Title: Redesigning Electroweak Theory: Does the Higgs Particle Exist?

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Abstract: An electroweak model in which the masses of the W and Z bosons and the fermions are generated by quantum loop graphs through a symmetry breaking of the vacuum is investigated. The model is based on a regularized quantum field theory in which the quantum loop graphs are finite to all orders of perturbation theory and the massless theory is gauge invariant, PoincarÃ© invariant, and unitary to all orders. The breaking of the electroweak symmetry SUL(2) Ã— UY (1) is achieved without a Higgs particle. A fundamental energy scale Î›W (not to be confused with a naive cutoff) enters the theory through the regularization of the Feynman loop diagrams. The finite regularized theory with Î›W allows for a fitting of low energy electroweak data. Î›W ~ 542 GeV is determined at the Z pole by fitting it to the Z mass mZ, and anchoring the value of sinÂ²Î¸w to its experimental value at the Z pole yields a prediction for the W mass mW that is accurate to about 0.5% without radiative corrections. The scattering amplitudes for WLWL â†' WLWL and e+eâˆ' â†' W+L Wâˆ'L processes do not violate unitarity at high energies due to the suppression of the amplitudes by the running of the coupling constants at vertices. There is no Higgs hierarchy fine-tuning problem in the model. The unitary tree level amplitudes for WLWL â†' WLWL scattering and e+eâˆ' â†' W+L Wâˆ'L annihilation, predicted by the finite electroweak model are compared with the amplitudes obtained from the standard model with Higgs exchange. These predicted amplitudes can be used to distinguish at the LHC between the standard electroweak model and the Higgsless model.

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1. The Standard Electroweak Model with a Higgs Particle

- The standard electroweak (EW) model gains mass for the W and Z bosons, while keeping the photon massless by introducing a classical scalar field into the action. This scalar degree of freedom is assumed to transform as an isospin doublet, spontaneously breaking the $SU_L(2) \times U_Y(1)$ by a Higgs mechanism at the purely classical tree graph level.

- The origin of the symmetry breaking mechanism remains elusive after almost 50 years. The standard and commonly accepted explanation is a spontaneous symmetry breaking framework in which the symmetry $SU_L(2) \times U_Y(1)$ is not broken by the interactions but is "softly" broken by the asymmetry of the ground state (vacuum state).

• Despite its phenomenological success, theoretical problems prompt searching for an alternative to the standard EW theory. There is the serious Higgs hierarchy problem (unstable Higgs mass) and the cosmological constant problem. The spontaneous breaking of SU(2) X U(1) generates a value $\langle 0|V_{vac}|0\rangle = -\mu^4/\lambda$ for the vacuum density, which is some 10^{56} times larger in magnitude than the observed value $\rho^{vac}_{obs} \sim (0.0024\ \text{eV})^4$ and has the wrong sign.

• The tree-level (bare) Higgs mass receives quadratically-divergent corrections from the Higgs loop diagrams.

• Discovering a satisfactory alternative has proved to be highly non-trivial. Proposed alternatives face severe problems. New particle contributions at less than 1 or 2 TeV level can affect precision EW data that can generate unacceptably large effects; significant fine-tuning may be required at least at the 1-percent level. These models include MSSM, Little Higgs, pseudo-Goldstone bosons. Extensions of the standard EW model such as technicolor, and other composite models can face unacceptably large flavor changing contributions and CP violation.

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• The minimal EW model with a Higgs doublet is consistent with the experimental bounds on flavor changing neutral currents and CP violation.

• The primary target of an EW global fit is the prediction of the Higgs mass. The *complete fit* represents the most accurate estimation of M_H considering all available data (arXiv:0811.0009[CERN]). The result is $M_H = 116.4+18.3-1.3$ GeV where the error accounts for both experimental and theoretical uncertainties . The 2σ and 3σ allowed regions of M_H, including all errors, are [114, 145] GeV and [[113, 168] and [180, 225]] GeV, respectively. The result for the *standard fit* without the direct Higgs searches is $M_H = 80+30-23$ GeV and the 2σ and 3σ intervals are, respectively, [39, 155] GeV and [26, 209] GeV. The 3σ upper limit is tighter than for the complete fit because of the increase of the best fit value of M_H in the *complete fit*.

• We conclude from this that the minimal EW model requires a light Higgs.

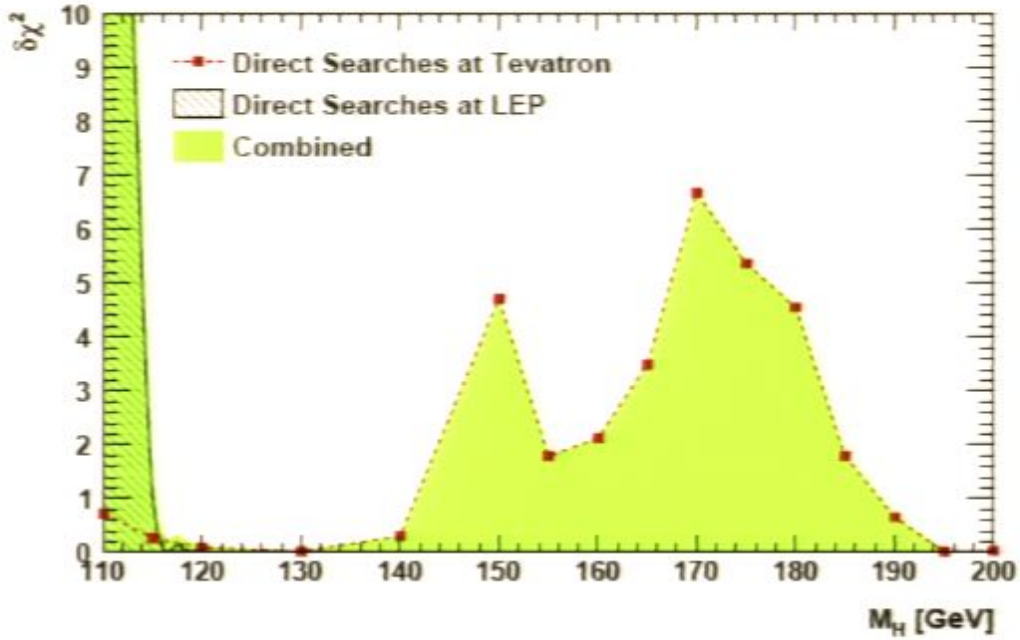
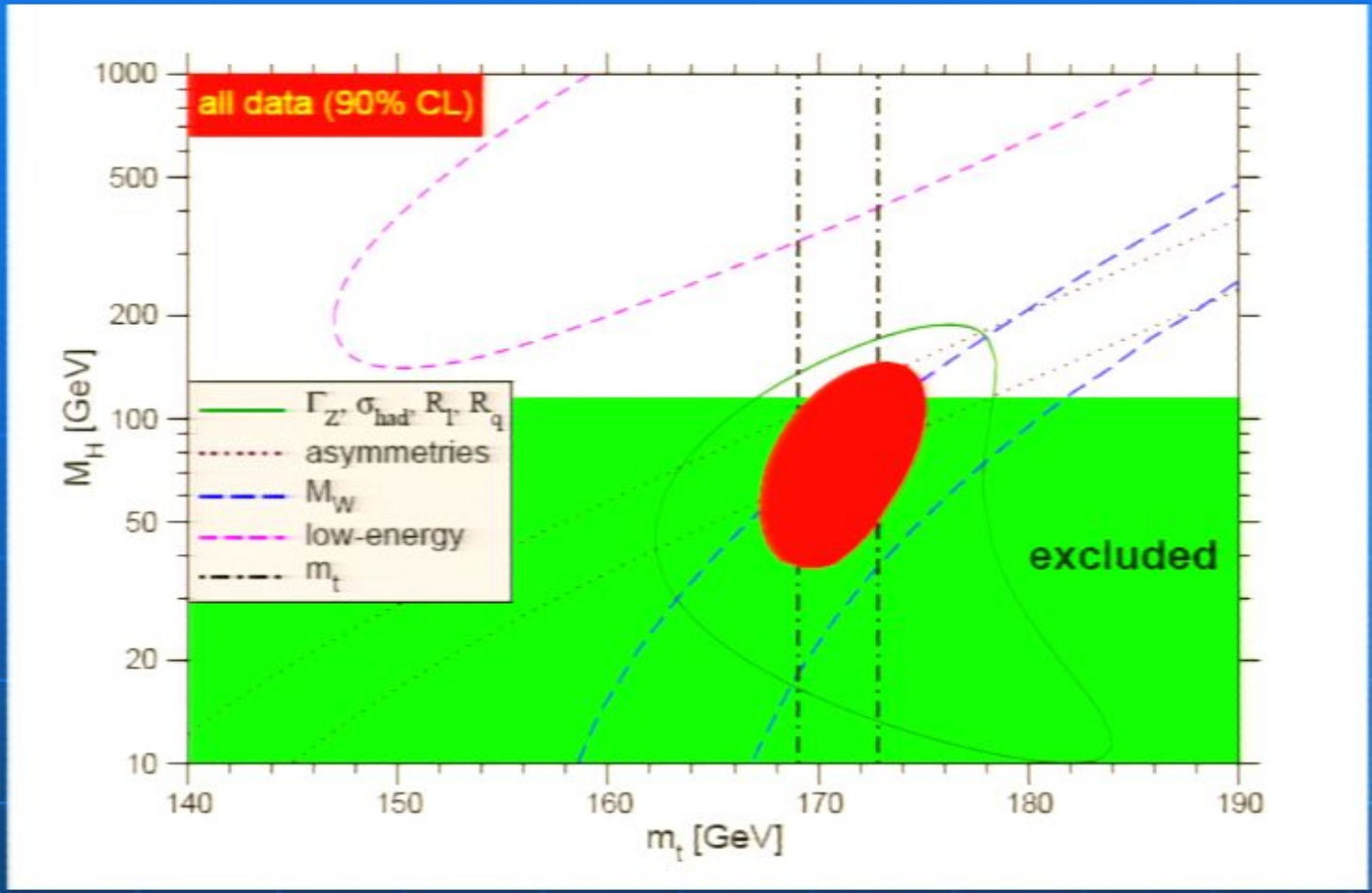
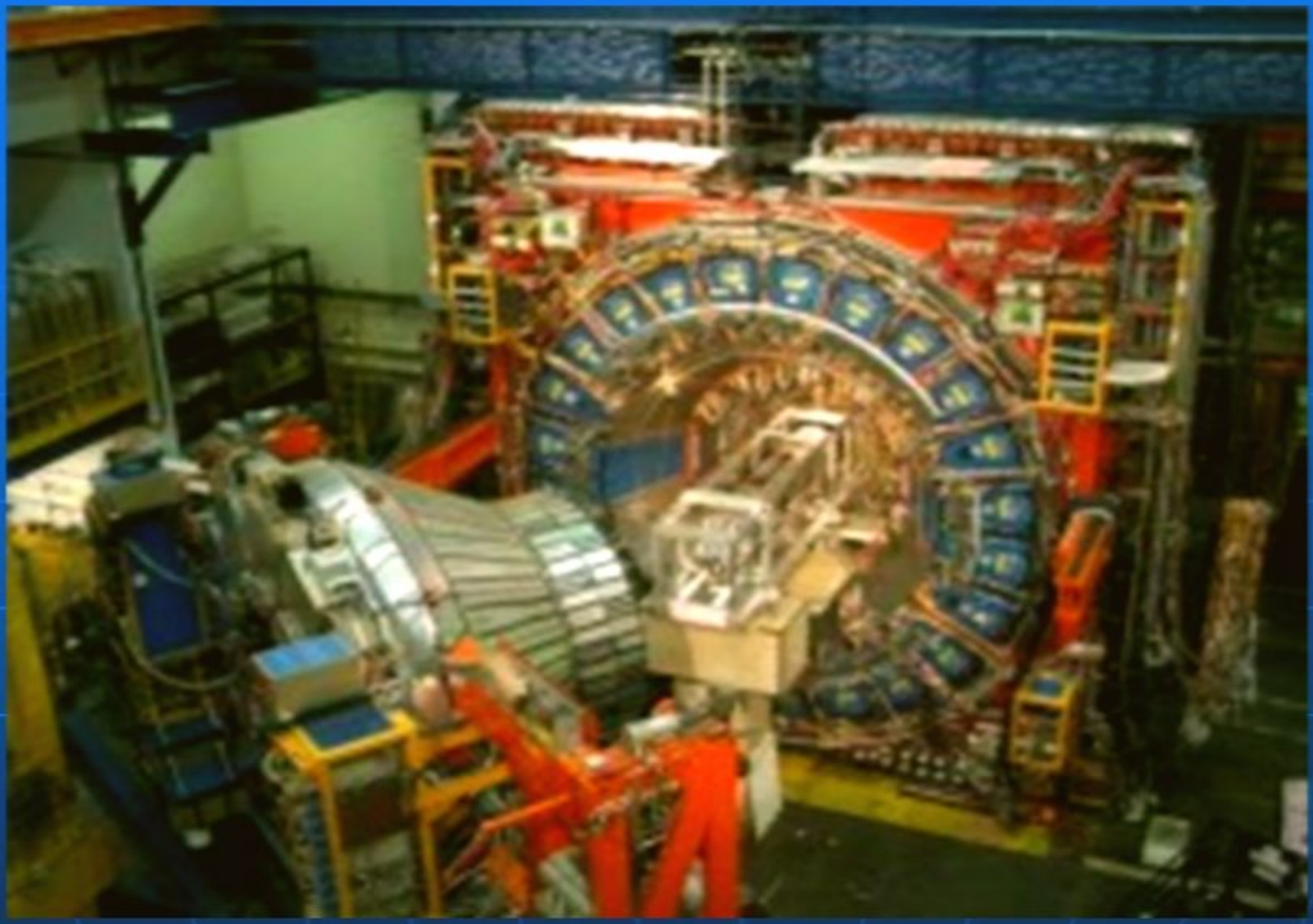


