

Title: Introduction to the Bosonic String

Date: Feb 27, 2009 10:00 AM

URL: <http://pirsa.org/09030005>

Abstract: This course provides a thorough introduction to the bosonic string based on the Polyakov path integral and conformal field theory. We introduce central ideas of string theory, the tools of conformal field theory, the Polyakov path integral, and the covariant quantization of the string. We discuss string interactions and cover the tree-level and one loop amplitudes. More advanced topics such as T-duality and D-branes will be taught as part of the course. The course is geared for M.Sc. and Ph.D. students enrolled in Collaborative Ph.D. Program in Theoretical Physics. Required previous course work: Quantum Field Theory (AM516 or equivalent). The course evaluation will be based on regular problem sets that will be handed in during the term. The primary text is the book: 'String theory. Vol. 1: An introduction to the bosonic string. J. Polchinski (Santa Barbara, KITP) . 1998. 402pp. Cambridge, UK: Univ. Pr. (1998) 402 p.' All interested students should contact Alex Buchel at abuchel@uwo.ca as soon as possible.

bc CFT

• a holomorphic $\frac{bc}{CFT}$.

$$S = \frac{1}{2\pi} \int dz d\bar{z} b \bar{c}$$

$$b = b(z, \bar{z})$$

$$S = \frac{1}{2\pi} \int d^2z \quad b \bar{\partial} c$$

$$b = b(z, \bar{z}) \quad c = c(z, \bar{z}).$$

FOM

$$\bar{\partial} c = 0$$

• a holomorphic CFT.

$$S = \frac{1}{2\pi} \int d^2z \quad b \bar{\partial} c$$

$$b = b(z, \bar{z}) \quad c = c(z, \bar{z}).$$

EOM

$$\bar{\partial} c = 0 \quad ; \quad \bar{\partial} b = 0$$

• a holomorphic $\frac{bc}{CFT}$.

$$b.c = -c.b$$

$$S = \frac{1}{2\pi} \int d^2z \, b \bar{\partial} c$$

$$b = b(z, \bar{z}) \quad c = c(z, \bar{z})$$

FOH

$$\bar{\partial} c = 0 \quad ; \quad \bar{\partial} b =$$

• a holomorphic \overline{bc} CFT.

$$b \cdot c = -c \cdot b$$

$$S = \frac{1}{2\pi} \int d^2z \, b \bar{\partial} c$$

$$(h_b, 0) =$$

$$b = b(z, \bar{z}) \quad c = c(z, \bar{z}).$$

FOH

$$\bar{\partial} c = 0 \quad ; \quad \bar{\partial} b = 0$$

• a holomorphic \overline{bc} CFT.

$$S = \frac{1}{2\pi} \int d^2z \ b \bar{\partial} c$$

$$b = b(z, \bar{z}) \quad c = c(z, \bar{z})$$

Eqn
 $\bar{\partial} c = 0 \quad ; \quad \bar{\partial} b = 0$

$$b \cdot c = -c \cdot b$$

$$(h_b, 0) = (\lambda, 0)$$

$$(h_c, 0) = (1-\lambda, 0)$$

Quantum EOM:

$$= \int [db][dc] \frac{\delta}{\delta c} \left[\begin{matrix} -s \\ e \\ c \end{matrix} \right]$$

Quantum EOM:

$$\partial = (c) \frac{\partial}{\partial c} \left[e^{-S} \right] = -\frac{1}{2\pi} \bar{\partial} b \cdot c + \delta^2(z, \bar{z})$$

Quantum EOM:

$$0 = \int [db] [dc] \frac{\delta}{\delta c} \left[e^{-S} c \right] = -\frac{1}{2\pi} \bar{\partial} b \cdot c + \delta^2(z, \bar{z})$$

$$:bc: = bc$$

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$$:bc: = bc - \frac{1}{2i2}$$

Quantum EOM:

$$0 = \int [db] [dc] \frac{\delta}{\delta c} \left[e^{-S} \right] = -\frac{1}{2\pi} \bar{\partial} b \cdot c + \delta^2(z, \bar{z})$$

$$:bc: = bc - \frac{1}{2i2}$$

$$b(z_1)c(z_2) \sim \frac{1}{z_{12}}$$

$$c(z_1)b(z_2) \sim \frac{1}{z_{12}}$$

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$$z_{12} \equiv z_1 - z_2$$

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$$bb \sim O(z_{12})$$

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$$bb \sim O(z_{12})$$

$$cc \sim O(z_{12})$$

$$b(z_1)c(z_2) \sim \frac{1}{z_{12}}$$

$$c(z_1)b(z_2) \sim \frac{1}{z_{12}}$$

$$z_{12} \equiv z_1 - z_2$$

$$bb \sim O(z_{12})$$

$$cc \sim O(z_{12})$$

$$T(z) = :abc: - \lambda :bc:$$

$$\mathbb{T}(z) = :T(z) - \lambda \delta(z) :$$

$$\tilde{\mathbb{T}}(\bar{z}) = 0$$



$$\mathbb{T} O_h \sim \frac{h}{z^2} O + \frac{1}{z} \partial O + \text{non-singular}$$

$$\mathbb{T}(z)O_h(z) \sim \frac{h}{z^2} O + \frac{1}{z} \partial O + \text{non-singular}$$

$\mathbb{T}b$

$$\mathbb{T}(z)O_h(z) \sim \frac{h}{z^2} O + \frac{1}{z} \partial O + \text{regular}$$

$$\mathbb{T}b = \left(\partial bc - \lambda \partial [bc] \right) b$$

\uparrow
 $\mathbb{T}(z)$

$$\mathbb{T}(z) O_h(z) \sim \frac{h}{z^2} O + \frac{1}{z} \partial O + \text{non-singular}$$

$$\mathbb{T} b = \left(\frac{\partial b}{\partial z} - \lambda \partial [bc] \right) \underline{b}$$

\uparrow
 $\mathbb{T}(z)$



$$\mathbb{T}(z) O_{\frac{1}{2}}(z) \sim \frac{\hbar}{z^2} O + \frac{1}{z} \partial O + \text{non-singular}$$

$$\mathbb{T} b = \left(\frac{\partial b c}{\partial z} - \lambda \partial [b c] \right) \underline{b} = \frac{\partial b(z)}{\partial z}$$

\uparrow
 $\mathbb{T}(z)$

$$\mathbb{T}(z) O_h(z) \sim \frac{h}{z^2} O + \frac{1}{z} \partial O + \text{non-singular}$$

$$\mathbb{T} b = \left(\frac{\partial b}{\partial z} - \lambda \partial [bc] \right) \underline{b} = \frac{\partial b(z)}{\partial z} - \lambda \partial b(z)$$

\uparrow
 $\mathbb{T}(z)$

$$T(z)O_h(z) \sim \frac{h}{z^2} O + \frac{1}{z} \partial O + \text{non-singular}$$

$$T b = \left(\frac{\partial b}{\partial z} - \lambda \partial [bc] \right) \underline{b} = \frac{\partial b(z)}{\partial z} - \lambda \partial b(z) \cdot \frac{1}{z}$$

\uparrow
 $T(z)$

$$\mathbb{T}(z) O_h(z) \sim \frac{h}{z^2} O + \frac{1}{z} \partial O + \text{non-singular}$$

$$\mathbb{T} b = \left(\frac{\partial b}{\partial z} - \lambda \partial [b] \right) \underline{b} = \frac{\partial b(z)}{\partial z} - \lambda \partial b(z) \cdot \frac{1}{z}$$

\uparrow
 $\mathbb{T}(z)$

$$\mathbb{T}(z)O_h(b) \sim \frac{h}{z^2} O + \frac{1}{z} \partial O + \text{non-singular}$$

$$\mathbb{T}b = \left(\frac{\partial b}{\partial z} - \lambda \partial [bc] \right) \underline{b} = \frac{\partial b(z)}{\partial z} - \lambda \partial b(z) \cdot \frac{1}{z}$$

\uparrow
 $\mathbb{T}(z)$

$$= \frac{1}{z} \partial b [1 - \lambda z]$$

$$\mathbb{T}(z)O_h(b) \sim \frac{h}{z^2} O + \frac{1}{z} \partial O + \text{non-singular}$$

$$\mathbb{T}b = \left(\frac{\partial bc}{\partial z} - \lambda \partial [bc] \right) \underline{b} = \frac{\partial b(z)}{\partial z} - \lambda \partial b(z) \cdot \frac{1}{z}$$

$$\begin{matrix} \uparrow \\ \mathbb{T}(z) \end{matrix} \quad \rightarrow \quad \left[\frac{1}{z} \right] \partial [bc] - \lambda \partial [bc]$$

$$\rightarrow \left[\lambda - 1 \right] \partial \frac{1}{z} = \frac{1}{z^2} \partial b$$

$$\mathbb{T}(z)O_h(z) \sim \frac{h}{z^2} O + \frac{1}{z} \partial O + \text{non-singular}$$

$$\begin{aligned} \mathbb{T}b &= \left(\frac{\partial bc}{\partial z} - \lambda \partial [bc] \right) \underline{b} = \frac{\partial b(z)}{\partial z} - \lambda \partial b(z) \cdot \frac{1}{z} \\ &= \frac{1}{z} \partial b [1 - \lambda] - \lambda [b + z \partial b] \left(\frac{1}{z} \right) \end{aligned}$$

$$\mathbb{T}(z)O_h(z) \sim \frac{h}{z^2} O + \frac{1}{z} \partial O + \text{non-singular}$$

$$\mathbb{T}b = \left(\frac{\partial bc}{\partial z} - \lambda \frac{\partial [bc]}{\partial z} \right) \underline{b} = \frac{\partial b(z)}{\partial z} - \lambda \frac{\partial b(z)}{\partial z} \cdot \frac{1}{z}$$

$$= \frac{1}{z} \frac{\partial b}{\partial z} [1 - \lambda] - \lambda [b + z \frac{\partial b}{\partial z}] \left(\frac{1}{z} \right)$$

$$\sim \frac{\lambda}{\lambda^2} \cdot b + \frac{1}{\lambda^2} q c [1 - \lambda + \lambda] //$$

$$\sim \left[\frac{k}{z^2} b + \frac{1}{z^2} q b [1 - \lambda + \lambda] \right] "$$

$$\sim \frac{k}{z^2} b + \frac{1}{z^2} q b$$

$$\sim \left[\frac{\lambda}{z^2} b + \frac{1}{z} a [1 - \lambda + \lambda] \right] //$$

$$\sim \frac{\lambda}{z^2} b + \frac{1}{z} a \quad \boxed{\lambda = 1}$$



$$\mathbb{T}(z) O_h^{(0)} \sim \frac{h}{z^2} O + \frac{1}{z} \partial O + \text{non-singular}$$

$$\mathbb{T} b = \left(\frac{\partial b c}{\partial c} - \lambda \partial [b c] \right) \underline{b} = \frac{\partial b^{(0)}}{z} - \lambda \partial b^{(0)} \cdot \frac{1}{z}$$

$$\begin{aligned} & \begin{array}{c} \uparrow \\ \mathbb{T}(z) \end{array} \quad \begin{array}{c} \left[\frac{-1}{z} \right] c \\ - \lambda b(z) \partial \left[\frac{-1}{z} \right] \end{array} \\ & = \frac{1}{z} \partial b \left[1 - \lambda \right] \underbrace{\left[q r z + q \right]}_{\left[\frac{-1}{z} \right]} \end{aligned}$$

$$\mathbb{T}(z)O_h^{(0)} \sim \frac{h}{z^2} O + \frac{1}{z} \partial O + \text{non-singular}$$

$$\mathbb{T}b = \left(\frac{\partial bc}{\partial z} - \lambda \partial [bc] \right) \underline{b} = \frac{\partial b^{(0)}}{z} - \lambda \partial b^{(0)} \cdot \frac{1}{z}$$

$$\begin{aligned} & \begin{array}{c} \uparrow \\ \mathbb{T}(z) \end{array} \\ & \left[\frac{-1}{z} \right] \partial [b(z)q] - \lambda b(z) \partial \left[\frac{-1}{z} \right] \\ & \left(\frac{-1}{z} \right) \underbrace{\left[q \partial z + q \right]}_{\mathbb{T}(z)} \left[\frac{-1}{z} \right] q \frac{1}{z} = \end{aligned}$$

$$\sim \frac{k}{z^2} b + \frac{1}{z^2} q b [1-k]$$

$$\sim \left(\frac{k}{z^2} b \right) + \frac{1}{z^2} q b$$

$$\nabla(\lambda) : C = [abc - \lambda \partial(bc)] \cdot c$$

$$\nabla(z) : C = [abc - \lambda \partial(bc)] \cdot e$$

~ -1

$$\mathbb{T}(z) : C = [abc - \lambda \partial(bc)] \cdot c$$

$$\sim \left\{ -1 \quad \partial \frac{1}{z} \right\}$$

$$\Pi(\lambda) : C = \left[abc - \lambda \partial(bc) \right] \cdot c$$

$$\sim \left\{ -1 \partial \frac{1}{2} + \lambda \partial \frac{1}{2} \right\} c$$



$$\Pi(\lambda) : C = \left[abc - \lambda \partial(bc) \right] \cdot c$$

$$\sim \left\{ -1 \cdot \frac{1}{2} + \lambda \frac{1}{2} \right\} c$$

$$\sim \left(-\lambda \cdot \frac{1}{2} + \frac{1}{2} \right) c \sim \frac{1-\lambda}{2} c + \dots$$

$$\Pi(z) : C = \left[abc - \lambda \partial(bc) \right] \cdot a$$

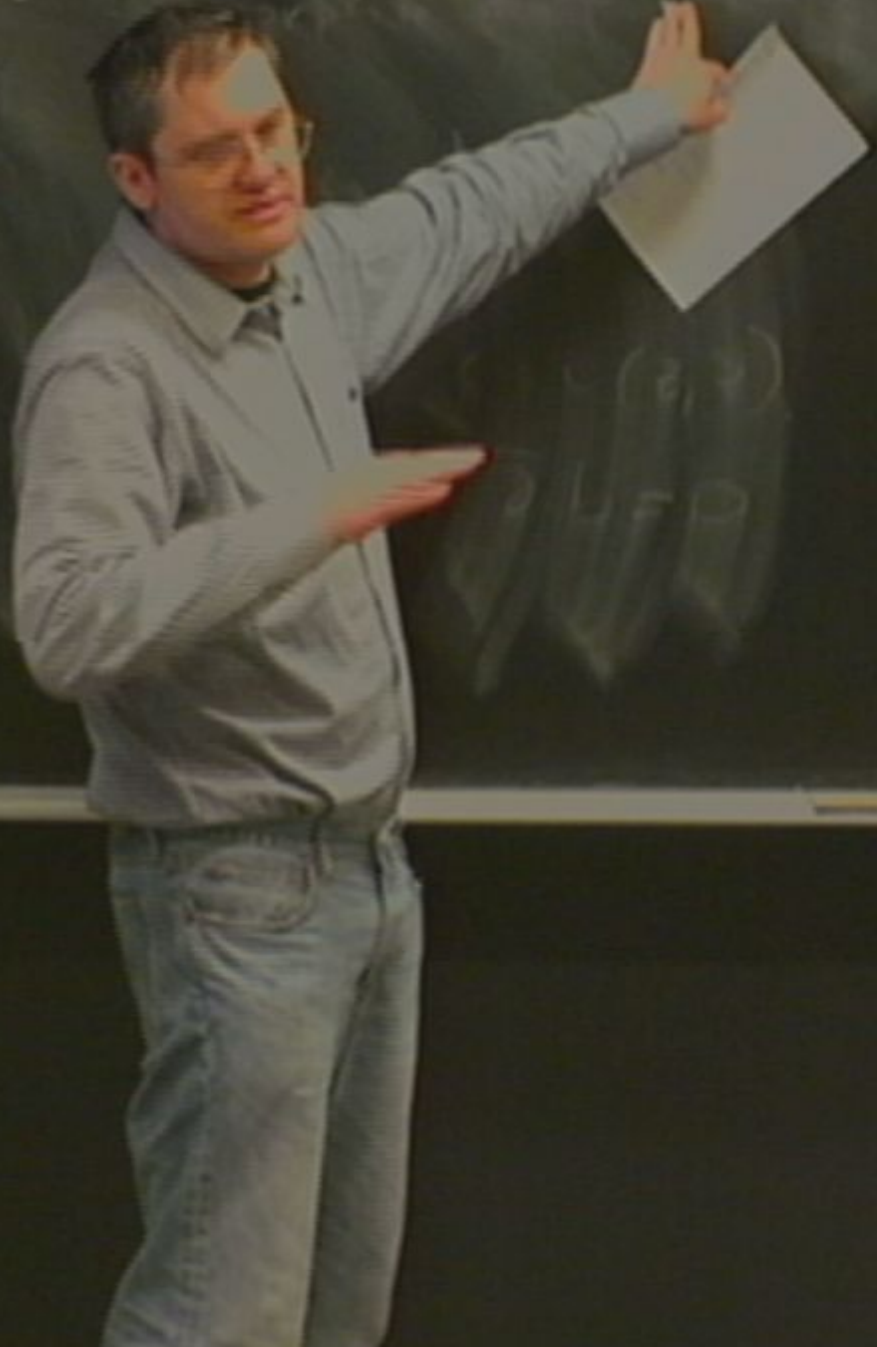
$$\sim \left\{ -1 \partial \frac{1}{z} + \lambda \partial \frac{1}{z} \right\}$$

$$\sim \left(-\lambda \cdot \frac{1}{z^2} + \frac{1}{z^2} \right) C$$

$$\sim \frac{1-\lambda}{z^2} C + \dots$$

$\lambda = 1$

D-free fields on a world sheet



D-free fields on a world sheet

$$\begin{aligned} \alpha_0 &= b \bar{a} c \\ \alpha_1 &= \sqrt{2} \alpha \\ \alpha_2 &= \sqrt{2} \alpha \end{aligned}$$

$$\begin{aligned} \alpha_0 &= b \bar{c} a \\ \alpha_1 &= \sqrt{2} \alpha \\ \alpha_2 &= \sqrt{2} \alpha \end{aligned}$$

D-free fields on a world sheet

$$c = -D$$

D-free fields on a world sheet

$$\underline{\underline{c=D}}$$

D-free fields on a world sheet

$$\underline{\underline{c=D}}$$

\Rightarrow Q: what is c of σ ?

