


Title: Introduction to the Bosonic String

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URL: <http://pirsa.org/09030006>

Abstract: This course provides a thorough introduction to the bosonic string based on the Polyakov path integral and conformal field theory. We introduce central ideas of string theory, the tools of conformal field theory, the Polyakov path integral, and the covariant quantization of the string. We discuss string interactions and cover the tree-level and one loop amplitudes. More advanced topics such as T-duality and D-branes will be taught as part of the course. The course is geared for M.Sc. and Ph.D. students enrolled in Collaborative Ph.D. Program in Theoretical Physics. Required previous course work: Quantum Field Theory (AM516 or equivalent). The course evaluation will be based on regular problem sets that will be handed in during the term. The primary text is the book: 'String theory. Vol. 1: An introduction to the bosonic string. J. Polchinski (Santa Barbara, KITP) . 1998. 402pp. Cambridge, UK: Univ. Pr. (1998) 402 p.' All interested students should contact Alex Buchel at abuchel@uwo.ca as soon as possible.

$j(z)$ $\tilde{j}(z)$



corresponding charges

$$[Q_1, Q_2] = \oint \frac{dz_c}{z_c} \operatorname{Res}_{z_1 \rightarrow z_2} j_1(z_1) j_2(z_2)$$

$$[Q_1, Q_2] = \oint_{\gamma} \frac{dz}{z} \operatorname{Res}_{z_1 \rightarrow z_2} j_1(z_1) j_2(z_2)$$

$$j_m = z^{m+1} T(z)$$

$$[Q_1, Q_2] = \oint_{\gamma} \frac{dz}{z} \operatorname{Res}_{z_1 \rightarrow z_2} j_1(z) j_2(z)$$

$$j_m = z^{m+1} T(z)$$

$$L_m = \int \frac{dz}{2\pi i} z^{m+1} T(z)$$

$$[Q_1, Q_2] = \oint \frac{dz}{2\pi i} \operatorname{Res}_{z_1 \rightarrow z_2} j_1(z_1) j_2(z_2)$$

$$j_m = z^{m+1} T(z)$$

$$\textcircled{L_m} = \int \frac{dz}{2\pi i} z^{m+1} T(z)$$

$$[Q_1, Q_2] = \oint \frac{dz}{2\pi i} \operatorname{Res}_{z_1 \rightarrow z_2} j_1(z_1) j_2(z_2)$$

$$j_m = z^{m+1} T(z)$$

$$L_m = \int \frac{dz}{2\pi i} z^{m+1} T(z)$$

⇒

$$[Q_1, Q_2] = \oint \frac{dz}{2\pi i} \operatorname{Res}_{z_1 \rightarrow z_2} j_1(z_1) j_2(z_2)$$

$$j_m = z^{m+1} T(z)$$

$$L_m = \int \frac{dz}{2\pi i} z^{m+1} T(z)$$

\Rightarrow Suppose we have a primary field

$$[Q_1, Q_2] = \oint \frac{dz}{2\pi i} \operatorname{Res}_{z_1 \rightarrow z_2} j_1(z_1) j_2(z_2)$$

$$j_m = z^{m+1} T(z)$$

$$L_m = \int \frac{dz}{2\pi i} z^{m+1} T(z)$$

\Rightarrow Suppose we have a primary field $O(z)$

$$[Q_1, Q_2] = \oint \frac{dz}{2\pi i} \text{Res}_{z_1 \rightarrow z_2} j_1(z_1) j_2(z_2)$$

$$j_m = z^{m+1} T(z)$$

$$L_m = \int \frac{dz}{2\pi i} z^{m+1} T(z)$$

\Rightarrow Suppose we have a primary field $\mathcal{O}(z)$

$(h, 0)$

$$[Q_1, Q_2] = \oint \frac{dz_1}{2\pi i} \text{Res}_{z_1 \rightarrow z_2} j_1(z_1) j_2(z_2)$$

$$j_m = z^{m+1} T(z)$$

$$L_m = \int \frac{dz}{2\pi i} z^{m+1} T(z)$$

\Rightarrow Suppose we have a primary field $\mathcal{O}(z)$

$$\mathcal{O} = \sum_{m=-\infty}^{+\infty} a_{-m}$$

$$[Q_1, Q_2] = \oint \frac{dz_1}{2\pi i} \operatorname{Res}_{z_1 \rightarrow z_2} j_1(z_1) j_2(z_2)$$

$$j_m = z^{m+1} T(z)$$

$$L_m = \int \frac{dz}{2\pi i} z^{m+1} T(z)$$

\Rightarrow Suppose we have a primary field $O(z)$

$$O = \sum_{m=-\infty}^{+\infty} \frac{O_m}{z^{m+h}}$$

$$[Q_1, Q_2] = \oint \frac{dz}{2\pi i} \operatorname{Res}_{z_1 \rightarrow z_2} j_1(z) j_2(z)$$

$$j_m = z^{m+1} T(z)$$

$$L_m = \int \frac{dz}{2\pi i} z^{m+1} T(z)$$

\Rightarrow Suppose we have a primary field $O(z)$ (4, 0)
 $O = \sum_{m=-\infty}^{+\infty} \frac{O_m}{z^{m+h}}$ "O_m behave as charges"

$$[L_m, O_n] = \left(\binom{m+n}{m} - n \right) O_{m+n}$$

$$[L_m, O_n] = \left(\frac{m-n}{2} \right) O_{m+n}$$

\Rightarrow look @ X-CFT

$$[L_m, O_n] = \left(\left(\frac{m-n}{2} \right) m - n \right) O_{m+n}$$

⇒ look @ X-CFT

⇒ We talk about isomorphisms between operators and states in a CFT

Mode expansion for X-CFT

∂X^m

Mode expansion for X-CFT

Mode expansion for X-CFT

∂X^μ

is

holomorphic

$(1, 0)$

Mode expansion for X-CFT

$$\underbrace{\partial X^m}_{\text{is holomorphic}} = -i \left(\frac{\alpha'}{2} \right)^{1/2}$$

is holomorphic

(1,0)

Mode expansion for X-CFT

$$\underbrace{\partial X^\mu}_{\text{is holomorphic}}(1,0) = -i \left(\frac{\alpha'}{2}\right)^{1/2} \sum_{m=-\infty}^{\infty} \frac{\alpha_m^\mu}{z^{m+1}}$$

Mode expansion for X-CFT

$$\underbrace{\partial X^\mu}_{\text{is holomorphic}} = -i \left(\frac{\alpha'}{2} \right)^{1/2} \sum_{m=-\infty}^{+\infty} \frac{\alpha_m^\mu}{z^{m+1}}$$

$$\bar{\partial} X^\mu = -i \left(\frac{\alpha'}{2} \right)^{1/2} \sum_{m=-\infty}^{+\infty} \frac{\bar{\alpha}_m^\mu}{\bar{z}^{m+1}}$$

\Rightarrow Single valuedness of $\chi^m(b, \bar{z})$



\Rightarrow Single valuedness of $\chi^m(b, \bar{z})$

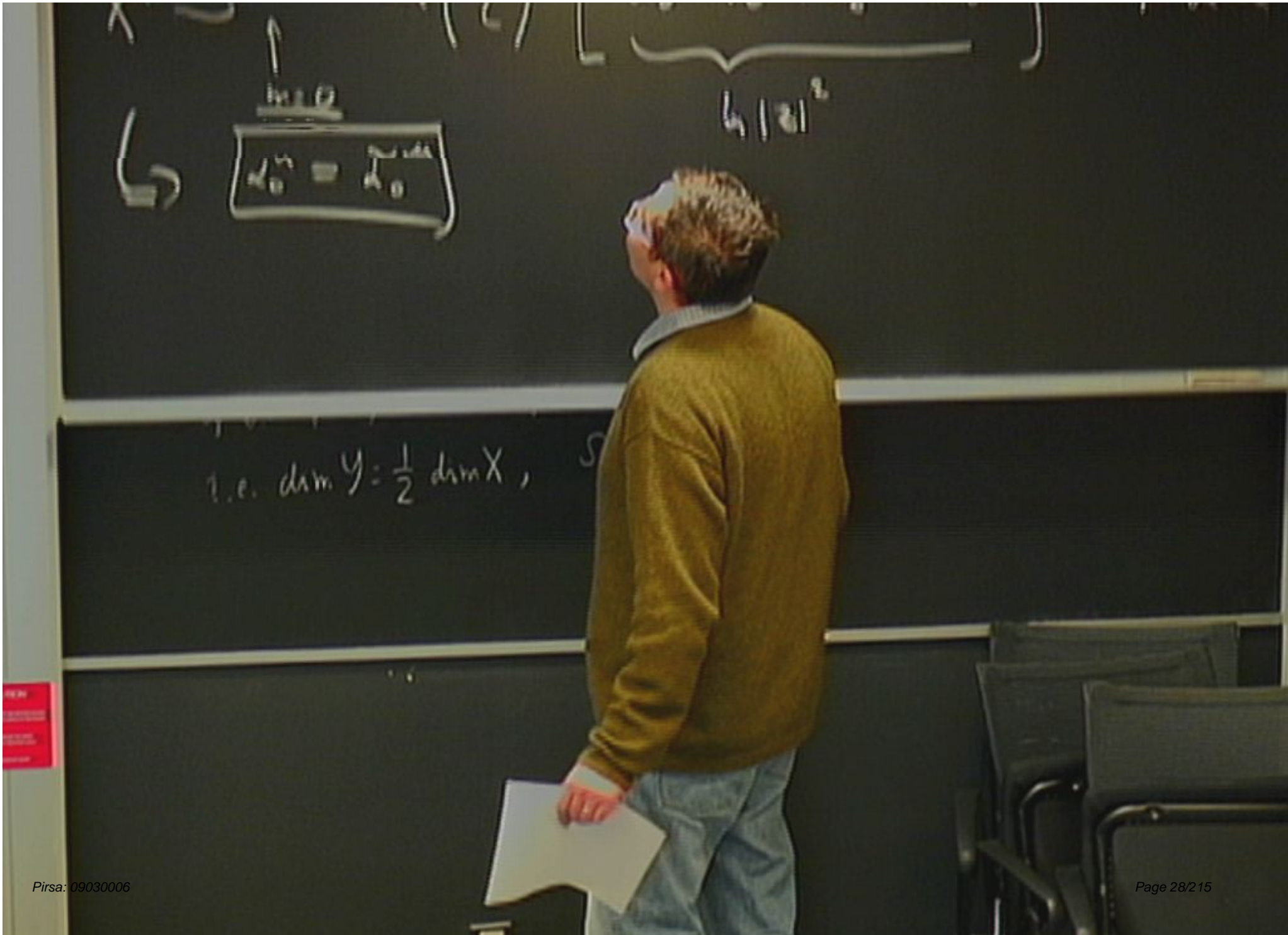
$\chi^m \curvearrowright$

⇒ Single valuedness of $\chi^m(z, \bar{z})$

$$\chi^m \sim \underset{\substack{\uparrow \\ m=0}}{-i \left(\frac{d'}{z} \right)^{1/2} \left[\alpha^m \ln z + \tilde{\alpha}_0^m \ln \bar{z} \right]} + \text{single cuts'}$$

⇒ Single valuedness of $\chi^m(z, \bar{z})$

$$\chi^m \sim \underset{\substack{\uparrow \\ m=0}}{-i \left(\frac{d'}{l} \right)^{1/2} \left[\underbrace{\alpha_0^m \ln z + \tilde{\alpha}_0^m \ln \bar{z}}_{\ln |z|^2} \right] + \text{single terms}}$$

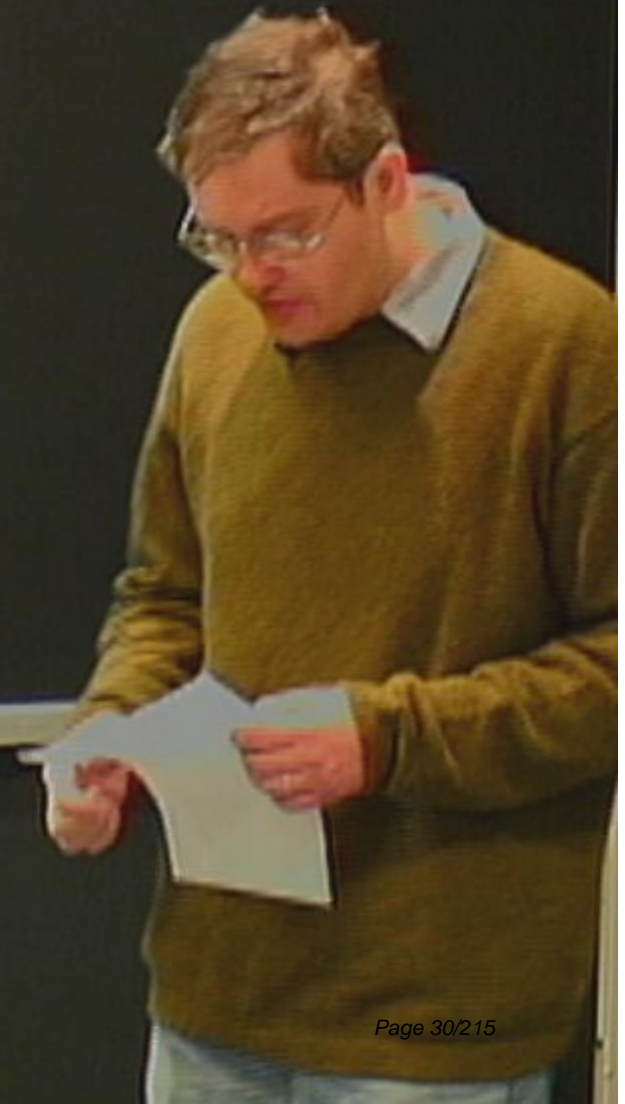


Recall

X_{in}

Recall

$$X^m \rightarrow X^m + a^m$$



Recall

$$X^m \rightarrow X^m + a^m \Leftrightarrow j_a = i \partial_a X^m \cdot \frac{1}{2^m}$$

Recall

$$X^{\mu} \rightarrow X^{\mu} + a^{\mu} \Leftrightarrow j_{\mu} = i \partial_{\alpha} X^{\mu} \cdot \frac{1}{2} \dot{\alpha}^{\alpha}$$

$$j^{\mu} = \frac{1}{2} \dot{\alpha}^{\mu} \partial X^{\nu} \quad ; \quad \bar{j}^{\mu} = \frac{1}{2} \dot{\alpha}^{\mu} \bar{\partial} X^{\nu}$$

Recall

$$X^{\mu} \rightarrow X^{\mu} + a^{\mu} \iff j_{\mu} = i \partial_{\alpha} X^{\mu} \cdot \frac{1}{2^{\frac{1}{2}}}$$

$$j^{\mu} = \frac{i}{2^{\frac{1}{2}}} \partial X^{\mu} \quad ; \quad \bar{j}^{\mu} = \frac{i}{2^{\frac{1}{2}}} \bar{\partial} X^{\mu}$$

A conserved charge corresponding to j_{μ} must be P^{μ}

$$P^m = \oint \frac{dz}{2\pi i}$$

$$P^m = \oint \frac{dz}{2\pi i} \sqrt{z} - \oint \frac{d\bar{z}}{2\pi i} \sqrt{\bar{z}}$$

$$P^m = \oint \frac{dz}{2\pi i} \sqrt{z} - \oint \frac{d\bar{z}}{2\pi i} \sqrt{\bar{z}} = \left(\frac{1}{2\pi i}\right)^{1/2} \left[\dots \right]$$

$$P^m = \oint \frac{dz}{2\pi i} \sqrt{z} - \oint \frac{d\bar{z}}{2\pi i} \sqrt{\bar{z}} = \left(\frac{1}{2}\right)^{1/2} \left[\mathcal{L}_-^m + \mathcal{L}_0^m \right]$$

$$P^m = \oint \frac{dz}{2\pi i} \sqrt{z} - \oint \frac{d\bar{z}}{2\pi i} \sqrt{\bar{z}} = \left(\frac{1}{2\alpha'}\right)^{1/2} \left[\alpha_{-m} + \tilde{\alpha}_0^m \right]$$

$$P^m = \left(\frac{2}{\alpha'}\right)^{1/2} \alpha_0^m = \left(\frac{2}{\alpha'}\right)^{1/2} \tilde{\alpha}_0^m$$

$p_m = \left(\frac{z}{z-i} \right)^m \alpha_0 = \left(\frac{z}{z-i} \right)^m \alpha_0$
 , what is the algebra of α_m α_n ?

