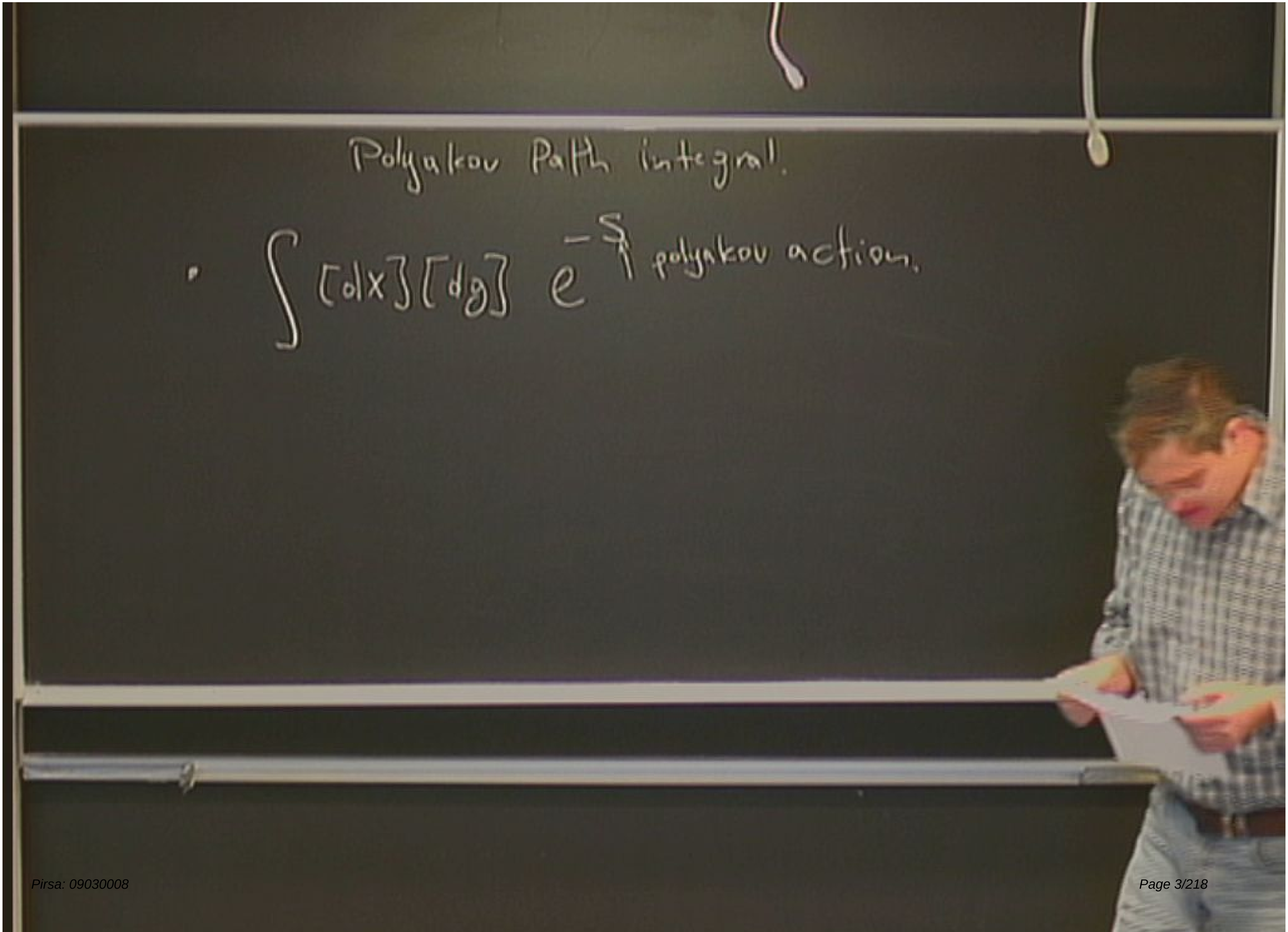
Title: Introduction to the Bosonic String

Date: Mar 20, 2009 10:00 AM

URL: http://pirsa.org/09030008

Abstract: This course provides a thorough introduction to the bosonic string based on the Polyakov path integral and conformal field theory. We introduce central ideas of string theory, the tools of conformal field theory, the Polyakov path integral, and the covariant quantization of the string. We discuss string interactions and cover the tree-level and one loop amplitudes. More advanced topics such as T-duality and D-branes will be taught as part of the course. The course is geared for M.Sc. and Ph.D. students enrolled in Collaborative Ph.D. Program in Theoretical Physics. Required previous course work: Quantum Field Theory (AM516 or equivalent). The course evaluation will be based on regular problem sets that will be handed in during the term. The primary text is the book: 'String theory. Vol. 1: An introduction to the bosonic string. J. Polchinski (Santa Barbara, KITP) . 1998. 402pp. Cambridge, UK: Univ. Pr. (1998) 402 p.' All interested students should contact Alex Buchel at abuchel@uwo.ca as soon as possible.

Polyakov Path integral.

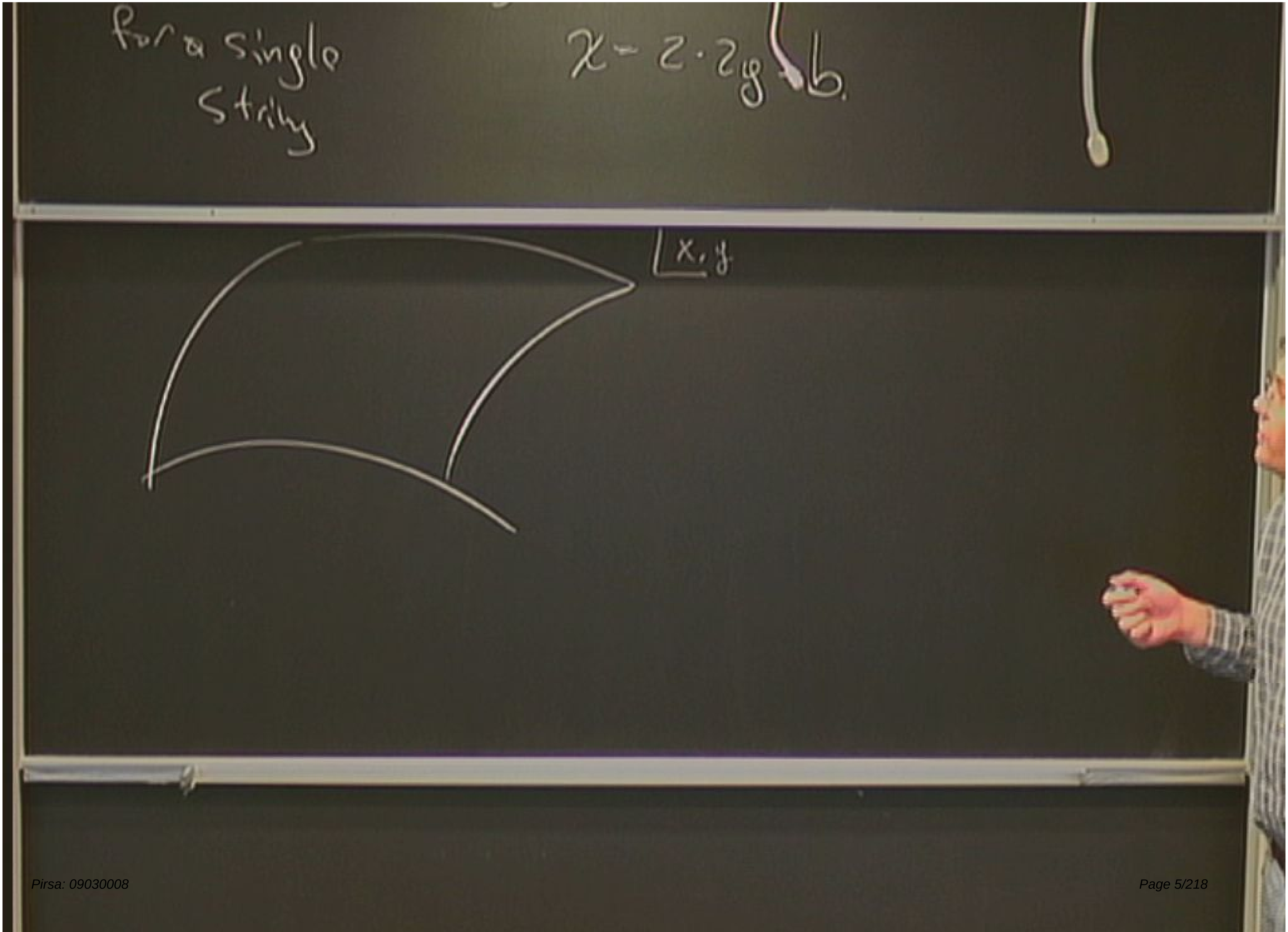

Polyakov Path integral.
• $\int [dx][dg]\, e^{-S}$
polyakov action.

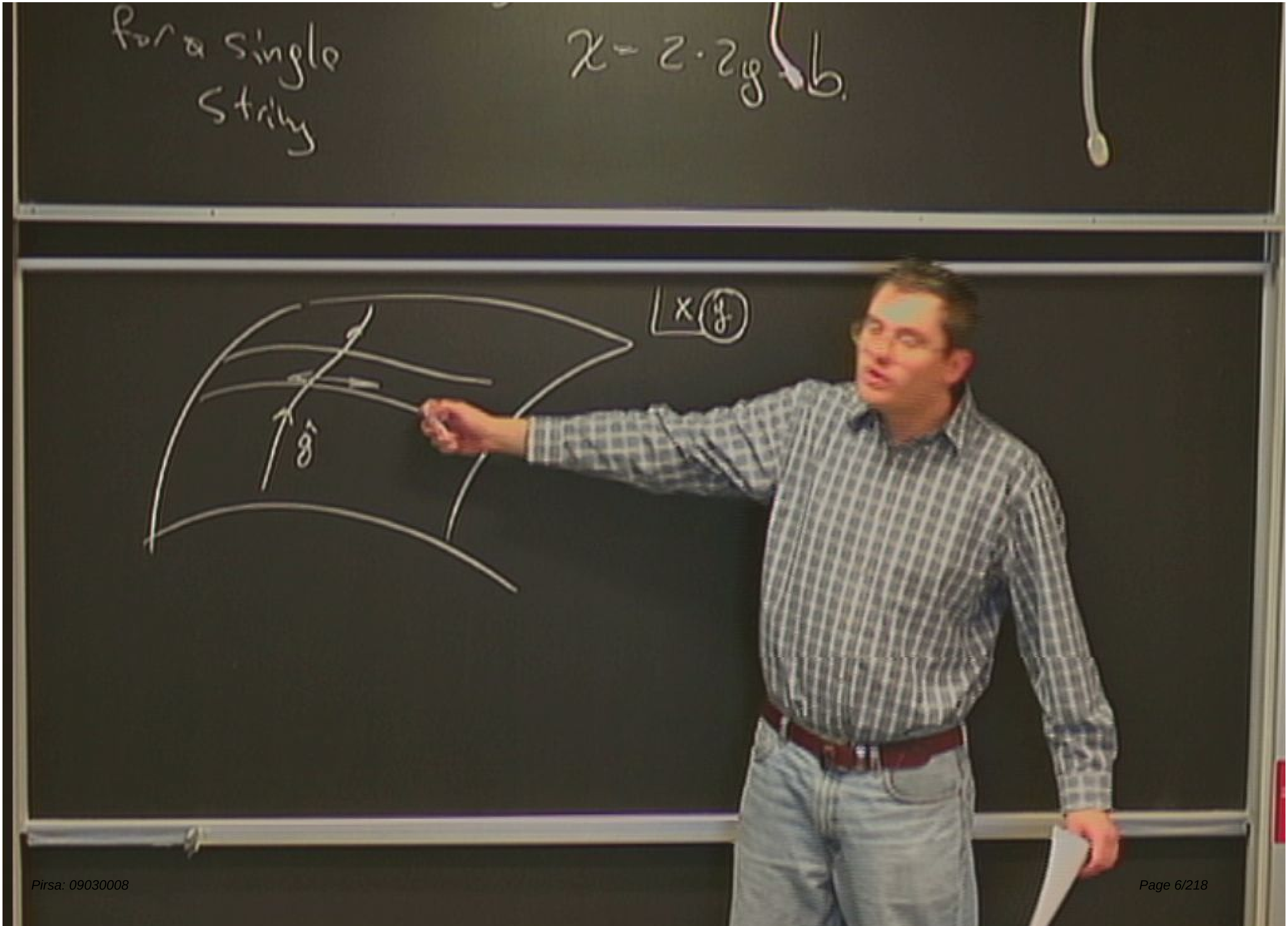
$$Z = \int [dX][dg]\, e^{-S_X}$$ polyakov action.

a partition function for a single string

$$S_X = \frac{1}{4\pi\alpha'}\int d^2\sigma \sqrt{g}\, \partial_a X^\mu \partial^a X_\mu + \lambda \chi$$

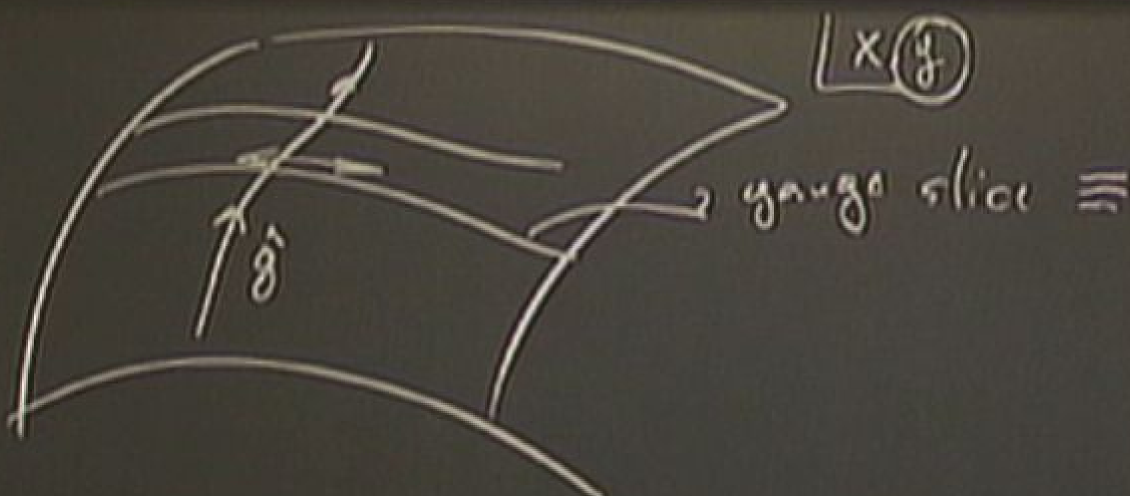
$$\chi = 2 \cdot 2g - b.$$


for a single string
$\chi = 2 \cdot 2g \cdot b.$
x, y


for a single string
x = Z · Zg · b.
x g
ĝ

for a single string

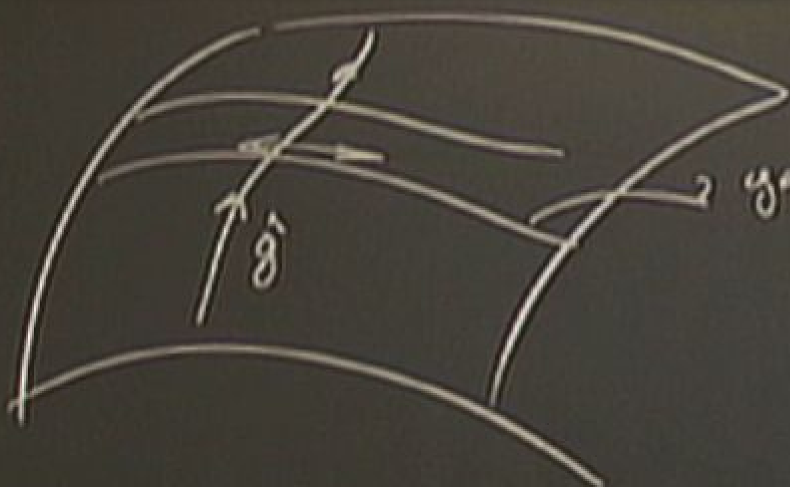
$$x = z \cdot z_g \cdot b.$$



for a single string

$\chi = 2 - 2g - b$

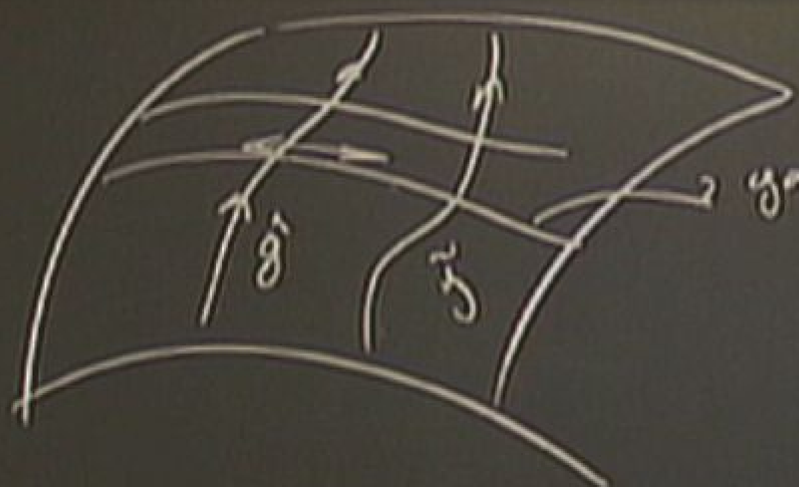
$L \times g$



→ gauge slice $\equiv$ diffeo $\times$ Weyl

for a single string

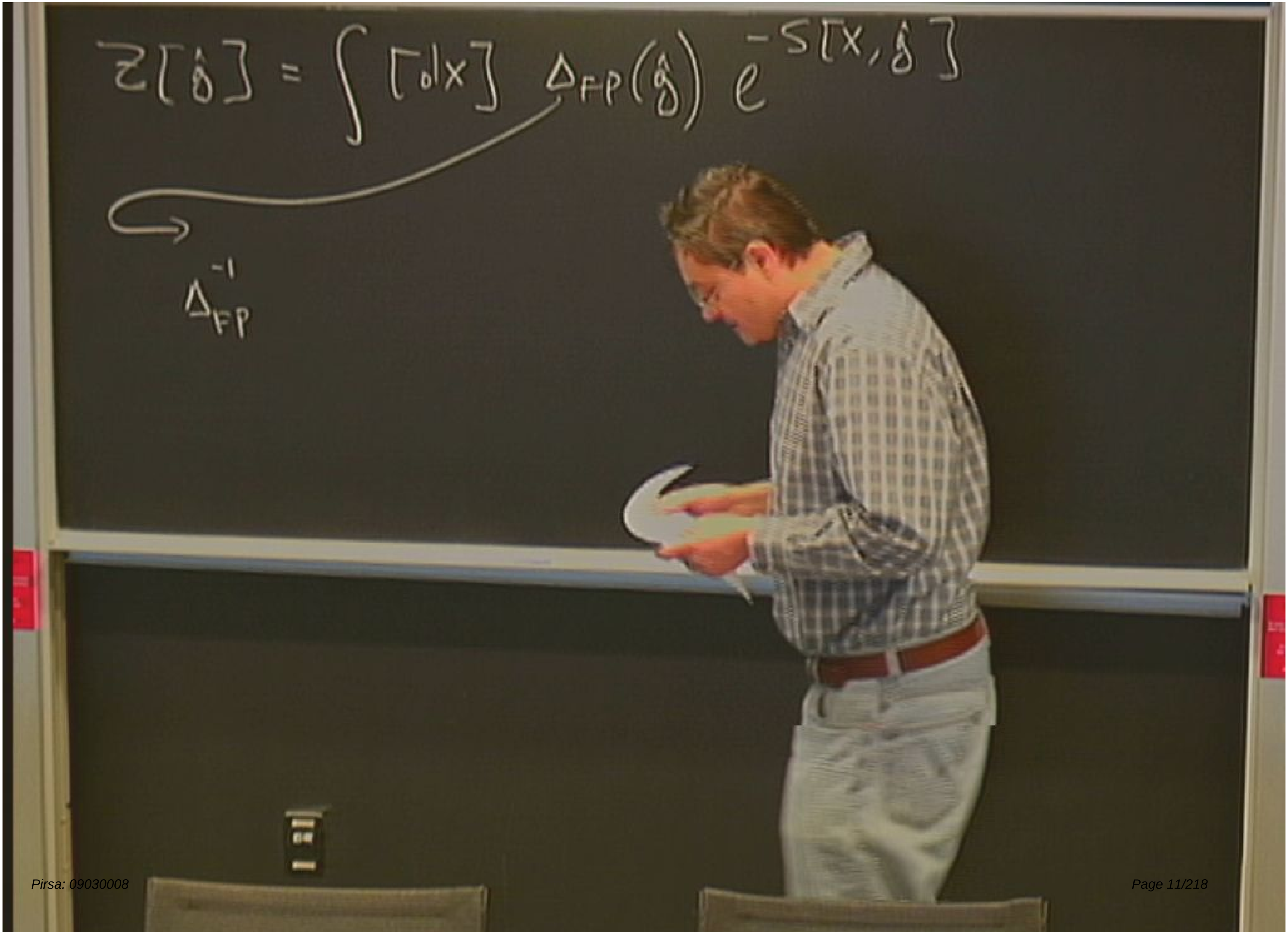
$\chi = 2 - 2g - b.$

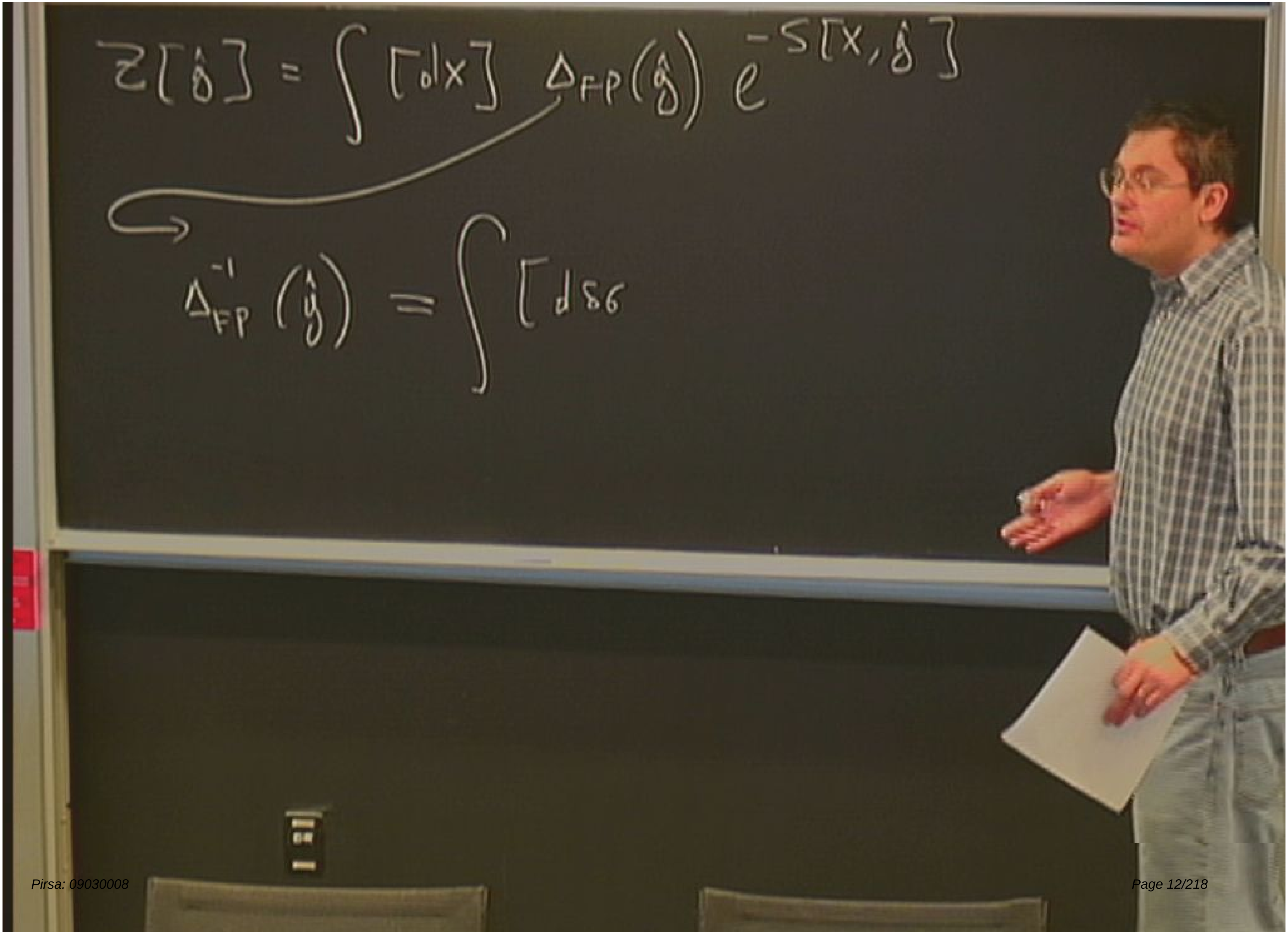


$\lfloor x \, (g)$

gauge slice $\equiv$ diffeo $\times$ Weyl.

$$Z[\hat{g}] = \int [dx] \; \Delta_{FP}(\hat{g}) \; e^{-S[x,\hat{g}]}$$


$Z[\hat{g}] = \int [dx]\ \Delta_{FP}(\hat{g})\ e^{-S[x,\hat{g}]}$
Δ_{FP}^{-1}


$Z[\hat{g}] = \int [dx]\, \Delta_{FP}(\hat{g})\, e^{-S[x,\hat{g}]}$
$\Delta_{FP}^{-1}(\hat{g}) = \int [d\delta\sigma$

$$Z[\hat{g}] = \int [dx]\, \Delta_{FP}(\hat{g})\, e^{-S[x,\hat{g}]}$$

$$\Delta_{FP}^{-1}(\hat{g}) = \int_{\beta'^{\,n}_{n} = 0} [d\delta\sigma\, d\beta']$$

$$Z[\hat{g}] = \int [dx] \; \Delta_{FP}(\hat{g}) \; e^{-S[x,\hat{g}]}$$

$$\Delta_{FP}^{-1}(\hat{g}) = \int_{\beta'_{n}{}^{n} = 0} [d\delta\sigma \; d\beta'] \; e^{4\pi i \int d^2\sigma \sqrt{\hat{g}} \; \beta'^{ab} (P_1 \delta\sigma)_{ab}}$$

$$(P_1\,\delta\sigma)_{ab} = \frac{1}{2}\left[\nabla_a\,\delta\sigma_b + \nabla_b\,\delta\sigma_a - \hat{g}_{ab}\nabla_c\,\delta\sigma^c\right]$$

$$(P_1 \delta\sigma)_{ab} = \frac{1}{2}\left[\nabla_a \delta\sigma_b + \nabla_b \delta\sigma_a - \hat{g}_{ab} \nabla_c \delta\sigma^c\right]$$

Aside (intro to Grassmann variable calculus)

$$(P_1\,\delta\sigma)_{ab} = \frac{1}{2}\left[\nabla_a \delta\sigma_b + \nabla_b \delta\sigma_a - \hat{g}_{ab}\nabla_c \delta\sigma^c\right]$$

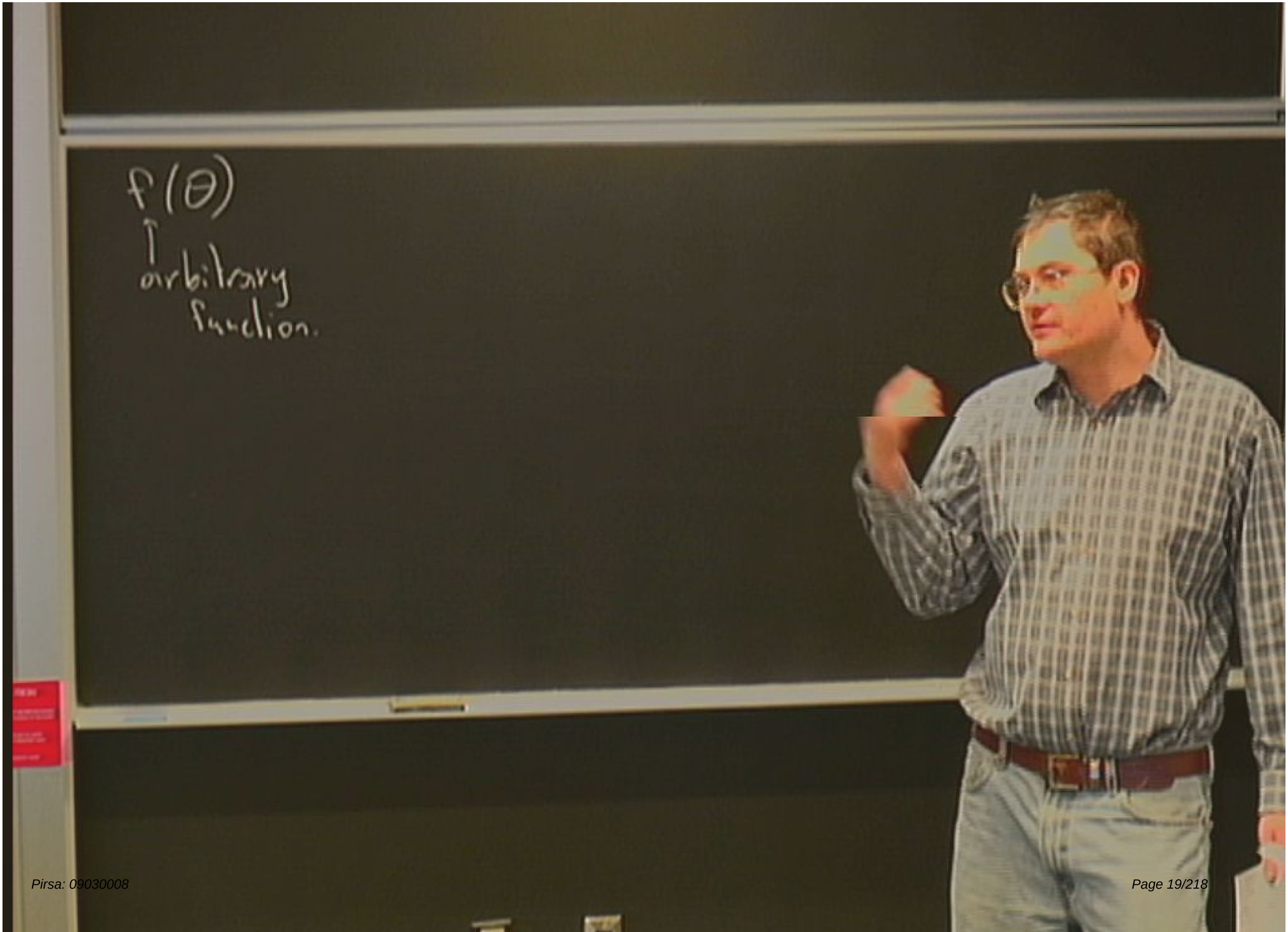
<u>Aside</u> (intro to Grassmann variable <u>calculus</u>)

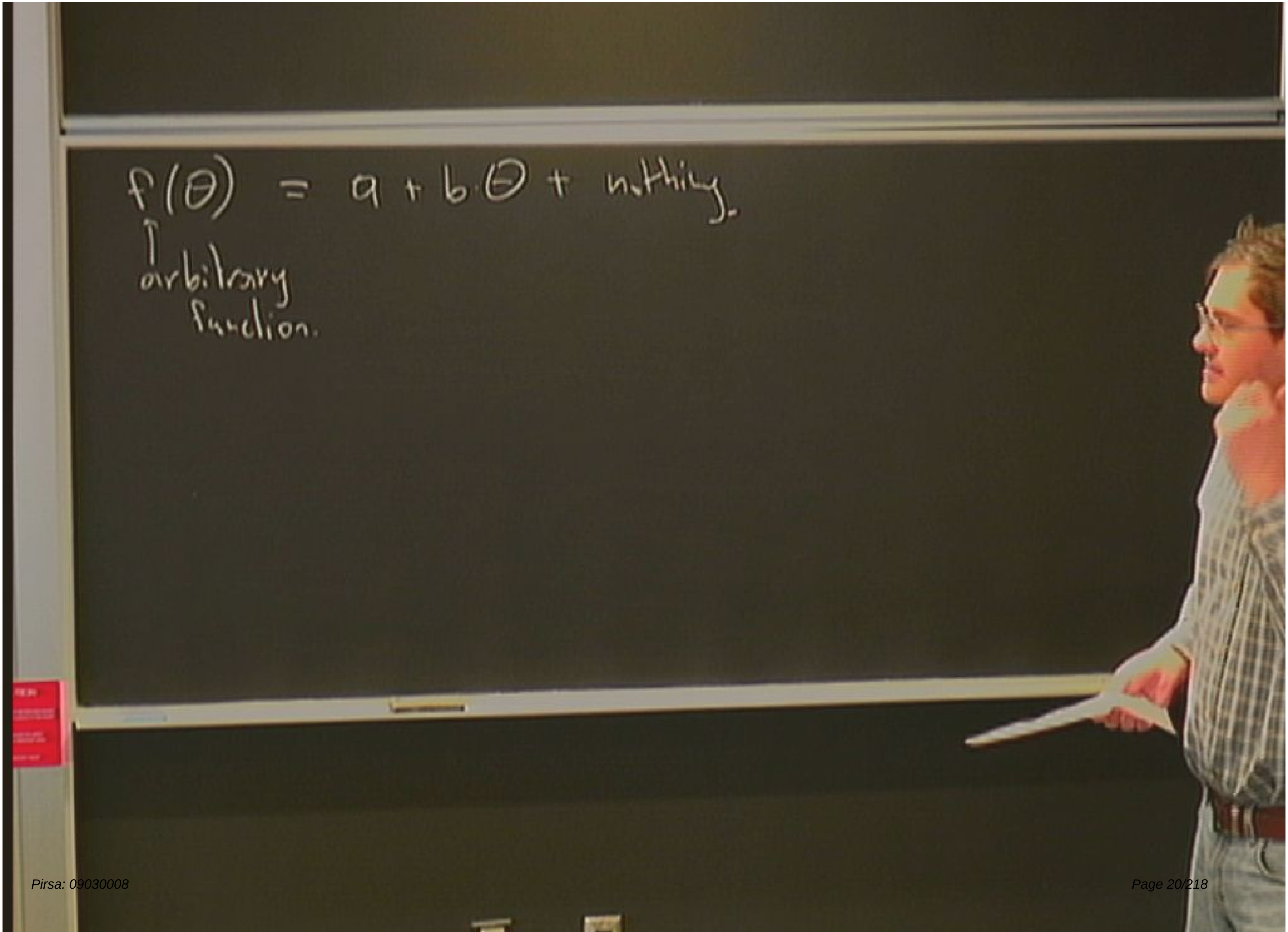
$$\theta_1 \theta_2 = -\theta_2 \theta_1 \quad \Rightarrow$$

