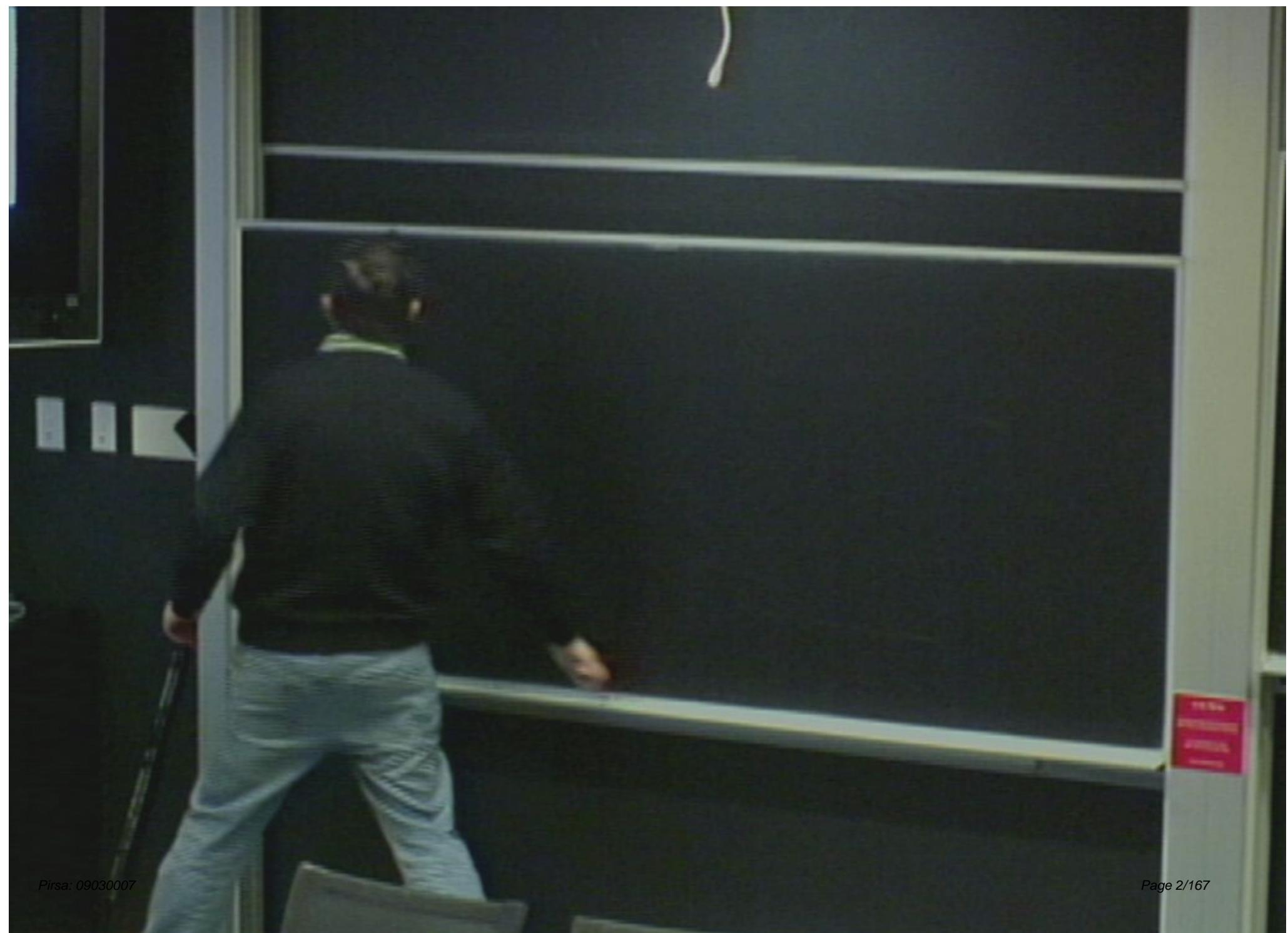


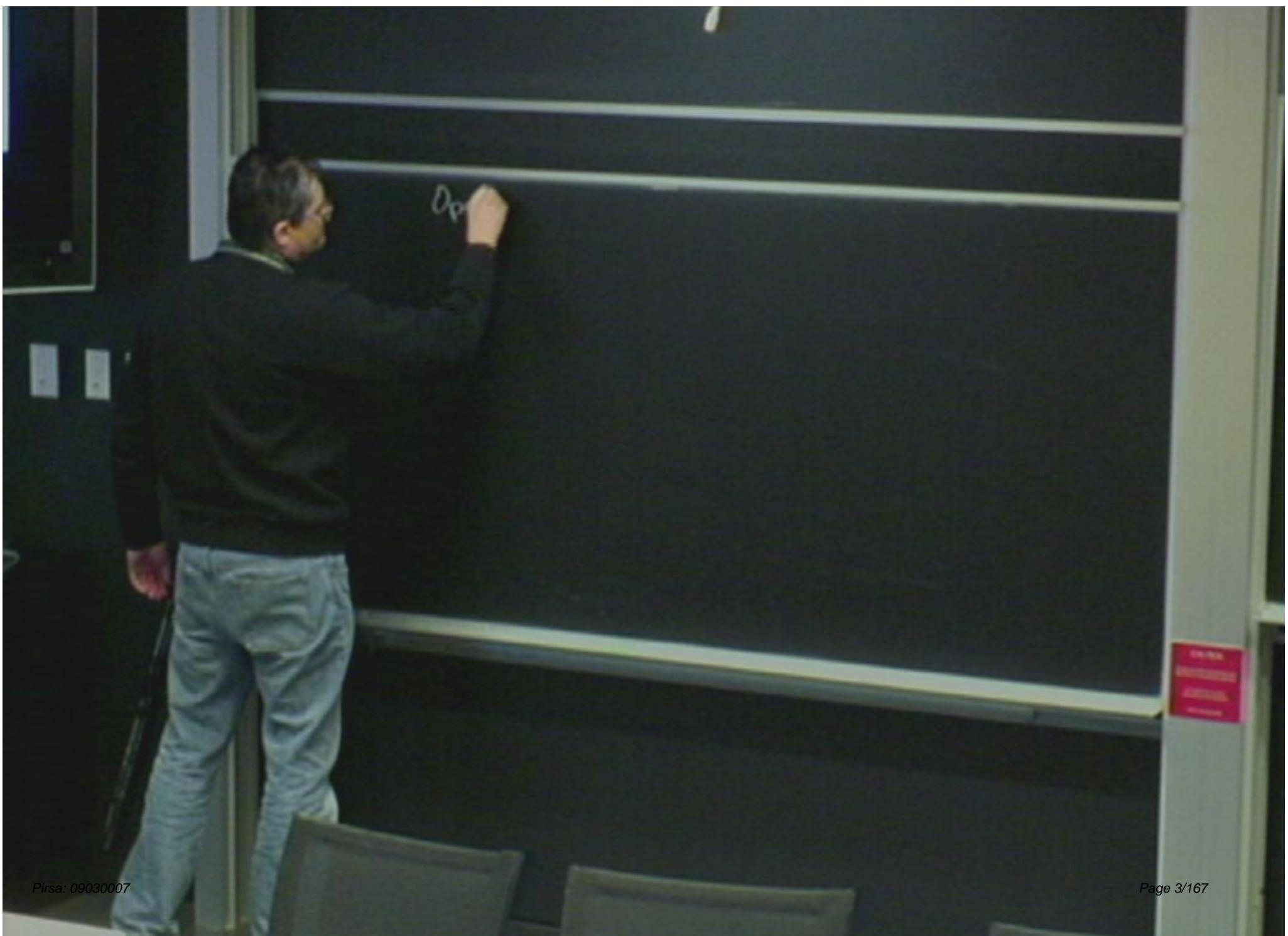
Title: Introduction to the Bosonic String

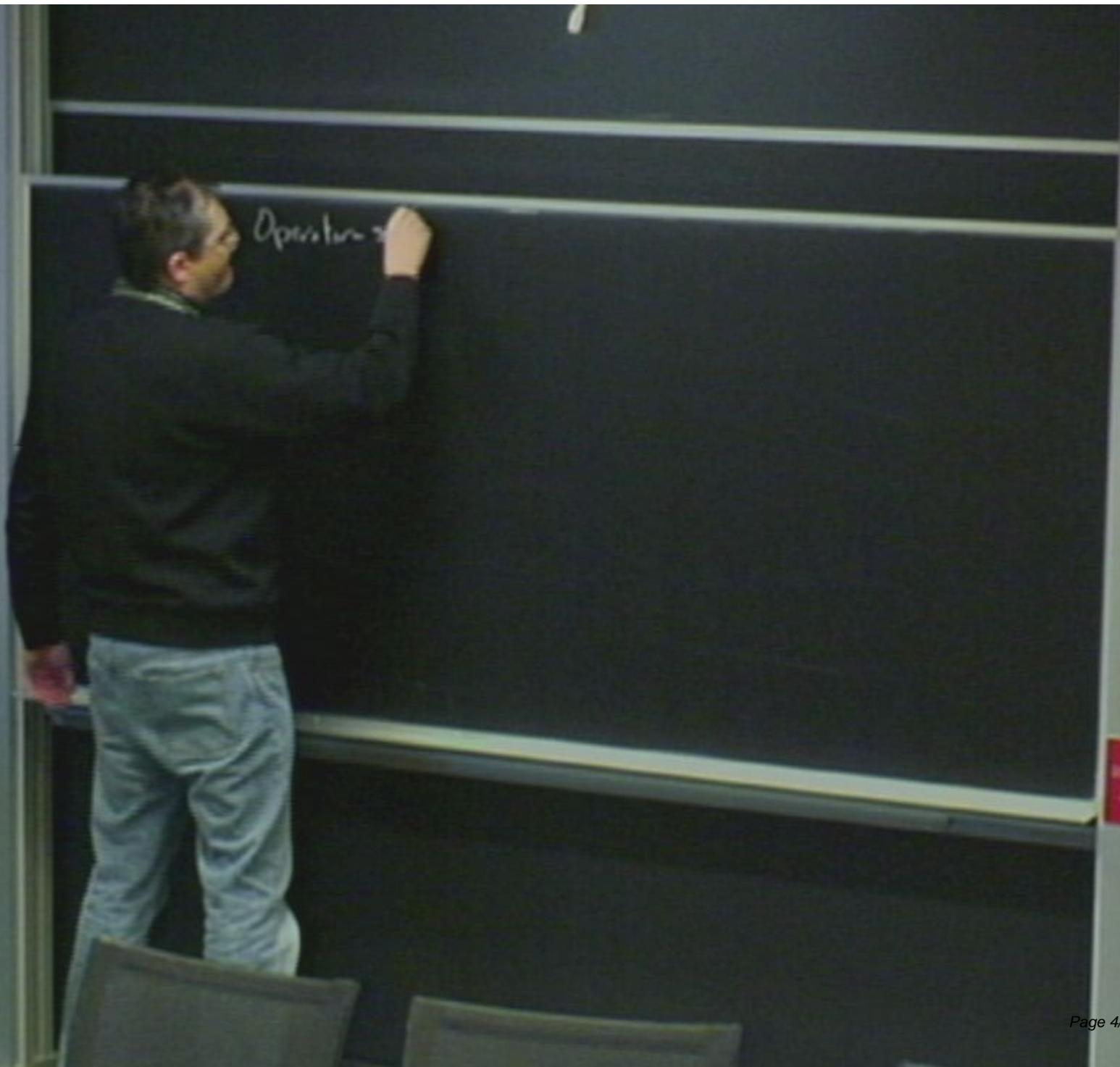
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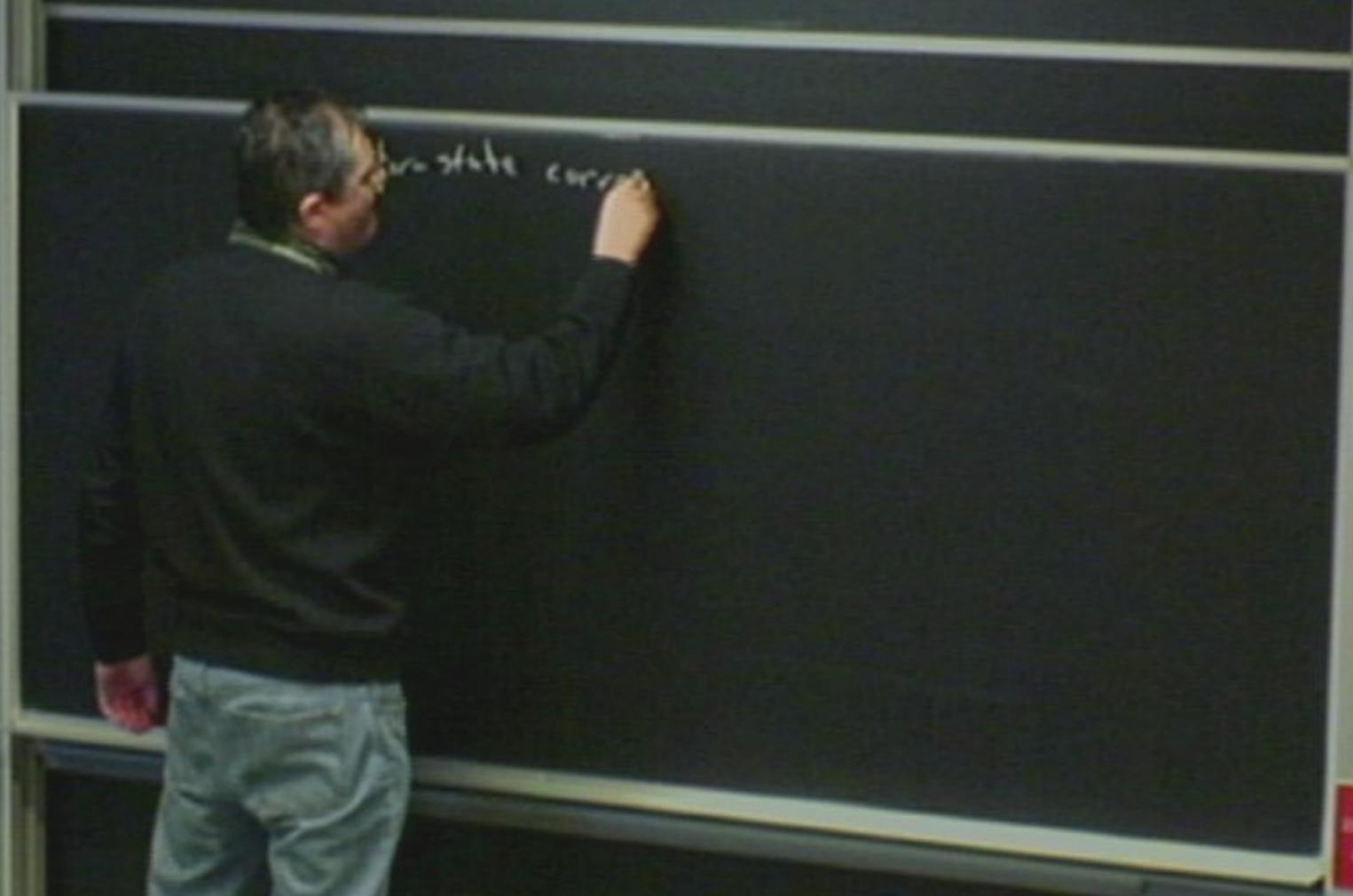
URL: <http://pirsa.org/09030007>

Abstract: This course provides a thorough introduction to the bosonic string based on the Polyakov path integral and conformal field theory. We introduce central ideas of string theory, the tools of conformal field theory, the Polyakov path integral, and the covariant quantization of the string. We discuss string interactions and cover the tree-level and one loop amplitudes. More advanced topics such as T-duality and D-branes will be taught as part of the course. The course is geared for M.Sc. and Ph.D. students enrolled in Collaborative Ph.D. Program in Theoretical Physics. Required previous course work: Quantum Field Theory (AM516 or equivalent). The course evaluation will be based on regular problem sets that will be handed in during the term. The primary text is the book: 'String theory. Vol. 1: An introduction to the bosonic string. J. Polchinski (Santa Barbara, KITP) . 1998. 402pp. Cambridge, UK: Univ. Pr. (1998) 402 p.' All interested students should contact Alex Buchel at abuchel@uwo.ca as soon as possible.









Operator-state  
Responsibility

Operator-state  $\rightarrow$  conservation.

Operator-state correspondence.

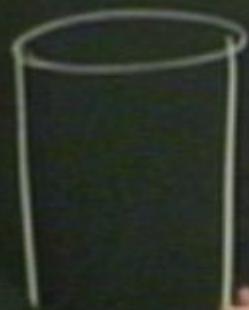
Operator-state correspondence.

operator-state correspondence.

Operator-state correspondence.



Operator-state correspondence.



Operator-state correspondence.



Operator-state correspondence.



Operator-state correspondence.



