Title: Landscape of holographic superconductors

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Abstract: Holographic superconductors provide tractable models for the onset of superconductivity in strongly coupled theories. They have some features in common with experimentally studied nonconventional superconductors. I will review the physics of holographic superconductors and go on to show that many such models are to be found in the string landscape of AdS_4 vacua.

Landscape of superconducting membranes

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Work with: **Frederik Denef, Chris Herzog and Gary Horowitz**

March 09 – Perimeter Institute

Plan of talk

Background

1. Motivation I: Nonconventional superconductors
2. Motivation II: A take on the string landscape

Holographic superconductors

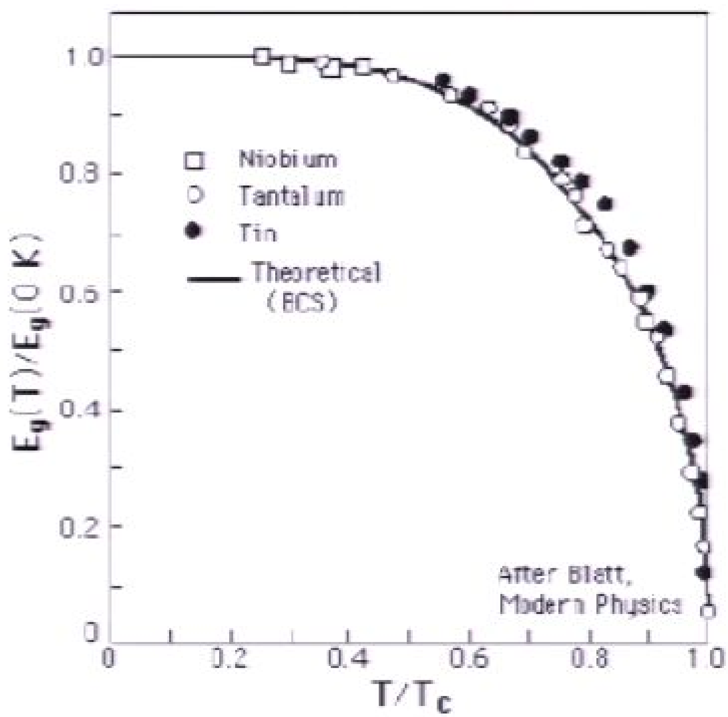
1. Ingredients for a holographic superconductor
2. Black hole instabilities
3. Electrical conductivity
4. Landscape of superconducting membranes

Background

1. BCS superconductivity
2. Nonconventional superconductors
3. What to do with the string landscape?

BCS theory (1957)

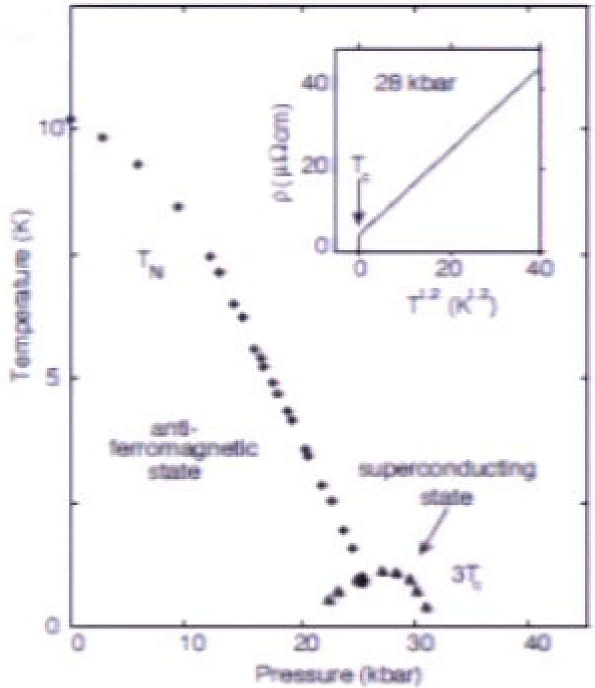
- Repulsive Coulomb force between electrons is screened. Attractive force due to the lattice phonons. Instability of Fermi surface.
- Spontaneous breaking of $U(1)_{\mathrm{EM}}$ by condensation of Cooper pairs of (dressed) electrons: $\mathcal{O} \sim \Psi^\dagger \Psi^\dagger$.
- Anderson-Higgs mechanism: $\langle j \rangle = \frac{\delta S}{\delta A} \sim A = \frac{iE}{\omega}$.
- Many predictions, for instance $E_g(0) \approx 3.5\, T_c$.
- Data:

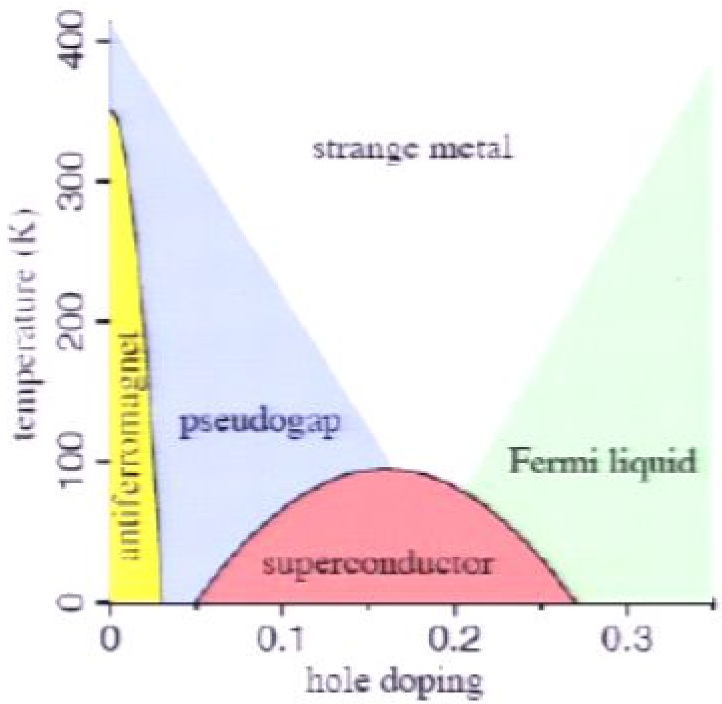


Two senses of non-BCS

- Many superconductors not described by BCS theory (eg. high-T_c).
- Non-BCS superconductivity might mean:
  - weak: 'pairing mechanism' does not involve phonons but a different 'glue' such as paramagnons (mediate spin-spin forces).
  - strong: inherently strongly coupled. No charged quasiparticles to 'pair' in the first place.
- Candidates for the stronger case if superconductivity close to a quantum critical point.

Two phase diagrams:



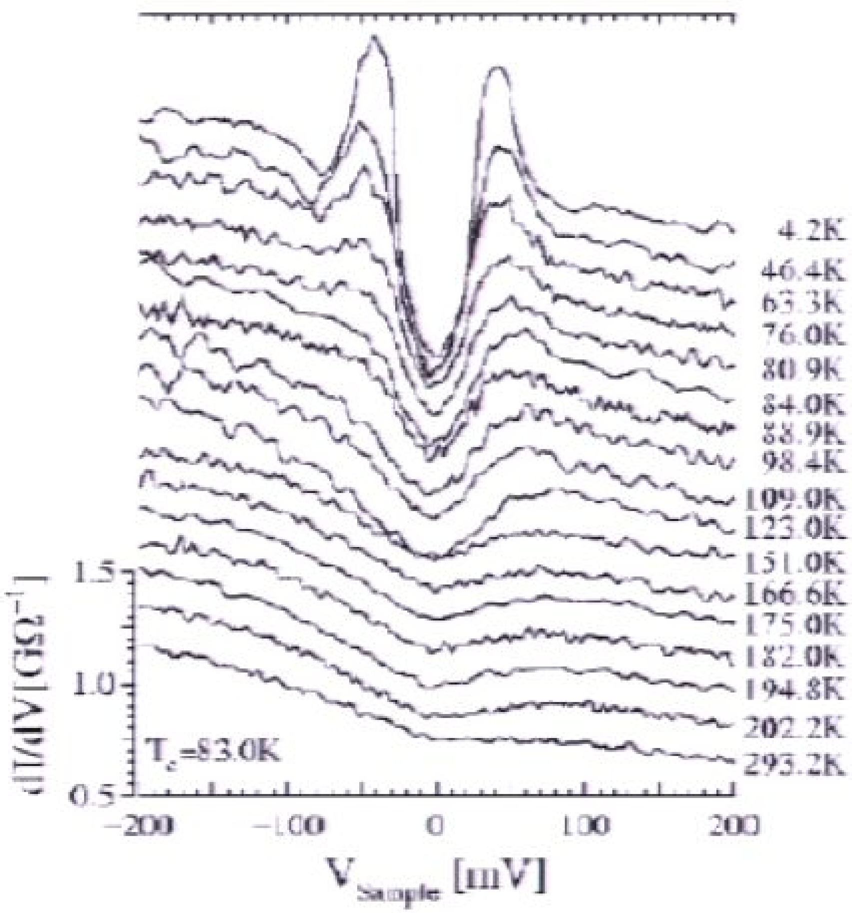


- Left: 'heavy fermion' compound $CePd_2Si_2$ [Mathur et al. '98]. Clear connection to quantum critical point.
- Right: high-T_c cuprate. Experimental evidence for nearby Valence Bond Solid (VBS - a.k.a. 'stripes') order and possible quantum critical point.

