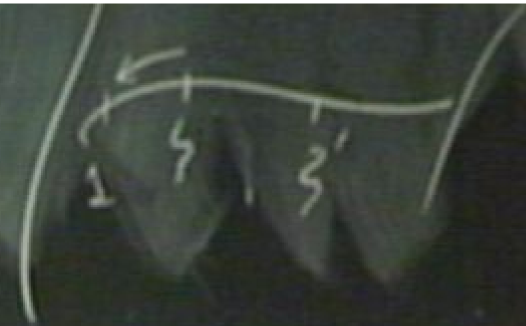

$$\Delta_{FP}^{-1}(g^{\xi}) = \int [d\xi']\, \delta(g^{\xi} - g^{\xi'}) = \int$$

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$$\Delta_{FP}(g) = \Delta_{FP}(g^{\xi})$$

$$\Delta_{FP}^{-1}(g^{\xi}) = \int [d\xi']\, \delta\left(g^{\xi} - g^{\xi'}\right) = \int [d\xi']\, \delta(g$$

$$\Delta_{FP}^{-1}(g^{\xi}) = \int [d\xi']\, \delta\left(g^{\xi} - g^{\xi'}\right) = \int [d\xi']\, \delta\left(g - g^{\xi^{-1}\xi'}\right)$$

$$\Delta_{FP}(g) = \Delta_{FP}(g^{\xi})$$

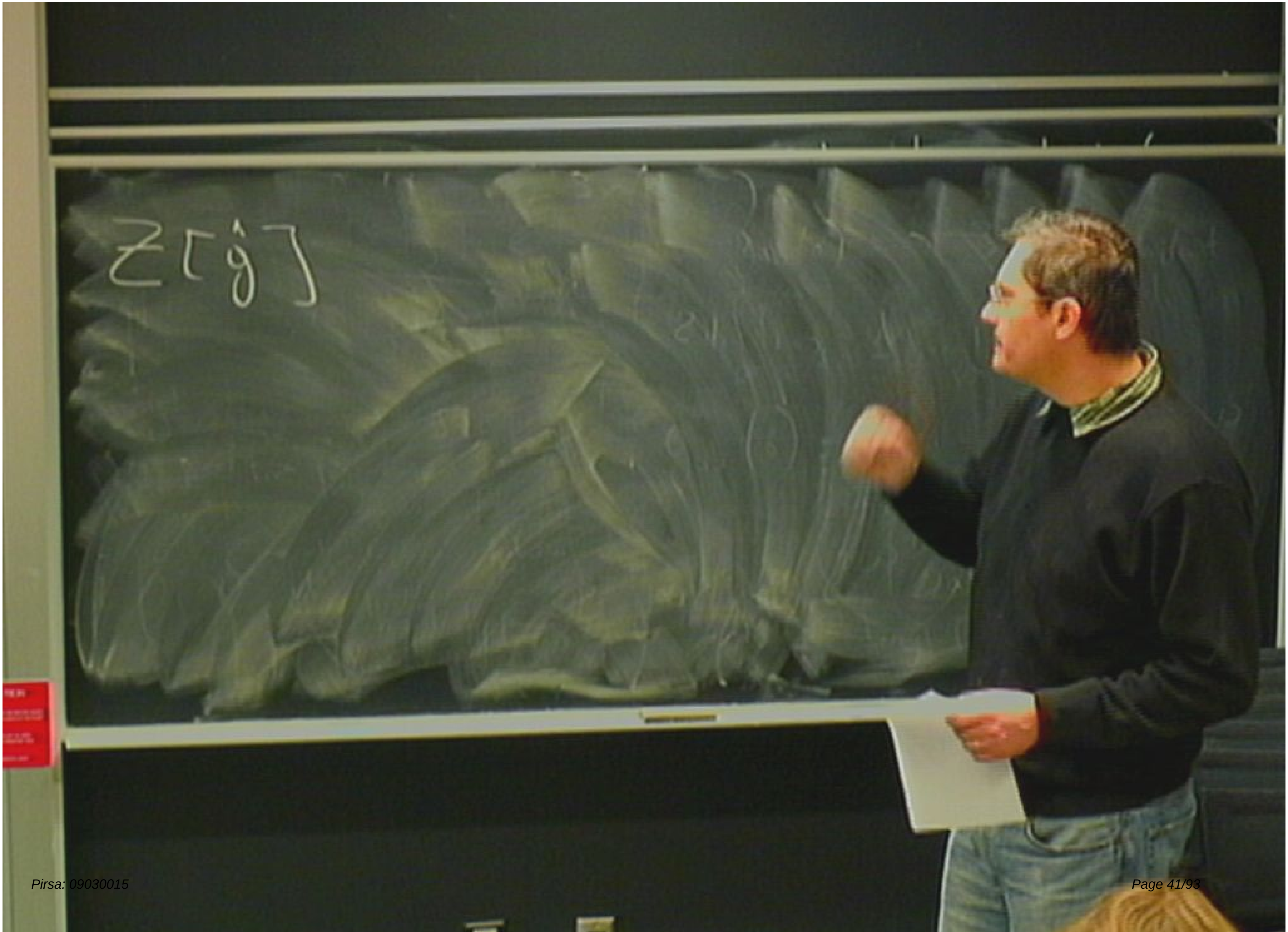
$$\Delta_{FP}^{-1}(g^{\xi}) = \int [d\xi']\, \delta\left(g^{\xi} - g^{\xi'}\right) = \int [d\xi']\, \delta\left(g - g^{\xi^{-1}\xi'}\right)$$

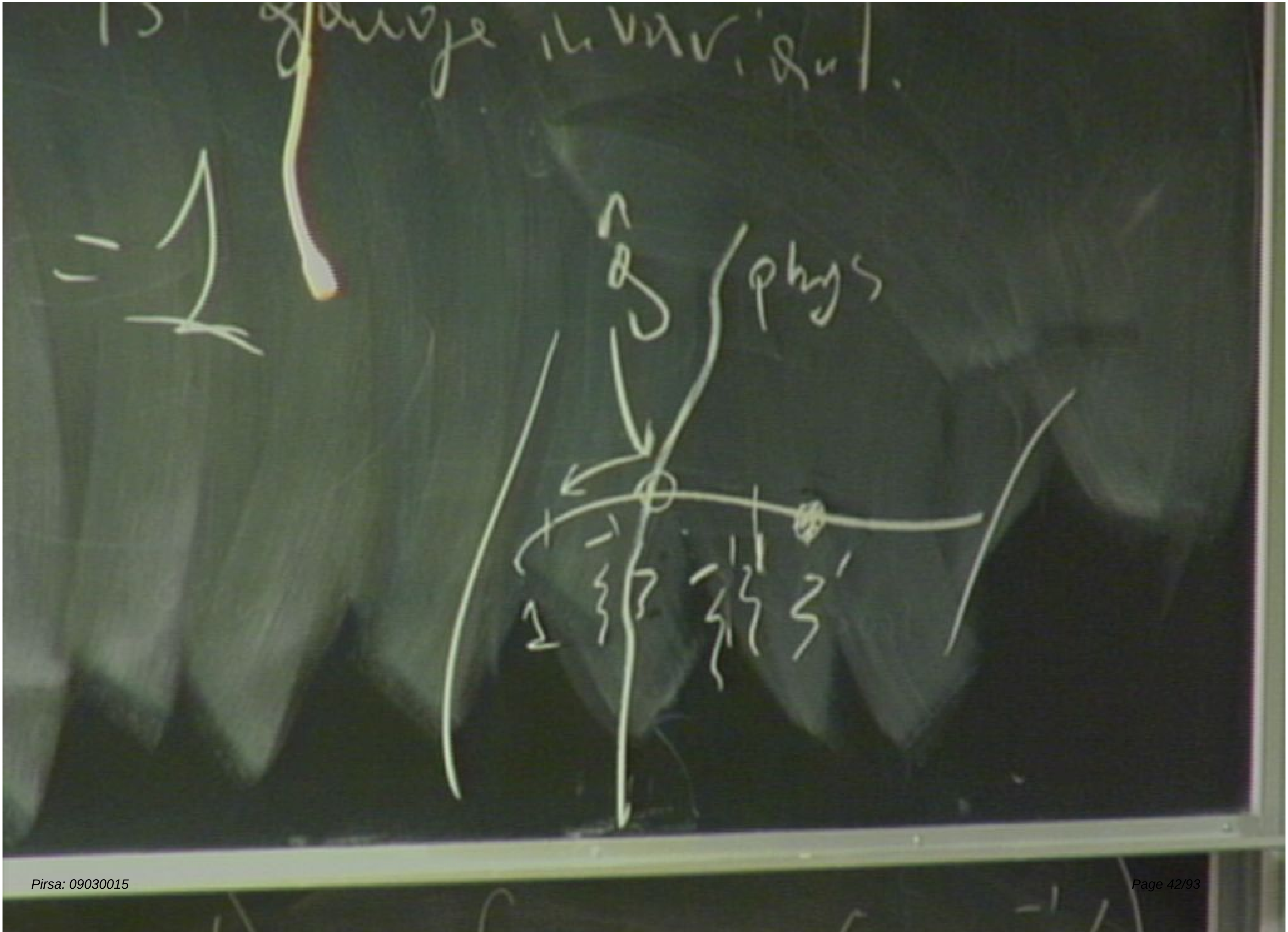
$$= \int [d\xi'']\, \delta\left(g - g^{\xi''}\right)$$

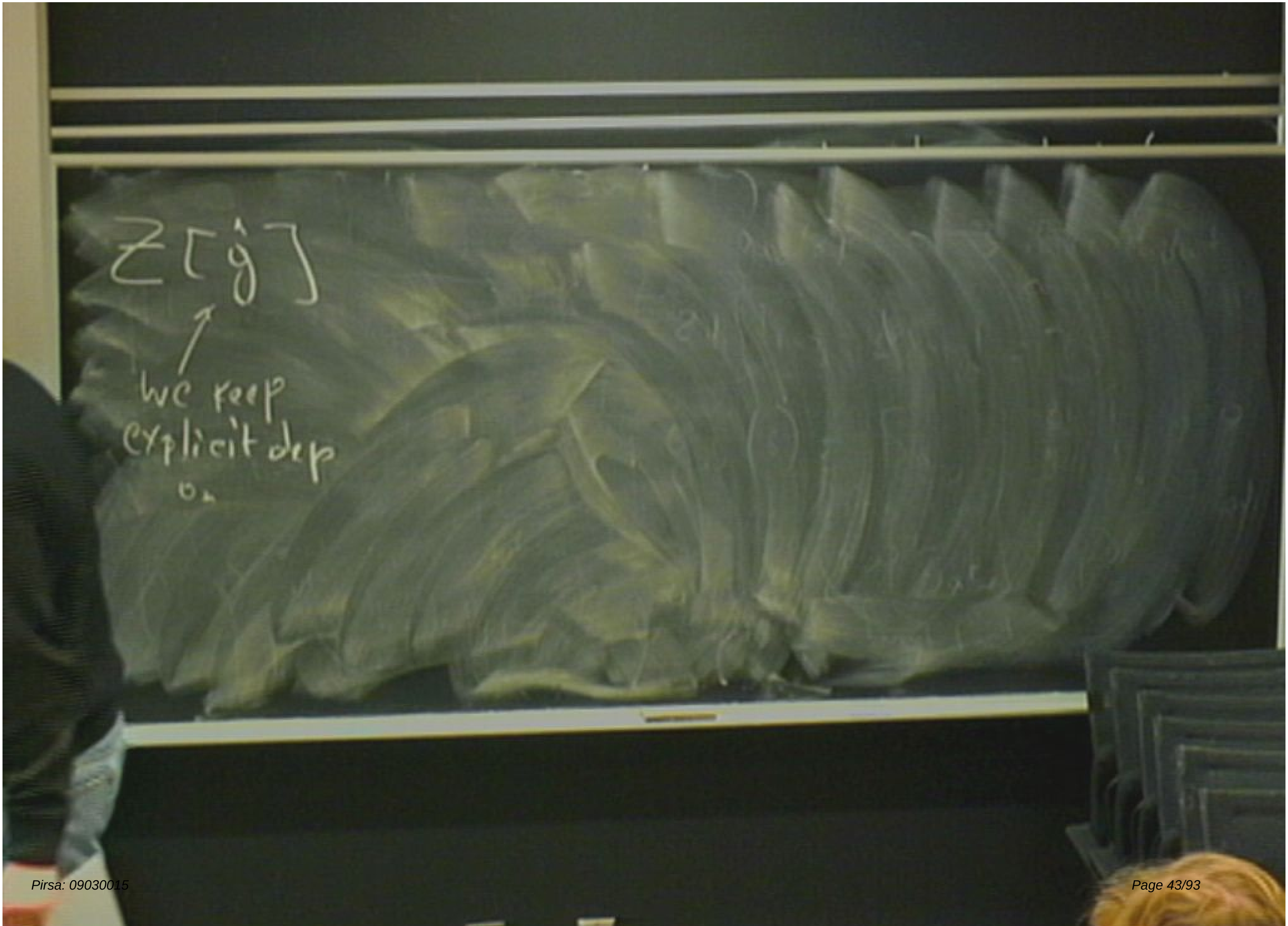
$$\Delta_{FP}(g) = \Delta_{FP}(g)$$

$$\Delta^{-1}_{FP}(g^{\xi}) = \int [d\xi'] \, \delta\left(g^{\xi} - g^{\xi'}\right) = \int [d\xi'] \, \delta\left(g - g^{\overbrace{\xi^{-1}\xi'}^{}_{\xi''}}\right)$$