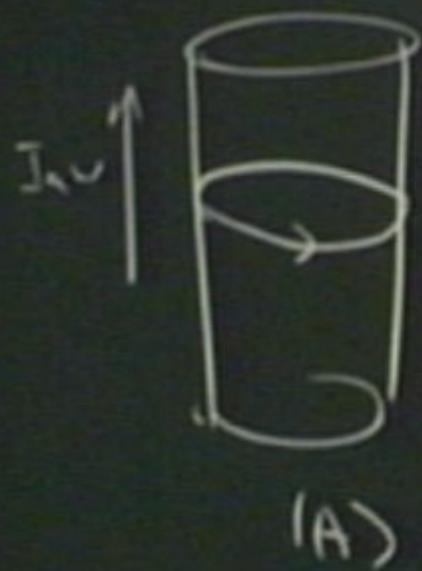
(A)

Operator-state correspondence.



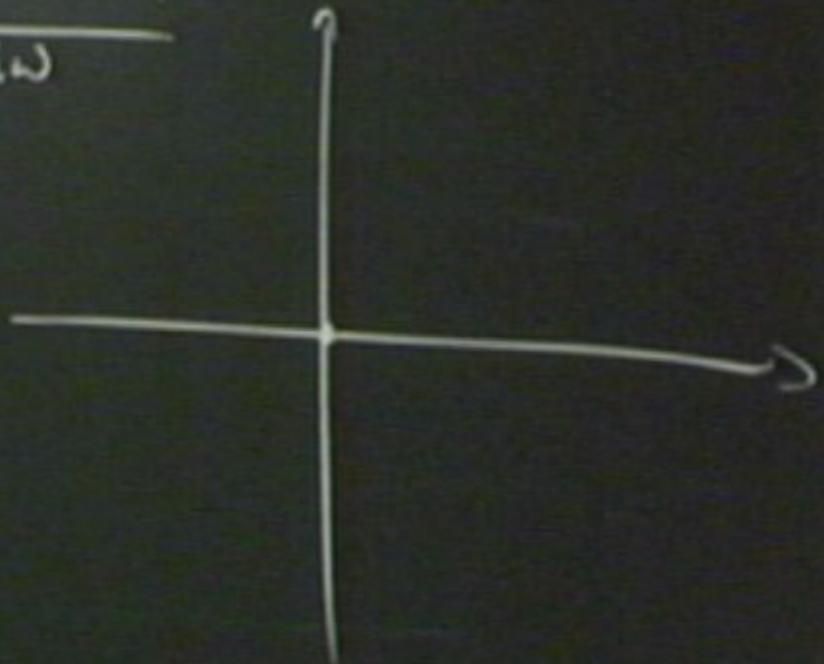
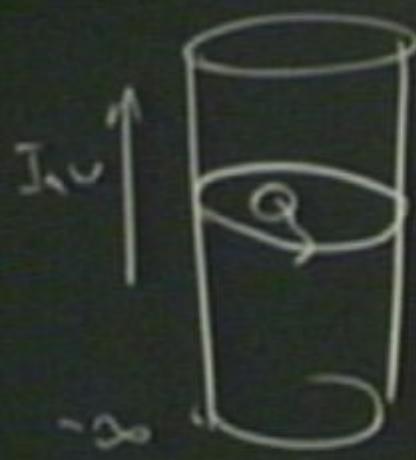
$|A\rangle$

Operator-state correspondence.



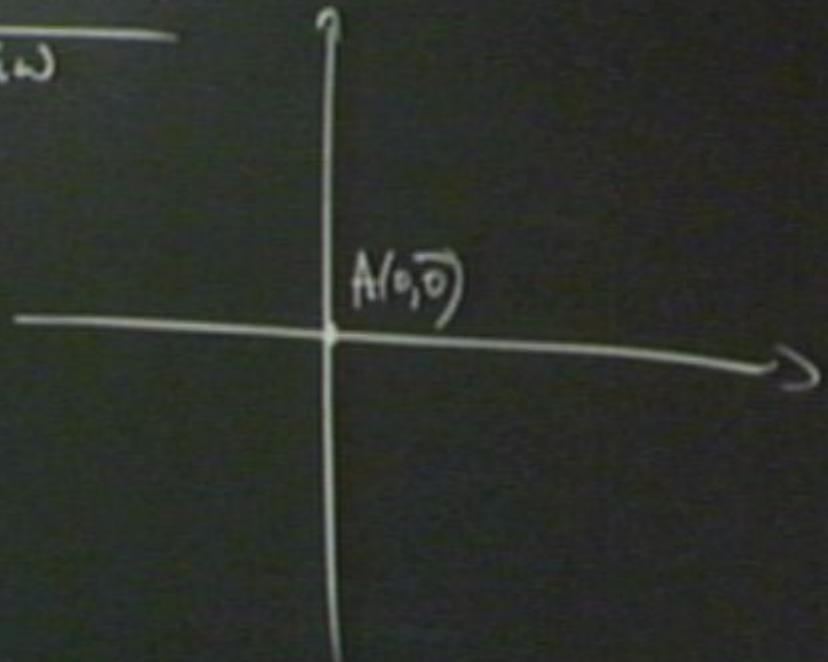
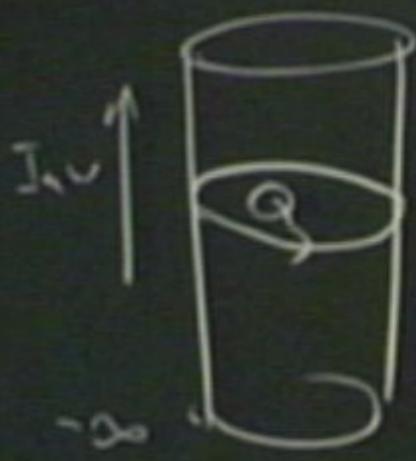
Operator-state correspondence.

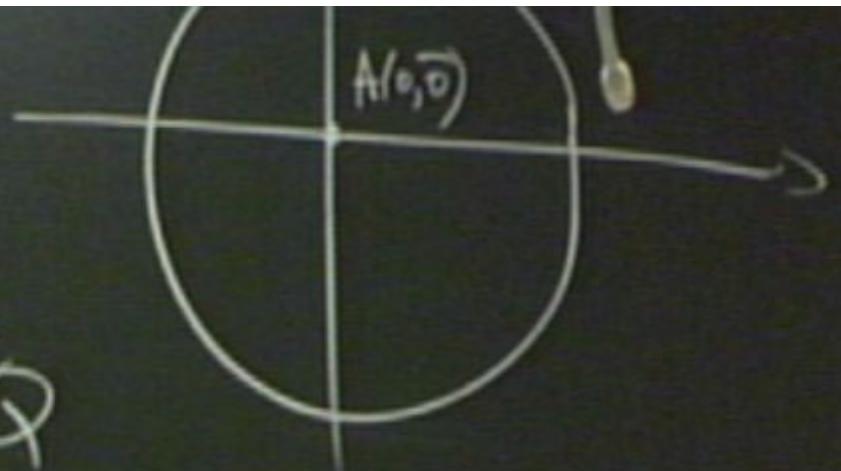
$$z = e^{-i\omega}$$



Operator-state correspondence.

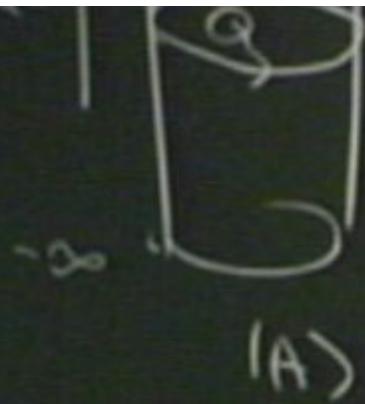
$$z = e^{-i\omega}$$



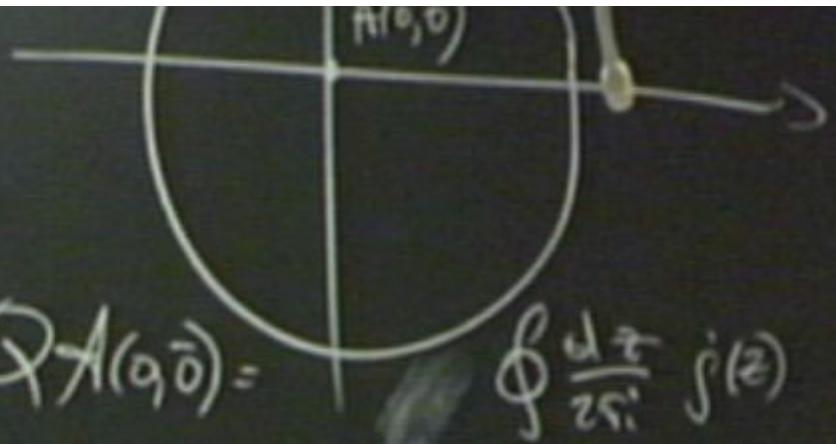


$$Q|A\rangle \Leftrightarrow Q$$



 $Q|A\rangle \Leftrightarrow$ 

$$Q_A(z, \bar{z}) = \oint \frac{dz}{2\pi i} f(z)$$



$$(A) \quad \stackrel{\sigma(A)}{\leftrightarrow} \text{map state} \rightarrow \text{state.} \quad Q\hat{A}(q, \bar{o}) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \delta(q - q') \delta(\bar{o} - \bar{o}') \hat{A}(q', \bar{o}')$$



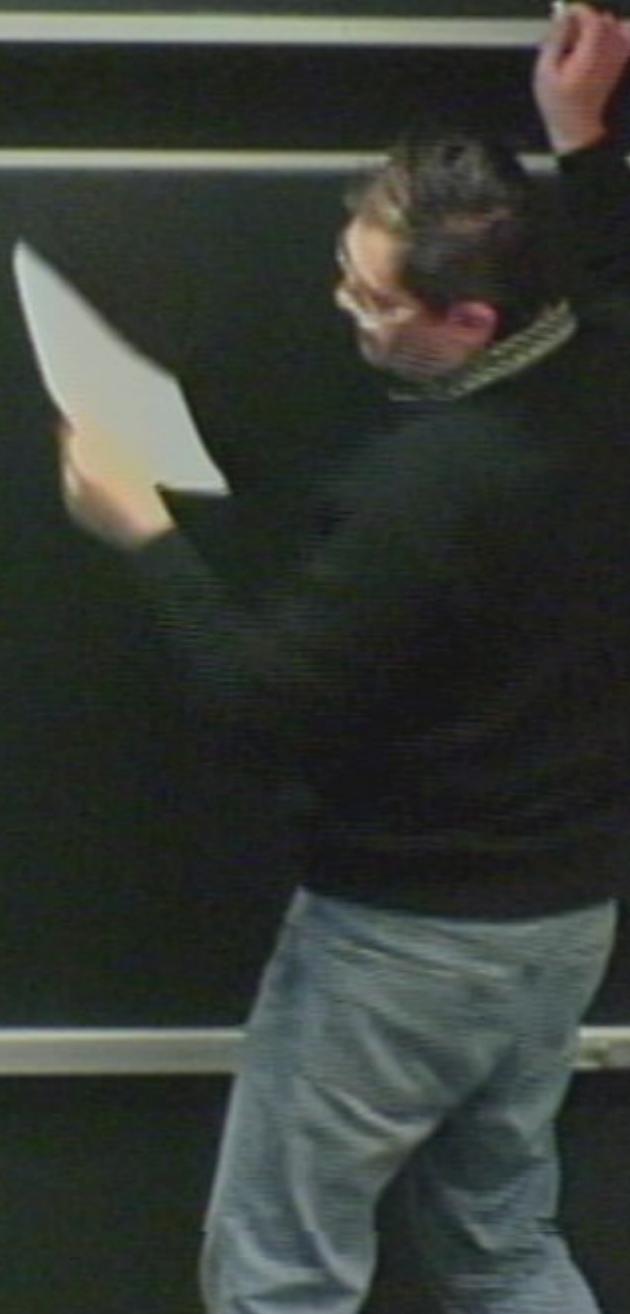
map state  $\rightarrow$  state.

$\mathcal{G}(t)(s) =$

new local operator

$\mathcal{G}^{\text{op}}(t)(s)$

operator  $\rightarrow$  operator



map state  $\rightarrow$  state.

$\tilde{\psi}(q_0) = 1$

new local operator

$\Psi \hat{F} \circ f(\tilde{\psi}(q))$

operator  $\rightarrow$  operator

$$\underbrace{L_n |A\rangle}_{\text{state}} = \oint \frac{dz}{2\pi i z} z^{n+1} \Psi(z) \hat{F}(q_0).$$



$\sim \infty$    
 $|A\rangle$

$\xrightarrow{\text{fins}}$   
to state

$Q|A\rangle \xleftrightarrow{|} \text{map state} \rightarrow \text{state}$

$Q|A(z, \bar{z})\rangle =$   
new local operator  $\int \frac{dz}{2\pi i} j(z) \delta(z)$   
 $\rightarrow$  operators  $\rightarrow$  operator

$\underbrace{j(z)}$



map state  $\rightarrow$  state.

$$\langle \psi | \psi(0) = 1$$

new local operator

$$\psi \hat{A} : \mathcal{F}(D)(z)$$

operator  $\rightarrow$  operator

$$|A\rangle \underset{\text{state}}{\equiv} \oint \frac{dz}{2\pi i} \underbrace{z^m \tau(z)}_{j(z)} \hat{A}(z) = \sum_n a_n \cdot \hat{A}(n)$$

$$\hat{A}(z) = \sum_{k=-\infty}^{\infty} z^{k-2} \langle k \cdot | A(z) \rangle$$



map state  $\rightarrow$  state

$$\langle \psi | \psi(0) \rangle = 1$$

new local operator

$$\langle \psi | \hat{A}(z, \bar{z}) | \psi(0) \rangle$$

operator  $\rightarrow$  operator

$$\underbrace{\langle L_n | A \rangle}_{\text{state to state}} \approx \oint \frac{dz}{2\pi i} z^{m_n} \underbrace{\langle \psi(z) | A(z, \bar{z}) | \psi(0) \rangle}_{J(z)} = L_n \cdot A(z, \bar{z})$$

$$\langle \psi(z) | A(z, \bar{z}) | \psi(0) \rangle = \sum_{k=-\infty}^{\infty} z^{-k-2} \underbrace{\int \frac{dz}{2\pi i} z^{m_n} z^{-k-2} L_k \cdot A(z, \bar{z})}_{\delta_{m_n k} \cdot 1}$$

map state  $\rightarrow$  state.

$$\mathcal{L}(z) = 1$$

new local operator

$$\mathcal{O} \leftarrow \mathcal{O}(z)$$

operator  $\rightarrow$  operator

$$\underbrace{\mathcal{L}_n |A\rangle}_{\text{state to state}} \equiv \oint \frac{dz}{2\pi i} z^{m_n} \underbrace{\mathcal{T}(z)}_{J(z)} A(z) = \mathcal{L}_n \cdot A(z)$$

$$\mathcal{T}(z) A(z) = \sum_{k=-\infty}^{\infty} z^{-k-2} \underbrace{\mathcal{L}_k \cdot A(z)}_{\text{def}} \\ \int \frac{dz}{2\pi i} z^{m_n} z^{-k-2} = \delta_{m_n k} \cdot 1$$

Recall

$$\nabla(\vec{r}) \cdot \vec{A}(0, \vec{r})$$

Recall

$$\mathcal{T}(\tau) A(0, \bar{z}) \sim \sum_{n=0}^{\infty} \frac{1}{\tau^{n+1}} \underbrace{A^{(n)}(0, \bar{z})}_{\hat{A}^{(n)}(0, \bar{z})}$$

$$\nabla A(t, \bar{z}) = -\epsilon \sum_{n=0}^{\infty} \frac{1}{n!} \left[ \partial_z V(t) \hat{A}^{(n)}(t, \bar{z}) + \overline{\partial} V(t) \hat{A}^{(n)}(t, \bar{z}) \right]$$

Recall

$$\nabla(z) A(0, \bar{z}) \sim \sum_{n=0}^{\infty} \frac{1}{z^{n+1}} \underbrace{f^n(0, \bar{z})}_{}$$

$$\nabla A(z, \bar{z}) = -\varepsilon \sum_{n=0}^{\infty} \frac{1}{n!} \left[ \partial z(z) A^{(n)}(z, \bar{z}) + \overline{\partial} \bar{z}(\bar{z}) \tilde{A}^{(n)}(z, \bar{z}) \right]$$

$$z \rightarrow z + \varepsilon U(z)$$

$$\bar{z} \rightarrow \bar{z} + \varepsilon \bar{U}(\bar{z})$$

$\epsilon \rightarrow \{U(\bar{z})\}$

If  $A$  is an operator on  $(L, h)$

$\epsilon \rightarrow \{U(\bar{z})\}$

If  $A$  is an operator of a given  $(L, T)$

$$T(R)A = -\frac{\hbar^2}{2} \Delta A(0) + \frac{1}{2} \nabla A(0) + \text{nonsingular terms}$$

$\zeta \rightarrow \{ U(\bar{z}) \}$

If  $A$  is an operator of a gison  $(h, T)$

$$T(R)A = - - - - \underbrace{\frac{h}{2^2} A(0) + \frac{1}{2} \partial A(0)}_{\text{Hun...}} + \text{nonsingular}$$