D-free fields on a world sheet

$$\underline{\underline{c=D}}$$

what is c of 'bc' system?

$$\underline{\underline{C=D}}$$

\Rightarrow Q: what is c of 'bc' system?

$$\mathbb{T}(0) \sim \frac{c}{z^4}$$

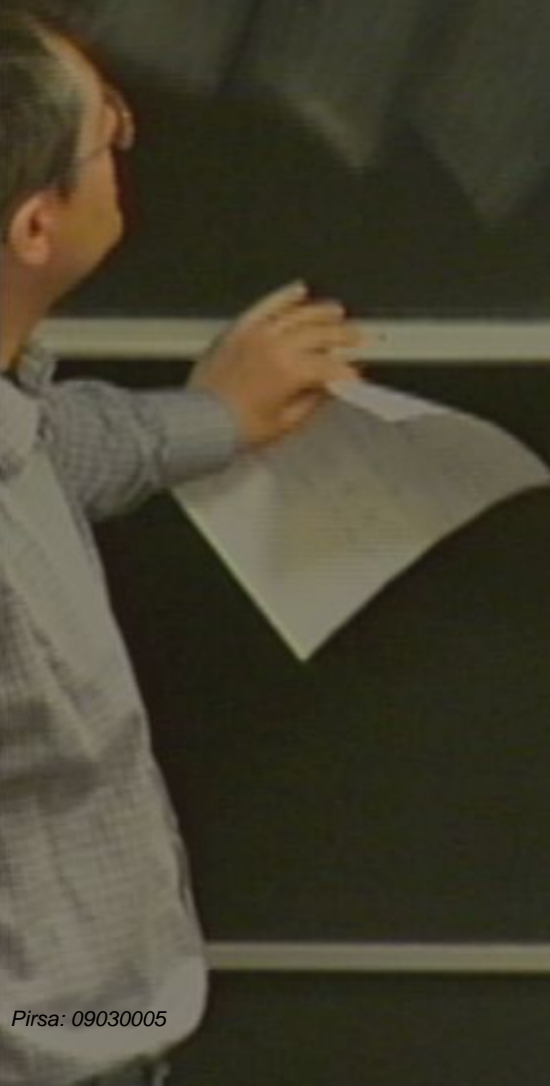
$$\underline{\underline{c = D}}$$

\Rightarrow Q: what is c of 'bc' system?

$$\Gamma(\sigma) \sim \frac{c}{z^4} + \frac{2\Gamma}{z^2} + \frac{1}{z} \sigma \Gamma + \dots$$

System

$$\Gamma(z)\Gamma(0) \sim \left(\frac{c}{z^4}\right) + \left\{ \frac{2\pi}{z^2} + \frac{1}{z} \right\} + \dots$$



$$\Pi' \Pi = \left[\partial b c - \lambda \alpha(b c) \right] \left[\partial b c - \lambda \alpha(b c) \right]$$

$$\left[\partial_{bc} - \lambda \partial(bc) \right] \left[\partial_{bc} - \lambda \partial(bc) \right]$$

$\underbrace{\hspace{15em}}_2$
 $\underbrace{\hspace{15em}}_0$

$$\mathbb{T}'\mathbb{T} = \underbrace{\left[\partial b c - \lambda \partial(bc) \right]}_2 \underbrace{\left[\partial b c - \lambda \partial(bc) \right]}_0$$

① : $(\partial b c)(\partial b c)$

$$-2\lambda (\partial b c)(\partial b c)$$

$$\mathbb{T}'\mathbb{T} = \underbrace{\left[\partial b c - \lambda \partial(bc) \right]}_2 \underbrace{\left[\partial b c - \lambda \partial(bc) \right]}_0$$

① : $(\partial b c)(\partial b c)$

② $-2\lambda (\partial b c)(\partial bc)$

③ $\lambda^2 (\partial bc)(\partial bc)$

$$\textcircled{1} = (\underline{abc})(\underline{abc})$$

CAUTION
PLEASE DO NOT TOUCH THE BOARD
OR THE EQUIPMENT IN THE ROOM
OR THE EQUIPMENT IN THE ROOM

$$\textcircled{1} = (\textcircled{2b.c}) (\textcircled{0b.c}) \rightsquigarrow (\textcircled{+}) \quad \frac{1}{2}$$

[Faded handwritten notes on the chalkboard, including the word "matrix" and some illegible symbols.]



CAUTION
 AVOID CONTACT WITH THE SURFACE OF THE BOARD
 TO PREVENT INJURY
 AND DAMAGE TO THE BOARD

$$\textcircled{1} = \textcircled{a|b|c} \textcircled{a|b|c} \rightsquigarrow \textcircled{+} \textcircled{a} \frac{1}{2}$$



CAUTION
 BEWARE OF HOT SURFACES
 AND ELECTRICAL WIRING
 IN THIS AREA

$$\textcircled{1} = (abc) \textcircled{abc} \sim \textcircled{+} \textcircled{+} \textcircled{\frac{1}{2}} \textcircled{\frac{1}{2}} \textcircled{+} \textcircled{+} \textcircled{\frac{1}{2}} \textcircled{\frac{1}{2}}$$

[The rest of the chalkboard is heavily scribbled out with white chalk.]

CAUTION
 Be careful of the sharp edges
 of the board when using it.
 Do not touch the board
 when it is hot.

$$\textcircled{1} = \begin{pmatrix} a & b & c \\ 0 & b & c \end{pmatrix} \sim \begin{pmatrix} + & 0 & \frac{1}{2} \\ - & 0 & \frac{1}{2} \end{pmatrix}$$

[Faded handwritten notes on the chalkboard]



CAUTION
 AVOID TO OPEN THE BOARD COVER
 TO PREVENT DAMAGE TO THE BOARD
 AND TO PREVENT INJURY

$$\mathbb{1} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} \sim \begin{pmatrix} + & 0 \\ 0 & - \end{pmatrix} \cdot \begin{pmatrix} \frac{1}{2} & \\ & \frac{1}{2} \end{pmatrix}$$

$$\textcircled{1} = \begin{pmatrix} a & b & c \\ 0 & b & c \\ 0 & 0 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$① = \begin{pmatrix} a & b & c \\ 0 & b & c \\ 0 & 0 & 0 \end{pmatrix} \sim \begin{pmatrix} + & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$a_2 \frac{1}{2} z_{12}$$

$$① = \begin{pmatrix} a & b & c \\ 0 & b & c \\ 0 & 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$a_2 \frac{1}{a_{12}} = -a_{21} = -\frac{1}{a_{12}}$$

EATK
 UNIVERSITÄT
 KÖLN
 INSTITUT FÜR
 MATHEMATIK
 LEHRSTUHL FÜR
 ALGEBRA
 UND
 ZAHLENLEHRE
 PROF. DR. GERT-MARTIN SEGER

$$① = \begin{pmatrix} a & b & c \\ 0 & b & c \\ 0 & 0 & 1 \end{pmatrix} \sim \begin{pmatrix} (+) & 0 & \frac{1}{2} \\ (-) & 0 & \frac{1}{2} \end{pmatrix}$$

$$a_2 \frac{1}{2} = -a_2 \frac{1}{2}$$



EATER
 UNIVERSITÄT
 ...
 ...

$$① = \begin{pmatrix} a & b & c \\ 0 & b & c \\ 0 & 0 & c \end{pmatrix} \sim \begin{pmatrix} a & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} a & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$a_2 \frac{1}{a_{12}} = -a_{21} = \frac{1}{a_{12}}$$

LATER
 ...
 ...
 ...

$$\Pi' \Pi = \underbrace{\left[\partial b c - \lambda \partial(bc) \right]}_2 \underbrace{\left[\partial b c - \lambda \partial(bc) \right]}_0$$

① : $(\partial b c)(\partial b c)$

② $-2\lambda (\partial b c)(\partial bc)$

③ $\lambda^2 (\partial bc)(\partial bc)$

$$= \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{pmatrix} \cdot \frac{1}{2} \begin{pmatrix} + \\ - \end{pmatrix} \sim \begin{pmatrix} 0 & b \\ a & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} \begin{pmatrix} 0 & b \\ a & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \textcircled{1}$$

$$\frac{1}{2} \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} = -\frac{1}{2} \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$$

$$\textcircled{2} - 2\lambda \begin{bmatrix} -\frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$$

$$\Pi' \Pi = \underbrace{\left[\partial b c - \lambda \partial(bc) \right]}_2 \underbrace{\left[\partial b c - \lambda \partial(bc) \right]}_0$$

① : $(\partial b c)(\partial b c)$

② $\neq -2\lambda (\partial b c)(\partial b c)$

③ $= (\partial b c)(\partial b c)$

$$= \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{pmatrix} \cdot \frac{1}{2} \begin{pmatrix} + & 0 \\ 0 & + \end{pmatrix} \sim \begin{pmatrix} \frac{1}{4} & 0 \\ 0 & -\frac{1}{4} \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \textcircled{1}$$

$$\frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = -\frac{1}{2} \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\textcircled{2} - 2\lambda \begin{bmatrix} -\frac{1}{2} & +\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$$

$$= \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{pmatrix} \cdot \frac{1}{2} \begin{pmatrix} + \\ - \end{pmatrix} \sim \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \textcircled{1}$$

$$\frac{1}{2} \begin{pmatrix} 1 \\ 2 \end{pmatrix} = - \frac{1}{2} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\textcircled{2} - 2 \lambda \left[\begin{array}{c} -\frac{1}{2^4} + \frac{1}{2} \cdot \frac{1}{2} \\ -\frac{1}{2} \end{array} \right]$$

$$\begin{pmatrix} 9 & 2 + 9 \\ -17 & 9 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\textcircled{1} = \begin{pmatrix} a & b & c \\ c & a & b \\ b & c & a \end{pmatrix} \sim \begin{pmatrix} + & - & - \\ - & + & - \\ - & - & + \end{pmatrix} \cdot \frac{1}{2} = \begin{pmatrix} \frac{1}{2} & - & - \\ - & \frac{1}{2} & - \\ - & - & \frac{1}{2} \end{pmatrix}$$

$$a_2 \frac{1}{2} = - \frac{1}{2} a_{12}$$

$$\textcircled{2} - 2\lambda \left[-\frac{1}{2^4} + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot (-1) \right] = \frac{12 \times}{2 \cdot 2^4}$$

$$\textcircled{3} \quad - \frac{12\lambda^2}{2\lambda^4}$$

$$C = -2 + 12\lambda - 12\lambda^2 = -3(2\lambda - 1)^2 + 1$$

$$\textcircled{3} \quad -\frac{12\lambda^2}{22^4}$$

$$C = -2 + 12\lambda - 12\lambda^2 = -3(2\lambda - 1)^2 + 1$$

$\vec{\eta} \vec{\eta}$

$$\textcircled{3} \quad -\frac{12\lambda^2}{2z^4}$$

$$C = -2 + 12\lambda - 12\lambda^2 = -3(2\lambda - 1)^2 + 1$$

$$\vec{\pi} \vec{\pi} \Rightarrow \tilde{C} = 0$$

$$\textcircled{3} \quad -\frac{12\lambda^2}{224}$$

$$c \neq \tilde{c}$$

$$c = -2 + 12\lambda - 12\lambda^2 = -3(2\lambda - 1)^2 + 1$$

$$\tilde{c} \neq c \Rightarrow \tilde{c} = 0$$

$$\textcircled{3} \quad -\frac{12\lambda^2}{22^4}$$

$$c \neq \tilde{c}$$

$$c = -2 + 12\lambda - 12\lambda^2 = -3(2\lambda - 1)^2 + 1$$

$$\tilde{c} \Rightarrow \tilde{c} = 0$$

bc system has a ghost # symmetry

$$\delta b = -i \epsilon b$$

bc system has a ghost # symmetry

$$\left. \begin{aligned} \delta b &= -i \epsilon b \\ \delta c &= +i \epsilon c \end{aligned} \right\} \Rightarrow$$

bc system has a ghost # symmetry

$$\left. \begin{aligned} \delta b &= -i \epsilon b \\ \delta c &= +i \epsilon c \end{aligned} \right\} \Rightarrow j = -i b c i$$

bc system has a ghost # symmetry

$$\delta b = -i \epsilon b$$

$$\delta c = +i \epsilon c$$

} \Rightarrow

$$j = -i b c$$

$$\left. \begin{aligned} \delta b &= -i \xi b \\ \delta c &= +i \xi c \end{aligned} \right\} \Rightarrow j = -i b c i$$

$$\mathcal{T}(z) j(0) = \left[\partial b c - \lambda \delta b c \right] \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

bc system has a ghost # symmetry

$$\left. \begin{aligned} \delta b &= -i \xi b \\ \delta c &= +i \xi c \end{aligned} \right\} \Rightarrow j = - : b c :$$

$$\langle T(z) j(0) \rangle = \left[\partial b c - \lambda \delta b c \right] (-1) : b c$$

~

bc system has a ghost # symmetry

$$\left. \begin{aligned} \delta b &= -i \epsilon b \\ \delta c &= +i \epsilon c \end{aligned} \right\} \Rightarrow j = - : b c :$$

$$\langle T(z) j(0) \rangle = \left[\partial b^c - \lambda \delta b c \right] (-1) : b c :$$

$$\sim (-1)$$

bc system has a ghost # symmetry

$$\left. \begin{aligned} \delta b &= -i \xi b \\ \delta c &= +i \xi c \end{aligned} \right\} \Rightarrow j = - :bc:$$

$$\langle T(z) j(0) \rangle = \left[\partial b^c - \lambda \delta b^c \right] (-1) :bc:$$

$$\sim (-1) \left[c^{\frac{1}{2}} \cdot \frac{1}{z} - \lambda \right]$$

bc system has a ghost # symmetry

$$\left. \begin{aligned} \delta b &= -i \epsilon b \\ \delta c &= +i \epsilon c \end{aligned} \right\} \Rightarrow j = - : b c :$$

$$\langle T(z) j(0) \rangle = \left[\frac{1}{z^2} - \lambda \frac{1}{z} \right] (-1) : b c :$$

$$\sim (-1) \left[\frac{1}{z^2} - \frac{1}{z} \right]$$

bc system has a ghost # symmetry

$$\left. \begin{aligned} \delta b &= -i \xi b \\ \delta c &= +i \xi c \end{aligned} \right\} \Rightarrow J = - : b c :$$

$$\langle T(z) j(0) \rangle = \left[\langle b^{\dagger} c^{\dagger} - \lambda \delta^{\dagger} c^{\dagger} \right] (-1) : b c :$$

$$\sim (-1) \left[z^{\frac{1}{2}} e^{-\frac{1}{2} \lambda} - \frac{1}{z} \right] (-1)$$

bc system has a ghost # symmetry

$$\left. \begin{aligned} \delta b &= -i \epsilon b \\ \delta c &= +i \epsilon c \end{aligned} \right\} \Rightarrow J = - : b c :$$

$$\langle T(z) j(w) \rangle = \langle [c^{\dagger}(z) b^{\dagger}(w) - \lambda \delta(z-w)] (-1) : b c : \rangle$$

$$\sim (-1) \int c^{\dagger} \frac{1}{z} \frac{1}{z} - \frac{1}{z} \frac{1}{z} e^{-\lambda} (-1)$$

bc system has a ghost # symmetry

$$\left. \begin{aligned} \delta b &= -i \xi b \\ \delta c &= +i \xi c \end{aligned} \right\} \Rightarrow j = - : b c :$$

$$\Gamma(z) j(0) = [: b c : - \lambda \delta b c] (-1) : b c :$$

$$\sim (-1) \left[\frac{1}{z} c \frac{1}{z} b - \frac{1}{z} \frac{1}{z} c b - \frac{1}{z} \frac{1}{z} c b \right] (-1)$$

bc system has a ghost # symmetry

$$\left. \begin{aligned} \delta b &= -i \xi b \\ \delta c &= +i \xi c \end{aligned} \right\} \Rightarrow j = - : b c :$$

$$\Gamma(z) j(0) = \left[\frac{1}{z} : b c : - \lambda \right] (-1) : b c :$$

$$\sim (-1) \left[\frac{1}{z} : b c : - \frac{1}{z} \right]$$

bc system has a ghost # symmetry

$$\left. \begin{aligned} \delta b &= -i \xi b \\ \delta c &= +i \xi c \end{aligned} \right\} \Rightarrow j = - : b c :$$

$$\Gamma(z) j(0) = \left[\frac{1}{z} \frac{1}{z} - \lambda \frac{1}{z} \right] (-1) : b c :$$

$$\sim (-1) \left[\frac{1}{z} \frac{1}{z} - \frac{1}{z} \frac{1}{z} - \frac{1}{z} \frac{1}{z} \right] \sim \frac{1-z}{z^2}$$

$$\left(\frac{1}{z^2} + \frac{1}{z} \right) \left(\frac{1}{z} \right) + \dots$$

bc system has a gain of 1

$$\left. \begin{aligned} \delta b &= -i \varepsilon b \\ \delta c &= +i \varepsilon c \end{aligned} \right\} \rightarrow j = -i b c i$$

$$\Gamma(z) j(0) = \begin{bmatrix} a & b \\ c & -\lambda \end{bmatrix} \begin{bmatrix} (-1) \\ b \end{bmatrix} \sim \begin{bmatrix} \frac{1}{2} c & \frac{1}{2} \lambda - \frac{1}{2} \frac{1}{z} \\ \frac{1}{2} c & \frac{1}{2} \lambda - \frac{1}{2} \frac{1}{z} \end{bmatrix} \sim \frac{1-\lambda}{z^2} + \frac{\lambda}{z} j(0) + \frac{1}{z}$$

$$\sim \left(-\lambda \cdot \frac{1}{z^2} + \frac{1}{z} \right) \sim \frac{1-\lambda}{z^2} + \frac{\lambda}{z}$$

$$T(z)j(0) = [abc - \lambda \lambda b^2] \begin{pmatrix} 1 \\ -1 \end{pmatrix} \cdot bc$$

$$\sim \begin{pmatrix} 1 \\ -1 \end{pmatrix} \left[c \frac{1}{z} - \frac{1}{z} - \frac{1}{z} e^{-\lambda} - \frac{1}{z} e^{-\lambda} \right] \sim \frac{1-z}{z^2} + \frac{1}{z^2} j(0) + \frac{1}{z}$$

$$+ \frac{K}{z^2} \left(\frac{1}{z^2} + \frac{1}{z} \right)$$

$K=1$

$$c \left[\frac{1}{z} \right] \sim \frac{1-z^2}{z^3} + \frac{1}{z^2} j'(0) + \frac{1}{z} j''(0)$$

) is almost a $(1, 0)$

) is almost a $(1, 0)$

) is almost a $(1, 0)$

π Transformation Law

$$\varepsilon \delta_j$$

) is almost a $(1, 0)$

π Transformation Law

$$s_3 = -\omega_0 j - j \omega_0$$

γ is almost a $(1, 0)$

π Transformation Law

$$\varepsilon \delta_j = -\partial \partial_j - j \partial \partial + \frac{2\lambda - 1}{2} \partial^2 \partial$$

Finite form of conformal transformations.

Finite form of conformal transformations.

$$z' = f(z) \quad j'_f(z)$$

Finite form of conformal transformations.

$$\left(\frac{\partial z'}{\partial z}\right) j'_w(z') = j_w(z)$$

Finite form of conformal transformations

$$\left(\frac{\partial z'}{\partial z}\right) j'_1(z) = j_1(z) + \frac{2\lambda - 1}{z} \frac{\partial^2 z'}{\partial z^2}$$

Finite form of conformal transformations.

$$\left(\partial_z z'\right) j'_z(z') = j_z(z) + \frac{2\lambda - 1}{z} \frac{\partial_z^2 z'}{\partial_z z'}$$

⇒ What that system is good for?

Finite form of conformal transformations.

$$\left(\frac{\partial z'}{\partial z}\right) j'_z(z) = j_z(z) + \frac{2\lambda - 1}{z} \frac{\partial^2 z'}{\partial z^2}$$

⇒ What that system is good for?

D

Finite form of conformal transformations.

$$\left(\frac{\partial z'}{\partial z}\right) j'_1(z) = j_1(z) + \frac{2\lambda - 1}{z} \frac{\partial^2 z'}{\partial z^2}$$

⇒ What that system is good for?

D

Finite form of conformal transformations.

$$\left(\frac{\partial z'}{\partial z}\right) j'_i(z) = j_i(z) + \frac{2\lambda - 1}{z} \frac{\partial^2 z'}{\partial z^2}$$

⇒ What that system is good for?

Calcl

D

Finite form of conformal transformations.

$$\left(\frac{\partial z'}{\partial z}\right) j'_z(z') = j_z(z) + \frac{2\lambda^{-1}}{z} \frac{\partial_z^2 z'}{\partial_z z'}$$

→ what that system is good for?

Soln =

D

Finite form of conformal transformations.

$$\left(\frac{\partial z'}{\partial z}\right) j'_z(z') = j_z(z) + \frac{2\lambda^{-1}}{z} \frac{\partial^2 z'}{\partial z^2}$$

What that system is good for?

$\lambda = 1$

$\cdot D$

Finite form of conformal transformations.

$$\left(\frac{\partial z'}{\partial z}\right) j'_1(z') = j_1(z) + \frac{2\lambda - 1}{2} \frac{\partial^2 z'}{\partial z^2}$$

⇒ What that system is good for?

