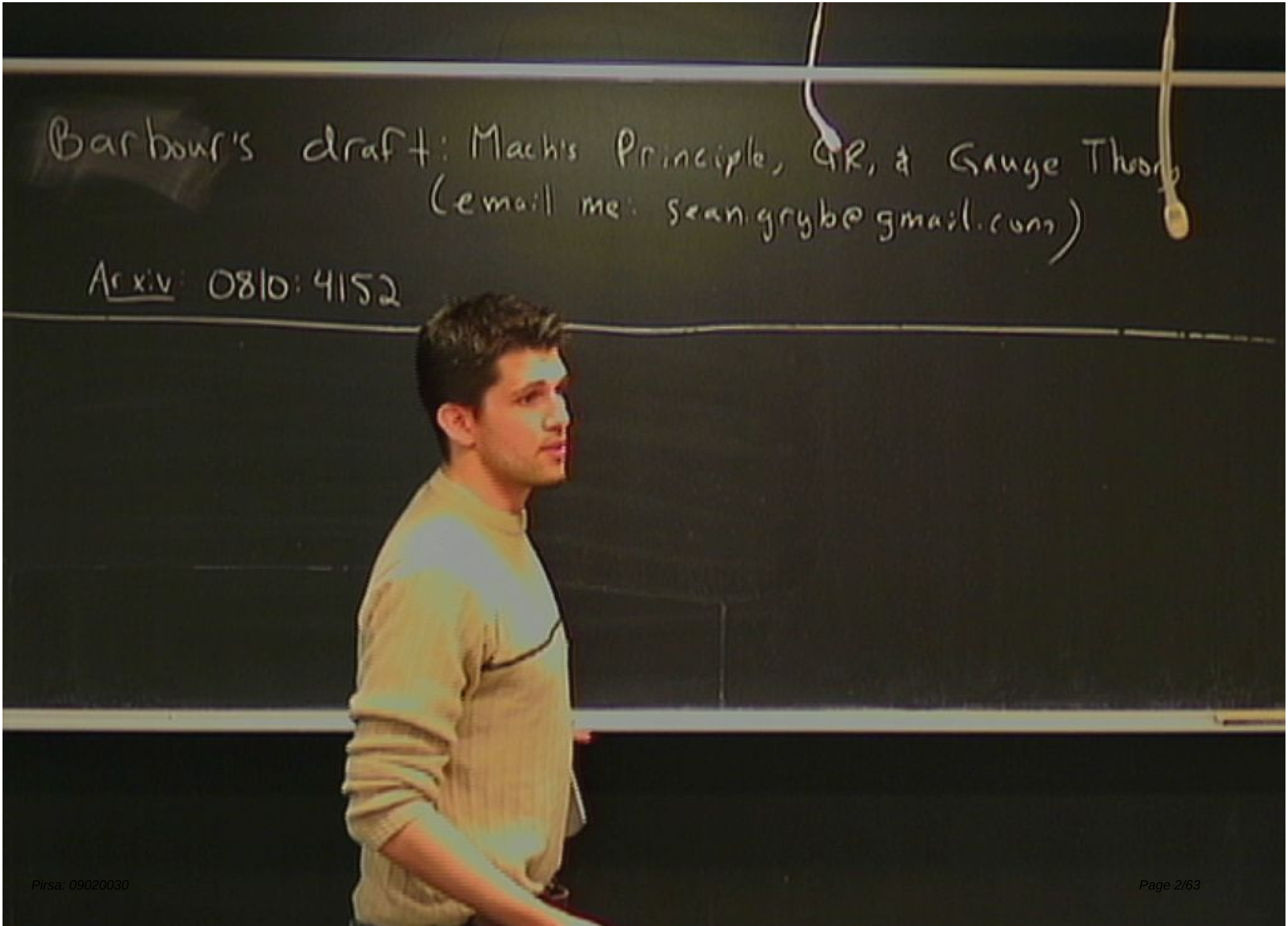
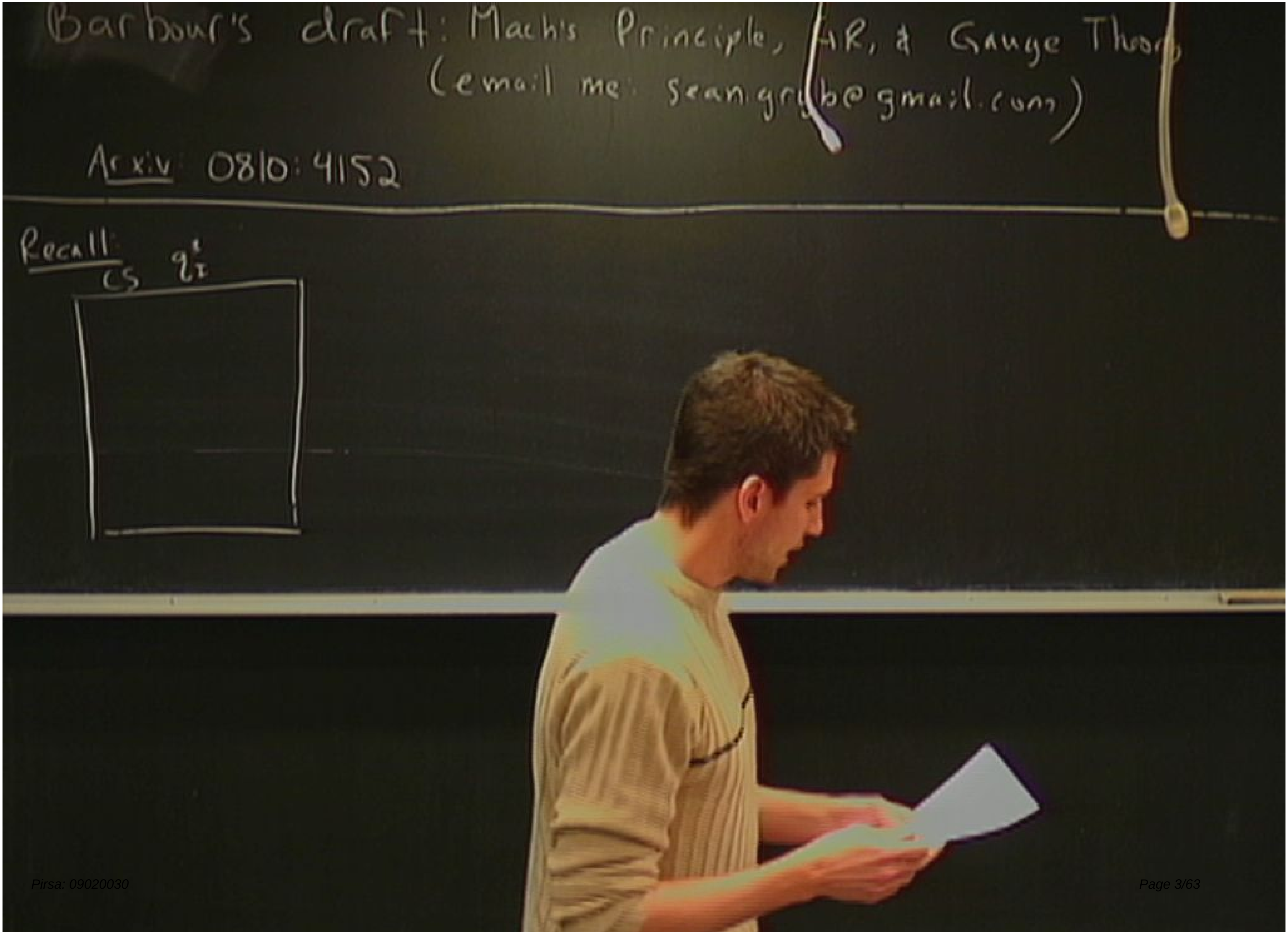
Title: Mini-Course on Mach's Principle - Lecture 2

Date: Feb 11, 2009 10:30 AM

URL: http://pirsa.org/09020030

Abstract:


Barbour's draft: Mach's Principle, GR, & Gauge Theory
(email me: sean.gryb@gmail.com)
Arxiv: 0810:4152



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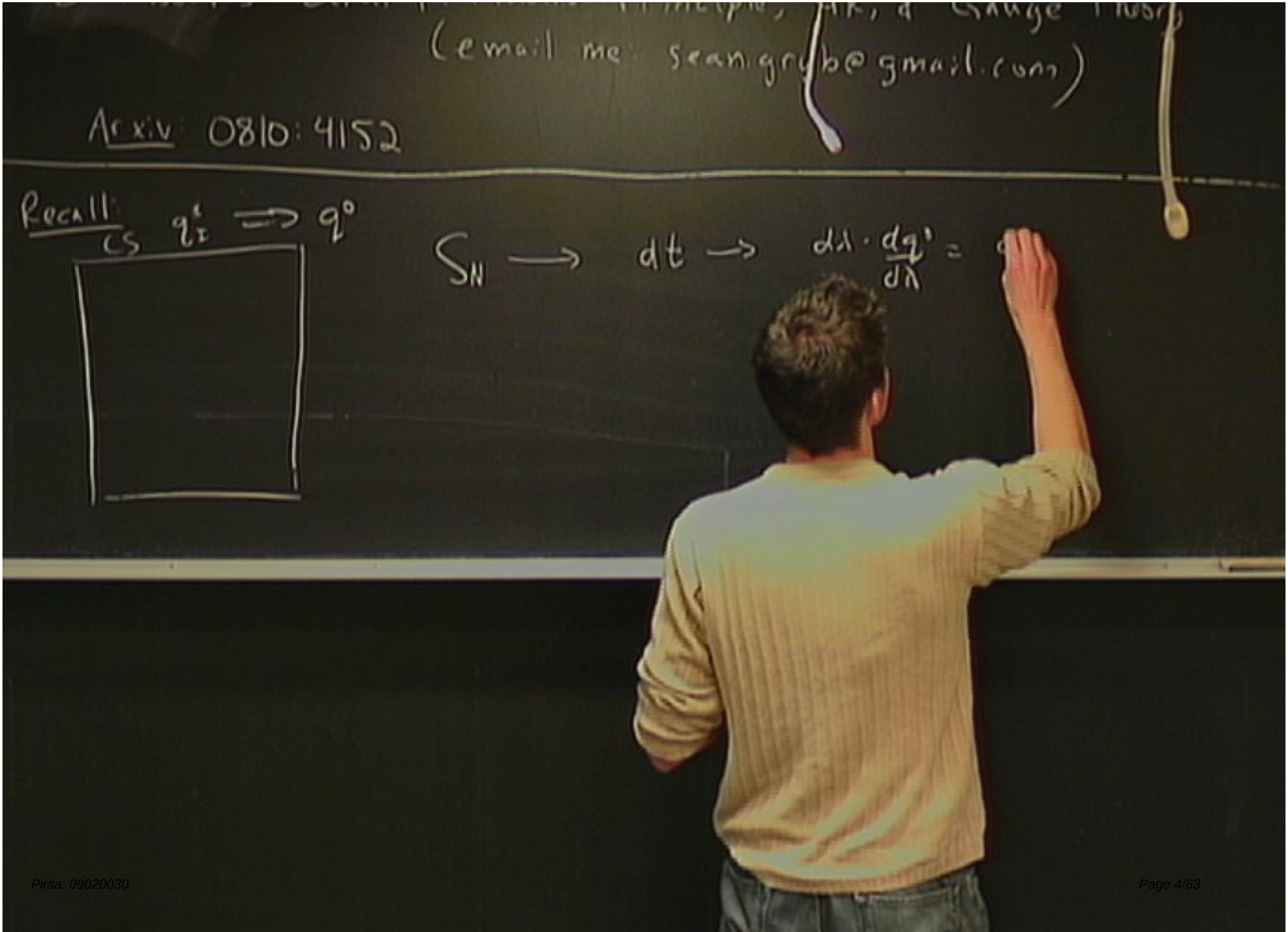
Recall:
CS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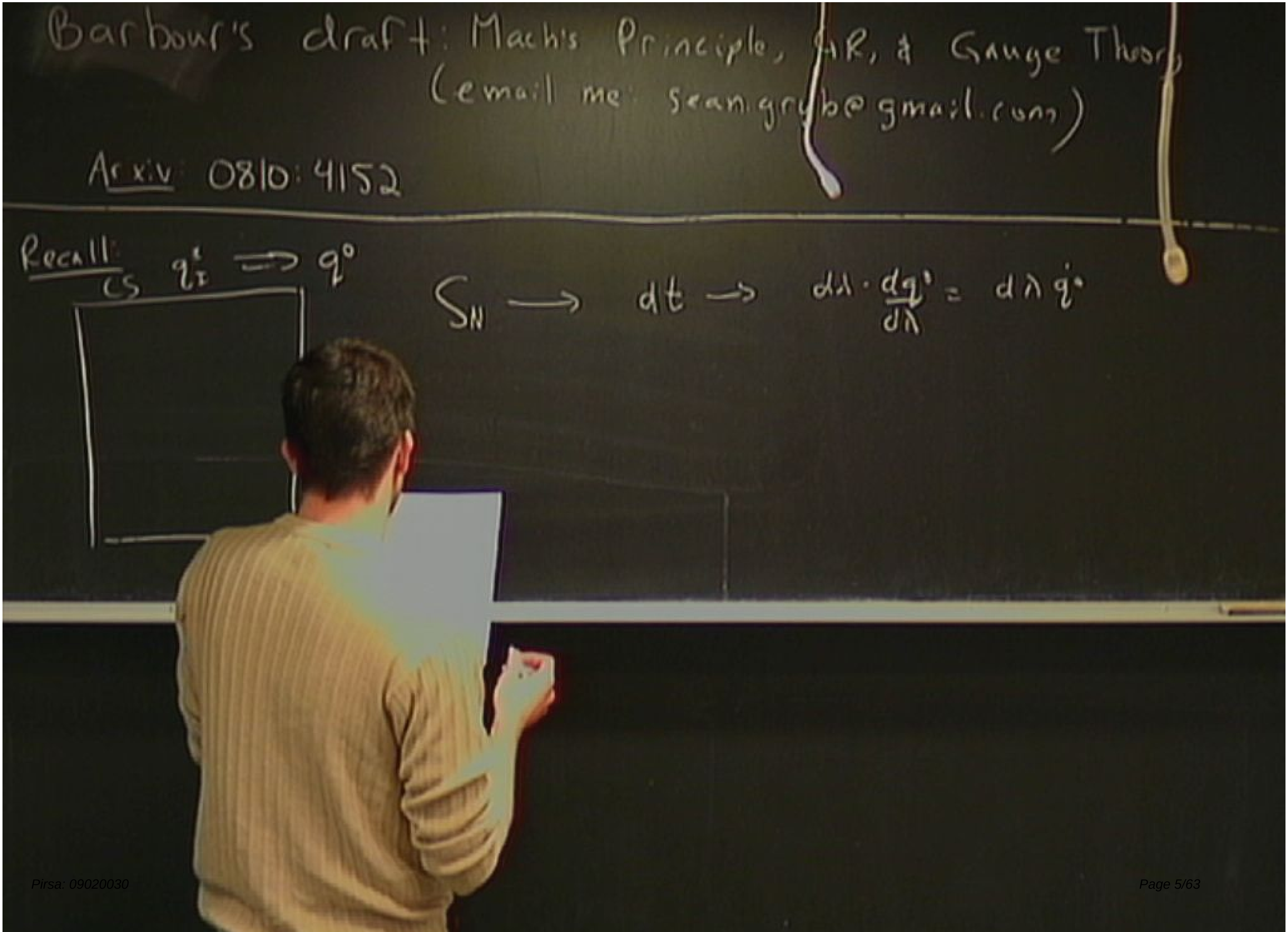
Arxiv: 0810:4152
Recall:
CS
⟹
Sn → dt → dλ · dq⁰/dλ =


