

Title: Mini-Course on Mach's Principle

Date: Feb 04, 2009 10:30 AM

URL: <http://www.pirsa.org/09020029>

Abstract:

"It is utterly beyond our power to measure the changes of things by time. Quite the contrary, time is an abstraction, at which we arrive by means of the changes of things; made because we are not restricted to any one definite measure, all being interconnected."

- E. Mach in *The Mechanics*, 1872.

"It is utterly beyond our power to measure the changes of things by time. Quite the contrary, time is an abstraction, at which we arrive by means of the changes of things; made because we are not restricted to any one definite measure, all being interconnected."

- E. Mach in *The Mechanics*, 1872.

Outline:

① Temporal Relationalism

-> Jacobi's Principle

Outline:

① Temporal Relationalism

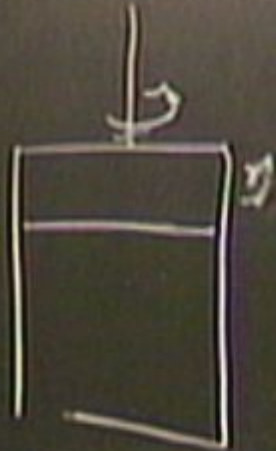
↳ Jacobi's Principle

↳ Para. Particle Mechanics

“When we say that a body K alters its direction and velocity solely through the influence of another body K', we have inserted a conception that is impossible to come at unless other bodies A, B, C... are present with reference to which the motion of the body K has been estimated.”

- E. Mach in *The Mechanics*, 1872.





sm

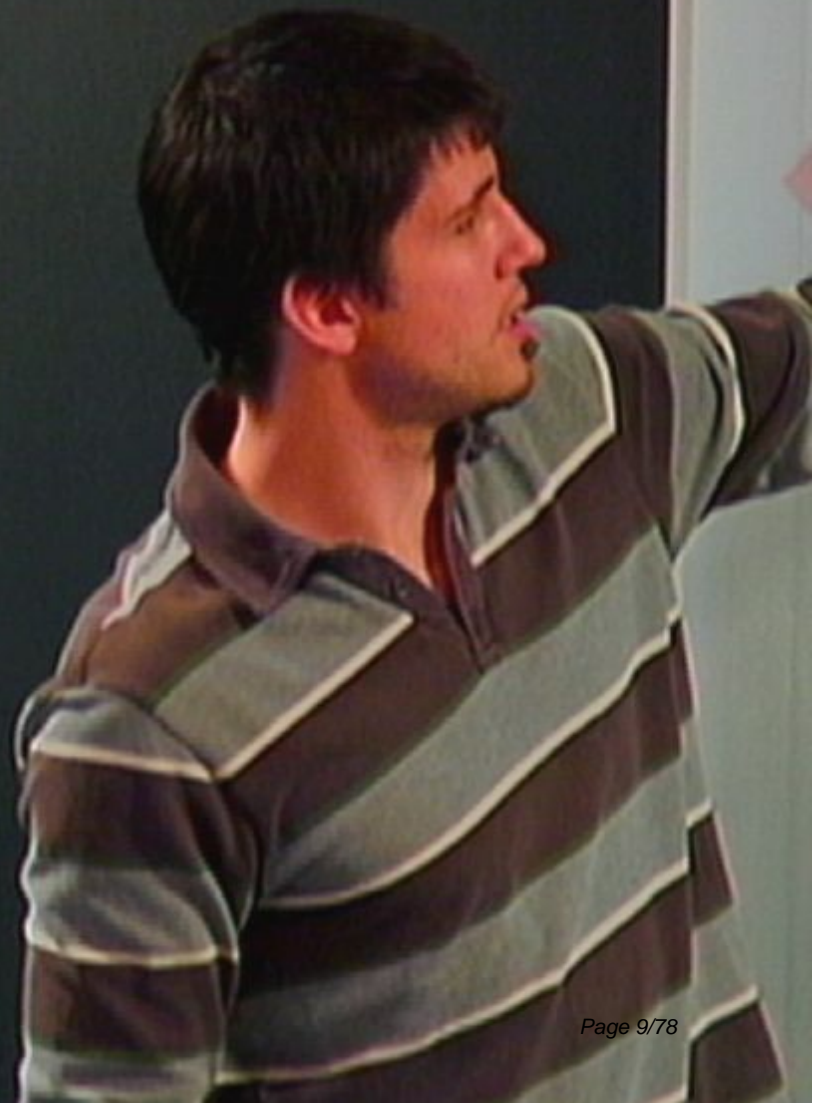
principle

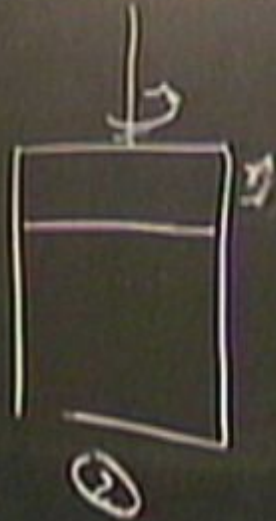
Particle Mechanics

Ψ

Rel Mt

no



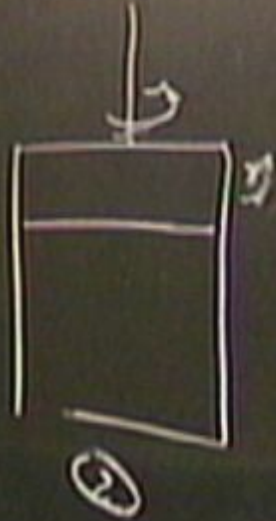


sm

principle

Particle Mechanics

| | Rel Mt | Flat |
|----------|--------|------|
| ψ | no | yes |
| ω | yes | yes |