$$= \int [d\xi''] \, \delta\left(g - g^{\xi''}\right) = \Delta_{FP}(g)$$


Z[ĝ]


phys


Z[ĝ]
we keep
explicit dep

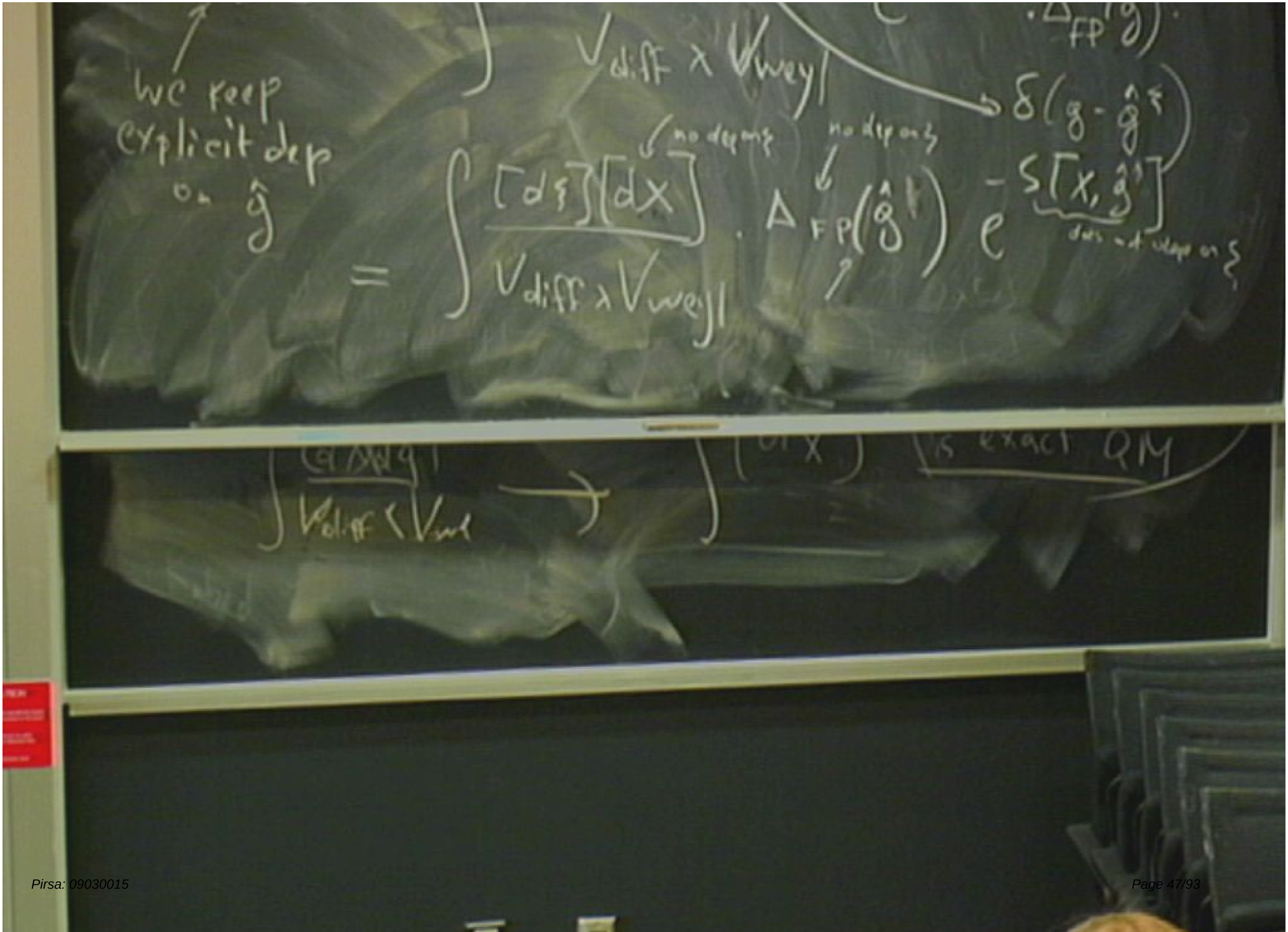
$$Z[\hat{g}] = \int \frac{[dX][dg]}{V_{\text{diff}} \times V_{\text{weyl}}} \, e^{-S[X,g]}$$

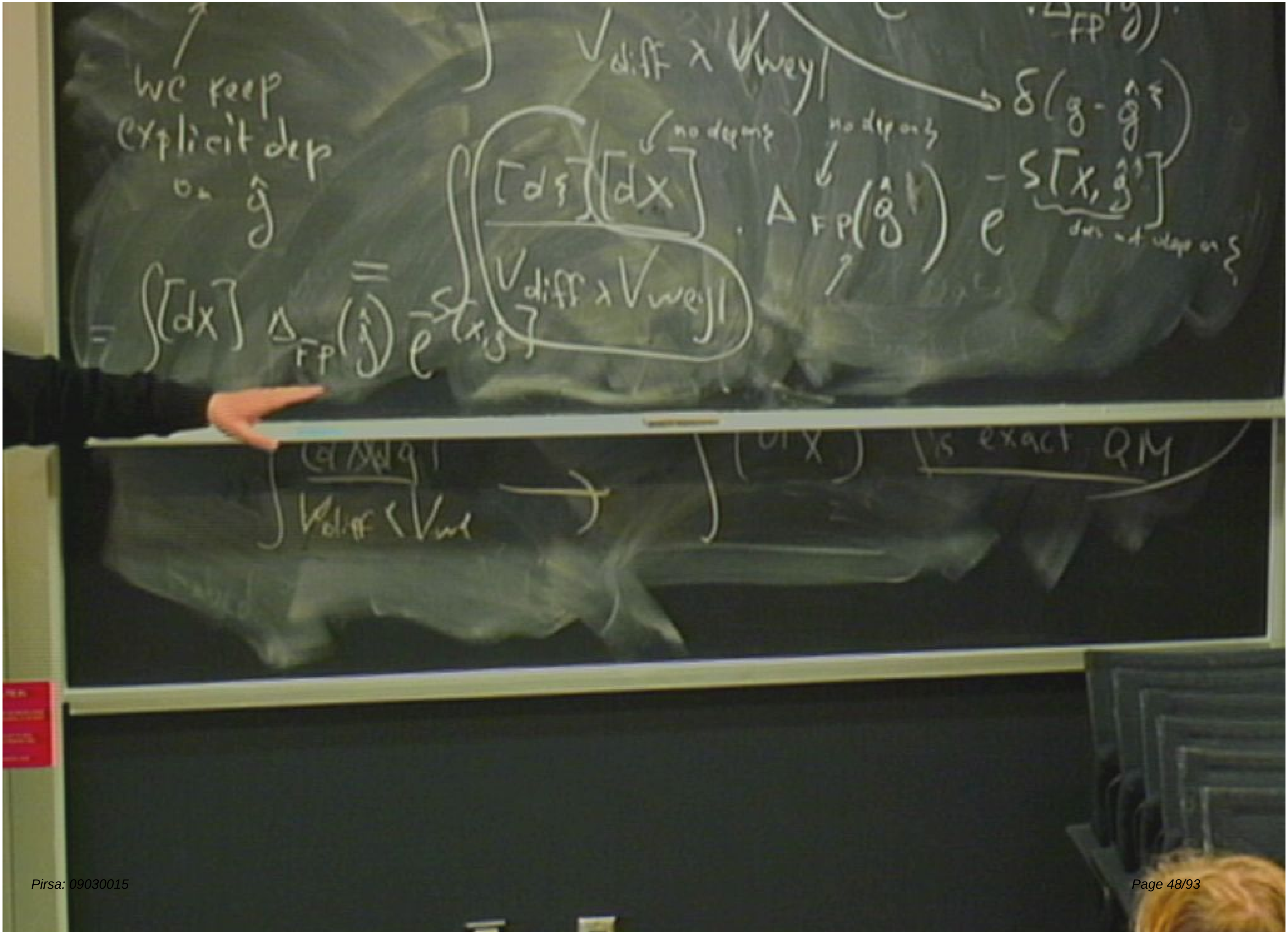
we keep explicit dep on $\hat{g}$

$$Z[\hat{g}] = \int \frac{[dX][dg][d\xi]}{V_{diff} \times V_{weyl}} e^{-S[X,g]} \cdot \Delta_{FP}(g) \cdot \delta(g - g^{\xi})$$

we keep explicit dep on $\hat{g}$


we keep
explicit dep
on ĝ
V diff x V weyl
δ(g - g
[dξ][dx]
V diff x V weyl
Δ FP(


we keep explicit dep on ĝ
no dep on ξ
no dep on ξ
does not dep on ξ
is exact QM


we keep
explicit dep
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V diff x V weyl
no dep on ξ
no dep on ξ
does not depend on ξ
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is exact QM

$$= \int [dx]\, \Delta_{FP}(\hat{g})\, e^{\dots}$$

$\Rightarrow$ we want to compute $\Delta_{FP}(\hat{g})$

$$\Delta_{FP}^{-1}(\hat{g}) = \int [d\xi]\, \delta\left(g - g^{\xi}\right)$$

$\Rightarrow$ we want to compute $\Delta_{FP}(\hat{g})$

$$\Delta_{FP}^{-1}(\hat{g}) = \int [d\xi]\, \delta\left(\hat{g} - \hat{g}^{\xi}\right)$$

$$\hat{g} \to \hat{g}^{\xi}$$

$\Rightarrow$ we want to compute $\Delta_{FP}(\hat{g})$

$$\Delta_{FP}^{-1}(\hat{g}) = \int [d\xi]\; \delta\left(\hat{g} - \hat{g}^{\xi}\right)$$

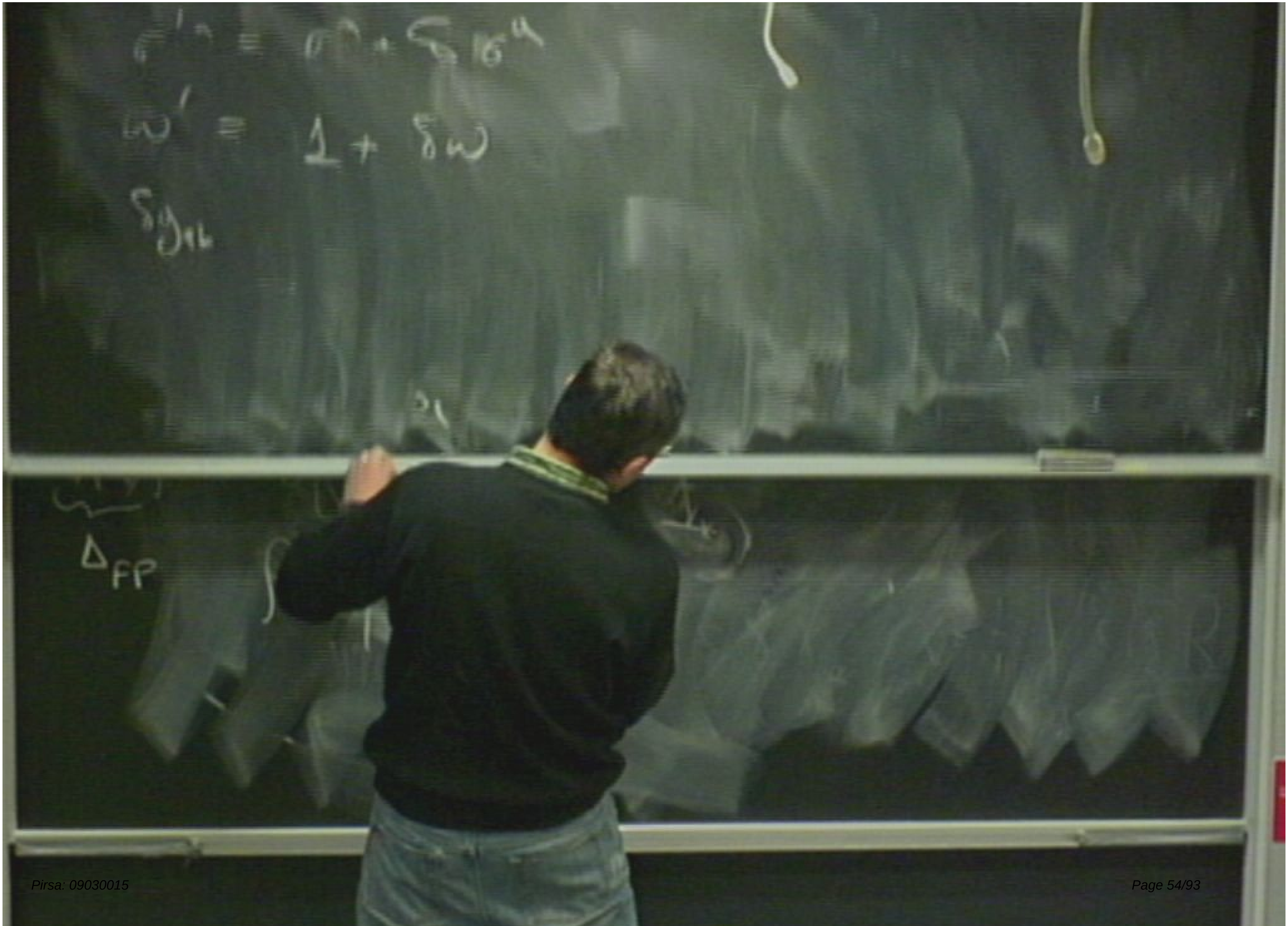
$$\hat{g} = \hat{g}^{\xi} \Rightarrow \xi = \mathbb{1}$$

