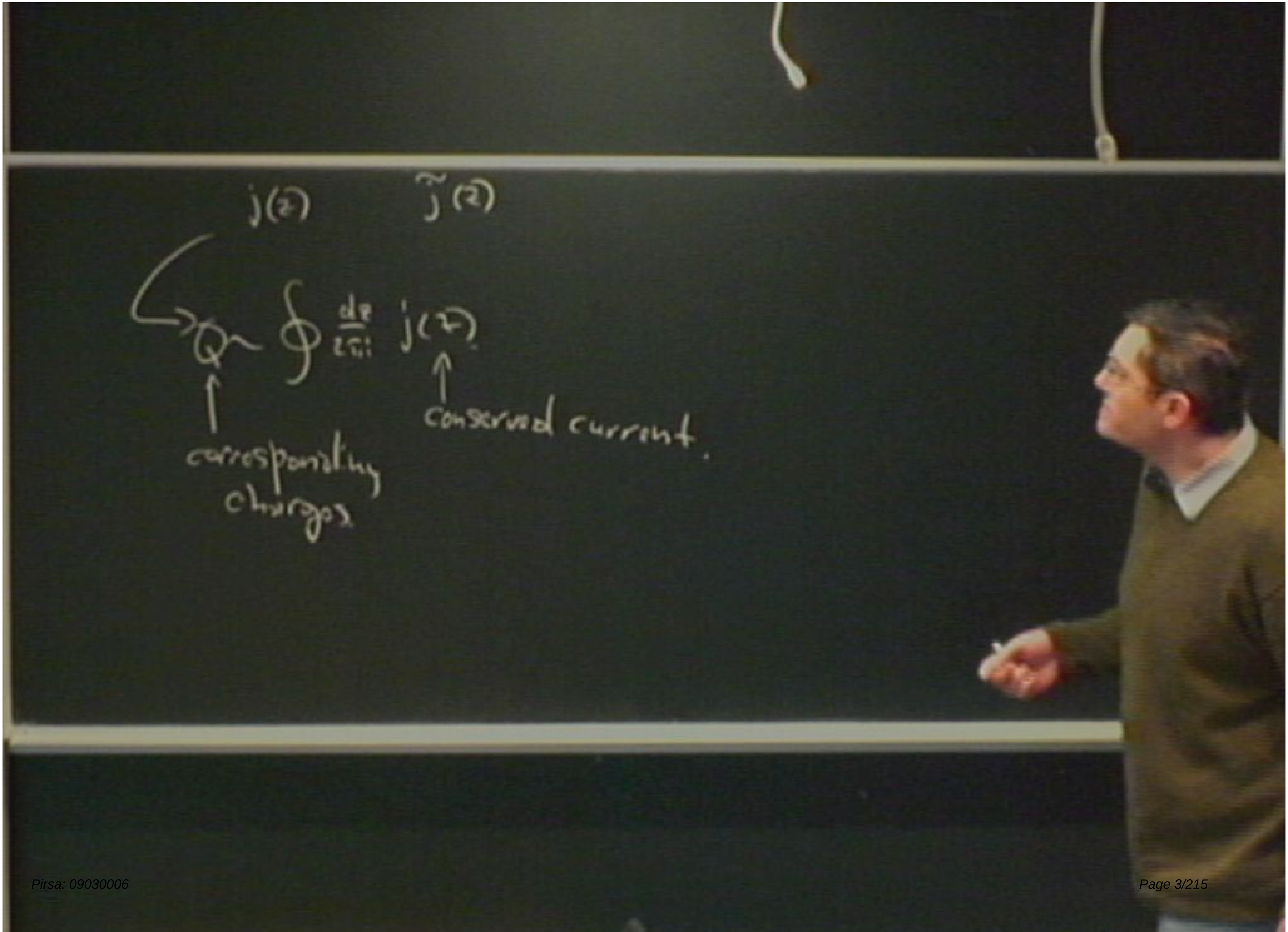
Title: Introduction to the Bosonic String

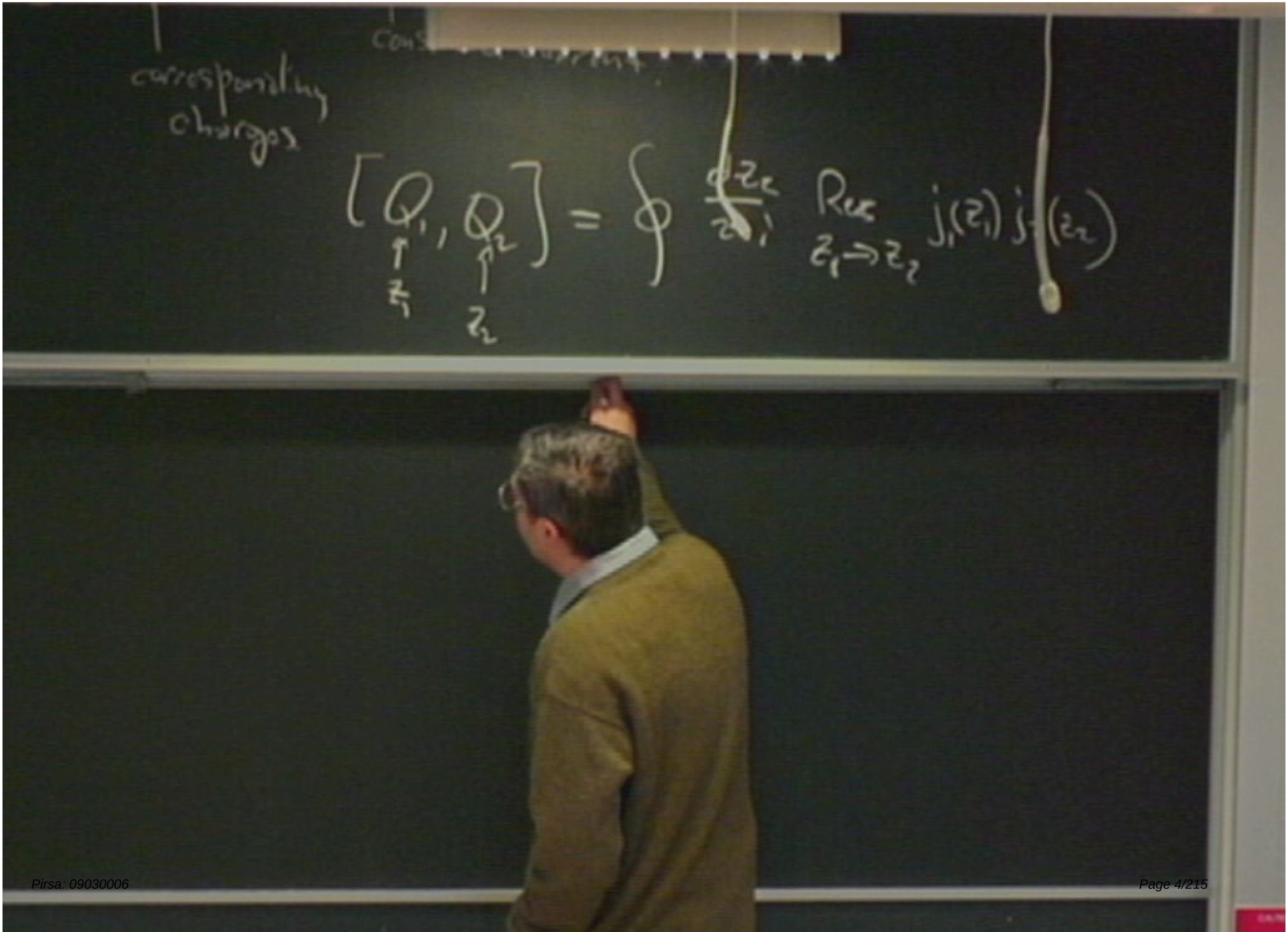
Date: Mar 06, 2009 10:00 AM

URL: http://pirsa.org/09030006

Abstract: This course provides a thorough introduction to the bosonic string based on the Polyakov path integral and conformal field theory. We introduce central ideas of string theory, the tools of conformal field theory, the Polyakov path integral, and the covariant quantization of the string. We discuss string interactions and cover the tree-level and one loop amplitudes. More advanced topics such as T-duality and D-branes will be taught as part of the course. The course is geared for M.Sc. and Ph.D. students enrolled in Collaborative Ph.D. Program in Theoretical Physics. Required previous course work: Quantum Field Theory (AM516 or equivalent). The course evaluation will be based on regular problem sets that will be handed in during the term. The primary text is the book: 'String theory. Vol. 1: An introduction to the bosonic string. J. Polchinski (Santa Barbara, KITP) . 1998. 402pp. Cambridge, UK: Univ. Pr. (1998) 402 p.' All interested students should contact Alex Buchel at abuchel@uwo.ca as soon as possible.

$j(z)$ $\tilde{j}(z)$


j(z)   j̃(z)
Q ~ ∮ dz/2πi j(z)
corresponding charges
conserved current.


corresponding
charges
$[Q_1, Q_2] = \oint \frac{dz_2}{2\pi i} \operatorname{Res}_{z_1 \to z_2} j_1(z_1)\, j_2(z_2)$
z_1
z_2

$$[Q_1, Q_2] = \oint \frac{dz_2}{2\pi i} \operatorname*{Res}_{z_1 \to z_2} j_1(z_1)\, j_2(z_2)$$

$$\underset{z_1}{\uparrow} \quad \underset{z_2}{\uparrow}$$

$$j_m = z^{m+1} T(z)$$

$$[Q_1, Q_2] = \oint \frac{dz_2}{2\pi i} \operatorname*{Res}_{z_1 \to z_2} j_1(z_1) j_2(z_2)$$

$$Q_1 \leftarrow z_1, \quad Q_2 \leftarrow z_2$$

$$j_m = z^{m+1} T(z)$$

$$L_m = \int \frac{dz}{2\pi i} z^{m+1} T(z)$$

$$[Q_1, Q_2] = \oint \frac{dz_2}{2\pi i} \operatorname*{Res}_{z_1 \to z_2} j_1(z_1) j_2(z_2)$$

$Q_1 \uparrow z_1$, $Q_2 \uparrow z_2$

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$$L_m = \oint \frac{dz}{2\pi i} z^{m+1} T(z)$$

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$\Rightarrow$

$$[Q_1, Q_2] = \oint \frac{dz_2}{2\pi i} \operatorname*{Res}_{z_1 \to z_2} j_1(z_1) j_2(z_2)$$

$Q_1 \leftarrow z_1$, $Q_2 \leftarrow z_2$

$$j_m = z^{m+1} T(z)$$

$$L_m = \int \frac{dz}{2\pi i} z^{m+1} T(z)$$

$\Rightarrow$ Suppose we have a primary field

$$[Q_1, Q_2] = \oint \frac{dz_2}{2\pi i} \operatorname*{Res}_{z_1 \to z_2} j_1(z_1) j_2(z_2)$$

$$\underset{z_1}{\uparrow} \quad \underset{z_2}{\uparrow}$$

$$j_m = z^{m+1} T(z)$$

$$L_m = \int \frac{dz}{2\pi i} z^{m+1} T(z)$$

$\Rightarrow$ Suppose we have a primary field $\mathcal{O}(z)$

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$\Rightarrow$ Suppose we have a primary field $\mathcal{O}(z)$ $(h, 0)$

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$Q_1 \leftarrow z_1$, $Q_2 \leftarrow z_2$

$$j_m = z^{m+1} T(z)$$

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$\Rightarrow$ Suppose we have a primary field $O(z)$ $(h, 0)$

$$O = \sum_{m=-\infty}^{+\infty}$$

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$\Rightarrow$ Suppose we have a primary field $O(z)$ $(h, 0)$

$$O = \sum_{m=-\infty}^{+\infty} \frac{O_m}{z^{m+h}}$$

$$[Q_1, Q_2] = \oint \frac{dz_2}{2\pi i} \operatorname*{Res}_{z_1 \to z_2} j_1(z_1) j_2(z_2)$$

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$$j_m = z^{m+1} T(z)$$

$$L_m = \int \frac{dz}{2\pi i} z^{m+1} T(z)$$

$\Rightarrow$ Suppose we have a primary field $\mathcal{O}(z)$ $(h, 0)$

$$\mathcal{O} = \sum_{m=-\infty}^{+\infty} \frac{\mathcal{O}_m}{z^{m+h}}$$

"$\mathcal{O}_m$ behave as charges"

$$[L_m, O_n] = \left((h-1)m - n\right) O_{m+n}$$

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$\Rightarrow$ look @ X - CFT

$$[L_m, O_n] = \left((h-1)m - n\right) O_{m+n}$$

$\Rightarrow$ look @ X - CFT

$\Rightarrow$ we talk about isomorphisms between operators and states in a CFT

Mode expansion for X-CFT

Mode expansion for X-CFT

∂X^{μ}

Mode expansion for X-CFT

$\underbrace{\partial X^{\mu}}$

is holomorphic

(1, 0)

Mode expansion for X-CFT

$$\underbrace{\partial X^\mu}_{\text{is holomorphic } (1,0)} = -i\left(\frac{\alpha'}{2}\right)^{1/2}$$

Mode expansion for X-CFT

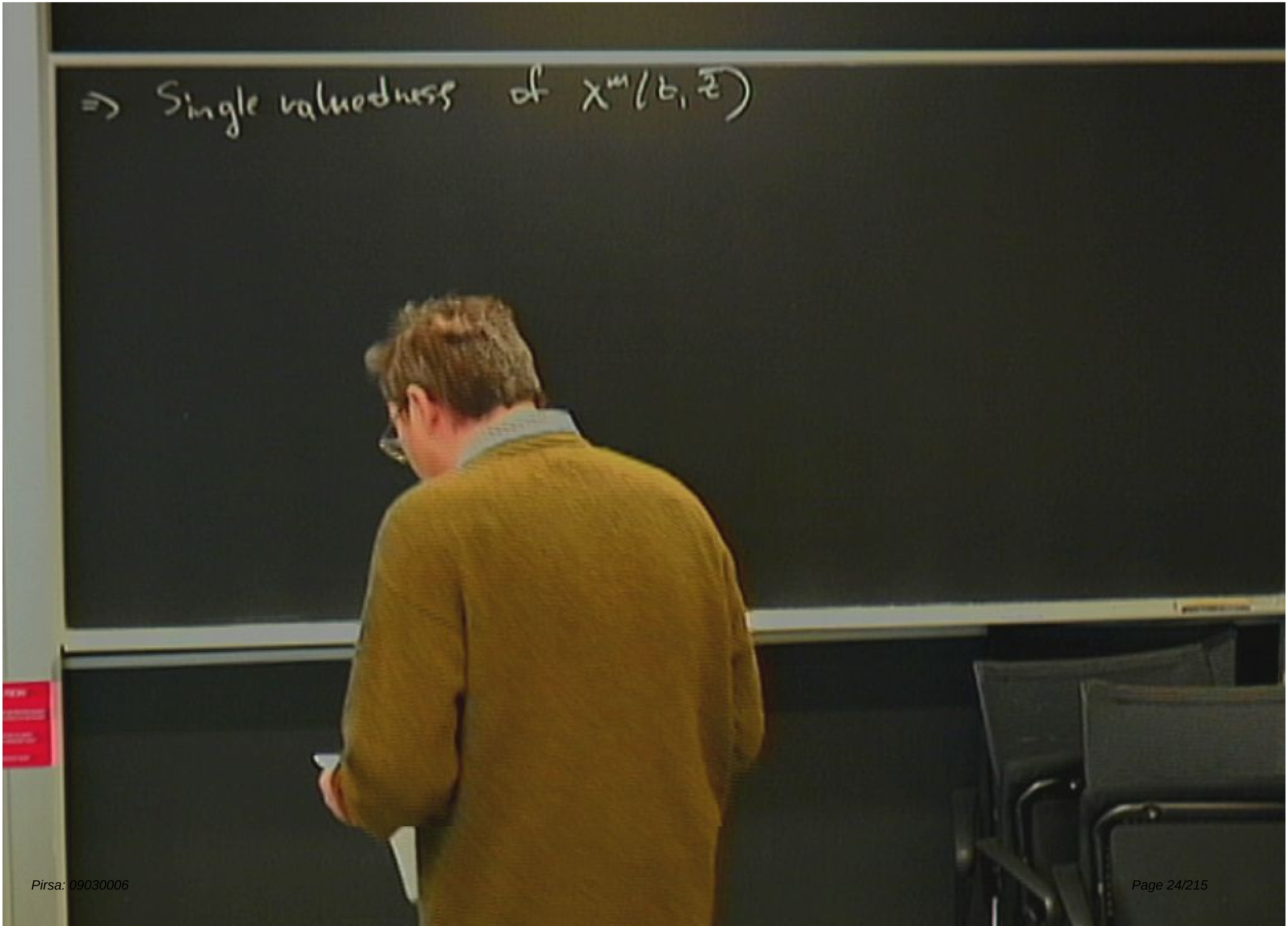
$$\underbrace{\partial X^{\mu}}_{\substack{\text{is holomorphic} \\ (1,0)}} = -i\left(\frac{\alpha'}{2}\right)^{1/2} \sum_{m=-\infty}^{+\infty} \frac{\alpha_m^{\mu}}{z^{m+1}}$$

Mode expansion for X-CFT

$$\underbrace{\partial X^{\mu}}_{\text{is holomorphic}} = -i\left(\frac{\alpha'}{2}\right)^{1/2} \sum_{m=-\infty}^{+\infty} \frac{\alpha_m^{\mu}}{z^{m+1}}$$

(1, 0)

$$\bar{\partial} X^{\mu} = -i\left(\frac{\alpha'}{2}\right)^{1/2} \sum_{m=-\infty}^{+\infty} \frac{\tilde{\alpha}_m^{\mu}}{\bar{z}^{m+1}}$$


$\Rightarrow$ Single valuedness of $X^m(z,\bar{z})$

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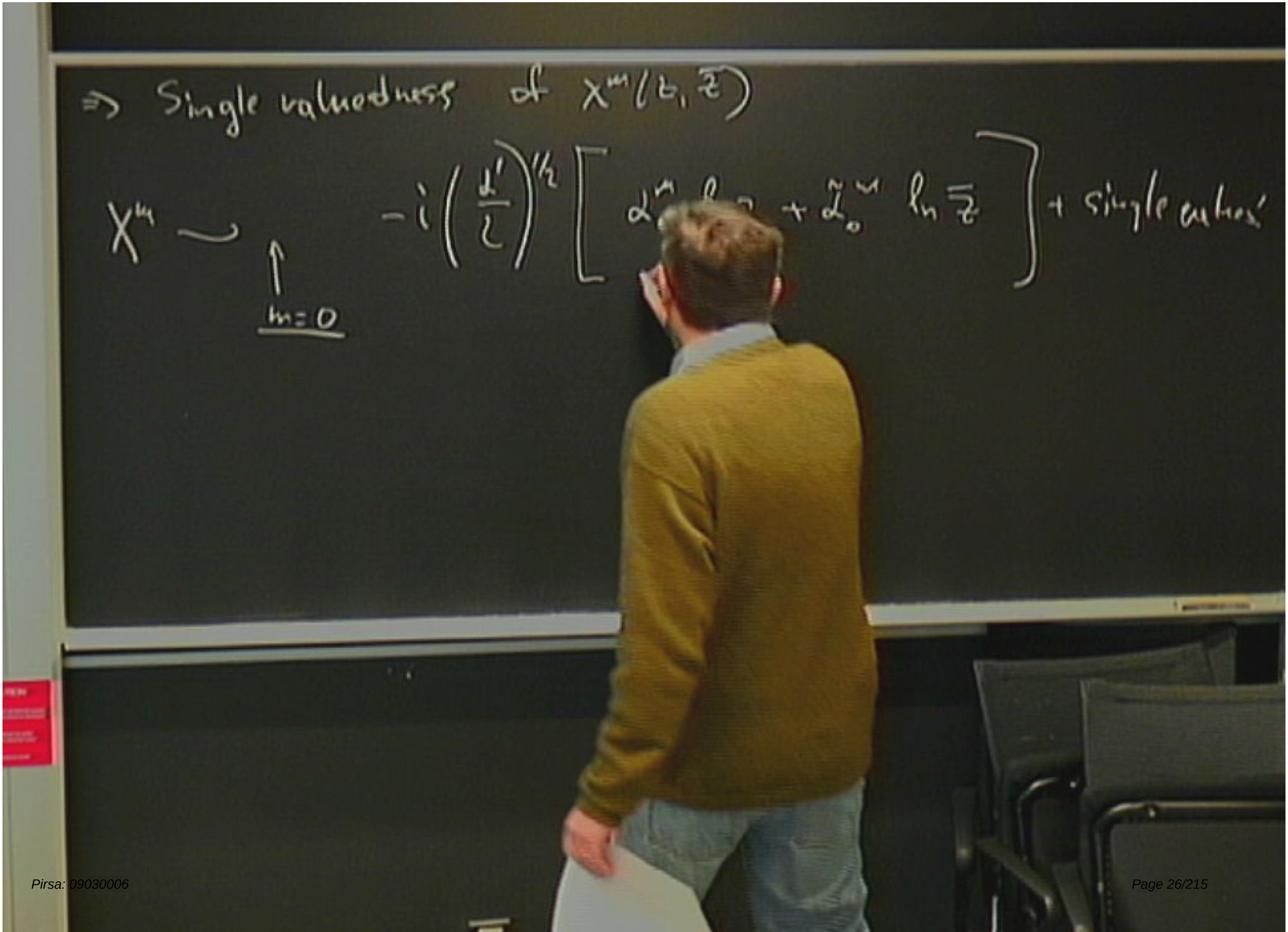
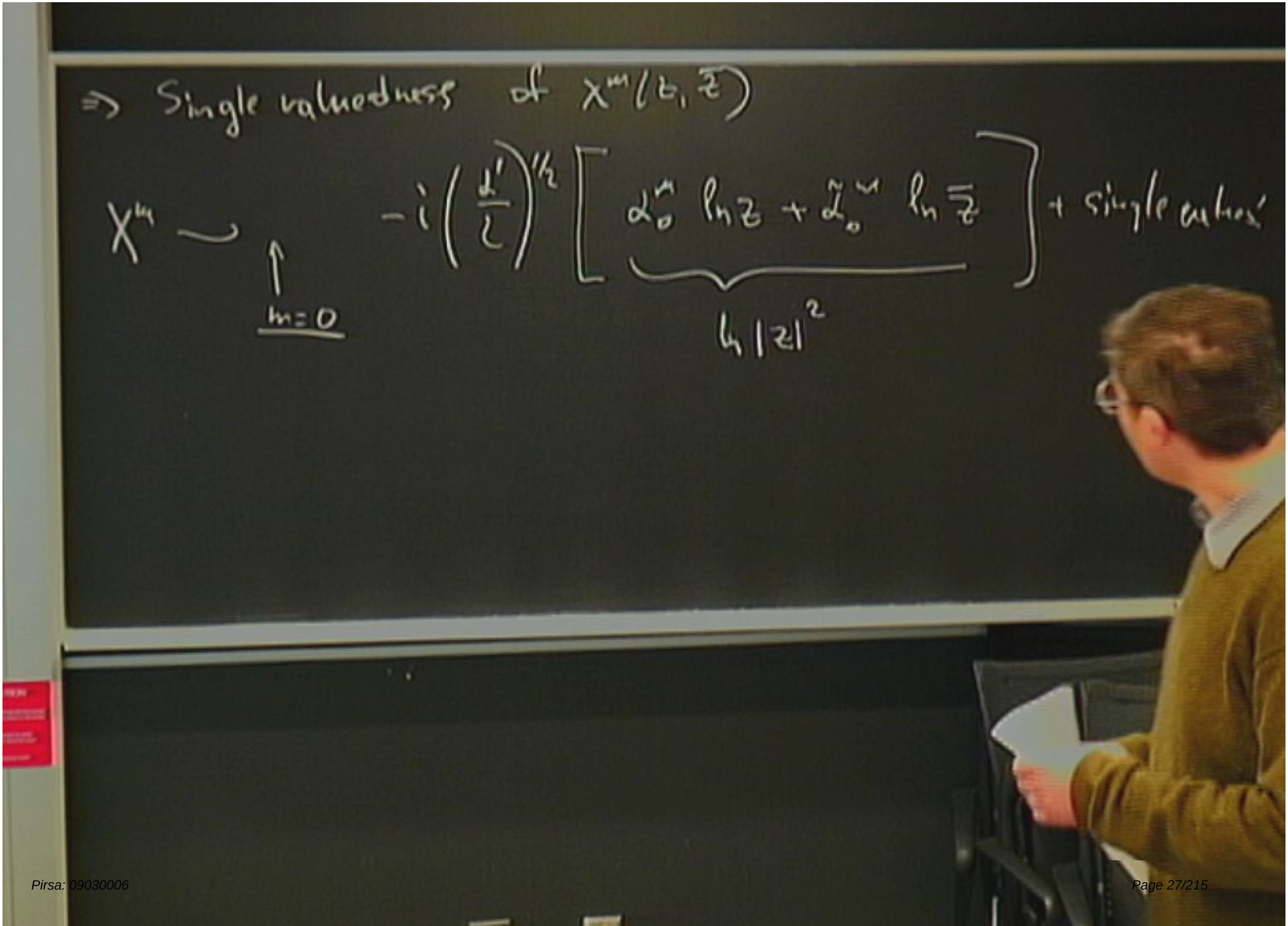
$X^m \sim$


⇒ Single valuedness of $X^\mu(z,\bar{z})$
$X^\mu \sim -i\left(\frac{\alpha'}{2}\right)^{1/2}\left[\alpha_0^\mu \ln z + \tilde{\alpha}_0^\mu \ln \bar{z}\right]$ + single valued
m=0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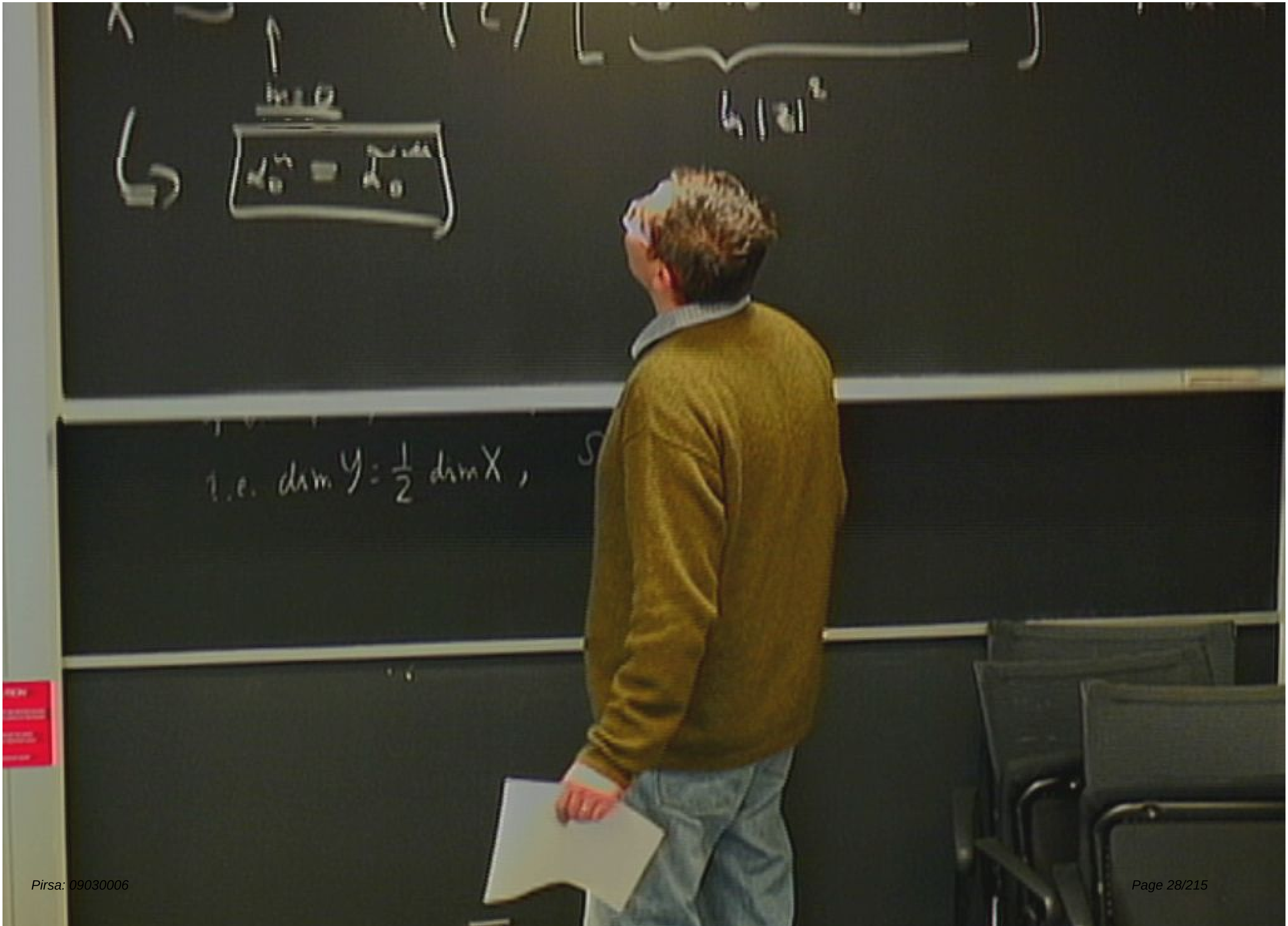
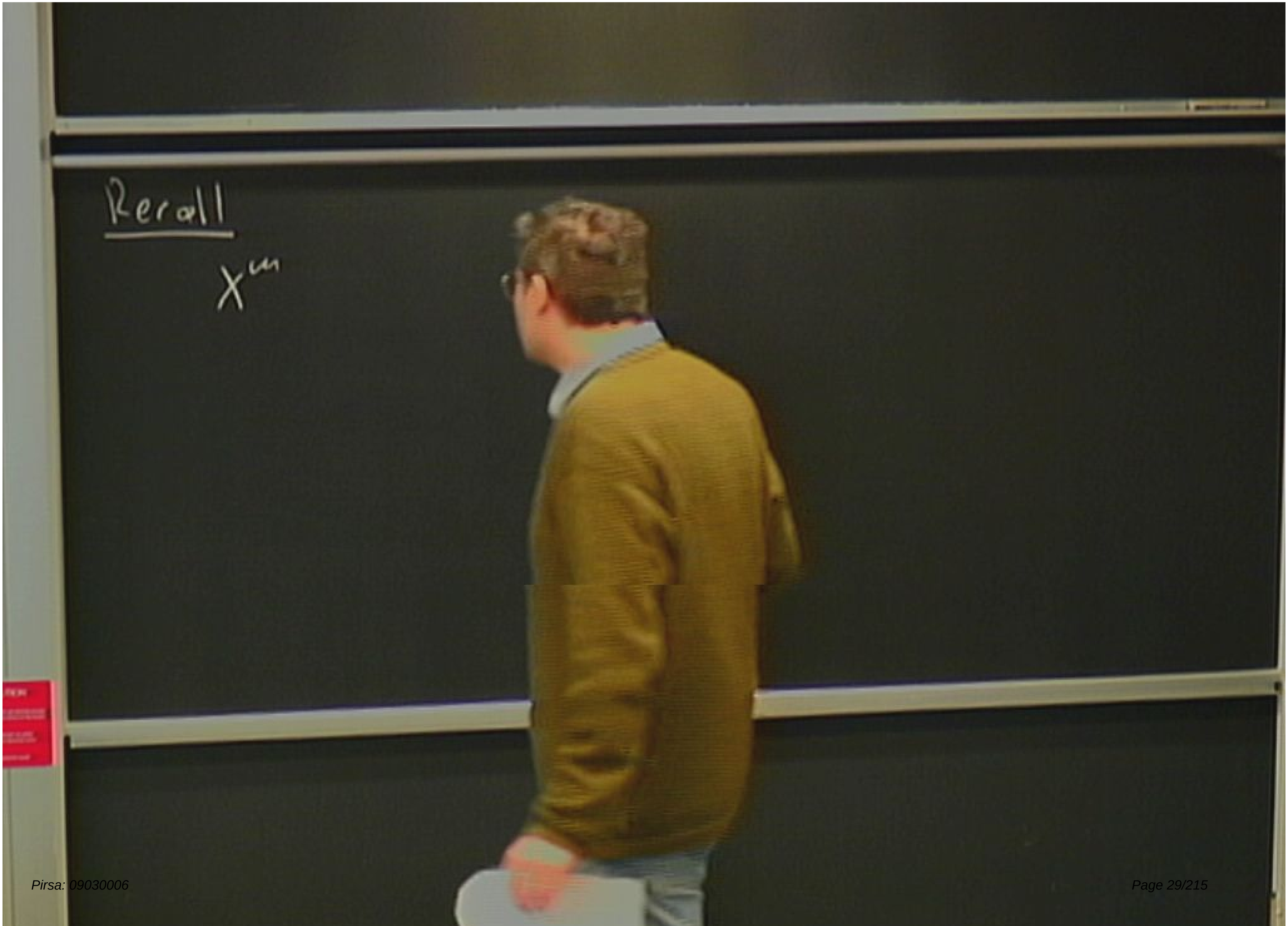
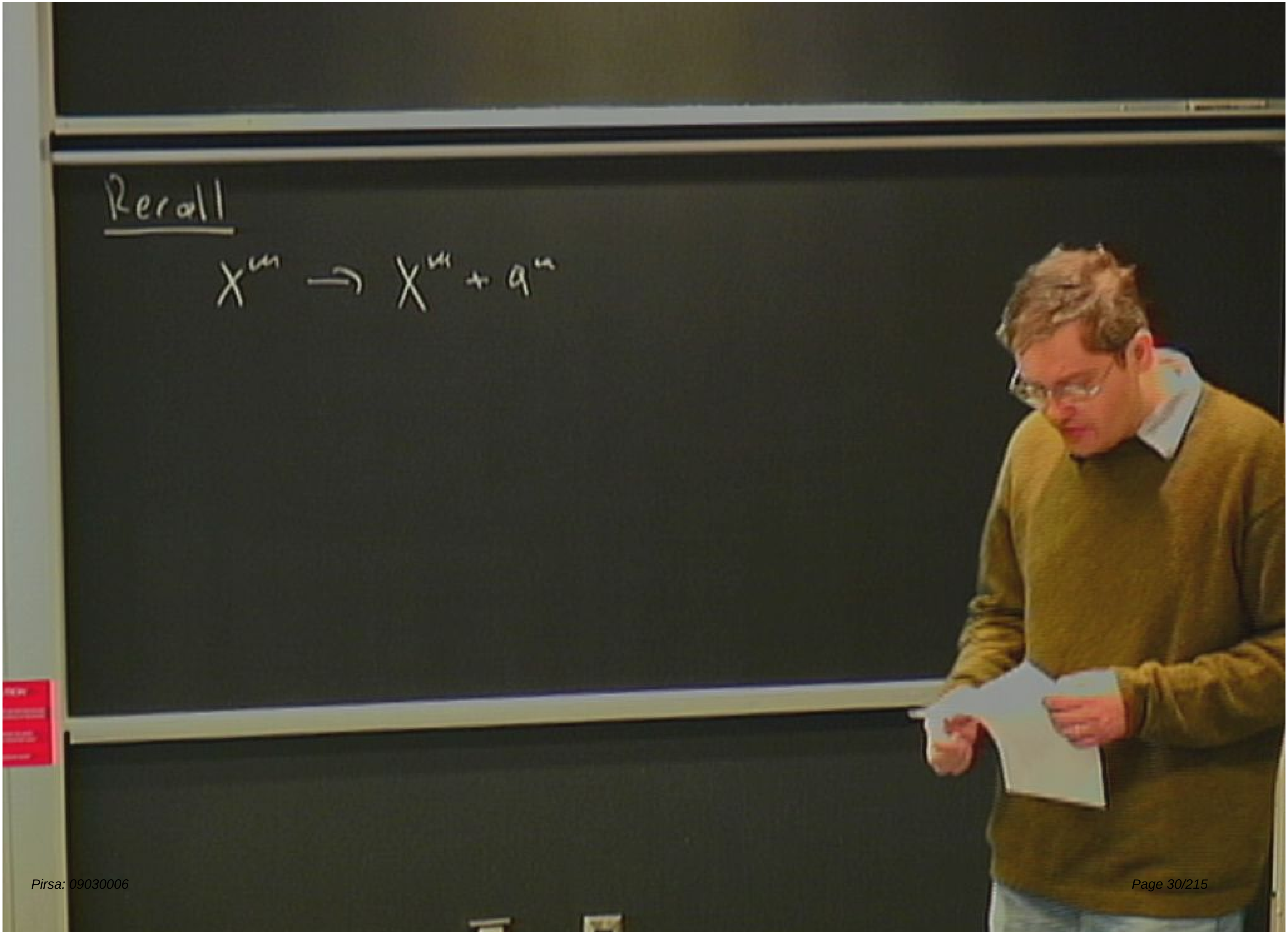
$m=0$

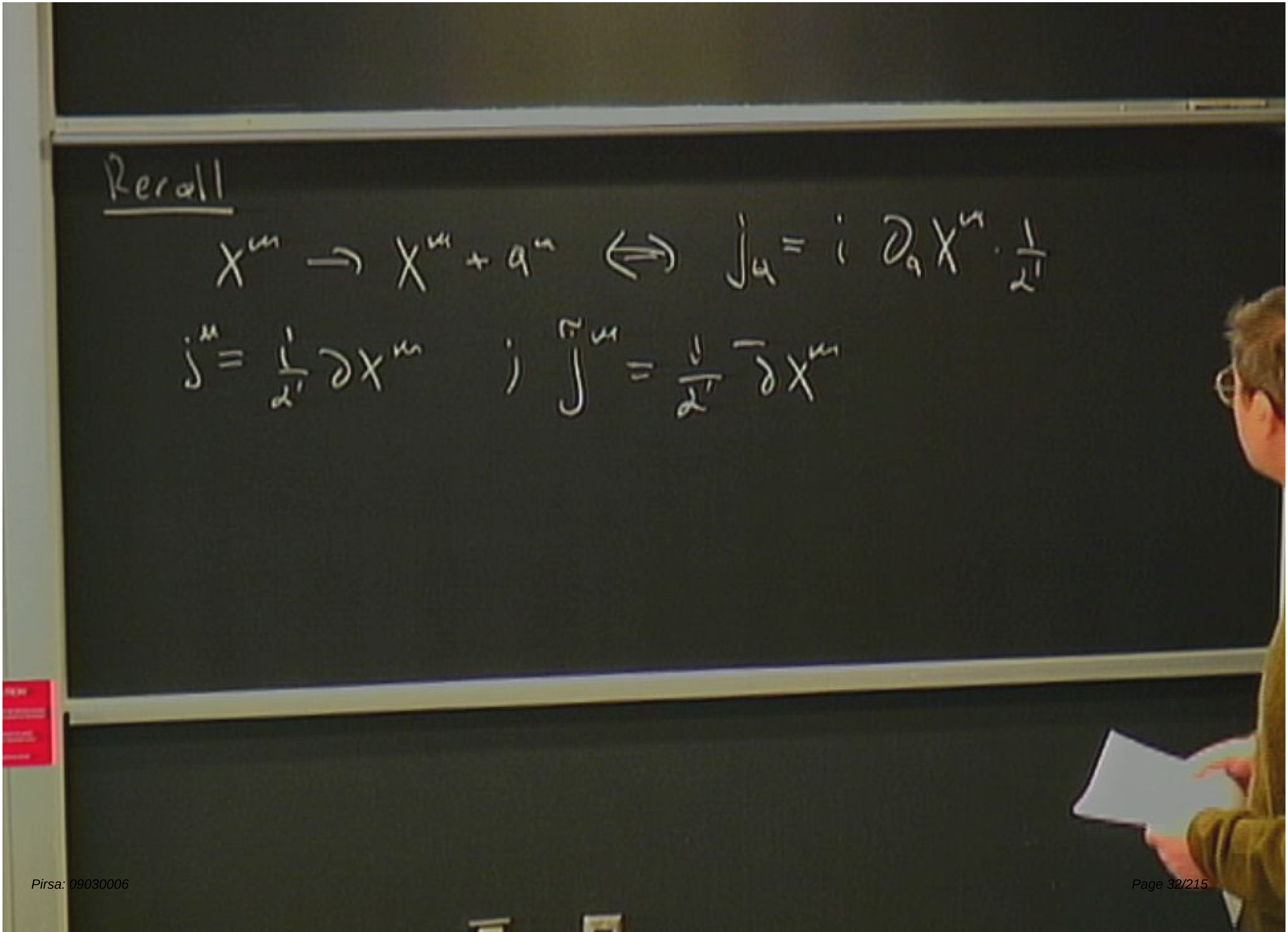

i.e. dim Y = 1/2 dim X ,


Recall
X^{m}


Recall
$X^\mu \to X^\mu + a^\mu$

Recall

$$X^{\mu} \to X^{\mu} + a^{\mu} \iff j_a = i\,\partial_a X^{\mu} \cdot \frac{1}{\alpha'}$$


Recall
$X^\mu \to X^\mu + a^\mu \iff j_a = i\,\partial_a X^\mu \cdot \frac{1}{\alpha'}$
$j^\mu = \frac{i}{\alpha'}\partial X^\mu \quad ; \quad \tilde{j}^\mu = \frac{i}{\alpha'}\bar{\partial} X^\mu$

