

Title: Mini-Course on Mach's Principle - Lecture 2

Date: Feb 11, 2009 10:30 AM

URL: <http://pirsa.org/09020030>

Abstract:

Barbour's draft: Mach's Principle, GR, & Gauge Theory
(email me: sean.gryb@gmail.com)

Arxiv: 0810.4152

Barbour's draft: Mach's Principle, GR, & Gauge Theory
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Recall:
CS $g^{\mu\nu}$



(email me: sean.gryb@gmail.com)

Arxiv: 0810.4152

Recall: $CS \int_{\Sigma} q^i \Rightarrow q^0$



$$S_H \rightarrow dt \rightarrow d\lambda \cdot \frac{dq^i}{d\lambda} =$$

Barbour's draft: Mach's Principle, GR, & Gauge Theory
(email me: sean.grubb@gmail.com)

Arxiv: 0810.4152

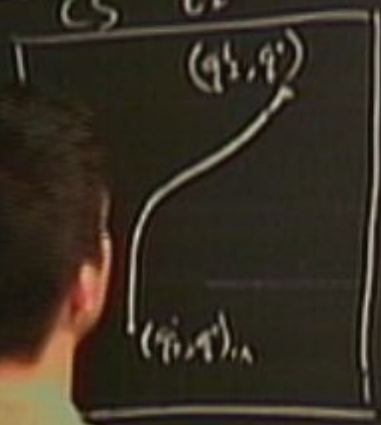
Recall:
CS $q_i^e \Rightarrow q^o$

$$S_H \rightarrow dt \rightarrow d\lambda \cdot \frac{dq^o}{d\lambda} = d\lambda \dot{q}^o$$

Barbour's draft: Mach's Principle, GR, & Gauge Theory
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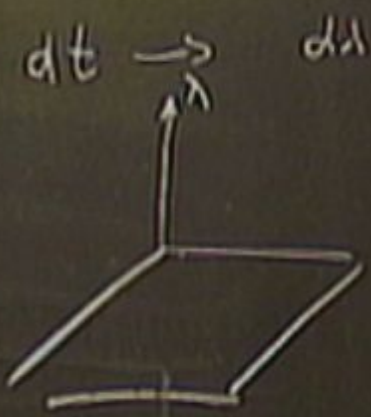
Arxiv: 0810.4152

Recall: $CS \int_{\Sigma} \dot{q}^i \Rightarrow q^0$



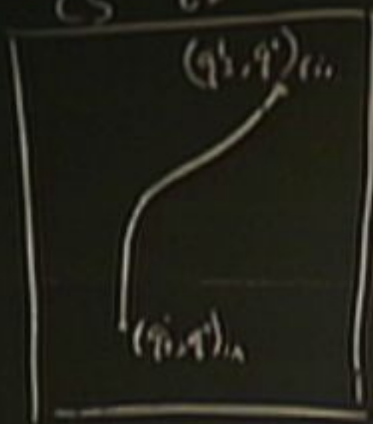
$S_H \rightarrow dt$

$\lambda \rightarrow f(\lambda)$



$$d\lambda \cdot \frac{dq^i}{d\lambda} = d\lambda \dot{q}^i$$

Recall CS $q^i \Rightarrow q^0$



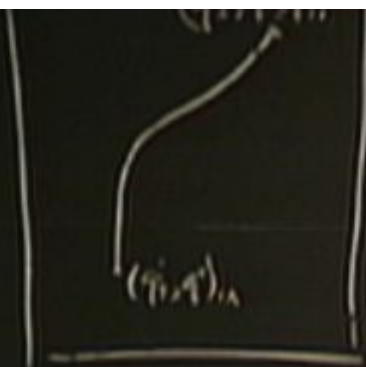
$S_N \rightarrow dt$

$\rightarrow d\lambda$

$$d\lambda \cdot \frac{dq^i}{d\lambda} = d\lambda \dot{q}^i$$

$\lambda \rightarrow f(\lambda)$





$$\lambda \rightarrow f(\lambda)$$



$$K = \dots + p + v = 0$$

$$H_0 = \dots \quad H_T = N X$$

$\delta q_i \delta \Rightarrow$ Newton's Law (2^o)

$$\delta q_i \delta \Rightarrow E = T' + U$$

$$T = (q'_{i..} - q^v) = \int d\lambda \sqrt{\frac{I}{E-v}}$$



$$\lambda \rightarrow f(\lambda)$$



$$\mathcal{H} = \sum_j \frac{(p_j^i)^2}{2m_j} + p_0 + V = 0$$

$\delta q_i \delta t \Rightarrow$ Newton's Law (2^o)

$$H_C = 0$$

$$H_T = N \mathcal{H}$$

$$\delta q_i \delta t \Rightarrow E = T' + U$$

$$T = (q_{i, in}^i - q_{i, out}^i) = \int d\lambda \sqrt{\frac{I}{E - V}}$$

- Quantize PPM

- Jacobi B15

Quantization:

$$\{q_I^i, p_J^{\alpha}\} = \delta^i_{\alpha} \delta^J_I \Rightarrow \{\hat{q}_I^i, \hat{p}_J^{\alpha}\} = i \delta^i_{\alpha} \delta^J_I$$