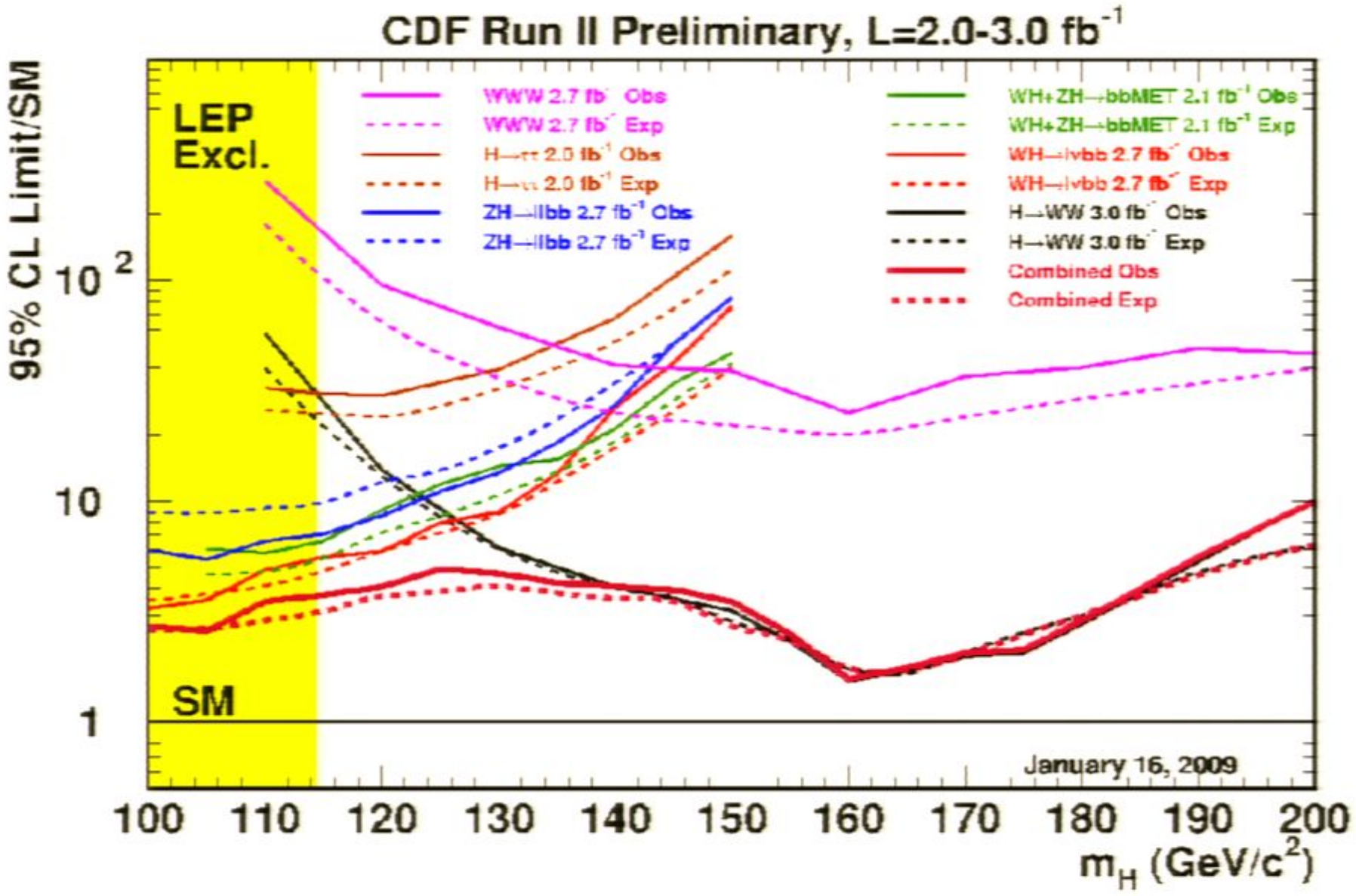
Figure 1: The contribution to the χ^2 estimator versus M_H derived from the experimental information on direct Higgs boson searches made available by the LEP Higgs Boson and the Tevatron New Phenomena and Higgs Boson Working Groups [71–73]. The solid dots indicate the Tevatron measurements. Following the original figure they have been interpolated by straight lines for the purpose of presentation and in the fit. See text for a description of the method applied.



1σ allowed regions in M_H vs m_t and the 90% cl global fit region from precision data, compared with the direct exclusion limits from LEP 2. Plot courtesy of the Particle Data Group.