Barbour's draft: Mach's Principle, GR, & Gauge Theory
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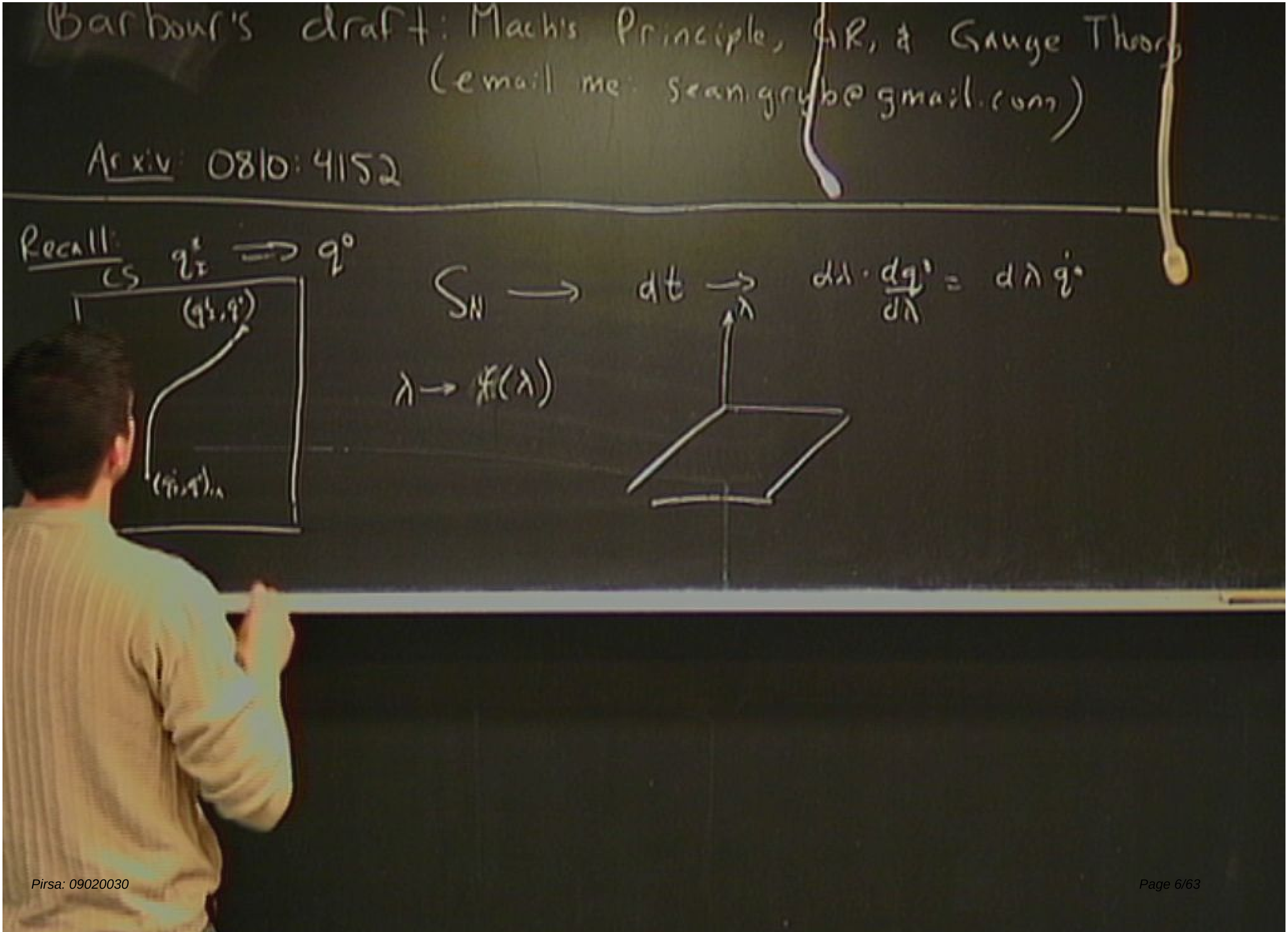
Arxiv: 0810:4152
Recall: CS
q^i_I ⇒ q^0
S_N → dt → dλ · dq^0/dλ = dλ q̇^0


Barbour's draft: Mach's Principle, GR, & Gauge Theory
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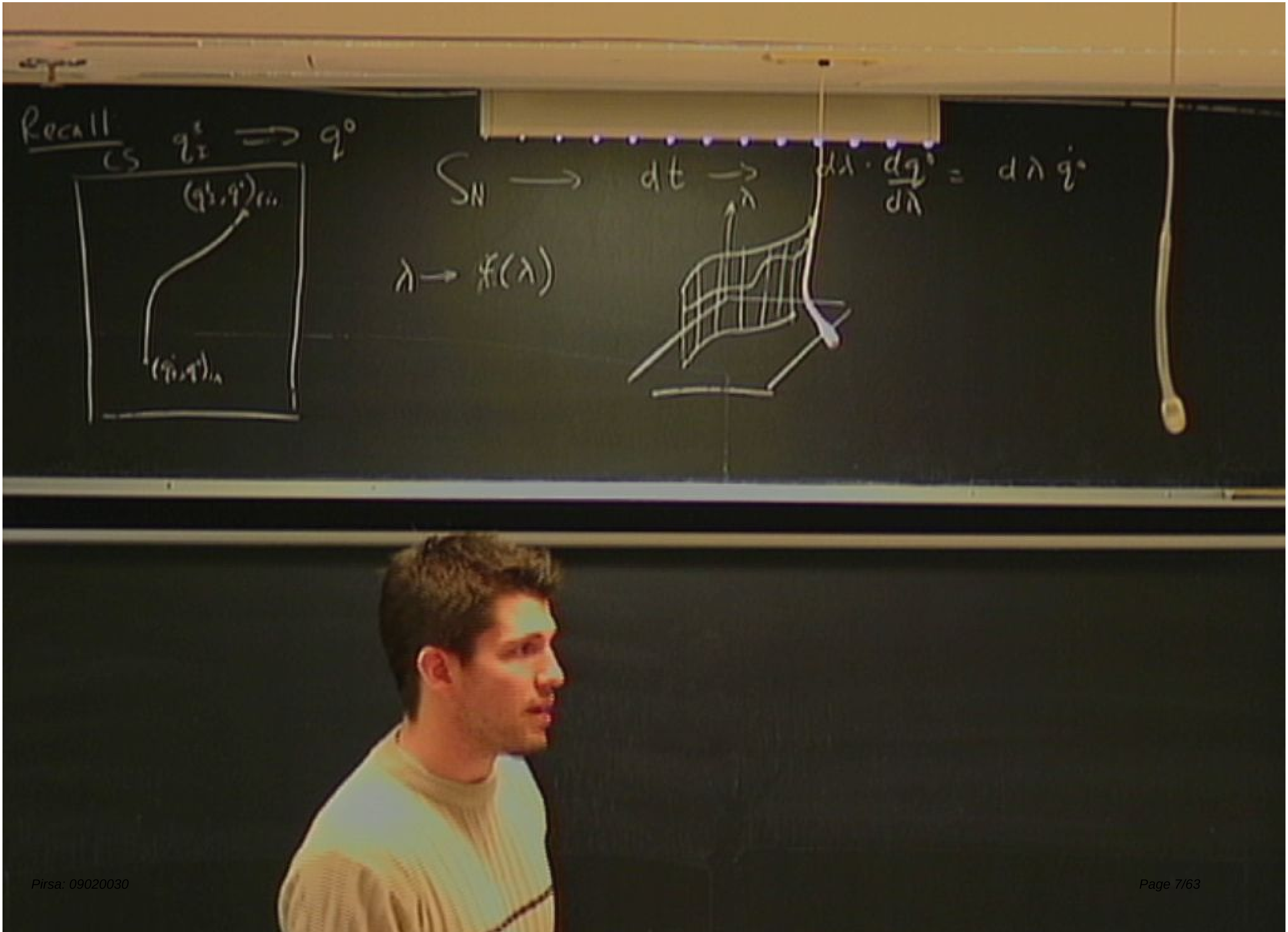
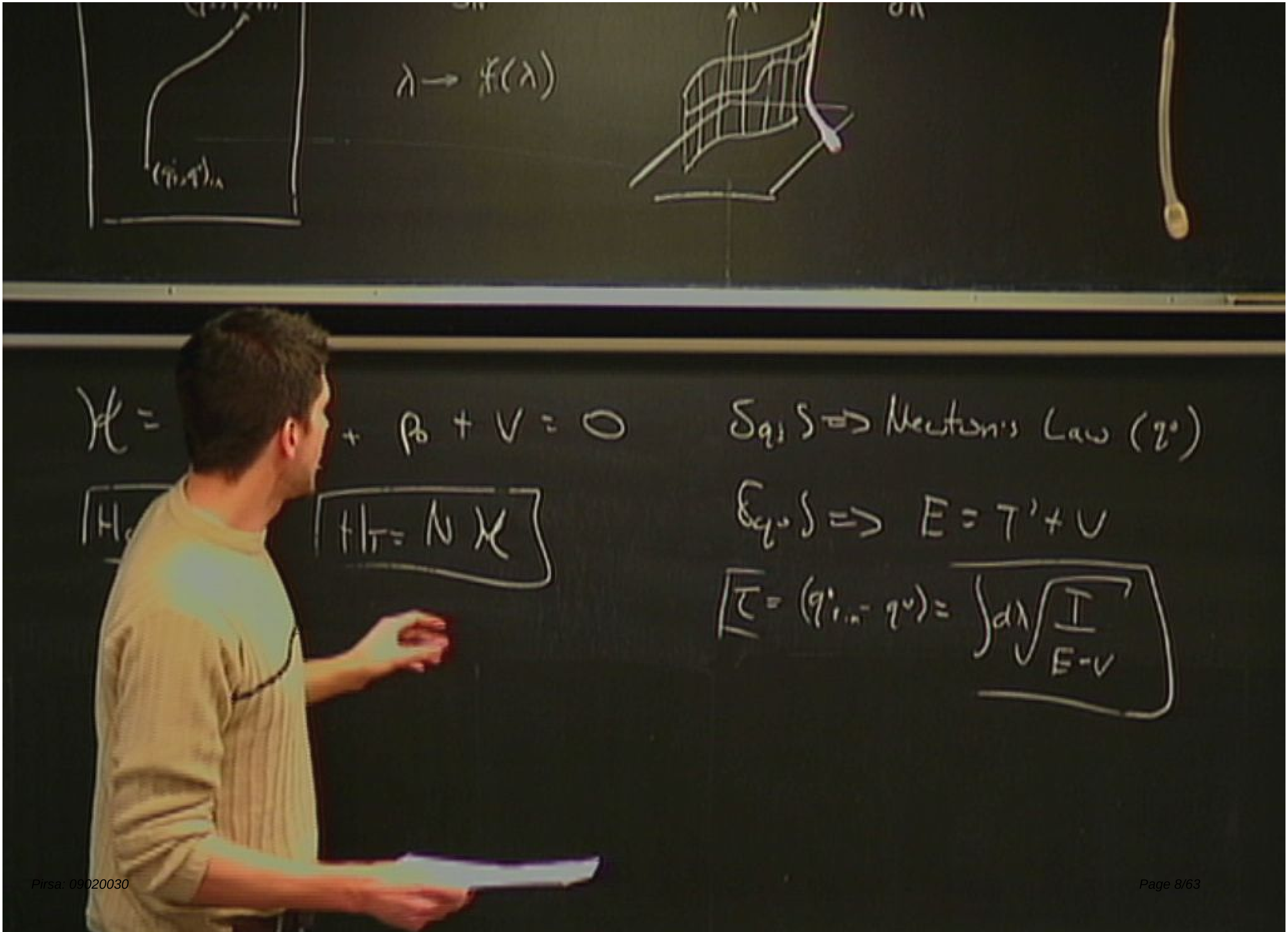
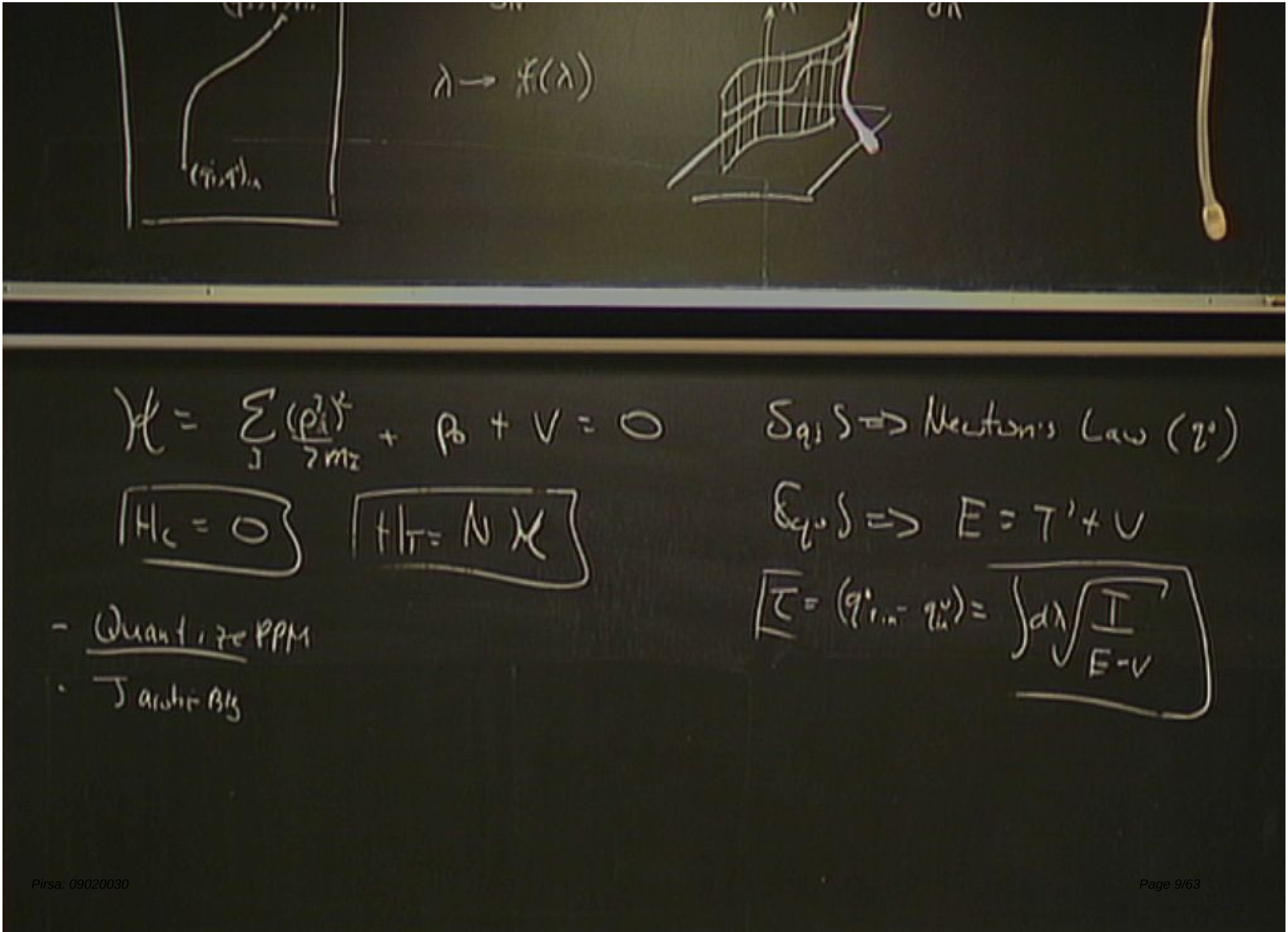
Arxiv: 0810:4152
Recall:
CS
$S_N \longrightarrow$
$dt \longrightarrow$
$d\lambda \cdot \frac{dq^0}{d\lambda} = d\lambda\, \dot{q}^0$
$\lambda \to f(\lambda)$


Recall
CS
$S_N \longrightarrow dt \longrightarrow d\lambda \cdot \frac{dq^0}{d\lambda} = d\lambda\, \dot{q}^0$
λ
$\lambda \rightarrow f(\lambda)$


λ → ℋ(λ)
ℋ = + p₀ + V = 0
H_T = N ℋ
δ_q S ⇒ Newton's Law (q^i)
δ_q S ⇒ E = T' + V
τ = (q^i_in − q^v) = ∫dλ √(I / (E − V))



λ → H(λ)
H = Σ_j (p_j)²/2m_j + p_0 + V = 0
H_c = 0
H_T = N H
- Quantize PPM
· Jacobi-Big
δ_{q_i} S ⇒ Newton's Law (q^0)
δ_{q^0} S ⇒ E = T' + V
τ = (q^0_{fin} − q^0_{in}) = ∫dλ √(I/(E−V))

Quantization:

$$\{q^i_I, p^J_j\} = \delta^i_j \delta^J_I \Rightarrow [\hat{q}^i_I, \hat{p}^J_j] = i \delta^i_j \delta^J_I$$

