Title: Mini-Course on Mach's Principle

Date: Feb 04, 2009 10:30 AM

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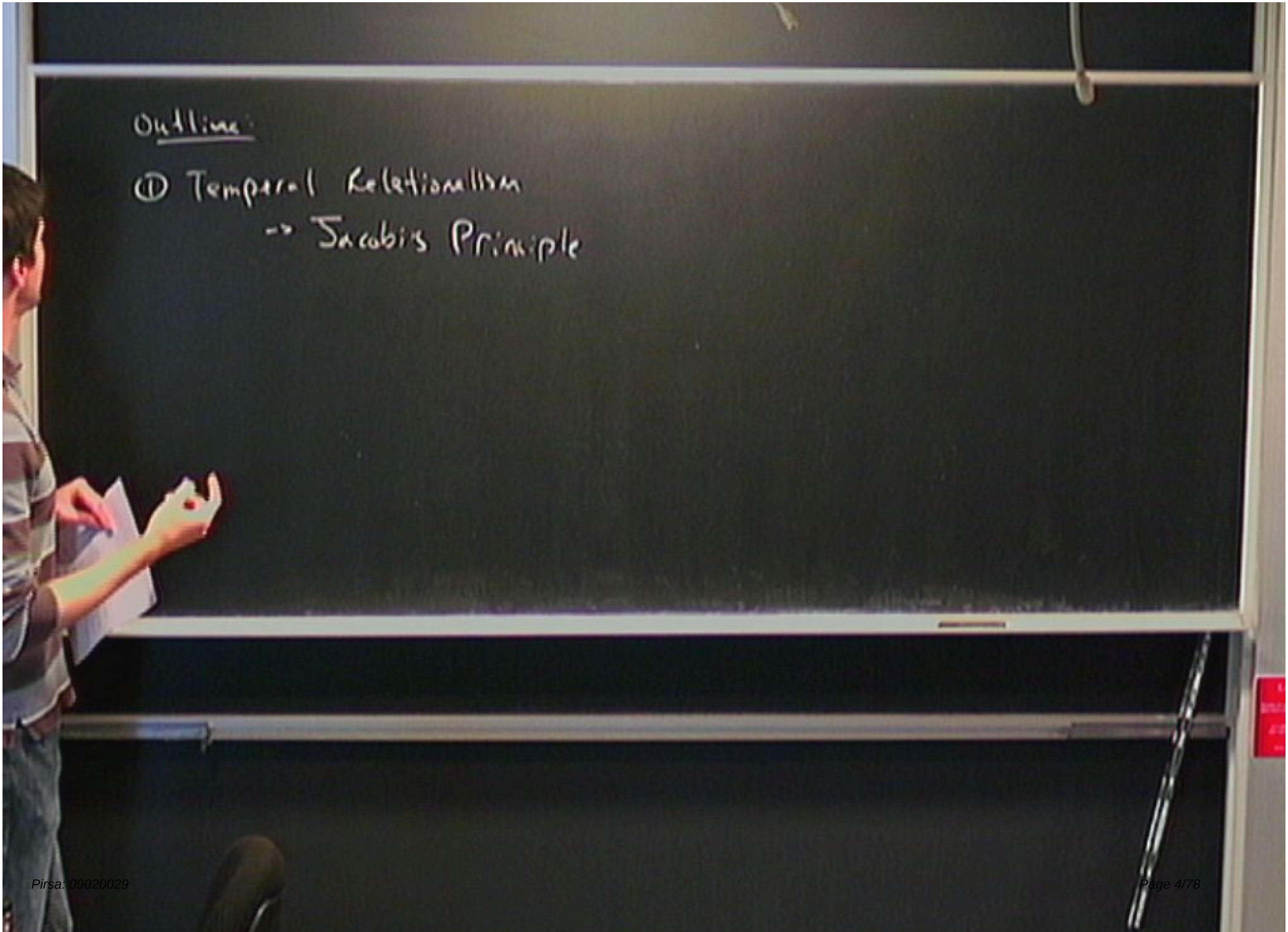
Abstract:

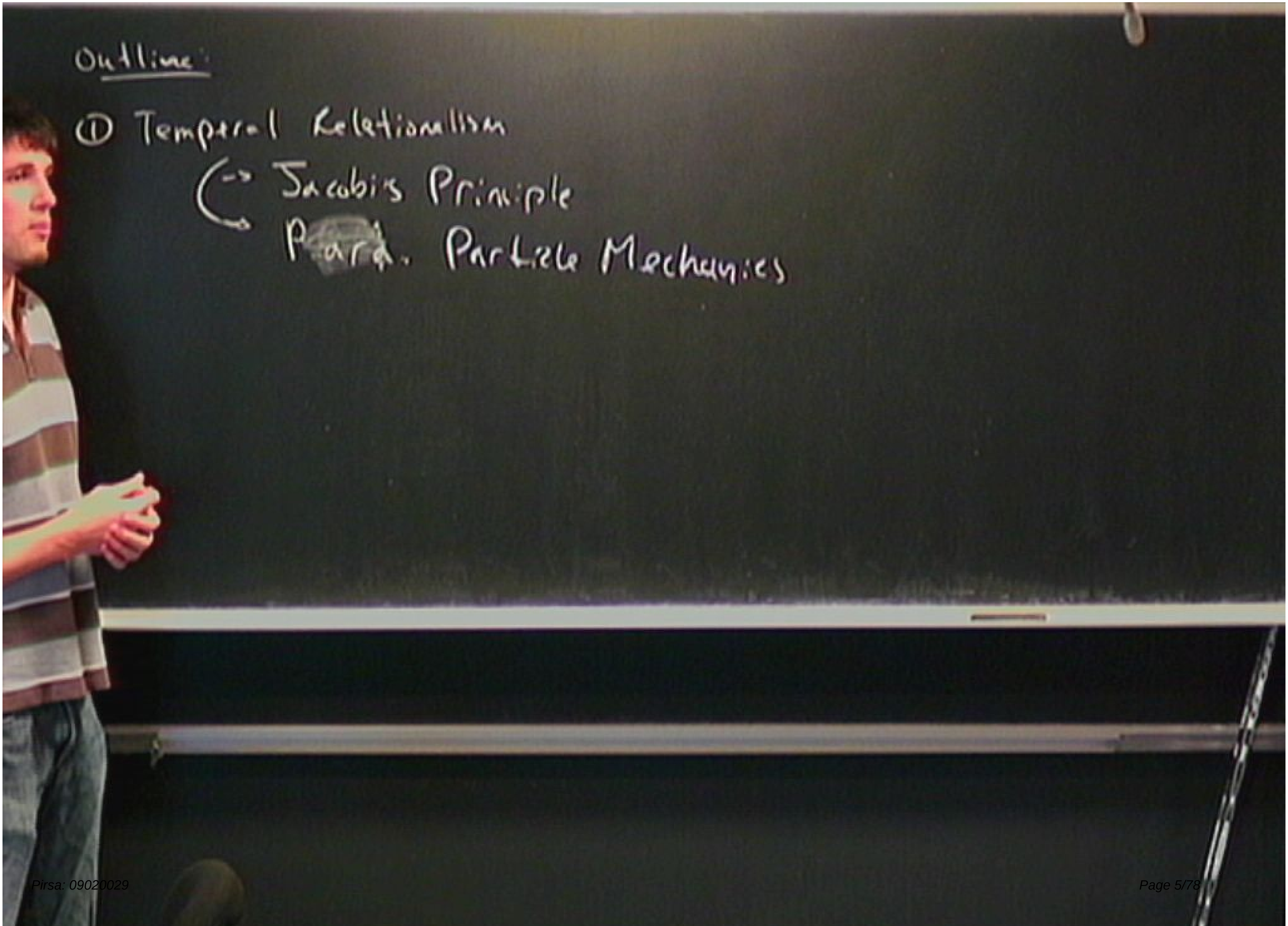
"It is utterly beyond our power to measure the changes of things by time. Quite the contrary, time is an abstraction, at which we arrive by means of the changes of things; made because we are not restricted to any one definite measure, all being interconnected."

- E. Mach in The *Mechanics*, 1872.

“It is utterly beyond our power to measure the changes of things by time. Quite the contrary, time is an abstraction, at which we arrive by means of the changes of things; made because we are not restricted to any one definite measure, all being interconnected.”

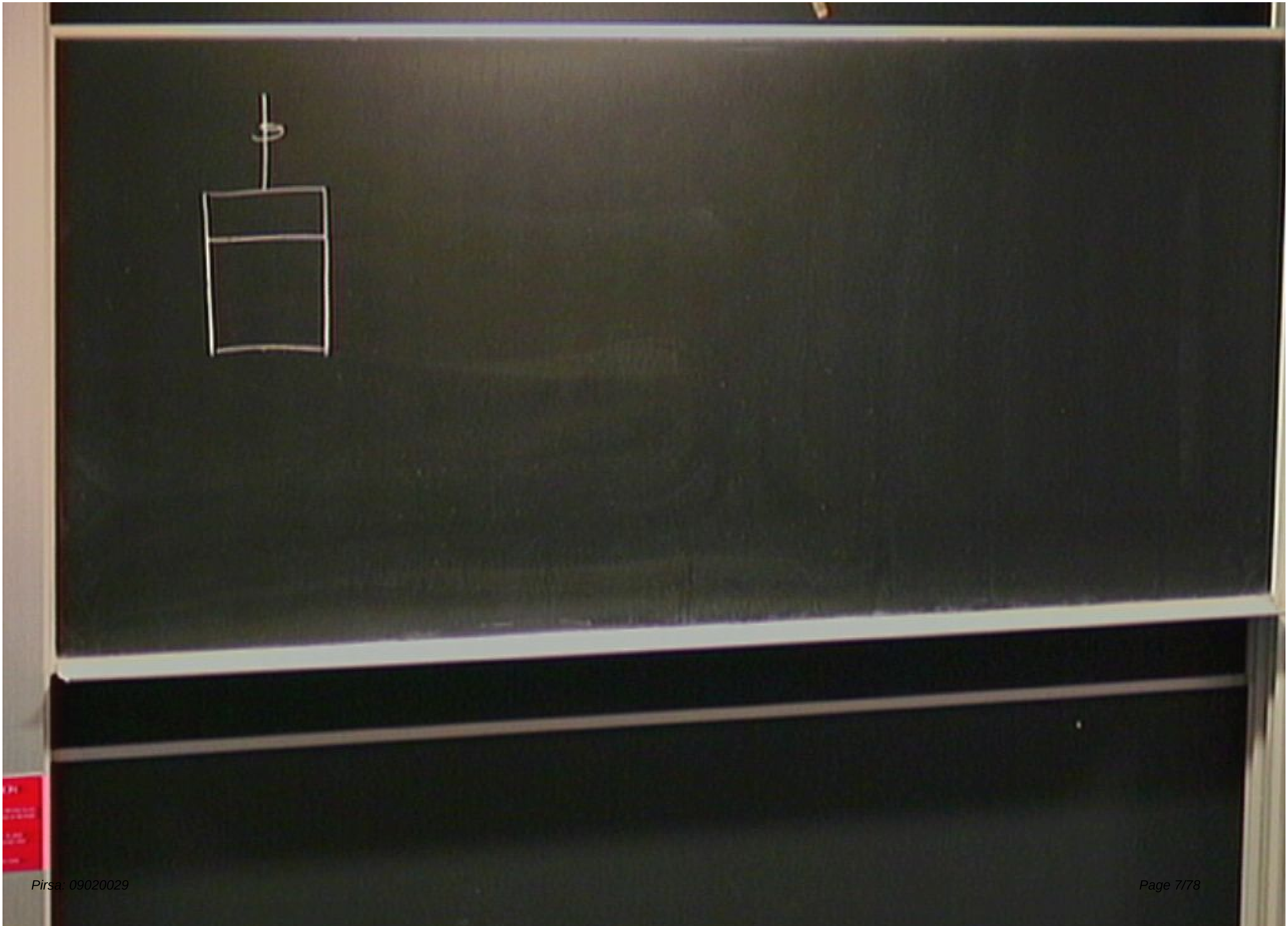
- E. Mach in The *Mechanics*, 1872.

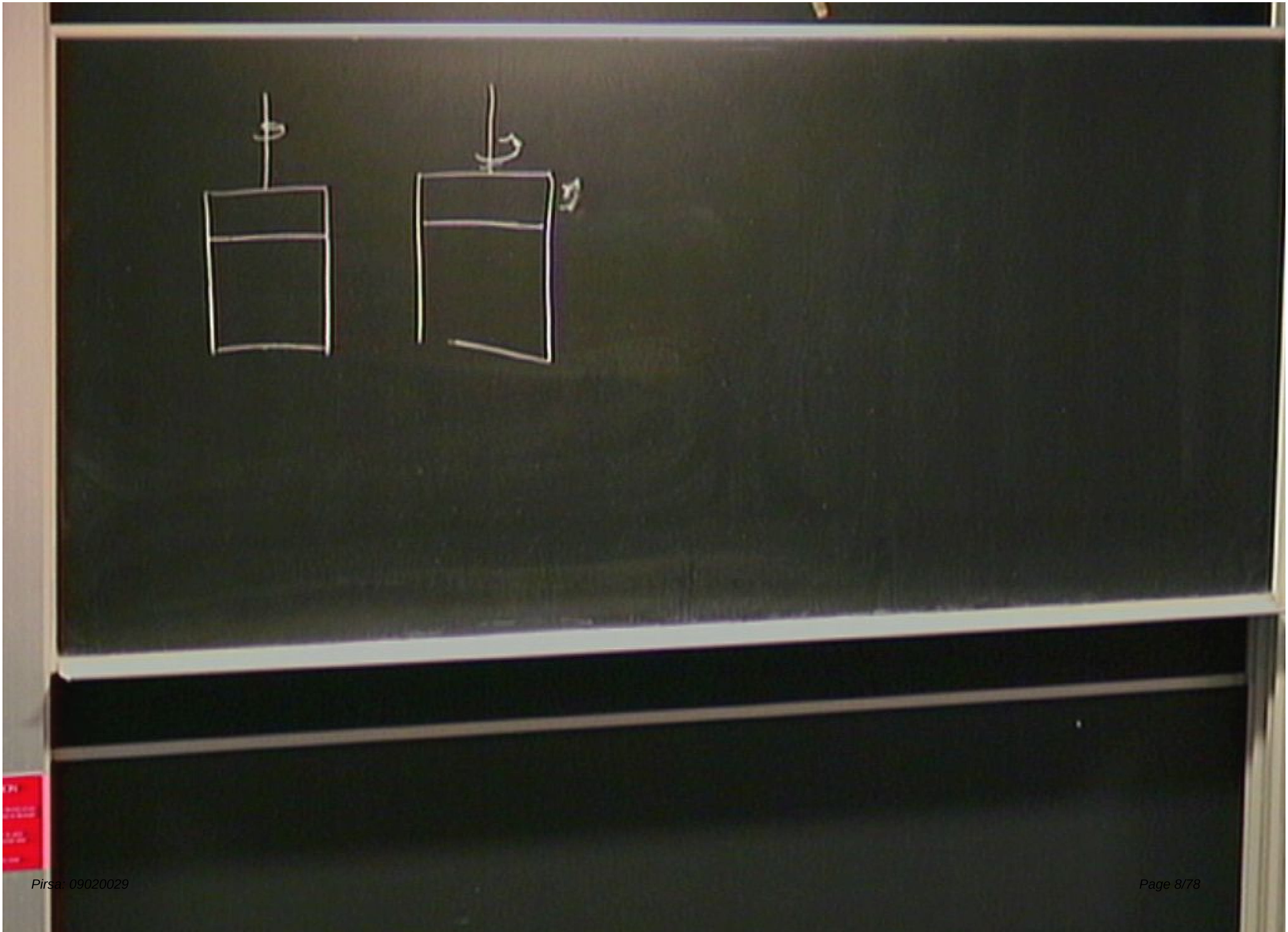

Outline:
① Temporal Relationalism
→ Jacobi's Principle

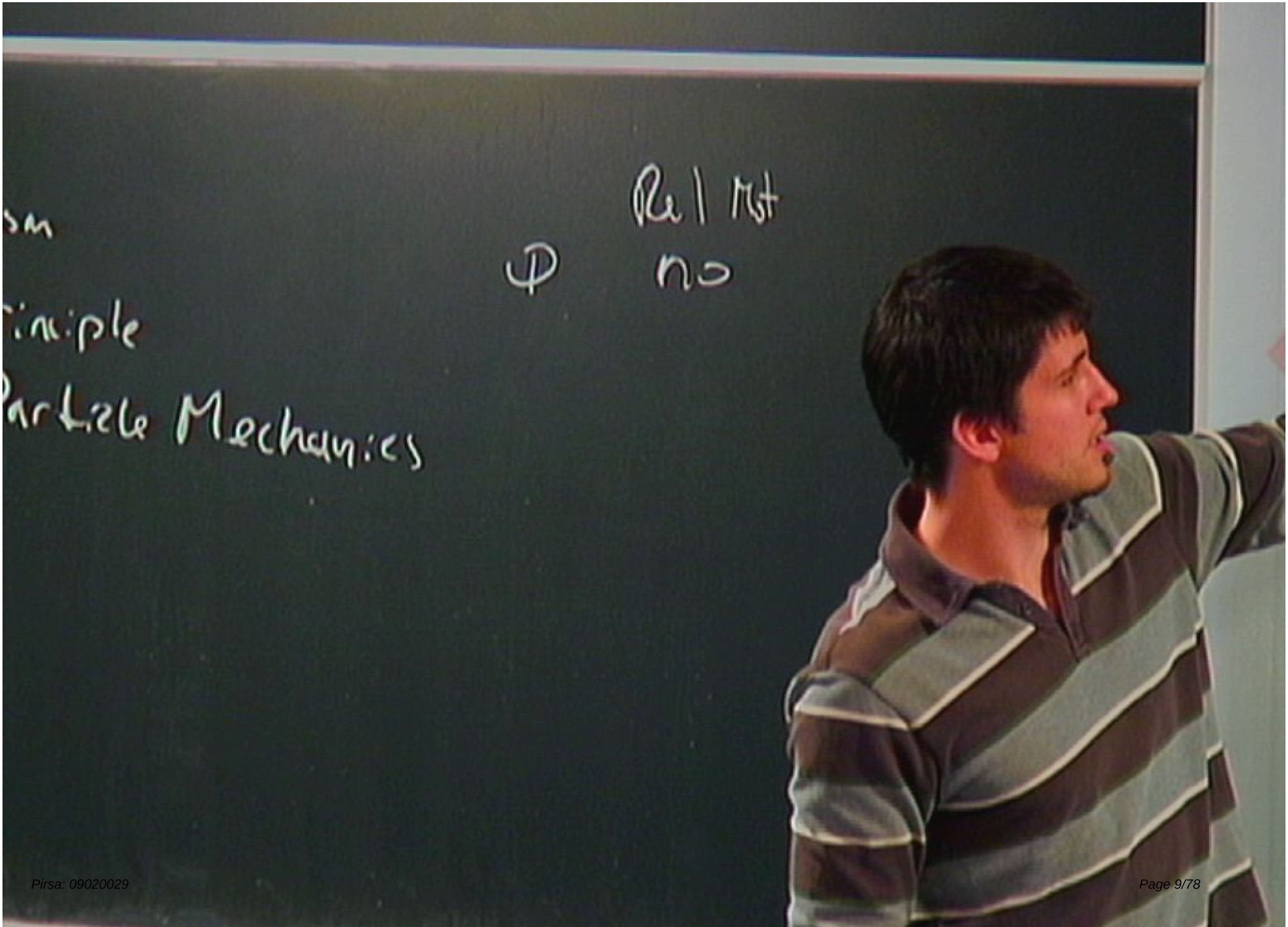

Outline:
① Temporal Relationalism
→ Jacobi's Principle
Particle Mechanics

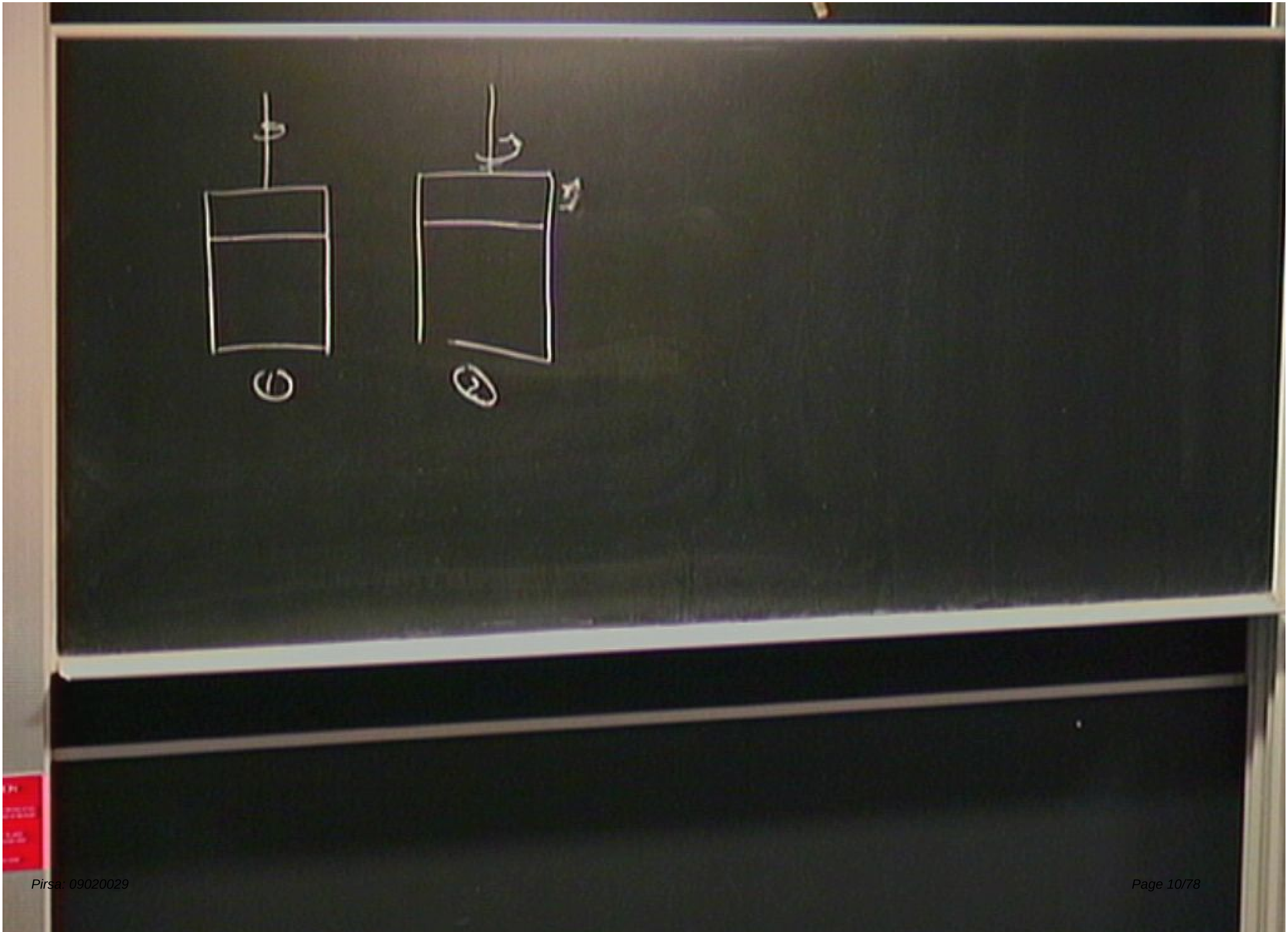
"When we say that a body K alters its direction and velocity solely through the influence of another body K', we have inserted a conception that is impossible to come at unless other bodies A, B, C... are present with reference to which the motion of the body K has been estimated."

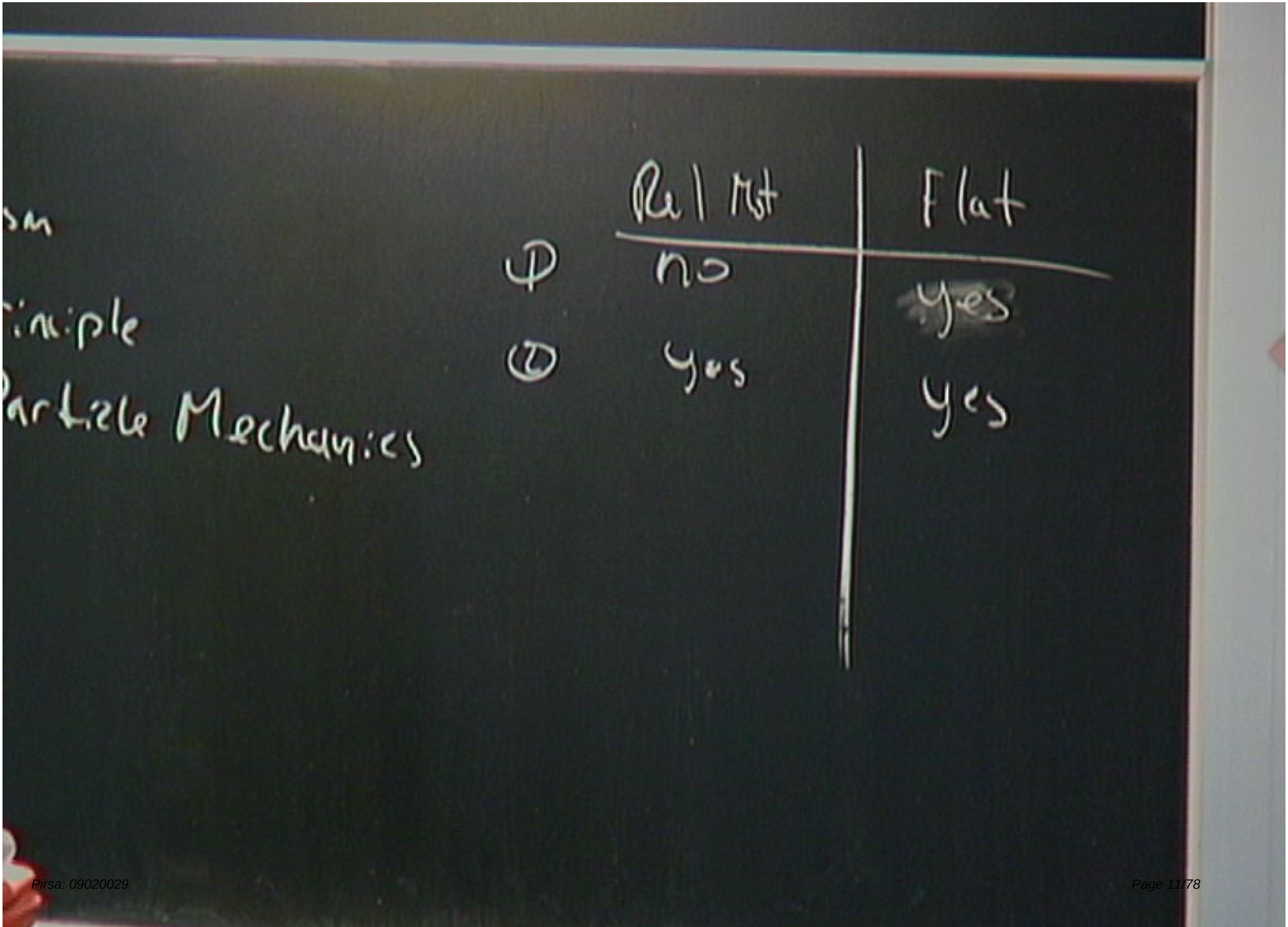
- E. Mach in The *Mechanics*, 1872.