CDF Tevatron detector at Fermi Laboratory


CDF Run II Preliminary, L=2.0-3.0 fb^{-1}
95% CL Limit/SM
LEP Excl.
SM
WWW 2.7 fb^{-1} Obs
WWW 2.7 fb^{-1} Exp
H→ττ 2.0 fb^{-1} Obs
H→ττ 2.0 fb^{-1} Exp
ZH→llbb 2.7 fb^{-1} Obs
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WH+ZH→bbMET 2.1 fb^{-1} Obs
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WH→lνbb 2.7 fb^{-1} Obs
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H→WW 3.0 fb^{-1} Obs
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Combined Obs
Combined Exp
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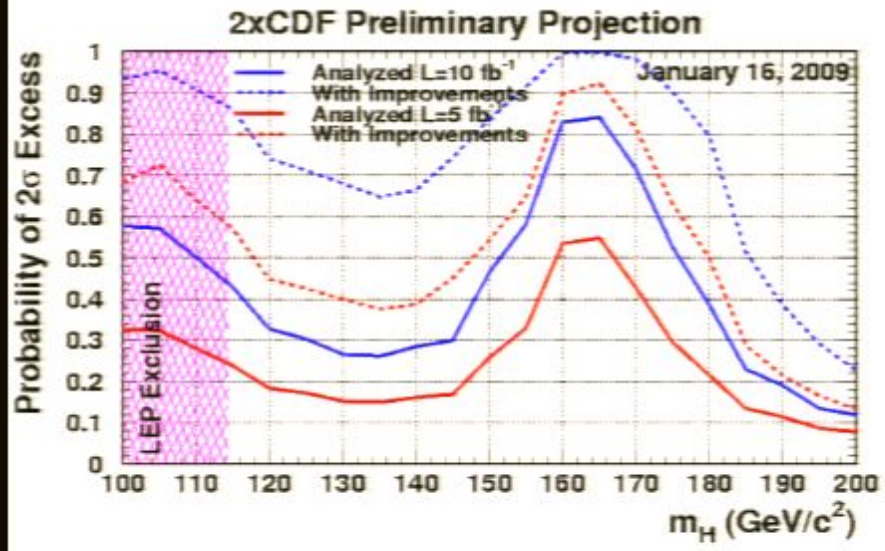
10^2
10
1
100 110 120 130 140 150 160 170 180 190 200
m_H (GeV/c^2)


2xCDF Preliminary Projection
Probability of 2σ Excess
Analyzed L=10 fb-1
With Improvements
Analyzed L=5 fb-1
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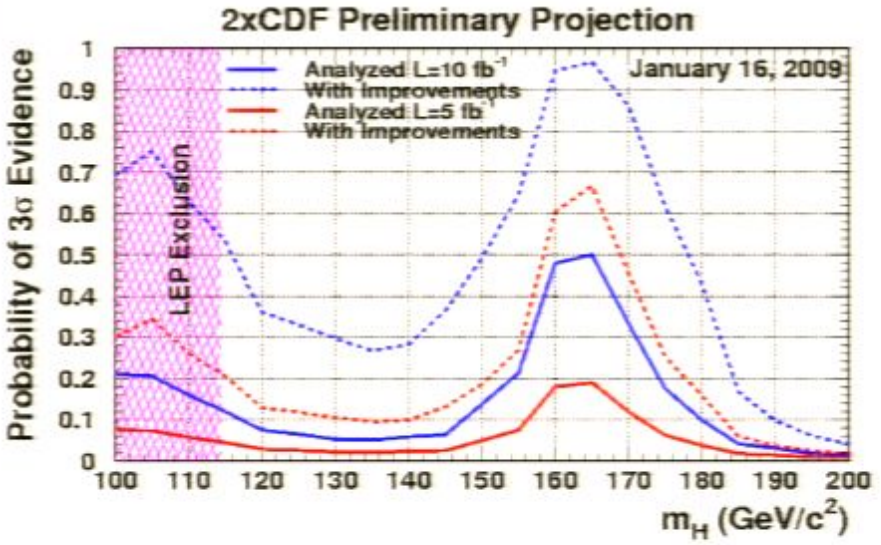
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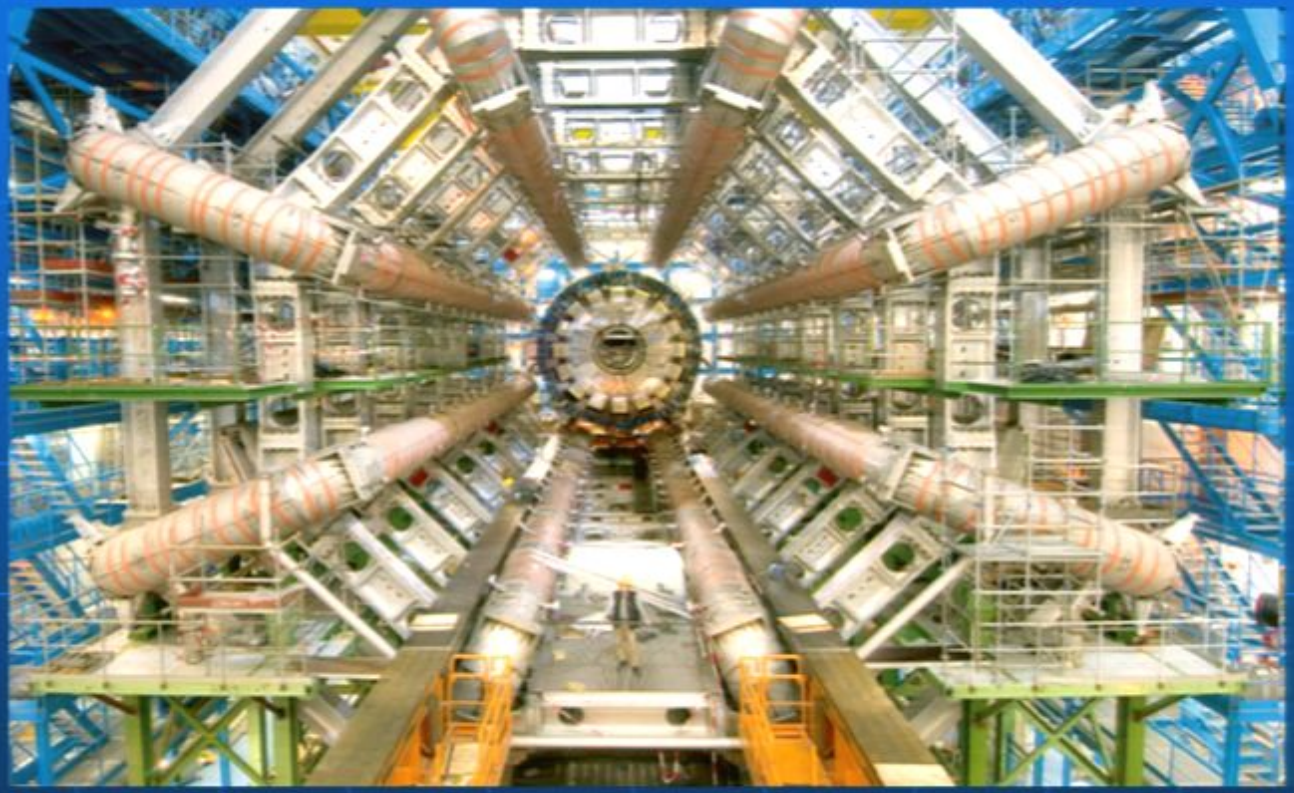
The ATLAS experiment at CERN. (Courtesy CERN)



2. The Gauge Invariant Local EW Theory

• We shall use the metric convention, $\eta_{\mu\nu}$= diag(+1,-1,-1,-1), and set ħ= c = 1. The theory is based on the local $SU_L(2)$ X $U_Y(1)$ invariant Lagrangian that includes leptons and quarks (with the color degree of freedom of the strong interaction group $SU_c(3)$) and the boson vector fields that arise from gauging the global $SU_L(2)$ X $U_Y(1)$ symmetries:

$$L_{\text{local}} = L_F + L_W + L_B + L_I$$

• L_F is the free fermion Lagrangian consisting of massless kinetic terms for each fermion:

$$L_F = \sum_{\psi} \bar{\psi} i\partial\!\!\!/ \psi = \sum_{q^L} \bar{q}^L i\partial\!\!\!/ q^L + \sum_{f} \bar{\psi}^R i\partial\!\!\!/ \psi^R$$

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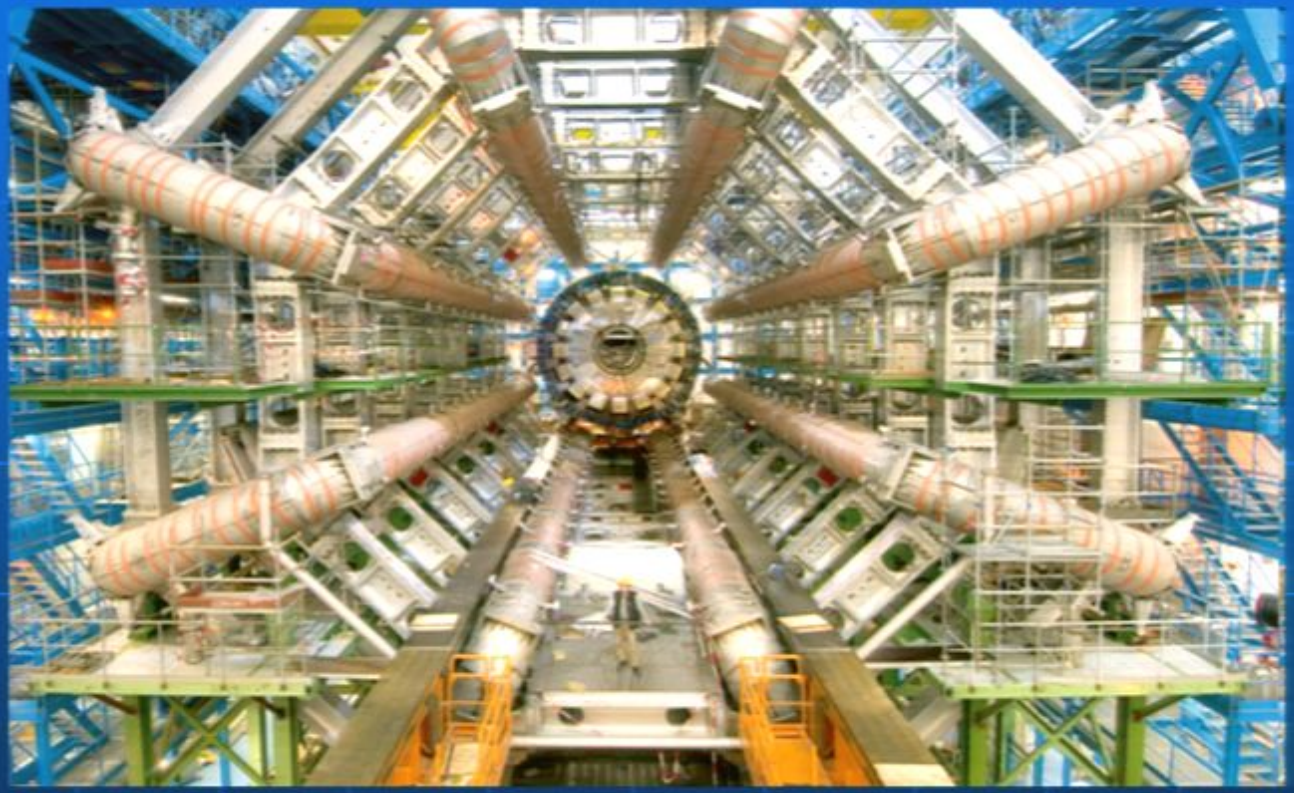