Quantization:

$$\{ q^i_I , p^J_j \} = \delta^i_j \delta^J_I \Rightarrow \{ \hat{q}^i_I , \hat{p}^J_j \} = i \delta^i_j \delta^J_I$$

$$\{ q^0 , p_0 \} = \text{[illegible]} \Rightarrow [\hat{q}^0 , \hat{p}_0] = i$$

Quantization:

$$\{q^i_I, p^J_j\} = \delta^i_j \delta^J_I \Rightarrow \{\hat{q}^i_I, \hat{p}^J_j\} = i\,\delta^i_j \delta^J_I$$

$$\{q^0, p_0\} = \quad \Rightarrow \quad [\hat{q}^0, \hat{p}_0] = i$$

Quantization:

$$\{ q^i_I , p^J_j \} = \delta^i_j \delta^J_I \Rightarrow [\hat{q}^i_I , \hat{p}^J_j] = i \delta^i_j \delta^J_I$$

$$\{ q^0 , p_0 \} = [illegible] \Rightarrow [\hat{q}^0 , \hat{p}_0] = i$$

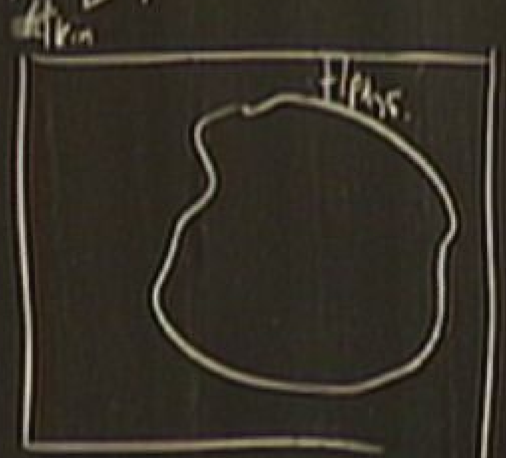
$$\psi = \psi (q^0 , q^i)$$

Quantization:

$$\{ q^i_I , p^J_j \} = \delta^i_j \delta^J_I \Rightarrow [\hat{q}^i_I , \hat{p}^J_j] = i \delta^i_j \delta^J_I$$

$$\{ q^0 , p_0 \} = \quad \Rightarrow [\hat{q}^0 , \hat{p}_0] = i$$

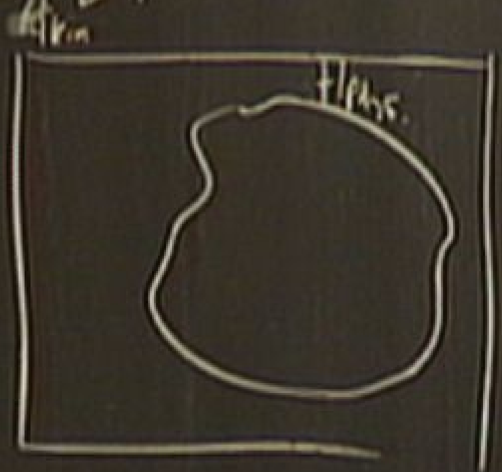
$$\psi = \psi(q^i_I\, q^i)$$



Quantization:

$$\{ q^i_I , p^J_j \} = \delta^i_j \delta^J_I \Rightarrow [\hat{q}^i_I , \hat{p}^J_j] = i \delta^i_j \delta^J_I$$

$$\{ q^0 , p_0 \} = \text{[illegible]} \Rightarrow [\hat{q}^0 , \hat{p}_0] = i$$

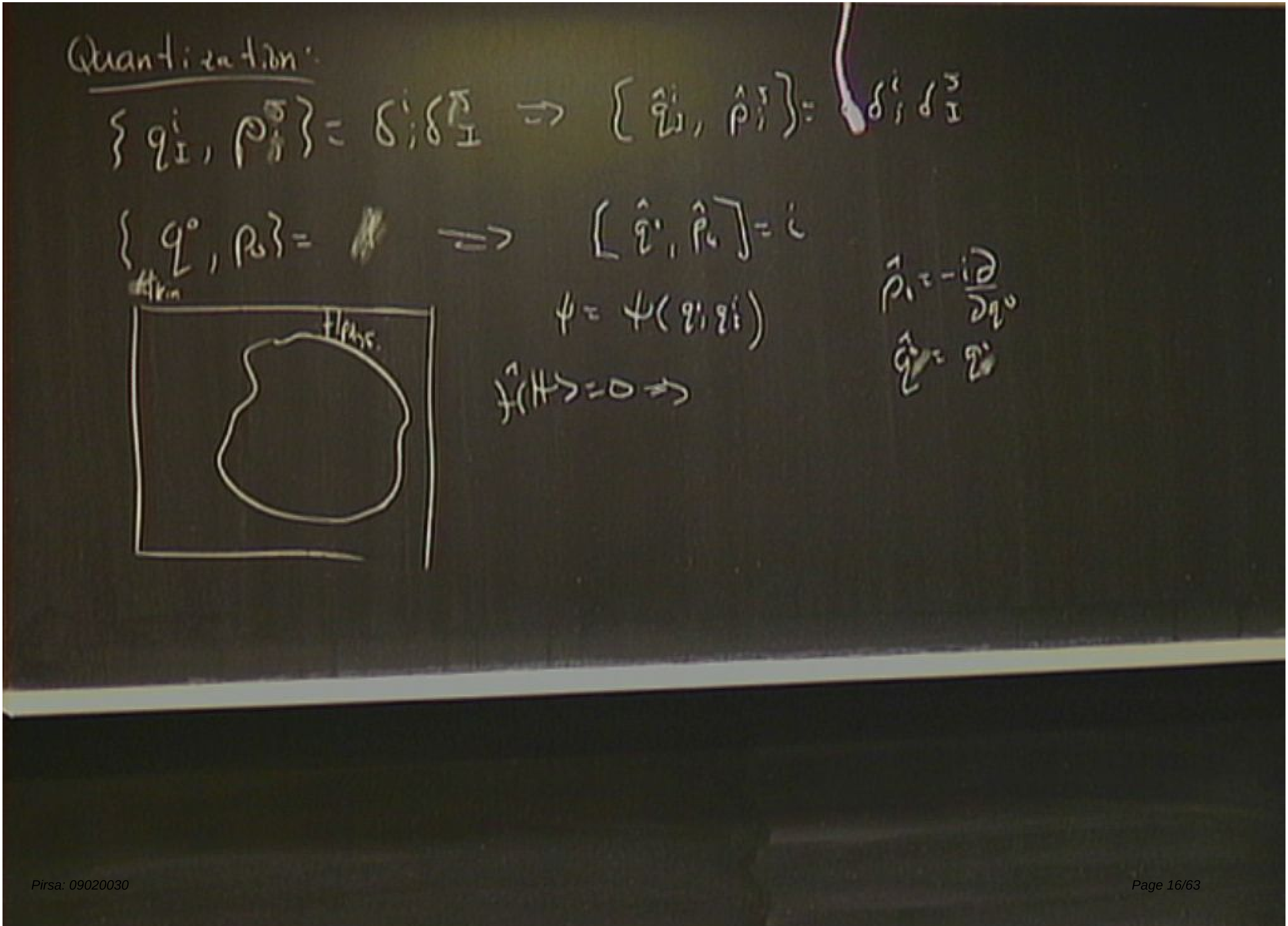


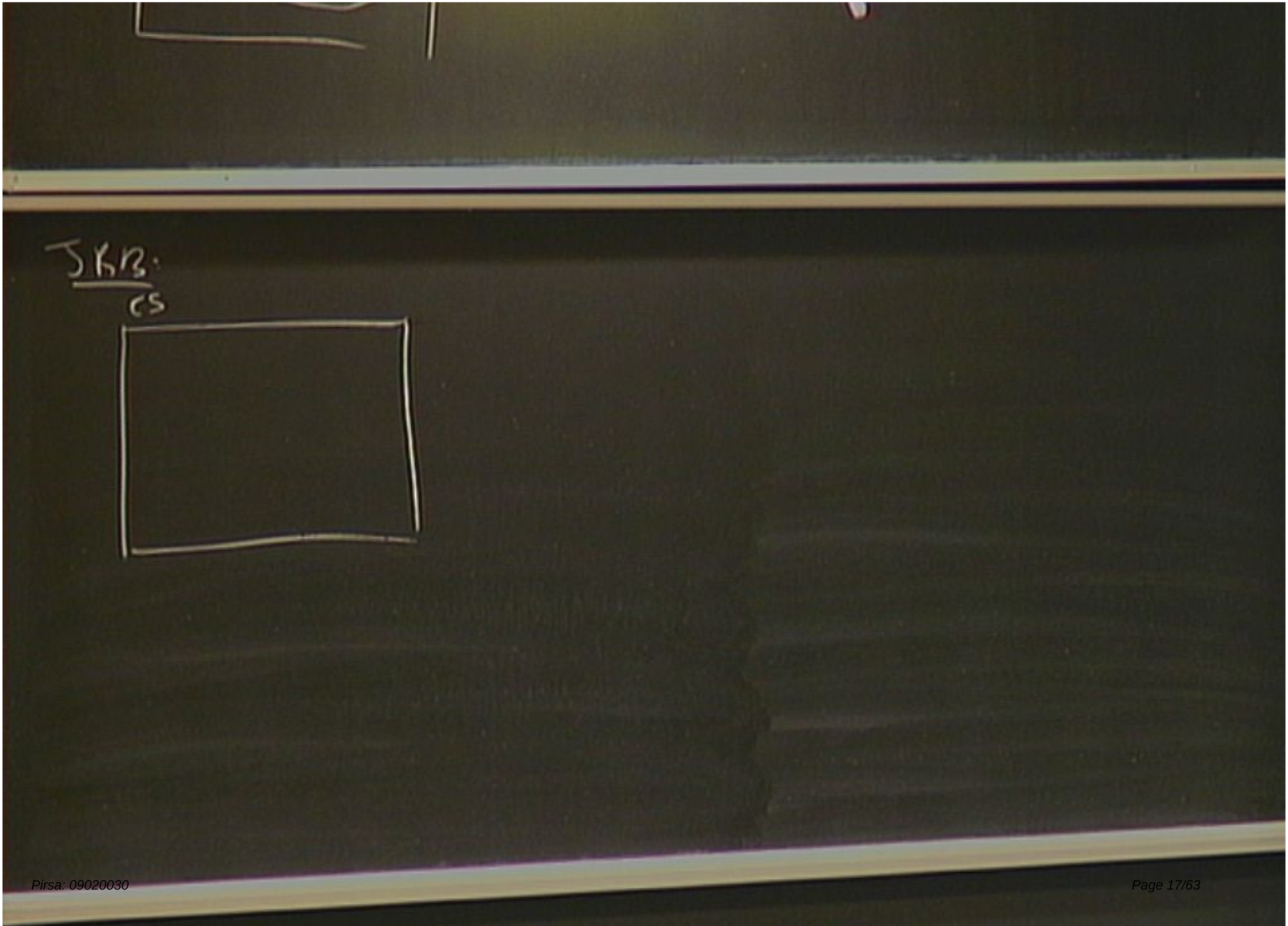
$$\psi = \psi (q^i_I , q^i)$$

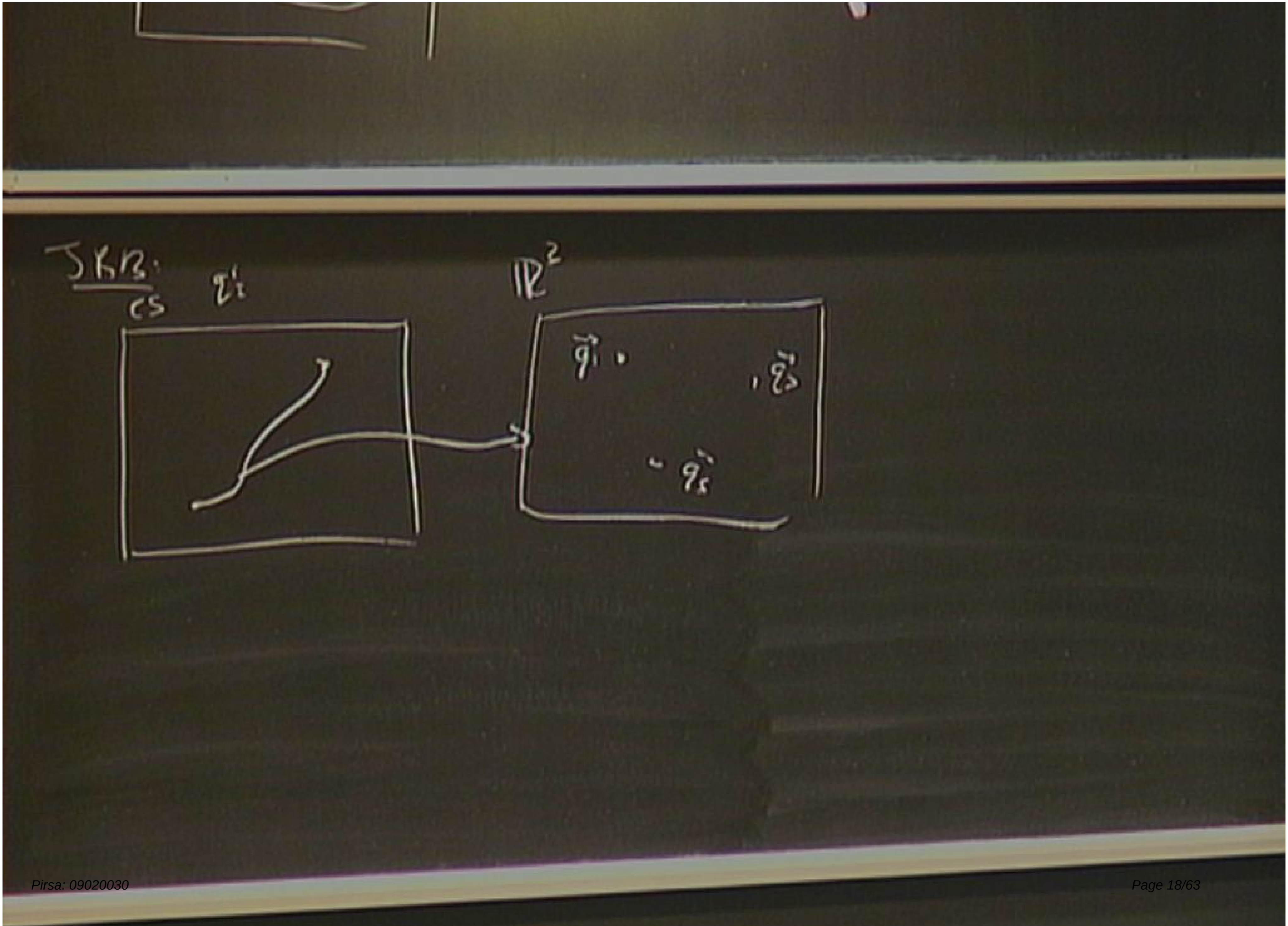
$$\hat{f} | \psi \rangle = 0 \Rightarrow$$

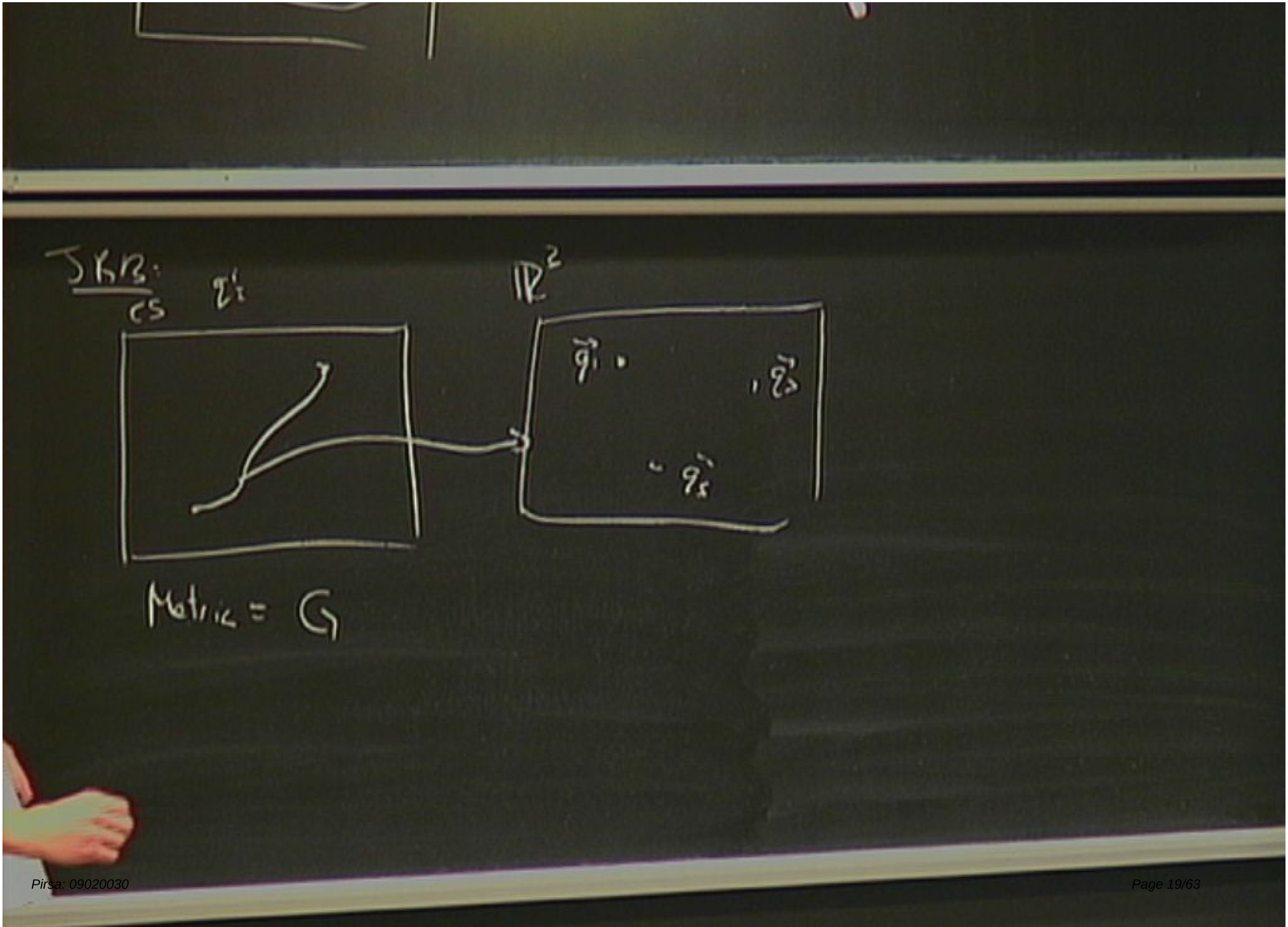
$$\hat{p}_0 = -i \frac{\partial}{\partial q^0}$$