\Rightarrow Suppose we
 $\theta = \sum_{m=-\infty}^{+\infty} \alpha_m z^m$

$$\frac{\theta}{z^m}$$

primary field $\mathcal{O}(z)$
 behave as charges

$$[Q_1, Q_2] = \int \frac{dz_2}{2\pi i} \text{Res}_{z_1 \rightarrow z_2} f(z) \psi(z)$$

$$[Q_1, Q_2] = \int \frac{dz_2}{2\pi i} \operatorname{Res}_{z_1 \rightarrow z_2} j_1(z_1) j_2(z_2)$$

$$[Q_1, Q_2] = \int \frac{dz_2}{2\pi i} \text{Res}_{z_1 \rightarrow z_2} j_1(z_1) j_2(z_2)$$

$$d_m^m = \frac{1}{2\pi i} \oint \frac{dz}{z^{m+1}} \partial X^m(z)$$

$$[Q_1, Q_2] = \int \frac{dz_2}{2\pi i} \text{Res}_{z_1 \rightarrow z_2} j_1(z_1) j_2(z_2)$$

$$\alpha_m = \frac{1}{2\pi i} \oint dz z^m \partial X^m(z)$$

$$j_1 = \frac{1}{2\pi i} z^m \partial X^m(z)$$

$$[Q_1, Q_2] = \int \frac{dz_2}{2\pi i} \text{Res}_{z_1 \rightarrow z_2} j_1(z_1) j_2(z_2)$$

$$a_m^{\dagger} = \frac{1}{2\pi i} \oint dz z^m \partial X^m(z)$$

$$j_1 = \frac{1}{2\pi i} \oint dz z^m \partial X^m(z)$$

$$[d_m^u, d_n^v] = [d_m^u, d_n^v] = m \delta_{m, -n} \gamma^{uv}$$

$$[d_m^{\omega}, d_n^{\omega}] = [y_m^{\omega}, z_n^{\omega}] = m \delta_{m,-n} \hbar \omega$$

$$[x^{\omega}, p^{\omega}] = i \hbar$$

$$[d_m^w, d_n^d] = [y_m^w, \tilde{d}_n^d] = m \delta_{m, -n} \langle y^w \rangle$$

$$[X^w, P^d] = i \langle y^w \rangle$$

↑

~~X^w~~

$$[d_m^{\omega}, d_n^{\omega}] = [d_m^{\omega}, d_n^{\omega}] = m \delta_{m,-n} \hbar \omega$$

$$[X^{\omega}, P^{\omega}] = i \hbar$$

$$X^{\omega} = X^{\omega}$$

$$[d_m^u, d_n^v] = [y_m^u, \tilde{y}_n^v] = m \delta_{m, -n} y^{\omega \nu}$$

$$[X^u, P^v] = i \delta^{uv} y^{\omega \nu}$$

↑

$$X^m = x^m - i \left(\frac{d^1}{2} \right) P^m \ln |z|^2 + i \left(\frac{d^1}{2} \right) \sum_{\substack{n=-\infty \\ n \neq 0}}^{+\infty} m \neq 0$$

$$[d_m^u, d_n^v] = [y_m^u, y_n^v] = m \delta_{m, -n} y^{\omega}$$

$$[X^u, P^v] = i y^{\omega}$$

↑

$$X^u = X^u - i \left(\frac{d^u}{2} \right) P^u \ln |z|^2 + i \left(\frac{d^u}{2} \right) \sum_{\substack{m=-\infty \\ m \neq 0}}^{+\infty} \frac{1}{m} \left[\frac{d_m^u}{2} + \frac{2}{m} \frac{d_m^u}{2} \right]$$



$$\sum_{m=-2}^2 \frac{1}{m \neq 0} \left[\dots \right]$$

$|0,0\rangle$

$$\left. \begin{array}{l} \dots \\ \dots \\ \dots \end{array} \right\} \begin{array}{l} m = -2 \\ m \neq 0 \end{array}$$

$$L_n^+ |0,0\rangle = 0, \quad n > 0.$$

$$P_n^+ |0,0\rangle = 0$$



$$L_{-}^{n} |0,0\rangle = 0, \quad n > 0.$$

$$P_{-}^{n} |0,0\rangle = 0$$

↳ we can get excited states.

$$d_{+n}^{\text{op}} |0,0\rangle = 0, \quad n > 0.$$

$$P_{-n}^{\text{op}} |0,0\rangle = 0$$

↳ we can get excited states $d_{-n}^{\text{op}} |0,0\rangle$

$$\langle \psi_n | 10, 0 \rangle = 0, \quad n > 0$$

$$P_{n0} | 0, 0 \rangle = 0$$

↳ we can get excited states $d_{-n} | 0, 0 \rangle$

L_m in terms of d_m
 \tilde{L}_m
 L_m



L_m in terms of z^{-1}

$$\tilde{L}_m$$

$$\prod(z)$$

L_m in terms of α_{-m}
 \tilde{L}_m

$$\mathbb{T}(z) \sim \alpha_{-m} \partial X_m$$

L_m in terms of $\frac{d^m}{dz^m}$
 \tilde{L}_m

$$\Pi(z) \sim \alpha X_m \partial X_m$$

$$L_m = \frac{1}{2\pi i} \oint \phi(z) z^{m+1} \Pi(z)$$

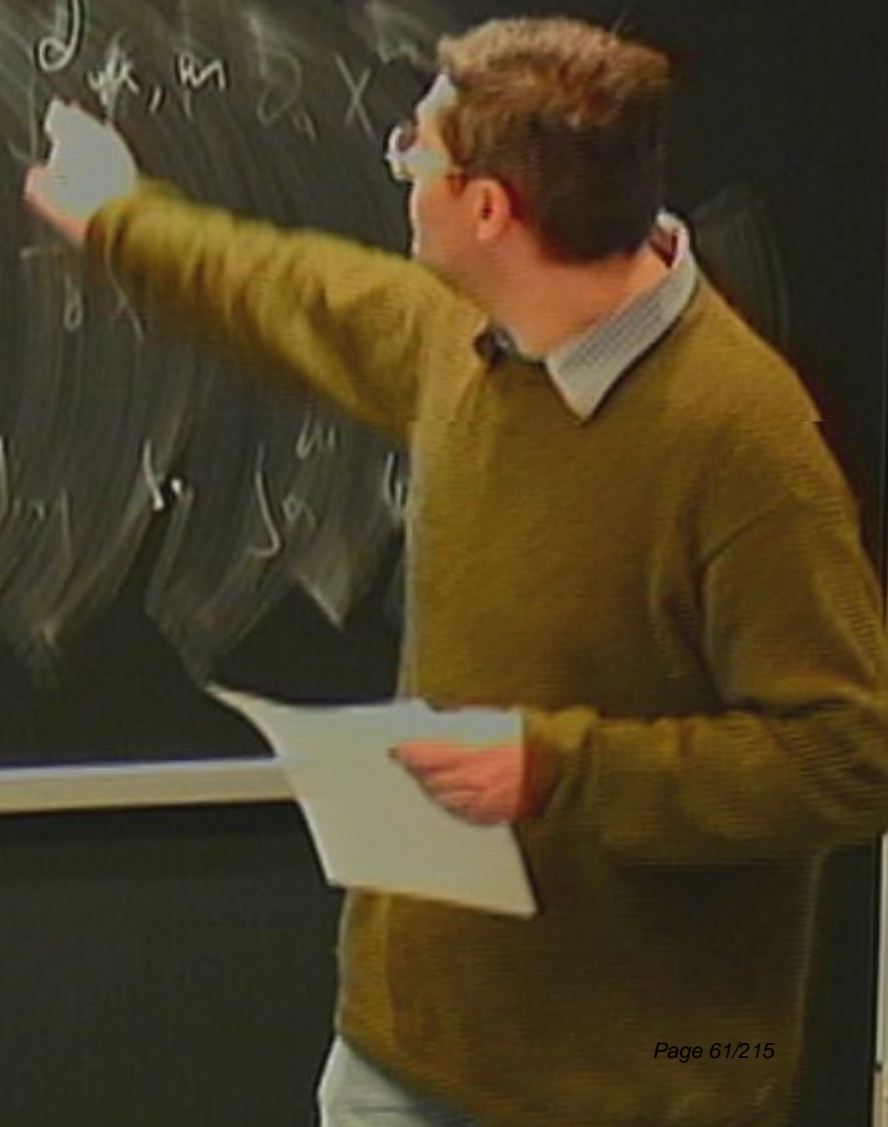


$$X_{m'} = x_m - i \left(\frac{d_{m'}}{2} \right) P_{m'} \left[\frac{1}{2} |z|^2 + i \left(\frac{d_{m'}}{2} \right) \left(\frac{d_{m'}}{2} + \sum_{n=1}^m \frac{d_n}{2} \right) \right]$$

can get excited states $d_{-n} = 1, 0, 0$



$$L_m \sim \frac{1}{2} \sum_{n=-\infty}^{+\infty} d_{m-n} d_{n, m}$$



$L_m \sim \frac{1}{2} \sum_{n=-\infty}^{+\infty} d_{m-n} d_{n, m}$

corresponding to J_n terms be P_n

$$L_m \sim \frac{1}{2} \sum_{n=-\infty}^{+\infty} \alpha_{m-n} \alpha_{n,m}$$

↳ ignored ordering of operators

$\sum_{m=-\infty}^{+\infty} \frac{1}{2}$

ignored ordering of operations

$n \neq 0$

$\sum_{k=1}^n \frac{1}{k}$

$\sum_{k=1}^n \frac{1}{k^2}$

$\sum_{k=1}^n \frac{1}{k^3}$

$\sum_{k=1}^n \frac{1}{k^4}$

$\sum_{k=1}^n \frac{1}{k^5}$

$\sum_{k=1}^n \frac{1}{k^6}$

$\sum_{k=1}^n \frac{1}{k^7}$

$\sum_{k=1}^n \frac{1}{k^8}$

$\sum_{k=1}^n \frac{1}{k^9}$

$\sum_{k=1}^n \frac{1}{k^{10}}$

$\sum_{k=1}^n \frac{1}{k^{11}}$

$\sum_{k=1}^n \frac{1}{k^{12}}$

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$\sum_{k=1}^n \frac{1}{k^{95}}$

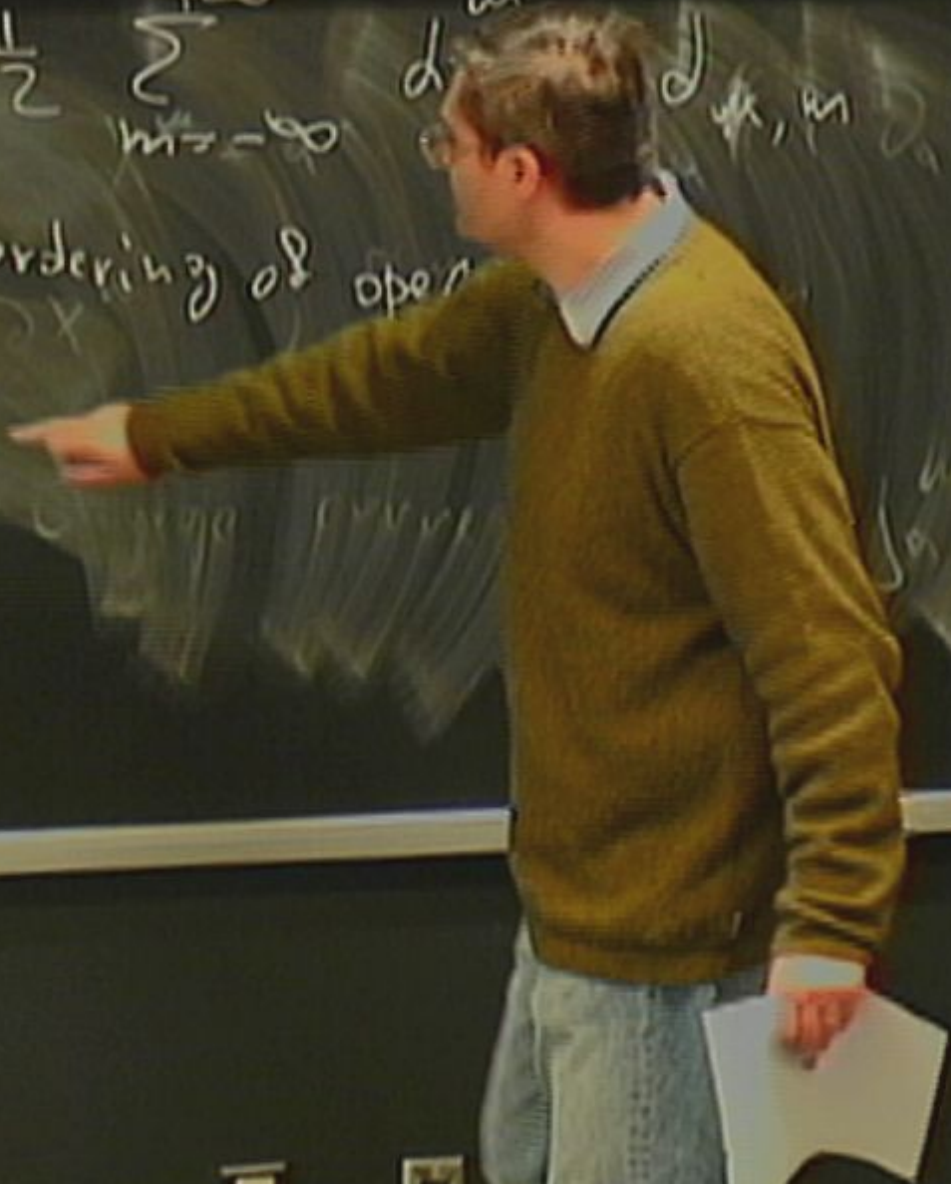
$\sum_{k=1}^n \frac{1}{k^{96}}$

$\sum_{k=1}^n \frac{1}{k^{97}}$

$\sum_{k=1}^n \frac{1}{k^{98}}$

$\sum_{k=1}^n \frac{1}{k^{99}}$

$\sum_{k=1}^n \frac{1}{k^{100}}$



$\int_{-\infty}^{+\infty} \frac{1}{z} dz$

\rightarrow ignored ordering of operators

$n \neq 0$

... corresponding to J_0 must be P_{12}

$$L_0 = \frac{\alpha' p^2}{4} + \sum_{n=1}^{\infty} \alpha_n \cdot \alpha_{-n}$$

$$L_0 = \frac{\alpha' p^2}{4} + \sum_{n=1}^{\infty} \alpha_{-n} \cdot \alpha_n + \alpha' x$$

normal ordering constant.

