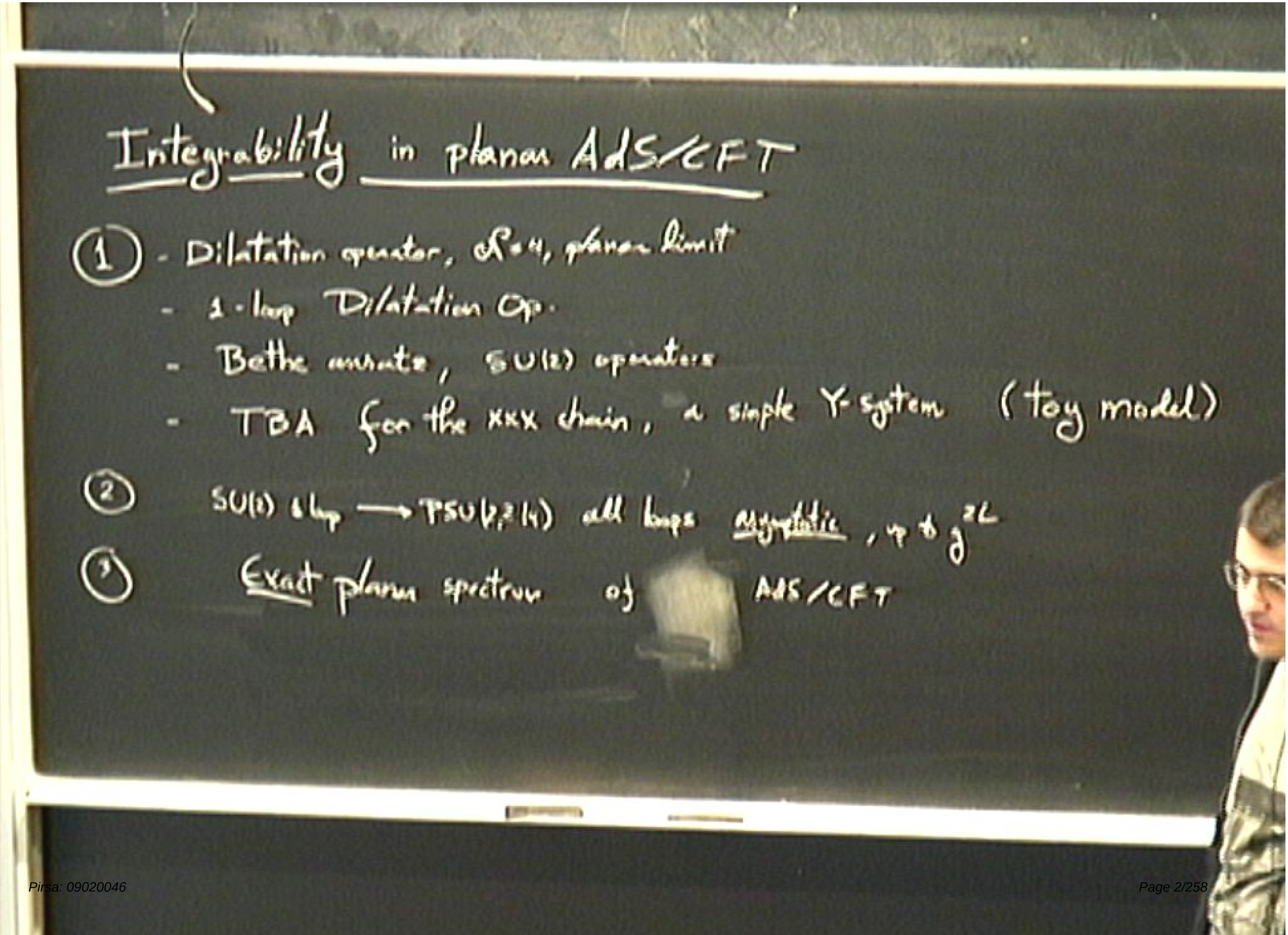
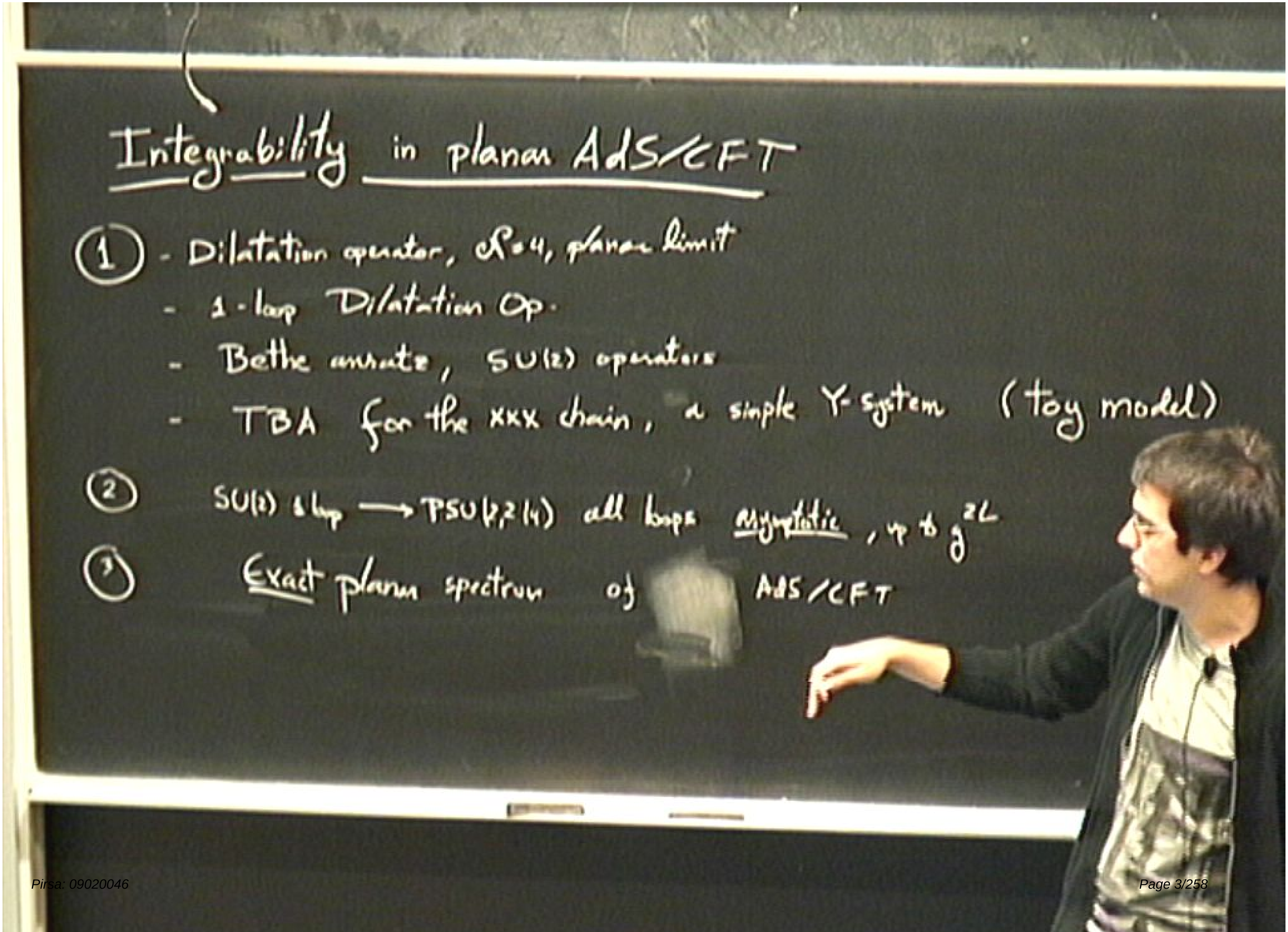
Title: Integrability and planar AdS/CFT - Lecture 1

Date: Feb 27, 2009 02:30 PM

URL: http://pirsa.org/09020046

Abstract: Lecture 1 - The one loop dilatation operator of N=4 SYM - Integrable models, Bethe ansatz - One-loop Bethe ansatz for N=4 SYM, coordinate and algebraic approach


Integrability in planar AdS/CFT
1 - Dilatation operator, N=4, planar limit
- 1-loop Dilatation Op.
- Bethe ansatz, SU(2) operators
- TBA for the XXX chain, a simple Y-system (toy model)
2 SU(2) 1 loop → PSU(2,2|4) all loops asymptotic, up to g^{2L}
3 Exact planar spectrum of AdS/CFT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Integrability in planar AdS/CFT

① - Dilatation operator, $\mathcal{N}=4$, planar limit
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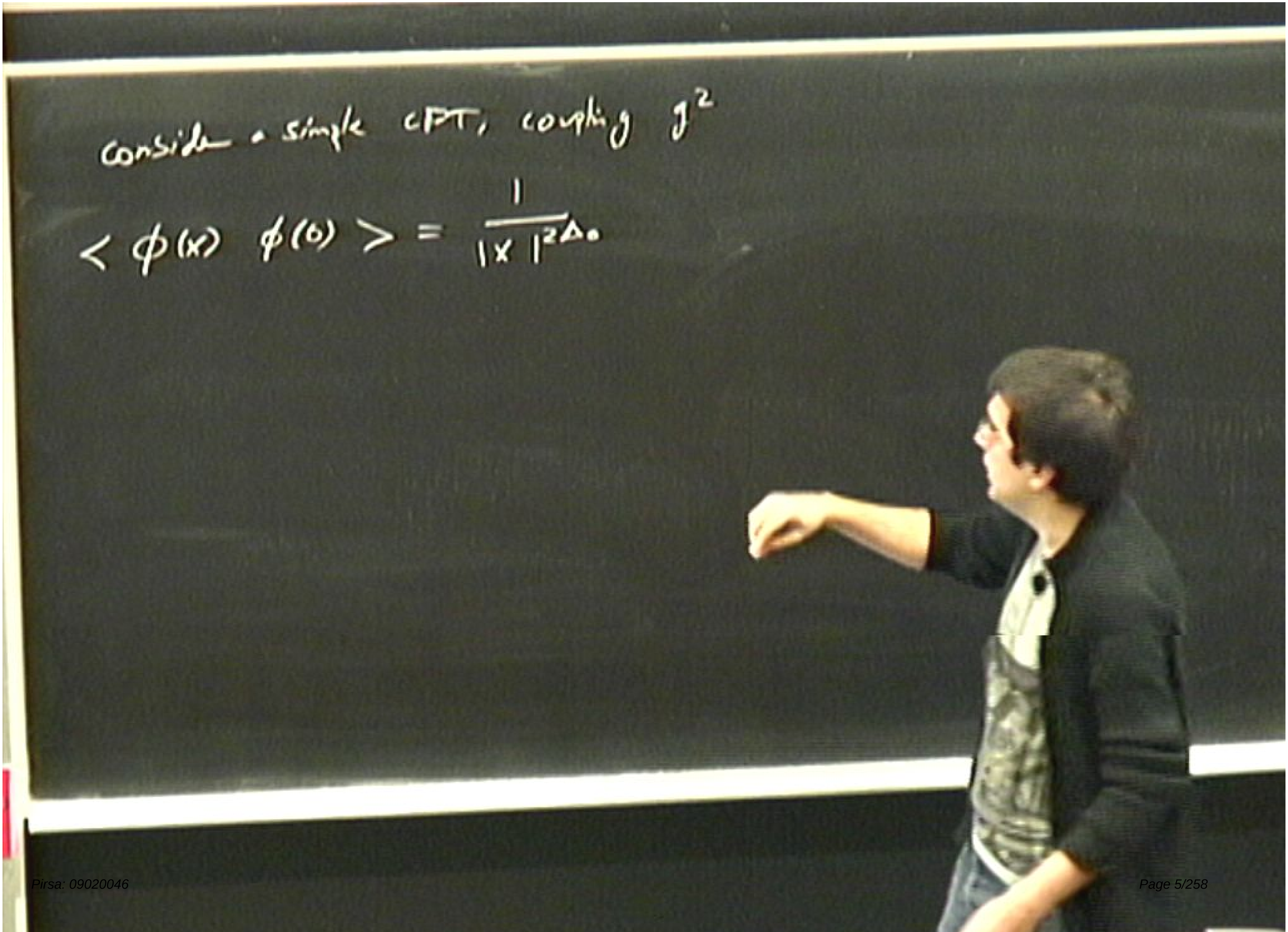
③ Exact planar spectrum of AdS/CFT


consider a simple CFT, coupling g^2
< φ(x) φ(0) > = 1 / |x|^{2Δ_0}

consider a simple CFT, coupling g^2

$$\langle \phi(x) \, \phi(0) \rangle = \frac{1}{|x|^{2\Delta_0}} \left(1 - \gamma g^2 \right.$$

consider a simple CFT, coupling g^2

$$< \phi(x) \, \phi(0) > = \frac{1}{|x|^{2\Delta_0}} \left(1 - \gamma g^2 \log |x|^2 \Lambda^2 \right)$$

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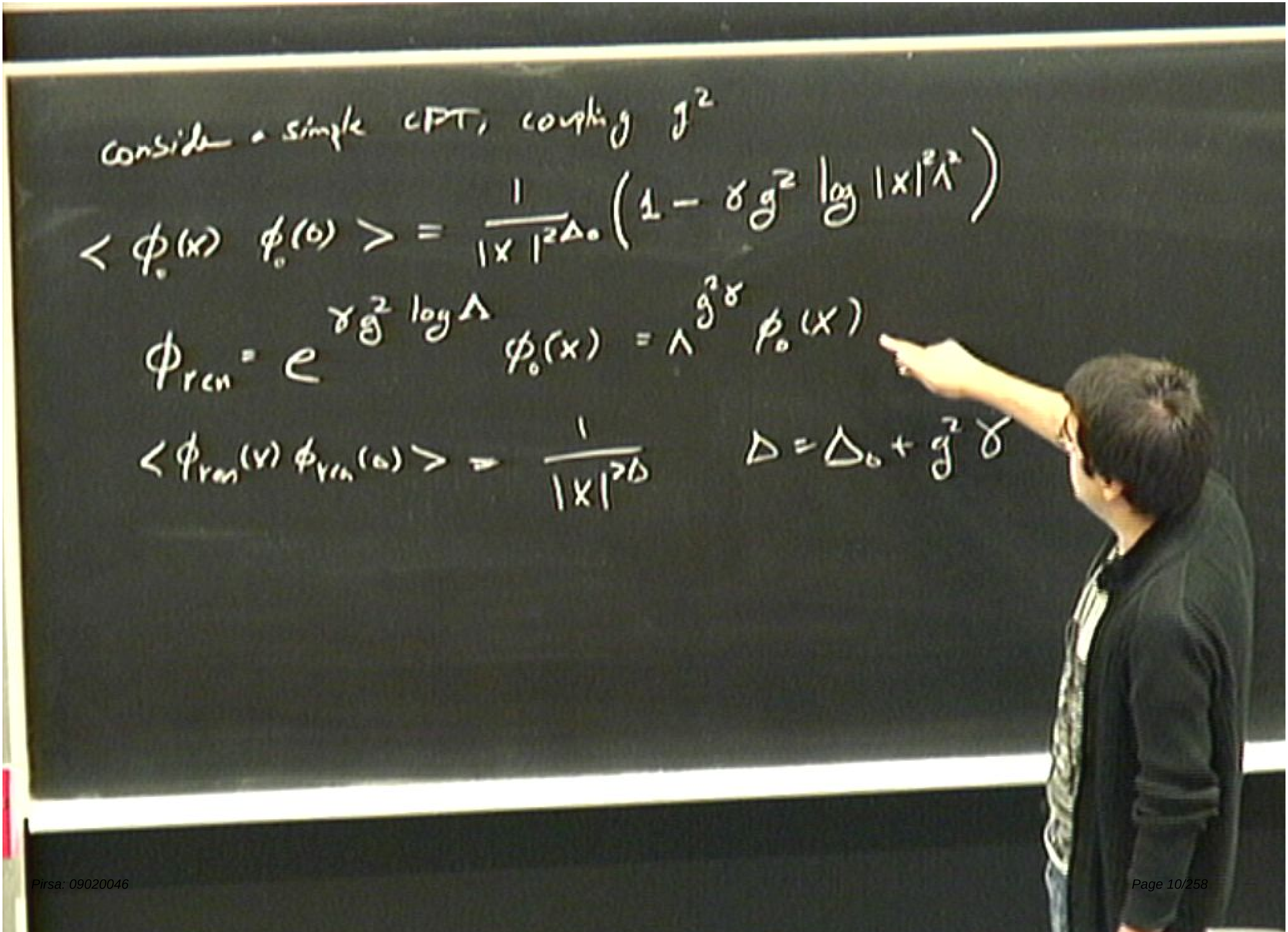
$$\langle \phi_0(x)\ \phi_0(0) \rangle = \frac{1}{|x|^{2\Delta_0}}\left(1 - \gamma g^2 \log |x|^2 \Lambda^2\right)$$

$$\phi_{ren} = e^{\gamma g^2 \log \Lambda}\ \phi_0(x)$$

$$\langle \phi_{ren}(x)\, \phi_{ren}(0) \rangle = \frac{1}{|x|^{2\Delta}} \qquad \Delta = \Delta_0 + g^2 \gamma$$


consider a simple CFT, coupling g^2
$\langle \phi_0(x)\ \phi_0(0) \rangle = \frac{1}{|x|^{2\Delta_0}}\left(1 - \gamma g^2 \log |x|^2\Lambda^2\right)$
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$\mathcal{N} = 4$

$L =$

consider a simple CFT, coupling g^2

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$\mathcal{N} = 4$

$$\mathcal{L}_{YM} = \mathrm{Tr}\left(\frac{1}{2} F_{\mu\nu}^2 + (D_\mu \Phi_i)^2 + \frac{1}{2}[\Phi_i, \Phi_j]^2 + \bar{\Psi} \not{D} \psi + \ldots\right)$$

ghosts + gauge-fixing

consider a simple CFT, coupling g^2

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$$\frac{1}{g_{YM}^2} \mathrm{Tr}\left(\frac{1}{2} F_{\mu\nu}^2 + (D_\mu \Phi_i)^2 - \frac{1}{2}[\Phi_i, \Phi_j]^2 + \bar{\Psi} \not{D} \Psi + \text{ghosts + gauge fixing} \right)$$

consider a simple CFT, coupling g^2

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$i = 1, \dots, 6$

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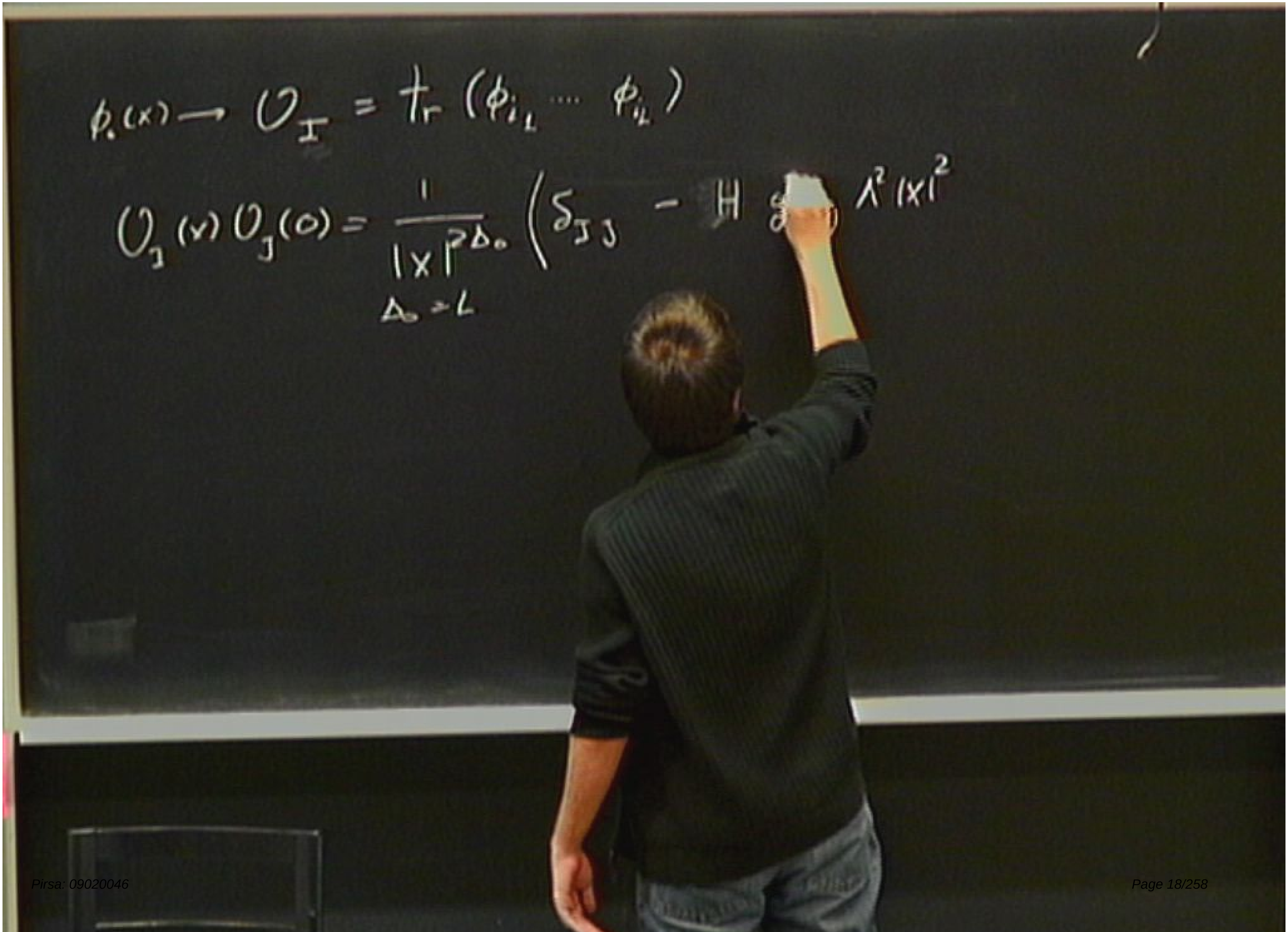
$$\phi_i(x) \longrightarrow \mathcal{O}_I = \mathrm{tr}\,(\phi_{i_1} \cdots \phi_{i_L})$$


$\phi_i(x) \longrightarrow \mathcal{O}_I = \mathrm{tr}(\phi_{i_1} \cdots \phi_{i_L})$
$\mathcal{O}_I(x)\,\mathcal{O}_J(0) = \frac{1}{|x|^{2\Delta_0}} \left(\delta_{IJ} - H \right.$ $\lambda^2 |x|^2$
$\Delta_0 = L$

$$\phi_a(x) \longrightarrow \mathcal{O}_I = \mathrm{tr}\,(\phi_{i_1} \cdots \phi_{i_L})$$

$$\mathcal{O}_I(x)\,\mathcal{O}_J(0) = \frac{1}{|x|^{2\Delta_0}}\left(\delta_{IJ} - H\, g^2 \log \Lambda^2 |x|^2\right)$$

$$\Delta_0 = L$$

$$g^2 = \frac{g_{YM}^2 N}{16\pi^2}$$

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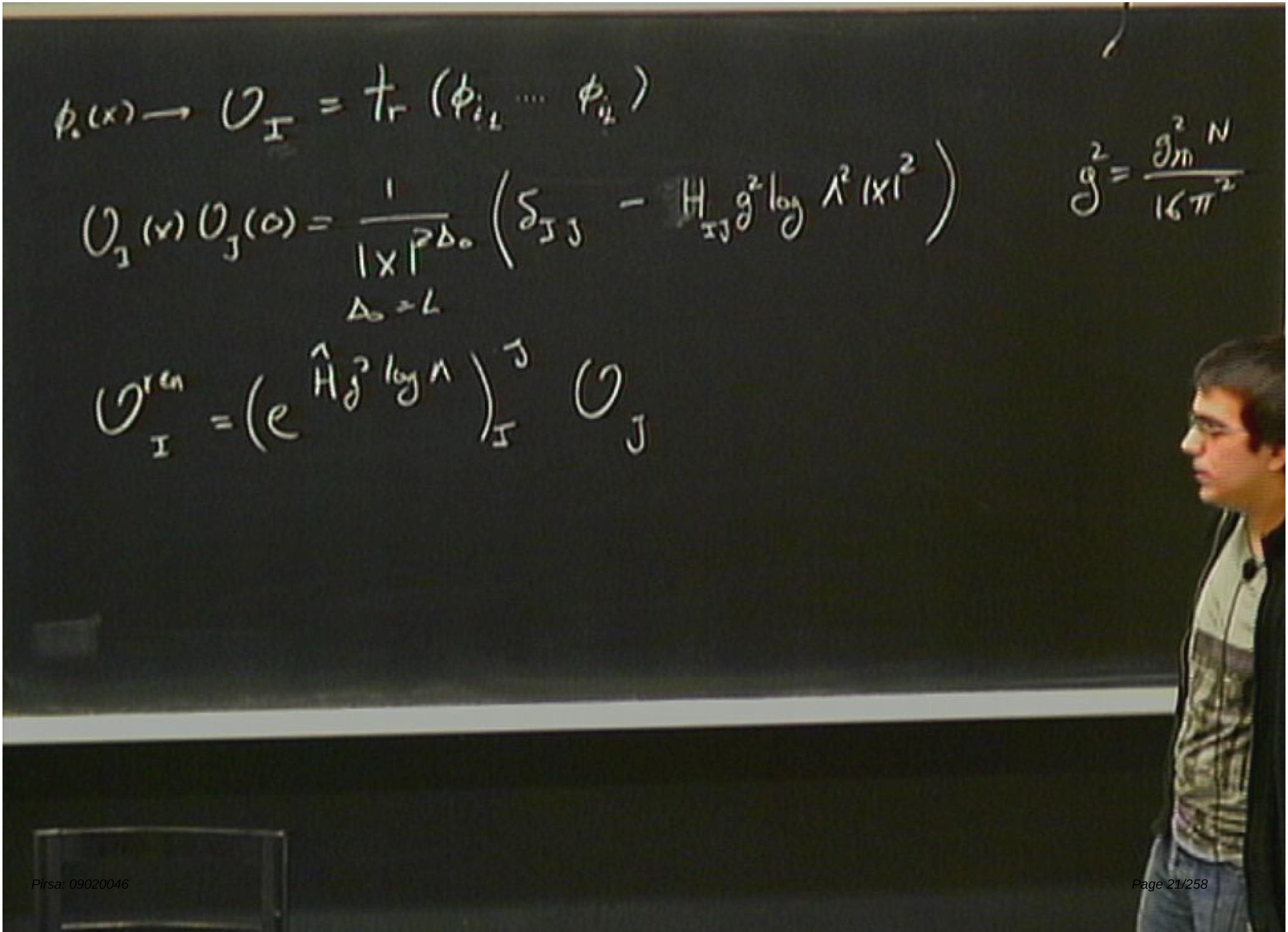
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$\Delta_0 = L$
$\mathcal{O}^{ren}_I = \left(e^{\hat{H} g^2 \log \Lambda}\right)_I^{\ J} \mathcal{O}_J$
$g^2 = \frac{g_m^2 N}{16\pi^2}$

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$\Delta_0 = L$

$\hat{H} \longrightarrow$ eigenvalues $\{\Delta\}$

eigenvectors

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$$\Psi_{i_1 \cdots i_L}\, \mathrm{tr}(\phi_{i_1} \cdots \phi_{i_L})$$

$$\phi_a(x) \longrightarrow \mathcal{O}_I = \mathrm{tr}\,(\phi_{i_1} \cdots \phi_{i_L})$$

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$\Delta_0 = L$

$$\mathcal{O}_I^{ren} = \left(e^{\hat{H} g^2 \log \Lambda}\right)_I^{\ J} \mathcal{O}_J$$

$\hat{H} \longrightarrow$ eigenvalues $\{\delta\Delta\}$

eigenvectors $\delta\Delta = \gamma g^2$

$$\Psi_{i_1 \cdots i_L}\, \mathrm{tr}(\phi_{i_1} \cdots \phi_{i_L}) = \mathcal{O}_\Delta$$

$$\phi_a(x) \longrightarrow \mathcal{O}_I = \mathrm{tr}\,(\phi_{i_1} \cdots \phi_{i_L})$$

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$$\hat{H} \longrightarrow \text{eigenvalues } \{\delta\Delta\}$$

$$\text{eigenvectors} \quad \delta\Delta = \gamma g^2$$

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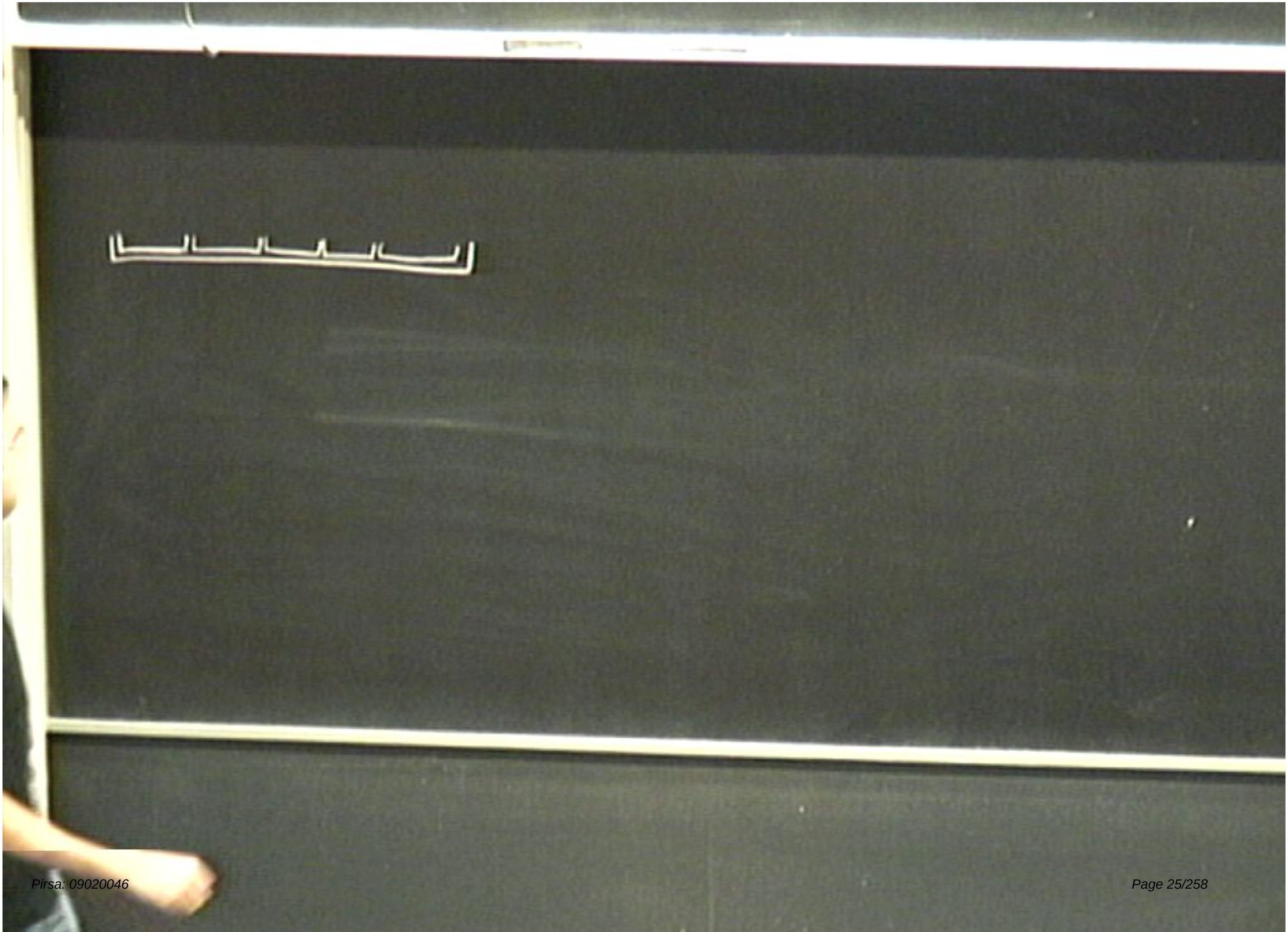
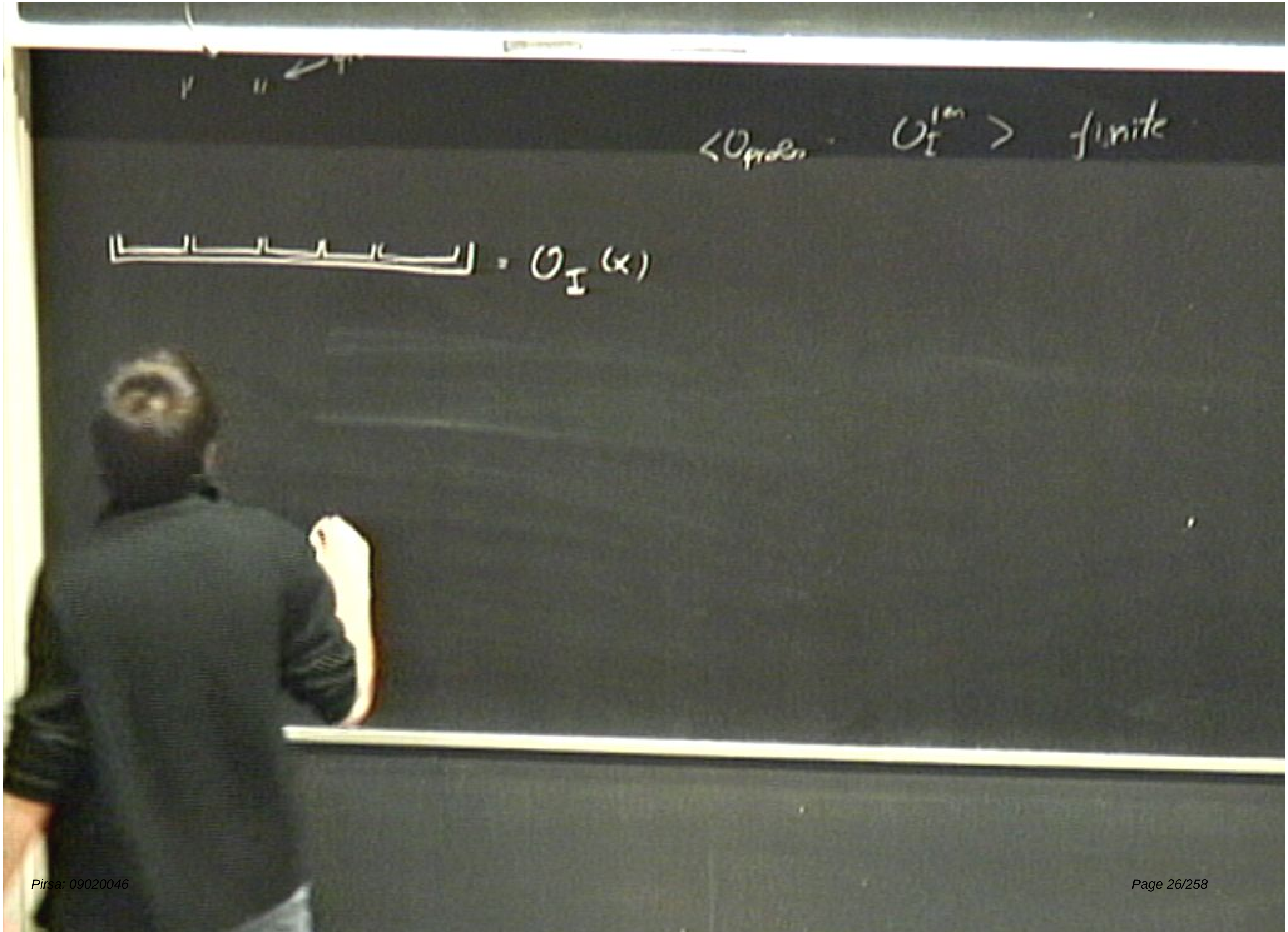
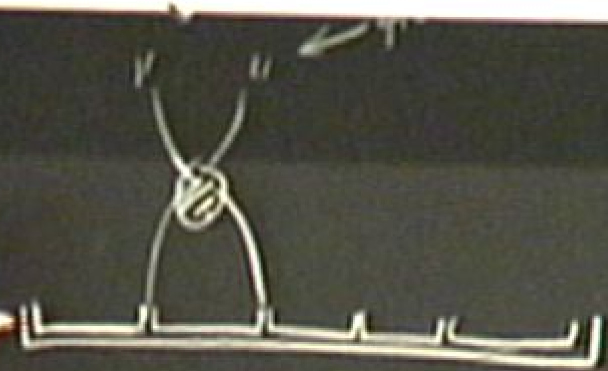



finite
= O_I (x)

$\langle O_{probe} \cdot O_I^{int} \rangle$ finite

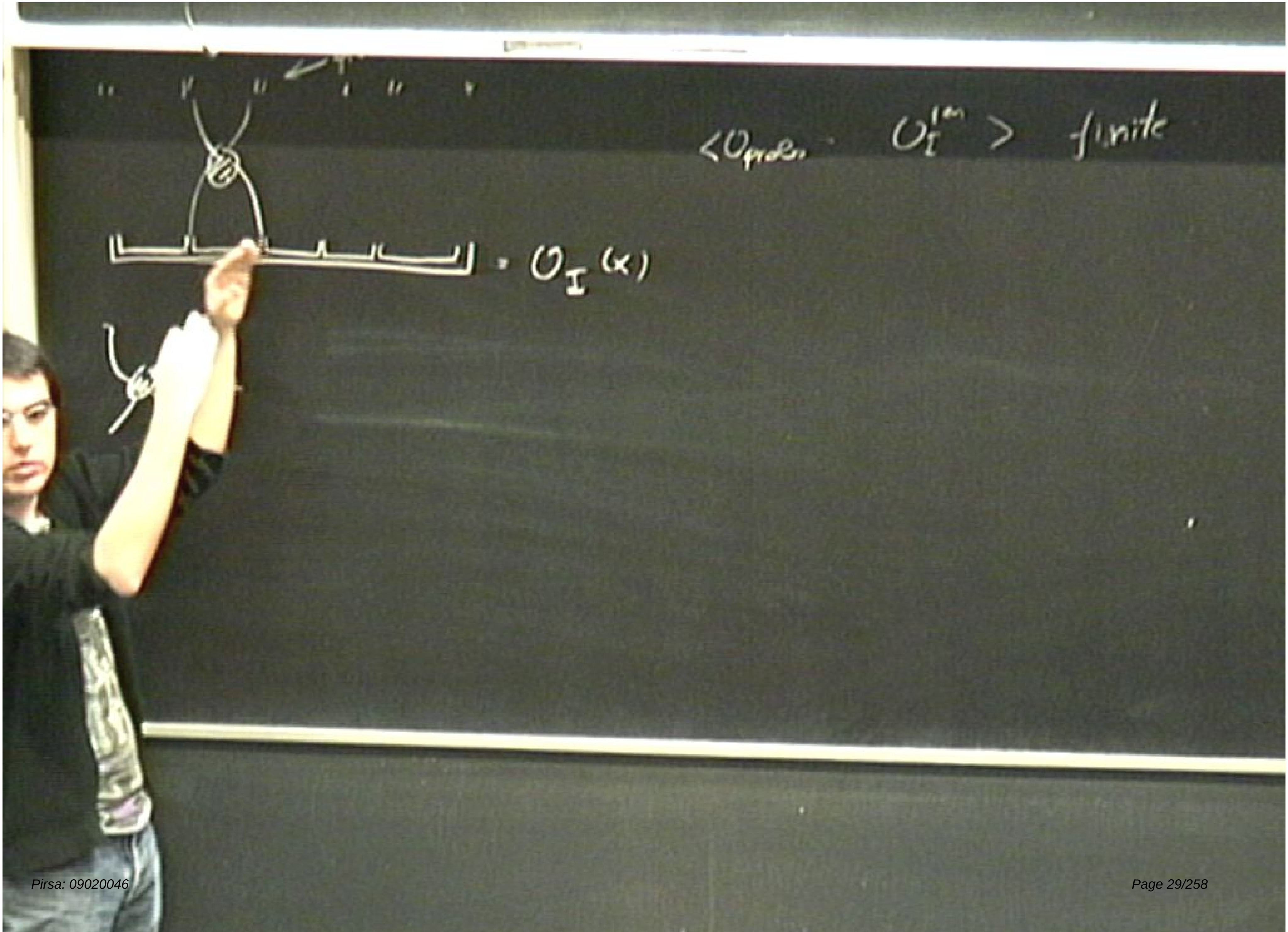
$\sqcup\!\sqcup\!\sqcup\!\sqcup\!\sqcup = O_I(x)$

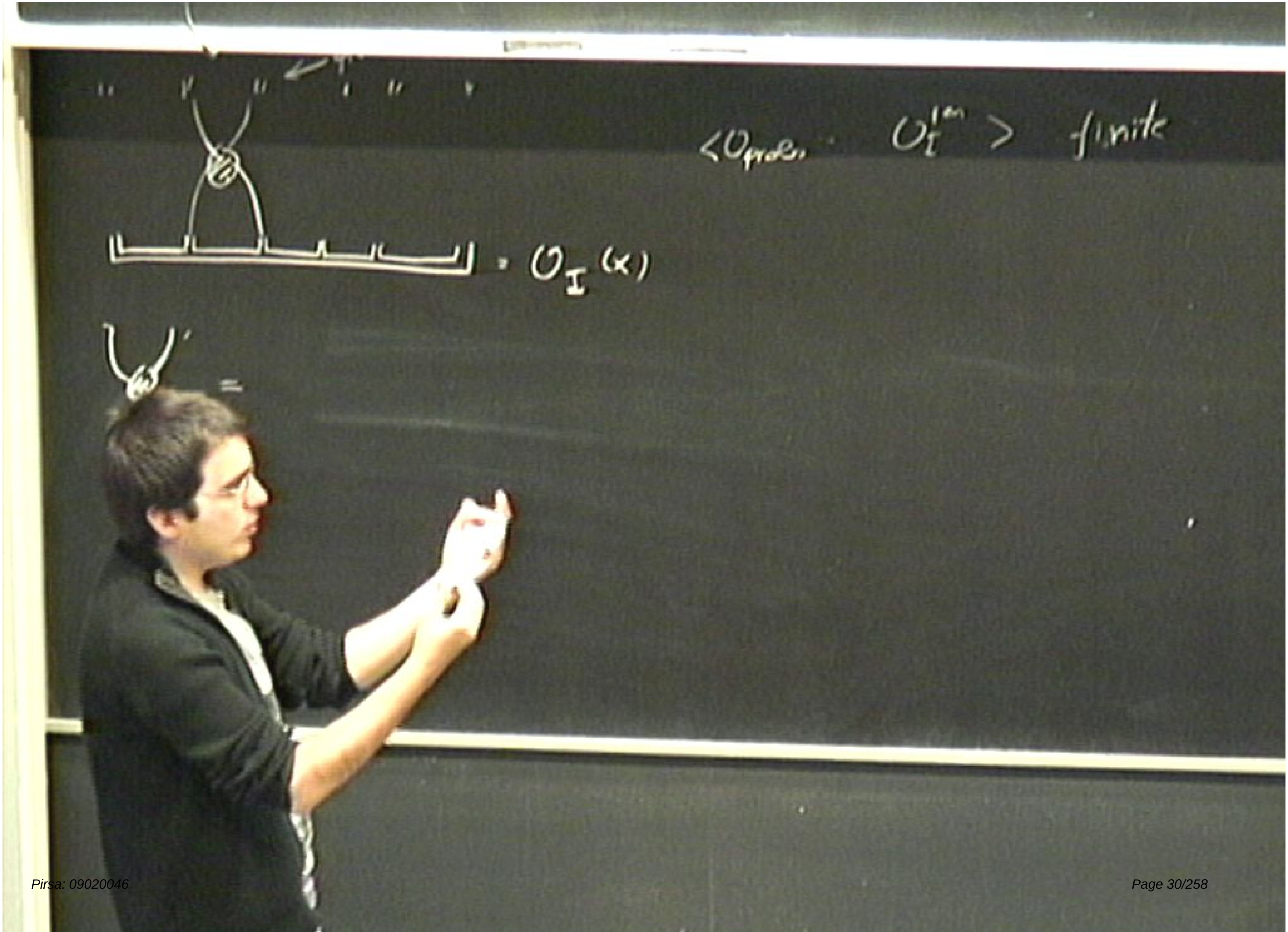
$\langle O_{probe} \cdot O_I^{1m} \rangle$ finite

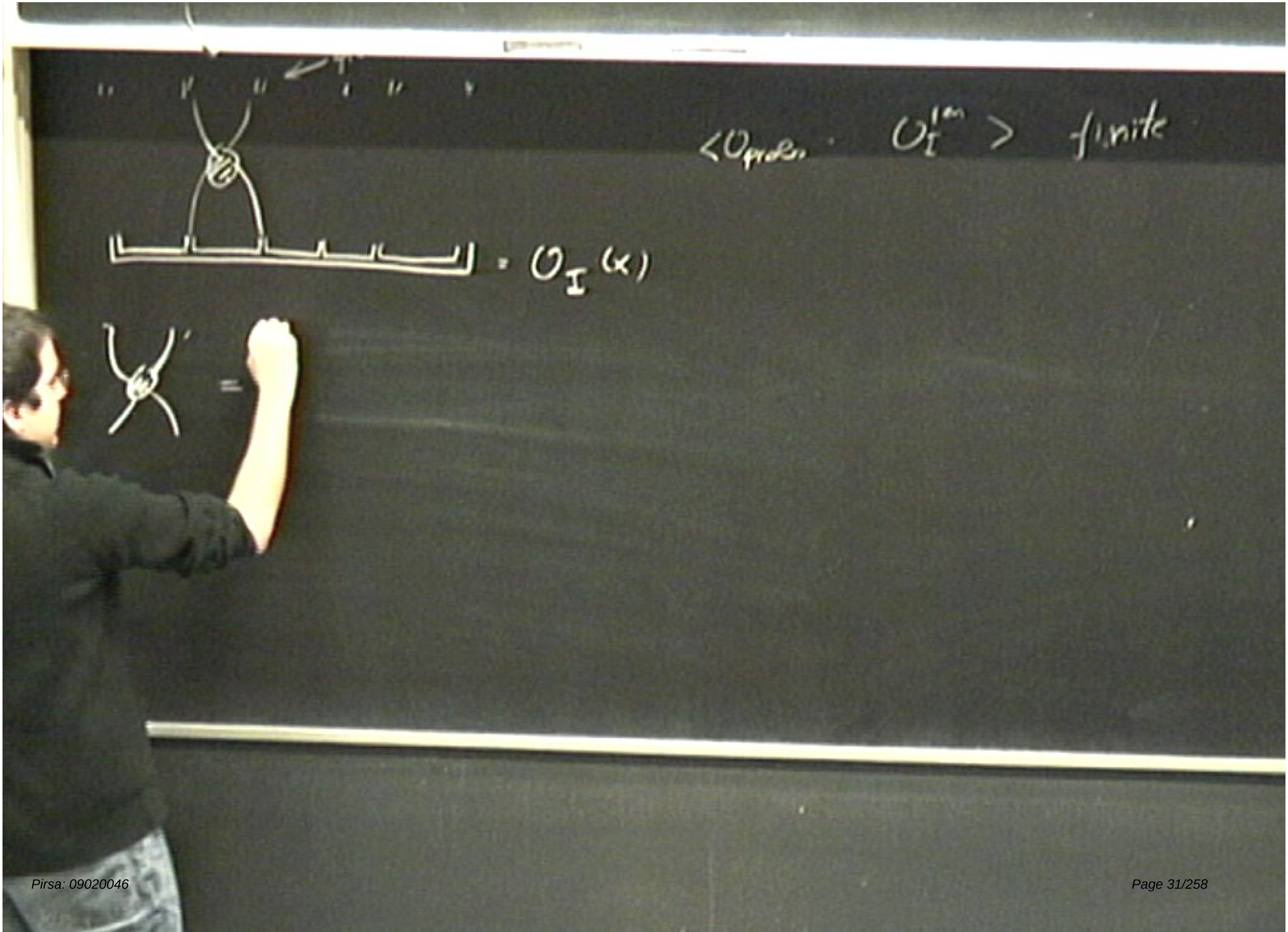


$= O_I(x)$

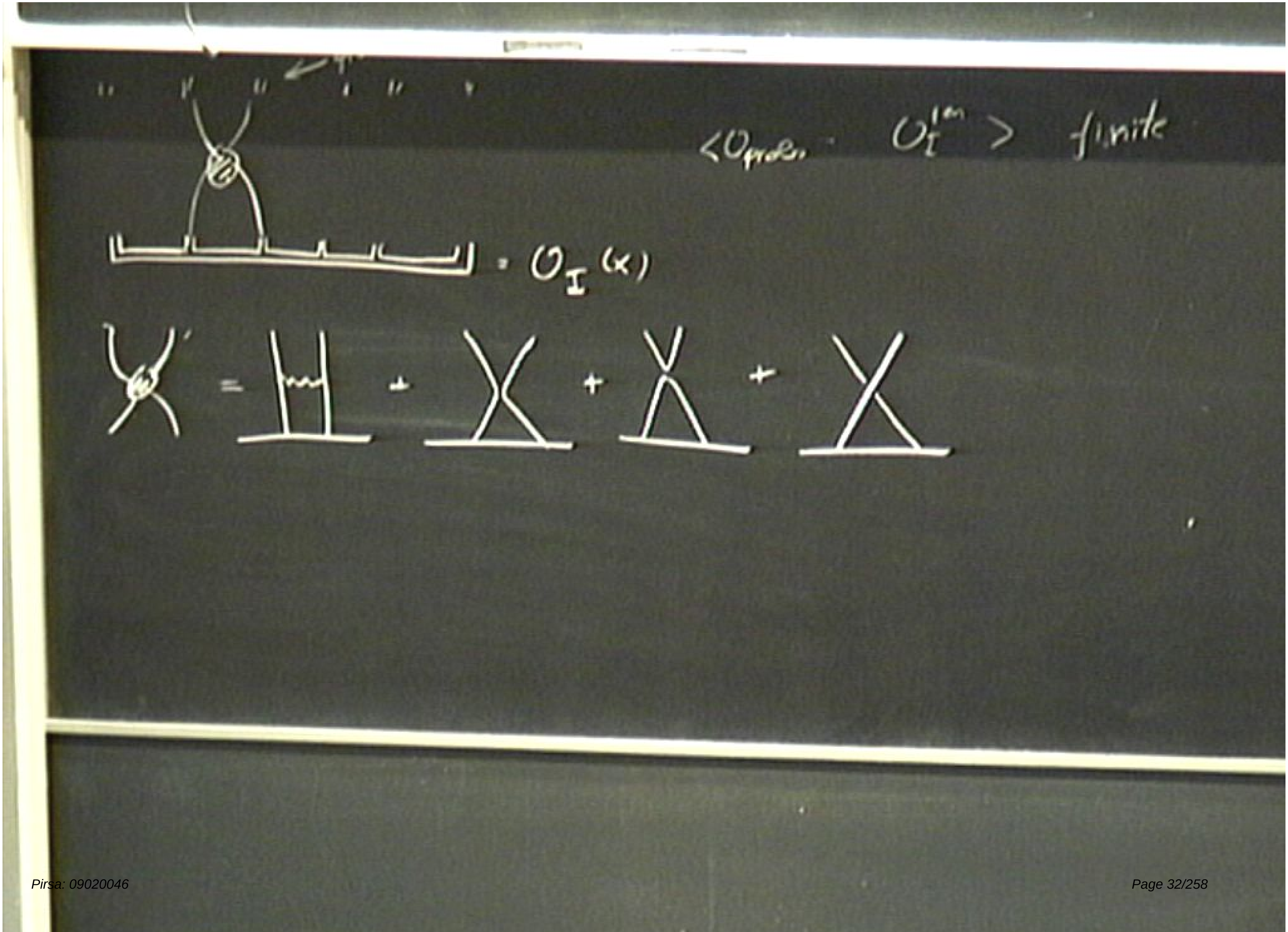
$=$

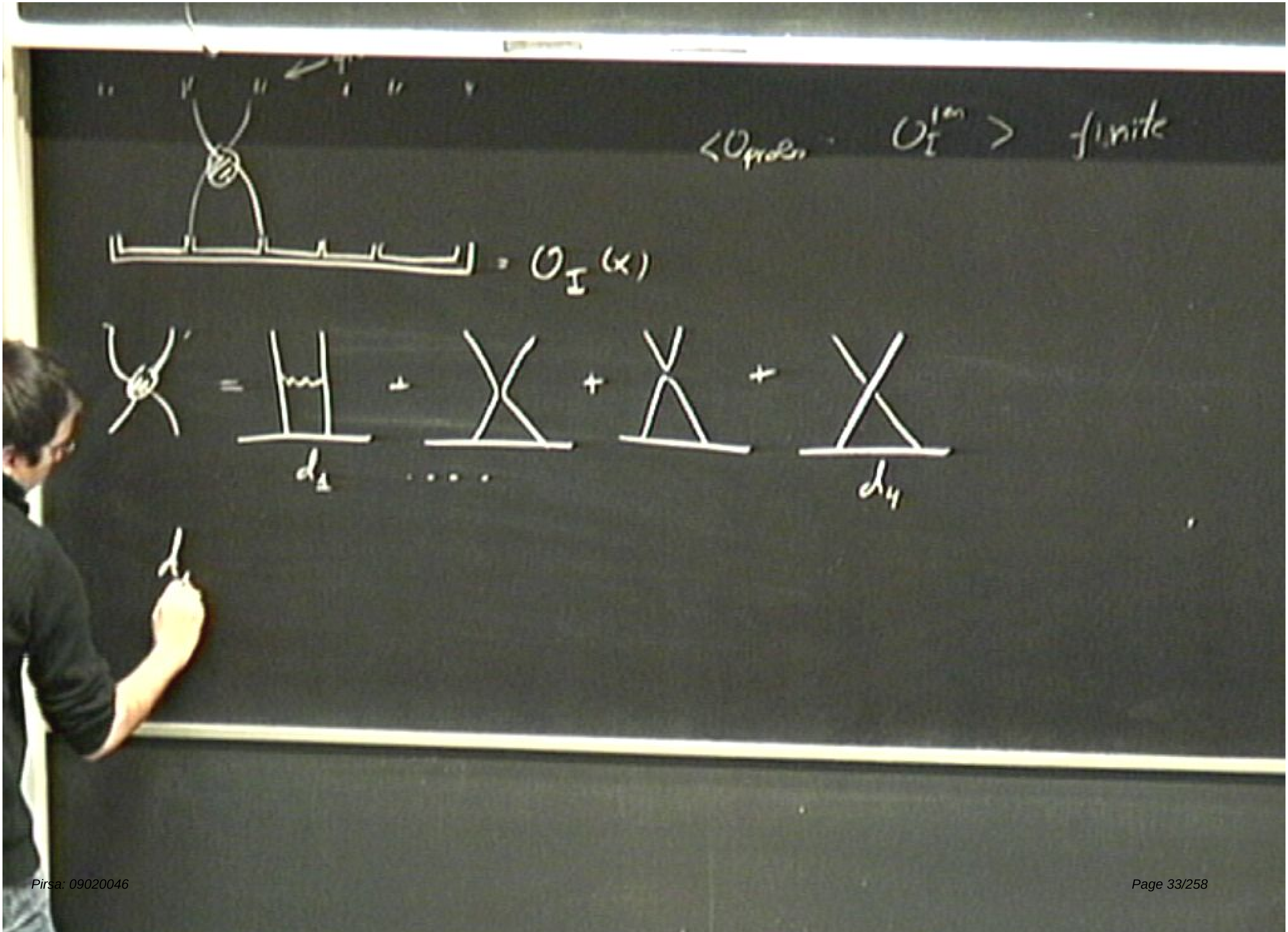

finite
= O_I (x)

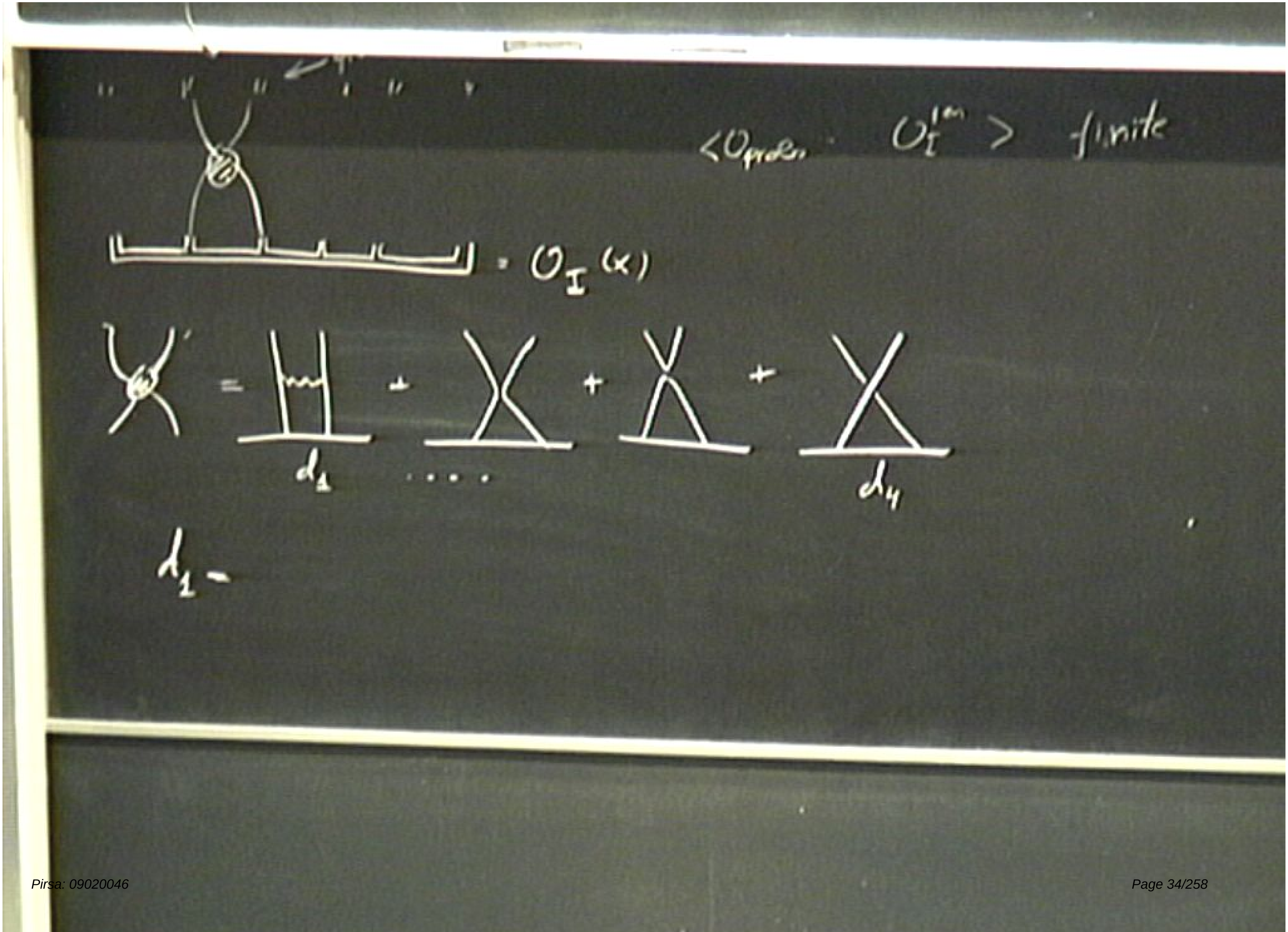

<O_probe · O_I > finite
= O_I (x)



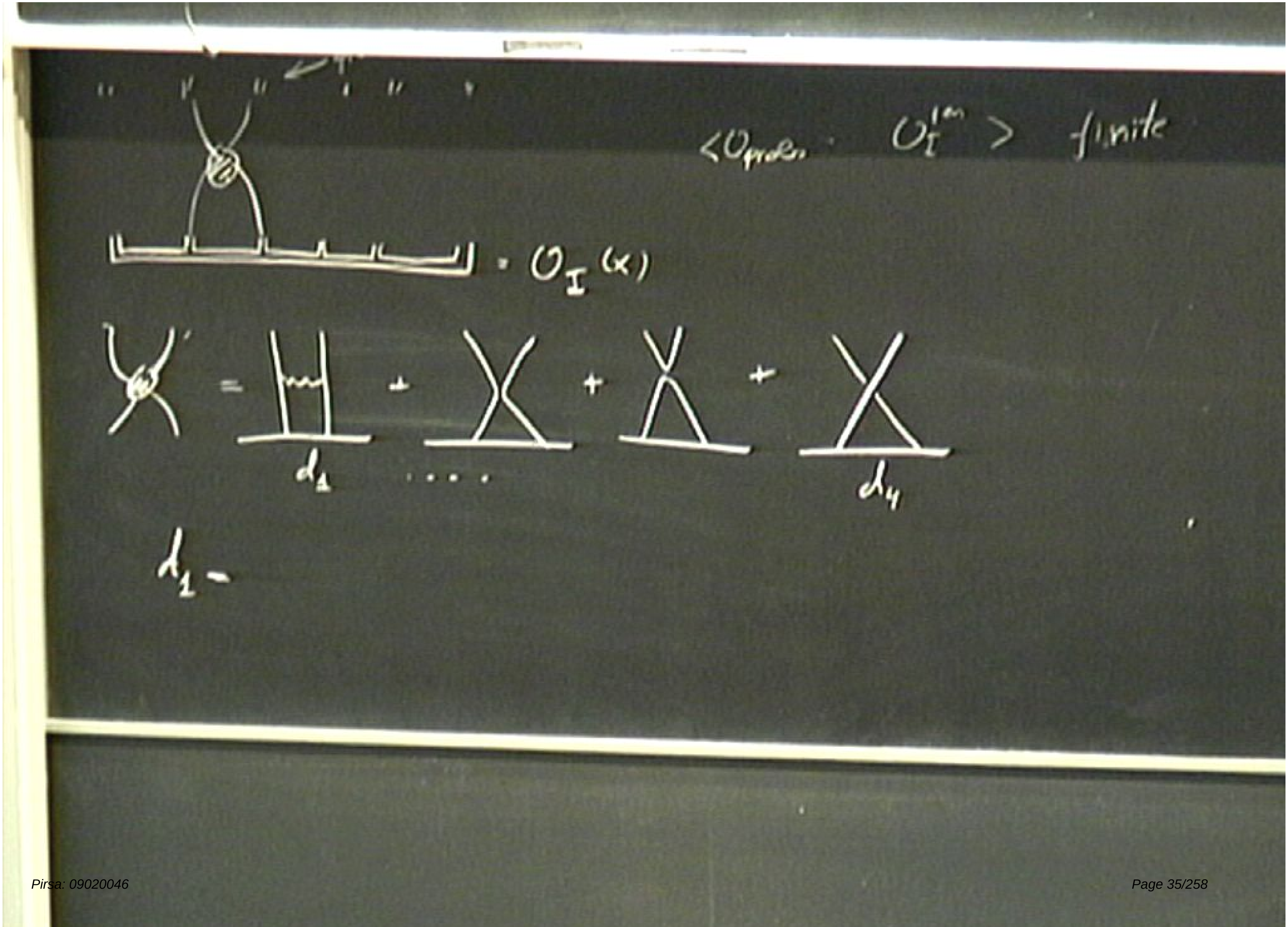
<Oproton · O_I^{1m} > finite
= O_I(x)
=

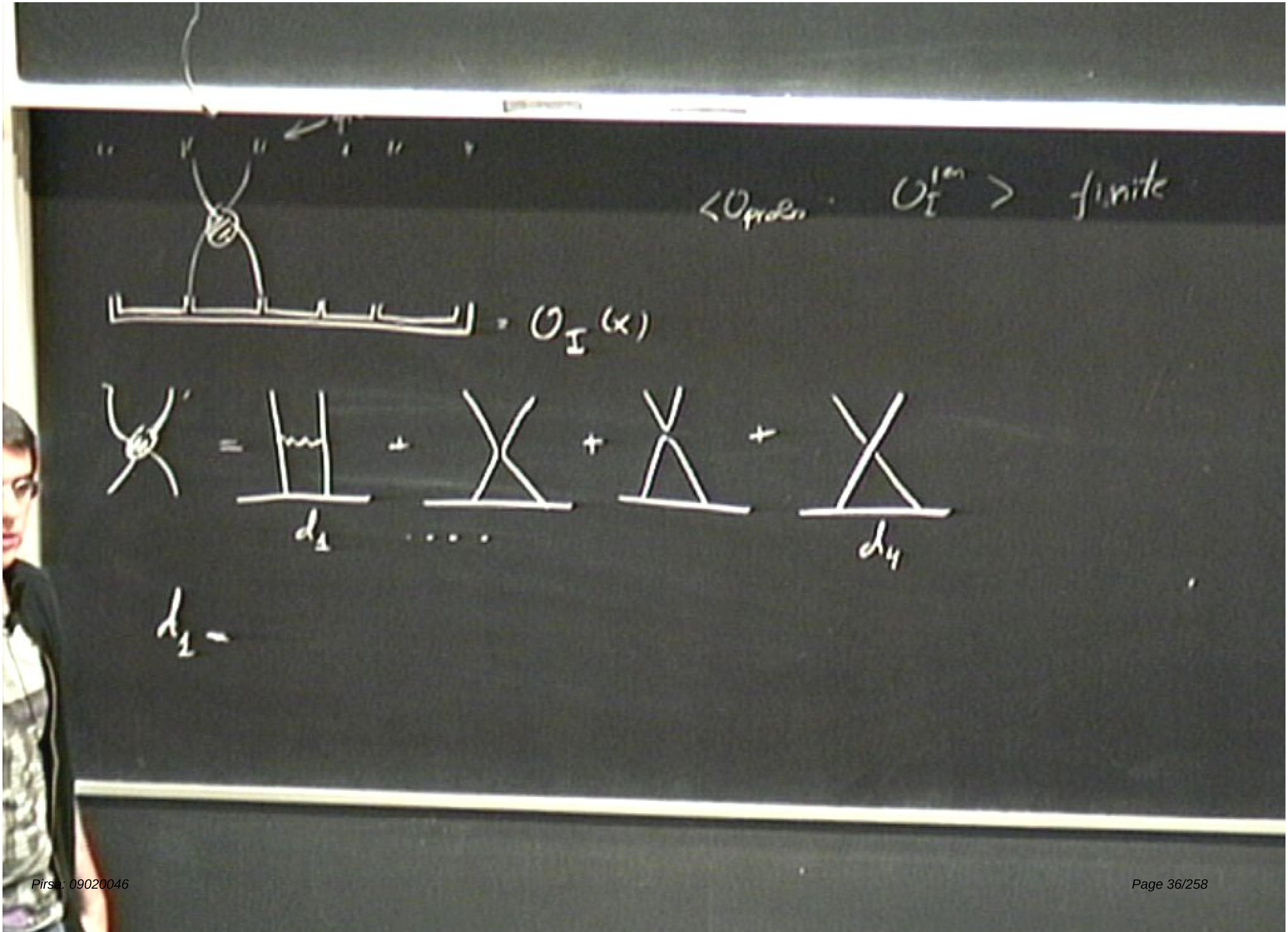

finite
= $O_I(x)$


finite
= O_I(x)
d_1
d_4

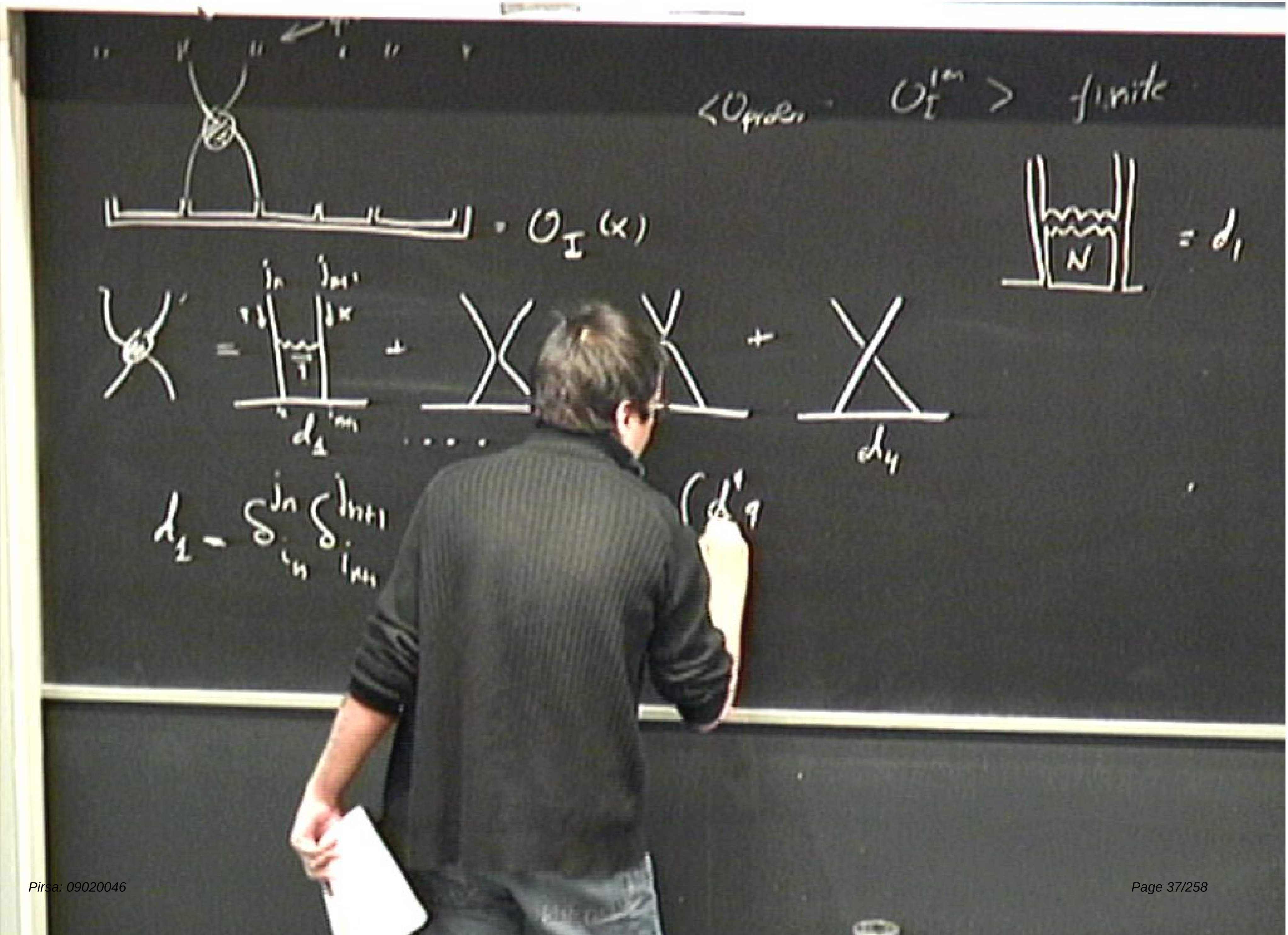


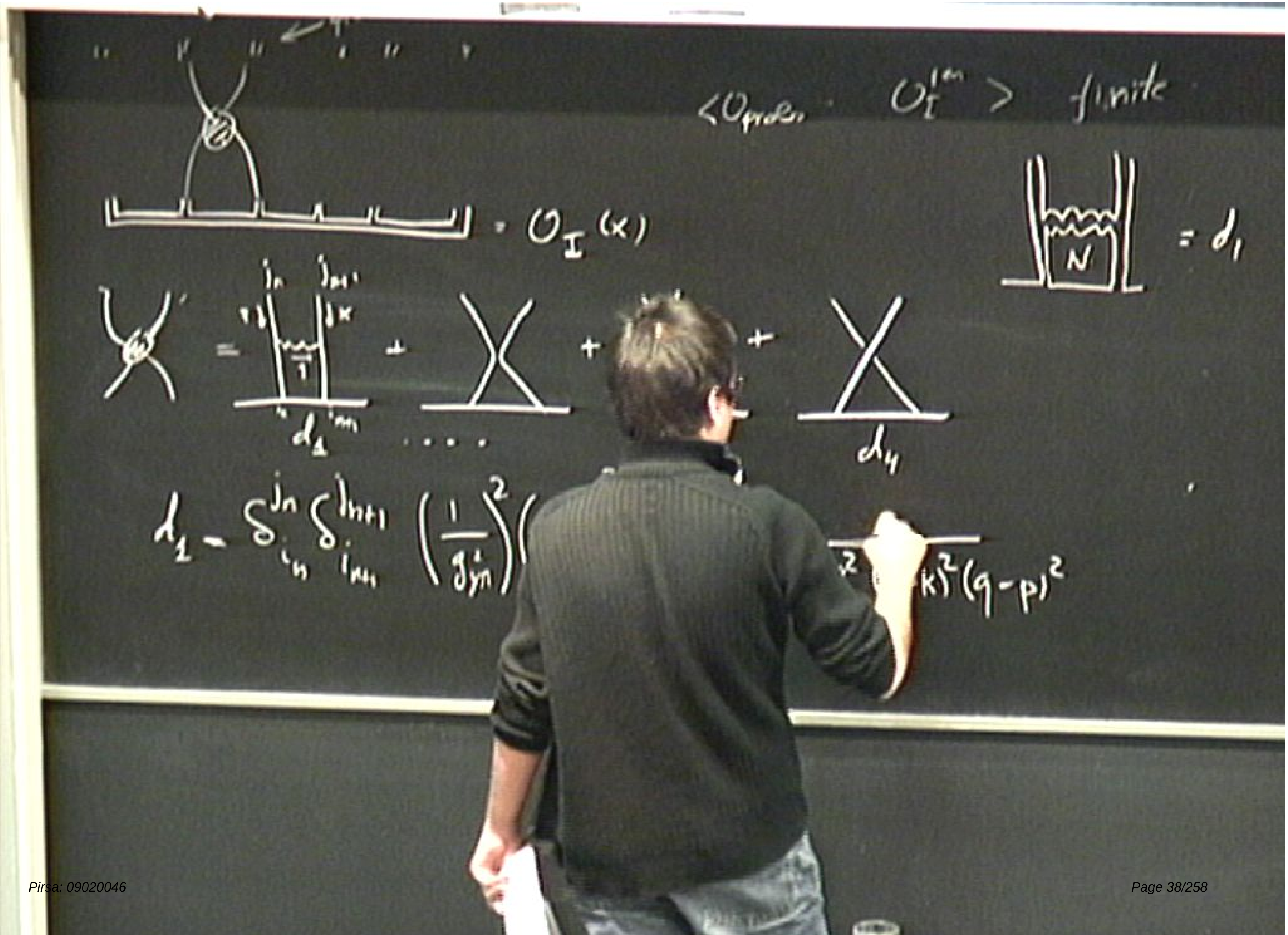
<O_prob · O_I^im > finite
= O_I (x)
λ_1
λ_4
λ_1 =


⟨O_proton · O_I⟩ finite
= O_I(x)
=
+
+
+
d_1
d_4
d_1 =

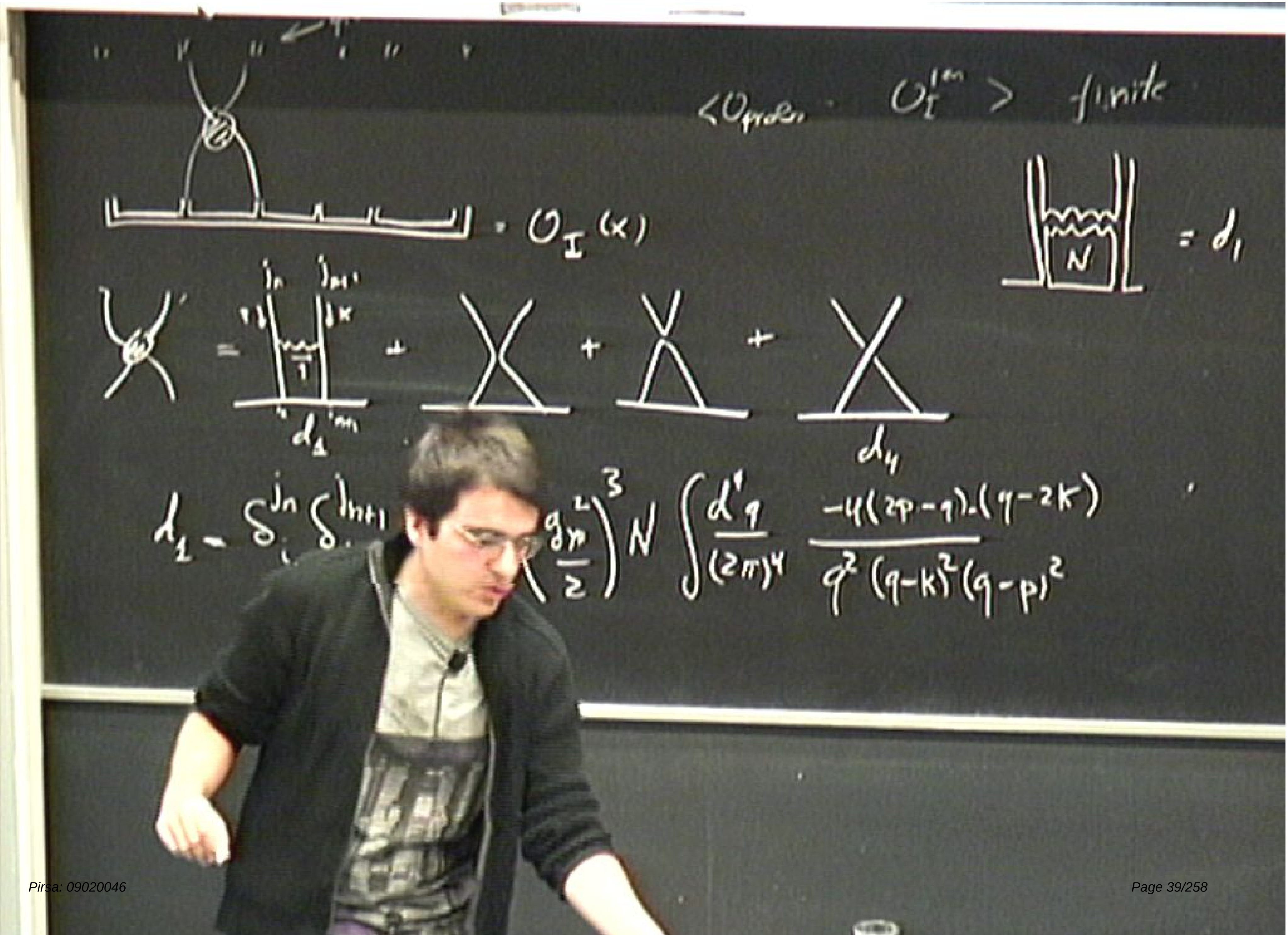


⟨O_probe · O_I^†m⟩ finite
= O_I(x)
λ_1
λ_4
λ_1 =

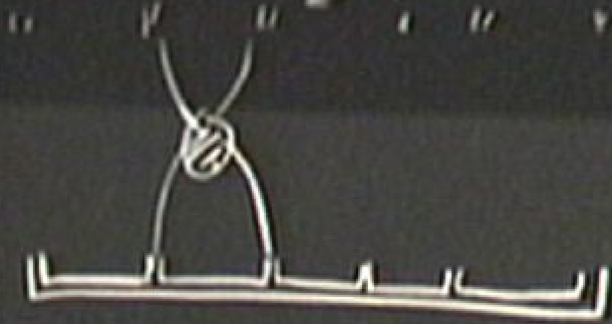

finite
N
= d_1
d_4



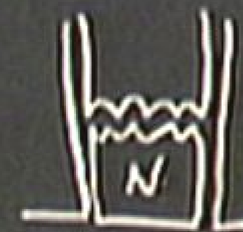
finite
= d1
= O_I (x)
d4


$\langle O_{probe} \cdot O_I^{lm} \rangle$ finite
$= O_I(x)$
$= d_1$
N
d_1^{lm}
d_4
$d_1 = \delta_i^{j_n} \delta^{j_{n+1}} \left(\frac{g_{YM}^2}{2}\right)^3 N \int \frac{d^4 q}{(2\pi)^4} \frac{-4(2p-q)\cdot(q-2k)}{q^2 (q-k)^2 (q-p)^2}$