$\epsilon \rightarrow \{U(\bar{z})\}$

If  $A$  is an operator of a given  $(L, T)$

$\check{\vee} \quad T(R)A = - - - - \underbrace{\frac{h}{2^2} A(0) + \frac{1}{2} \partial A(0)}_{\text{non singular}} + \text{higher}$

$L_1 A = \partial A$

$\epsilon \rightarrow \{U(z)\}$

If  $A$  is an operator of a gison  $(L, h)$

$$\check{\nabla}(k) \neq -\frac{h}{2^2} A(0) + \frac{1}{2} \partial A(0) + \text{nonsingular terms}$$

$$L_{-1} \neq \partial A$$

$$L_0 \neq h \cdot A$$

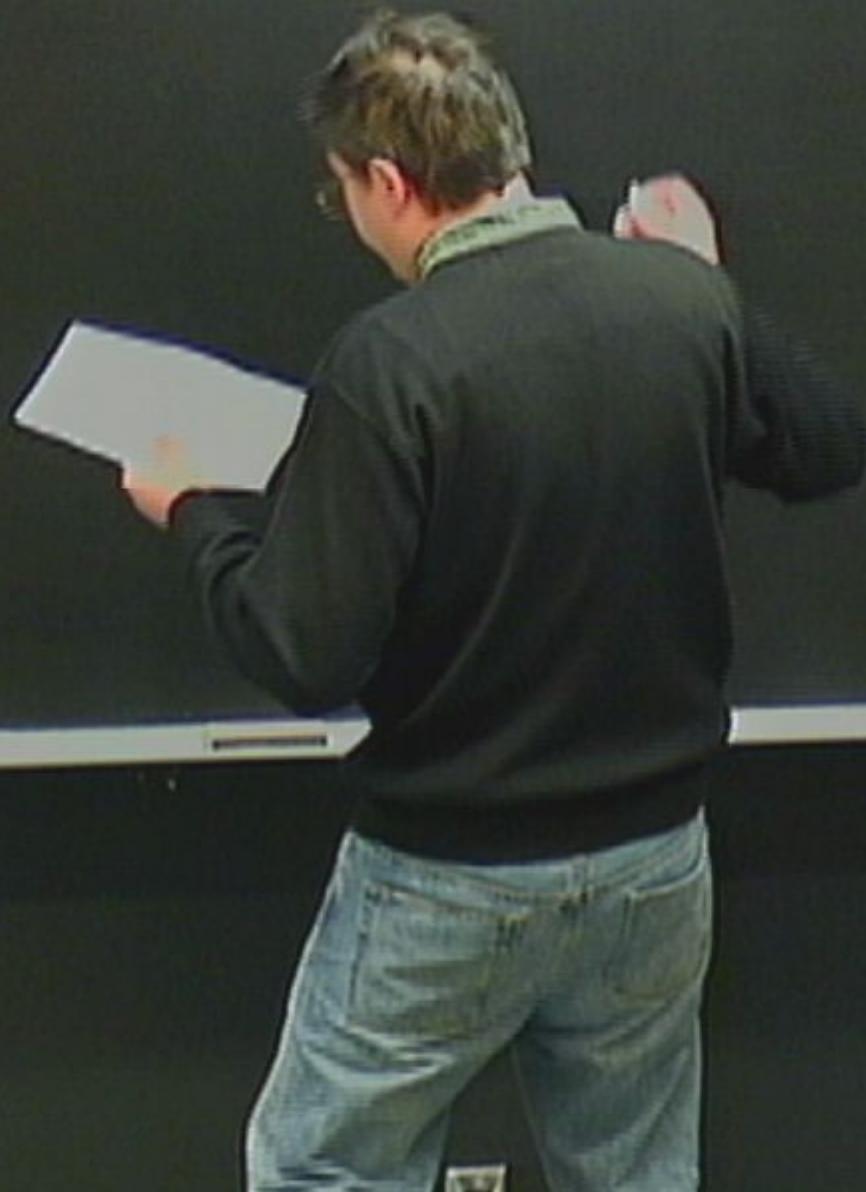
$$z \rightarrow \{U(z)\}$$

If  $A$  is an operator of a gison  $(L, h)$

$$\check{\nabla}(k)A = - - - \frac{h}{2^2} A(0) + \frac{1}{2} \partial A(0) + \text{nonsingular terms}$$

$$\left. \begin{array}{l} L_{-1} A = \partial A \\ L_0 A = h \cdot A \end{array} \right\} \quad \left. \begin{array}{l} \tilde{L}_{-1} A = \bar{\partial} A \\ \tilde{L}_0 A = \tilde{h} \cdot A \end{array} \right.$$

Primary fields  $\sigma$



Primary fields  $\sigma(\zeta)$

$$\langle T(z) \sigma \rangle = \frac{h}{z^L}$$



Primary fields  $O$  ( $h, \bar{h}$ )

$$T(z) O = \frac{h}{z^2} O + \frac{1}{z} \partial O + \dots$$

additional singular terms are missing

$$Q|A\rangle \leftrightarrow |A\rangle$$

map state  $\rightarrow$  state.

$$Q\gamma A(z\bar{z}) = \sum_{m,n} \frac{1}{z^m \bar{z}^n} \int(z) \partial^m \bar{\partial}^n A(z\bar{z})$$

new local operator    operators  $\rightarrow$  operator

$L_m |A\rangle$       State  $\rightarrow$  S'       $\oint \frac{dz}{2\pi i} z^{m+1} \Pi(z) \gamma A(z\bar{z}) = L_m \cdot \gamma A(z\bar{z})$

$\gamma A(z\bar{z}) = \sum_{k=-\infty}^{\infty} z^{k-1} L_k \cdot \gamma A(z\bar{z})$

$z^{m+1} z^{-k-1} = \delta_{m,k} \cdot 1$

Primary fields  $O$  ( $h, \zeta$ )

$$T(z) O = \frac{h}{z^2} O + \frac{1}{z} \partial O + \dots$$

additional regular terms are missing

$$L_n O = 0$$

$$h > 0$$

Primary fields  $\phi$  ( $h, \zeta$ )

$$T(z)\phi = \frac{h}{z^2}\phi + \frac{1}{z}\partial\phi + \dots$$

additional singular terms are omitted

$$L_n \phi = 0$$

$$n > 0 \quad (\text{for } n=0)$$

$$\frac{1}{z^{1-n}} L_n \phi$$



(for  $n=1$ )  $L_0 \circ L_1 \circ$

$O$  is called a highest weight operator.

If  $L_n O = 0 \quad n > 0$



$O$  is called a highest weight operator.

If  $L_n O = 0 \quad \underline{n \geq 0}$

Note

$O$  is called a highest weight operator.

If  $L_n O = O \quad \underline{n \geq 0}$

Note  $L_0 |h\rangle = h |A\rangle$

$$L_0(L_n |x\rangle) = (h - n) |x\rangle$$

$\mathcal{O}$  is called a highest weight operator.

If  $L_n \mathcal{O} = \mathcal{O}$   $n > 0$  If we act with

Note  $L_0 |h\rangle = h |h\rangle$   
 $L_0 (L_n |x\rangle) = (h - n) |x\rangle$  we finally reach the highest weight state

$O$  is called a highest weight operator.

If  $L_n O = 0 \quad n > 0$  If we act with

Note  $L_0 |h\rangle = h |A\rangle$  bunch of  $L_n (n > 0)$   
 $L_0 (L_n |x\rangle) = (h - n) |x\rangle$  we finally reach  
the highest weight state  
Implicitly  $L_0$  spectrum is bounded from below

## Unitary CFT's.

Unitary CFT:

$$\langle \mathbb{I} \rangle$$

a positive linear  
product on a  
Hilbert space.

is

$$L_m^* = L_{-m}$$

dof o D o  
unitary  
CFT.



## Unitary CFT's.

Unitary CFT:

$\langle \cdot | \cdot \rangle$   
is  
a positive linear  
product on a  
Hilbert space.

$$\underline{L_m^+ = L_{-n}}$$

dof of a  
unitary  
CFT

$$\langle \alpha | A \beta \rangle =$$

## Unitary CFT's.

Unitary CFT:

$$\langle \cdot | \cdot \rangle$$

is

$$L_m^+ = L_{-n}$$

dof of a  
unitary  
CFT

a positive linear  
product on a  
Hilbert space.

$$\langle \alpha | A \beta \rangle = \langle A^\dagger \alpha | \beta \rangle$$

a positive linear

product on a

Hilbert space.

$\langle A | \alpha \rangle$

i)  $h_0 > 0$  in a unitary CFT(1+1)

0

a positive linear  
product on a  
Hilbert space

(i)  $h_0 > 0$  in a unitary CFT ( $\omega$ )

(ii)  $U$  is (anti-)holomorphic iff  $(h, \tilde{h})_{(h=0)} = 0$

$$\langle 0 | 2L_0 | 0 \rangle = 2^4 \sigma \cdot \langle 0 | 0 \rangle$$

$$\langle 0 | z L_0 | 0 \rangle = 2 \hbar \sigma \cdot \langle 0 | 0 \rangle$$

||

$$\langle 0 | [L_i, L_j] | 0 \rangle$$

$$\langle \uparrow | 2L_0 | \downarrow \rangle = 2^4 \cdot \langle 0|0 \rangle$$

||  $\rightarrow$  is a highest weight state

$$\langle \uparrow | [L_-, L_+] | \downarrow \rangle =$$

$$\langle \Psi | 2L_0 | \Psi \rangle = 2h_F \cdot \langle 0 | 0 \rangle$$

||  $\rightarrow$  is a highest weight state

$$\langle \Psi | [L_-, L_+] | \Psi \rangle = \langle \Psi | L_- L_+$$

$$\langle \star | 2L_0 | \star \rangle = 2L_0 \cdot \langle 0 | 0 \rangle$$

||  $\rightarrow$  is a highest weight state

$$\langle \star | [L_f, L_b] | \star \rangle = \langle \star | L_f L_b | \star \rangle - \langle \star | L_b L_f | \star \rangle$$

$$\langle \uparrow | 2L_0 | \downarrow \rangle = 2 \hbar \uparrow \cdot \langle 0 | 0 \rangle$$

||  $\rightarrow$  is a highest weight state

$$\begin{aligned} \langle \uparrow | [L_x, L_z] | \downarrow \rangle &= \langle \uparrow | L_+ L_- | \downarrow \rangle - \langle \uparrow | L_- L_+ | \downarrow \rangle \\ &= |L_-| \uparrow \rangle|^2 \end{aligned}$$

$$\langle \star | 2L_0 | \star \rangle = 2h_F \cdot \langle \star | \star \rangle$$

||  $\rightarrow$  is a highest weight state

$$\begin{aligned} \langle \star | [L_r, L_s] | \star \rangle &= \langle \star | L_r L_s | \star \rangle - \langle \star | L_s L_r | \star \rangle \\ &= |L_r| \star \rangle^2 \geq 0 \quad \Rightarrow h_F = \end{aligned}$$

If  $A$  is an operator of a gison  $(L, h)$

$$\check{V}(\lambda)A = \frac{\lambda}{2^2} A(0) + \frac{1}{2} \partial A(0) + \text{nonsingular term.}$$

$$\left. \begin{array}{l} L_{-1} \tau = \partial A \\ L_0 \tau = h \cdot A \end{array} \right\} \quad \left. \begin{array}{l} \tilde{L}_{-1} A = \bar{\partial} A \\ \tilde{L}_0 A = \tilde{h} A \end{array} \right.$$

$$= |L_+|k\rangle^2 \geq 0 \Rightarrow k \geq 0$$

operator  $\mathcal{L}$  given  $(L, h)$   $\mathbb{C} \rightarrow \mathbb{C}$

$$\mathcal{L} = -\frac{\hbar^2}{2} \Delta + \frac{1}{2} \nabla A(\phi) + \text{nonsingular term}$$

$$\begin{cases} \tilde{L}_- A = \tilde{\nabla} A \\ \tilde{L}_+ A = \tilde{h} A \end{cases}$$

(iii) Suppose  $\{x_n\}$  is bounded above.

$$\exists M > 0$$

$$\forall n \in \mathbb{N}$$

$$x_n \leq M$$

(ii) Suppose  $h_1 = 0$  &  $\lambda \neq 0$

$\lambda$  is a root of  $f(\lambda)$

$f(\lambda) = 0$

$\lambda^2 + \lambda - 2 = 0$

$\lambda^2 + \lambda + 1 = 1$

$\lambda^2 + \lambda + 1 = 1$

(ii) Suppose  $h_1 = 0 \Rightarrow$  In a unitary CFT  
 $g_1$  is anti-holomorphic

(ii) Suppose  $h_A = 0 \Rightarrow$  In a unitary CFT  
 $g : \underline{\text{antiholomorphic}}$

$$z h_A (A|z) = L_{-1}.$$

(ii) Suppose  $h_A = 0 \Rightarrow$  In a unitary CFT  
 $g$  is anti-holomorphic

$$z h_A (c) |A\rangle = |L_{-1} \cdot A|^2$$

(iii) Suppose  $h_A = 0 \Rightarrow$  In a unitary CFT  
 $g : \simeq \underline{\text{anti-holomorphic}}$

$$0 = \langle h_A | A | A \rangle = | L_{-1} \cdot A |^2 \Rightarrow L_{-1} \cdot A = 0$$



(ii) Suppose  $h_A = 0 \Rightarrow$  In a unitary CFT

$g : \Sigma$  anti-holomorphic

$$\tilde{O} = \langle h_A | g A | d \rangle = \langle L_{-1} \cdot A | A^2 | d \rangle \Rightarrow \underbrace{\langle L_{-1} \cdot A |}_{\partial A} = 0$$

(ii) Suppose  $h_A = 0 \Rightarrow$  In a unitary CFT.

$g$  is anti-holomorphic

$$\langle \cdot | = z h_A |A| \cdot \cdot \rangle = |L_{-1} \cdot A|^2 \Rightarrow L_{-1} \cdot A = 0$$

$$\text{If } A = 0 \Rightarrow A = h(\bar{z})$$

$$A_i(z, \bar{z}) A_j(\bar{z}, \bar{z})$$

A<sub>i</sub>



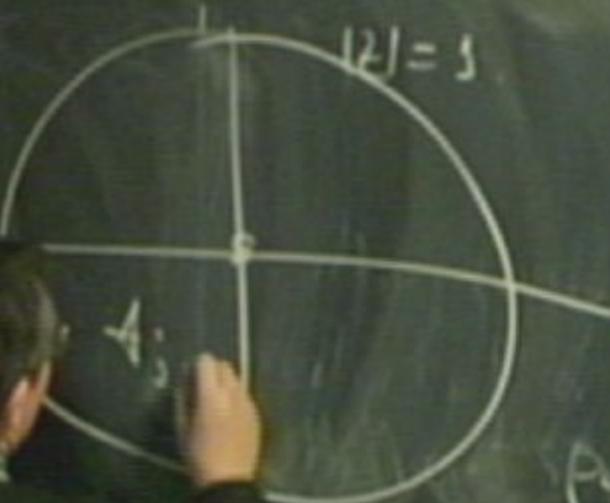
$$A_i(z, \bar{z}) A_j(\bar{z}, \bar{z}) = \sum_k C_{ijk} A_k(z, \bar{z})$$

some  $z, \bar{z}$  dependence



$$A_1(z, \bar{z}) A_1(\bar{z}, z) = -1 \quad (\text{for } z \neq 0)$$

$$|z| < 1.$$



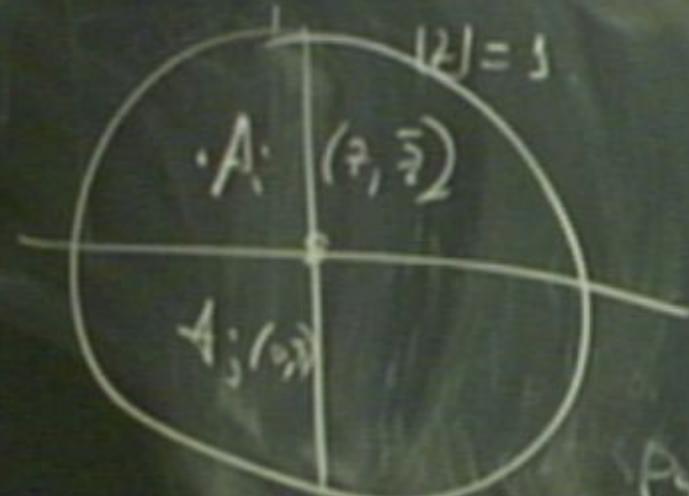
some  $z, \bar{z}$  dependence

anti-holomorphic iff  $(h=0) \quad h=0$

$$A_1(z, \bar{z}) A_1(\bar{z}, \bar{z}) = -1 \quad (\text{ijk} \rightarrow k(0,0))$$