relationalism

→ Jacobi's Principle

→ ~~Part.~~ Particle Mechanics

①

Rel Mt

no

Flat

yes

②

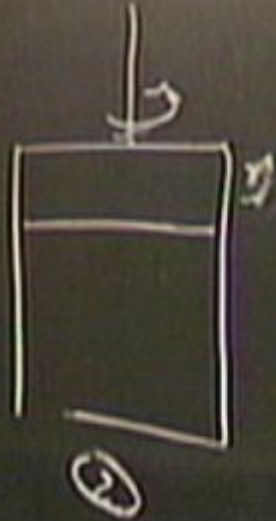
yes

yes

③

no

no

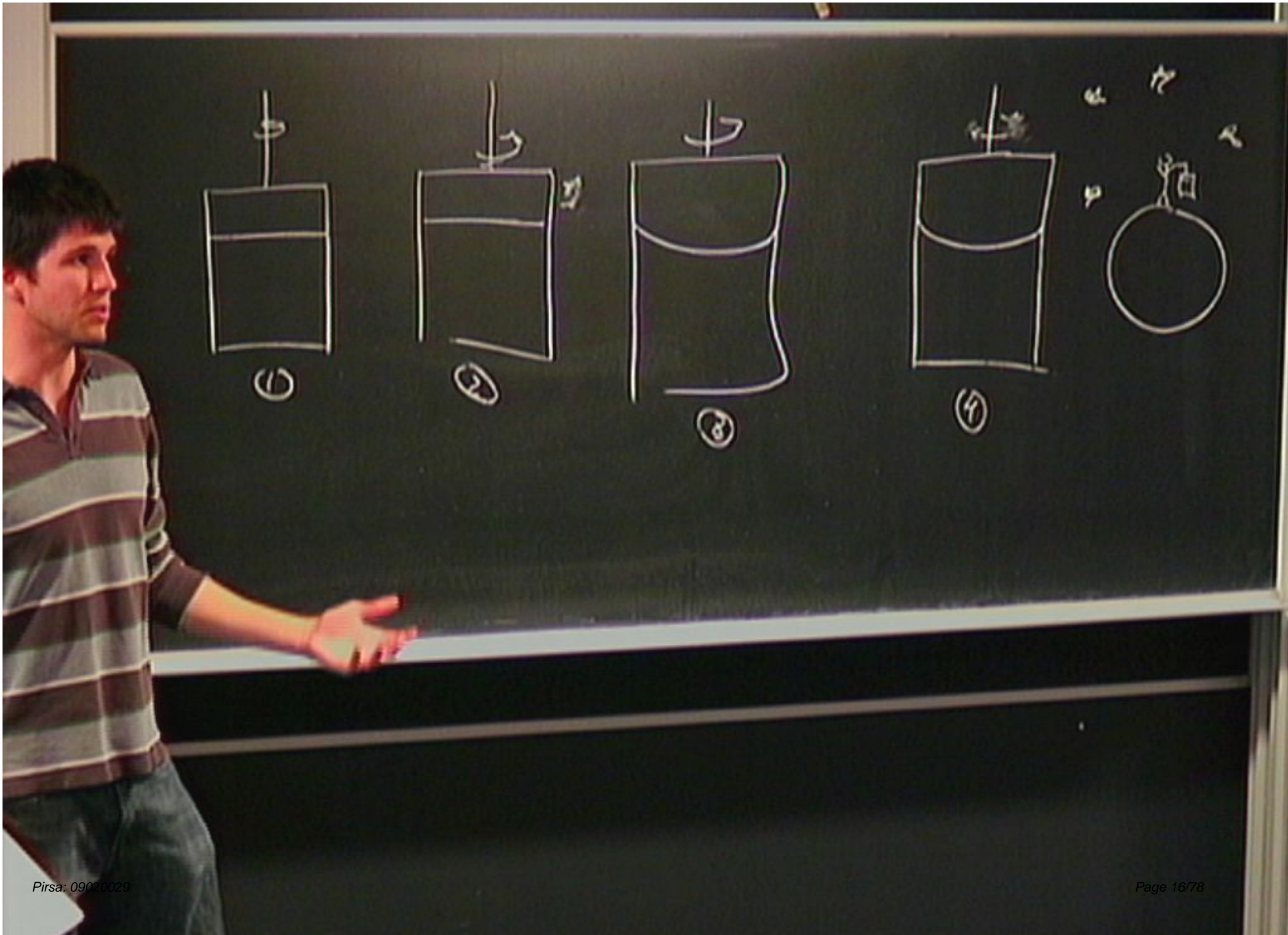


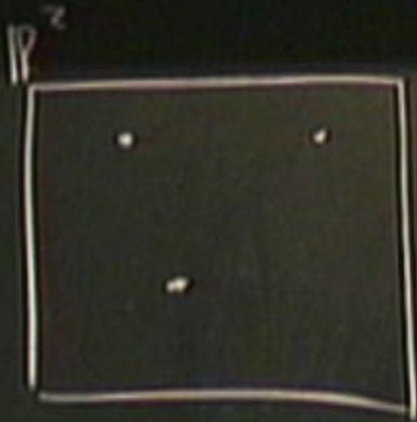
relationalism

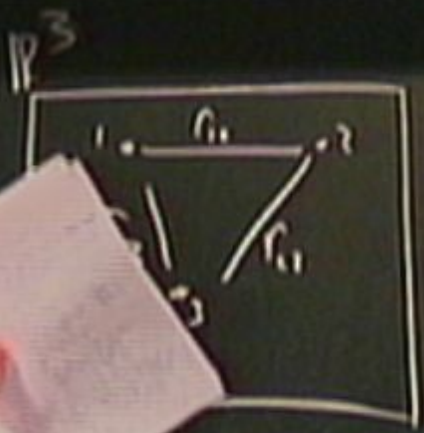
Jacobis Principle

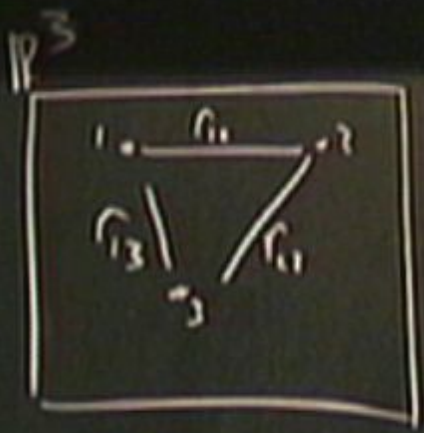
Classical Particle Mechanics

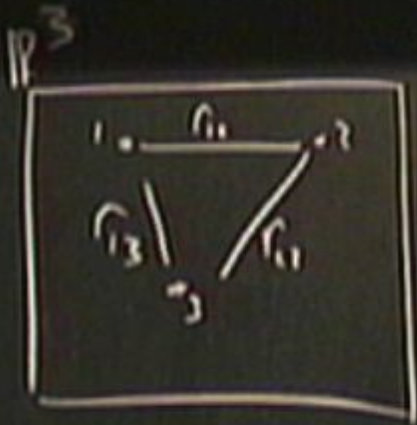
| | Rel Mt | Flat |
|---|--------|------|