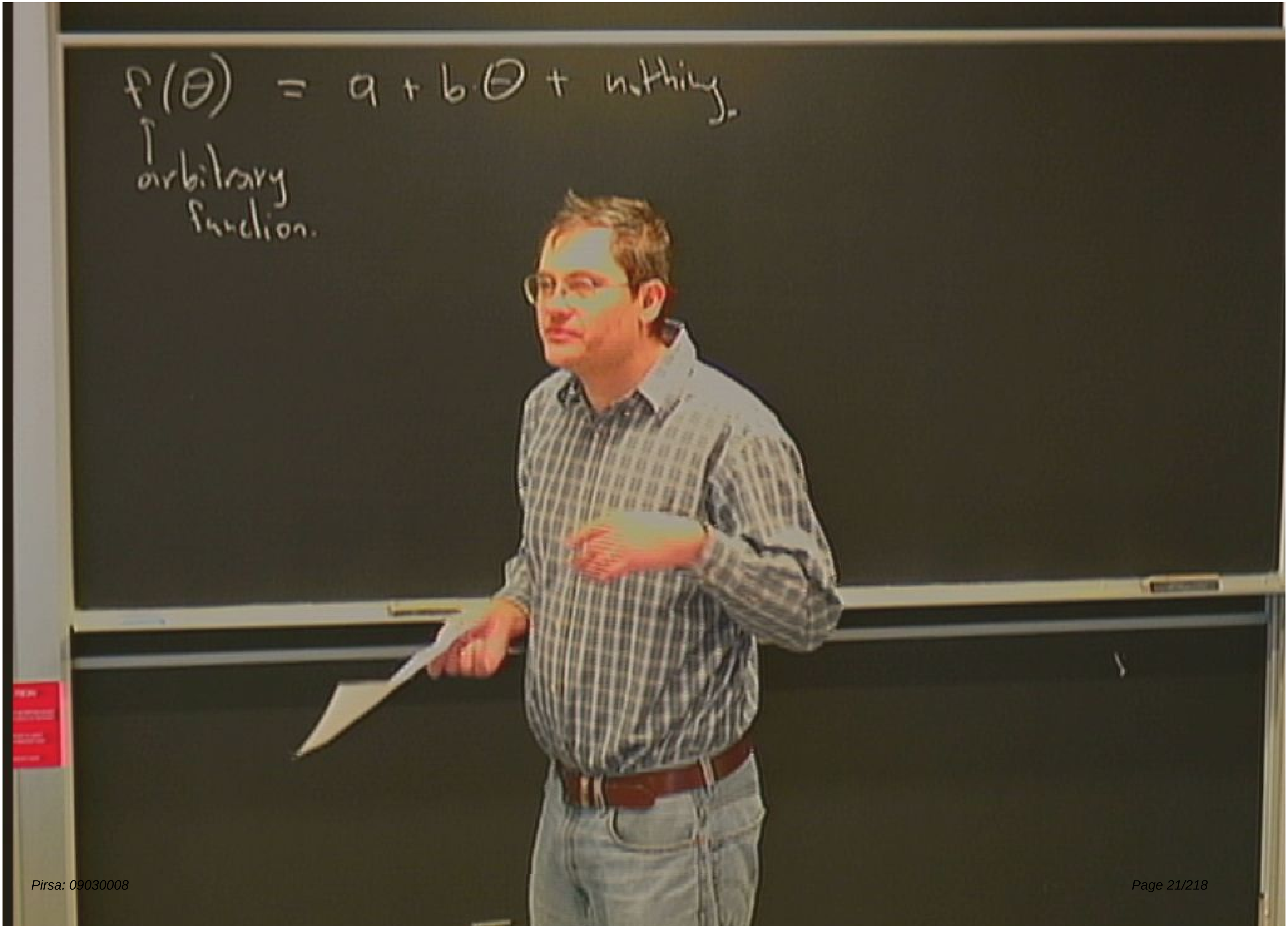
$$(P_1\,\delta\sigma)_{ab} = \frac{1}{2}\left[\nabla_a \delta\sigma_b + \nabla_b \delta\sigma_a - \hat{g}_{ab}\nabla_c \delta\sigma^c\right]$$

<u>Aside</u> (intro to Grassmann variable <u>calculus</u>)

$$\theta_1\theta_2 = -\theta_2\theta_1 \Rightarrow \theta^2 = 0$$


f(θ)
arbitrary
function.


f(Θ) = a + b·Θ + nothing.
arbitrary function.


f(θ) = a + b·θ + nothing.
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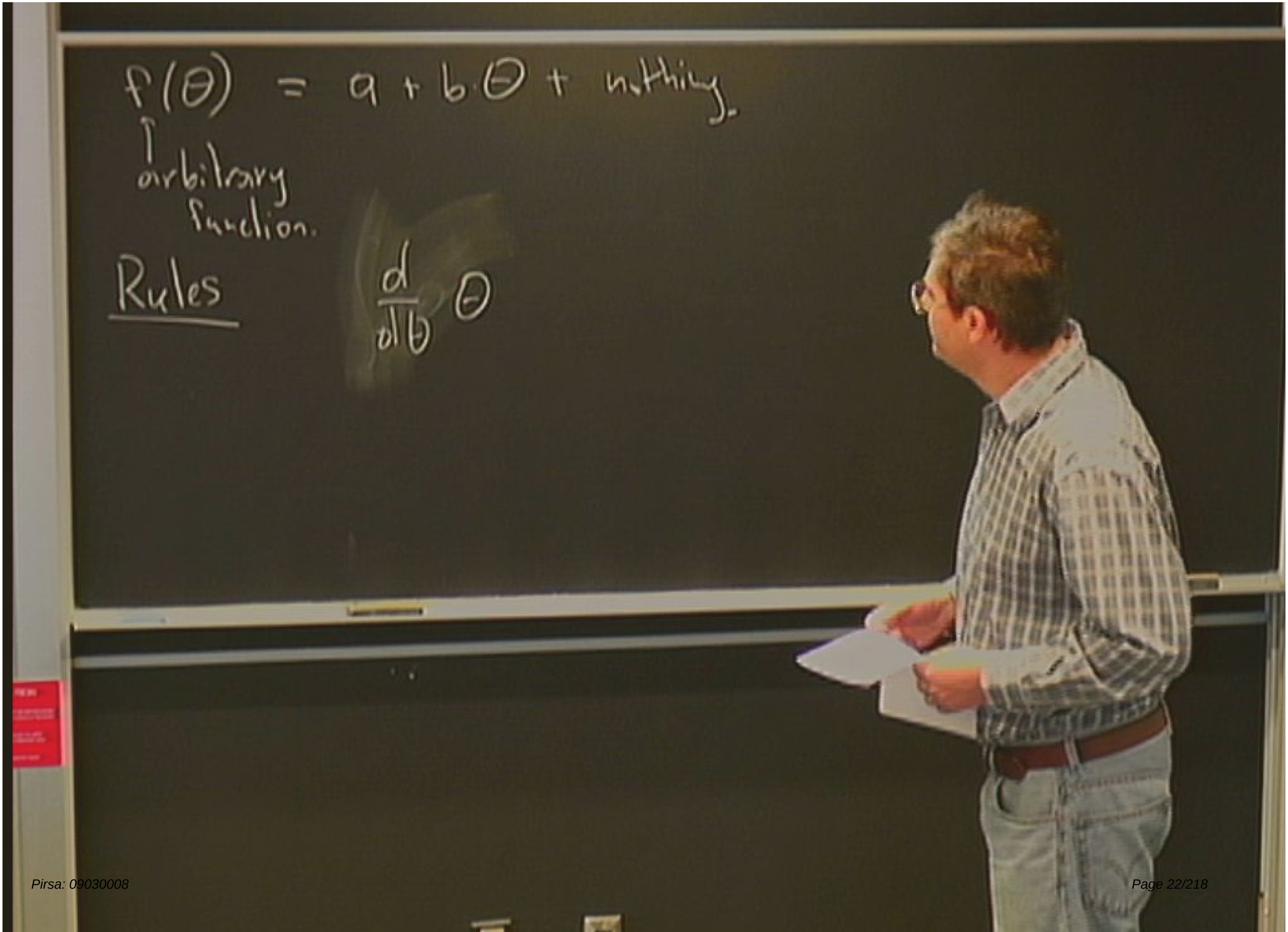
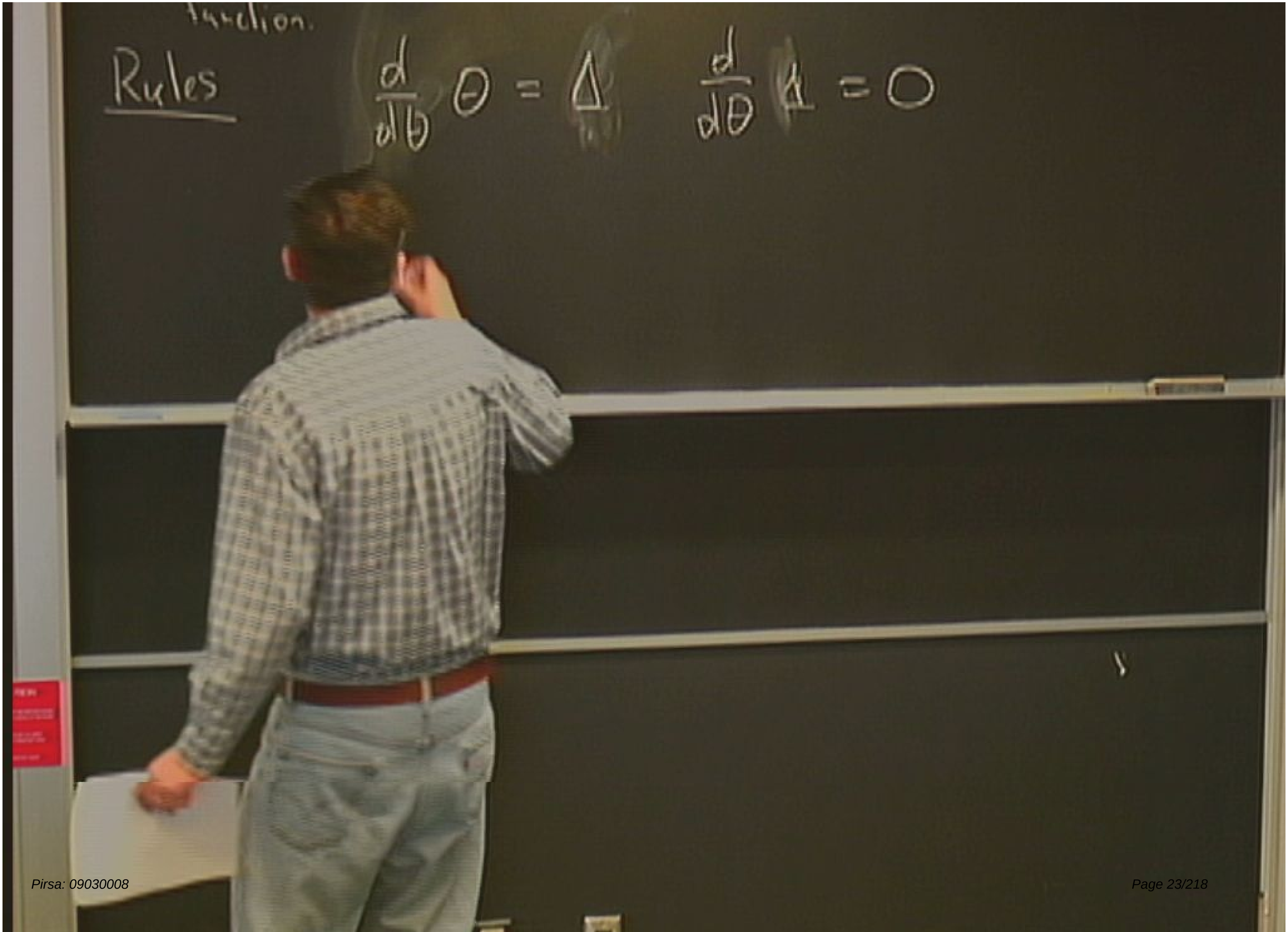
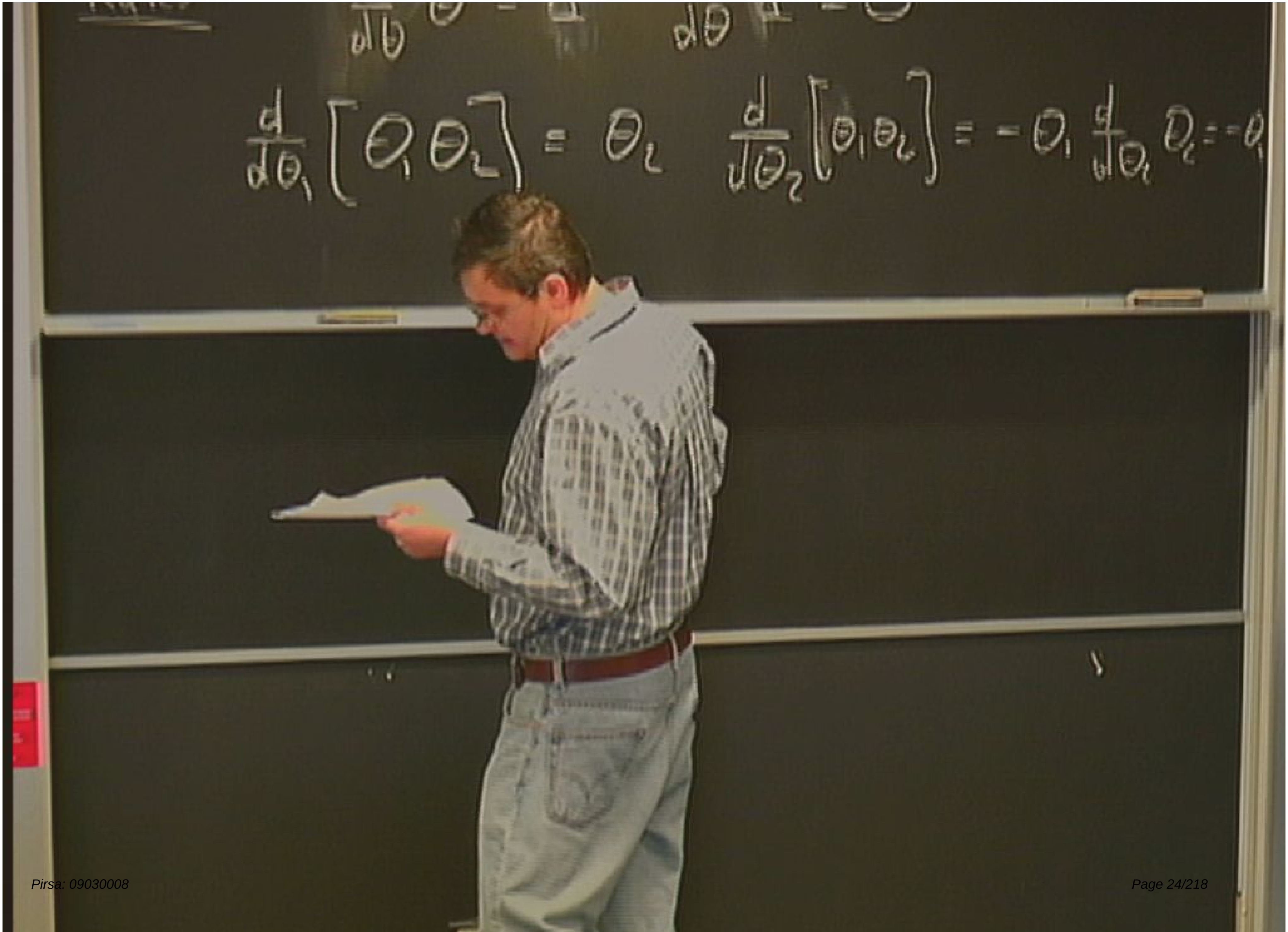
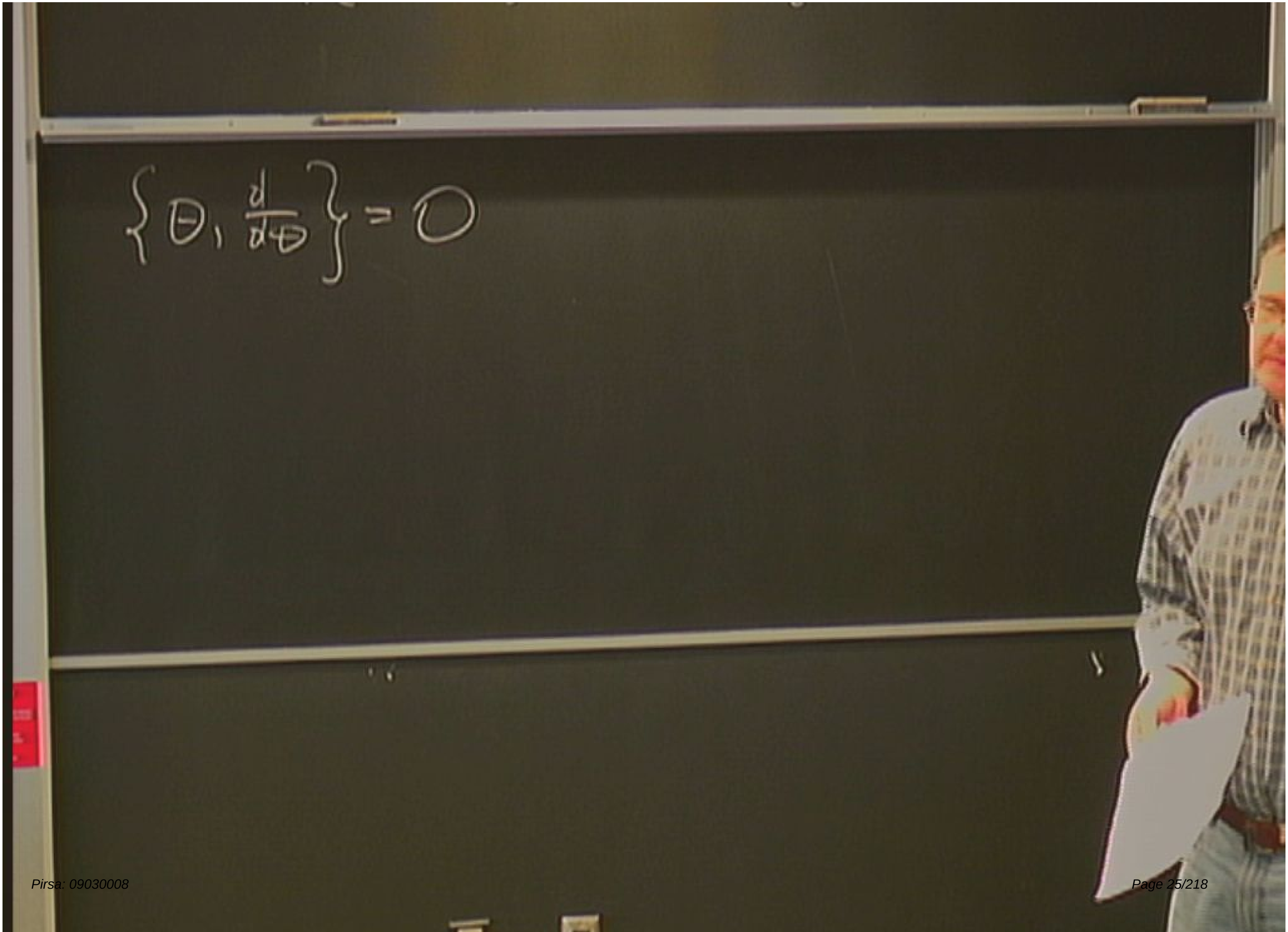

$f(\theta) = a + b\cdot\theta + \text{nothing}.$
arbitrary function.
Rules
$\frac{d}{d\theta}\theta$


function.
Rules
$\frac{d}{d\theta}\Theta = \Delta$
$\frac{d}{d\Theta}\Delta = 0$


$\frac{d}{d\theta_1}[\theta_1\theta_2] = \theta_2$
$\frac{d}{d\theta_2}[\theta_1\theta_2] = -\theta_1$


$\left\{\Theta, \frac{d}{d\Theta}\right\} = 0$

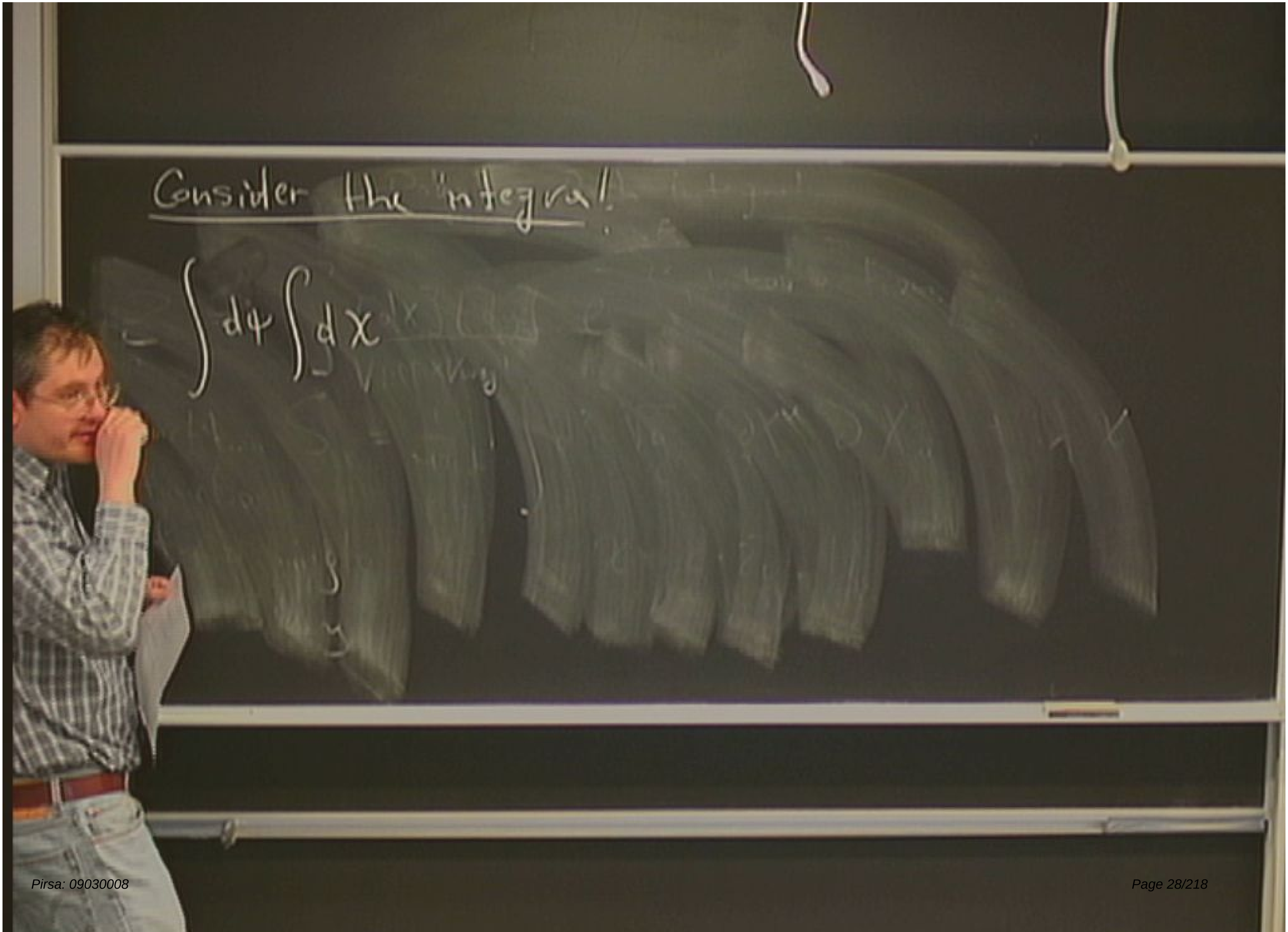
$$\left\{\Theta, \frac{d}{d\Theta}\right\} = 0$$

$$\int d\Theta \cdot 1 = 0 \qquad \int d\Theta \cdot \Theta = -\int \Theta \, d\Theta$$

$$\left\{\theta, \frac{d}{d\theta}\right\} = 0$$

$$\int d\theta \cdot \underline{1} = 0 \qquad \int d\theta \cdot \theta = -\int \theta \, d\theta = \underline{1}$$

Integration is the same as differentiation!


Consider the "integral"
∫dφ ∫dx

Consider the "integral"

$$\int d\psi \int dx \; e^{\lambda x \psi}$$

$$\psi x = -x\psi$$

Consider the 'integral'

$$\int d\psi \int dx \; e^{\lambda x \psi} = \int d\psi \, dx$$

$$\psi x = -x\psi$$

Consider the integral!

$$-\int d\psi \int dx \; e^{\lambda x \psi} = \int d\psi \, dx \, [1 + \lambda x \psi$$

$$\psi x = -x\psi$$

Consider the 'integral'

$$\int d\psi \int dx \, e^{\lambda x \psi} = \int d\psi \, dx \left[1 + \lambda x \psi + \frac{\lambda}{2!} (\lambda x \psi)^2 + \cdots \right.$$

$$\psi x = -x\psi$$

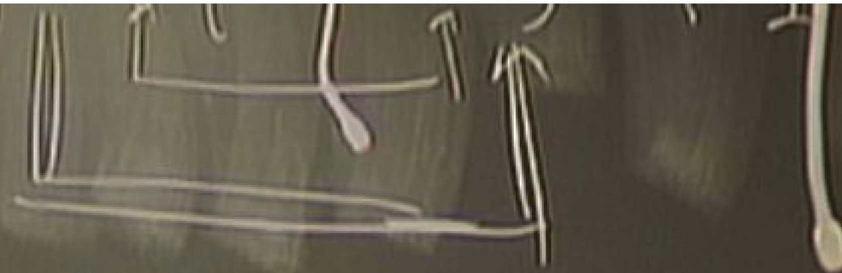
Consider the "integral"

$$\int d\psi \int dx\; e^{\lambda x \psi} = \int d\psi\, dx \left[1 + \lambda x \psi + \cancel{\frac{1}{2!}(\lambda x \psi)^2} + \cdots \right]$$

$= 0$, $x^2 = 0$

$$\psi x = - x \psi$$

$$= \int d\psi\, dx \left[1 + \lambda x \psi \right] = \lambda \cdot 1$$



$$\int_{-\infty}^{+\infty} dx \int_{-\infty}^{+\infty} dy \; e^{2\pi i \lambda \cdot xy}$$

$$xy = yx$$

$$\int_{-\infty}^{+\infty} dx \int_{-\infty}^{+\infty} dy \; e^{2\pi i \lambda \cdot xy} = \int_{-\infty}^{+\infty} dx$$

$$xy = yx$$

$$\int_{-\infty}^{+\infty} dx \int_{-\infty}^{+\infty} dy \; e^{2\pi i \lambda \cdot xy} = \int_{-\infty}^{+\infty} dx \; \delta(\lambda \cdot x) = \frac{1}{\lambda}$$

$$xy = yx$$

$$\int_{-\infty}^{+\infty} dx \int_{-\infty}^{+\infty} dy \; e^{2\pi i \lambda \cdot xy} = \int_{-\infty}^{+\infty} dx \; \delta(\lambda \cdot x) = \frac{1}{\lambda}$$

$$xy = yx$$

$$\boxed{\int dx\,dy \; e^{2\pi i \lambda xy} = \left[\int d\psi \, dx \; e^{\lambda x \psi} \right]^{-1}}$$

$$\beta'_{\alpha}{}^{n} = 0$$

$$\delta\sigma \cdot \beta' = \beta' \delta\sigma$$

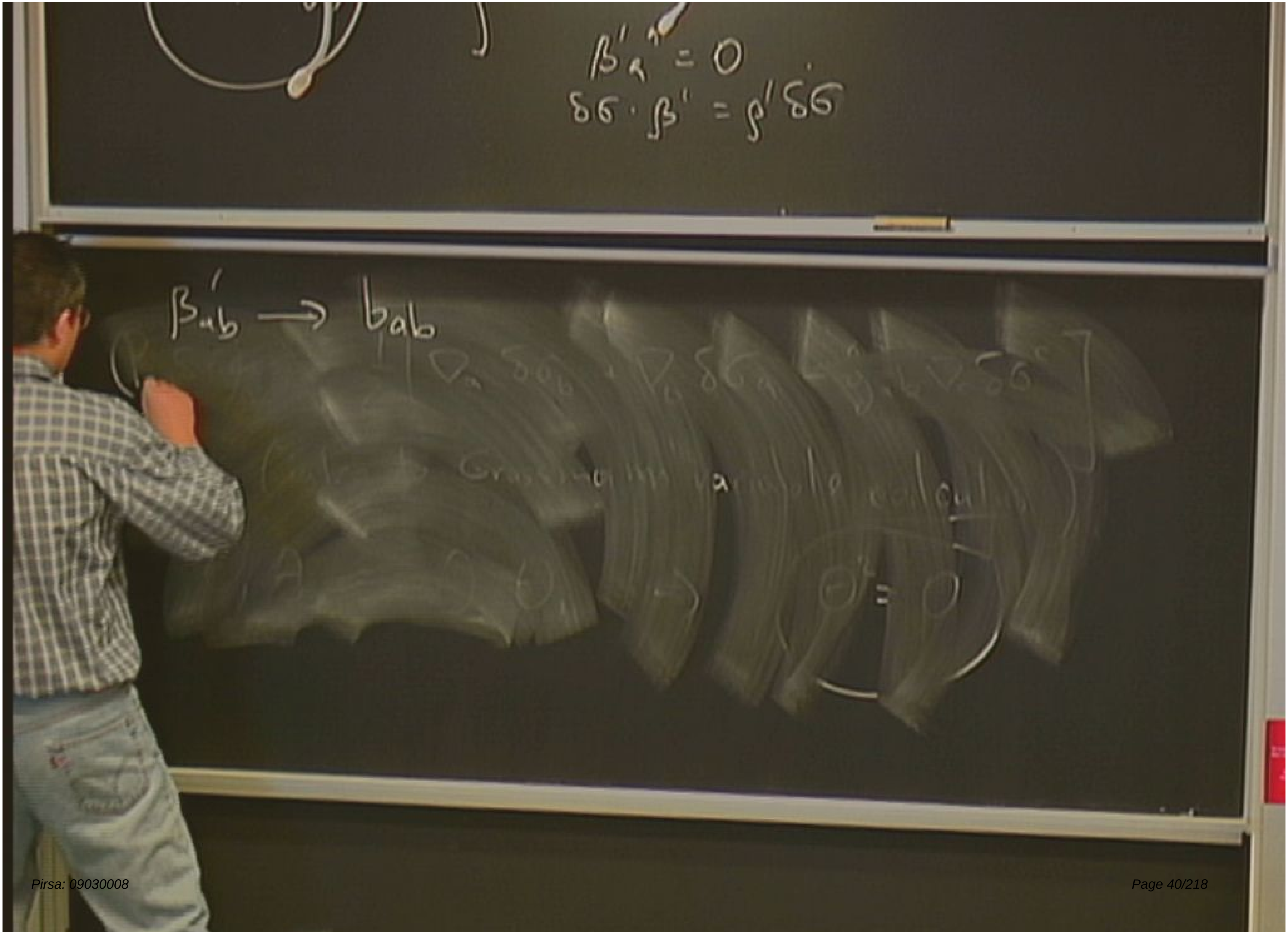
Aside (intro to Grassmann variable calculus)

$$\theta_1 \theta_2 = -\theta_2 \theta_1 \Rightarrow \theta^2 = 0$$

$$\int_{-\infty}^{+\infty} dx \int_{-\infty}^{+\infty} dy \; e^{2\pi i \lambda \cdot xy} = \int_{-\infty}^{+\infty} dx \; \delta(\lambda \cdot x) = \frac{1}{\lambda}$$

$$xy = yx$$

$$\boxed{\int dx\,dy\; e^{2\pi i \lambda xy} = \left[\int d\psi\, d\chi\; e^{\lambda \chi \psi}\right]^{-1}}$$


$\beta'_a{}^a = 0$
$\delta\sigma \cdot \beta' = \beta' \delta\sigma$
$\beta'_{ab} \longrightarrow b_{ab}$

$$\beta'_a = 0$$

$$\delta\sigma \cdot \beta' = \beta' \delta\sigma$$

$$\left.\begin{array}{l} \beta'_{ab} \longrightarrow b_{ab} \\ \delta\sigma^a \longrightarrow c^a \end{array}\right\} \text{Grassmann ghost fields}$$

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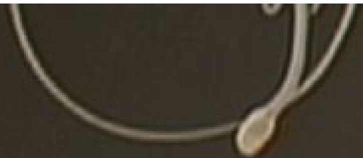

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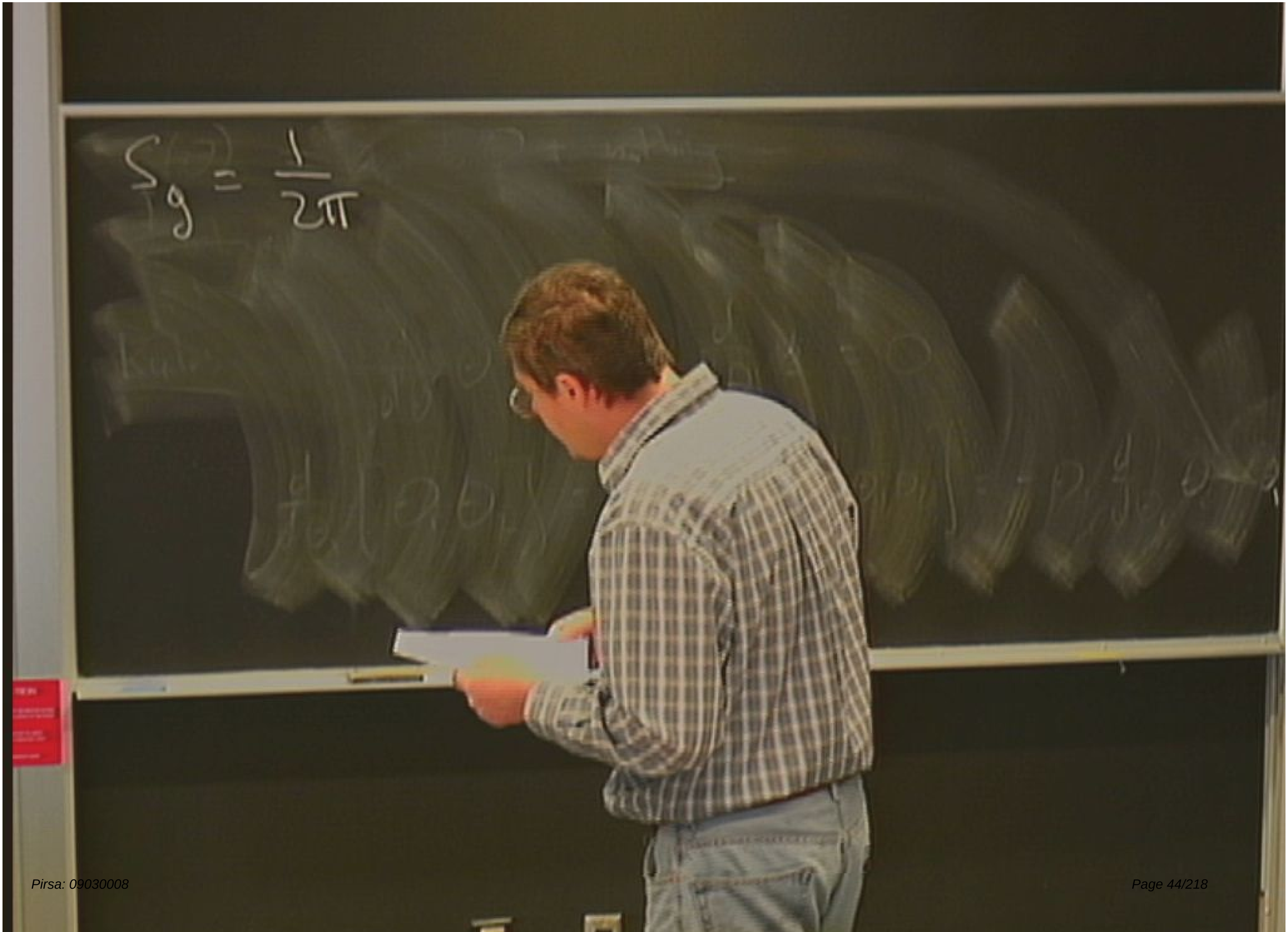
$$\Delta^{+1}_{FP}(\hat{g}) = \int [db\, dc]$$


