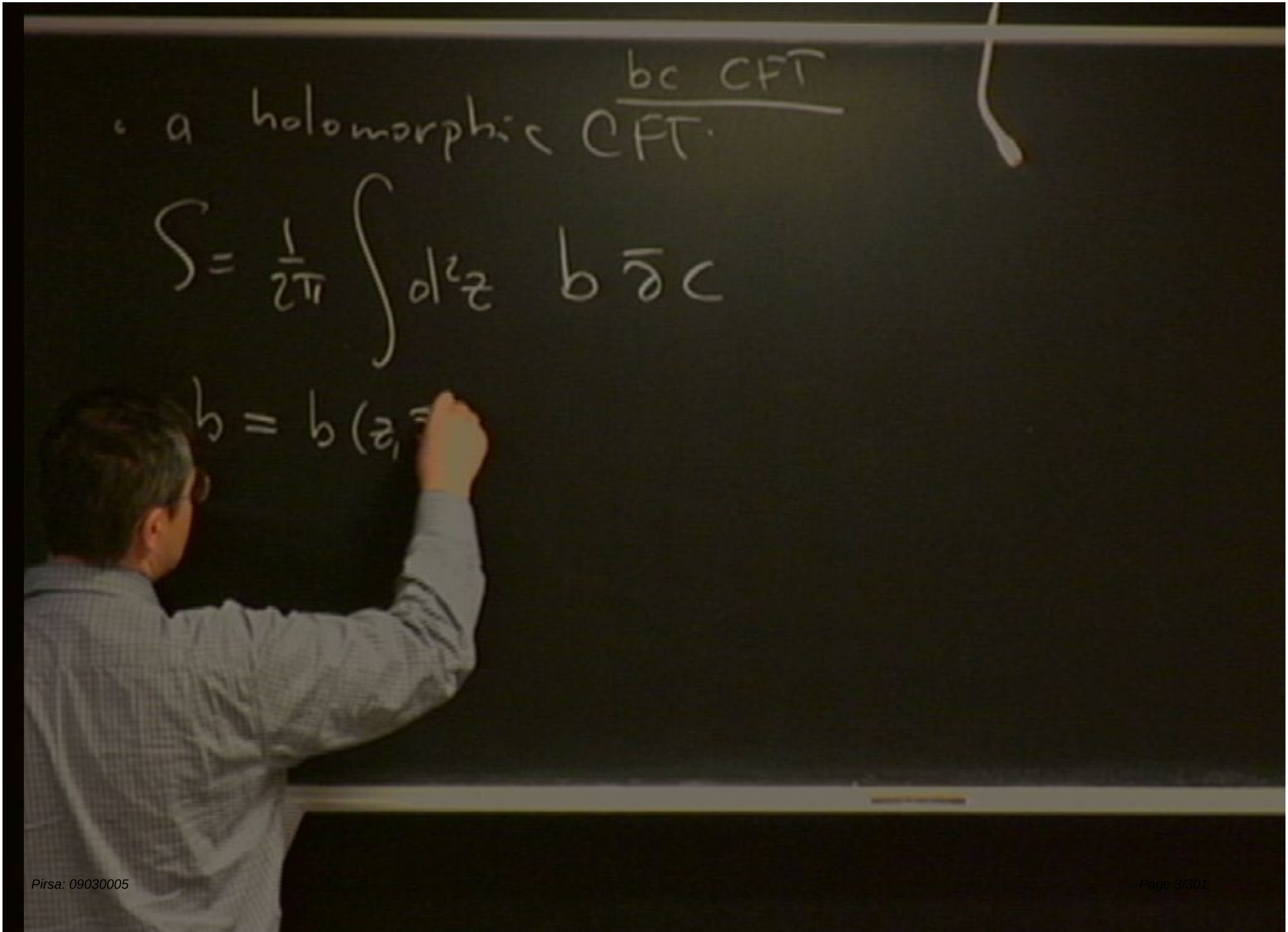
Title: Introduction to the Bosonic String

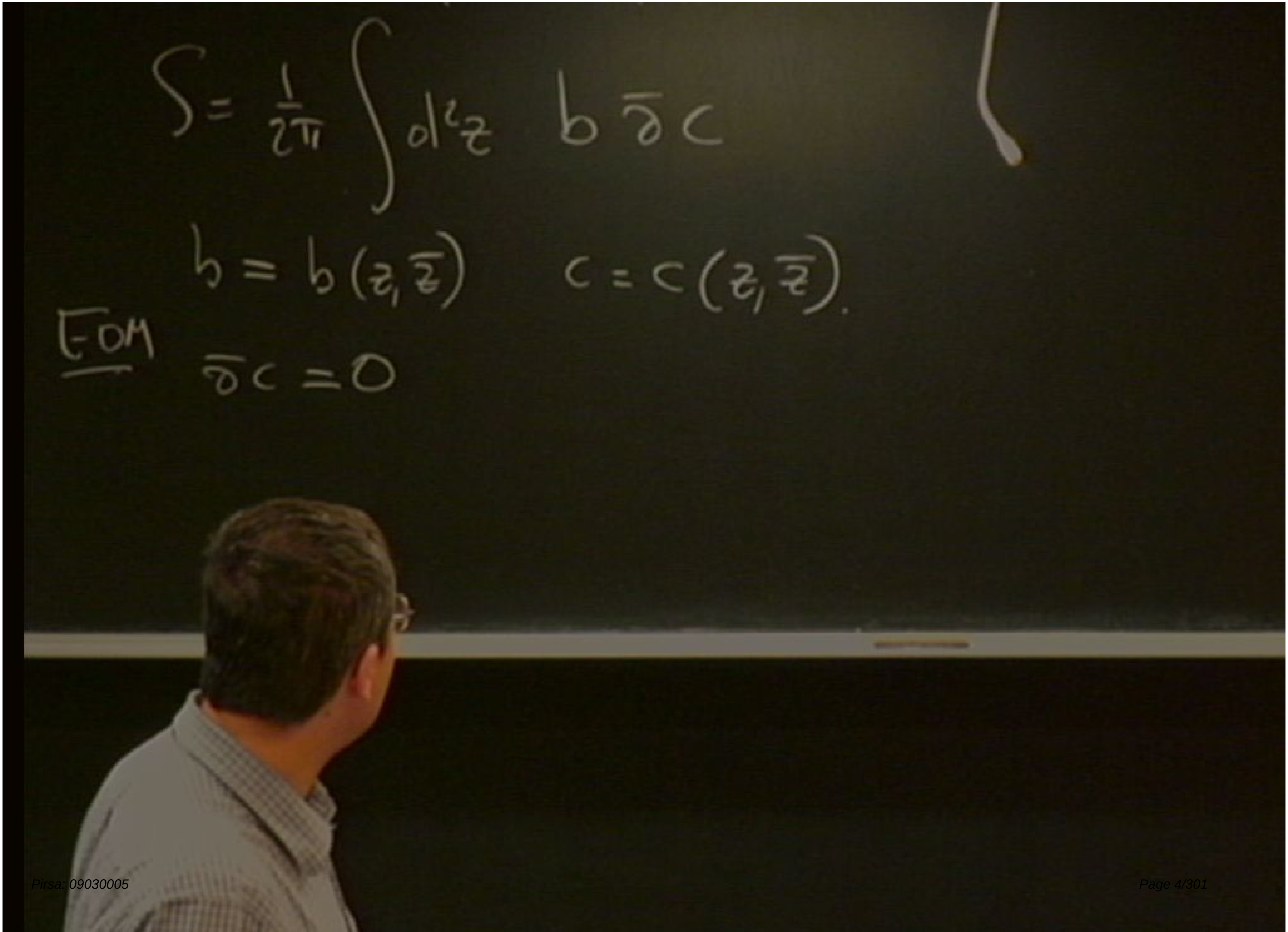
Date: Feb 27, 2009 10:00 AM

URL: http://pirsa.org/09030005

Abstract: This course provides a thorough introduction to the bosonic string based on the Polyakov path integral and conformal field theory. We introduce central ideas of string theory, the tools of conformal field theory, the Polyakov path integral, and the covariant quantization of the string. We discuss string interactions and cover the tree-level and one loop amplitudes. More advanced topics such as T-duality and D-branes will be taught as part of the course. The course is geared for M.Sc. and Ph.D. students enrolled in Collaborative Ph.D. Program in Theoretical Physics. Required previous course work: Quantum Field Theory (AM516 or equivalent). The course evaluation will be based on regular problem sets that will be handed in during the term. The primary text is the book: 'String theory. Vol. 1: An introduction to the bosonic string. J. Polchinski (Santa Barbara, KITP) . 1998. 402pp. Cambridge, UK: Univ. Pr. (1998) 402 p.' All interested students should contact Alex Buchel at abuchel@uwo.ca as soon as possible.

bc CFT


bc CFT
a holomorphic CFT.
$S = \frac{1}{2\pi} \int d^2z \; b \bar{\partial} c$
$b = b(z,$

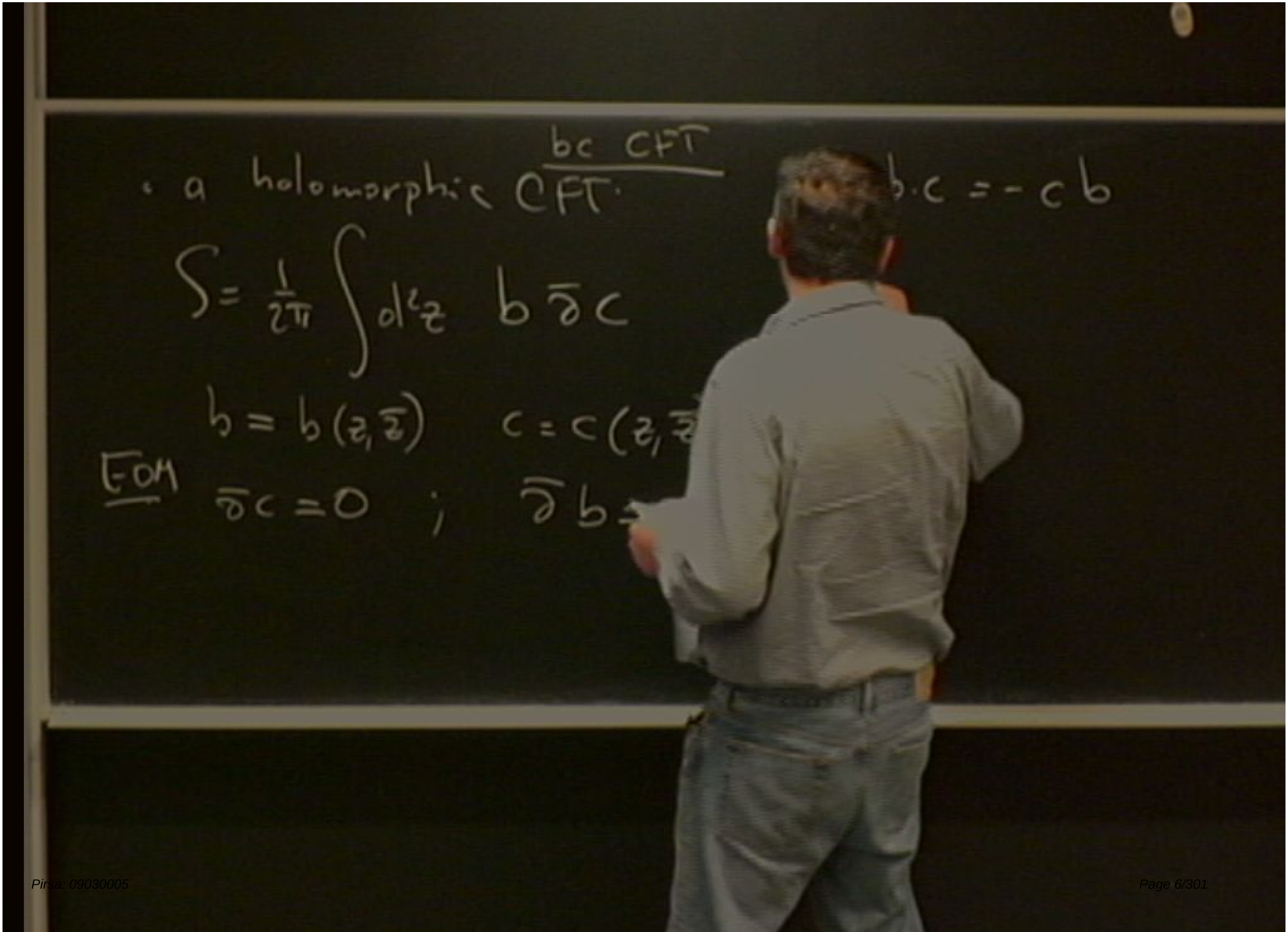

$S = \frac{1}{2\pi} \int d^2z \; b \bar{\partial} c$
$b = b(z, \bar{z}) \qquad c = c(z, \bar{z})$.
EOM
$\bar{\partial} c = 0$

· a holomorphic CFT:

$$S = \frac{1}{2\pi} \int d^2 z \; b \bar{\partial} c$$

$$b = b(z, \bar{z}) \qquad c = c(z, \bar{z}).$$

EOM $\quad \bar{\partial} c = 0 \quad ; \quad \bar{\partial} b = 0$


bc CFT
· a holomorphic CFT.
$b \cdot c = - c b$
$S = \frac{1}{2\pi} \int d^2 z \; b \bar{\partial} c$
$b = b(z, \bar{z})$ $c = c(z, \bar{z}$
EOM $\bar{\partial} c = 0$; $\bar{\partial} b =$

bc CFT

- a holomorphic CFT.

$b \cdot c = -cb$

$$S = \frac{1}{2\pi} \int d^2z \; b \bar{\partial} c$$

$(h_b, 0) =$

$b = b(z, \bar{z}) \qquad c = c(z, \bar{z}).$

EOM $\quad \bar{\partial} c = 0 \quad ; \quad \bar{\partial} b = 0$

bc CFT

· a holomorphic CFT.

$$S = \frac{1}{2\pi} \int d^2z \; b \bar{\partial} c$$

$b = b(z, \bar{z}) \qquad c = c(z, \bar{z})$.

EOM $\quad \bar{\partial} c = 0 \quad ; \quad \bar{\partial} b = 0$

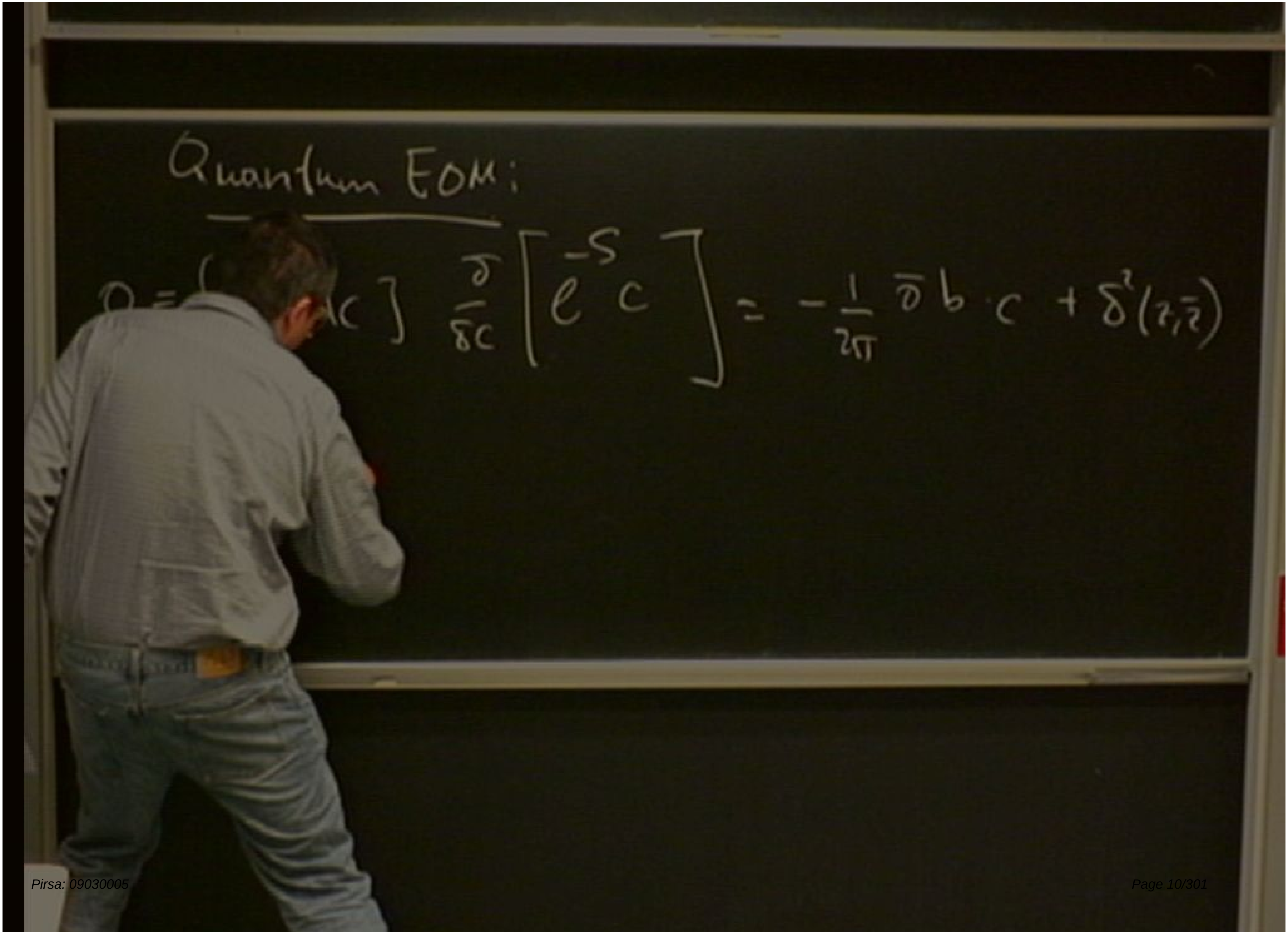
$b \cdot c = - c b$

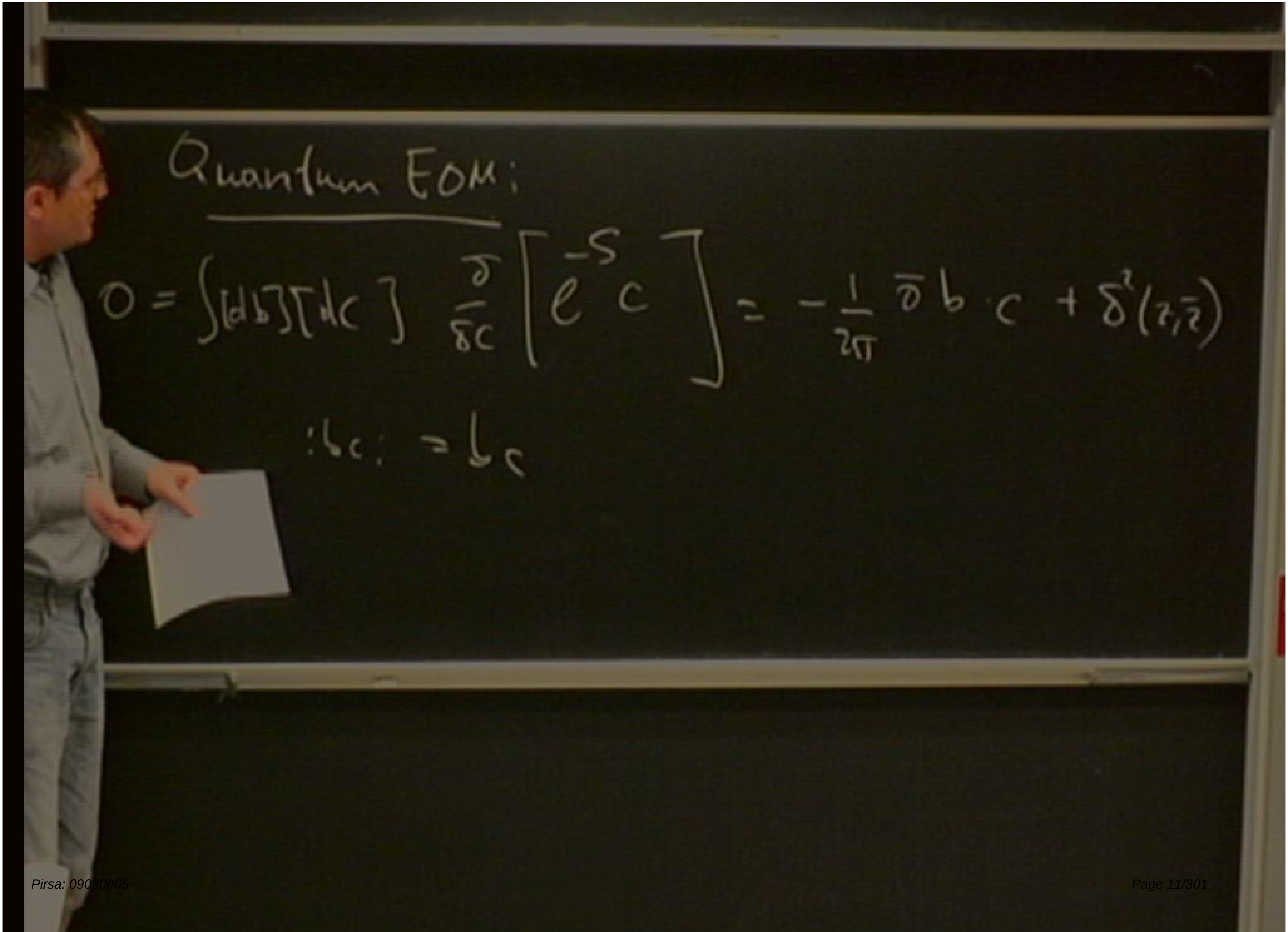
$(h_b, 0) = (\lambda, 0)$.

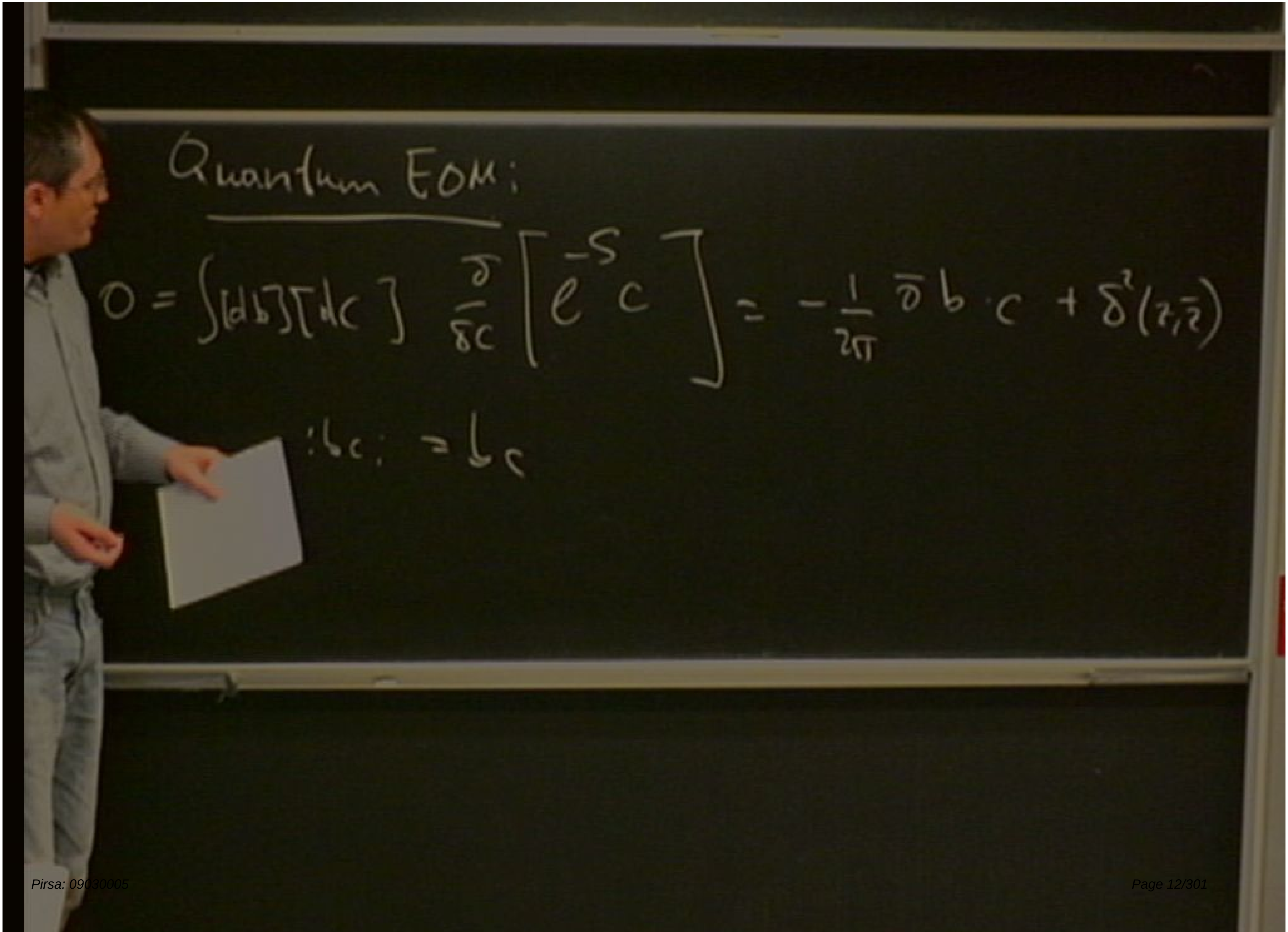
$(h_c, 0) = (1 - \lambda, 0)$

Quantum EOM:

$$= \int [db][dc] \, \frac{\delta}{\delta c} \left[e^{-S} c \right]$$


Quantum EOM:
$0 = \int [dc] \; \frac{\delta}{\delta c} \left[e^{-S} c \right] = -\frac{1}{2\pi} \bar{\partial} b \cdot c + \delta^2(z,\bar{z})$


Quantum EOM:
$0 = \int [db][dc] \frac{\delta}{\delta c}\left[e^{-S} c \right] = -\frac{1}{2\pi} \bar{\partial} b \cdot c + \delta^2(z,\bar{z})$
$:bc: = bc$



Quantum EOM:
$0 = \int [db][dc] \frac{\delta}{\delta c}\left[e^{-S} c \right] = -\frac{1}{2\pi} \bar{\partial} b \cdot c + \delta^2(z,\bar{z})$
$:bc: = bc$

Quantum EOM:

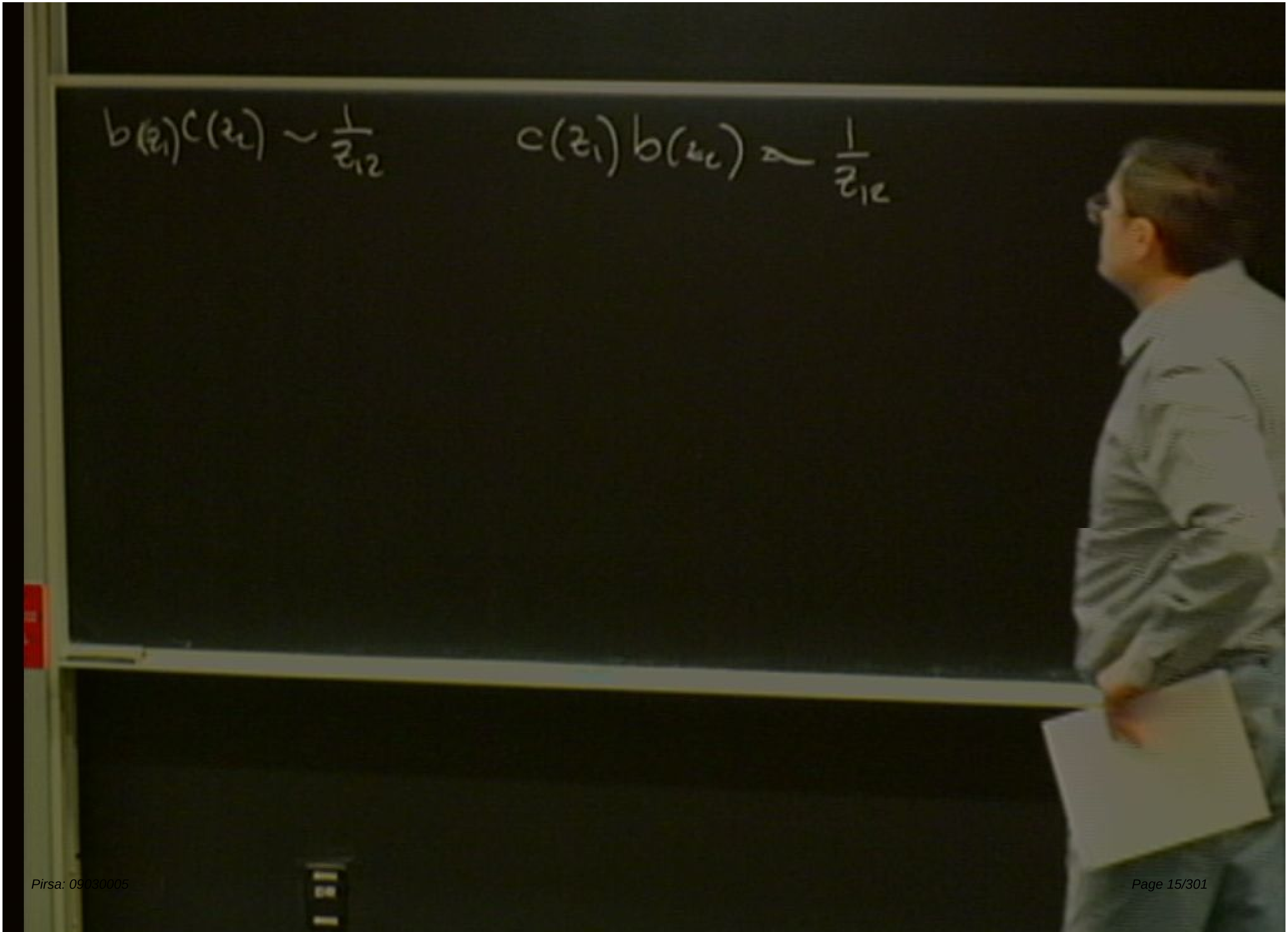
$$0 = \int [db][dc] \frac{\delta}{\delta c}\left[e^{-S} c \right] = -\frac{1}{2\pi} \bar{\partial} b \cdot c + \delta^2(z,\bar{z})$$

$$:bc: = bc - \frac{1}{z_{12}}$$

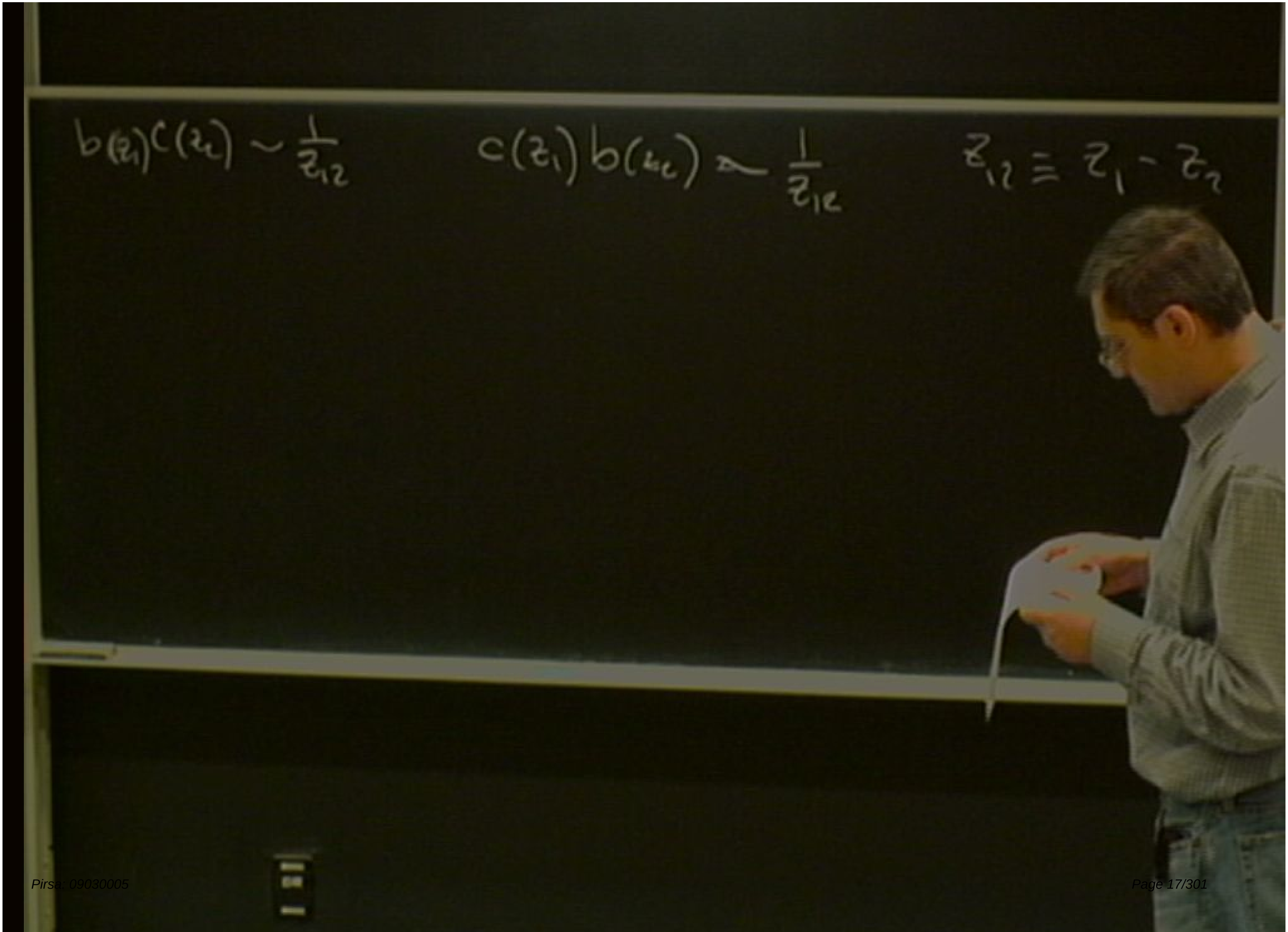
Quantum EOM:

$$0 = \int [db][dc]\, \frac{\delta}{\delta c}\left[e^{-S} c \right] = -\frac{1}{2\pi} \bar\partial b \cdot c + \delta^2(z,\bar z)$$

$$:bc: = bc - \frac{1}{z_{12}}$$


$b(z_1)c(z_2) \sim \frac{1}{z_{12}}$
$c(z_1)b(z_2) \sim \frac{1}{z_{12}}$

$$b(z_1)c(z_2) \sim \frac{1}{z_{12}} \qquad c(z_1)b(z_2) \sim \frac{1}{z_{12}}$$


$b(z_1)c(z_2) \sim \frac{1}{z_{12}}$
$c(z_1)b(z_2) \sim \frac{1}{z_{12}}$
$z_{12} \equiv z_1 - z_2$

$$b(z_1)c(z_2) \sim \frac{1}{z_{12}} \qquad c(z_1)b(z_2) \sim \frac{1}{z_{12}} \qquad z_{12} \equiv z_1 - z_2$$

$$bb \sim O(z_{12})$$

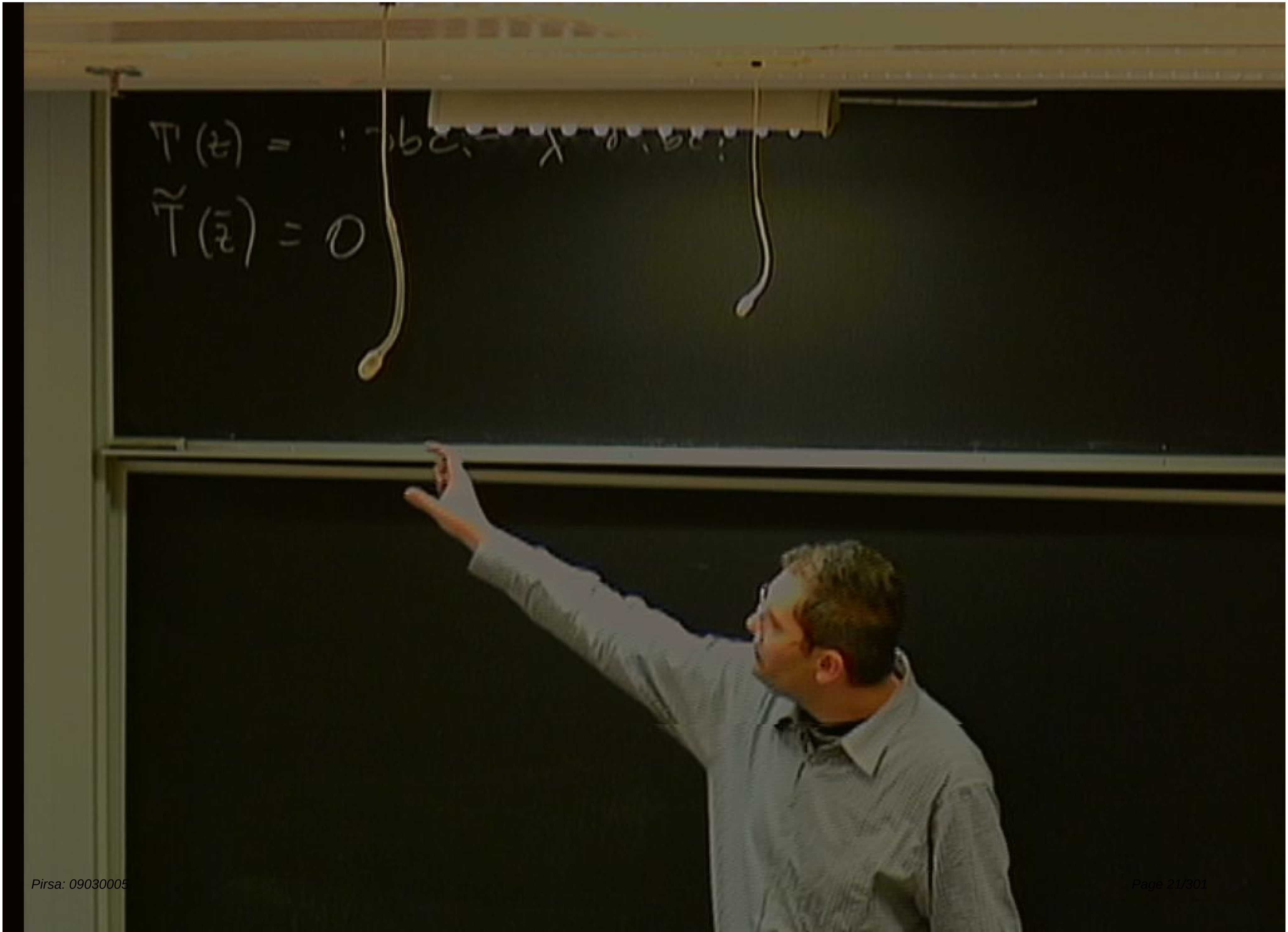
$$b(z_1)\,c(z_2) \sim \frac{1}{z_{12}} \qquad c(z_1)\,b(z_2) \sim \frac{1}{z_{12}} \qquad z_{12} \equiv z_1 - z_2$$

$$b\,b \sim O(z_{12}) \qquad c\,c \sim O(z_{12})$$

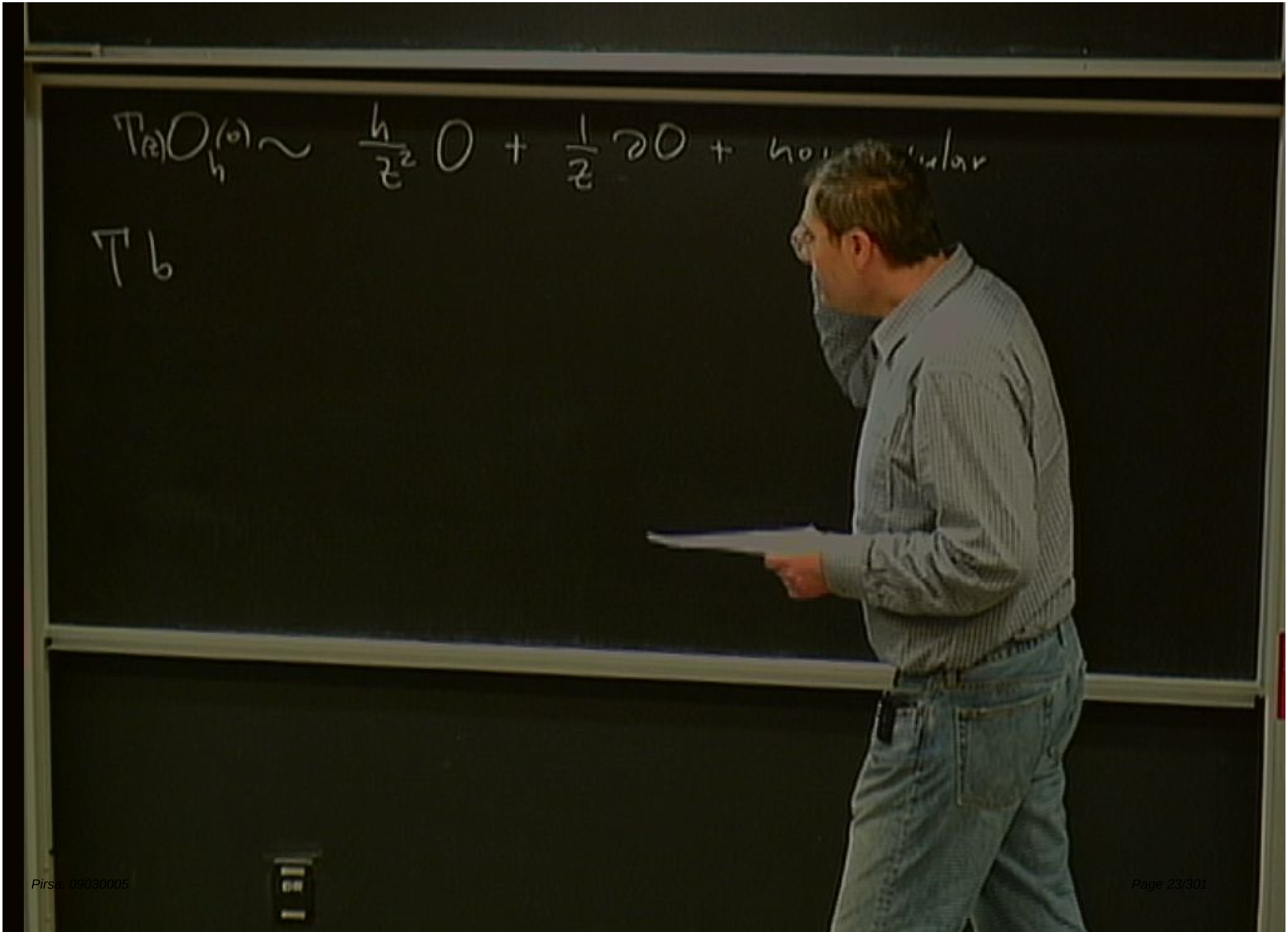
$$b(z_1)\,c(z_2) \sim \frac{1}{z_{12}} \qquad c(z_1)\,b(z_2) \sim \frac{1}{z_{12}} \qquad z_{12} \equiv z_1 - z_2$$

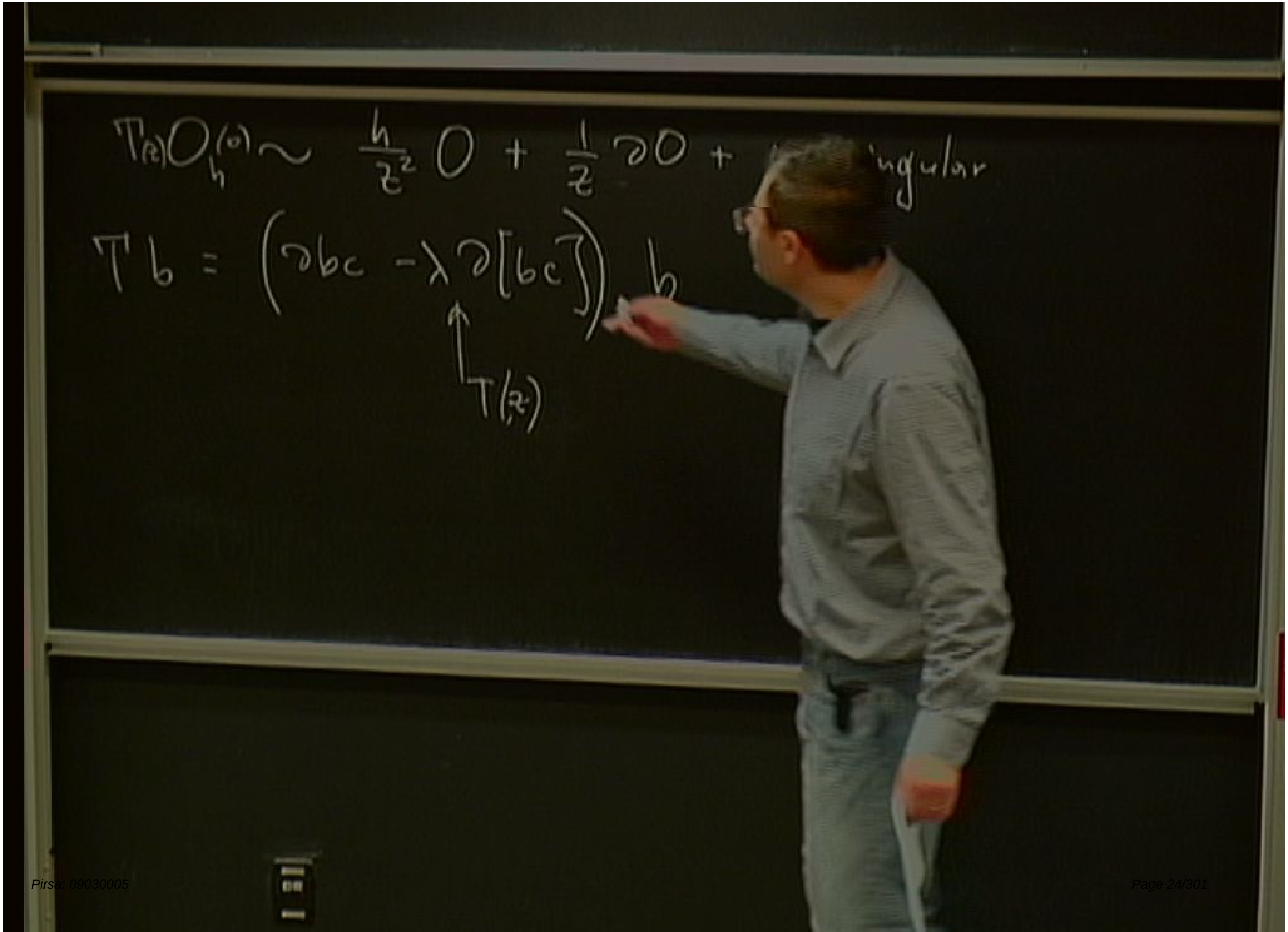
$$bb \sim O(z_{12}) \qquad cc \sim O(z_{12})$$

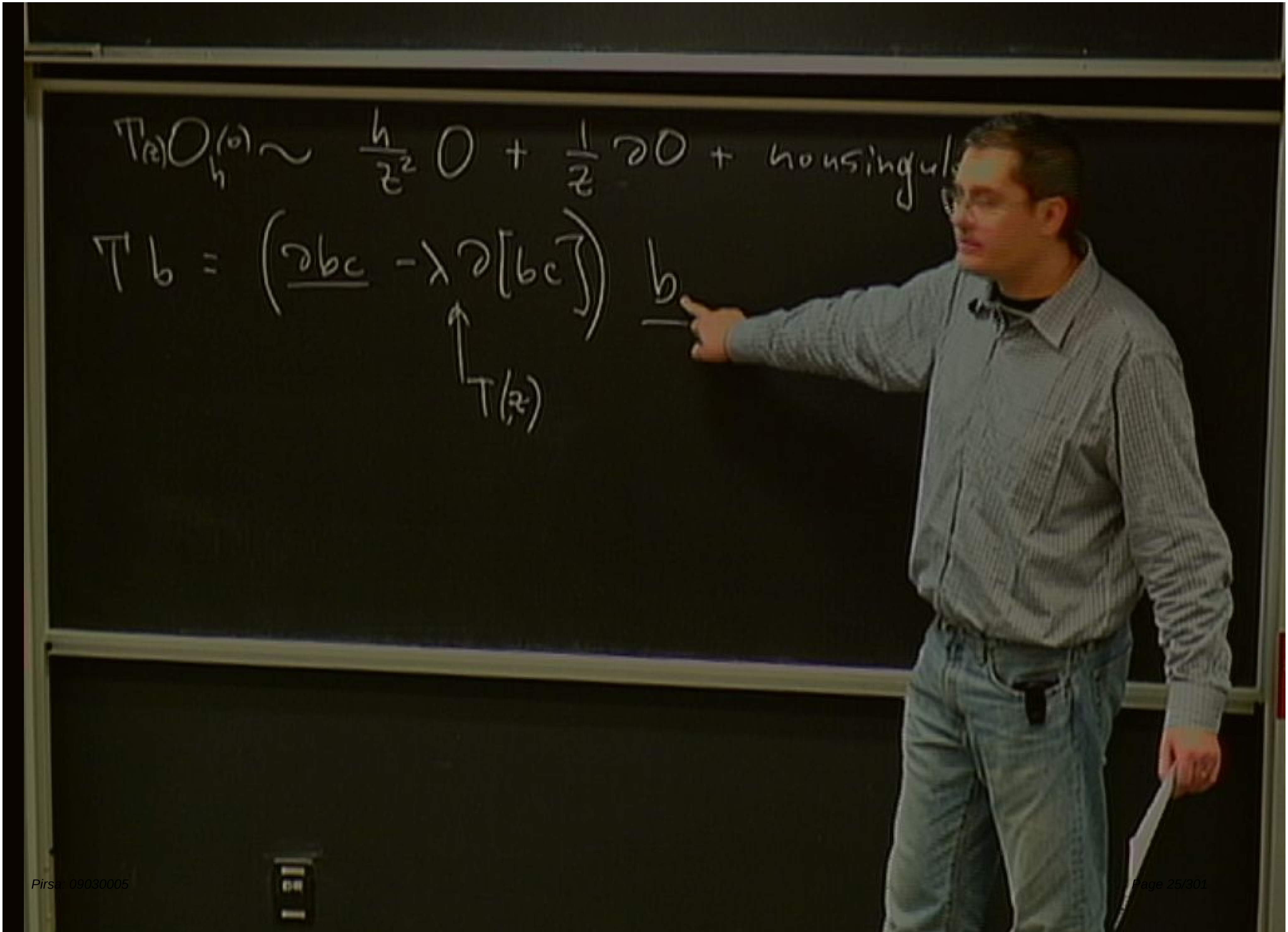
$$T(z) = \,:\partial b c - \lambda\, \partial :bc:$$


T(z) =
T̃(z̄) = 0

$$T\,O_h \sim \frac{h}{z^2} O + \frac{1}{z} \partial O + \text{nonsingular}$$


$T(z)O_h(0) \sim \frac{h}{z^2} O + \frac{1}{z} \partial O +$ no ular
$T\,b$


$T(z)\mathcal{O}_h(0) \sim \frac{h}{z^2}\mathcal{O} + \frac{1}{z}\partial\mathcal{O} +$
$T b = \left(\partial b c - \lambda \partial[bc]\right) b$
$T(z)$



$T(z)\mathcal{O}_h(0) \sim \frac{h}{z^2}\mathcal{O} + \frac{1}{z}\partial\mathcal{O} + \text{nonsingul}$
$Tb = (\partial bc - \lambda\partial[bc])\, b$
$T(z)$

$$T(z)\mathcal{O}_h(0) \sim \frac{h}{z^2}\mathcal{O} + \frac{1}{z}\partial\mathcal{O} + \text{nonsingular}$$

$$T\,b = \left(\underline{\partial b c} - \lambda\,\partial[bc]\right)\underline{b} = \frac{\partial b(0)}{z}$$

$T(z)$

$$T(z)\,O_h(0) \sim \frac{h}{z^2}\,O + \frac{1}{z}\,\partial O + \text{nonsingular}$$

$$T b = \left(\underline{\partial b c} - \lambda\,\partial[bc]\right)\underline{b} = \frac{\partial b(0)}{z} - \lambda\,\partial b(0)$$

$T(z)$

$$T(z)\mathcal{O}_h(0) \sim \frac{h}{z^2}\mathcal{O} + \frac{1}{z}\partial\mathcal{O} + \text{nonsingular}$$

$$Tb = \Big(\underline{\partial b c} - \lambda \partial[bc]\Big)\,\underline{b} = \frac{\partial b(0)}{z} - \lambda\,\partial b(0)\cdot\frac{1}{z}$$

$T(z)$

$$-\lambda\, b(z)$$

$$T(z)\mathcal{O}_h(0) \sim \frac{h}{z^2}\mathcal{O} + \frac{1}{z}\partial\mathcal{O} + \text{nonsingular}$$

$$Tb = \left(\underline{\partial b c} - \lambda\partial[bc]\right)\underline{b} = \frac{\partial b(0)}{z} - \lambda\,\partial b(0)\cdot\frac{1}{z}$$

$T(z)$

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$$T(z)\,O_h(0) \sim \frac{h}{z^2}\,O + \frac{1}{z}\,\partial O + \text{nonsingular}$$

$$T\,b = \left(\underline{\partial b c} - \lambda\,\partial[bc]\right)\,\underline{b} = \frac{\partial b(0)}{z} - \lambda\,\partial b(0)\cdot\frac{1}{z}$$

$T(z)$

$$- \lambda\,\underline{b(z)}\,\partial\left[\frac{1}{z}\right]$$

$$= \frac{1}{z}\,\partial b\,[1-\lambda]$$

$$T(z)\,O_h(0) \sim \frac{h}{z^2}\,O + \frac{1}{z}\,\partial O + \text{nonsingular}$$

$$T\,b = \left(\underline{\partial b\, c} - \lambda\, \partial[bc]\right)\,\underline{b} = \frac{\partial b(0)}{z^2} - \lambda\, \partial b(0)\cdot\frac{1}{z}$$

$$T(z) \qquad\qquad - \lambda\, \underline{b(z)}\, \partial\left[\frac{1}{z}\right]$$

$$= \frac{1}{z}\,\partial b\left[1-\lambda\right] \quad -\lambda$$

$$T(z)\,\mathcal{O}_h(0) \sim \frac{h}{z^2}\,\mathcal{O} + \frac{1}{z}\,\partial\mathcal{O} + \text{nonsingular}$$

$$T\,b = \left(\underline{\partial b c} - \lambda\,\partial[bc]\right)\underline{b} = \frac{\partial b(0)}{z} - \lambda\,\partial b(0)\cdot\frac{1}{z}$$

$T(z)$

$$-\lambda\,\underline{b(z)}\,\partial\left[\frac{1}{z}\right]$$

$$= \frac{1}{z}\,\partial b\,[1-\lambda] - \lambda\,[b + z\,\partial b]\left(-\frac{1}{z^2}\right)$$

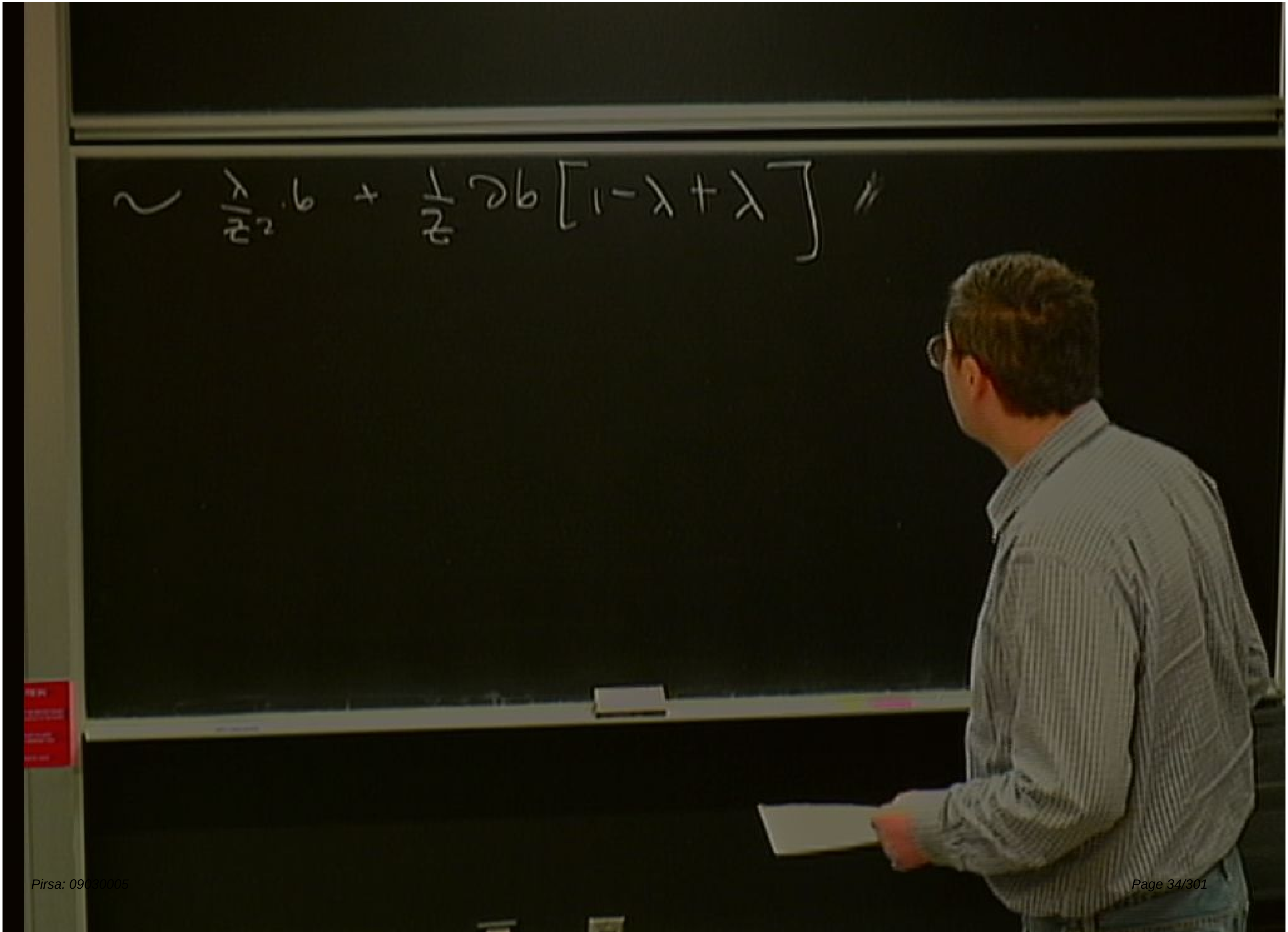
$$T(z)\,O_h(0) \sim \frac{h}{z^2}\,O + \frac{1}{z}\,\partial O + \text{nonsingular}$$

$$T b = \left(\underline{\partial b c} - \lambda\, \partial[bc]\right) \underline{b} = \frac{\partial b(0)}{z} - \lambda\, \partial b(0) \cdot \frac{1}{z}$$

$T(z)$

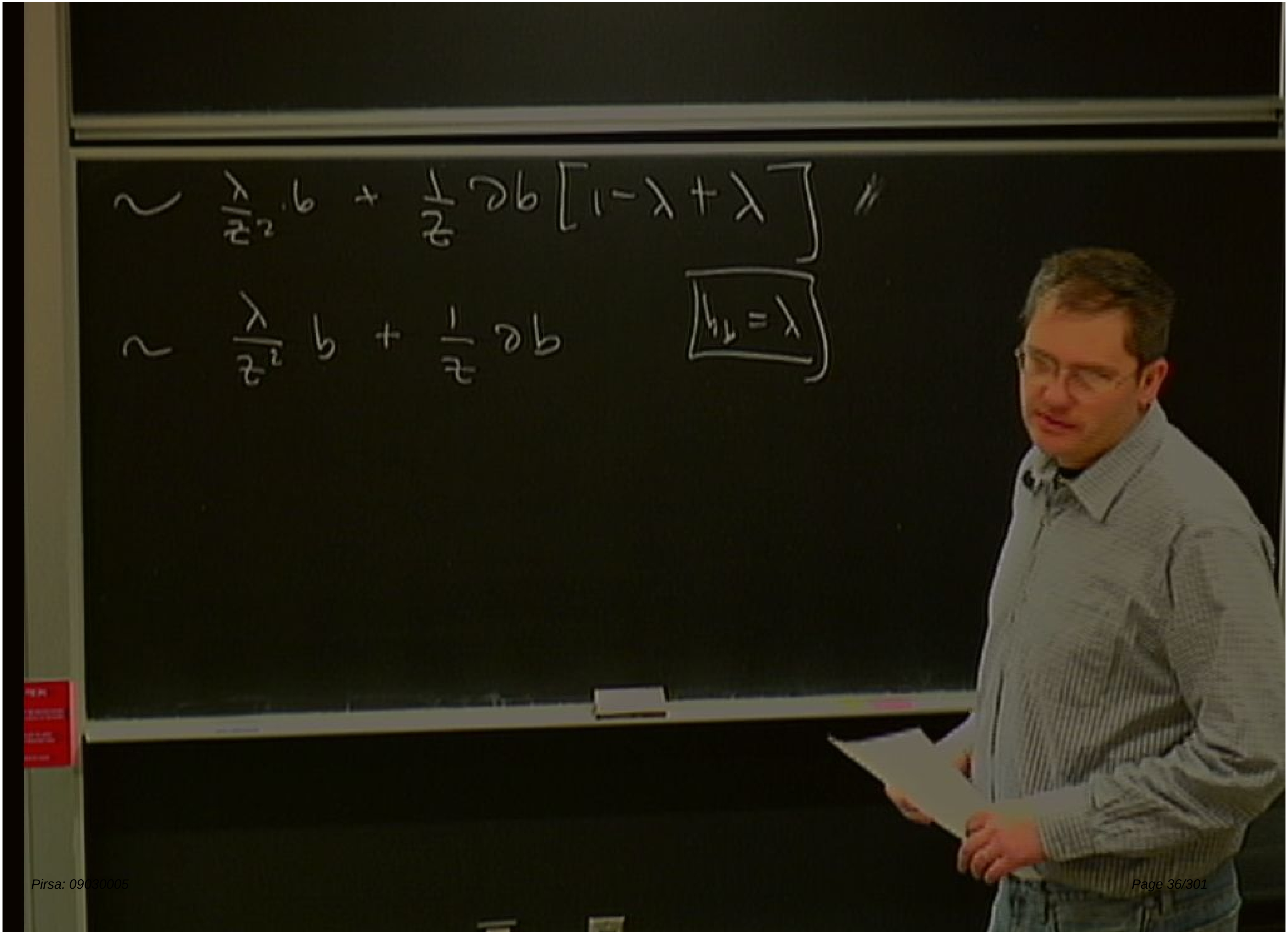
$$- \lambda\, \underline{b(z)}\; \partial\left[\frac{1}{z}\right]$$

$$= \frac{1}{z}\,\partial b\,[1-\lambda] - \lambda\,[b + \underline{z\,\partial b}]\left(\underline{-\frac{1}{z^2}}\right)$$


$\sim \frac{\lambda}{\epsilon^2} \cdot 6 + \frac{1}{\epsilon} \partial 6 \left[1 - \lambda + \lambda \right] //$

$$\sim \frac{\lambda}{z^2}\,b + \frac{1}{z}\,\partial b\left[1-\lambda+\lambda\right] \;//$$

$$\sim \frac{\lambda}{z^2}\,b + \frac{1}{z}\,\partial b$$



$\sim \frac{\lambda}{z^2} b + \frac{1}{z} \partial b \left[1 - \lambda + \lambda \right]$ //
$\sim \frac{\lambda}{z^2} b + \frac{1}{z} \partial b$
$\boxed{h_b = \lambda}$

$$T(z)\,\mathcal{O}_h(0) \sim \frac{h}{z^2}\mathcal{O} + \frac{1}{z}\partial\mathcal{O} + \text{nonsingular}$$

$$T\,b = \left(\underline{\partial b c} - \lambda \underbrace{\partial[bc]}_{T(z)}\right)\underline{b} = \frac{\partial b(0)}{z} - \lambda\,\partial b(0)\cdot\frac{1}{z}$$

$$- \lambda\,\underline{b(z)}\,\partial\left[\frac{1}{z}\right]$$

$$= \frac{1}{z}\partial b\,[1-\lambda] - \lambda\,\underbrace{[b + \underline{z\partial b}]}\left(\underline{-\frac{1}{z^2}}\right)$$

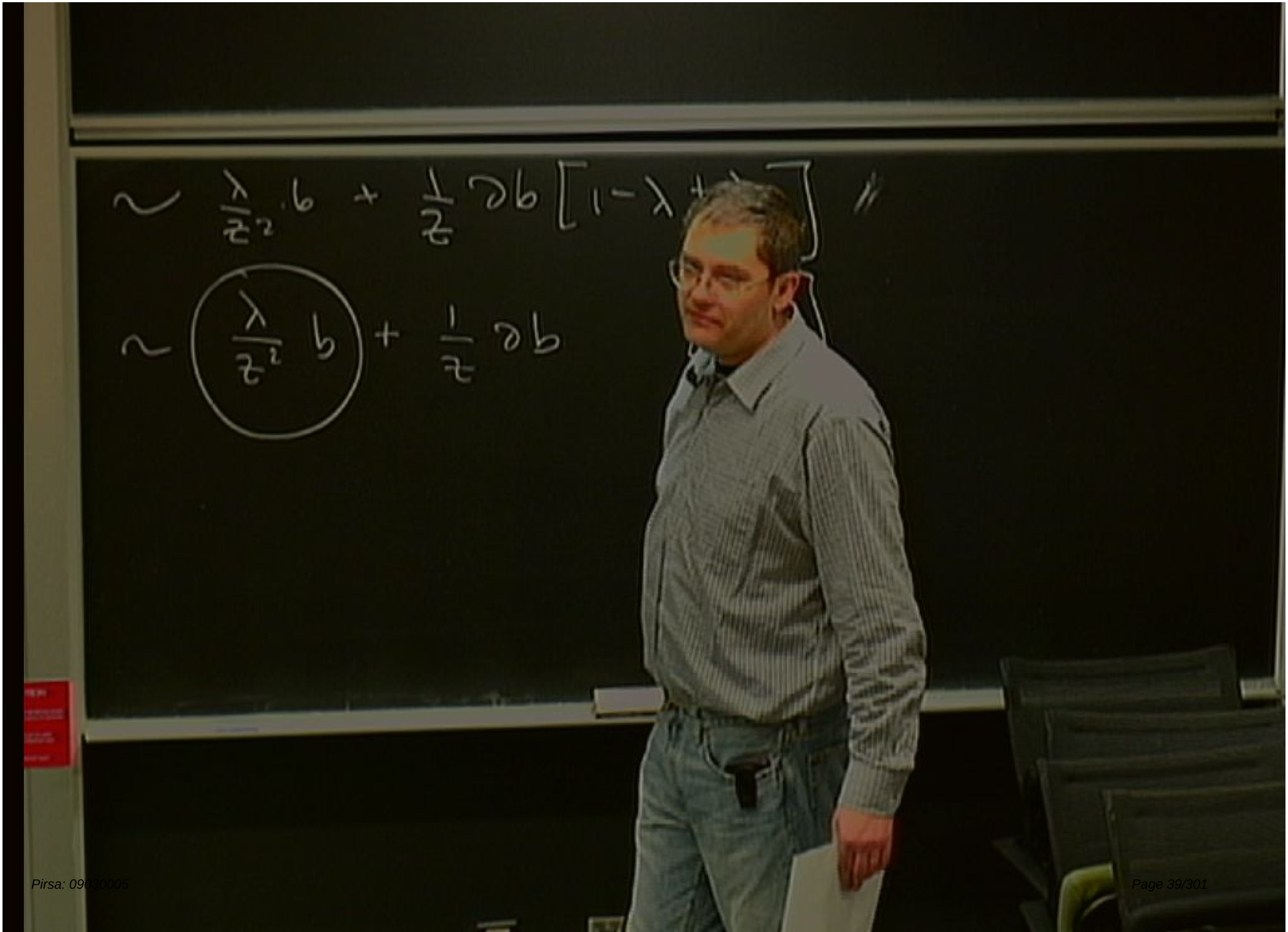
$$T(z)\,\mathcal{O}_h(0) \sim \frac{h}{z^2}\,\mathcal{O} + \frac{1}{z}\,\partial\mathcal{O} + \text{nonsingular}$$

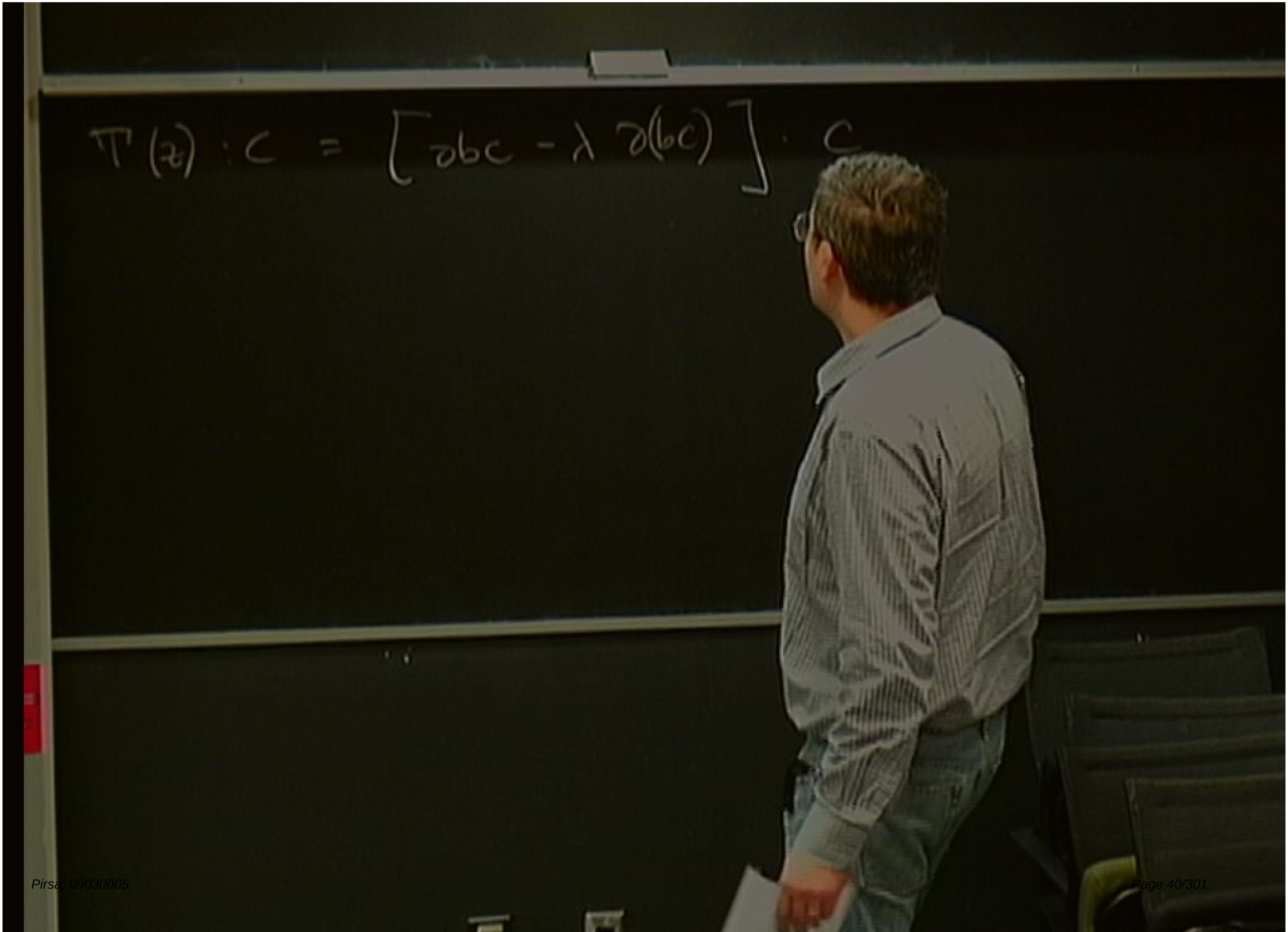
$$T\,b = \left(\underline{\partial b c} - \lambda\,\partial[bc]\right)\underline{b} = \frac{\partial b(0)}{z} - \lambda\,\partial b(0)\cdot\frac{1}{z}$$