Quantization:

$$\{q_I^i, p_J^{\alpha}\} = \delta^i_{\alpha} \delta^J_I \Rightarrow \{\hat{q}_I^i, \hat{p}_J^{\alpha}\} = i \delta^i_{\alpha} \delta^J_I$$

$$\{q^0, p_0\} = \# \Rightarrow [\hat{q}^0, \hat{p}_0] = i$$

Quantization:

$$\{q_I^i, p_J^j\} = \delta^i_j \delta_{IJ} \Rightarrow \{\hat{q}_I^i, \hat{p}_J^j\} = i \delta^i_j \delta_{IJ}$$

$$\{q^0, p_0\} = \# \Rightarrow [\hat{q}^0, \hat{p}_0] = i$$



Quantization:

$$\{q_I^i, p_J^{\alpha}\} = \delta^i_{\alpha} \delta^J_I \Rightarrow \{\hat{q}_I^i, \hat{p}_I^{\alpha}\} = i \delta^i_{\alpha} \delta^J_I$$

$$\{q^0, p_0\} = \# \Rightarrow [\hat{q}^0, \hat{p}_0] = i$$

$$\psi = \psi(q_i, p_i)$$

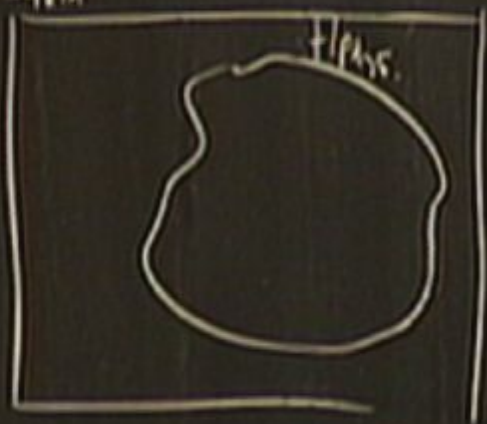


Quantization:

$$\{q_I^i, p_J^j\} = \delta^i_j \delta_{IJ} \Rightarrow [\hat{q}_I^i, \hat{p}_J^j] = i \delta^i_j \delta_{IJ}$$

$$\{q^0, p_0\} = \# \Rightarrow [\hat{q}^0, \hat{p}_0] = i$$

$$\psi = \psi(q_i, p_i)$$



Quantization:

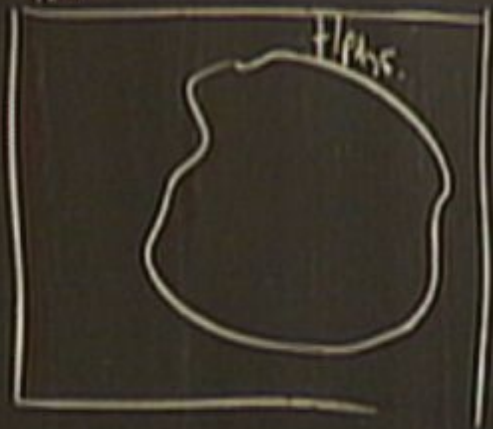
$$\{q_i^i, p_j^i\} = \delta_{ij} \delta_{ij}^T \Rightarrow \{\hat{q}_i^i, \hat{p}_j^i\} = \delta_{ij} \delta_{ij}^T$$

$$\{q^0, p_0\} = \# \Rightarrow [\hat{q}^0, \hat{p}_0] = i$$

$$\psi = \psi(q^i, q_i)$$

$$\hat{p}_0 = -i \frac{\partial}{\partial q^0}$$

$$\hat{q}^i = q^i$$



$$\hat{H}|\psi\rangle = 0 \Rightarrow$$

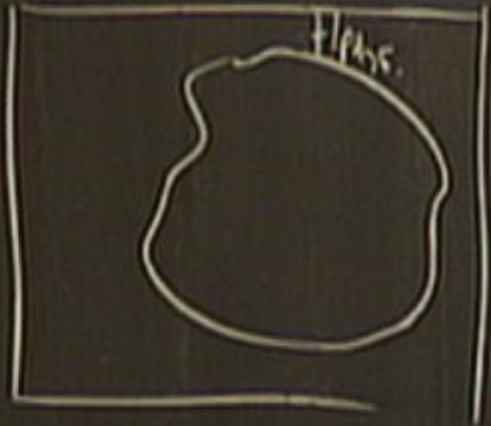
Quantization:

$$\{q_I^i, p_I^j\} = \delta^i \delta^j \Rightarrow \{\hat{q}_I^i, \hat{p}_I^j\} = \delta^i \delta^j$$