| ① | no | yes |
| ② | yes | yes |
| ③ | no | no |
| ④ | yes | no |



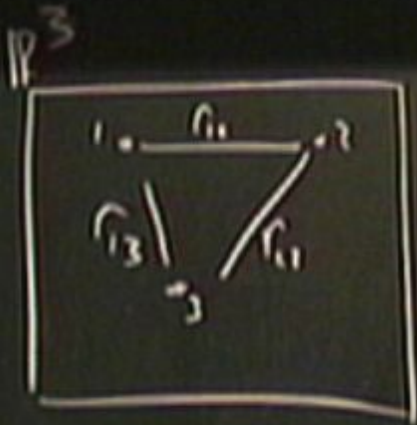




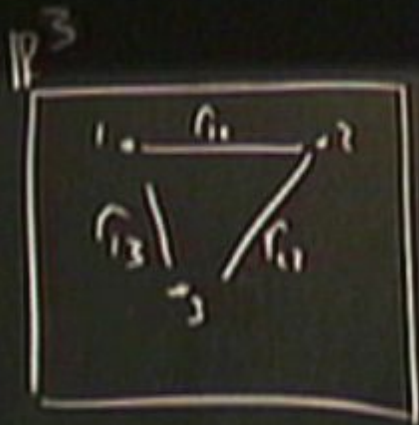




$\frac{g_1}{g_2}, \frac{g_1}{g_3}$



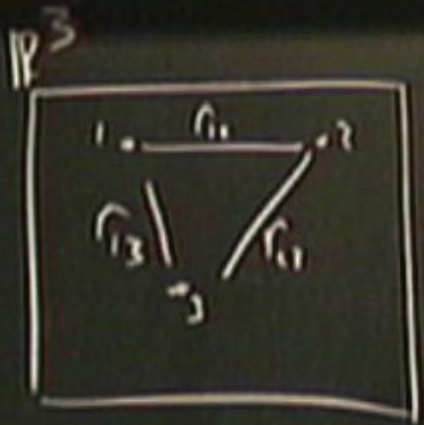
$\frac{g_1}{g_2}, \frac{g_1}{g_3}$



$\frac{r_1}{r_3}, \frac{r_2}{r_3}$



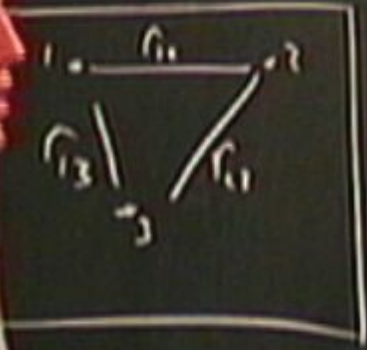
I - particle 4
in space



A hand-drawn circle containing the fractions $\frac{r_1}{r_2}$ and $\frac{r_1}{r_3}$.

