

Title: Gravitational Lensing

Date: Mar 17, 2009 03:30 PM

URL: <http://pirsa.org/09030013>

Abstract: Weak lensing has emerged as a powerful probe of fundamental physics such as dark energy and dark matter. After briefly reviewing the standard argument for the power of lensing, I present a variety of surprises: some quantities that are supposedly simple measures of cosmic shear are actually polluted by other effects and some quantities apparently unrelated to lensing are contaminated by lensing. These effects may lead to opportunities to strengthen the constraints lensing will place on dark energy.

I. THE PROGRAM

~~B~~

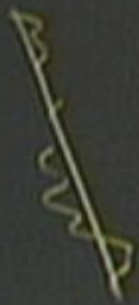
$$\frac{d^2 x^i}{d\lambda^2} + \Gamma^i_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda} = 0$$

Perturbed
FRW metric

$$\frac{d^2}{d\lambda^2} \begin{bmatrix} \chi \\ \theta' \end{bmatrix} = 2\bar{\Phi}_{,i}$$

comoving
distance

I. THE PROGRAM



$$\frac{d^2 x^i}{d\lambda^2} + \Gamma^i_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda} = 0$$

Perturbed
FRW metric

$$\frac{d^2}{d\lambda^2} [\chi \theta'] = 2 \bar{\Phi}_{,i}$$

comoving
distance

Newtonian
Potential

I. THE PROGRAM

~~FRW~~

$$\frac{d^2 x^i}{d\lambda^2} + \Gamma^i_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda} = 0$$

Perturbed
FRW metric

$$\frac{d^2}{d\lambda^2} [x \quad \theta^i] = 2 \bar{\Phi}_{,i}$$

comoving
distance

Angular
diameter

Newtonian
Potential

I. THE PROGRAM

~~FRW~~

$$\frac{d^2 x^i}{d\lambda^2} + \Gamma^i_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda} = 0$$

Perturbed
FRW metric

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I. THE PROGRAM

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$$\frac{d^2 x^i}{d\lambda^2} + \Gamma^i_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda} = 0$$

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$$\frac{d^2}{d\lambda^2} [\chi \theta^i] = 2 \bar{\Phi}_{,i}$$

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I. THE PROGRAM

~~FRW~~

$$\frac{d^2 x^i}{d\lambda^2} + \Gamma^i_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda} = 0$$

Perturbed
FRW metric

$$\frac{d^2}{d\lambda^2} [x \theta^i] = 2 \bar{\Phi}_{,i}$$

comoving
distance

Angular
diameter

Newtonian
Potential

I. THE PROGRAM

~~Bardeen~~

$$\frac{d^2 x^i}{d\lambda^2} + \Gamma^i_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda} = 0$$

Perturbed
FRW metric

$$\frac{d^2}{d\lambda^2} [\chi \theta^i] = 2 \bar{\Phi}_{,i}$$

comoving distance Angular distance Newtonian Potential

$\delta\theta^i$

I. THE PROGRAM

~~Background~~

$$\frac{d^2 x^i}{dx^2} + \Gamma^i_{\mu\nu} \frac{dx^\mu}{dx} \frac{dx^\nu}{dx} = 0$$

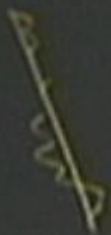
Perturbed
FRW metric

$$\frac{d^2}{dx^2} [\chi \theta^i] = 2 \bar{\Phi}_{,i}$$

comoving distance Amplitude
Newtonian Potential

$$S\theta^i = \int_0^{x_5} dx W(x, x_i)$$

I. THE PROGRAM



$$\frac{d^2 x^i}{d\lambda^2} + \Gamma^i_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda} = 0$$

Perturbed
FRW metric

$$\frac{d^2}{d\lambda^2} [x \theta^i] = 2 \bar{\Phi}_{,i}$$

Comoving
distance

Angular
diameter

Newtonian
Potential

$$S\theta^i = \int_0^{x_0} dx W(x, x_i) \bar{\Phi}_{,i}$$

I. THE PROGRAM

$\frac{d^2 \chi}{dx^2}$

$$\frac{d^2 \chi^i}{dx^2} + \Gamma^i_{\mu\nu} \frac{dx^\mu}{dx} \frac{dx^\nu}{dx} = 0$$