$$\{q^0, p_0\} = \# \Rightarrow [\hat{q}^0, \hat{p}_0] = i$$

$$\psi = \psi(q^i, q_i)$$

$$\hat{p}_0 = -i \frac{\partial}{\partial q^0}$$
$$\hat{q}^0 = q^0$$

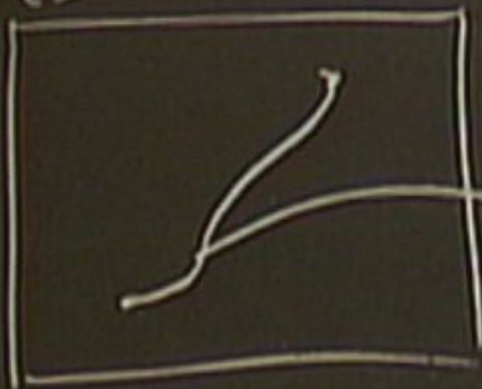


$$\hat{H}|\psi\rangle = 0 \Rightarrow$$

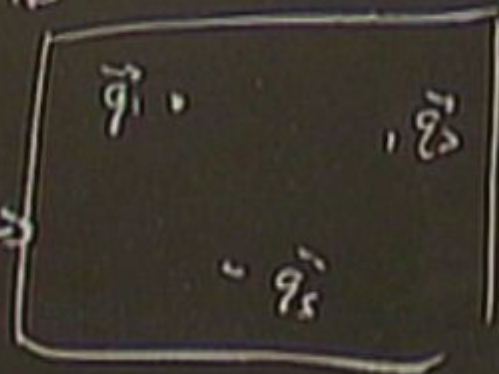
JKB.
CS



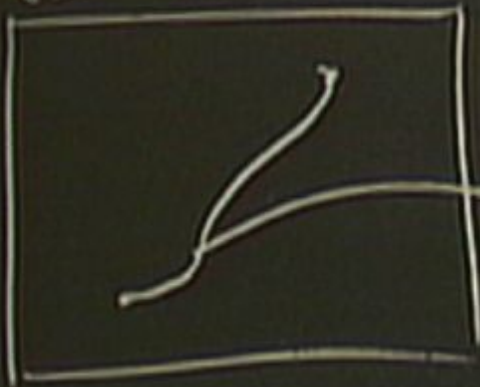
JKB:
CS \mathbb{R}^2



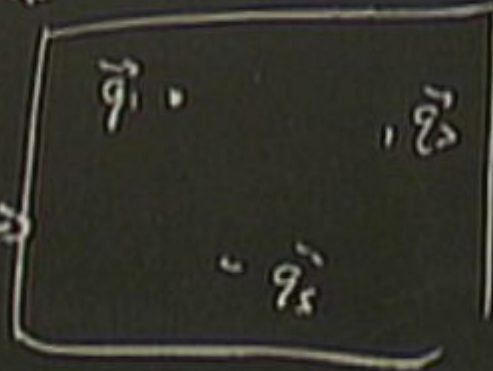
\mathbb{R}^2



JKB: \mathbb{R}^2



\mathbb{R}^2



Metric = G

JKB:
CS \vec{e}_i



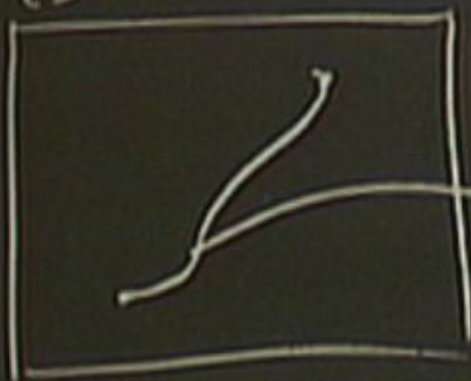
Metric = G

\mathbb{R}^2



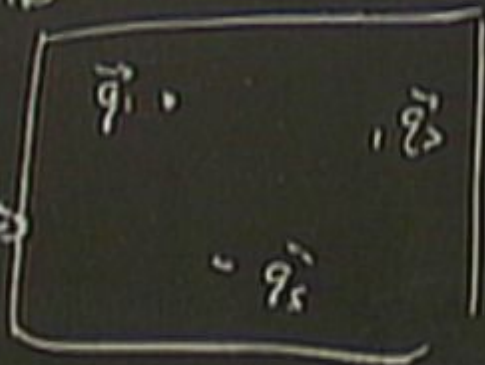
$$S_{\text{JKB}} = \int_{q_h}^{q_h} \sqrt{G(dq_i, dq_j)}$$

SKB:
CS z_i



Metric = G

\mathbb{R}^2

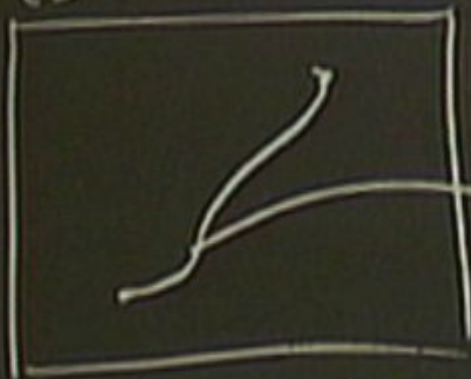


$$G = -V(q_i) G' \quad (V \leq 0)$$

$V = \text{conf fact.} = \text{Potential}$

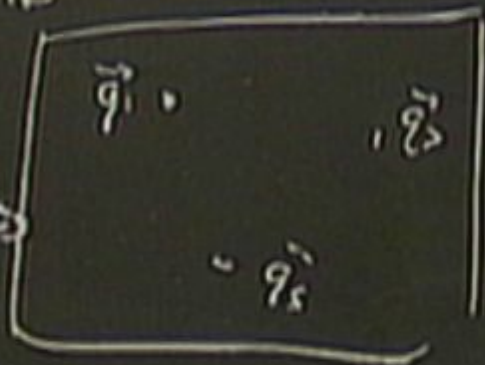
$$S_{\text{KB}} = \int_{q_n}^{q_{fin}} \sqrt{G_1 (dq_i, dq_j)}$$

JKB:
CS z_i



Metric = G

\mathbb{R}^2



$$G = -V(q_i) G' \quad (V \leq 0)$$

$V = \text{conf fact.} = |\text{potential}|$

$$S_{\text{KB}} = \int_{q_n}^{q_{fin}} \sqrt{G} (dq_i, dq_j)$$

$$G'(\dot{q}, \dot{q}) = 4 \cdot \sum_I \frac{1}{2} m_i (\dot{q}_i^I)^2 = 4T$$

Metric = G

$S_{\text{ADM}} = \int_{q_h}^{q_{\text{fin}}} \sqrt{G(q_i, dq_i)} \sim \int d\lambda \sqrt{-V \sqrt{G'(q_i, q_i)}}$

$$Q'(q, \dot{q}) = 4 \cdot \sum_I \frac{1}{2} m_i (\dot{q}_i)^2 = \underline{4T}$$