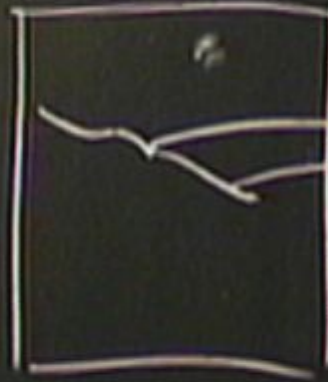


\mathbb{I} - particle #
i - space

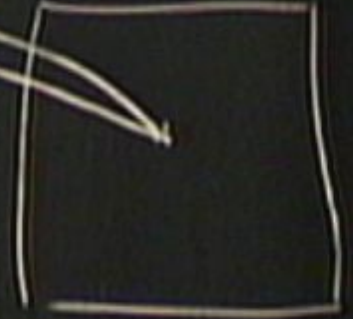


$$\frac{r_2}{r_3}, \frac{r_1}{r_3}$$

ACS, gE



I - particle H
in space RCS



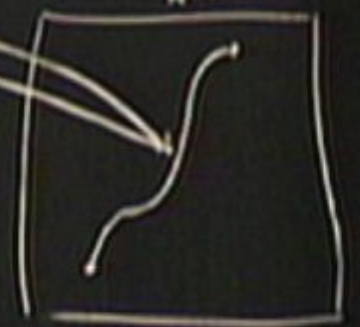
\mathbb{P}^3



ACS, \mathbb{P}^2



\mathbb{I} - particle in space \mathbb{R}^3



\mathbb{P}^3

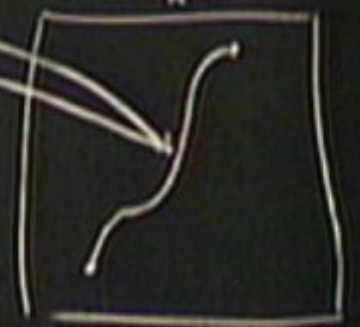


$\frac{g_1}{g_2}, \frac{g_2}{g_3}$

ACS, gE



\mathbb{I} - particle H
i - space RCS



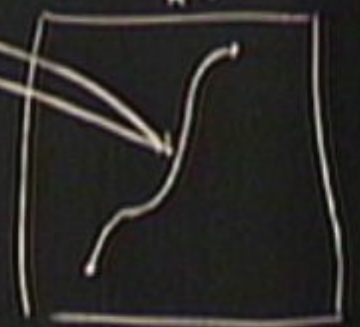


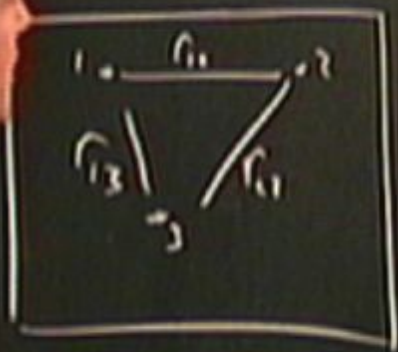
$\frac{r_{12}}{r_{13}}, \frac{r_{12}}{r_{23}}$

ACS, $g \in \mathbb{R}^N$



I - particle H
i - space RCS



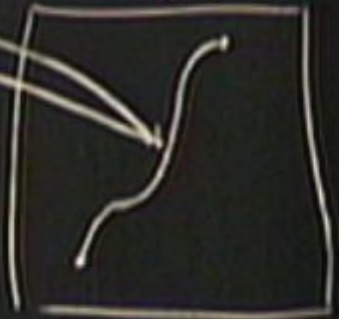


$$\frac{r_{12}}{r_{13}}, \frac{r_{12}}{r_{23}}$$

ACS, $q \in \mathbb{R}^N$



I - particle i
 i - space RCS



constraints

\mathbb{R}^3

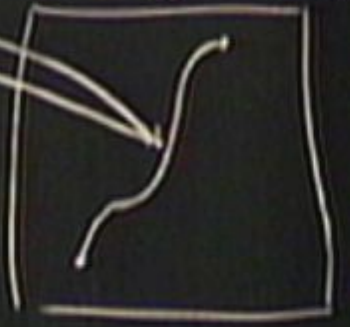


$\frac{r_1}{r_2}, \frac{r_2}{r_3}$

ACS, $g \in dN$



\mathbb{I} - particle u
in space ACS $dN-7$



constraints

Outline:

① Temporal Relationalism

→ Jacobi's Principle

→ ~~Para~~ Particle Mechanics

② Spatial Relationalism:

→ Best-matching

→ Gauge theory

→

①

Rel Mt

no

②

yes

③

no

④

yes

Fl

yes

yes

no

no

Outline:

① Temporal Relationalism

→ Jacobi's Principle

→ ~~Para.~~ Particle Mechanics

② Spatial Relationalism:

→ Best-matching

→ Gauge theory

→ time-gauge PPM

①

②

③

④

Rel Mt

no

yes

no

yes

Fl

yes

yes

no

no

Outline:

① Temporal Relationalism

→ Jacobi's Principle

→ ~~Para.~~ Particle Mechanics

①

Rel Mt

no

②

yes

③

no

② Spatial Relationalism:

→ Best-matching

→ Gauge theory

→ time-gauge PPM

① GR

→ Wam (BPM field theory)

→ GR

④

yes

Fl

ye

ye

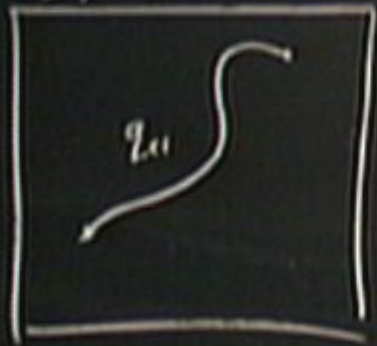
no

no

PPM:



PPM:
 $\frac{CS, q_i^i}{}$



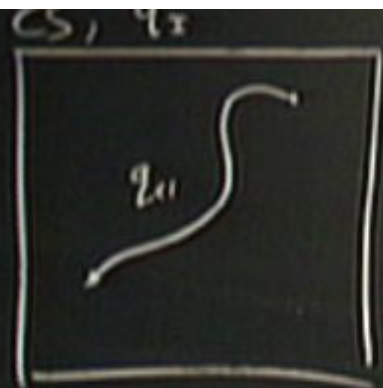
$$S_N = \int dt (T' - V)$$

ppm.
CS, q:



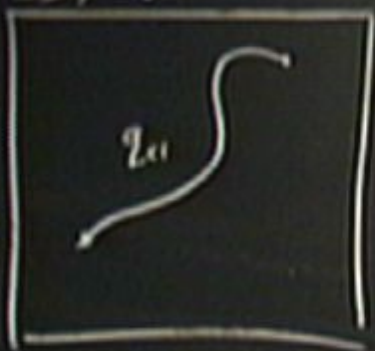
$$S_N = \int dt (T - V)$$

$$T = \frac{1}{2} \sum_I m_I \left(\frac{d\mathbf{r}_I}{dt} \right)^2$$



$$\int \delta W = \int dt (T - V)$$

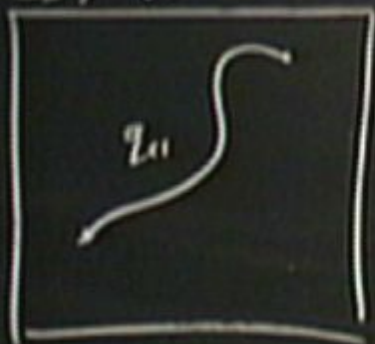
$$T = \frac{1}{2} \sum_i m_i \left(\frac{dq_i}{dt} \right)^2 ; V = V(q_i)$$



$$T = \frac{1}{2} \sum_i m_i \left(\frac{dq_i}{dt} \right)^2 ; V = V(q_i)$$

$$d\tau \rightarrow d\lambda \frac{dq^i}{d\lambda}$$

ppm.
CS, q_i



$$S_N = \int dt (T' - V)$$

$$T' = \frac{1}{2} \sum_I m_I \left(\frac{d\mathbf{q}_I}{dt} \right)^2 ; V = V(q_i)$$

$$\left[d\lambda \frac{dq^i}{d\lambda} = \dot{q}^i d\lambda \right]$$

$$S_N \rightarrow S_{PPM} =$$

$$S_N \rightarrow S_{PPM} = \int d\Omega \dot{q}_0 \left[\frac{T}{\dot{q}_0^2} - v \right]$$

$$S_N \rightarrow S_{PPM} = \int dt \dot{q}_0 \left[\frac{T}{\dot{q}_0^2} - V \right] \quad T = \sum_i \frac{1}{2} m_i \dot{q}_i^2$$

$$S_W \rightarrow \left[S_{ppm} = \int d\lambda \dot{q}_0 \left[\frac{I}{\dot{q}_0^2} - V \right] \right. \quad T = \sum_i \frac{1}{2} m_i \dot{q}_i^2$$

$$\left. = \int d\lambda \left[\frac{I}{\dot{q}_0} - V \dot{q}_0 \right] \right]$$

"It is utterly beyond our power to measure the changes of things by time. Quite the contrary, time is an abstraction, at which we arrive by means of the changes of things; made because we are not restricted to any one definite measure, all being interconnected."

- E. Mach in *The Mechanics*, 1872.

$$S_N \rightarrow \underbrace{S_{PPM} = \int d\lambda \dot{q}_0 \left[\frac{I}{\dot{q}_0^2} - V \right]}_{= \int d\lambda \left[\frac{I}{\dot{q}_0} - v \dot{q}_0 \right]} \quad T = \sum_i \frac{1}{2} m_i \dot{q}_i^2$$