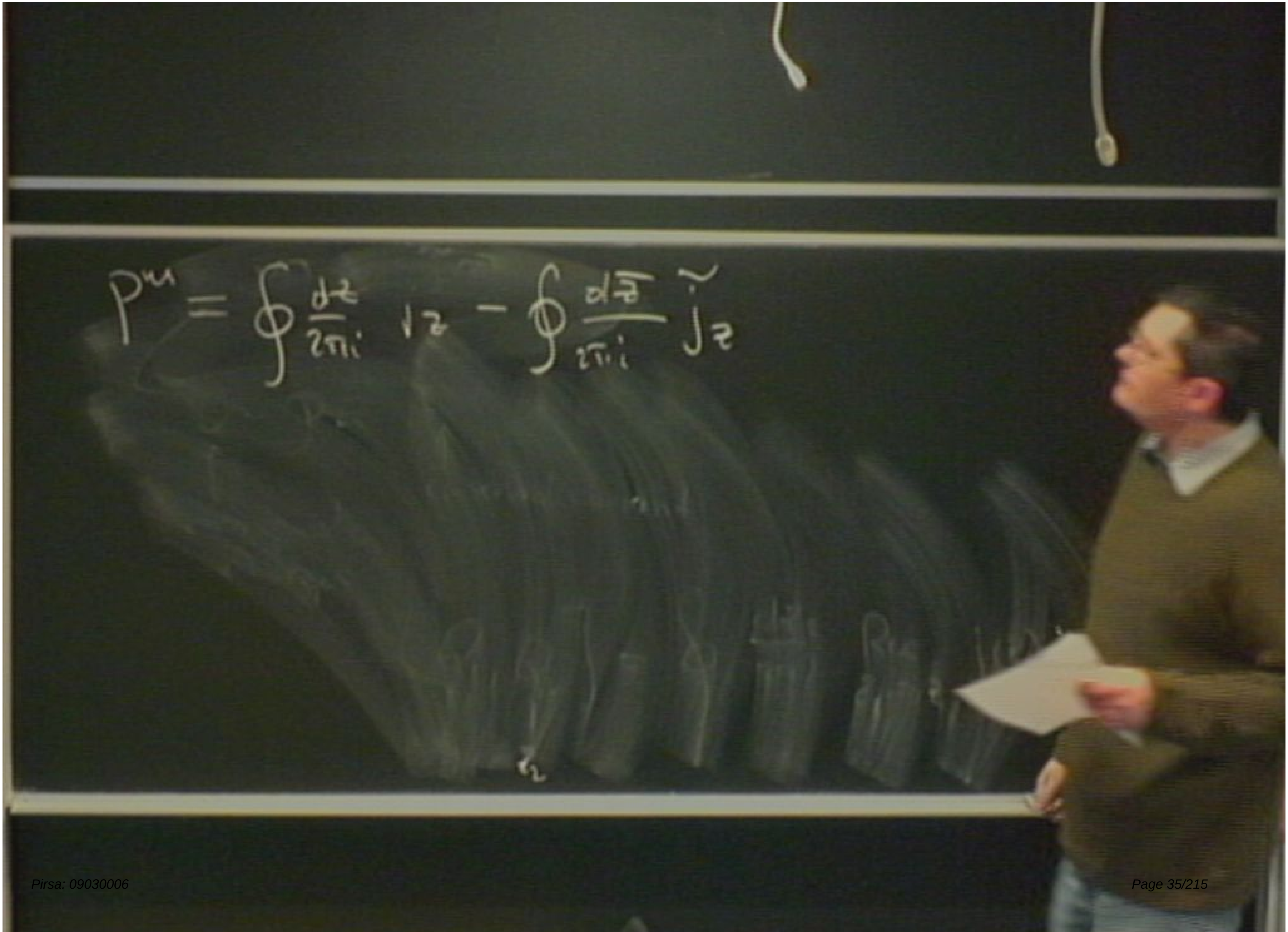
Recall

$$X^\mu \to X^\mu + a^\mu \iff j_a = i\,\partial_a X^\mu \cdot \frac{1}{\alpha'}$$

$$j^\mu = \frac{i}{\alpha'}\partial X^\mu \quad ; \quad \tilde{j}^\mu = \frac{i}{\alpha'}\bar{\partial} X^\mu$$

a conserved charge corresponding to j_a^μ must be p^μ

$$p^\mu = \oint \frac{dz}{2\pi i}$$


$p^\mu = \oint \frac{dz}{2\pi i} j_z - \oint \frac{d\bar{z}}{2\pi i} \tilde{j}_{\bar{z}}$

$$p^{\mu} = \oint \frac{dz}{2\pi i}\, j_z - \oint \frac{d\bar{z}}{2\pi i}\, \tilde{j}_{\bar{z}} = \left(\frac{1}{2\alpha'}\right)^{1/2} \left[\, \alpha_0^{\mu}\right.$$

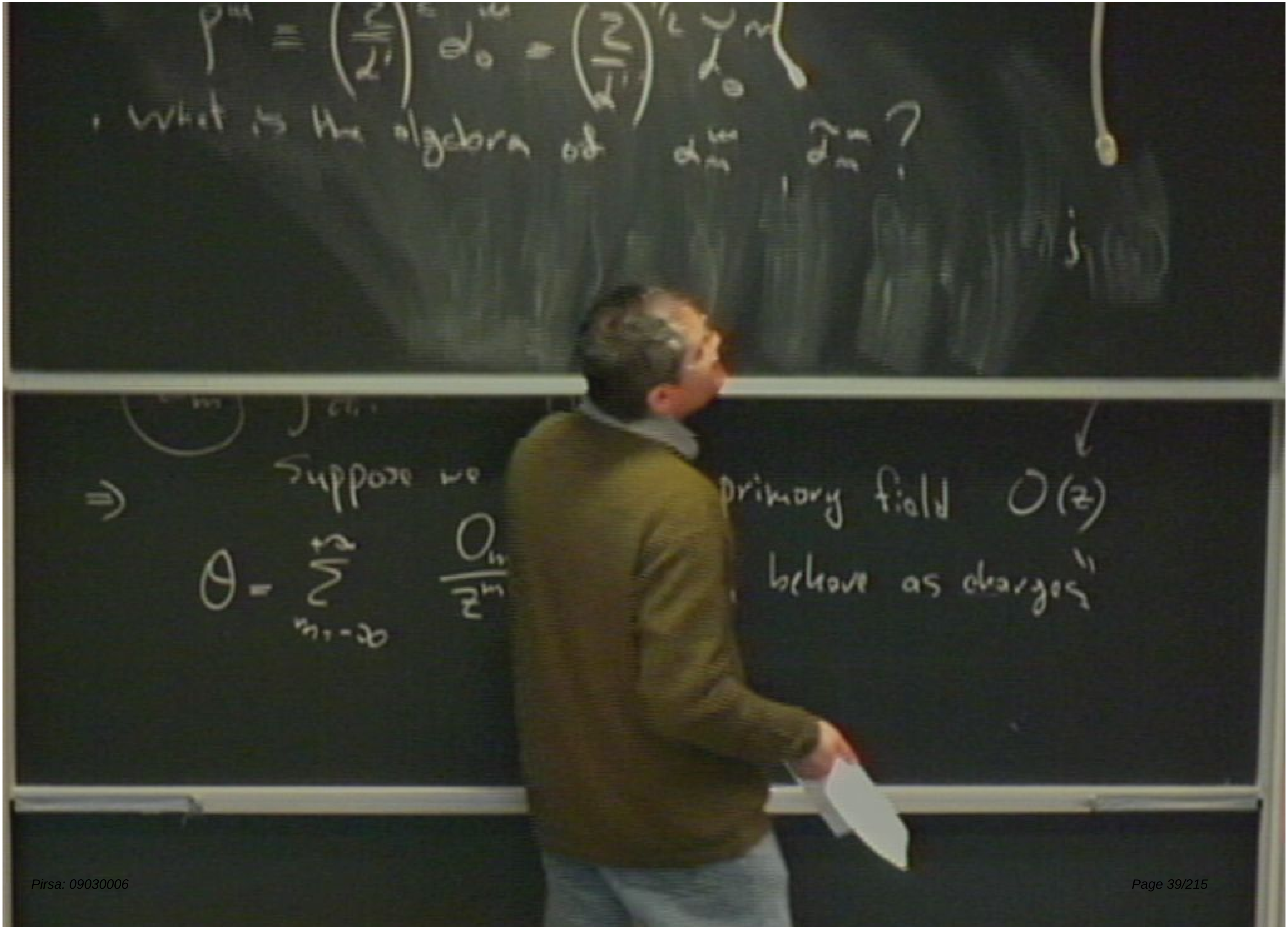
$$p^{\mu} = \oint \frac{dz}{2\pi i} j_z - \oint \frac{d\bar{z}}{2\pi i} \tilde{j}_{\bar{z}} = \left(\frac{1}{2\alpha'}\right)^{1/2} \left[\alpha_0^{\mu} + \tilde{\alpha}_0^{\mu} \right]$$

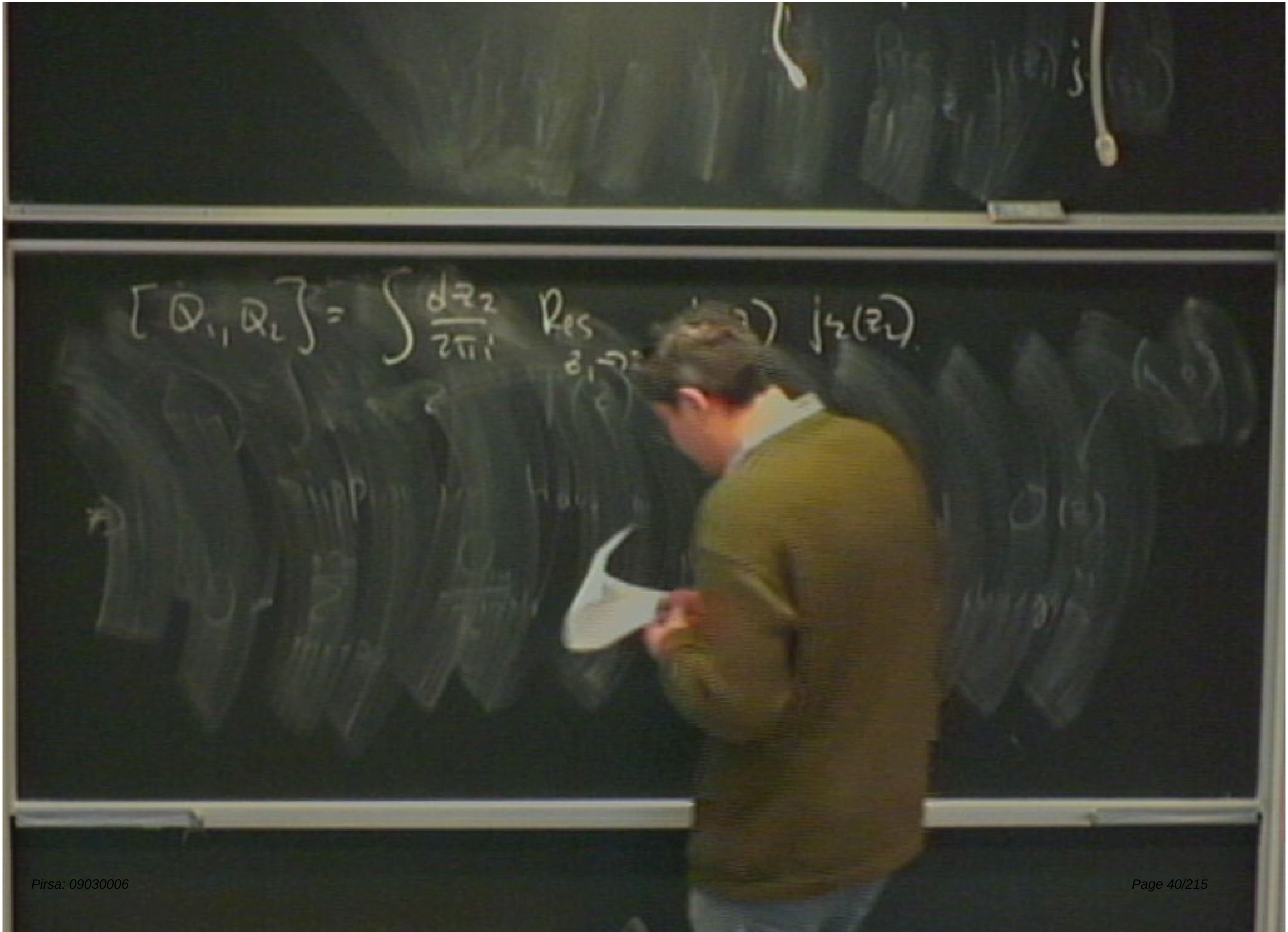
$j_1(z_1)$

z_2

$$p^\mu = \oint \frac{dz}{2\pi i}\, j_z - \oint \frac{d\bar{z}}{2\pi i}\, \tilde{j}_{\bar{z}} = \left(\frac{1}{2\alpha'}\right)^{1/2}\left[\alpha_0^\mu + \tilde{\alpha}_0^\mu\right]$$

$$p^\mu = \left(\frac{2}{\alpha'}\right)^{1/2}\alpha_0^\mu = \left(\frac{2}{\alpha'}\right)^{1/2}\tilde{\alpha}_0^\mu$$


What is the algebra of d_m^m $\tilde{d}_m^m$?
Suppose we
primary field $O(z)$
$\Theta = \sum_{m=-\infty}^{+\infty}$
behave as charges"


$[Q_1, Q_2] = \int \frac{dz_2}{2\pi i} \operatorname{Res}_{z_1 \to z_2} \; j_2(z_2)$

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$$[Q_1, Q_2] = \int \frac{dz_2}{2\pi i} \operatorname*{Res}_{z_1 \to z_2} j_1(z_1)\, j_2(z_2)$$

$$\alpha_m^\mu = \frac{2}{\alpha'} \oint \frac{dz}{2\pi i} z^m \partial X^\mu(z)$$

$$[Q_1, Q_2] = \int \frac{dz_2}{2\pi i} \operatorname{Res}_{z_1 \to z_2} j_1(z_1)\, j_2(z_2)$$

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$$j_1 = \frac{2}{\alpha'} z_1^m \partial X^\mu(z_1)$$

$$[\alpha_m^\mu, \alpha_n^\nu] = [\tilde{\alpha}_m^\mu, \tilde{\alpha}_n^\nu] = m\,\delta_{m,-n}\,\eta^{\mu\nu}$$

$$[\alpha_m^\mu, \alpha_n^\nu] = [\tilde{\alpha}_m^\mu, \tilde{\alpha}_n^\nu] = m\,\delta_{m,-n}\,\eta^{\mu\nu}$$

$$[x^\mu, p^\nu] = i\,\eta^{\mu\nu}$$

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$$X^\mu$$

$$[\alpha_m^\mu, \alpha_n^\nu] = [\tilde{\alpha}_m^\mu, \tilde{\alpha}_n^\nu] = m\,\delta_{m,-n}\,\eta^{\mu\nu}$$

$$[x^\mu, p^\nu] = i\eta^{\mu\nu}$$

$$X^\mu = x^\mu$$

$$[\alpha_m^\mu, \alpha_n^\nu] = [\tilde{\alpha}_m^\mu, \tilde{\alpha}_n^\nu] = m\,\delta_{m,-n}\,\eta^{\mu\nu}$$

$$[x^\mu, p^\nu] = i\eta^{\mu\nu}$$

$$X^\mu = x^\mu - i\left(\frac{\alpha'}{2}\right) p^\mu \ln|z|^2 + i\left(\frac{\alpha'}{2}\right) \sum_{\substack{m=-\infty \\ m\neq 0}}^{+\infty}$$

$$[\alpha_m^\mu, \alpha_n^\nu] = [\tilde{\alpha}_m^\mu, \tilde{\alpha}_n^\nu] = m\,\delta_{m,-n}\,\eta^{\mu\nu}$$

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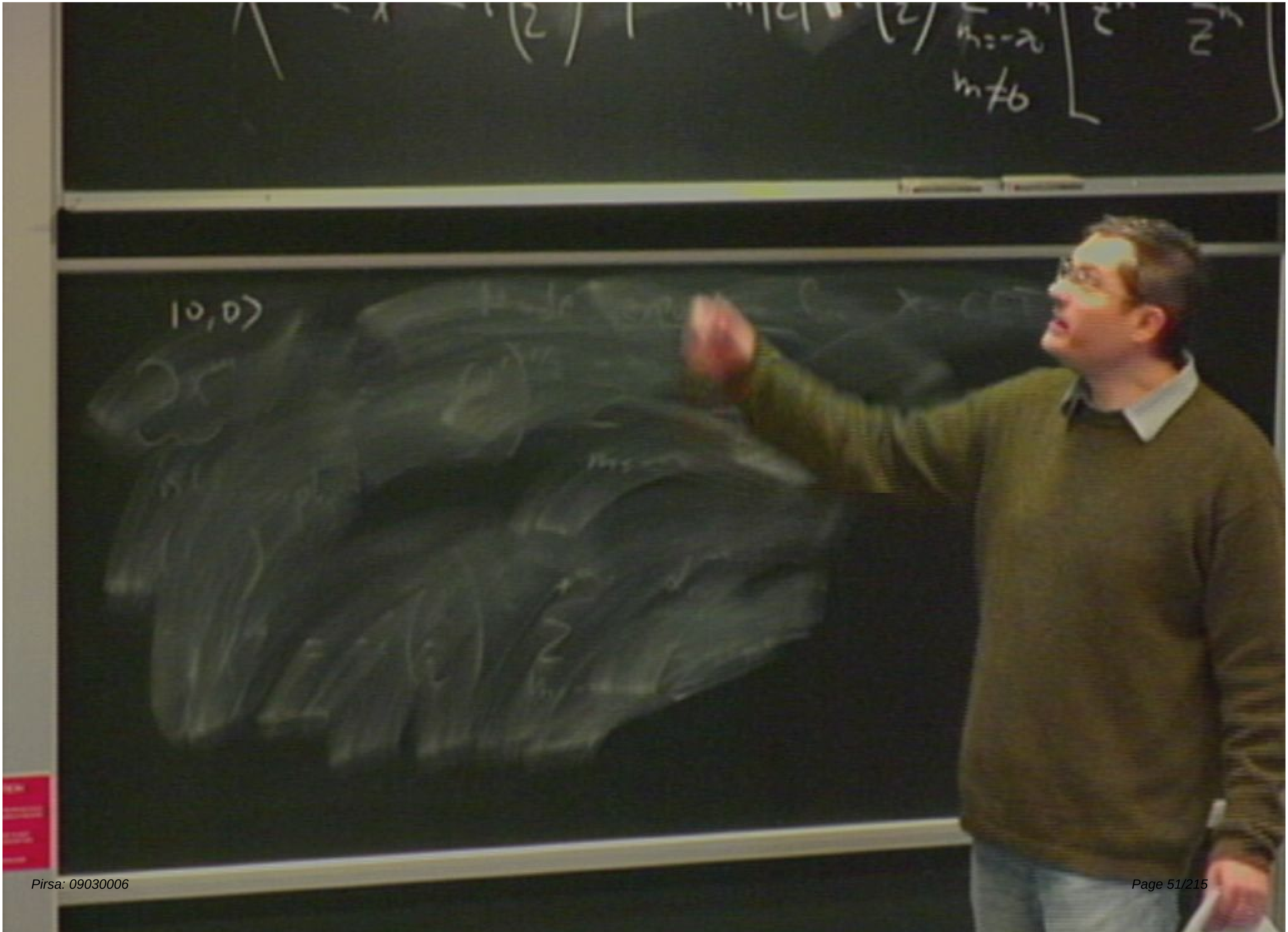
$$X^\mu = x^\mu - i\left(\frac{\alpha'}{2}\right) p^\mu \ln|z|^2 + i\left(\frac{\alpha'}{2}\right)^{1/2} \sum_{\substack{m=-\infty \\ m\neq 0}}^{+\infty} \frac{1}{m}\left[\frac{\alpha_m^\mu}{z^m} + \frac{\tilde{\alpha}_m^\mu}{\bar{z}^m}\right]$$


|0,0⟩

$\alpha_n^{\mu}|0,0\rangle = 0\ ,\ n>0$

$p^{\mu}|0,0\rangle = 0$

$\alpha_n^{\mu}|0,0\rangle = 0 \, , \; n>0$

$p^{\mu}|0,0\rangle = 0$

↳ we can get excited states.

$\alpha^{\mu}_{n} |0,0\rangle = 0 \ , \ n>0$

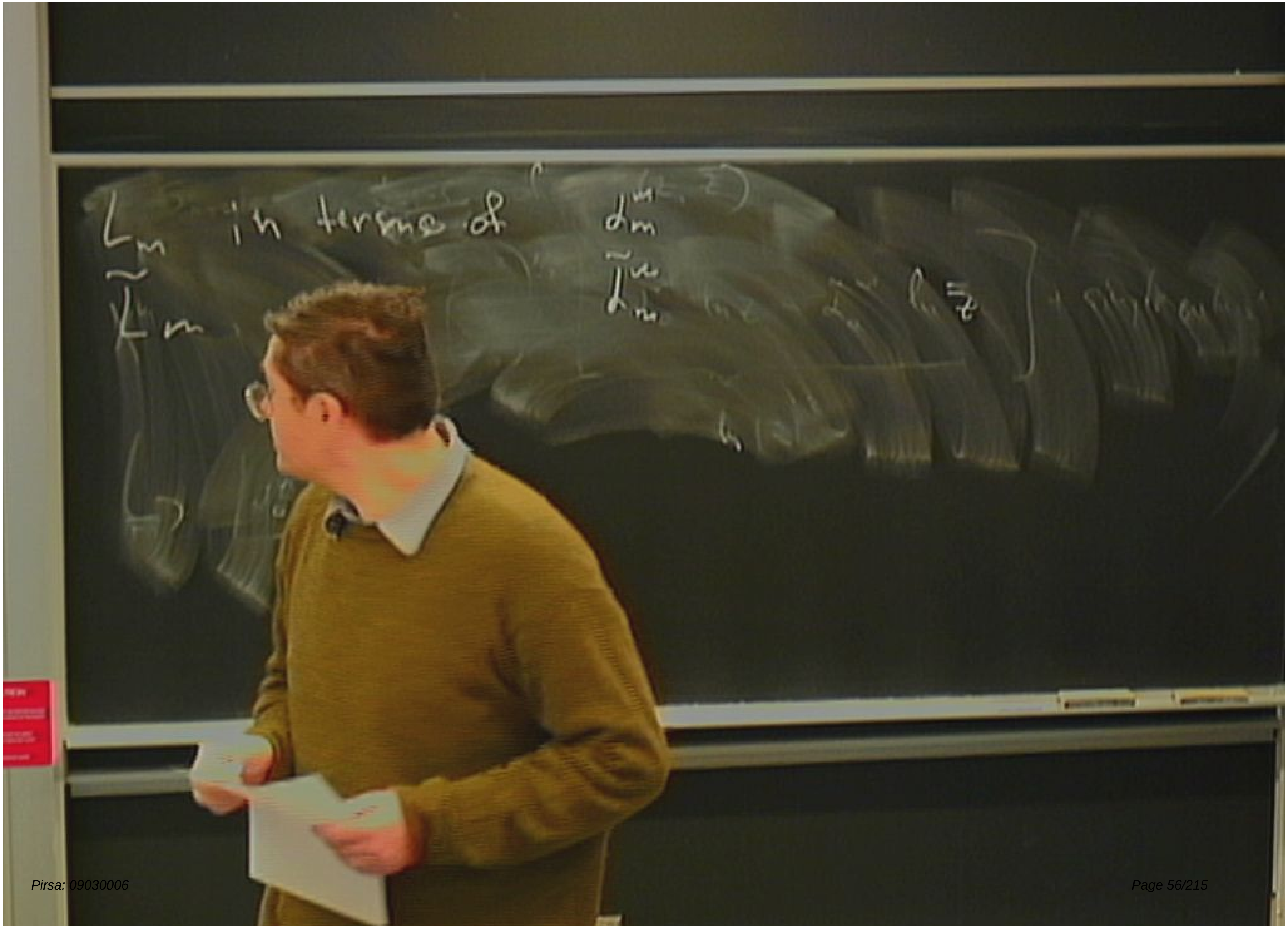
$p^{\mu} |0,0\rangle = 0$

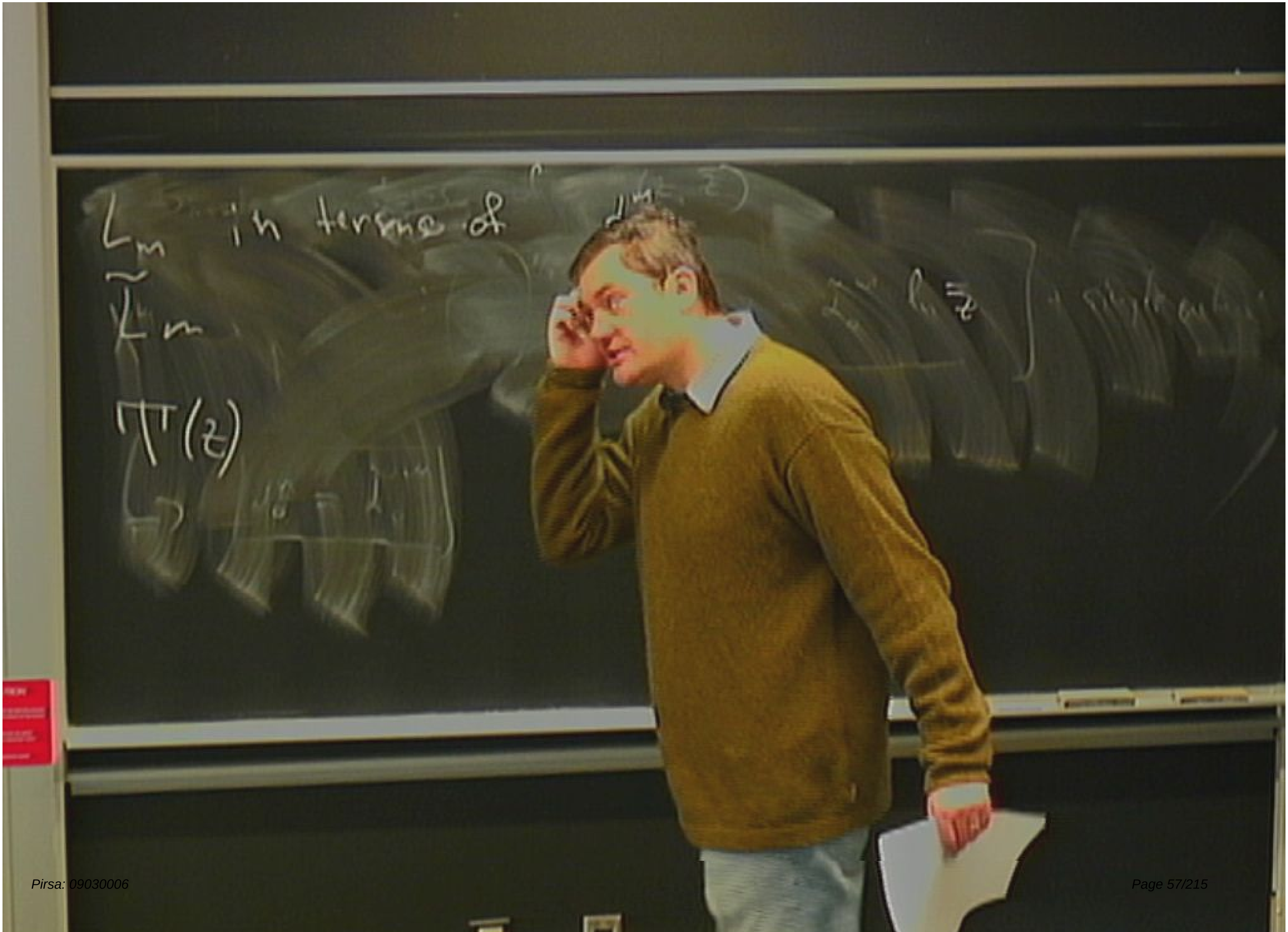
$\hookrightarrow$ we can get excited states $\alpha^{\mu}_{-n} |0,0\rangle$

$$\alpha_n^{\mu} |0,0\rangle \overset{\text{def}}{=} 0 \ , \ n > 0$$

$$p^{\mu} |0,0\rangle = 0$$

$\hookrightarrow$ we can get excited states. $\alpha_{-n}^{\mu} |0,0\rangle$

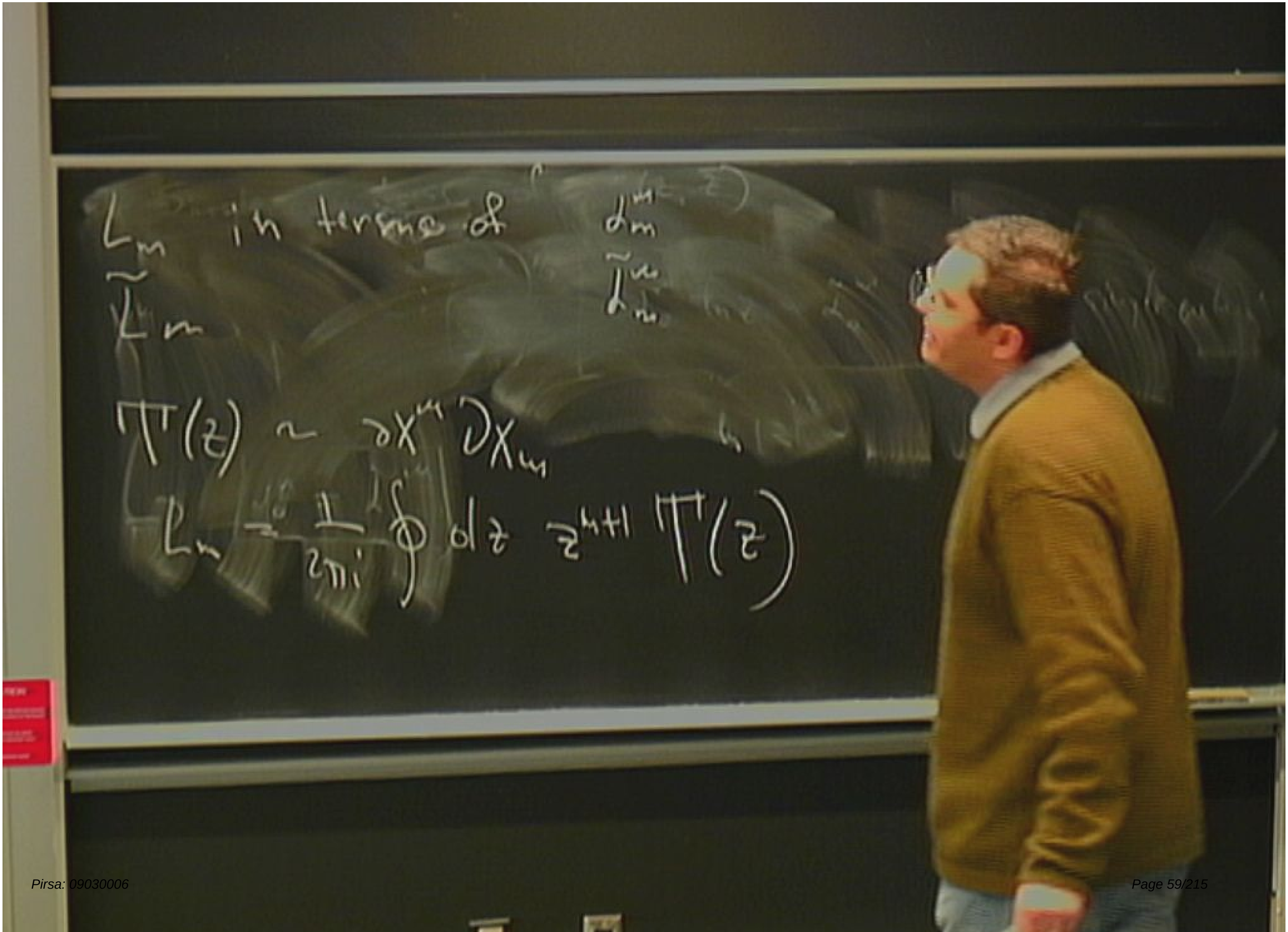

in terms of

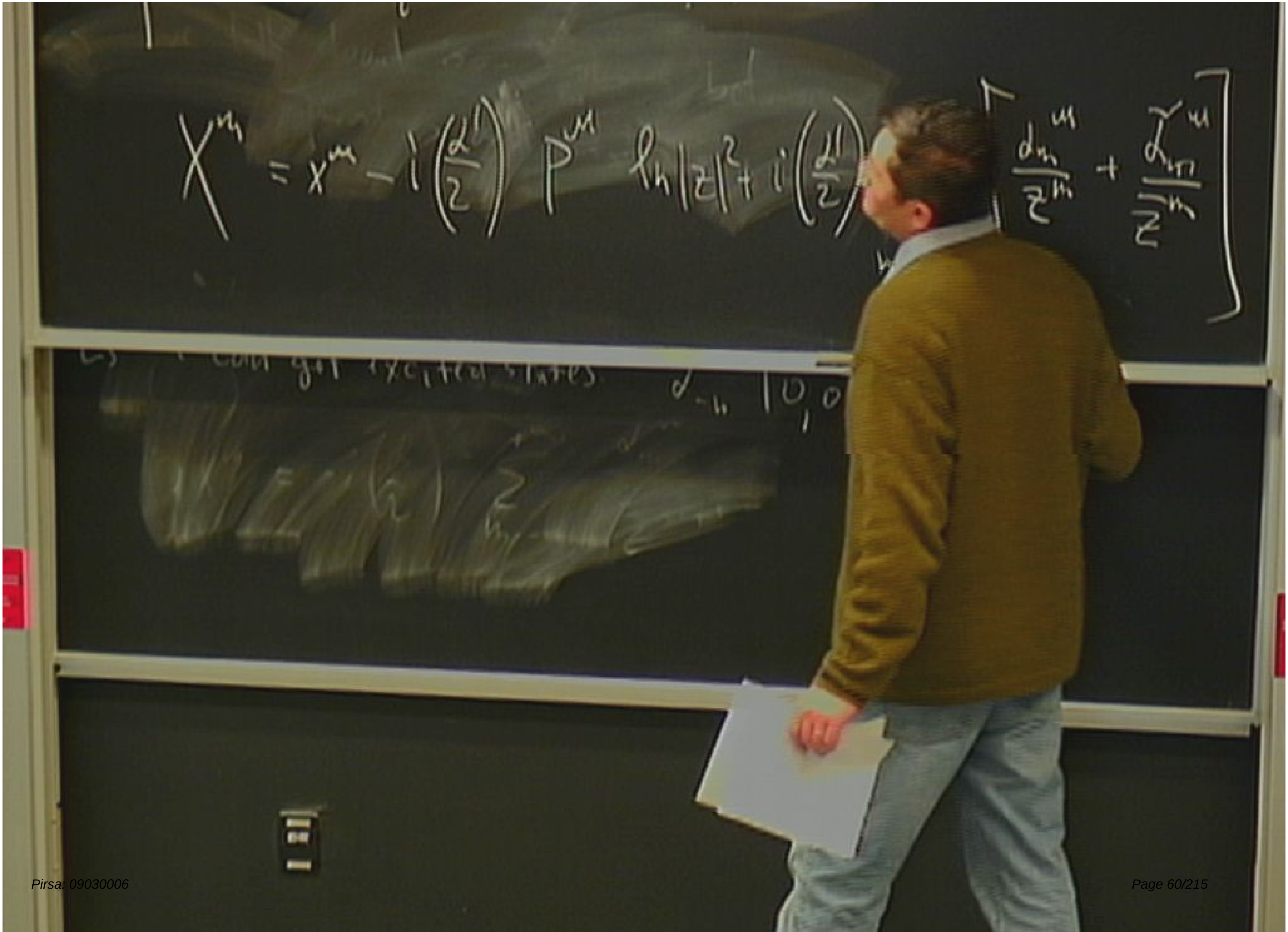

Lm in terms of
T(z)

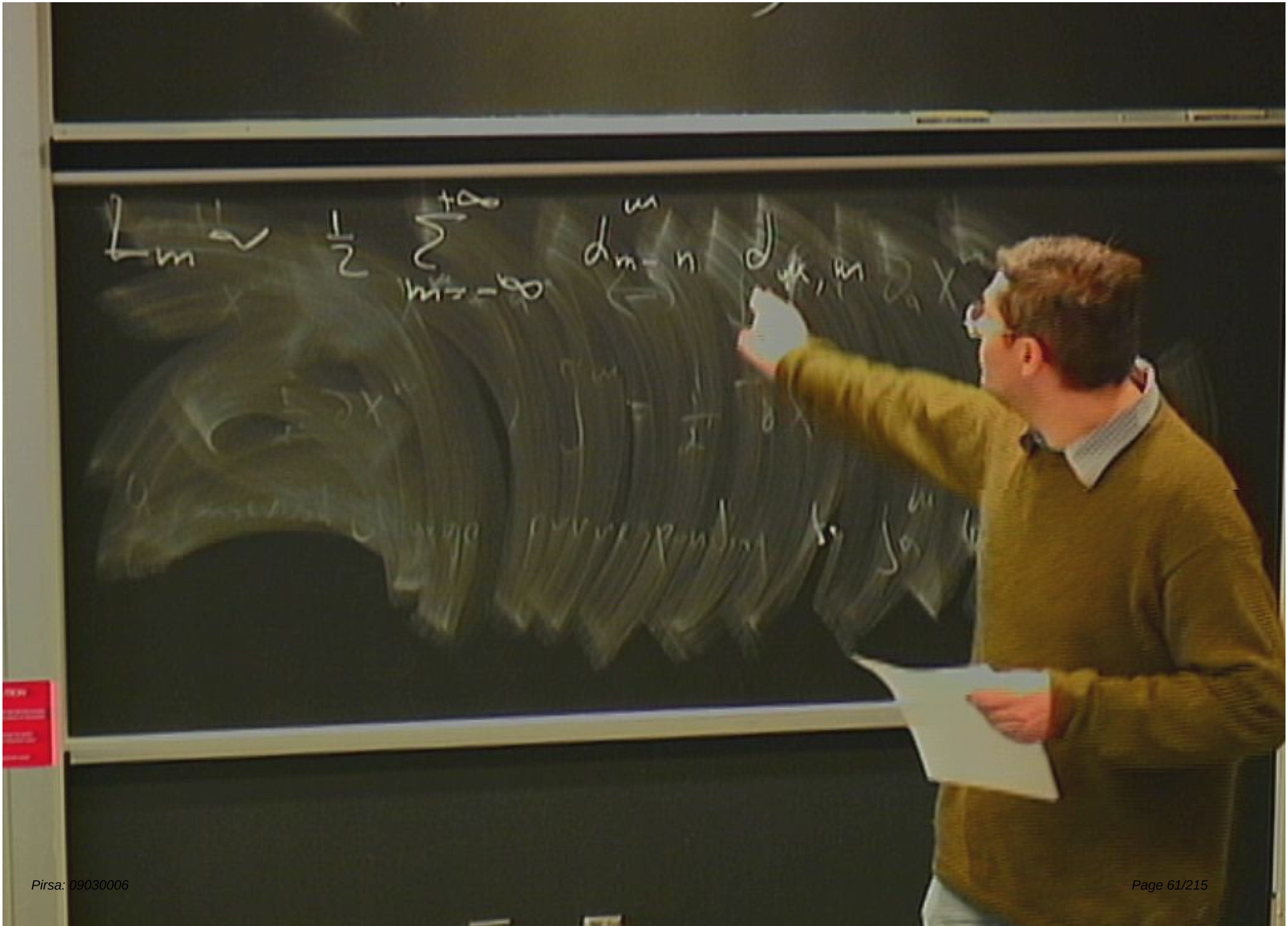
L_m in terms of α_m^μ

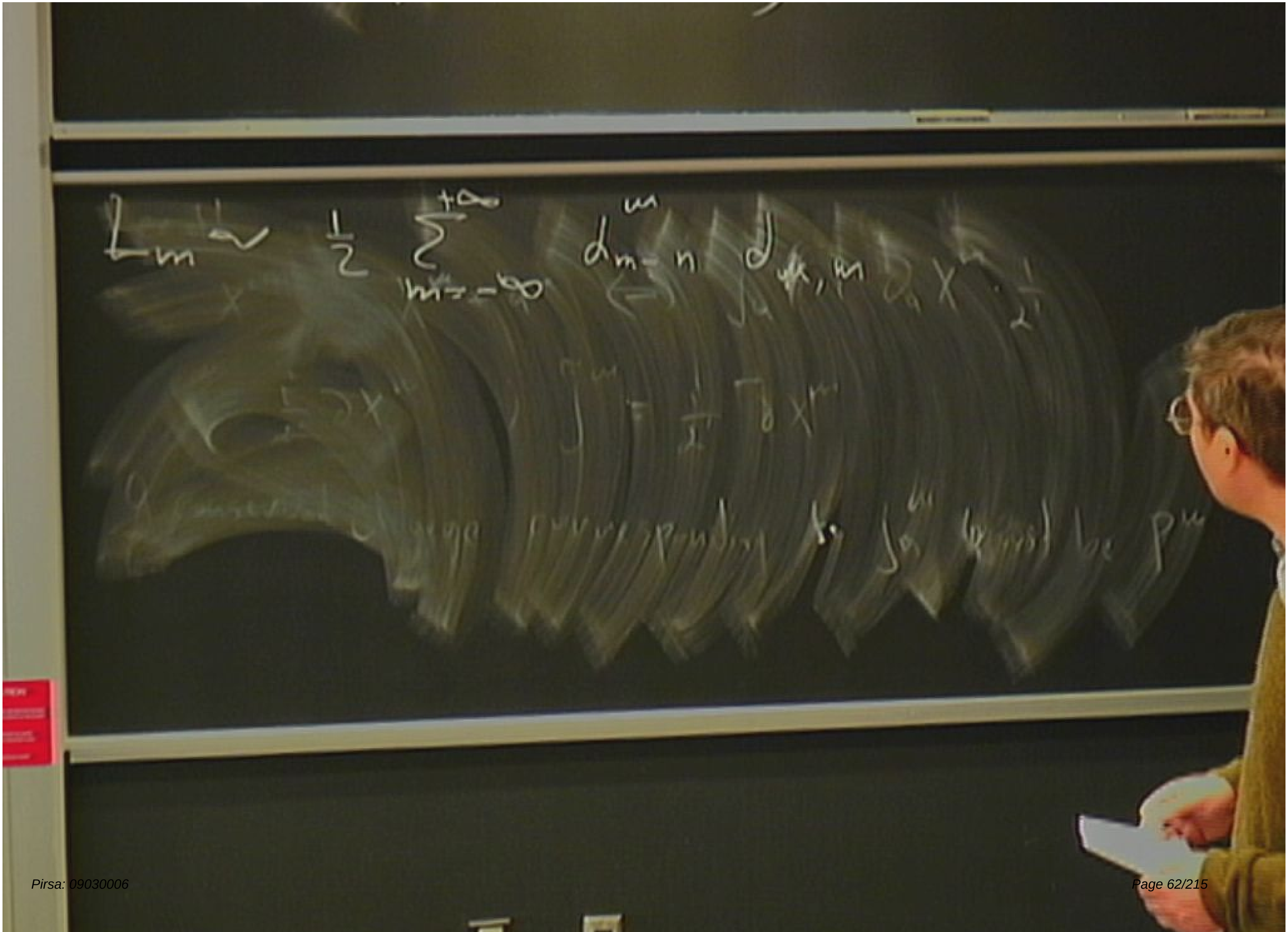
$\tilde{L}_m$ $\tilde{\alpha}_m^\mu$

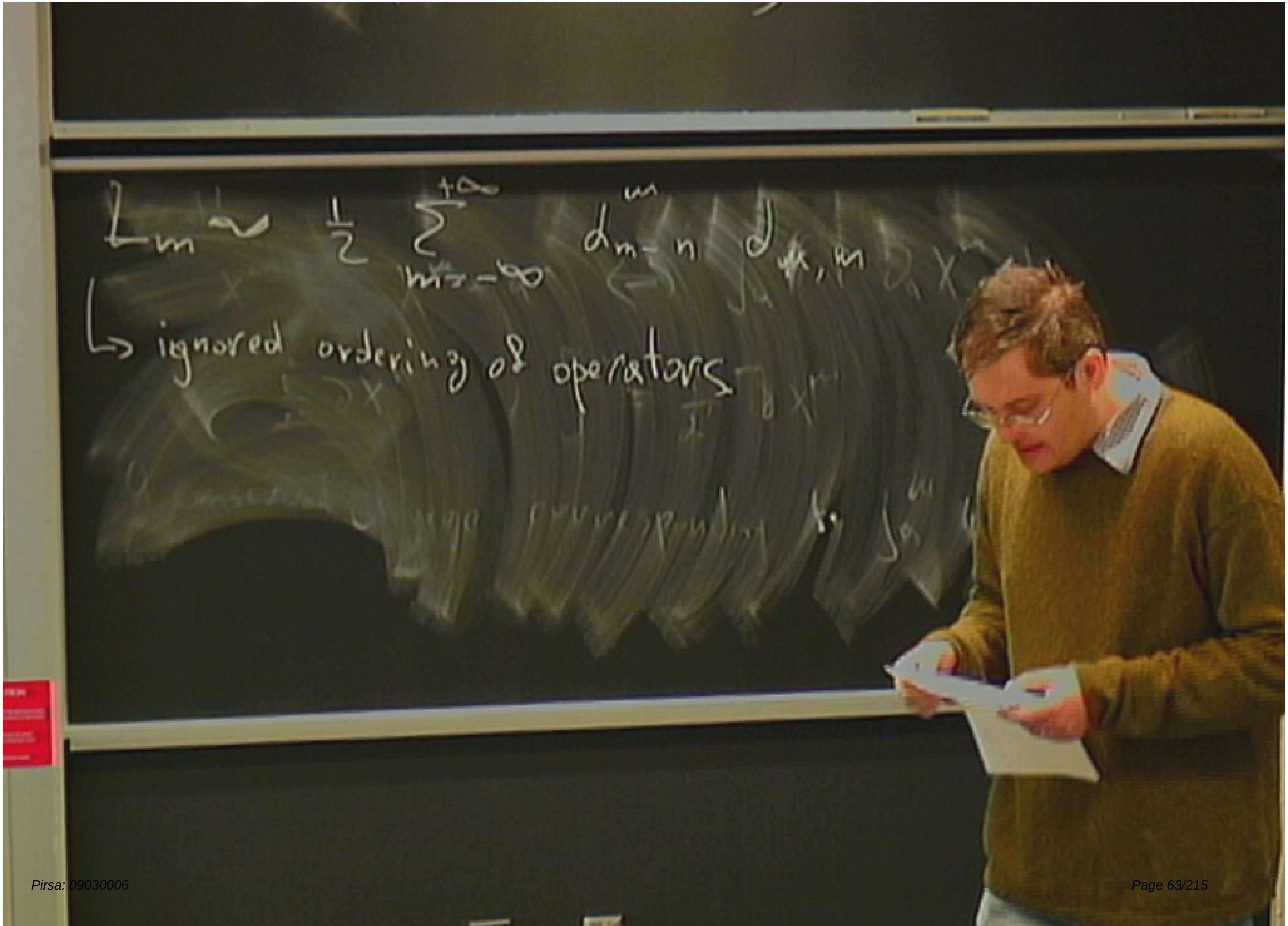
$T(z) \sim \partial X^\mu \partial X_\mu$