$$V = V_0 + (\dots) \Rightarrow$$

$$\rightarrow (\dots) = V \quad E = -V_0$$



$$Q'(i, i) = 4 \cdot \sum_i \frac{1}{2} m_0 (\dot{q}_i')^2 = \overline{4T}$$

$$V = V_0 + (\dots) \Rightarrow$$

$$\rightarrow (\dots) = V \quad E = -V_0$$

$$\int_{\text{STATS}} = \int d\lambda \, 2\sqrt{T} \sqrt{E - V}$$

$$Q'(q, \dot{q}) = 4 \cdot \sum_i \frac{1}{2} m_i (\dot{q}_i)^2 = \underline{4T}$$

$$V = V_0 + (\dots) \Rightarrow$$

$$\rightarrow (\dots) = V \quad E = -V_0$$

$$\int_{S_{TAS}} = \int d\lambda \, 2\sqrt{T} \sqrt{E - V}$$

Jacobi

CAUTION

$$Q'(q, \dot{q}) = 4 \cdot \sum_i \frac{1}{2} m_0 (\dot{q}_i)^2 = \overbrace{4T}$$

$$V = V_0 + (\dots) \Rightarrow$$

$$\rightarrow (\dots) = V \quad E = -V_0$$

$$\int_{S_{MS}} = \int d\lambda \, 2\sqrt{T} \sqrt{E - V}$$

Jacobi ~~kinetic~~

$$G'(\dot{q}, q) = 4 \cdot \sum_i \frac{1}{2} m_i (\dot{q}_i)^2 = \underbrace{4T}_{dq}$$

$$\left. \begin{aligned}
 V &= V_0 + (\dots) \Rightarrow \\
 \rightarrow (\dots) &= V \quad E = -V_0
 \end{aligned} \right\} \int_{S_{MS}} = \int dq \, 2\sqrt{T} \sqrt{E - V}$$

↙ Jacobi ↘ Minimus



$$G'(\dot{q}, q) = 4 \cdot \sum_I \frac{1}{2} m_i (\dot{q}_i')^2 = 4T \quad dq$$

$$V = V_0 + (\dots) \Rightarrow$$

$$\rightarrow (\dots) = V \quad E = -V_0$$

$$\int_{S_1}^{S_2} ds = \int dq \, 2\sqrt{T} \sqrt{E - V}$$

Jacobi timeless

Routhian Procedure:

$$C_1(q, \dot{q}) = \frac{1}{2} \dot{q}^T M \dot{q} + \dots$$

$$V = V_0 + (\dots) \Rightarrow$$

$$\rightarrow (\dots) = V \quad E = -V_0$$

$$S_{\text{sys}} = \int dt \, 2\sqrt{T}\sqrt{E-V}$$

Jacobi Timeless

Routhian Procedure:

$$S_{\text{rpm}} = \int dt \left[\frac{1}{2} \dot{q}^2 - V(q) \right]$$

$$C_1(q, \dot{q}) = \frac{1}{2} \dot{q}^2 + \dots$$

$$V = V_0 + (\dots) \Rightarrow$$

$$\rightarrow (\dots) = V \quad E = -V_0$$

$$S_{\text{sys}} = \int dt \, 2\sqrt{T}\sqrt{E-V}$$

Jacobi timeless

Routhian Procedure:

$$S_{\text{RPN}} = \int dt \left[\frac{1}{2} \dot{q}^2 - V(q) \right]$$

$$V = V_0 + (\dots) \Rightarrow$$

$$\rightarrow (\dots) = V \quad E = -V_0$$

$$S_{\text{sep}} = \int d\lambda \, 2\sqrt{T}\sqrt{E-V}$$

Jacobi ~~Final~~

Routhian Procedure:

$$S_{\text{sep}} = \int d\lambda \left[\frac{I}{2} \dot{q}^2 - V(q) \right]$$

$$E = \frac{I}{2} \dot{q}^2 + V$$

$$\dot{q} = \sqrt{\frac{2}{I}(E-V)}$$

$$S_{ppn} = \int d\lambda \left[\frac{I}{\dot{q}} - v \dot{q} \right]$$

$$E \Rightarrow \frac{I}{\dot{q}^2} + v$$
$$\dot{q} = \sqrt{\frac{I}{E-v}}$$

$$S_{ppn} \rightarrow S = \int d\lambda \left[\frac{I}{\dot{q}} - \dot{q} v + E \dot{q} \right]_{\dot{q} = \sqrt{\frac{I}{E-v}}}$$

Koutherian Procedure.

$$S_{ppn} = \int d\lambda \left[\frac{I}{\dot{q}} - v \dot{q} \right]$$

$$E = \frac{I}{\dot{q}^2} + v$$
$$\dot{q} = \sqrt{\frac{I}{E-v}}$$

$$S_{ppn} \rightarrow \left[S_{ppn} = \int d\lambda \left[\frac{I}{\dot{q}} - \dot{q}v + E\dot{q} \right] \right. \\ \left. \dot{q} = \sqrt{\frac{I}{E-v}} \right]$$
$$= \int d\lambda \left[\frac{I}{\sqrt{\frac{I}{E-v}}} - \sqrt{\frac{I}{E-v}}v + E\sqrt{\frac{I}{E-v}} \right]$$
$$= \int d\lambda \left[2\sqrt{Iv(E-v)} \right]$$