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$$q^L \in \left[\begin{pmatrix} \nu^L \\ e^L \end{pmatrix} \cdot \begin{pmatrix} u^L \\ d^L \end{pmatrix}_{r,g,b} \right]$$

$$L_B = -\frac{1}{4} B^{\mu\nu} B_{\mu\nu}$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

$$L_W = -\frac{1}{4} W^a_{\mu\nu} W^{a\mu\nu}$$

$$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu - g f^{abc} W^b_\mu W^c_\nu$$

- Note that there is no classical scalar field contribution in the Lagrangian.
- The SU(2) generators satisfy

$$[T^a . T^b] = i f^{abc} T^c . \quad \text{with} \quad T^a = \frac{1}{2} \sigma^a$$

- The fermion gauge-boson interaction terms are

$$L_I = -g J^{a\mu} W^a_\mu - g' J^\mu_Y B_\mu$$

$$J^{a\mu} = \sum_{q^L} \bar{q}^L \gamma^\mu T^a q^L . \quad \text{and} \quad J^\mu_Y = \sum_\psi \frac{1}{2} Y_\psi \bar{\psi} \gamma^\mu \psi$$

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$$Y(q^L_{\text{lepton}}) = -1. \quad Y(q^L_{\text{quark}}) = \frac{1}{3}. \quad Y(e^R) = -2. \quad Y(\nu^R) = 0. \quad Y(u^R) = \frac{4}{3}. \quad Y(d^R) = \frac{2}{3}.$$

- L is invariant under the local gauge transformations (order g,g′):

$$\delta\psi^L = -\left(igT^a\theta^a + ig'\frac{Y_\psi}{2}\beta\right)\psi^L. \qquad \delta\psi^R = -ig'\frac{Y_\psi}{2}\beta\psi^R.$$

$$\delta W^a_\mu = \partial_\mu\theta^a + gf^{abc}\theta^b W^c_\mu. \qquad \delta B_\mu = \partial_\mu\beta$$

- L is an $SU(2)_L$ X $U(1)_Y$ invariant Lagrangian.
- Quantization is accomplished via the path integral formalism:

$$\langle T(\mathcal{O}[\phi])\rangle \propto \int [D\bar{\psi}][D\psi][DW][DB]\mu_{\text{inv}}[\bar{\psi}.\psi.B.W]\mathcal{O}[\phi]\exp\left(i\int d^4x L_{\text{local}}\right)$$

- In the local case the invariant measure μ_{inv} is the trivial one.
- We have to gauge fix the Lagrangian:

$$L_{GF} = -\frac{1}{2\xi}(\partial_\mu B^\mu)^2 - \frac{1}{2\xi}(\partial_\mu W^{a\mu})^2$$

- We look at diagonalizing the charged sector and mixing in the neutral boson sector. If we write

$$W^\pm = \frac{1}{\sqrt{2}}(W^1 \mp iW^2)$$

then we get the fermion interaction terms:

$$-\frac{g}{\sqrt{2}}(J^+_\mu W^{+\mu} + J^-_\mu W^{-\mu}).$$

$$J^\pm_\mu = J^\pm_{1\mu} \pm iJ^\pm_{2\mu} = \sum_{q_L} \bar{q}^L \gamma_\mu T^\pm q^L$$

$$J^+_\mu = \sum_{q_L} (\bar{\nu}^L \gamma_\mu e^L + \bar{u}^L \gamma_\mu d^L).$$

- In the neutral sector, we can mix the fields in the usual way:

$$Z_\mu = c_w W_\mu^3 - s_w B_\mu \quad \text{and} \quad A_\mu = c_w B_\mu + s_w W_\mu^3.$$

$$s_w = \sin\theta_w \qquad c_w = \cos\theta_w$$

$$s_w^2 = \frac{g'^2}{g^2 + g'^2} \quad \text{and} \quad c_w^2 = \frac{g^2}{g^2 + g'^2}.$$

- The neutral current fermion interaction terms now look like:

$$-gJ^{3\mu}W_\mu^3 - g' J_Y^\mu B_\mu = -(gs_w J^{3\mu} + g' c_w J_Y^\mu)A_\mu - (gc_w J^{3\mu} - g' s_w J_Y^\mu)Z_\mu.$$

- We have the unification condition: $e = gs_w = g' c_w$

$$J_{\text{em}}^\mu = J^{3\mu} + J_Y^\mu. \qquad J_{\text{NC}}^\mu = J^{3\mu} - s_w J_{\text{em}}^\mu.$$

$$L_I = -\frac{g}{\sqrt{2}}(J_\mu^+ W^{+\mu} + J_\mu^- W^{-\mu}) - gs_w J_{\text{em}}^\mu A_\mu - \frac{g}{c_w} J_{\text{NC}}^\mu Z_\mu$$

3. The Gauge Invariant Regularized Theory

• To regularize the fields, we write the non-local (smeared) fields as a convolution of the local fields with a function whose Euclidean momentum space Fourier transform is an entire function. This function can be related to a Lorentz invariant operator distribution as

$$\Phi(x) = \int d^4y G(x-y)\phi(y) = G\left(\frac{\Box}{\Lambda_W^2}\right)\phi(x).$$

$$G\left(\frac{\Box}{\Lambda_W^2}\right) \equiv \mathcal{E}_m = \exp\left(-\frac{\Box + m^2}{2\Lambda_W^2}\right).$$

• We now write the initial Lagrangian in non-local form:

$$L_{\rm reg} = L[\phi]_F + \mathcal{L}[\Phi]_I.$$

where $\mathcal{L}[\Phi]_I$ indicates smearing of the interacting fields.

• An essential feature of the regularized, non-local field theory is the requirement that the classical tree graph theory remain local, giving us a well defined classical limit in the gauge invariant case.

JWM, Phys. Rev. D 41, 1177 (1990); D. Evens, JWM, G. Kleppe, and R. P. Woodard, Phys. Rev. D 43, 499 (1991); JWM, ArXiv 0709.4269 [hep-ph] (2007); JWM and V. T. Toth, ArXiv 0812.1991 [hep-ph] (2008).

• We first note that we must alter the quantized form of the theory by generalizing the path integral:

$$\langle T^*(\mathcal{O}[\Phi])\rangle \propto \int [D\bar{\psi}][d\psi][DW][DB][D\bar{\eta}][D\eta][D\bar{c}][Dc]\mathcal{O}[\Phi]\exp(iS_0[\phi]+iS_I[\Phi]).$$

$$W[\mathcal{J}] = \ln(Z[\mathcal{J}]) = \ln\left(\int [D\phi]\exp\left(i\int dx\{L_F[\phi]+\mathcal{L}_I[\Phi]+\mathcal{J}(x)\Phi(x)\}\right)\right).$$

• We note that in momentum space, the smeared fields are related one-to-one to the local fields:

$$\Phi(p) = G(p^2)\phi(p) = \exp\left(\frac{p^2-m^2}{2\Lambda_W^2}\right)\phi(p).$$

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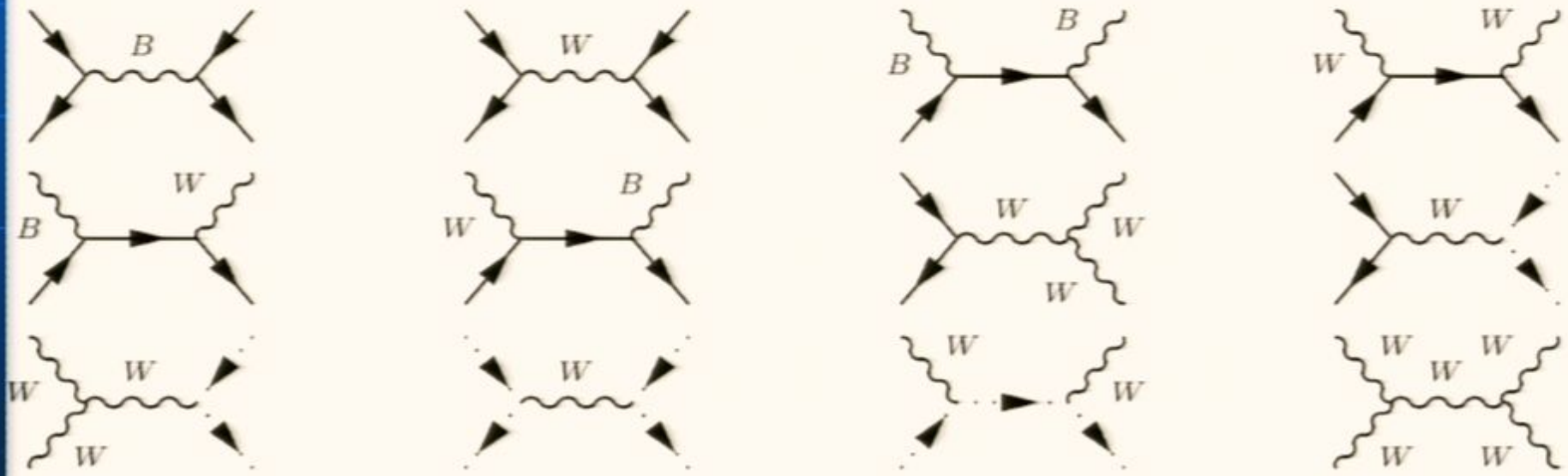