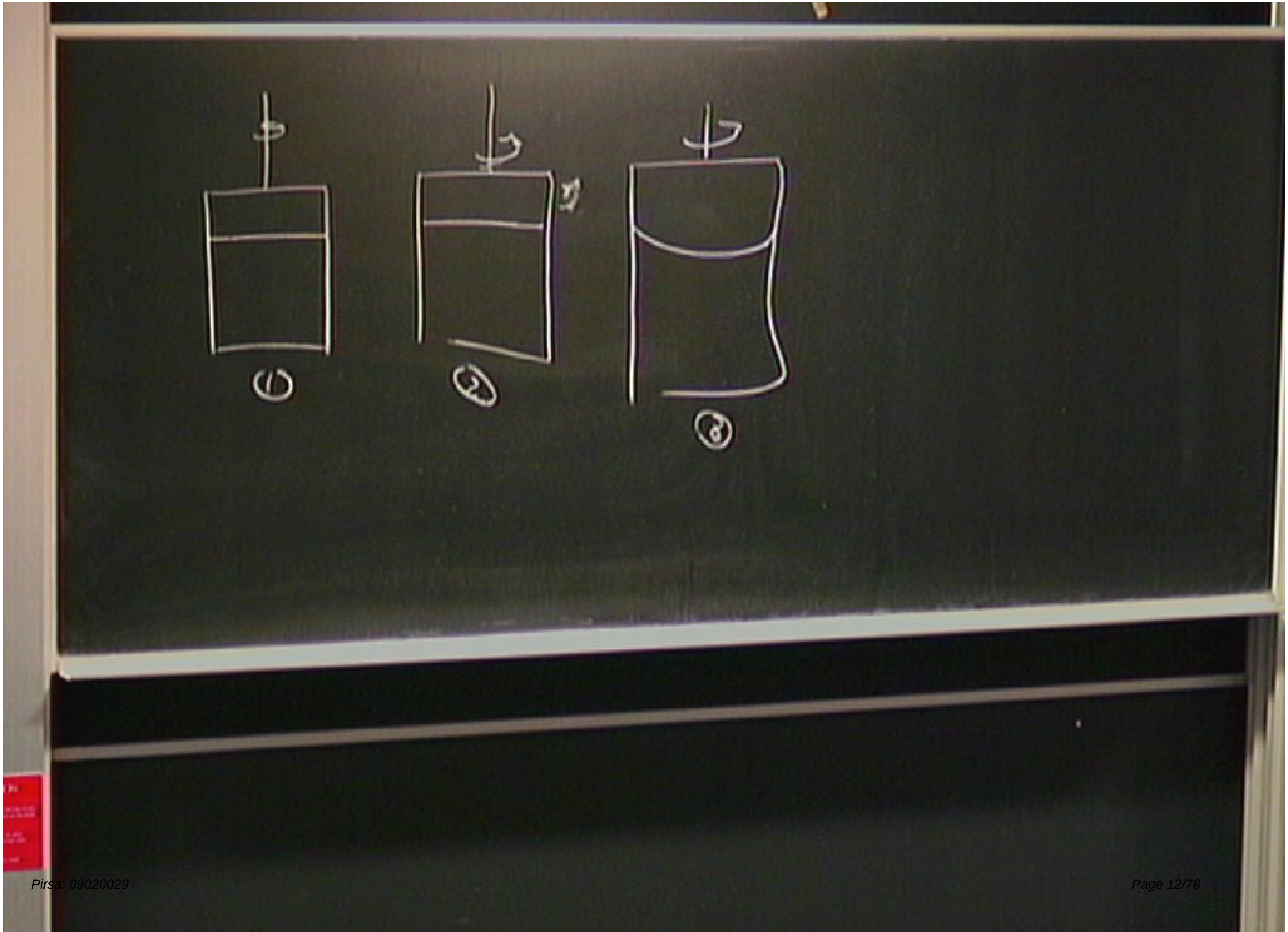




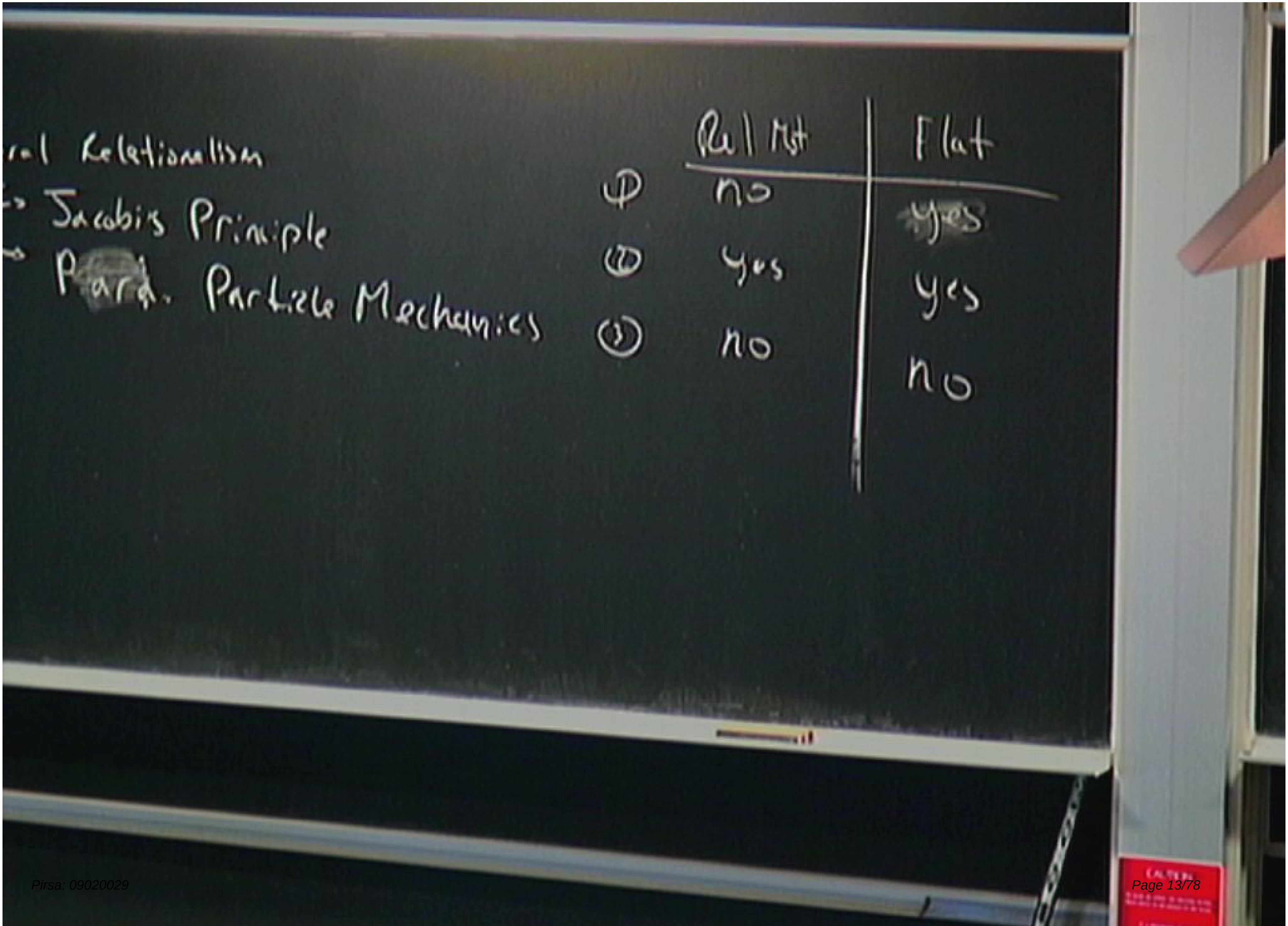

sm
inciple
article Mechanics
no

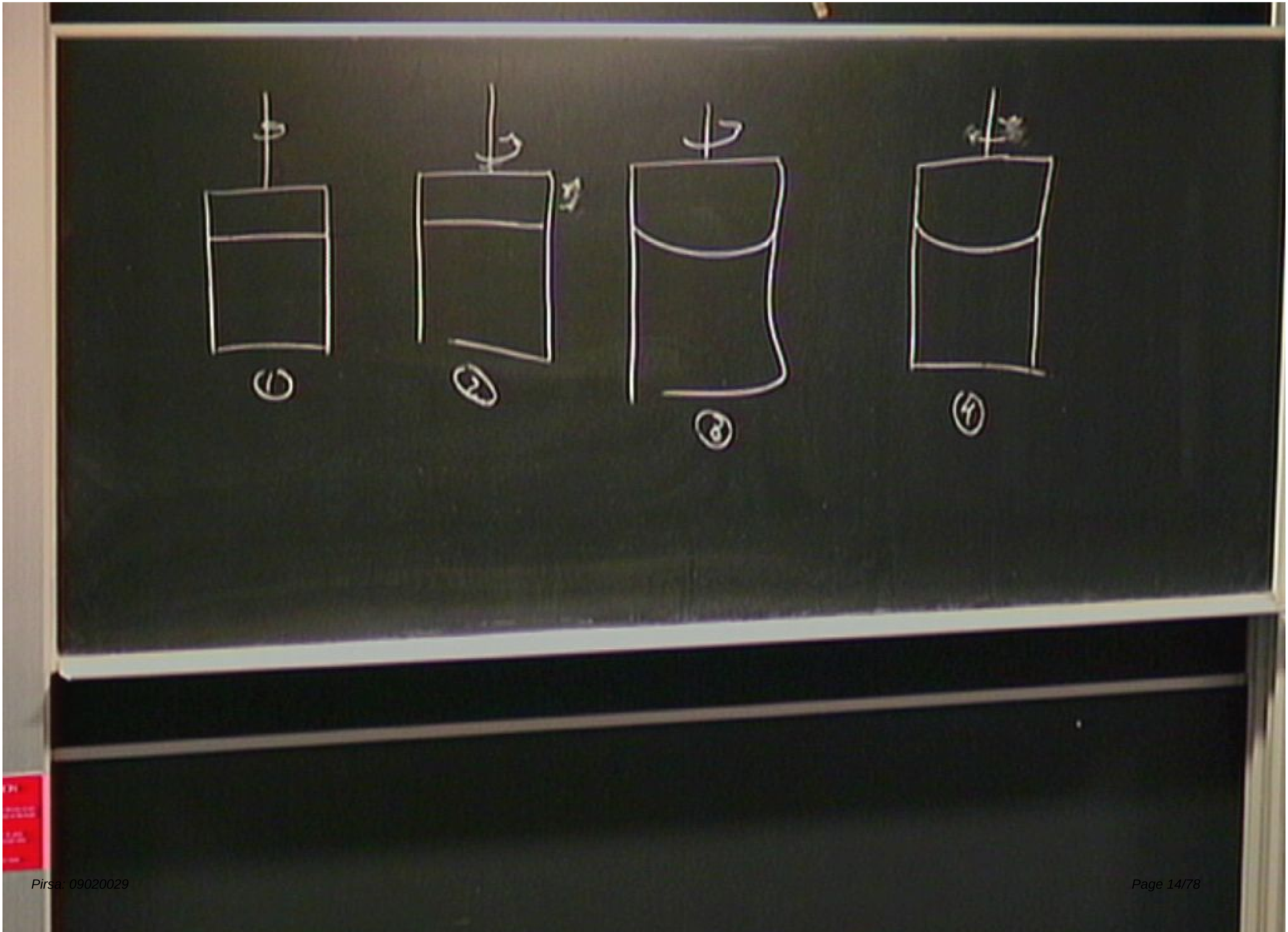

1
2

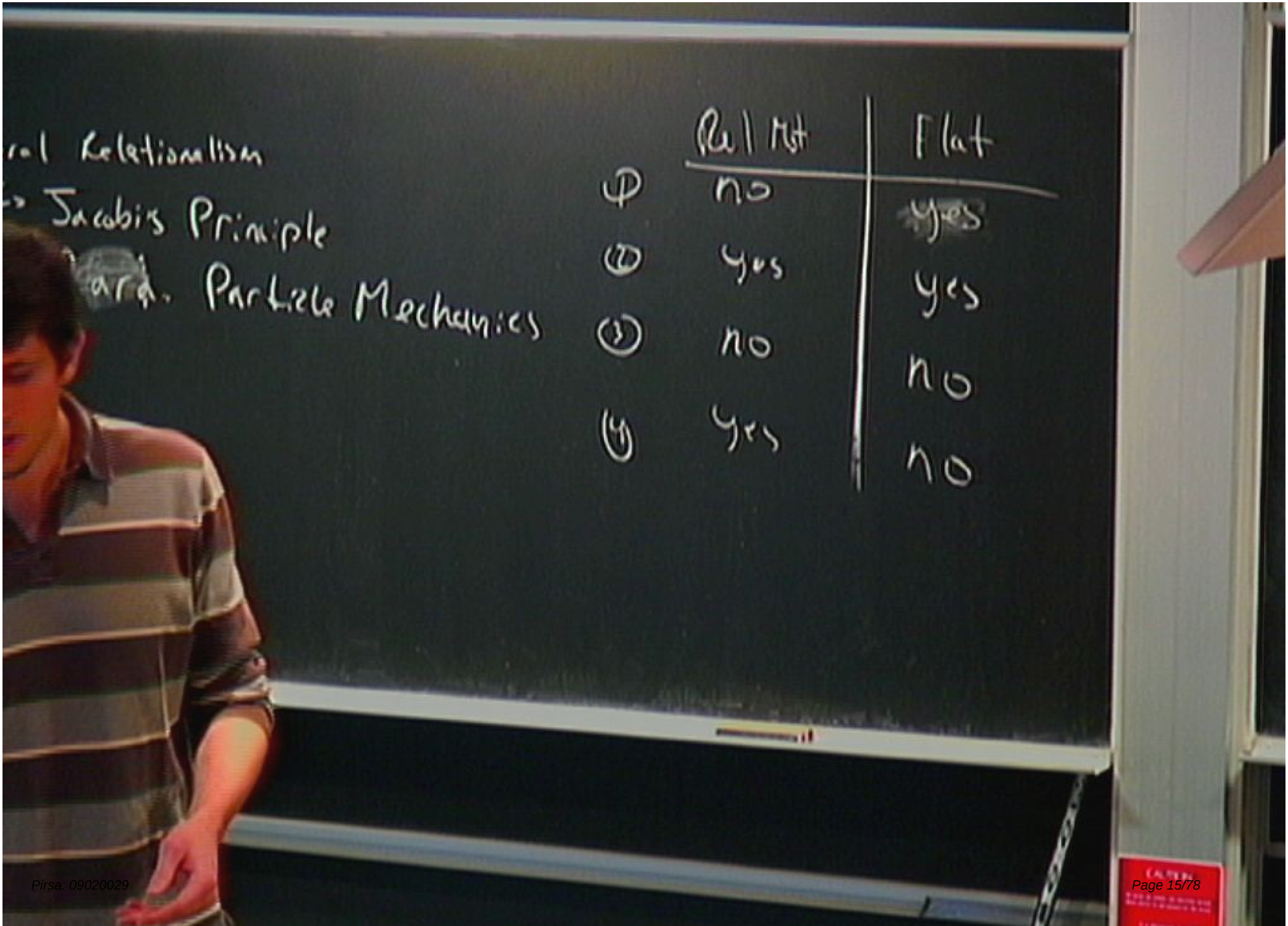

Rel Mot
Flat
no
yes
yes
yes
Mechanics

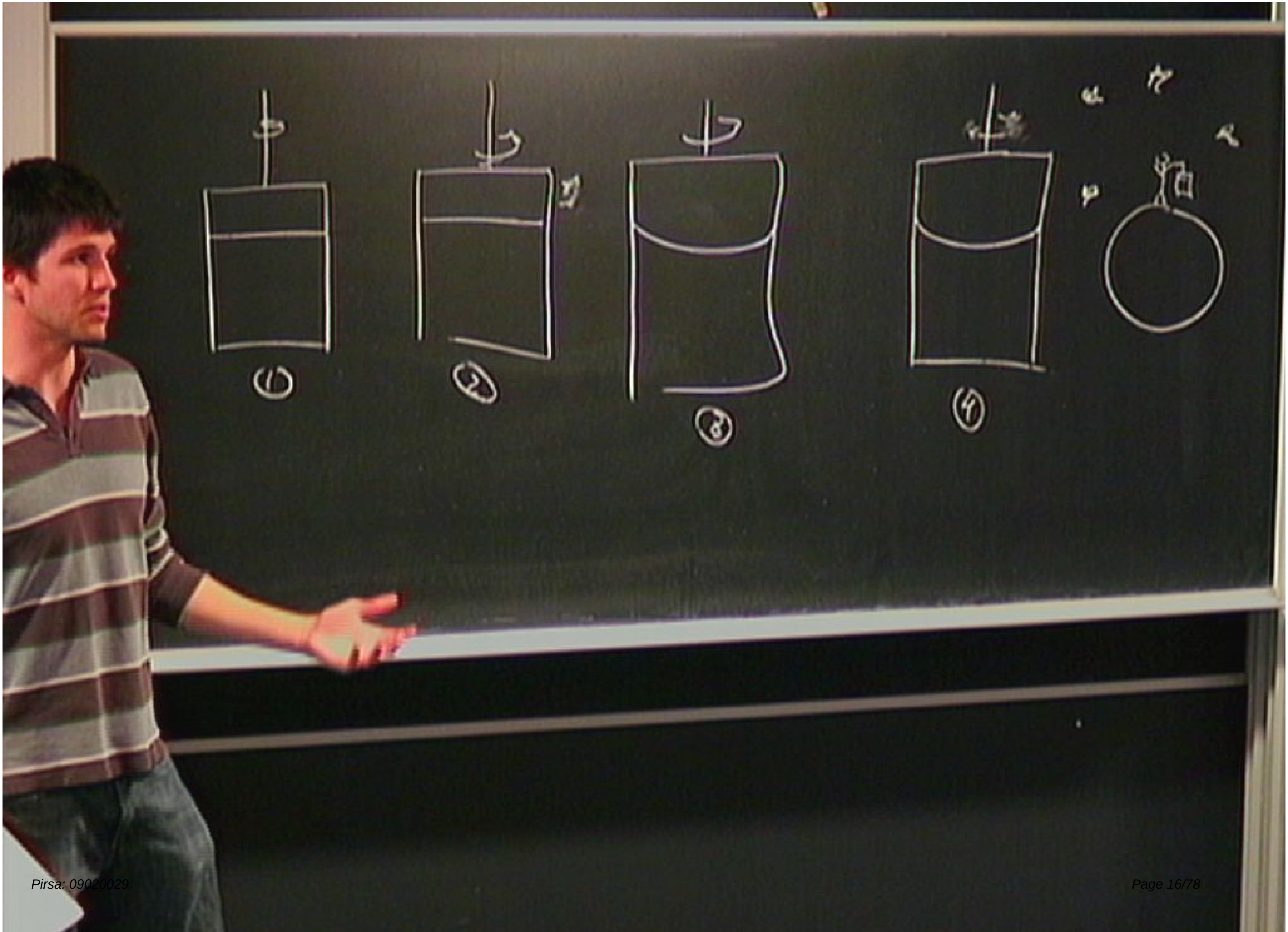


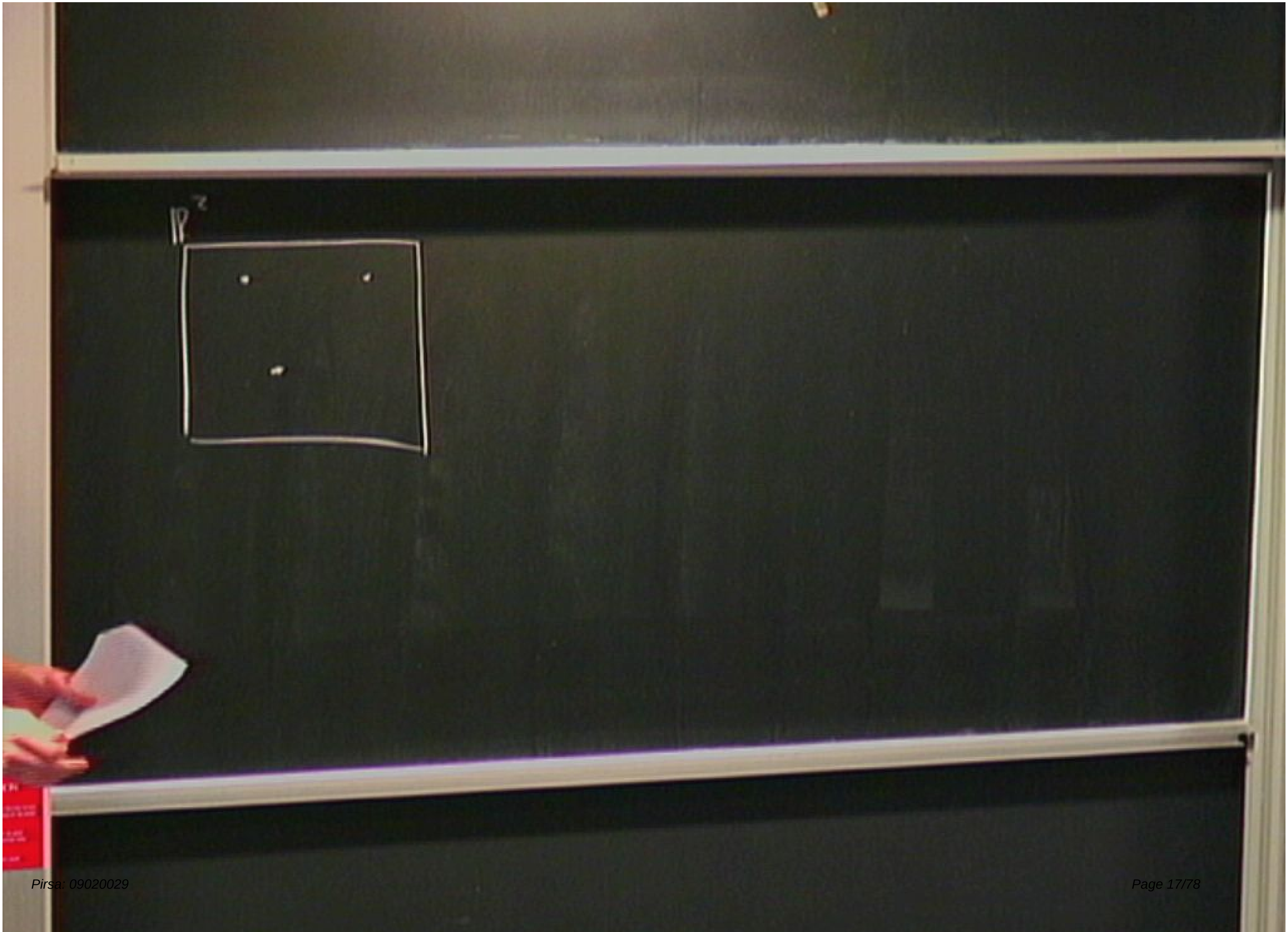
1
2
3

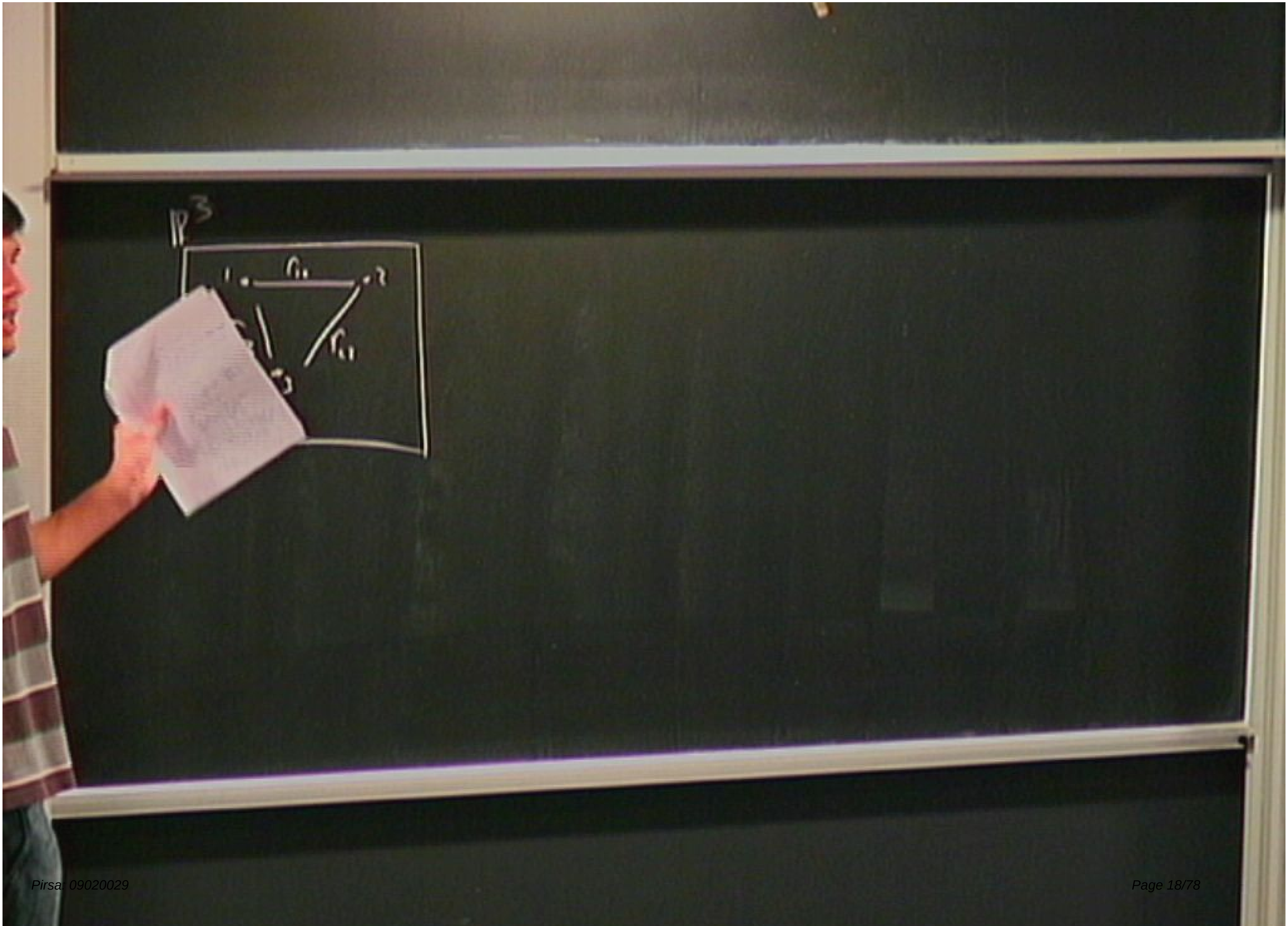

Relationalism
Jacobi's Principle
Particle Mechanics
Rel Mot
Flat
① no yes
② yes yes
③ no no