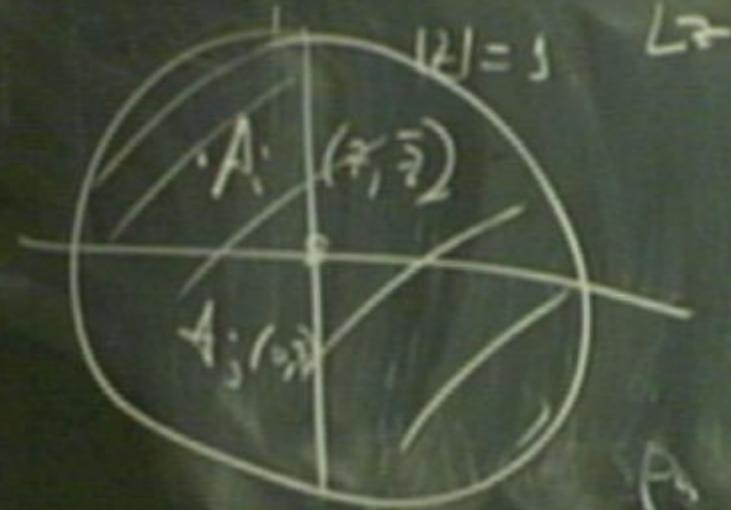
+ 2, \bar{2} \text{ dependence}

$|z| < 1.$



$(h, \tilde{h})$  holomorphic iff  $(h = f)$

$$|z| < 1$$

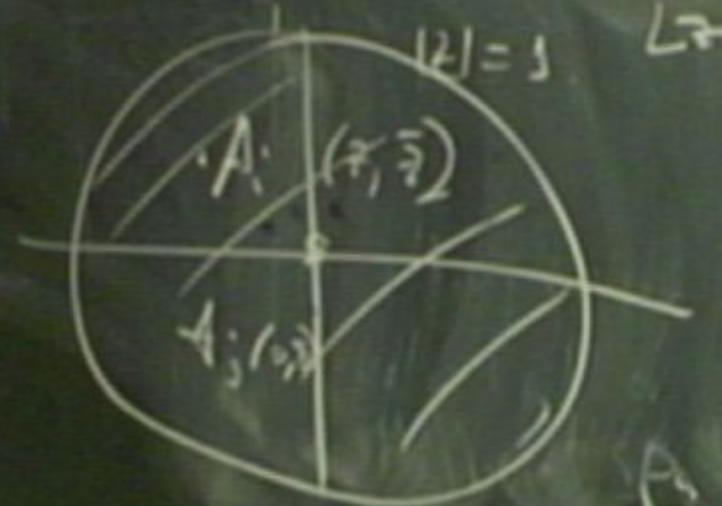


sum  $z, \bar{z}$  dependence



$(h, \tilde{h})$  holomorphic iff  $f'(h=0) = 0$

$$|z| < 1$$



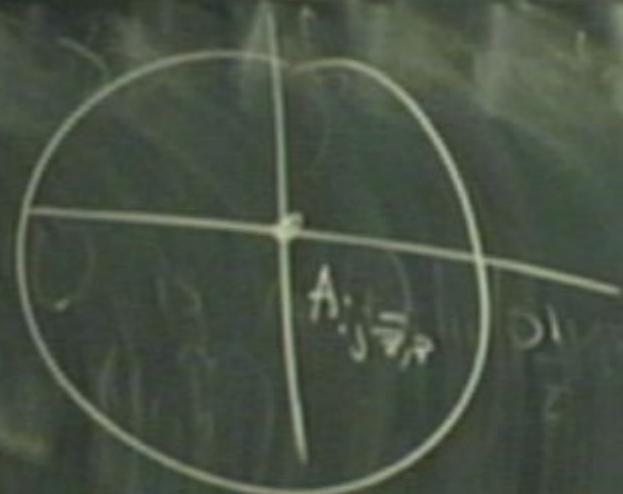
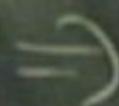
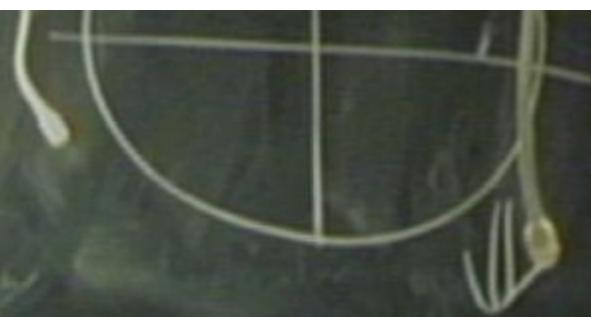
$\rightarrow$   $z, \bar{z}$  dependence,

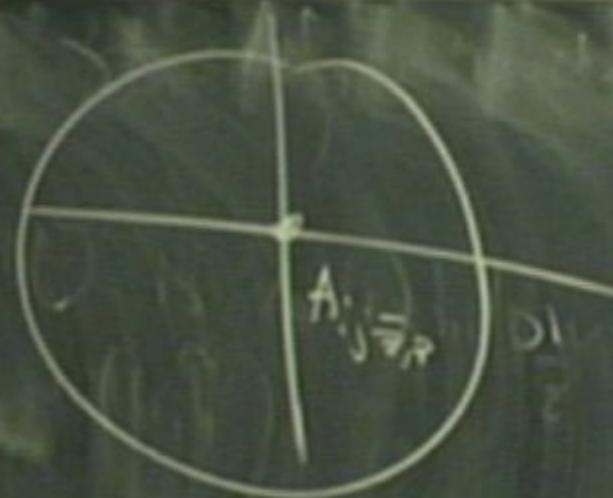


holomorphic if  $(h, \bar{h}) = (0, 0)$



P<sub>1</sub>



$t_{ij}(r)$  $P_s$  $\Rightarrow$ 

$$A_{ij} \approx \sum$$

$$c_{ij} A$$

$t_{ij}(0,0)$  $P_3$  $\Rightarrow$ 

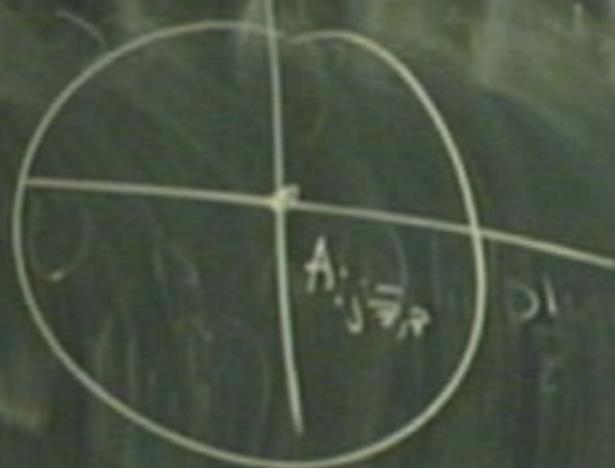
$$A_{ij} z, \bar{z} = \sum c_j^k A_k(0,0)$$
$$\bar{z} \tilde{h}_k - \tilde{h}_j - \tilde{h}_i$$

$$t_1(1,2)$$

$$\cancel{t_2'(2,3)}$$

A<sub>2</sub>

$\Rightarrow$



$$A_{ij} \in \mathbb{R} = \sum_{k=1}^n c_{ij}^k A_k(0,0)$$

Below the equation, there is a diagram showing two vectors originating from the same point. One vector is labeled  $\tilde{h}_k - \tilde{h}_i - \tilde{h}_j$ .

Polyakov Path integral.

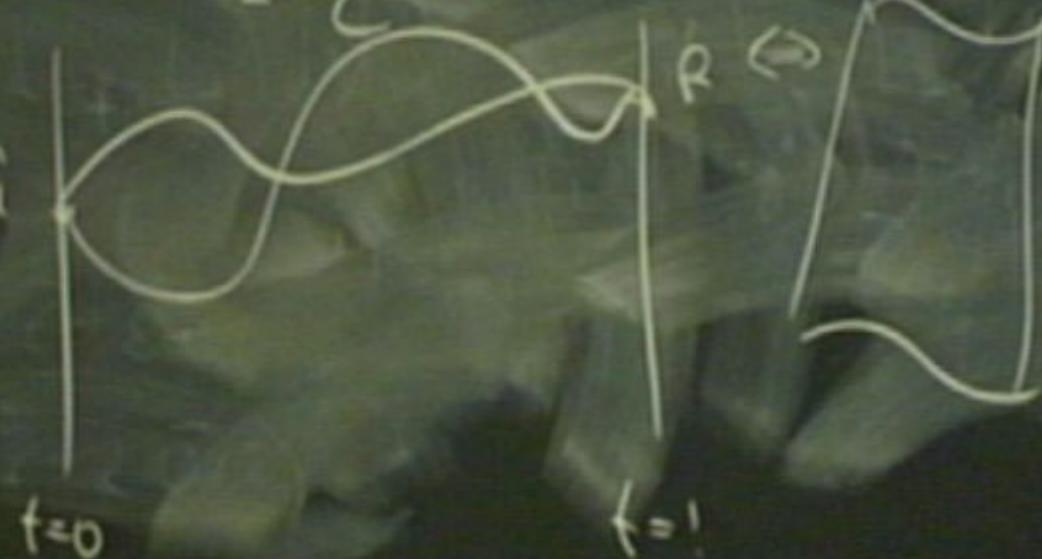
$$\int \mathcal{D}\vec{x} e^{iS_{\text{eff}}/\hbar} = \frac{\text{Polyakov Path integral.}}{\mathcal{Z}}$$

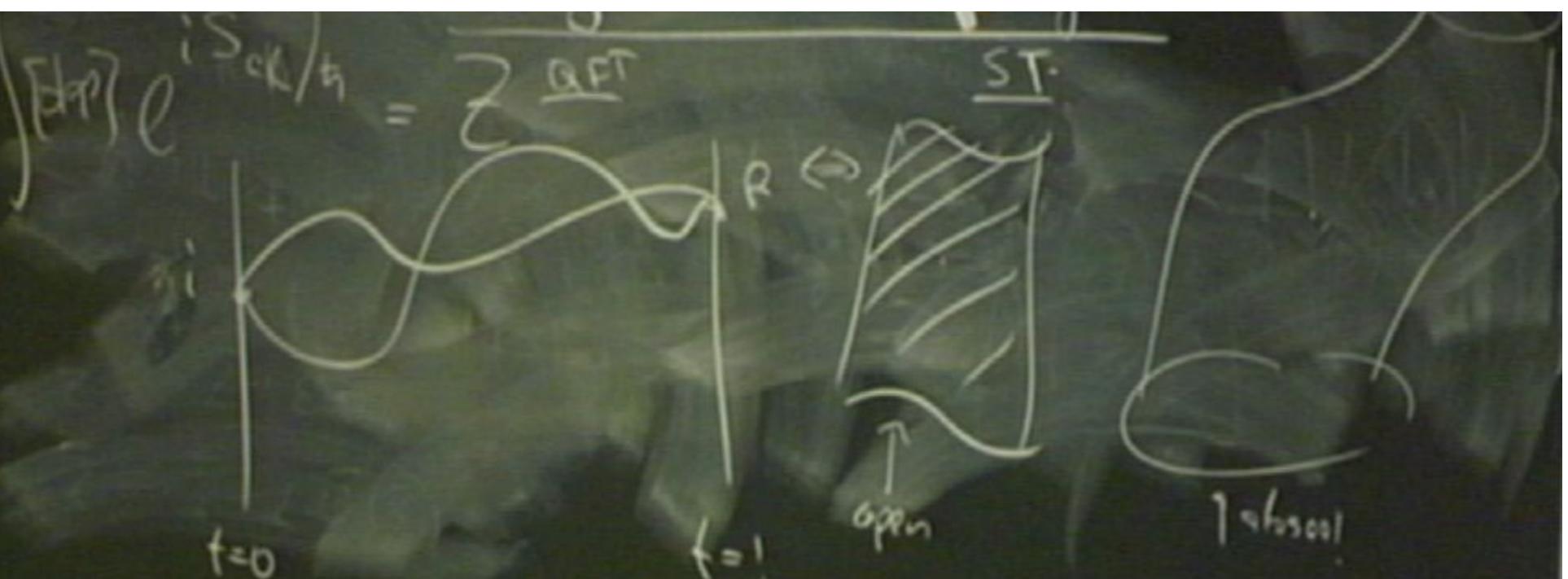
$$\int [dp] e^{i S_{\text{cl}}/\hbar} = \frac{\text{Polyakov Path integral.}}{Z}$$

$$\int_{\Gamma} \mathcal{D}\vec{r}(t) e^{i S_{\text{cl}}/\hbar} = \frac{\text{Path integral}}{Z}$$



$$\int_{\text{Polyakov}} \mathcal{D}[\bar{\phi}] e^{i S_{\text{eff}}/\hbar} = \frac{\mathcal{Z}_{\text{QFT}}}{\mathcal{Z}_{\text{ST}}}$$





$$T L^{-1} \stackrel{?}{=} \Rightarrow \frac{\partial L^{-1}}{\partial A} = 0 \\ \partial A = 0 \Rightarrow A = h(\bar{z})$$

$f=0$  $f=1$  open

1 = basal

⇒ Interactions are implicit in

$f=0$

$f=1$  open

1st class

→ Interactions are implicit in weak ol. strat. topologies.

$f=0$

$f=1$  open

1st

⇒ Interactions are implicit in weakly coupled hydrodynamics.

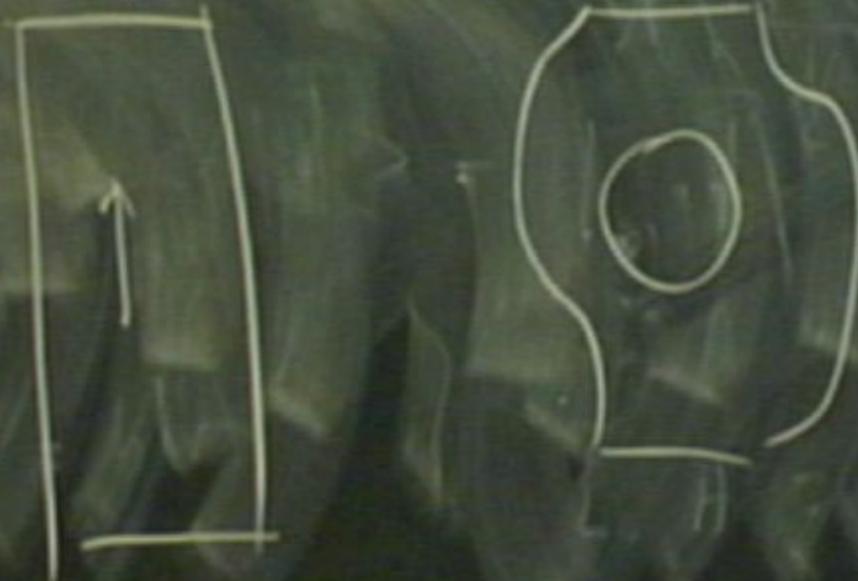


$f=0$

$f=1$  open

labeled

⇒ Interactions are implicit in workflow logic



$f=0$

$f=1$  open

lens

⇒ Interactions are implicit in most short logic gates

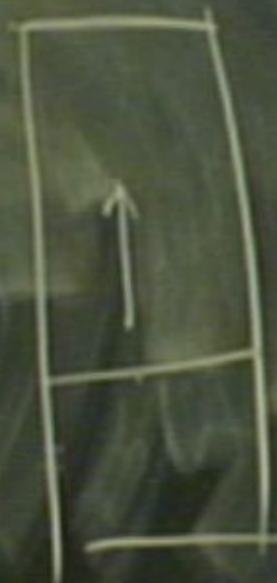


$f=0$

$f=1$  open

1 absool.

⇒ Interactions are implicit in most short epidemiology.



$f=0$  $f=1$  open

1st

⇒ Interactions are implicit in worldsheet topology.