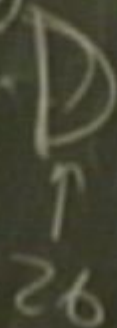
$|c_1| = 0$

Finite form of conformal transformations.

$$(\partial_z z') j'_z(z') = j_z(z) + \frac{2\lambda - 1}{z} \frac{\partial_z^2 z'}{\partial_z z'}$$

⇒ what that system is good for?

$C_{\text{total}} = 0$



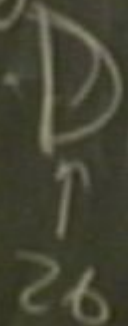
$C =$

Finite form of conformal transformations.

$$(\partial_z z') j'_z(z') = j_z(z) + \frac{2\lambda - 1}{z} \frac{\partial_z^2 z'}{\partial_z z'}$$

⇒ What that system is good for?

$C_{tot} = 0$



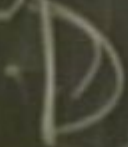
$C =$
with sector 26

Finite form of conformal transformations.

$$\left(\partial_z z'\right) j'_\mu(z') = j_\mu(z) + \frac{2\lambda^{-1}}{z} \frac{\partial_z^2 z'}{\partial_z z'}$$

⇒ What that system is good for?

total = 0

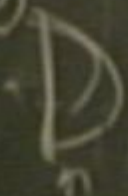


$\xi =$
with sector 26
+

$(abc)(abc)$

⇒ What that system is good for?

$$|G(z)| = 0$$



$C =$
with sector 26

+

0

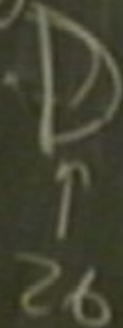
(bc)

(bc)

$$\left(\partial_z z'\right) j'_1(z) = j_1(z) + \frac{2\lambda-1}{2} \frac{\partial_z^2 z'}{\partial_z z}$$

⇒ what that system is good for?

$C_{total} = 0$



$C =$
with sector 26

$+ C_{ghost} = 'bc'$ with $\lambda = 2$

$$\neq -2\lambda \left(\partial_z^2 z'\right) \left(\partial_z z'\right)$$

$$\lambda^2 \left(\partial_z^2 z'\right) \left(\partial_z z'\right)$$

Finite form of conformal transformations

$$\left(\partial_z z'\right) j'_z(z') = j_z(z) + \frac{2\lambda - 1}{2} \frac{\partial_z^2 z'}{\partial_z z'}$$

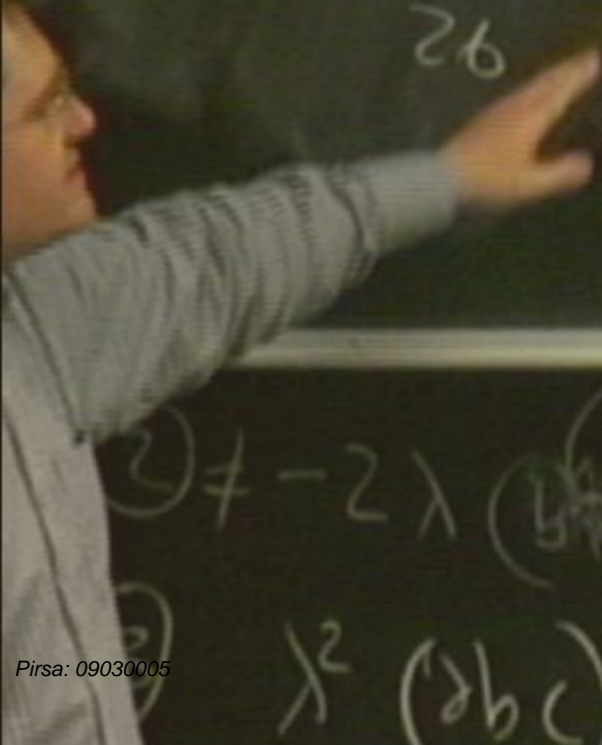
⇒ What that system is good for?

$|c_{gh}| = 0$



$c =$
with sector 26

$+ c_{ghost} = 'bc'$ with $\lambda = 2$



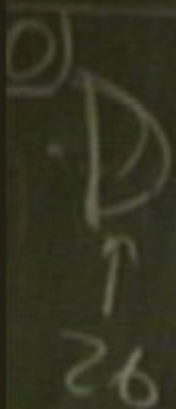
$$\partial_z \neq -2\lambda (\partial_z z') (\partial_z z')$$

$$\partial_z^2 z' (\partial_z z')$$

$\vec{d} z \tau$

$$\underline{\lambda = 2}$$

$$C_{\alpha}(\lambda = 2) = -26$$



$C =$
with sector 26

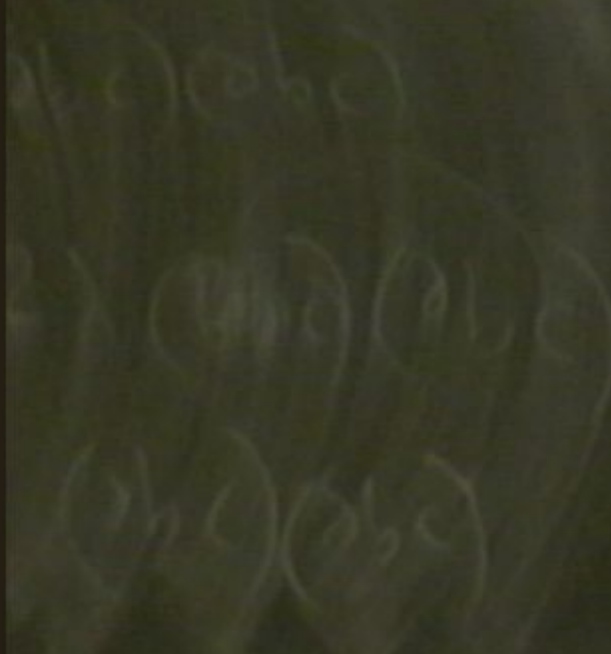
$+ C_{\text{total}} = \underline{\text{'bc' with } \lambda = 2}$

$$C_{\text{bc}}(\lambda=2) = -26$$

$\frac{1}{2}$

$(a|b|c)$
 $(a|b|c)$
 $(a|b|c)$

$$h_b = h_c = \frac{1}{2}$$



$$h_b = h_c = \frac{1}{2}$$

$$z = 4$$

$$\lambda = \frac{1}{2}$$

$$h_b = h_c = \frac{1}{2}$$

$$b = \psi = \frac{1}{\sqrt{2}} \begin{bmatrix} \psi_1 + i\psi_2 \end{bmatrix}$$

(a) (b) (c)

(1) (2) (3) (4) (5) (6)

(7) (8) (9) (10)

$$\lambda = \frac{1}{2}$$

$$h_b = h_c = \frac{1}{2}$$

$$b = \psi = \frac{1}{\sqrt{2}} [\psi_1 + i\psi_2]$$

$$c = \psi =$$

$$\lambda = \frac{1}{2}$$

$$h_b = h_c = \frac{1}{2}$$

$$b = 4 = \frac{1}{\sqrt{2}} [4_1 + i4_2]$$

$$c = 4 = \frac{1}{\sqrt{2}} [4_1(z) + i4_2(z)]$$

$$\lambda = \frac{1}{2}$$

$$b_b = b_c = \frac{1}{2}$$

$$S_{bc} = \frac{1}{4\pi} \int d^2z [\psi_1 \bar{\partial} \psi_1 + \psi_2 \bar{\partial} \psi_2]$$

$$b = \psi = \frac{1}{\sqrt{2}} [\psi_1 + i\psi_2]$$

$$c = \bar{\psi} = \frac{1}{\sqrt{2}} [\psi_1(\bar{z}) - i\psi_2(\bar{z})]$$

$$\lambda = \frac{1}{2}$$

$$h_b = h_c = \frac{1}{2}$$

$$b = \psi = \frac{1}{\sqrt{2}} [\psi_1 + i\psi_2]$$

$$c = \bar{\psi} = \frac{1}{\sqrt{2}} [\psi_1(\bar{z}) - i\psi_2(\bar{z})]$$

$$S_{bc} = \frac{1}{4\pi} \int d^2z [\psi_1 \bar{\partial} \psi_1 + \psi_2 \bar{\partial} \psi_2]$$

π

$$\lambda = \frac{1}{2} \quad b_b = b_c = \frac{1}{2}$$

$$b = \psi = \frac{1}{\sqrt{2}} [\psi_1 + i\psi_2]$$

$$c = \bar{\psi} = \frac{1}{\sqrt{2}} [\psi_1(\bar{z}) - i\psi_2(\bar{z})]$$

$$S_{bc} = \frac{1}{4\pi} \int d^2z [\psi_1 \bar{\partial} \psi_1 + \psi_2 \bar{\partial} \psi_2]$$

$$\mathbb{T} = -\frac{1}{2} \psi_1 \partial \psi_1 - \frac{1}{2} \psi_2 \partial \psi_2$$

$$\lambda = \frac{1}{2} \quad h_b = h_c = \frac{1}{2}$$

$$b = \psi = \frac{1}{\sqrt{2}} [\psi_1 + i\psi_2]$$

$$c = \bar{\psi} = \frac{1}{\sqrt{2}} [\psi_1(\bar{z}) - i\psi_2(\bar{z})]$$

$$S_{bc} = \frac{1}{4\pi} \int d^2z [\psi_1 \bar{\partial} \psi_1 + \psi_2 \bar{\partial} \psi_2]$$

$$\mathbb{T} = -\frac{1}{2} \psi_1 \partial \psi_1 - \frac{1}{2} \psi_2 \partial \psi_2$$

$$G_{bc}(\lambda = \frac{1}{2}) = -3(2\lambda - 1)^2 + 1$$

$$\lambda = \frac{1}{2} \quad h_b = h_c = \frac{1}{2}$$

$$b = \psi = \frac{1}{\sqrt{2}} [\psi_1 + i\psi_2]$$

$$c = \bar{\psi} = \frac{1}{\sqrt{2}} [\psi_1(\bar{z}) - i\psi_2(\bar{z})]$$

$$c_{\psi_1} = \frac{1}{2}$$

$$S_{bc} = \frac{1}{4\pi} \int d^2z [\psi_1 \bar{\partial} \psi_1 + \psi_2 \bar{\partial} \psi_2]$$

$$\mathbb{T} = -\frac{1}{2} \psi_1 \partial \psi_1 - \frac{1}{2} \psi_2 \partial \psi_2$$

$$h_{bc}(\lambda = \frac{1}{2}) = -3(2\lambda - 1)^2 + 1 = 1$$

$$\lambda = \frac{1}{2} \quad h_b = h_c = \frac{1}{2}$$

$$b = \psi = \frac{1}{\sqrt{2}} [\psi_1 + i\psi_2]$$

$$c = \bar{\psi} = \frac{1}{\sqrt{2}} [\psi_1(\bar{z}) - i\psi_2(\bar{z})]$$

$$c_{\psi_1} = \frac{1}{2}$$

$$S_{bc} = \frac{1}{4\pi} \int d^2z [\psi_1 \bar{\partial} \psi_1 + \psi_2 \bar{\partial} \psi_2]$$

$$\pi = -\frac{1}{2} \psi_1 \partial \psi_1 - \frac{1}{2} \psi_1 \partial \psi_2$$

$$h_{bc}(\lambda = \frac{1}{2}) = -3(2g-1)^2 + |g-1|$$

$$\lambda = \frac{1}{2} \quad h_b = h_c = \frac{1}{2}$$

$$b = \psi = \frac{1}{\sqrt{2}} [\psi_1 + i\psi_2]$$

$$c = \bar{\psi} = \frac{1}{\sqrt{2}} [\psi_1(\bar{z}) - i\psi_2(\bar{z})]$$

$$c_{\psi_1} = \frac{1}{2}$$

$$S_{bc} = \frac{1}{4\pi} \int d^2z [\psi_1 \bar{\partial} \psi_1 + \psi_2 \bar{\partial} \psi_2]$$

$$\mathbb{T} = -\frac{1}{2} \psi_1 \partial \psi_1 - \frac{1}{2} \psi_2 \partial \psi_2$$

$$G_{bc}(\lambda = \frac{1}{2}) = -3(2\lambda - 1)^2 + 1 = 1$$

supersymmetry
matter

$$\lambda = \frac{1}{2} \quad h_b = h_c = \frac{1}{2}$$

$$b = \psi = \frac{1}{\sqrt{2}} [\psi_1 + i\psi_2]$$

$$c = \bar{\psi} = \frac{1}{\sqrt{2}} [\psi_1(\bar{z}) - i\psi_2(\bar{z})]$$

$$c_{\psi_1} = \frac{1}{2}$$

$$S_{bc} = \frac{1}{4\pi} \int d^2z [\psi_1 \bar{\partial} \psi_1 + \psi_2 \bar{\partial} \psi_2]$$

$$\mathbb{T} = -\frac{1}{2} \psi_1 \partial \psi_1 - \frac{1}{2} \psi_2 \partial \psi_2$$

$$C_{bc}(\lambda = \frac{1}{2}) = -3(2\lambda - 1)^2 + 1 = 1$$

$$\text{supersymmetry matter} = 1 + \frac{1}{2} = \frac{3}{2}$$

$$\lambda = \frac{1}{2} \quad h_b = h_c = \frac{1}{2}$$

$$b = \psi = \frac{1}{\sqrt{2}} [\psi_1 + i\psi_2]$$

$$c = \bar{\psi} = \frac{1}{\sqrt{2}} [\psi_1(\bar{z}) - i\psi_2(\bar{z})]$$

$$c_{\psi_1} = \frac{1}{2}$$

$$S_{bc} = \frac{1}{4\pi} \int d^2z [\psi_1 \bar{\partial} \psi_1 + \psi_2 \bar{\partial} \psi_2]$$

$$\mathbb{T} = -\frac{1}{2} \psi_1 \partial \psi_1 - \frac{1}{2} \psi_2 \partial \psi_2$$

$$G_{bc}(\lambda = \frac{1}{2}) = -3(2\lambda - 1)^2 + 1 = 1$$

$$\text{supersymmetry matter} = 1 + \frac{1}{2} = \frac{3}{2}$$

$$\lambda = \frac{1}{2} \quad h_b = h_c = \frac{1}{2}$$

$$b = \psi = \frac{1}{\sqrt{2}} [\psi_1 + i\psi_2]$$

$$c = \bar{\psi} = \frac{1}{\sqrt{2}} [\psi_1(\bar{z}) - i\psi_2(\bar{z})]$$

$$c_{\psi_1} = \frac{1}{2}$$

$$S_{bc} = \frac{1}{4\pi} \int d^2z [\psi_1 \bar{\partial} \psi_1 + \psi_2 \bar{\partial} \psi_2]$$

$$\pi' = -\frac{1}{2} \psi_1 \partial \psi_1 - \frac{1}{2} \psi_1 \partial \psi_2$$

$$C_{bc}(\lambda = \frac{1}{2}) = -3(2-1)^2 + 1 = 1$$

$$\text{Anomalous dimension} = 1 + \frac{1}{2} = \frac{3}{2}$$

BY - CFT

$$\lambda = 2$$

$$C_{bc}(\lambda=2) = -26$$

$$S_{bc} = \frac{1}{4\pi} \int d^2z \left[\psi_1 \bar{\partial} \psi_1 + \psi_2 \bar{\partial} \psi_2 \right]$$

BY - CFT

B γ - CFT

B γ is a bosonic ghost CFT

$\beta\gamma$ - CFT

$\beta\gamma$ is a bosonic ghost CFT

$$S = \frac{1}{2\pi} \int d^2z \beta \bar{\partial} \gamma$$

$$\beta\gamma = \gamma\beta$$

$P_0 = CFT$
 $\beta\gamma$ is a bosonic ghost CFT

$$S = \frac{1}{2\pi} \int d^2z \beta \bar{\partial} \gamma$$

$$\beta\gamma = \gamma\beta$$

$$\bar{\partial}\beta = 0 \quad \partial\gamma = 0$$

$$\beta\gamma \sim -\frac{1}{z}$$

$$\gamma\beta \sim \frac{1}{z}$$

$$\tilde{\pi}\pi \Rightarrow \tilde{c} = 0$$

$\beta\gamma$ is a bosonic ghost CFT

$$S = \frac{1}{2\pi} \int d^2z \beta \bar{\partial} \gamma$$

$$\bar{\partial} \beta = 0 \quad \partial \gamma = 0$$

$$\beta\gamma \sim -\frac{1}{z} \quad \gamma\beta \sim \frac{1}{z}$$

$$\mathbb{T} = \partial(\beta\gamma) - \lambda \partial(\beta\gamma)$$

$$\beta\gamma = \gamma\beta$$

$$c = 3(2\lambda - 1)^2 - 1$$

\mathbb{I}_4

\mathbb{Z}_2

$$c = -2 + 12\lambda - 12\lambda^2$$

$$\tilde{\mathbb{T}} \tilde{\mathbb{T}} \Rightarrow \tilde{c} = 0$$

$\beta\gamma$ - CFT

$\beta\gamma$ is a bosonic ghost CFT

$$S = \frac{1}{2\pi} \int d^2z \beta \bar{\partial} \gamma$$

$$\beta\gamma = \gamma\beta$$

$$\bar{\partial}\beta = 0 \quad \partial\gamma = 0$$

$$c = 3(2\lambda - 1)^2 - 1 \quad \tilde{c} = 0$$

$$\beta\gamma \sim -\frac{1}{z} \quad \gamma\beta \sim \frac{1}{z}$$

In superstrings

$$\pi = \partial\beta\gamma - \lambda\partial(\beta\gamma) \quad \tilde{\pi} = 0$$

$$\lambda = \frac{3}{2}$$

$$c = -2 + 12\lambda - 12\lambda^2 = -3(2\lambda - 1) + 1$$

$$\tilde{\pi}\tilde{\pi} \Rightarrow \tilde{c} = 0$$

$$\Rightarrow \Gamma(z) \Rightarrow \int_0^{\infty} t^{z-1} e^{-t} dt = -\psi(z) \Gamma(z)$$

$$\Rightarrow \Psi(z) \Rightarrow \int = -\Psi(z) \Psi'(z) \Rightarrow$$

$$\Psi(z) \Rightarrow$$

$$j = -U(z)\Psi'(z) \Rightarrow$$

Infinite set
of conserved
charges.

$j = -\psi(z) \nabla(z) \Rightarrow$ Infinite set
of conserved
charges.

$$T(z) \Rightarrow j = -U(z) T'(z) \Rightarrow \text{Infinitesimal conformal transformations}$$

Virasoro algebra

$\Rightarrow T(z) \Rightarrow j = -\psi(z) \Rightarrow$ Infinite set of conserved charges.

Virasoro

open string

$\Rightarrow T(z) \Rightarrow j = -\alpha(z) T(z) \Rightarrow$ Infinite set of conserved charges.

Virasoro algebra

open string

closed string

(σ_1, τ_2)

$$\Rightarrow T(z) \Rightarrow j = -\alpha(z) T(z) \Rightarrow \text{Infinite set of conserved charges.}$$

Virasoro algebra

open string

closed string

space
 (σ, τ)

↑ time

$\Rightarrow T(z) \Rightarrow j = -\alpha(z) T(z) \Rightarrow$ Infinite set of conserved charges.

Virasoro algebra

open string

closed string

space

(σ, τ)

time

$\Rightarrow T(z) \Rightarrow j = -U(z)T(z) \Rightarrow$ Infinite set of conserved charges.

Virasoro algebra

space
 (σ_1, σ_2)

open string

closed string

↑ time

$0 < \sigma' < \pi$

$0 < \sigma' < 2\pi$

$-\infty < \sigma < +\infty$

$\Rightarrow T(z) \Rightarrow j = -U(z)T(z) \Rightarrow$ Infinite set of conserved charges.