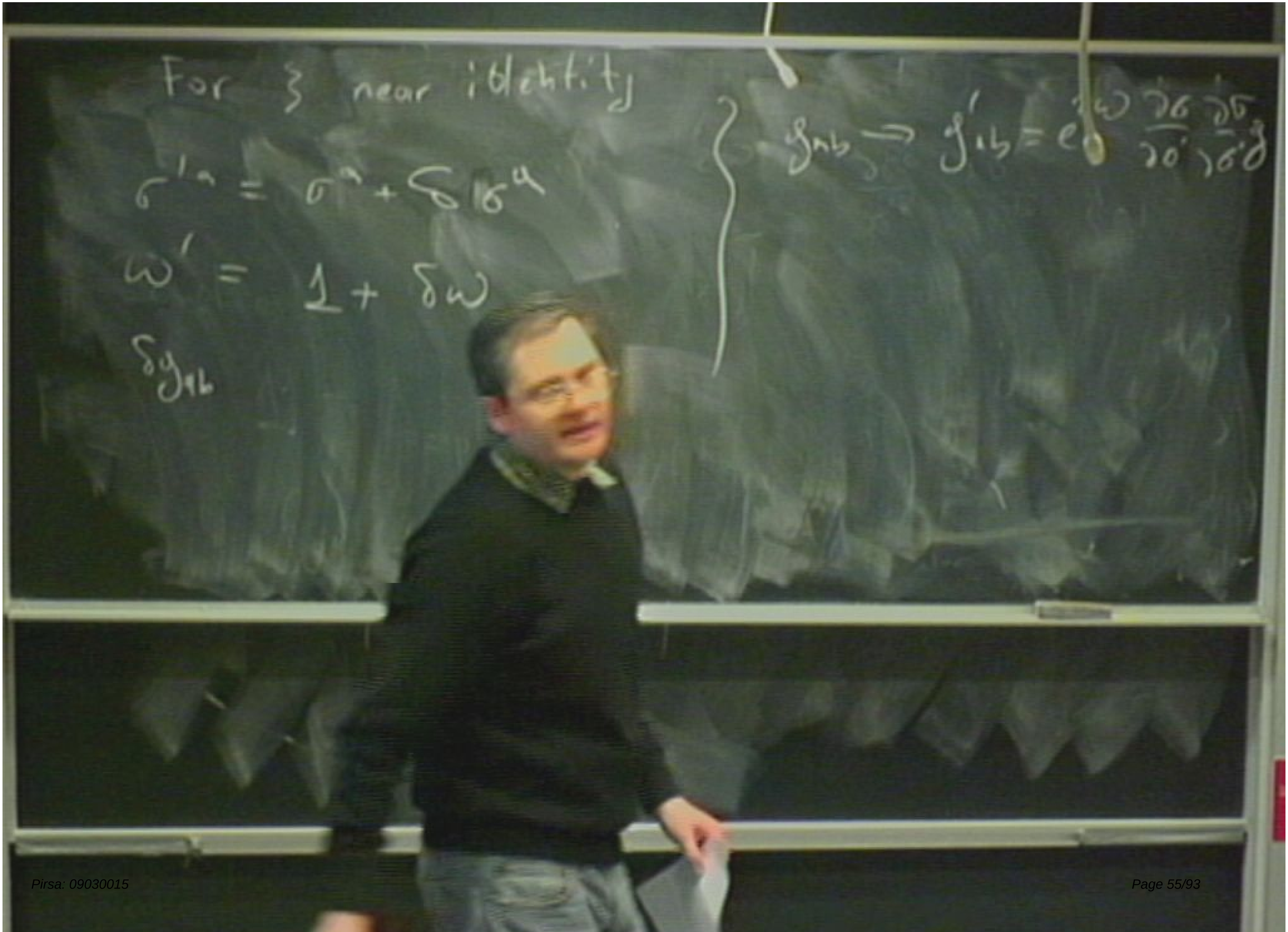

For ξ near identity

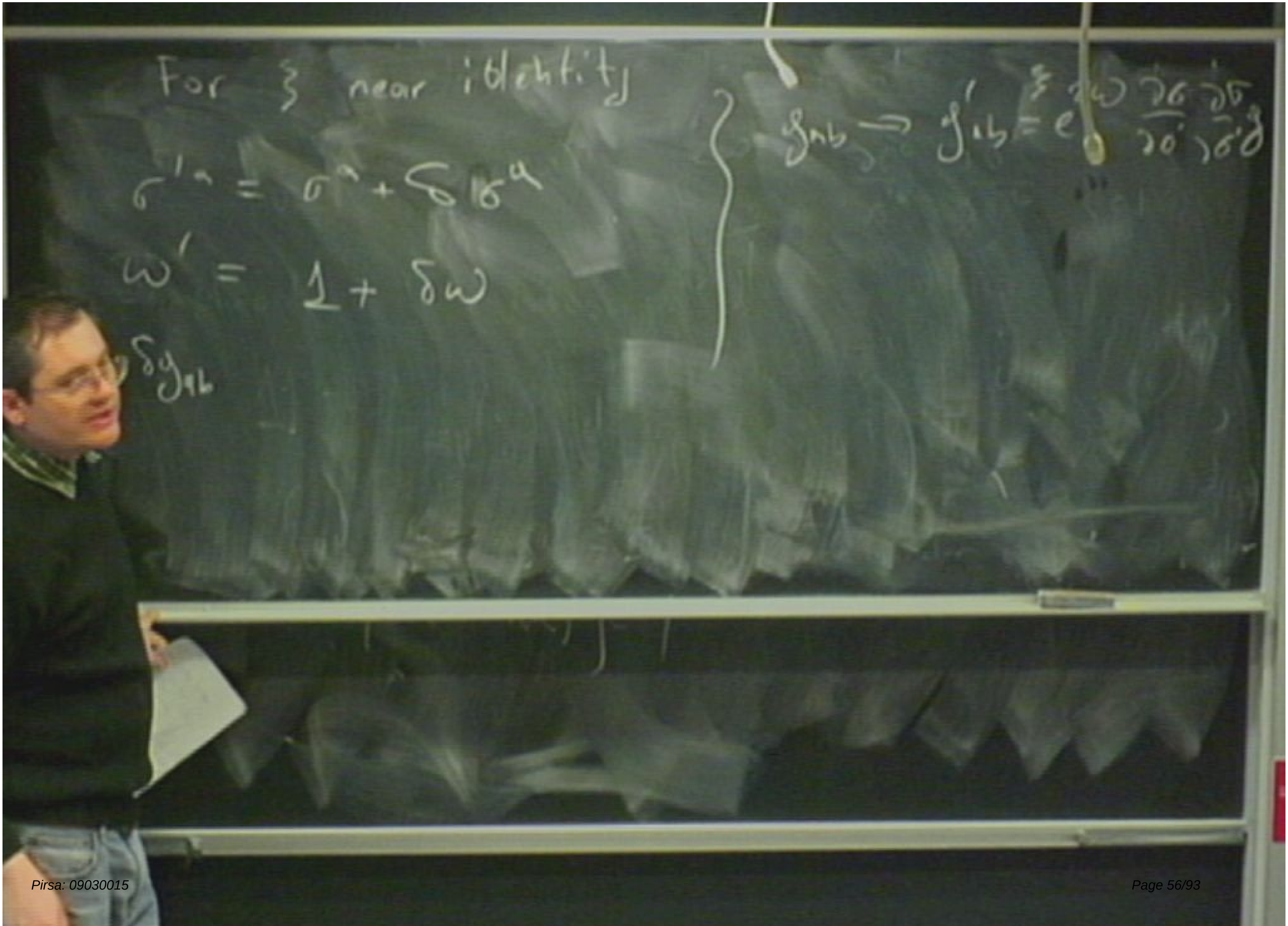
For ξ near identity

$$\sigma'^{a} = \sigma^{a} + \delta\sigma^{a}$$

$$\omega' = 1$$


ω' = 1 + δω
δg_ab
Δ_FP


For ξ near identity
σ'^a = σ^a + δσ^a
ω' = 1 + δω
δg_ab
g_ab ⟹ g'_ab = e^{2ω} ∂σ/∂σ' ∂σ/∂σ' g

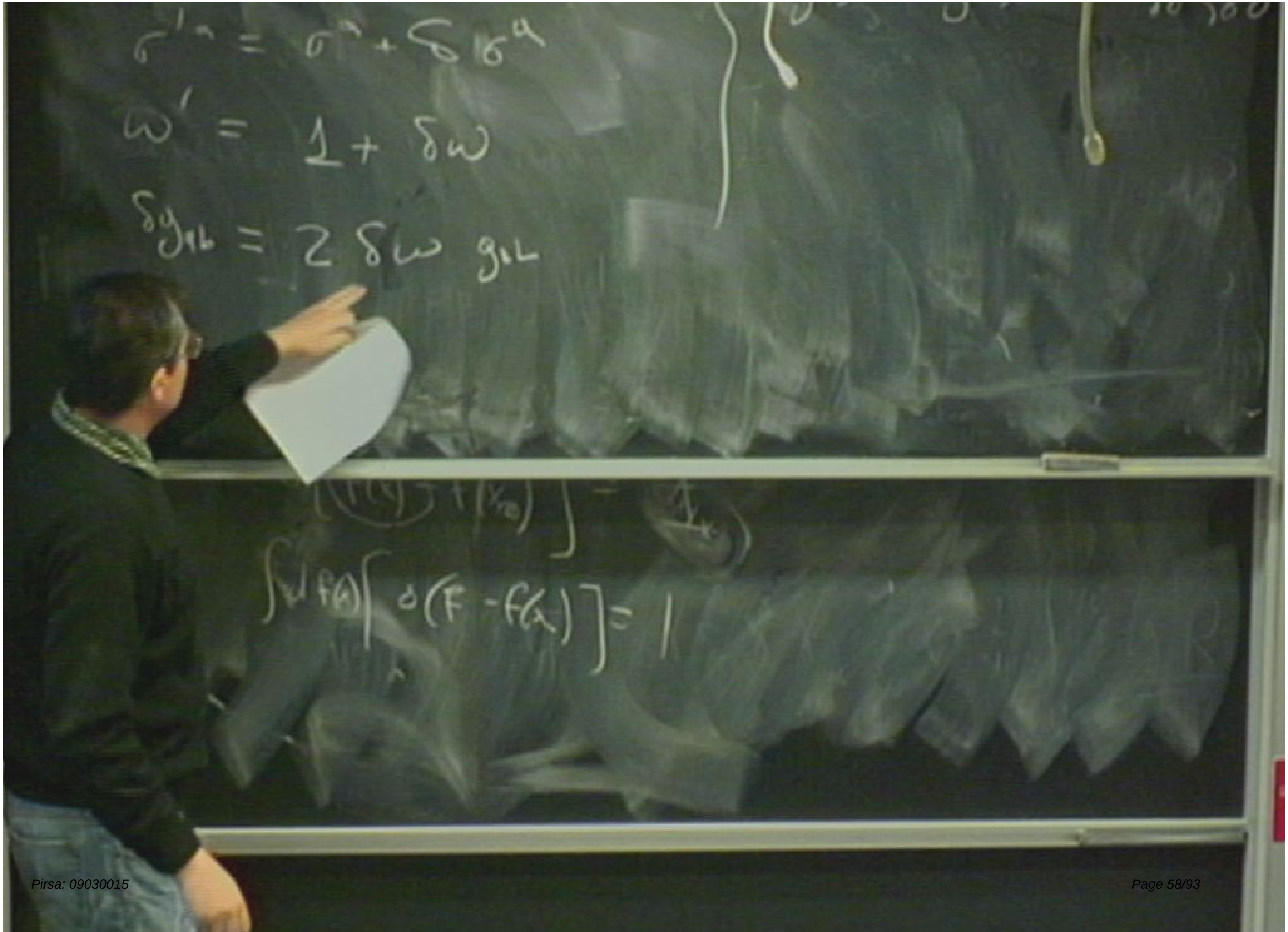

For ξ near identity
$\sigma'^a = \sigma^a + \delta\sigma^a$
$\omega' = 1 + \delta\omega$
δg_{ab}
$g_{ab} \Rightarrow g'_{ab} = e^{\xi\omega} \frac{\partial\sigma}{\partial\sigma'} \frac{\partial\sigma}{\partial\sigma'} g$