$S_{ppm} = \int d\lambda \left[\dot{q}^0 \right]$
 $\dot{q}^0 = \sqrt{\frac{I}{E-v}}$

$$\begin{aligned}
 S_{ppm} &\rightarrow \int_{\lambda_1}^{\lambda_2} d\lambda \left[\dot{q}^0 - \dot{q}^0 v + E \dot{q}^0 \right] \quad \dot{q}^0 = \sqrt{\frac{I}{E-v}} \\
 &= \int d\lambda \left[\sqrt{\frac{I}{E-v}} - \sqrt{\frac{I}{E-v}} v + E \sqrt{\frac{I}{E-v}} \right] \\
 &= \int d\lambda z \sqrt{TE-v}
 \end{aligned}$$

Hamiltonian:

momenta:
$$p_i^j = \frac{\partial L_{\text{RHS}}}{\partial \dot{q}_i^j} = \frac{\partial}{\partial \dot{q}_i^j} \left(2 \sqrt{\frac{1}{2} \sum_j m_j (\dot{q}_j^i)^2} (E - V) \right)$$
$$= \sqrt{\frac{E - V}{1}} \cdot m_j \dot{q}_i^j$$

Hamiltonian:

momenta:

$$p_i^j = \frac{\partial L_{\text{rel}}}{\partial \dot{q}_i^j} = \frac{\partial}{\partial \dot{q}_i^j} \left(2 \sqrt{\frac{1}{2} \sum m_i (\dot{q}_i^j)^2} (E - V) \right)$$
$$= \sqrt{\frac{E - V}{1}} \cdot m_i \dot{q}_i^j \quad \left(\sqrt{\frac{E - V}{1}} = \dot{q}_i^0 \right)$$

Hamiltonian:

momenta: $\overline{p}_i = \frac{\partial L_{\text{rel}}}{\partial \dot{q}_i} = \frac{\partial}{\partial \dot{q}_i} \left(2 \sqrt{\frac{1}{2} \sum_j m_j (\dot{q}_j^i)^2} (E - V) \right)$

$$= \sqrt{\frac{E - V}{1}} \cdot m_i \dot{q}_i \quad \left(\sqrt{\frac{E - V}{1}} = \dot{q}_0 \right)$$

Hamiltonian:

momenta: $\vec{p}_i = \frac{\partial L_{\text{rel}}}{\partial \dot{q}_i} = \frac{\partial}{\partial \dot{q}_i} \left(2 \sqrt{\frac{1}{2} \sum m_i (\dot{q}_i)^2} (E - V) \right)$

$$\frac{\dot{q}_i}{\sqrt{\sum (\dot{q}_i)^2}} = \frac{\vec{q}_i}{|\vec{q}_i|} = \sqrt{\frac{E - V}{T}} \cdot m_i \dot{q}_i \quad \left(\sqrt{\frac{E - V}{T}} = \dot{q}_0 \right)$$

Hamiltonian:

momenta: $\vec{p}_i = \frac{\partial L_{\text{rel}}}{\partial \dot{q}_i} = \frac{\partial}{\partial \dot{q}_i} \left(2 \sqrt{\frac{1}{2} \sum_j m_j (\dot{q}_j)^2} (E - V) \right)$

$$\frac{\dot{q}_i}{\sqrt{\sum_j (\dot{q}_j)^2}} = \frac{\vec{q}_i}{|\vec{q}_i|} = \sqrt{\frac{E - V}{T}} \cdot m_i \dot{q}_i \quad \left(\sqrt{\frac{E - V}{T}} = \dot{q}_i' \right)$$

Hamiltonian:

momenta: $\vec{p}_i = \frac{\partial L_{\text{res}}}{\partial \dot{q}_i} = \frac{\partial}{\partial \dot{q}_i} \left(2 \sqrt{\frac{1}{2} \sum_j m_j (\dot{q}_j)^2} (E - V) \right)$

$$\frac{\dot{q}_i}{\sqrt{\sum_j (\dot{q}_j)^2}} = \frac{\vec{q}_i}{|\vec{q}_i|} = \cos(\dots) = \sqrt{\frac{E-V}{T}} \cdot m_i \dot{q}_i \quad \left(\sqrt{\frac{E-V}{T}} = \dot{q}_i' \right)$$

$$\mathcal{H} = \sum_i \frac{(p_i)^2}{2m_i} + V - E = 0$$

$$\mathcal{H} = \sum_i \frac{(p_i)^2}{2m_i} + V - E = 0$$

check:

$$\mathcal{H} = \sum_{\mathbf{r}} \frac{(p_{\mathbf{r}}^i)^2}{2m_{\mathbf{r}}} + V - E = 0$$

check: $\left(\frac{E - V}{\hbar} \right) \sum_{\mathbf{r}}$

$$\mathcal{H} = \sum_I \frac{(p_i^I)^2}{2m_I} + V - E = 0$$

check: $\left(\frac{E - V}{T} \right) \underbrace{\sum_I \frac{m_I}{2} (\dot{q}_i^I)^2}_T + V - E = 0 \quad \checkmark$