Virasoro algebra

space
 (σ, τ)

time

open string

closed string

$0 < \sigma' < \pi$

$0 < \sigma' < 2\pi$

$-\infty < \tau < +\infty$

$\Rightarrow T(z) \Rightarrow j = -U(z)T(z) \Rightarrow$ Infinite set of conserved charges.

Virasoro algebra

space (σ, τ)
 \uparrow time

open string

$0 < \sigma' < \pi$

$-\infty < \tau < \infty$

closed string

$0 < \sigma' < 2\pi$

$\Rightarrow T(z) \Rightarrow j = -U(z)T(z) \Rightarrow$ Infinite set of conserved charges.

Virasoro algebra

space
 (σ, τ)
 ↑ time

open string
 $0 < \sigma' < \pi$
 $0 < \sigma < +\infty$

closed string
 $0 < \sigma' < 2\pi$

$$\omega = \delta' + i\delta^2$$

$$z = e^{-i\omega} \Rightarrow e^{-i\sigma_1 + \sigma_2}$$

$$w = \sigma' + i\sigma''$$

$$z = e^{-iw} \Rightarrow e^{-i(\sigma' + i\sigma'')} = e^{-i\sigma' + \sigma''}$$

conformal transformations.

$$w = \sigma' + i\sigma''$$

$\text{Im} w$



$$z = e^{-iw} \Rightarrow e^{-i\sigma' + \sigma''}$$

conformal transformations.

$$w = \sigma' + i\sigma''$$

Im w
↑ time

0 → spac.

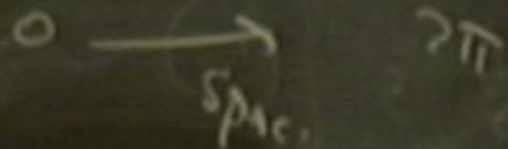
2π

$$z = e^{-iw} = e^{-i(\sigma' + i\sigma'')} = e^{-i\sigma' + \sigma''}$$

conformal transformations.

$$\omega = \sigma' + i\sigma^2$$

Im ω
↑
line
Hamiltonian
is Im ω
translation



$$z = e^{-i\omega} \Rightarrow e^{-i\sigma_1 + \sigma_2}$$

conformal transformations.

$$\omega = \delta' + i\delta^2$$

Im ω
↑ time

Hamiltonian
is Im ω
translation

0 → Spac.

2π

$$e^{-i\omega} \Rightarrow e^{-i\delta_1 + \delta_2}$$

transformations.

$$\omega = \sigma' + i\sigma^2$$

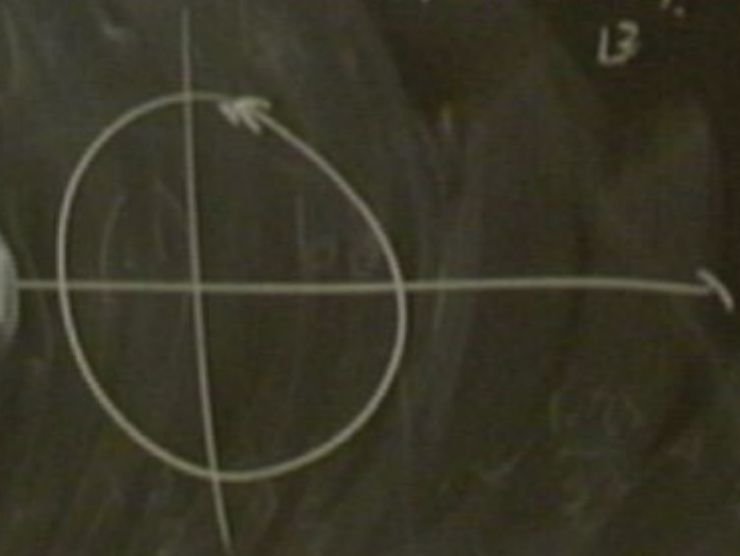
Im ω
↑
line

Hamiltonian
is Im ω
transf

0

$$z = e^{-i\omega} \Rightarrow e^{-i(\sigma_1 + i\sigma_2)} = e^{-i\sigma_1 + \sigma_2}$$

conformal transformations.



$$\omega = \sigma' + i\sigma^2$$

Im ω
↑
time

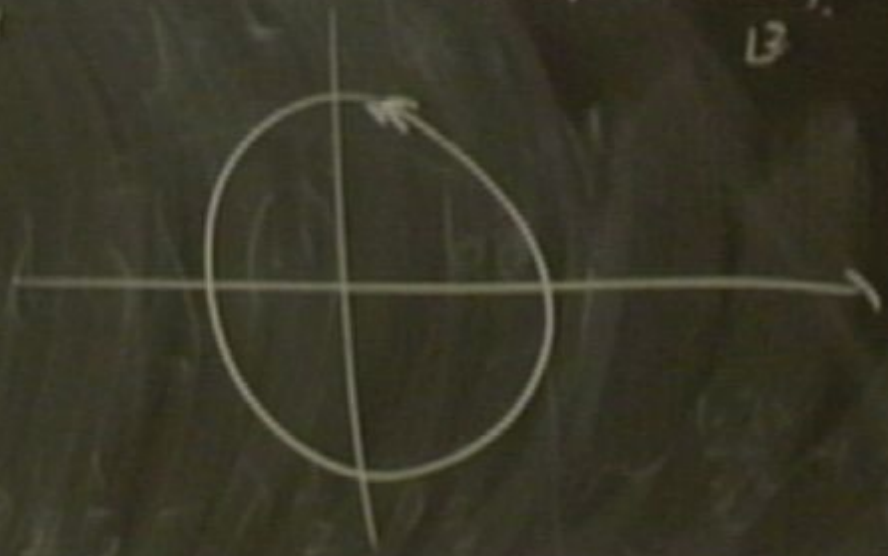
Hamiltonian
is Im ω
translation

0 → 2π
Spac.

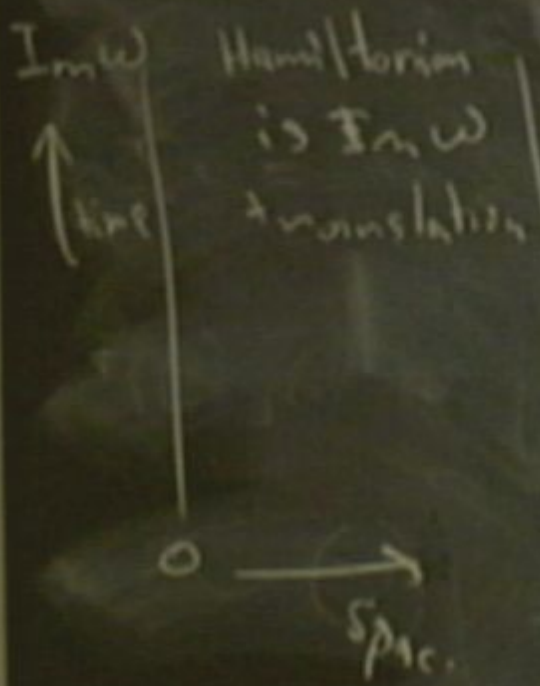
angle

$$z = e^{-i\omega} = e^{-i(\sigma' + i\sigma^2)} = e^{-i\sigma' + \sigma^2}$$

conformal transformations.



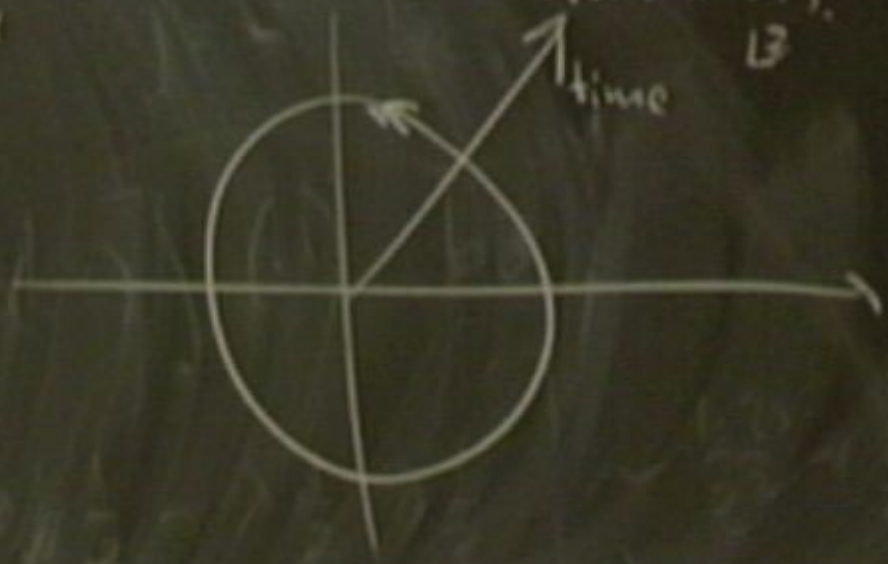
$$\omega = \sigma' + i\sigma^2$$



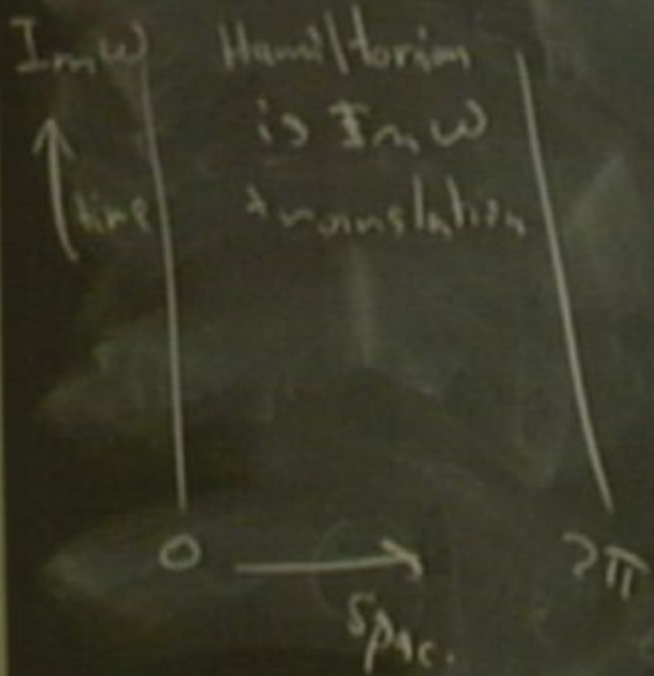
angle

$$z = e^{-i\omega} = e^{-i\sigma_1 + \sigma_2}$$

conformal transformations.

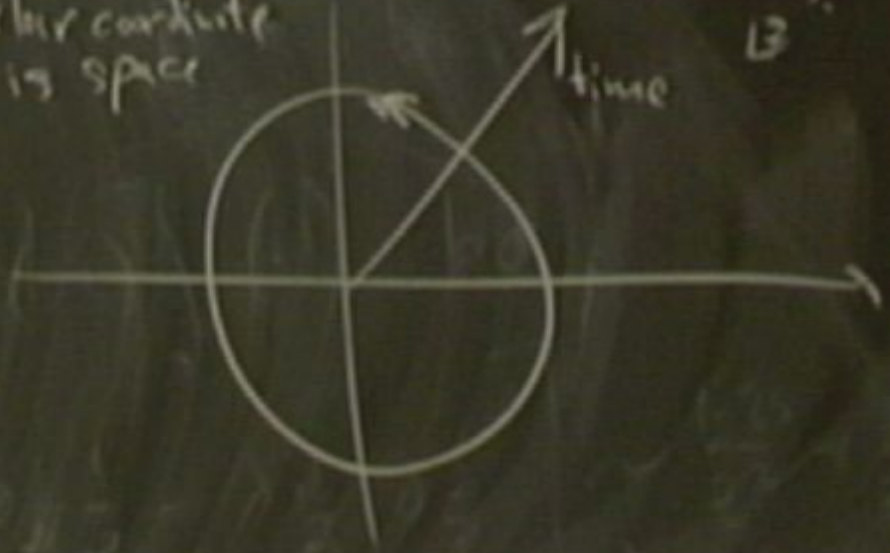


$$\omega = \sigma' + i\sigma^2$$

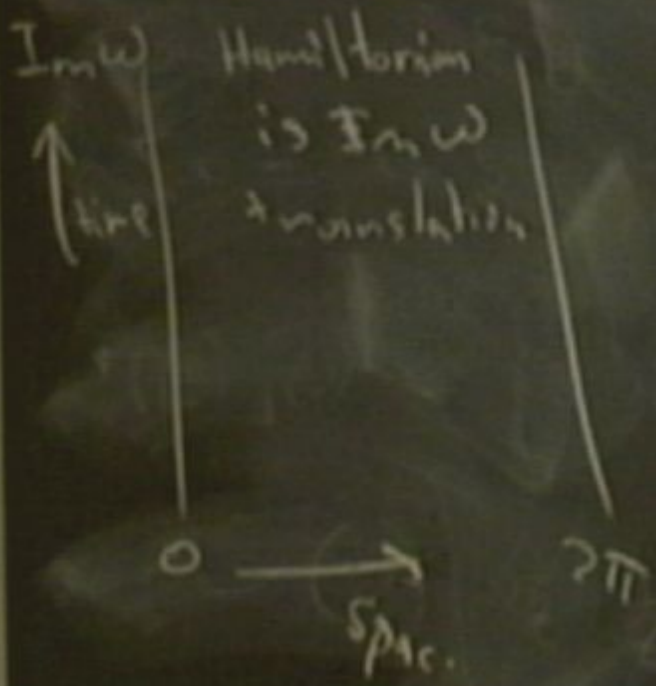


$$z = e^{-i\omega} = e^{-i\sigma_1 + \sigma_2}$$

conformal transformations.
 angular coordinate is space
 time

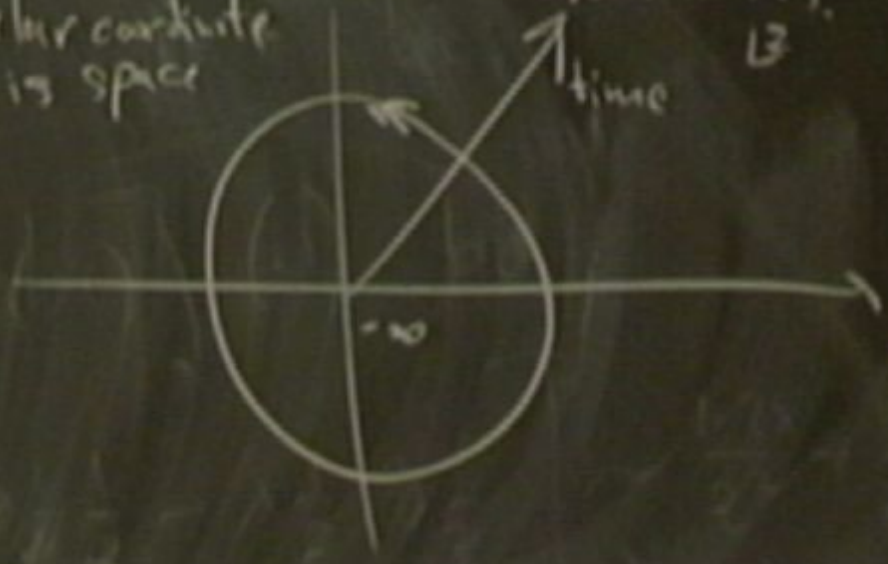


$$\omega = \delta' + i\delta^2$$

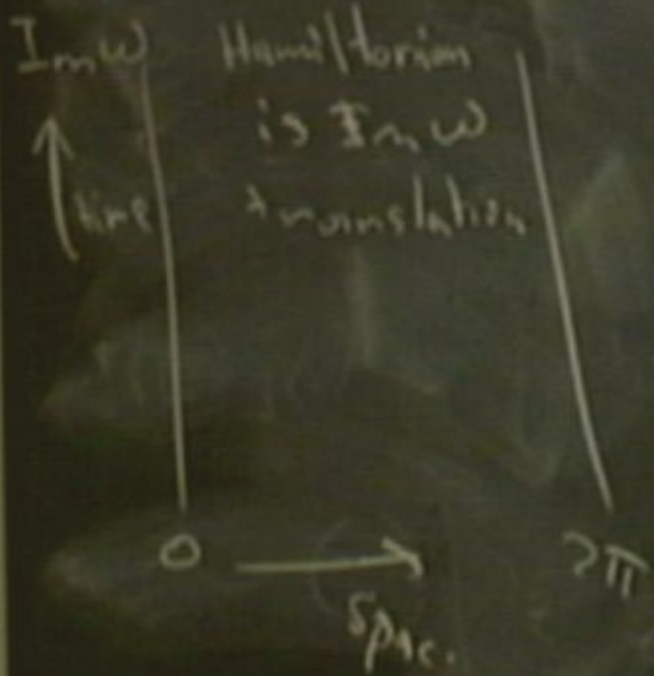


$$z = e^{-i\omega} = e^{-i\delta_1 + \delta_2}$$

conformal transformations.
 angular coordinate is space

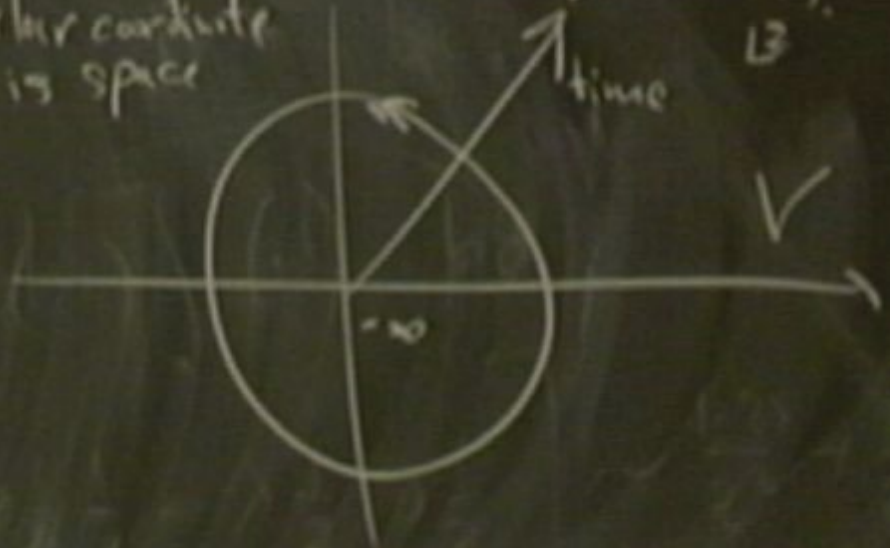


$$\omega = \sigma_1 + i\sigma_2$$



$$z = e^{-i\omega t} = e^{-i\sigma_1 t + \sigma_2 t}$$

conformal transformations.
 angular coordinate is space



$$T'(z) \cdot C = [abc - \lambda z(bc)] \cdot C$$

$T'(z)$

$(1, z)$

$$T(z) = \sum_{m=-\infty}^{+\infty} (1, a)$$

$$\mathcal{T}(z) = \sum_{m=-\infty}^{+\infty} \frac{L_m}{z^{m+2}}$$



$$T(z) = \sum_{m=-\infty}^{+\infty} \frac{L_m}{z^{m+2}}$$

L_m are coefficients of a exp.

||

$$T(z) \stackrel{\text{def}}{=} \sum_{m=-\infty}^{+\infty} \frac{L_m}{z^{m+2}}$$

L_m are coefficients of a Laurent exp.

||
Virasoro generators

$$T(z) \stackrel{\text{def}}{=} \sum_{m=-\infty}^{+\infty} \frac{L_m}{z^{m+2}}$$

$$L_m =$$

L_m are coefficients of a Laurent exp.

|| Virasoro generators

$$T(z) \stackrel{\text{def}}{=} \sum_{m=-\infty}^{+\infty} \frac{L_m}{z^{m+2}}$$

$$L_m = \oint_C \frac{dz}{2\pi i} z^{m+2} T(z)$$

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$$\boxed{\tilde{T}, \tilde{L}}$$