$\Rightarrow$ we want to compute $\Delta_{FP}(\hat{g})$

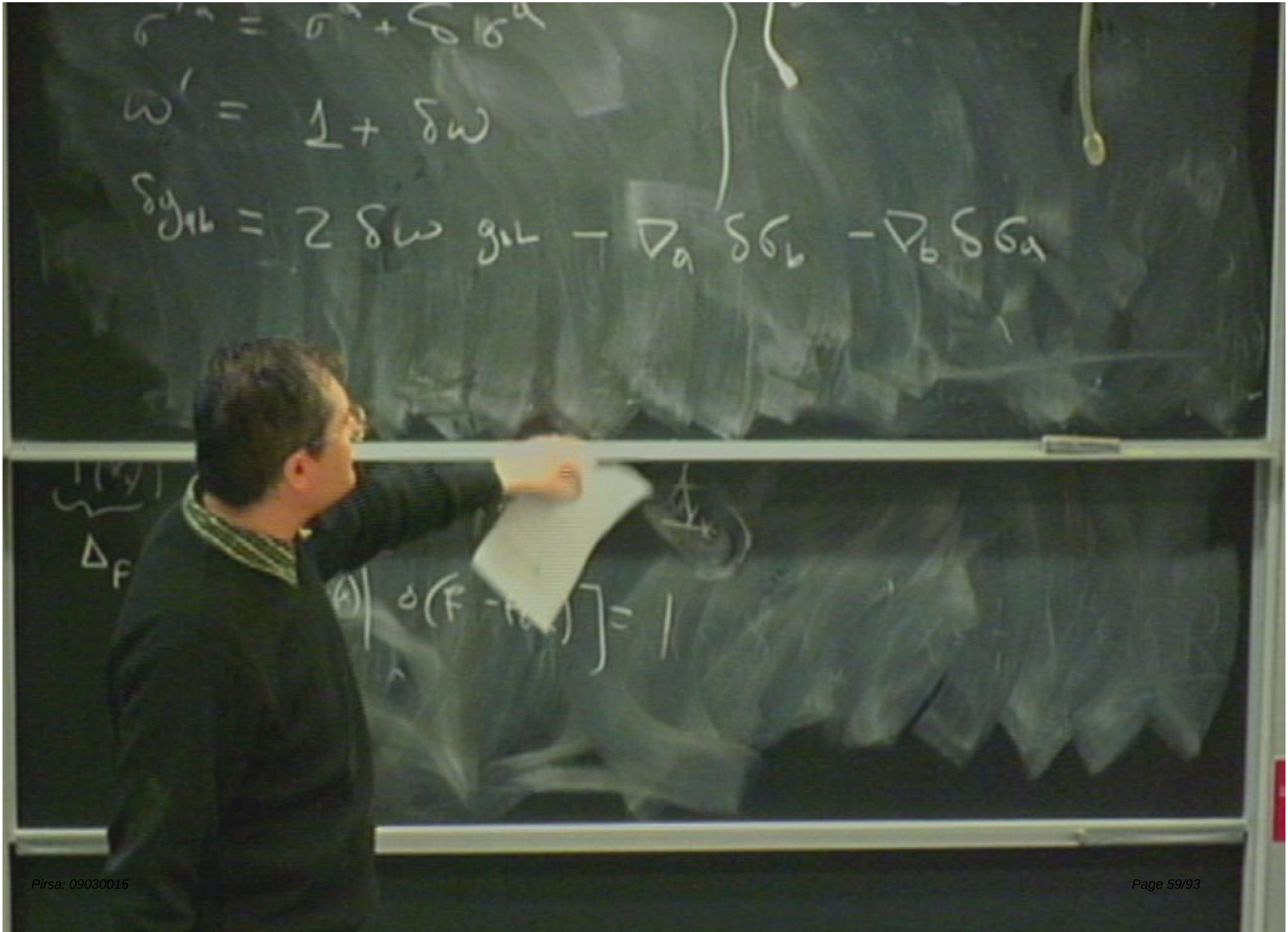
$$\Delta_{FP}^{-1}(\hat{g}) = \int [d\xi]\, \delta(\hat{g} - \hat{g}^{\xi})$$

