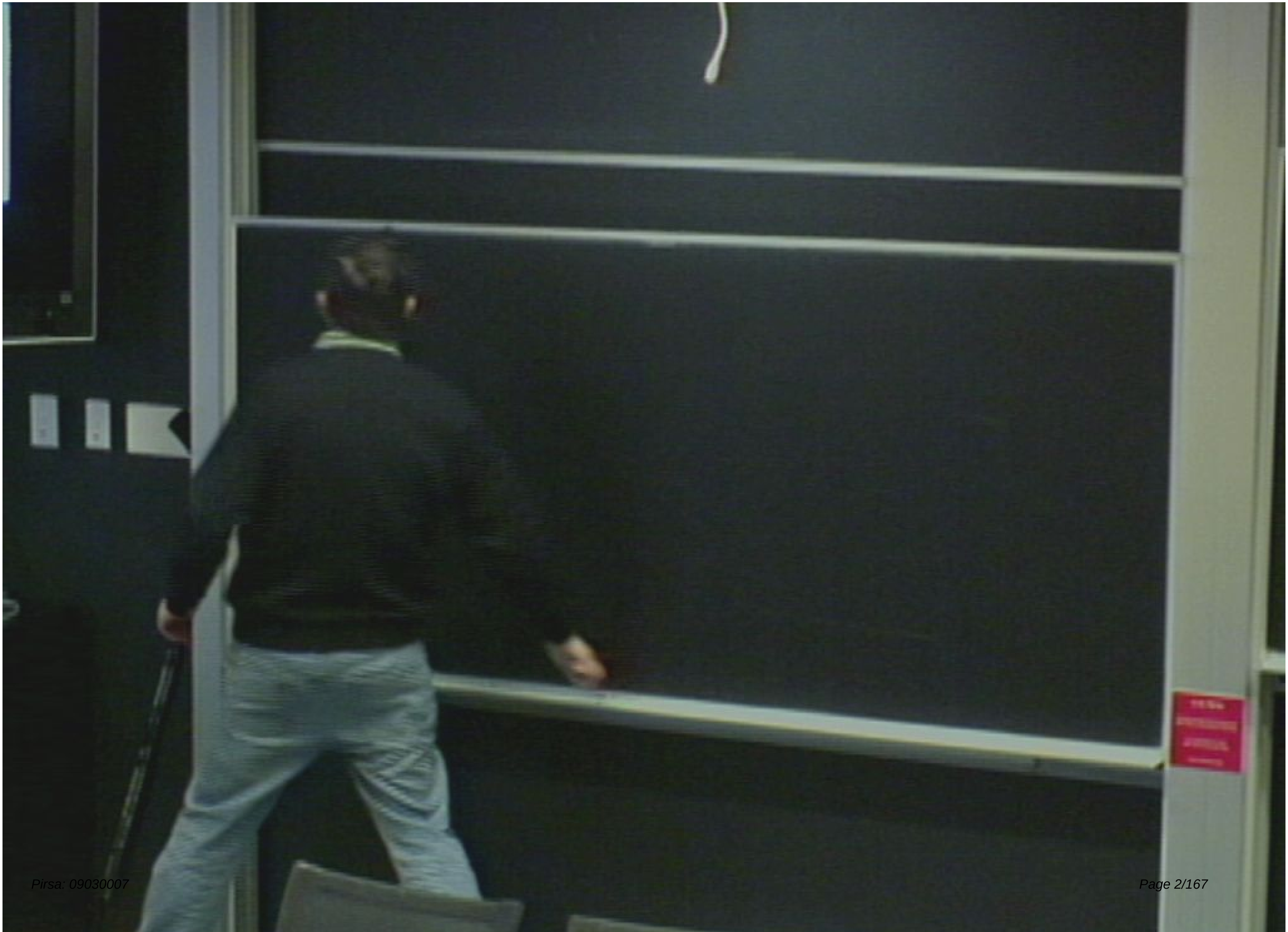
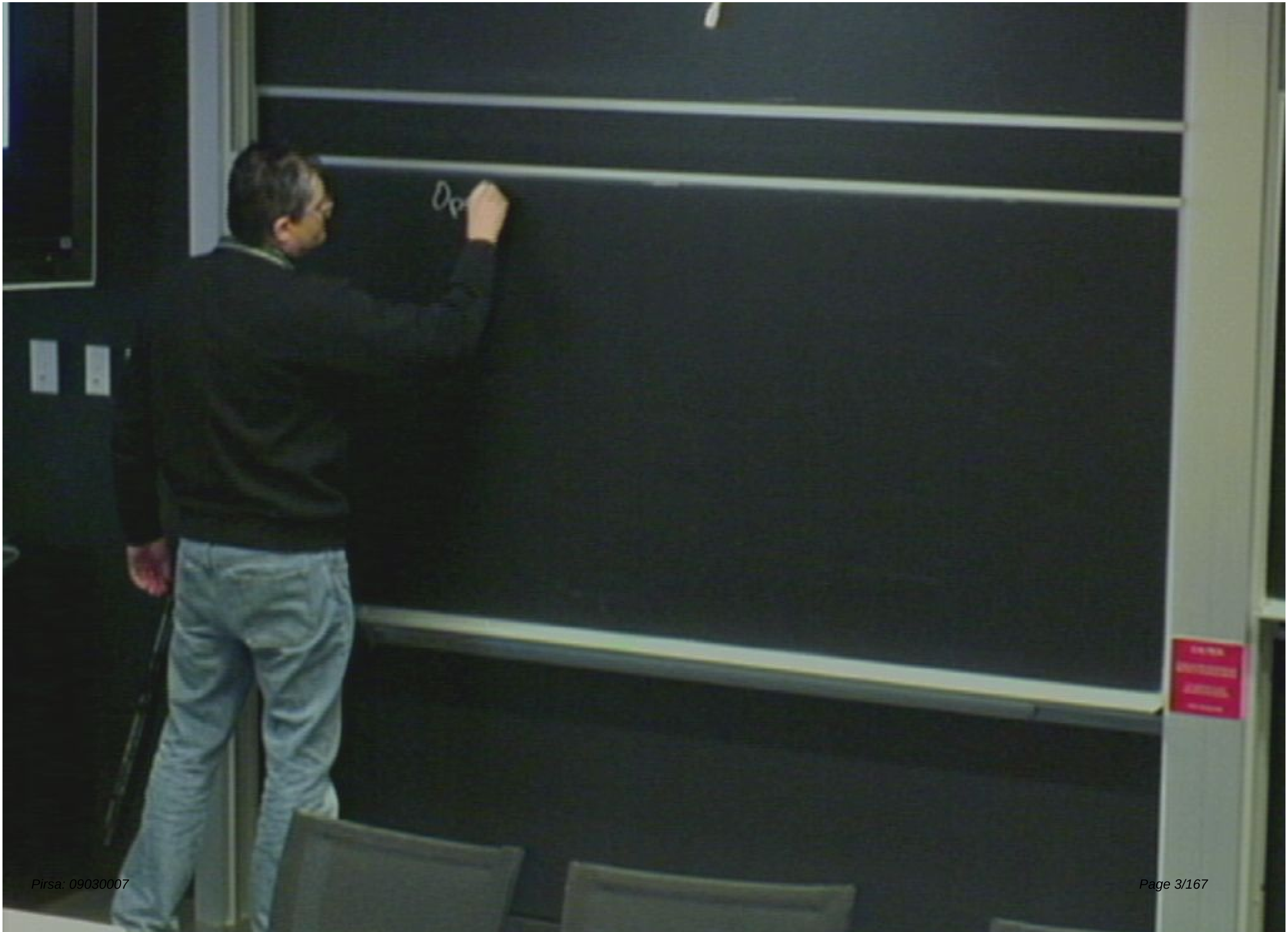
Title: Introduction to the Bosonic String

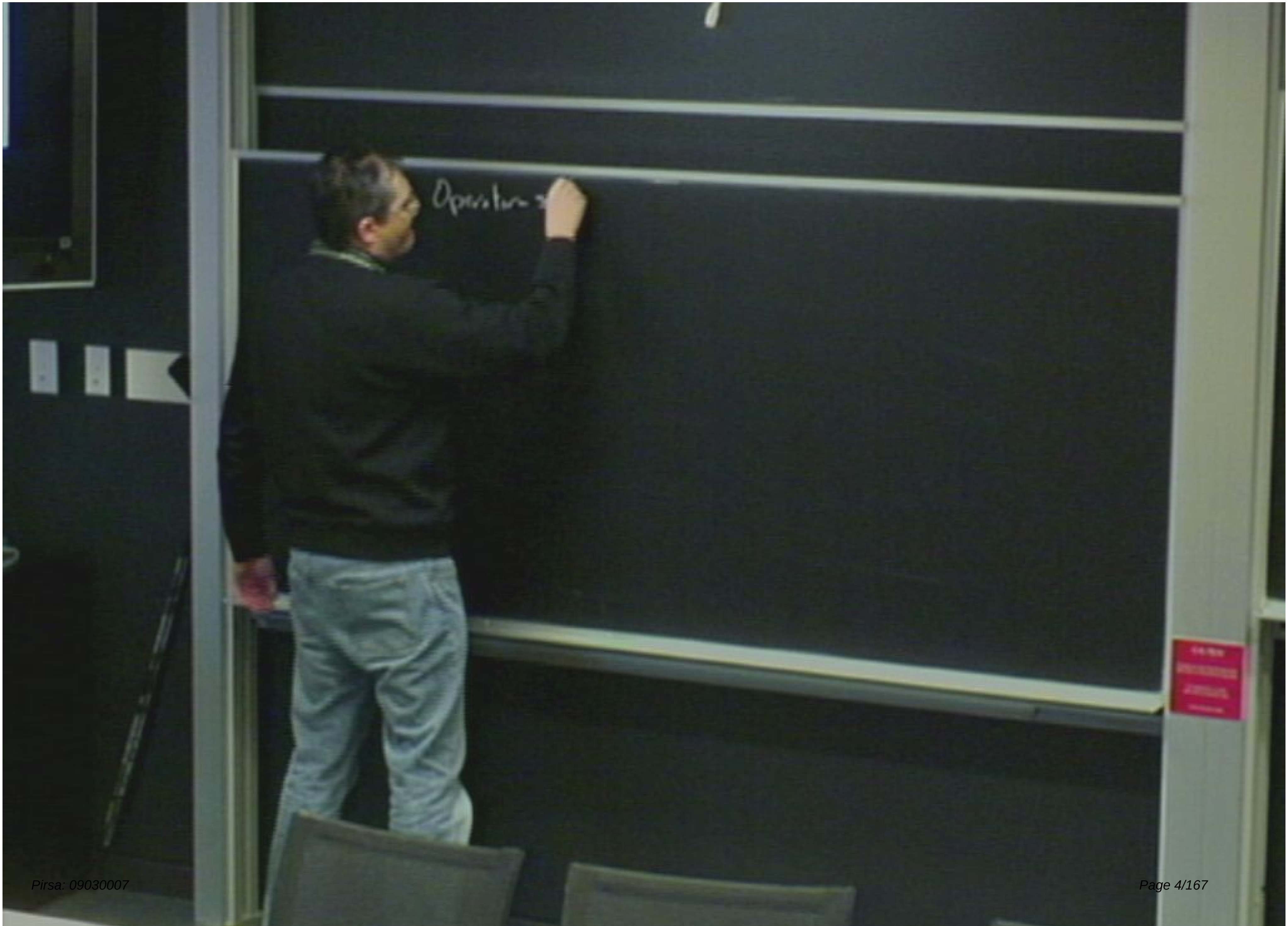
Date: Mar 13, 2009 10:00 AM

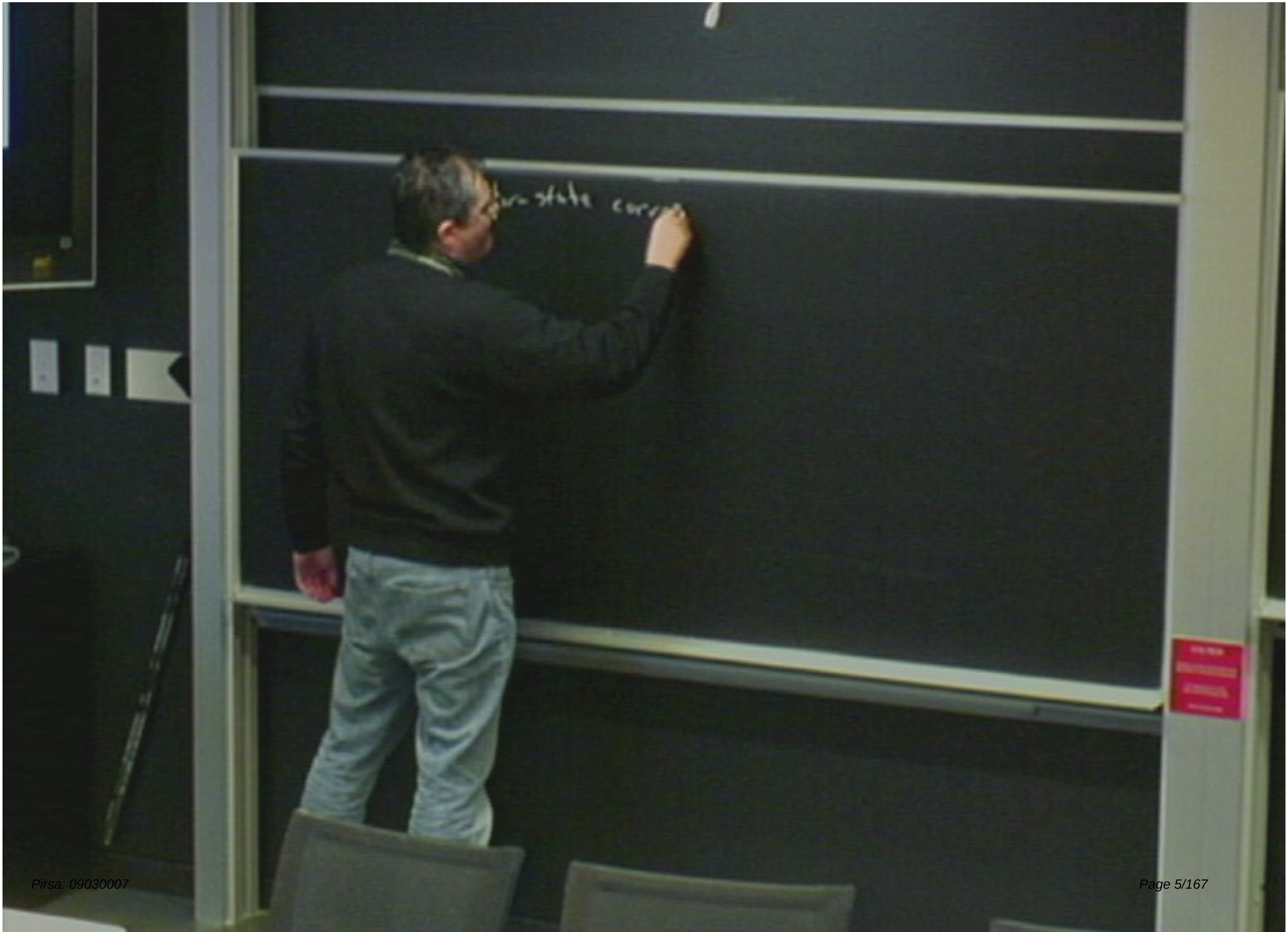
URL: http://pirsa.org/09030007

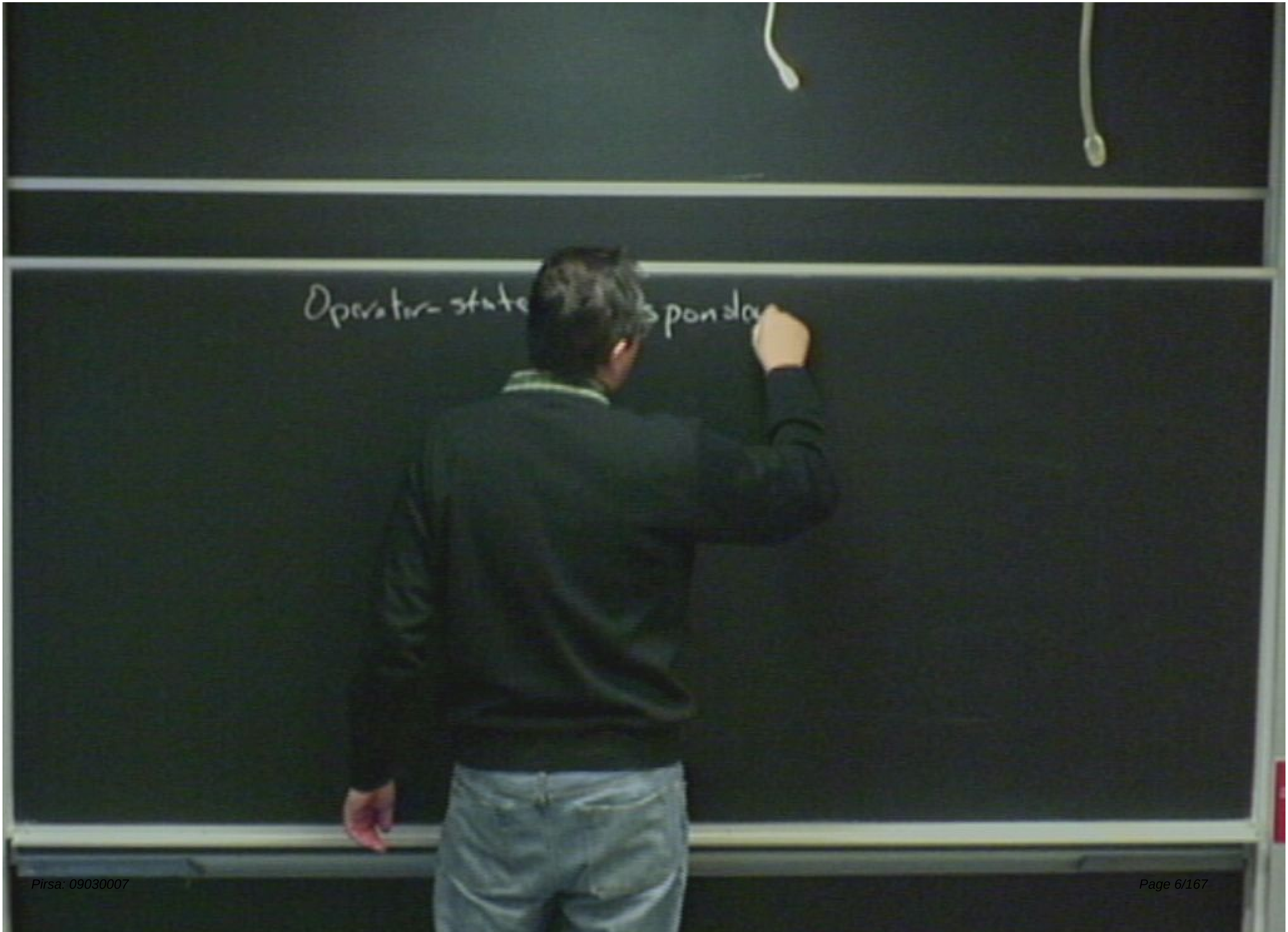
Abstract: This course provides a thorough introduction to the bosonic string based on the Polyakov path integral and conformal field theory. We introduce central ideas of string theory, the tools of conformal field theory, the Polyakov path integral, and the covariant quantization of the string. We discuss string interactions and cover the tree-level and one loop amplitudes. More advanced topics such as T-duality and D-branes will be taught as part of the course. The course is geared for M.Sc. and Ph.D. students enrolled in Collaborative Ph.D. Program in Theoretical Physics. Required previous course work: Quantum Field Theory (AM516 or equivalent). The course evaluation will be based on regular problem sets that will be handed in during the term. The primary text is the book: 'String theory. Vol. 1: An introduction to the bosonic string. J. Polchinski (Santa Barbara, KITP) . 1998. 402pp. Cambridge, UK: Univ. Pr. (1998) 402 p.' All interested students should contact Alex Buchel at abuchel@uwo.ca as soon as possible.

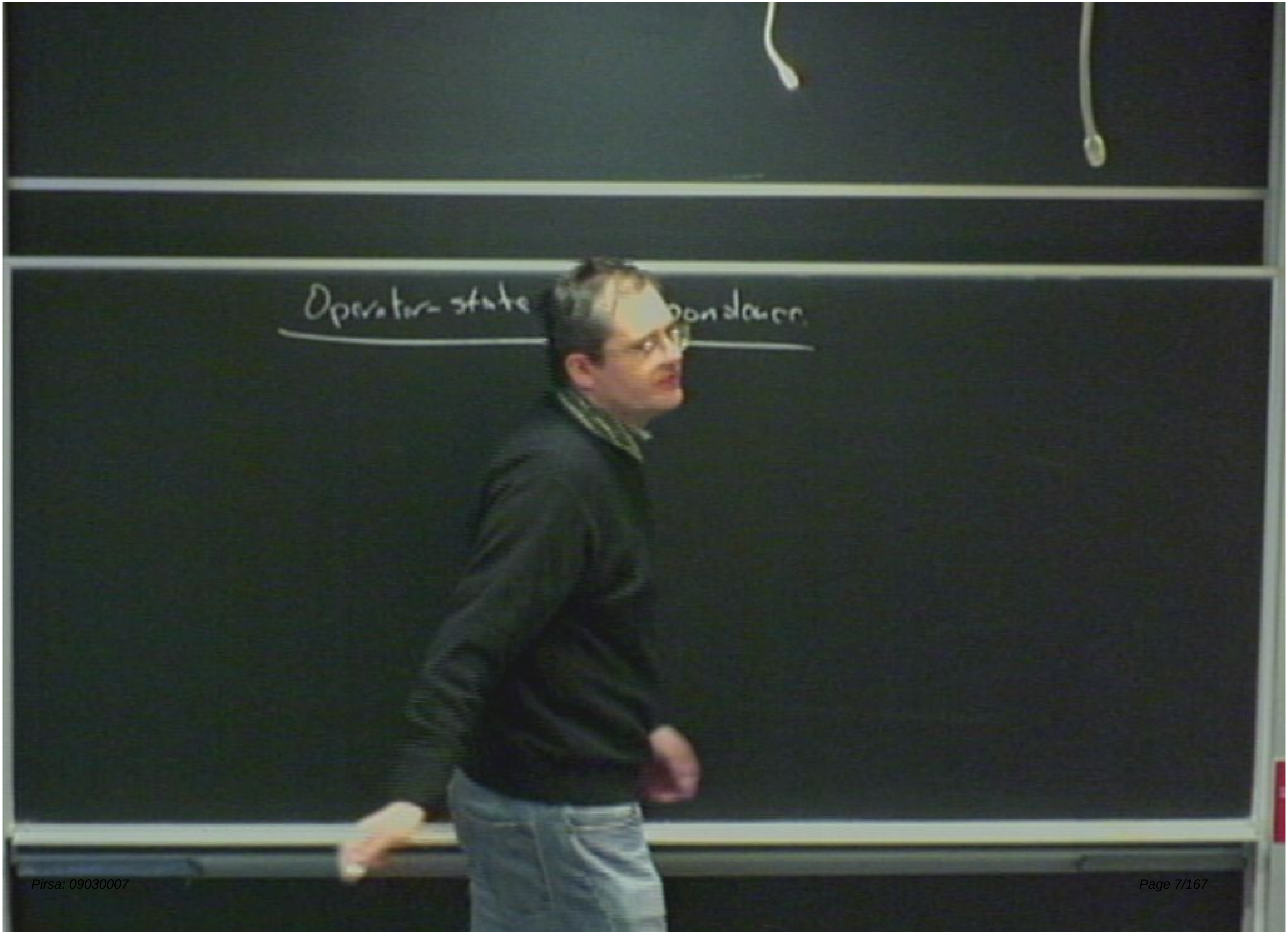









Operator-state


Operator-state
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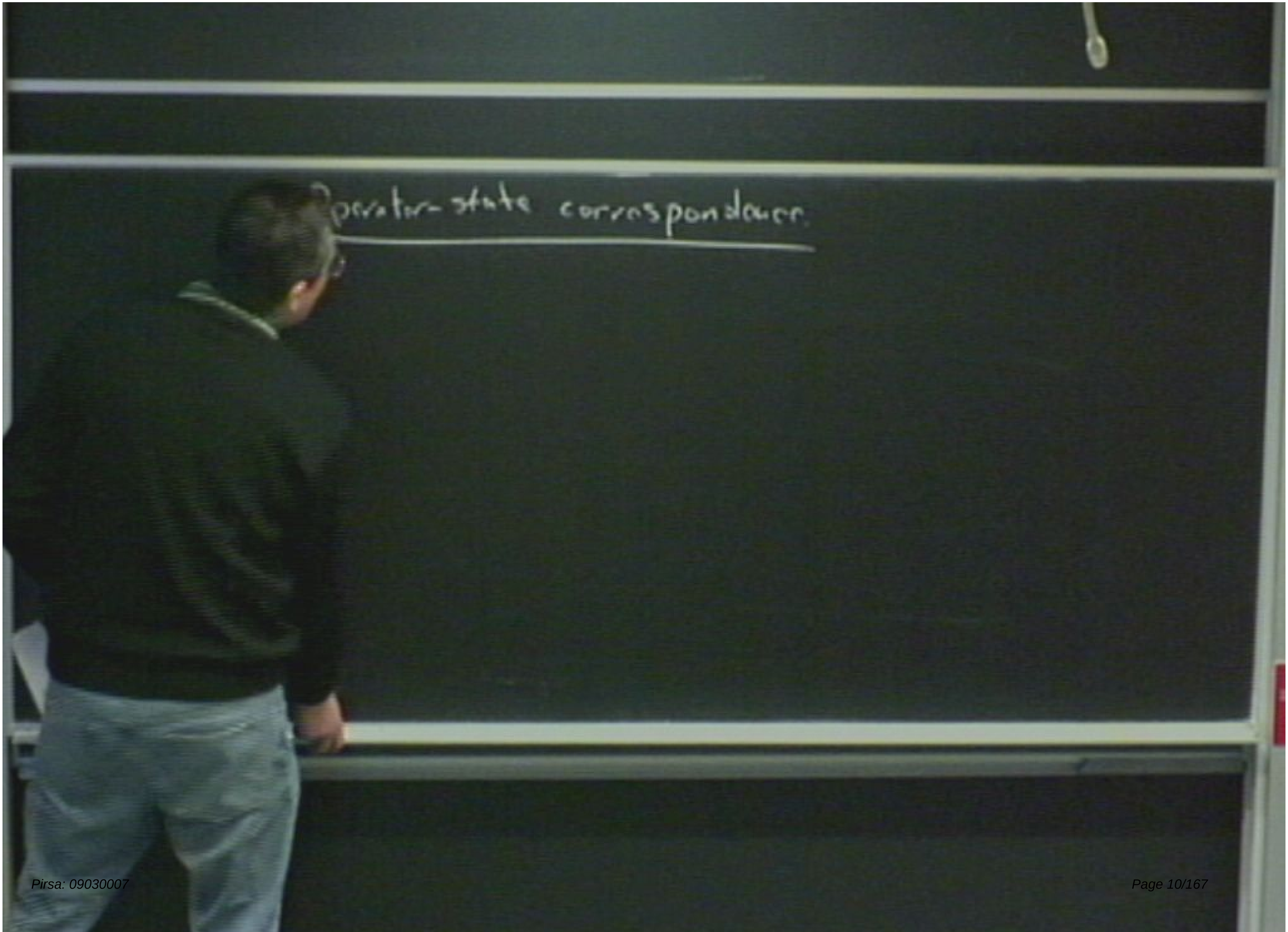
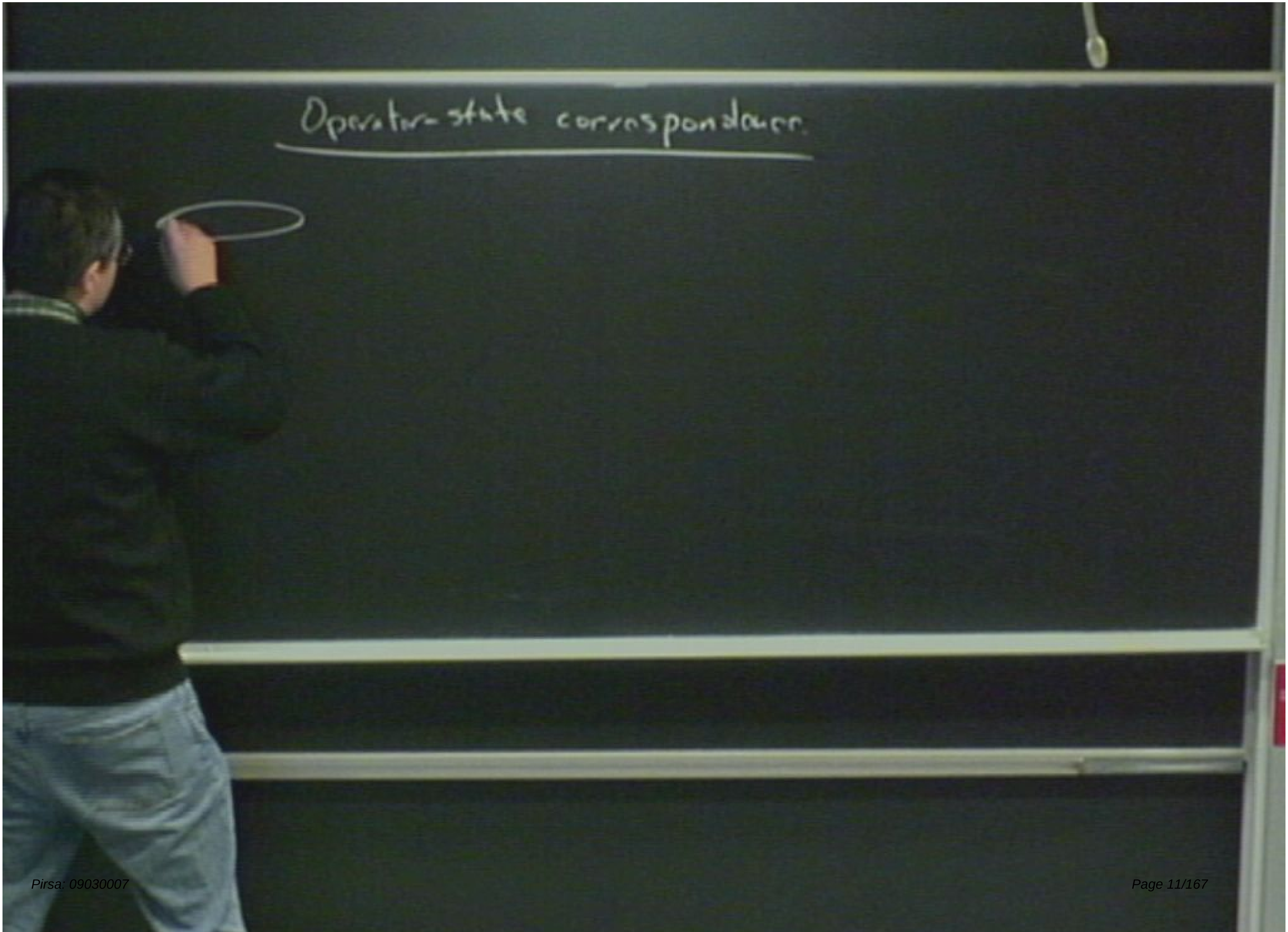
Operator-state correspondence

Operator-state correspondence


Operator-state correspondence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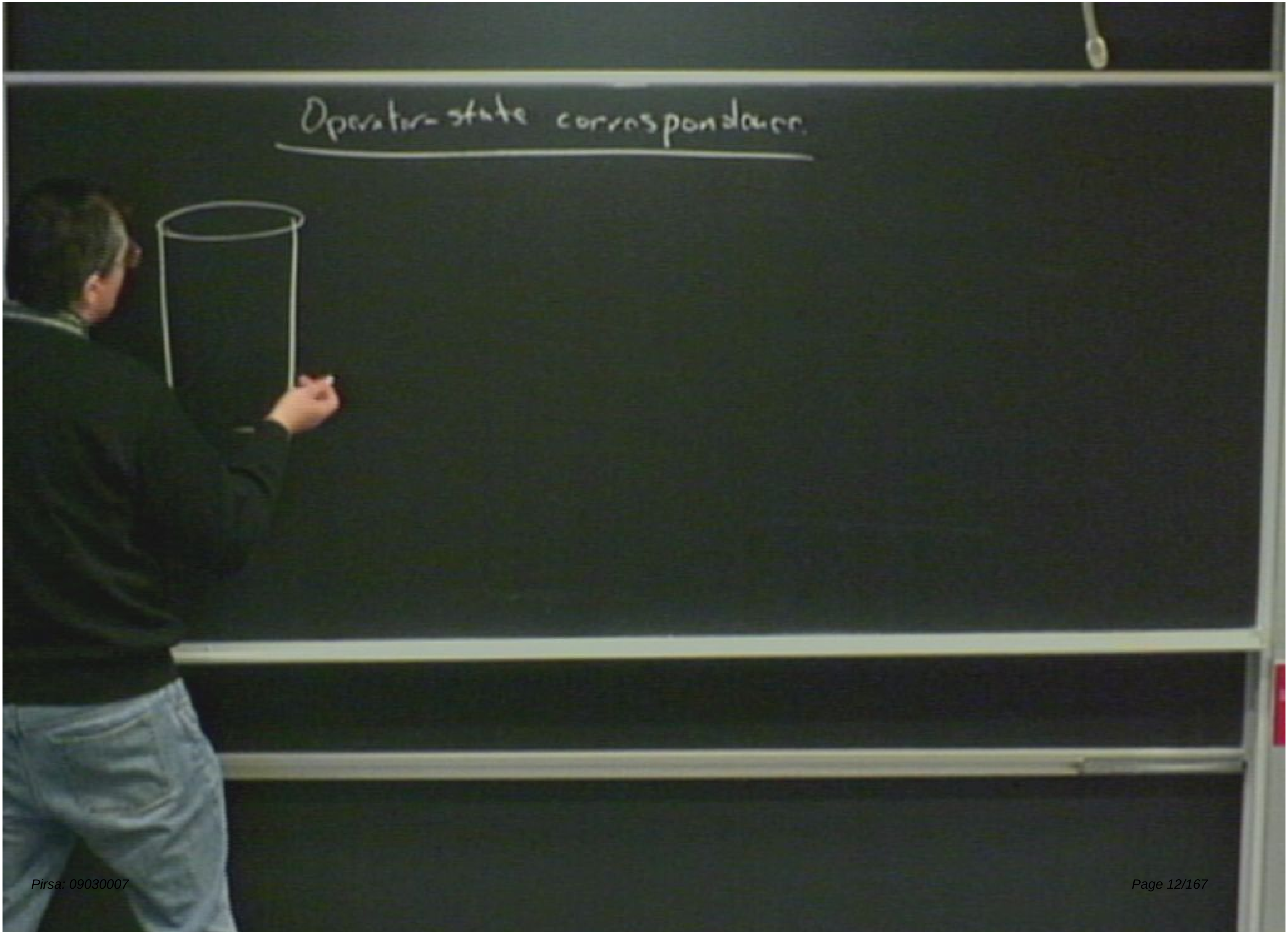
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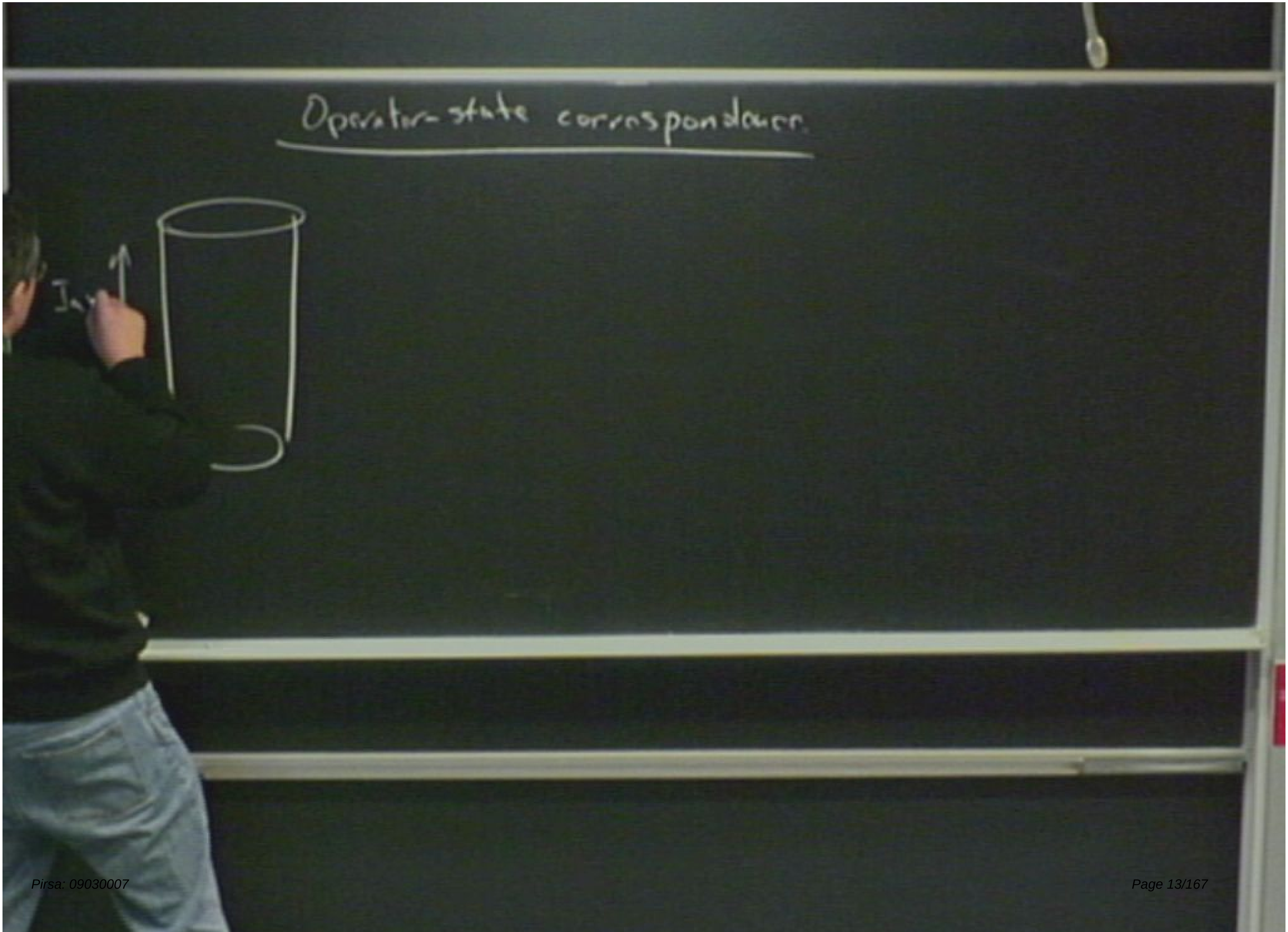
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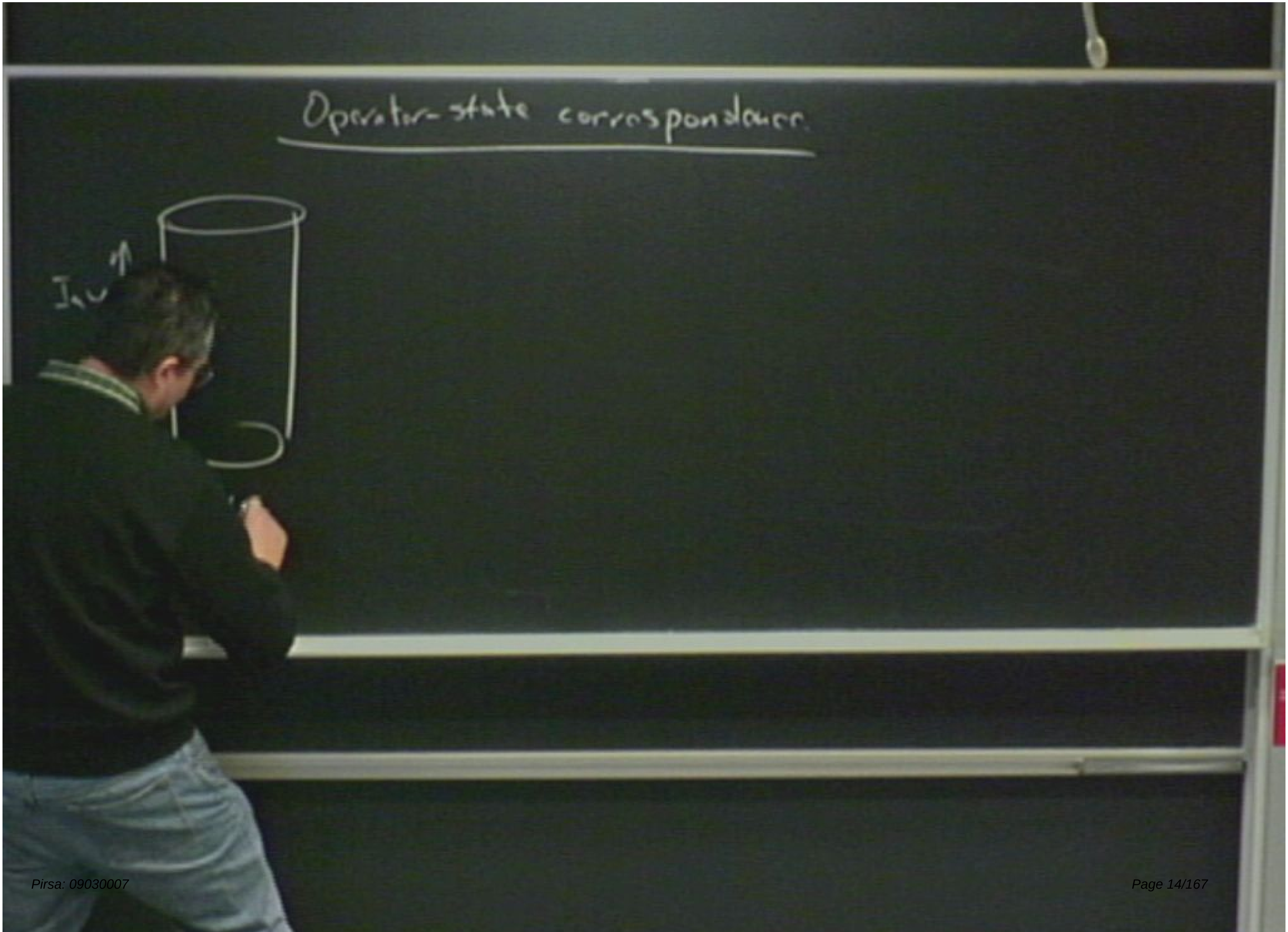
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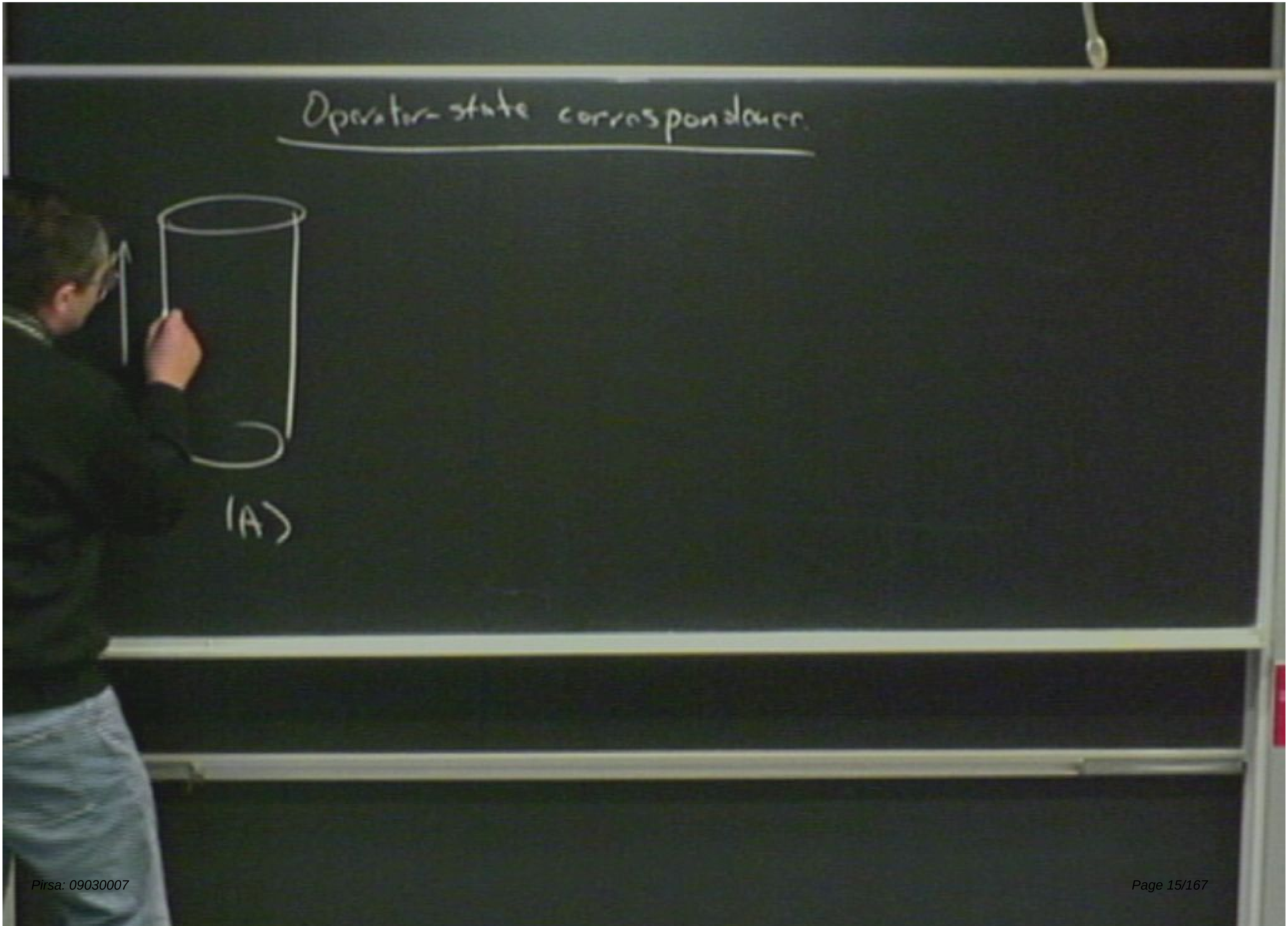
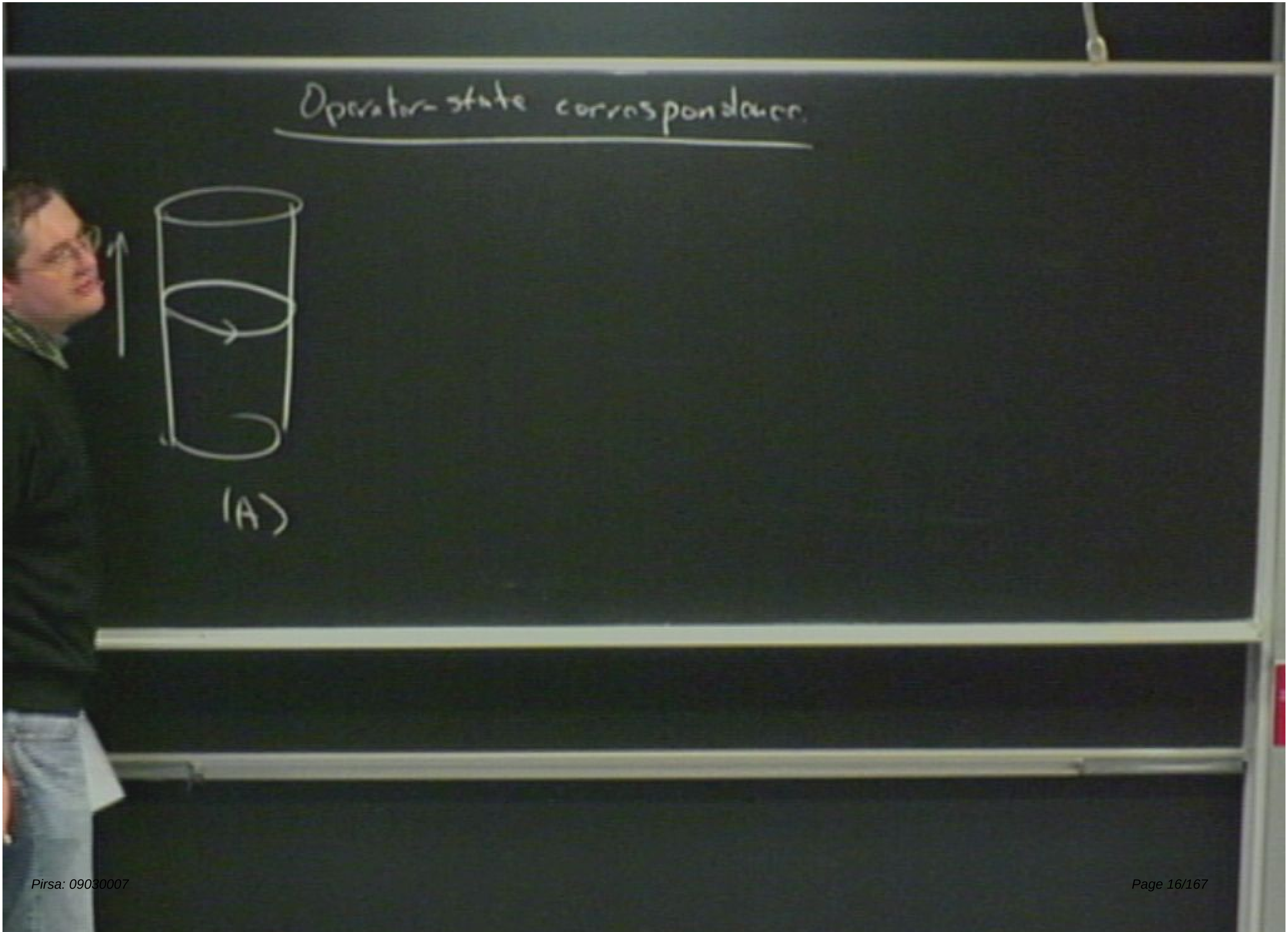