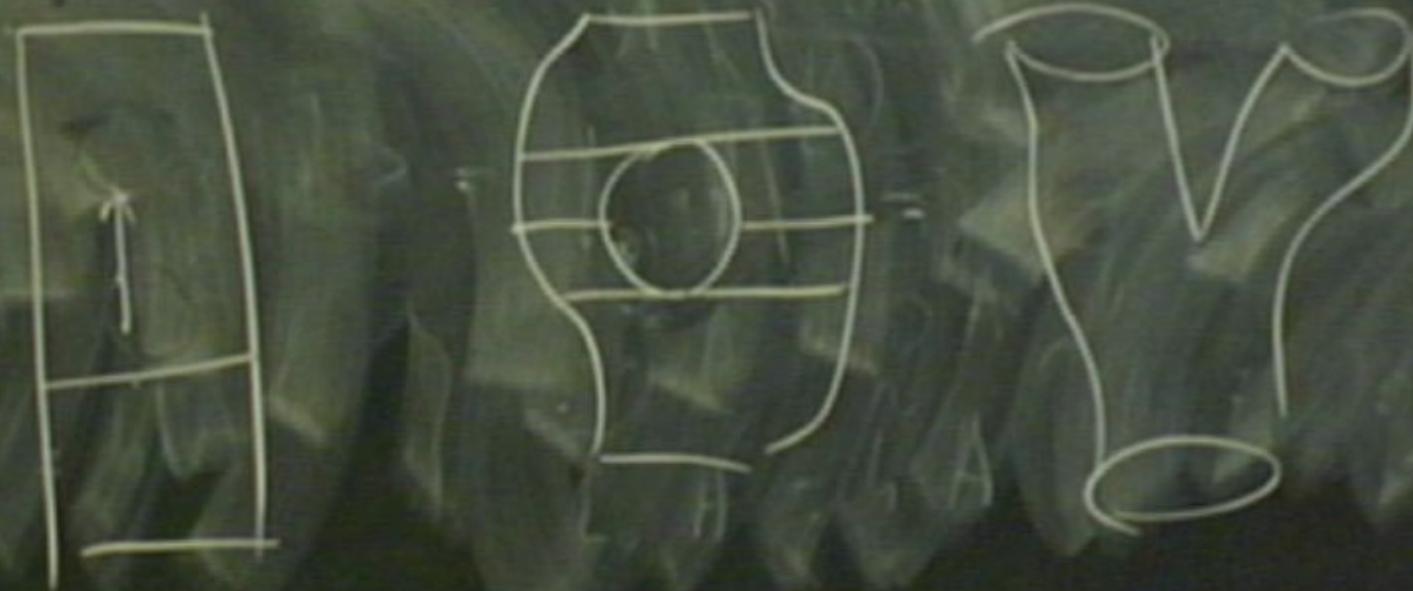


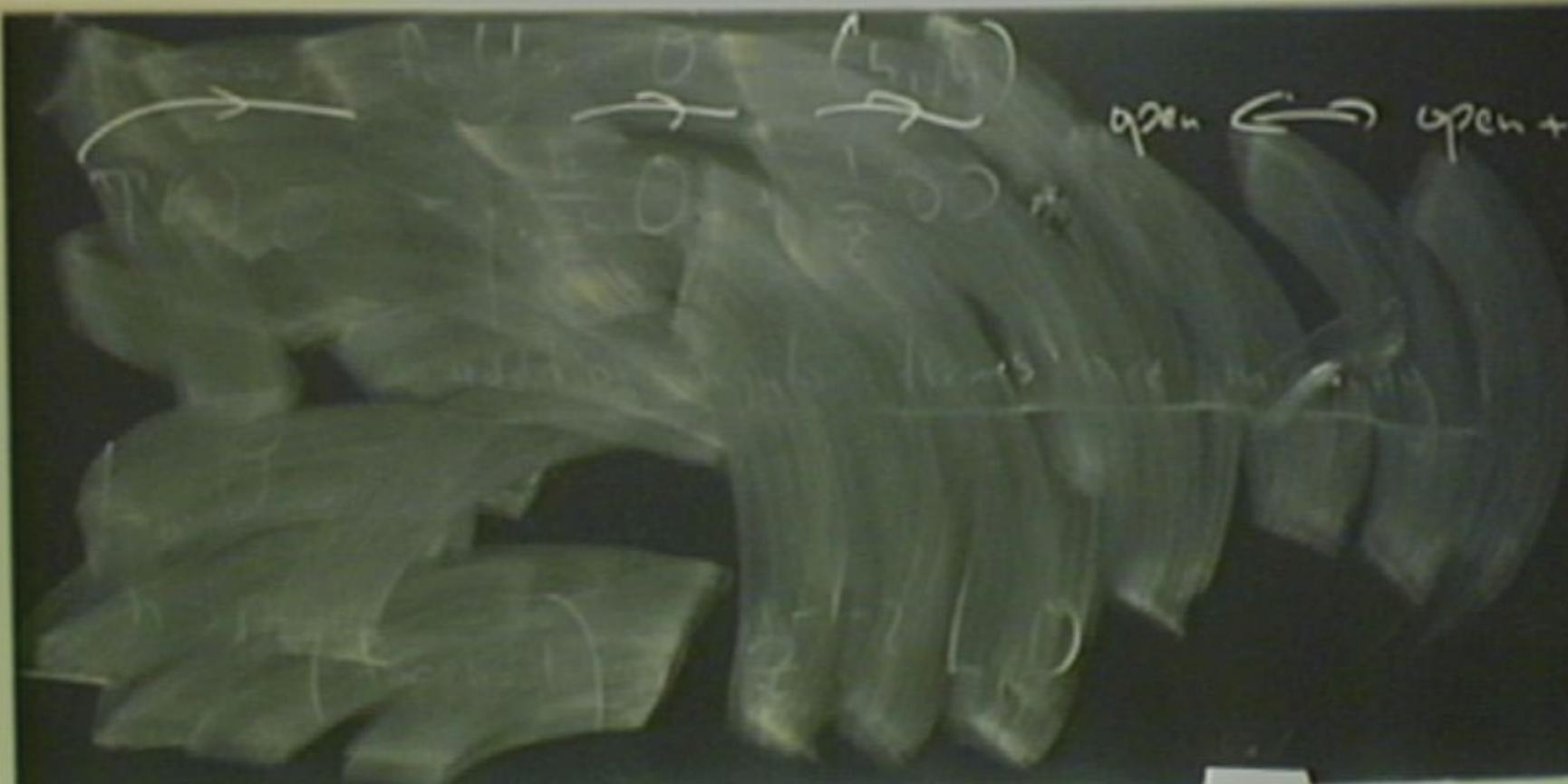
$f=0$

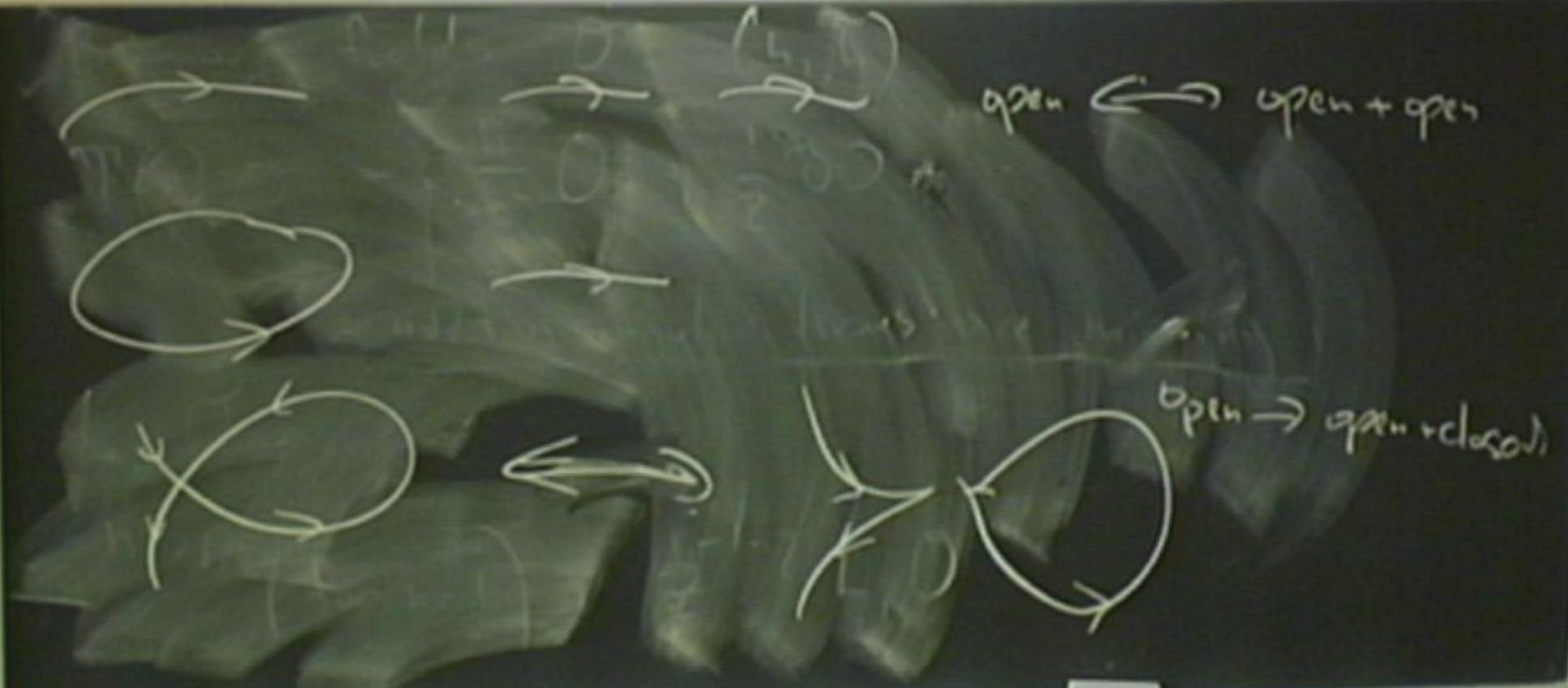
$f=1$  open

labeled

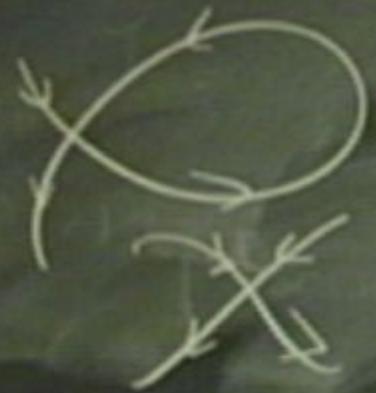
⇒ Interactions are implicit in world sheet topology











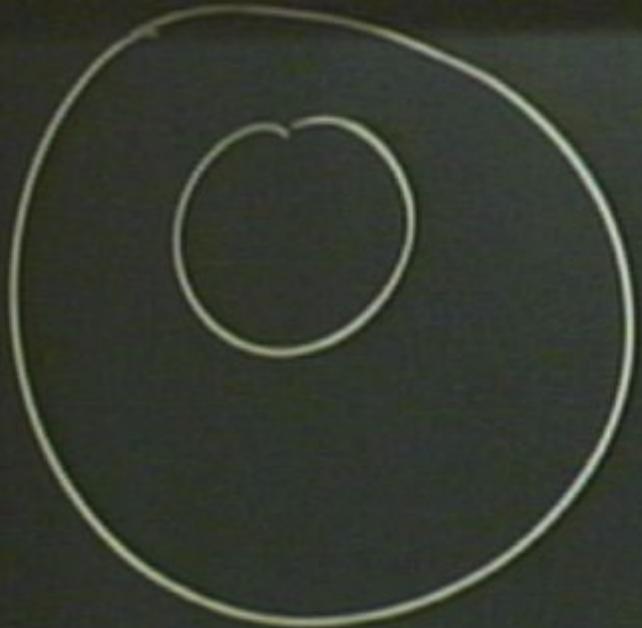
$b_{pin} \rightarrow open + closed$

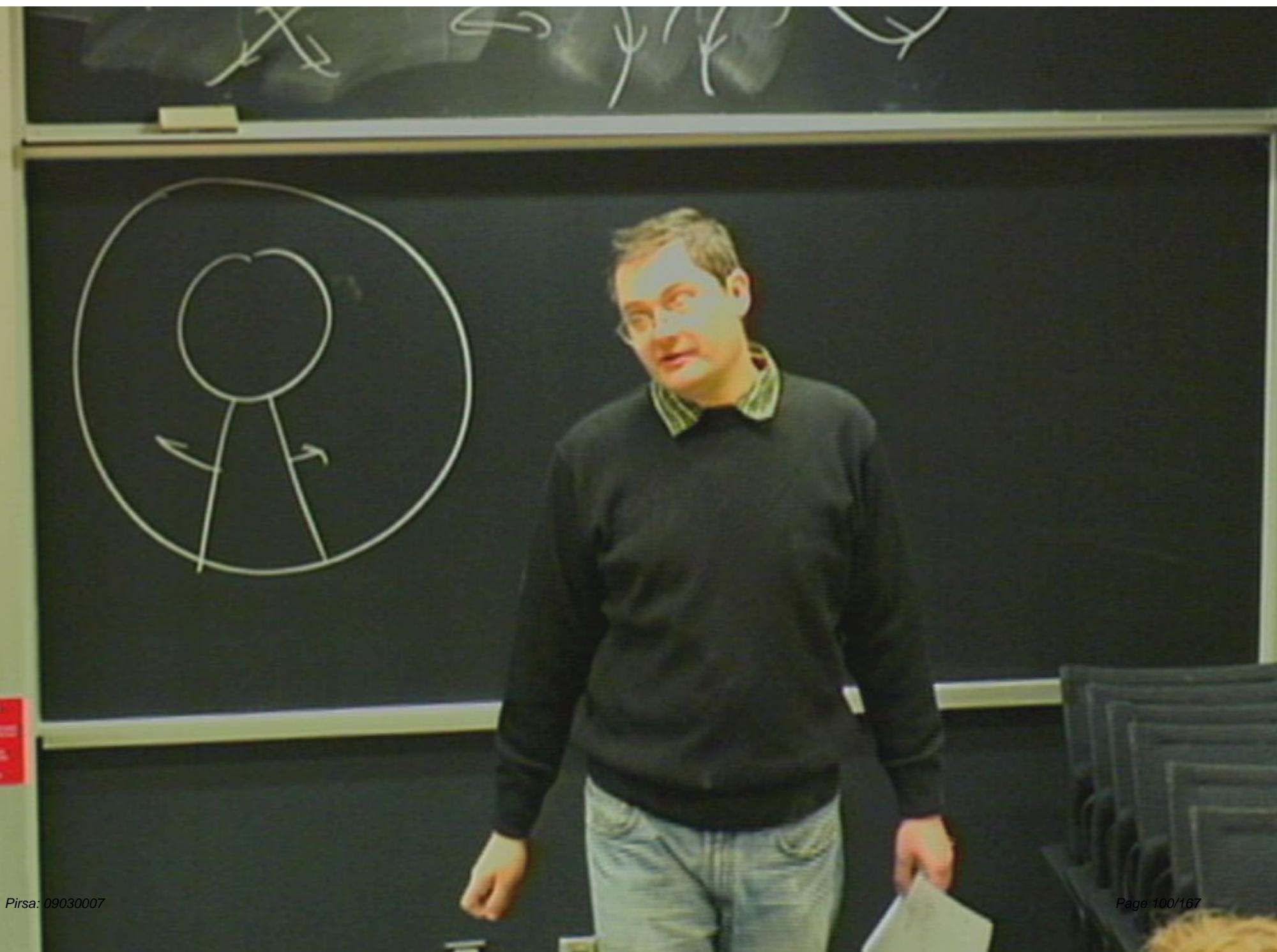
Note

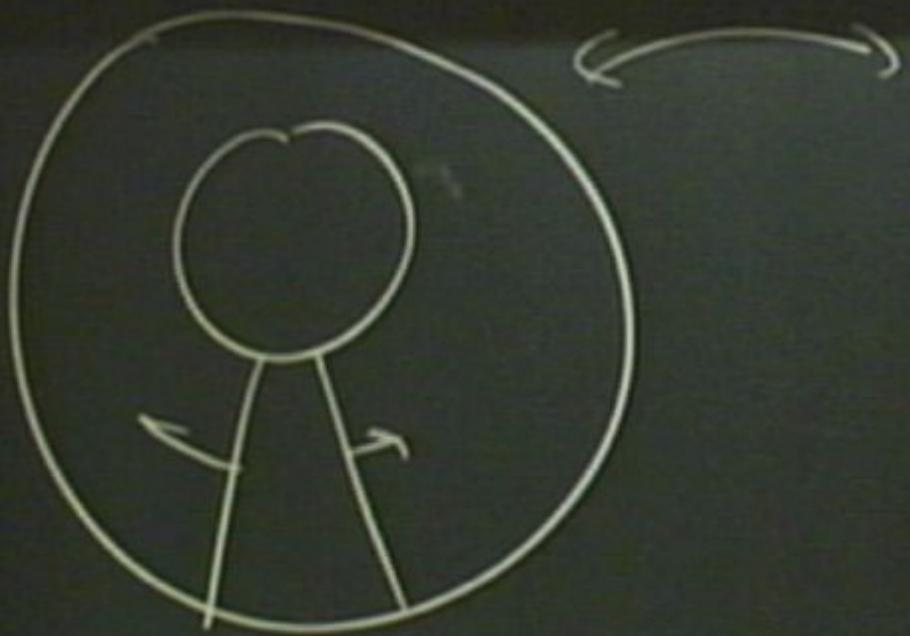
$$L_0 |A\rangle = h |A\rangle$$

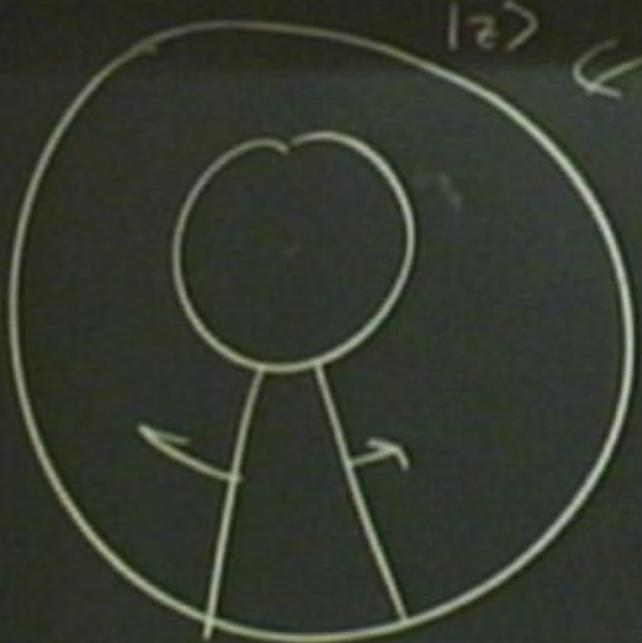
$$L_0 (L_n |A\rangle) = (h - h) |A\rangle$$