$$\hat{g} = \hat{g}^{\xi} \Rightarrow \xi = 1$$

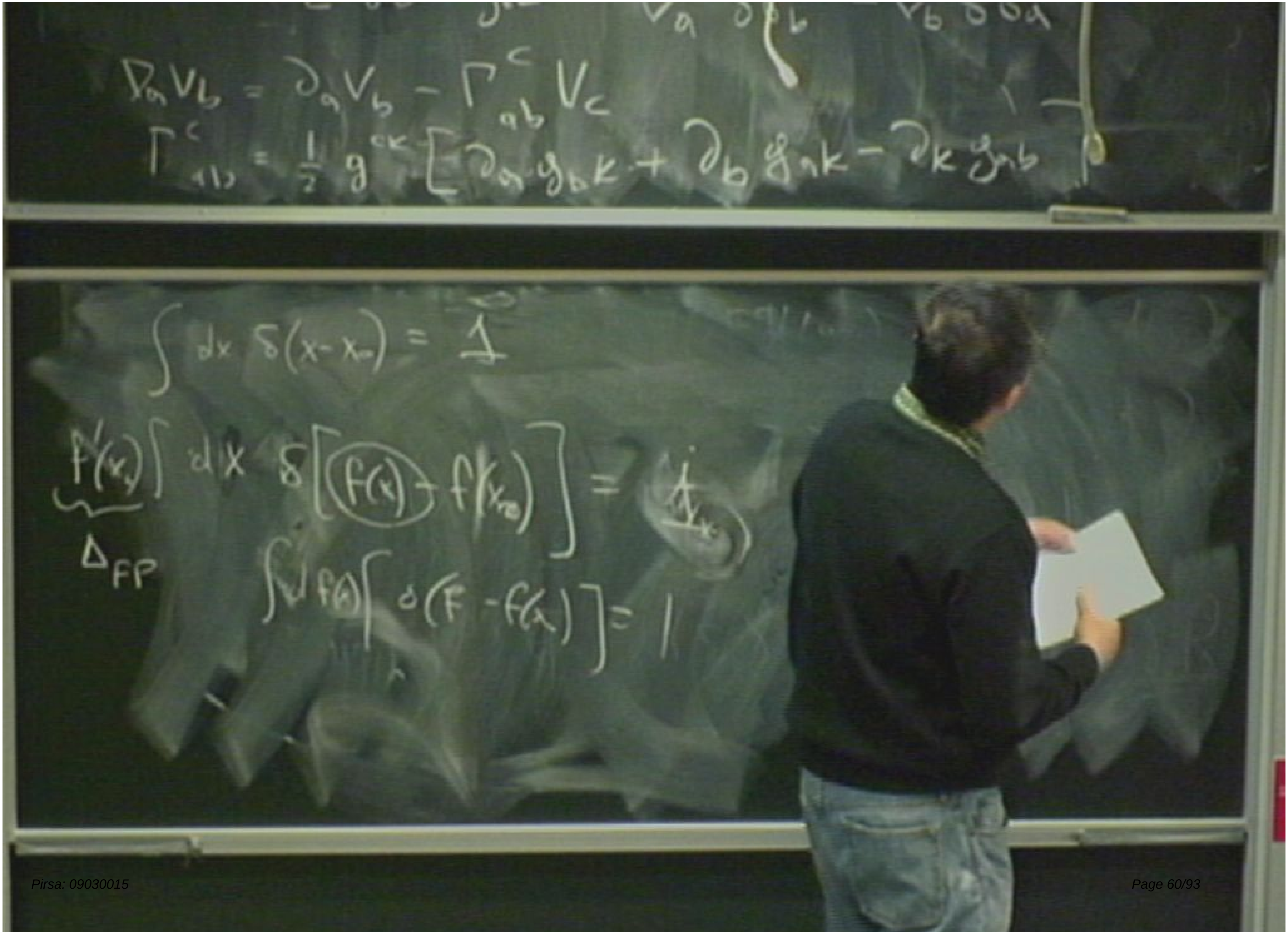
$\int dv \delta[f(v) - f(x_0)]$

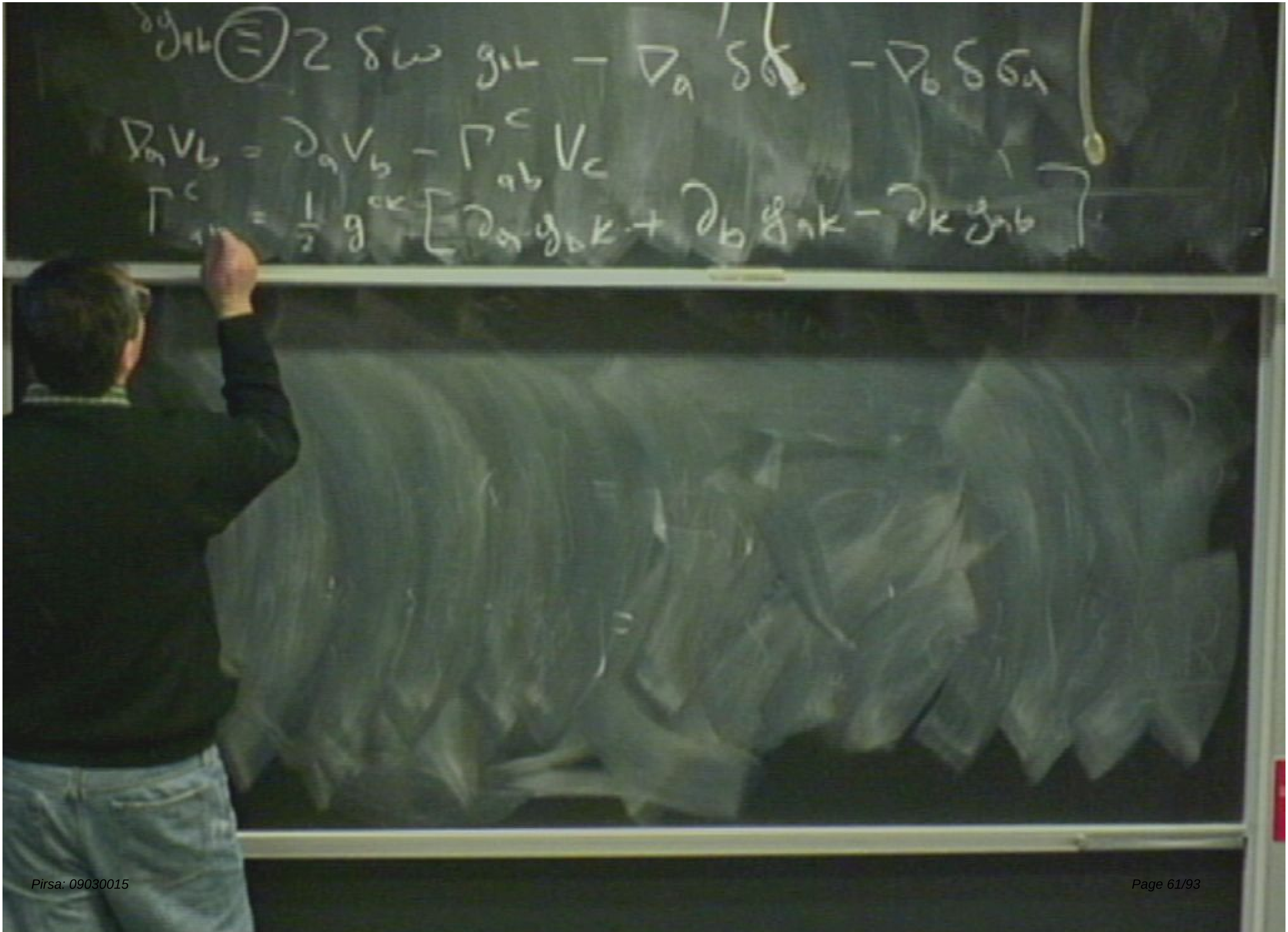


σ'^a = σ^a + δσ^a
ω' = 1 + δω
δg_ab = 2 δω g_ab

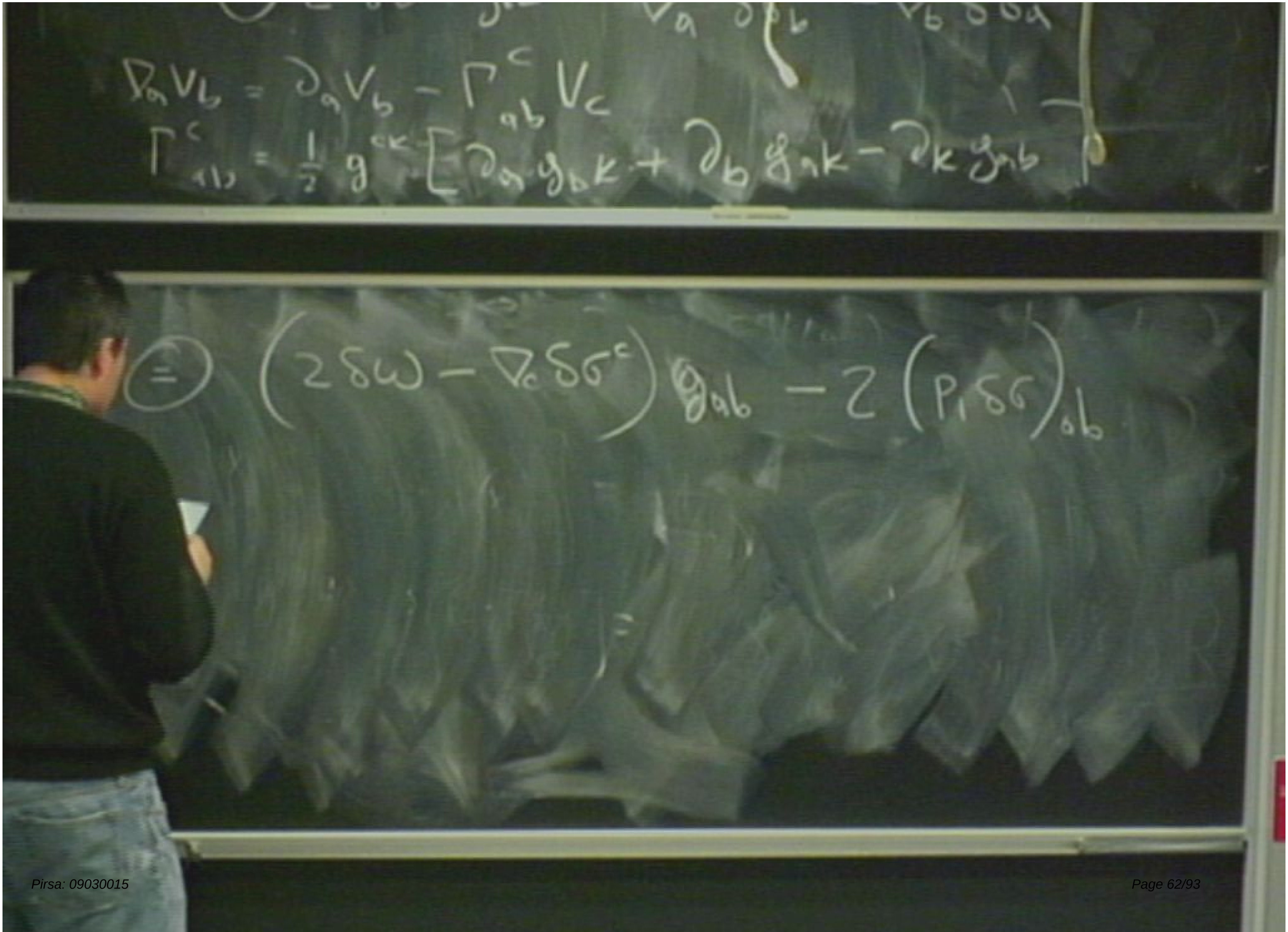


$\omega' = 1 + \delta\omega$
$\delta g_{ab} = 2\,\delta\omega\, g_{ab} - \nabla_a \delta\sigma_b - \nabla_b \delta\sigma_a$


$\nabla_a V_b = \partial_a V_b - \Gamma^c_{ab} V_c$
$\Gamma^c_{ab} = \frac{1}{2} g^{ck}[\partial_a g_{bk} + \partial_b g_{ak} - \partial_k g_{ab}]$
$\int dx\, \delta(x-x_0) = 1$
$\underbrace{f'(x_0)}_{\Delta_{FP}} \int dx\, \delta[f(x) - f(x_0)] = 1$
$\int df(x) [\delta(F - f(x))] = 1$



$\delta g_{ab} = 2\,\delta\omega\, g_{ab} - \nabla_a \delta\sigma_b - \nabla_b \delta\sigma_a$
$\nabla_a V_b = \partial_a V_b - \Gamma^c_{ab} V_c$
$\Gamma^c_{ab} = \frac{1}{2} g^{ck} [\partial_a g_{bk} + \partial_b g_{ak} - \partial_k g_{ab}]$


$\nabla_a V_b = \partial_a V_b - \Gamma^c_{ab} V_c$
$\Gamma^c_{ab} = \frac{1}{2} g^{ck} [\partial_a g_{bk} + \partial_b g_{ak} - \partial_k g_{ab}]$
$= (2\delta\omega - \nabla_c \delta\sigma^c) g_{ab} - 2 (P_1 \delta\sigma)_{ab}$

$$\nabla_a V_b = \partial_a V_b - \Gamma^c_{ab} V_c$$

$$\Gamma^c_{ab} = \frac{1}{2} g^{ck}\left[\partial_a g_{bk} + \partial_b g_{ak} - \partial_k g_{ab}\right]$$

$$= \left(2\delta\omega - \nabla_c \delta\sigma^c\right) g_{ab} - 2\left(P_1 \delta\sigma\right)_{ab}$$

$$\left(P_1 \delta\sigma\right)_{ab} = \frac{1}{2}\left[\nabla_a \delta\sigma_b + \nabla_b \delta\sigma_a - g_{ab} \nabla_c \delta\sigma^c\right]$$

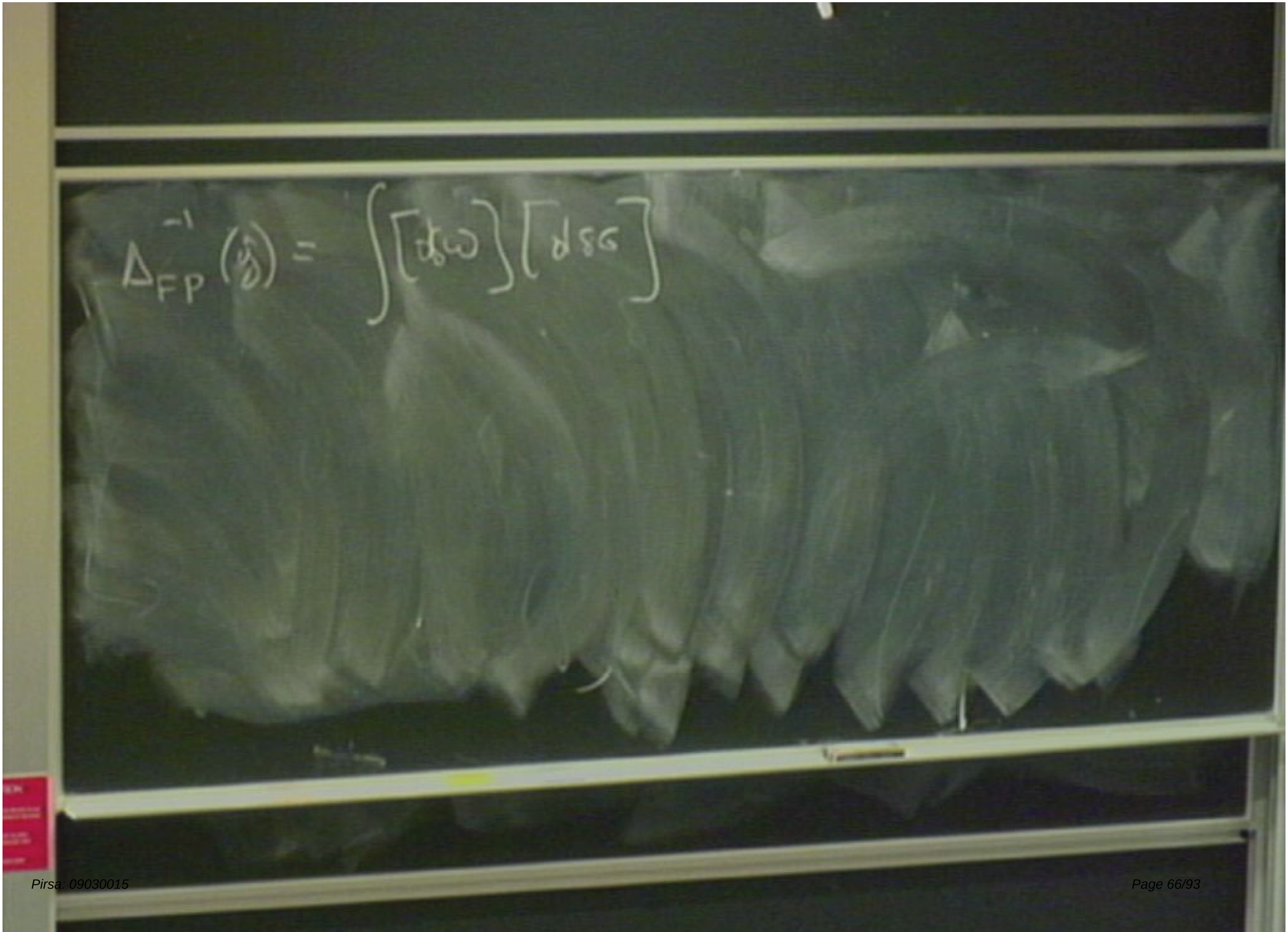
$$\nabla_a V_b = \partial_a V_b - \Gamma^c_{ab} V_c$$

$$\Gamma^c_{ab} = \frac{1}{2} g^{ck} \left[\partial_a g_{bk} + \partial_b g_{ak} - \partial_k g_{ab} \right]$$

$$= (2\delta\omega - \nabla_c \delta\sigma^c) g_{ab} - 2(P_1 \delta\sigma)_{ab}$$

$$(P_1 \delta\sigma)_{ab} = \frac{1}{2} \left[\nabla_a \delta\sigma_b + \nabla_b \delta\sigma_a - g_{ab} \nabla_c \delta\sigma^c \right]$$