Operator-state correspondence
|A⟩


Operator-state correspondance
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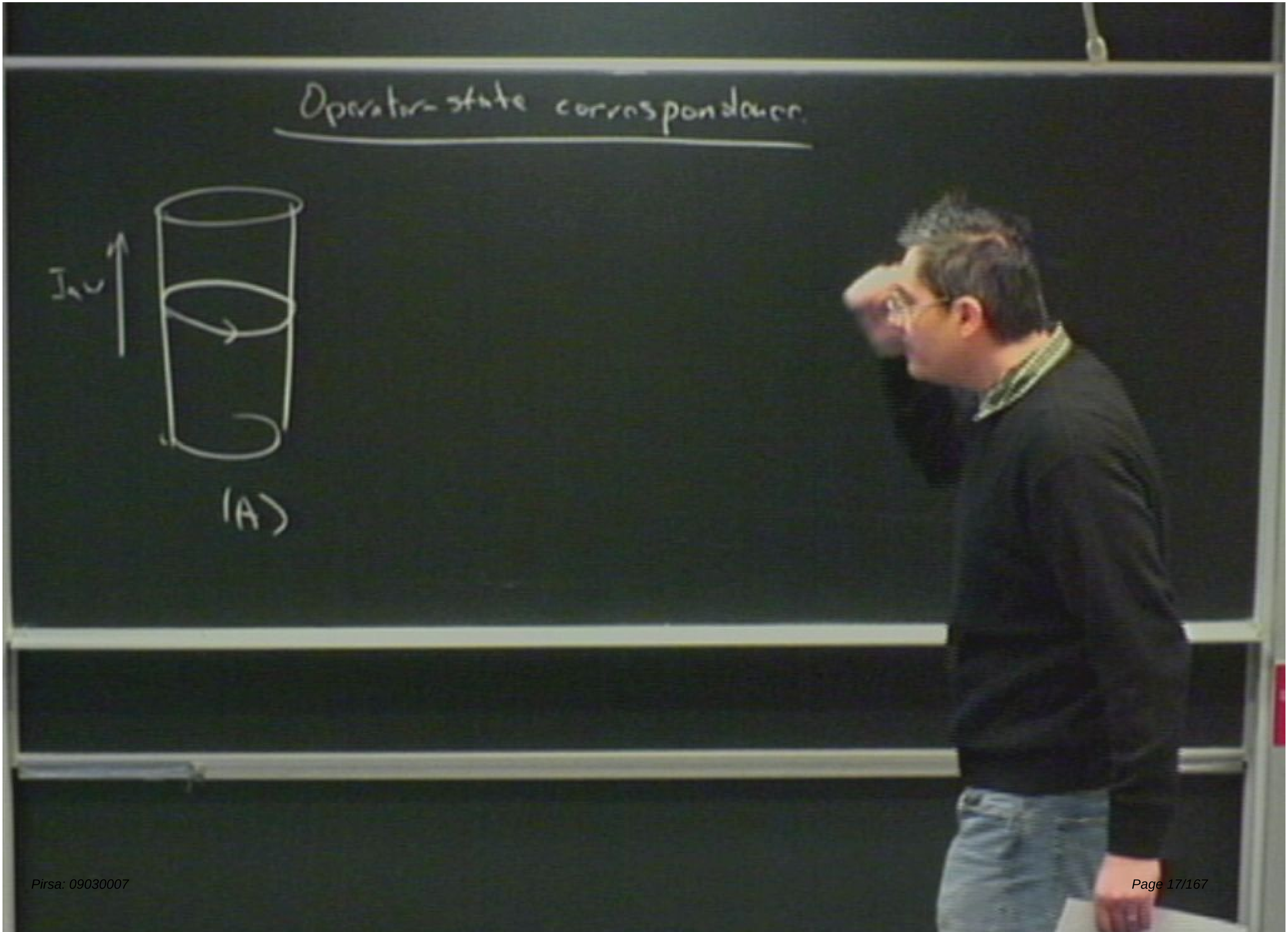
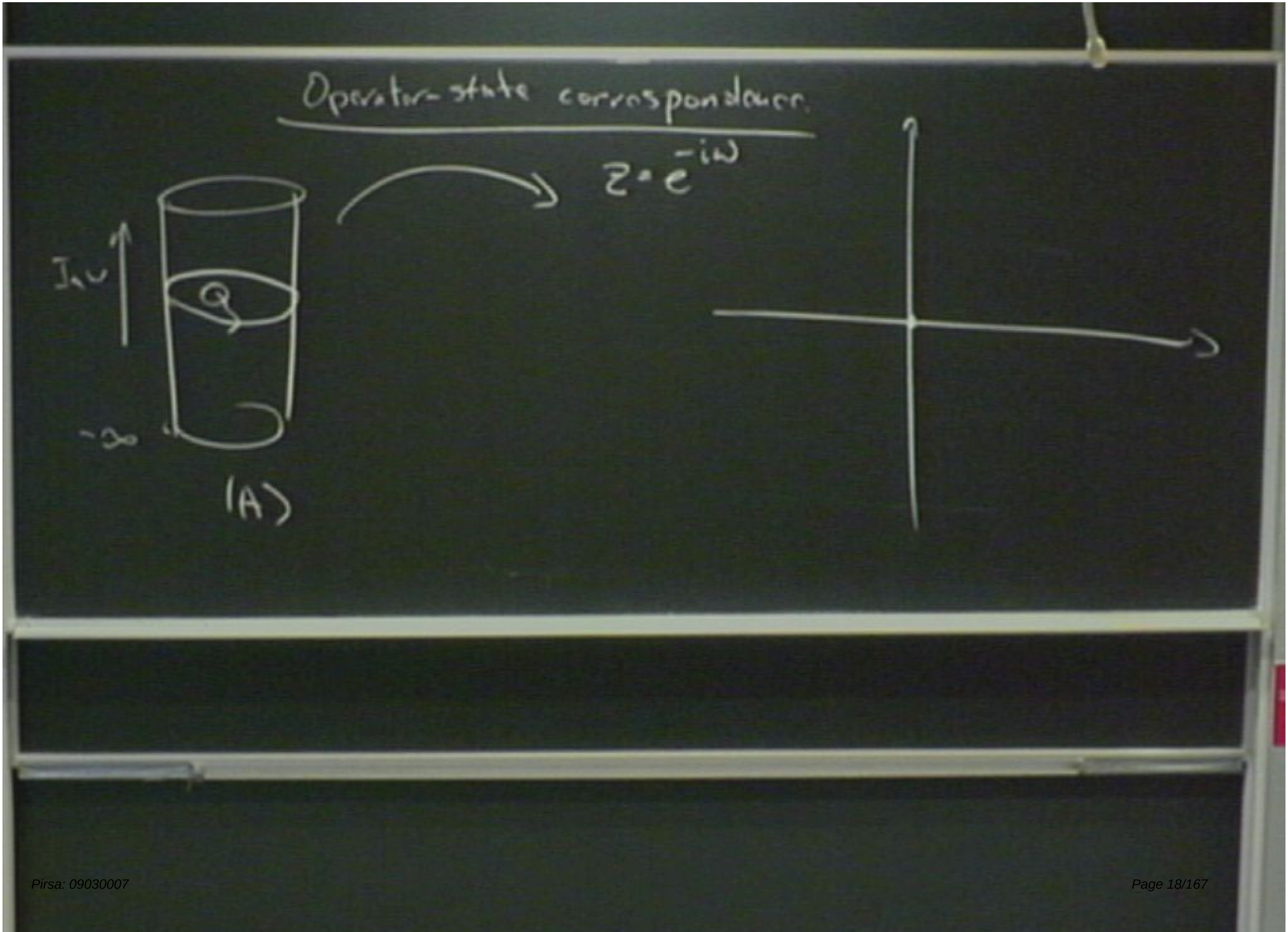
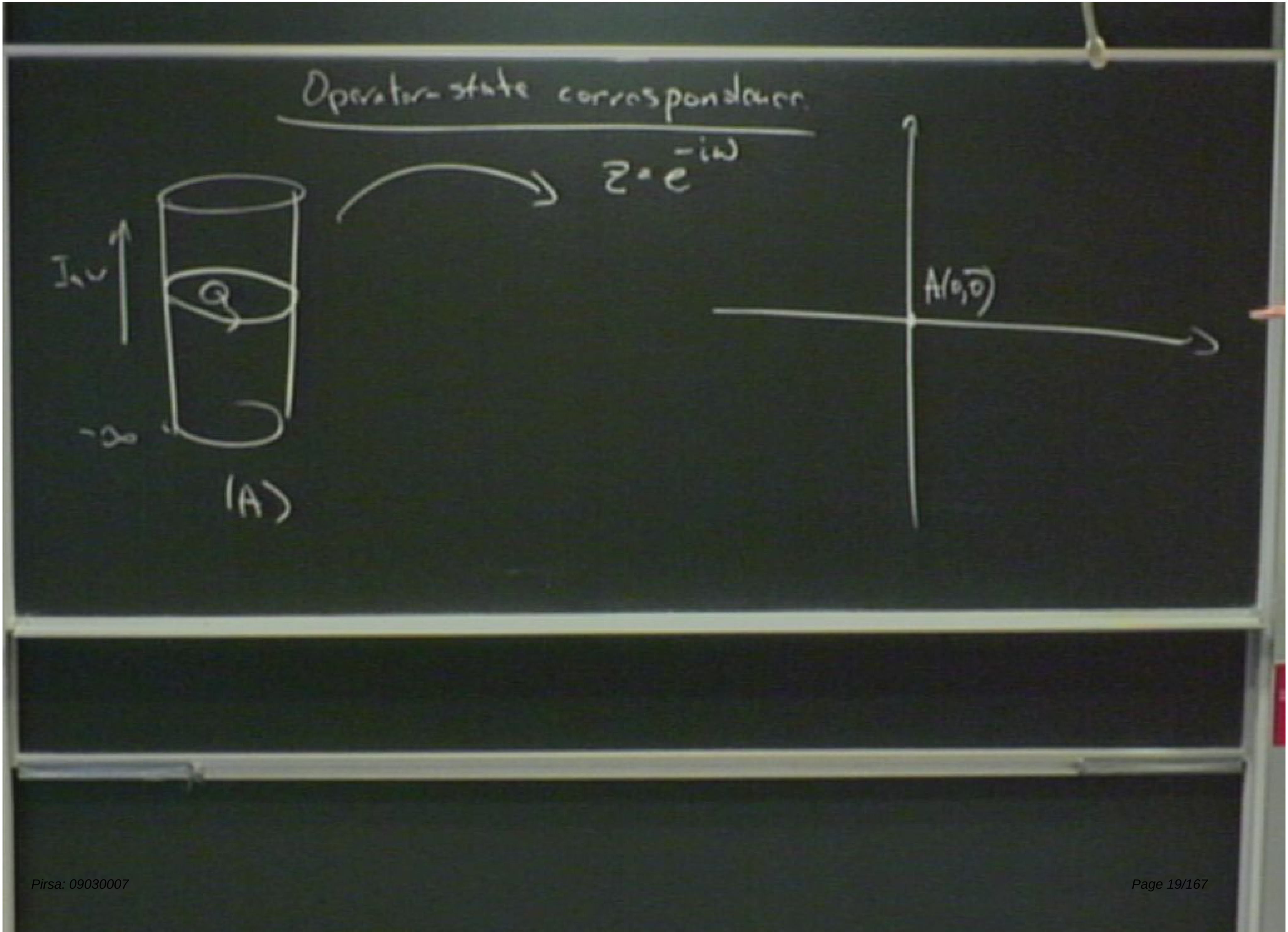
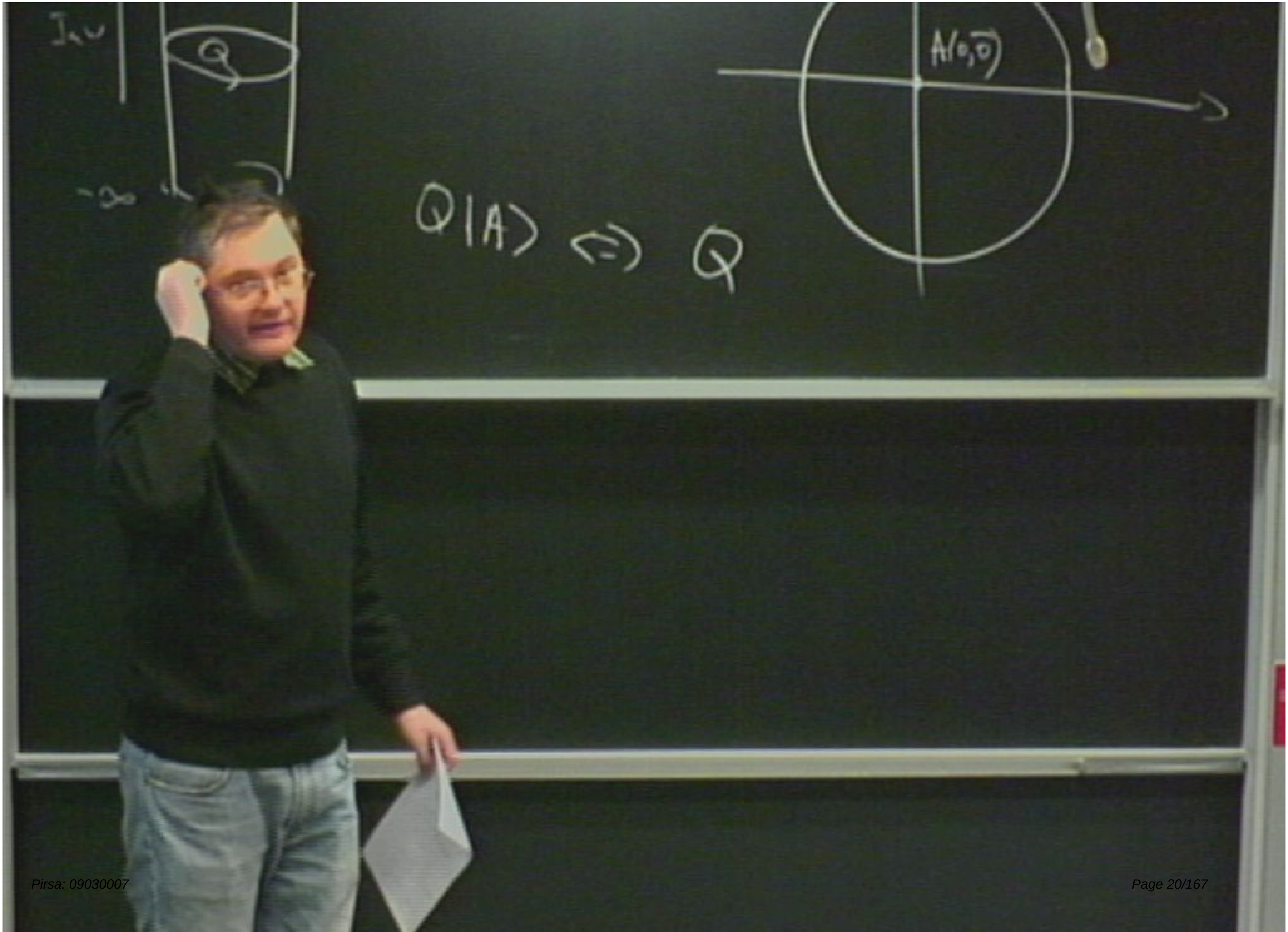

Operator-state correspondence
|A⟩

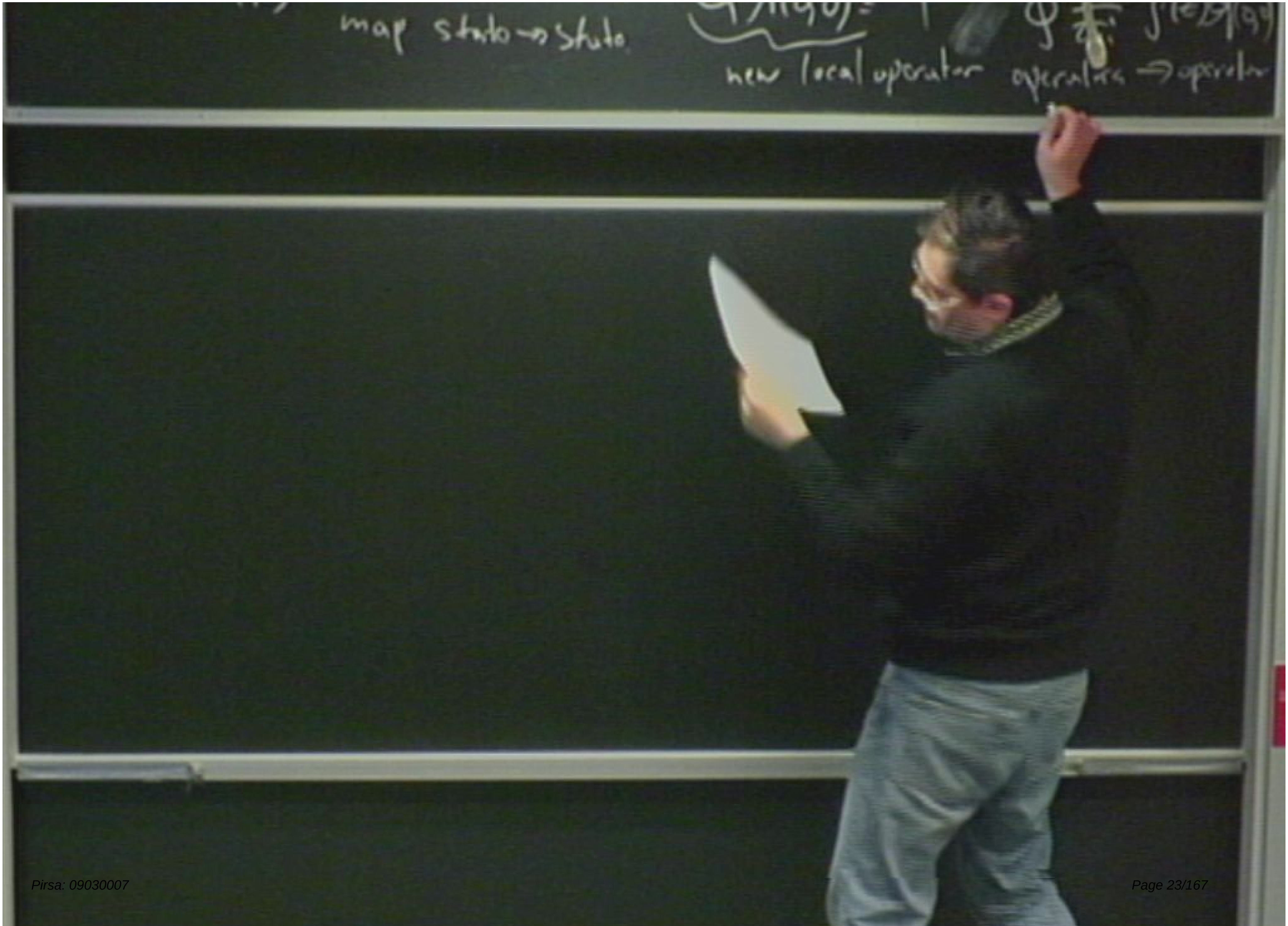

Operator-state correspondence
z = e^{-iw}
Im w
-∞
|A⟩

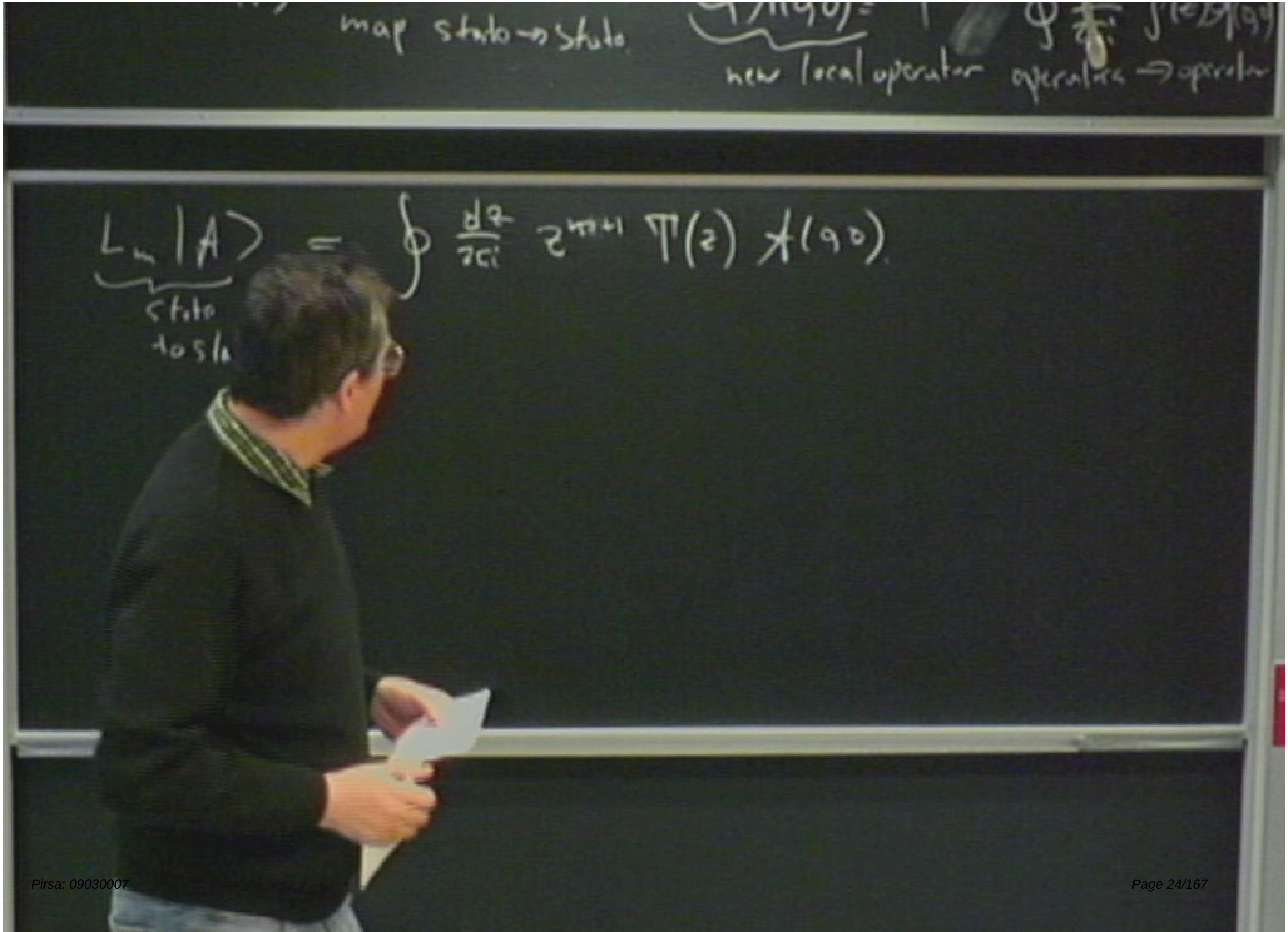

Operator-state correspondence
$z = e^{-iw}$
Im w
$-\infty$
$|A\rangle$
$A(0,\bar{0})$

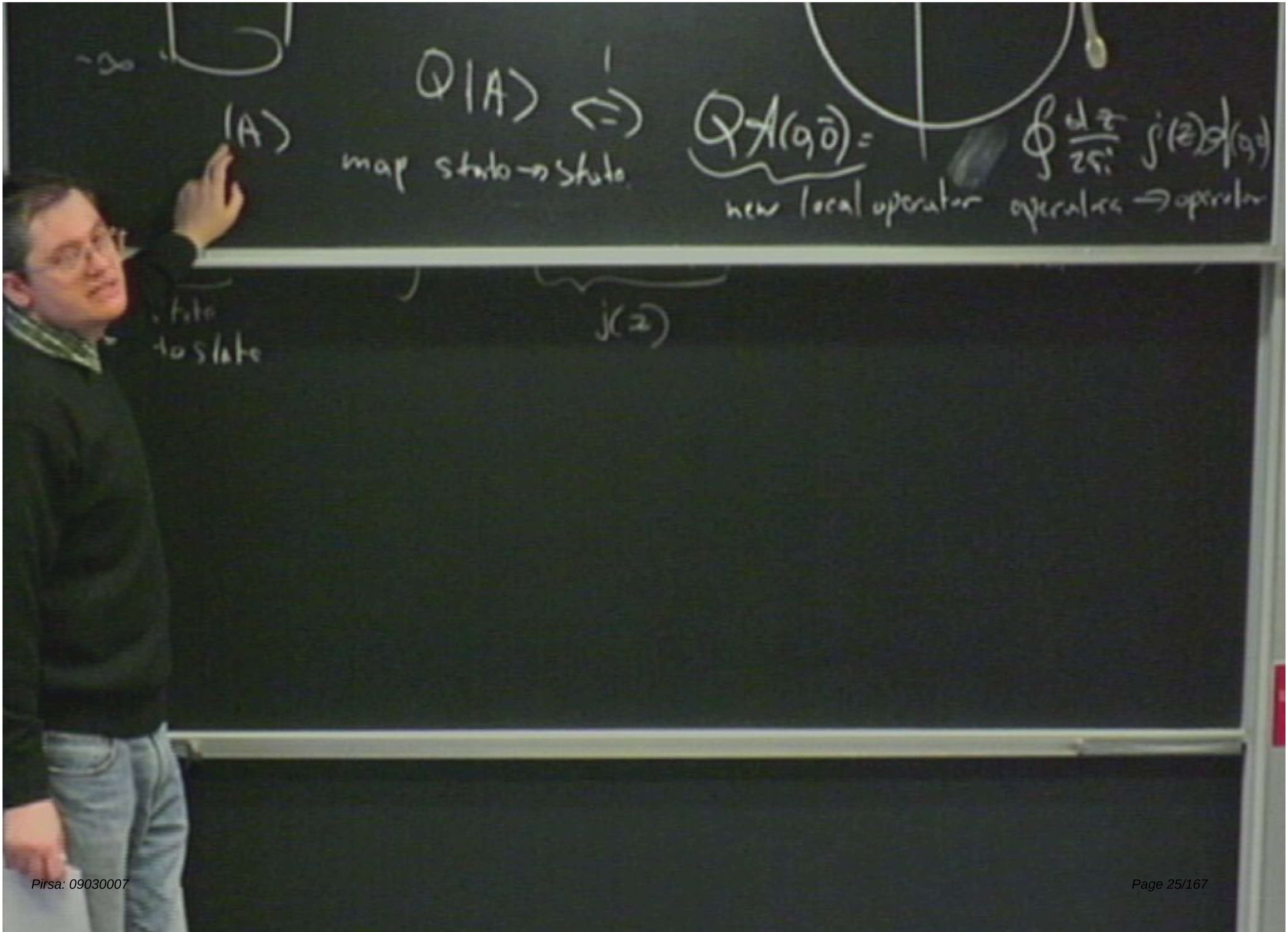

Q|A⟩ ⟺ Q
A(0,0̄)
-∞


$-\infty$
$|A\rangle$
$Q|A\rangle \Leftrightarrow QA(0,\bar{0}) = \oint \frac{dz}{2\pi i} j(z)$
$A(0,0)$

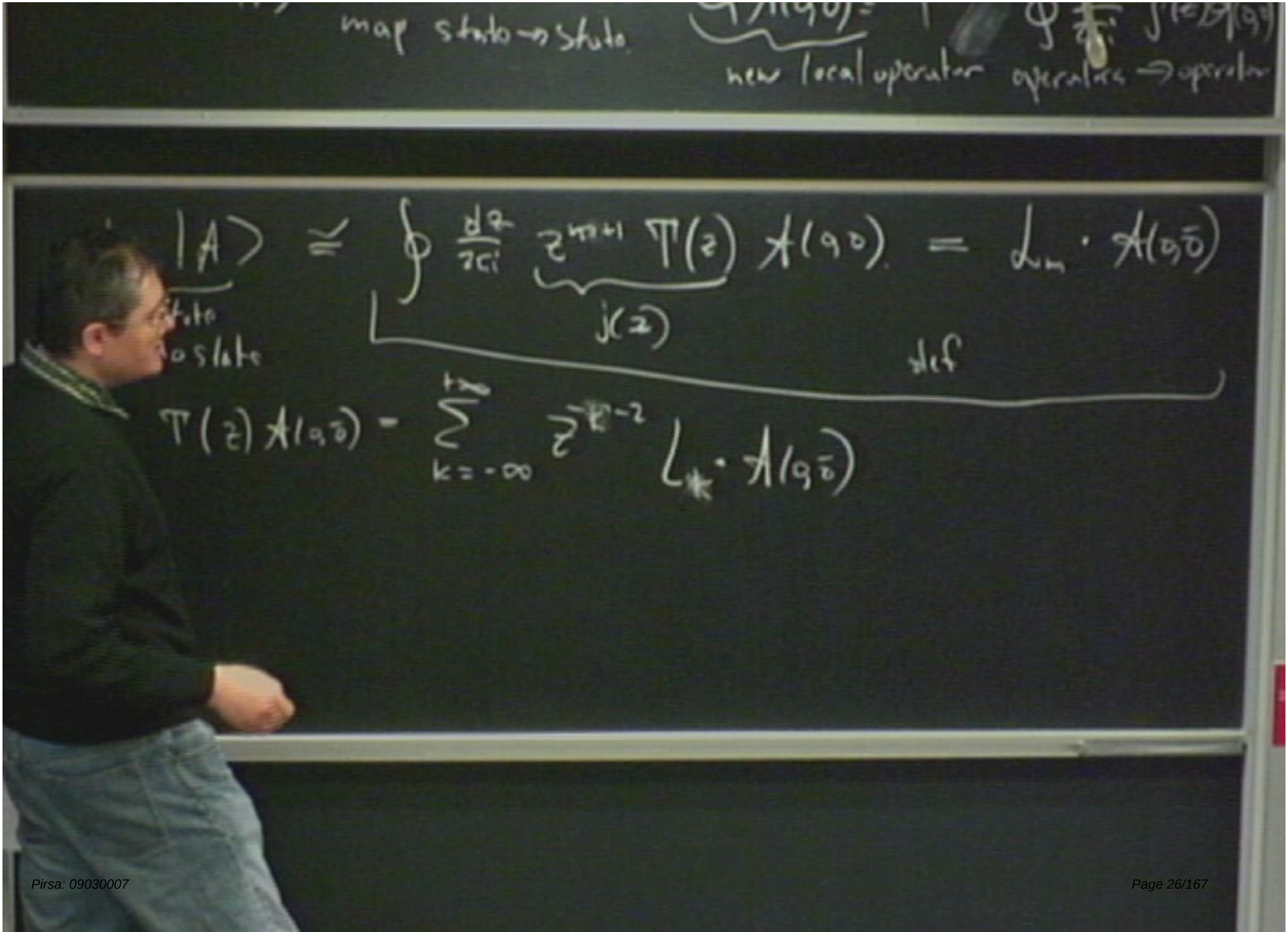

|A⟩
map state → state
QA(0,0̄) =


map state → state
new local operator
operators → operator


map state → state
new local operator
operators → operator
L_m |A⟩ = ∮ dz/2πi z^{m+1} T(z) A(0)



$-\infty$
$Q|A\rangle$
$|A\rangle$
map state → state
$QA(0,\bar{0}) = \oint \frac{dz}{2\pi i} j(z) A(0,0)$
new local operator
operators → operator
$j(z)$
state
to state


map state → state
new local operator
operators → operator
$|A\rangle \cong \oint \frac{dz}{2\pi i} z^{n+1} T(z) A(0,0) = L_n \cdot A(0,\bar{0})$
$j(z)$
state
state
def
$T(z) A(0,\bar{0}) = \sum_{k=-\infty}^{+\infty} z^{-k-2} L_k \cdot A(0,\bar{0})$

map state → state

new local operator

operators → operator

$$\underbrace{L_m |A\rangle}_{\text{state to state}} \cong \oint \frac{dz}{2\pi i} \underbrace{z^{m+1} T(z)}_{j(z)} A(0,0) = L_m \cdot A(0,\bar{0})$$

def

$$T(z) A(0,\bar{0}) = \sum_{k=-\infty}^{+\infty} z^{-k-2} \, L_k \cdot A(0,\bar{0})$$

$$\oint \frac{dz}{2\pi i} z^{m+1} z^{-k-2} = \delta_{m,k} \cdot 1$$

map state $\to$ state

new local operator operators $\to$ operator

$$\underbrace{L_m |A\rangle}_{\substack{\text{state} \\ \text{to state}}} \cong \oint \frac{dz}{2\pi i} \underbrace{z^{m+1} T(z)}_{j(z)} A(0,0) = L_m \cdot A(0,\bar{0})$$

def

$$T(z) A(0,\bar{0}) = \sum_{k=-\infty}^{+\infty} z^{-k-2} \, L_k \cdot A(0,\bar{0})$$

$$\int \frac{dz}{2\pi i} z^{m+1} z^{-k-2} = \delta_{m,k} \cdot 1$$

Recall

$T(z)\, A(0,\bar{0})$

Recall

$$T(z)\,A(0,\bar{0}) \sim \sum_{n=0}^{\infty} \frac{1}{z^{n+1}} \underbrace{A^{n}(0,\bar{0})}$$

$$\delta A(z,\bar{z}) = -\varepsilon \sum_{n=0}^{\infty} \frac{1}{n!} \Big[\partial^{n} v(z)\, A^{(n)}(z,\bar{z}) + \bar{\partial} \bar{v}(\bar{z})\, \tilde{A}^{n}(z,\bar{z}) \Big)$$

Recall

$$T(z)\,A(0,\bar 0) \sim \sum_{n=0}^{\infty} \frac{1}{z^{n+1}} \underbrace{A^{n}(0,\bar 0)}$$

$$\delta A(z,\bar z) = -\varepsilon \sum_{n=0}^{\infty} \frac{1}{n!} \left[\partial^{n} v(z)\, A^{(n)}(z,\bar z) + \bar\partial \bar v(\bar z)\, \tilde A^{n}(z,\bar z) \right]$$

$$z \to z + \varepsilon v(z)$$

$$\bar z \to \bar z + \varepsilon \bar v(\bar z)$$

$\bar{z} \to \bar{z} + \epsilon\, \bar{v}(\bar{z})$

If A is an operator of a given $(h, \bar{h})$

$\varepsilon \ldots \{+\{\,\bar{U}(\bar{z})$

If A is an operator of a given $(h, \bar{h})$

$$T(z)A = \ldots\ldots \frac{h}{z^2}A(0) + \frac{1}{z}\partial A(0) + \text{nonsingular terms}$$

If A is an operator of a given $(h, \bar{h})$

$$T(z)A = \cdots \frac{h}{z^2}A(0) + \frac{1}{z}\partial A(0) + \text{nonsingular terms}\ldots$$

If A is an operator of a given $(h, \bar{h})$

$T(z) A = \cdots \frac{h}{z^2} A(0) + \frac{1}{z} \partial A(0) +$ nonsingular term..

$L_{-1} A = \partial A$

$\ldots + \epsilon\, U(\bar{z})$

If A is an operator of a given $(h, \bar{h})$

$T(z)A = \ldots \frac{h}{z^2}A(0) + \frac{1}{z}\partial A(0) +$ nonsingular terms..

$L_{-1}A = \partial A$

$L_0 A = h \cdot A$

$\{ + \{ O(\bar{z})$

If A is an operator of a given $(h, \bar{h})$

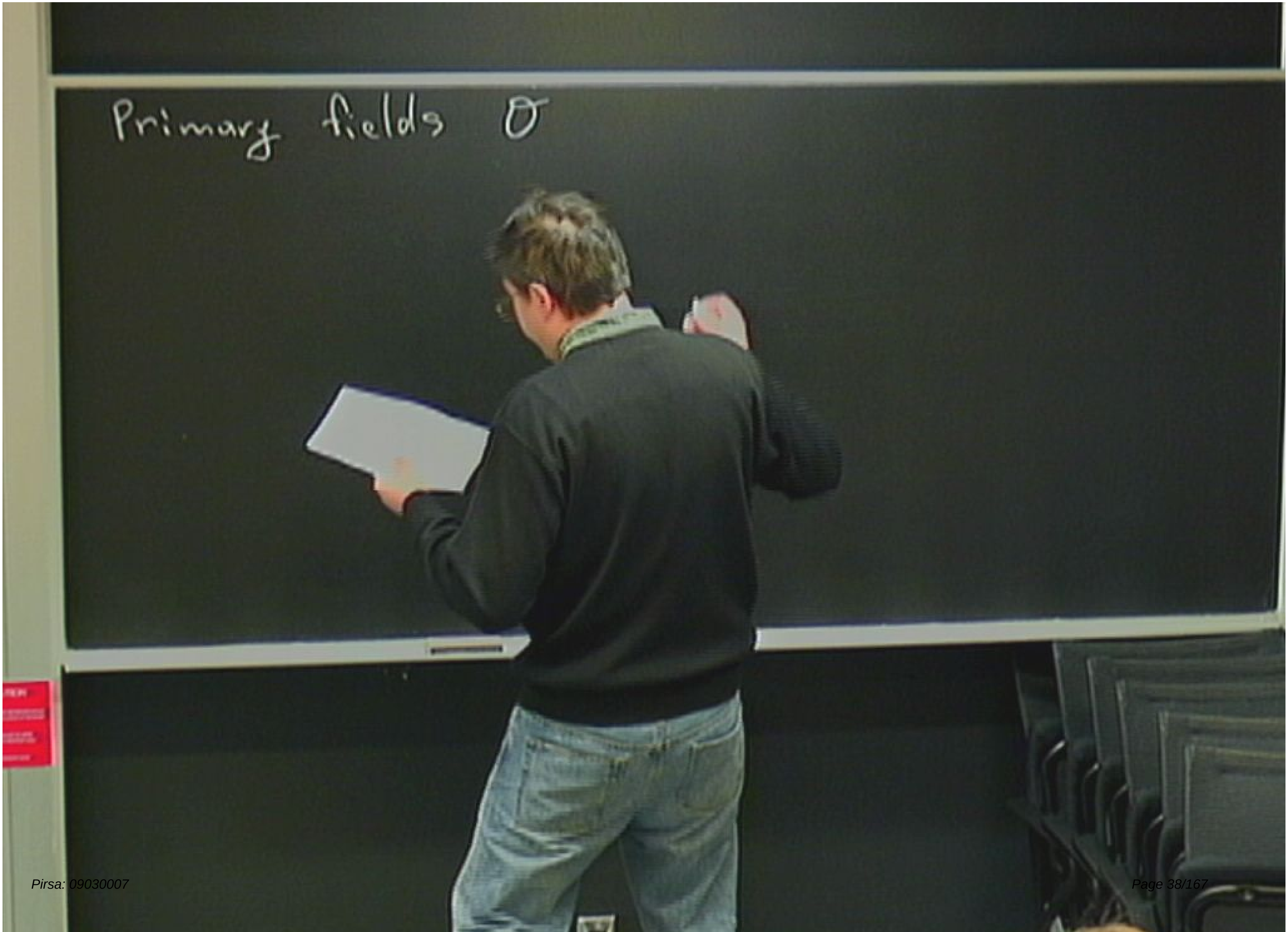
$\checkmark$ $T(z) A = \cdots \frac{h}{z^2} A(0) + \frac{1}{z} \partial A(0)$ + nonsingular term..