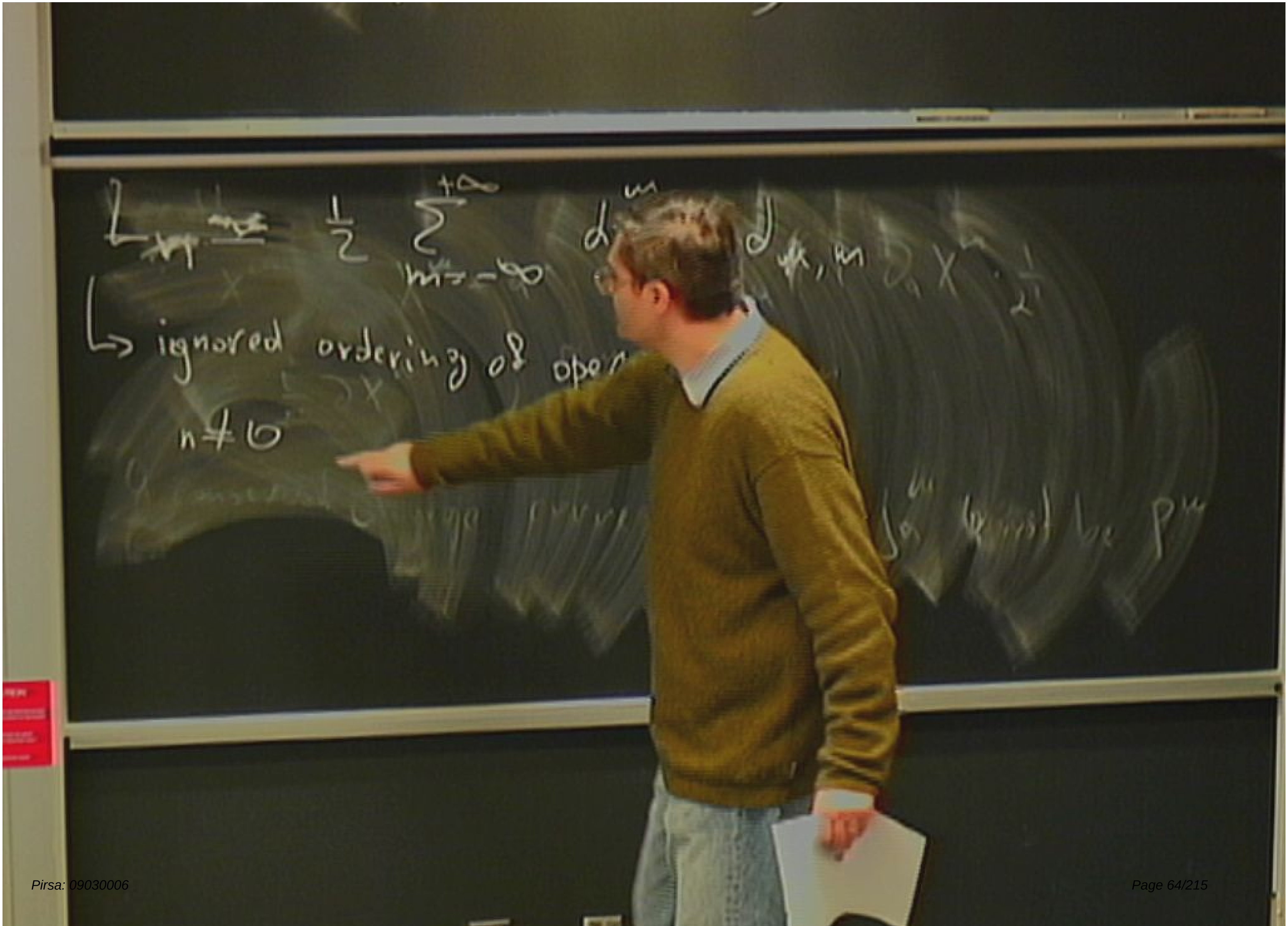

L_m in terms of α_m^μ
$\tilde{L}_m$ $\tilde{\alpha}_m^\mu$
$T(z) \sim \partial X^\mu \partial X_\mu$
$L_m = \frac{1}{2\pi i} \oint dz\, z^{m+1}\, T(z)$


$X^\mu = x^\mu - i\left(\frac{\alpha'}{2}\right) p^\mu \ln|z|^2 + i\left(\frac{\alpha'}{2}\right) \left[\frac{\alpha_m^\mu}{z^m} + \frac{\tilde{\alpha}_m^\mu}{\bar{z}^m}\right]$
excited states


$L_m \sim \frac{1}{2} \sum_{n=-\infty}^{+\infty} \alpha^{\mu}_{m-n} \alpha_{\mu,n}$


$L_m \sim \frac{1}{2}\sum_{m=-\infty}^{+\infty} \alpha^{\mu}_{m-n}\,\alpha_{\mu,n}$
corresponding to
be p^μ


$L_m \sim \frac{1}{2} \sum_{n=-\infty}^{+\infty} \alpha^{\mu}_{m-n} \alpha_{\mu, n}$
ignored ordering of operators



ignored ordering of ope
n≠0

$$L_n = \frac{1}{2} \sum_{-\infty}^{+\infty} \alpha^{\mu} \alpha X^{\mu} \cdot \frac{1}{2}$$

↳ ignored ordering of operators

$n \neq 0$

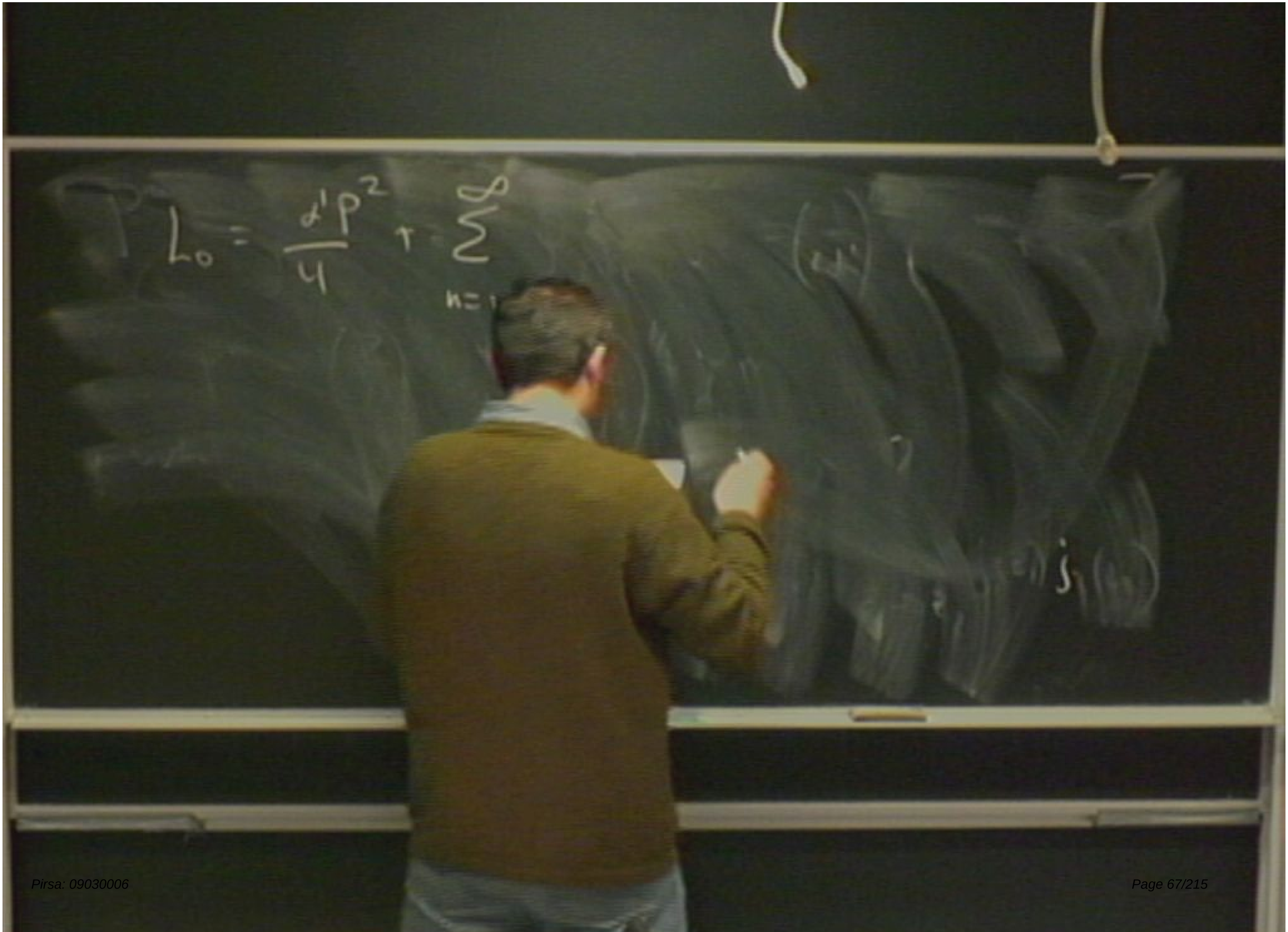
$$L_m = \frac{1}{2} \sum_{n=-\infty}^{+\infty} d^m \underbrace{\;}_{m'} \; d \underbrace{\;}_{n'} \; \partial X^m \cdot \frac{1}{2}$$

↳ ignored ordering of operators

$m \neq 0$

$\underline{m' \neq n'}$

↑ the ordering is not important.


$L_0 = \frac{\alpha' p^2}{4} + \sum^{\infty}$
$n=$


$L_0 = \frac{\alpha' p^2}{4} + \sum_{n=1}^{\infty} :\alpha^{\mu}_{-n} \alpha_{\mu n}: + a^X$
normal ordering constant.

$$L_0 = \frac{\alpha' p^2}{4} + \sum_{n=1}^{\infty} :\alpha^{\mu}_{-n}\,\alpha_{\mu n}: + \, a$$

conformal normal ordering

normal ordering constant.

annihilation

$$L_0 = \frac{\alpha' p^2}{4} + \sum_{n=1}^{\infty} :\alpha^{\mu}_{-n}\, \alpha_{\mu n}: + a$$

creation operator

conformal normal ordering

normal ordering constant.

$$L_0 = \frac{\alpha' p^2}{4} + \sum_{n=1}^{\infty} {:}\alpha_{-n}^{\mu}\,\alpha_{\mu n}{:} + a^X$$

annihilation

creation

operator

conformal normal ordering

normal ordering constant.

↳ Recall that Virasoro algebra implies

$$L_0 = \frac{\alpha' p^2}{4} + \sum_{n=1}^{\infty} :\alpha^{\mu}_{-n}\,\alpha_{\mu n}: + a$$

annihilation

normal ordering

conformal normal ordering

normal ordering constant.

↳ recall that Virasoro algebra implies L_m

$$2L_0 = [L_1, L_{-1}] = 0$$

$2L_0 = [L_1, L_-]$

→ Previously we constructed our Hilbert space

$L_n|0\rangle = 0, \quad n > 0.$

$p^m|0\rangle = 0$

$$L_m = \frac{1}{2} \sum_{-\infty}^{+\infty} \alpha^{\mu}_{m'} \, \alpha_{\mu\, n'}$$

↳ ignored ordering of operators

$m \neq 0$

$\underline{m' \neq n'}$

↑ the ordering is not important.

$2L_0 = [L_1, L_-]$

→ Previously we constructed our Hilbert space

$L_n |0\rangle = 0, \; n > 0. \iff \alpha^\mu_n |0\rangle = 0, \; n > 0.$

$p^\mu |0\rangle = 0$

$2L_0 = [L_1, L_{-1}]$

$\Rightarrow$ Previously we constructed our Hilbert space

$L_n|0\rangle = 0$, $n > 0$. $\Longleftrightarrow$ $\alpha^\mu_n|0\rangle = 0$, $n > 0$.

$p^\mu|0\rangle = 0$

$L_{-1}|0\rangle = 0$ $\Rightarrow$ $2L_0|0\rangle$

$2L_0 = [L_1, L_-]$

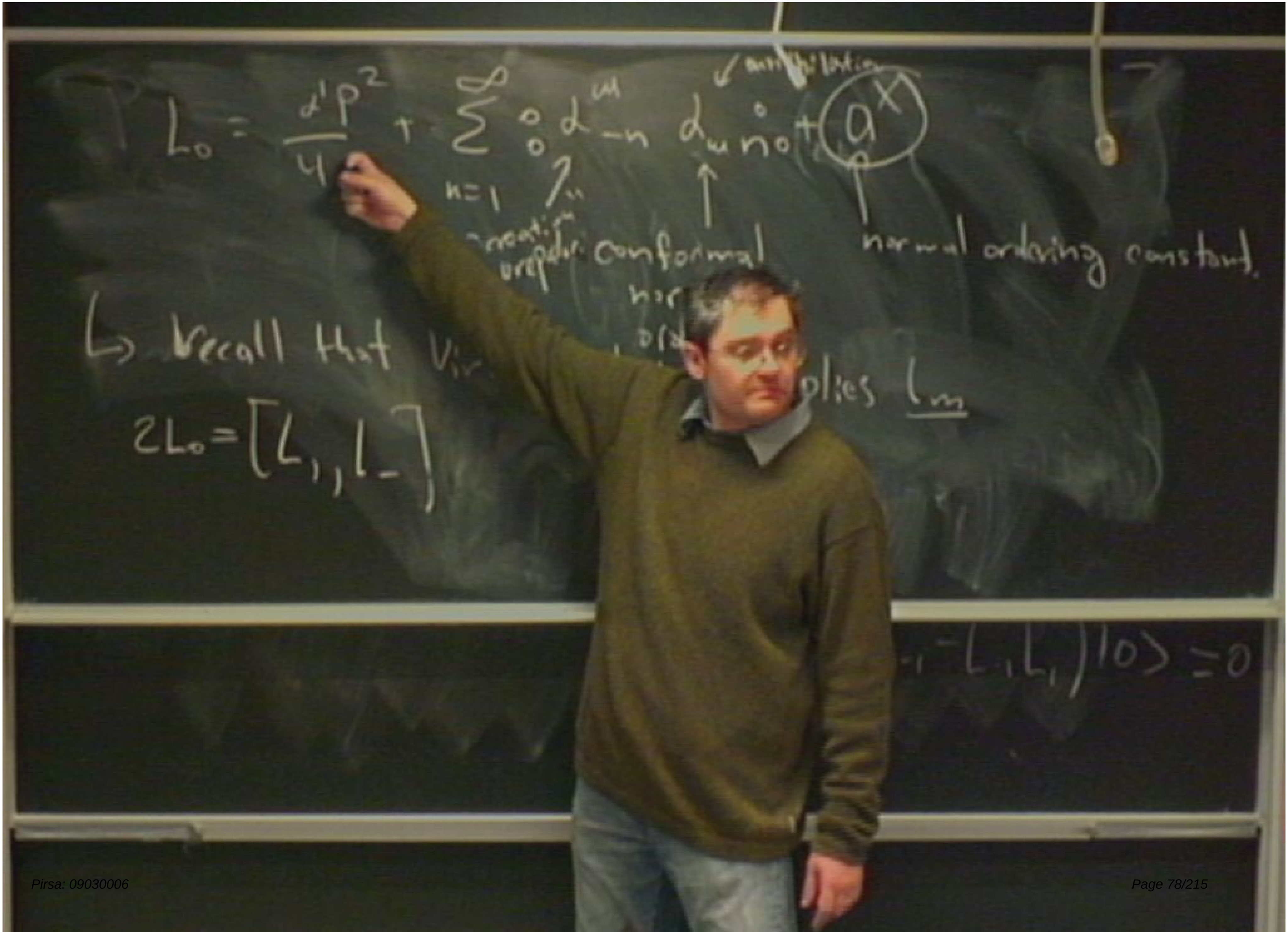
→ Previously we constructed our Hilbert space

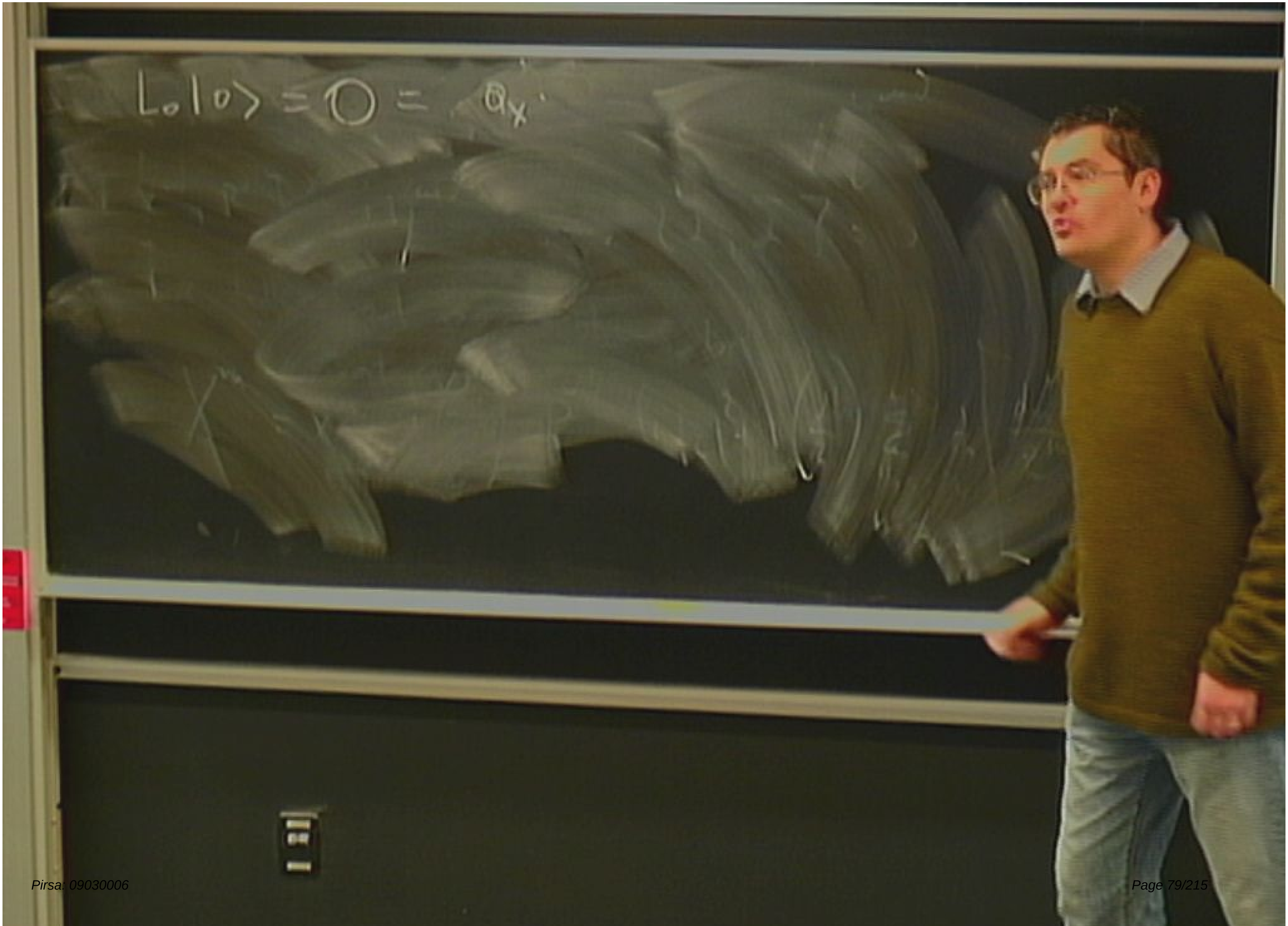
$L_n|0\rangle = 0$, $n > 0$. $\Longleftrightarrow$ $\alpha^\mu_n|0\rangle = 0$, $n > 0$.

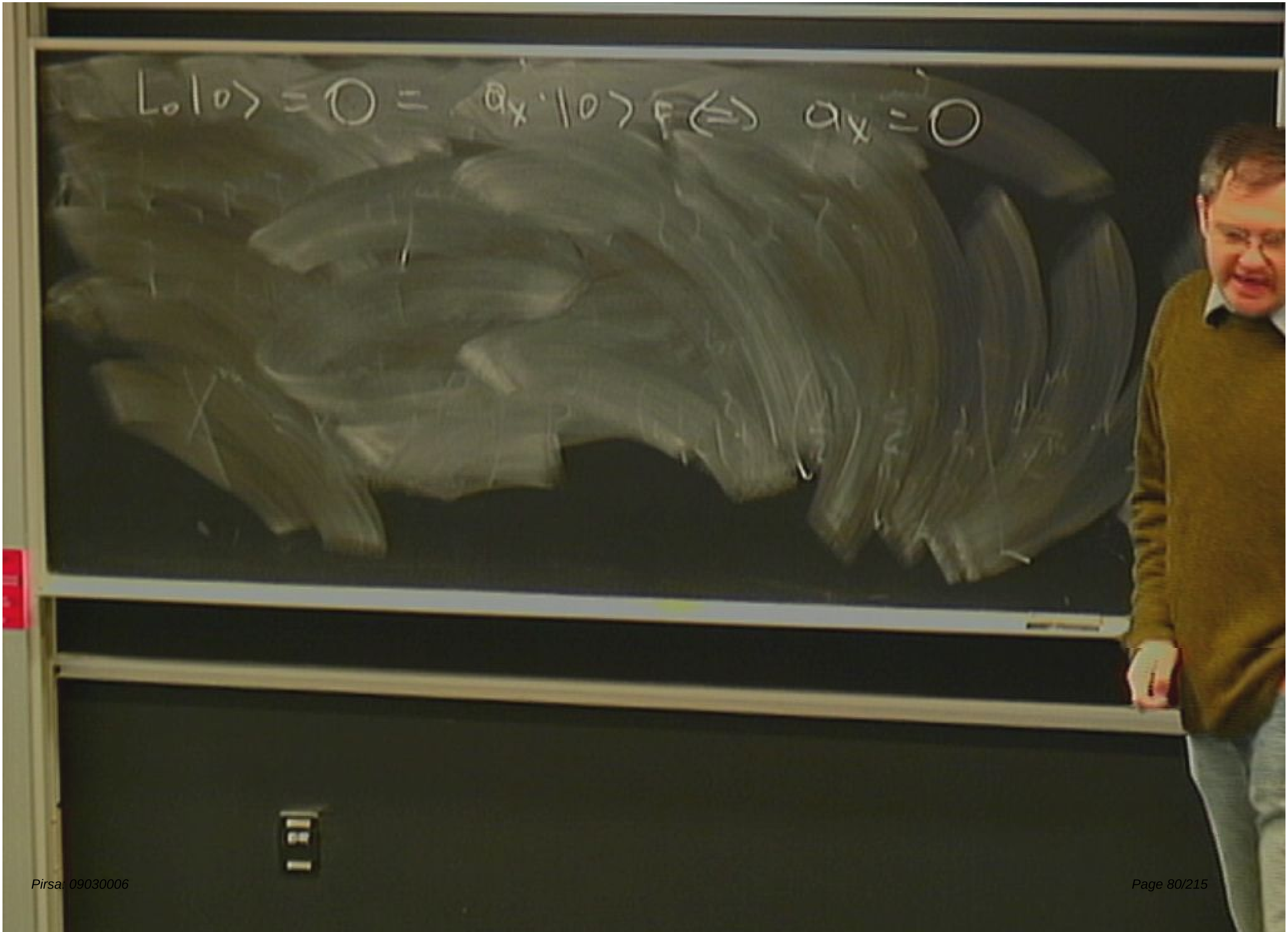
$p^\mu|0\rangle = 0$

$L_{-1}|0\rangle = 0$ $\Rightarrow$ $2L_0|0\rangle = 2[L_1, L_{-1}]|0\rangle$

$= 2(L_1 L_{-1} - L_{-1} L_1)|0\rangle = 0$


annihilation
$L_0 = \frac{\alpha' p^2}{4} + \sum_{n=1}^{\infty} :\alpha_{-n}\alpha_n: + a$
creation operator
conformal
normal ordering constant.
recall that Vir
plies L_m
$2L_0 = [L_1, L_-]$
$|0\rangle = 0$


$L_0|0\rangle = 0 = Q_x$

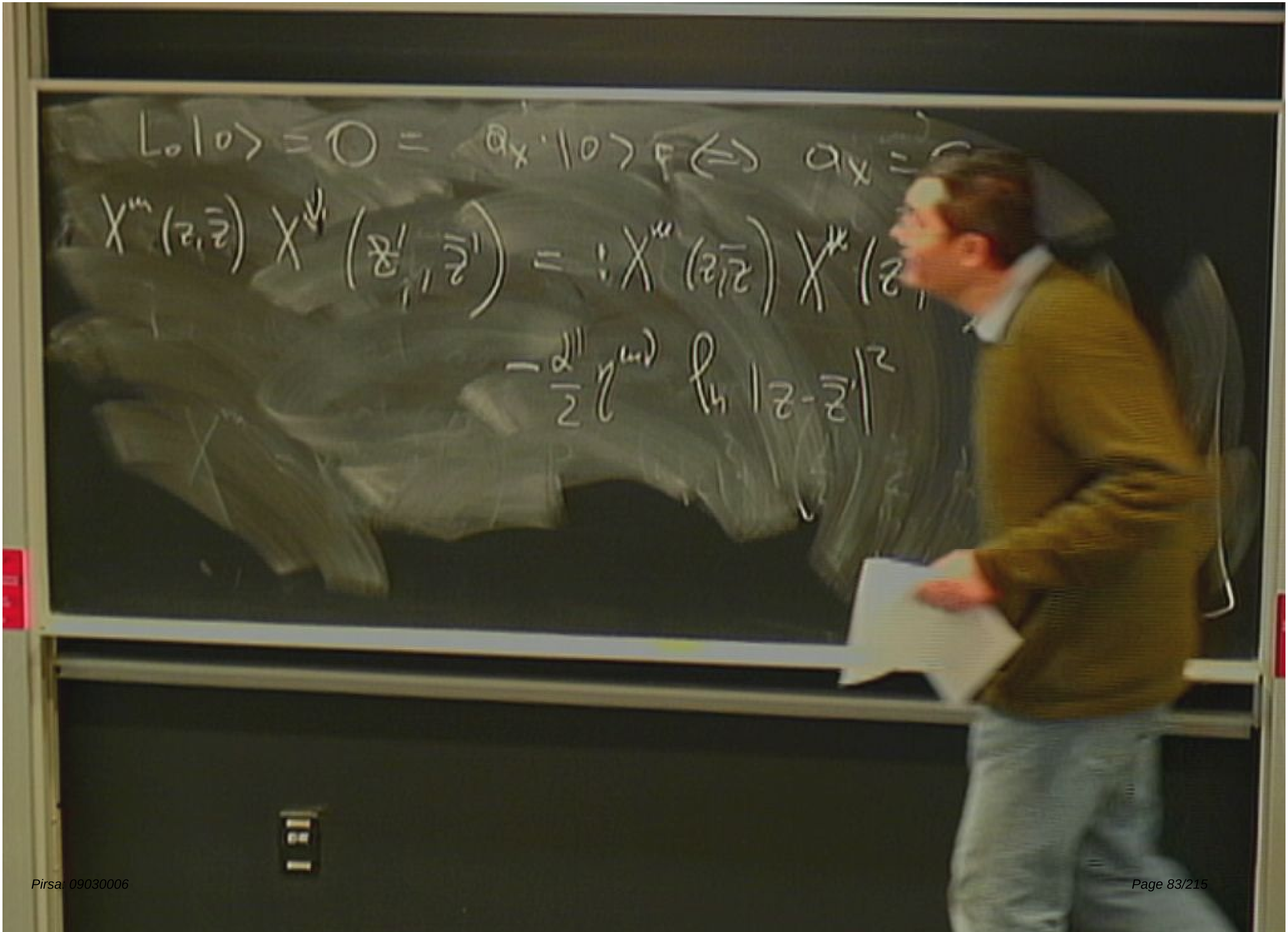

$L_0|0\rangle = 0 = a_x \cdot |0\rangle_F \Leftrightarrow a_x = 0$

$$L_0|0\rangle = 0 = a_x \cdot |0\rangle_F \Leftrightarrow a_x = 0$$

$$X^\mu(z,\bar z)\, X^\nu(z', \bar z')$$

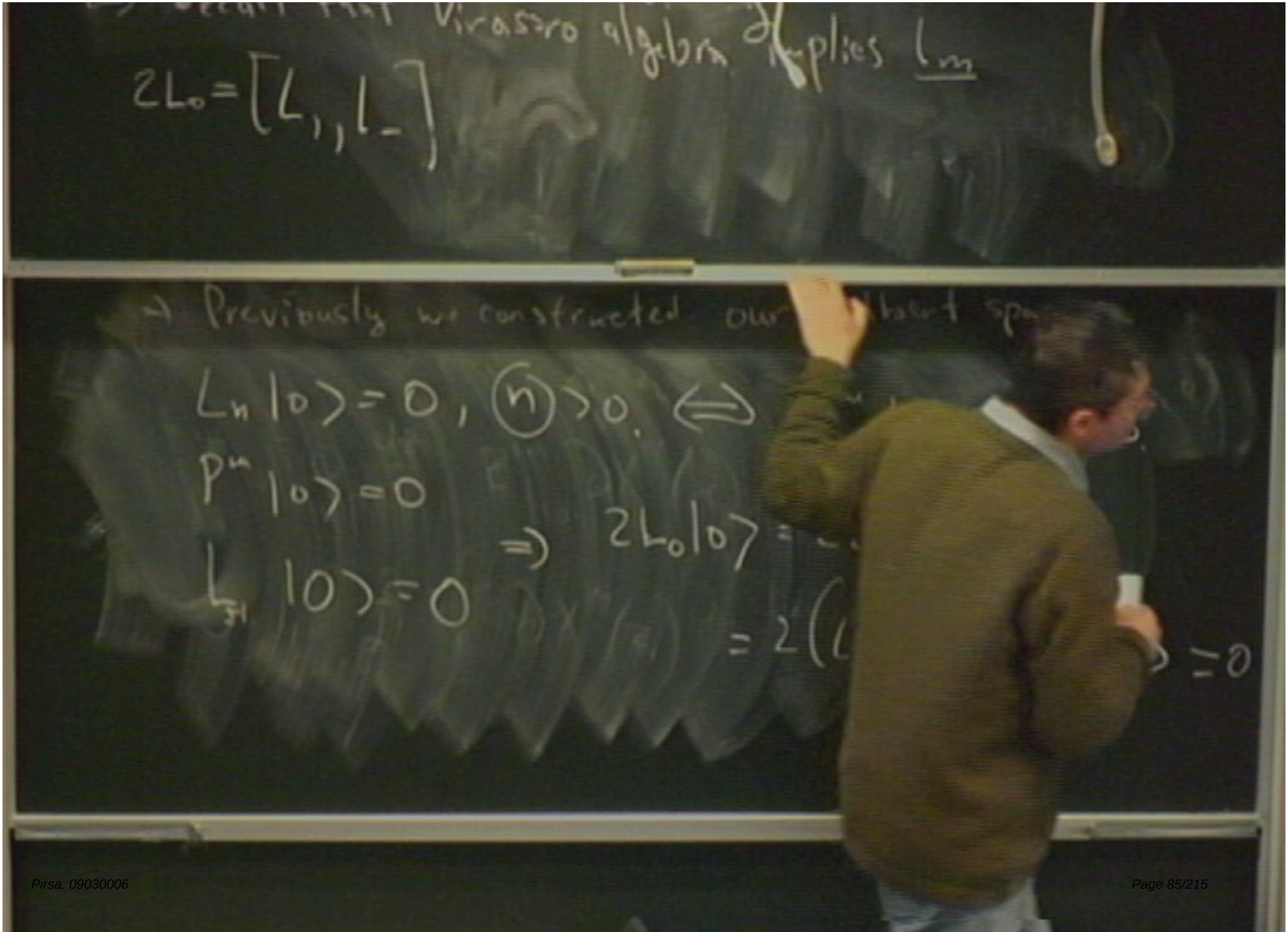
$$L_0|0\rangle = 0 = a_x \cdot |0\rangle \Leftrightarrow a_x = 0$$

$$X^{\mu}(z,\bar{z})\, X^{\nu}(z',\bar{z}') = :X^{\mu}(z,\bar{z})\, X^{\nu}(z',\bar{z}'):$$


L₀|0> = 0 = a_x · |0> ⇔ a_x =
X^μ(z, z̄) X^ν(z', z̄') = :X^μ(z, z̄) X^ν(z
− α'/2 η^μν ln |z − z̄'|²

$$L_0|0\rangle = 0 = a_x \cdot |0\rangle \Leftrightarrow a_x = 0$$

$$X^\mu(z,\bar z)\, X^\nu(z',\bar z') = :X^\mu(z,\bar z)\, X^\nu(z',\bar z'): - \frac{\alpha'}{2}\eta^{\mu\nu} \ln|z-\bar z'|^2$$

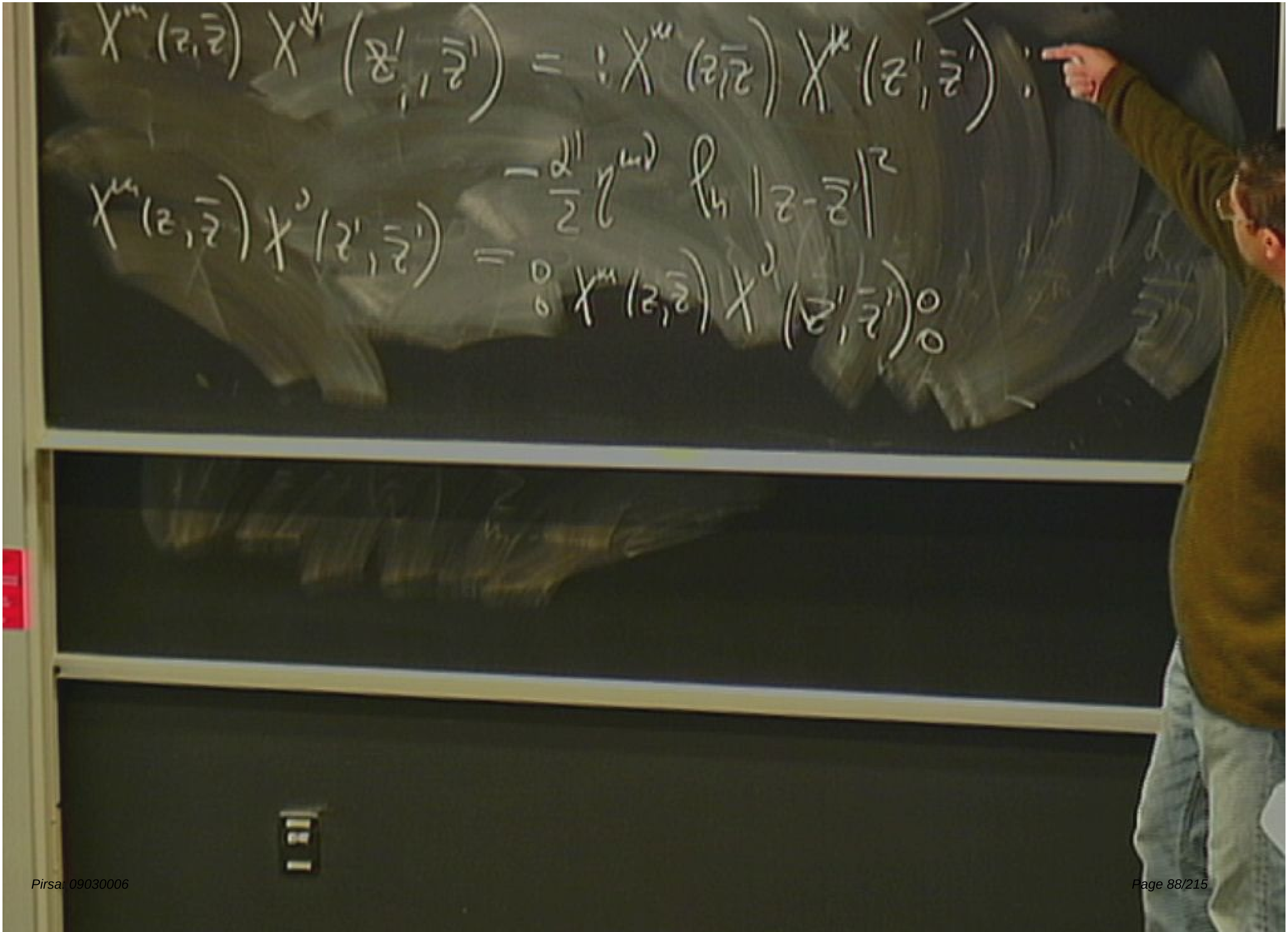

Virasoro algebra
plies L_m
2L_0 = [L_1, L_-]
Previously we constructed our
L_n |0> = 0, n > 0
p^m |0> = 0
L_±1 |0> = 0
=> 2L_0 |0>
= 2(
= 0

$$L_0|0\rangle = 0 = a_x \cdot |0\rangle_F \Leftrightarrow a_x = 0$$

$$X^\mu(z,\bar z)\, X^\nu(z',\bar z') = :X^\mu(z,\bar z)\, X^\nu(z',\bar z'): - \frac{\alpha'}{2}\eta^{\mu\nu} \ln|z-\bar z'|^2$$

$$X^\mu(z,\bar z)\, X^\nu(z',\bar z') = :X^\mu(z,\bar z)\, X^\nu(z',\bar z'):$$

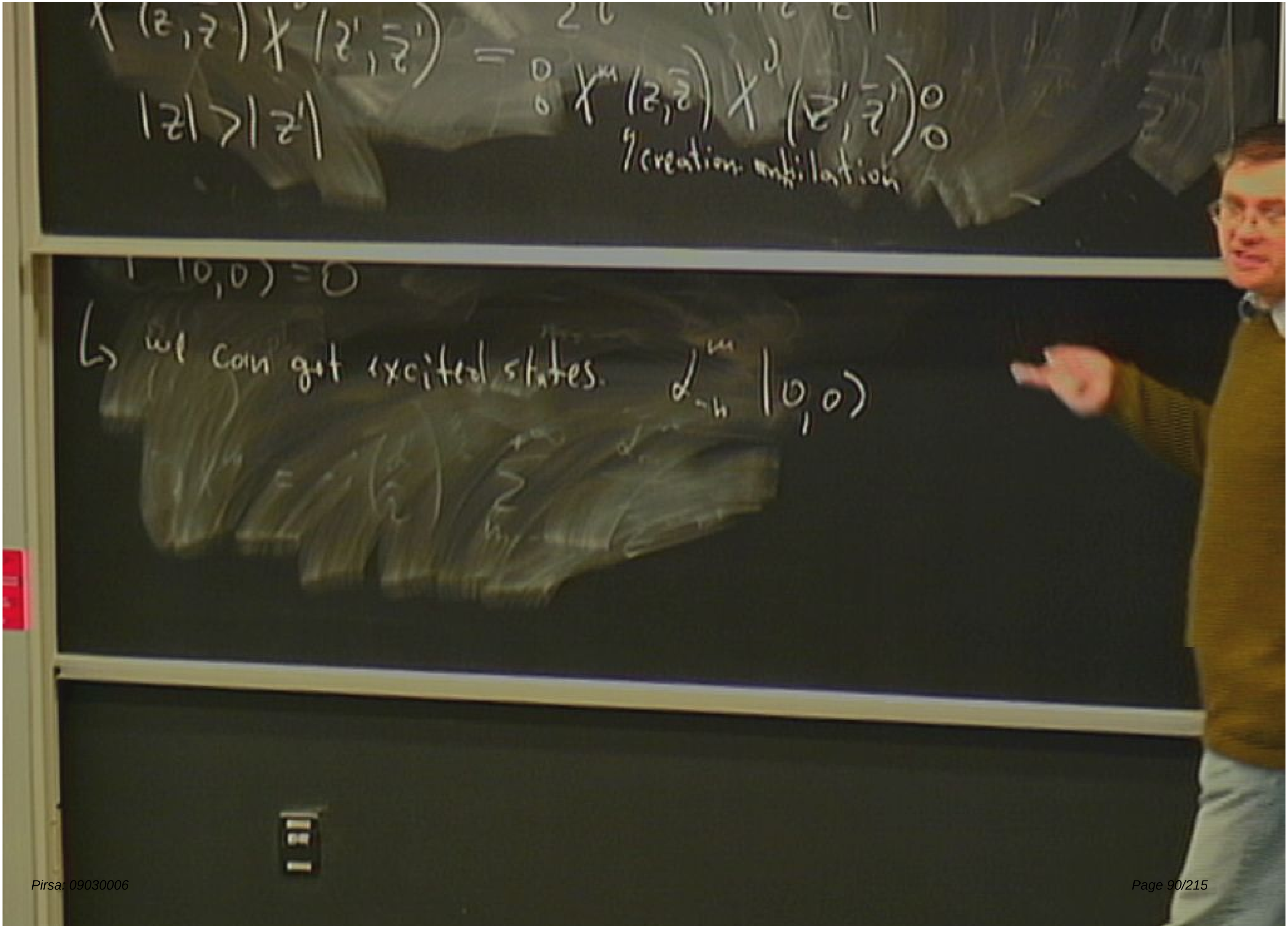
$$X^\mu(z,\bar z)\, X^\nu(z',\bar z') \quad -\frac{\alpha'}{2}\eta^{\mu\nu} \ln |z-z'|^2$$