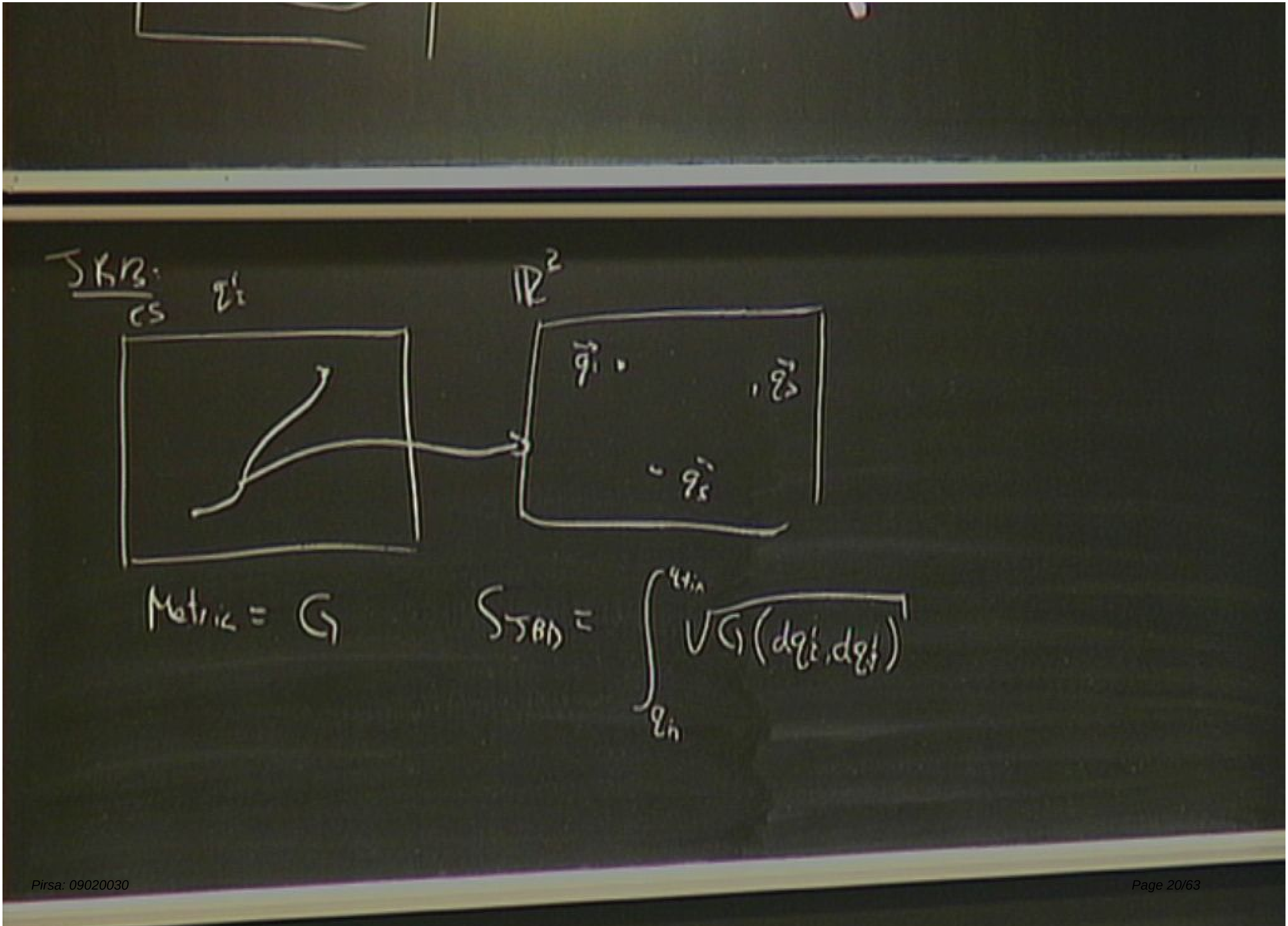
$$\hat{q}^i = q^i$$

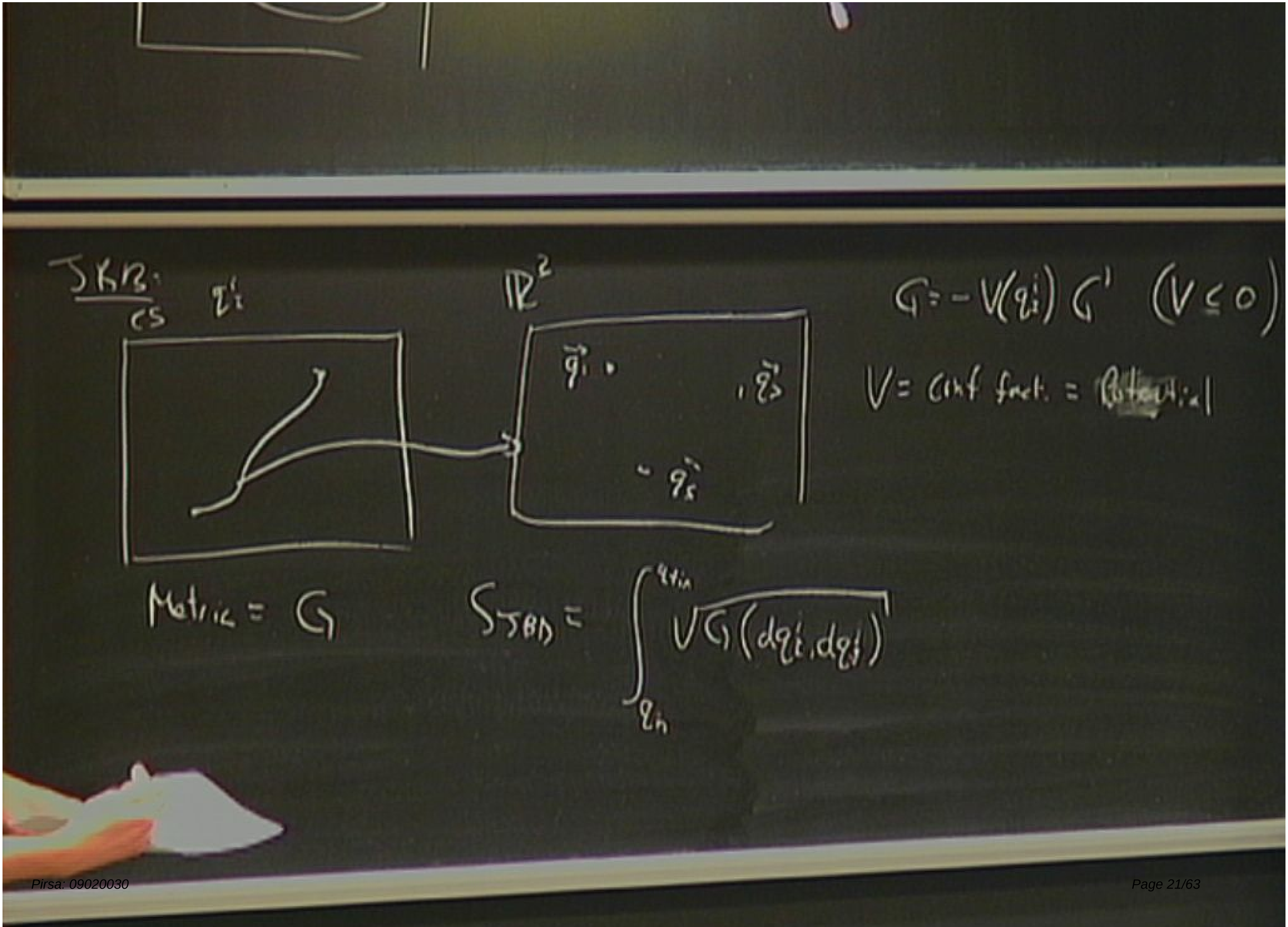

Quantization:
{ q^i_I , p^J_j } = δ^i_j δ^J_I ⇒ { q̂^i_I , p̂^J_j } = δ^i_j δ^J_I
{ q^0 , p_0 } = ⇒ [q̂^0 , p̂_0] = i
ψ = ψ(q^i_I , q^i)
p̂_0 = −i ∂/∂q^0
q̂^i = q^i
H_kin
H_phys

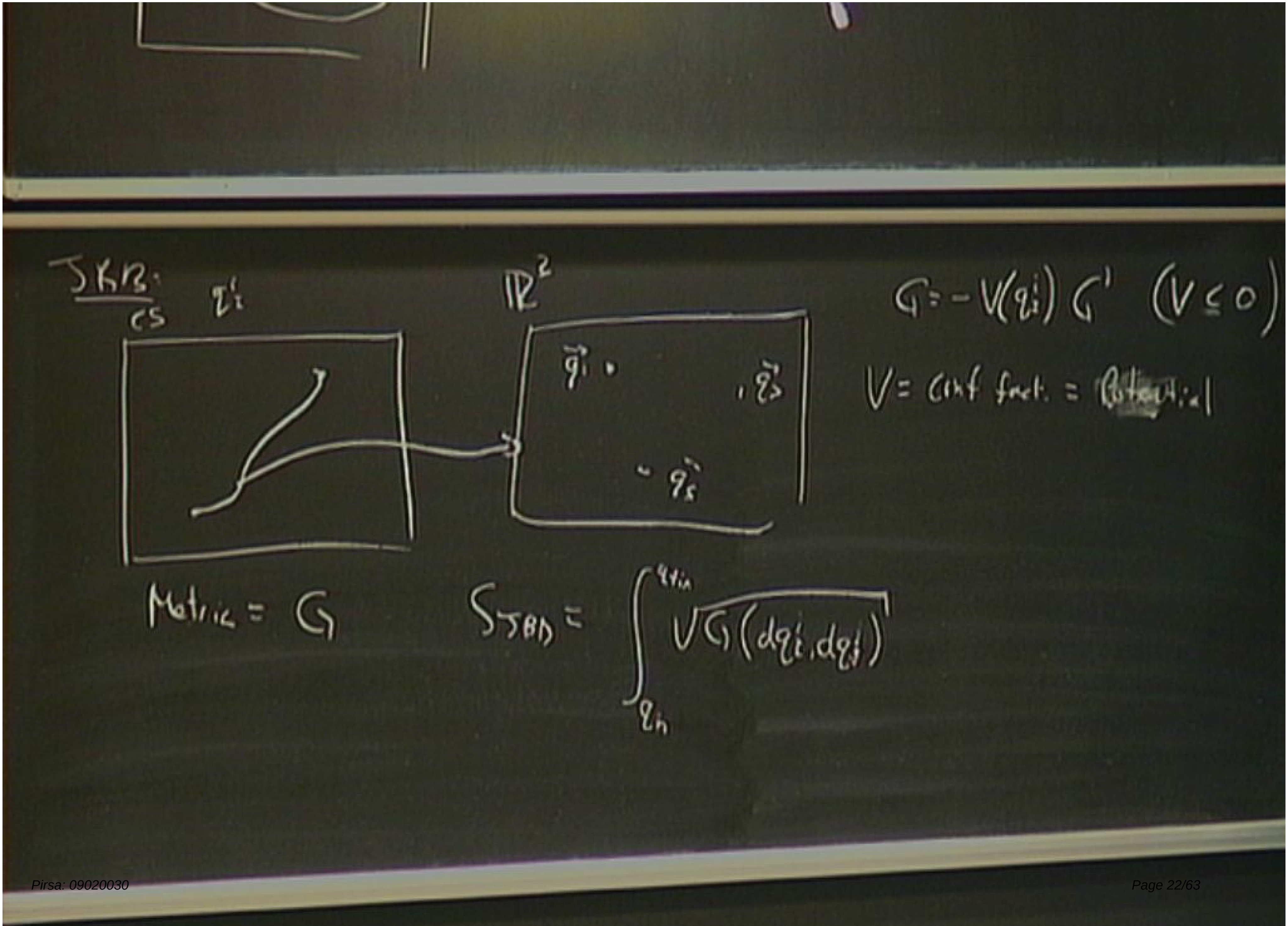

JKB:
CS


JKB:
CS
$\mathbb{R}^2$
$\vec{q}_1$
$\vec{q}_5$

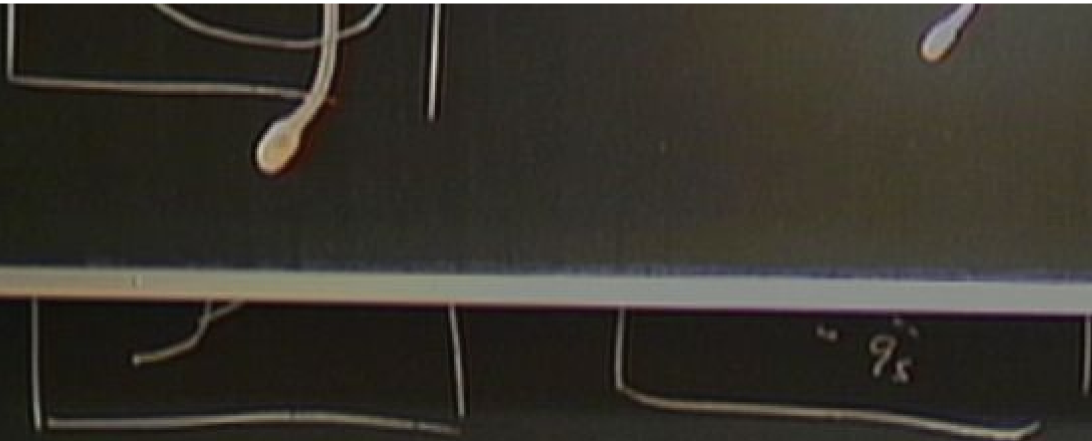

ℝ²
Metric = G


JBB:
CS
ℝ²
Metric = G
S_JBD =


JBB: CS
ℝ²
G = −V(qᵢ) G′ (V ≤ 0)
V = conf fact. = Potential
Metric = G
S_JBD = ∫ √G(dqᵢ, dqᵢ)

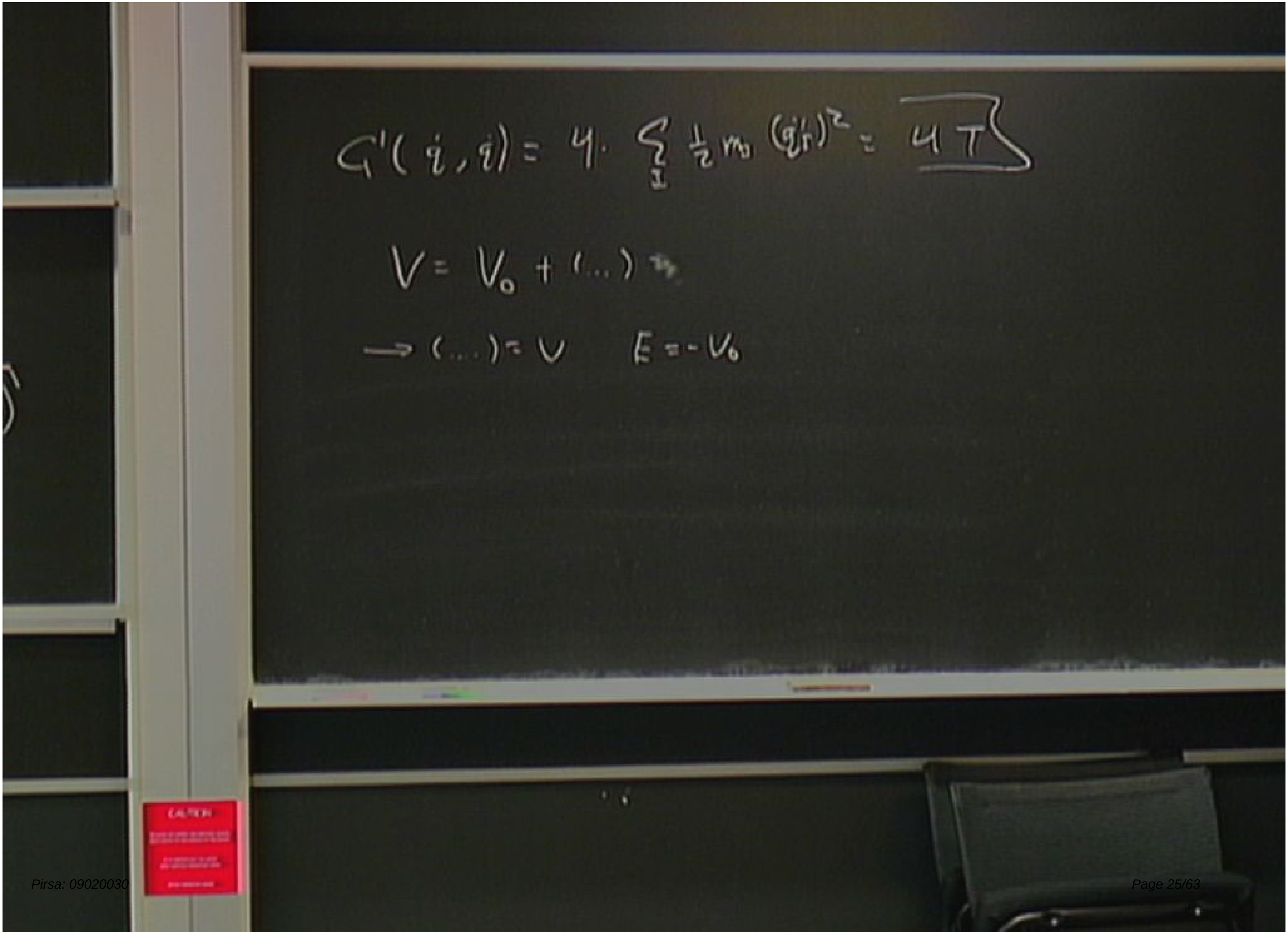

JBB: CS
q_i
$\mathbb{R}^2$
$\vec{q}_1$
$\vec{q}_2$
$\vec{q}_3$
$G = -V(q_i)\, G' \quad (V \leq 0)$
V = conf fact. = Potential
Metric = G
$S_{JBD} = \int_{q_{in}}^{q_{fin}} \sqrt{G(dq_i, dq_i)}$

$$G'(\dot{q}, \dot{q}) = 4 \cdot \sum_i \frac{1}{2} m_i (\dot{q}^i)^2 = 4T$$



$$\text{Metric} = G$$

$$S_{JBD} = \int_{q_{in}}^{q_{fin}} \sqrt{G(dq_t, dq_t)} \sim \int d\lambda \sqrt{-V}\sqrt{G'(\dot{q}_t, \dot{q}_t)}$$