The pseudogap in $Bi_2Sr_2CaCu_2O_{8+\delta}$

[Renner et al. '98]

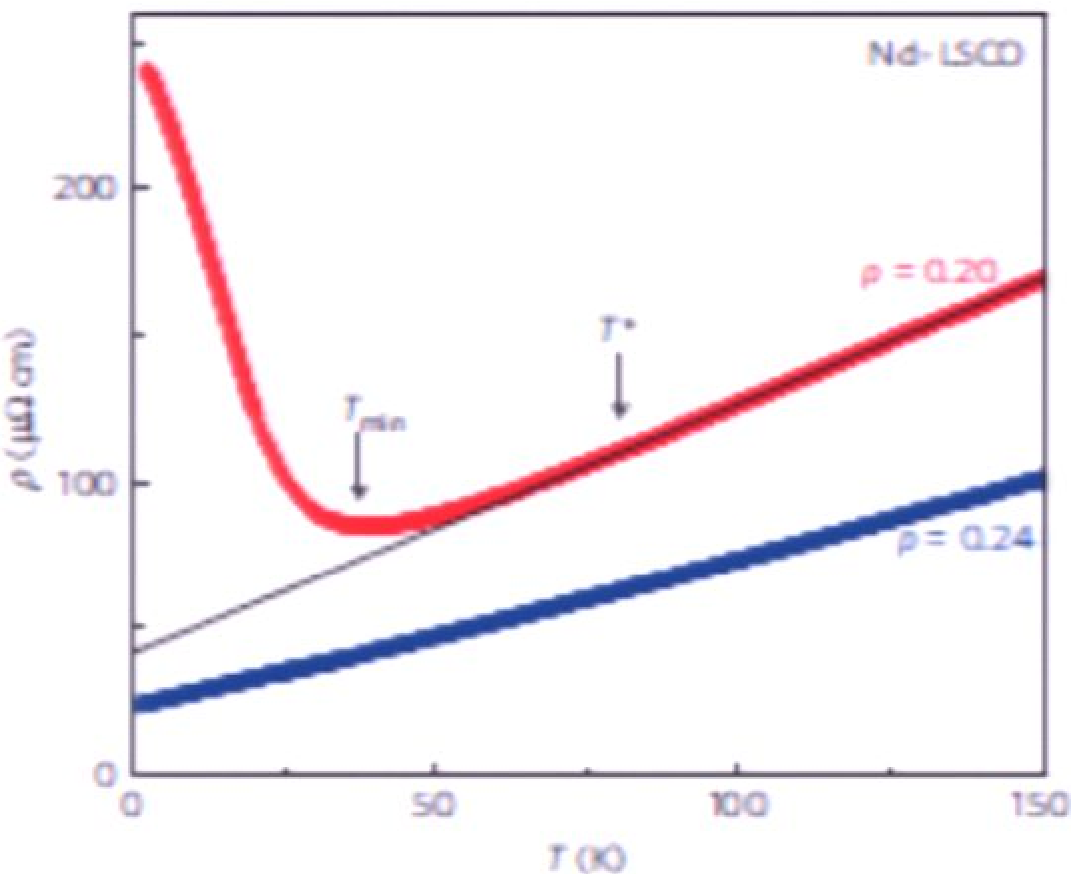
- Depletion of states at Fermi energy continuous across $T_c \approx 83K$.



Resistance linear in temperature ($La_{1.6-x}Nd_{0.4}Sr_xCuO_4$)

[Daou et al. '08]

- Resistivity in 'normal phase' linear in temperature (anomalous).
- Applying a large magnetic field shows persistance down to $T = 0$ at critical doping.



Uses for the string landscape?

- There appear to be many vacua of string theory.
- Methodological problems for stringy cosmology or stringy particle physics (only do experiments in one universe)?
- Resonates well with atomic physics, which also has a landscape:

	Atomic Landscape	String Landscape
Microscopic theory	Standard Model	M theory
Fundamental excitations	Leptons, quarks, photons, etc.	??
Typical vacuum	Atomic lattice	Compactification
Low energy excitations	Dressed electrons, phonons, spinons, triplons, etc.	Gravitons, gauge bosons, moduli, intersectons, etc.
Low energy theory	Various QFTs	Various supergravities

- Logic: Let's look at statistical properties of string AdS_4 vacua from the dual perspective as quantum critical points.
- Find many superconductors.

Holographic superconductors

1. Minimal ingredients for a holographic superconductor
2. Black hole instabilities
3. Hairy black holes
4. Electrical conductivties
5. Critical magnetic fields
6. Landscape of superconducting membranes

Minimal ingredients for a holographic superconductor

- Minimal ingredients
  - Continuum theory $\Rightarrow$ have $T^{\mu\nu}$ $\Rightarrow$ need bulk g_{ab}.
  - Conserved charge $\Rightarrow$ have J^{μ} $\Rightarrow$ need bulk A_a.
  - 'Cooper pair' operator $\Rightarrow$ have $\mathcal{O}$ $\Rightarrow$ need bulk ϕ.
- Write a minimal 'phenomenological' bulk Lagrangian

$$\mathcal{L}_{1+3} = \frac{1}{2\kappa^2}R + \frac{3}{L^2\kappa^2} - \frac{1}{4g^2}F_{\mu\nu}F^{\mu\nu} - |\nabla\phi - iqA\phi|^2 - m^2|\phi|^2 \,.$$

There are four dimensionless quantities in this action.

- The central charge of the CFT is $c = 192L^2/\kappa^2$.
- DC conductivity $\sigma_{xx} = \frac{1}{g^2}$.
- $\Delta(\Delta - 3) = (mL)^2$. Either root admissible if $\Delta \geq \frac{1}{2}$.
- The charge q is the charge of the dual operator $\mathcal{O}$.

Two instabilities of a charged AdS black hole

- To get a critical temperature, need a scale. Will work at constant chemical potential μ. By dimensional analysis $T_c \propto \mu$.
- The dual geometry is therefore Reissner-Nordstrom-AdS.
- RN-AdS can be unstable against a (charged) scalar for two reasons.
- Reason 1 [Gubser '08]: Background charge shifts mass:

$$m^2_{\text{eff.}} \sim m^2 - q^2 A_t^2 \,.$$

- Reason 2 [SAH-Herzog-Horowitz '08]: Near extremality AdS_2 throat with

$$m^2_{\text{BF-2}} = -\frac{1}{4L_2^2} = -\frac{3}{2L^2} > -\frac{9}{4L^2} = m^2_{BF-4} \,.$$

- Precise criterion for instability at $T = 0$ [Denef-SAH '09, Gubser '08]

$$q^2\gamma^2 \geq 3 + 2\Delta(\Delta - 3) \,, \qquad \gamma^2 = \frac{2g^2L^2}{\kappa^2} \,.$$

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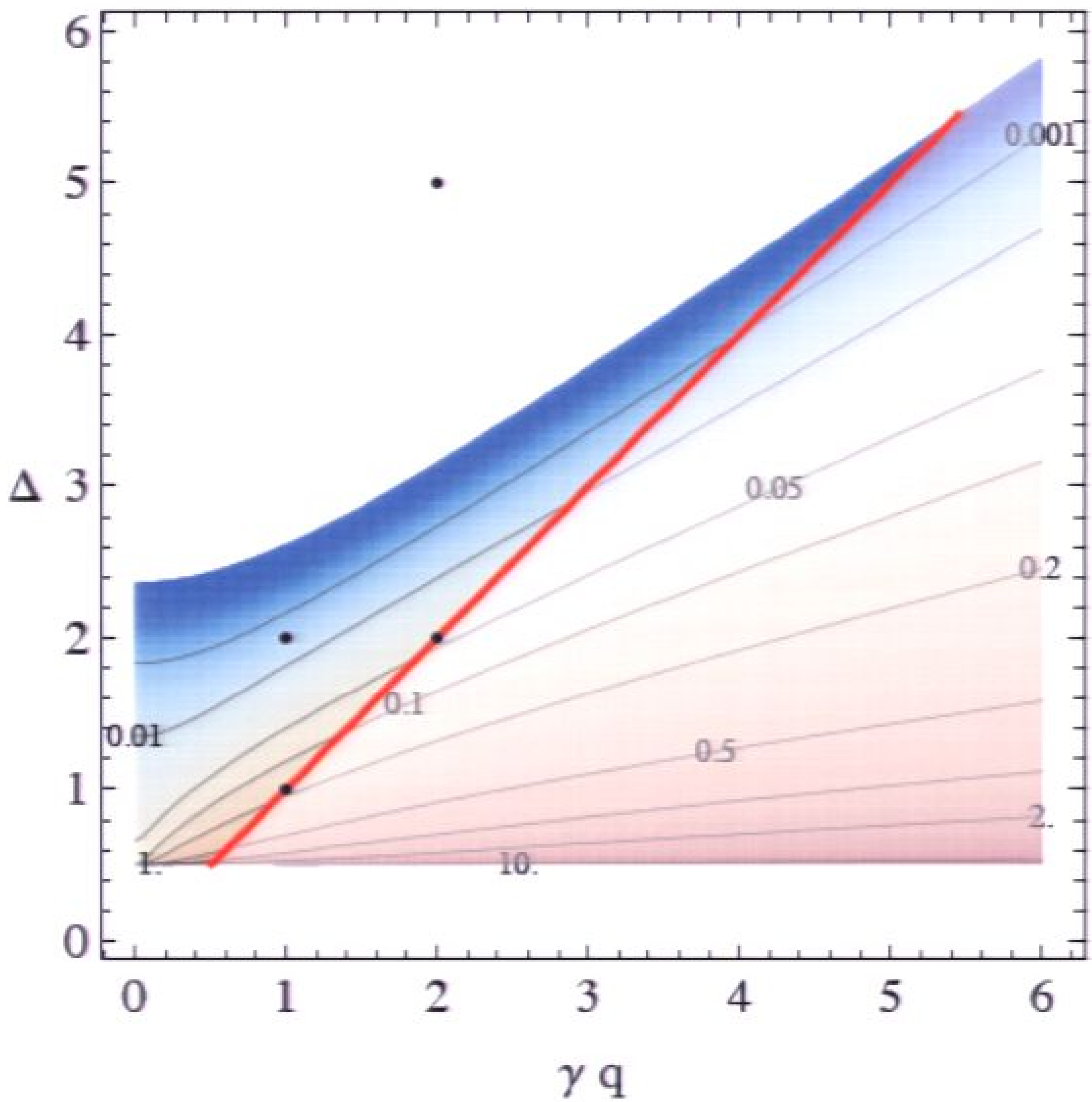