$S_g = \frac{1}{2\pi}$

$$4\pi i \int d^2\sigma \sqrt{\hat g}\, \beta^{ab} (P_1 \delta\sigma)_{ab}$$

$$\downarrow$$

$$\beta'_{ab} (P_1 \delta\sigma)^{ab}$$

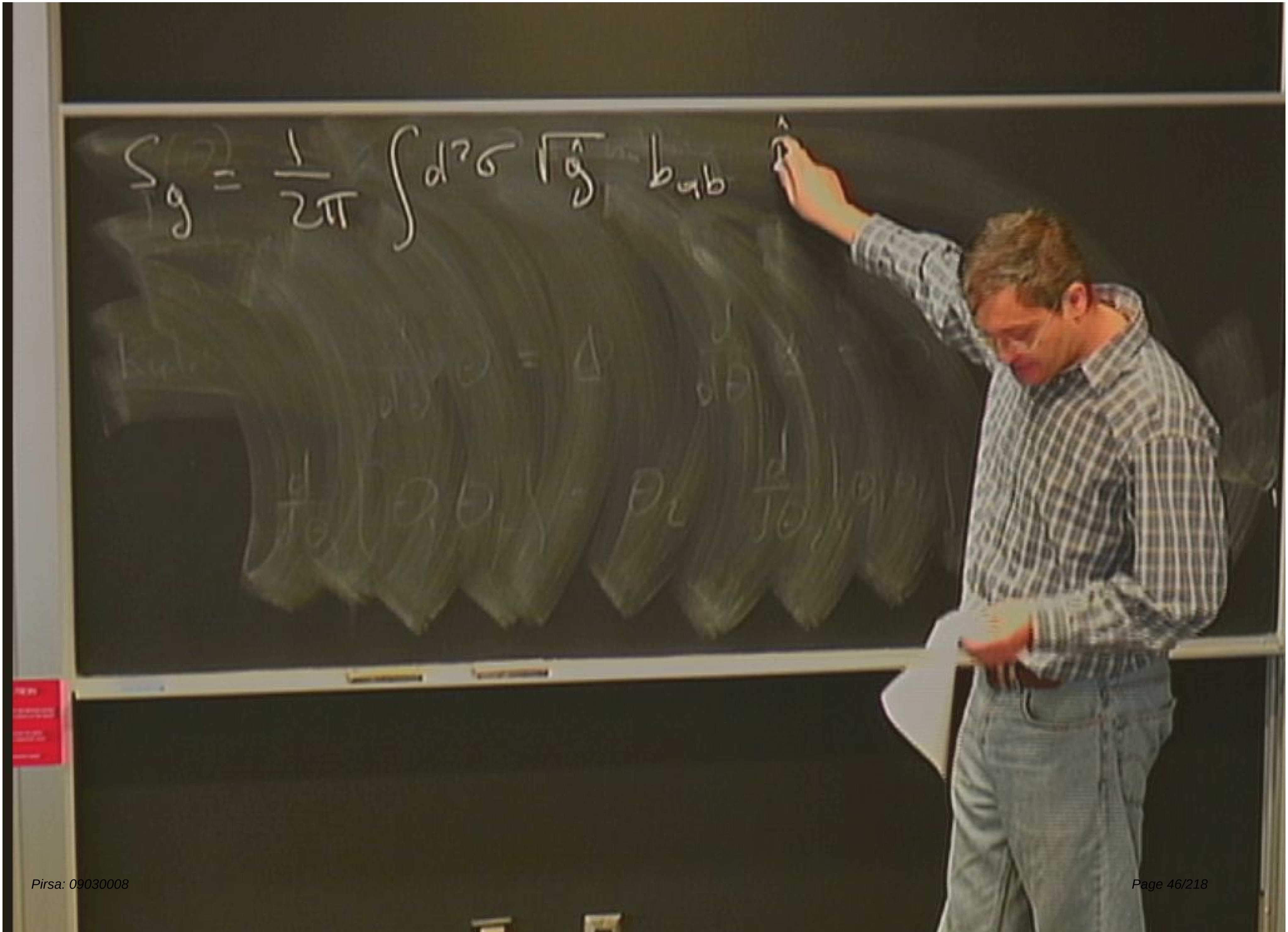
$$\Delta_{FP}^{-1}(\hat g) = \int [d\delta\sigma\, d\beta']\, e$$

$$\beta'_a{}^a = 0$$

$$\delta\sigma \cdot \beta' = \beta' \delta\sigma$$

$$\delta\sigma \longrightarrow c^a$$

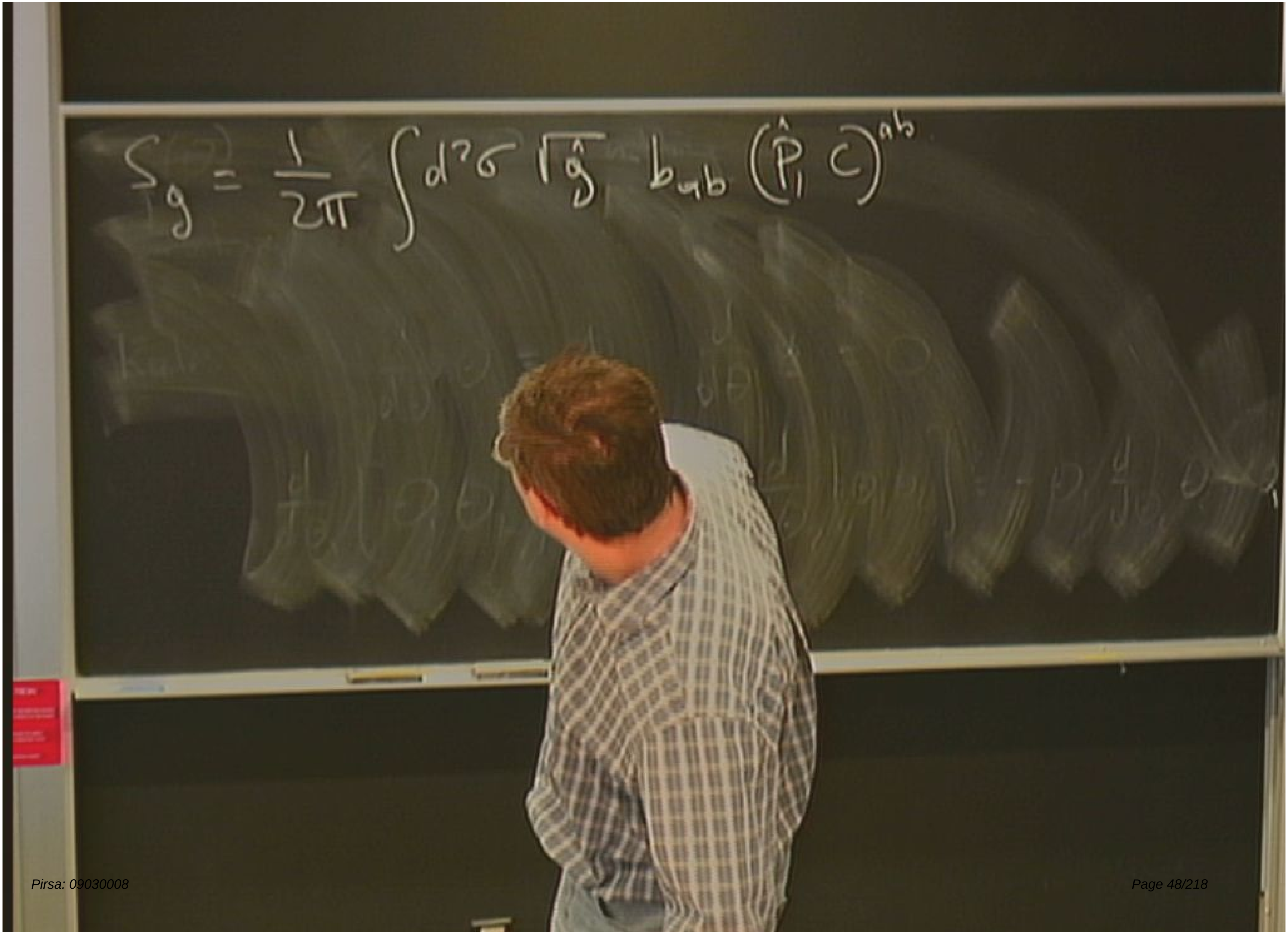
$$\Delta_{FP}^{+1}(\hat g) = \int [db\, dc]\, e^{-S_g}$$


$S_g = \frac{1}{2\pi} \int d^2\sigma \sqrt{\hat{g}}\, b_{ab}$

$$\delta\sigma \cdot \beta' = \beta' \delta\sigma$$

$$\left.\begin{array}{l} \beta'_{ab} \longrightarrow \# b_{ab} \\ \delta\sigma^a \longrightarrow \# c^a \end{array}\right\} \text{Grassmann ghost fields}$$

$$\Delta_{FP}^{+1}(\hat{g}) = \int [db\, dc]\, e^{-S_g}$$


$S_g = \frac{1}{2\pi}\int d^2\sigma \sqrt{\hat{g}}\; b_{ab} (\hat{P}_1 C)^{ab}$

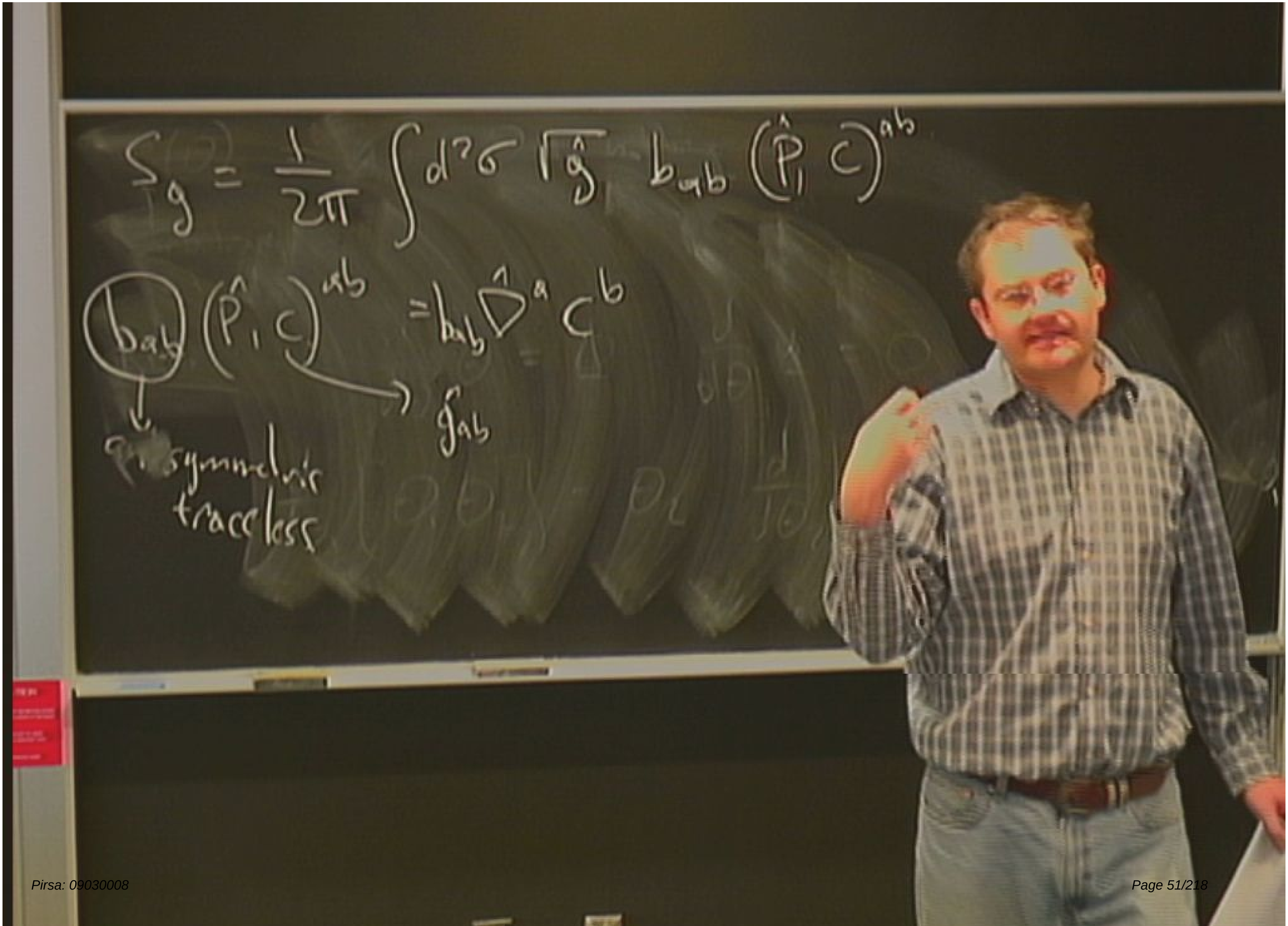
$$S_g = \frac{1}{2\pi}\int d^2\sigma \sqrt{\hat{g}}\; b_{ab}\,(\hat{P}_1 C)^{ab}$$

$$(\hat{P}_1 C)^{ab} = \hat{\nabla}^a C^b$$

$$S_g = \frac{1}{2\pi} \int d^2\sigma \sqrt{\hat{g}}\, b_{ab} (\hat{P}_1 C)^{ab}$$

$$b_{ab} (\hat{P}_1 C)^{ab} = b_{ab} \hat{\nabla}^a C^b$$

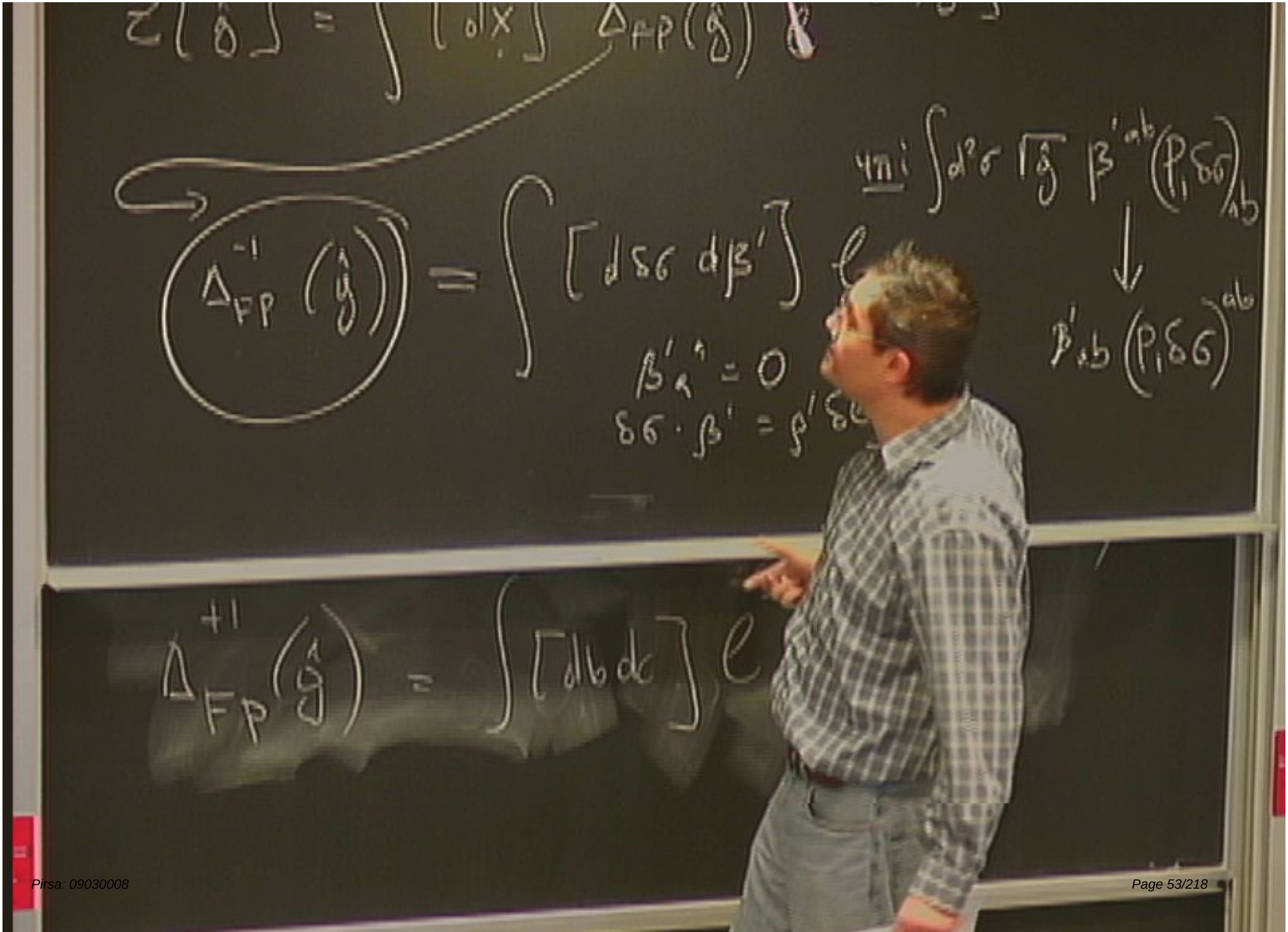
antisymmetric
traceless

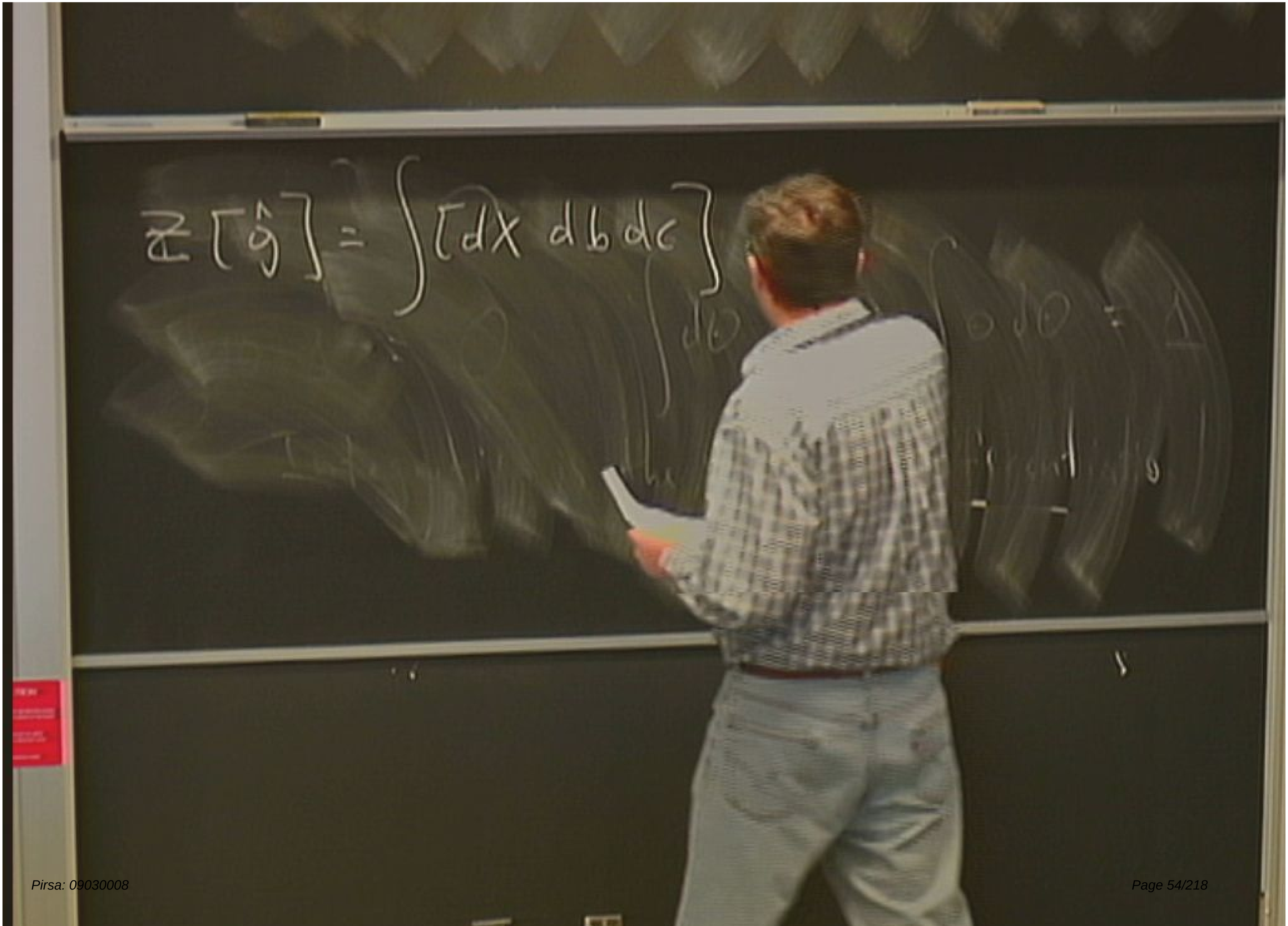

$S_g = \frac{1}{2\pi} \int d^2\sigma \sqrt{\hat{g}}\, b_{ab} (\hat{P}_1 C)^{ab}$
$b_{ab} (\hat{P}_1 C)^{ab} = b_{ab} \hat{\nabla}^a C^b$
$\hat{g}_{ab}$
symmetric
traceless

$$\delta\sigma \cdot \beta' = \beta' \delta\sigma$$

$$\left.\begin{array}{l} \beta'_{ab} \longrightarrow \# b_{ab} \\ \delta\sigma^a \longrightarrow \# c^a \end{array}\right\}$$ Grassmann ghost fields

$$\Delta^{+1}_{FP}(\hat{g}) = \int [db\,dc]\, e^{-S_g}$$


$Z[\hat{g}] = \int [dX] \, \Delta_{FP}(\hat{g})$
$4\pi i \int d^2\sigma \sqrt{\hat{g}} \, \beta'^{ab} (P_1 \delta\sigma)_{ab}$
$\Delta_{FP}^{-1}(\hat{g}) = \int [d\delta\sigma \, d\beta'] \, e$
$\beta'_{a}{}^{a} = 0$
$\delta\sigma \cdot \beta' = \beta' \delta\sigma$
$\beta'_{ab} (P_1 \delta\sigma)^{ab}$
$\Delta_{FP}^{+1}(\hat{g}) = \int [db \, dc] \, e$

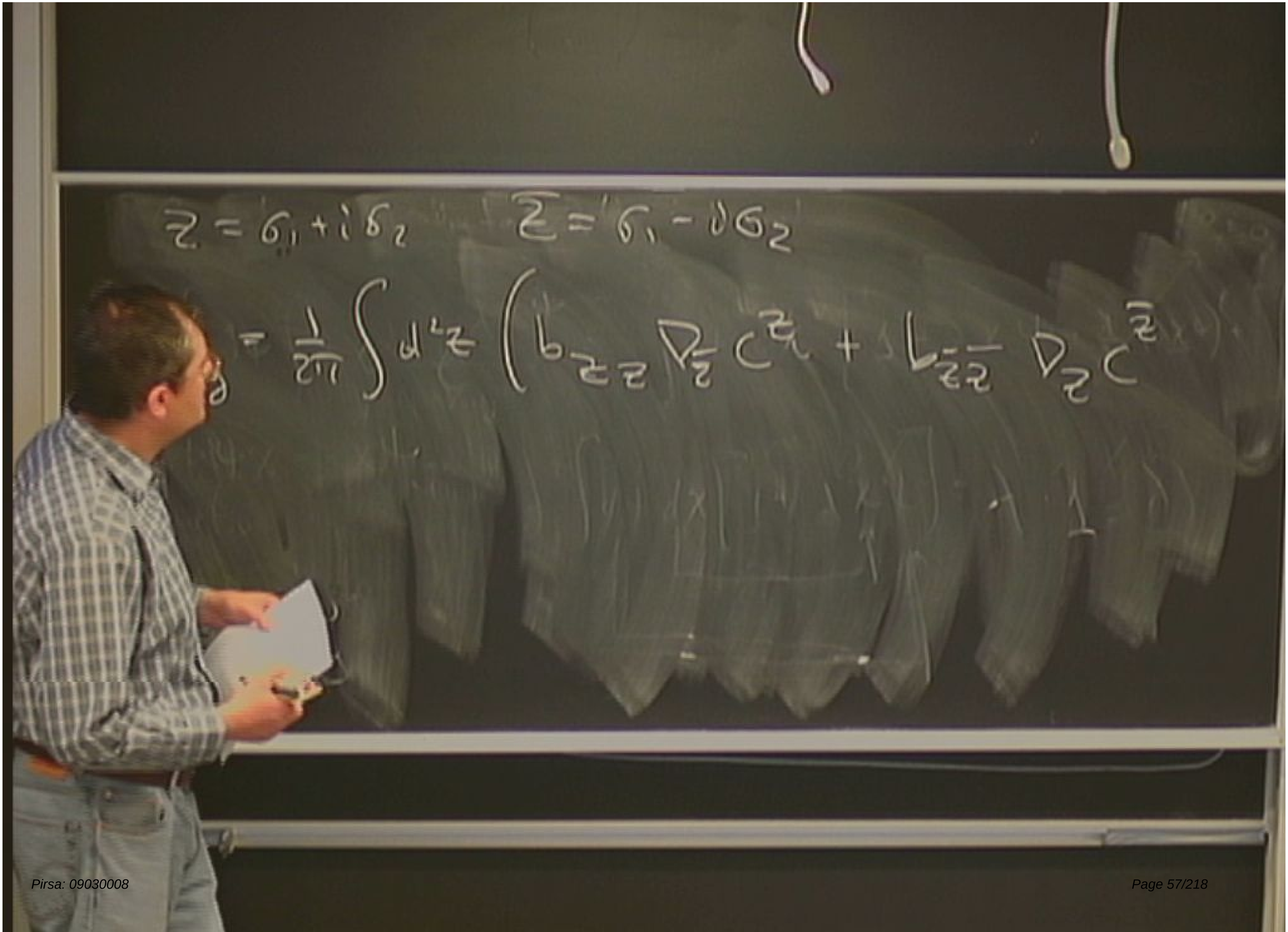

$Z[\hat{g}] = \int [dx\, db\, dc]$

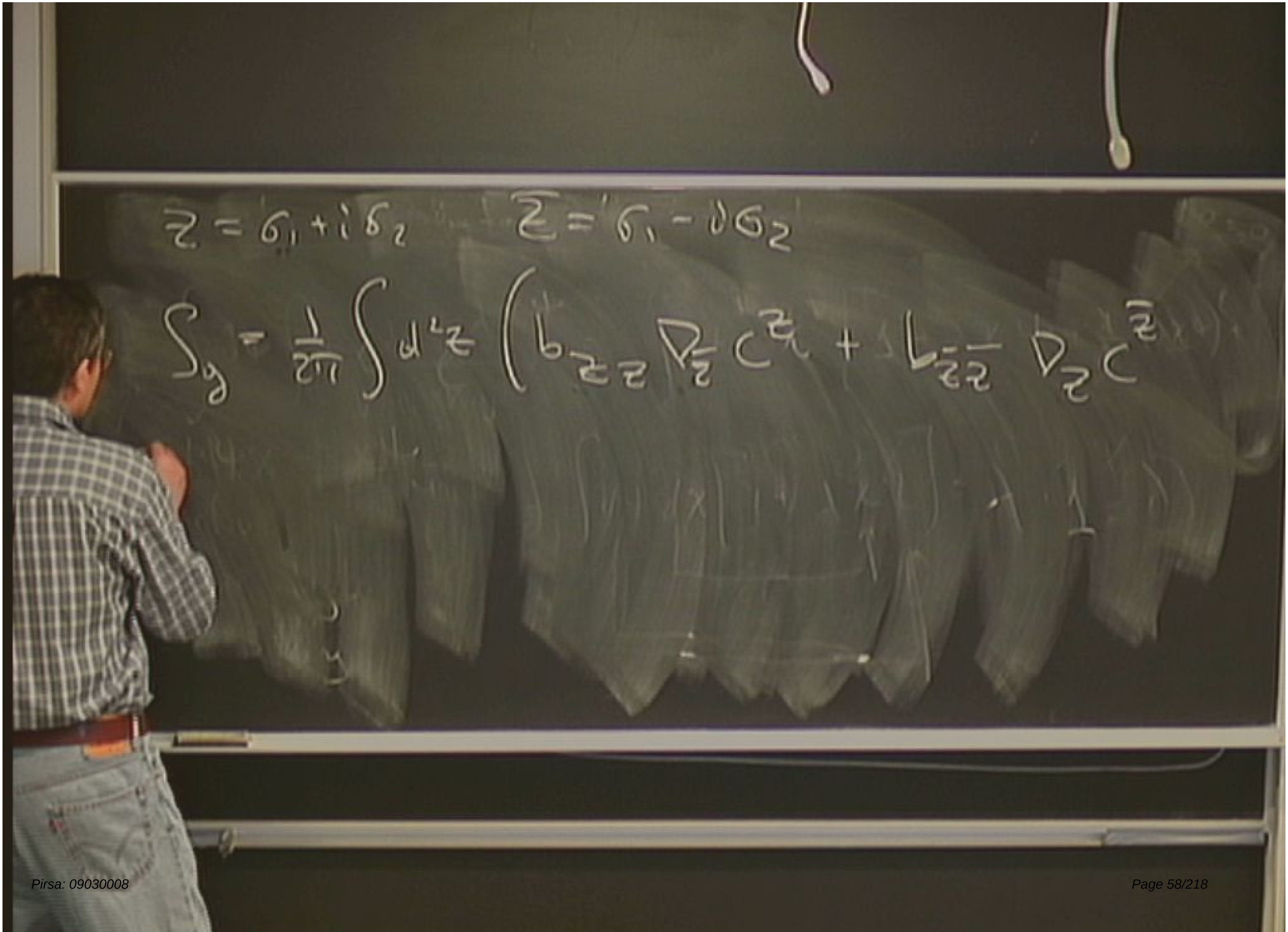
$$Z[\hat{g}] = \int [dX\, db\, dc]\, e^{-S_X - S_g}$$

$$Z[\hat{g}] = \int [dX\, db\, dc]\, e^{-S_X - S_g}$$

Let's go to "conformal gauge"

$$\hat{g}_{ab} = e^{2\omega} \delta_{ab}$$


$z = \sigma_1 + i\sigma_2 \qquad \bar{z} = \sigma_1 - i\sigma_2$
$= \frac{1}{2\pi}\int d^2z \left(b_{zz} \nabla_{\bar{z}} c^{z} + b_{\bar{z}\bar{z}} \nabla_{z} c^{\bar{z}}\right.$


$z = \sigma_1 + i\sigma_2 \qquad \bar{z} = \sigma_1 - i\sigma_2$
$S_g = \frac{1}{2\pi}\int d^2z \, \Big(b_{zz} \nabla_{\bar{z}} c^{z} + b_{\bar{z}\bar{z}} \nabla_{z} c^{\bar{z}}$

$z = \sigma_1 + i\sigma_2 \qquad \bar{z} = \sigma_1 - i\sigma_2$

$$S_g = \frac{1}{2\pi}\int d^2z \left(b_{zz} \nabla_{\bar{z}} c^{z} + b_{\bar{z}\bar{z}} \nabla_z c^{\bar{z}} \right.$$

$\Rightarrow$ In conformal gauge.