$$L_0 = \frac{\alpha' p^2}{4} + \sum_{n=1}^{\infty} \alpha_{-n} \cdot \alpha_n + a$$

conformal
normal
ordering

normal ordering constant,

$$L_0 = \frac{\alpha' p^2}{4} + \sum_{n=1}^{\infty} \left(\overset{\text{creation operator}}{\alpha_{-n}} \overset{\text{annihilation}}{\alpha_n} + \overset{\text{normal ordering constant}}{a} \right)$$

The diagram shows the following annotations:

- An arrow points from the text "creation operator" to the α_{-n} term in the sum.
- An arrow points from the text "annihilation" to the α_n term in the sum.
- An arrow points from the text "normal ordering constant" to the a term.
- The text "conformal normal ordering" is written below the sum, with an arrow pointing to the $\alpha_{-n} \alpha_n$ product.

$$L_0 = \frac{\alpha' p^2}{4} + \sum_{n=1}^{\infty} \alpha_{-n} \cdot \alpha_n$$

\swarrow creation operator \nwarrow annihilation
 conformal normal ordering normal ordering constant

↳ Recall that Virasoro algebra implies

$$L_0 = \frac{\alpha' p^2}{4} + \sum_{n=1}^{\infty} \alpha_{-n} \alpha_n + \alpha' X$$

\swarrow creation op. \swarrow annihilation
 \nearrow conformal normal ordering \nearrow normal ordering constant

↳ Recall that Virasoro algebra implies L_m

$$2L_0 = [L_+, L_-] = 0$$

$$2L_0 = [L_+, L_-]$$

→ Previously we constructed our Hilbert space

$$L_n |0\rangle = 0, \quad n > 0.$$

$$P^n |0\rangle = 0$$

$$2L_0 = [L_+, L_-]$$

→ Previously we constructed our Hilbert space

$$L_n |0\rangle = 0, \quad \textcircled{n} > 0, \quad \Leftrightarrow \quad L_{-n} |0\rangle = 0, \quad n > 0.$$

$$P^n |0\rangle = 0$$

$$2L_0 = [L_+, L_-]$$

→ Previously we constructed our Hilbert space

$$L_n |0\rangle = 0, \quad n > 0. \quad \Leftrightarrow \quad L_{-n} |0\rangle = 0, \quad n > 0.$$

$$P^0 |0\rangle = 0$$

$$\Rightarrow 2L_0 |0\rangle = 0$$

$$L_{-1} |0\rangle = 0$$

$$2L_0 = [L_+, L_-]$$

→ Previously we constructed our Hilbert space

$$L_n |0\rangle = 0, \quad \textcircled{n} > 0, \quad \Leftrightarrow \quad L_n^* |0\rangle = 0, \quad n > 0.$$

$$P^+ |0\rangle = 0$$

$$\Rightarrow 2L_0 |0\rangle = 2[L_+, L_-] |0\rangle$$

$$L_{-1} |0\rangle = 0$$

$$= 2(L_+ L_{-1} - L_- L_1) |0\rangle = 0$$

$$L_0 = \frac{\alpha' p^2}{4} + \sum_{n=1}^{\infty} \alpha_{-n} \cdot \alpha_n$$

α_{-n} creation operator
 α_n annihilation operator
 normal ordering constant

Recall that Vir

$$2L_0 = [L_+, L_-]$$

implies L_m

$$[L_-, L_+] |0\rangle = 0$$

$$L_0 |0\rangle = 0 = \alpha_x$$



$$\langle 0|0\rangle = 0 = \langle a_x|0\rangle \Leftrightarrow a_x = 0$$



$$\langle 0|0\rangle = 0 = \langle a_x | 0\rangle \Leftrightarrow a_x = 0$$

$$X^m (z, \bar{z}) \quad X^n (z, \bar{z}) \quad \begin{pmatrix} 1 \\ z \\ \bar{z} \end{pmatrix}$$

$$\langle 0|0\rangle = 0 = \langle a_x | 0\rangle \Leftrightarrow a_x = 0$$

$$X^m(z, \bar{z}) X^{\dagger n}(z', \bar{z}') = :X^m(z, \bar{z}) X^{\dagger n}(z', \bar{z}'):$$

$$\langle 0|0\rangle = 1 = \langle a_x | 10 \rangle \Leftrightarrow a_x = 0$$

$$X^m(z, \bar{z}) X^{\#n}(z, \bar{z}) = :X^m(z, \bar{z}) X^{\#n}(z, \bar{z}):$$

$$-\frac{\alpha'}{2} \eta^{\mu\nu} p_{\mu} |z - \bar{z}|^2$$

$$\langle 0|0\rangle = 0 = \langle a_x | 0\rangle \Leftrightarrow a_x = 0$$

$$X^m(z, \bar{z}) X^{\#n}(z', \bar{z}') = :X^m(z, \bar{z}) X^{\#n}(z', \bar{z}'):$$

$$-\frac{\alpha'}{2} \ln |z - \bar{z}'|^2$$

Virasoro algebra implies L_m

$$2L_0 = [L_+, L_-]$$

→ Previously we constructed our highest state

$$L_n |0\rangle = 0, \quad n > 0. \quad \Leftrightarrow$$

$$P^\mu |0\rangle = 0$$

$$L_{-1} |0\rangle = 0 \Rightarrow 2L_0 |0\rangle = \dots$$

$$= 2(L_0) |0\rangle = 0$$

$$\langle 0|0\rangle = 0 = \langle a_x | 0\rangle \Leftrightarrow a_x = 0$$

$$X^m(z, \bar{z}) X^{\#n}(z', \bar{z}') = :X^m(z, \bar{z}) X^{\#n}(z', \bar{z}'):$$

$$-\frac{\alpha'}{2} \eta^{\mu\nu} p_{\mu} |z - \bar{z}'|^2$$

$$X^m(z, \bar{z}) X^k(z', \bar{z}') = X^m(z, \bar{z}) X^k(z', \bar{z}') :$$

$$X^m(z, \bar{z}) X^k(z', \bar{z}') - \frac{\alpha' m k}{2} \ln |z - \bar{z}'|^2$$

$$X^m(z, \bar{z}) X^j(z', \bar{z}') = \delta_{m, -j} X^m(z, \bar{z}) X^j(z', \bar{z}') + \dots$$

"creation, annihilation"

↳ we can get excited states $d_{-h}^m |0, 0\rangle$

$$\langle \alpha, \alpha | X | \alpha', \alpha' \rangle = 0 \quad \langle \alpha, \alpha | X^m | \alpha', \alpha' \rangle = 0$$

creation, annihilation

$$|\alpha\rangle > |\alpha'\rangle$$

$$\langle 1, 0, 0 | = 0$$

↳ we can get excited states. $d_{-h}^m |0, 0\rangle$



$$\left[+ \frac{d'}{2} y^{(m)} \right] - \ln|z|^2 + \sum_{m=1}^{\infty} \frac{1}{m} \left[\left(\frac{z}{|z|} \right)^m + \left(\frac{|z|}{z} \right)^m \right]$$

$$\left[+ \frac{d'}{2} y^{(m)} \right] - \ln |z|^2 + \sum_{m=1}^{\infty} \frac{1}{3m} \left[\binom{2m}{m} + \binom{2m}{m} \right]$$

$$- \frac{1}{2} y^{(m)} \ln |z|^2$$

$$+ \frac{d'}{2} \gamma^{(m)} \left[-\ln|z|^2 + \sum_{m=1}^{\infty} \frac{1}{m} \left[\binom{-m}{m} + \binom{-m}{m} \right] \right]$$

$$- \frac{d'}{2} \gamma^{(m)} \left[-\ln|z|^2 + \sum_{m=1}^{\infty} \frac{1}{m} \left[\binom{-m}{m} + \binom{-m}{m} \right] \right]$$

$$\begin{aligned}
 & + \frac{d'}{2} \gamma^{(m)} \left[-\ln|z|^2 + \sum_{m=1}^{\infty} \frac{1}{m} \left[\left(\frac{|z|^m}{|z|^m} \right)^m + \left(\frac{|z|^m}{|z|^m} \right)^m \right] \right] \\
 & = -\frac{1}{2} \ln|z|^2 - \sum_{m=1}^{\infty} \frac{1}{m} \ln|z|^2 = -\frac{1}{2} \ln|z|^2 - \ln|z|^2 = -\frac{3}{2} \ln|z|^2
 \end{aligned}$$

$$\begin{aligned}
 & + \frac{d'}{2} \gamma^{(m)} \left[-\ln |z|^2 + \sum_{m=1}^{\infty} \frac{1}{m} \left[\left(\frac{z^m}{m} \right) + \left(\frac{z^{-m}}{m} \right) \right] \right] \\
 & - \frac{1}{2} \gamma^{(m)} \left[\ln |z|^2 - \sum_{m=1}^{\infty} \frac{1}{m} \left(\ln |z|^2 + \ln \left| \frac{z^m}{m} \right| + \ln \left| \frac{z^{-m}}{m} \right| \right) \right]
 \end{aligned}$$

$$L_{\text{m}} = \frac{1}{2} \sum_{\alpha} \dot{d}_{\alpha} \dot{d}_{\alpha}$$

Conformal normal ordering \Leftrightarrow

Conformal normal ordering \Leftrightarrow creation-annihilation h.o.

Conformal normal ordering \Leftrightarrow creation-annihilation h.o.

not always true!

p_{μ}

Conformal normal ordering \Leftrightarrow creation-annihilation h.o.

not always true!

p_{μ}

→ We struggle about us an annihil. operator

$$p^{\mu} |0\rangle = 0$$

$$\left(\frac{1}{z} \right) \left(\frac{1}{z} \right)$$

creation annihilation

$$\begin{aligned}
 & + \frac{d'}{2} \psi^{(w)} \left[-\ln|z|^2 + \sum_{m=1}^{\infty} \frac{1}{m} \left[\left(\frac{z'}{z} \right)^m + \left(\frac{\bar{z}'}{\bar{z}} \right)^m \right] \right] \\
 & - \frac{d'}{2} \psi^{(w)} \left[\ln|z|^2 + (n-1) \ln|z|^2 + \frac{2n}{z} \right]
 \end{aligned}$$

$$\left(\frac{1}{z} \right) \left(\frac{1}{z} \right)$$

creation annihilation

$$+ \frac{d'}{2} \psi^{(w)} \left[-\ln|z|^2 + \sum_{m=1}^{\infty} \frac{1}{m} \left[\left(\frac{z'}{z} \right)^m + \left(\frac{z'}{z} \right)^m \right] \right]$$

$$= -\frac{d'}{2} \psi^{(w)} \left(\ln|z|^2 + (n-1) \ln \left(\frac{z'}{z} \right) \right)$$

$$\rightarrow -\frac{d'}{2} \psi^{(w)} \left(\ln|z|^2 + (n-1) \ln \left(\frac{z'}{z} \right) \right)$$

bc = CFT

creation-annihilation

$$b(z) = \sum_{h=-\infty}^{+\infty} \frac{b_m}{z^{m+1}}$$

$$c(z) = \sum_{m=-\infty}^{+\infty} \frac{c_m}{z^{m+1}}$$



bc = CFT

creation-annihilation

$$b(z) = \sum_{h=-\infty}^{+\infty} \frac{b_m}{z^{m+1}}$$

$$c(z) = \sum_{m=-\infty}^{+\infty} \frac{c_m}{z^{m+1}}$$

$\lambda = \frac{1}{2}$



bc - CFT

creation-annihilation ops

$$b(z) = \sum_{h=-\infty}^{+\infty} \frac{b_m}{z^{m+1}}$$

$$c(z) = \sum_{m=-\infty}^{+\infty} \frac{c_m}{z^{m+1}}$$

$\lambda = \frac{1}{2} \rightarrow$ is important for a superstring

bc = CFT

$$b(z) = \sum_{h=-\infty}^{+\infty} \frac{b_m}{z^{m+\lambda}}$$

creation-annihilation, ρ

$$c(z) = \sum_{m=-\infty}^{+\infty} \frac{c_m}{z^{m+\lambda}}$$

$\lambda = \frac{1}{2} \rightarrow$ is important for a superstring

$\lambda = 2$

When λ is integer

$$bc = -bc$$

to

to

pw

$$bc = -bc$$

$$b(z, c)$$

$$bc = -bc$$

$$b(z_1)c(z_2) \sim \frac{1}{z_{12}}$$

$$bc = -bc$$

$$b(z_1)c(z_2) \sim \frac{1}{z_{12}}$$

$$[Q_1, Q_2] =$$

$$bc = -bc$$

$$b(z_1)c(z_2) \sim \frac{1}{z_{12}}$$

$$[Q_1, Q_2]_+ = \oint \frac{dz_2}{2\pi i} \text{Res}_{z_1 \rightarrow z_2} j_1(z) j_2(z)$$

$$bc = -cb$$

$$b(z_1)c(z_2) \sim \frac{1}{z_{12}}$$

$$[Q_1, Q_2] = \oint \frac{dz_2}{2\pi i} \text{Res}_{z_1 \rightarrow z_2} j_1(z) j_2(z)$$

$$\{Q_1, Q_2\} = Q_1 Q_2 + Q_2 Q_1$$

$$bc = -bc$$

$$b(z_1)c(z_2) = \frac{1}{z_{12}}$$

$$\Rightarrow \{b_m, c_n\} = \delta_{m,-n}$$

$$[Q_1, Q_2] = \oint \frac{dz}{2\pi i} \text{Res}_{z_1 \rightarrow z_2} j_1(z) j_2(z)$$

$$\Downarrow$$
$$\{Q_1, Q_2\} = Q_1 Q_2 + Q_2 Q_1$$



$l=m=0 \Rightarrow$
↳ the ground state is degenerate.