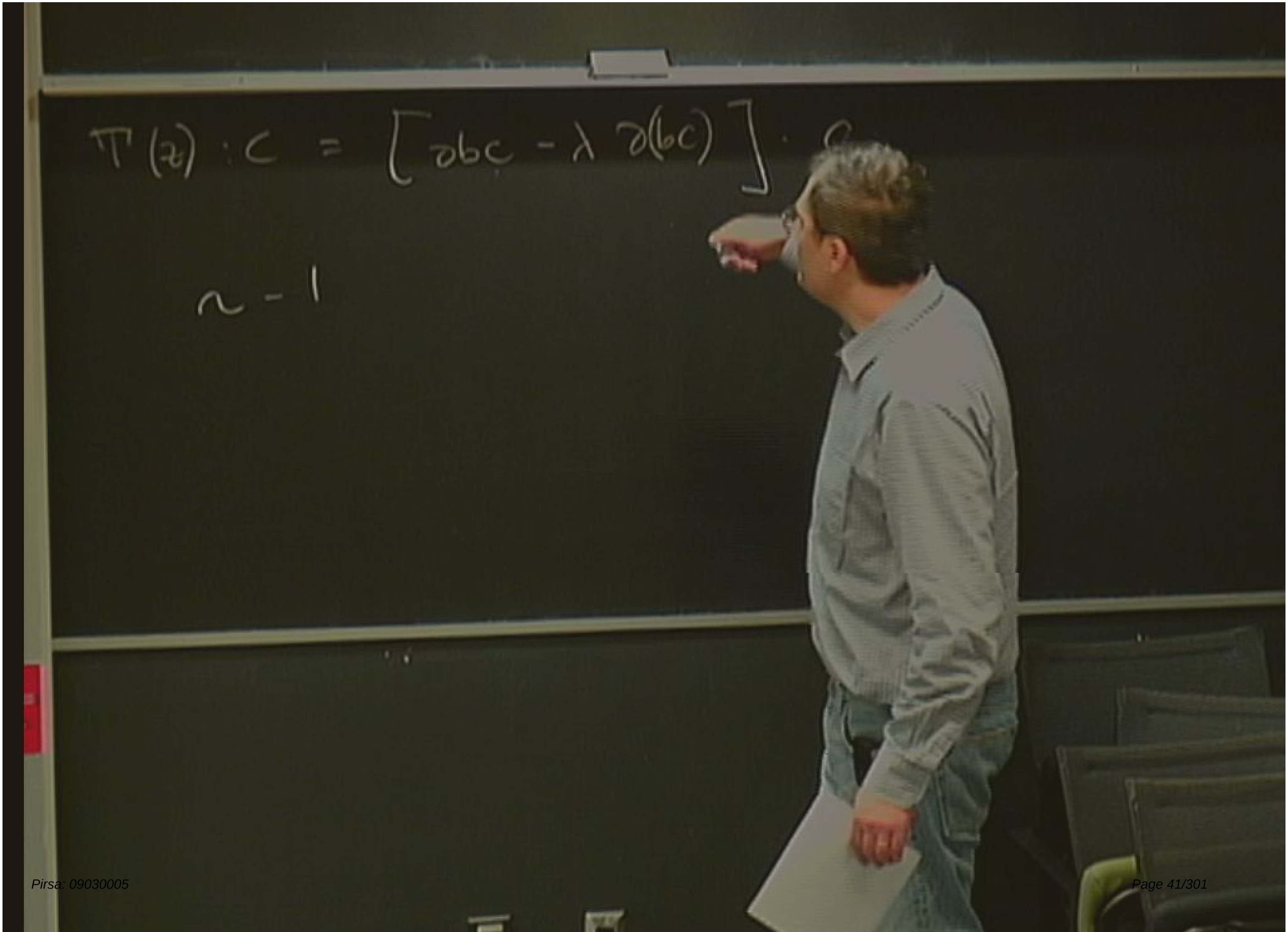
$T(z)$

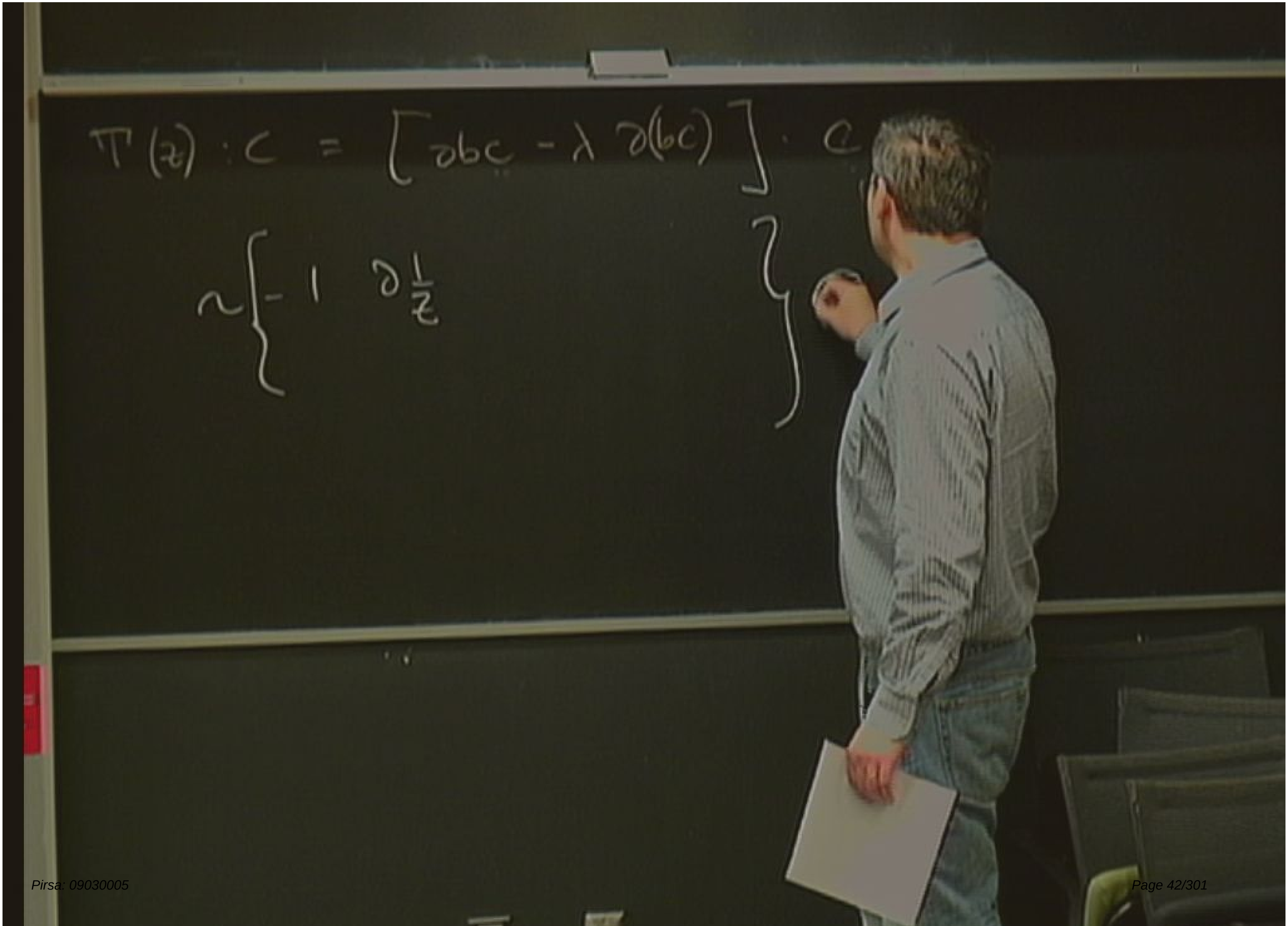
$$-\lambda\,\underline{b(z)}\,\partial\left[\frac{1}{z}\right]$$

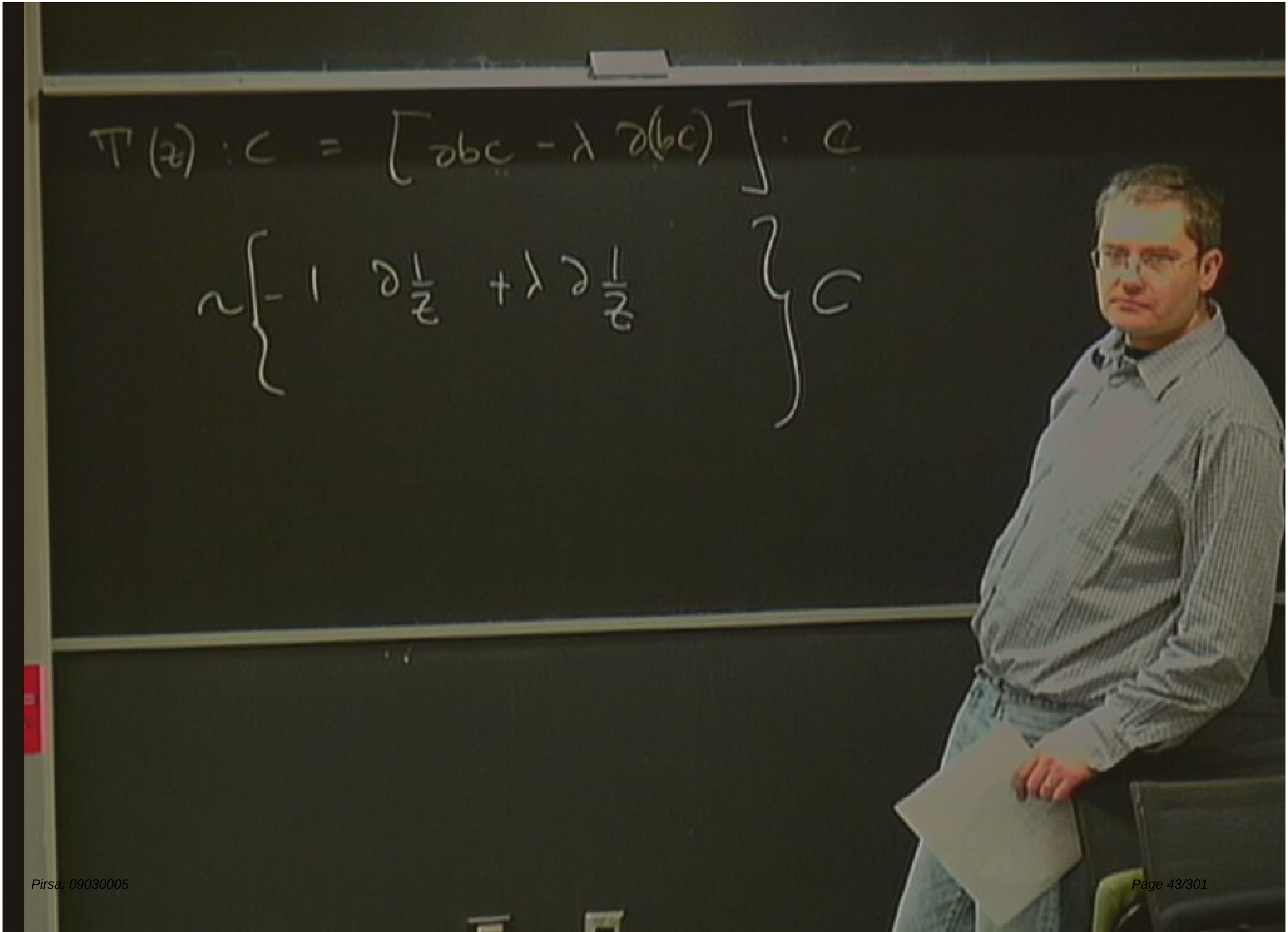
$$= \frac{1}{z}\,\partial b\,[1-\underline{\underline{\lambda}}] - \lambda\,[b + \underline{z\,\partial b}]\left(\underline{-\frac{1}{z^2}}\right)$$


~ λ/z² b + 1/z ∂b


$\pi'(z) \cdot c = \left[\partial bc - \lambda\, \partial(bc) \right] \cdot c$




$T(z) \cdot c = [\partial b c - \lambda \partial(bc)] \cdot c$
$\sim \{ - 1 \quad \partial \frac{1}{z}$


$\pi(z) : c = [\partial bc - \lambda \partial(bc)] \cdot c$
$\sim \left\{ -1 \ \partial \frac{1}{z} + \lambda \partial \frac{1}{z} \right\} c$

$$T(z) : c = \left[\partial b c - \lambda\, \partial(bc) \right] \cdot c$$

$$\sim \left\{ -1 \ \partial \frac{1}{z} + \lambda\, \partial \frac{1}{z} \right\} c$$

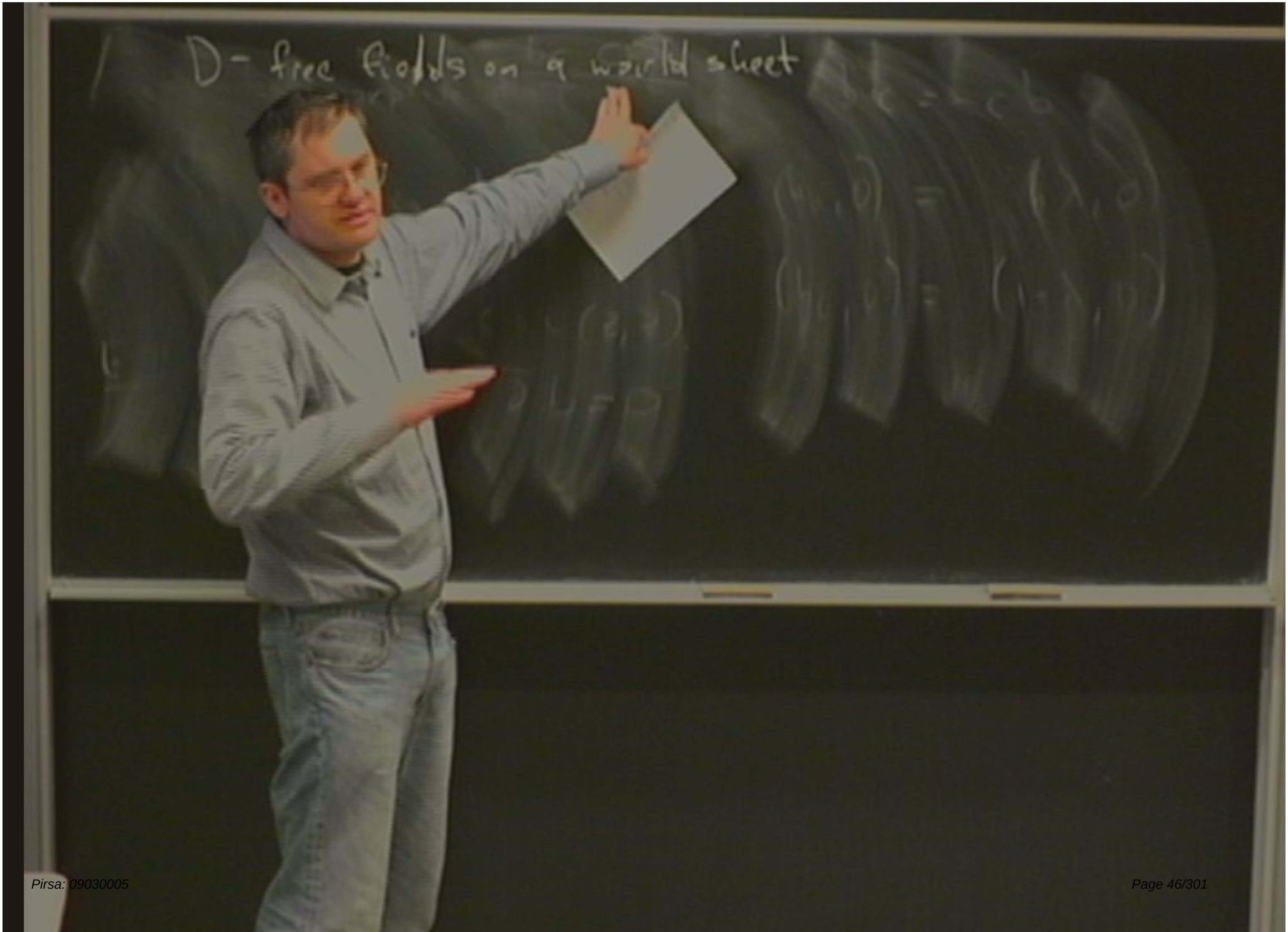
$$\sim \left(-\lambda \cdot \frac{1}{z^2} + \frac{1}{z^2} \right) c \sim \frac{1-\lambda}{z^2}\, c + \cdot \quad \cdot$$

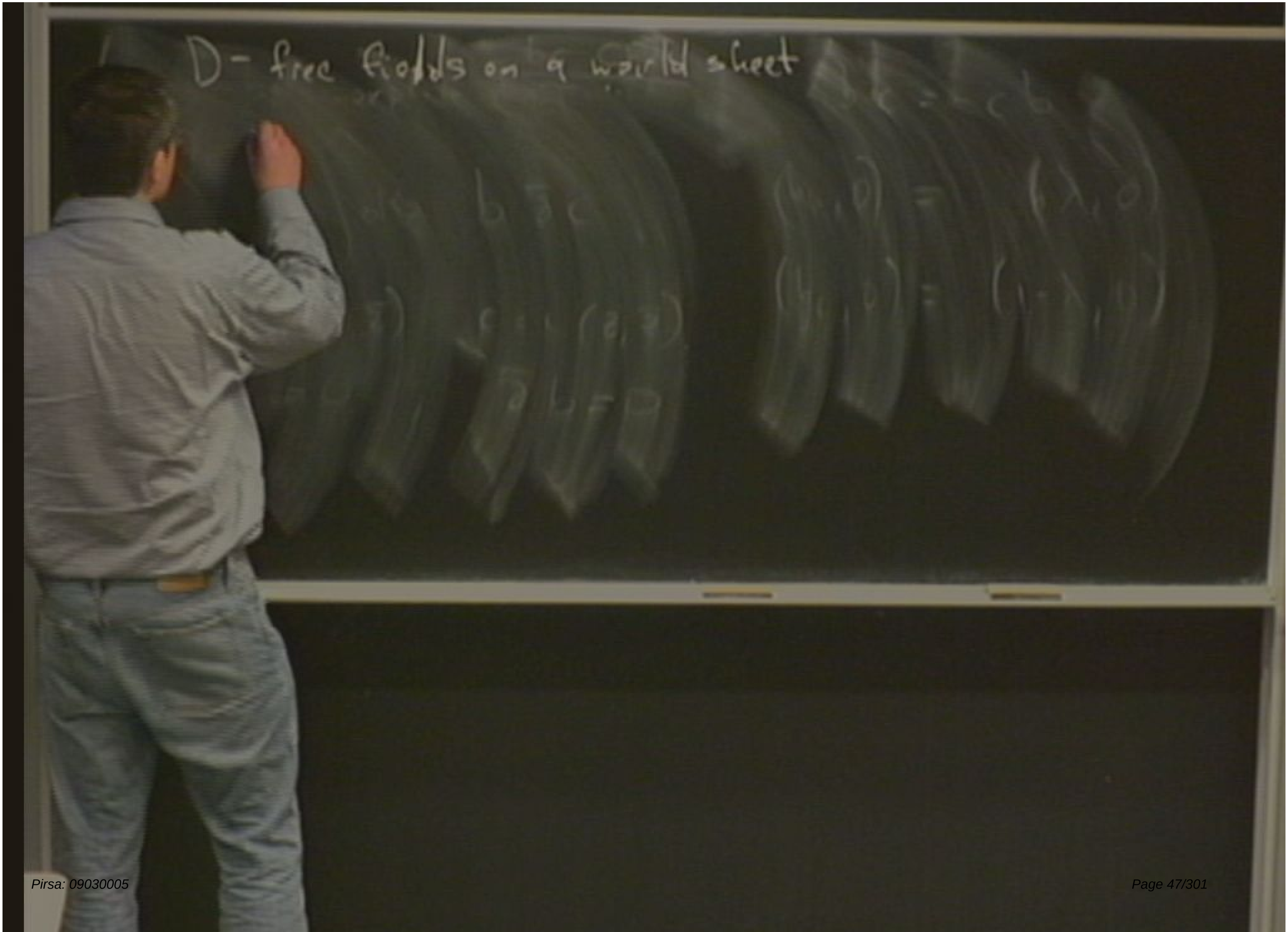
$$T(z) : c = \left[\partial b c - \lambda \, \partial(bc) \right] \cdot c$$

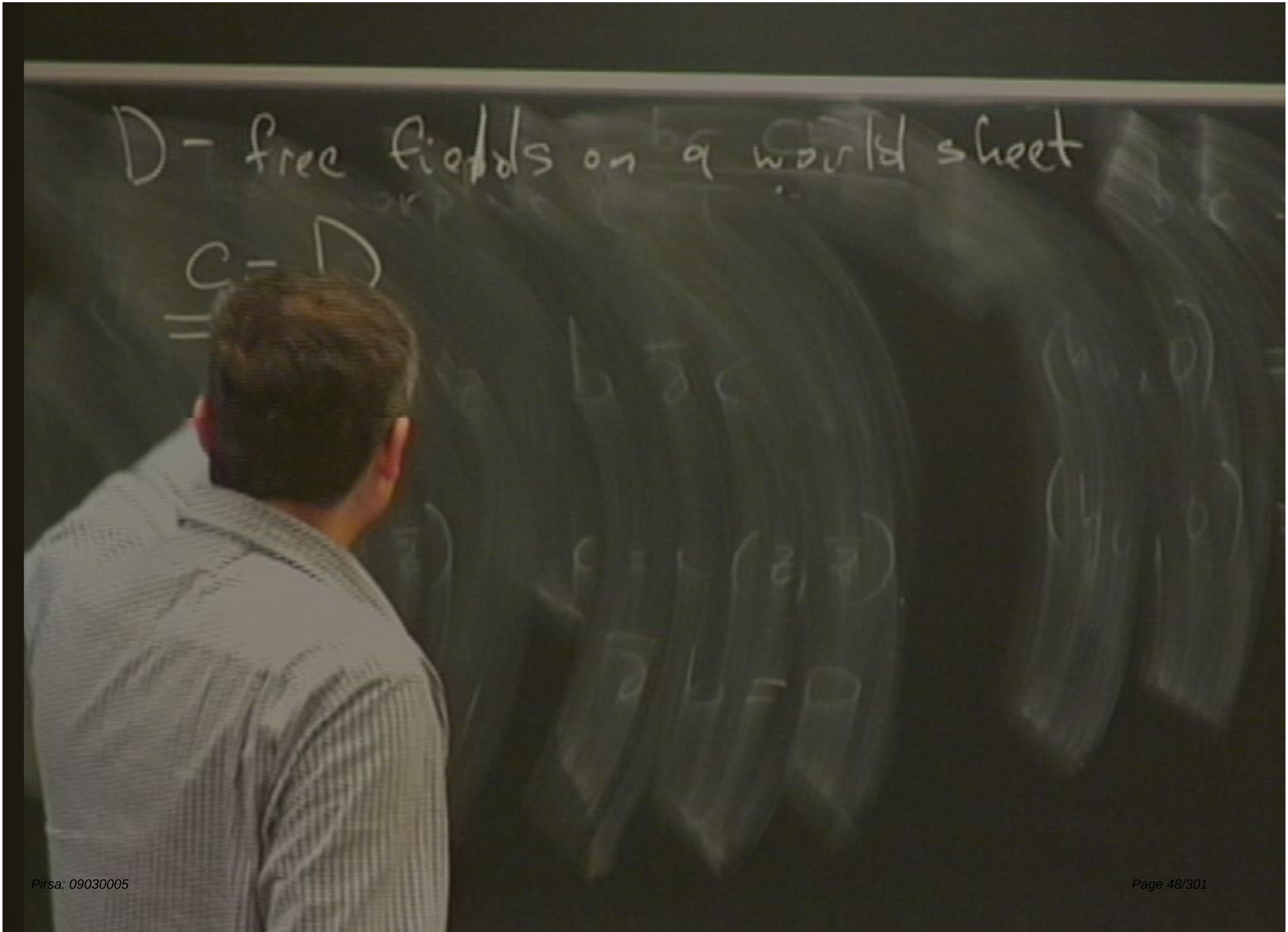
$$\sim \left\{ -1 \; \partial \frac{1}{z} + \lambda \, \partial \frac{1}{z} \right\} c$$

$$h_c = 1 - \lambda$$

$$\sim \left(-\lambda \cdot \frac{1}{z^2} + \frac{1}{z^2} \right) c \sim \frac{1-\lambda}{z^2} c + \cdots$$

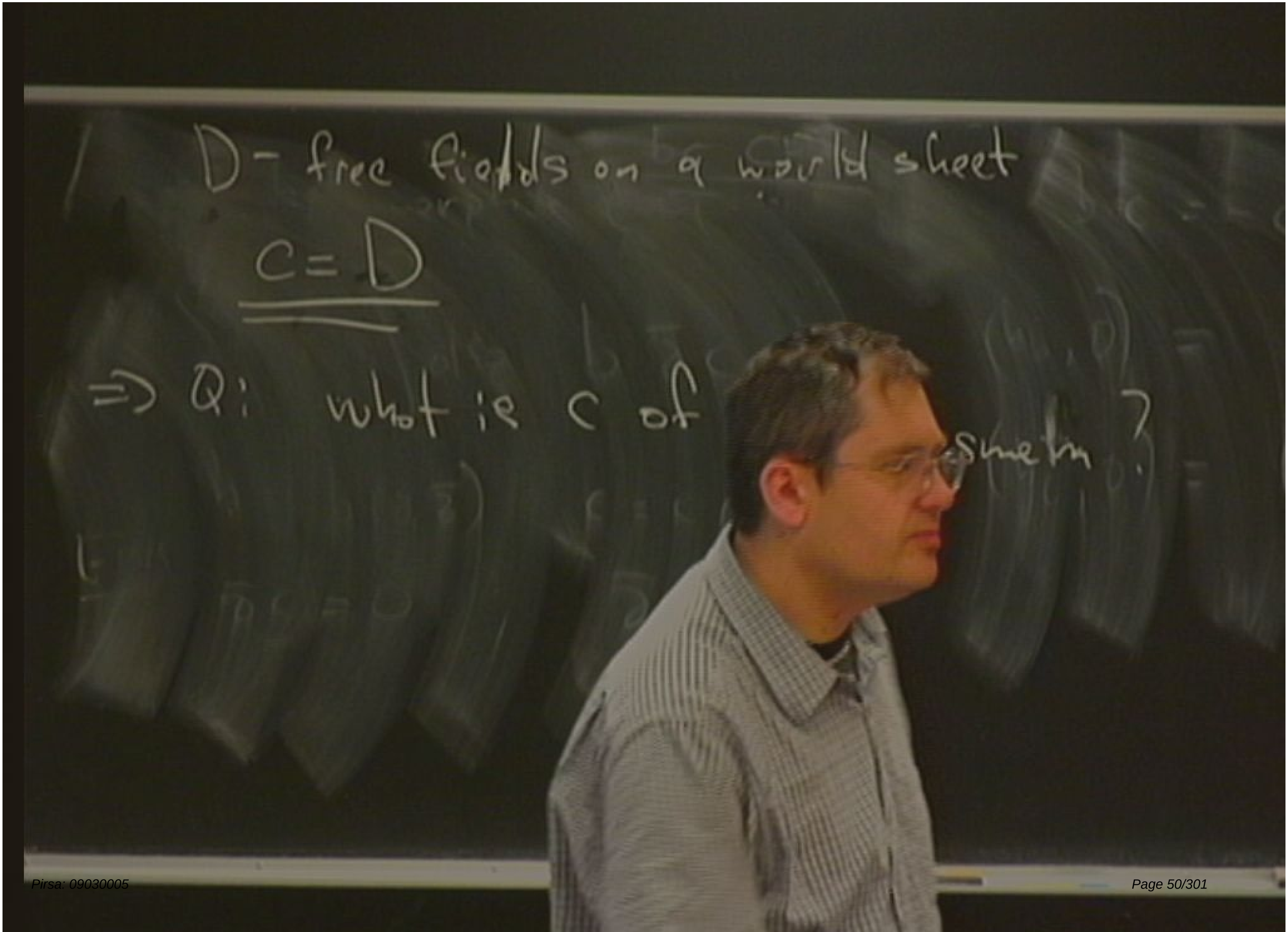

D- free fields on a world sheet

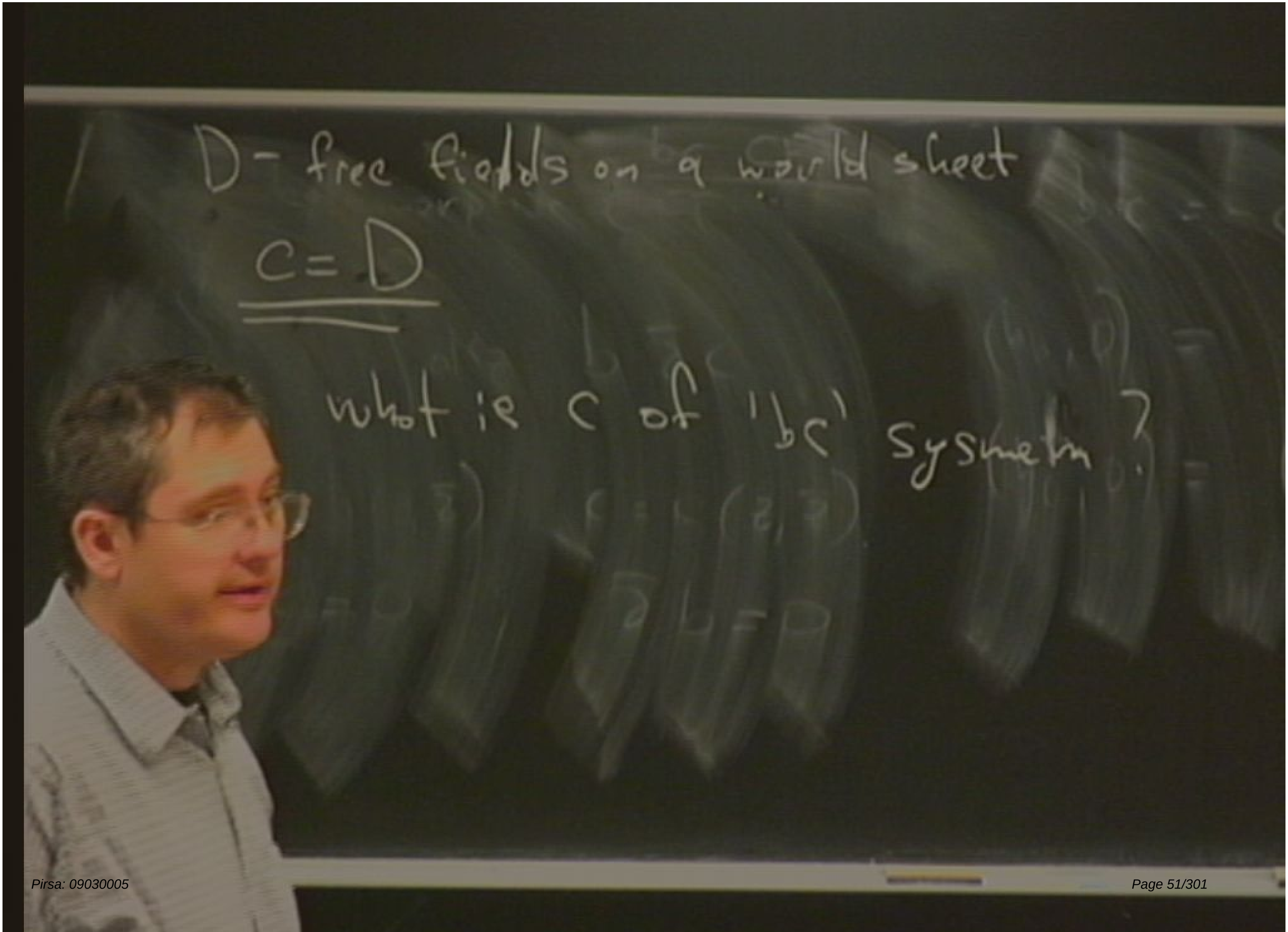

D- free fields on a world sheet


D - free fields on a world sheet
c = D

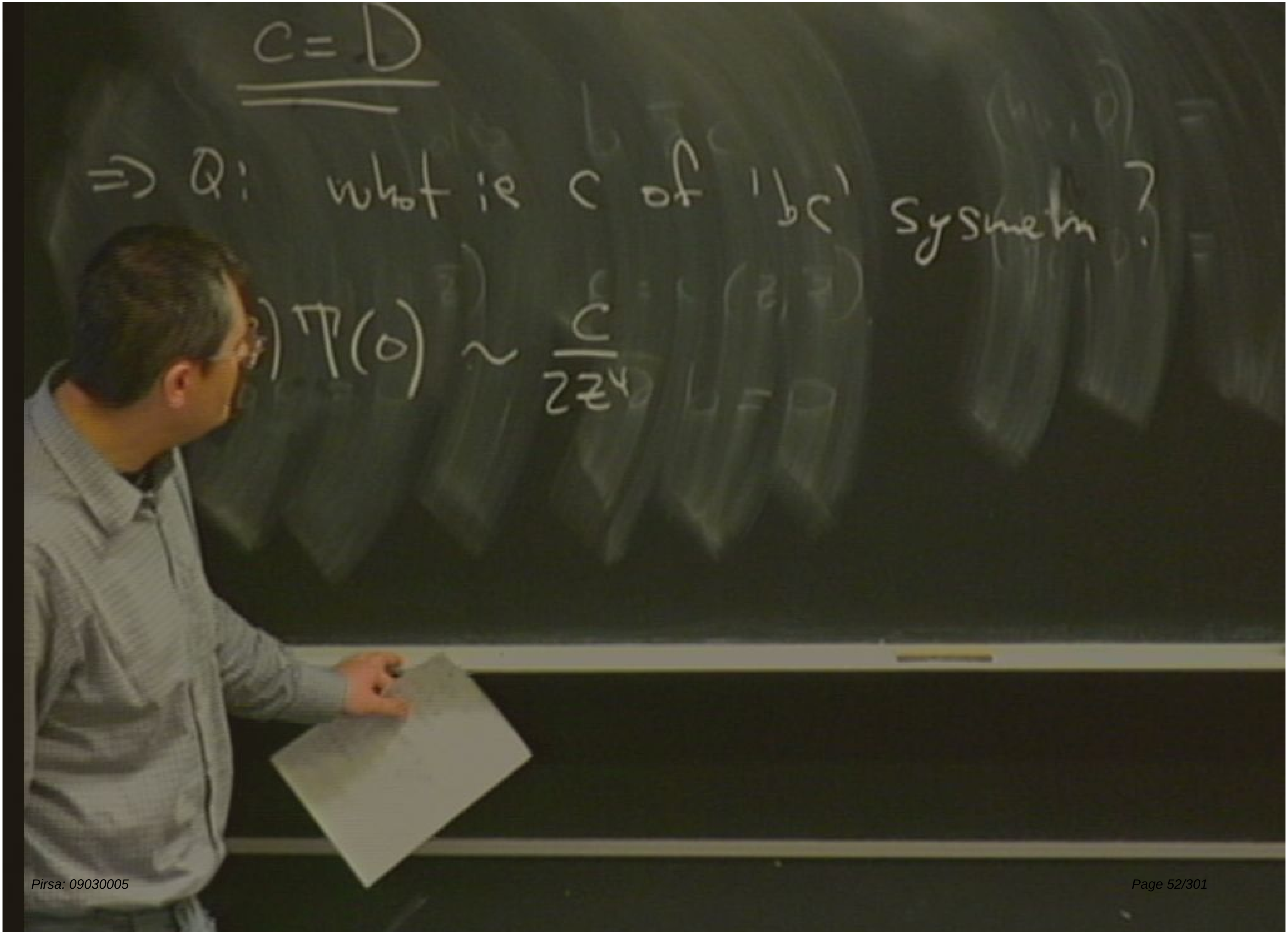
D - free fields on a world sheet

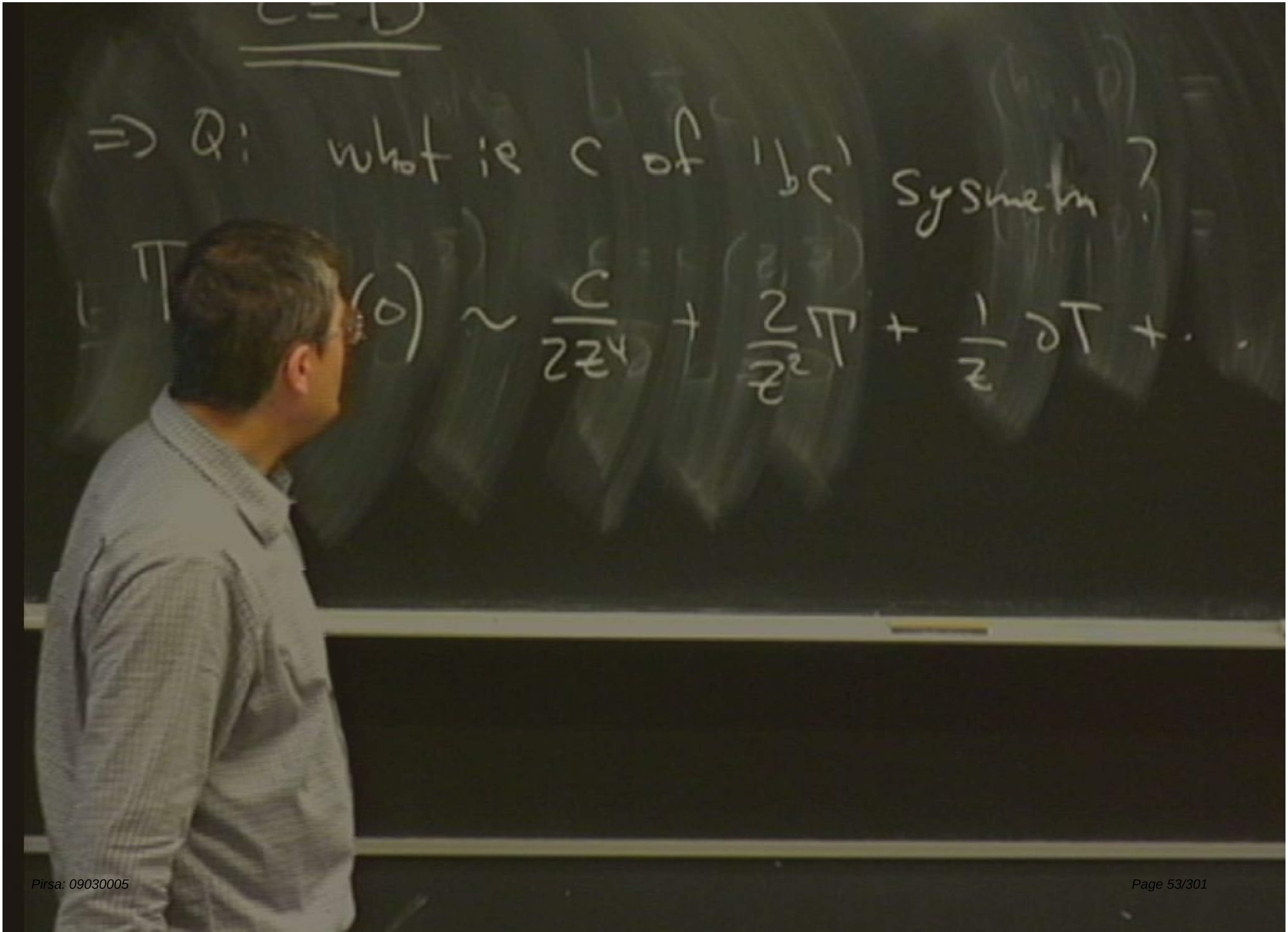
$c = D$

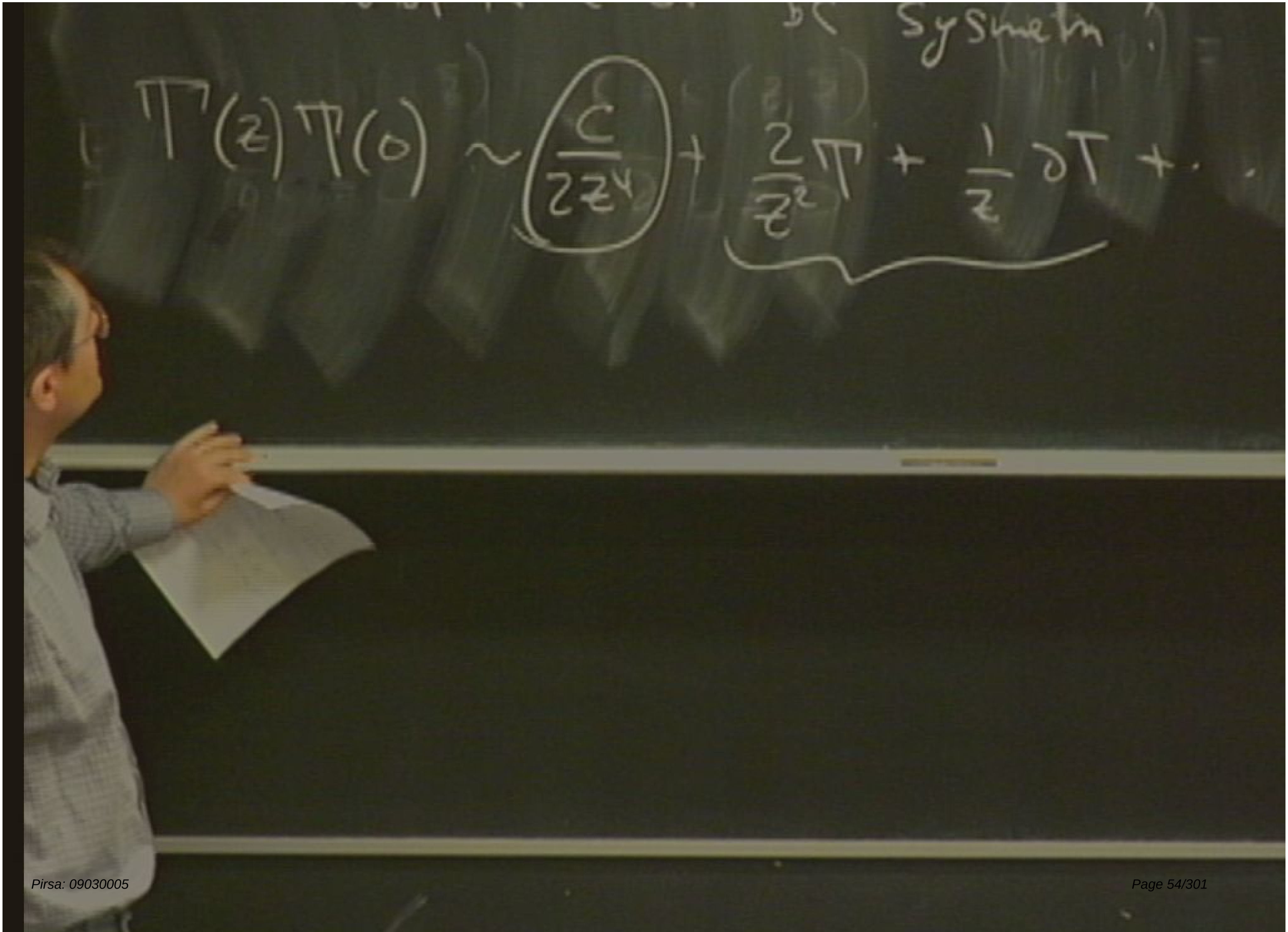

D - free fields on a world sheet
c = D
⇒ Q: what is c of



D - free fields on a world sheet
c = D
what is c of 'bc' system ?

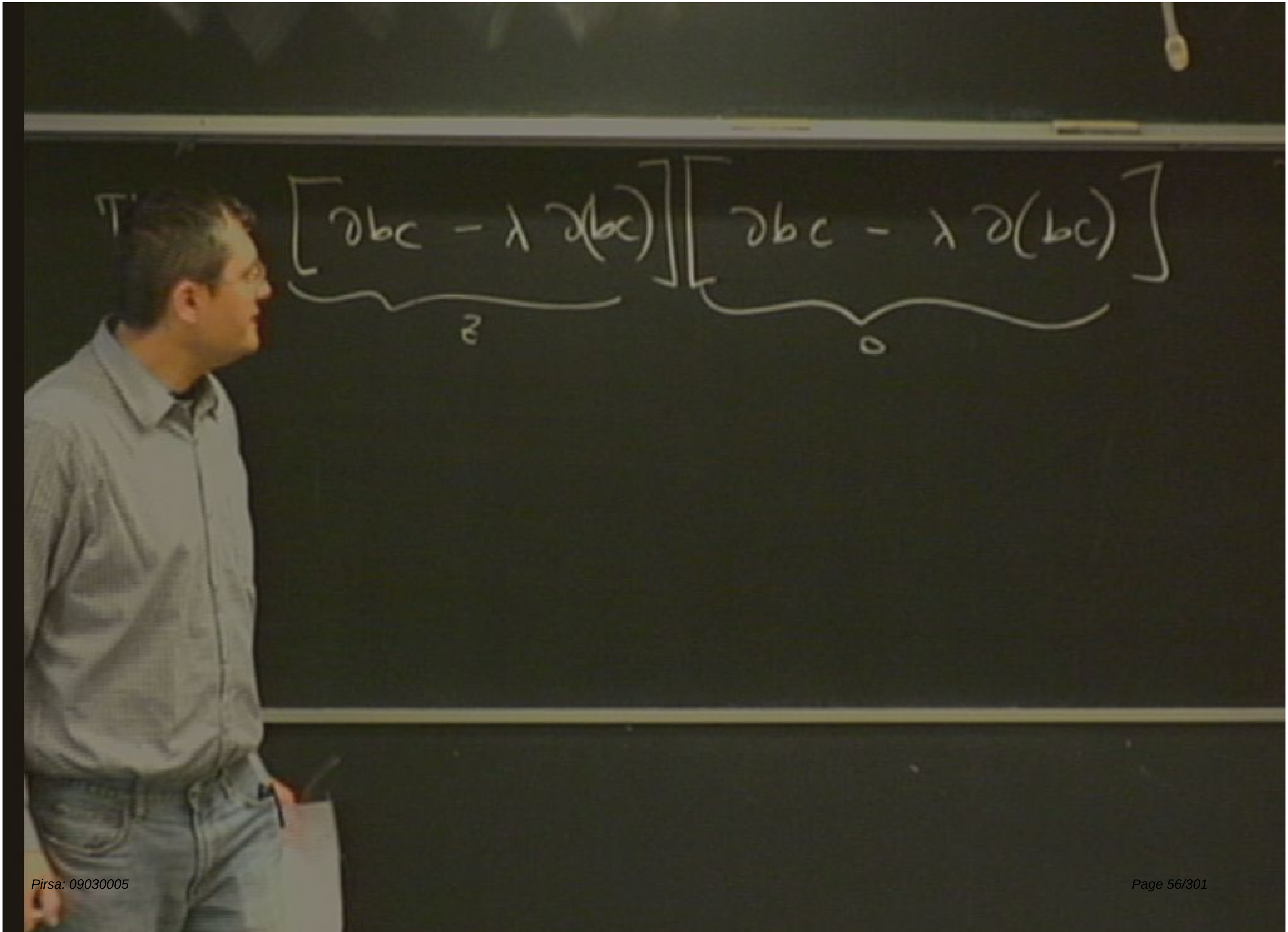

c = D
⇒ Q: what is c of 'bc' sysmetm ?
) T(0) ~ c / 2z^4


⇒ Q: what is c of 'bc' system ?
(0) ~ c/2z^4 + 2/z^2 T + 1/z ∂T + . . .



Symmetry
$T(z)T(0) \sim \frac{c}{2z^4} + \underbrace{\frac{2}{z^2}T + \frac{1}{z}\partial T + \cdots}$

$$T'T = \left[\partial b c - \lambda \partial(bc)\right]\left[\partial b c - \lambda \partial(bc)\right]$$


[∂bc − λ ∂(bc)] [∂bc − λ ∂(bc)]
0

$$T'T = \underbrace{\left[\partial b c - \lambda\, \partial(bc)\right]}_{z} \underbrace{\left[\partial b c - \lambda\, \partial(bc)\right]}_{0}$$

$$① : (\partial b c)(\partial b c)$$

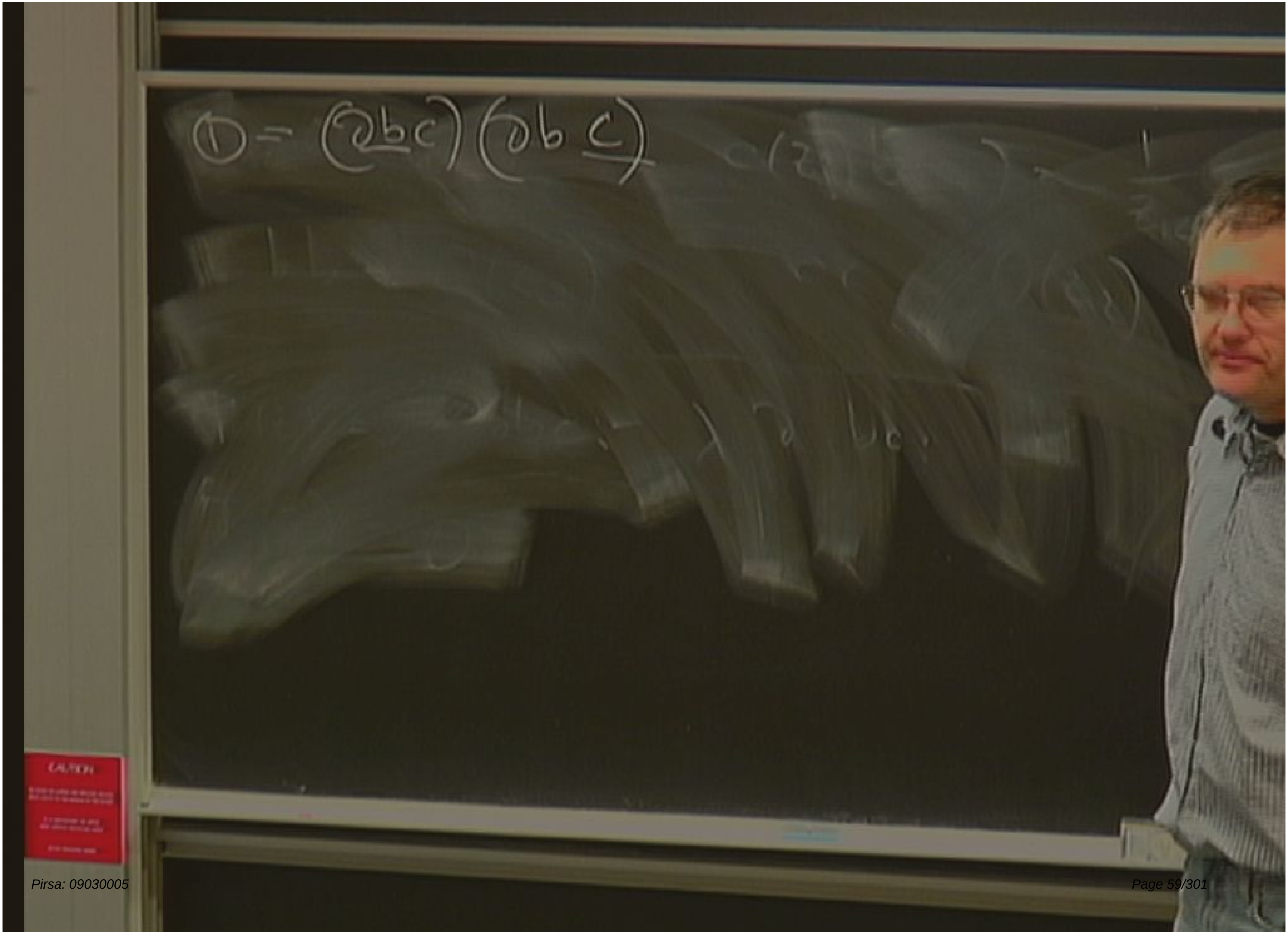
$$-2\lambda (\partial b c)(\partial b c)$$

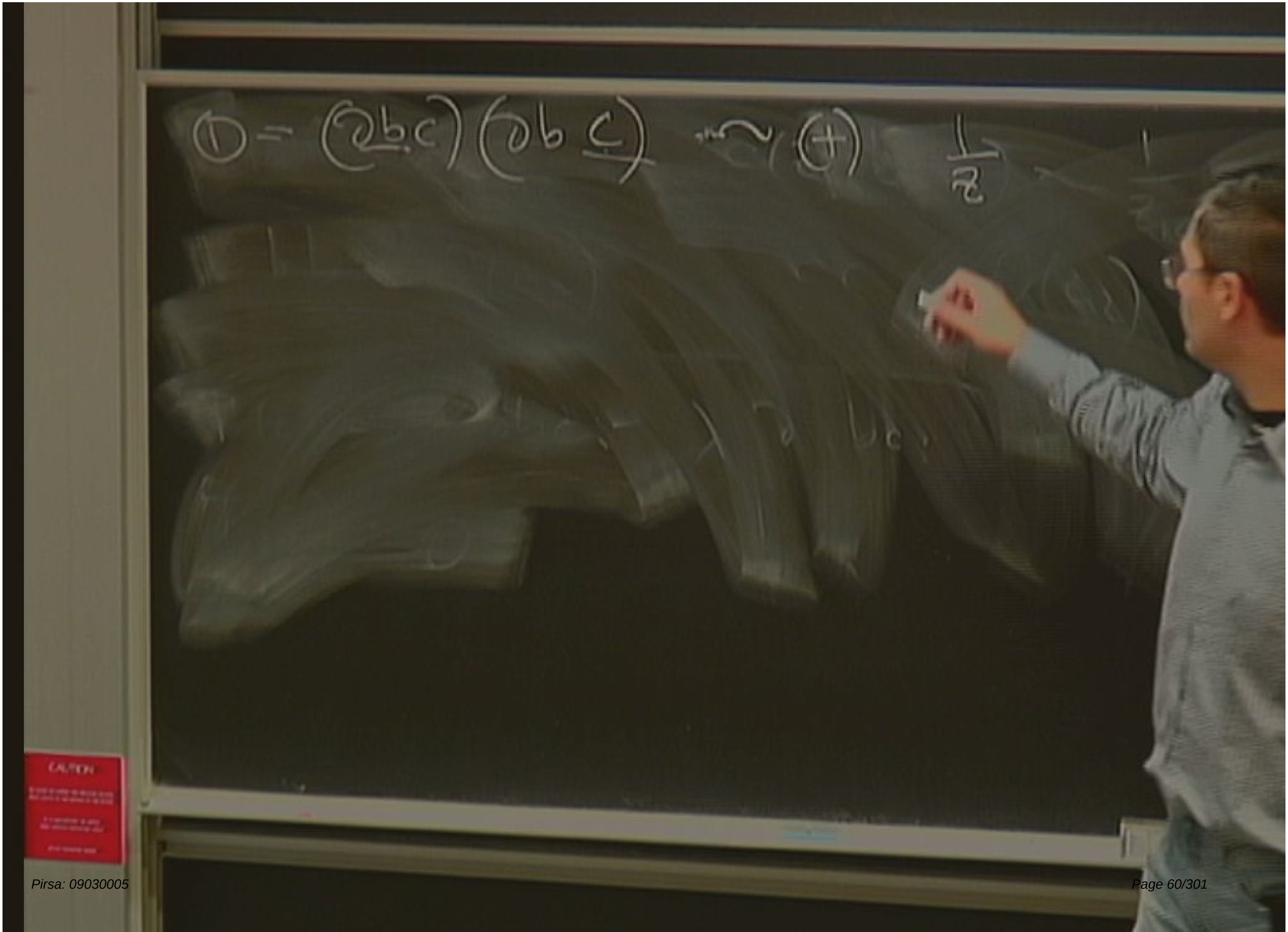
$$T'T = \underbrace{\left[\partial bc - \lambda\,\partial(bc)\right]}_{z}\underbrace{\left[\partial bc - \lambda\,\partial(bc)\right]}_{0}$$

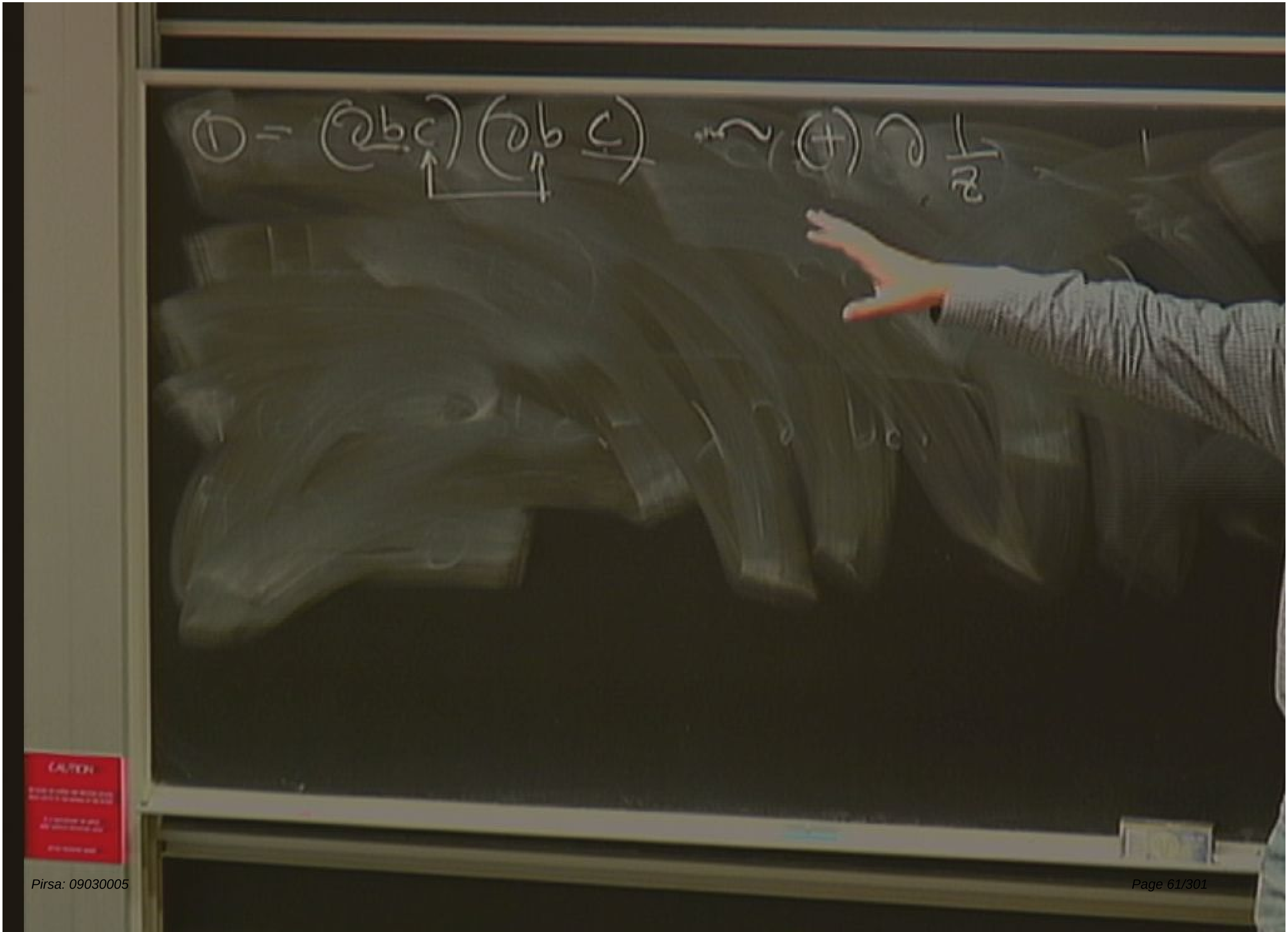
① : $(\partial bc)(\partial bc)$

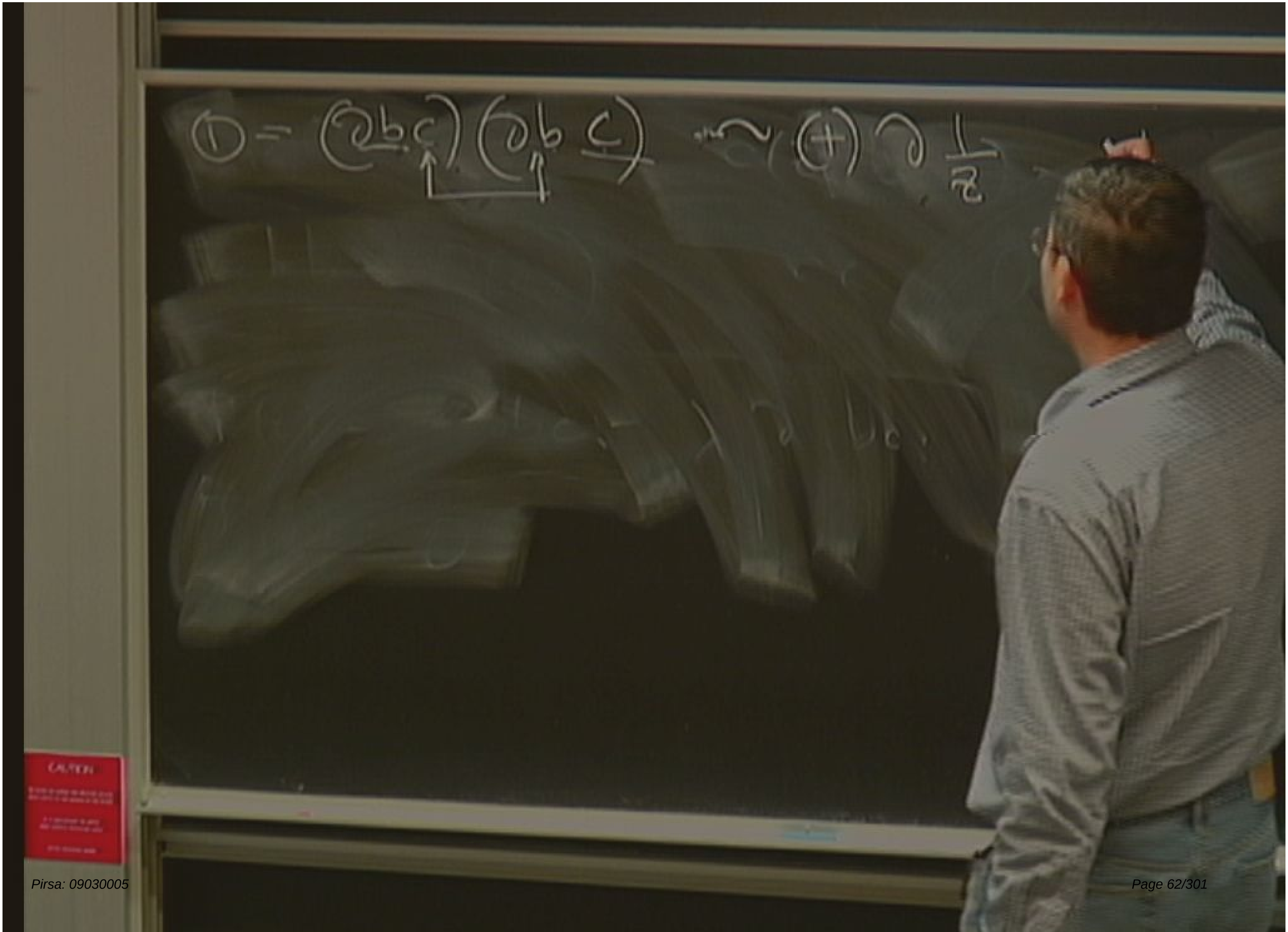
② $-2\lambda(\partial bc)(\partial bc)$

③ $\lambda^2(\partial bc)(\partial bc)$

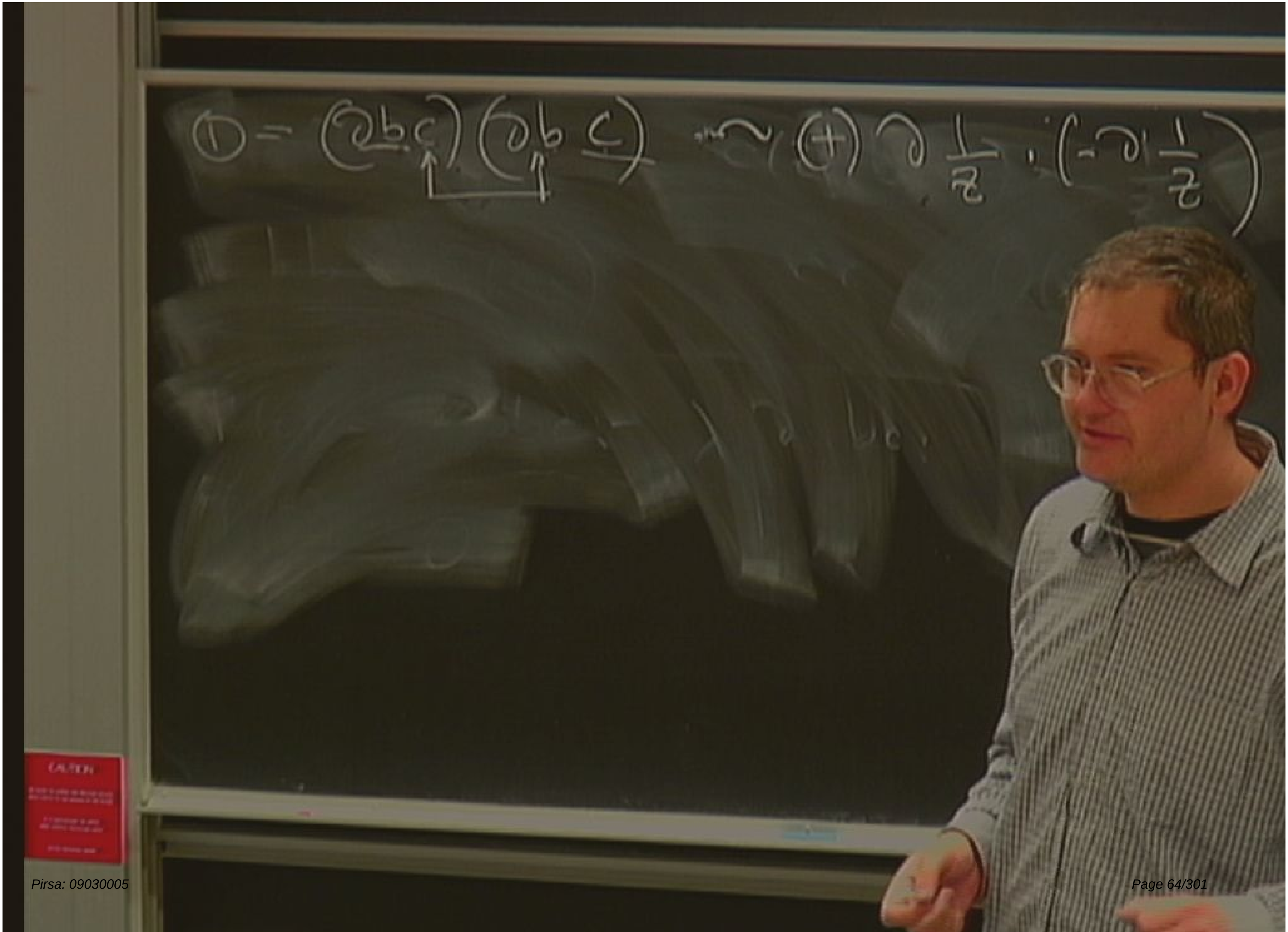


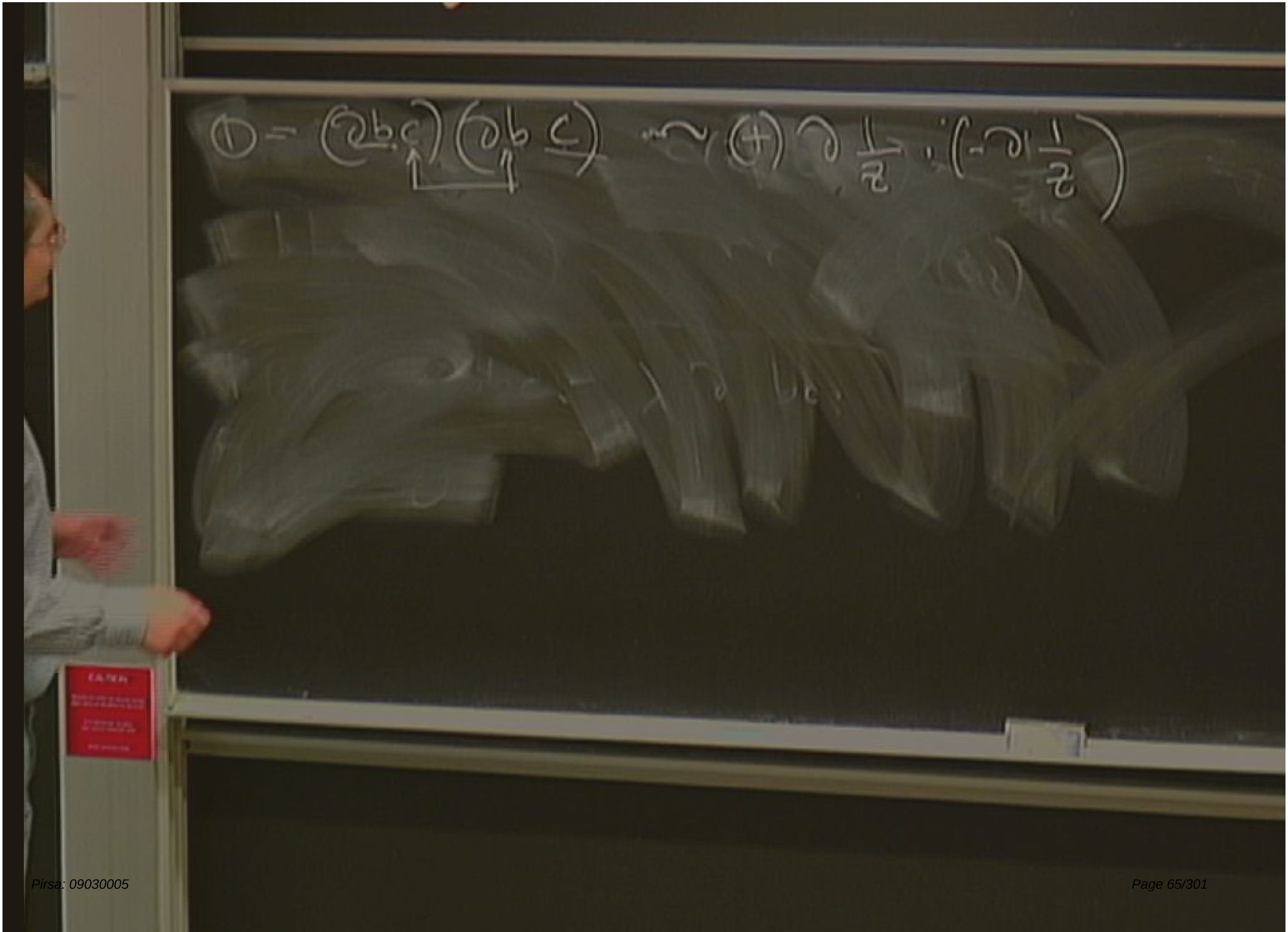


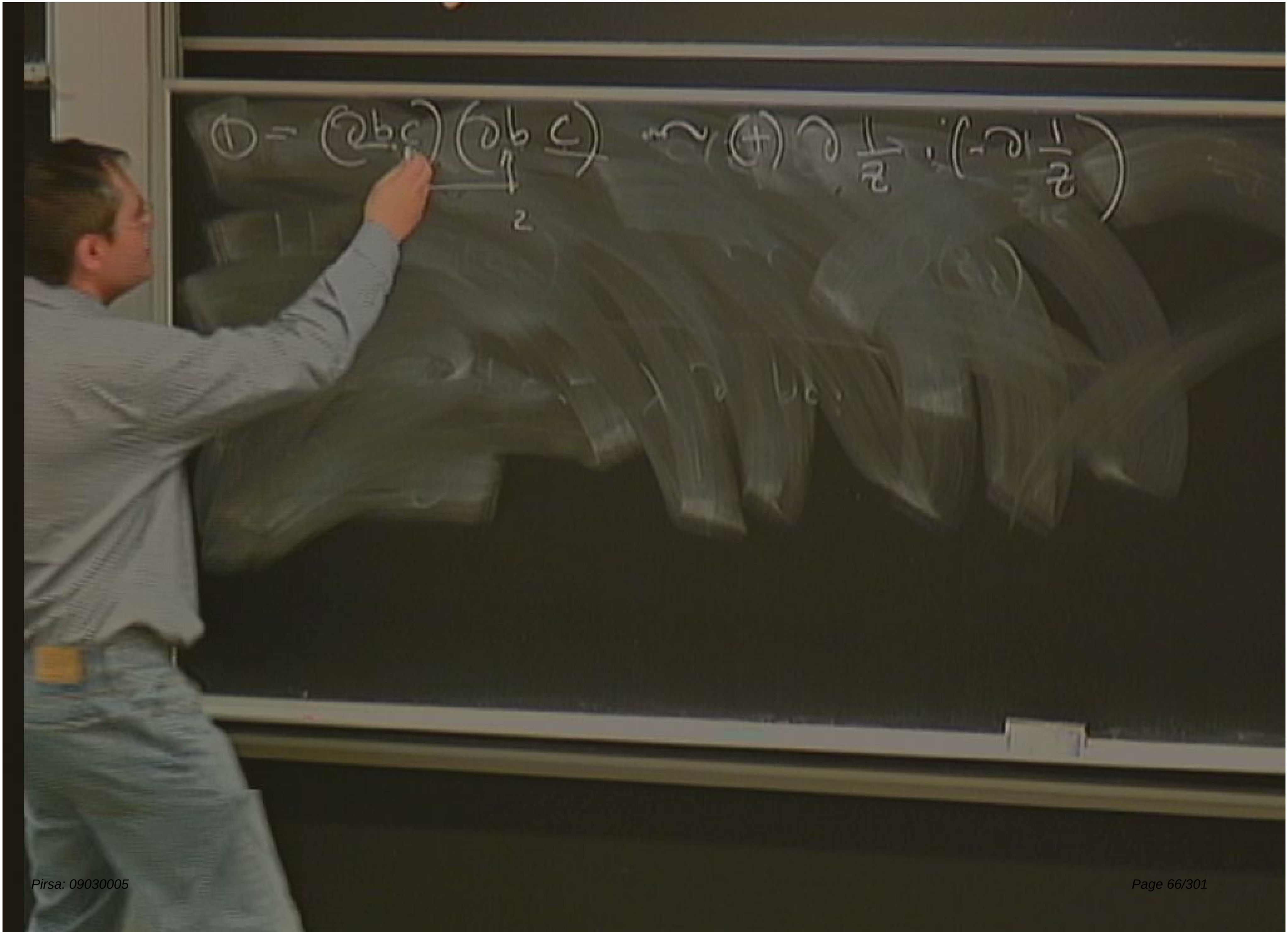

CAUTION

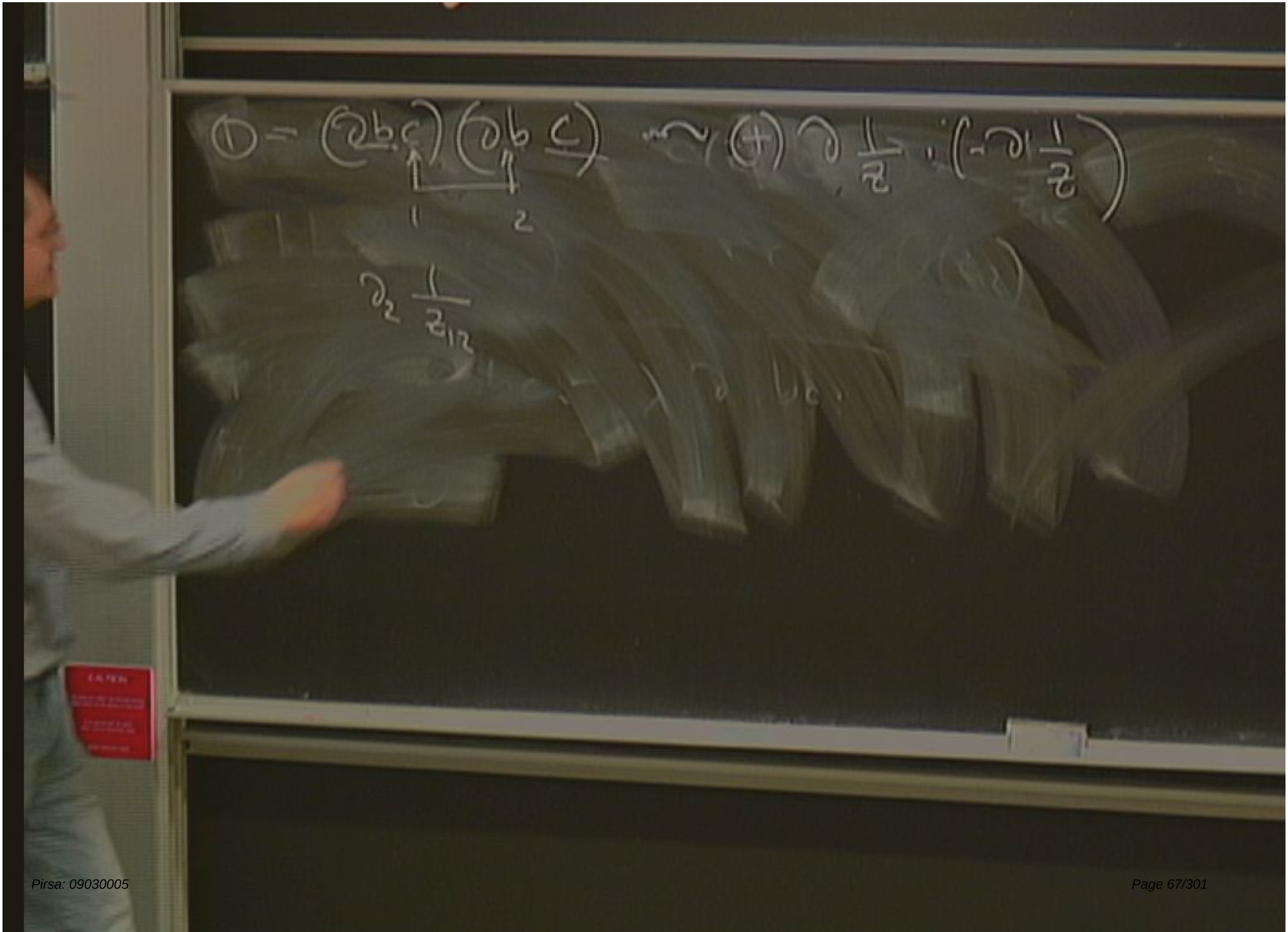


$$① = (\partial b c)(\partial b c) \sim (+)\, \partial \frac{1}{z} \cdot \left(-\partial \frac{1}{z}\right)$$