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L_m are coefficients of a Laurent exp.

|| Virasoro generators

$$L_m = \oint_C \frac{dz}{2\pi i} z^{m+2} T(z)$$

$$\boxed{\tilde{T}, \tilde{L}}$$

$$\mathbb{T} = \mathbb{T}_{27} \rightarrow \mathbb{T}_{www}$$

$$\mathbb{T} = \mathbb{T}_{zz} \rightarrow \mathbb{T}_{ww} = (\partial_w z)^2 \mathbb{T}_{zz}$$

\bar{z}_{12}

$$\pi = \pi'_{29} \rightarrow \pi_{ww} = (\partial_w z)^2 \pi'_{22} +$$

Schwarzian derivative

$\bar{z}_1 z$

$$\pi = \pi_{27} \rightarrow \pi_{ww} = (\partial_w z)^2 \pi_{zz} + \frac{1}{24}$$

Scharzhin derivat;



\bar{z}

$$\pi = \pi_{z\bar{z}} \rightarrow \pi_{ww} = (\partial_w \bar{z})^2 \pi_{z\bar{z}} + \frac{1}{24}$$

Laurent expansion in $z \iff$

Schwarzian derivative



\bar{z}

$$\pi = \pi_{zz} \rightarrow \pi_{ww} = (\partial_w z)^2 \pi_{zz} + \frac{\partial^2 z}{\partial w^2}$$

Laurent expansion in $z \iff$ Fourier expansion in w Scharzhin derivat;

\bar{z}

$$\pi = \pi_{z\bar{z}} \rightarrow \pi_{w\bar{w}} = (\partial_w \bar{z})^2 \pi_{z\bar{z}} + \dots$$

Laurent expansion in $z \iff$ Fourier expansion in w . Scharzhin derivat;

$$\pi_{z\bar{z}}(w) =$$



\bar{z}

$$\pi = \pi_{z\bar{z}} \rightarrow \pi_{w\bar{w}} = (\partial_w \bar{z})^2 \pi_{z\bar{z}} + \frac{1}{24}$$

Laurent expansion in $z \iff$ Fourier expansion in w Scharzhin derivat.

$$f(w) =$$



\bar{z}

$$\Gamma = \Gamma_{z\bar{z}} \rightarrow \Gamma_{w\bar{w}} = (\partial_w \bar{z})^2 \Gamma_{z\bar{z}} + \dots$$

Laurent expansion in $z \iff$ Fourier expansion in w Scharzhin derivat;

$$\Gamma_{w\bar{w}}(w) =$$

$$\Gamma_{w\bar{w}}(w) = \dots$$

$$\pi = \pi_{zz} \rightarrow \pi_{ww} = (\partial_w z)^2 \pi_{zz} + \frac{1}{24}$$

Laurent expansion in $z \iff$ Fourier expansion in w Scharzhin derivat.

$$\pi_{ww}(w) = - \sum_{m=-\infty}^{+\infty} e^{im\sigma' - m\sigma^2} \pi_m$$



$$y_1 = \frac{1}{2}$$

$$\text{Charakter} = 1 + \frac{1}{2} = \frac{3}{2}$$

$\bar{z}|z$

$$\pi = \pi_{z\bar{z}} \rightarrow \pi_{w\bar{w}} = (\partial_w \bar{z})^2 \pi_{z\bar{z}} + \frac{c}{24}$$

Laurent expansion in $z \iff$ Fourier expansion in w . Scharzhin derivat;

$$\pi_{w\bar{w}}(w) = - \sum_{n=-\infty}^{+\infty} e^{i n \sigma' - w \sigma^2} \pi_n$$

$$\pi_n = L_n - \delta_{n,0} \frac{c}{24} \quad \tilde{\pi}_n = \tilde{L}_n - \frac{c}{24} \quad (n \neq -26)$$

\bar{z}

$$\mathbb{T} = \mathbb{T}'_{2\tau} \rightarrow \mathbb{T}'_{w\omega} = (2wz)^2 \mathbb{T}'_{zz} + \frac{c}{24}$$

Laurent expansion in $z \iff$ Fourier expansion in w Scharzhin derivat

$$\mathbb{T}'_{w\omega}(w) = - \sum_{n=-\infty}^{+\infty} e^{i n \sigma' - w \sigma^2} \mathbb{T}'_n$$

$$\mathbb{T}'_n = L_n - \delta_{n,0} \frac{c}{24} \quad \tilde{\mathbb{T}}_n = \tilde{L}_n - \frac{c}{24} \delta_{n,0}$$

$$(z) = \sum_{m=-\infty}^{+\infty} \frac{L_m}{z^{m+2}}$$

$$L_m = \oint_C \frac{dz}{2\pi i} z^{m+2} T(z)$$

L_m are coefficients of a Laurent series

|| Virasoro generators

$\bar{z}|z$

$$\mathbb{T} = \mathbb{T}'_{zz} \rightarrow \mathbb{T}'_{ww} = (\partial_w z)^2 \mathbb{T}'_{zz} + \frac{c}{24}$$

Laurent expansion in $z \iff$ Fourier expansion in w Scharzhin derivat;

$$\mathbb{T}'_{ww}(w) = - \sum_{n=-\infty}^{+\infty} e^{i n \sigma' - w \sigma^2} \mathbb{T}'_n$$

$$\mathbb{T}'_n = L_n - \delta_{n,0} \frac{c}{24} \quad \tilde{\mathbb{T}}_n = \tilde{L}_n - \frac{\tilde{c}}{24} \quad (n=0) = -26$$

\bar{z}_{12}

$$\mathbb{T} = \mathbb{T}'_{2\tau} \rightarrow \mathbb{T}'_{w\omega} = (\partial_w z)^2 \mathbb{T}'_{z\bar{z}} + \frac{c}{24}$$

Laurent expansion in $z \iff$ Fourier expansion in w Scharzhin derivat;

$$\mathbb{T}'_{w\omega}(w) = - \sum_{m=-\infty}^{+\infty} e^{im\sigma' - m\sigma^2} \mathbb{T}'_m$$

$$\mathbb{T}'_m = L_m - \delta_{m,0} \frac{c}{24} \quad \tilde{\mathbb{T}}_m = \tilde{L}_m - \frac{c}{24}$$

$\frac{1}{z^2}$

$$\mathbb{T} = \mathbb{T}_{z^2} \rightarrow \mathbb{T}_{ww} = (2wz)^{\frac{3}{2}} \mathbb{T}_{zz} + \frac{3}{24}$$

Laurent expansion in $z \iff$ Fourier Scharzhin w

$$\mathbb{T}_{ww}(w) = - \sum_{m=-\infty}^{+\infty} e^{im\sigma' - w\sigma^2} \mathbb{T}_m$$

$$\mathbb{T}_m = L_m - \delta_{m,0} \frac{c}{24} \quad \tilde{\mathbb{T}}_m = \tilde{L}_m - \frac{c}{24}$$

\bar{z}, z

$$\mathbb{T} = \mathbb{T}_{z\bar{z}} \rightarrow \mathbb{T}_{w\bar{w}} = \left(\partial_w z\right)^2 \mathbb{T}_{z\bar{z}} + \frac{1}{24}$$

$\partial_w z = -iz$

Laurent expansion in $z \iff$ Fourier expansion in w Scharzhin derivat;

$$\mathbb{T}_{w\bar{w}}(w) = - \sum_{m=-\infty}^{+\infty} e^{im\sigma - m\sigma^2} \mathbb{T}_m$$

$$\mathbb{T}_m = L_m - \delta_{m,0} \frac{c}{24} \quad \tilde{\mathbb{T}}_m = \tilde{L}_m - \frac{\tilde{c}}{24} \quad (h, \tilde{h}) = -26$$

\bar{z}, z

$$\mathbb{T} = \mathbb{T}'_{z\bar{z}} \rightarrow \mathbb{T}'_{w\bar{w}} = \left(\partial_w z\right)^2 \mathbb{T}'_{z\bar{z}} + \frac{1}{24}$$

$\partial_w z = -iz$

Laurent expansion in $z \iff$ Fourier expansion in w . Scharzhin derivat;

$$\mathbb{T}'_{w\bar{w}}(w) = - \sum_{m=-\infty}^{+\infty} e^{im\sigma' - m\sigma^2} \mathbb{T}'_m$$

$$\mathbb{T}'_m = L_m - \delta_{m0} \frac{c}{24} \quad \tilde{\mathbb{T}}_m = \tilde{L}_m - \frac{\tilde{c}}{24} \quad c(\sigma) = -26$$

$$(z) = \sum_{m=-\infty}^{+\infty} \frac{L_m}{z^{m+2}}$$

L_m are coefficients of a Laurent exp.

|| Virasoro generators

$$L_m = \oint_C \frac{dz}{2\pi i} z^{m+2} T(z)$$

$$\boxed{\tilde{T}, \tilde{L}}$$

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$$\mathbb{T} = \mathbb{T}'_{z\bar{z}} \rightarrow \mathbb{T}'_{w\bar{w}} = \left(\partial_w z\right)^2 \mathbb{T}'_{z\bar{z}} + \frac{1}{24}$$

$\partial_w z = -iz$

Laurent expansion in $z \iff$ Fourier expansion in w Scharzhin derivat

$$\mathbb{T}'_{w\bar{w}}(w) = - \sum_{m=-\infty}^{+\infty} e^{im\sigma' - m\sigma^2} \mathbb{T}'_m$$

$$\mathbb{T}'_m = L_m - \delta_{m,0} \frac{c}{24} \quad \tilde{\mathbb{T}}_m = \tilde{L}_m - \frac{\tilde{c}}{24} \quad (m, \tilde{m}) = -26$$

\bar{z}_{12}

$$\mathbb{T} = \mathbb{T}'_{z\bar{z}} \rightarrow \mathbb{T}'_{w\bar{w}} = \left(\partial_w \bar{z}\right)^2 \mathbb{T}'_{z\bar{z}} + \frac{0}{24}$$

$\partial_w \bar{z} = -i\bar{z}$

Laurent expansion in $z \iff$ Fourier expansion in w Scharzkin derivative

$$\mathbb{T}'_{w\bar{w}}(w) = \ominus \sum_{n=-\infty}^{+\infty} e^{im\sigma' - m\sigma^2} \mathbb{T}'_m$$

$$\mathbb{T}'_n = L_n - \delta_{n,0} \frac{c}{24} \quad \tilde{\mathbb{T}}_m = \tilde{L}_m - \frac{c}{24} \quad (h=0) \Rightarrow c=26$$

Commutation relations of conserved charges

[Faint, mostly illegible handwritten notes on a chalkboard, likely containing mathematical derivations and equations.]



Commutation relations of conserved charges

$j(z)$ - is conserved current.

Commutation relations of conserved charges

$j(z)$ - is conserved current.

$Q \{ c \}$

Commutation relations of conserved charges

$j(z)$ - is conserved current.

$$Q\{c\} = \oint_C \frac{dz}{2\pi i} j(z)$$

Commutation relations of conserved charges

$j(z)$ - is conserved current.

$$Q \{c\} = \oint_{\tilde{C}} dz j'(z)$$

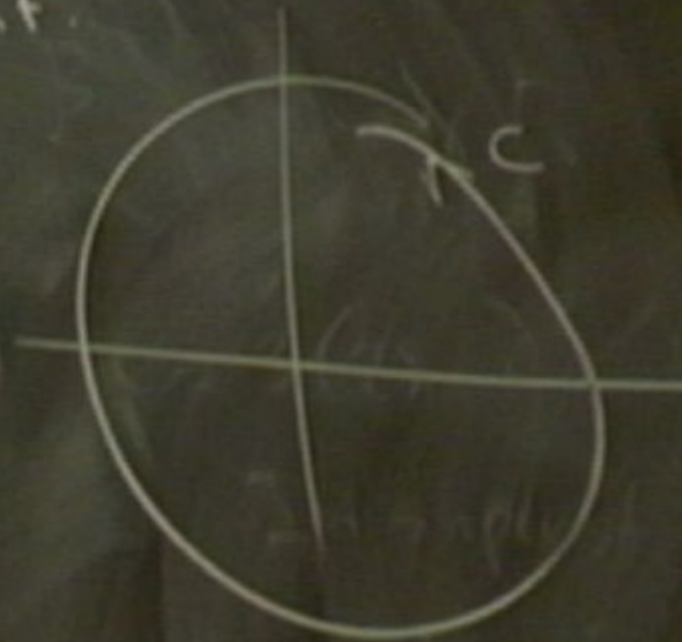
↑ conserved charge

Commutation relations of conserved charges

$j(z)$ - is conserved current.

$$Q \{c\} = \oint_{\tilde{C}} dz j(z)$$

↑ conserved charge



Commutation relations of conserved charges

$j(z)$ - is conserved current

$$Q \{ c \} = \oint_C \frac{dz}{2\pi i} j(z)$$

↑ conserved charge



Commutation relations of conserved charges

$j(z)$ - is conserved current

$$Q\{C\} = \oint_C \frac{dz}{2\pi i} j(z)$$

Q ↑ conserved charge



$$Q \{c_1\} = Q \{c_2\} = Q \{c_3\}$$

$$Q \{c_1\}_{t_1} = Q \{c_2\}_{t_2} = Q \{c_3\}_{t_3}$$

$$Q \{c_1\} = Q \{c_2\} = Q \{c_3\}$$

$$t_1 < t_2 < t_3$$

$j: (2)$

$$Q \{c_1\} = Q \{c_2\} = Q \{c_3\}$$

$$t_1 < t_2 < t_3$$

$$j: (z) \quad i=1,2 \Rightarrow Q_1, Q_2$$

$$Q \{c_1\} = Q \{c_2\} = Q \{c_3\}$$

$$t_1 < t_2 < t_3$$

$$j: (\bar{z}) \quad i=1,2 \Rightarrow Q_1, Q_2 \quad \underline{Q'} \quad [a$$

$$Q \{c_1\} = Q \{c_2\} = Q \{c_3\}$$

$$t_1 < t_2 < t_3$$

$$j: (z) \quad i=1,2 \Rightarrow Q_1, Q_2 \quad \underline{Q} [Q_1, Q_2] = ?$$

$$Q \{c_1\} = Q \{c_2\} = Q \{c_3\}$$

$$t_1 < t_2 < t_3$$

$$j: (z) \quad i=1,2 \Rightarrow Q_1, Q_2 \quad \underline{Q} [\hat{Q}_1, \hat{Q}_2] = ?$$

$$Q \{c_1\} = Q \{c_2\} = Q \{c_3\}$$

$$t_1 < t_2 < t_3$$

$$j: (z) \quad i=1,2 \Rightarrow Q_1, Q_2 \quad \underline{Q} \quad [\hat{Q}_1, \hat{Q}_2] = ?$$

Relation between Path integral and Hilbert Space

Relation between Path integral and Hilbert Space

Relation between Path integral and Hilbert Space

of a single $q(t)$

26

Relation between Path integral and Hilbert Space

of a single $q(t)$

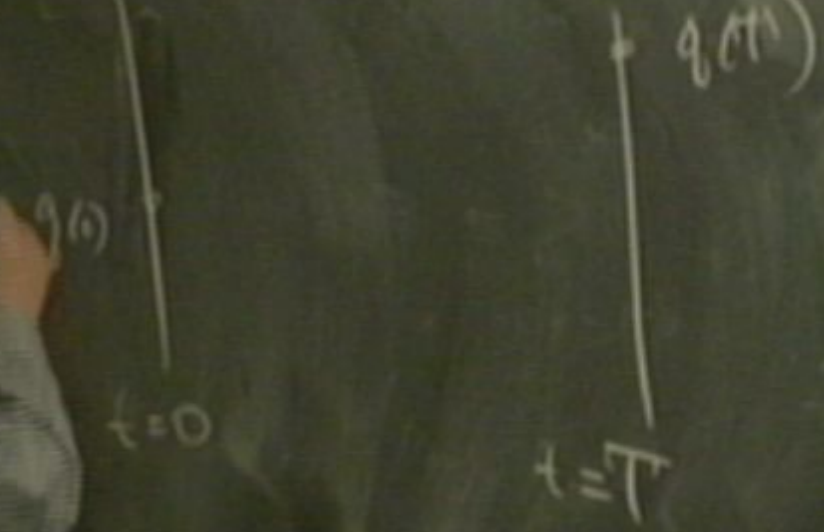
$t=0$

$t=T$

26

Relation between Path integral and Hilbert Space

QM of a single $q(t)$



Relation between Path integral and Hilbert Space

QM of a single $q(t)$

$$q(t) = q_T$$

$$q_0 = q(t)$$

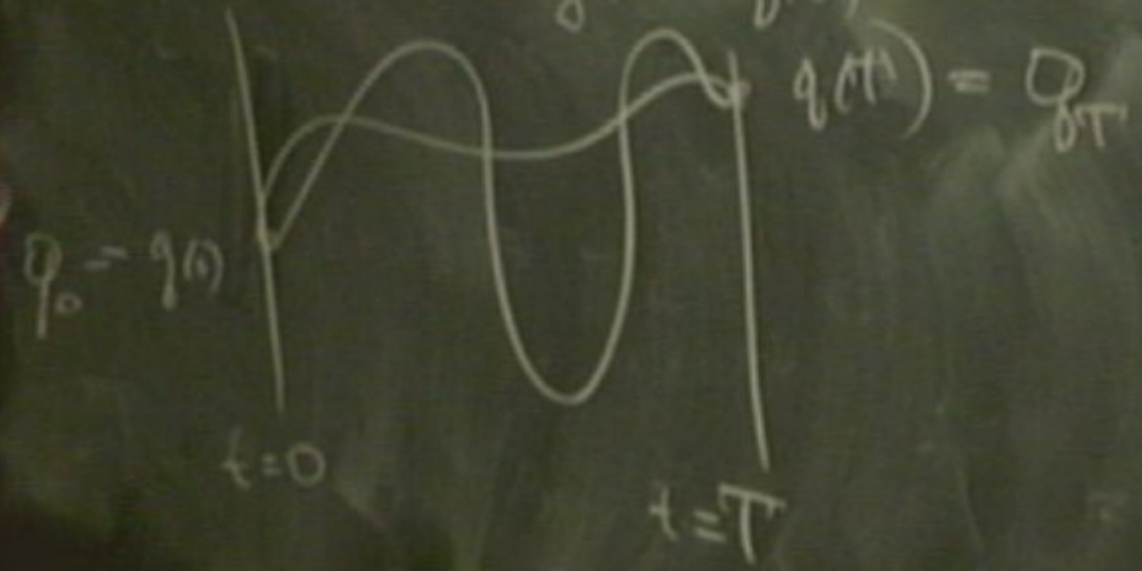
$t=0$

$t=T$

9-26

Relation between Path integral and Hilbert Space

QM of a single $q(t)$



Relation between Path Integral and Hilbert Space

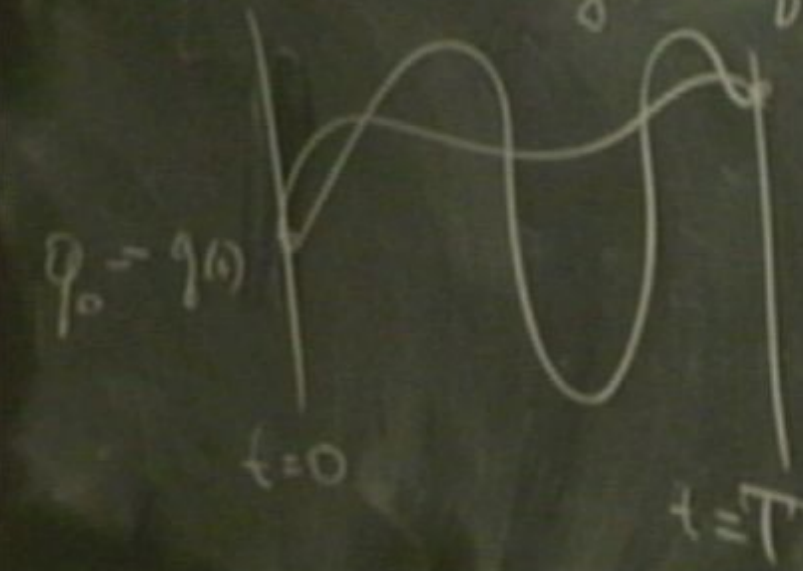
QM of a single $q(t)$

$$q(t) = q_T$$

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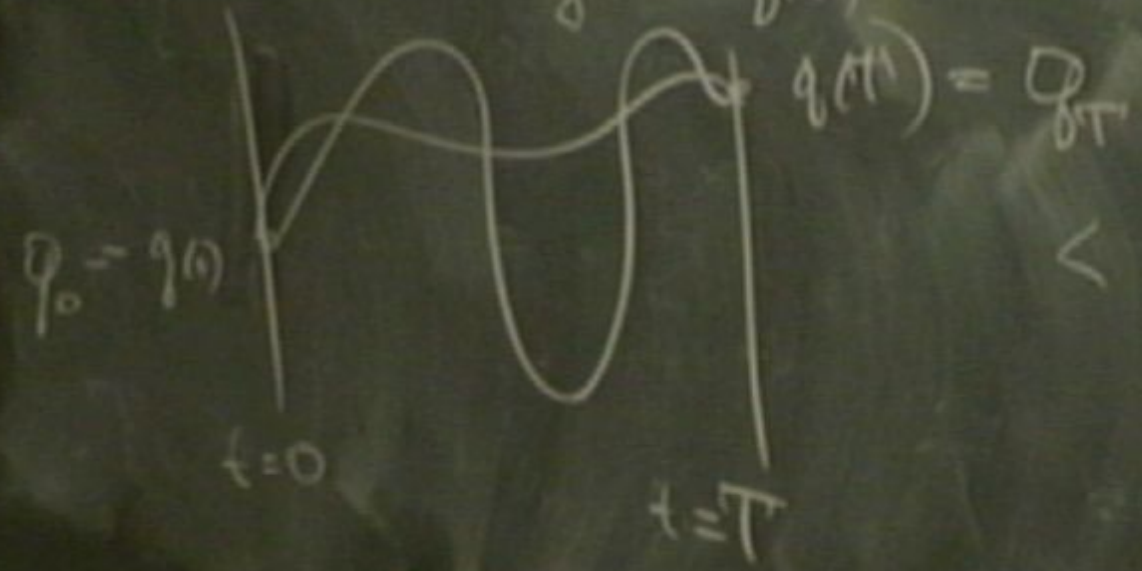
$$t=0$$