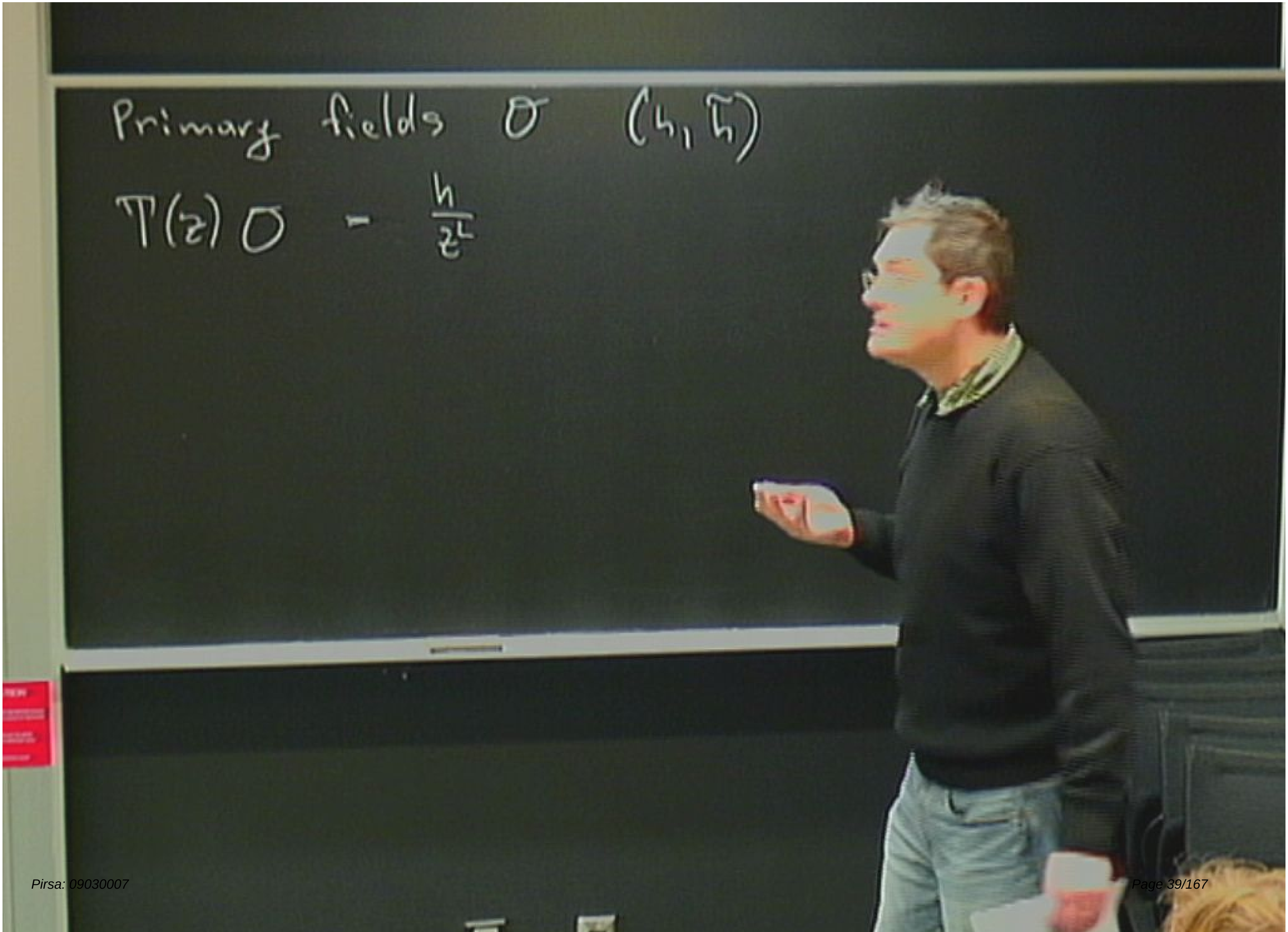
$L_{-1} A = \partial A$

$L_0 A = h \cdot A$

$\tilde{L}_{-1} A = \bar{\partial} A$

$\tilde{L}_0 A = \tilde{h} A$


Primary fields O

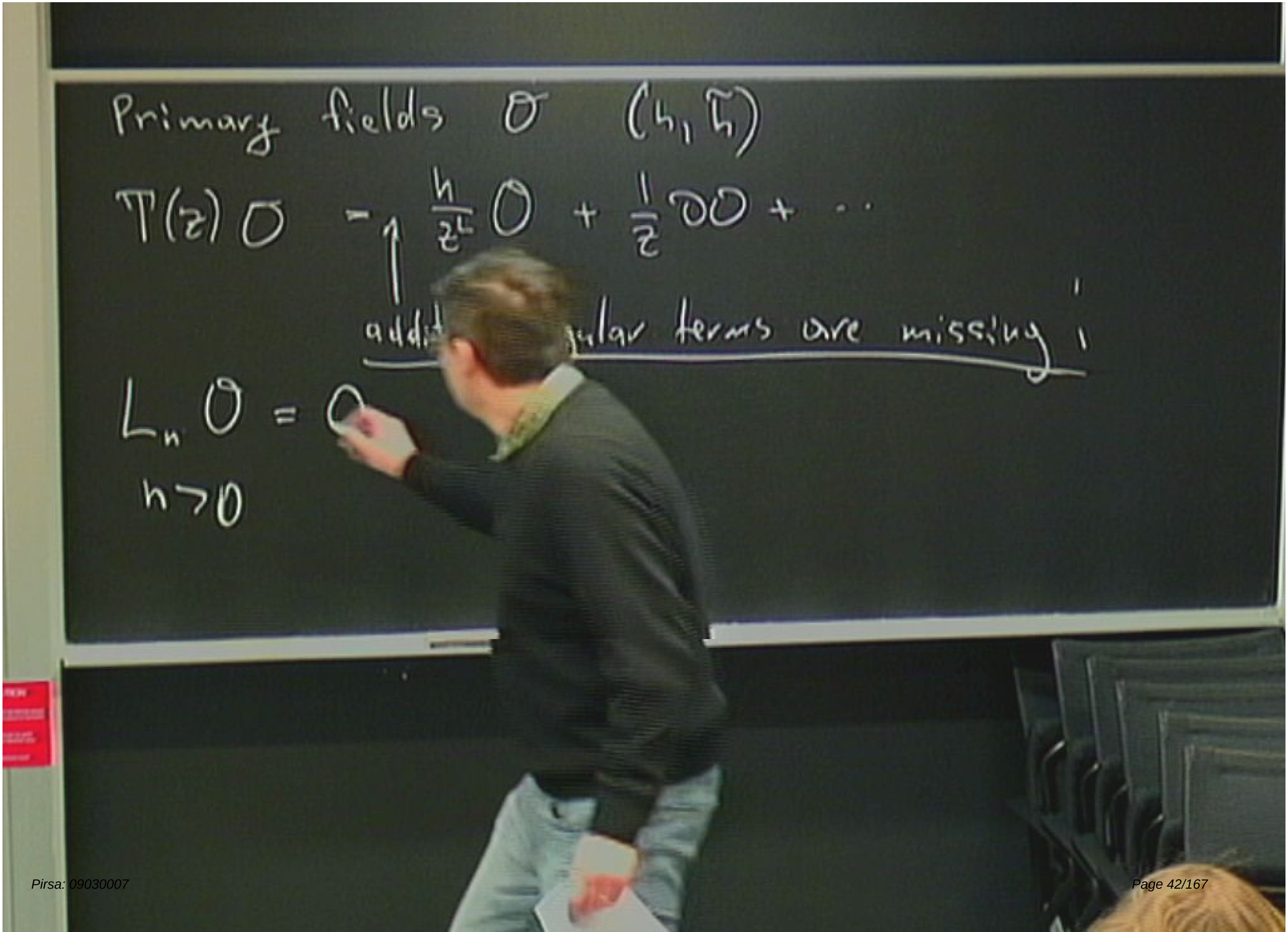

Primary fields $\mathcal{O}$ $(h, \bar{h})$
$T(z)\,\mathcal{O} \sim \frac{h}{z^2}$

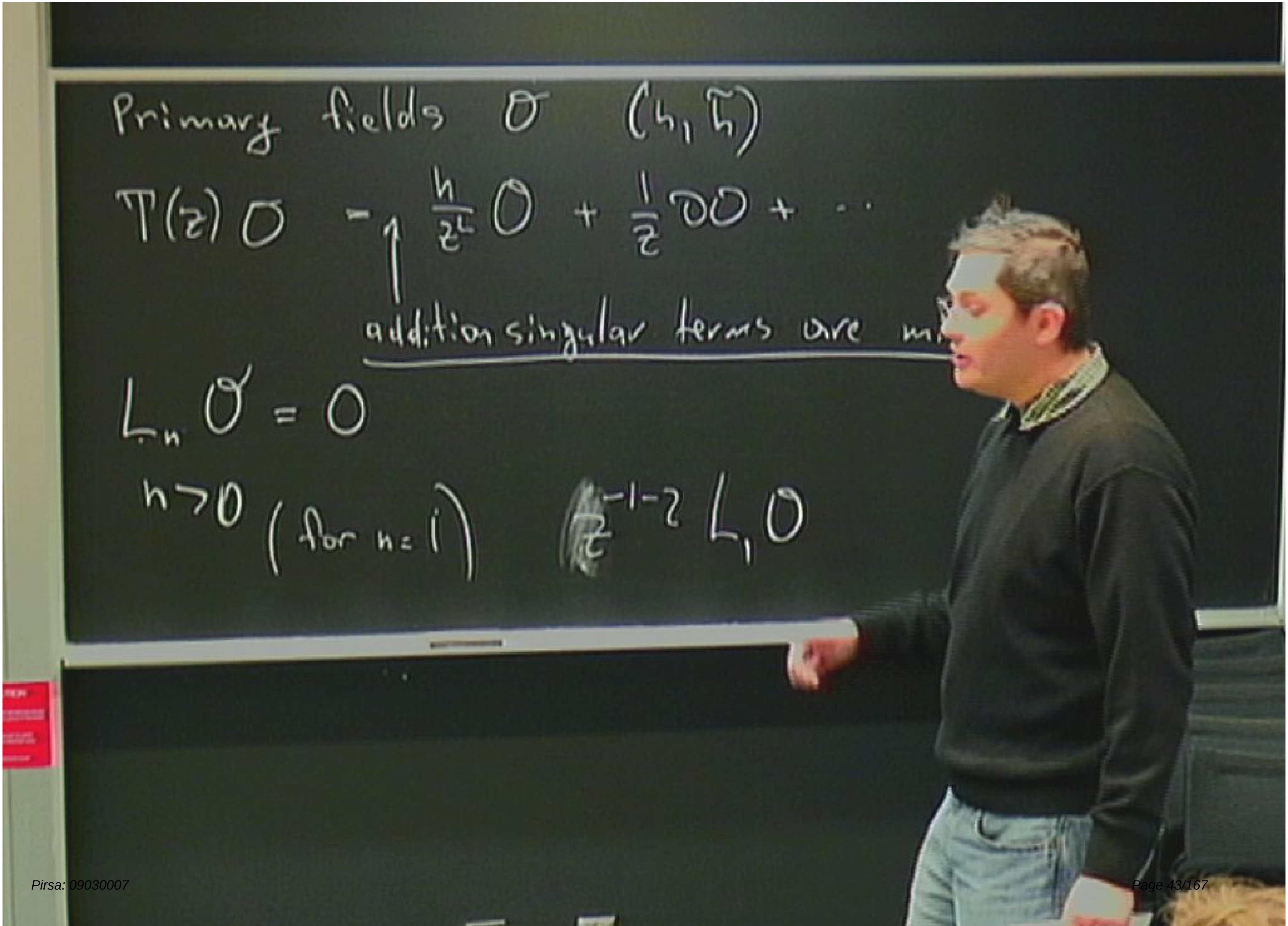
Primary fields $\mathcal{O}$ $(h, \bar{h})$

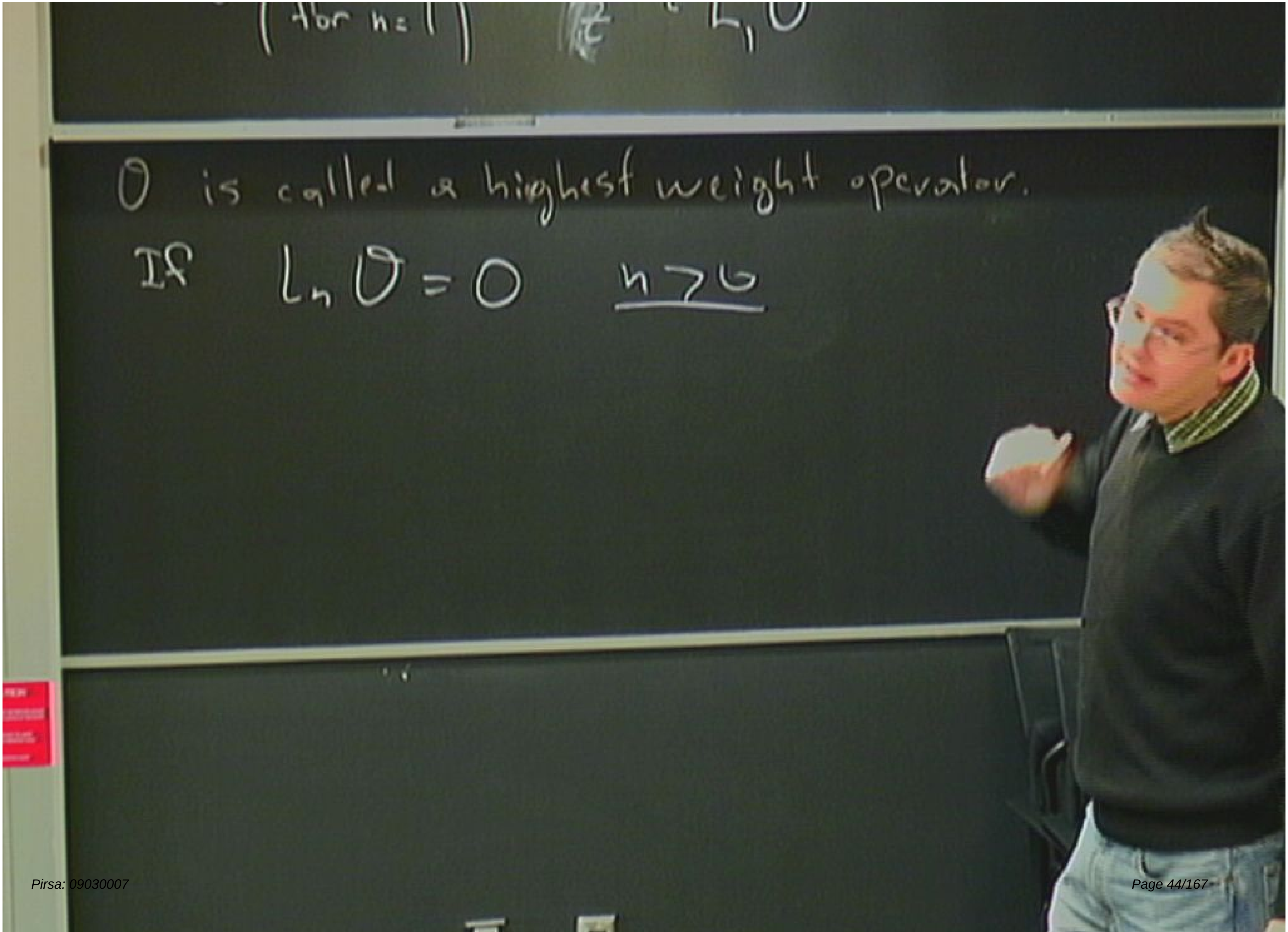
$$T(z)\,\mathcal{O} \sim \frac{h}{z^2}\,\mathcal{O} + \frac{1}{z}\,\partial\mathcal{O} + \cdots$$

addition singular terms are missing !


$Q|A\rangle \Leftrightarrow Q \cdot A(0,\bar{0}) = \oint \frac{dz}{2\pi i} j(z) A(0,\bar{0})$
$|A\rangle$
map state → state
new local operator
operators → operator
$L_m |A\rangle$ state to s'
$\oint \frac{dz}{2\pi i} z^{m+1} T(z) A(0,0) = L_m \cdot A(0,\bar{0})$
$j(z)$
def
$\sum_{k=-\infty}^{+\infty} z^{-k-2} L_k \cdot A(0,\bar{0})$
$z^{m+1} z^{-k-2} = \delta_{m,k} \cdot 1$


Primary fields $\mathcal{O}$ $(h, \bar{h})$
$T(z)\,\mathcal{O} \sim \frac{h}{z^2}\mathcal{O} + \frac{1}{z}\partial\mathcal{O} + \cdots$
addit
gular terms are missing !
$L_n \mathcal{O} = 0$
$n > 0$


Primary fields O (h, h̃)
T(z) O ~ h/z² O + 1/z ∂O + ...
addition singular terms are m
Lₙ O = 0
n>0 (for n=1)
z⁻¹⁻² L₁ O


(for n=1)
L, O
O is called a highest weight operator.
If $L_n \mathcal{O} = 0$ $n > 0$

(for $n = 1$) $\quad L_1 \mathcal{O}$

$\mathcal{O}$ is called a highest weight operator.

If $\quad L_n \mathcal{O} = 0 \quad \underline{n > 0}$

Note

(for $n=1$) $\quad L_1 \mathcal{O}$

$\mathcal{O}$ is called a highest weight operator.

If $L_n \mathcal{O} = 0 \quad \underline{n > 0}$

<u>Note</u> $L_0 |A\rangle = h |A\rangle$

$L_0 (L_n |A\rangle) = (h-n) |A\rangle$

(for $n=1$) $\quad L_1 \mathcal{O}$

$\mathcal{O}$ is called a highest weight operator.

If $\quad L_n \mathcal{O} = 0 \quad \underline{n > 0}$

If we act with bunch of L_n $(n>0)$ we finally reach the highest weight state

<u>Note</u>

$$L_0 |A\rangle = h |A\rangle$$

$$L_0 (L_n |A\rangle) = (h-n) |A\rangle$$

$(\text{for } n=1)$ $\quad L_1 \mathcal{O}$

$\mathcal{O}$ is called a highest weight operator.

If $L_n \mathcal{O} = 0$ $\quad \underline{n>0}$

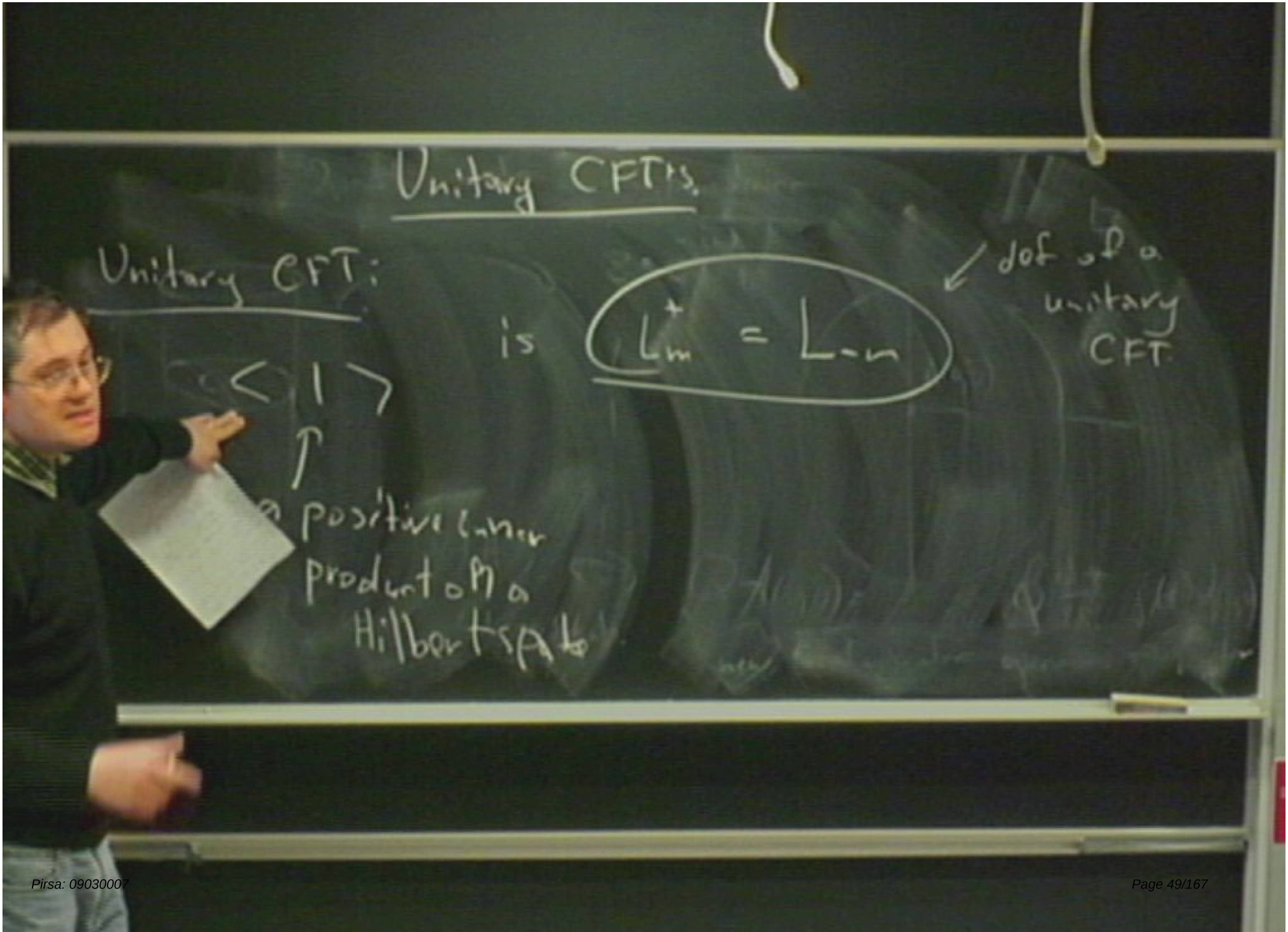
If we act with bunch of L_n $(n>0)$ we finally reach the highest weight state

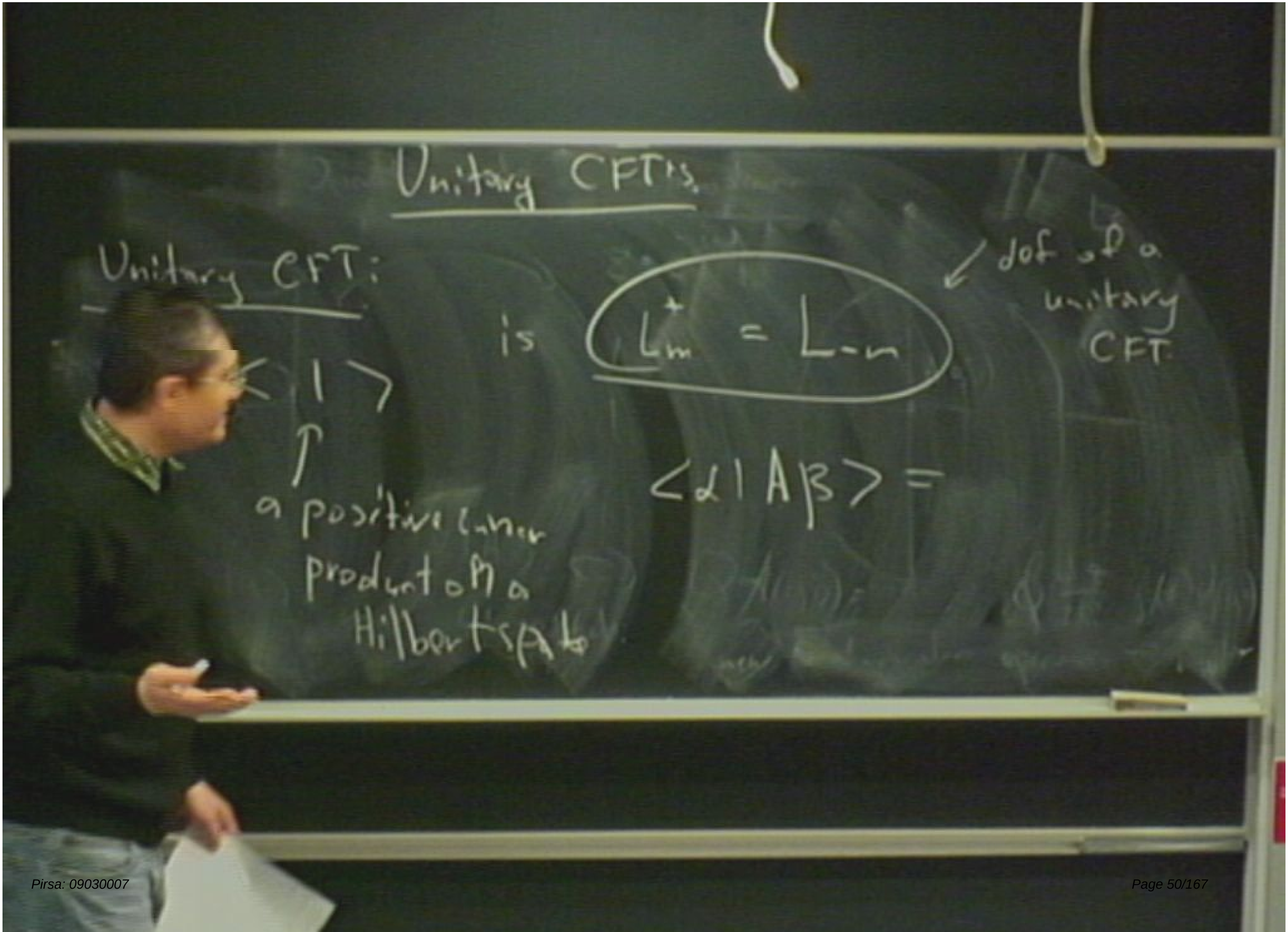
Note

$$L_0 |A\rangle = h |A\rangle$$

$$L_0 (L_n |A\rangle) = (h-n) |A\rangle$$

Implicitly L_0 spectrum is bounded from below


Unitary CFT's
Unitary CFT:
is
L_m^† = L_-m
dof of a unitary CFT
< | >
a positive inner product of a Hilbert space


Unitary CFT's
Unitary CFT:
is
$L_m^\dagger = L_{-m}$
dof of a unitary CFT
$\langle \, | \, \rangle$
a positive inner product on a Hilbert space
$\langle \alpha | A \beta \rangle =$

Unitary CFT's

Unitary CFT:

$\langle \ | \ \rangle$

a positive inner product on a Hilbert space

is $L_m^\dagger = L_{-m}$

dof of a unitary CFT

$$\langle \alpha | A \beta \rangle = \langle A^\dagger \alpha | \beta \rangle$$

a positive inner product on a Hilbert space

$\langle A \alpha | \beta \rangle$

ii) $h_0 > 0$ in a unitary CFT

a positive inner
product on a
Hilbert space

$\langle A\alpha | \beta \rangle$

(i) $h_0 > 0$ in a unitary CFT

(ii) $\mathcal{O}$ is (anti) holomorphic iff $(h = 0)$ $\tilde{h} = 0$
$(h, \tilde{h})$

$$\langle 0|2L_0|0\rangle = 2h_\sigma \cdot \langle 0|0\rangle$$

$\langle 0 | 2L_0 | 0 \rangle = 2h_\sigma \langle 0 | 0 \rangle$
$\langle 0 | [L_-, L_+] | 0 \rangle$

$$\langle A | 2L_0 | A \rangle = 2h_A \langle 0|0 \rangle$$

is a highest weight state

$$\parallel$$

$$\langle A | [L_-, L_+] | A \rangle =$$

$$\langle A | 2L_0 | A \rangle = 2h_A \langle 0|0\rangle$$

$\hookrightarrow$ is a highest weight state

$\|$

$$\langle A | [L_-, L_+] | A \rangle = \langle A | L_- L_+$$

$$\langle A | 2L_0 | A \rangle = 2h_A \langle 0|0\rangle$$

is a highest weight state

$$\parallel$$

$$\langle A | [L_+, L_-] | A \rangle = \langle A | L_+ L_- | A \rangle - \langle A | L_- L_+ | A \rangle$$

$$\langle A | 2L_0 | A \rangle = 2h_A \langle 0|0\rangle$$

$\hookrightarrow$ is a highest weight state

$$\parallel$$

$$\langle A | [L_+, L_-] | A \rangle = \langle A | L_+ L_- | A \rangle - \langle A | L_- L_+ | A \rangle$$

$$= \left| L_- |A\rangle \right|^2$$

$$\langle A | 2L_0 | A \rangle = 2h_A \langle A | A \rangle$$

is a highest weight state

$$\|$$

$$\langle A | [L_+, L_-] | A \rangle = \langle A | L_+ L_- | A \rangle - \langle A | \cancel{L_- L_+} | A \rangle$$

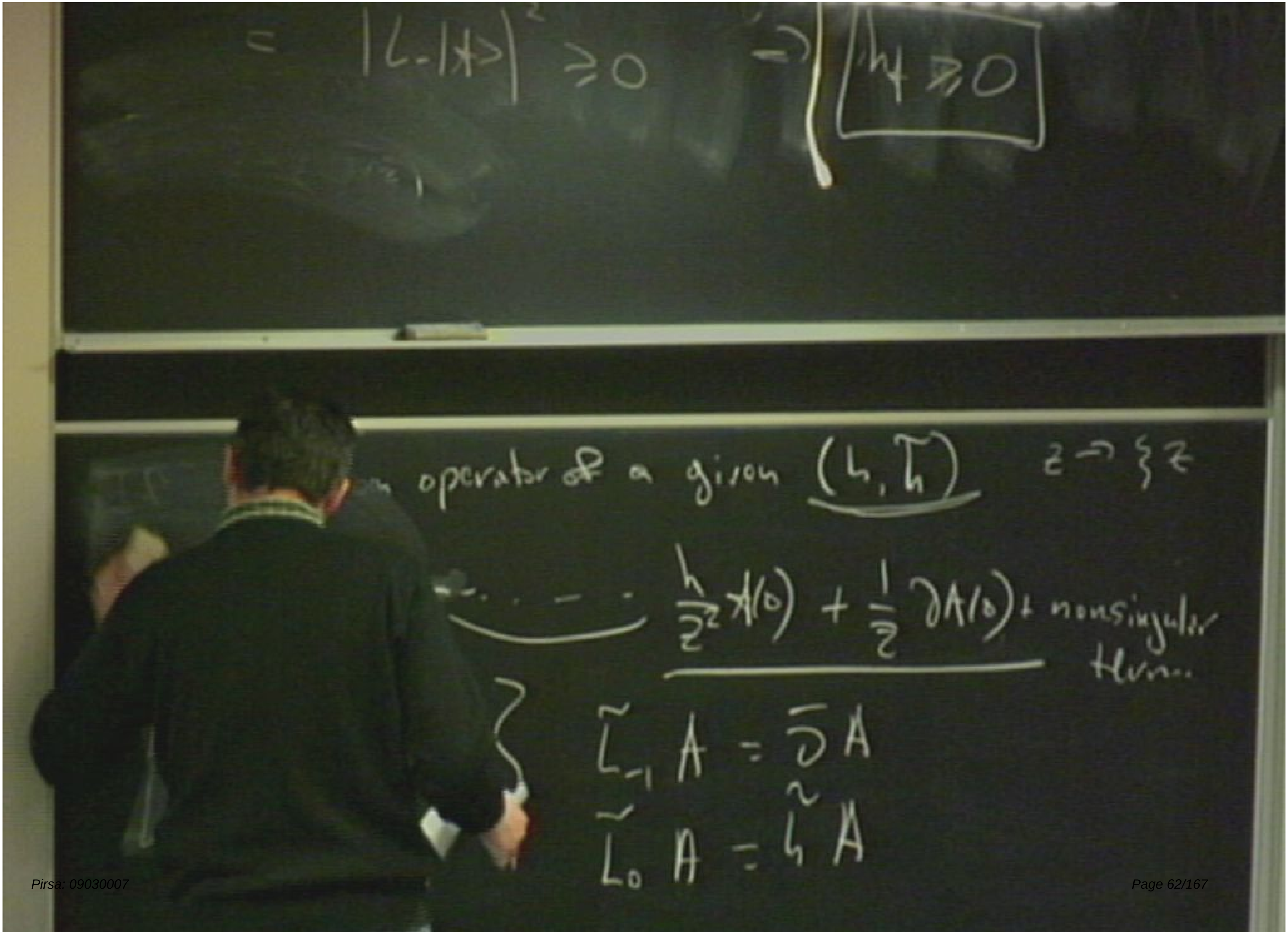
$$= \left| L_- | A \rangle \right|^2 \geq 0 \Rightarrow h_A =$$

If A is an operator of a given $(h, \bar{h})$ $\quad z \to \xi z$

$\checkmark \quad T(z) A = \cdots\cdots \frac{h}{z^2} A(0) + \frac{1}{z} \partial A(0) +$ nonsingular terms

$$L_{-1} A = \partial A \qquad \tilde{L}_{-1} A = \bar{\partial} A$$

$$L_0 A = h \cdot A \qquad \tilde{L}_0 A = \tilde{h} A$$


$= |L_{-1}|h\rangle|^2 \geq 0 \Rightarrow \boxed{h \geq 0}$
operator of a given $(h, \bar{h})$
$z \to \xi z$
$\frac{h}{z^2} A(0) + \frac{1}{z} \partial A(0)$ + nonsingular terms..
$\tilde{L}_{-1} A = \bar{\partial} A$
$\tilde{L}_0 A = \tilde{h} A$

(ii) Suppose

(ii) Suppose $h_A = 0$

(iii) Suppose $h_A = 0 \Rightarrow$ in a unitary CFT.

A is antiholomorphic

(iii) Suppose $h_A = 0 \Longrightarrow$ in a unitary CFT.

A is anti-holomorphic

$$2h_A \langle A|A\rangle = L_{-1}\cdot$$

(iii) Suppose $h_A = 0 \Rightarrow$ in a unitary CFT.

A is anti-holomorphic

$$2h_A \langle A|A\rangle = |L_{-1} \cdot A|^2$$

(ii) Suppose $h_A = 0 \Rightarrow$ in a unitary CFT

A is antiholomorphic

$$0 = 2h_A \langle A|A\rangle = |L_{-1}\cdot A|^2 \Rightarrow L_{-1}\cdot A = 0$$

(iii) Suppose $h_A = 0 \Rightarrow$ in a unitary CFT

A is antiholomorphic

$$0 = 2h_A \langle A|A\rangle = |L_{-1}\cdot A|^2 \Rightarrow \underbrace{L_{-1}\cdot A}_{\partial A} = 0$$

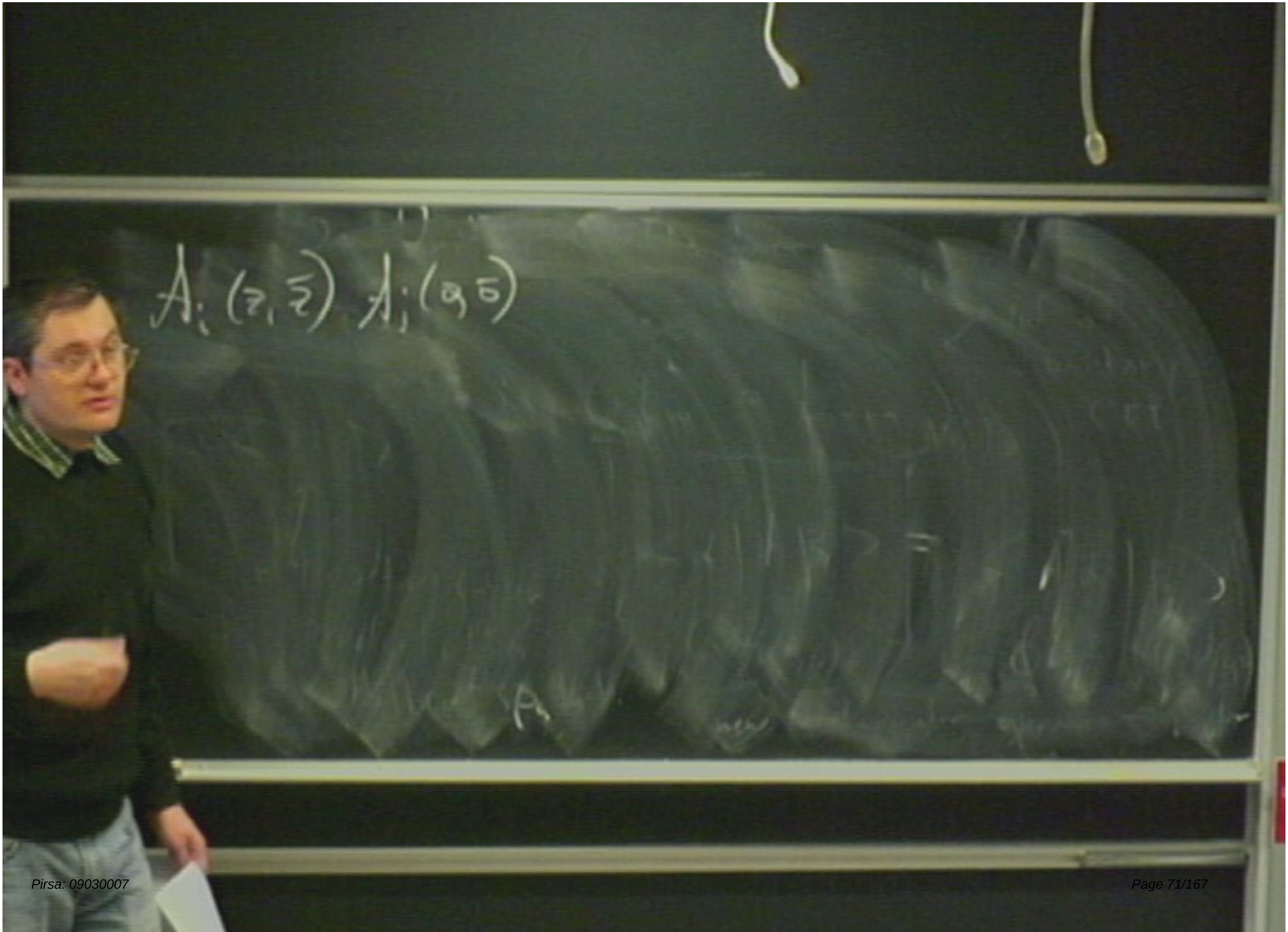
$$\partial A = 0$$

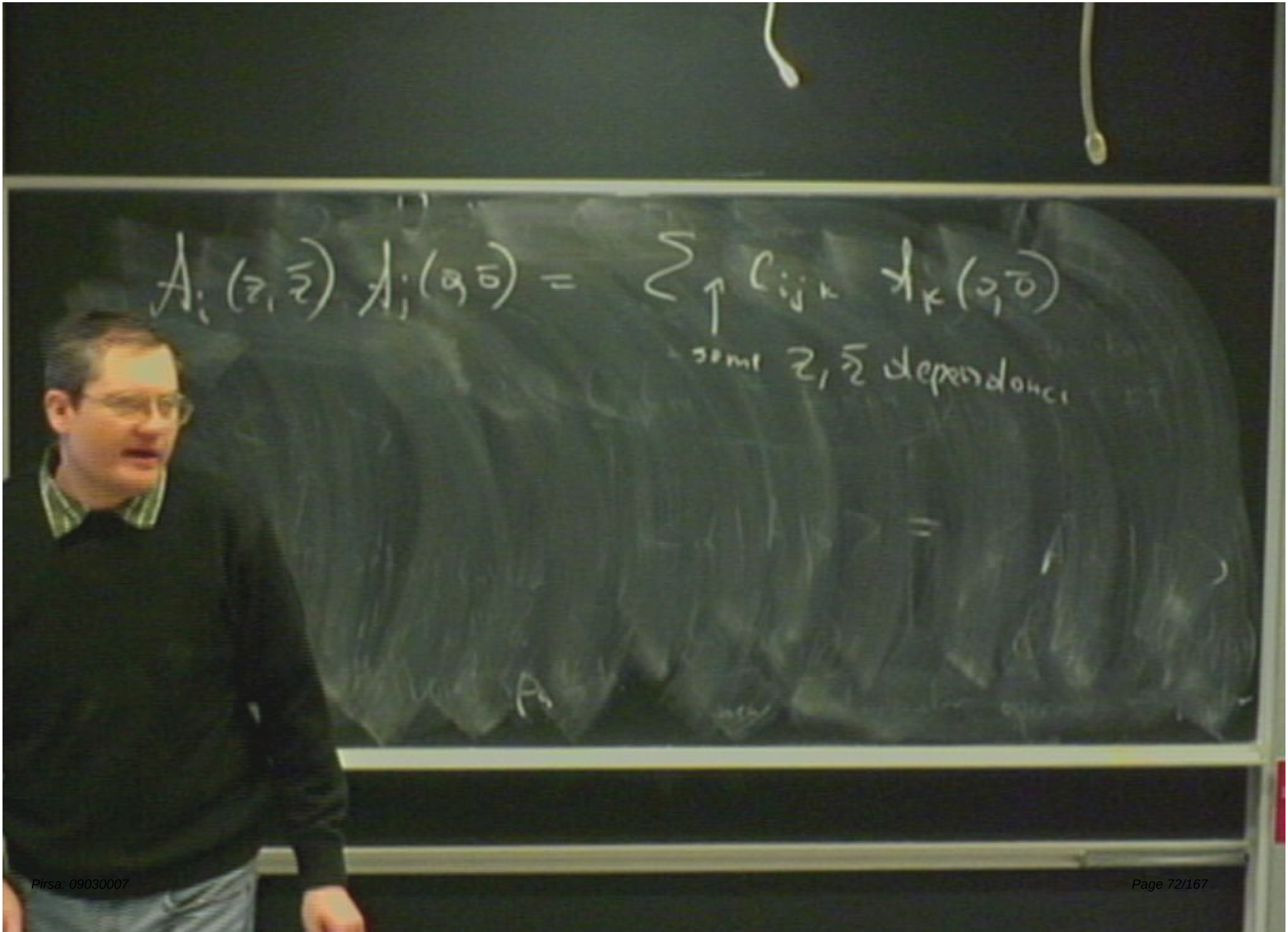
(iii) Suppose $h_A = 0 \Rightarrow$ in a unitary CFT

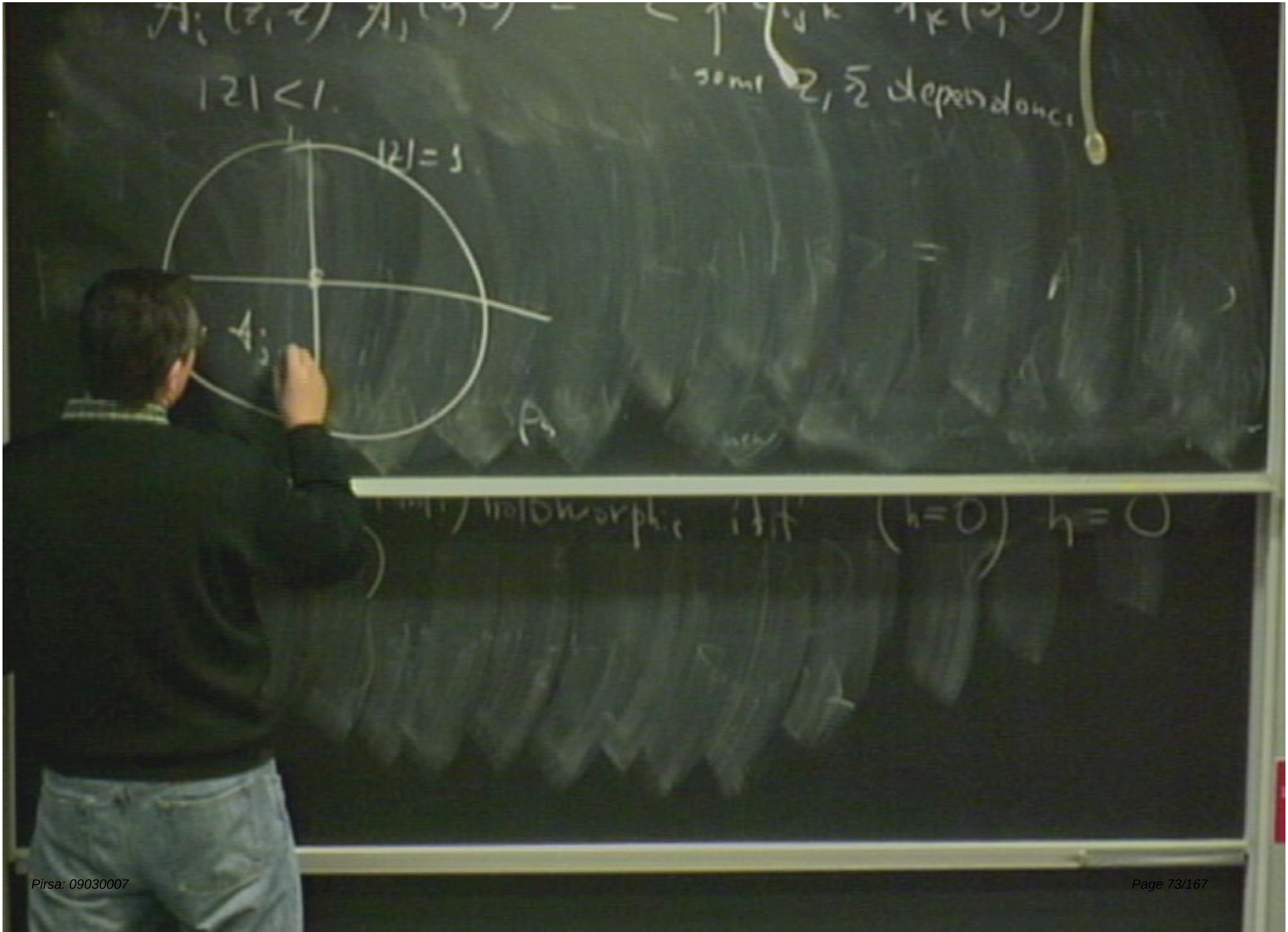
A is antiholomorphic

$$0 = 2 h_A \langle A | A \rangle = | L_{-1} \cdot A |^2 \Rightarrow L_{-1} \cdot A = 0$$

$$L_{-1} \cdot A = \partial A = 0 \Rightarrow A = A(\bar z)$$


$A_i(z,\bar{z})\ A_j(w,\bar{w})$

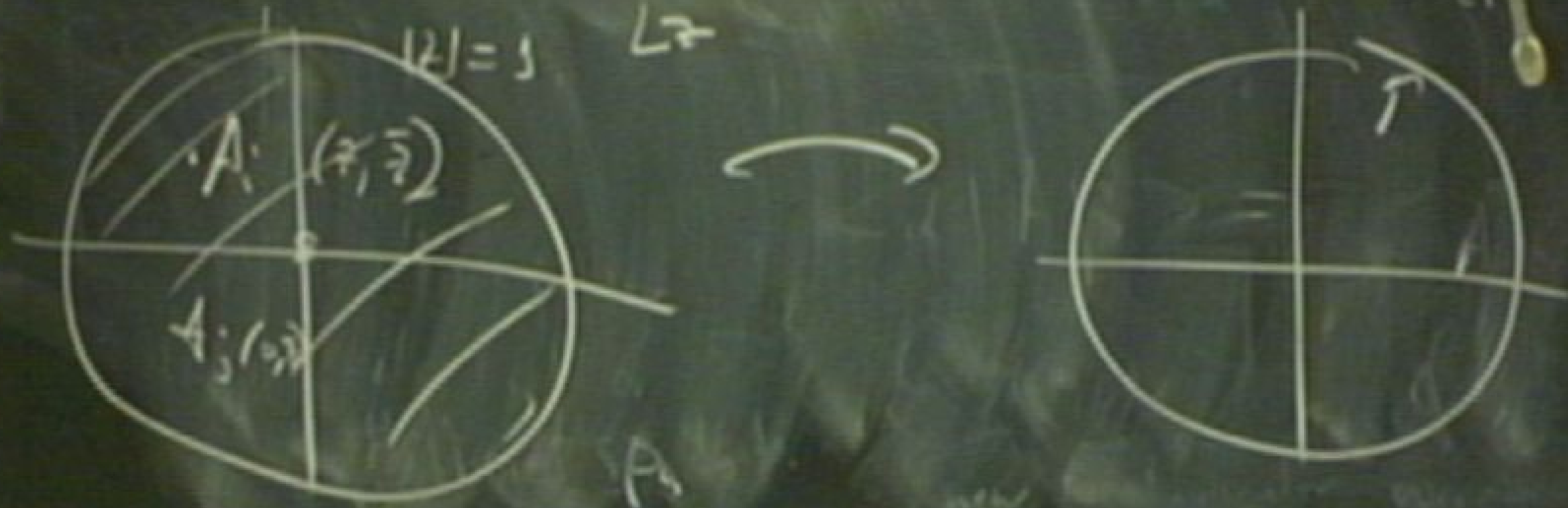

$A_i(z,\bar z)\, A_j(0,\bar 0) = \sum C_{ijk}\, A_k(0,\bar 0)$
some $z, \bar z$ dependence


|z| < 1
some z, z̄ dependence
|z| = 1
holomorphic iff (h=0) h=0


|z| < 1.
some z, z̄ dependence
|z| = s
holomorphic iff
(h, h̃)