$X^\mu(z,\bar{z})\, X^\nu(z',\bar{z}') = :X^\mu(z,\bar{z})\, X^\nu(z',\bar{z}'):$
$-\frac{\alpha'}{2}\eta^{\mu\nu} \ln|z-z'|^2$
$X^\mu(z,\bar{z})\, X^\nu(z',\bar{z}') = {}^\circ_\circ X^\mu(z,\bar{z})\, X^\nu(z',\bar{z}')\, {}^\circ_\circ$

$$X^{\mu}(z,\bar{z})\, X^{\nu}(z',\bar{z}') = \,{}^{\circ}_{\circ} X^{\mu}(z,\bar{z})\, X^{\nu}(z',\bar{z}')\, {}^{\circ}_{\circ}$$

creation annihilation

↳ we can get excited states $\alpha^{\mu}_{-n} |0,0\rangle$


$X^\mu(z,\bar z)\,X^\nu(z',\bar z') = {:}X^\mu(z,\bar z)\,X^\nu(z',\bar z'){:}$
$|z|>|z'|$
creation annihilation
$|0,0\rangle = 0$
↳ we can get excited states. $\alpha^\mu_{-n}|0,0\rangle$

$$+\frac{\alpha'}{2}\eta^{\mu\nu}\left[-\ln|z|^2+\sum_{m=1}^{\infty}\frac{1}{m}\left[\left(\frac{z'}{z}\right)^m+\left(\frac{\bar{z}'}{\bar{z}}\right)^m\right]\right]$$

$$+\frac{\alpha'}{2}\eta^{\mu\nu}\left[-\ln|z|^2+\sum_{m=1}^{\infty}\frac{1}{m}\left[\left(\frac{z'}{z}\right)^m+\left(\frac{\bar z'}{\bar z}\right)^m\right]\right]$$

$$\|$$

$$-\frac{\alpha'}{2}\eta^{\mu\nu}\ln|z-z'|^2$$

$$+\frac{\alpha'}{2}\gamma^{(w)}\left[-\ln|z|^2+\sum_{m=1}^{\infty}\frac{1}{m}\left[\left(\frac{z'}{z}\right)^m+\left(\frac{\bar{z}'}{\bar{z}}\right)^m\right]\right]$$

$$\|$$

$$-\frac{\alpha'}{2}\gamma^{(w)}\ln|z-\bar{z}'|^2$$

$$+\frac{\alpha'}{2}\eta^{\mu\nu}\left[-\ln|z|^2+\sum_{m=1}^{\infty}\frac{1}{m}\left[\left(\frac{z'}{z}\right)^m+\left(\frac{\bar{z}'}{\bar{z}}\right)^m\right]\right]$$

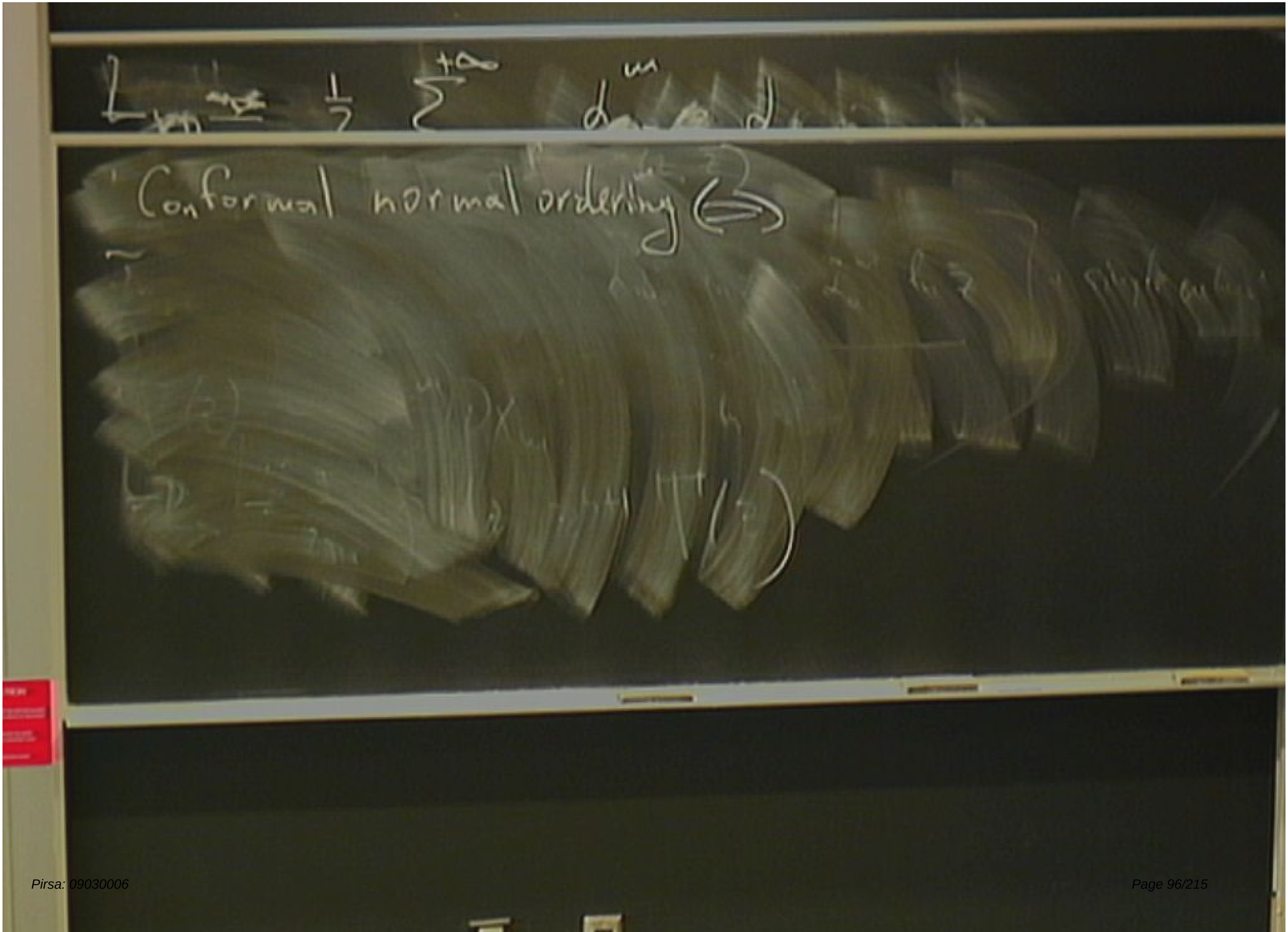
$$\parallel$$

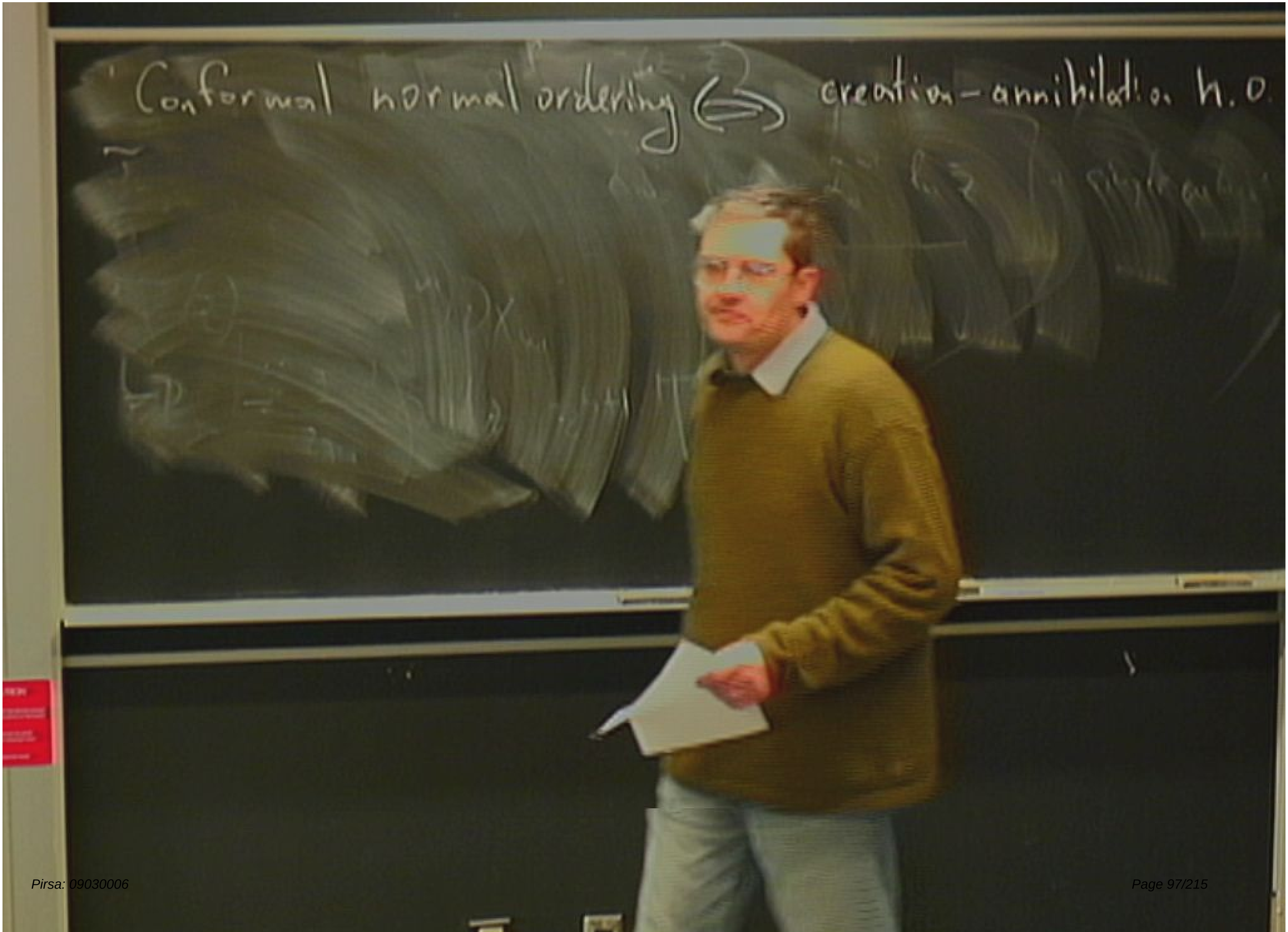
$$-\frac{\alpha'}{2}\eta^{\mu\nu}\ln|z-z'|^2=-\frac{\alpha'}{2}\eta^{\mu\nu}\left(\ln|z|+\ln\left|1-\frac{z'}{z}\right|\right)^2$$

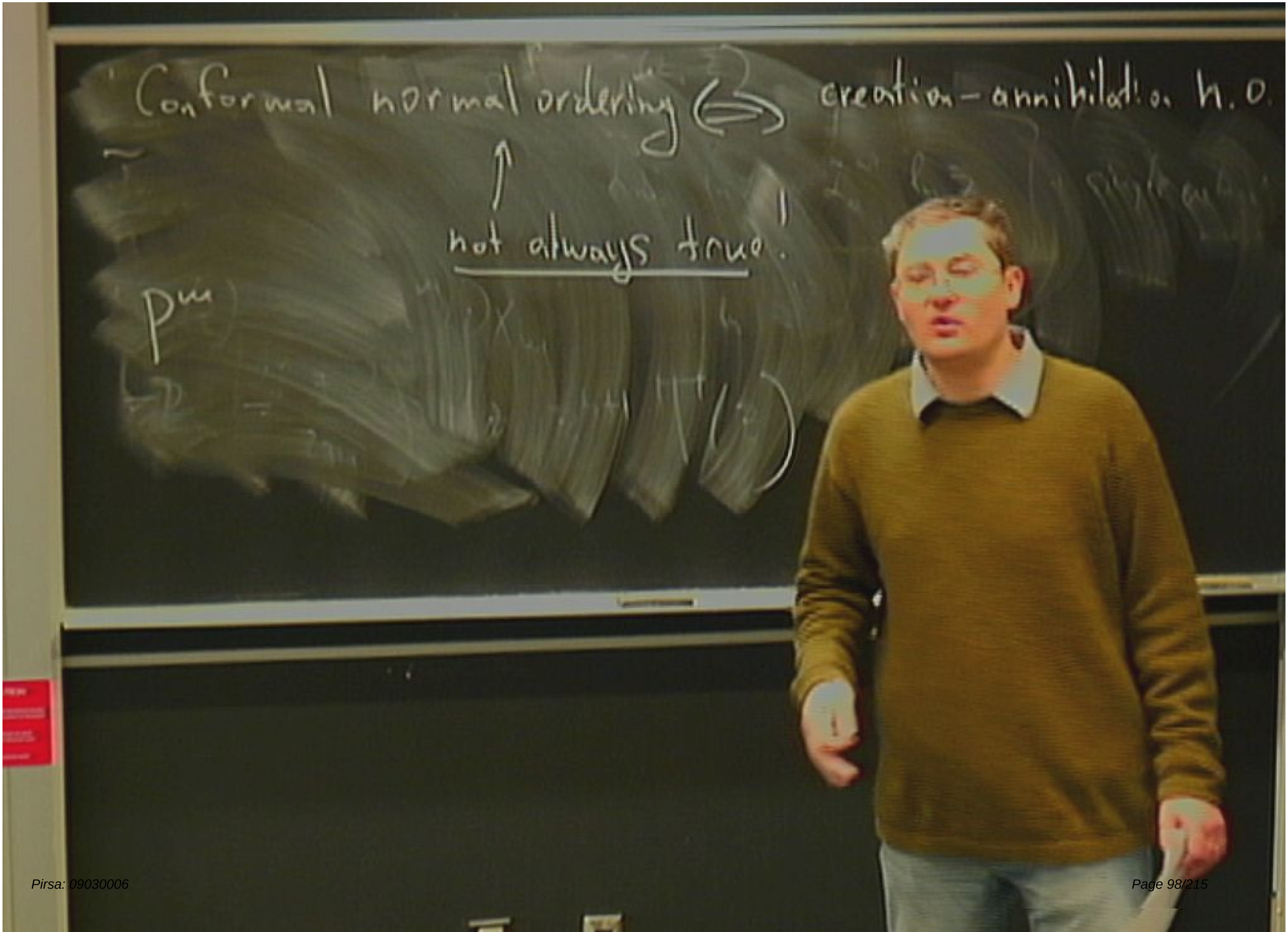
$$+\frac{d'}{2}\eta^{(\omega)}\left[-\ln|z|^2+\sum_{m=1}^{\infty}\frac{1}{m}\left[\left(\frac{z'}{z}\right)^m+\left(\frac{\bar{z}'}{\bar{z}}\right)^m\right]\right]$$

$$\| $$

$$-\frac{d'}{2}\eta^{(\omega)}\ln|z-z'|^2=-\frac{d'}{2}\eta^{(\omega)}\left(\ln|z|^2+\ln\left|1-\frac{z'}{z}\right|^2\right)$$


Conformal normal ordering


Conformal normal ordering ⟺ creation-annihilation n.o.


Conformal normal ordering ⟺ creation-annihilation n.o.
not always true!

Conformal normal ordering $\Leftrightarrow$ creation - annihilation n.o.

not always true!

p^μ

we thought about as an annihilation operator

$p^\mu |0\rangle = 0$

$$|z| > |z'|$$

creation anihilation $(z, z')_0$

$$+\frac{\alpha'}{2}\eta^{\mu\nu}\left[-\ln|z|^2 + \sum_{m=1}^{\infty}\frac{1}{m}\left[\left(\frac{z'}{z}\right)^m + \left(\frac{\bar{z}'}{\bar{z}}\right)^m\right]\right]$$

$$\|$$

$$-\frac{\alpha'}{2}\eta^{\mu\nu}\ln|z-z'|^2 = -\frac{\alpha'}{2}\eta^{\mu\nu}\left(\ln|z|^2 + \ln\left|1-\frac{z'}{z}\right|^2\right)$$

$|z| > |z'|$

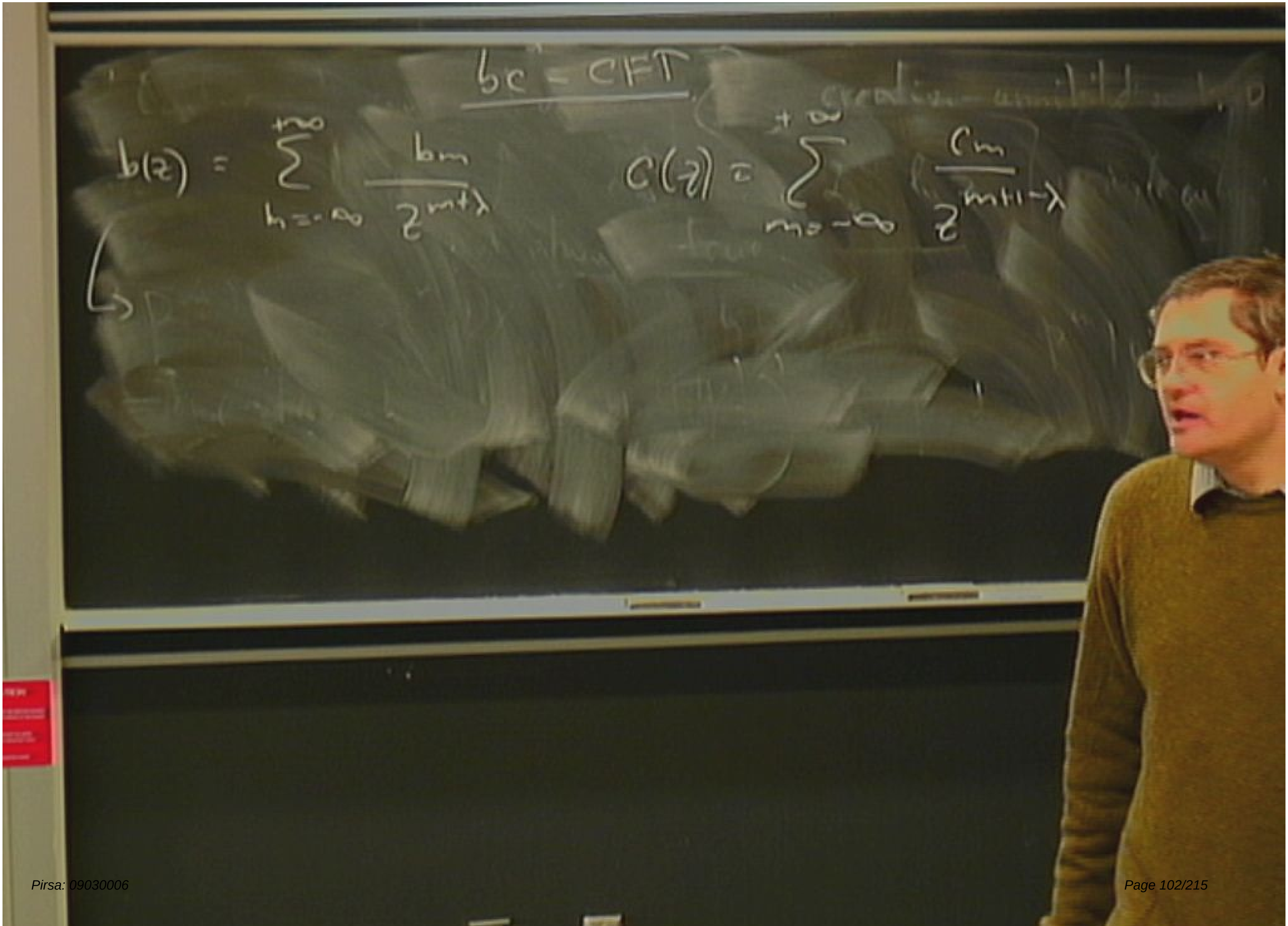
creation anihilation

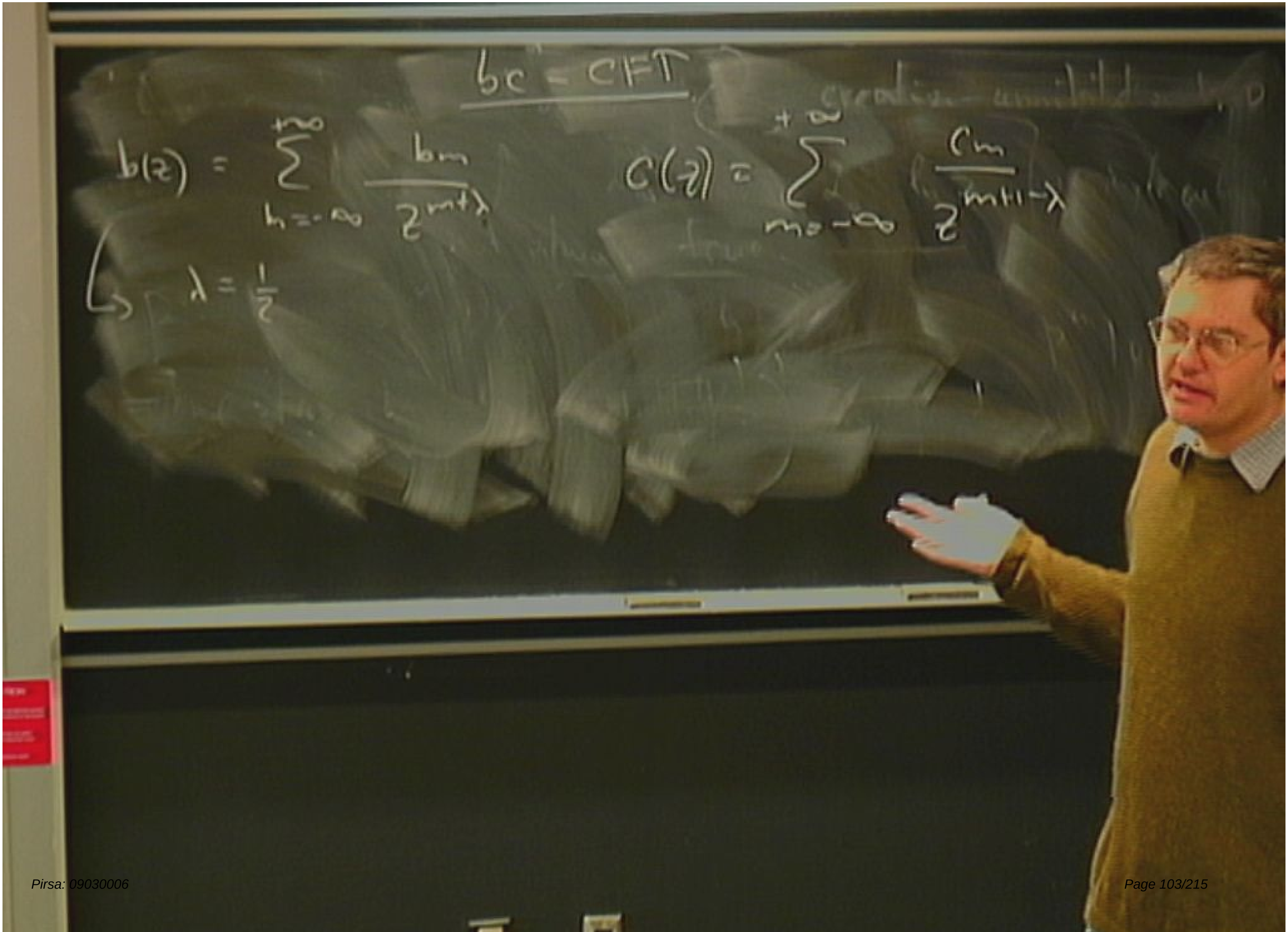
$$+\frac{\alpha'}{2}\eta^{\mu\nu}\left[-\ln|z|^2 + \sum_{m=1}^{\infty}\frac{1}{m}\left[\left(\frac{z'}{z}\right)^m + \left(\frac{\bar{z}'}{\bar{z}}\right)^m\right]\right]$$

$$\|$$

$$-\frac{\alpha'}{2}\eta^{\mu\nu}\ln|z - z'|^2 = -\frac{\alpha'}{2}\eta^{\mu\nu}\left(\ln|z|^2 + \ln\left|1 - \frac{z'}{z}\right|^2\right)$$

$$z - z' = z\left(1 -\right.$$

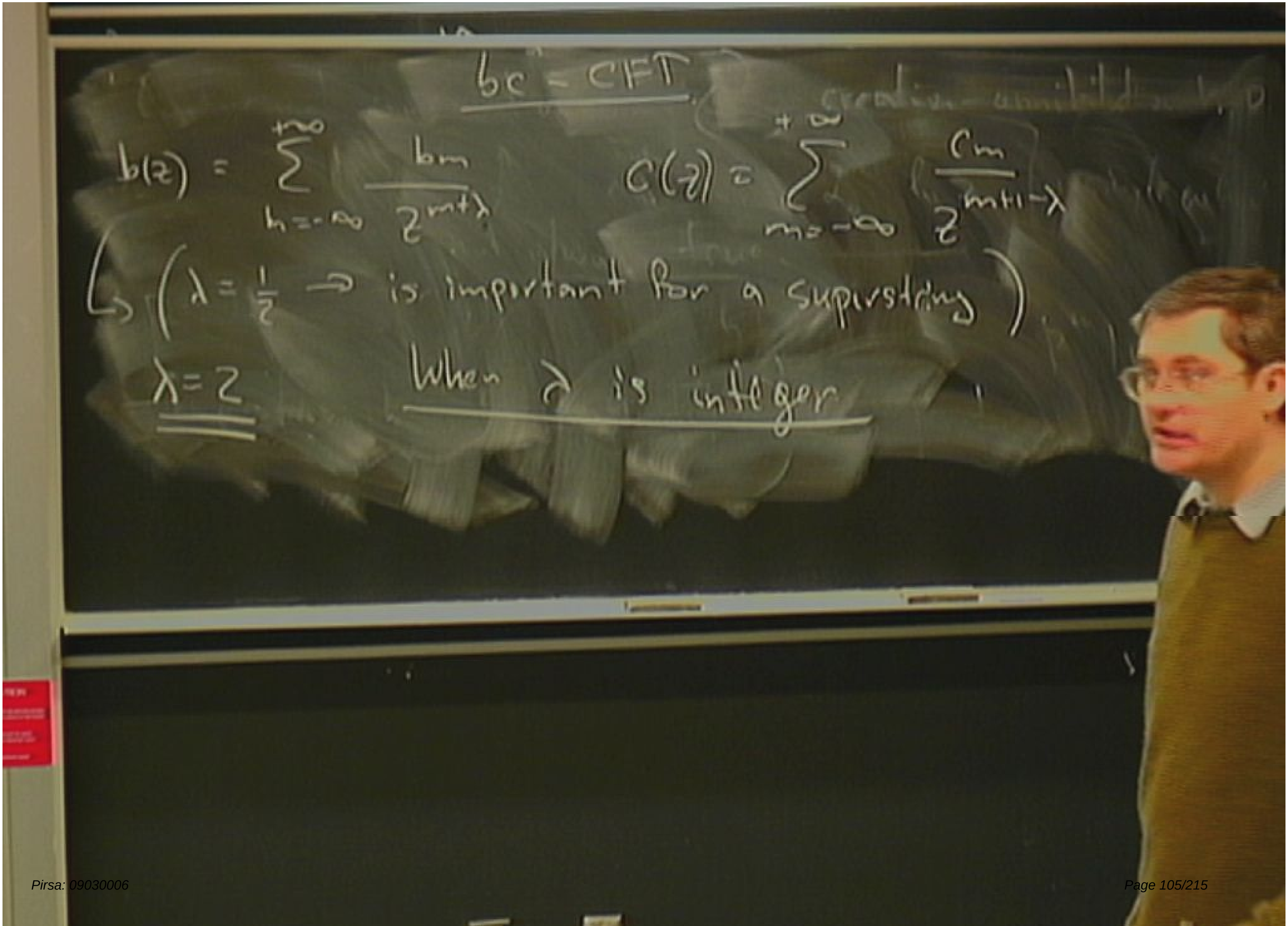

bc - CFT
$b(z) = \sum_{n=-\infty}^{+\infty} \frac{b_m}{z^{m+\lambda}}$
$c(z) = \sum_{m=-\infty}^{+\infty} \frac{c_m}{z^{m+1-\lambda}}$


bc - CFT
$b(z) = \sum_{n=-\infty}^{+\infty} \frac{b_m}{z^{m+\lambda}}$
$c(z) = \sum_{m=-\infty}^{+\infty} \frac{c_m}{z^{m+1-\lambda}}$
$\lambda = \frac{1}{2}$

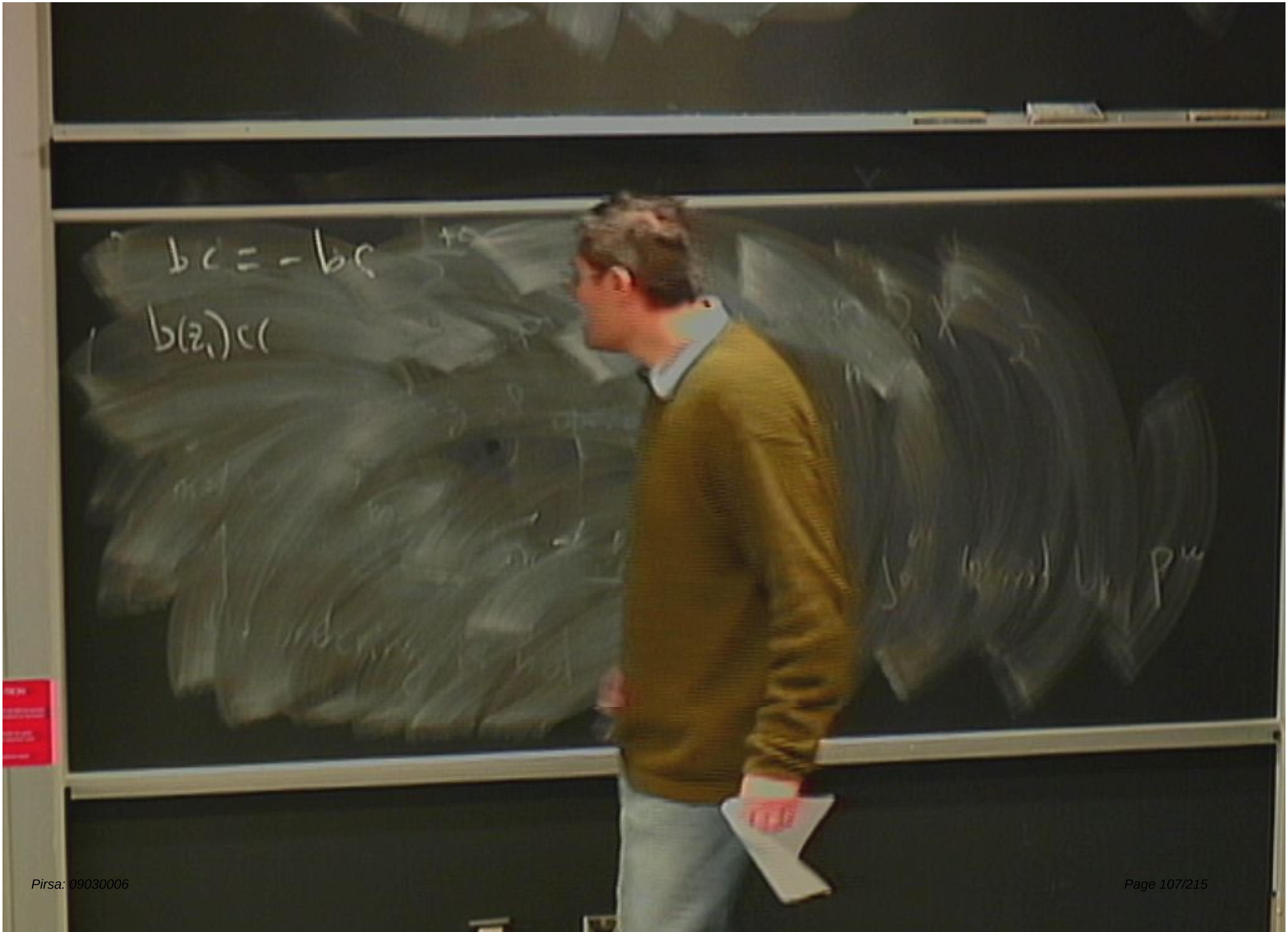
bc - CFT

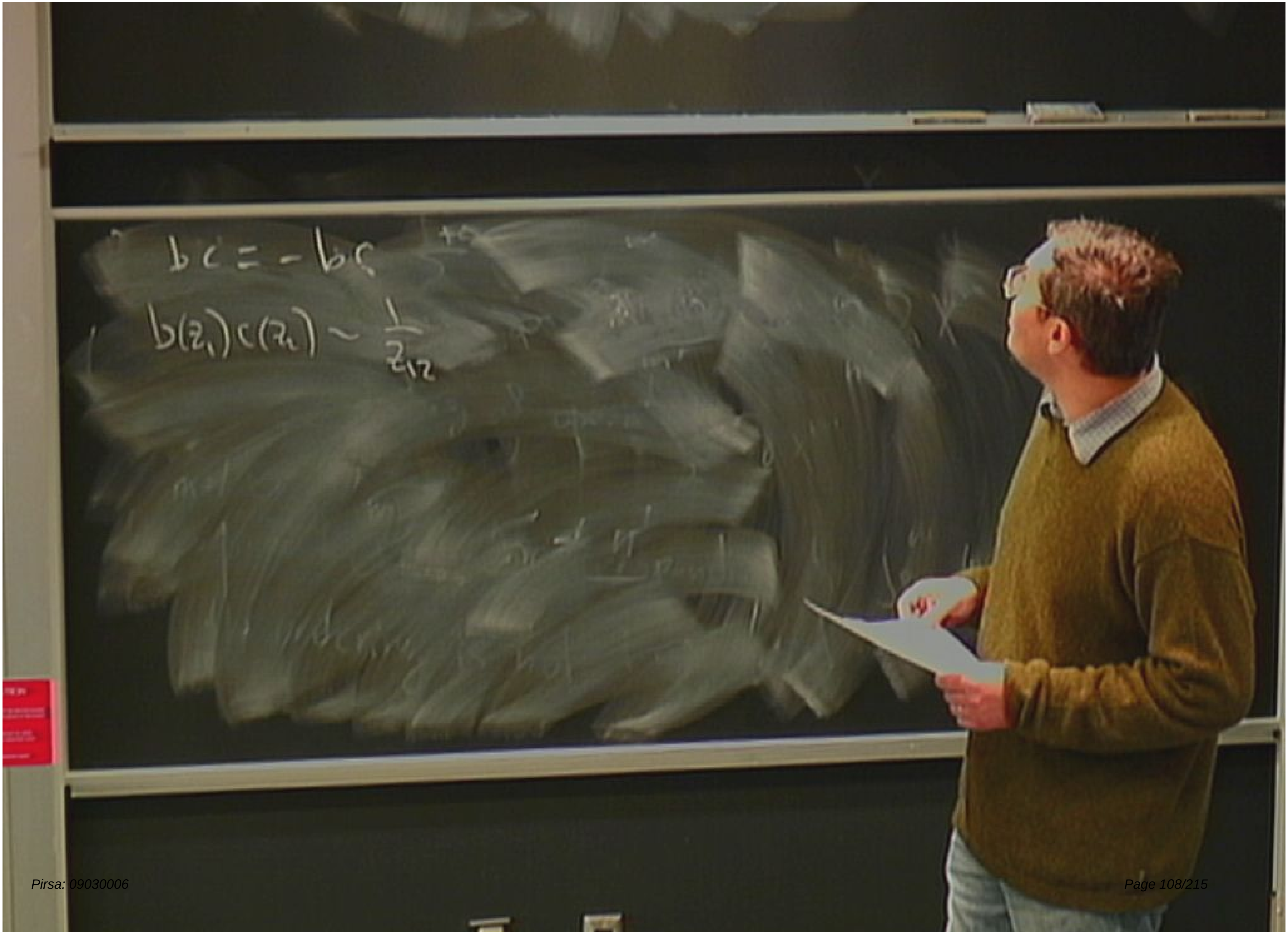
$$b(z) = \sum_{n=-\infty}^{+\infty} \frac{b_m}{z^{m+\lambda}} \qquad c(z) = \sum_{m=-\infty}^{+\infty} \frac{c_m}{z^{m+1-\lambda}}$$

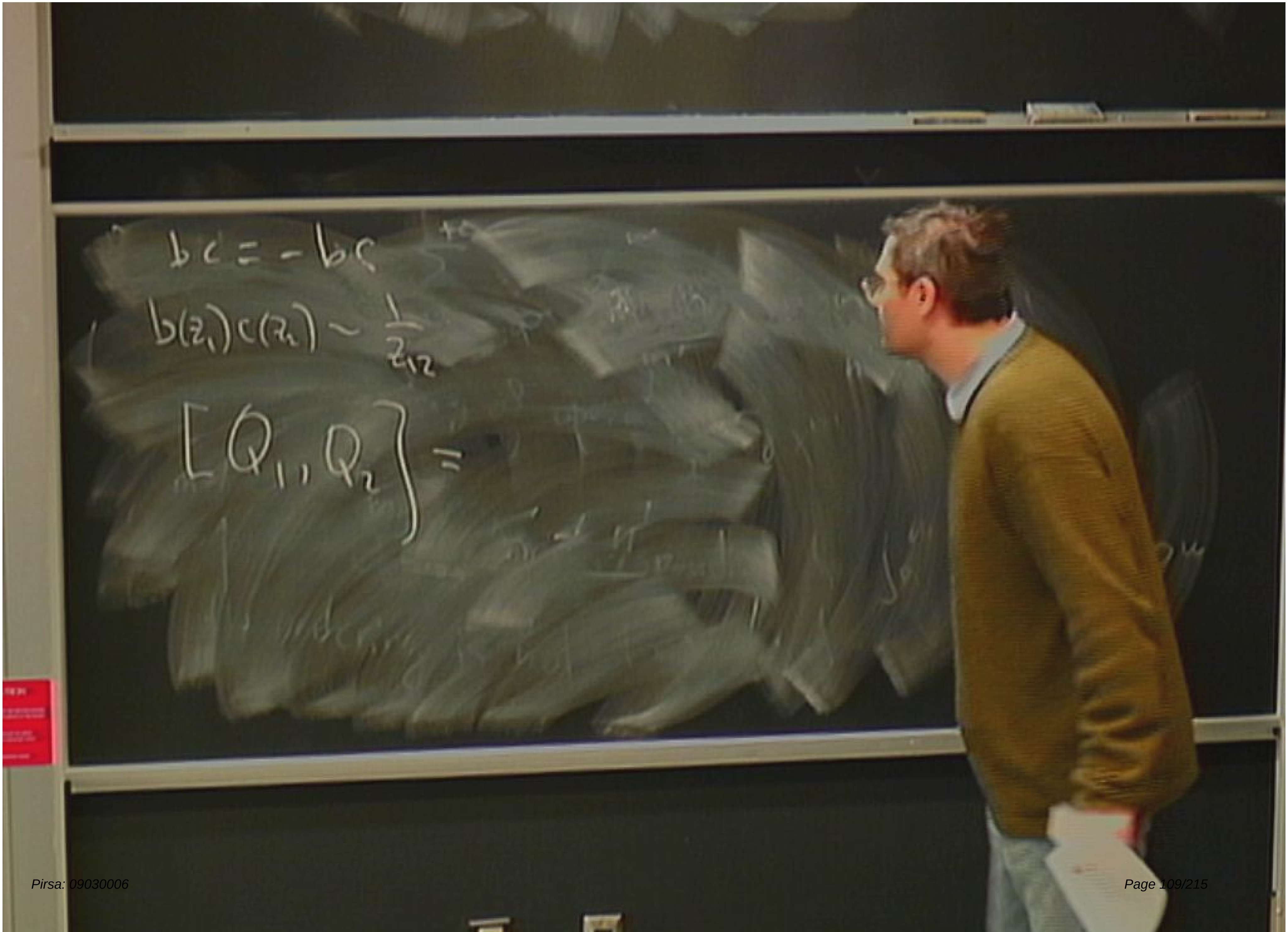
$\left(\lambda = \frac{1}{2} \Rightarrow \right.$ is important for a superstring $\left.\right)$


bc - CFT
$b(z) = \sum_{n=-\infty}^{+\infty} \frac{b_m}{z^{m+\lambda}}$
$c(z) = \sum_{m=-\infty}^{+\infty} \frac{c_m}{z^{m+1-\lambda}}$
$\left(\lambda = \frac{1}{2} \Rightarrow\right.$ is important for a superstring $\left.\right)$
$\lambda = 2$
When λ is integer