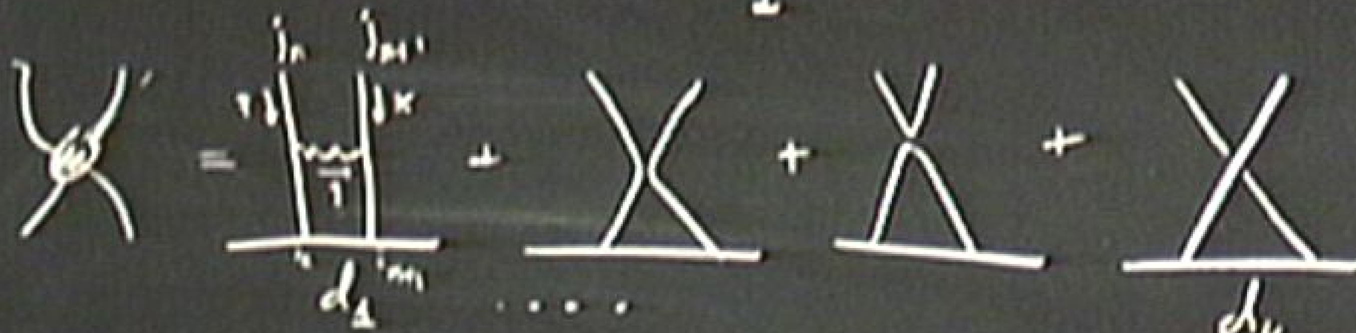
$\langle O_{probe}\; O_I^{\,im} \rangle$ finite



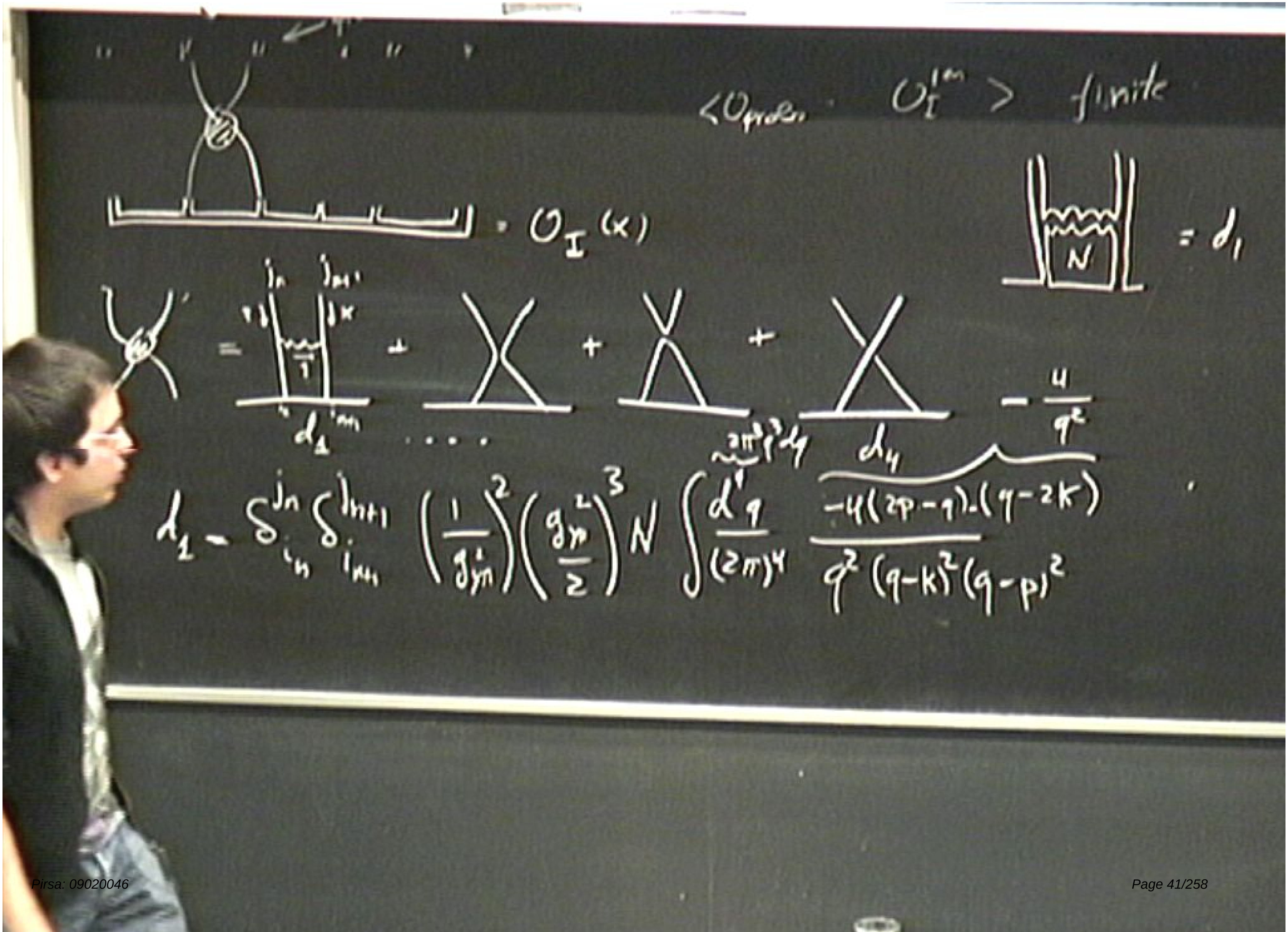
$= O_I(x)$



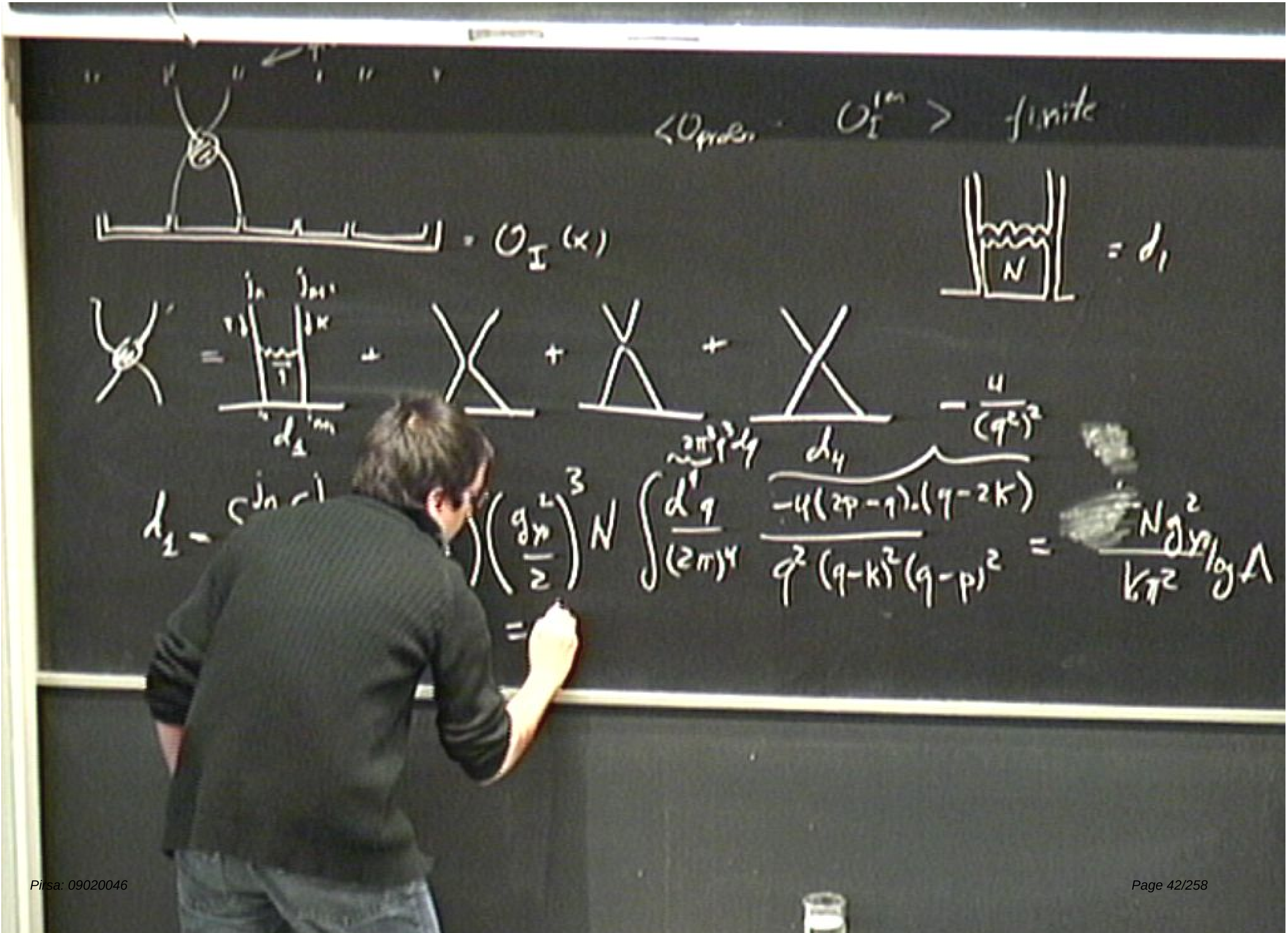
$= d_1$

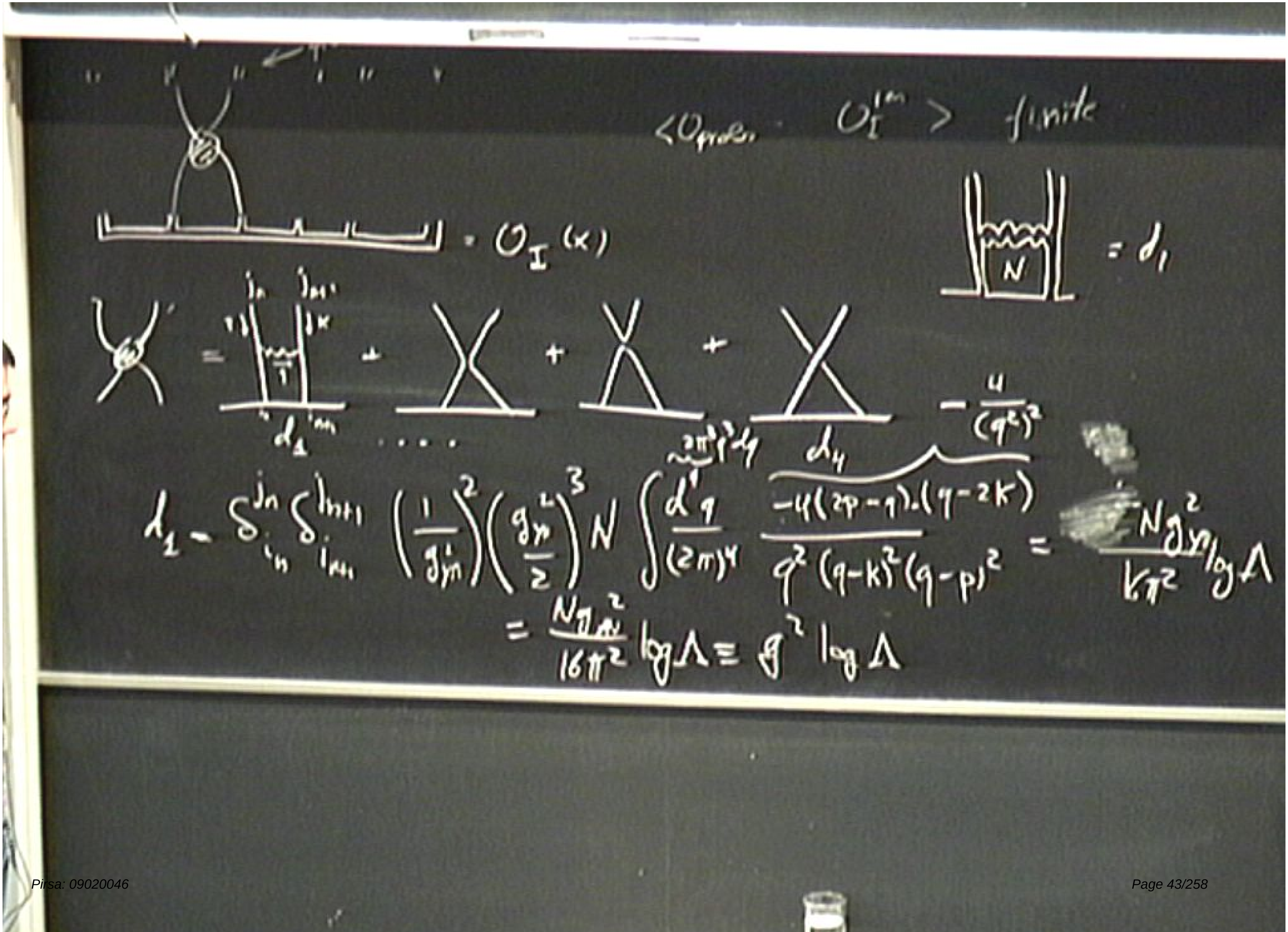


$$d_1 = \delta^{j_n}_{i_n}\,\delta^{j_{n+1}}_{i_{n+1}} \left(\frac{1}{g_{ym}^2}\right)^2 \left(\frac{g_{ym}^2}{2}\right)^3 N \int \frac{d^4 q}{(2\pi)^4}\, \frac{-4(2p-q)\cdot(q-2k)}{q^2 (q-k)^2 (q-p)^2}$$

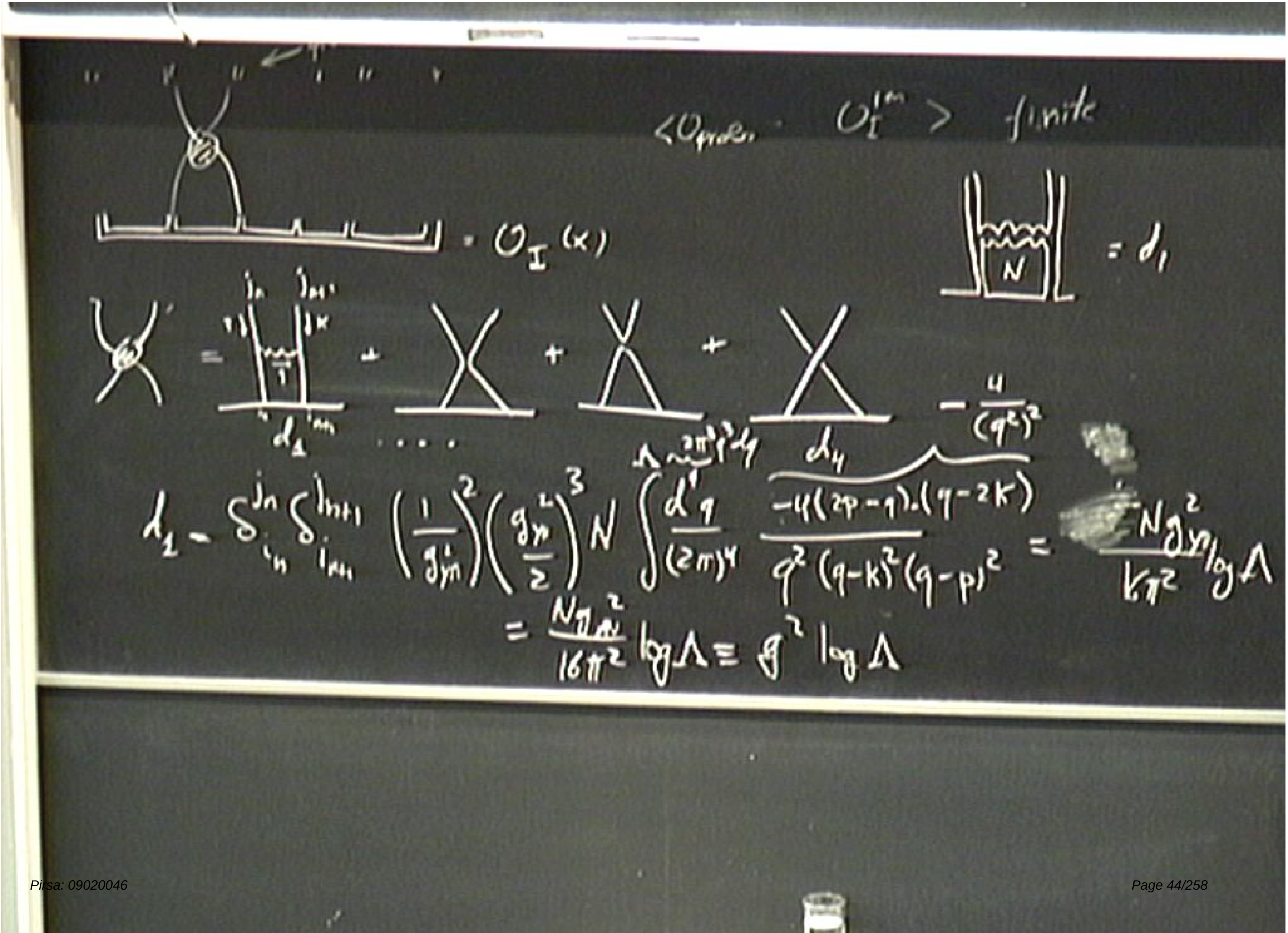


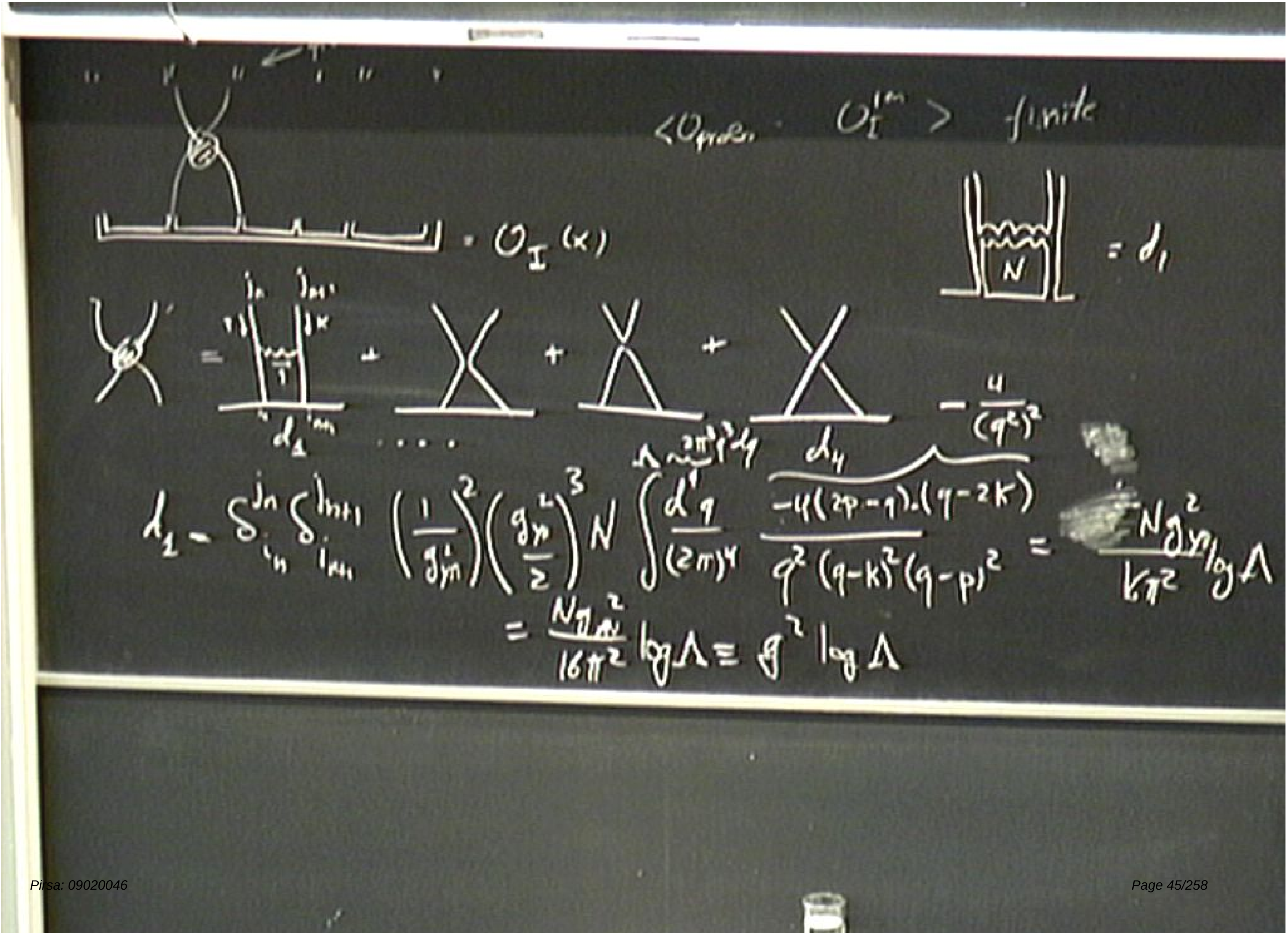
finite
= O_I (x)
= d_1
N
d_4


finite
= O_I (x)
= d_1
N
d_1
d_4

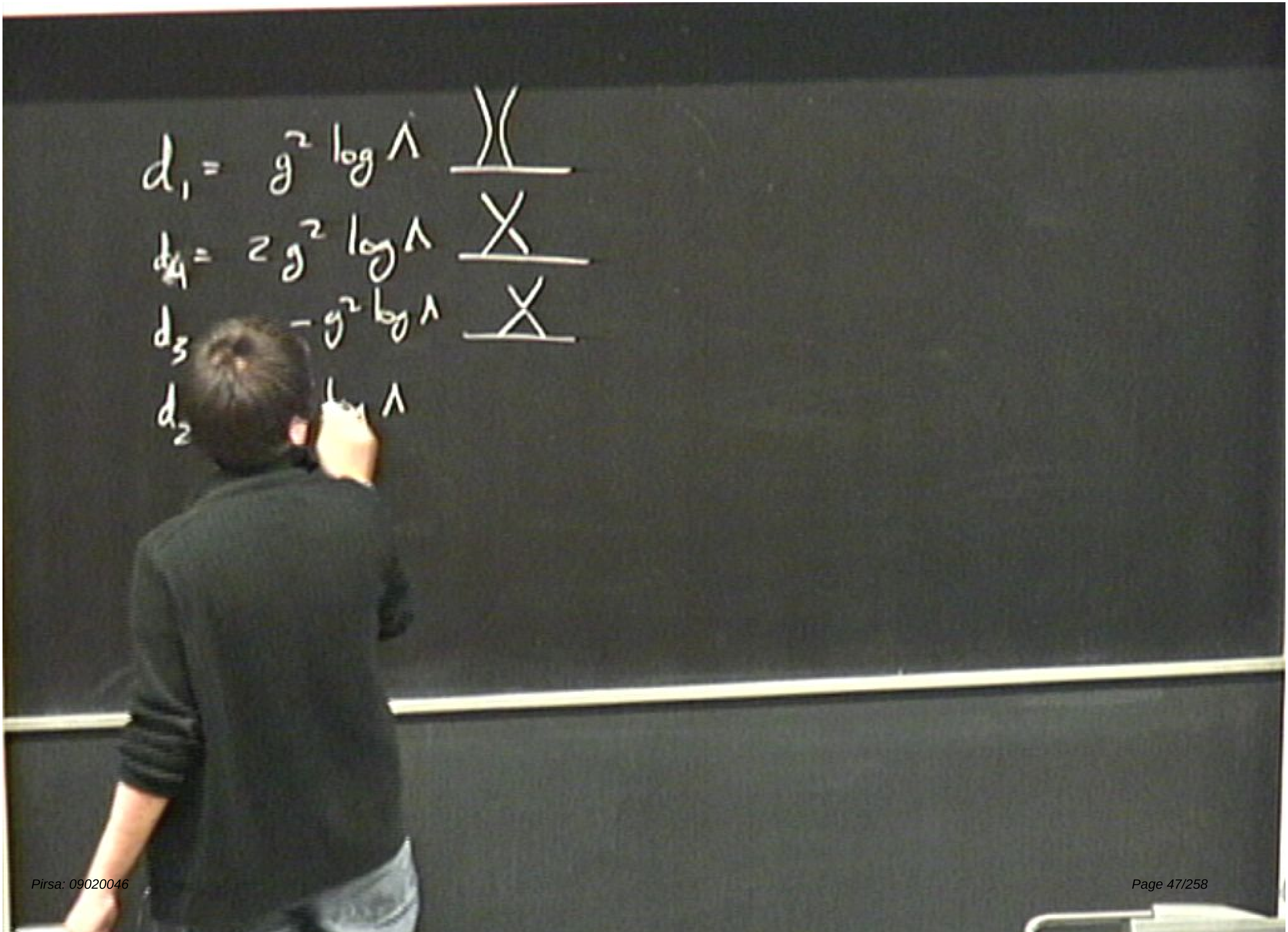


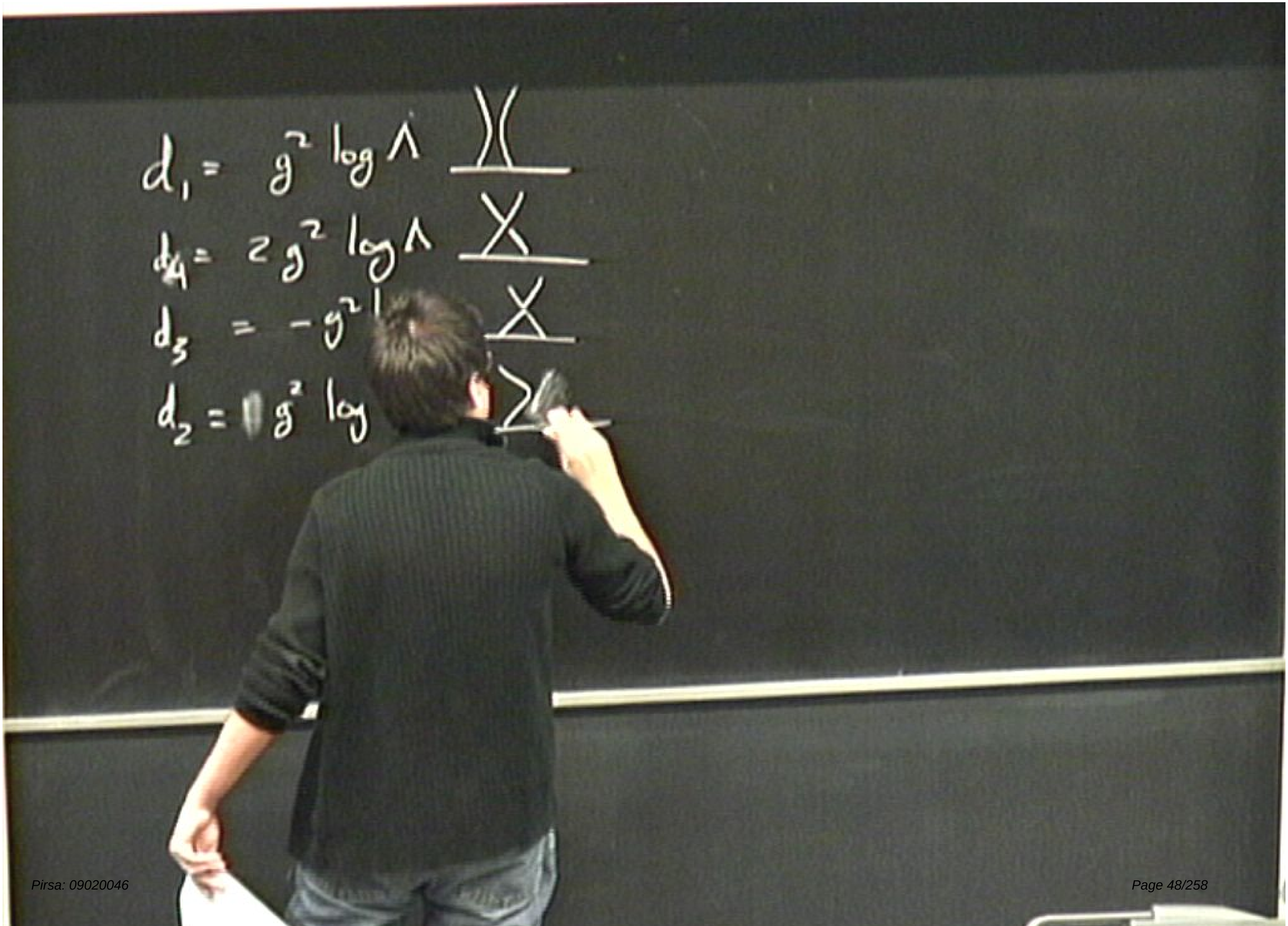
finite
= d_1
= O_I (x)
= N g_YM^2 / 16π^2 log Λ ≡ g^2 log Λ

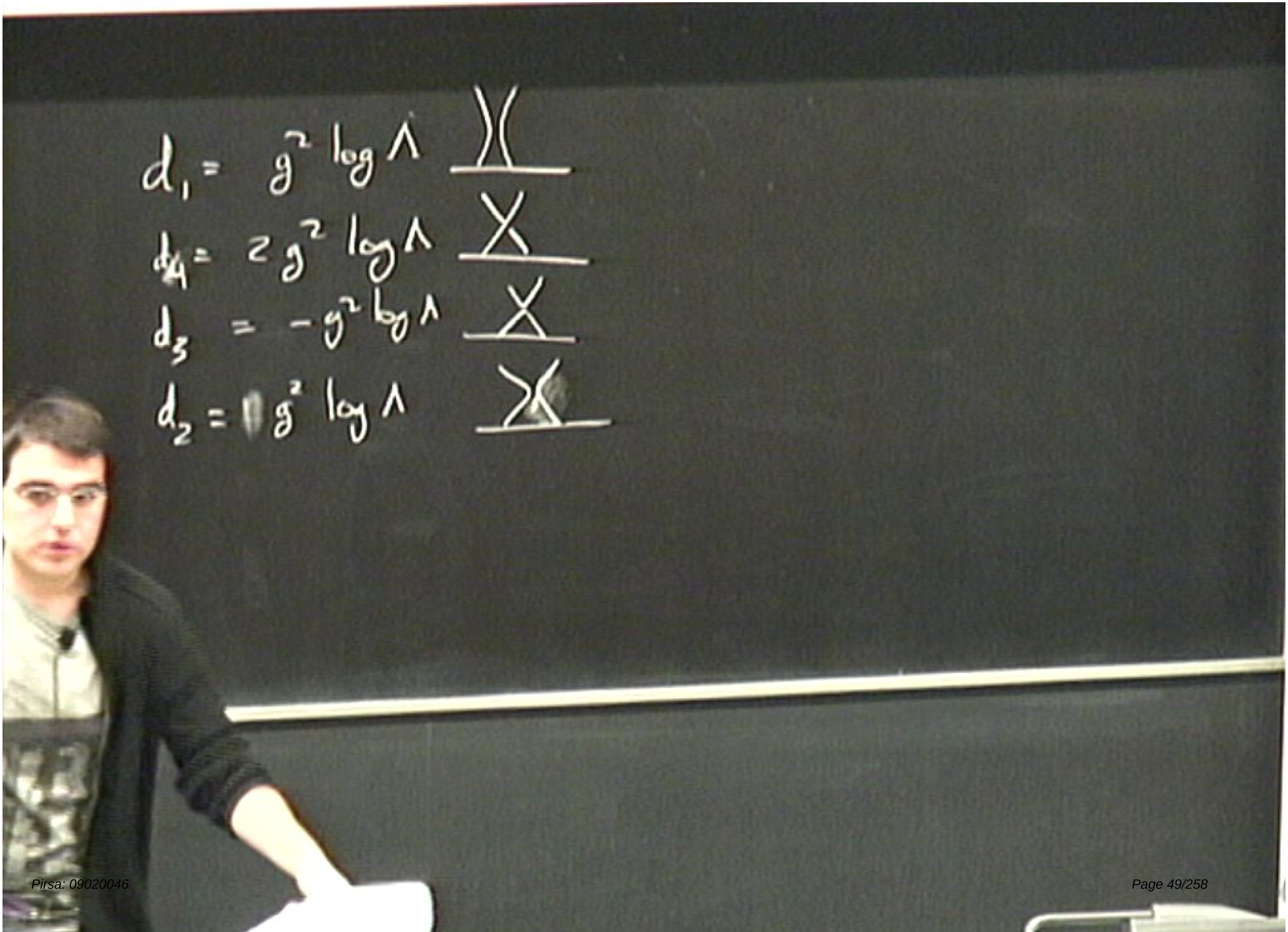

⟨O_prob · O_I^{im}⟩ finite
= O_I(x)
= d_1
N
d_1
d_4
= N g^2/16π^2 log Λ ≡ g^2 log Λ


<O_prob · O_I^{lm} > finite
= O_I(x)
= d_1
d_1
d_4
d_1 = δ^{in}_{in} δ^{in+1}_{in+1} (1/g_ym^2)^2 (g_ym^2/2)^3 N ∫ d^4q/(2π)^4 · −4(2p−q)·(q−2K) / q^2(q−k)^2(q−p)^2 = N g_ym^2/16π^2 log Λ
= N g_ym^2/16π^2 log Λ ≡ g^2 log Λ

$$d_1 = g^2 \log \Lambda$$ 


$d_1 = g^2 \log \Lambda$
$d_4 = 2 g^2 \log \Lambda$
d_3
$-g^2 \log \Lambda$
d_2
Λ


d1 = g2 log Λ
d4 = 2 g2 log Λ
d3 = − g2 l
d2 = g2 log


$d_1 = g^2 \log \Lambda$
$d_4 = 2 g^2 \log \Lambda$
$d_3 = -g^2 \log \Lambda$
$d_2 = g^2 \log \Lambda$

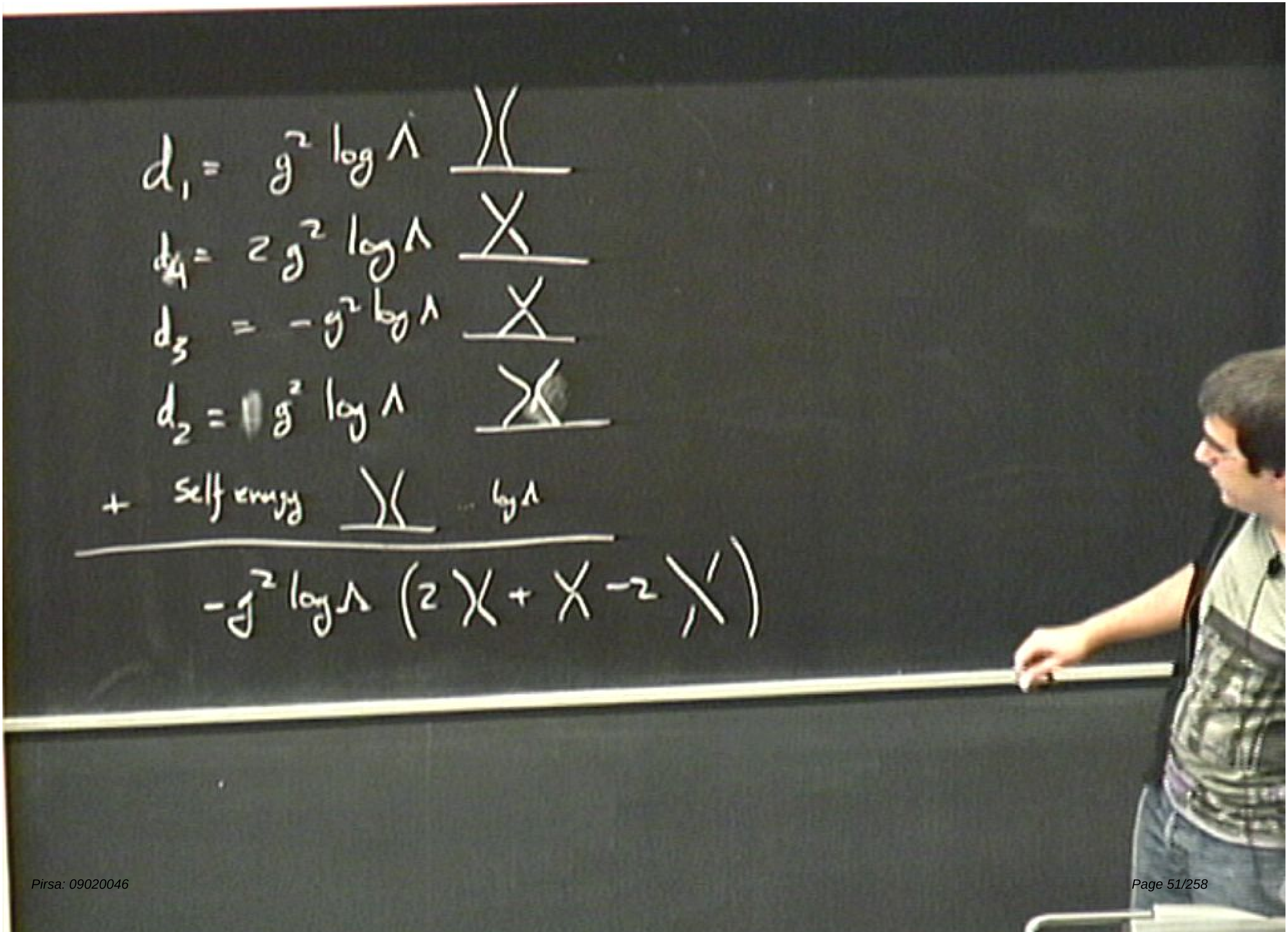
$d_1 = g^2 \log \Lambda$ 

$d_4 = 2 g^2 \log \Lambda$ 

$d_3 = - g^2 \log \Lambda$ 

$d_2 = g^2 \log \Lambda$ 

$+$ Self energy  $\sim \log \Lambda$


d1 = g2 log Λ
d4 = 2 g2 log Λ
d3 = − g2 log Λ
d2 = g2 log Λ
+ Self energy
log Λ
− g2 log Λ (2
+
− 2

$d_1 = g^2 \log \Lambda$

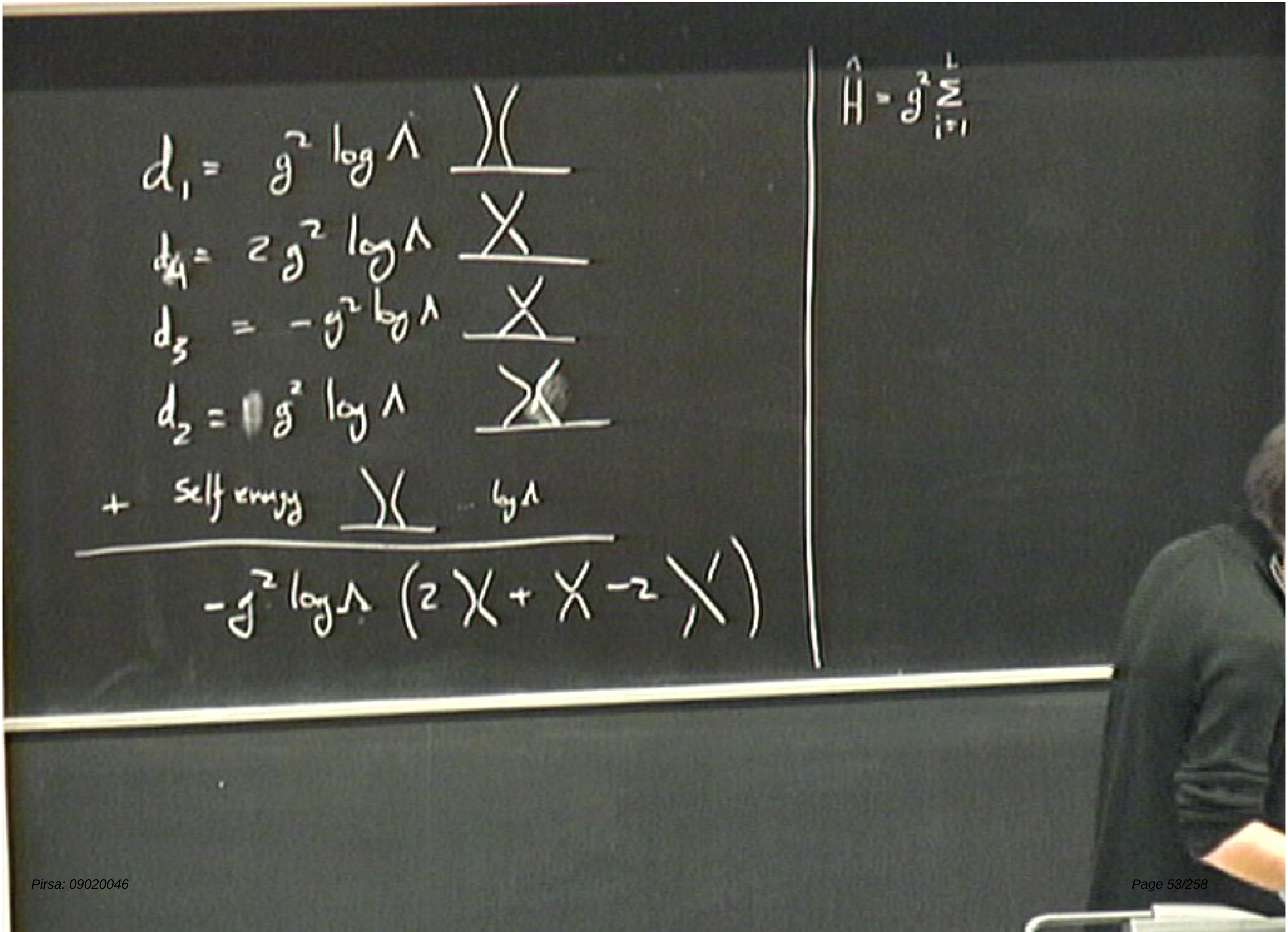
$d_4 = 2 g^2 \log \Lambda$

$d_3 = -g^2 \log \Lambda$

$d_2 = g^2 \log \Lambda$

$+$ Self energy $\log \Lambda$

$-g^2 \log \Lambda \left(2 \,\chi + \chi - 2 \,\chi' \right)$



$d_1 = g^2 \log \Lambda$
$d_4 = 2 g^2 \log \Lambda$
$d_3 = -g^2 \log \Lambda$
$d_2 = g^2 \log \Lambda$
+ Self energy $\log \Lambda$
$-g^2 \log \Lambda \,(2 \ldots + \ldots - 2 \ldots)$
$\hat{H} = g^2 \sum_{i=1}^{L}$

$d_1 = g^2 \log \Lambda$ 

$d_4 = 2 g^2 \log \Lambda$ 

$d_3 = -g^2 \log \Lambda$ 

$d_2 =$ [illegible] $g^2 \log \Lambda$ 

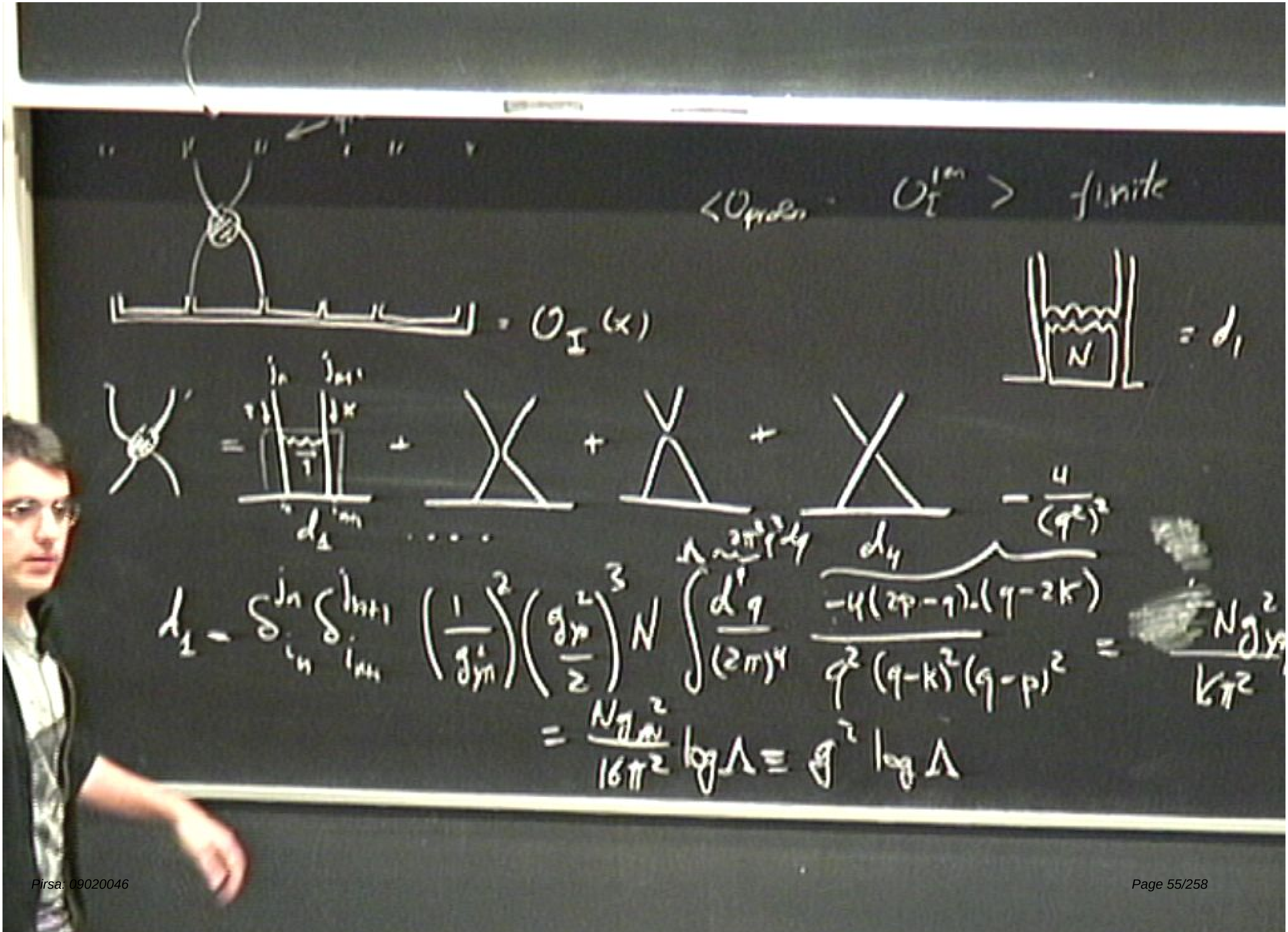
$+$ Self energy  ... $\log \Lambda$

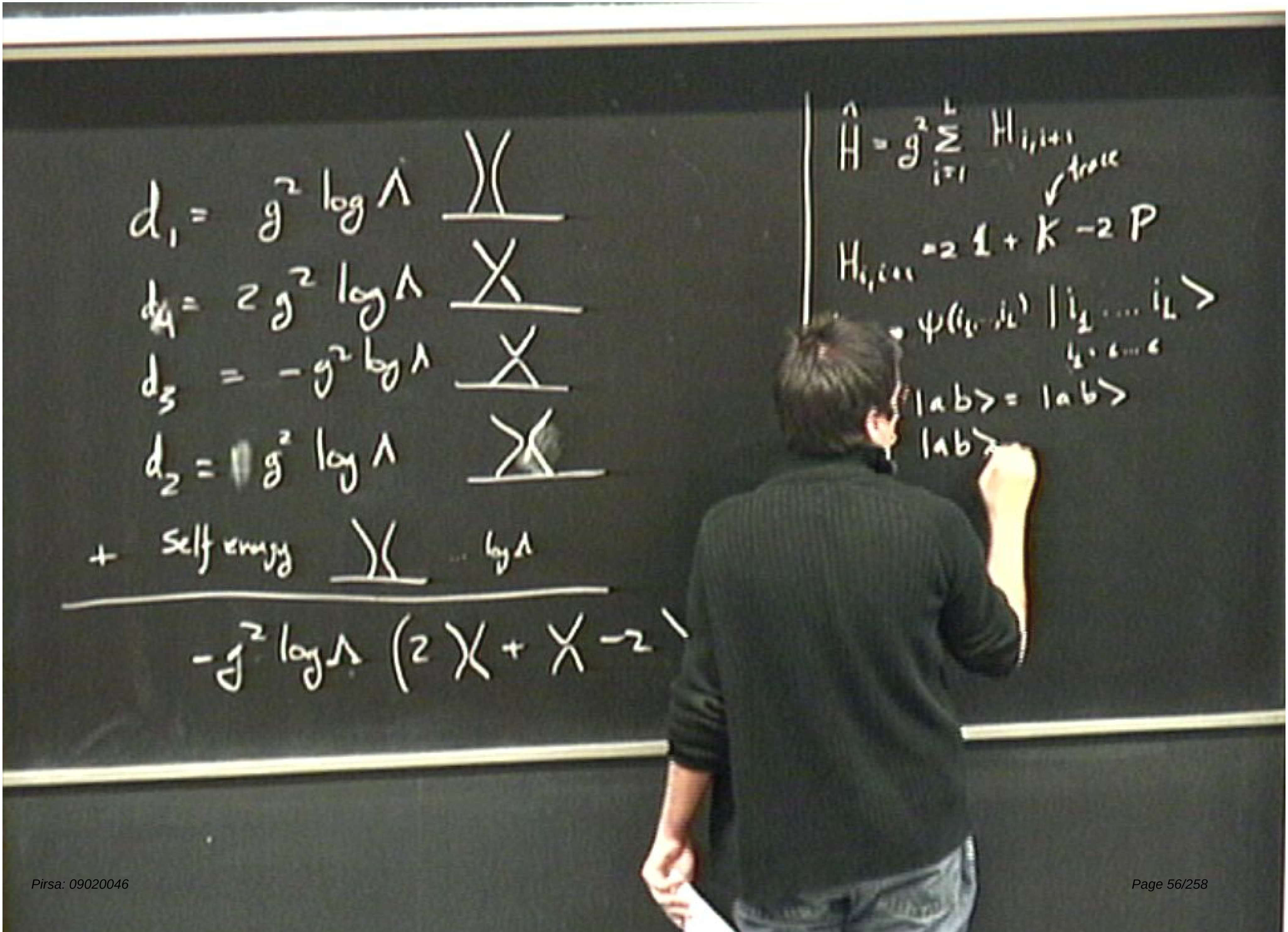
$-g^2 \log \Lambda \, (2$  $+$  -2  $)$

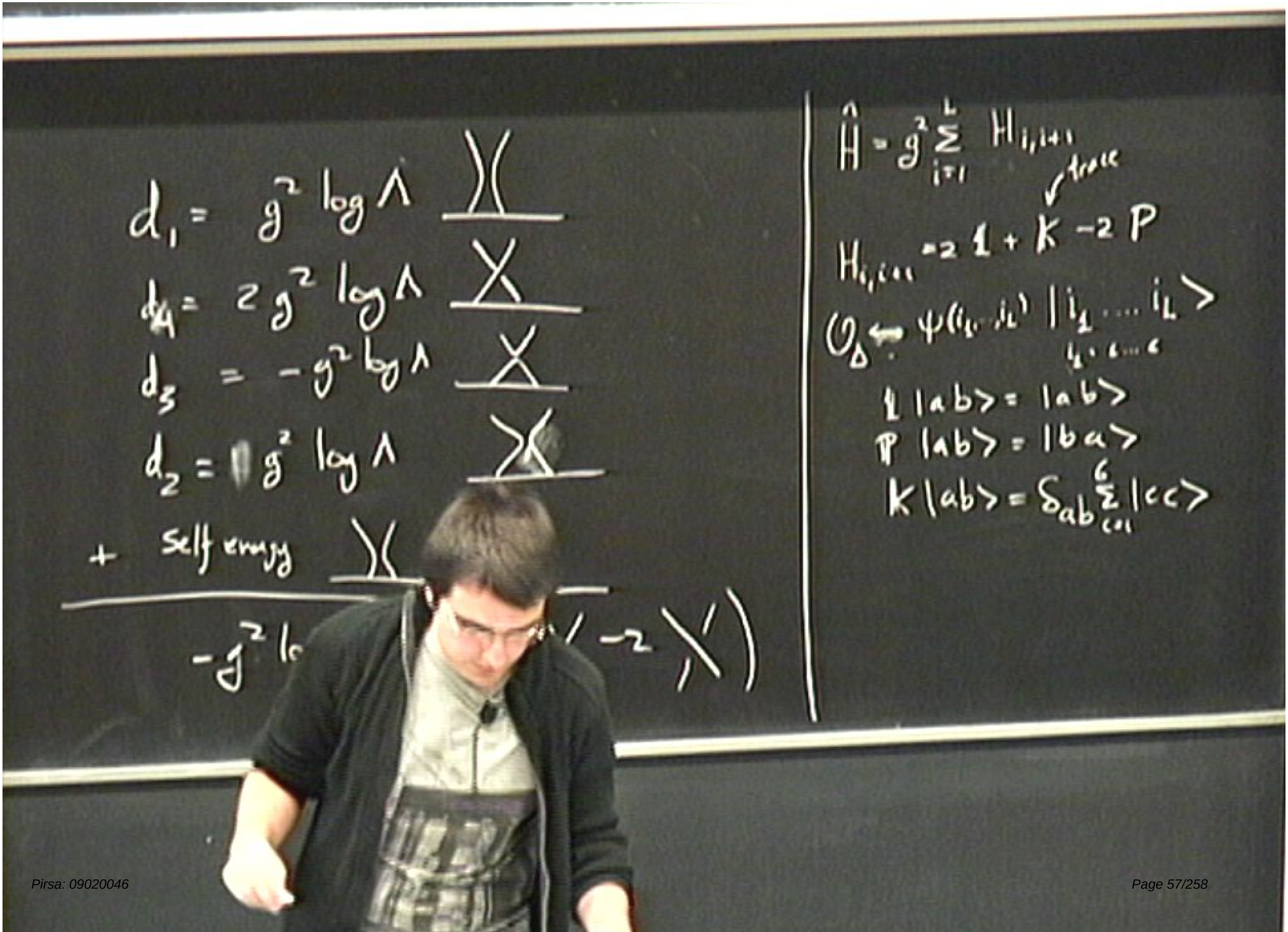
$$\hat{H} = g^2 \sum_{i=1}^{L} H_{i,i+1}$$

trace

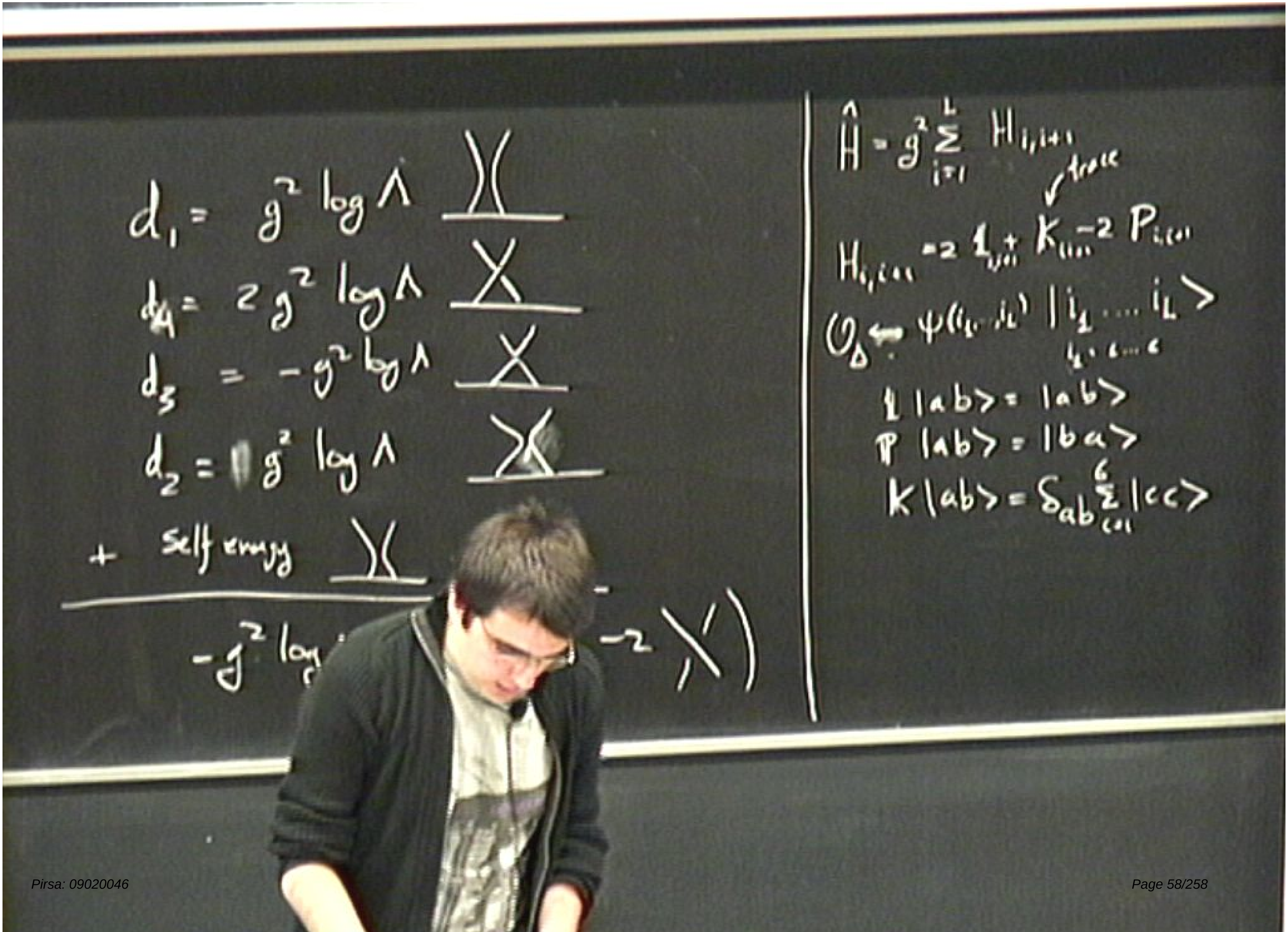
$$H_{i,i+1} = 2\,\mathbb{1} + K - 2P$$


⟨O_proben · O_I^{1m}⟩ finite
= O_I(x)
= d_1
d_1 = δ^{jn}_{in} δ^{jn+1}_{in+1} (1/g_ym)^2 (g_ym/2)^3 N ∫ d^4q/(2π)^4 · −4(2p−q)·(q−2k) / q^2(q−k)^2(q−p)^2
= N g_ym^2/16π^2 log Λ ≡ g^2 log Λ


d1 = g2 log Λ
d4 = 2 g2 log Λ
d3 = − g2 log Λ
d2 = g2 log Λ
+ Self energy ... log Λ
− g2 log Λ (2 + − 2
Ĥ = g2 Σ H i,i+1
trace
H i,i+1 = 2 1 + K − 2 P
ψ(i1 ... iL) | i1 .... iL >
|ab> = |ab>
|ab>



d1 = g2 log Λ
d4 = 2 g2 log Λ
d3 = − g2 log Λ
d2 = g2 log Λ
+ Self energy
Ĥ = g2 Σ H
trace
H = 2 1 + K − 2 P
1 |ab> = |ab>
P |ab> = |ba>
K |ab> = δab Σ |cc>


$d_1 = g^2 \log \Lambda$
$d_4 = 2 g^2 \log \Lambda$
$d_3 = -g^2 \log \Lambda$
$d_2 = g^2 \log \Lambda$
+ self energy
$\hat{H} = g^2 \sum_{i=1}^{L} H_{i,i+1}$
trace
$H_{i,i+1} = 2\,\mathbb{1}_{i,i+1} + K_{i,i+1} - 2 P_{i,i+1}$
$\mathcal{O}_\Delta \Leftrightarrow \psi(i_1 \ldots i_L)\, |i_1 \ldots i_L\rangle$
$\mathbb{1}\,|ab\rangle = |ab\rangle$
$\mathbb{P}\,|ab\rangle = |ba\rangle$
$K\,|ab\rangle = \delta_{ab} \sum_{c=1}^{6} |cc\rangle$

$d_1 = g^2 \log \Lambda$

$d_4 = 2 g^2 \log \Lambda$

$d_3 = - g^2 \log \Lambda$

$d_2 = g^2 \log \Lambda$

$+$ self energy $\ldots \log \Lambda$

$-g^2 \log \Lambda \, (2 \, \ldots + \ldots - 2 \, \ldots)$

$\hat{H} = g^2 \sum_{i=1}^{L} \hat{H}_{i,i+1}$

trace

$\hat{H}_{i,i+1} = 2\, \mathbb{1}_{i,i+1} + K_{i,i+1} - 2 P_{i,i+1}$

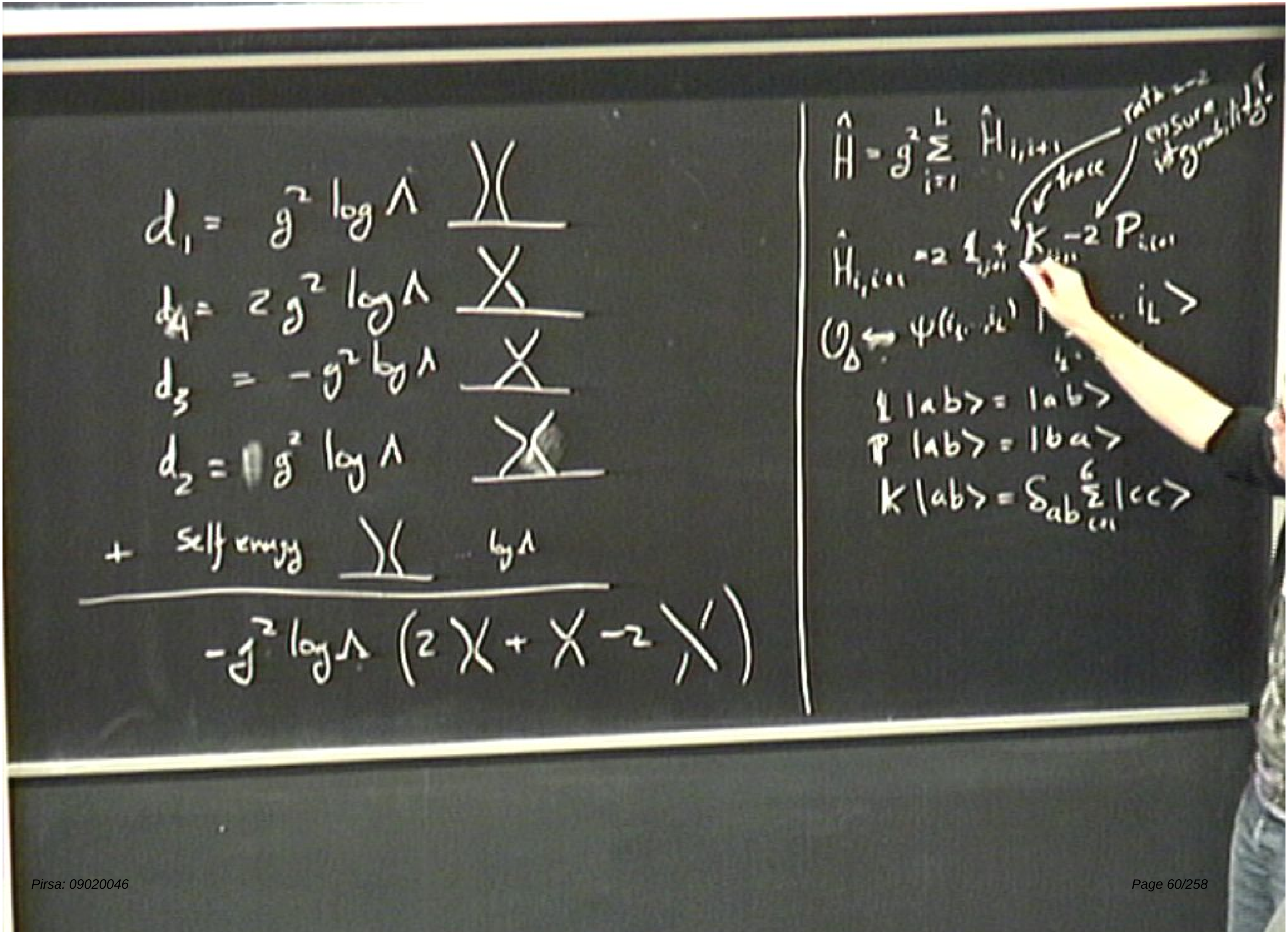
$\mathcal{O}_\Delta \Leftrightarrow \psi(i_1 \ldots i_L) \, | i_1 \ldots i_L \rangle$

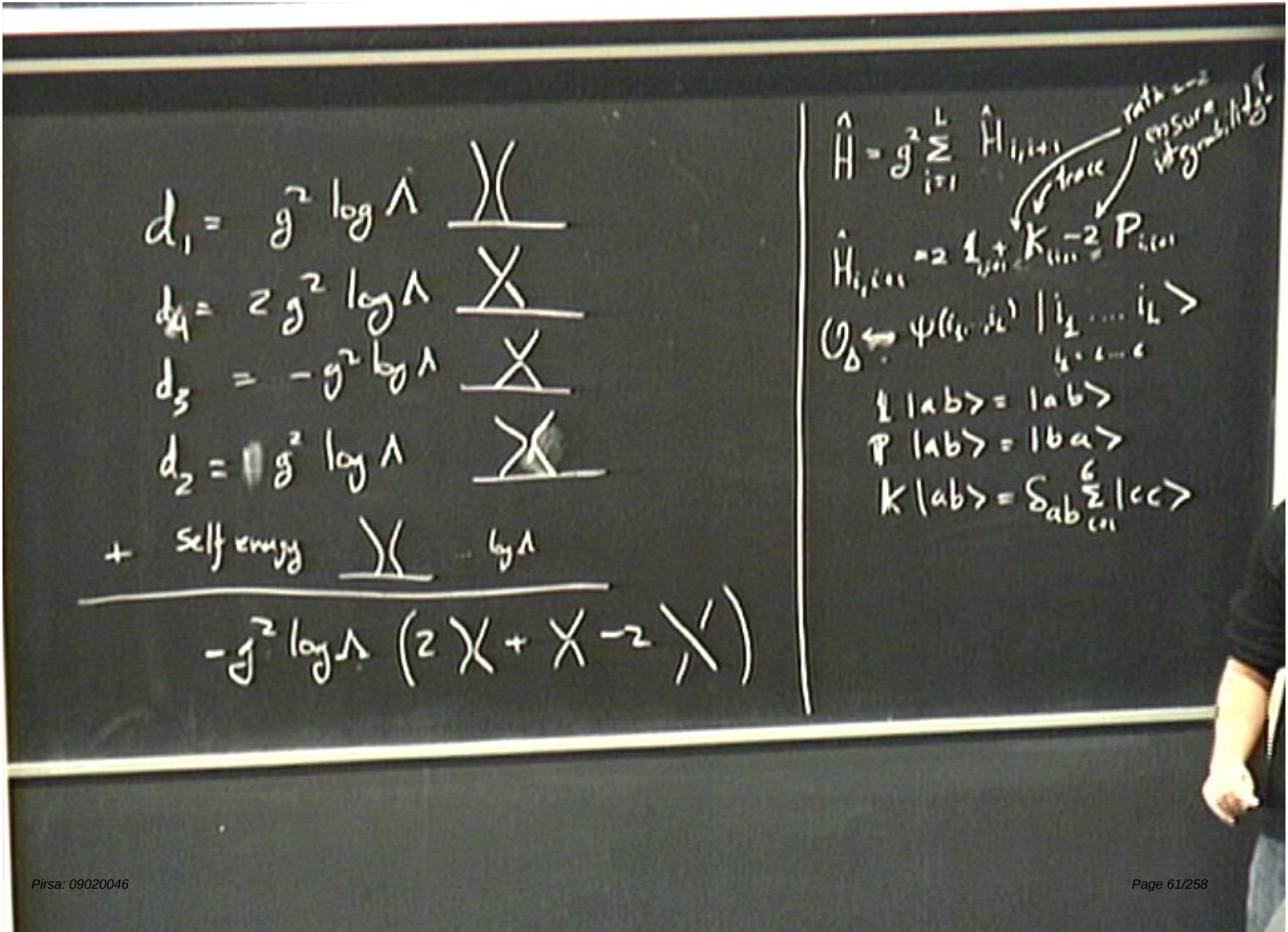
$i_1 = 1 \ldots 6$

$\mathbb{1} \, | ab \rangle = | ab \rangle$

$\mathbb{P} \, | ab \rangle = | ba \rangle$

$K \, | ab \rangle = \delta_{ab} \sum_{c=1}^{6} | cc \rangle$


d1 = g2 log Λ
d4 = 2 g2 log Λ
d3 = − g2 log Λ
d2 = g2 log Λ
+ Self energy ... log Λ
− g2 log Λ (2 + − 2)
Ĥ = g2 Σ Ĥ
Ĥ = 2 1 + K − 2 P
rather ensure integrability?
trace
1 |ab> = |ab>
P |ab> = |ba>
K |ab> = δab Σ |cc>



d1 = g^2 log Λ
d4 = 2 g^2 log Λ
d3 = −g^2 log Λ
d2 = g^2 log Λ
+ Self energy ... log Λ
−g^2 log Λ (2 … + … − 2 …)
Ĥ = g^2 Σ_{i=1}^{L} Ĥ_{i,i+1}
trace
ensure integrability
Ĥ_{i,i+1} = 2 1_{i,i+1} + K_{i,i+1} − 2 P_{i,i+1}
O_Δ ⟺ ψ(i1 … iL) |i1 … iL⟩
1 |ab⟩ = |ab⟩
P |ab⟩ = |ba⟩
K |ab⟩ = δ_ab Σ_{c=1}^{6} |cc⟩

$d_1 = g^2 \log \Lambda$

$d_4 = 2 g^2 \log \Lambda$

$d_3 = -g^2 \log \Lambda$

$d_2 = g^2 \log \Lambda$

$+$ self energy $\dots \log \Lambda$

$-g^2 \log \Lambda \left(2 \, \mathbb{1} + K - 2 P \right)$

$\hat{H} = g^2 \sum_{i=1}^{L} \hat{H}_{i,i+1}$ — ratio and / ensure integrability!