After all this we will  
reach the highest weight state  
of  $L_n (n > 0)$   
we finally reach  
the highest weight state  
Implicitly  $L_0$  spectrum is  
bounded from below









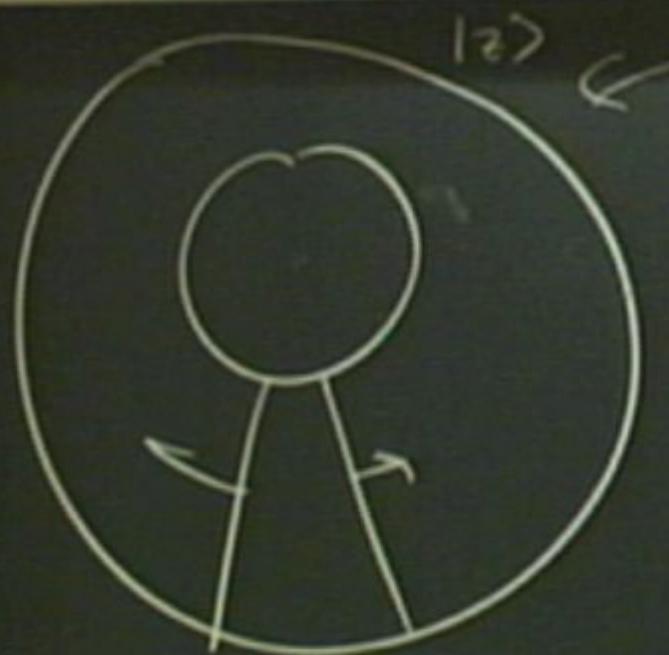
<21

مساند  
 $e = 2$

$T_{\text{orb}} = 0$



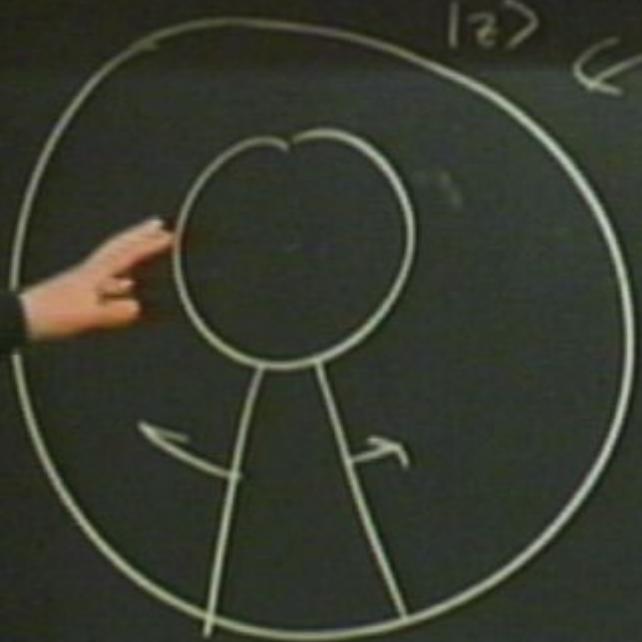
X ↪ Y ↪ Z



$$\begin{aligned} \text{مساند} \\ I_2 = e \end{aligned}$$

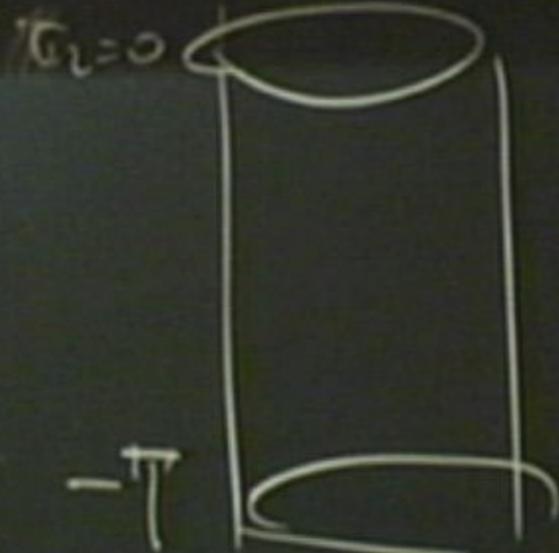
$$T_{21} = 0$$

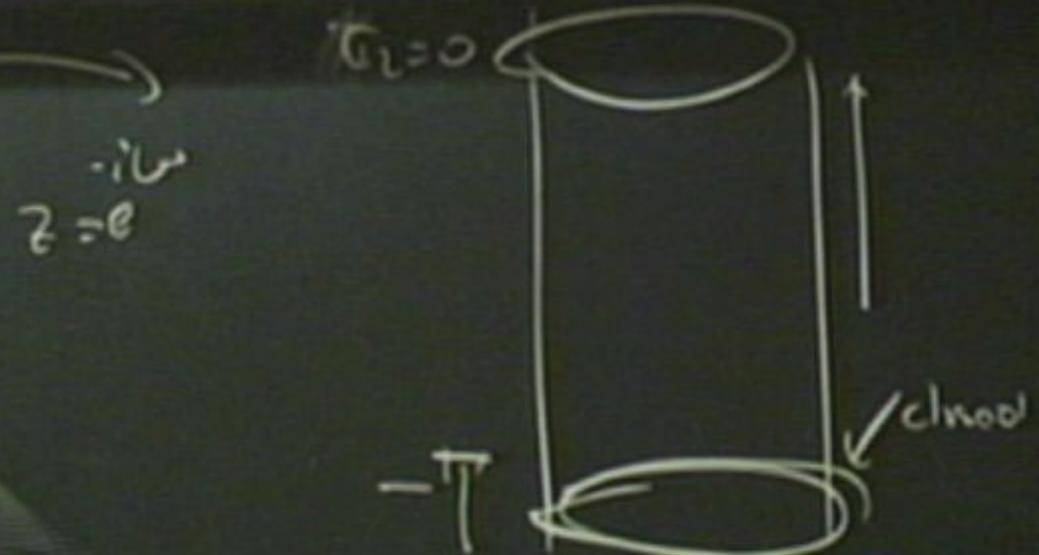


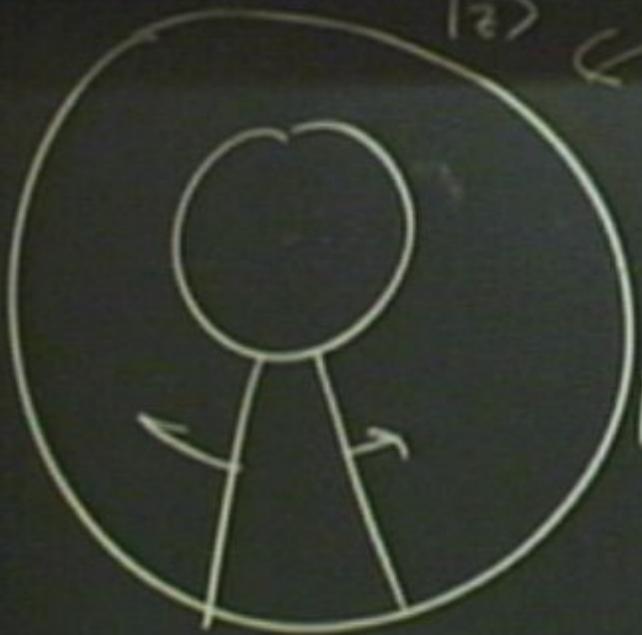


$I_2 > I_1$

مساند  
 $I = e$







(2)

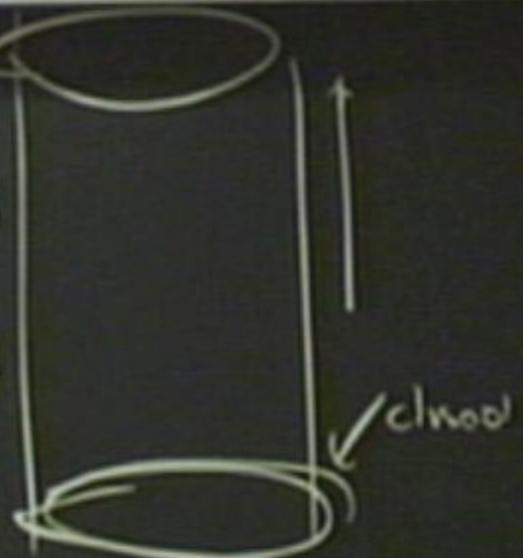
loop-open  
string  
diagramm

$$Z = e^{i\pi}$$

$$G_2 = 0$$

tree-level  
closed  
string

$$-T$$



Polyakov path integral (closed) string

A

B

Polyakov path integral (closed) string

⇒ Show that  $D = 26$

→ Show that  $D = 26$

⇒ Euclidean path integrals

$P_0$

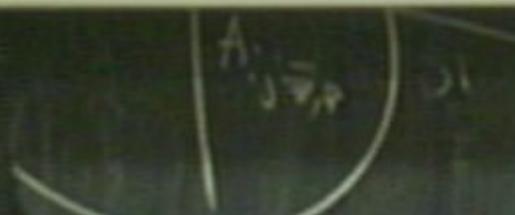
$$\bar{\mathcal{Z}}^{\sum h_k \cdot h_i - h_j}$$

$\Rightarrow$  Show that  $D = 26$

$\Rightarrow$  Euclidean path integrals

$$e^{iS/h} \rightarrow \bar{e}^{-S}$$

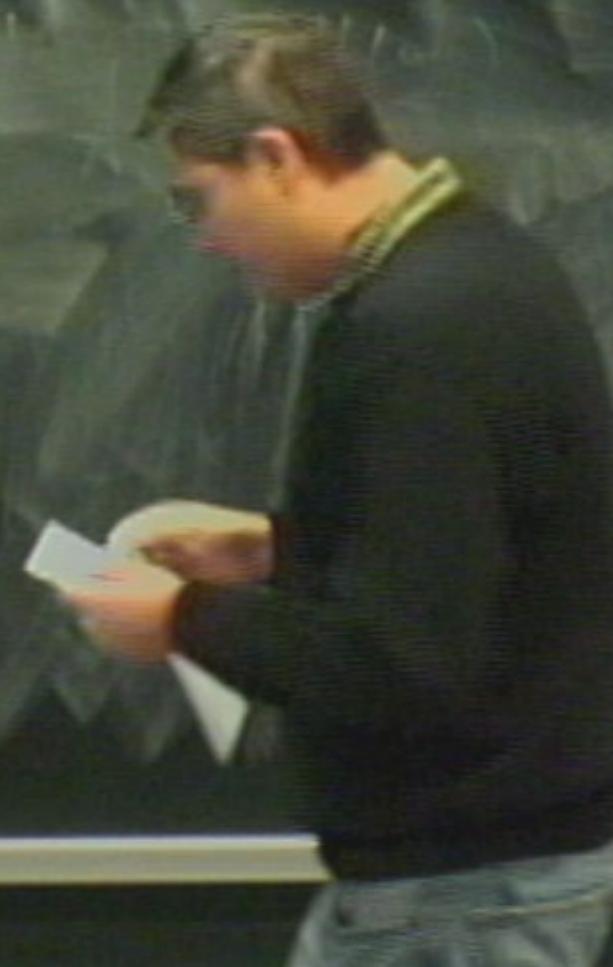
$P_s$



$$\bar{\Sigma}^{\tilde{l}_k - \tilde{l}_i - \tilde{l}_j}$$

$$e \rightarrow e$$

$$\int [dx]$$



$$\ell \xrightarrow{\quad} \ell$$

$$\int [dx] [dg] e^{-S}$$

$$S = S_x + \lambda$$

$$\int [dx] [dg] e^{-S} \quad \text{Enter character of worldsheet}$$
$$S = S_x + \lambda \chi$$

$$\int [dx] [dg] e^{-S}$$

Enter character of or worldsheet

$$S = S_x + \lambda \chi$$

$$S_x = \frac{1}{4\pi k} \int d^4x \sqrt{g} \partial_\mu X^\nu \delta^\alpha_\nu X_\alpha$$

$$\int [dx][dg] e^{-S}$$

Enter character of worldsheet

$$S = S_x + \lambda \chi$$

$$\mu = 0 \dots D-1$$

$$S_x = \frac{1}{4\pi k} \int_M d^{\mu} \sqrt{g} \partial_{\mu} X^{\nu} \delta^{\alpha}_{\nu} X_{\alpha} / \chi = \frac{1}{4\pi} \int_M d^{\mu} \sqrt{g} R$$

$$\int [dx] [dg] e^{-S}$$

Enter character of pr worldsheet

$$S = S_x + \lambda \chi$$

$$u=0 \dots D-1$$

$$S_x = \frac{1}{4\pi k} \int_M d^4x \sqrt{g} \partial_\mu X^\alpha \partial^\mu X_\alpha$$