$z = \sigma_1 + i\sigma_2 \qquad \bar{z} = \sigma_1 - i\sigma_2$

$$S_g = \frac{1}{2\pi}\int d^2z \left(b_{zz} \nabla_{\bar{z}} c^{z} + b_{\bar{z}\bar{z}} \nabla_{z} c^{\bar{z}} \right.$$

$\Rightarrow$ In conformal gauge

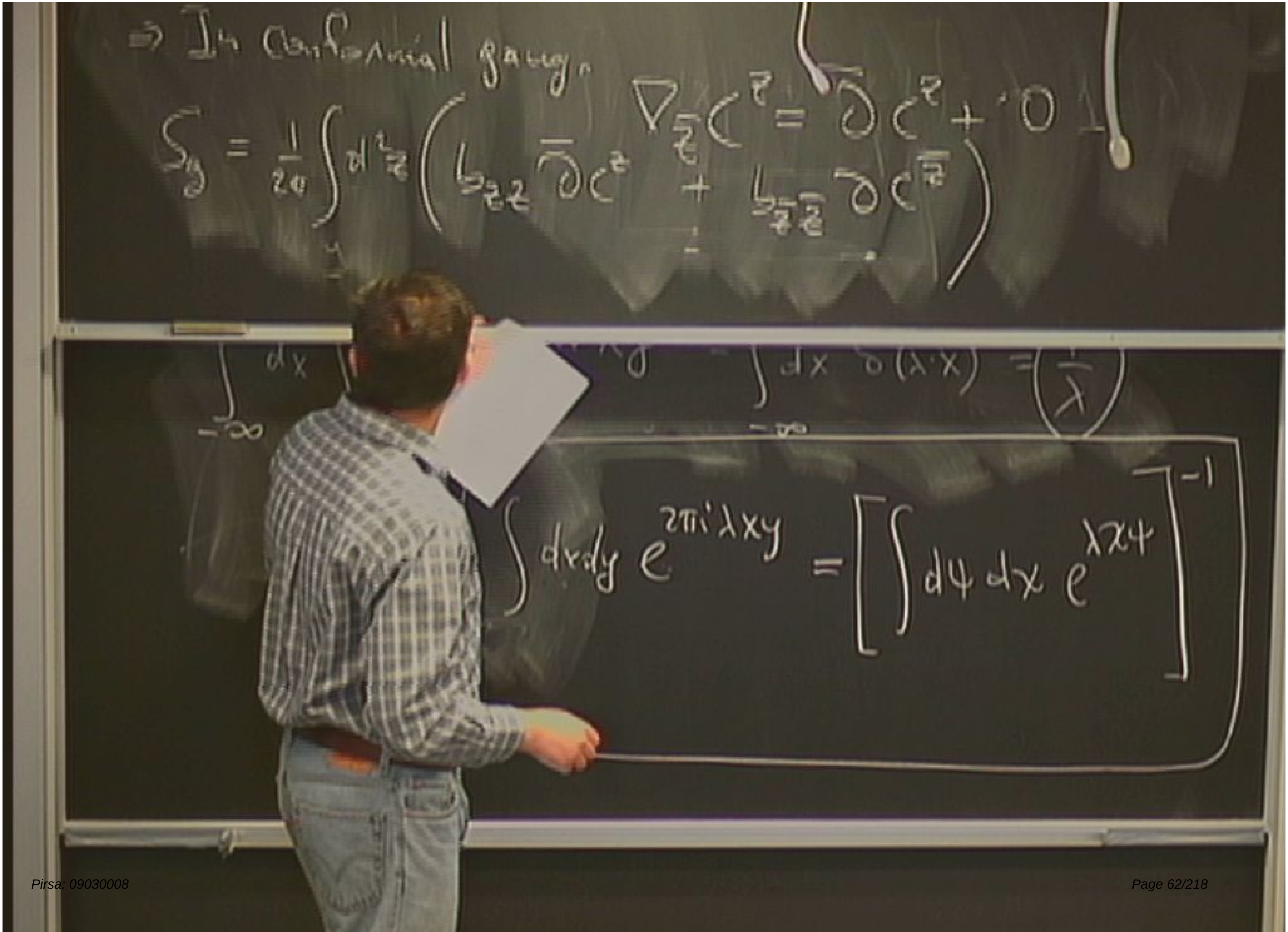
$$\nabla_{\bar{z}} c^{z} = \bar{\partial} c^{z} + \quad 1$$

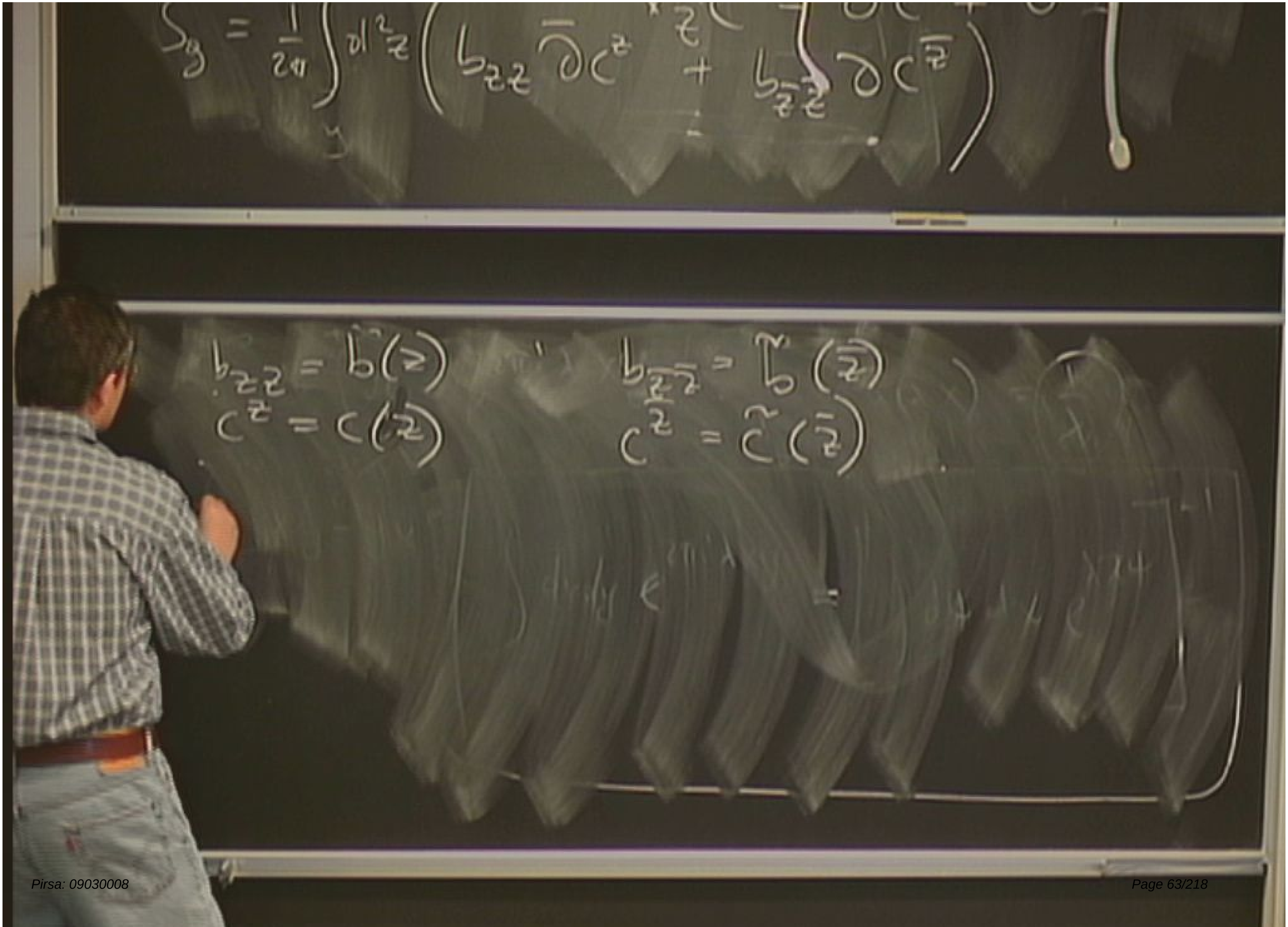
$z = \sigma_1 + i\sigma_2 \quad \bar{z} = \sigma_1 - i\sigma_2$

$$S_g = \frac{1}{2\pi}\int d^2z \left(b_{zz} \nabla_{\bar{z}} c^z + b_{\bar{z}\bar{z}} \nabla_z c^{\bar{z}} \right.$$

$\Rightarrow$ In conformal gauge,

$$\nabla_{\bar{z}} c^z = \bar{\partial} c^z + 0$$


⇒ In conformal gauge,
$S_g = \frac{1}{2\pi}\int d^2z \left(b_{zz}\bar{\partial}c^z + b_{\bar{z}\bar{z}}\partial c^{\bar{z}} \right)$
$\nabla_{\bar{z}} c^z = \bar{\partial} c^z + 0$
$\int dx\, \delta(\lambda \cdot x) = \frac{1}{\lambda}$
$\int dx\,dy\, e^{2\pi i \lambda x y} = \left[\int d\psi\, d\chi\, e^{\lambda \chi \psi} \right]^{-1}$


$S_g = \frac{1}{2\pi}\int d^2z \left(b_{zz}\,\bar\partial c^{z} + b_{\bar z\bar z}\,\partial c^{\bar z} \right)$
$b_{zz} = b(z)$
$c^{z} = c(z)$
$b_{\bar z\bar z} = \tilde b(\bar z)$
$c^{z} = \tilde c(\bar z)$

$$S_g = \frac{1}{2\pi}\int d^2z \left(b_{zz}\,\bar\partial c^z + b_{\bar z\bar z}\,\partial c^{\bar z} \right)$$

$$b_{zz} = b(z) \qquad b_{\bar z\bar z} = \tilde b(\bar z)$$

$$c^z = c(z) \qquad c^{\bar z} = \tilde c(\bar z)$$

$$S_g = \frac{1}{2\pi}\int d^2z \left[b\bar\partial c + \tilde b\,\partial\tilde c \right]$$

$$S_g = \frac{1}{2\pi}\int d^2z \left(b_{zz}\bar\partial c^z + b_{\bar z\bar z}\partial c^{\bar z} \right)$$

$b_{zz} = b(z)$ $\qquad$ $b_{\bar z\bar z} = \tilde b(\bar z)$

$c^z = c(z)$ $\qquad$ $c^{\bar z} = \tilde c(\bar z)$

$$S_g = \frac{1}{2\pi}\int d^2z \left[b\bar\partial c + \tilde b \partial \tilde c \right]$$

does not depend on w

$$S_g = \frac{1}{2\pi}\int d^2z \left(b_{zz}\bar{\partial} c^z + b_{\bar{z}\bar{z}}\partial c^{\bar{z}} \right)$$

$$b_{zz} = b(z) \qquad b_{\bar{z}\bar{z}} = \tilde{b}(\bar{z})$$

$$c^z = c(z) \qquad c^{\bar{z}} = \tilde{c}(\bar{z})$$

does not depend on w

$$S_g = \frac{1}{2\pi}\int d^2z \left[b\bar{\partial}c + \tilde{b}\partial\tilde{c} \right]$$

$$(h_b, h_c) = (\lambda, 1-\lambda)$$

$$S_g = \frac{1}{2\pi}\int d^2z \left(b_{zz}\bar{\partial} c^z + b_{\bar{z}\bar{z}}\partial c^{\bar{z}} \right)$$

$h_b = 2 \Leftrightarrow b_{zz} = b(z)$ $\qquad b_{\bar{z}\bar{z}} = \tilde{b}(\bar{z})$

$h_c = -1 = c^z = c(z)$ $\qquad c^{\bar{z}} = \tilde{c}(\bar{z})$

does not depend on w

$$S_g = \frac{1}{2\pi}\int d^2z \left[b\bar{\partial}c + \tilde{b}\partial\tilde{c} \right]$$

$$(h_b, h_c) = (\lambda, 1-\lambda) \Rightarrow$$

$$S_g = \frac{1}{2\pi}\int d^2z \left(b_{zz}\bar{\partial} c^z + b_{\bar{z}\bar{z}} \partial c^{\bar{z}} \right)$$

$h_b = 2 \Leftrightarrow b_{zz} = b(z)$, $b_{\bar{z}\bar{z}} = \tilde{b}(\bar{z})$

$h_c = -1 = c^z = c(z)$ $\quad c^{\bar{z}} = \tilde{c}(\bar{z})$

does not depend on w

$$S_g = \frac{1}{2\pi}\int d^2z \left[b\bar{\partial}c + \tilde{b}\partial\tilde{c} \right]$$

$\lambda = 2$

$(h_b, h_c) = (\lambda, 1-\lambda) \Rightarrow$

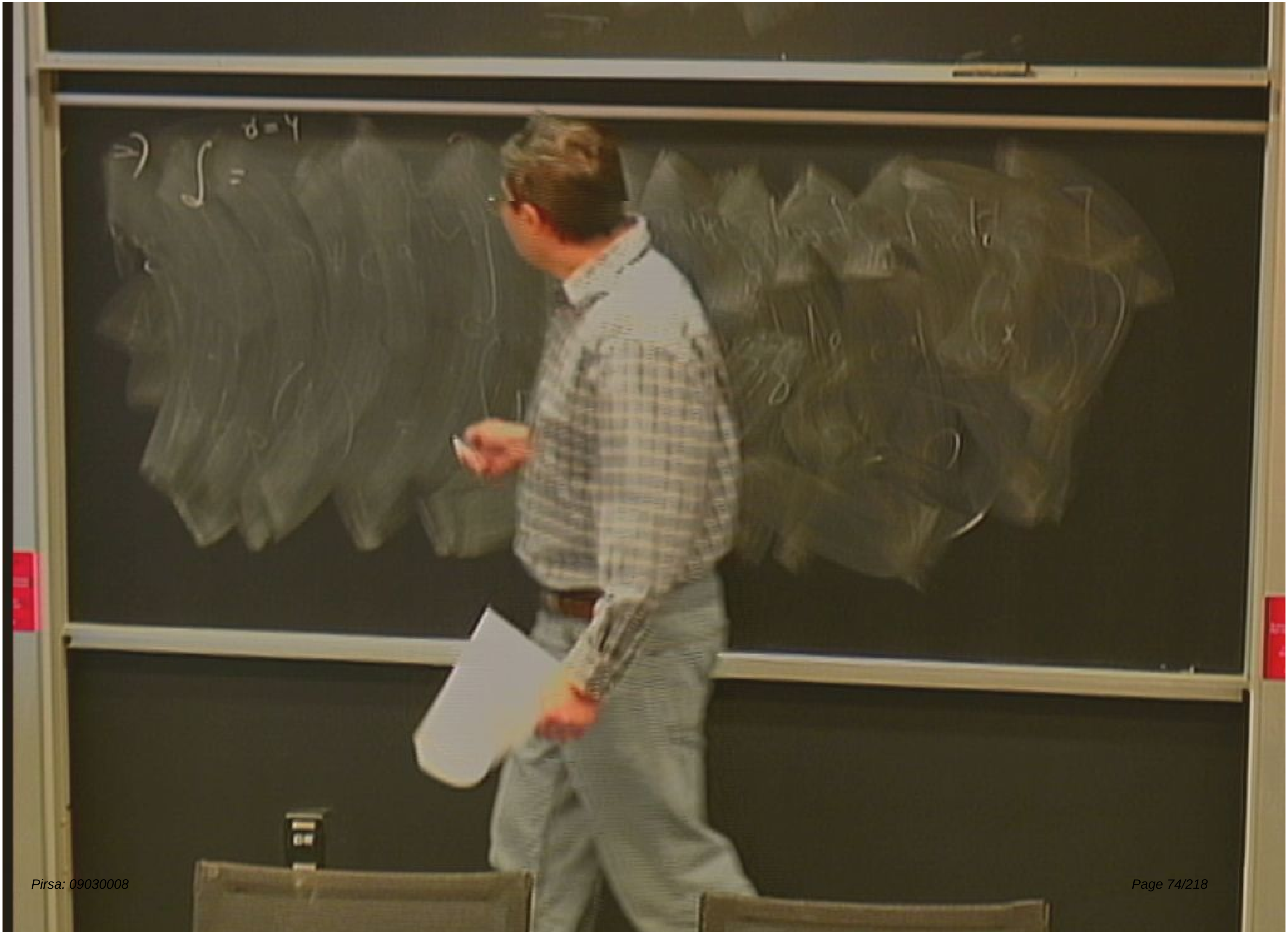
$$Z[\hat{g}] = Z[\bar{g}]$$

$$Z[\hat{g}] = Z[\tilde{g}]$$

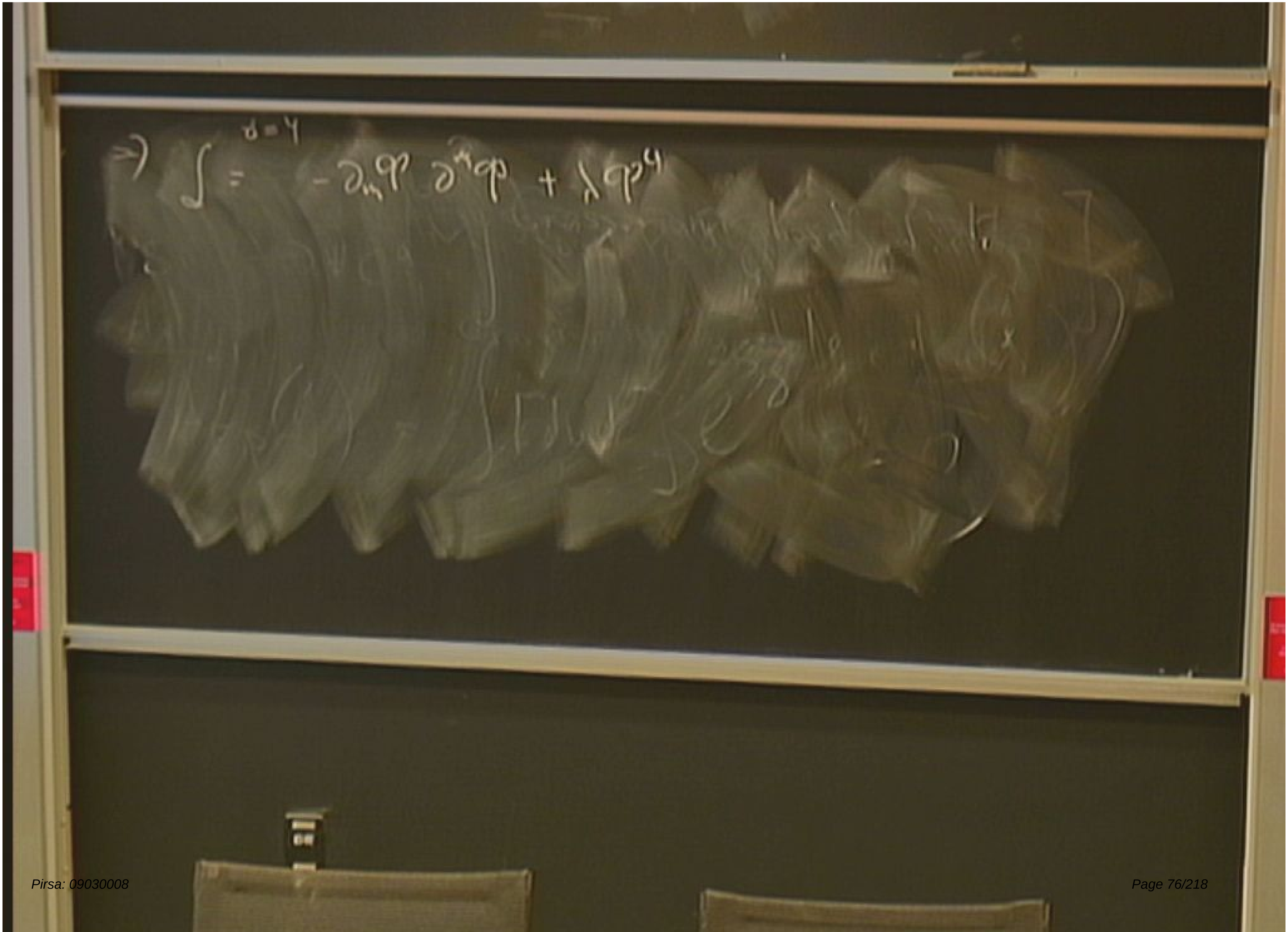
diffeo × Weyl is a classical symmetry of S_x, S_g

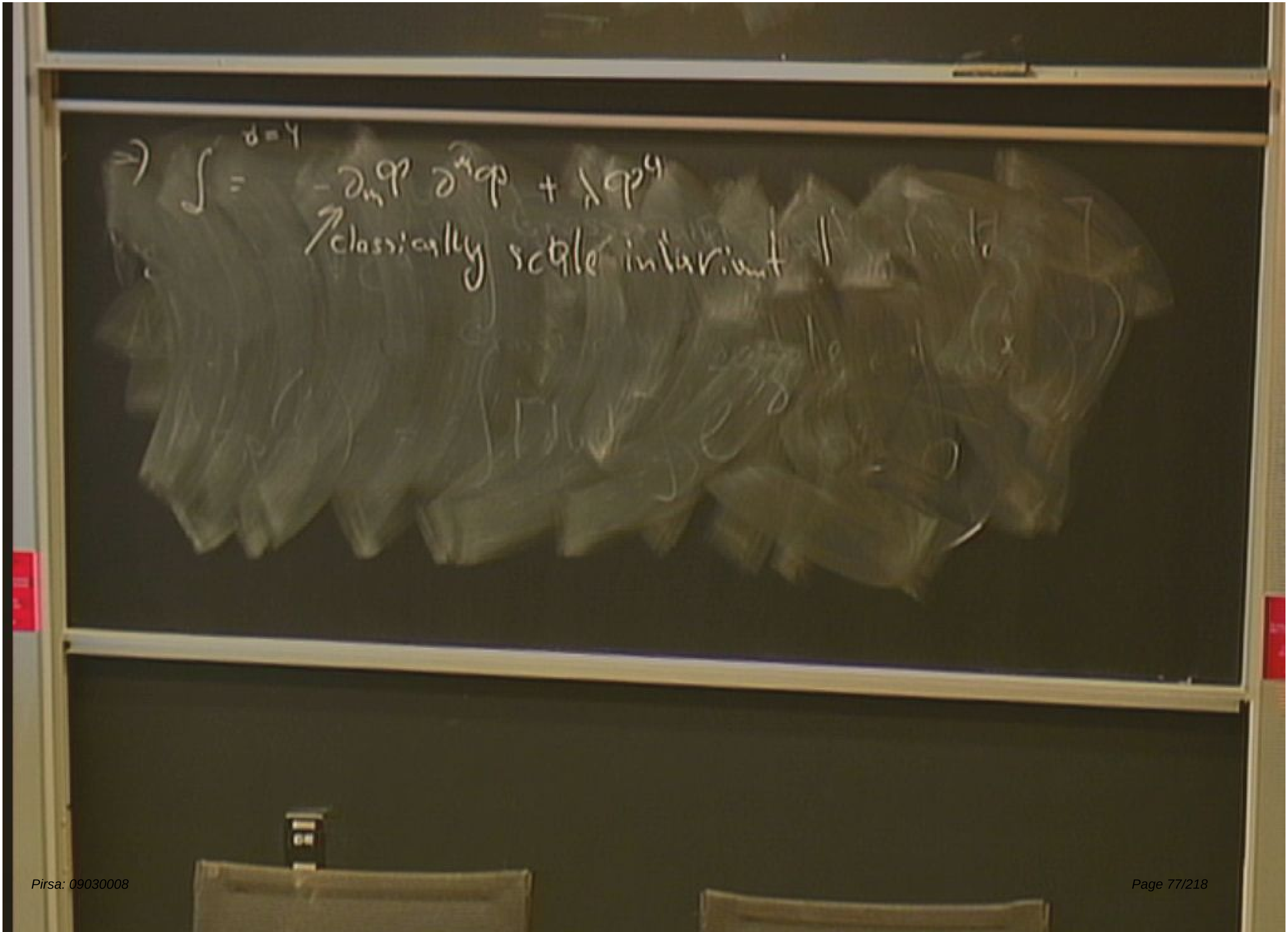
Is diffeo × Weyl a symmetry QM?

$$\Delta_{FP}(\hat{g}) = \int [db\, dc]\, e^{\cdots}$$

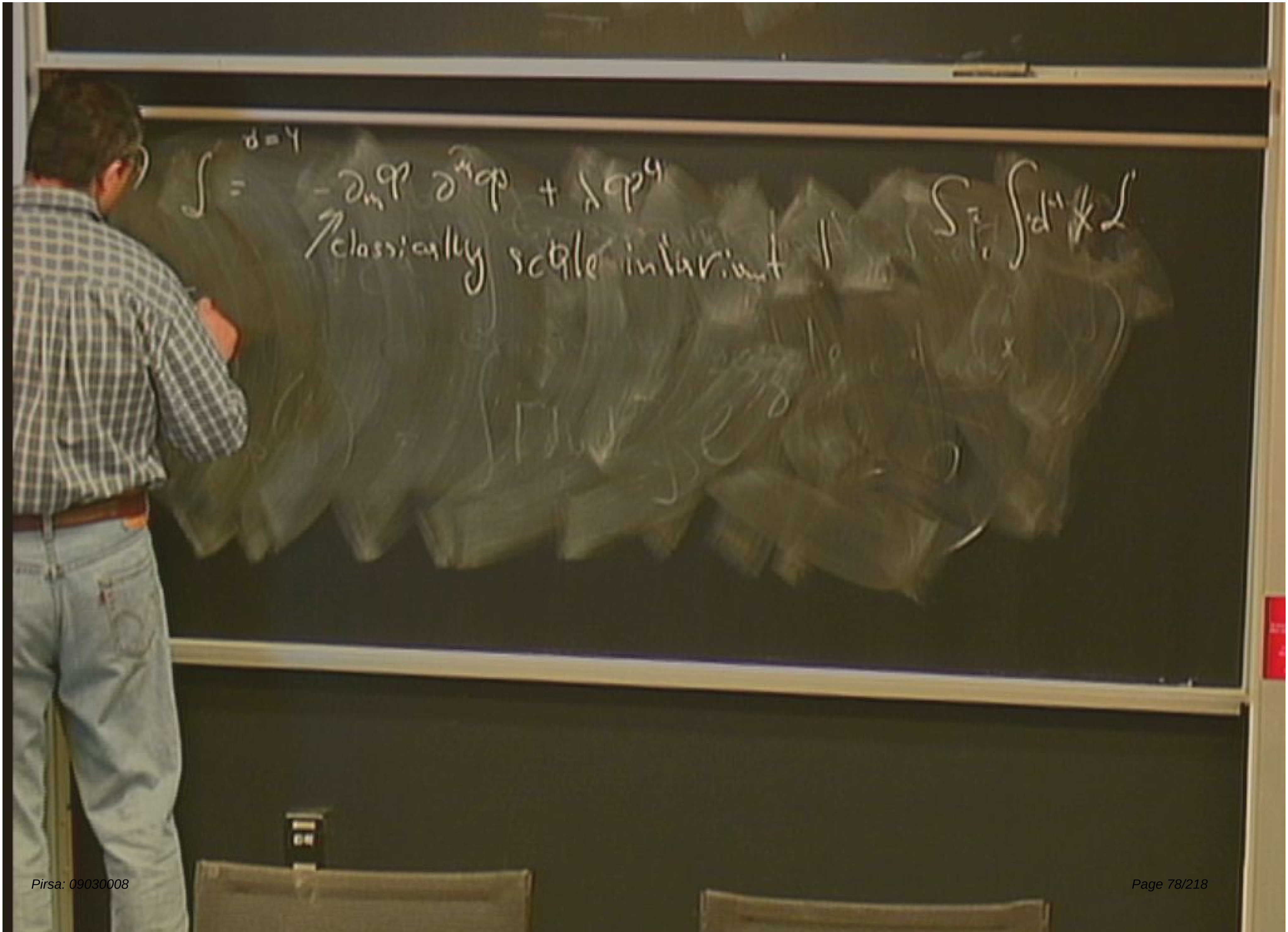



d = 4
⇒ L = - ∂μφ ∂μφ


d = 4
$\Rightarrow \mathcal{L} = -\partial_\mu \varphi \, \partial^\mu \varphi + \lambda \varphi^4$



d=4
⇒ ℒ = −∂_μφ ∂^μφ + λφ^4
classically scale invariant!


d=4
$\mathcal{L} = -\partial_m\varphi\,\partial^m\varphi + \lambda\varphi^4$
classically scale invariant!
$S \int d^4x\,\mathcal{L}$

$d=4$

$$\Rightarrow \mathcal{L} = -\partial_\mu \varphi \, \partial^\mu \varphi + \lambda \varphi^4 \qquad S = \int d^4x \, \mathcal{L}$$

classically scale invariant !

$[\varphi] = +1$

$[\partial] = +1$

$[\lambda] = 0$

$d=4$

$\Rightarrow \mathcal{L} = -\partial_\mu\varphi\,\partial^\mu\varphi + \lambda\varphi^4 \qquad S = \int d^4x\,\mathcal{L}$

classically scale invariant!

$[\varphi] = +1$

$[\partial] = +1$

$[\lambda] = 0$

$$\boxed{\frac{d}{d\ln\mu}\lambda = 0}$$

$d=4$

$\Rightarrow \mathcal{L} = -\partial_\mu\varphi\,\partial^\mu\varphi + \lambda\varphi^4$

classically scale invariant !

$S = \int d^4x\,\mathcal{L}$

$[\varphi] = +1$

$[\partial] = +1$

$[\lambda] = 0$

$$\boxed{\frac{d}{d\ln\mu}\lambda = \beta}$$

Non-abelian YM

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + g[A_\mu, A_\nu]$$

Non-abelian YM

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$[A] = +1$

$[F] = +2$

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$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + g[A_\mu, A_\nu]$$

$$[A] = +1 \qquad [\mathcal{L}] = 4 \Rightarrow [g] = 0$$

$$[F] = +2$$

$$\frac{d}{d\ln\mu} g^2(\mu) \neq 0$$

Classically

$$T^{ab} = \frac{4\pi}{\sqrt{g}} \frac{\delta S}{\delta g_{ab}}$$

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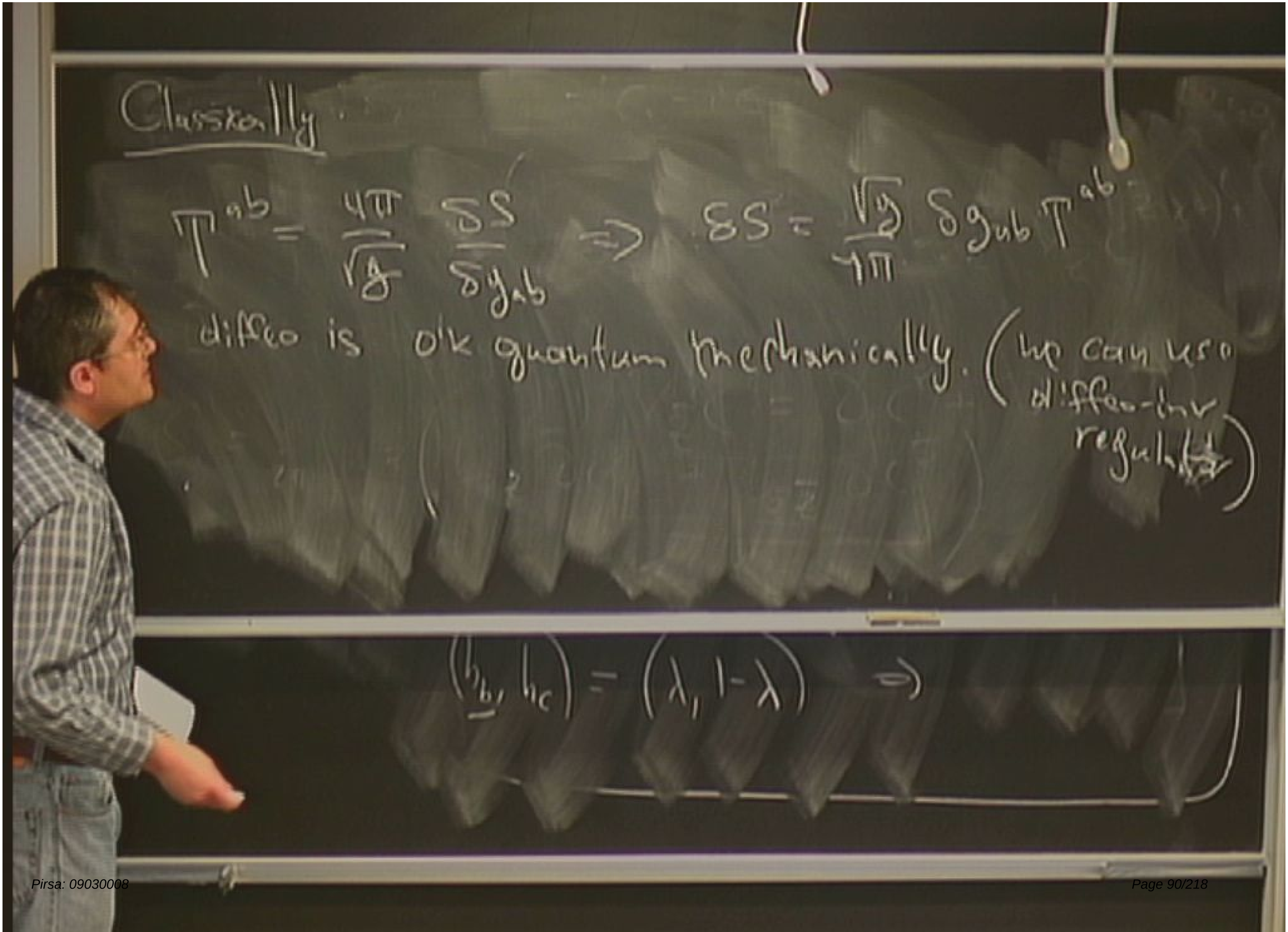
Classically

$$T^{ab} = \frac{4\pi}{\sqrt{g}} \frac{\delta S}{\delta g_{ab}} \Rightarrow \delta S = \frac{\sqrt{g}}{4\pi} \delta g_{ab} T^{ab}$$

$\hookrightarrow$


Classically
$T^{ab} = \frac{4\pi}{\sqrt{g}} \frac{\delta S}{\delta g_{ab}} \Rightarrow \delta S = \frac{\sqrt{g}}{4\pi} \delta g_{ab} T^{ab}$
diffeo is o'k quantum mechanically. (we can use diffeo-inv regulator)
$(h_b, h_c) = (\lambda, 1-\lambda) \Rightarrow$

↳ diffeo is ok quantum mechanically. (we can use diffeo-inv. regulator)

↳ only problem can come from Weyl transformation

$h_b = 2 \Leftrightarrow b_{zz} = b(z)$; $b_{\bar{z}\bar{z}} = \tilde{b}(\bar{z})$

$h_c = -1 = c^{z} = c(z)$; $c^{\bar{z}} = \tilde{c}(\bar{z})$

$$S_g = \frac{1}{2\pi}\int d^2z \left[b\bar{\partial}c + \tilde{b}\partial\tilde{c} \right]$$

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↳ only problem can come from Weyl transformation (regulator)

$\delta_W Z[\hat{g}]$

↳ $\hat{g} \to e^{2\omega} \hat{g}$

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↳ only problem can come from Weyl transformation (regulator)

$\delta_W Z[\hat{g}] \Leftrightarrow \delta_W \langle \dots \rangle_{\tilde{g}}$

↳ $\hat{g} \to e^{2\omega}\hat{g}$

↳ only problem can come from Weyl transformation (regulator)

$$\delta_W Z[\hat{g}] \Leftrightarrow \delta_W \langle \ldots \rangle_{\tilde{g}} = 0$$

↳ $\hat{g} \to e^{2\omega} \hat{g}$

if we want exact Weyl symmetry.