CHECK: $\left(\frac{E - V}{T} \right) \left[\sum_I \frac{m_I (q_i)}{2} \right] + V - E = 0 \quad \checkmark$

$$H_c = \sum_I p_i^T q_i^L - L(q, p)$$

$$p_{\text{eff}} = \sum_i p_i \left(\frac{q_i}{q} \right) \quad q = \sqrt{\frac{H}{E-V}}$$

$$H_c = \sum_i p_i^2 q_i^2 - L(q, p) = \sqrt{\frac{E-V}{T}} \underbrace{\sum_i m_i (q_i^2)^2}_{2T} - 2\sqrt{T}\sqrt{E-V} = 0$$

$$\text{ppm} = \left[\frac{1}{2} \dot{q}_i^2 \right] \quad \dot{q}_i = \sqrt{\frac{2(E-V)}{m}}$$

$$\mathcal{L} = \sum_i \frac{(p_i)^2}{2m_i} + V - E = 0$$

check: $\left(\frac{E-V}{T} \right) \sum_i \frac{m_i (\dot{q}_i)^2}{2} + V - E = 0 \checkmark$

$$\boxed{H_c} = \sum_i p_i \dot{q}_i - L(q, p) = \sqrt{\frac{E-V}{T}} \underbrace{\sum_i m_i (\dot{q}_i)^2}_{2T} - 2\sqrt{T}\sqrt{E-V} = 0$$

$$S_{ppm} = \int \dot{q}^i$$

$$\dot{q}^i = \sqrt{\frac{2}{E-V}}$$

$$H_c = \sum_i \frac{p_i^2}{2m_i} + V = E = 0$$

check: $\left(\frac{E-V}{T}\right) \sum_i \frac{m_i (\dot{q}_i)^2}{2} + V = E = 0 \checkmark$

$$H_c = \sum_i p_i^2 q_i^{\frac{1}{2}} - L(q, p) = \sqrt{\frac{E-V}{T}} \underbrace{\sum_i m_i (q_i^{\frac{1}{2}})^2}_{2T} - 2\sqrt{T}\sqrt{E-V} = 0$$

$$H_1 = \frac{H_0}{0} + N(p, q) \quad \text{K}$$

$$= N \left[\sum_{j=1}^n \frac{(p_j)^2}{2^{n_j}} + v - \text{FI} \right]$$

$$H_T = H_K + N(p, q) \psi$$

$$= N \left[\sum_i \frac{(p_i)^2}{2m} + V - \mu \right]$$

$$H_T = \frac{1}{2} \sum_i \dot{q}_i^2 + N(p, q) \mathcal{H}$$

$$\text{P.A. } \{q_i^i, p_j^j\} = \delta_i^j \delta_{ij}^s$$

$$= N \left[\sum_i \frac{(p_i)^2}{2m} + V - E \right]$$

$$H_T = H_0 + N(p, q) \mathcal{H}$$

$$= N \left[\sum_i \frac{(p_i)^2}{2m_i} + V - E \right]$$

$$[q_i] = \{ q_i, H_T \} = N \frac{p_i}{m_i}$$

$$[p_i, q_j] = \delta_{ij} \delta_{ij}$$

$$H_i = \frac{1}{2} \dot{q}_i^2 + N(p, q) \mathcal{H}$$

PA. $\{q_i^j, p_j^k\} = \delta_i^j \delta_k^l$

$$= N \left[\sum_j \frac{(p_j^j)^2}{2m_j} + V - E \right]$$

$$\dot{p}_i^j = \{p_i^j, H_i\} =$$

$$\boxed{\dot{q}_i^j = \{q_i^j, H_i\} = N \frac{p_i^j}{m_j}}$$



Metric = G

$S_{TAD} = \int \sqrt{G} dt$

$$H_T = \frac{1}{2} \dot{q}^2 + N(p, q) \quad \text{---}$$

$$= N \left[\frac{1}{2} \frac{(p)^2}{m} + V - E \right]$$

$$\{q_i, p_j\} = \delta_{ij} \delta^3$$

$$\dot{p}_i = \{p_i, H_T\} = - \frac{\partial V}{\partial q_i} \cdot N$$

$$\dot{q}_i = \{q_i, H_T\} = N \frac{p_i}{m} \Rightarrow \frac{1}{N} \frac{d}{dt} \left(\frac{1}{N} m \dot{q}_i \right) = - \frac{\partial V}{\partial q_i}$$



Metric = G

V = Conf. fact. = Potential

$$G(dq_i, dt) = \int \dots$$

$$H_T = \frac{1}{2} \dot{q}^2 + N(p, q) \dot{q}$$

$$= N \left[\sum \frac{p_i^2}{2m_i} + V - E \right]$$

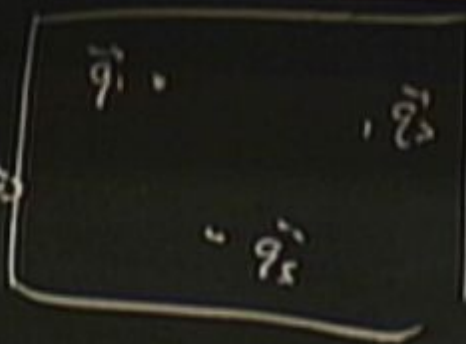
$$\{q_i, p_j\} = \delta_{ij} \delta^3$$

$$\dot{p}_i = \{p_i, H_T\} = - \frac{\partial V}{\partial q_i} \cdot N$$

$$\boxed{\dot{q}_i = \{q_i, H_T\} = N \frac{p_i}{m_i}}$$

$$p_i = \frac{m_i \dot{q}_i}{N}$$

$$\Rightarrow \frac{1}{N} \frac{d}{dt} \left(\frac{1}{N} m_i \frac{dq_i}{dt} \right) = - \frac{\partial V}{\partial q_i}$$



$V = \text{conf. fact.} = \text{Potential}$

Metric = G

$$S_{\text{GR}} = \int \sqrt{G} (dq_i \cdot dq_i)$$

$$= N \left[\sum_i \frac{p_i^2}{2m_i} + V - E \right]$$

$$p_i = \left\{ p_{i,1}, p_{i,2} \right\} = - \frac{\partial V}{\partial q_i} \cdot N$$