$\Delta_{FP}^{-1}(g) = \int [\partial\omega]$


$\Delta_{FP}^{-1}(\hat{g}) = \int [d_{g}\omega][d\delta G]$

$$\Delta_{FP}^{-1}(\hat{g}) = \int [d\delta\omega][d\delta\sigma]\ \delta\left[-\delta g_{ab}\right]$$

$$= \int [d\delta\omega\, d\delta\sigma]\ \delta\left[-\left(2\delta\omega - \nabla_c \delta\sigma^c\right) g_{ab}\right.$$

$$\Delta_{FP}^{-1}(\hat{g}) = \int [d\delta\omega][d\delta\sigma]\; \delta\left[-\delta g_{ab}\right]$$

$$= \int [d\delta\omega\, d\delta\sigma]\; \delta\left[-\left(2\delta\omega - \nabla_c\delta\sigma^c\right)g_{ab} + 2P_1\delta\sigma\right]$$

$$\sigma'^a = \sigma^a + \delta\sigma^a$$

$$\omega' = 1 + \delta\omega$$

$$\delta g_{ab} = 2\,\delta\omega\, g_{ab} - \nabla_a \delta\sigma_b - \nabla_b \delta\sigma_a$$

$$\nabla_a V_b = \partial_a V_b - \Gamma^c_{ab} V_c$$

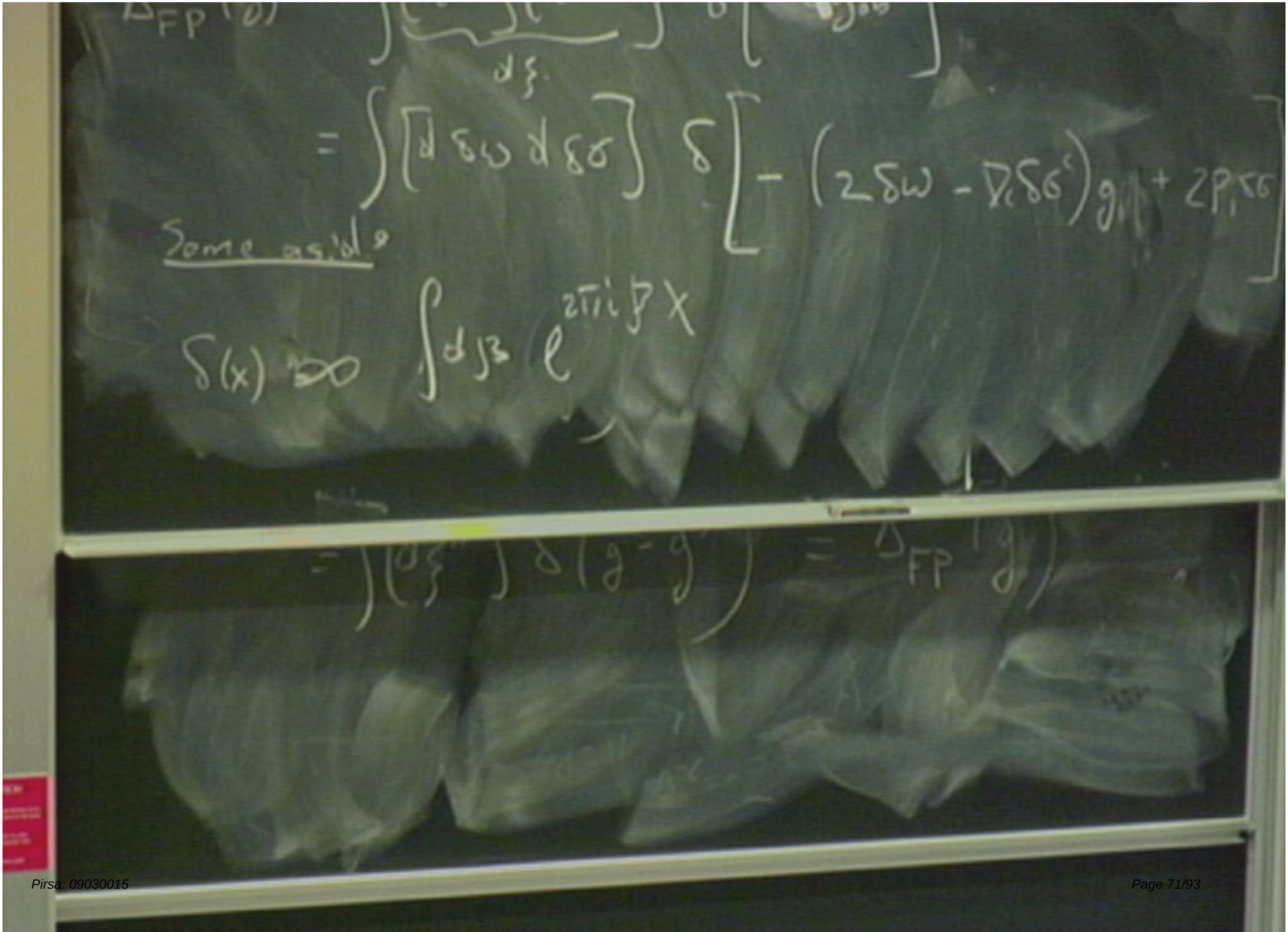
$$\Gamma^c_{ab} = \frac{1}{2} g^{ck}\left[\partial_a g_{bk} + \partial_b g_{ak} - \partial_k g_{ab}\right]$$

$$(P_1\delta\sigma)_{ab} = \frac{1}{2}\left[\nabla_a \delta\sigma_b + \nabla_b \delta\sigma_a - g_{ab}\nabla_c \delta\sigma^c\right]$$

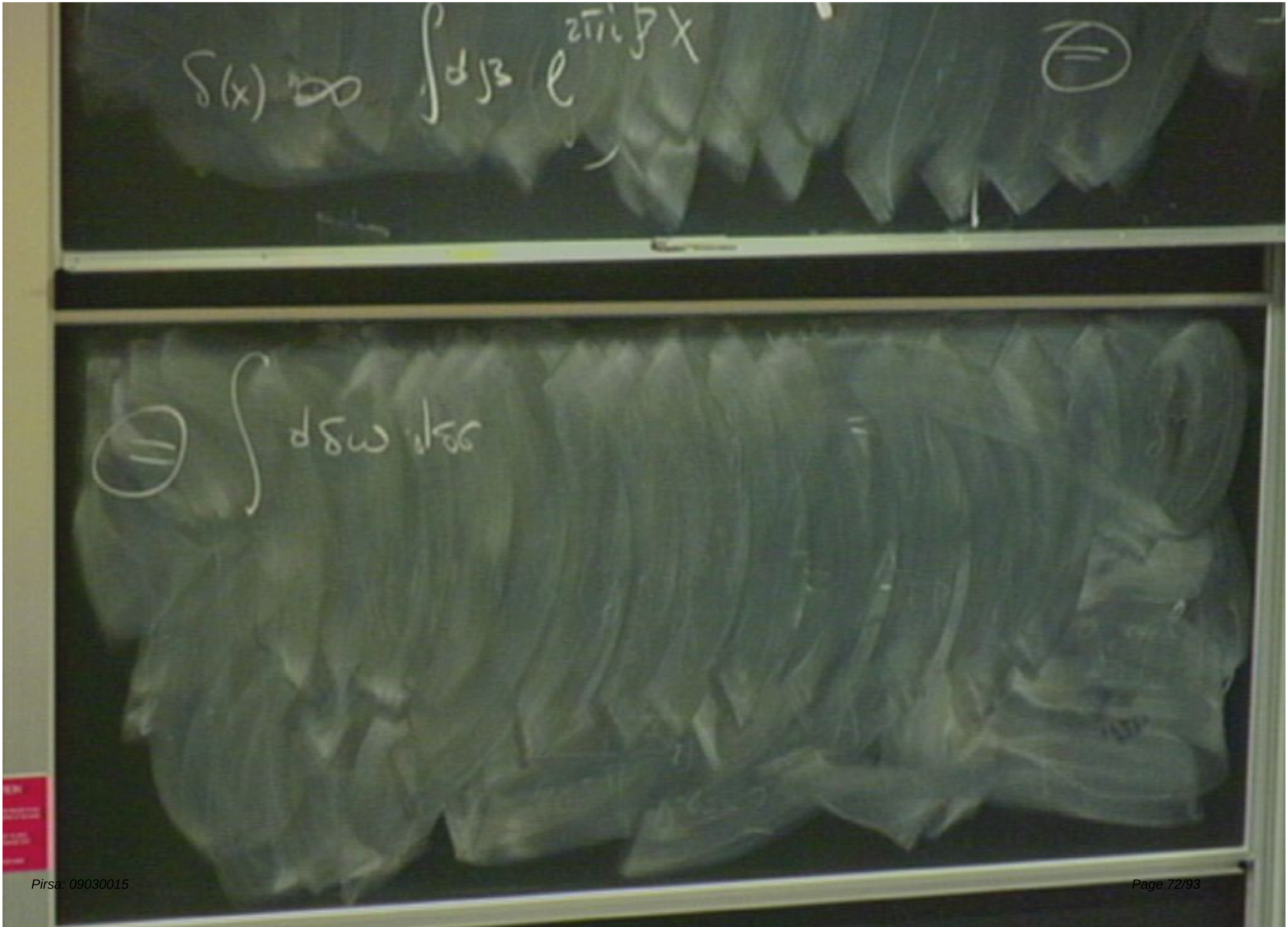
$$= \int [d\delta\omega \, d\delta\sigma] \, \delta\left[-(2\delta\omega - D_c\delta\sigma^c) g_{ab} + 2P_1\delta\sigma\right]$$

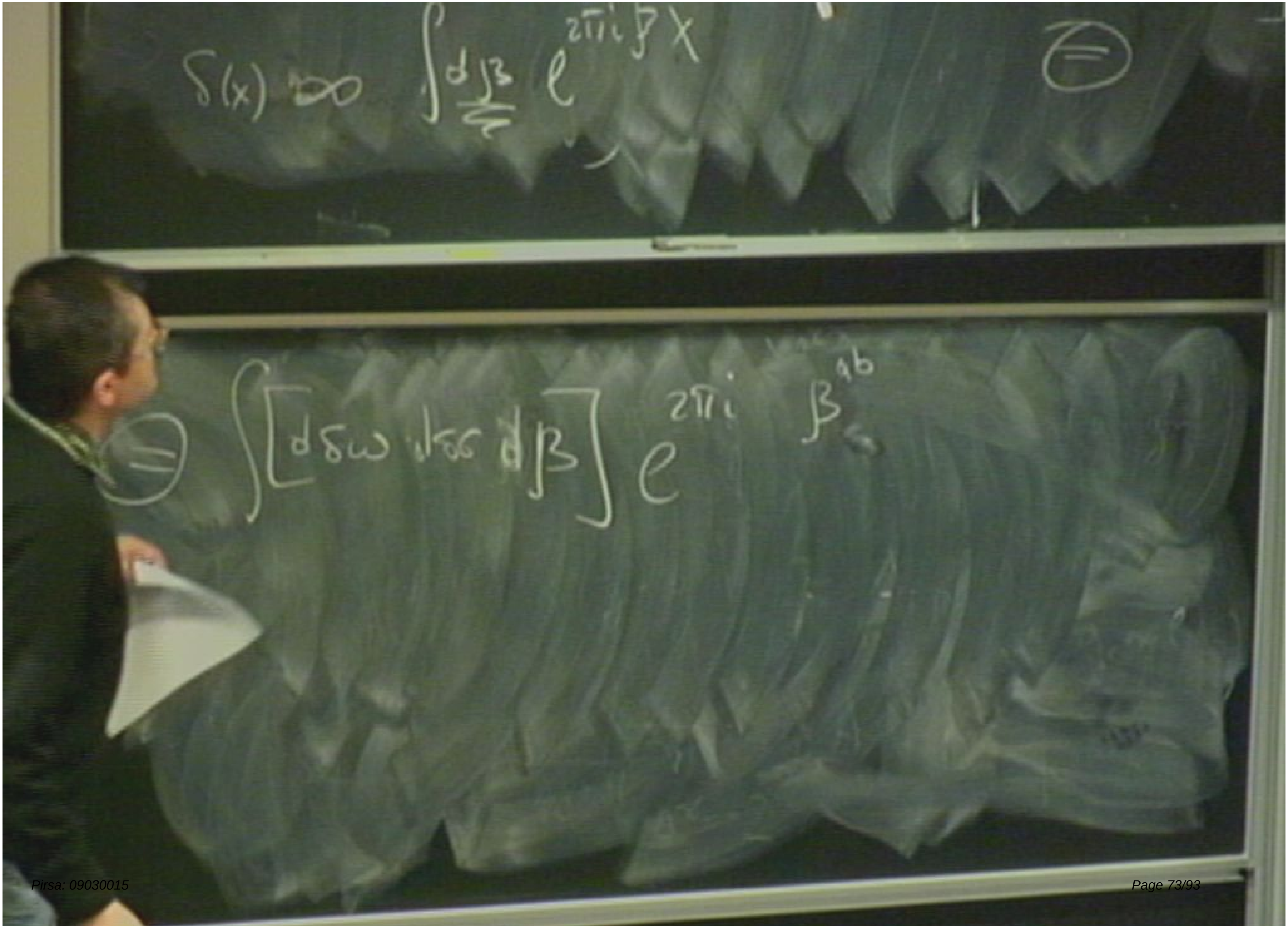
Some aside:

$$= \int (d\xi) \, \delta(g - g') = \Delta_{FP}(g)$$



Some aside
$$= \int [d\,\delta\omega\, d\,\delta\sigma]\, \delta\left[-(2\delta\omega - \nabla_i \delta\sigma^i) g_{ij} + 2P_1 \delta\sigma\right]$$
$$\delta(x) \propto \int d\beta\, e^{2\pi i \beta x}$$
$$= \int [d\xi] \, \delta(g - g^{\xi}) = \Delta_{FP}(g)$$