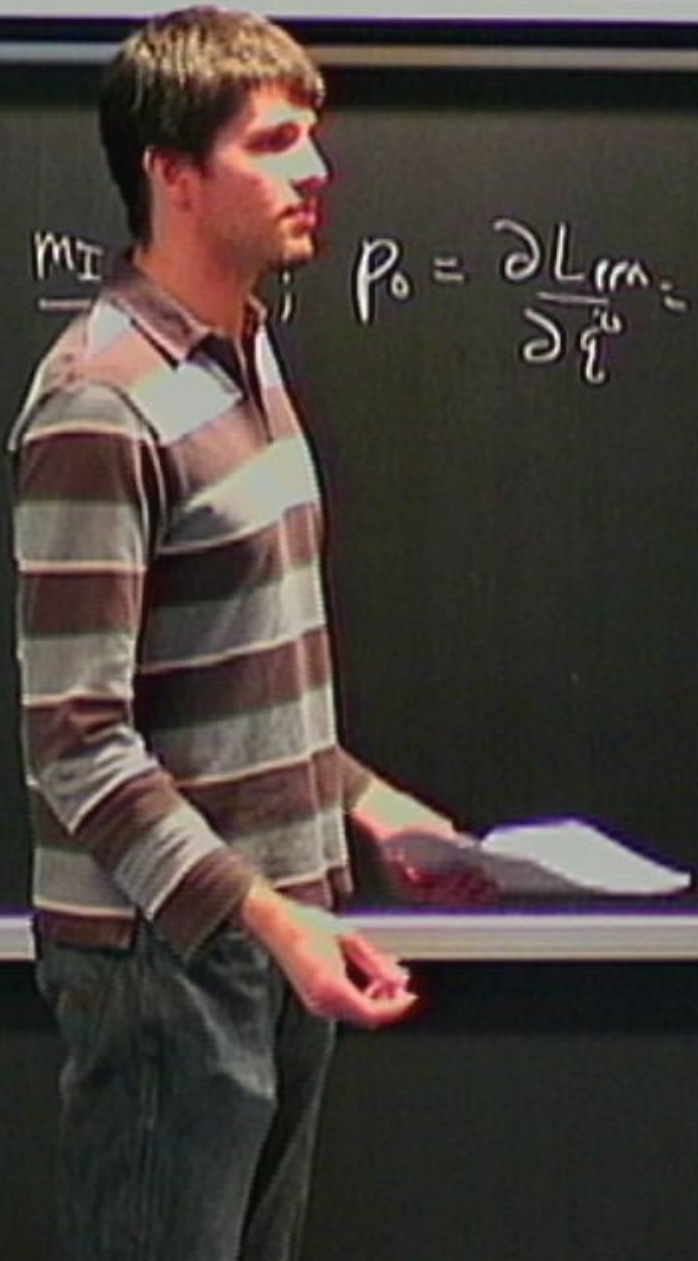
Time Gauge PPM

OK

Ham:

$$p_i^I = \frac{\partial L_{rm}}{\partial q^i} = \frac{m\dot{q}^i}{1}$$

$$p_0 = \frac{\partial L_{rm}}{\partial \dot{q}^0} = -\frac{T}{q^0} - V$$



Time: 5:40 P.M

Ham:

$$p_i^I = \frac{\partial L_{rm}}{\partial \dot{q}^i} = \frac{mI \dot{q}_i^i}{q_0} ; p_0 = \frac{\partial L_{rm}}{\partial \dot{q}^0} = \frac{I}{q_0^2} \dot{q}_0 - V$$

Time: 5 min PPM

$$\frac{\partial L_{\text{rm}}}{\partial q^i} = \frac{m_I \dot{q}^i}{q^i} ; p_0 = \frac{\partial L_{\text{rm}}}{\partial \dot{q}^0} = -\frac{T}{\dot{q}^0} - V$$

$$\psi = \sum_i \frac{p_i^2}{2m_I} + V + p_0 = 0 \Rightarrow$$

Time: 5:45 PPM

Ham:

$$p_i^I = \frac{\partial L_{\text{rm}}}{\partial \dot{q}^i} \quad ; \quad p_0 = \frac{\partial L_{\text{rm}}}{\partial \dot{q}^0} = -\frac{I}{\dot{q}^0} - V$$

$$\mathcal{H} = \dots + p_0 = 0 \Rightarrow \frac{1}{2} \sum m_i \dot{q}_i^2 + V - \frac{I}{\dot{q}^0} - V = 0$$

$$p_i^I = \frac{\partial L_{\text{rm}}}{\partial \dot{q}^i} = \frac{m_I \dot{q}_i^I}{\dot{q}_i^I} ; p_0 = \frac{\partial L_{\text{rm}}}{\partial \dot{q}^0} = -\frac{T}{\dot{q}_0^I} - V$$

$$\mathcal{H} = \sum_i \frac{p_i^{I2}}{2m_I} + V + p_0 = 0 \Rightarrow \frac{1}{2} \sum_i \frac{m_I \dot{q}_i^I^2}{m_I} + V - \frac{T}{\dot{q}_0^I} - \mathcal{H} = 0$$

$$h_c = \sum_i p_i^2 q_i$$

$$H_c = \sum_I p_I^2 \dot{z}_I + \bar{p}_0 \dot{z}^0 - L_{\text{eff}}(z, \dot{z})$$

$$H_c = \sum_I p_I^2 \dot{q}_I + \bar{p}_0 \dot{q}^0 - L_{\text{eff}}(q, p)$$

$$= \sum_I$$

$$H_c = \sum_I p_I^2 \dot{q}_I^2 + \bar{p}_0 \dot{q}^0 - L_{\text{eff}}(q, p)$$

$$= \sum_I \frac{m_I \dot{q}_I^2}{2}$$

$$\sum_i \dot{q}_i^2 + \beta_0 q^0 - L_{\text{eff}}(q, p)$$

$$\sum_i \frac{m_i \dot{q}_i^2}{2} + -\left(\frac{I}{q_{i2}} + U\right) \dot{q}_i$$

$$H_c = \sum_I p_I^2 \dot{q}_I + p_0 \dot{q}^0 - L_{\text{eff}}(q, p)$$

$$= \sum_I \frac{m_I \dot{q}_I^2}{2} + - \left(\frac{I}{q^{1,2}} + U \right) \dot{q}_I -$$

$$H_c = \sum_I p_I^2 \dot{q}_I + \bar{p}_0 \dot{q}^0 - L_{\text{eff}}(q, p)$$

$$= \sum_I \frac{m_I \dot{q}_I^2}{2} + - \left(\frac{I}{q^{12}} + U \right) q_I - \left[\frac{I}{q} - \dot{q}^0 \right]$$

$$\overline{H}_c = \sum_I p_I^2 \dot{q}_I^2 + p_0 \dot{q}^0 - L_{\text{eff}}(q, p)$$

$$= \sum_I \frac{m_I \dot{q}_I^2}{2} + - \left(\frac{I}{q^{12}} + U \right) q_I - \left[\frac{I}{q} - \dot{q}^0 \right]$$