Perturbed
FRW metric

$$\frac{d^2}{dx^2} [\chi \theta^i] = 2 \bar{\Phi}_{,i}$$

comoving
distance Angular
disturbance Newtonian
Potential

$$S\theta^i = \int_0^{x_3} dx W(x, x_i) \Gamma_{,i}$$

Typically, we cannot detect $S\theta^i$

I. THE PROGRAM

$\frac{d^2 x^i}{d\lambda^2}$

$$\frac{d^2 x^i}{d\lambda^2} + \Gamma^i_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda} = 0$$

Perturbed FRW metric

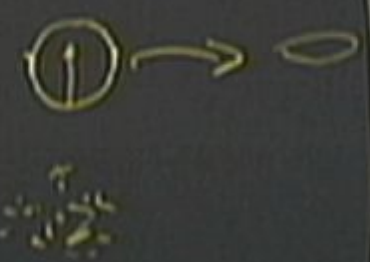
$$\frac{d^2}{d\lambda^2} [x \theta^i] = 2 \bar{\Phi}_{,i}$$

comoving distance Angular distance Newtonian Potential

$$\delta\theta^i = \int_0^{x_s} dx W(x, x_s) \bar{\Gamma}_{,i}$$

Typically, we cannot detect $\delta\theta^i$

We can measure $\delta\vec{\theta}(\vec{\theta}')$



$$\psi_{ij} = \frac{\partial \delta\theta^i}{\partial \theta^j}$$

I. THE PROGRAM



$$\frac{d^2 x^i}{d\lambda^2} + \Gamma^i_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda} = 0$$

Perturbed
FRW metric

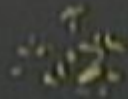
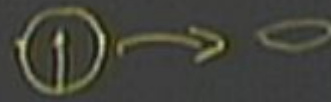
$$\frac{d^2}{d\lambda^2} [X \Theta^i] = 2 \bar{\Phi}_{,i}$$

comoving distance Angular distance Newtonian Potential

$$\delta \Theta^i = \int_0^{x_3} dx W(x, x_i) \mathcal{L}_{,i}$$

Typically, we cannot detect $\delta \Theta^i$

We can measure $\delta \vec{\Theta}(\vec{\Theta}^i)$



$$\Psi_{ij} = \frac{\delta \delta \Theta^i}{\delta \Theta^j}$$

$$\equiv \begin{bmatrix} -X - Y_1 & -Y_2 \\ -Y_2 & -X + Y_1 \end{bmatrix}$$

We can measure:

$$\epsilon_1 = \frac{\int d^3\theta I(\vec{\theta}) (\theta_x^2 - \theta_y^2)}{\int d^3\theta I(\vec{\theta}) (\theta_x^2 + \theta_y^2)}$$

— $\epsilon_1 > 0$

| $\epsilon_1 < 0$

$\epsilon_2 > 0$ /

$\epsilon_2 < 0$ \

We can measure:

$$\epsilon_1 = \frac{\int d^3\theta I(\theta) (\theta_x^2 - \theta_y^2)}{\int d^3\theta I(\theta) (\theta_x^2 + \theta_y^2)}$$

ϵ_1

— $\epsilon_1 > 0$

| $\epsilon_1 < 0$

$$\epsilon_1 = 2\chi_1$$

$\epsilon_2 > 0$

$\epsilon_2 < 0$

Measure $(\epsilon_1, \epsilon_2) \rightarrow (\chi_1, \chi_2)$

χ

We can measure:

$$\epsilon_1 = \frac{\int d^3\theta \mathcal{I}(\theta) (\theta_x^2 - \theta_y^2)}{\int d^3\theta \mathcal{I}(\theta) (\theta_x^2 + \theta_y^2)}$$

— $\epsilon_1 > 0$

| $\epsilon_1 < 0$

$\epsilon_2 > 0$ /

$\epsilon_2 < 0$ \

$$\epsilon_i = 2\gamma_i$$

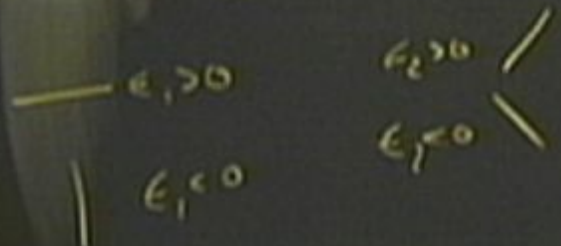
Measure $(\epsilon_1, \epsilon_2) \rightarrow (\gamma_1, \gamma_2)$