$bc = -bc$


bc = -bc
b(z1)c(

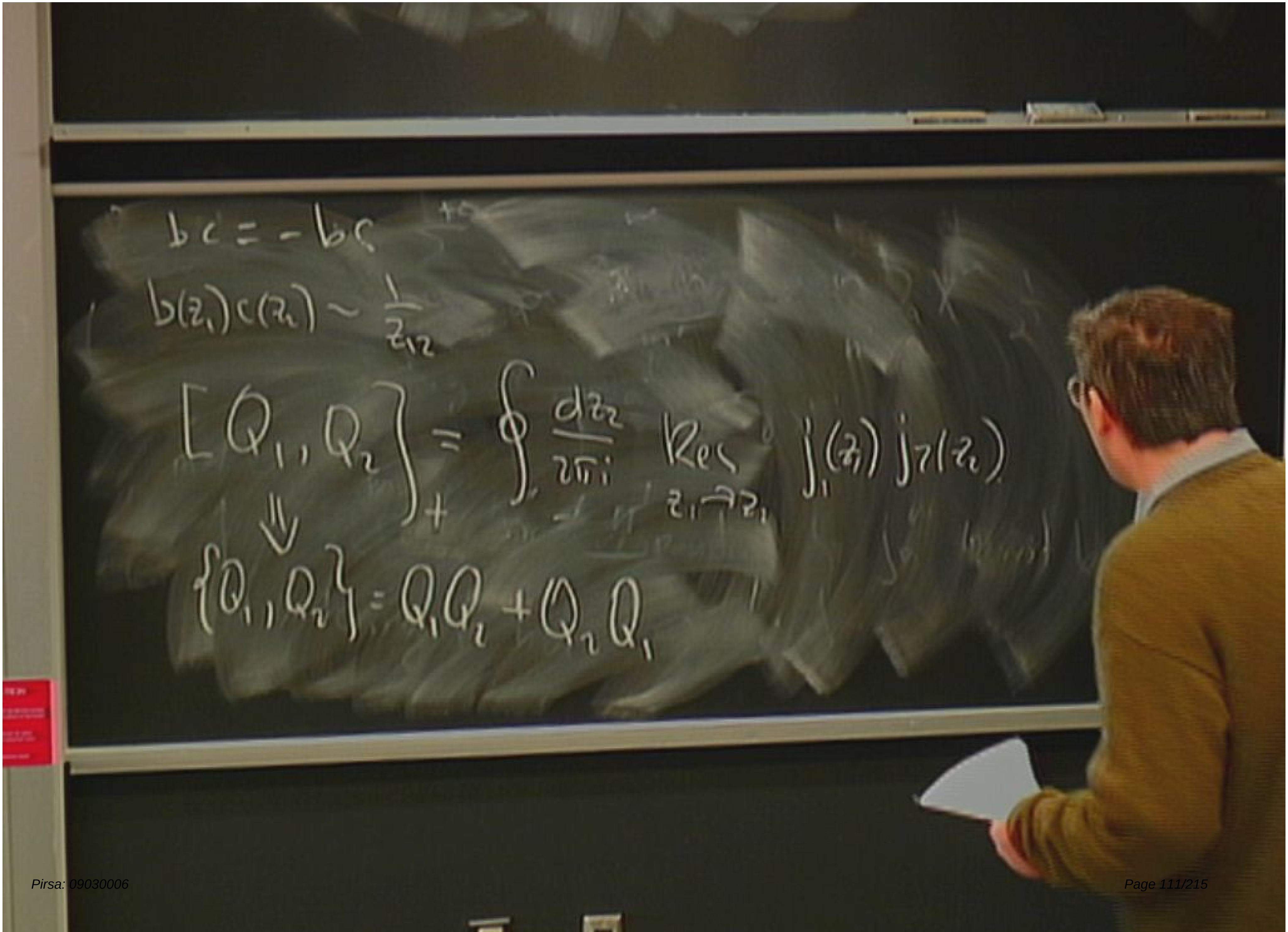

$bc = -bc$
$b(z_1)c(z_2) \sim \frac{1}{z_{12}}$


bc = -bc
b(z₁)c(z₂) ~ 1/z₁₂
[Q₁, Q₂] =

$$bc = -bc$$

$$b(z_1)c(z_2) \sim \frac{1}{z_{12}}$$

$$[Q_1, Q_2]_+ = \oint \frac{dz_2}{2\pi i} \operatorname*{Res}_{z_1 \to z_2} j_1(z_1)\, j_2(z_2)$$


$bc = -bc$
$b(z_1)c(z_2) \sim \frac{1}{z_{12}}$
$[Q_1, Q_2]_+ = \oint \frac{dz_2}{2\pi i} \operatorname{Res}_{z_1 \to z_2} j_1(z_1)\, j_2(z_2)$
$\Downarrow$
$\{Q_1, Q_2\} = Q_1 Q_2 + Q_2 Q_1$

$$bc = -bc$$

$$b(z_1)c(z_2) \sim \frac{1}{z_{12}} \Rightarrow \{b_m, c_n\} = \delta_{m,-n}$$

$$[Q_1, Q_2]_+ = \oint \frac{dz_2}{2\pi i} \operatorname*{Res}_{z_1 \to z_2} j_1(z_1)\, j_2(z_2)$$

$$\Downarrow$$

$$\{Q_1, Q_2\} = Q_1 Q_2 + Q_2 Q_1$$

$n = m = 0 \Rightarrow$

$\hookrightarrow$ the ground state is degenerate:

$|\uparrow\rangle \qquad |\downarrow\rangle$

$n = m = 0 \Rightarrow$

the ground state is degenerate:

$|\uparrow\rangle \quad |\downarrow\rangle$

$b_0|\downarrow\rangle \neq 0 \qquad c_0|\uparrow\rangle \neq 0$

$b_0|\uparrow\rangle = |\downarrow\rangle \qquad c_0|\downarrow\rangle = |\uparrow\rangle$

$n = m = 0 \Rightarrow$

↳ the ground state is degenerate:

$$|\uparrow\rangle \qquad |\downarrow\rangle$$

$$b_0 |\downarrow\rangle = 0 \qquad c_0 |\uparrow\rangle = 0$$

$$b_0 |\uparrow\rangle = |\downarrow\rangle \qquad c_0 |\downarrow\rangle = |\uparrow\rangle$$

$$b_n |\downarrow\rangle = b_n |\uparrow\rangle = c_n |\uparrow\rangle = c_n |\downarrow\rangle = 0, \; n > 0$$

$$b_n|\downarrow\rangle = b_n|\uparrow\rangle = c_n|\uparrow\rangle = c_n|\downarrow\rangle = 0, \quad n>0$$

→ Previously we constructed our Hilbert sp...

$$L_n|0\rangle = 0, \quad n>0 \iff \alpha^\mu_n|0\rangle = 0, \quad n>0.$$

$$p^\mu|0\rangle = 0$$

$$L_{\pm 1}|0\rangle = 0 \implies 2L_0|0\rangle = 2[L_1, L_{-1}]|0\rangle$$

$$= 2(L_1 L_{-1} - L_{-1}L_1)|0\rangle = 0$$

$$b_n|\downarrow\rangle = b_n|\uparrow\rangle = c_n|\uparrow\rangle = c_n|\downarrow\rangle = 0, \quad n > 0$$

Given mode expansion for $b(z)$, $c(z)$ $\rightarrow$

$|0\rangle = 0$

$$b_n|\downarrow\rangle = b_n|\uparrow\rangle = c_n|\uparrow\rangle = c_n|\downarrow\rangle = 0, \ n \geq 0$$

Given mode expansion for $b(z)$, $c(z)$ $\Rightarrow$

$$L_m = \sum_{n=-\infty}^{+\infty} (m\lambda - n) \, \vdots$$

$|0\rangle = 0$

$$b_n|\downarrow\rangle = b_n|\uparrow\rangle = c_n|\uparrow\rangle = c_n|\downarrow\rangle = 0, \ n > 0$$

Given mode expansion for $b(z), c(z) \Rightarrow$

$$L_m = \sum_{n=-\infty}^{+\infty} (m\lambda - n) \,{}^{\circ}_{\circ}\, b_n c_{m-n} \,{}^{\circ}_{\circ} + \delta_{m,0}\, a_g$$

$|0\rangle = 0$

$$b_n|\downarrow\rangle = b_n|\uparrow\rangle = c_n|\uparrow\rangle = c_n|\downarrow\rangle = 0, \quad n > 0$$

Given mode expansion for $b(z)$, $c(z)$ $\longrightarrow$

normal ordering constant

$$L_m = \sum_{n=-\infty}^{+\infty} (m\lambda - n) :b_n c_{m-n}: + \delta_{m,0}\, a_g$$

$|0\rangle = 0$

$$b_n|\downarrow\rangle = b_n|\uparrow\rangle = c_n|\uparrow\rangle = c_n|\downarrow\rangle = 0, \; n > 0$$

Given mode expansion for $b(z), c(z) \rightarrow$

$$L_m = \sum_{n=-\infty}^{+\infty} (m\lambda - n) :b_n c_{m-n}: + \delta_{m,0}\, a_g$$

normal ordering constant

$|0\rangle = 0$

$$b_n|\Downarrow\rangle = b_n|\Uparrow\rangle = c_n|\Uparrow\rangle = c_n|\Downarrow\rangle = 0, \quad n > 0$$

Given mode expansion for $b(z)$, $c(z)$ $\longrightarrow$

$$L_m = \sum_{n=-\infty}^{+\infty} (m\lambda - n) :b_n c_{m-n}: + \delta_{m,0}\, a_g$$

normal ordering constant

$|0\rangle = 0$

$$b_n|\downarrow\rangle = b_n|\uparrow\rangle = c_n|\uparrow\rangle = c_n|\downarrow\rangle = 0, \; n > 0$$

Given mode expansion for $b(z)$, $c(z)$ $\longrightarrow$

$$L_m = \sum_{n=-\infty}^{+\infty} (m\lambda - n) :b_n c_{m-n}: + \delta_{m,0}\, a_g$$

normal ordering constant

$$2L_0 = [L_1, L_{-1}]$$

$|0\rangle = 0$

$$b_n|\downarrow\rangle = b_n|\uparrow\rangle = c_n|\uparrow\rangle = c_n|\downarrow\rangle = 0, \quad n \geq 0$$

Given mode expansion for $b(z)$, $c(z)$ $\longrightarrow$

normal ordering constant

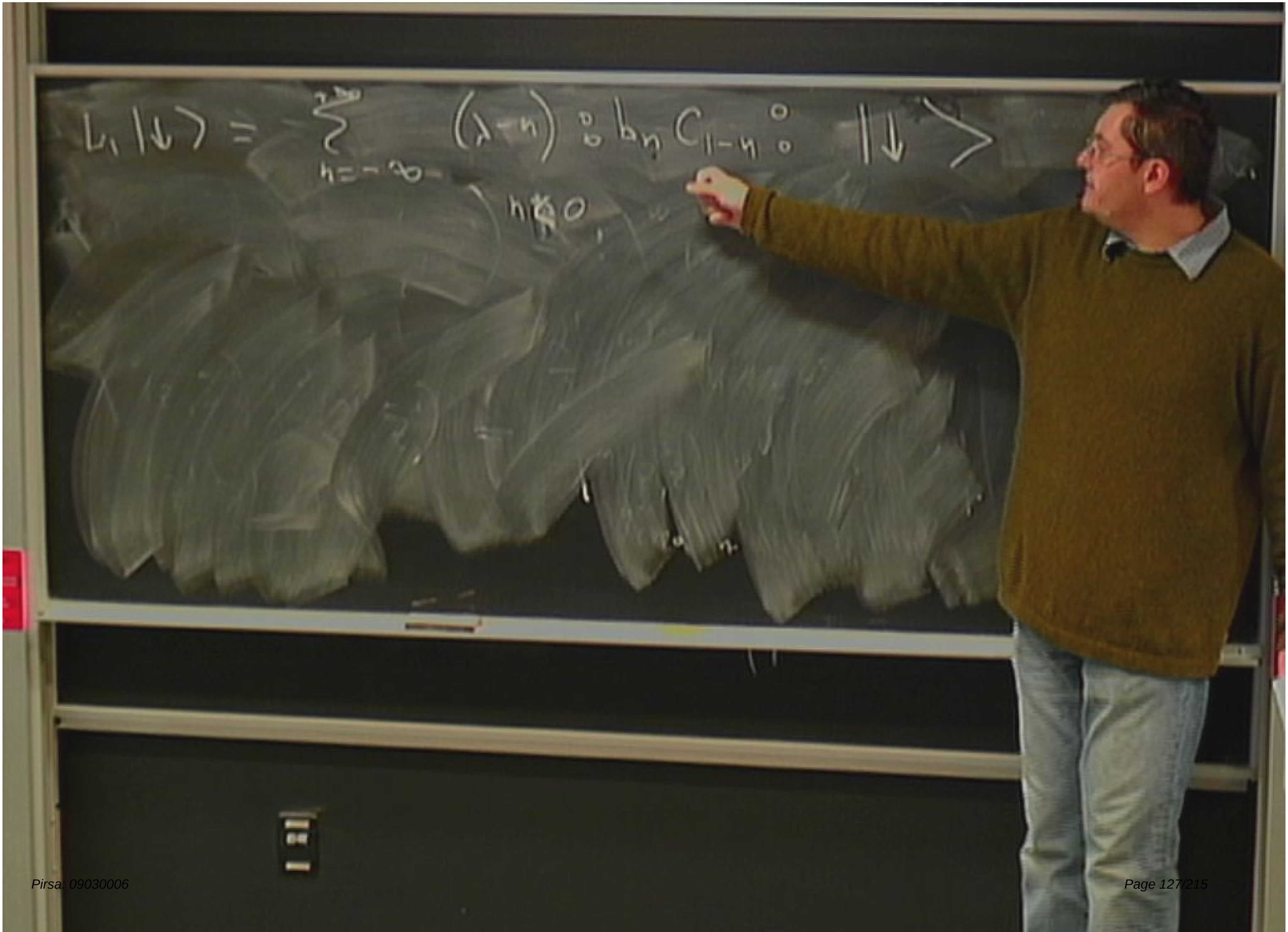
$$L_m = \sum_{n=-\infty}^{+\infty} (m\lambda - n) :b_n c_{m-n}: + \delta_{m,0}\, a_g$$

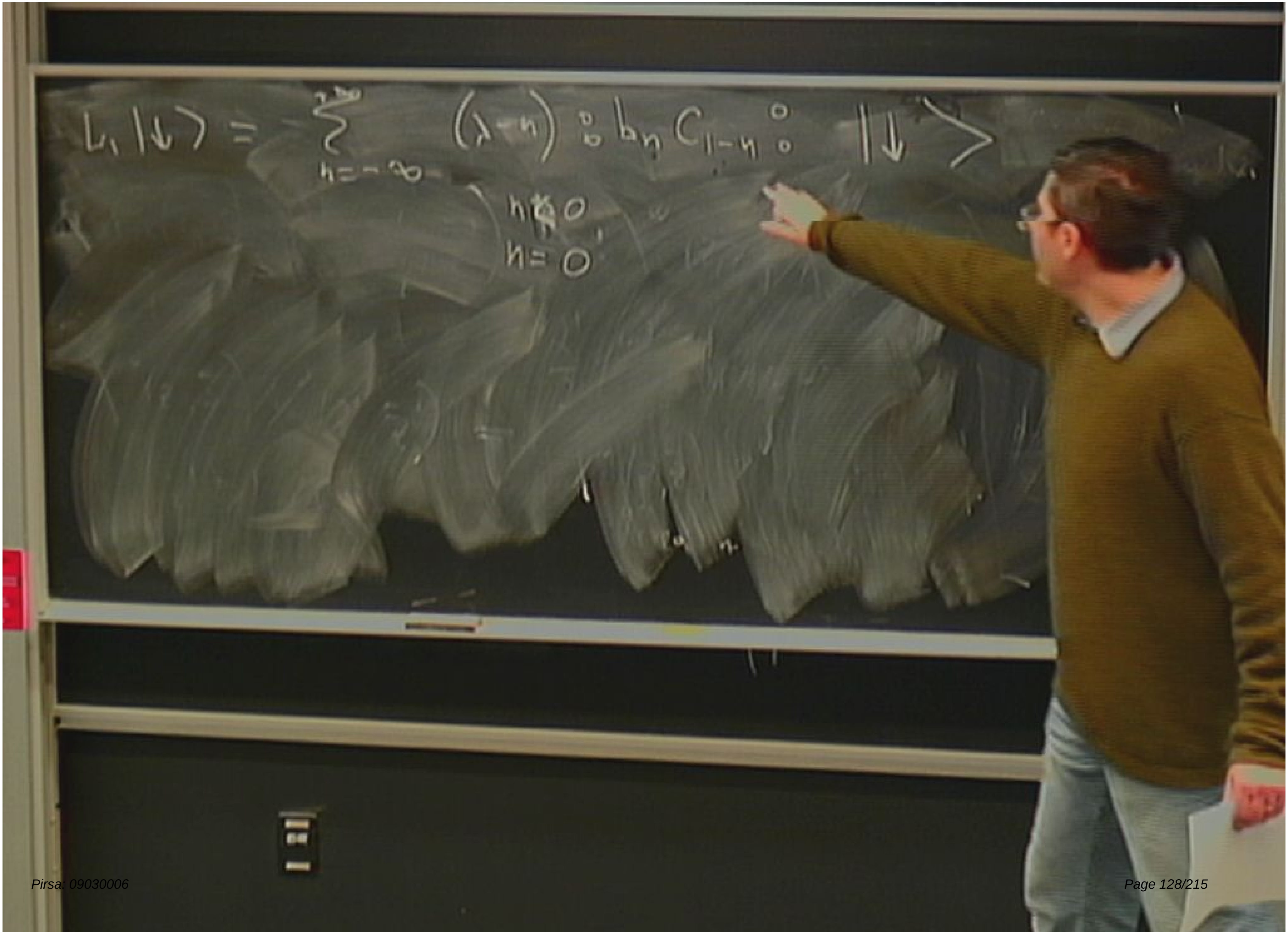
$$2L_0 = [L_1, L_{-1}]$$

$\hookrightarrow$ study $|\downarrow\rangle$

$|0\rangle = 0$

$$L_1 |\Downarrow\rangle = \sum_{n=-\infty}^{\infty} (\lambda - n) : b_n c_{1-n} : \, |\Downarrow\rangle$$


$L_1 |\downarrow\rangle = \sum_{n=-\infty}^{+\infty} (\lambda - n) :b_n C_{1-n}: |\downarrow\rangle$
$n \neq 0$


L₁ |↓⟩ = Σ_{n=−∞}^{+∞} (λ−n) :b_n C_{1−n}: |↓⟩
n≠0
n=0

$$L_1 |\downarrow\rangle = \sum_{n=-\infty}^{\infty} (\lambda - n) :b_n c_{1-n}: |\downarrow\rangle$$

$n \neq 0$

$n = 0$ $\quad :b_0 c_1:$

$$L_1 |\downarrow\rangle = \sum_{n=-\infty}^{\infty} (\lambda - n) :b_n c_{1-n}: |\downarrow\rangle$$

$n \neq 0$

$n = 0$ $\quad :b_0 c_1: |\downarrow\rangle = 0$

$$L_1 |\downarrow\rangle = \sum_{n=-\infty}^{+\infty} (\lambda - n) :b_n c_{1-n}: |\downarrow\rangle$$

$n < 0$

$n = 0$ $\quad :b_0 c_1: |\downarrow\rangle = 0$

$n > 0$ $\quad :b_n c_{1-n}: = c_{1-n} b_n |\downarrow\rangle = 0$

$$b_n|\Downarrow\rangle = b_n|\Uparrow\rangle = c_n|\Uparrow\rangle = c_n|\Downarrow\rangle = 0, \; n > 0$$

Given mode expansion for $b(z)$, $c(z)$ $\Rightarrow$

$$L_m = \sum_{n=-\infty}^{+\infty} (m\lambda - n) \, {}^{\circ}_{\circ} b_n c_{m-n} {}^{\circ}_{\circ} + \delta_{m,0}\, a_g$$

normal ordering constant

$$2L_0 = [L_1, L_{-1}] = L_1 L_{-1} - L_{-1} L_1$$

$\hookrightarrow$ study $|\Downarrow\rangle$

$n \neq 0$

$n = 0 \quad :b_0 c_{1}: |\downarrow\rangle = 0$

$n > 0 \quad :b_n c_{1-n}: = c_{1-n} b_n |\downarrow\rangle = 0$

$$L_1 L_{-1} |\downarrow\rangle = L_1 \sum_{n=-\infty}^{+\infty} (-\lambda - n)\, b_n c_{-1-n}$$

$$= -\frac{1}{2}\eta \left(\ln |z| + \ln \left| 1 - \frac{z'}{z} \right| \right)$$

$z - z' = z(1-$

$n \neq 0$

$n = 0 \quad {}^{\circ}_{\circ} b_0 c_{1+n} {}^{\circ}_{\circ} |\Downarrow\rangle = 0$

$n > 0 \quad {}^{\circ}_{\circ} b_n c_{1-n} {}^{\circ}_{\circ} = c_{1-n} b_n |\Downarrow\rangle = 0$

$$L_1 L_{-1} |\Downarrow\rangle = L_1 \sum_{n=-\infty}^{+\infty} (-\lambda - n)\, b_n c_{-1-n} |\Downarrow\rangle$$

$$z - z' = z\left(1 - \right.$$

$n \neq 0$

$n = 0 \quad : b_0 c_{1+p} : |\downarrow\rangle = 0$

$n > 0 \quad : b_n c_{1-n} : = c_{1-n} b_n |\downarrow\rangle = 0$

$$L_1 L_{-1} |\downarrow\rangle = L_1 \sum_{n=-\infty}^{+\infty} (-\lambda - n)\, b_n \, c_{-1-n} |\downarrow\rangle$$

$$= L_1 \, b_{-1} c_0 |\downarrow\rangle$$

$$\ln|z - z'| = -\frac{\alpha'}{2}\eta \left(\ln|z| + \ln\left|1 - \frac{z'}{z}\right| \right)$$

$z - z' = z(1 -$

$n \neq 0$

$n=0 \quad :b_0 c_1: |\downarrow\rangle = 0$

$n>0 \quad :b_n c_{1-n}: = c_{1-n} b_n |\downarrow\rangle = 0$

$$L_1 L_{-1} |\downarrow\rangle = L_1 \sum_{n=-\infty}^{+\infty} (-\lambda - n)\, b_n c_{-1-n} |\downarrow\rangle$$

$n=-1$

$$= L_1 b_{-1} c_0 |\downarrow\rangle =$$

$$2\,(\ln|z-z'| = -\frac{1}{2}\eta\left(\ln|z| + \ln\left|1-\frac{z'}{z}\right|\right)$$

$$z - z' = z(1-$$

$$L_1 |\downarrow\rangle = \sum_{n=-\infty} (\lambda - n) :b_n C_{1-n}: |\downarrow\rangle = 0$$

$n \neq 0$

$n = 0$ $\quad :b_0 C_1: |\downarrow\rangle = 0$

$n > 0$ $\quad :b_n C_{1-n}: = C_{1-n} b_n |\downarrow\rangle = 0$

$$L_1 L_{-1} |\downarrow\rangle = L_1 \sum_{n=-\infty}^{+\infty} (-\lambda - n)\, b_n C_{-1-n} |\downarrow\rangle$$

$n = -1$

$$= L_1\, b_{-1} C_0 |\downarrow\rangle = \lambda\, b_0 C_1$$

$$z' \quad \ln|z - z'| = -\frac{1}{2}\eta \left(\ln |z| + \ln\left|1 - \frac{z'}{z}\right| \right)$$

$$z - z' = z\left(1 - \right.$$

$$L_1 |\downarrow\rangle = \sum_{n=-\infty} (\lambda - n) :b_n c_{1-n}: |\downarrow\rangle = 0$$

$n \neq 0$

$n = 0 \quad :b_0 c_1: |\downarrow\rangle = 0$

$n > 0 \quad :b_n c_{1-n}: = c_{1-n} b_n |\downarrow\rangle = 0$

$$L_1 L_{-1} |\downarrow\rangle = L_1 \sum_{n=-\infty}^{+\infty} (-\lambda - n)\, b_n c_{-1-n} |\downarrow\rangle$$

$n = -1$

$$= L_1\, b_{-1} c_0 |\downarrow\rangle = \lambda\, b_0 c_1\, b_{-1} c_0 |\downarrow\rangle$$

$$\ln|z - z'| = -\frac{1}{2}\eta\left(\ln|z| + \ln\left|1 - \frac{z'}{z}\right|\right)$$

$z - z' = z(1 -$

$$L_1 |\downarrow\rangle = \sum_{n=-\infty}^{\infty} (\lambda - n) :b_n C_{1-n}: |\downarrow\rangle = 0$$

$n \neq 0$

$n = 0 \quad :b_0 C_{1}: |\downarrow\rangle = 0$

$n > 0 \quad :b_n C_{1-n}: = C_{1-n} b_n |\downarrow\rangle = 0$

$$L_1 L_{-1} |\downarrow\rangle = L_1 \sum_{n=-\infty}^{+\infty} (-\lambda - n)\, b_n C_{-1-n} |\downarrow\rangle$$

$n = -1$

$$= L_1 (-1-\lambda) b_{-1} C_0 |\downarrow\rangle = \lambda (-1-\lambda) b_0 C_1 b_{-1} C_0 |\downarrow\rangle$$

$$z' \quad \ln|z - z'| = -\frac{\alpha'}{2}\eta \left(\ln|z| + \ln\left|1 + \frac{z'}{z}\right| \right)$$

$$z - z' = z(1 -$$

$$= L_1(-\lambda)\, b_{-1} c_0 |\downarrow\rangle = \lambda\, b_0 c_1\, b_{-1} c_0 |\downarrow\rangle$$

$$= (1-\lambda)\,\lambda\; b_0 c_1\, b_{-1} c_0 |\downarrow\rangle$$

$$= \lambda\, b_0 c_1\, b_{-1} c_0 |\downarrow\rangle$$

$$= (1-\lambda)\lambda\, b_0 \underbrace{c_1 b_{-1}} c_0 |\downarrow\rangle = (1-\lambda)\lambda$$

$$\{c_1, b_{-1}\} = 1$$

$$c_1 b_{-1} = 1 - b_{-1} c_1$$

$$= \ldots b_{-1} c_0 |\downarrow\rangle = \lambda\, b_0 c_1 b_{-1} c_0 |\downarrow\rangle$$

$$= (1-\lambda)\lambda\; b_0 \underbrace{c_1 b_{-1}} c_0 |\downarrow\rangle \quad = (1-\lambda)\lambda$$

$$\{c_1, b_{-1}\} = 1$$

$$c_1 b_{-1} = 1 - b_{-1} c_1$$

$$= \lambda b_0 c_1 b_{-1} c_0 | \Downarrow \rangle$$

$$= (1-\lambda)\lambda \; b_0 c_1 \underbrace{b_{-1}} c_0 | \Downarrow \rangle \quad = (1-\lambda)\lambda \; b_0 c_0 | \Downarrow \rangle$$

$$\{c_1, b_{-1}\} = 1$$

$$c_1 b_{-1} = 1 - b_{-1} c_1$$

$$= L_1 (1-\lambda) b_{-1} c_0 |\Downarrow\rangle = \lambda b_0 c_1 b_{-1} c_0 |\Downarrow\rangle$$

$$= (1-\lambda)\lambda\, b_0 \underbrace{c_1 b_{-1}} c_0 |\Downarrow\rangle = (1-\lambda)\lambda\, b_0 c_0 |\Downarrow\rangle$$

$$\{c_1, b_{-1}\} = 1$$

$$c_1 b_{-1} = 1 - b_{-1} c_1$$

$$= (1-\lambda)\lambda\, |\Downarrow\rangle$$

$$= L_1 (1-\lambda) b_{-1} c_0 |\downarrow\rangle = \lambda b_0 c_1 b_{-1} c_0 |\downarrow\rangle$$

$$= (1-\lambda)\lambda\, b_0 c_1 b_{-1} c_0 |\downarrow\rangle = (1-\lambda)\lambda\, b_0 c_0 |\downarrow\rangle$$

$$\{c_1, b_{-1}\} = 1 \qquad = (1-\lambda)\lambda |\downarrow\rangle \equiv$$

$$c_1 b_{-1} = 1 - b_{-1} c_1 \qquad \equiv 2L_0 |\downarrow\rangle$$

$$= L_1 (1-\lambda) b_{-1} c_0 |\downarrow\rangle = \lambda b_0 c_1 b_{-1} c_0 |\downarrow\rangle$$

$$= (1-\lambda)\lambda\, b_0 \underbrace{c_1 b_{-1}} c_0 |\downarrow\rangle = (1-\lambda)\lambda\, \cancel{b_0} c_0 |\downarrow\rangle$$

$$\{c_1, b_{-1}\} = 1$$

$$c_1 b_{-1} = 1 - b_{-1} c_1$$

$$= (1-\lambda)\lambda |\downarrow\rangle \equiv$$

$$\equiv 2L_0 |\downarrow\rangle$$

$$= 2q^{g} |\downarrow\rangle$$

$$= L_1 (1-\lambda) b_{-1} c_0 |\Downarrow\rangle = \lambda b_0 c_1 b_{-1} c_0 |\Downarrow\rangle$$

$$= (1-\lambda)\lambda\, b_0 \underbrace{c_1 b_{-1}} c_0 |\Downarrow\rangle = (1-\lambda)\lambda\, b_0 c_0 |\Downarrow\rangle$$

$$\{c_1, b_{-1}\} = 1$$

$$c_1 b_{-1} = 1 - b_{-1} c_1$$

$$= \underline{\underline{(1-\lambda)\lambda}}\, |\Downarrow\rangle \equiv 2L_0 |\Downarrow\rangle = \underline{\underline{2a^g}}\, |\Downarrow\rangle$$

$$a^g = \frac{1}{2}\lambda(1-\lambda)$$

$$= L_1 (1-\lambda) b_{-1} c_0 |\downarrow\rangle = \lambda b_0 c_1 b_{-1} c_0 |\downarrow\rangle$$

$$= (1-\lambda)\lambda \, b_0 c_1 b_{-1} c_0 |\downarrow\rangle = (1-\lambda)\lambda \, b_0 c_0 |\downarrow\rangle$$

$$\{c_1, b_{-1}\} = 1$$

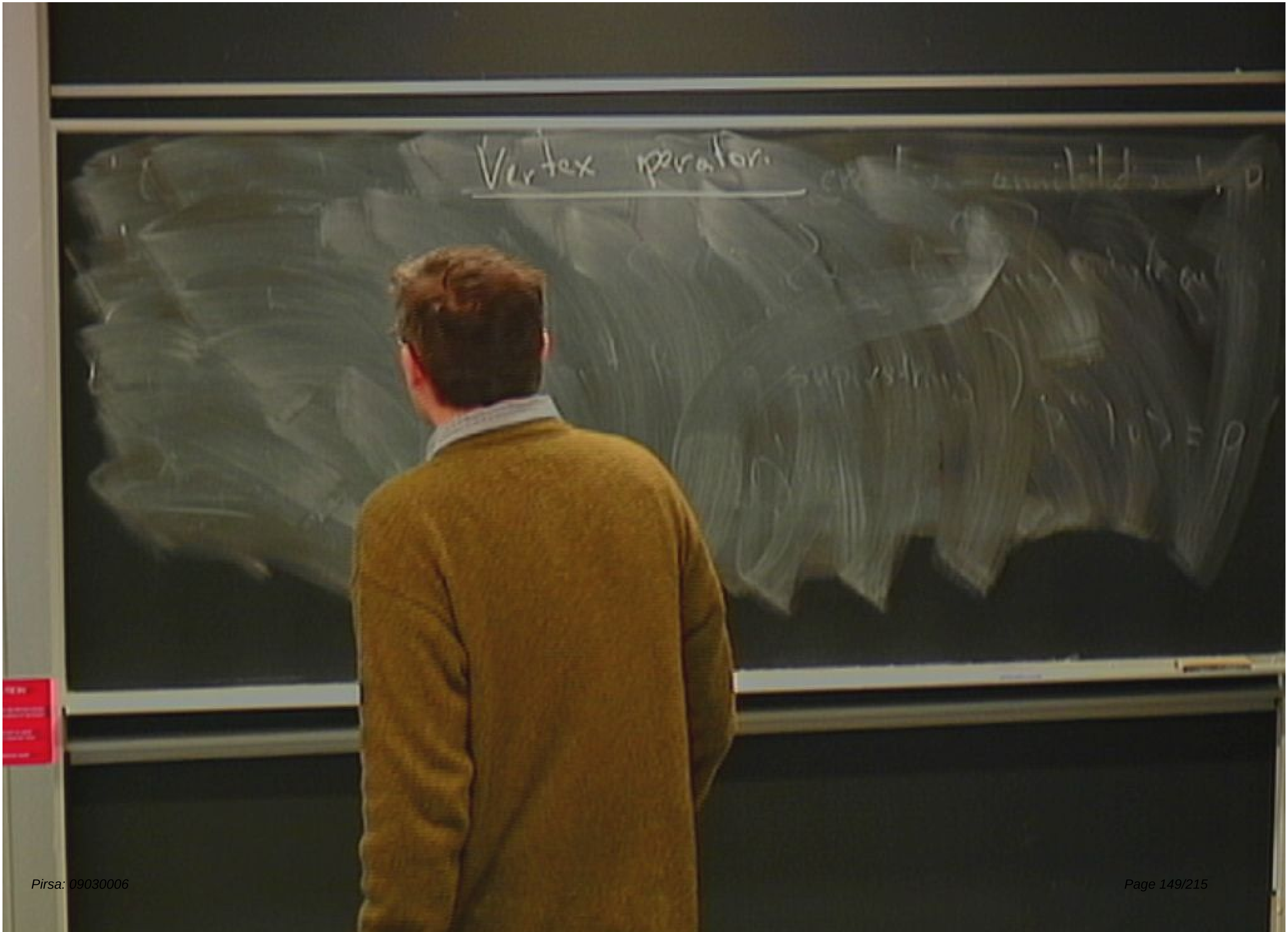
$$c_1 b_{-1} = 1 - b_{-1} c_1$$

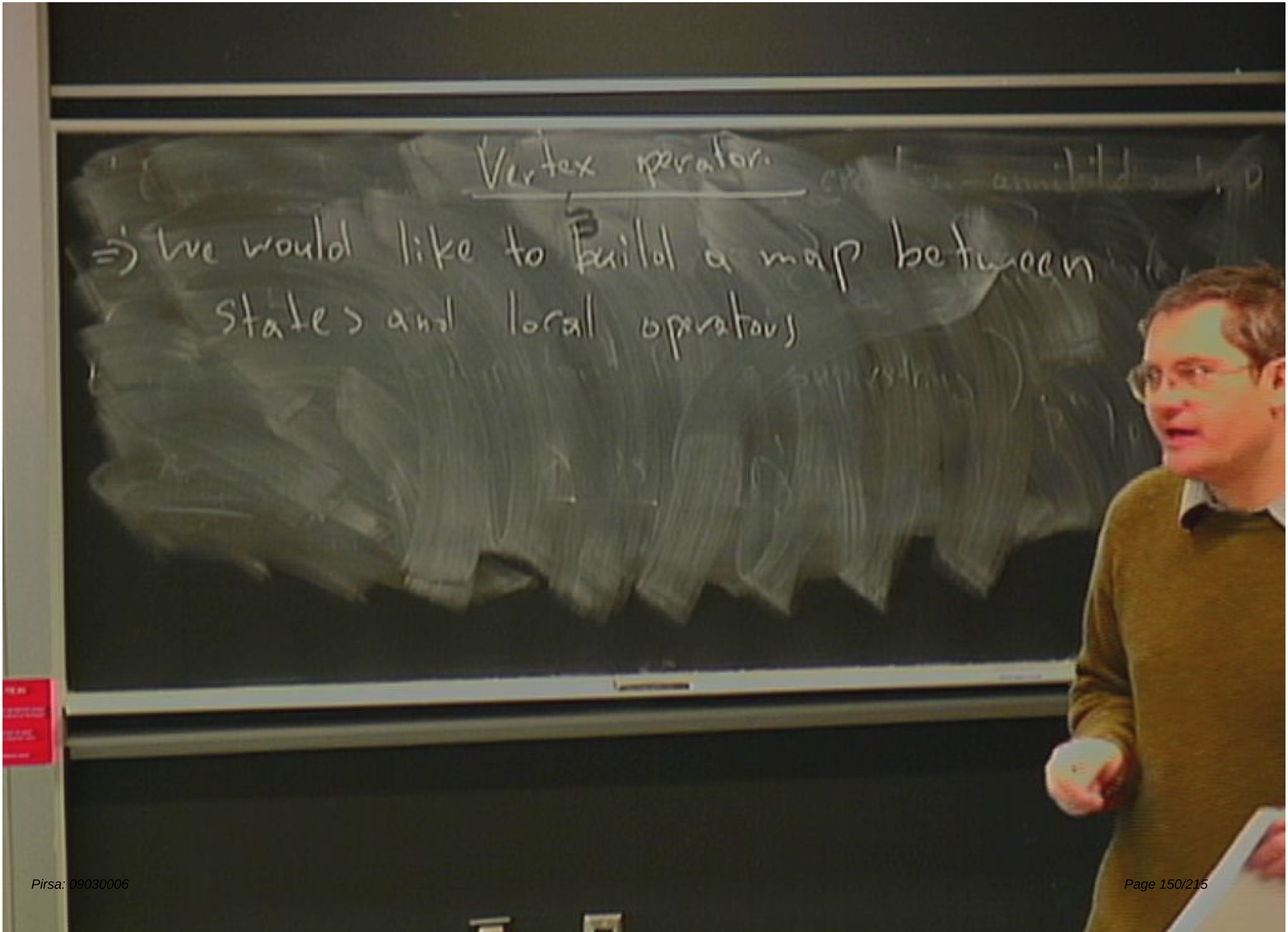
$$= \underline{\underline{(1-\lambda)\lambda}} |\downarrow\rangle \equiv$$

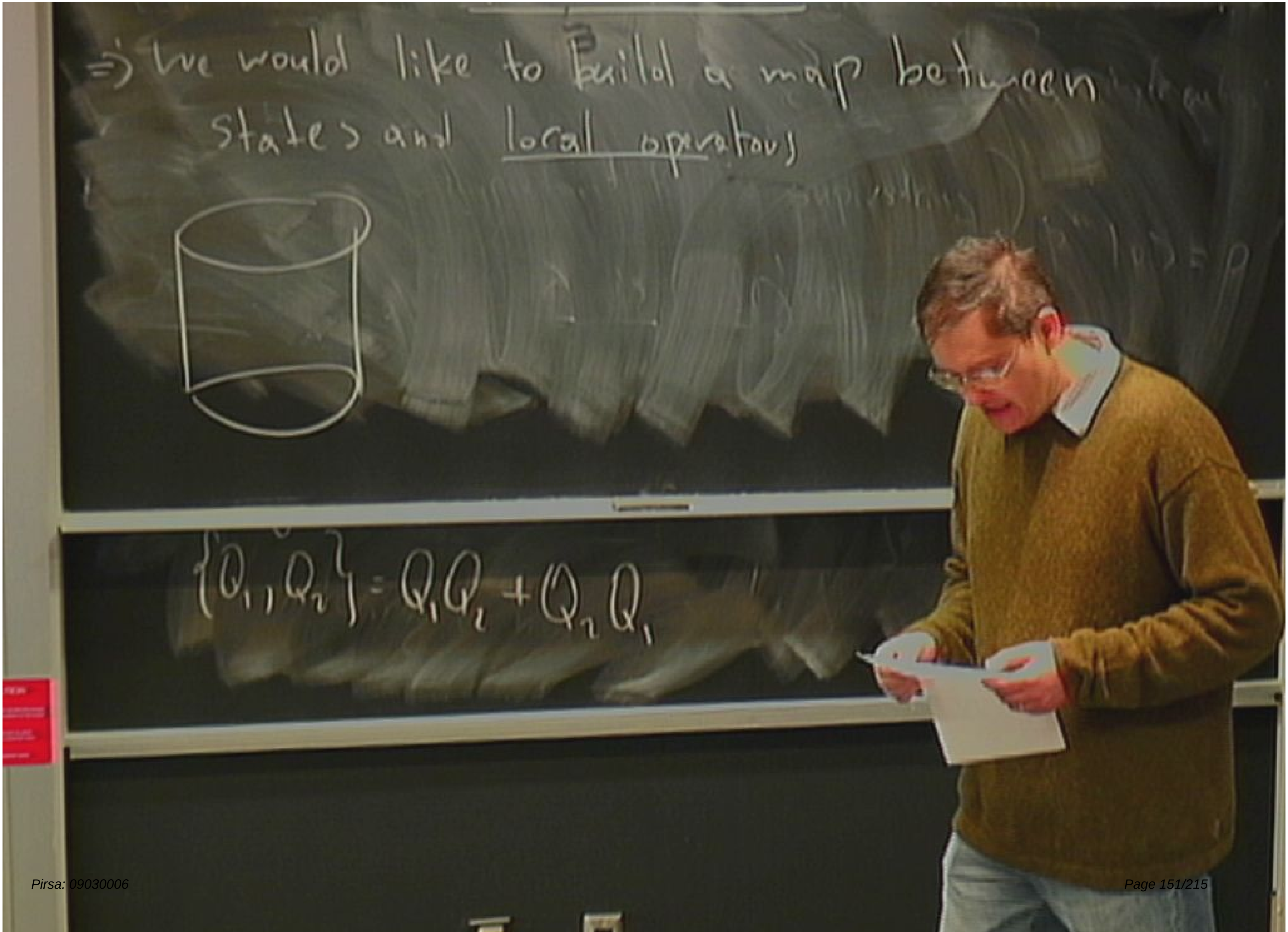
$$\equiv 2L_0 |\downarrow\rangle$$

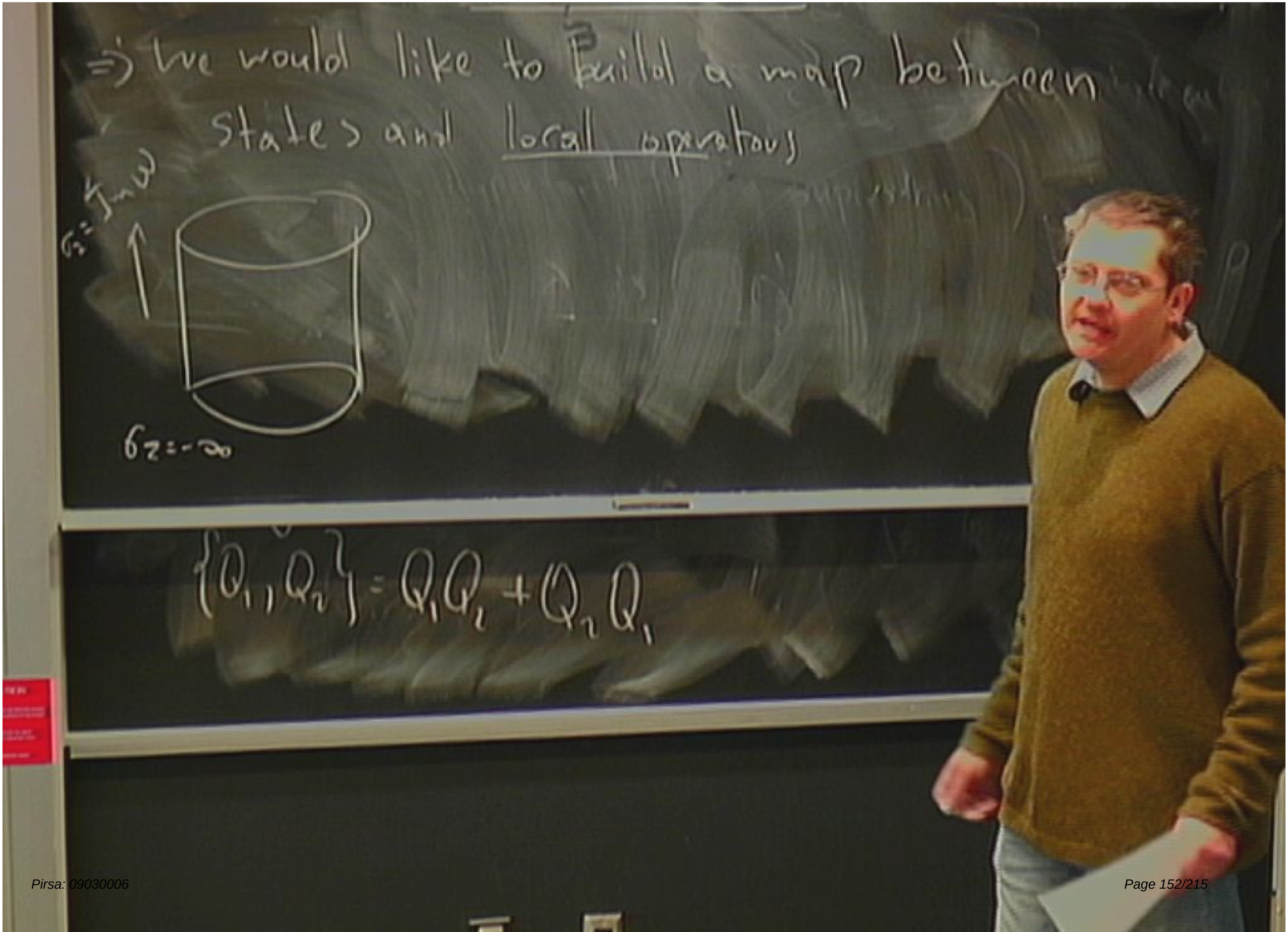
$$= 2\underline{\underline{a^g}} |\downarrow\rangle$$