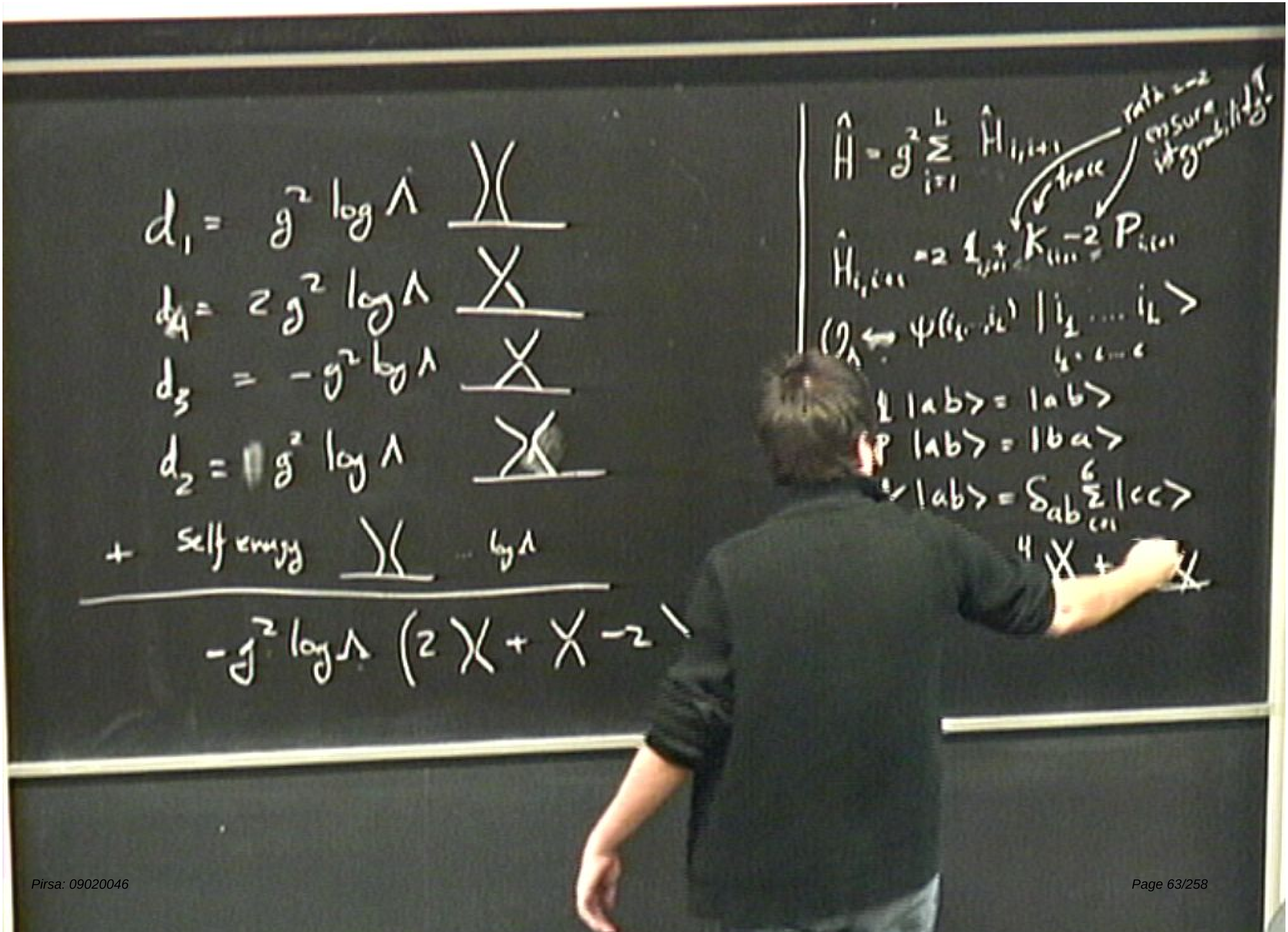
$\hat{H}_{i,i+1} = 2 \, \mathbb{1}_{i,i+1} + K_{i,i+1} - 2 P_{i,i+1}$ (trace)

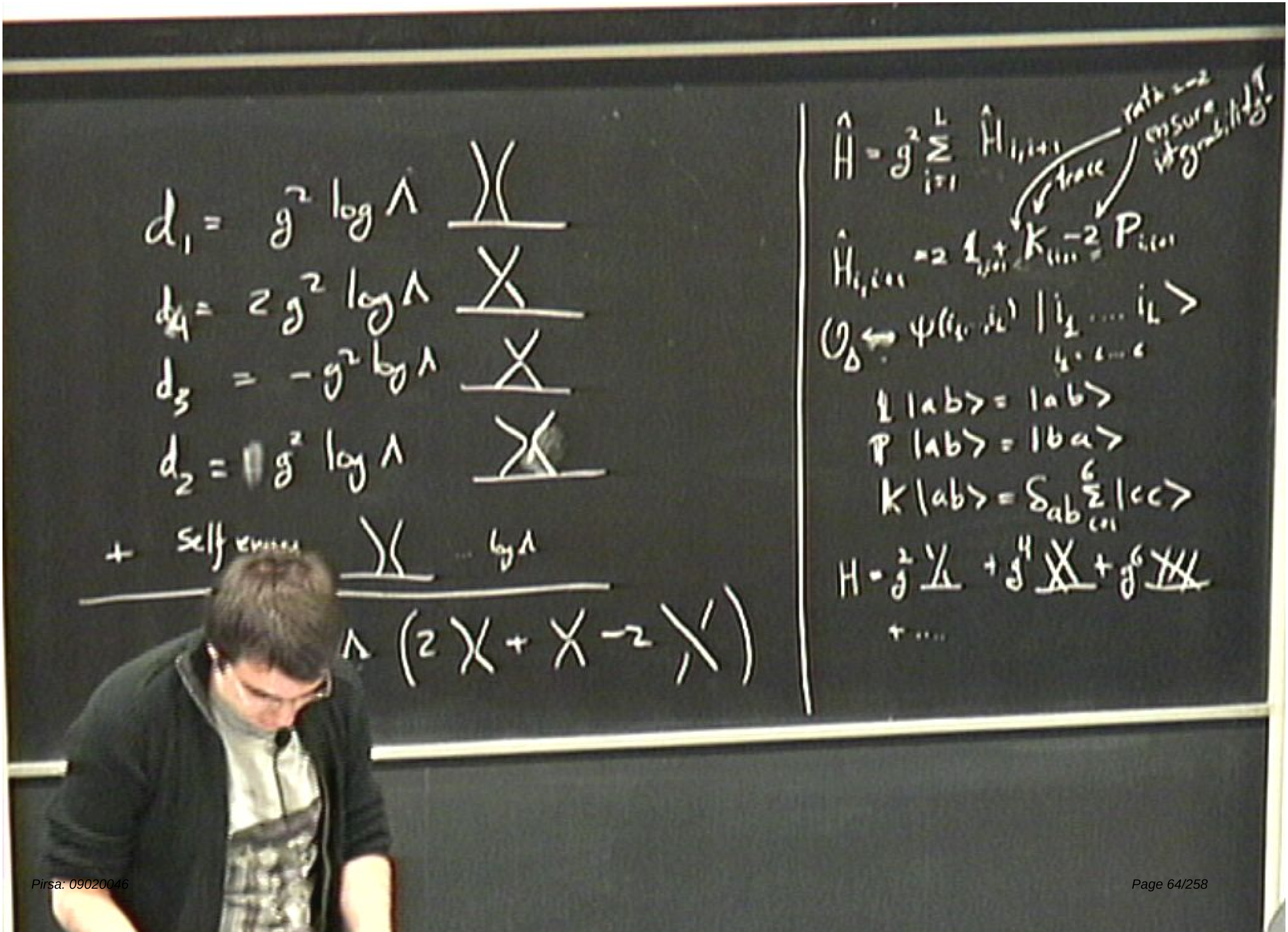
$\mathcal{O}_\Delta \leftrightarrow \psi(i_1 \dots i_L) \, | i_1 \dots i_L \rangle$, $i_k = 1 \dots 6$

$\mathbb{1} | ab \rangle = | ab \rangle$

$P | ab \rangle = | ba \rangle$

$K | ab \rangle = \delta_{ab} \sum_{c=1}^{6} | cc \rangle$


d1 = g² log Λ
d4 = 2 g² log Λ
d3 = −g² log Λ
d2 = g² log Λ
+ Self energy … log Λ
−g² log Λ (2 + −2
Ĥ = g² Σ Ĥ
trace
ensure integrability
Ĥ = 2 1 + K − 2 P
|ab> = |ab>
|ab> = |ba>
|ab> = δab Σ |cc>


d1 = g2 log Λ
d4 = 2g2 log Λ
d3 = -g2 log Λ
d2 = g2 log Λ
+ Self energy ... log Λ
Λ (2 X + X - 2 X)
Ĥ = g2 Σ Ĥi,i+1
rational
ensure integrability
trace
Ĥi,i+1 = 2 1i,i+1 + Ki,i+1 - 2 Pi,i+1
OΔ ⇔ ψ(i1...iL) | i1 iL
1 |ab> = |ab>
P |ab> = |ba>
K |ab> = δab Σ |cc>
H = g2 + g4 + g6
+ ....

$$d_1 = g^2 \log\Lambda$$

$$d_4 = 2 g^2 \log\Lambda$$

$$d_3 = -g^2 \log\Lambda$$

$$d_2 = g^2 \log\Lambda$$

+ self energy ... $\log\Lambda$

$$-g^2 \log\Lambda \left(2 \; + \; - 2 \right)$$

$$\hat{H} = g^2 \sum_{i=1}^{L} \hat{H}_{i,i+1}$$

trace — ensure integrability?

$$\hat{H}_{i,i+1} = 2\,\mathbb{1}_{i,i+1} + K_{i,i+1} - 2 P_{i,i+1}$$

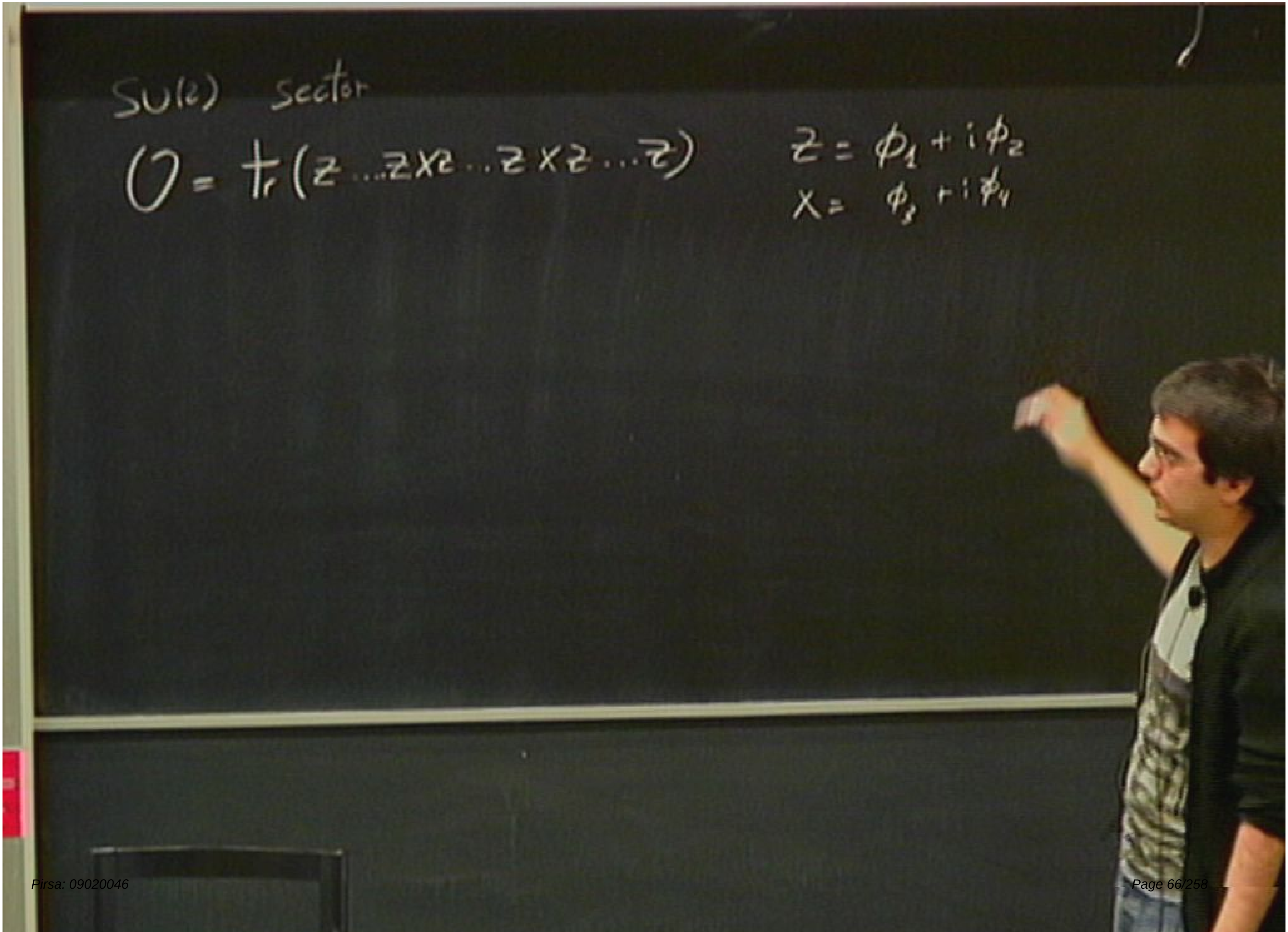
$$\mathcal{O}_\Delta \Leftrightarrow \psi(i_1 \ldots i_L)\, |i_1 \ldots i_L\rangle, \quad i_k = 1 \ldots 6$$

$$\mathbb{1}\,|ab\rangle = |ab\rangle$$

$$P\,|ab\rangle = |ba\rangle$$

$$K\,|ab\rangle = \delta_{ab} \sum_{c=1}^{6} |cc\rangle$$

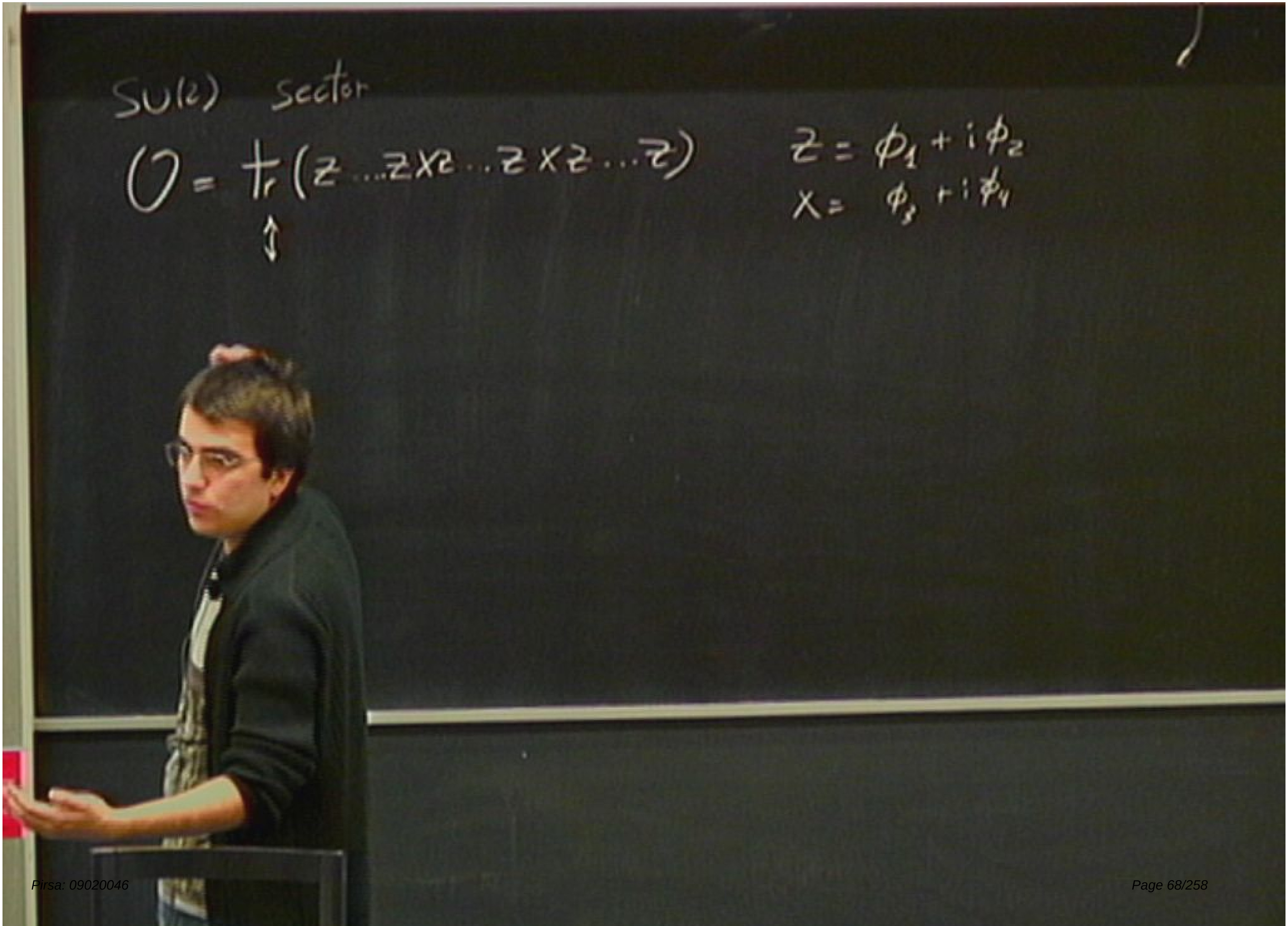
$$H = g^2 \; + g^4 \; + g^6 \; + \ldots$$

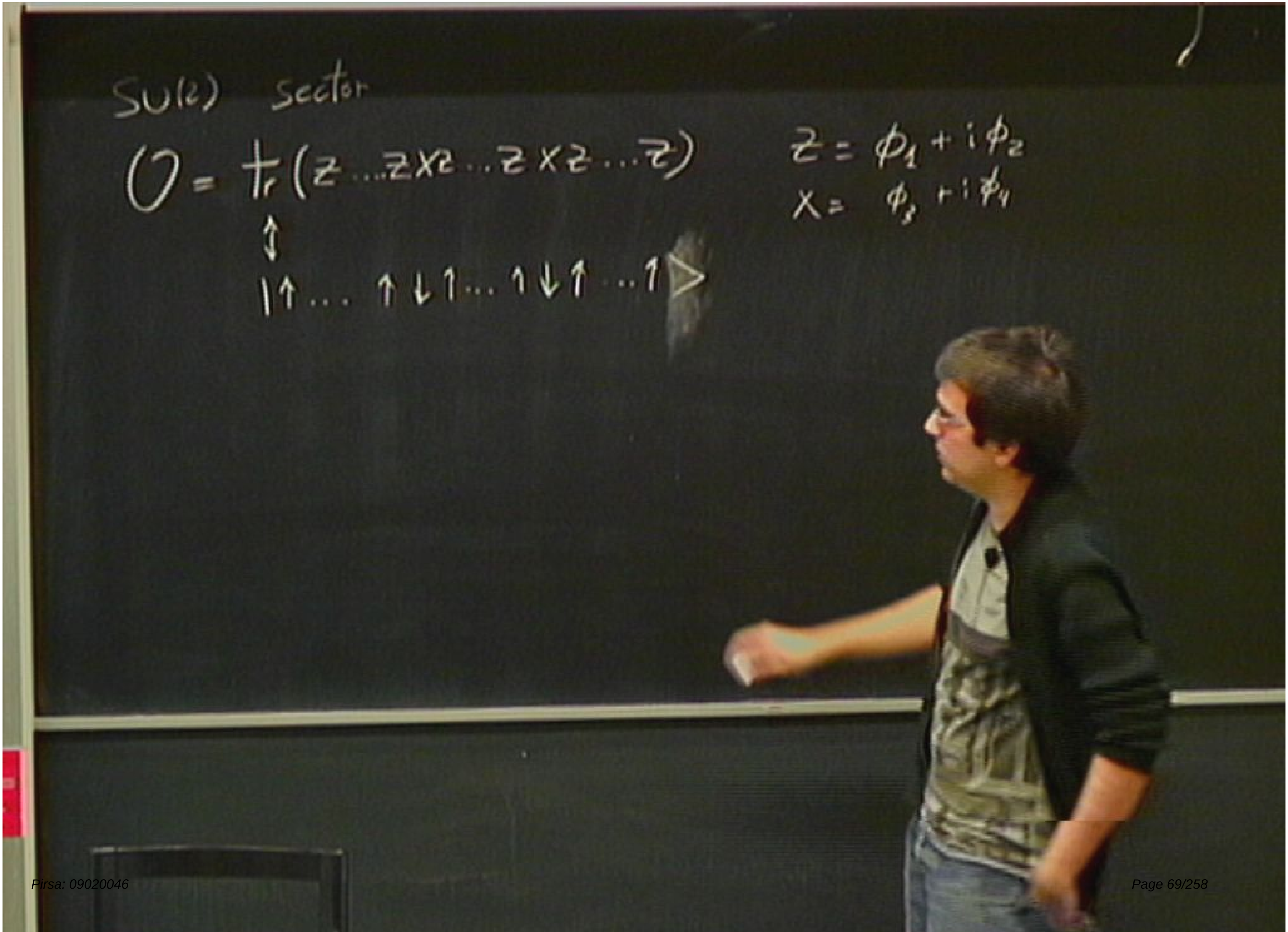

SU(2) sector
O = tr(Z ... ZXZ ... Z X Z ... Z)
Z = φ1 + iφ2
X = φ3 + iφ4

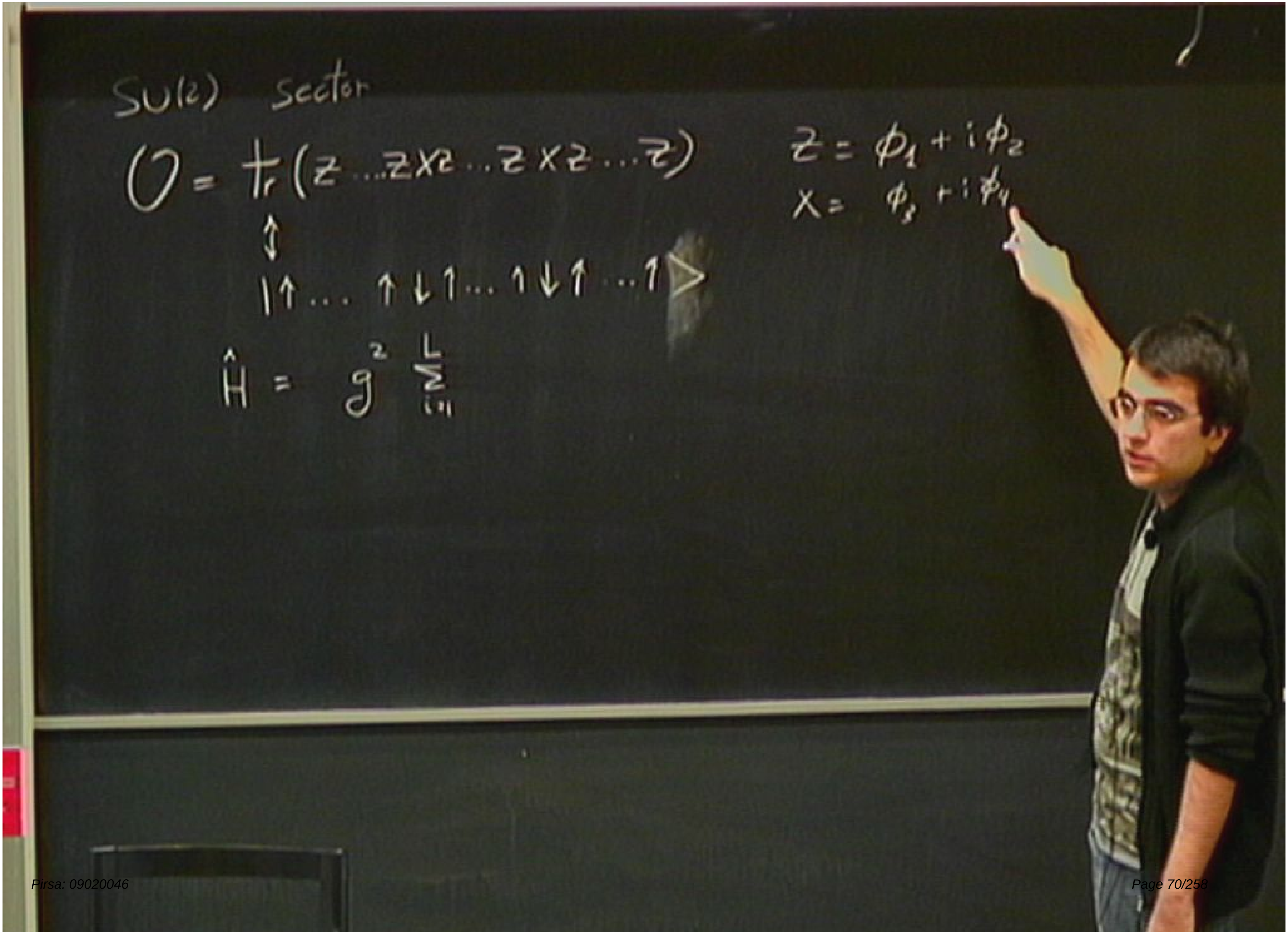
SU(2) sector

$$\mathcal{O} = \mathrm{tr}(Z \ldots Z X Z \ldots Z X Z \ldots Z)$$

$$Z = \phi_1 + i\phi_2$$
$$X = \phi_3 + i\phi_4$$


SU(2) sector
O = tr(Z ....ZXZ ... Z X Z ... Z)
Z = φ1 + i φ2
X = φ3 + i φ4


SU(2) sector
O = tr(Z ....ZXZ ... ZXZ ... Z)
|↑ .... ↑↓↑ ... ↑↓↑ ... ↑>
Z = φ1 + iφ2
X = φ3 + iφ4


SU(2) sector
O = tr(Z ...ZXZ ...ZXZ ...Z)
Z = φ1 + iφ2
X = φ3 + iφ4
|↑ ... ↑↓↑ ... ↑↓↑ ... ↑>
Ĥ = g² Σ i=1 L

SU(2) sector

$\mathcal{O} = \mathrm{tr}(Z \ldots ZXZ \ldots ZXZ \ldots Z)$

$\updownarrow$

$|\uparrow \ldots \uparrow\downarrow\uparrow \ldots \uparrow\downarrow\uparrow \ldots \uparrow\rangle$

$\hat{H} = g^2 \sum_{i=1}^{L}$

$Z = \phi_1 + i\phi_2$

$X = \phi_3 + i\phi_4$

$K|ZZ\rangle = K|\phi_1\phi_1\rangle$

$- K|\phi_2\phi_2\rangle$

SU(2) sector

$$\mathcal{O} = \mathrm{tr}(Z \ldots ZXZ \ldots ZXZ \ldots Z)$$

$$\updownarrow$$

$$|\uparrow \ldots \uparrow\downarrow\uparrow \ldots \uparrow\downarrow\uparrow \ldots \uparrow\rangle$$

$$\hat{H} = g^2 \sum_{i=1}^{L}$$

$$Z = \phi_1 + i\phi_2$$

$$X = \phi_3 + i\phi_4$$

$$K|ZZ\rangle = K|\phi_1\phi_1\rangle - K|\phi_2\phi_2\rangle = 0$$

$$K|ZX\rangle = 0$$

SU(2) sector

$$\mathcal{O} = \mathrm{tr}(Z \ldots ZXZ \ldots ZXZ \ldots Z)$$

$$\updownarrow$$

$$|\uparrow \ldots \uparrow\downarrow\uparrow \ldots \uparrow\downarrow\uparrow \ldots \uparrow\rangle$$

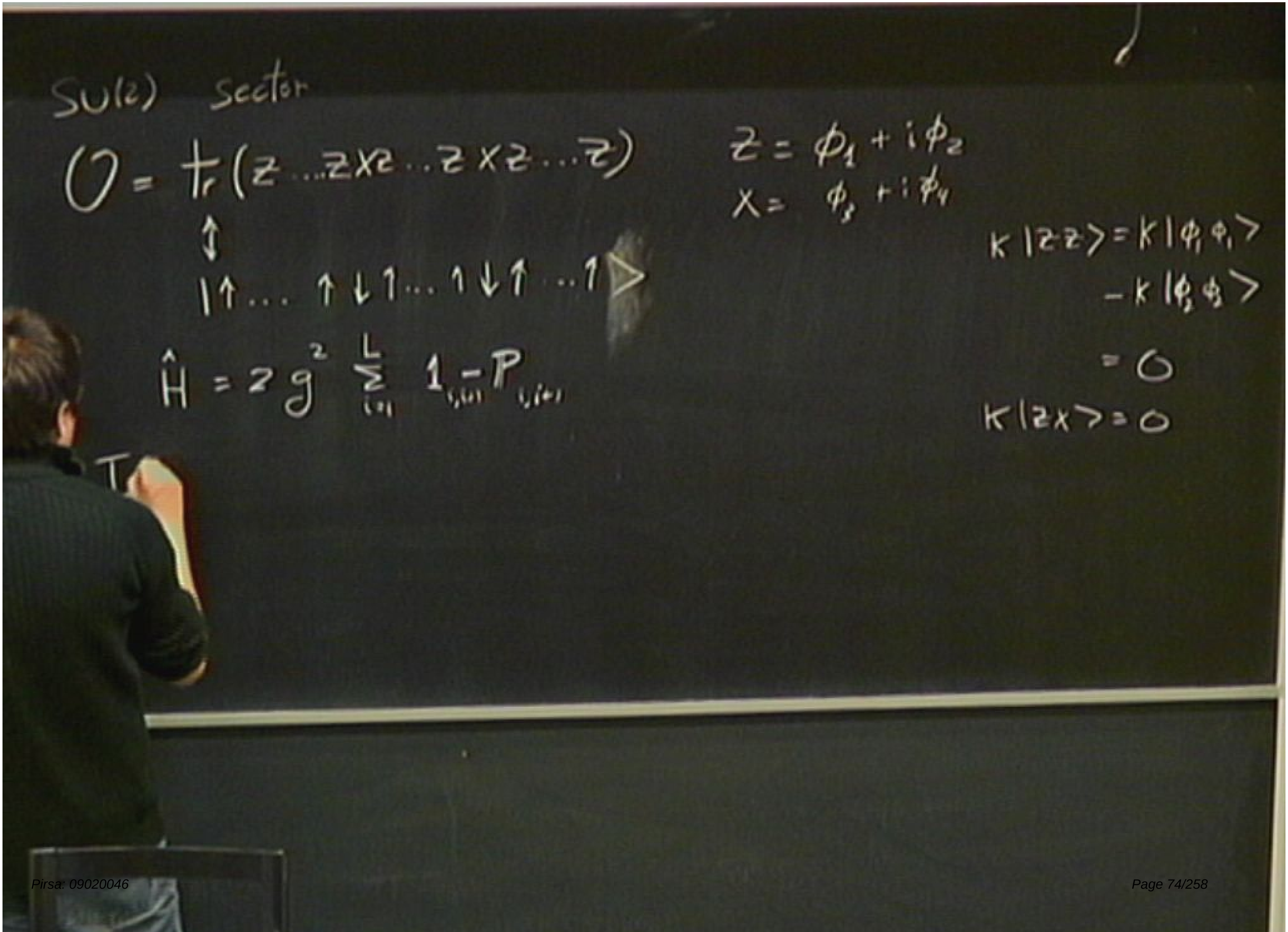
$$\hat{H} = 2g^2 \sum_{l=1}^{L} 1_{l,l+1} - P_{l,l+1}$$

$$Z = \phi_1 + i\phi_2$$

$$X = \phi_3 + i\phi_4$$

$$K|ZZ\rangle = K|\phi_1\phi_1\rangle - K|\phi_2\phi_2\rangle = 0$$

$$K|ZX\rangle = 0$$


SU(2) sector
$\mathcal{O} = \mathrm{tr}(Z \ldots ZXZ \ldots ZXZ \ldots Z)$
$\updownarrow$
$|\uparrow \ldots \uparrow\downarrow\uparrow \ldots \uparrow\downarrow\uparrow \ldots \uparrow\rangle$
$\hat{H} = 2g^2 \sum_{l=1}^{L} 1_{l,l+1} - P_{l,l+1}$
$Z = \phi_1 + i\phi_2$
$X = \phi_3 + i\phi_4$
$K|ZZ\rangle = K|\phi_1\phi_1\rangle - K|\phi_2\phi_2\rangle$
$= 0$
$K|ZX\rangle = 0$

SU(2) sector

$$\mathcal{O} = \mathrm{tr}(Z \dots ZXZ \dots ZXZ \dots Z)$$

$$\updownarrow$$

$$|\uparrow \dots \uparrow \downarrow \uparrow \dots \uparrow \downarrow \uparrow \dots \uparrow\rangle$$

$$\hat{H} = 2g^2 \sum_{i=1}^{L} 1_{i,i+1} - P_{i,i+1}$$

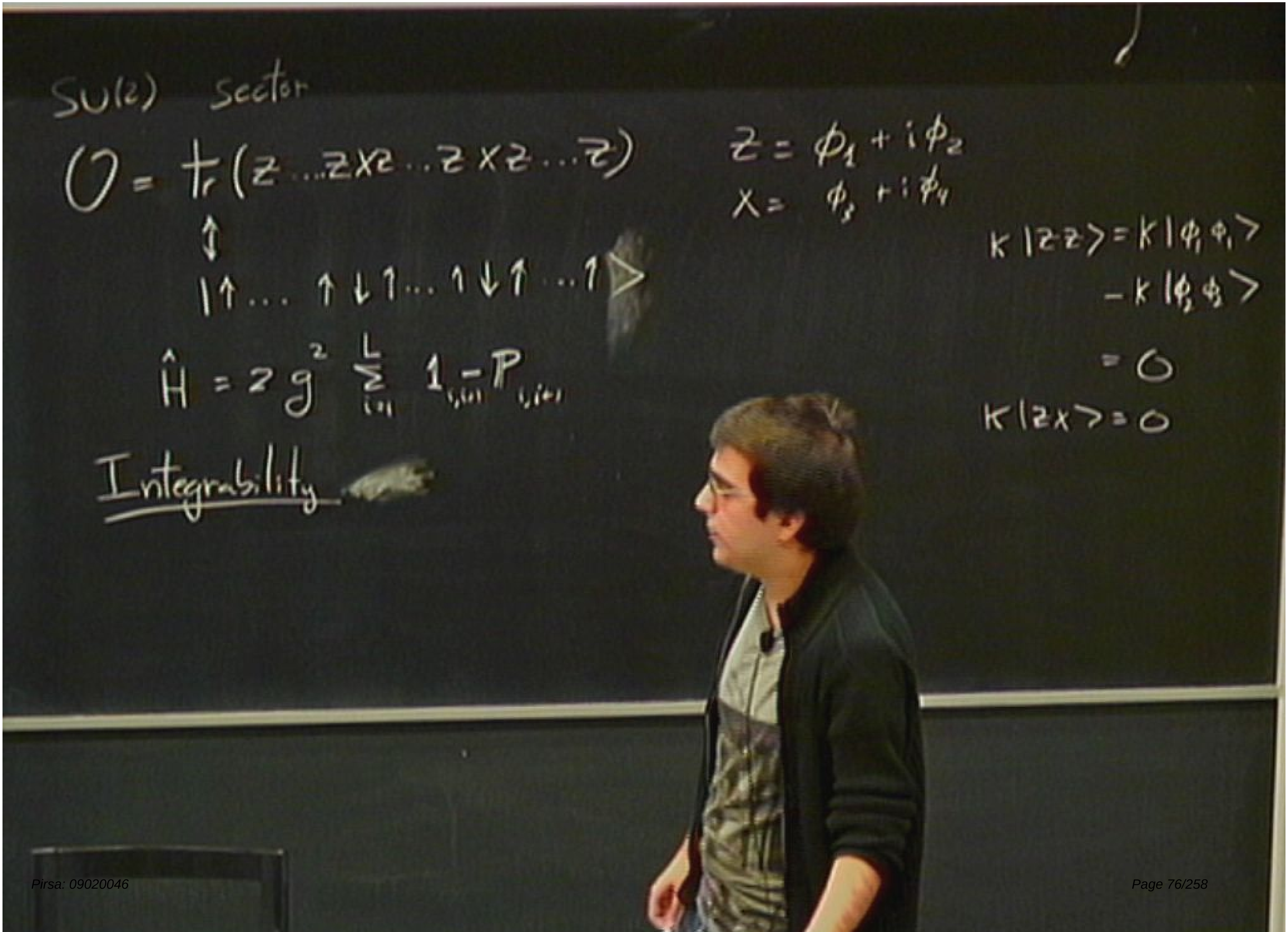
<u>Integrability</u>

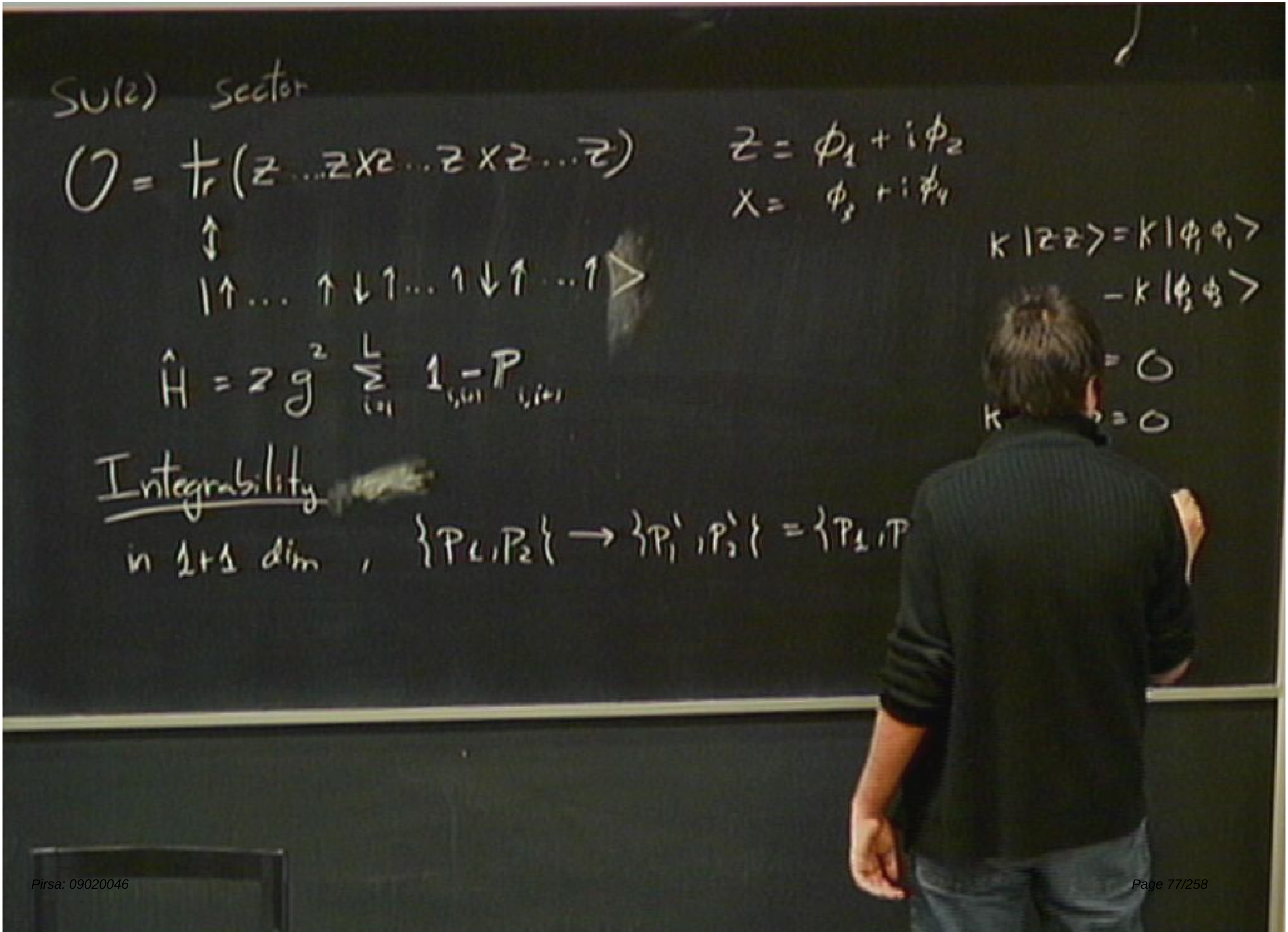
$$Z = \phi_1 + i\phi_2$$

$$X = \phi_3 + i\phi_4$$

$$K|ZZ\rangle = K|\phi_1\phi_1\rangle - K|\phi_2\phi_2\rangle = 0$$

$$K|ZX\rangle = 0$$


SU(2) Sector
O = tr(Z ...ZXZ ...Z X Z ...Z)
|↑ ... ↑↓↑... ↑↓↑ ...↑>
Ĥ = 2g² Σ 1 − P
Integrability
Z = φ1 + iφ2
X = φ3 + iφ4
K|ZZ> = K|φ1 φ1>
− K|φ2 φ2>
= 0
K|ZX> = 0


SU(2) sector
O = tr(Z ...ZXZ...ZXZ...Z)
Z = φ1 + iφ2
X = φ3 + iφ4
|↑... ↑↓↑...↑↓↑ ...↑>
K |ZZ> = K |φ1 φ1>
− K |φ3 φ3>
Ĥ = 2g² Σ 1 − P
Integrability
in 1+1 dim , {P1, P2} → {P1', P2'} = {P1, P

SU(2) sector

$$\mathcal{O} = \mathrm{tr}(Z \ldots ZXZ \ldots ZXZ \ldots Z)$$

$$\updownarrow$$

$$|\uparrow \ldots \uparrow\downarrow\uparrow \ldots \uparrow\downarrow\uparrow \ldots \uparrow\rangle$$

$$\hat{H} = 2g^2 \sum_{l=1}^{L} 1_{l,l+1} - P_{l,l+1}$$

$Z = \phi_1 + i\phi_2$

$X = \phi_3 + i\phi_4$

$K|ZZ\rangle = K|\phi_1\phi_1\rangle - K|\phi_2\phi_2\rangle = 0$

$K|ZX\rangle = 0$

Integrability

in 1+1 dim, $\{p_1, p_2\} \to \{p_1', p_2'\} = \{p_1, p_2\} \leftarrow \exists\, Q_1 = \sum p_j$

$Q_2 = \sum p_j^2$

$\left(\sum \sin^2 \frac{p}{2} \ldots\right)$

SU(2) sector

$$\mathcal{O} = \mathrm{tr}(Z \ldots Z X Z \ldots Z X Z \ldots Z) \qquad Z = \phi_1 + i\phi_2$$

$$X = \phi_3 + i\phi_4$$

$$\updownarrow$$

$$| \uparrow \ldots \uparrow \downarrow \uparrow \ldots \uparrow \downarrow \uparrow \ldots \uparrow \rangle$$

$$K|ZZ\rangle = K|\phi_1\phi_1\rangle - K|\phi_2\phi_2\rangle = 0$$

$$\hat{H} = 2g^2 \sum_{l=1}^{L} 1_{l,l+1} - P_{l,l+1}$$

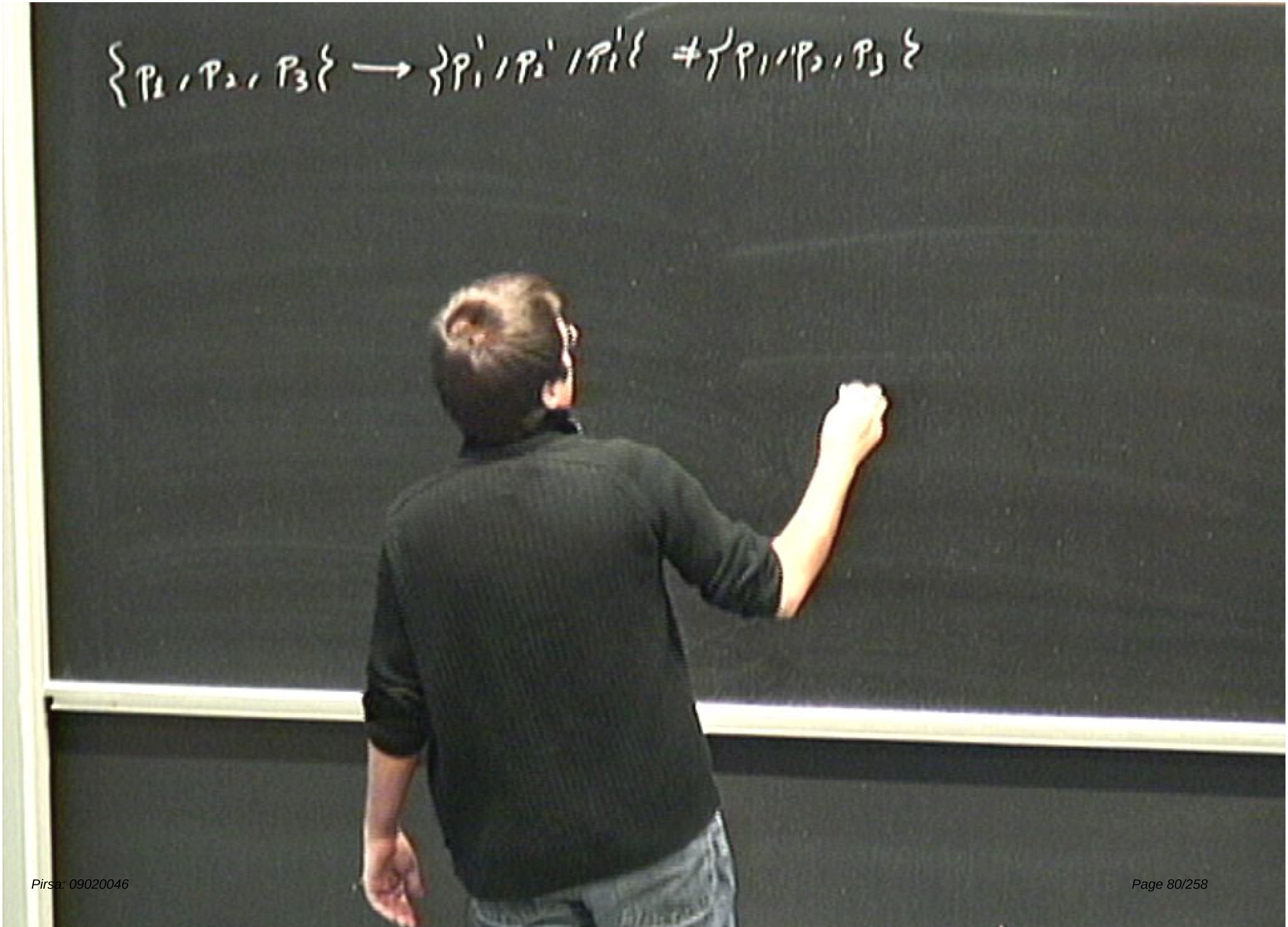
$$K|ZX\rangle = 0$$

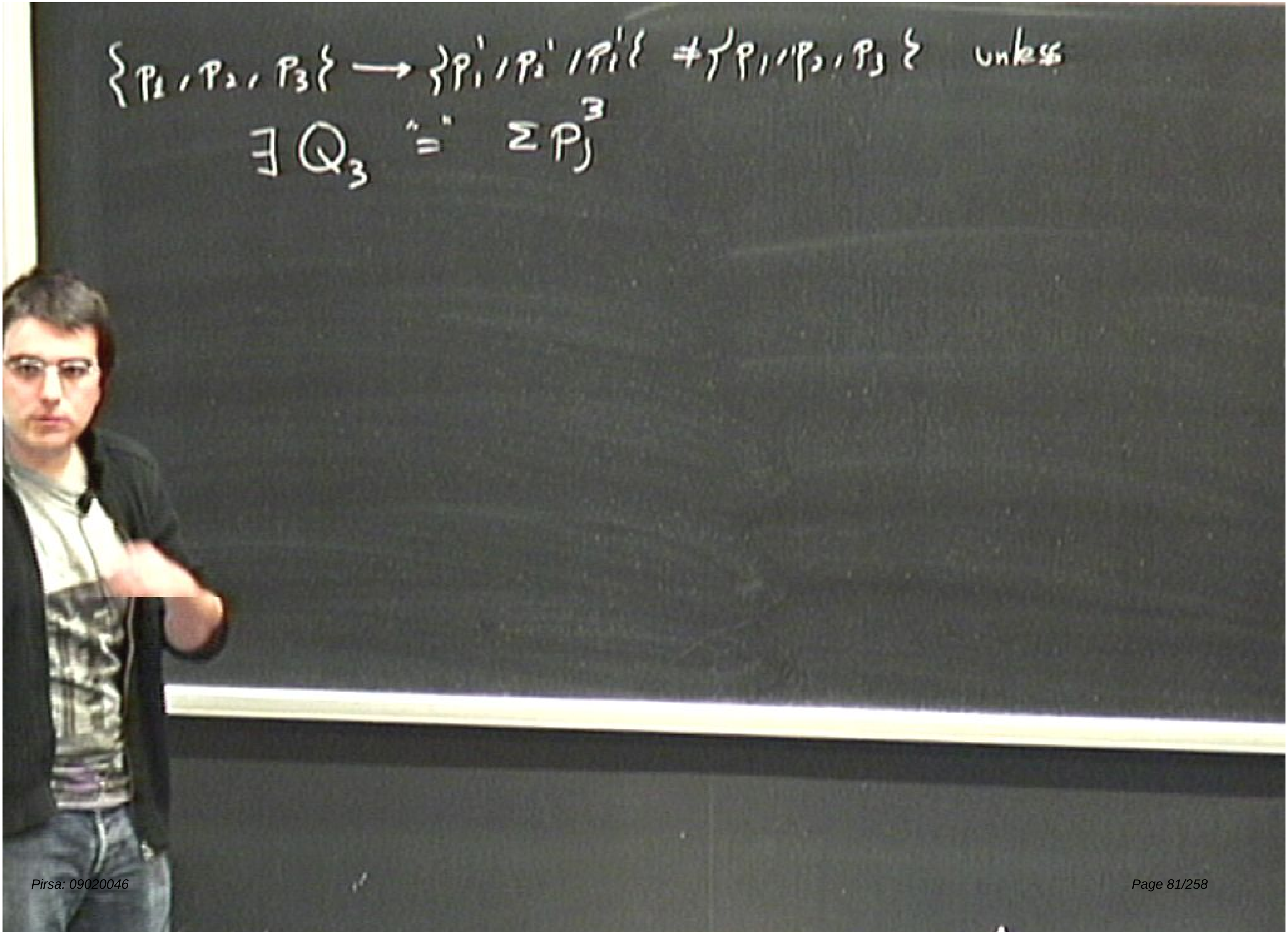
Integrability

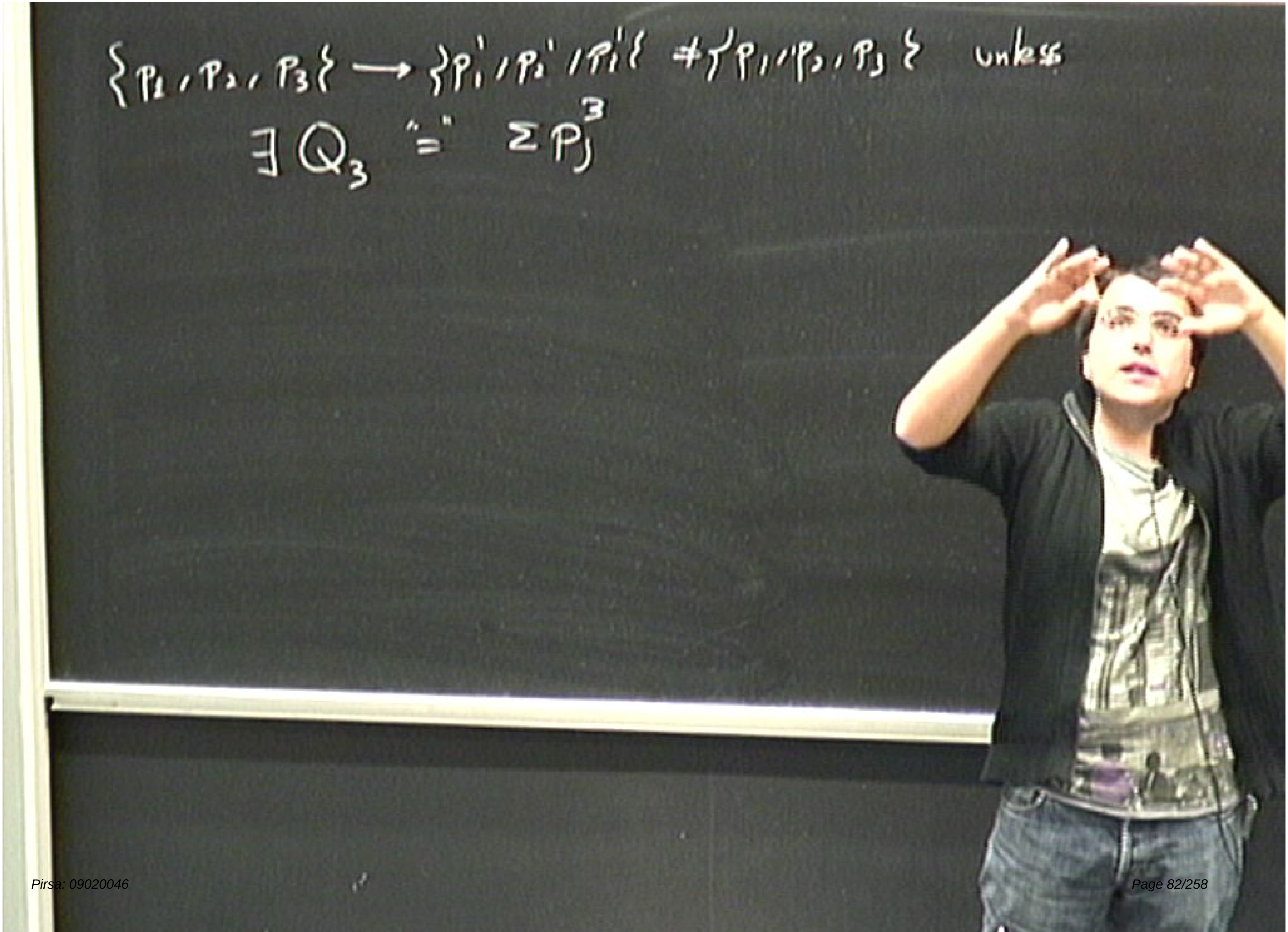
in 1+1 dim , $\{p_1, p_2\} \to \{p_1', p_2'\} = \{p_1, p_2\}$ $\Leftarrow$ $\exists\, Q_1 = \sum p_j$

$$Q_2 = \sum p_j^2$$

$$\left(\sum \sin^2 \tfrac{p}{2} \text{ in lattice}\right)$$


{p1, p2, p3} → {p1', p2', p3'} ≠ {p1, p2, p3}

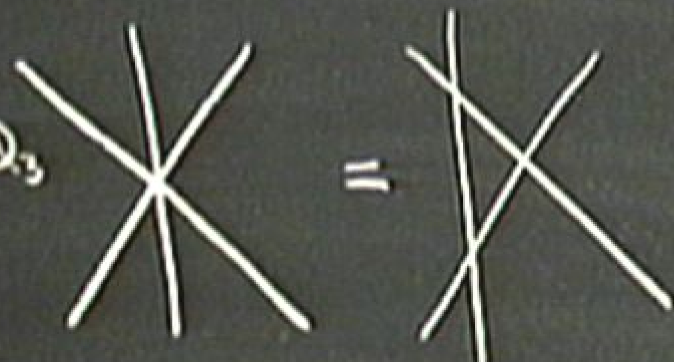

$\{p_1, p_2, p_3\} \longrightarrow \{p_1', p_2', p_3'\} \neq \{p_1, p_2, p_3\}$ unless
$\exists\, Q_3$ "=" $\sum p_j^3$


$\{p_1, p_2, p_3\} \longrightarrow \{p_1', p_2', p_3'\} \neq \{p_1, p_2, p_3\}$ unless
$\exists\, Q_3$ "=" $\Sigma\, p_j^3$

$\{p_1, p_2, p_3\} \longrightarrow \{p_1', p_2', p_3'\} \neq \{p_1, p_2, p_3\}$ unless

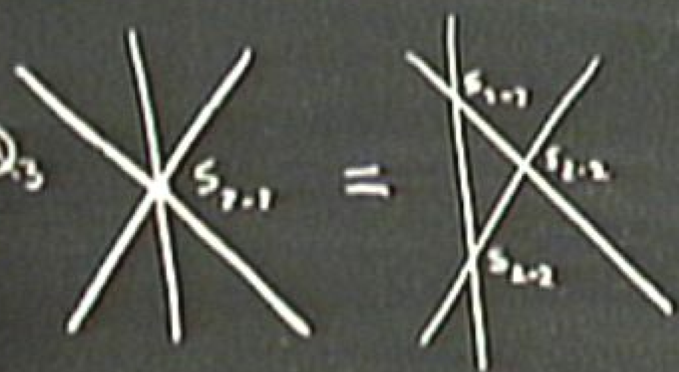
$\exists\, Q_3$ "=" $\sum p_j^3$

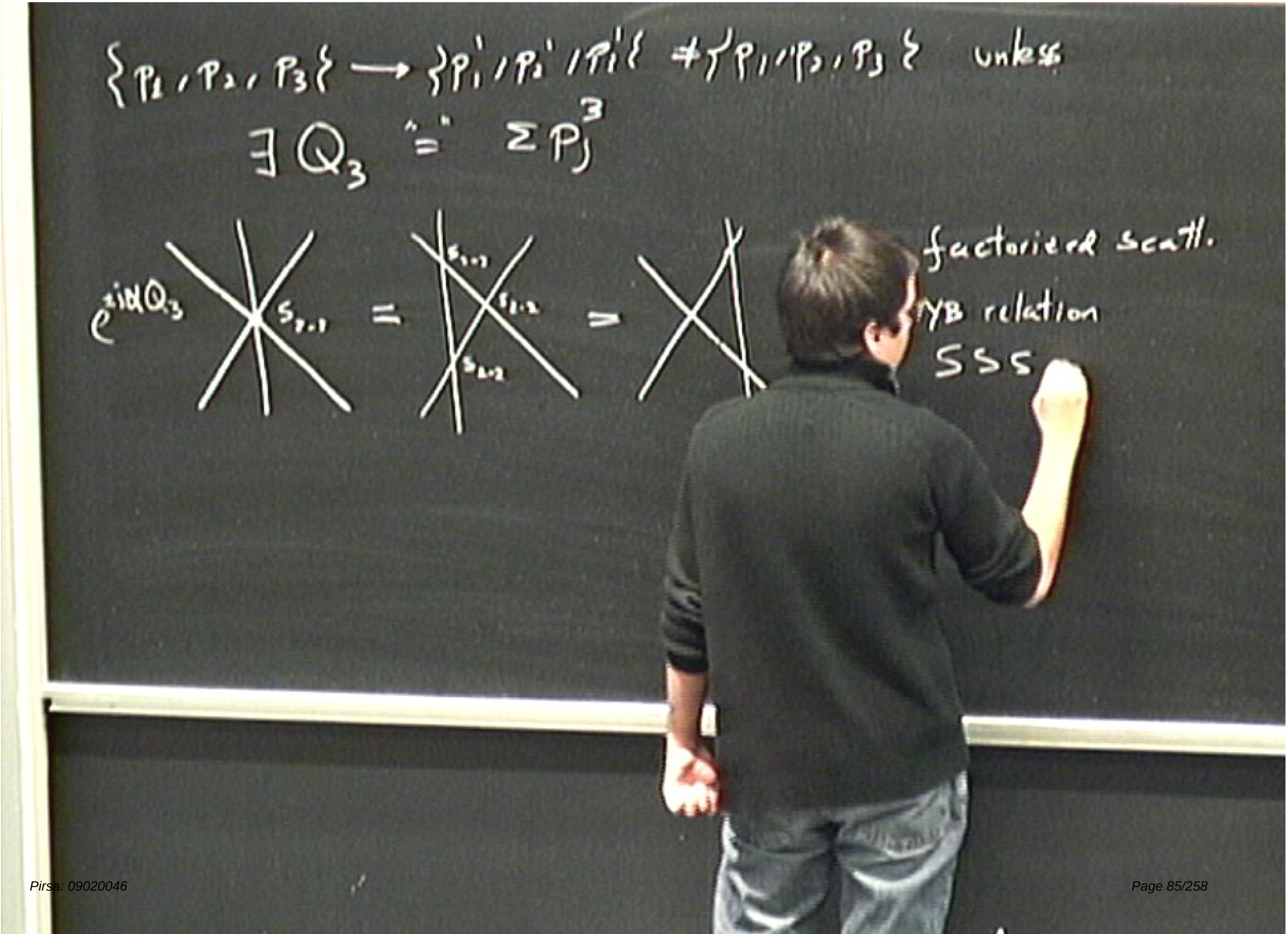
$e^{i\alpha Q_3}$

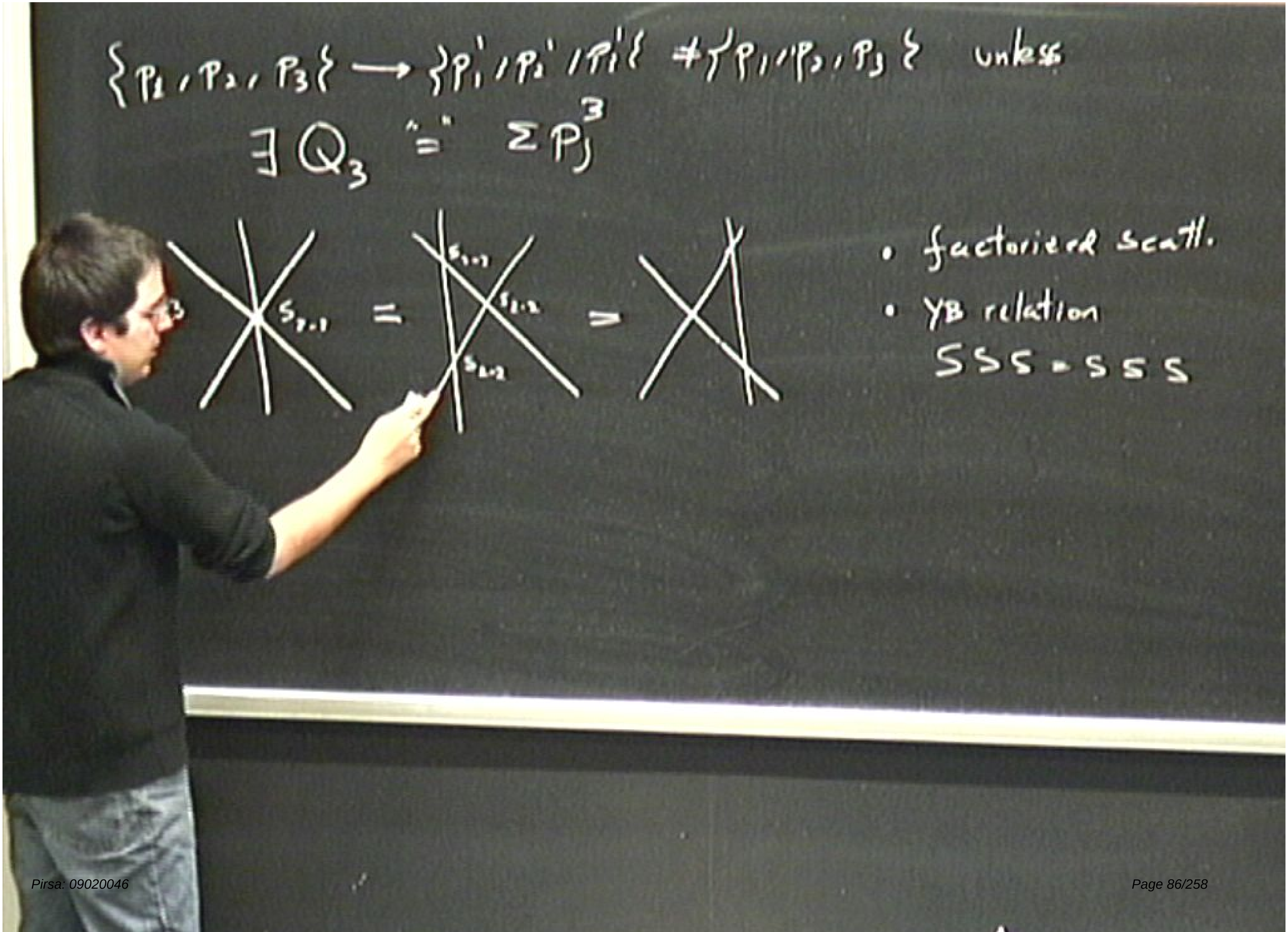


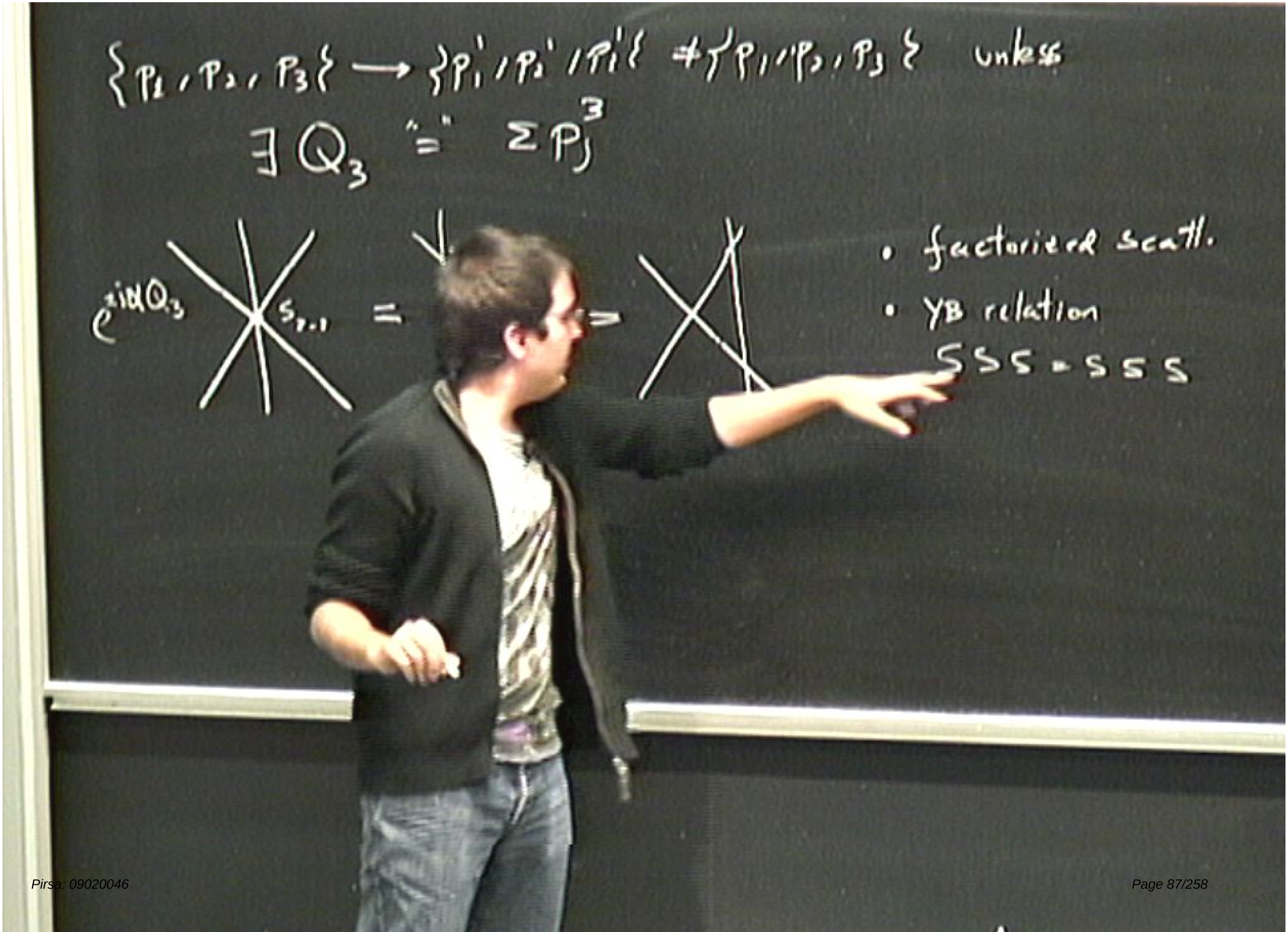
$$\{p_1, p_2, p_3\} \longrightarrow \{p_1', p_2', p_3'\} \neq \{p_1, p_2, p_3\} \text{ unless}$$

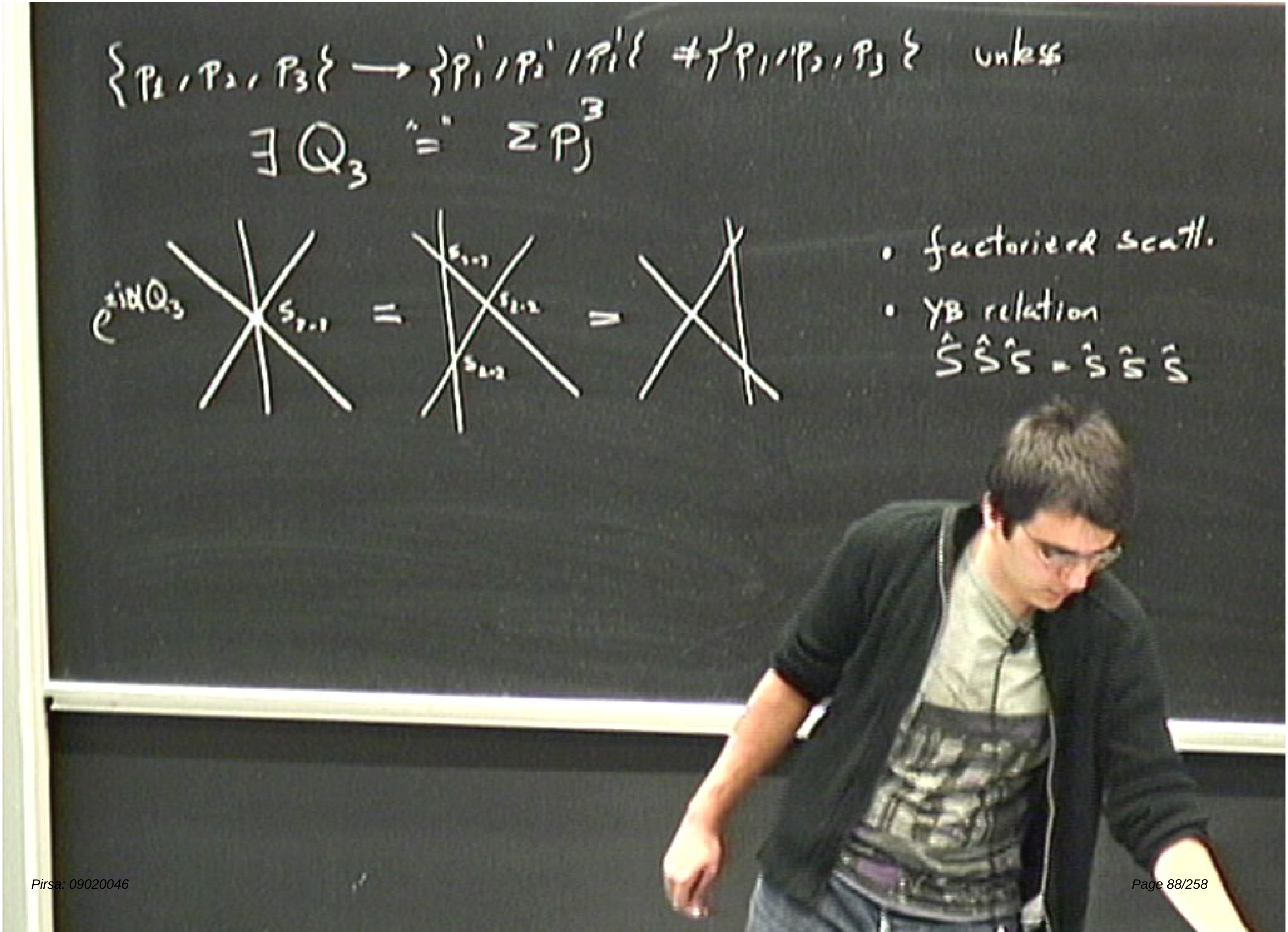
$$\exists\, Q_3 \text{ “=” } \Sigma p_j^3$$

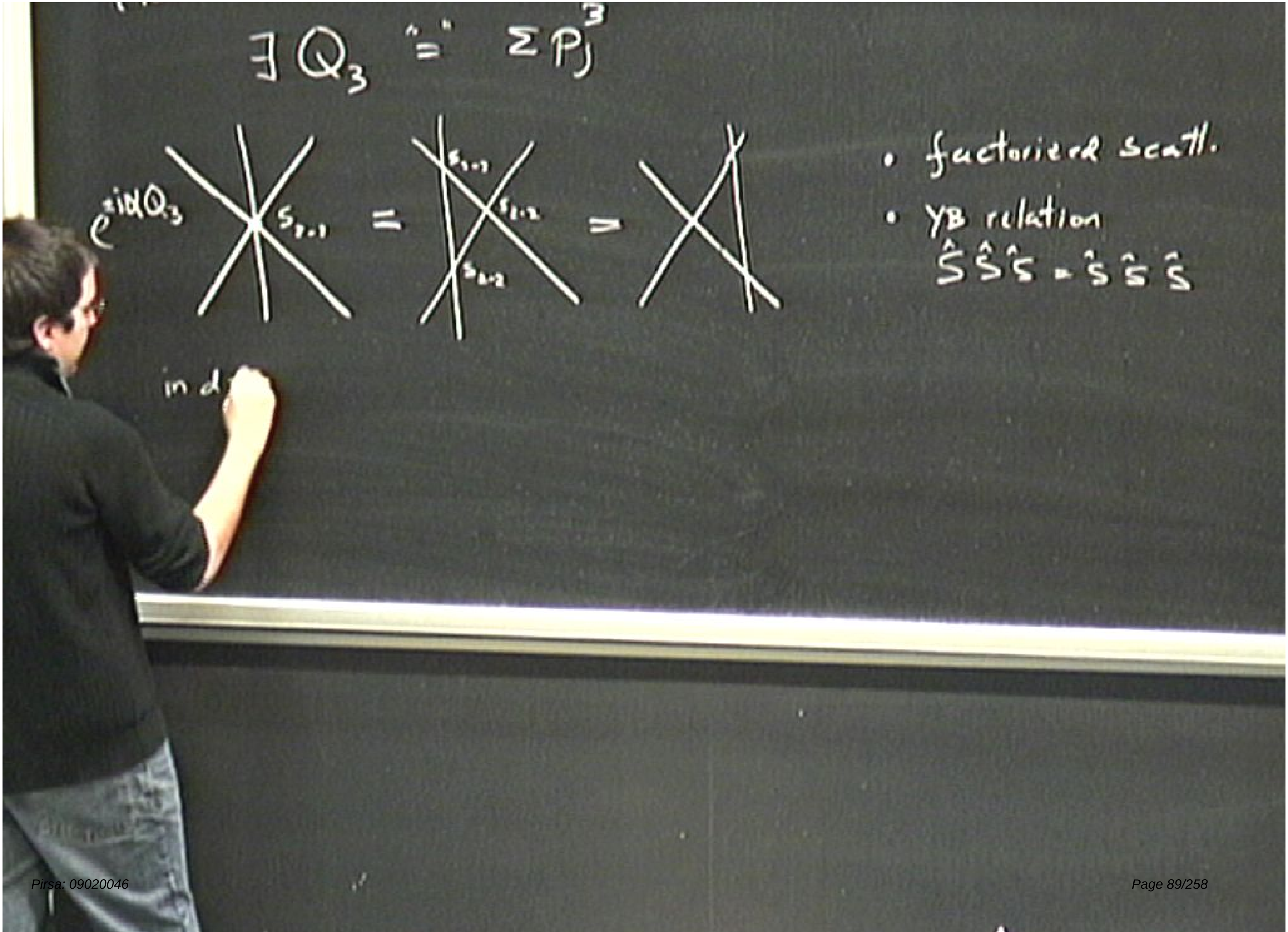



{p1, p2, p3} → {p1', p2', p3'} ≠ {p1, p2, p3} unless
∃ Q3 "=" Σ pj^3
e^{iαQ3}
S2,1
=
=
factorized scatt.
YB relation
SSS


{p1, p2, p3} → {p1', p2', p3'} ≠ {p1, p2, p3} unless
∃ Q3 "=" Σ pj^3
S
=
=
• factorized scatt.
• YB relation
SSS = SSS


{p1, p2, p3} → {p1', p2', p3'} ≠ {p1, p2, p3} unless
∃ Q3 "=" Σ pj^3
• factorized scatt.
• YB relation
SSS = SSS

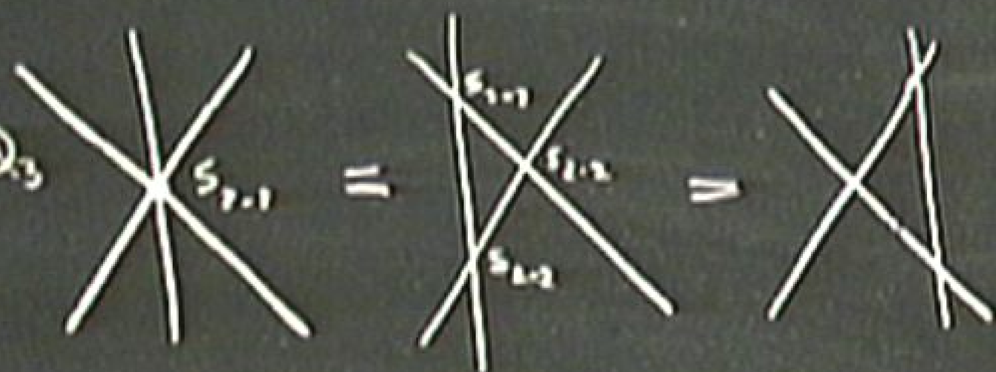

{p1, p2, p3} ⟶ {p1', p2', p3'} ≠ {p1, p2, p3} unless
∃ Q3 "=" Σ pj^3
e^{iαQ3}
S1,2 = =
• factorized scatt.
• YB relation
Ŝ Ŝ Ŝ = Ŝ Ŝ Ŝ


∃ Q3 "=" Σ Pj3
e^±iαQ3
S
=
=
• factorized scatt.
• YB relation
ŜŜŜ = ŜŜŜ
in d

$\{p_1, p_2, p_3\} \longrightarrow \{p_1', p_2', p_3'\} \neq \{p_1, p_2, p_3\}$ unless

$$\exists\, Q_3 \text{ "=" } \Sigma p_j^3$$

$e^{2i\alpha Q_3}$

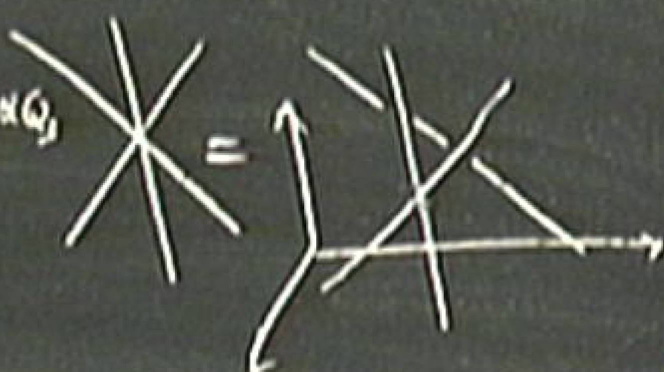


- factorized scatt.
- YB relation

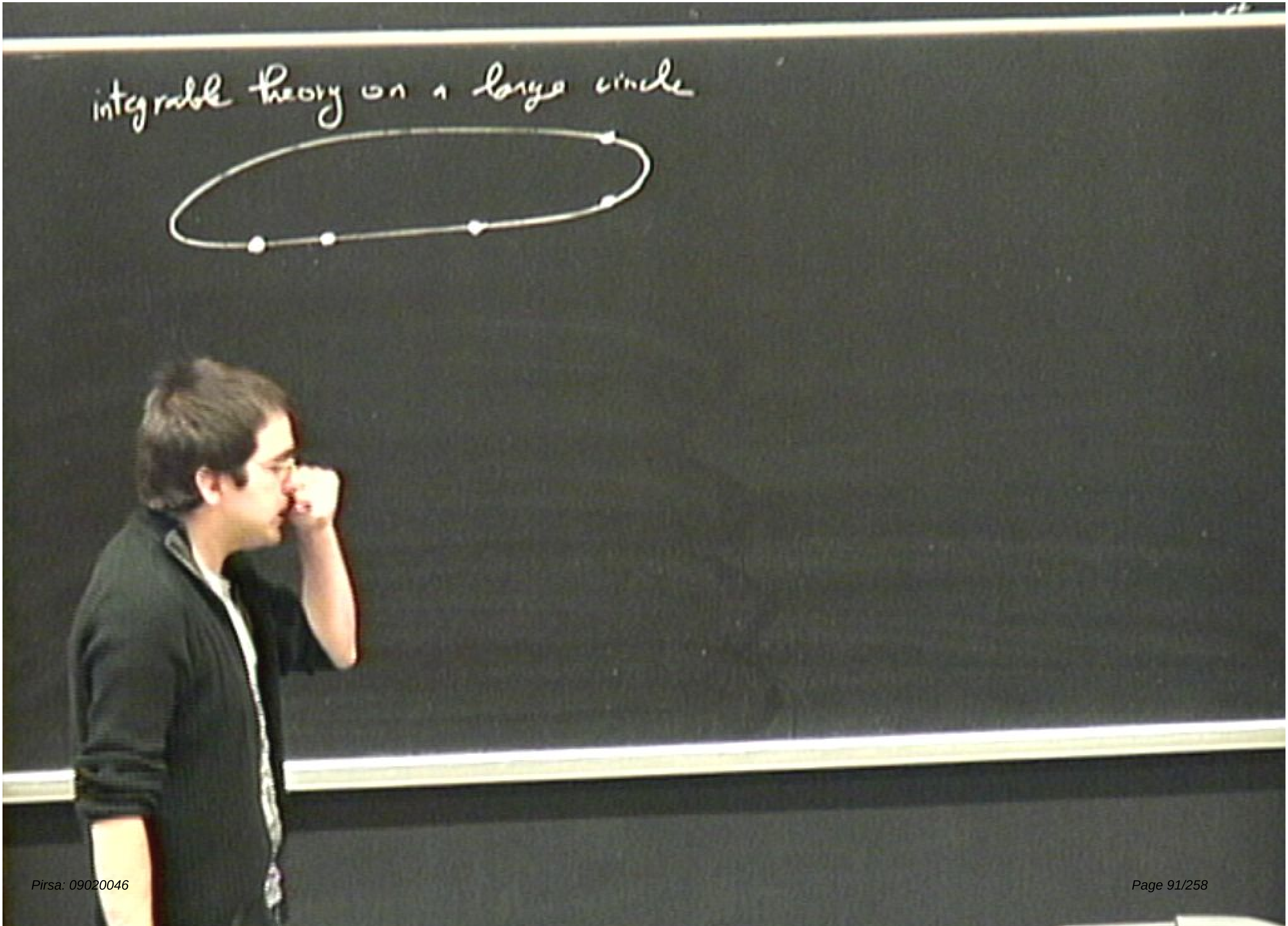
$$\hat{S}\hat{S}\hat{S} = \hat{S}\hat{S}\hat{S}$$