[Denef-SAH '09]

Endpoint – hairy black holes

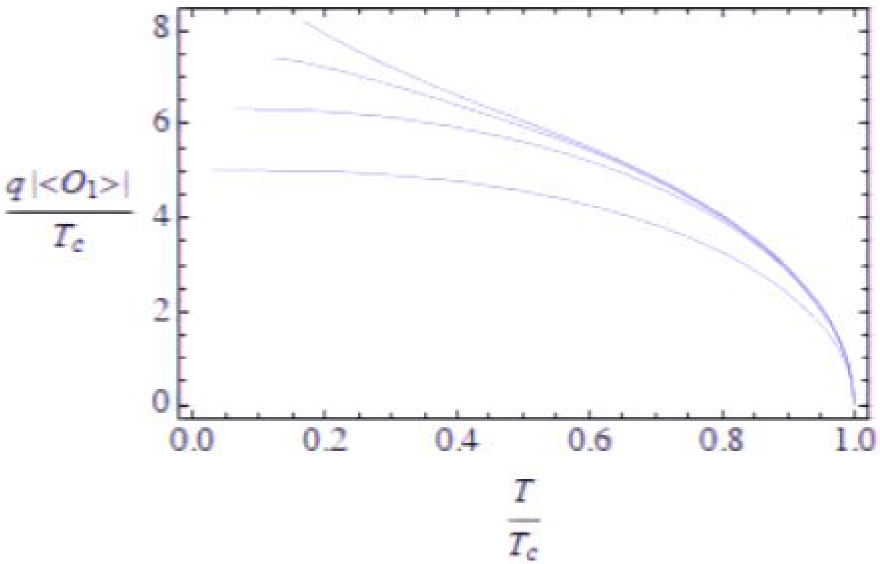
[SAH-Herzog-Horowitz '08]

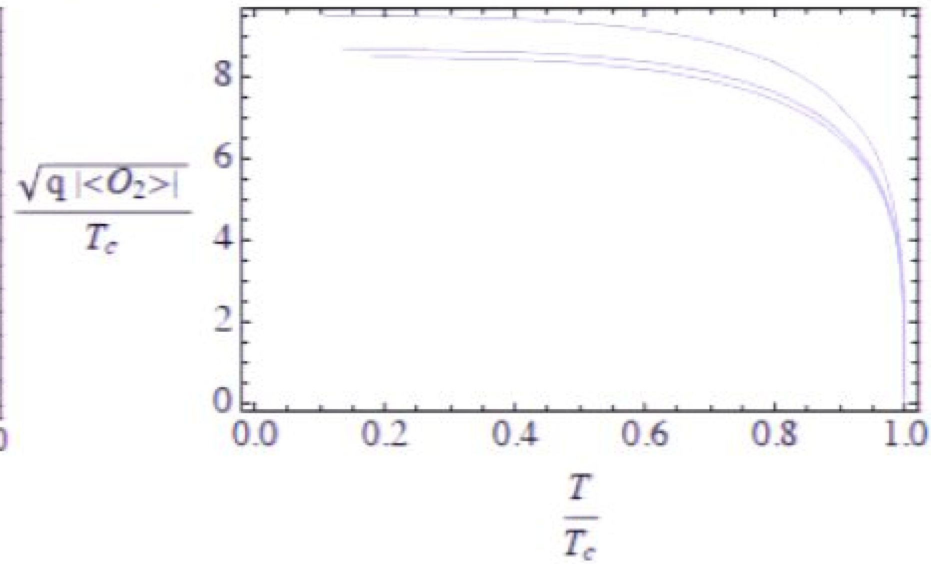
- Endpoint of instability is a hairy black hole:

$$ds^2 = -g(r)e^{-\chi(r)}dt^2 + \frac{dr^2}{g(r)} + \frac{L^2}{r^2}\left(dx^2 + dy^2\right) ,$$

$$A = A_t(r)dt , \qquad \phi = \phi(r) .$$

- Solve numerically (take $m^2 = -2/L^2$). Can obtain $\langle \mathcal{O} \rangle$:





- Compare 8 to ∼ 3.5 for BCS and ∼ 5 − 8 for High-T_C.

Electrical conductivity - some experimental expectations

- BCS theory prediction and some data

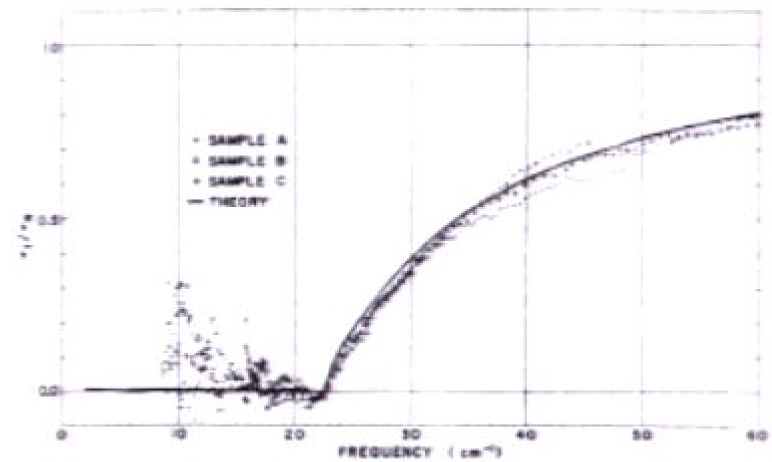
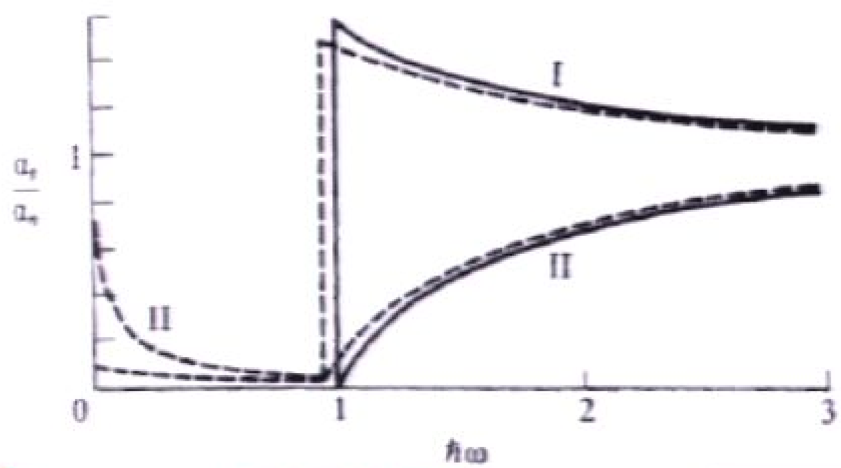


Fig. 2. Measured and calculated values of the conductivity ratio σ_1/σ_n of three superconducting lead films as a function of the photon frequency. [After Palmer (39).]

- A figure from a texbook (Tinkham)



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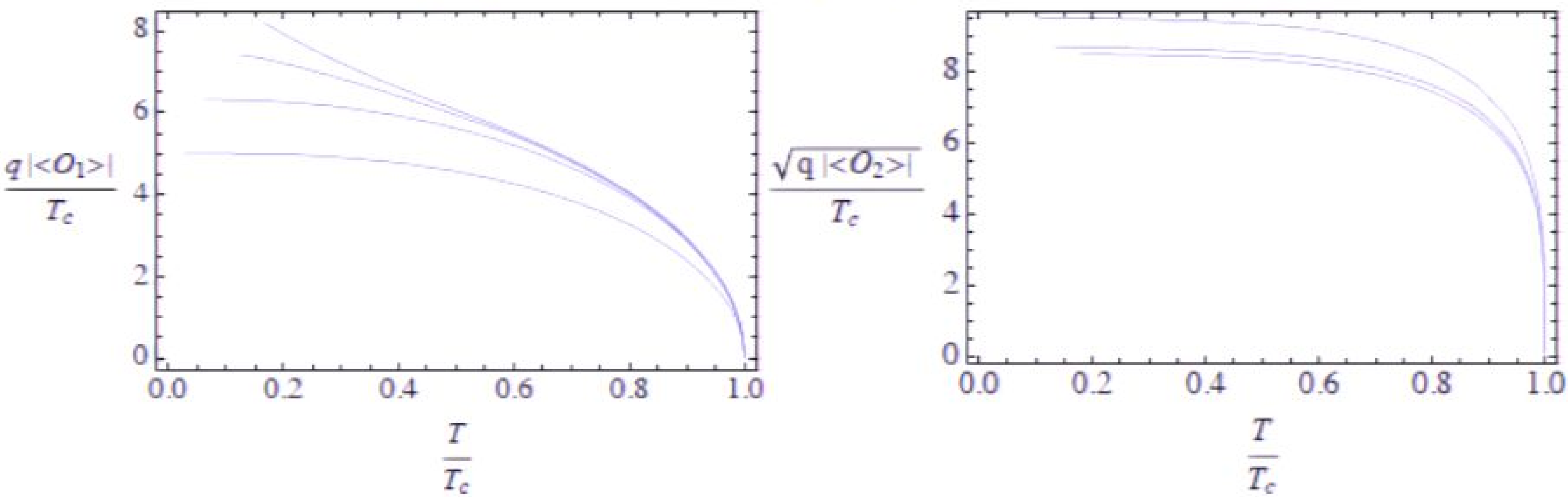