χ

$$\chi = \int_0^{\chi_0} d\chi \sqrt{V(\chi, \chi_0)} \nabla^2 \Phi$$

We can measure:

$$\epsilon_1 = \frac{\int d^3\theta I(\theta) (\theta_x^2 - \theta_y^2)}{\int d^3\theta I(\theta) (\theta_x^2 + \theta_y^2)}$$



$$\epsilon_1 = 2\gamma_1$$

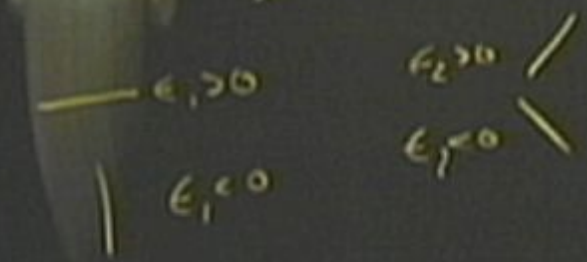
Measure $(\epsilon_1, \epsilon_2) \rightarrow (\gamma_1, \gamma_2)$

$$\begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix} = \begin{pmatrix} \cos 2\theta_1 \\ \sin 2\theta_1 \end{pmatrix} \chi$$

$$\chi = \int_0^{\chi_0} d\chi W(\chi, \chi_0) \nabla^2 \Psi$$

We can measure:

$$\epsilon_1 = \frac{\int d\theta I(\theta) (\theta_2^2 - \theta_1^2)}{\int d\theta I(\theta) (\theta_2^2 + \theta_1^2)}$$



$$\epsilon_1 = 2\gamma_1$$

Measure $(\epsilon_1, \epsilon_2) \rightarrow (\gamma_1, \gamma_2)$

$$\begin{aligned} (\epsilon_1^2 - \epsilon_2^2) \frac{1}{2} &\sim \cos 2\theta, \chi \\ (\epsilon_1^2 + \epsilon_2^2) \frac{1}{2} &\sim \end{aligned}$$

$$\chi = \int_0^{\chi_0} d\chi W(\chi, \chi_0) \nabla^2 \Phi \rightarrow \gamma$$

$$-\chi > 0$$

$$-\chi^2 > - \left[L^2 k^2 \frac{1}{2} \right]^2 - \left(\frac{L}{\lambda} \right)^4 \frac{1}{2} \chi_{im}^2$$

We can measure:

$$\epsilon_1 = \frac{\int d\theta I(\theta) (\theta_2^2 - \theta_1^2)}{\int d\theta I(\theta) (\theta_2^2 + \theta_1^2)}$$

$$\begin{array}{l} \epsilon_1 > 0 & \epsilon_2 > 0 \\ \epsilon_1 < 0 & \epsilon_2 < 0 \end{array}$$

$$\epsilon_1 = 2\gamma_1$$

Measure $(\epsilon_1, \epsilon_2) \rightarrow (\gamma_1, \gamma_2)$

$$\begin{aligned} (\epsilon_1 - \epsilon_2) \frac{L}{\lambda} &= \cos 2\theta_1 \chi \\ (\epsilon_1 + \epsilon_2) \frac{L}{\lambda} &= \dots \\ f &= \frac{k L^2 \frac{dk}{k^2}}{k^2 \frac{dk}{k^2}} = \frac{1}{kL} \end{aligned}$$

$$\chi = \int_0^{\chi_0} d\chi W(\chi, \chi_0) \nabla^2 \Phi \rightarrow \dots$$

$$\begin{aligned} -\chi &> 0 \\ -\chi^2 &> \dots \left[L^2 k^2 \frac{dk}{k^2} \right]^2 = \left(\frac{L}{\lambda} \right)^4 \frac{1}{k^2} f \sim 10^{-4} \end{aligned}$$

Tomography

Evolution of Structure

Scott Adelman $\bar{b}(z)$

$X(z)$

$C_I(z)$
 $P_I(z)$



Tomography

Evolution of Structure

Scott

Model

$\bar{\rho}(z)$

$\chi(z)$

$C_p(z)$
 $P_E(z)$



Both of these are very sensitive to dark energy.