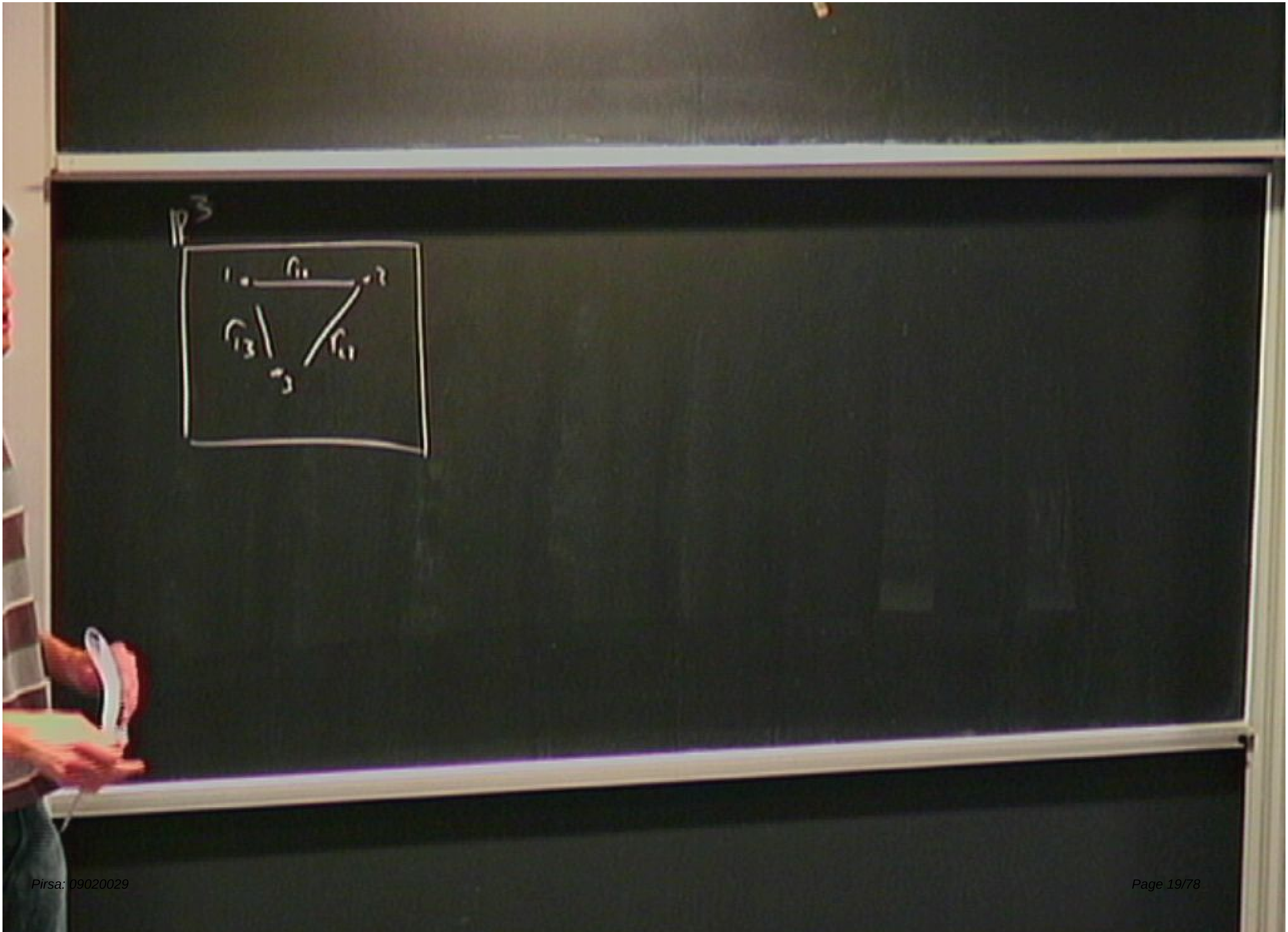


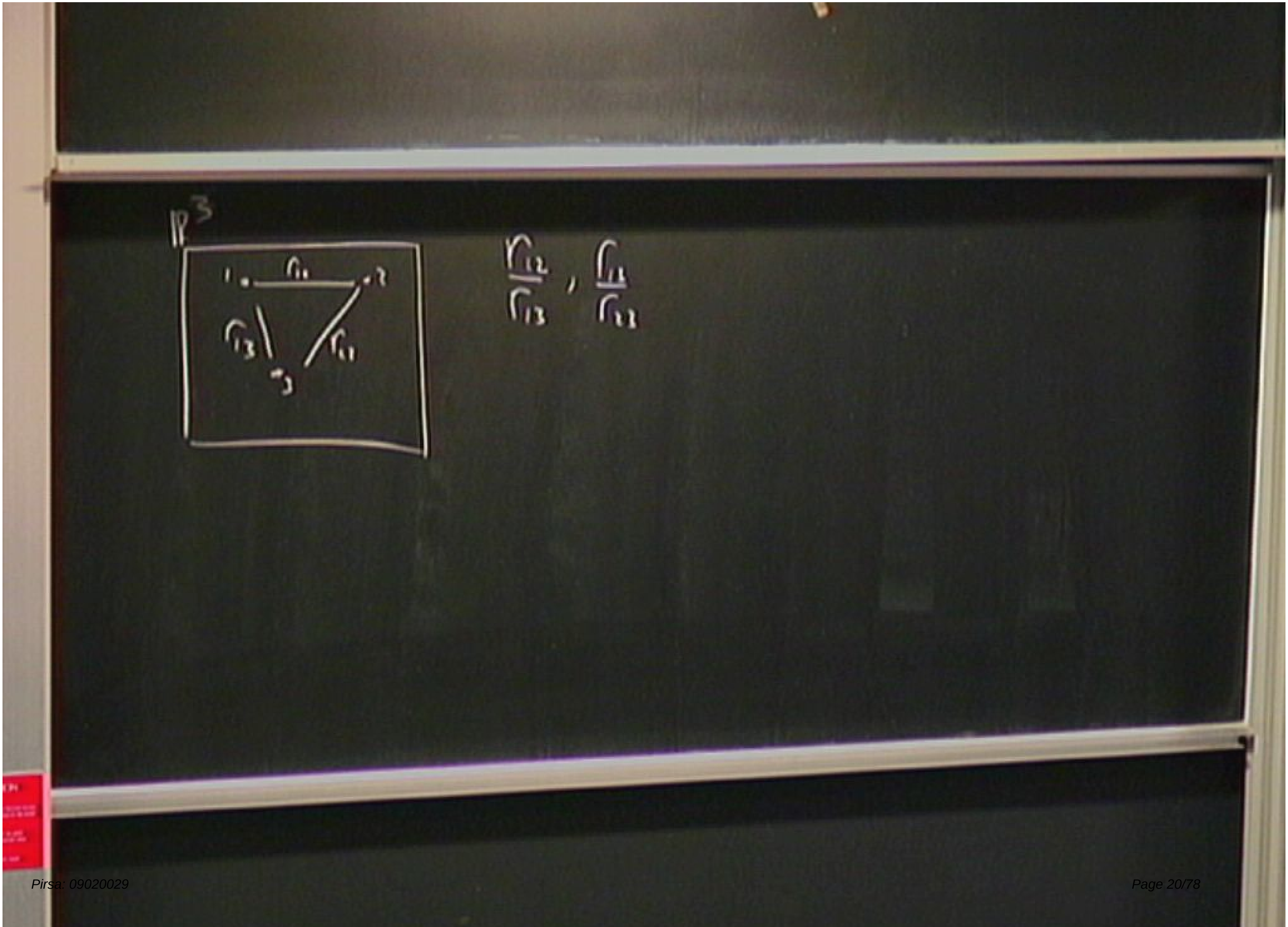

Jacobi's Principle
Particle Mechanics
Rel Mt
Flat
no
yes
yes
yes
no
no
yes
no




$\mathbb{P}^2$






1
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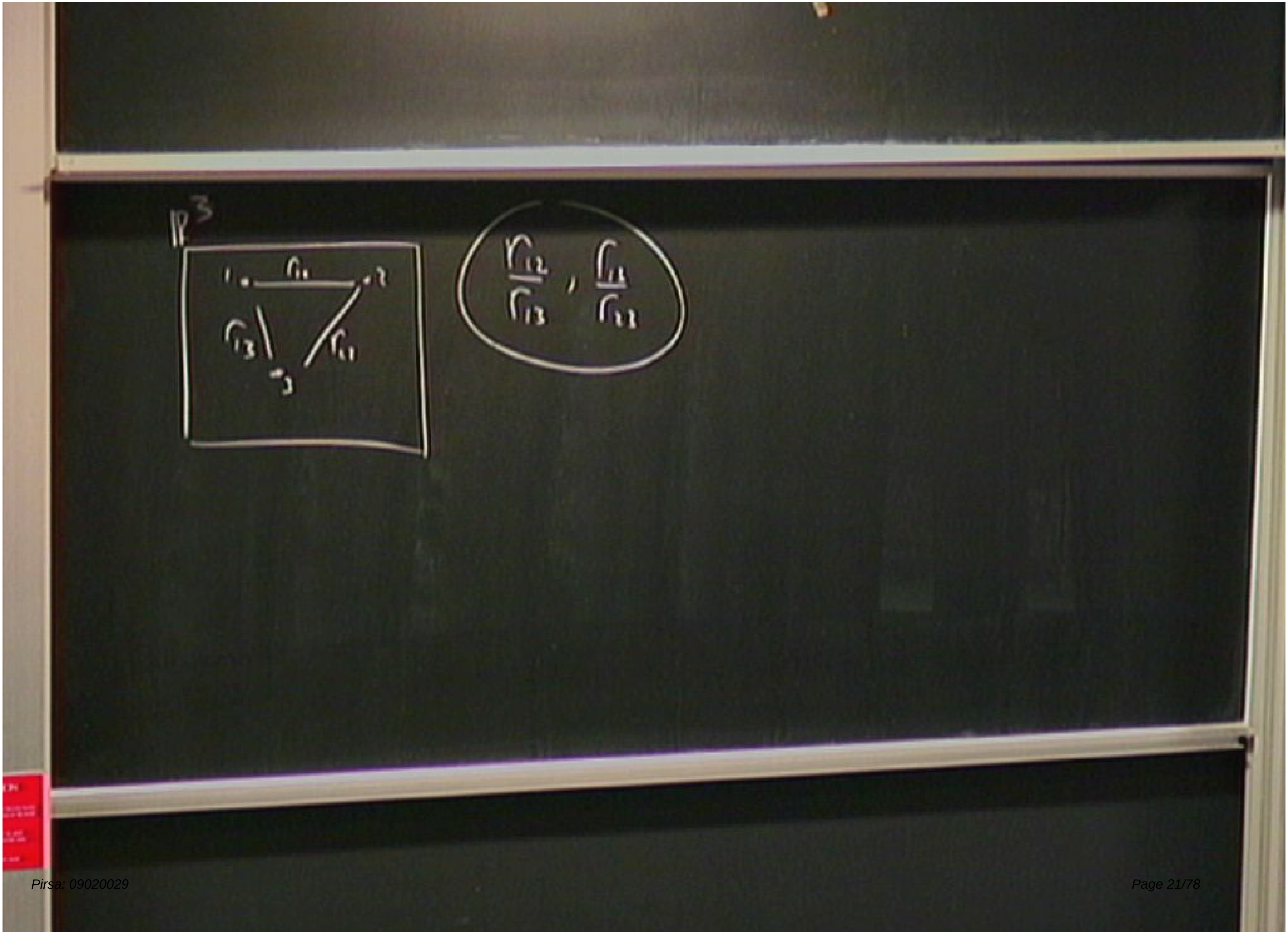
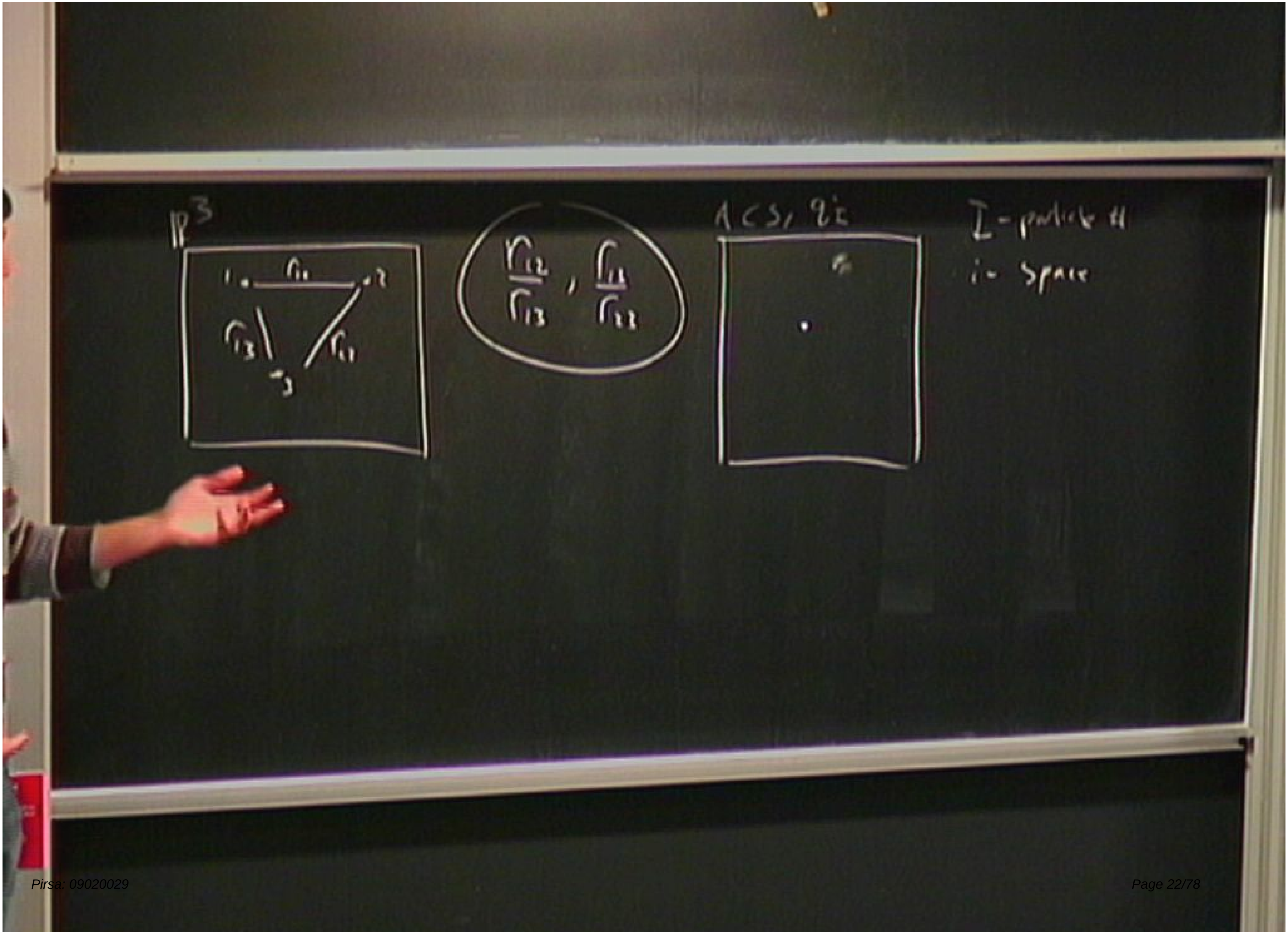
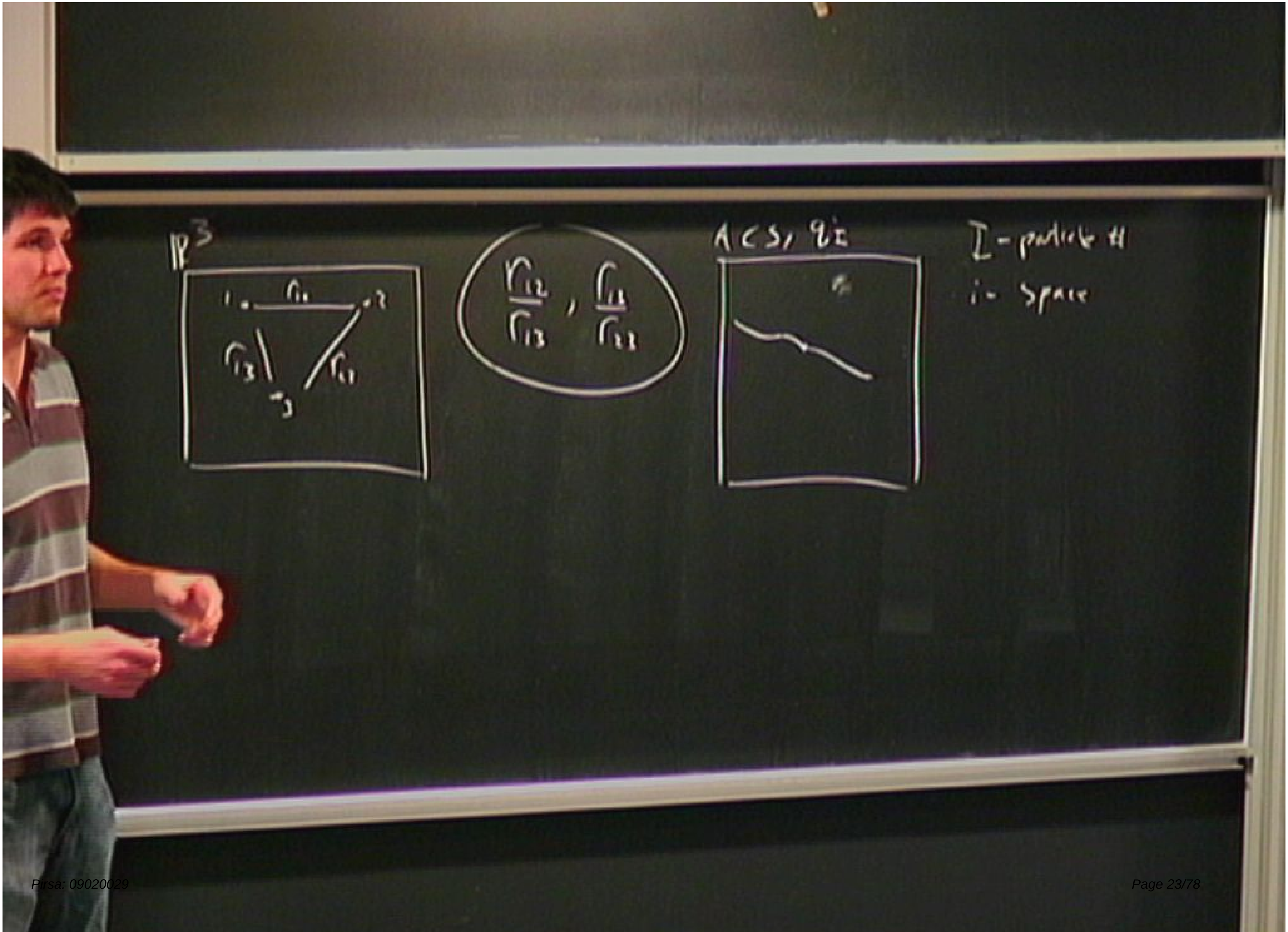
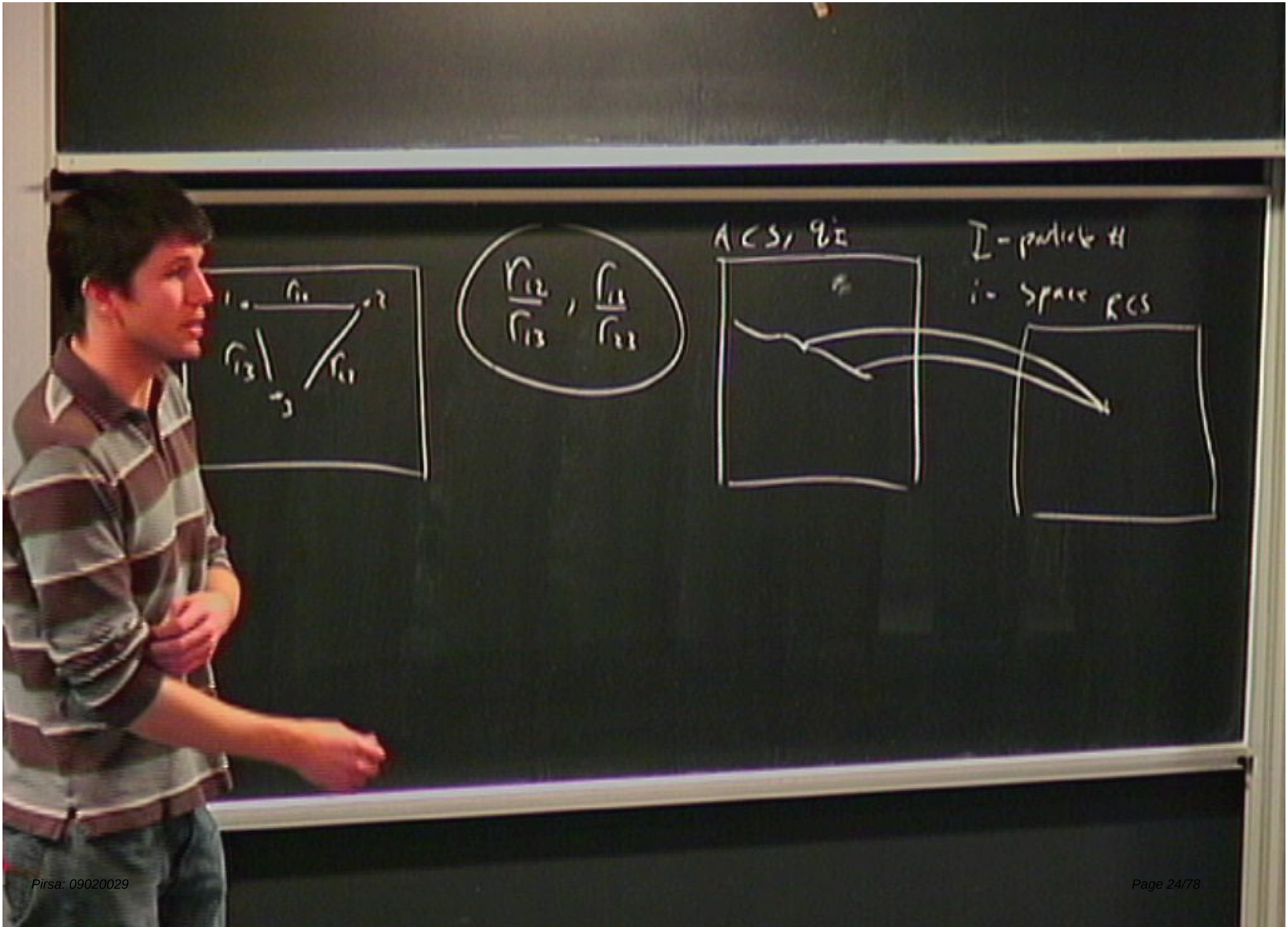
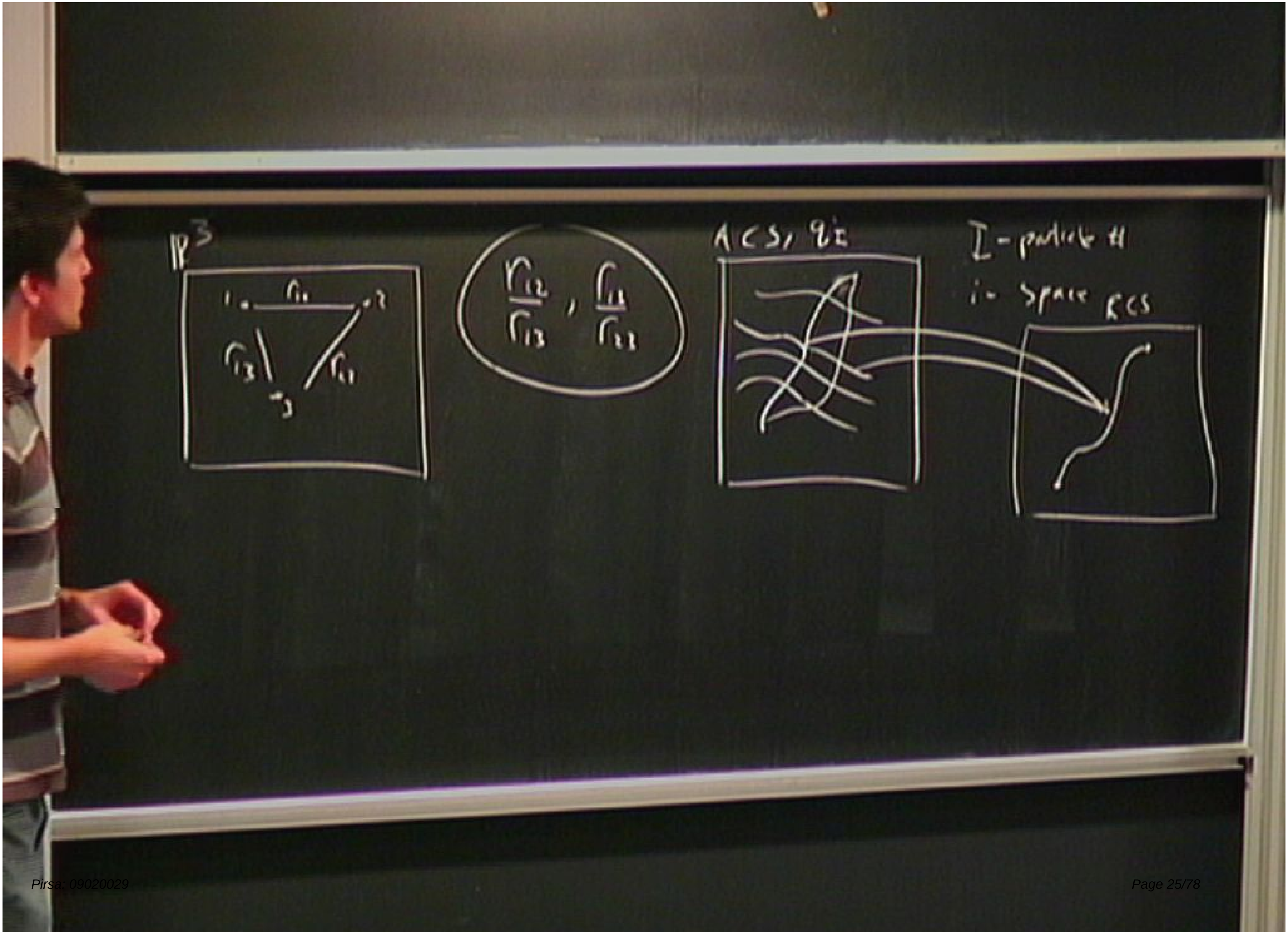
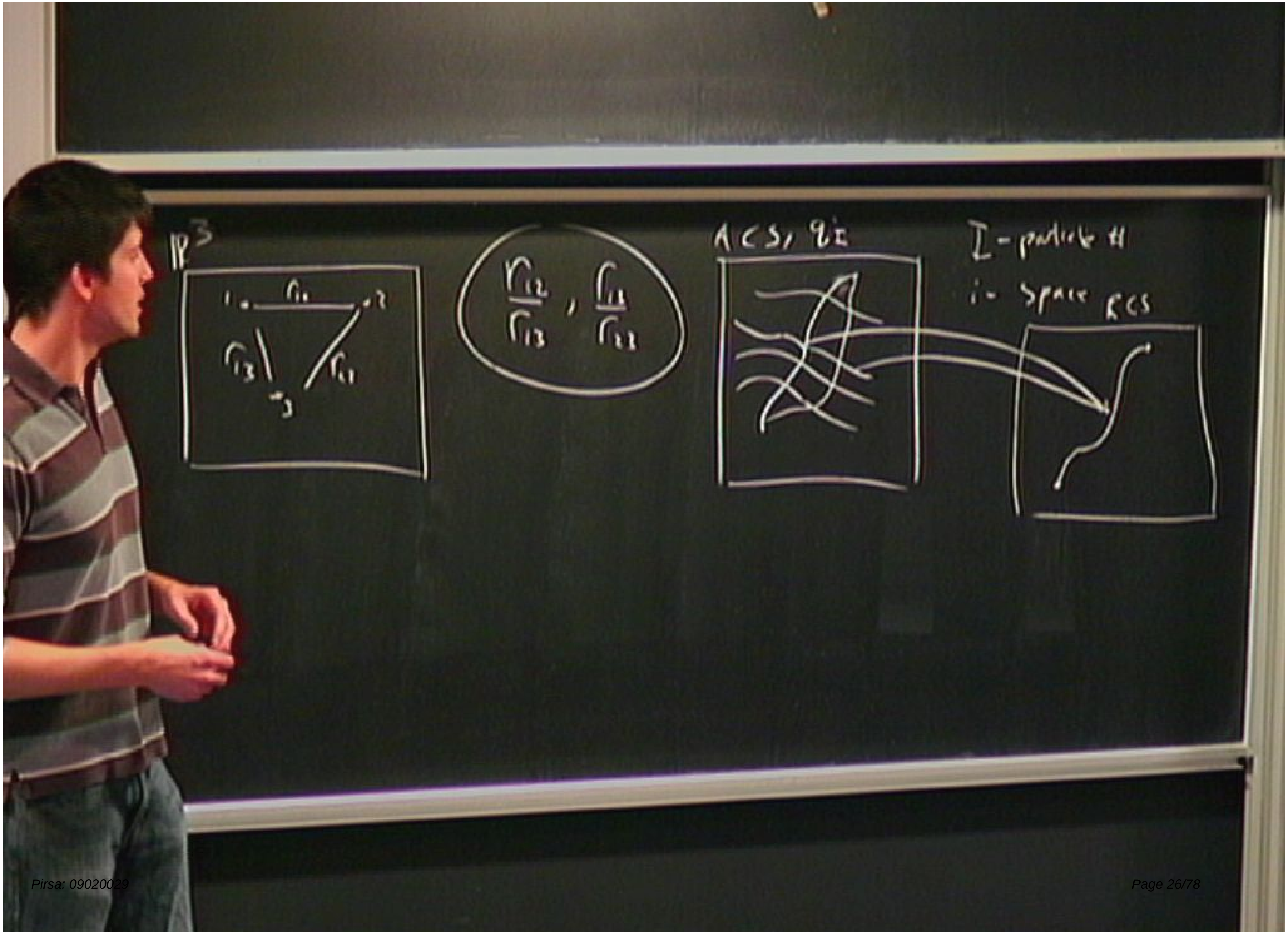

$\mathbb{P}^3$
1
2
3
r_{12}
r_{13}



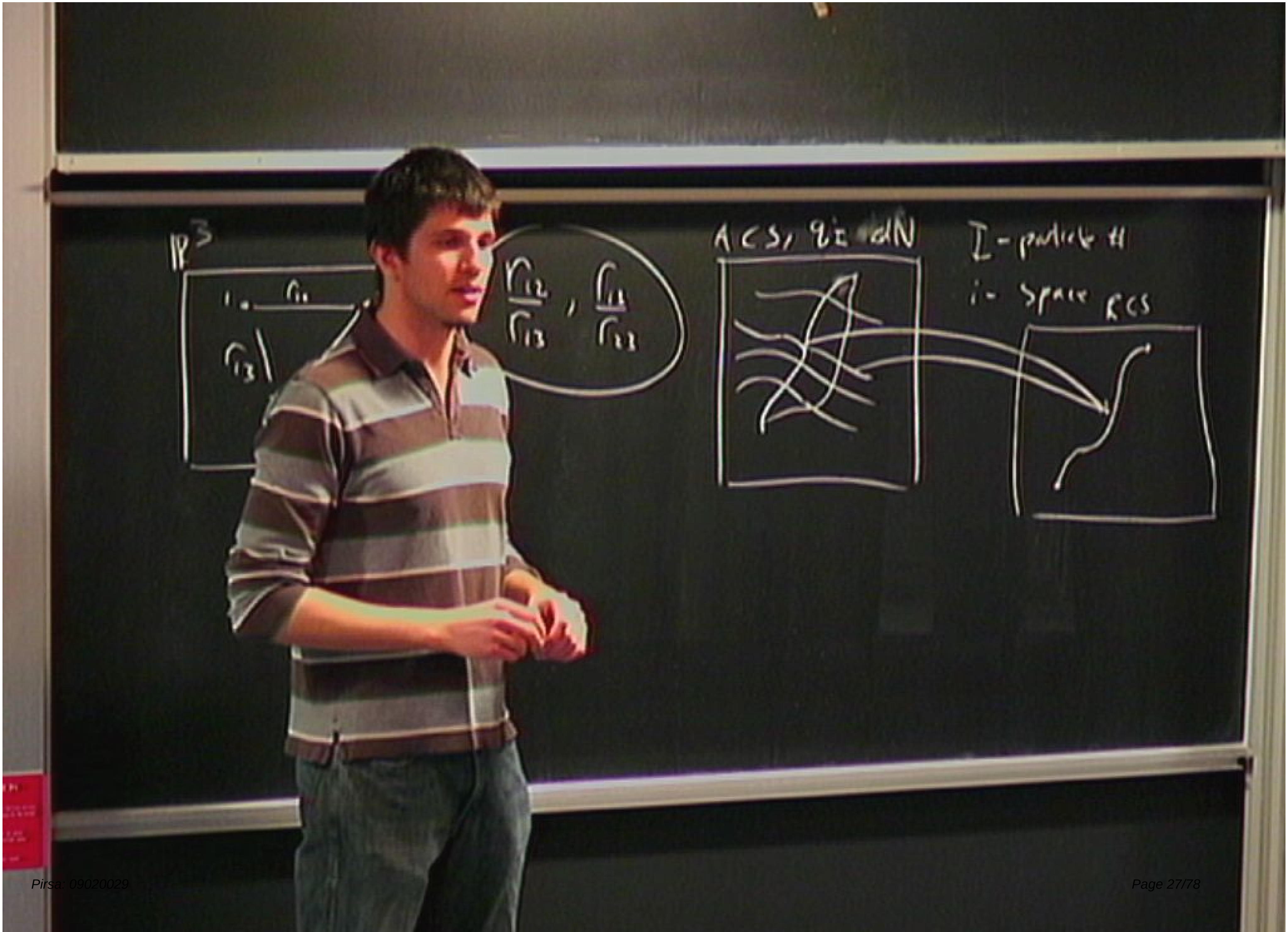

I - particle #
i - space

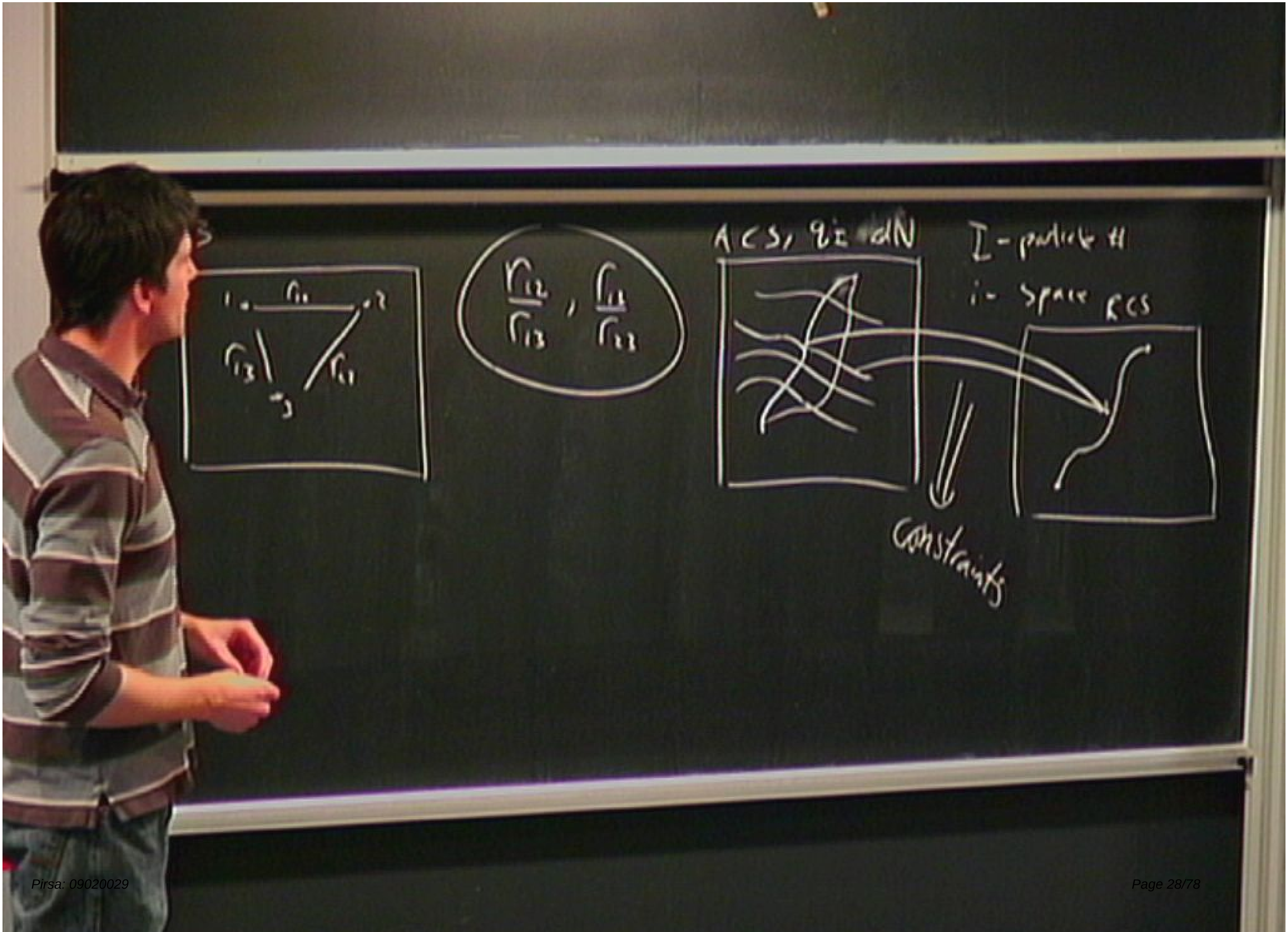

I - particle #
i - space
RCS

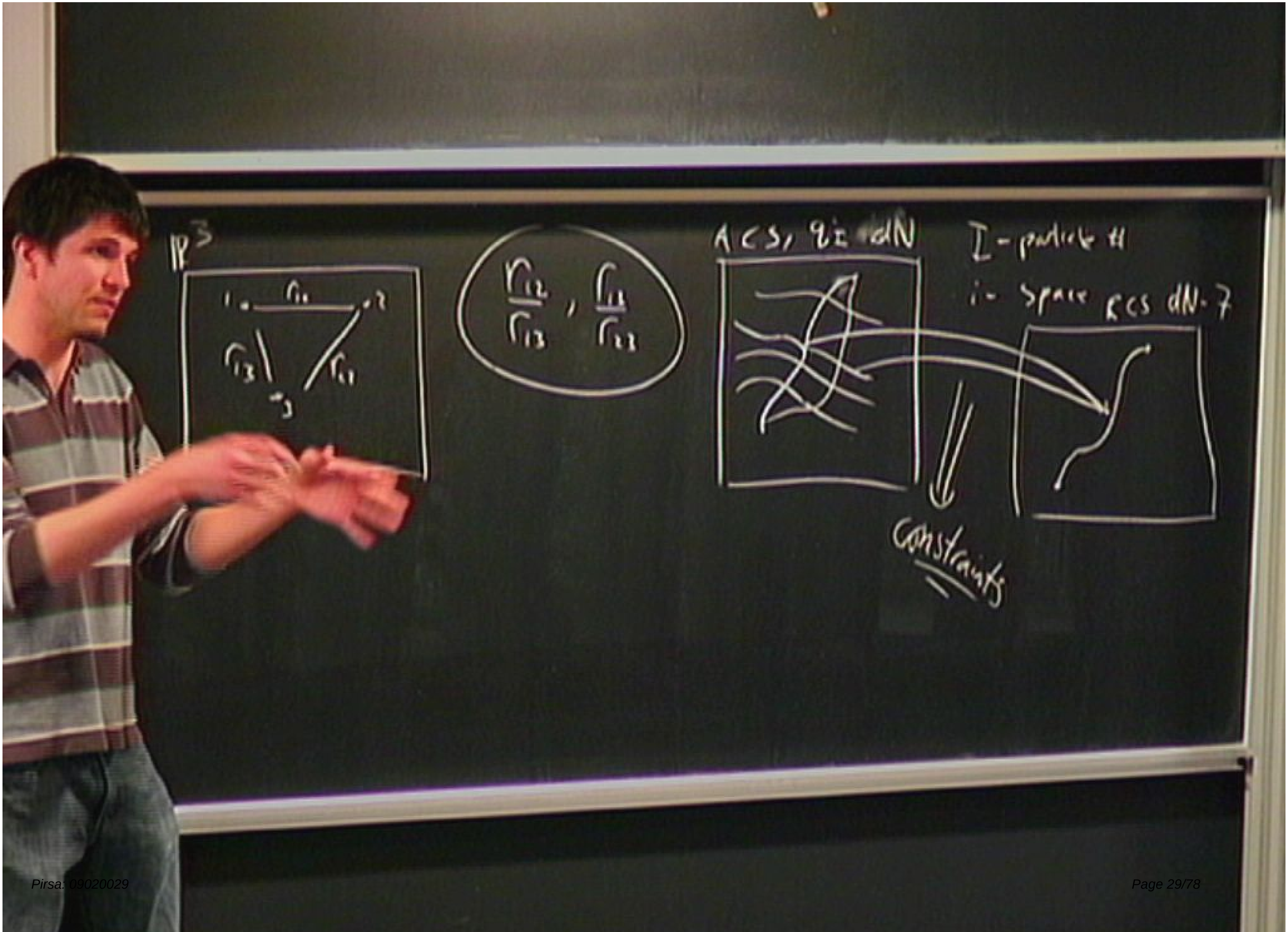

in space

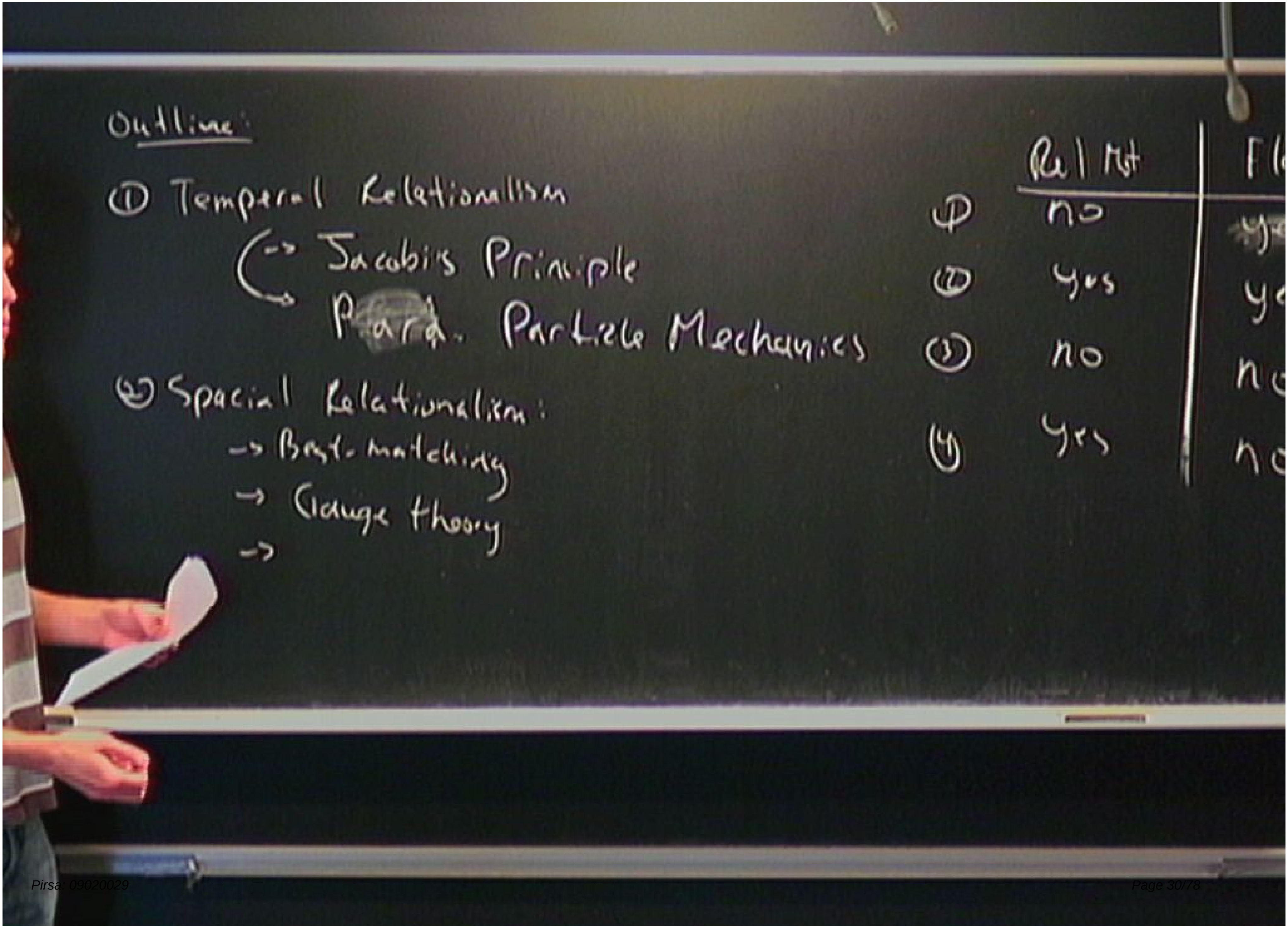


A ⊂ S, q_I
I - particle #
i - space

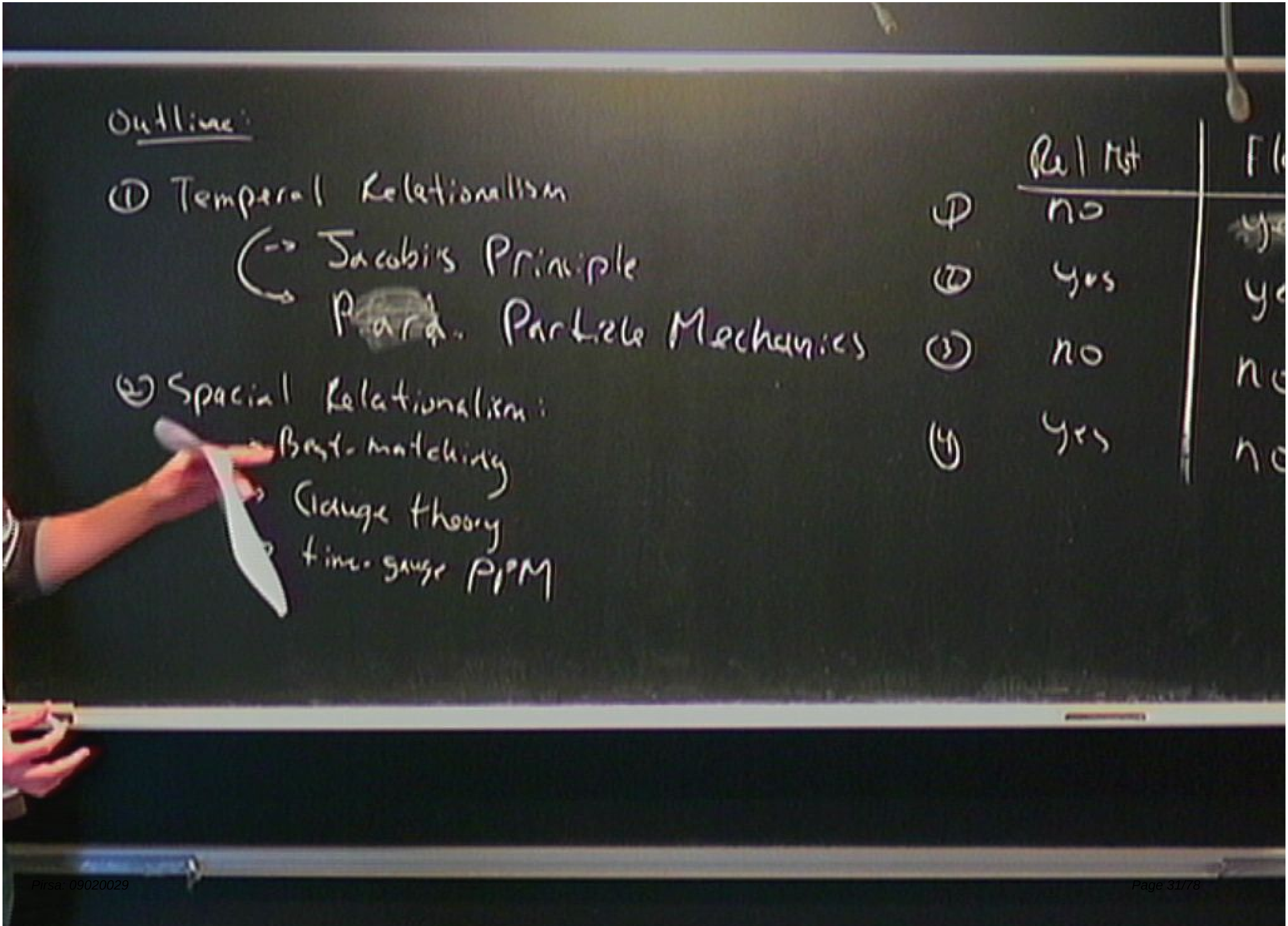



constraints





Outline:
① Temporal Relationalism
→ Jacobi's Principle
Particle Mechanics
② Spacial Relationalism:
→ Best-matching
→ Gauge theory
Rel Mot
① no
② yes
③ no
④ yes