$\delta(x) \propto \int d\beta \, e^{2\pi i \beta x}$
$= \int d\delta\omega$


$\delta(x) = \int d\beta \, e^{2\pi i \beta X}$
$= \int [d\delta\omega \, d\delta e \, d\beta] \, e^{2\pi i \beta^{ab}}$

$$\delta(x) \propto \int \frac{d\beta}{2\pi}\, e^{2\pi i \beta x}$$

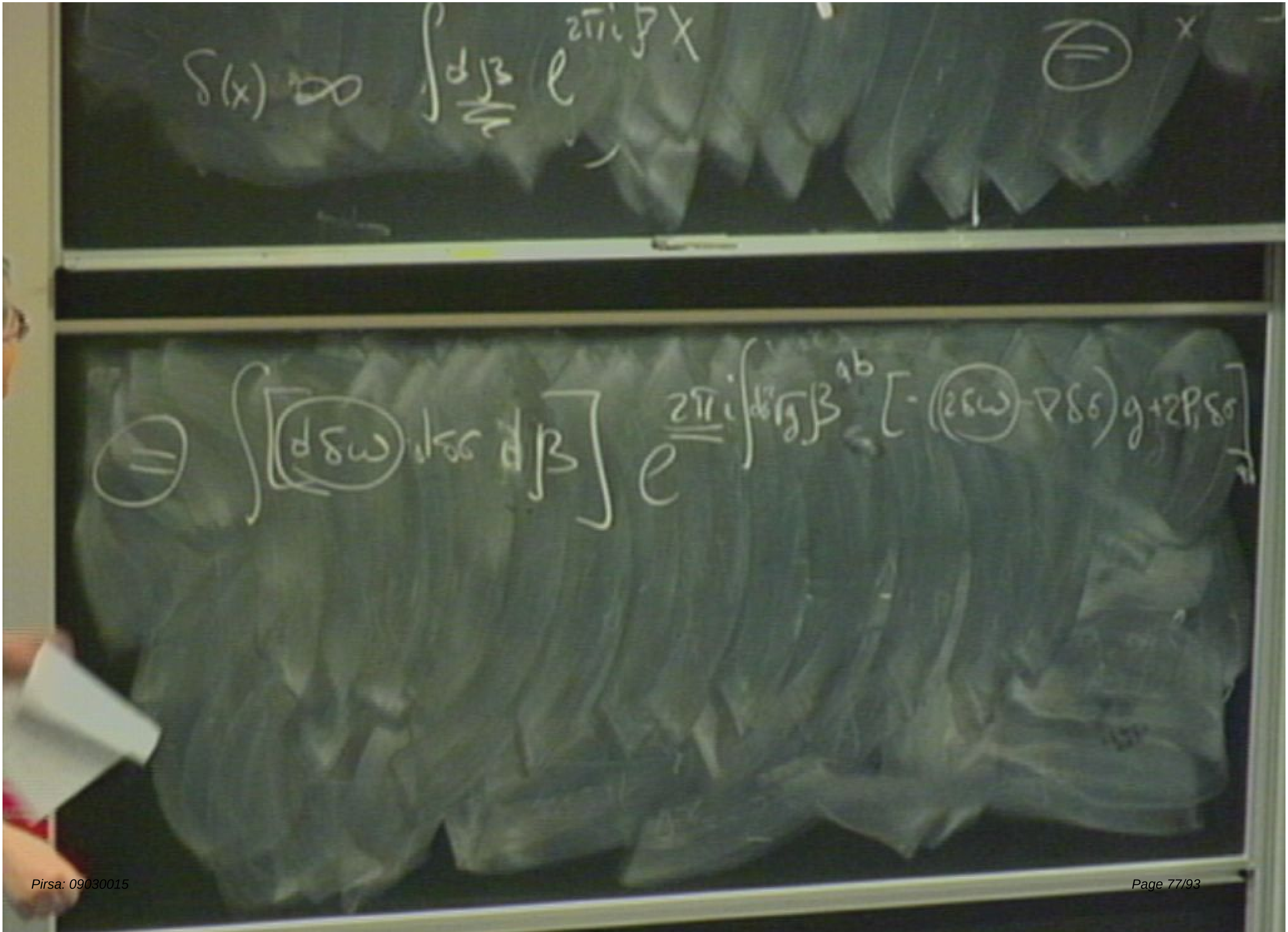
$$= \int \left[d\delta\omega \; d\delta\sigma \; d\beta\right] e^{2\pi i \; \beta^{ab}\left[-(2\delta\omega - \nabla\delta\sigma) g + 2P_1 \delta\sigma\right]}$$

$$\delta(x) \propto \int \frac{d\beta}{2\pi} e^{2\pi i \beta x} \quad = \quad x$$

$$= \int [d\delta\omega \, d\delta\sigma \, d\beta] \, e^{2\pi i \int d^4 \sqrt{g}\, \beta^{ab} [-(2\delta\omega - \nabla\delta\sigma) g + 2 P_1 \delta\sigma]}$$

$$\delta(x) \propto \int \frac{d\beta}{2\pi}\, e^{2\pi i \beta x}$$

$$= \int \left[d\delta\omega\, d\delta\sigma\, d\beta \right] e^{2\pi i \int d^3\!\sigma\, \beta^{ab} \left[-\left(d\delta\omega - \nabla\delta\sigma \right) g + 2 P_i\, \delta\sigma \right]}$$


$\delta(x) \propto \int \frac{d\beta}{2\pi} e^{2\pi i \beta x}$
$= \int [d\delta\omega \, d\delta\sigma \, d\beta] \, e^{2\pi i \int d^4x \sqrt{g} \beta^{ab} [-(d\delta\omega - \nabla\delta\sigma) g + 2P_i \delta\sigma]}$

$$\delta(x) \propto \int \frac{d\beta}{2\pi}\, e^{2\pi i \beta x} = x$$

$$= \int [d\delta\omega\, d\delta\sigma\, d\beta]\, e^{2\pi i \int d^4 r_g \beta^{ab} [-((2\delta\omega - \nabla\delta\sigma) g + 2 P_1 \delta\sigma]_{ab}}$$

$$\delta(x) \propto \int \frac{d\beta}{\,} e^{2\pi i \beta x} \qquad = \quad x$$

$$= \int [d\delta\omega][d\delta\sigma][d\beta]\, e^{2\pi i \int d^4x \sqrt{g}\, \beta^{ab}\left[-\left((\delta\omega) - \nabla\delta\sigma\right) g + 2P_1 \delta\sigma\right]_{ab}}$$

$$\delta[\beta^{ab}$$

$$\delta(x) \propto \int \frac{d\beta}{2\pi}\, e^{2\pi i \beta x} \quad = \quad x$$

$$= \int [d\delta\omega][d\delta\sigma][d\beta]\, e^{2\pi i \int d^4x \sqrt{g}\, \beta^{ab}\left[-(2\delta\omega - \nabla\delta\sigma)\, g_{ab} + 2P_{ab}\,\delta\sigma\right]}$$

$$\delta\left[\beta^{ab}\, g_{ab}\right]$$

$$\delta(x) \propto \int d\beta \; e^{2\pi i \beta x} \qquad = \qquad x$$

$$= \int [d\delta\omega \, d\delta\sigma \, d\beta] \; e^{2\pi i \int d^2\sigma \sqrt{g}\, \beta^{ab} \left[-(2\delta\omega - \nabla\delta\sigma) g_{ab} + 2P_1 \delta\sigma \right]_{ab}}$$

$$\delta\left[2\beta^{ab} \cdot g_{ab} \right]$$

$$\delta(x) \propto \int \frac{d\beta}{2\pi}\, e^{2\pi i \beta x}$$

$$= \int [d\delta\omega][d\delta\sigma][d\beta]\, e^{2\pi i \int d^4x \sqrt{g}\, \beta^{ab}\left[-(\nabla\delta\omega - \nabla\delta\sigma) g_{ab} + 2P_{ab}\delta\sigma\right]}$$

$$\delta\left[2\beta^{ab} g_{ab}\right] \Rightarrow \beta^{ab}$$

$$\delta(x) \propto \int \frac{d\beta}{2\pi}\, e^{2\pi i \beta x} \;=\; x$$

$$= \int [d\delta\omega \; d\delta\sigma \; d\beta]\, e^{2\pi i \int d^d\sigma \sqrt{g}\, \beta^{ab} [-(2\delta\omega - \nabla\delta\sigma) g_{ab} + 2 P_1 \delta\sigma]_{ab}}$$

$$\delta[2\beta^{ab} \cdot g_{ab}] \Rightarrow \beta^{ab} g_{ab} = \beta^{a}{}_{a} = 0$$

$$\delta(x) \propto \int \frac{d\beta}{2\pi}\, e^{2\pi i \beta x}$$

$$= \int [d\delta\omega][d\delta\sigma][d\beta]\; e^{2\pi i \int d^3x \sqrt{g}\, \beta^{ab} \left[-\left((2\delta\omega) - P\delta\sigma\right) g_{ab} + 2P_{ab}\delta\sigma\right]}$$

traceless

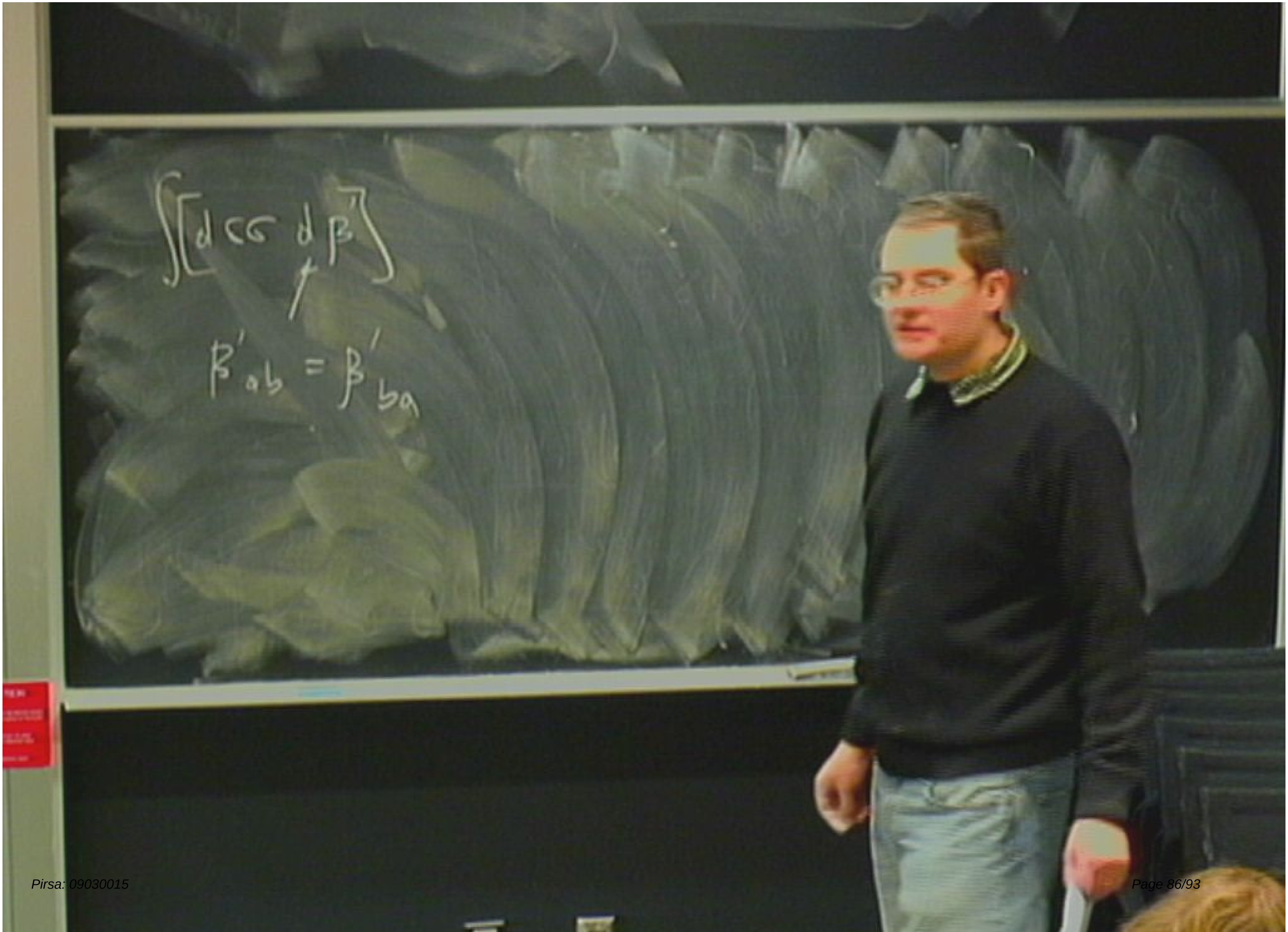
$$\delta\left[2\beta^{ab} g_{ab}\right] \Rightarrow \beta^{ab} g_{ab} = \beta^{a}{}_{a} = 0$$