CAUTION



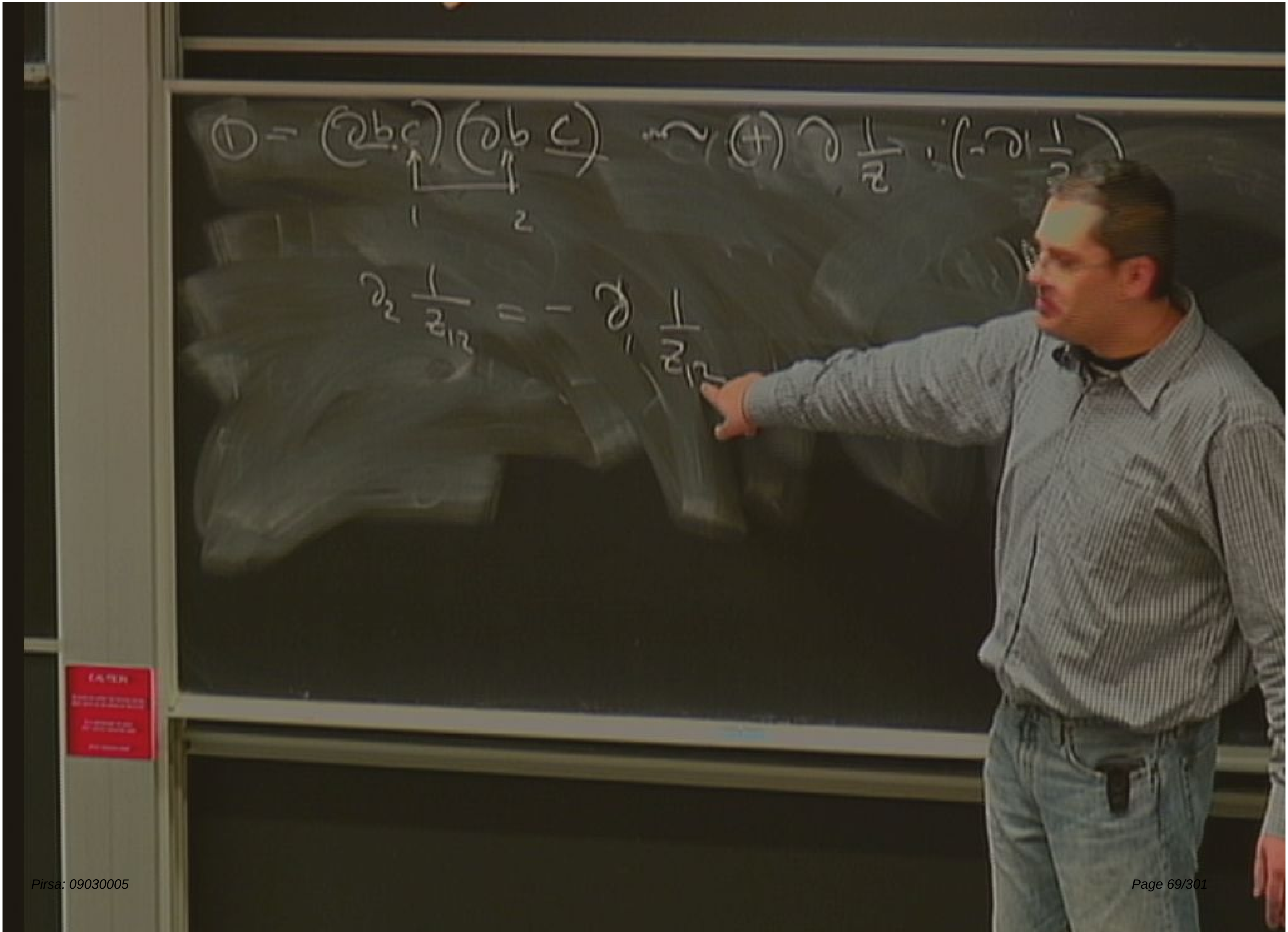



① = (∂b c)(∂b c) ~ (+) ∂ 1/z · (−∂ 1/z)
1 2
∂₂ 1/z₁₂

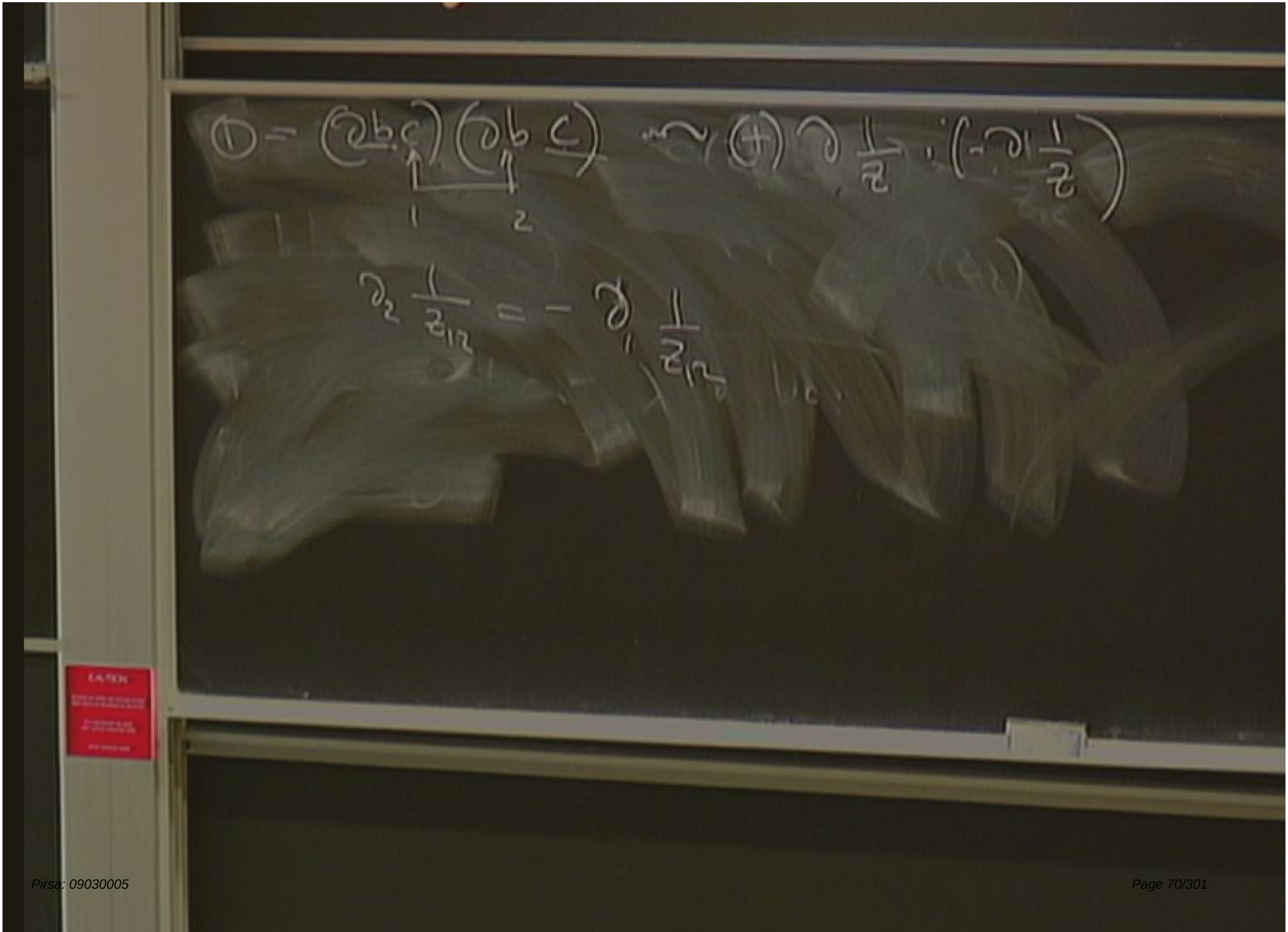
$$① = (\partial b\, c)(\partial b\, c) \sim (+)\, \partial \frac{1}{z} \cdot \left(-\partial \frac{1}{z}\right)$$

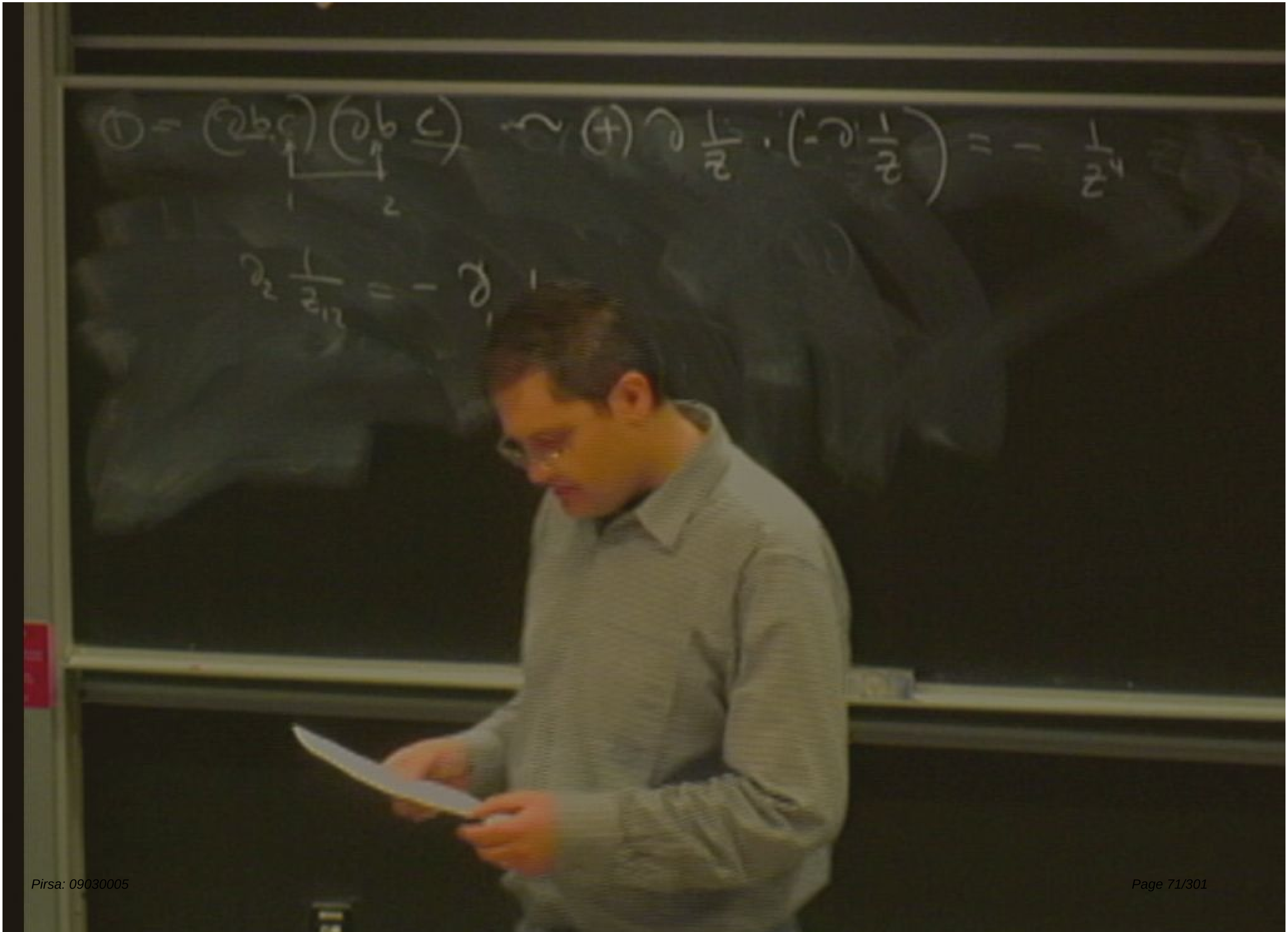
1 2

$$\partial_2 \frac{1}{z_{12}} = -\partial_1 \frac{1}{z_{12}}$$



$\partial_2 \frac{1}{z_{12}} = -\partial_1 \frac{1}{z_{12}}$


$\partial_2 \frac{1}{z_{12}} = -\partial_1 \frac{1}{z_{12}}$


① = (∂b c)(∂b c) ~ (+) ∂ 1/z · (−∂ 1/z) = − 1/z^4
∂_2 1/z_12 = − ∂_1

$$T'T = \underbrace{\left[\partial bc - \lambda\,\partial(bc)\right]}_{z}\underbrace{\left[\partial bc - \lambda\,\partial(bc)\right]}_{0}$$

① : $(\partial bc)(\partial bc)$

② $-2\lambda\,(\partial b\partial)(\partial bc)$

③ $\lambda^2\,(\partial bc)(\partial bc)$

$$① = (\partial b\, c)(\partial b\, c) \sim (+)\, \partial \frac{1}{z} \cdot \left(-\partial \frac{1}{z}\right) =$$

$$\partial_2 \frac{1}{z_{12}} = -\partial_1 \frac{1}{z_{12}}$$

$$② \; -2\lambda \left[-\frac{1}{z^4} \right]$$

$$\frac{1}{z}\partial b\,[1-\lambda]$$

$$\partial(b + z\partial b$$

$$T'T = \underbrace{\left[\partial b c - \lambda\, \partial(bc)\right]}_{z}\underbrace{\left[\partial b c - \lambda\, \partial(bc)\right]}_{0}$$

① : $(\partial b c)(\partial b c)$

② $\neq -2\lambda$ ([illegible] ∂) $(\partial b c)$

$(\partial b c)(\partial b c)$

$$① = (\partial b\, c)(\partial b\, c) \sim (+)\, \partial \frac{1}{z} \cdot \left(-\partial \frac{1}{z}\right) =$$

$$\partial_2 \frac{1}{z_{12}} = -\partial_1 \frac{1}{z_{12}}$$

$$② \; -2\lambda \left[-\frac{1}{z^4} + \frac{1}{z} \right]$$

$$= \frac{1}{z} \partial b \,[1-\lambda]$$

$$\partial b + z\, \partial b$$

$$① = (\partial b\, c)(\partial b\, c) \sim (+)\, \partial \frac{1}{z} \cdot \left(-\partial \frac{1}{z}\right) =$$

$$\partial_2 \frac{1}{z_{12}} = -\partial_1 \frac{1}{z_{12}}$$

$$② \; -2\lambda \left[-\frac{1}{z^4} + \frac{1}{z} \cdot \frac{1}{z} \right]$$

$$= \frac{1}{z} \partial b \,[1 - \lambda] \qquad \partial (b + z\, \partial b$$

$$① = (\partial b\, c)(\partial b\, c) \sim (+)\, \partial \frac{1}{z} \cdot \left(-\partial \frac{1}{z}\right) =$$

$$\partial_2 \frac{1}{z_{12}} = -\partial_1 \frac{1}{z_{12}}$$

$$② \; -2\lambda \left[-\frac{1}{z^4} + \frac{1}{z} \cdot \partial^2 \frac{1}{z} (-1) \right] = \frac{12\lambda}{2z^4}$$

(3) $-\frac{12\lambda^2}{2z^4}$

$$C = -2 + 12\lambda - 12\lambda^2 = -3(2\lambda-1)^2 + 1$$

(3) $-\frac{12\lambda^2}{2z^4}$

$$C = -2 + 12\lambda - 12\lambda^2 = -3(2\lambda-1)^2 + 1$$

$\widetilde{\Pi}^\dagger \widetilde{\Pi}$

(3) $-\frac{12\lambda^2}{2z^4}$

$$C = -2 + 12\lambda - 12\lambda^2 = -3(2\lambda-1)^2 + 1$$

$$\tilde{\pi}^\dagger \tilde{\pi} \Rightarrow \tilde{C} = 0$$

③ $-\frac{12\lambda^2}{2z^4}$

$c \neq \tilde{c}$

$c = -2 + 12\lambda - 12\lambda^2 = -3(2\lambda-1)^2 + 1$

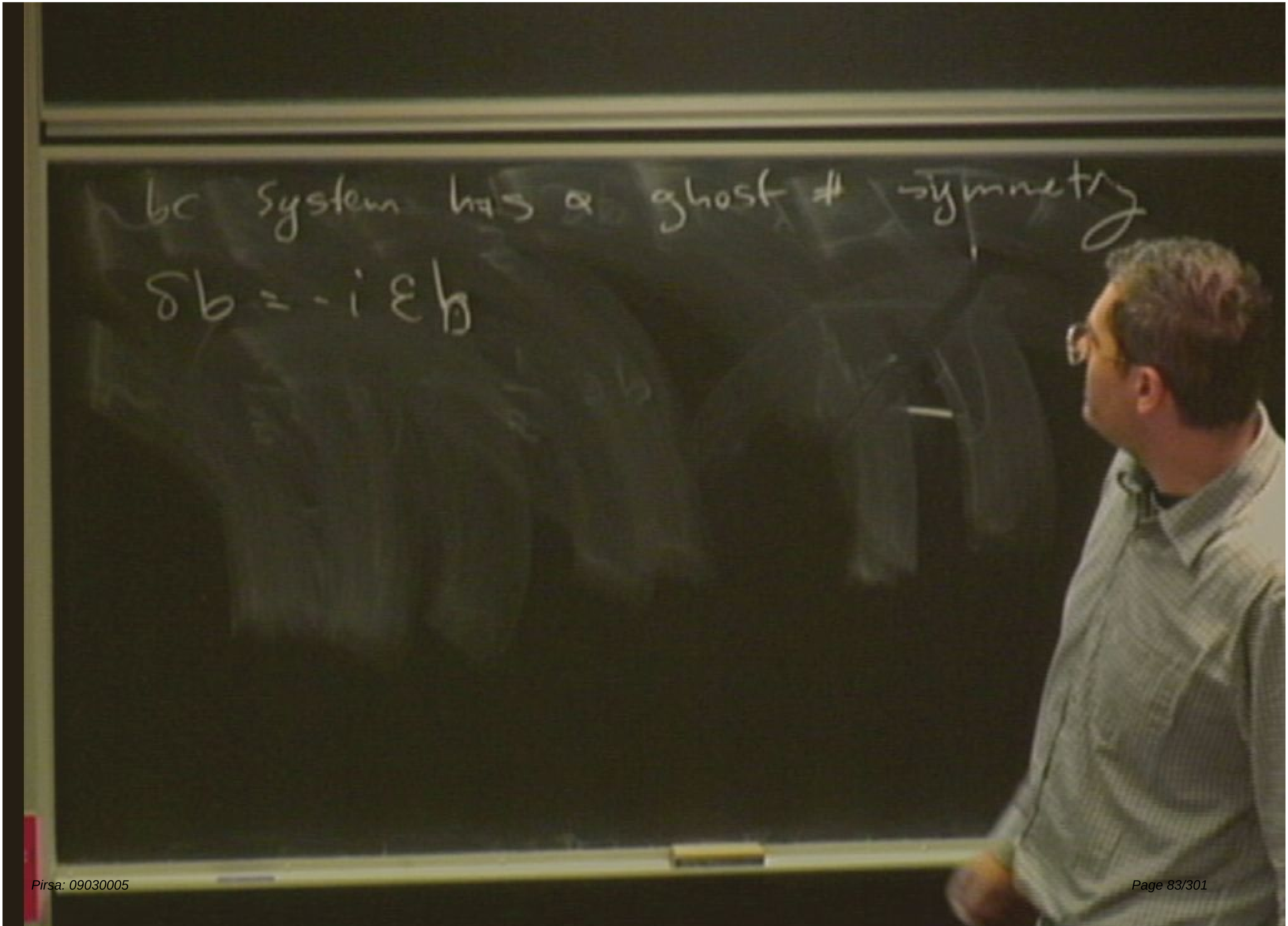
$\tilde{\pi}\tilde{\pi} \Rightarrow \tilde{c} = 0$

(3) $-\dfrac{12\lambda^2}{2z^4}$

$c \neq \tilde{c}$

$$c = -2 + 12\lambda - 12\lambda^2 = -3(2\lambda-1)^2 + 1$$

$\widetilde{\pi}\,\widetilde{\pi} \Rightarrow \tilde{c} = 0$


bc system has a ghost # symmetry
δb = -iεb

bc system has a ghost # symmetry

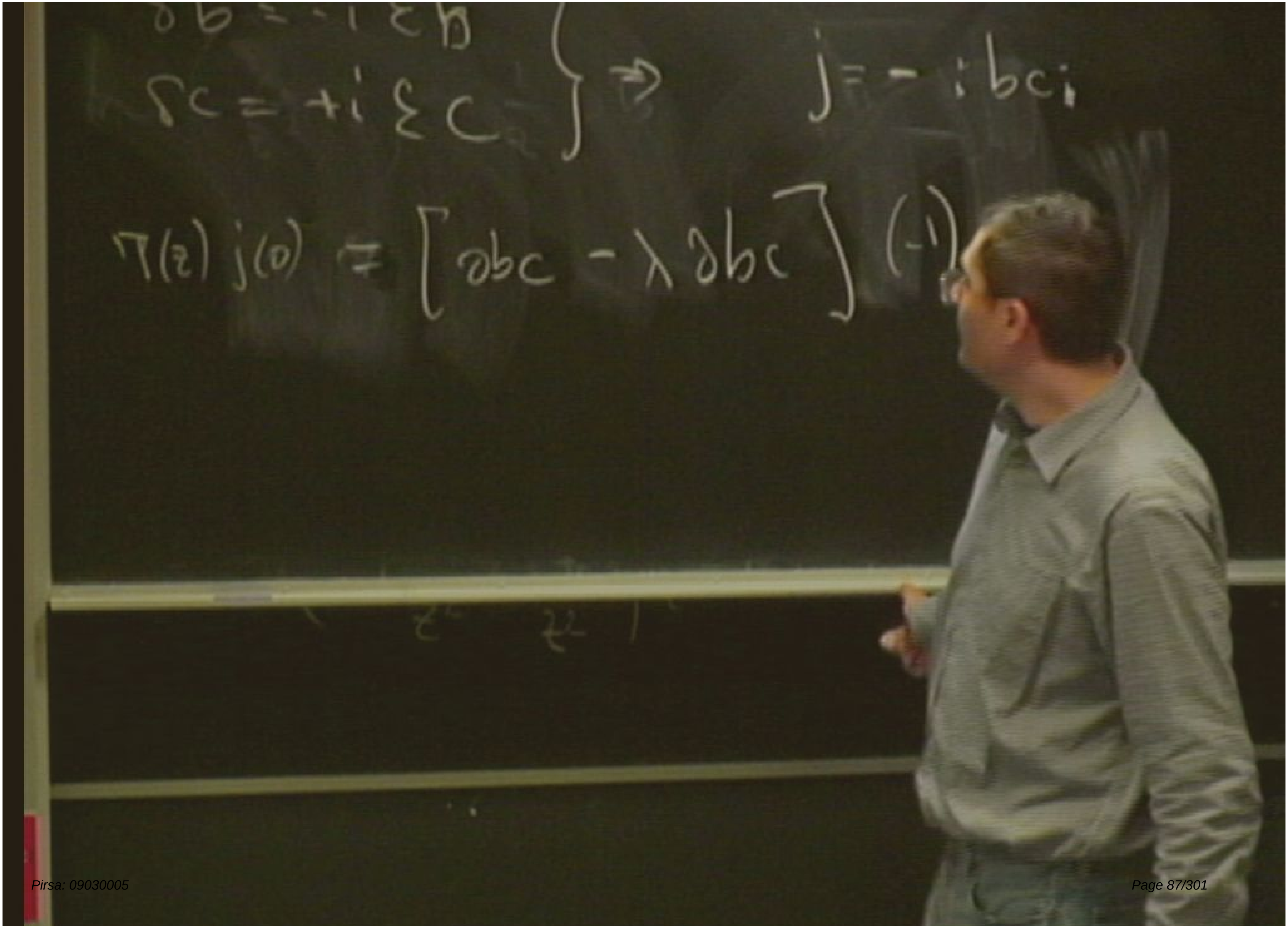
$$\left.\begin{aligned} \delta b &= -i\varepsilon b \\ \delta c &= +i\varepsilon c \end{aligned}\right\} \Rightarrow$$

bc system has a ghost # symmetry

$$\left.\begin{aligned} \delta b &= -i\varepsilon b \\ \delta c &= +i\varepsilon c \end{aligned}\right\} \Rightarrow \quad j = -i\, bc$$

bc system has a ghost # symmetry

$$\left.\begin{aligned} \delta b &= -i\varepsilon b \\ \delta c &= +i\varepsilon c \end{aligned}\right\} \Rightarrow \quad j = -i\, bc$$


δc = +i ε c
j = − i b c
T(z) j(0) ≈ [∂bc − λ ∂bc] (−1)

bc system has a ghost # symmetry

$$\left.\begin{aligned} \delta b &= -i\varepsilon b \\ \delta c &= +i\varepsilon c \end{aligned}\right\} \Rightarrow \quad j = -:bc:$$

$$T(z)\, j(0) = \left[\partial b c - \lambda \partial b c\right] (-1) :bc$$

$$\sim$$

$$z^2 \qquad z^2 \qquad \frac{(1-\lambda)}{z^2}\, c +$$

bc system has a ghost # symmetry

$$\left.\begin{aligned} \delta b &= -i\varepsilon b \\ \delta c &= +i\varepsilon c \end{aligned}\right\} \Rightarrow \quad j = -:bc:$$

$$T(z)\,j(0) = \left[\partial b c - \lambda \partial b c\right](-1):bc$$

$$\sim (-1)$$

$$\frac{}{z^2} \qquad \frac{}{z^2} \qquad \frac{(1-\lambda)}{z^2}\,c +$$

bc system has a ghost # symmetry

$$\left.\begin{aligned}\delta b &= -i\varepsilon b\\ \delta c &= +i\varepsilon c\end{aligned}\right\} \Rightarrow \quad j = -:bc:$$

$$T(z)\,j(0) = \left[\partial b\, c - \lambda\, \partial b\, c\right](-1)\,:bc:$$

$$\sim (-1)\left[\partial \frac{1}{z}\cdot\frac{1}{z} - \lambda\right.$$

$$\frac{\quad}{z^2} \quad \frac{\quad}{z^2} \qquad \frac{(1-\lambda)}{z^2}\, c +$$

bc system has a ghost # symmetry

$$\left.\begin{array}{l}\delta b = -i\varepsilon b \\ \delta c = +i\varepsilon c\end{array}\right\} \Rightarrow \quad j = -:bc:$$

$$T(z)\, j(0) = \left[\partial b c - \lambda \partial(bc)\right] (-1) :bc$$

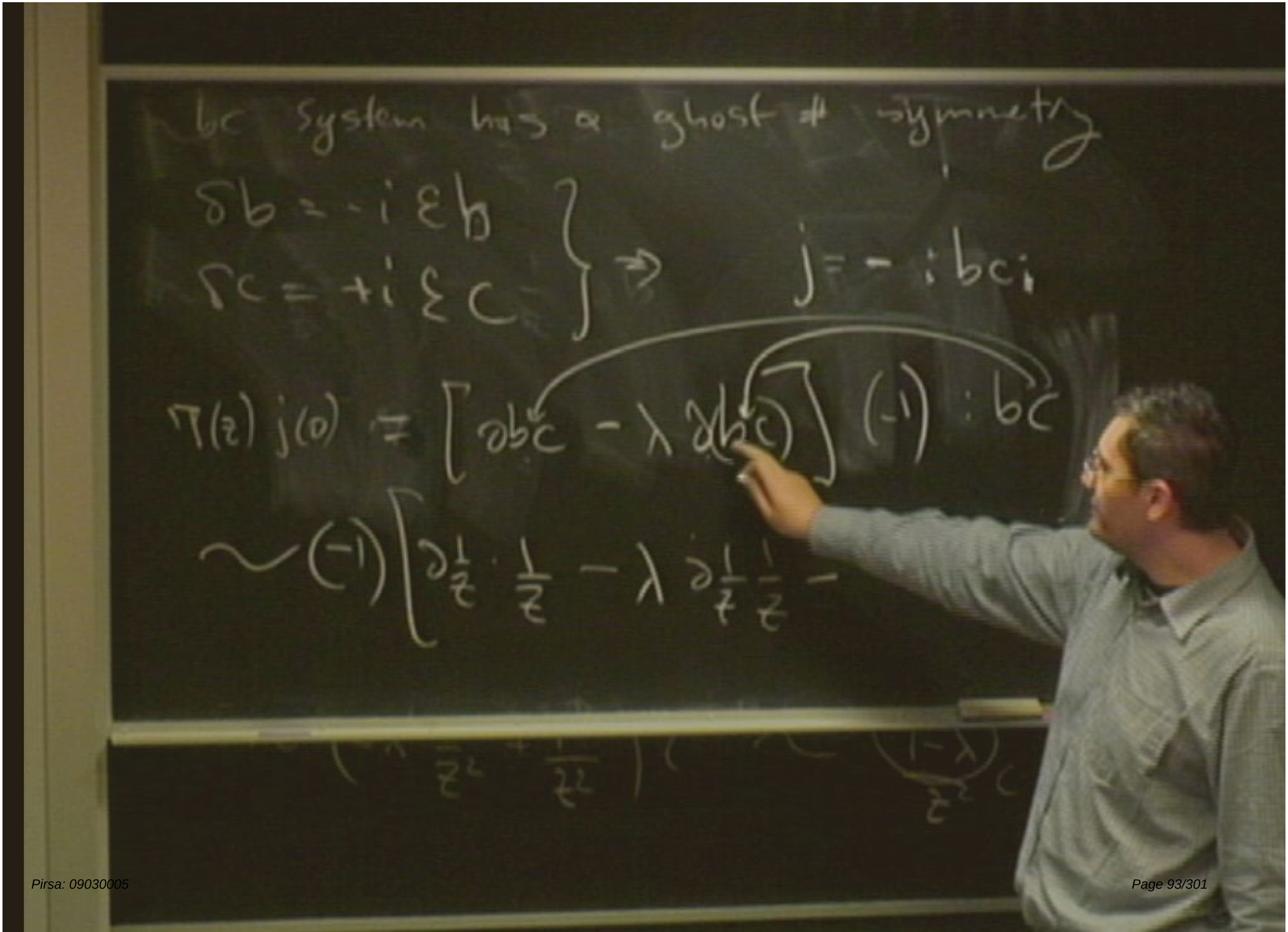
$$\sim (-1)\left[\partial \frac{1}{z} \cdot \frac{1}{z} - \lambda\right.$$

bc system has a ghost # symmetry

$$\left.\begin{array}{l}\delta b = -i\varepsilon b\\ \delta c = +i\varepsilon c\end{array}\right\} \Rightarrow \quad j = -:bc:$$

$$T(z)\, j(0) = \left[\partial b c - \lambda\, \partial(bc)\right] (-1)\, :bc:$$

$$\sim (-1)\left[\partial \frac{1}{z} \cdot \frac{1}{z} - \lambda\, \partial \frac{1}{z} \frac{1}{z}\right.$$


bc system has a ghost # symmetry
δb = -iεb
δc = +iεc
j = - :bc:
T(z) j(0) = [∂bc - λ ∂(bc)] (-1) :bc:
~ (-1)[∂1/z · 1/z - λ ∂1/z 1/z -

bc system has a ghost # symmetry

$$\left.\begin{aligned}\delta b &= -i\varepsilon b \\ \delta c &= +i\varepsilon c\end{aligned}\right\} \Rightarrow \quad j = -:bc:$$

$$T(z)\,j(0) = \left[\partial b c - \lambda\,\partial(bc)\right](-1) :bc$$

$$\sim (-1)\left[\partial\frac{1}{z}\cdot\frac{1}{z} - \lambda\,\partial\frac{1}{z}\frac{1}{z} - \lambda\frac{1}{z}\,\partial\frac{1}{z}\right]$$

$$\left(\ldots\frac{}{z^2} + \frac{1}{z^2}\right)\left(\ldots\lambda c \ldots (1-\lambda)\frac{}{z^2}\,c + \ldots\right)$$

bc system has a ghost # symmetry

$$\left.\begin{aligned} \delta b &= -i\varepsilon b \\ \delta c &= +i\varepsilon c \end{aligned}\right\} \Rightarrow \quad j = -:bc:$$

$$T(z)\, j(0) = \left[\partial b c - \lambda\, \partial(bc)\right] (-1) :bc:$$

$$\sim (-1)\left[\partial \frac{1}{z} \cdot \frac{1}{z} - \lambda\, \partial\frac{1}{z}\frac{1}{z} - \lambda \frac{1}{z}\, \partial \frac{1}{z}\right]$$

bc system has a ghost # symmetry

$$\left.\begin{aligned}\delta b &= -i\varepsilon b\\ \delta c &= +i\varepsilon c\end{aligned}\right\} \Rightarrow \quad j = -:bc:$$

$$T(z)\,j(0) = \left[\partial b c - \lambda\,\partial(bc)\right](-1):bc:$$

$$\sim(-1)\left[\partial\frac{1}{z}\cdot\frac{1}{z} - \lambda\,\partial\frac{1}{z}\frac{1}{z} - \lambda\frac{1}{z}\,\partial\frac{1}{z}\right] \sim \frac{1-2\lambda}{z^3}$$

$$\frac{}{z^2} + \frac{1}{z^2} \qquad \frac{(1-\lambda)}{z^2}\,c +$$

bc system has a g...

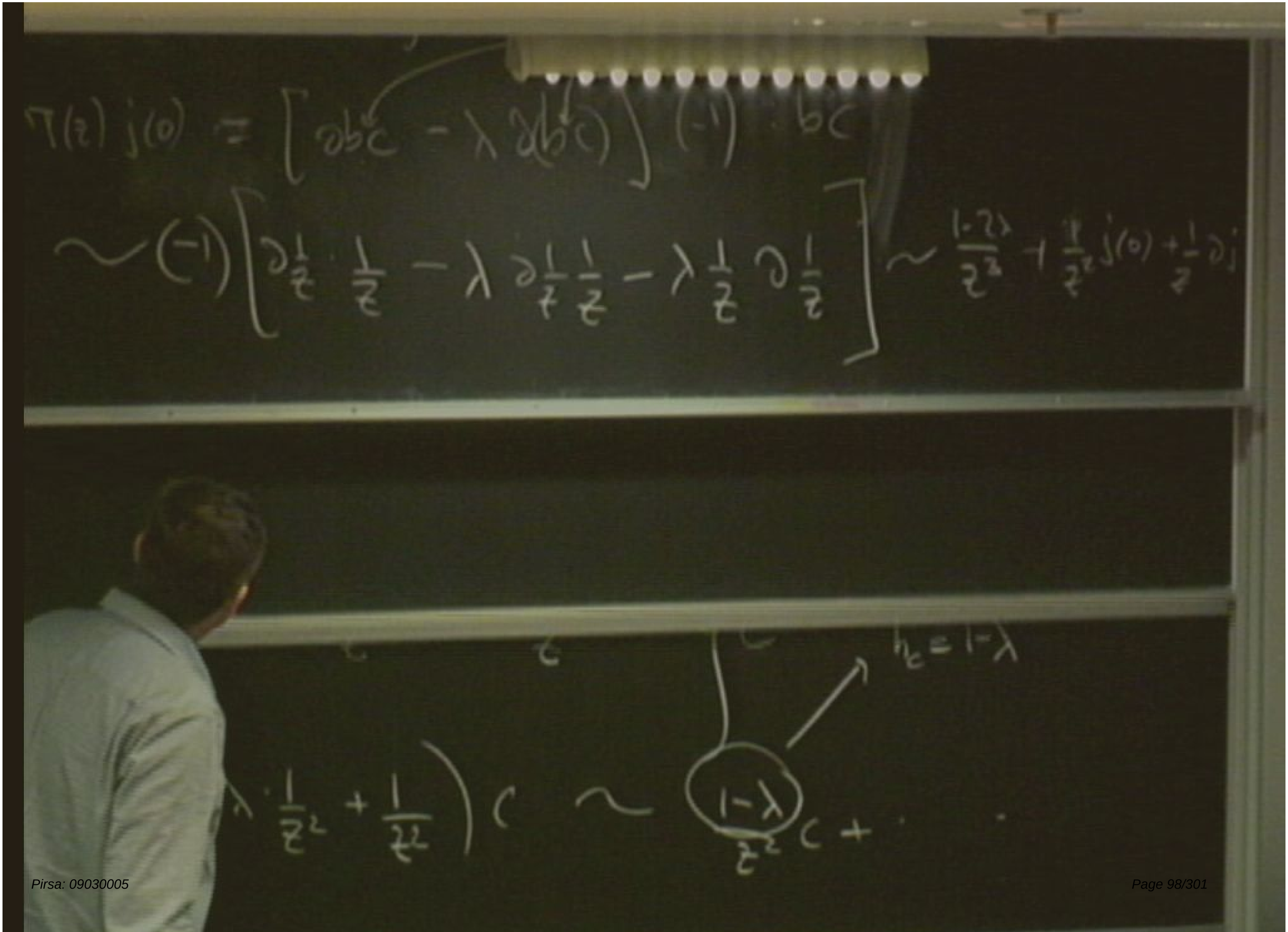
$$\left.\begin{aligned}\delta b &= -i\varepsilon b\\ \delta c &= +i\varepsilon c\end{aligned}\right\} \Rightarrow \quad j = -:bc:$$

$$T(z)\,j(0) = \left[\partial b c - \lambda\,\partial(bc)\right](-1)\,:bc$$

$$\sim(-1)\left[\partial\frac{1}{z}\cdot\frac{1}{z} - \lambda\,\partial\frac{1}{z}\frac{1}{z} - \lambda\frac{1}{z}\,\partial\frac{1}{z}\right] \sim \frac{1-2\lambda}{z^3} + \frac{1}{z^2}j(0) + \frac{1}{z}\partial j$$

$$\sim\left(-\lambda\cdot\frac{1}{z^2} + \frac{1}{z^2}\right)c \sim \frac{1-\lambda}{z^2}c + \cdots$$

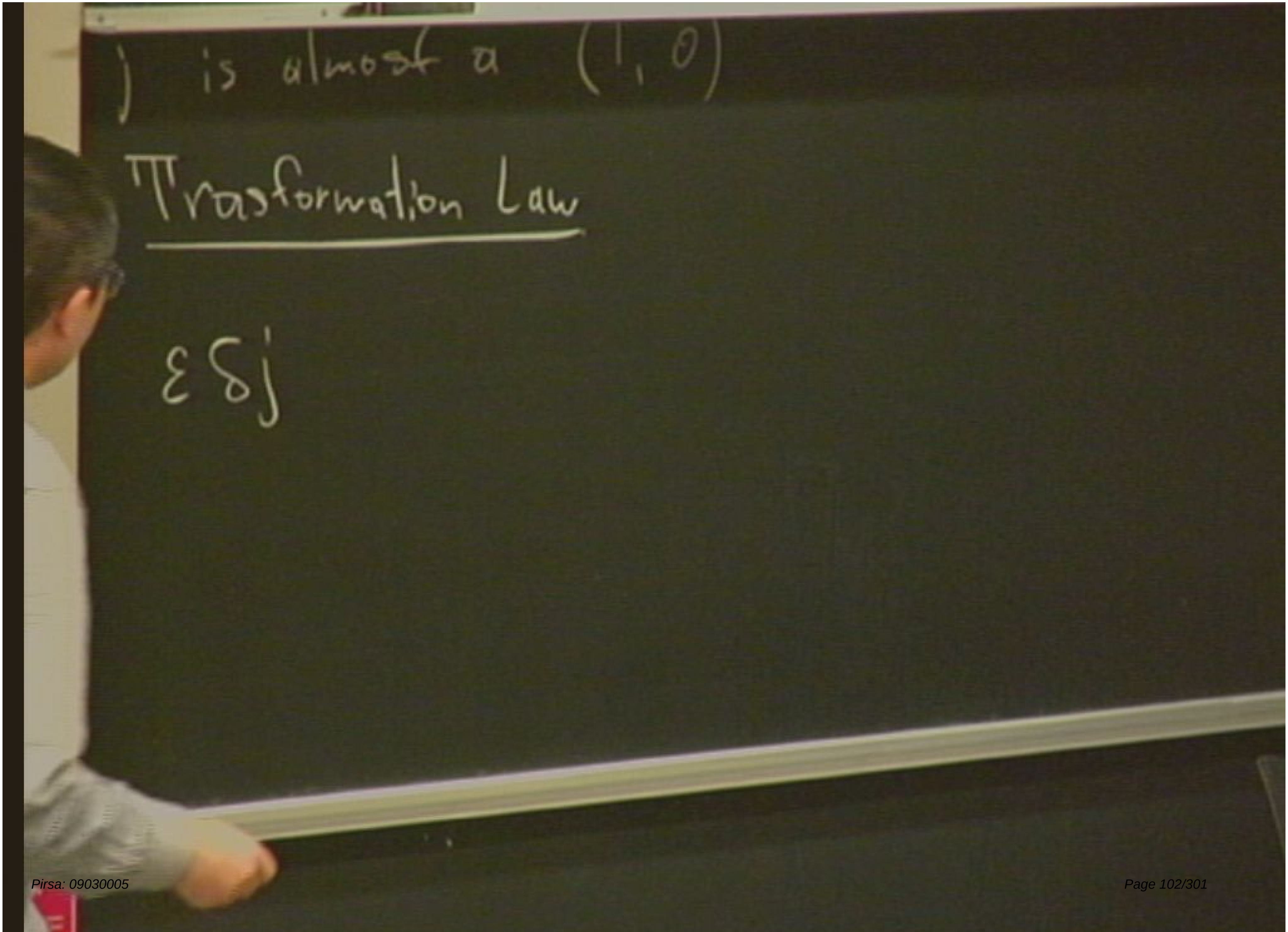
$h_c = 1-\lambda$


T(z) j(0) = [∂bc − λ ∂(bc)] (−1) : bc
~ (−1)[∂1/z 1/z − λ ∂1/z 1/z − λ 1/z ∂1/z] ~ (1−2λ)/z³ + 1/z² j(0) + 1/z ∂j
h_c = 1−λ
λ 1/z² + 1/z²) c ~ (1−λ)/z² c +

$$\partial \frac{1}{z} \Big] \sim \frac{1-2\lambda}{z^3} + \frac{1}{z^2} j(0) + \frac{1}{z} \partial j$$

) is almost a $(1,0)$

j is almost a $(1, 0)$


) is almost a (1, 0)
Trasformation Law
ε δj

j is almost a (1,0)

Trasformation Law

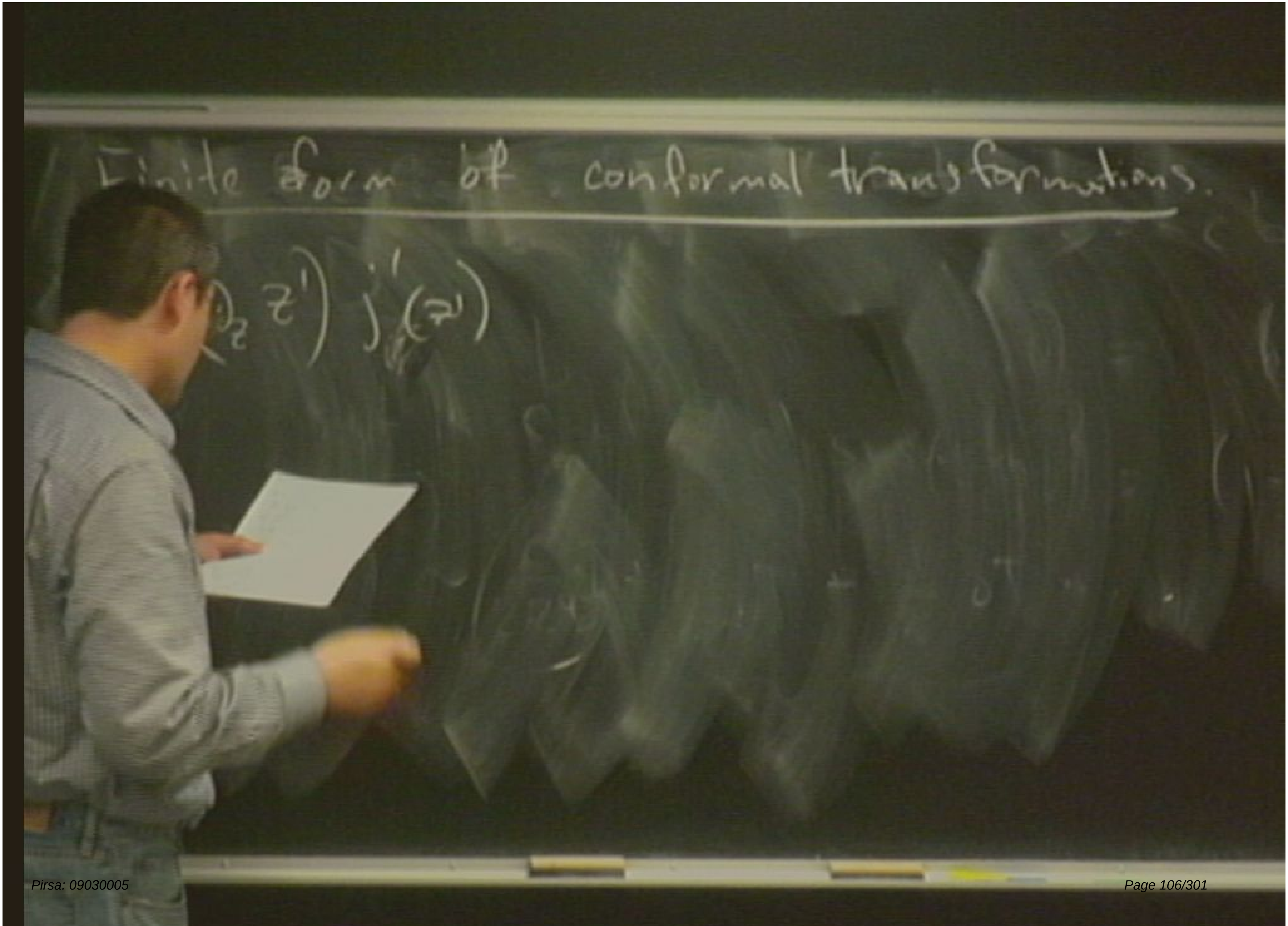
$$\varepsilon\,\delta j = -v\,\partial j - j\,\partial v$$

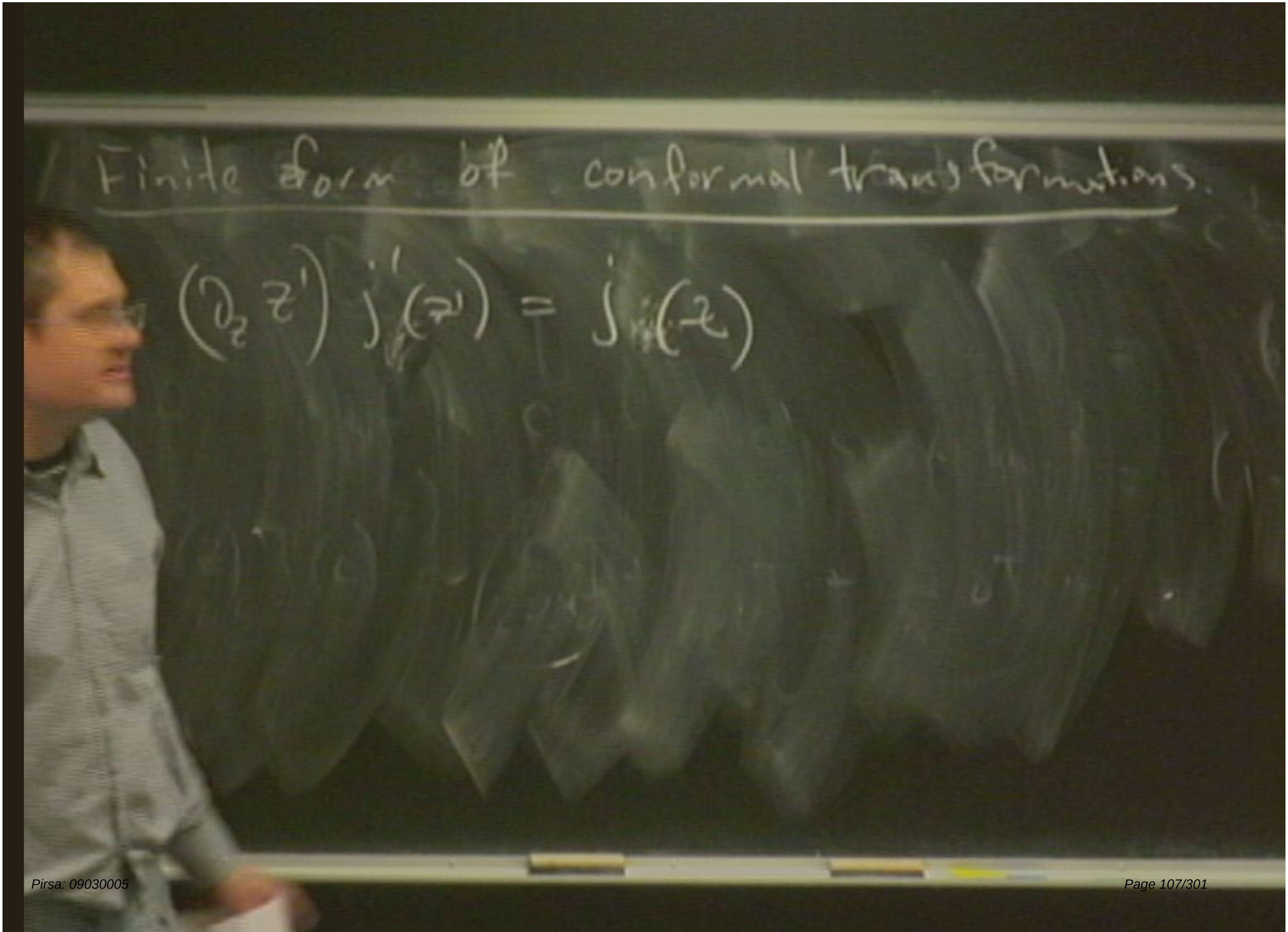
j is almost a $(1,0)$

Transformation Law

$$\varepsilon\,\delta j = -v\,\partial j - j\,\partial v + \frac{2\lambda-1}{2}\,\partial^2 v$$

Finite form of conformal transformations.


Finite form of conformal transformations.
$\partial_z z')\, j'(z')$


Finite form of conformal transformations.
$(\partial_z z') \, j'(z') = j(z)$

Finite form of conformal transformations

$$(\partial_z z')\, j'(z') = j(z) + \frac{2\lambda - 1}{2} \frac{\partial_z^2 z'}{\partial_z z'}$$

Finite form of conformal transformations.

$$(\partial_z z')\, j'(z') = j(z) + \frac{2\lambda - 1}{2}\, \frac{\partial_z^2 z'}{\partial_z z'}$$

$\Rightarrow$ What that system is good for?

Finite form of conformal transformations.

$$(\partial_z z')\, j'(z') = j(z) + \frac{2\lambda - 1}{2}\, \frac{\partial_z^2 z'}{\partial_z z'}$$

$\Rightarrow$ What that system is good for?

Finite form of conformal transformations.

$$(\partial_z z')\, j'(z') = j(z) + \frac{2\lambda - 1}{2} \frac{\partial_z^2 z'}{\partial_z z'}$$

$\Rightarrow$ What that system is good for?

$\cdot D$

Finite form of conformal transformations.

$$(\partial_z z')\, j'(z') = j(z) + \frac{2\lambda - 1}{2}\, \frac{\partial_z^2 z'}{\partial_z z'}$$

$\Rightarrow$ What that system is good for?

Global

Finite form of conformal transformations.

$$(\partial_z z')\, j'(z') = j(z) + \frac{2\lambda - 1}{2}\,\frac{\partial_z^2 z'}{\partial_z z'}$$

⇒ What that system is good for?

Central:

Finite form of conformal transformations.

$$(\partial_z z')\, j'(z') = j(z) + \frac{2\lambda-1}{2} \frac{\partial_z^2 z'}{\partial_z z'}$$

What that system is good for?

Finite form of conformal transformations.

$$(\partial_z z')\, j'(z') = j(z) + \frac{2\lambda - 1}{2} \frac{\partial_z^2 z'}{\partial_z z'}$$

$\Rightarrow$ what that system is good for?

$\boxed{c_{bc}(y) = 0}$

Finite form of conformal transformations.

$$(\partial_z z')\, j'(z') = j(z) + \frac{2\lambda-1}{2}\,\frac{\partial_z^2 z'}{\partial_z z'}$$

$\Rightarrow$ what that system is good for?

$\boxed{c_{total} = 0}$

D

$\uparrow$

26

$c =$

Finite form of conformal transformations.

$$(\partial_z z')\, j'(z') = j(z) + \frac{2\lambda-1}{2}\,\frac{\partial_z^2 z'}{\partial_z z'}$$

$\Rightarrow$ what that system is good for?

$\boxed{c_{total} = 0}$

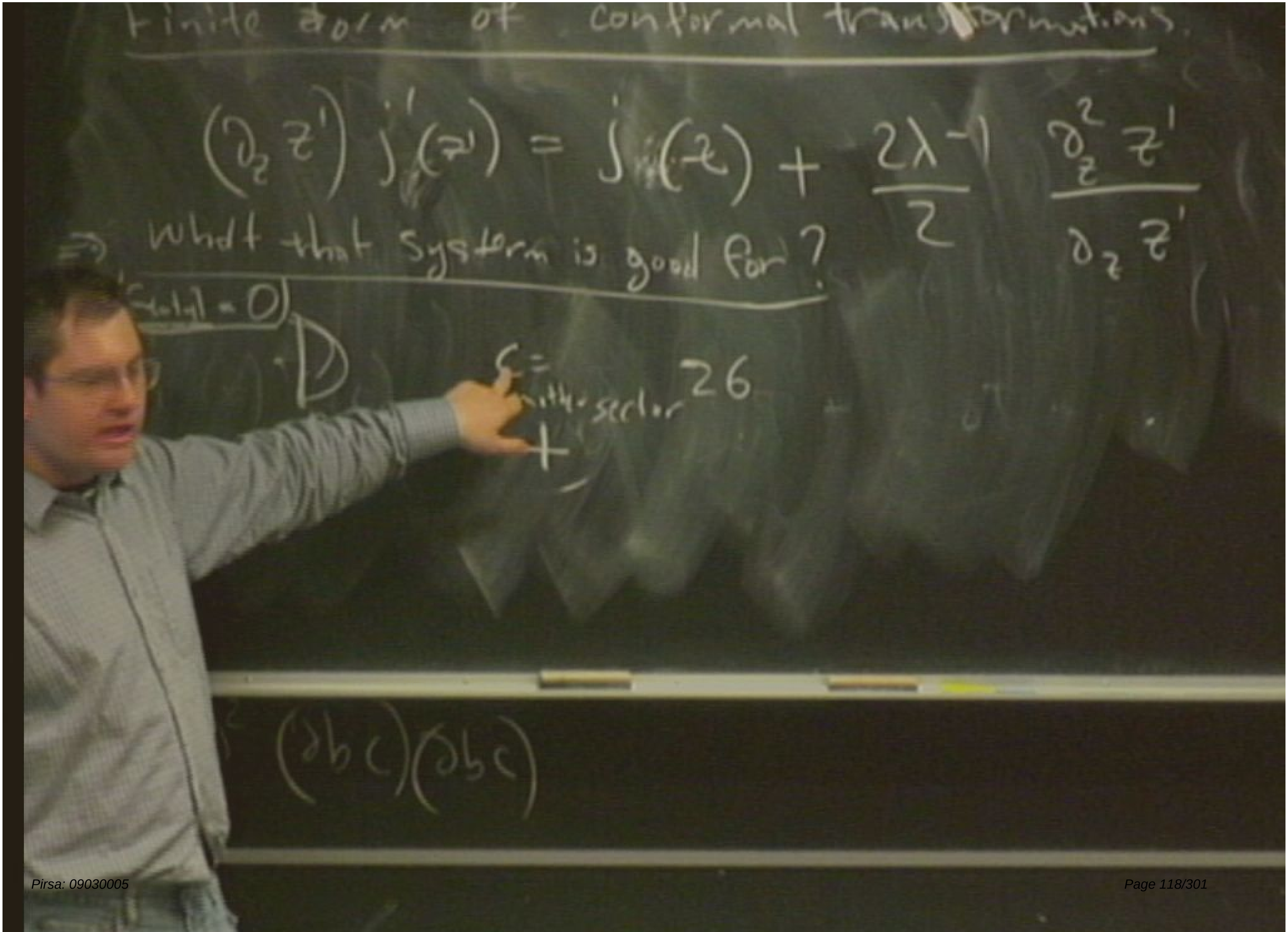
D

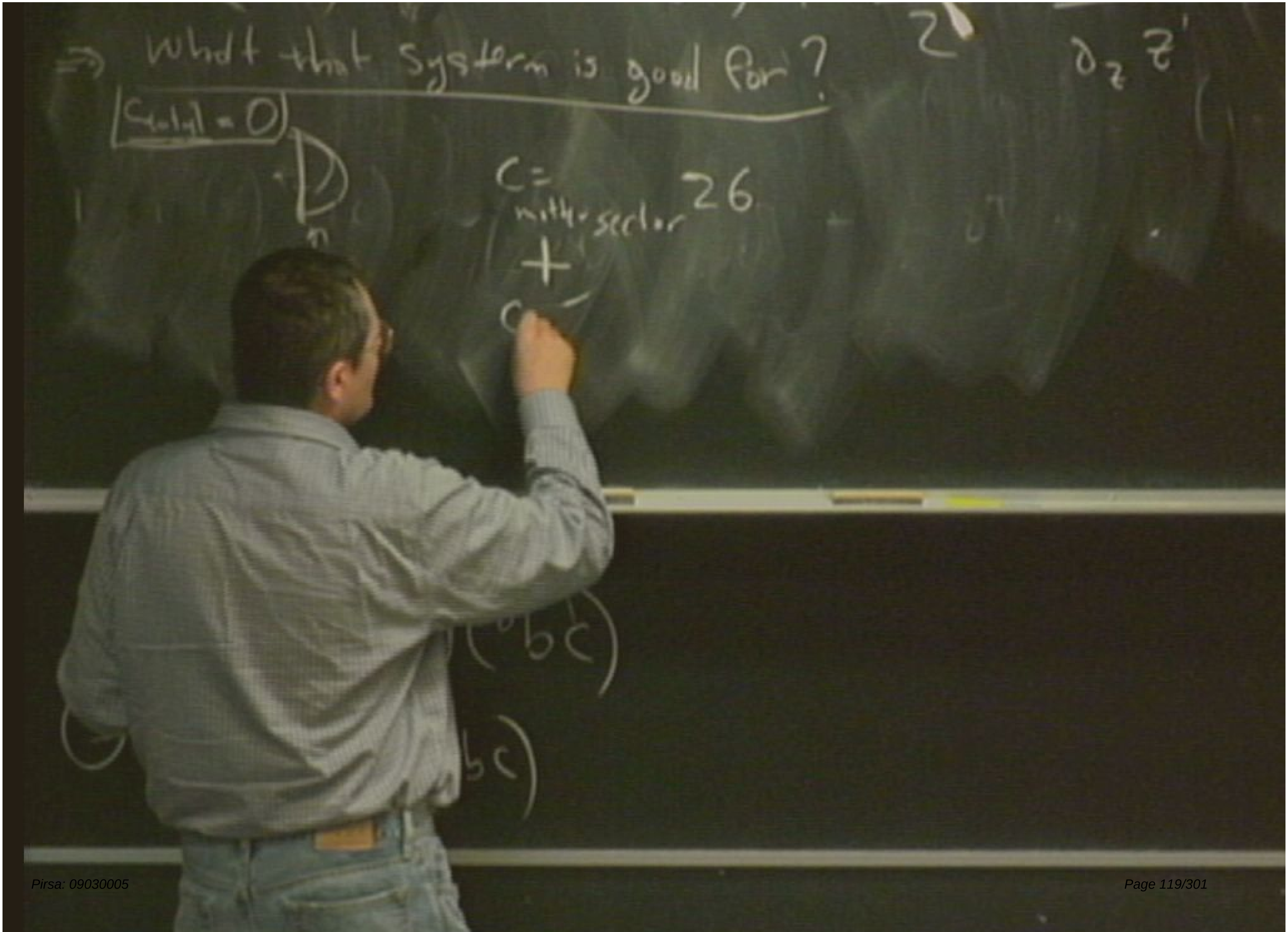
$\uparrow$

26

$c_{\text{matter sector}} = 26$

$\lambda\ (\partial b c)(\partial b c)$


Finite form of conformal transformations.
$(\partial_z z')\, j'(z') = j(z) + \frac{2\lambda-1}{2} \frac{\partial_z^2 z'}{\partial_z z'}$
What that system is good for?
$c =$ 26
matter sector
+
$(\partial b c)(\partial b c)$


What that system is good for ?
c=
26
matter sector
+

$$(\partial_z z')\, j'(z') = j(z) + \frac{2\lambda-1}{2}\,\frac{\partial_z^2 z'}{\partial_z z'}$$

$\Rightarrow$ What that system is good for?

$\boxed{c_{total} = 0}$

D

$\uparrow$

26

$c_{\text{matter sector}} = 26$

$+$

c_{ghost} — 'bc' with $\lambda = 2$

$\neq -2\lambda\,(\partial b)(\partial b c)$

$\lambda^2\,(\partial b c)(\partial b c)$

Finite form of conformal transformations.

$$(\partial_z z')\, j'(z') = j(z) + \frac{2\lambda - 1}{2}\,\frac{\partial_z^2 z'}{\partial_z z'}$$

$\Rightarrow$ What that system is good for?