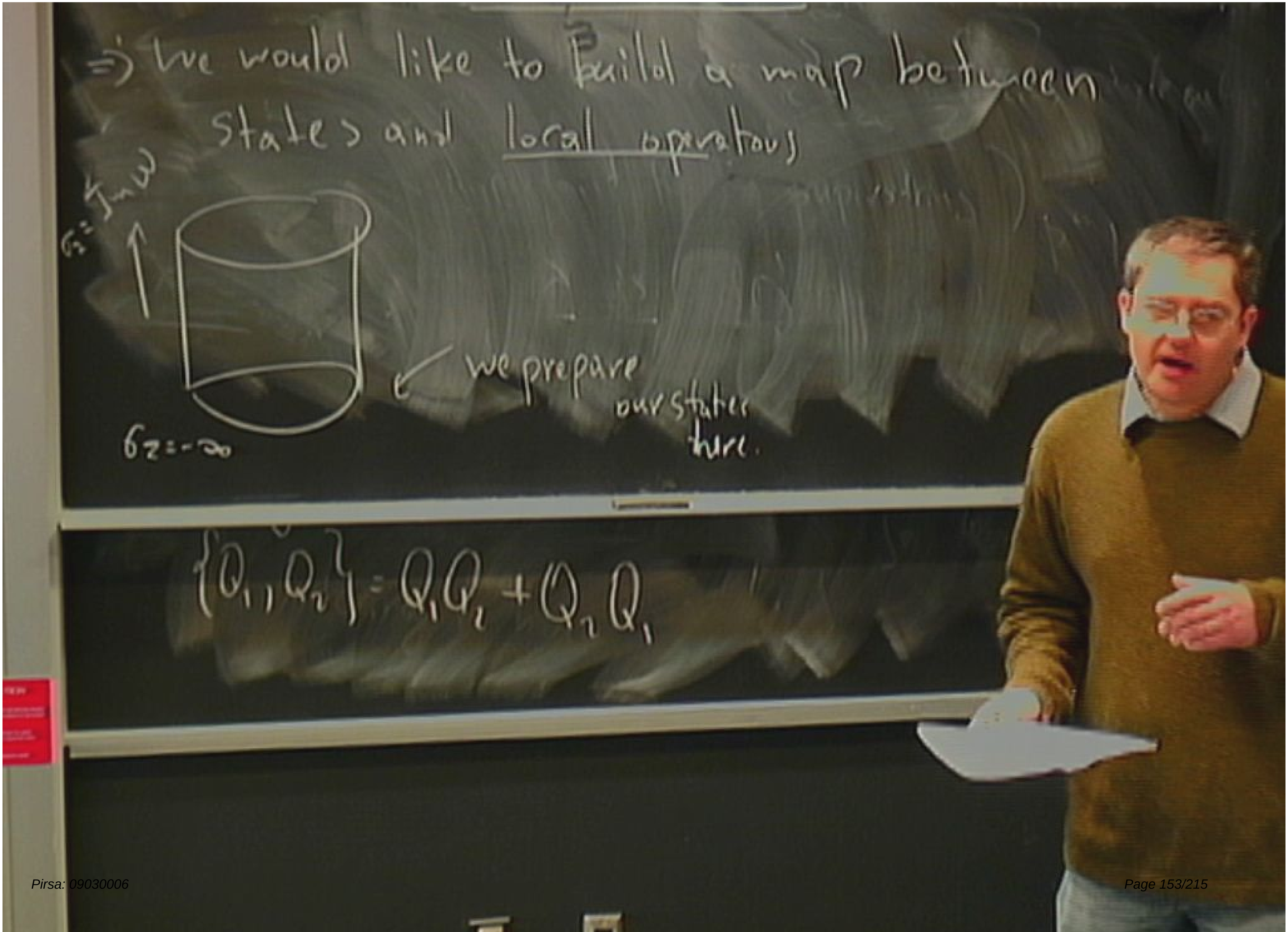
$$a^g = \frac{1}{2}\lambda(1-\lambda)$$

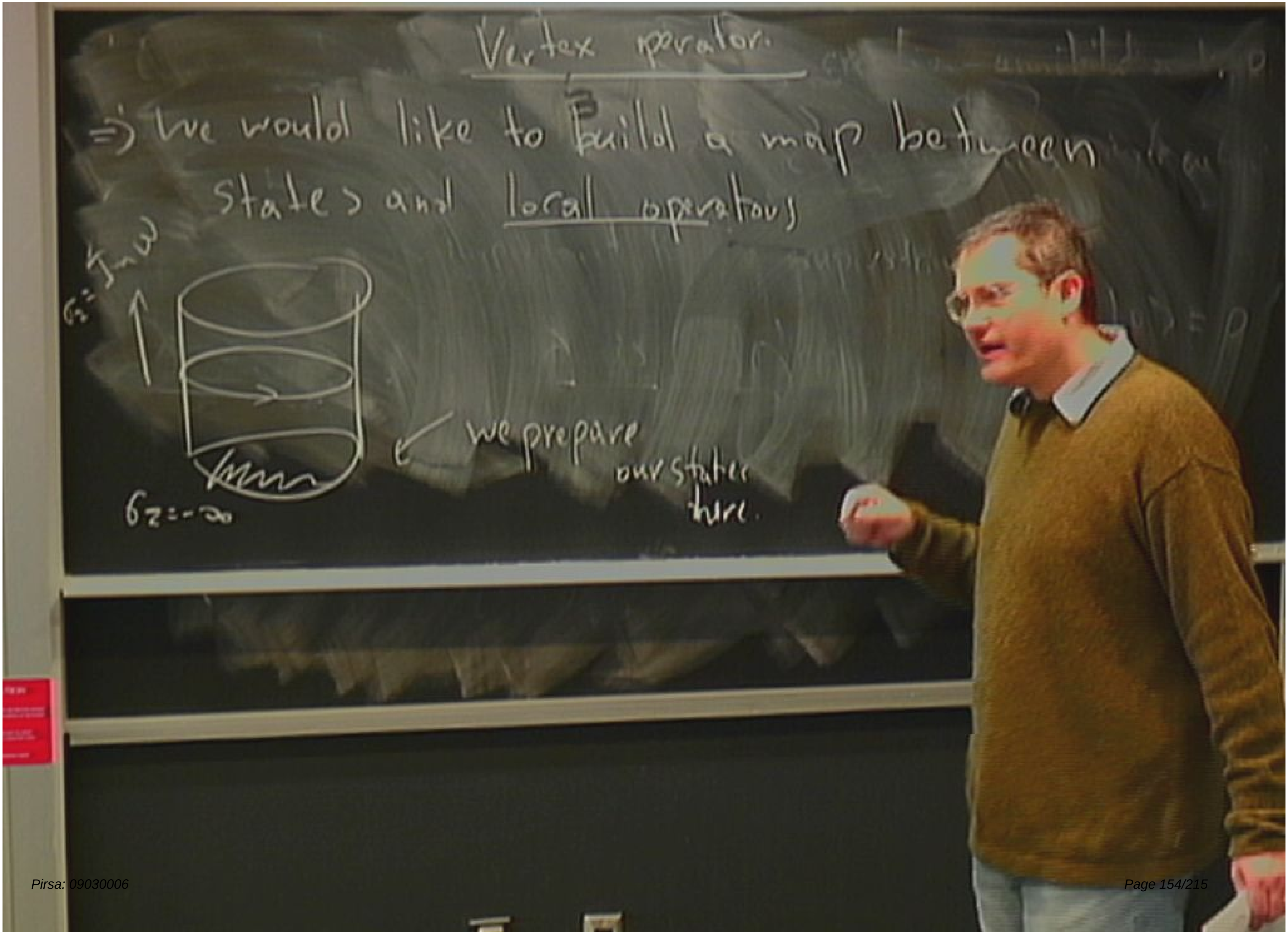

Vertex operator.

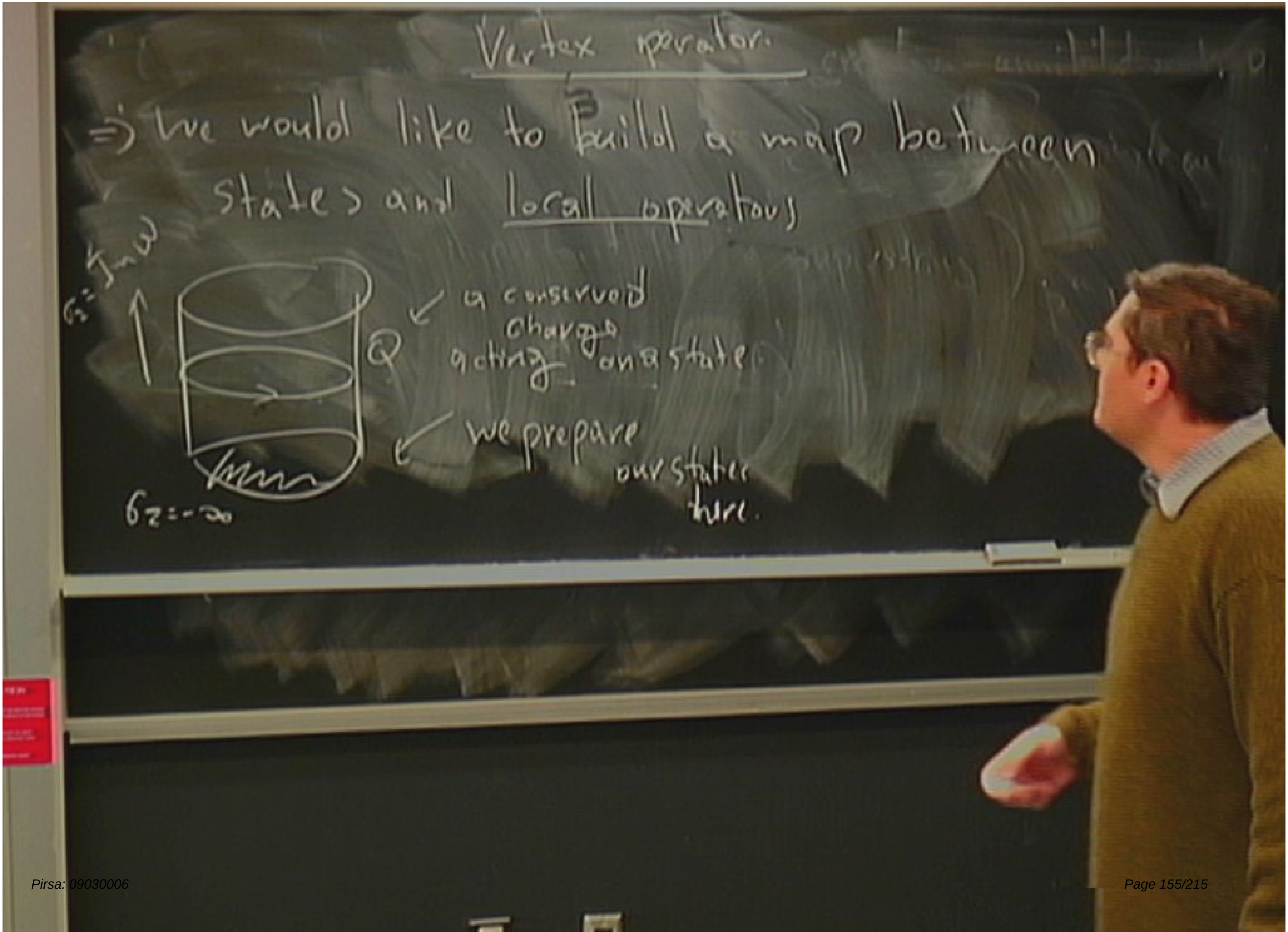

Vertex operator
⇒ we would like to build a map between
states and local operators


⇒ we would like to build a map between
states and local operators
{Q1, Q2} = Q1Q2 + Q2Q1


⇒ we would like to build a map between
states and local operators
σ₂ = −∞
{Q₁, Q₂} = Q₁Q₂ + Q₂Q₁


⇒ we would like to build a map between
states and local operators
$\sigma_2 = \mathrm{Im}\, w$
we prepare
our states
here.
$\sigma_2 = -\infty$
$\{Q_1, Q_2\} = Q_1 Q_2 + Q_2 Q_1$

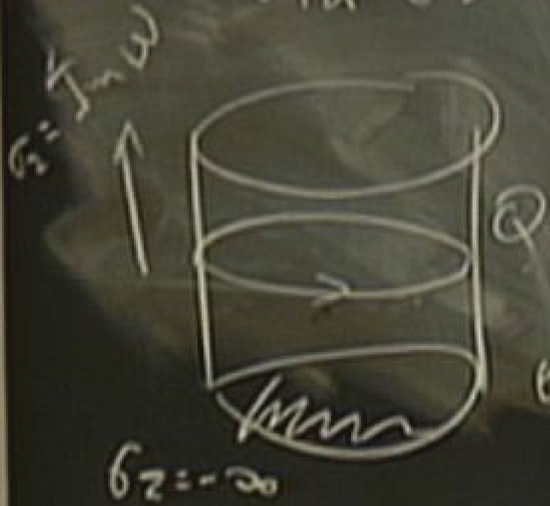

Vertex operator.
⇒ we would like to build a map between
states and local operators
we prepare
our states
here.


Vertex operator.
⇒ we would like to build a map between
states and local operators
σ₂ = Im w
a conserved charge
Q acting on a state.
we prepare our states here.
σ₂ = −∞

Vertex operator.

$\Rightarrow$ we would like to build a map between states and local operators

$z = e^{-iw}$



a conserved charge acting on a state.

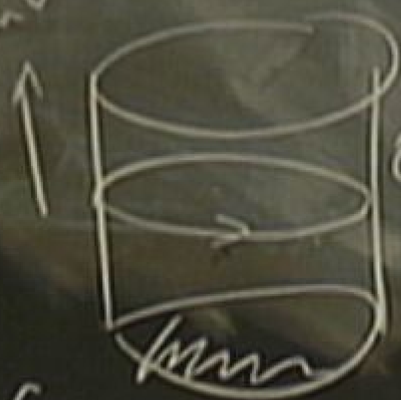
we prepare our states here.

Vertex operator

⇒ we would like to build a map between states and local operators

$w = \sigma_1 + i\sigma_2$, $\sigma_1 \sim \sigma_1 + 2\pi$ $\quad z = e^{-iw}$

$\sigma_2 = \mathrm{Im}\, w$

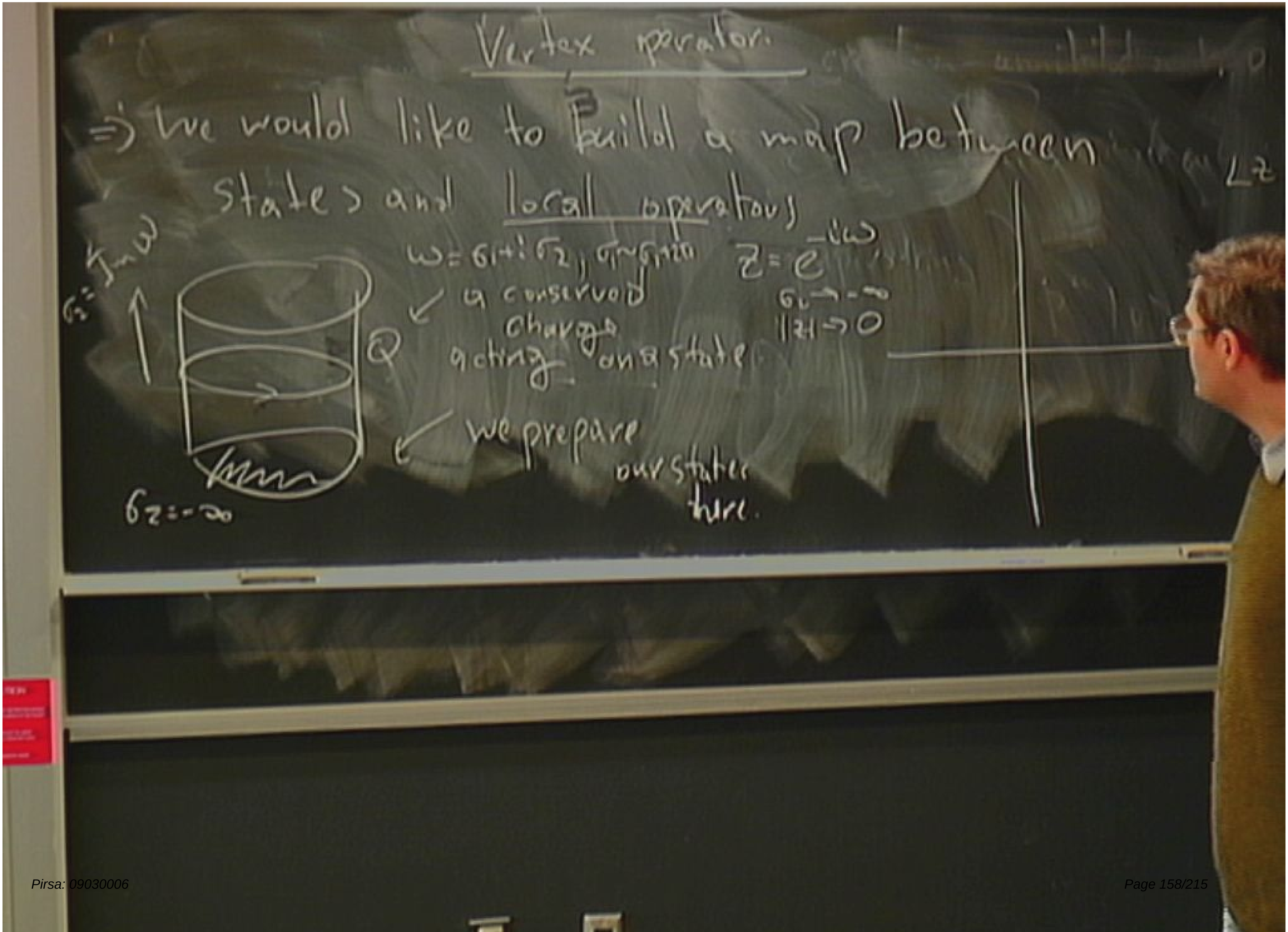


Q ↙ a conserved charge acting on a state

↙ we prepare our states here

$\sigma_2 = -\infty$

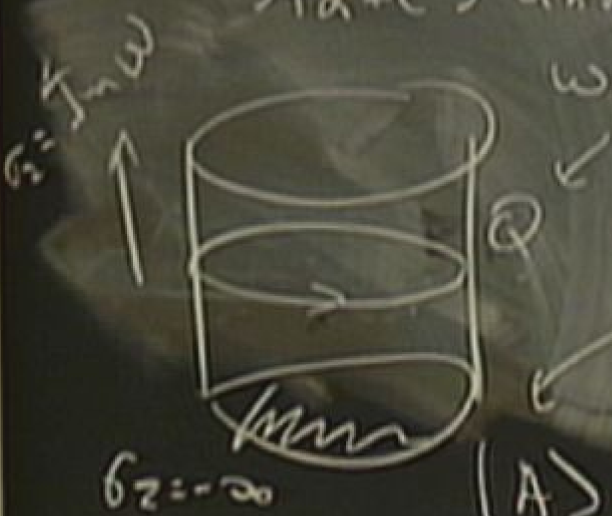
z


Vertex operator.
⇒ we would like to build a map between
states and local operators
ω = σ1 + iσ2, σ1 ~ σ1+2π
z = e^{-iω}
σ2 = Im ω
a conserved charge
Q
acting on a state
σ2 → -∞
|z| → 0
we prepare our states here.
σ2 = -∞

Vertex operator.

⇒ We would like to build a map between states and local operators

$\omega = \sigma_1 + i\sigma_2$, $\sigma_1 \sim \sigma_1 + 2\pi$ $\quad z = e^{-i\omega}$

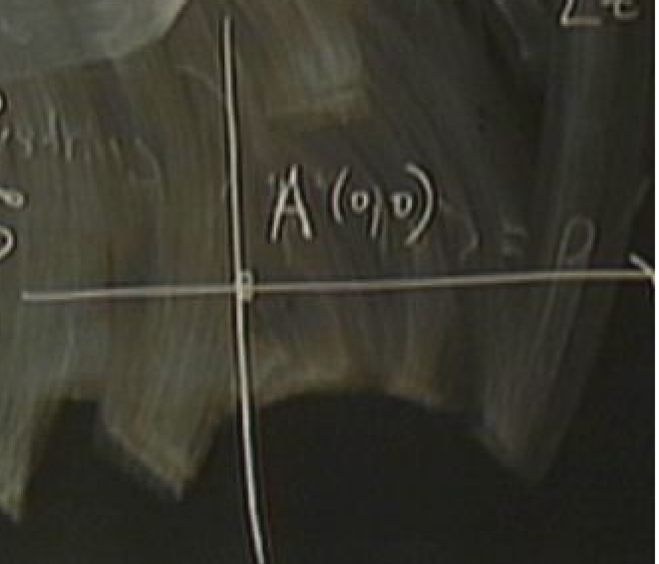


a conserved charge Q acting on a state.

$\sigma_2 \to -\infty$

$|z| \to 0$

we prepare our states here.

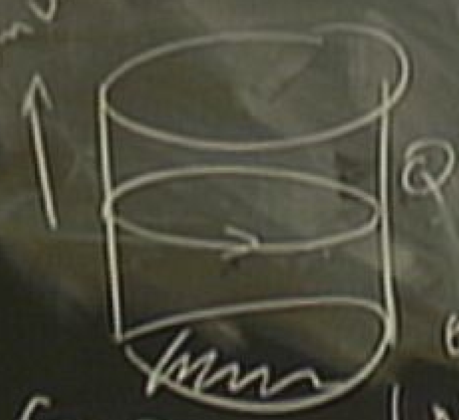


Vertex operator

⇒ We would like to build a map between states and local operators

$\omega = \sigma_1 + i\sigma_2$, $\sigma_1 \sim \sigma_1 + 2\pi$ $\quad z = e^{-i\omega}$

$\sigma_2 = \mathrm{Im}\,\omega$



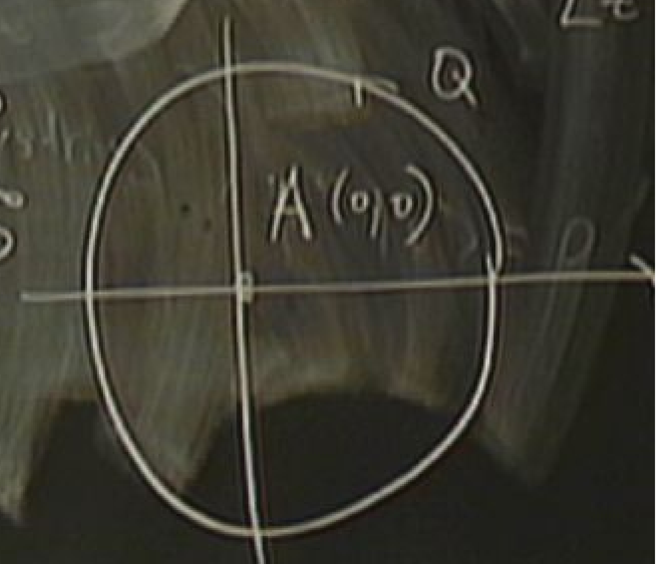
a conserved charge Q acting on a state

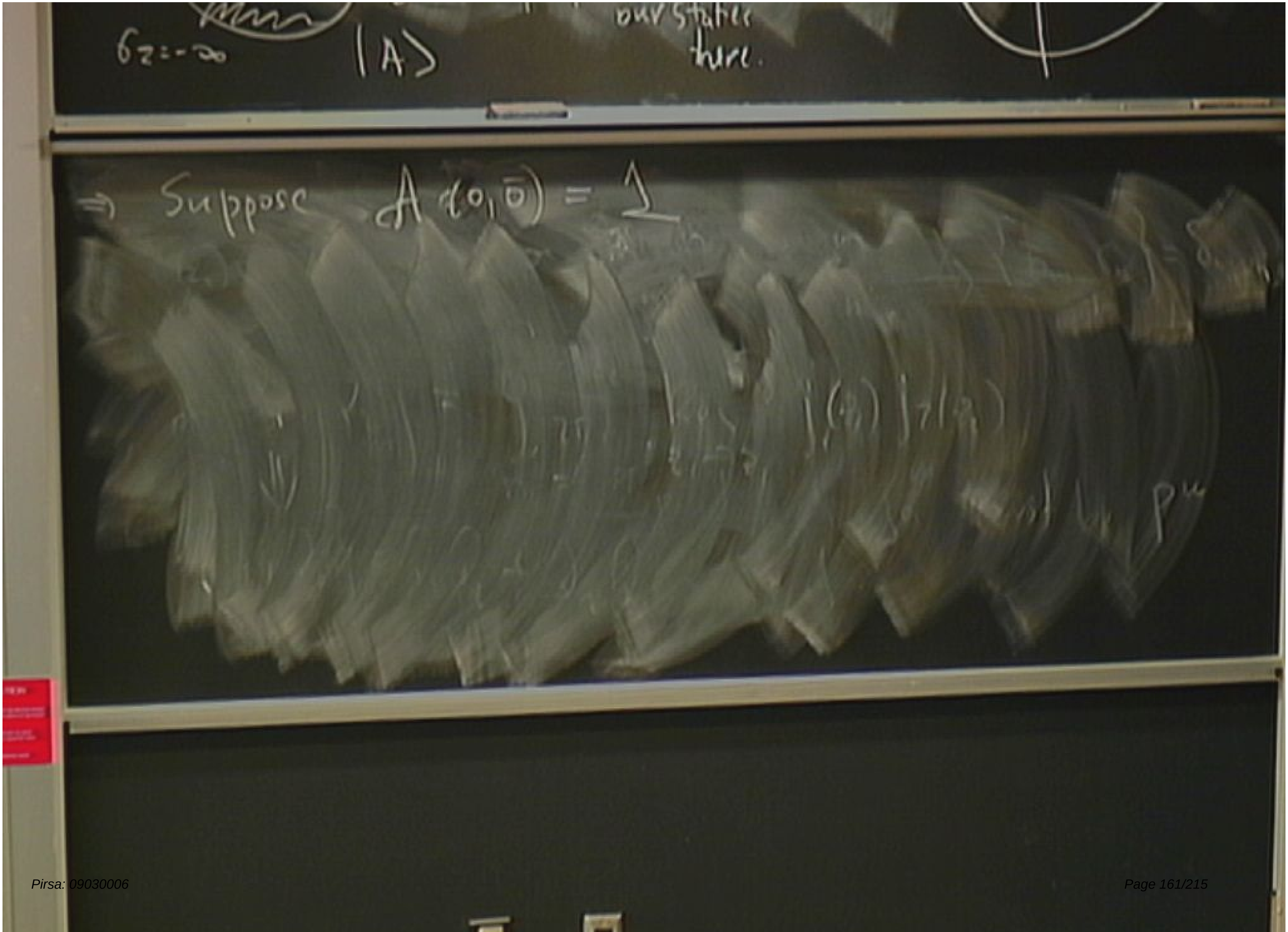
we prepare our states here.

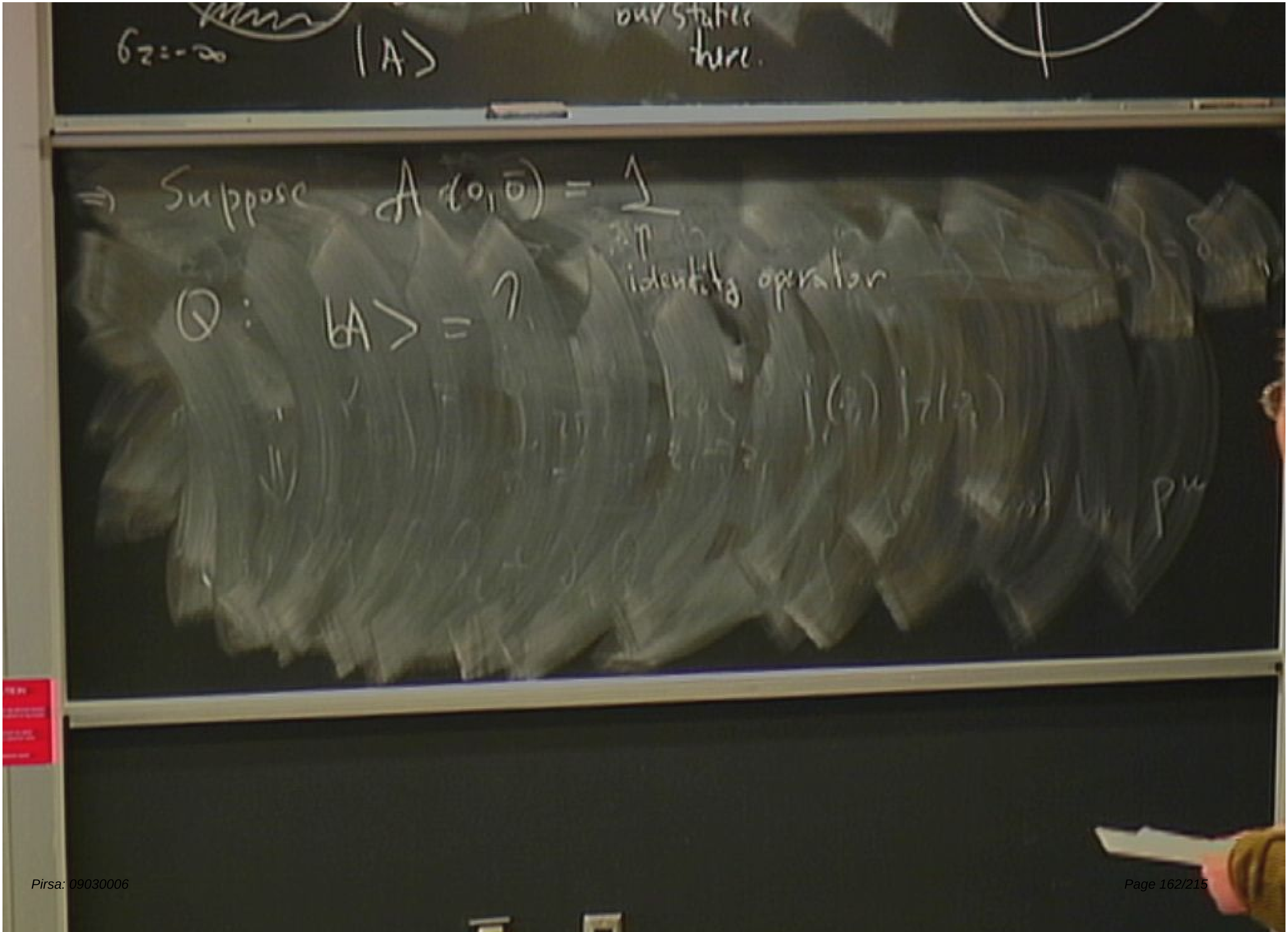
$\sigma_2 = -\infty$ $\quad |A\rangle$

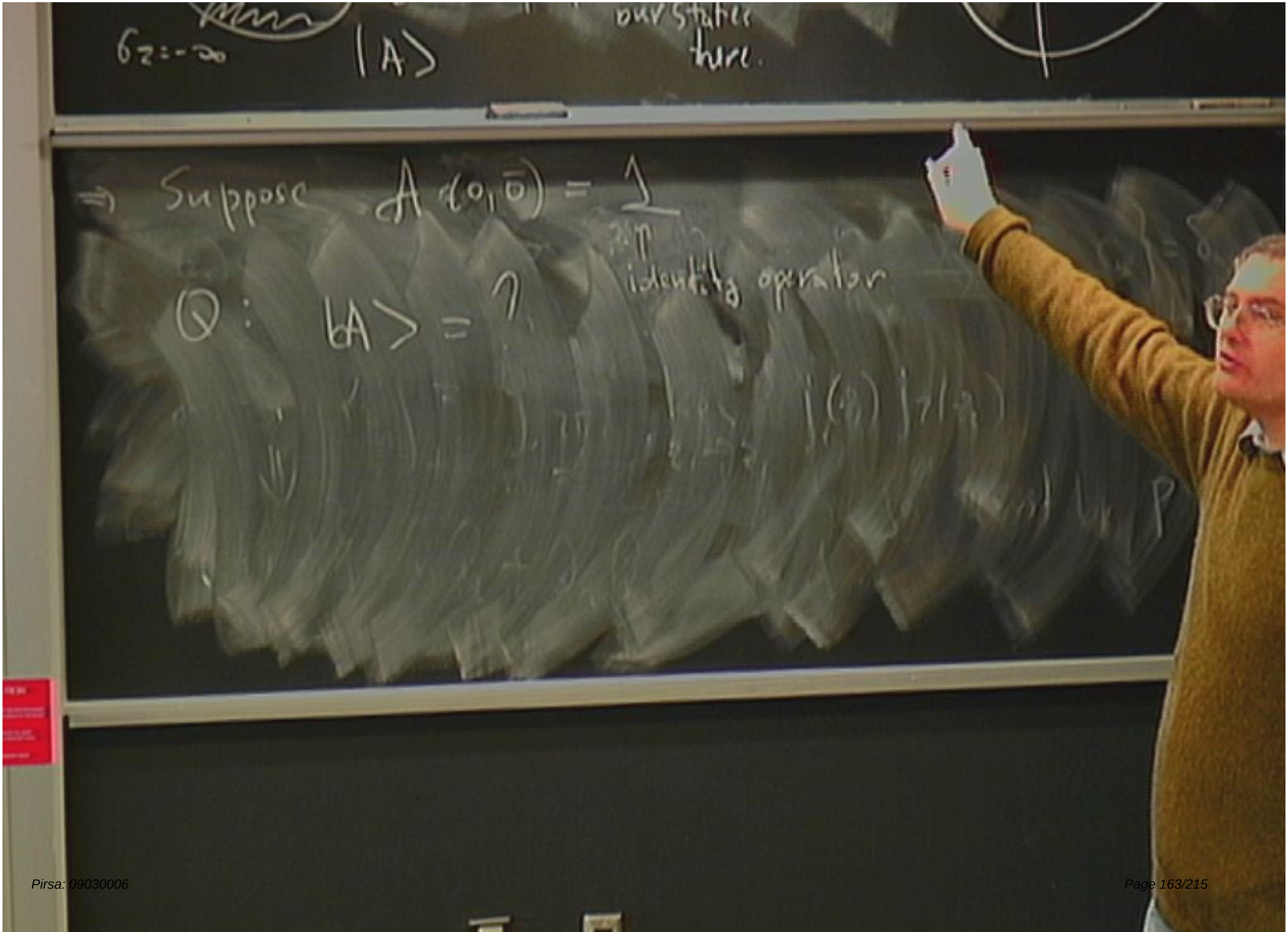
$\sigma_2 \to -\infty$

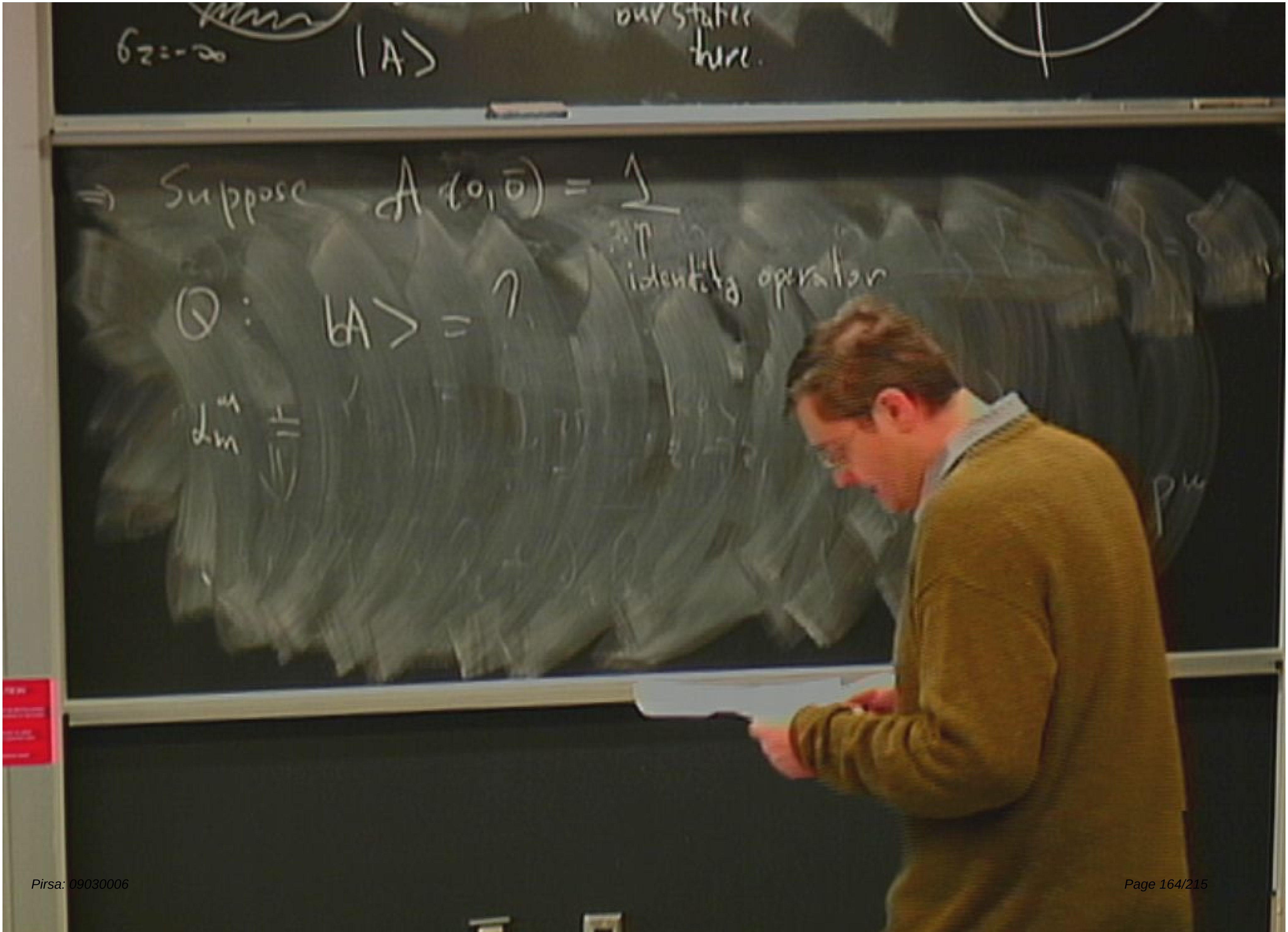
$|z| \to 0$

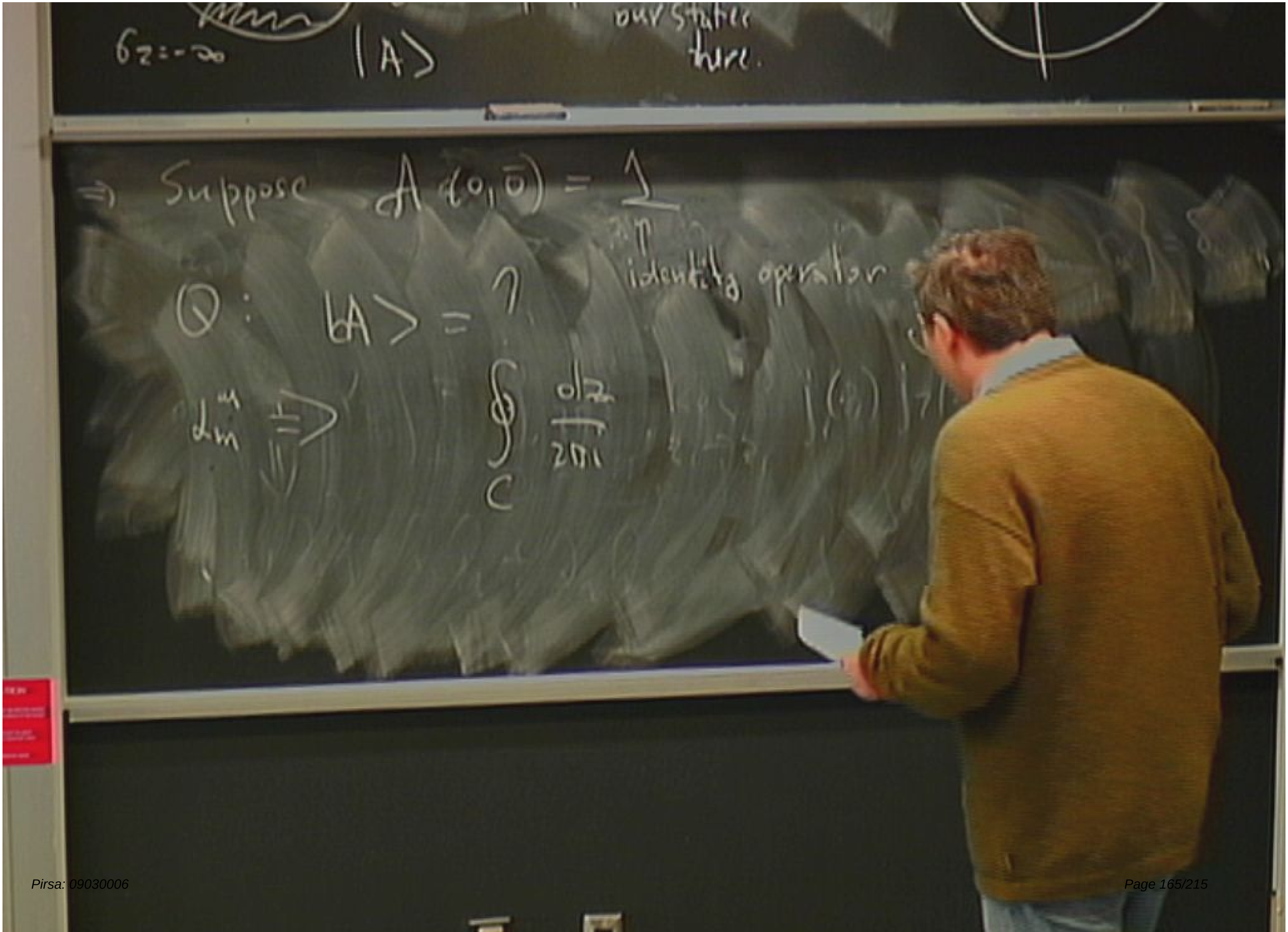



6z=-∞
|A⟩
our states
here.
⇒ Suppose A(0, 0̄) = Σ


σz = -∞
|A⟩
our states
here.
⇒ Suppose A(0, 0̄) = 1
identity operator
Q: |A⟩ = ?
j(z₁) j(z₂)


our states
here.
|A>
Suppose A(0,0̄) = 1
identity operator
Q: |A> = ?


|A>
our states here.
Suppose A(0, 0̄) = 1
identity operator
Q: |A> = ?


our states
there.
|A⟩
Suppose A(0,0̄) = 1
identity operator
Q: |A⟩ = ?