Outline:
① Temporal Relationalism
→ Jacobi's Principle
→ Rel. Particle Mechanics
② Spacial Relationalism:
→ Best-matching
→ Gauge theory
→ time-gauge RPM
Rel Mot
① no
② yes
③ no
④ yes

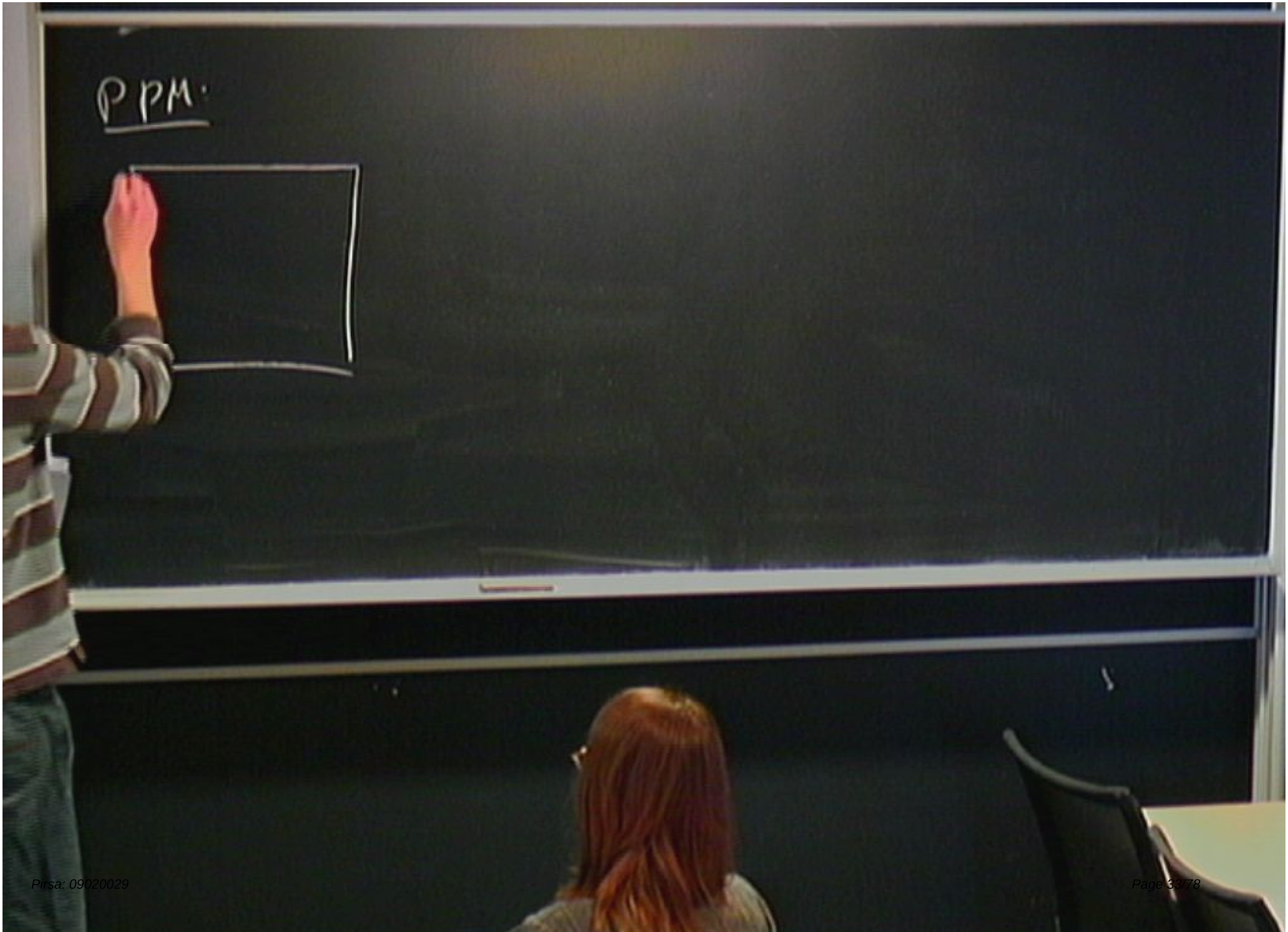
Outline:

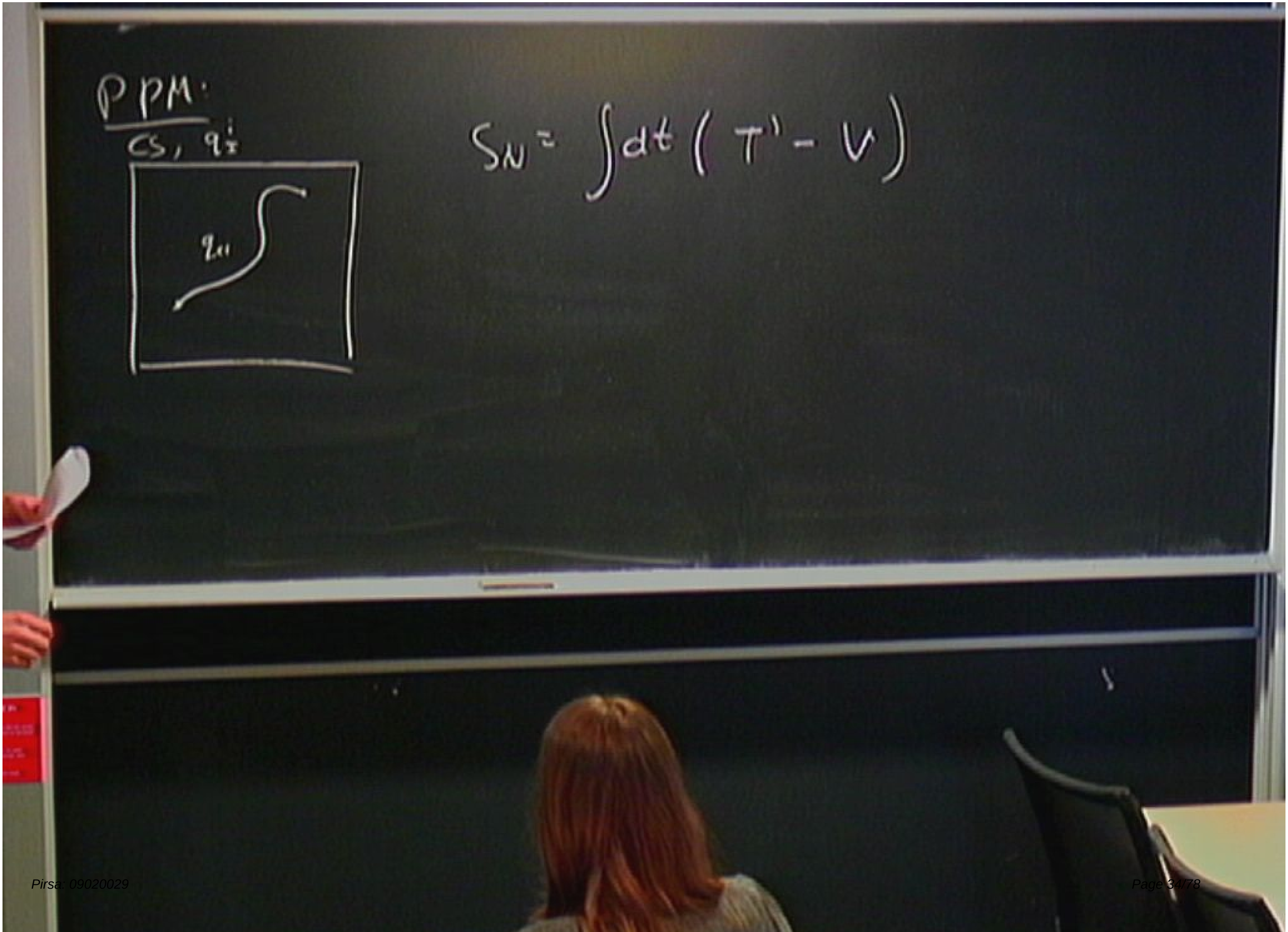
① Temporal Relationalism
- → Jacobi's Principle
- → Para. Particle Mechanics

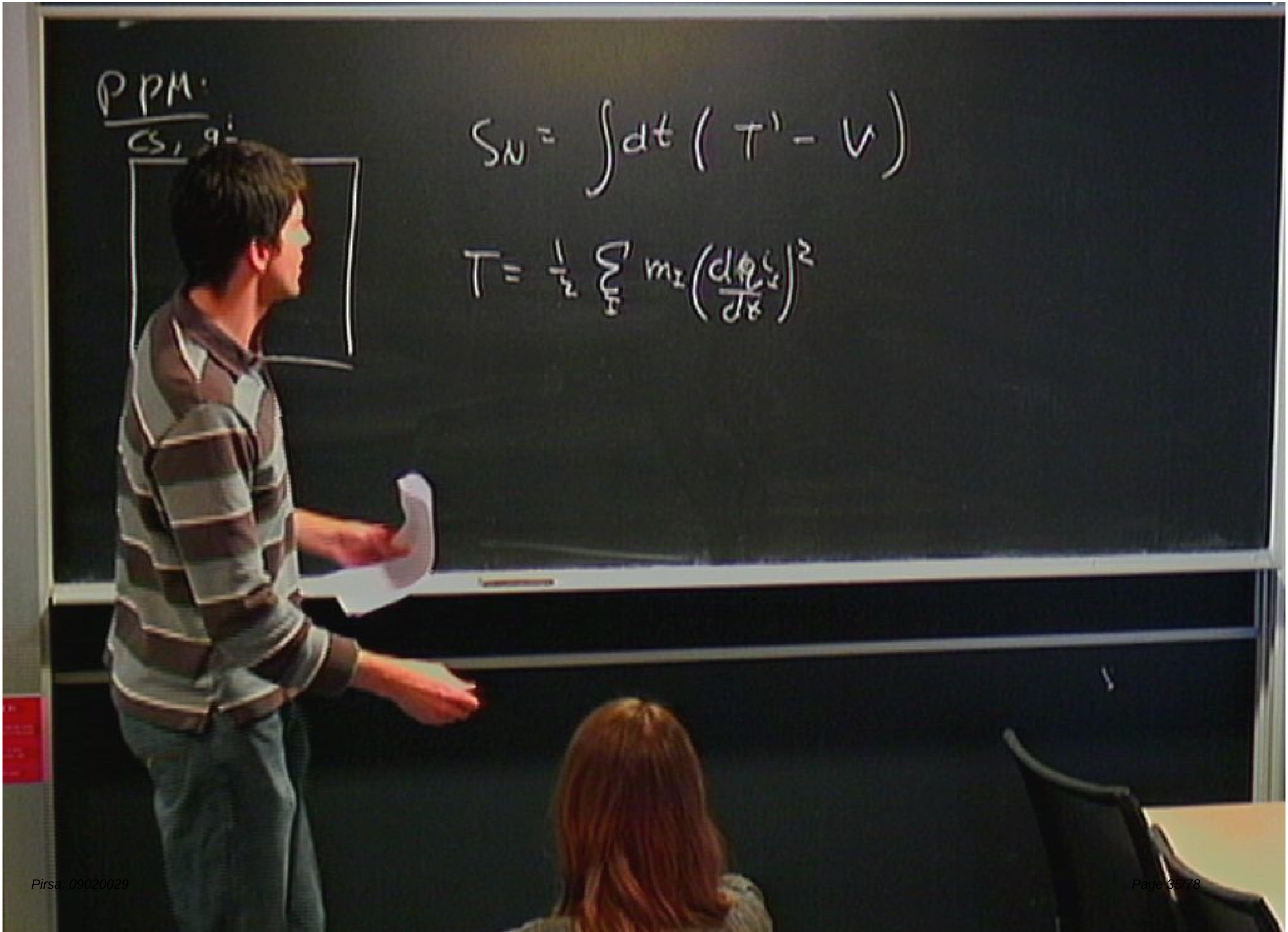
② Spacial Relationalism:
- → Best-matching
- → Gauge theory
- → time-gauge PPM

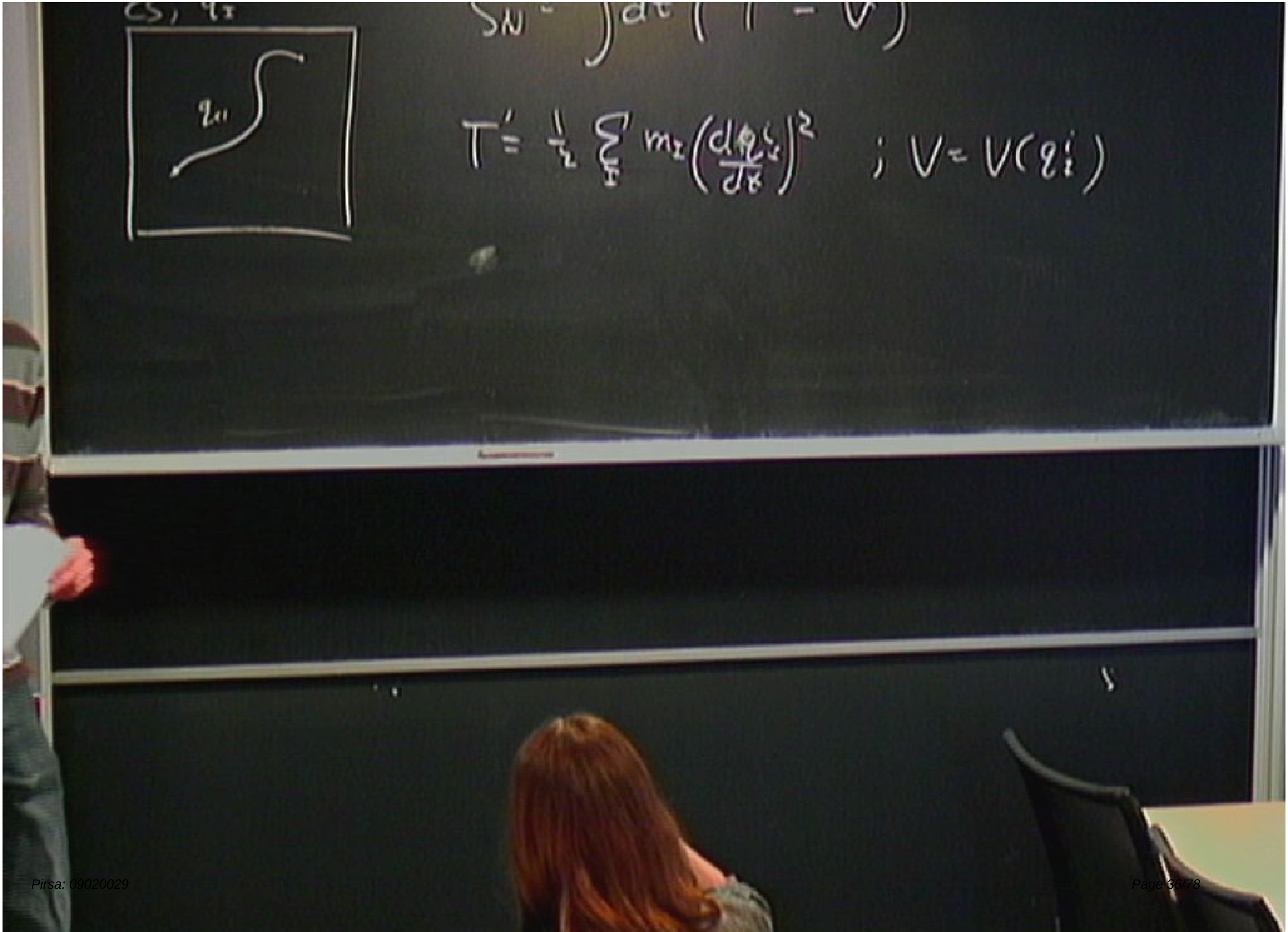
① GR
- → Warm (BM field theory)
- → GR

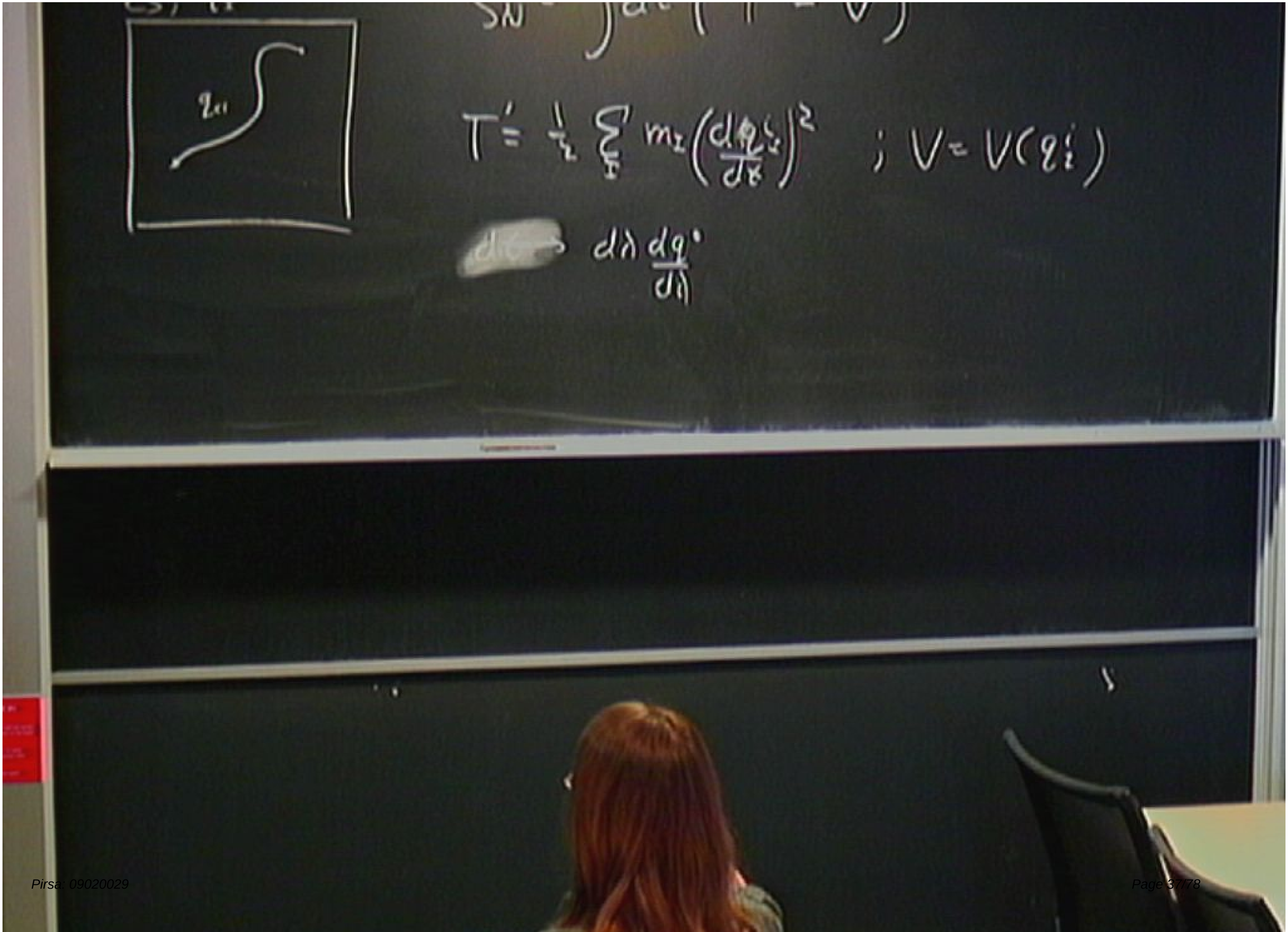
	Rel Mot	Fl
①	no	ye
②	yes	ye
③	no	no
④	yes	no

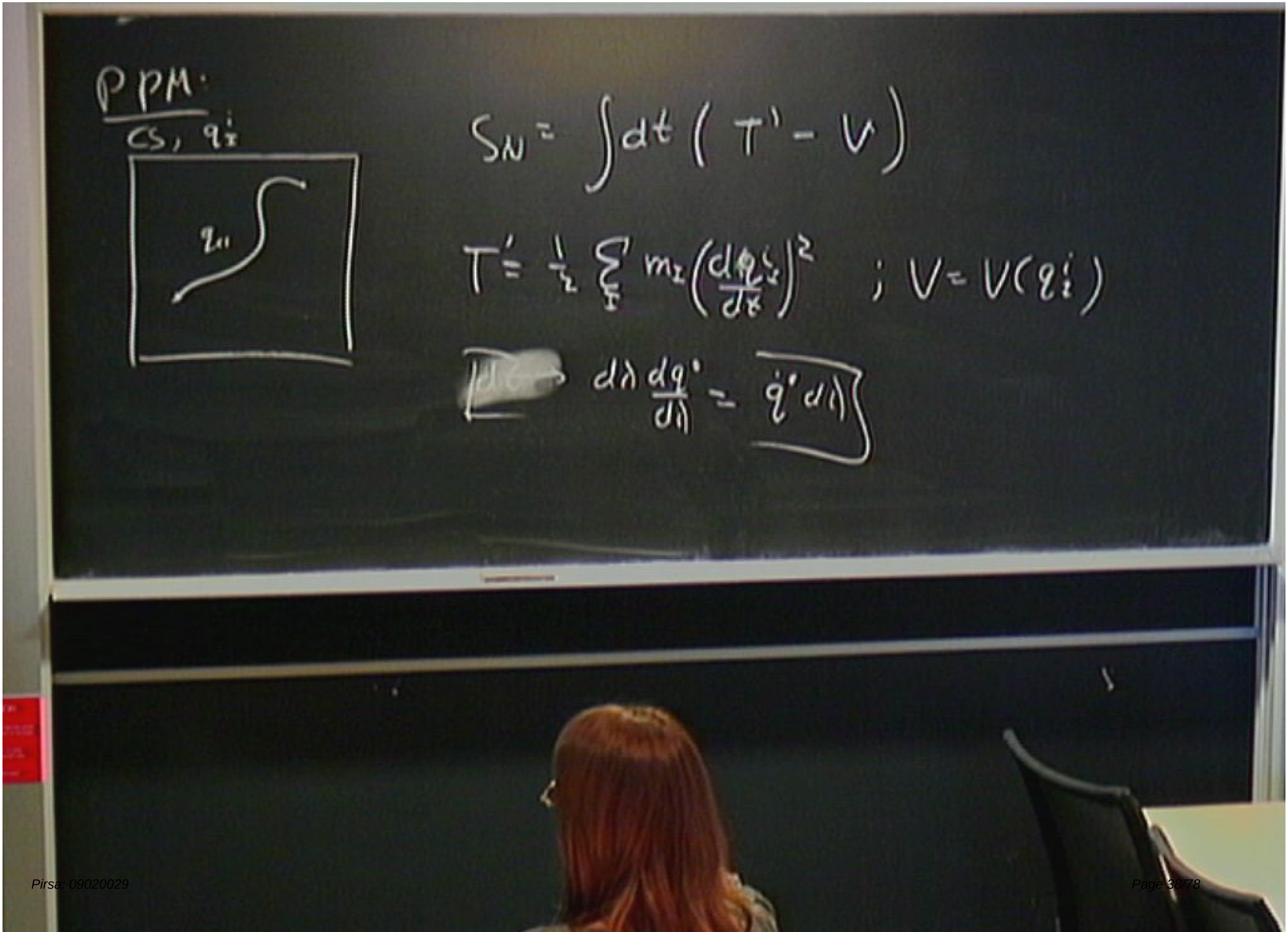

PPM:

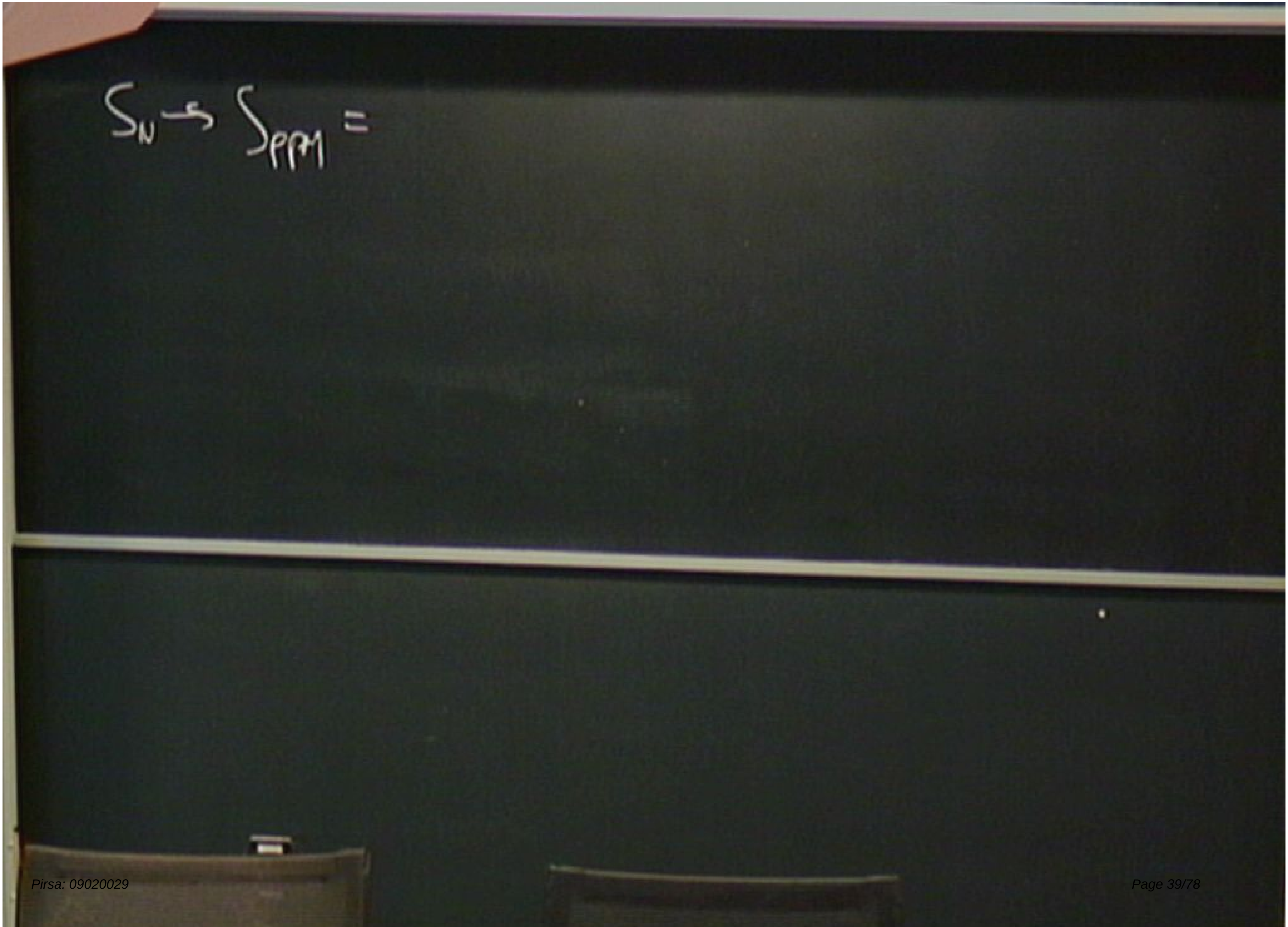

PPM:
CS,
S_N = ∫dt (T' - V)