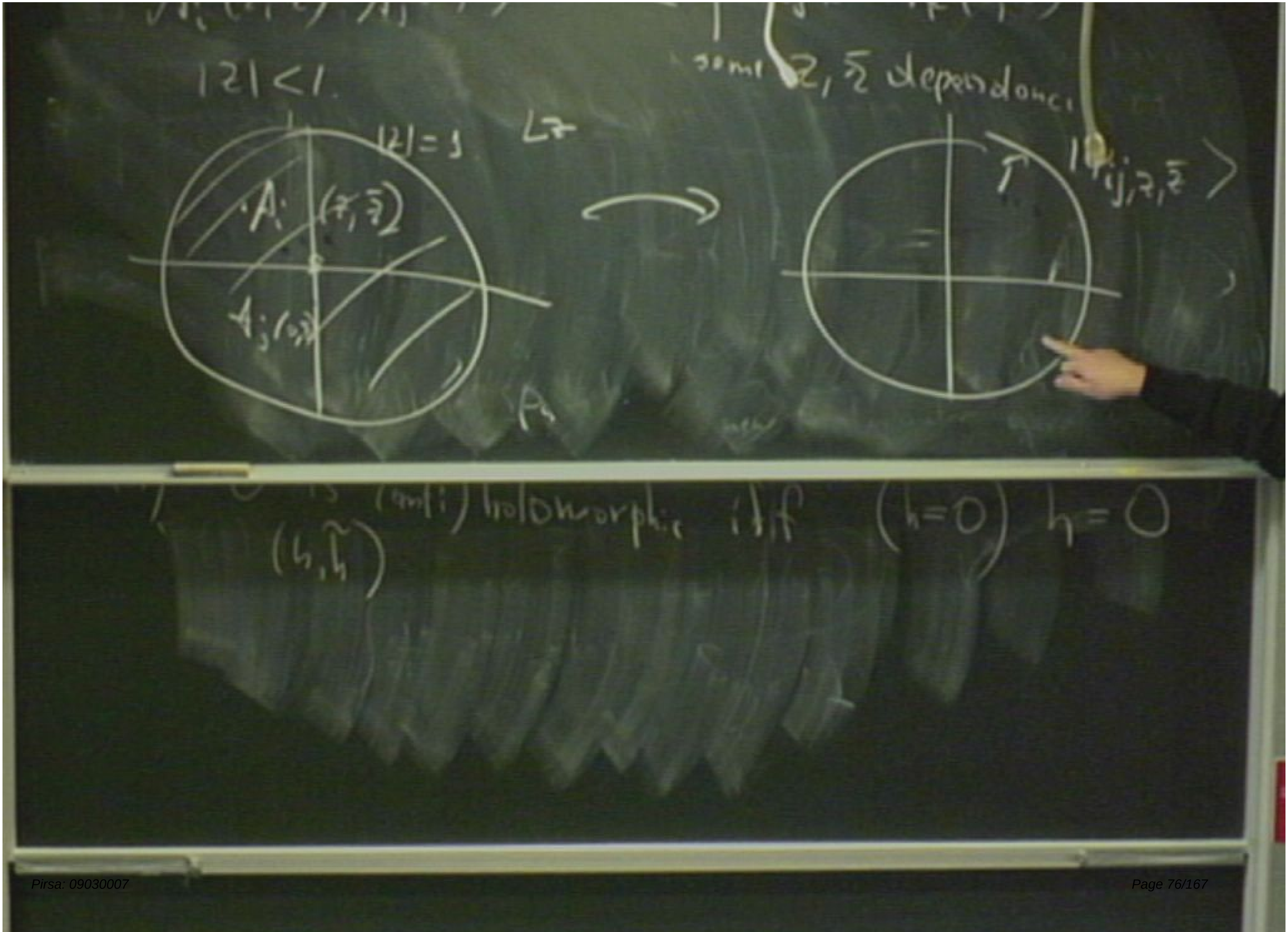
$|z| < 1.$

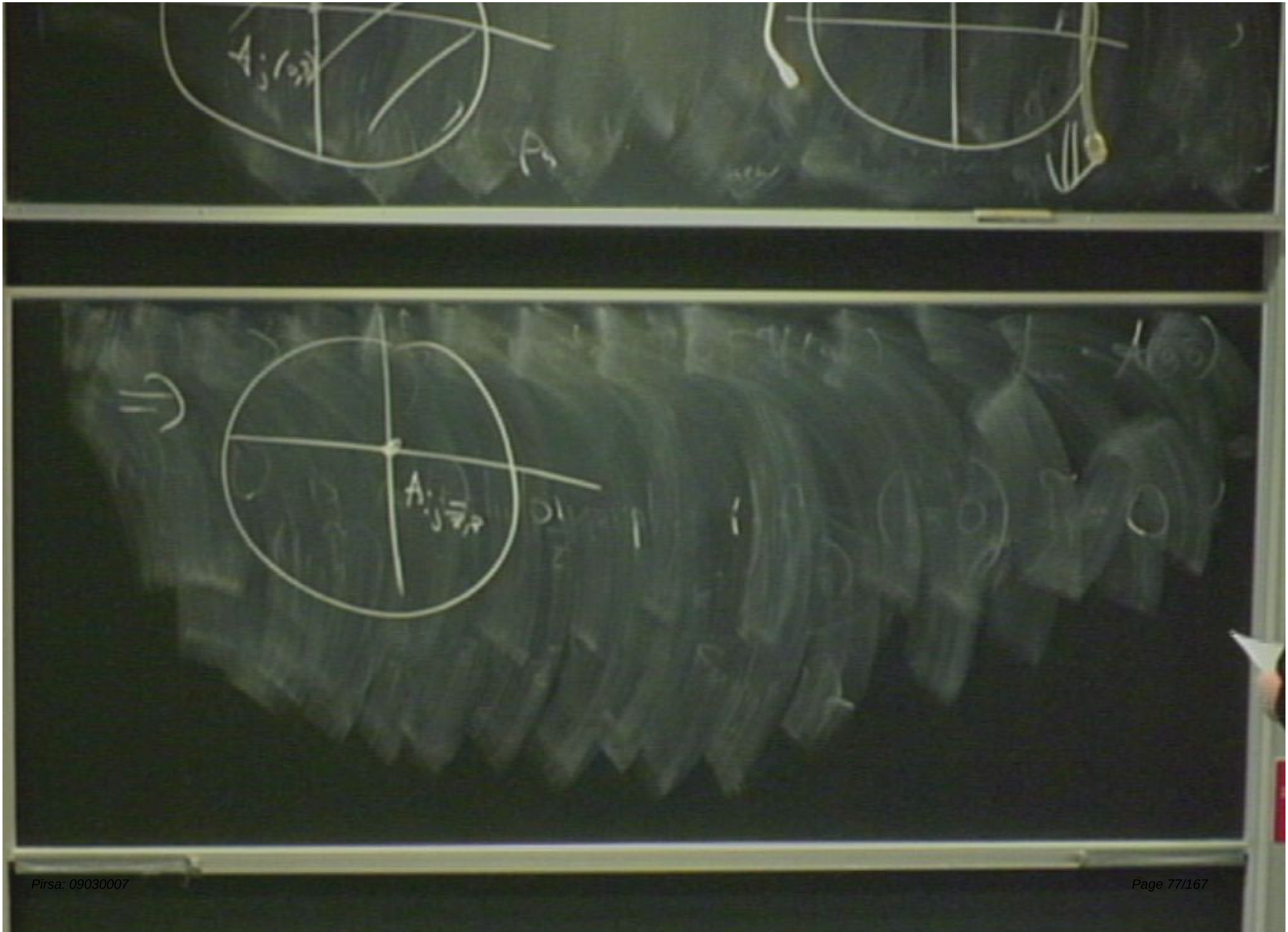
some $z, \bar{z}$ dependence

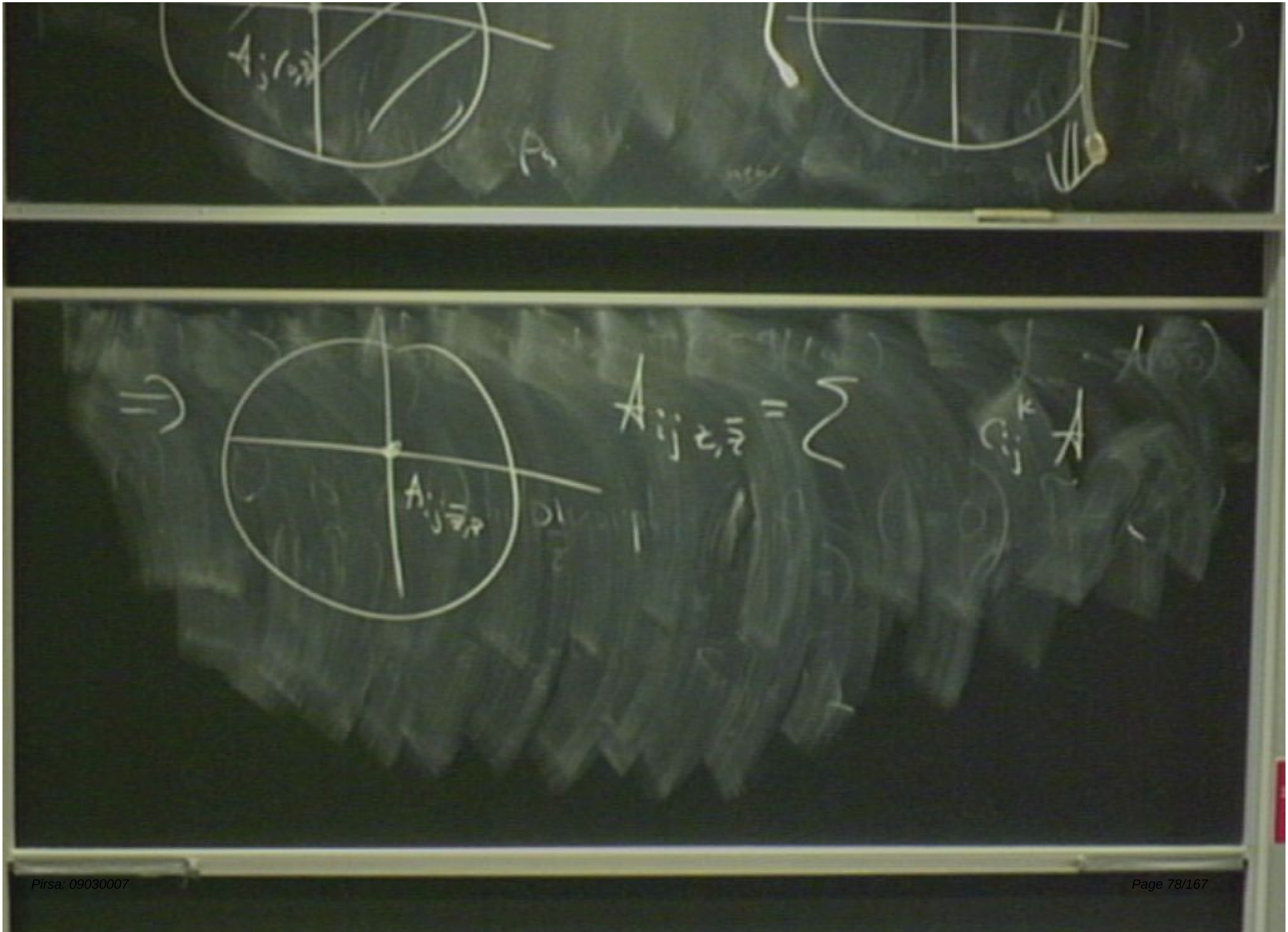


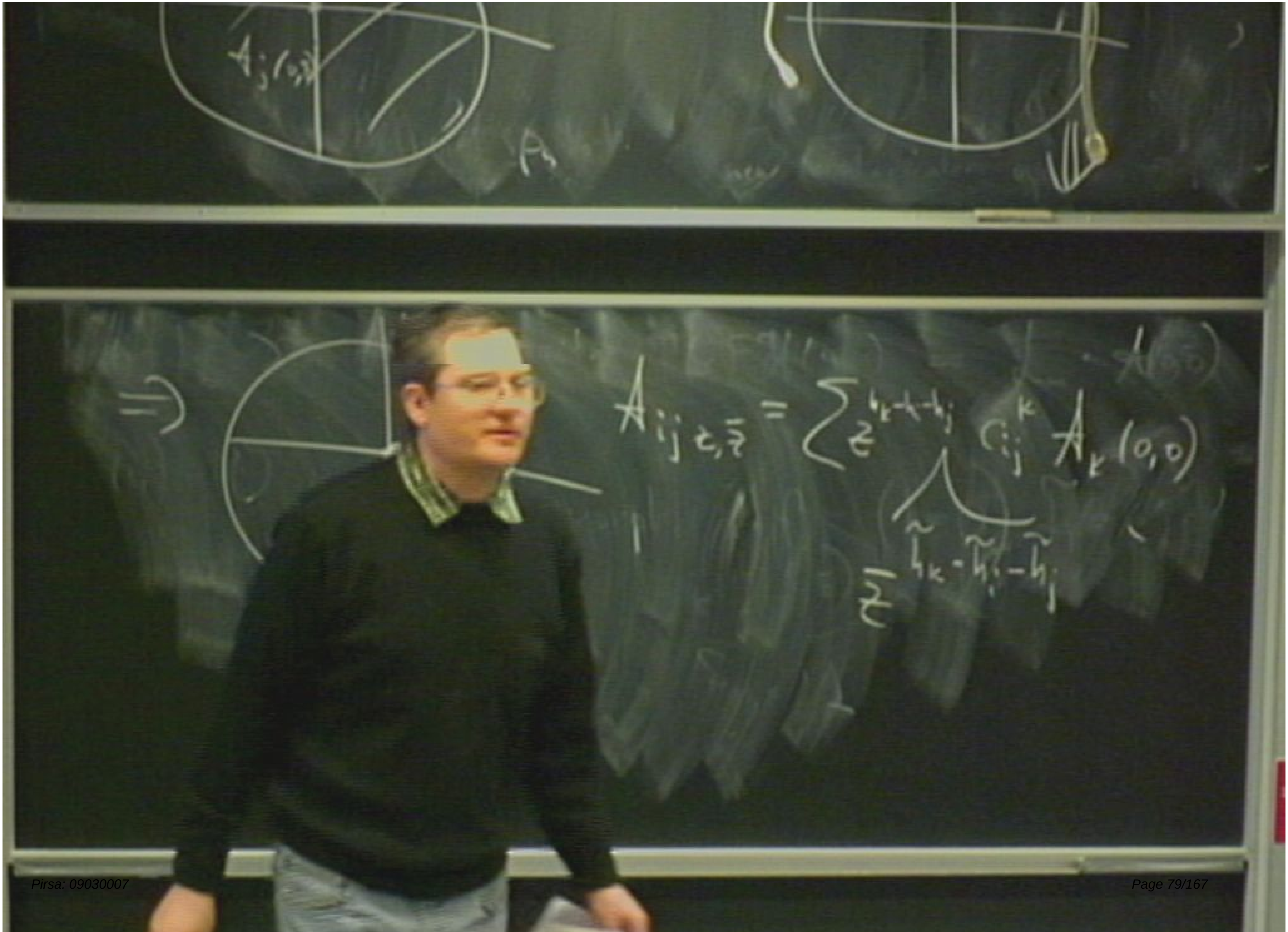
is (anti) holomorphic iff $(h=0)$ $\bar{h}=0$

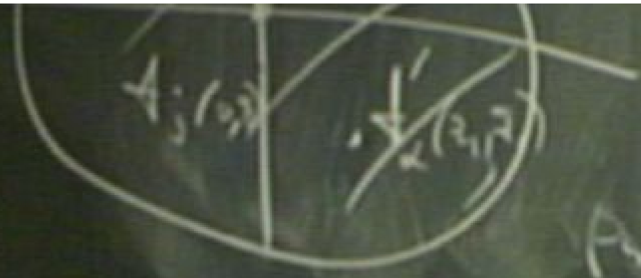
$(h, \bar{h})$

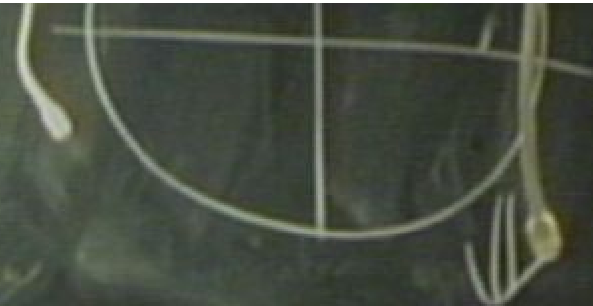

|z| < 1
some z, z̄ dependence
|z| = 1
A_i (z, z̄)
A_j (0,0)
|A_j, z, z̄ >
(anti) holomorphic iff (h=0) h̄=0
(h, h̄)




$A_{ij\,z,\bar z} = \sum C_{ij}^{k} A$


A_ij z,z̄ = Σ z^(h_k-h_i-h_j) C_ij^k A_k (0,0)
z̄^(h̃_k-h̃_i-h̃_j)





$\Rightarrow$

$$A_{ij}(z,\bar z) = \sum z^{h_k-h_i-h_j}\, \underbrace{C_{ij}^{k}}_{\bar z^{\tilde h_k-\tilde h_i-\tilde h_j}}\, A_k(0,0)$$

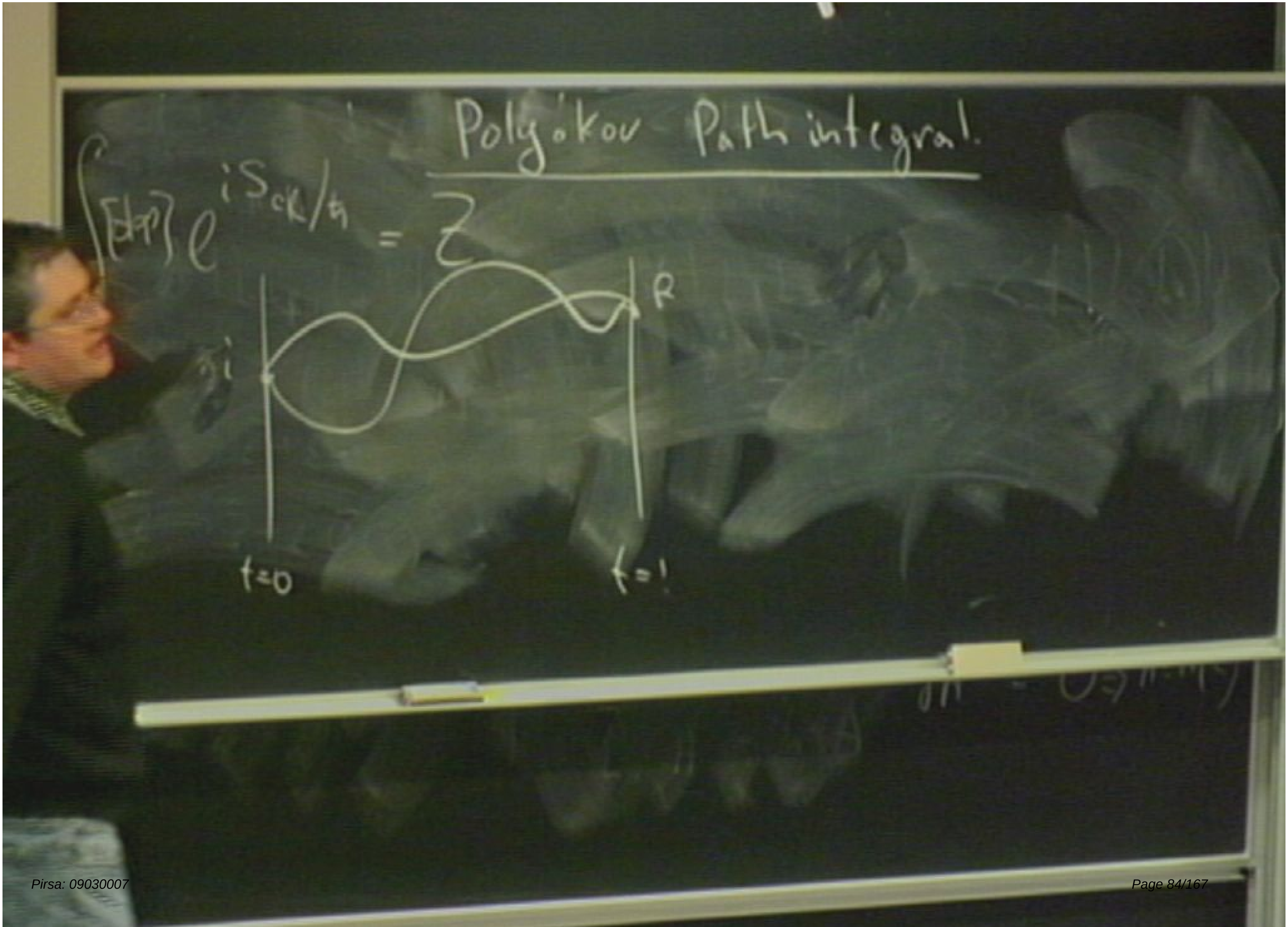

Polyakov Path integral.

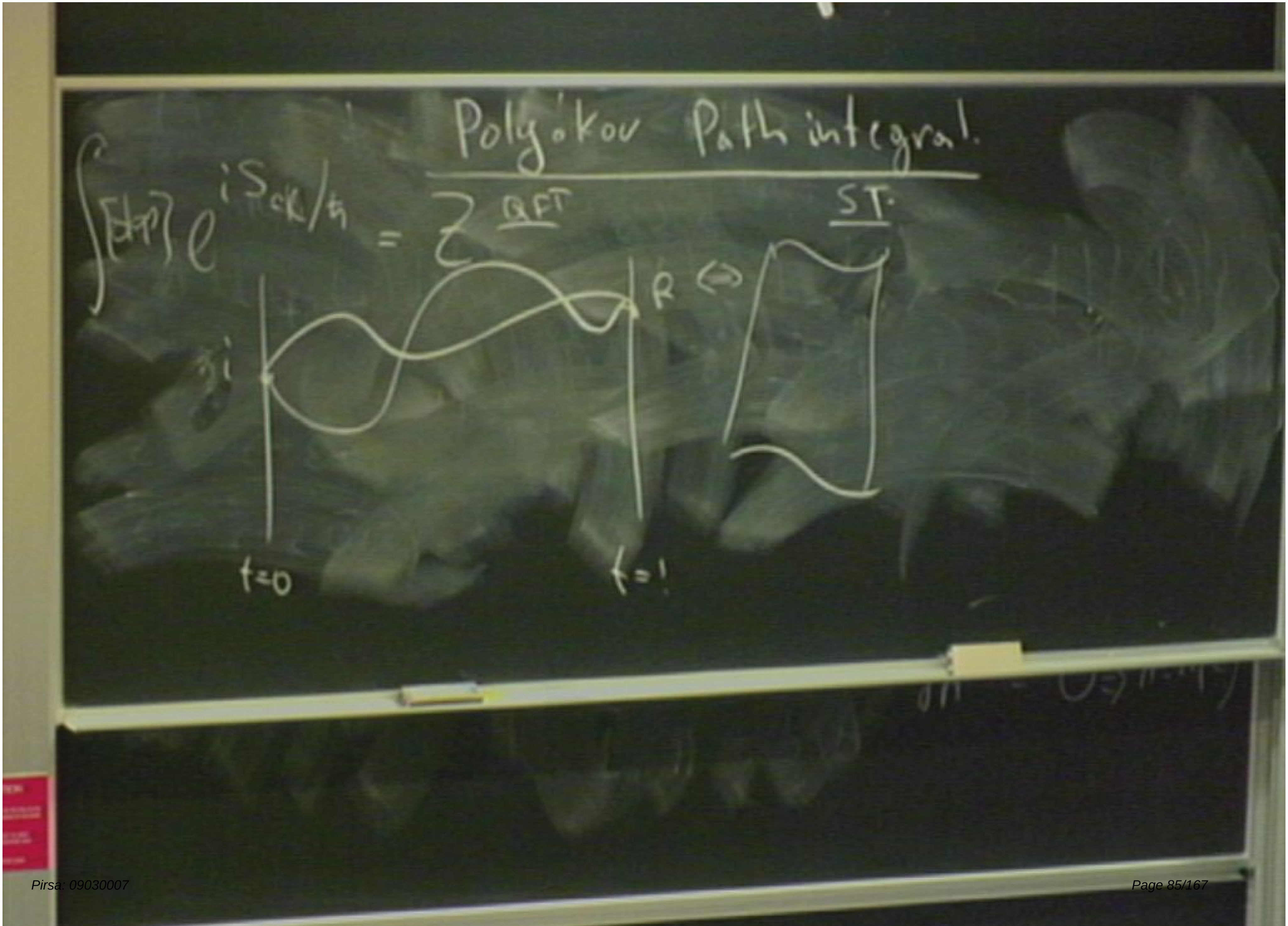
Polyakov Path integral.

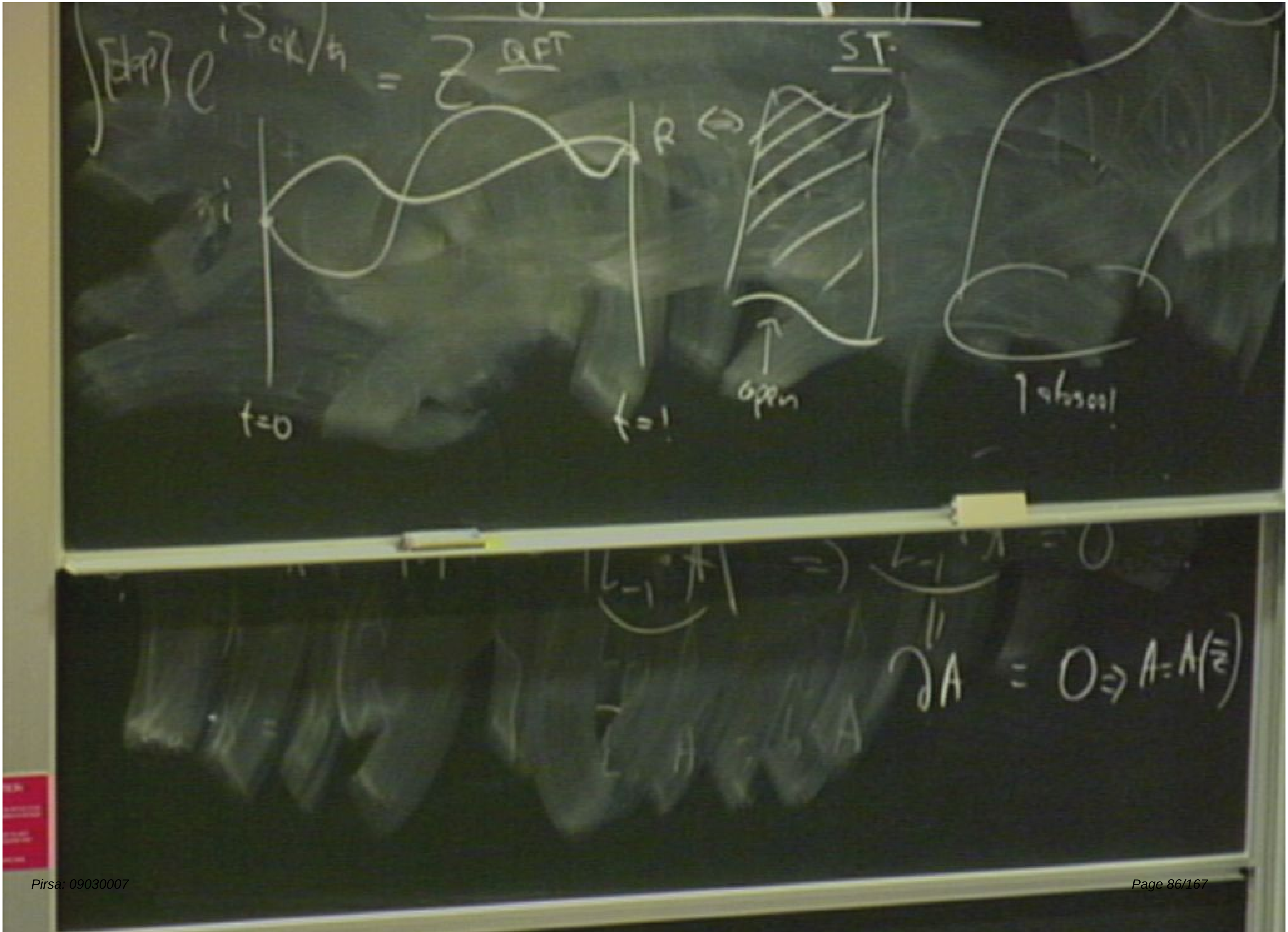
$$\int [\mathcal{D}\varphi]\, e^{iS_{cl}/\hbar} = Z$$

Polyakov Path integral.

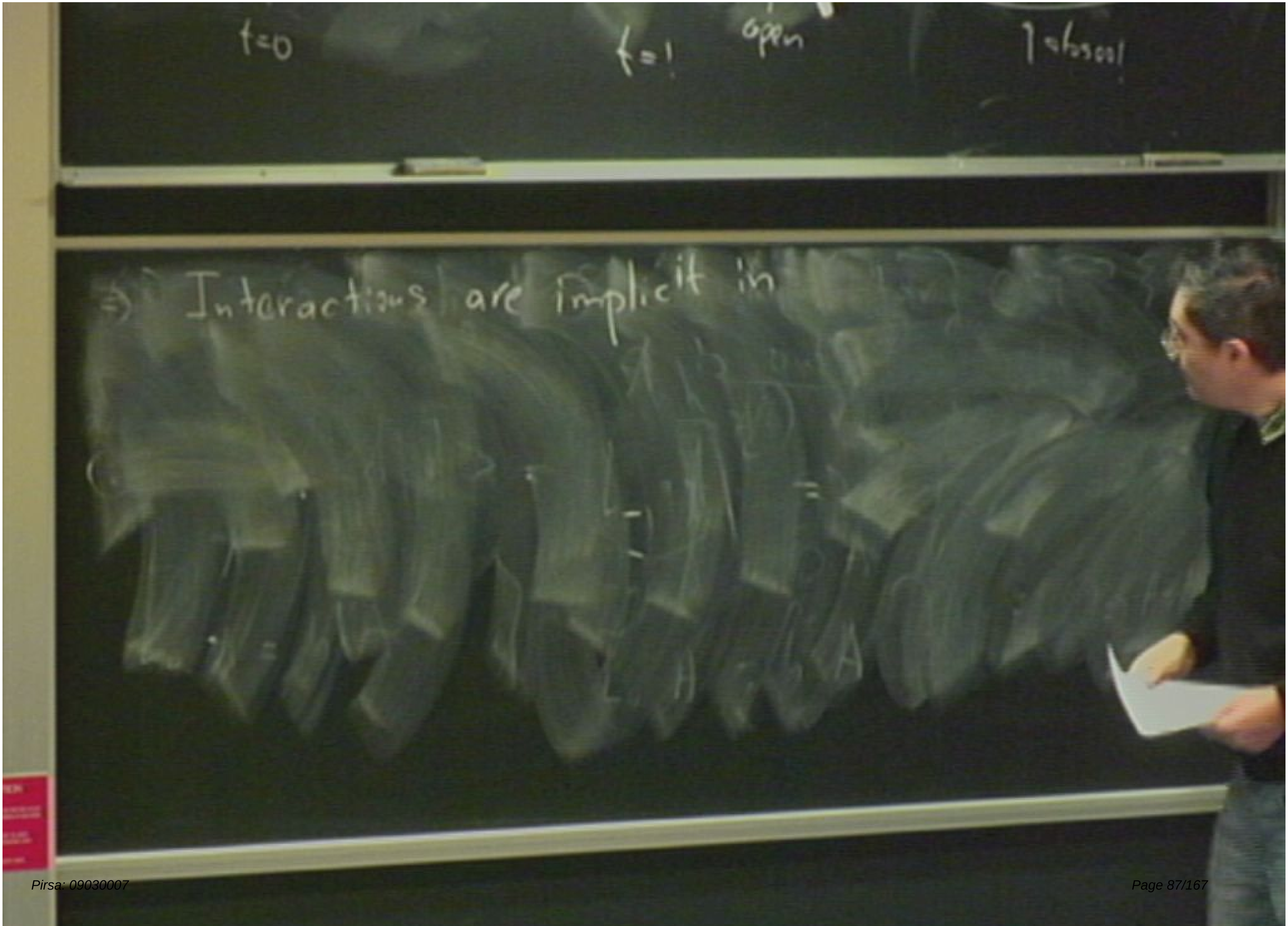
$$\int [dx^\mu]\, e^{iS_{cl}/\hbar} = Z$$