$\sigma_2 = -\infty$ $|A\rangle$

our states there.

$\Rightarrow$ Suppose $A(0,\bar{0}) = \underline{1}$

identity operator

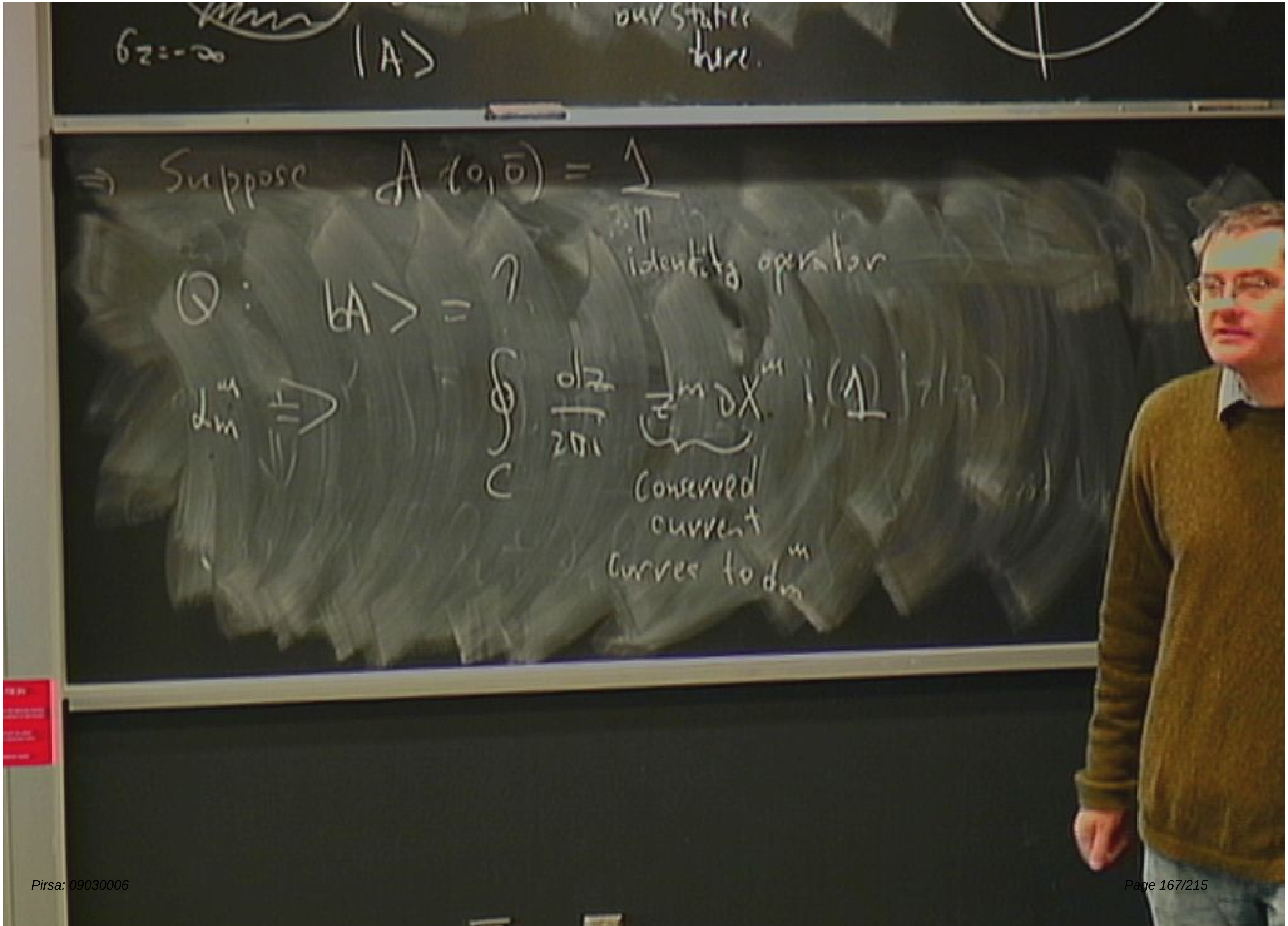
Q: $|A\rangle = ?$

$\alpha^\mu_m \Rightarrow \oint_C \frac{dz}{2\pi i} \underbrace{z^m \partial X^\mu}$

Conserved current

Curves to α^μ_m

p^μ


our states
there.
|A>
Suppose A(0,0̄) = 1
identity operator
Q: |A> = ?
Conserved
current

$\sigma_z = -\infty$ $|A\rangle$ our states there.

$\Rightarrow$ Suppose $A(0,\bar{0}) = \mathbb{1}$

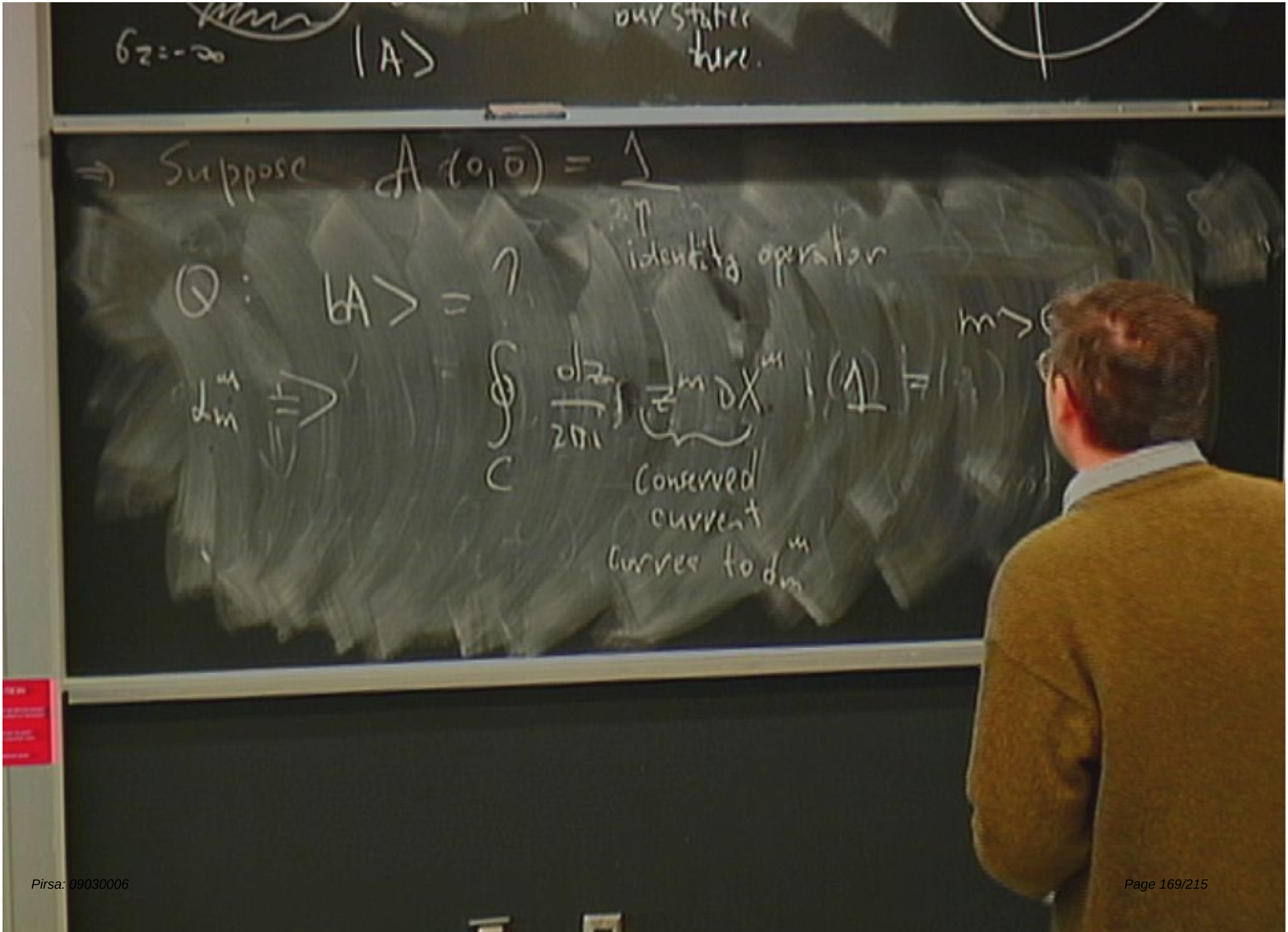
identity operator

Q: $|A\rangle = ?$

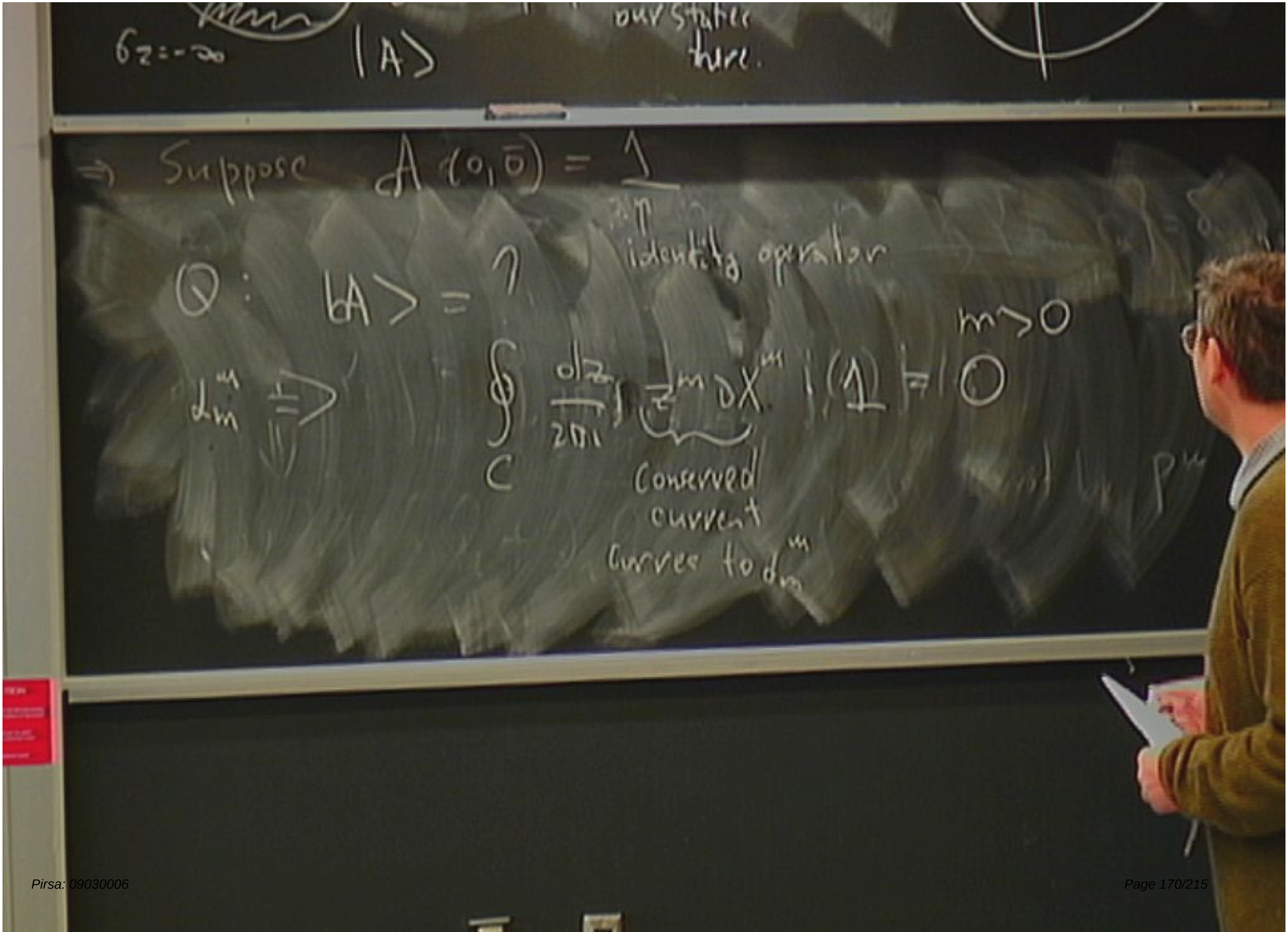
$$\alpha_m^\mu \Rightarrow \oint_C \frac{dz}{2\pi i} \underbrace{z^m \partial X^\mu}_{\text{conserved current}} \, (\mathbb{1}) \, |0\rangle$$

curves to α_m^μ

p^μ



σ_z = -∞
|A⟩
our states
there.
⇒ Suppose A(0, 0̄) = 1
identity operator
Q: |A⟩ = ?
m > 0
Conserved
current
Curves to α_m


our states
there.
|A⟩
⇒ Suppose A(0,0̄) = 1
identity operator
Q: bA⟩ = ?
m>0
∮ dz/2πi z^m ∂X^μ |(1)⟩ = 0
C
Conserved
current
curves to α_m^μ

$\sigma_z = -\infty$ $|A\rangle$

our states there.

identity operator

Q: $|A\rangle = ?$

$m > 0$

$$L_m^\mu \Rightarrow \oint \frac{dz}{2\pi i} \underbrace{z^m \partial X^\mu}_{\substack{\text{conserved current}\\ \text{curves to } L_m^\mu}} \cdot (\mathbb{1}) = 0$$

$$\boxed{L_m^\mu |A\rangle = 0, \; m > 0}$$

$\sigma_z = -\infty$ $\quad |A\rangle$

our states there.

identity operator

Q: $|A\rangle = ?$

$m > 0$

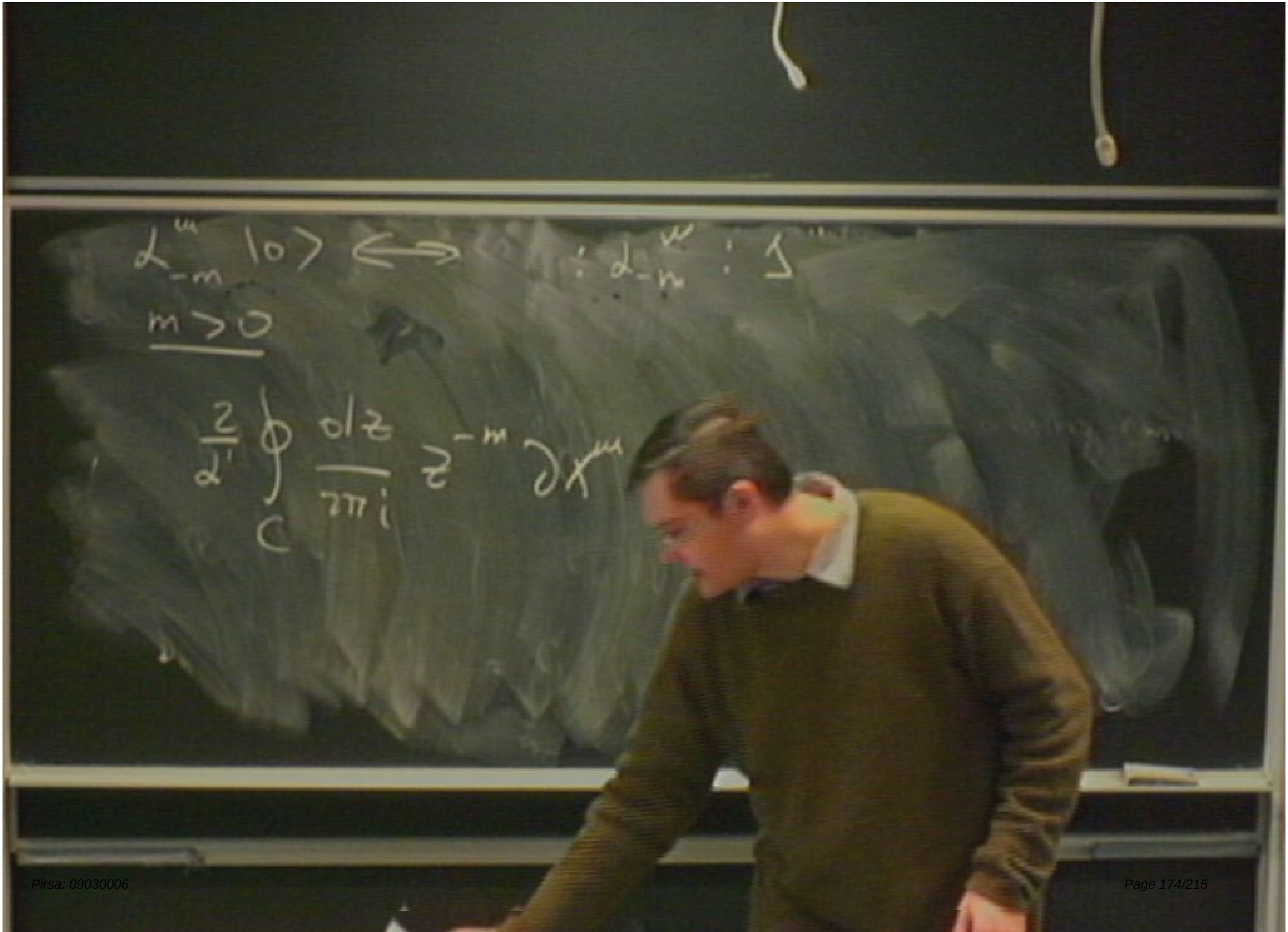
$\alpha_m^\mu \Rightarrow \oint \frac{dz}{2\pi i} \underbrace{z^m \partial X^\mu}_{\text{conserved current}} \, (\mathbb{1}) = 0$

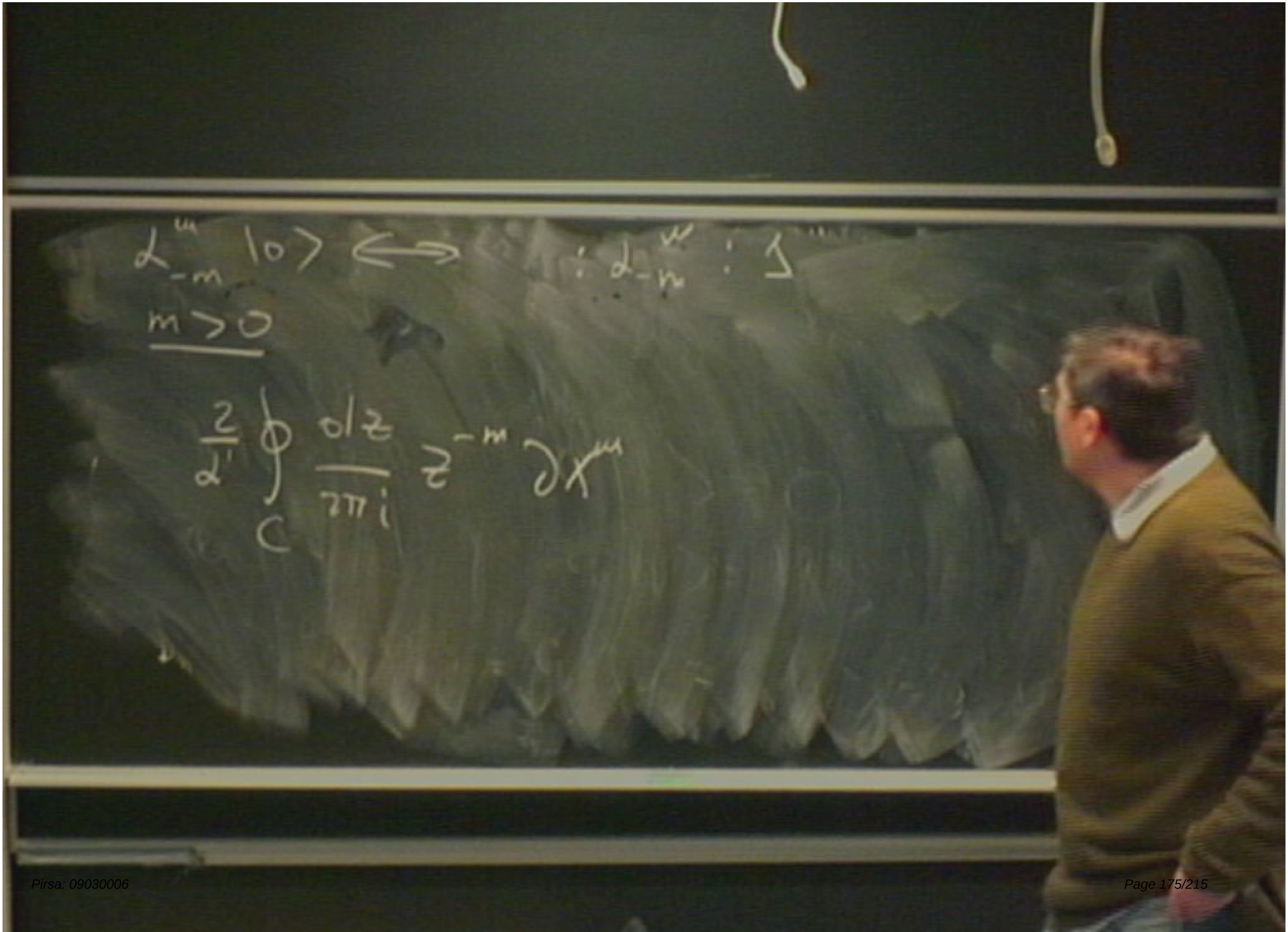
curves to α_m^μ

$$\boxed{\alpha_m^\mu |A\rangle = 0, \quad m > 0}$$

$|0,0\rangle \Longleftrightarrow \mathbb{1}$

$\alpha^{\mu}_{-m}|0\rangle \Longleftrightarrow \; :\alpha^{\mu}_{-n}: \int$


$\alpha^\mu_{-m}|0\rangle \Longleftrightarrow$
m > 0
$\frac{2}{\alpha'}\oint_C \frac{dz}{2\pi i} z^{-m} \partial X^\mu$


α^μ_{-m} |0⟩ ⟺ :α^μ_{-m}: 1
m > 0
(2/α') ∮_C dz/2πi z^{-m} ∂X^μ

$\alpha^{\mu}_{-m}|0\rangle \Longleftrightarrow \; :\alpha^{\mu}_{-m}: \;\mathbb{1}$

$\underline{m>0}$

$$\frac{2}{\alpha'}\oint_C \frac{dz}{2\pi i} z^{-m} \partial X^{\mu} \rightarrow \left(\frac{2}{\alpha'}\right)^{1/2} \frac{i}{(m-1)!}$$

$\alpha^{\mu}_{-m} |0\rangle \Longleftrightarrow \; :\alpha^{\nu}_{-n}: \; \Delta$

$\underline{m > 0}$

$\partial X^{\mu}(z)$

$$\frac{2}{\alpha'} \oint_C \frac{dz}{2\pi i} z^{-m} \partial X^{\mu} \rightarrow \left(\frac{2}{\alpha'}\right)^{1/2} \frac{i}{(m-1)!}$$

$$\alpha_{-m}^{\mu} |0\rangle \Longleftrightarrow \, :\alpha_{-m}^{\mu}: \, \mathcal{I}$$

$\underline{m > 0}$

$$\partial X^{\mu}(z) = \partial X^{\mu}(0) + \frac{z}{1!}\partial^2 X^{\mu} + \frac{z^{m-1}}{(m-1)!}$$

$$\frac{2}{\alpha'} \oint_C \frac{dz}{2\pi i} z^{-m} \partial X^{\mu} \Rightarrow \left(\frac{2}{\alpha'}\right)^{1/2} \frac{i}{(m-1)!}$$

$$\alpha^{\mu}_{-m} |0\rangle \Longleftrightarrow \; :\alpha^{\mu}_{-m}: \, \mathcal{J}$$

$\underline{m > 0}$

$$\partial X^{\mu}(z) = \partial X^{\mu}(0) + \frac{z}{1!} \partial^2 X^{\mu} + \frac{z^{m-1}}{(m-1)!} \partial^{m} X$$

$$\frac{2}{\alpha'} \oint_C \frac{dz}{2\pi i} z^{-m} \partial X^{\mu} \Rightarrow \left(\frac{2}{\alpha'}\right)^{1/2} \frac{i}{(m-1)!}$$

$$\alpha^{\mu}_{-m}|0\rangle \Longleftrightarrow :\alpha^{\mu}_{-m}: \Delta$$

$\underline{m>0}$

$$\partial X^{\mu}(z) = \partial X^{\mu}(0) + \frac{z}{1!}\partial^2 X^{\mu} + \frac{z^{m-1}}{(m-1)!}\partial^m X$$

$$\frac{2}{\alpha'}\oint_C \frac{dz}{2\pi i} z^{-m}\,\partial X^{\mu} \Rightarrow \left(\frac{2}{\alpha'}\right)^{1/2} \frac{i}{(m-1)!}\partial^m X(0)$$


$m > 0$
$\frac{2}{\alpha'} \oint_C \frac{dz}{2\pi i} z^{-m} \partial X^\mu \Rightarrow \left(\frac{2}{\alpha'}\right)^{1/2} \frac{i}{(m-1)!} \partial^m X(0)$
$\alpha^\mu_{-m} |0\rangle \Longleftrightarrow \left(\frac{2}{\alpha'}\right)^{1/2} \frac{i}{(m-1)!}$
$2L_0 = [L_1, L_{-1}]$
study $|\downarrow\rangle$

$m > 0$

$\partial X^{\mu}(z) = \partial X^{\mu}(0) + \dots + \frac{z^{m-1}}{(m-1)!} \partial X \dots$

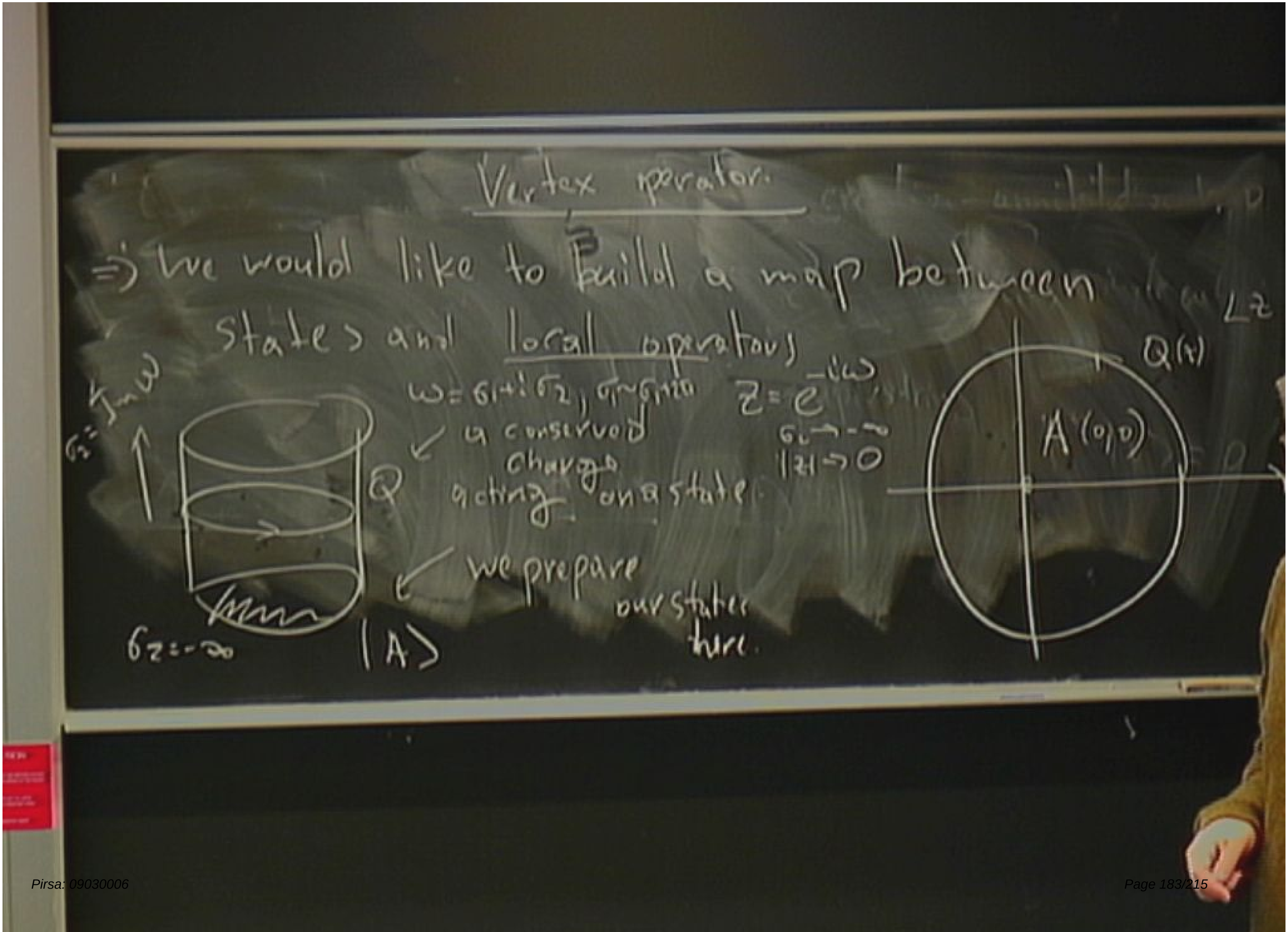
$$\frac{2}{\alpha'} \oint_C \frac{dz}{2\pi i} z^{-m} \partial X^{\mu} \Rightarrow \left(\frac{2}{\alpha'}\right)^{1/2} \frac{i}{(m-1)!} \partial^m X(0)$$

$$\alpha_{-m}^{\mu} |0\rangle \Longleftrightarrow \left(\frac{2}{\alpha'}\right)^{1/2} \frac{i}{(m-1)!} \partial^m X^{\mu}(0)$$

$$2L_0 = [L_1, L_{-1}] = L_1 L_{-1} - \cancel{L_{-1}} L_1$$

$L_1|0\rangle = 0$

$\hookrightarrow$ study $|\downarrow\rangle$

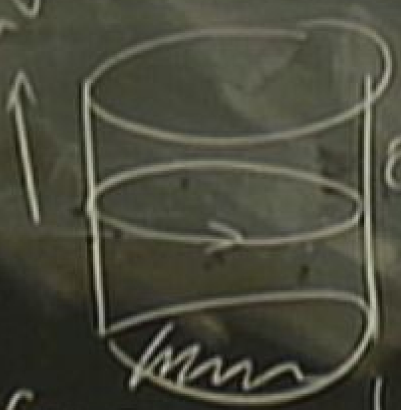

Vertex operator.
⇒ we would like to build a map between
states and local operators
$\omega = \sigma_1 + i\sigma_2$
$z = e^{-i\omega}$
$\sigma_2 = \mathrm{Im}\,\omega$
a conserved charge
Q
acting on a state.
$\sigma_1 \to -\infty$
$|z| \to 0$
Q(x)
A(0,0)
we prepare our states here.
$\sigma_2 = -\infty$
$|A\rangle$

Vertex operator.

⇒ we would like to build a map between states and local operators

$\omega = \sigma_1 + i\sigma_2$, $\sigma_1 \sim \sigma_1 + 2\pi$ $z = e^{-i\omega}$

$\sigma_2 = \mathrm{Im}\,\omega$



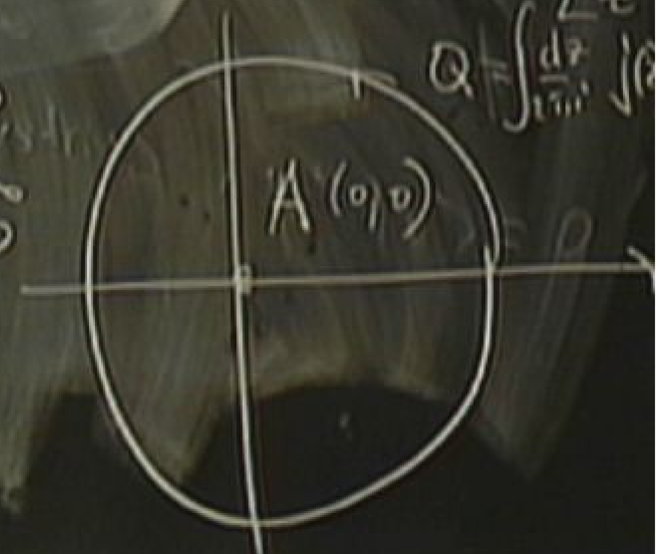
Q ↙ a conserved charge acting on a state.

↙ we prepare our states here.

$\sigma_2 = -\infty$ $|A\rangle$

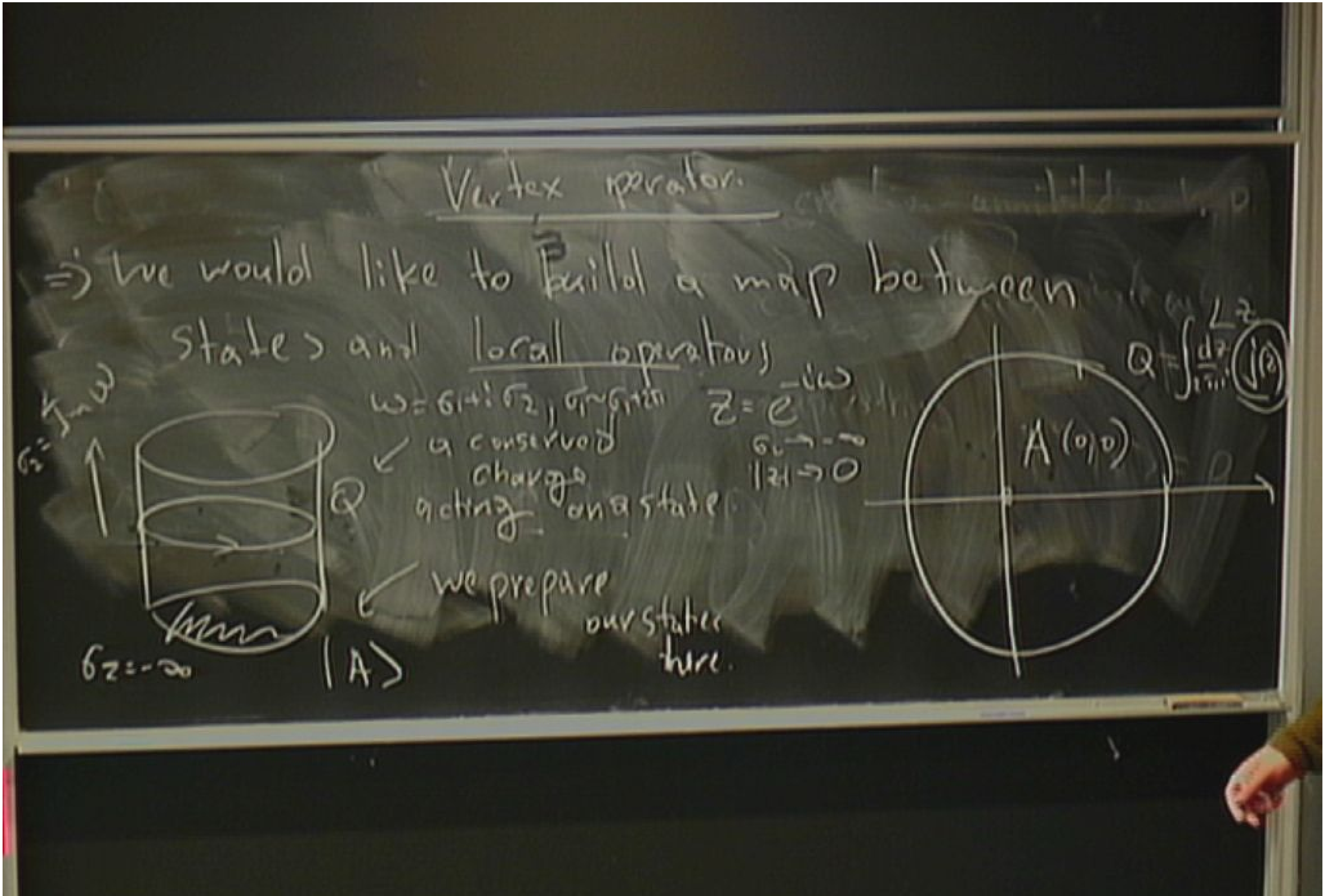
$\sigma_2 \to -\infty$

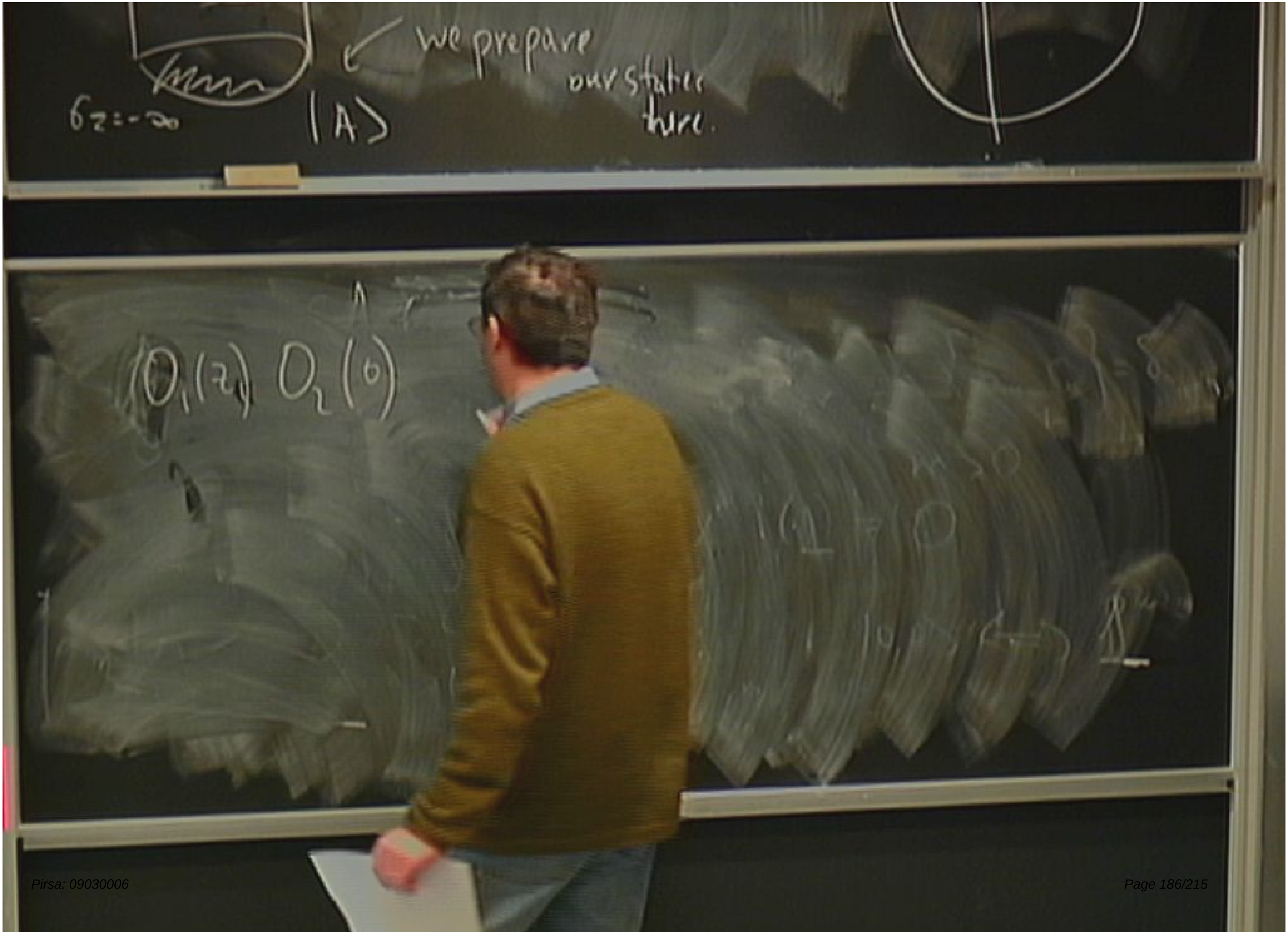
$|z| \to 0$

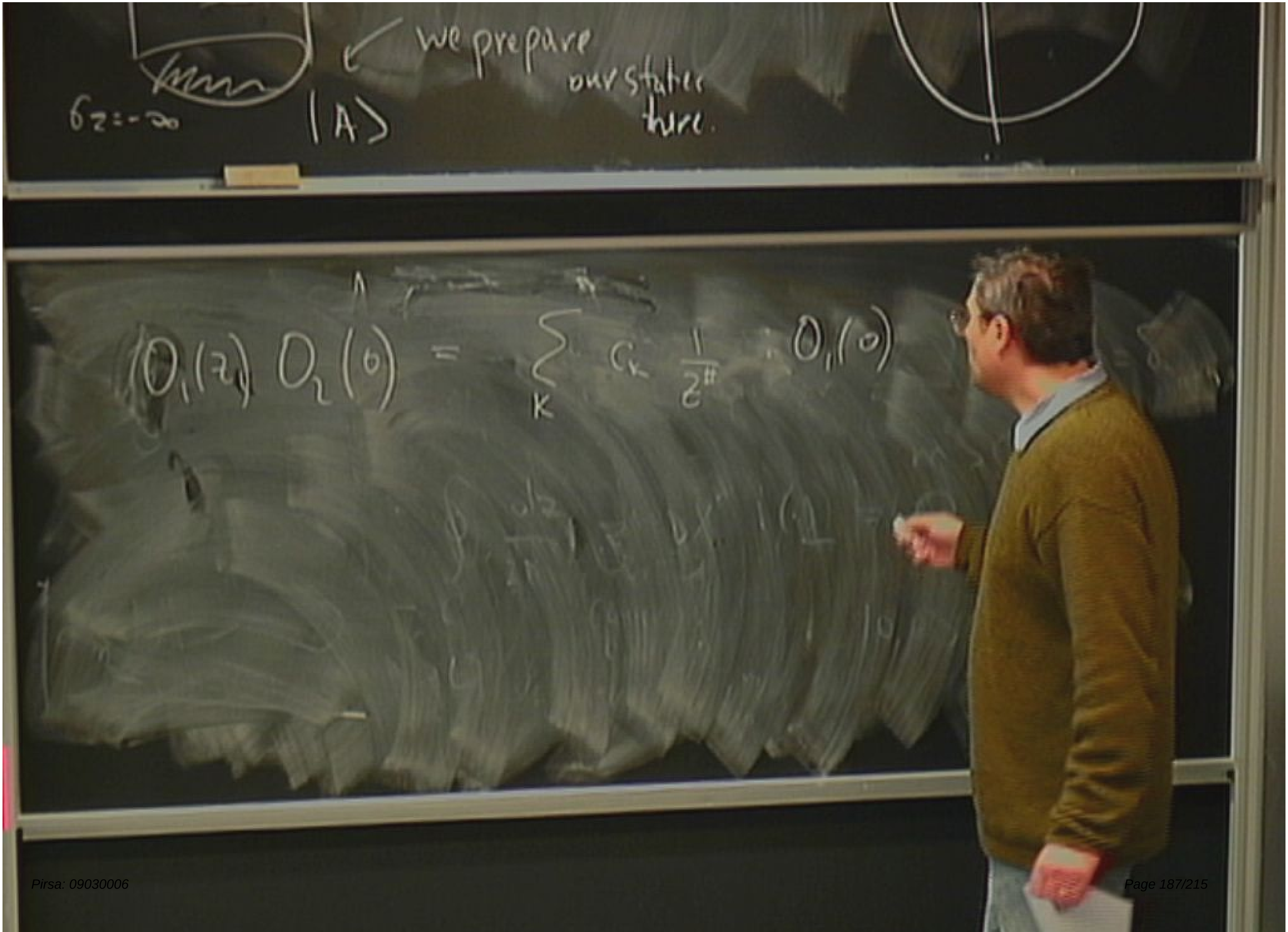


A(0,0)