$|\uparrow\rangle$

$|\downarrow\rangle$

$h=m=0 \Rightarrow$
↳ the ground state is degenerate.

$$\begin{array}{l} |\uparrow\rangle \quad |\downarrow\rangle \\ b. |\downarrow\rangle \neq 0 \quad c. |\uparrow\rangle = 0 \\ d. |\uparrow\rangle = |\downarrow\rangle \quad e. |\downarrow\rangle = |\uparrow\rangle \end{array}$$

$$l = m = 0 \Rightarrow$$

↳ the ground state is degenerate:

$$|\uparrow\rangle$$

$$|\downarrow\rangle$$

$$b_0 |\downarrow\rangle \neq 0$$

$$c_0 |\uparrow\rangle \neq 0$$

$$b_0 |\uparrow\rangle = |\downarrow\rangle$$

$$c_0 |\downarrow\rangle = |\uparrow\rangle$$

$$b_n |\downarrow\rangle = b_n |\uparrow\rangle = c_n |\uparrow\rangle = c_n |\downarrow\rangle = 0, n > 0$$

$$b_n |\downarrow\rangle = b_n |\uparrow\rangle = c_n |\uparrow\rangle = c_n |\downarrow\rangle = 0, \quad n > 0$$

→ Previously we constructed our Hilbert space

$$L_n |0\rangle = 0, \quad (n) > 0, \Leftrightarrow L_n^u |0\rangle = 0, \quad n > 0.$$

$$P^u |0\rangle = 0$$

$$L_{-n} |0\rangle = 0 \Rightarrow 2L_0 |0\rangle = 2[L_1, L_{-1}] |0\rangle = 2(L_1 L_{-1} - L_{-1} L_1) |0\rangle = 0$$

$$b_n | \downarrow \rangle = b_n | \uparrow \rangle = c_n | \uparrow \rangle = c_n | \downarrow \rangle = 0, \quad n > 0$$

Given mode expansion for $b(z)$, $c(z) \rightarrow$

$$b_n |\downarrow\rangle = b_n |\uparrow\rangle = c_n |\uparrow\rangle = c_n |\downarrow\rangle = 0, \quad n > 0$$

Given mode expansion for $b(z)$, $c(z) \rightarrow$

$$L_m = \sum_{n=-\infty}^{+\infty} (m-n) \begin{matrix} 0 \\ 0 \end{matrix}$$

$$b_n |\downarrow\rangle = b_n |\uparrow\rangle = c_n |\uparrow\rangle = c_n |\downarrow\rangle = 0, \quad n > 0$$

Given mode expansion for $b(z)$, $c(z) \rightarrow$

$$L_m = \sum_{n=-\infty}^{+\infty} (m-n) \begin{pmatrix} 0 & 0 \\ 0 & b_n c_{m-n} \end{pmatrix} + \delta_{m,0} qg$$

$$b_n |\downarrow\rangle = b_n |\uparrow\rangle = c_n |\uparrow\rangle = c_n |\downarrow\rangle = 0, \quad n \neq 0$$

Given mode expansion for $b(z), c(z) \rightarrow$

$$L_m = \sum_{n=-\infty}^{+\infty} (m-n) \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} b_n c_{m-n} + \delta_{m,0} \text{normal ordering constant } qg$$

normal ordering constant

$$b_n | \downarrow \rangle = b_n | \uparrow \rangle = c_n | \uparrow \rangle = c_n | \downarrow \rangle = 0, \quad n > 0$$

Given mode expansion for $b(z)$, $c(z) \rightarrow$

$$L_m = \sum_{n=-\infty}^{+\infty} (m-n) \begin{pmatrix} 0 & 0 \\ 0 & b_n c_{m-n} \end{pmatrix} + \delta_{m,0} \text{normal ordering constant } qg$$

$$b_n |\downarrow\rangle = b_n |\uparrow\rangle = c_n |\uparrow\rangle = c_n |\downarrow\rangle = 0, \quad n > 0$$

Given mode expansion for $b(z), c(z) \rightarrow$

$$L_m = \sum_{n=-\infty}^{+\infty} (m-n) \begin{pmatrix} 0 & 0 \\ 0 & b_n c_{n-m} \end{pmatrix} + \delta_{m,0} \text{ normal ordering constant}$$

$$b_n |\downarrow\rangle = b_n |\uparrow\rangle = c_n |\uparrow\rangle = c_n |\downarrow\rangle = 0, \quad n > 0$$

Given mode expansion for $b(z), c(z) \rightarrow$

$$L_n = \sum_{m=-\infty}^{+\infty} (m\lambda - n) \begin{pmatrix} 0 & 0 \\ 0 & b_m c_{m-n} \end{pmatrix} + \delta_{m,0} qg$$

normal ordering constant

$$2L_0 = [L_{11}, L_{-1}]$$

$$L_0 |0\rangle = 0$$

$$b_n |\downarrow\rangle = b_n |\uparrow\rangle = c_n |\uparrow\rangle = c_n |\downarrow\rangle = 0, \quad n \geq 0$$

Given mode expansion for $b(z), c(z) \rightarrow$

$$L_n = \sum_{m=-\infty}^{+\infty} (m\lambda - n) \begin{pmatrix} 0 & 0 \\ 0 & b_m c_{n-m} \end{pmatrix} + \delta_{m,0} qg$$

normal ordering constant

$$2L_0 = [L_{11}, L_{-1}]$$

\hookrightarrow study $|\downarrow\rangle$

$$L_1 | \downarrow \rangle = \sum_{n=-\infty}^{\infty} (\lambda - n) b_n C_{\lambda-n} | \downarrow \rangle$$

$$L_1 |\downarrow\rangle = \sum_{n=-\infty}^{\infty} (\lambda - n) b_n c_{1-n} |\downarrow\rangle$$

$n \neq 0$



$$L_1 | \downarrow \rangle = \sum_{n=-\infty}^{\infty} (\lambda - n) b_n c_{1-n} | \downarrow \rangle$$

$$n \neq 0, \\ n = 0$$

$$L_1 | \downarrow \rangle = \sum_{n=-\infty}^{\infty} (\lambda - n) \begin{matrix} 0 & b_n & 0 \\ 0 & 0 & 0 \end{matrix} C_{|\lambda-n|} \begin{matrix} 0 \\ \downarrow \\ 0 \end{matrix} \rangle$$

$$\begin{matrix} n \neq 0 \\ n = 0 \end{matrix} \begin{matrix} 0 & b_n & 0 \\ 0 & 0 & 0 \end{matrix} C_{|\lambda-n|} \begin{matrix} 0 \\ \downarrow \\ 0 \end{matrix} \rangle$$

$$L_1 | \downarrow \rangle = \sum_{n=-\infty}^{\infty} (\lambda - n) \begin{matrix} 0 \\ 0 \end{matrix} b_n \begin{matrix} 0 \\ 0 \end{matrix} C_{1-n} \begin{matrix} 0 \\ 0 \end{matrix} | \downarrow \rangle$$

$$\begin{matrix} n \neq 0 \\ n = 0 \end{matrix} \begin{matrix} 0 \\ 0 \end{matrix} b_0 \begin{matrix} 0 \\ 0 \end{matrix} C_{1-0} \begin{matrix} 0 \\ 0 \end{matrix} | \downarrow \rangle = 0$$

$$L_1 | \downarrow \rangle = \sum_{n=-\infty}^{\infty} (\lambda - n) b_n c_{1-n} | \downarrow \rangle$$

$$n \leq 0, \quad b_0 c_{1-0} | \downarrow \rangle = 0$$

$$n > 0, \quad b_n c_{1-n} = c_{1-n} b_n | \downarrow \rangle = 0$$

$$b_n |\downarrow\rangle = b_n |\uparrow\rangle = c_n |\uparrow\rangle = c_n |\downarrow\rangle = 0, \quad n > 0$$

Given mode expansion for $b(z)$, $c(z) \rightarrow$

$$L_m = \sum_{n=-\infty}^{+\infty} (m-n) \begin{pmatrix} 0 & 0 \\ b_n & c_{n-n} \end{pmatrix} + \delta_{m,0} qg$$

normal ordering constant

$$2L_0 = [L_1, L_{-1}] = L_1 L_{-1} - L_{-1} L_1$$

\hookrightarrow study $|\downarrow\rangle$

$n \neq 0$
 $n = 0$
 $n > 0$

$$\begin{pmatrix} 0 & b_0 c_{1-p} \\ 0 & 0 \end{pmatrix} \downarrow = 0$$

$$\begin{pmatrix} 0 & b_n c_{1-n} \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} c_{1-n} b_n \\ 0 \end{pmatrix} \downarrow = 0$$

$$L, L_{-1} \downarrow = L, \sum_{n=-\infty}^{+\infty}$$

$$(-)^{n-k} b_n c_{1-n} \downarrow$$

$$\begin{pmatrix} n/2p \\ n/2 + 1/2 | n \end{pmatrix} = \begin{pmatrix} n/2 \\ n/2 - 2/4 \end{pmatrix}$$

$n \geq 0$
 $n = 0$
 $n < 0$

$$0 \begin{matrix} 0 & b_0 & c_{10} \\ 0 & 0 & 0 \end{matrix} \downarrow \rangle = 0$$

$$L, L_{-1} \downarrow \rangle = L \sum_{n=-\infty}^{+\infty} (-1)^{n-k} b_n c_{1-n} = c_{1-n} b_n \downarrow \rangle = 0$$

$$= L \downarrow \rangle = c_{10} b_0 \downarrow \rangle = 0$$

$$\frac{1}{\sqrt{2}} \left(|n\rangle + |21n\rangle \right)$$

$$\frac{1}{\sqrt{2}} \left(|2\rangle - |21n\rangle \right)$$

$$L_1 | \downarrow \rangle = \sum_{n=-\infty}^{\infty} (\lambda - n) b_n c_{1-n} | \downarrow \rangle = 0$$

$$\begin{matrix} n \neq 0 \\ n = 0 \end{matrix} \quad b_0 c_{1-0} | \downarrow \rangle = 0$$

$$L_1 L_{-1} | \downarrow \rangle = L_1 \sum_{n=-\infty}^{\infty} (-\lambda - n) b_n c_{1-n} | \downarrow \rangle = 0$$

$$= L_1 b_0 c_{1-0} | \downarrow \rangle = \lambda b_0 c_{1-0} | \downarrow \rangle = 0$$

$$\begin{aligned} & \frac{1}{\sqrt{2}} (|n\rangle + |2-n\rangle) \\ & \frac{1}{\sqrt{2}} (|2\rangle + |0\rangle) \end{aligned}$$

$$L_n | \downarrow \rangle = \sum_{k=-\infty}^{n-1} (n-k) b_k C_{n-k} | \downarrow \rangle = 0$$

$$n \neq 0, \quad n=0, \quad b_0 C_1 | \downarrow \rangle = 0$$

$$L_n L_{-1} | \downarrow \rangle = L_n \sum_{k=-\infty}^{n+1} (-k-n) b_k C_{-k-n} | \downarrow \rangle = C_{1-n} b_n | \downarrow \rangle = 0$$

$$= L_n b_0 C_1 | \downarrow \rangle = \lambda b_0 C_1 | \downarrow \rangle = \lambda b_0 C_1 | \downarrow \rangle = 0$$

$$\begin{aligned} & \left(\frac{n}{2} + 1 \right) | \downarrow \rangle + \left(\frac{n}{2} + 1 \right) | \downarrow \rangle \\ & \left(\frac{n}{2} + 1 \right) | \downarrow \rangle = \left(\frac{n}{2} + 1 \right) | \downarrow \rangle \\ & \left(\frac{n}{2} + 1 \right) | \downarrow \rangle = \left(\frac{n}{2} + 1 \right) | \downarrow \rangle \end{aligned}$$

$$L_1 | \downarrow \rangle = \sum_{n=-\infty}^{\infty} (\lambda - n) b_n c_{1-n} | \downarrow \rangle = 0$$

$$\begin{matrix} n \neq 0 \\ n = 0 \end{matrix} \quad \begin{matrix} 0 \\ 0 \end{matrix} b_n c_{1-n} | \downarrow \rangle = 0$$

$$L_1 L_{-1} | \downarrow \rangle = L_1 \sum_{n=-\infty}^{\infty} (-\lambda - n) b_n c_{-1-n} | \downarrow \rangle = 0$$

$$= L_1 (b_{-1} c_0 | \downarrow \rangle) = \lambda b_{-1} c_0 | \downarrow \rangle = 0$$

$$\begin{matrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{matrix} \begin{matrix} |n\rangle + |21\rangle \\ |n\rangle \end{matrix} = \frac{1}{\sqrt{2}} \begin{matrix} |2\rangle \\ |2-2\rangle \end{matrix}$$

$$\langle \downarrow | \rho_1' | \downarrow \rangle = \lambda \langle \downarrow | \rho_0 | \downarrow \rangle = \lambda \langle \downarrow | \rho_1 | \downarrow \rangle$$

$$= (1-\lambda) \lambda \langle \downarrow | \rho_0 | \downarrow \rangle$$

$$= \langle \downarrow | \rho_1^{-1} b_{-1} \rho_1^{-1} \downarrow \rangle = \lambda \langle \downarrow | \rho_1^{-1} b_{-1} \rho_1^{-1} \downarrow \rangle$$

$$= (1-\lambda) \lambda \langle \downarrow | \underbrace{\rho_1^{-1} b_{-1} \rho_1^{-1}} \downarrow \rangle = (1-\lambda) \lambda$$

$$\{c_1, b_{-1}\} = 1$$

$$c_1 b_{-1} = 1 - b_{-1} c_1$$



$$= \langle \downarrow | \rho_1^{-1} \rho_0^{-1} b_{-1} c_1 | \downarrow \rangle = \lambda \langle \downarrow | \rho_1^{-1} \rho_0^{-1} b_{-1} c_1 | \downarrow \rangle$$

$$= (1-\lambda) \lambda \langle \downarrow | \rho_1^{-1} \rho_0^{-1} b_{-1} c_1 | \downarrow \rangle = (1-\lambda) \lambda$$

$$\{c_1, b_{-1}\} = 1$$

$$c_1 b_{-1} = \textcircled{1} - \textcircled{b_{-1} c_1}$$

$$= \langle \downarrow | \rho_0 \rangle = \langle \downarrow | \rho_0 \rangle = \langle \downarrow | \rho_0 \rangle$$

$$= (1-\lambda) \lambda \langle \downarrow | \rho_0 \rangle = (1-\lambda) \lambda \langle \downarrow | \rho_0 \rangle$$

$$\{c_i, b_{-i}\} = 1$$

$$c_i b_{-i} = \textcircled{1} - \textcircled{b_{-i} c_i}$$

$$= \langle \downarrow | (1-\lambda) b_{-1} c_0 | \downarrow \rangle = \lambda \langle \downarrow | b_0 c_1 b_{-1} c_0 | \downarrow \rangle$$

$$= (1-\lambda) \lambda \langle \downarrow | b_0 c_1 b_{-1} c_0 | \downarrow \rangle = (1-\lambda) \lambda \langle \downarrow | b_0 c_0 | \downarrow \rangle$$