Polyakov Path integral.
∫[dp] e^{iS_cl/ħ} = Z
i
R
t=0
t=1


Polyakov Path integral.
QFT
ST.
=
R ⟺
t=0
t=1

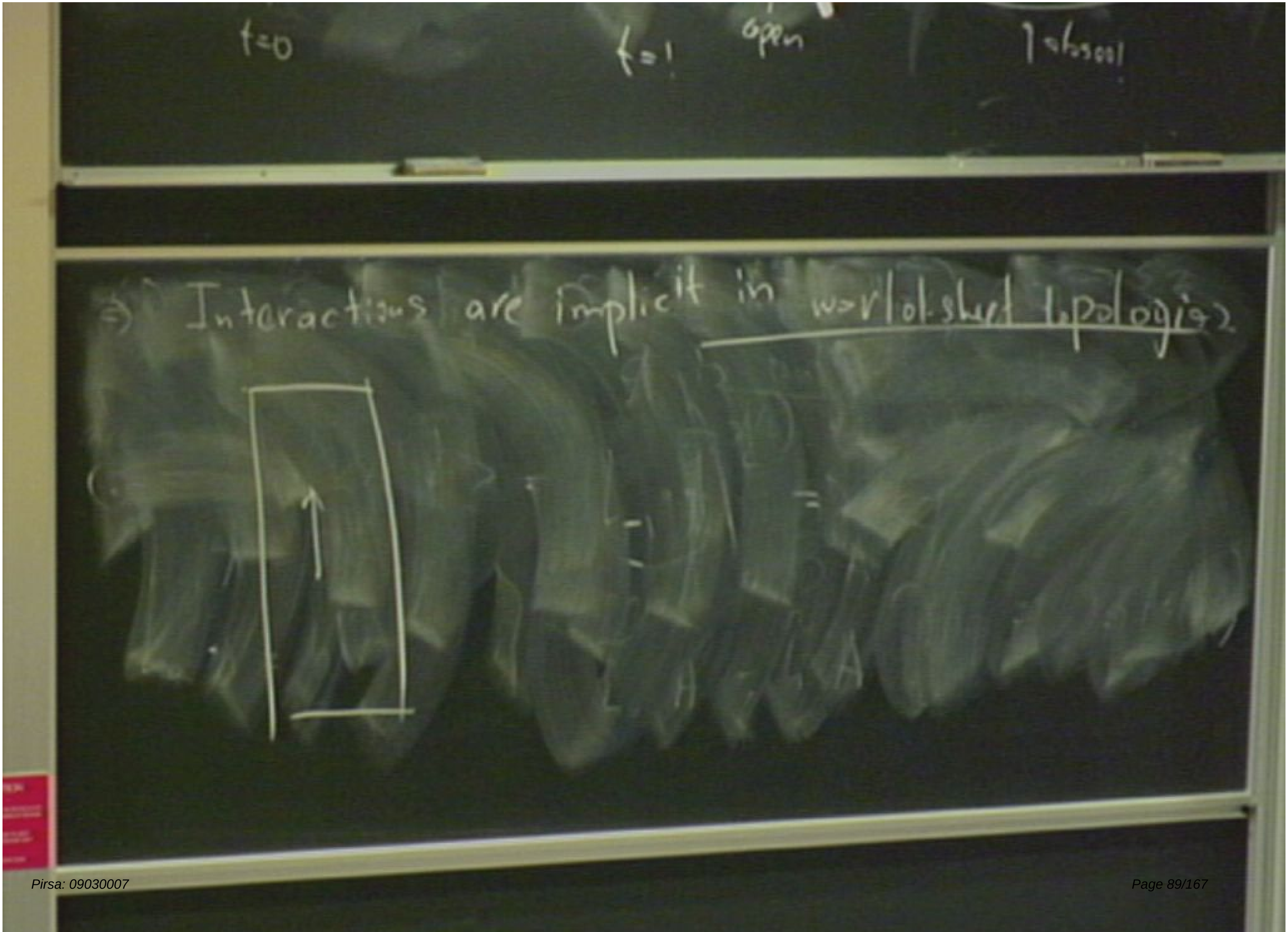


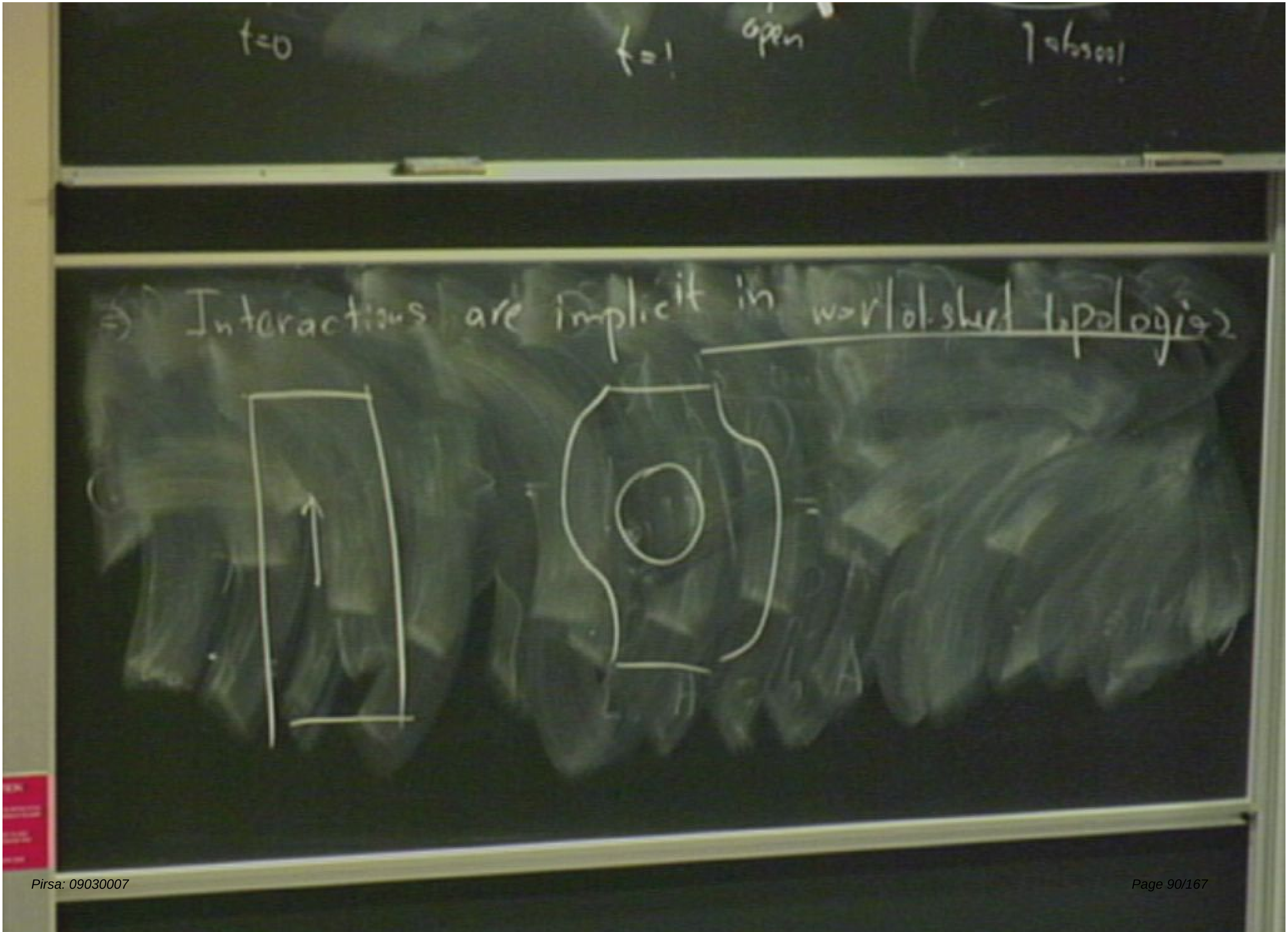
QFT
ST.
t=0
t=1
open

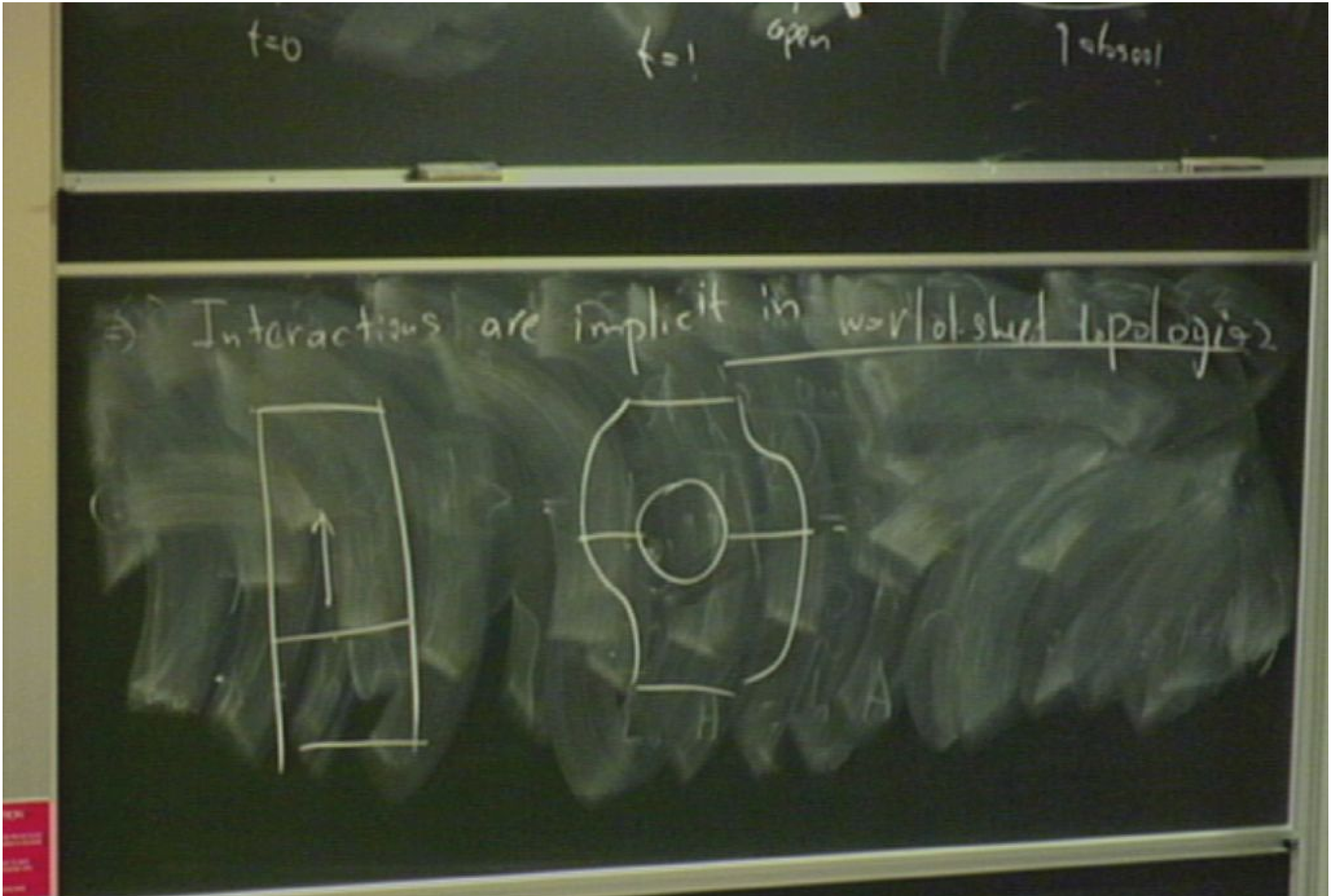

t=0
t=1
open
⇒ Interactions are implicit in

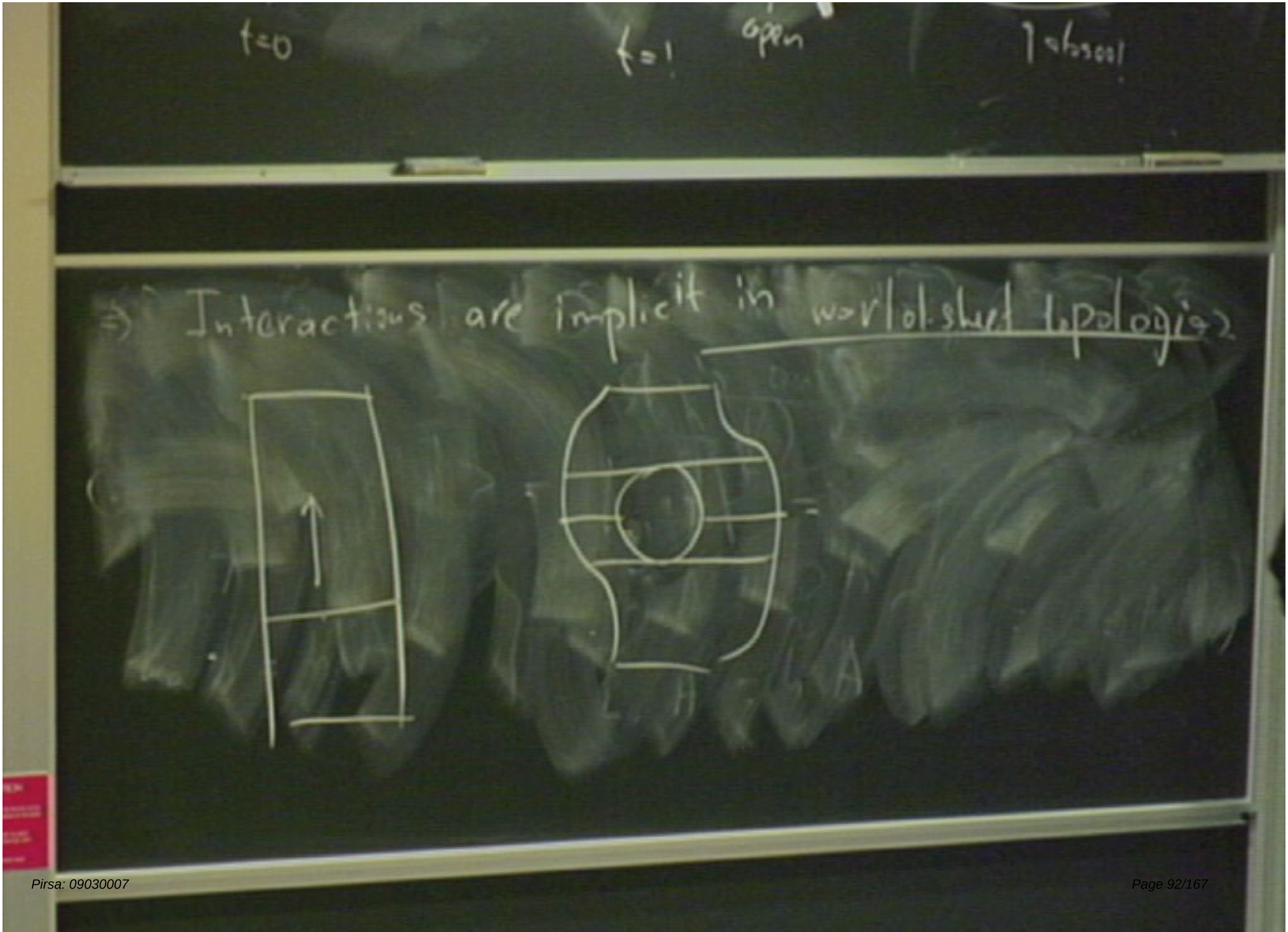
$t=0$ $t=1$ open

$\Rightarrow$ Interactions are implicit in worldsheet topologies

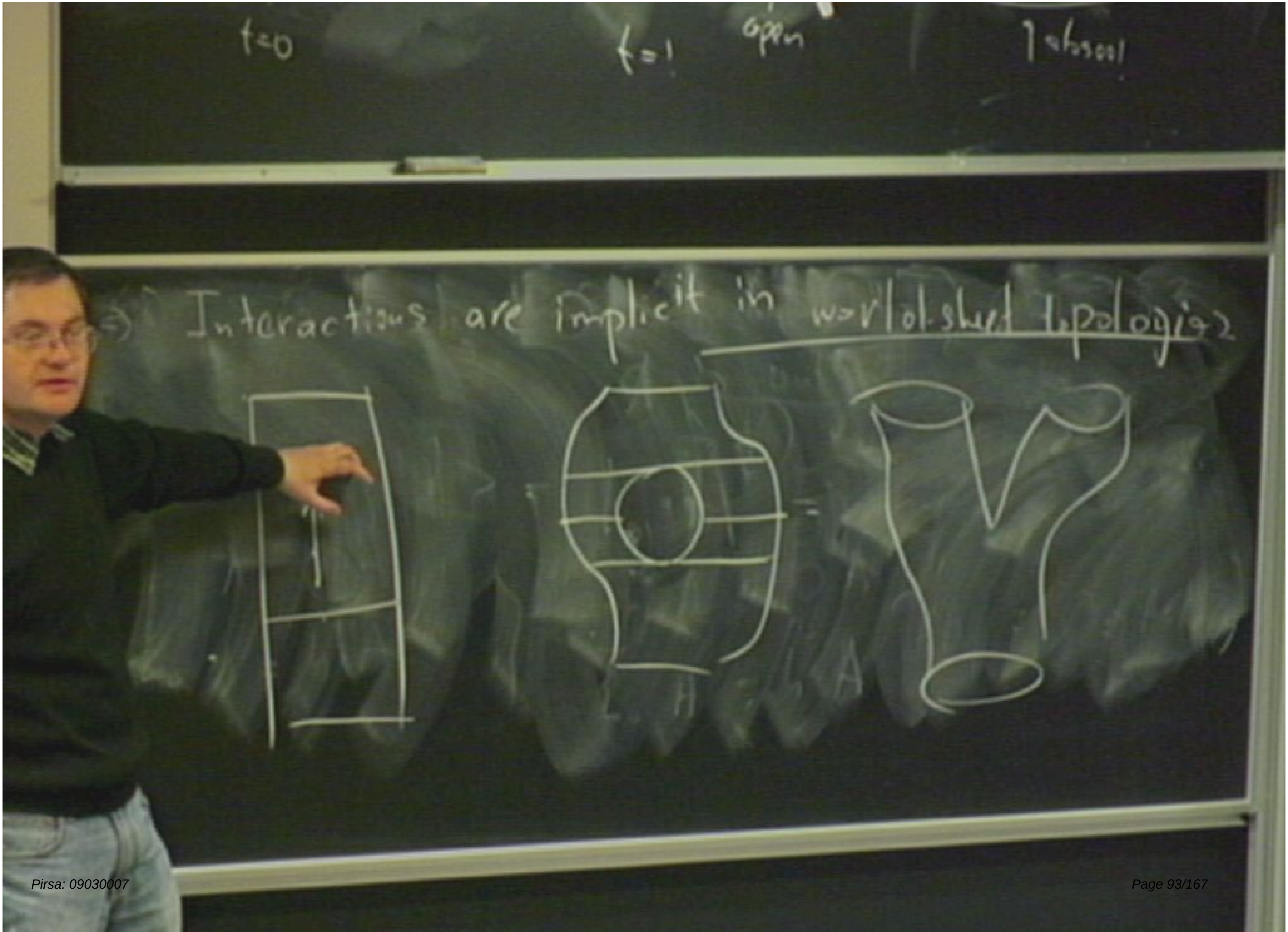

t=0
t=1
open
⇒ Interactions are implicit in worldsheet topologies

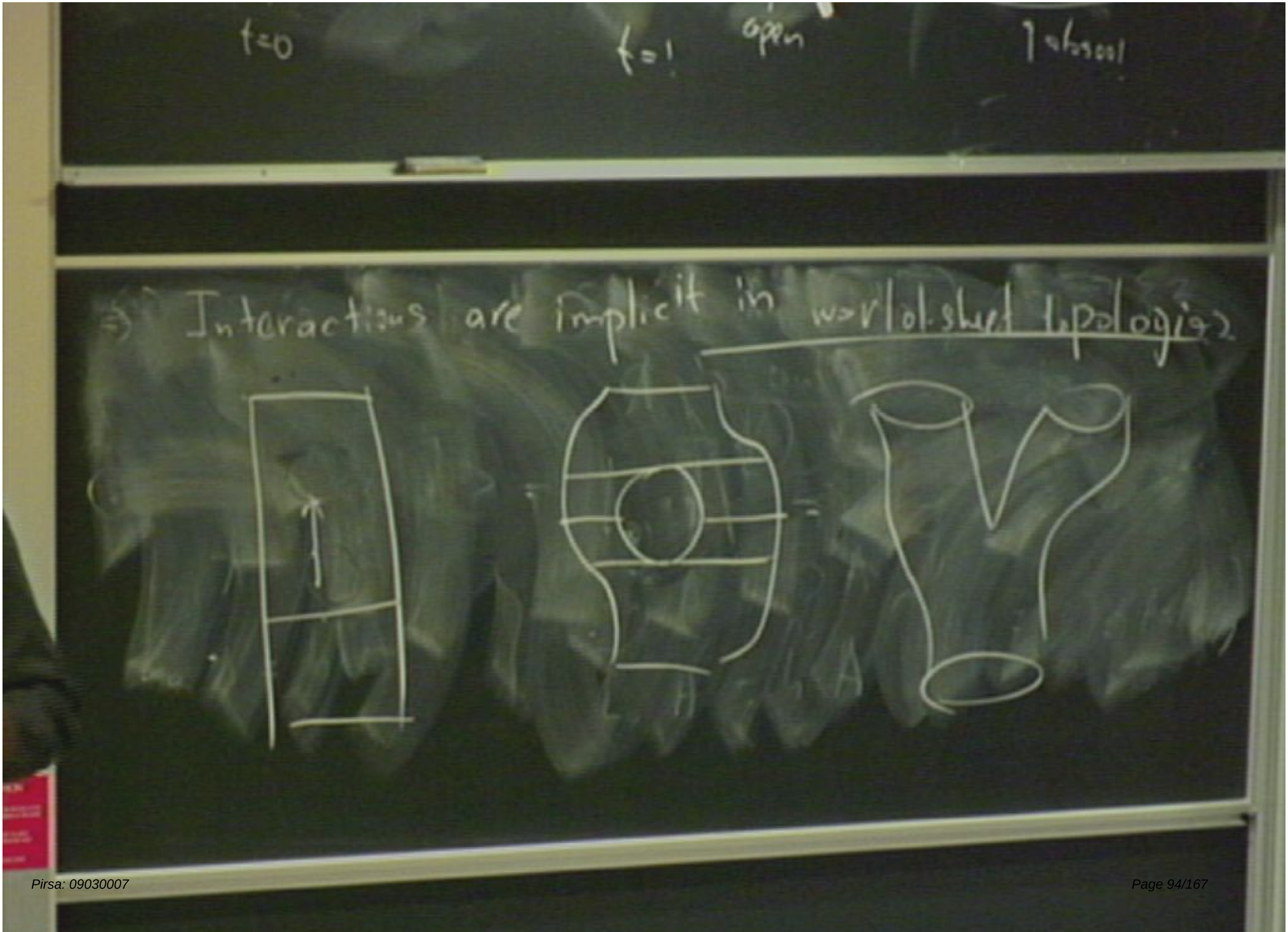

t=0
t=1
open
⇒ Interactions are implicit in worldsheet topologies

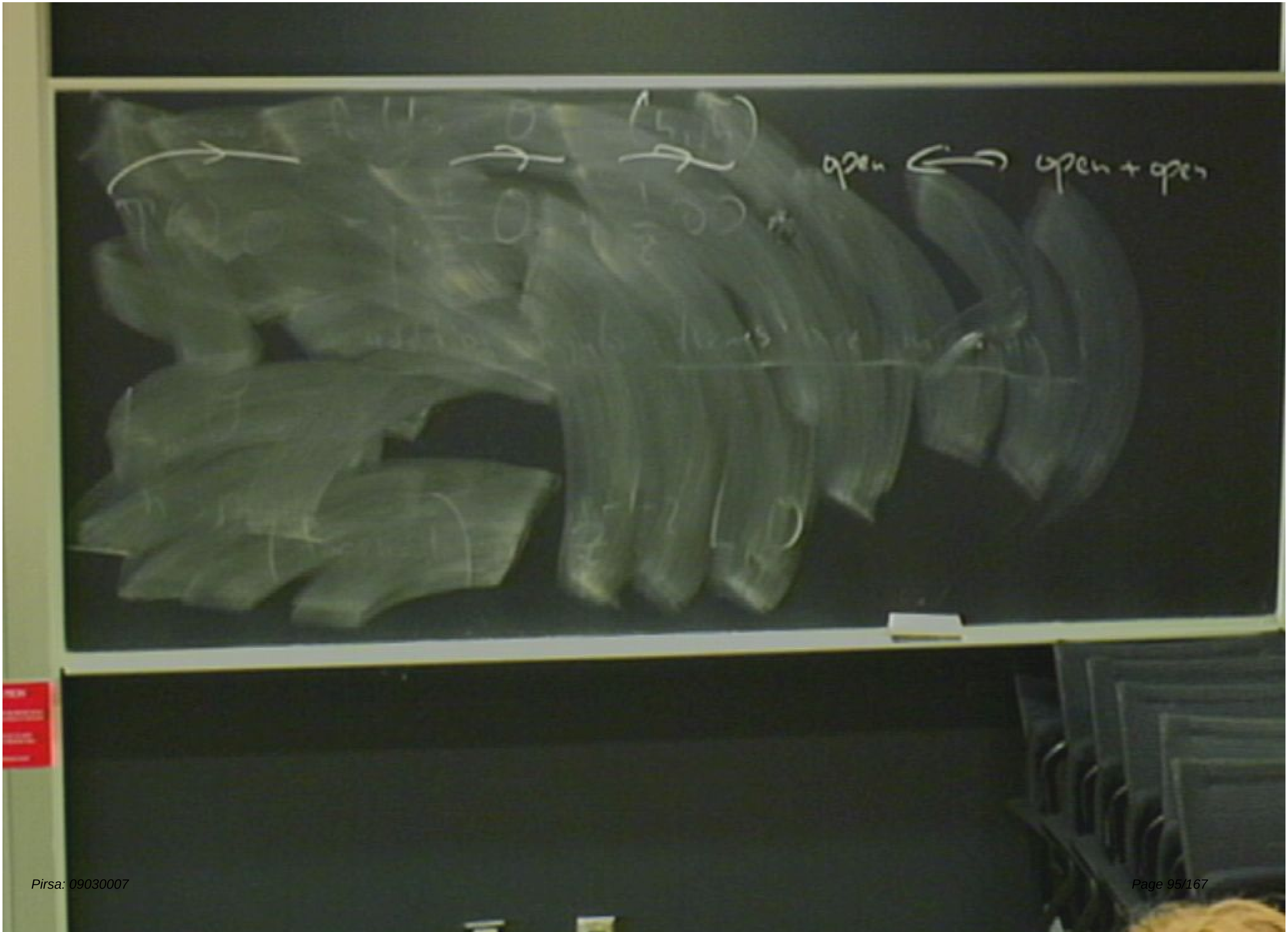

t=0
t=1
open
⇒ Interactions are implicit in worldsheet topologies

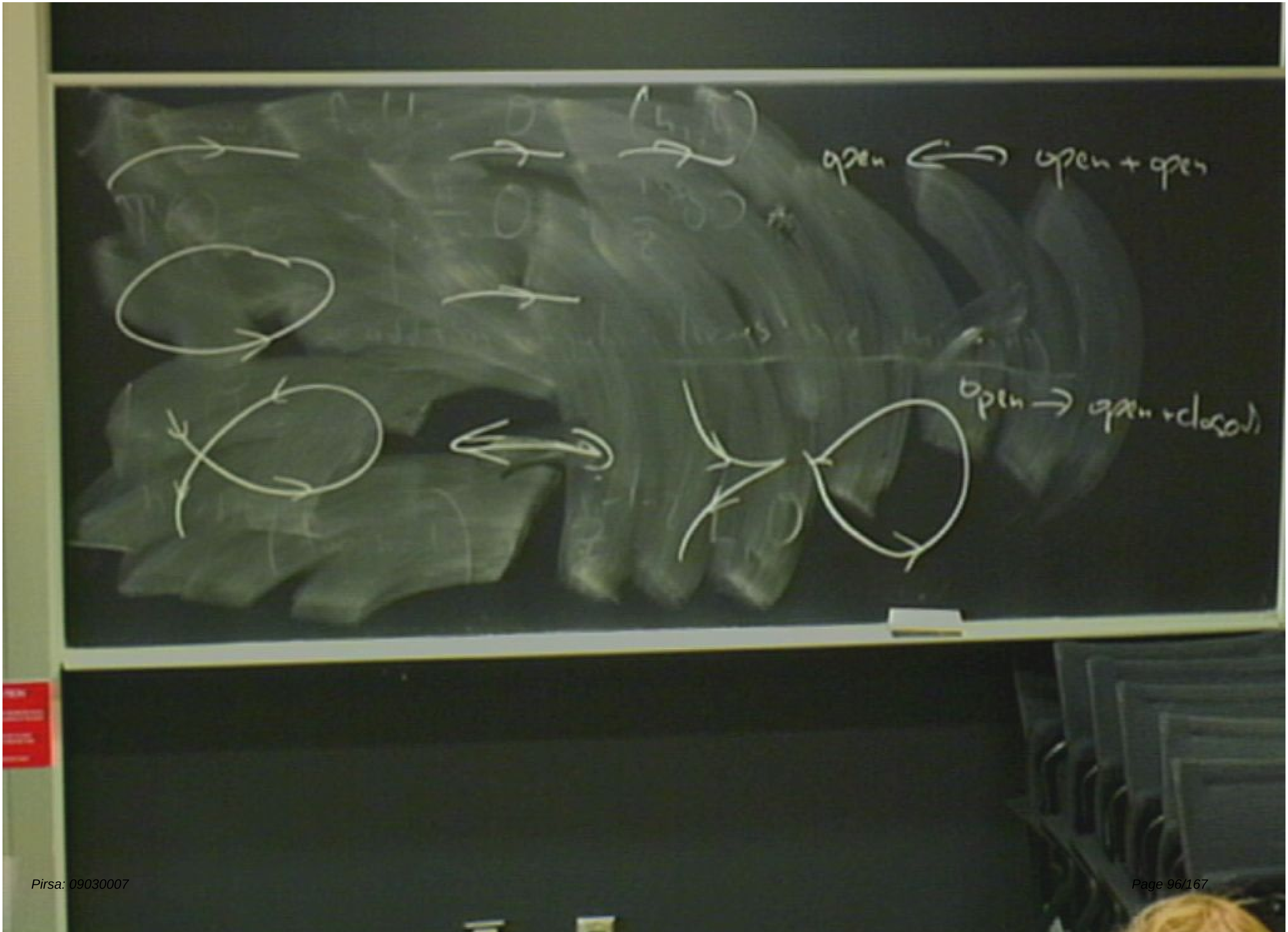


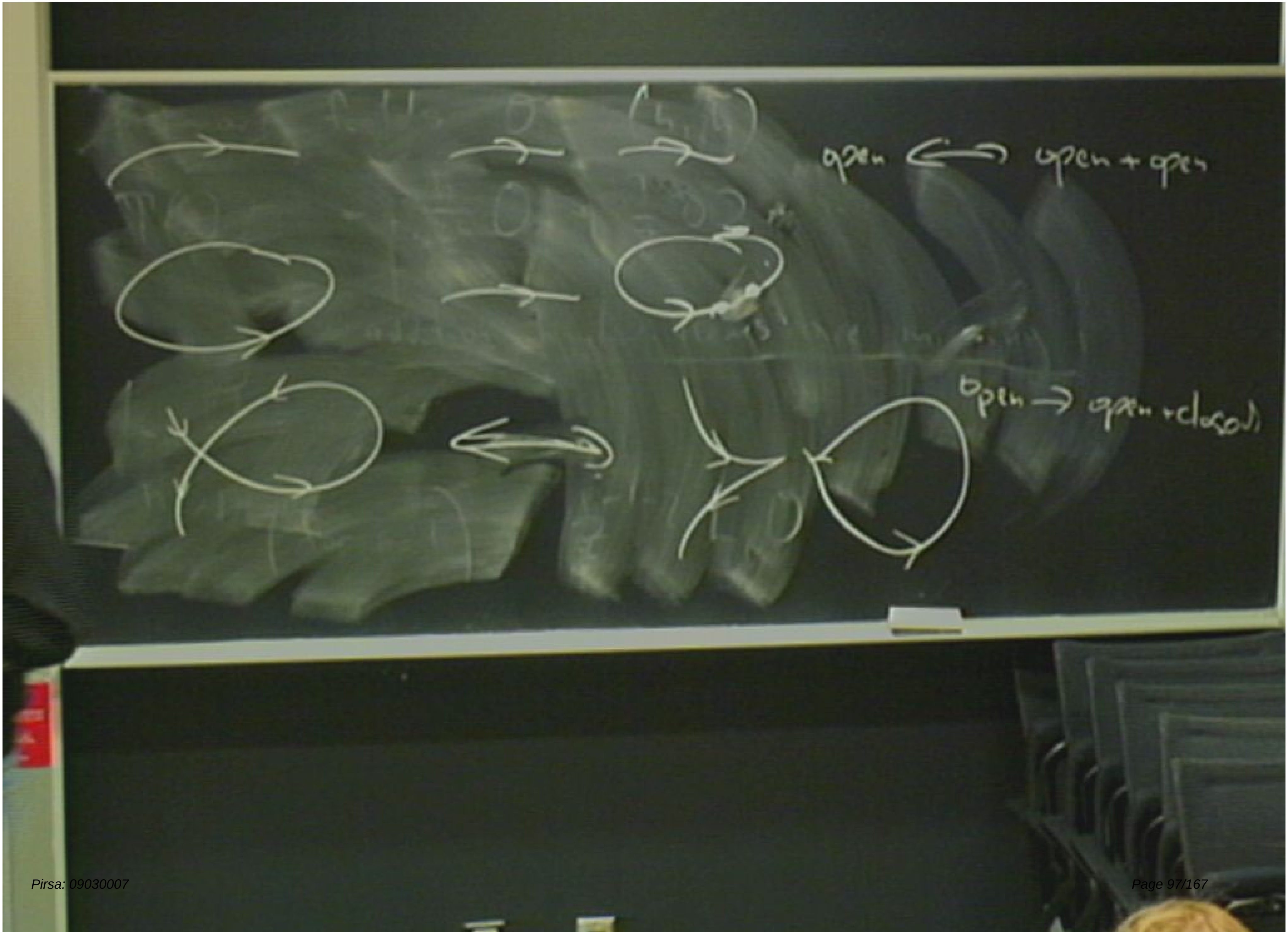
t=0
t=1
open
⇒ Interactions are implicit in worldsheet topologies

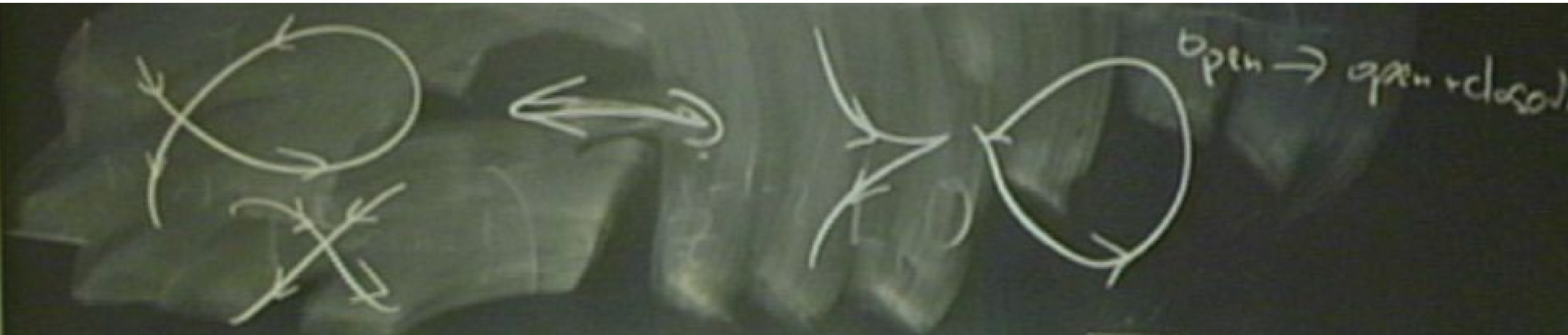

t=0
t=1
open
Interactions are implicit in worldsheet topologies


t=0
t=1
open
⇒ Interactions are implicit in worldsheet topologies


open ⟺ open + open


open ⟷ open + open
open → open + closed


open ⟷ open + open
open → open + closed



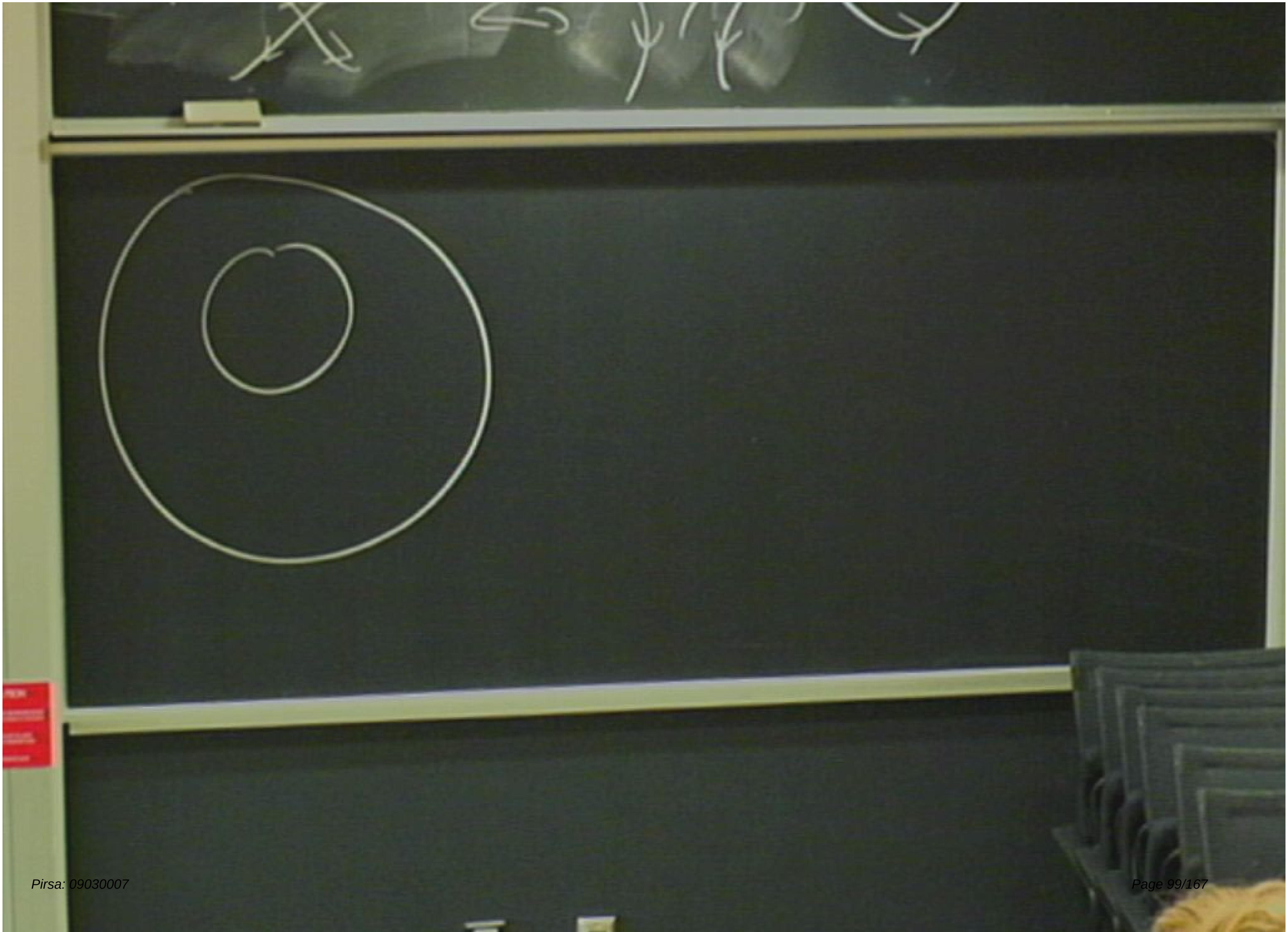
Note

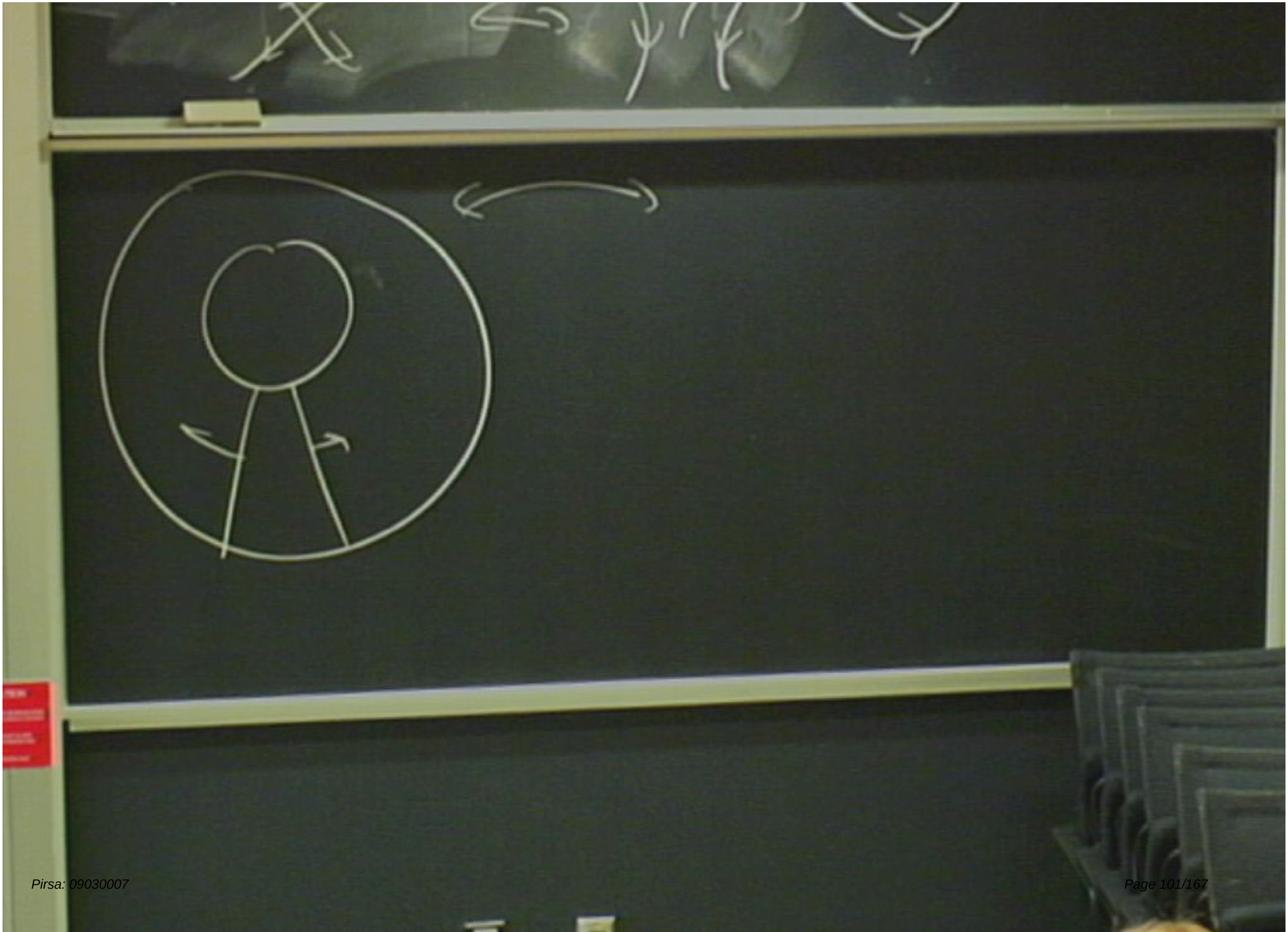
$$L_0 |A\rangle = h |A\rangle$$

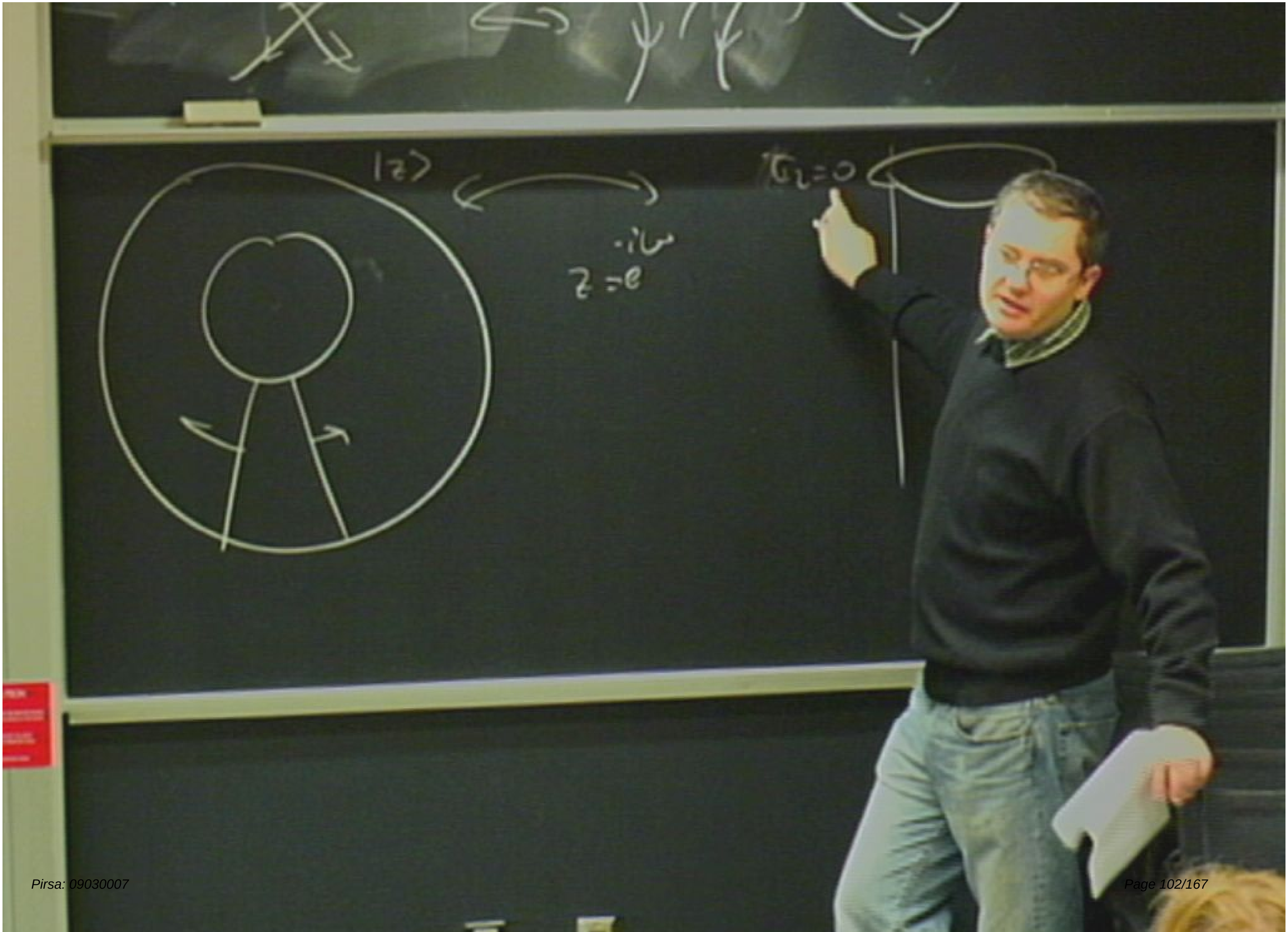
$$L_0 (L_n |A\rangle) = (h-n) |A\rangle$$

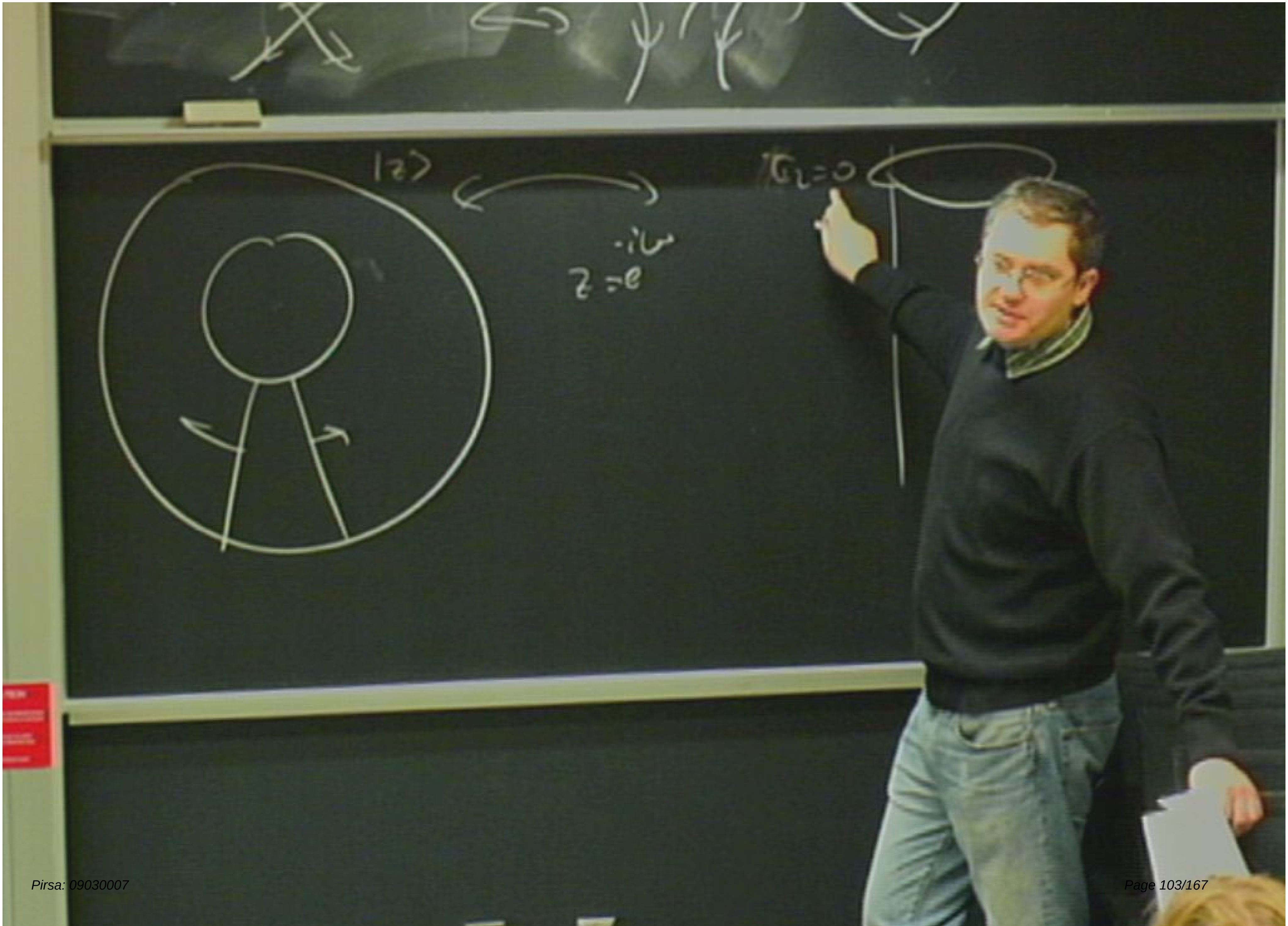
If we act with
bunch of L_n ($n>0$)
we finally reach
the highest weight state

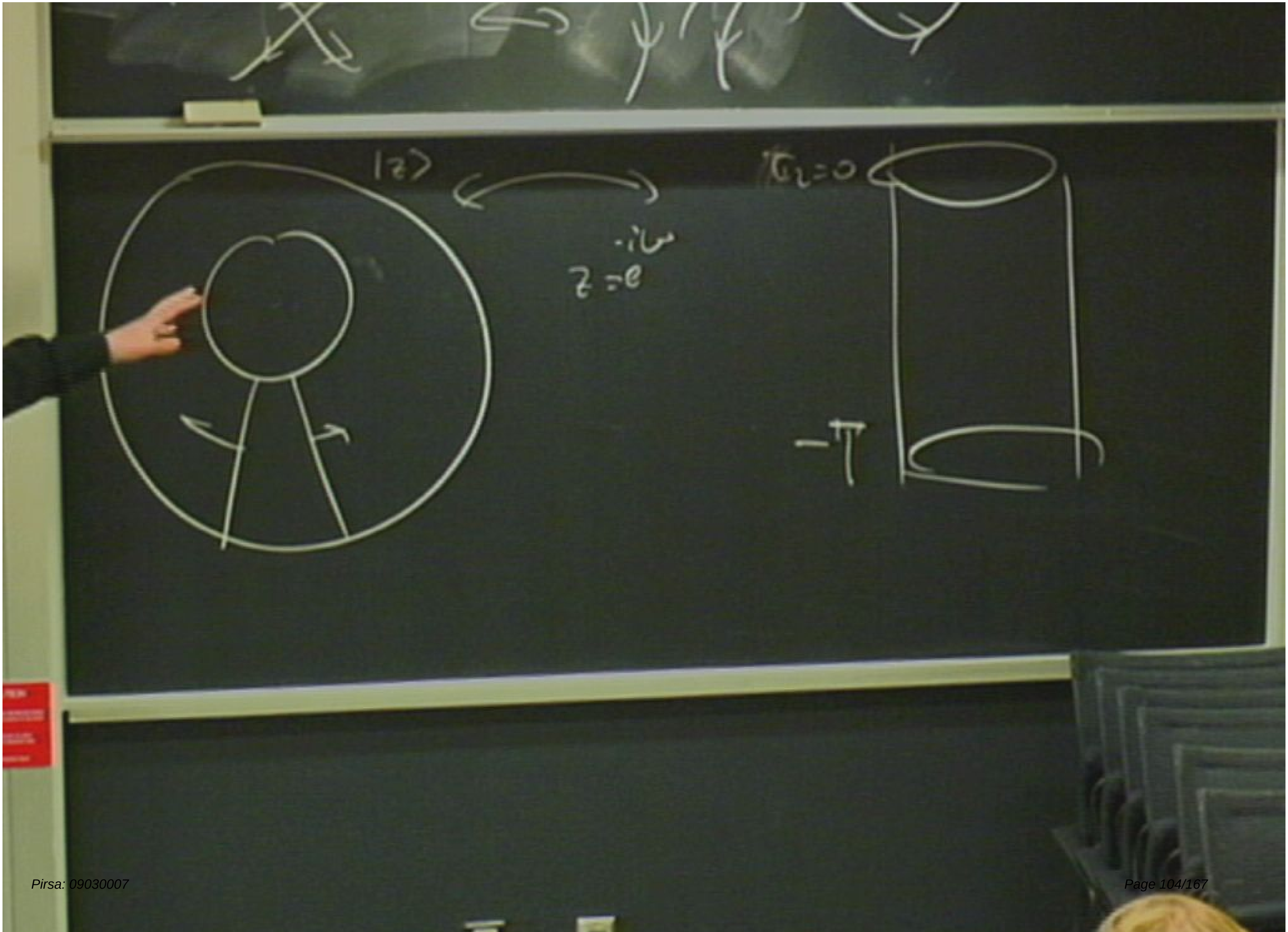
Implicitly L_0 spectrum is
bounded from below



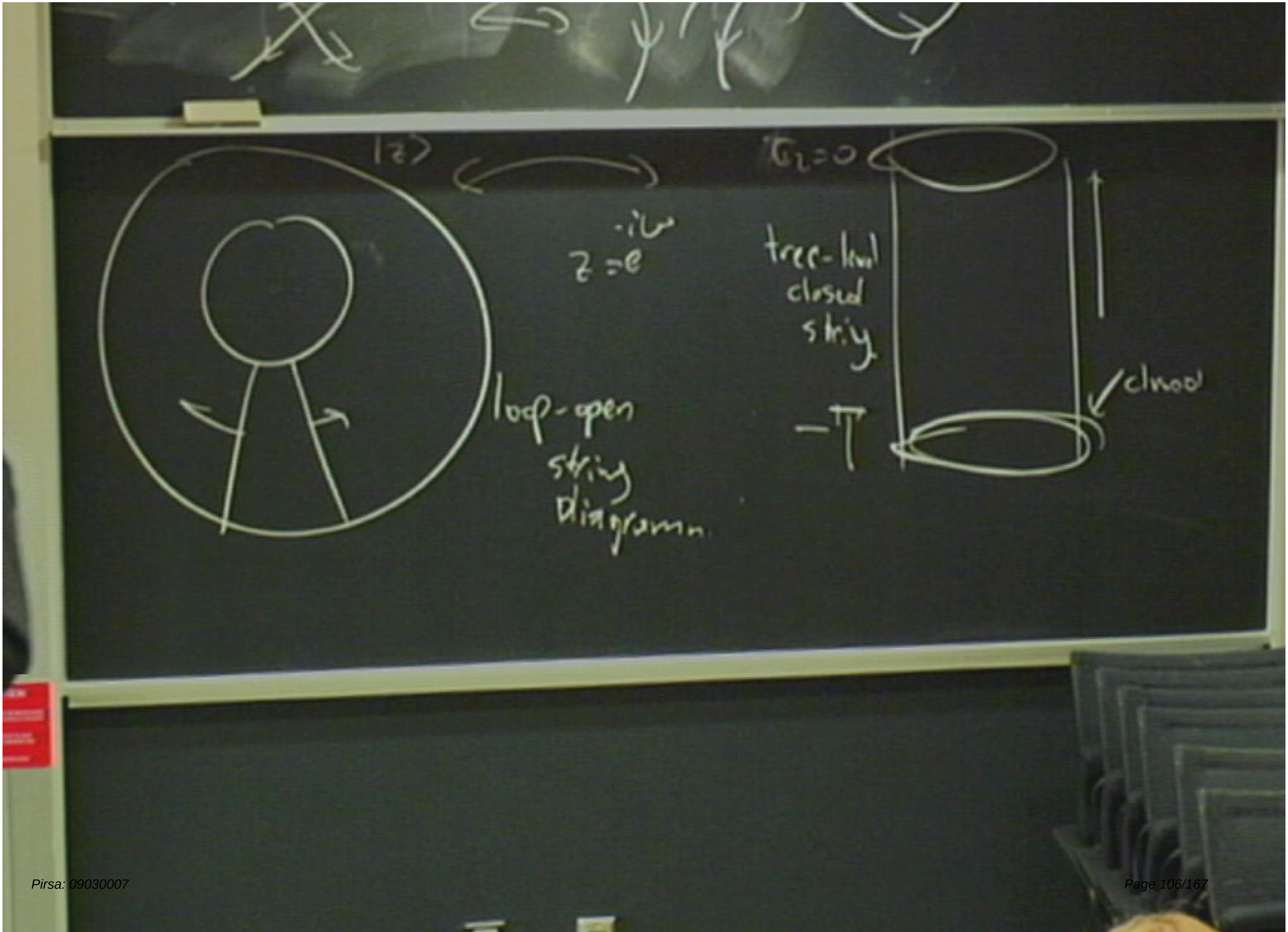



|z>
σ2=0
z = e^-iω




|z>
-π


|z⟩
$z = e^{-i\omega}$
$\tau_2 = 0$
$-\pi$
clnod


|z⟩
z = e^{-iw}
loop-open
string
diagramm
σ_2 = 0
tree-level
closed
string
closed
-π

Polyakov path integral (closed) strings

Polyakov path integral (closed) strings

$\Rightarrow$ show that $\underline{D = 26}$

$\Rightarrow$ show that $\underline{D = 26}$

$\Rightarrow$ Euclidean path integrals

$\bar{z}^{\,h_k - h_i - h_j}$


⇒ show that D=26
⇒ Euclidean path integrals
e^{iS/ħ} → e^{-S}


∫[dx]

$$\int [dx][dg]\, e^{-S}$$

$$S = S_x + \lambda$$

$$\int [dx][dg]\, e^{-S}$$

$$S = S_x + \lambda \chi$$

Euler character of or worldsheet

$$\int [dX][dg]\, e^{-S}$$

Euler character of or worldsheet

$$S = S_X + \lambda \chi$$

$$S_X = \frac{1}{4\pi\alpha'} \int_M d^2\sigma \sqrt{g}\, \partial_a X^\mu \partial^a X_\mu$$

$$\int [dX][dg]\, e^{-S}$$

Euler character of worldsheet

$$S = S_X + \lambda \chi \qquad \mu = 0 \ldots D-1$$

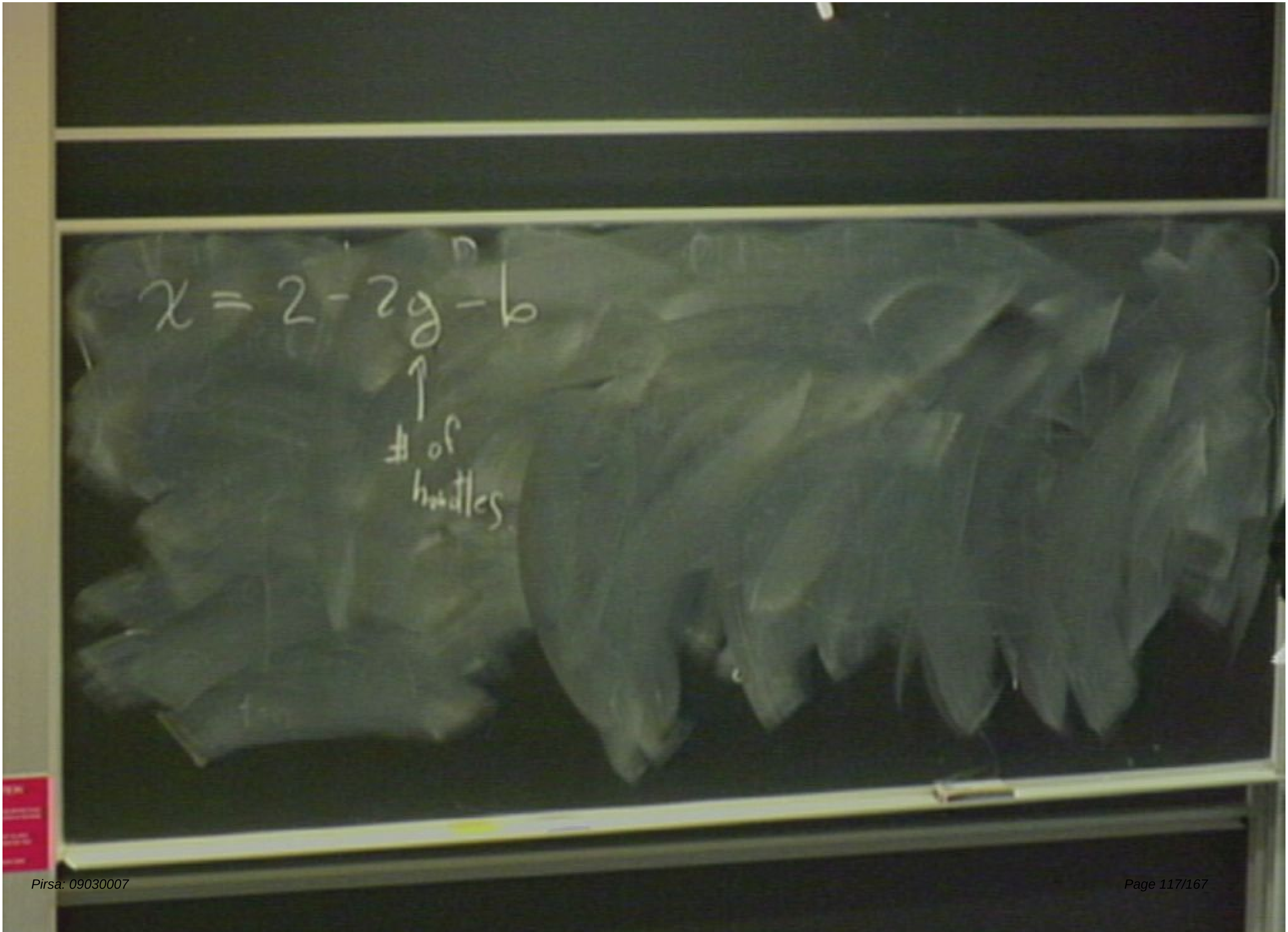
$$S_X = \frac{1}{4\pi\alpha'}\int_M d^2\sigma \sqrt{g}\, \partial_a X^\mu \partial^a X_\mu \,, \quad \chi = \frac{1}{4\pi}\int_M d^2\sigma \sqrt{g}\, R$$

$$\int [dx][dg]\, e^{-S}$$

Euler character of or worldsheet

$$S = S_X + \lambda \chi \qquad \mu = 0 \ldots D-1$$

$$S_X = \frac{1}{4\pi\alpha'} \int_M d^2\sigma \sqrt{g}\, \partial_a X^\mu \partial^a X_\mu \qquad \chi = \frac{1}{4\pi} \int_M d^2\sigma \sqrt{g}\, R$$