Tomography

Evolution of Structure

Scott Lodelson $\bar{\rho}(z)$

$\chi(z)$

$C_p(z)$
 $\bar{P}_{\bar{I}}(\frac{z}{2})$

Both of these are very sensitive to dark energy.
Not only DE but also distinguish
DE \leftrightarrow MG $\psi - \bar{\rho}, \delta$



Tomography

Evolution of Structure

$z_1 \dots z_n$

Scott & Liddle

$\bar{\rho}(z)$

$X(z)$

$C_p(z_i)$
 $P_E(z_i)$



Both of these are very sensitive to dark energy.
Not only DE but also distinguish DE \rightarrow M16 ψ - $\bar{\rho}$, δ

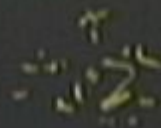
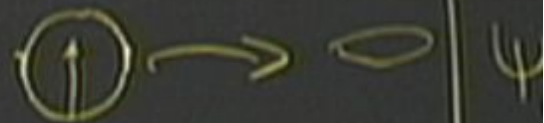
III. YOU THINK YOU'RE MEASURING
X, BUT

- Beyond Born

$$S\theta^i = \int_0^{x_s} dx W(\theta)$$

Typically, we can

We can measure S



II. YOU THINK YOU'RE MEASURING X , BUT - -

- Beyond Born

$$\left(\int dx W(x) \nabla^2 \Psi(x, t) - \int dx W(x) \left[U^2 \Psi(x, t) + \frac{\partial^2 \Psi}{\partial x^2} \right] \right)$$

$$\begin{aligned} \hat{C}_1 &= C_2 + \langle Y | X^2 | \Psi \rangle \\ &= \int dx W_1(x) \Psi(x) + \int dx B(x) W_2(x) \end{aligned}$$

- $\Phi = \frac{2Y}{1-X}$ (cross/wed shear)
- $\rightarrow 2X + 2$
- $< Y | X^2 | \Psi \rangle$

II. YOU THINK YOU'RE MEASURING
 X , BUT - - -

• Beyond Born

$$\left(\int dx W(x) \nabla^2 \Psi(x, t) - \int dx W(x) \left[D^2 \Psi(x, t) + \frac{\partial^2 \Psi(x, t)}{\partial x^2} \right] \right)$$

$$\hat{C}_x = C_x + \langle Y X X^2 \rangle$$

$$= \int dx W(x) \Psi(x) + \int dx B(x) W(x)$$

• $\Phi = \frac{2Y}{1-X}$ (rad/wave shear)

$$\rightarrow 2\delta_x + 2\delta_x X$$

$$\langle Y X X^2 \rangle$$

• $\Phi \rightarrow$



II. YOU THINK YOU'RE MEASURING κ , BUT - -

- Beyond Born

$$\left(\int dx W(x) \nabla^2 \Phi(x, \kappa) - \int dx W(x) \left[\nabla^2 \Phi(x, \kappa) + \frac{\partial^2 \Phi(x, \kappa)}{\partial \kappa^2} \right] \right)$$

$$\hat{C}_\kappa = C_\kappa + \langle Y \kappa^2 \rangle = \int dk W(k) A(k) + \int dk B(k) W(k)$$

- $\Phi = \frac{2Y}{1-\kappa}$ (reduced shear)

$$\rightarrow 2\delta_\kappa + 2\delta_\kappa \kappa$$

- $\delta \rightarrow \delta_{HL} - \delta_L + f(\delta_L)$



II. YOU THINK YOU'RE MEASURING K , BUT

- Beyond Born

$$\left(\int dx W(x) \nabla^2 \Phi(x, x_0) - \int dx W(x) \left[\nabla^2 \Phi(x, x_0) + \frac{\partial^2 \Phi(x, x_0)}{\partial x^2} \right] \right)$$

$$\hat{C}_1 = C_2 + \langle Y | K | X \rangle$$

$$= \int dk W_1(k) P(k) + \int dk B(k) W(k)$$

- $\delta \epsilon = \frac{2Y}{1-K}$ (reduced shear)

$$\rightarrow 2\delta_{\epsilon} + 2\delta_{\epsilon} K$$

$$\langle Y | K | X \rangle$$

- $\delta \rightarrow \delta_{NL} - \delta_L + f(\delta_L)$

Recall: we started from the 2nd order

II. YOU THINK YOU'RE MEASURING χ , BUT - - -

- Beyond Born

$$\left. \begin{aligned} & \int dx W(x) \nabla^2 \Phi(x_{min}) \\ & - \int dx W(x) \left(\nabla^2 \Phi(x_{min}) + \frac{\partial^2 \Phi}{\partial x^2} \right) \end{aligned} \right\}$$

$$\begin{aligned} \hat{C}_x &= C_x + \langle YK^2 \rangle \\ &= \int dk W(k) \tilde{P}(k) + \int dk B(k) W(k) \end{aligned}$$

- $\Phi = \frac{2Y}{1-K}$ (residual shear)
- $\rightarrow 2\delta_{ij} + 2\delta_{ij} K$
- $\langle YK^2 \rangle$

- $\delta \rightarrow \delta_{NL} - \delta_L + f(\delta_L)$

Recall: we started from the 2nd order growth.