$$= (1-\lambda) \lambda \langle \downarrow | \downarrow \rangle$$

$$\{c_i, b_{-i}\} = 1$$

$$c_i b_{-i} = \textcircled{1} - \textcircled{b_{-i} c_i}$$

$$= \langle \downarrow | \rho_{-1} b_{-1} c_0 | \downarrow \rangle = \lambda \langle \downarrow | \rho_{-1} b_{-1} c_0 | \downarrow \rangle$$

$$= (1-\lambda) \lambda \langle \downarrow | \rho_{-1} b_{-1} c_0 | \downarrow \rangle = (1-\lambda) \lambda \langle \downarrow | \rho_{-1} c_0 | \downarrow \rangle$$

$$= (1-\lambda) \lambda \langle \downarrow | \downarrow \rangle \equiv \langle \downarrow | \downarrow \rangle$$

$$\{c_i, b_{-i}\} = 1$$

$$c_i b_{-i} = \textcircled{1} - \textcircled{b_{-i} c_i}$$

$$= \langle \downarrow | \rho_{-1} b_{-1} c_0 | \downarrow \rangle = \lambda \langle \downarrow | \rho_{-1} b_{-1} c_0 | \downarrow \rangle$$

$$= (1-\lambda) \lambda \langle \downarrow | \rho_{-1} b_{-1} c_0 | \downarrow \rangle = (1-\lambda) \lambda \langle \downarrow | \rho_{-1} c_0 | \downarrow \rangle$$

$$\{c_i, b_{-i}\} = 1$$

$$c_i b_{-i} = \underbrace{1}_{\text{circled}} - \underbrace{b_{-i} c_i}_{\text{circled}}$$

$$= (1-\lambda) \lambda \langle \downarrow | \downarrow \rangle \equiv$$

$$\equiv 2 \langle \downarrow | \downarrow \rangle$$

$$= 2 \langle \downarrow | \downarrow \rangle$$

$$= \langle \downarrow | \rho_{-1} b_{-1} c_0 | \downarrow \rangle = \lambda \langle \downarrow | \rho_{-1} b_{-1} c_0 | \downarrow \rangle$$

$$= (1-\lambda) \lambda \langle \downarrow | \rho_{-1} b_{-1} c_0 | \downarrow \rangle = (1-\lambda) \lambda \langle \downarrow | \rho_{-1} c_0 | \downarrow \rangle$$

$$= \underline{\underline{(1-\lambda) \lambda}} \langle \downarrow | \downarrow \rangle \equiv$$

$$\{c_i, b_{-i}\} = 1$$

$$c_i b_{-i} = \textcircled{1} - \textcircled{b_{-i} c_i}$$

$$\equiv 2 \langle \downarrow | \downarrow \rangle$$

$$q^y = \frac{1}{2} \lambda (1-\lambda) = \underline{\underline{2 q^y}} \langle \downarrow | \downarrow \rangle$$

$$= \langle \downarrow | \rho_{-1} b_{-1} c_0 | \downarrow \rangle = \lambda \langle \downarrow | \rho_{-1} b_{-1} c_0 | \downarrow \rangle$$

$$= (1-\lambda) \lambda \langle \downarrow | \underbrace{c_1 b_{-1}}_{=1} c_0 | \downarrow \rangle = (1-\lambda) \lambda \langle \downarrow | b_0 c_0 | \downarrow \rangle$$

$$\{c_1, b_{-1}\} = 1$$

$$c_1 b_{-1} = \textcircled{1} - \textcircled{b_{-1} c_1}$$

$$= \underline{\underline{(1-\lambda) \lambda}} \langle \downarrow | \downarrow \rangle \equiv$$

$$\equiv 2 \langle \downarrow | \downarrow \rangle$$

$$q^y = \frac{1}{2} \lambda (1-\lambda) = \underline{\underline{2 q^y}} \langle \downarrow | \downarrow \rangle$$

Vertex operator

creation-annihilation operators

superstring

$\alpha_{-1} \cdot \alpha_1 = 1$

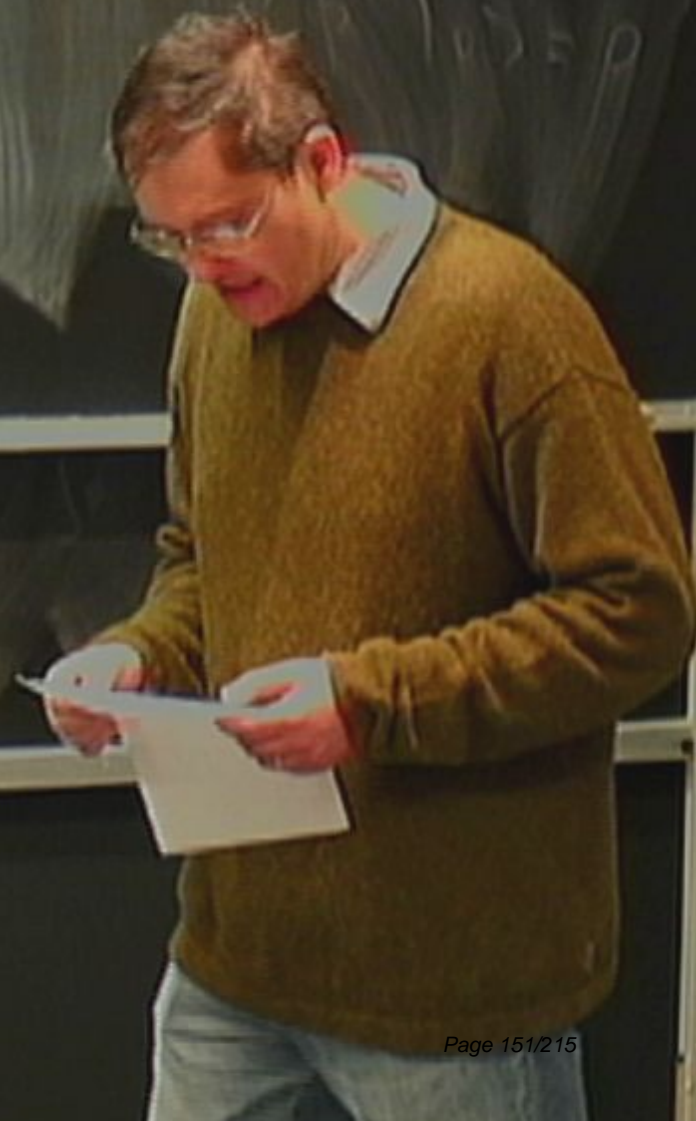
Vertex operator

\Rightarrow we would like to build a map between
states and local operators

⇒ we would like to build a map between
states and local operators



$$\{Q_1, Q_2\} = Q_1 Q_2 + Q_2 Q_1$$



⇒ we would like to build a map between
states and local operators

$\mathbb{R}^2 = \mathbb{R} \times \mathbb{R}$



$z = -\infty$

$$\{Q_1, Q_2\} = Q_1 Q_2 + Q_2 Q_1$$

⇒ we would like to build a map between states and local operators

$\mathbb{R}^3 = \mathbb{R} \times \mathbb{W}$



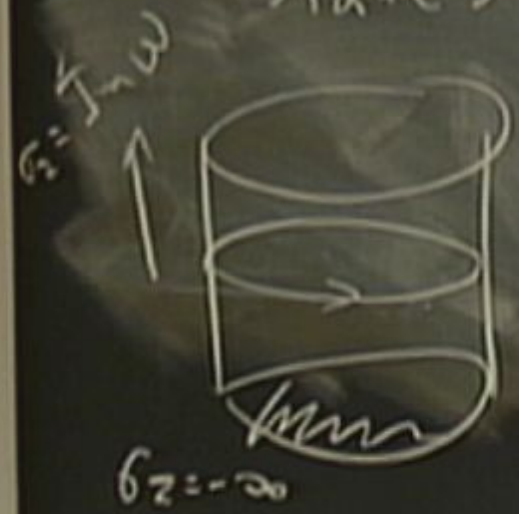
$z = -\infty$

we prepare our states here.

$$\{Q_1, Q_2\} = Q_1 Q_2 + Q_2 Q_1$$

Vertex operator.

⇒ we would like to build a map between states and local operators

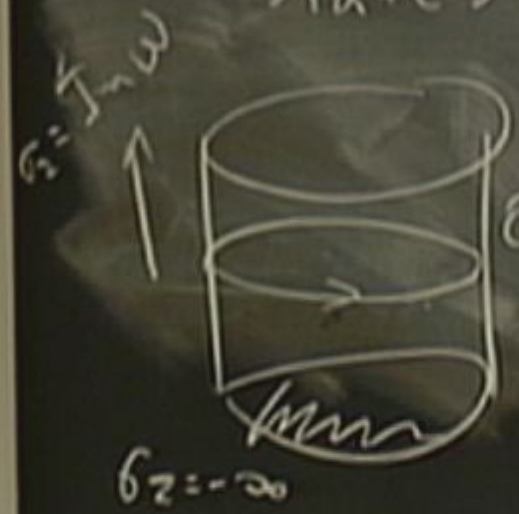


we prepare our states here.



Vertex operator

⇒ we would like to build a map between states and local operators



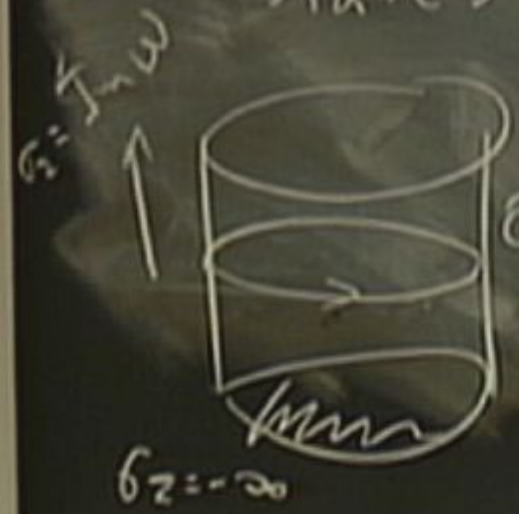
← a conserved charge acting on a state.

← we prepare our states here.

Vertex operator

⇒ we would like to build a map between states and local operators

$$z = e^{-i\omega}$$



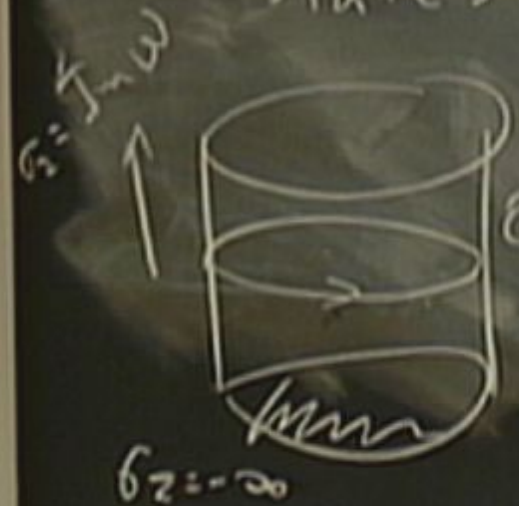
← a conserved charge acting on a state.

← we prepare our states here.



Vertex operator

⇒ we would like to build a map between states and local operators



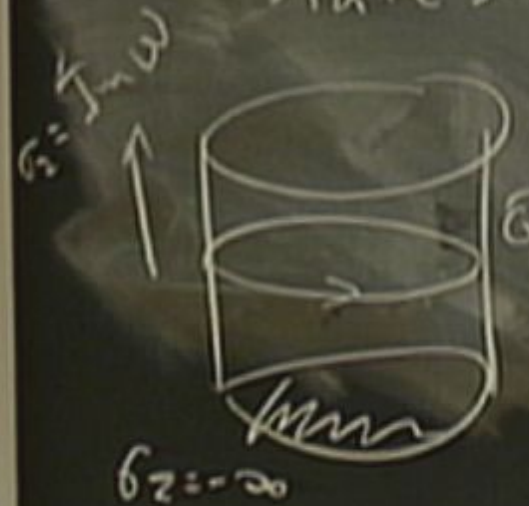
$\omega = \sigma_1 + i\sigma_2$, $z = e^{-i\omega}$
a conserved charge acting on a state.

we prepare our states here.



Vertex operator

⇒ we would like to build a map between states and local operators



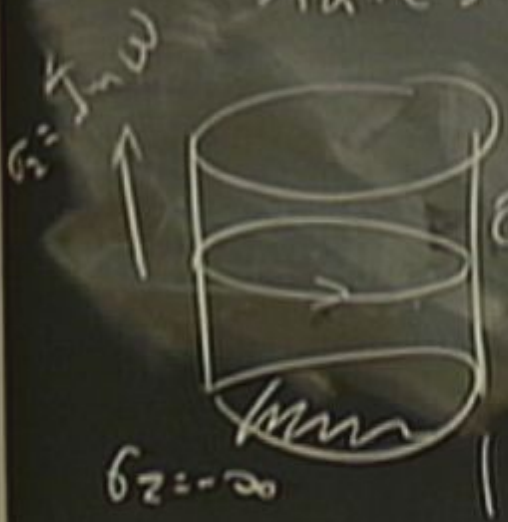
$\omega = \sigma_1 + i\sigma_2$, origin $z = e^{-i\omega}$
 a conserved charge acting on a state.
 $\sigma_1 \rightarrow -\sigma_1$
 $|\downarrow\rangle \rightarrow 0$

we prepare our states here.



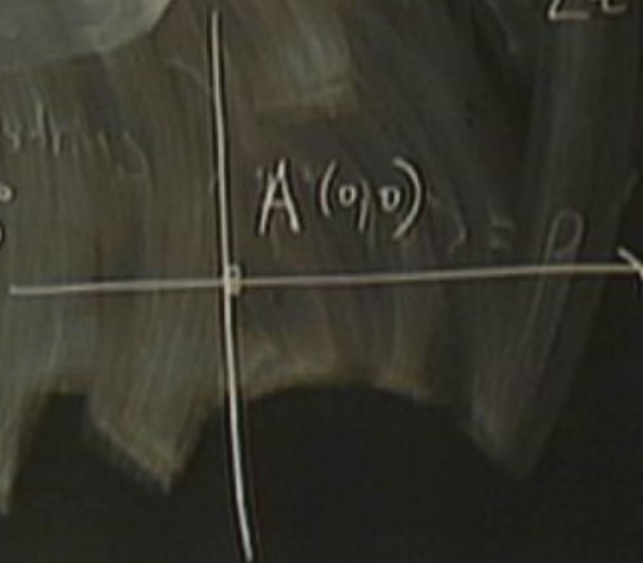
Vertex operator

⇒ we would like to build a map between
states and local operators



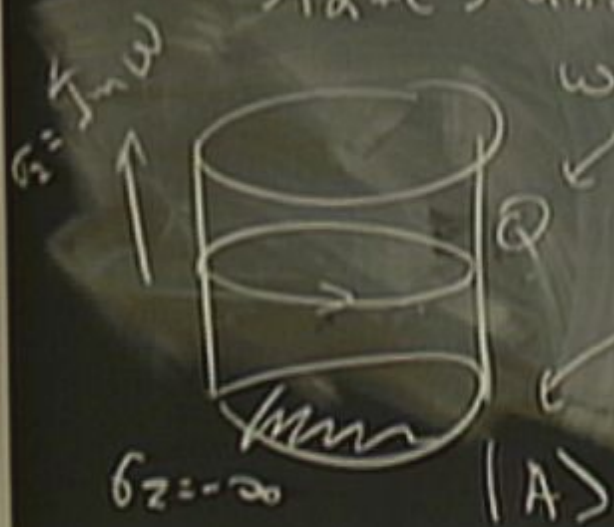
$w = \sigma_1 + i\sigma_2$, origin $z = e^{-iw}$
 a conserved charge acting on a state.
 $\sigma_1 \rightarrow -\infty$
 $|z| \rightarrow 0$

we prepare our states here.