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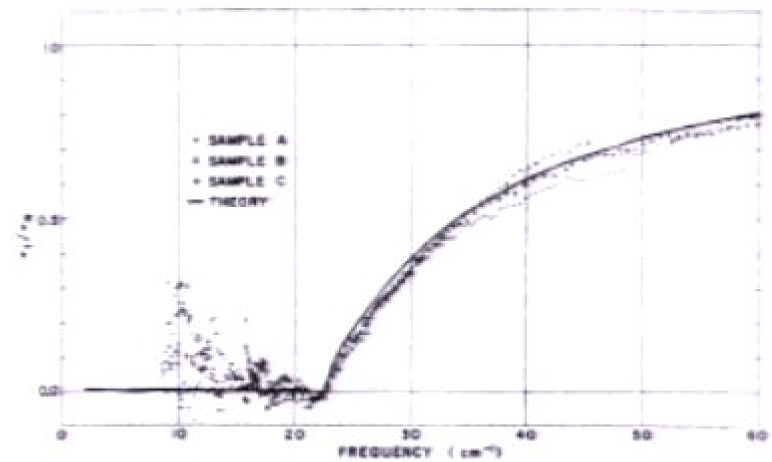

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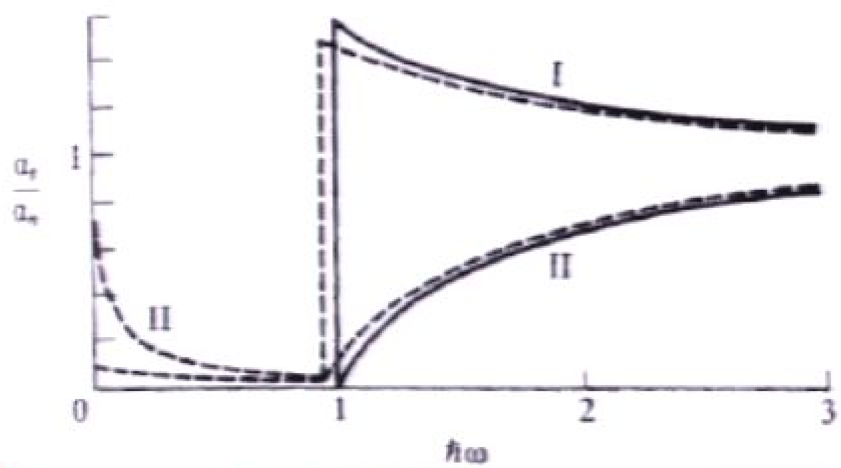

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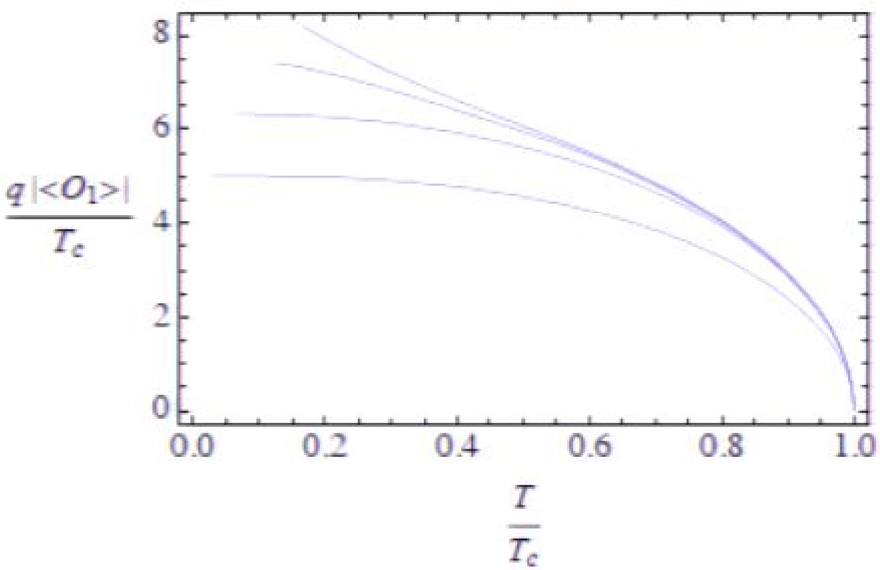

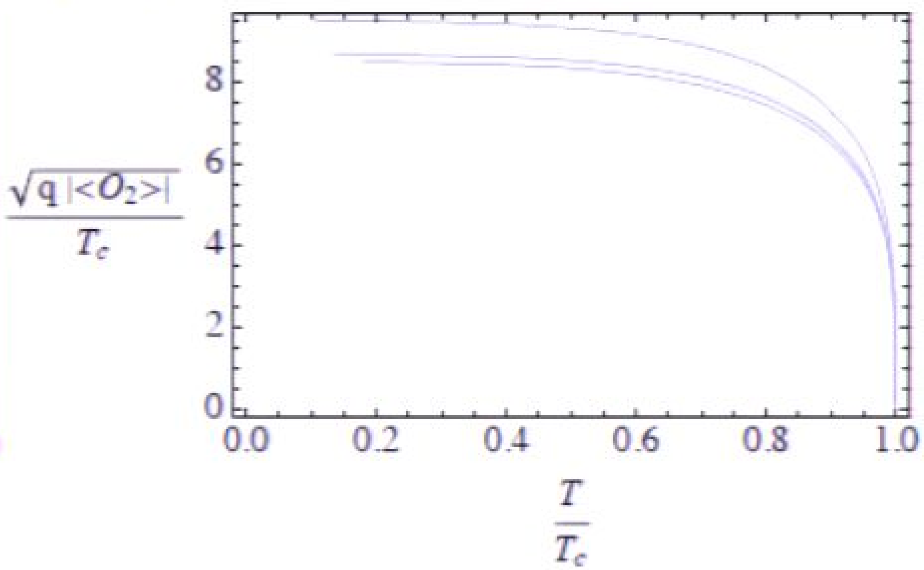

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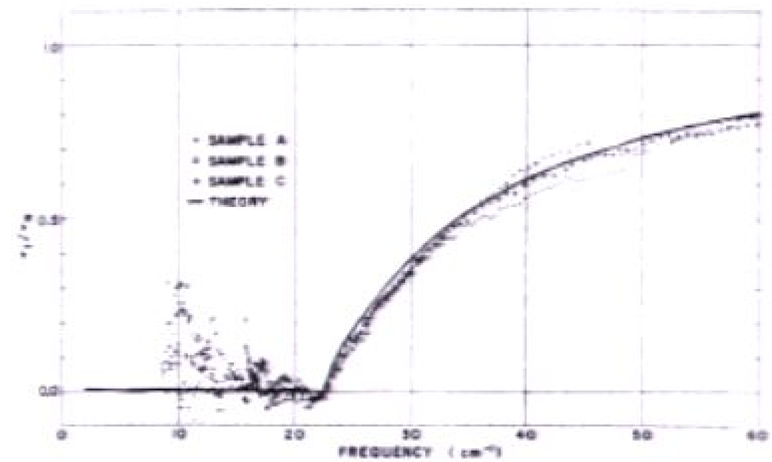

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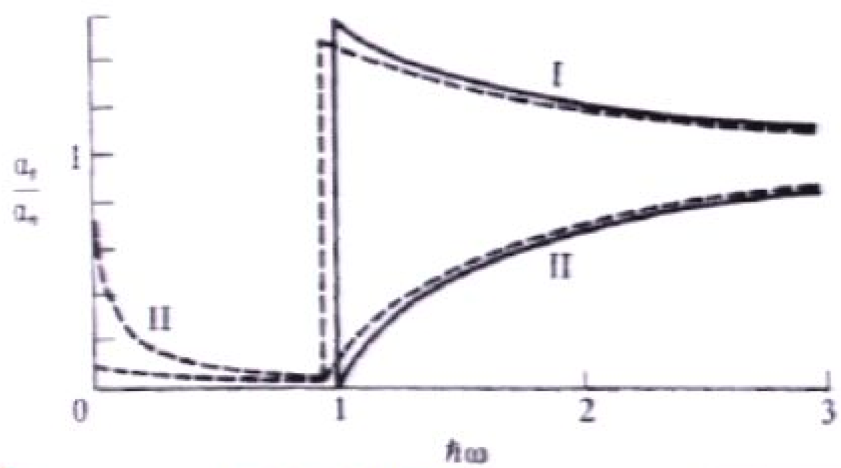

Electrical conductivity - AdS/CFT

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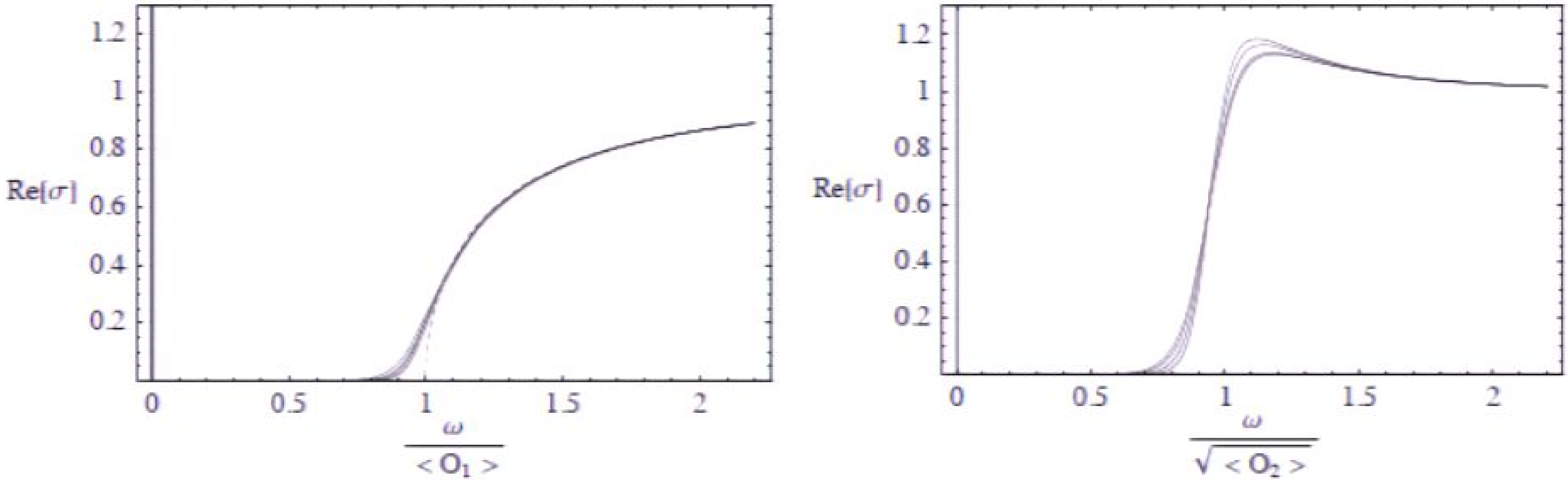
- Let's focus on the probe limit ($q \to \infty$) for simplicity.
- We computed the conductivity (2 cases). At $T \sim 0$:



- If the gap is 2Δ then we found that

$$\mathrm{Re}\,\sigma(\omega \to 0) \sim e^{-\alpha\Delta/T}.$$

- $\alpha = 1$ as $q \to \infty$, as in BCS theory, weakly coupled?
- Puzzle: why is there an exact gap?

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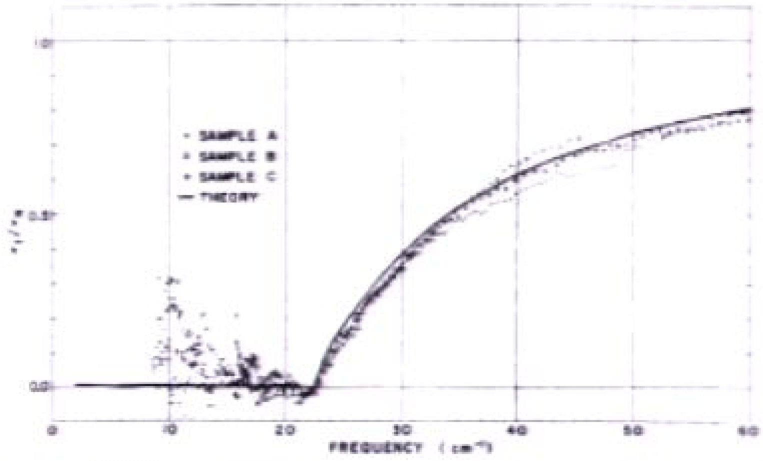

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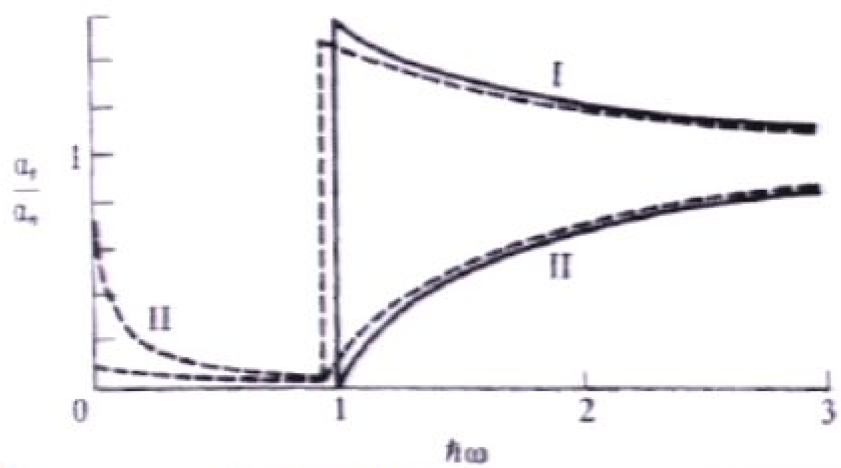

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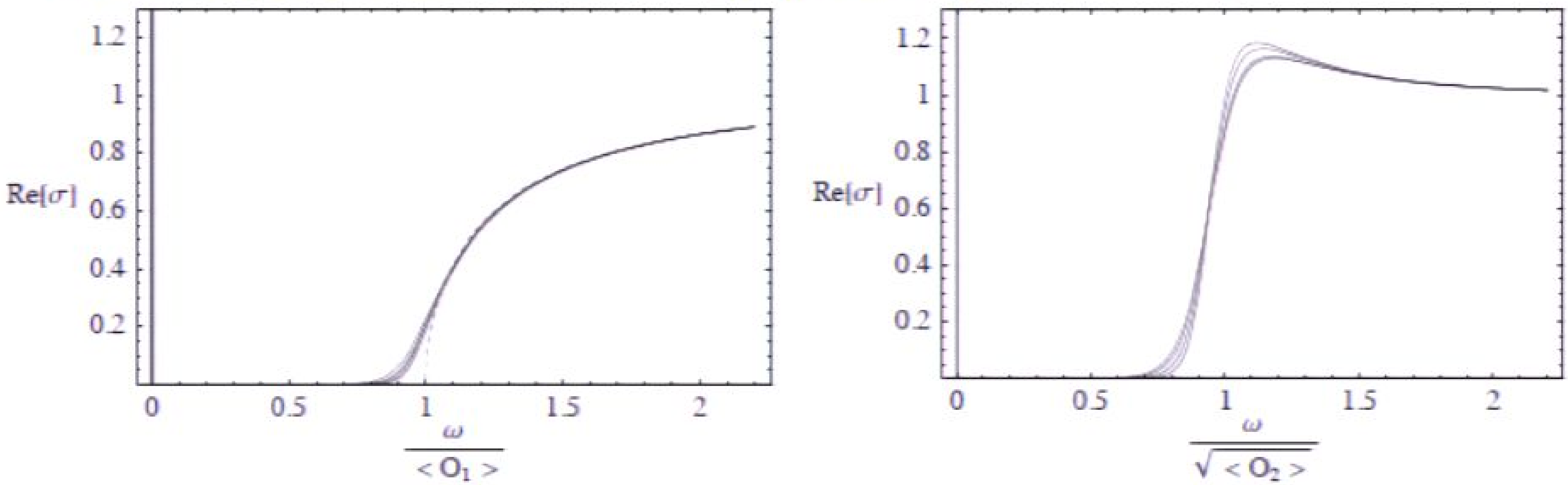

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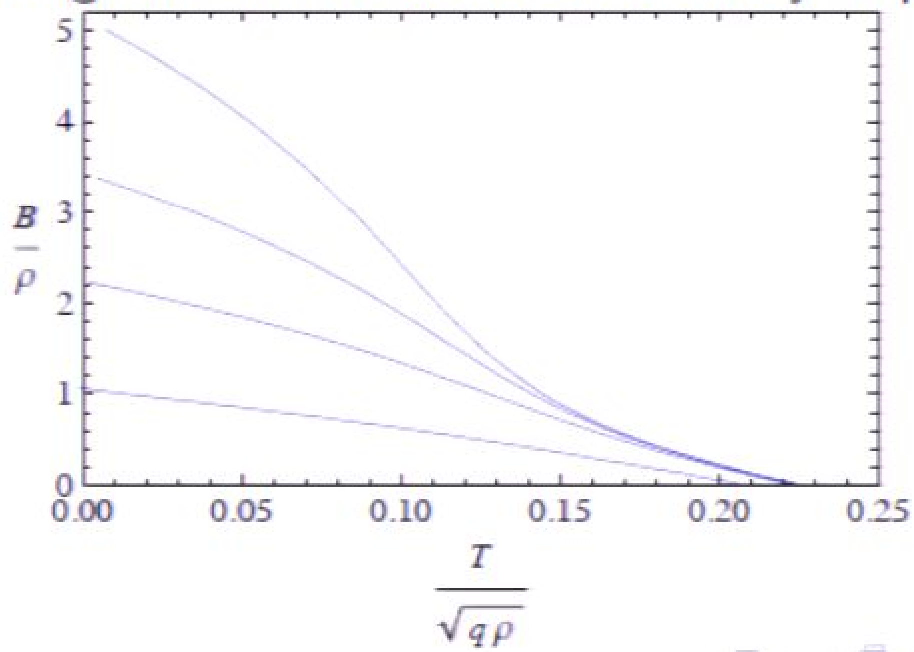
Critical magnetic fields

[Albash-Johnson '08, SAH-Herzog-Horowitz '08]

- The pole in the conductivity implies the London equation

$$j = \sigma E \sim \frac{i\#}{\omega} E = -\# A \,.$$

  Screening currents: attempt to expel an applied magnetic field.
- No dynamical photon $\rightarrow$ extreme type II (appropriate for films).
- Compute critical magnetic field: vortices destroy superconductivity