$\boxed{c_{total} = 0}$

D

$\uparrow$

26

$c_{\text{matter sector}} = 26$

$+$

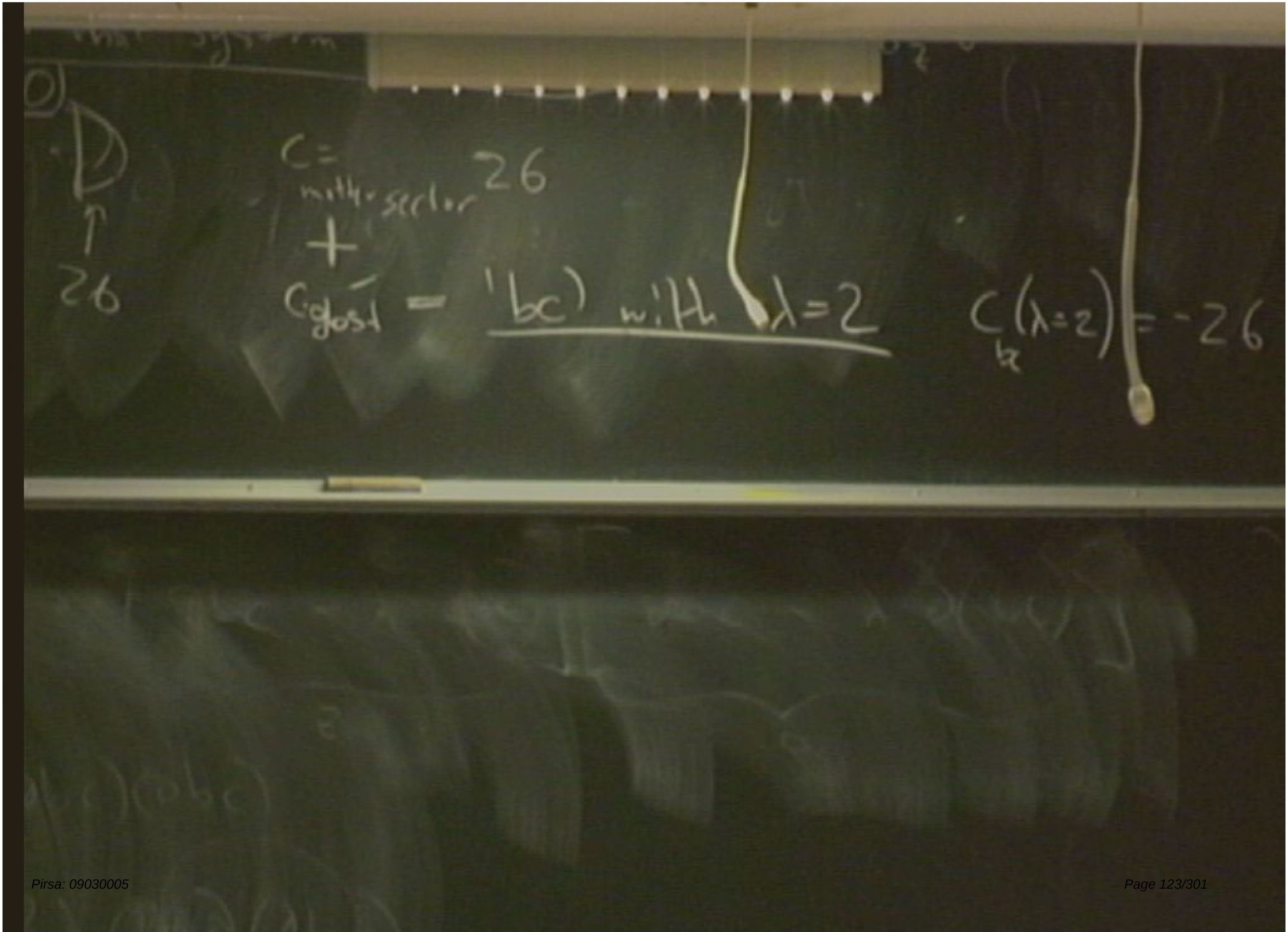
c_{ghost} — 'bc' with $\lambda = 2$

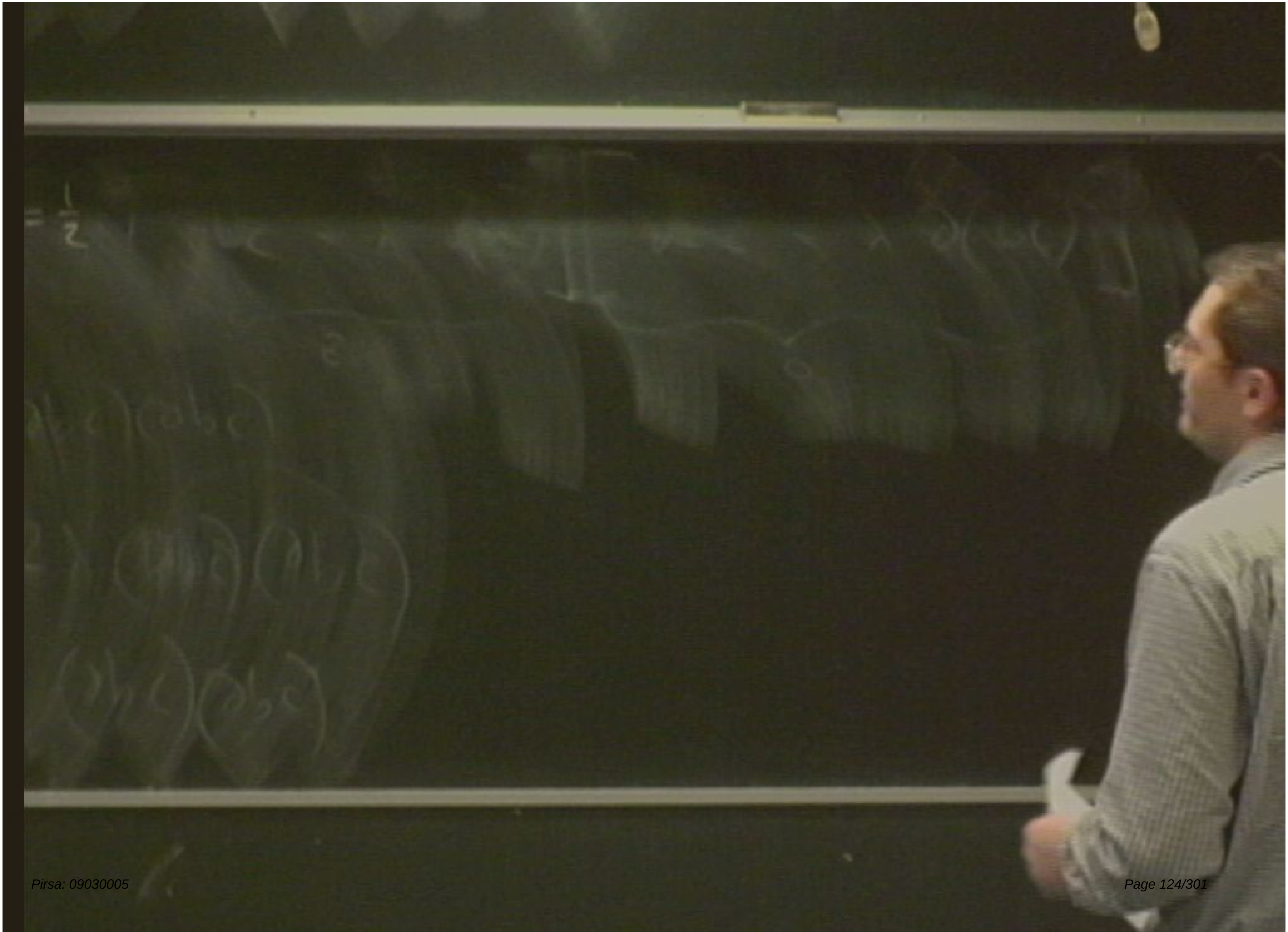
$\neq -2\lambda\, (bc)(\partial b c)$

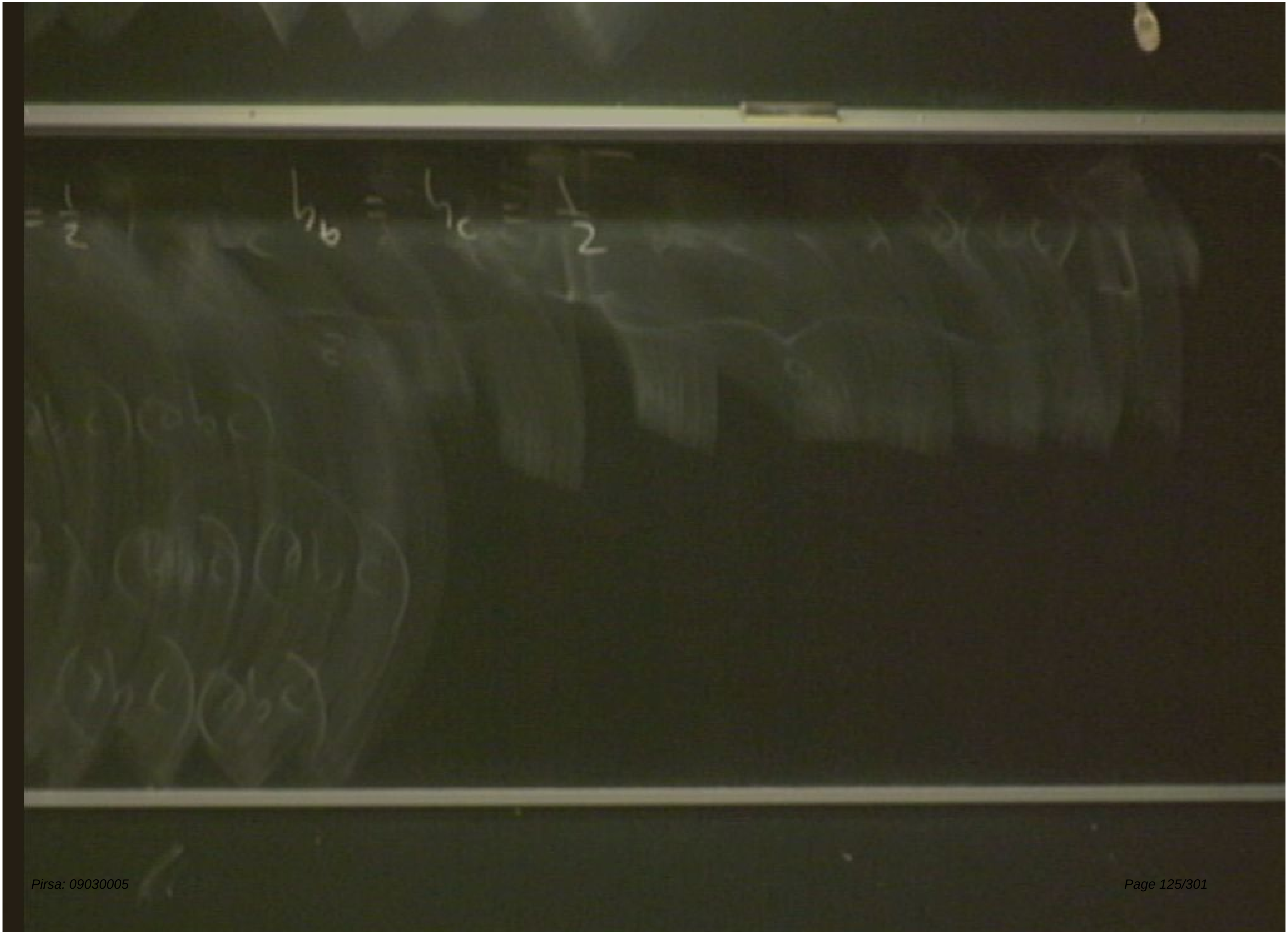
$\lambda^2\, (\partial b c)(\partial b c)$

$\partial_z \, t$

$\underline{\lambda = 2}$ $\qquad$ $c_{bc}(\lambda = 2) = -26$

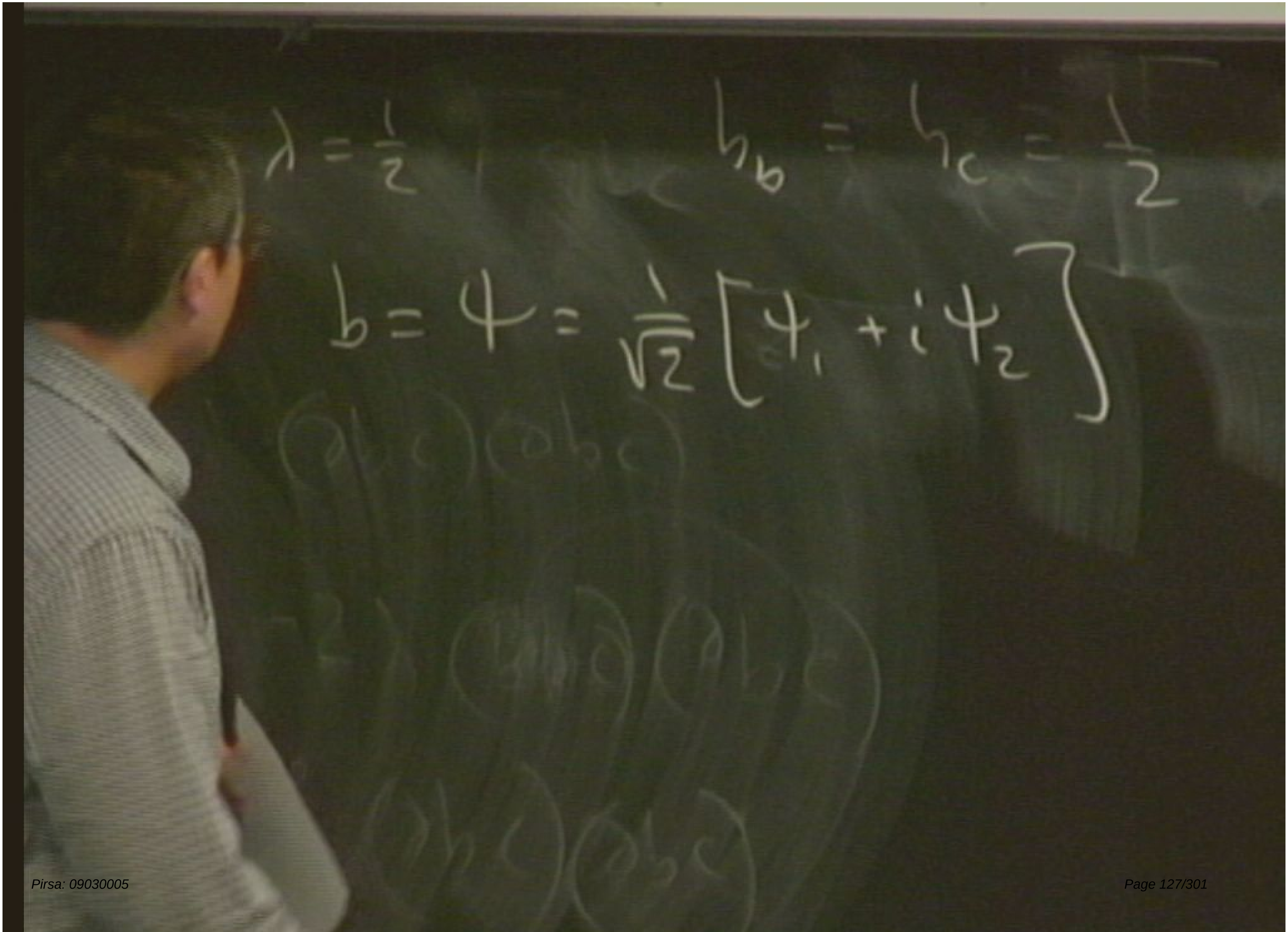

c = 26
matter sector
+
c ghost = bc with λ=2
c bc (λ=2) = -26
26

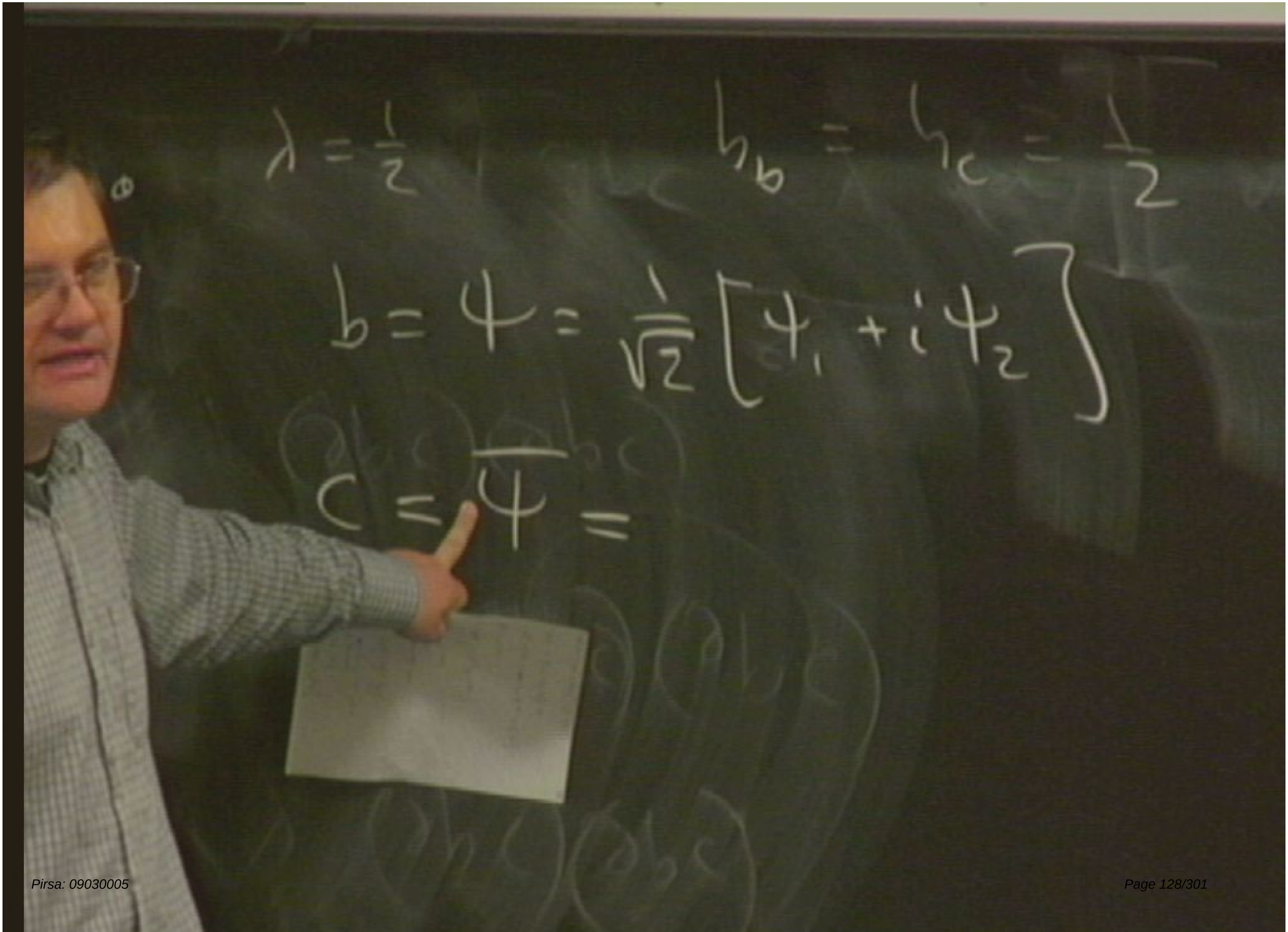




h_b = h_c = 1/2


$h_b = h_c = \frac{1}{2}$


$\lambda = \frac{1}{2}$
$h_b = h_c = \frac{1}{2}$
$b = \psi = \frac{1}{\sqrt{2}}\left[\psi_1 + i\psi_2\right]$


$\lambda = \frac{1}{2}$
$h_b = h_c = \frac{1}{2}$
$b = \psi = \frac{1}{\sqrt{2}}[\psi_1 + i\psi_2]$
$c = \bar{\psi} =$

$$\lambda = \frac{1}{2} \qquad h_b = h_c = \frac{1}{2}$$

$$b = \psi = \frac{1}{\sqrt{2}}\left[\psi_1 + i\psi_2\right]$$

$$c = \bar{\psi} = \frac{1}{\sqrt{2}}\left[\psi_1(z) - i\psi_2(z)\right]$$

$\lambda = \frac{1}{2}$ $\quad h_b = h_c = \frac{1}{2}$ $\quad S_{bc} = \frac{1}{4\pi}\int d^2z \left[\psi_1 \bar{\partial} \psi_1 + \psi_2 \bar{\partial} \psi_2\right]$

$$b = \psi = \frac{1}{\sqrt{2}}\left[\psi_1 + i\psi_2\right]$$

$$c = \bar{\psi} = \frac{1}{\sqrt{2}}\left[\psi_1(z) - i\psi_2(z)\right]$$

$\lambda = \frac{1}{2}$ $h_b = h_c = \frac{1}{2}$

$$S_{bc} = \frac{1}{4\pi}\int d^2z \left[\psi_1 \bar{\partial}\psi_1 + \psi_2 \bar{\partial}\psi_2\right]$$

$$b = \psi = \frac{1}{\sqrt{2}}\left[\psi_1 + i\psi_2\right]$$

π

$$c = \bar{\psi} = \frac{1}{\sqrt{2}}\left[\psi_1(z) - i\psi_2(z)\right]$$

$\lambda = \frac{1}{2}$ $\qquad h_b = h_c = \frac{1}{2}$

$$S_{bc} = \frac{1}{4\pi}\int d^2z \left[\psi_1 \bar{\partial} \psi_1 + \psi_2 \bar{\partial} \psi_2\right]$$

$$b = \psi = \frac{1}{\sqrt{2}}\left[\psi_1 + i\psi_2\right]$$

$$T = -\frac{1}{2}\psi_1 \partial \psi_1 - \frac{1}{2}\psi_1 \partial \psi_2$$

$$c = \bar{\psi} = \frac{1}{\sqrt{2}}\left[\psi_1(z) - i\psi_2(z)\right]$$

$\lambda = \frac{1}{2}$ $\quad h_b = h_c = \frac{1}{2}$

$b = \psi = \frac{1}{\sqrt{2}}\left[\psi_1 + i\psi_2\right]$

$c = \bar{\psi} = \frac{1}{\sqrt{2}}\left[\psi_1(z) - i\psi_2(z)\right]$

$S_{bc} = \frac{1}{4\pi}\int d^2z\left[\psi_1\bar{\partial}\psi_1 + \psi_2\bar{\partial}\psi_2\right]$

$T = -\frac{1}{2}\psi_1\partial\psi_1 - \frac{1}{2}\psi_1\partial\psi_2$

$c_{bc}\left(\lambda = \frac{1}{2}\right) = -3(2\lambda - 1)^2 + 1$

$$\lambda = \frac{1}{2} \qquad h_b = h_c = \frac{1}{2}$$

$$S_{bc} = \frac{1}{4\pi}\int d^2z \left[\psi_1 \bar{\partial} \psi_1 + \psi_2 \bar{\partial} \psi_2\right]$$

$$b = \psi = \frac{1}{\sqrt{2}}\left[\psi_1 + i\psi_2\right]$$

$$T = -\frac{1}{2}\psi_1 \partial \psi_1 - \frac{1}{2}\psi_1 \partial \psi_2$$

$$c = \bar{\psi} = \frac{1}{\sqrt{2}}\left[\psi_1(z) - i\psi_2(z)\right]$$

$$c_{bc}\left(\lambda = \frac{1}{2}\right) = -3(2\lambda - 1)^2 + 1 = 1$$

$$c_{\psi_1} = \frac{1}{2}$$

$\lambda = \frac{1}{2}$ $\qquad h_b = h_c = \frac{1}{2}$

$$b = \psi = \frac{1}{\sqrt{2}}\left[\psi_1 + i\psi_2\right]$$

$$c = \overline{\psi} = \frac{1}{\sqrt{2}}\left[\psi_1(z) - i\psi_2(z)\right]$$

$$c_{\psi_1} = \frac{1}{2}$$

$$S_{bc} = \frac{1}{4\pi}\int d^2z\left[\psi_1\bar{\partial}\psi_1 + \psi_2\bar{\partial}\psi_2\right]$$

$$T = -\frac{1}{2}\psi_1\partial\psi_1 - \frac{1}{2}\psi_1\partial\psi_2$$

$$c_{bc}\left(\lambda = \frac{1}{2}\right) = -3\left(2\lambda - 1\right)^2 + 1 = 1$$

$$\lambda = \frac{1}{2} \qquad h_b = h_c = \frac{1}{2}$$

$$S_{bc} = \frac{1}{4\pi}\int d^2z\left[\psi_1 \bar\partial \psi_1 + \psi_2 \bar\partial \psi_2\right]$$

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$$c^{\text{superstring}}_{\text{matter}}$$

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$$\boxed{c^{\text{superstring}}_{\text{matter}} = 1 + \frac{1}{2} = \frac{3}{2}}$$

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$$c_{bc}\left(\lambda = \frac{1}{2}\right) = -3(2\lambda - 1)^2 + 1 = 1$$

$$\boxed{c^{\text{superstring}}_{\text{matter}} = 1 + \frac{1}{2} = \frac{3}{2}}$$


βγ - CFT

$$\underline{\lambda = 2} \qquad c_{bc}(\lambda = 2) = -26$$

$$S_{bc} = \frac{1}{4\pi}\int d^2z\left[\psi_1\bar{\partial}\psi_1 + \psi_2\bar{\partial}\psi_2\right]$$

$\beta\gamma$ - CFT

$\beta\gamma$ - CFT

$\beta\gamma$ is a bosonic ghost CFT

$\beta\gamma$ - CFT

$\beta\gamma$ is a bosonic ghost CFT

$$S = \frac{1}{2\pi}\int d^2z \ \beta \bar{\partial} \gamma \qquad \beta\gamma = \gamma\beta$$

$\beta\gamma$ – CFT

$\beta\gamma$ is a bosonic ghost CFT

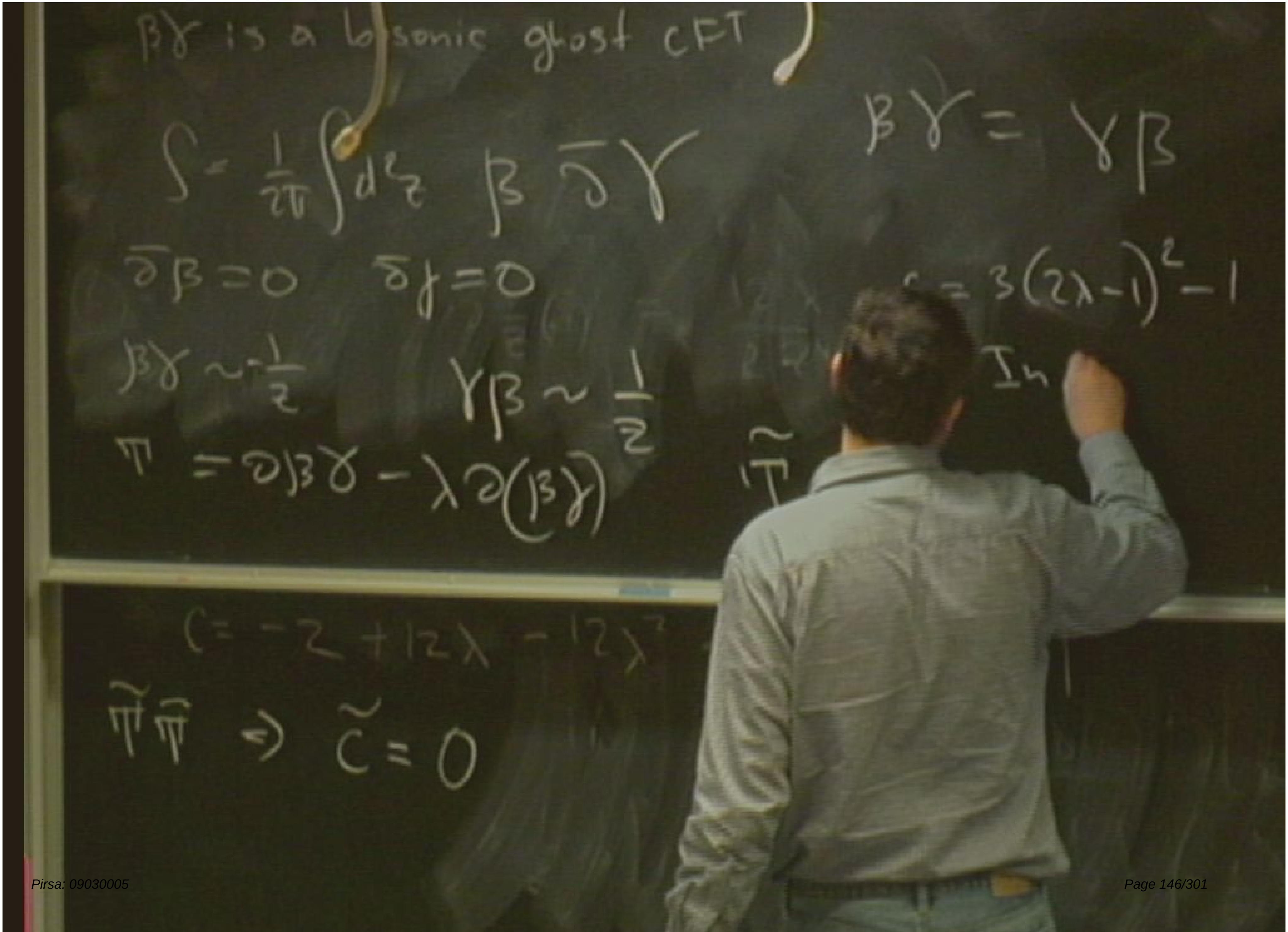
$$S = \frac{1}{2\pi}\int d^2z \; \beta\,\bar\partial\gamma$$

$$\beta\gamma = \gamma\beta$$

$$\bar\partial\beta = 0 \qquad \bar\partial\gamma = 0$$

$$\beta\gamma \sim -\frac{1}{z} \qquad \gamma\beta \sim \frac{1}{z}$$

$$\tilde\pi^{\dagger}\tilde\pi \Rightarrow \tilde{c} = 0$$


$\beta\gamma$ is a bosonic ghost CFT
$S = \frac{1}{2\pi}\int d^2z\, \beta\bar{\partial}\gamma$
$\beta\gamma = \gamma\beta$
$\bar{\partial}\beta = 0 \quad \bar{\partial}\gamma = 0$
$c = 3(2\lambda-1)^2 - 1$
$\beta\gamma \sim -\frac{1}{z} \qquad \gamma\beta \sim \frac{1}{z}$
In
$T = \partial\beta\gamma - \lambda\partial(\beta\gamma)$
$\tilde{T}$
$c = -2 + 12\lambda - 12\lambda^2$
$\tilde{T}\tilde{T} \Rightarrow \tilde{c} = 0$

$\beta\gamma$ - CFT

$\beta\gamma$ is a bosonic ghost CFT

$$S = \frac{1}{2\pi}\int d^2z \; \beta \bar{\partial} \gamma \qquad \beta\gamma = \gamma\beta$$

$$\bar{\partial}\beta = 0 \quad \bar{\partial}\gamma = 0 \qquad c = 3(2\lambda-1)^2 - 1 \qquad \tilde{c} = 0$$

$$\beta\gamma \sim -\frac{1}{z} \qquad \gamma\beta \sim \frac{1}{z}$$

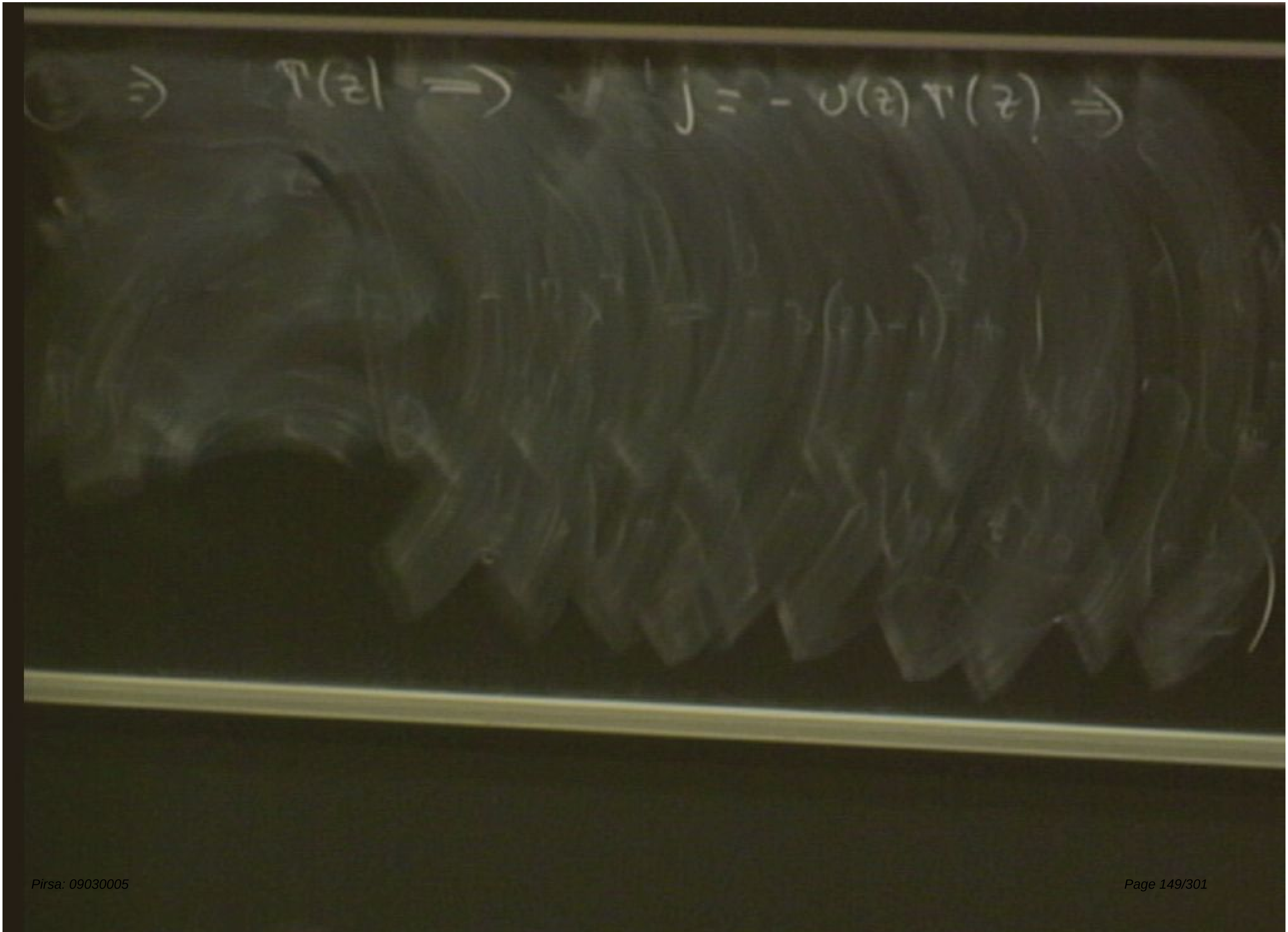
In superstrings

$$T = \partial\beta\,\gamma - \lambda\partial(\beta\gamma) \qquad \tilde{T} = 0 \qquad \lambda = \frac{3}{2}$$

$$c = -2 + 12\lambda - 12\lambda^2 = -3(2\lambda-1)^2 + 1$$

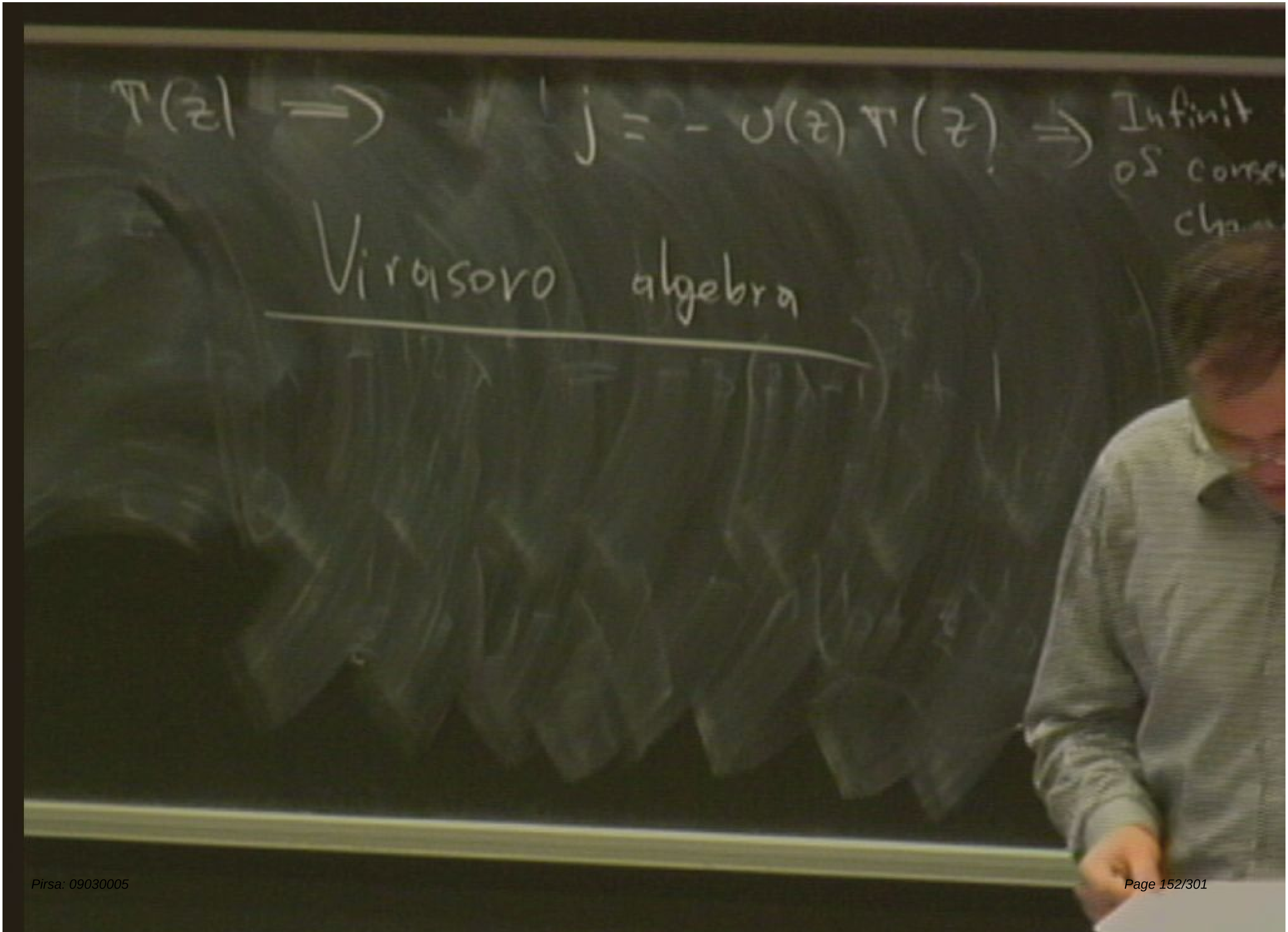
$$\tilde{T}\tilde{T} \Rightarrow \tilde{c} = 0$$

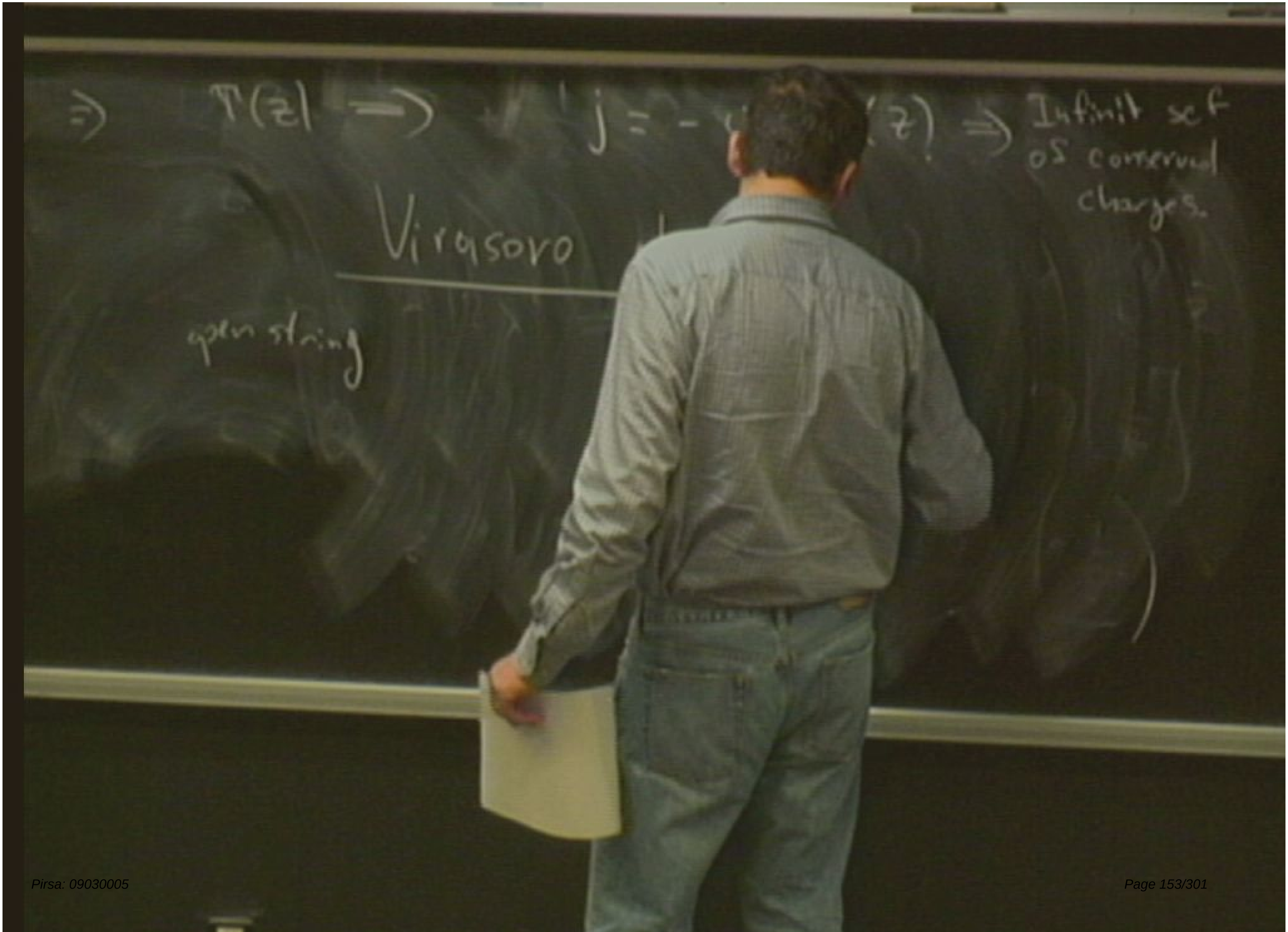
$$\Rightarrow \quad \psi(z) \longrightarrow \quad j = -\,\upsilon(z)\,\psi(z)$$

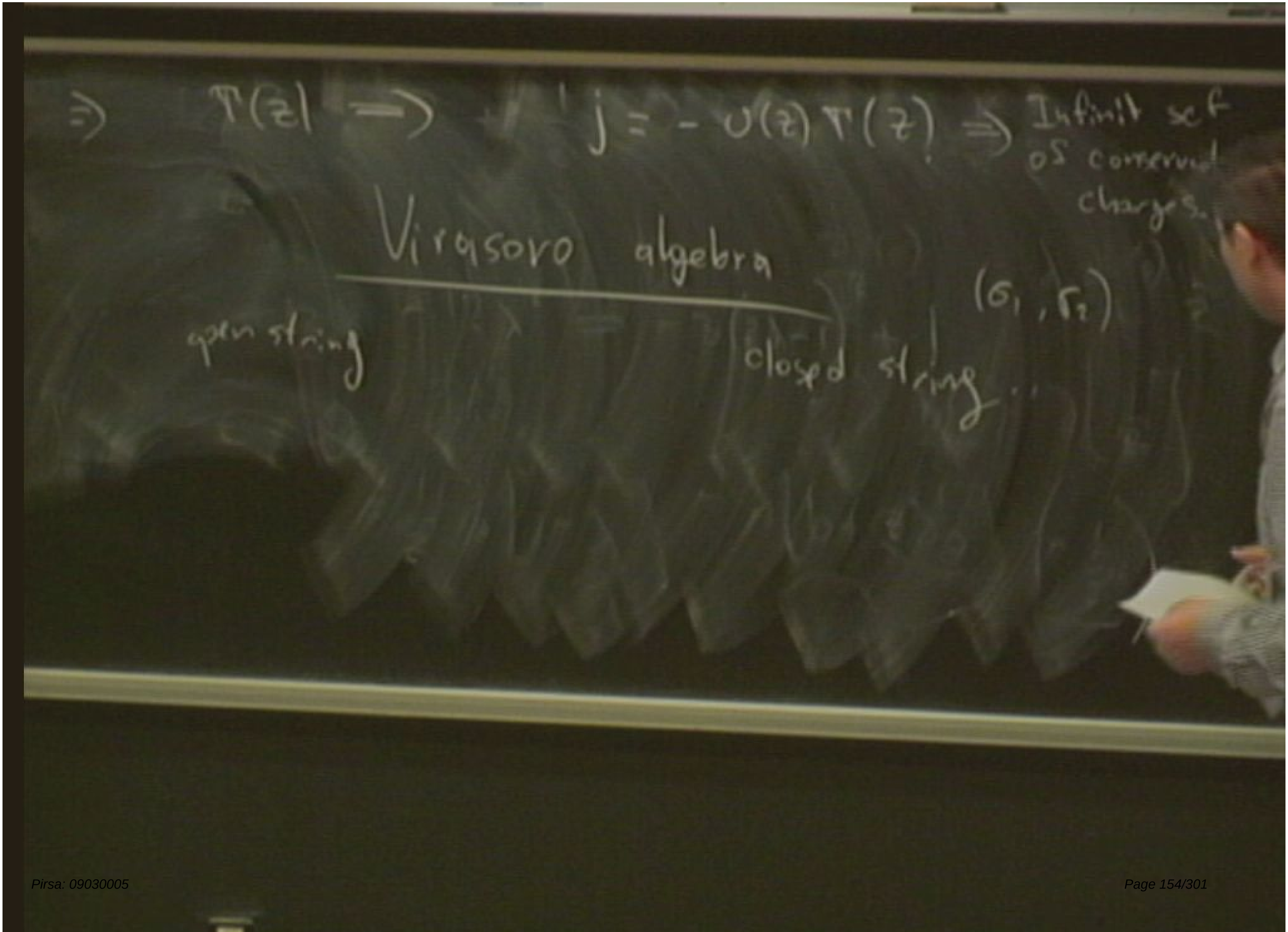

j = - υ(z) Ψ'(z) ⇒

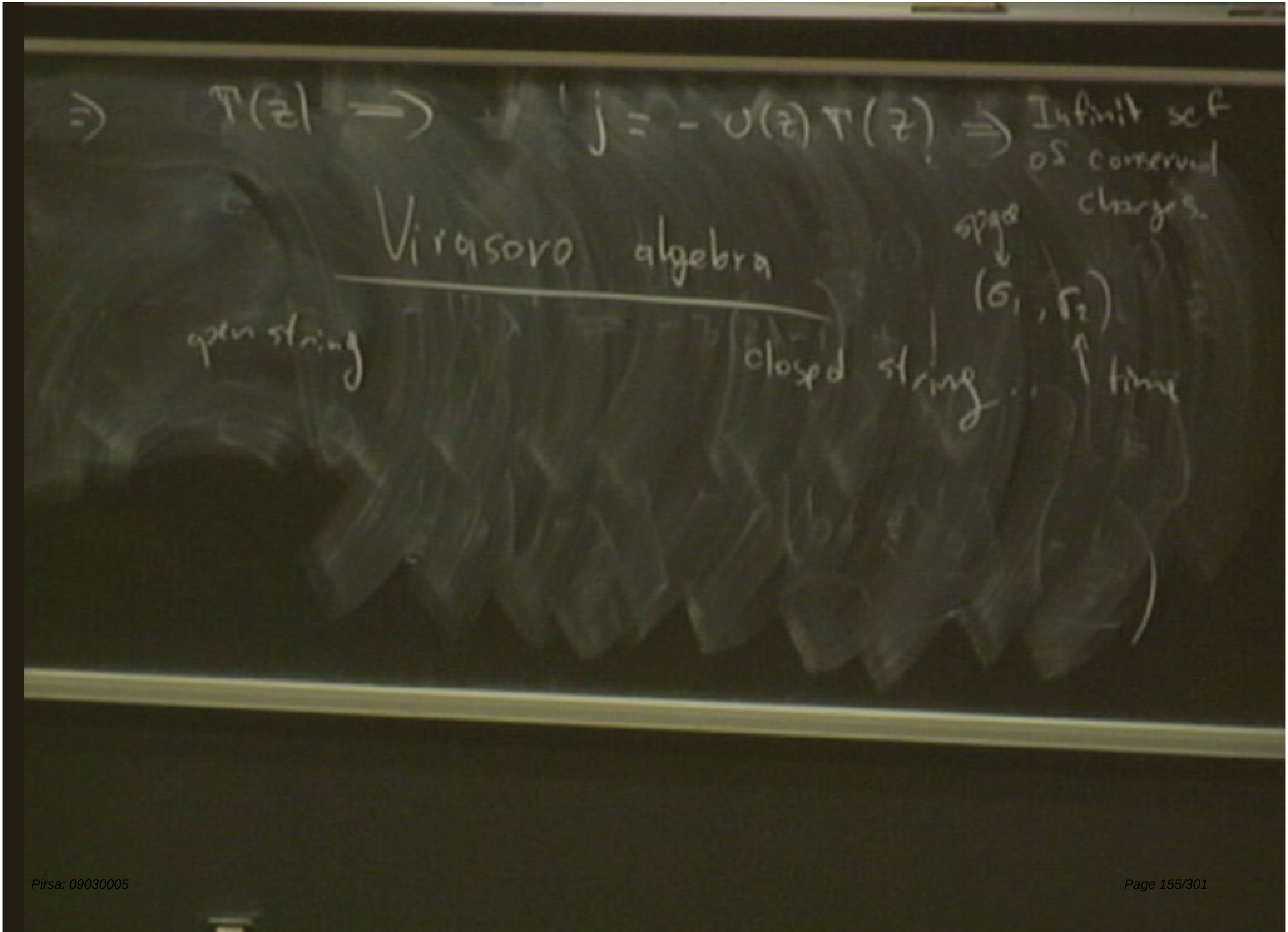
$T(z) \Rightarrow$ $j = -U(z)T(z) \Rightarrow$ Infinit set of conserved charges.

$$j = -\,U(z)\,T(z) \Rightarrow$$ Infinit sef of conserved charges.


T(z) ⟹ j = - v(z) T(z) ⟹
Infinit
of conser
Virasoro algebra


Infinite set
of conserved
charges.
Virasoro
open string


⇒ T(z) ⇒ j = − v(z) T(z) ⇒ Infinite set of conserved charges.
Virasoro algebra
open string
closed string
(σ1, σ2)


⇒ T(z) ⇒ j = - v(z) T(z) ⇒ Infinit set of conserved charges.
Virasoro algebra
space
(σ1 , σ2)
time
open string
closed string

$\Rightarrow \quad T(z) \longrightarrow \quad j = -v(z)\,T(z) \Rightarrow$ Infinite set of conserved charges.

Virasoro algebra

open string

closed string

space

(σ_1, σ_2)

time

$\Rightarrow \quad T(z) \Longrightarrow \quad j = -v(z)\,T(z) \Rightarrow$ Infinite set of conserved charges.

Virasoro algebra

space

(σ_1, σ_2)

time

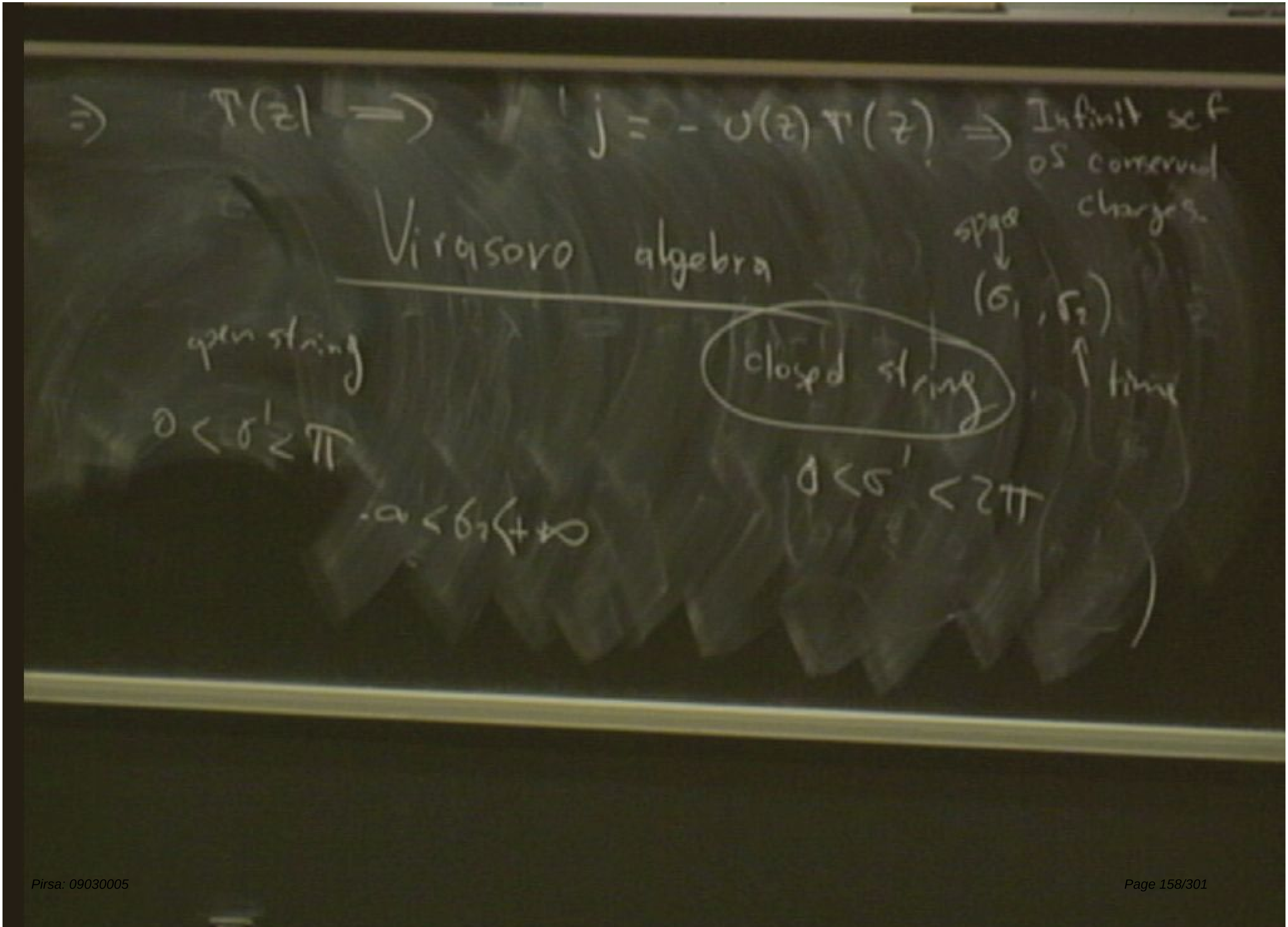
open string

$0 < \sigma_1 < \pi$

$-\infty < \sigma_2 < +\infty$

closed string

$0 < \sigma_1 < 2\pi$


⇒ T(z) ⟹ j = − v(z) T(z) ⇒ Infinite set of conserved charges
Virasoro algebra
space
(σ1, σ2)
time
open string
closed string
0 < σ' < π
0 < σ' < 2π
-∞ < σ2 < +∞

$\Rightarrow \quad T(z) \Longrightarrow \quad j = -v(z)\,T(z) \Rightarrow$ Infinite set of conserved charges.

Virasoro algebra

space
(σ_1, σ_2)
time

open string
$0 < \sigma_1 < \pi$
$-\infty < \sigma_2 < +\infty$

closed string
$0 < \sigma_1 < 2\pi$

$\Rightarrow \quad T(z) \longrightarrow \quad j = -v(z)\,T(z) \Rightarrow$ Infinit set of conserved charges.

Virasoro algebra

open string

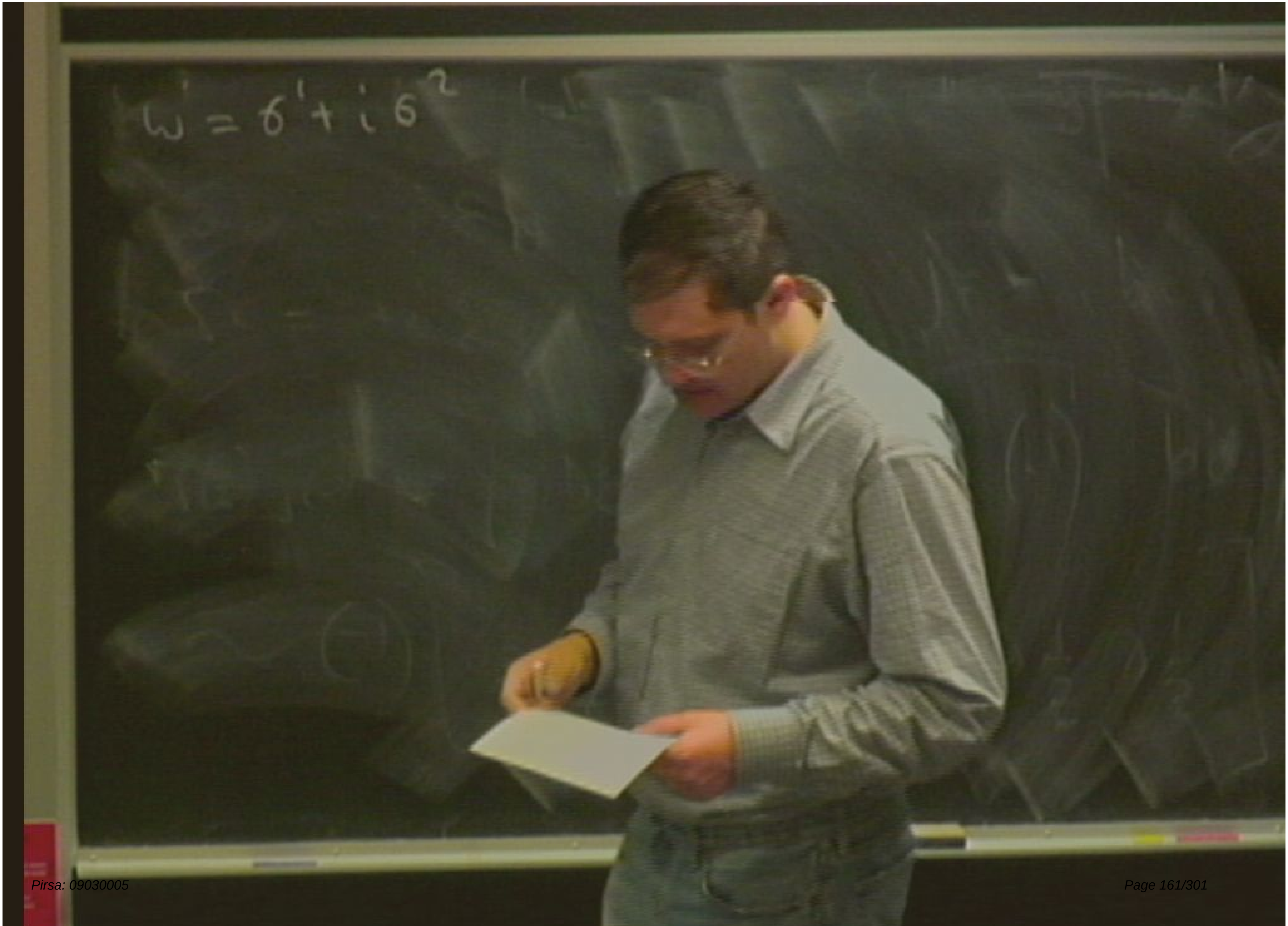
$0 < \sigma^1 < \pi$

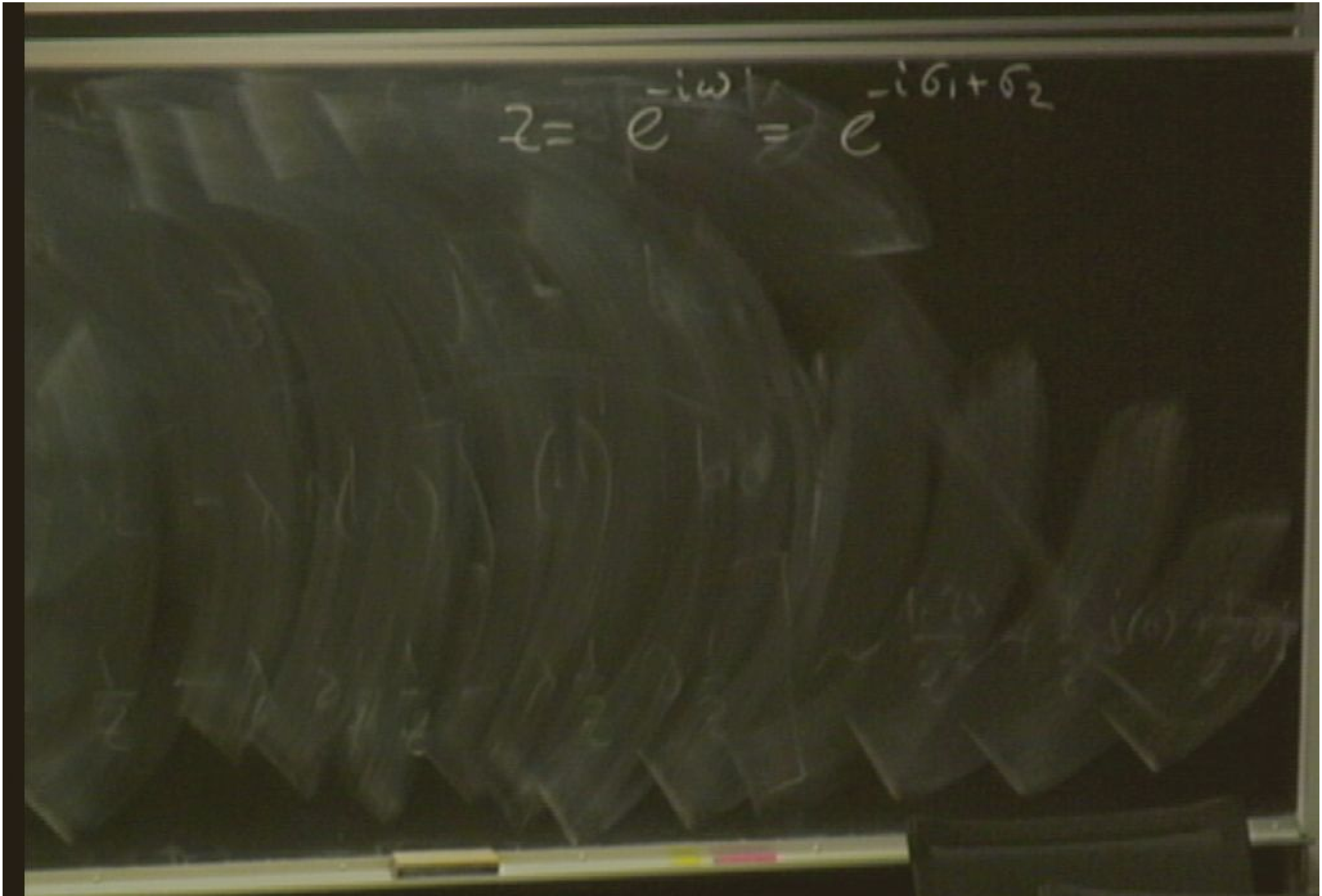
$-\infty < \sigma_2 < +\infty$

closed string

$0 < \sigma^1 < 2\pi$

space → (σ_1, σ_2) ← time


ω' = σ' + i σ²


$z = e^{-i\omega t} = e^{-i\sigma_1+\sigma_2}$

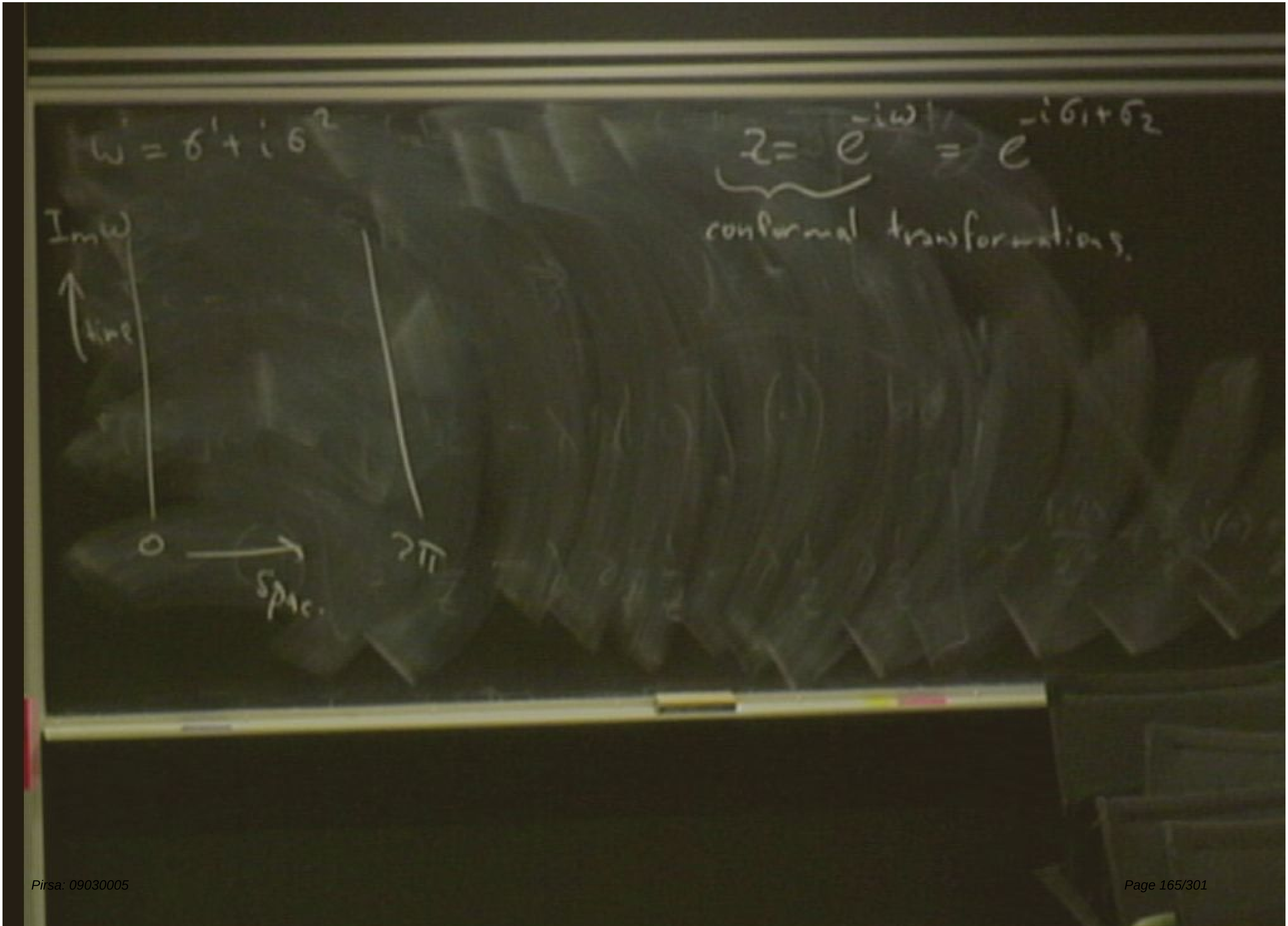
$$w^1 = \sigma^1 + i\sigma^2$$

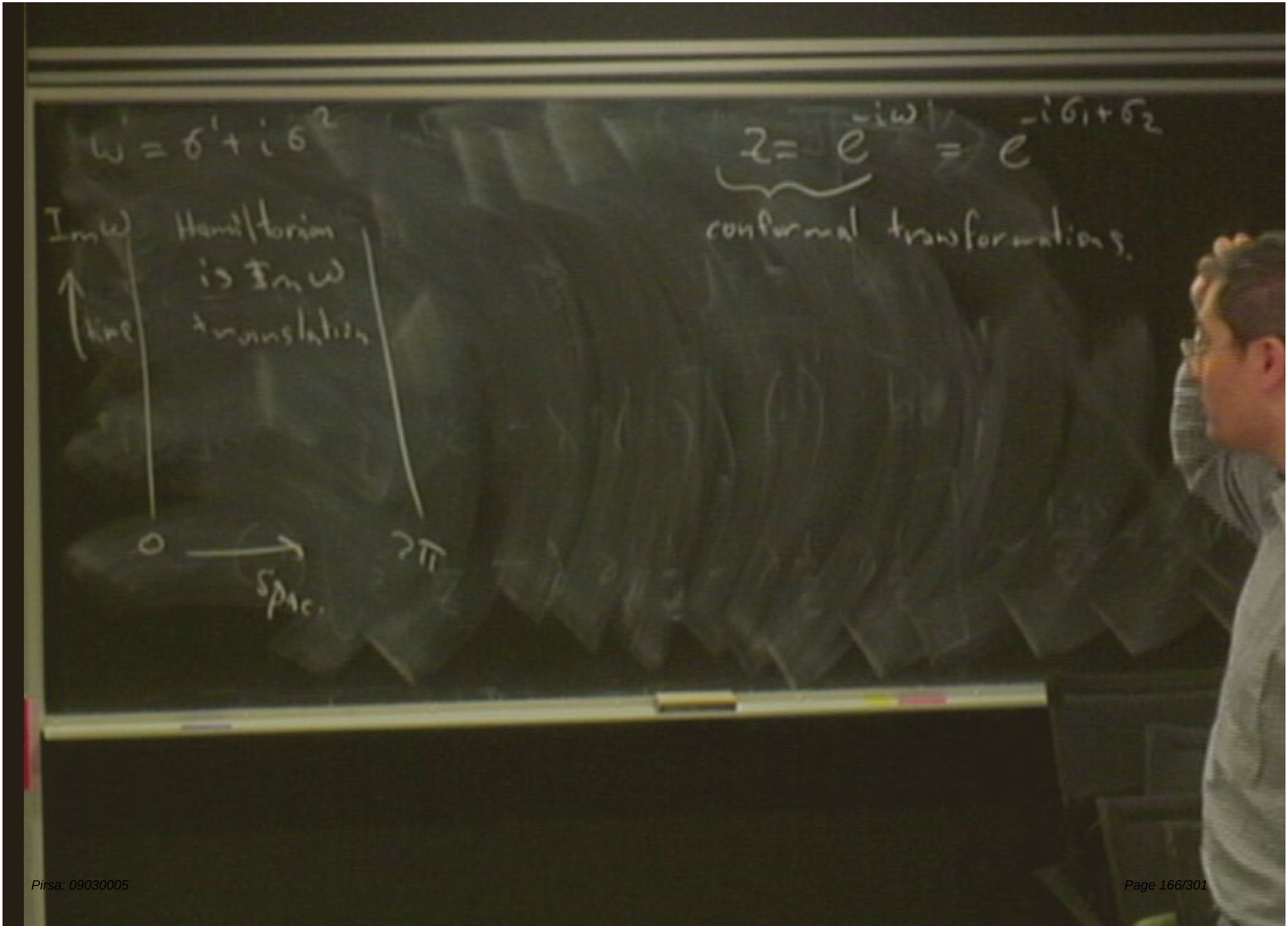
$$\underbrace{z = e^{-iw^1}}_{\text{conformal transformations.}} = e^{-i\sigma_1 + \sigma_2}$$

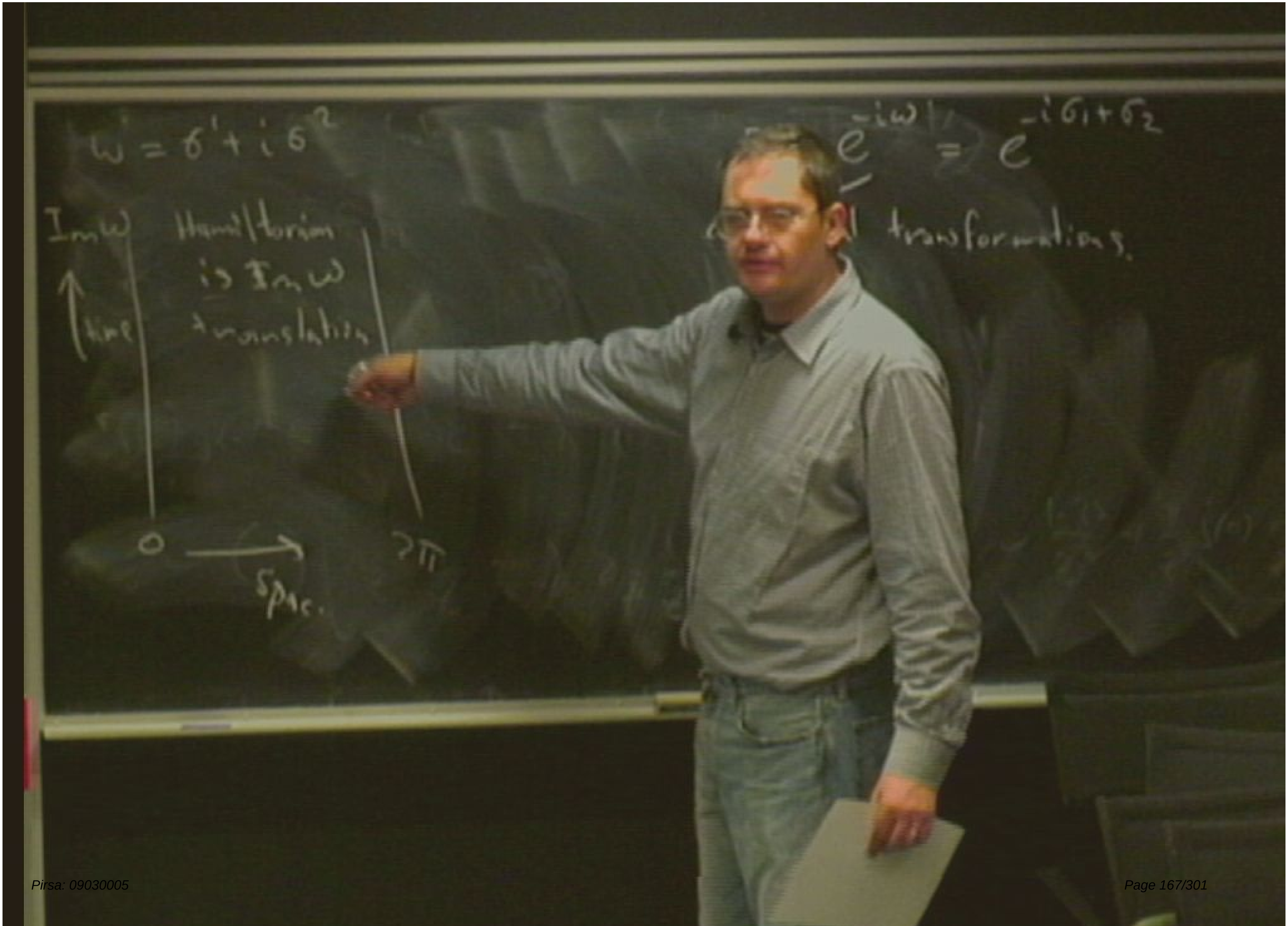
$$w = \sigma^1 + i\sigma^2$$

$$\underbrace{z = e^{-iw}}_{\text{conformal transformations.}} = e^{-i\sigma_1 + \sigma_2}$$

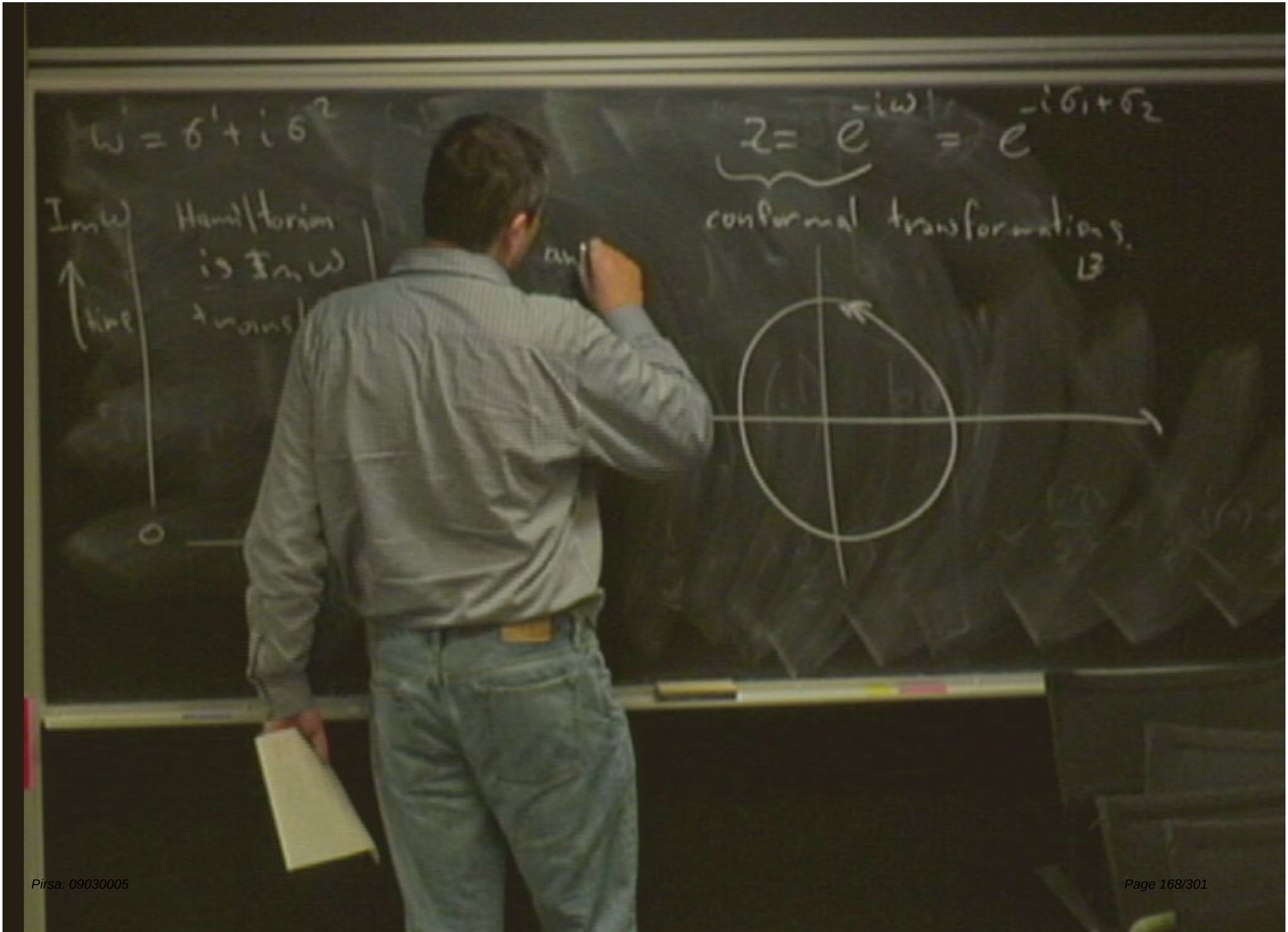
Im w

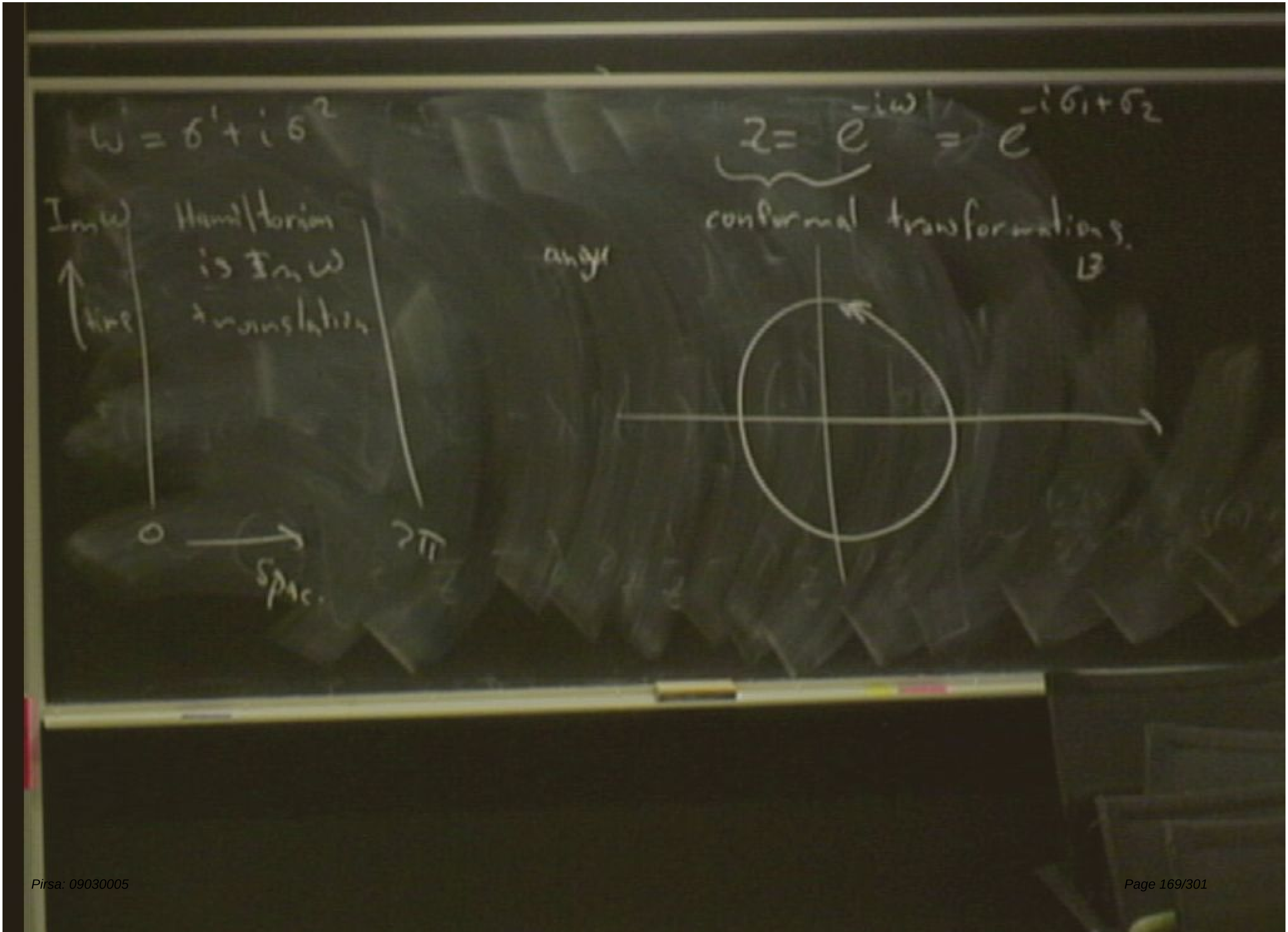

$w = \sigma^1 + i\sigma^2$
$\underbrace{z = e^{-iw}}$ $= e^{-i\sigma_1 + \sigma_2}$
conformal transformations.
Im w
time
0
2π
spac.


w = σ¹ + iσ²
z = e^{-iw} = e^{-iσ₁+σ₂}
conformal transformations
Im w
time
Hamiltonian
is Im w
translation
0
2π
space

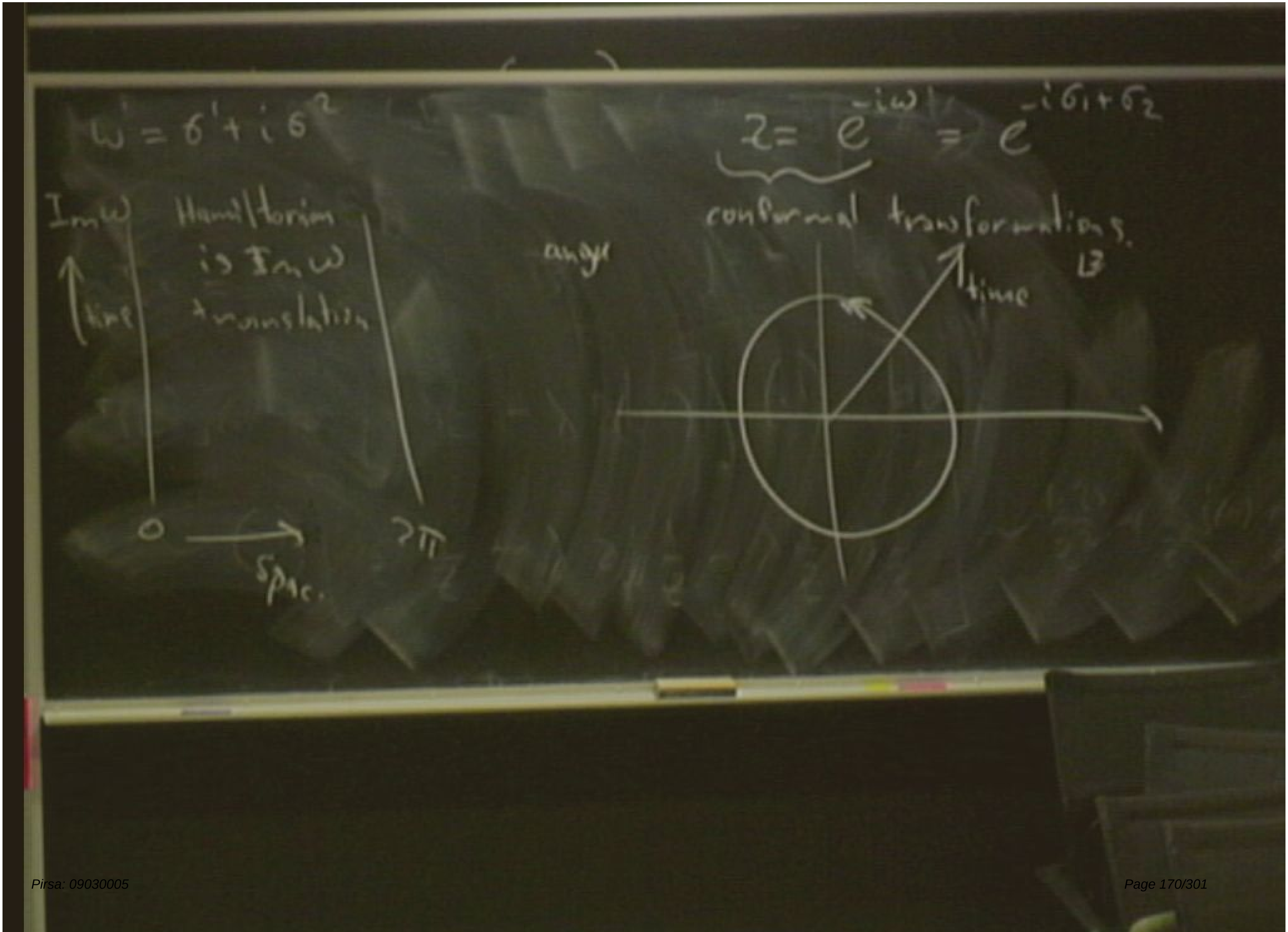


ω = σ¹ + i σ²
Im ω
Hamiltonian is Im ω translation
time
0
2π
space
e^{-iω} = e^{-iσ₁+σ₂}
transformations.