II. YOU THINK YOU'RE MEASURING χ , BUT

• Beyond Born

$$\left. \begin{aligned} & \int d^3x W(x) \nabla^2 \Phi(x_{in}) \\ & - \int d^3x W(x) \left[\nabla^2 \Phi(x_{in}) + \frac{\partial^2 \Phi(x_{in})}{\partial x^2} \right] \end{aligned} \right\}$$

$$\begin{aligned} \hat{C}_x &= C_x + \langle \chi | K | \chi \rangle \\ &= \int d^3k W(k) |P(k)| + \int d^3k B(k) |V(k)| \end{aligned}$$

• $\Theta = \frac{2\chi}{1-\chi}$ (redwood shear)

$\rightarrow 2\delta_L + 2\delta_L \chi$
 $\langle \chi | K | \chi \rangle$

• $\delta \rightarrow \delta_{NL} = \delta_L + f(\delta_L)$

Recall: we started from the 2nd order geodesic.

Only-known corrections
contribute, at the present
level.

Ignoring those corrections,

⇒ systematic errors
 ↳ statistical errors.

Measure (μ, σ) → (\bar{x}, s)
 $x = \mu + \sigma \cdot z$
 $\sigma^2 = \text{var}(x)$
 $\sigma^2 = \text{var}(\mu + \sigma \cdot z)$
 $\sigma^2 = \sigma^2 \cdot \text{var}(z)$
 $\sigma^2 = \sigma^2 \cdot 1$
 $\sigma^2 = \sigma^2$

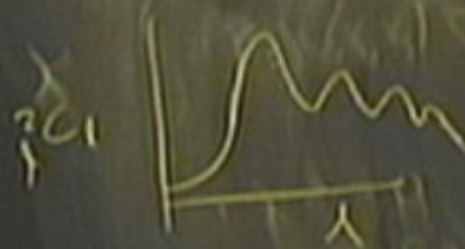
Only known corrections
contribute, at the percent
level.

Ignoring these corrections

⇒ systematic errors
→ statistical errors.

III YOU THINK YOU'RE MEASURING Ω
BUT

• CMB Lensing



BAO

Environment Structure



III. You T

• Beyond

$$\hat{C}_1 = C_2$$

CAUTION
Do not touch the chalkboard
when it is hot or when it is
being cleaned.

BAO

Evolution of Structure



$\delta(\mathbf{q})$
 $H(\mathbf{q})$



III. You T

• Bayo

CAUTION
DO NOT TOUCH THE BOARD
OR THE EQUIPMENT
IF YOU HAVE ANY
QUESTIONS
PLEASE ASK THE
LECTURER

II. Number of Galaxies



$$S_{obs} = S_d + \beta_n X$$

$$\Phi = \frac{2Y}{1-X}$$

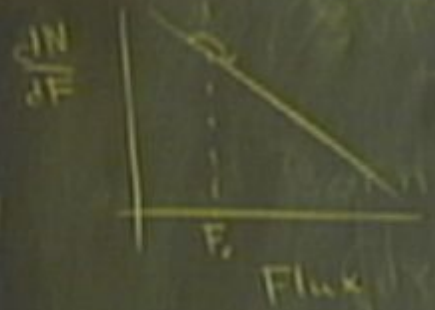
$$T_{vir} \rightarrow 2\alpha_i + \dots$$

$$S \rightarrow S_{MC}$$

call we start
order good

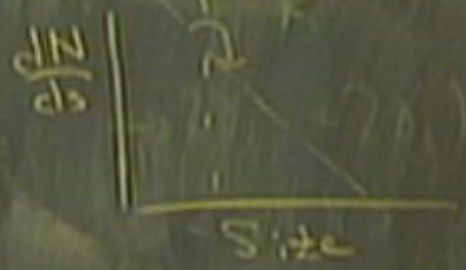
CA/TA
UNIVERSITY OF CALIFORNIA
SAN DIEGO