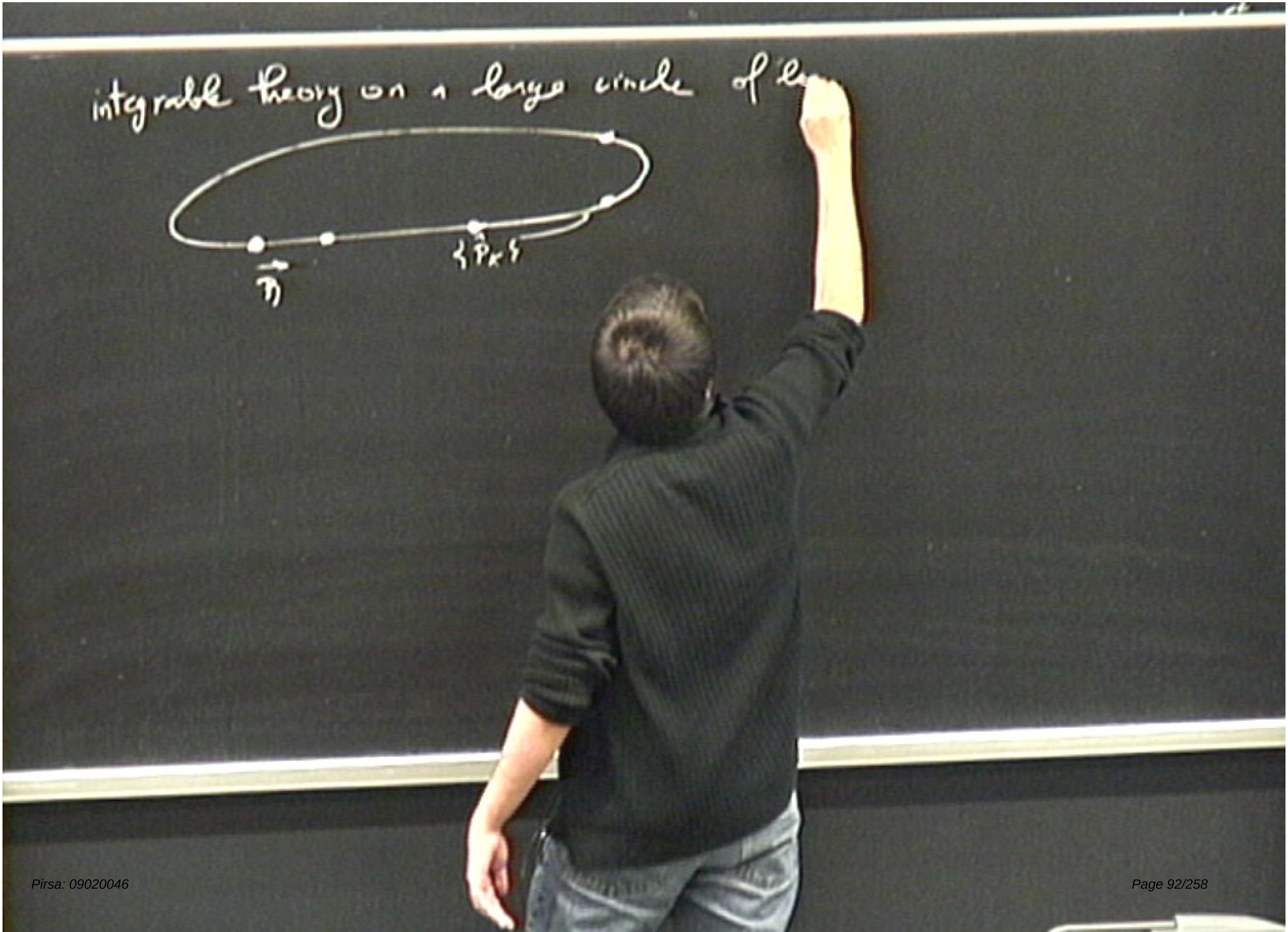
in $d>2$

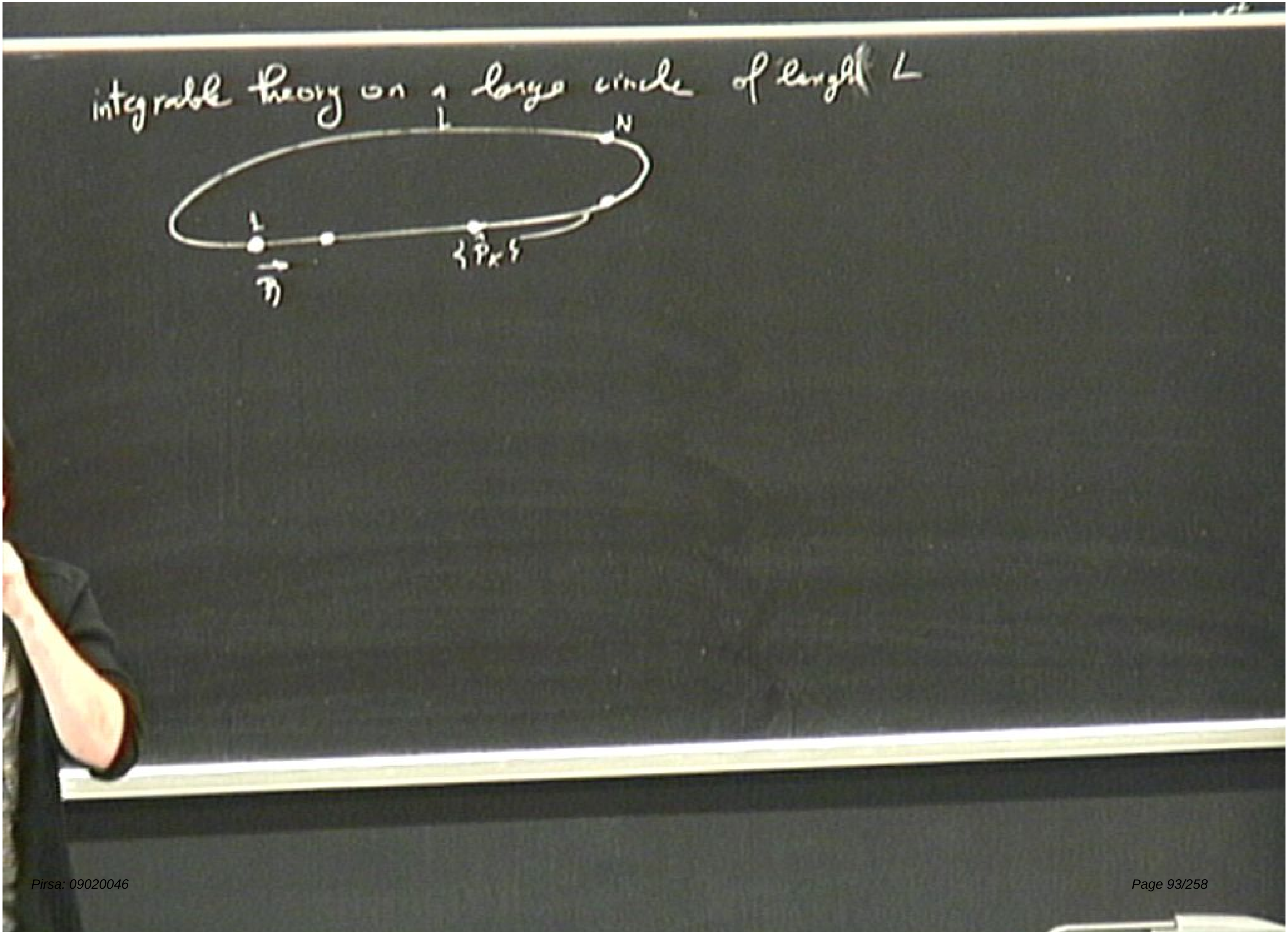
$e^{i\alpha Q_3}$

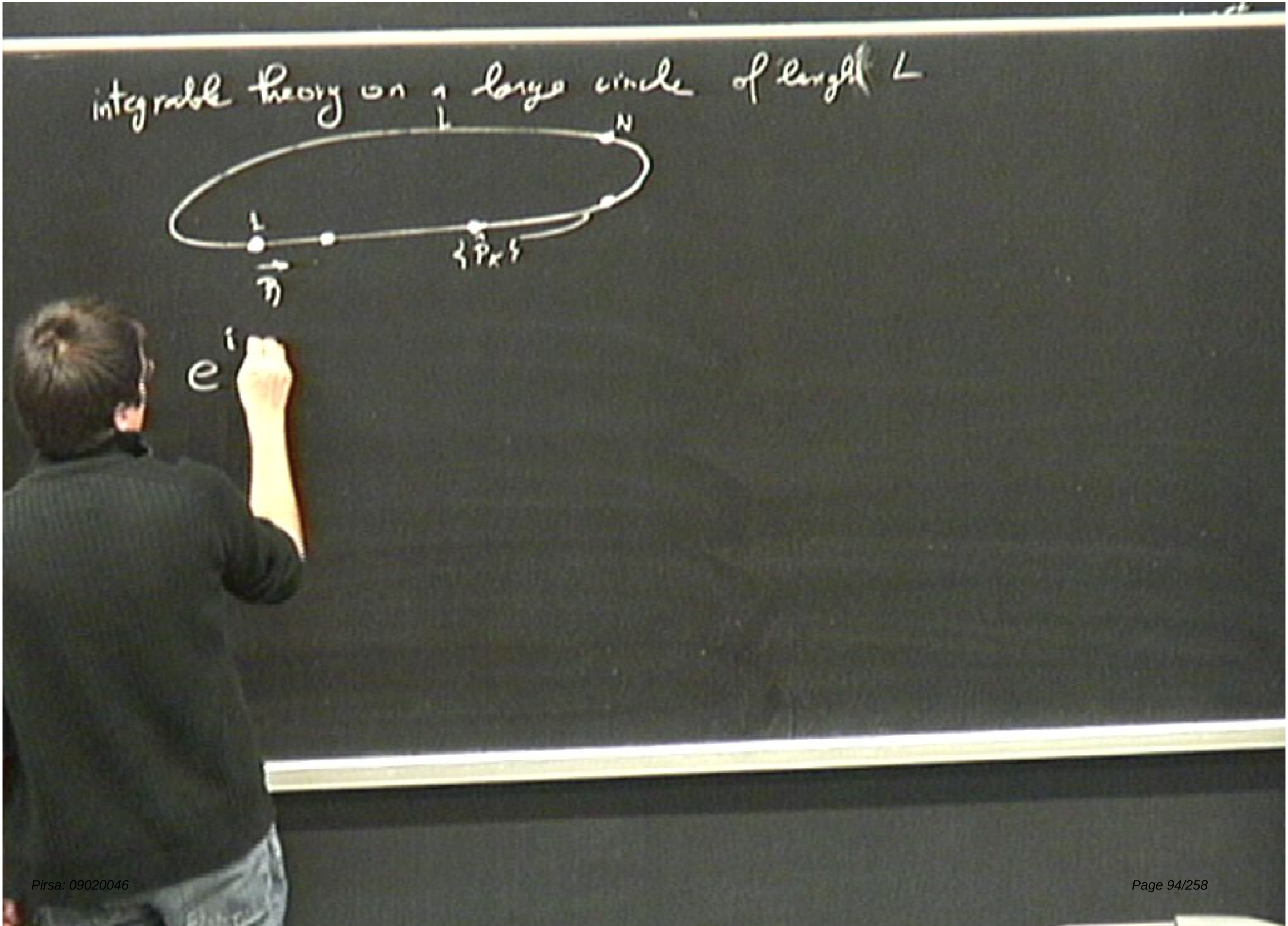


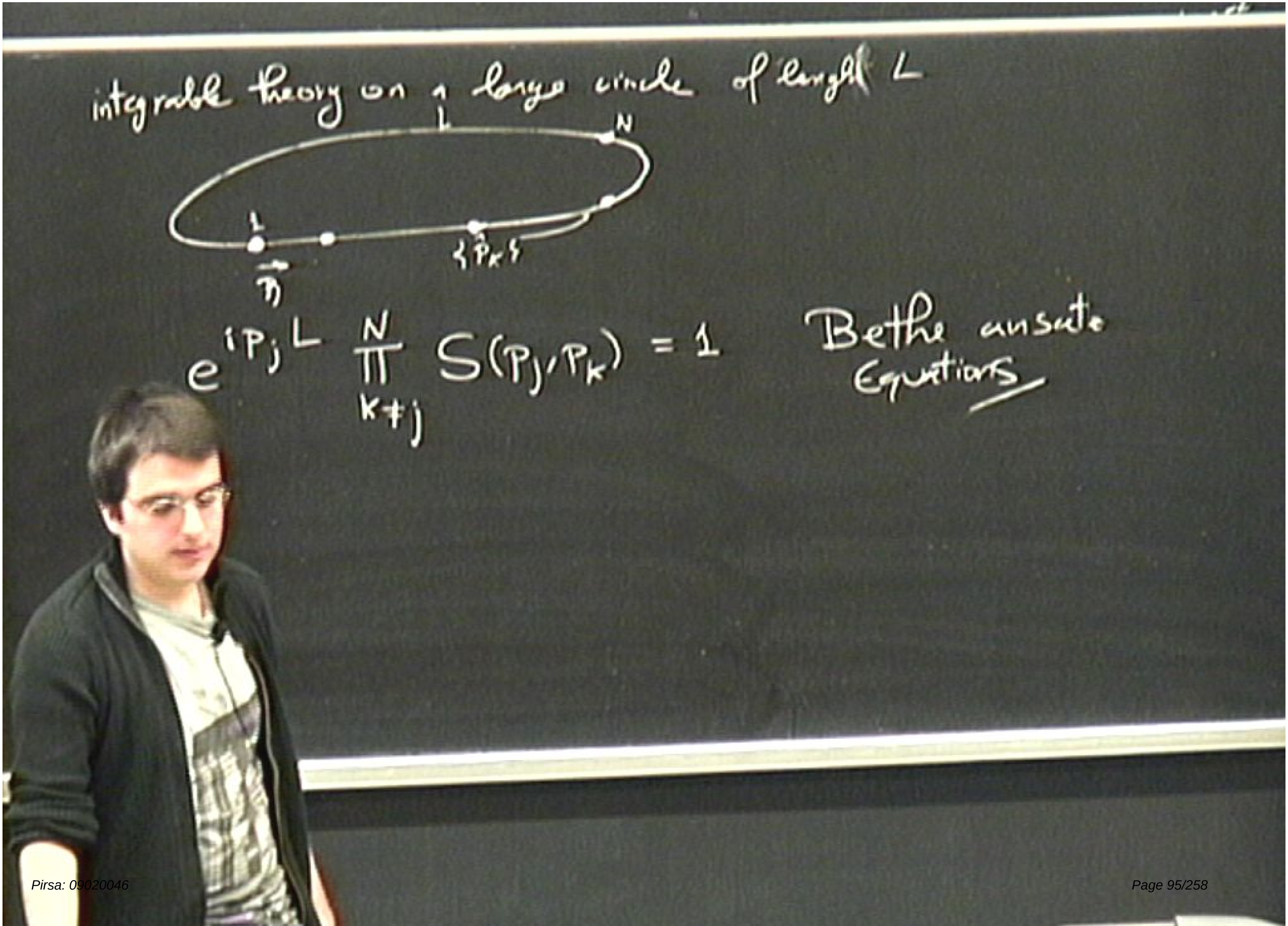
$$\hat{S} = 1$$


integrable theory on a large circle

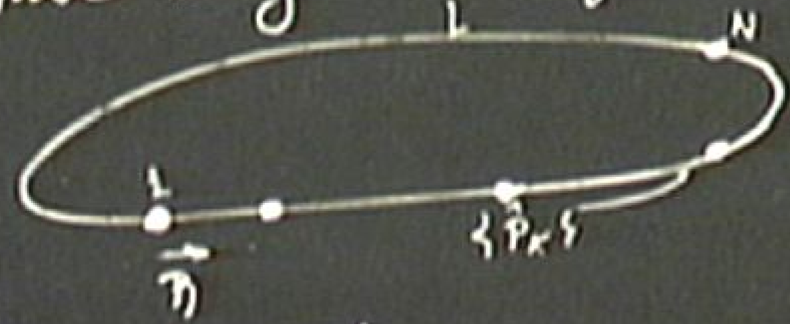

integrable theory on a large circle of
{p_k}
η


integrable theory on a large circle of length L
1
N
1
$\{\vec{p}_k\}$
$\vec{n}$


integrable theory on a large circle of length L
N
{p_k}
e

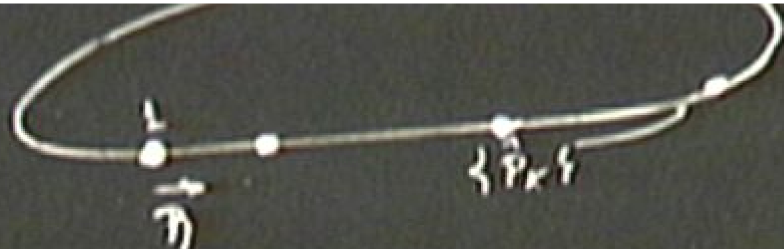

integrable theory on a large circle of length L
1
N
1
{p_k}
$e^{ip_j L} \prod_{k\neq j}^{N} S(p_j, p_k) = 1$
Bethe ansatz
Equations

integrable theory on a large circle of length L



$$e^{ip_j L} \prod_{k \neq j}^{N} S(p_j, p_k) = 1$$

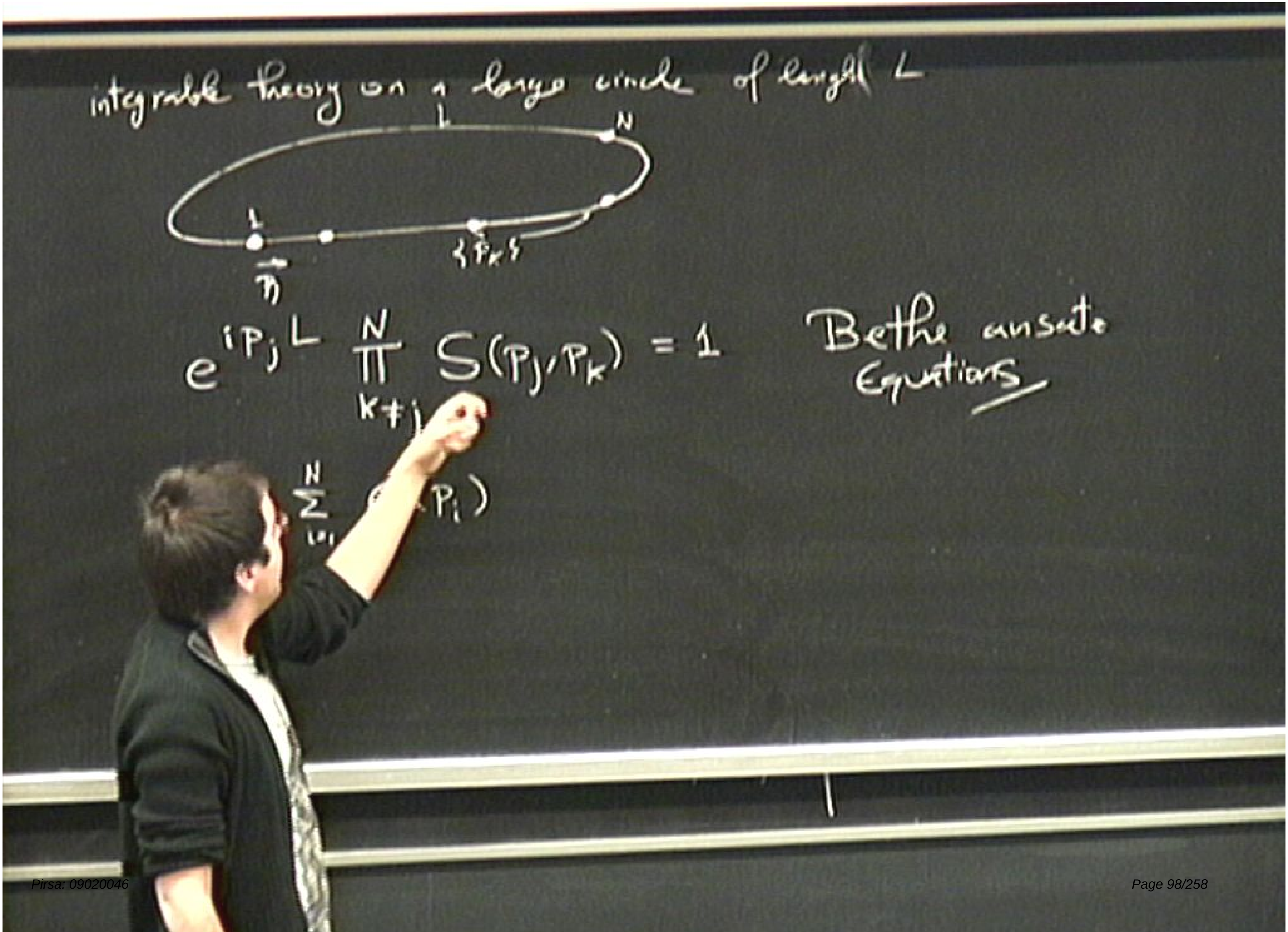
Bethe ansatz Equations

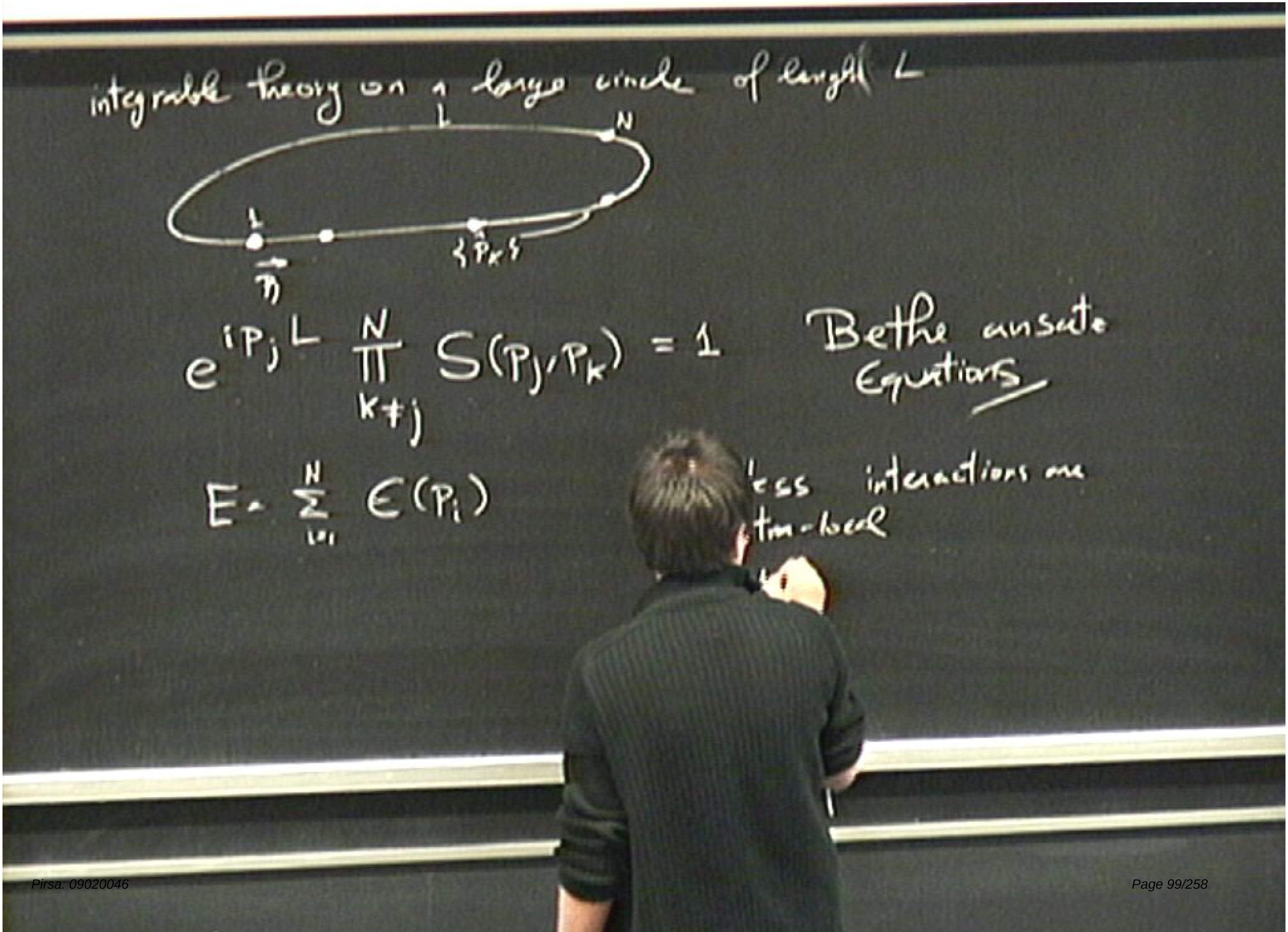


$$e^{ip_jL}\prod_{k\neq j}^{N} S(p_j,p_k)=1$$

Bethe ansatz Equations

$$E=\sum_{i=1}^{N}\epsilon(p_i)$$


integrable theory on a large circle of length L
L
N
L
$\{p_k\}$
$\vec{n}$
$e^{ip_j L} \prod_{k\neq j}^{N} S(p_j,p_k) = 1$
Bethe ansatz Equations
$\sum_{i=1}^{N}$


integrable theory on a large circle of length L
L
N
L
{p_k}
$e^{ip_j L} \prod_{k\neq j}^{N} S(p_j,p_k) = 1$
Bethe ansatz Equations
$E = \sum_{i=1}^{N} \epsilon(p_i)$
interactions are

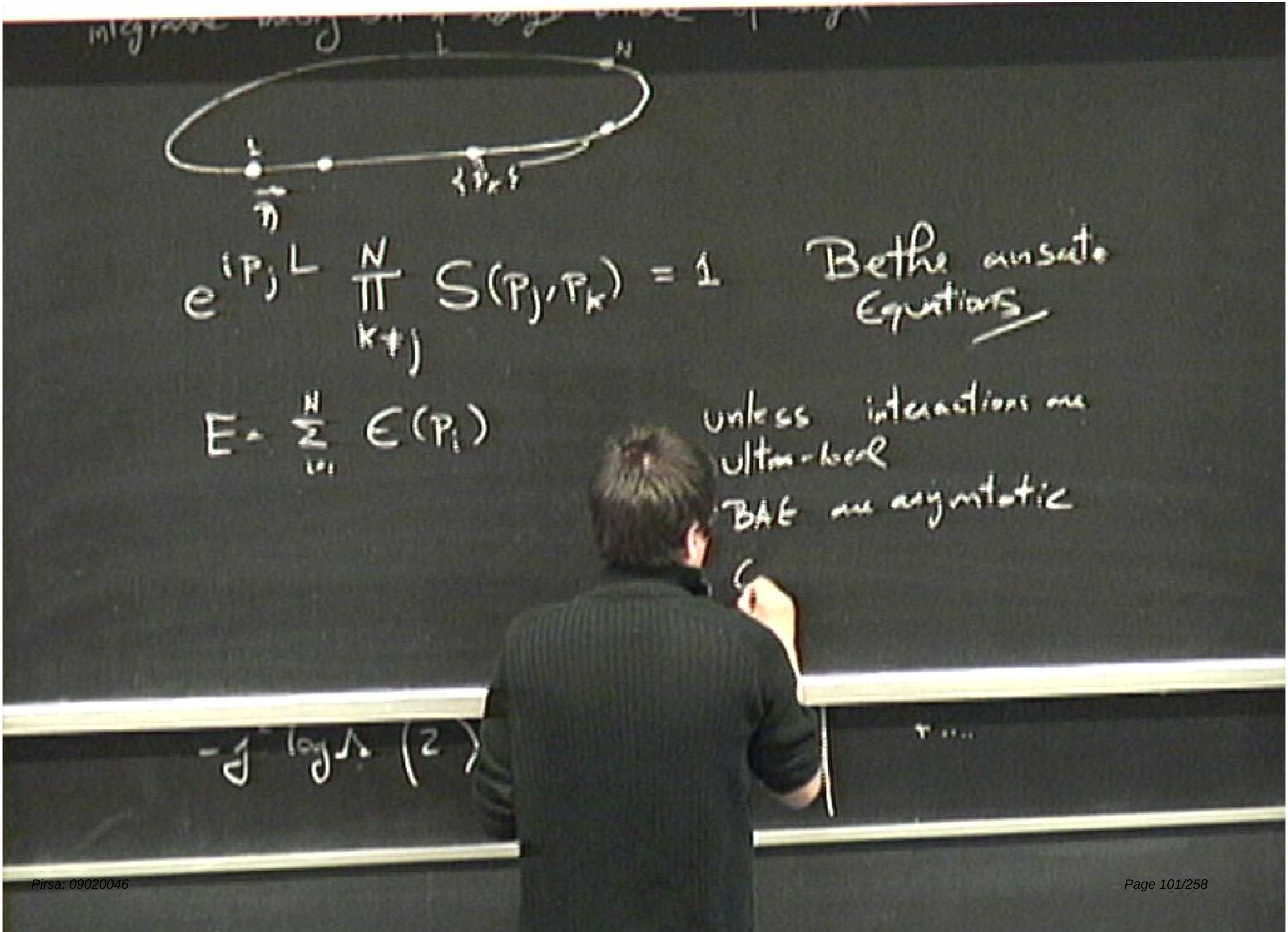
integrable theory on a large circle of length L

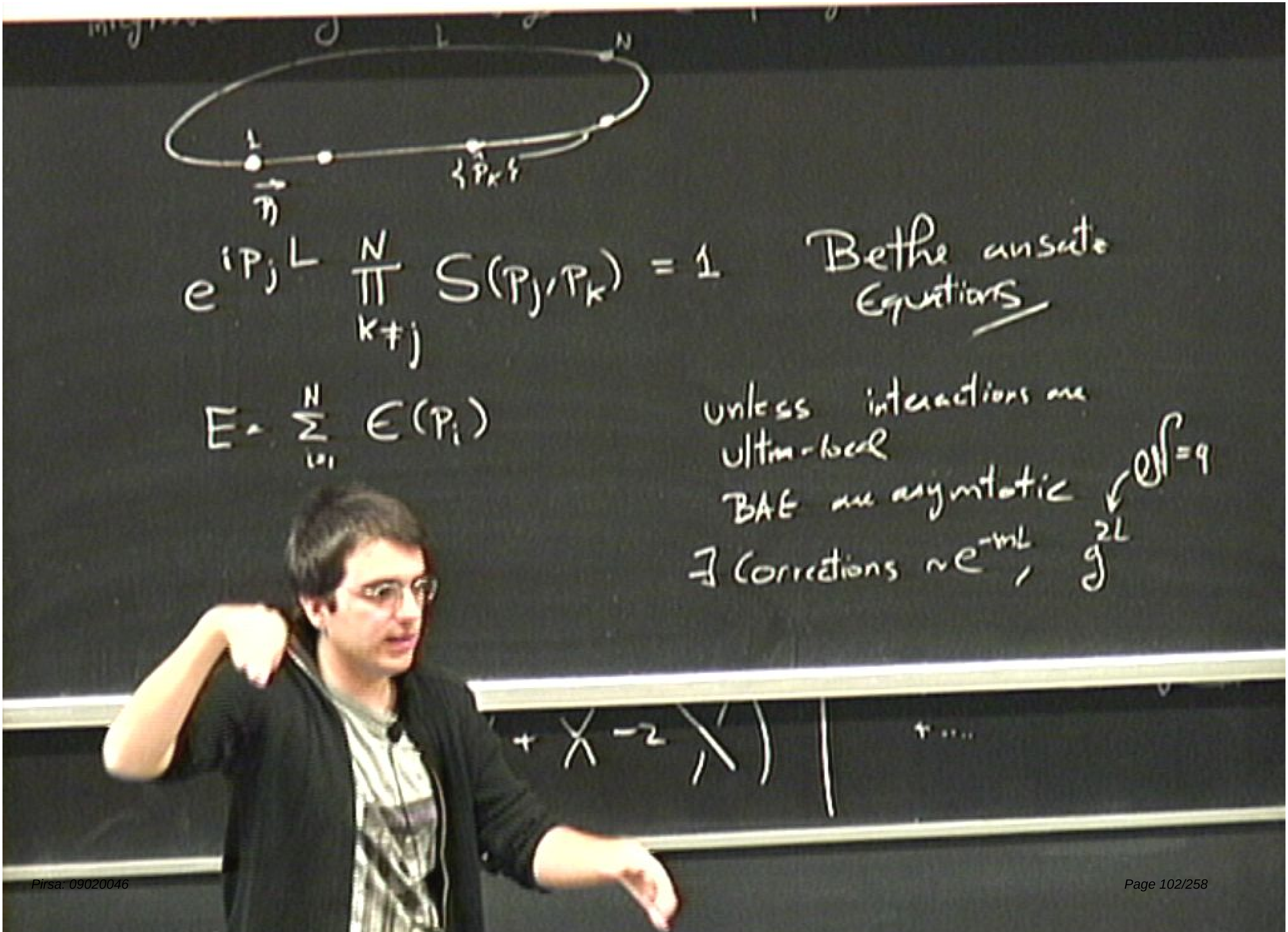
$$e^{ip_j L} \prod_{k \neq j}^{N} S(p_j, p_k) = 1$$

Bethe ansatz Equations

$$E = \sum_{i=1}^{N} \epsilon(p_i)$$

unless interactions are ultra-local
BAE are asymptotic

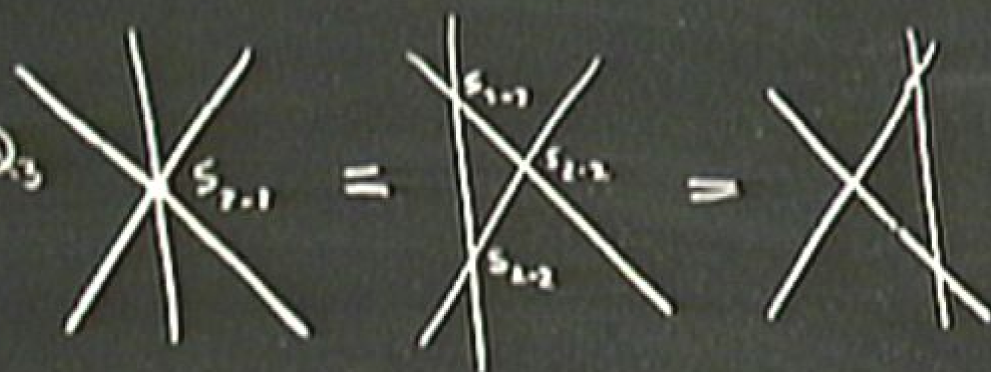

{p_k}
e^{ip_j L} ∏_{k≠j}^{N} S(p_j, p_k) = 1
Bethe ansatz Equations
E = Σ_{i=1}^{N} ε(p_i)
unless interactions are
ultra-local
BAE are asymptotic


L
N
1
$\vec{\pi}$
$\{p_k\}$
$e^{ip_jL}\prod_{k\neq j}^{N} S(p_j,p_k) = 1$
Bethe ansatz Equations
$E = \sum_{i=1}^{N} \epsilon(p_i)$
unless interactions are ultra-local
BAE are asymptotic
$\exists$ Corrections $\sim e^{-mL}$, g^{2L}
$e^{\ell}=q$

$\{p_1, p_2, p_3\} \longrightarrow \{p_1', p_2', p_3'\} \neq \{p_1, p_2, p_3\}$ unless

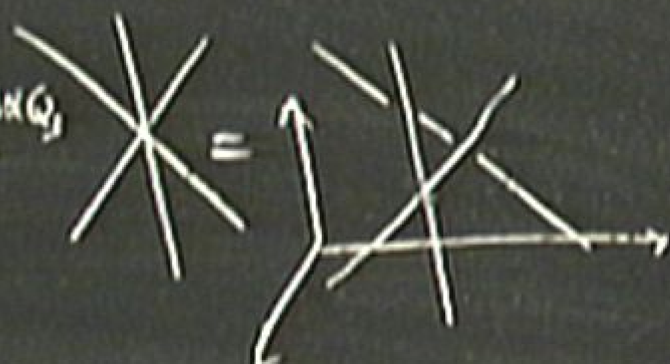
$\exists\, Q_3$ "=" $\sum p_j^3$

$e^{2i\alpha Q_3}$

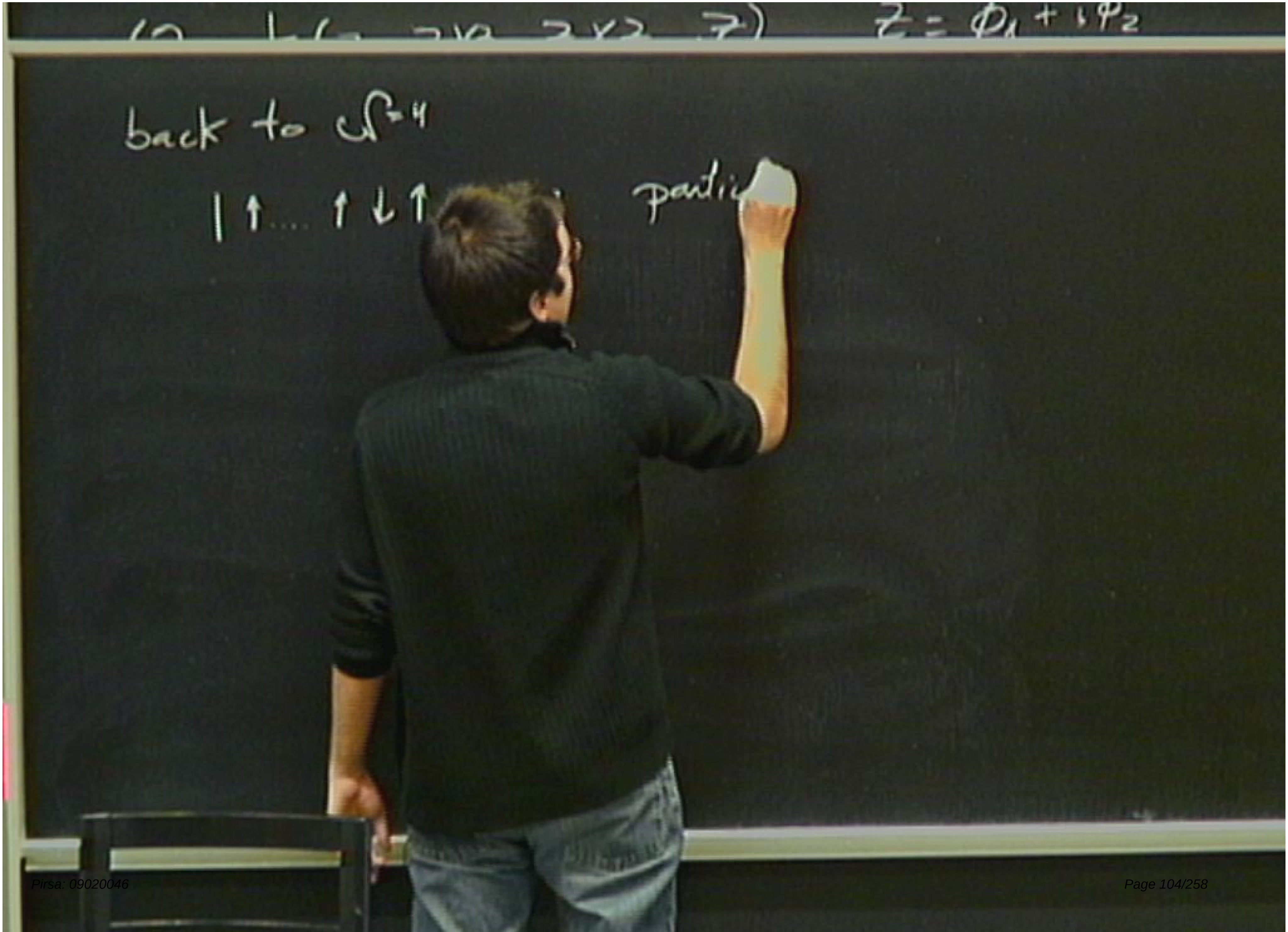


- factorized scatt.
- YB relation
  $\hat{S}\hat{S}\hat{S} = \hat{S}\hat{S}\hat{S}$

in $d>2$ $e^{i\alpha Q_3}$



$\hat{S} = 1$


back to $\mathcal{N}=4$
$Z = \Phi_1 + i\varphi_2$

back to $\mathcal{N}=4$

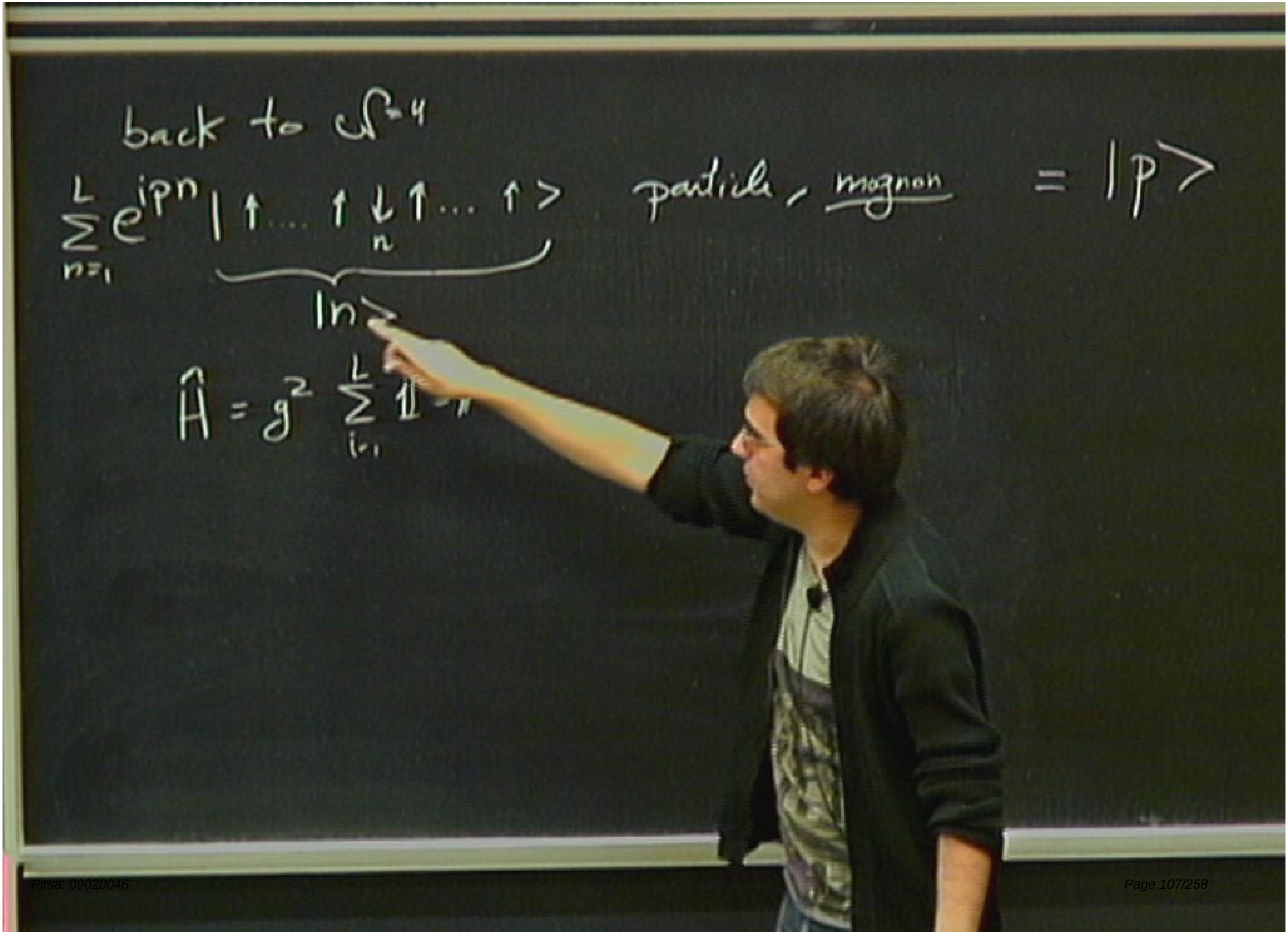
$$\sum_{n=1}^{L} e^{ipn} \underbrace{|\uparrow \dots \uparrow \underset{n}{\downarrow} \uparrow \dots \uparrow>}_{|n>}$$

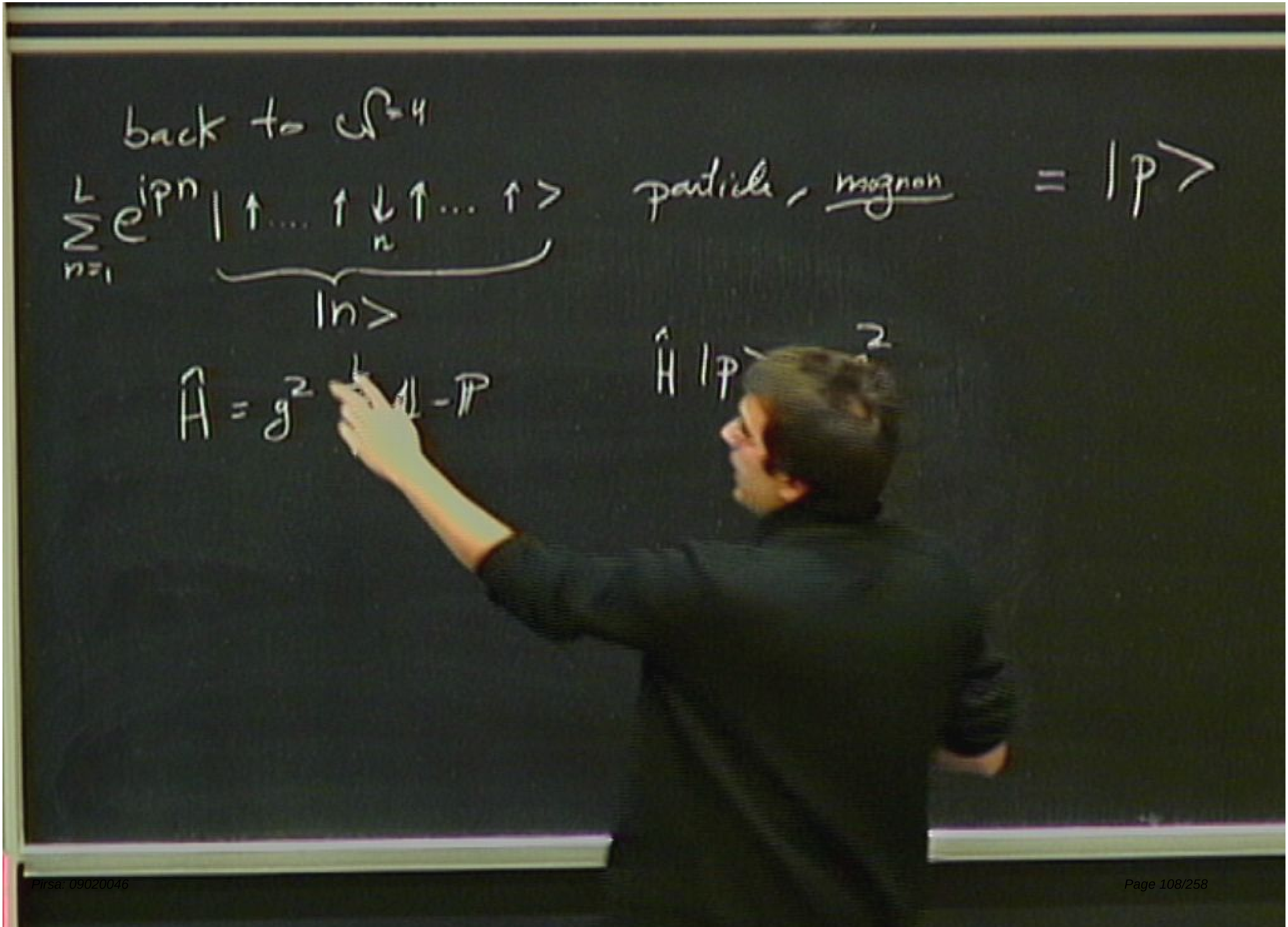
particle, magnon

$$\sum_{n=1}^{L} e^{ipn} \underbrace{| \uparrow \ldots \uparrow \underset{n}{\downarrow} \uparrow \ldots \uparrow >}_{|n>}$$

particle, magnon

$$\hat{H} = g^2 \sum_{l=1}^{L} \mathbb{1} - \mathbb{P}$$


back to N=4
Σ_{n=1}^{L} e^{ipn} |↑ .... ↑ ↓_n ↑ ... ↑ >
particle, magnon
= |p>
|n>
Â = g² Σ_{l=1}^{L} 𝟙


back to N=4
$\sum_{n=1}^{L} e^{ipn} |\uparrow \ldots \uparrow \downarrow_n \uparrow \ldots \uparrow\rangle$ particle, magnon $= |p\rangle$
$|n\rangle$
$\hat{H} = g^2 \sum^{L} \mathbb{1} - \mathbb{P}$
$\hat{H}\,|p\rangle$

back to $\mathcal{N}=4$

$$\sum_{n=1}^{L} e^{ipn} \underbrace{|\uparrow \dots \uparrow \downarrow_n \uparrow \dots \uparrow\rangle}_{|n\rangle}$$

particle, magnon $= |p\rangle$

$$\hat{H} = g^2 \sum_{l=1}^{L} \mathbb{1} - \mathbb{P}$$

$$\hat{H}|p\rangle = g^2 (1 - e^{ip}$$

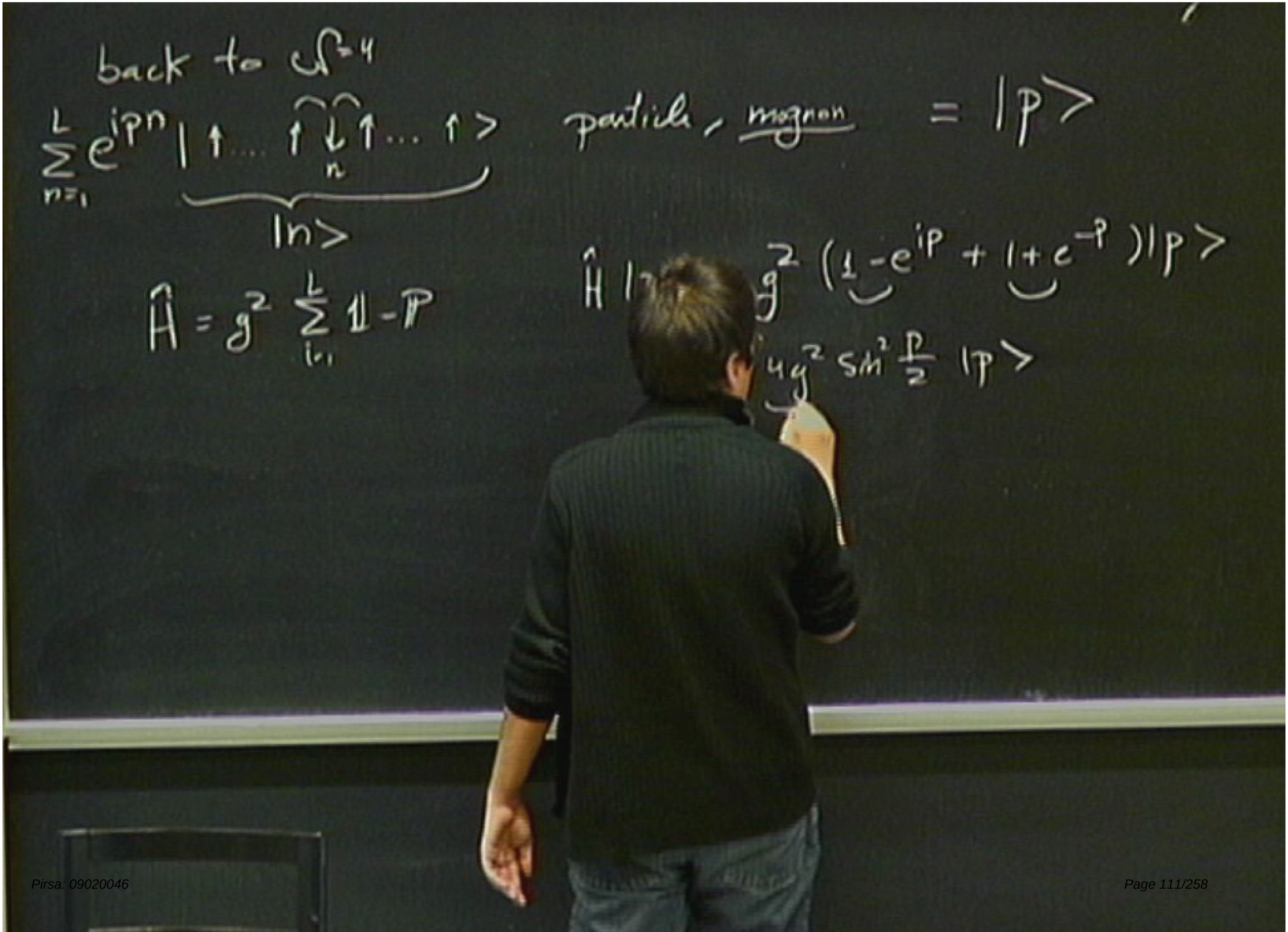
back to $\mathcal{N}=4$

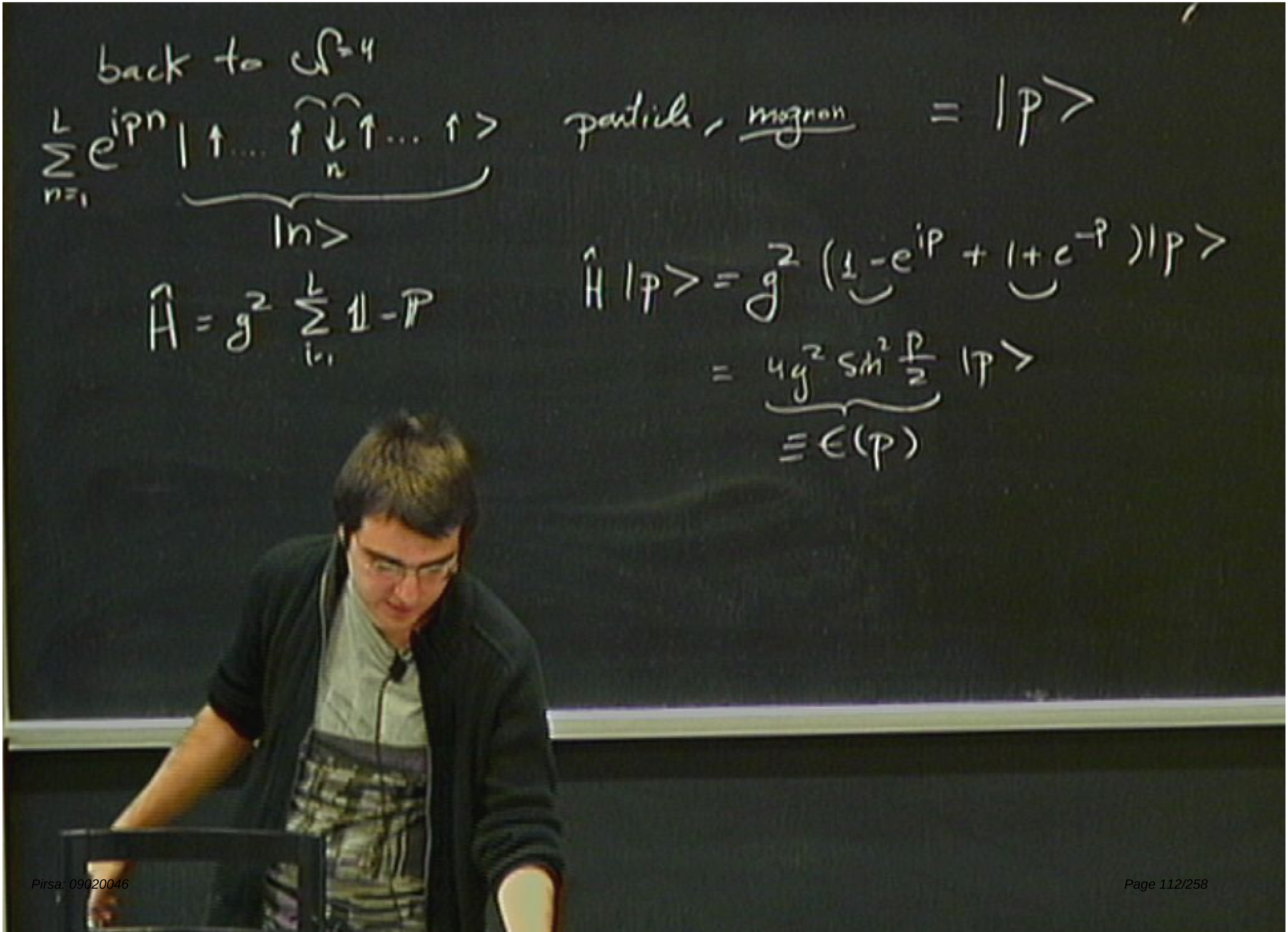
$$\sum_{n=1}^{L} e^{ipn} \underbrace{|\uparrow \ldots \uparrow \underset{n}{\downarrow} \uparrow \ldots \uparrow\rangle}_{|n\rangle}$$

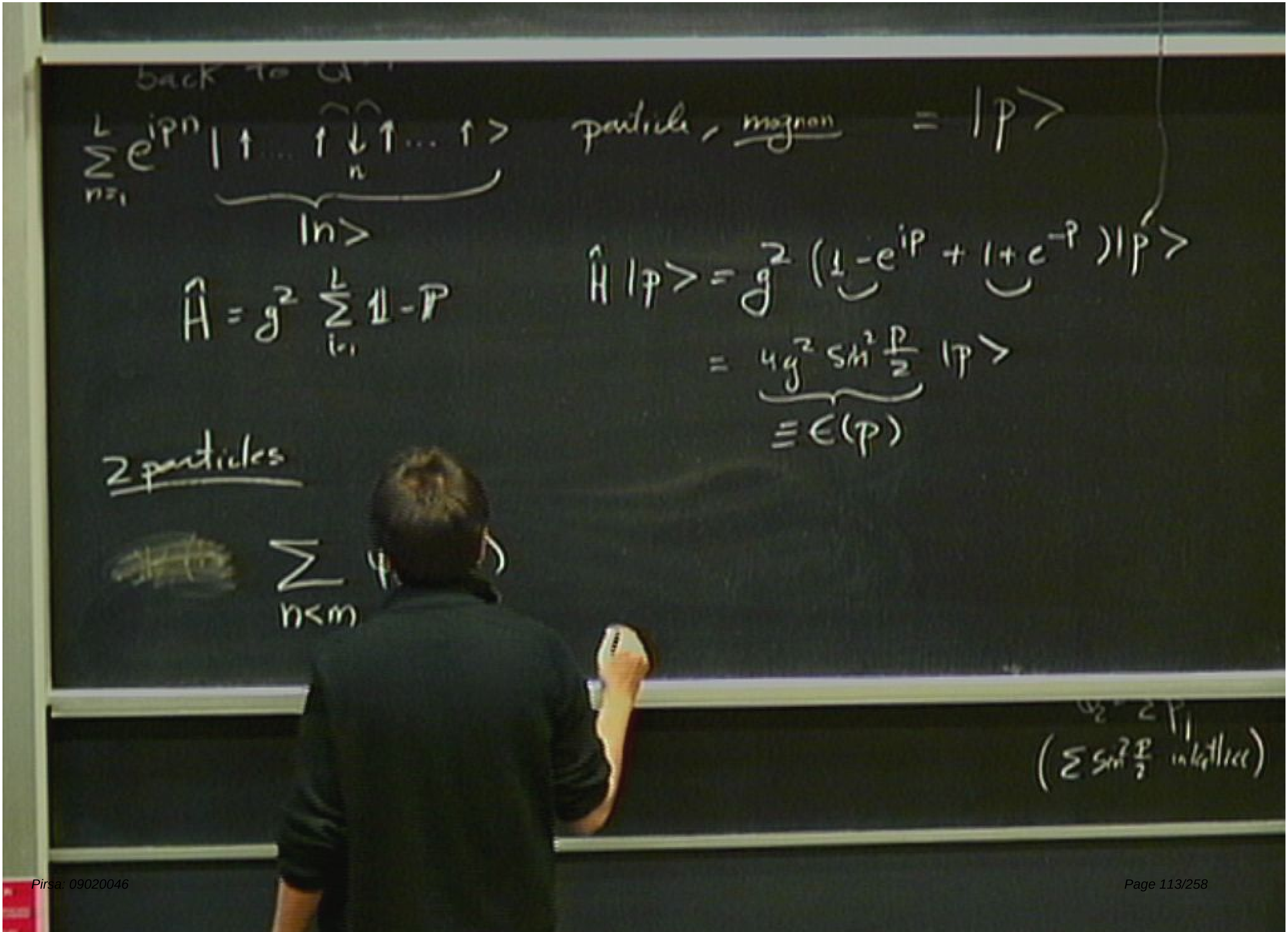
particle, magnon $= |p\rangle$

$$\hat{H} = g^2 \sum_{l=1}^{L} \mathbb{1} - \mathbb{P}$$

$$\hat{H}\,|p\rangle = g^2 \left(1 - e^{ip} + 1 + e^{-ip} \right.$$


back to N=4
$\sum_{n=1}^{L} e^{ipn} |\uparrow \ldots \uparrow \downarrow \uparrow \ldots \uparrow\rangle$
$|n\rangle$
particle, magnon $= |p\rangle$
$\hat{H} |p\rangle = g^2 (1 - e^{ip} + 1 + e^{-ip}) |p\rangle$
$\hat{H} = g^2 \sum_{l=1}^{L} \mathbb{1} - \mathbb{P}$
$= 4g^2 \sin^2 \frac{p}{2} |p\rangle$


back to N=4
particle, magnon = |p>
$\sum_{n=1}^{L} e^{ipn} |\uparrow \ldots \uparrow \downarrow \uparrow \ldots \uparrow>$
|n>
$\hat{H} = g^2 \sum_{i=1}^{L} 1 - P$
$\hat{H}|p> = g^2 (1 - e^{ip} + 1 - e^{-ip})|p>$
$= 4g^2 \sin^2 \frac{p}{2} |p>$
$\equiv \epsilon(p)$


back to Q
$\sum_{n=1}^{L} e^{ipn} |\uparrow \ldots \uparrow \downarrow_n \uparrow \ldots \uparrow\rangle$
$|n\rangle$
particle, magnon
$= |p\rangle$
$\hat{H} = g^2 \sum_{l=1}^{L} \mathbb{1} - \mathbb{P}$
$\hat{H} |p\rangle = g^2 (1 - e^{ip} + 1 - e^{-ip}) |p\rangle$
$= 4g^2 \sin^2 \frac{p}{2} |p\rangle$
$\equiv \epsilon(p)$
2 particles
$\sum_{n<m}$
$(\sum \sin^2 \frac{p}{2}$ in lattice$)$

back to Q...

$$\sum_{n=1}^{L} e^{ipn} \underbrace{|\uparrow \ldots \uparrow \overset{\frown\frown}{\underset{n}{\downarrow}} \uparrow \ldots \uparrow\rangle}_{|n\rangle}$$

particle, magnon $= |p\rangle$

$$\hat{H} = g^2 \sum_{l=1}^{L} \mathbb{1} - \mathbb{P}$$

$$\hat{H}|p\rangle = g^2 (1 - e^{ip} + 1 - e^{-ip})|p\rangle$$

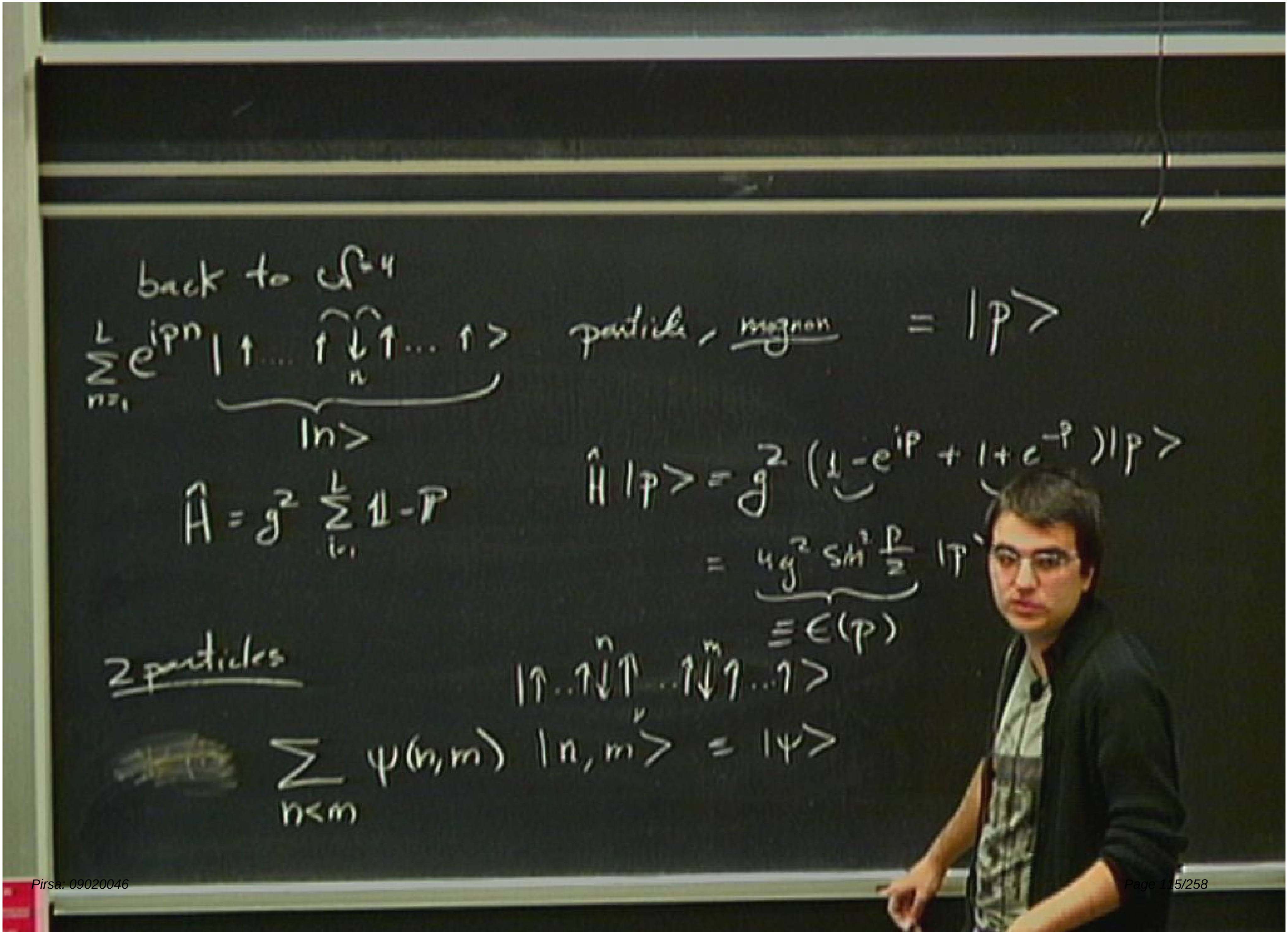
$$= \underbrace{4g^2 \sin^2\frac{p}{2}}_{\equiv \epsilon(p)} |p\rangle$$

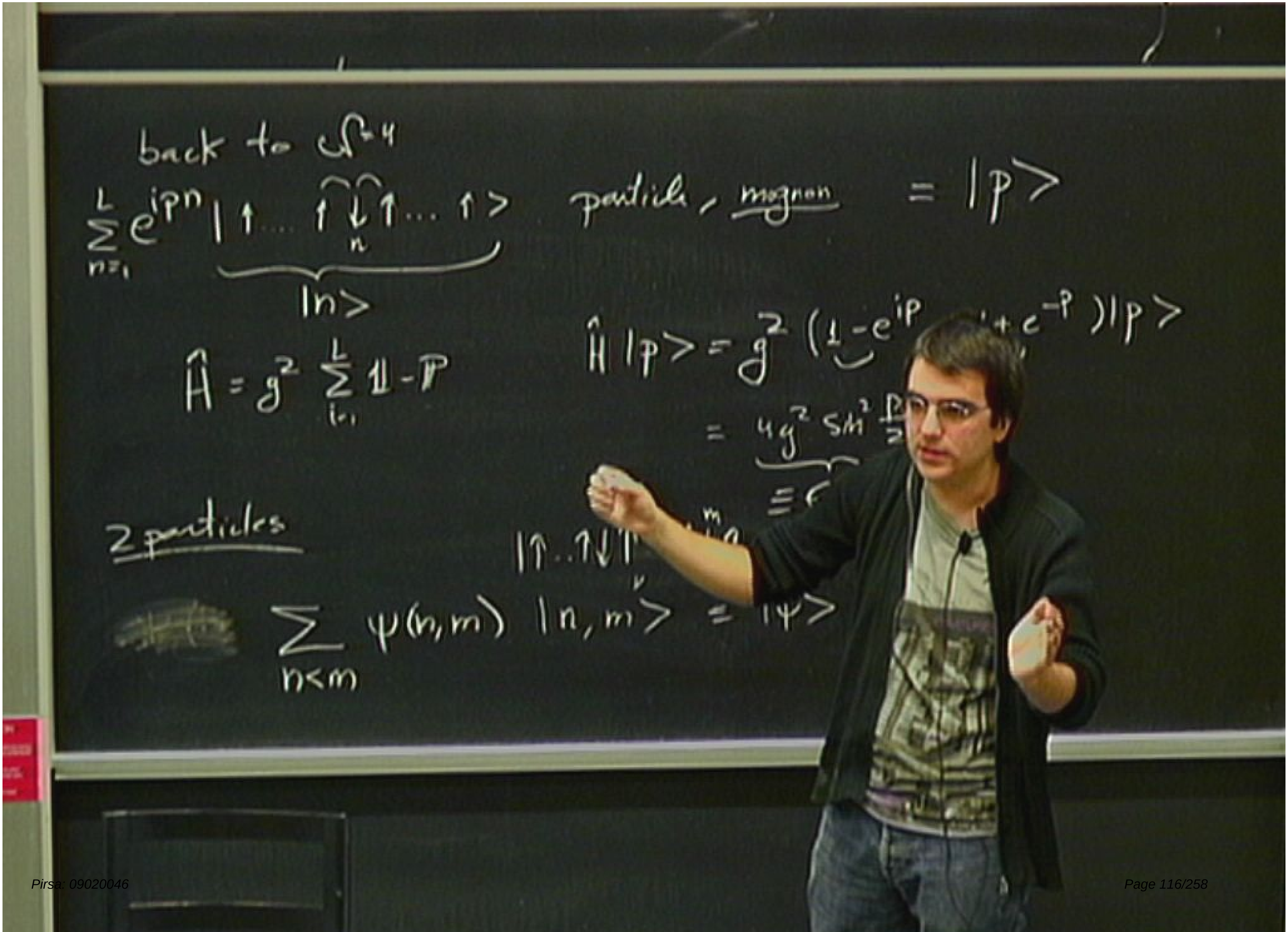
2 particles

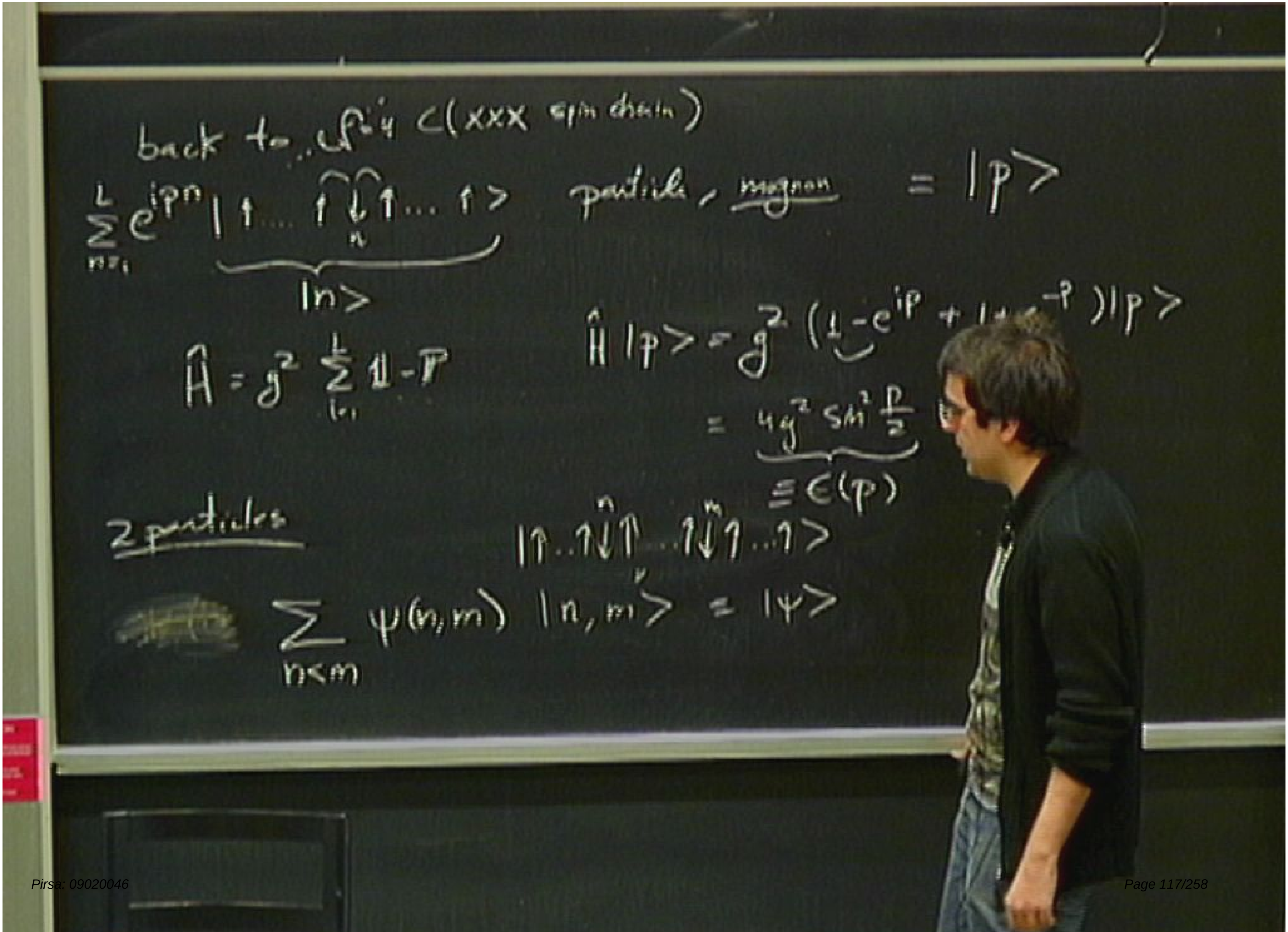
$$|\uparrow \ldots \uparrow \overset{n}{\downarrow} \uparrow \ldots \uparrow \overset{m}{\downarrow} \uparrow \ldots \uparrow\rangle$$

$$\sum_{n<m} \psi(n,m)\, |n,m\rangle$$

$(\Sigma \sin^2$


back to $\mathcal{N}=4$
$\sum_{n=1}^{L} e^{ipn} |\uparrow \ldots \uparrow \downarrow \uparrow \ldots \uparrow\rangle$ particle, magnon $= |p\rangle$
$|n\rangle$
$\hat{H} = g^2 \sum_{i=1}^{L} \mathbb{1} - \mathbb{P}$
$\hat{H}|p\rangle = g^2 (1 - e^{ip} + 1 + e^{-ip})|p\rangle$
$= 4g^2 \sin^2 \frac{p}{2} |p\rangle$
$\equiv \epsilon(p)$
2 particles
$|\uparrow \ldots \uparrow \downarrow \uparrow \ldots \uparrow \downarrow \uparrow \ldots \uparrow\rangle$
$\sum_{n<m} \psi(n,m) |n,m\rangle = |\psi\rangle$


back to N=4
particle, magnon = |p>
|n>
Ĥ = g² Σ 1 - P
Ĥ |p> = g² (1 - e^ip
= 4g² sin² p/2
2 particles
|↑..↑↓↑
Σ ψ(n,m) |n, m> = |ψ>
n<m


back to (XXX spin chain)
particle, magnon
= |p>
|n>
2 particles
n<m
= |ψ>

SU(2) sector

back to [illegible] (XXX spin chain)

$$\sum_{n=1}^{L} e^{ipn} \underbrace{|\uparrow \dots \uparrow \underset{n}{\downarrow} \uparrow \dots \uparrow\rangle}_{|n\rangle}$$

particle, magnon $= |p\rangle$

$$\hat{H} = g^2 \sum_{l=1}^{L} \mathbb{1} - P$$

$$\hat{H}|p\rangle = g^2 (1 - e^{ip} + 1 - e^{-ip})|p\rangle$$

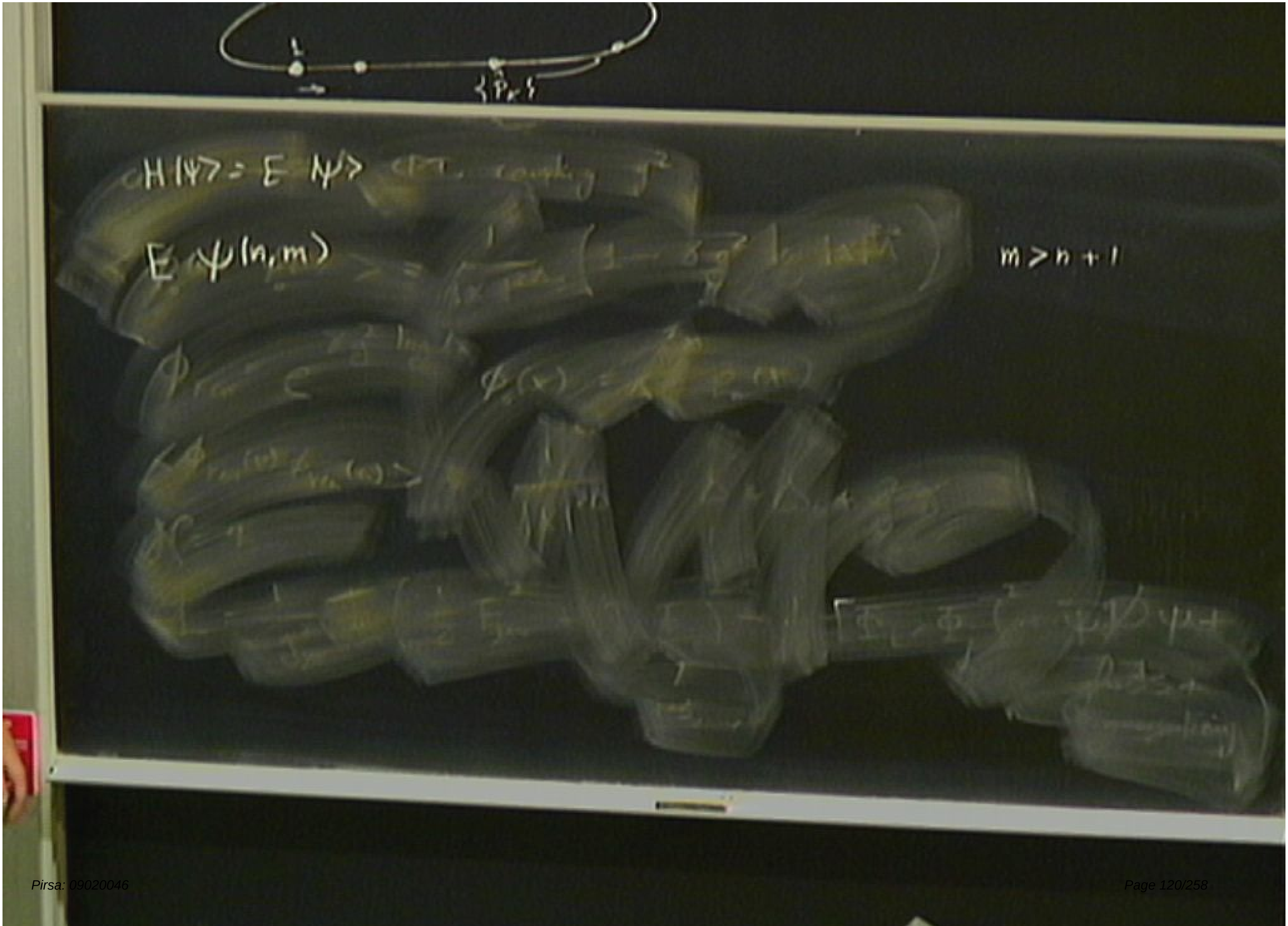
$$= \underbrace{4g^2 \sin^2\frac{p}{2}}_{\equiv \epsilon(p)} |p\rangle$$