χ = 2 - 2g - b
# of handles

$$\chi = 2 - 2g - b$$

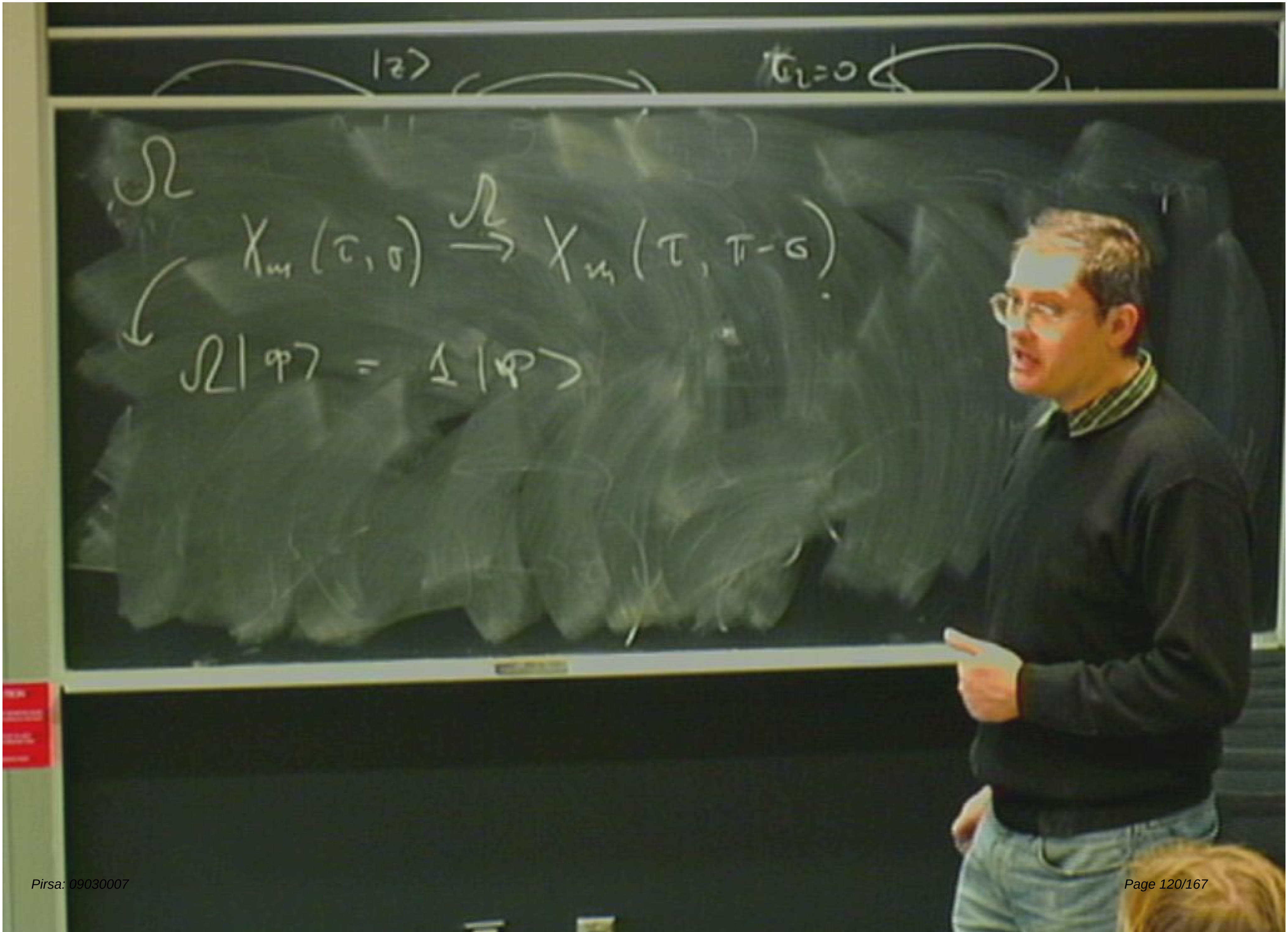
# of boundary components

# of handles

$|z\rangle$ $\qquad \tau_2 = 0$

Ω

$$X_m(\tau, \sigma) \xrightarrow{\Omega} X_m(\tau, \pi - \sigma)$$

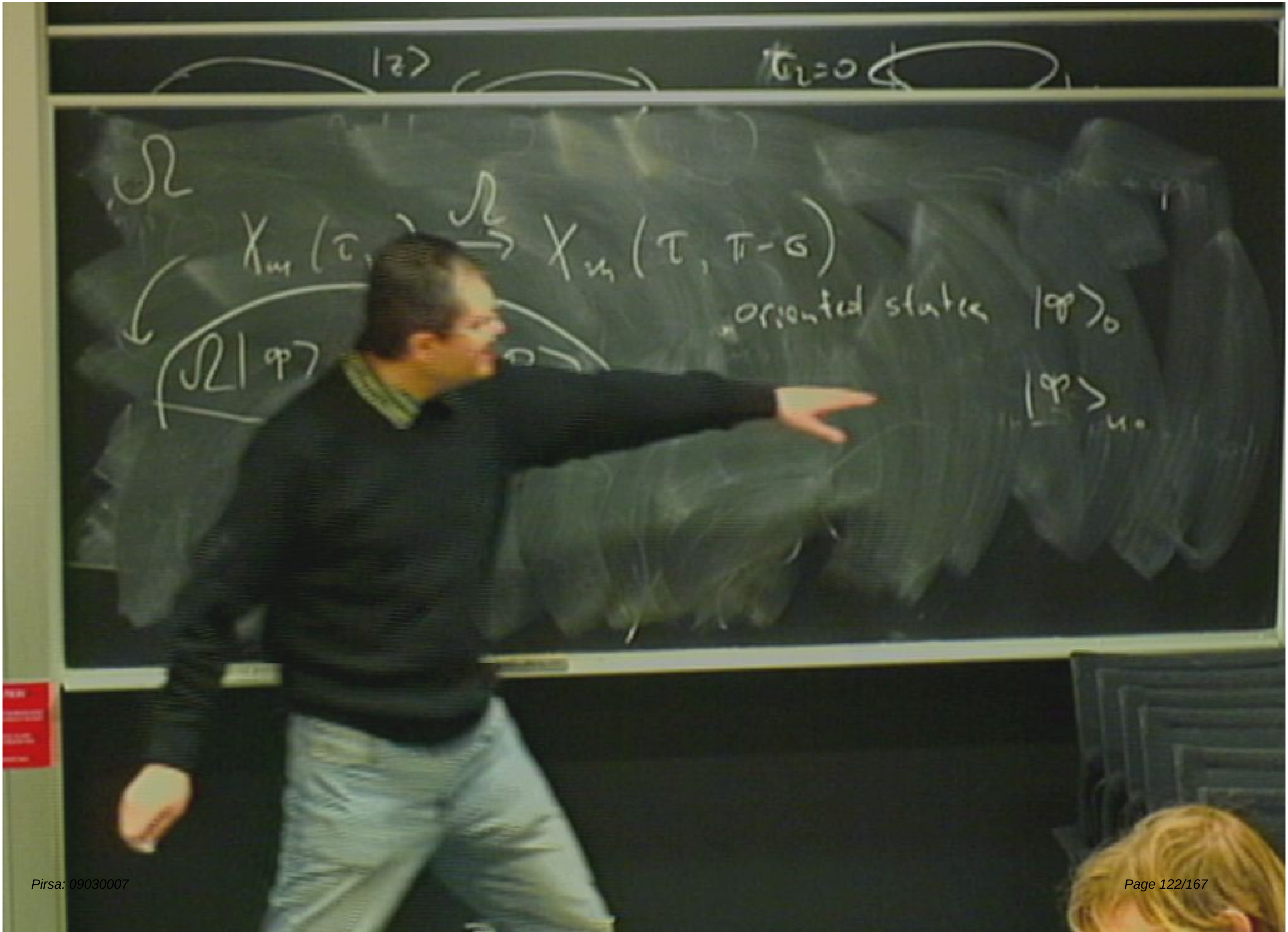

Ω
X_m (τ, σ) → X_m (τ, π − σ)
Ω|φ> = ±|φ>

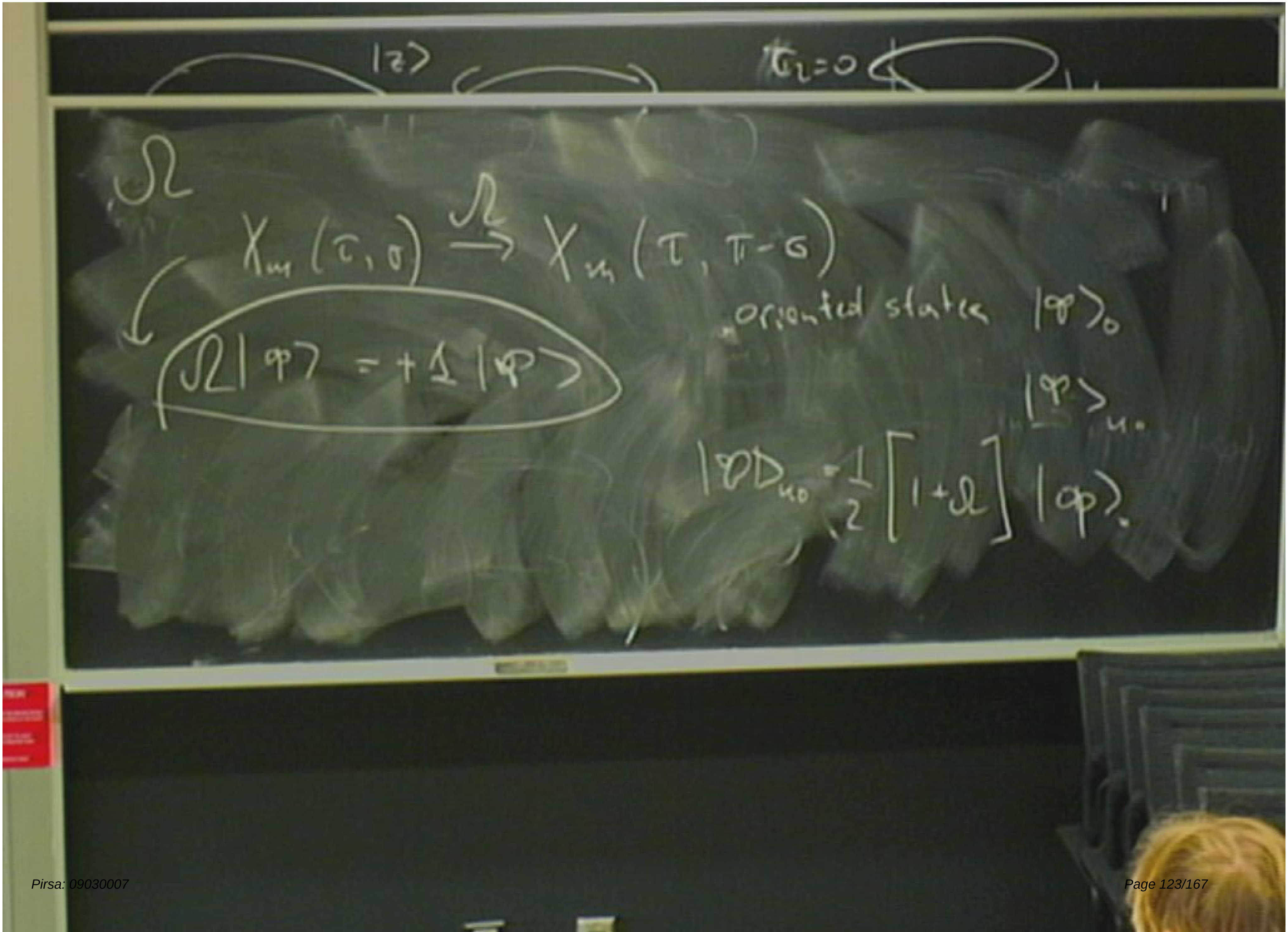
$|z\rangle$ $\tau_2 = 0$

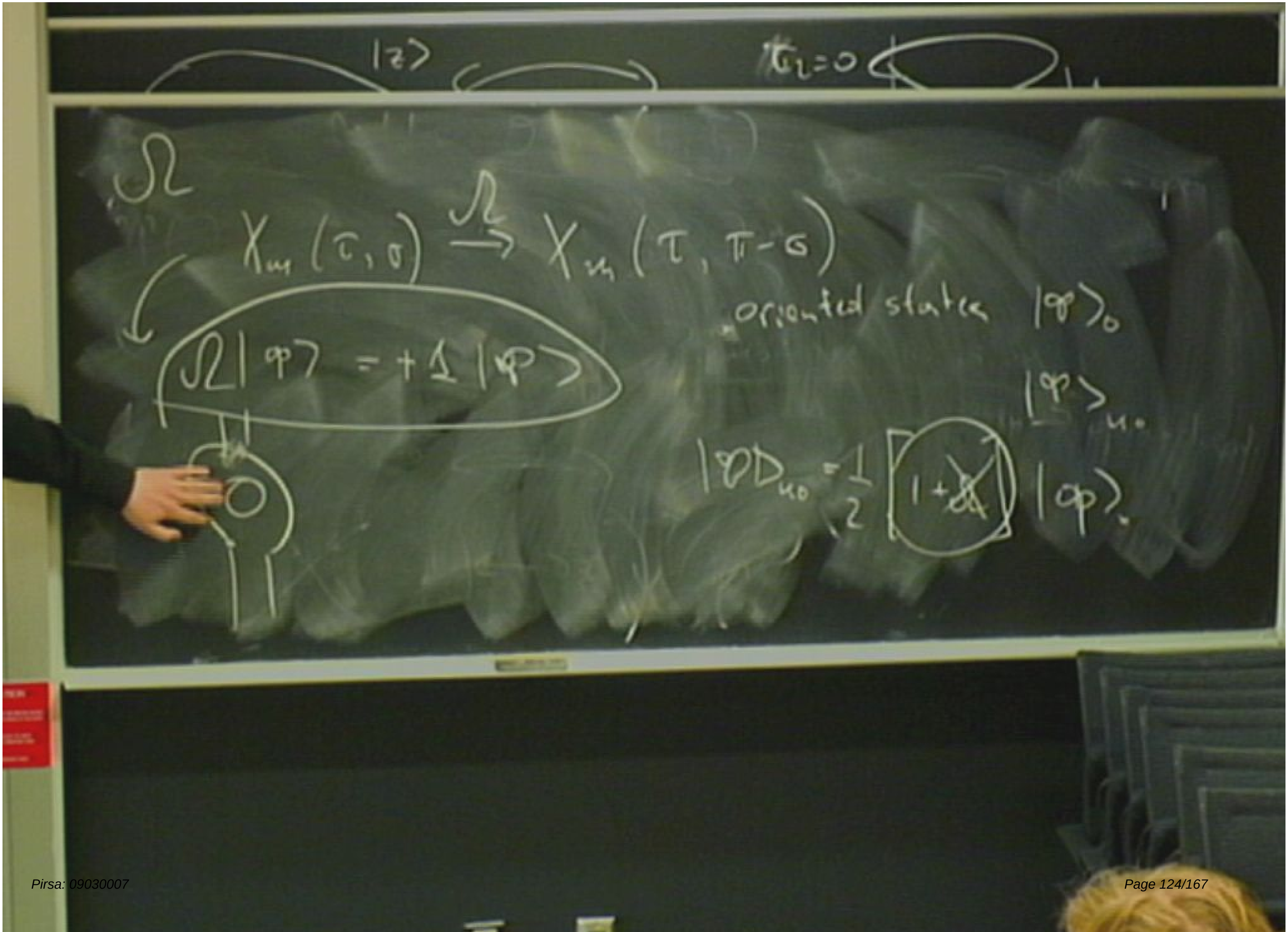
Ω

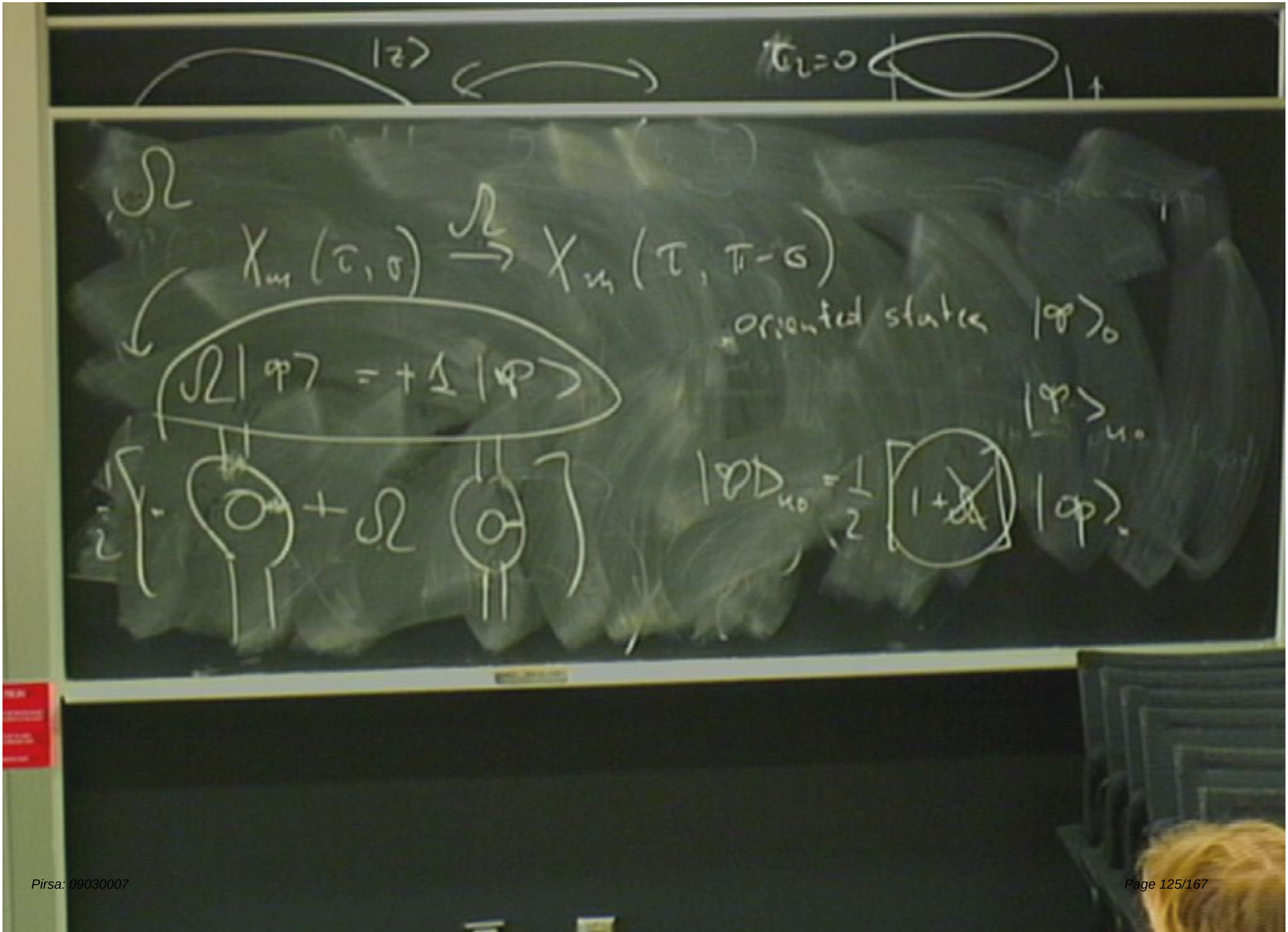
$$X_m(\tau, \sigma) \xrightarrow{\Omega} X_m(\tau, \pi - \sigma)$$

$$\Omega|\varphi\rangle = +1\,|\varphi\rangle$$

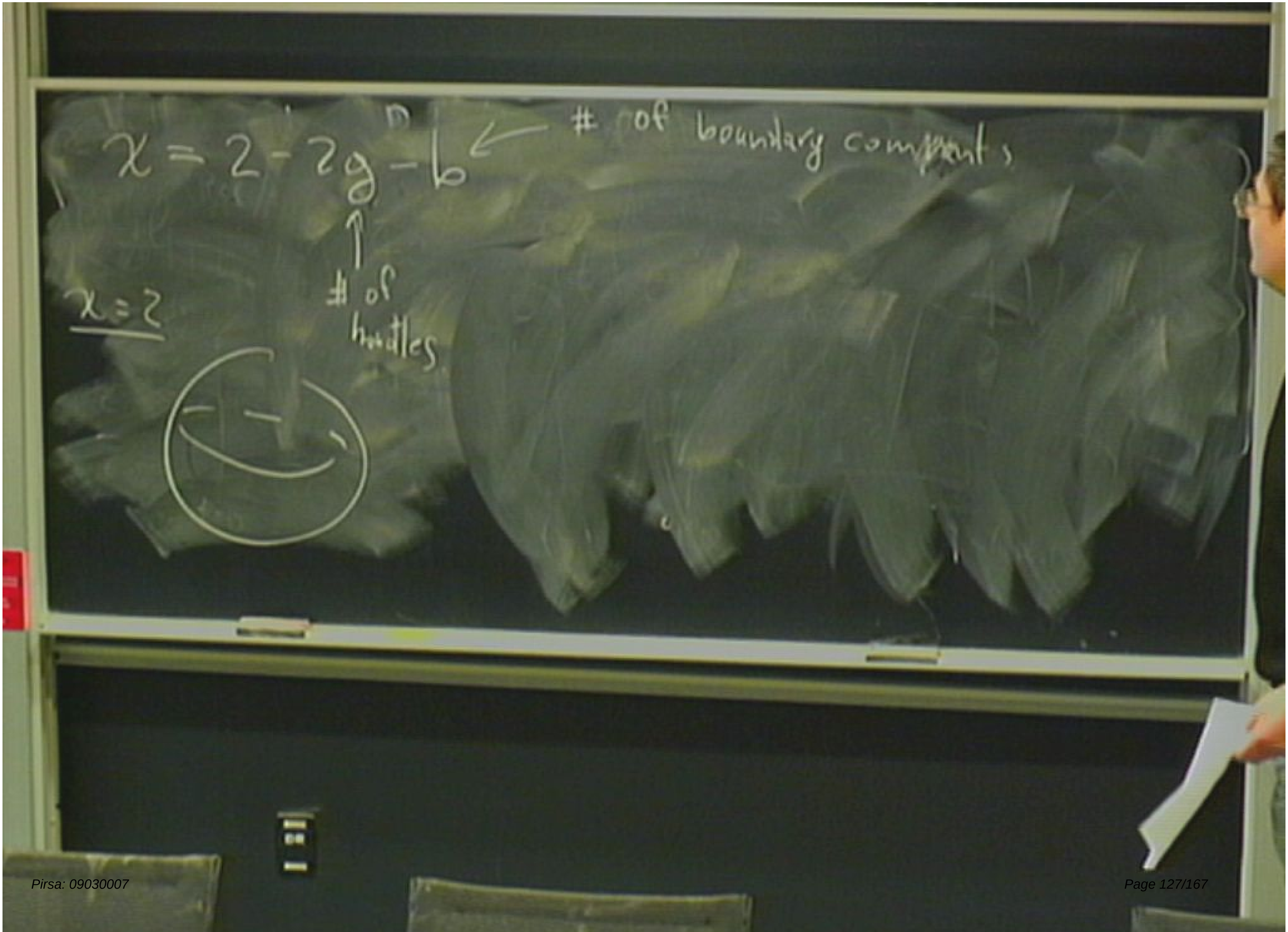

|z>
oriented states


|z⟩
τ₂=0
Ω
X_m(τ, σ) → X_m(τ, π−σ)
oriented states |φ⟩_o
Ω|φ⟩ = +1|φ⟩
|φ⟩_u.o.
|φ⟩_uo = 1/2 [1+Ω] |φ⟩_o

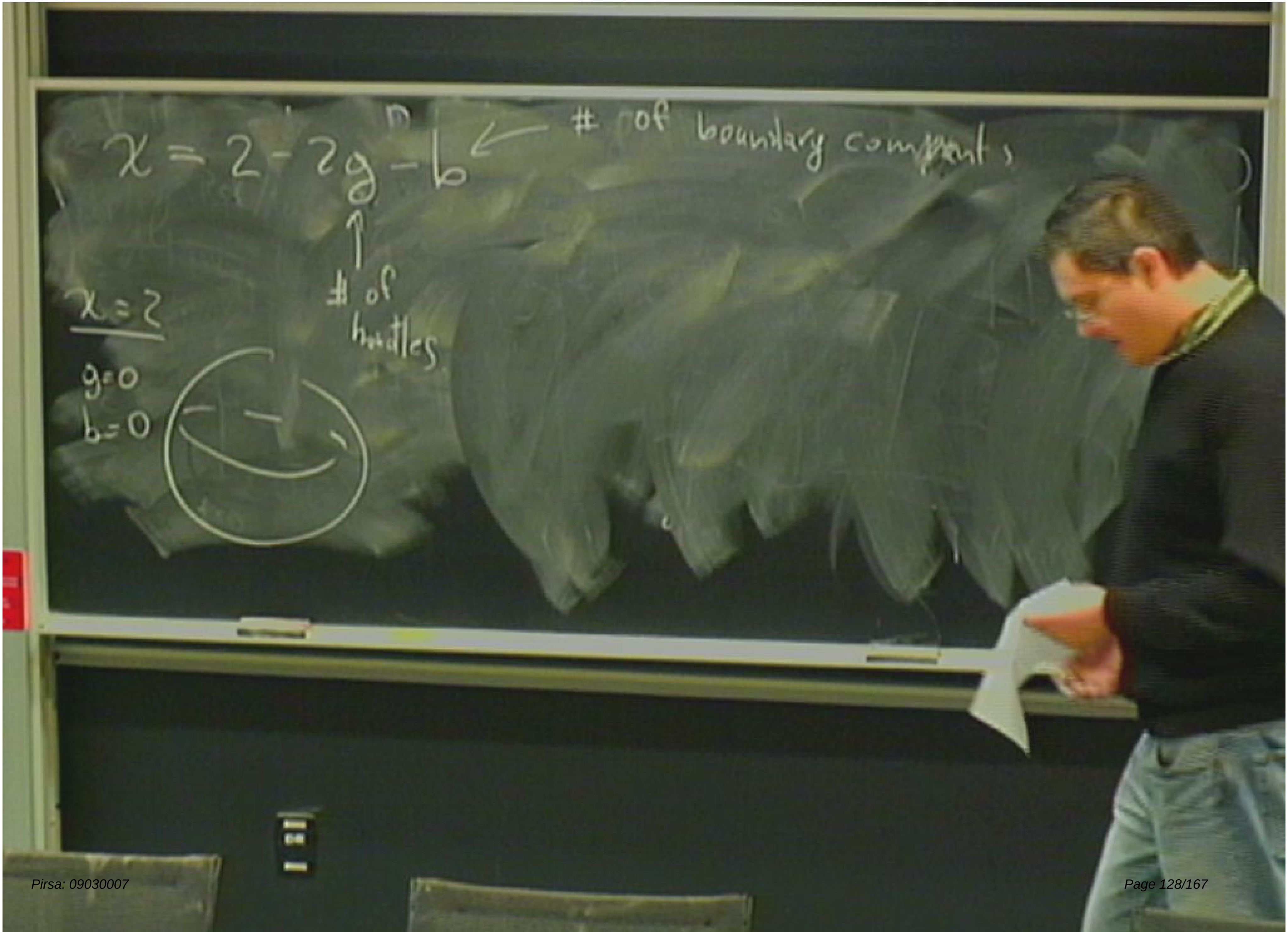

|z>
Ω
X_m (τ, σ) → X_m (τ, π − σ)
oriented states |φ>_o
Ω |φ> = +1 |φ>
|φ>_uo
|φ>_uo = 1/2 [1 + Ω] |φ>_o

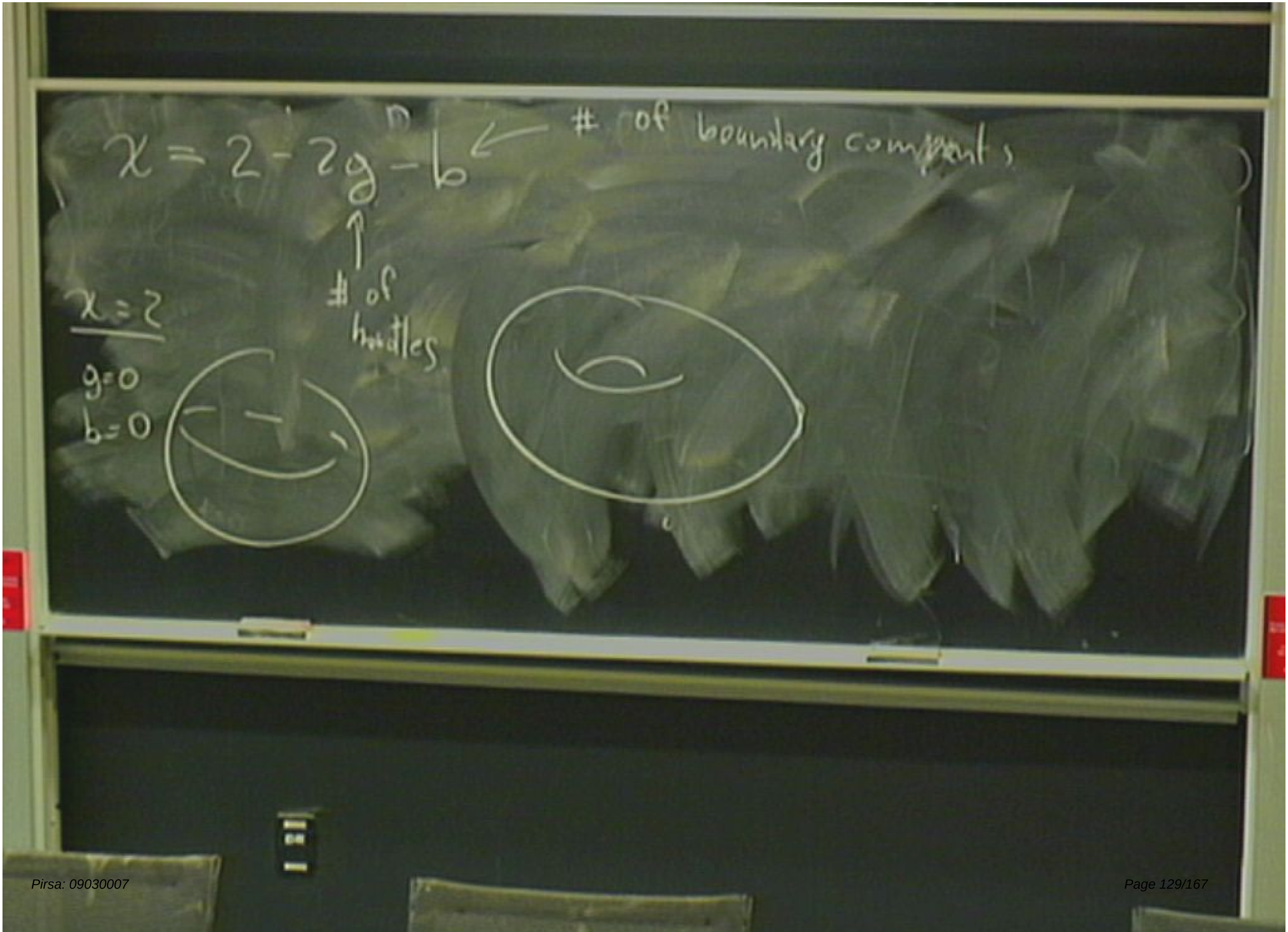

|z⟩
τ₂=0
Ω
X_m (τ, σ) → X_m (τ, π − σ)
oriented states |φ⟩_o
Ω|φ⟩ = +1 |φ⟩
|φ⟩_u.o.
|φ⟩_u.o. = 1/2 [1 + Ω] |φ⟩_o

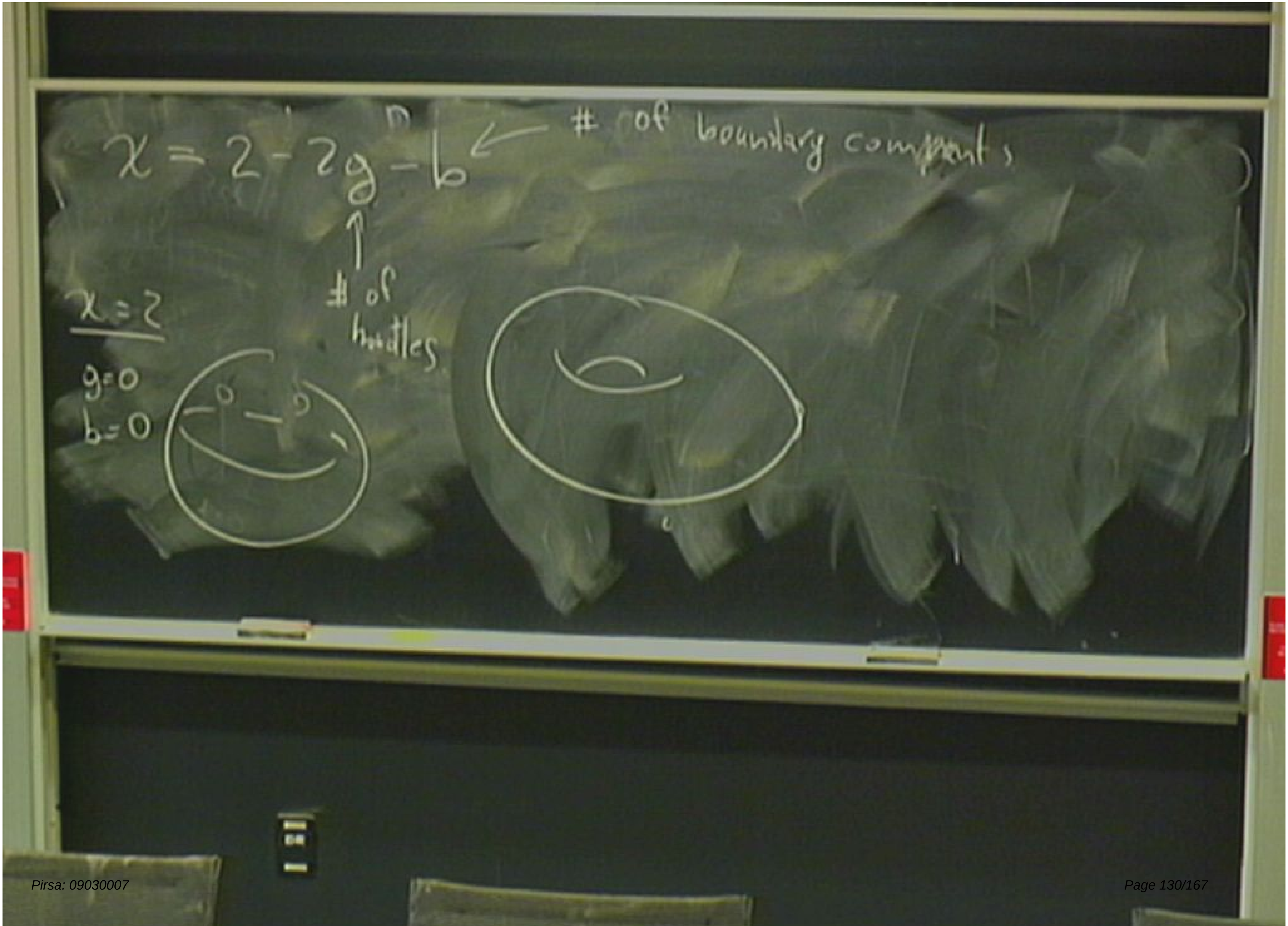

$\int [dx][dg]\, e^{-S}$
$S = S_X + \lambda \chi$
worldsheet
D-1
$\chi = \frac{1}{4\pi}\int_M d^2\sigma \sqrt{g} R$

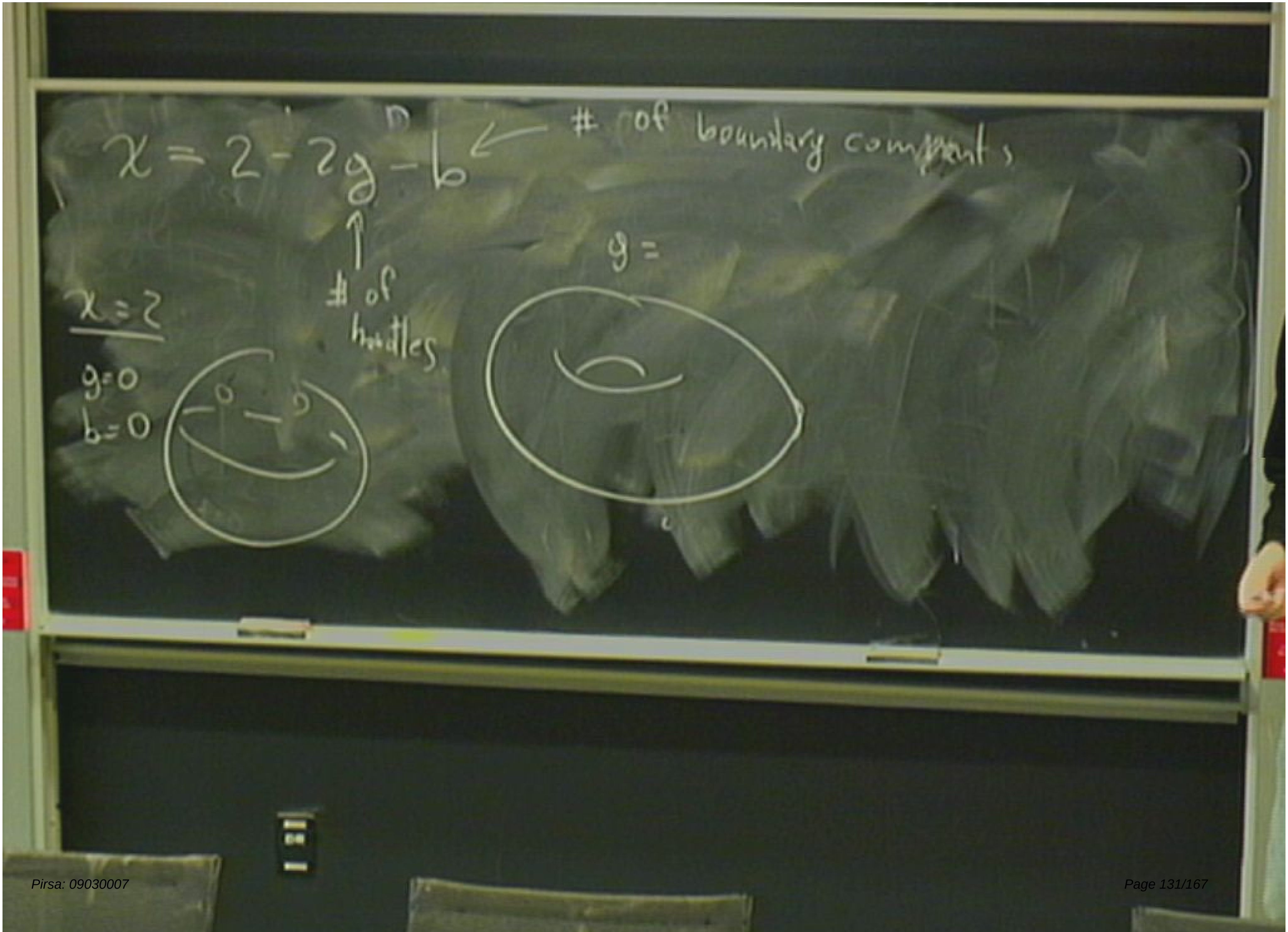


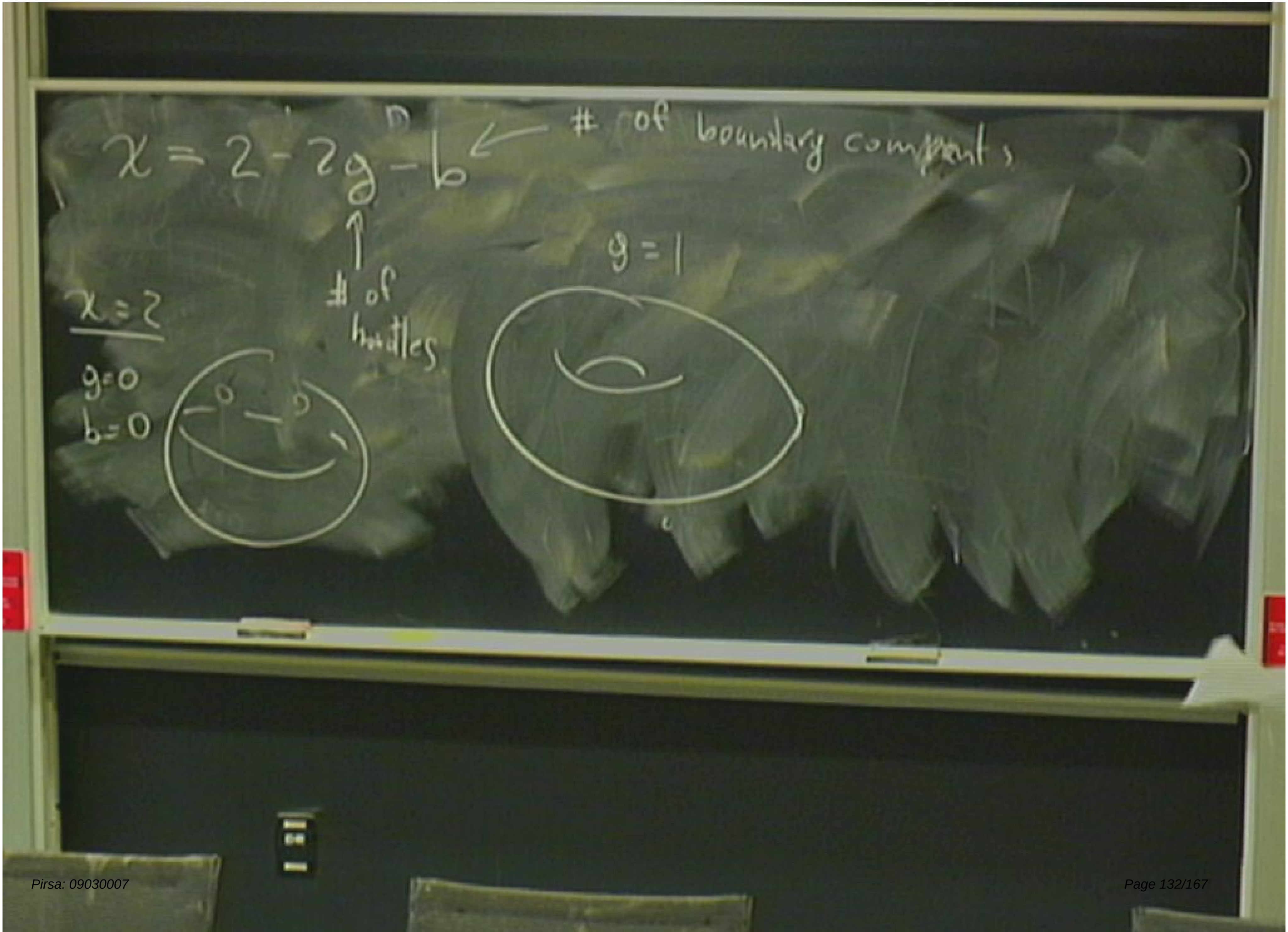
$\chi = 2 - 2g - b$
# of boundary components
# of handles
$\chi = 2$


$\chi = 2 - 2g - b$
# of boundary components
# of handles
$\chi = 2$
$g = 0$
$b = 0$

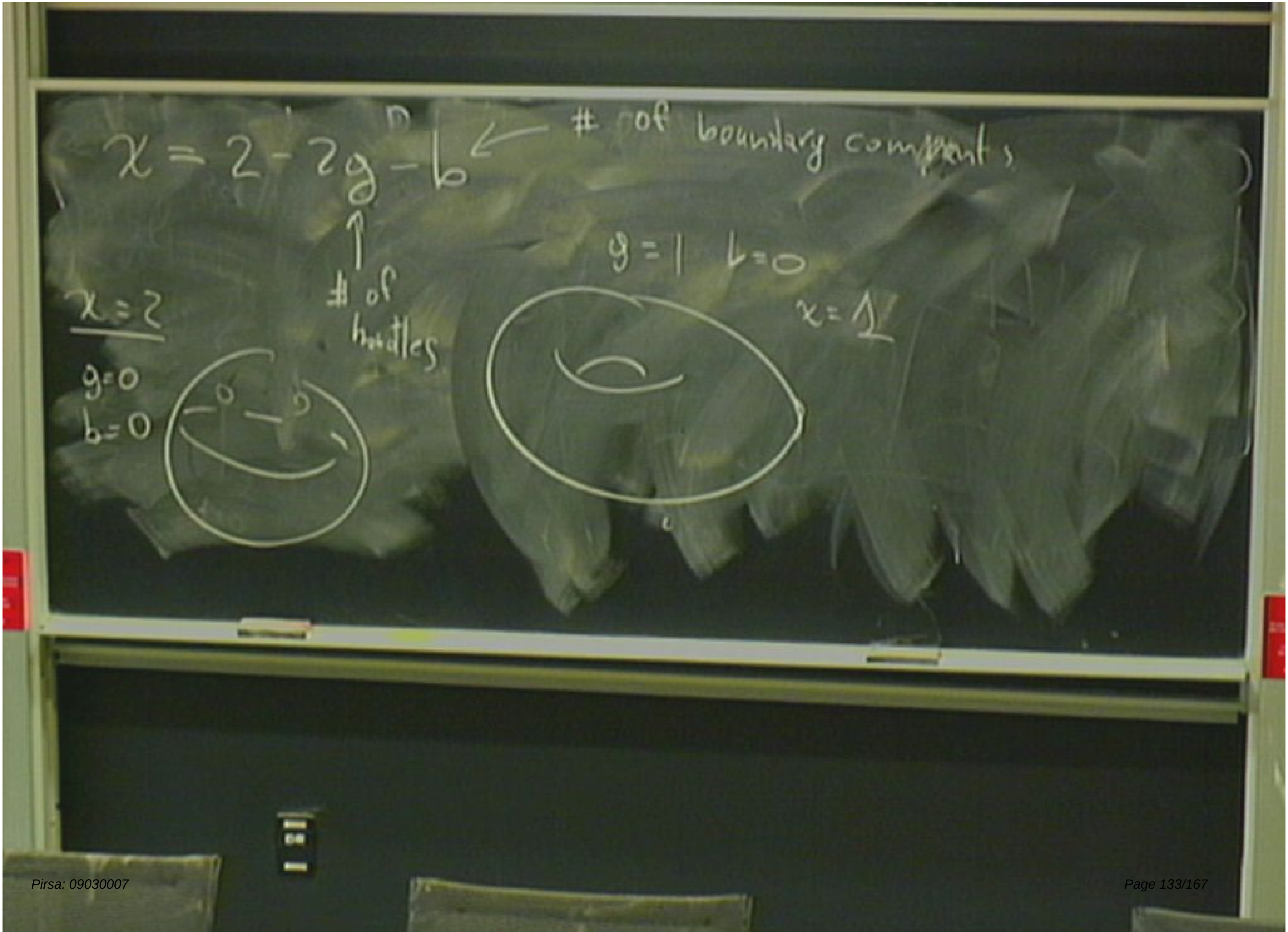

$\chi = 2 - 2g - b$
# of boundary components
# of handles
$\chi = 2$
$g = 0$
$b = 0$


$\chi = 2 - 2g - b$
# of boundary components
# of handles
$\chi = 2$
$g = 0$
$b = 0$