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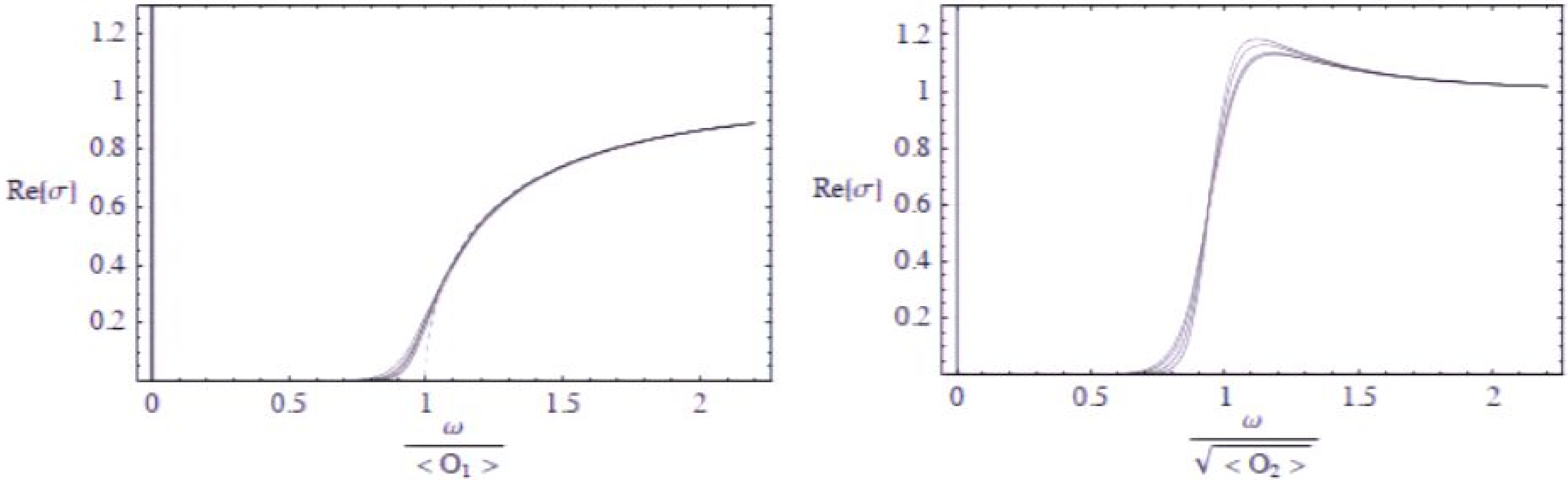

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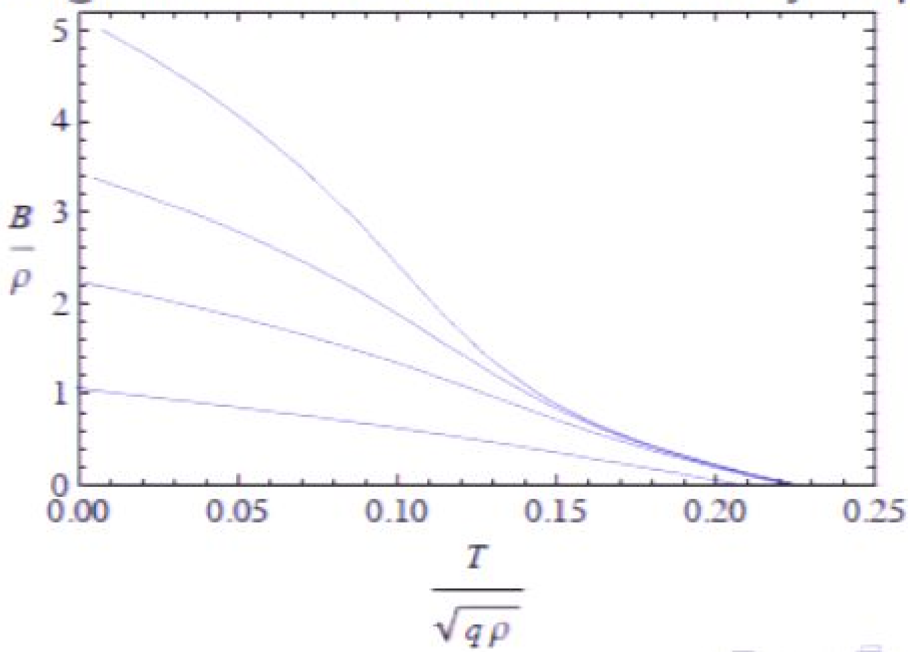


Superconducting M2 branes

[Denef-SAH '09]

- Embed holographic superconductors into string theory. Two reasons:
  1. May give explicit field theory action ('quiver Chern-Simons' theory) [Martelli-Sparks '08]

$$
\begin{aligned}
S &= \sum_i \frac{k_i}{4\pi} \int d^3x \mathrm{Tr} \left(A_i \wedge dA_i + \frac{2}{3} A_i \wedge A_i \wedge A_i + \text{superpartners} \right) \\
&+ \sum_a \int d^3x \left(|D\phi_a|^2 - \left| \frac{\partial W}{\partial \phi_a} \right|^2 + \text{superpartners} \right) .
\end{aligned}
$$

  2. Can ask a new question of the landscape: how many superconductors?
- Examples will be M2 branes placed at the tip of a Calabi-Yau cone.
- Dual geometry: $AdS_4 \times X^7$.

- Studied the stability of $AdS_4 \times X_7$ vacua for Sasaki-Einstein X_7.
- If X_7 has moduli: can build a 3-form mode that linearly decouples from all other modes, even in a background electric field.
- Calabi-Yau cone over X_7 has an anti-self dual closed (3,1)-form:

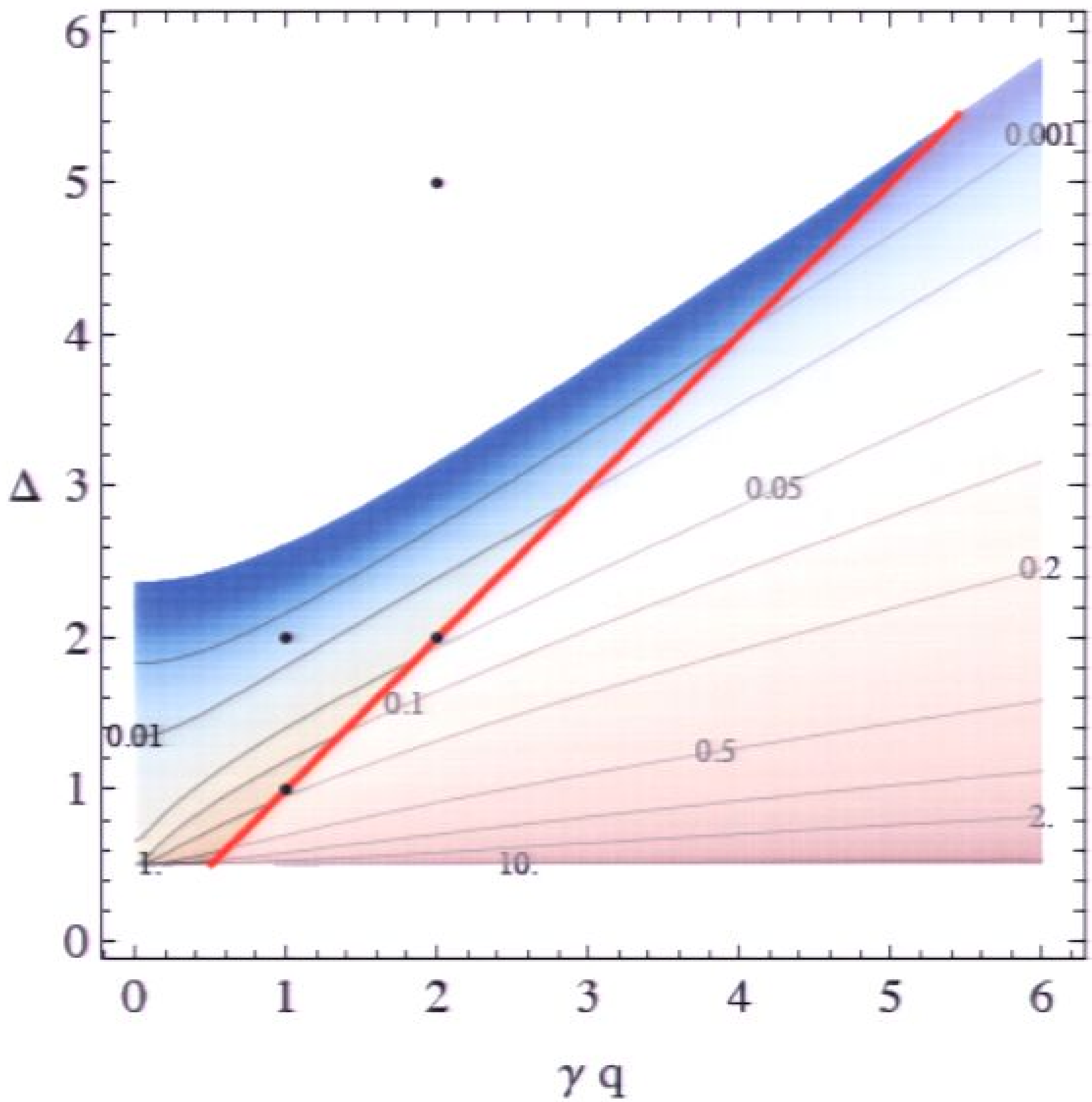
$$\hat{Y}_4 \equiv \delta g_{\bar{a}\bar{e}} \, \hat{\Omega}_4^{\bar{e}}{}_{bcd} \, d\bar{z}^{\bar{a}} \wedge dz^b \wedge dz^c \wedge dz^d \, .$$

- Contracting with $r\partial_r$ get 3-form on Sasaki-Einstein. Mode is

$$\delta C = \phi Y_3 + \text{c.c.} \, .$$

- This mode is always unstable ($\Delta = 2, \gamma q = 2$) and leads to superconductivity:

$$T_c \approx 0.0416 \frac{\mu}{\gamma} \, .$$



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Distribution of critical temperatures