II. Number of Galaxies = MEASURING

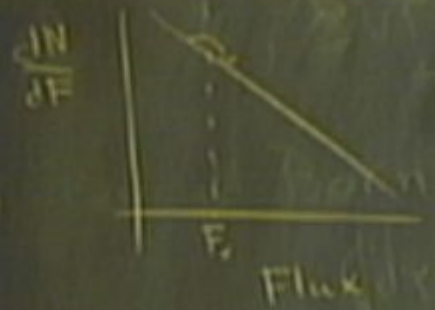


$$S_{obs} = S_d + B_n \times X$$

III. Size of Galaxies

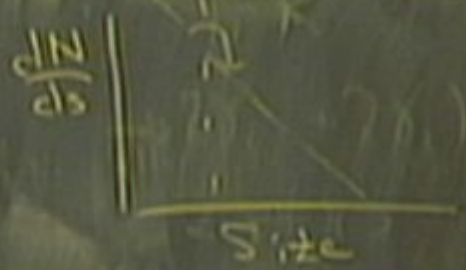


Number of Galaxies



$$\delta_{obs} = \delta_g + \beta_n X$$

Size of Galaxies



$$\delta_{obs} = \delta_{gal} + q X$$

IV. THIS UPSETS THE PROGRAM

III YOU THINK YOU'RE MEASURING BUT ...

• CMB Lensing



IV. THIS UPSETS THE PROGRAM

$$\hat{X} = \sum X_i \eta_i$$



$$\begin{aligned} \rightarrow \hat{X} &= X(1 + \delta_{\text{lens}}) \\ &= X + X\delta_g + g X^{1/2} \end{aligned}$$

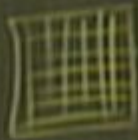
III YOU THINK YOU'RE MEASURING BUT ...

• CMB Lensing



IV. THIS UPSETS THE PROGRAM

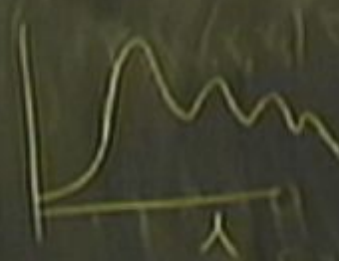
$$\hat{X} = \sum X_i \eta_i$$



$$\begin{aligned} \rightarrow \hat{X} &= X(1 + \delta_{obs}) \\ &= X + X\delta_g + q \end{aligned}$$

III YOU THINK YOU'RE MEASURING BUT ...

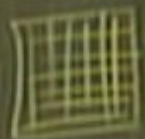
• CMB Lensing



CAUTION
DO NOT TOUCH THE BOARD
OR THE CHALK

IV. THIS UPSETS THE PROGRAM

$$\hat{X} = \sum X_i n_i$$



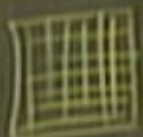
$$\begin{aligned} \rightarrow \hat{X} &= X(1 + \delta_{\text{loss}}) \\ &= X + X\delta_g + qX^2 \end{aligned}$$

Faded handwritten notes on the right side of the chalkboard, including the words "Lensing" and "X-ray" which are partially obscured by shadows.



IV. THIS UPSETS THE PROGRAM

$$\hat{X} = \sum X_i n_i$$



$$\begin{aligned} \rightarrow \hat{X} &= X(1 + \delta_{obs}) \\ &= X + X\delta_g + qX^{1/2} \end{aligned}$$

$$\Delta C_A = (4-5\%)C_A$$

V. CAN WE USE THIS?

IV. THIS UPSETS THE PROGRAM

$$\dot{X} = \sum X_i n_i$$



$$\begin{aligned} \rightarrow \hat{X} &= X(1 + \delta_{\text{loss}}) \\ &= X + X\delta_g + q X^{1/2} \end{aligned}$$

$$\Delta C_A \approx (4-5\%) C_A$$

V. CAN WE USE THIS?

$$X_{\text{new}} = m X_{\text{old}} + a$$

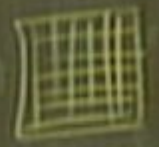
$$\langle X_{\text{new}} \rangle = m^2 \langle X_{\text{old}} \rangle$$

$$\langle X_{\text{new}} \rangle$$

CAUTION

IV. THIS UPSETS THE PROGRAM

$$\hat{X} = \sum X_i \eta_i$$



$$\begin{aligned} \rightarrow \hat{X} &= X(1 + \delta_{005}) \\ &= X + X\delta_0 + qX^2 \end{aligned}$$

IV. $\Delta C_A = (4.5\%)C_A$

V. CAN WE USE THIS?

$$\begin{aligned} X_{new} &= mX_{old} + a \\ \langle X_{new} | X_{new} \rangle &= m^2 \langle X_{old} | X_{old} \rangle \\ \langle X_{new} | S \rangle &= m \langle X_{old} | S \rangle \end{aligned}$$