$$t=T$$



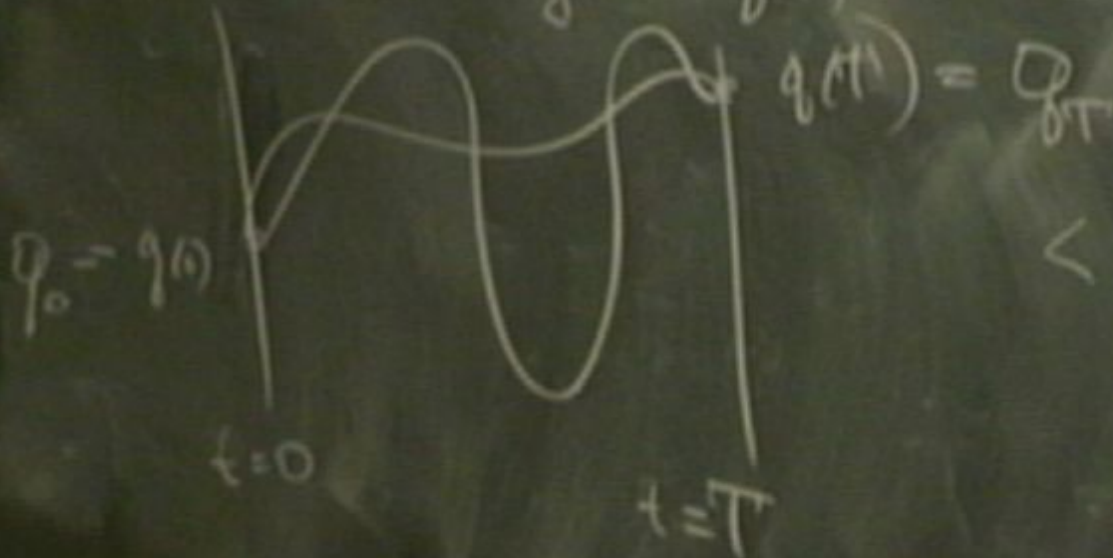
Relation between Path Integral and Hilbert Space

QM of a single $q(t)$



Relation between Path Integral and Hilbert Space

QM of a single $q(t)$



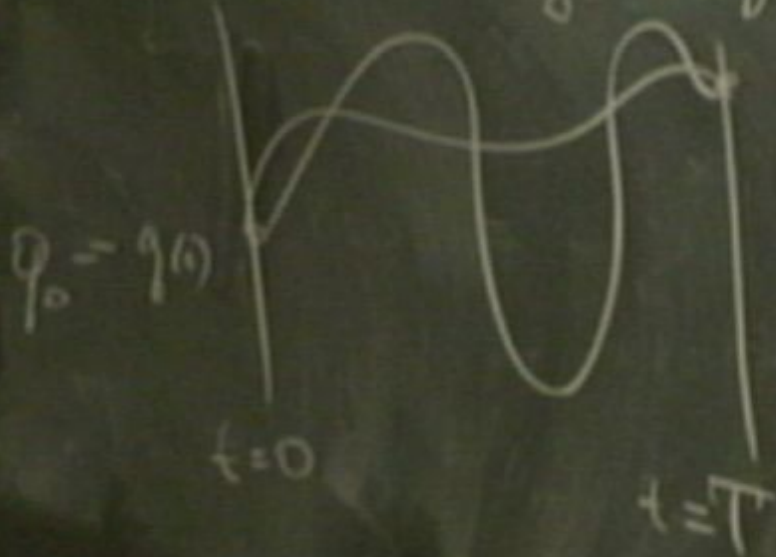
$$\langle \cdot | \cdot \rangle$$

Relation between Path Integral and Hilbert Space

QM of a single $q(t)$

$$q(T) = q_0$$

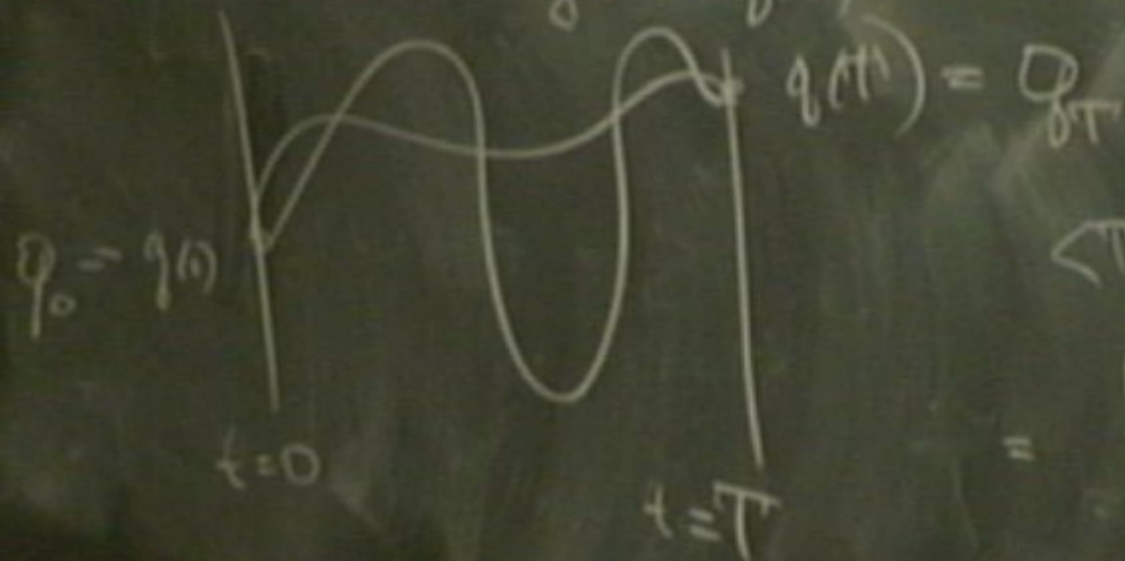
$$\langle T, q_T | 0, q_0 \rangle$$



2

Relation between Path Integral and Hilbert Space

QM of a single $q(t)$



$$\langle \pi, q_T | 0; q_0 \rangle$$

$$= \int [dq(t)]$$

$$q_0 = q(t)$$

$t=0$

$t=T$

$$\langle \pi, q_{\pi} | \sigma, q_0 \rangle$$

$$= \int [dq(t)] e^{-S[q]}$$

$$c = \psi = \frac{1}{\sqrt{2}} (\psi_1(z) + i\psi_2(z))$$

$$c_{\psi_1} = \frac{1}{2}$$

$$c_{\psi_2}(\lambda = \frac{1}{2}) = -3(2\lambda - 1)^2 + 1 = 1$$

$$\text{supernormalizing constant} = 1 + \frac{1}{2} = \frac{3}{2}$$

$q_0 = q(t)$
 $t=0$



$\langle \pi, q_{\pi} | 0, q_0 \rangle$

$$= \int [dq(\omega)] e^{-S[q]}$$

$q(t)|_{t=0} = q_0, q(t) \neq q_T$

$$c = \bar{\psi} = \frac{1}{\sqrt{2}} [\psi_1(z) + i\psi_2(z)]$$

$$c_{\psi_1} = \frac{1}{2}$$

$$b_c(\lambda = \frac{1}{2}) = -3(2 - 1)^2 + | -1$$

$$\text{supernormal matter} = 1 + \frac{1}{2} = \frac{3}{2}$$

$$q_0 = q(t)$$

$$t=0$$

$$t=T$$

$$= \int_{\mathcal{C}} [dq(t)] e^{-S[q]}$$

$$\mathcal{C} = \{q(t) | q(0) = q_0, q(T) = q_T\}$$

$$h_b = h_c = \frac{1}{2}$$

$$\psi = \frac{1}{\sqrt{2}} [\psi_1 + i\psi_2]$$

$$= \frac{1}{\sqrt{2}} [\psi_1(z) + i\psi_2(z)]$$

$$c_{\psi_1} = \frac{1}{2}$$

$$S_{bc} = \frac{1}{4\pi} \int d^2z [\psi_1 \bar{\partial} \psi_1 + \psi_2 \bar{\partial} \psi_2]$$

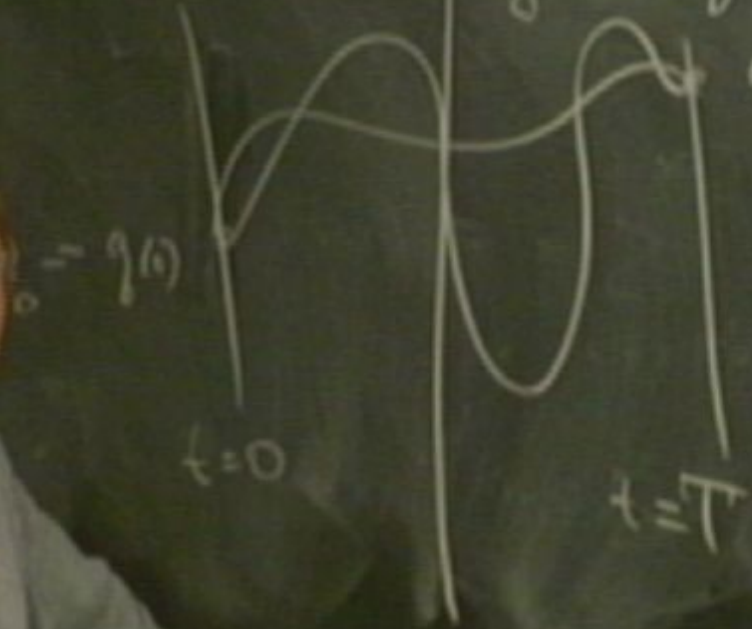
$$\mathbb{T} = -\frac{1}{2} \psi_1 \partial \psi_1 - \frac{1}{2} \psi_1 \partial \psi_2$$

$$G_{bc}(\lambda = \frac{1}{2}) = -3(z-1)^2 + 1 = 1$$

$$\text{supersymmetry matter} = 1 + \frac{1}{2} = \frac{3}{2}$$

Relation between Path Integral and Hilbert Space

QM of a single $q(t)$



$$q(T) = q_T$$

$$\langle \pi, q_T | 0, q_0 \rangle$$

$$= \int [dq(p)] e^{-S[q]}$$

$$q(t)|_{t=0} = q_0, q(t)|_{t=T} = q_T$$

$$q(T) = q_T$$

$$q_0 = q(0)$$

$t=0$

$t=T$

$t=0$

$$\langle \pi, q_T | 0, q_0 \rangle$$

$$= \int [dq(\omega)] e^{-S[q]}$$

$$q(t)|_{t=0} = q_0, \quad q(t)|_{t=T} = q_T$$

Relation between Path Integral and Hilbert Space

QM of a single $q(t)$

$$q(t) = q_T$$

$$q_0 = q(0)$$

$$t=0$$

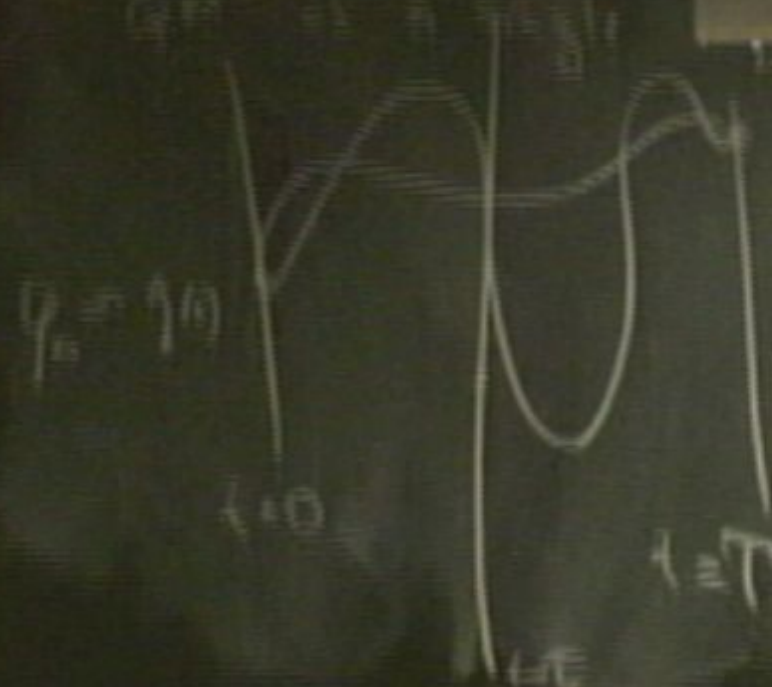
$$t=T$$

$$t=0$$

$$\langle \pi, q_T | 0, q_0 \rangle$$

$$= \int [dq(\omega)] e^{-S[q]}$$

$$q(0)|_{t=0} = q_0, \quad q(T) = q_T$$



$$y(t) = \frac{1}{T} \int_0^T y(\tau) e^{-j\omega(t-\tau)} d\tau$$

$$= \int_{-\infty}^{\infty} [\dots] e^{-j\omega t} d\omega$$

$$= \int_{-\infty}^{\infty} [\dots] e^{-j\omega t} d\omega$$

$$y(t) = \frac{1}{T} \int_0^T y(\tau) e^{-j\omega(t-\tau)} d\tau$$

$$\lim_{t \rightarrow \infty} \frac{1}{t} \int_0^t \dots$$

$$\langle f | i \rangle = \int \int d^3q \int d^3q'$$

$$f(t) \quad f(t) \Big|_{t=0} = q_0, \quad f(t) \Big|_{t=T} = q_T$$

$$\int \int dg(t)$$

$$\int dg(t) e^S$$



$$\psi(H)|_{H=0} = g_0, \quad \psi(T) = g_T$$

$$\langle f | i \rangle =$$

$$g_0$$

$$\int (dg/dt) e^S$$

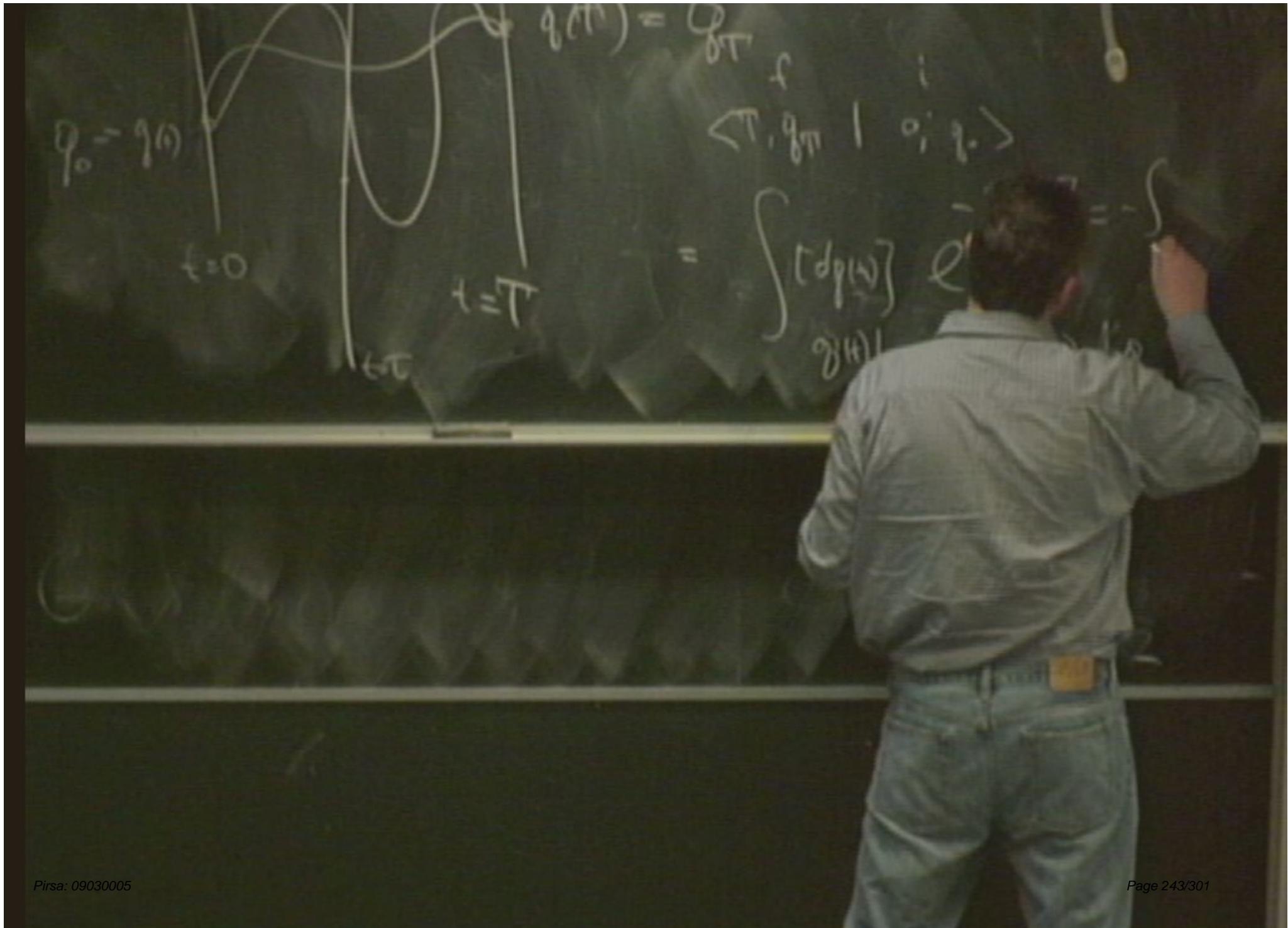
$$t \rightarrow \infty \quad \psi(t) |_{t \rightarrow \infty} = \psi_0, \quad \psi(t) \neq \psi_T$$