"0"

$$H_T = H_C + N(\lambda) H_C = N(\lambda) \left(\sum_i \frac{p_i^2}{2m_i} + V + P_\omega \right)$$

$$H_T = H_C + N(\lambda) H_C - N(\lambda) \left(\sum_i \frac{p_i^2}{2m_i} + V + P_\omega \right)$$

$$q_i^i = \{q_i^i, H_i\} = p_i^2 N$$

$$H_T = H_C + N(\lambda) H_C - N(\lambda) \left(\sum_i \frac{p_i^2}{2m_z} + V + P_C \right)$$

$$q_i^2 = \{q_i^2, H_C\} = \frac{p_i^2}{m_z} N$$

$$H_T = H_c + N(\lambda) H(\lambda) = N(\lambda) \left(\sum_i \frac{p_i^2}{2m_i} + V + P_\omega \right)$$

$$\overline{[q^i]^2} = \langle q^i, H_T \rangle = \frac{p_i^2 N}{m_i}$$

$$\overline{[q^i]^2} = \langle q^i, H_T \rangle = N$$

$$H_T = H_C + N(\lambda) \left(\sum_i \frac{p_i^2}{2m_i} + V + p_\lambda \right)$$

$$\overline{q_i} = \{0\}$$

$$\overline{p_i} = \left\{ \frac{p_i^2}{m_i} N \right\}$$

$$\overline{p_i} = \{p_i, H_T\} = - \frac{\partial V}{\partial q_i} N$$

$$\overline{p_\lambda} = \{p_\lambda, H_T\} = 0$$

$$(p_\lambda = -E)$$

$$\begin{aligned}
 & \left(\sum_i \frac{p_i^2}{2m_i} + V + p_\omega \right) \\
 \overline{[q^i]_t} = \{q^i, H_t\} &= \overline{\left[\frac{p_i^2}{m_i} N \right]} \quad \overline{[p^i]_t} = \{p^i, H_t\} = - \overline{\left[\frac{\partial V}{\partial q^i} N \right]} \quad \overline{[p^i]_t} = \{p^i, H_t\} = 0 \\
 \overline{[q^i]_t} = \{q^i, H_t\} &= \overline{N} \quad (p^i = -E)
 \end{aligned}$$

$$H_T = H_C + N(\lambda) H' - N(\lambda) \left(\sum_i \frac{p_i^2}{2m_z} + V + p_u \right)$$

$$\overline{[q^i]_T} = \{q^i, H_T\} = \left[\frac{p_i^2}{m_z} N \right]$$

$$\overline{[q^i]_T} = \{q^i, H_T\} = N$$

$$\overline{[p^i]_T} = \{p^i, H_T\} = - \frac{\partial V}{\partial q^i} \quad \overline{[p^i]_T} = \{p^i, H_T\} = 0$$

$$(p^i = -E)$$

$$p^i = m_z \dot{q}^i \Rightarrow \frac{1}{N} \frac{d}{dt} \left(\frac{N_j q^i}{N} \right) = - \frac{\partial V}{\partial q^i}$$

$$\sum_i N d\lambda = \frac{dq^0}{dt} \alpha \lambda - dq^0$$

$$\{ \mathcal{H}_T \} = N$$

$$p_i^t = m \dot{q}_i^t \Rightarrow \frac{1}{N} \frac{d}{dt} \left(\frac{m \dot{q}_i^t}{N} \right) = - \frac{\partial V}{\partial q_i^t}$$

$$(p^t = -E)$$

$$m \frac{d^2 q^i}{dt^2} = - \frac{\partial V}{\partial q^i}$$

$$\int \lambda d\lambda = \frac{dq^0}{dt} d\lambda - dq^0$$

$$\frac{dq^i}{dt} = \frac{N}{m}$$

$$p_i = m \frac{dq^i}{dt} = \frac{1}{N} \frac{d}{dt} \left(\frac{N m q^i}{N} \right) = - \frac{\partial V}{\partial q^i}$$

$$p = -E$$

$$m_z \frac{d^2 q_i^z}{dt^2} = - \frac{\partial V}{\partial q_i^z}$$

$$\overline{q^z} = \{q_i^z, H_i^z\} = \left[\frac{p_i^z}{m_z}, N \right]$$

$$\overline{q^z} = \{q_i^z, H_i^z\} = N$$

$$\overline{p^z} = \{p_i^z, H_i^z\} = - \frac{\partial V}{\partial q_i^z}$$

$$\overline{p^z} = \{p_i^z, H_i^z\} = 0$$

($p^z = -E$)

$$p_i^z = m_z \dot{q}_i^z$$

$$\Rightarrow \left[\frac{1}{N} \frac{d}{dt} \left(\frac{m_z q_i^z}{N} \right) = - \frac{\partial V}{\partial q_i^z} \right]$$

$$m_z \frac{d^2 q_i^z}{dt^2} = - \frac{\partial V}{\partial q_i^z}$$

$$H_{cl} = \sum q_i^z, H_{cl} = \frac{p_i^z{}^2}{m_z} N$$

$$\overline{\dot{q}^z} = \sum_{N=1} \{ \dot{q}_i^z, H_{cl} \} = \overline{N}$$

$N=1$

$$H_{cl} = \sum p_i^z, H_{cl} = - \frac{\partial V}{\partial q_i^z} \quad \left(\overline{p^z} = - \overline{E} \right)$$

$$p_i^z = m_z \dot{q}_i^z = \frac{1}{N} \frac{d}{dt} \left(\frac{m_z q_i^z}{N} \right) = - \frac{\partial V}{\partial q_i^z}$$

$$\sum_i \lambda \delta q_i = \frac{dq_i}{dt} \delta \lambda - dq_i$$

$$m_i \frac{d^2 q_i}{dt^2} = - \frac{\partial V}{\partial q_i}$$

$$T + V = E$$

$$= \sum_i \frac{m_i \dot{q}_i^2}{2} + - \left(\frac{I}{2} + V \right) q_i - \left[\frac{I}{2} - q_i \right]$$

$$\sum_I m_I \left(\frac{dq_i}{dq_0} \right)^2 + V + \mathcal{M}$$

(1)