↳ only problem can come from Weyl transformation (regulator)

$\delta_W Z[\hat{g}] \Leftarrow \delta_W \langle \ldots \rangle_{\tilde{g}} = 0$

↳ $\hat{g} \to e^{2\omega} \hat{g}$

$\int [dX] \, e^{-S_X} \ldots$

if we want exact Weyl symmetry.

$$\delta_{\delta g} \langle \cdots \rangle_g = -\frac{1}{4\pi} \int d^2\sigma \sqrt{g}\, \delta g_{ab} \langle$$

$$\frac{\delta}{\delta g}$$

$$\delta_{\delta g} \langle \cdots \rangle_g = -\frac{1}{4\pi} \int d^2\sigma \sqrt{g}\, \delta g_{ab} \langle T^{ab} \cdots \rangle_g$$

$$\delta_{\delta g}\langle \cdots \rangle_g = -\frac{1}{4\pi}\int d^2\sigma \sqrt{g}\,\delta g_{ab}\,\langle T^{ab} \cdots \rangle_{\delta g}$$

insertions to prepare a state

$$\delta_{\delta g} \langle \cdots \rangle_g = -\frac{1}{4\pi} \int d^2\sigma \sqrt{g}\, \delta g_{ab} \langle T^{ab} \cdots \rangle_g$$

insertions to prepare a state

$\delta g =$

$$\delta_{\delta g}\langle \cdots \rangle_g = -\frac{1}{4\pi}\int d^2\sigma \sqrt{g}\, \delta g_{ab} \langle T^{ab} \cdots \rangle_g$$

$$\delta g = 2\omega \cdot g_{ab}$$

insertions to prepare a state

$$\delta_{\delta g} \langle \cdots \rangle_g = -\frac{1}{4\pi} \int d^2\sigma \sqrt{g}\, \delta g_{ab} \langle T^{ab} \cdots \rangle_g$$

Insertions to prepare a state

$$\delta g_{ab} = 2\delta\omega\, g_{ab}$$

$$\delta_W \langle \cdots \rangle =$$

$$\delta_{\delta g} \langle \cdots \rangle_g = -\frac{1}{4\pi} \int d^2\sigma \sqrt{g}\, \delta g_{ab} \langle T^{ab} \cdots \rangle_g$$

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$$\delta g_{ab} = 2\delta\omega\, g_{ab}$$

$$\delta_W \langle \cdots \rangle = -\frac{1}{2\pi} \int d^2\sigma \sqrt{g}\, \delta\omega \langle T^a{}_a \cdots \rangle_g$$

$$\delta_{\delta g}\langle \cdots \rangle_g = -\frac{1}{4\pi}\int d^2\sigma \sqrt{g}\, \delta g_{ab} \langle T^{ab} \cdots \rangle_g$$

Insertions to prepare a state

$$\delta g_{ab} = 2\delta\omega\, g_{ab}$$

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$$\delta_{\delta g} \langle \cdots \rangle_g = -\frac{1}{4\pi}\int d^2\sigma \sqrt{g}\, \delta g_{ab} \langle T^{ab} \cdots \rangle_g$$

Insertions to prepare a state

$$\delta g_{ab} = 2\,\delta\omega\, g_{ab}$$

$$0 = \delta_W \langle \cdots \rangle_g = -\frac{1}{2\pi}\int d^2\sigma \sqrt{g}\, \delta\omega \langle T^a{}_a \cdots \rangle_g$$

$$\boxed{T^a{}_a = 0}$$

$$\delta_{\delta g}\langle \cdots \rangle_g = -\frac{1}{4\pi}\int d^2\sigma \sqrt{g}\, \delta g_{ab} \langle T^{ab} \cdots \rangle_g$$

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as an operator statement.

$$\delta_{\delta g} \langle \cdots \rangle_g = -\frac{1}{4\pi} \int d^2\sigma \sqrt{g}\, \delta g_{ab} \langle T^{ab} \cdots \rangle_g$$

Insertions to prepare a state

$$\delta g_{ab} = 2\delta\omega\, g_{ab}$$

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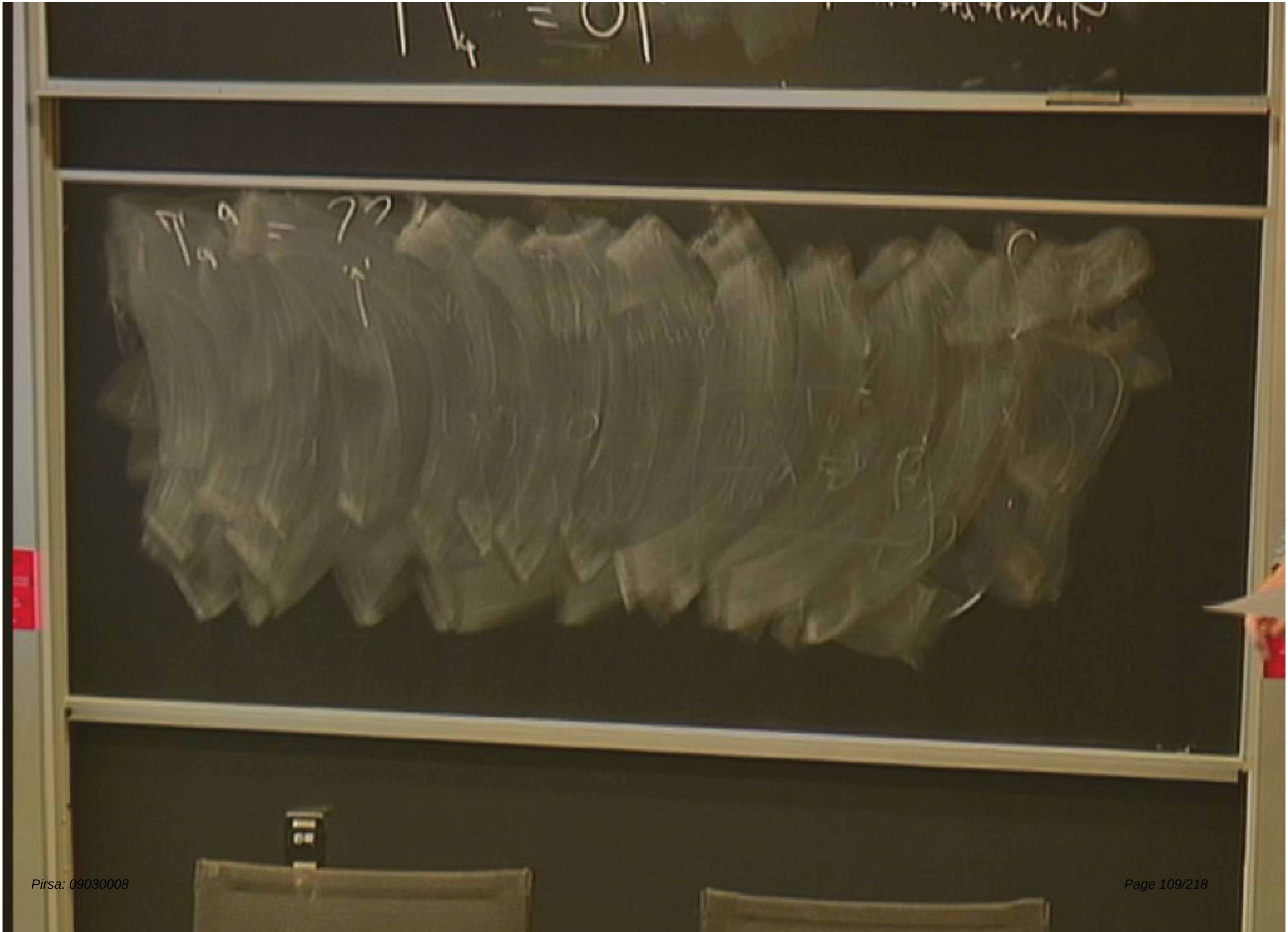
$$\boxed{\underbrace{T^a{}_a}_{\text{LHS}} = \underbrace{0}_{\text{RHS}}}$$

as an operator statement.



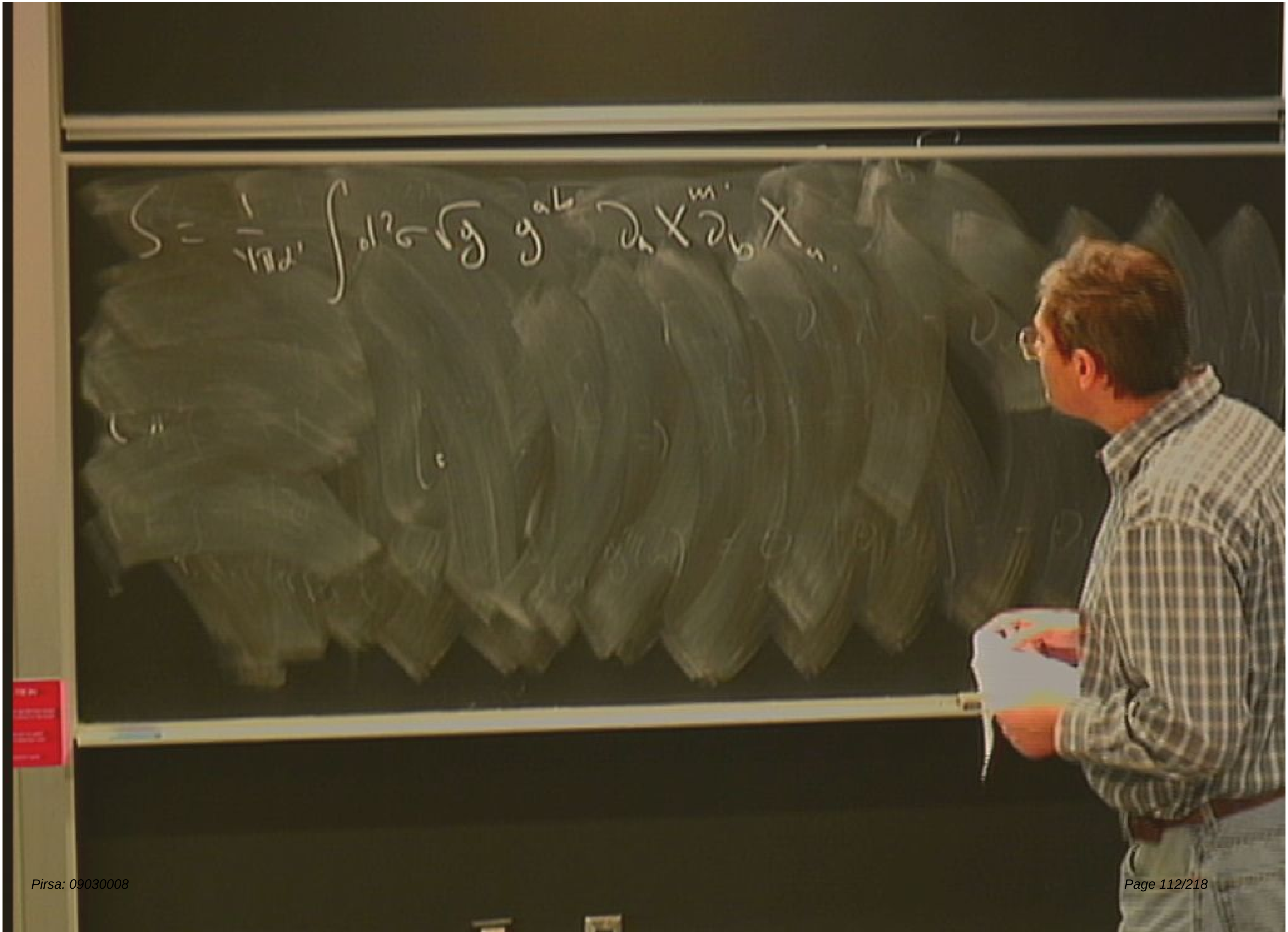
$$\Pi_{kp} = 0$$

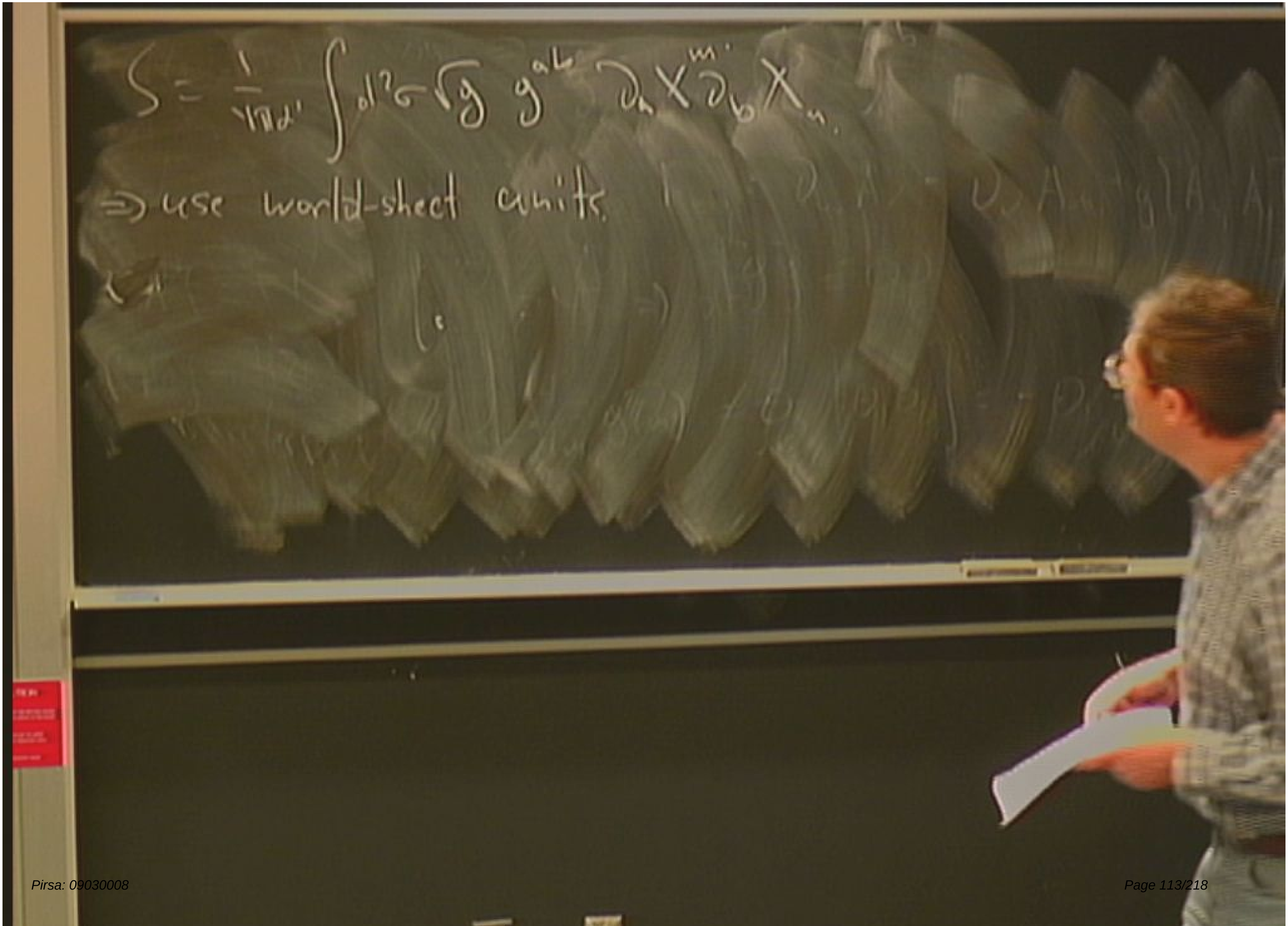
$$\Pi_a{}^a = ??$$

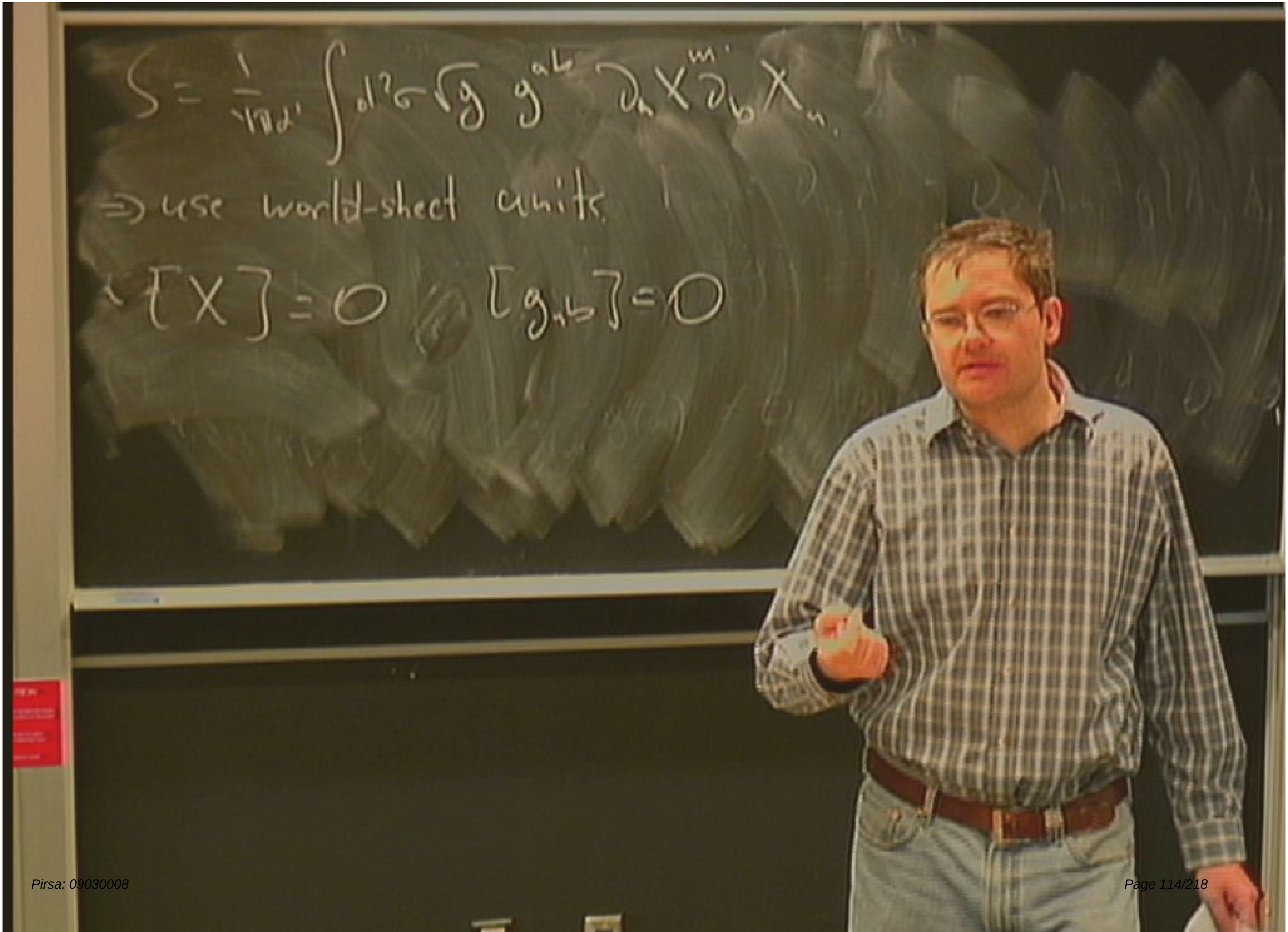
$\hat{T}$ must be invariant under world-sheet diffeo

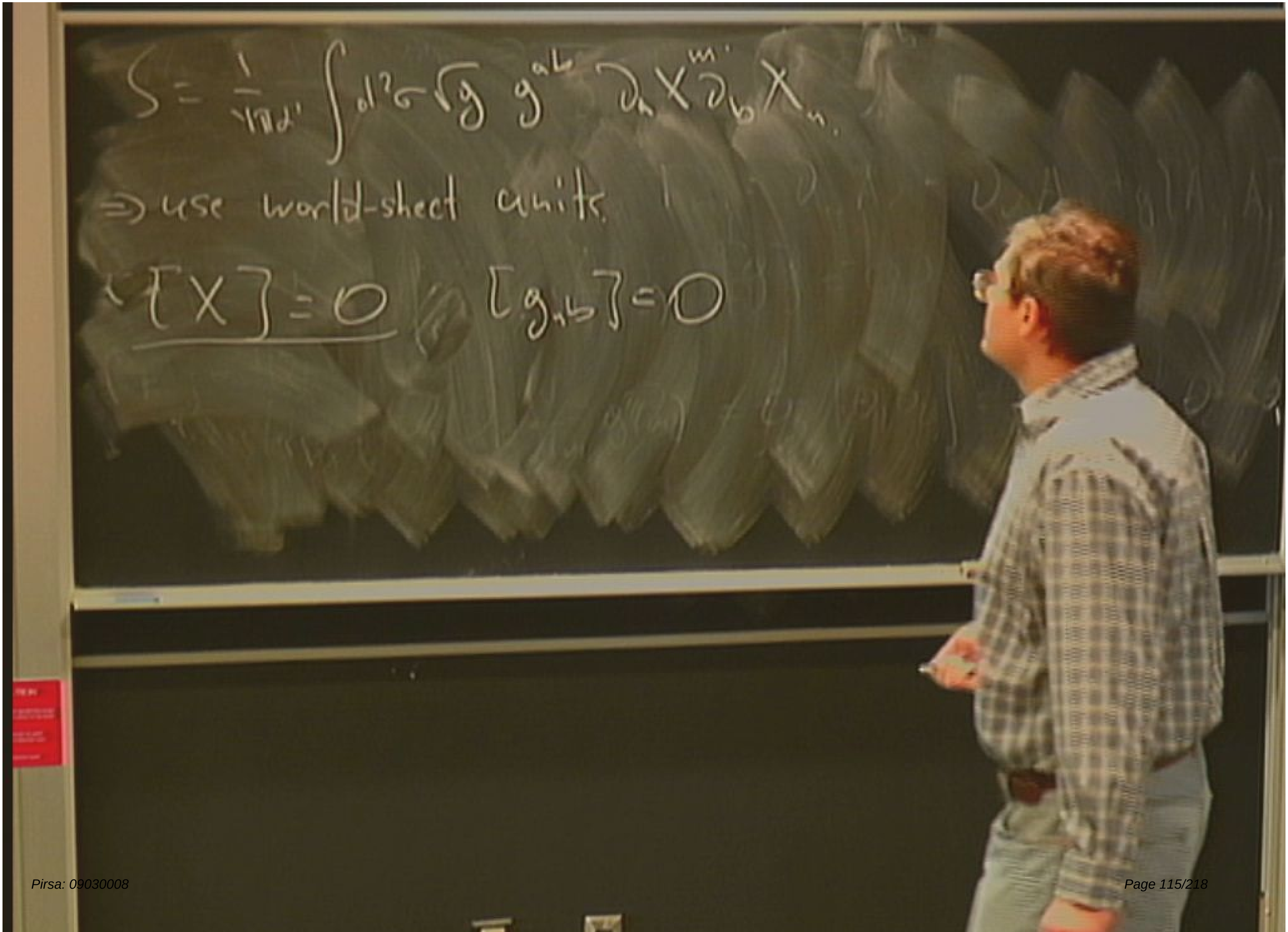
$T_a{}^a = ??$

- $\hat{T}$ must be invariant under world-sheet diffeo
- scaling dims on LHS = scaling dim on RHS


$S = \frac{1}{4\pi\alpha'} \int d^2\sigma \sqrt{g}\, g^{ab}\, \partial_a X^m \partial_b X_m$


$S = \frac{1}{4\pi\alpha'}\int d^2\sigma \sqrt{g}\, g^{ab}\, \partial_a X^m \partial_b X_m$
$\Rightarrow$ use world-sheet units


S = 1/4πα' ∫ d²σ √g g^ab ∂_a X^m ∂_b X_m
⇒ use world-sheet units
[X] = 0   [g_ab] = 0


S = 1/(4πα') ∫ d²σ √g g^ab ∂_a X^μ ∂_b X_μ
⇒ use world-sheet units
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$$S = \frac{1}{4\pi\alpha'} \int d^2\sigma \sqrt{g}\, g^{ab}\, \partial_a X^m \partial_b X_m$$

$\Rightarrow$ use world-sheet units

$[X] = 0 \qquad [g_{ab}] = 0 \qquad [\sigma_1] = [\sigma_2] = -1$

$$S = \frac{1}{4\pi\alpha'} \int d^2\sigma \sqrt{g}\, g^{ab}\, \partial_a X^{\mu}\, \partial_b X_{\mu}$$

$\Rightarrow$ use world-sheet units

$$\underline{[X] = 0} \qquad [g_{ab}] = 0 \qquad [\sigma_1] = [\sigma_2] = -1$$

$$[\partial_a] = +1$$

$$S = \frac{1}{4\pi\alpha'} \int d^2\sigma \sqrt{g}\, g^{ab}\, \partial_a X^m \partial_b X_m$$

$\Rightarrow$ use world-sheet units

$$\underline{[X] = 0} \qquad [g_{ab}] = 0 \qquad [\sigma_1] = [\sigma_2] = -1$$

$$[\partial_a] = +1$$

$$[d\sigma] = -1$$

$$\underbrace{\left. T \right|_{ku}}_{\text{LHS}} = \underbrace{0}_{\text{RHS}}$$

as an operator statement

$T_a{}^a = ??$

- "T" must be invariant under world-sheet diffeo
- scaling dims on LHS = scaling dim on RHS

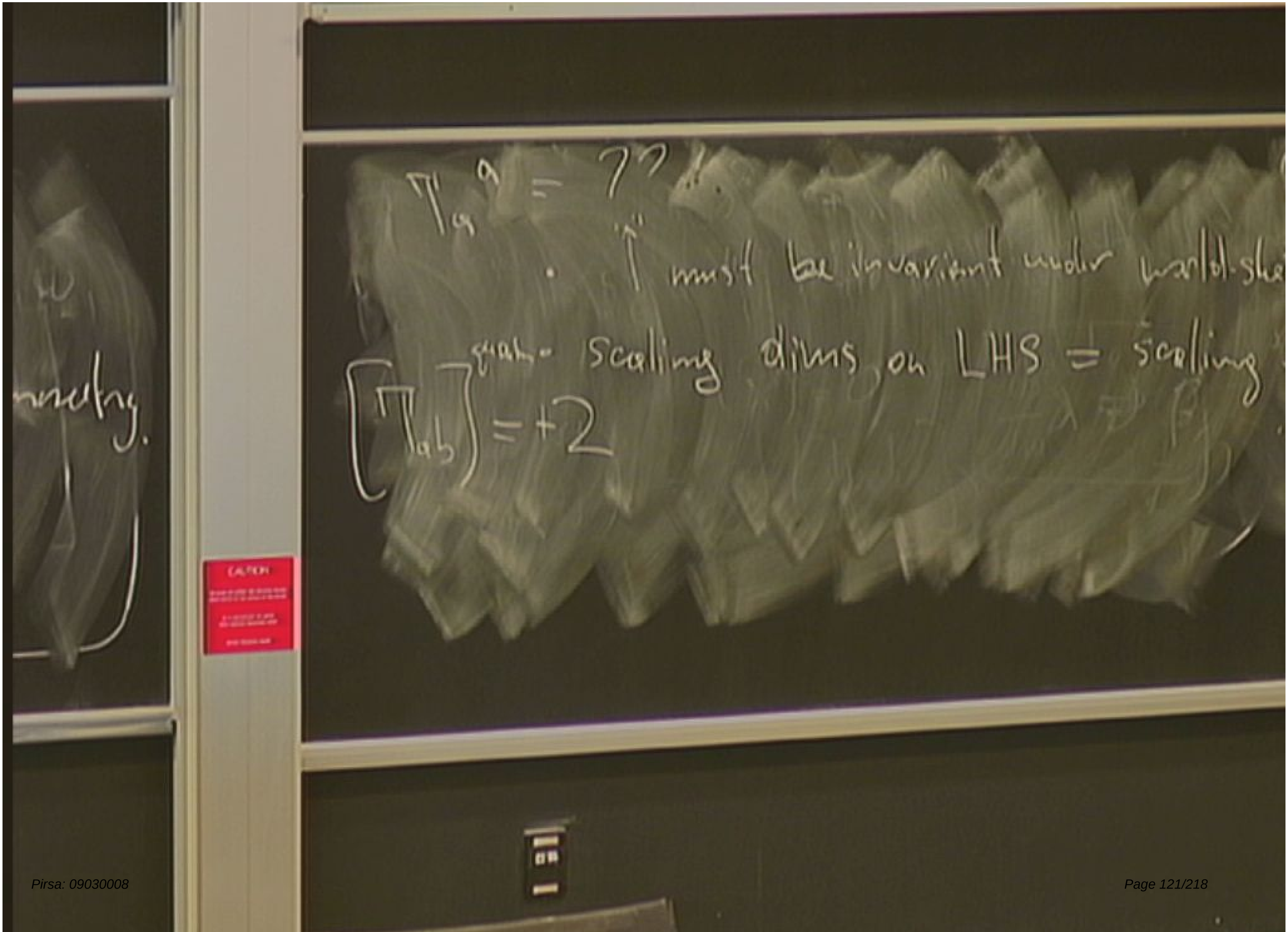
$[T_{ab}] =$

$$\boxed{\begin{matrix} \text{LHS} & & \text{RHS} \\ T_{\phi}{}^{a} & = & 0 \end{matrix}}$$ as an operator statement.

$T_a{}^a = ??$

- "T" must be invariant under world-sheet diffeo
- scaling dims on LHS = scaling dim on RHS

$[T_{ab}] =$



$T_a^{\ a}$ = ??
• T must be invariant under world-she
quantum scaling dims on LHS = scaling
$[T_{ab}] = +2$
mmetry.

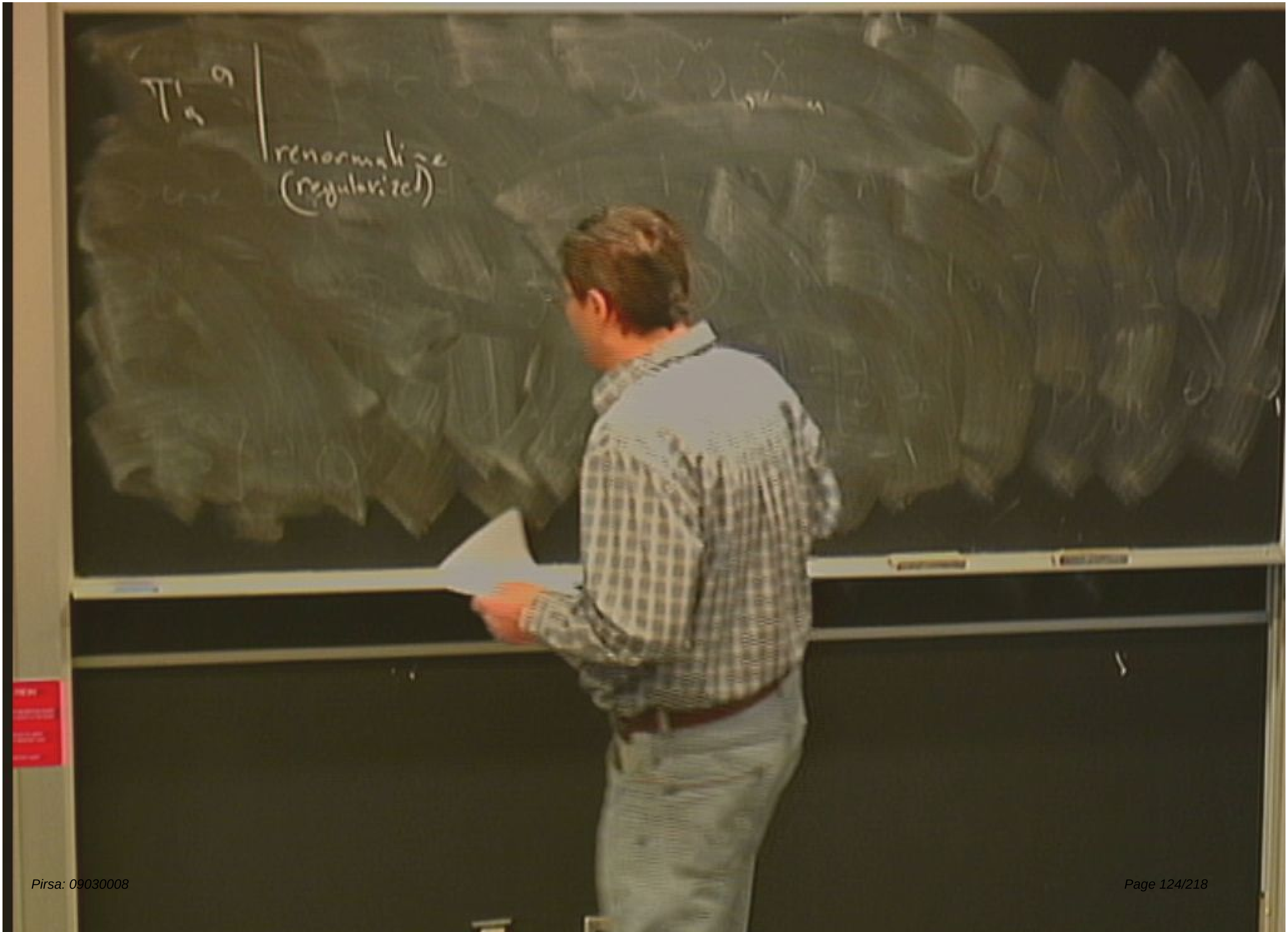
$$\boxed{\frac{\text{LHS}}{T_a{}^a} \;\; = \;\; \overset{\text{RHS}}{0}}$$ as an operator statement.