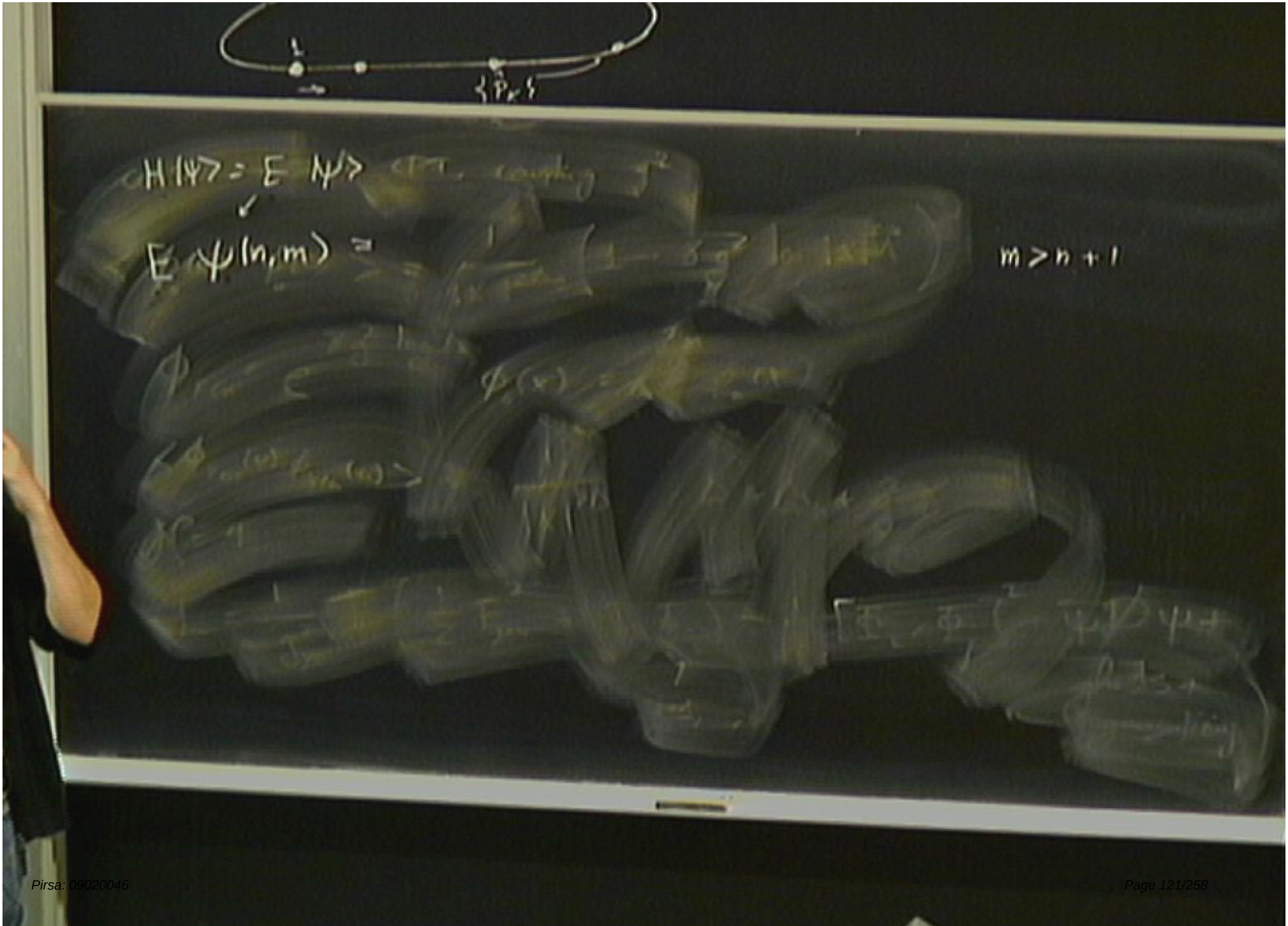
2 particles

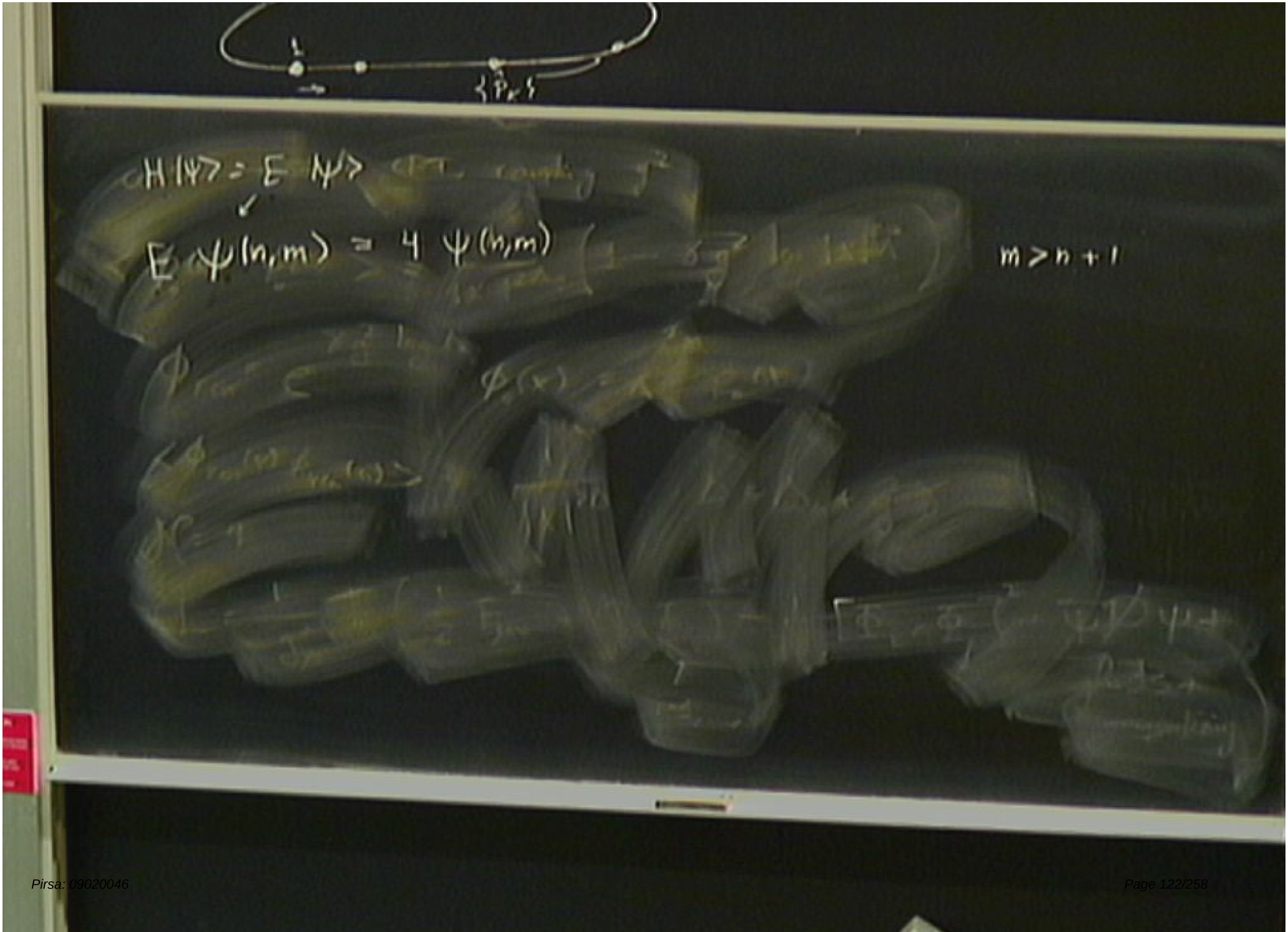
$$|\uparrow \dots \uparrow \overset{n}{\downarrow} \uparrow \dots \uparrow \overset{m}{\downarrow} \uparrow \dots \uparrow\rangle$$

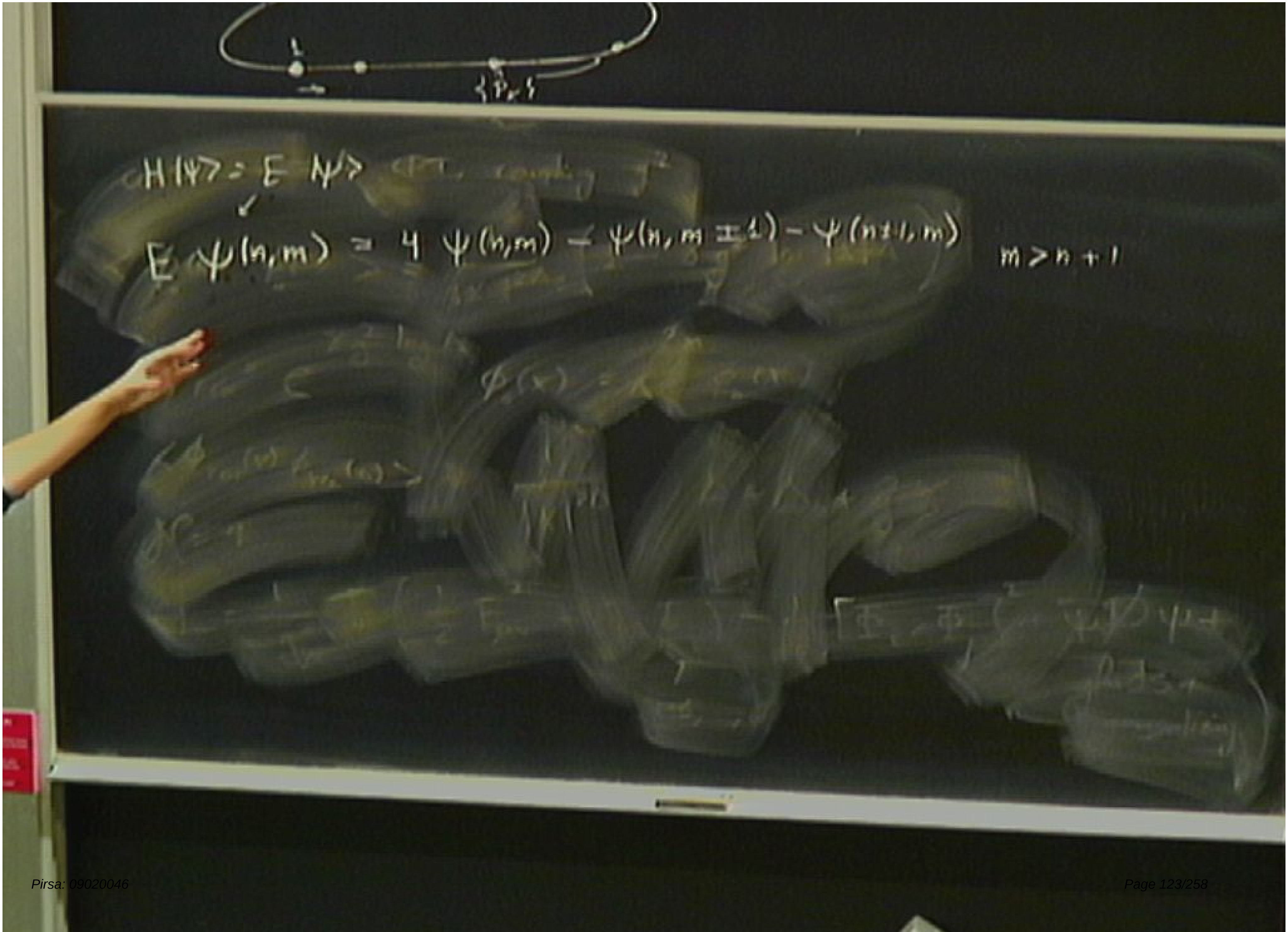
$$\sum_{n<m} \psi(n,m)\, |n,m\rangle = |\psi\rangle$$

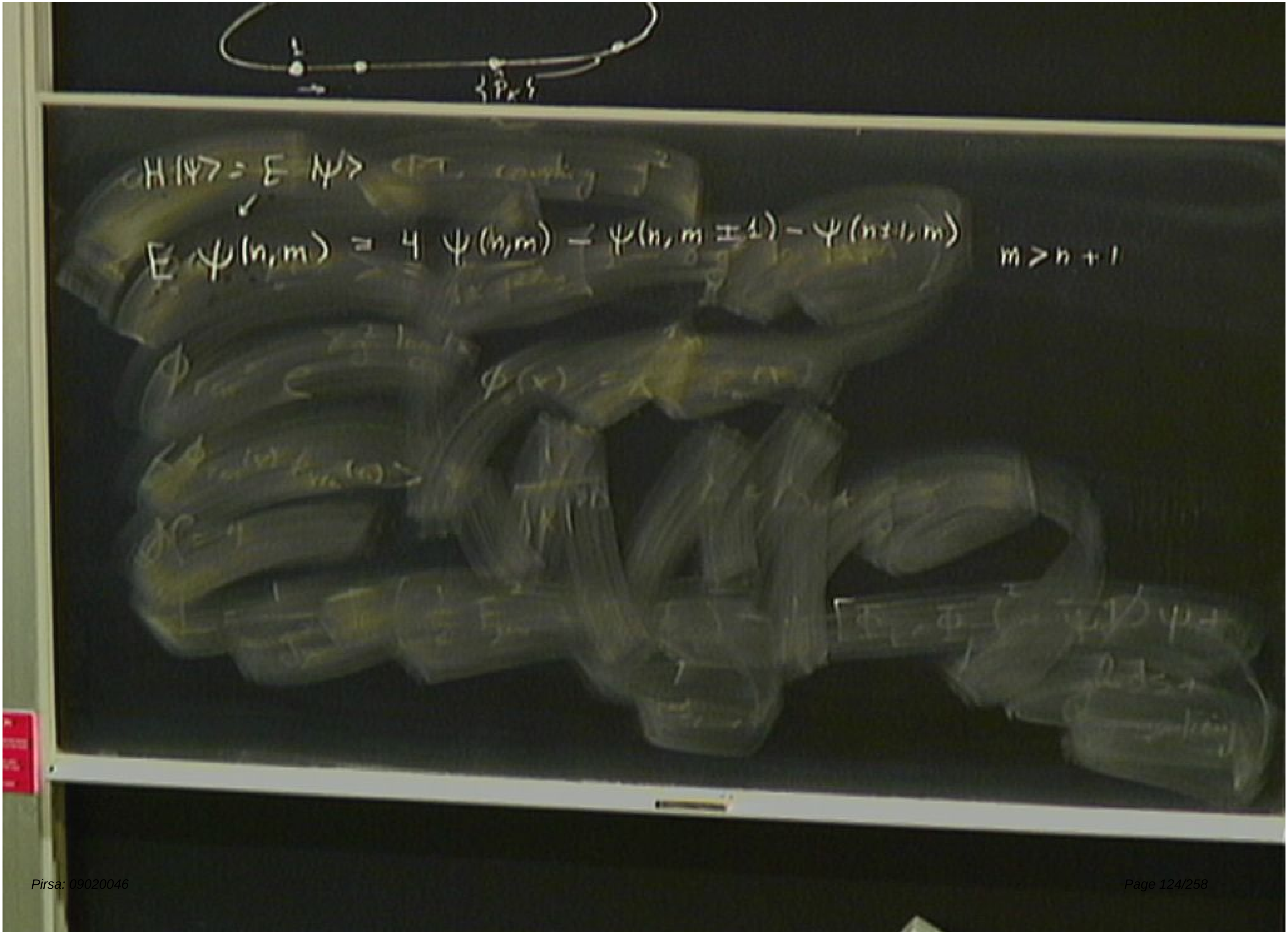
$H|\psi\rangle = E|\psi\rangle$

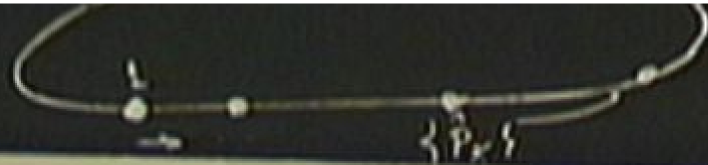

H|ψ> = E |ψ>
E ψ(n,m)
m > n + 1


H|ψ> = E |ψ>
E ψ(n,m) =
m > n + 1


H|ψ> = E |ψ>
E ψ(n,m) = 4 ψ(n,m)
m > n + 1


H|ψ> = E |ψ>
E ψ(n,m) = 4 ψ(n,m) − ψ(n, m ± 1) − ψ(n±1, m)
m > n + 1


{P_x}
H|ψ> = E |ψ>
E ψ(n,m) = 4 ψ(n,m) − ψ(n, m ± 1) − ψ(n±1, m)
m > n + 1

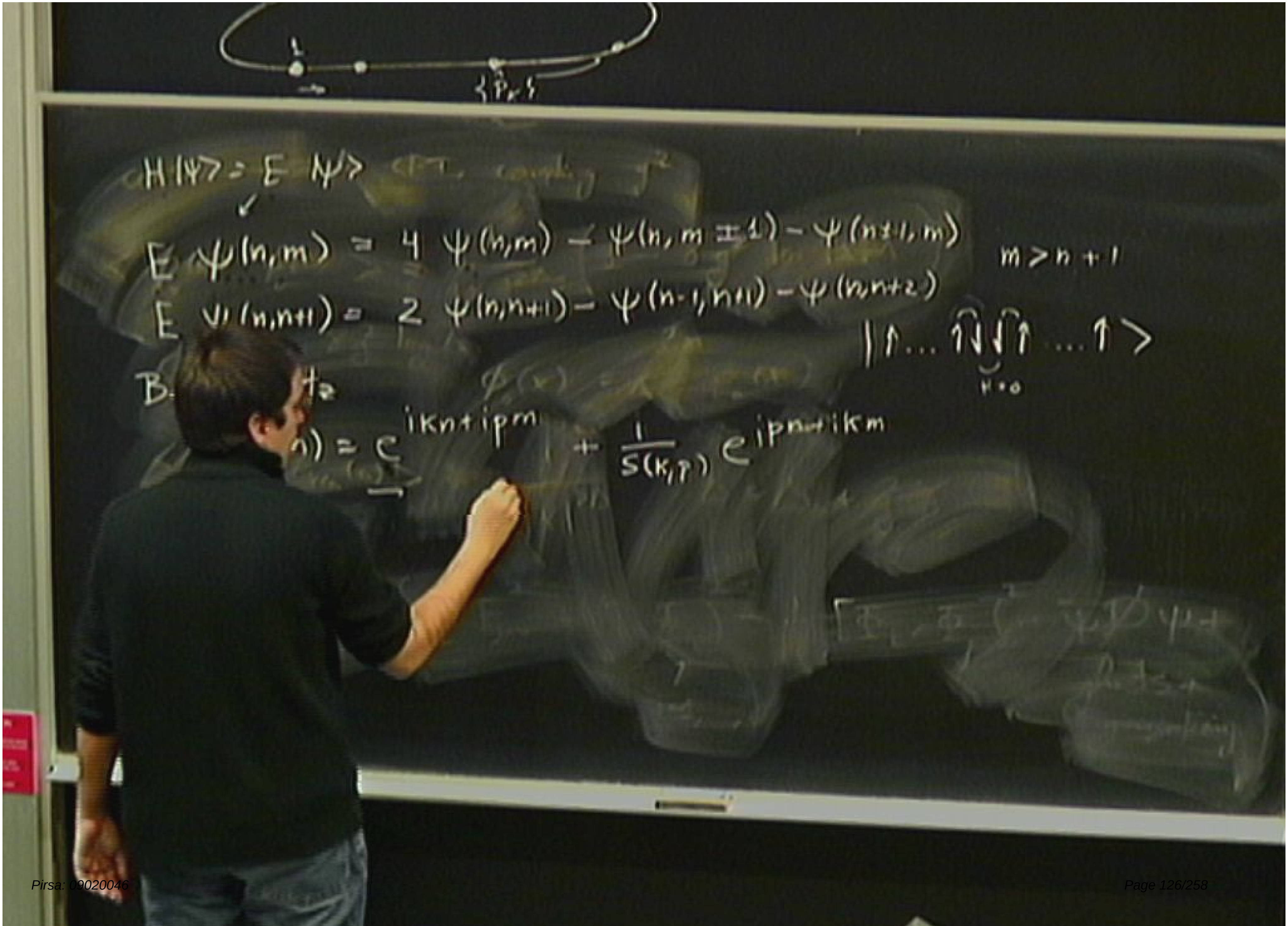


$$H|\psi\rangle = E|\psi\rangle$$

$$E\,\psi(n,m) = 4\,\psi(n,m) - \psi(n,m\pm1) - \psi(n\pm1,m) \qquad m > n+1$$

$$E\,\psi(n,n+1) = 2\,\psi(n,n+1) - \psi(n-1,n+1) - \psi(n,n+2)$$

$$|\uparrow \ldots \uparrow\downarrow\downarrow\uparrow \ldots \uparrow\rangle$$


H|ψ> = E |ψ>
E ψ(n,m) = 4 ψ(n,m) − ψ(n, m ± 1) − ψ(n±1, m)
m > n + 1
E ψ(n,n+1) = 2 ψ(n,n+1) − ψ(n−1, n+1) − ψ(n,n+2)
|↑... ↑↓↓↑ ...↑>
e^{ikn+ipm}
+ 1/S(k,p) e^{ipn+ikm}

$\{p_k\}$

$H|\psi\rangle = E|\psi\rangle$

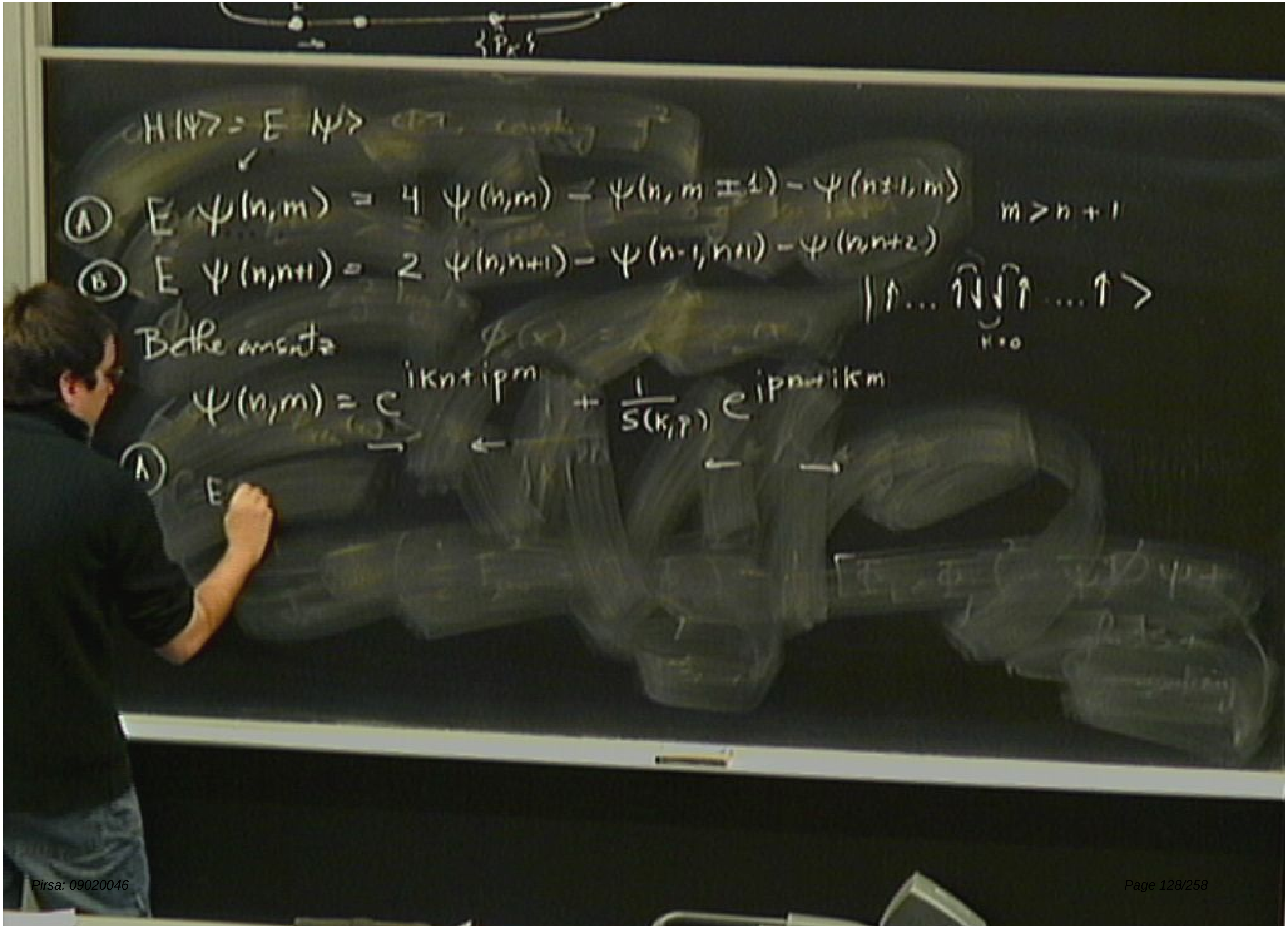
$E\,\psi(n,m) = 4\,\psi(n,m) - \psi(n, m\pm 1) - \psi(n\pm 1, m)$ $m > n+1$

$E\,\psi(n,n+1) = 2\,\psi(n,n+1) - \psi(n-1,n+1) - \psi(n,n+2)$

$|\uparrow \ldots \uparrow\downarrow\downarrow\uparrow \ldots \uparrow\rangle$

Bethe ansatz

$$\psi(n,m) = e^{ikn+ipm} + \frac{1}{S(k,p)}\, e^{ipn+ikm}$$


H|ψ> = E |ψ>
(A) E ψ(n,m) = 4 ψ(n,m) − ψ(n, m ± 1) − ψ(n ± 1, m)
m > n + 1
(B) E ψ(n,n+1) = 2 ψ(n,n+1) − ψ(n-1,n+1) − ψ(n,n+2)
|↑ ... ↑↓↓↑ ... ↑>
Bethe ansatz
ψ(n,m) = e^{ikn+ipm} + 1/S(k,p) e^{ipn+ikm}
(A)
E

$$H|\psi\rangle = E|\psi\rangle$$

(A) $$E\,\psi(n,m) = 4\,\psi(n,m) - \psi(n, m\pm 1) - \psi(n\pm 1, m) \qquad m > n+1$$

(B) $$E\,\psi(n,n+1) = 2\,\psi(n,n+1) - \psi(n-1,n+1) - \psi(n,n+2)$$

$$|\uparrow \ldots \uparrow\downarrow\downarrow\uparrow \ldots \uparrow\rangle$$

Bethe ansatz

$$\psi(n,m) = e^{ikn+ipm} + \frac{1}{S(k,p)}\, e^{ipn+ikm}$$

(A) $$E = \epsilon(p) + \epsilon(k)$$

$$H|\psi\rangle = E|\psi\rangle$$

(A) $$E\,\psi(n,m) = 4\,\psi(n,m) - \psi(n, m \pm 1) - \psi(n \pm 1, m) \qquad m > n+1$$

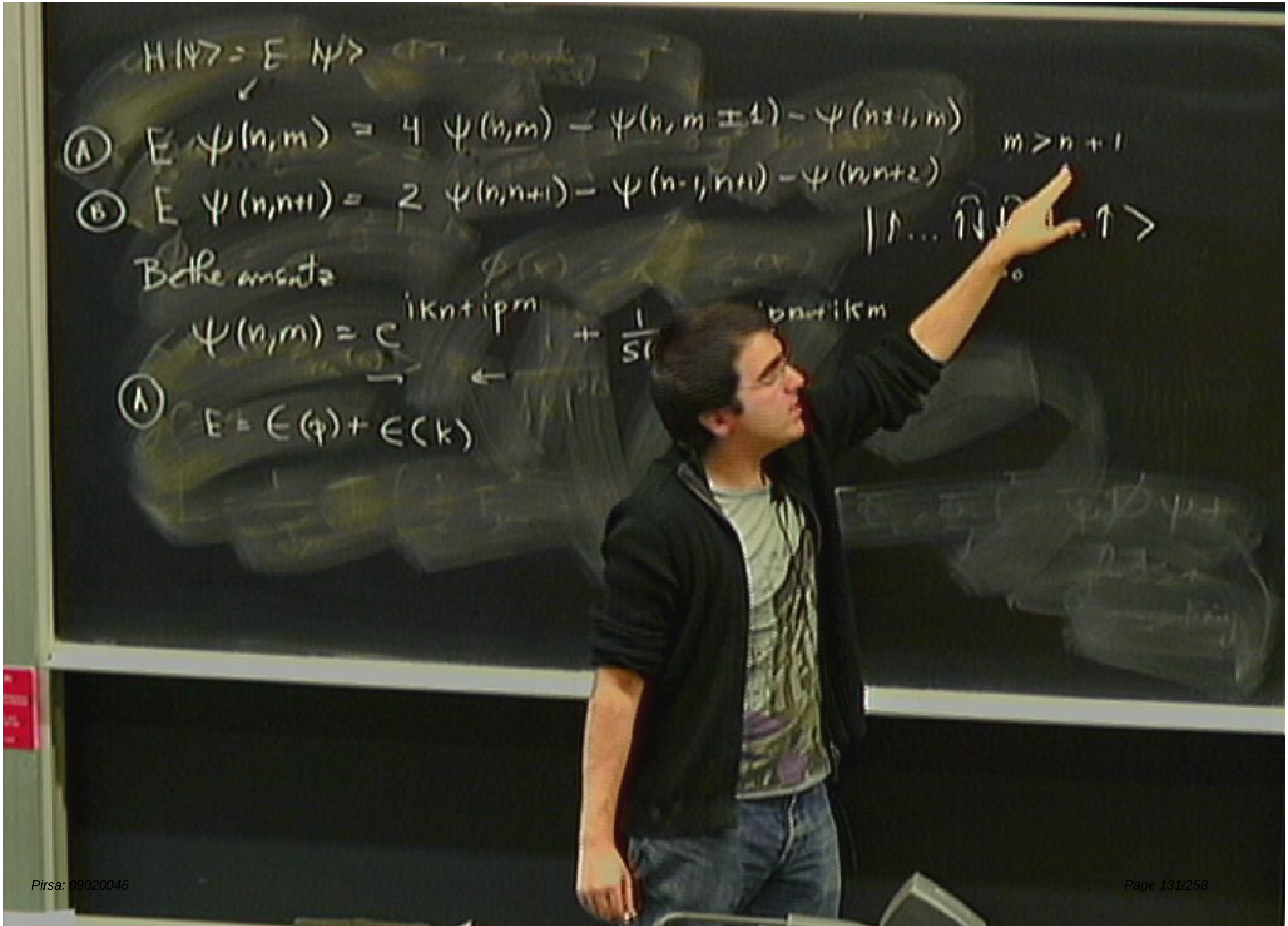
(B) $$E\,\psi(n,n+1) = 2\,\psi(n,n+1) - \psi(n-1,n+1) - \psi(n,n+2)$$

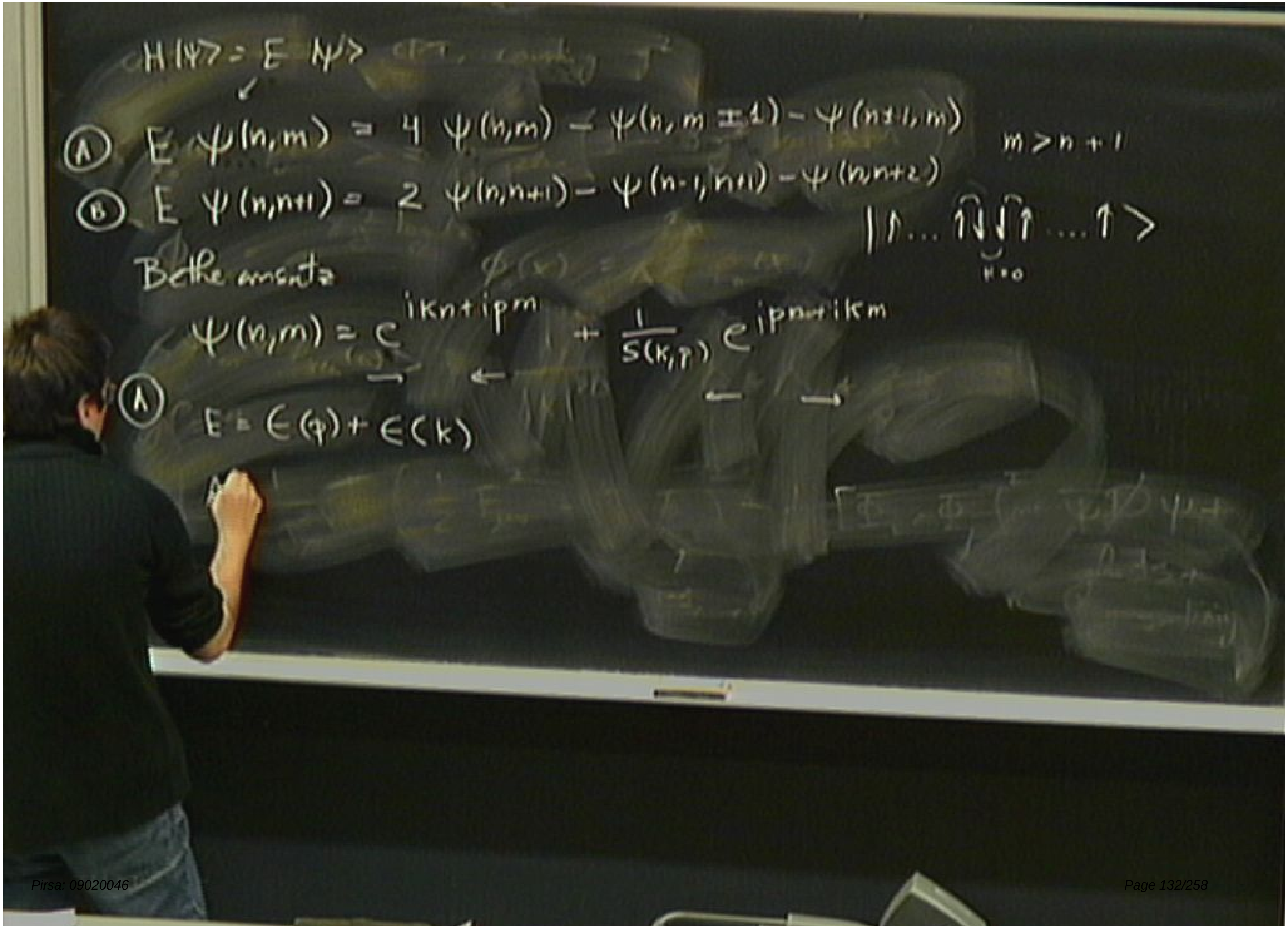
$$|\uparrow \ldots \uparrow\downarrow\downarrow\uparrow \ldots \uparrow\rangle$$

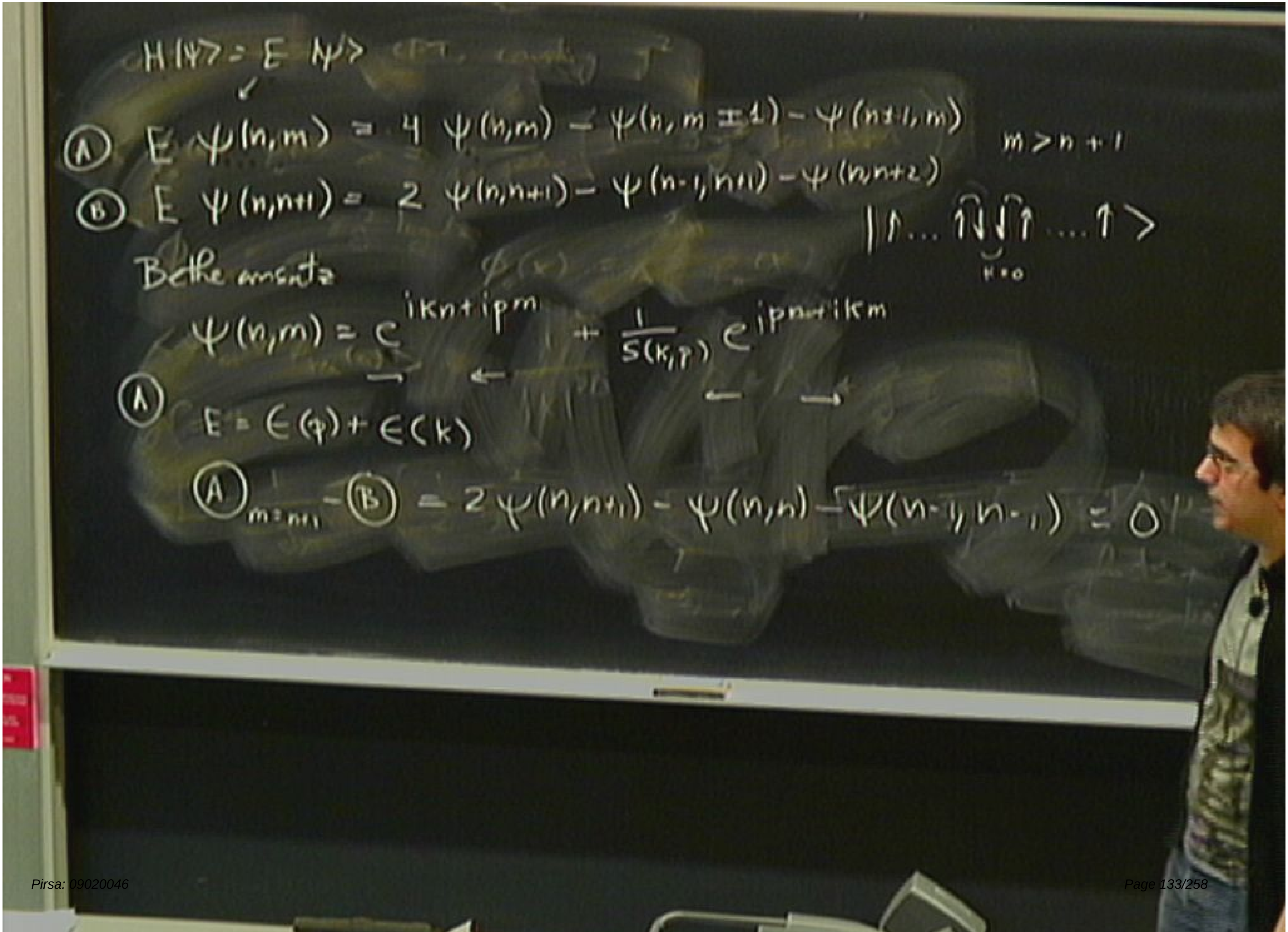
Bethe ansatz

$$\psi(n,m) = e^{ikn+ipm} + \frac{1}{S(k,p)}\, e^{ipn+ikm}$$

$$E = \epsilon(p) + \epsilon(k)$$


H|ψ> = E |ψ>
Ⓐ E ψ(n,m) = 4 ψ(n,m) − ψ(n, m ± 1) − ψ(n±1, m)
m > n + 1
Ⓑ E ψ(n,n+1) = 2 ψ(n,n+1) − ψ(n-1,n+1) − ψ(n,n+2)
Bethe ansatz
ψ(n,m) = e^ikn+ipm
Ⓐ
E = ∈(p) + ∈(k)


H|ψ> = E |ψ>
(A) E ψ(n,m) = 4 ψ(n,m) − ψ(n, m ± 1) − ψ(n±1, m)
m > n + 1
(B) E ψ(n,n+1) = 2 ψ(n,n+1) − ψ(n−1,n+1) − ψ(n,n+2)
|↑ ... ↑↓↓↑ ... ↑>
Bethe ansatz
ψ(n,m) = e^(ikn+ipm) + 1/S(k,p) e^(ipn+ikm)
(A) E = ε(p) + ε(k)


H|ψ> = E |ψ>
(A) E ψ(n,m) = 4 ψ(n,m) − ψ(n, m ± 1) − ψ(n ± 1, m)
m > n + 1
(B) E ψ(n,n+1) = 2 ψ(n,n+1) − ψ(n−1,n+1) − ψ(n,n+2)
|↑ ... ↑↓↓↑ ... ↑>
Bethe ansatz
ψ(n,m) = e^{ikn+ipm} + 1/S(k,p) e^{ipn+ikm}
(A) E = ε(p) + ε(k)
(A)_{m=n+1} − (B) = 2ψ(n,n+1) − ψ(n,n) − ψ(n+1,n+1) = 0

$H|\psi\rangle = E|\psi\rangle$

(A) $E\,\psi(n,m) = 4\,\psi(n,m) - \psi(n, m\pm 1) - \psi(n\pm 1, m)$ $\quad m > n+1$

(B) $E\,\psi(n,n+1) = 2\,\psi(n,n+1) - \psi(n-1,n+1) - \psi(n,n+2)$

$|\uparrow \dots \uparrow\downarrow\downarrow\uparrow \dots \uparrow\rangle$

Bethe ansatz

$$\psi(n,m) = e^{ikn+ipm} + \frac{1}{S(k,p)}\,e^{ipn+ikm}$$

$$S(k,p) = \frac{\frac{1}{2}\cot\frac{k}{2} - \frac{1}{2}\cot\frac{p}{2} - i}{\frac{1}{2}\cot\frac{k}{2} - \frac{1}{2}\cot\frac{p}{2} + i}$$

(A) $E = \epsilon(p) + \epsilon(k)$

$$\text{(A)}_{m=n+1} - \text{(B)} = 2\,\psi(n,n+1) - \psi(n,n) - \psi(n+1,n+1) = 0$$

$H|\psi\rangle = E|\psi\rangle$

(A) $E\,\psi(n,m) = 4\,\psi(n,m) - \psi(n, m\pm 1) - \psi(n\pm 1, m)$ $\quad m > n+1$

(B) $E\,\psi(n,n+1) = 2\,\psi(n,n+1) - \psi(n-1,n+1) - \psi(n,n+2)$

$|\uparrow \ldots \uparrow\downarrow\downarrow\uparrow \ldots \uparrow\rangle$

Bethe ansatz

$$\psi(n,m) = e^{ikn+ipm} + \frac{1}{S(k,p)} e^{ipn+ikm}$$

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(A) $E = \epsilon(p) + \epsilon(k)$

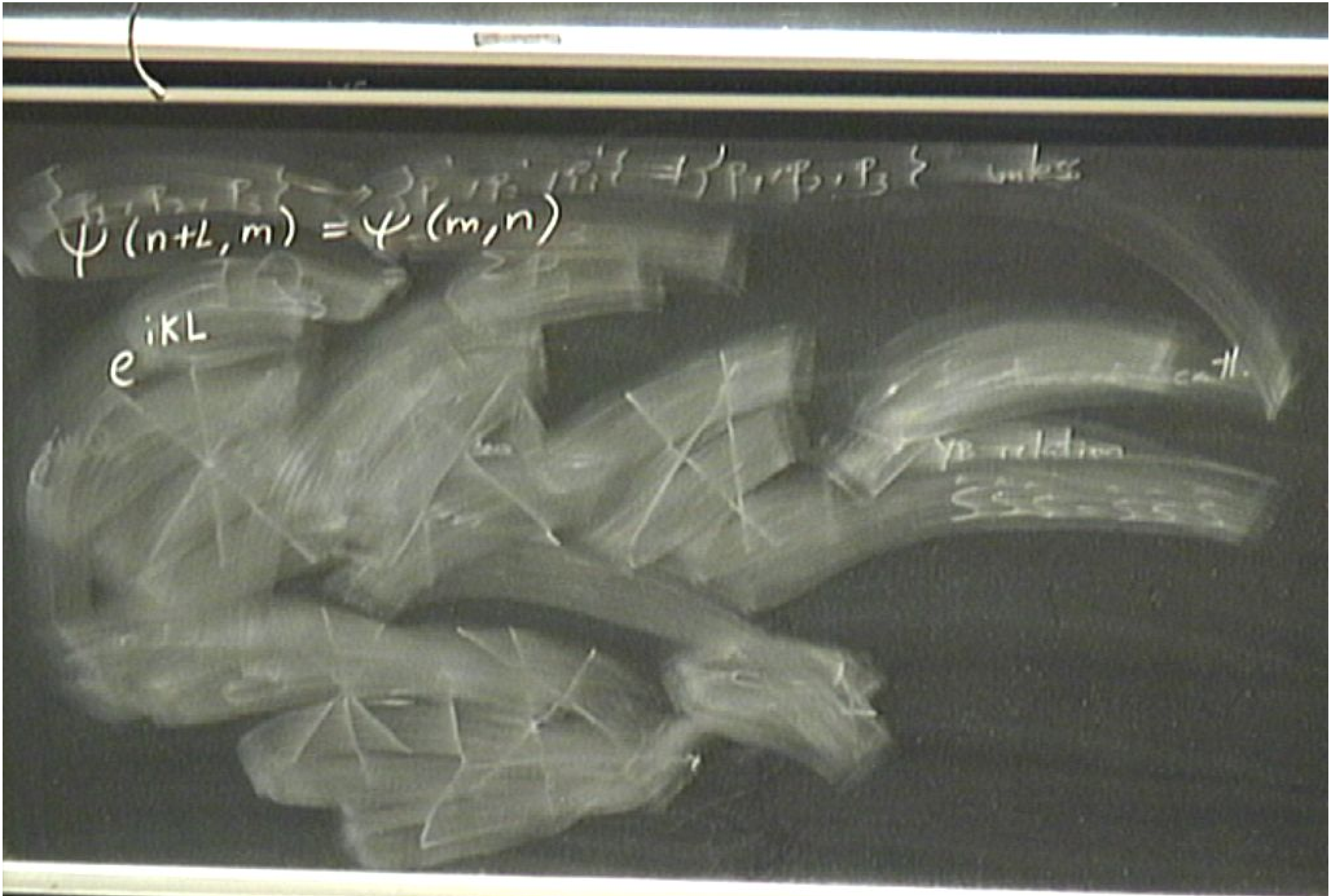
$(A)_{m=n+1} - (B) = 2\,\psi(n,n+1) - \psi(n,n) - \psi(n+1,n+1) = 0$

$\{p_1, p_2, p_3\} \to \{p_1', p_2', p_3'\} = \{p_1, p_2, p_3\}$ unless

$\psi(n+L, m) = \psi(m, n)$

... no relation

$\hat{S}\hat{S}\hat{S} = \hat{S}\hat{S}\hat{S}$



ψ (n+L, m) = ψ (m, n)
e^{iKL}
unless
YB relation

$\{p_1, p_2, p_3\} \to \{p_1', p_2', p_3'\} = \{p_1, p_2, p_3\}$ unless

$$\psi(n+L, m) = \psi(m, n)$$

$$e^{iKL} S(K,p) = 1$$

BAE

YB relation

$SSS = SSS$

$$\psi(n+L, m) = \psi(m, n)$$

$$e^{iKL} S(K,p) = 1$$

BAE

$$e^{ipL} S(p,K) = 1$$

$\psi(n+L, m) = \psi(m, n)$

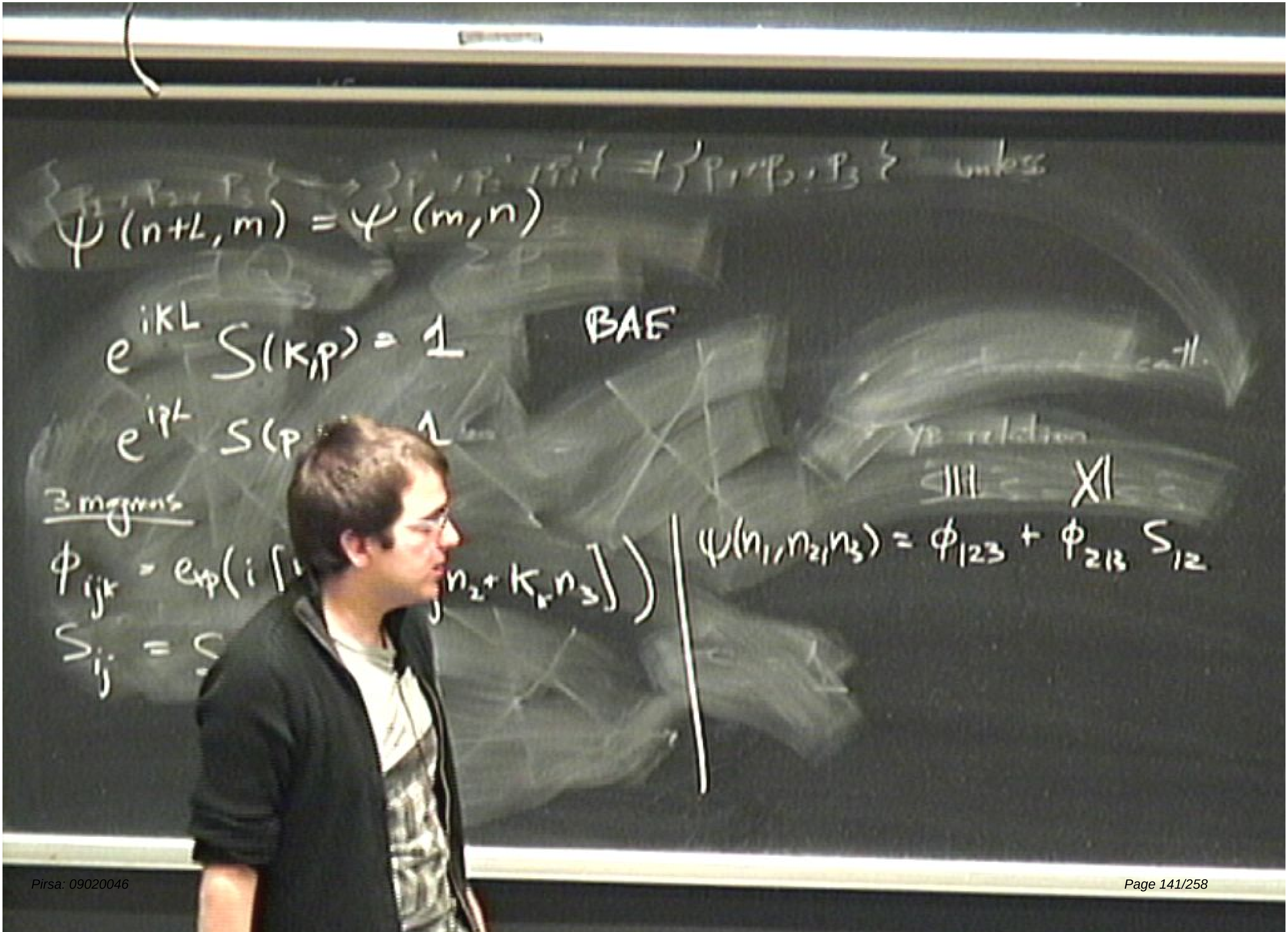
$e^{iKL} S(k,p) = 1$ BAE

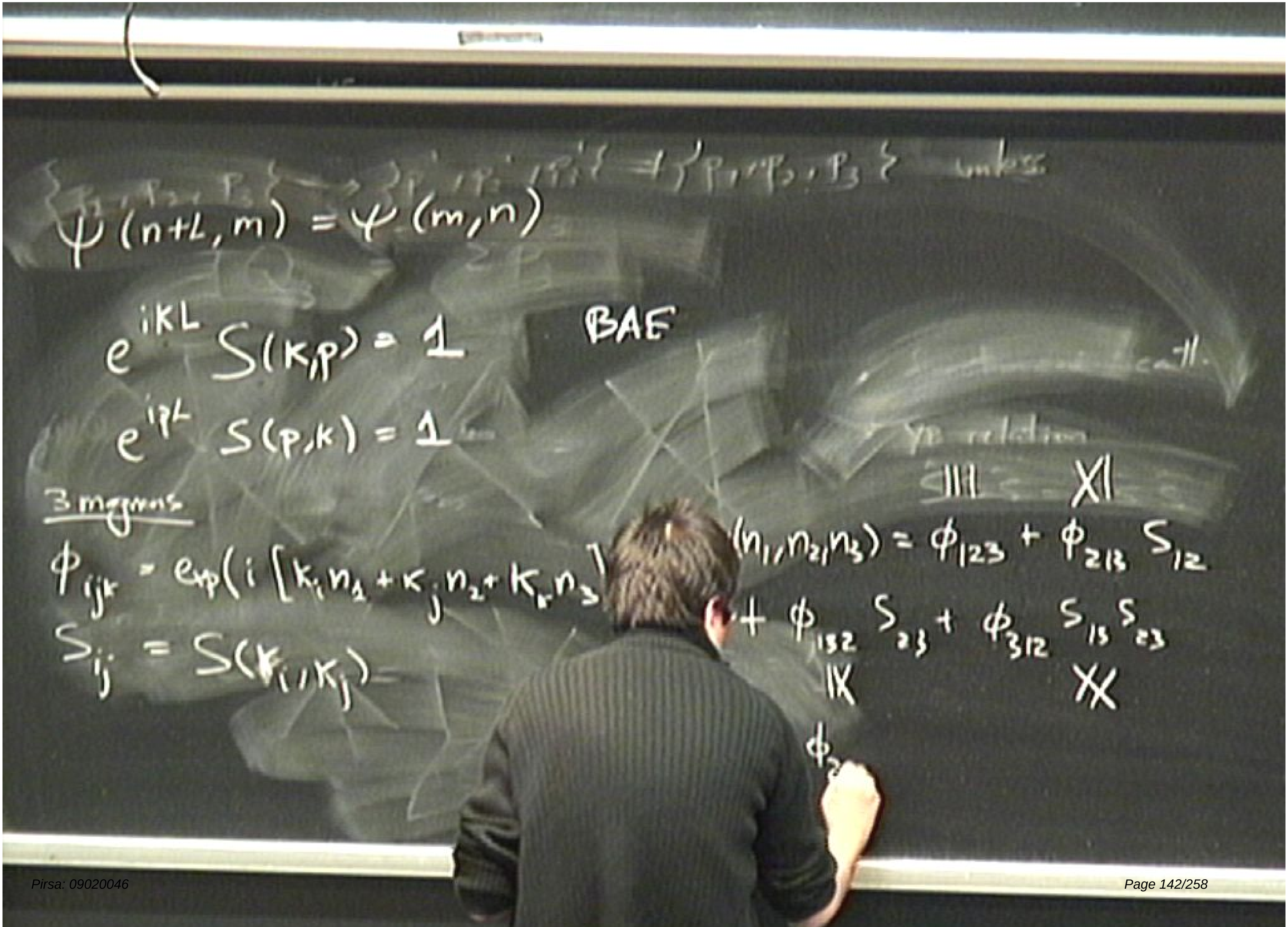
$e^{ipL} S(p,k) = 1$

3 magnons

$\phi_{ijk} = \exp(i[k_i n_1 + k_j n_2 + k_k n_3])$

YB relation


ψ(n+L, m) = ψ(m, n)
e^{iKL} S(K,p) = 1
BAE
3 magnons
ψ(n1, n2, n3) = φ123 + φ213 S12


ψ (n+L, m) = ψ (m, n)
e^{iKL} S(K,p) = 1
BAE
e^{ipL} S(p,K) = 1
3 magnons
φ_{ijk} = exp(i [k_i n_1 + k_j n_2 + k_k n_3]
S_{ij} = S(k_i, k_j)
(n_1, n_2, n_3) = φ_123 + φ_213 S_12
+ φ_132 S_23 + φ_312 S_13 S_23
φ

$$\psi(n+L, m) = \psi(m, n)$$

$$e^{iKL} S(K,p) = 1$$ BAE

$$e^{ipL} S(p,K) = 1$$

YB relation

3 magnons

ϕ [illegible] $\exp(i[k_i n_1 + k_j n_2 + k_k n_3]))$

$$S_{ij} = S(k_i, k_j)$$

$$\psi(n_1, n_2, n_3) = \phi_{123} + \phi_{213} S_{12} + \phi_{132} S_{23} + \phi_{312} S_{13} S_{23} + \phi_{231} S_{13} S_{12} + \phi_{312} S_{23} S_{11} S_{1[illegible]}$$

$d_1 = g^2 \log\Lambda$

$d_1 = 2 g^2 \log\Lambda$

$\hat{H}_{i,i+1} = 2\, 1_{i,i+1} + K_{i,i+1} - 2 P_{i,i+1}$

integrable theory on a large circle of length L

$$e^{ip_j L} = \prod_{k \neq j}^{N} 1/S(p_j, p_k)$$

$$e^{ip_j L} \prod_{k \neq j}^{N} S(p_j, p_k) = 1$$

Bethe ansatz Equations

$$E = \sum_{i=1}^{N} \epsilon(p_i)$$

unless interactions are ultra-local

BAE are asymptotic

$\exists$ Corrections $\sim e^{-mL}$, g^{2L}

$\mathcal{N} = 4$

$d_1 = g^2 \log \Lambda$

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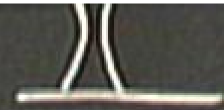
Bethe ansatz Equations

$$E = \sum_{i=1}^{N} \epsilon(p_i)$$

unless interactions are ultra-local

BAE are asymptotic

$\exists$ Corrections $\sim e^{-mL}$, g^{2L} ($\mathcal{N}=4$)

$d_1 = g^2 \log \Lambda$ 

$d_1 = 2 g^2 \log \Lambda$ 

$\hat{H}_{i,i+1} = 2\, 1_{i,i+1} + K_{i,i+1} - 2 P_{i,i+1}$

integrable theory on a large circle of length L

$$e^{ip_j L} = \prod_{k \neq j}^{N} 1/S(p_j, p_k)$$

Why? Where is Q_3?

$$e^{ip_j L} \prod_{k \neq j}^{N} S(p_j, p_k) = 1$$

Bethe ansatz Equations

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BAE are asymptotic $\mathcal{N}=4$

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Bethe ansatz Equations

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unless interactions are ultra-local

BAE are asymptotic

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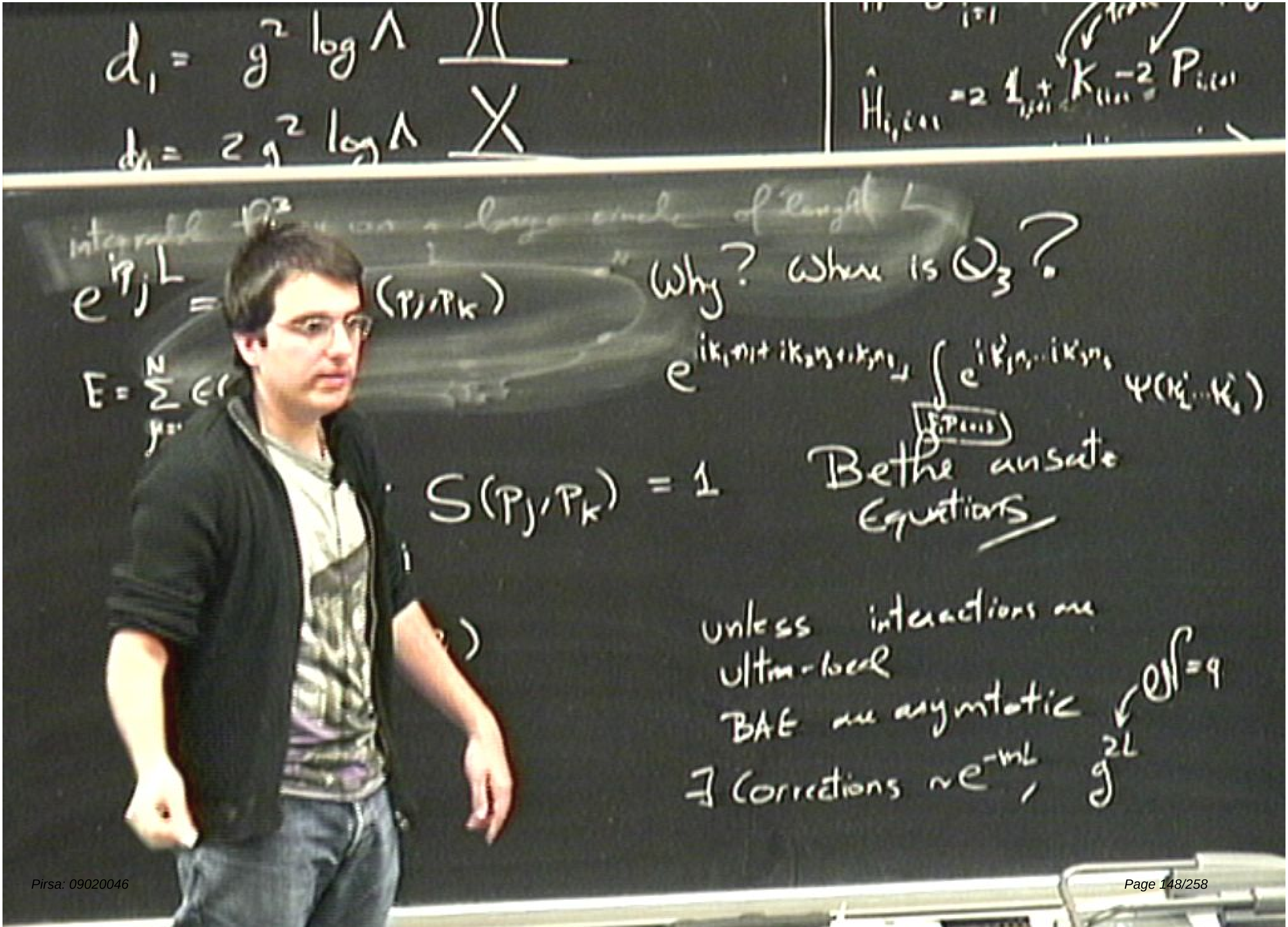
$\mathcal{N}=4$



$d_1 = g^2 \log \Lambda$
$d_1 = 2 g^2 \log \Lambda$
Why? Where is Q_3?
$S(p_j, p_k) = 1$
Bethe ansatz Equations
unless interactions are ultra-local
BAE are asymptotic
∃ Corrections $\sim e^{-mL}$, g^{2L}

$$e^{ip_j L} = \prod_{k \neq j} S(p_j, p_k)$$

$$E = \sum_{j=1}^{N} \epsilon(k_j)$$

$$e^{ik_1 n_1 + ik_2 n_2 + \dots k_N n_N} \int_{\{P\dots\}} e^{ik'_1 n_1 \dots ik'_N n_N} \psi(k'_1 \dots k'_N)$$

$$u = \frac{1}{2}\cot\frac{k}{2}, \quad \frac{u + i/2}{u - i/2} = e^{ip}$$

$$\left(\frac{u_j + i/2}{u_j - i/2}\right)^L = \prod_{k \neq j} \frac{u_j - u_k + i}{u_j - u_k - i}, \quad \epsilon(u) = \frac{1}{u^2 + 1/4}$$

$$e^{ip_j L} = \prod_{k \neq j}^{N} 1/S(p_j, p_k)$$

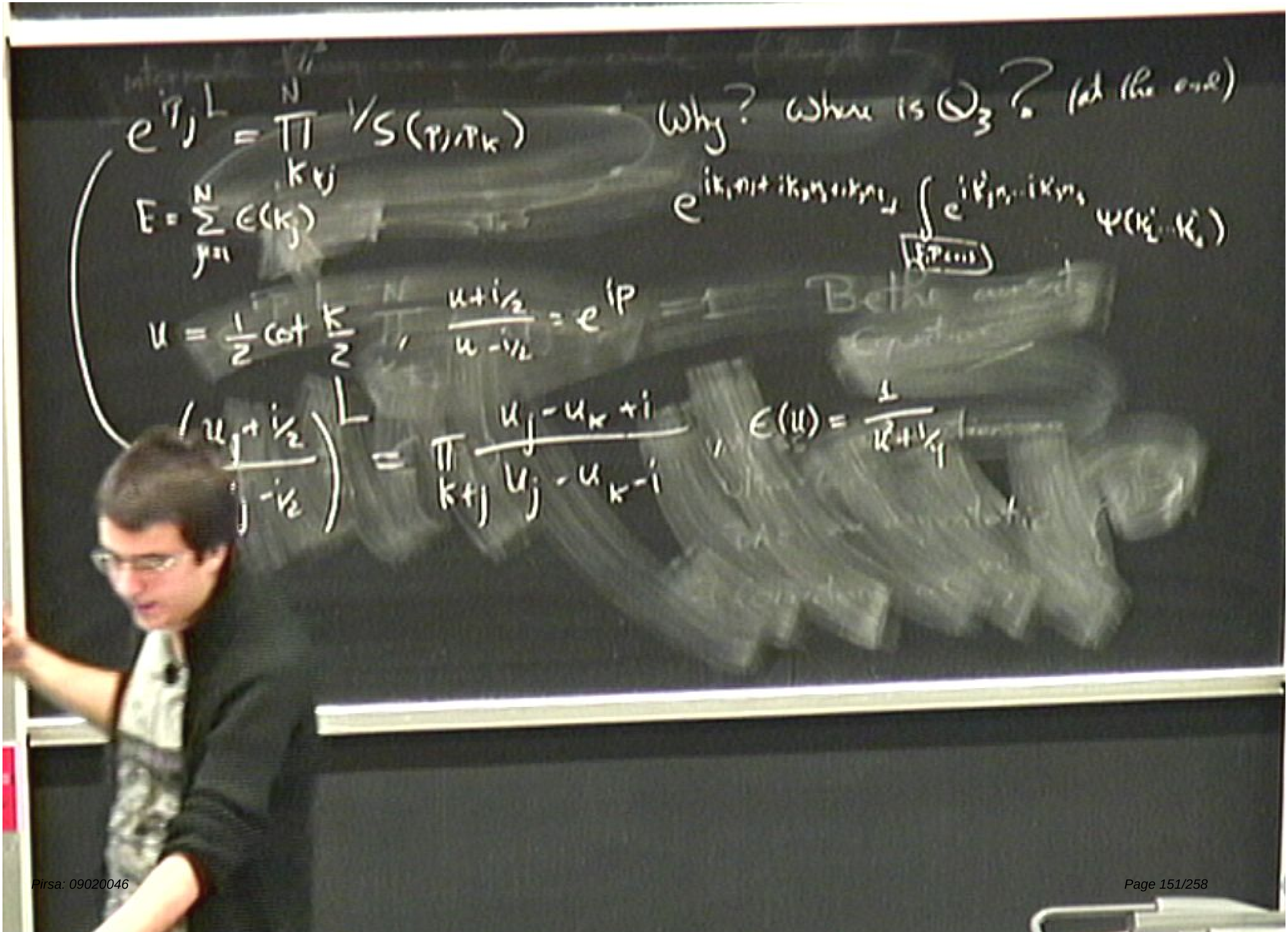
Why? Where is Q_3?

$$E = \sum_{j=1}^{N} \epsilon(k_j)$$

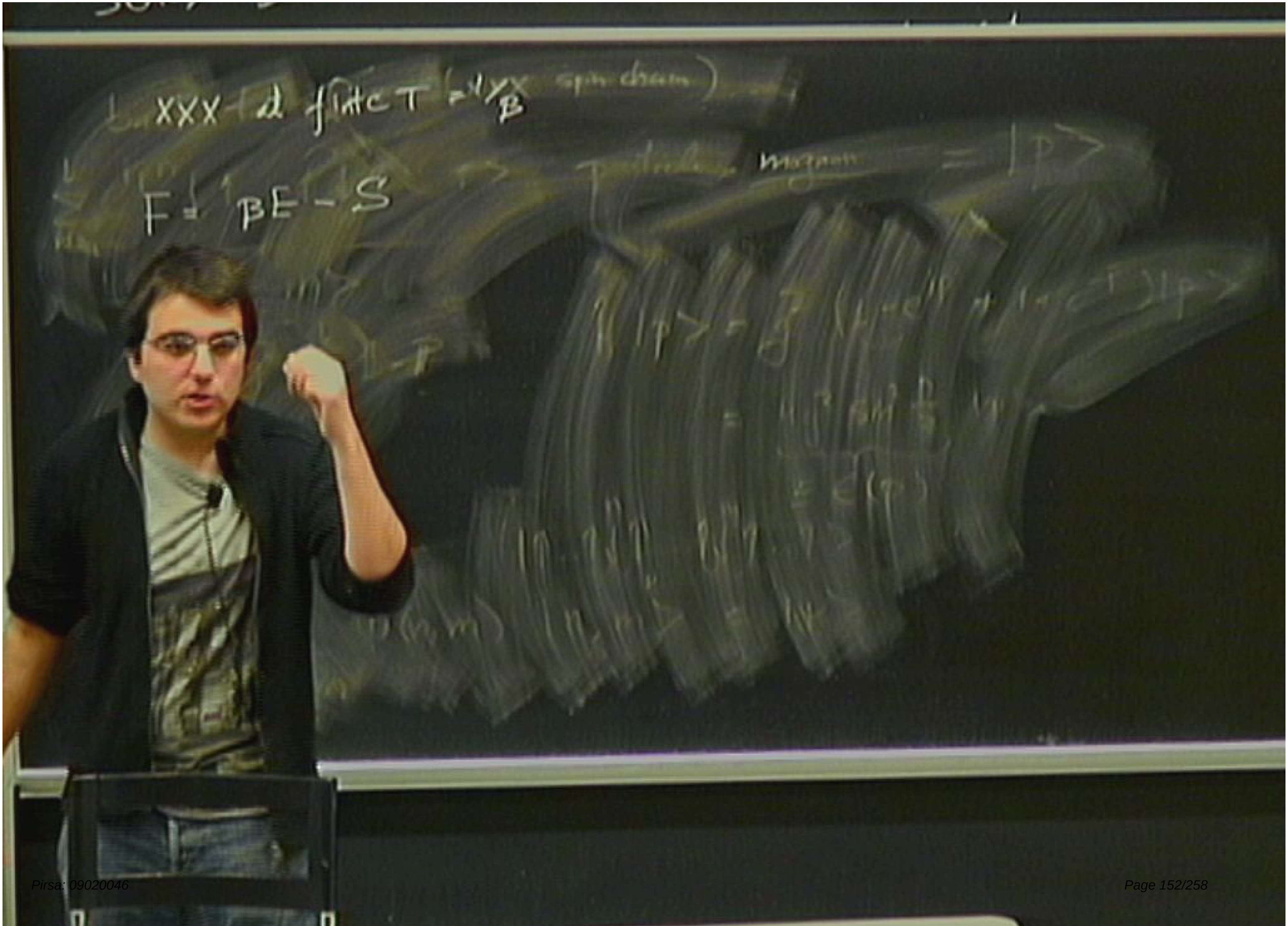
$$e^{ik_1 n_1 + ik_2 n_2 + \dots + ik_N n_N} \dots \int_{\{iP\dots\}} e^{ik'_1 n_1 \dots ik'_N n_N} \, \psi(k'_1 \dots k'_N)$$

$$u = \frac{1}{2}\cot\frac{k}{2}, \quad \frac{u + i/2}{u - i/2} = e^{ip}$$

$$\left(\frac{u_j + i/2}{u_j - i/2}\right)^L = \prod_{k \neq j} \frac{u_j - u_k + i}{u_j - u_k - i}, \quad \epsilon(u) = \frac{1}{u^2 + 1/4}$$



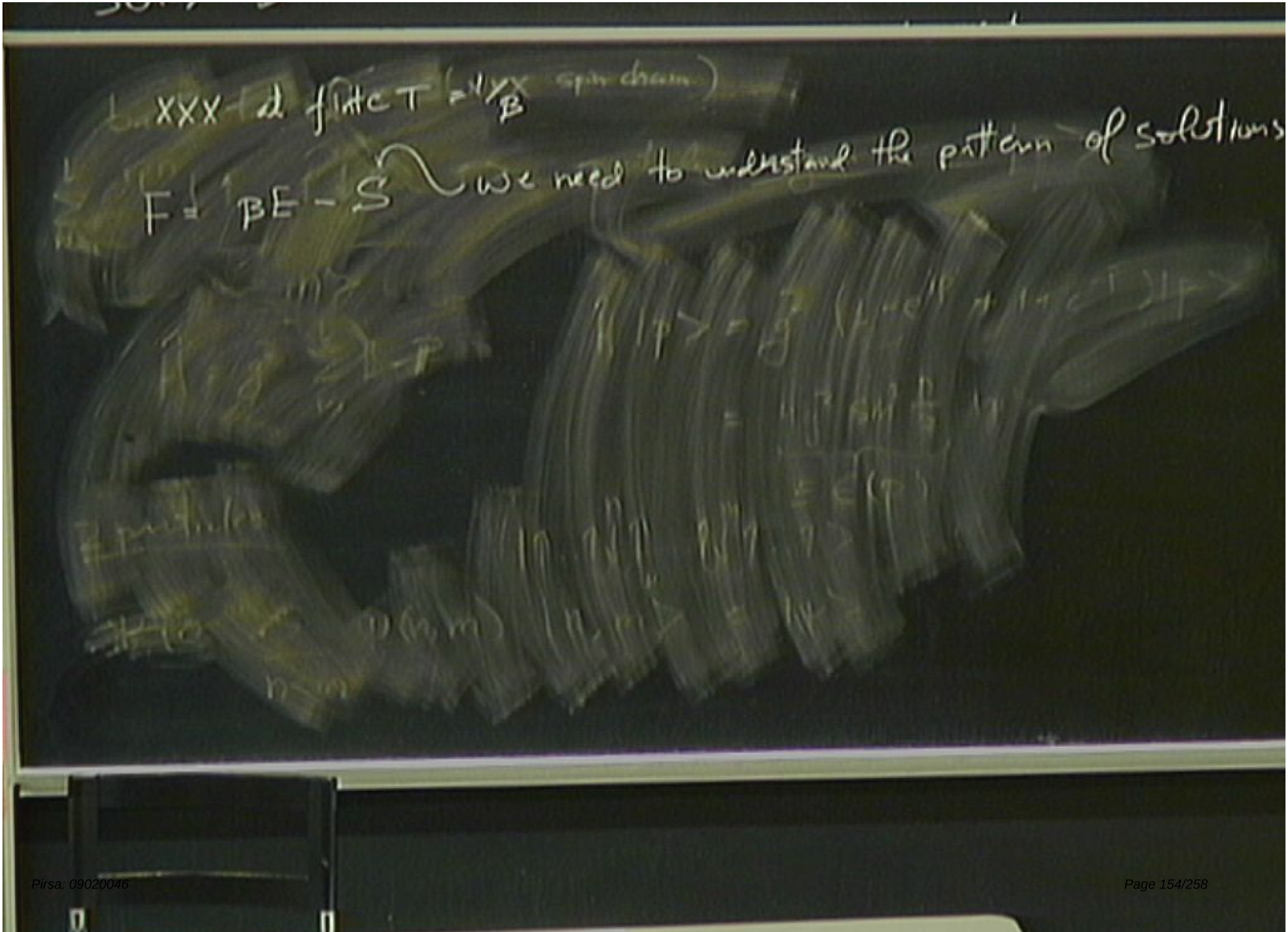
Why? Where is Q3? (at the end)
e^{ip_j L} = ∏_{k≠j}^{N} 1/S(p_j, p_k)
E = ∑_{j=1}^{N} ε(k_j)
u = 1/2 cot k/2 , ∏ (u + i/2)/(u − i/2) = e^{ip}
((u_j + i/2)/(u_j − i/2))^L = ∏_{k≠j} (u_j − u_k + i)/(u_j − u_k − i) , ε(u) = 1/(u²+1/4)

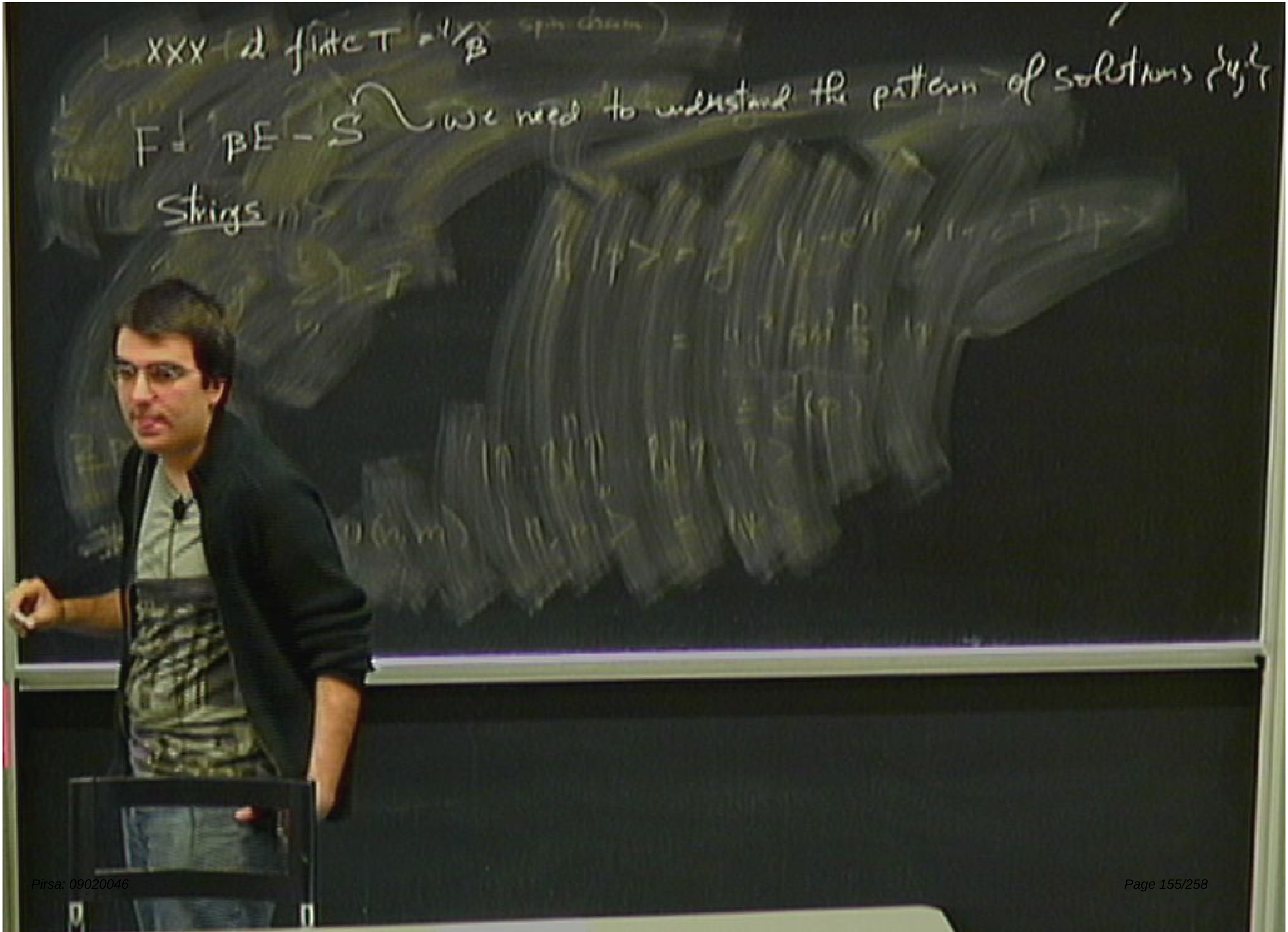

F = βE − S

XXX (& fin. T = 1/β spin chain)

$F = \beta E - S$

magnon $= |P\rangle$


XXX at finite T (=1/β XXX spin chain)
F = βE − S ~ we need to understand the pattern of solutions


XXX + d finite T
F = βE − S
we need to understand the pattern of solutions
Strings

XXX λ finite $T = 1/\beta$

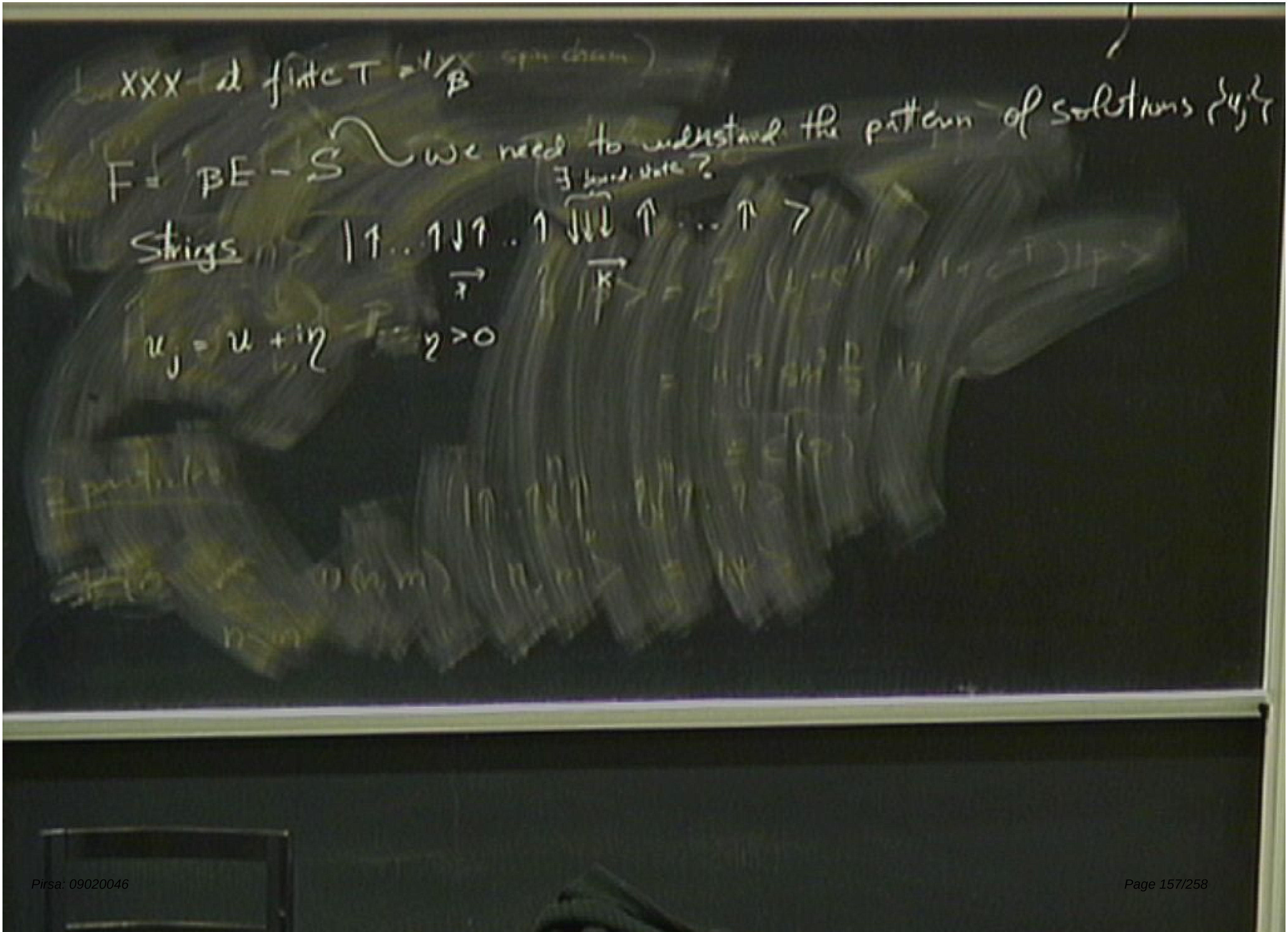
$F = \beta E - S$ ~ we need to understand the pattern of solutions $\{u_j\}$

$\exists$ bound state?