(2)

(3)

(4)

↓
constraints

$$\sum_I m_I \left(\frac{dq_i}{dt} \right)^2 + V + \mathcal{H} \quad (3)$$

$$\dot{q}_i = \sqrt{\frac{\sum_I m_I \dot{q}_i^2}{E - V}} \quad (4)$$

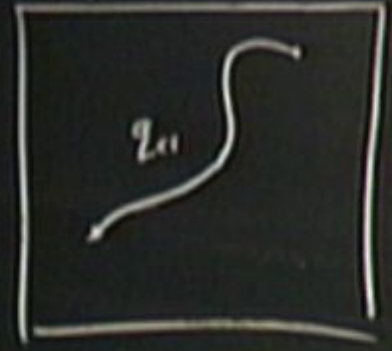
constraints

$$\sum_I m_I \left(\frac{dq_i}{dq_0} \right)^2 + V = E \quad (3)$$

$$dq_0 = \sqrt{\frac{\sum_I m_I dq_i^2}{E - V}} \quad (4)$$

constraints

CS, q_i



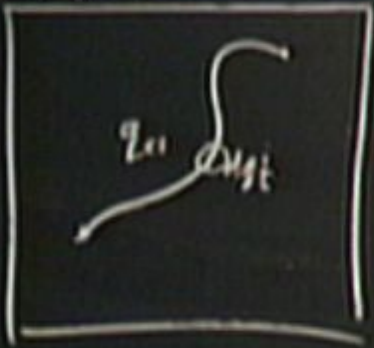
$$S_N = \int dt (T' - V)$$

~~$T = \sum_i m_i \left(\frac{dx_i}{dt} \right)^2$
 $V = V(x_i)$
 $\int \sum_i m_i dx_i$
 $\int \sum_i m_i dx_i$~~

$$m_i \frac{d^2 q_i}{dt^2} = - \frac{\partial V}{\partial q_i}$$

$$T + V = E$$

CS, q_i



$$S_N = \int dt (T' - V)$$

~~$\sum_i m_i \left(\frac{1}{2} \dot{q}_i^2 - V(q_i) \right)$~~
 ~~$V = V(q_i)$~~
 ~~$\int N d\lambda F, \frac{dq^0}{\partial \lambda} = dq^0$~~
 ~~$T' + V = E$~~

$$m_i \frac{d^2 q_i}{dt^2} = - \frac{\partial V}{\partial q_i}$$

$$T' + V = E$$

$$\sum_i m_i \left(\frac{dq_i}{dt} \right)^2 + V = E \quad (3)$$

$$dq_0 = \sqrt{\frac{\sum_i m_i q_i^2}{E - V}} \quad (4)$$

Constraints

PPM.
CS, q_i, q^0



$$S_N = \int dt (T' - V)$$

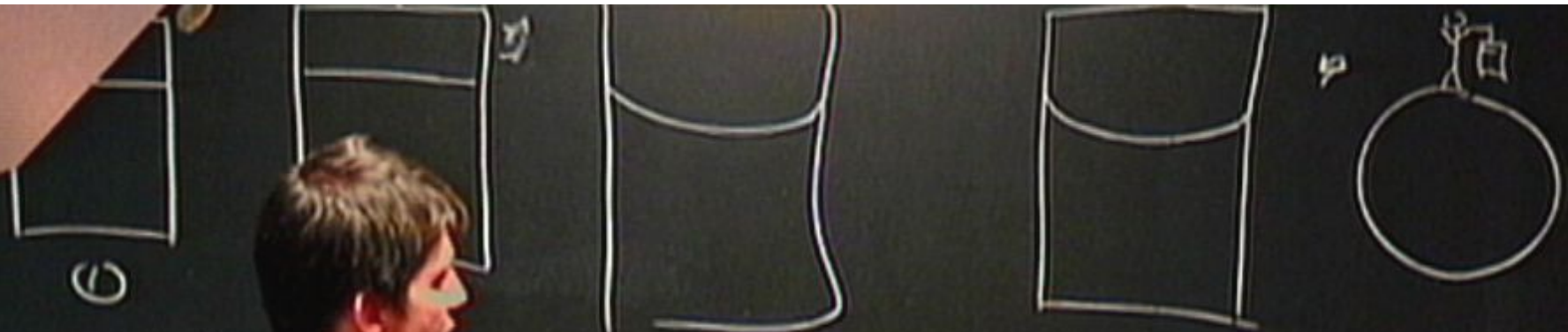
$$m_i \frac{d^2 q_i}{dt^2} = - \frac{\partial V}{\partial q_i}$$



$$\sum_I m_I (dq_i)^2$$

$$\sqrt{\sum_I \frac{m_I}{2} \dot{q}_i^2} = \dot{q}_m - \dot{q}_n$$

constraints



$$\sum_{i=1}^n m_i z_i (dq_i)$$

III

↓

$$\int_{q_{in}}^{q_{out}} \sqrt{\frac{\sum_{i=1}^n m_i z_i^2 \dot{q}_i^2}{E - V}} = q_{out}^0 - q_{in}^0$$

Constraints