$G'(\dot{q}, \dot{q}) = 4 \cdot \sum_i \frac{1}{2} m_i (\dot{q}^i)^2 = 4T$
$V = V_0 + (\ldots)$
$\longrightarrow (\ldots) = V \quad E = -V_0$

$$G'(\dot{q},\dot{q}) = 4 \cdot \sum_i \frac{1}{2} m_i (\dot{q}^i)^2 = 4T$$

$$V = V_0 + (\ldots)$$

$$\rightarrow (\ldots) = V \qquad E = -V_0$$

$$S_{JBB} = \int d\lambda \, 2\sqrt{T}\sqrt{E-V}$$

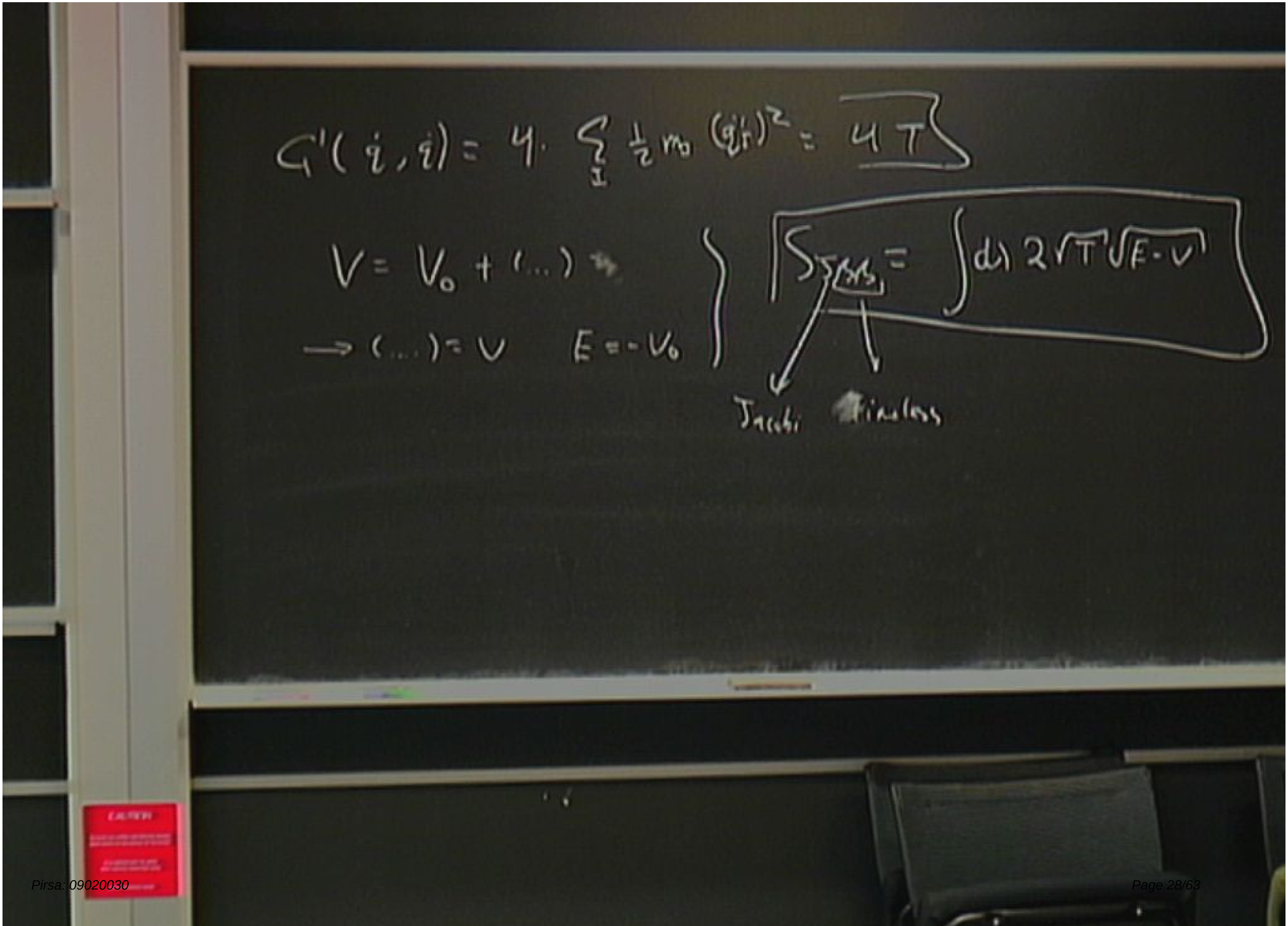
$$G'(\dot{q},\dot{q}) = 4 \cdot \sum_I \tfrac{1}{2} m_I (\dot{q}^i)^2 = 4T$$

$$V = V_0 + (\ldots)$$

$$\rightarrow (\ldots) = V \qquad E = -V_0$$

$$S_{JBB} = \int d\lambda \, 2\sqrt{T}\sqrt{E-V}$$

Jacobi



$G'(\dot q, \dot q) = 4 \cdot \sum_I \frac{1}{2} m_I (\dot q^I)^2 = 4T$
$V = V_0 + (\ldots)$
$\rightarrow (\ldots) = V \quad E = -V_0$
$S_{Jacobi} = \int d\lambda\, 2\sqrt{T}\sqrt{E - V}$
Jacobi
CAUTION

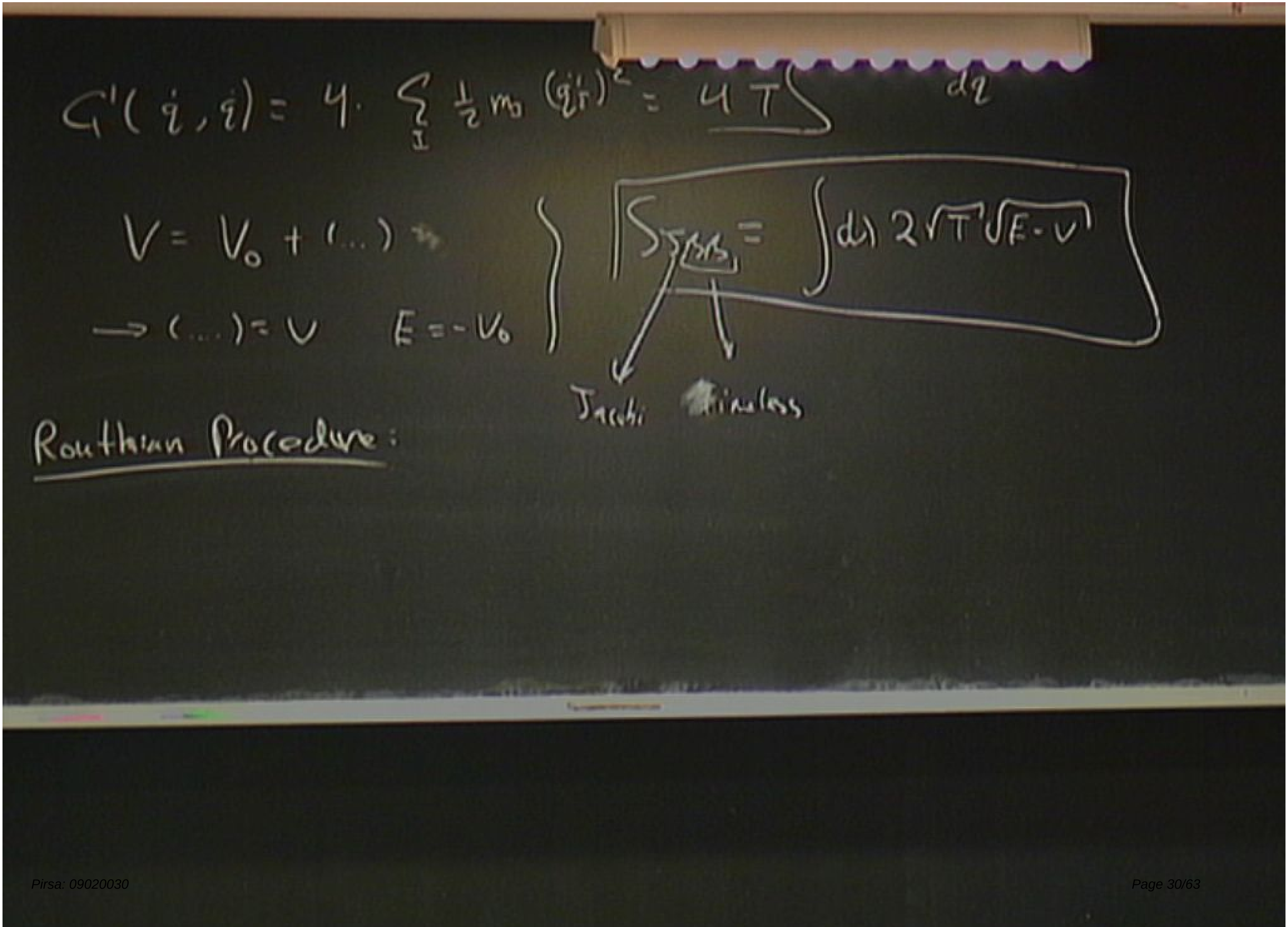
$$G'(\dot{q},\dot{q}) = 4 \cdot \sum_I \tfrac{1}{2} m_I (\dot{q}^I)^2 = 4T \qquad dq$$

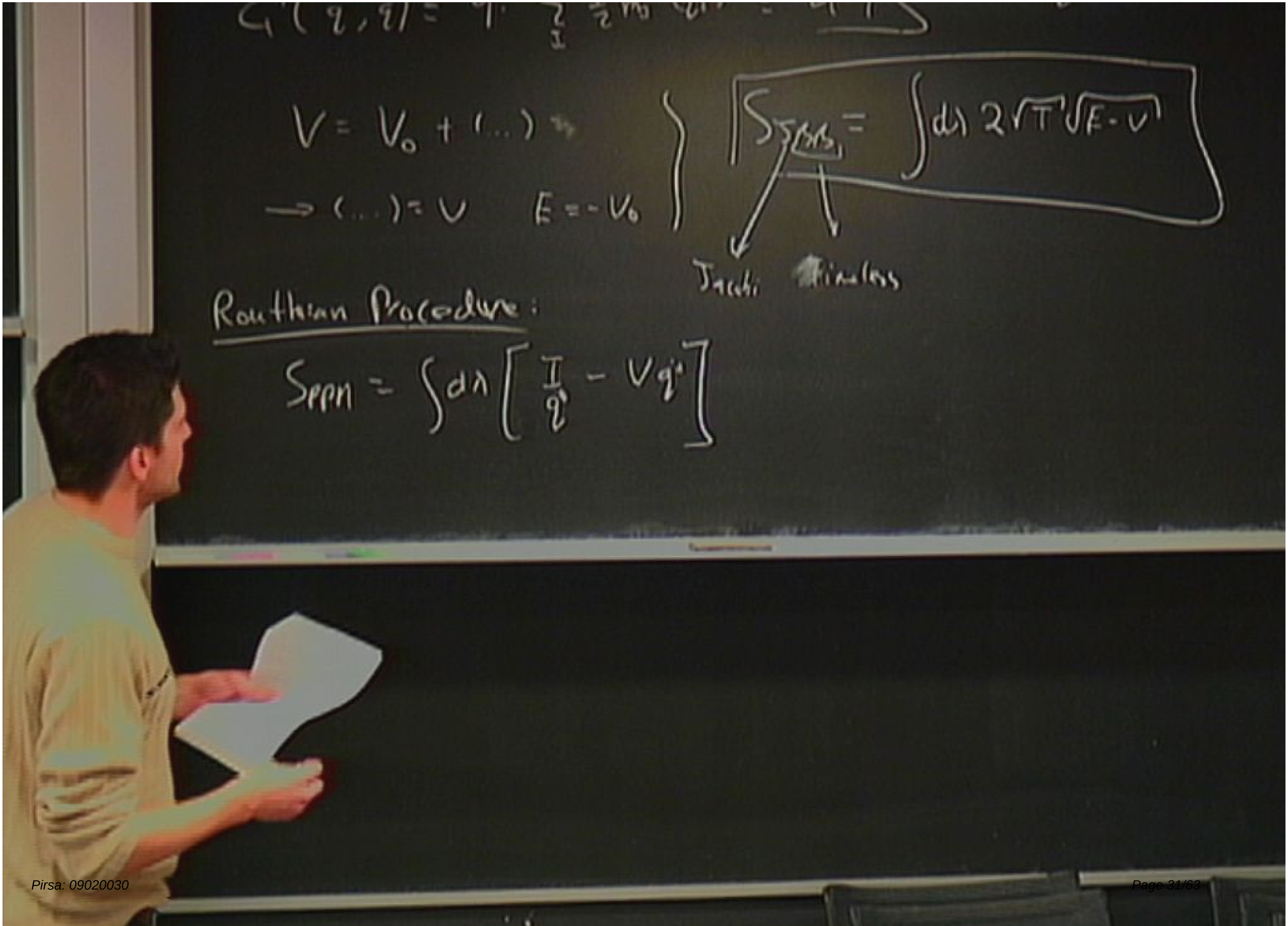
$$V = V_0 + (\ldots)$$

$$\rightarrow (\ldots) = V \qquad E = -V_0$$

$$S_{JBB} = \int d\lambda \, 2\sqrt{T}\sqrt{E - V}$$

Jacobi Timeless


G'(q̇, q̇) = 4 · Σ_I ½ m_I (q̇^i_I)² = 4T
dq
V = V_0 + (...)
→ (...) = V   E = -V_0
S_Jacobi = ∫ dλ 2√T √(E-V)
Jacobi
timeless
Routhian Procedure:


V = V0 + (...)
→ (...) = V   E = - V0
S = ∫dλ 2√T √E - V
Jacobi
Routhian Procedure:
Srth = ∫dλ [I/q̇ - V q̇]

$$V = V_0 + (\dots)$$

$$\longrightarrow (\dots) = V \quad E = -V_0$$

$$S_{JMS} = \int d\lambda\, 2\sqrt{T}\sqrt{E-V}$$

Jacobi Finsler

Routhian Procedure:

$$S_{PPN} = \int d\lambda \left[\frac{I}{\dot{q}} - V\dot{q} \right]$$

$V = V_0 + (\ldots)$

$\longrightarrow (\ldots) = V \quad E = -V_0$

$$S_{Jacobi} = \int d\lambda\, 2\sqrt{T}\sqrt{E-V}$$

Jacobi Timeless

Routhian Procedure:

$$S_{eph} = \int d\lambda \left[\frac{I}{\dot q} - V\dot q \right]$$

$$E = \frac{I}{\dot q^2} + V$$

$$\dot q = \sqrt{\frac{I}{E-V}}$$

$$S_{PPN} = \int d\lambda \left[\frac{I}{\dot{q}^0} - V\dot{q}^0 \right]$$

$$E = \frac{I}{\dot{q}^{0\,2}} + V$$

$$\dot{q}^0 = \sqrt{\frac{I}{E-V}}$$

$$S_{PPN} \longrightarrow S = \int d\lambda \left[\frac{I}{\dot{q}^0} - \dot{q}^0 V + E\dot{q}^0 \right]_{\dot{q}^0 = \sqrt{\frac{I}{E-V}}}$$