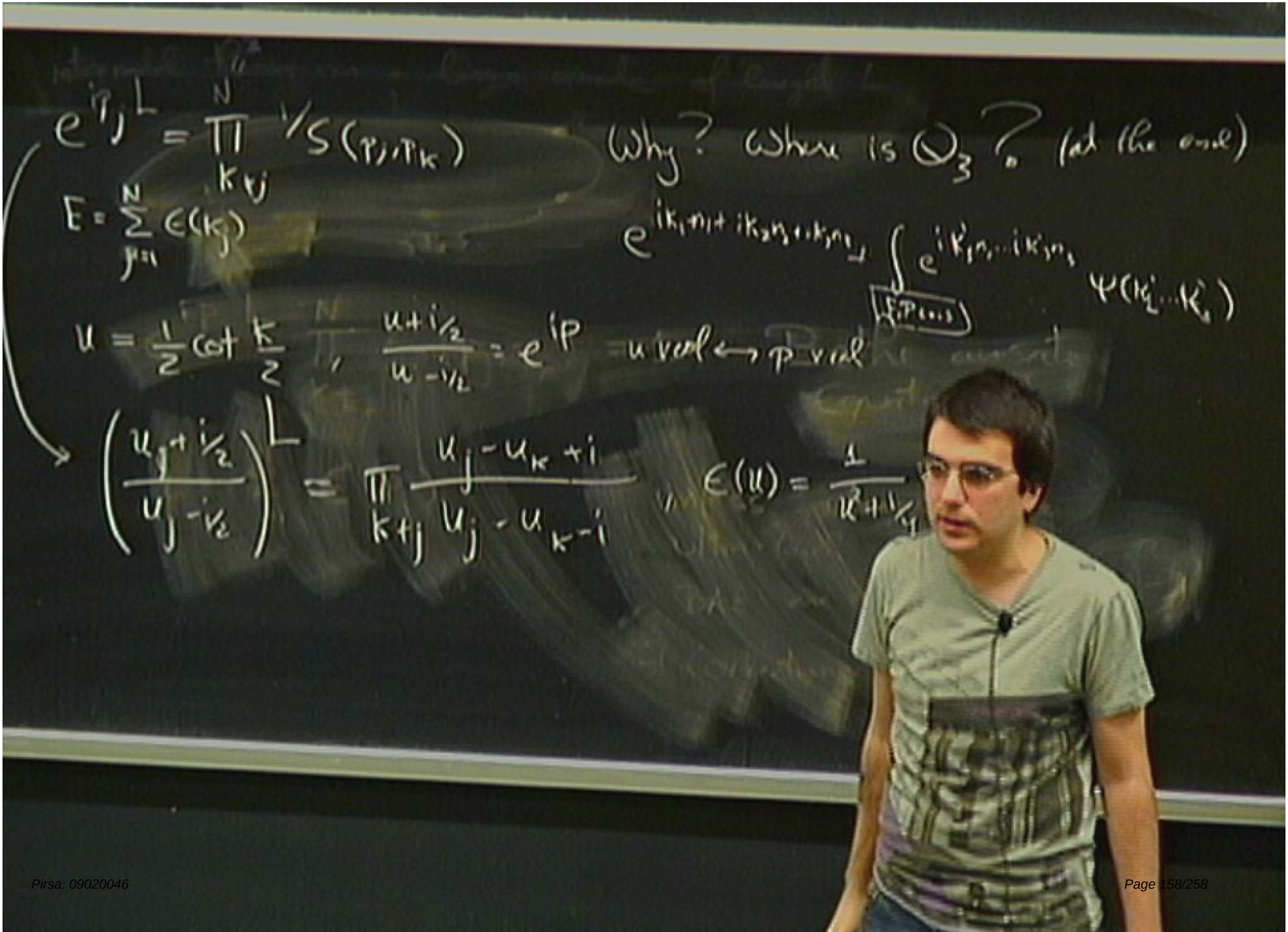
Strings $|\uparrow \ldots \uparrow\downarrow\uparrow \ldots \uparrow \downarrow\downarrow\downarrow \uparrow \ldots \uparrow \rangle$

$\vec{x}$ $\vec{k}$

$u_j = u + i\eta$



XXX
F = βE − S
we need to understand the pattern of solutions
∃ bound state?
Strings
u_j = u + iη
η > 0



$e^{ip_j L} = \prod_{k \neq j}^{N} 1/S(p_j, p_k)$
Why? Where is Q_3? (at the end)
$E = \sum_{j=1}^{N} \epsilon(k_j)$
$u = \frac{1}{2}\cot\frac{k}{2}$, $\frac{u+i/2}{u-i/2} = e^{ip}$ u real ⟷ p real
$\left(\frac{u_j+i/2}{u_j-i/2}\right)^L = \prod_{k \neq j} \frac{u_j-u_k+i}{u_j-u_k-i}$, $\epsilon(u) = \frac{1}{u^2+1/4}$

$$e^{ip_jL} = \prod_{k\neq j}^{N} 1/S(p_j,p_k)$$

Why? Where is Q_3? (at the end)

$$E = \sum_{j=1}^{N} \epsilon(k_j)$$

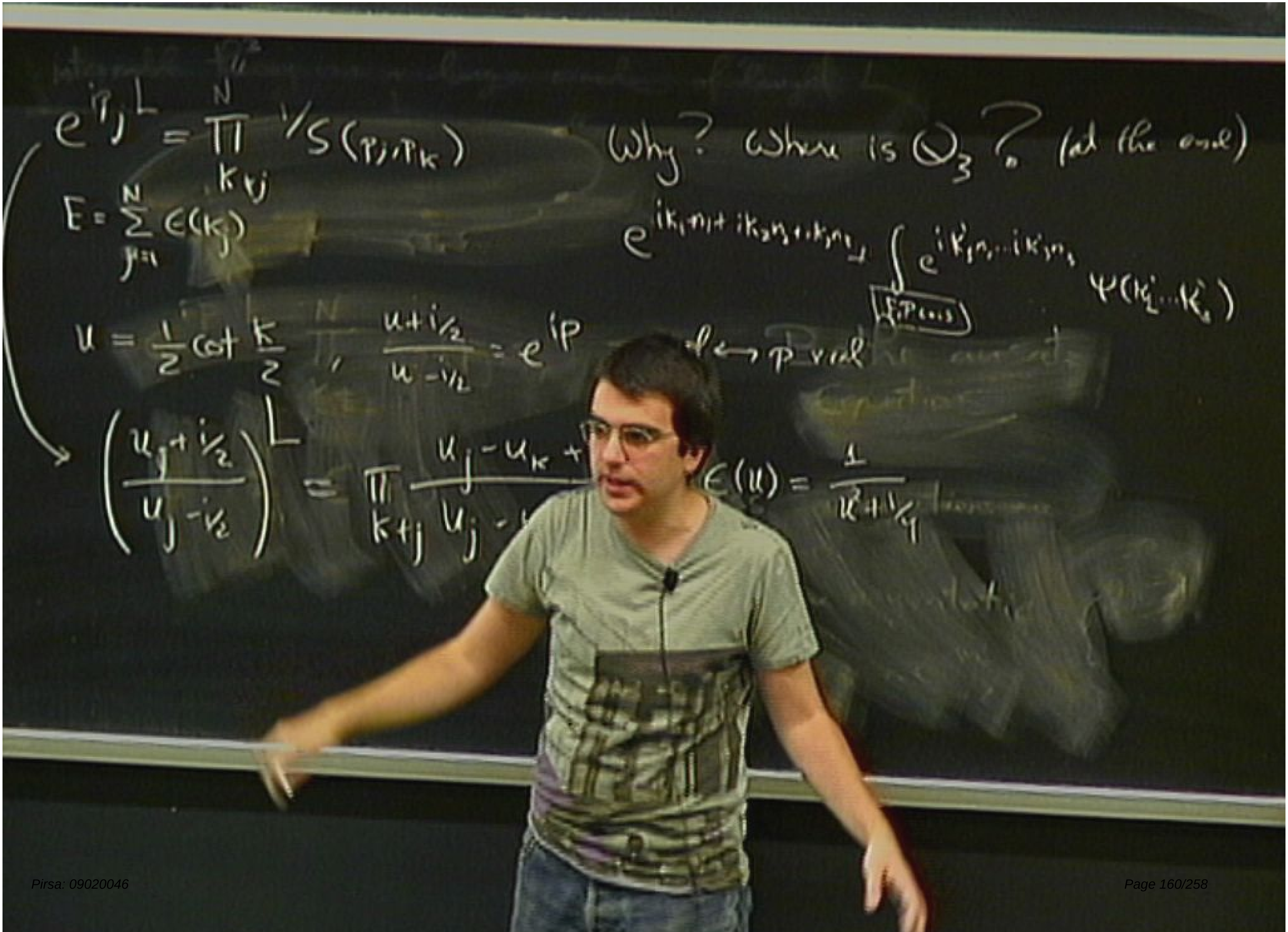
$$e^{ik_1n_1+ik_2n_2+ik_3n_3} \int e^{ik_1'n_1 \ldots ik_3'n_3} \psi(k_1' \ldots k_3')$$

$\Sigma p \ldots$

$$u = \frac{1}{2}\cot\frac{k}{2}, \quad \frac{u+i/2}{u-i/2} = e^{ip}$$

u real $\leftrightarrow$ p real

$$\left(\frac{u_j+i/2}{u_j-i/2}\right)^L = \prod_{k\neq j} \frac{u_j-u_k+i}{u_j-u_k-i}, \quad \epsilon(u) = \frac{1}{u^2+1/4}$$


Why? Where is Q3 ? (at the end)
E = Σ ε(kj)
u = 1/2 cot k/2
p real

$$e^{ip_j L} = \prod_{k \neq j}^{N} 1/S(p_j, p_k)$$

Why? Where is Q_3? (at the end)

$$E = \sum_{j=1}^{N} \epsilon(k_j)$$

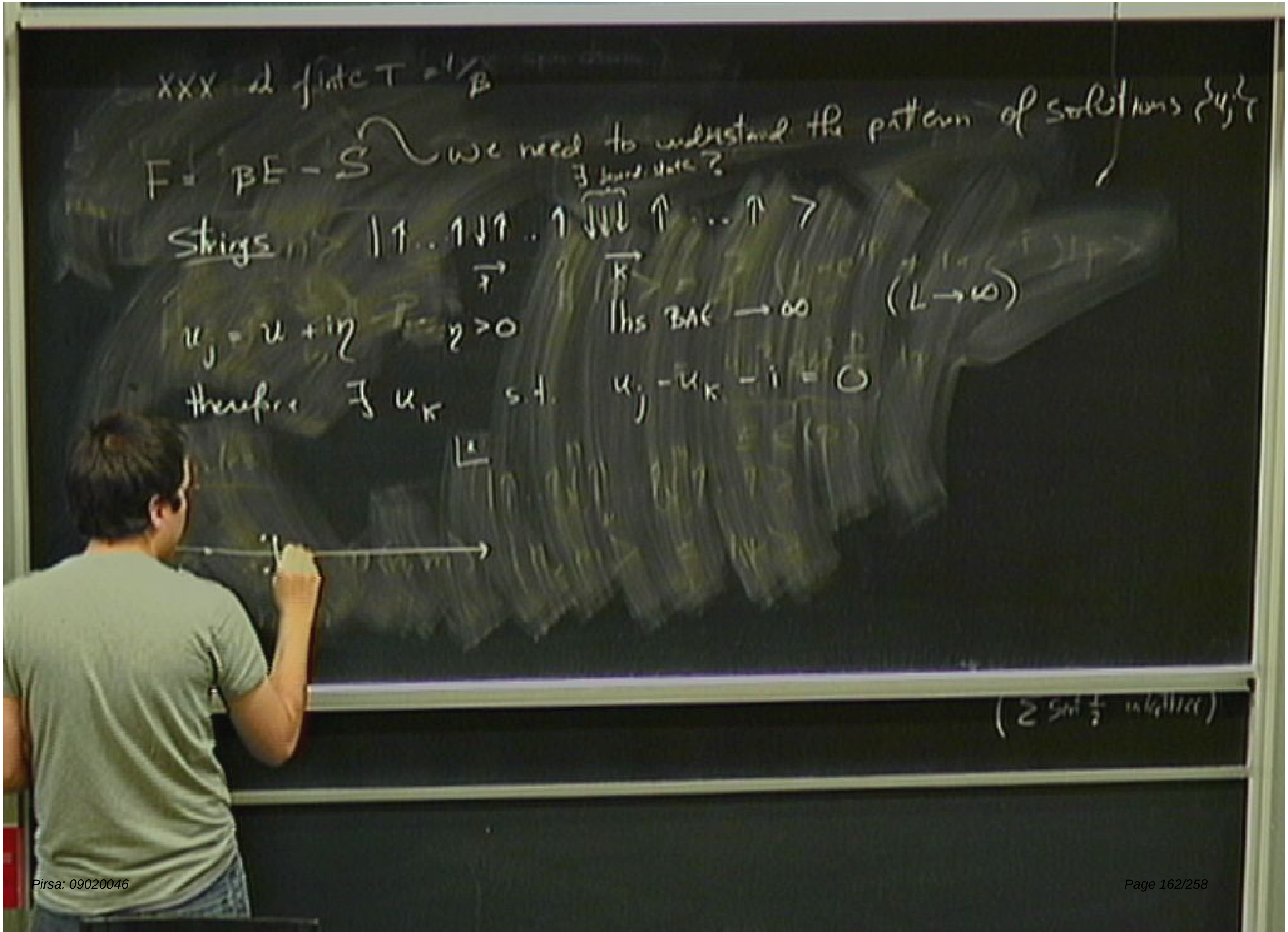
$$e^{ik_1 n_1 + ik_2 n_2 + \dots + ik_j n_j} + \int e^{ik_1' n_1 \dots ik_j' n_j} \, \Psi(k_1' \dots k_j')$$

$$u = \frac{1}{2} \cot \frac{k}{2} \, , \quad \frac{u + i/2}{u - i/2} = e^{ip}$$

u real ⟷ p real

$$\left(\frac{u_j + i/2}{u_j - i/2} \right)^L = \prod_{k \neq j} \frac{u_j - u_k + i}{u_j - u_k - i} \quad \text{BAE}$$

$$\epsilon(u) = \frac{1}{u^2 + 1/4}$$


XXX at finite T = 1/β
F = βE − S ~ we need to understand the pattern of solutions {u_j}
∃ bound state?
Strings |↑ .. ↑↓↑ .. ↑↓↓↓ ↑ ... ↑ >
lhs BAE → ∞ (L → ∞)
u_j = u + iη   η > 0
therefore ∃ u_k s.t. u_j − u_k − i = 0

XXX at finite $T = 1/\beta$

$F = \beta E - S$ ~ we need to understand the pattern of solutions $\{u_j\}$

∃ bound state?

Strings $|\uparrow \ldots \uparrow \downarrow \uparrow \ldots \uparrow \downarrow\downarrow\downarrow \uparrow \ldots \uparrow\rangle$

$\vec{x}$ $\vec{k}$

$u_j = u + i\eta$ $\quad \eta > 0$ $\quad$ lhs BAE $\to \infty$ $\quad (L \to \infty)$

therefore ∃ u_k s.t. $u_j - u_k - i = 0$

(Σ spin ½ integrable)

XXX at finite T = 1/β

F = βE − S ~ we need to understand the pattern of solutions $\{u_j\}$

∃ bound state?

Strings |↑ .. ↑↓↑ .. ↑ ↓↓↓ ↑ ... ↑ ⟩

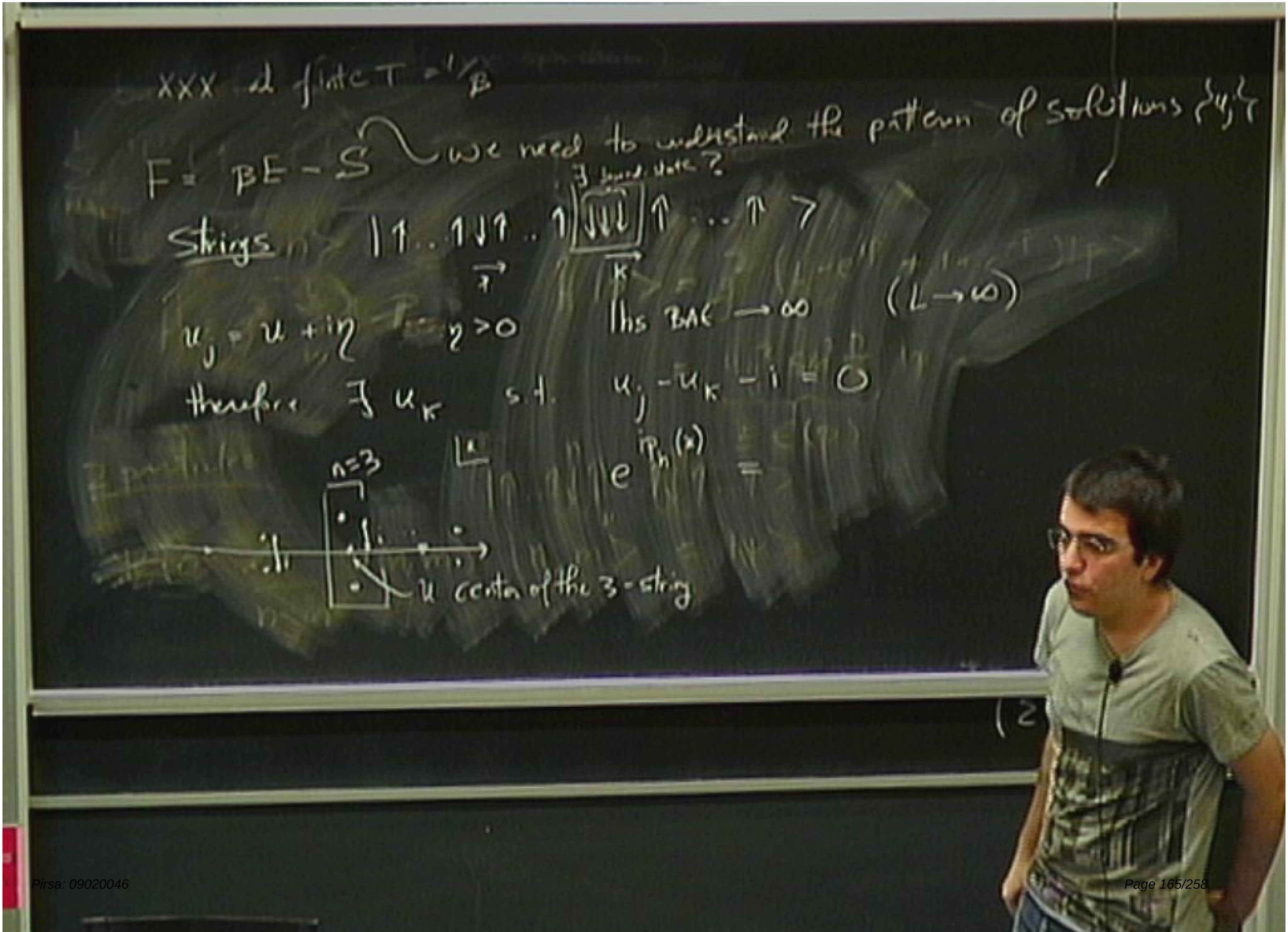
$u_j = u + i\eta$ $\eta > 0$ lhs BAE → ∞ (L → ∞)

therefore ∃ u_k s.t. $u_j - u_k - i = 0$

n=3

e

(∑ Spin ½ integrable)


XXX at finite T = 1/β
F = βE − S ~ we need to understand the pattern of solutions {uj}
∃ bound state?
Strings
(L → ∞)
this BAE → ∞
uj = u + iη   η > 0
therefore ∃ uk s.t. uj − uk − i = 0
n=3
u center of the 3-string

XXX at finite $T = 1/\beta$

$F = \beta E - S$ ~ we need to understand the pattern of solutions $\{u_j\}$

∃ bound state?

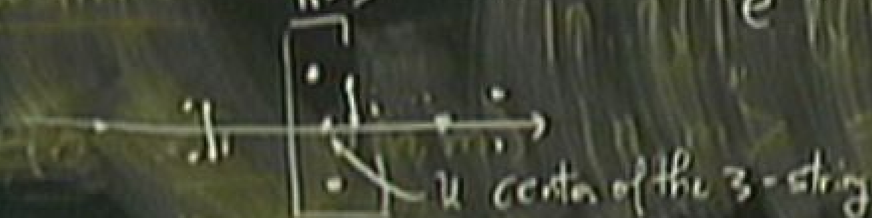
Strings $|\uparrow \dots \uparrow\downarrow\uparrow \dots \uparrow \downarrow\downarrow\downarrow \uparrow \dots \uparrow\rangle$

$\vec{x}$ $\vec{k}$

$u_j = u + i\eta$ $\eta > 0$ lhs BAE $\to \infty$ $(L \to \infty)$

therefore ∃ u_k s.t. $u_j - u_k - i = 0$

$n = 3$ $e^{i p_n(x)} =$



$(\Sigma$ spin $\frac{1}{2}$ integrable$)$

$F = \beta E - S$ ~ we need to understand the pattern of solutions $\{u_j\}$

∃ bound state?

Strings $|\uparrow \ldots \uparrow\downarrow\uparrow \ldots \uparrow \boxed{\downarrow\downarrow\downarrow} \uparrow \ldots \uparrow\rangle$

$u_j = u + i\eta \quad \eta > 0 \qquad$ lhs BAE $\to \infty \quad (L \to \infty)$

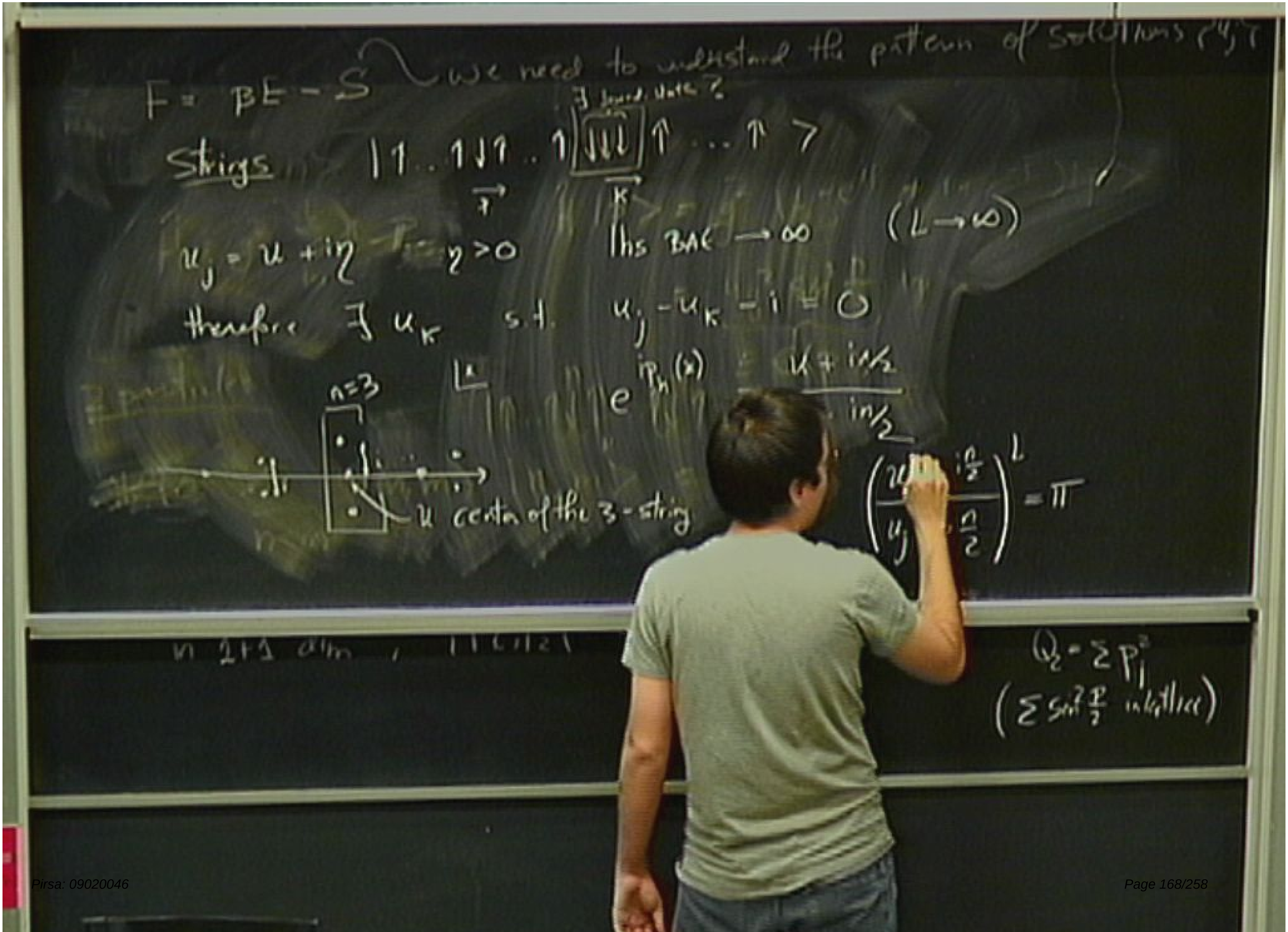
therefore $\exists\, u_k$ s.t. $u_j - u_k - i = 0$

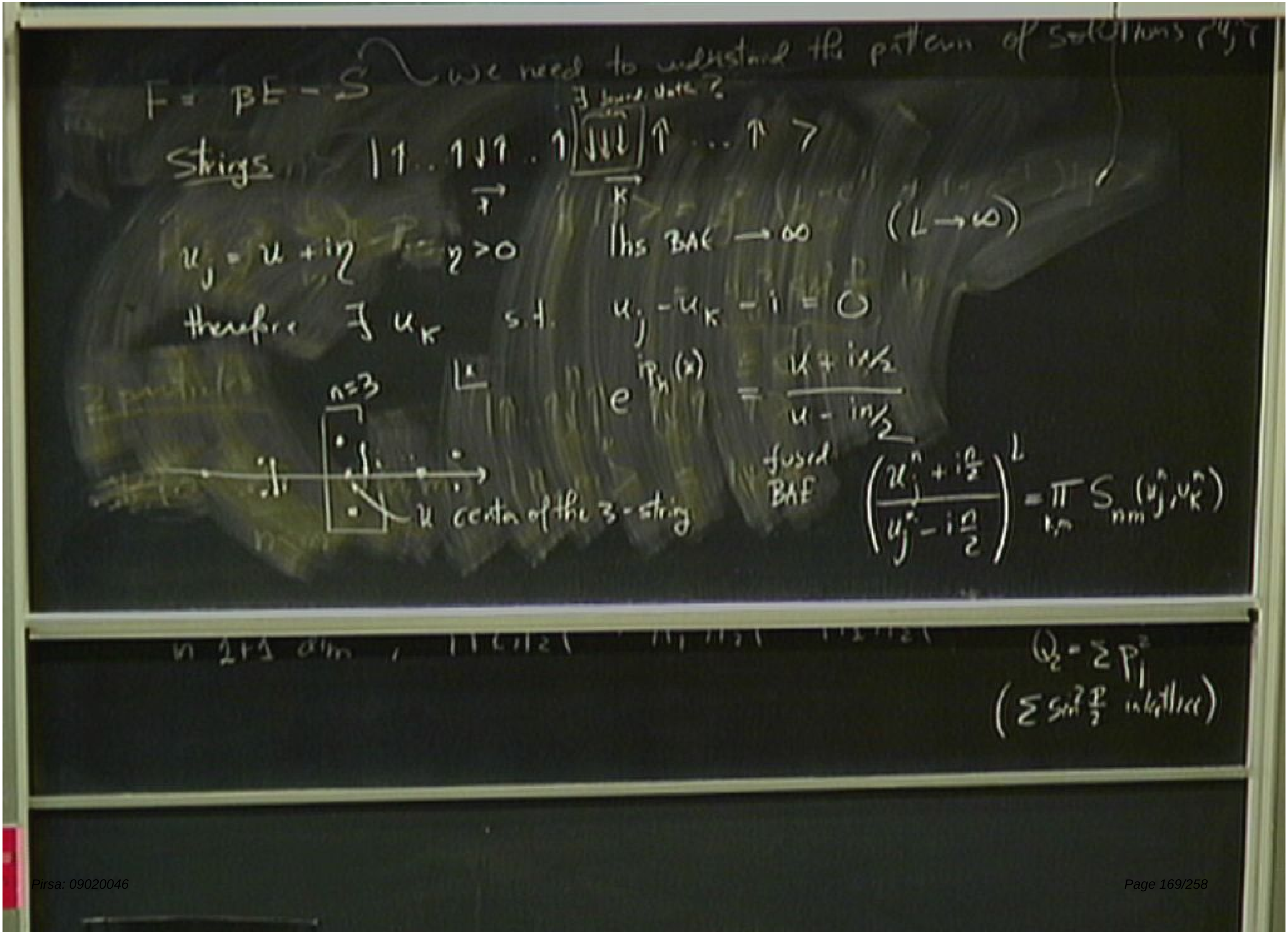
$n=3 \qquad e^{ip_n(x)} = \frac{u + in/2}{u - in/2}$

u center of the 3-string

fused BAE

in 2+1 dim ... $\sum p_j^2$ ($\sum \sin^2 \frac{p}{2}$ in lattice)


F = βE − S
we need to understand the pattern of solutions
Strings
∃ bound state?
ln s BAE → ∞
(L → ∞)
η > 0
therefore ∃ u_K s.t.
n=3
center of the 3-string
Q_2 = Σ P_j^2
(Σ sin² p/2 in lattice)



F = βE − S ~ we need to understand the pattern of solutions {u_j}
∃ bound state?
Strings |↑ .. ↑↓↑ .. ↑ ↓↓↓ ↑ ... ↑ ⟩
u_j = u + iη   η > 0
lhs BAE → ∞ (L → ∞)
therefore ∃ u_k s.t. u_j − u_k − i = 0
n=3
e^{ip_n(x)} = (u + in/2)/(u − in/2)
u centre of the 3-string
fused BAE
((u_j^n + i n/2)/(u_j^n − i n/2))^L = Π S_nm(u_j^n, u_k^m)
Q_2 = Σ p_j^2
(Σ sin² p/2 in lattice)

$F = \beta E - S$ ~ we need to understand the pattern of solutions $\{u_j\}$

∃ bound state?

Strings $|\uparrow \ldots \uparrow\downarrow\uparrow \,.\, \uparrow \boxed{\downarrow\downarrow\downarrow} \uparrow \ldots \uparrow \rangle$

$\vec{p}$ $\quad \vec{k}$

$u_j = u + i\eta, \quad \eta > 0$ lhs BAE $\to \infty$ $(L \to \infty)$

therefore $\exists\, u_k$ s.t. $u_j - u_k - i = 0$

$n=3$

u center of the 3-string

$$e^{i p_n(x)} = \frac{u + in/2}{u - in/2}$$

fused BAE

$$\left(\frac{u_j^n + i\frac{n}{2}}{u_j^n - i\frac{n}{2}}\right)^L = \prod_{k,m} S_{nm}(u_j^n, u_k^m)$$

in 2+1 dim ,

$Q_2 = \sum p_j^2$

$\left(\sum \sin^2\frac{p}{2} \text{ in lattice}\right)$

$F = \beta E - S$ — we need to understand the pattern of solutions $\{u_j\}$

∃ bound state?

Strings $|\uparrow \ldots \uparrow\downarrow\uparrow \ldots \uparrow \boxed{\downarrow\downarrow\downarrow} \uparrow \ldots \uparrow\rangle$

$u_j = u + i\eta \qquad \eta > 0 \qquad$ lhs BAE $\to \infty \qquad (L \to \infty)$

therefore $\exists\, u_K$ s.t. $u_j - u_K - i = 0$

$n=3$

$e^{ip_n(x)} = \frac{u + in/2}{u - in/2}$

u center of the 3-string

fused BAE
$$\left(\frac{u_j^n + i\frac{n}{2}}{u_j^n - i\frac{n}{2}}\right)^L = \prod_{k,m} S_{nm}(u_j^n, u_K^m)$$

in 1+1 dim

$Q_2 = \sum p_j^2$

$\left(\sum \sin^2 \frac{p}{2} \text{ in lattice}\right)$

XXX at finite T $= 1/\beta$

F = BE − S ~ we need to understand the pattern of solutions $\{u_j\}$

∃ bound state?

Strings $|\uparrow \ldots \uparrow \downarrow \uparrow \ldots \uparrow \downarrow\downarrow\downarrow \uparrow \ldots \uparrow\rangle$

$u_j = u + i\eta \quad \eta > 0 \quad$ lhs BAE $\to \infty \quad (L \to \infty)$

therefore ∃ u_k s.t. $u_j - u_k - i = 0$

$n = 3$

$e^{ip_n(x)} = \frac{u + in/2}{u - in/2}$

fused BAE $\left(\frac{u_j^n + i\frac{n}{2}}{u_j^n - i\frac{n}{2}}\right)^L = \prod_{k,m} S_{nm}(u_j \ldots$

XXX at finite $T = 1/\beta$

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$u_j = u + i\eta \qquad \eta > 0 \qquad$ lhs BAE $\to \infty \qquad (L \to \infty)$

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$$e^{ip_n(x)} = \frac{u + in/2}{u - in/2}$$

n=3

u center of the 3-string

fused BAE
$$\left(\frac{u_j^n + i\frac{n}{2}}{u_j^n - i\frac{n}{2}}\right)^L = \prod_{k,m} S_{nm}(u_j^n, u_k^m)$$

$$e^{ip_jL} = \prod_{k\neq j}^{N} 1/S(p_j,p_k)$$

Why? Where is Q_3? (at the end)

$$E = \sum_{j=1}^{N} \epsilon(k_j)$$

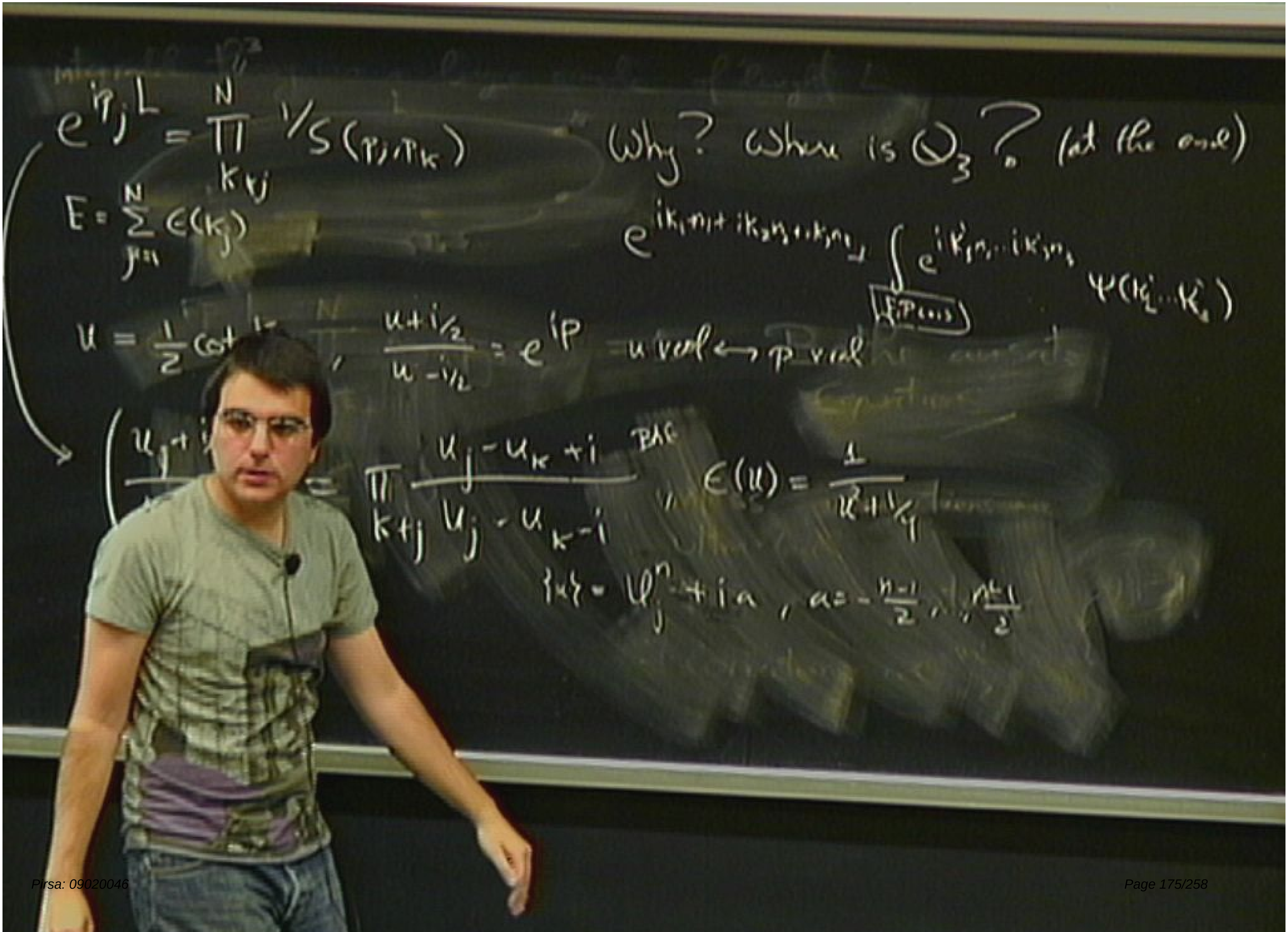
$$e^{ik_1n_1+ik_2n_2+\ldots k_Nn_N} \int_{\{P\ldots\}} e^{ik_1'n_1\ldots ik_N'n_N} \Psi(k_1'\ldots k_N')$$

$$u = \frac{1}{2}\cot\frac{k}{2}, \quad \frac{u+i/2}{u-i/2} = e^{ip}$$

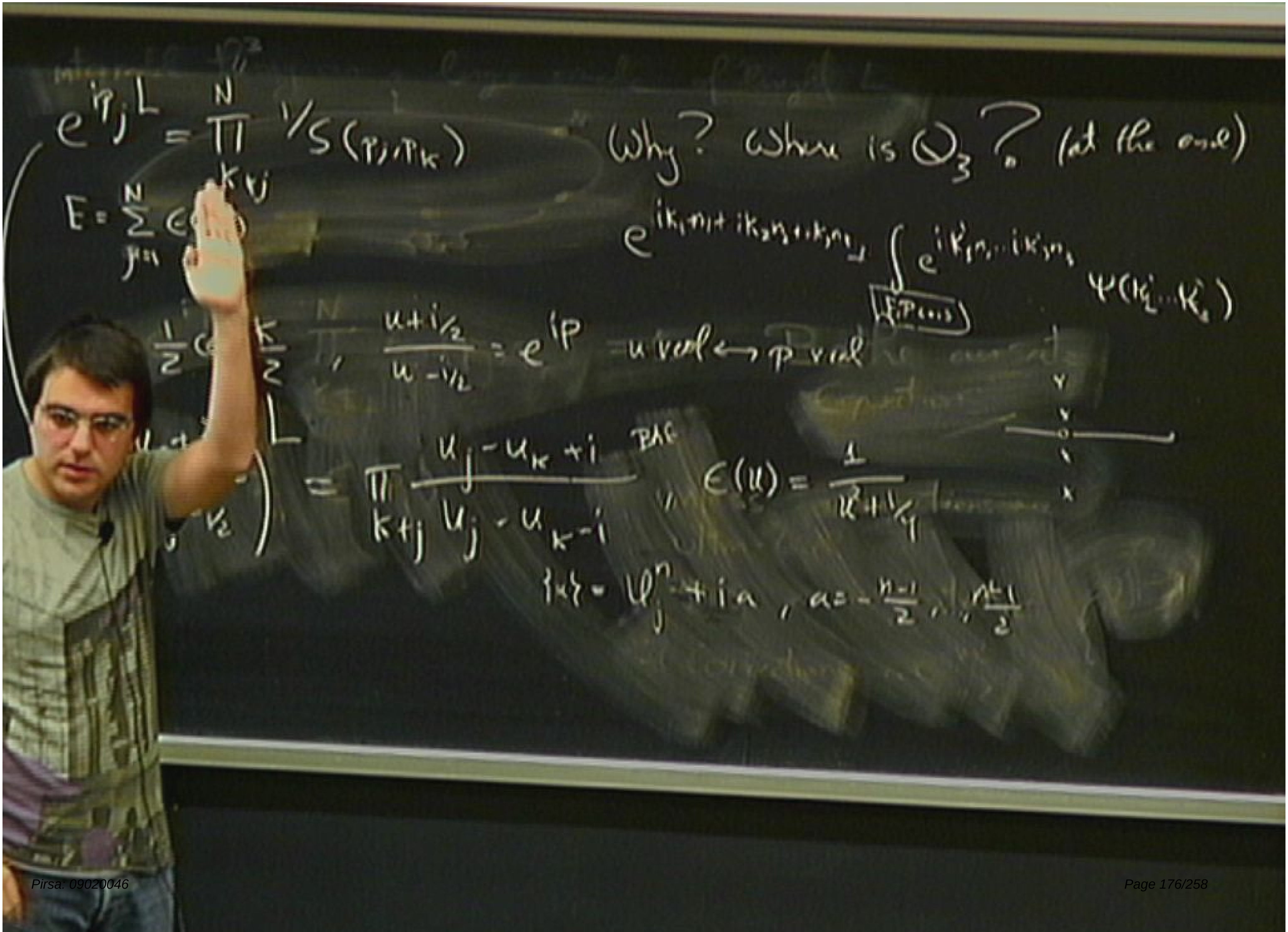
u real $\leftrightarrow$ p real

$$\left(\frac{u_j+i/2}{u_j-i/2}\right)^L = \prod_{k} \frac{u_j \ldots}{\ldots}, \quad \epsilon(u) = \frac{1}{u^2+1/4}$$

$e_j^n - u \ldots$



$e^{ip_j L} = \prod_{k \neq j}^{N} 1/S(p_j, p_k)$
$E = \sum_{j=1}^{N} \epsilon(k_j)$
Why? Where is Q_3? (at the end)
$u = \frac{1}{2}\cot$
$\frac{u+i/2}{u-i/2} = e^{ip}$
u real ⟷ p real
BAE
$\epsilon(u) = \frac{1}{u^2+1/4}$


Why? Where is Q3? (at the end)
u real ⟷ p real
PAC

$$e^{ip_j L} = \prod_{k \neq j}^{N} 1/S(p_j, p_k)$$

Why? Where is Q_3? (at the end)

$$E = \sum_{j=1}^{N} \epsilon(k_j)$$

$$e^{ik_1 n_1 + ik_2 n_2 + \dots + k_n n_{..}} \int e^{ik_1' n_1 \dots ik_3' n_3} \quad \psi(k_1' \dots k_3')$$

$\{P \dots\}$

$$u = \frac{1}{2} \cot \frac{k}{2} \,, \quad \frac{u + i/2}{u - i/2} = e^{ip}$$

u real $\leftrightarrow$ p real

$$\left(\frac{u_j + i/2}{u_j - i/2} \right)^L = \prod_{k \neq j} \frac{u_j - u_k + i}{u_j - u_k - i} \,, \quad \epsilon(u) = \frac{1}{u^2 + 1/4}$$

BAE

$$\{u\} = u_j^n + ia \,, \quad a = -\frac{n-1}{2}, \dots, \frac{n-1}{2}$$

XXX & finite T $= 1/\beta$

$F \leftarrow BE - S$ ~ we need to understand the pattern of solutions $\{u_j\}$

∃ bound state ?

<u>Strings</u> $|\uparrow \ldots \uparrow \downarrow \uparrow \ldots \uparrow \boxed{\downarrow\downarrow\downarrow} \uparrow \ldots \uparrow \rangle$ ($\vec{j}$, $\vec{k}$)

$u_j = u + i\eta$, $\eta > 0$ lhs BAE $\to \infty$ $(L \to \infty)$

therefore $\exists\, u_k$ s.t. $u_j - u_k - i = 0$

$$e^{ip_n(x)} = \frac{u + in/2}{u - in/2}$$

fused BAE

$$\left(\frac{u_j^n + i\frac{n}{2}}{u_j^n - i\frac{n}{2}}\right)^L = \prod_{k,m} S_{nm}(u_j^n, u_k^m)$$

$$e^{ip_jL} = \prod_{k\neq j}^{N} 1/S(p_j,p_k)$$

Why? Where is Q_3? (at the end)

$$E = \sum_{j=1}^{N} \epsilon(k_j)$$

$$e^{ik_1n_1+ik_2n_2+ik_3n_3} + \int_{[illegible]} e^{ik_1'n_1 \cdots ik_3'n_3}\, \psi(k_1' \cdots k_3')$$

$$u = \frac{1}{2}\cot\frac{k}{2}, \quad \frac{u+i/2}{u-i/2} = e^{ip}$$

u real $\leftrightarrow$ p real

$$\left(\frac{u_j+i/2}{u_j-i/2}\right)^L = \prod_{k\neq j} \frac{u_j-u_k+i}{u_j-u_{k}-i} \quad \text{BAE}, \quad \epsilon(u) = \frac{1}{u^2+1/4}$$

$$\{u\} = u_j^n + ia, \quad a = -\frac{n-1}{2}, \dots, \frac{n-1}{2}$$

$$e^{ip_j L} = \prod_{k \neq j}^{N} 1/S(p_j, p_k)$$

Why? Where is Q_3? (at the end)

$$E = \sum_{j=1}^{N} \epsilon(k_j)$$

$$e^{ik_1 n_1 + ik_2 n_2 + \dots + ik_m n_m} \int_{\{P_{\dots}\}} e^{ik'_1 n_1 \dots ik'_s n_s} \psi(k'_1 \dots k'_s)$$

$$u = \frac{1}{2}\cot\frac{k}{2}, \quad \frac{u + i/2}{u - i/2} = e^{ip} \quad u \text{ real} \leftrightarrow p \text{ real}$$

$$\left(\frac{u_j + i/2}{u_j - i/2}\right)^L = \prod_{k \neq j} \frac{u_j - u_k + i}{u_j - u_k - i} \quad \text{BAE}, \quad \epsilon(u) = \frac{1}{u^2 + 1/4}$$

$$\{u\} = u_j^n + ia, \quad a = -\frac{n-1}{2}, \dots, \frac{n-1}{2}$$

$$\mathcal{O} = \mathrm{tr}(ZDZ \dots ZDZ)$$

$$e^{ip_j L} = \prod_{k\neq j}^{N} 1/S(p_j, p_k)$$

Why? Where is Q_3? (at the end)

$$E = \sum_{j=1}^{N} \epsilon(k_j)$$

$$e^{ik_1 n_1 + ik_2 n_2 + ik_3 n_3} \int_{\{\hat{p}_{k=1}\}} e^{ik'_1 n_1 \dots ik'_3 n_3} \Psi(k'_1 \dots k'_3)$$

$$u = \frac{1}{2}\cot\frac{k}{2}, \quad \frac{u+i/2}{u-i/2} = e^{ip} \quad u \text{ real} \leftrightarrow p \text{ real}$$

$$\left(\frac{u_j + i/2}{u_j - i/2}\right)^L = \prod_{k\neq j} \frac{u_j - u_k + i}{u_j - u_k - i} \quad \text{BAE}, \quad \epsilon(u) = \frac{1}{u^2 + 1/4}$$

$$\{u\} = u_j^n + ia, \quad a = -\frac{n-1}{2}, \dots, \frac{n-1}{2}$$

$\mathcal{O} = \text{tr}(ZDZ \dots ZDZ)$ SL(2) sector

$$e^{ip_j L} = \prod_{k \neq j}^{N} 1/S(p_j, p_k)$$

Why? Where is Q_3? (at the end)

$$E = \sum_{j=1}^{N} \epsilon(k_j)$$

$$e^{ik_1 n_1 + ik_2 n_2 + \dots + k_N n_N} + \int e^{ik'_1 n_1 \dots ik'_N n_N} \Psi(k'_1 \dots k'_N)$$

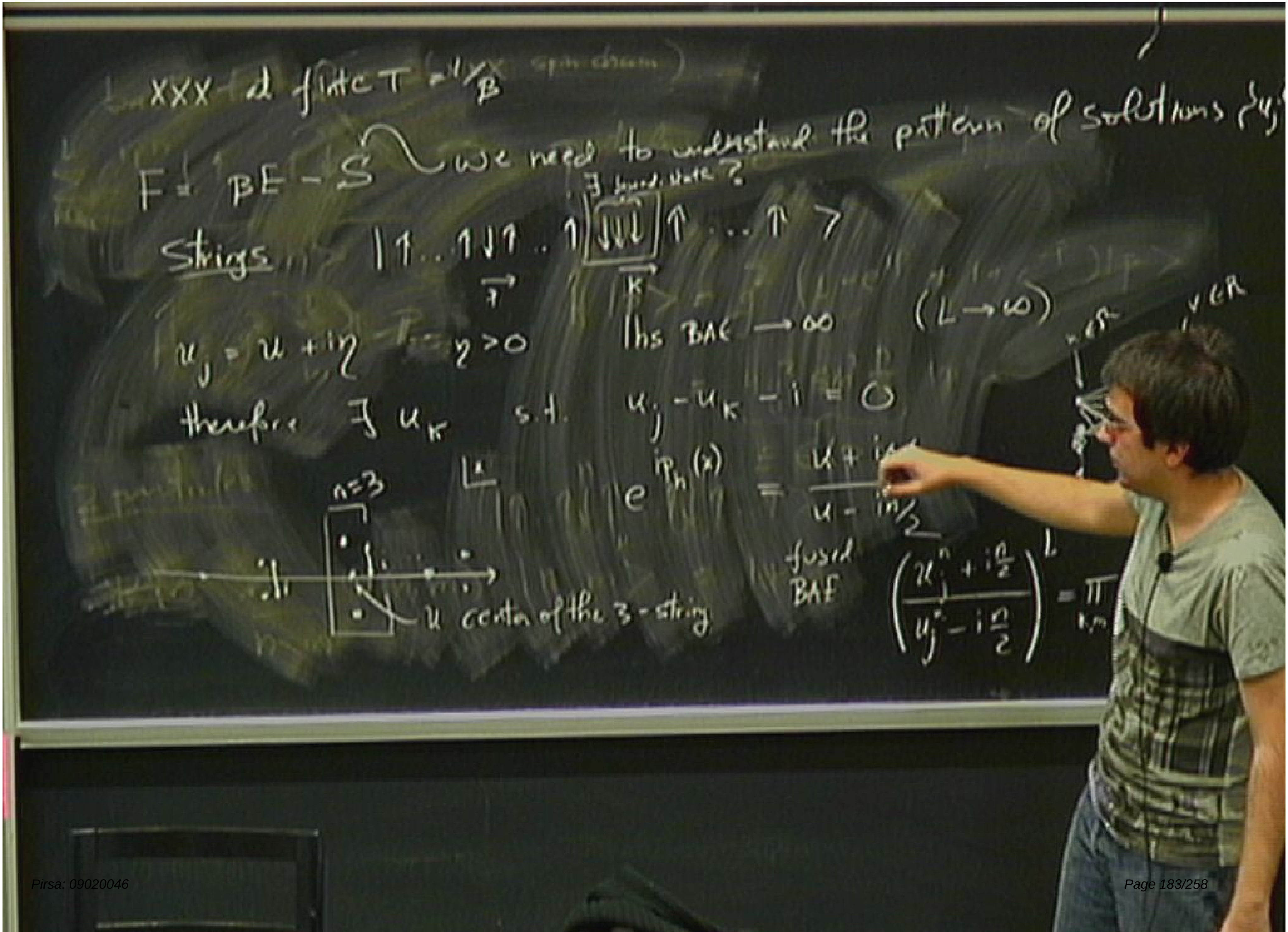
$$u = \frac{1}{2}\cot\frac{k}{2}, \quad \frac{u + i/2}{u - i/2} = e^{ip}$$

u real $\leftrightarrow$ p real

$$\left(\frac{u_j + i/2}{u_j - i/2}\right)^L = \prod_{k \neq j} \frac{u_j - u_k + i}{u_j - u_k - i}, \quad \epsilon(u) = \frac{1}{u^2 + 1/4}$$

$$\{u\} = u_j^n + ia, \quad a = -\frac{n-1}{2}, \dots, \frac{n-1}{2}$$

$\mathcal{O} = \mathrm{tr}(ZDZ \dots ZDZ)$ SL(2) Sector, all solutions are real!


XXX
we need to understand the pattern of solutions
BE - S
∃ fused state ?
Strings
lhs BAE → ∞
(L → ∞)
therefore ∃ u_K s.t. u_j - u_K - i = 0
n=3
u center of the 3 - string
fused BAE

XXX fd finite T $\sim 1/\beta$ (spin chain)

F $\leftarrow$ BE – S $\sim$ we need to understand the pattern of solutions $\{u_j\}$

$\exists$ bound state?

Strings $|\uparrow \ldots \uparrow \downarrow \uparrow \ldots \uparrow \downarrow\downarrow\downarrow \uparrow \ldots \uparrow\rangle$

$u_j = u + i\eta \quad \eta > 0$ $\qquad$ lhs BAE $\to \infty$ $\quad (L \to \infty)$

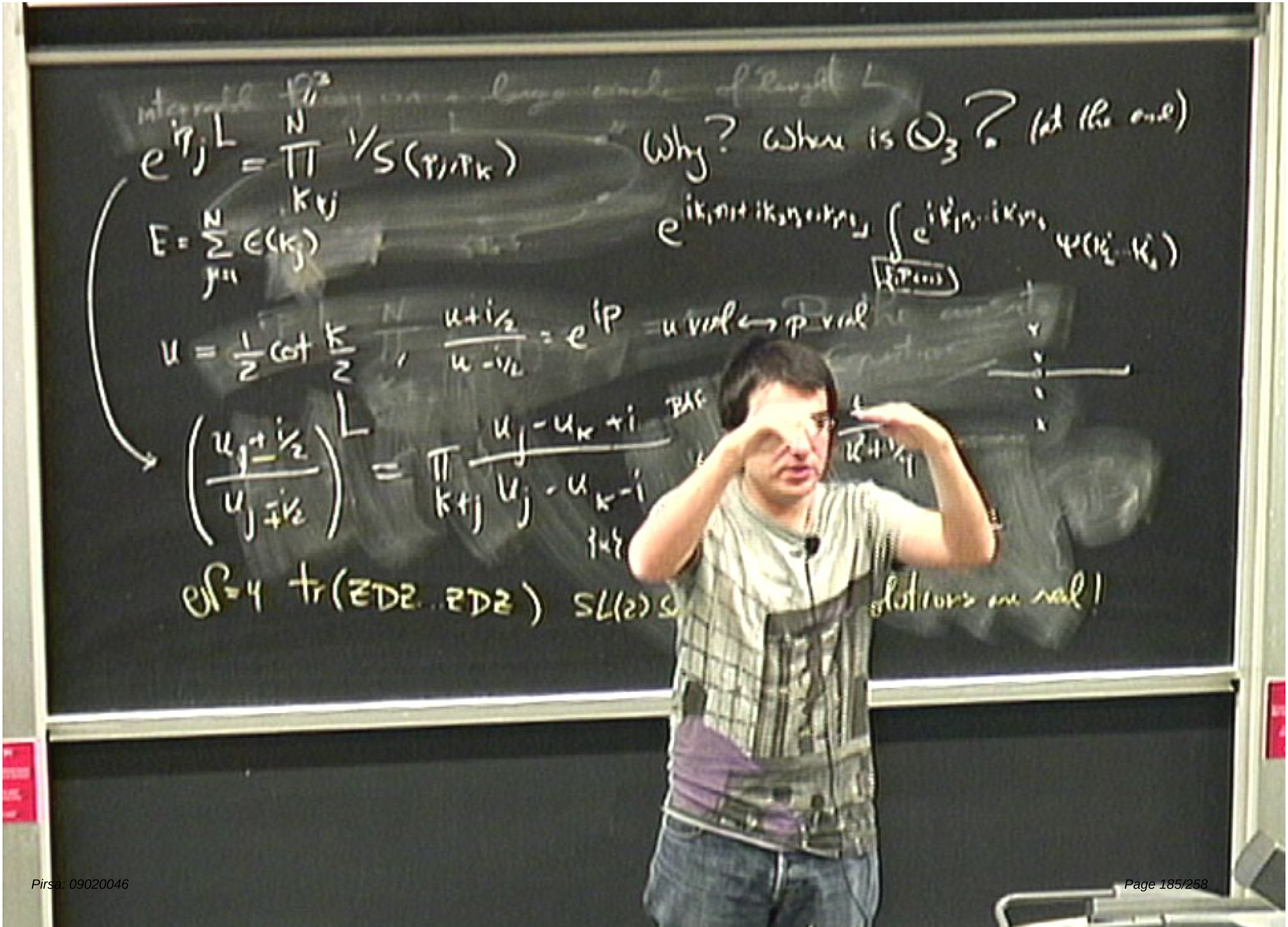
therefore $\exists\, u_K$ s.t. $u_j - u_K - i = 0$

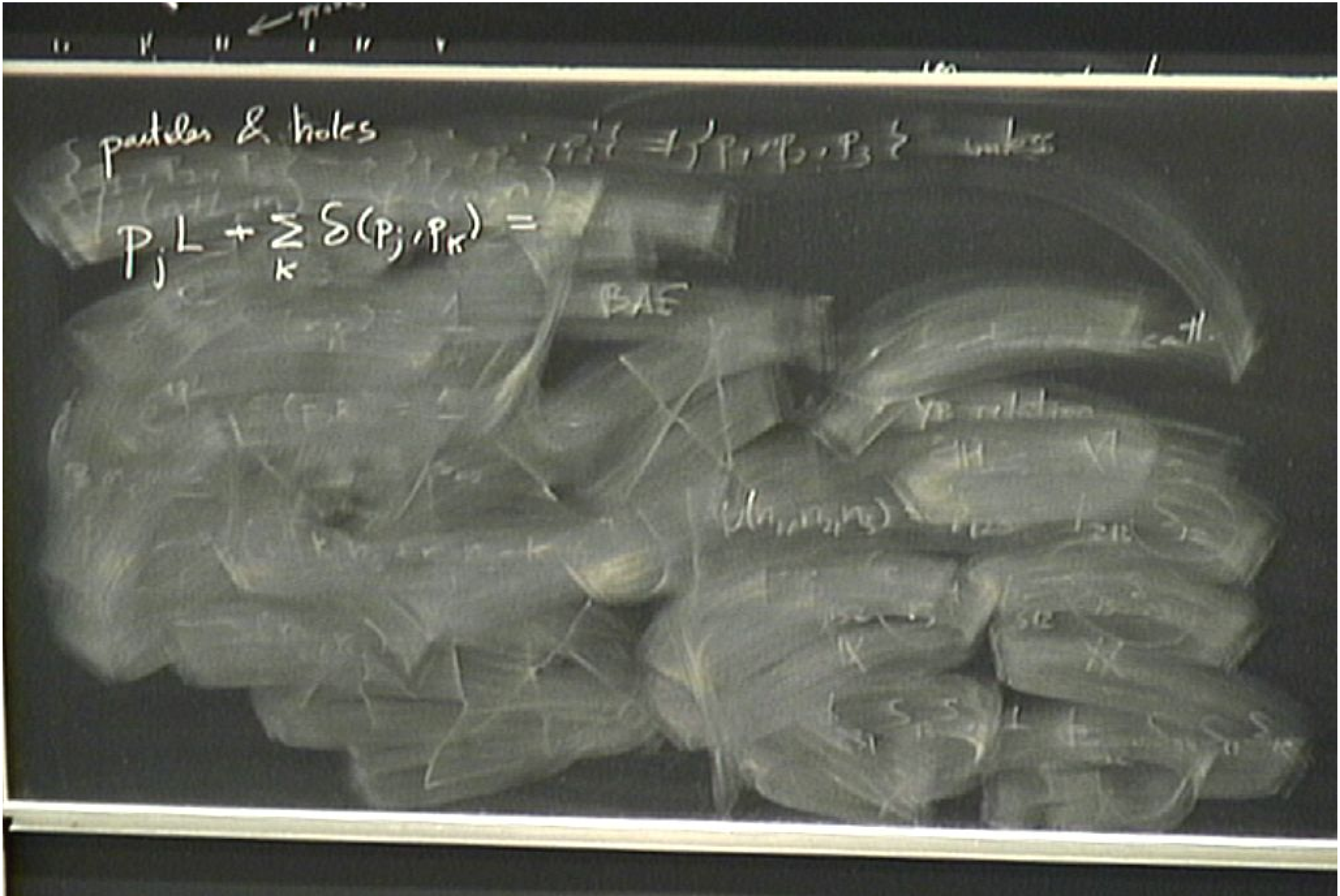
$n=3$

$e^{ip_n(x)} = \dfrac{u + in/2}{u - in/2}$

u center of the 3-string

fused BAE $\left(\dfrac{u_j^n + i\frac{n}{2}}{u_j^n - i\frac{n}{2}}\right)^L = \prod_{k,m} S_{nm}(u_j^n, u_k^m)$

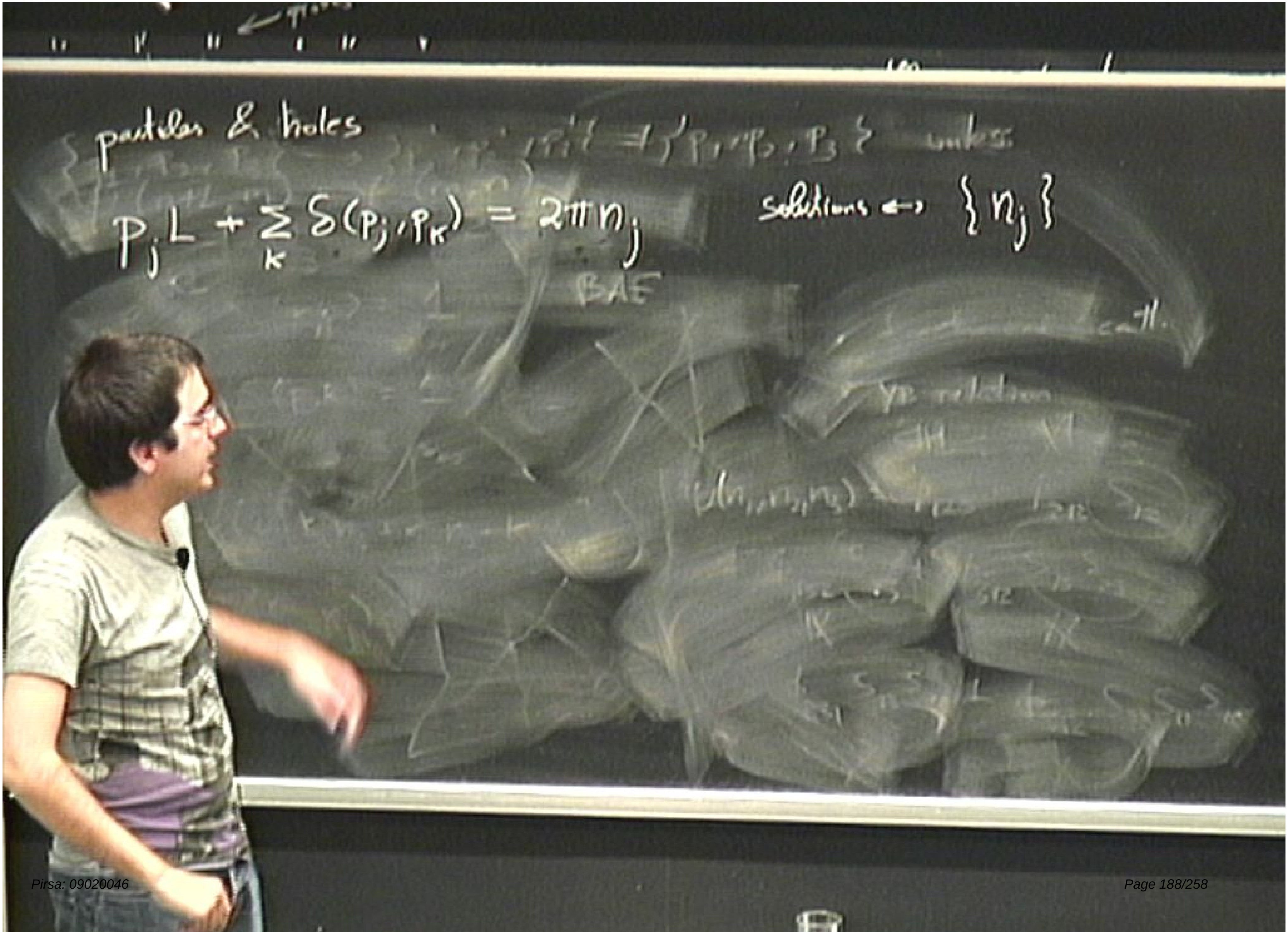

e^{ip_j L} = ∏_{k≠j}^{N} 1/S(p_j, p_k)
Why? Where is Q_3 ? (at the end)
E = ∑_{j=1}^{N} ε(k_j)
u = 1/2 cot k/2 , (u + i/2)/(u − i/2) = e^{ip}
u real ↔ p real
((u_j + i/2)/(u_j − i/2))^L = ∏_{k≠j} (u_j − u_k + i)/(u_j − u_k − i)
tr(ZDZ...ZDZ) SL(2)
solutions are real!


particles & holes
$p_j L + \sum_k \delta(p_j, p_k) =$
BAE

particles & holes

$$p_j L + \sum_k \delta(p_j, p_k) = 2\pi n_j$$

BAE

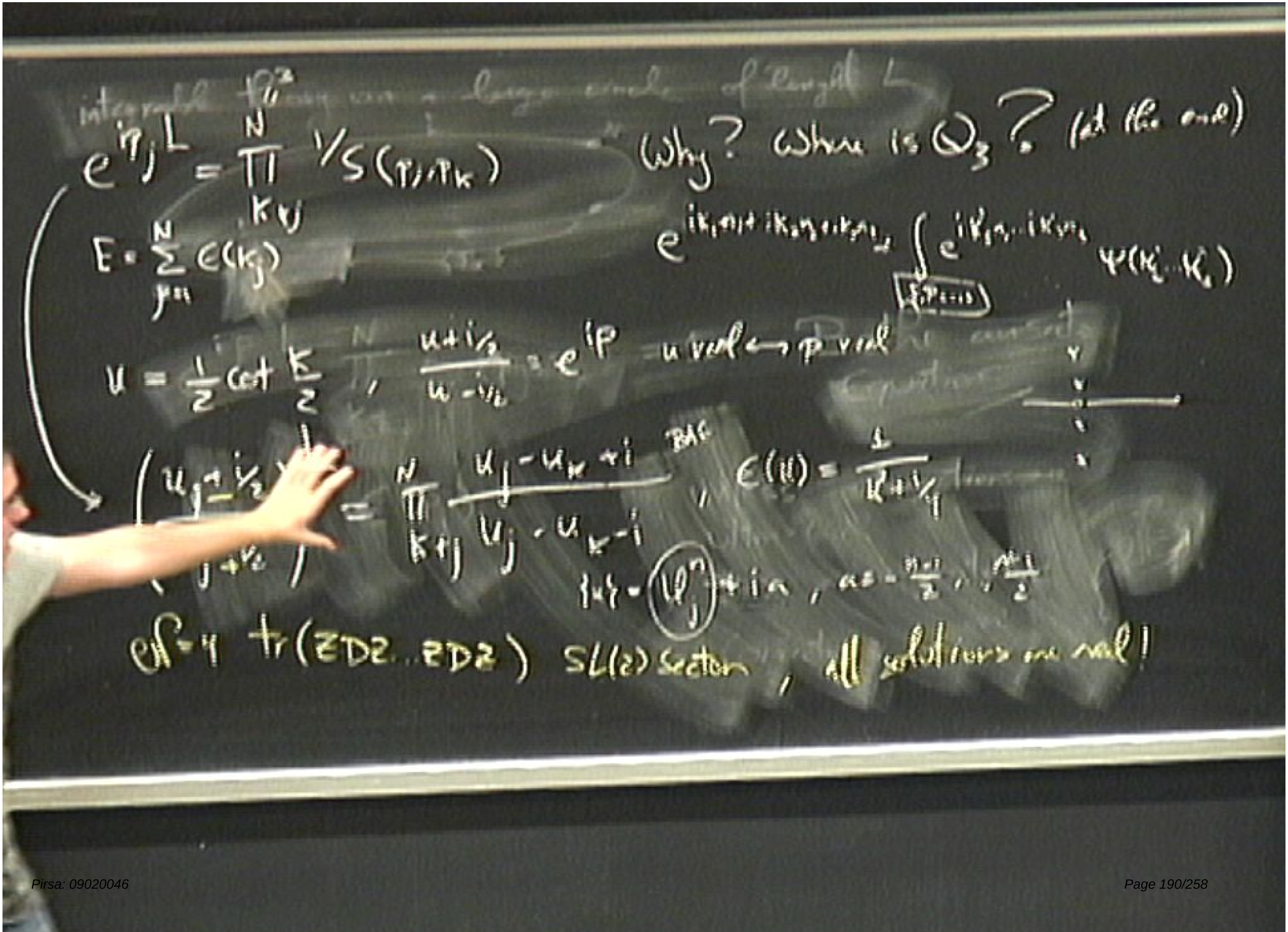

particles & holes
{p1', p2', p3'} ≠ {p1, p2, p3} unless
pj L + Σk δ(pj, pk) = 2π nj
solutions ↔ {nj}
BAE