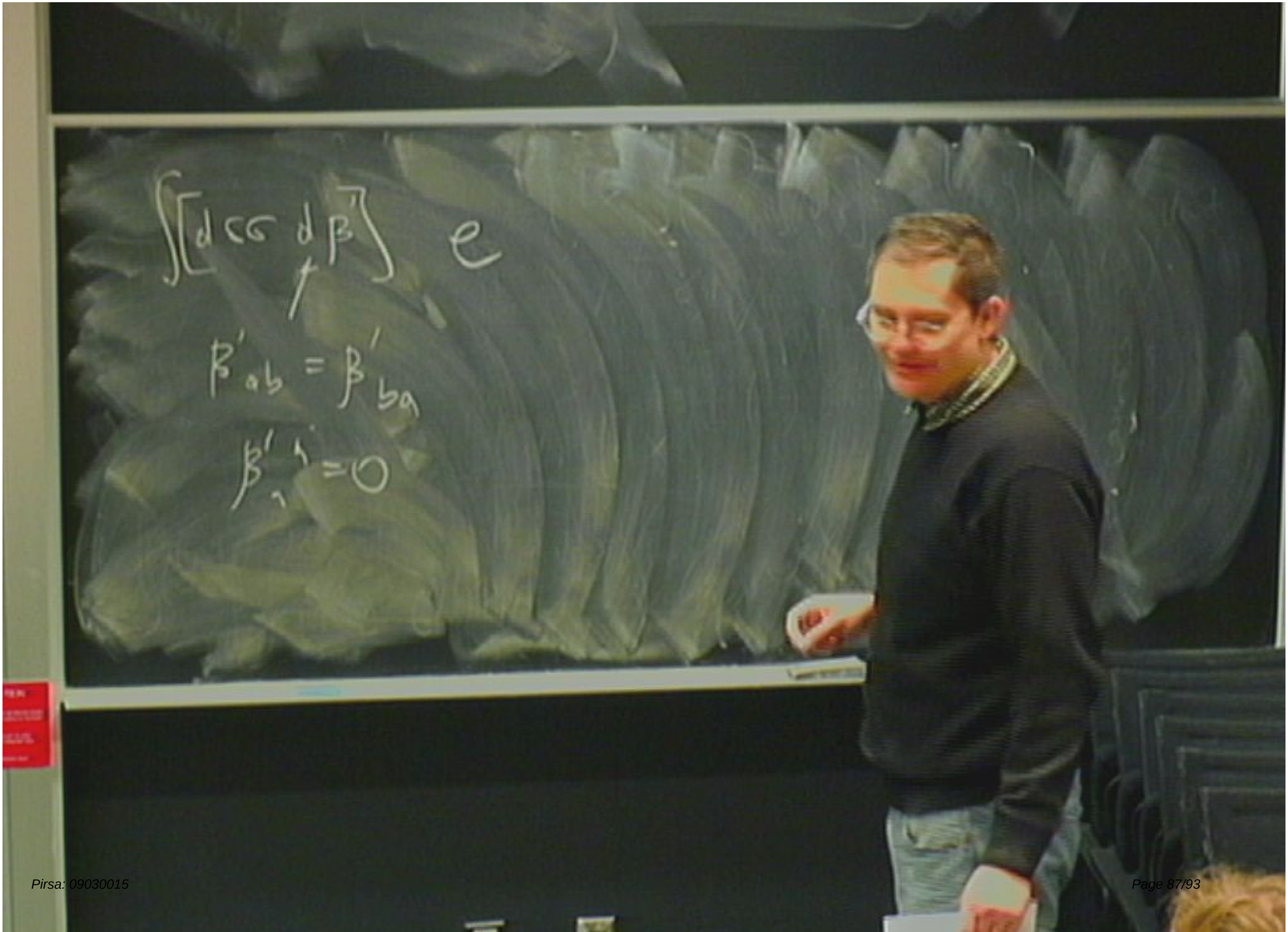
$$\delta(x) \propto \int \frac{d\beta}{2\pi}\, e^{2\pi i \beta x} = X$$

$$= \int [d\delta\omega][d\delta\sigma][d\beta]\; e^{\frac{2\pi i}{\hbar}\int d^4x \sqrt{g}\,\beta^{ab}\left[-\cancel{(2\delta\omega - \nabla\delta\sigma)\, g_{ab}} + 2P_1\delta\sigma\right]_{ab}}$$

traceless

$$\delta\left[2\beta^{ab} \cdot g_{ab}\right] \Rightarrow \beta^{ab} g_{ab} = \beta^{a}{}_{a} = 0$$


$\int [d\sigma \, d\beta']$
$\beta'_{ab} = \beta'_{ba}$


$\int [d\sigma \, d\beta'] \; e$
$\beta'_{ab} = \beta'_{ba}$
$\beta'^{\,a}_{a} = 0$

$$\ldots \; 2\,\delta\omega\, g_{ab} - \nabla_a \delta\sigma_b - \nabla_b \delta\sigma_a$$

$$\nabla_a V_b = \partial_a V_b - \Gamma^c_{ab} V_c$$

$$\Gamma^c_{ab} = \frac{1}{2} g^{ck}\left[\partial_a g_{bk} + \partial_b g_{ak} - \partial_k g_{ab}\right]$$

$$= \left(2\delta\omega - \nabla_c \delta\sigma^c\right) g_{ab} - 2\left(P_1 \delta\sigma\right)_{ab}$$

$$\left(P_1 \delta\sigma\right)_{ab} = \frac{1}{2}\left[\nabla_a \delta\sigma_b + \nabla_b \delta\sigma_a - g_{ab} \nabla_c \delta\sigma^c\right]$$

$$\int [d\delta\sigma \, d\beta'] \, e^{4\pi i \int d^2\sigma \, \beta'^{ab} (\hat{P}_1 \delta\sigma)_{ab}} = \Delta_{FP}(\hat{g})$$

$$\beta'_{ab} = \beta'_{ba}$$

$$\beta'^{\,a}_{a} = 0$$

$$\int [d\sigma\, d\beta'] \; e^{4\pi i \int d^2\sigma \, \beta'^{ab} (\hat{P}_1 \delta\sigma)_{ab}} = \Delta_{FP}^{-1}(\hat{g})$$

$$\beta'_{ab} = \beta'_{ba}$$

$$\beta'^{a}_{a} = 0$$

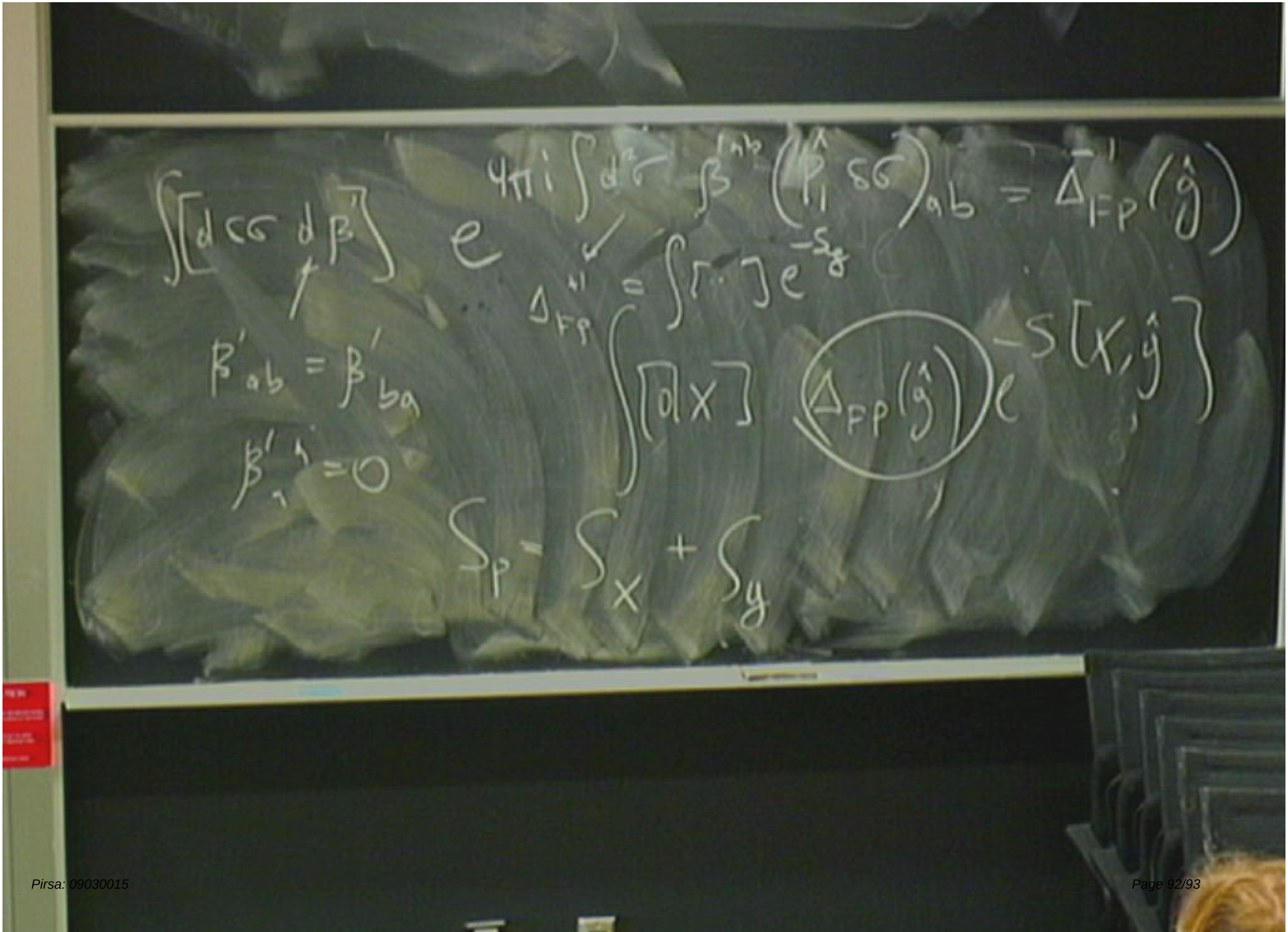
$$\int [dX] \; \Delta_{FP}(\hat{g}) \; e^{-S[X,\hat{g}]}$$

$$\int [d\delta\sigma \, d\beta'] \, e^{4\pi i \int d^2\sigma \, \beta'^{ab} (\hat{P}_1 \delta\sigma)_{ab}} = \Delta_{FP}^{-1}(\hat{g})$$

$$\beta'_{ab} = \beta'_{ba}$$

$$\beta'^{a}_{a} = 0$$

$$\int [dX] \, \Delta_{FP}(\hat{g}) \, e^{-S[X, \hat{g}]}$$



$\int [d c \sigma \, d\beta'] \, e^{4\pi i \int d^2\sigma \, \beta'^{ab} (\hat{P}_1 \delta\sigma)_{ab}} = \Delta_{FP}^{-1}(\hat{g})$
$\Delta_{FP}^{-1} = \int [\cdots] e^{-S_g}$
$\int [dX] \, \Delta_{FP}(\hat{g}) \, e^{-S[X, \hat{g}]}$
$\beta'_{ab} = \beta'_{ba}$
$\beta'^{a}_{a} = 0$
$S_p = S_X + S_y$

$$\int [d\delta\sigma \, d\beta'] \, e^{4\pi i \int d^2\sigma \, \beta'^{ab} (\hat{P}_1 \delta\sigma)_{ab}} = \Delta_{FP}^{-1}(\hat{g})$$

$$\Delta_{FP}^{-1} = \int [\cdots] e^{-S_g}$$

$$\beta'_{ab} = \beta'_{ba}$$

$$\beta'^{a}_{a} = 0$$

$$\int [dX] \, \Delta_{FP}(\hat{g}) \, e^{-S[X,\hat{g}]}$$

$$S_p = S_x + S_y$$