Vertex operator.

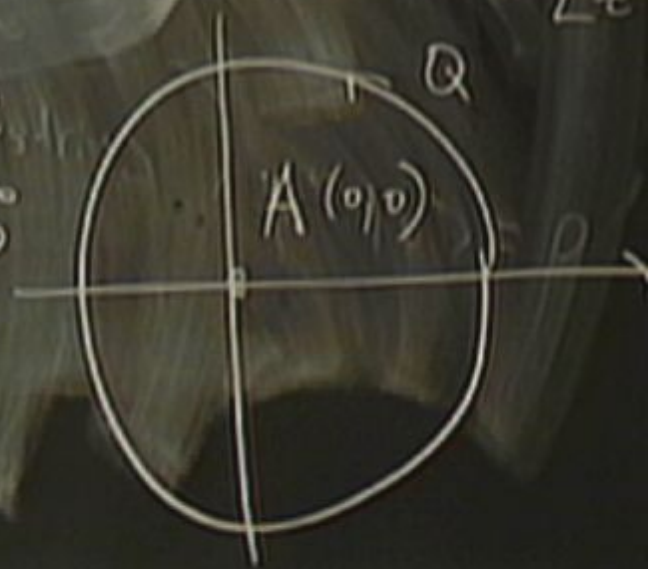
\Rightarrow we would like to build a map between states and local operators



$\omega = \omega_1 + i\omega_2$, $z = e^{-i\omega}$
 $\omega \rightarrow -\omega$
 $|z| \rightarrow 0$

\leftarrow a conserved charge acting on a state.

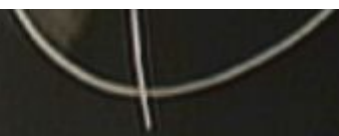
\leftarrow we prepare our states here.



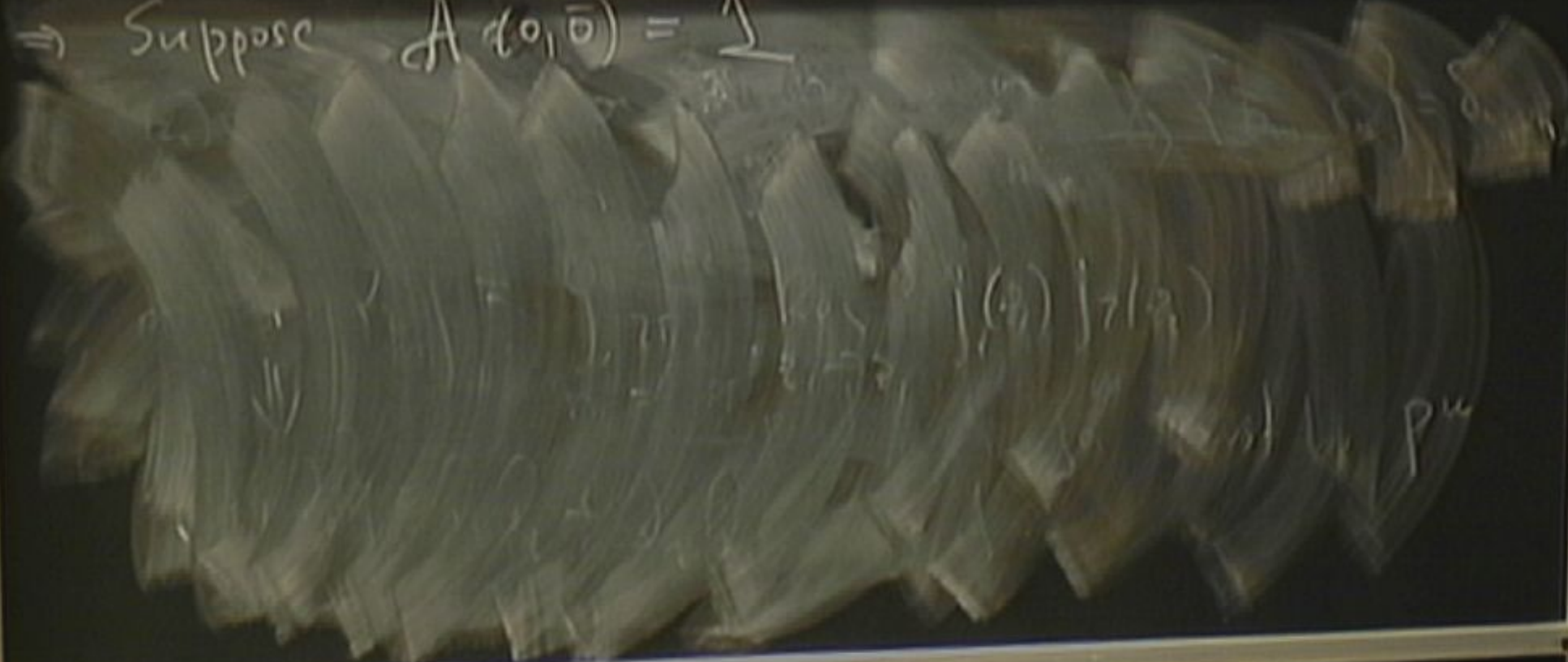
$\infty = -\infty$

$|A\rangle$

our states here.



\Rightarrow Suppose $A(0, \bar{0}) = \uparrow$



$\alpha = -\infty$

$|A\rangle$

our states here.

\Rightarrow Suppose $A(0,0) = \mathbb{1}$

\uparrow
identity operation

Q: $|A\rangle = ?$

$i(0) |7/2\rangle$

P^u

$\alpha = -\infty$ $|A\rangle$

our states
here.

\Rightarrow Suppose $A|0, \bar{0}\rangle = \mathbb{1}$

$\mathbb{1}$
identity operator

Q: $|A\rangle = ?$

$\lim_{z \rightarrow -\infty}$
 $|A\rangle$

our states
here.

\Rightarrow Suppose $A(0,0) = \mathbb{1}$

$\mathbb{1}$
identity operator

Q: $|A\rangle = ?$

$\dim \equiv$

$\infty = -\infty$

$|A\rangle$

our states here.

\Rightarrow Suppose $A|0, \bar{0}\rangle = \uparrow$

identity operator

Q: $|A\rangle = ?$

in dm \Rightarrow

$\int \frac{d\psi}{2\pi i}$

paper

$$b_2 = -\infty$$

$|A\rangle$

our states here.

\Rightarrow Suppose $A(0, \vec{0}) = \uparrow$

identity operation

①: $b|A\rangle = \uparrow$

$\int_{dm} \Rightarrow$

$$\int_C \frac{dz}{2\pi i}$$

$$\oint_{\mathbb{Z}^m} dx^m$$

Conserved current
curves to dm

$z = -\infty$

$|A\rangle$

our states here.

⇒ Suppose $A(z, \bar{z}) = \mathbb{1}$

identity operator

Ⓚ: $|A\rangle = \int$

$\int_{\mathcal{C}} \Rightarrow$

$\int_{\mathcal{C}} \frac{dz}{2\pi i}$

$\int_{\mathcal{C}} dz \partial X^{\mu} |A\rangle$

Conserved current
curves to $d\mu$

$$G_2 = -\infty$$

$$|A\rangle$$

our states here.

⇒ Suppose $A(0, \bar{0}) = \uparrow$

↑ identity operator

Ⓚ: $|A\rangle = \uparrow$

$$\int_{\mathcal{C}} \Rightarrow$$

$$\int_{\mathcal{C}} \frac{dz}{2\pi i} \oint_{\mathcal{C}} dx^{\mu} i(\uparrow) |z\rangle$$

Conserved current
curves to d_m

ψ
 $\psi = -\infty$ $|A\rangle$

our states here.

\Rightarrow Suppose $A(0,0) = \uparrow$

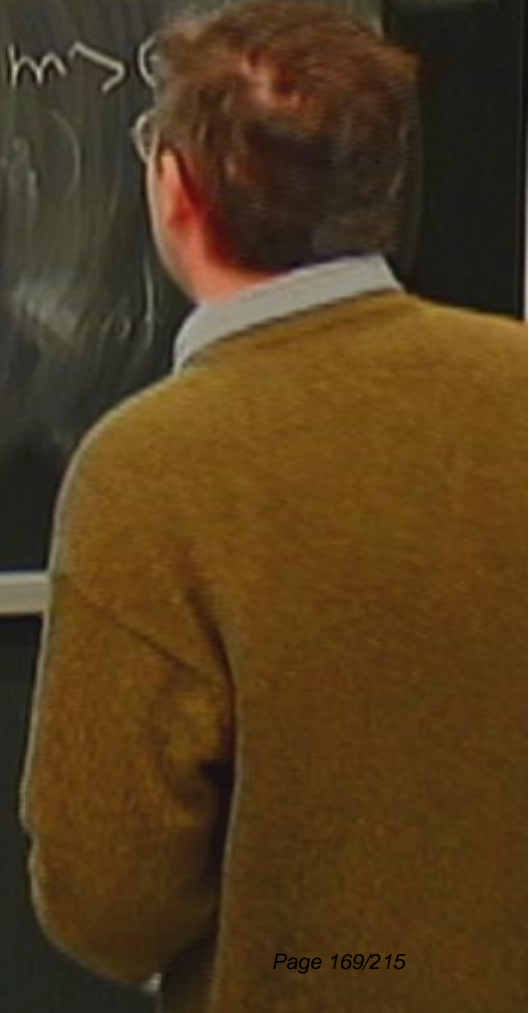
identity operator

Q: $|A\rangle = \uparrow$

\Rightarrow

$$\oint_C \frac{dz}{2\pi i} \int_{\mathbb{R}^m} dX^m |(\uparrow) = \dots$$

Conserved current
 curves to d^m



$z = -\infty$

$|A\rangle$

our states here.

\Rightarrow Suppose $A(0,0) = \uparrow$

identity operator

Q: $|A\rangle = ?$

$\int_{dm} \Rightarrow$

\oint_C

$\frac{dz}{2\pi i}$

$\int_{dm} dx^m$

$|\uparrow\rangle$

$m > 0$

$\neq 0$

Conserved current
curves to dm

PK

$\infty = -\infty$

$|A\rangle$

our states here.

Q: $|A\rangle = ?$

identity operation

$\frac{d}{dt} \Rightarrow$

$$\int \frac{dz}{2\pi i} \oint \frac{1}{z} \partial X^m |A\rangle = |0\rangle$$

$$\oint \frac{1}{z} \partial X^m |A\rangle = 0, m > 0$$

Conserved current
curves to $\frac{d}{dt}$

$\infty = -29$

$|A\rangle$

our states here.

Q: $|A\rangle = \uparrow$

identity operator

$\frac{d}{dm} \Rightarrow$

$\int \frac{d\alpha}{2\pi i} \sum_m \alpha X^m |A\rangle = |0\rangle$

$m > 0$

$L_m |A\rangle = 0, m > 0$

Conserved current
curves to d_m

$|0,0\rangle \leftrightarrow \uparrow$

$$\alpha_{-m}^{\mu} |0\rangle \leftrightarrow |0\rangle : \alpha_{-m}^{\nu} : \Delta^{\mu\nu}$$

$$\alpha_{-m}^{\mu} |0\rangle \leftrightarrow |0\rangle : \alpha_{-m}^{\nu} : \Delta^{\mu\nu}$$

$m > 0$

$$\oint_C \frac{z}{z-i} dz = 2\pi i$$

$$\oint_C \frac{z^{-m}}{z-i} dz = 2\pi i (-i)^{-m-1}$$

$$\alpha_{-m} |0\rangle \leftrightarrow \alpha_{-m}^\dagger |0\rangle$$

$m > 0$

$$\oint_C \frac{z}{z-i} z^{-m} dz$$

$$\alpha_{-m}^{\mu} |0\rangle \leftrightarrow \langle 0| : \alpha_{-m}^{\nu} : \Delta^{\mu\nu}$$

$m > 0$

$$\oint_C \frac{z}{z'} \phi \left(\frac{z}{z'} \right) z^{-m} dz \rightarrow \left(\frac{z}{z'} \right)^{1/2} \frac{1}{(m-1)!}$$

$$\alpha_{-m}^m |0\rangle \leftrightarrow \alpha_{-m}^m : \Delta^m$$

$m > 0$

$$\partial X^m(z)$$

$$\oint_C \frac{z}{\alpha} \phi \frac{dz}{z} z^{-m} \partial X^m \rightarrow \left(\frac{z}{\alpha}\right)^{1/2} \frac{1}{(m-1)!}$$

$$\alpha_{-m} |0\rangle \leftrightarrow \alpha_{-m} : \Delta^m$$

$$\underline{m > 0}$$

$$\partial X^m(z) = \partial X^m(0) + \frac{z}{i} \partial^2 X^m + \dots + \frac{z^{m-1}}{(m-1)!} \partial^m X^m$$

$$\oint_{C'} \frac{dz}{2\pi i} z^{-m} \partial X^m(z) \rightarrow \left(\frac{z}{i}\right)^{m-1} \frac{1}{(m-1)!} \partial^m X^m$$

$$\alpha_{-m} |0\rangle \leftrightarrow \alpha_{-m} : \Delta$$

$$m > 0$$

$$\partial X^m(z) = \partial X^m(0) + \frac{z}{1} \partial^2 X^m$$

$$\oint_{C'} \frac{dz}{2\pi i} z^{-m} X^m \rightarrow \left(\frac{z}{2}\right)^{1/2} \left(\frac{1}{2}\right)^{1/2} \frac{1}{(m-1)!} \partial^{m-1} X^m$$

$$\alpha_{-m} |0\rangle \leftrightarrow \alpha_{-m} : \Delta^m$$

$m > 0$

$$\partial X^m(z) = \partial X^m(0) + \frac{z}{i} \partial^2 X^m + \dots + \frac{z^{m-1}}{(m-1)!} \partial^m X^m$$

$$\oint_C \frac{z}{z-i} \partial X^m(z) \rightarrow \left(\frac{z}{z-i}\right)^{1/2} \left(\frac{z}{z-i}\right)^{m-1/2} \partial X^m(0) + \dots + \frac{z^{m-1}}{(m-1)!} \partial^m X^m$$

$$\underline{m > 0}$$

$$\partial \left(\frac{z}{z'} \right)^m = \partial X^m(0) + \dots + \partial X^m$$

$$\oint_C \frac{z}{z'} \frac{1}{z} z^{-m} \partial X^m \rightarrow \left(\frac{z}{z'} \right)^{1/2} \frac{i}{(m-1)!} \partial X^m$$

$$d_{-m}^w |0\rangle \iff \left(\frac{z}{z'} \right)^{1/2} \frac{i}{(m-1)!}$$

$$2L_0 = [L_{11}, L_{-11}]$$

↳ states \downarrow

$m > 0$

$$\partial^m X(z) = \partial^m X(0) + \dots + \partial^m X^{(m-1)}(0) z$$

$$\oint_C \frac{z^{-m}}{2\pi i} \partial^m X(z) dz \rightarrow \left(\frac{z}{z'}\right)^{1/2} \frac{i}{(m-1)!} \partial^m X(0)$$