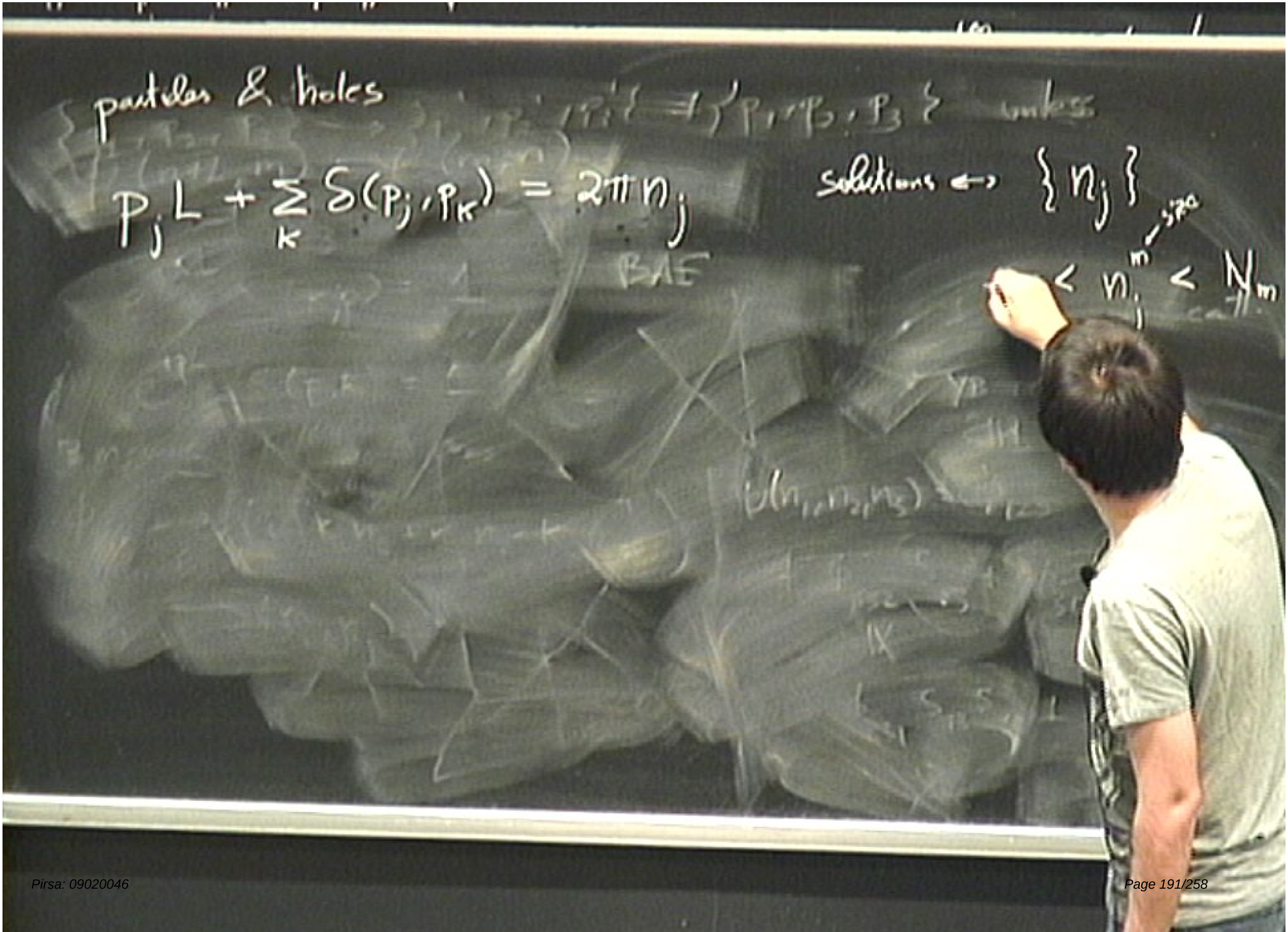
particles & holes

$$p_j L + \sum_k \delta(p_j, p_k) = 2\pi n_j$$

solutions $\leftrightarrow$ $\{n_j\}$

BAE


Why? Where is Q3 ? (at the end)
u real ⟷ p real
BAE
SL(2) Sector
all solutions are real!


particles & holes
$p_j L + \sum_k \delta(p_j, p_k) = 2\pi n_j$
BAE
solutions ↔ $\{n_j\}$
$< n_j^m < N_m$

particles & holes

$$p_j L + \sum_k \delta(p_j, p_k) = 2\pi n_j$$

solutions $\leftrightarrow$ $\{n_j\}$

$-N_m < n_j^m < N_m$

particles & holes

$\{p_1, p_2, p_3\}$ .... rules

$$p_j L + \sum_k \delta(p_j, p_k) = 2\pi n_j$$

BAE

solutions $\leftrightarrow$ $\{n_j\}$

$-N_m < n_j^m < N_m$

$\# = 2^L$

particles & holes

$\{p_1, p_2, p_3\}$ unless

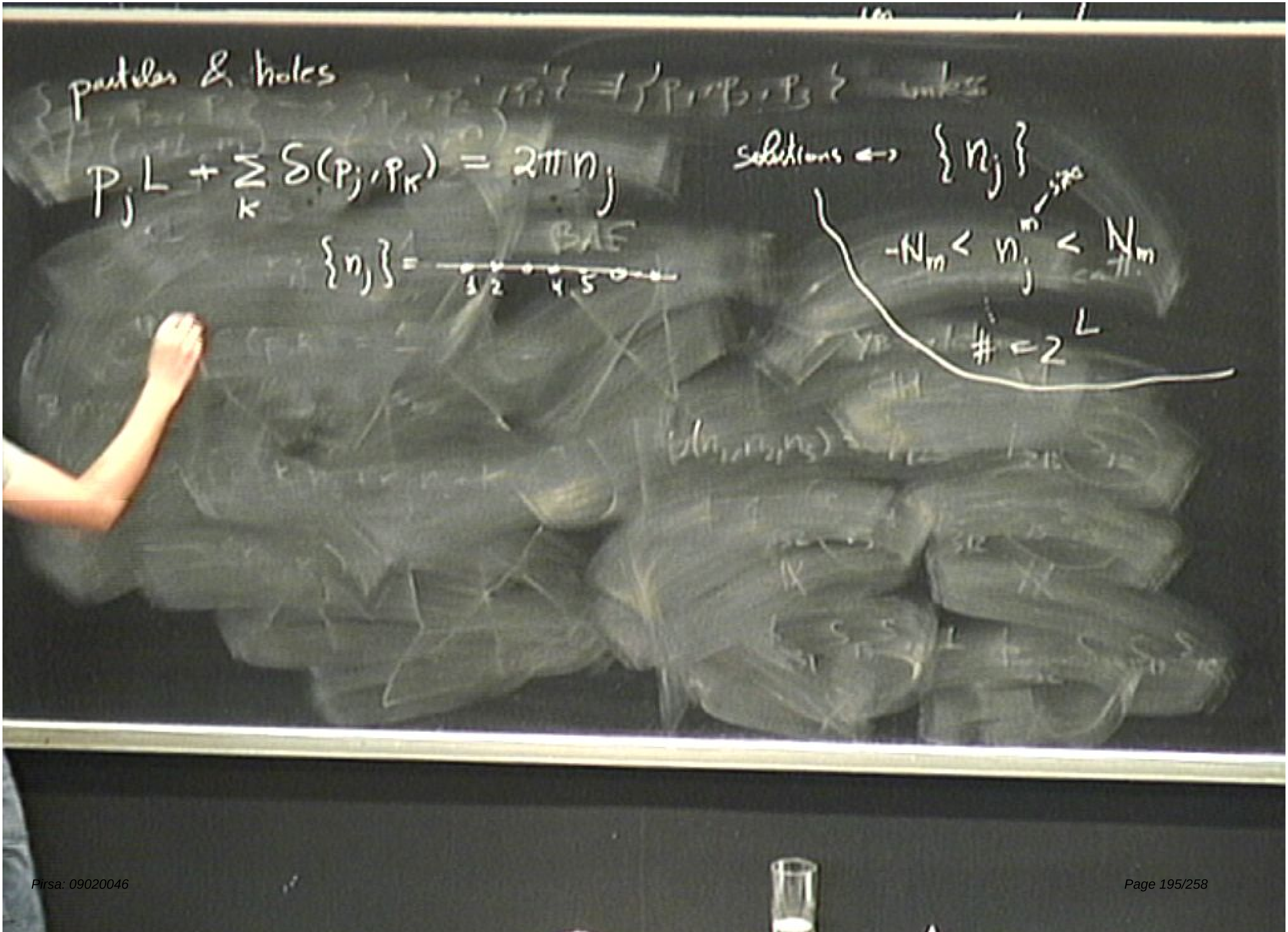
$$p_j L + \sum_k \delta(p_j, p_k) = 2\pi n_j$$

BAE

solutions $\leftrightarrow$ $\{n_j\}$

$-N_m < n_j^m < N_m$

$\# = 2^L$


particles & holes
p_j L + Σ_k δ(p_j, p_k) = 2π n_j
BAE
{n_j} =
1 2 4 5
solutions ↔ {n_j}
-N_m < n_j^m < N_m
= 2^L

particles & holes

$$p_j L + \sum_k \delta(p_j, p_k) = 2\pi n_j$$

e.g. $\{n_j\} = $ 1 2 4 5

counting function

$$p L + \sum_{k=1}^{N} \delta(p, p_k)$$

solutions $\leftrightarrow$ $\{n_j\}$

$$-N_m < n_j^m < N_m$$

$$\# = 2^L$$

particles & holes

$$p_j L + \sum_k \delta(p_j, p_k) = 2\pi n_j$$

BAE

e.g. $\{n_j\} =$ — • — • — • — • — • — • — • —
1 2 4 5

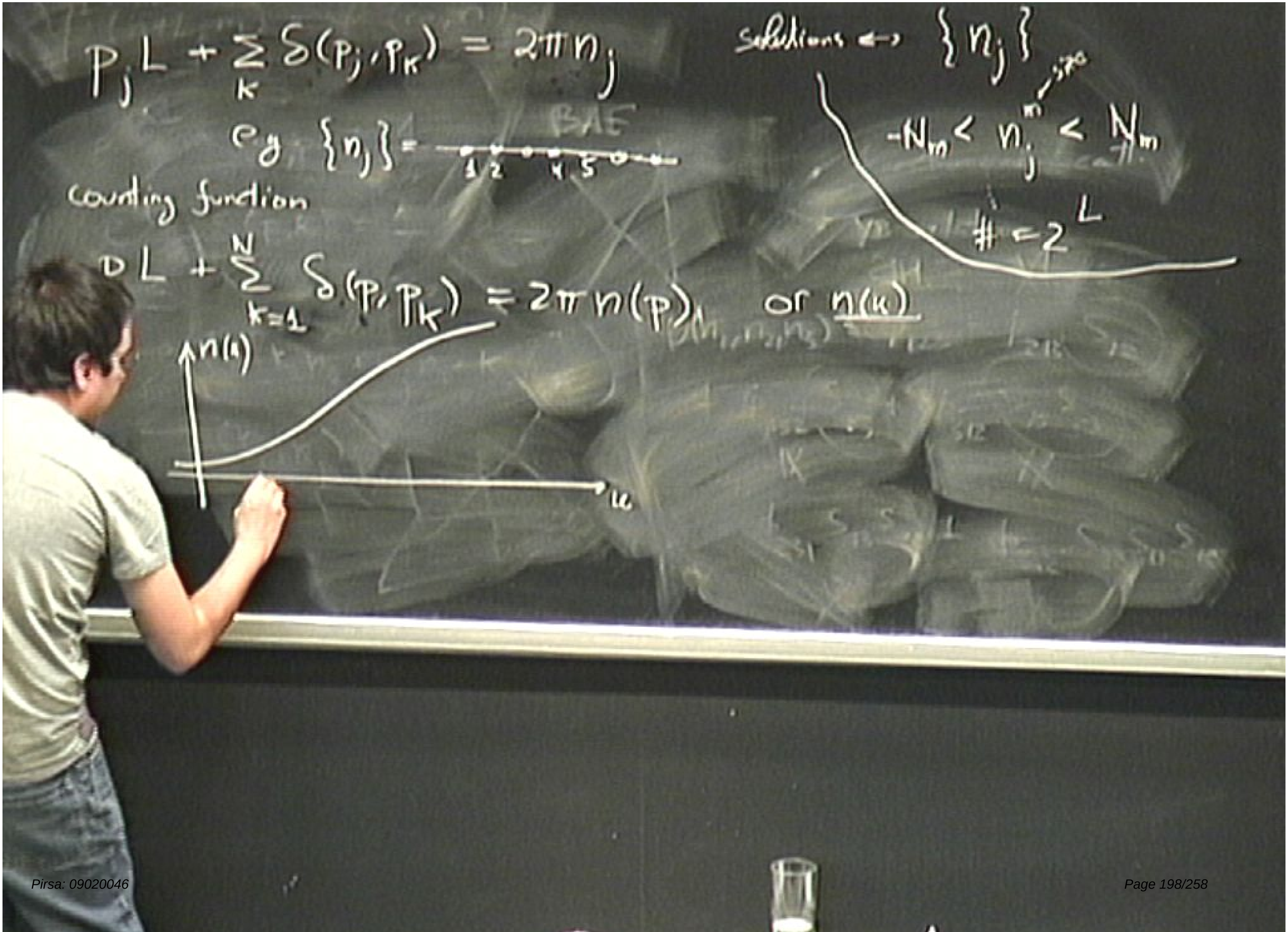
counting function

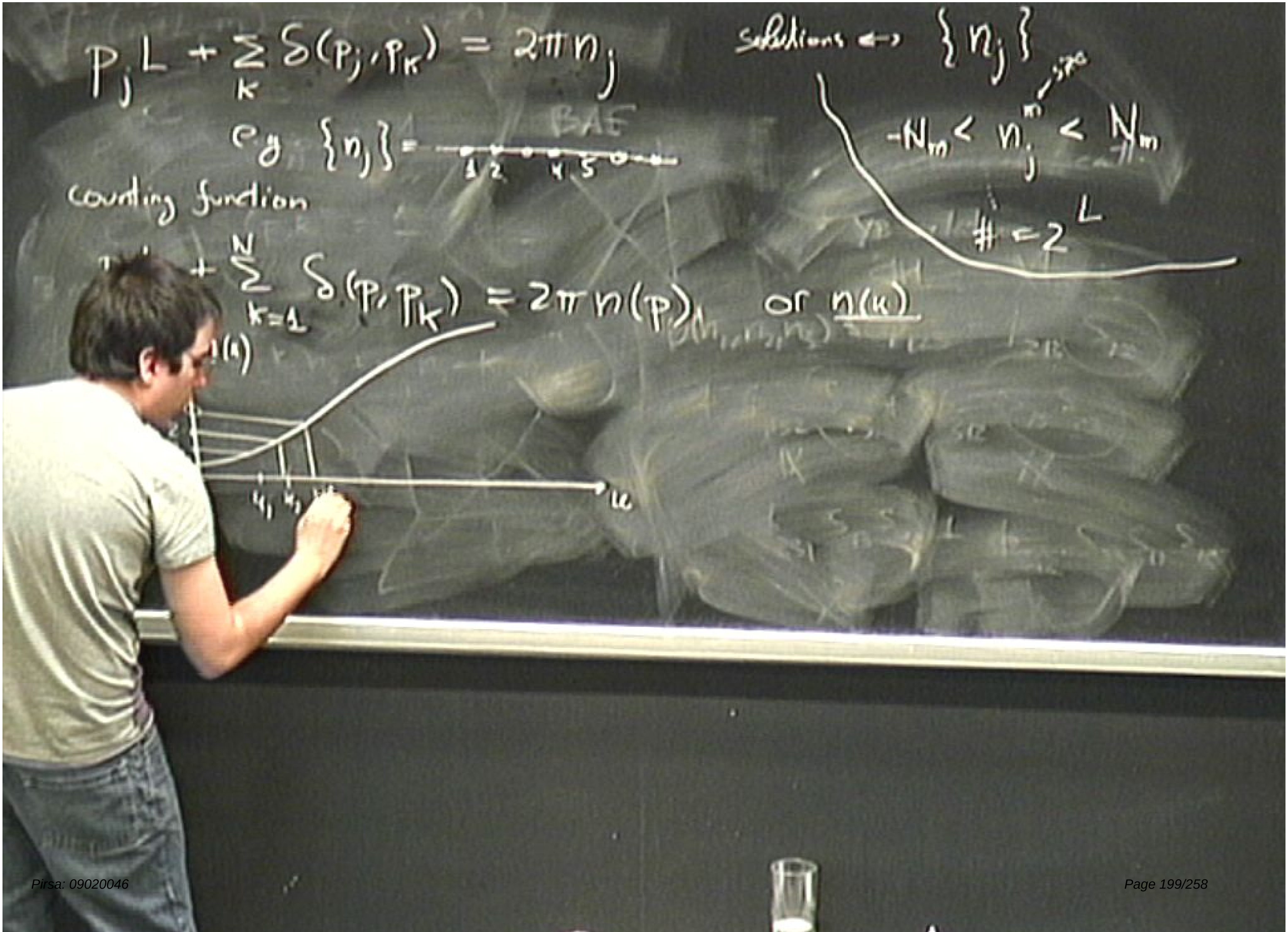
$$p L + \sum_{k=1}^{N} \delta(p, p_k) = 2\pi n(p)$$

solutions ⟷ $\{n_j\}$

$$-N_m < n_j^m < N_m$$

$$\# = 2^L$$


$p_j L + \sum_k \delta(p_j, p_k) = 2\pi n_j$
e.g. $\{n_j\} =$
1 2 4 5
counting function
$pL + \sum_{k=1}^{N} \delta(p, p_k) = 2\pi n(p)$, or $n(k)$
$n(k)$
k
solutions ⟷ $\{n_j\}$
$-N_m < n_j^m < N_m$
$\# \sim 2^L$


p_j L + Σ_k δ(p_j, p_k) = 2π n_j
e.g. {n_j} =
1 2 4 5
counting function
δ(p, p_k) = 2π n(p), or n(k)
k=1
Solutions ↔ {n_j}
-N_m < n_j^m < N_m
= 2^L

$$p_j L + \sum_k \delta(p_j, p_k) = 2\pi n_j$$

solutions $\leftrightarrow$ $\{n_j\}$

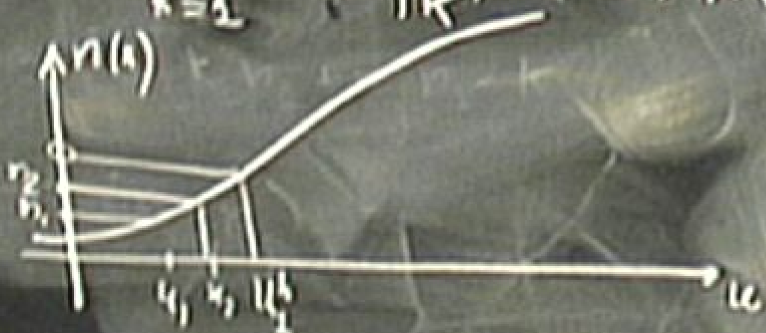
e.g. $\{n_j\} =$ 1 2 4 5

$-N_m < n_j^m < N_m$

$\# = 2^L$

counting function

$$pL + \sum_{k=1}^{N} \delta(p, p_k) = 2\pi n(p) \quad \text{or } n(k)$$



$$p_j L + \sum_k \delta(p_j, p_k) = 2\pi n_j$$

e.g. $\{n_j\} =$ • • · • • · • • (1 2 · 4 5)

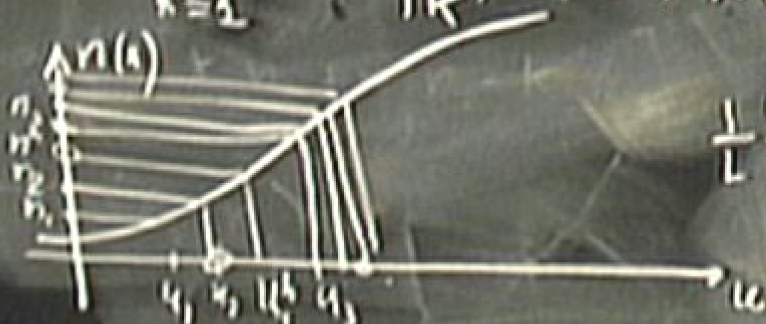
solutions $\leftrightarrow$ $\{n_j\}$

$$-N_m < n_j^m < N_m$$

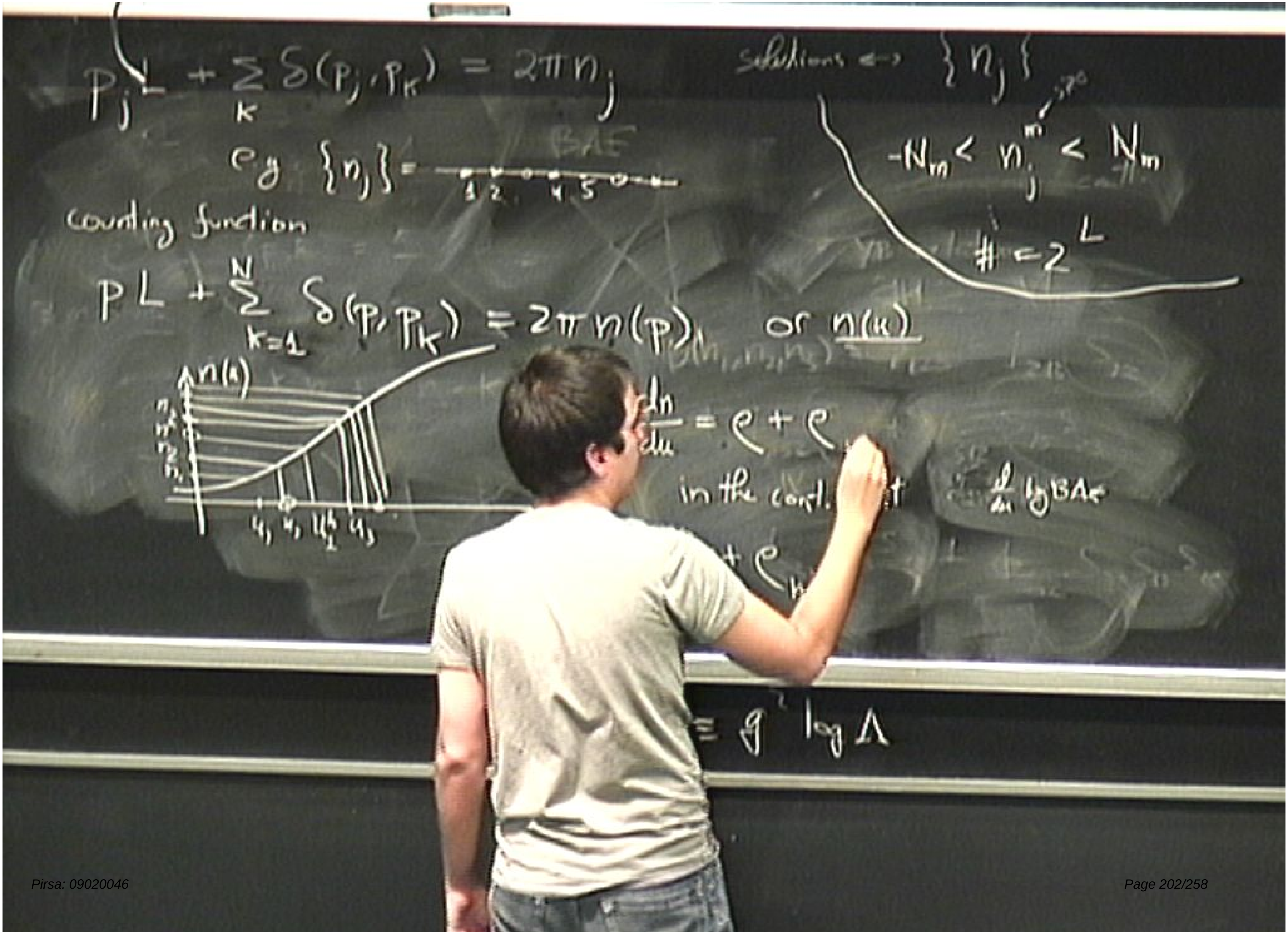
$\# = 2^L$

counting function

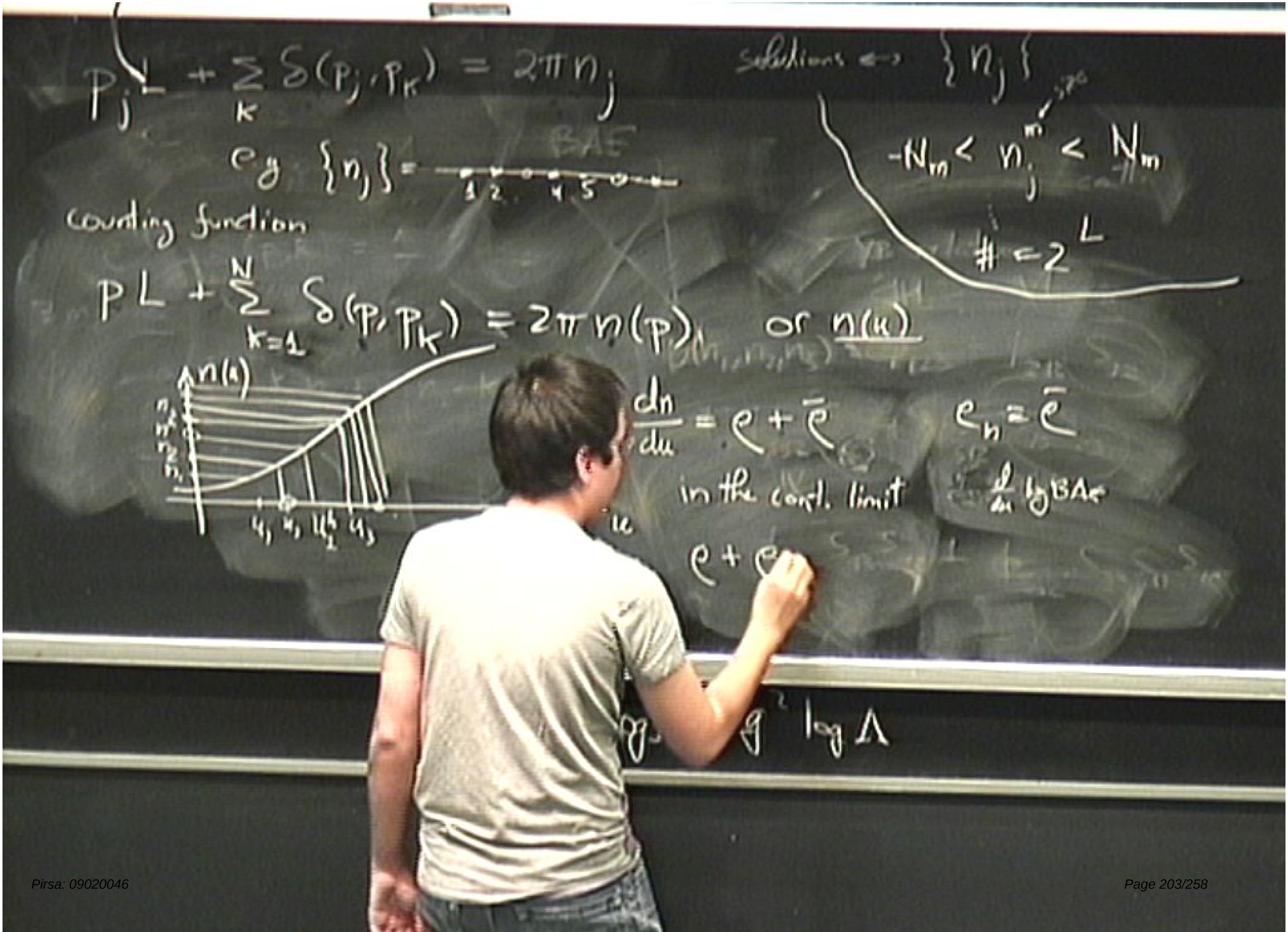
$$pL + \sum_{k=1}^{N} \delta(p, p_k) = 2\pi n(p), \quad \text{or } \underline{n(k)}$$



$$\frac{1}{L}\frac{dn}{du} = \rho + \rho_h$$



$p_j L + \sum_k \delta(p_j, p_k) = 2\pi n_j$
solutions ↔ $\{n_j\}$
BAE
e.g. $\{n_j\}$ =
1 2 4 5
$-N_m < n_j^m < N_m$
$\# = 2^L$
counting function
$p L + \sum_{k=1}^{N} \delta(p, p_k) = 2\pi n(p)$, or $n(u)$
$n(u)$
u_1 u_2 u_1^h u_3
$\frac{dn}{du} = \rho + \rho$
in the cont.
$\frac{d}{du} \log \mathrm{BAE}$
$+ \rho_h$
$= g^2 \log \Lambda$


p_j L + Σ_k δ(p_j, p_k) = 2π n_j
solutions ⟷ {n_j}
e.g. {n_j} =
-N_m < n_j^m < N_m
= 2^L
counting function
p L + Σ_{k=1}^N δ(p, p_k) = 2π n(p), or n(u)
n(u)
dn/du = ρ + ρ̄
ρ_h = ρ̄
in the cont. limit
ρ + ρ
log Λ

$$p_j L + \sum_k \delta(p_j, p_k) = 2\pi n_j$$

solutions $\leftrightarrow$ $\{n_j\}$

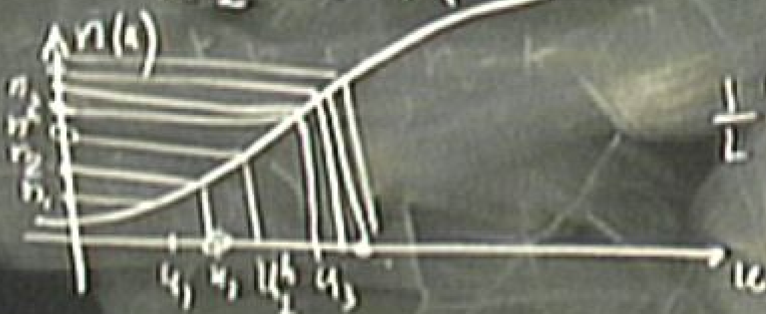
$$-N_m < n_j^m < N_m$$

$$\# = 2^L$$

e.g. $\{n_j\} =$ 1 2 4 5

counting function

$$pL + \sum_{k=1}^{N} \delta(p, p_k) = 2\pi n(p) \quad \text{or } n(u)$$

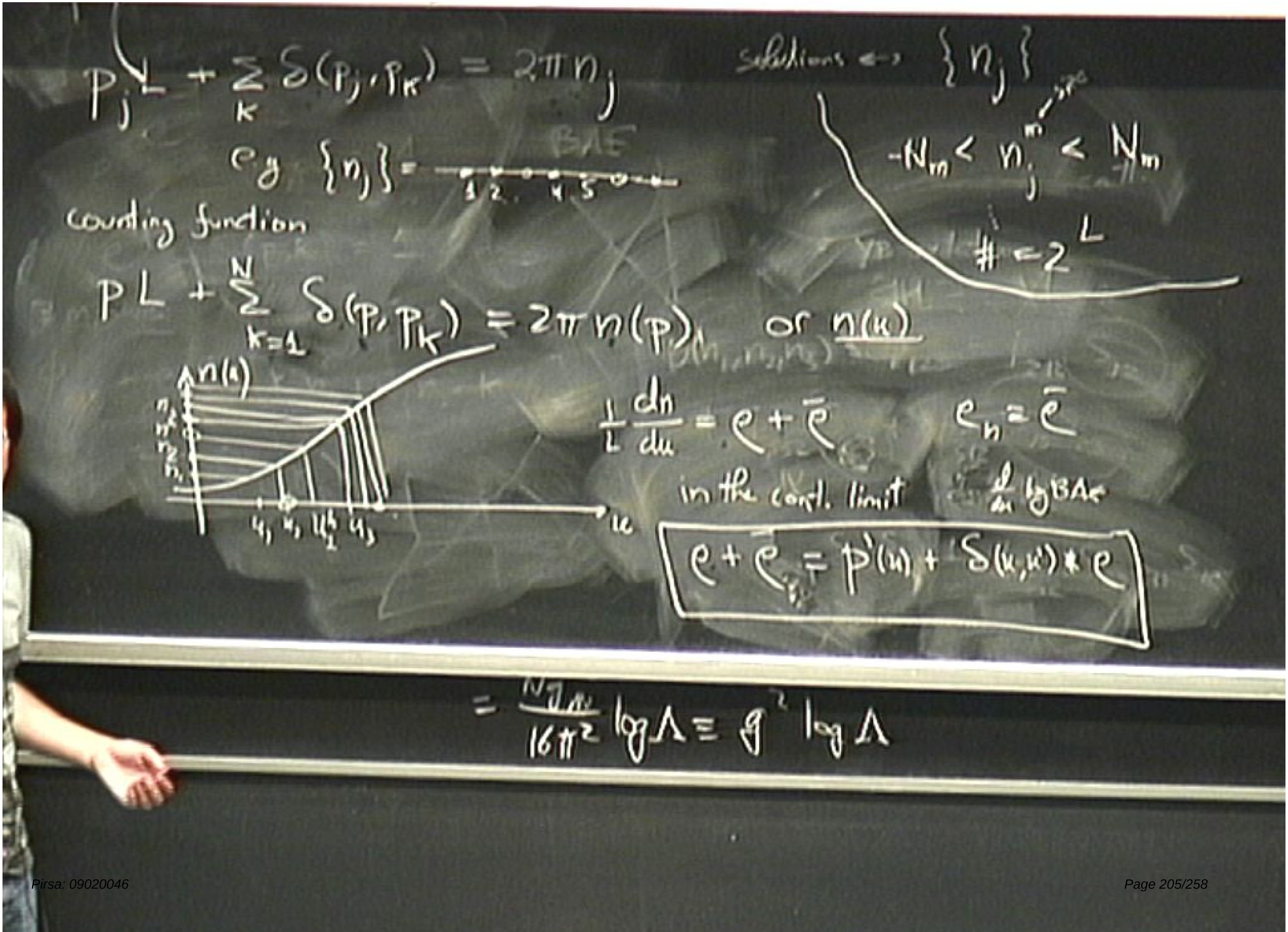


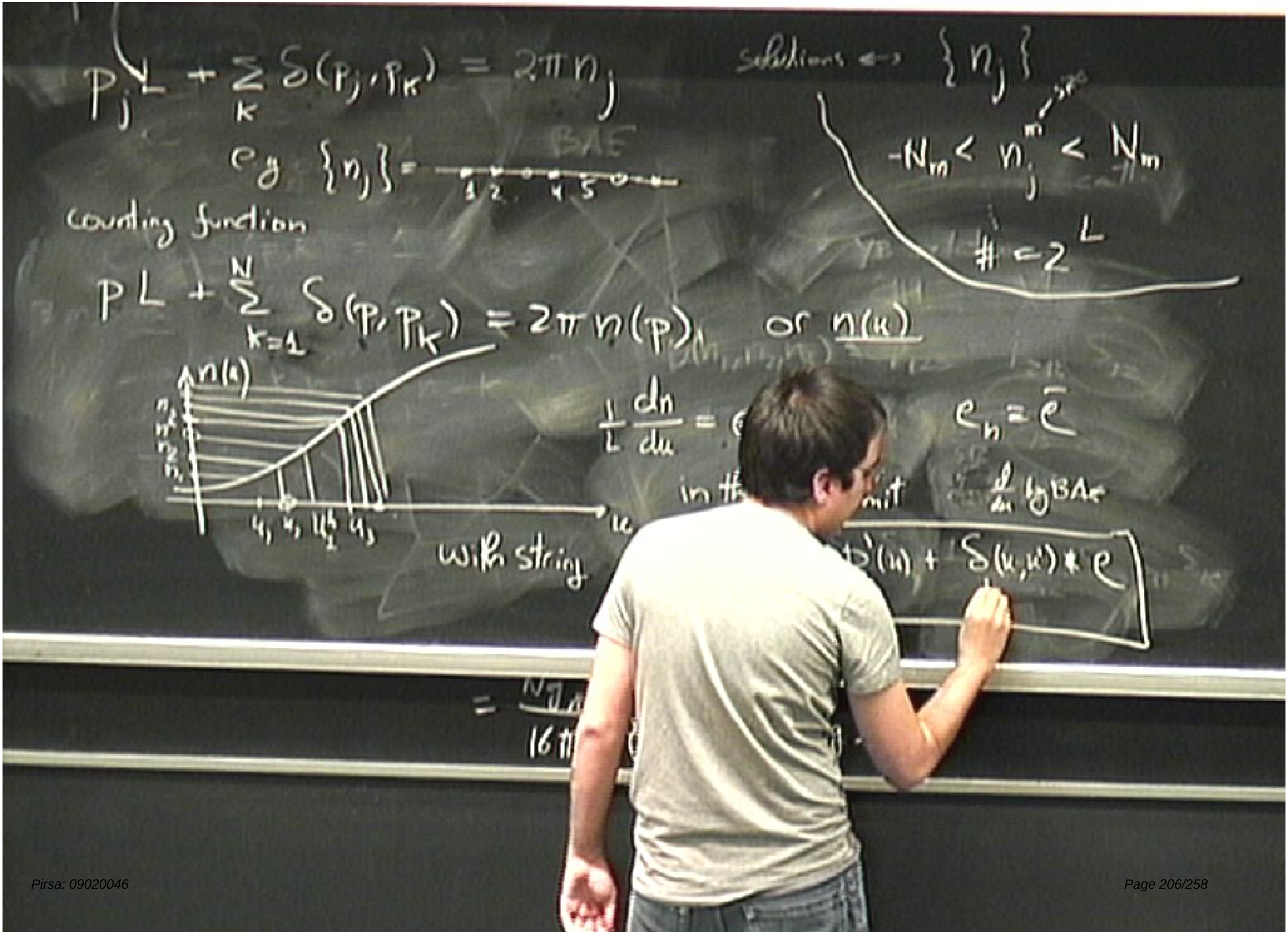
$$\frac{1}{L}\frac{dn}{du} = \rho + \bar{\rho} \qquad \rho_h = \bar{\rho}$$

in the cont. limit $\quad \frac{d}{du} \log \text{BAE}$

$$\rho + \bar{\rho} = p'(u) + \delta(u,u') * \rho$$

$$= \frac{N g_{YM}^2}{16\pi^2} \log \Lambda \equiv g^2 \log \Lambda$$


p_j L + Σ_k δ(p_j, p_k) = 2π n_j
solutions ↔ {n_j}
e.g. {n_j} =
1 2 4 5
-N_m < n_j^m < N_m
= 2^L
counting function
pL + Σ_{k=1}^N δ(p, p_k) = 2π n(p), or n(u)
n(u)
u
1/L dn/du = ρ + ρ̄
ρ_h = ρ̄
in the cont. limit
ρ + ρ̄ = p'(u) + δ(u,u') * ρ
16π² log Λ ≡ g² log Λ



solutions ⟷ {n_j}
-N_m < n_j^m < N_m
= 2^L
counting function
or n(k)
n(k)
with string

$$p_j L + \sum_k \delta(p_j, p_k) = 2\pi n_j$$

solutions $\leftrightarrow$ $\{n_j\}$

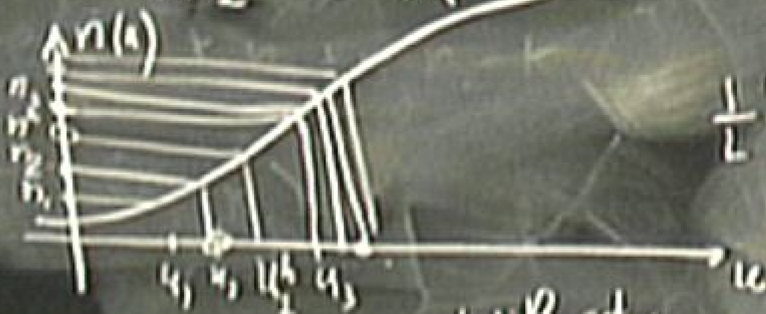
e.g. $\{n_j\} =$ 1 2 4 5

$-N_m < n_j^m < N_m$ (string)

$\# = 2^L$

counting function

$$pL + \sum_{k=1}^{N} \delta(p, p_k) = 2\pi n(p) \quad \text{or } \underline{n(u)}$$



$$\frac{1}{L}\frac{dn}{du} = \rho + \bar{\rho} \qquad \rho_n = \bar{\rho}$$

in the cont. limit $\quad \frac{d}{du}\log \text{BAE}$

with string

$$\rho_n + \bar{\rho}_n = p_n'(u) + \sum_{m=1}^{\infty} \delta(u,u') * \rho_m$$

$$= \frac{N g_{YM}^2}{16\pi^2} \log\Lambda \equiv g^2 \log\Lambda$$

$$p_j L + \sum_k \delta(p_j, p_k) = 2\pi n_j$$

solutions $\leftrightarrow$ $\{n_j\}$

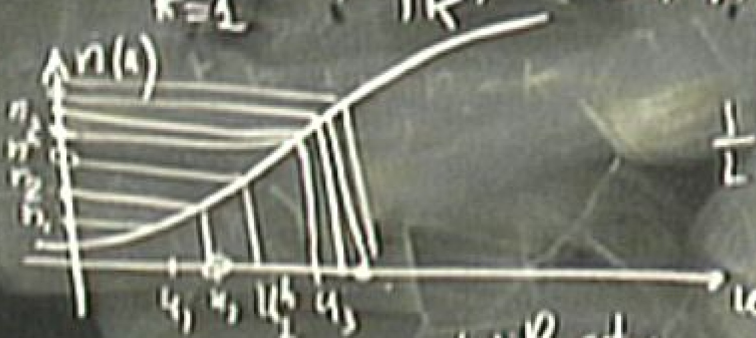
e.g. $\{n_j\} =$ 1 2 4 5

$-N_m < n_j^m < N_m$

$\# = 2^L$

counting function

$$p L + \sum_{k=1}^{N} \delta(p, p_k) = 2\pi n(p) \quad \text{or } \underline{n(u)}$$



$$\frac{1}{L}\frac{dn}{du} = \rho + \bar{\rho} \qquad \rho_n = \bar{\rho}$$

in the cont. limit $\quad \frac{d}{du}$ log BAE

with string

$$\rho_n + \bar{\rho}_n = p_n'(u) + \sum_{m=1}^{\infty} \delta(u, u') * \rho_m$$

$$= \frac{N g_{YM}}{16\pi^2} \log \Lambda \equiv g^2 \log \Lambda$$

XXX at finite $T = 1/\beta$

$F = \beta E - S$ ~ we need to understand the pattern of solutions $\{u_j\}$

Strings $|\uparrow \ldots \uparrow \downarrow \uparrow \ldots \uparrow \boxed{\downarrow\downarrow\downarrow} \uparrow \ldots \uparrow\rangle$ ∃ bound state?

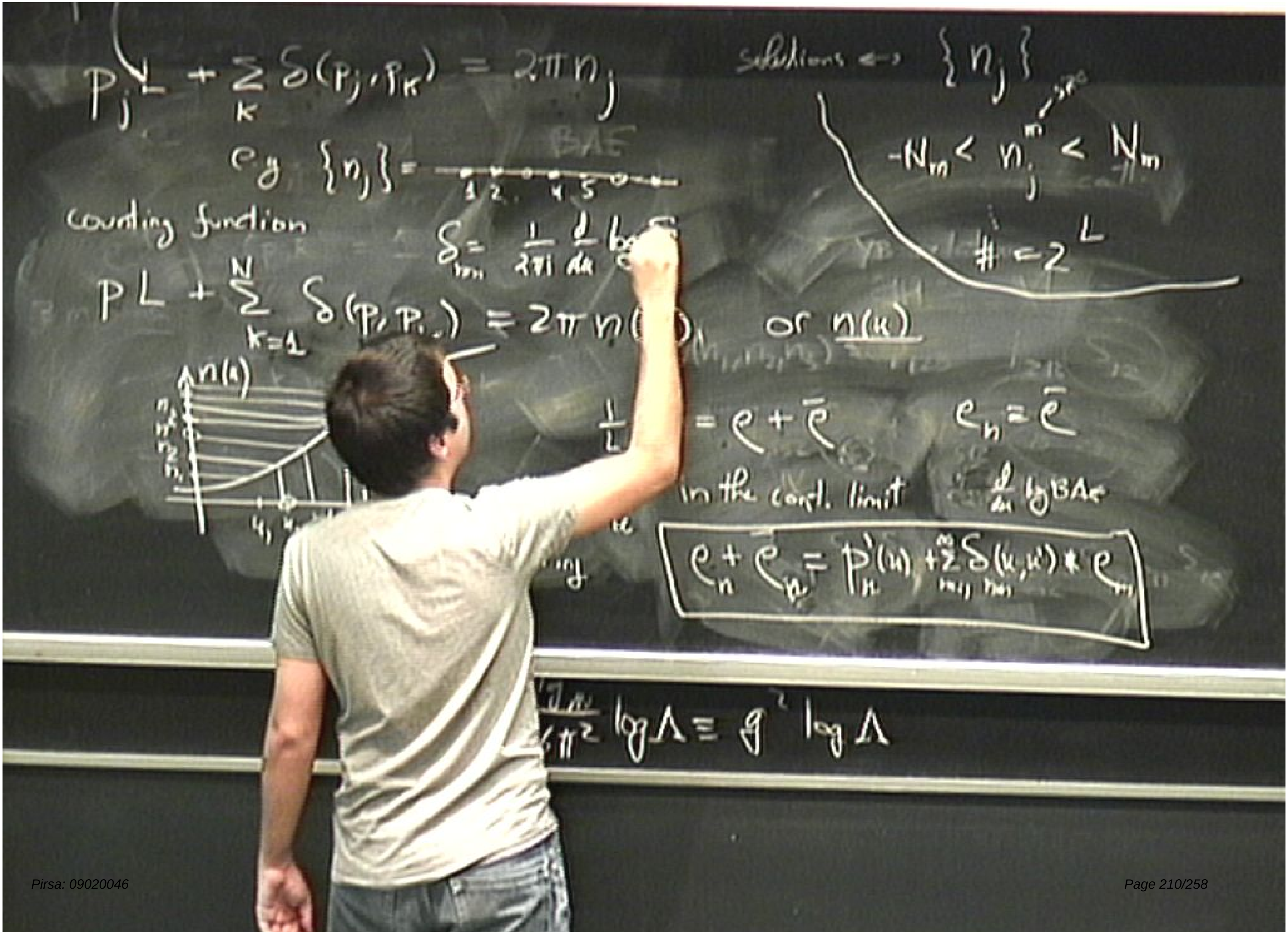
$u_j = u + i\eta$, $\eta > 0$ lhs BAE $\to \infty$ $(L \to \infty)$

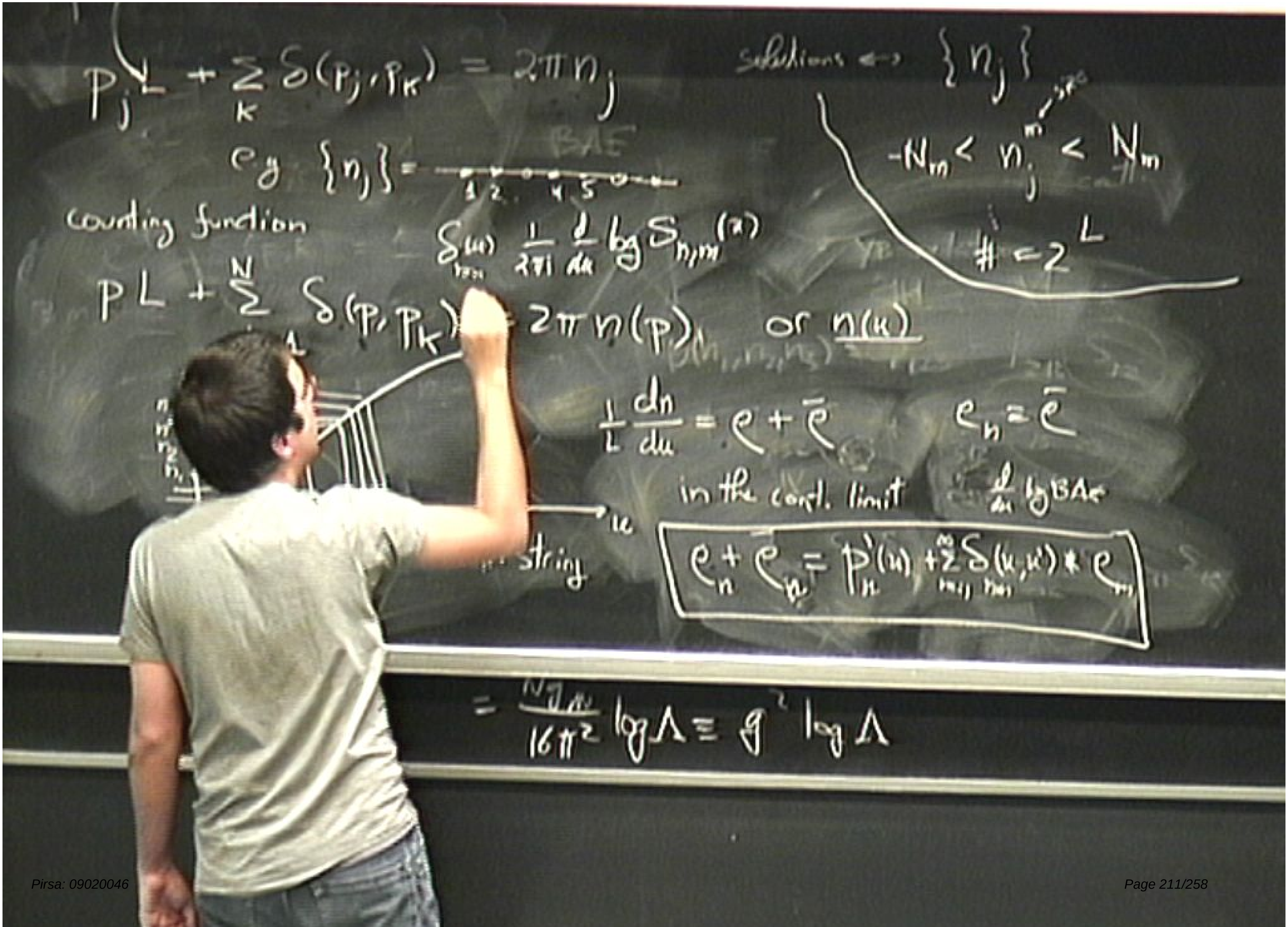
therefore $\exists\, u_K$ s.t. $u_j - u_K - i = 0$

$$e^{ip_n(x)} = \frac{u + in/2}{u - in/2}$$

$n=3$

fused BAE
$$\left(\frac{u_j^n + i\frac{n}{2}}{u_j^n - i\frac{n}{2}}\right)^L = \prod_{k,m} S_{nm}(u_j^n, u_k^n)$$


solutions ⟷ {n_j}
counting function
or n(k)
in the cont. limit
d/dk log BAe


solutions ↔ {n_j}
e.g. {n_j} =
1 2 4 5
counting function
in the cont. limit
string

$$p_j L + \sum_k \delta(p_j, p_k) = 2\pi n_j$$

solutions $\leftrightarrow$ $\{n_j\}$

e.g. $\{n_j\} =$ 1 2 4 5

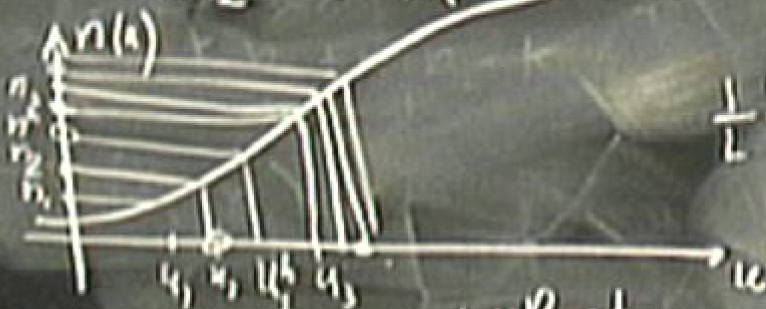
$$-N_m < n_j^m < N_m$$

counting function

$$\delta_{nm}(u) = \frac{1}{2\pi i}\frac{d}{du}\log S_{n,m}(x)$$

$$\# = 2^L$$

$$pL + \sum_{k=1}^{N} \delta(p, p_k) = 2\pi n(p), \quad \text{or } n(u)$$



$$\frac{1}{L}\frac{dn}{du} = \rho + \bar{\rho} \qquad \rho_h = \bar{\rho}$$

in the cont. limit $\quad \frac{d}{du}\log \text{BAE}$

with string

$$\rho_n + \bar{\rho}_n = p_n'(u) + \int \sum_{m=1}^{\infty} \delta_{nm}(u-u')\, \rho_m(u')\, du'$$

$$= \frac{N g_{YM}^2}{16\pi^2}\log\Lambda \equiv g^2 \log\Lambda$$

$$p_j L + \sum_k \delta(p_j, p_k) = 2\pi n_j$$

solutions $\leftrightarrow$ $\{n_j\}$

e.g. $\{n_j\}$ = (1, 2, 4, 5)

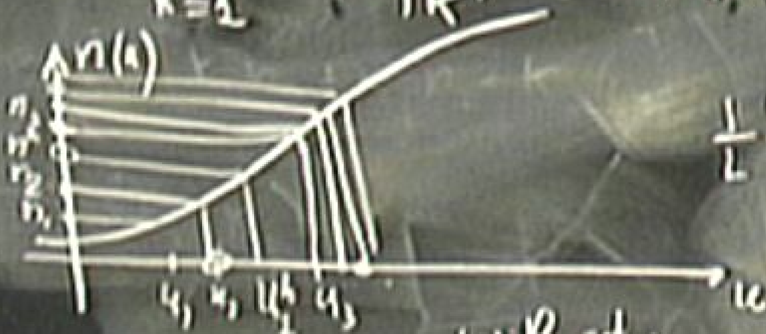
$-N_m < n_j^m < N_m$

counting function

$\delta_{nm}(u) \; \frac{1}{2\pi i} \frac{d}{du} \log S_{n,m}(x)$

$\# = 2^L$

$$pL + \sum_{k=1}^{N} \delta(p, p_k) = 2\pi n(p), \quad \text{or } n(u)$$



$$\frac{1}{L}\frac{dn}{du} = \rho + \bar{\rho} \qquad \rho_n = \bar{\rho}$$

in the cont. limit $\frac{d}{du} \log \text{BAE}$

with string

$$\rho_n + \bar{\rho}_n = p_n'(u) + \int \sum_{m=1}^{\infty} \delta(u-u')_{nm} \, \rho_m(u')\, du'$$

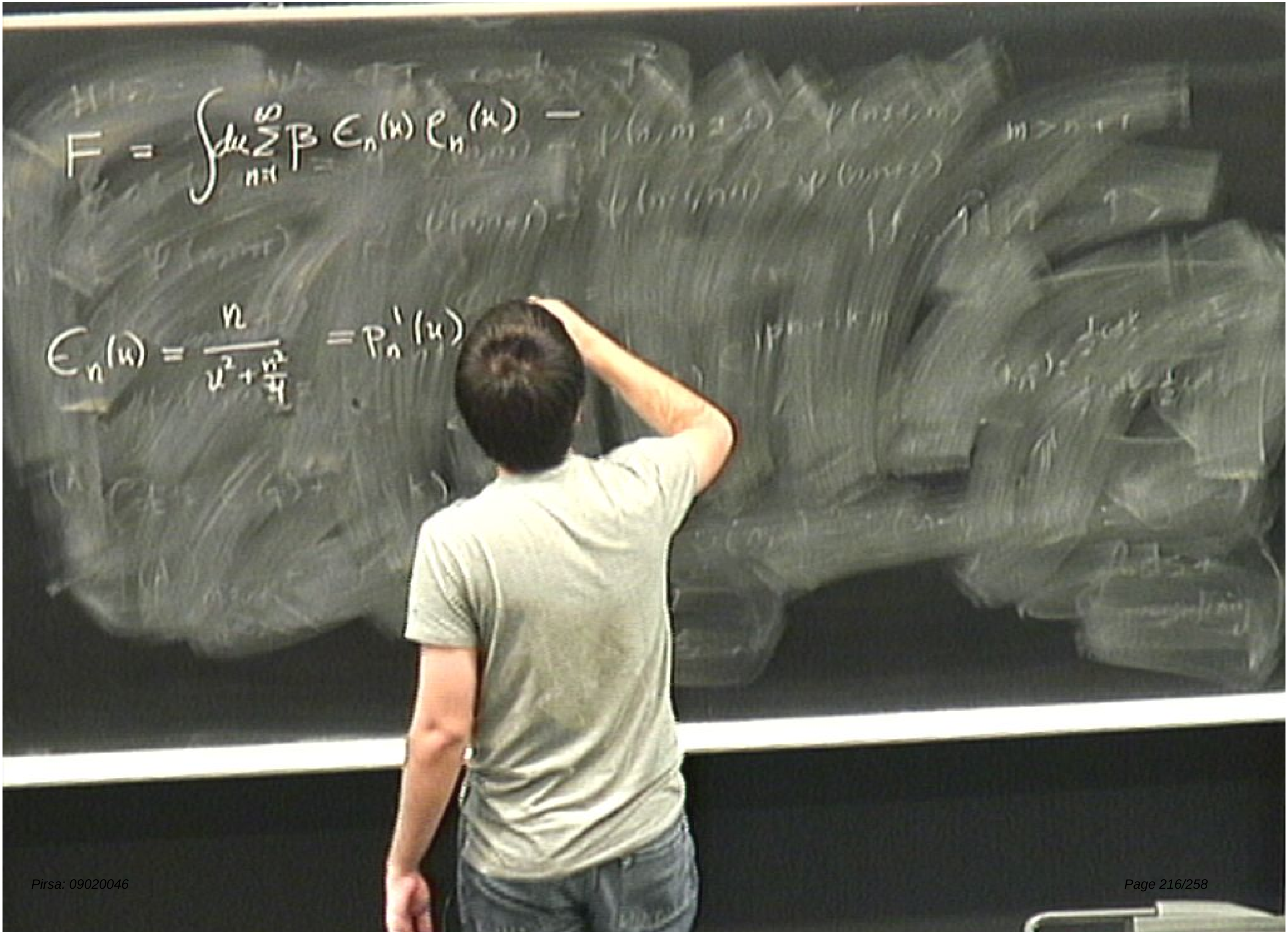
$$= \frac{N g_{YM}}{16\pi^2} \log \Lambda \equiv g^2 \log \Lambda$$

$F =$

$$\epsilon_n(u) = \frac{n}{u^2 + \frac{n^2}{2}} = p_n'(u)$$

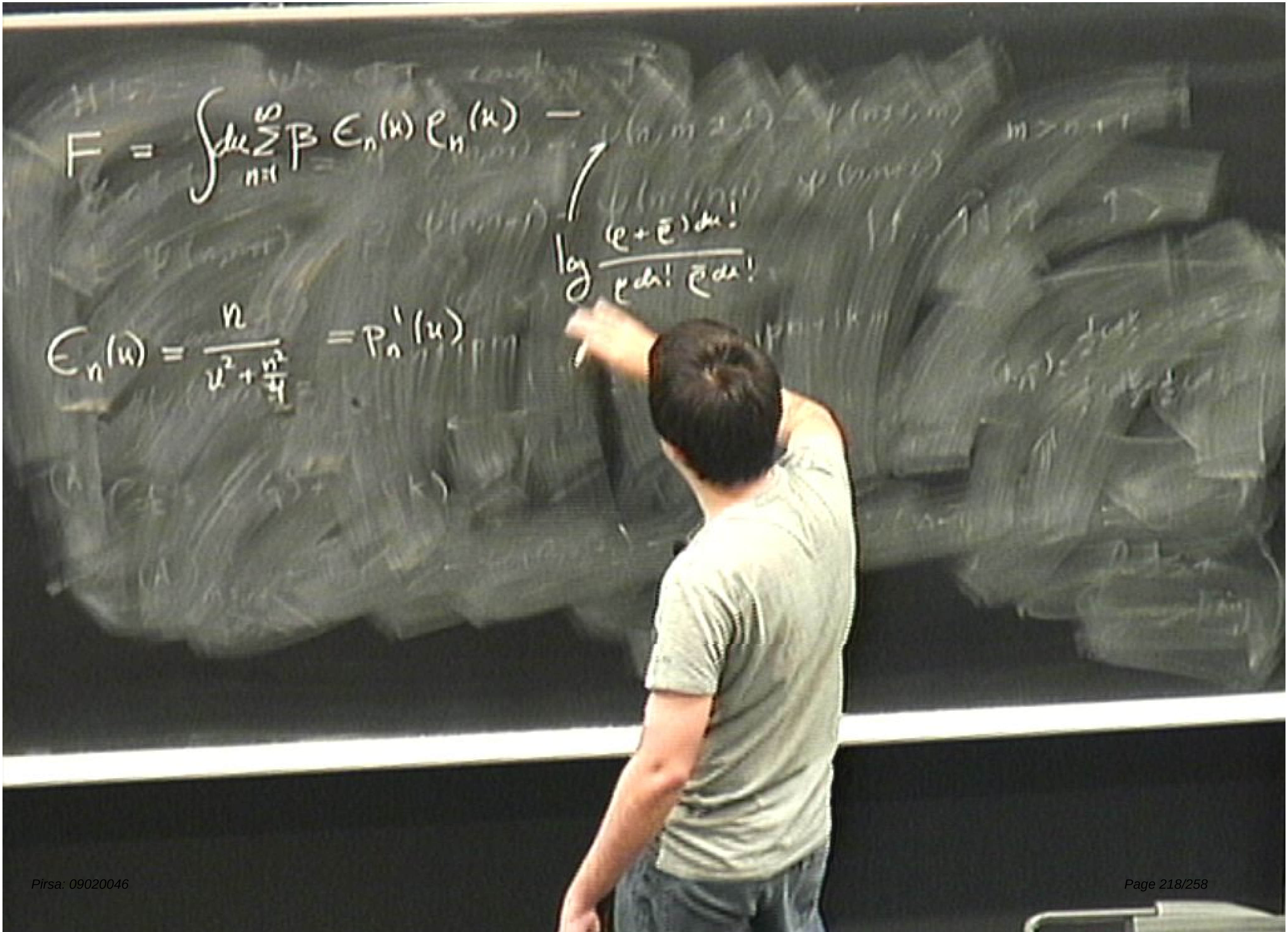
$F =$

$$\epsilon_n(u) = \frac{n}{u^2 + \frac{n^2}{4}} = p_n'(u)$$


$F = \int du \sum_{n=1}^{\infty} \beta \, \epsilon_n(u) \, \rho_n(u) \; -$
$\epsilon_n(u) = \frac{n}{u^2 + \frac{n^2}{4}} = p_n'(u)$

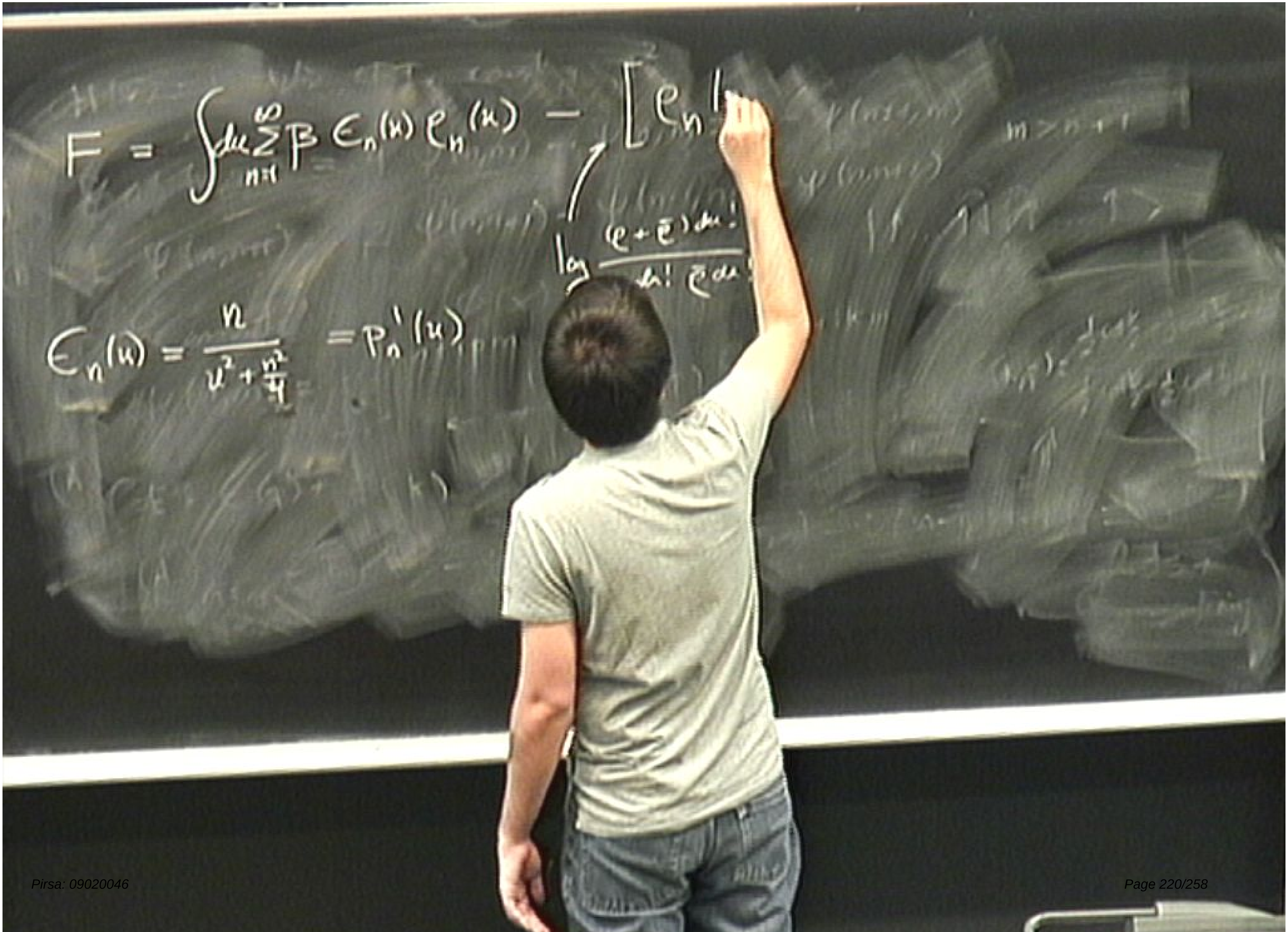
$$F = \int du \sum_{n=1}^{\infty} \beta\, \epsilon_n(u)\, \rho_n(u) - \log$$

$$\epsilon_n(u) = \frac{n}{u^2 + \frac{n^2}{4}} = p_n'(u)$$


F = ∫du Σ_{n=1}^∞ β ε_n(u) ρ_n(u) −
log (ρ+ρ̄)du! / ρdu! ρ̄du!
ε_n(u) = n / (u² + n²/4) = p_n'(u)

$$F = \int du \sum_{n=1}^{\infty} \beta\, \epsilon_n(u)\, \rho_n(u) - \log \frac{(\rho + \bar{\rho})\, du!}{\rho\, du!\; \bar{\rho}\, du!}$$

$$\epsilon_n(u) = \frac{n}{u^2 + \frac{n^2}{4}} = p_n'(u)$$


F = ∫du Σ_{n=1}^{∞} β ε_n(u) ρ_n(u) − [ρ_n
log (ρ + ρ̄) du! / ρ du! ρ̄ du!
ε_n(u) = n / (u² + n²/4) = p_n'(u)

$$F = \int du \sum_{n=1}^{\infty} \beta \, \epsilon_n(u) \rho_n(u) - \left[\rho_n \log\left(1 + \frac{\bar{\rho}_n}{\rho_n}\right) + \bar{\rho}_n \log\left(1 + \frac{\rho_n}{\bar{\rho}_n}\right) \right]$$

$$\log \frac{((\rho + \bar{\rho})du)!}{(\rho du)! \, (\bar{\rho} du)!}$$

$$\epsilon_n(u) = \frac{n}{u^2 + \frac{n^2}{4}} = p_n'(u)$$

$$F = \int du \sum_{n=1}^{\infty} \beta\, \epsilon_n(u)\, \rho_n(u) - \left[\rho_n \log\left(1+\frac{\bar{\rho}_n}{\rho_n}\right) + \bar{\rho}_n \log\left(1+\frac{\rho_n}{\bar{\rho}_n}\right) \right]$$

$$\log \frac{(\rho+\bar{\rho})du!}{\rho du!\ \bar{\rho} du!}$$

$$\epsilon_n(u) = \frac{n}{u^2+\frac{n^2}{4}} = p_n'(u)$$

$$F_{\mathcal{I}} = \int du \sum_{n=1}^{\infty} \beta\, \epsilon_n(u)\, \rho_n(u) - \left[\rho_n \log\left(1 + \frac{\bar{\rho}_n}{\rho_n}\right) + \bar{\rho}_n \log\left(1 + \frac{\rho_n}{\bar{\rho}_n}\right) \right]$$

$$\log \frac{(\rho + \bar{\rho})\, du!}{\rho\, du!\; \bar{\rho}\, du!}$$

$$\epsilon_n(u) = \frac{n}{u^2 + \frac{n^2}{4}} = p_n'(u)$$

$$\ldots = F_L = \int du \sum_{n=1}^{\infty} \beta\, \epsilon_n(u)\, \rho_n(u) - \left[\rho_n \log\left(1 + \frac{\bar{\rho}_n}{\rho_n}\right) + \bar{\rho}_n \log\left(1 + \frac{\rho_n}{\bar{\rho}_n}\right) \right]$$

$$\log \frac{(\rho + \bar{\rho})\, du!}{\rho\, du!\; \bar{\rho}\, du!}$$

$$\epsilon_n(u) = \frac{n}{u^2 + \frac{n^2}{4}} = p_n'(u)$$

$$f = F_{\text{I}} = \int du \sum_{n=1}^{\infty} \beta\, \epsilon_n(u)\, \rho_n(u) - \left[\rho_n \log\left(1+\frac{\bar{\rho}_n}{\rho_n}\right) + \bar{\rho}_n \log\left(1+\frac{\rho_n}{\bar{\rho}_n}\right) \right]$$

$$\log \frac{(\rho+\bar{\rho})\,du!}{\rho\,du!\;\bar{\rho}\,du!}$$

$$\epsilon_n(u) = \frac{n}{u^2+\frac{n^2}{4}} = p_n'(u)$$

Saddle point $\delta f = 0$, $f_{\delta f=0} = ?$

$$F_I = \int du \sum_{n=1}^{\infty} \beta\, \epsilon_n(u)\, \rho_n(u) - \left[\rho_n \log\left(1 + \frac{\bar{\rho}_n}{\rho_n}\right) + \bar{\rho}_n \log\left(1 + \frac{\rho_n}{\bar{\rho}_n}\right) \right]$$

$$\log \frac{((\rho + \bar{\rho})\, du)!}{(\rho\, du)!\, (\bar{\rho}\, du)!}$$

$$\epsilon_n(u) = \frac{n}{u^2 + \frac{n^2}{4}} = p_n'(u)$$

Saddle point $\delta f = 0$ $\quad f_{\delta f = 0} = ?$

$$f = F_{\Sigma} = \int du \sum_{n=1}^{\infty} \beta \, \epsilon_n(u)\, \rho_n(u) - \left[\rho_n \log\left(1 + \frac{\bar{\rho}_n}{\rho_n}\right) + \bar{\rho}_n \log\left(1 + \frac{\rho_n}{\bar{\rho}_n}\right) \right]$$

$$\log \frac{(\rho + \bar{\rho})\, du!}{\rho\, du!\ \bar{\rho}\, du!}$$

$$\epsilon_n(u) = \frac{n}{u^2 + \frac{n^2}{4}} = p_n'(u)$$

Saddle point $\delta f = 0$, $f_{\delta f = 0} = ?$

$$f = F_I = \int du \sum_{n=1}^{\infty} \beta\, \epsilon_n(u)\, \rho_n(u) - \left[\rho_n \log\left(1 + \frac{\bar{\rho}_n}{\rho_n}\right) + \bar{\rho}_n \log\left(1 + \frac{\rho_n}{\bar{\rho}_n}\right) \right]$$

$$\log \frac{((\rho + \bar{\rho})\, du)!}{(\rho\, du)!\, (\bar{\rho}\, du)!}$$

$$\epsilon_n(u) = \frac{n}{u^2 + \frac{n^2}{4}} = p_n'(u)$$

Saddle point $\delta f = 0$, $f_{\delta f = 0} = ?$

$$\delta f = 0$$

$$f = F_{I} = \int du \sum_{n=1}^{\infty} \beta \, \epsilon_n(u) \rho_n(u) - \left[\rho_n \log\left(1 + \frac{\bar{\rho}_n}{\rho_n}\right) + \bar{\rho}_n \log\left(1 + \frac{\rho_n}{\bar{\rho}_n}\right) \right]$$

$$\log \frac{(\rho + \bar{\rho})\, du!}{\rho\, du! \; \bar{\rho}\, du!}$$

$$\epsilon_n(u) = \frac{n}{u^2 + \frac{n^2}{4}} = p_n'(u)$$

Saddle point $\delta f = 0$, $f_{\delta f = 0} = ?$

$$\delta f = 0$$

$$\uparrow$$

$$\delta \bar{\rho}_n = -\delta \rho_n + \delta_{nm} \, \delta \rho_m$$

$$f = F_I = \int du \sum_{n=1}^{\infty} \beta\, \epsilon_n(u)\, \rho_n(u) - \left[\rho_n \log\left(1+\frac{\bar{\rho}_n}{\rho_n}\right) + \bar{\rho}_n \log\left(1+\frac{\rho_n}{\bar{\rho}_n}\right) \right]$$

$$\log \frac{(\rho+\bar{\rho})\,du!}{\rho\,du!\;\bar{\rho}\,du!}$$

$$\epsilon_n(u) = \frac{n}{u^2+\frac{n^2}{4}} = p_n'(u)$$

Saddle point $\delta f = 0$, $f_{\delta f = 0} = ?$

$$\delta f = 0 = \int \delta\rho_n(u) \left[\beta\, p_n'(u) - \log\frac{\rho_n}{\bar{\rho}_n} - \delta_{mn} * \log\left(1+\frac{\rho_m}{\bar{\rho}_m}\right) \right]$$

$$\delta\bar{\rho}_n = -\delta\rho_n + \delta_{nm} * \rho_m$$

$$f = F_I = \int du \sum_{n=1}^{\infty} \beta\, \epsilon_n(u)\, \rho_n(u) - \left[\rho_n \log\left(1+\frac{\bar\rho_n}{\rho_n}\right) + \bar\rho_n \log\left(1+\frac{\rho_n}{\bar\rho_n}\right) \right]$$

$$\log \frac{(\rho+\bar\rho)\,du!}{\rho\,du!\;\bar\rho\,du!}$$

$$\epsilon_n(u) = \frac{n}{u^2+\frac{n^2}{4}} = p_n'(u)$$

Saddle point $\delta f = 0$, $f_{\delta f = 0} = ?$

$$\delta f = 0 = \int \delta\rho_n(u) \left[\beta\, p_n'(u) - \log\frac{\rho_n}{\bar\rho_n} - \delta_{mn} * \log\left(1+\frac{\rho_m}{\bar\rho_m}\right) \right]$$

$$= 0$$

$$\delta\bar\rho_n = -\delta\rho_n + \delta_{nm} * \rho_m$$

$$= F_{\mathcal{I}} = \int du \sum_{n=1}^{\infty} \beta\, \epsilon_n(u)\, \rho_n(u) - \left[\rho_n \log\left(1 + \frac{\bar\rho_n}{\rho_n}\right) + \bar\rho_n \log\left(1 + \frac{\rho_n}{\bar\rho_n}\right) \right]$$

$$\log \frac{(\rho + \bar\rho)\, du!}{\rho\, du!\ \bar\rho\, du!}$$

$$\epsilon_n(u) = \frac{n}{u^2 + \frac{n^2}{4}} = p_n'(u)$$

Saddle point $\delta f = 0$, $f_{\delta f = 0} = ?$

$$\delta f = 0 = \int \delta\rho_n(u) \left[\beta\, p_n'(u) - \log \frac{\rho_n}{\bar\rho_n} - \delta_{mn} * \log\left(1 + \frac{\rho_m}{\bar\rho_m}\right) \right] = 0$$

$$\delta\bar\rho_n = -\delta\rho_n + \delta_{nm} * \rho_m$$

$$\frac{\bar\rho_n}{\rho_n} = Y_n$$

$$f = F_{\mathcal{I}} = \int du \sum_{n=1}^{\infty} \beta\, \epsilon_n(u)\, \rho_n(u) - \left[\rho_n \log\left(1 + \frac{\bar{\rho}_n}{\rho_n}\right) + \bar{\rho}_n \log\left(1 + \frac{\rho_n}{\bar{\rho}_n}\right) \right]$$

$$\log \frac{(\rho + \bar{\rho})\,du!}{\rho\,du!\;\bar{\rho}\,du!}$$

$$\epsilon_n(u) = \frac{n}{u^2 + \frac{n^2}{4}} = p_n'(u)$$

Saddle point $\delta f = 0$, $f_{\delta f = 0} = ?$

$$\delta f = 0 = \int \delta\rho_n(u) \left[\beta\, p_n'(u) - \log \frac{\rho_n}{\bar{\rho}_n} - \delta_{mn} * \log\left(1 + \frac{\rho_m}{\bar{\rho}_m}\right) \right]$$

$$\delta \bar{\rho}_n = -\delta\rho_n + \delta_{nm} * \rho_m$$

$$= 0 \qquad \frac{\bar{\rho}_n}{\rho_n} = Y_n$$

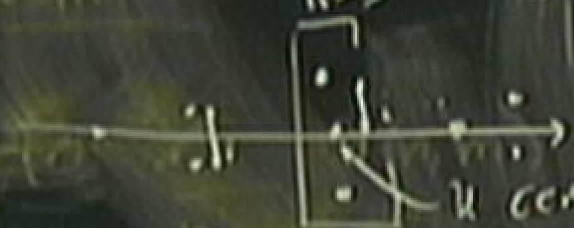
$$\log Y_n = \beta p_n' + \delta_{mn} * \log(1 + 1/Y_m)$$