$Q = \oint \frac{dz}{2\pi i}\, j(z)$


Vertex operator.
⇒ we would like to build a map between
states and local operators
ω = σ1 + i σ2 , σ1 ~ σ1 + 2π
z = e^{-iω}
σ2 → -∞
|z| → 0
a conserved charge
acting on a state.
Q
we prepare
our states
here.
σ2 = Im ω
σ2 = -∞
|A⟩
A (0,0)
Q = ∮ dz/2πi j(z)

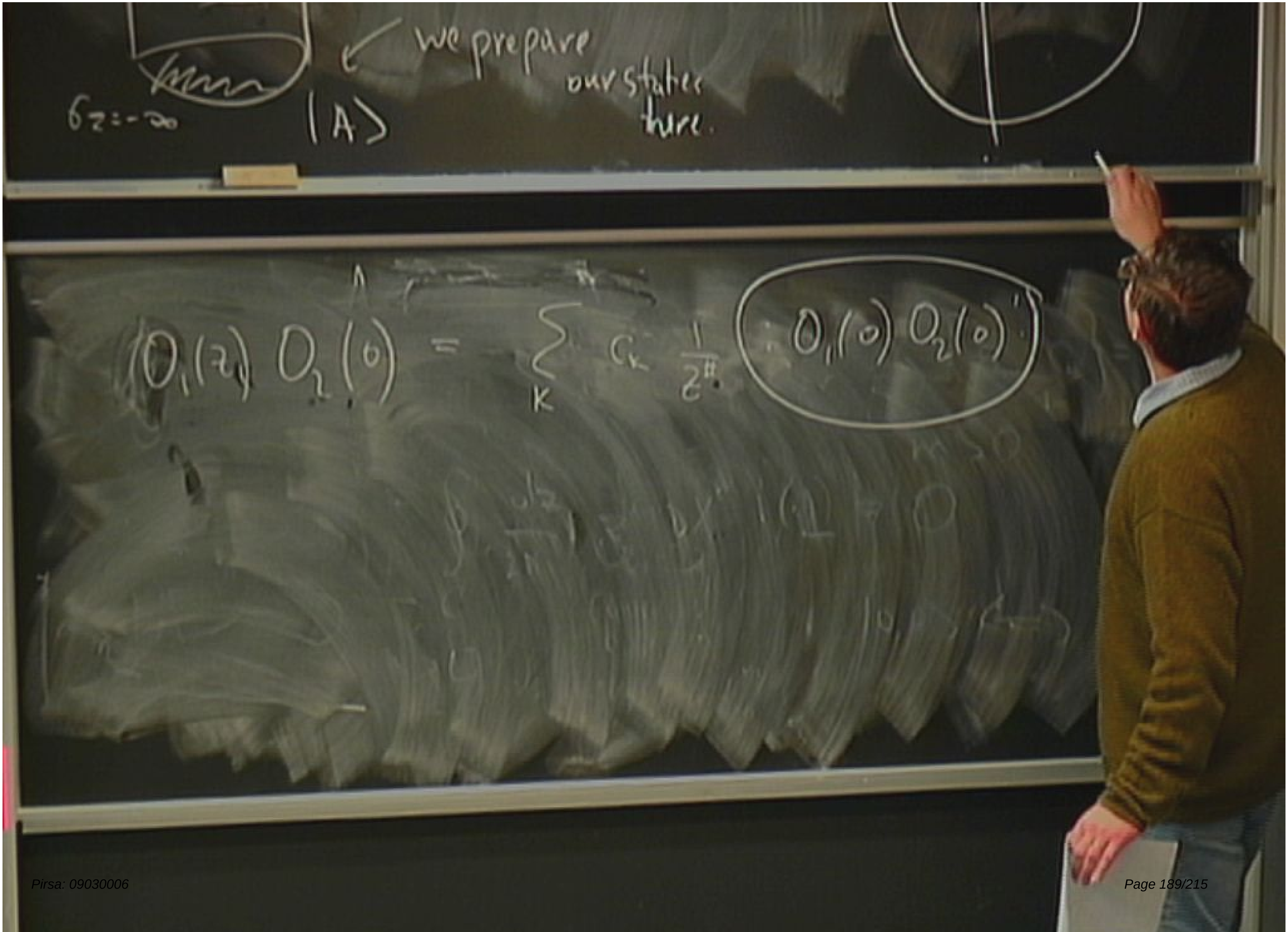

we prepare
our states
there.
|A>
$O_1(z)\, O_2(0)$

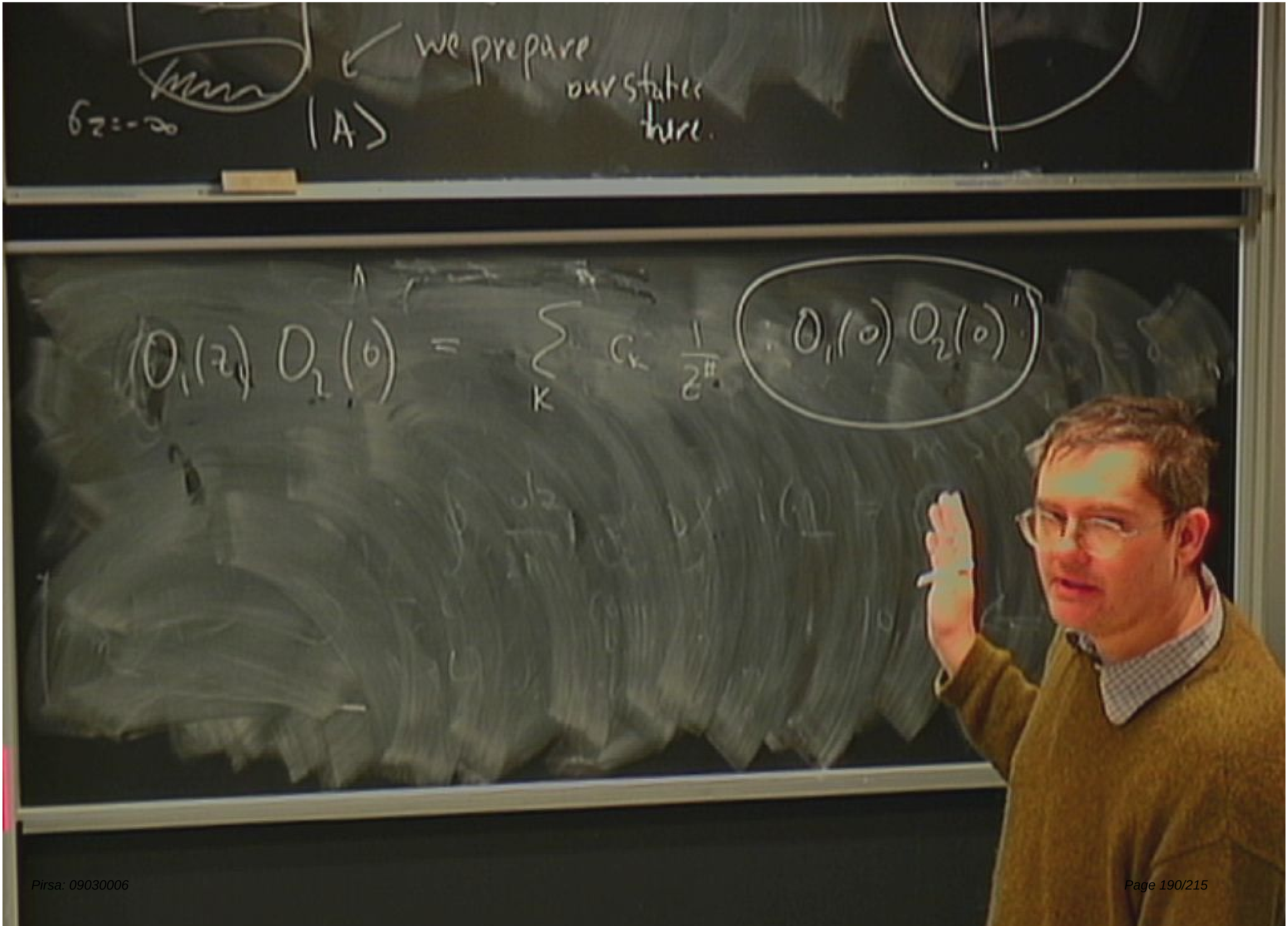

we prepare
our states
there.
|A⟩
z = -∞
O_1(z) O_2(0) = Σ_K C_K 1/z^# O_1(0)

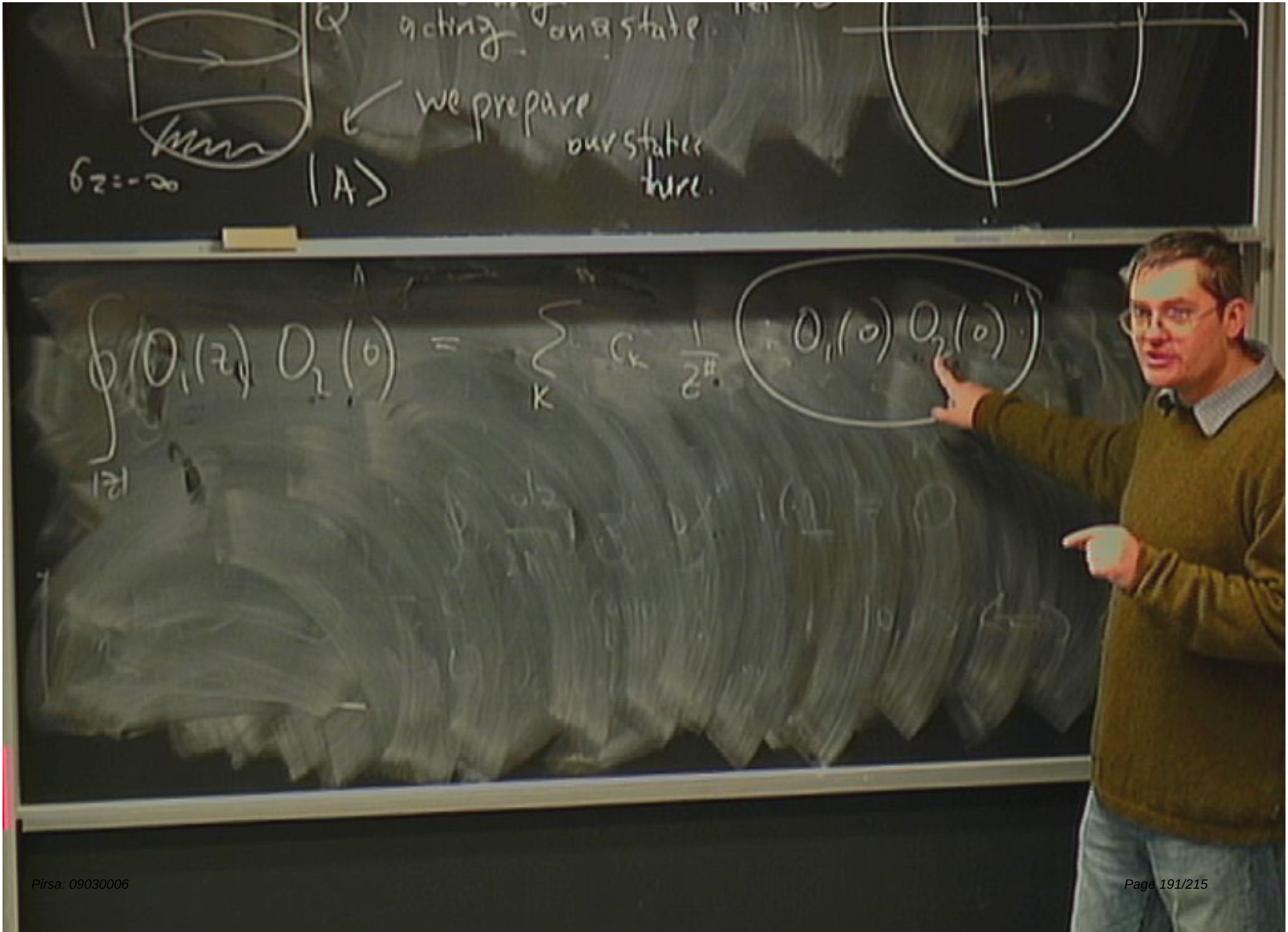
we prepare our states there.

$6z=-\infty$ $|A\rangle$

$$O_1(z)\, O_2(0) = \sum_K C_k \frac{1}{z^{\#}}\, O_1(0)\, O_2(0)$$


we prepare our states there.
$\tau = -\infty$
$|A\rangle$
$O_1(z)\, O_2(0) = \sum_K C_K \frac{1}{z^{\#}} \, :O_1(0)\, O_2(0):$


we prepare
our states
there.
|A>
O1(z) O2(0) = Σ_K C_K 1/z^# :O1(0) O2(0):


acting on a state.
we prepare
our states
here.
|A⟩
$\oint_{|z|} \mathcal{O}_1(z)\, \mathcal{O}_2(0) = \sum_K C_K \frac{1}{z^{\#}} \mathcal{O}_1(0)\, \mathcal{O}_2(0)$

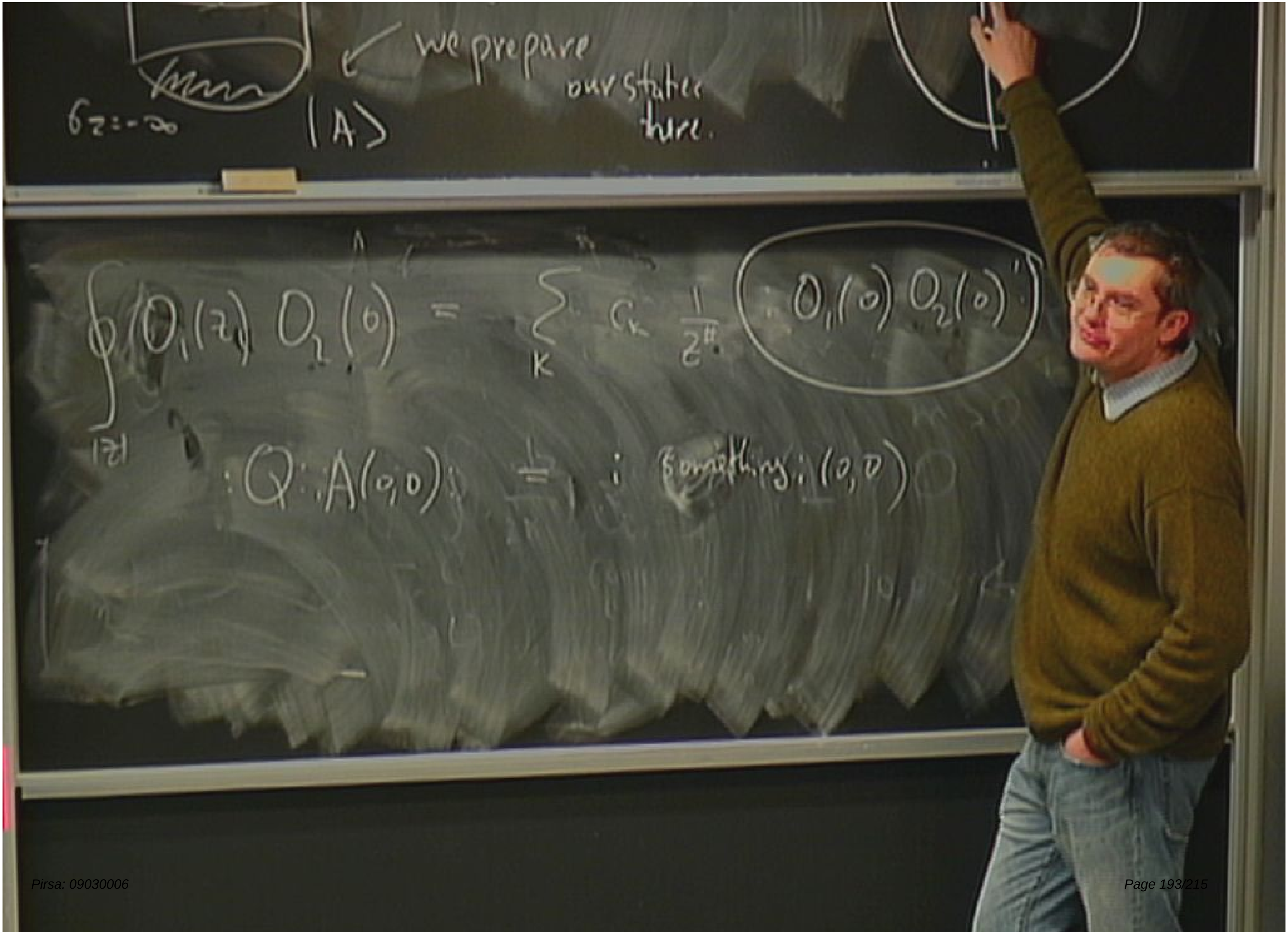
$$\alpha^{\mu}_{-m}|0\rangle \Longleftrightarrow \; :\alpha^{\mu}_{-m}\,\mathbb{1}: \;\Longleftrightarrow\; :\alpha^{\mu}_{-m}\,\mathbb{1}:$$

$\underline{m>0}$

$$\partial X^{\mu}(z) = \partial X^{\mu}(0) + \frac{z}{1!}\,\partial^2 X^{\mu} + \cdots + \frac{z^{m-1}}{(m-1)!}\,\partial^{m} X^{\mu}$$

$$\frac{2}{\alpha'}\oint_C \frac{dz}{2\pi i}\, z^{-m}\,\partial X^{\mu} \rightarrow \left(\frac{2}{\alpha'}\right)^{1/2} \frac{i}{(m-1)!}\,\partial^{m} X(0)$$

$$\alpha^{\mu}_{-m}|0\rangle \Longleftrightarrow \left(\frac{2}{\alpha'}\right)^{1/2} \frac{i}{(m-1)!}\,\partial^{m} X^{\mu}(0)$$


we prepare our states there.
|A⟩
∮ O₁(z) O₂(0) = Σ_K C_K 1/z^# :O₁(0) O₂(0):
|z|
:Q::A(0,0): = :something:(0,0)

we prepare our states there.

$\sigma_{z=-\infty}$ $|A\rangle$

$$\oint_{|z|} \mathcal{O}_1(z)\, \mathcal{O}_2(0) = \sum_K C_K \frac{1}{z^{\#}} :\mathcal{O}_1(0)\,\mathcal{O}_2(0):$$

$$:\underline{Q : A(0,0)}: \quad : \text{something} : (0,0)$$

$$\alpha^{\mu}_{-m}|0\rangle \Longleftrightarrow :\alpha^{\mu}_{-m}: \mathcal{J} \Longleftrightarrow :\alpha^{\mu}_{-m}\mathcal{J}:$$

$\underline{m>0}$

$$\partial X^{\mu}(z) = \partial X^{\mu}(0) + \frac{z}{1!}\partial^2 X^{\mu} + \frac{z^{m-1}}{(m-1)!}\partial^{m-1} X$$

$$\frac{2}{\alpha'}\oint_C \frac{dz}{2\pi i} z^{-m} \underline{\partial X^{\mu}} \Rightarrow \left(\frac{2}{\alpha'}\right)^{1/2} \frac{i}{(m-1)!}\partial^m X(0)$$

$$\alpha^{\mu}_{-m}|0\rangle \Longleftrightarrow \left(\frac{2}{\alpha'}\right)^{1/2} \frac{i}{(m-1)!}\partial^m X^{\mu}(0)$$

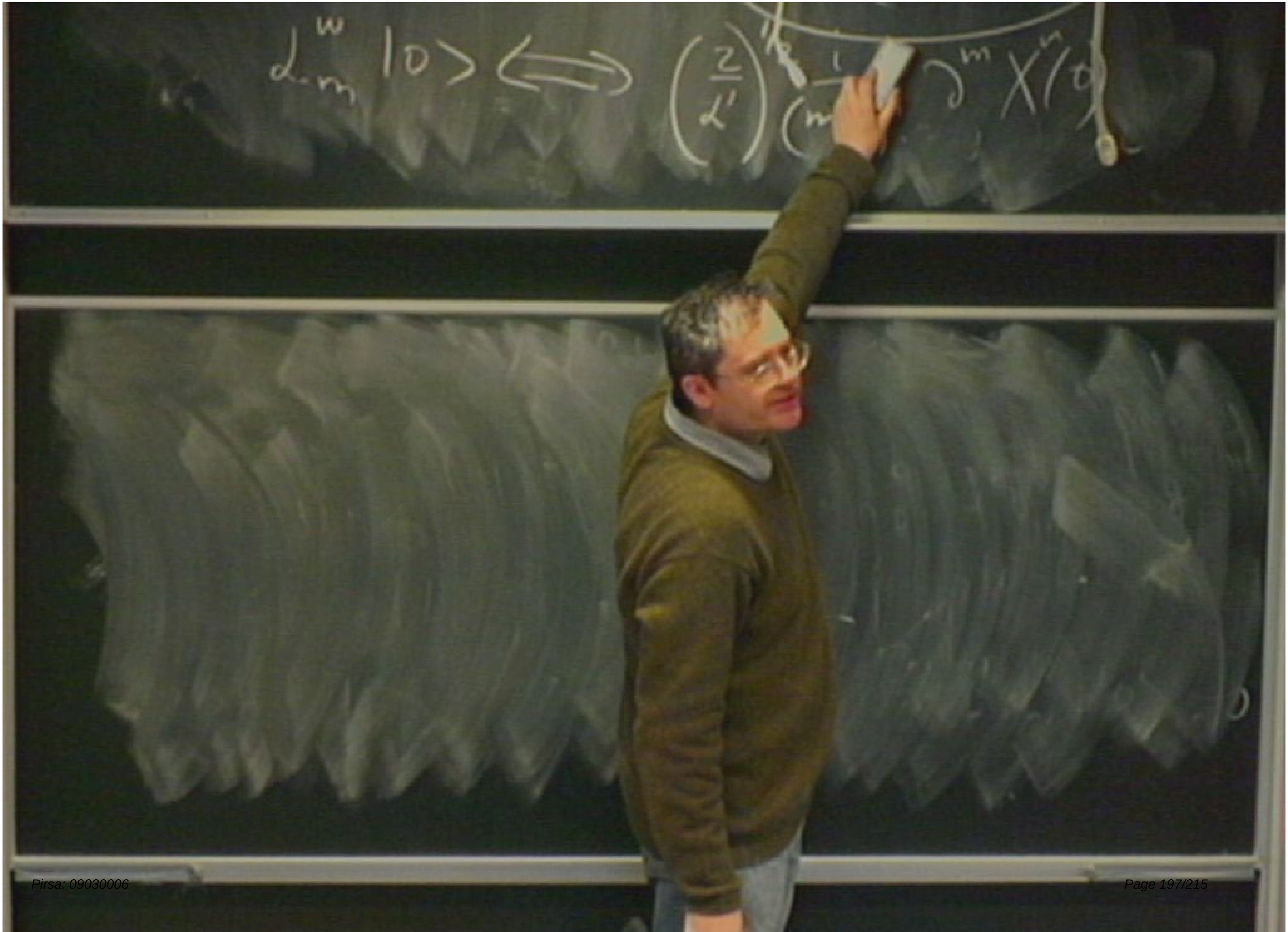
$$\alpha^{\mu}_{-m}|0\rangle \Longleftrightarrow :\alpha^{\mu}_{-m}:\mathbb{1} \Longleftrightarrow :\alpha^{\mu}_{-m}\mathbb{1}:$$

$\underline{m>0}$

$$\partial X^{\mu}(z) = \partial X^{\mu}(0) + \frac{z}{1!}\partial^2 X^{\mu} + \frac{z^{m-1}}{(m-1)!}\partial^{m-1}X$$

$$\frac{2}{\alpha'}\oint_C \frac{dz}{2\pi i} z^{-m}\,\underline{\partial X^{\mu}} \Rightarrow \left(\frac{2}{\alpha'}\right)^{1/2}\frac{i}{(m-1)!}\partial^m X(0)$$

$$\alpha^{\mu}_{-m}|0\rangle \Longleftrightarrow \left(\frac{2}{\alpha'}\right)^{1/2}\frac{i}{(m-1)!}\partial^m X^{\mu}(0)$$



$$\alpha^{w}_{-m}|0\rangle \Longleftrightarrow \left(\frac{2}{\alpha'}\right)^{1/2} \frac{1}{(m-1)!}\,\partial^{m} X^{w}(0)$$

$$\alpha^{m}_{-m}|A\rangle$$

$$\alpha_{-m}^{\mu}|0\rangle \Longleftrightarrow \left(\frac{2}{\alpha'}\right)^{1/2} \frac{i}{(m-1)!}\,\partial^m X^\mu(0)$$

$$\underbrace{\alpha_{-m}^{\mu}|A\rangle} \Rightarrow$$

$$\alpha^{w}_{-m}|0\rangle \Longleftrightarrow \left(\frac{2}{\alpha'}\right)^{1/2}\frac{1}{(m-1)!}\partial^{m}X^{n}(0)$$

$$\underbrace{\alpha^{m}_{-m}|A\rangle}_{\text{a new state}} \Rightarrow$$

$$\alpha^{\mu}_{-m}|0\rangle \Longleftrightarrow \left(\frac{2}{\alpha'}\right)^{1/2} \frac{i}{(m-1)!}\,\partial^{m} X^{\mu}(0)$$

$$\underbrace{\alpha^{\mu}_{-m}|A\rangle}_{\text{new state}} \Rightarrow \underbrace{\left(\frac{2}{\alpha'}\right)^{1/2}\oint\frac{dz}{2\pi}\, z^{m}\,\partial X^{\mu}(z)}_{\alpha^{\mu}_{-m}}$$

$$\alpha_{-m}^{\mu}|0\rangle \Longleftrightarrow \left(\frac{2}{\alpha'}\right)^{1/2}\frac{i}{(m-1)!}\partial^m X^\mu(0)$$

$$\underbrace{\alpha_{-m}^{\mu}|A\rangle}_{\text{a new state}} \Rightarrow \underbrace{\left(\frac{2}{\alpha'}\right)^{1/2}\oint\frac{dz}{2\pi}\, z^{m}\,\partial X^\mu(z)}_{\alpha_{-m}^{\mu}}\,\mathcal{A}(0,0)$$

$$\alpha_{-m}^{\mu}|0\rangle \Longleftrightarrow \left(\frac{2}{\alpha'}\right)^{1/2}\frac{i}{(m-1)!}\partial^m X^{\mu}(0)$$

$$\underbrace{\alpha_{-m}^{\mu}|A\rangle}_{\text{a new state}} \Rightarrow \underbrace{\left(\frac{2}{\alpha'}\right)^{1/2}\oint\frac{dz}{2\pi}\, z^m}_{\alpha_{-m}^{\mu}}\,\partial X^{\mu}(z)\,\mathcal{A}(0,0)$$

$$=$$

$$\alpha^{\mu}_{-m}|0\rangle \Longleftrightarrow \left(\frac{2}{\alpha'}\right)^{1/2}\frac{i}{(m-1)!}\partial^{m}X^{\mu}(0)$$

$$\underbrace{\alpha^{\mu}_{-m}|A\rangle}_{\text{a new state}} \Rightarrow \underbrace{\left(\frac{2}{\alpha'}\right)^{1/2}\oint\frac{dz}{2\pi}\, z^{-m}}_{\alpha^{\mu}_{-m}}\partial X^{\mu}(z)\,\mathcal{A}(0,0)$$

$=$

$$\alpha^{\mu}_{-m}|0\rangle \Longleftrightarrow \left(\frac{2}{\alpha'}\right)^{1/2} \frac{i}{(m-1)!}\,\partial^m X^{\mu}(0)$$

$$\underbrace{\alpha^{\mu}_{-m}|A\rangle}_{\text{a new state}} \Rightarrow \underbrace{\left(\frac{2}{\alpha'}\right)^{1/2}\oint\frac{dz}{2\pi}\, z^{-m}\,\partial X^{\mu}(z)}_{\alpha^{\mu}_{-m}}\; \mathcal{A}(0,0)$$

can contain $X^{\mu}(0,0)$

$=$

$$\alpha^{\mu}_{-m}|0\rangle \Longleftrightarrow \left(\frac{2}{\alpha'}\right)^{1/2} \frac{i}{(m-1)!}\,\partial^m X^{\mu}(0)$$

$$\underbrace{\alpha^{\mu}_{-m}|A\rangle}_{\text{a new state}} \Rightarrow \underbrace{\left(\frac{2}{\alpha'}\right)^{1/2}\oint\frac{dz}{2\pi}\,z^{-m}}_{\alpha^{\mu}_{-m}}\,\partial X^{\mu}\underbrace{(z)\,\mathcal{A}}_{\mathcal{O}(|z|^2)}(0,0)$$

can contain $X^{\mu}(0,\bar{0})$

$=$

$$\alpha^{\mu}_{-m}|0\rangle \Longleftrightarrow \left(\frac{2}{\alpha'}\right)^{1/2} \frac{i}{(m-1)!}\,\partial^m X^\mu(0)$$

$$\underbrace{\alpha^{\mu}_{-m}|A\rangle}_{\text{a new state}} \Rightarrow \underbrace{\left(\frac{2}{\alpha'}\right)^{1/2}\oint\frac{dz}{2\pi}\, z^{-m}}_{\alpha^{\mu}_{-m}}\,\underbrace{\partial X^\mu(z)\,\mathcal{A}(0,0)}_{\partial\partial \ln|z|^2}$$

$\partial X(0,0)$

can contain $X^\mu(0,\bar{0})$

$=$

$$\alpha^{\mu}_{-m}|0\rangle \Longleftrightarrow \left(\frac{2}{\alpha'}\right)^{1/2}\frac{i}{(m-1)!}\partial^m X^{\mu}(0)$$

$$\underbrace{\alpha^{\mu}_{-m}|A\rangle}_{\text{a new state}} \Rightarrow \underbrace{\left(\frac{2}{\alpha'}\right)^{1/2}\oint\frac{dz}{2\pi}}_{\alpha^{\mu}_{-m}} z^{-m}\,\partial X^{\mu}(z)\,\mathcal{A}(0,0)$$

$\partial X(z)$

$\partial\bar{\partial}\ln|z|^2$

can contain $X^{\mu}(0,\bar{0})$

$=$

$$\alpha_{-m}^{\mu}|0\rangle \Longleftrightarrow \left(\frac{2}{\alpha'}\right)^{1/2} \frac{i}{(m-1)!}\partial^m X^\mu(0)$$

$$\underbrace{\alpha_{-m}^{\mu}|A\rangle}_{\text{a new state}} \Longrightarrow \underbrace{\left(\frac{2}{\alpha'}\right)^{1/2}\int\frac{dz}{2\pi}\,z^{-m}}_{\alpha_{-m}^{\mu}}\,\partial X^\mu(z)\,\mathcal{A}(0,0)$$

$\partial X(z)$

can contain $X^\mu(0,0)$

$\partial\partial \ln|z|^2$

$\frac{1}{z}$

$$= \int\frac{dz}{z^{m+1}}$$

$$\alpha^{\mu}_{-m}|0\rangle \Longleftrightarrow \left(\frac{2}{\alpha'}\right)^{1/2}\frac{i}{(m-1)!}\partial^m X^{\mu}(0)$$

$$\underbrace{\alpha^{\mu}_{-m}|A\rangle}_{\text{a new state}} \Rightarrow \underbrace{\left(\frac{2}{\alpha'}\right)^{1/2}\oint\frac{dz}{2\pi}}_{\alpha^{\mu}_{-m}} z^{-m}\,\partial X^{\mu}(z)\,\mathcal{A}(0,0)$$

$\partial X(z)$

can contain $X^{\mu}(0,0)$

$\partial\bar\partial\ln|z|^2$

$$=$$

$$\oint\frac{dz}{z^{m+1}}=0$$

$$\alpha_{-m}^{\mu}|0\rangle \Longleftrightarrow \left(\frac{2}{\alpha'}\right)^{1/2}\frac{i}{(m-1)!}\partial^m X^\mu(0)$$

$$\underbrace{\alpha_{-m}^{\mu}|A\rangle}_{\text{a new state}} \Longrightarrow \underbrace{\left(\frac{2}{\alpha'}\right)^{1/2}\oint\frac{dz}{2\pi}}_{\alpha_{-m}^{\mu}} z^{-m}\, \partial X^\mu(z)\, A(0,0):$$

$\partial X(z)$

can contain $X^\mu(0,0)$

$\partial^2 \ln|z|^2$

$\frac{1}{z}$

$$\partial X^\mu(z) =$$

$$\oint \frac{dz}{z^{m+1}} = 0$$

$$= \partial X(0) + \frac{z}{1!}\partial^2 X^\mu(0) + \cdots + \frac{\partial^m X^\mu(0)\, z^{m-1}}{(m-1)!}$$

$$\alpha^{\mu}_{-m}|0\rangle \Longleftrightarrow \left(\frac{2}{\alpha'}\right)^{1/2} \frac{i}{(m-1)!}\partial^m X^{\mu}(0)$$

$$\underbrace{\alpha^{\mu}_{-m}|A\rangle}_{\text{a new state}} \Rightarrow \underbrace{\left(\frac{2}{\alpha'}\right)^{1/2}\oint \frac{dz}{2\pi}}_{\alpha^{\mu}_{-m}} z^{-m}\,\partial X^{\mu}(z)\,A(0,0):$$

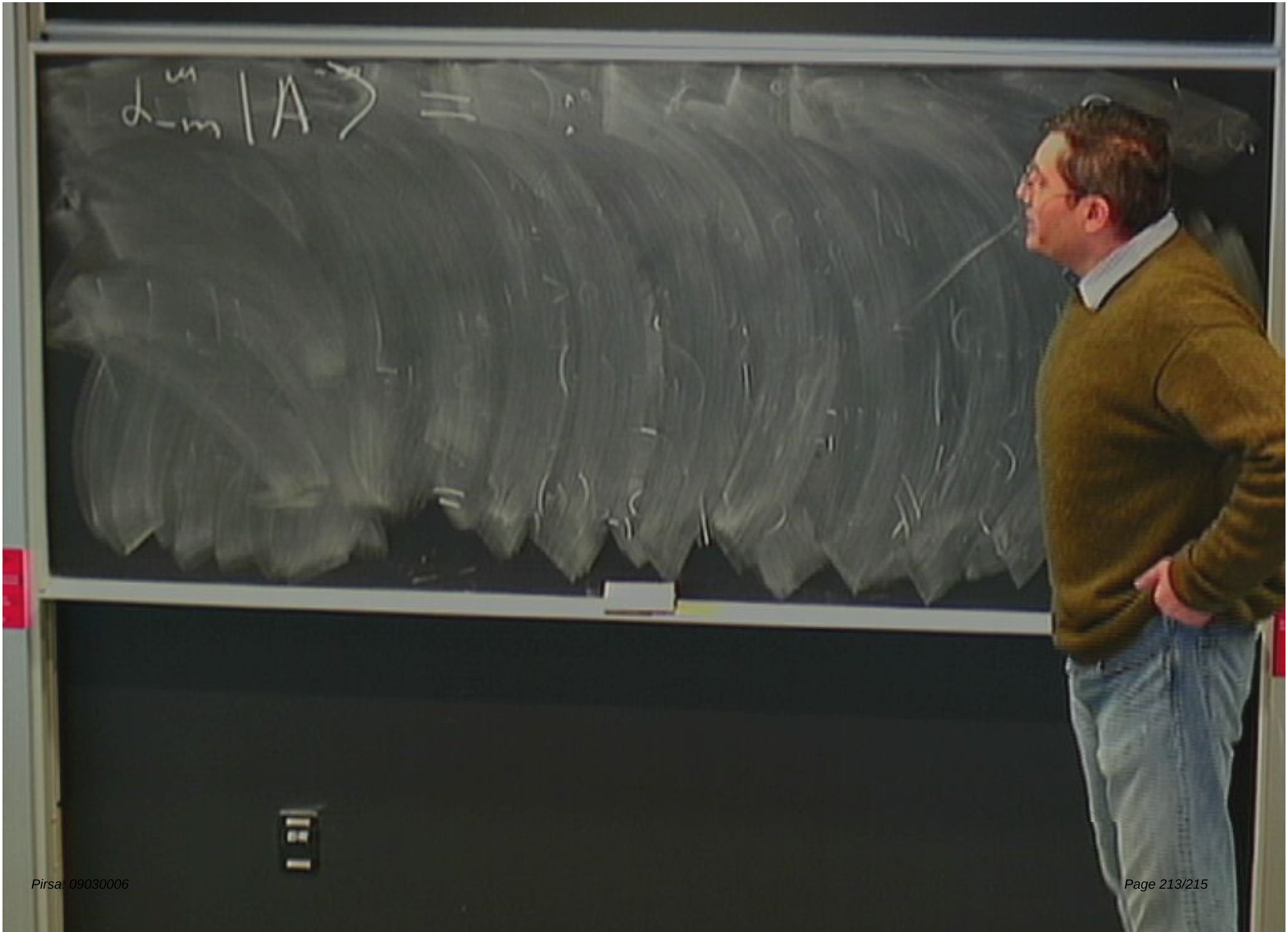
$\partial X(z)$

can contain $X^{\mu}(0,0)$

$\partial\bar{\partial}\ln|z|^2$

$$\partial X^{\mu}(z) = \quad \oint \frac{dz}{z^{m+1}} = 0$$

$$= \partial X^{\mu}(0) + \frac{z}{1!}\partial^2 X^{\mu}(0) + \cdots + \frac{\partial^m X^{\mu}(0)\, z^{m-1}}{(m-1)!}$$


$\alpha^{m}_{-m}|A\rangle =$

$$\alpha_{-m}^{\mu}|A\rangle = \left(\frac{2}{\alpha'}\right)^{1/2} \frac{i}{(m-1)!} : \partial^{m} X^{\mu}(0)\, A(0,\bar{0}) :$$

$$\alpha_{-m}^{\mu} |A\rangle = \left(\frac{2}{\alpha'}\right)^{1/2} \frac{i}{(m-1)!} :\partial^m X^\mu(0)\, A(0,\bar{0}):$$