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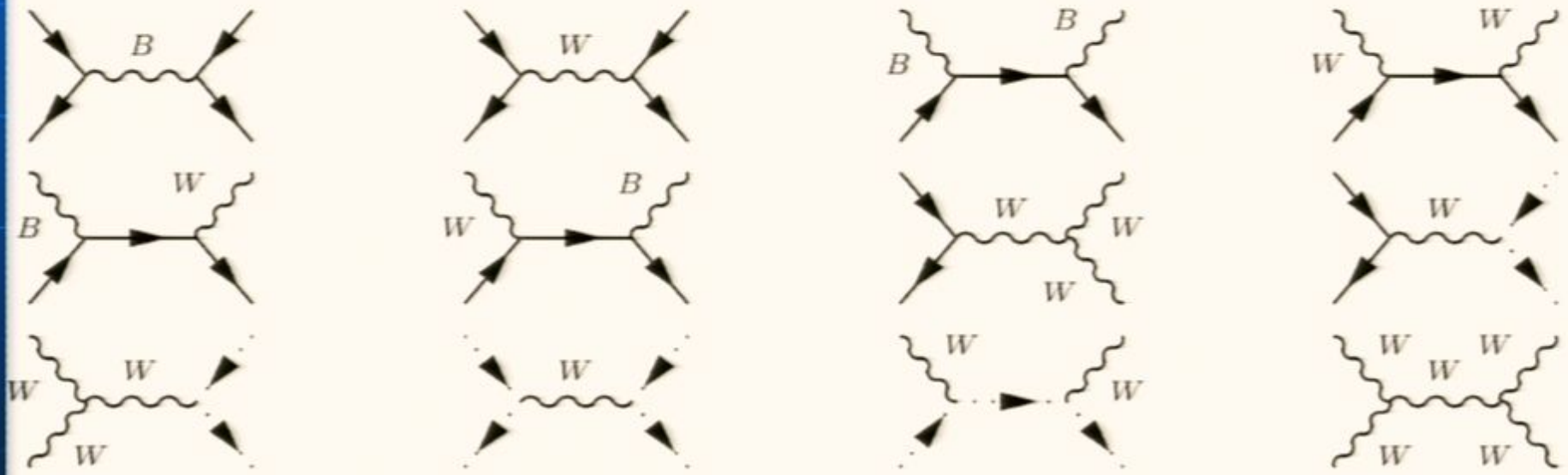

FIG. 2: Tree graphs fixed by the nonlocal theory.

- For convenience later on, we will define another set of propagators

$$i\bar{\Delta} = (1 - G^2) i\Delta. \qquad \text{etc..}$$

so that the sum of the tree propagators with these give the causal propagators of local point theory. This is useful when calculating tree graphs, since one can merely replace the smeared propagator with the barred one in the amplitude, and then add the appropriate term to the interaction Lagrangian. This procedure guarantees that all calculated tree graphs are local and point-like to all orders of perturbation theory.

- Along with the interaction terms of the local theory that now look identical after having made the field redefinition, we have to second order in coupling additional terms coming from fixing the tree graphs.

$$\mathcal{L}_I = -\frac{1}{2}g'^2 J_Y^\mu \bar{D}_{\mu\nu} J_Y^\mu - \frac{1}{2}g^2 J^{a\mu} \bar{D}^{ab}_{\mu\nu} J^{b\nu} - g'^2 \sum_\psi \left(\frac{Y_\psi}{2}\right)^2 \bar{\psi} \not{B} \bar{S} \not{B} \psi$$
$$- g^2 \sum_{q^L} \bar{q}^L \not{W} S \not{W} q^L - gg' \sum_{q^L} \frac{Y_q}{2} \bar{q}^L \not{W} \bar{S} \not{B} q^L - gg' \sum_{q^L} \frac{Y_q}{2} \bar{\psi}^L \not{B} \bar{S} \not{W} \psi^L$$
$$- ig^2 J^{a\mu} \bar{D}^{ab}_{\mu\nu} C^{b\nu} - ig^2 f^{acd} J^{b\mu} \bar{D}^{ab}_{\mu\nu} \partial^\nu \bar{c}^c c^d + g^2 f^{acd} C^{b\mu} \bar{D}^{ab}_{\mu\nu} \partial^\nu \bar{c}^c c^d$$
$$+ \frac{1}{2}g^2 f^{abc} f^{def} \partial^\mu \bar{c}^b c^c \bar{D}^{ad}_{\mu\nu} \partial^\nu \bar{c}^e c^f - g^2 f^{abc} f^{def} \partial^\mu \bar{c}^b W^c_\mu \bar{\Delta}^{ad} \partial^\nu (W^e_\nu c^f)$$
$$+ \frac{1}{2}g^2 C^{a\mu} \bar{D}^{ab}_{\mu\nu} C^{b\nu}.$$

$$C^a_\mu = f^{abc}(2W^c_\nu \partial^\nu W^b{}_\mu - W^c_\nu \partial_\mu W^{b\nu} - W^c_\mu \partial_\nu W^{b\nu}).$$

$$iS = \frac{-i}{\not{p} + i\epsilon} = \frac{-i\not{p}}{p^2 + i\epsilon}, \quad i\Delta = \frac{-i}{p^2 + i\epsilon}$$

$$iD_{\mu\nu} = \frac{-i}{p^2 + i\epsilon}\left(\eta_{\mu\nu} + (\xi - 1)\frac{p_\mu p_\nu}{p^2}\right). \quad i\Delta^{ab} = i\Delta\delta^{ab}, \quad iD^{ab}_{\mu\nu} = iD_{\mu\nu}\delta^{ab}.$$

The regularized propagators are G^2 multiplied by those given in the local theory.

- One can then show that to second order in coupling, the non-local $L_{non\text{-}local}$ is invariant under the following non-linear gauge transformations:

$$\delta W_\mu^a = \lambda\xi\partial_\mu c^a + g\lambda\xi f^{abc}c^b W_\mu^c - g^2\xi\lambda f^{abc}G^2[c^b\bar{D}_{\mu\nu}C^{c\nu} + c^b\bar{D}_{\mu\nu}J^{c\nu}]$$
$$- g^2\xi\lambda f^{abc}f^{cde}G^2[c^b\bar{D}_{\mu\nu}\partial^\nu\bar{c}^d c^e + W_\mu^b\bar{\Delta}\partial_\nu(W^{d\nu}c^e),$$

$$\delta B_\mu = \lambda_0\xi\partial_\mu\eta,$$

$$\delta\psi^L = G^2\left[-ig\xi\lambda c - ig'\frac{Y}{2}\xi\lambda_0\eta + ig^2\xi\lambda c\bar{S}\not{W}\right.$$
$$\left. + igg'\frac{Y}{2}\xi\lambda c\bar{S}\not{B} + igg'\frac{Y}{2}\xi\lambda_0\eta\bar{S}\not{W} + ig'^2\left(\frac{Y}{2}\right)^2\xi\lambda_0\eta\bar{S}\not{B}\right]\psi^L,$$

$$\delta\psi^R = G^2\left[-ig'\xi\lambda_0\frac{Y}{2}\eta + ig'^2\left(\frac{Y}{2}\right)^2\xi\lambda_0\eta\bar{S}\not{B}\right]\psi^R,$$

$$\delta c^a = -\frac{\xi}{2}\lambda g f^{abc}G^2c^b c^c - \xi\lambda g^2 f^{abc}f^{cde}G^2c^b\bar{\Delta}\partial^\mu(W_\mu^d c^e), \qquad \delta\bar{c}^a = \lambda\partial_\mu W^{a\mu},$$

$$\delta\eta = 0, \qquad \delta\bar{\eta} = -\lambda_0\partial_\mu B^\mu.$$

- The non-local action $S_{\text{non-loc}}$ has a Becchi, Rouet, Stora, Tyutin (BRST) invariance, assuring us that we have a correctly quantized theory to second order.

- We will derive the measure by requiring that the theory remain gauge invariant to all orders in the loop expansion. This is equivalent, at second order, to ensuring that nothing picks up a mass term at one loop. We work in the Feynman gauge ($\xi = 1$), for it is much simpler operationally to work with, but it should be kept in mind that unphysical degrees of freedom will occur. The simplest self-energy is that of the ghost in Euclidean momentum space:

$$-i\Sigma^{ad}_{\text{ghost}} = \frac{-ig^2 p_E^2}{(4\pi)^2} f^{abc} f^{dbc} \int_0^{\frac{1}{2}} d\tau E_1\left(\tau \frac{p_E^2}{\Lambda_W^2}\right)$$
$$= \frac{-ig^2 p_E^2}{(4\pi)^2} f^{abc} f^{dbc} \left\{ \frac{\Lambda_W^2}{p_E^2}\left[1 - e^{-p_E^2/2\Lambda_W^2}\right] + \frac{1}{2} E_1\left(\frac{p_E^2}{2\Lambda_W^2}\right)\right\}.$$

$$E_n(x) = \int_1^\infty e^{-xt} y^{-n} dy = \frac{1}{n-1}[e^{-x} - xE_{n-1}(x)] \qquad (x > 0).$$

• Next we can compute the fermion self-energies with massless fermions:

$$-i\Sigma_{\text{fermion}} = \frac{2ie^2}{(4\pi)^2}\Gamma_5^+ \not{p} \int_0^{\frac{1}{T}} d\tau E_1\left(\frac{\tau p_E^2}{\Lambda_W^2}\right) = \frac{2ie^2}{(4\pi)^2}\Gamma_5^+ \not{p}\left\{\frac{\Lambda_W^2}{p_E^2}\left[1 - e^{-p_E^2/2\Lambda_W^2}\right] + \frac{1}{2}E_1\left(\frac{p_E^2}{2\Lambda_W^2}\right)\right\}$$

FIG. 4: Fermion self-energy.