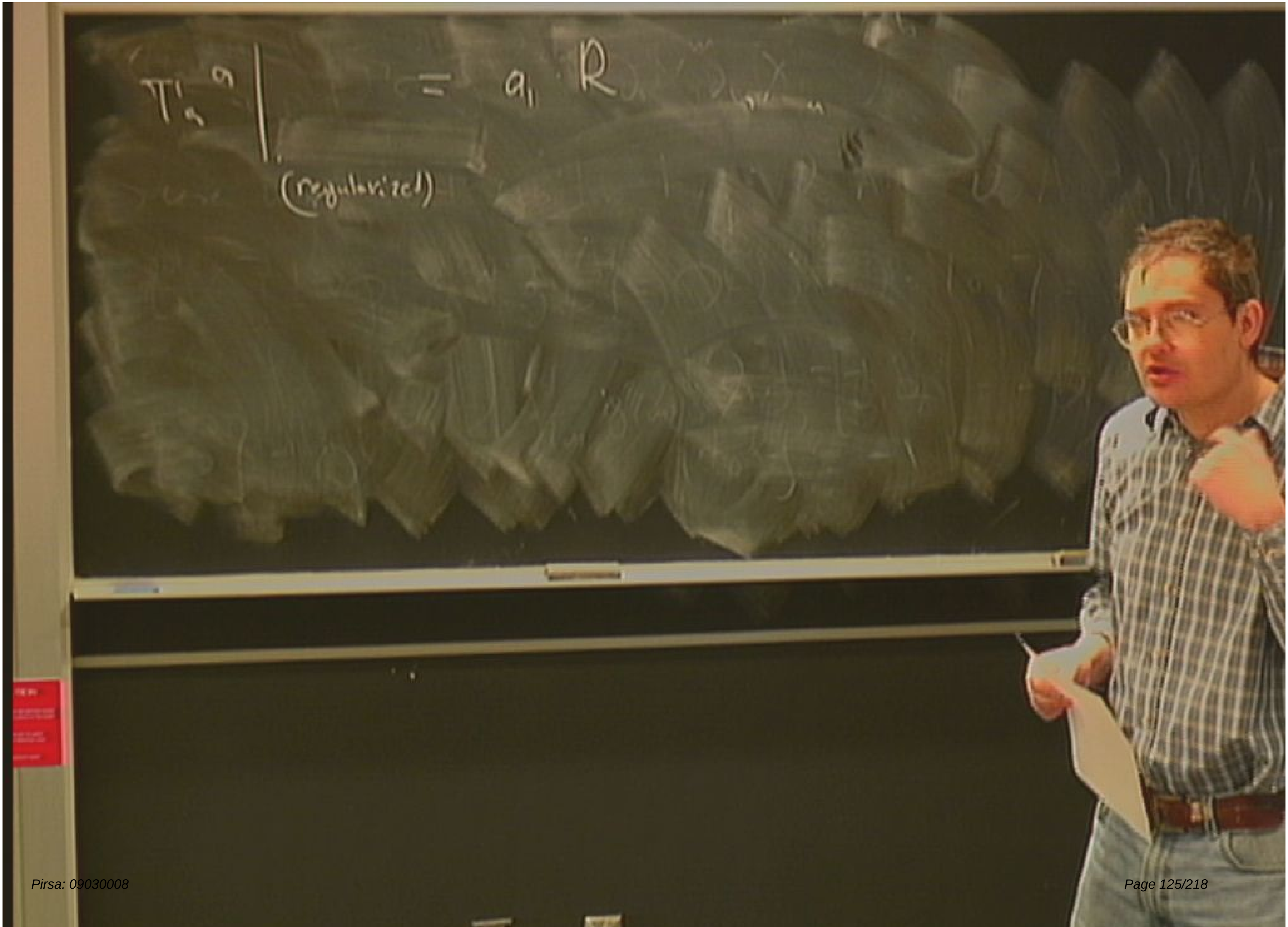
$T_a{}^a = ??$

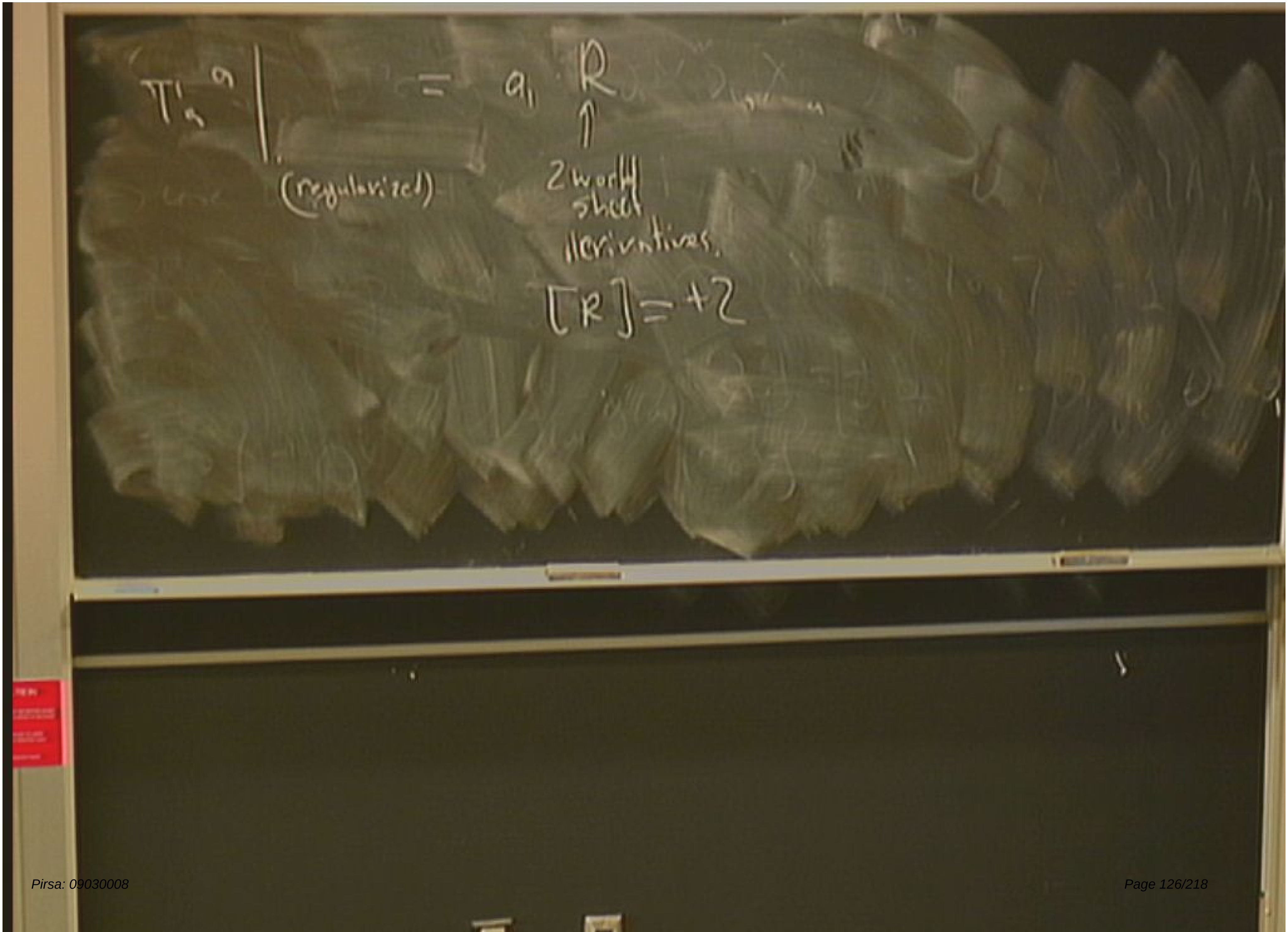
• "T" must be invariant ...

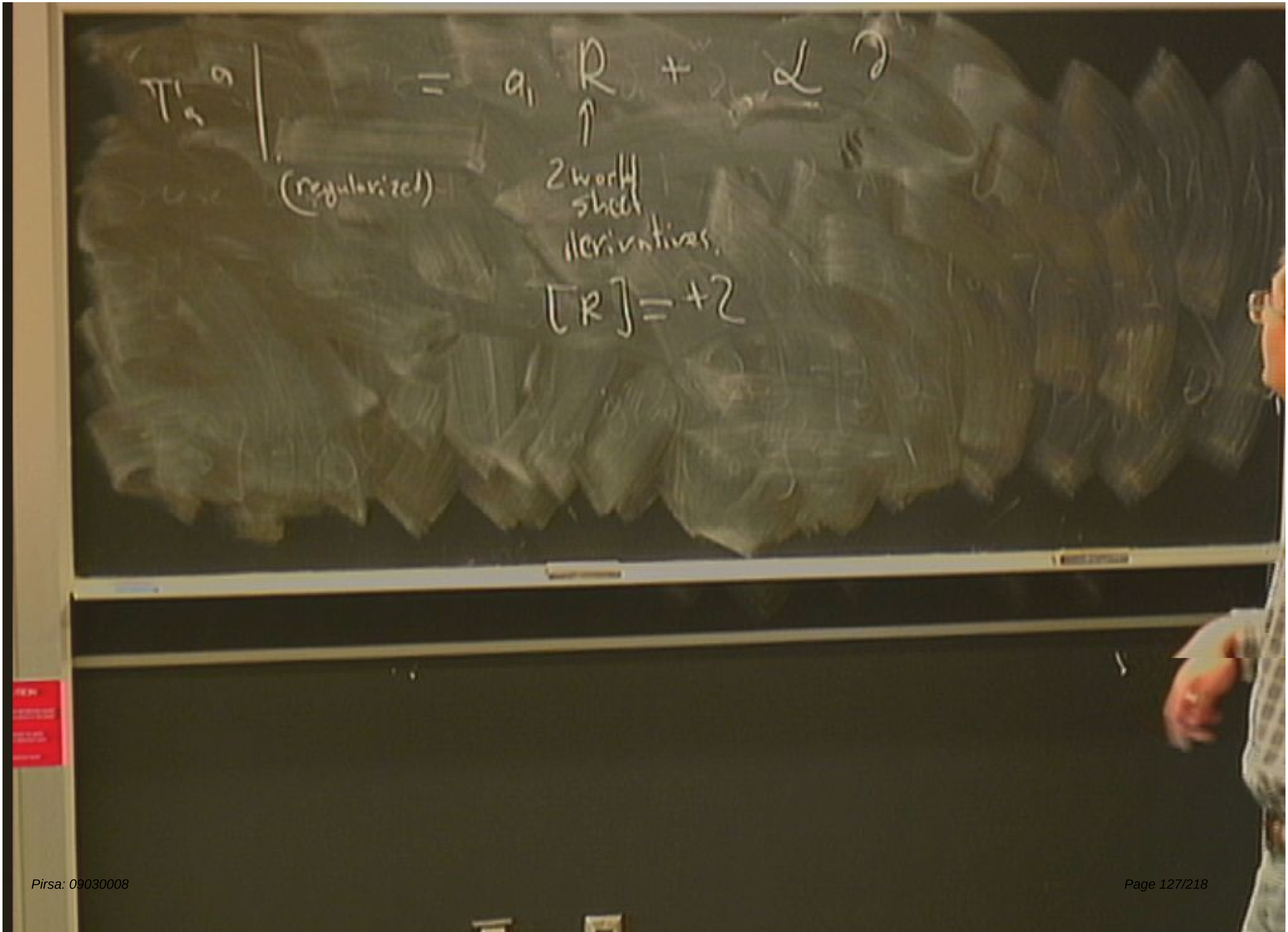
quantum scaling dims on LHS = scaling dim o...

$[T_{ab}] = +2$


renormalize
(regularized)


$\pi'_a{}^a \Big|_{\text{(regularized)}} = a_1 R$
(regularized)


$T'^{a}_{a} \Big|$ (regularized) $= a_1 R$
2 world sheet derivatives
$[R] = +2$


T'_a^a | (regularized) = a_1 R + ℒ ∂
2 world sheet derivatives
[R] = +2

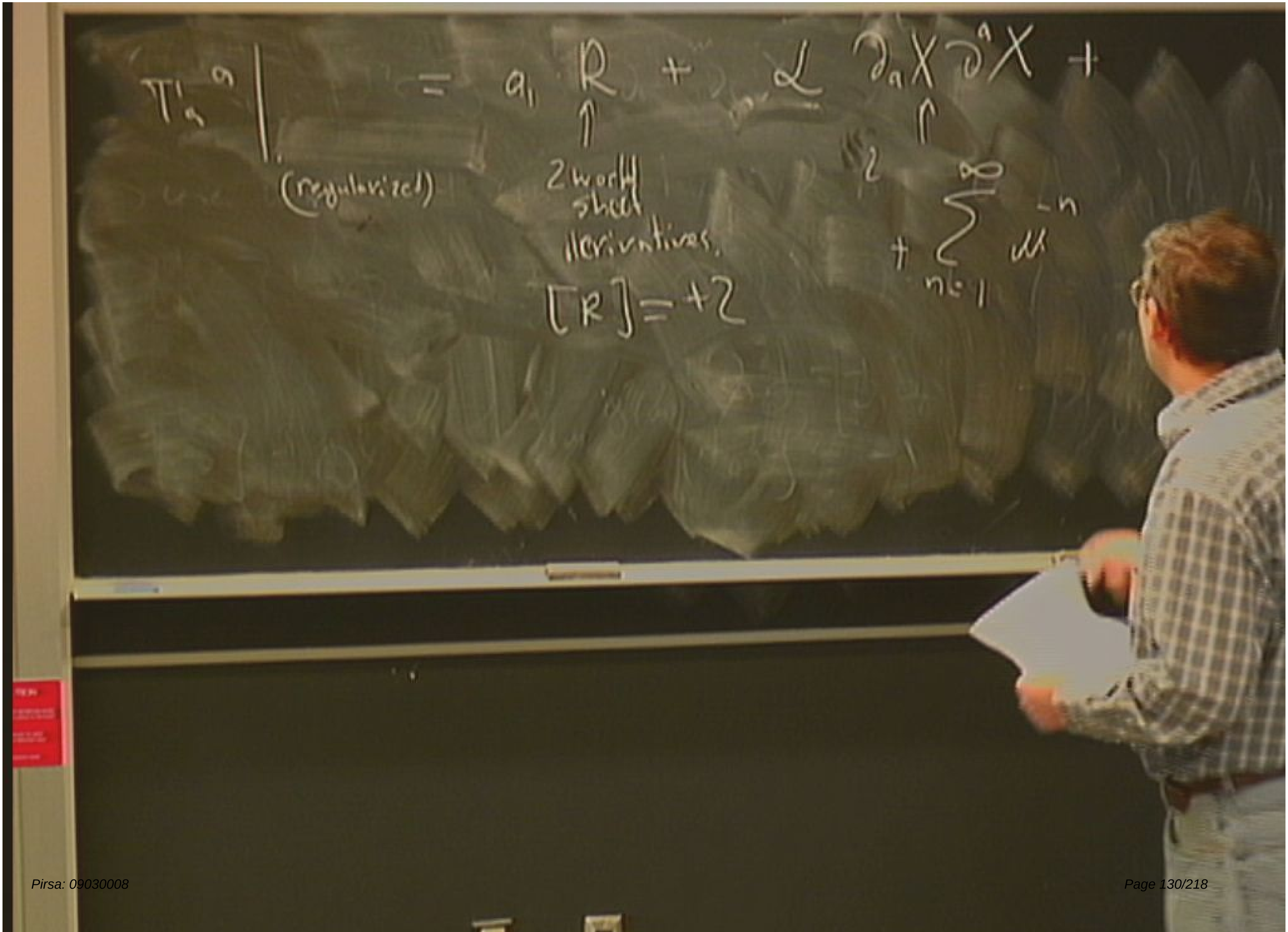
$$T'{}_a{}^a \Big|_{\text{(regularized)}} = a_1 \underset{\substack{\Uparrow \\ \text{2 world} \\ \text{sheet} \\ \text{derivatives}}}{R} + \alpha\, \partial_a X \partial^a X$$

$$[R] = +2$$

$$T'_a{}^a \Big|_{\text{(regularized)}} = a_1 \, R + \alpha \, \partial_a X \, \partial^a X$$

2 world sheet derivatives

$[R] = +2$


$T'^{a}_{a} \Big|_{\text{(regularized)}} = a_1 R + \alpha \, \partial_a X \partial^a X +$
2 world sheet derivatives.
$[R] = +2$
ℓ^2
$+ \sum_{n=1}^{\infty}$

$$T'^{a}_{a}\Big|_{\text{(regularized)}} = a_1 \underset{\substack{\uparrow \\ \text{2 world sheet derivatives.}}}{R} + \alpha\, \underset{\substack{\uparrow \\ 2}}{\partial_a X}\, \partial^a X + \sum_{n=1}^{\infty} \mu^{-n}\, \mathcal{F}_{2+n}$$

$$[R] = +2$$

$$[\mu] = +1$$

$$T'^{a}_{a}\Big|_{\text{(regularized)}} = a_1 R + \alpha\, \partial_a X \partial^a X + \sum_{n=1}^{\infty} \mu^{-n} \mathcal{J}_{2+n}$$

2 world sheet derivatives

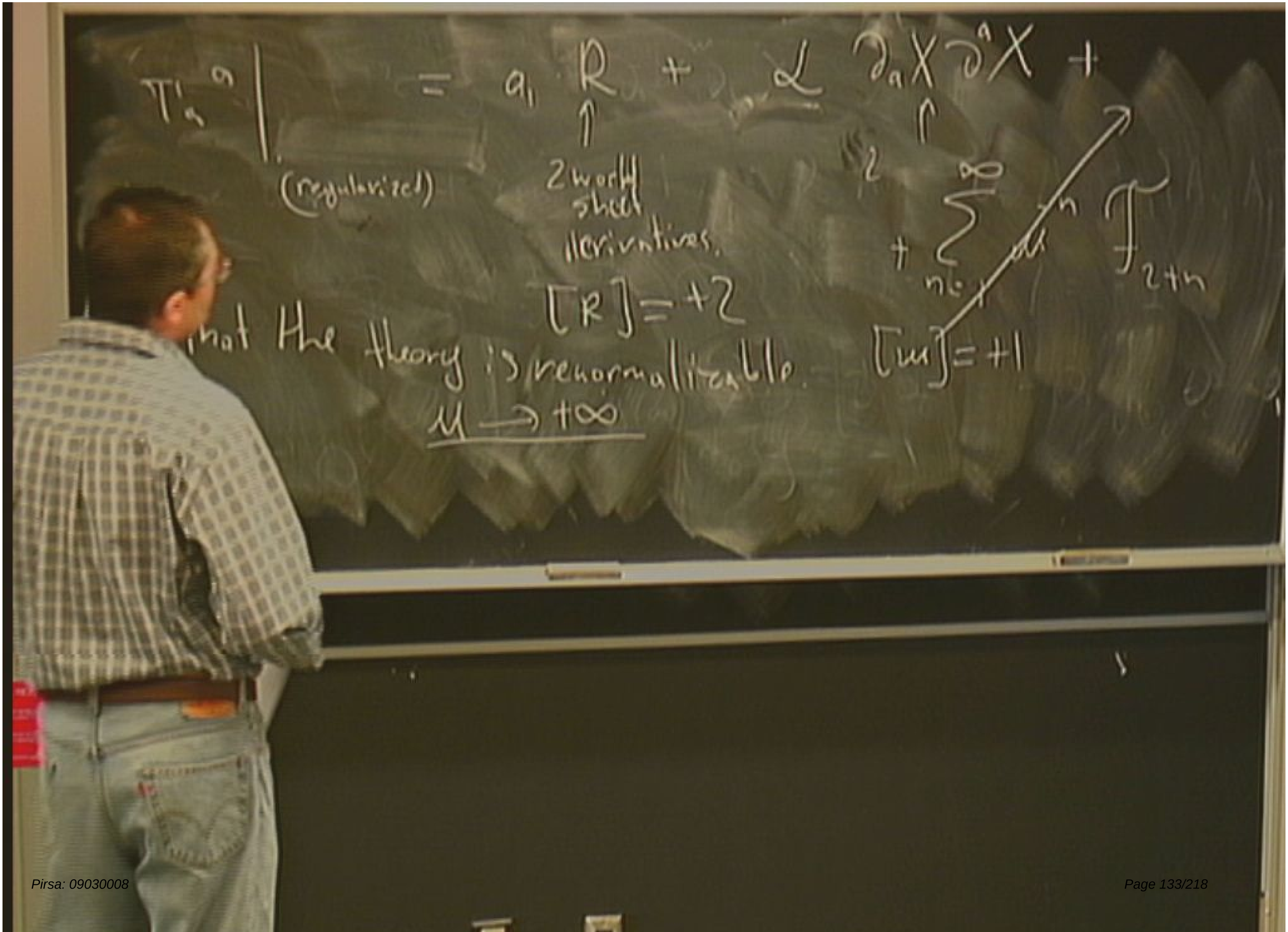
2

$[R] = +2$

$[\mu] = +1$

That the theory is renormalizable

$\mu \to +\infty$


$T'^{a}_{a}\Big|_{\text{(regularized)}} = a_1 R + \alpha\, \partial_a X \partial^a X + \sum_{n=1}^{\infty} \mu^n \mathcal{F}_{2+n}$
2 world sheet derivatives
2
$[R] = +2$
$[\mu] = +1$
that the theory is renormalizable
$\mu \to +\infty$

$$T'^{a}_{a}\Big|_{\text{(regularized)}} = a_1 R + \alpha\, \partial_a X \partial^a X + \sum_{n=1}^{\infty} \mu^n \mathcal{F}_{2+n}$$

2 world sheet derivatives

2

$[R] = +2$

$[\mu] = +1$

That the theory is renormalizable

$\underline{\mu \to +\infty}$

$$T_a{}^a\Big|_{\text{regularized}} = a_1 R + \alpha (\partial X)^2$$

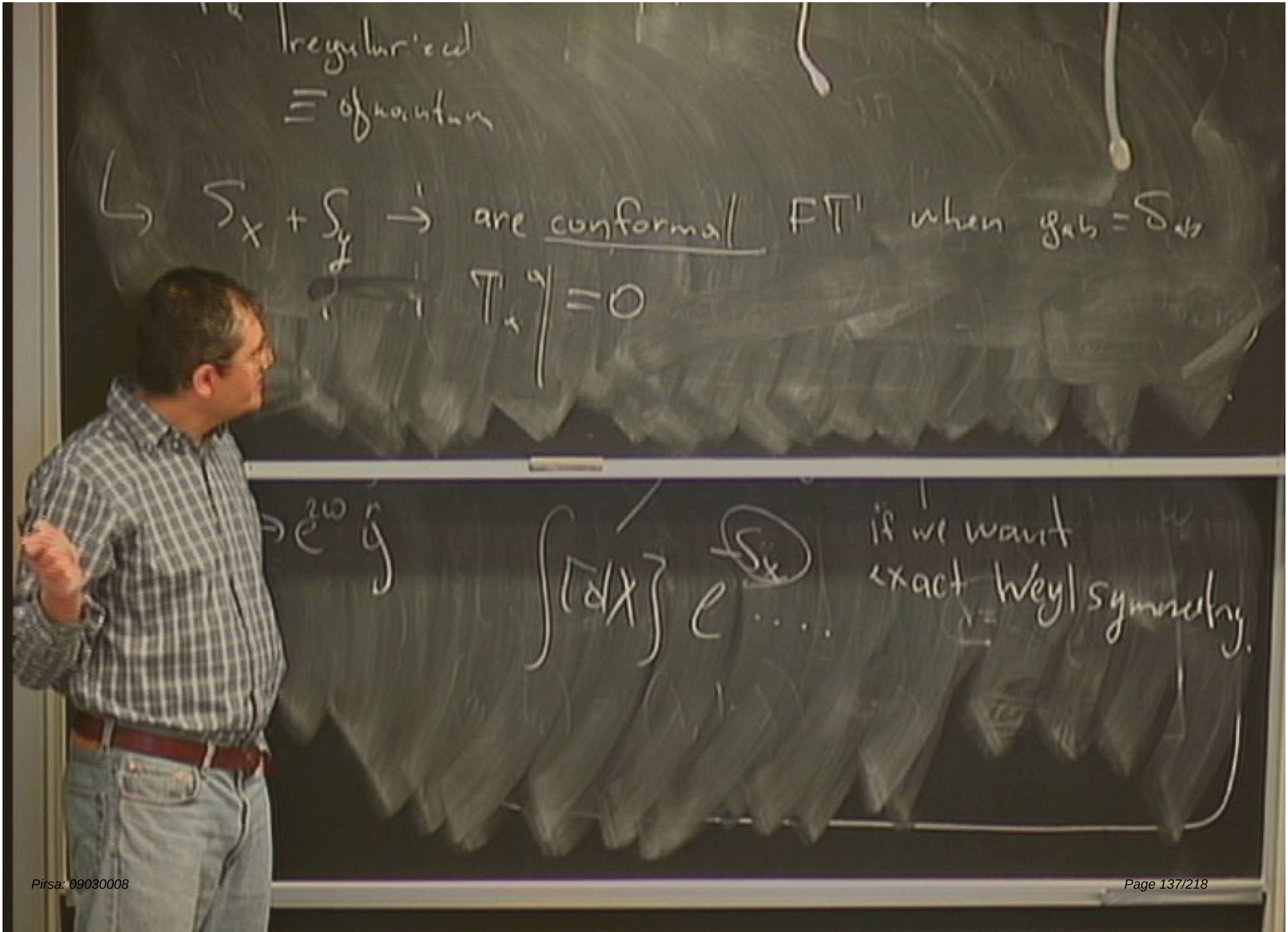
$\equiv$ of quantum

$$T_a{}^a\Big|_{\text{regularized}} = a_1 R + a(\partial X)^2$$

$\equiv$ of momentum

$\hookrightarrow S_X + S_g$

$\int (dX) \int e \ldots$ exact Weyl symmetry.



regular'ized
≡ of momentum
$S_X + S_y \to$ are conformal FT' when $g_{ab} = \delta_{ab}$
$T_a{}^a = 0$
$e^{2\omega} g$
$\int [dX] \, e^{-S_X} \ldots$
if we want exact Weyl symmetry.

regular'ed

Ξ of quantum

$\hookrightarrow$ $S_X + S_g \rightarrow$ are conformal FT' when $g_{ab} = \delta_{ab}$

$\rightarrow$ $T_\alpha^{\ \alpha}\Big|_{g_{ab}=\delta_{ab}} = 0$

$\hookrightarrow \hat{g} \rightarrow e^{2\omega}\,\hat{g}$

$\int [dX]\, e^{-S_X} \ldots$

if we want exact Weyl symmetry.

regular'ed

Ξ of quantum

$\hookrightarrow$ $S_X + S_g \to$ are <u>conformal</u> FT' when $g_{ab} = \delta_{ab}$

$\to T_a^{\ a}\Big|_{g_{ab}=\delta_{ab}} = 0 \Rightarrow R = 0.\quad T_a^{\ a}\Big|_{R=0} = 0.$

$\hookrightarrow \hat{g} \to e^{2\omega}\hat{g}$

$\int [dX]\, e^{-S_X} \ldots$

if we want exact Weyl symmetry.

$$T_a{}^a\Big|_{\text{regularized}} = a_1 R + \underset{0}{\underset{\parallel}{\alpha}}(\partial X)^2$$

$\equiv$ of quantum

$\hookrightarrow$ $S_X + S_g \rightarrow$ are <u>conformal</u> FT' when $g_{ab} = \delta_{ab}$

$$\rightarrow T_a{}^a\Big|_{g_{ab}=\delta_{ab}} = 0 \Rightarrow R = 0. \quad T_a{}^a\Big|_{R=0} = 0$$

$T_a{}^a\big|_{g_{ab}=\delta_{ab}} = 0 \Rightarrow R = 0.$ $\quad T_a{}^a \;\; = 0$

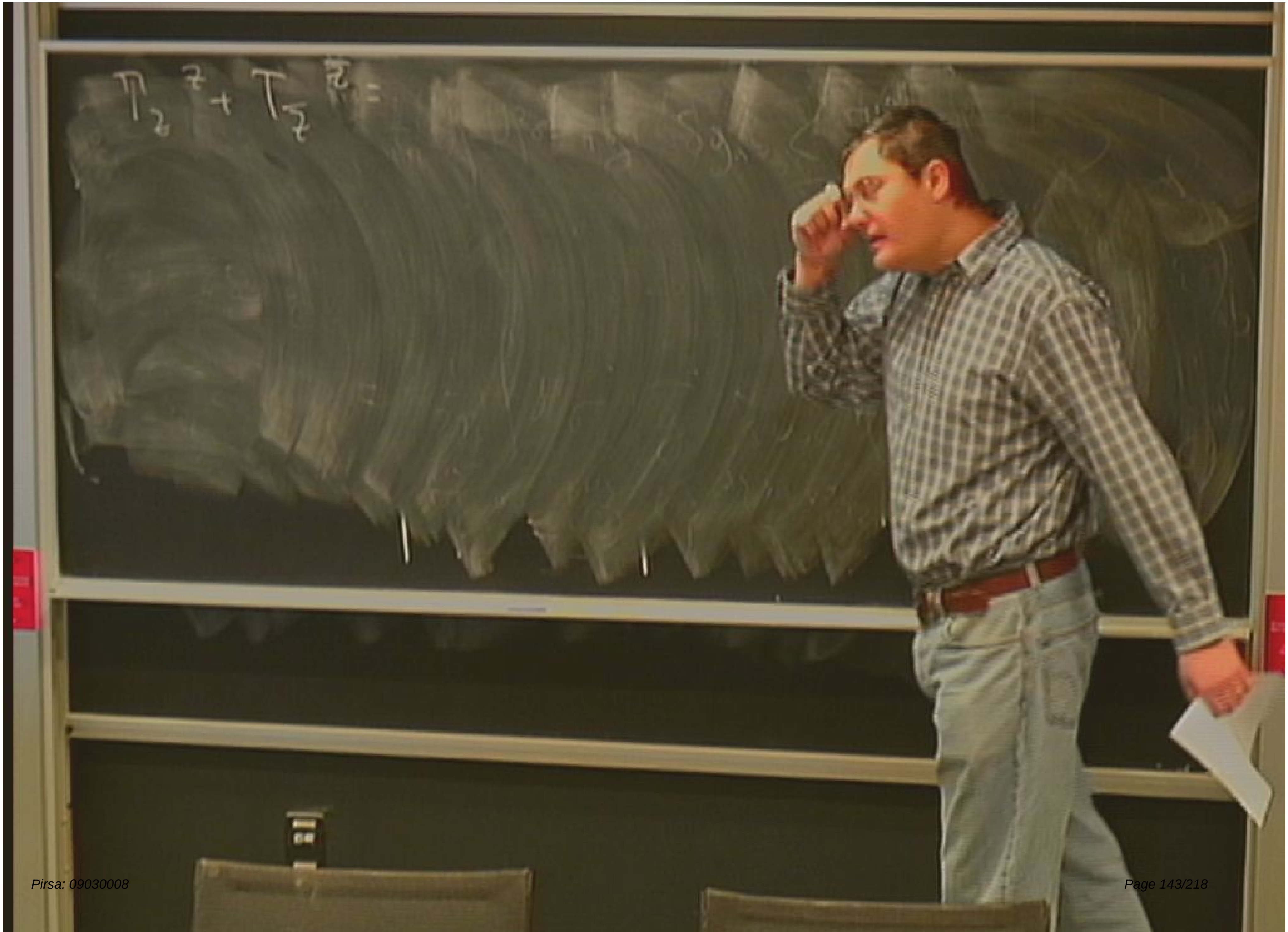
$R = 0$

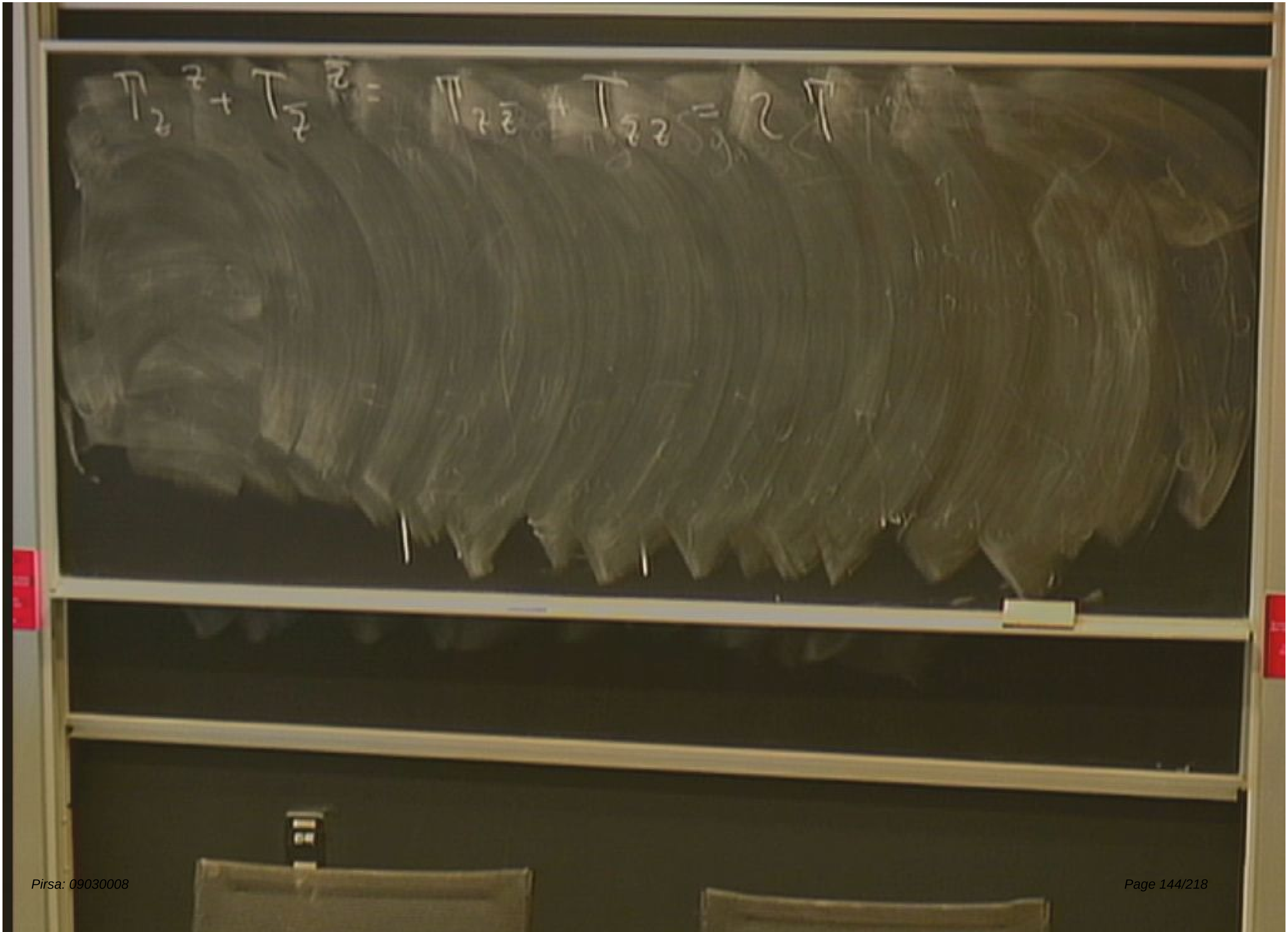
$T_a{}^a = a.\, R$

$$\Pi_a{}^a\Big|_{g_{ab}=\delta_{ab}} = 0 \Rightarrow R = 0. \quad \Pi_a{}^a\Big|_{R=0} = 0$$

$$\Pi_a{}^a = a_1 R$$

$\hookrightarrow$ we going to explicitly compute a_1


$T_z z + T_{\bar z} \bar z =$


$T_z z + T_{\bar z} \bar z = T_{z\bar z} + T_{\bar z z} = 2T$

$$T_z{}^z + T_{\bar z}{}^{\bar z} = g^{z\bar z}\left(T_{z\bar z} + T_{\bar z z}\right) = 2\left(T_{z\bar z}\right) g^{z\bar z} = a_1 R$$

$$T_{z\bar z} = \frac{1}{2} a_1 R \cdot g_{z\bar z}$$

$$T_{z\bar{z}} = \frac{1}{2}\phi_1 R \cdot g_{z\bar{z}}$$

$$\nabla^{\bar{z}} T_{\bar{z}z} = \frac{\phi_1}{2}\nabla^{\bar{z}}$$

$(T_{ab}) = +2$ dim on RHS

$$T_{z\bar z} = \frac{1}{2}\Phi_1 R \cdot g_{z\bar z}$$

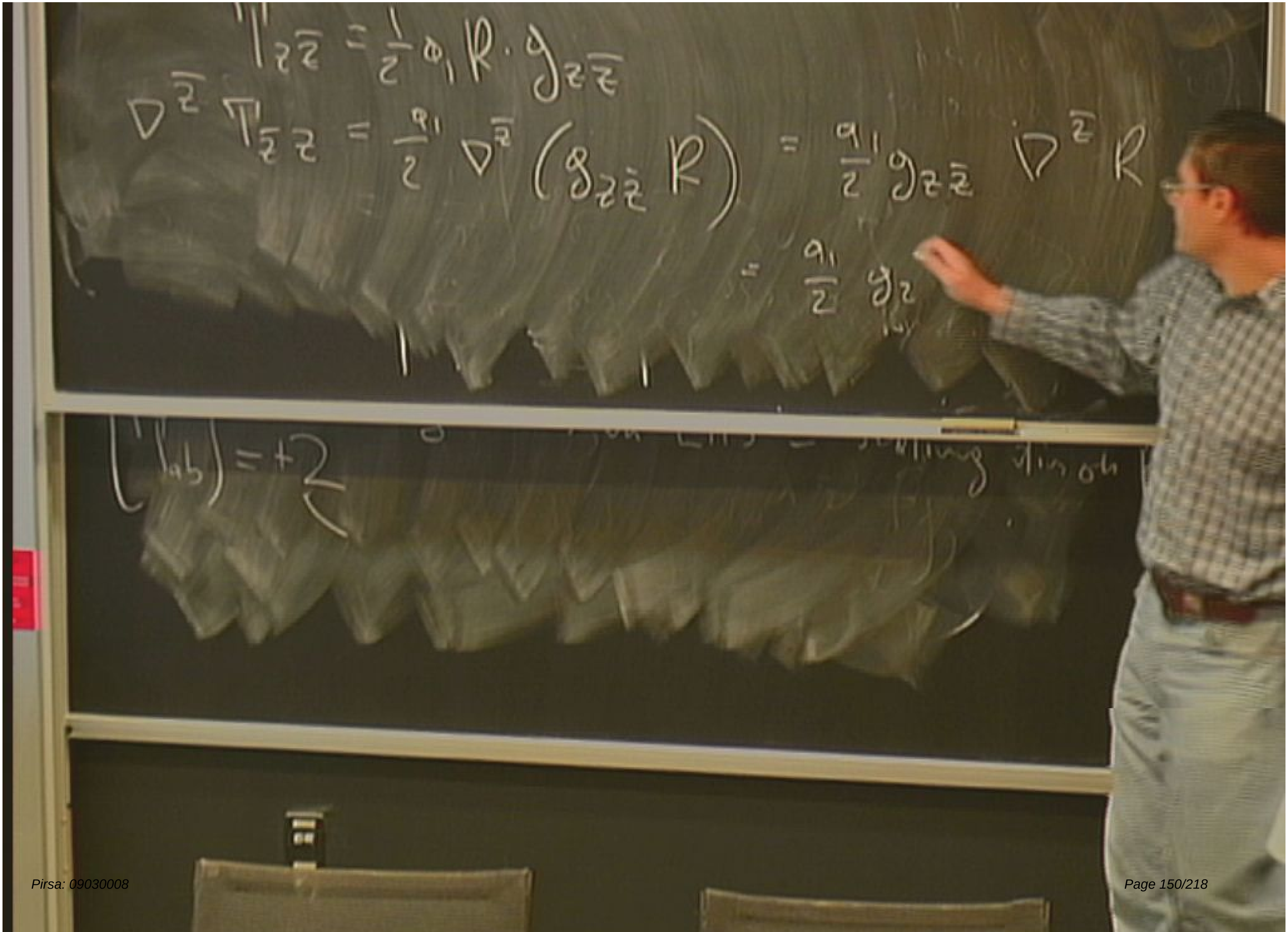
$$\nabla^{\bar z} T_{\bar z z} = \frac{\Phi_1}{2} \nabla^{\bar z} \left(g_{z\bar z} R \right)$$