PPM:
CS, q^i
S_N = ∫dt (T' − V)
T = ½ Σ_I m_I (dq^i_I/dτ)^2

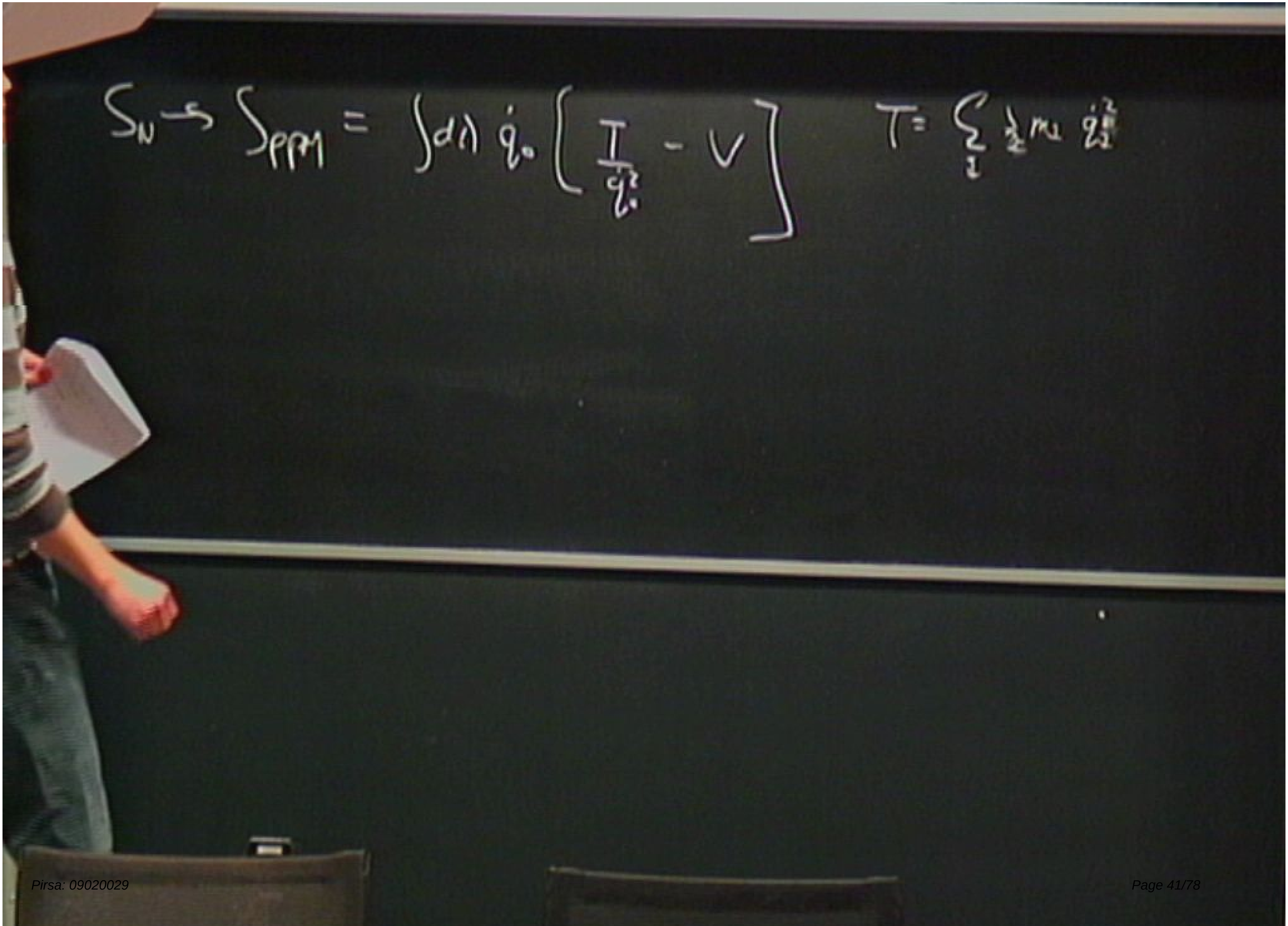

$S_N = \int dt\,(T - V)$
q_{I1}
$T' = \frac{1}{2}\sum_I m_I \left(\frac{dq^i_I}{dt}\right)^2 \; ; \; V = V(q^i_I)$


T' = ½ Σ m_i (dq^i/dτ)^2 ; V = V(q^i)
dλ dq^i/dλ


PPM:
CS, q^i_I
S_N = ∫dt (T' − V)
T' = ½ Σ_I m_I (dq^i_I/dτ)² ; V = V(q^i_I)
dλ dq^a/dλ = q̇^a dλ


$S_N \to S_{PPM} =$

$$S_N \to S_{PPM} = \int d\lambda\, \dot{q}_0 \left[\frac{I}{\dot{q}_0^2} - V \right]$$


S_N → S_PPM = ∫dλ q̇_0 [T/q̇_0² − V]
T = Σ_i ½ m_i q̇_i²

$$S_N \to \boxed{S_{PPM}} = \int d\lambda\, \dot{q}_0 \left[\frac{T}{\dot{q}_0^2} - V \right] \qquad T = \sum_i \tfrac{1}{2} m_i \dot{q}_i^2$$

$$= \int d\lambda \left[\frac{T}{\dot{q}_0} - V \dot{q}_0 \right]$$

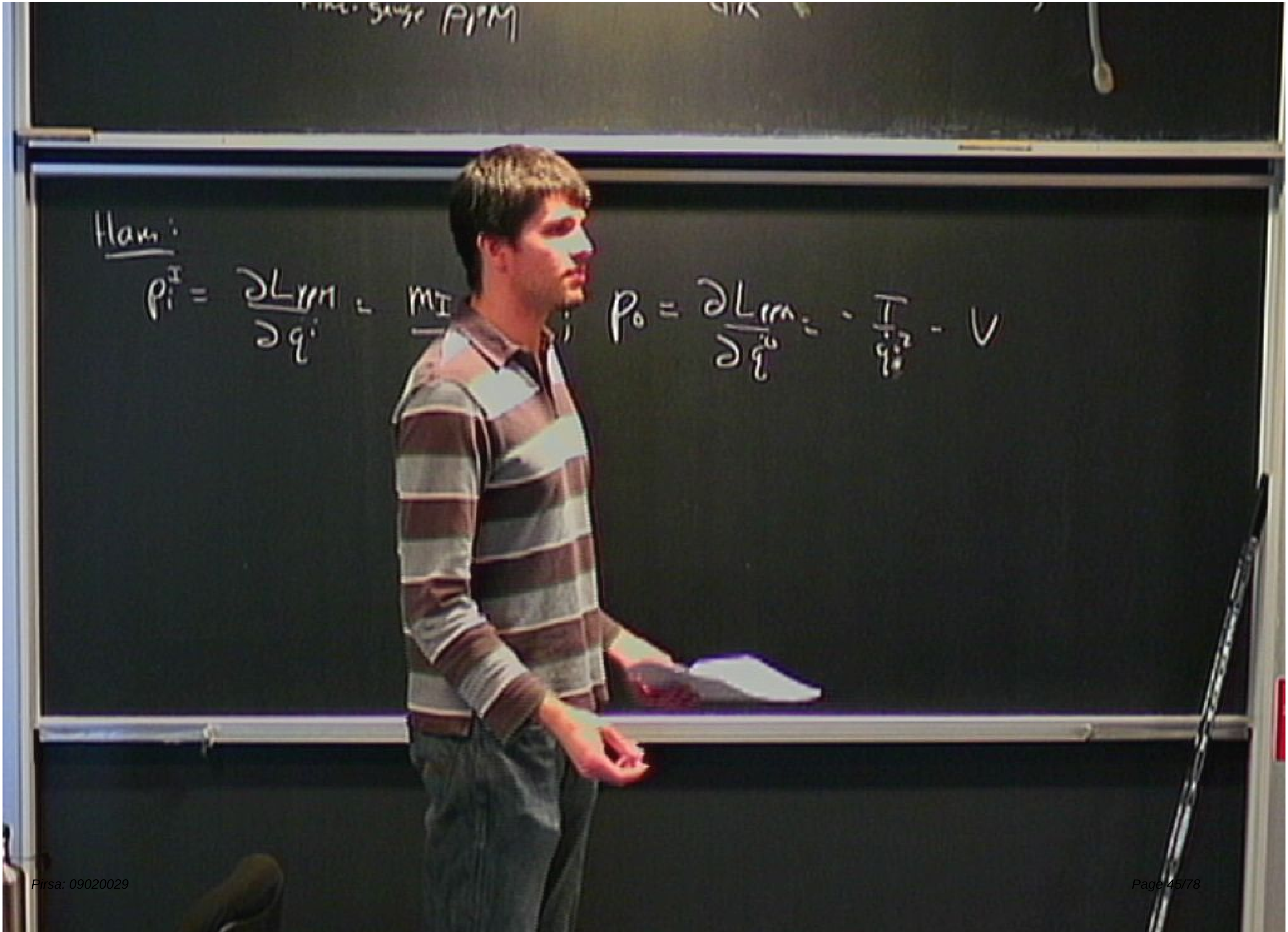
"It is utterly beyond our power to measure the changes of things by time. Quite the contrary, time is an abstraction, at which we arrive by means of the changes of things; made because we are not restricted to any one definite measure, all being interconnected."

- E. Mach in The *Mechanics*, 1872.

$$S_N \to S_{PPM} = \int d\lambda\, \dot{q}_0 \left[\frac{T}{\dot{q}_0^2} - V \right]$$

$$= \int d\lambda \left[\frac{T}{\dot{q}_0} - V \dot{q}_0 \right]$$

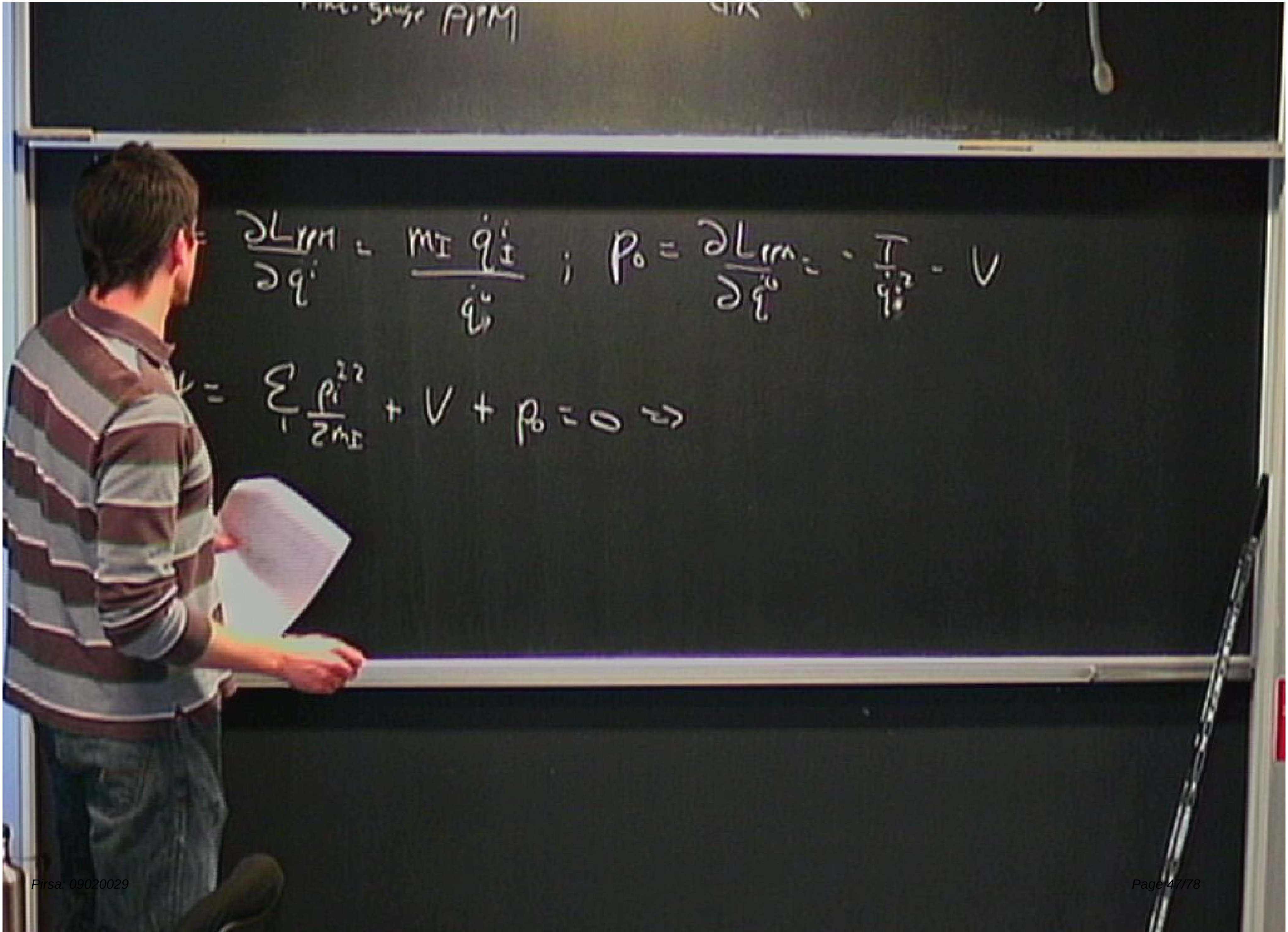
$$T = \sum_i \frac{1}{2} m_i \dot{q}_i^2$$


Ham:

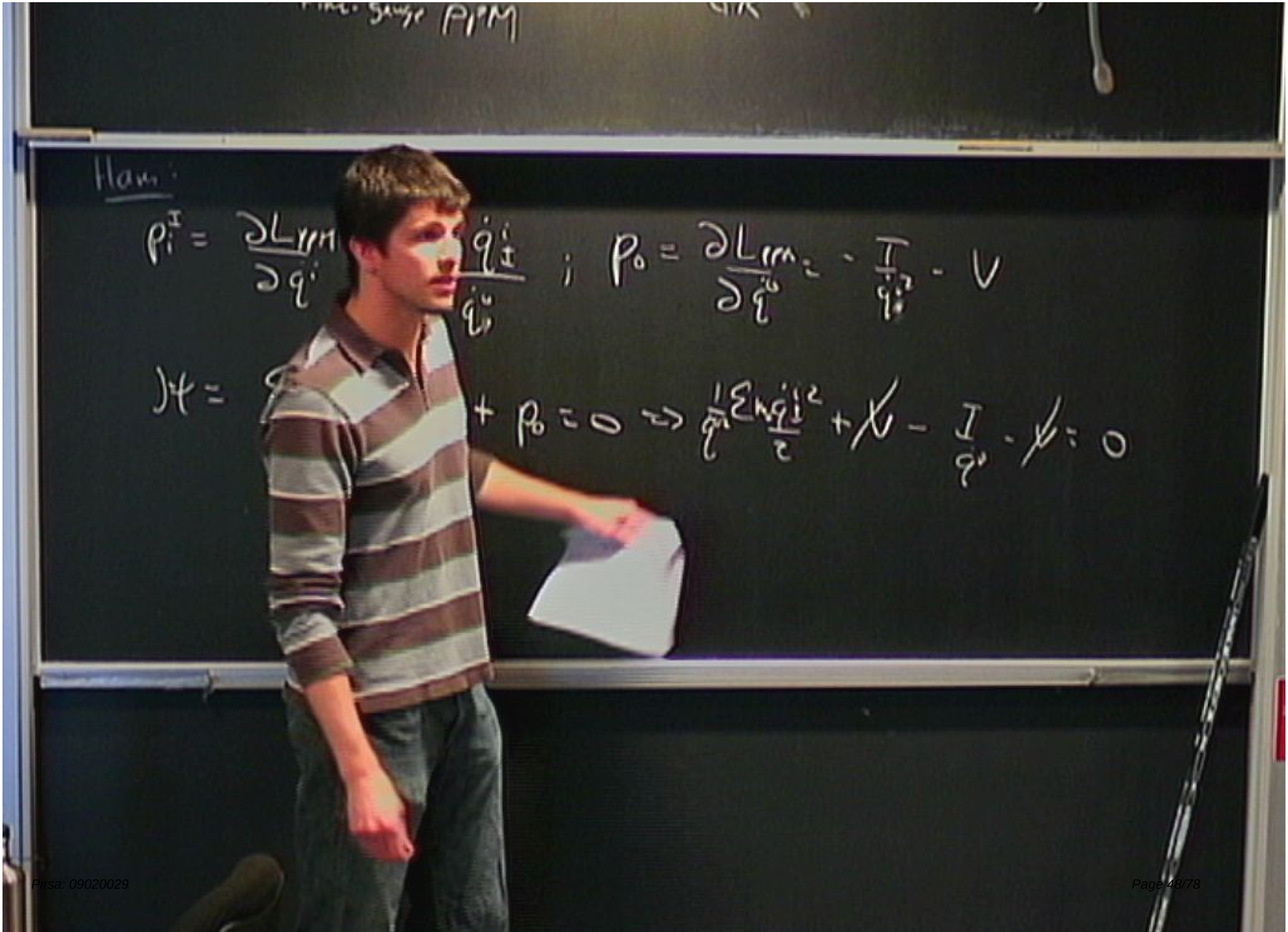
... gauge PRM

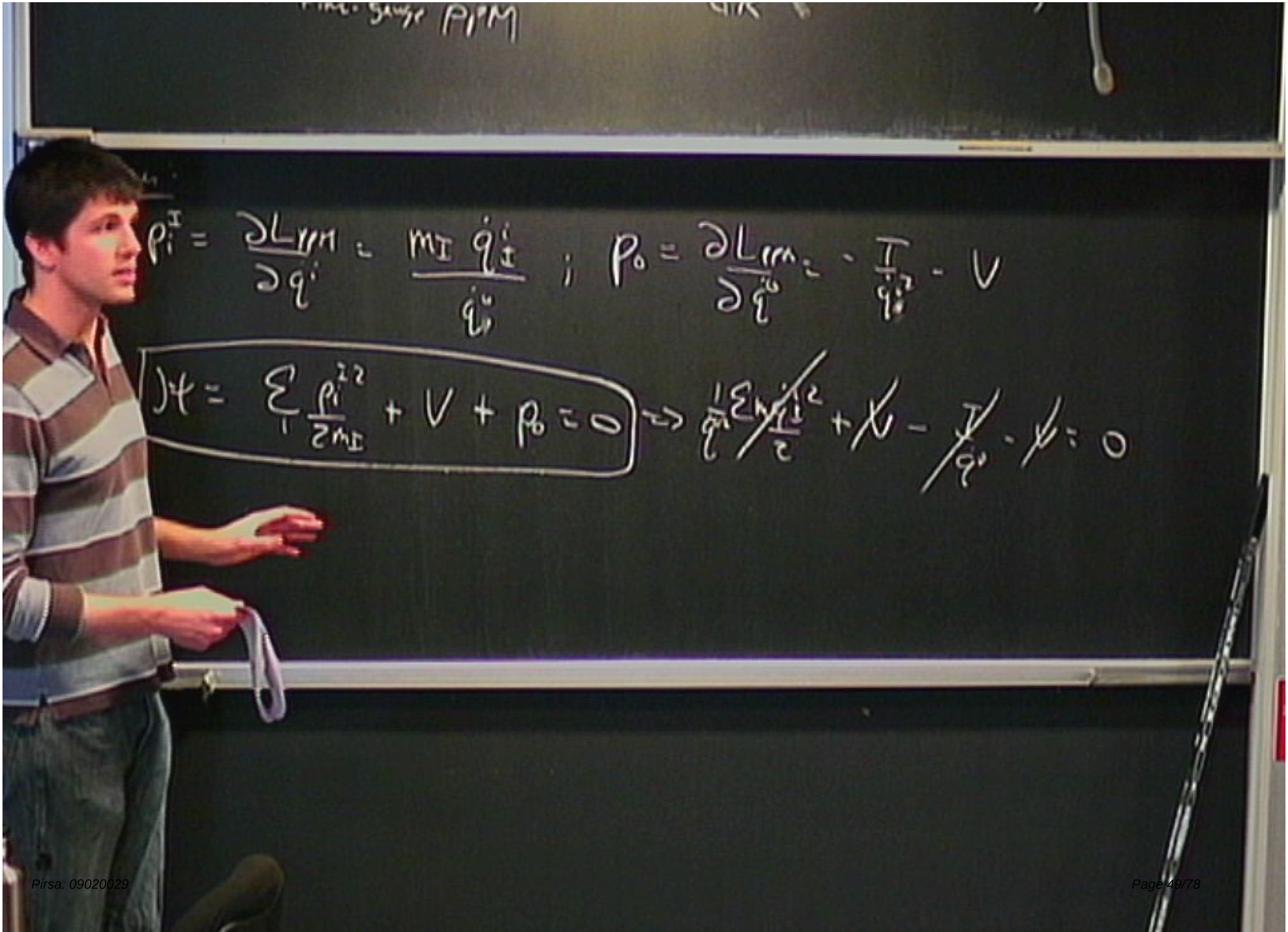
Ham:

$$p_i^I = \frac{\partial L_{PRM}}{\partial \dot{q}^i} = \frac{m_I \dot{q}^i_I}{\dot{q}_0} \; ; \quad p_0 = \frac{\partial L_{PRM}}{\partial \dot{q}^0} = -\frac{T}{\dot{q}_0^2} - V$$

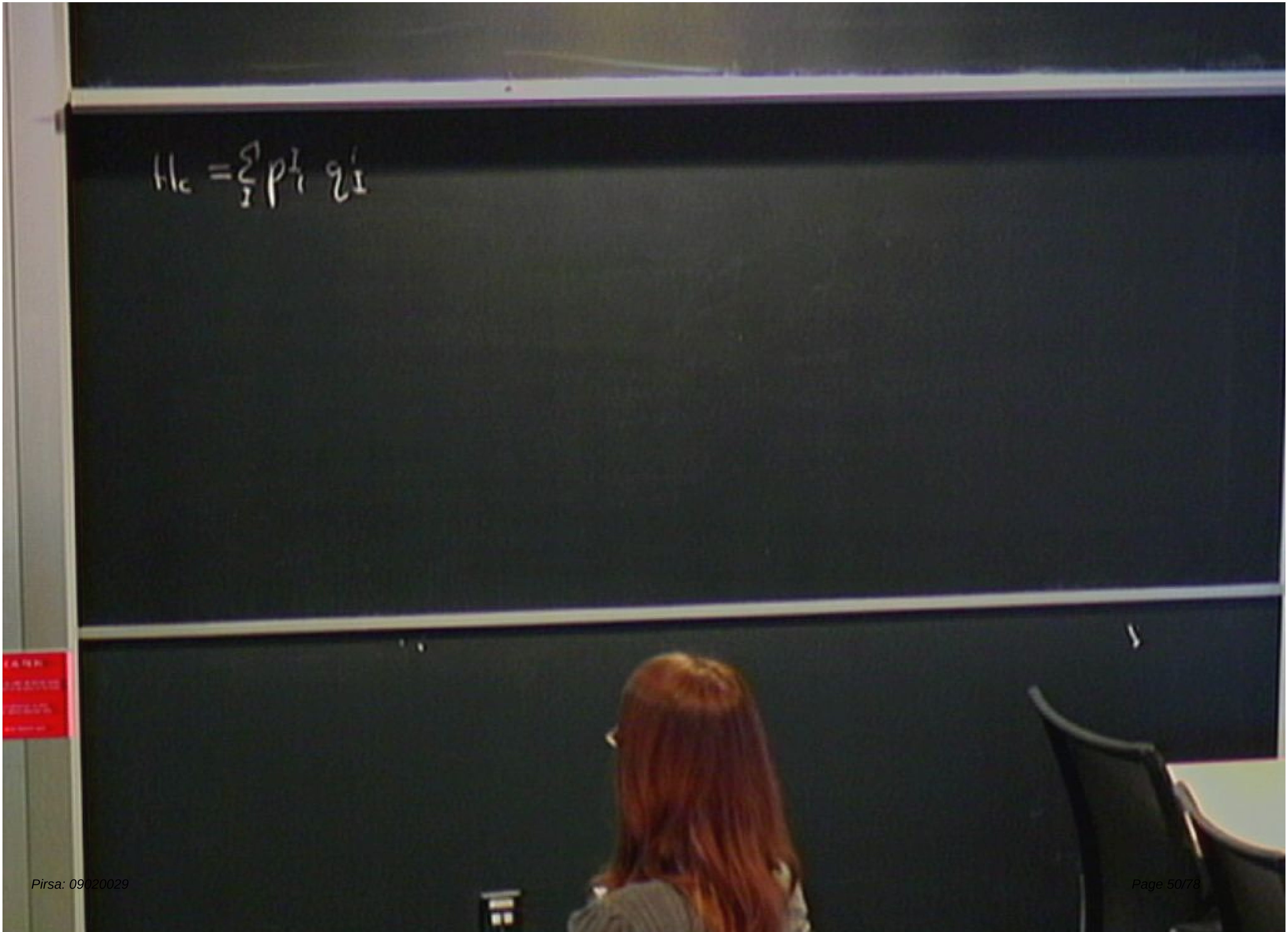


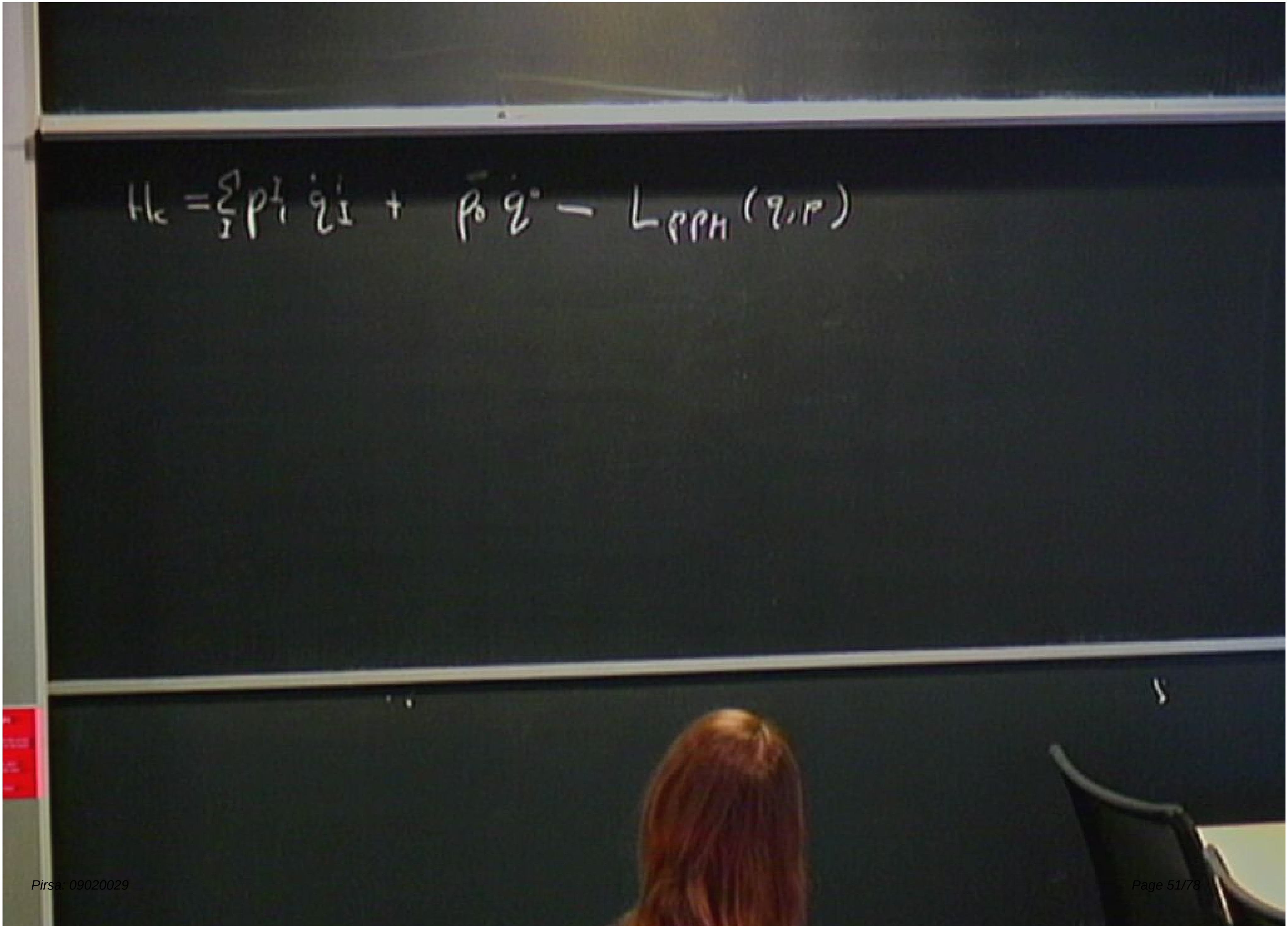
RPM
$\frac{\partial L_{rpm}}{\partial q^i} = \frac{m_I \dot{q}^i_I}{\dot{q}^0}$; $p_0 = \frac{\partial L_{rpm}}{\partial \dot{q}^0} = -\frac{T}{\dot{q}^{0\,2}} - V$
$= \sum_i \frac{p_i^2}{2m_I} + V + p_0 = 0 \Rightarrow$

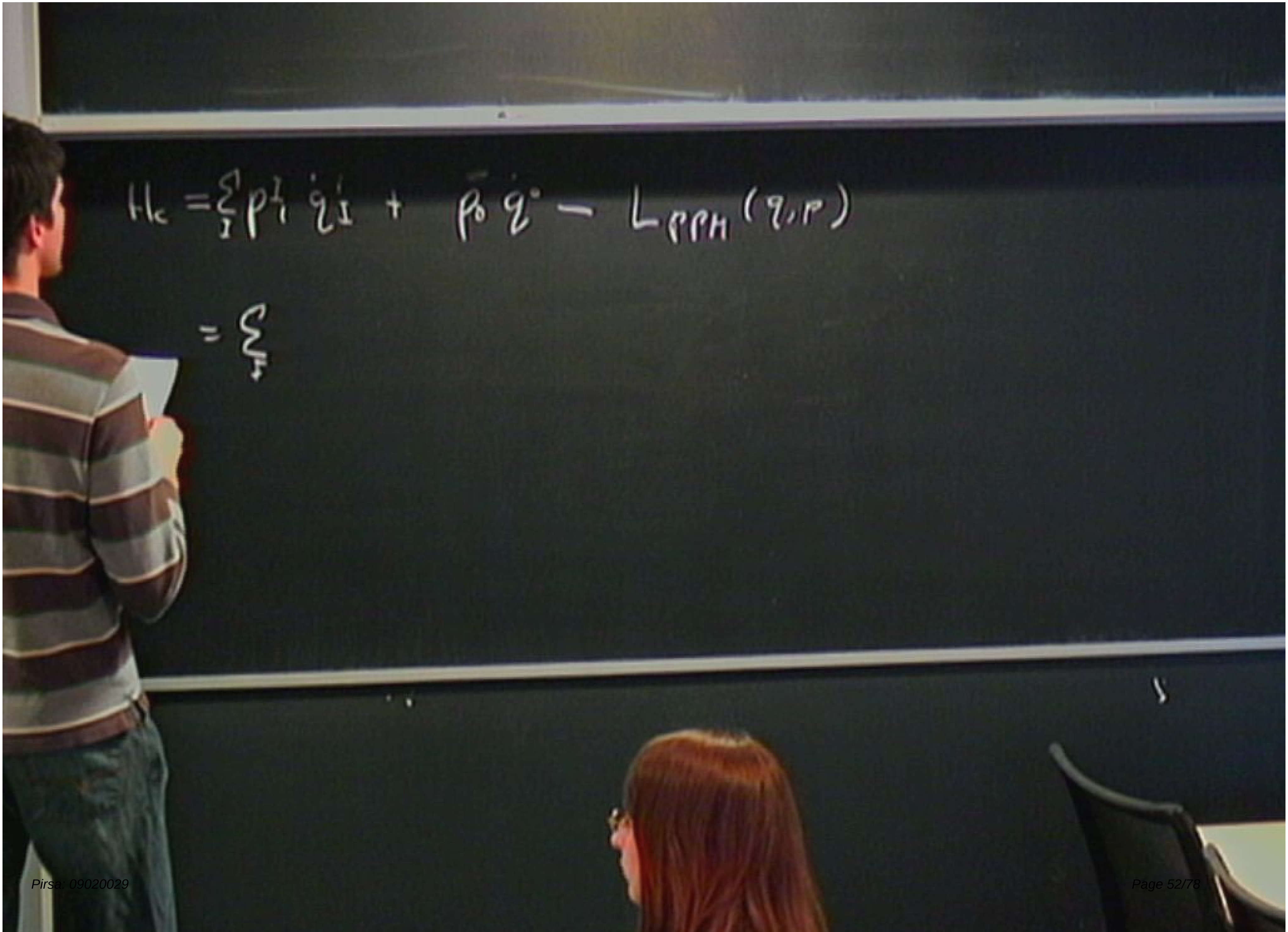

Ham:

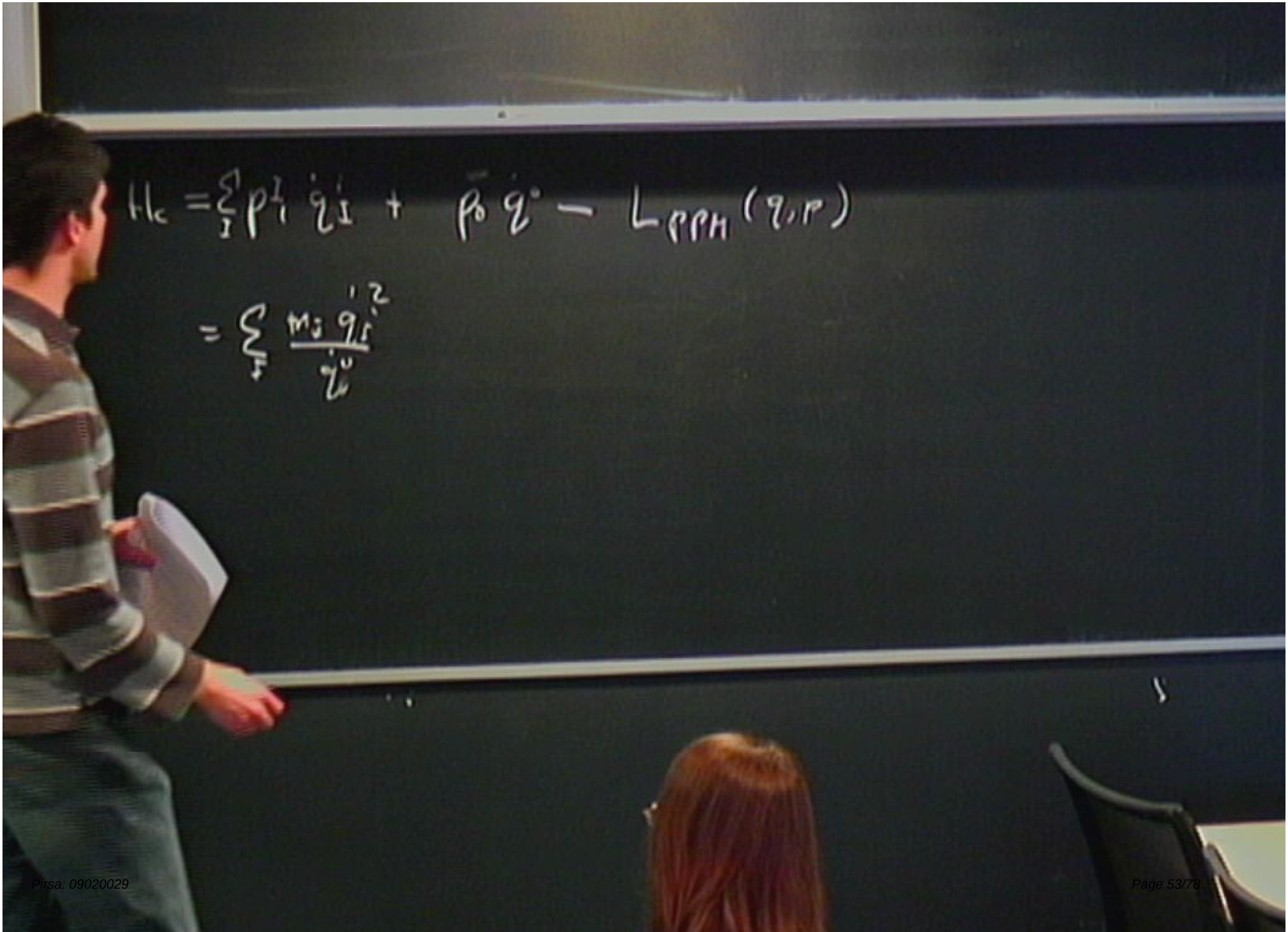


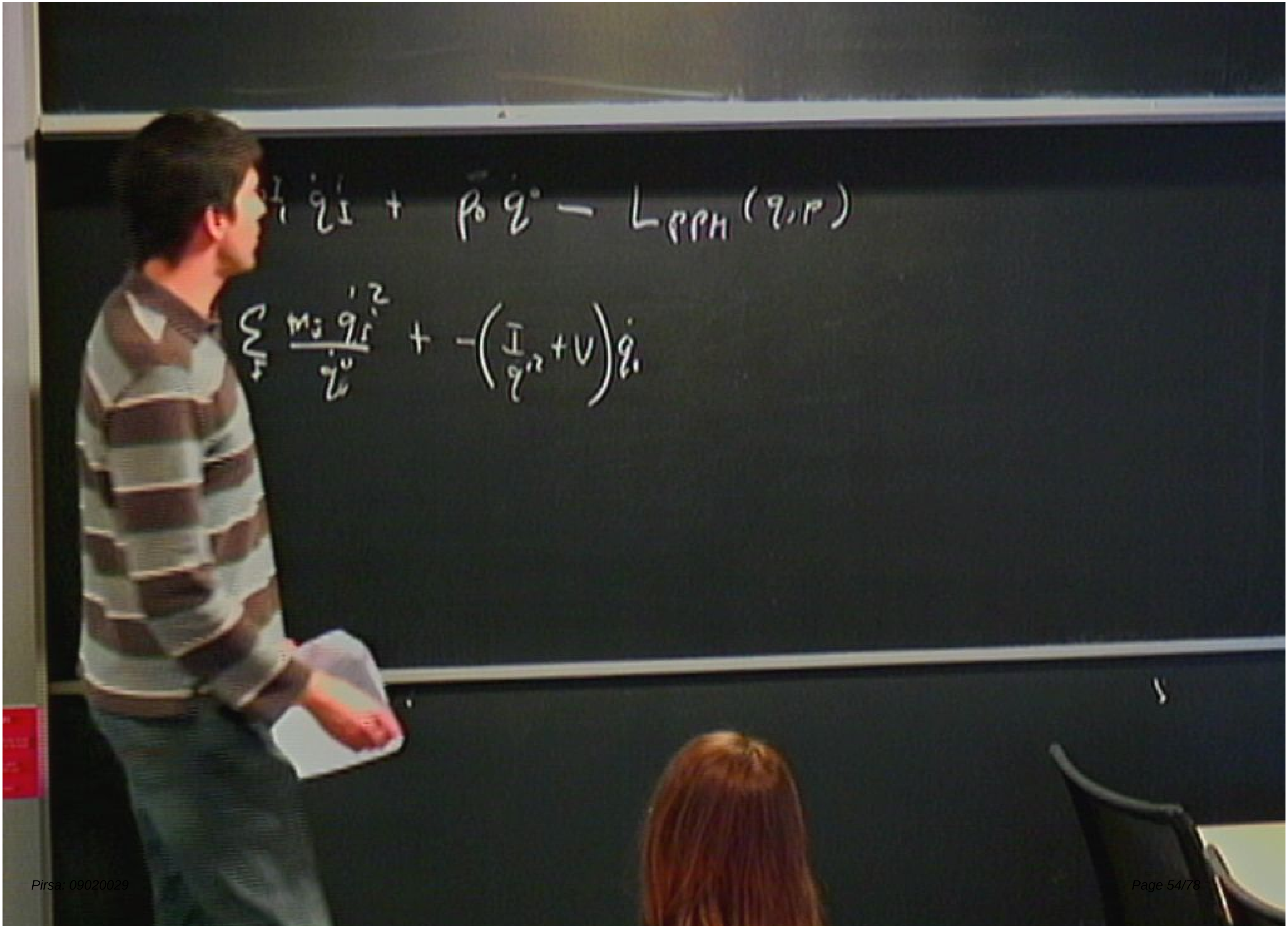
p_i^I = ∂L_RPM / ∂q^i = m_I q̇_I^i / q̇^0 ; p_0 = ∂L_RPM / ∂q̇^0 = - T / q̇^0² - V
H = Σ_i p_i^I² / 2m_I + V + p_0 = 0

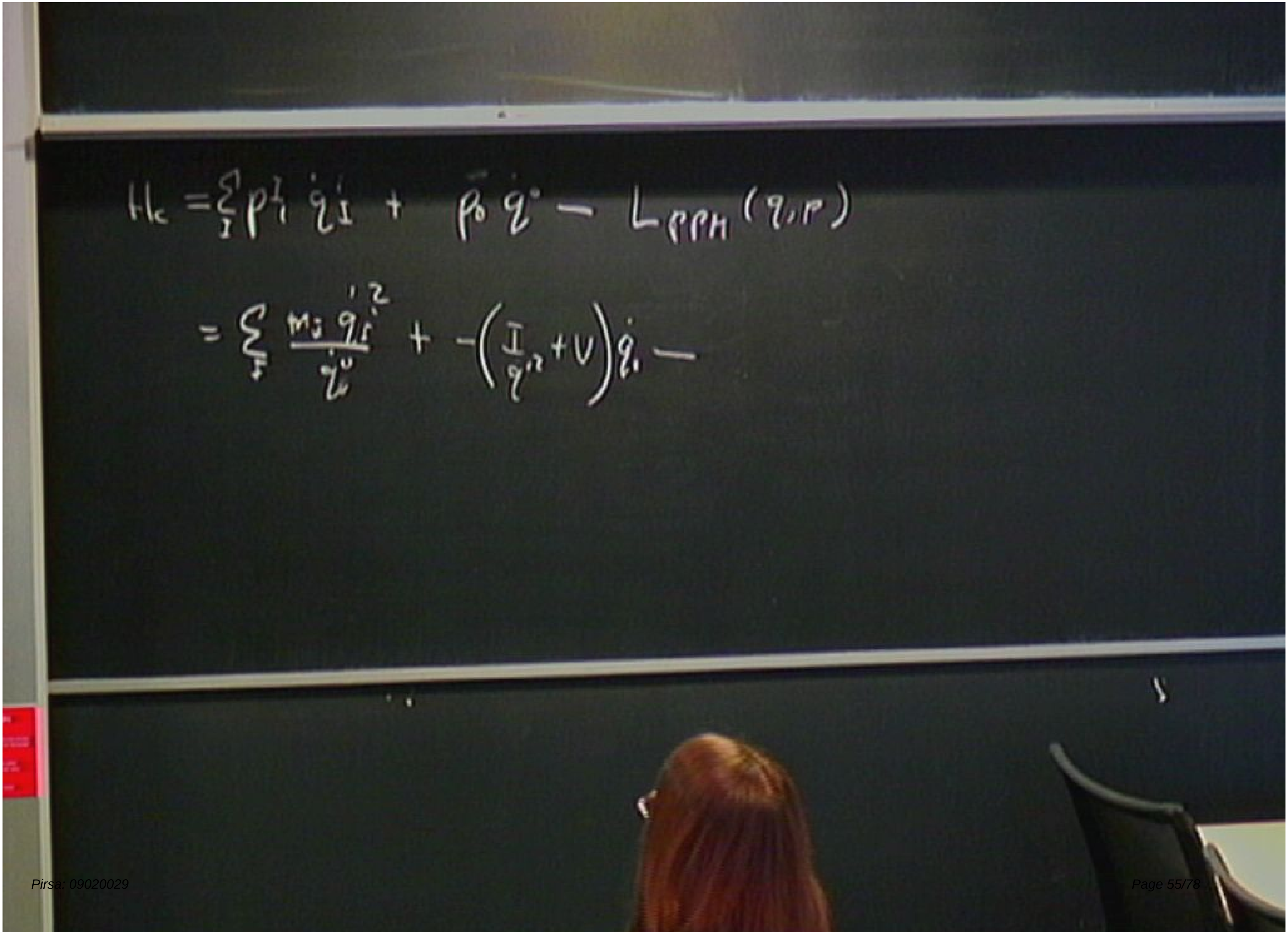

$H_c = \sum_I p^I_i q^i_I$

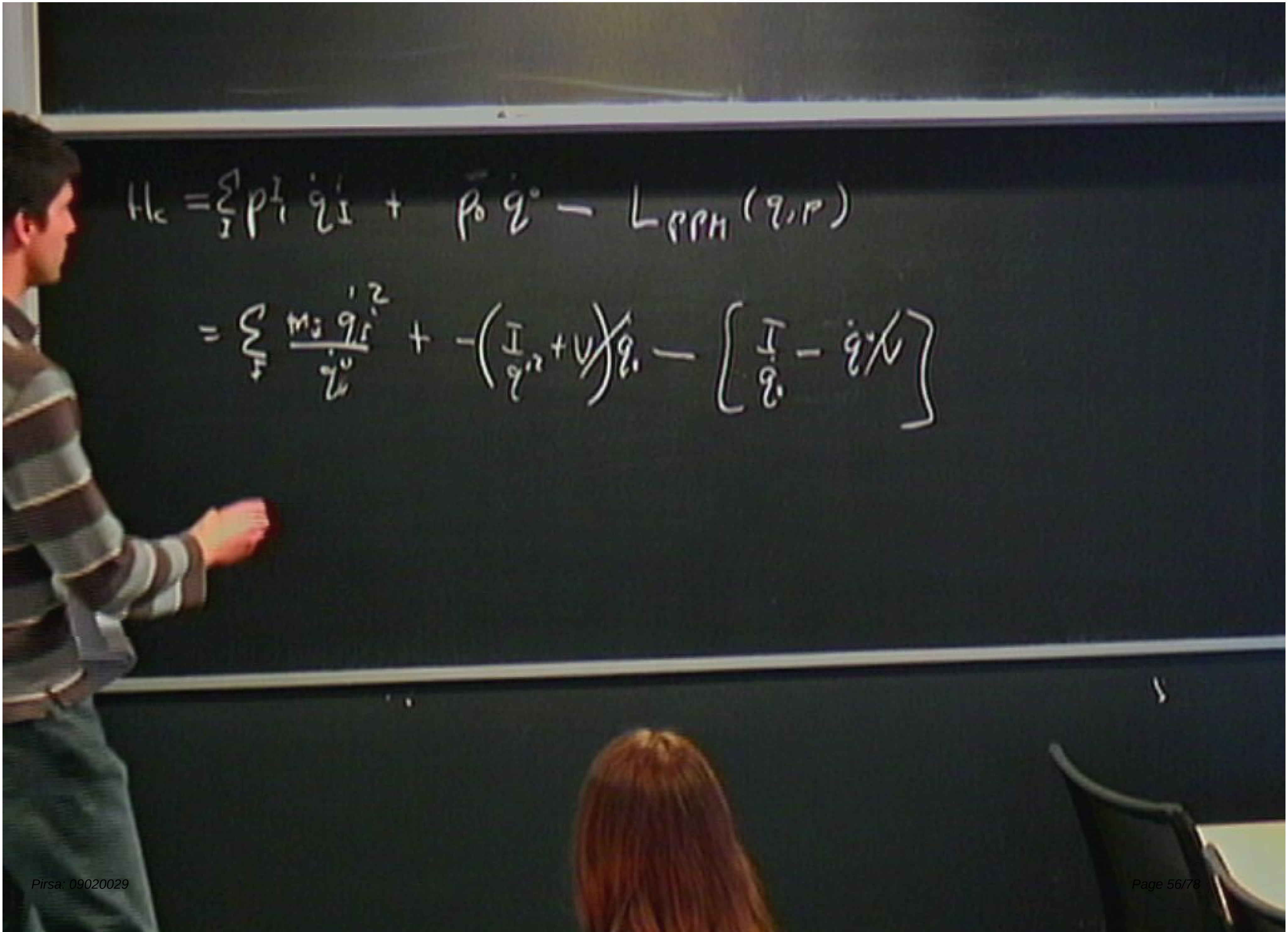

$H_c = \sum_i p^i_i \dot{q}^i + \bar{p}_0 \dot{q}^0 - L_{PPH}(q, p)$

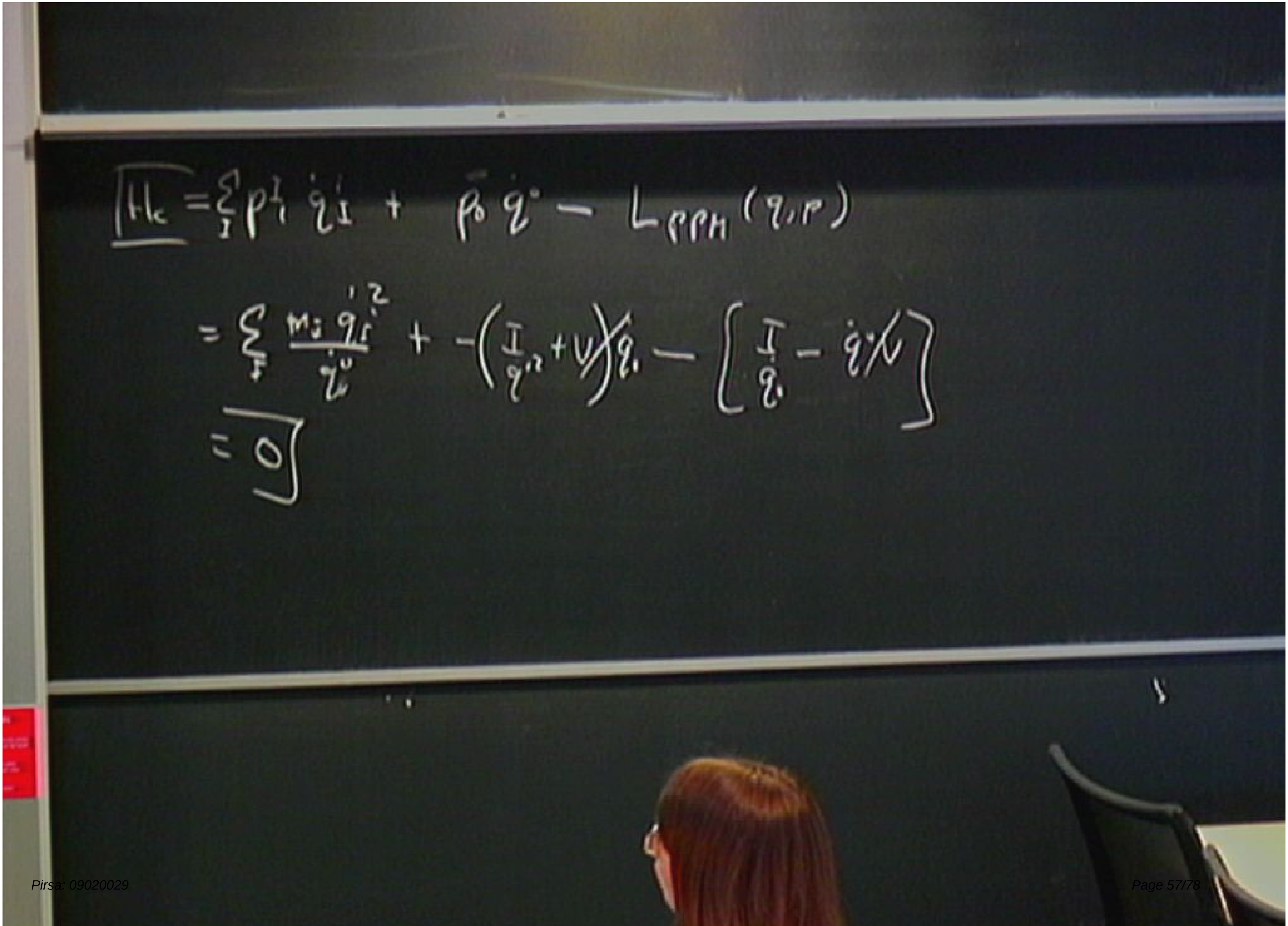

Hc = Σ p q̇ + p0 q̇0 − L_PPM (q,p)
= Σ


Hc = Σi pii q̇ii + p0 q̇0 − LPPN (q,p)
= Σi mi q̇i2 / q̇0

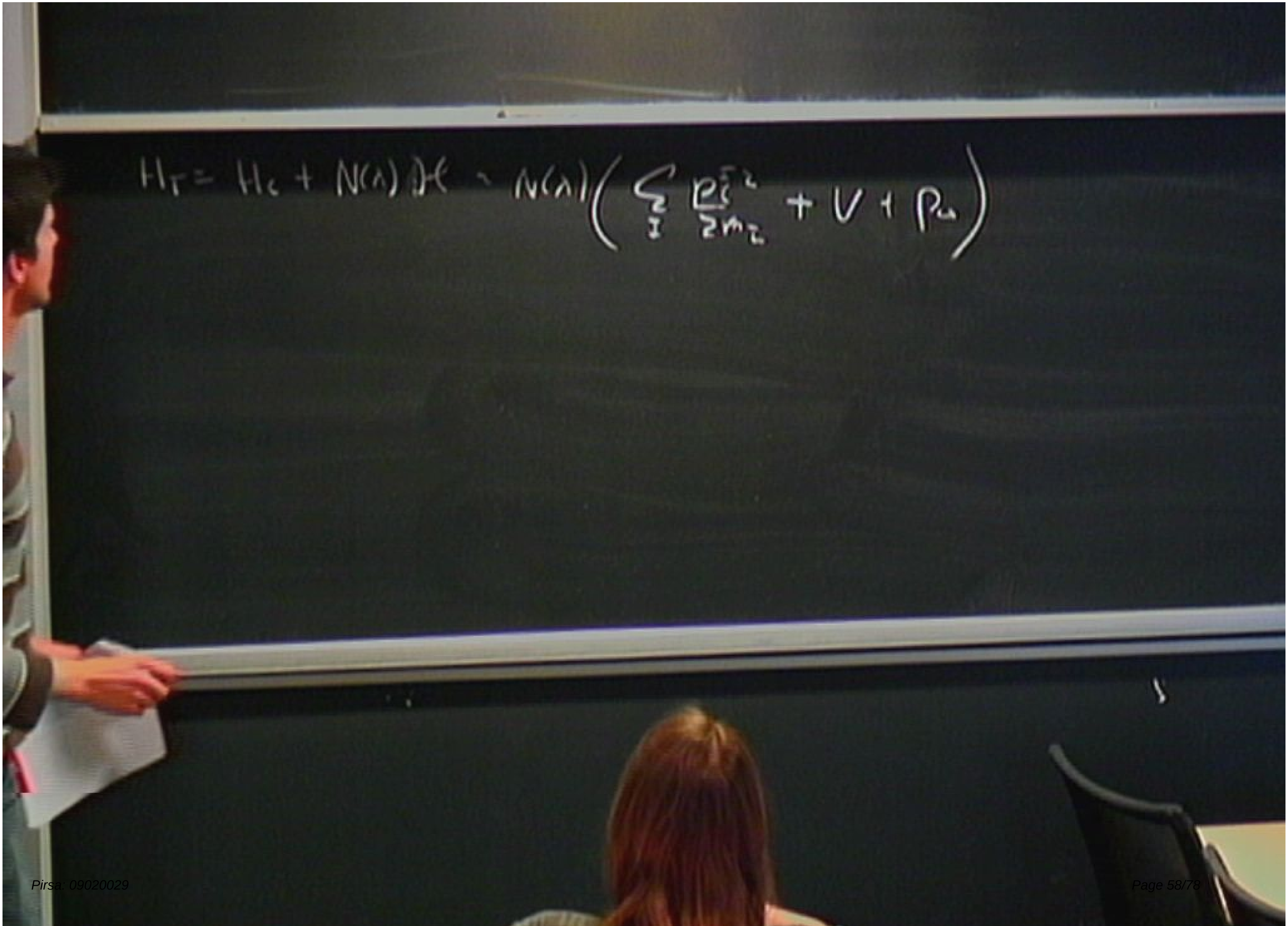


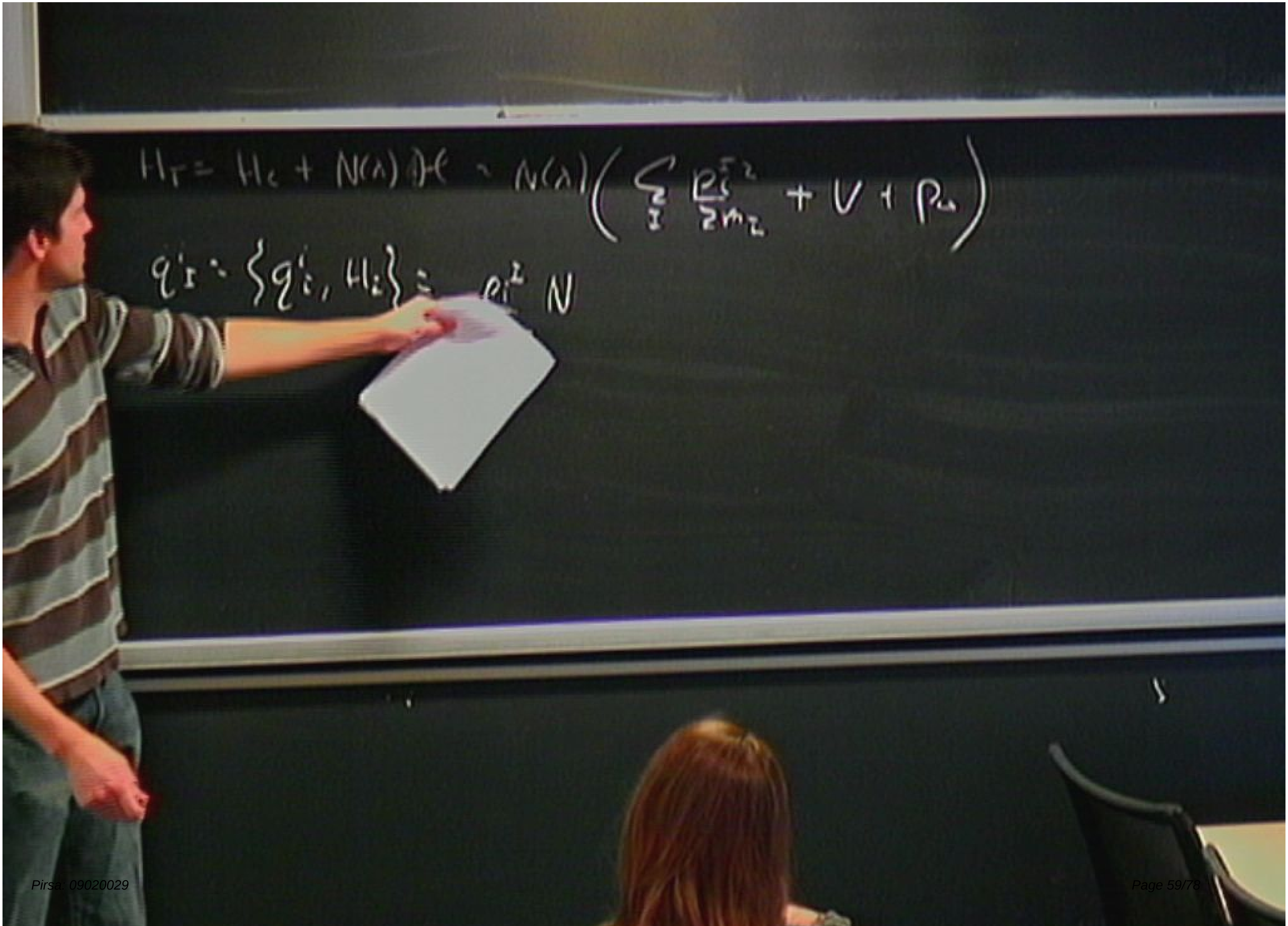

$H_c = \sum_i p^i_i \dot{q}^i_i + p_0 \dot{q}^0 - L_{PPN}(q, p)$
$= \sum_i \frac{m_i \dot{q}_i^{2}}{\dot{q}^0} + -\left(\frac{I}{q^{0 2}} + V\right)\dot{q}_0 -$

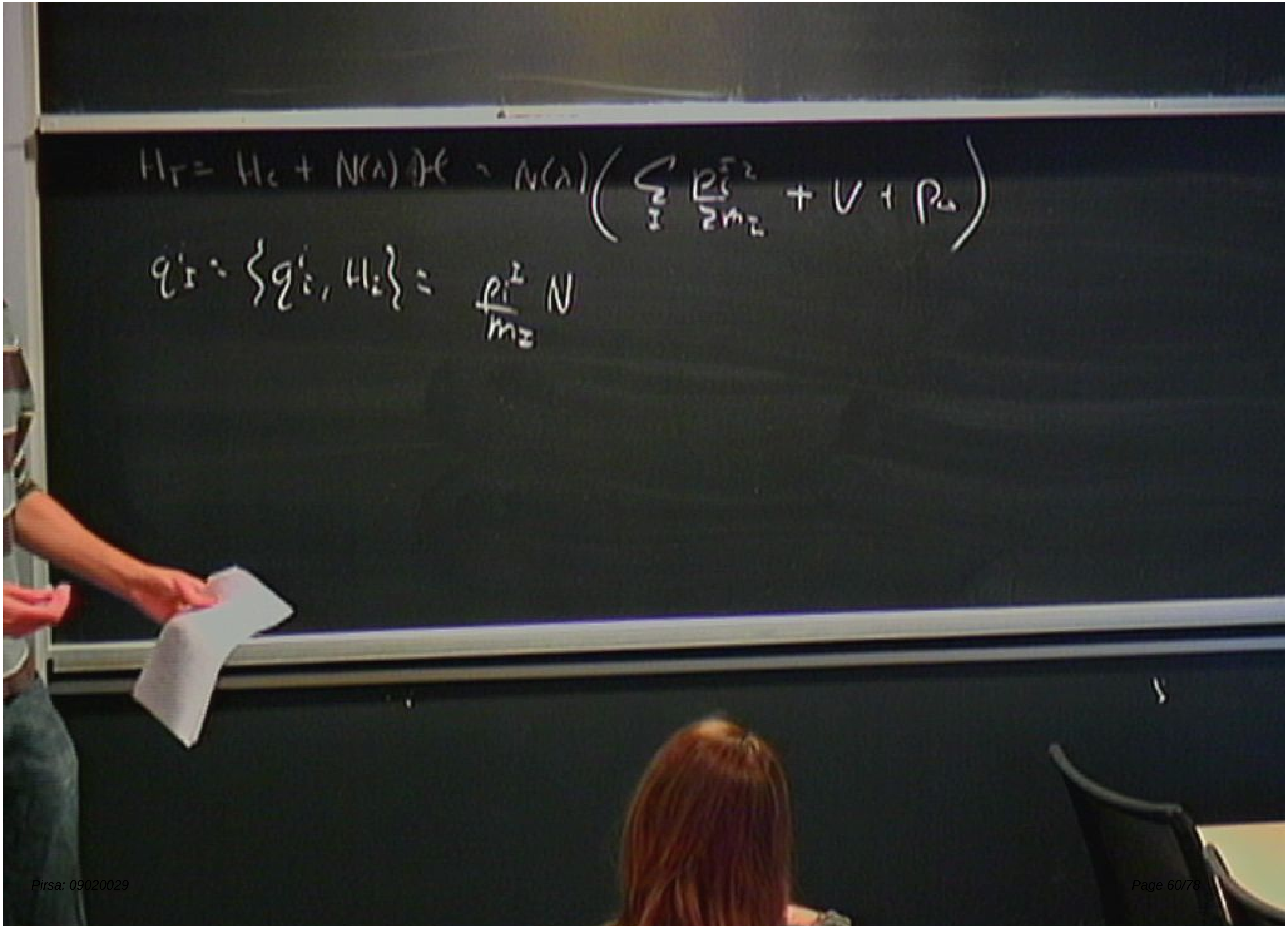

Hc = Σ p q̇ + p0 q̇° − L(q,p)