- Examples supplied by (some) Brieskorn-Pham cones [Boyer-Galicki '03]:

$$z_1^{m_1} + \cdots + z_5^{m_5} = 0\,.$$

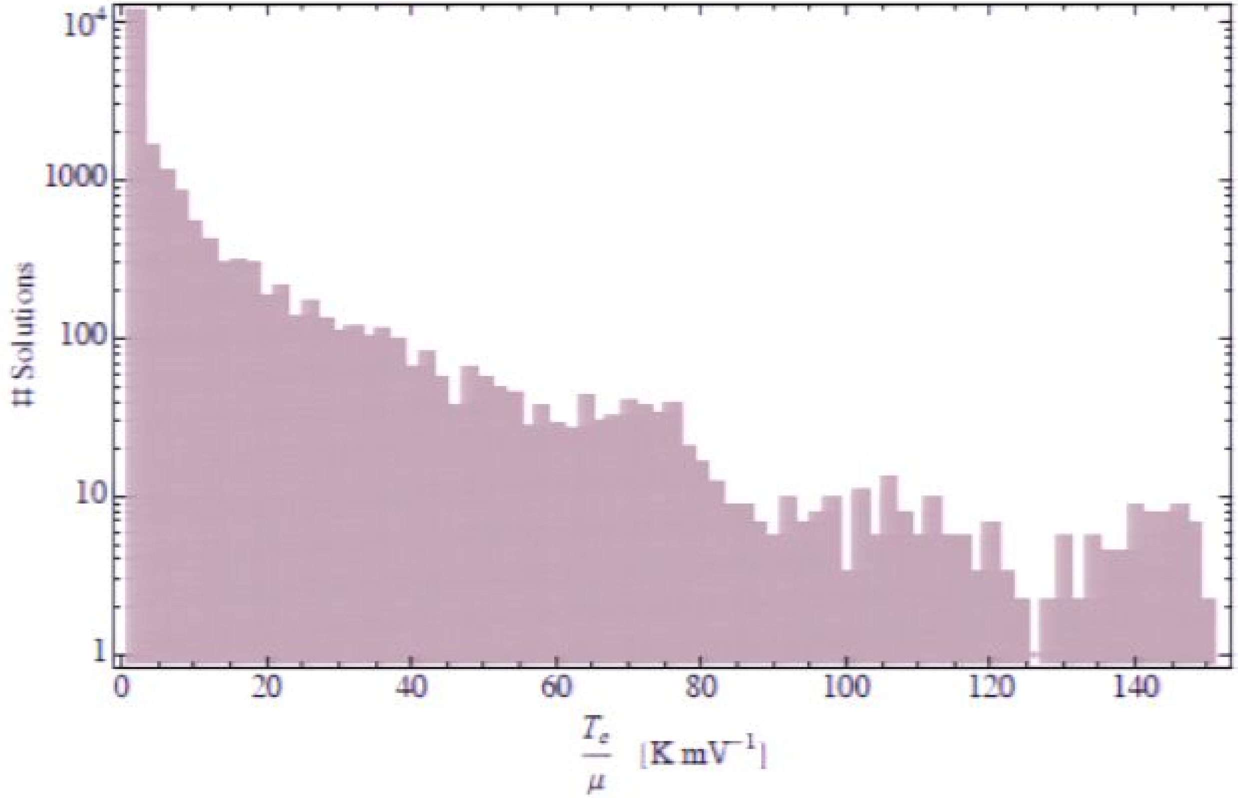
- For Sasaki-Einstein compactifications of M theory

$$\gamma = \frac{1}{2|\mathrm{Reeb}|}\,.$$

  Can be computed algebraically from the $\{m_i\}$ [Bergman-Herzog '01].

- Simple scan over $m_i \leq 100$, together with Sasaki-Einstein preserving quotients by $\mathbb{Z}_k$, found 11,821 backgrounds.

- Distribution of critical temperatures



- There are infinite families but T_c bounded above.
- $X_7 = S^7$ has a similar instability $\quad (\hat{Y}_4 = d\bar{z}_1 \wedge dz_2 \wedge dz_3 \wedge dz_4)$
  $\rightarrow$ ABJM theory has a superconducting phase.
  (modulo Gubser-Mitra instability).

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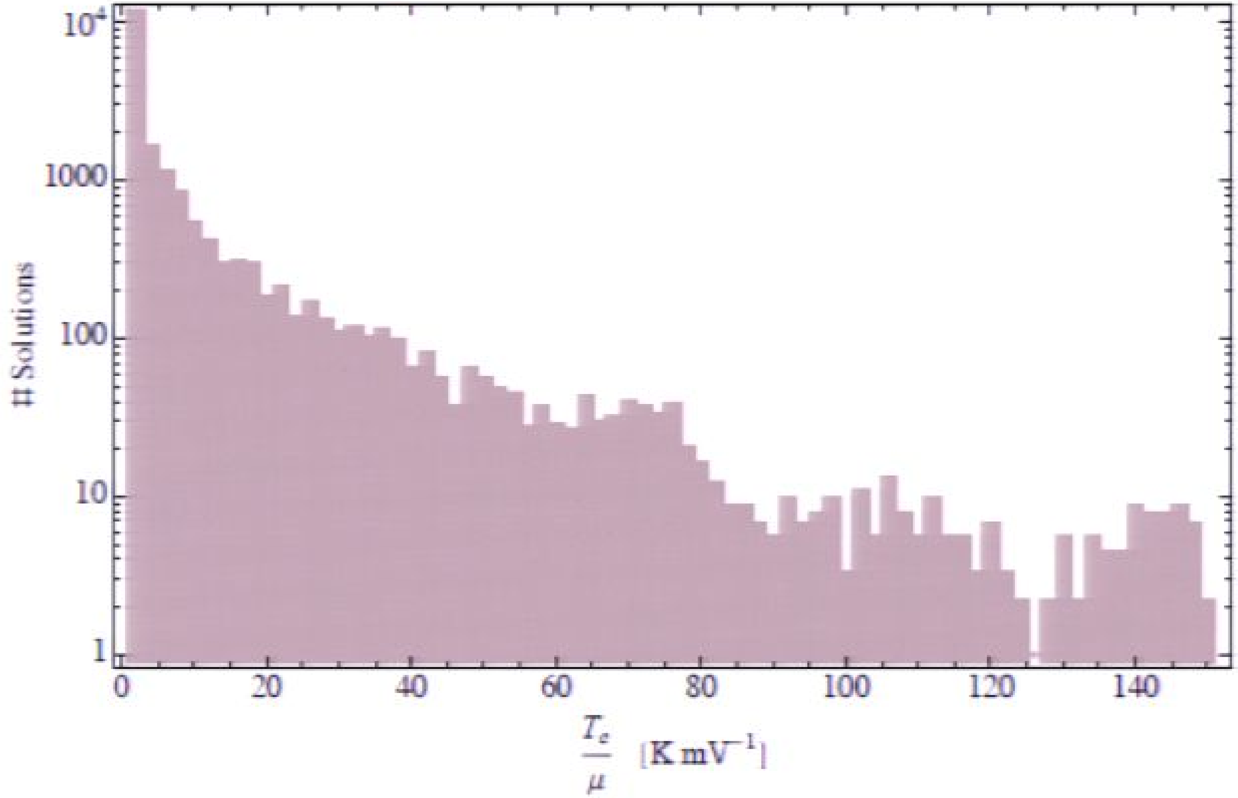


- There are infinite families but T_c bounded above.
- $X_7 = S^7$ has a similar instability $(\hat{Y}_4 = d\bar{z}_1 \wedge dz_2 \wedge dz_3 \wedge dz_4)$

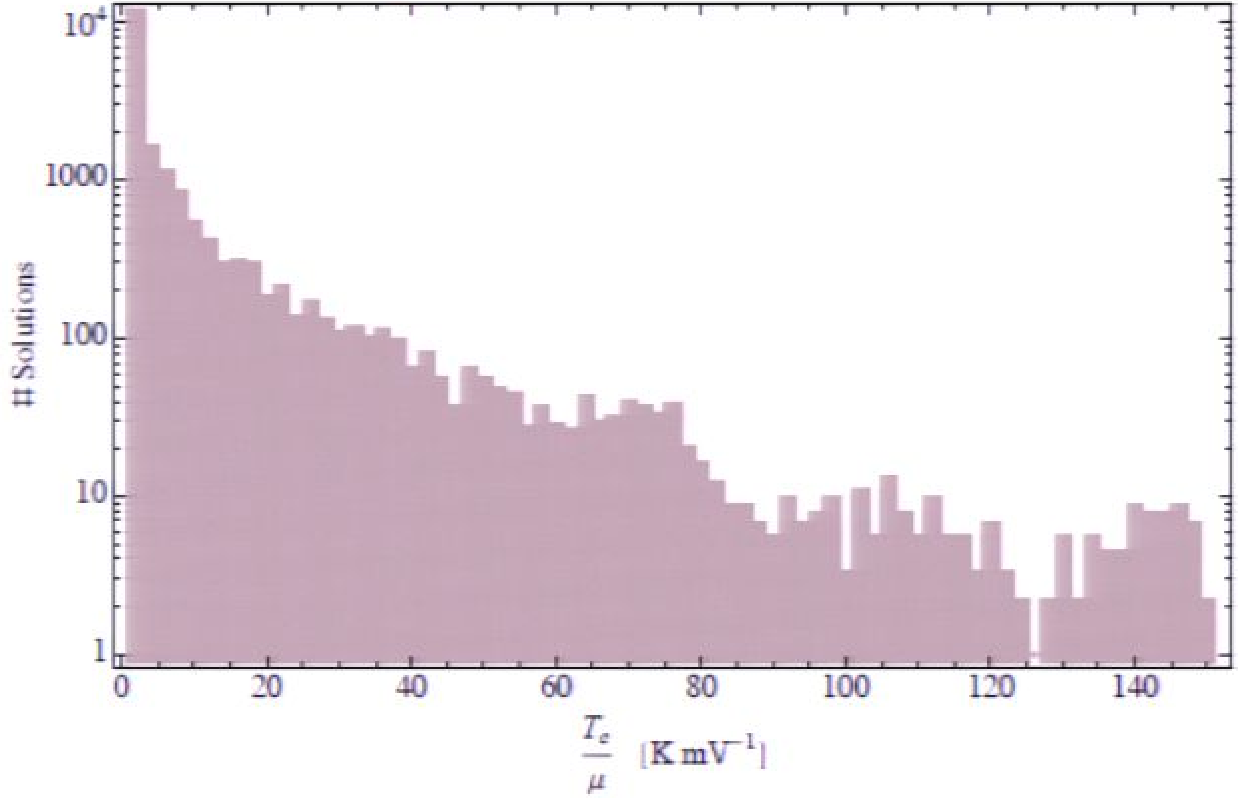
  $\rightarrow$ ABJM theory has a superconducting phase.