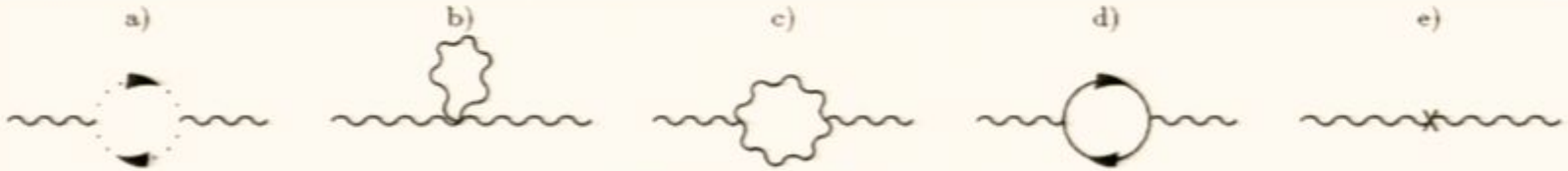

FIG. 5: Boson self-energy.

- We split the vacuum polarization tensor into longitudinal and transverse pieces:

$$-i\Pi_{\mu\nu} = -i\Pi^T \left(\eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) - i\Pi^L \frac{p_\mu p_\nu}{p^2}$$

- We turn our attention to the fermionic sector:

$$i\Pi_f^L = -\frac{4iee'\Lambda_W^2}{(4\pi)^2} g_+ K.$$

$$-i\Pi_f^T = -\frac{4iee'\Lambda_W^2}{(4\pi)^2} g_+ (K + 2P).$$

$$g_\pm = g_v g'_v \pm g_a g'_a.$$

$$K = 2\int_0^{\frac{1}{2}} d\tau (1-\tau) \exp\left(-\tau \frac{p_E^2}{\Lambda_W^2}\right) = -\left(\frac{\Lambda_W^2}{p_E^2} + 2\frac{\Lambda_W^4}{p_E^4}\right) e^{-p_E^2/2\Lambda_W^2} + 2\frac{\Lambda_W^4}{p_E^4}.$$

$$P = -2\frac{p_E^2}{\Lambda_W^2} \int_0^{\frac{1}{2}} d\tau \tau (1-\tau) E_1\left(\tau \frac{p_E^2}{\Lambda_W^2}\right)$$

$$= -\frac{1}{6}\frac{p_E^2}{\Lambda_W^2} E_1\left(\frac{p_E^2}{2\Lambda_W^2}\right) + \frac{1}{3}\left(1 + \frac{\Lambda_W^2}{p_E^2} - 4\frac{\Lambda_W^4}{p_E^4}\right) e^{-p_E^2/2\Lambda_W^2} - \frac{\Lambda_W^2}{p_E^2} + \frac{4}{3}\frac{\Lambda_W^4}{p_E^4}.$$

- To rid ourselves of the longitudinal degrees of freedom, we include a measure contribution for each diagram:

$$\ln(\mu_{\rm inv}[AA']_{\rm ferm}) = \int d^4x A^\mu \Upsilon^{AA'}_{\mu\nu} A'^\nu.$$

$$\Upsilon^{AA'}_{\mu\nu} = -S\frac{4iee'\Lambda_W^2}{(4\pi)^2} g_+ \eta_{\mu\nu} K.$$

$S = \frac{1}{2}$ if $A_\mu = A'_\mu$ and $S = 1$ otherwise.

This leaves just the transverse piece:

$$-i\Pi^T_f = -\frac{8iee'\Lambda_W^2}{(4\pi)^2} g_+ P.$$

- For the B - B sector we have

$$\Upsilon^{BB}_{\mu\nu} = -\frac{2ig'^2\Lambda_W^2}{(4\pi)^2}\eta_{\mu\nu}K\sum_\psi((Q-T_3)^2+Q^2) = -20\frac{ig'^2\Lambda_W^2}{(4\pi)^2}\eta_{\mu\nu}K.$$

$$-i\Pi^T_{BBf} = -\frac{8ig'^2\Lambda_W^2}{(4\pi)^2}P\sum_\psi((Q-T_3)^2+Q^2) = -80\frac{ig'^2\Lambda_W^2}{(4\pi)^2}K.$$

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- For the W_3 - W_3 sector, we find

$$\Upsilon_{\mu\nu}^{W^3W^3} = -\frac{ig^2\Lambda_W^2}{(4\pi)^2}\eta_{\mu\nu}K\sum_{q^L}1 = -12\frac{ig^2\Lambda_W^2}{(4\pi)^2}\eta_{\mu\nu}K,$$

$$-i\Pi_{WWf}^T = -\frac{4ig^2\Lambda_W^2}{(4\pi)^2}P\sum_{q^L}1 = -48\frac{ig^2\Lambda_W^2}{(4\pi)^2}P.$$

When we diagonalize the W_1 - W_2 sector into the physical $W^\pm$ fields, we get:

$$\Upsilon_{\mu\nu}^{W^\pm} = -\frac{2ig^2\Lambda_W^2}{(4\pi)^2}\eta_{\mu\nu}K\sum_{q^L}1 = -24\frac{ig^2\Lambda_W^2}{(4\pi)^2}\eta_{\mu\nu}K,$$

$$-i\Pi_{W^\pm f}^T = -\frac{4ig^2\Lambda_W^2}{(4\pi)^2}P\sum_{q^L}1 = -48\frac{ig^2\Lambda_W^2}{(4\pi)^2}P.$$

- For the W_3 - B mixing sector we get

$$\Upsilon^{WB}_{\mu\nu} = -\frac{igg'\Lambda_W^2}{(4\pi)^2}\eta_{\mu\nu}K\sum_{q^L} YT^3.$$

$$-i\Pi^T_{WBf} = -\frac{2igg'\Lambda_W^2}{(4\pi)^2}P\sum_{q^L} YT^3.$$

- The sum of the above two terms is zero in the gauge invariant case. The invariant measure is then given by the product of each piece generated above. We also note that the BRST invariance implies Slavnov-Taylor identities analogous to those in the local case, which also must be satisfied to all orders for a valid perturbation theory.

4. Symmetry Breaking

- We shall now address the problem of how we break the gauge symmetry. Let us consider the simple problem of breaking the U(1) gauge symmetry of the massless photon Lagrangian:

$$L_0 = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} = \frac{1}{2}A_\mu(\Box\eta^{\mu\nu} - \partial^\mu\partial^\nu)A_\nu \qquad \partial_\mu A^\mu = 0.$$

The propagator in momentum space for the massless (spin-1) vector particle is

$$iD_{\mu\nu} = \frac{-i}{q^2 + i\epsilon}\left[\eta_{\mu\nu} - (1-\xi)\frac{q_\mu q_\nu}{q^2}\right]$$

In the Feynman gauge $\xi = 1$ we obtain

$$iD_{\mu\nu} = -\frac{i\eta_{\mu\nu}}{q^2 + i\epsilon}.$$

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In the Feynman gauge $\xi = 1$ we obtain

$$iD_{\mu\nu} = -\frac{i\eta_{\mu\nu}}{q^2 + i\epsilon}.$$

- We now add a photon mass term

$$L = L_0 + m_\gamma^2 A_\mu(x) A^\mu(x).$$

- This mass term in the Lagrangian explicitly breaks the U(1) gauge invariance. However, we now introduce a proper self-energy contribution: $\Pi = \Pi(q^2)$

$$(\Box + m_\gamma^2) A_\mu(x) = 0. \quad \partial_\mu A^\mu(x) = 0.$$

$$-q^2 A_\mu(q) + m_{\gamma 0}^2 A_\mu(q) + \Pi(q^2) A_\mu(q) = 0.$$

$$m_\gamma^2 = m_{\gamma 0}^2 + \delta m_\gamma^2$$

- The trivial perturbative solution leads to a renormalization of the photon self-energy by canceling the quadratically divergent self-energy and leaving the photon massless (renormalization).

- We now break the vacuum symmetry and the U(1) gauge invariance and find the non-perturbative, self-consistent solution:

$$m_\gamma^2 = \Pi(0) \neq 0 \text{ and } m_{\gamma 0} = 0.$$

- The propagator solution is now given by

$$iD_{\mu\nu}(q) \to iD_{\mu\nu}(q) + iD_{\mu\lambda}(q) i\Pi^{\lambda\rho}(q^2) iD_{\rho\nu}(q) + iD_{\mu\lambda}(q) i\Pi^{\lambda\rho}(q^2) iD_{\rho\sigma}(q) i\Pi^{\sigma\kappa}(q^2) iD_{\kappa\nu}(q) + \ldots$$

$$-\frac{i}{q^2+i\epsilon} \to -\frac{i}{q^2}\left[1 + \Pi(q^2)\frac{1}{q^2} + \Pi(q^2)^2\left(\frac{1}{q^2}\right)^2 + \ldots\right] = -\frac{i}{q^2 - \Pi(q^2) + i\epsilon}.$$

$$i\Pi_{\mu\nu}(q^2) = -\frac{ie^2}{(2\pi)^4}\mathrm{Tr}\int d^4p \left[i\gamma_\mu \frac{i}{(\not{p}+\not{q}-m_f)} i\gamma_\nu \frac{i}{(\not{p}-m_f)}\right]$$

- This integral is quadratically divergent.