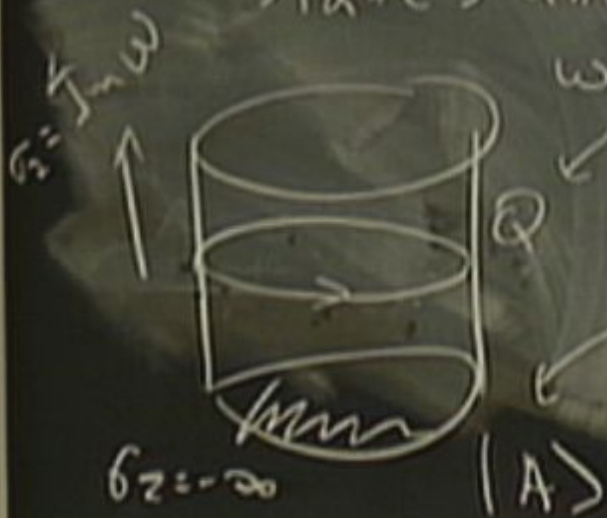
$$d_{-m}^w |0\rangle \Leftrightarrow \left(\frac{z}{z'}\right)^{1/2} \frac{i}{(m-1)!} \partial^m X(0)$$

$$2L_0 = [L_{11}, L_{-1}] = L_1 L_{-1} - L_{-1} L_1$$

Study \Downarrow

Vertex operator.

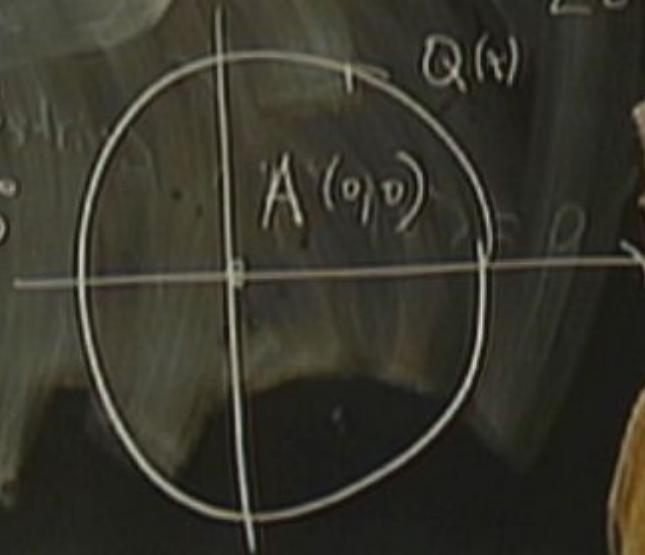
⇒ we would like to build a map between states and local operators



$\omega = \sigma_1 + i\sigma_2$, σ_3 is a conserved charge acting on a state.

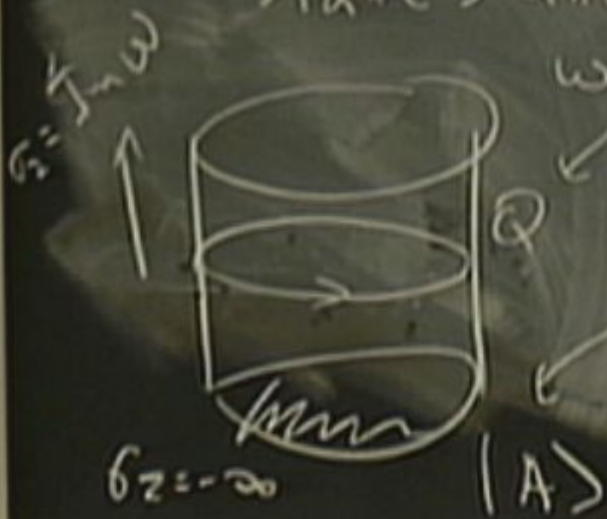
$z = e^{-i\omega}$
 $\sigma_z \rightarrow -\sigma_z$
 $|z| \rightarrow 0$

we prepare our states here.



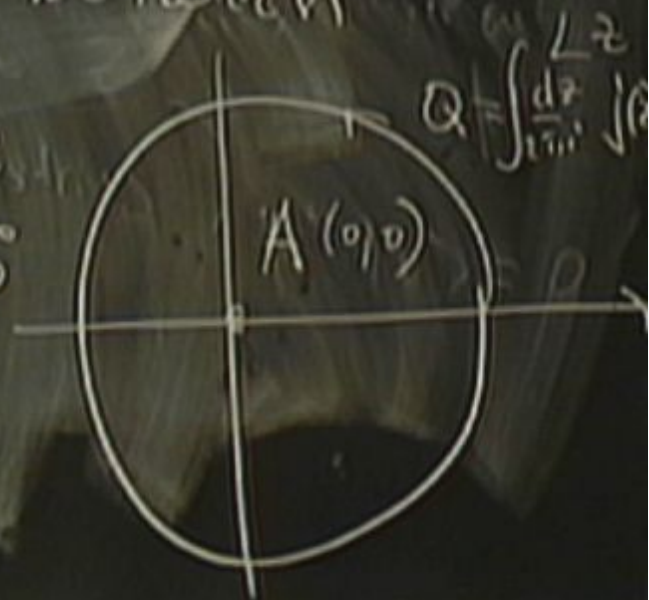
Vertex operator.

⇒ we would like to build a map between states and local operators



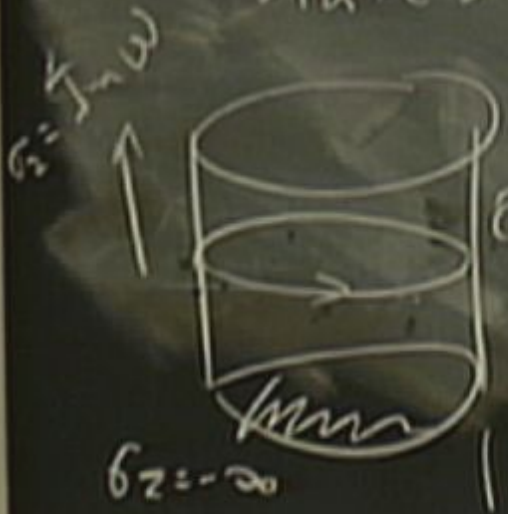
$\omega = \sigma_1 + i\sigma_2, \sigma_1, \sigma_2$
 a conserved charge acting on a state.
 $z = e^{-i\omega}$
 $\sigma_1 \rightarrow -\sigma_1$
 $|\sigma_1| \rightarrow 0$

we prepare our states here.

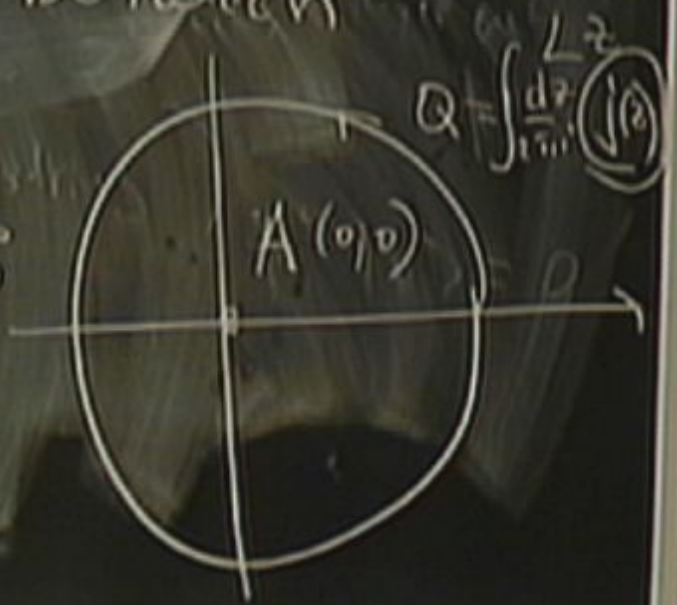


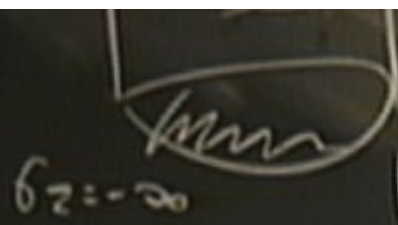
Vertex operator.

⇒ we would like to build a map between states and local operators



$\omega = \sigma_1 + i\sigma_2, \sigma_1, \sigma_2$
 $z = e^{-i\omega}$
 $\sigma_1 \rightarrow -\infty$
 $|\sigma_1| \rightarrow 0$
 a conserved charge acting on a state.
 we prepare our states here.

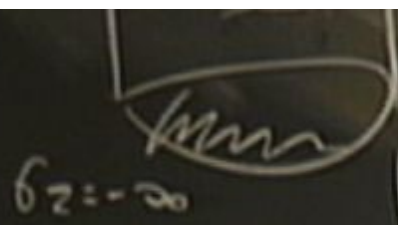




we prepare our states here.
 $|A\rangle$



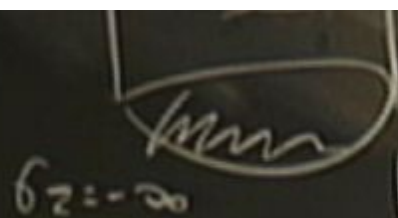
$$D_1(z) \quad D_2(b)$$



we prepare our states here.
 $|A\rangle$

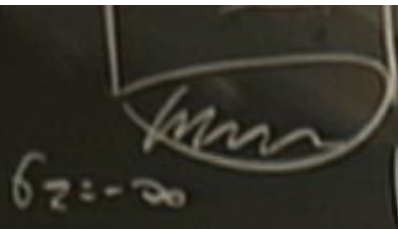
$$|O_1(z)\rangle |O_2(0)\rangle = \sum_K c_K \frac{1}{z^\#} |O_1(0)\rangle$$





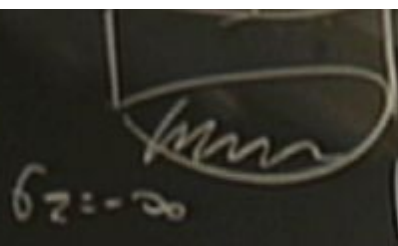
we prepare our states here.

$$|O_1(z) O_2(0)\rangle = \sum_K c_K \frac{1}{z^H} |O_1(0) O_2(0)\rangle$$



we prepare our states here.

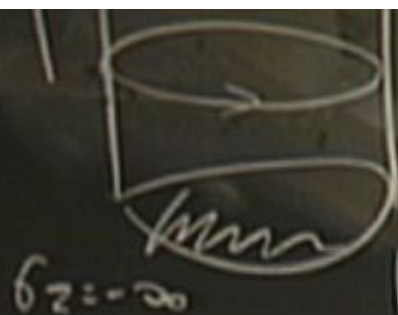
$$|O_1(z) O_2(0)\rangle = \sum_K c_K \frac{1}{2^{\#}} |O_1(0) O_2(0)\rangle$$



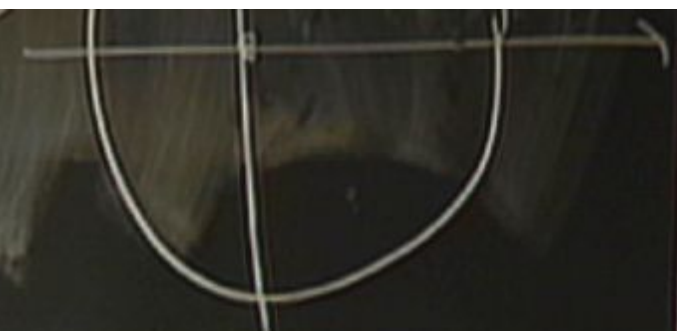
we prepare our states here.
 $|A\rangle$

$$|O_1(z)\rangle |O_2(b)\rangle = \sum_K c_K \frac{1}{z^H} | : O_1(0) O_2(0) \rangle$$

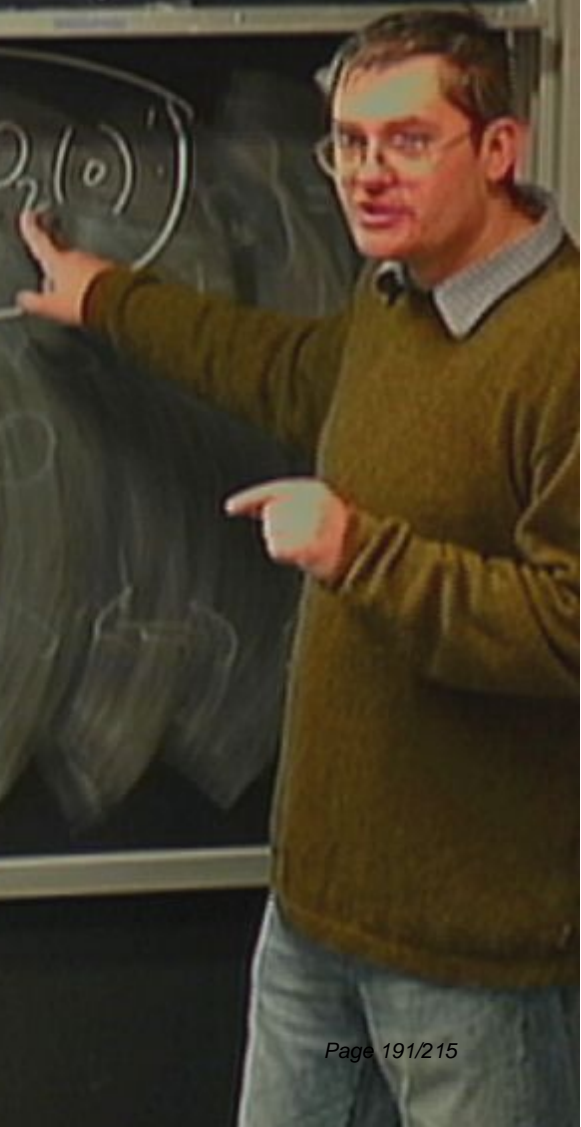




acting on a state.
 we prepare our states here.
 $|A\rangle$



$$|z\rangle = \int \langle 0, z | O_1(0) = \sum_k c_k \frac{1}{z^{\#}} \circlearrowleft \begin{matrix} O_1(0) & O_2(0) \end{matrix}$$