$(T_{ab}) = +2$ … dim on RHS

$$T_{z\bar{z}} = \frac{1}{2} a_1 R \cdot g_{z\bar{z}}$$

$$\nabla^{\bar{z}} T_{\bar{z} z} = \frac{a_1}{2} \nabla^{\bar{z}} \left(g_{z\bar{z}} R \right) = \frac{a_1}{2} g_{z\bar{z}} \nabla^{\bar{z}} R$$

$\langle T_{ab} \rangle = +2$ ... RHS


$T_{z\bar{z}} = \frac{1}{2} a_1 R \cdot g_{z\bar{z}}$
$\nabla^{\bar{z}} T_{\bar{z}z} = \frac{a_1}{2} \nabla^{\bar{z}} (g_{z\bar{z}} R) = \frac{a_1}{2} g_{z\bar{z}} \nabla^{\bar{z}} R$
$= \frac{a_1}{2} g_{z}$
$= +2$

$$T_{z\bar z} = \frac{1}{2} a_1 R \cdot g_{z\bar z}$$

$$\nabla^{\bar z} T_{\bar z z} = \frac{a_1}{2} \nabla^{\bar z} \left(g_{z\bar z} R \right) = \frac{a_1}{2} g_{z\bar z} \nabla^{\bar z} R$$

$$= \frac{a_1}{2} \nabla_z R$$

$$\left(T_{ab} \right) = + \Sigma$$

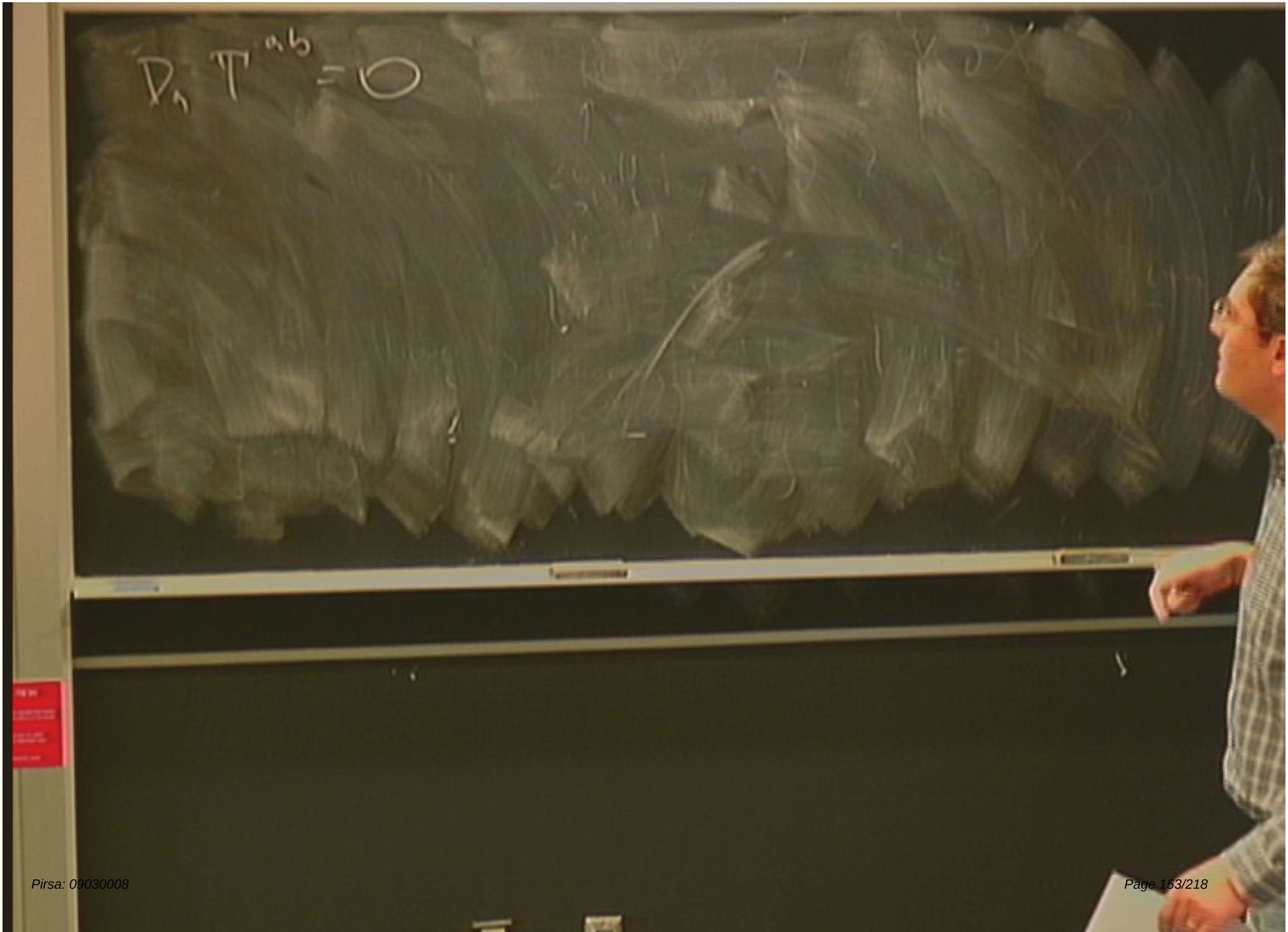
... LHS ... setting ... on RHS

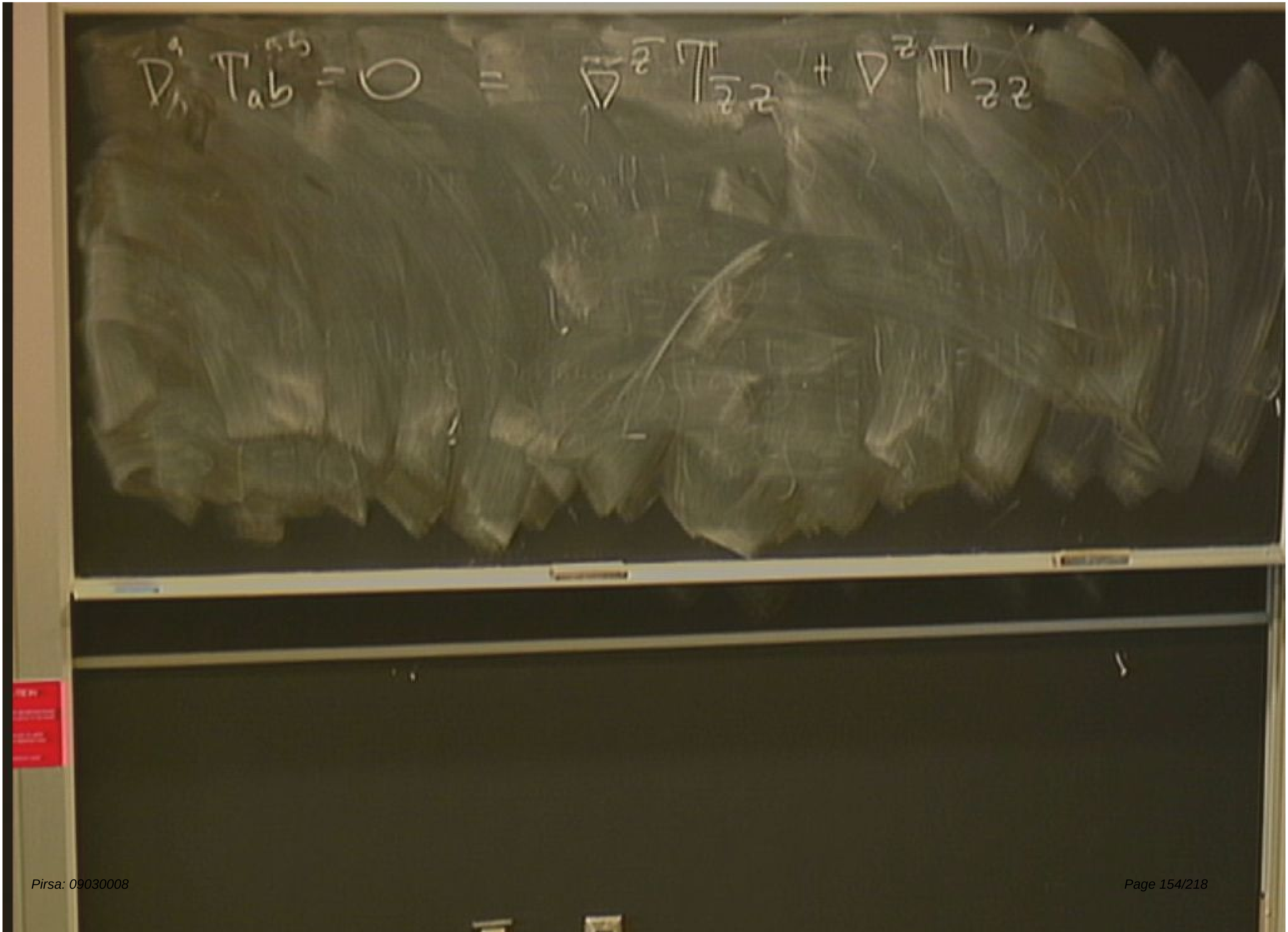
$$T_{z\bar z} = \frac{1}{2} a_1 R \cdot g_{z\bar z}$$

$$\nabla^{\bar z} T_{\bar z z} = \frac{a_1}{2} \nabla^{\bar z} \left(g_{z\bar z} R \right) = \frac{a_1}{2} g_{z\bar z} \nabla^{\bar z} R$$

$$= \frac{a_1}{2} \nabla_z R = \frac{a_1}{2} \partial R$$

$\left(T_{ab} \right) = +2$ ... on RHS


$D_a T^{ab} = 0$


$\nabla^a_h T_{ab}^{55} = 0 = \bar{\nabla}^{\bar{z}} T_{\bar{z}z} + \nabla^z T_{\bar{z}z}$

$$T_z + T_{\bar z} \quad g^{z\bar z}(T_{z\bar z} + T_{\bar z z}) = \langle T_{z\bar z}\rangle g^{z\bar z} = a_1 R$$

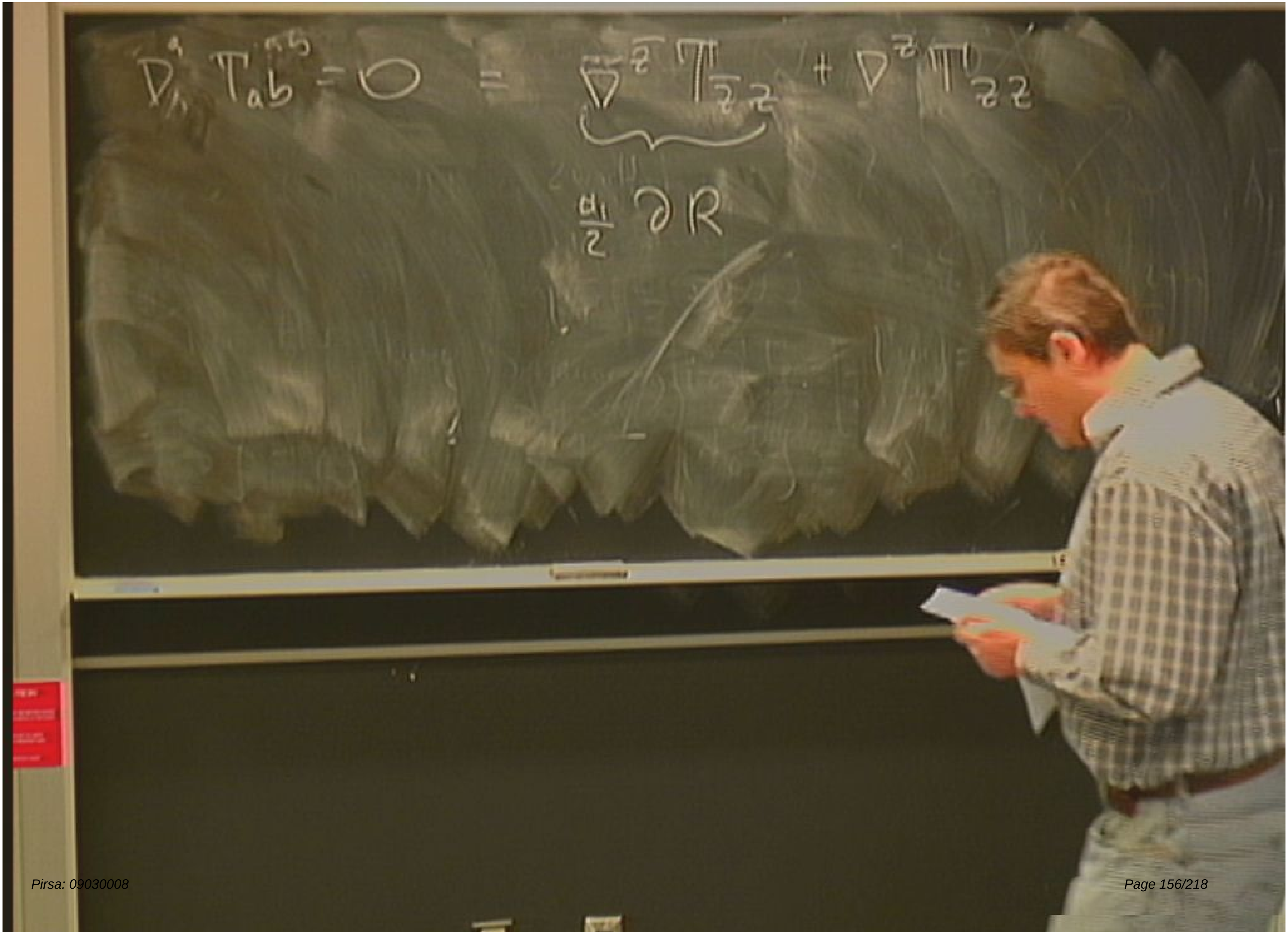
$$T_{z\bar z} = \frac{1}{2} a_1 R \cdot g_{z\bar z}$$

$$\nabla^{\bar z} T_{\bar z z} = \frac{a_1}{2} \nabla^{\bar z} \left(g_{z\bar z} R\right) = \frac{a_1}{2} g_{z\bar z} \nabla^{\bar z} R$$

$$= \frac{a_1}{2} \nabla_z R = \frac{a_1}{2} \partial R$$

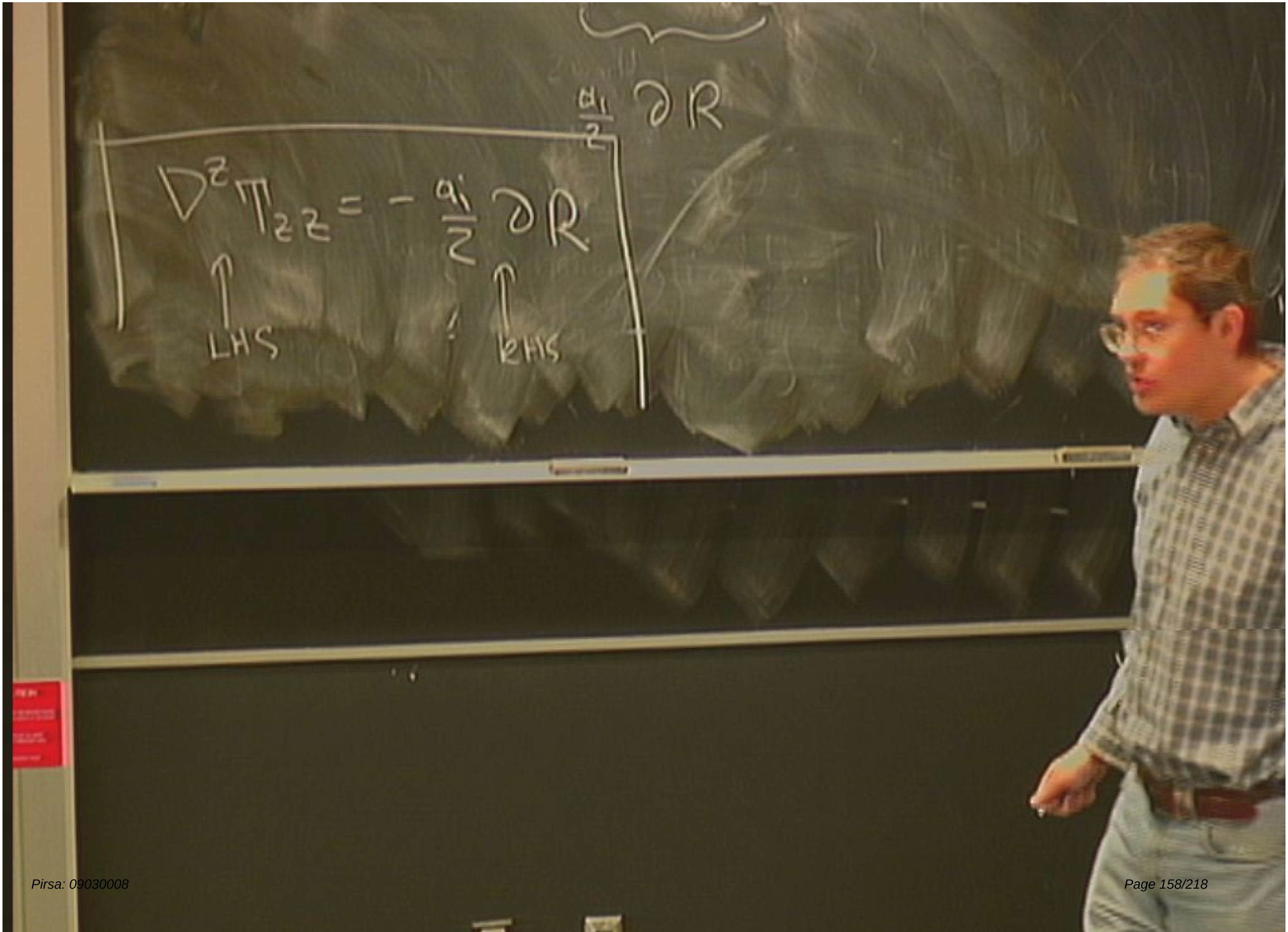
on LHS = scaling dim on RHS

$$\langle T_{ab}\rangle = +2$$


∇^a T_ab = 0 = ∇^z̄ T_z̄z + ∇^z T_zz
a_1/2 ∂R

$$\nabla^a T_{ab} = 0 = \underbrace{\nabla^{\bar z} T_{\bar z z}}_{\frac{a_1}{2}\partial R} + \nabla^z T_{zz}$$

$$\boxed{\nabla^z T_{zz} = -\frac{a_1}{2}\partial R}$$


$\frac{a_1}{2}\partial R$
$D^z T_{zz} = -\frac{a_i}{2}\partial R$
LHS
RHS

$\frac{a_1}{2} \partial R$

$$\boxed{\nabla^z T_{zz} = -\frac{a_i}{2} \partial R}$$

LHS RHS

$\Rightarrow$ we act with Weyl transformations on LHS & RHS

$\Rightarrow$ we act with Weyl transformations on LHS & RHS

$\Rightarrow$ RHS

$$g_{ab} \rightarrow g'_{ab} = e^{2\omega} g_{ab}$$

$\Rightarrow$ we act with Weyl transformations on LHS & RHS

$$g_{ab} \rightarrow g'_{ab} = e^{2\omega} g_{ab}$$

$$\sqrt{g'}\, R'$$

$\Rightarrow$ we act with Weyl transformations on LHS & RHS

$$g_{ab} \rightarrow g'_{ab} = e^{2\omega} g_{ab}$$

$$\sqrt{g'} R' = \sqrt{g} \left(R - 2 \nabla^2 \omega \right)$$

$\Rightarrow$ we act with Weyl transformations on LHS & RHS

$$g_{ab} \to g'_{ab} = e^{2\omega} g_{ab}$$

$$\sqrt{g'} R' = \sqrt{g} \left(R - 2 \nabla^2 \omega \right)$$

$\hookrightarrow$ weyl transformation about $g_{ab} = \delta_{ab}$

$\Rightarrow$ we act with Weyl transformations on LHS & RHS

g_{ab} g_{ab} g_{ab}

$$\sqrt{g'}\,R' = \sqrt{g}\left(R - 2\nabla^2\omega\right)$$

↳ Weyl transformation about $g_{ab} = \delta_{ab}$ $\Rightarrow$ $R = 0$

$$R' = e^{-2\omega}$$

$\Rightarrow$ we act with Weyl transformations on LHS & RHS

$$\sqrt{g'}\, R' = \sqrt{g}\left(R - 2\nabla^2 \omega\right)$$

$\hookrightarrow$ Weyl transformation about $g_{ab} = \delta_{ab}$ $\Rightarrow$ $R = 0$

$$R' = e^{-2\omega}\left(-2\nabla^2 \omega\right)$$

$\Rightarrow$ we act with Weyl transformations on LHS & RHS

$$g_{ab} \to g'_{ab} = e^{2\omega} g_{ab}$$

$$\sqrt{g'} R' = \sqrt{g} \left(R - 2\nabla^2 \omega \right)$$

$\hookrightarrow$ weyl transformation about $g_{ab} = \delta_{ab} \Rightarrow R = 0$

$$\delta R = R' - R = e^{-2\omega} \left(-2\nabla^2 \omega \right)$$

$\Rightarrow$ we act with Weyl transformations on LHS & RHS

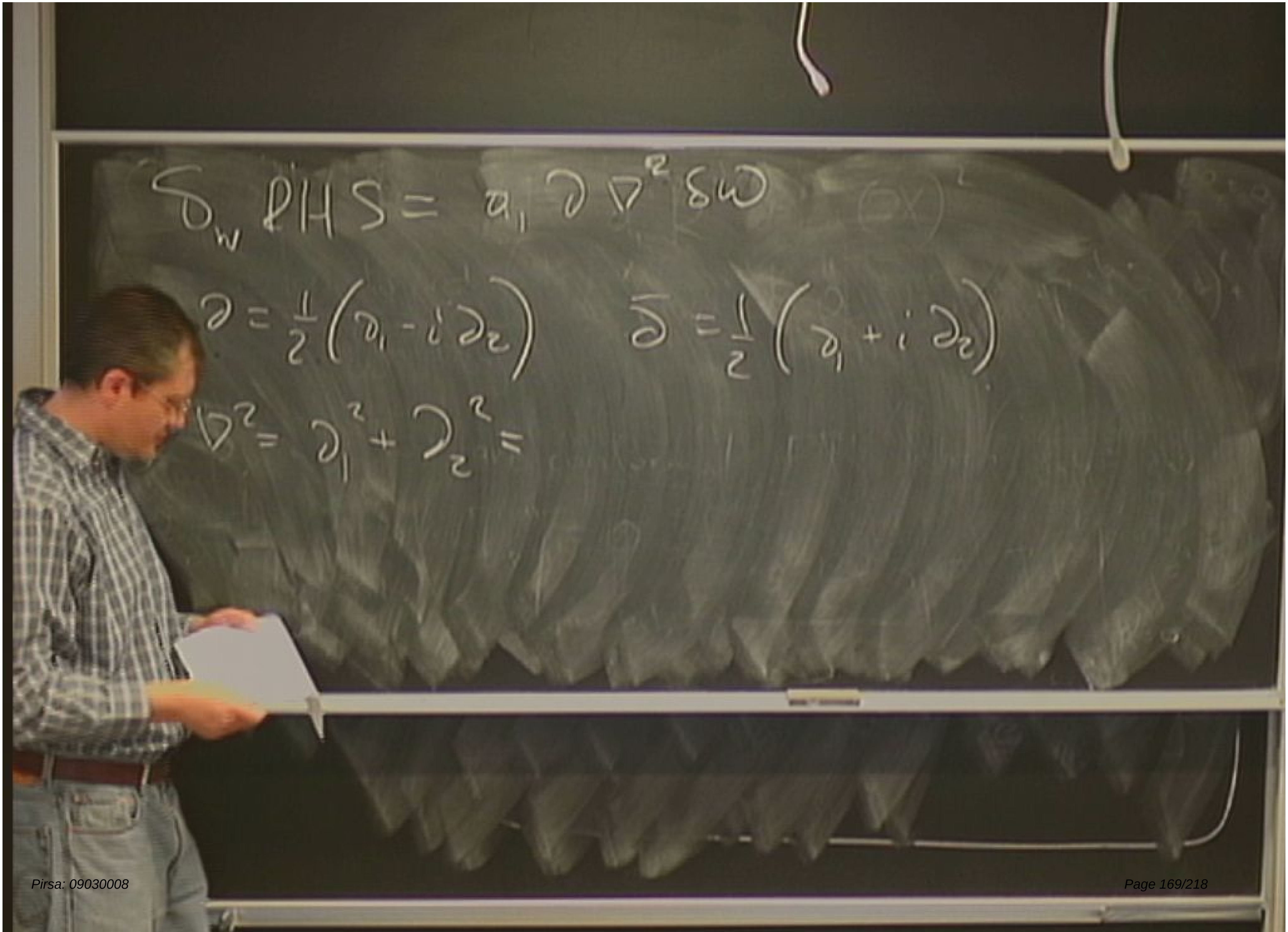
$$g_{ab} \rightarrow g'_{ab} = e^{2\omega} g_{ab}$$

$$\sqrt{g'} R' = \sqrt{g}\left(R - 2\nabla^2 \omega\right)$$

↳ weyl transformation about $g_{ab} = \delta_{ab} \Rightarrow R = 0$

$$\delta R = R' - R = e^{-2\omega}\left(-2\nabla^2 \delta\omega\right) = \boxed{-2\nabla^2 \delta\omega \equiv \delta R}$$

$$\delta_w \, \ell HS = a_1 \, \partial \, \nabla^2 \, \delta\omega$$


δ_w RHS $= a_1 \partial \nabla^2 \delta\omega$
$\partial = \frac{1}{2}(\partial_1 - i\partial_2)$
$\bar{\partial} = \frac{1}{2}(\partial_1 + i\partial_2)$
$\nabla^2 = \partial_1^2 + \partial_2^2 =$

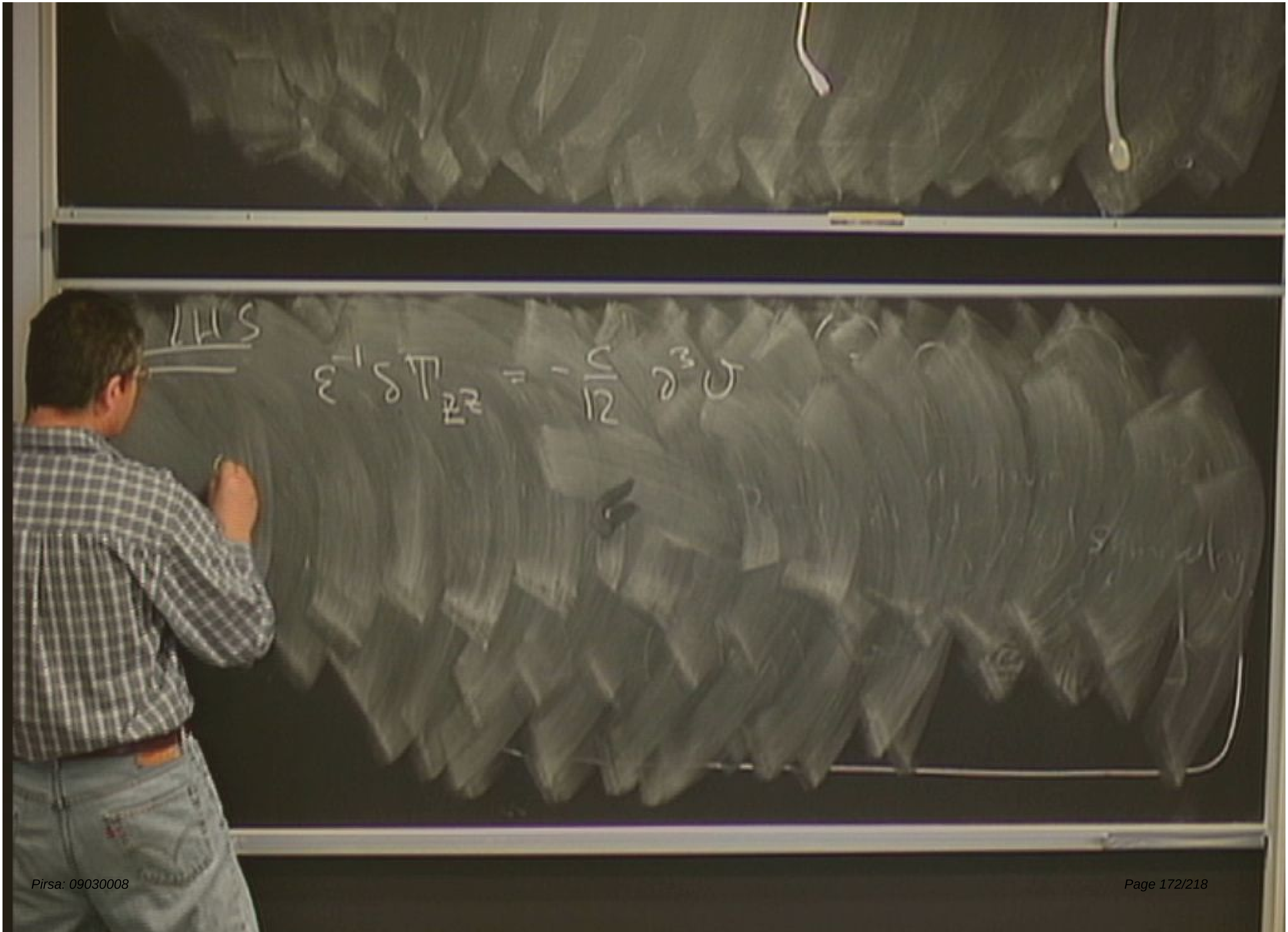
$$\delta_w \, \mathrm{LHS} = a_1 \partial \nabla^2 \delta \omega = 4 a_1 \partial^2 \bar{\partial} \, \delta \omega + O(\delta \omega^2)$$

$$\partial = \frac{1}{2}(\partial_1 - i \partial_2) \qquad \bar{\partial} = \frac{1}{2}(\partial_1 + i \partial_2)$$

$$\nabla^2 = \partial_1^2 + \partial_2^2 = 4 \partial \bar{\partial}$$

LHS

$\varepsilon^{-1}\delta\pi_{zz}$


LHS
$\varepsilon^{-1}\delta T_{zz} = -\frac{c}{12}\partial^3 v$

LHS

$$\varepsilon^{-1}\,\delta T_{zz} = -\frac{c}{12}\,\partial^3 v$$

$$\delta z = \varepsilon\, v(z)$$

$$\delta \bar{z} = \varepsilon\, \bar{v}$$

LHS

$$\varepsilon^{-1}\delta T_{zz} = \underbrace{-\frac{c}{12}\,\partial^3 v}_{\text{anomalous term}} - \dots$$

$\delta z = \varepsilon v(z)$

$\delta \bar{z} = \varepsilon \bar{v}$

LHS

$$\varepsilon^{-1}\delta T_{zz} = \underbrace{-\frac{c}{12}\partial^3 v}_{\text{anomalous term}} - 2\partial v\, T_{zz} - v\,\partial_z T_{zz}$$

$\delta z = \varepsilon v(z)$

$\delta \bar{z} = \varepsilon \bar{v}$

LHS

$$\varepsilon^{-1}\delta T_{zz} = \underbrace{-\frac{c}{12}\partial^3 v}_{\text{anomalous term}} \underbrace{- 2\partial v\, T_{zz} - v\,\partial_z T_{zz}}$$

$\delta z = \varepsilon v(z)$

$\delta \bar{z} = \varepsilon \bar{v}$

LHS

$$\varepsilon^{-1}\delta T_{zz} = \underbrace{-\frac{c}{12}\partial^3 v}_{\text{anomalous term}} \underbrace{- 2\partial v\, T_{zz} - v\,\partial_z T_{zz}}_{\text{due to coordinate transformations of}}$$

$\delta z = \varepsilon v(z)$

$\delta \bar{z} = \varepsilon \bar{v}$

LHS

$$\varepsilon^{-1}\delta T_{zz} = \underbrace{-\frac{c}{12}\,\partial^3 v}_{\text{anomalous term}} \underbrace{-\,2\,\partial v\, T_{zz} - v\,\partial_z T_{zz}}_{\text{due to coordinate transformations of } M_{zz}}$$