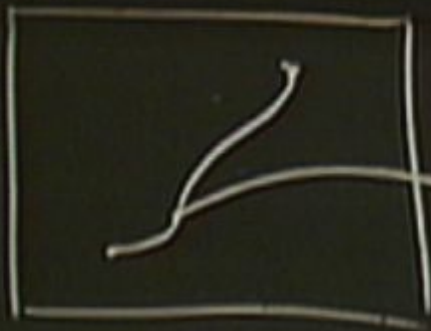
$$\boxed{\vec{q}_i = \{ q_i, \lambda_i \} = N \frac{p_i}{m_i}}$$

$$p_i = \frac{m_i q_i}{N}$$

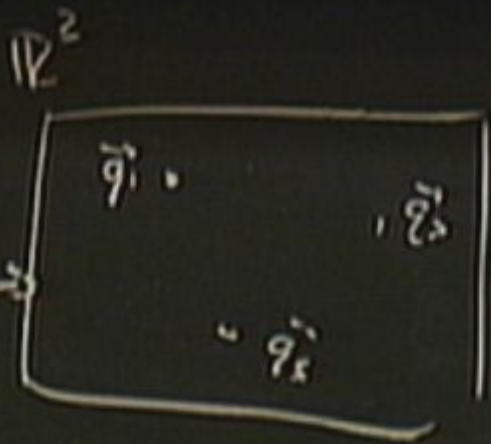
$$\Rightarrow \frac{1}{N} \frac{d}{dt} \left(\frac{1}{N} m_i \frac{dq_i}{dt} \right) = - \frac{\partial V}{\partial q_i}$$

$$\left(\frac{1}{N} \frac{d}{dt} = \frac{d}{dt_{eff}} \right)$$

$\frac{5 \text{ Hz} \cdot \text{cs}}{2^i}$



Metric = G



START =

$$\sqrt{G} (dq_i, dt) = \dots$$

$$G = -V(q_i) G' \quad (V \leq 0)$$

V = Conf. fact. = Potential

$$= N \left[\sum_i \frac{p_i^2}{2m_i} + V - E \right]$$

$$\frac{\partial}{\partial p_i} \left\{ p_i^2, \dots \right\} = - \frac{\partial V}{\partial q_i} \cdot N$$

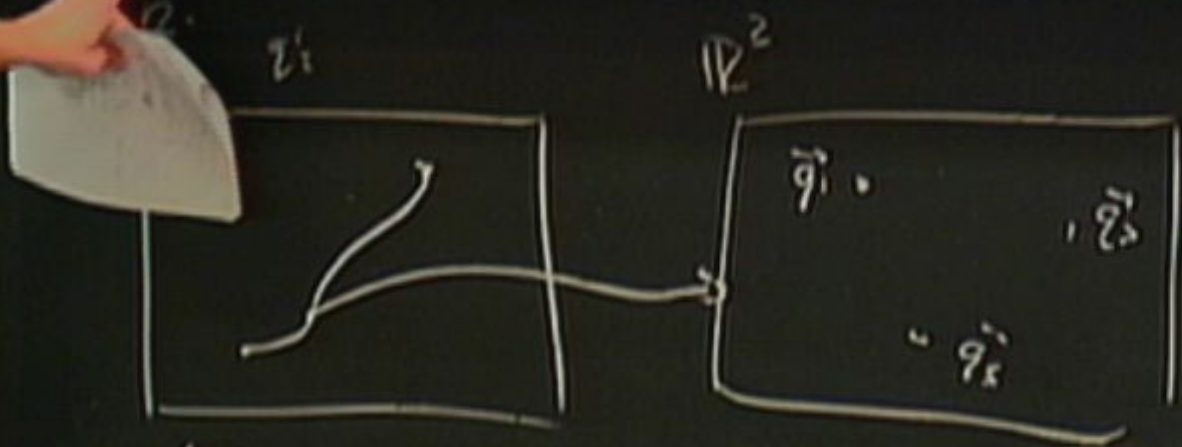
$$\vec{q}_i = \{ q_i, \dots \} = N \frac{p_i}{m_i}$$

$$p_i = \frac{m_i q_i}{N}$$

$$\Rightarrow \frac{1}{N} \frac{d}{dt} \left(\frac{1}{N} m_i \frac{dq_i}{dt} \right) = - \frac{\partial V}{\partial q_i}$$

$$\left(\frac{1}{N} \frac{d}{dt} = \frac{d}{dt_{eff}} \right)$$

$$2) H \approx 0 \quad \frac{1}{N} \sum_i \frac{m_i (q_i^2)}{2}$$



$$G = -V(q_i) G' \quad (V \leq 0)$$

$V = \text{conf. fact.} = \text{Potential}$

Metric = G

STRAD =

$$\sqrt{G} (dq_i, da_i) = \dots$$

$$= N \left(\sum_i \frac{p_i^2}{2m_i} + V - E \right) \quad p_i = \{ p_{i1}, p_{i2} \} = - \frac{\partial V}{\partial q_i} \cdot N$$

$$\{ \dot{q}_i \} = \{ \dot{q}_i, \dots \} = N \frac{p_i}{m_i} \quad p_i = \frac{m_i \dot{q}_i}{N} \Rightarrow \frac{1}{N} \frac{d}{dt} \left(\frac{1}{N} m_i \frac{dq_i}{dt} \right) = - \frac{\partial V}{\partial q_i}$$