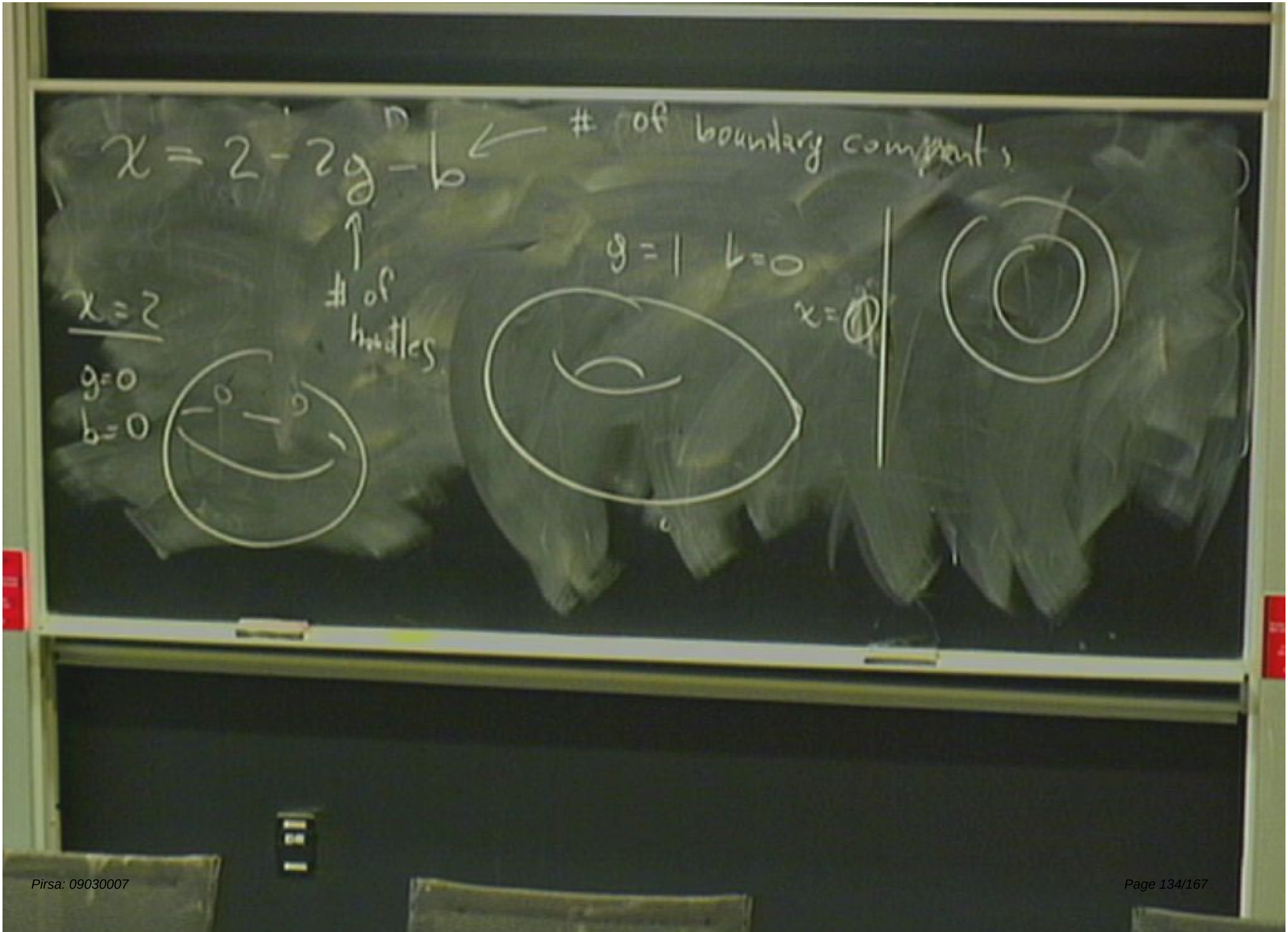

χ = 2 - 2g - b
# of boundary components
# of handles
χ = 2
g = 0
b = 0
g =

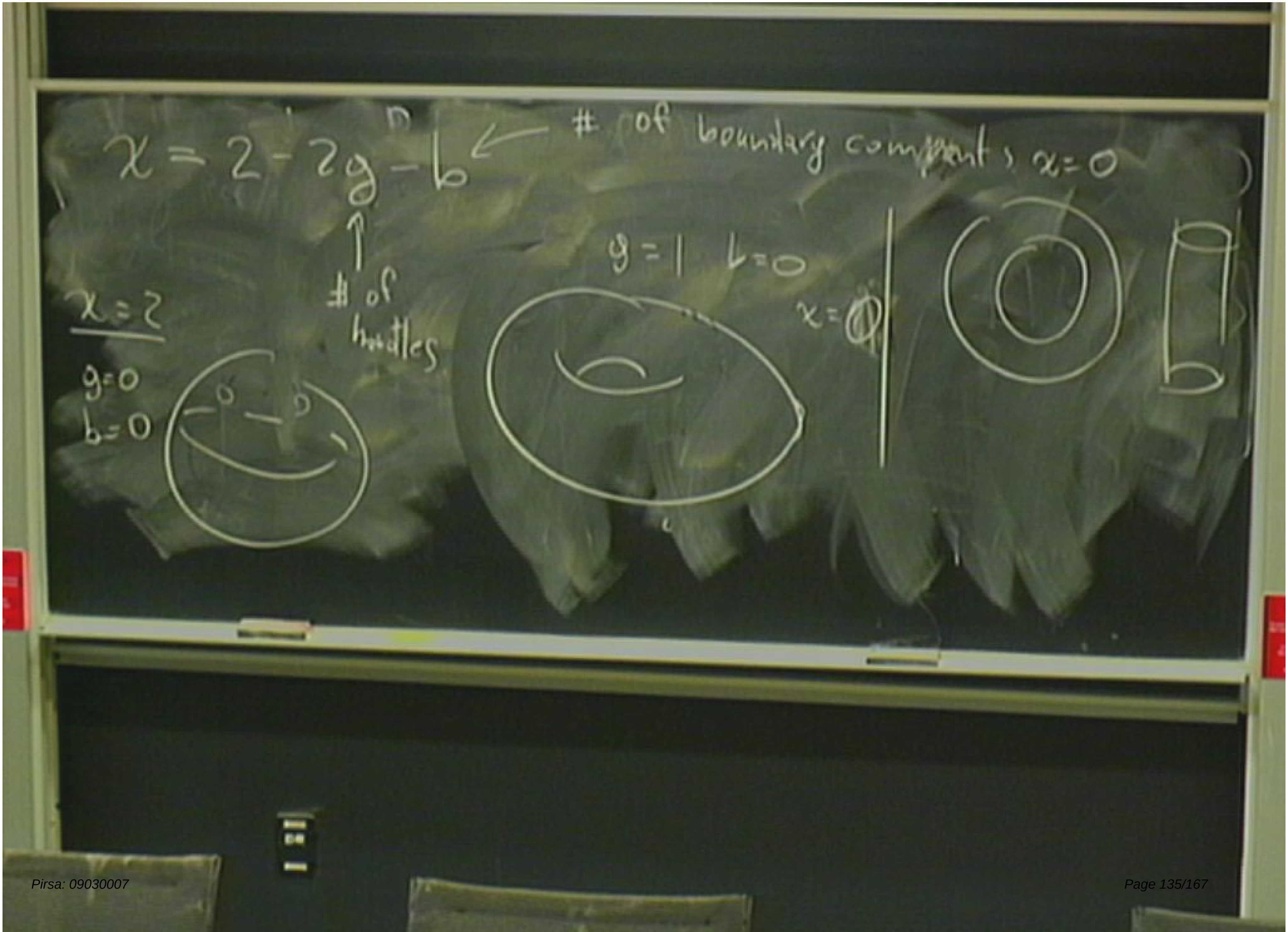


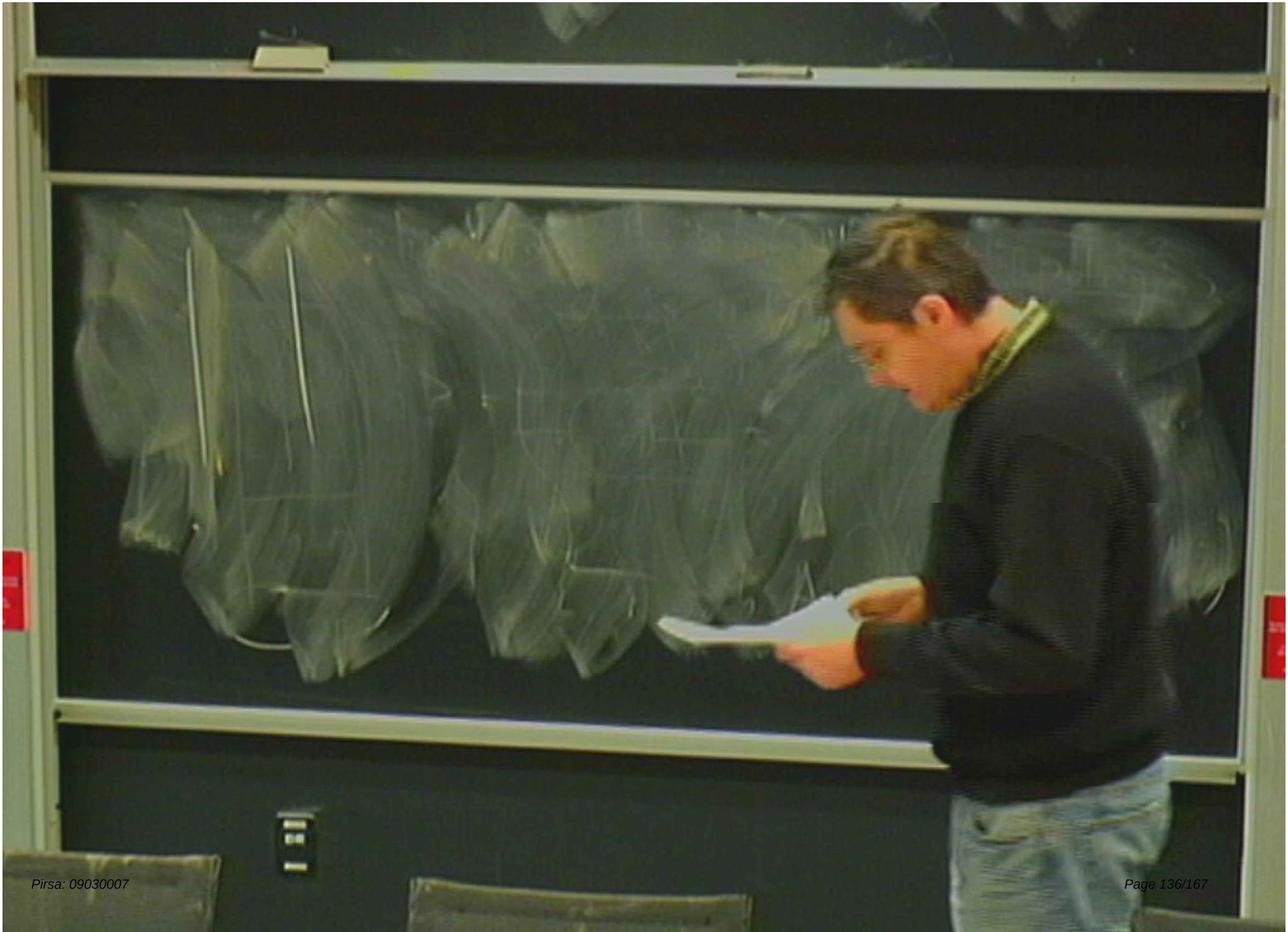
χ = 2 - 2g - b
# of boundary
# of handles
χ = 2
g=0
b=0
g = 1

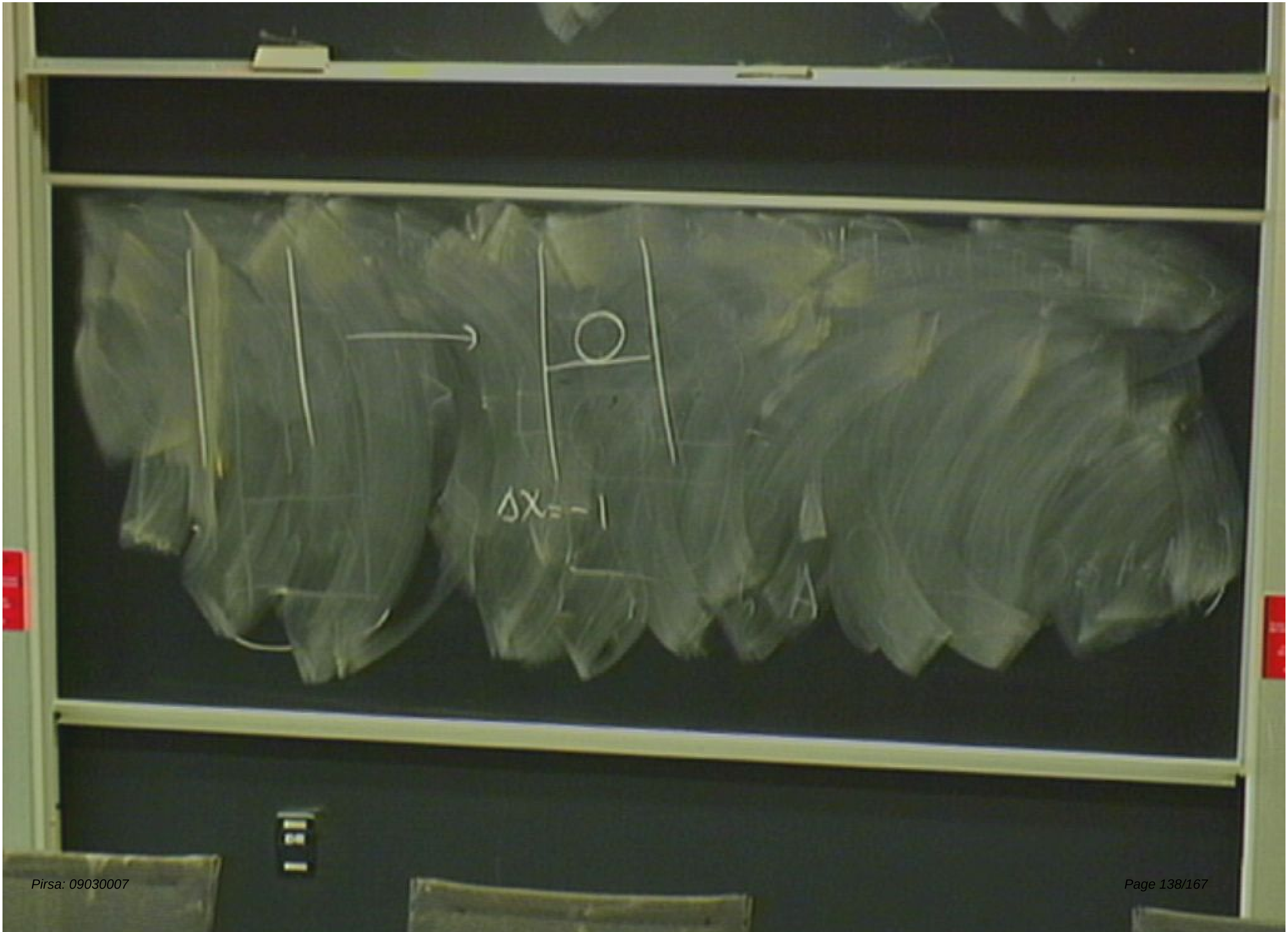


χ = 2 − 2g − b
# of boundary components
# of handles
χ = 2
g=0
b=0
g=1 b=0

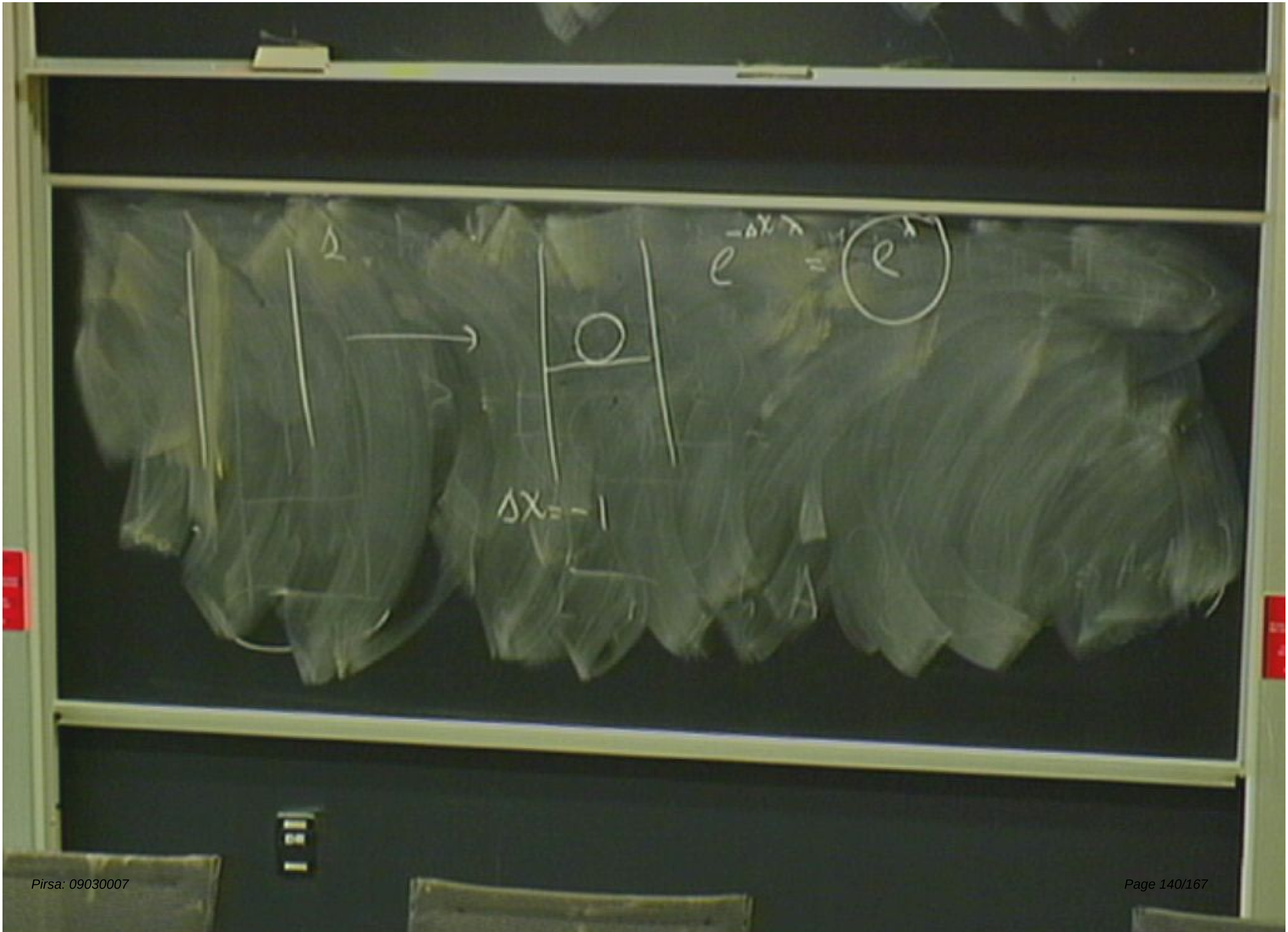

χ = 2 − 2g − b
# of boundary components
# of handles
χ = 2
g=0
b=0
g = 1 b = 0
χ = 0

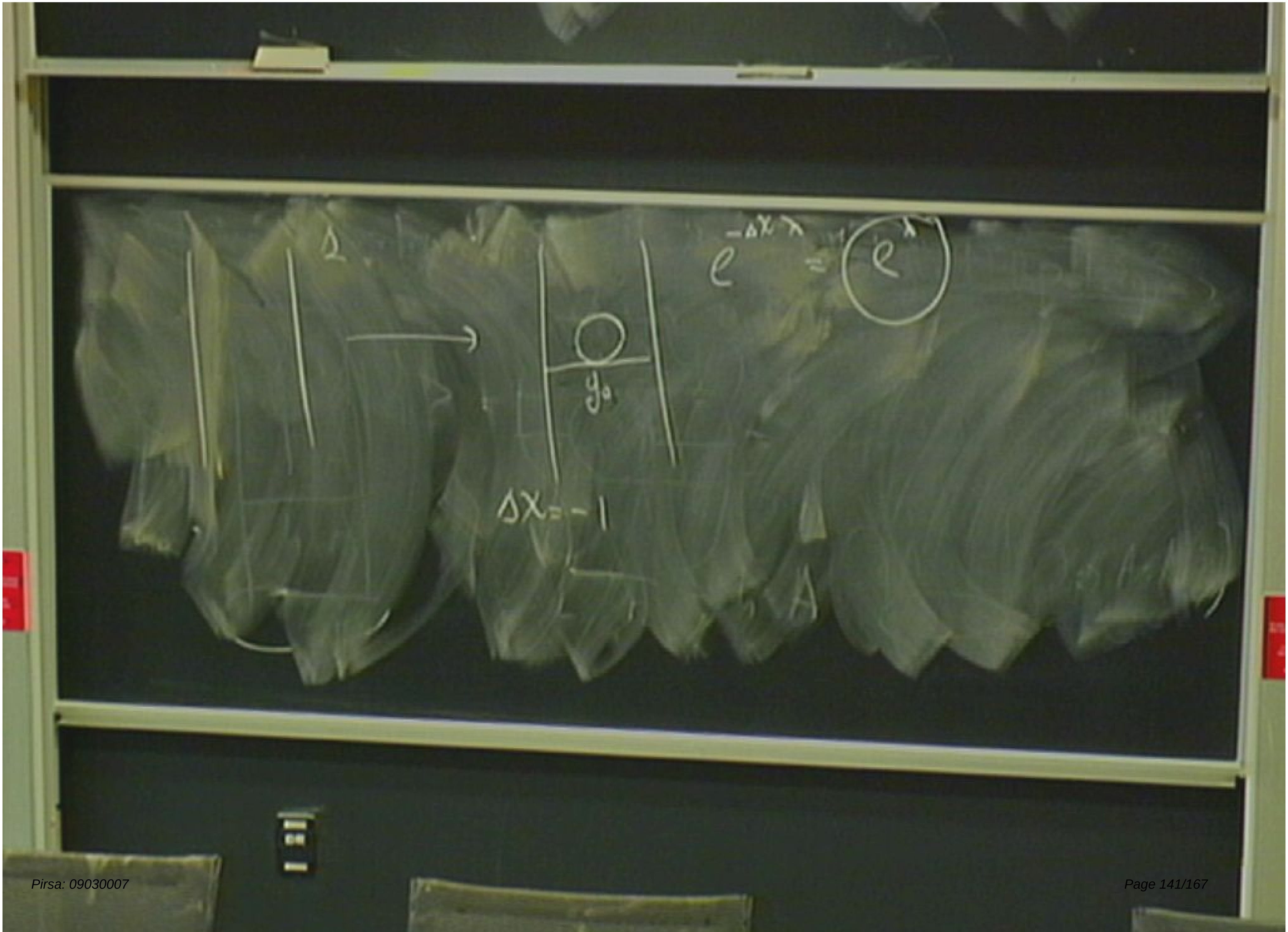

χ = 2 - 2g - b
# of boundary components, χ = 0
# of handles
χ = 2
g = 0
b = 0
g = 1 b = 0
χ = 0

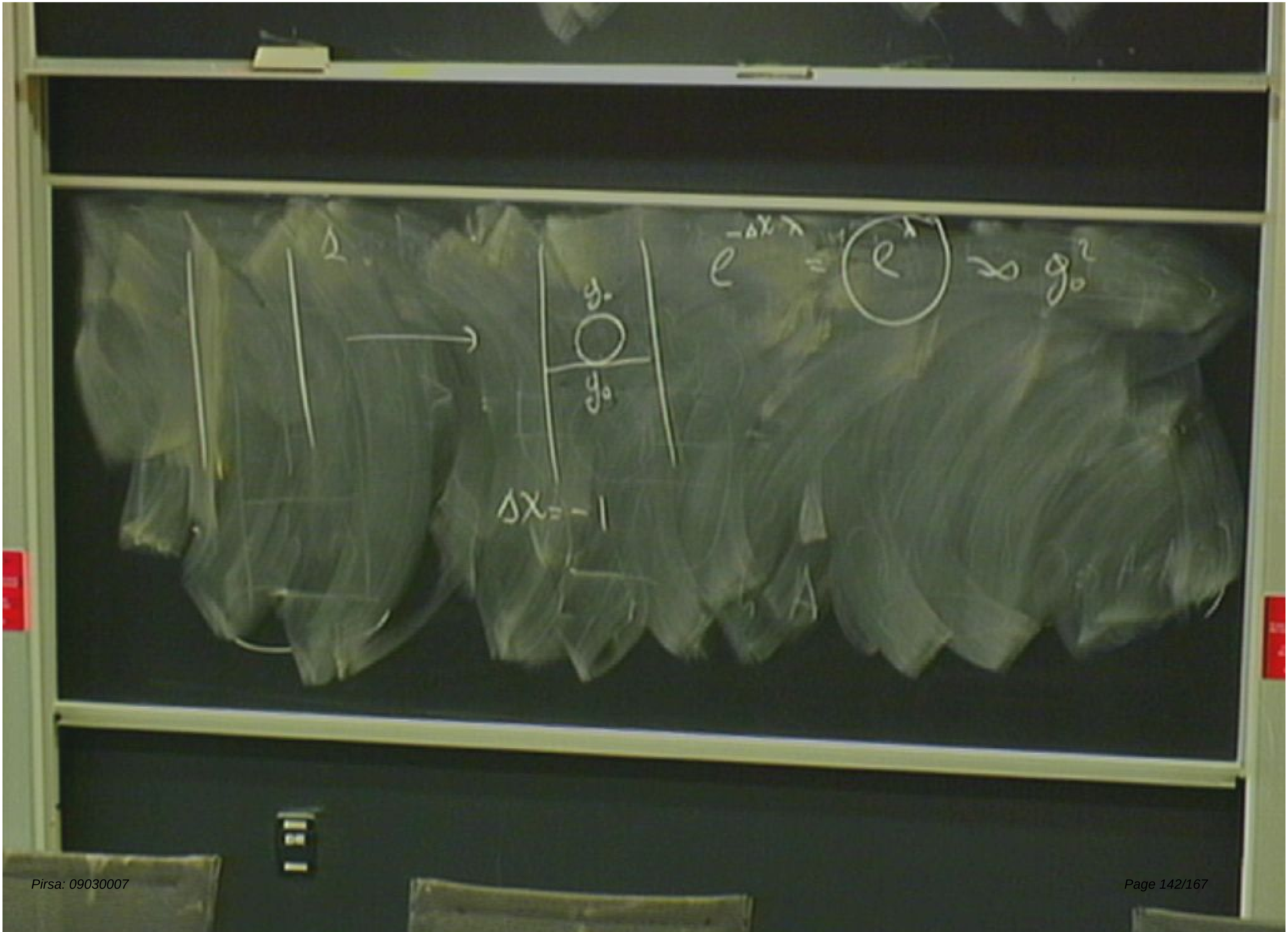


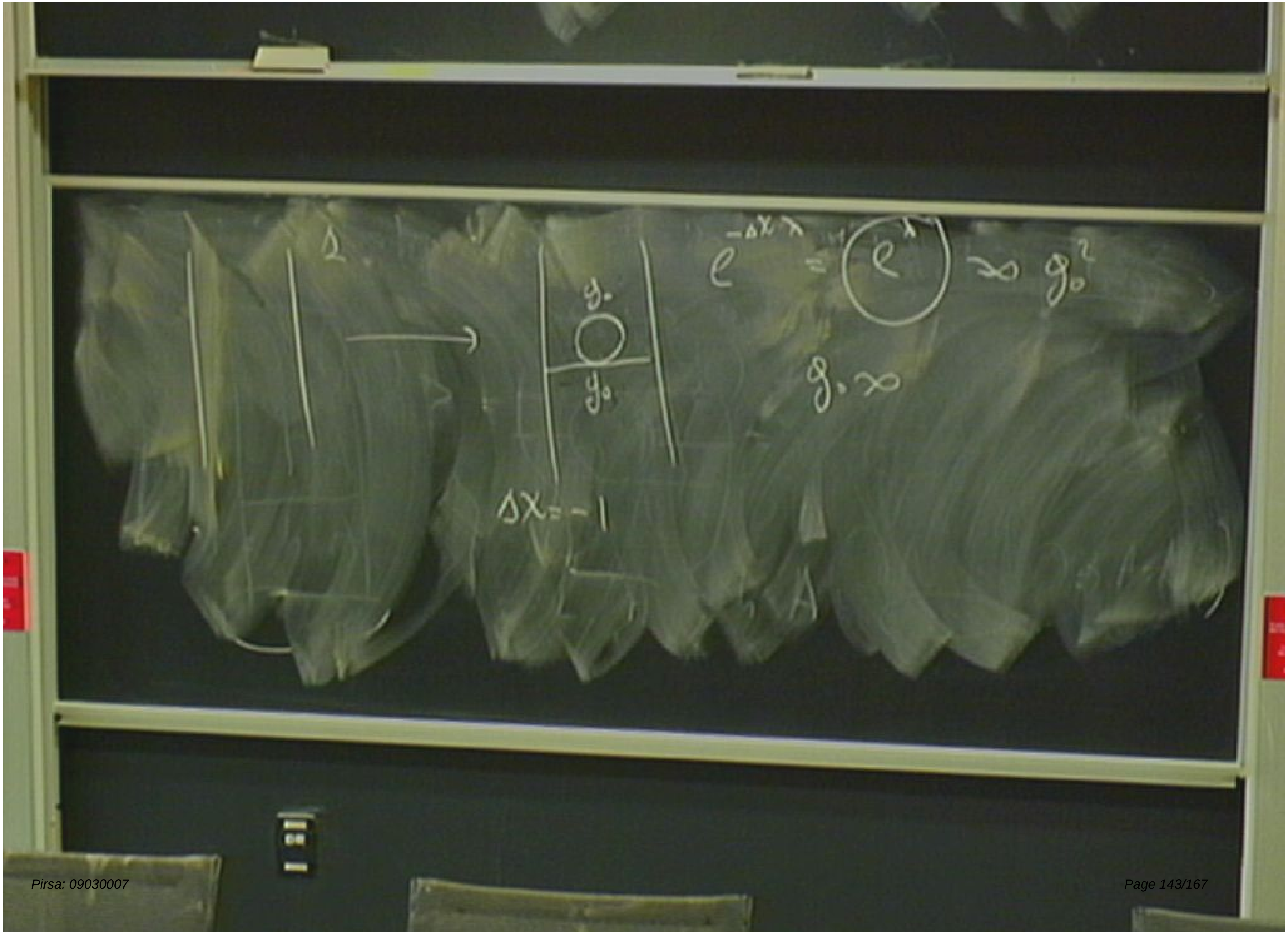

$\Delta X = -1$

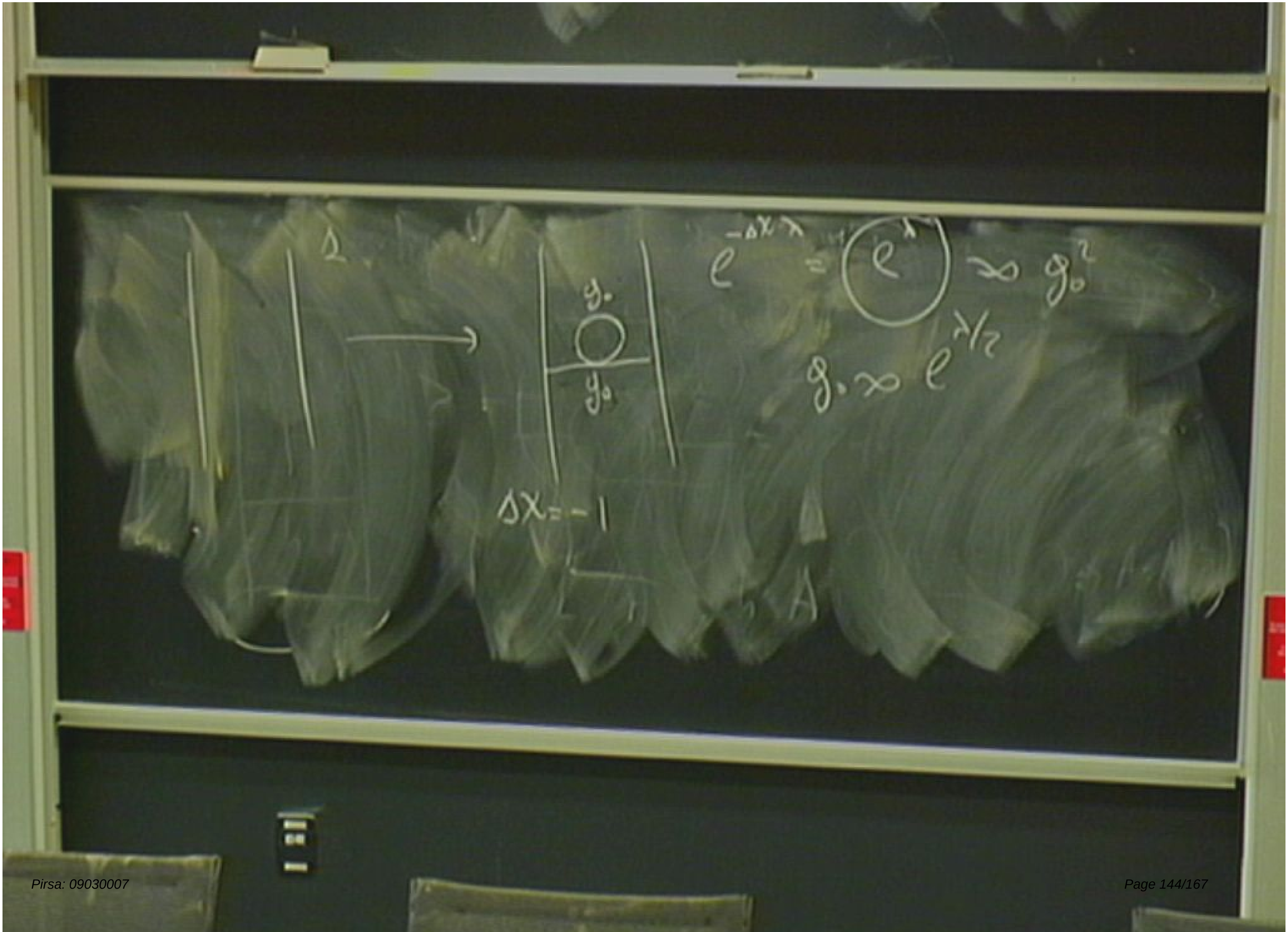

Δx=-1

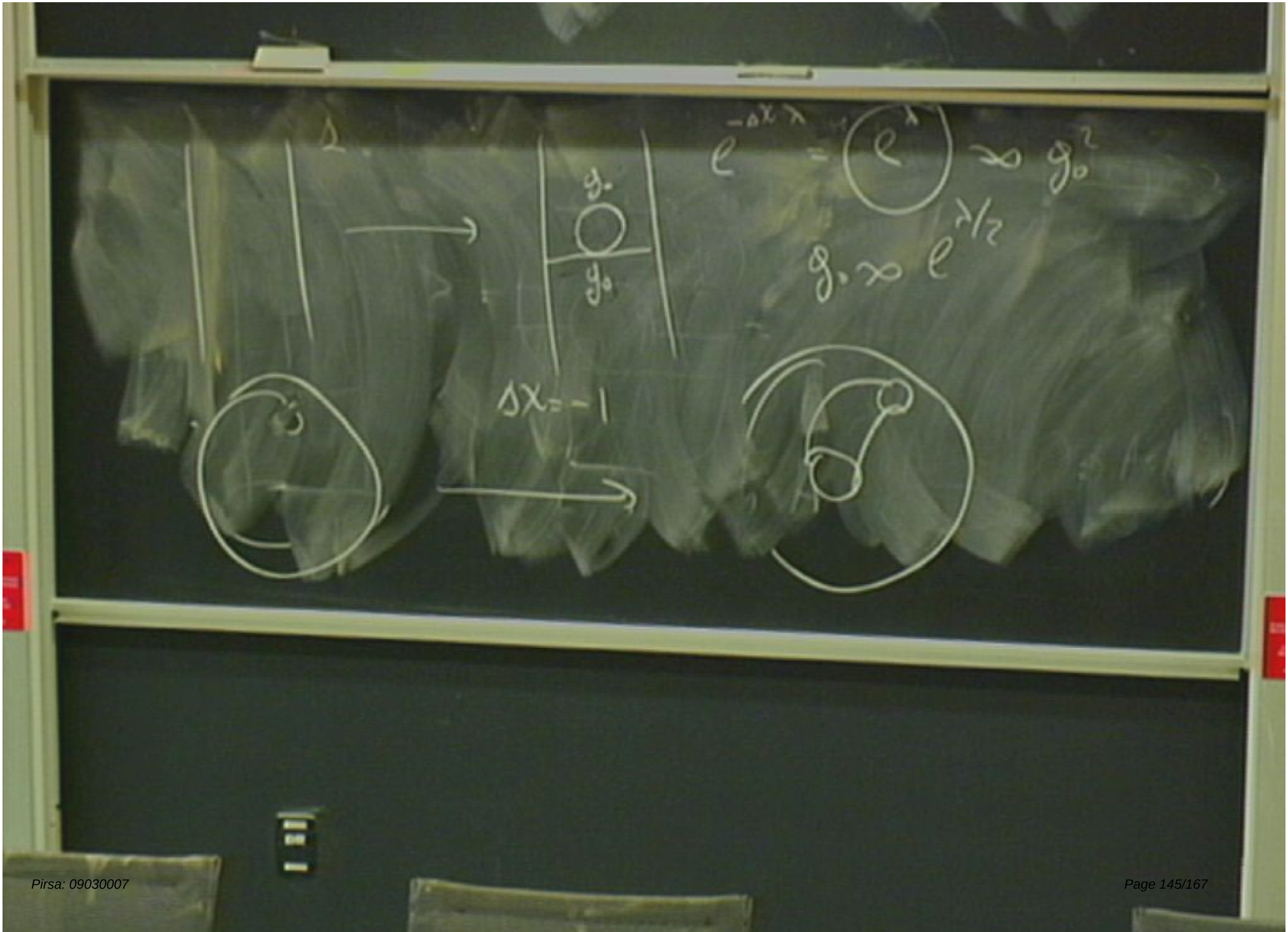

ΔX = -1

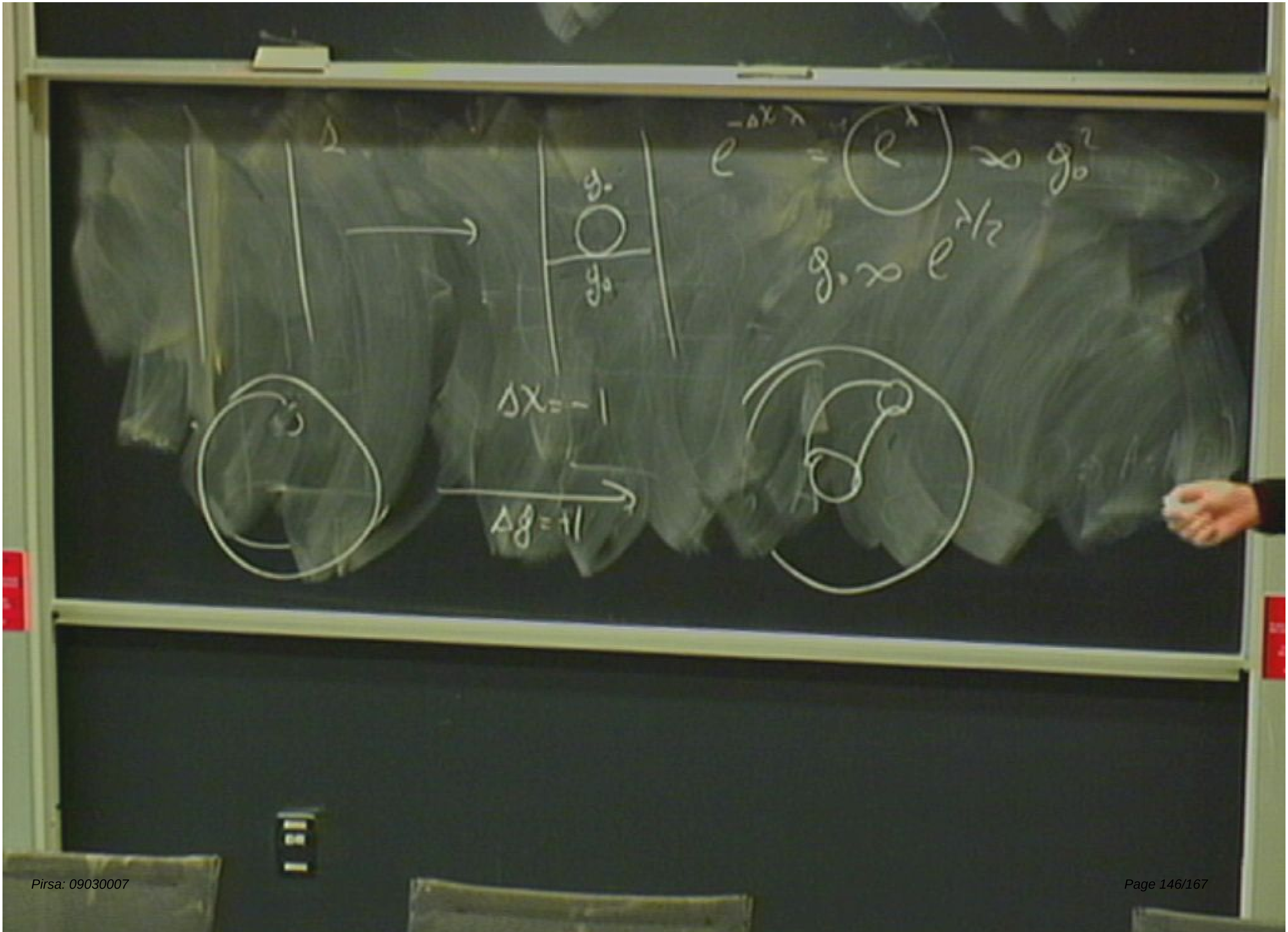

y_0
$\Delta x = -1$

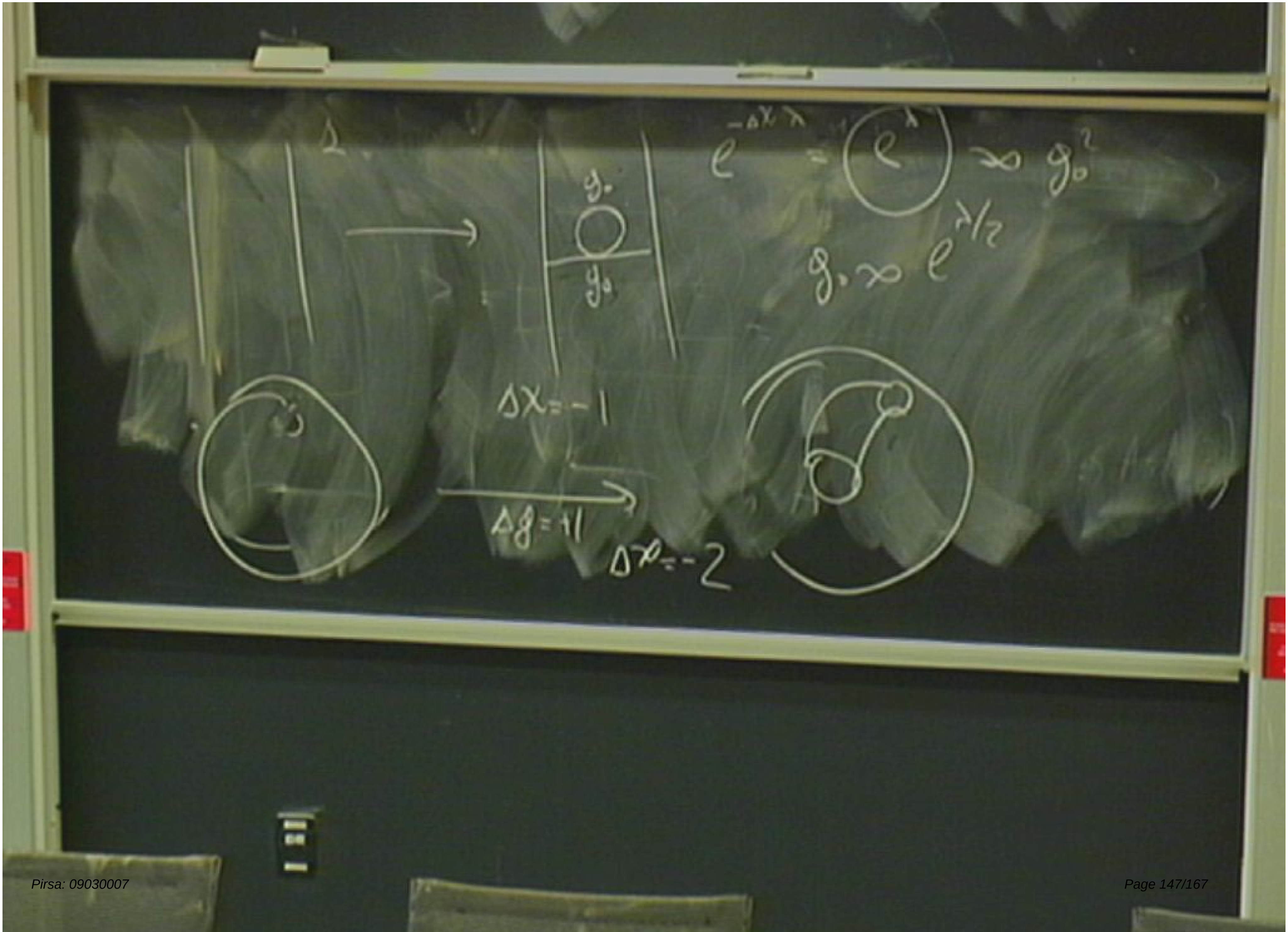

$e^{-\Delta x \lambda} = e^{\lambda} \propto g_0^2$
g_0
g_0
$\Delta x = -1$

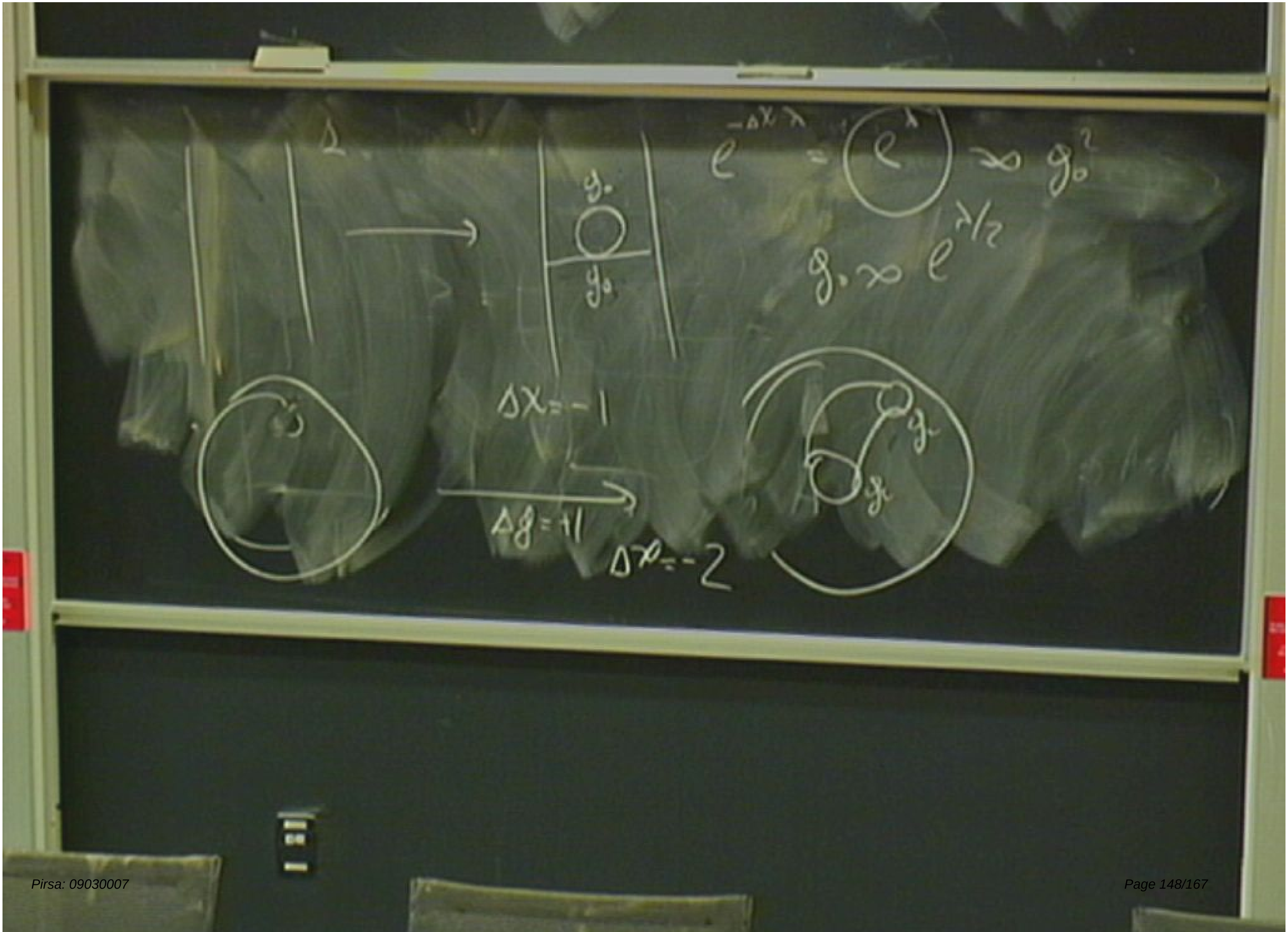

$e^{-\Delta\chi\,\lambda} = e^{\lambda} \propto g_0^2$
g_0
g_0
$g_0 \propto$
$\Delta\chi = -1$