$u_j = u + i\eta$ $\quad \eta > 0$ $\quad$ lhs BAE $\to \infty$ $\quad (L \to \infty)$

therefore $\exists\, u_k$ s.t. $u_j - u_k - i = 0$

$$e^{ip_n(x)} = \frac{u + in/2}{u - in/2}$$

n=3



fused BAE

$$\left(\frac{u_j^n + i\frac{n}{2}}{u_j^n - i\frac{n}{2}}\right)^L = \prod_{k,m} S_{nm}(u_j^n, u_k^m)$$

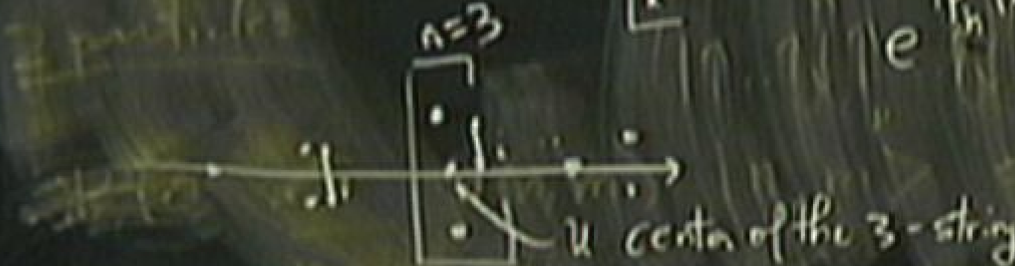
$$\log Y_n = \beta p_n' + S_{mn} * \log\left(1 + 1/Y_m\right) \qquad \text{TBA eqs}$$

$$f(\beta) = -\sum_{n\geq 1} \int p_n'(x) \log\left(1 + 1/Y_n\right)$$

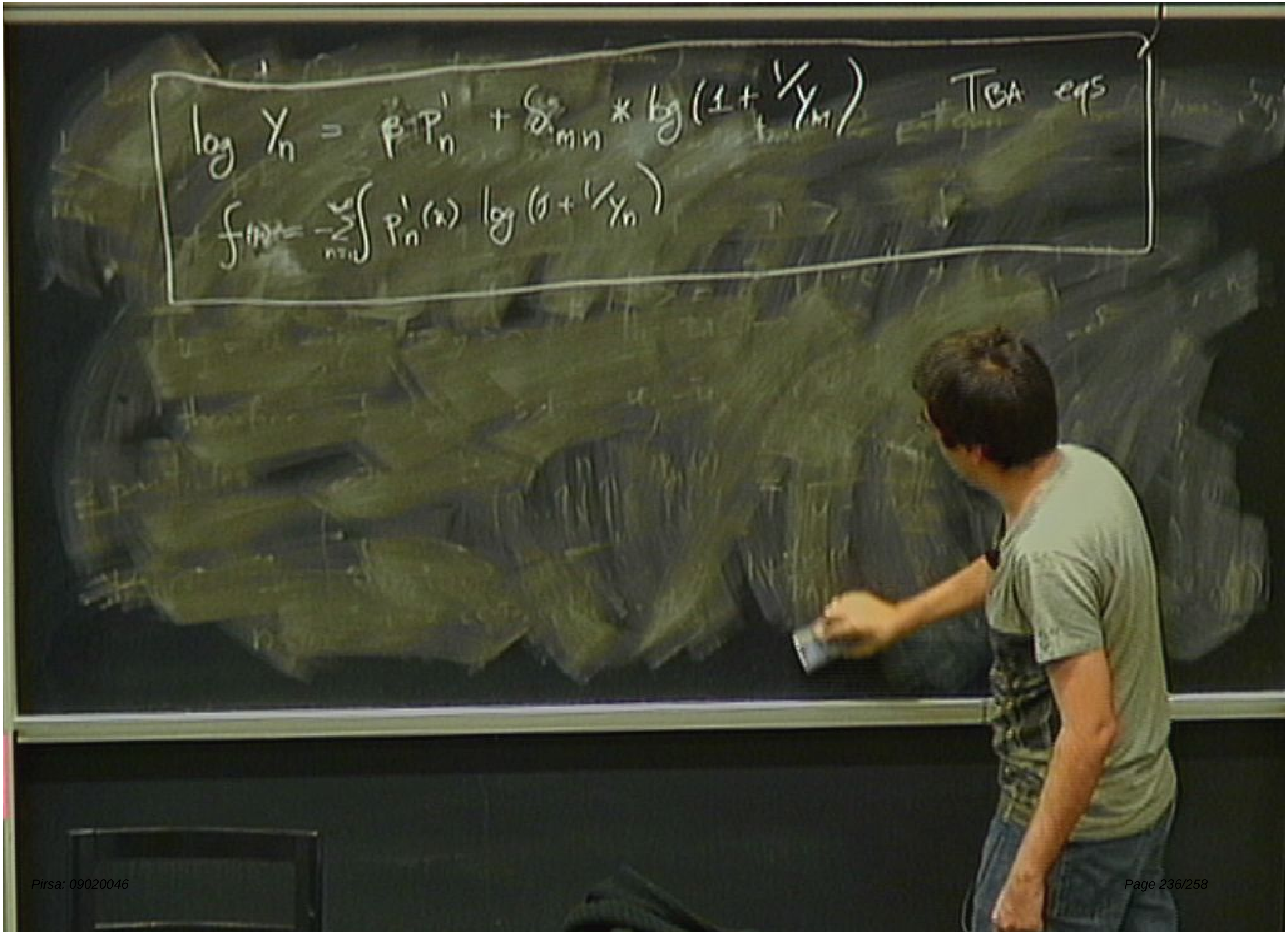
$u_j = u + i\eta \qquad \eta > 0 \qquad$ lhs BAE $\to \infty \qquad (L \to \infty)$

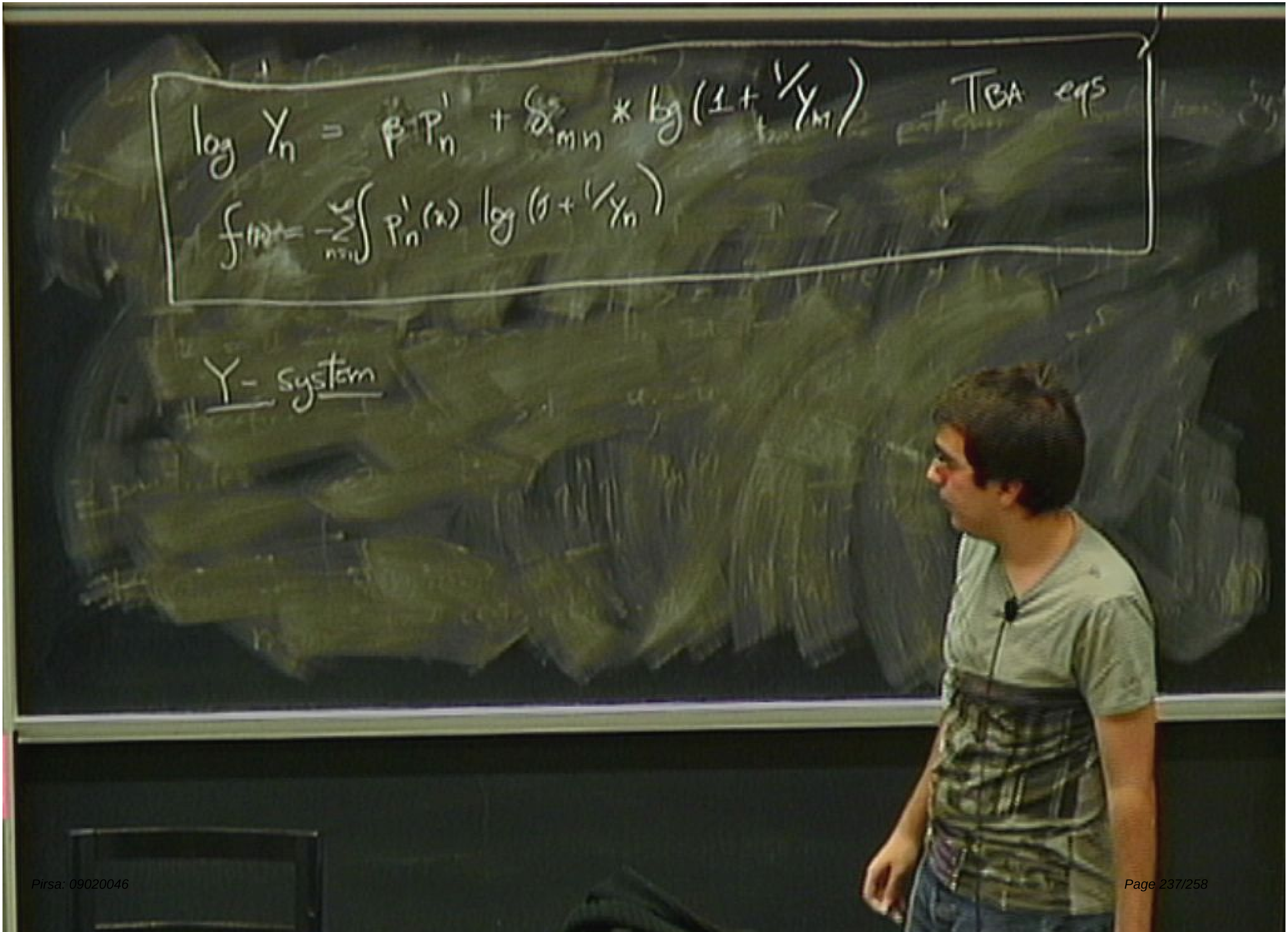
therefore $\exists\, u_k$ s.t. $u_j - u_k - i = 0$

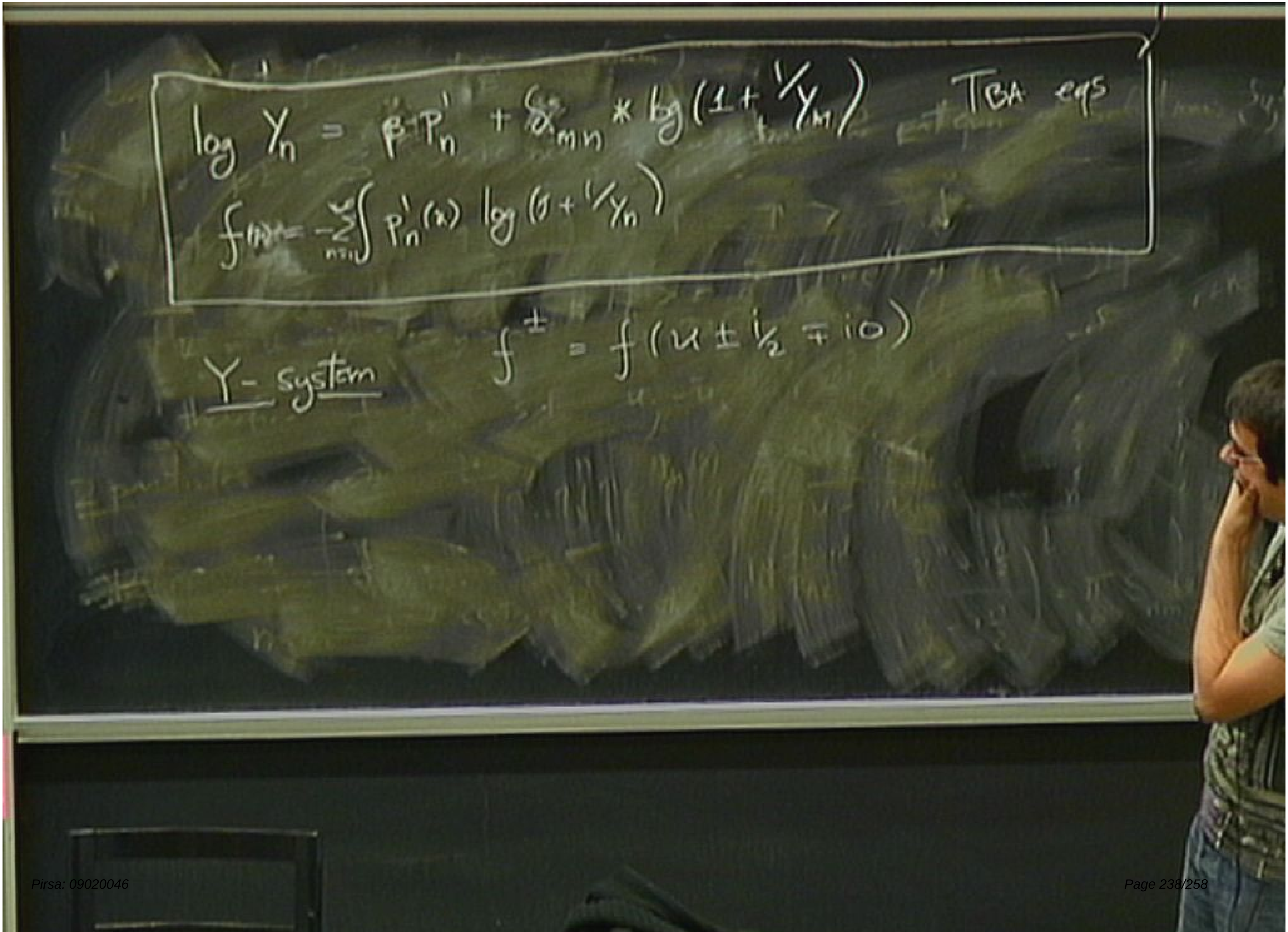
$$e^{i p_n(x)} = \frac{u + in/2}{u - in/2}$$

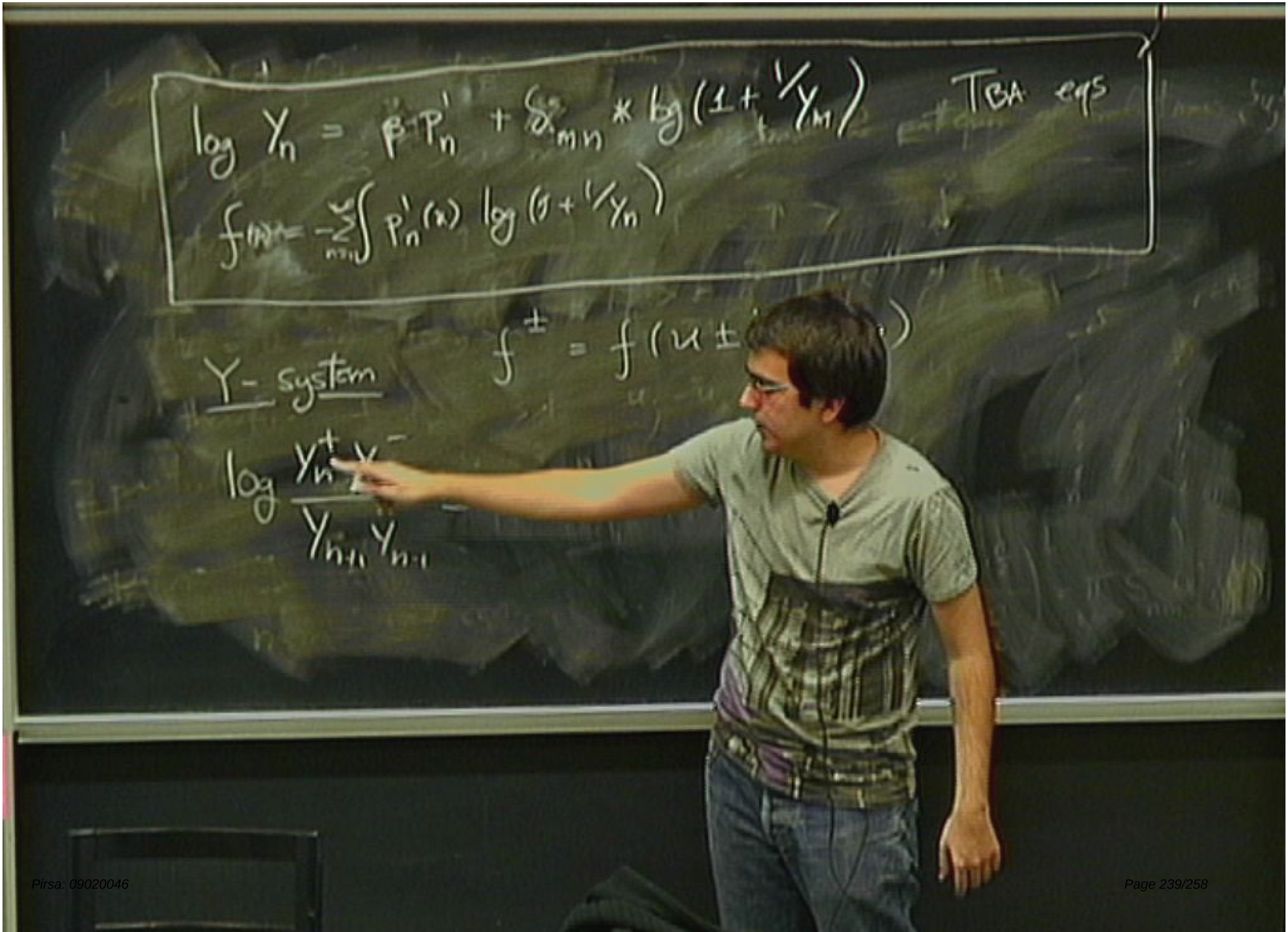


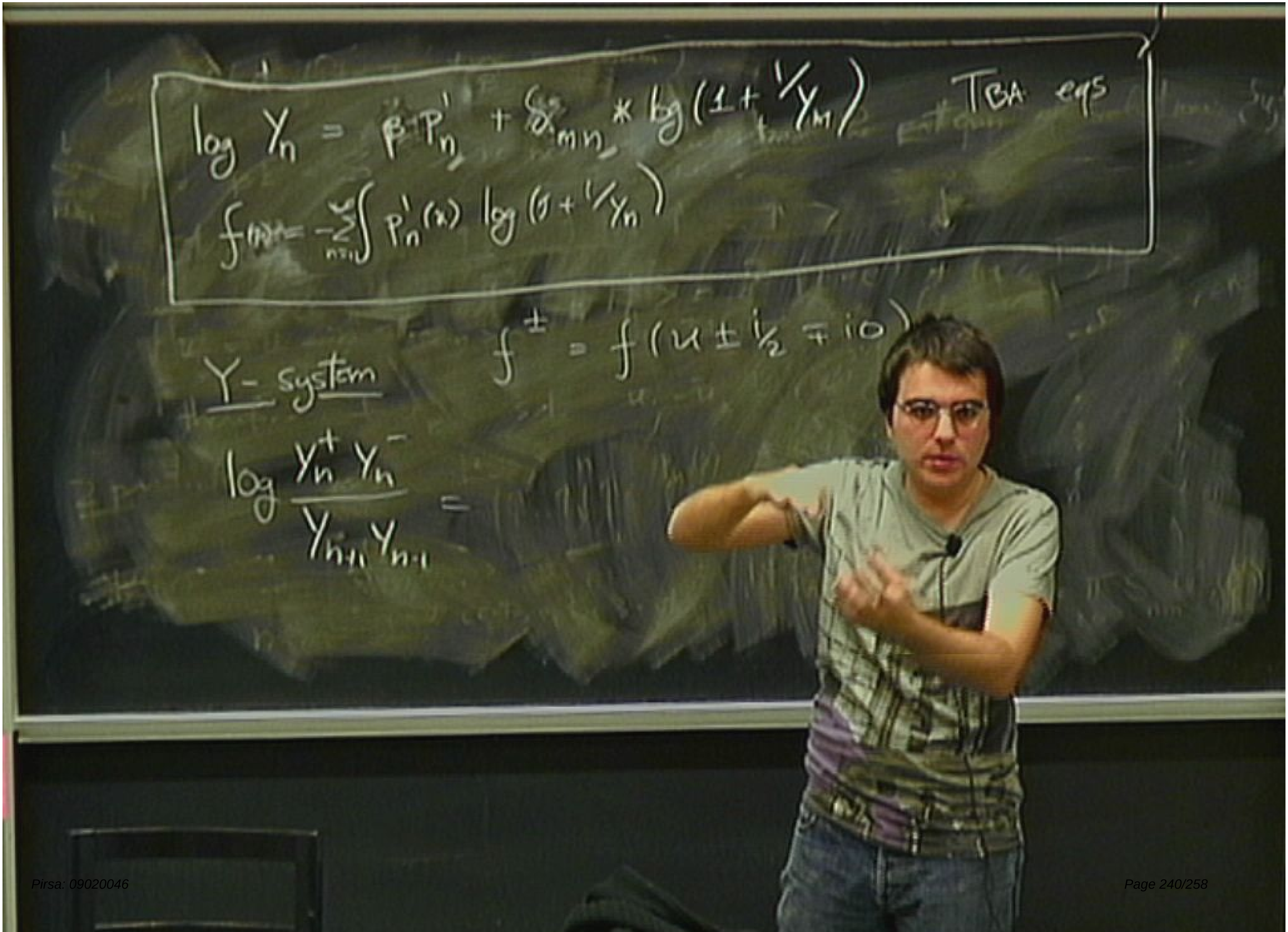
fused BAE
$$\left(\frac{u_j^n + i\frac{n}{2}}{u_j^n - i\frac{n}{2}}\right)^L = \prod_{k,m} S_{nm}(u_j^n, u_k^m)$$

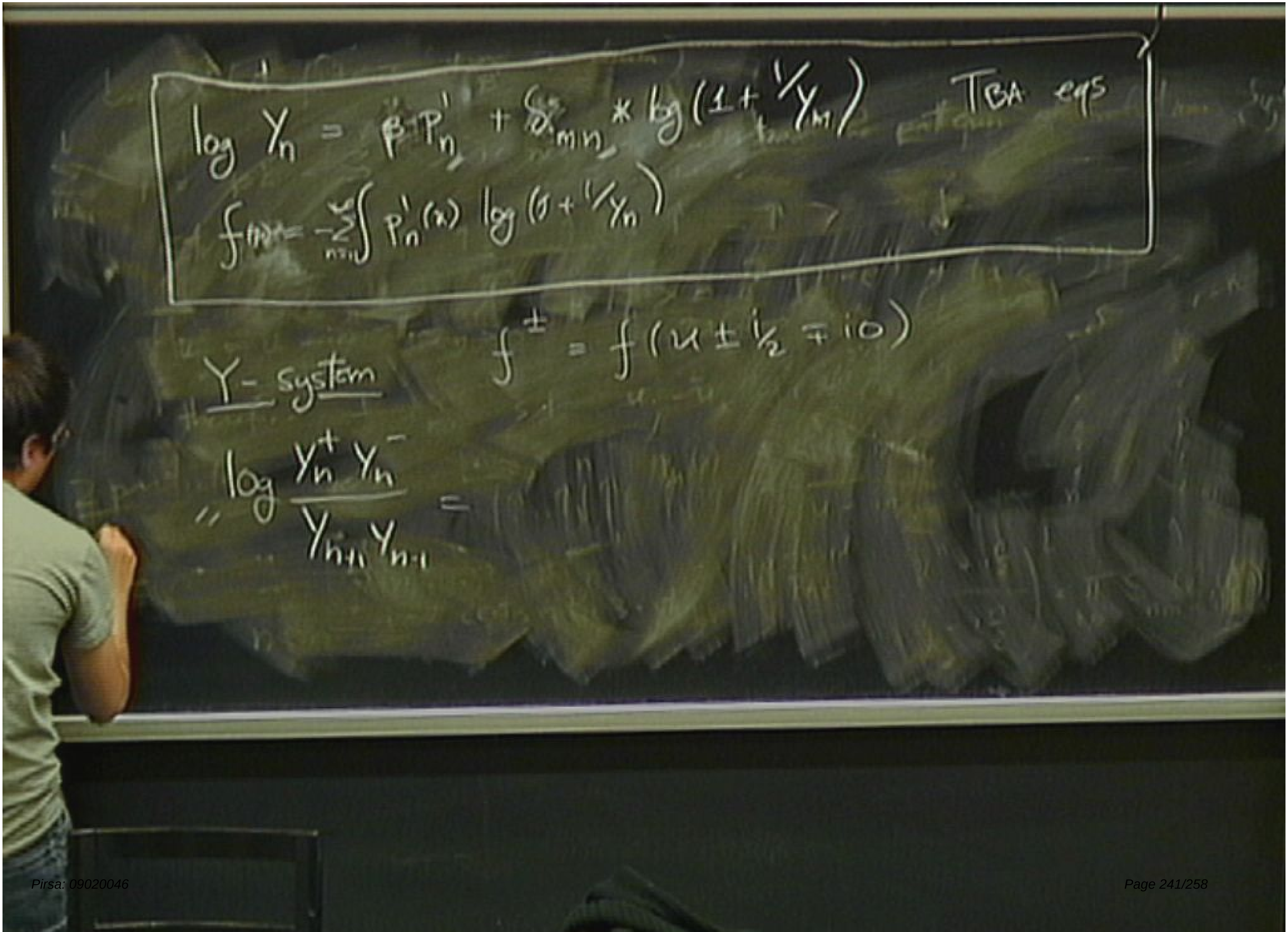

log Y_n = β p'_n + φ_mn * log(1 + 1/Y_m)
TBA eqs
f(β) = -Σ ∫ p'_n(x) log(1 + 1/Y_n)

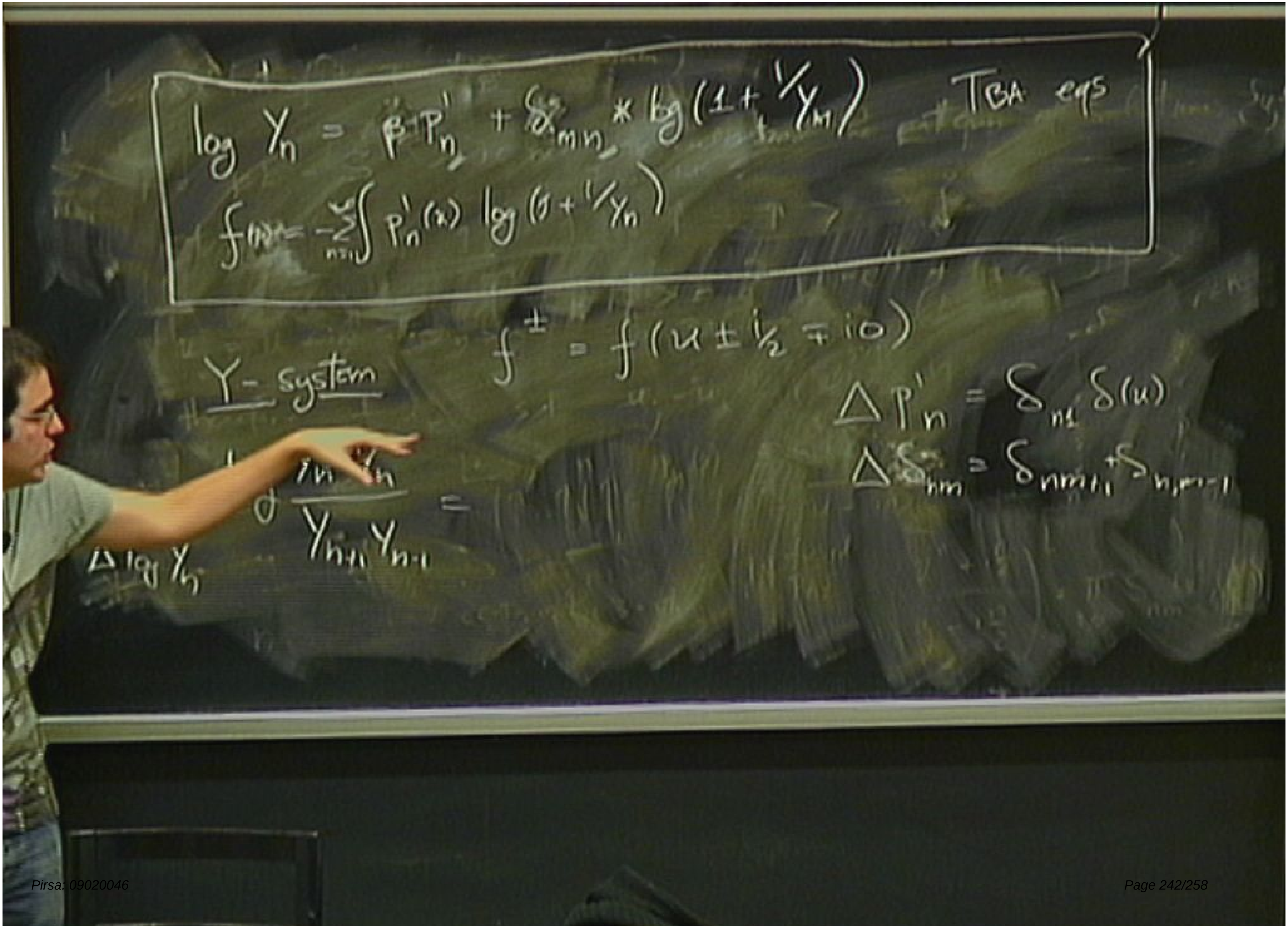

log Y_n = β P'_n + φ_mn * log(1 + 1/Y_m)
TBA eqs
f(β) = -Σ ∫ p'_n(x) log(1 + 1/Y_n)
Y- system


log Yn = β P'n + Kmn * log(1 + 1/Ym)
TBA eqs
f = -Σ ∫ p'n(u) log(1 + 1/Yn)
f± = f(u ± i/2 ∓ i0)
Y-system


TBA eqs
Y- system


log Y_n = β P'_n + Φ_mn * log(1 + 1/Y_m)
TBA eqs
f(u) = −Σ ∫ P'_n(x) log(1 + 1/Y_n)
f^± = f(u ± i/2 ∓ i0)
Y-system
log (Y_n^+ Y_n^−)/(Y_{n+1} Y_{n−1}) =


log Y_n = β P_n' + δ_mn * log(1 + 1/Y_m)   TBA eqs
f(n) = −Σ_{n=1} ∫ p_n'(x) log(1 + 1/Y_n)
f^± = f(u ± i/2 ∓ i0)
Y-system
log (Y_n^+ Y_n^-)/(Y_{n+1} Y_{n-1}) =


TBA eqs
Y-system
Δ log Y
$f^{\pm} = f(u \pm \frac{i}{2} \mp i0)$
$\Delta p'_n = \delta_{n1}\,\delta(u)$

TBA eqs

$$\log Y_n = \beta p_n' + \sum_{m=1}^{\infty} \delta_{mn} * \log\left(1 + \frac{1}{Y_m}\right)$$

$$f(u) = -\sum_{n=1}^{\infty} \int p_n'(x) \log\left(1 + \frac{1}{Y_n}\right)$$

Y- system

$$f^{\pm} = f\left(u \pm \tfrac{i}{2} \mp i0\right)$$

$$\Delta p_n' = \delta_{n1}\,\delta(u)$$

$$\Delta \delta_{nm} = \delta_{n,m+1} + \delta_{n,m-1}$$

$$\Delta \log Y_n = \log \frac{Y_n^+ Y_n^-}{Y_{n+1} Y_{n-1}} = \beta\, \delta_{n1}\,\delta(u) + \log\left(\left(1+\frac{1}{Y_{n+1}}\right)\left(1+\frac{1}{Y_{n-1}}\right)\right)$$

$$\log Y_n = \beta\, p'_n + \sum_{m=1}^{\infty} \mathcal{S}_{mn} * \log\left(1 + 1/Y_m\right) \qquad \text{TBA eqs}$$

$$f(\beta) = -\sum_{n=1}^{\infty} \int p'_n(x) \log\left(1 + 1/Y_n\right)$$

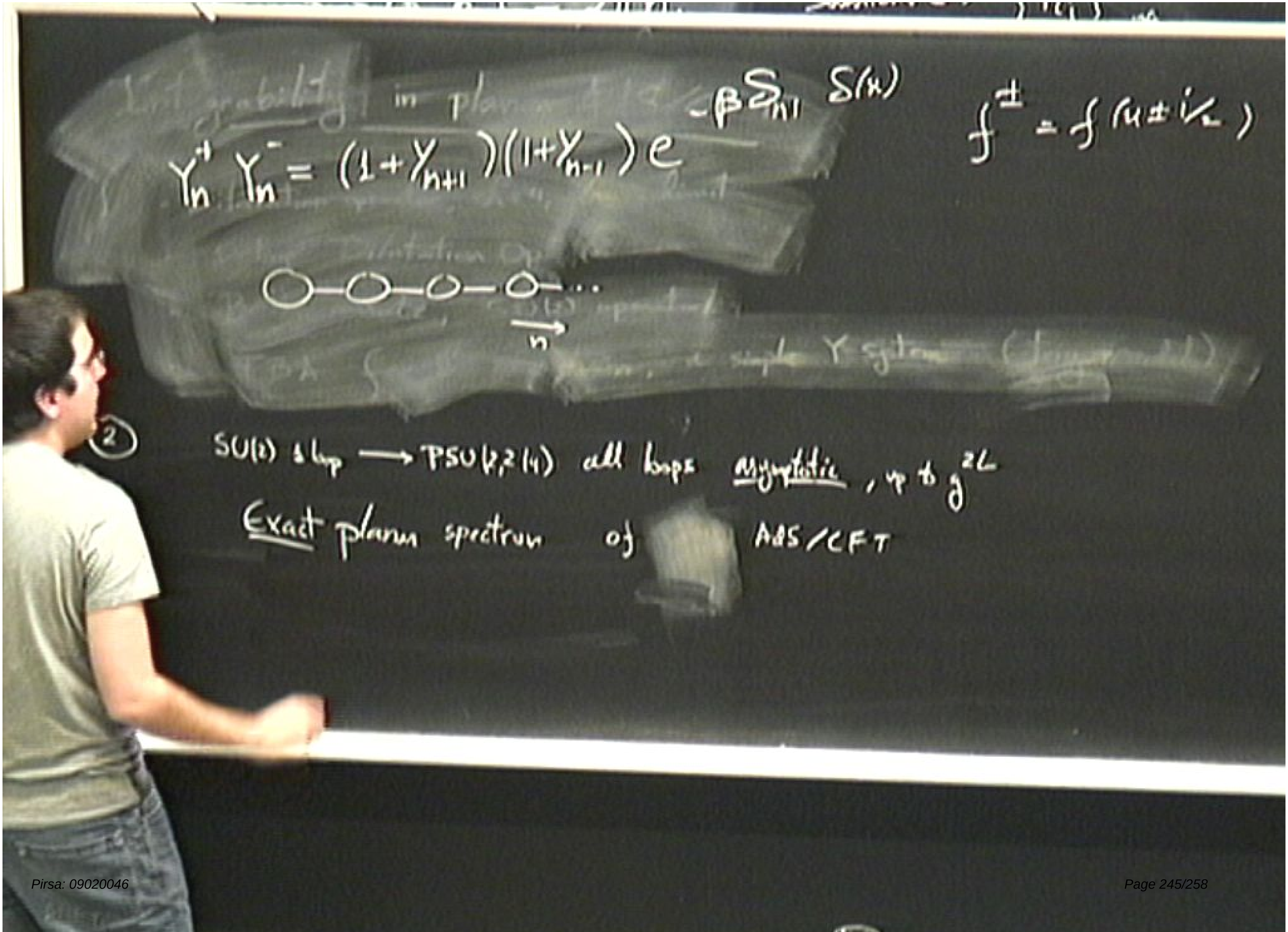
$$f^{\pm} = f(u \pm i/2 \mp i0)$$

Y-system

$$\Delta p'_n = \delta_{n1}\, \delta(u)$$

$$\Delta \mathcal{S}_{nm} = \delta_{n,m+1} + \delta_{n,m-1}$$

$$\Delta \log Y_n \text{ ''} \log \frac{Y_n^+ Y_n^-}{Y_{n+1} Y_{n-1}} = \beta\, \delta_{n1}\, \delta(u) + \log\left((1 + 1/Y_{n+1})(1 + 1/Y_{n-1})\right)$$


Y_n^+ Y_n^- = (1+Y_{n+1})(1+Y_{n-1}) e^{-βS_{n1}} δ(x)
f^± = f(u ± i/2)
n
2
SU(2) 1 loop → PSU(2,2|4) all loops asymptotic, up to g^{2L}
Exact planar spectrum of AdS/CFT

$$Y_n^+ Y_n^- = (1+Y_{n+1})(1+Y_{n-1})\, e^{-\beta \delta_{n1}\, \delta(x)}$$

$$f^\pm = f(u \pm i/2)$$

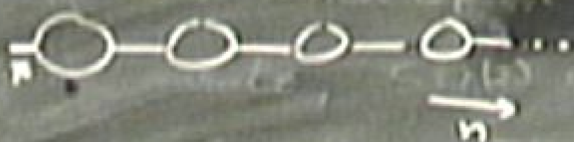
Y-system

(2) SU(2) 1 loop $\longrightarrow$ PSU(2,2|4) all loops asymptotic, up to g^{2L}

(3) Exact planar spectrum of AdS/CFT

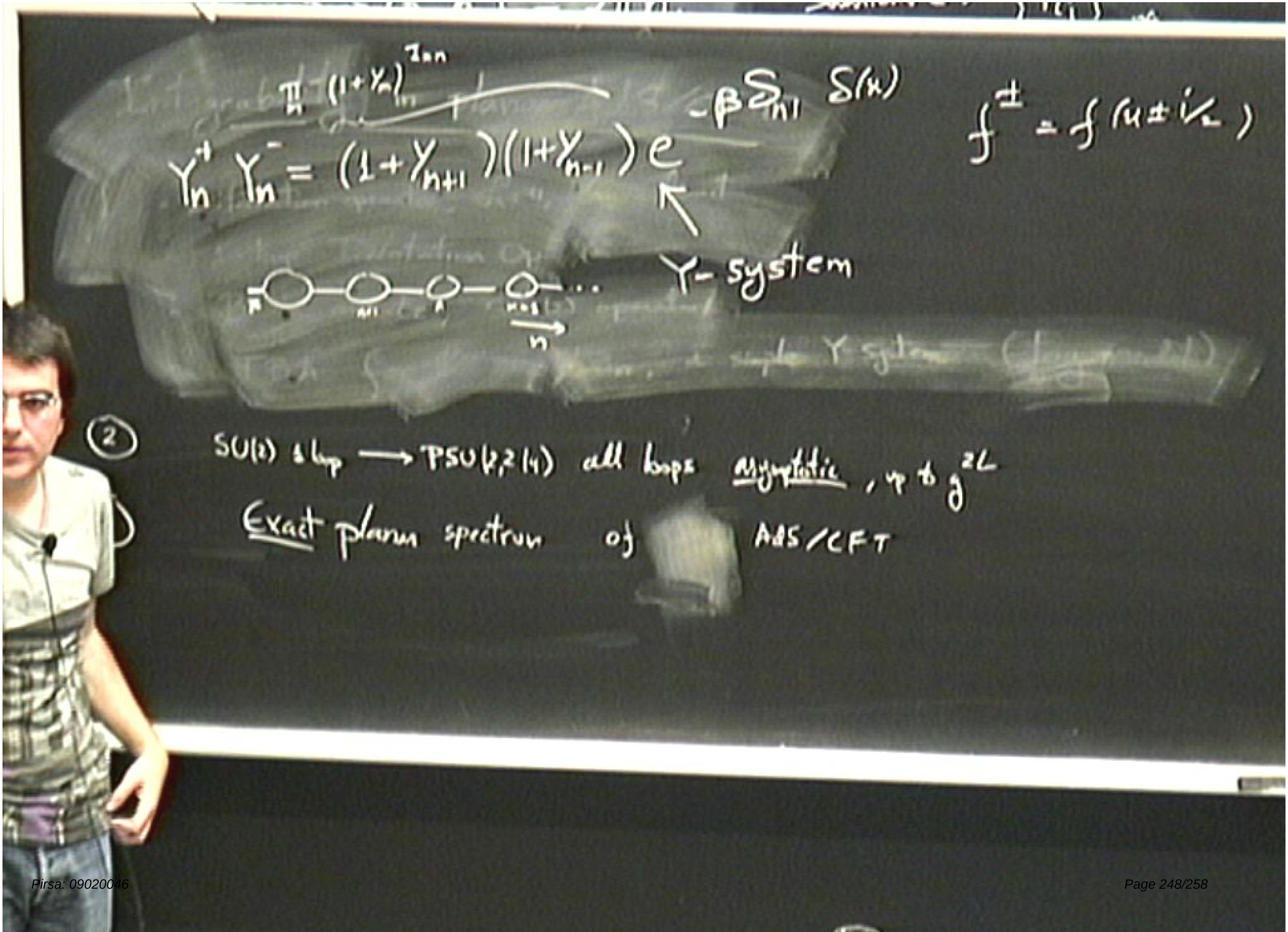
$$Y_n^+ Y_n^- = (1+Y_{n+1})(1+Y_{n-1})\, e^{-\beta S_{n1}\, \delta(u)}$$

$$f^\pm = f(u \pm i/2)$$

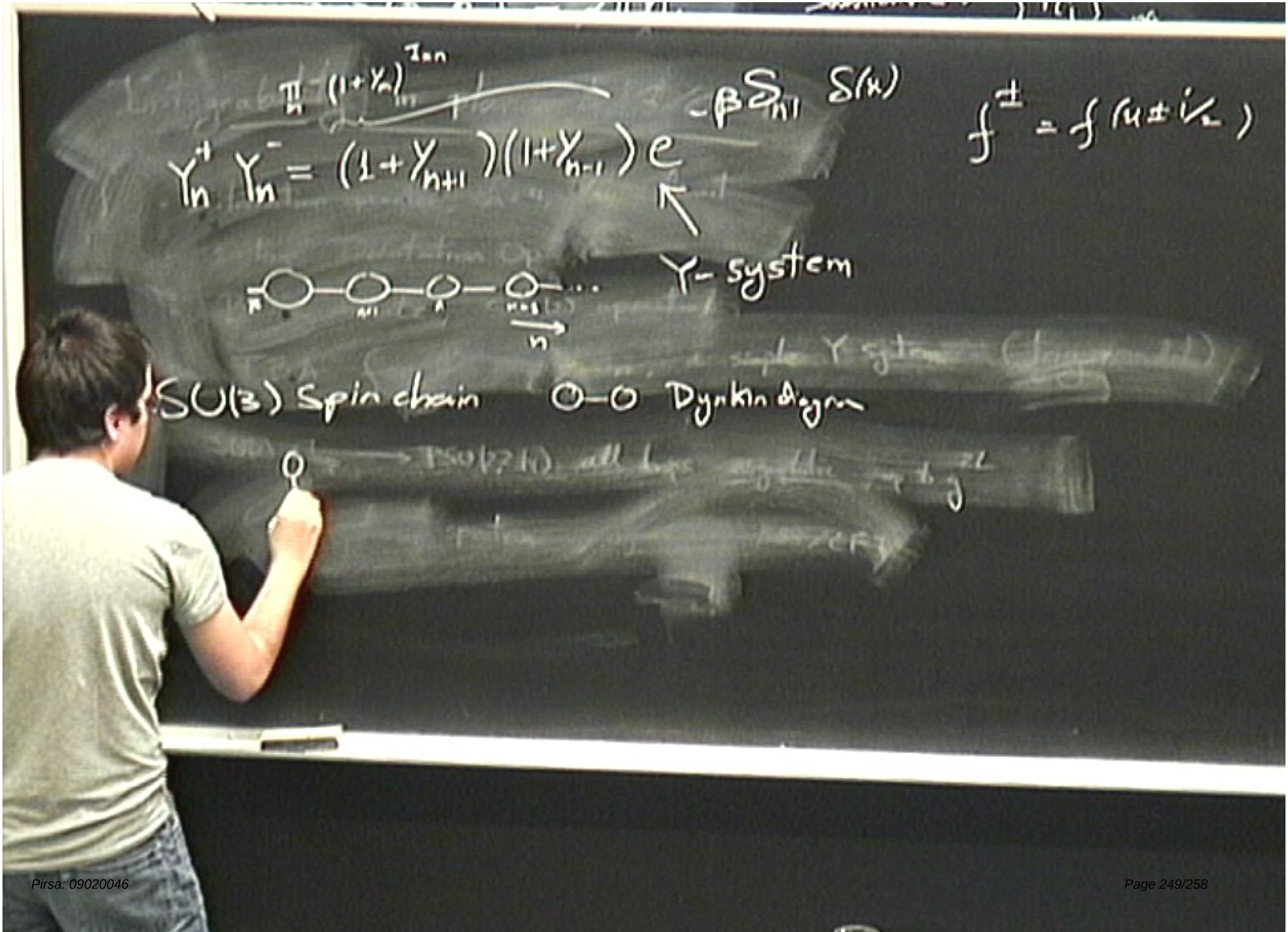


(2) SU(2) 1 loop $\longrightarrow$ PSU(2,2|4) all loops asymptotic, up to g^{2L}

(3) Exact planar spectrum of AdS/CFT



$\prod_m (1+Y_m)^{I_{mn}}$
$\delta(u)$
$Y_n^+ Y_n^- = (1+Y_{n+1})(1+Y_{n-1})\, e^{-\beta S_{n1}}$
$f^\pm = f(u \pm i/2)$
Y- system
n
2
SU(2) 1 loop ⟶ PSU(2,2|4) all loops asymptotic, up to g^{2L}
Exact planar spectrum of AdS/CFT

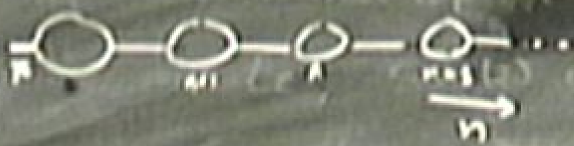

$Y_n^+ Y_n^- = (1+Y_{n+1})(1+Y_{n-1}) e^{-\beta S_{n1}}$
$S(x)$
$f^\pm = f(u \pm i/2)$
Y-system
SU(3) Spin chain
Dynkin Diagram

$\prod_n (1+Y_n)^{\alpha_{nn}}$

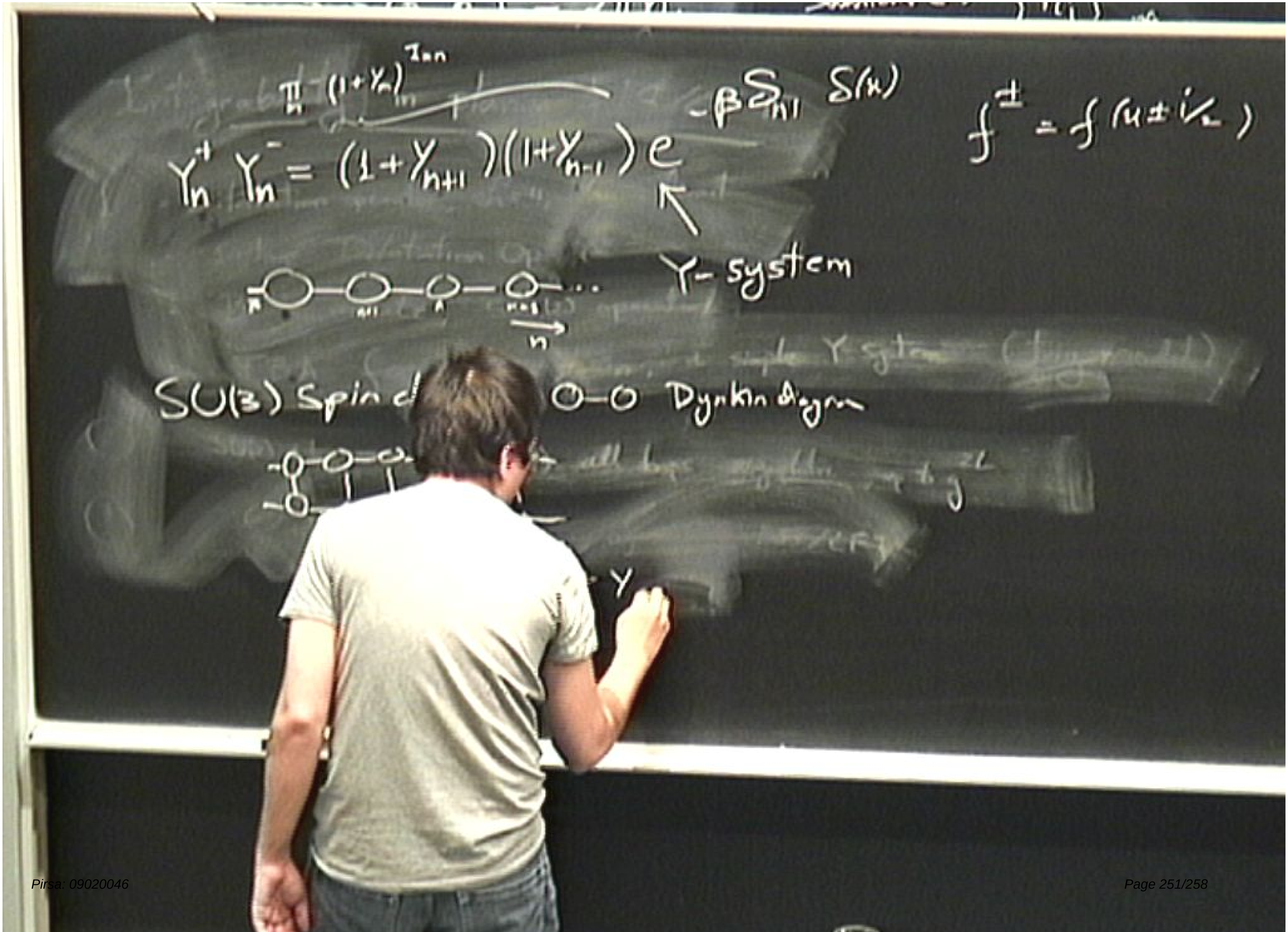
$$Y_n^+ Y_n^- = (1+Y_{n+1})(1+Y_{n-1})\, e^{-\beta \delta_{n1} \delta(x)}$$

$$f^\pm = f(u \pm i/2)$$

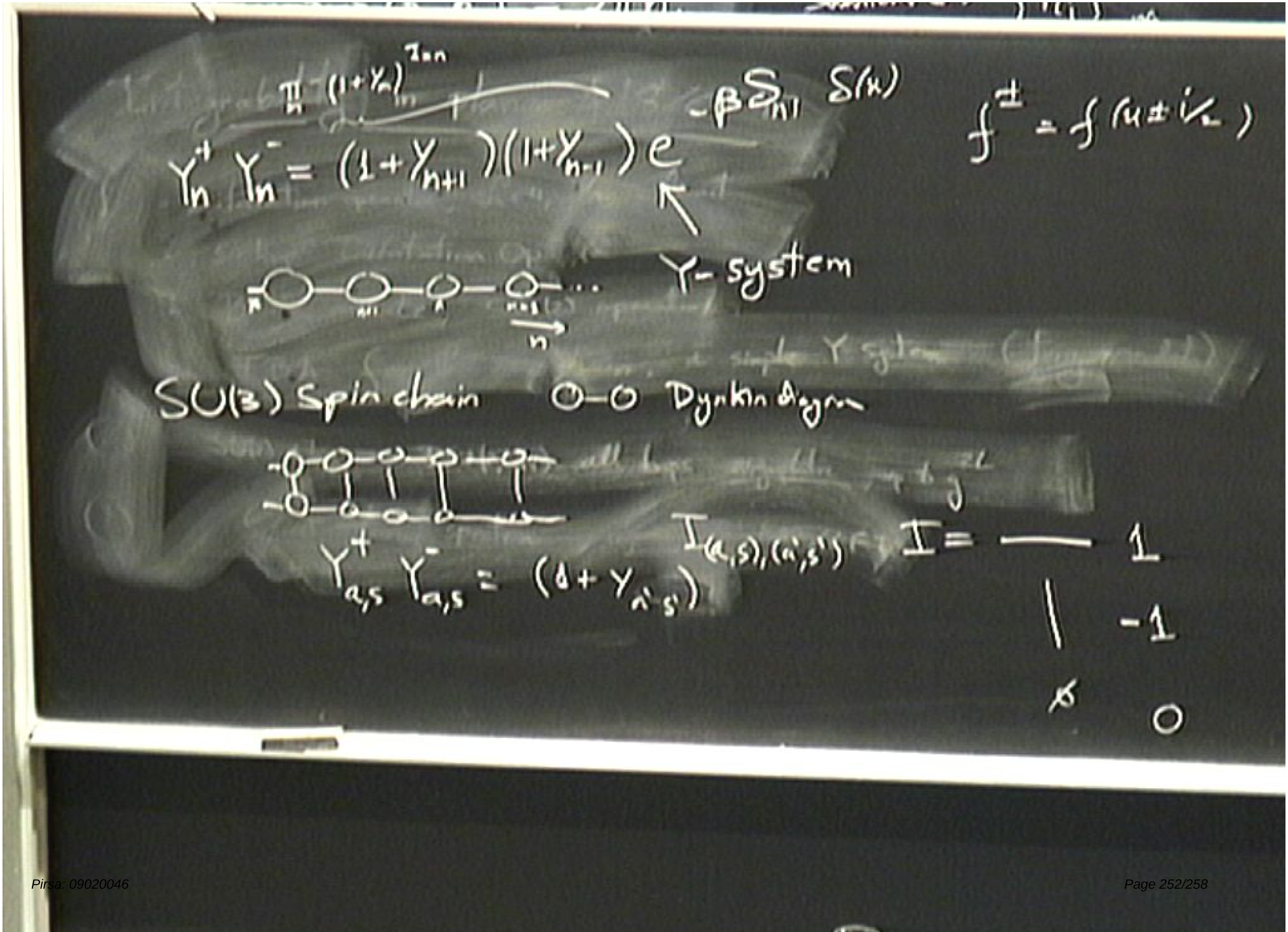
Y- system



SU(3) Spin chain  Dynkin diagram



$\prod_n (1+Y_n)^{T_{nn}}$
$Y_n^+ Y_n^- = (1+Y_{n+1})(1+Y_{n-1}) e^{-\beta S_{n1}}$
$S(u)$
$f^{\pm} = f(u \pm i/2)$
Y- system
SU(3) Spin
Dynkin diagram

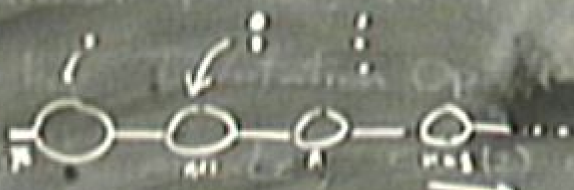

$\prod_n (1+Y_n)^{I_{nn}}$
$Y_n^+ Y_n^- = (1+Y_{n+1})(1+Y_{n-1}) e^{-\beta S_{n1}}$ $S(x)$
$f^\pm = f(u \pm i/2)$
Y-system
SU(3) Spin chain
Dynkin diagram
$I_{(a,s),(a',s')}$
$Y^+_{a,s} Y^-_{a,s} = (1+Y_{a',s'})$
I = — 1
| -1
0

$$\prod_n (1+Y_n)^{I_{kn}}$$

$$Y_n^+ Y_n^- = (1+Y_{n+1})(1+Y_{n-1})\, e^{-\beta S_{n1}} \quad S(x)$$

$$f^{\pm} = f(u \pm i/2)$$

Y-system



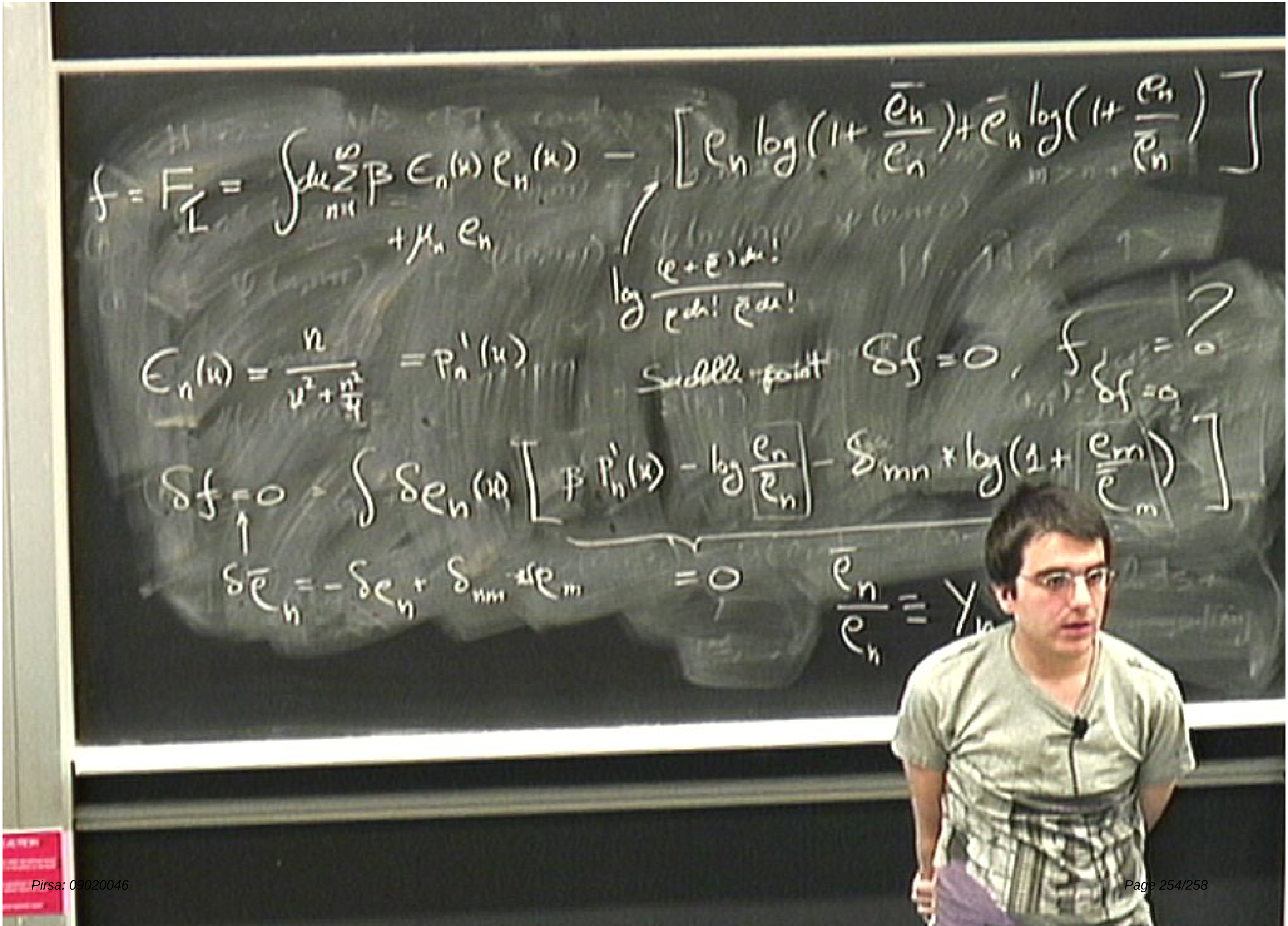
SU(3) Spin chain O–O Dynkin Diagram

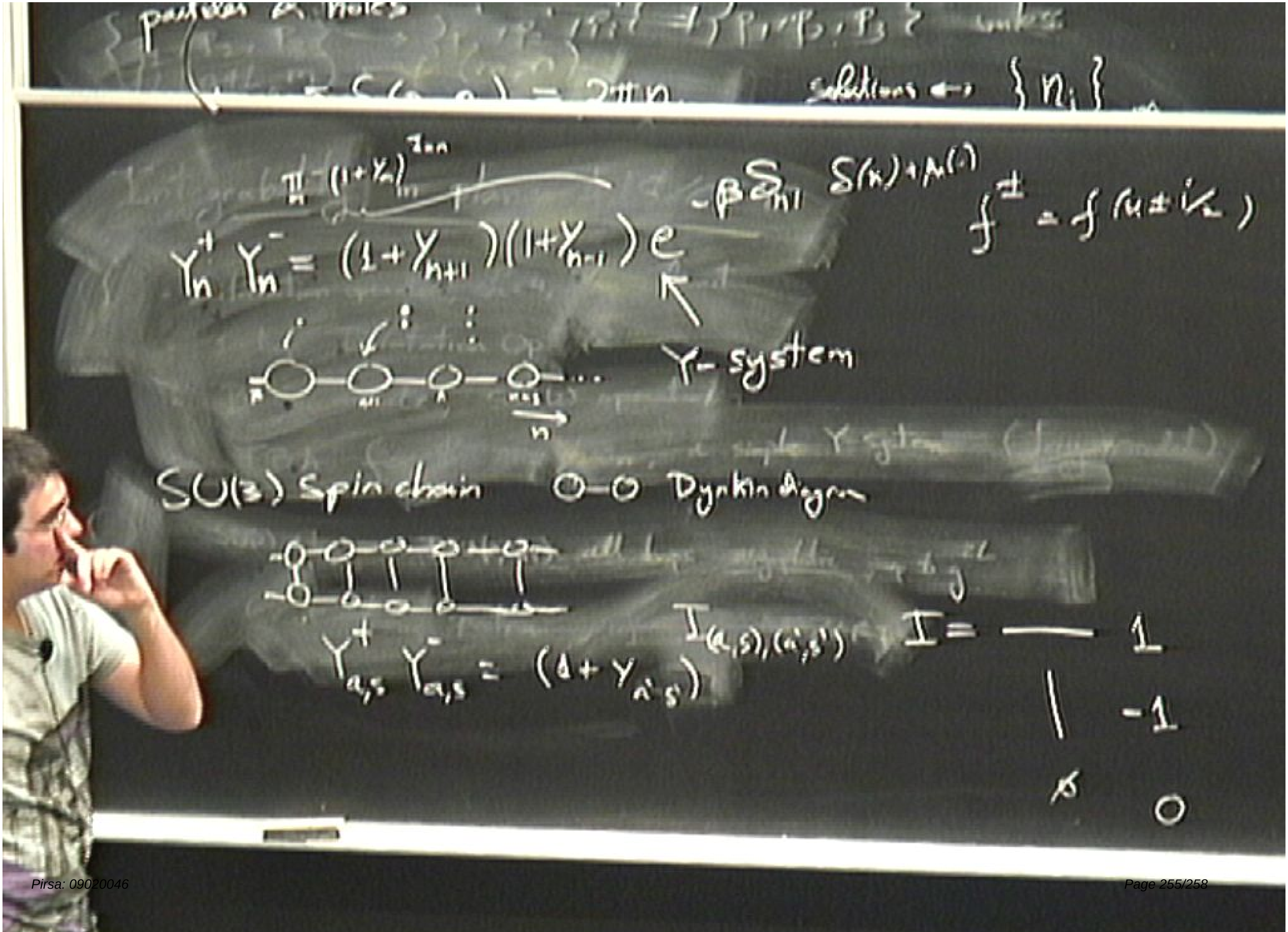
$$Y_{a,s}^+ Y_{a,s}^- = (1+Y_{a',s'})^{I_{(a,s),(a',s')}}$$

$I =$ — 1

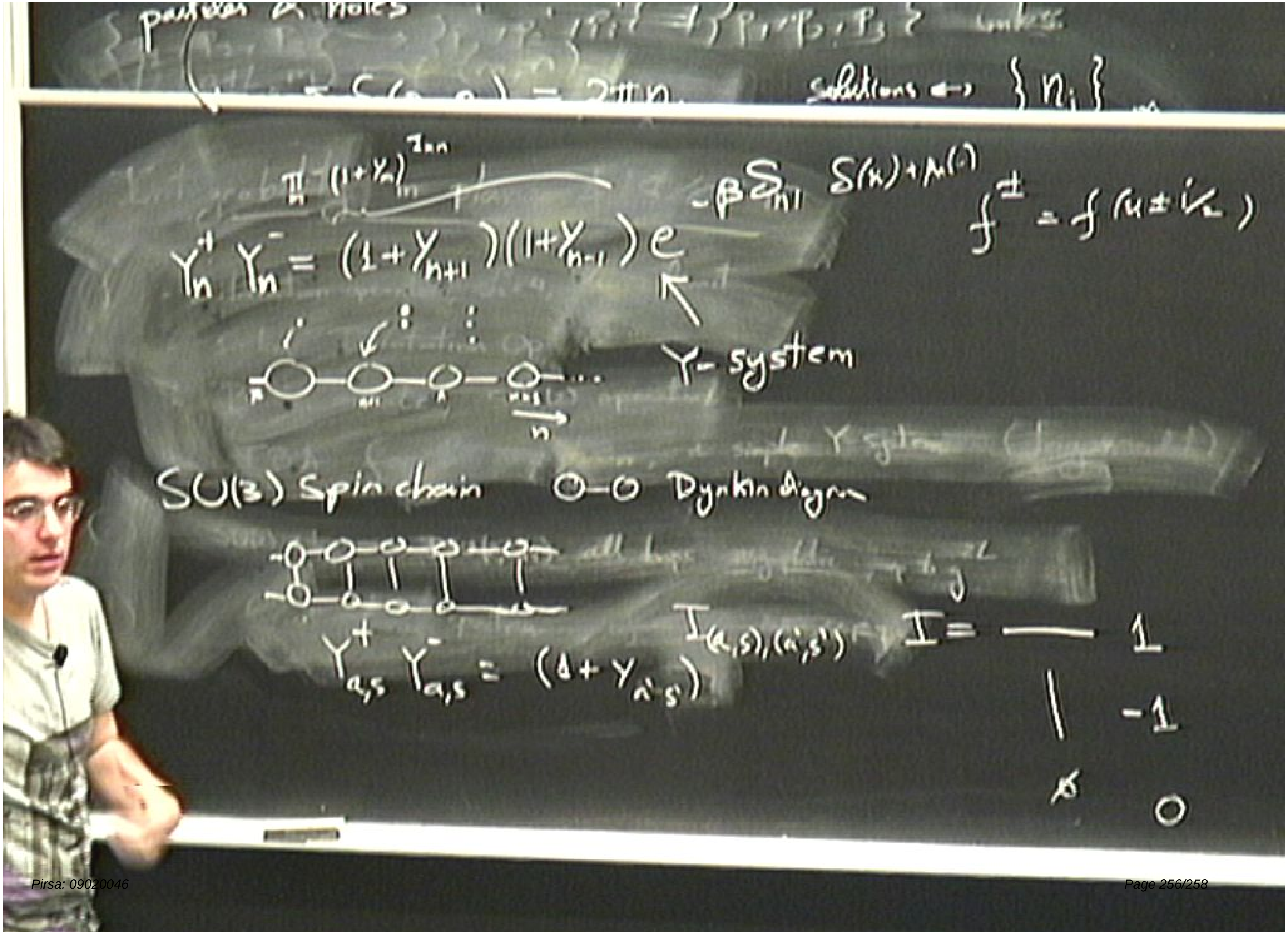
| -1

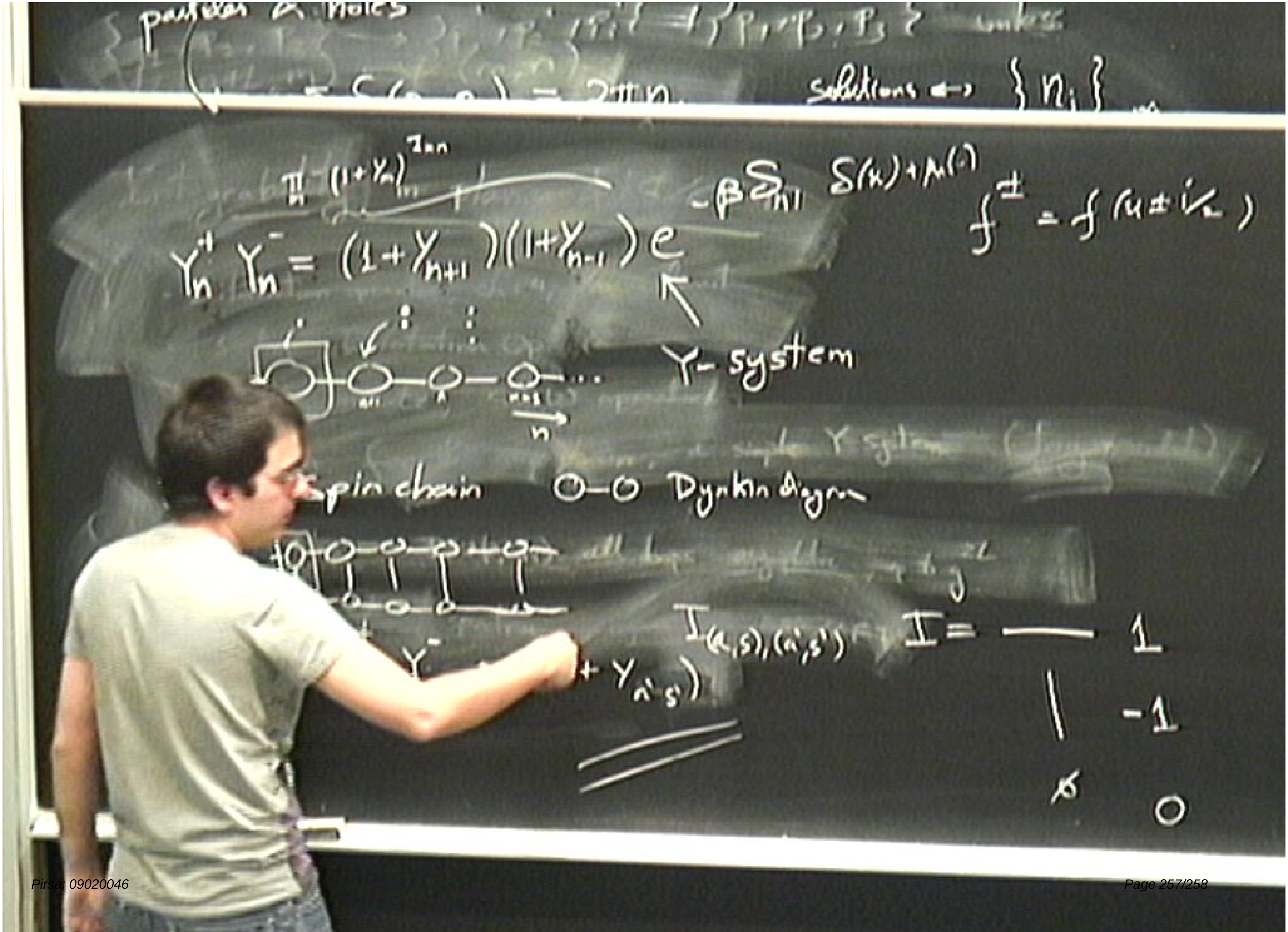
✗ 0

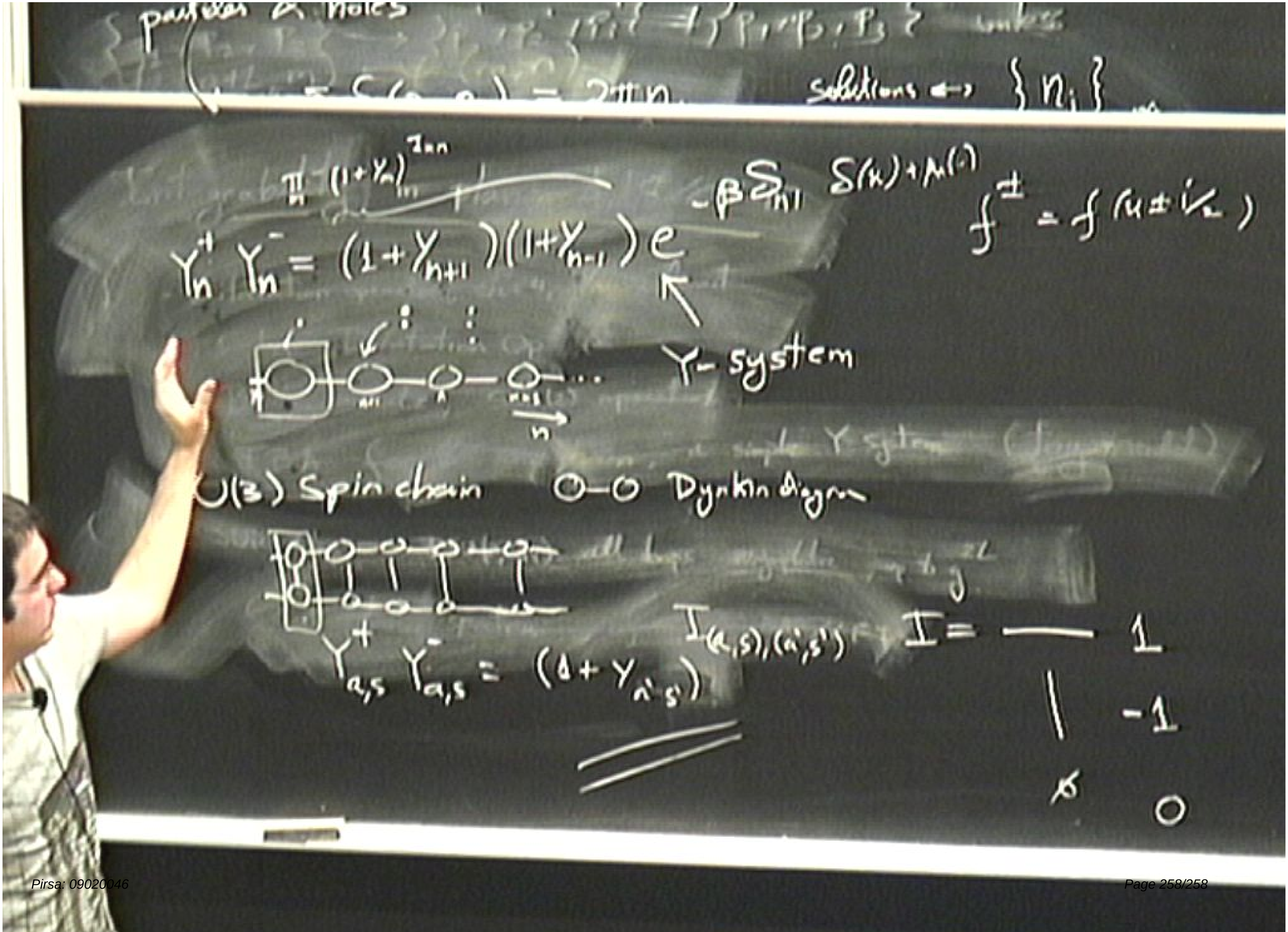

Saddle point δf = 0 , f_{δf=0} = ?
=0



Y-system
SU(3) Spin chain
Dynkin diagram


solutions ↔ {n_i}
Y_n^+ Y_n^- = (1+Y_{n+1})(1+Y_{n-1}) e^{-β S_{n1}}
f^± = f(u ± i/2)
Y- system
SU(3) Spin chain
Dynkin diagram
Y^+_{a,s} Y^-_{a,s} = (1+Y_{a',s'})
I_{(a,s),(a',s')}
I =
1
-1
0


solutions ↔ {n_i}
f^± = f(u ± i/2)
Y_n^+ Y_n^- = (1+Y_{n+1})(1+Y_{n-1}) e^{-βS_{n1}}
Y-system
spin chain
O–O Dynkin diagram
I = —— 1
| -1
0


particles & holes
solutions ↔ {n_i}
Y_n^+ Y_n^- = (1+Y_{n+1})(1+Y_{n-1}) e
f^± = f(u ± i/2)
Y-system
U(3) Spin chain
Dynkin diagram
Y_{a,s}^+ Y_{a,s}^- = (1+Y_{a',s'})
I_{(a,s),(a',s')}
I = 1
-1
0