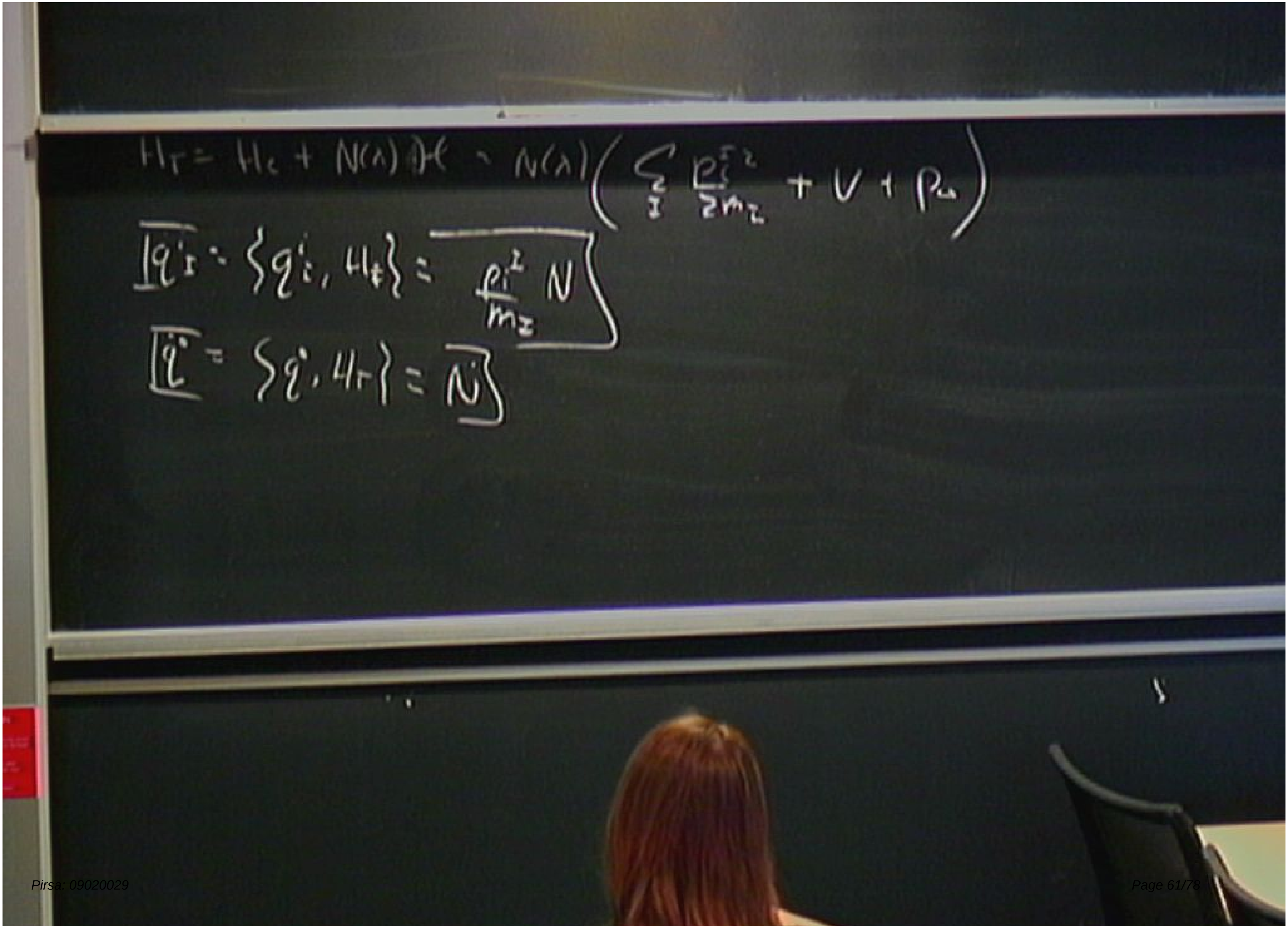


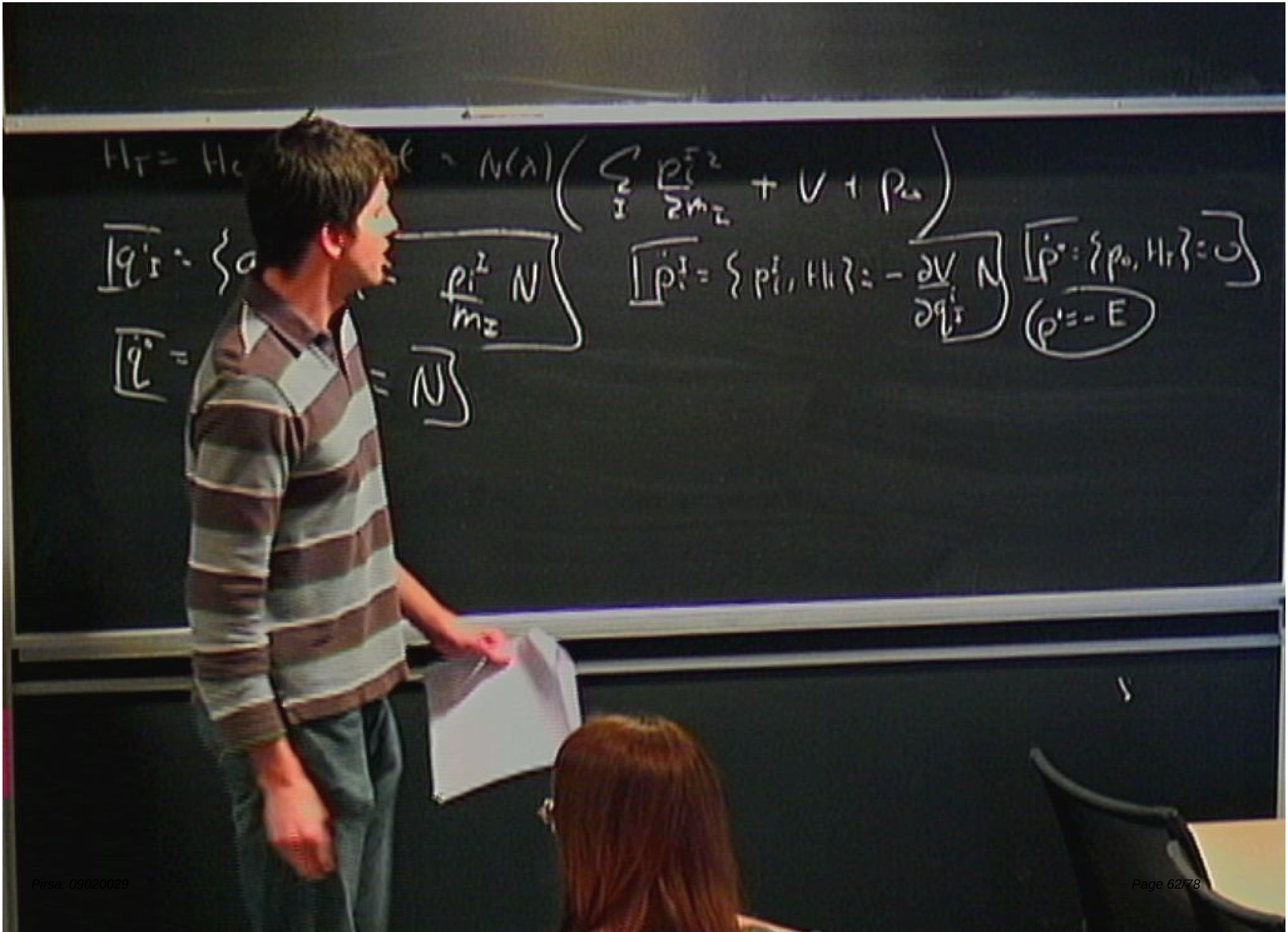
$H_c = \sum_i p^i_i \dot{q}^i_i + p_0 \dot{q}^0 - L_{PPH}(q,p)$
$= \sum_i \frac{m_i \dot{q}_i^2}{\dot{q}^0} + -\left(\frac{I}{\dot{q}^{0\,2}} + V\right)\dot{q}^0 - \left[\frac{I}{\dot{q}^0} - \dot{q}^0 V\right]$
$= 0$

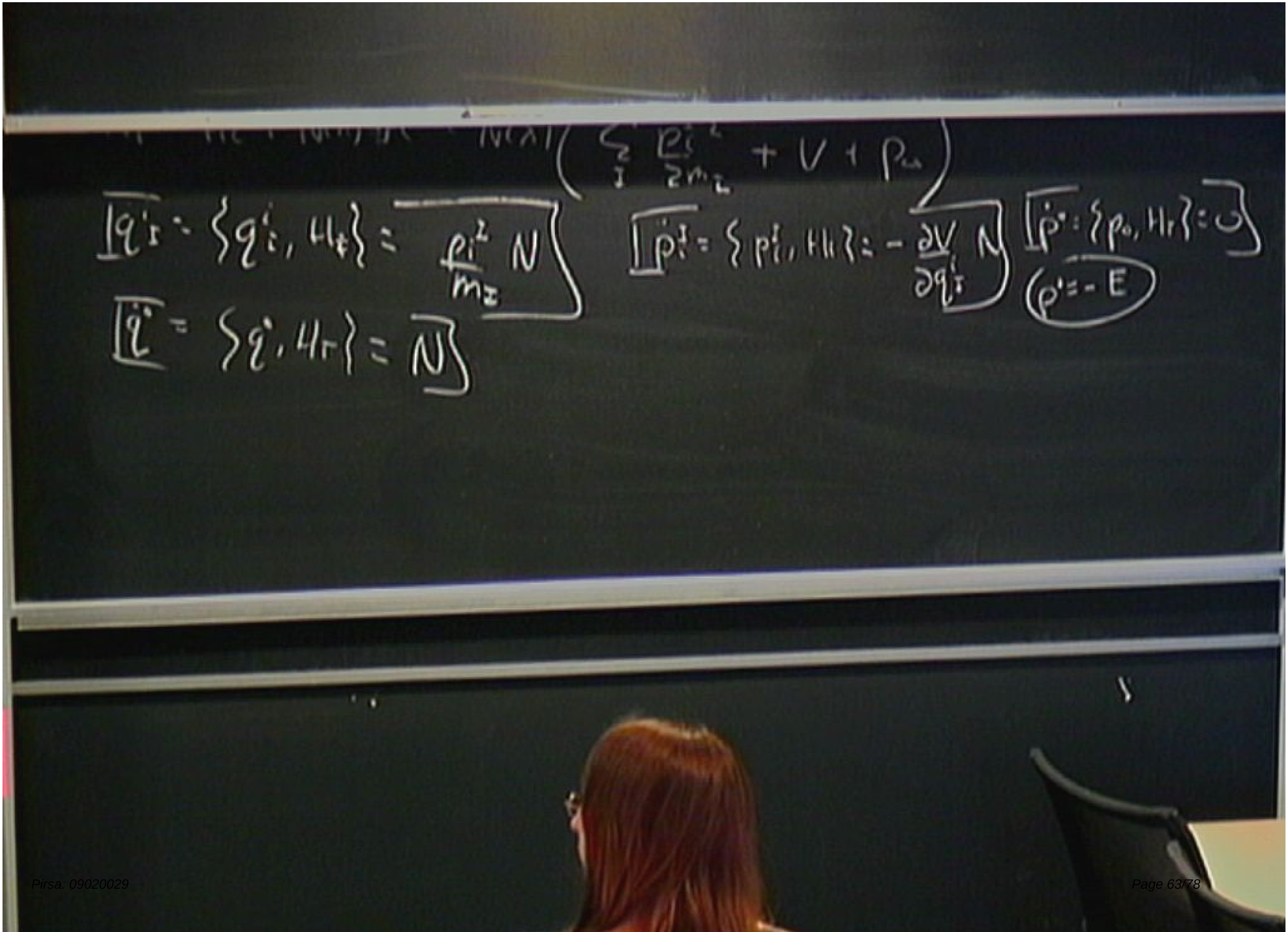

$H_T = H_c + N(\lambda)\mathcal{H} = N(\lambda)\left(\sum_i \frac{p_i^2}{2m_i} + V + p_\omega\right)$

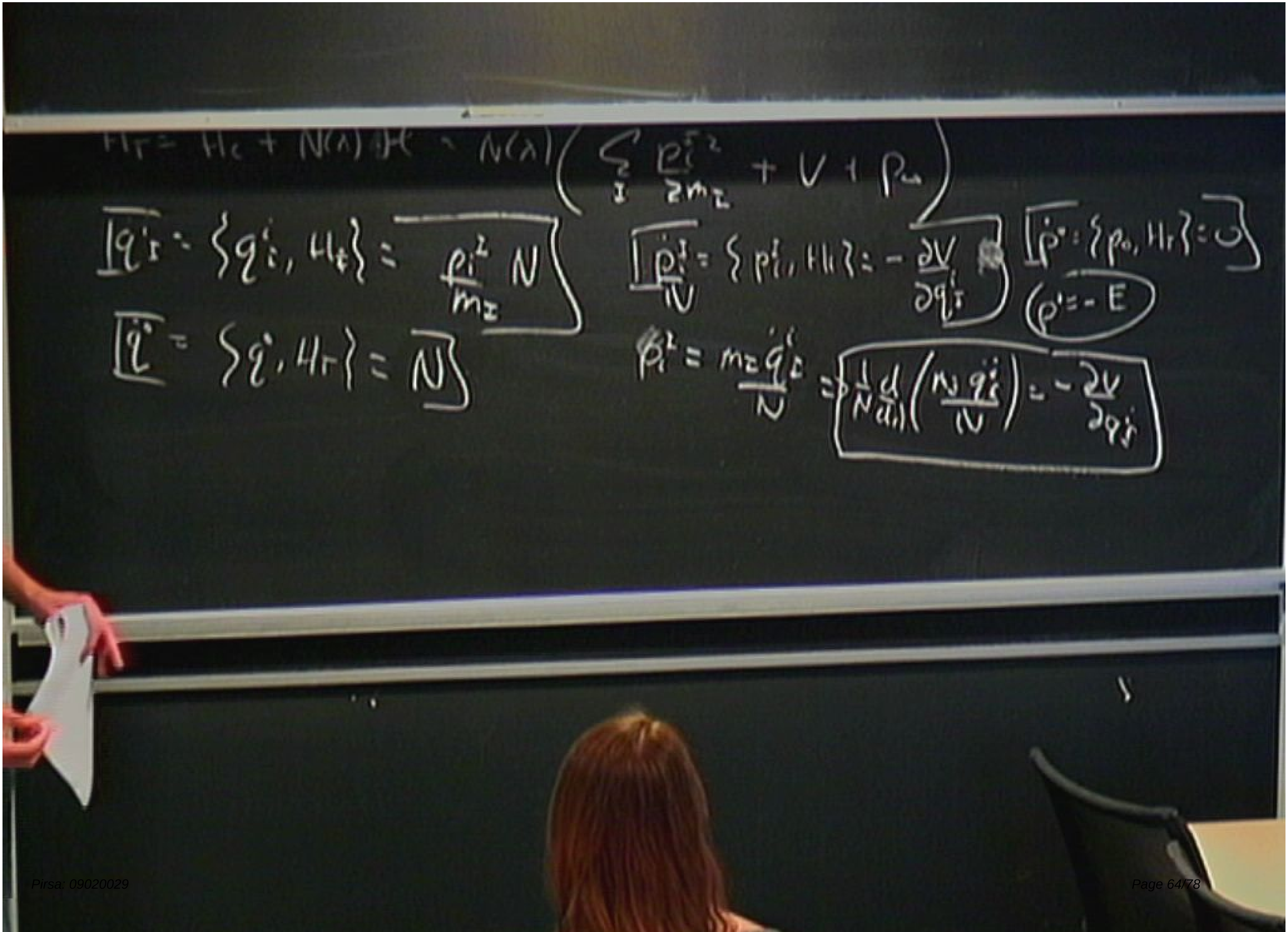

$H_T = H_c + N(\lambda)\mathcal{H} = N(\lambda)\left(\sum_I \frac{p_I^2}{2m_I} + V + p_\lambda\right)$
$\dot q_I = \{q_I, H_T\} = p_I^2 N$

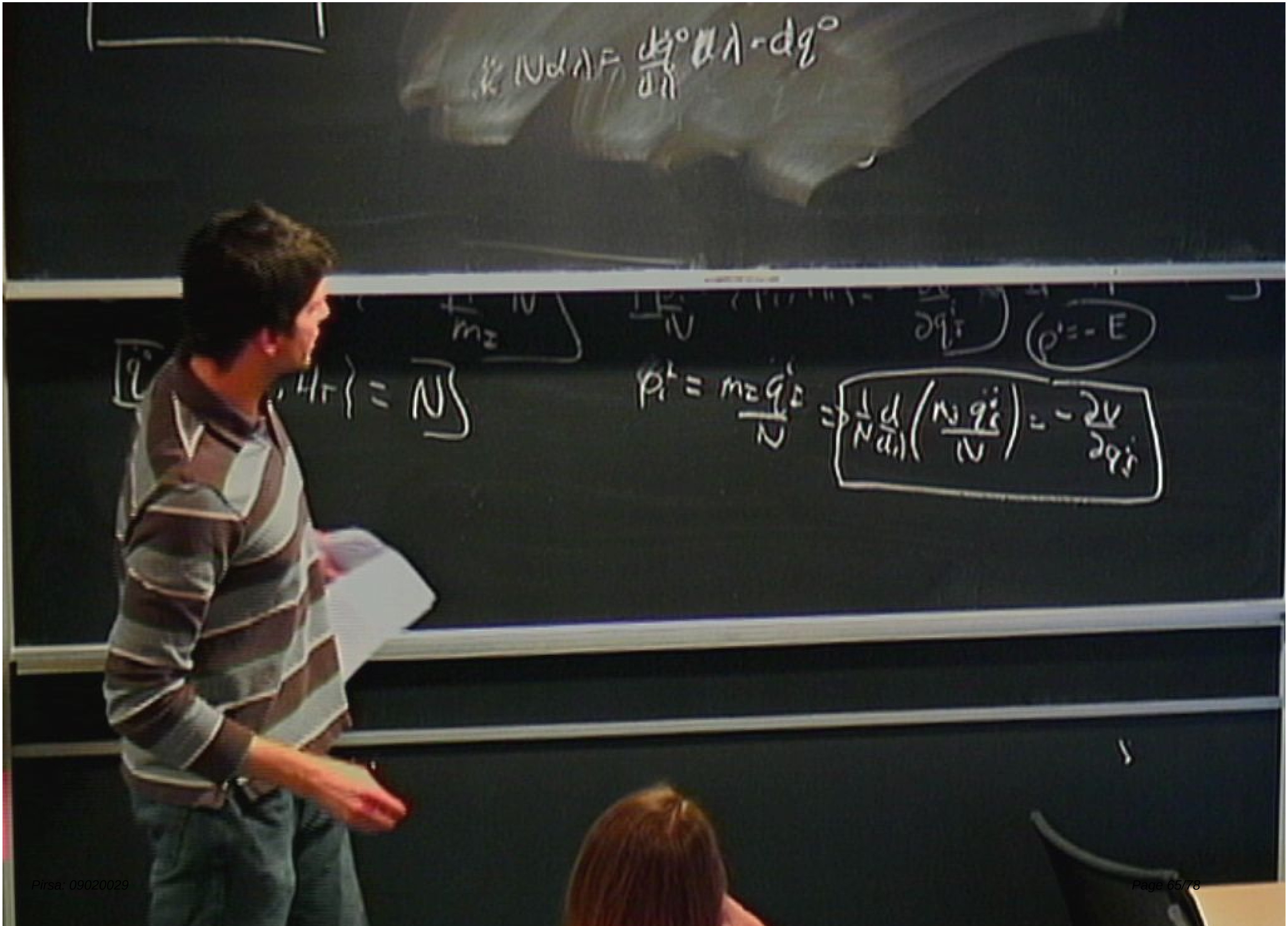

$H_T = H_c + N(\lambda)\mathcal{H} = N(\lambda)\left(\sum_I \frac{p_I^{i\,2}}{2m_I} + V + p_\lambda\right)$
$\dot{q}^i_I = \{q^i_I, H_T\} = \frac{p_I^i}{m_I} N$

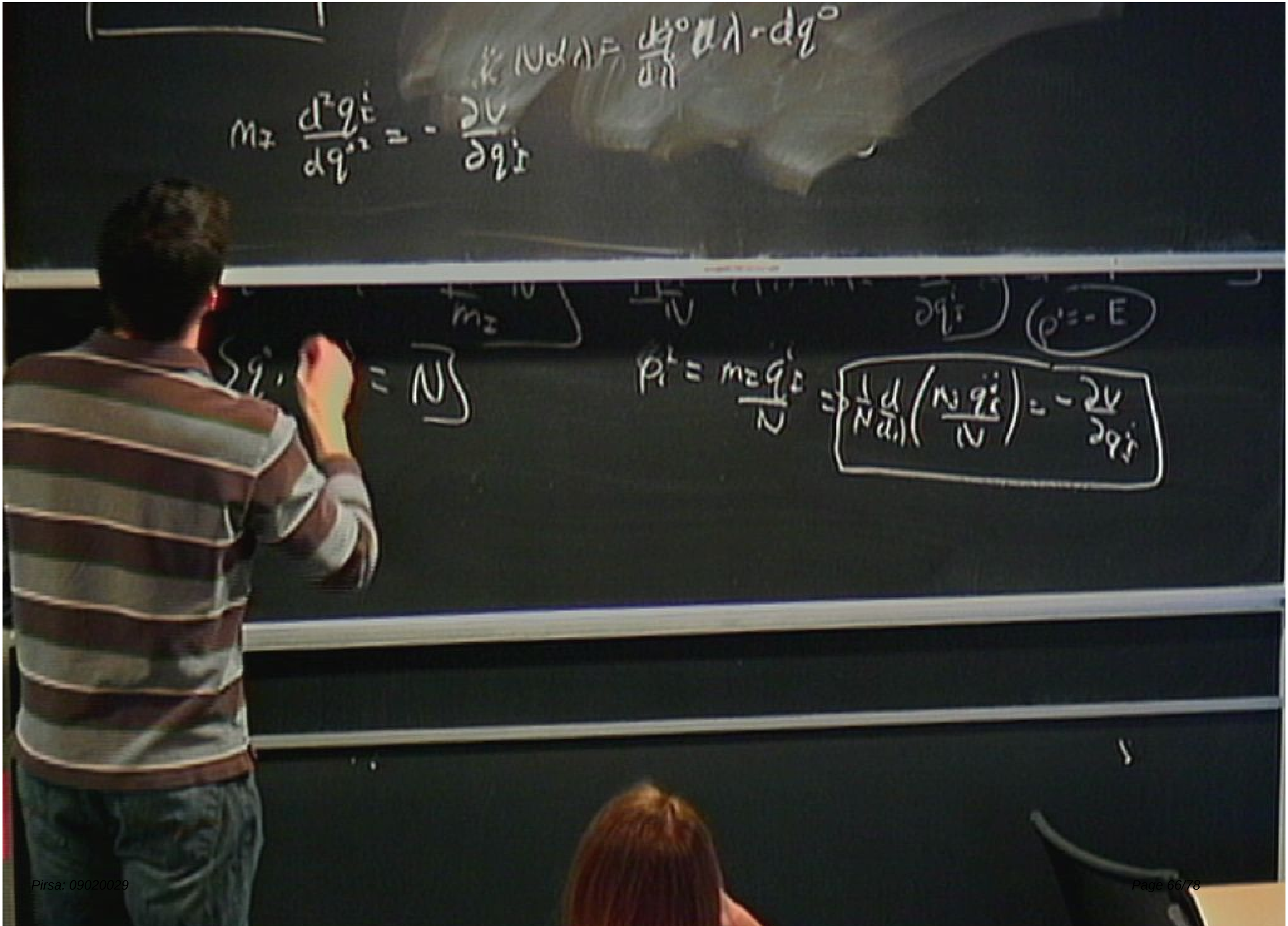

$H_T = H_c + N(\lambda)\mathcal{H} = N(\lambda)\left(\sum_I \frac{p^{I\,2}}{2m_I} + V + p_\omega \right)$
$\dot{q}^i_I = \{q^i_I, H_T\} = \frac{p^i_I}{m_I} N$
$\dot{q}^0 = \{q^0, H_T\} = N$


ṗ⁰ = {p₀, H_T} = 0
p⁰ = −E
ṗ^I = {p^I, H_T} = − ∂V/∂q^I N


q̇^i_I = {q^i_I, H_T} = p_i^I / m_I N
q̇ = {q, H_T} = N
ṗ^i_I = {p^i_I, H_T} = − ∂V/∂q^i_I N
ṗ_0 = {p_0, H_T} = 0
p_0 = − E


H_T = H_c + N(λ) H = N(λ) (Σ_I p_I^i2 / 2m_I + V + p_0)
q̇^i_I = {q^i_I, H_T} = p_I^i N / m_I
q̇^0 = {q^0, H_T} = N
ṗ^i_I / N = {p^i_I, H_T} = − ∂V/∂q^i_I
ṗ_0 = {p_0, H_T} = 0
p_0 = − E
p^i_I = m_I q̇^i_I / N ⇒ 1/N d/d(λ) (m_I q̇^i_I / N) = − ∂V/∂q^i_I


p_i = m_i q̇^i / N ⇒ (1/N) d/dλ (m_i q̇^i / N) = −∂V/∂q^i
p' = −E


$m_I \frac{d^2 q^i}{dq^{0\,2}} = - \frac{\partial V}{\partial q^i}$
$p^0 = -E$
$p^i = m_I \frac{\dot{q}^i}{N}$
$\frac{1}{N}\frac{d}{d\lambda}\left(\frac{m_I \dot{q}^i}{N}\right) = -\frac{\partial V}{\partial q^i}$

$$m_I \frac{d^2 q^i_I}{dq^{0\,2}} = -\frac{\partial V}{\partial q^i_I}$$

$$\dot q^i_I = \{q^i_I, H_T\} = \frac{p_i^I}{m_I} N$$

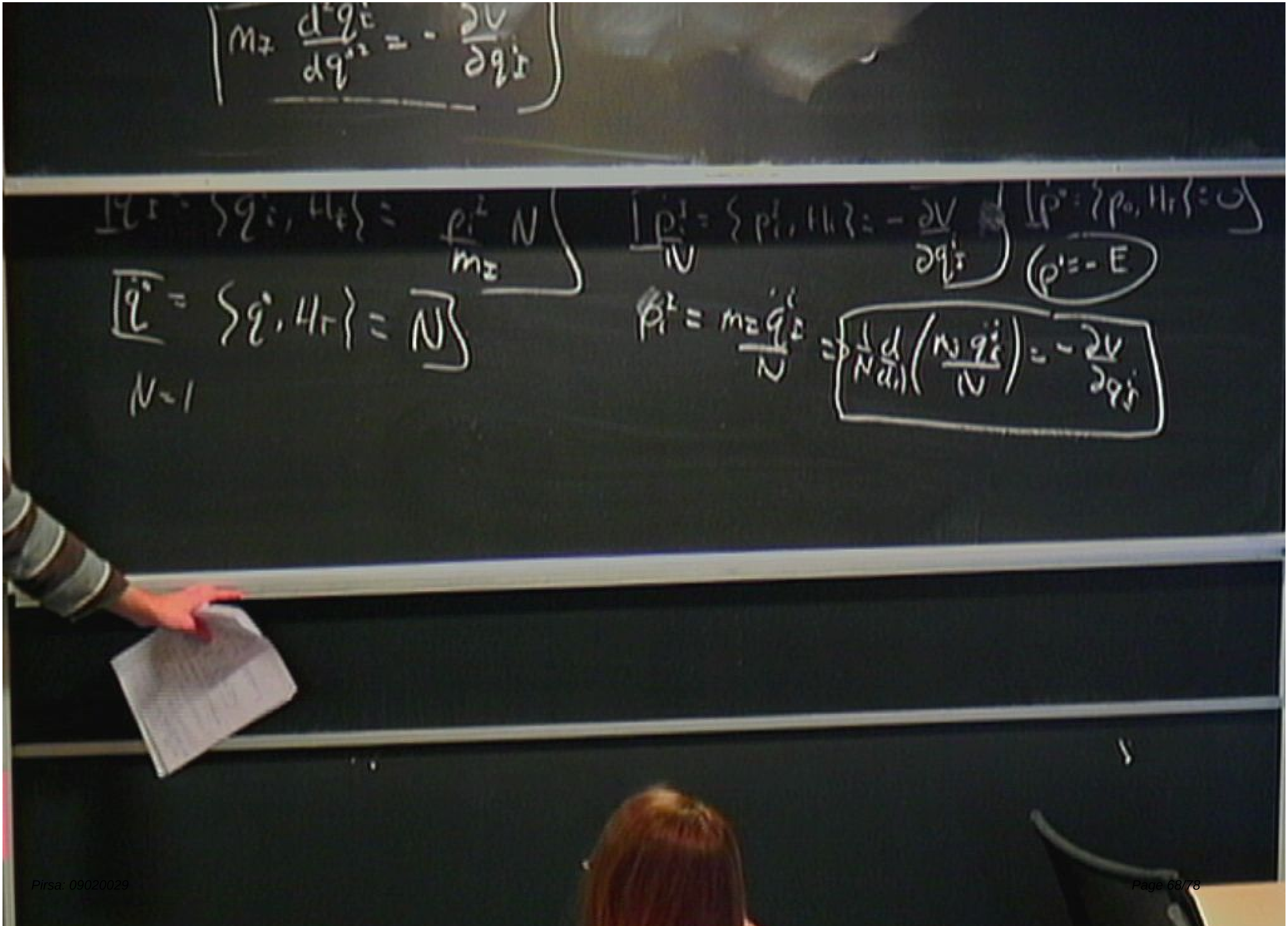
$$\dot q^0 = \{q^0, H_T\} = N$$

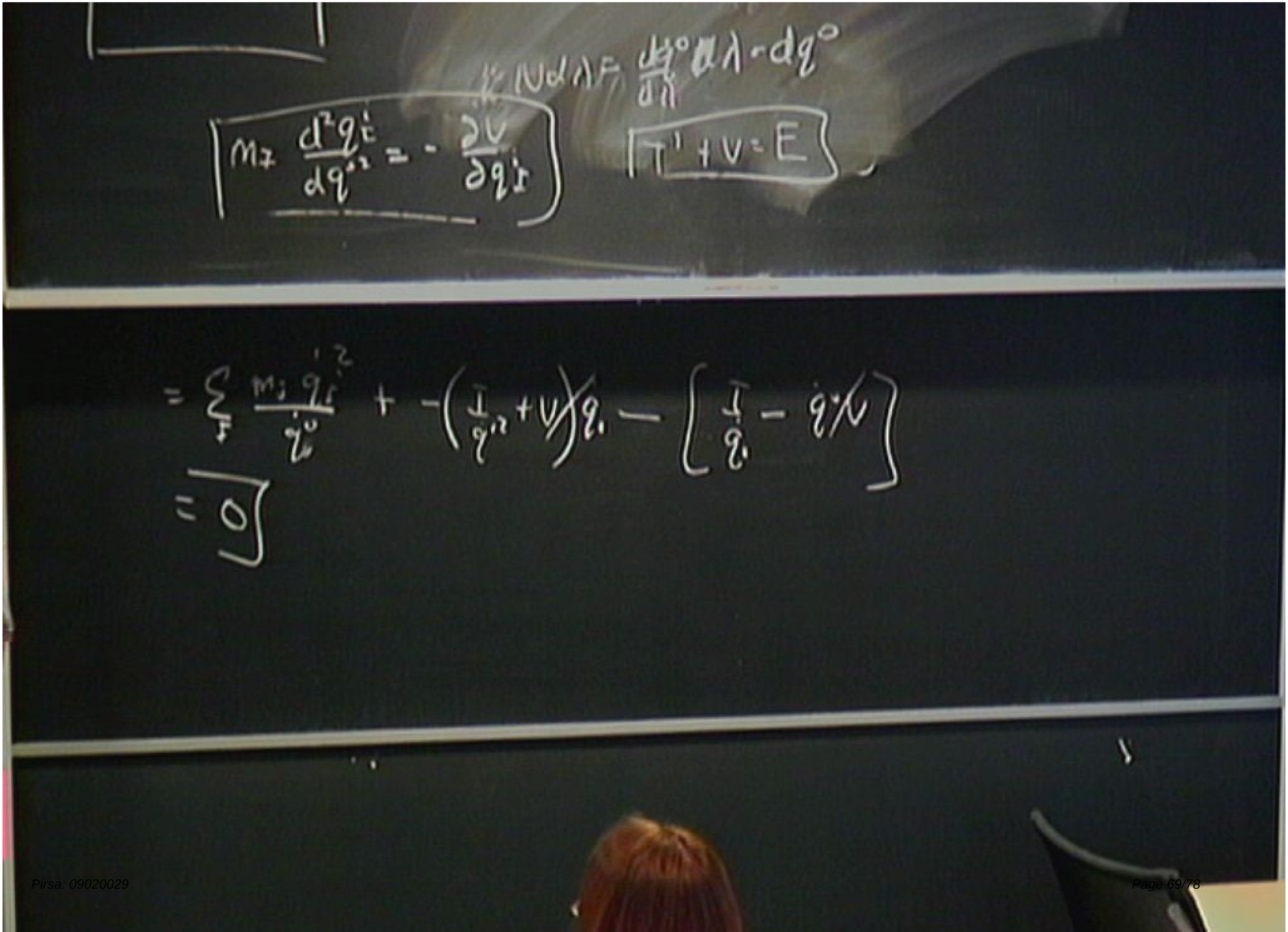
$$\frac{\dot p^I_i}{N} = \{p^I_i, H_T\} = -\frac{\partial V}{\partial q^i_I}$$