Routhian Procedure:

$$S_{PPN} = \int d\lambda \left[\frac{I}{\dot{q}^0} - V\dot{q}^0 \right] \qquad E = \frac{I}{\dot{q}^{0\,2}} + V \qquad \dot{q}^0 = \sqrt{\frac{I}{E-V}}$$

$$S_{PPN} \longrightarrow \left\{ S_{JBN} = \int d\lambda \left[\frac{I}{\dot{q}^0} - \dot{q}^0 V + E\dot{q}^0 \right]_{\dot{q}^0 = \sqrt{\frac{I}{E-V}}} \right.$$

$$= \int d\lambda \left\{ \frac{I}{\sqrt{\frac{I}{E-V}}} - \sqrt{\frac{I}{E-V}}\,V + E\sqrt{\frac{I}{E-V}} \right\}$$

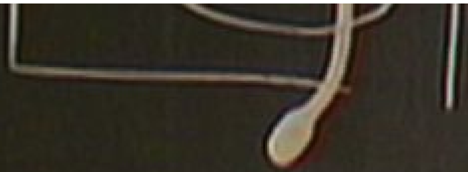
$$= \int d\lambda\, 2\sqrt{I}\sqrt{E-V}$$

$$S_{PPM} = \int \ldots \qquad \dot q^0 = \sqrt{\frac{I}{E-V}}$$

$$S_{PPM} \longrightarrow S_{JBM} = \int d\lambda \left[\frac{I}{\dot q^0} - \dot q^0 V + E \dot q^0 \right]_{\dot q^0 = \sqrt{\frac{I}{E-V}}}$$

$$= \int d\lambda \left\{ \frac{I}{\sqrt{\frac{I}{E-V}}} - \sqrt{\frac{I}{E-V}}\, V + E \sqrt{\frac{I}{E-V}} \right\}$$

$$= \int d\lambda\, 2 \sqrt{I} \sqrt{E-V}$$



Hamiltonian:

momenta: $p^I_i = \frac{\partial L_{BB}}{\partial \dot{q}^i_I} = \frac{\partial}{\partial \dot{q}^i_I}\left(2\sqrt{\frac{1}{2}\sum_J m_J (\dot{q}^i_J)^2}\,(E-V)^{\frac{1}{2}}\right)$

$$= \sqrt{\frac{E-V}{T}} \cdot m_I \dot{q}^i_I$$

Hamiltonian:

momenta: $$p^I_i = \frac{\partial L_{BB}}{\partial \dot{q}^i_I} = \frac{\partial}{\partial \dot{q}^i_I}\left(2\sqrt{\frac{1}{2}\sum_I m_I (\dot{q}^i_I)^2}\,(E-V)^{1/2}\right)$$

$$= \sqrt{\frac{E-V}{T}}\cdot m_I \dot{q}^i_I \qquad \left(\sqrt{\frac{E-V}{T}} = \dot{q}^0\right)$$

Hamiltonian:

momenta: $$p^i_I = \frac{\partial L_{BB}}{\partial \dot{q}^i_I} = \frac{\partial}{\partial \dot{q}^i_I}\left(2\sqrt{\frac{1}{2}\sum_I m_I (\dot{q}^i_I)^2}\,(E-V)^{1/2}\right)$$

$$= \sqrt{\frac{E-V}{T}} \cdot m_I \dot{q}^i_I \qquad \left(\sqrt{\frac{E-V}{T}} = \dot{q}^0\right)$$

Hamiltonian:

momenta: $\boxed{p_i^j} = \dfrac{\partial L_{BB}}{\partial \dot{q}_I^i} = \dfrac{\partial}{\partial \dot{q}_I^i}\left(2\sqrt{\frac{1}{2}\sum_J m_J (\dot{q}_J^i)^2}\,\left(E - V\right)^{1/2}\right)$

$\dfrac{\dot{q}_I^i}{\sqrt{\sum_J (\dot{q}_J^i)^2}} = \dfrac{\vec{\dot{q}}_I}{|\vec{\dot{q}}_I|}$

$= \boxed{\sqrt{\dfrac{E-V}{T}} \cdot m_I \dot{q}_I^i} \quad \left(\sqrt{\dfrac{E-V}{T}} = \dot{t}^0\right)$

Hamiltonian:

momenta: $\boxed{p^i_I} = \frac{\partial L_{BB}}{\partial \dot{q}^i_I} = \frac{\partial}{\partial \dot{q}^i_I}\left(2\sqrt{\frac{1}{2}\sum_J m_J(\dot{q}^i_J)^2}\,(E-V)^{1/2}\right)$

$\frac{\dot{q}^i_I}{\sqrt{\sum_J(\dot{q}^i_J)^2}} = \frac{\vec{\dot{q}}_I}{|\vec{\dot{q}}_I|}$ $\qquad = \sqrt{\frac{E-V}{T}}\cdot m_I \dot{q}^i_I \quad \left(\sqrt{\frac{E-V}{T}} = \dot{q}^{*i}\right)$

Hamiltonian:

momenta: $\boxed{p^i_I} = \frac{\partial L_{BB}}{\partial \dot q^i_I} = \frac{\partial}{\partial \dot q^i_I}\left(2\sqrt{\frac{1}{2}\sum_J m_J(\dot q^i_J)^2}\,(E-V)^{1/2}\right)$

$\frac{\dot q^i_I}{\sqrt{\sum_J(\dot q^i_J)^2}} = \frac{\vec{\dot q}_I}{|\vec{\dot q}_I|} = \cos(\ldots)$ $= \boxed{\sqrt{\frac{E-V}{T}}\cdot m_I\,\dot q^i_I}$ $\left(\sqrt{\frac{E-V}{T}} = \dot q^{0}\right)$

$$\mathcal{H} = \sum_i \frac{(p_i)^2}{2m_i} + V - E = 0$$

$$\mathcal{H} = \sum_i \frac{(p_i)^2}{2m_i} + V - E = 0$$

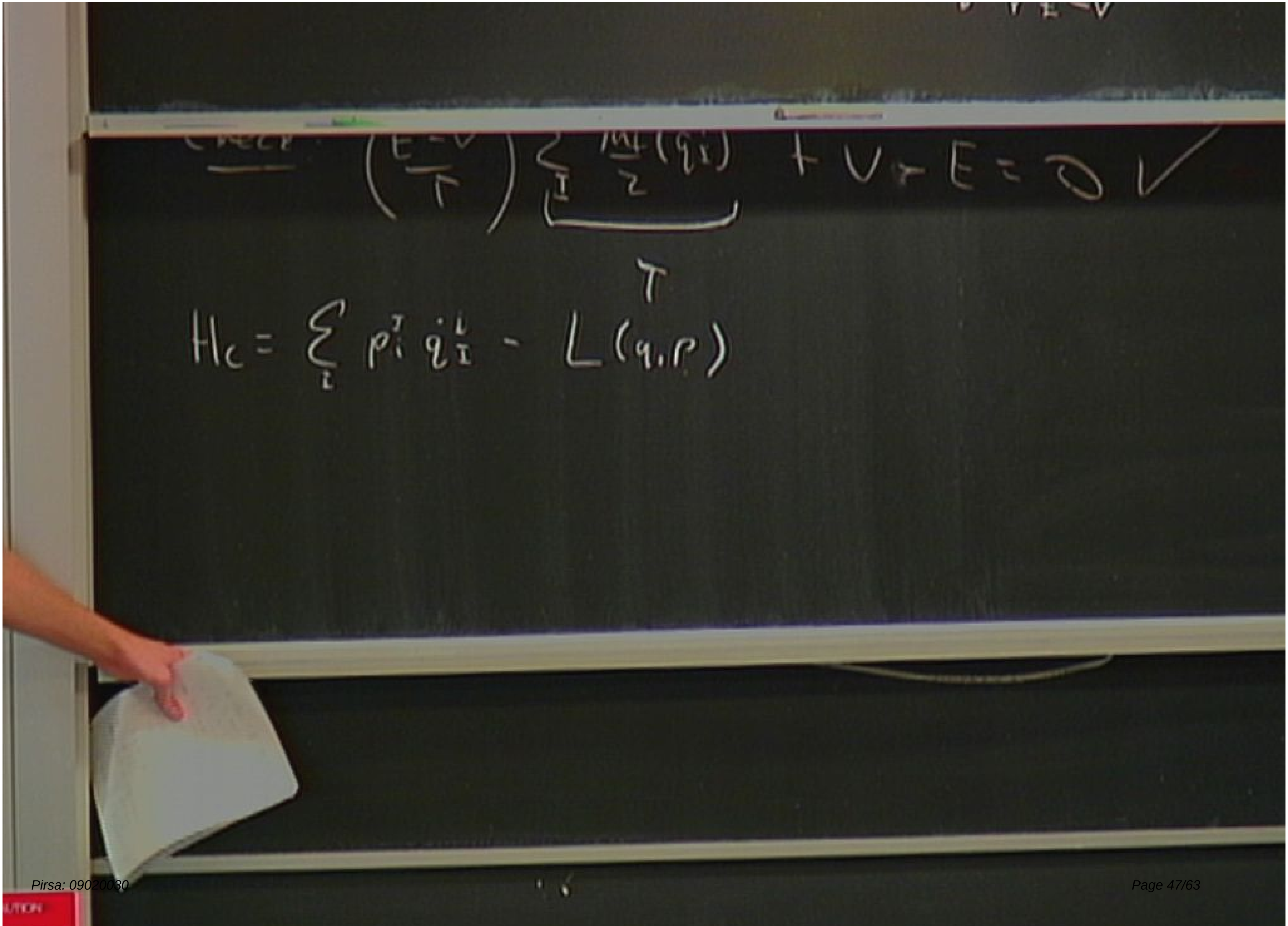
Check:

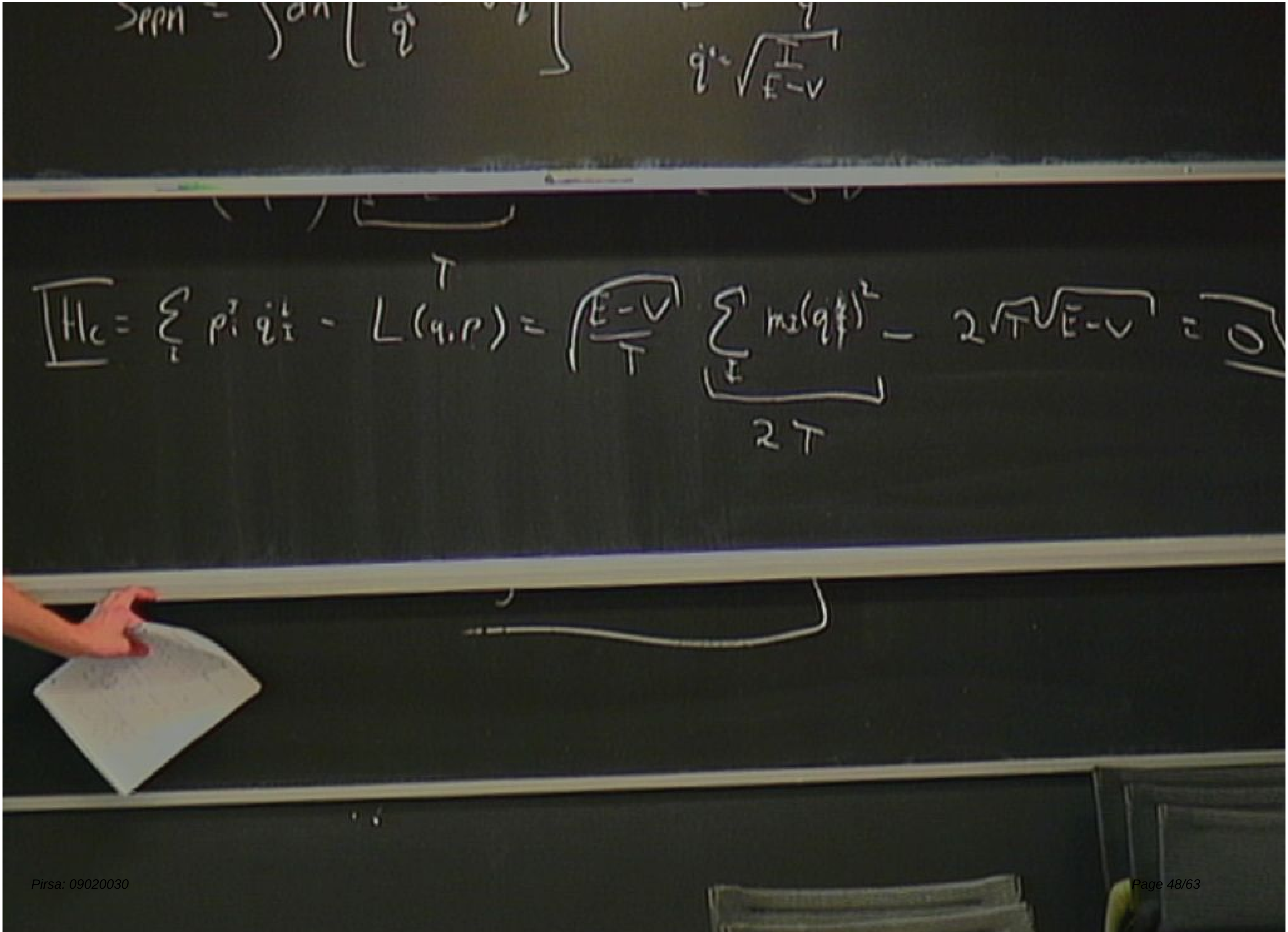
$$\mathcal{H} = \sum_{I} \frac{(p^i_I)^2}{2m_I} + V - E = 0$$

<u>Check</u>: $\left(\frac{E-V}{T}\right)\sum_{I}$

$$\mathcal{H} = \sum_i \frac{(p_i^i)^2}{2m_i} + V - E = 0$$

<u>Check</u>: $\left(\frac{E-V}{T}\right) \underbrace{\sum_I \frac{m_I}{2} (\dot{q}_I^i)^2}_{T} + V - E = 0 \checkmark$


$\left(\frac{E-V}{T}\right) \sum_I \frac{m_I}{2}(\dot{q}_I)^2 + V - E = 0$ ✓
T
$H_c = \sum_i p^i_i \dot{q}^i_I - L(q, p)$


$S_{ppn} = \int d\lambda \left[\ldots \right]$
$\dot{q}^i = \sqrt{\frac{I}{E-V}}$
$H_c = \sum_i p^i \dot{q}^i - L(q,p) = \sqrt{\frac{E-V}{T}} \sum_i m_i (\dot{q}^i)^2 - 2\sqrt{T}\sqrt{E-V} = 0$
T
2T

$S_{JBB} = \int d\lambda \left[\ldots \right]$ $\qquad \dot{q}^i = \sqrt{\frac{T}{E-V}}$

$$\mathcal{H} = \sum_i \frac{(p^i_i)^2}{2m_i} + V - E = 0$$

<u>Check:</u> $\left(\frac{E-V}{T}\right) \underbrace{\sum_i \frac{m_i}{2} (\dot{q}^i)^2}_{T} + V - E = 0$ ✓

$$\boxed{H_c = \sum_i p^i_i \dot{q}^i_i - L(q,p) = \sqrt{\frac{E-V}{T}} \underbrace{\sum_i m_i (\dot{q}^i)^2}_{2T} - 2\sqrt{T}\sqrt{E-V} = 0}$$

$$S_{PPN} = \int dx \left[\frac{1}{\dot{q}} \; \cdots \right] \qquad \dot{q} = \sqrt{\frac{T}{E-V}}$$

$$\mathcal{H} = \sum_i \frac{(p_i)^2}{2m_i} + V - E = 0$$

Check: $$\left(\frac{E-V}{T}\right) \underbrace{\sum_i \frac{m_i}{2}(\dot{q}^i)^2}_{T} + V - E = 0 \;\checkmark$$