• In the present context of a simple pedagogical description of the symmetry breaking mechanism, we can use a cutoff to make the integral convergent at high energies. However, we will implement the non-local, regularization field theory to make the theory ultraviolet (UV) complete. This UV completion introduces a fundamental electroweak energy scale Λ_W, that is not equivalent to a simple (naive) cutoff, for it preserves gauge invariance, unitarity and Poincaré invariance in the massless limit. Moreover, as will be shown, it does not lead to a conflict with low energy electroweak precision data in the $SU_L(2) \times U_Y(1)$ broken symmetry case, and suppresses all higher-dimensional operators.

• An alternative to the standard perturbative renormalization method is to identify the photon self-energy with the photon mass. The photon creates a virtual fermion-anti-fermion pair which in turn creates a photon, producing the photon self-energy diagram. The fermion-anti-fermion pair can be pictured as a virtual fermion "condensate".

• Thus, the photon acquires a mass through a self-consistent mass equation.

- Let us now consider a non-Abelian gauge vector field W^a_μ. We assume that W^a_μ is an SU(2) isospin vector which transforms as

$$W_\mu \rightarrow A_\mu + i\theta^a[T^a, W_\mu],$$

- Our action now picks up a quadratic term from the lowest order non-Abelian self-energy diagram:

$$g^2 \Pi \mathrm{Tr}[T^a, T^b] W^a_\mu W^{\mu b}.$$

- The gauge boson mass squared are determined by the eigenvalues of the 3 by 3 matrix:

$$g^2 \Pi \mathrm{Tr}[T^a, T^b].$$

- Let us consider the symmetry group G which is broken down to the subgroup H. We find that N(G) - N(H) Nambu-Goldstone bosons will be generated. We start with N(G) massless gauge bosons, one for each generator.

• Upon symmetry breaking of the vacuum, the N(G) - N(H) Nambu-Goldstone bosons are eaten by N(G) - N(H) gauge bosons, leaving N(H) massless gauge bosons. For the case of $SU_L(2) \times U_Y(1)$, we have N(G) = 4 and N(H) = 1 and we end up with one massless gauge boson, namely, the photon. In our Lagrangian after the breaking of the vacuum symmetry:

$$L_m = \frac{1}{2} g^2 \Pi [T^a \cdot T^b] W^{\mu a} W^b_\mu = \frac{1}{2} W^{\mu a} (m^2)^{ab} W^b_\mu$$

$$(m^2)^{ab} = g^2 \Pi [T^a \cdot T^b]$$

• We now diagonalize $(m^2)_{ab}$ to obtain the masses of the gauge bosons.

• The mass matrix $(m^2)_{ab}$ is a 4 by 4 matrix with 1 zero eigenvalue for our group $SU_L(2) \times U_Y(1)$. Since U(1) remains unbroken by the breaking of the vacuum symmetry, the generator T_c associated with the U(1) symmetry satisfies $T_c \Pi = 0$, leaving the photon massless.

$$L_m = \frac{1}{4} g^2 \Pi W^+_\mu W^{-\mu} + \frac{1}{8} \Pi (g W^3_\mu - g' B_\mu)^2.$$

$$A_\mu = s_w W_\mu^3 + c_w B_\mu.$$

$$Z_\mu = c_w W_\mu^3 - s_w B_\mu$$

$$m_Z^2 = \Pi(g^2 + g'^2)/4$$

- We now obtain the standard tree graph result

$$\rho = \frac{m_W^2}{m_Z^2 c_w^2} = 1$$

- Let us introduce the spin-1 vector V^a_μ and from the fermion-anti-fermion loop graph, we obtain the mass matrix:

$$V_\mu^\alpha = W_\mu^a \text{ and } V_\mu^0 = B_\mu.$$

$$SU_L(2) \times U_Y(1) \to U_{\rm em}(1)$$

$$m^2_{\alpha\beta} = \begin{pmatrix} m_W^2 & & & \\ & m_W^2 & & \\ & & m_3^2 & m^2 \\ & & m^2 & m_0^2 \end{pmatrix}$$

• The unbroken electromagnetic gauge invariance that guarantees a massless photon dictates that the upper left 2 by 2 block of the matrix be proportional to the unit matrix. Moreover, it also says that the upper-right and the lower-left blocks must vanish. The vanishing of one of the eigenvalues guarantees a massless photon, which corresponds to:

$$\det \begin{pmatrix} m_3^2 & m^2 \\ m^2 & m_0^2. \end{pmatrix} = m_3^2 m_0^2 - m^4 = 0.$$

$$\tan\theta_w = \frac{m^2}{m_3^2} = \left|\frac{m_0}{m_3}\right|$$

$$m_Z^2 = \mathrm{tr} \begin{pmatrix} m_3^2 & m^2 \\ m^2 & m_0^2 \end{pmatrix} = m_0^2 + m_3^2 = m_3^2(1 + \tan^2\theta_w) = m_3^2 \sec^2\theta_w.$$

$$\frac{m_W}{m_Z} = \frac{g}{\sqrt{g^2 + g'^2}} = c_w.$$

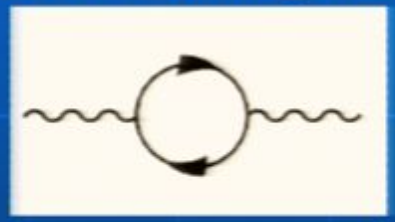
5. Breaking The Symmetry With A Path Integral Measure

- We break $SU_L(2)$ X $U_Y(1)$ down to $U_{em}(1)$ not at the classical level as is done in the standard model, which generates boson masses at tree level, but in the quantum regime, so that all the effects show up at loop order (which is where the non-locality shows up as well, as both are quantum effects). This means leaving the action gauge invariant and modifying the measure, which alters the quantization of the theory, in order to produce the desired results.

- The symmetry breaking measure in our path integral generates three new degrees of freedom as scalar Nambu-Goldstone bosons that give the $W^{\pm}$ and Z^0 bosons longitudinal modes, which makes them massive while retaining a massless photon.

- Since we want to mix the W_3 and B to get a massive Z and a photon, we need to work with the measure in a sector which is common to all gauge bosons. This implies working with the fermion contributions and leaving the bosonic and ghost contributions invariant.

- The self-energy contribution coming from



is given by

$$-i\Pi_f^L = -\frac{4iee'\Lambda_W^2}{(4\pi)^2}[g_+(K_{m_1m_2} - L_{m_1m_2}) + g_- M_{m_1m_2}],$$

$$-\Pi_f^T = -\frac{4iee'\Lambda_W^2}{(4\pi)^2}[g_+(K_{m_1m_2} - L_{m_1m_2} + 2P_{m_1m_2}) + g_- M_{m_1m_2}],$$

where we define

$$
\begin{aligned}
K_{m_1m_2} &= \int_0^{\frac{1}{2}} d\tau(1-\tau)\left[\exp\left(-\tau\frac{p_E^2}{\Lambda_W^2}-f_{m_1m_2}\right)+\exp\left(-\tau\frac{p_E^2}{\Lambda_W^2}-f_{m_2m_1}\right)\right],\\
P_{m_1m_2} &= -\frac{p_E^2}{\Lambda_W^2}\int_0^{\frac{1}{2}} d\tau\tau(1-\tau)\left[E_1\left(\tau\frac{p_E^2}{\Lambda_W^2}+f_{m_1m_2}\right)+E_1\left(\tau\frac{p_E^2}{\Lambda_W^2}+f_{m_2m_1}\right)\right],\\
L_{m_1m_2} &= \int_0^{\frac{1}{2}} d\tau(1-\tau)\left[f_{m_1m_2}E_1\left(\tau\frac{p_E^2}{\Lambda_W^2}+f_{m_1m_2}\right)+f_{m_2m_1}E_1\left(\tau\frac{p_E^2}{\Lambda_W^2}+f_{m_2m_1}\right)\right],\\
M_{m_1m_2} &= \frac{m_1m_2}{\Lambda_W^2}\int_0^{\frac{1}{2}} d\tau\left[E_1\left(\tau\frac{p_E^2}{\Lambda_W^2}+f_{m_1m_2}\right)+E_1\left(\tau\frac{p_E^2}{\Lambda_W^2}+f_{m_2m_1}\right)\right],
\end{aligned}
$$

$$
f_{m_1m_2} = \frac{m_1^2}{\Lambda_W^2}+\frac{\tau}{1-\tau}\frac{m_2^2}{\Lambda_W^2}.
$$

- If we insert this into the quadratic terms and invert, we get the corrected propagators (in a general gauge):

$$
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$$

- When the longitudinal piece Π_L is nonzero in the unitary gauge (where only the physical particle spectrum remains), we have no unphysical poles in the longitudinal sector. In this way, we can assure ourselves that we are not introducing spurious degrees of freedom into the theory.