ω = σ¹ + iσ²
z = e^{-iω} = e^{-iσ₁+σ₂}
conformal transformations
Im ω
Hamiltonian
is Im ω
time
0

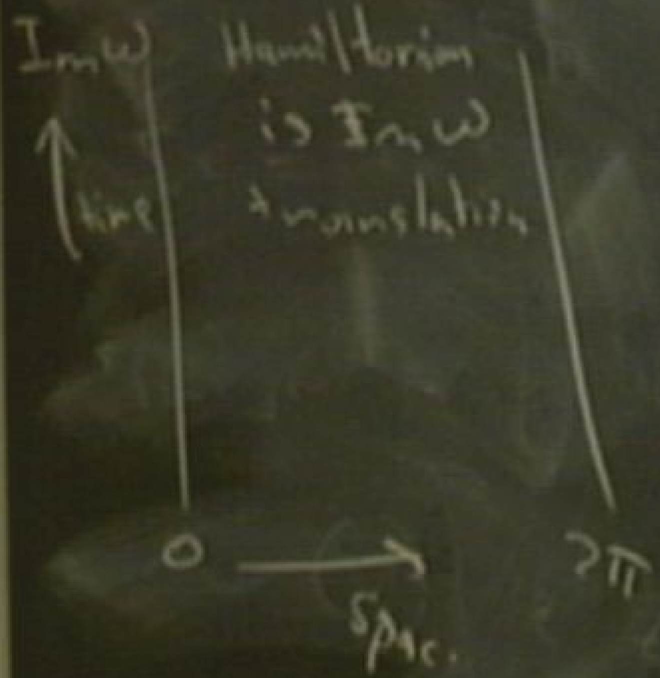


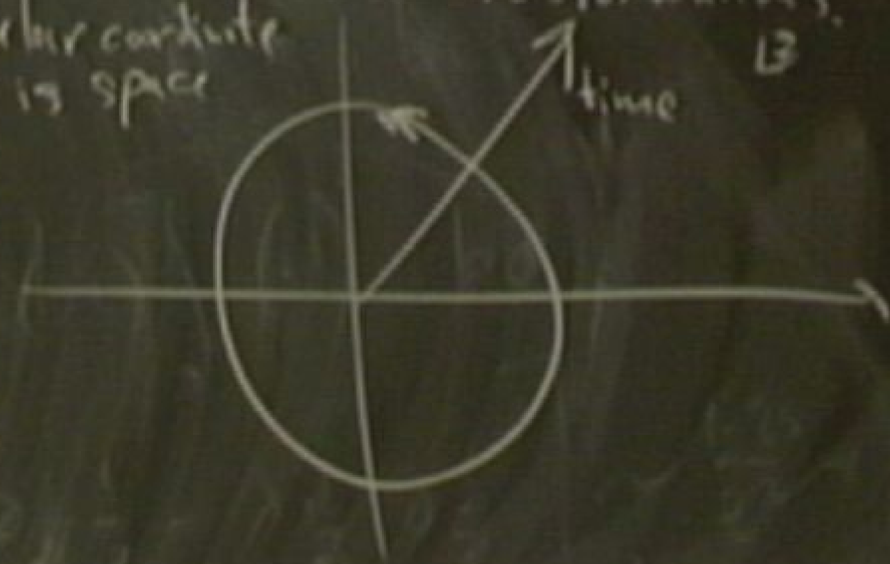
$w = \sigma^1 + i\sigma^2$
$z = e^{-iw} = e^{-i\sigma_1 + \sigma_2}$
conformal transformations
Im w
Hamiltonian
is Im w
translation
time
angle
0
2π
space

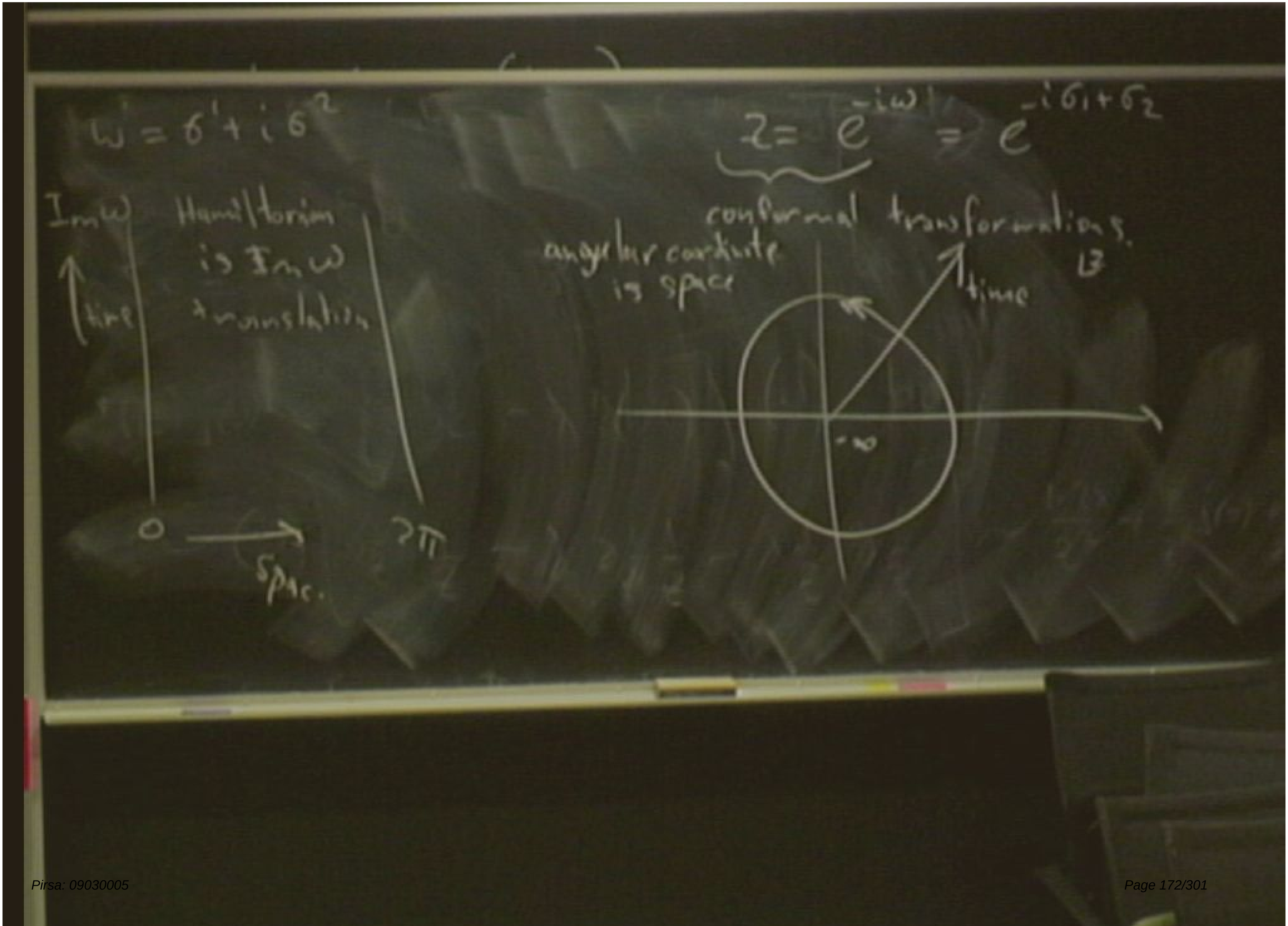

$\omega = \sigma^1 + i\sigma^2$
$z = e^{-i\omega} = e^{-i\sigma_1 + \sigma_2}$
conformal transformations
Im ω
Hamiltonian
is Im ω
translation
time
angle
time
0
2π
space

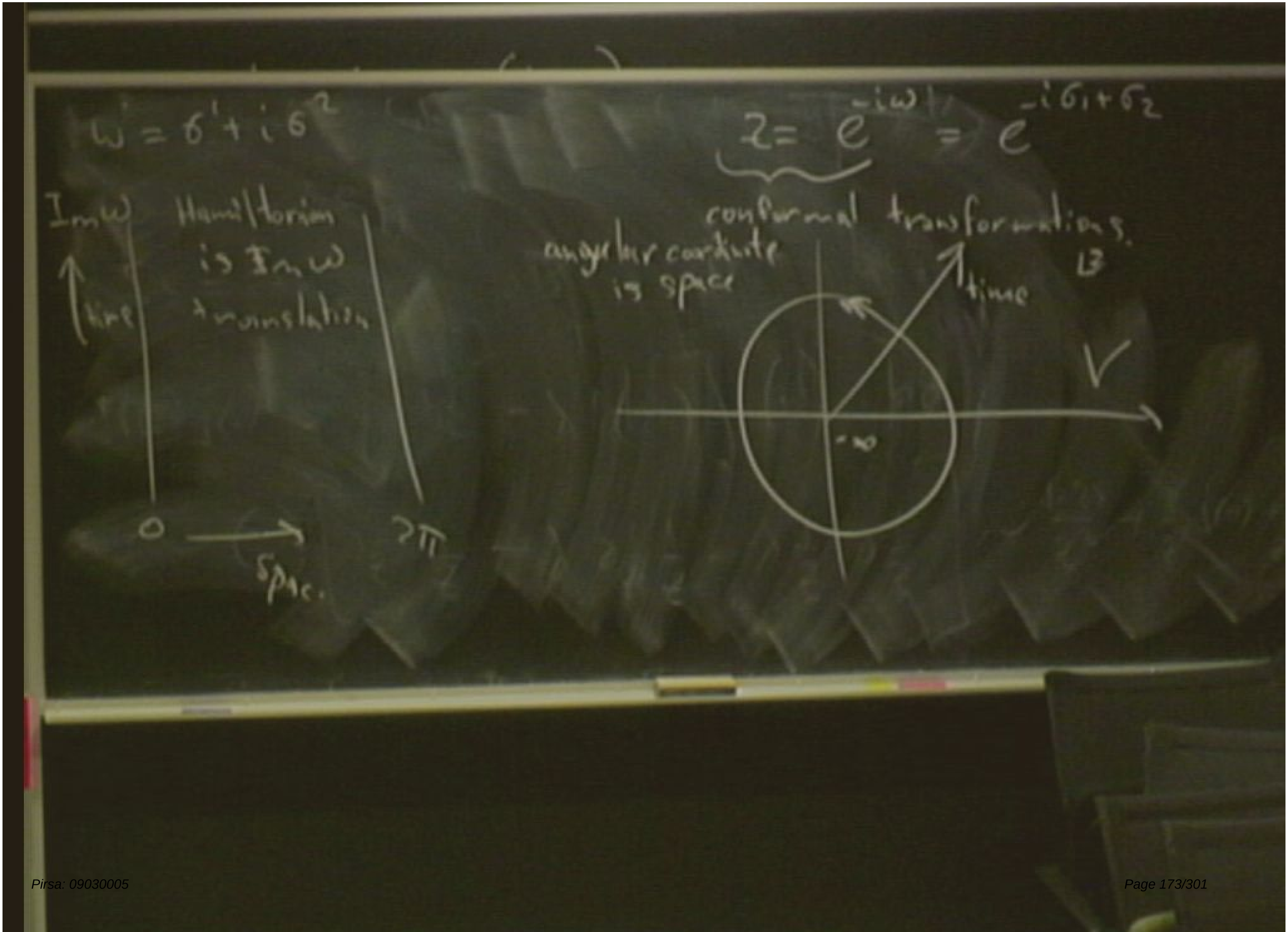
$w = \sigma^1 + i\sigma^2$

$$\underbrace{z = e^{-iw}}_{\text{conformal transformations.}} = e^{-i\sigma_1 + \sigma_2}$$

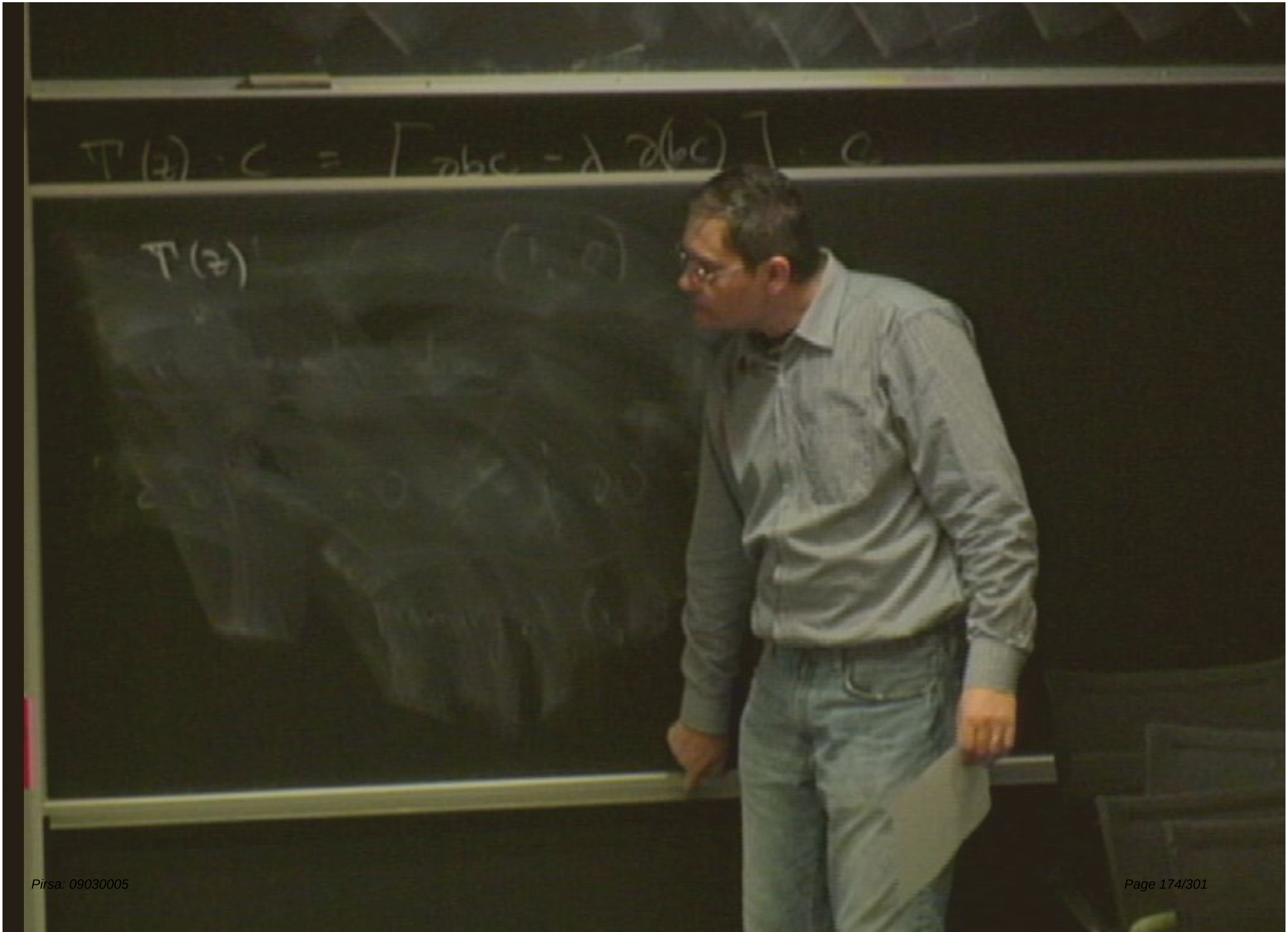


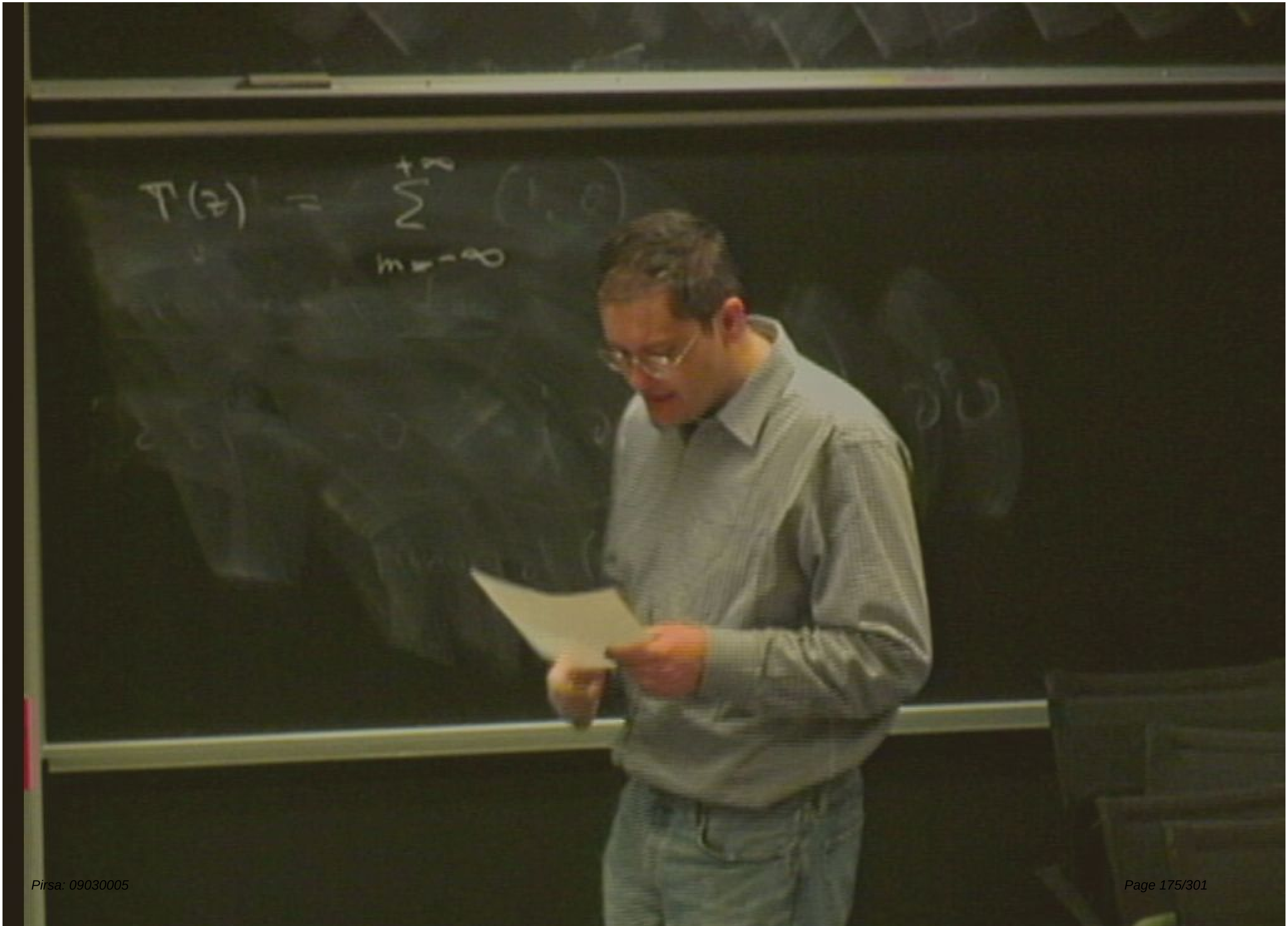



$\omega = \sigma^1 + i\sigma^2$
$z = e^{-i\omega} = e^{-i\sigma_1 + \sigma_2}$
conformal transformations.
Im ω
Hamiltonian is Im ω translation
time
0
space
2π
angular coordinate is space
time
-∞

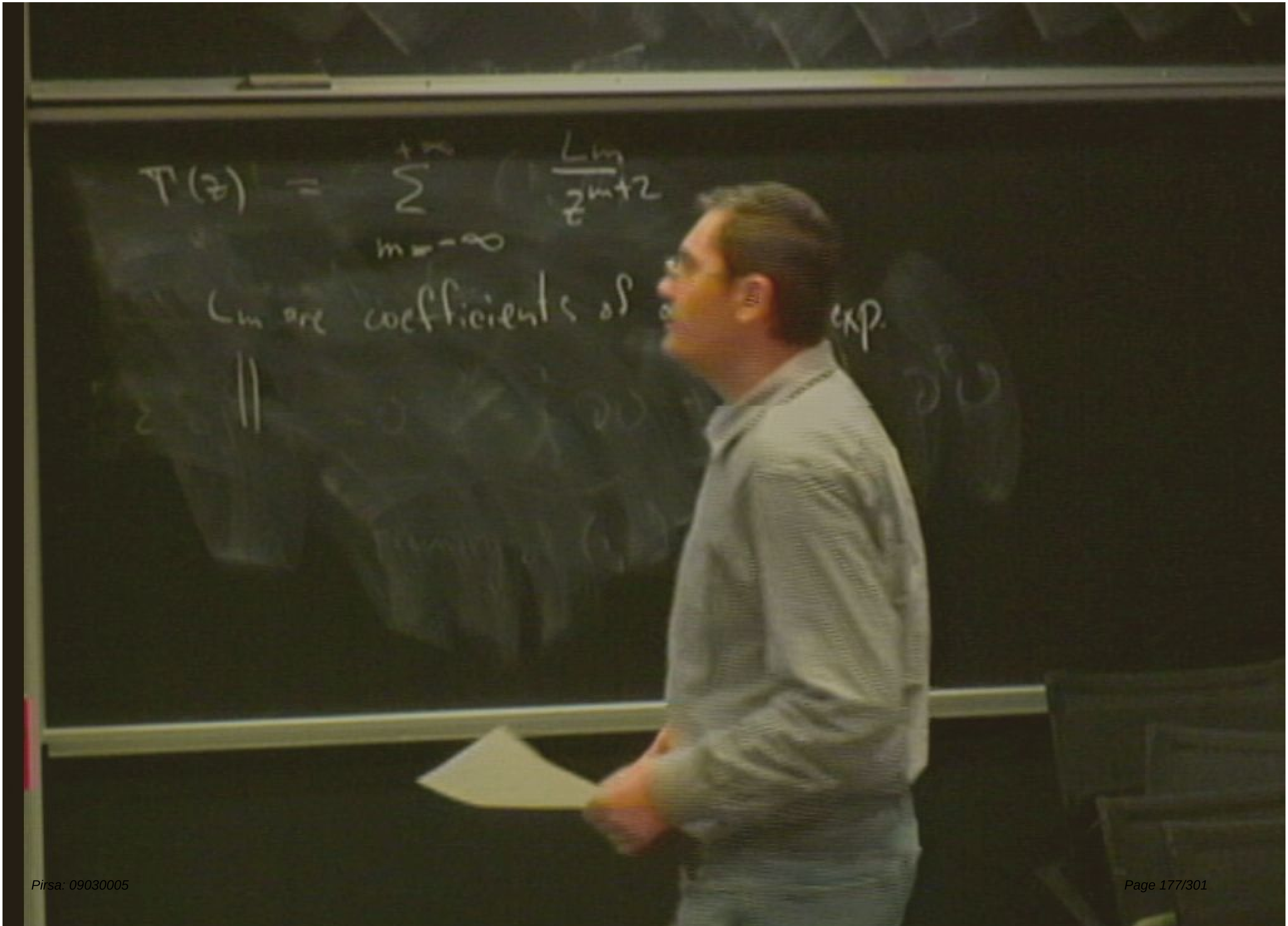


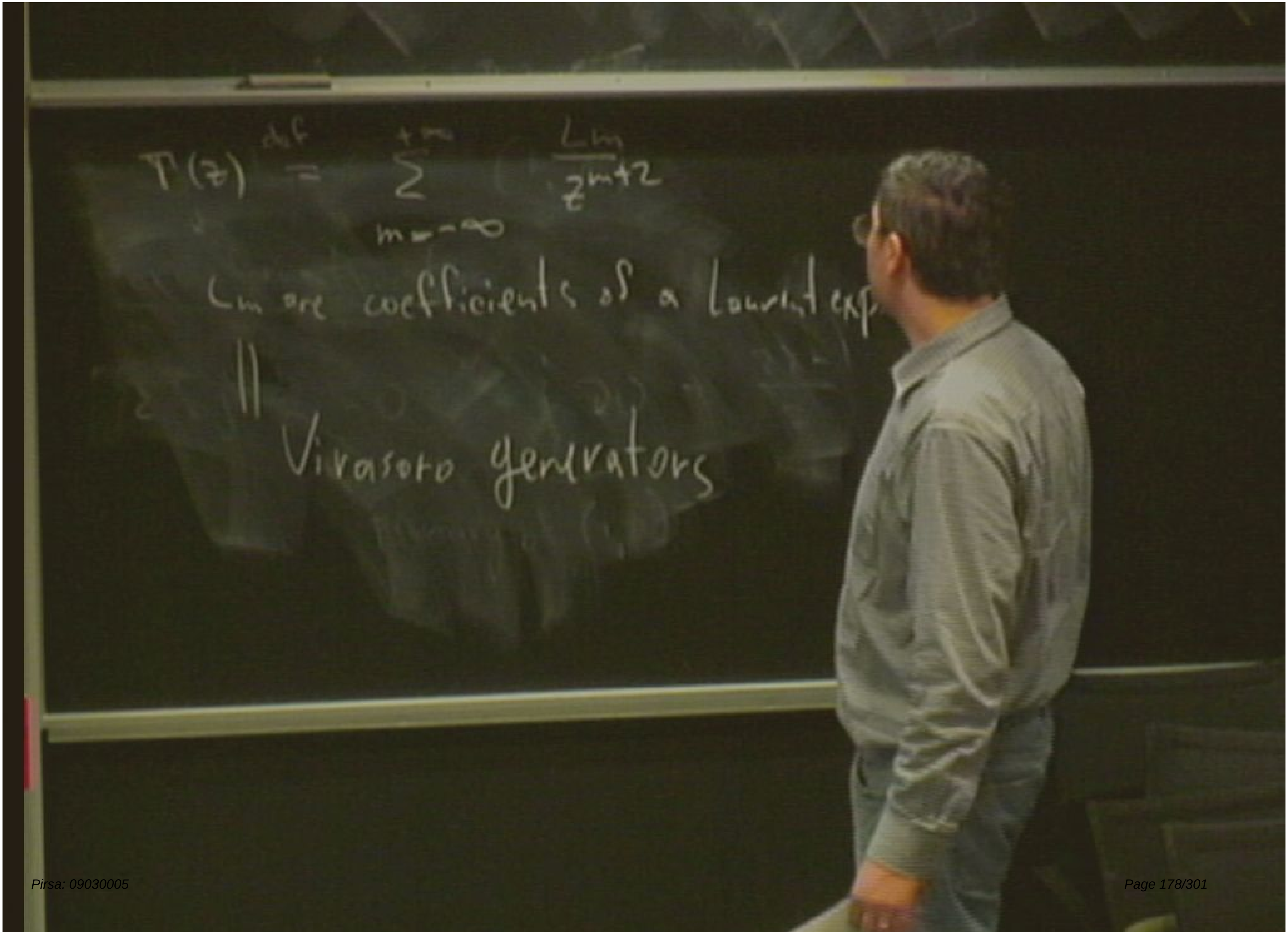
w = σ¹ + iσ²
z = e^{-iw} = e^{-iσ₁+σ₂}
conformal transformations
Im w
Hamiltonian
is Im w
translation
time
angular coordinate
is space
time
0
2π
space


T(z)




$T(z) = \sum_{m=-\infty}^{+\infty} \frac{L_m}{z^{m+2}}$


T(z) = Σ_{m=-∞}^{+∞} L_m / z^{m+2}
L_m are coefficients of
exp.


$T(z) \overset{def}{=} \sum_{m=-\infty}^{+\infty} \frac{L_m}{z^{m+2}}$
L_m are coefficients of a Laurent exp.
||
Virasoro generators

$$T(z) \stackrel{def}{=} \sum_{m=-\infty}^{+\infty} \frac{L_m}{z^{m+2}} \qquad L_m =$$

L_m are coefficients of a Laurent exp.

$\parallel$

Virasoro generators

$$T(z) \overset{\text{def}}{=} \sum_{m=-\infty}^{+\infty} \frac{L_m}{z^{m+2}} \qquad L_m = \oint_C \frac{dz}{2\pi i\, z}\, z^{m+2}\, T$$

L_m are coefficients of a Laurent exp.

||

Virasoro generators

$$T(z) \overset{def}{=} \sum_{m=-\infty}^{+\infty} \frac{L_m}{z^{m+2}} \qquad L_m = \oint_C \frac{dz}{2\pi i\, z} z^{m+2}\, T(z)$$

L_m are coefficients of a Laurent exp.

$\|$

Virasoro generators

$$\boxed{\bar{T},\ \tilde{L}}$$

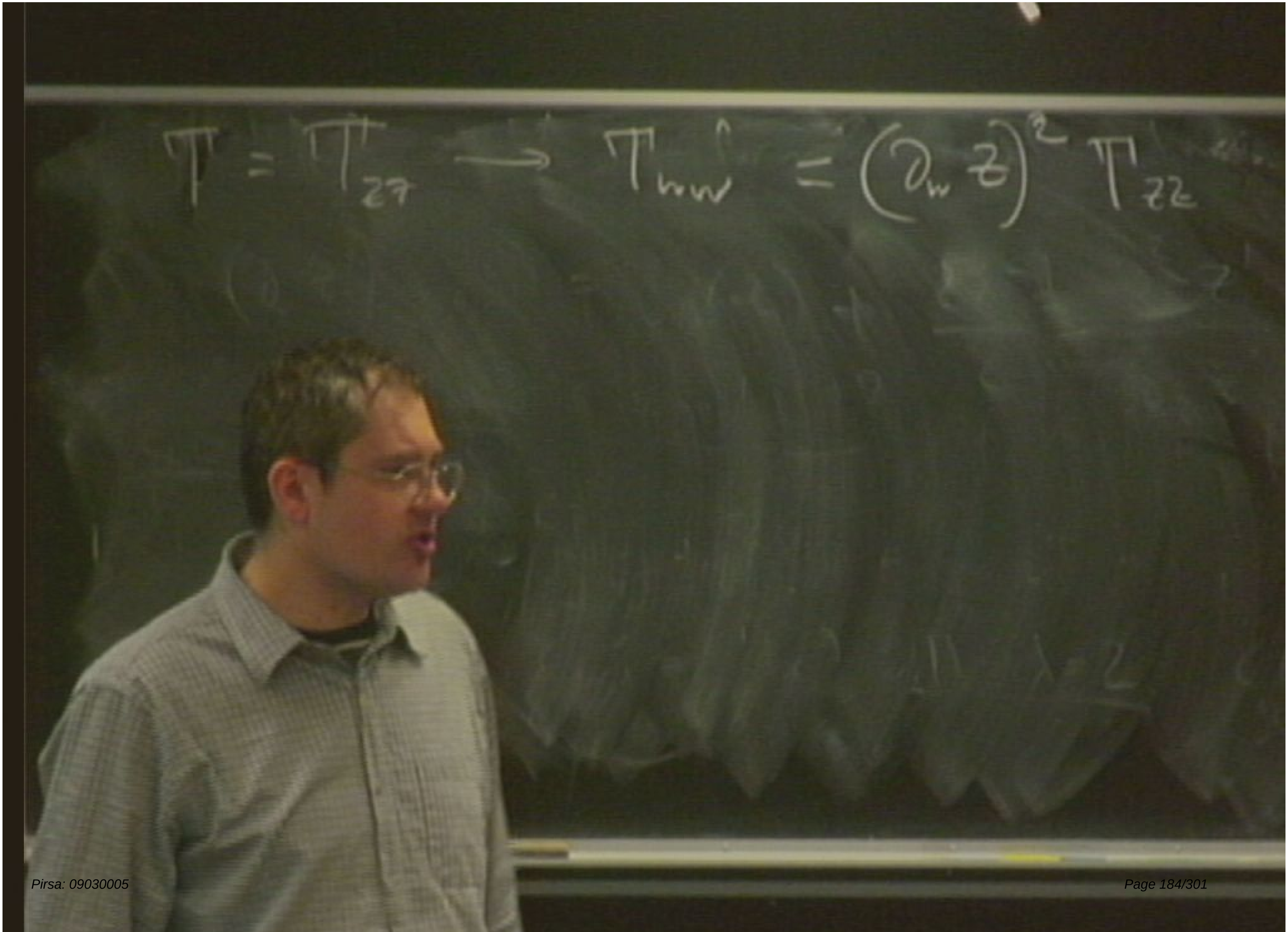
$$T(z) \stackrel{\text{def}}{=} \sum_{m=-\infty}^{+\infty} \frac{L_m}{z^{m+2}} \qquad L_m = \oint_C \frac{dz}{2\pi i\, z} z^{m+2}\, T(z)$$

L_m are coefficients of a Laurent exp.

$\Vert$

Virasoro generators

$$\boxed{\tilde{\bar{T}},\ \tilde{L}}$$

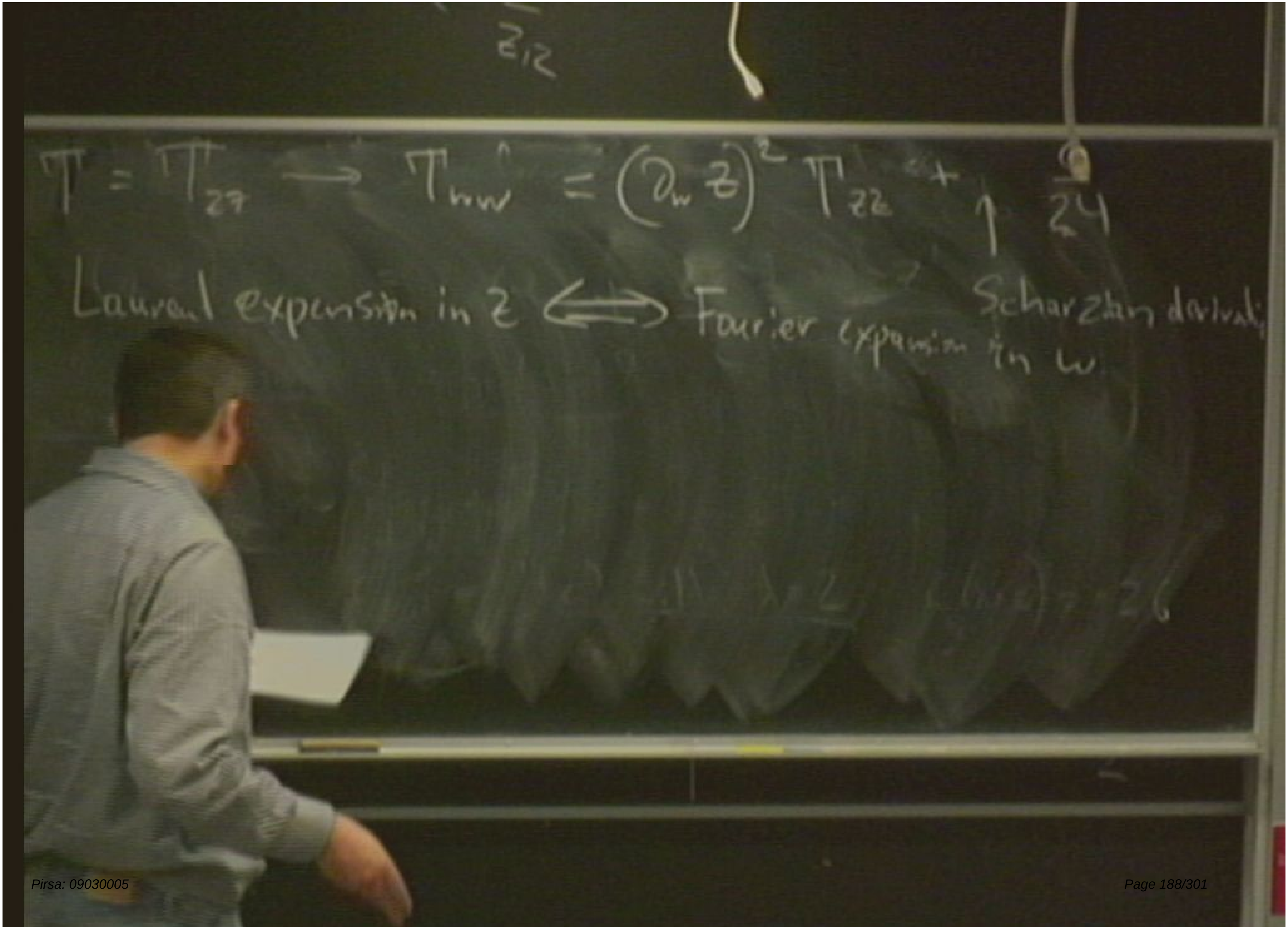

$T = T_{zz} \longrightarrow T_{ww} = (\partial_w z)^2 T_{zz}$

z_{12}

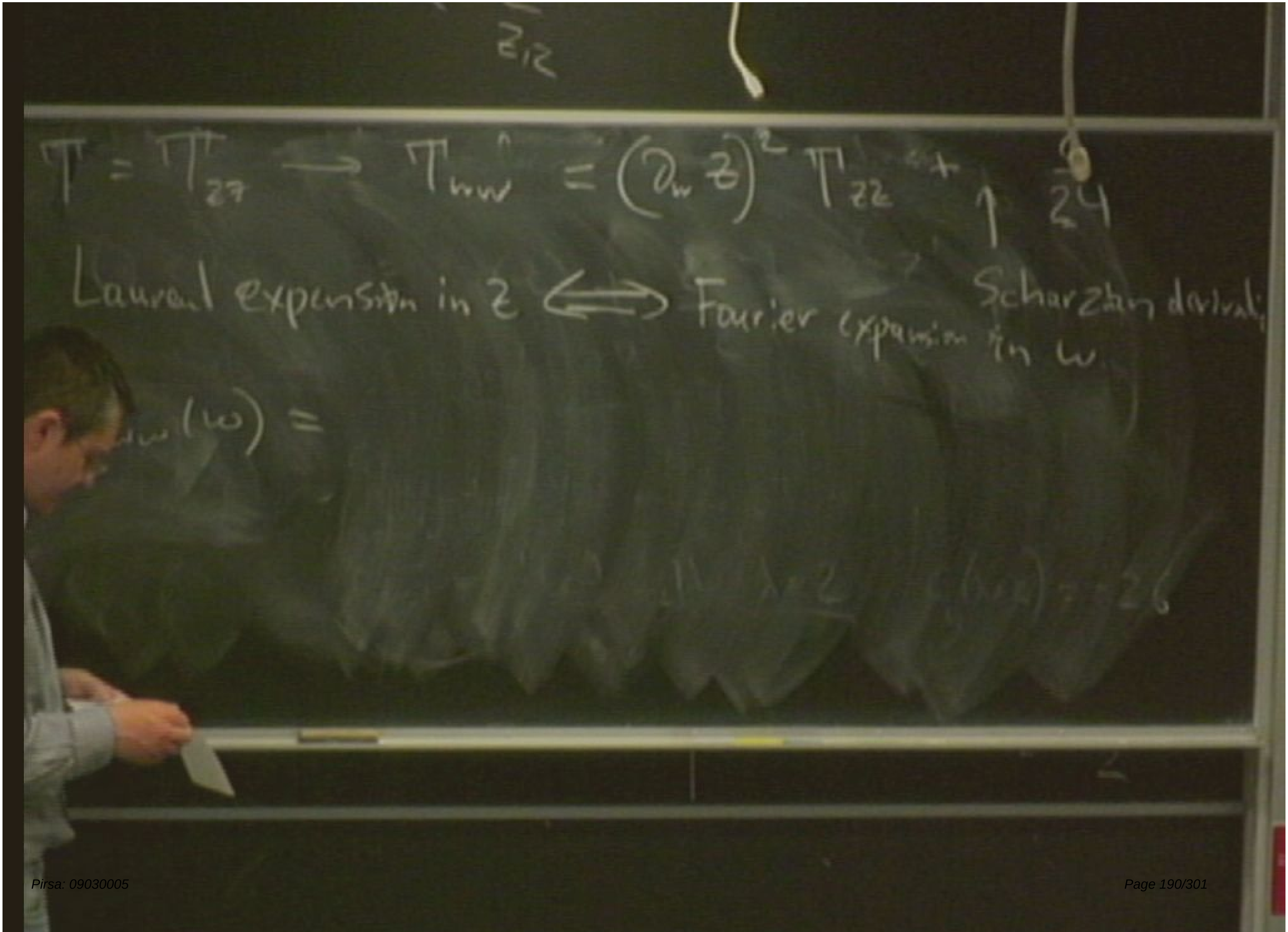
$$T = T_{zz} \longrightarrow T_{ww} = (\partial_w z)^2 \, T_{zz} + \uparrow$$

Scharzian derivati


Laurent expansion in z


T = T_zz → T_ww = (∂_w z)^2 T_zz + c/24
Schwarzian derivative
Laurent expansion in z ⟺ Fourier expansion in w


Laurent expansion in z ⟺ Fourier expansion in w.
24


$T = T_{zz} \longrightarrow T_{ww} = (\partial_w z)^2 T_{zz} + \frac{c}{24}$
Scharzian derivati
Laurent expansion in z ⟺ Fourier expansion in w.
$T_{ww}(w) =$

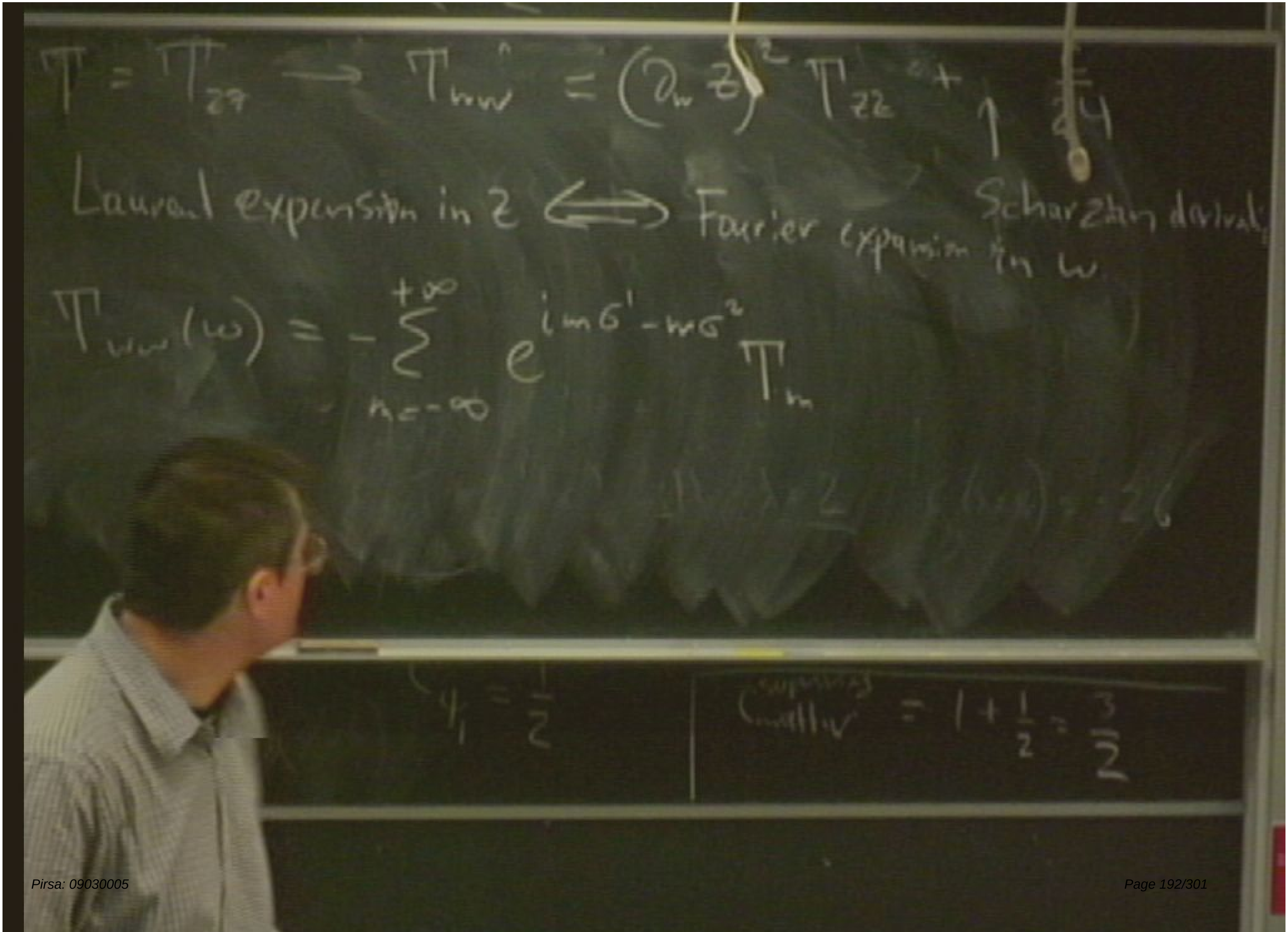
z_{12}

$$T = T_{zz} \longrightarrow T_{ww} = (\partial_w z)^2 \, T_{zz} + \frac{c}{24}$$

Scharzian derivati

Laurent expansion in z $\Longleftrightarrow$ Fourier expansion in w

$$T_{ww}(w) =$$


T = T_zz → T_ww = (∂_w z)^2 T_zz + ...
Scharzian derivative
Laurent expansion in z ⟺ Fourier expansion in w.
T_ww(w) = − Σ_{n=−∞}^{+∞} e^{inσ¹ − nσ²} T_n
= 1 + 1/2 = 3/2

z_{12}

$$T = T_{zz} \longrightarrow T_{ww} = (\partial_w z)^2 T_{zz} + \frac{c}{24}$$

Scharzian derivati[ve]

Laurent expansion in z $\Longleftrightarrow$ Fourier expansion in w.

$$T_{ww}(w) = -\sum_{m=-\infty}^{+\infty} e^{im\sigma^1 - m\sigma^2} T_m$$

$$T_m = L_m - \delta_{m,0}\frac{c}{24} \qquad \tilde{T}_m = \tilde{L}_m - \frac{\tilde{c}}{24}$$

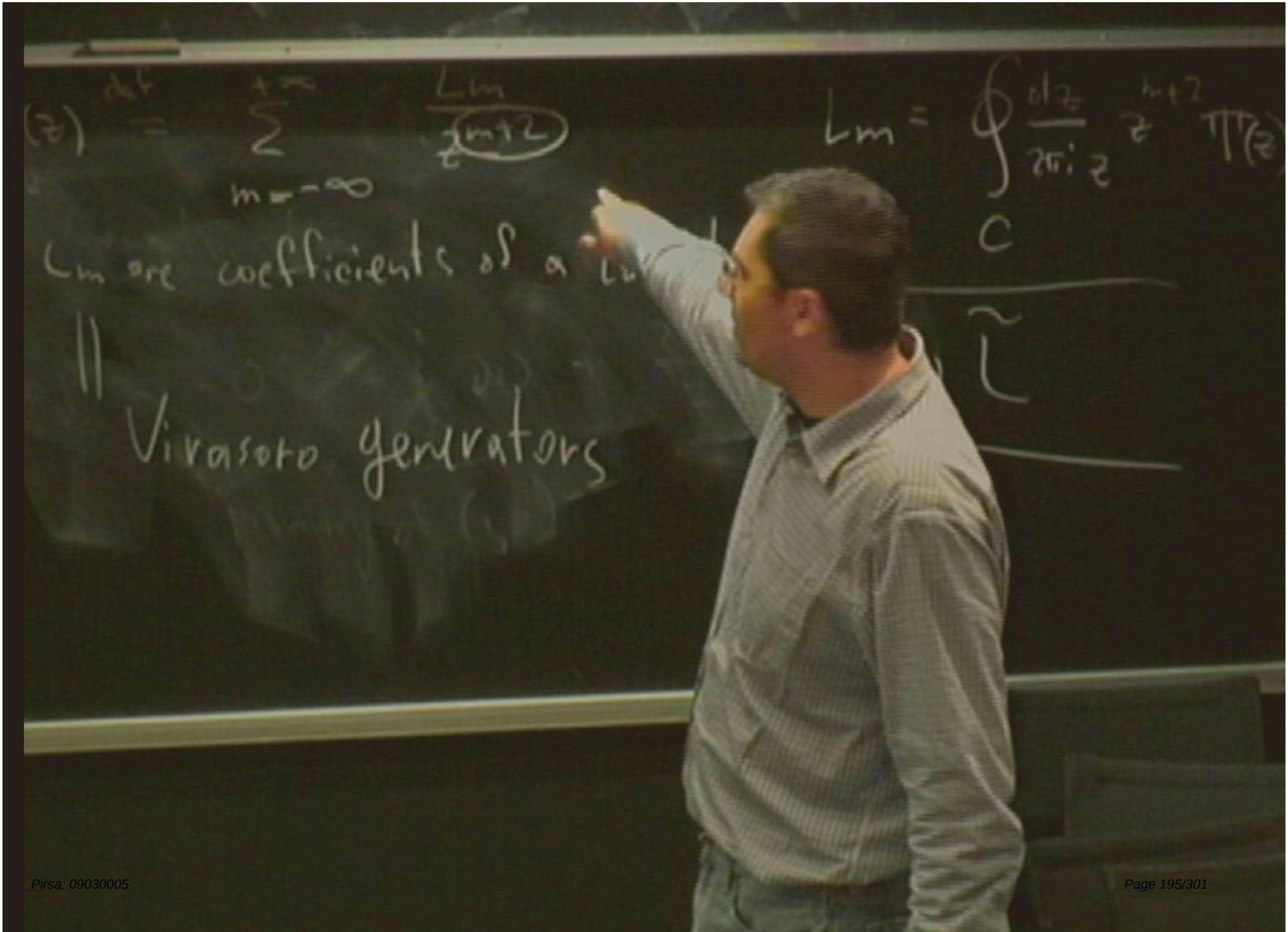
z_{12}

$$T = T_{zz} \longrightarrow T_{ww} = (\partial_w z)^2\, T_{zz} + \uparrow \frac{c}{24}$$

Laurent expansion in $z \Longleftrightarrow$ Fourier expansion in w. Scharzian derivati

$$T_{ww}(w) = -\sum_{m=-\infty}^{+\infty} e^{im\sigma^1 - m\sigma^2}\, T_m$$

$$T_m = L_m - \delta_{m0}\frac{c}{24} \qquad \tilde{T}_m = \tilde{L}_m - \frac{\tilde{c}}{24}$$



$(z) \overset{def}{=} \sum_{m=-\infty}^{+\infty} \frac{L_m}{z^{m+2}}$
$L_m = \oint_C \frac{dz}{2\pi i\, z} z^{m+2} T(z)$
Lm are coefficients of a
||
Virasoro generators

$T = T_{zz} \longrightarrow T_{ww} = (\partial_w z)^2\, T_{zz} + \frac{c}{24}$

Scharzian derivati

Laurent expansion in $z \Longleftrightarrow$ Fourier expansion in w.

$$T_{ww}(w) = -\sum_{n=-\infty}^{+\infty} e^{in\sigma^1 - n\sigma^2}\, T_n$$

$$T_n = L_n - \delta_{n0}\frac{c}{24} \qquad \tilde{T}_n = \tilde{L}_n - \frac{\tilde{c}}{24}$$

z_{12}

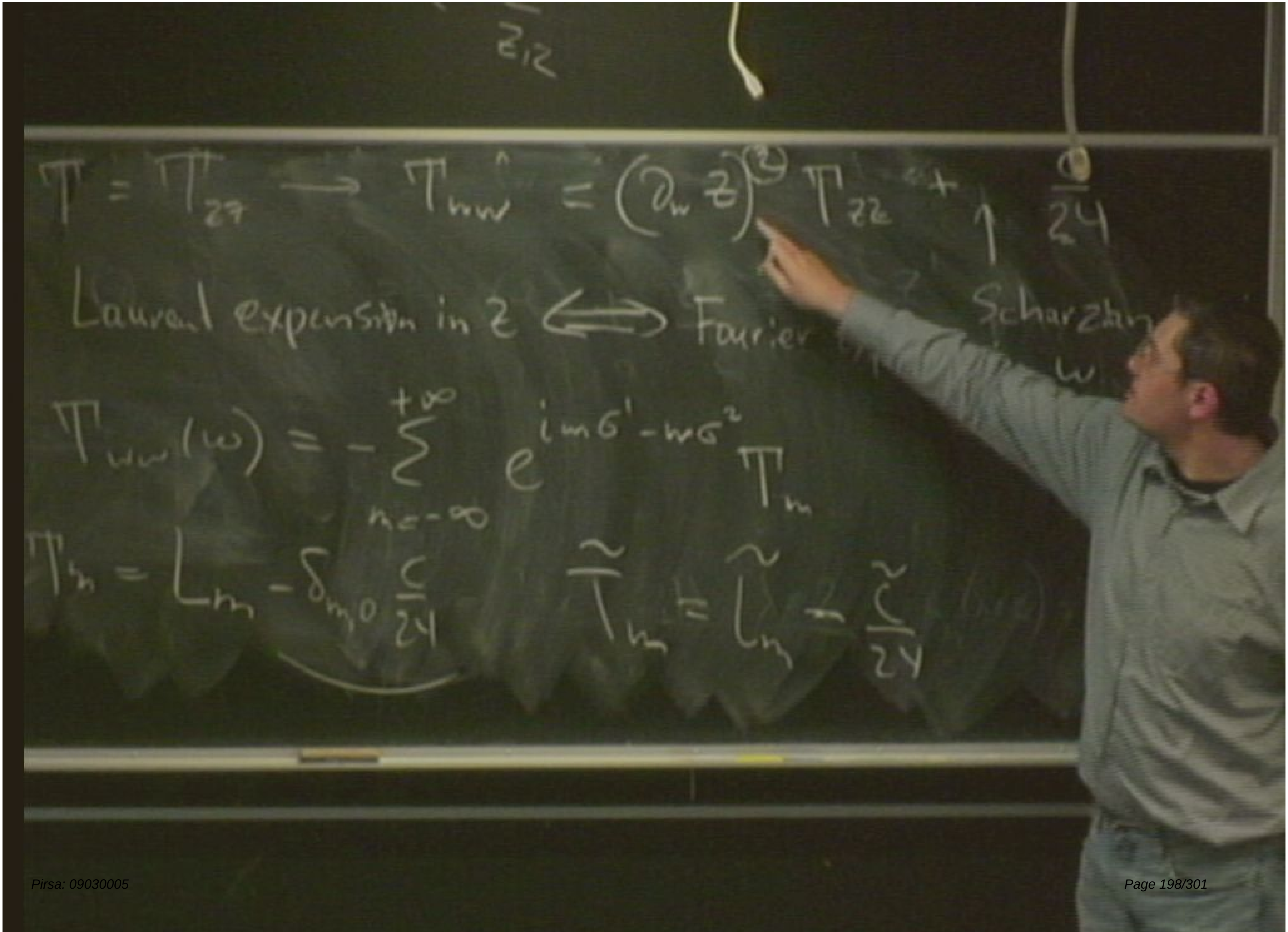
$$T = T_{zz} \longrightarrow T_{ww} = (\partial_w z)^2 \, T_{zz} + \frac{c}{24}\,\{\ \}$$

Scharzian derivati

Laurent expansion in $z \Longleftrightarrow$ Fourier expansion in w.

$$T_{ww}(w) = -\sum_{m=-\infty}^{+\infty} e^{im\sigma^1 - m\sigma^2}\, T_m$$

$$T_m = L_m - \delta_{m,0}\frac{c}{24} \qquad \tilde{T}_m = \tilde{L}_m - \frac{\tilde{c}}{24}$$


$T = T_{zz} \longrightarrow T_{ww} = (\partial_w z)^2 T_{zz} + \frac{c}{24}$
Scharzian
Laurent expansion in z ⟺ Fourier
w
$T_{ww}(w) = -\sum_{m=-\infty}^{+\infty} e^{im\sigma^1 - w\sigma^2} T_m$
$T_m = L_m - \delta_{m,0} \frac{c}{24}$
$\tilde{T}_m = \tilde{L}_m - \frac{\tilde{c}}{24}$

z_{12}

$$T = T_{zz} \longrightarrow T_{ww} = (\partial_w z)^2 \, T_{zz} + \frac{c}{24}$$

$\partial_w z = -iz$

Scharzian derivati

Laurent expansion in $z \Longleftrightarrow$ Fourier expansion in w.

$$T_{ww}(w) = -\sum_{m=-\infty}^{+\infty} e^{im\sigma^1 - m\sigma^2} \, T_m$$

$$T_m = L_m - \delta_{m0} \frac{c}{24} \qquad \tilde{T}_m = \tilde{L}_m - \frac{\tilde{c}}{24}$$

$\mathbb{T} = \mathbb{T}_{zz} \longrightarrow \mathbb{T}_{ww} = (\partial_w z)^{2} \, \mathbb{T}_{zz} + \frac{c}{24}$

$\partial_w z = -i z$

Schwarzian derivative

Laurent expansion in $z \Longleftrightarrow$ Fourier expansion in w.

$$\mathbb{T}_{ww}(w) = -\sum_{n=-\infty}^{+\infty} e^{i n \sigma^1 - n \sigma^2} \, \mathbb{T}_n$$

$$\mathbb{T}_n = L_n - \delta_{n0} \frac{c}{24} \qquad \tilde{\mathbb{T}}_n = \tilde{L}_n - \frac{\tilde{c}}{24}$$

$$(z) \overset{def}{=} \sum_{m=-\infty}^{+\infty} \frac{L_m}{z^{m+2}}$$

$$L_m = \oint_C \frac{dz}{2\pi i \, z} z^{m+2} T(z)$$

L_m are coefficients of a Laurent exp.

||

Virasoro generators

$$\boxed{\tilde{T}, \tilde{L}}$$

z_{12}

$$T = T_{zz} \longrightarrow T_{ww} = (\partial_w z)^2\, T_{zz} + \frac{c}{24}$$

$\partial_w z = -i z$ Scharzian derivative

Laurent expansion in $z \Longleftrightarrow$ Fourier expansion in w.

$$T_{ww}(w) = -\sum_{n=-\infty}^{+\infty} e^{in\sigma^1 - n\sigma^2}\, T_n$$

$$T_n = L_n - \delta_{n0}\frac{c}{24} \qquad \tilde{T}_n = \tilde{L}_n - \frac{\tilde{c}}{24}$$

z_{12}

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Scharzian derivati

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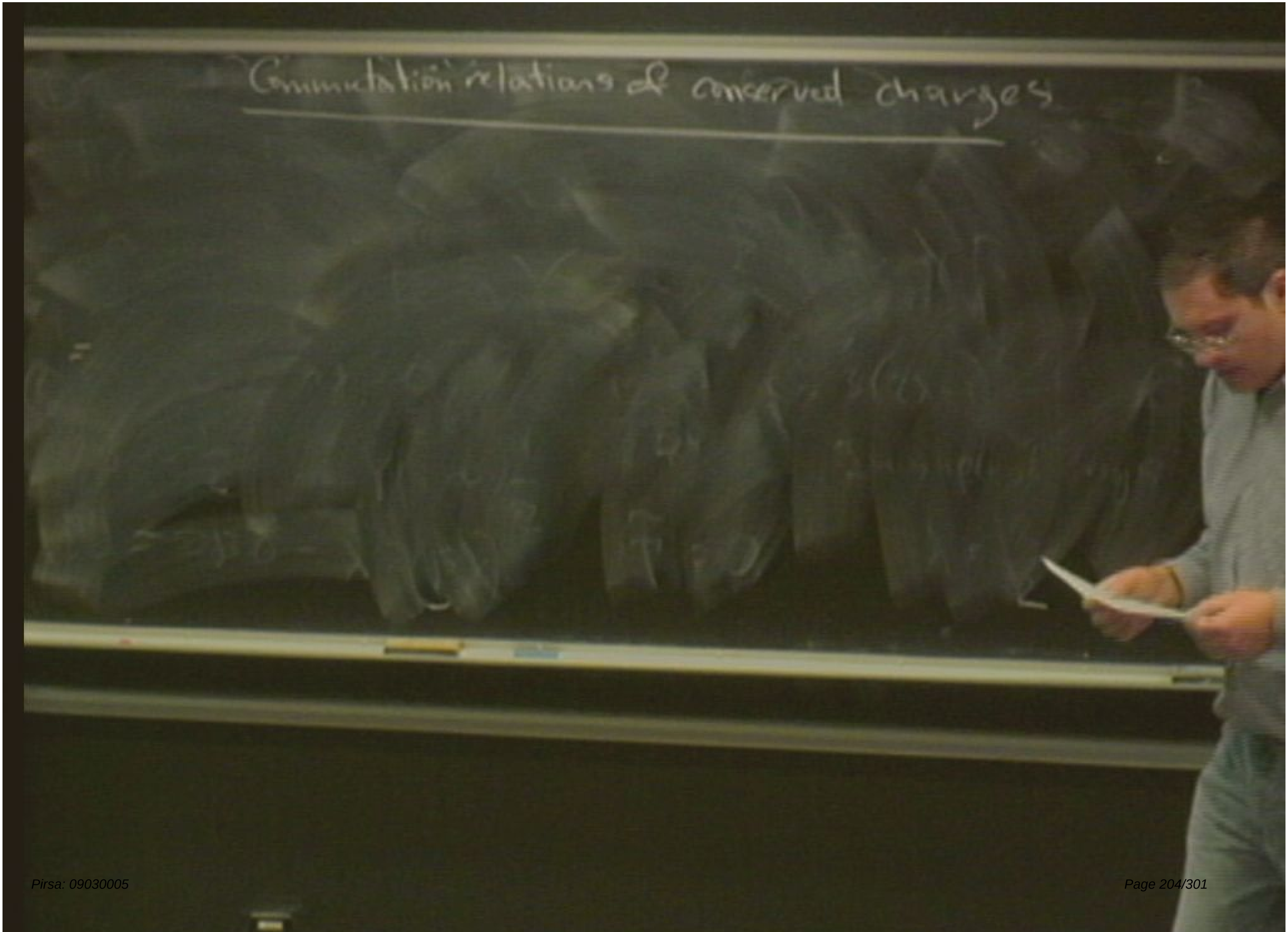
Commutation relations of conserved charges

Commutation relations of conserved charges

$j(z)$ – is conserved current.