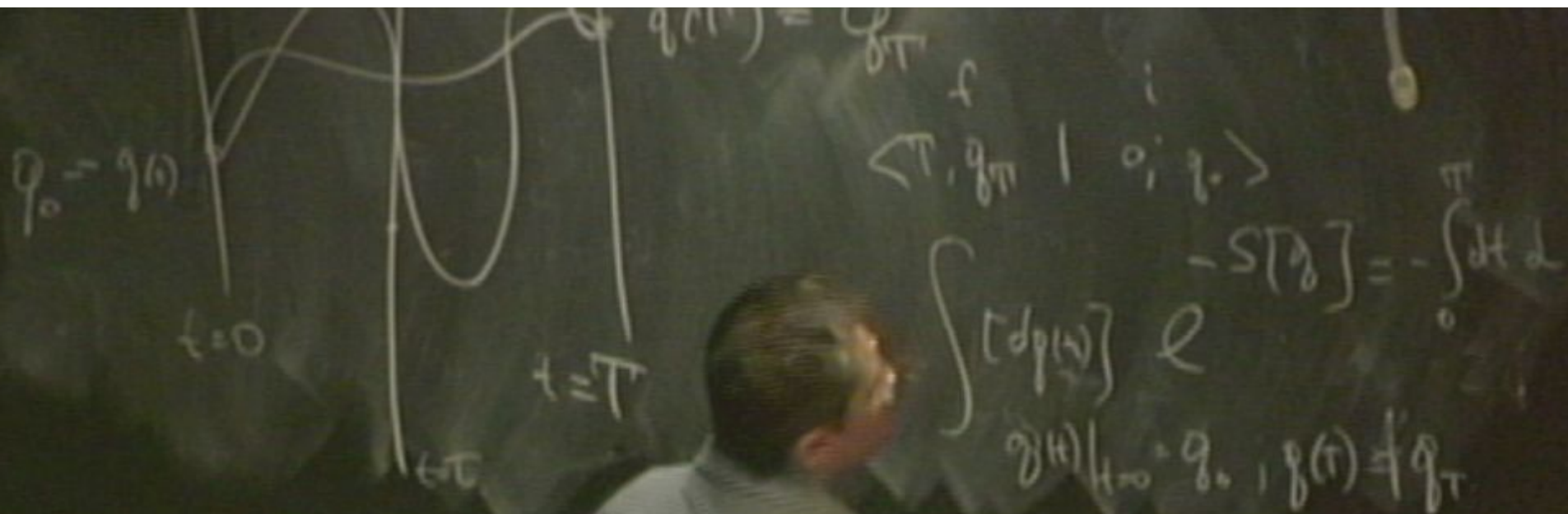
$$\langle f | i \rangle$$

$$\int d\psi(t)$$

$$\int (d\psi(t)) e^{S}$$

$$\psi(0) = \psi_0$$
$$\psi(t)$$





$$|t\rangle$$

$$\psi(t)|_{t=0} = q_0, \psi(t) = q_T$$

$$\langle f|i \rangle = \int dq(q)$$

$$\int \left[\frac{dq(t)}{dt} \right] e^{\int dt L}$$

$$q(0) = q_0$$
$$q(t)$$

$$\psi(H)|_{H=0} = \psi_0, \quad \psi(T) = \psi_T$$

$$\langle f | i \rangle = \int \int d^3q \psi(q)$$

$$\int \int d^3q \psi(q) e^{i \mathbf{q} \cdot \mathbf{r}}$$
$$\psi(0) = \psi_0$$
$$\psi(T)$$



$$y(t) \Big|_{t=0} = y_0, \quad y(t) \Big|_{t=T} = y_T$$

$\langle f | i \rangle$

$$\int dy(t)$$

$$\int_0^T dy(t) e^{-\int_0^t \dots}$$

$$y(0) = y_0$$

$$y(T) = y_T$$

$$\psi(t) \Big|_{t=0} = \psi_0, \quad \psi(t) \Big|_{t=T} = \psi_T$$

$$\langle f | \psi \rangle =$$

$$\int dq(q) e^{i \dots}$$

$$\int (dq(t)) e^{i \dots}$$

$$\begin{aligned} \psi(0) &= \psi_0 \\ \psi(T) &= \psi_T \end{aligned}$$

$$\psi(H)|_{t=0} = \psi_0, \quad \psi(H)|_{t=T} = \psi_T$$

$$\langle f|i \rangle = \int \int_{\mathcal{D}} d\varphi e^{\int_0^T dt L}$$

$$\int \int_{\mathcal{D}} [d\varphi(t)] e^{\int_0^T dt L}$$

$$\varphi(0) = \varphi_0$$

$$\varphi(T) = \varphi_T$$

$$\psi(H)|_{H=0} = g_0 + g(T) + g_T$$

$$\langle f | i \rangle = \int_{-\infty}^{\infty} [dg(q)] e^{i \int dq L}$$

$$g(0) = q_0$$

$$\int_{-\infty}^{\infty} [dg(t)] e^{i \int dt L}$$

$$g(0) = q_0$$

$$g(T) = q_T$$

$$\psi(t) \Big|_{t=0} = \psi_0, \quad \psi(t) \Big|_{t=T} = \psi_T$$

$$f(t) = \int_{t_0}^T \left[\frac{d\psi(t)}{dt} \right] e^{-\lambda t} dt$$

$$\psi(t_0) = \psi_0$$

$$\psi(T) = \psi_T$$

$$f(t) = \int_0^T \left[\frac{d\psi(t)}{dt} \right] e^{-\lambda t} dt$$

$$\psi(0) = \psi_0$$

$$\psi(T) = \psi_T$$



QM of a single $q(t)$

$$q(\pi) = q_T$$

$$q_0 = q(0)$$

$t=0$

$t=T$

$t=\tau$

$$\langle \pi, q_{\pi} | 0; q_0 \rangle$$

$$= \int [dq(\omega)] e^{-S[q]}$$

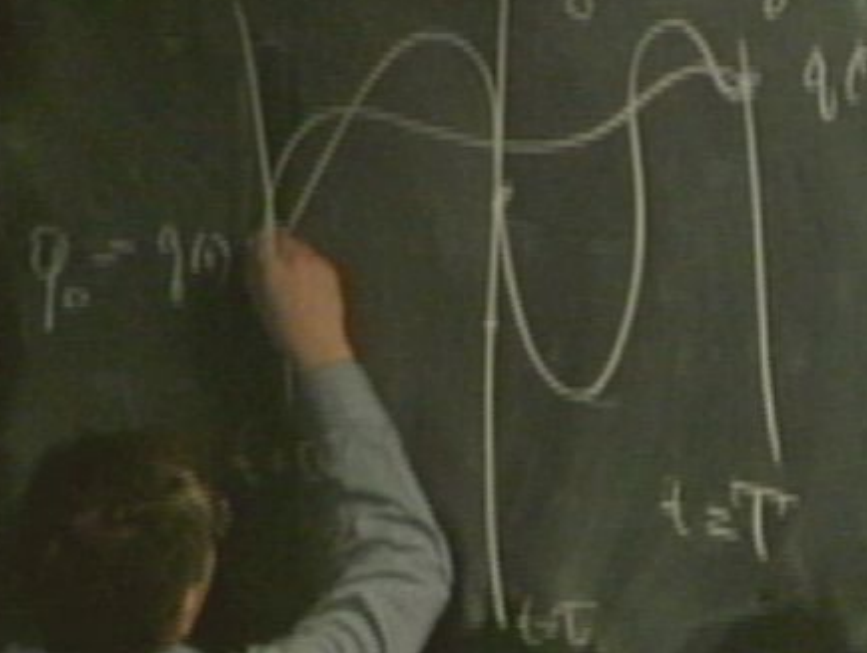
$$-S[q] = -\int_0^{\pi} dt L$$

$$q(t)|_{t=0} = q_0, q(t) \neq q_T$$

$$q(\pi) = q_T$$

$$q(\tau) = q_{\tau}$$

QM of a particle $q(t)$



$$q(t) = q_T$$

$\langle T, q_m | 0, q_0 \rangle$

$$= \int [dq(\omega)] \mathcal{L}$$

$$-S[q] = -\int_0^T dt L$$

$$q(t)|_{t=0} = q_0, q(t)|_{t=T} = q_T$$

Relation between Path Integral and Hilbert Space

QM of a single $q(t)$

$$q(t) = q_T$$

$$q_0 = q(0)$$

$$t = T$$

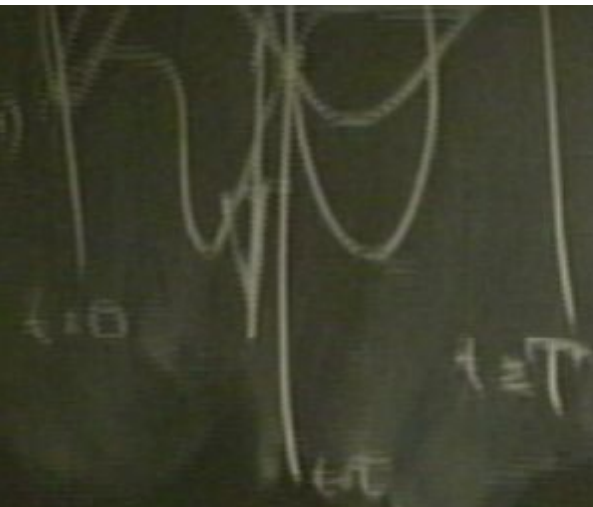
$$\langle T, q_T | 0, q_0 \rangle$$

$$= \int [dq(t)] e^{-S[q]} = - \int_0^T dt L$$

$$q(t)|_{t=0} = q_0, q(t)|_{t=T} = q_T$$



$$p = q(t)$$



$$= \int_0^T [dq(t)] e^{i\omega t} = \int_0^T dq(t) e^{i\omega t}$$

$$q(0) = q_0, \quad q(T) = q_T$$

$$f(t) = \int [dq(t)] e^{i\omega t}$$

$$q(0) = q_0$$

$$q(T) = q_T$$

$$\int [dq(t)] e^{i\omega t}$$

$$q(0) = q_0$$

$$q(T) = q_T$$

$$\begin{aligned}
 & \int_{t=0}^{t=T} \dots \\
 & = \int \left[\frac{dq(t)}{dt} \right] e^{-st} dt \\
 & q(t) \Big|_{t=0} = q_0, \quad q(T) = q_T
 \end{aligned}$$

$$\begin{aligned}
 & \int_{-\infty}^{\infty} \left[\frac{dq(t)}{dt} \right] e^{-st} dt \\
 & q(\tau) = q_0 \\
 & q(\pi) = q_T
 \end{aligned}$$

$$\begin{aligned}
 & \int \left[\frac{dq(t)}{dt} \right] e^{-st} dt \\
 & q(0) = q_0 \\
 & q(\pi) = q_T
 \end{aligned}$$

$t=0$

$t=T$

$$= \int_{q_0}^{q_T} [dq(t)] e^{-\int_0^T dt L}$$

$q(t)|_{t=0} = q_0, q(t)|_{t=T} = q_T$

$$\langle f|i \rangle = \int_{q_0}^{q_T} [dq(t)] e^{-\int_0^T dt L}$$

$q(0) = q_0$
 $q(T) = q_T$

$$\int_{q_0}^{q_T} [dq(t)] e^{-\int_0^T dt L}$$

$q(0) = q_0$
 $q(T) = q_T$

$$t \rightarrow \tau \quad y(t)|_{t=0} = y_0, \quad y(\tau) = y_T$$

$$= \int_{-\infty}^{\tau} \left[\frac{dy(t)}{dt} \right] e^{t/\tau} dt$$

$y(0) = y_0$
 $y(\tau) = y_T$

$$\int_0^{\tau} \left[\frac{dy(t)}{dt} \right] e^{-t/\tau} dt$$

$y(0) = y_0$
 $y(\tau) = y_T$

=

↗

$$t \rightarrow \tau \quad \mathcal{H}(t)|_{t=0} = g_0, \quad \mathcal{H}(t)|_{t=T} = g_T$$

$$\langle f | i \rangle = \int_{q_c}^{\mathcal{H}(T)} \left[\frac{d\mathcal{H}(t)}{dt} \right] e^{\int_{t_0}^t \mathcal{H}(t) dt}$$

$$g(0) = g_0$$

$$g(T) = g_T$$

$$\left[\frac{d\mathcal{H}(t)}{dt} \right] e^{\int_{t_0}^t \mathcal{H}(t) dt}$$

$$g(0) = g_0$$

$$g(T) = g_T$$

$$\langle \tau | i \rangle$$

$$1 \leftarrow \tau \qquad \psi(t) \Big|_{t=0} = q_0, \quad \psi(t) \Big|_{t=T} = q_T$$

$$= \int_{-\infty}^{\infty} dp_c \int_{-\infty}^{\infty} [dq(t)] e^{i \int dt L}$$

$$q(0) = q_0$$

$$q(T) = q_T$$

$$\int_{-\infty}^{\infty} [dq(t)] e^{i \int dt L}$$

$$q(0) = q_0$$

$$q(T) = q_T$$

$$\langle q_0 | q \rangle$$



$$|t=\tau \quad \psi(t)|_{t=0} = q_0, \quad \psi(t) \rightarrow q_T$$

$$|t=\tau \quad \psi(t) = \int_{-\infty}^{\infty} dp_k \left[\int_{-\infty}^{\infty} dq \right] e^{i p_k t}$$

$$q(0) = q_0$$

$$q(\tau) = q_T$$

$$\left[\int_{-\infty}^{\infty} dq(t) \right] e^{i p_k t}$$

$$q(0) = q_0$$

$$q(\tau) = q_T$$

$$= q_T \langle q_T | e^{i p_k t} \rangle$$

$$1 + \tau \quad \psi(H)|_{H=0} = g_0, \quad \psi(\tau) = g_T$$

$$\langle f | i \rangle$$

$$\int_0^{\tau} (dg/dt) e^{t} dt$$

$$\int_0^{\tau} (dg/dt) e^{-t} dt$$

$$g(\tau) = g_T$$

$$g(0) = g_0$$

$$g(\tau) = g_T$$

$$\langle f | g_T \rangle \quad \langle g_T | i \rangle$$

$t = \tau$

$\psi(H)|_{t=0} = \psi_0, \psi(t) \rightarrow \psi_T$

$$\langle f | i \rangle = \int_{q_0}^{q_T} [dq] e^{i \int_{t_0}^{t_1} L(q, \dot{q}, t) dt}$$

$$q(t_0) = q_0$$

$$q(t_1) = q_T$$

$$\int_{q_0}^{q_T} [dq] e^{i \int_{t_0}^{t_1} L(q, \dot{q}, t) dt}$$

$$q(t_0) = q_0$$

$$q(t_1) = q_T$$

$$\langle f | q_T \rangle \langle q_0 | i \rangle$$

$t = \tau$

$$\psi(H)|_{t=0} = \psi_0, \quad \psi(\tau) = \psi_T$$

$$\langle f | i \rangle = \int_{-\infty}^{\infty} dq_{\tau} \left[\int_{q_0}^{q_T} (dq) \right] e^{i \int_{q_0}^{q_T} p dq}$$

$$q(0) = q_0 \\ q(\tau) = q_T$$

$$\left[\int_{q_0}^{q_T} (dq) \right] e^{i \int_{q_0}^{q_T} p dq}$$

$$q(0) = q_0 \\ q(\tau) = q_T$$

$$\int dq_{\tau} \langle f | q_{\tau} \rangle \langle q_{\tau} | i \rangle$$

$t \rightarrow \tau$

$\psi(t)|_{t=0} = \psi_0, \psi(t)|_{t=\tau} = \psi_T$

$$\langle f | i \rangle = \int_{-\infty}^{\infty} dq_c \int_{q_c}^{q_T} [dq(q)] e^{iS}$$

$$q(0) = q_c$$

$$q(\tau) = q_T$$

$$\int_{q_0}^{q_c} [dq(q)] e^{iS}$$

$$q(0) = q_0$$

$$q(\tau) = q_c$$

complete set of states

$$\int dq_c \langle f | q_c \rangle \langle q_c | i \rangle = \langle f | \int dq | p \rangle \langle p | i \rangle$$

$$|t\rangle \quad \mathcal{H}|_{t=0} = q_0, \quad \mathcal{H}|_{t=T} = q_T$$

$$\langle f | i \rangle = \int_{-\infty}^{\infty} dq_c \int [dq(q)] e^{i \int_{t=0}^{t=T} \mathcal{L} dt}$$

$q(0) = q_0$
 $q(T) = q_T$

$$\int [dq(q)] e^{i \int_{t=0}^{t=T} \mathcal{L} dt}$$

$q(0) = q_0$
 $q(T) = q_T$

complete set of states

$$\int dq_c \langle f | q_c \rangle \langle q_c | i \rangle = \langle f | \int dq_c |q_c\rangle \langle q_c | i \rangle$$

$\| \int dq_c |q_c\rangle \langle q_c| = I$

$$\int \sin x e^{-x} dx$$



$$\int \sin e^{-\frac{1}{2}x} dx$$



$$\int [dq] e^{-\int_0^T dt \mathcal{L}} q(\tau)$$

$$\int [dq] e^{-\int_0^T dt L} q(\tau) = - \int d$$

$$\int [dq] e^{-\beta H(q, p)} g(q) = \int dq g$$

$$\int |f\rangle \langle f| e^{-\beta H} g(\tau) = \int dg \langle f|g\rangle g \langle$$

$$\int [dq] e^{-\int dt f(x)} q(t) = \int dq \langle f | q \rangle q \langle q | i \rangle$$

$$\int [Dq] e^{-\int_0^T dt L} q(t) = \int dq \underbrace{\langle f | q \rangle}_{\int_0^T dt} \underbrace{g \langle q | i \rangle}_{\int_0^T dt}$$

$$\int [dq] e^{-\beta H(q)} g(q) = \int dq \underbrace{\langle f | q \rangle}_{\text{Feyn}} \underbrace{g(q) \langle q | i \rangle}_{\text{Feyn}}$$

$$\int [dq] e^{-\int dt L} q(t) = \int dq \underbrace{\langle f | q \rangle}_{\int} \underbrace{q(t) \langle q | i \rangle}_{\int}$$

$|q\rangle$

$$\int [dq] e^{-i\int dt L} q(t) = \int dq \underbrace{\langle f | q \rangle}_{\mathbb{R}} \underbrace{q \langle q | i \rangle}_{\mathbb{R}}$$

$|q\rangle \rightarrow$ a state is a Hilbert space

$$\int [dq] e^{-i\int dt L} q(t) = \int dq \underbrace{\langle f | q \rangle}_{\text{H}} \underbrace{q \langle q | i \rangle}_{\text{H}}$$

$|q\rangle \rightarrow$ a state is a Hilbert space

\uparrow
 q

$$\int [dq] e^{-\int dt L} q(\tau) = \int dq \underbrace{\langle f | q \rangle}_{\int} \underbrace{q \langle q | i \rangle}_{\int}$$

$|q\rangle \rightarrow$ a state is a Hilbert space

$$\hat{q} |q\rangle = q |q\rangle$$

$$\int [dq] e^{-\int_0^{\tau} dt f(q)} q(\tau) = \int dq \underbrace{\langle f | q \rangle}_{\mathbb{F}} \underbrace{q \langle q | i \rangle}_{\mathbb{F}}$$