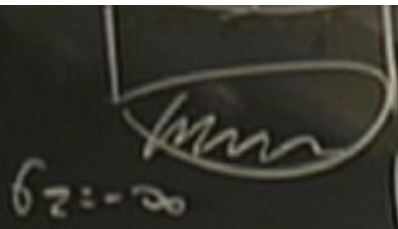


$$\alpha_{-m}^w |0\rangle \leftrightarrow \dots \Rightarrow \dots$$

$$\partial X^m(z) = \partial X^m(0) + \frac{z}{i} \partial^2 X^m + \dots$$

$$\oint \frac{dz}{2\pi i} z^{-m} \partial X^m \rightarrow \left(\frac{z}{i}\right)^{1/2} \frac{i}{(m-1)!} \partial^m X(0) + \dots$$

$$\alpha_{-m}^w |0\rangle \leftrightarrow \left(\frac{z}{i}\right)^{1/2} \frac{i}{(m-1)!} \partial^m X(0)$$



we prepare our states here.
 $|A\rangle$

$$\int_{|z\rangle} \phi(z) Q_1(z) Q_2(0) = \sum_K c_K \frac{1}{z^{\#}} \circlearrowleft : Q_1(0) Q_2(0) \circlearrowright$$

$$: Q : A(0,0) : = : \text{something} : (0,0)$$



$\lim_{t \rightarrow -\infty}$ $|A\rangle$ ← we prepare our states here.

$$\int_{|z\rangle} \phi(O_1(z)) O_2(0) = \sum_K c_K \frac{1}{z^\#} \circlearrowleft : O_1(0) O_2(0) \circlearrowright$$

$\underbrace{Q: A(0,0) :}_{\rightarrow |v\rangle} = : \text{something} : (0,0)$

$$\alpha_{-m}^w |0\rangle \iff \alpha_{-m}^w : \Delta^w \iff \alpha_{-m}^w |0\rangle$$

$m > 0$

$$\partial X^w(z) = \partial X^w(0) + \frac{z}{1} \partial^2 X^w + \frac{z^{m-1}}{(m-1)!} \partial^m X^w$$

$$\oint_C \frac{z}{z'} \frac{0/z}{2\pi i} z^{-m} \partial X^w \rightarrow \left(\frac{z}{z'}\right)^{1/2} \frac{i}{(m-1)!} \partial^m X^w(0)$$

$$\alpha_{-m}^w |0\rangle \iff \left(\frac{z}{z'}\right)^{1/2} \frac{i}{(m-1)!} \partial^m X^w(0)$$

$$d_{-m}^w |0\rangle \iff \dots \iff \boxed{d_{-m}^w |0\rangle}$$

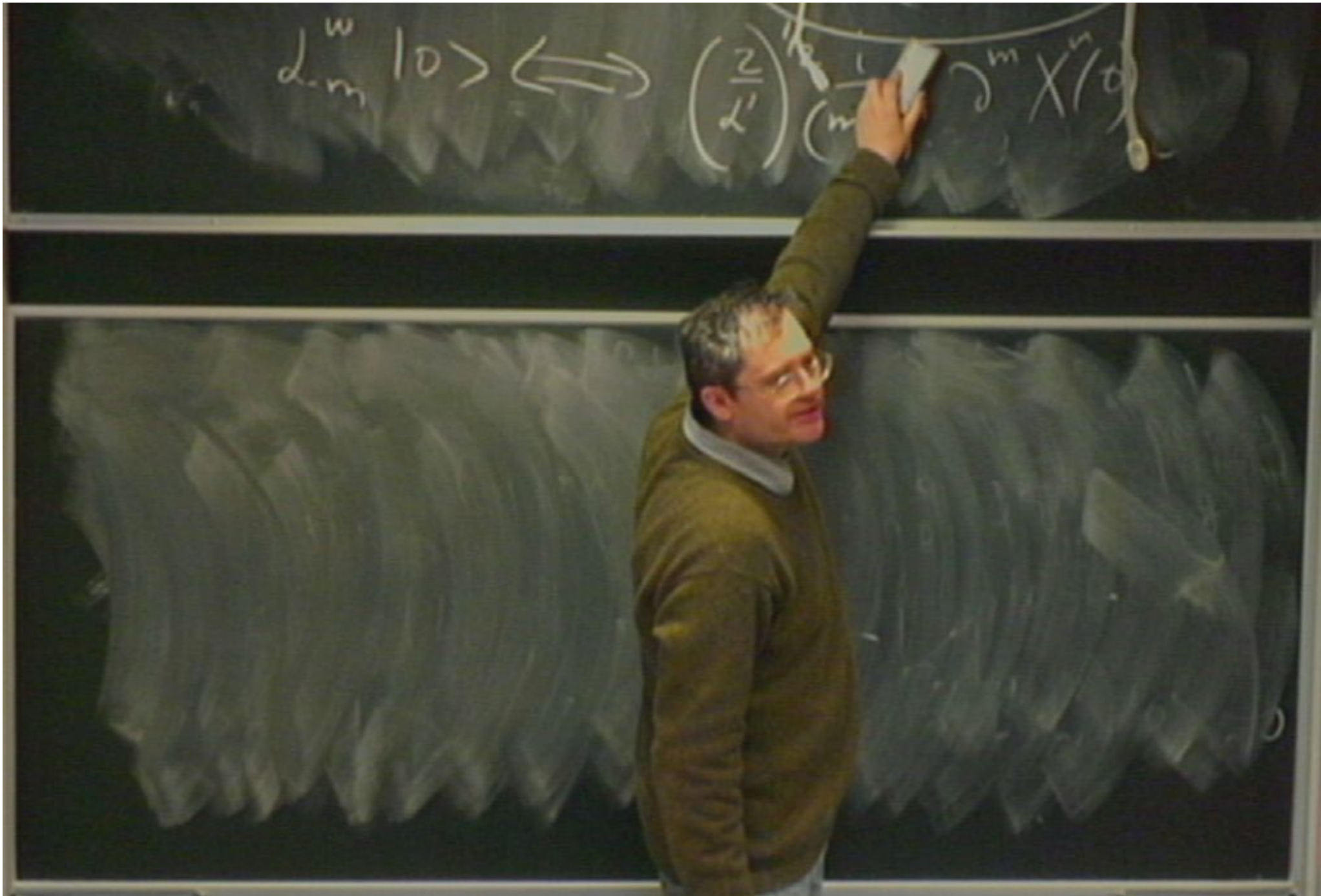
$m > 0$

$$\partial X^m(z) = \partial X^m(0) + \frac{z}{1} \partial^2 X^m + \dots + \frac{z^{m-1}}{(m-1)!} \partial^m X^m$$

$$\oint_C \frac{z}{d'} \frac{1}{z} \frac{1}{z} z^{-m} \partial X^m$$

$$\Rightarrow \left(\frac{z}{d'} \right)^{1/2} \frac{1}{(m-1)!} \partial^m X^m(0)$$

$$d_{-m}^w |0\rangle \iff$$



$$d_{-m}^w |0\rangle \iff \begin{pmatrix} 2 \\ d' \end{pmatrix} \begin{pmatrix} 1 \\ m \end{pmatrix} \begin{pmatrix} m \\ 0 \end{pmatrix} X \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$d_m^w |0\rangle \iff \binom{2}{d'} \binom{1}{m-1} \binom{m}{2} X |0\rangle$$

$$d_m^w |A\rangle$$

$$d_m^w |0\rangle \Leftrightarrow \binom{2}{d'} \binom{1}{m-1} \binom{m}{2} X \binom{0}{d}$$

$$d_m^w |A\rangle \Rightarrow$$

$$d_{-m}^w |0\rangle \iff \binom{2}{d'} \binom{1}{m-1} \binom{m}{2} X \binom{1}{0}$$

$$\underbrace{d_{-m}^w |A\rangle}_{\text{a new state}} \Rightarrow$$

$$d_m^w |0\rangle \iff \left(\frac{2}{d'}\right)^{1/2} \binom{m-1}{m} X^m |0\rangle$$

$$\underbrace{d_m^w |A\rangle}_{\text{new state}} = \underbrace{\left(\frac{2}{d'}\right)^{1/2} \int \frac{d^2z}{2\pi} X^m |z\rangle}_{d_m^w}$$

$$d_{-m}^w |0\rangle \iff \left(\frac{z}{z'}\right)^{1/2} \binom{m}{m-1} |0\rangle X^m |0\rangle$$

$$\underbrace{d_{-m}^m |A\rangle}_{\text{or new state}} = \underbrace{\left(\frac{z}{z'}\right)^{1/2} \int \frac{dz}{2\pi} z^m X^m(z) \phi(0,0)}_{d_{-m}^m}$$

$$d_m^w |0\rangle \Leftrightarrow \left(\frac{2}{d'}\right)^{1/2} \binom{1}{m-1} \binom{m}{2} X^m |0\rangle$$

$$\underbrace{d_m^w |A\rangle}_{\text{or new state}} = \underbrace{\left(\frac{2}{d'}\right)^{1/2} \int_{-\pi}^{\pi} \frac{d\phi}{2\pi} z^m X^m \binom{0}{1} \binom{0}{0} \binom{0}{0}}_{d_m^w}$$

=

$$d_{-m}^w |0\rangle \iff \left(\frac{2}{d'}\right)^{1/2} \binom{1}{m-1} \binom{m}{2} X^m |0\rangle$$

$$d_{-m}^w |A\rangle = \underbrace{\left(\frac{2}{d'}\right)^{1/2} \int \frac{d^2z}{2\pi} \binom{1}{m} X^m \binom{2}{z} \psi(0,0)}_{d_{-m}^w} \underbrace{\psi(0,0)}_{\text{or new state}}$$

=

$$d_{-m}^w |0\rangle \iff \left(\frac{2}{d'}\right)^{1/2} \frac{1}{(m-1)!} \int_0^{2\pi} X^m \psi(\theta)$$

$$d_{-m}^w |A\rangle =$$

or new state

$$\left(\frac{2}{d'}\right)^{1/2} \int_0^{2\pi} \frac{d\theta}{2\pi} \frac{1}{2^m} X^m \psi(\theta)$$

d_{-m}^w

$$\psi(\theta) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\psi(\theta) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

=

$$d_{-m}^w |0\rangle \iff \left(\frac{2}{d'}\right)^{1/2} \binom{1}{m-1} |0\rangle^m X^m |0\rangle$$

$$d_{-m}^w |A\rangle = \underbrace{\left(\frac{2}{d'}\right)^{1/2}}_{\text{or new state}} \int \frac{d^2z}{2\pi} \binom{1}{2} |z\rangle^m X^m \underbrace{\left(\frac{1}{2}\right)^m}_{|z|^2} |0,0\rangle$$

correlator
 $X^m |0,0\rangle$

$$d_m^w |0\rangle \iff \left(\frac{2}{d'}\right)^{1/2} \binom{1}{m-1} \binom{m}{2} X^m |0\rangle$$

$$\underbrace{d_m^w |A\rangle}_{\text{new state}} = \underbrace{\left(\frac{2}{d'}\right)^{1/2} \binom{1}{m-1} \binom{m}{2}}_{d_m^w} \int \frac{d^2z}{2\pi} \binom{0}{1} \binom{0}{0} \binom{0}{0} X^m |z\rangle$$

correlation
 $X^m |z\rangle$

$$d_m^w |0\rangle \Leftrightarrow \left(\frac{2}{d'}\right)^{1/2} \binom{1}{m-1} \binom{m}{2} X^m |0\rangle$$

$$\underbrace{d_m^w |A\rangle}_{\text{original state}} = \underbrace{\left(\frac{2}{d'}\right)^{1/2} \int \frac{d^2z}{2\pi} \binom{1}{m} \binom{m}{2} X^m \rho(z,0)}_{d_m^w} \underbrace{\binom{1}{m} \binom{m}{2} X^m \rho(z,0)}_{\text{correlation function } X^m(z,0)}$$

$$d_m^w |0\rangle \Leftrightarrow \left(\frac{z}{z'}\right)^{1/2} \binom{1}{m-1} \binom{m}{0} X^m \left(\frac{z}{z'}\right)$$

$$\underbrace{d_m^w |A\rangle}_{\text{or new state}} = \int \frac{dz}{2\pi} \left(\frac{z}{z'}\right)^{1/2} \binom{m}{0} \binom{m}{0} X^m \left(\frac{z}{z'}\right)$$

$\binom{m}{0} X^m \left(\frac{z}{z'}\right)$
 correlation
 $\binom{m}{0} X^m \left(\frac{z}{z'}\right)$

$$= \int \frac{dz}{2\pi} \frac{z^{m+1}}{z^{n+1}}$$

$$d_{-m}^w |0\rangle \iff \left(\frac{z}{z'}\right)^{\frac{1}{2}} \binom{1}{m-1} \binom{m}{0} X^m \left(\frac{z}{z'}\right)$$

$$d_{-m}^w |A\rangle = \underbrace{\left(\frac{z}{z'}\right)^{\frac{1}{2}}}_{\text{or new state}} \int \frac{dz}{z} \binom{1}{m} \binom{m}{0} X^m \left(\frac{z}{z'}\right) \left(\frac{z}{z'}\right)^{\frac{1}{2}}$$

correlation
 $X^m \left(\frac{z}{z'}\right)$

$$= \int \frac{dz}{z} \binom{1}{m} \binom{m}{0} X^m \left(\frac{z}{z'}\right) \left(\frac{z}{z'}\right)^{\frac{1}{2}}$$

$$d_m^w |0\rangle \iff \left(\frac{z}{2}\right)^{1/2} \left(\frac{1}{m-1}\right)^{1/2} \chi_m(z)$$

$$d_m^w |A\rangle =$$

or new state

$$\left(\frac{z}{2}\right)^{1/2} \int_{\mathbb{R}^m} \frac{d^m z}{2\pi} \chi_m(z) \psi(0,0)$$

$\chi_m(z)$
 $\psi(0,0)$
 $\psi(z,0)$

$$\chi_m(z) = \int_{\mathbb{R}^m} \frac{d^m p}{(2\pi)^m} \psi(p,0)$$

$$\int_{\mathbb{R}^m} \frac{d^m p}{(2\pi)^m} \psi(p,0)$$

$$= \chi_m(0) + \frac{z^2}{2} \chi_m''(0) + \dots + \frac{z^{m-1}}{(m-1)!} \chi_m^{(m-1)}(0)$$

$$\dim A \rightarrow$$

=

:

1

0

$$\begin{matrix} m \\ \downarrow \\ \text{dim } A \end{matrix} \rightarrow = \begin{pmatrix} 2 \\ \vdots \\ m-1 \end{pmatrix} : \partial^m X(\theta) A(\theta, \bar{D})$$

$$L_m | A \rangle = \left(\frac{2}{\alpha} \right)^{1/2} \frac{1}{(m-1)!} \partial^m X(\sigma) A(\sigma, \bar{\sigma})$$