$$\chi = \frac{1}{4\pi} \int_M d^4x \sqrt{g} R$$

$$\chi = 2 - 2g - b$$

$\uparrow$   
# of handles

$$\chi = 2 - 2g - b$$

$\uparrow$   
# of handles

# of boundary components

$\rightarrow$

$t_1=0$

$\mathcal{S}$

$$X_m(\tau, \sigma) \xrightarrow{\mathcal{N}} X_m(\tau, \tau - \sigma)$$

$|z\rangle$

$t_{tr}=0$

$\mathcal{S}$

$$X_m(\tau, \sigma) \xrightarrow{\mathcal{S}} X_m(\tau, \tau - \sigma)$$

$$\mathcal{S}|q\rangle = \Delta|q\rangle$$



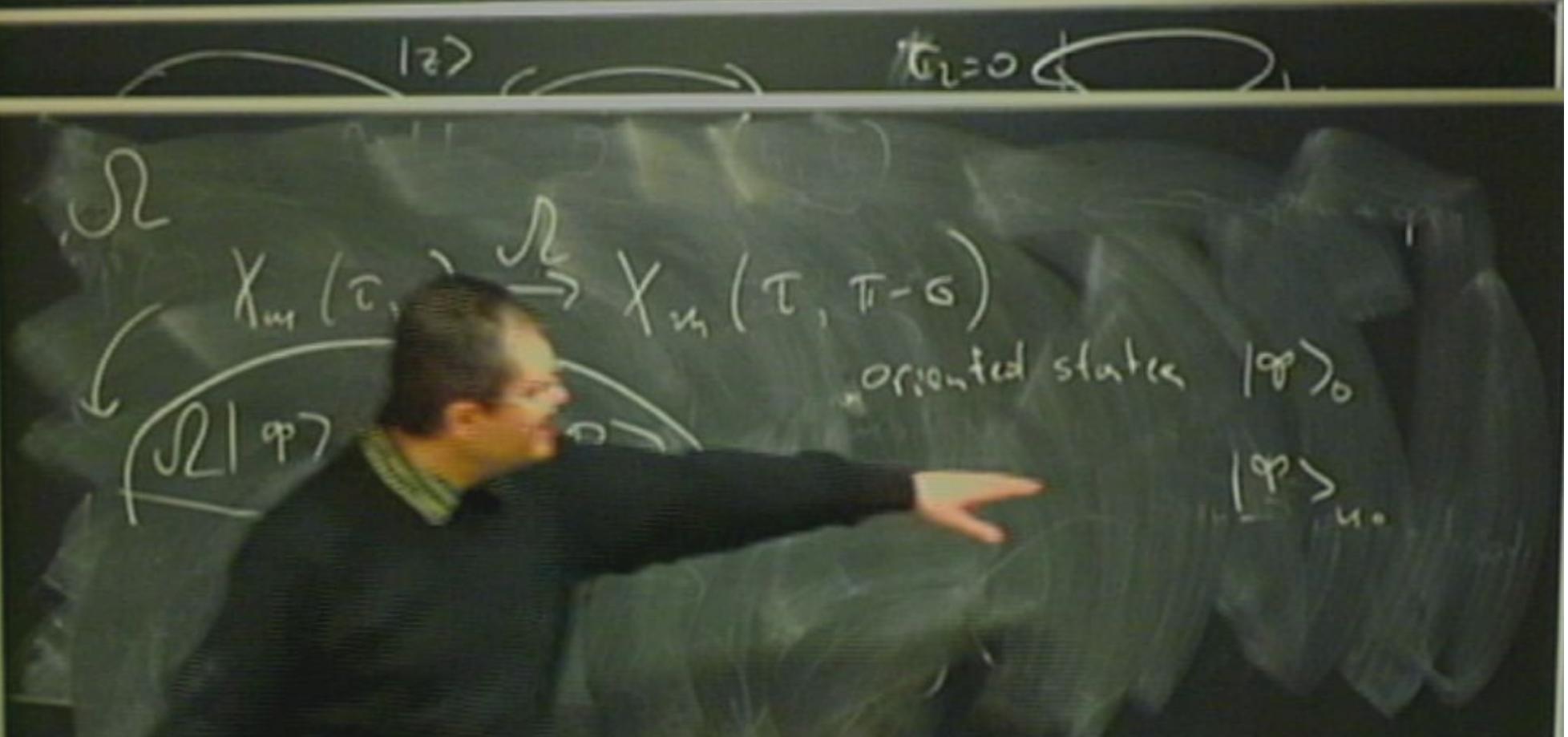
$|z\rangle$

$t_{\text{tr}}=0$

$\mathcal{S}$

$$X_m(\tau, \sigma) \xrightarrow{\mathcal{N}} X_m(\tau, \tau - \sigma)$$

$$\mathcal{R}|\Psi\rangle = +1|\Psi\rangle$$



$|z\rangle$

$t_1=0$

$\mathcal{S}$

$$X_m(\tau, \sigma) \xrightarrow{\mathcal{N}} X_m(\tau, \tau - \sigma)$$

$$\langle \mathcal{R} | \Psi \rangle = +\Delta |\Psi \rangle$$

oriented states  $|\Psi\rangle_0$

$$|\Psi_{u0}\rangle = \frac{1}{2} [1 + \Delta] |\Psi\rangle_0$$

$|\Psi\rangle_u$

$|z\rangle$

$\hat{G}_L=0$

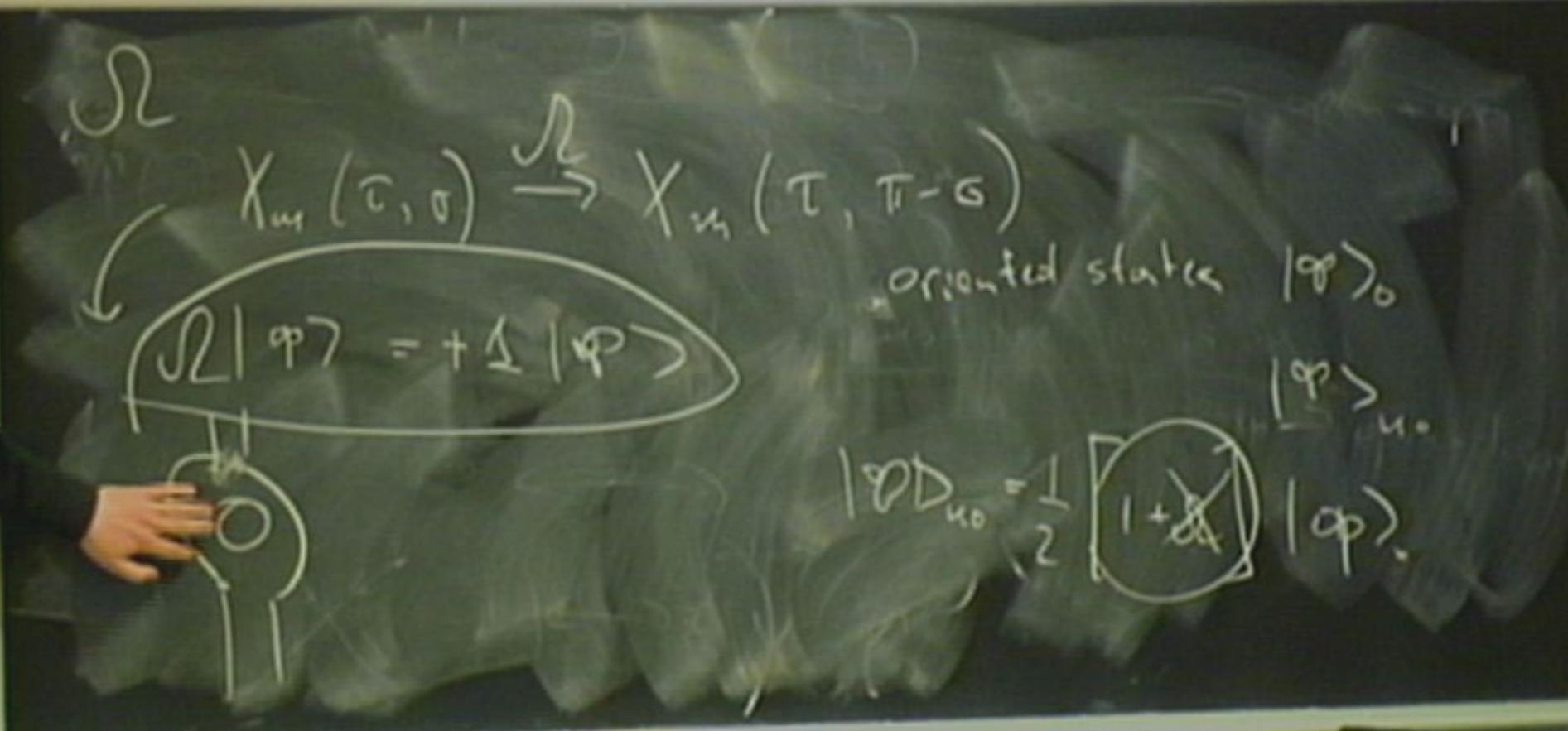
$\mathcal{S}$

$$X_m(\tau, \sigma) \xrightarrow{\mathcal{S}} X_m(\tau, \tau - \sigma)$$

$$\mathcal{S} | \Psi \rangle = +\Delta | \Psi \rangle$$

oriented states  $|\Psi\rangle_0$

$$|\Psi\rangle_{u_0} = \frac{1}{\sqrt{2}} \left[ |+\rangle_{\alpha} - |-\rangle_{\alpha} \right] |\Psi\rangle$$

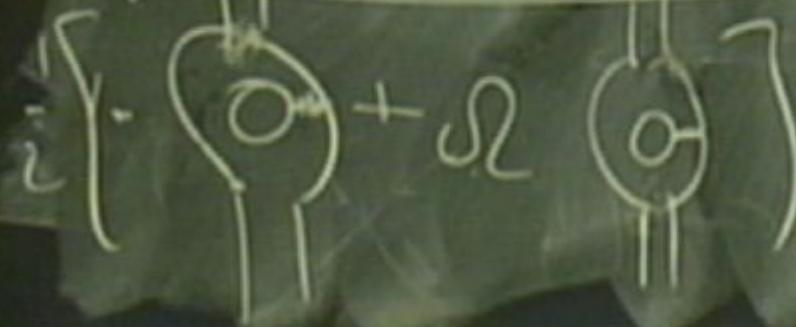


$\langle z_1 \rangle$  $t_{\text{tr}}=0$  $|z_1\rangle$ 

$$X_m(\tau, \sigma) \xrightarrow{\quad} X_m(\tau, \tau - \sigma)$$

oriented states  $|q\rangle_0$

$$\sqrt{2}|q\rangle = +1|q\rangle$$



$$|D\rangle_{u0} = \frac{1}{\sqrt{2}} \left[ |+\alpha\rangle - |-\alpha\rangle \right] |q\rangle_0$$

$$\int [dx] [dg] e^{-S}$$

$$S = S_x + \lambda \chi$$

$$S_x = \frac{1}{4\pi k} \int d^4x$$

or worldsheet

$$= 0 \dots D-1$$

$$\chi = \frac{1}{4\pi} \int_M d^4x \sqrt{g} R$$

$$\chi = 2 - 2g - b$$

↑  
# of handles

# of boundary components

$$\underline{\chi = 2}$$

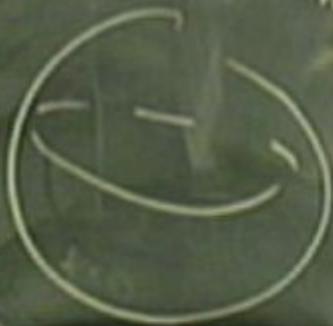


$$\chi = 2 - 2g - b \leftarrow \begin{matrix} \text{\# of boundary components} \\ \uparrow \\ \text{\# of handles} \end{matrix}$$

$$\chi = 2$$

$$g=0$$

$$b=0$$

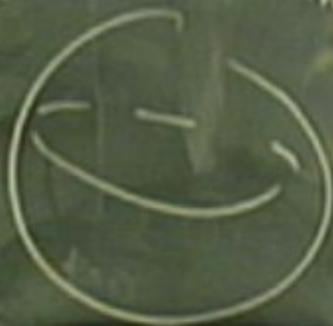


$$\chi = 2 - 2g - b \leftarrow \begin{matrix} \text{# of boundary components} \\ \uparrow \\ \text{# of handles} \end{matrix}$$