Commutation relations of conserved charges

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$Q\{C\}$

Commutation relations of conserved charges

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$$Q\{C\} = \oint_C \frac{dz}{2\pi i} j(z)$$

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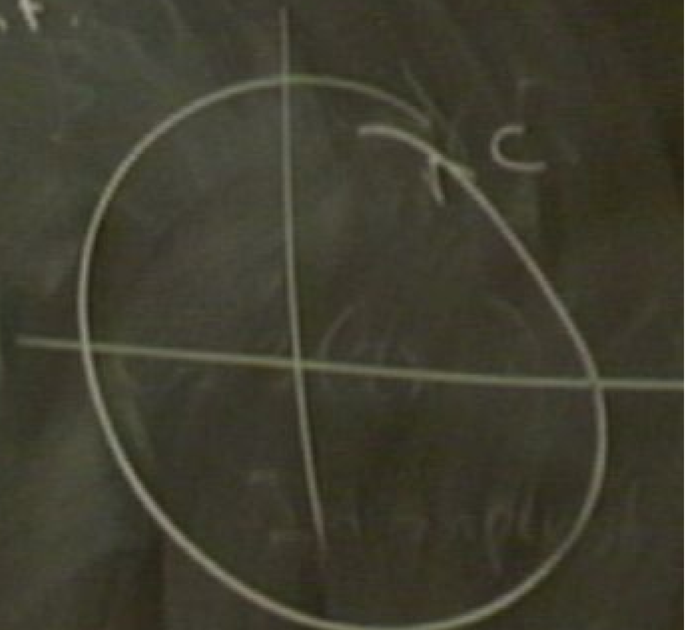
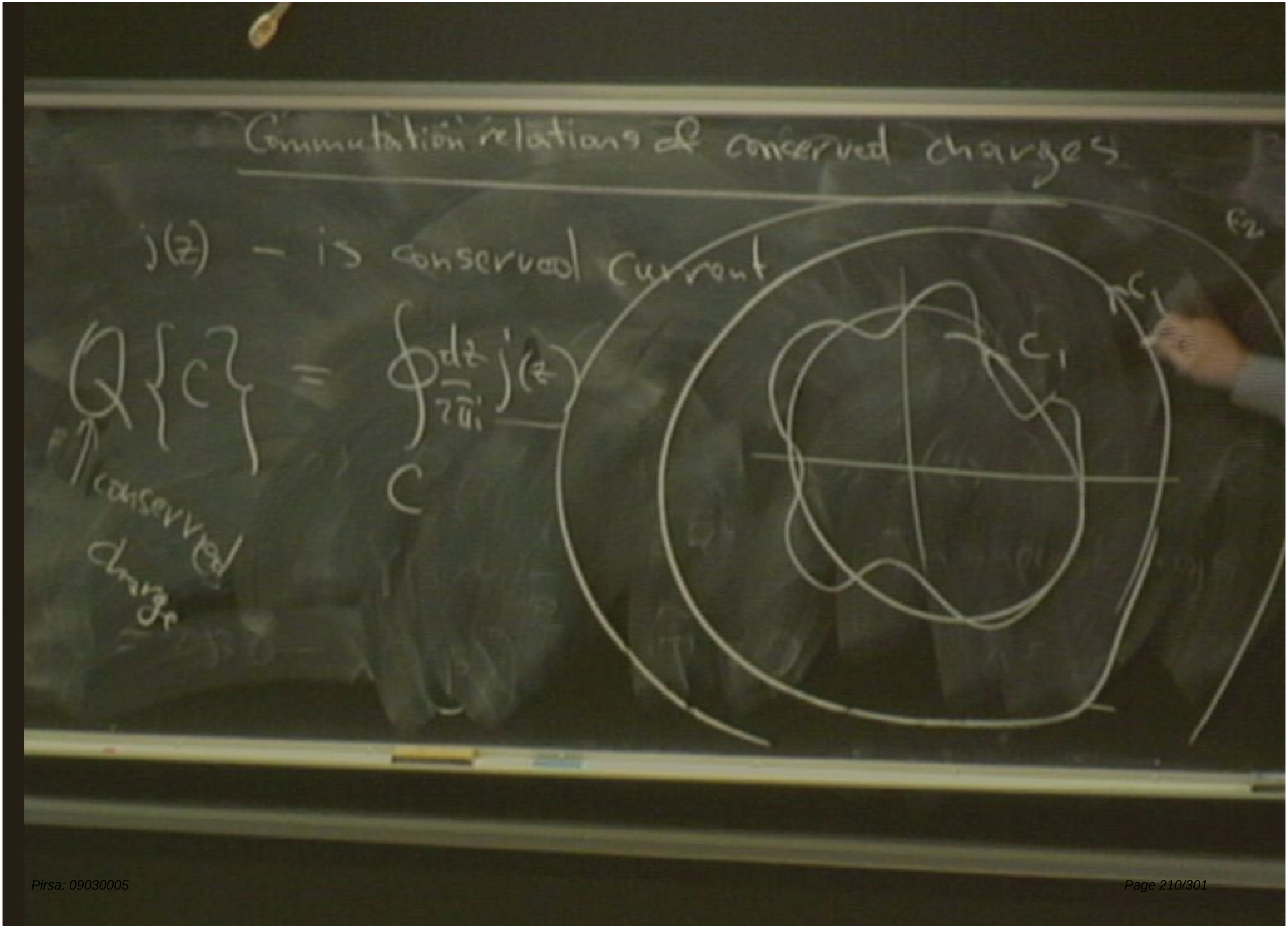




Commutation relations of conserved charges
j(z) - is conserved current
Q{C} = ∮ dz/2πi j(z)
C
conserved charge
C1
C2

Commutation relations of conserved charges

$j(z)$ – is conserved current

$$Q\{C\} = \oint_C \frac{dz}{2\pi i}\, j(z)$$

conserved charge

$$Q\{c_1\} = Q\{c_2\} = Q\{c_3\}$$

$$Q\{C_1\} = Q\{C_2\} = Q\{C_3\}$$

$$t_1 < t_2 < t_3$$

$$Q\{C_1\} = Q\{C_2\} = Q\{C_3\}$$

$$t_1 < t_2 < t_3$$

$j_i(z)$

$$Q\{C_1\} = Q\{C_2\} = Q\{C_3\}$$

$$t_1 < t_2 < t_3$$

$j_i(z)$ $\quad i = 1,2 \Rightarrow Q_1, Q_2$

$$Q\{C_1\} = Q\{C_2\} = Q\{C_3\}$$

$$t_1 < t_2 < t_3$$

$j_i(z)$ $\quad i = 1,2 \Rightarrow Q_1, Q_2$ $\quad \underline{Q'}$ $\quad \lceil Q$

$$Q\{C_1\} = Q\{C_2\} = Q\{C_3\}$$

$$t_1 < t_2 < t_3$$

$j_i(z)$ $\quad i = 1, 2 \Rightarrow Q_1, Q_2$

<u>Q:</u> $[Q_1, Q_2] = \,?$

$$Q\{C_1\} = Q\{C_2\} = Q\{C_3\}$$

$$t_1 < t_2 < t_3$$

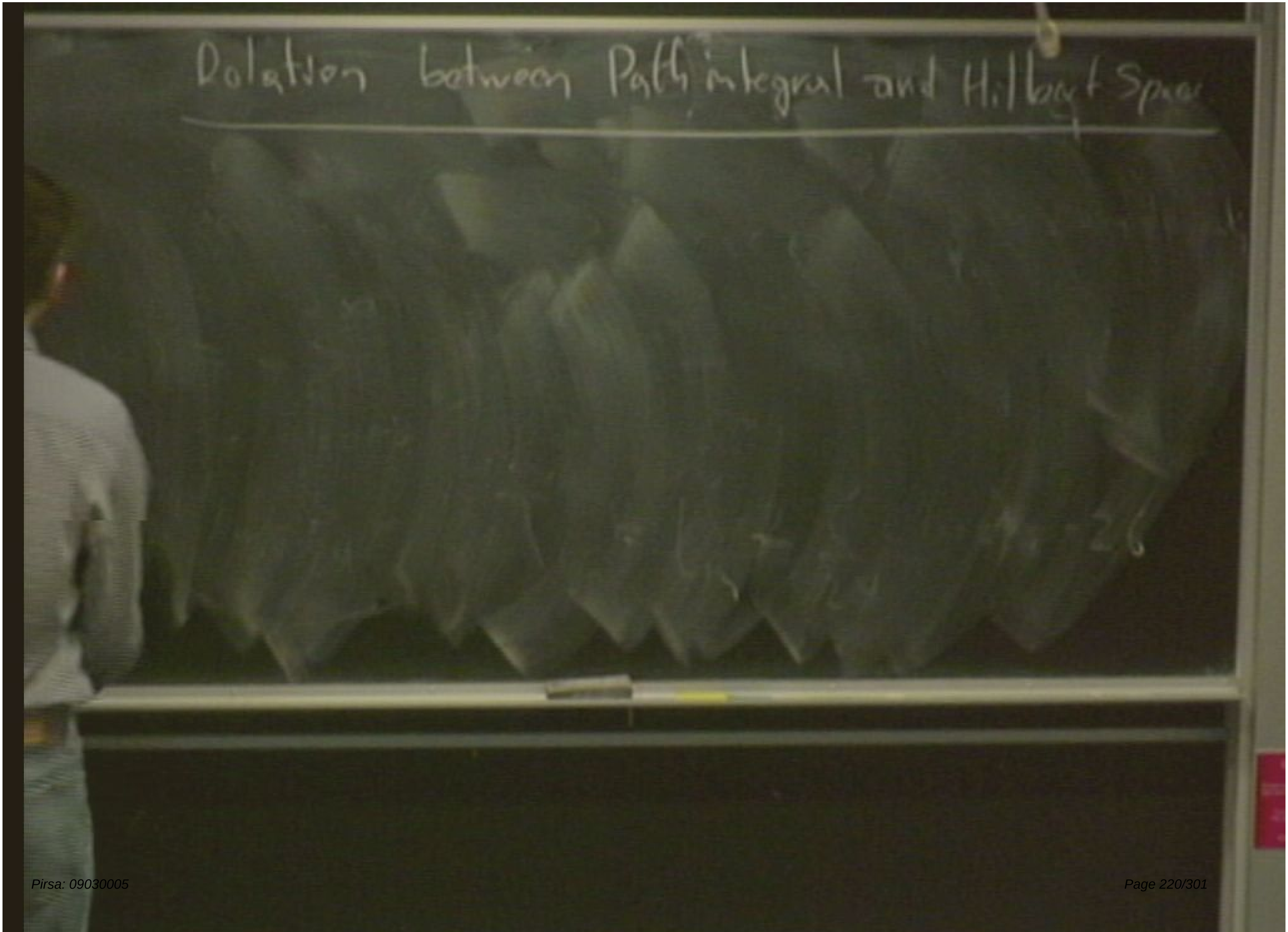
$j_i(z)$ $\quad i = 1,2 \Rightarrow Q_1, Q_2$ $\quad$ Q: $[\hat{Q}_1, \hat{Q}_2] = ?$

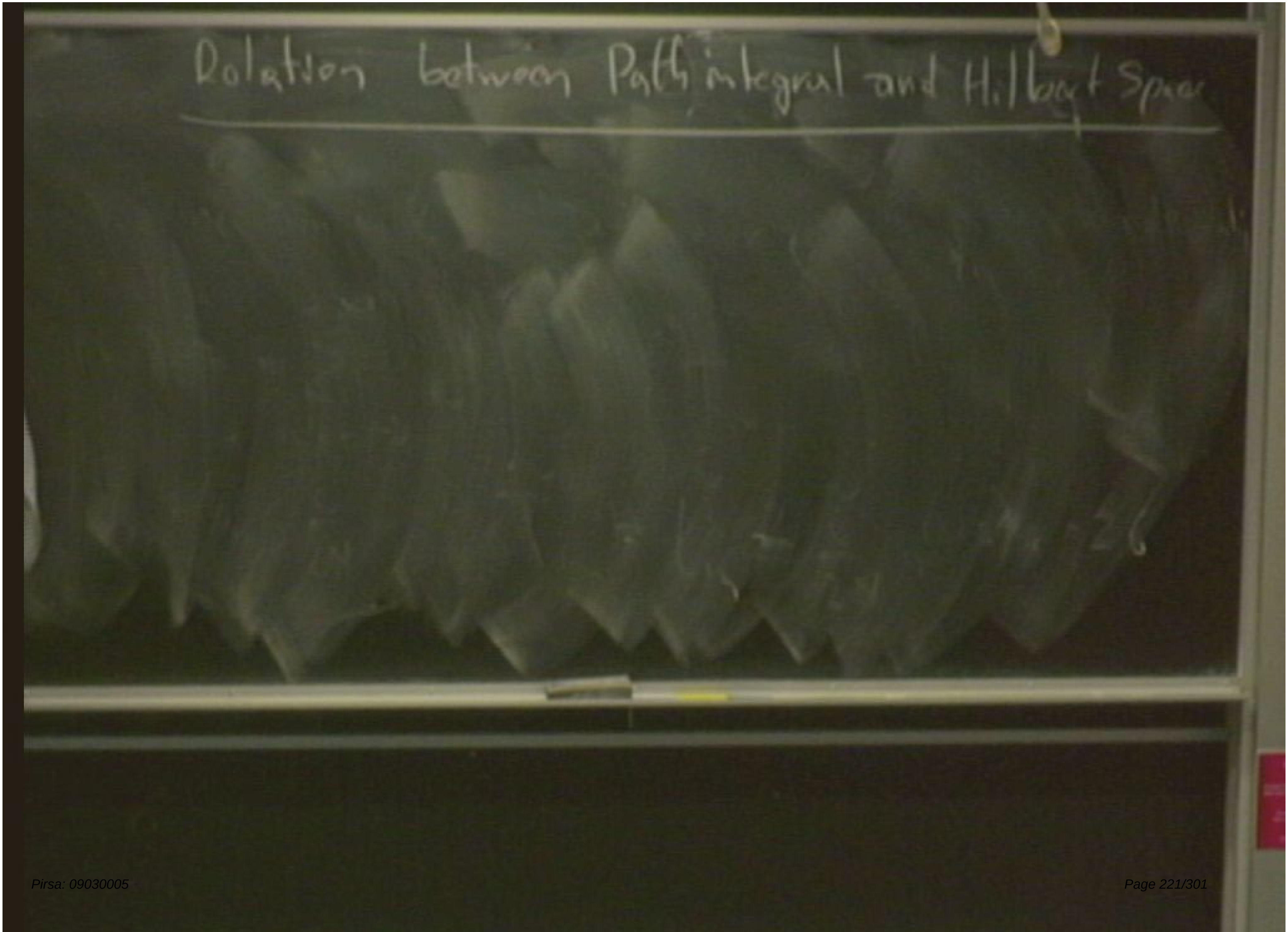
$$Q\{C_1\} = Q\{C_2\} = Q\{C_3\}$$

$$t_1 < t_2 < t_3$$

$j_i(z) \quad i = 1,2 \Rightarrow Q_1 \ , Q_2$

$\underline{Q:}$ $[\hat{Q}_1 , \hat{Q}_2] = ?$


Relation between Path integral and Hilbert Space


Relation between Path integral and Hilbert Space

Relation between Path integral and Hilbert Space

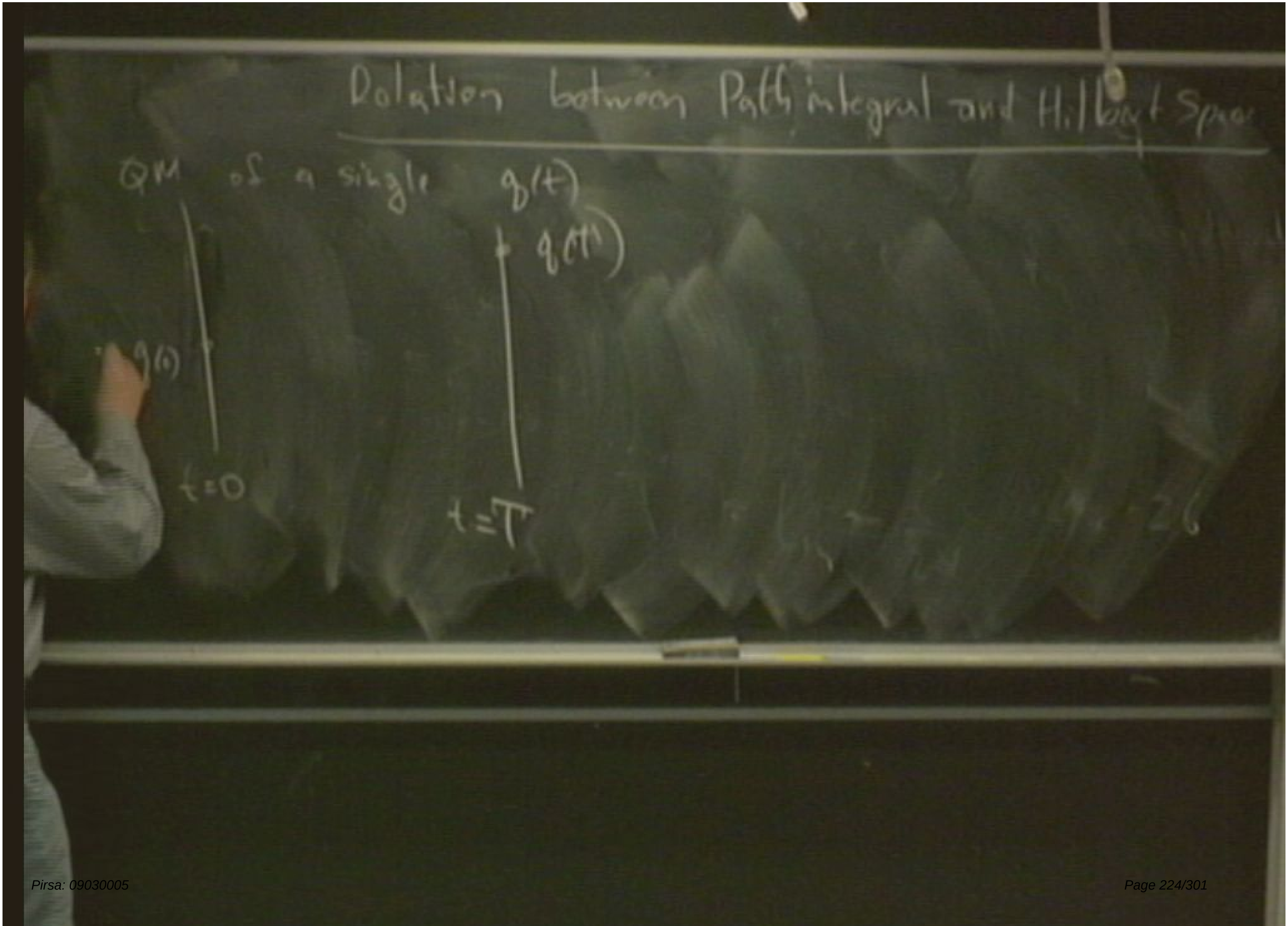
of a single $q(t)$

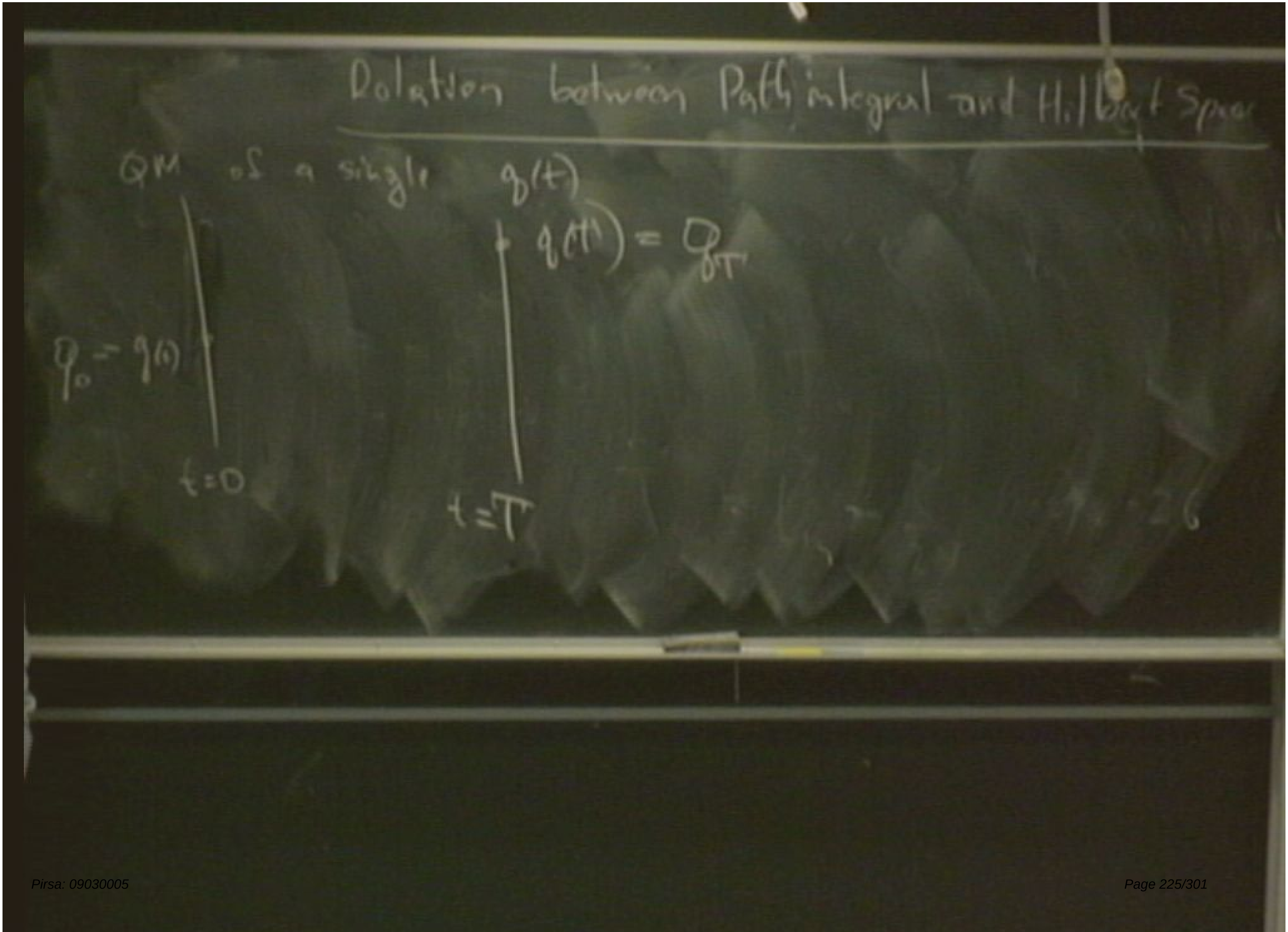
Relation between Path integral and Hilbert Space

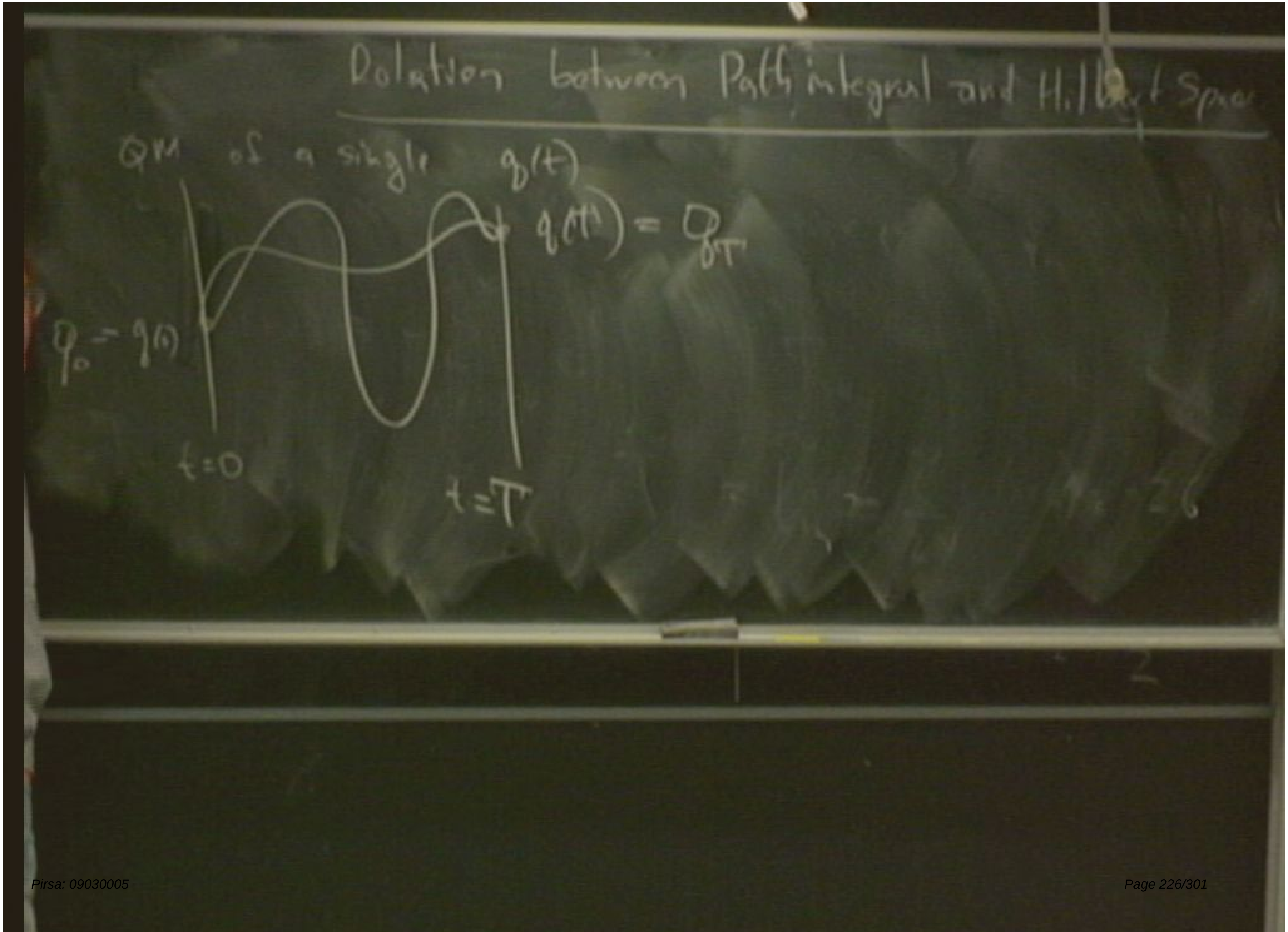
of a single $q(t)$

$t=0$

$t=T$


Relation between Path integral and Hilbert Spa
QM of a single q(t)
q(T)
q(0)
t=0
t=T


Relation between Path integral and Hilbert Space
QM of a single q(t)
q(T) = q_T
q_0 = q(0)
t = 0
t = T


Relation between Path integral and Hilbert Space
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q(T) = q_T
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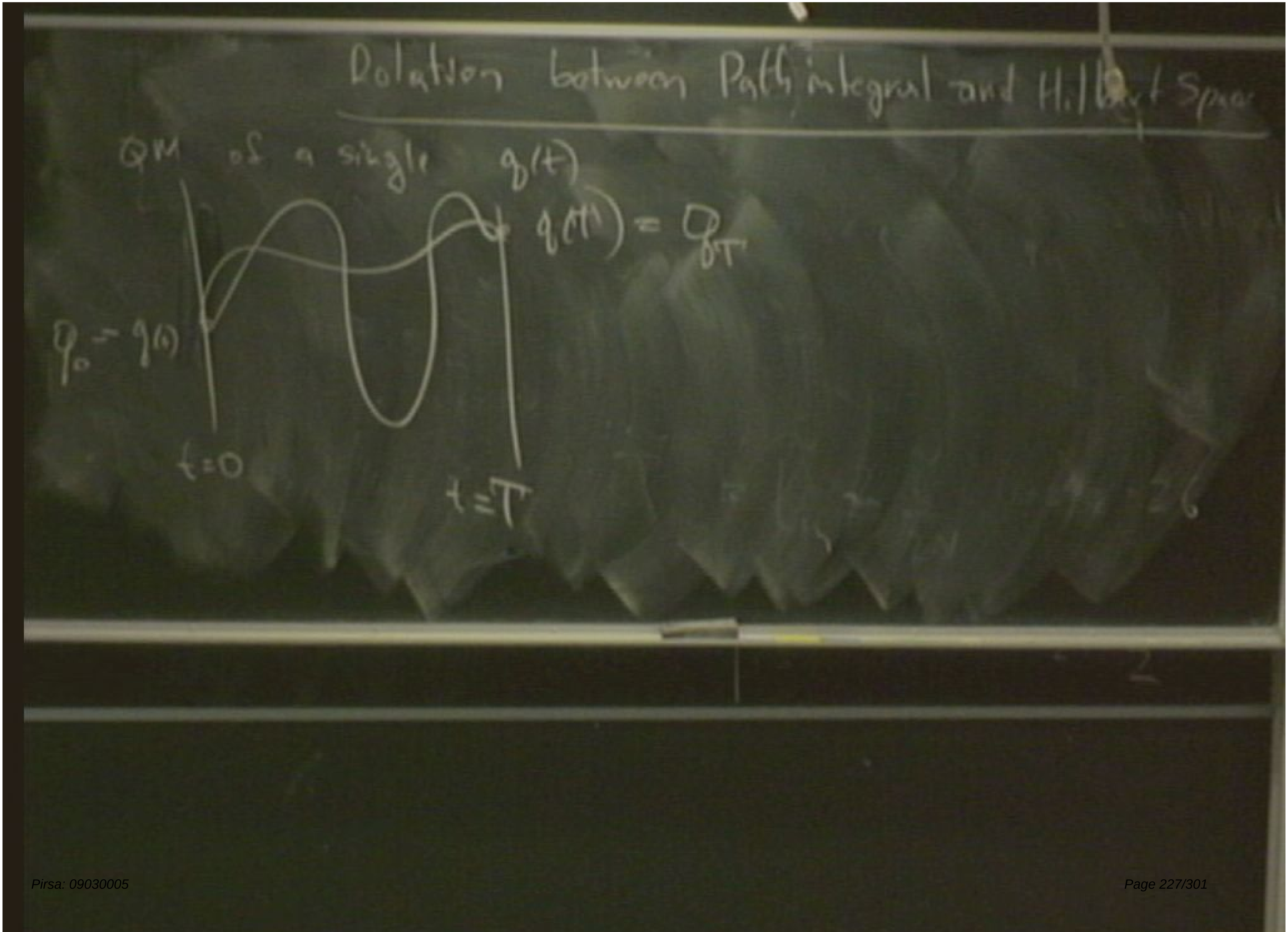

Relation between Path integral and Hilbert Spac
QM of a single q(t)
q(T) = q_T
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t=0
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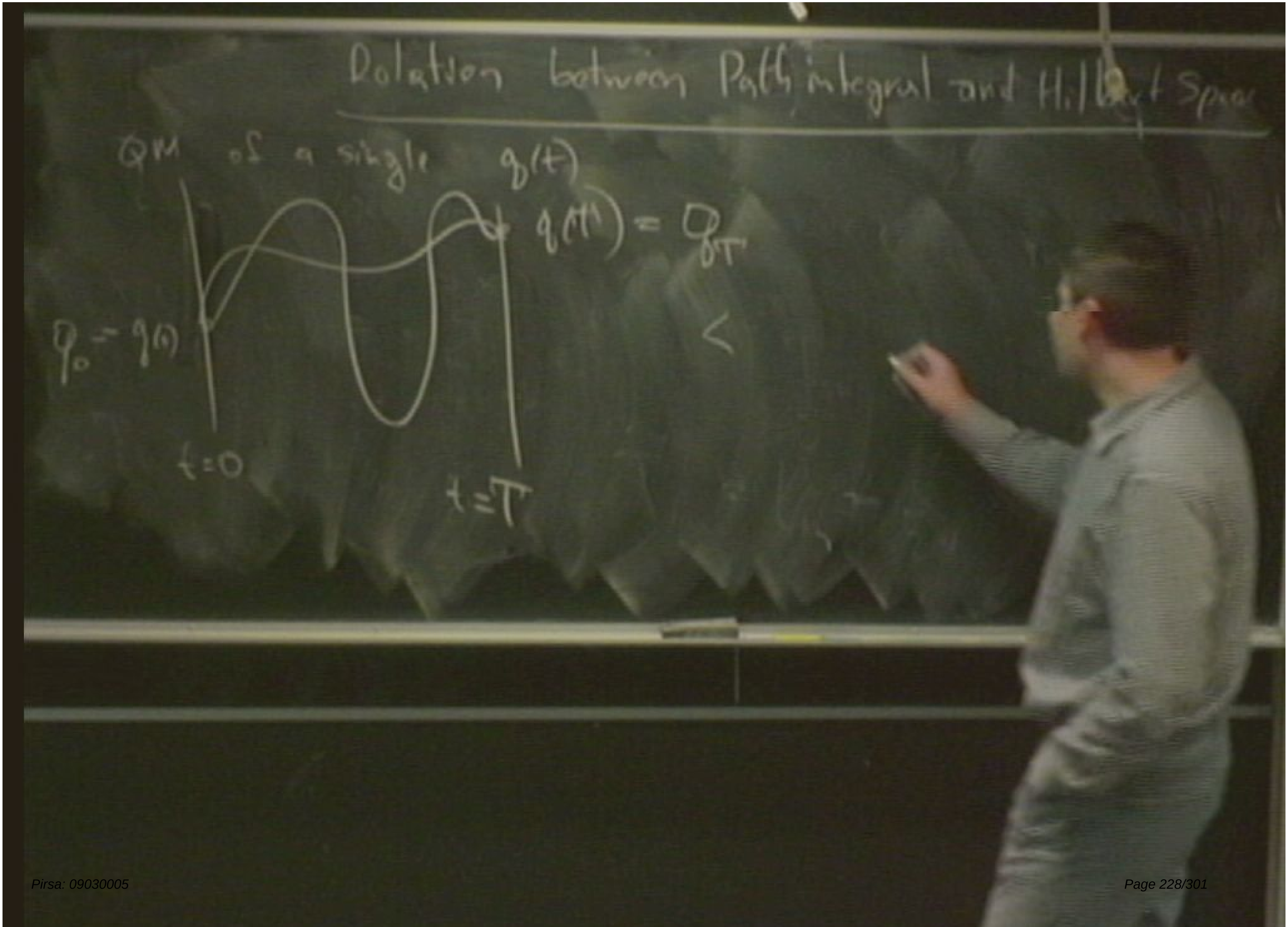
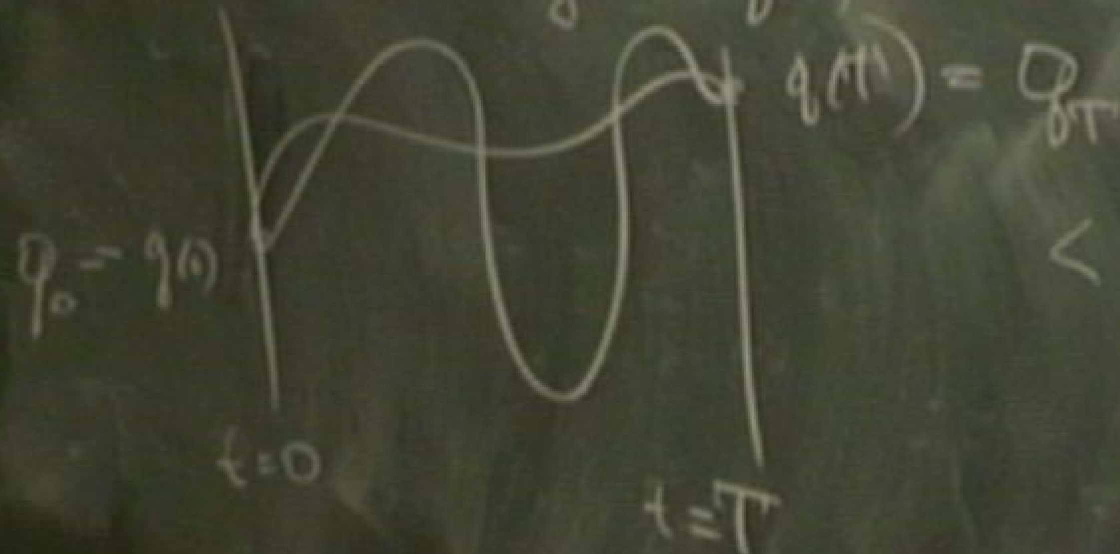

Relation between Path integral and Hilbert Space
QM of a single q(t)
q(T) = q_T
q_0 = q(0)
t=0
t=T

Relation between Path integral and Hilbert Spac

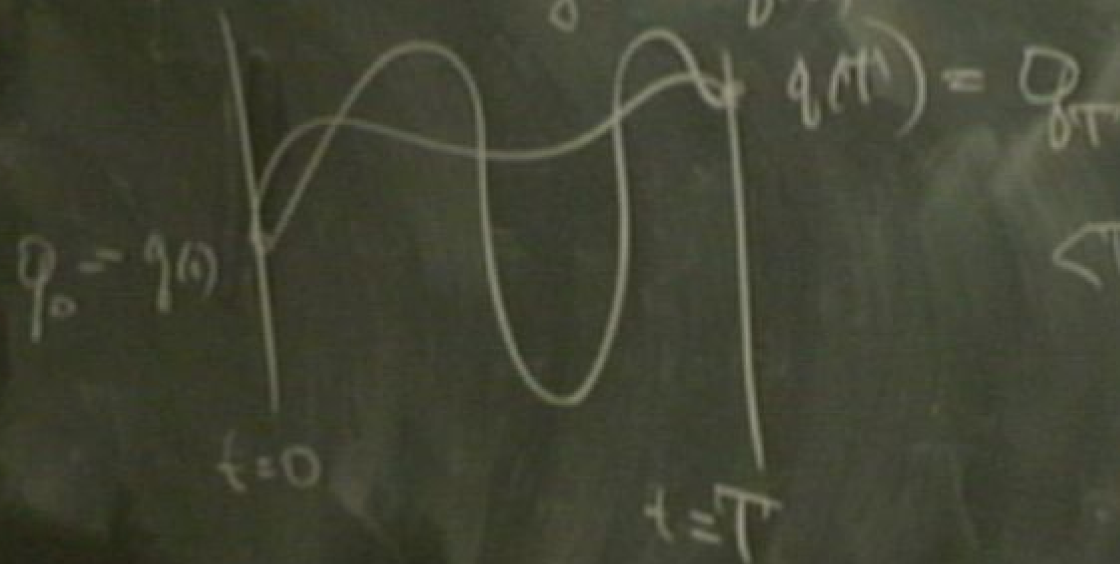
QM of a single $q(t)$



$\langle \quad | \; 0; q_0 \rangle$

Relation between Path integral and Hilbert Space

QM of a single $q(t)$



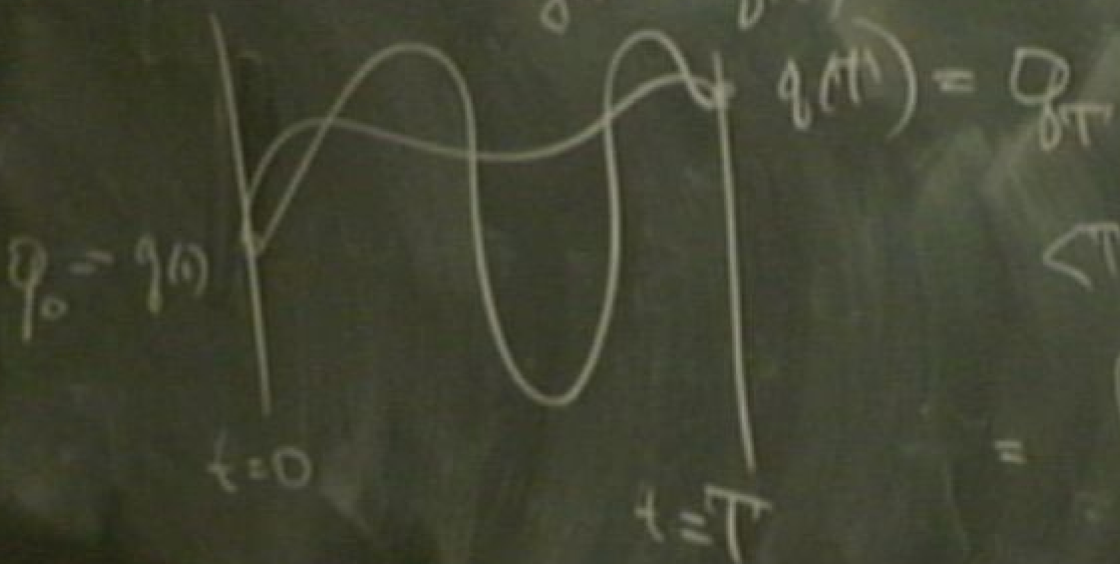
$\langle T, q_T \mid 0; q_0 \rangle$

26

2

Relation between Path integral and Hilbert Space

QM of a single $q(t)$



$$\langle T, q_T \mid 0; q_0 \rangle$$

$$= \int [dq(t)]$$

$q_0 = q(0)$

$t=0$ $\qquad$ $t=T$

$$\langle T, q_T \,|\, 0; q_0 \rangle = \int [dq(t)]\, e^{-S[q]}$$

$$c = \psi = \frac{1}{\sqrt{2}}\left(\psi_1(z) + i\psi_2(z)\right)$$

$$c_{\psi_1} = \frac{1}{2}$$

$$c_{bc}\left(\lambda = \frac{1}{2}\right) = -3(2\lambda-1)^2 + 1 = 1$$

$$\boxed{c^{\text{superstring}}_{\text{matter}} = 1 + \frac{1}{2} = \frac{3}{2}}$$

$q_0 = q(0)$

$t=0$ $t=T$

$\langle T, q_T \mid 0; q_0 \rangle$

$$= \int [dq(t)]\, e^{-S[q]}$$

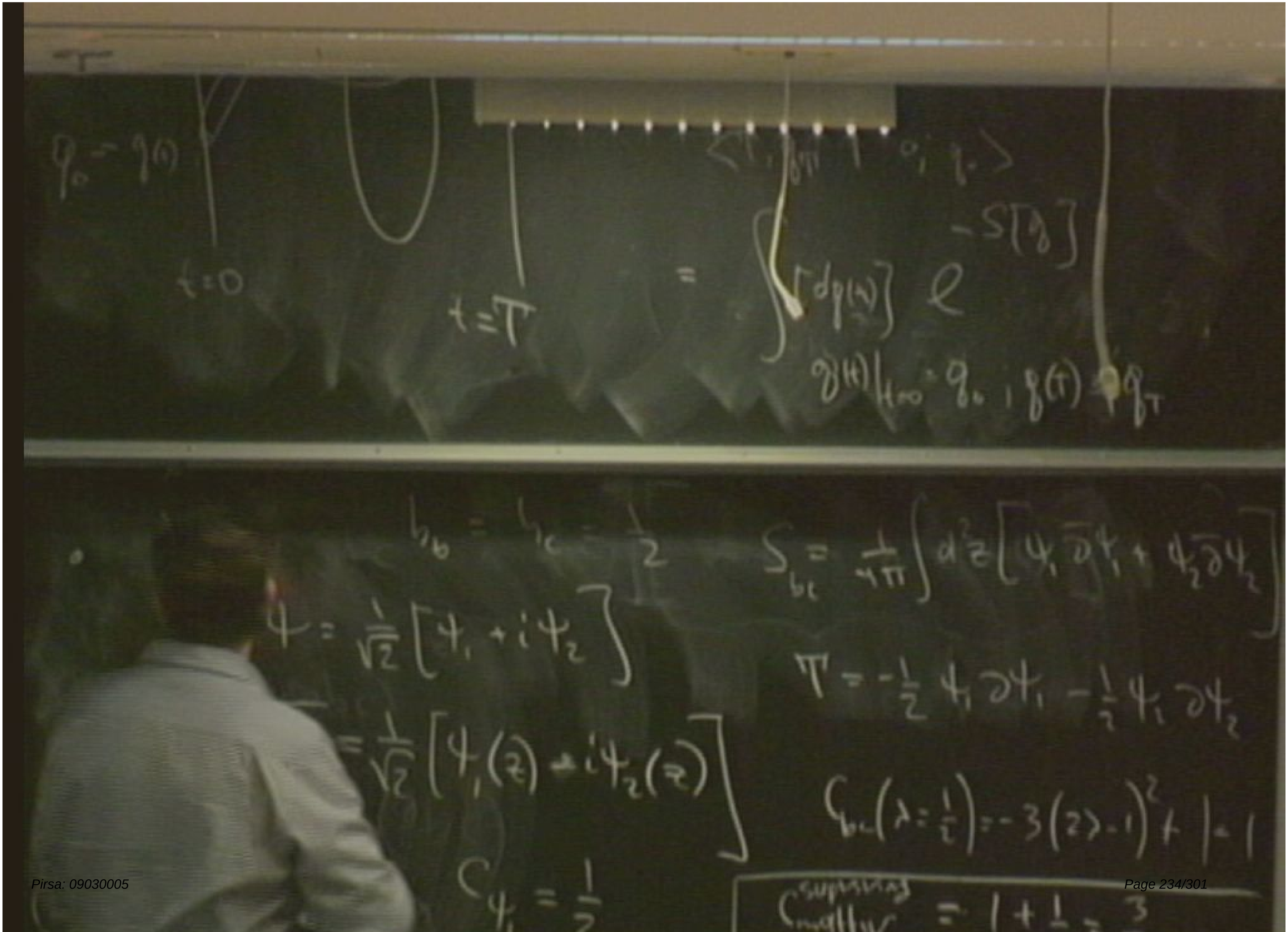
$q(t)|_{t=0} = q_0 \,;\, q(T) = q_T$

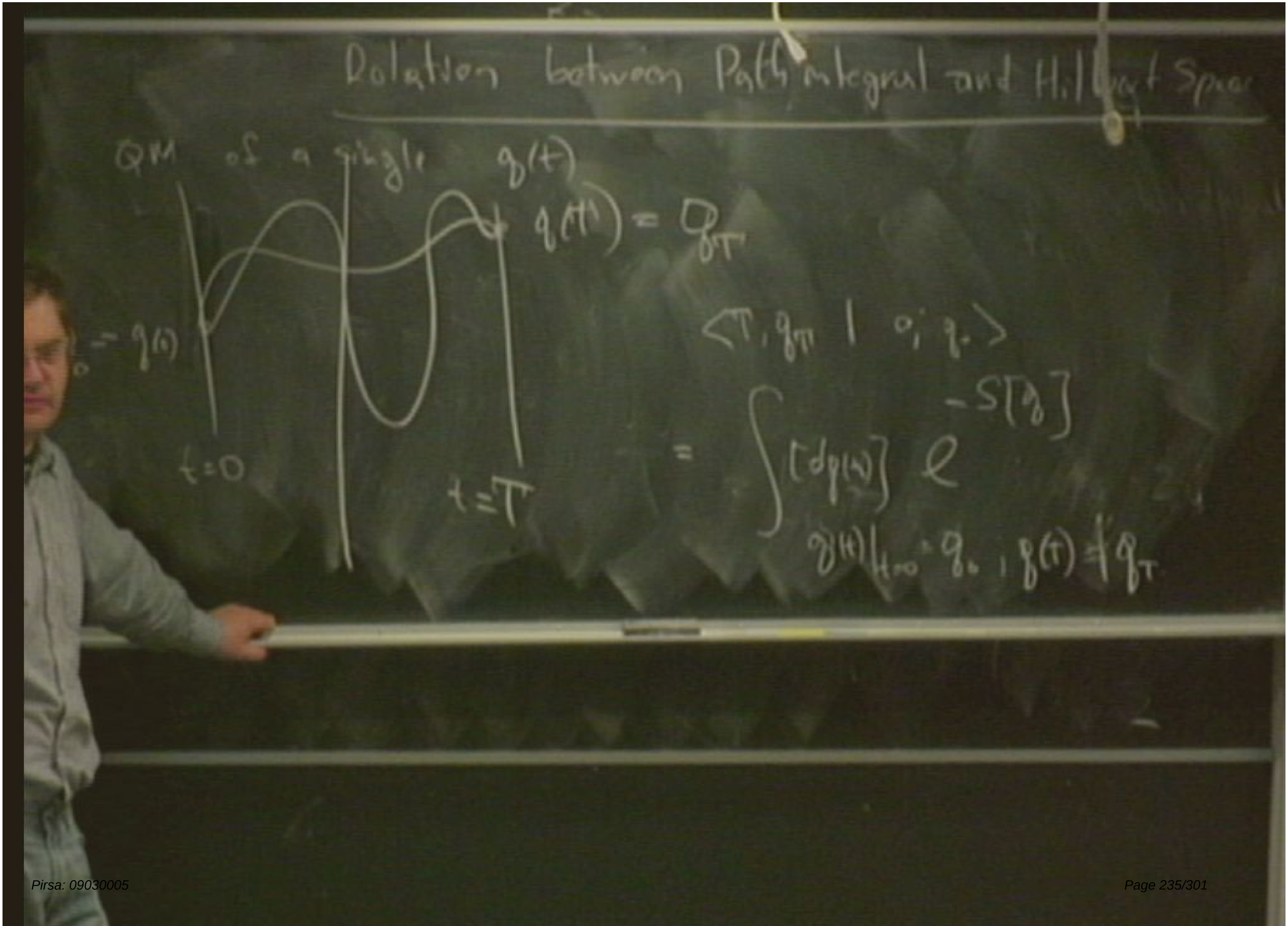
$$c = \overline{\psi} = \frac{1}{\sqrt{2}}\left[\psi_1(z) + i\psi_2(z)\right]$$

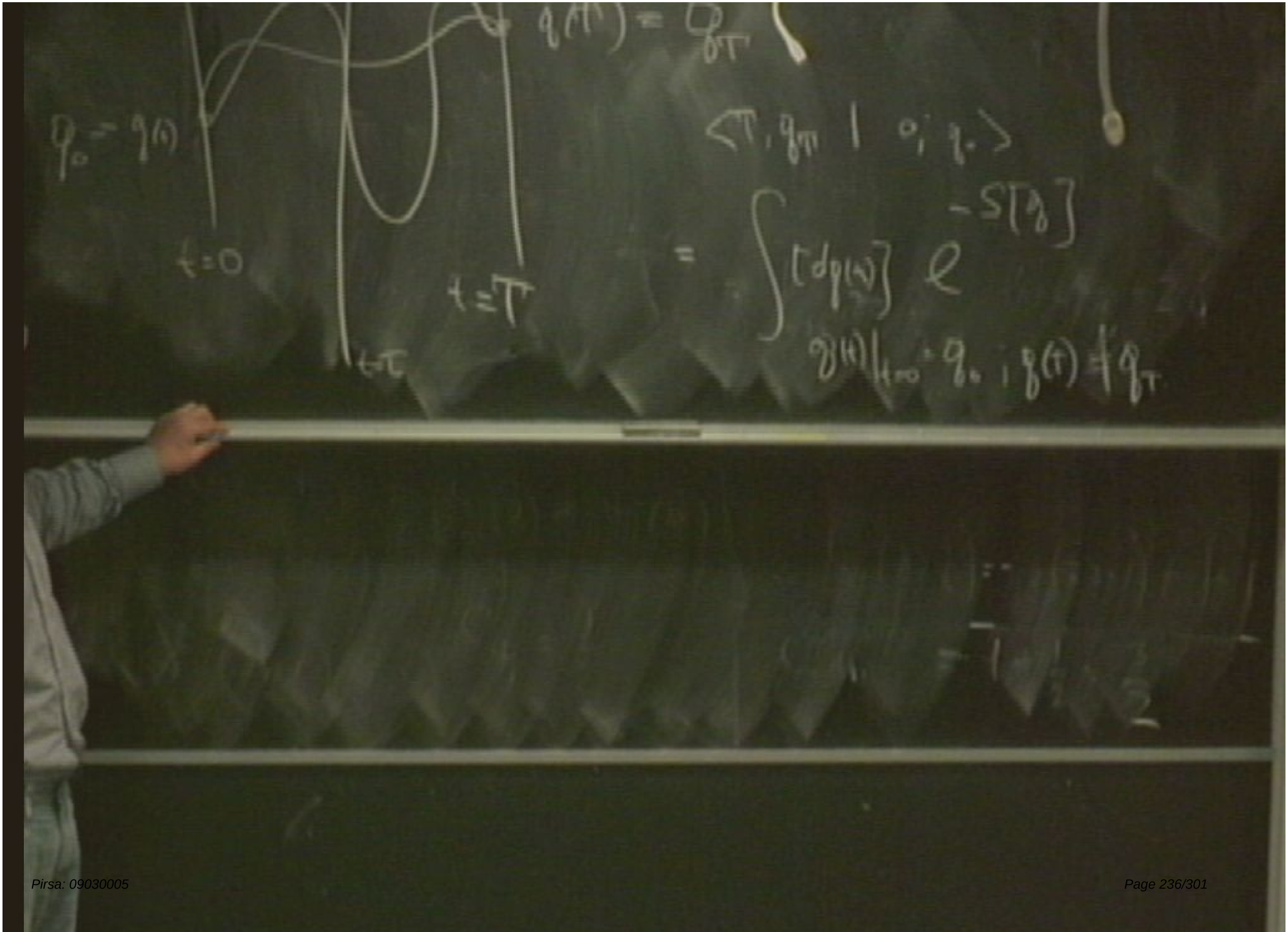
$$c_{\psi_1} = \frac{1}{2}$$

$$c_{bc}\left(\lambda = \tfrac{1}{2}\right) = -3(2\lambda - 1)^2 + 1 = 1$$

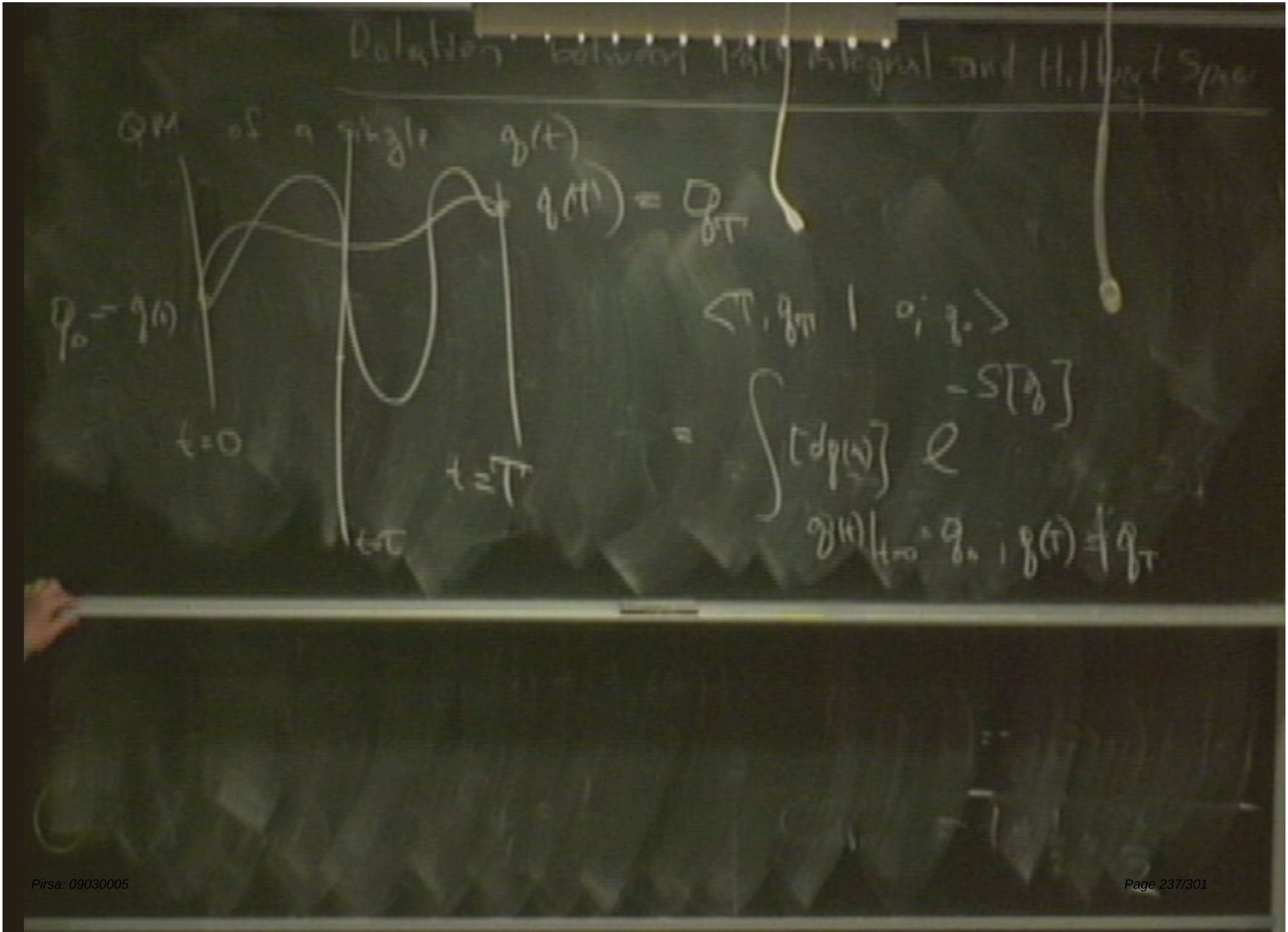
$$\boxed{c^{\text{superstring}}_{\text{matter}} = 1 + \frac{1}{2} = \frac{3}{2}}$$

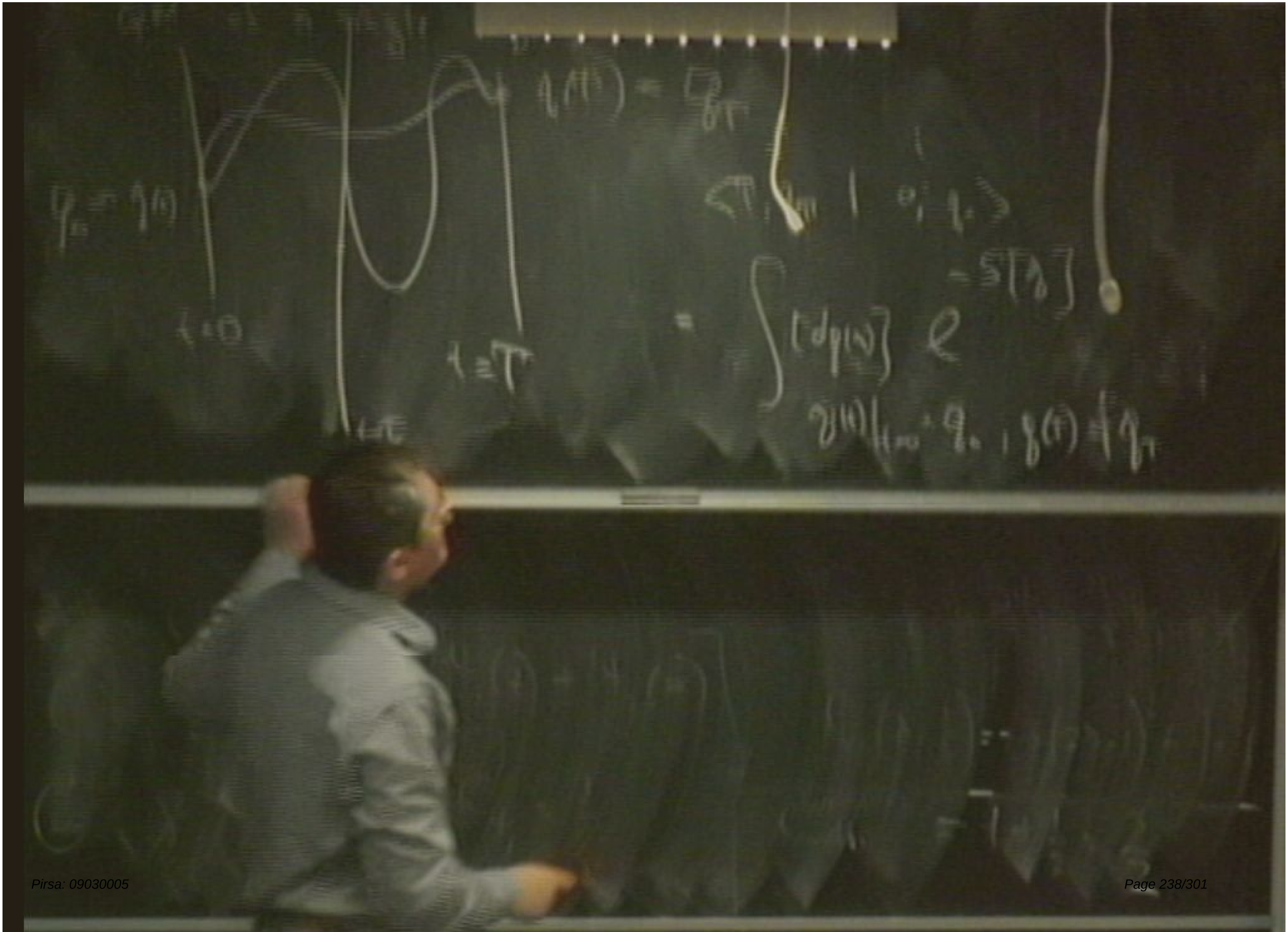

$q_0 = q(0)$
$t=0$
$t=T$
$\langle T, q_T | 0, q_0 \rangle = \int [dq(t)]\, e^{-S[q]}$
$q(t)|_{t=0} = q_0 \,;\, q(T) = q_T$
$h_b = h_c = \frac{1}{2}$
$S_{bc} = \frac{1}{4\pi}\int d^2z \left[\psi_1 \bar\partial \psi_1 + \psi_2 \bar\partial \psi_2\right]$
$\psi = \frac{1}{\sqrt{2}}\left[\psi_1 + i\psi_2\right]$
$= \frac{1}{\sqrt{2}}\left[\psi_1(z) - i\psi_2(z)\right]$
$T = -\frac{1}{2}\psi_1\partial\psi_1 - \frac{1}{2}\psi_2\partial\psi_2$
$c_{bc}(\lambda = \frac{1}{2}) = -3(2\lambda - 1)^2 + 1 = 1$
$c_{\psi_1} = \frac{1}{2}$


Relation between Path integral and Hilbert Space
QM of a single q(t)
q(T) = q_T
q_0 = q(0)
t=0
t=T
⟨T, q_T | 0, q_0⟩
= ∫[dq(t)] e^{-S[q]}
q(t)|_{t=0} = q_0 , q(T) = q_T



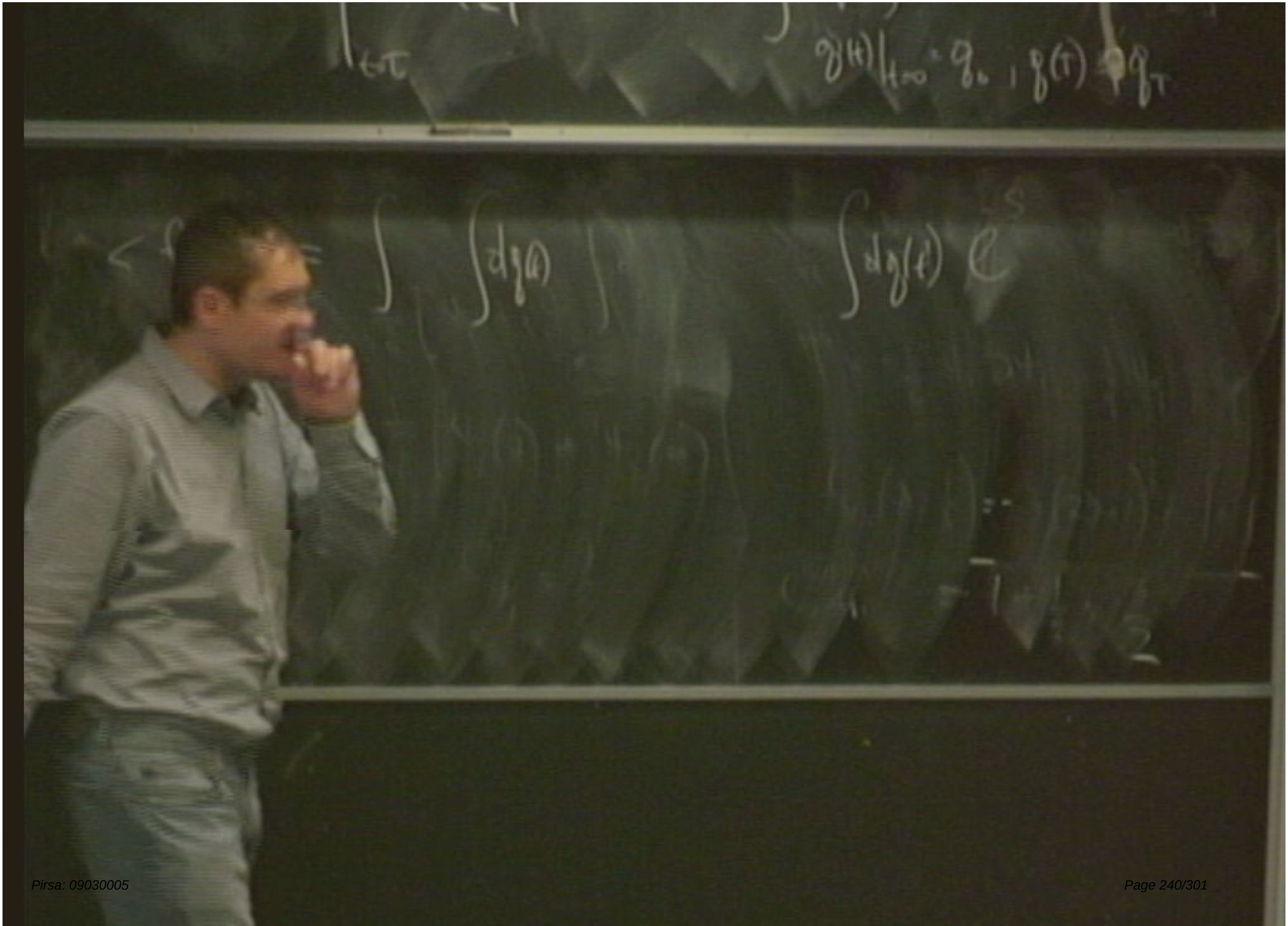
$q(T) = q_T$
$q_0 = q(0)$
$t=0$
$t=T$
$t=T$
$\langle T, q_T \mid 0; q_0 \rangle$
$= \int [dq(t)]\, e^{-S[q]}$
$q(t)|_{t=0} = q_0 \; ; \; q(T) = q_T$

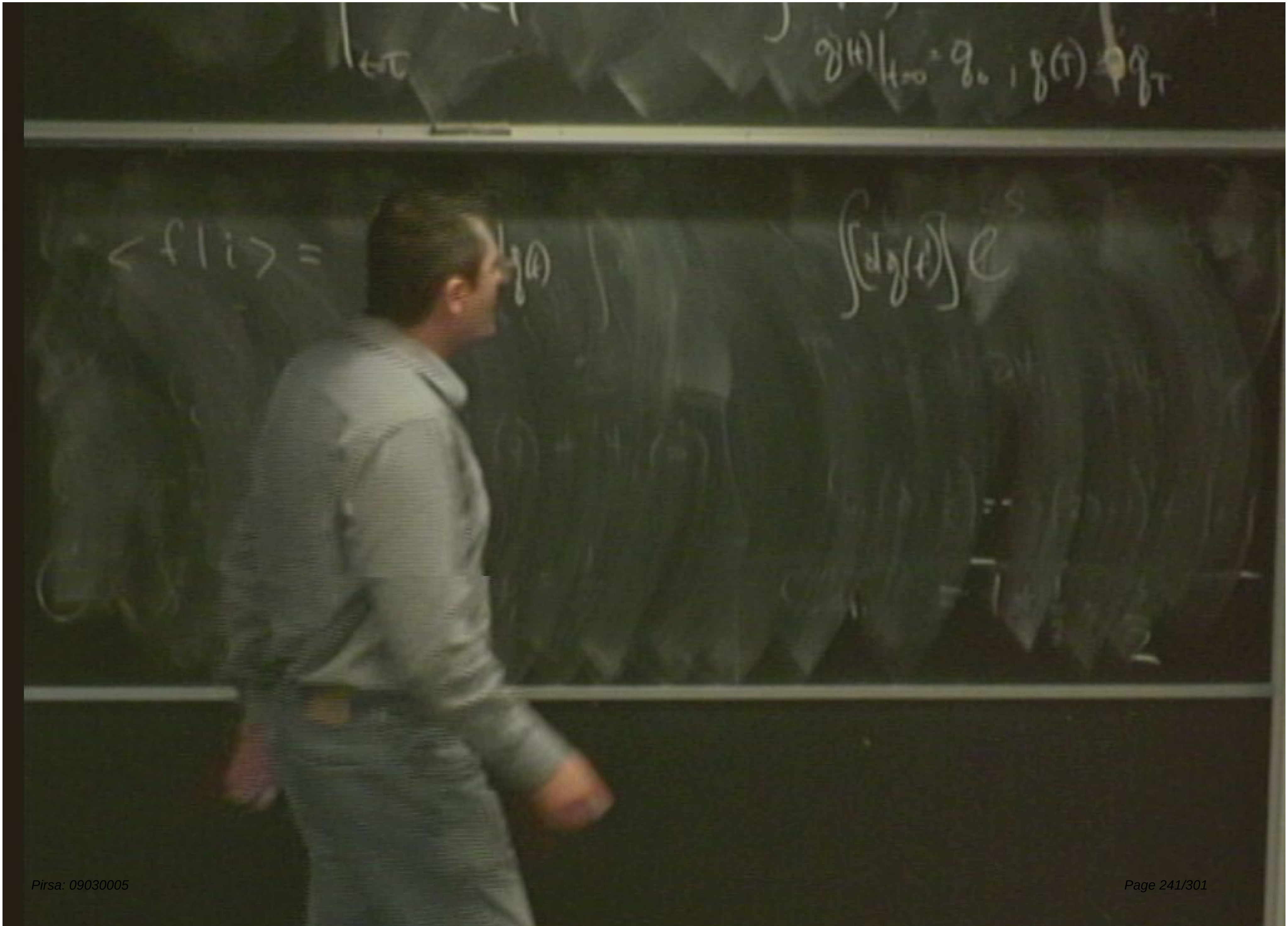

Relation between path integral and Hilbert Space
QM of a single q(t)
q(T) = q_T
q_0 = q(0)
t=0
t=T
⟨T, q_T | 0; q_0⟩
= ∫[dq(t)] e^{-S[q]}
q(t)|_{t=0} = q_0 , q(T) = q_T

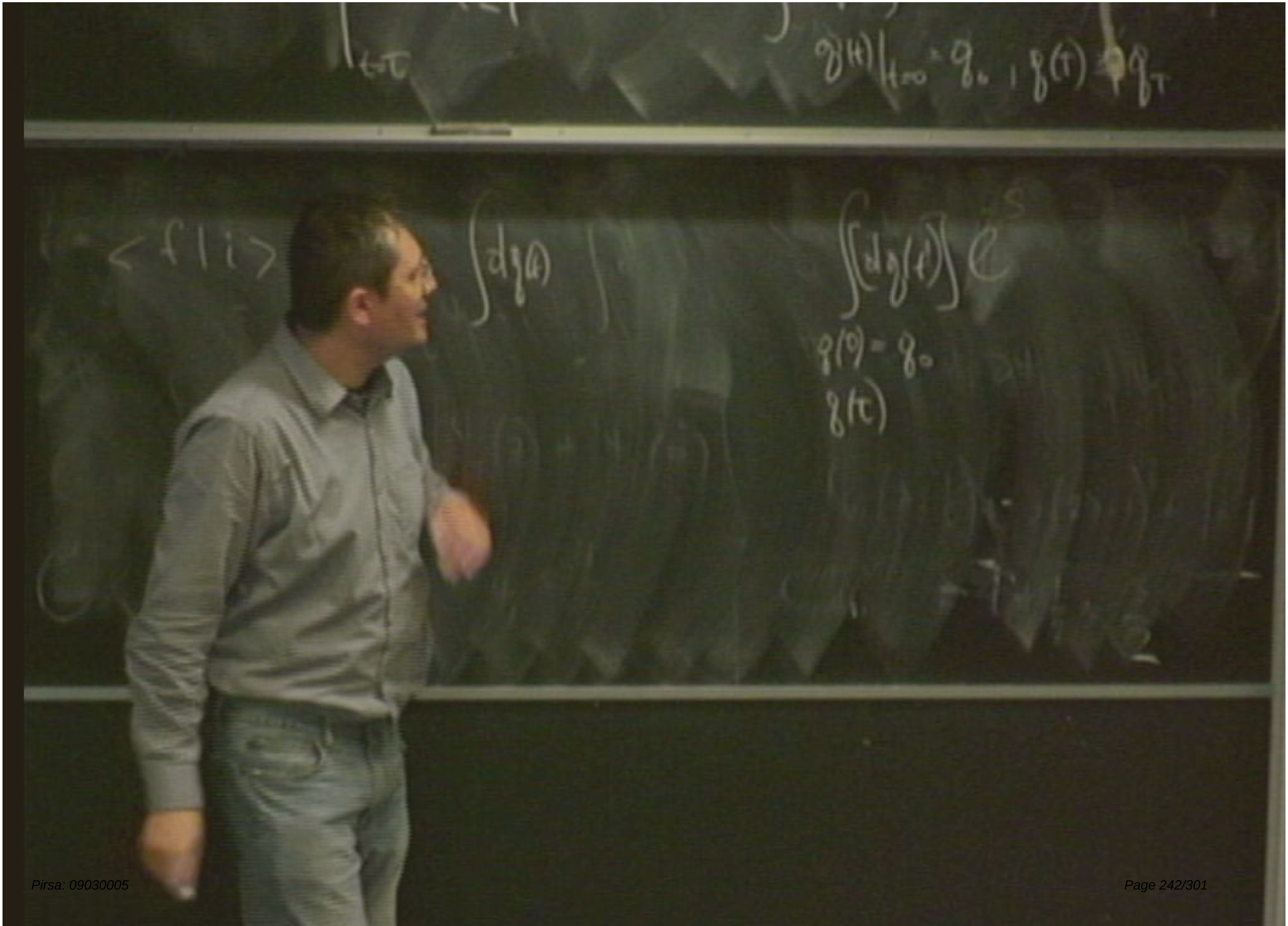


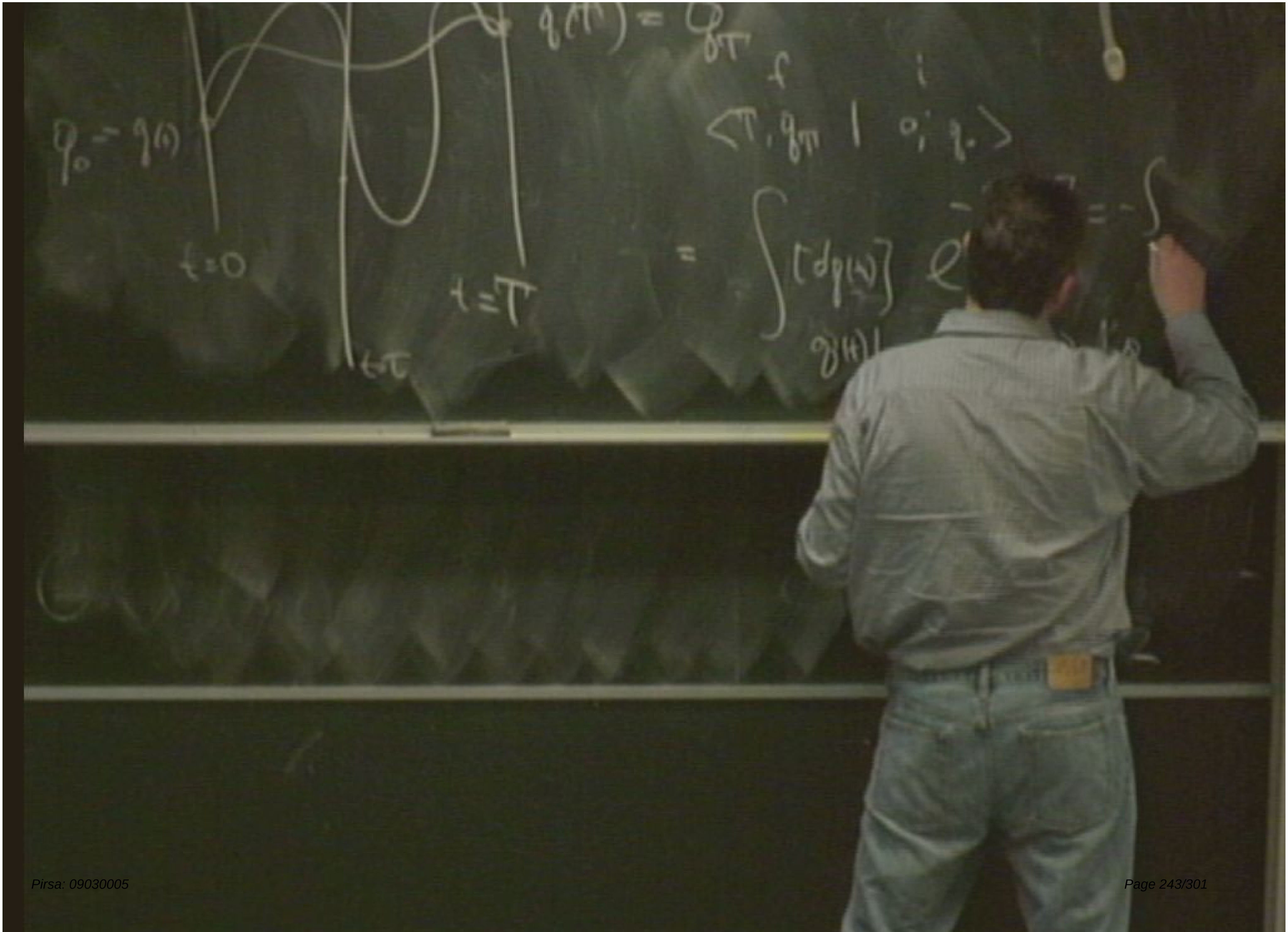
$\Big|_{t=T}$ $\quad q(t)\big|_{t=0} = q_0 \,,\ q(T) = q_T$