$\delta z = \varepsilon\, v(z)$

$\delta \bar{z} = \varepsilon\, \bar{v}$

LHS

$$\varepsilon^{-1}\delta T_{zz} = \underbrace{-\frac{c}{12}\,\partial^3 v}_{\text{anomalous term}} \underbrace{- 2\,\partial v\, T_{zz} - v\,\partial_z T_{zz}}_{\text{due to coordinate transformations of } M_{zz}}$$

$\delta z = \varepsilon\, v(z)$

$\delta \bar{z} = \varepsilon\, \bar{v}$

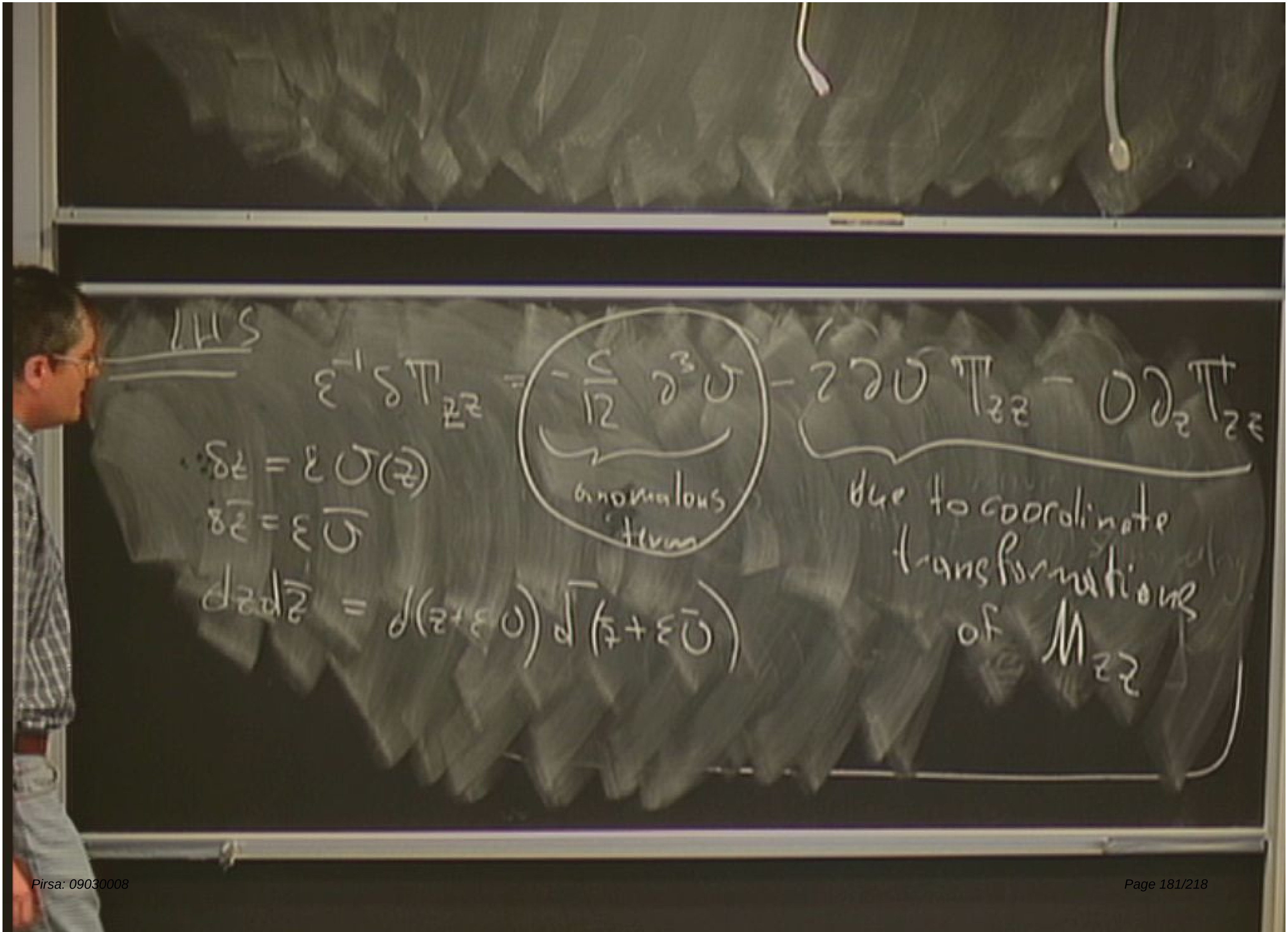
LHS

$$\varepsilon^{-1}\delta T_{zz} = \underbrace{-\frac{c}{12}\partial^3 v}_{\text{anomalous term}} \underbrace{- 2\partial v\, T_{zz} - v\,\partial_z T_{zz}}_{\text{due to coordinate transformations of } M_{zz}}$$

$\delta z = \varepsilon v(z)$

$\delta \bar{z} = \varepsilon \bar{v}$

$dz d\bar{z} =$


LHS
$\varepsilon^{-1}\delta T_{zz} = -\frac{c}{12}\partial^3 v - 2\partial v\, T_{zz} - v\,\partial_z T_{zz}$
anomalous term
due to coordinate transformations of M_{zz}
$\delta z = \varepsilon v(z)$
$\delta \bar{z} = \varepsilon \bar{v}$
$dz\,d\bar{z} = d(z+\varepsilon v)\,\bar{d}(\bar{z}+\varepsilon\bar{v})$

LHS

$$\varepsilon^{-1}\delta T_{zz} = -\frac{c}{12}\partial^3 v - 2\partial v\, T_{zz} - v\,\partial_z T_{zz}$$

anomalous term

due to coordinate transformations of M_{zz}

$\delta z = \varepsilon v(z)$

$\delta\bar{z} = \varepsilon\bar{v}$

$$dz\,d\bar{z} = d(z+\varepsilon v)\,d(\bar{z}+\varepsilon\bar{v}) =$$

$$= dz\,d\bar{z}\left(1+\varepsilon\partial v+\varepsilon\bar{\partial}\bar{v}\right)$$

conformal transf $z \to f(z) \neq$ coordinate trans +
+ Weyl rescaling

conformal transf $z \to f(z)$ $\neq$ coordinate trans +

+ weyl rescaling

$$2\omega = \varepsilon \partial v + \varepsilon \overline{\partial v}$$

conformal transf $z \to f(z)$ = coordinate trans + + Weyl rescaling

$$2\omega = \varepsilon \partial v + \xi \overline{\partial v}$$

$$\delta_w T'_{zz}$$

conformal transf $z \to f(z)$ $\Leftrightarrow$ coordinate trans + + weyl rescaling

$$2\omega = \varepsilon \partial v + \bar{\varepsilon}\, \overline{\partial v}$$

$$\delta_\omega T_{zz} = -\frac{c}{12}\partial_z^3 \omega -$$

conformal transf $z \to f(z)$ = coordinate trans + + Weyl rescaling

$$2\omega = \varepsilon\partial v + \bar{\varepsilon}\,\overline{\partial v}$$

$$\delta_\omega T_{zz} = -\frac{c}{12}\partial_\varepsilon^3\omega - -\frac{c}{12}\partial^2\left[\varepsilon\partial v\right.$$

∂R

conformal transf $z \to f(z)$ = coordinate trans + + Weyl rescaling

$$2\omega = \varepsilon \partial v + \bar{\varepsilon} \overline{\partial v}$$

$$\delta_\omega T_{zz} = -\frac{c}{12} \partial_z^3 \omega - \frac{c}{12} \partial^2 \left[\varepsilon \partial v + \bar{\varepsilon} \overline{\partial v} \right]$$

conformal transf $z \to f(z)$ ≡ coordinate trans + + Weyl rescaling

$$2\delta\omega = \varepsilon\,\partial v + \xi\,\overline{\partial v}$$

$$\delta_\omega T_{zz} = -\frac{c}{12}\,\partial_z^3\omega - \frac{c}{12}\,\partial^2\left[\underbrace{\varepsilon\,\partial v + \xi\,\overline{\partial v}}_{2\delta\omega}\right]$$

$$\delta_w T_{zz} = -\frac{c}{12}\partial_z^3\omega - \frac{c}{12}\partial^2\Big[\underbrace{\varepsilon'\partial\omega + \varepsilon\,\overline{\partial\omega}}_{2\partial\omega}\Big]$$

$$\delta_w T_{zz}$$

$$(T_{ab}) = +2$$

... RHS

$$\delta_w T_{zz} = -\frac{c}{12}\,\partial^3_\varepsilon \omega - \frac{c}{12}\,\partial^2\left[\underbrace{\varepsilon'\partial\omega + \varepsilon\,\overline{\partial\omega}}_{2\partial\delta\omega}\right]$$

$$\delta_w T_{zz} = -\frac{c}{6}\,\partial^2\delta\omega$$

$(T_{ab}) = +2$ ... scaling dim on RHS

$$\delta_w T'_{zz} = -\frac{c}{12}\partial_z^3\omega - \frac{c}{12}\partial^2\Big[\underbrace{\xi'\partial\omega + \xi\,\overline{\partial\omega}}_{2\delta\omega}\Big]$$

$$\boxed{\delta_w T_{zz} = -\frac{c}{6}\partial^2\delta\omega}$$

$[T_{ab}] = +2$ ... ing dim on RHS

$\delta_w[RHS] =$

$$\delta_w \left[RHS \right] = -\frac{c}{6}$$

$$\delta_w[RHS] = -\frac{c}{6}\nabla^2 \bar{\partial}^2 \delta\omega = -\frac{c}{6}\bar{\partial}\,\bar{\partial}^2 \delta\omega$$

$$\delta_\omega[\mathrm{RHS}] = -\frac{c}{6}\nabla^2 \partial^2 \delta\omega = -\frac{c}{6}\bar{\partial}\bar{\partial}^2 \delta\omega + \mathcal{O}(\delta\omega^2)$$

$$\delta_\omega[RHS] = -\frac{c}{6}\nabla^2\bar\partial^2\delta\omega = -\frac{c}{6}\bar\partial\bar\partial^2\delta\omega + \mathcal{O}(\delta\omega^2)$$

$$\delta_\omega[LHS] = \delta_\omega[RHS]$$

$$-\frac{c}{6}\bar\partial\bar\partial^2\delta\omega =$$

$$\delta_w[RHS] = -\frac{c}{6}\nabla^2\bar\partial\,\delta\omega = -\frac{c}{6}\bar\partial\,\bar\partial^2\,\delta\omega + O(\delta\omega^2)$$

$$\delta_w[LHS] = \delta_w[RHS]$$

$$-\frac{c}{6}\bar\partial\,\bar\partial^2\,\delta\omega = 4a_1\,\partial^2\bar\partial\,\delta\omega$$

$$\delta_\omega[RHS] = -\frac{c}{6}\nabla^2 \bar\partial^2 \delta\omega = -\frac{c}{6}\bar\partial\,\bar\partial^{-2}\delta\omega + O(\delta\omega^2)$$

$$\delta_\omega[LHS] = \delta_\omega[RHS]$$

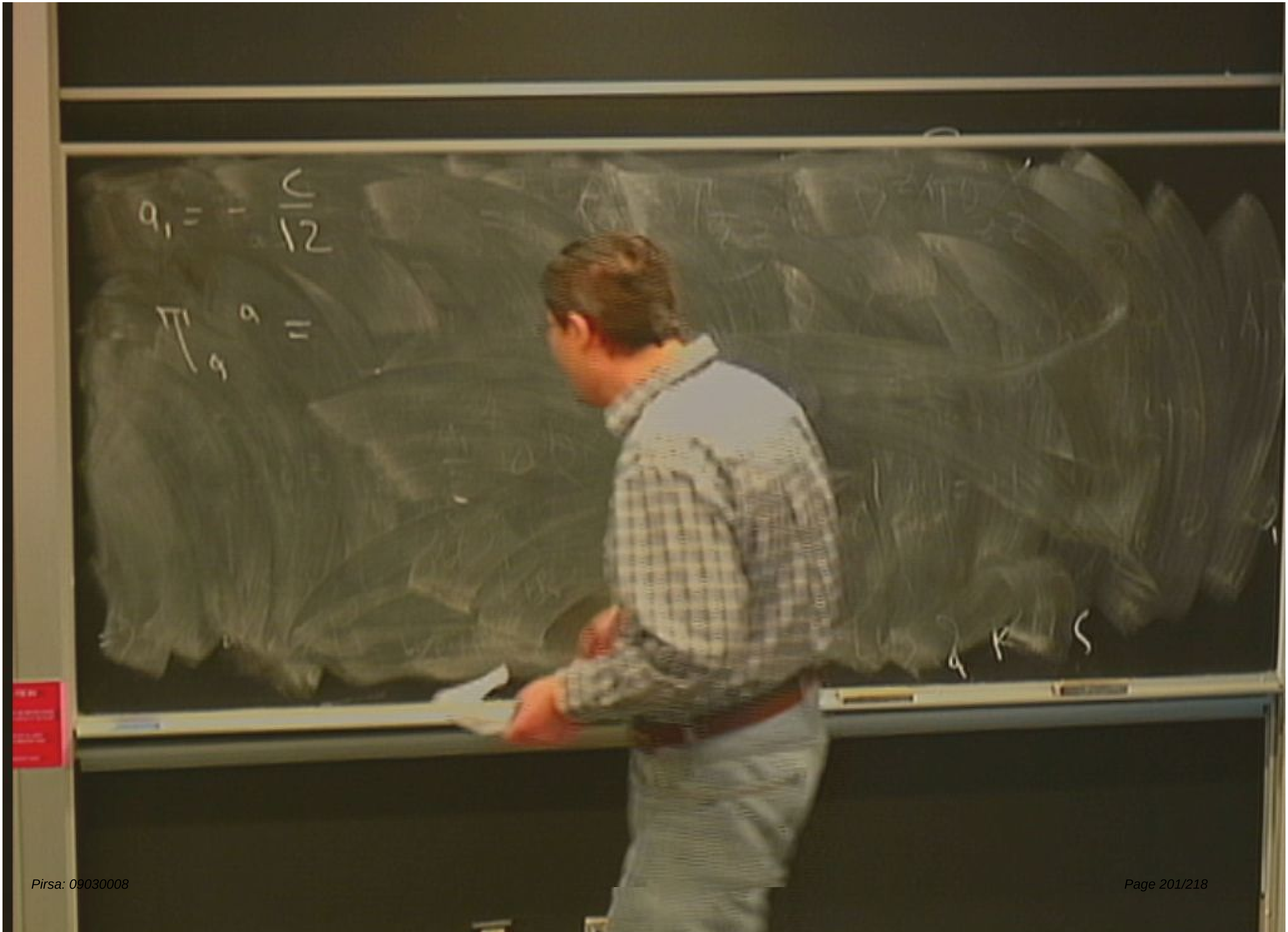
$$-\frac{c}{6}\bar\partial\,\bar\partial^{-2}\delta\omega = 4a_1\,\partial^2\bar\partial\,\delta\omega$$

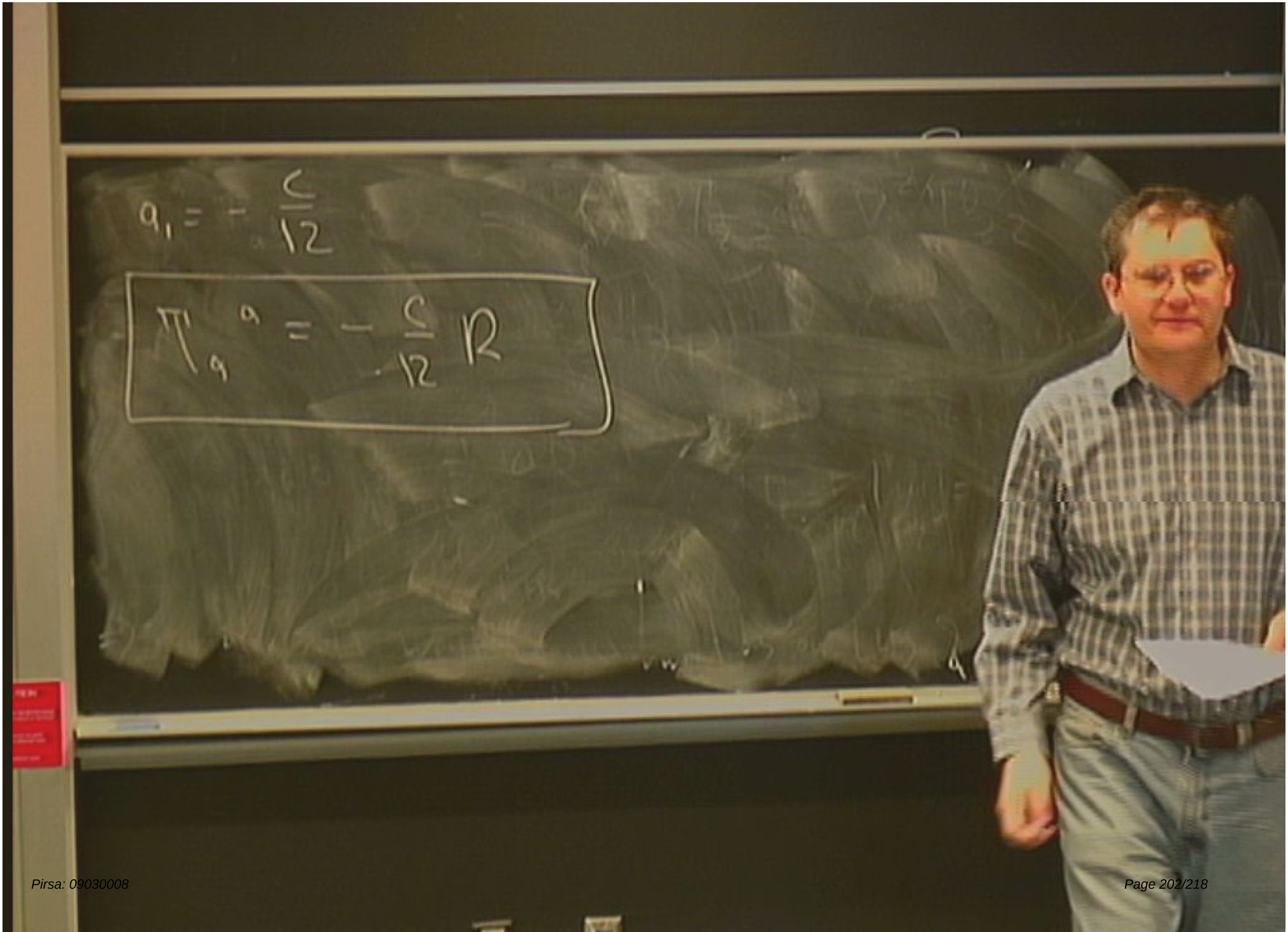
$$\delta_w T_{zz} = -\frac{c}{6}\,\partial\,\delta\omega$$

$$\delta_w[\mathrm{RHS}] = -\frac{c}{6}\nabla^2\partial^2\delta\omega = -\frac{c}{6}\bar\partial\bar\partial^{-2}\delta\omega + O(\delta\omega^2)$$

$$\delta_w[\mathrm{LHS}] = \delta_w[\mathrm{RHS}]$$

$$\left(2-\frac{c}{6}\right)\bar\partial\bar\partial^{-2}\delta\omega = 4a_1\,\partial^2\bar\partial\,\delta\omega$$


$a_1 = - \frac{c}{12}$
$\pi_a{}^a =$


$a_1 = -\frac{c}{12}$
$\boxed{T_a{}^a = -\frac{c}{12} R}$

$$a_1 = -\frac{c}{12}$$

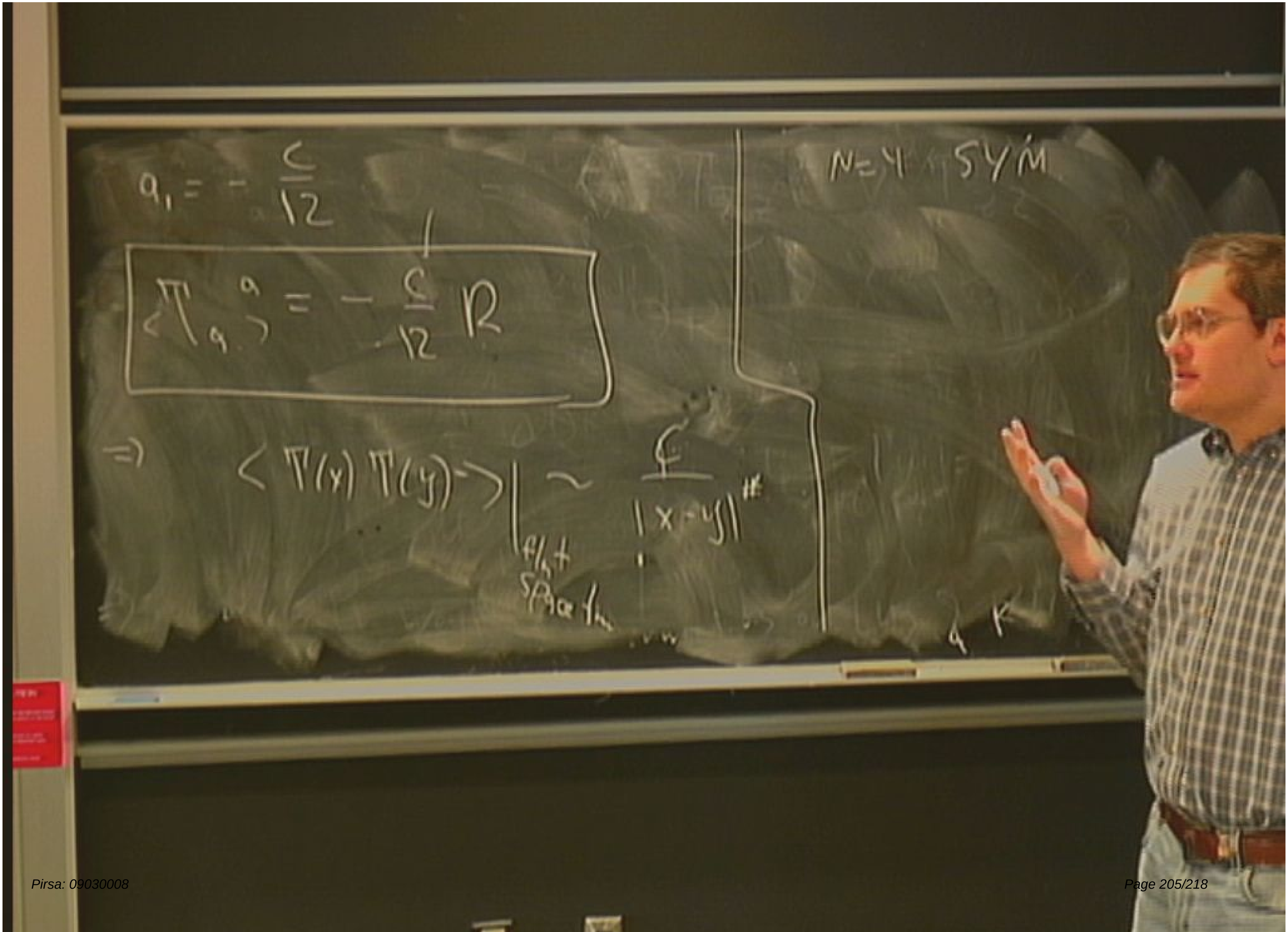
$$\boxed{\langle T_a{}^a \rangle = -\frac{c}{12} R}$$

$$\Rightarrow \quad \langle T(x)\, T(y) \rangle \sim \frac{1}{|x-y|^{\#}}$$

$a_1 = -\frac{c}{12}$

$$\boxed{\langle T_a{}^a \rangle = -\frac{c}{12} R}$$

$\Rightarrow \quad \langle T(x)\, T(y) \rangle \Big|_{\text{flat space time}} \sim \frac{c}{|x-y|^4}$


a1 = − c/12
<T_a^a> = − c/12 R
⇒ <T(x) T(y)> | ~ c/|x − y|^#
flat space
N=4 SYM

$$a_1 = -\frac{c}{12}$$

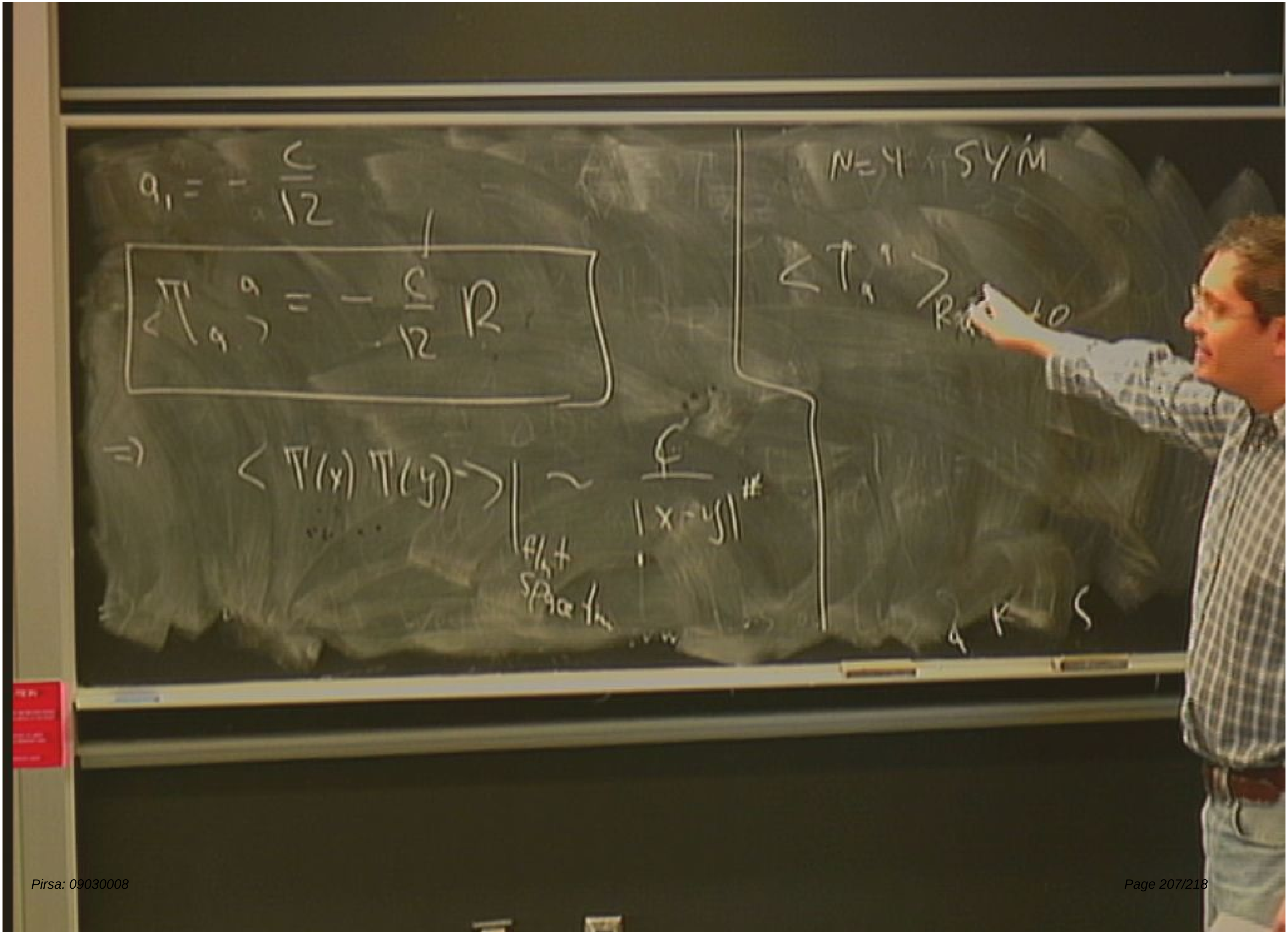
$$\boxed{\langle T_a{}^a \rangle = -\frac{c}{12} R}$$

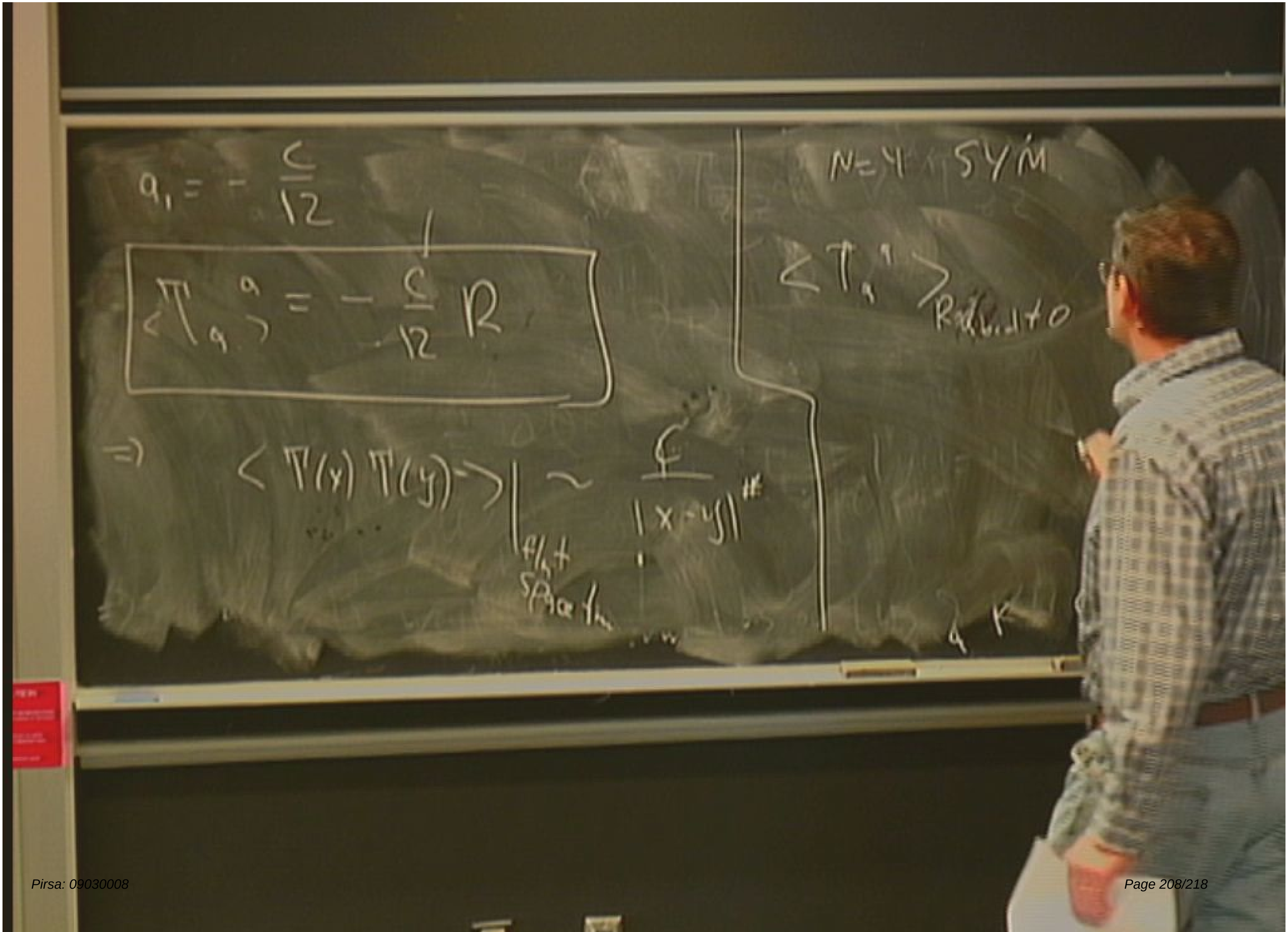
$$\Rightarrow \quad \langle T(x)\, T(y) \rangle \Big|_{\text{flat spacetime}} \sim \frac{c}{|x-y|^4}$$

$N=4$ SYM

$$\langle T_a{}^a \rangle$$

a K S


a1 = - c/12
<T_a^a> = - c/12 R
=> <T(x) T(y)> ~ c/|x-y|^4
N=4 SYM


a₁ = − c/12
⟨T_a^a⟩ = − c/12 R
⟨T(x) T(y)⟩ ~ c/|x − y|⁴
flat space
N=4 SYM
⟨T_a^a⟩

$a_1 = -\frac{c}{12}$

$$\boxed{\langle T_a{}^a \rangle = -\frac{c}{12} R}$$

$\Rightarrow \quad \langle T(x)\, T(y) \rangle \Big|_{\substack{\text{flat} \\ \text{space } t_m}} \sim \frac{c}{|x-y|^4}$

$N=4$ SYM

$\langle T_a{}^a \rangle_{R_{abcd} \neq 0} \propto$

$\propto c \left[R_{ab} R^{ab} - \frac{1}{3} R^2 \right]$

$a \quad K \quad S$

$$a_1 = -\frac{c}{12}$$

$$\boxed{\langle T_a{}^a \rangle = -\frac{c}{12} R}$$

$$\Rightarrow \quad \langle T(x)\, T(y) \rangle \Big|_{\text{flat space}} \sim \frac{c}{|x-y|^{4}}$$

$N=4$ SYM

$$\langle T_a{}^a \rangle_{R_{abcd} \neq 0} \propto c \left[R_{ab} R^{ab} - \frac{1}{3} R^2 \right]$$

$\mathbb{R}^{3,1}$

a K S

$$a_1 = -\frac{c}{12}$$

$$\boxed{\langle T_a{}^a \rangle = -\frac{c}{12} R}$$

$$\Rightarrow \quad \langle T(x)\, T(y) \rangle \Big|_{\text{flat space}} \sim \frac{c}{|x-y|^4}$$

$N=4$ SYM

$$\langle T_a{}^a \rangle_{R_{abcd} \neq 0} \propto c \left[R_{ab} R^{ab} - \frac{1}{3} R^2 \right]$$

$R^{3,1}$ $\quad R \times S^3$

$$a_1 = -\frac{c}{12}$$

$$\boxed{\langle T_a{}^a \rangle = -\frac{c}{12} R}$$

$$\Rightarrow \quad \langle T(x)\, T(y) \rangle \Big|_{\text{flat space}} \sim \frac{c}{|x-y|^4}$$

$N=4$ SYM

$$\langle T_a{}^a \rangle_{R_{ab} \neq 0} \propto c \left[R_{ab} R^{ab} - \frac{1}{3} R^2 \right]$$

$$R^{3,1} \rightarrow R \times S^3$$

Polyakov string is anomalous

unless $c = 0$

Polyakov string is anomalous

unless $c = 0 = c_X$

Polyakov string is anomalous

unless $c = 0 = c_x + c_g$

Polyakov string is anomalous

unless

$$c = 0 = c_X + c_g$$

$$\begin{array}{cc} c_X & c_g \\ \updownarrow & \updownarrow \\ D & (b, c)_{\lambda=2} \end{array}$$

Polyakov string is anomalous

unless

$$c = 0 = c_X + c_g$$

$c_X \updownarrow D$

$c_g \updownarrow (b, c)_{\lambda=2}$, $c_g = -26$

Polyakov string is anomalous

unless

$$c = 0 = c_X + c_g$$

$$c_X \leftrightarrow D \qquad c_g = -26 \leftrightarrow (b,c)_{\lambda=2}$$

$$c = D - 26 = 0 \Rightarrow \boxed{D = 26}$$