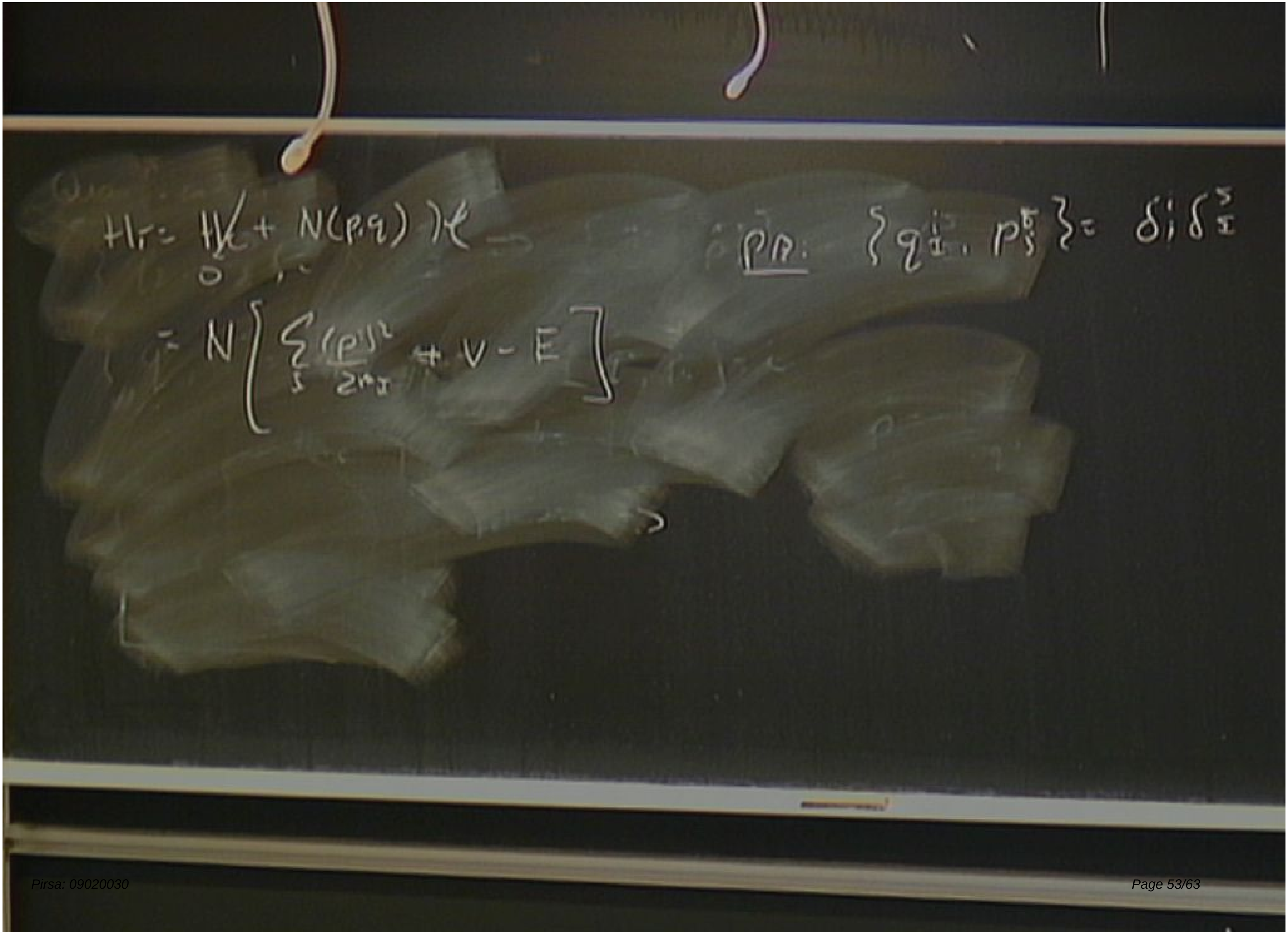
$$\boxed{H_c = \sum_i p_i \dot{q}^i - L(q,p) = \sqrt{\frac{E-V}{T}} \underbrace{\sum_i m_i (\dot{q}^i)^2}_{2T} - 2\sqrt{T}\sqrt{E-V} = 0}$$

$$H_T = \underbrace{H_K}_{0} + N(p,q)\,\mathcal{H}$$

$$= N\left[\sum_i \frac{(p^i)^2}{2m_I} + V - E\right]$$

$$H_T = \underbrace{H_c}_{0} + N(p,q)\, H_c$$

$$= N \left[\sum_i \frac{(p^i)^2}{2 m_i} + V - E \right]$$

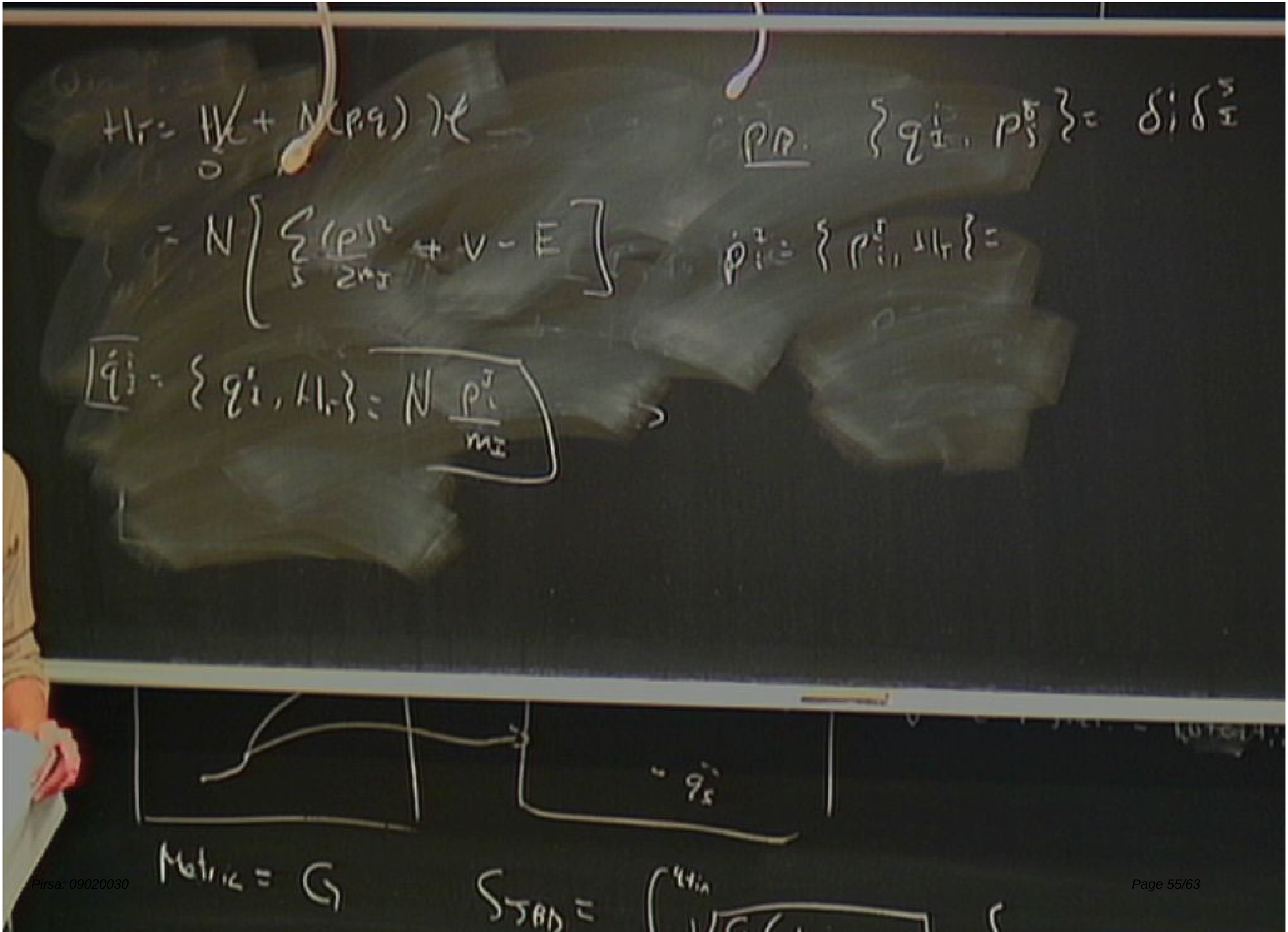

$H_T = H_0 + N(p,q)\, \mathcal{H}$
PR: $\{q^i_I, p^J_j\} = \delta_i \delta^J_I$
$= N \left[\sum_I \frac{(p_I)^2}{2m_I} + V - E \right]$

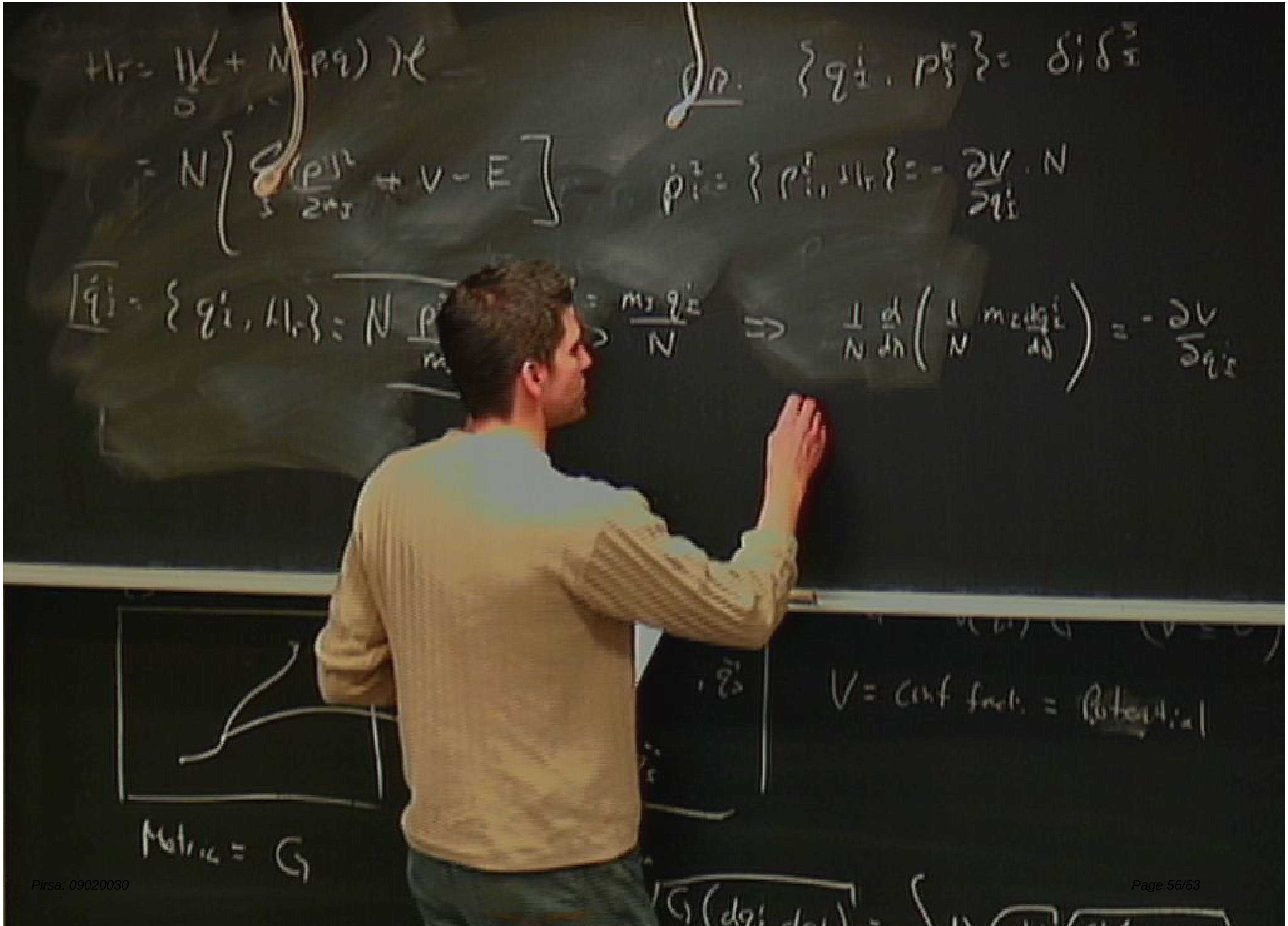
$$H_T = H_0 + N(p,q)\,\mathcal{H}$$

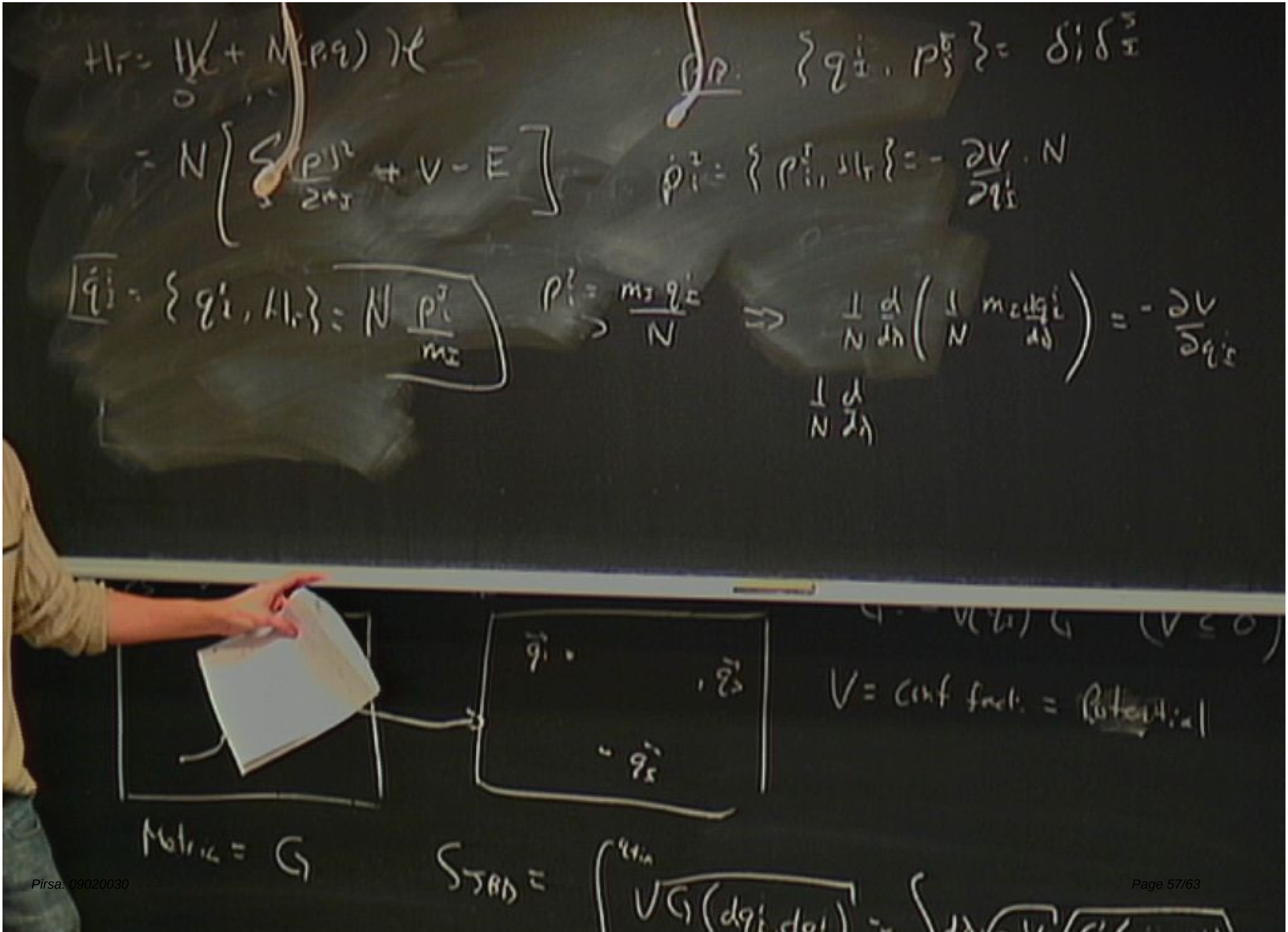
$$= N\left[\sum_I \frac{(p^I)^2}{2m_I} + V - E\right]$$

$$\boxed{\dot{q}^i} = \{q^i, H_T\} = N\,\frac{p^i}{m_I}$$

PB: $\{q^i_I, p^J_j\} = \delta^i_j\,\delta^J_I$


P.B.
Metric = G


V = conf fact. = Potential
Metric = G

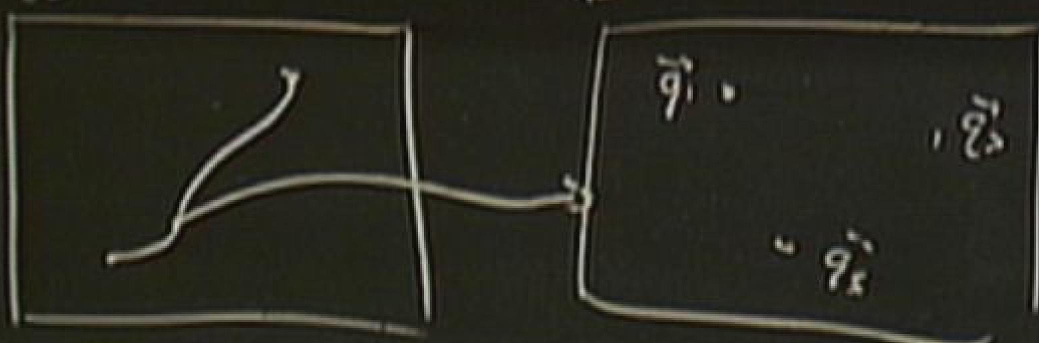

V = conf fact. = Potential
Metric = G

$$= N\left[\sum_i \frac{p_i^2}{2m_i} + V - E\right] \qquad \dot{p}_i = \{p_i, H_T\} = -\frac{\partial V}{\partial q_i}\cdot N$$

$$\boxed{\dot{q}_i} = \{q_i, H_T\} = N\frac{p_i}{m_i} \qquad p_i = \frac{m_i \dot{q}_i}{N} \Rightarrow \frac{1}{N}\frac{d}{d\lambda}\left(\frac{1}{N} m_i \frac{dq_i}{d\lambda}\right) = -\frac{\partial V}{\partial q_i}$$