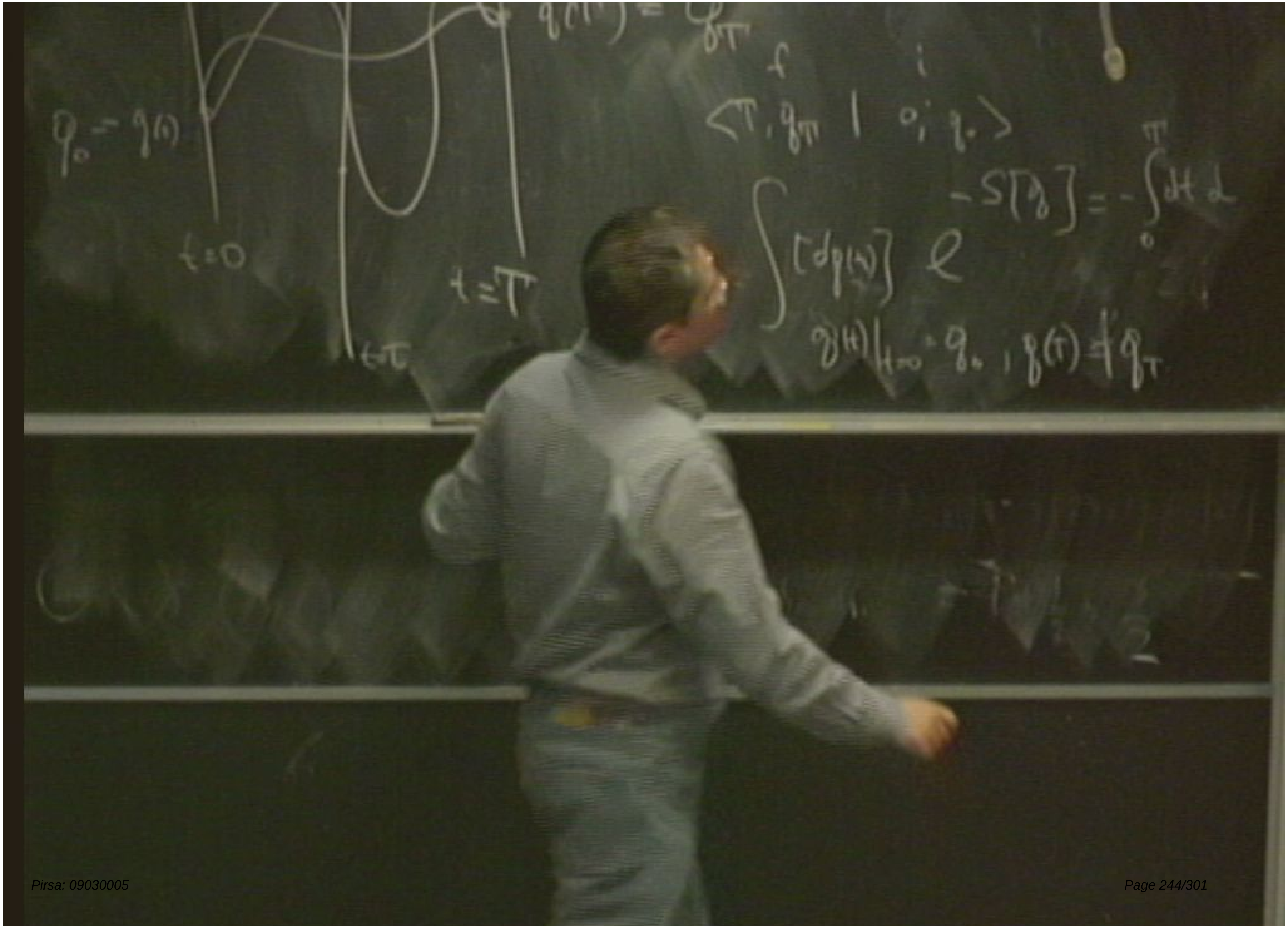
$$\langle f | i \rangle = \int \int dq(t) \int d\dot{q}(t)$$

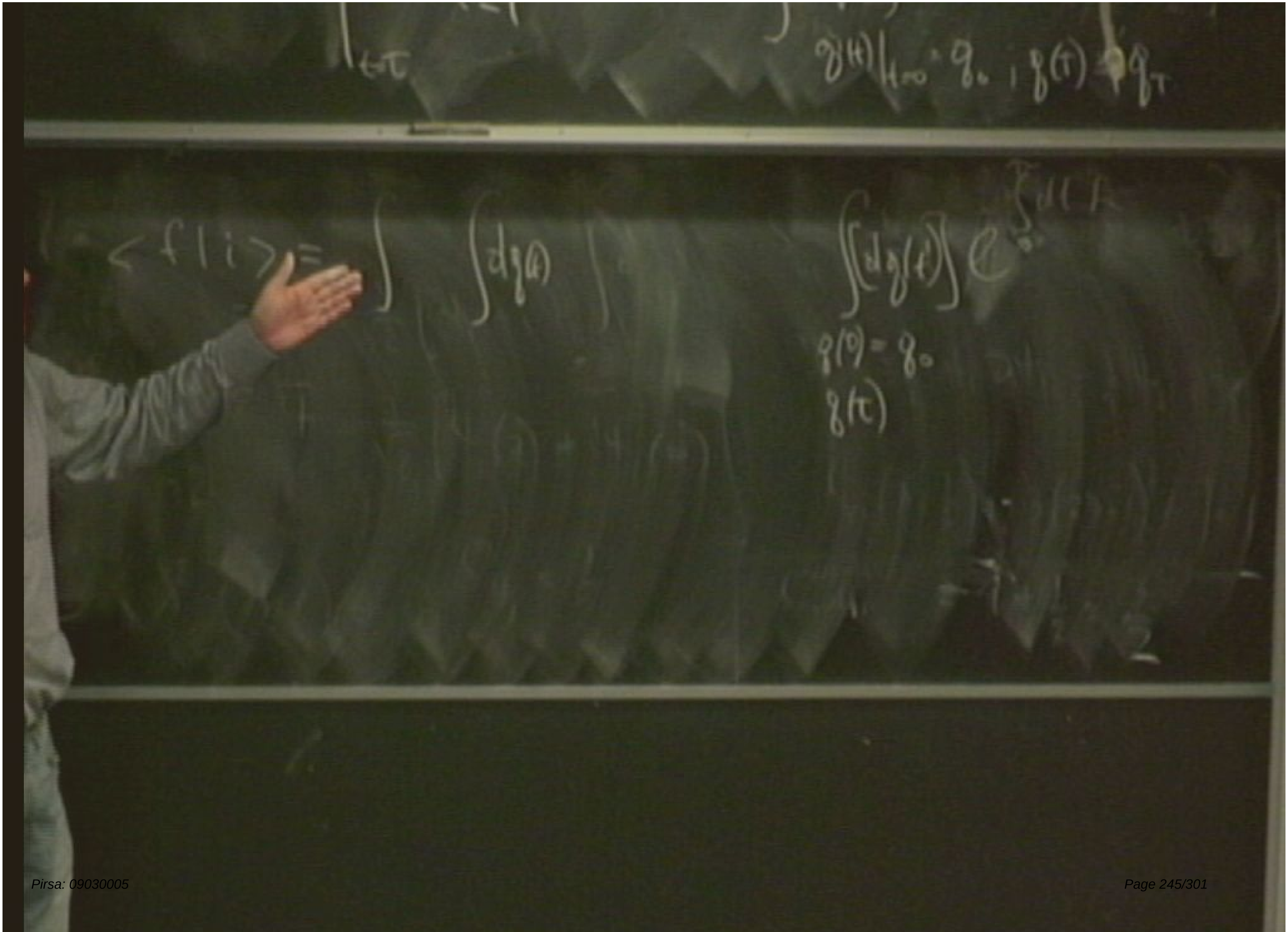


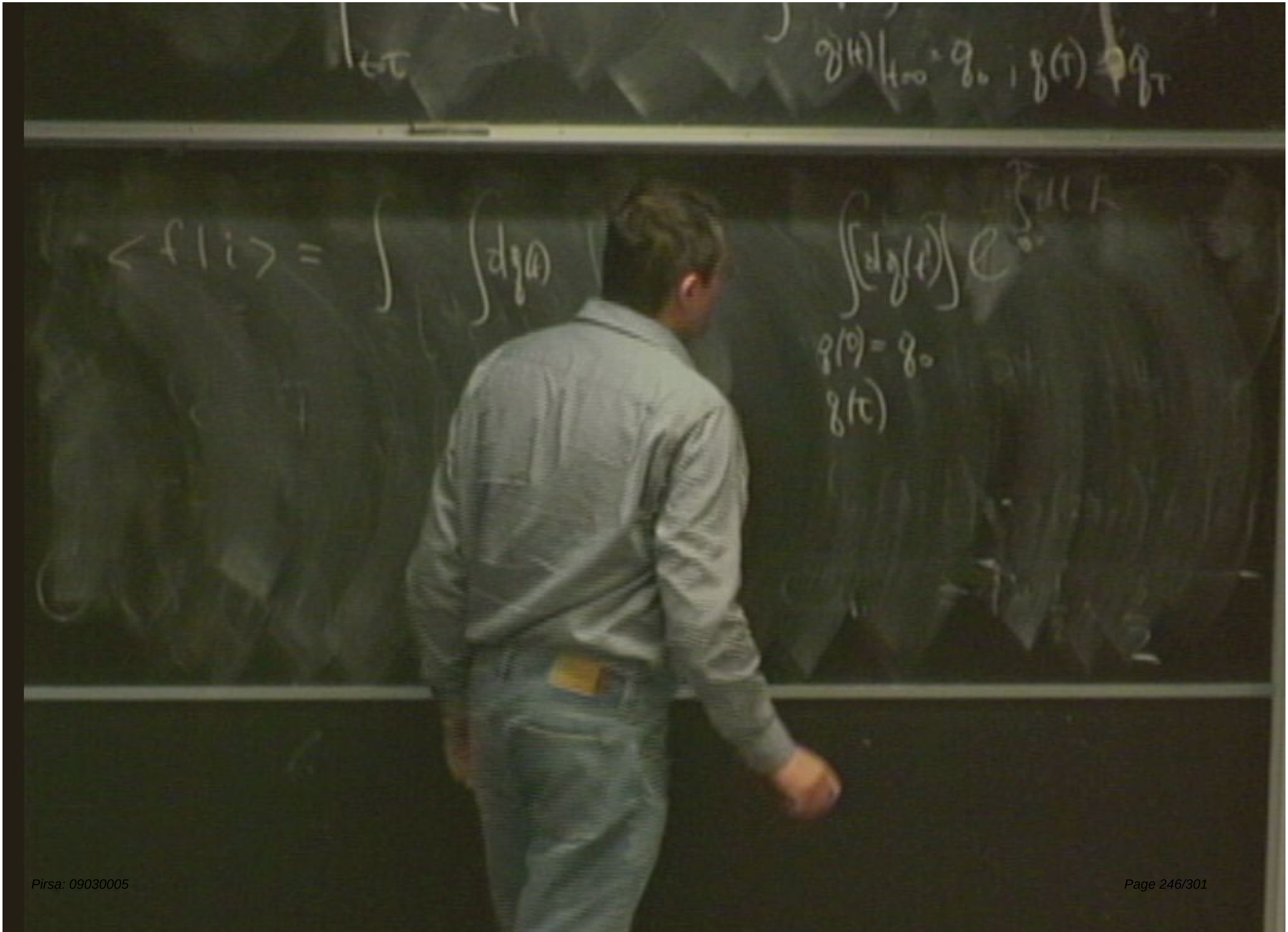

< f | i > =


q(t)|t=0 = q0 , q(T) = qT
<f|i>
∫dq(t)
∫[dq(t)] e^S
q(0) = q0
q(T)

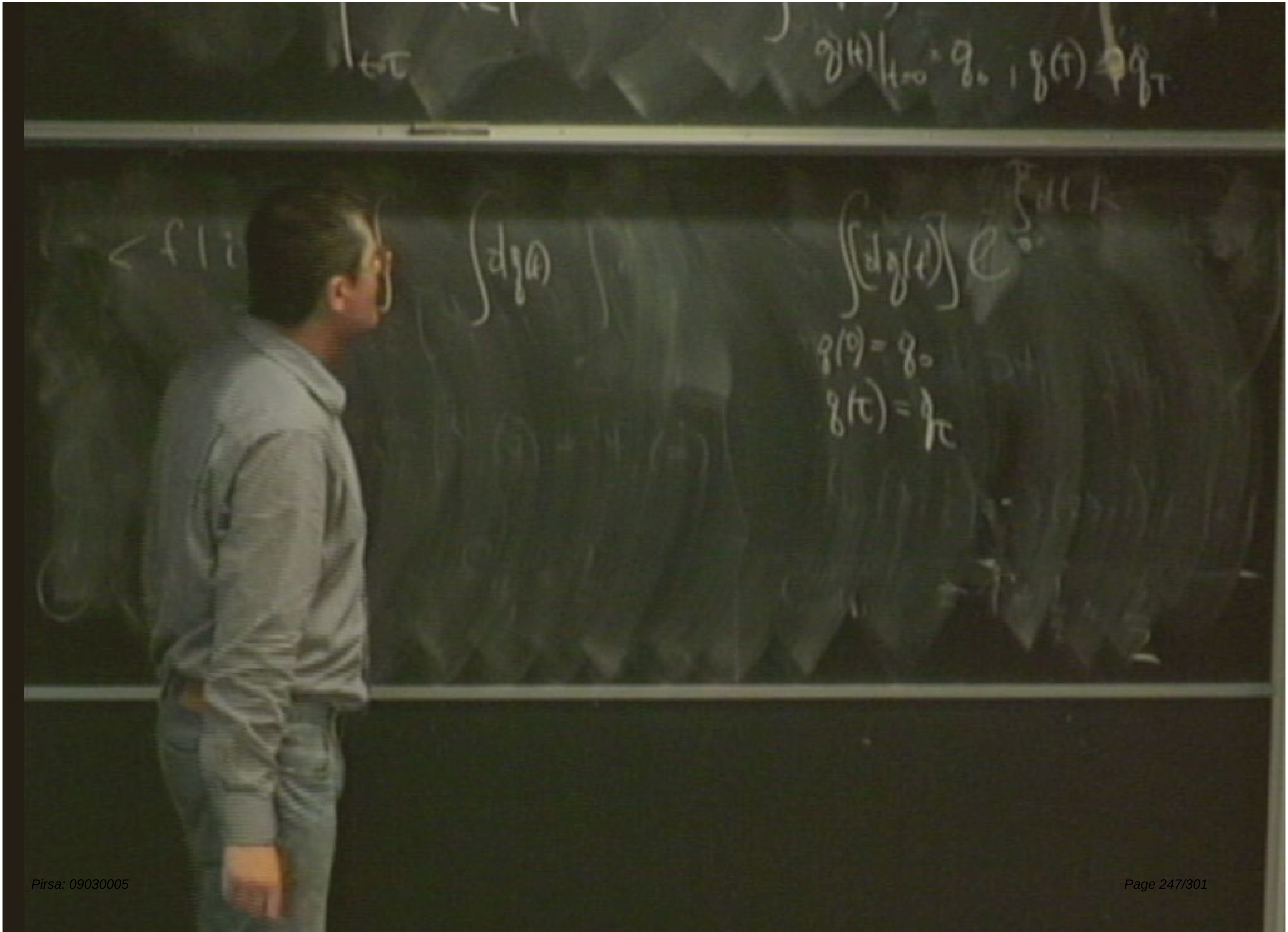

q(T) = q_T
f
i
<T, q_T | 0; q_0>
q_0 = q(0)
t=0
t=T
= ∫ [dq(t)] e

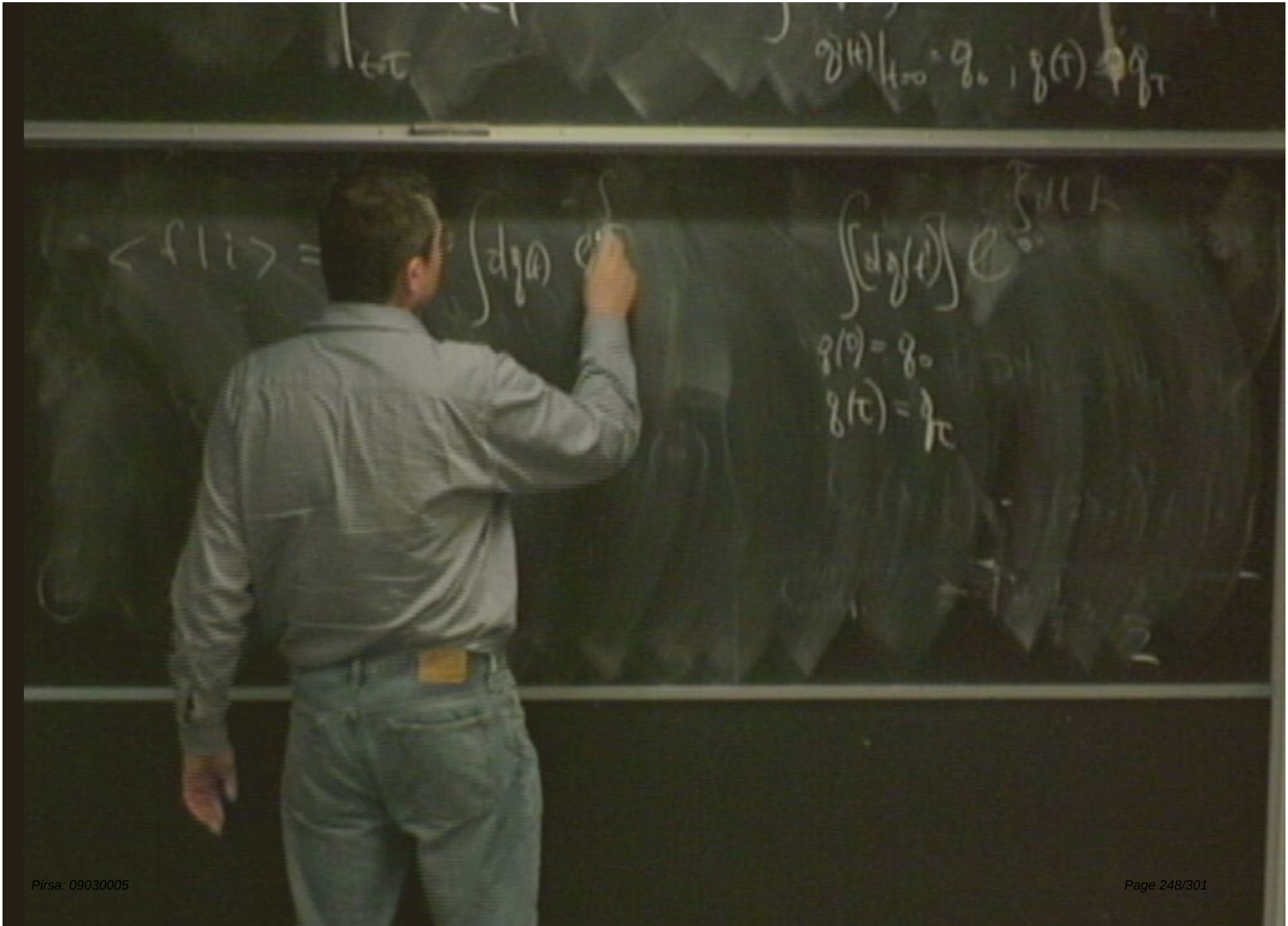

q(T) = q_T
f
i
<T, q_T | 0, q_0>
q_0 = q(0)
t=0
t=T
-S[q] = -∫ dt L
∫[dq(t)] e
q(t)|t=0 = q_0 ; q(T) = q_T

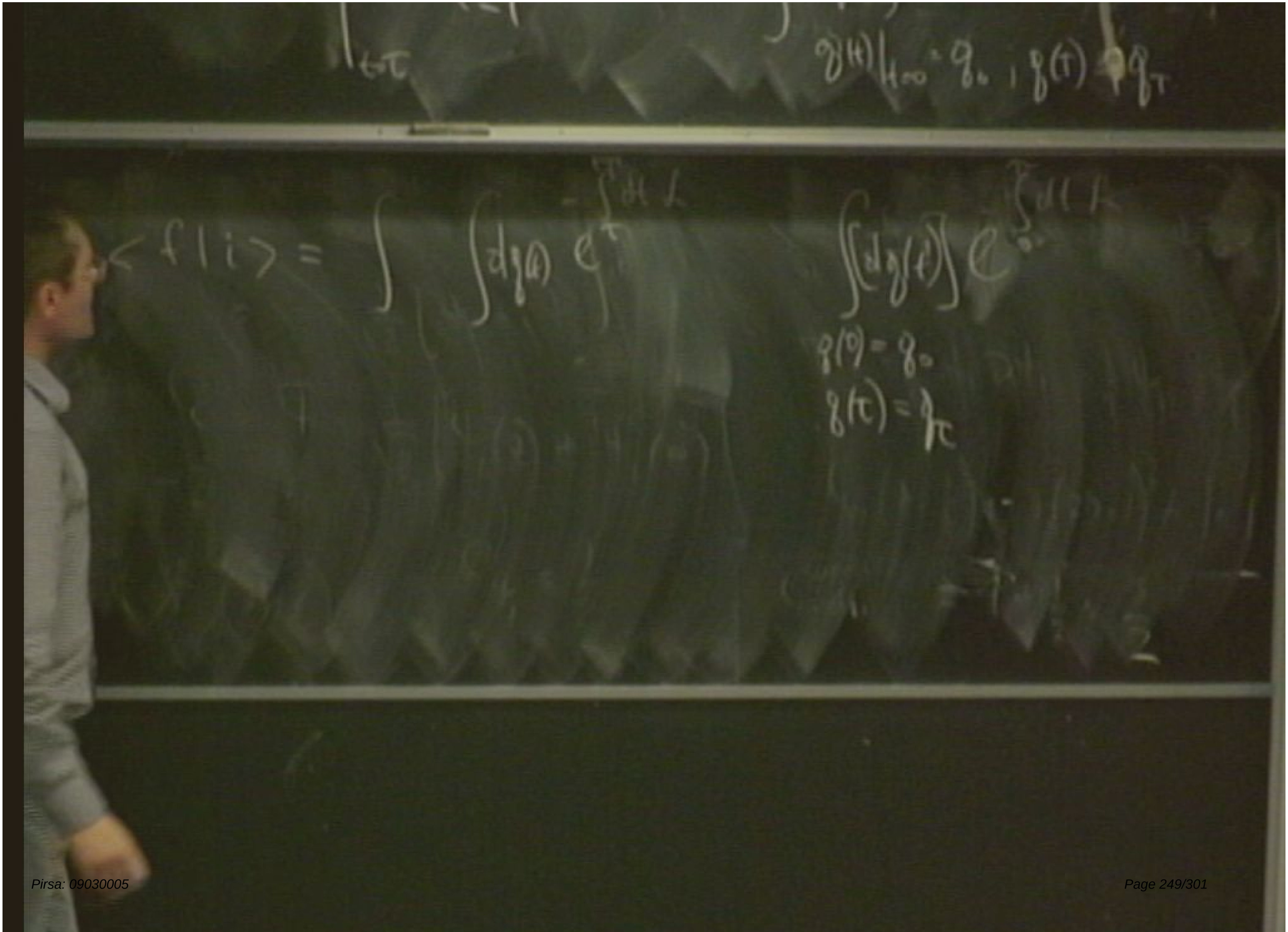

$q(t)|_{t=0} = q_0$, $q(T) = q_T$
$\langle f | i \rangle =$
$\int dq(t)$
$\int [dq(t)] \, e$
$q(0) = q_0$
$q(T)$

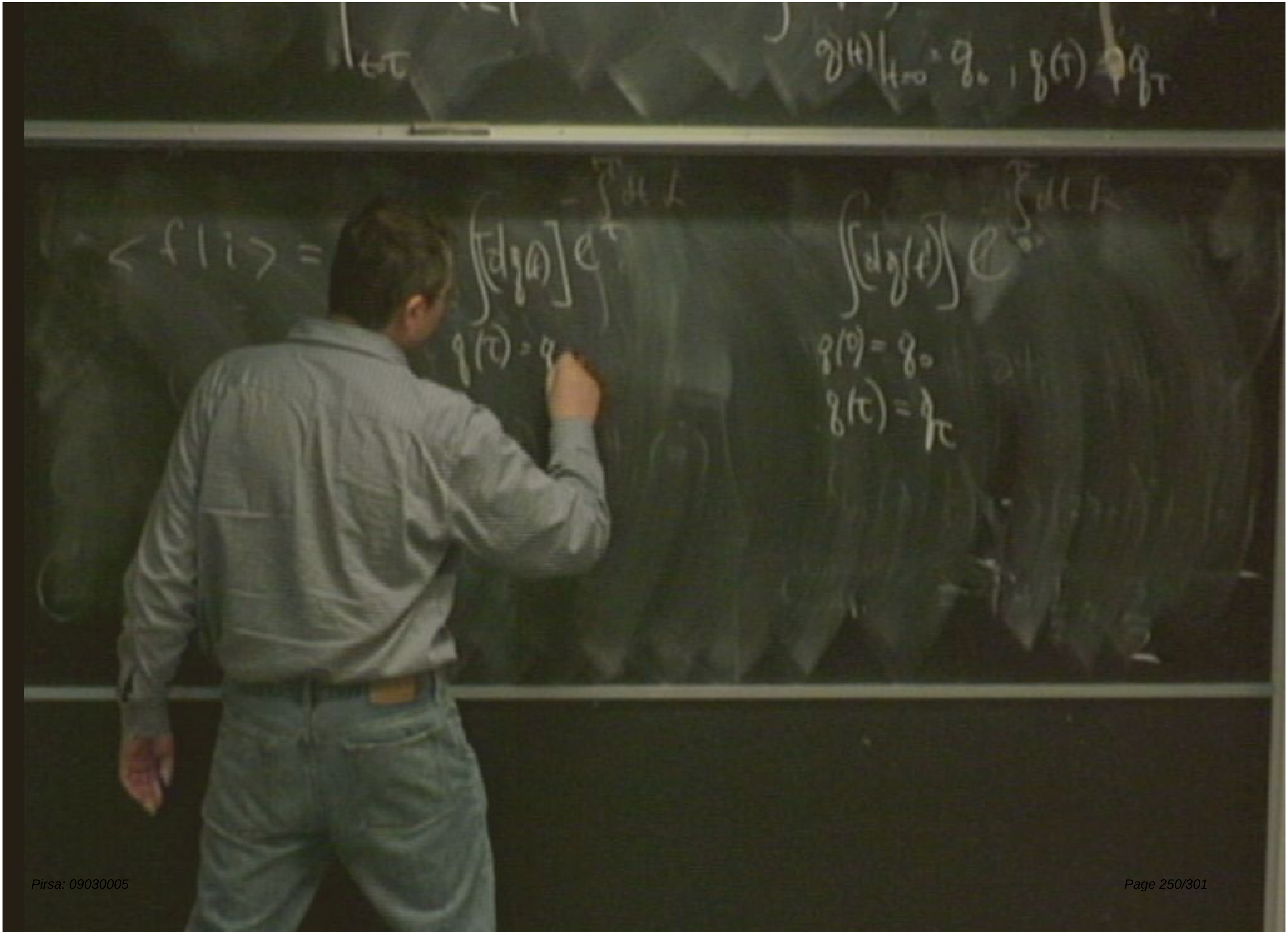


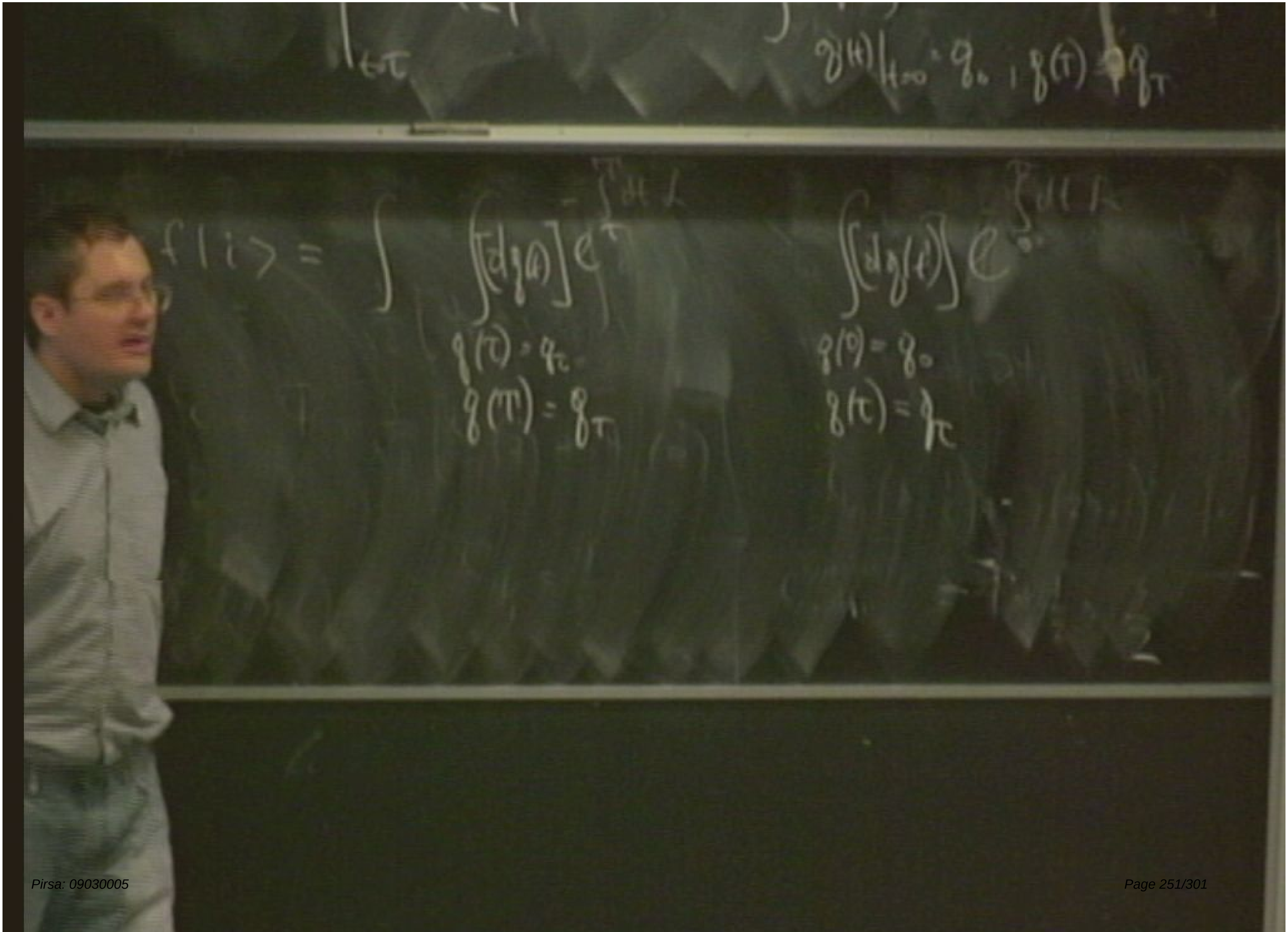
<f|i> =
q(t)|t=0 = q0 , q(T) = qT
q(0) = q0
q(T)

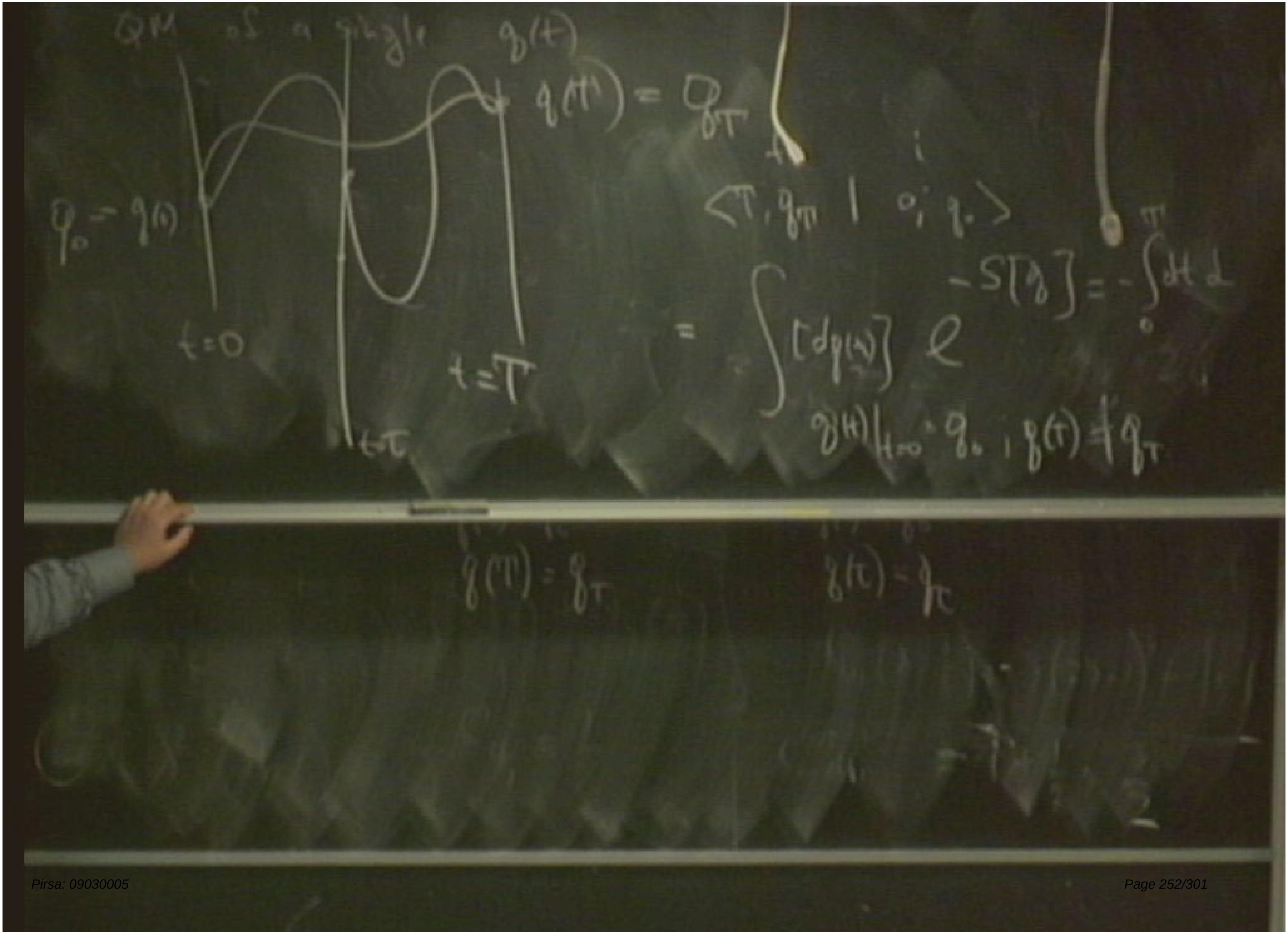

q(t)|t=0 = q0 , q(τ) = qτ
q(0) = q0
q(τ) = qτ

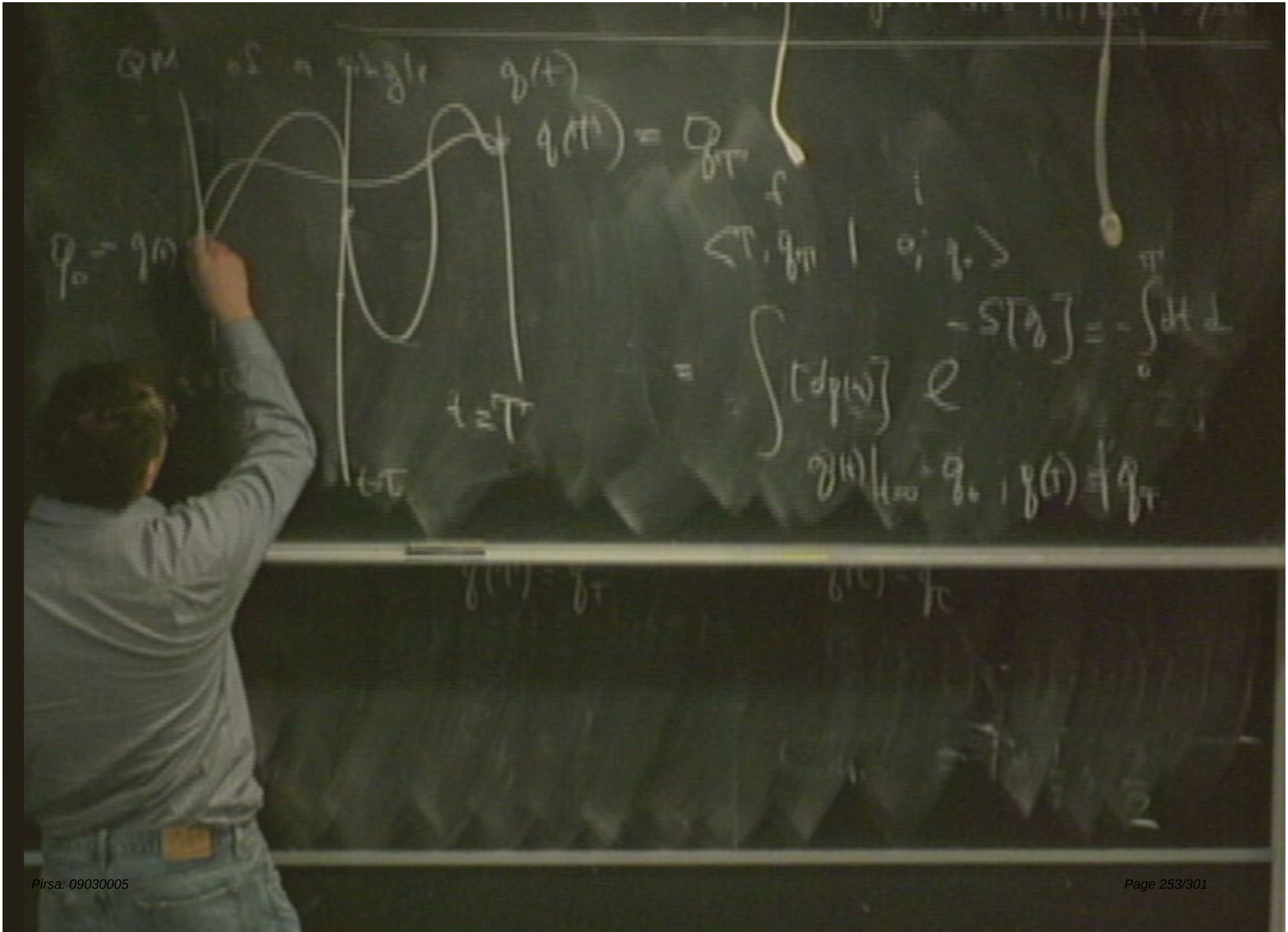

$\langle f | i \rangle =$
$q(0) = q_0$
$q(T) = q_T$

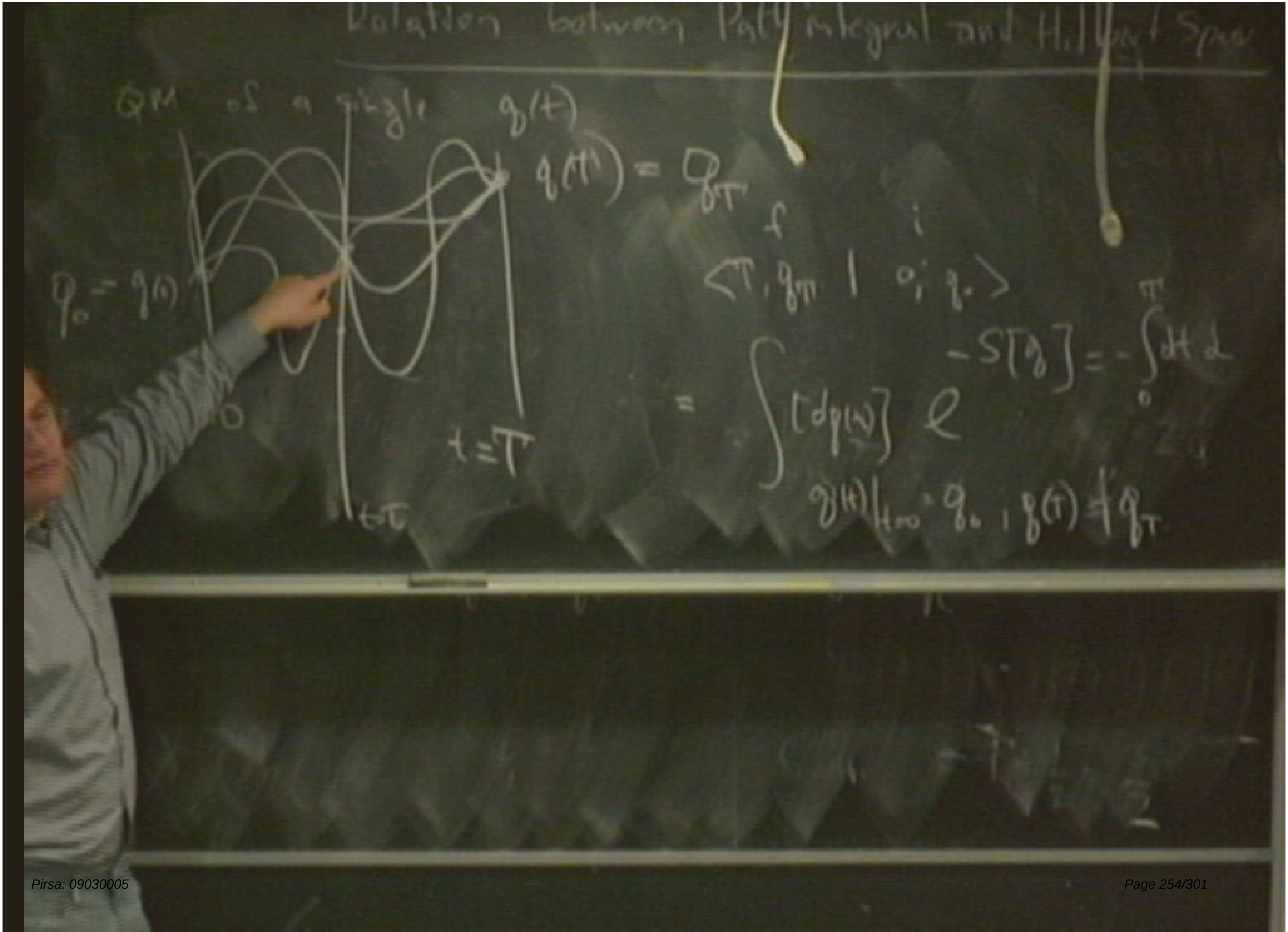

q(t)|t=0 = q0 , q(T) = qT
< f | i > =
q(0) = q0
q(T) = qT

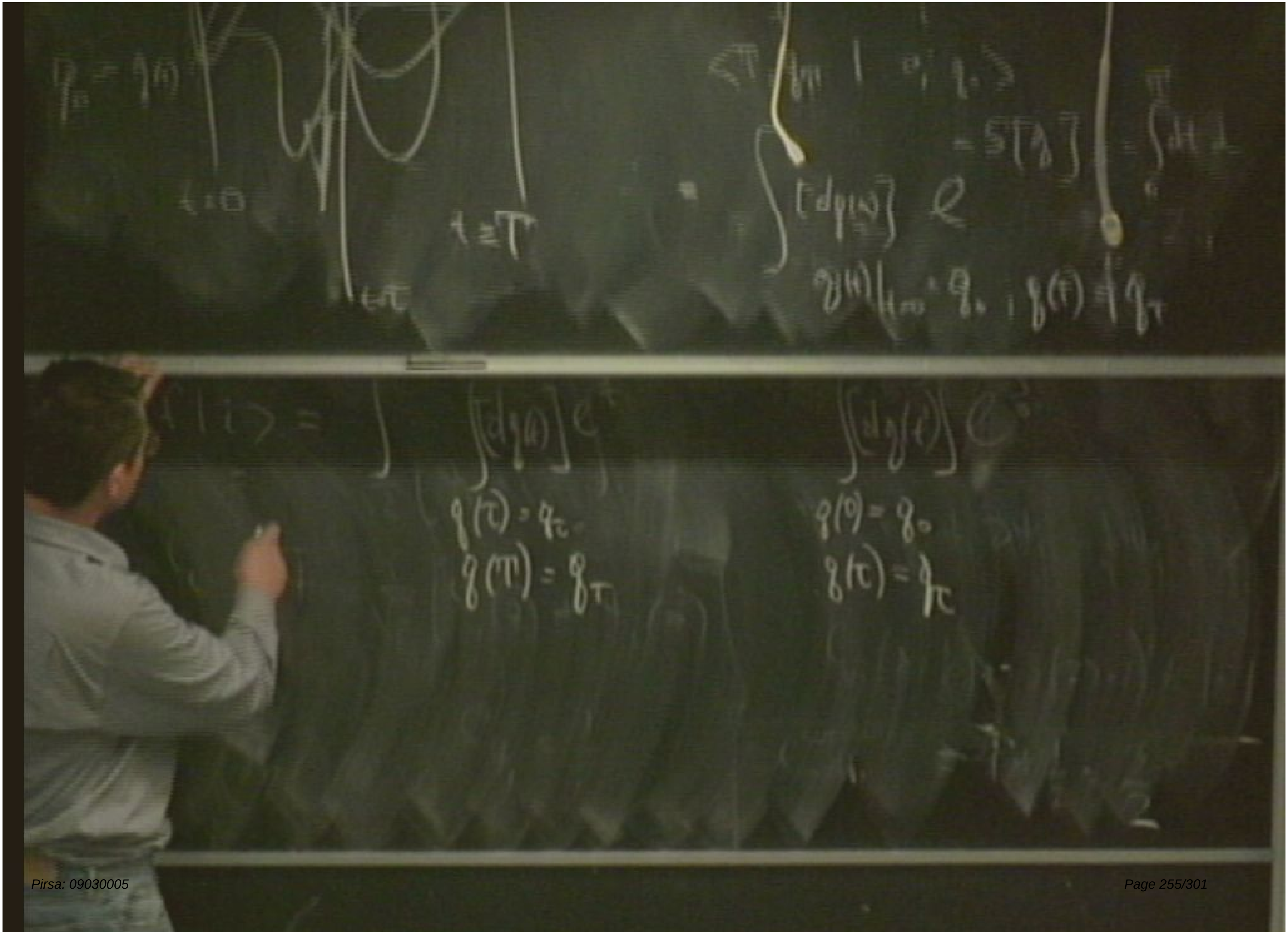

q(t)|t=0 = q0 , q(T) = qT
< f | i > =
q(T) = q
q(0) = q0
q(T) = qT

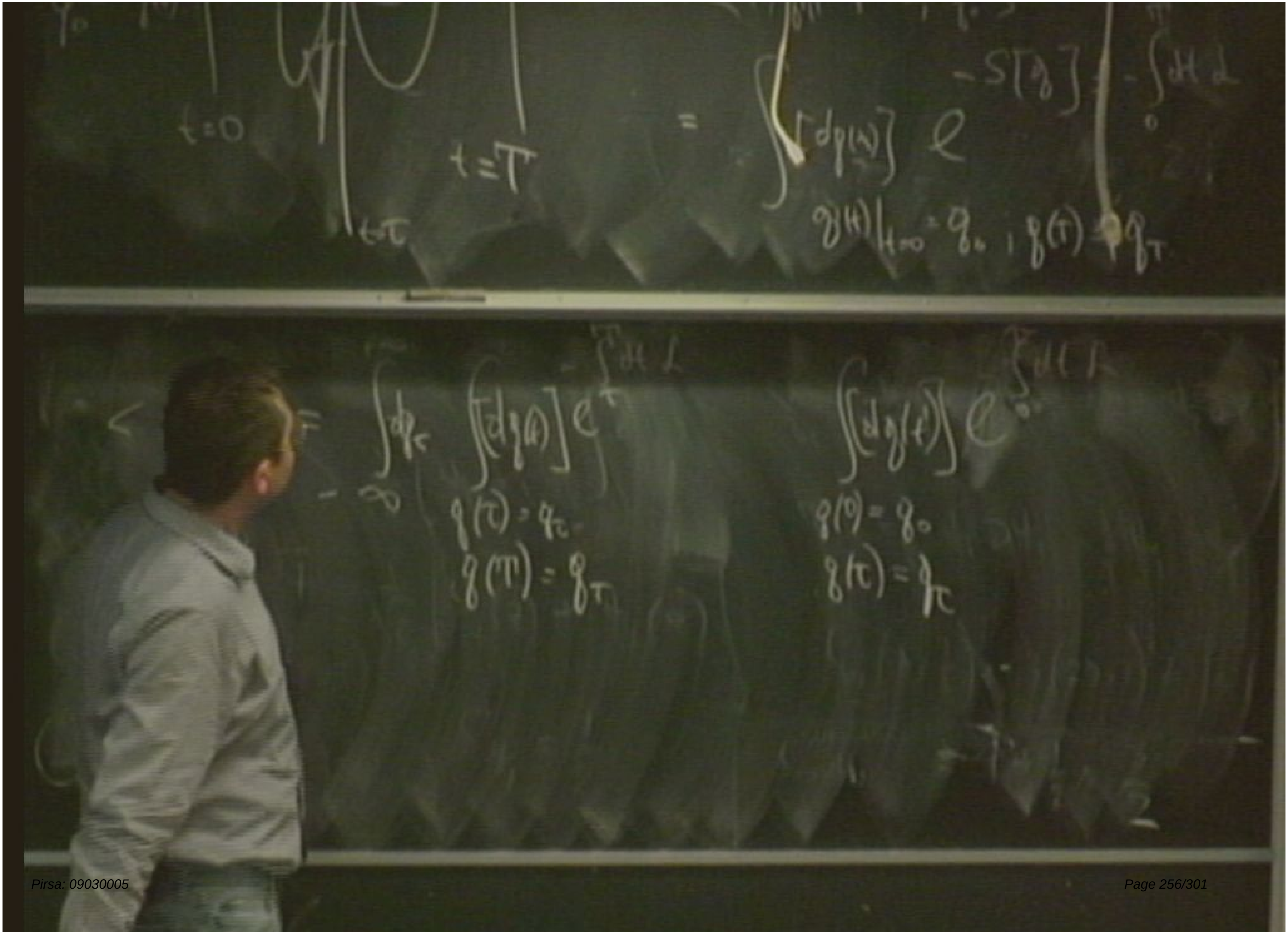

q(t)|_{t=0} = q_0 , q(T) = q_T
f | i > = ∫ ∫[dq(t)] e ∫[dq(t)] e
q(τ) = q_τ
q(T) = q_T
q(0) = q_0
q(τ) = q_τ

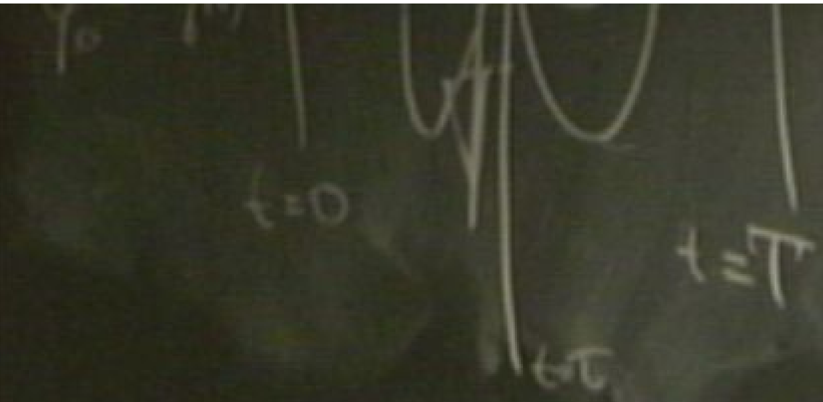

QM of a single q(t)
q(T) = q_T
q_0 = q(0)
t=0
t=T
t=τ
⟨T, q_T | 0; q_0⟩
= ∫[dq(t)] e
-S[q] = -∫_0^T dt L
q(t)|_{t=0} = q_0 ; q(T) = q_T
q(T) = q_T
q(τ) = q_τ


QM of a single q(t)
q(T) = q_T
q_0 = q(0)
t = T
t = 0
f i
⟨T, q_T | 0, q_0⟩
= ∫[dq(t)] e
-S[q] = -∫ dt L
q(t)|_{t=0} = q_0 , q(T) = q_T


Relation between Path integral and Hilbert Space
QM of a single $q(t)$
$q(T) = q_T$
$q_0 = q(0)$
0
$t = T$
f i
$\langle T, q_T \mid 0; q_0 \rangle$
$-S[q] = -\int_0^T dt\, L$
$= \int [dq(t)]\, e$
$q(t)|_{t=0} = q_0,\ q(T) = q_T$


t = 0
t = T
t = τ
= S[q]
q(τ) = q_τ
q(T) = q_T
q(0) = q_0
q(τ) = q_τ


t=0
t=T
t=τ
q(t)|t=0 = q0 , q(T) = qT
q(τ) = qτ
q(T) = qT
q(0) = q0
q(τ) = qτ

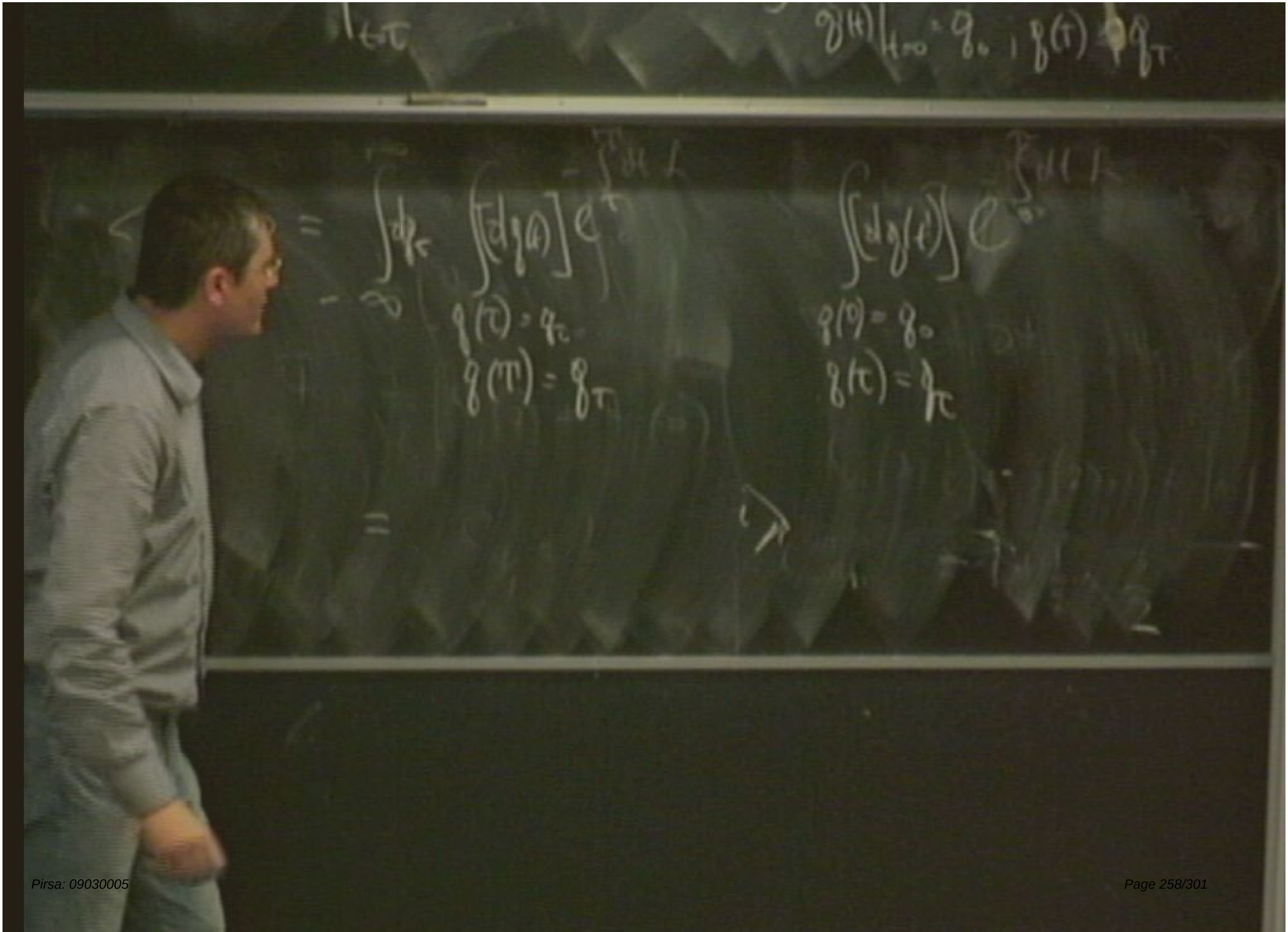


$$= \int [dq(t)]\, e^{-S[q] = -\int_0^T dt\, L}$$

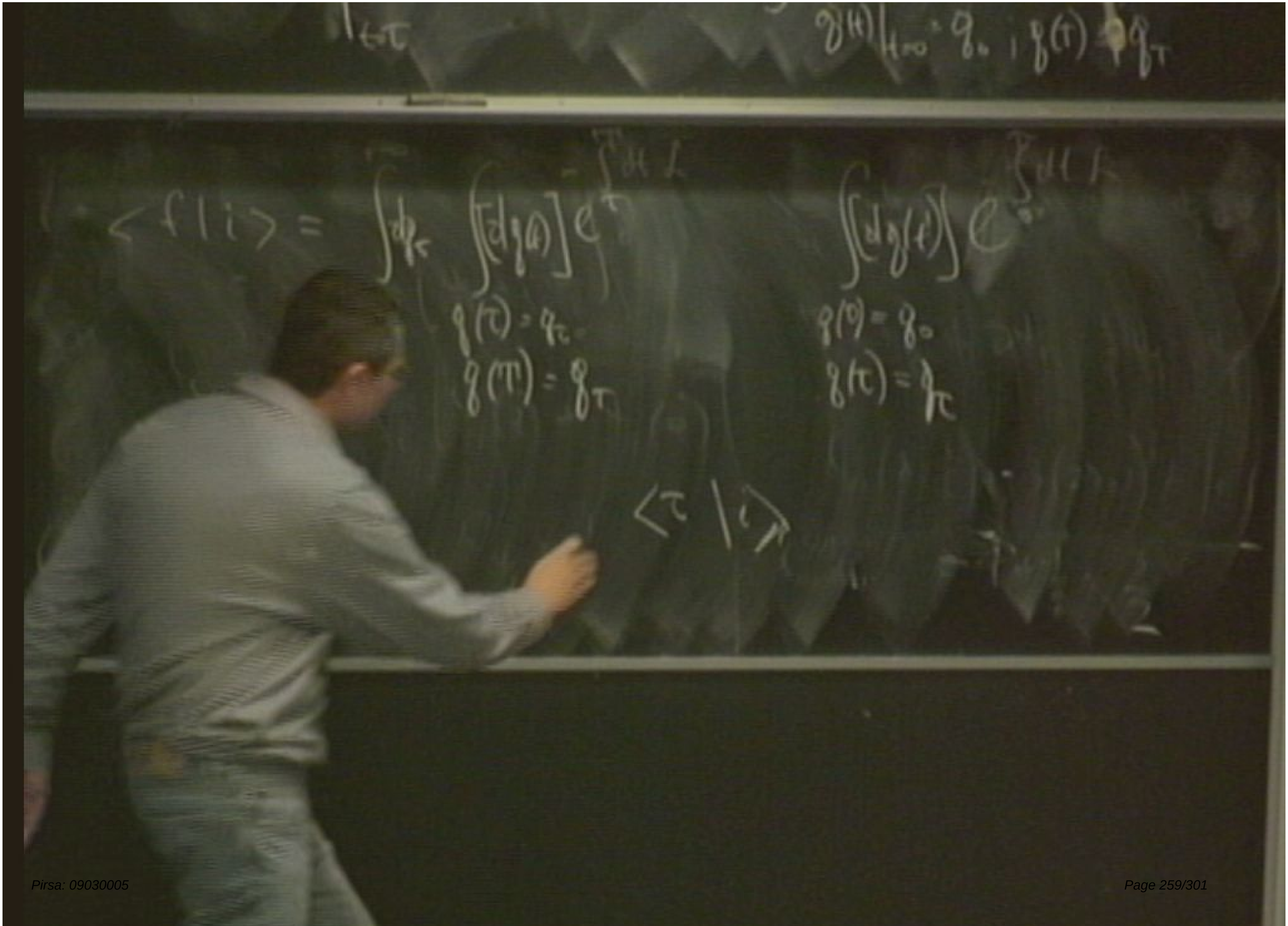
$$q(t)\big|_{t=0} = q_0 \,,\; q(T) = q_T$$

$$\langle f | i \rangle = \int_{-\infty}^{\infty} dq_\tau \int [dq(t)]\, e^{-\int_\tau^T dt\, L} \int [dq(t)]\, e^{-\int_0^\tau dt\, L}$$

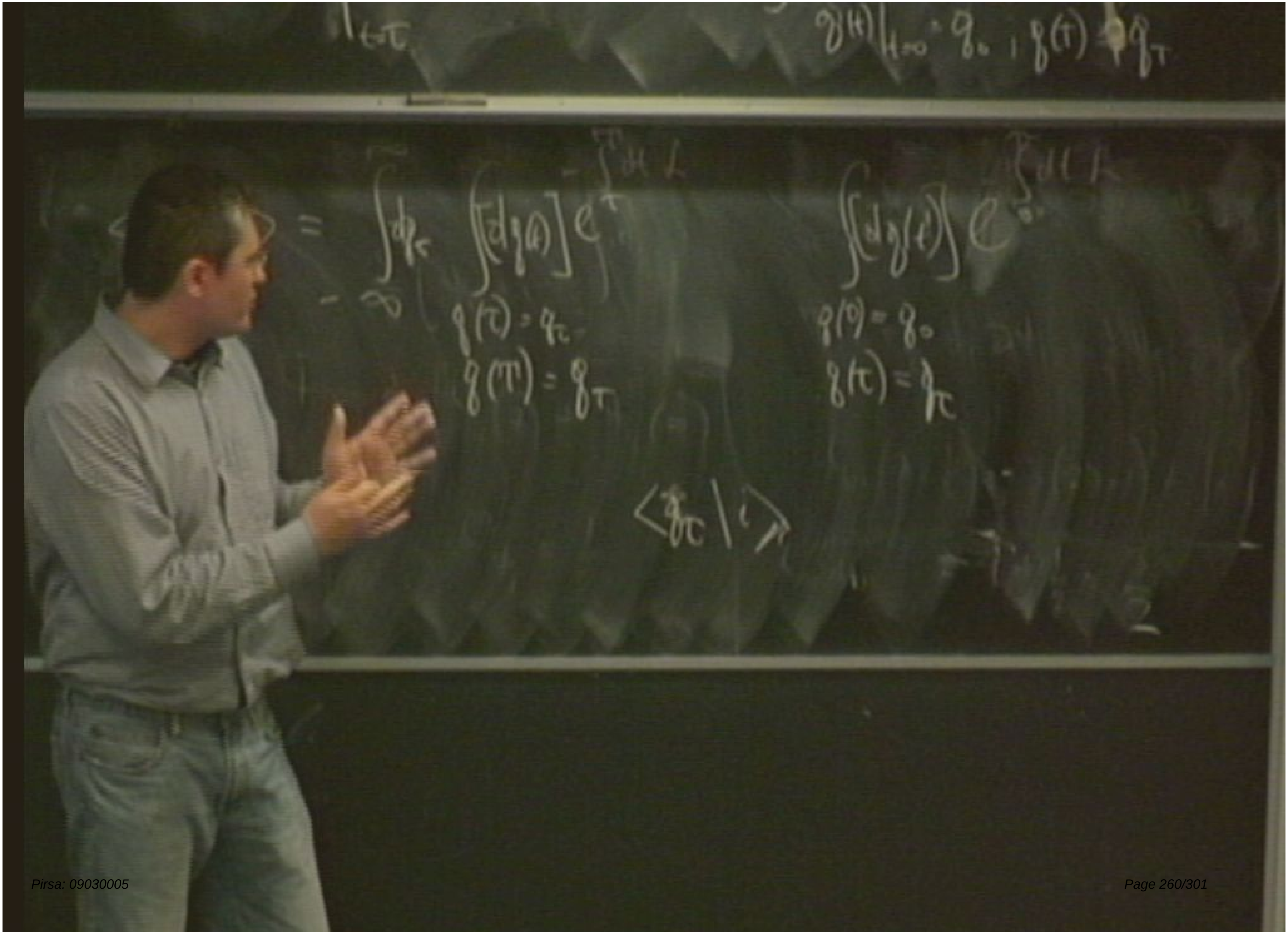
$q(\tau) = q_\tau$, $q(T) = q_T$ $\qquad$ $q(0) = q_0$, $q(\tau) = q_\tau$

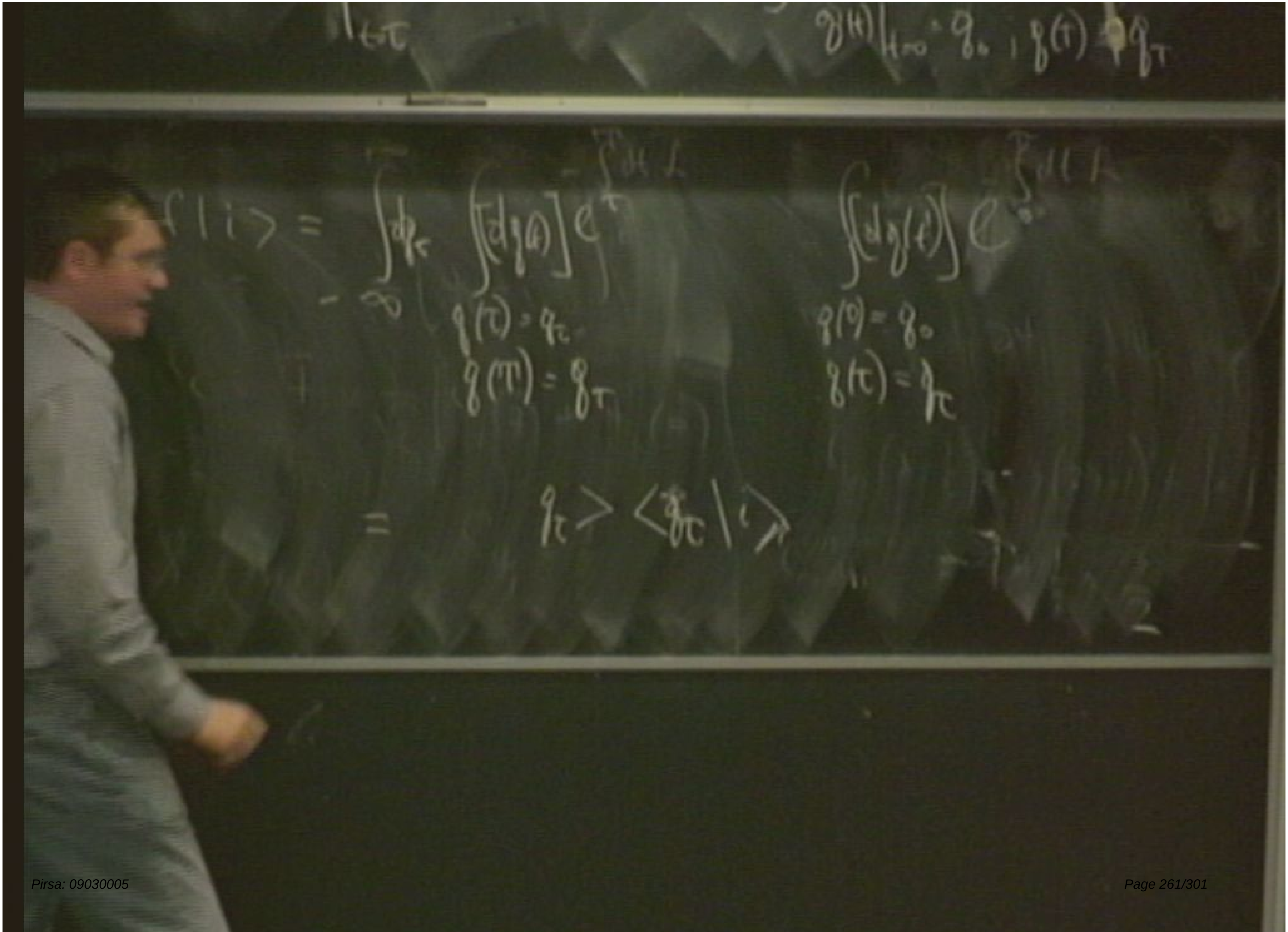


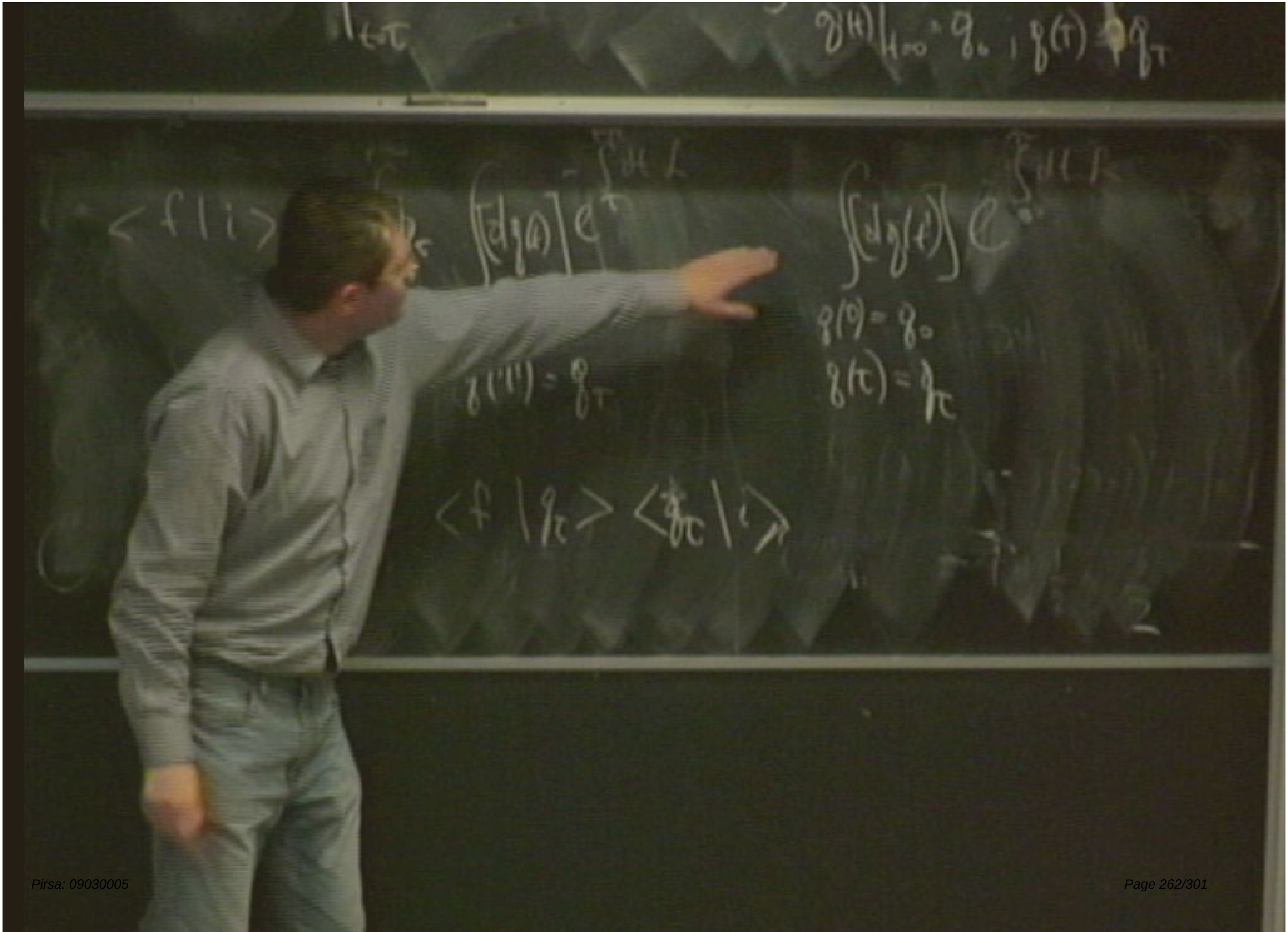
$q(t)|_{t=0} = q_0 , q(T) = q_T$
$q(\tau) = q_\tau$
$q(T) = q_T$
$q(0) = q_0$
$q(\tau) = q_\tau$