$|q\rangle \rightarrow$ a state is a Hilbert space \mathbb{F}

$$\hat{q} |q\rangle = q |q\rangle = \int dq q \langle f |$$

$$\int [dq] e^{-i\int dt L} q(\tau) = \int dq \underbrace{\langle f | q \rangle}_{\mathbb{F}} \underbrace{q \langle q | i \rangle}_{\mathbb{I}}$$

$|q\rangle \rightarrow$ a state is a Hilbert space \mathbb{F}

$$\hat{q}|q\rangle = q|q\rangle = \int dq' \langle f | \hat{q} | q' \rangle \langle q' | i \rangle$$

$$\int [dq] e^{-\int dt L} q(\tau) = \int dq \underbrace{\langle f | q \rangle}_{\int} \underbrace{q \langle q | i \rangle}_{\int}$$

$|q\rangle \rightarrow$ a state is a Hilbert space

$$\hat{q} |q\rangle = q |q\rangle$$

$$= \int dq \langle f | \hat{q} |q\rangle \langle q | i \rangle$$

$$\int [dq] e^{-\int dt L} q(\tau) = \int dq \underbrace{\langle f | q \rangle}_{\int} \underbrace{q | i \rangle}_{\int}$$

$|q\rangle \rightarrow$ a state is a Hilbert space

$$\hat{q} |q\rangle = q |q\rangle$$

$$= \int dq \langle f | \hat{q} |q\rangle \langle q | i \rangle$$

$$\int [dq] e^{-i\int dt L} q(\tau) = \int dq \underbrace{\langle f | q \rangle}_{\int} \underbrace{q | i \rangle}_{\int}$$

$|q\rangle \rightarrow$ a state is a Hilbert space

$$\hat{q} |q\rangle = q |q\rangle = \int dq' \langle f | \hat{q} |q\rangle \langle q' |$$

$$\int [dq] e^{-\int dt L} q(\tau) = \int dq \underbrace{\langle f | q \rangle}_{\text{Hilbert space}} \underbrace{\hat{q} | q \rangle}_{\text{Hilbert space}} \langle q | i \rangle$$

$|q\rangle \rightarrow$ a state is a Hilbert space

$$\hat{q} | q \rangle = q | q \rangle$$

$$= \int dq \langle f | \hat{q} | q \rangle \langle q | i \rangle$$

$$\int [dx] e^{-\int dt L} q(\tau) = \int dq \underbrace{\langle f | q \rangle}_{\text{Hilbert}} \underbrace{\hat{q} | q \rangle}_{\text{Hilbert}}$$

$|q\rangle \rightarrow$ a state is a Hilbert space

$$\hat{q} | q \rangle = q | q \rangle = \int dq' \langle f | \hat{q} | q' \rangle \langle q' | i \rangle$$

1. (c) $l=1,2 \Rightarrow Q_1, Q_2 \quad Q' \quad [\hat{Q}_1, \hat{Q}_2] = ?$

$$\int_{\mathbb{R}^n} e^{i\mathbf{p}\cdot\mathbf{q}} \hat{q}(\mathbf{p}) = \int d\mathbf{q} \underbrace{\langle \mathbf{f} | \mathbf{q} \rangle}_{\mathbb{R}^n} \underbrace{\hat{q}(\mathbf{p}) \langle \mathbf{q} | \mathbf{i} \rangle}_{\mathbb{R}^n}$$

$|\mathbf{q}\rangle \rightarrow$ a state is a Hilbert space

$$\hat{q}(\mathbf{p}) |\mathbf{q}\rangle = \mathbf{q} |\mathbf{q}\rangle$$

$$= \langle \mathbf{f} | \hat{q}(\mathbf{p}) | \mathbf{i} \rangle$$

$$= \int d\mathbf{q} \langle \mathbf{f} | \hat{q}(\mathbf{p}) | \mathbf{q} \rangle \langle \mathbf{q} | \mathbf{i} \rangle$$

$$i=1,2 \Rightarrow Q_1, Q_2 \quad Q' \quad [Q_1, Q_2] = ?$$

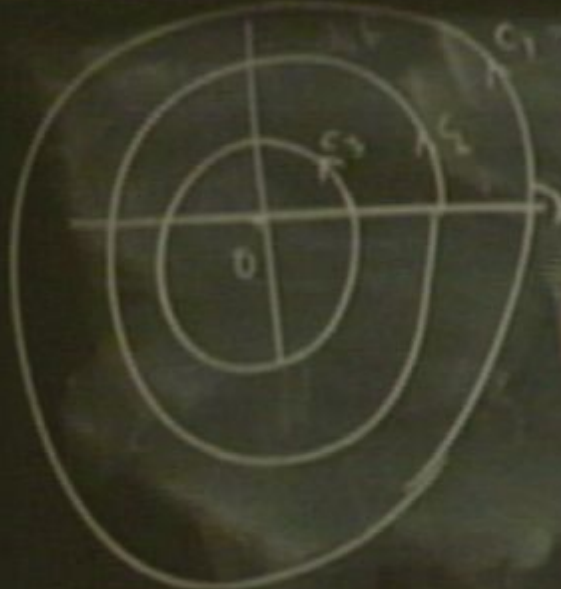
$$\int_{-\infty}^{\infty} e^{-\lambda t} q(t) q(t) dt \Rightarrow$$

$$\int [dq] e^{-\int dt L} q(t_1) q(t_2) = \langle F | T(q(t_1) q(t_2)) | i \rangle$$

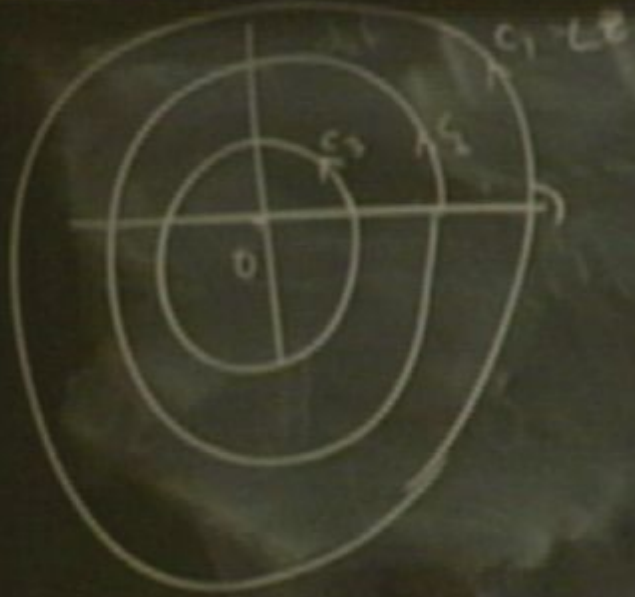
$$T(q(t_1) q(t_2)) = \Theta(t_1 - t_2) q(t_1) q(t_2) + \Theta(t_2 - t_1)$$

$$\int [dq] e^{-\int dt L} \circledast q(t_1) q(t_2) = \langle F | T(q(t_1) q(t_2)) | i \rangle$$

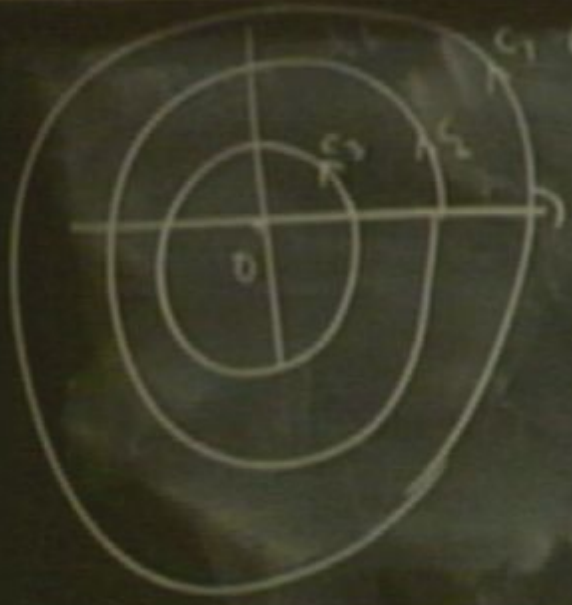
$$T(q(t_1) q(t_2)) = \Theta(t_1 - t_2) q(t_1) q(t_2) + \Theta(t_2 - t_1)$$



19c.

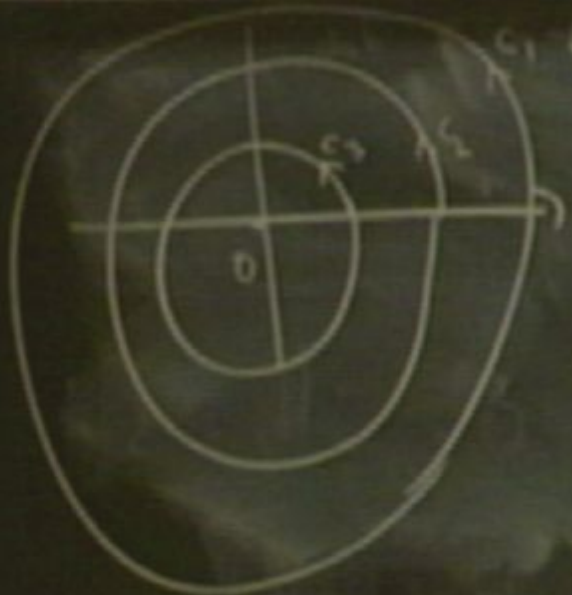


circles of a family



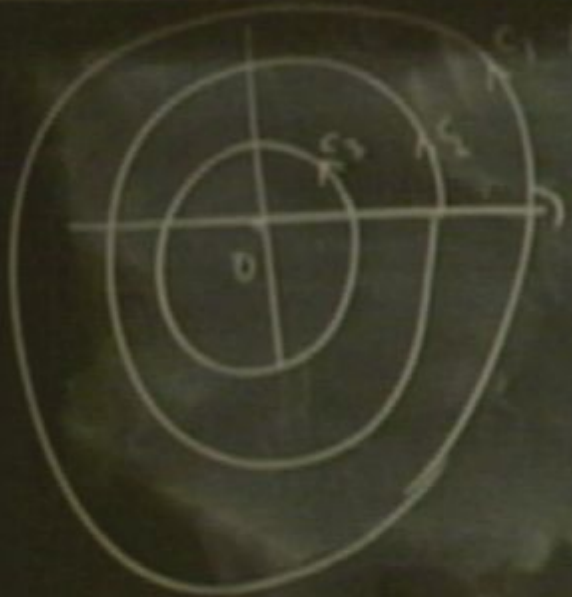
$$Q_1 \{c_1\} \quad Q_2 \{c_2\}$$





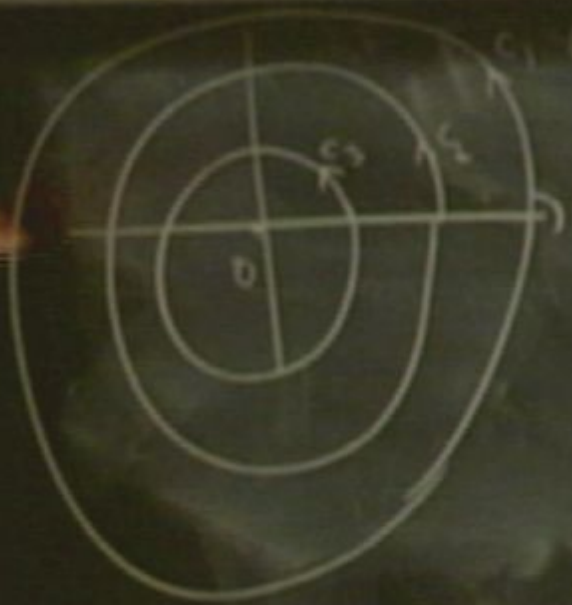
$Q_1 \{ C_1 \}$ $Q_2 \{ C_2 \}$
 ↑ ↑
 taken taken





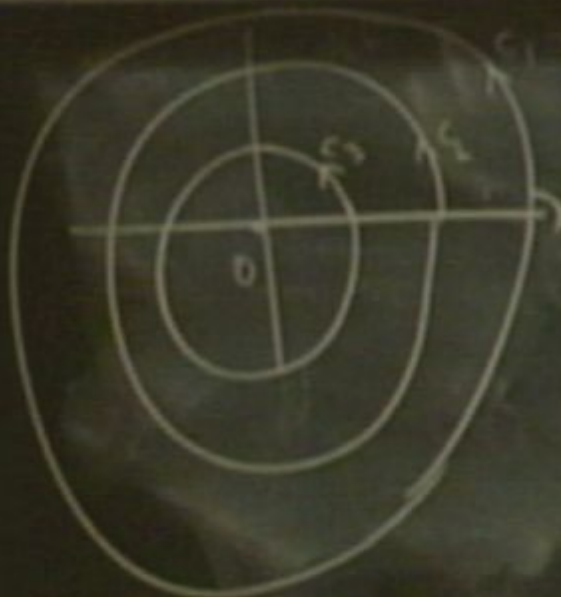
$$Q_1 \{ C_1 \} \quad Q_2 \{ C_2 \} - Q_3 \{ C_3 \} \quad Q_1 \{ C_3 \}$$

\uparrow \uparrow
 Inter capta



$$Q_1 \{ C_1 \} - Q_2 \{ C_2 \} = Q_2 \{ C_2 \} - Q_1 \{ C_1 \}$$

↑ later
 ↑ earlier
 ↑ later
 ↑ earlier

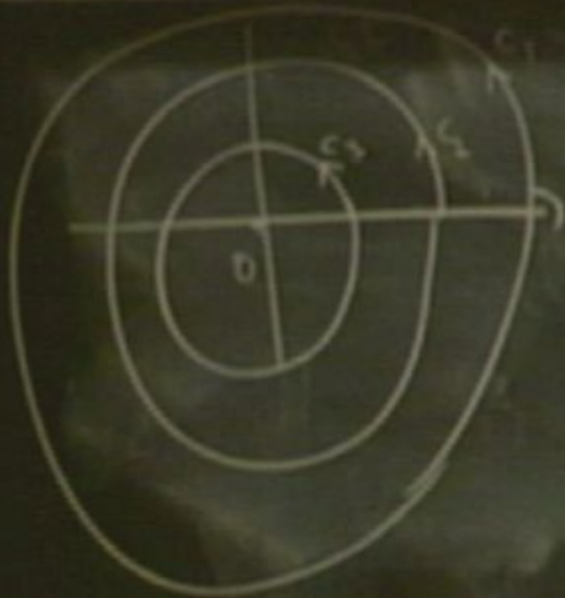


$$Q_1 \{ C_1 \} \quad Q_2 \{ C_2 \} = Q_1 \{ C_2 \} \quad Q_1 \{ C_1 \}$$

\uparrow taken \uparrow earlier \uparrow taken \uparrow earlier

$\Rightarrow Q_1$

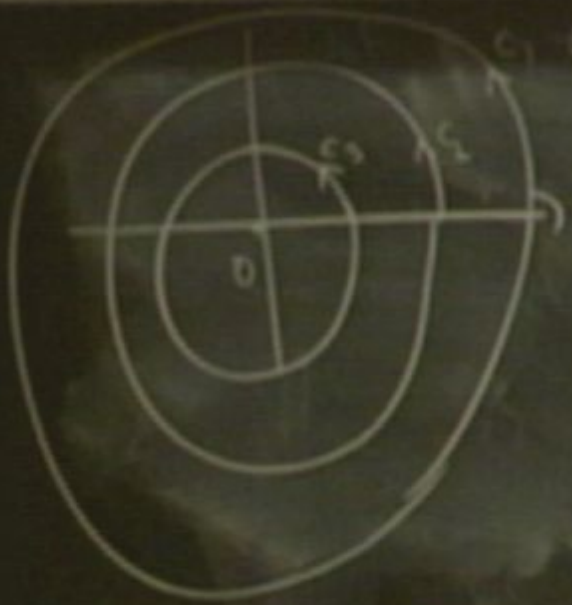




$$Q_1 \{ C_1 \} \quad Q_2 \{ C_2 \} = Q_2 \{ C_2 \} \quad Q_1 \{ C_1 \}$$

↑ later ↑ earlier ↑ later ↑ earlier

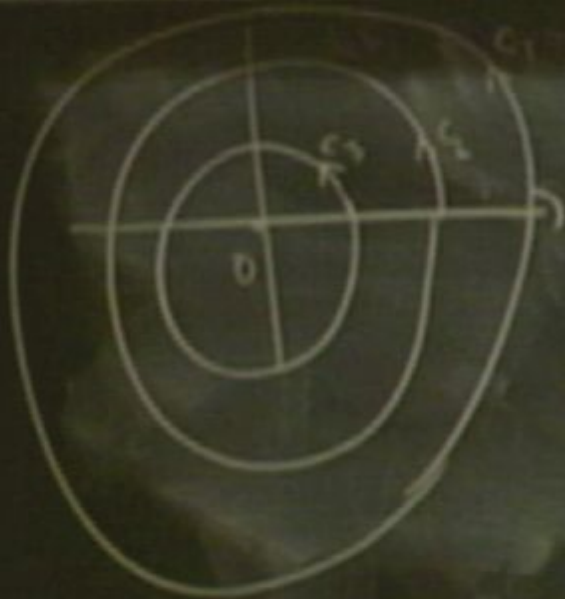
$$\Rightarrow \hat{Q}_1 \hat{Q}_2 - \hat{Q}_2 \hat{Q}_1 =$$



$$Q_1 \{C_1\} \quad Q_2 \{C_2\} - Q_1 \{C_2\} \quad Q_1 \{C_3\}$$

\uparrow later \uparrow earlier \uparrow later \uparrow earlier

$$\Rightarrow \hat{Q}_1 \hat{Q}_2 - \hat{Q}_2 \hat{Q}_1 = [\hat{Q}_1, \hat{Q}_2]$$

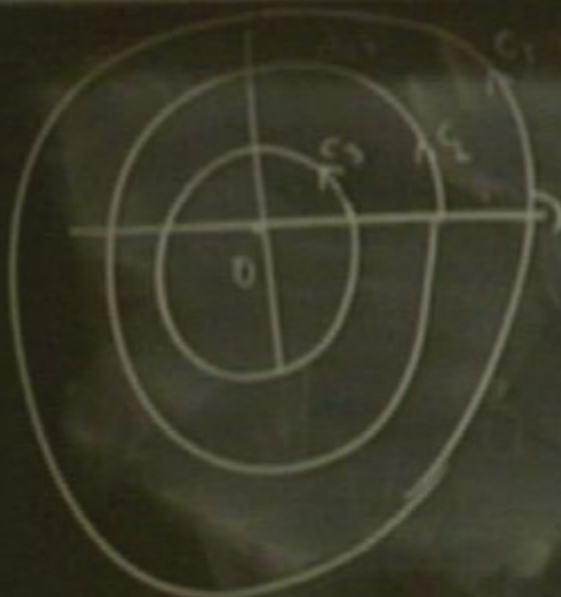


$\oint_C \mathbf{j} \frac{d\mathbf{r}}{|\mathbf{r}|^3}$

$\hat{Q}_1 \{C_1\} - \hat{Q}_2 \{C_2\} = \hat{Q}_2 \{C_2\} \hat{Q}_1 \{C_1\}$

(Labels: \hat{Q}_1 inner, \hat{Q}_2 outer, \hat{Q}_2 inner, \hat{Q}_1 outer)

$\Rightarrow \hat{Q}_1 \hat{Q}_2 - \hat{Q}_2 \hat{Q}_1 = [\hat{Q}_1, \hat{Q}_2]$



$\oint_C \mathbf{j} \cdot \frac{d\mathbf{s}}{c_1}$

$\hat{Q}_1 \{c_1\} \quad \hat{Q}_2 \{c_2\} - \hat{Q}_1 \{c_2\} \hat{Q}_2 \{c_1\}$

↑ later ↑ earlier ↑ later ↑ earlier

$\Rightarrow \hat{Q}_1 \hat{Q}_2 - \hat{Q}_2 \hat{Q}_1 = [\hat{Q}_1, \hat{Q}_2]$