  (modulo Gubser-Mitra instability).

Summary

- Superconductivity is due to spontaneous symmetry breaking.
- Established theories of superconductivity depend on having 'glue' and 'electrons' which are described by quasiparticles.
- Spontaneous symmetry breaking can occur naturally in AdS/CFT.
- Explored basic phenomenology of holographic superconductors.
- Noted a certain 'resonance' between string and condensed matter landscapes.
- Many $\mathcal{N} = 2$ theories with Sasaki-Einstein duals superconducting.

- Distribution of critical temperatures



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- Studied the stability of $AdS_4 \times X_7$ vacua for Sasaki-Einstein X_7.
- If X_7 has moduli: can build a 3-form mode that linearly decouples from all other modes, even in a background electric field.
- Calabi-Yau cone over X_7 has an anti-self dual closed (3,1)-form:

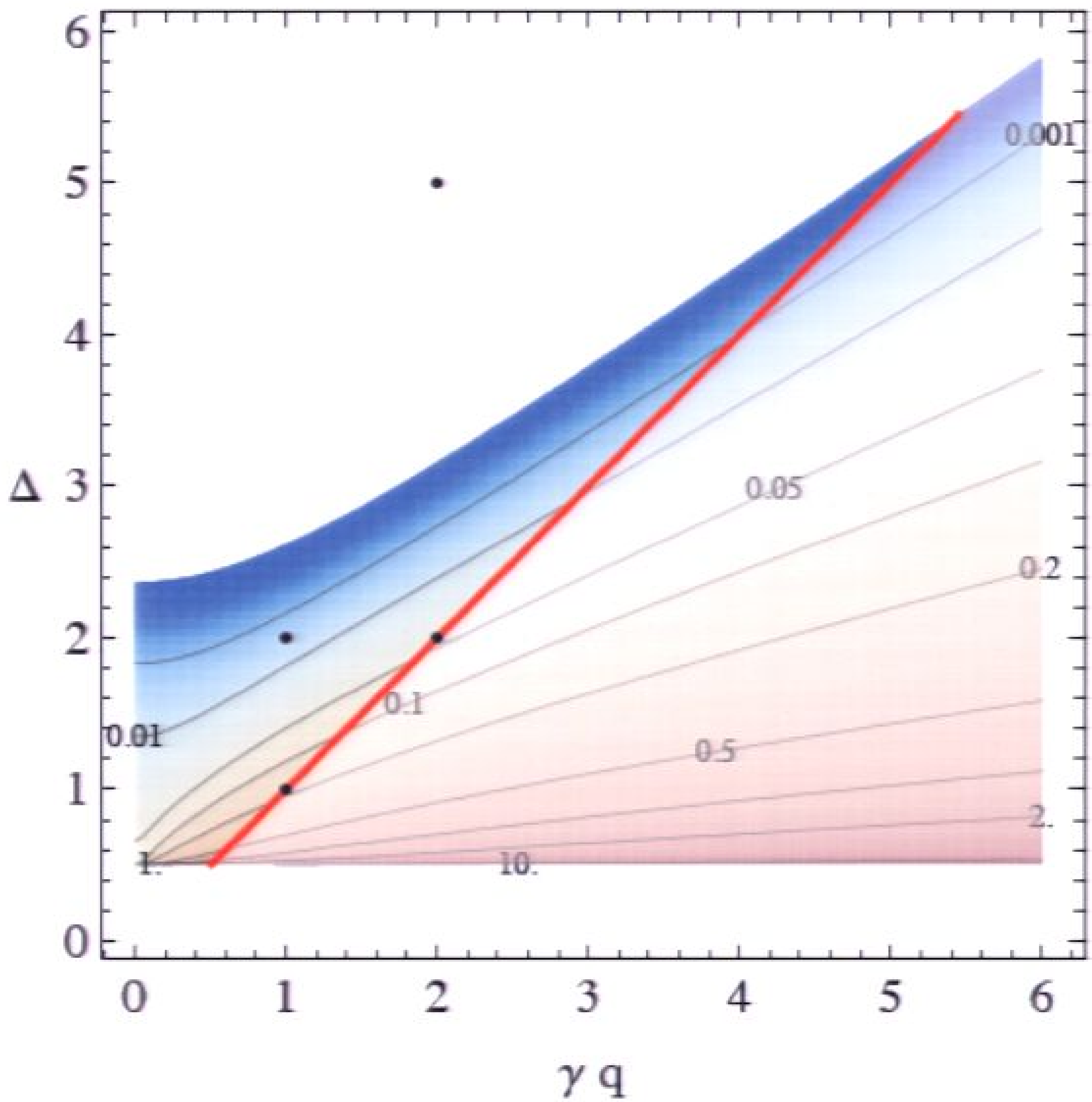
$$\hat{Y}_4 \equiv \delta g_{\bar{a}\bar{e}} \, \hat{\Omega}_4{}^{\bar{e}}{}_{bcd} \, d\bar{z}^{\bar{a}} \wedge dz^b \wedge dz^c \wedge dz^d \, .$$

- Contracting with $r\partial_r$ get 3-form on Sasaki-Einstein. Mode is

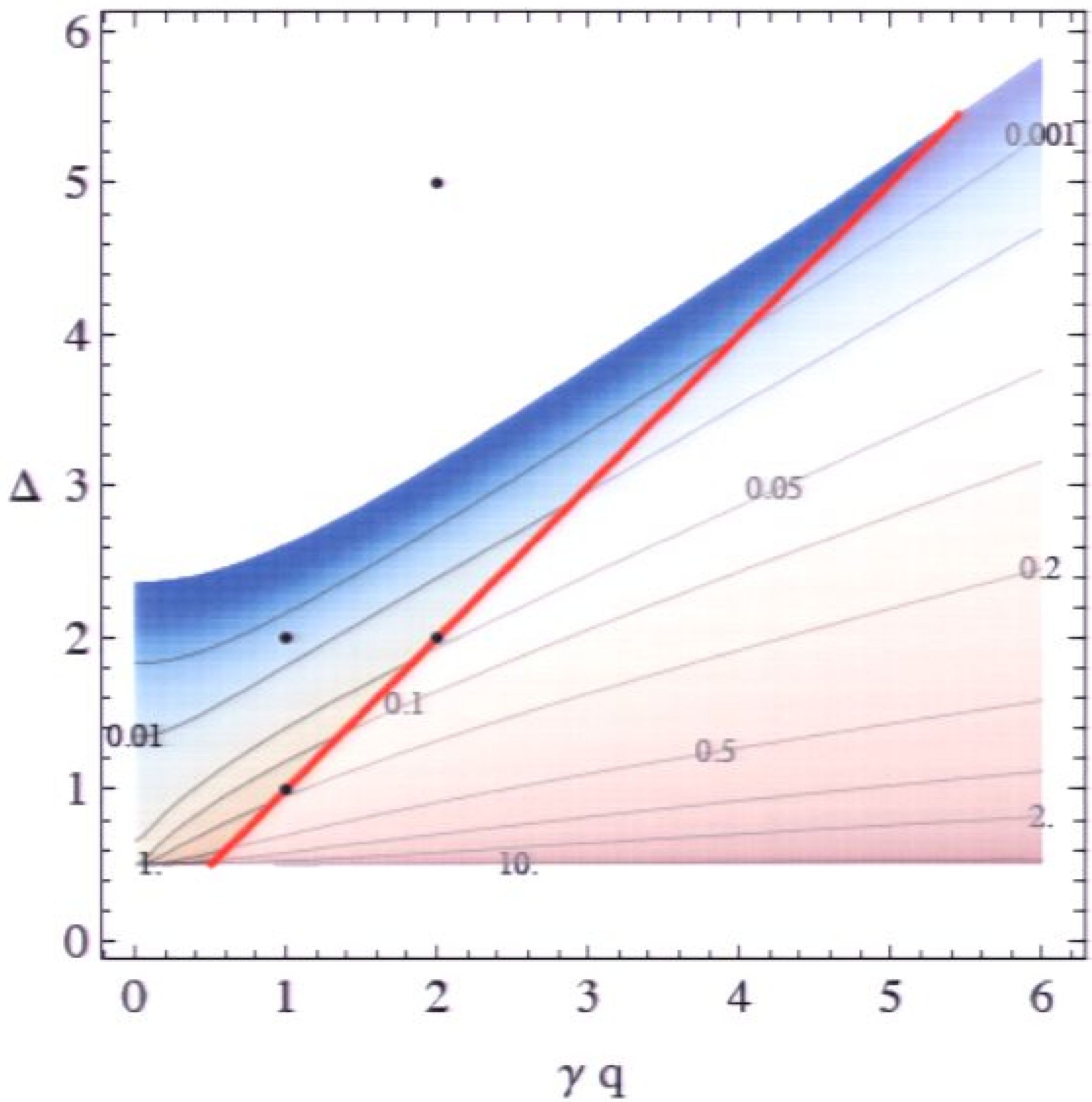
$$\delta C = \phi Y_3 + \text{c.c.} \, .$$

- This mode is always unstable $(\Delta = 2, \gamma q = 2)$ and leads to superconductivity:

$$T_c \approx 0.0416 \frac{\mu}{\gamma} \, .$$



[Denef-SAH '09]



[Denef-SAH '09]