$$\left(\frac{1}{N}\frac{d}{d\lambda} = \frac{d}{d\tau_{eph}}\right)$$

JBB. CS q_i $\qquad \mathbb{R}^2$

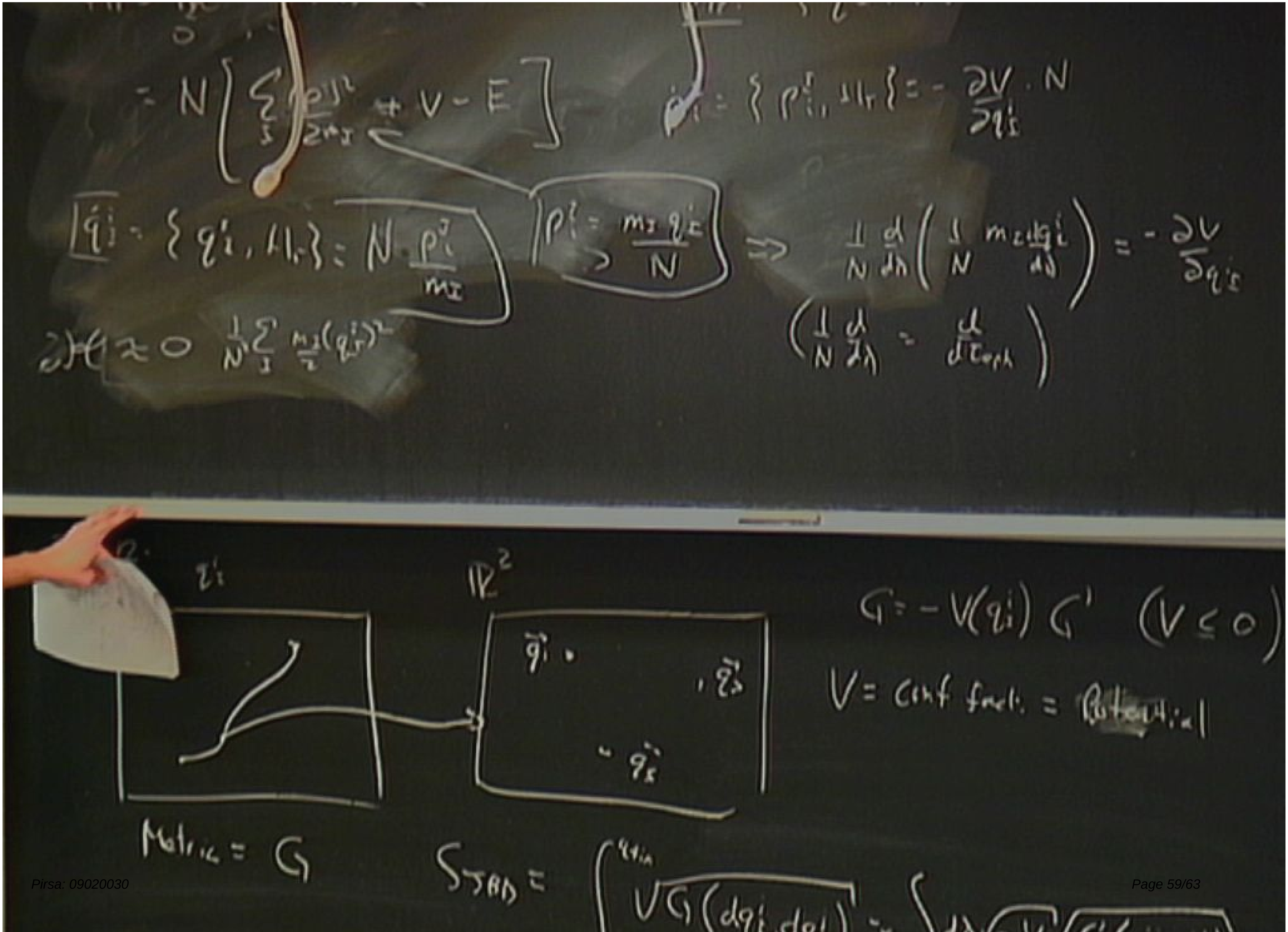


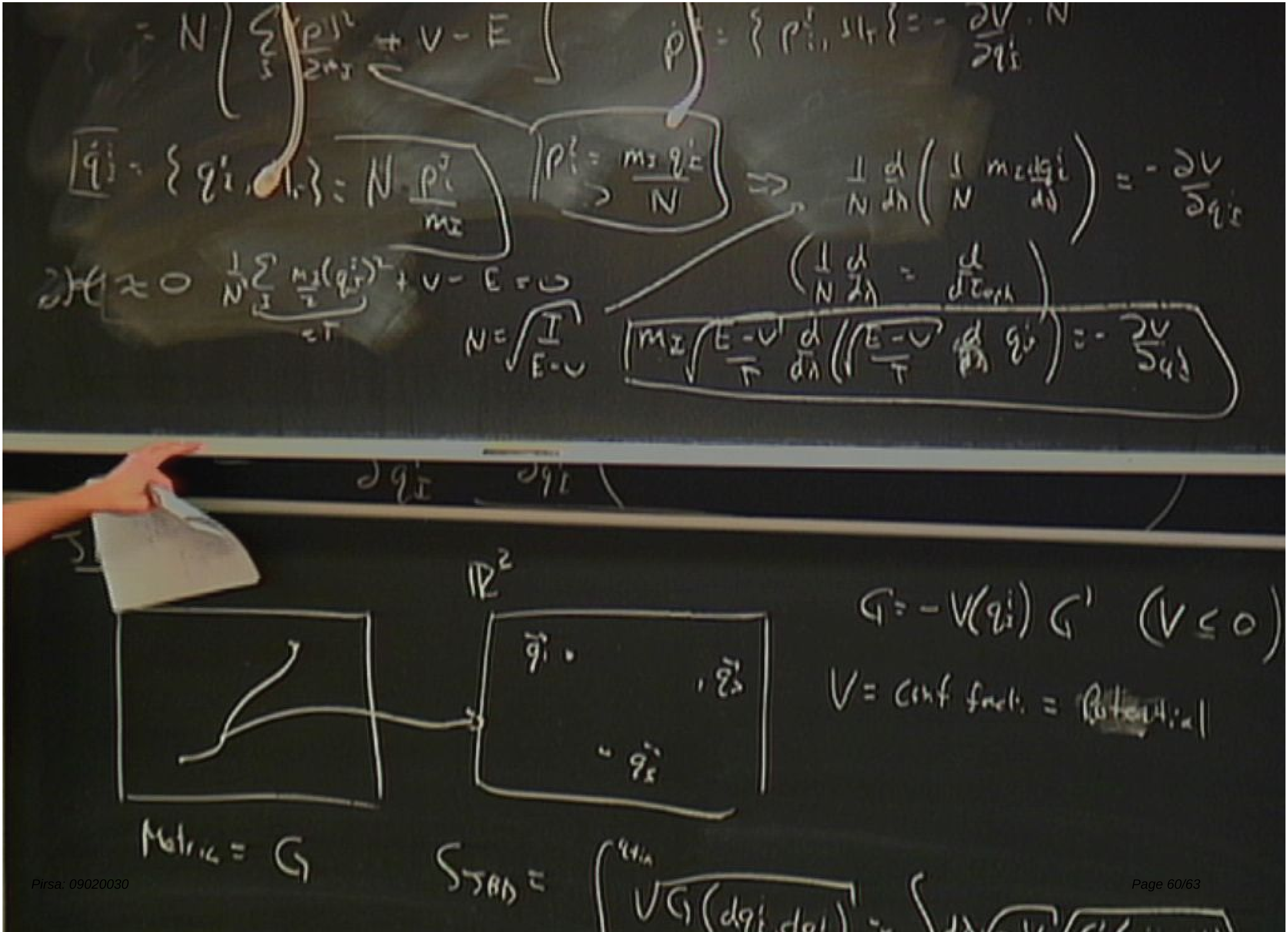
$G = -V(q_i)\, G' \quad (V \leq 0)$

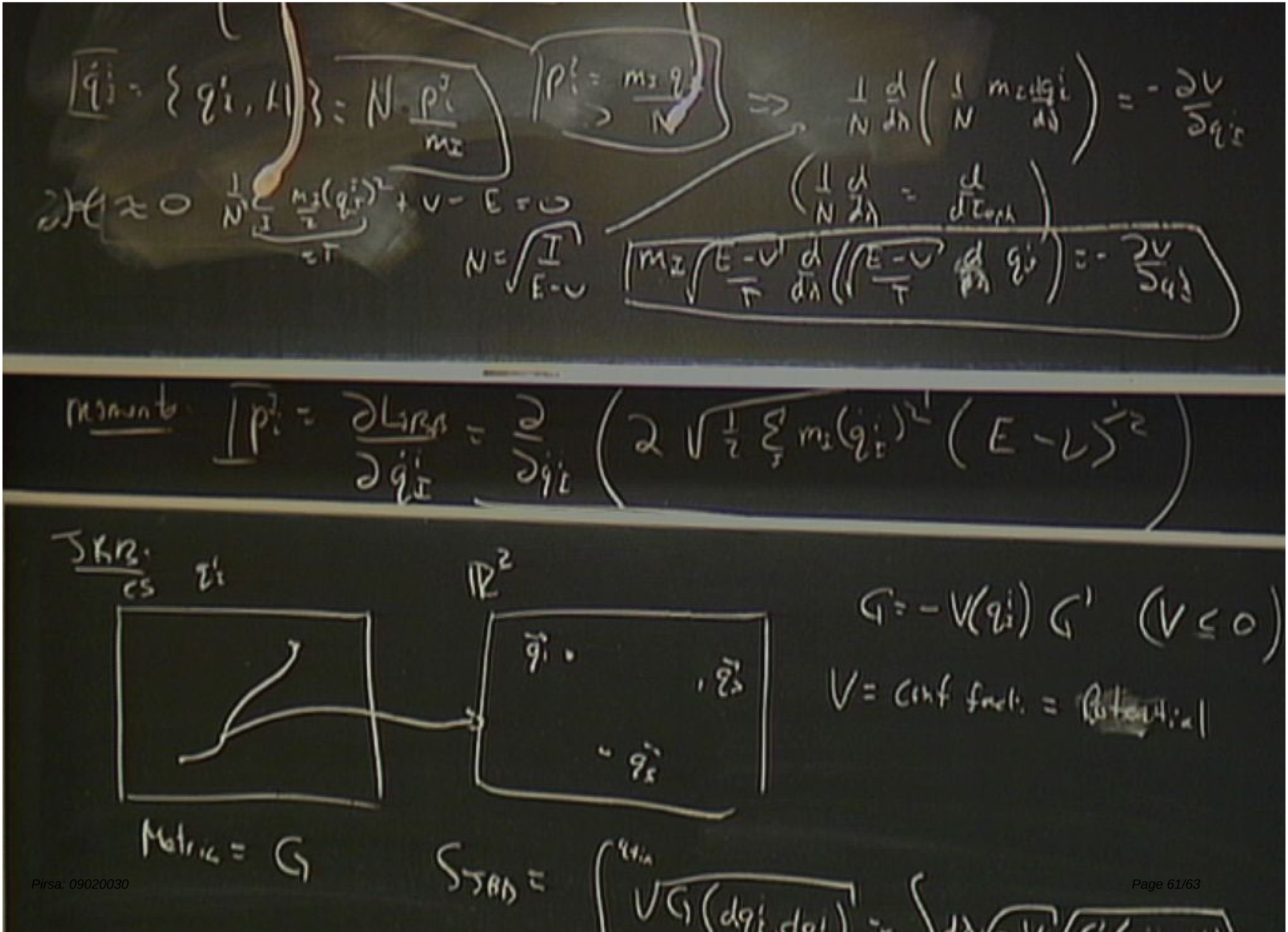
V = conf func. = Potential

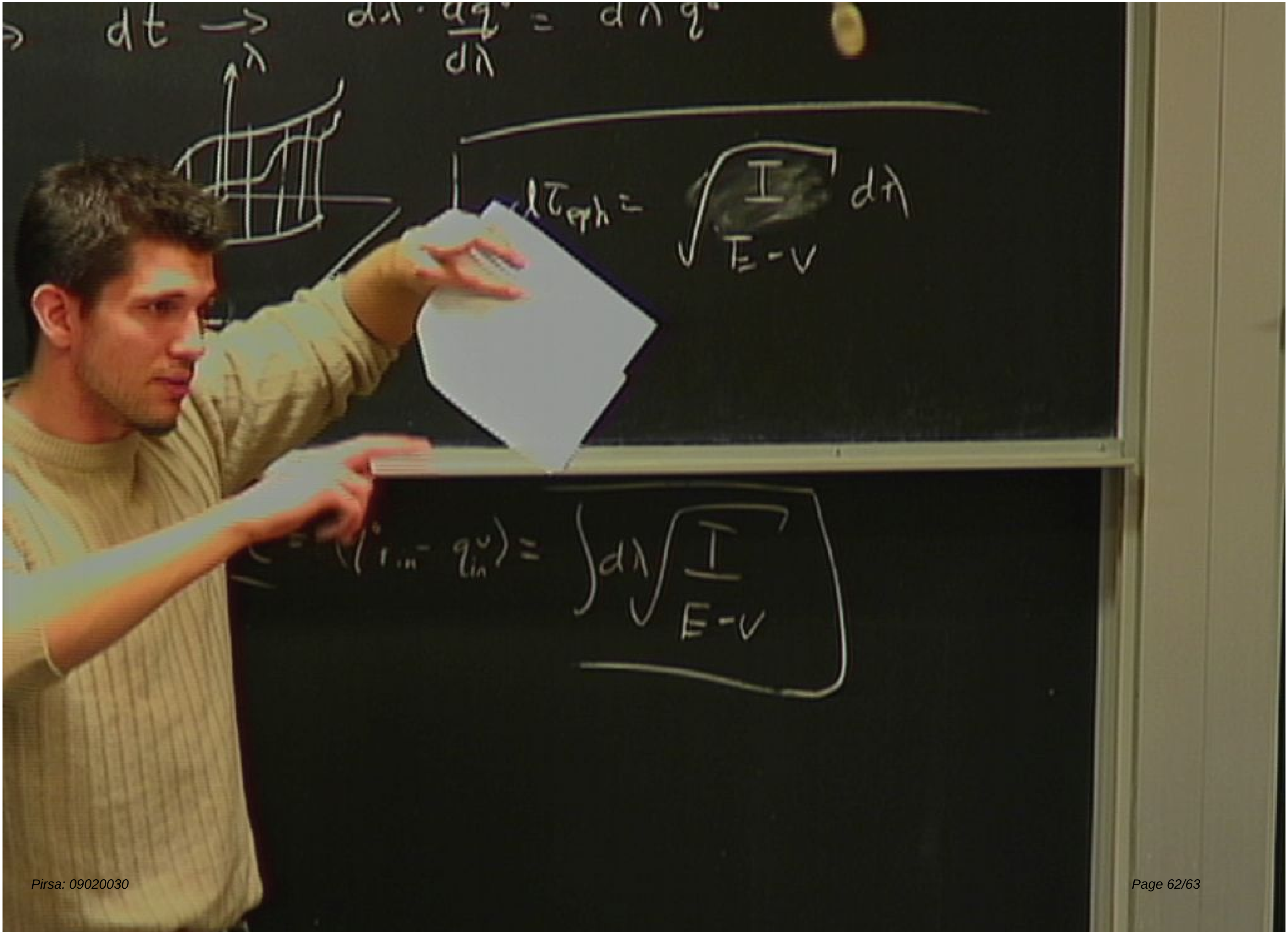
Metric = G

$$S_{JBB} = \int^{q_{fin}} \sqrt{G\left(dq_i\, dq_i\right)} = \int d\lambda \sqrt{-V}\sqrt{G'(\ldots)}$$


$= N \left[\sum_I \frac{p_I^2}{2m_I} + V - E \right]$
$\dot{p}^i_I = \{ p^i_I , H_T \} = -\frac{\partial V}{\partial q^i_I} \cdot N$
$\dot{q}^i_I = \{ q^i_I , H_T \} = N \frac{p^i_I}{m_I}$
$p^i_I = \frac{m_I \dot{q}^i_I}{N}$
$\Rightarrow \frac{1}{N}\frac{d}{d\lambda}\left(\frac{1}{N} m_I \frac{dq^i_I}{d\lambda} \right) = -\frac{\partial V}{\partial q^i_I}$
$\left(\frac{1}{N}\frac{d}{d\lambda} = \frac{d}{d\tau_{eph}} \right)$
$\mathbb{R}^2$
$G = -V(q^i_I)\, G'$ $(V \leq 0)$
V = Conf funct. = Potential
Metric = G


ℝ²
G = −V(qᵢ) G' (V ≤ 0)
V = conf funct. = Potential
Metric = G


momenta
Metric = G
V = conf. fact. = Potential
$\mathbb{R}^2$



$\rightarrow \quad dt \longrightarrow \quad d\lambda \cdot \frac{dq^0}{d\lambda} = d\lambda\, \dot{q}^0$

$$d\tau_{eph} = \sqrt{\frac{I}{E-V}}\, d\lambda$$

$$\tau = (q^0_{fin} - q^0_{in}) = \int d\lambda \sqrt{\frac{I}{E-V}}$$