$$\chi = 2$$

$$g = 0$$

$$b = 0$$



$$\chi = 2 - 2g - b \leftarrow \# \text{ of boundary components}$$

$$\chi = 2$$

$$g = 0$$

$$b = 0$$



# of handles



$$\chi = 2 - 2g - b$$

↑  
# of handles

# of boundary components

$$\chi = 2$$

$$g = 0$$

$$b = 0$$



$$g =$$

$$\chi = 2 - 2g - b \leftarrow \# \text{ of boundary components}$$

$$\chi = 2$$

$$g = 0$$

$$b = 0$$



$\uparrow$   
 $\#$  of handles

$$g = 1$$



$$\chi = 2 - 2g - b \leftarrow \# \text{ of boundary components}$$

$$\chi = 2$$

$$g = 0$$

$$b = 0$$



# of handles

$$g = 1$$



$$\chi = 1$$

$$\chi = 2 - 2g - b \leftarrow \# \text{ of boundary components}$$

$$\chi = 2$$

$$g=0$$

$$b=0$$



# of handles

$$g=1$$



$$\chi=0$$



$$\chi = 2 - 2g - b \leftarrow \# \text{ of boundary components } \alpha = 0$$

$$\chi = 2$$

$$g = 0$$

$$b = 0$$



# of handles

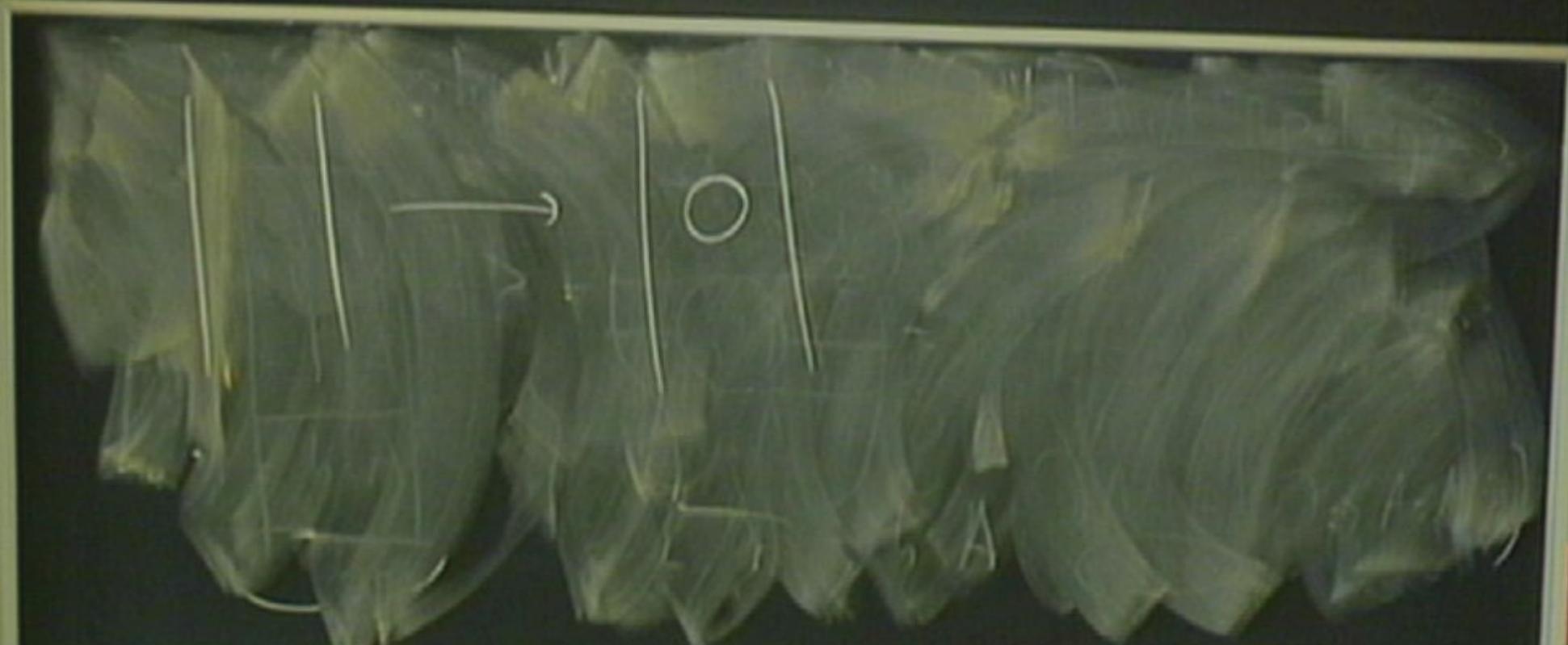
$$g = 1$$



$$\chi = 0$$

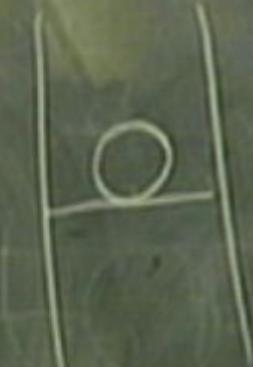


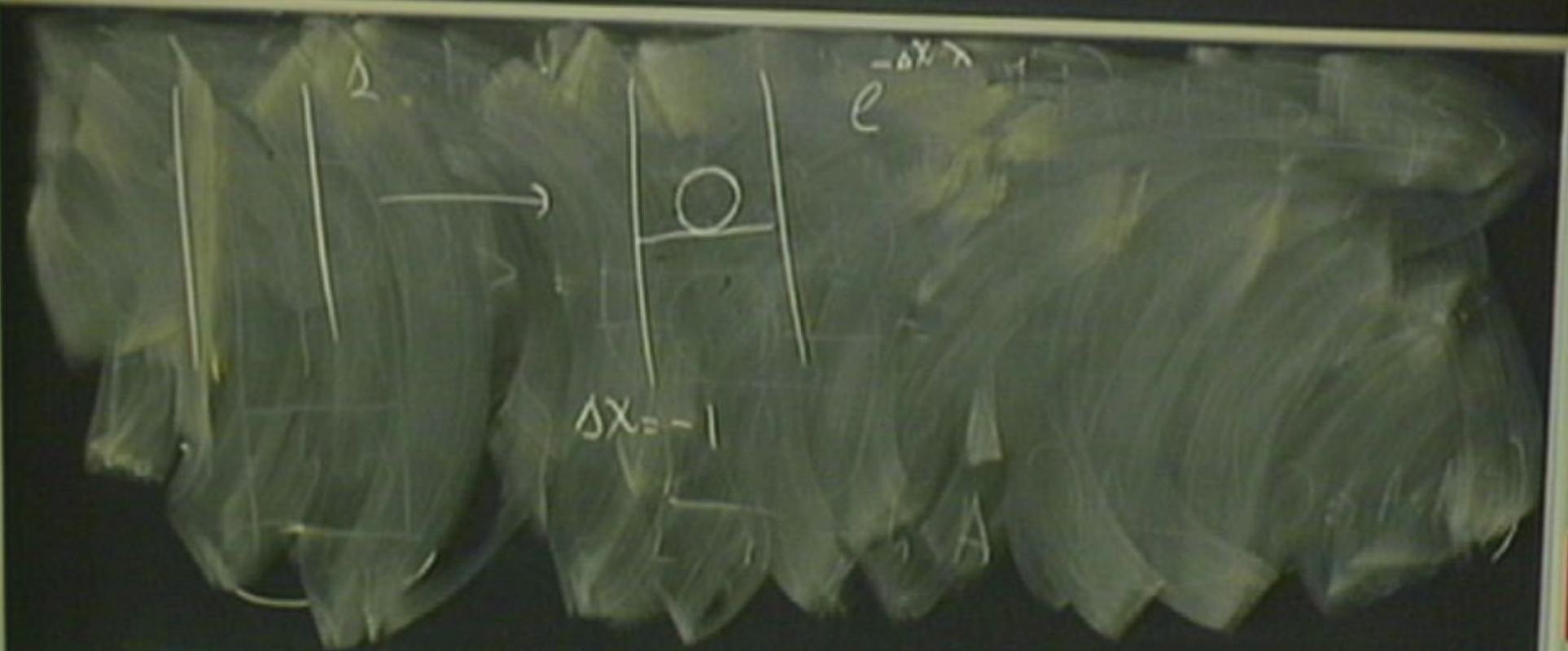






$$\Delta x = -1$$



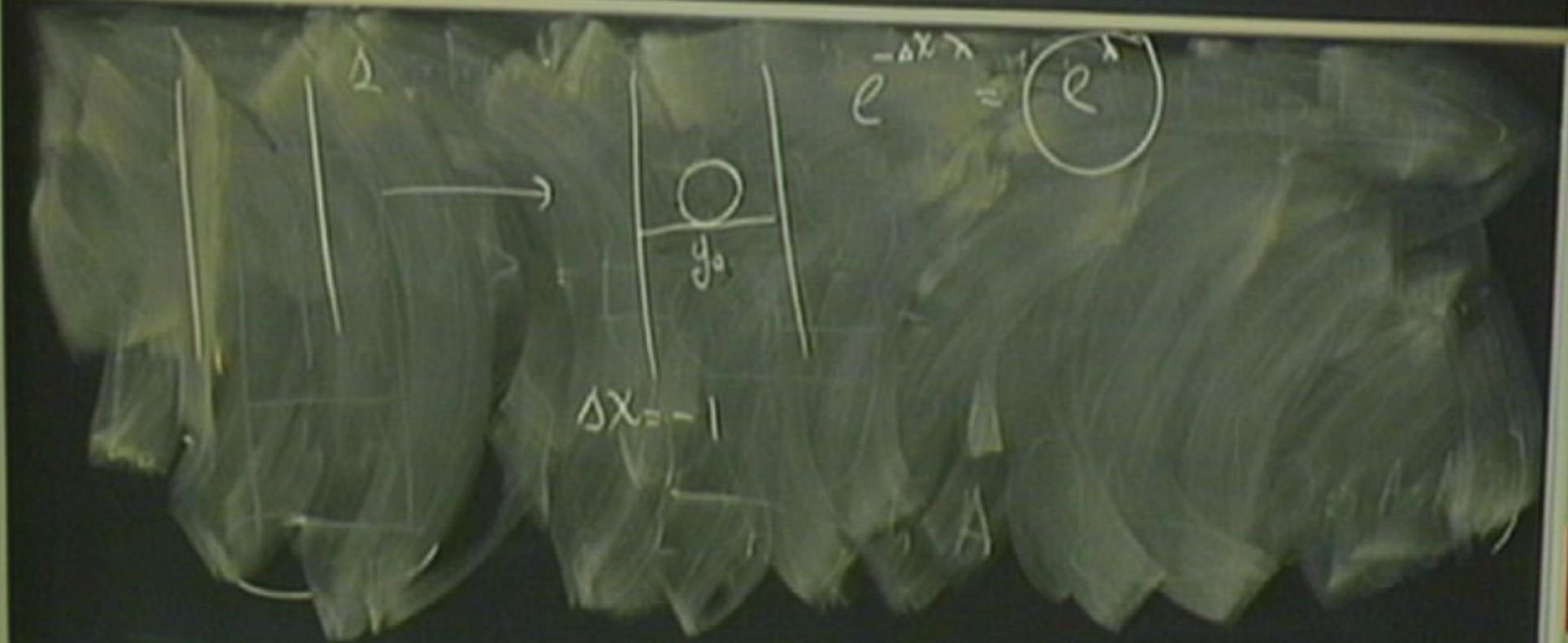


$\Delta$



$\Delta x = -1$

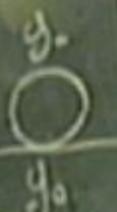
$$e^{-\Delta x} = e^{\Delta}$$



$\Delta$



$\Delta x = -1$



$$e^{-\Delta x \lambda} = e^{\lambda} \approx g_0^2$$

$\Delta$ 

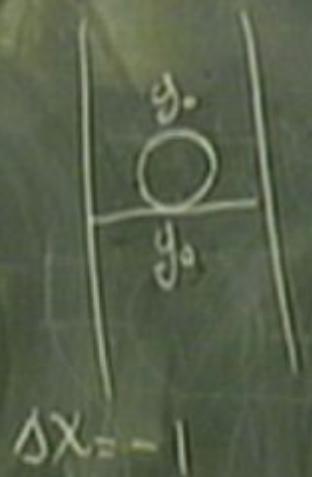
y.

 $y_0$  $\Delta x = -1$ 

$$e^{-\Delta x \gamma} = e^{\gamma} \approx g_0^2$$

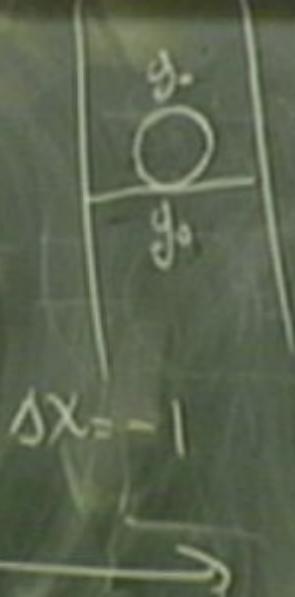
 $g_0 \infty$

$$\Delta \rightarrow$$



$$\Delta x = -1$$

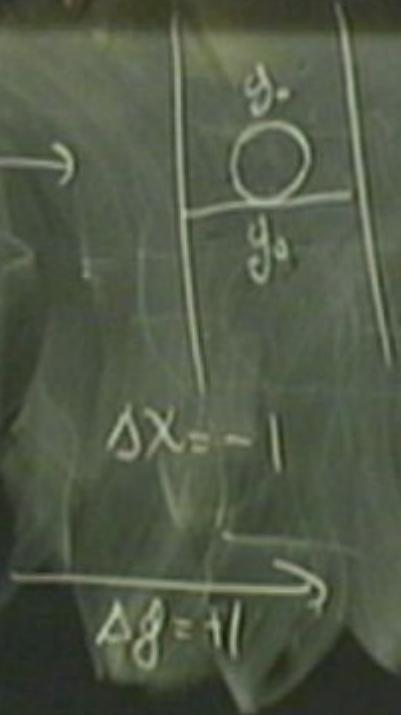
$$e^{-\Delta x \gamma} = \frac{e^{\gamma}}{r} \propto g_0^2$$
$$g_0 \propto e^{\gamma/2}$$



$$e^{-\frac{\Delta x \hbar}{\lambda}} = \left(\frac{R}{\lambda}\right) \propto g_0^2$$

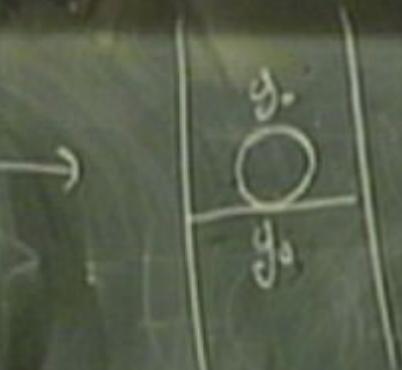
$$g_0 \propto e^{\frac{\Delta x \hbar}{2}}$$





$$e^{-\frac{\Delta x}{2}} = \left(\frac{r}{l}\right) \propto g_0^2$$
$$g_0 \propto e^{\frac{\Delta x}{2}}$$

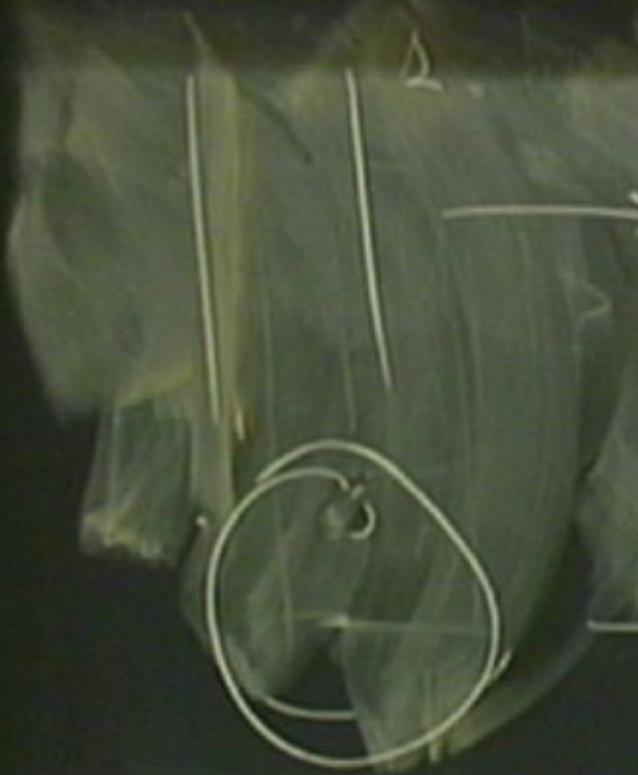




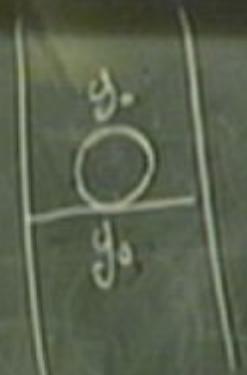
$$\begin{array}{c} \Delta x = -1 \\ \Delta y = +1 \\ \Delta \rho = -2 \end{array}$$

$$e^{-\Delta x} = \left(\frac{r}{r_0}\right) \propto g_0^2$$
$$g_0 \propto e^{\Delta x/2}$$





$$\Delta x = -1$$
$$\Delta g = +1$$
$$\Delta \theta = -2$$

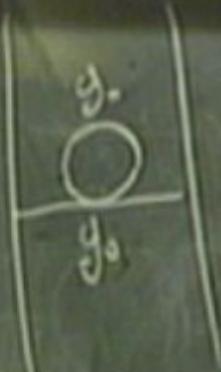


$$e^{-\Delta x} = (e^{\lambda})^{-1} \propto g_0^2$$
$$g_0 \propto e^{\lambda/2}$$



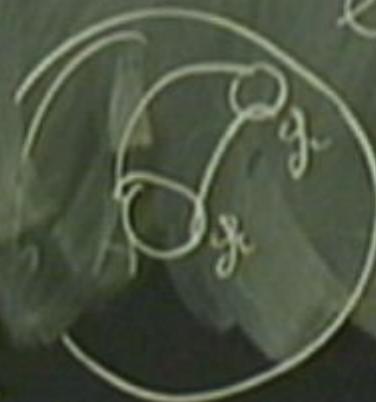


$$\begin{array}{c} \Delta x = -1 \\ \Delta g = +1 \\ \Delta \theta = -2 \end{array}$$



$$e^{-\Delta \theta} = (e^{\lambda}) \propto g_0^2$$

$$g_0 \propto e^{\lambda/2}$$



$$e^{2\lambda} = g_c^2$$

$$g_c \propto e^{\lambda}$$

$$Z = \int [dx dg] e^{-S_X}$$

$$Z = \int \frac{[d \times dg]}{V_{\text{eff}} \times V_{\text{mag}}} e^{-\beta_x}$$

$$Z = \int \frac{[dx \times dg]}{V_{bf} \times V_{ug}} e^{-Sx(t)}$$

single string  
partition function



|z>  $\rightarrow$   $t_{\text{c}} = 0$   $\curvearrowleft \alpha_1$

$$Z = \int \frac{[d \times d_g]}{V_{\text{bf}} \times V_{\text{reg}}} e^{-S_x} g^{\text{Ab}(o)}$$

single string  
partition function

$|z\rangle$

$t_{\nu=0}$

$$Z = \int \frac{[dx dg]}{V_{bf} \times V_{wgl}} e^{-Sx(g)}$$

$g_{ab}(s) \rightarrow 3$  independant components

$$2 \text{ diff. cos } + \text{Weyl} = 3$$

$$\hat{g}_{ab}(s) \equiv \delta_{ab}$$

single string partition function

partition function

$$g_{ab}(\sigma) = \sum_{\text{softon}} S_{ab}$$

$\hat{g}_{ab}$



loop-open  
string  
diagram.



$$\hat{g}_{ab}(\sigma) = e^{2\omega} \delta_{ab}$$

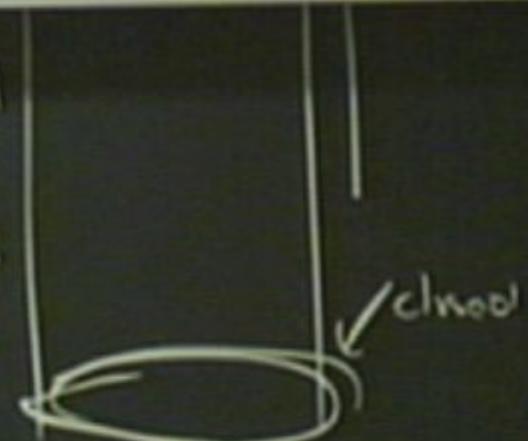
negl factor.



مسار  
 $\gamma = e$

loop-open  
string  
diagram

tree-level  
closed  
string  
 $-T$



use differentiation  
to bring

$$g_{ab}(\zeta) = e^{2\psi} \delta_{ab}$$

negl factor.

Use diffeomorphism  
to bring

$$g_{ab}(\epsilon) = e^{2\omega} \delta_{ab}$$

Weyl factor.

Let's do Weyl transform  $-g_{ab} \rightarrow g'_{ab} = e^{2\omega} g_{ab}$

use diffeomorphisms  
to bring

$$g_{ab}(\zeta) = e^{2\omega} \tilde{g}_{ab}$$

Weyl factor.

Let's do Weyl transf.  $\tilde{g}_{ab} \rightarrow g'_{ab} = e^{2\omega} g_{ab}$

$$\sqrt{\tilde{g}'} R' = \sqrt{g} [R - 2\nabla^2 \omega]$$

Use diffeomorphism  
to bring  $\rightarrow$

$$g_{ab}(\varphi) = e^{2\omega} \delta_{ab}$$

Weyl factor.

Let's do Weyl transf  $g_{ab} \rightarrow g'_{ab} = e^{2\omega} g_{ab}$

$$\sqrt{g'} R' = \sqrt{g} [R - 2\Delta^i \omega]$$

$$\boxed{\Delta^i \omega = \frac{1}{2} R}$$

Summary from today

Use diffeomorphism  
to bring  $\rightarrow$

$$g_{ab}(\epsilon) = e^{2\omega} \delta_{ab}$$

Weyl factor.

Let's do Weyl transf  $g_{ab} \rightarrow g'_{ab} = e^{\omega} g_{ab}$

$$\sqrt{g'} R' = \sqrt{g} [R - 2D'\omega]$$

$$R'_{abcd} \rightarrow R' [g_{ac} g'_{bd} - \boxed{D'\omega = \frac{1}{2} R}]$$

download from talker

⇒ note that the gauge (diffeo ×  $\text{Int}_\theta$ ) is not completely fixed

$$S_x = \frac{1}{4\pi k} \int d^4x \sqrt{g}$$

$$S = \frac{1}{4\pi} \int d^4x \underline{\sqrt{g}} R$$

$\Rightarrow$  note that the gauge (diffeo  $\times$  U(1)) is not completely fixed

$$f(\theta) = \sin\theta \times \Rightarrow$$

$$z = \sigma_1 + i\sigma_2$$

$$\bar{z} = \sigma_1 - i\sigma_2$$



$$\pi \rightarrow 0 \text{ as } \lambda \rightarrow \infty$$

$\Rightarrow$  note that the gauge (diffeo  $\times$  wrgl) is not completely fixed

$$f(\sigma_1 + \sigma_2) = \sigma_1 \Rightarrow z = \sigma_1 + i\sigma_2 \quad z \Rightarrow z' = f(z)$$
$$\bar{z} = \sigma_1 - i\sigma_2$$

$$f(z) = \sum a_n z^n \Rightarrow$$

$$z = \sigma_1 + i\sigma_2$$

$$\bar{z} = \sigma_1 - i\sigma_2$$

$$z \Rightarrow z' = f(z)$$

$$ds^2 \rightarrow (ds')^2 = |Df|^2 dz' d\bar{z}'$$

$$\begin{array}{c} || \\ z' \\ \hline \end{array}$$

$$x \quad \partial X$$

$m=0 \dots D-1$

$$-\frac{1}{4\pi k} \int_M d^4r \sqrt{g} \partial X^\mu \delta X^\nu \partial X_\nu$$
$$X = \frac{1}{4\pi} \int_M d^4r \underline{\sqrt{g} R}$$

$$J_{AB} = \nabla_A V \times \Rightarrow$$

$$Z = \sigma_1 + i\sigma_2$$

$$Z \Rightarrow Z' = f(Z)$$

$$\bar{Z} = \sigma_1 - i\sigma_2$$

$$ds^2 \rightarrow (ds')^2 = e^{2\omega} |Df|^2 dz' d\bar{z}'$$

$$\parallel dz' d\bar{z}'$$

$$+ \int \chi \left( \frac{d^r \sqrt{g}}{\sqrt{g}} \partial_\mu X^\mu \delta^r X_\mu \right) \quad u=0 \dots D-1$$
$$\chi = \frac{1}{4\pi} \int_M d^r \sqrt{g} R$$

$$f(z) = \sigma_1 + i\sigma_2 \Rightarrow z = \sigma_1 + i\sigma_2 \quad z \Rightarrow z' = f(z)$$

$$\bar{z} = \sigma_1 - i\sigma_2$$

$$ds^2 \rightarrow (ds')^2 = e^{2\omega} |df|^2 dz' d\bar{z}'$$

$$dz d\bar{z}$$

$$z \rightarrow f(z)$$

$$\omega = \ln |\partial f|$$

$$S = S_x + \lambda \tilde{\chi}$$

$$S_x = \frac{1}{4\pi k} \int_M d^4r \sqrt{g} \quad \partial_\mu X^\alpha \partial^\mu X_\alpha$$

$$u=0 \dots D-1$$

$$\tilde{\chi} = \frac{1}{4\pi} \int_M d^4r \sqrt{g} R$$