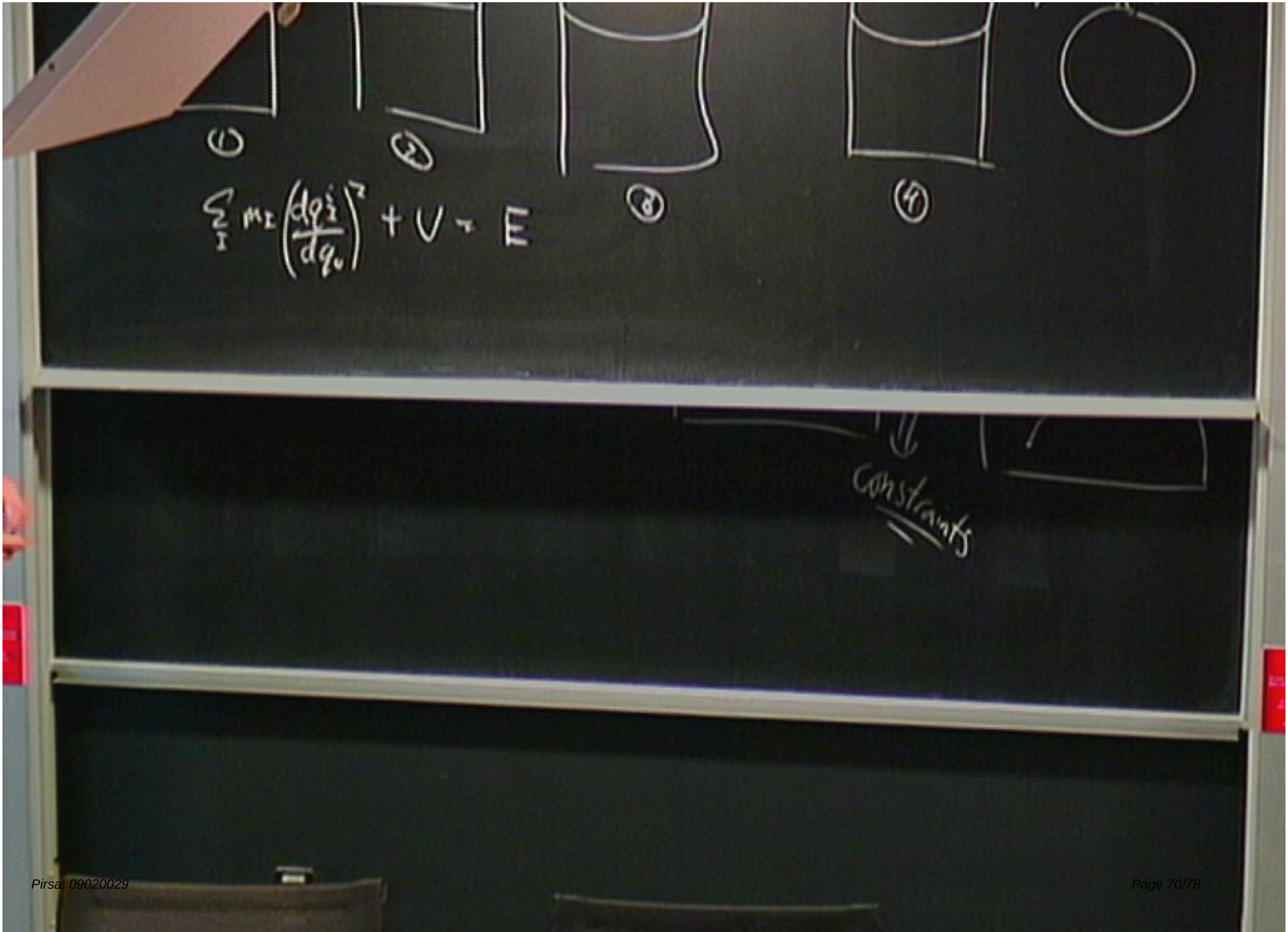
$$\dot p^0 = \{p_0, H_T\} = 0$$

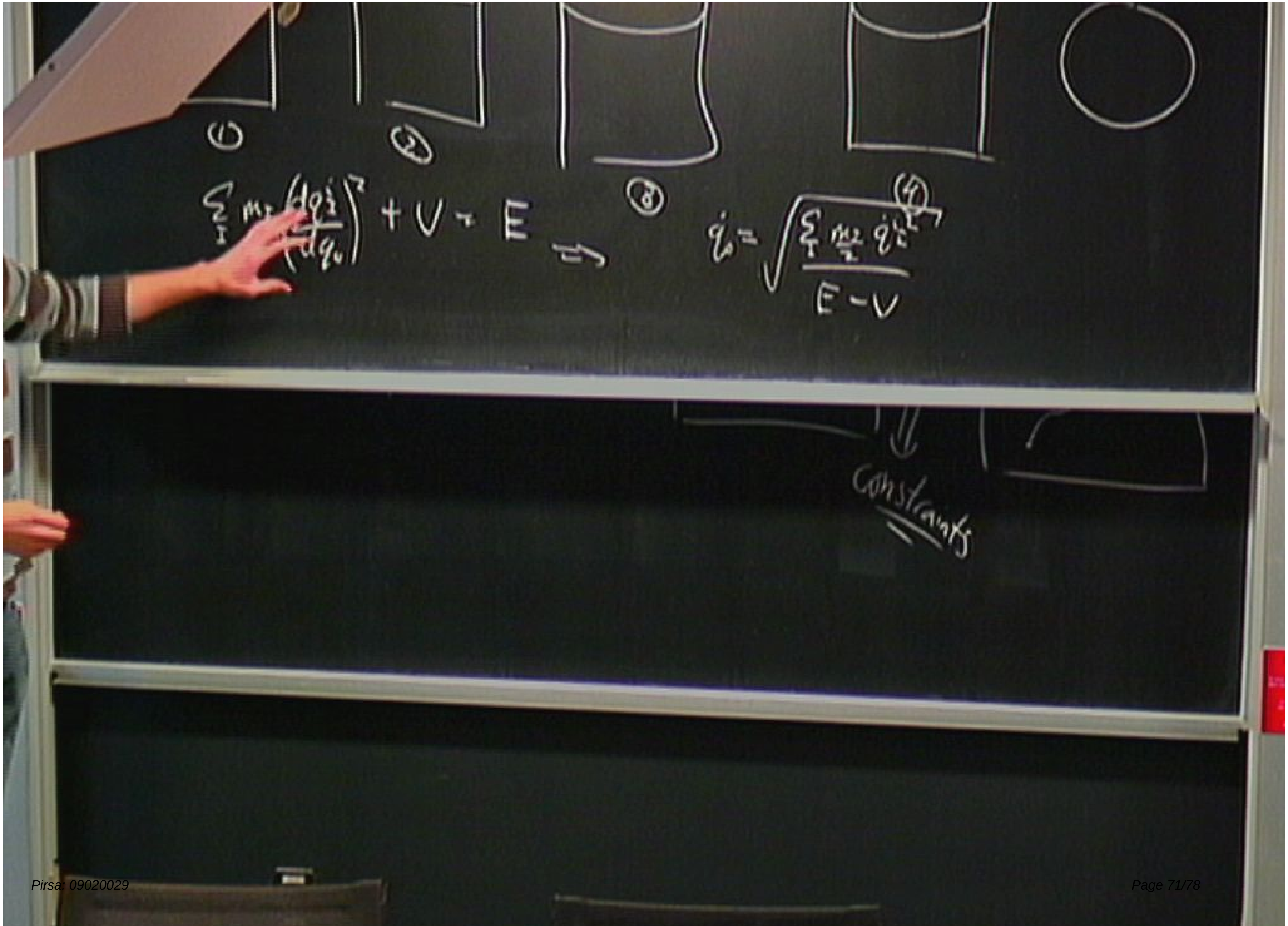
$$p^0 = -E$$

$$p^I_i = m_I \frac{\dot q^i_I}{N} \Rightarrow \frac{1}{N}\frac{d}{dt}\left(m_I \frac{\dot q^i_I}{N}\right) = -\frac{\partial V}{\partial q^i_I}$$




m_I d²q^i/dq'² = - ∂V/∂q^i
T' + V = E
= 0


1
2
3
4
$\sum_I m_I \left(\frac{dq^i_I}{dq_0}\right)^2 + V = E$
Constraints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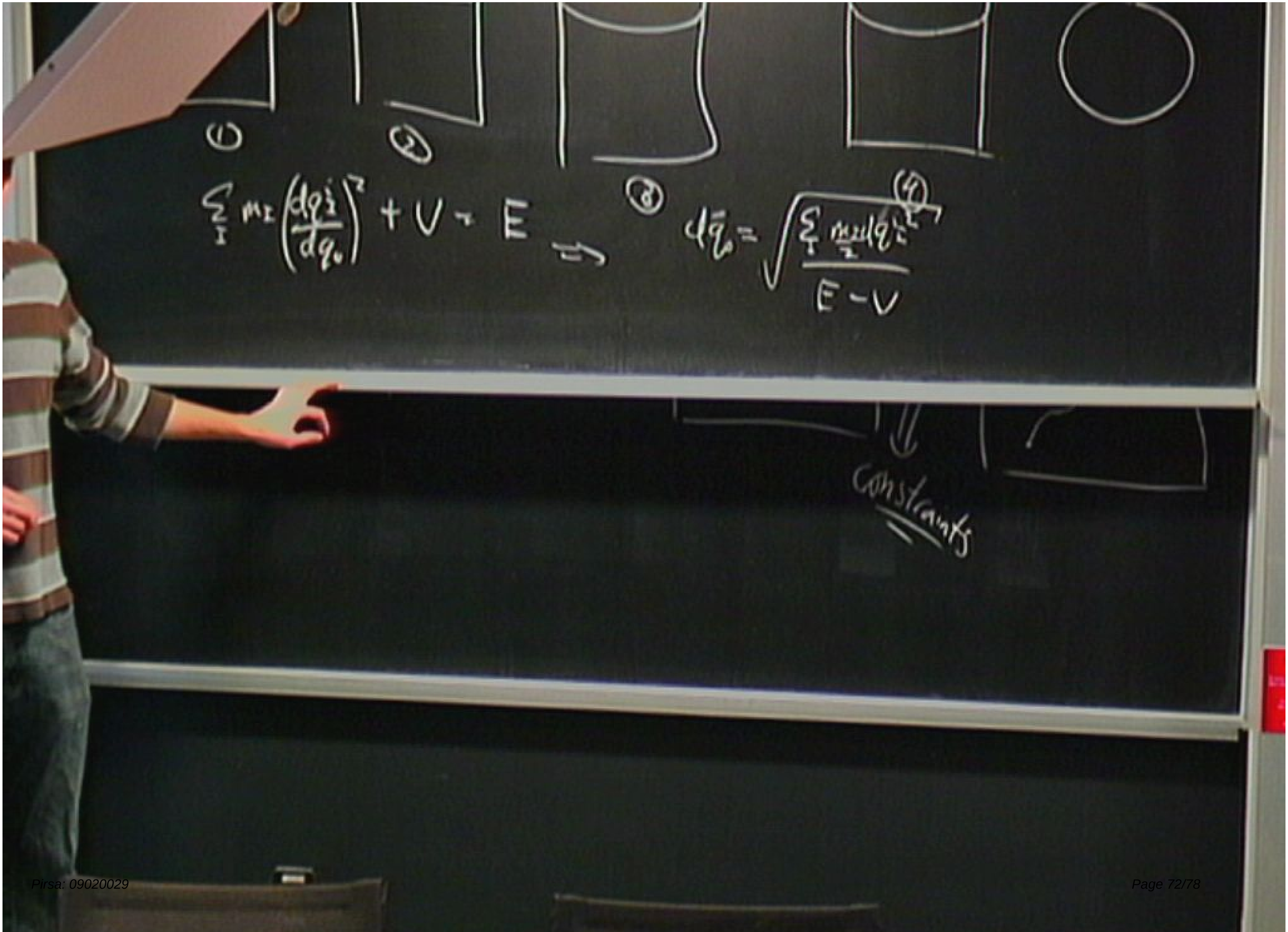
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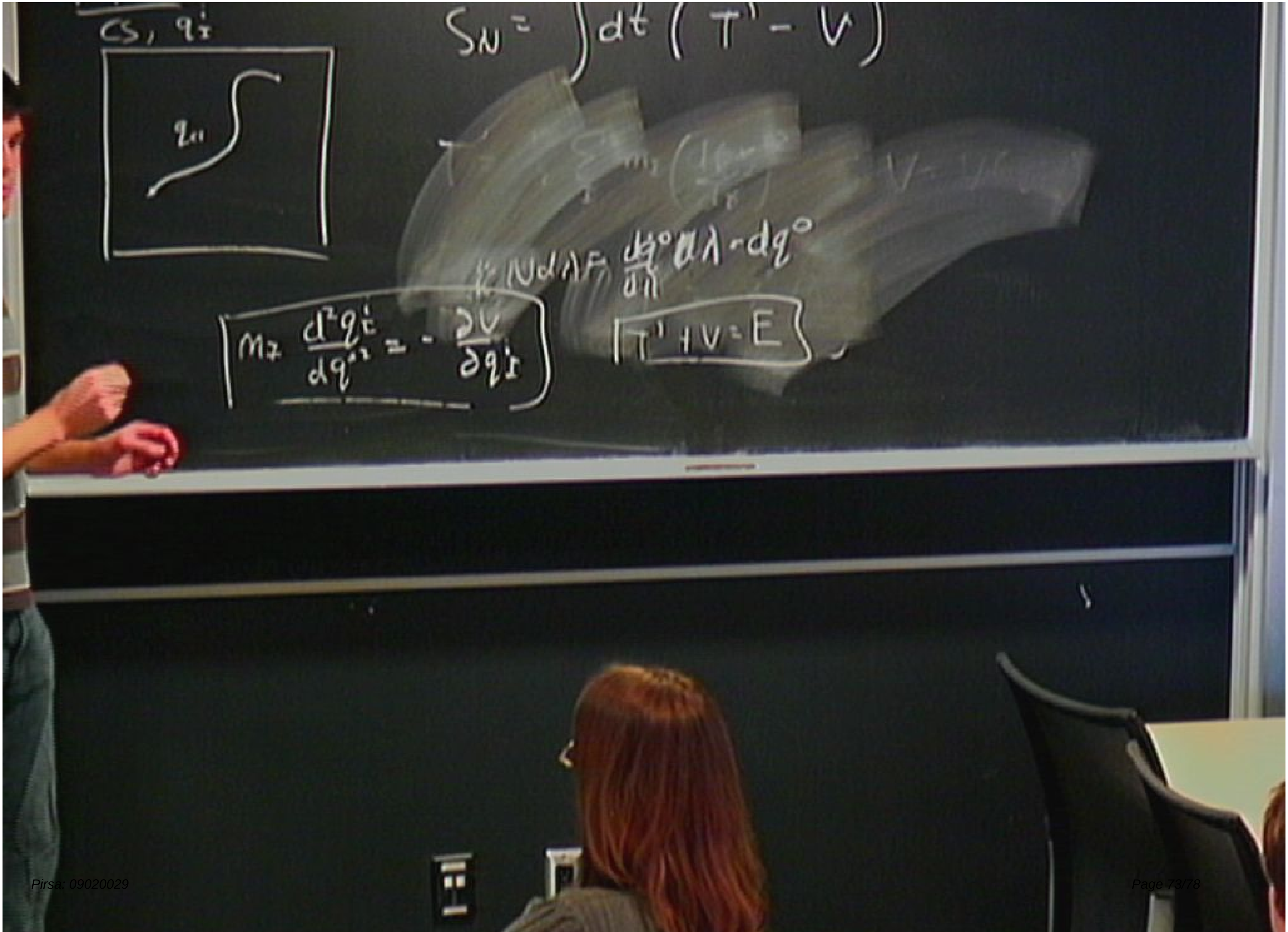
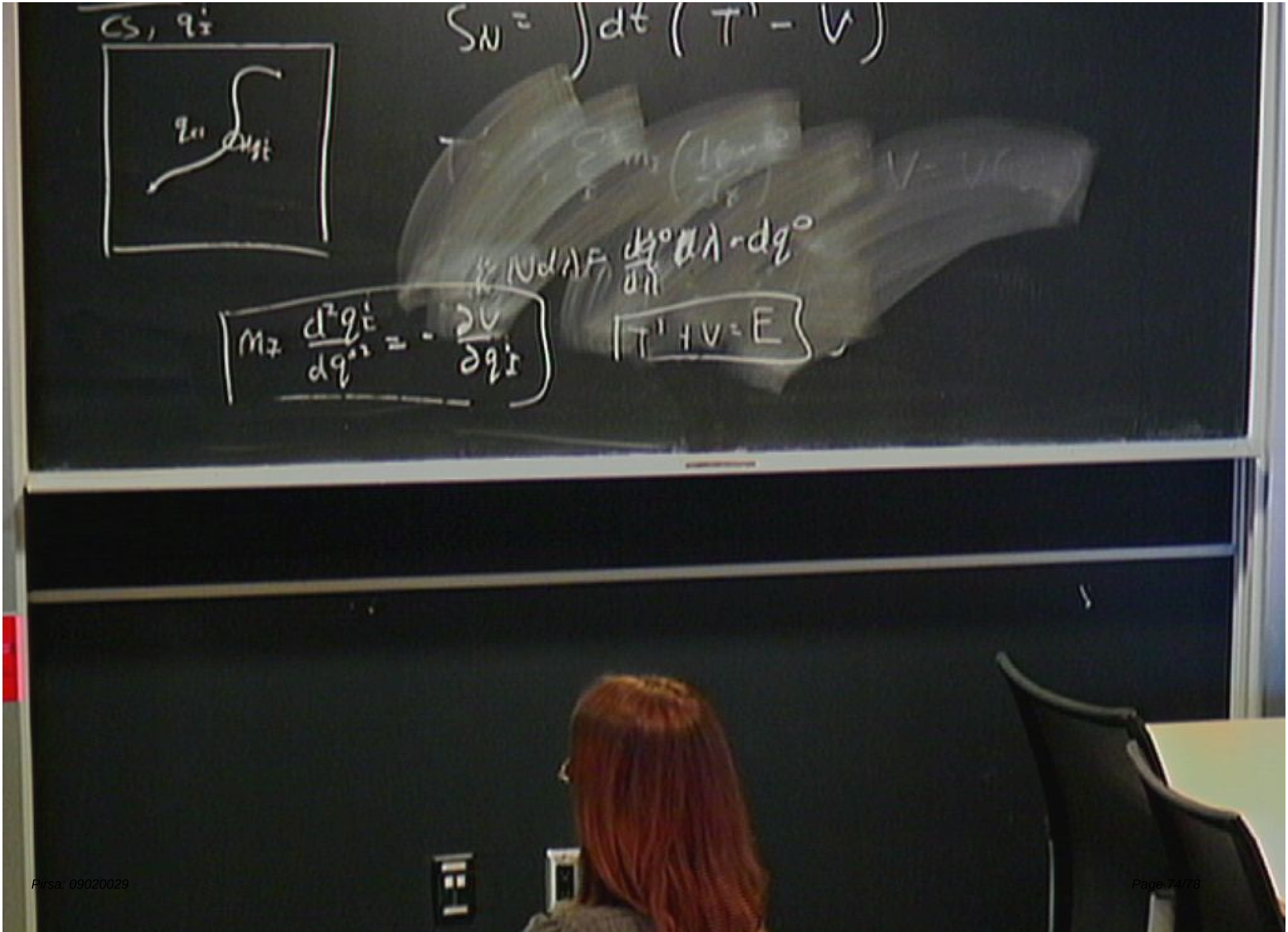
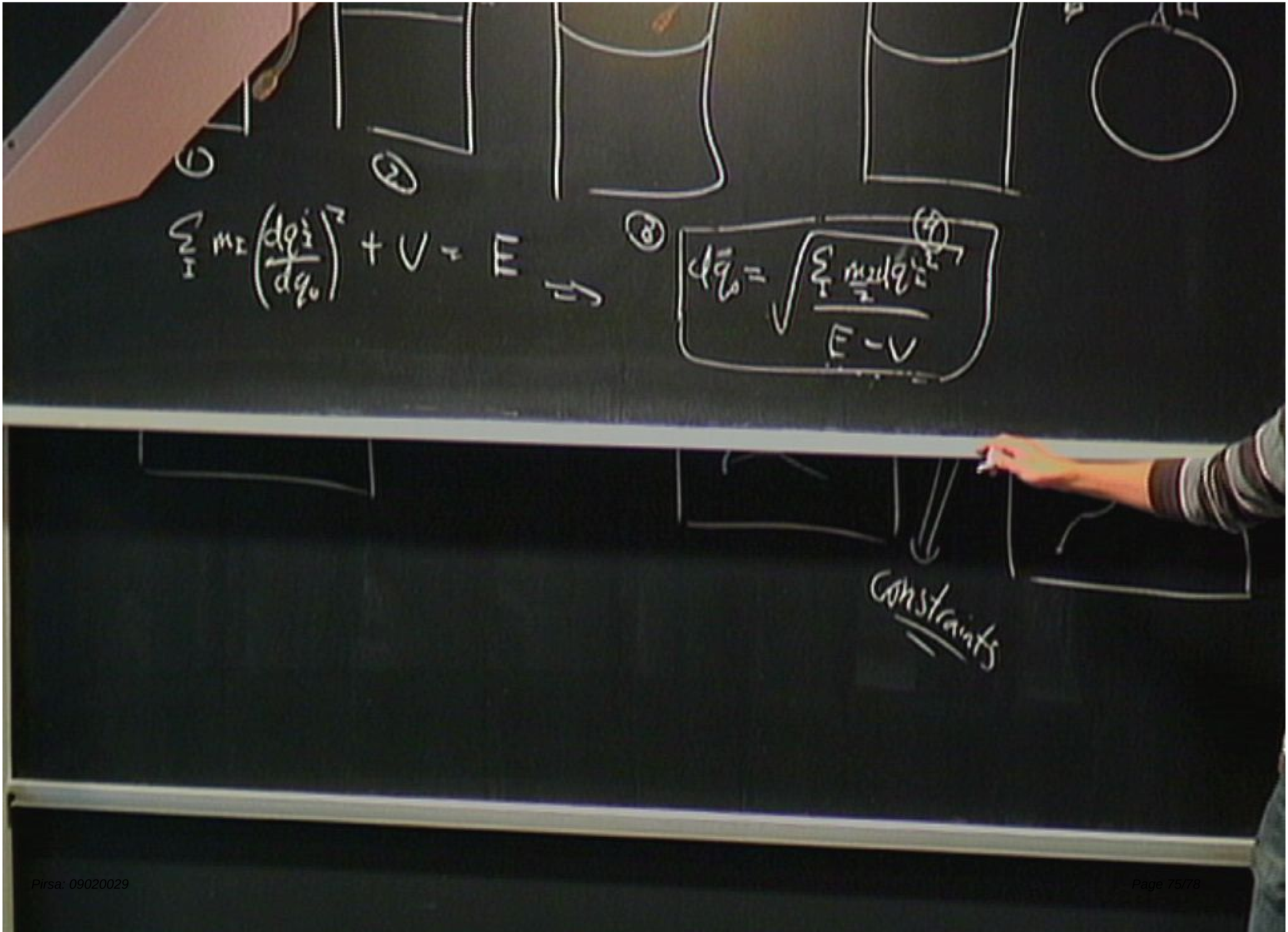
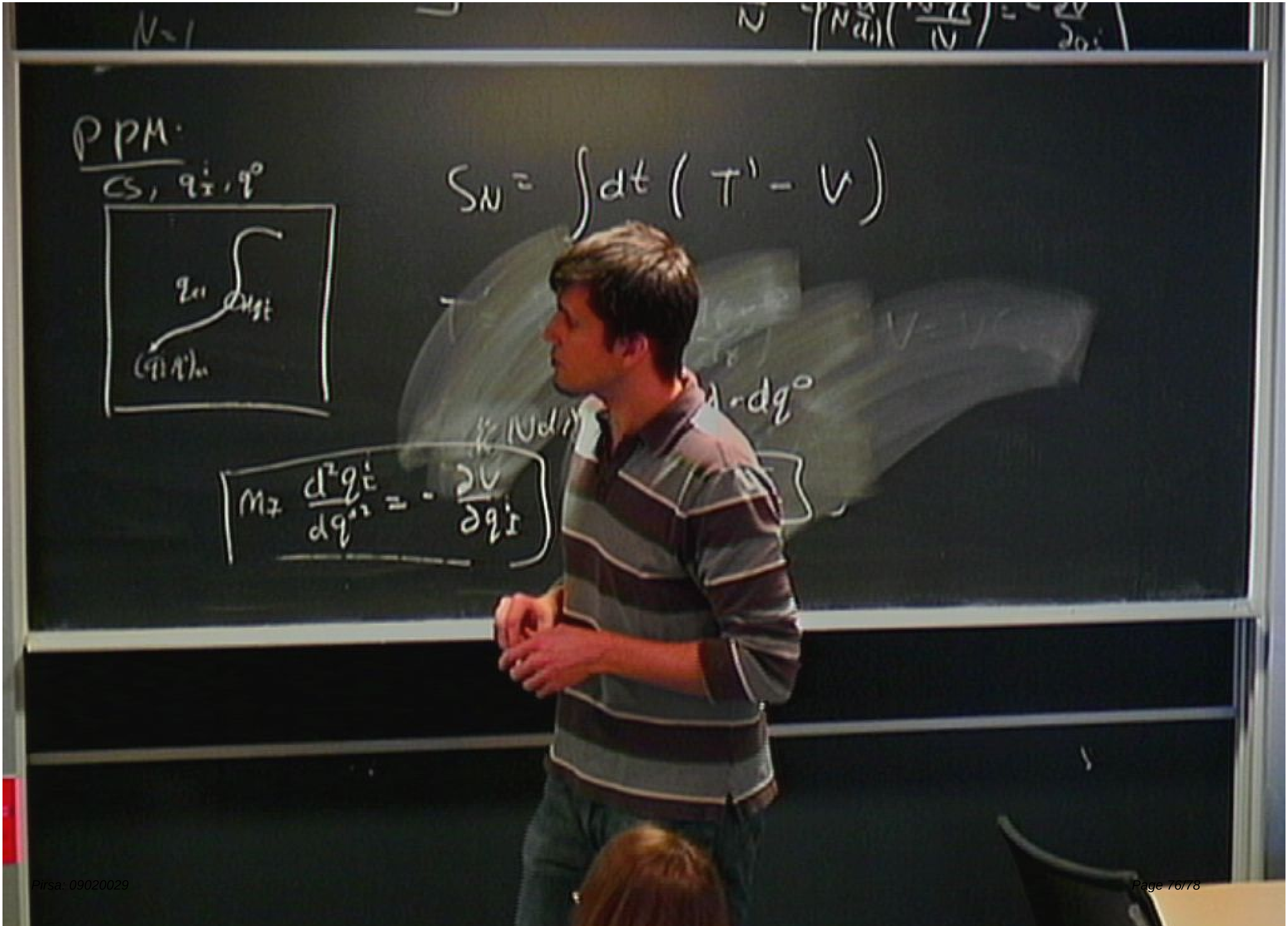
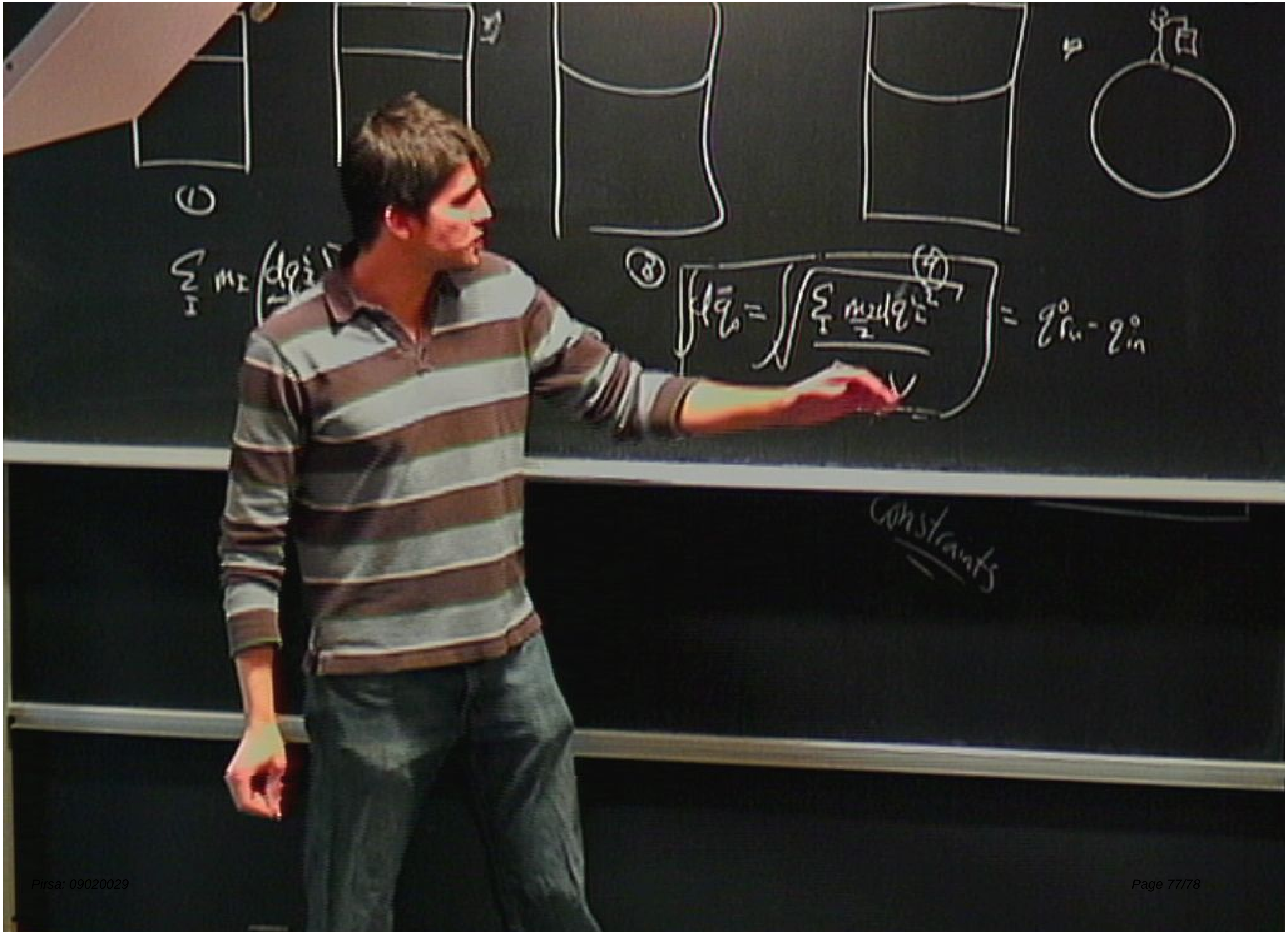

CS, q_I^i
$S_N = \int dt \, (T' - V)$
$m_I \frac{d^2 q_I^i}{dq^{o2}} = -\frac{\partial V}{\partial q_I^i}$
$T' + V = E$


CS, q_I^i
$S_N = \int dt \, (T' - V)$
$m_I \frac{d^2 q_I^i}{dq^{0\,2}} = - \frac{\partial V}{\partial q_I^i}$
$T' + V = E$


① ② ③ ④
$\sum_I m_I \left(\frac{dq^i}{dq_0}\right)^2 + V = E \Rightarrow$
$dq_0 = \sqrt{\frac{\sum_I m_I dq^{i\,2}}{E - V}}$
Constraints


PPM
CS, q^i_I, q^0
$S_N = \int dt \, (T' - V)$
$m_I \frac{d^2 q^i_I}{dq^{0\,2}} = - \frac{\partial V}{\partial q^i_I}$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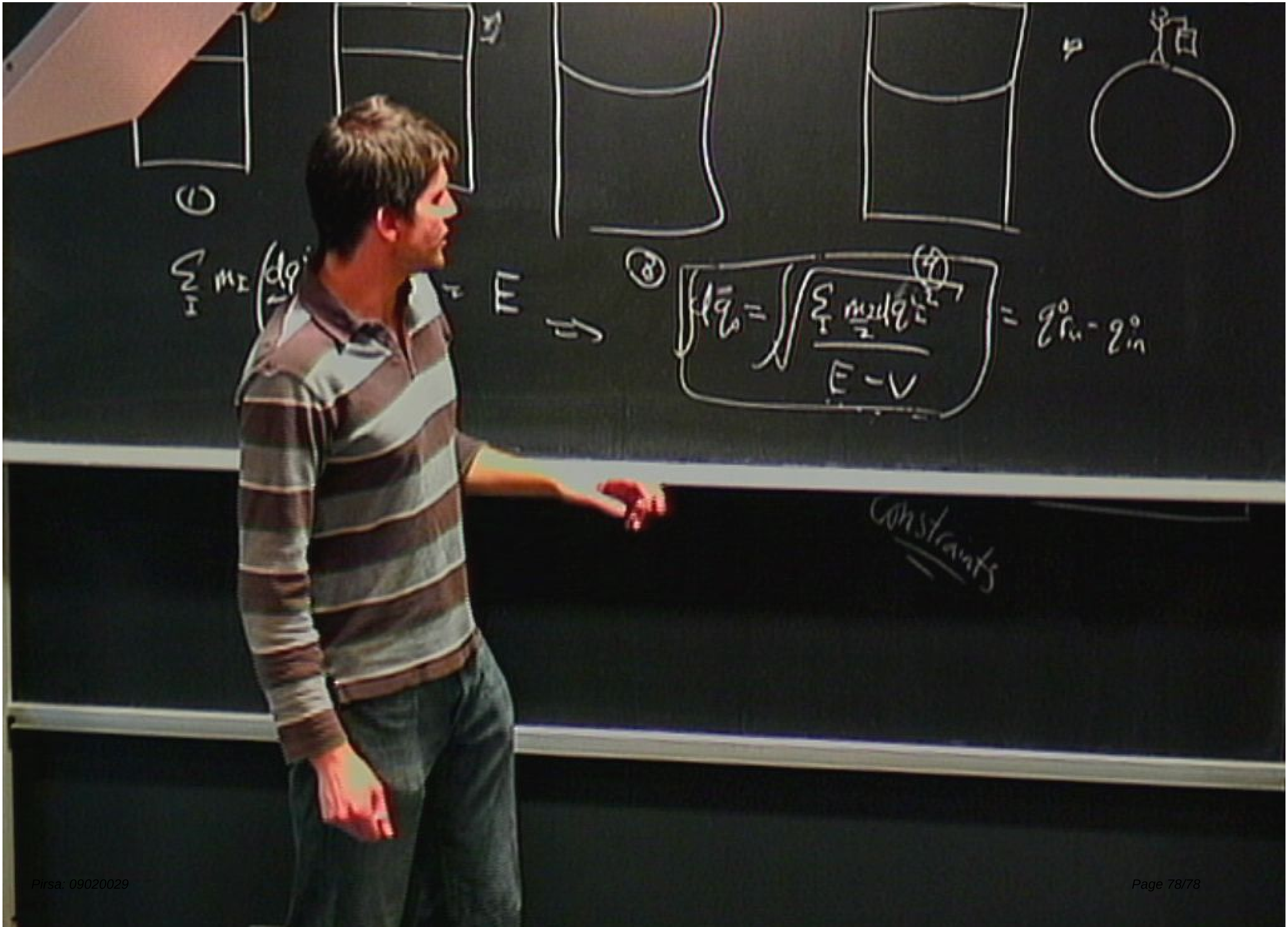
Constraints