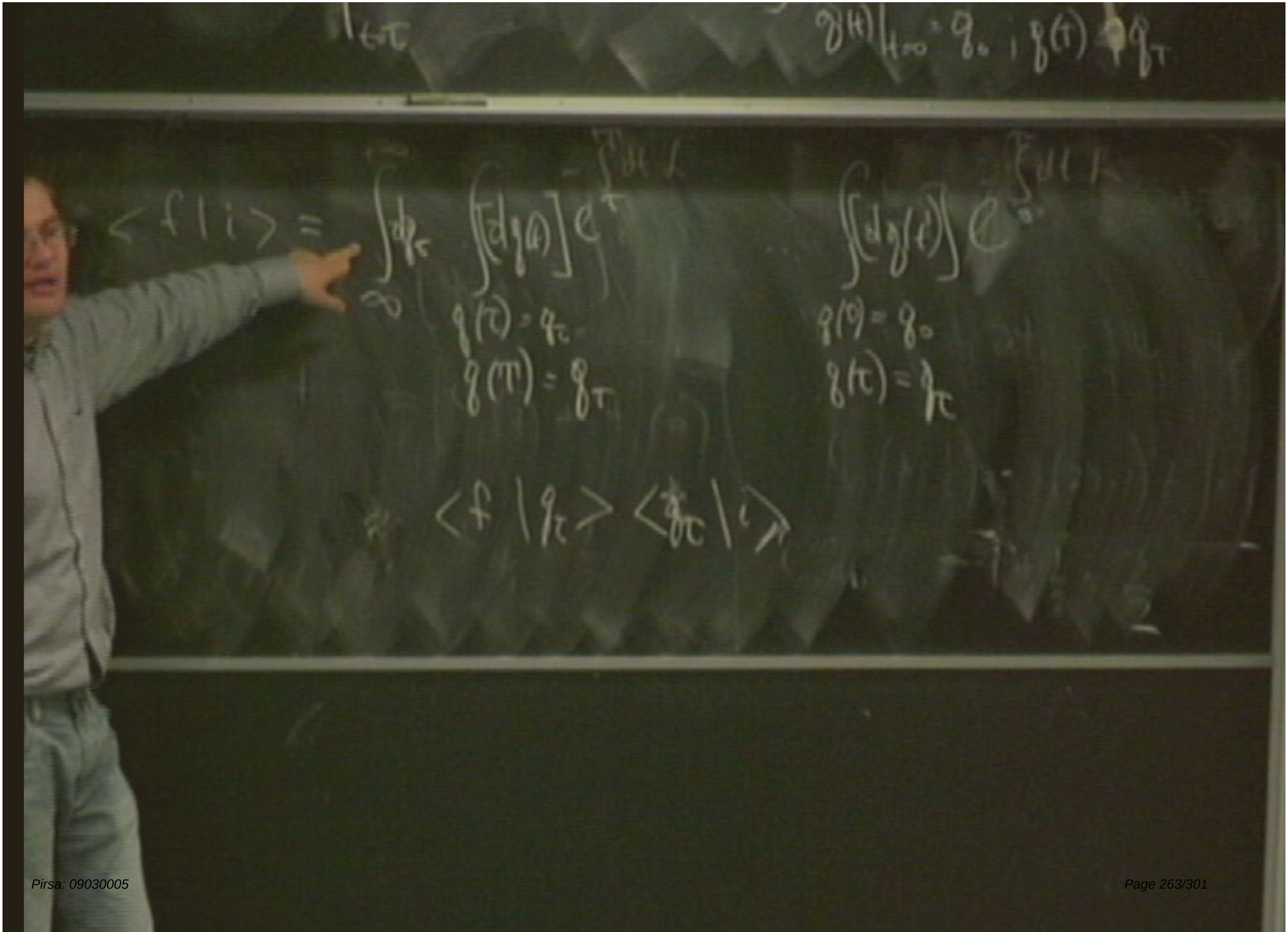


<f | i> =
q(τ) = q_τ
q(T) = q_T
q(0) = q_0
q(τ) = q_τ
<τ | i>


$q(\tau) = q_\tau$
$q(T) = q_T$
$q(0) = q_0$
$q(\tau) = q_\tau$
$\langle q_\tau | i \rangle$


q(t)|t=0 = q0 ; q(T) = qT
f | i > = ∫ dqτ ∫[dq(t)] e ∫ dt L ∫[dq(t')] e ∫ dt L
q(τ) = qτ
q(T) = qT
q(0) = q0
q(τ) = qτ
= qτ > < qτ | i >


$q(t)|_{t=0} = q_0 \; , \; q(T) = q_T$
$\langle f | i \rangle$
$q(0) = q_0$
$q(\tau) = q_\tau$
$\langle f | q_\tau \rangle \langle q_\tau | i \rangle$


<f | i> =
q(0) = q0
<f | qτ> <qτ | i>

$\Big|_{t=\tau}$ $\quad q(t)\big|_{t=0} = q_0 \ , \ q(T) = q_T$

$$\langle f | i \rangle = \int_{-\infty}^{\infty} dq_\tau \int [dq(t)]\, e^{-\int_\tau^T dt\, L} \cdots \int [dq(t')]\, e^{\int_0^\tau dt'\, L}$$

$q(\tau) = q_\tau$ $\quad q(0) = q_0$

$q(T) = q_T$ $\quad q(\tau) = q_\tau$

$$\int dq_\tau \, \langle f | q_\tau \rangle \langle q_\tau | i \rangle$$

$$\Big|_{t=\tau} \qquad q(t)\Big|_{t=0} = q_0 \;,\; q(T) = q_T$$

$$\langle f | i \rangle = \int_{-\infty}^{\infty} dq_\tau \int_{\substack{q(\tau)=q_\tau \\ q(T)=q_T}} [dq(t)]\, e^{-\int_\tau^T dt\, L} \cdots \int_{\substack{q(0)=q_0 \\ q(\tau)=q_\tau}} [dq(t')]\, e^{\int_0^\tau dt'\, L}$$

$$\int dq_\tau \langle f | q_\tau \rangle \langle q_\tau | i \rangle = \langle f | \underbrace{\int dq\, |q\rangle\langle q|}_{\text{complete set of states}} | i \rangle$$

$$q(t)\big|_{t=0} = q_0 \;;\; q(T) = q_T$$

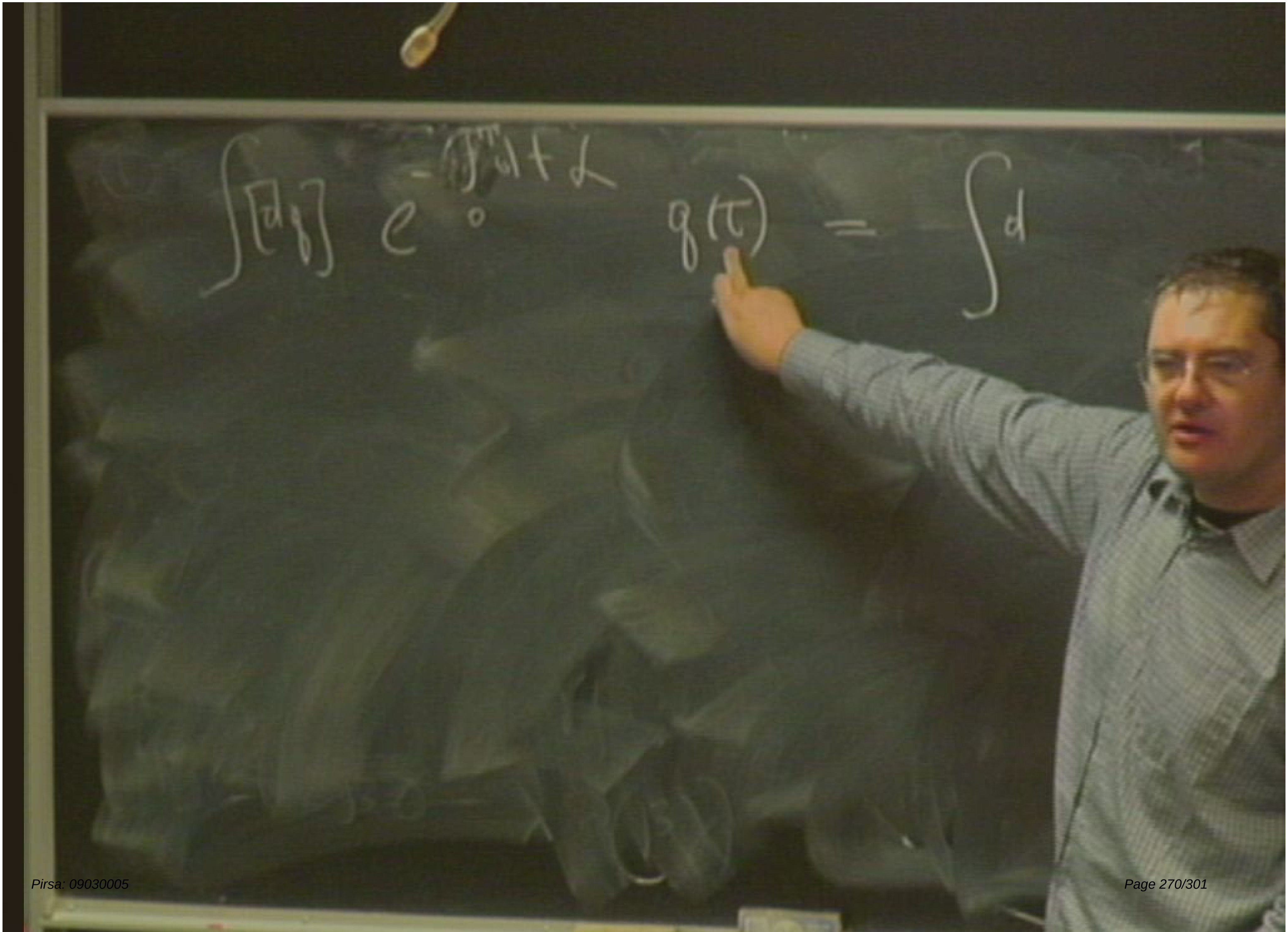
$$\langle f | i \rangle = \int_{-\infty}^{\infty} dq_\tau \int [dq(t)]\, e^{-\int_\tau^T dt\, L} \int [dq(t')]\, e^{-\int_0^\tau dt'\, L}$$

$q(\tau) = q_\tau$, $q(T) = q_T$; $q(0) = q_0$, $q(\tau) = q_\tau$

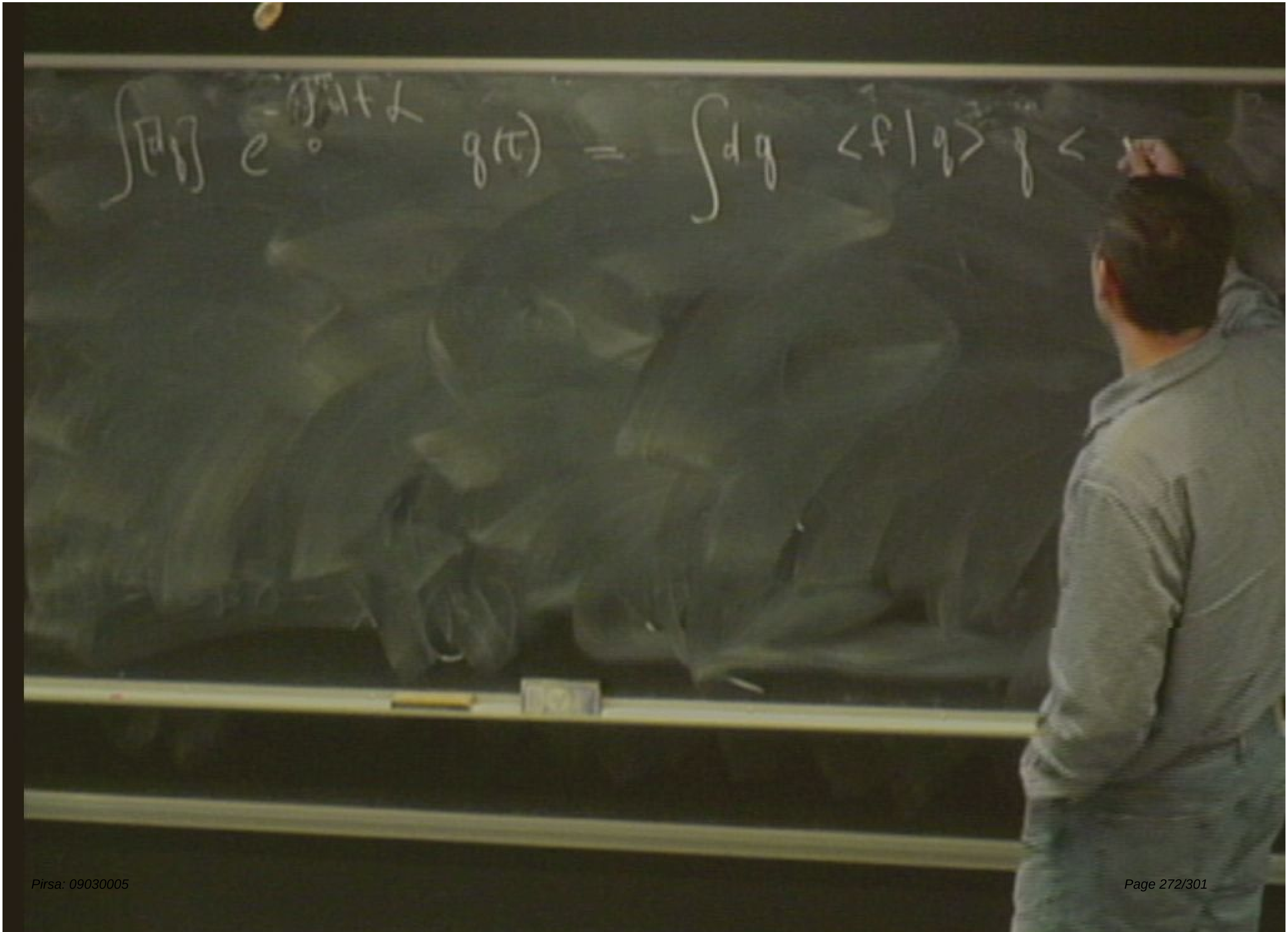
$$\int dq_\tau \langle f | q_\tau \rangle \langle q_\tau | i \rangle = \langle f | \underbrace{\int dq\, |q\rangle\langle q|}_{\text{complete set of states}} | i \rangle$$

$$\int dq\, |q\rangle\langle q| = I$$

$$\int [dq]\; e^{-\int_0^{T} dt\, L} \quad q(\tau)$$


$\int [dq]\, e^{-\int_0^T dt\, L}\, q(\tau) = \int d$

$$\int [dq]\, e^{-\int_0^{T} dt\, L}\, q(\tau) = \int dq$$


$\int [dq]\, e^{-\int_0^T dt\, L}\, q(\tau) = \int dq\, \langle f|q\rangle\, q\, \langle$

$$\int [dq]\, e^{-\int_0^T dt\, L}\, q(\tau) = \int dq \,\langle f | q \rangle\, q \,\langle q | i \rangle$$

$$\int [dq]\, e^{i\int_0^T dt\, L}\, q(\tau) = \int dq\, \underbrace{\langle f|q\rangle}_{\int_\tau^T}\, q\, \underbrace{\langle q|i\rangle}_{\int_0^\tau}$$

$$\int [dq]\, e^{-\int_0^{T} dt\, L}\, q(\tau) = \int dq\, \underbrace{\langle f | q \rangle}_{\int_\tau^{T}}\, q(\tau) \underbrace{\langle q | i \rangle}_{\int_0^{\tau}}$$

$$\int [dq]\, e^{-\int_0^T dt\, L}\, q(\tau) = \int dq\, \underbrace{\langle f | q \rangle}_{\int_\tau^T}\, q(\tau) \underbrace{\langle q | i \rangle}_{\int_0^\tau}$$

$|q\rangle$

$$\int [dq]\, e^{-\int_0^T dt\, L}\, q(\tau) = \int dq\, \underbrace{\langle f | q \rangle}_{\int_\tau^T}\, q(\tau) \underbrace{\langle q | i \rangle}_{\int_0^\tau}$$

$|q\rangle \rightarrow$ a state is a Hilbert space

$$\int [dq]\, e^{i\int_0^T dt\, L}\, q(\tau) = \int dq\, \underbrace{\langle f | q \rangle}_{\int_\tau^T}\, q(\tau) \underbrace{\langle q | i \rangle}_{\int_0^\tau}$$

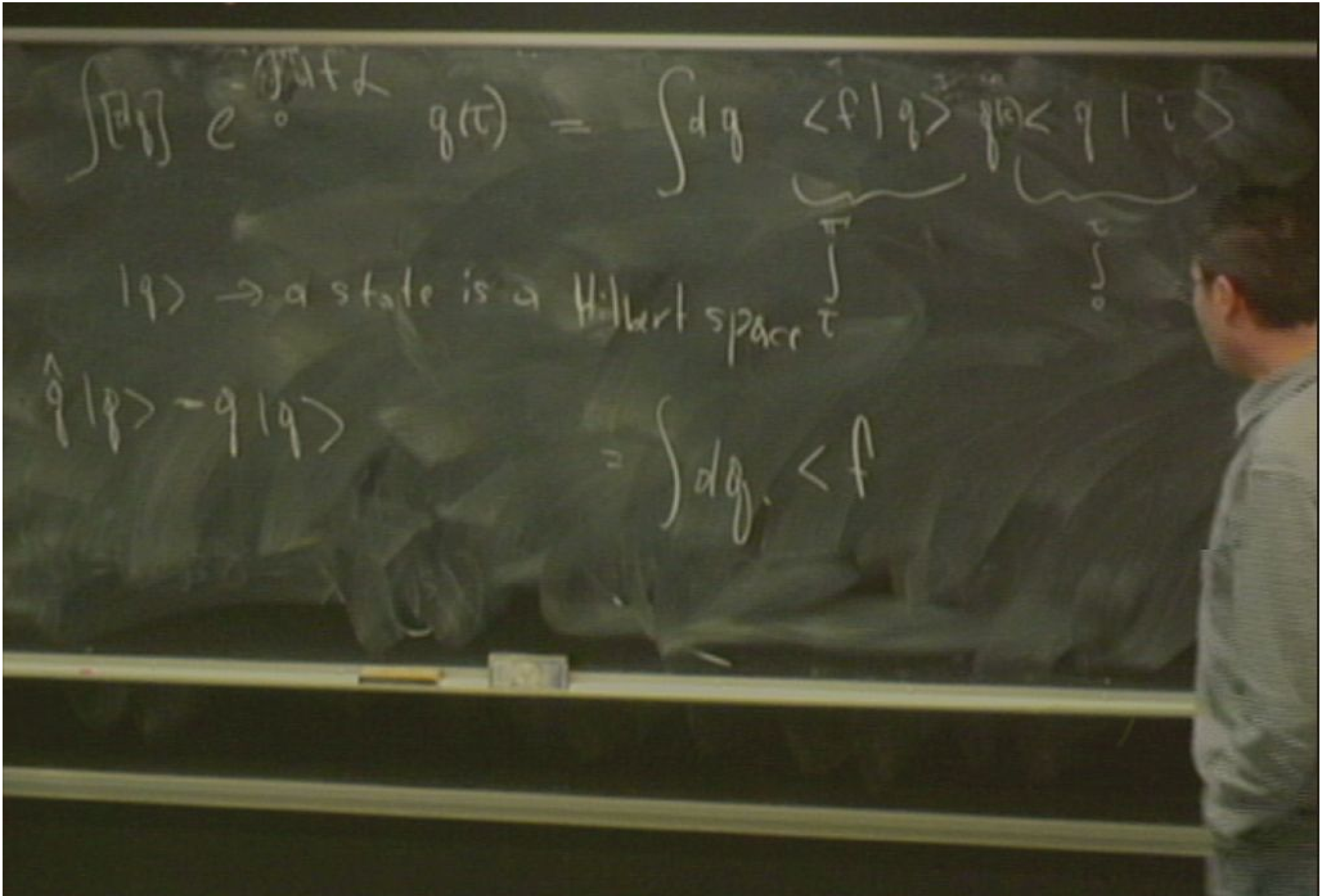
$|q\rangle \rightarrow$ a state is a Hilbert space

$\hat{q}$

$$\int [dq]\, e^{-\int_0^T dt\, L}\, q(\tau) = \int dq\, \underbrace{\langle f | q \rangle}_{\int_\tau^T}\, q(\tau) \underbrace{\langle q | i \rangle}_{\int_0^\tau}$$

$|q\rangle \rightarrow$ a state is a Hilbert space

$$\hat{q}\,|q\rangle = q\,|q\rangle$$


$\int [dq]\, e^{-\int_0^T dt\, L}\, q(\tau) = \int dq\, \underbrace{\langle f | q \rangle}_{\int_\tau^T} q(\tau) \underbrace{\langle q | i \rangle}_{\int_0^\tau}$
$|q\rangle \rightarrow$ a state is a Hilbert space
$\hat{q}\,|q\rangle = q\,|q\rangle$
$= \int dq\, \langle f$

$$\int [dq]\, e^{-\int_0^T dt\, L}\; q(\tau) = \int dq\; \underbrace{\langle f | q \rangle}_{\int_\tau^T} q(\tau) \underbrace{\langle q | i \rangle}_{\int_0^\tau}$$

$|q\rangle \rightarrow$ a state is a Hilbert space

$\hat{q}\,|q\rangle = q\,|q\rangle$

$$= \int dq\, \langle f | \hat{q} | q \rangle \langle q | i \rangle$$

$$\int [dq]\, e^{-\int_0^T dt\, L}\, q(\tau) = \int dq\, \underbrace{\langle f | q \rangle}_{\int_\tau^T} q(\tau) \underbrace{\langle q | i \rangle}_{\int_0^\tau}$$

$|q\rangle \rightarrow$ a state is a Hilbert space

$$\hat{q}\,|q\rangle = q\,|q\rangle$$

$$= \int dq\, \langle f | \hat{q}\, |q\rangle\langle q|\, i\rangle$$

$$\int [dq]\, e^{-\int_0^T dt\, L}\, q(\tau) = \int dq\, \underbrace{\langle f | q \rangle}_{\int_\tau^T} q(\tau) \underbrace{\langle q | i \rangle}_{\int_0^\tau}$$

$|q\rangle \rightarrow$ a state is a Hilbert space

$\hat{q}(0)|q\rangle = q\,|q\rangle$

$$= \int dq\, \langle f | \hat{q}\, (|q\rangle\langle q|)\, i \rangle$$

$$\int [dq]\, e^{i\int_0^T dt\, L}\, q(\tau) = \int dq\, \underbrace{\langle f | q \rangle}_{\int_\tau^T} q(\tau) \underbrace{\langle q | i \rangle}_{\int_0^\tau}$$

$|q\rangle \rightarrow$ a state is a Hilbert space

$$\hat{q}(\tau)|q\rangle = q|q\rangle$$

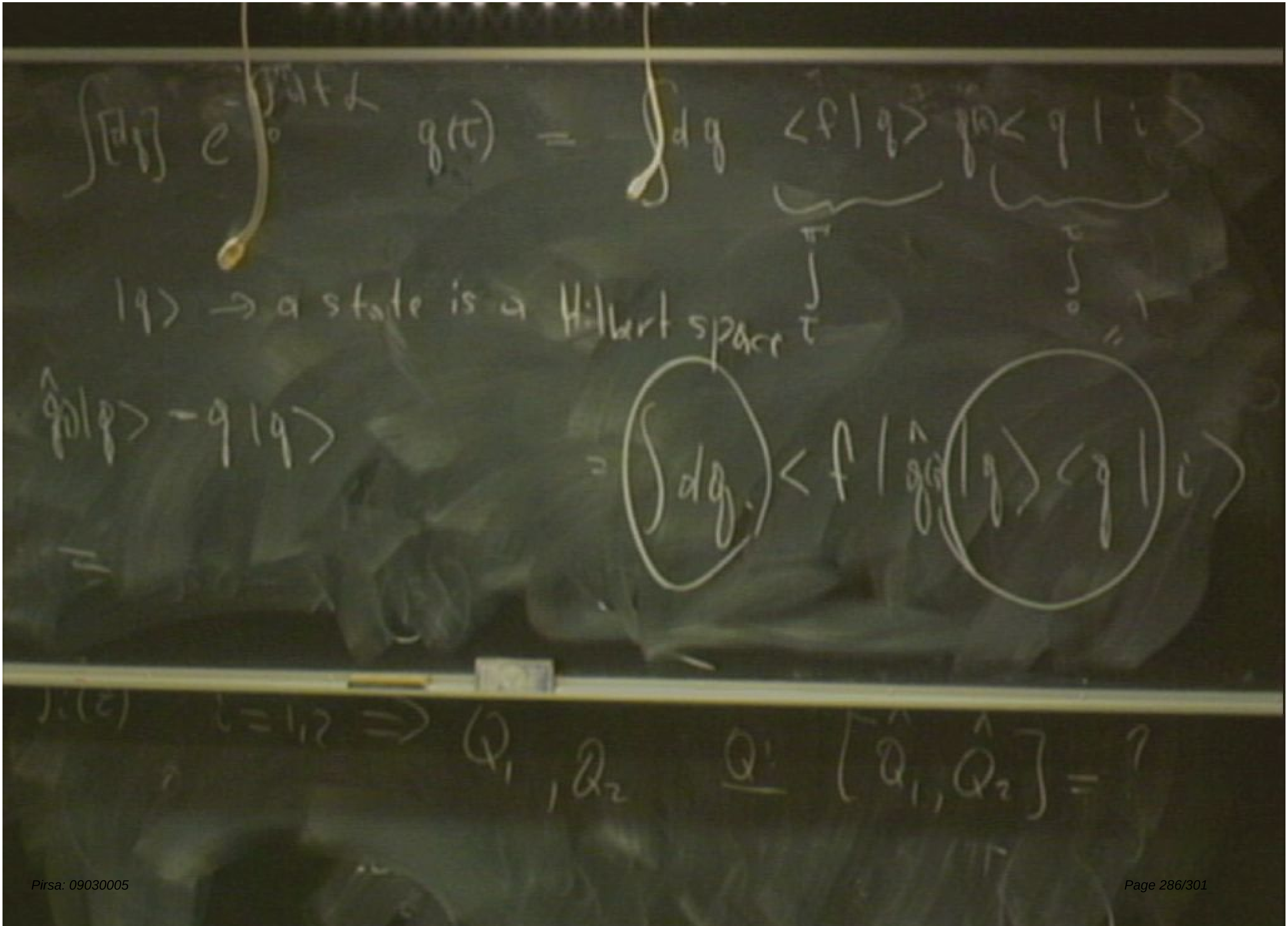
$$= \int dq \, \langle f | \hat{q}(\tau) | q \rangle \langle q |$$

$$\int [dq]\, e^{-\int_0^T dt\, L}\, q(\tau) = \int dq\, \underbrace{\langle f | q \rangle}_{\int_\tau^T}\, q(\tau) \underbrace{\langle q | i \rangle}_{\int_0^\tau}$$

$|q\rangle \rightarrow$ a state is a Hilbert space

$$\hat{q}(0)|q\rangle = q\,|q\rangle$$

$$= \int dq\, \langle f | \hat{q}(0) | q \rangle \langle q | i \rangle$$


|q> → a state is a Hilbert space
Q: [Q1, Q2] = ?

$$\int [dq]\, e^{-\int_0^{T} dt\, L}\; q(\tau) = \int dq\, \underbrace{\langle f | q \rangle}_{\int_\tau^{T}} q(\tau) \underbrace{\langle q | i \rangle}_{\int_0^{\tau}}$$

$|q\rangle \Rightarrow$ a state is a Hilbert space

$$\hat{q}(\tau)|q\rangle = q|q\rangle$$

$$= \int dq\, \langle f | \hat{q}(\tau) | q \rangle \langle q | i \rangle$$

$$= \langle f | \hat{q}(\tau) | i \rangle$$

$i = 1, 2 \Rightarrow Q_1, Q_2$ Q: $[\hat{Q}_1, \hat{Q}_2] = ?$

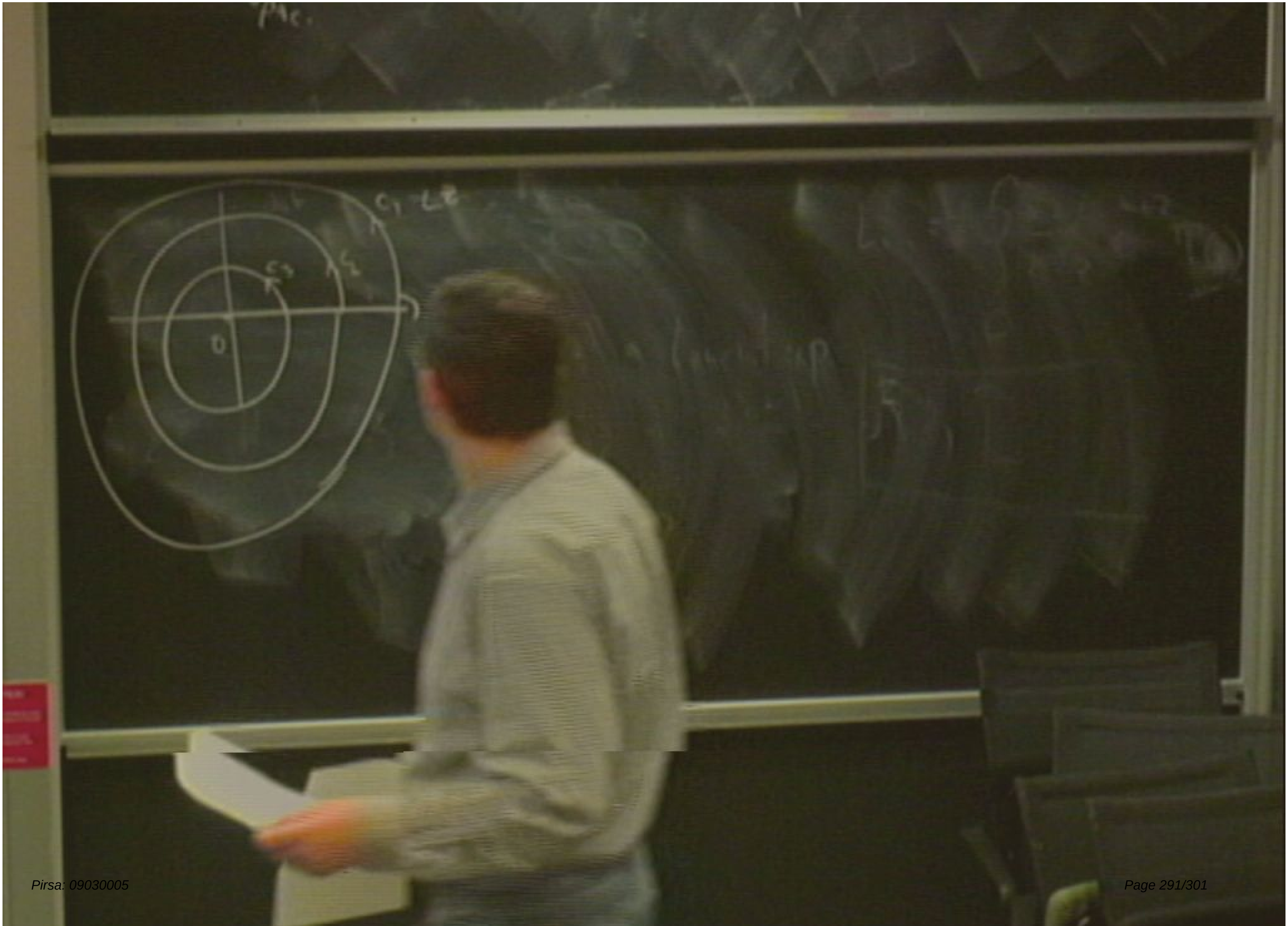
$$\int [Dq]\, e^{-\int_0^T dt\, L}\, q(t_1)\, q(t_2) =$$

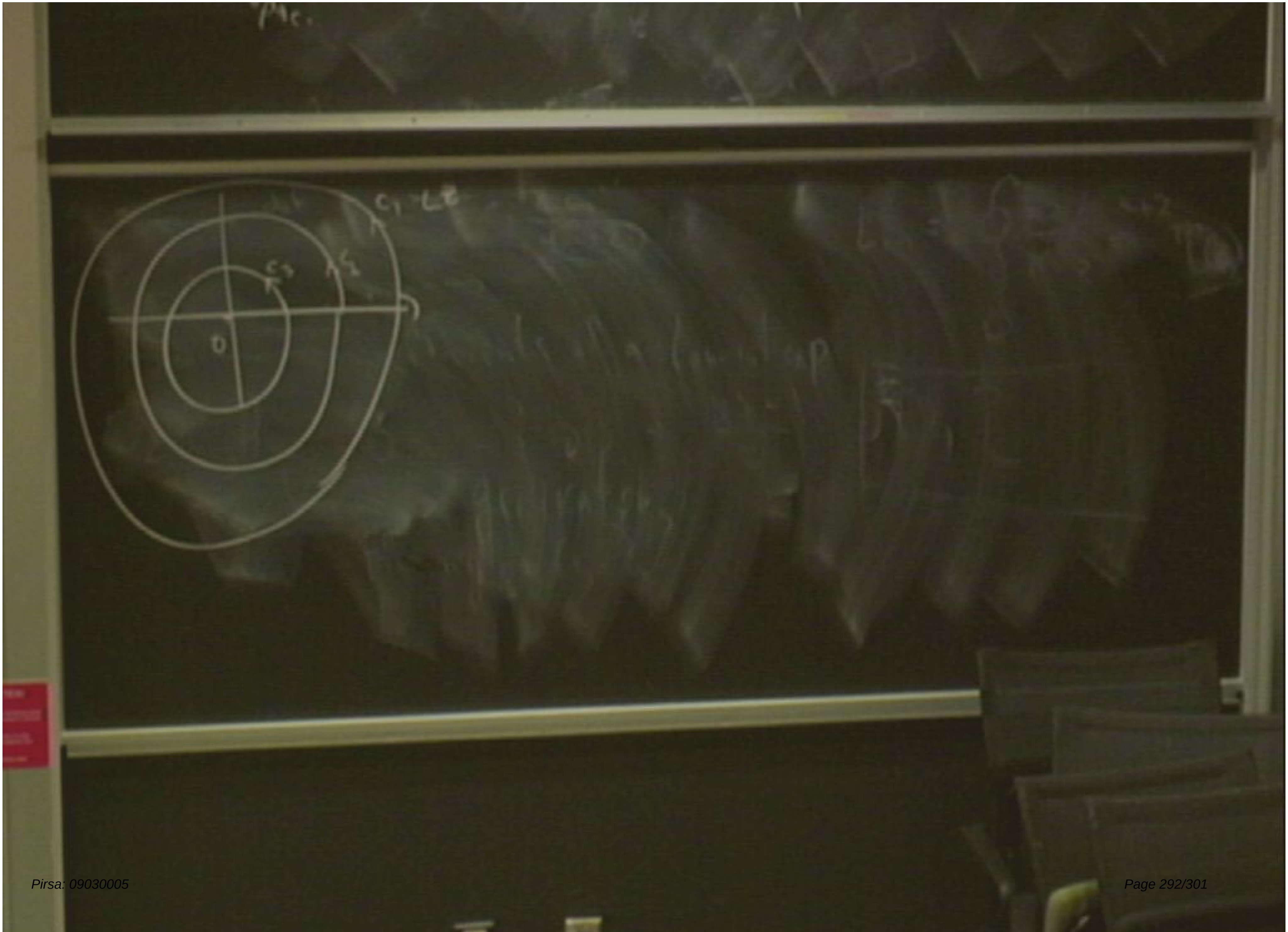
$$\int [dq]\, e^{-\int dt\, L}\, q(t_1)\, q(t_2) = \langle f \,|\, T\big(q(t_1)\, q(t_2)\big) \,|\, i \rangle$$

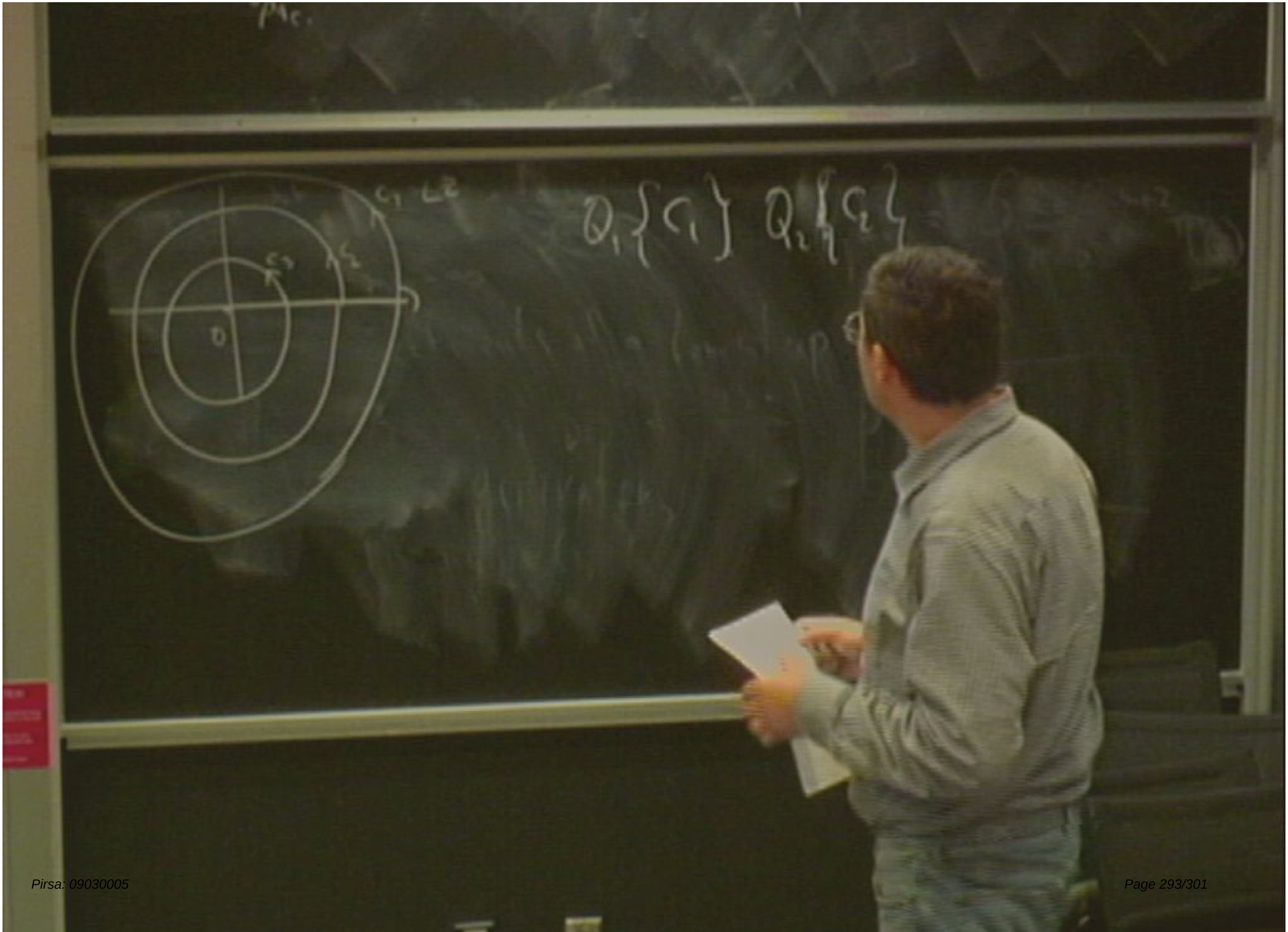
$$T\big(q(t_1)\, q(t_2)\big) = \Theta(t_1 - t_2)\, q(t_1)\, q(t_2) + \Theta(t_2 - t_1)$$

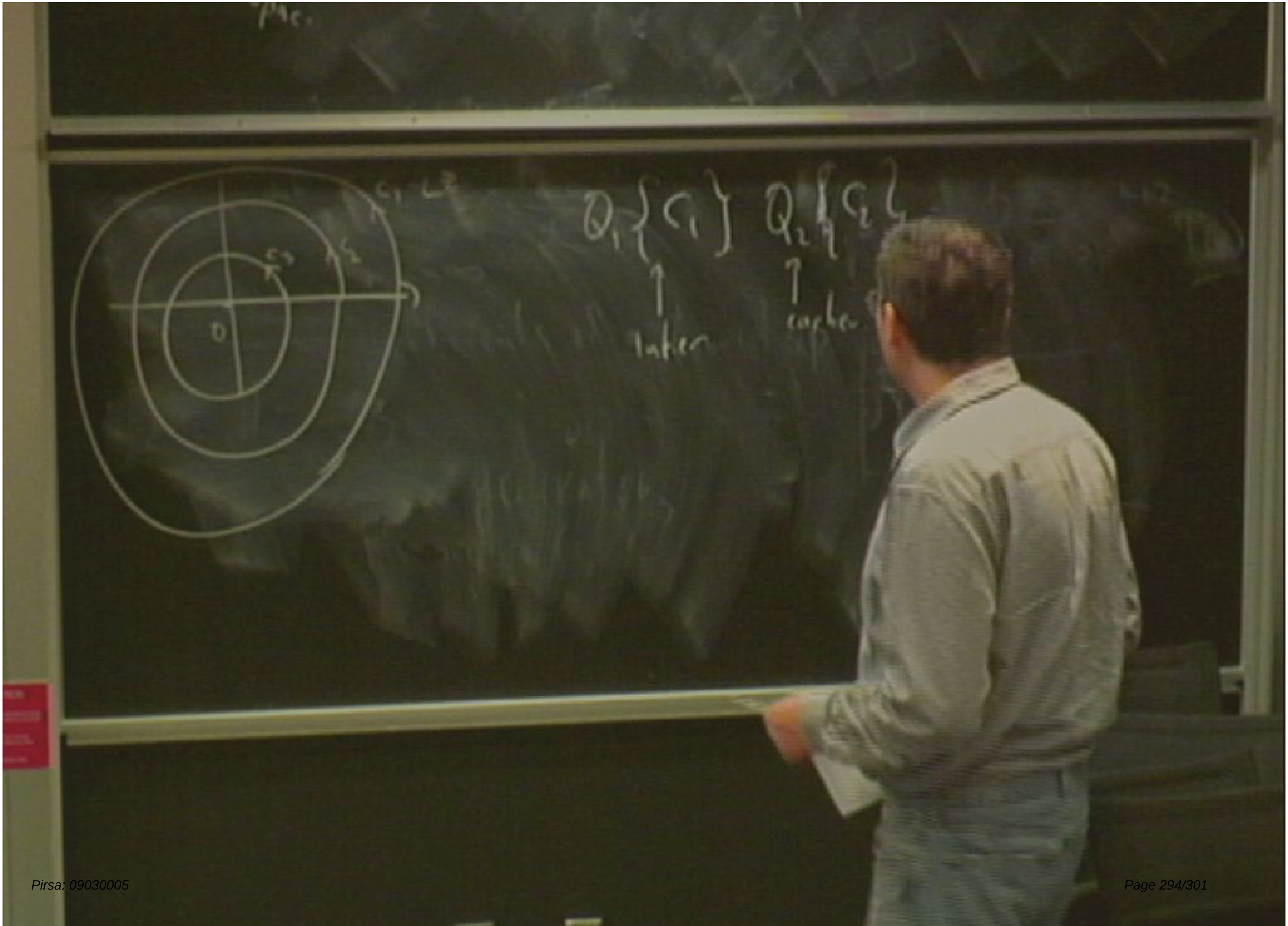
$$\int [dq]\, e^{-i\int dt\, L}\, q(t_1)\, q(t_2) = \langle f |\, T\big(q(t_1)\, q(t_2)\big) \,| i \rangle$$

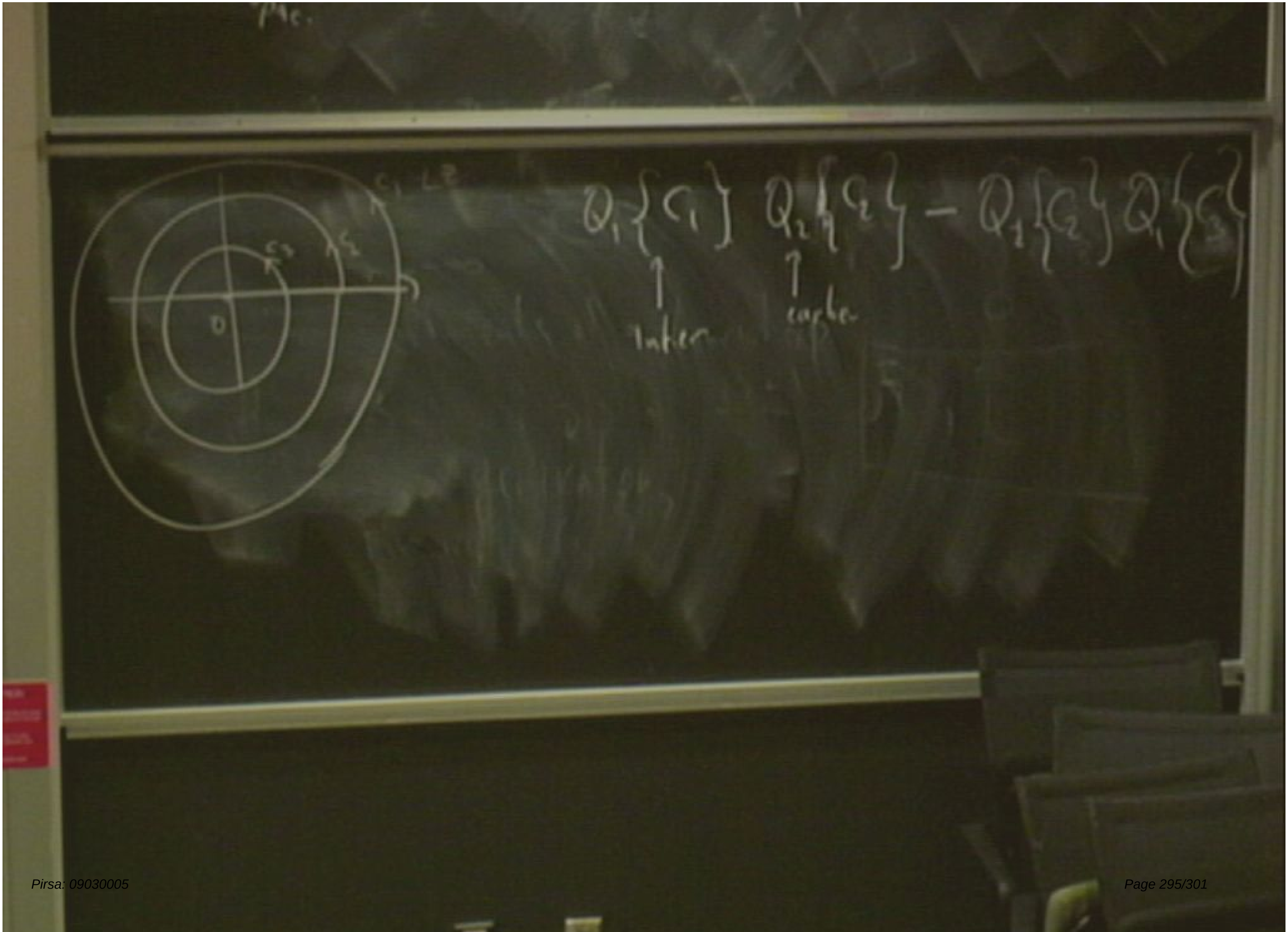
$$T\big(q(t_1)\, q(t_2)\big) = \Theta(t_1 - t_2)\, q(t_1)\, q(t_2) + \Theta(t_2 - t_1) \ldots$$

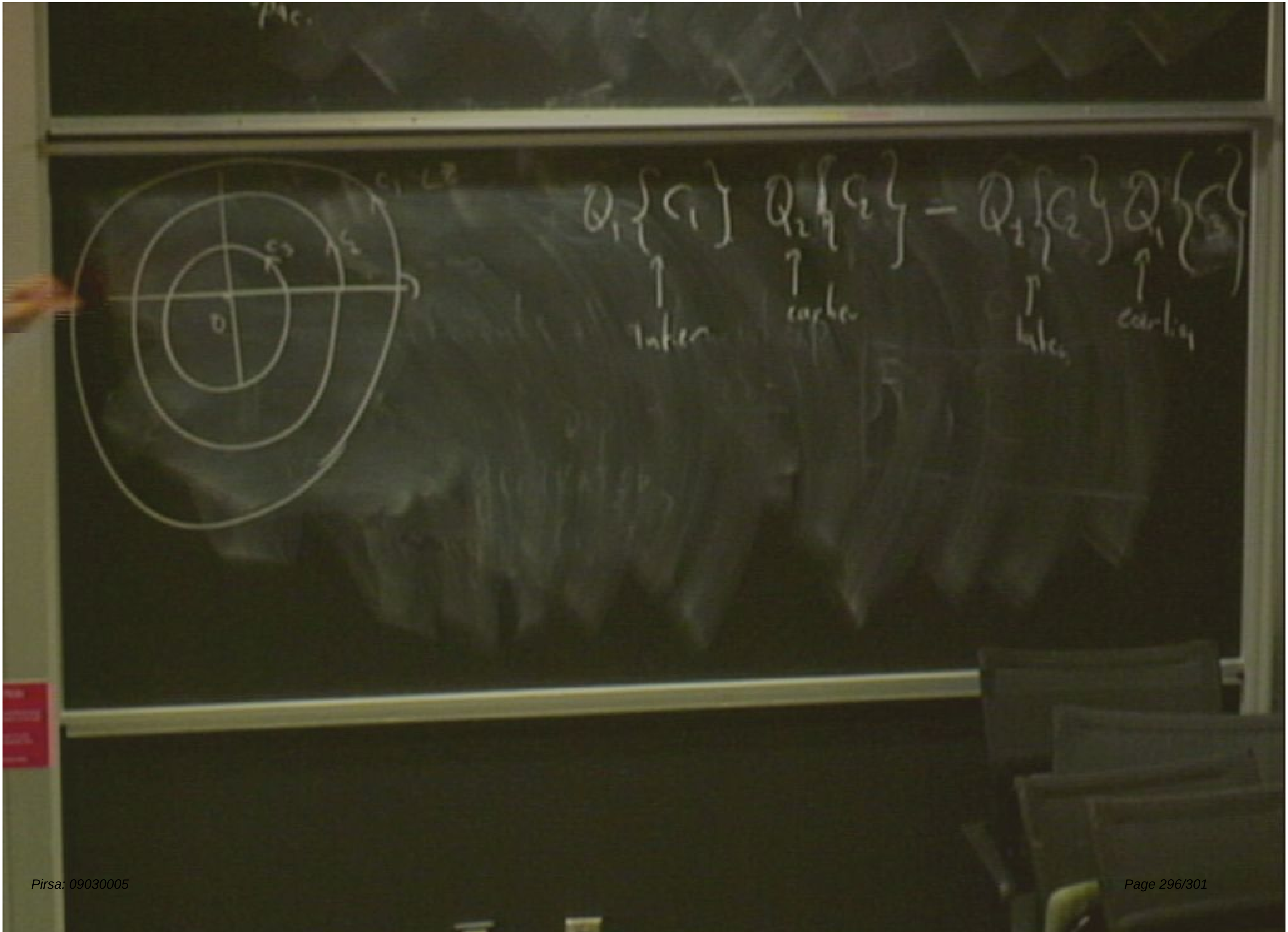


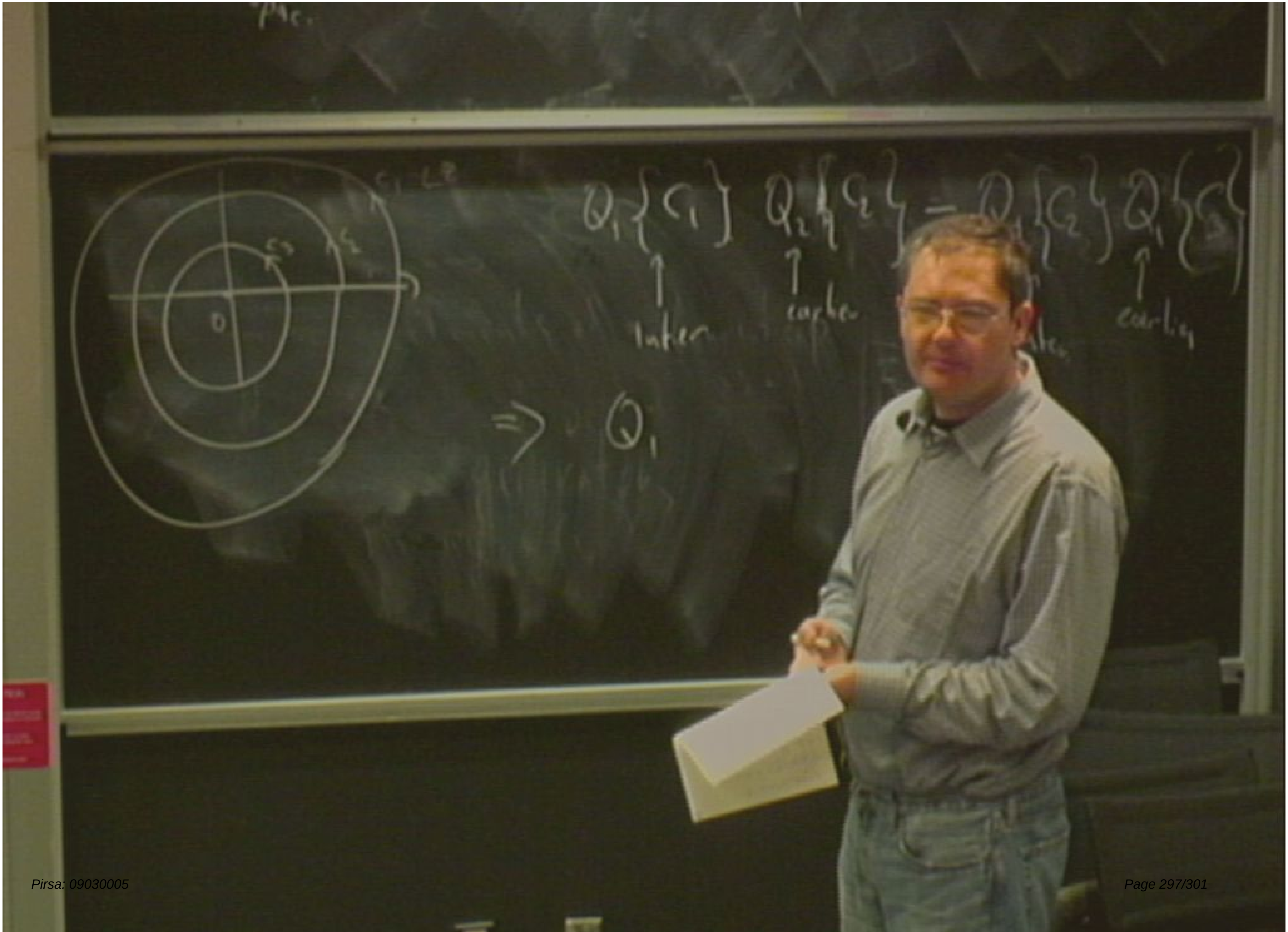



Q1 {C1} Q2 {C2}

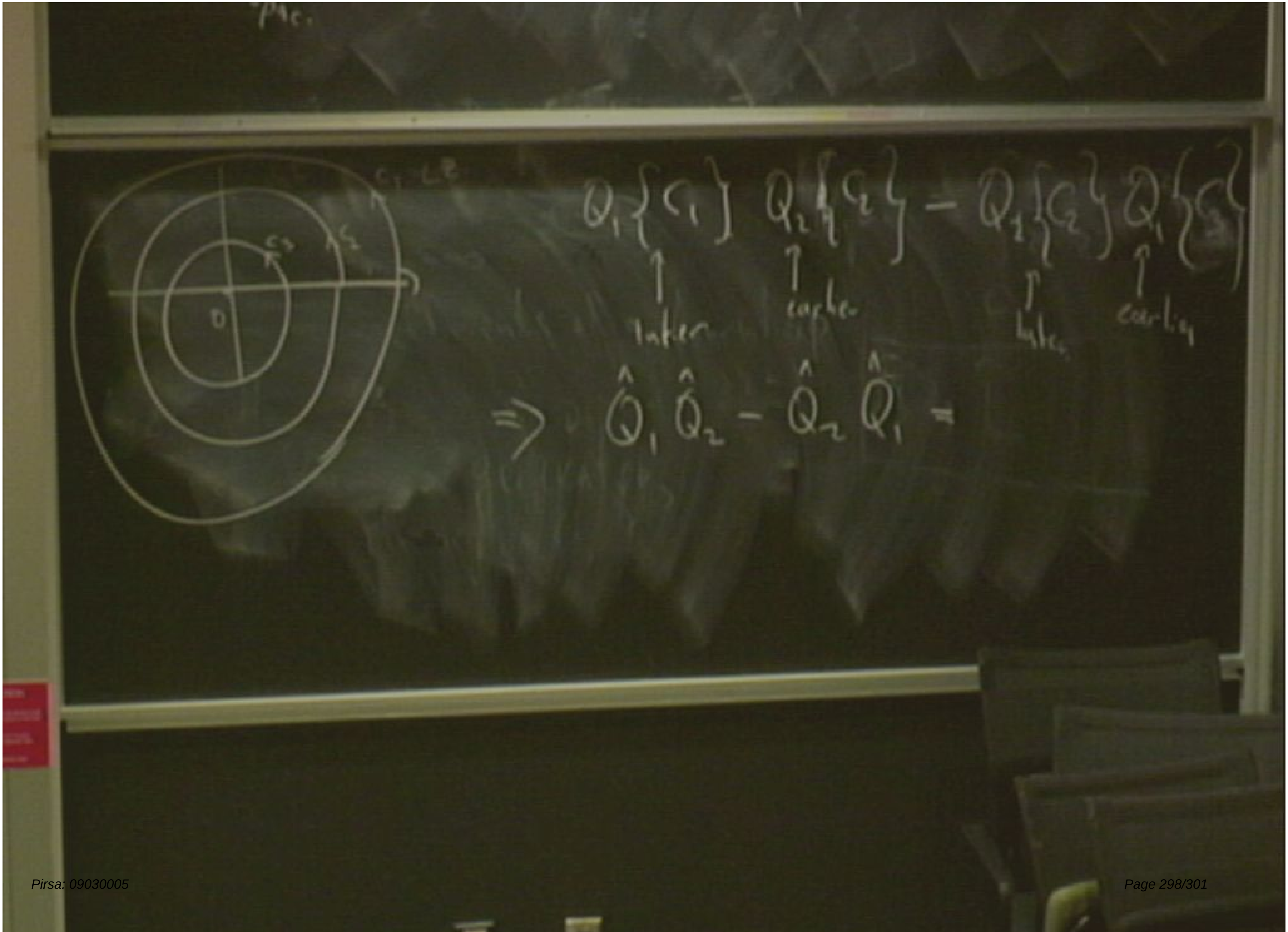

$Q_1 \{C_1\}$ $Q_2 \{C_2\}$
later
O

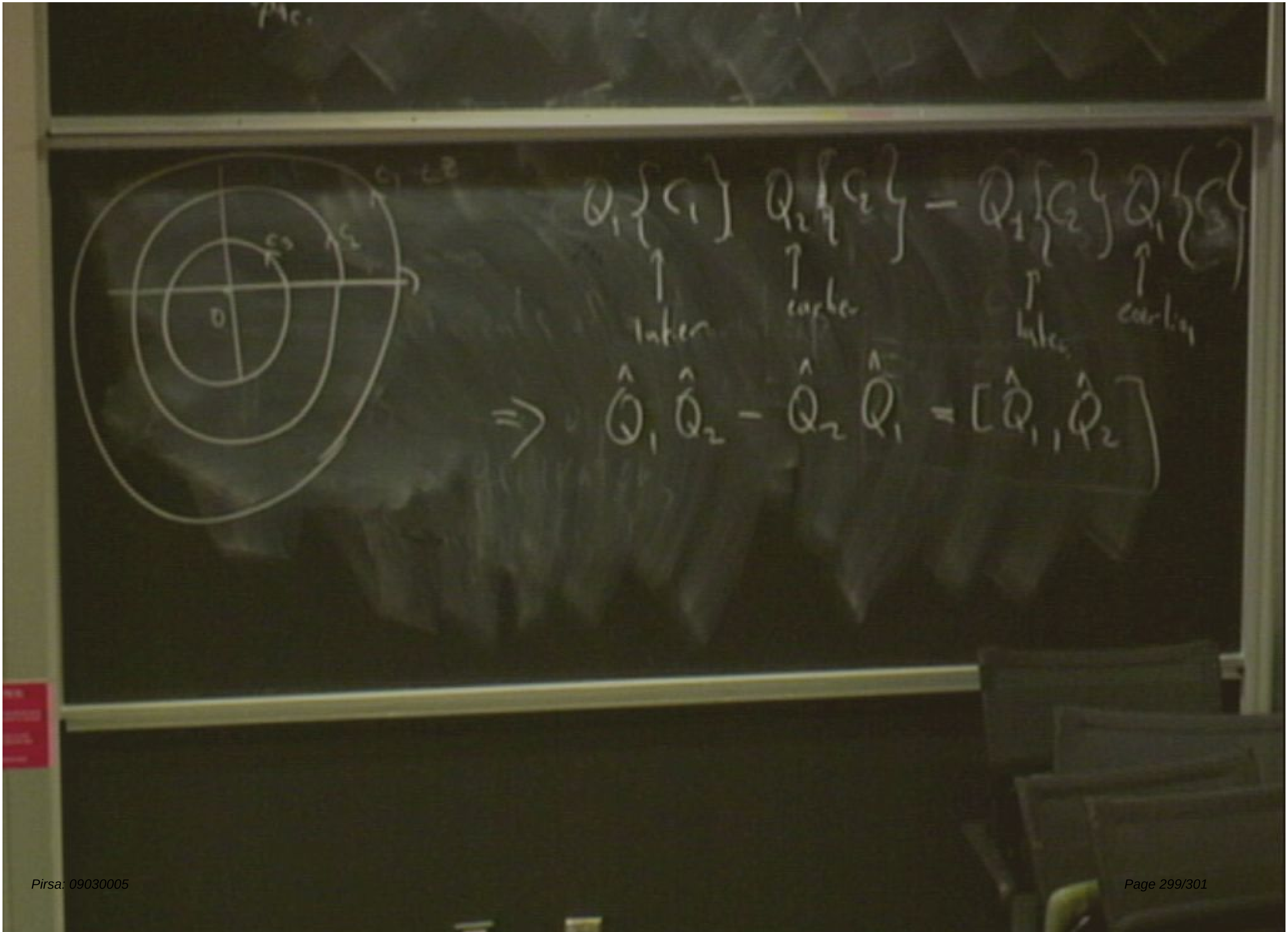

$Q_1\{C_1\}\ Q_2\{C_2\} - Q_1\{C_2\}\ Q_1\{C_3\}$
inner
earlier
C_1
C_2
C_3
0


Q1{C1} Q2{C2} = Q2{C2} Q1{C1}
later
earlier
later
earlier

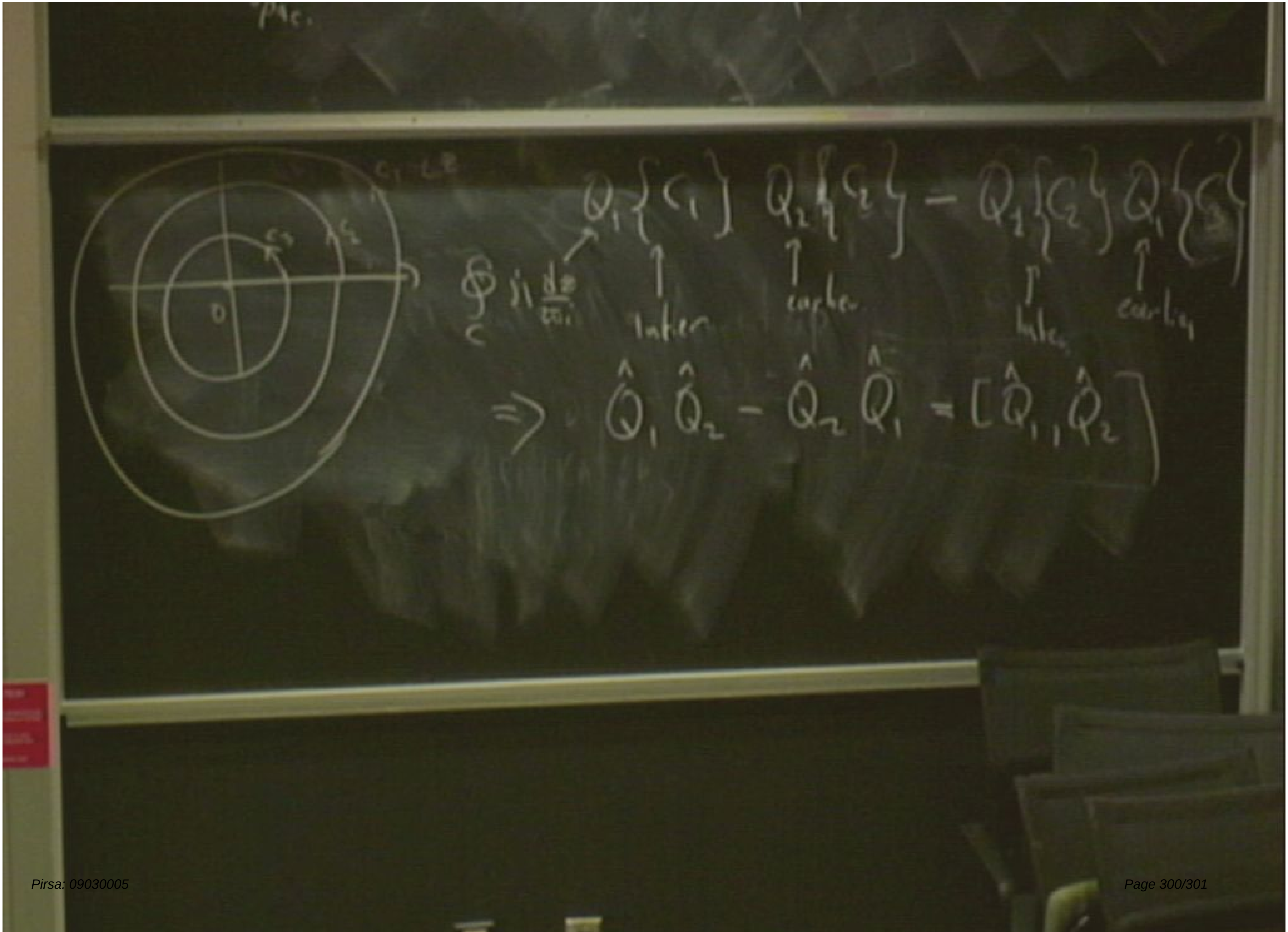


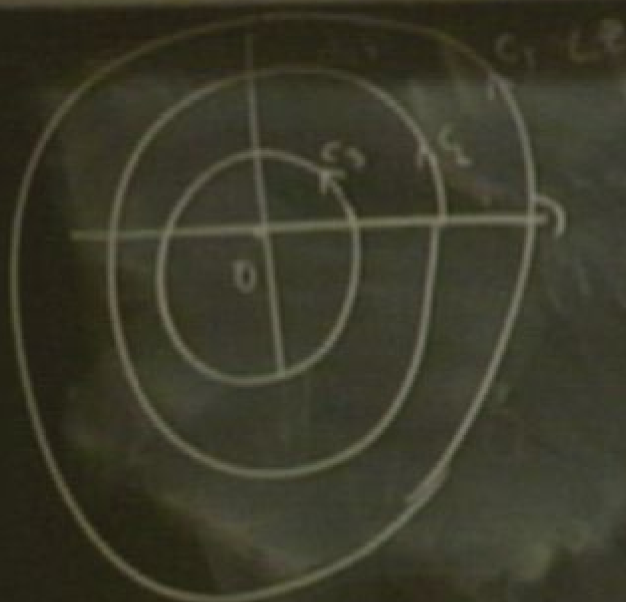
later
earlier
earlier


$Q_1\{C_1\}\ Q_2\{C_2\} - Q_1\{C_2\}\ Q_1\{C_3\}$
later
earlier
later
earlier
$\Rightarrow \hat{Q}_1 \hat{Q}_2 - \hat{Q}_2 \hat{Q}_1 =$



$Q_1\{C_1\}\ Q_2\{C_2\} - Q_1\{C_2\}\ Q_1\{C_3\}$
later
earlier
later
earlier
$\Rightarrow \hat{Q}_1 \hat{Q}_2 - \hat{Q}_2 \hat{Q}_1 = [\hat{Q}_1, \hat{Q}_2]$


$Q_1\{C_1\}\, Q_2\{C_2\} - Q_1\{C_2\}\, Q_1\{C_3\}$
$\oint_C j_1 \frac{d\sigma}{2\pi i}$
later
earlier
later
earlier
$\Rightarrow \hat{Q}_1 \hat{Q}_2 - \hat{Q}_2 \hat{Q}_1 = [\hat{Q}_1, \hat{Q}_2]$



$$\oint_C j_1 \frac{d\sigma}{\ldots} \rightarrow Q_1\{C_1\}\; Q_2\{C_2\} - Q_1\{C_2\}\; Q_1\{C_3\}$$

later, earlier, later, earlier

$$\Rightarrow \hat{Q}_1 \hat{Q}_2 - \hat{Q}_2 \hat{Q}_1 = [\hat{Q}_1, \hat{Q}_2]$$