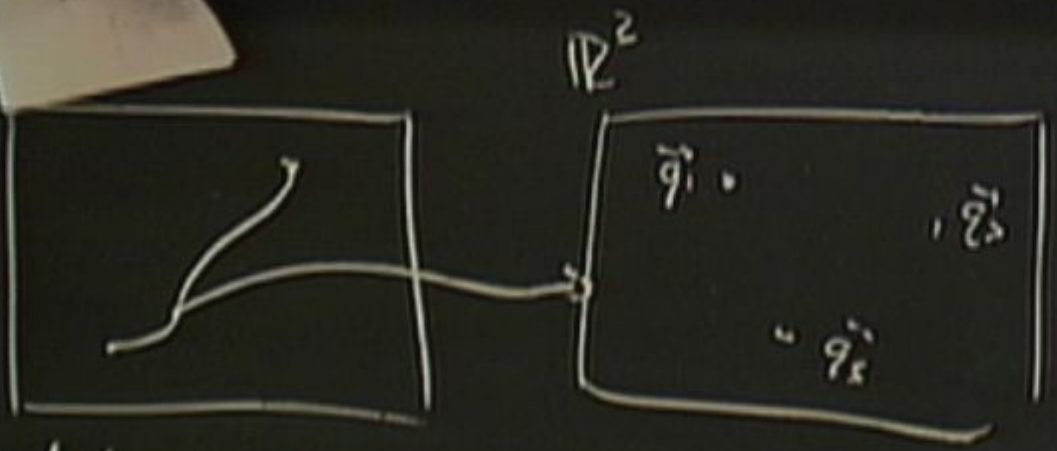
$$2) H \approx 0 \quad \frac{1}{N} \sum_i \frac{m_i (\dot{q}_i)^2}{2} + V - E = 0$$

$$N = \sqrt{\frac{I}{E - V}}$$

$$\left(\frac{1}{N} \frac{d}{dt} = \frac{d}{dt_{\text{eff}}} \right)$$

$$m_i \sqrt{\frac{E - V}{T}} \frac{d}{dt} \left(\sqrt{\frac{E - V}{T}} \frac{dq_i}{dt} \right) = - \frac{\partial V}{\partial q_i}$$

$\frac{\partial}{\partial q_i}$ $\frac{\partial}{\partial q_i}$



$$G = -V(q_i) G' \quad (V \leq 0)$$

$V = \text{const. fact.} = \text{Potential}$

Metric = G

$$S_{\text{path}} = \int_{q_{i1}}^{q_{i2}} \sqrt{G} (dq_i, da_i) = \dots$$

$$\boxed{\vec{q}_i = \{q_i, t\}} = N \frac{\vec{p}_i}{m_i} \quad \vec{p}_i = \frac{m_i \dot{q}_i}{N} \Rightarrow \frac{1}{N} \frac{d}{dt} \left(\frac{1}{N} m_i \frac{dq_i}{dt} \right) = - \frac{\partial V}{\partial q_i}$$

$$\mathcal{H} \approx 0 \quad \frac{1}{N} \sum_i \frac{m_i (q_i^i)^2}{2} + V - E = 0$$

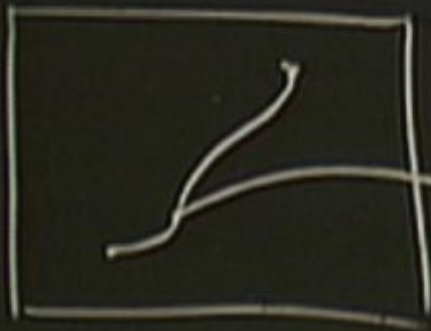
$$N = \sqrt{\frac{I}{E-V}}$$

$$\left(\frac{1}{N} \frac{d}{dt} = \frac{d}{d\tau_{\text{eff}}} \right)$$

$$\boxed{m_i \sqrt{\frac{E-V}{T}} \frac{d}{d\lambda} \left(\sqrt{\frac{E-V}{T}} \frac{dq_i}{d\lambda} \right) = - \frac{\partial V}{\partial q_i}}$$

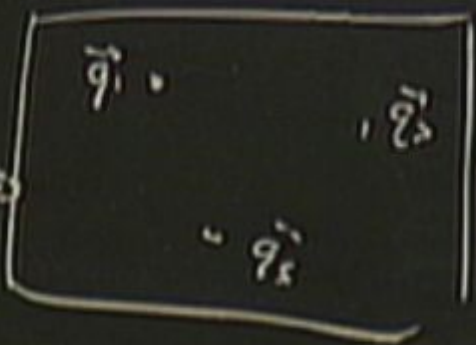
momenta $\vec{p}_i = \frac{\partial L_{\text{eff}}}{\partial \dot{q}_i} = \frac{\partial}{\partial \dot{q}_i} \left(2 \sqrt{\frac{1}{2} \sum_i m_i (q_i^i)^2} (E - V)^{1/2} \right)$

\mathbb{R}^2 \vec{q}_i



Metric = G

\mathbb{R}^2




$$G = -V(q_i) G' \quad (V \leq 0)$$

$V = \text{const. fact.} = \text{Potential}$

$S_{\text{eff}} =$

$$\int \sqrt{G} (dq_i, dt)$$

$\rightarrow dt \rightarrow \frac{d\lambda \cdot dq}{d\lambda} = d\eta q^u$



$d\tau_{\text{eff}} = \sqrt{\frac{I}{E-V}} d\lambda$

$(\tau_{\text{in}} - q_{\text{in}}^u) = \int d\lambda \sqrt{\frac{I}{E-V}}$

$$\rightarrow dt \rightarrow \frac{d\lambda \cdot dq^0}{d\lambda} = d\lambda q^0$$



$$dT_{\text{eff}} = \int \sqrt{\frac{I}{E-V}} d\lambda$$

$$T = (q^0_{\text{r.in}} - q^0_{\text{in}}) = \int d\lambda \sqrt{\frac{I}{E-V}}$$