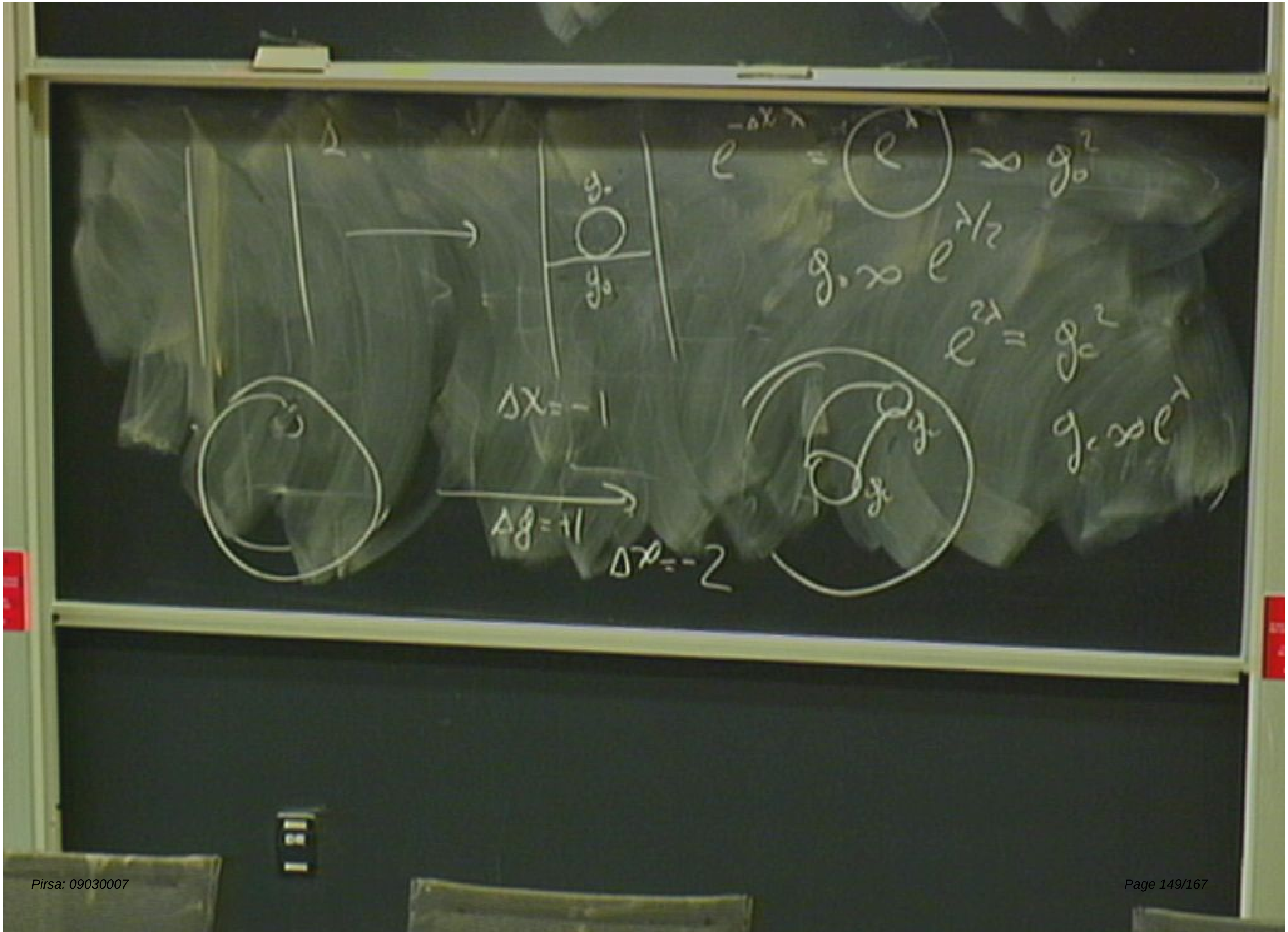

e^{-ΔX λ} = e^λ ∝ g0^2
g0
g0
g0 ∝ e^{λ/2}
ΔX = -1

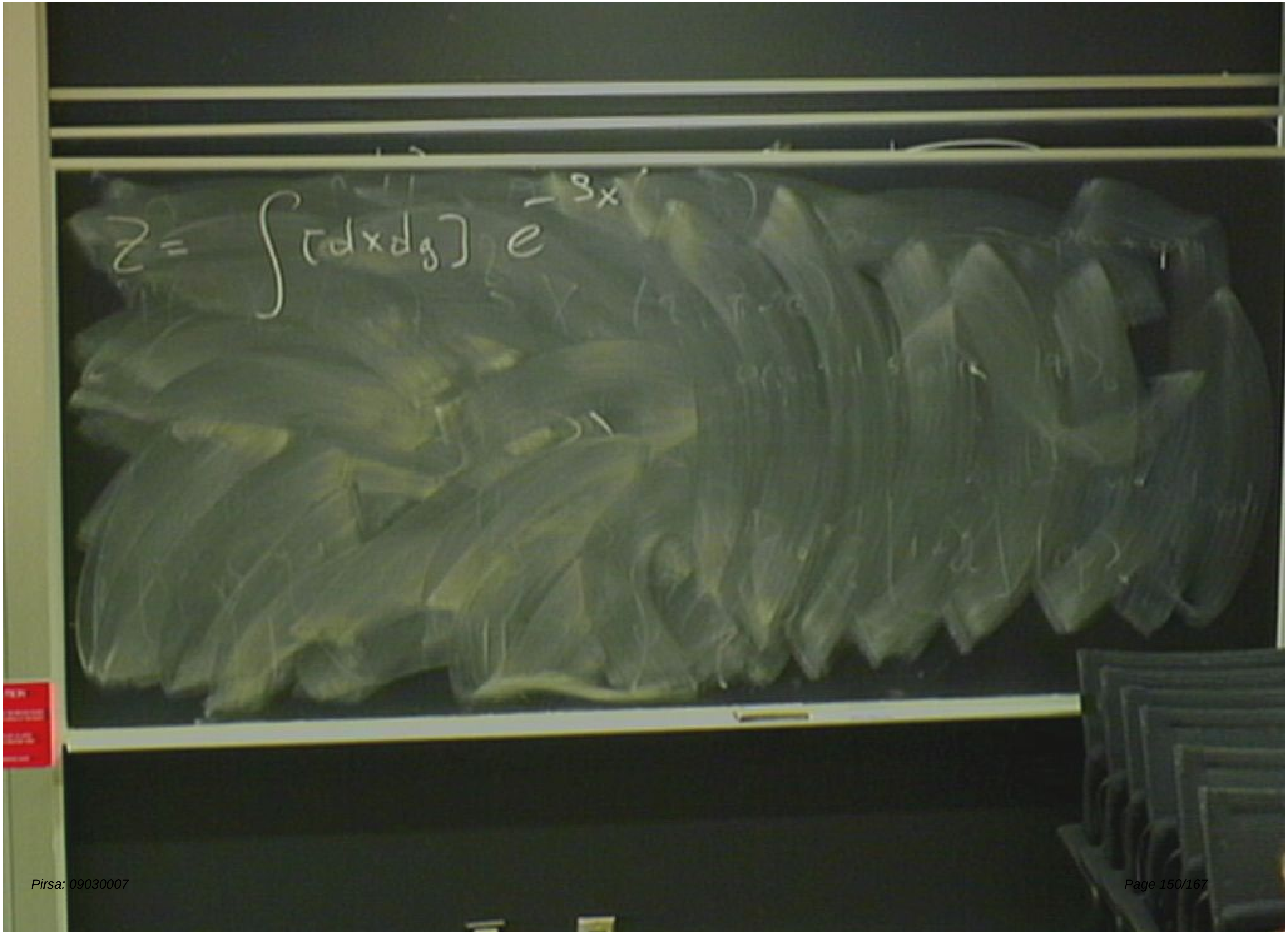

$e^{-\Delta\chi\lambda}$ = e^{λ} $\propto g_0^2$
g_0
g_0
$g_0 \propto e^{\lambda/2}$
$\Delta\chi = -1$

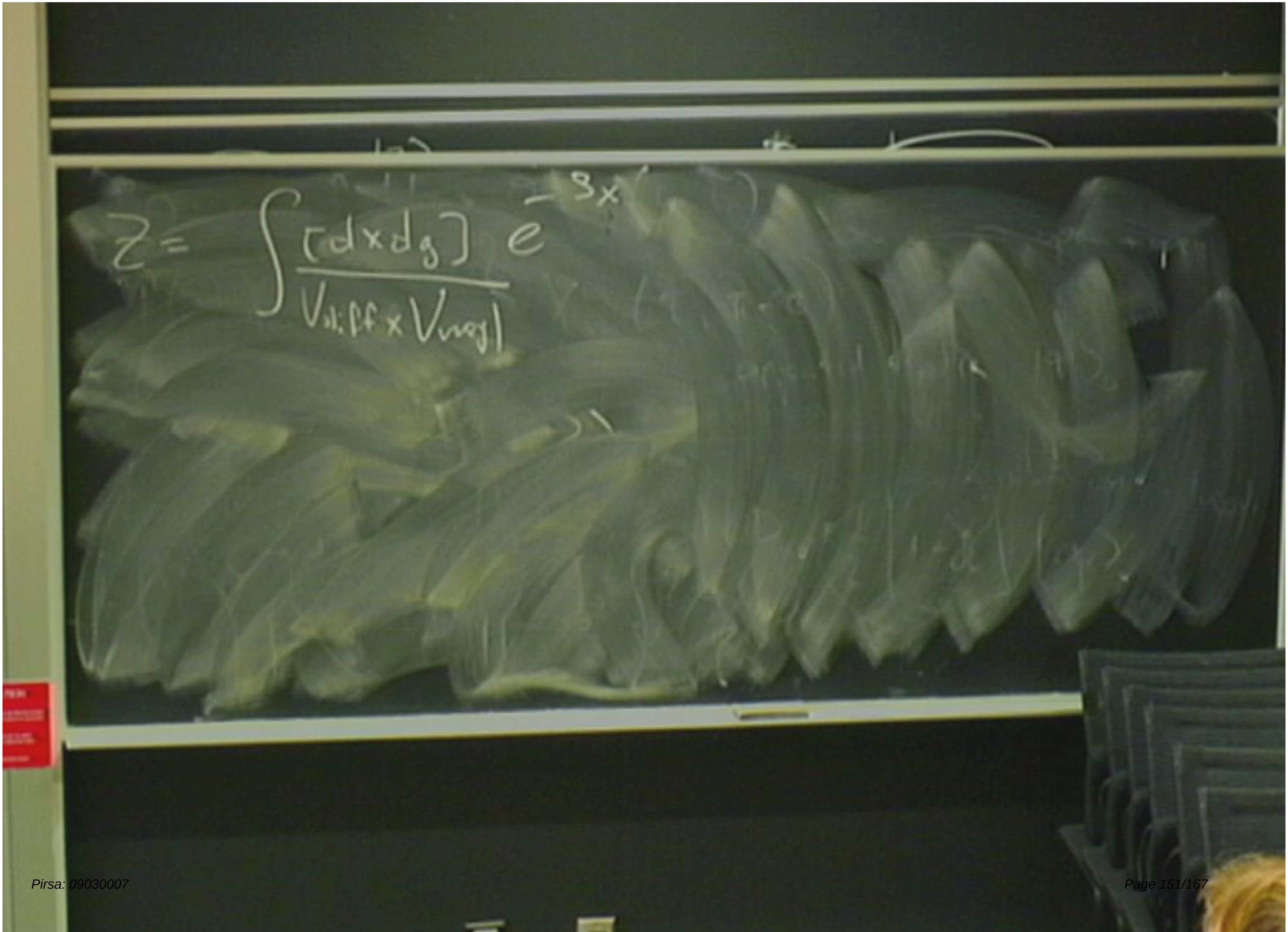

$e^{-\Delta\chi\lambda} = e^{\lambda} \propto g_0^2$
$g_0 \propto e^{\lambda/2}$
g_0
g_0
$\Delta\chi = -1$
$\Delta g = +1$

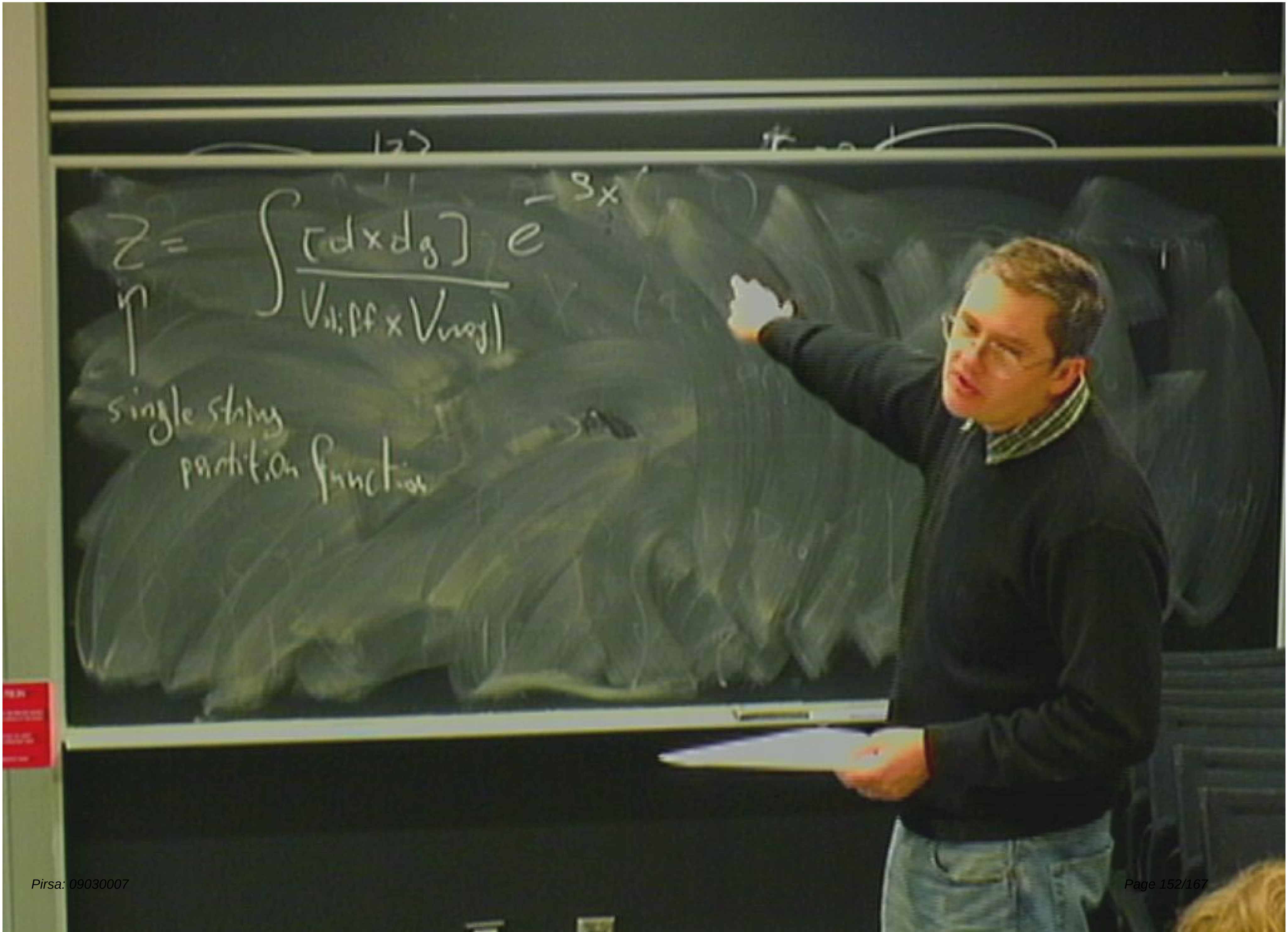

ΔX = -1
Δg = +1
ΔX = -2
g₀ ∝ e^{λ/2}
∝ g₀²

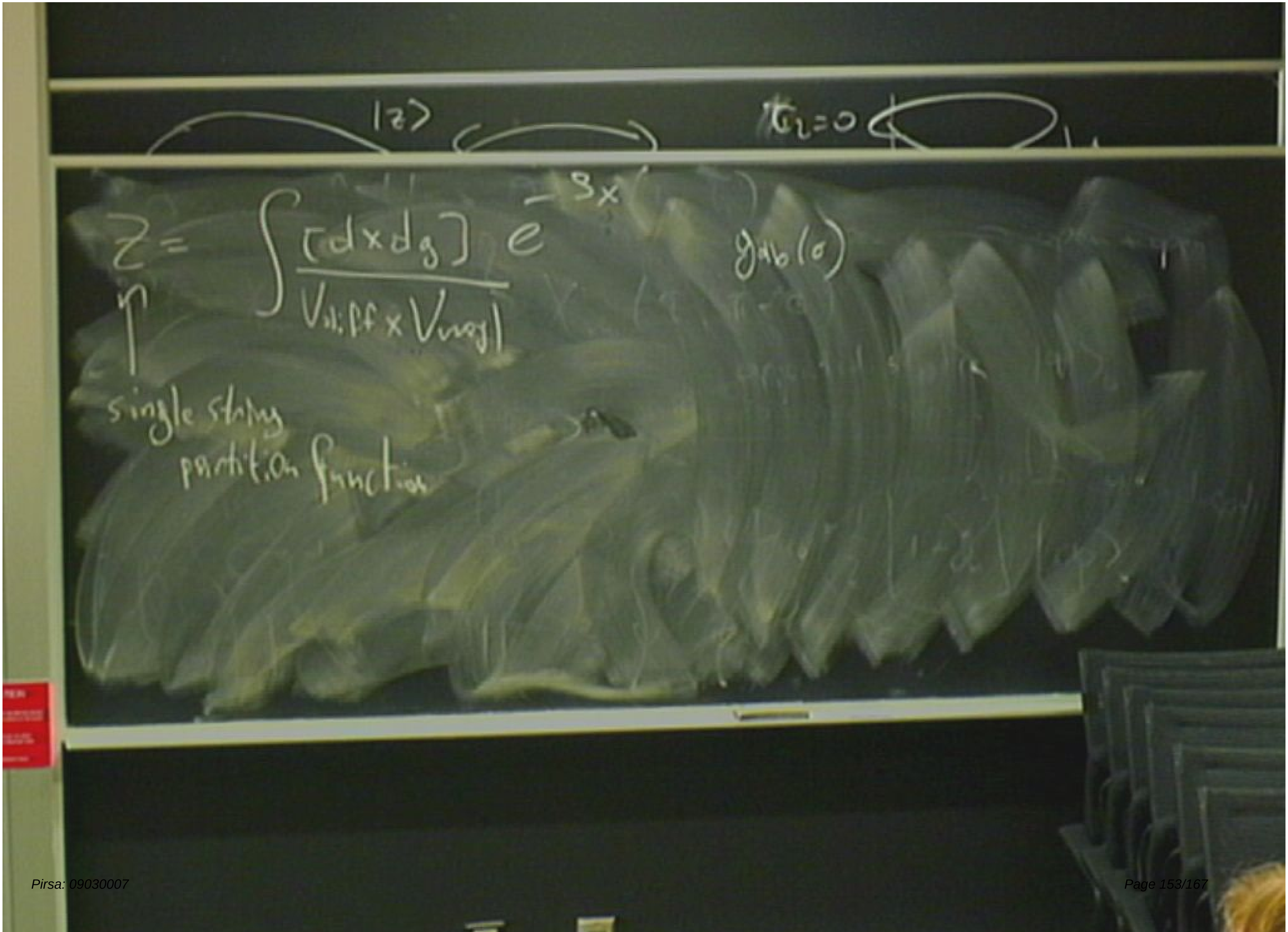

$e^{-\Delta\chi\,\lambda} = e^{\lambda} \propto g_0^2$
$g_0 \propto e^{\lambda/2}$
g_0
g_0
$\Delta\chi = -1$
$\Delta g = +1$
$\Delta\chi = -2$
g_c
g_c


e^{-ΔX λ} = e^λ ∝ g_o^2
g_o ∝ e^{λ/2}
e^{2λ} = g_c^2
g_c ∝ e^λ
ΔX = -1
Δg = +1
ΔX = -2
g_o
g_c


$Z = \int [dx\,dg]\, e^{-S_x}$


$Z = \int \frac{[dx\,dg]}{V_{diff} \times V_{weyl}} e^{-S_x}$


$Z = \int \frac{[dx\,dg]}{V_{diff} \times V_{weyl}} e^{-S_x}$
single string
partition function


|z⟩
τ₂=0
Z = ∫ [dx dg] / (Vdiff × Vweyl) e^{-S_x}
g_ab(σ)
single string
partition function

$|z\rangle$

$t_2 = 0$

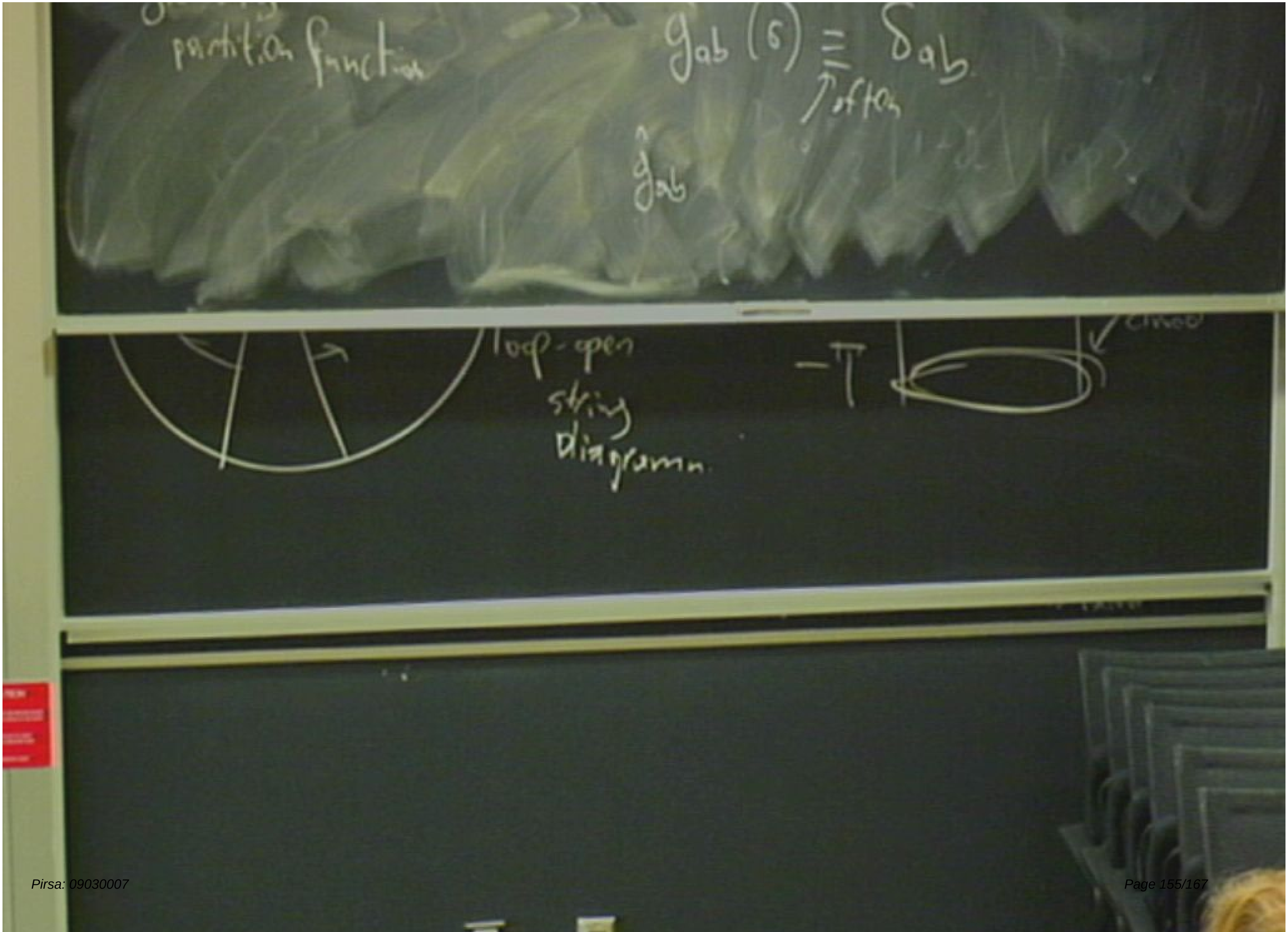
$$Z = \int \frac{[dx\, dg]}{V_{diff} \times V_{weyl}}\, e^{-S_x}$$

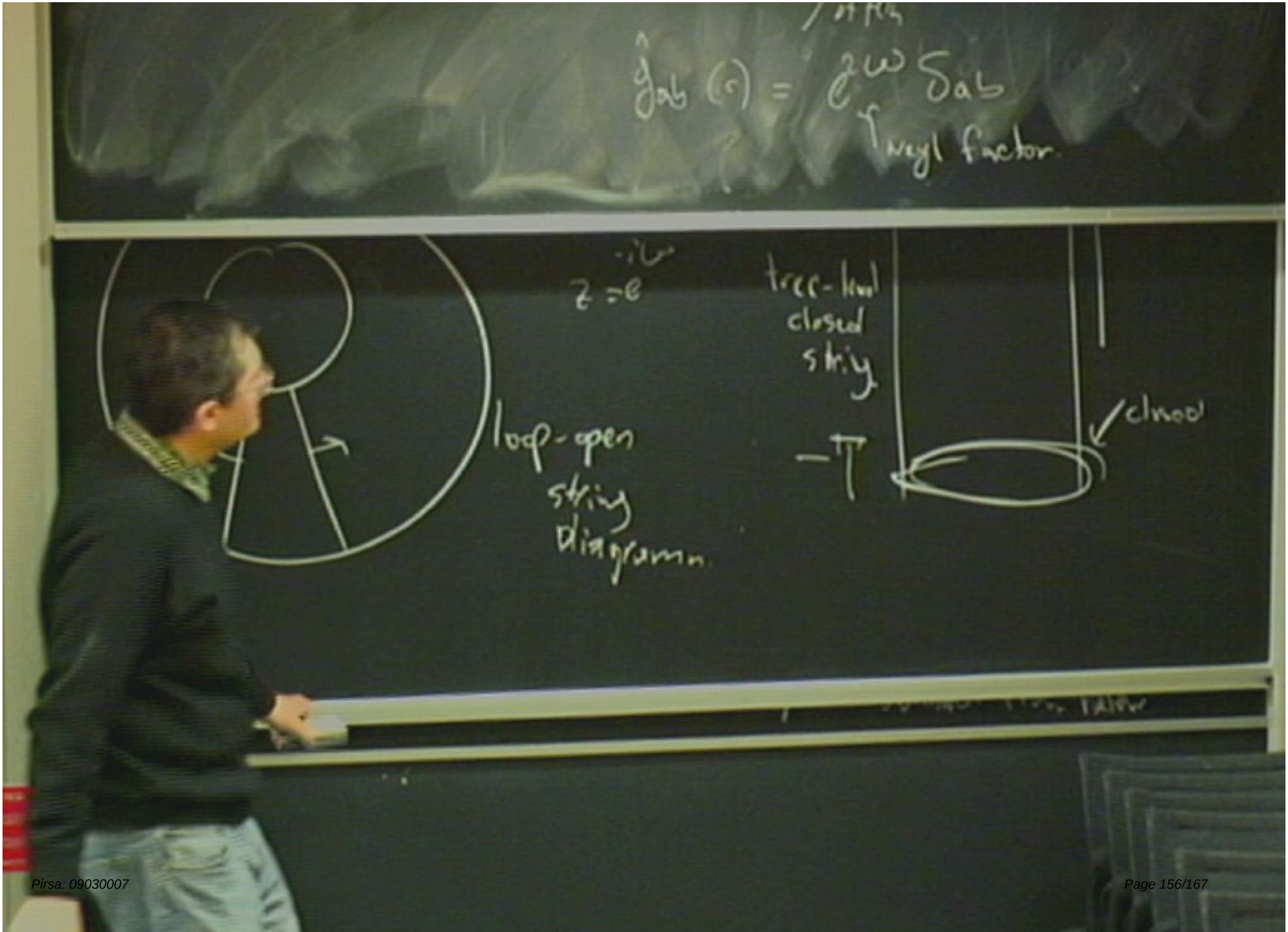
single string partition function

$g_{ab}(\sigma) \longrightarrow 3$ independent components

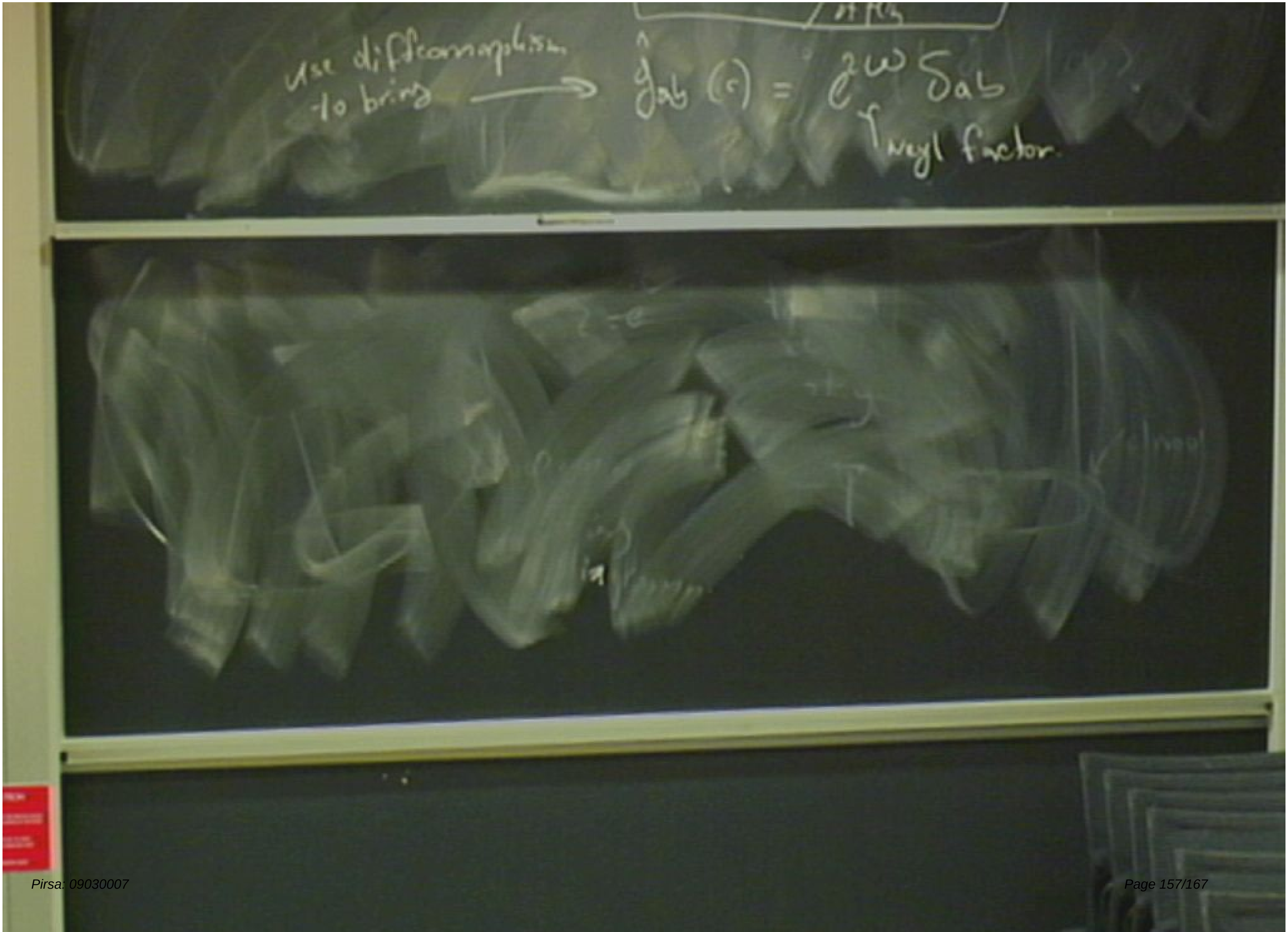
2 diffeos $+ 1$ weyl $= 3$

$$\hat{g}_{ab}(\sigma) \equiv \delta_{ab}$$


partition function
gab (σ) ≡ δab
often
ĝab
loop-open
string
diagramm
−π



$\hat{g}_{ab}(\sigma) = e^{2\omega}\delta_{ab}$
Weyl factor
$z = e^{-i\omega}$
tree-level closed string
loop-open string diagram
closed


use diffeomorphism
to bring
$\hat{g}_{ab}(\sigma) = e^{2\omega}\delta_{ab}$
Weyl factor

use diffeomorphism to bring $\longrightarrow$ $\hat{g}_{ab}(\sigma) = e^{2\omega}\delta_{ab}$

Weyl factor

Let's do Weyl transf $g_{ab} \to g'_{ab} = e^{2\omega} g_{ab}$

use diffeomorphism to bring $\longrightarrow$ $\hat{g}_{ab}(\sigma) = e^{2\omega}\delta_{ab}$

Weyl factor.

Let's do weyl transf $\quad g_{ab} \to g'_{ab} = e^{2\omega} g_{ab}$

$$\sqrt{g'}\,R' = \sqrt{g}\left[R - 2D^2\omega\right]$$

use diffeomorphism to bring $\longrightarrow$ $\hat{g}_{ab}(\sigma) = e^{2\omega} \delta_{ab}$

Weyl factor.

Let's do weyl transf $g_{ab} \to g'_{ab} = e^{2\omega} g_{ab}$

$$\sqrt{g'}\, R' = \sqrt{g}\left[R - 2D^2\omega\right]$$

0

$$\boxed{D^2\omega = \frac{1}{2}R}$$

use diffeomorphism to bring $\longrightarrow$ $\hat{g}_{ab}(\sigma) = e^{2\omega}\delta_{ab}$

Weyl factor

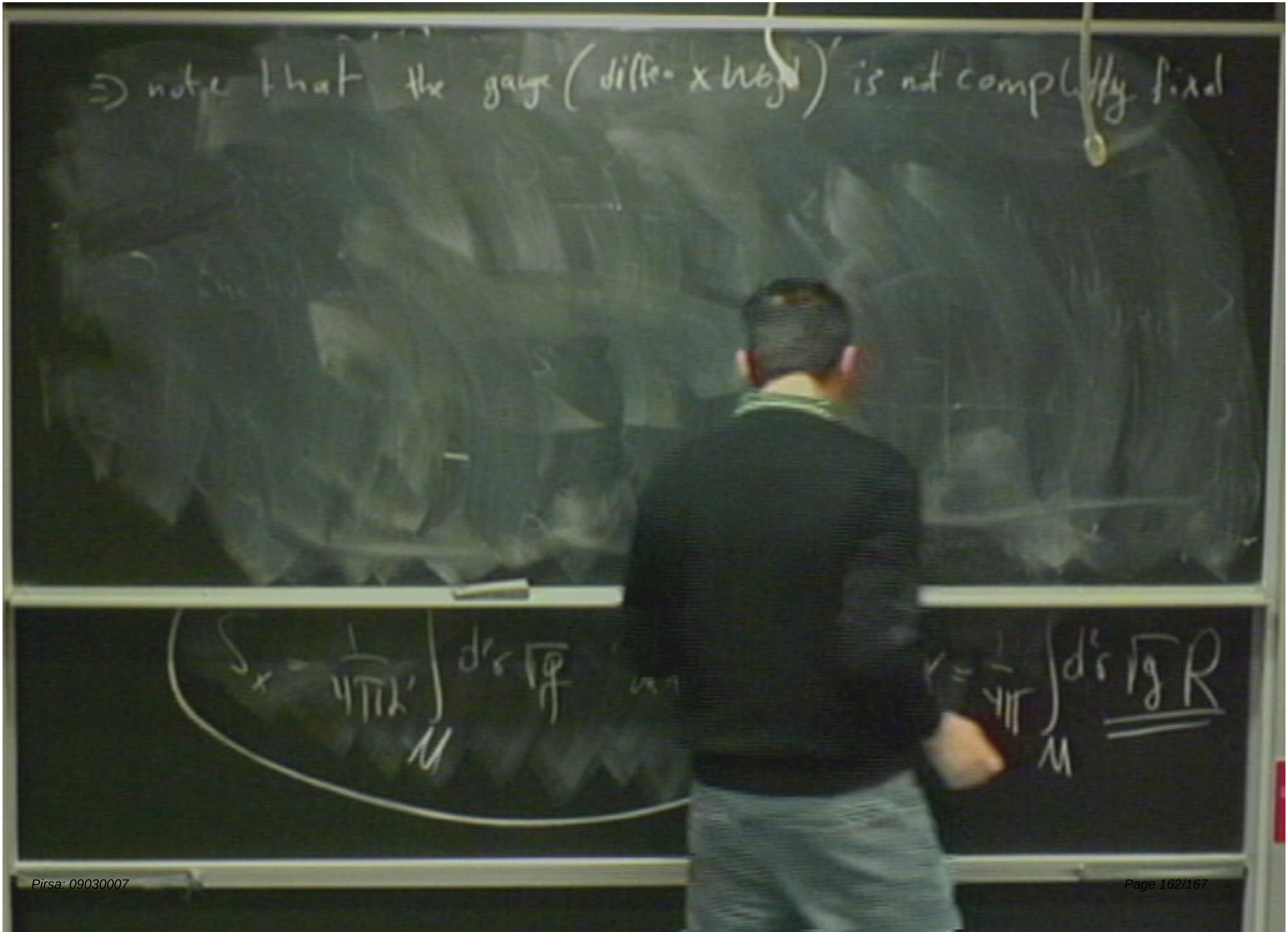
Let's do weyl transf $g_{ab} \to g'_{ab} = e^{2\omega} g_{ab}$

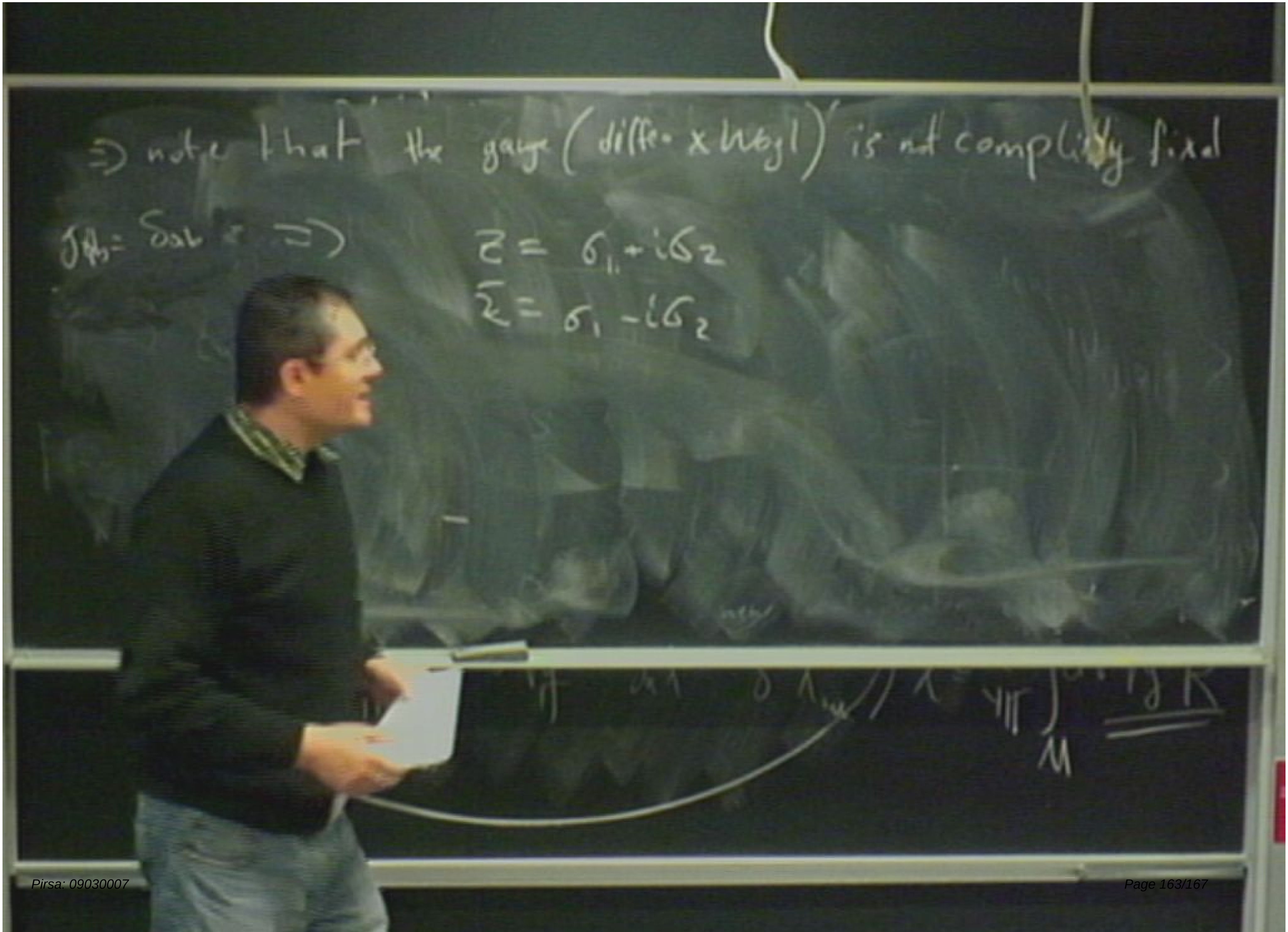
$$\sqrt{g'}\,R' = \sqrt{g}\left[R - 2D^2\omega\right]$$

0

$$\boxed{D^2\omega = \frac{1}{2}R}$$

$$R'_{abcd} \sim R'\left[g'_{ac}\,g'_{bd} - \right\}$$


⇒ note that the gauge (diffeo × Weyl) is not completely fixed
$S_X = \frac{1}{4\pi\lambda'}\int_M d^2\sigma\sqrt{g}$
$\frac{1}{4\pi}\int_M d^2\sigma\sqrt{g}\,R$


⇒ note that the gauge (diffeo x Weyl) is not completely fixed
z = σ1 + iσ2
z̄ = σ1 − iσ2

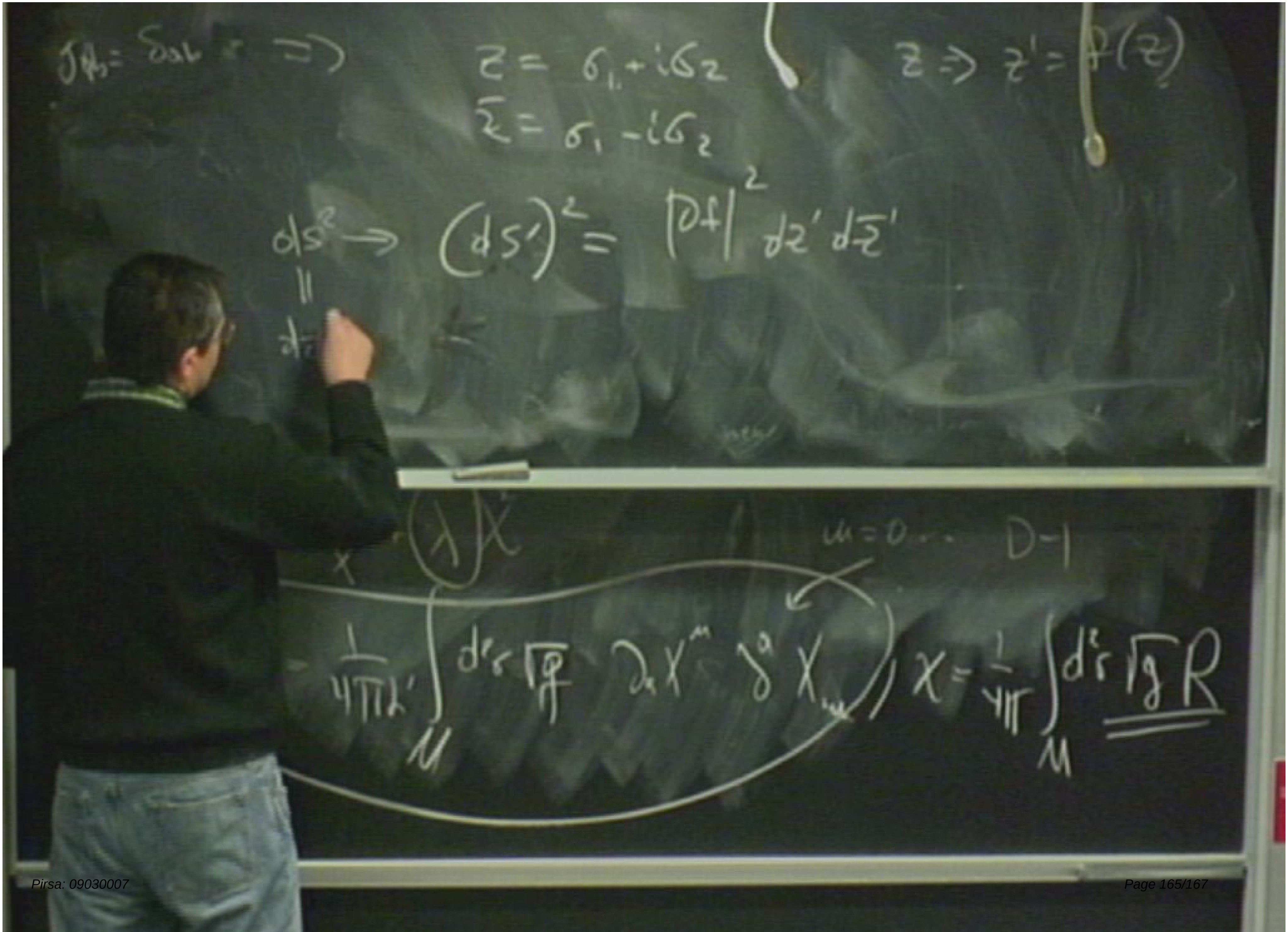
⇒ note that the gauge (diffeo × Weyl) is not completely fixed

$\gamma_{ab} = \delta_{ab} \Rightarrow$

$$z = \sigma_1 + i\sigma_2$$
$$\bar{z} = \sigma_1 - i\sigma_2$$

$$z \Rightarrow z' = f(z)$$

$\frac{1}{4\pi\lambda'}\int_M \qquad \chi = \frac{1}{4\pi}\int_M \sqrt{g}\,R$


z = σ1 + iσ2
z̄ = σ1 − iσ2
z ⇒ z' = f(z)
ds² → (ds')² = |∂f|² dz' dz̄'
μ = 0 ... D−1
χ = 1/4π ∫ d²σ √g R

$g_{ab} = \delta_{ab} \Rightarrow$

$z = \sigma_1 + i\sigma_2$

$\bar{z} = \sigma_1 - i\sigma_2$

$z \Rightarrow z' = f(z)$

$ds^2 \rightarrow (ds')^2 = e^{2\omega} |\partial f|^{-2} dz' d\bar{z}'$

$\|$

$2 dz d\bar{z}$

$z \to f(z)$

$\omega = \ln |\partial f|$

$S = S_X + \lambda \chi$

$\mu = 0 \ldots D-1$

$S_X = \frac{1}{4\pi\alpha'} \int_M d^2\sigma \sqrt{g}\, \partial_a X^\mu \partial^a X_\mu$

$\chi = \frac{1}{4\pi} \int_M d^2\sigma \sqrt{g}\, R$