- In the diagonalized $W^{\pm}$ sector, we get

$$-i\Pi^{L}_{W^{\pm}f} = -\frac{ig^2\Lambda_W^2}{(4\pi)^2}\sum_{q^L}(K_{m_1m_2} - L_{m_1m_2}),$$

$$-i\Pi^{T}_{W^{\pm}f} = -\frac{ig^2\Lambda_W^2}{(4\pi)^2}\sum_{q^L}(K_{m_1m_2} - L_{m_1m_2} + 2P_{m_1m_2}).$$

- We can now postulate the non-existence of a measure for the fermionic contribution to the W boson self-energies. This introduces three Nambu-Goldstone degrees of freedom into the W sector and the W bosons acquire a longitudinal part and a corresponding mass. To see this, note that

$$-i\Pi^{L}_{W^{\pm}f}\Big|_{p^2=0} = -i\Pi^{T}_{W^{\pm}f}\Big|_{p^2=0} = -\frac{ig^2\Lambda_W^2}{(4\pi)^2}\sum_{q^L}(K_{m_1m_2} - L_{m_1m_2})\Big|_{p^2=0} \neq 0.$$

- In the B sector we have

$$-i\Pi^L_{Bf} = -\frac{1}{2}\frac{ig'^2\Lambda_W^2}{(4\pi)^2}\sum_\psi[16(Q-T^3)^2(K_{mm}-L_{mm})+32Q(Q-T^3)M_{mm}],$$

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- We write the measure contribution as

$$\Upsilon^{BB}_{\mu\nu} = -\frac{ig'^2\Lambda_W^2}{(4\pi)^2}\eta_{\mu\nu}\sum_\psi\left[\left(\frac{1}{2}-8(Q-T^3)^2\right)(K_{mm}-L_{mm})-16Q(Q-T^3)M_{mm}\right]$$

Then we have

$$-i\Pi^L_{Bf} = -\frac{1}{2}\frac{ig'^2\Lambda_W^2}{(4\pi)^2}\sum_\psi(K_{mm}-L_{mm}),$$

$$-i\Pi^T_{Bf} = -\frac{1}{2}\frac{ig'^2\Lambda_W^2}{(4\pi)^2}\sum_\psi[(K_{mm}-L_{mm})+32(Q-T^3)^2P_{mm}].$$

- The B - W_3 mixing sector originally looks like

$$-i\Pi^{L}_{W^3Bf} = -\frac{4igg'\Lambda_W^2}{(4\pi)^2}\sum_{\psi}[T^3(Q-T^3)(K_{mm}-L_{mm})+QM_{mm}],$$

$$-i\Pi^{T}_{W^3Bf} = -\frac{4igg'\Lambda_W^2}{(4\pi)^2}\sum_{\psi}[T^3(Q-T^3)(K_{mm}-L_{mm}+2P_{mm})+QM_{mm}].$$

- Thus, to make the mass contributions look identical, we write

$$\Upsilon^{W^3B}_{\mu\nu} = -\frac{igg'\Lambda_W^2}{(4\pi)^2}\eta_{\mu\nu}\sum_{\psi}\left[\left(-\frac{1}{2}-4T^3(Q-T^3)\right)(K_{mm}-L_{mm})-4QM_{mm}\right].$$

- Then we have

$$-i\Pi^{L}_{W^3Bf} = \frac{1}{2}\frac{igg'\Lambda_W^2}{(4\pi)^2}\sum_{\psi}(K_{mm}-L_{mm}),$$

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- Only the diagonal Z - Z piece has a longitudinal part

$$-i\Pi^L_{Zf} = -\frac{1}{2}\frac{i(g^2+g'^2)\Lambda_W^2}{(4\pi)^2}\sum_\psi (K_{mm}-L_{mm}).$$

$$Z_\mu = c_w W^3_\mu - s_w B_\mu \quad \text{and} \quad A_\mu = c_w B_\mu + s_w W^3_\mu$$

- For the Z - Z part we get

$$-i\Pi^T_{Zf} = -\frac{1}{2}\frac{i(g^2+g'^2)\Lambda_W^2}{(4\pi)^2} \times \sum_\psi [(K_{mm}-L_{mm}) + P_{mm}(2c_w^4 + s_w^4 32(Q-T^3)^2 - 16 s_w^2 c_w^2 T^3(Q-T^3))].$$

- The pure photon sector gives

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\begin{aligned}
K_{m_1m_2} &= \int_0^{\frac{1}{2}} d\tau(1-\tau)\left[\exp\left(-\tau\frac{p_E^2}{\Lambda_W^2}-f_{m_1m_2}\right)+\exp\left(-\tau\frac{p_E^2}{\Lambda_W^2}-f_{m_2m_1}\right)\right],\\
P_{m_1m_2} &= -\frac{p_E^2}{\Lambda_W^2}\int_0^{\frac{1}{2}} d\tau\tau(1-\tau)\left[E_1\left(\tau\frac{p_E^2}{\Lambda_W^2}+f_{m_1m_2}\right)+E_1\left(\tau\frac{p_E^2}{\Lambda_W^2}+f_{m_2m_1}\right)\right],\\
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6. Calculation Of The ρ Parameter And Λ_W

- When we consider the scattering of longitudinally polarized vector bosons, the vector boson propagator reads

$$iD^{\mu\nu}(p^2) = \frac{-i\eta^{\mu\nu}}{p^2 - \Pi_f^T(p^2)},$$

This differs from the vector boson propagator of the standard model in that the squared mass m^2_V of the vector boson is replaced by the self-energy term $\Pi^T_{f.}$ For an on-shell vector boson, demanding agreement with the standard model requires that the following consistency equation be satisfied:

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the right-hand side of which contains Λ_W can be used to determine Λ_W. Using

$$g = 0.649, \quad \sin^2\theta_w = 0.2312, \quad m_t = 171.2\ \text{GeV}, \quad m_Z = 91.1876 \pm 0.0021$$

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$$m_W \simeq 80.05\ \text{GeV}.$$

• This result, which does not incorporate radiative corrections, is actually slightly closer to the experimental value m_W = 80.398 ± 0.025 GeV than the comparable tree-level standard model prediction m_W = 79.95 GeV, obtained using ρ = 1 where

$$\rho = \frac{m_W^2}{m_Z^2 \cos^2 \theta_w},$$

We get from our model

$$\rho \simeq 1.0023.$$

which agrees well with estimates from the experimental ratio of neutral to charged currents.

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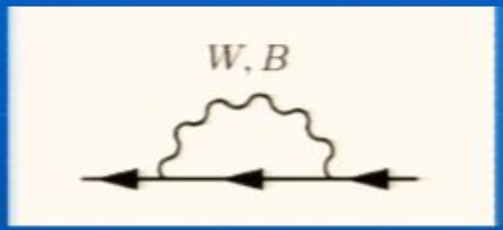
7. Fermion Masses

• We will generate fermion masses from the finite one-loop fermion self-energy graph:



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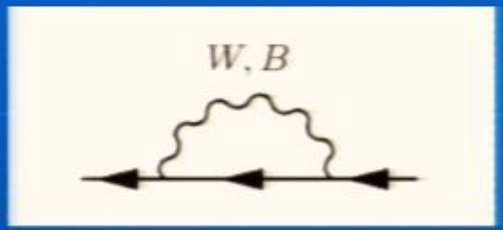

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Here, m_{0f} is the bare fermion mass, m_f is the observed fermion mass and $\Sigma(p)$ is the finite proper self-energy part. We have

$$m_f - m_{0f} = \Sigma(p, m_f, g, \Lambda_f)|_{\not{p}-m_f=0}.$$

where Λ_f denotes the energy scales for lepton and quark masses.

- A solution can be found by successive approximations starting from the bare mass m_{0f}, but we can also find a solution for $m_f \neq 0$ when $m_{0f} = 0$ for a broken symmetry vacuum state (Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122, 345 (1961)).

- The one-loop correction to the self-energy of a fermion with mass m_f in the regularized theory for a massive vector field is

$$-i\Sigma(p) = i\int \frac{d^4k}{(2\pi)^4} ig\gamma_\mu \frac{i}{\not{p}-\not{k}-m_f} ig\gamma_\nu \frac{-i\eta^{\mu\nu}}{k^2-m_V^2} \exp\left(\frac{p^2-m_f^2}{\Lambda_f^2} + \frac{(p-k)^2-m_f^2}{\Lambda_f^2} + \frac{k^2-m_V^2}{\Lambda_f^2}\right)$$

$$= \frac{-ig^2}{8\pi^4} \exp\left(\frac{p^2-m_f^2}{\Lambda_f^2}\right) \int d^4k \frac{-\not{p}+\not{k}+2m_f}{(p-k)^2-m_f^2} \frac{1}{k^2-m_V^2} \exp\left(\frac{(p-k)^2-m_f^2}{\Lambda_f^2} + \frac{k^2-m_V^2}{\Lambda_f^2}\right)$$

- Promoting the propagator to Schwinger proper time integrals:

$$\Sigma(p) = \frac{g^2}{8\pi^2} \exp\left(\frac{p^2 - m_f^2}{\Lambda_f^2}\right) \int_1^\infty d\tau_1 \int_1^\infty d\tau_2 \left(\frac{-\tau_2}{(\tau_1+\tau_2)^3} \not{p} + \frac{2}{(\tau_1+\tau_2)^2} m_f\right)$$

$$\exp\left(\frac{\tau_1\tau_2}{\tau_1+\tau_2}\frac{p^2}{\Lambda_f^2} - \tau_1\frac{m_f^2}{\Lambda_f^2} - \tau_2\frac{m_V^2}{\Lambda_f^2}\right)$$

At p = 0, this integral becomes

$$\Sigma(0) = \frac{g^2}{4\pi^2} \exp\left(\frac{-m_V^2}{\Lambda_f^2}\right) m_f \left[E_1\left(\frac{2m_f^2}{\Lambda_f^2}\right) - \frac{m_V^2}{\Lambda_f^2}\int_2^\infty d\tau \exp\left(\tau\frac{m_V^2 - m_f^2}{\Lambda_f^2}\right) E_1\left(\tau\frac{m_V^2}{\Lambda_f^2}\right)\right]$$

- We now identify the fermion mass as $m_f = \Sigma(0)$:

$$m_f = \frac{g^2}{4\pi^2} \exp\left(\frac{-m_V^2}{\Lambda_f^2}\right) m_f \left[E_1\left(\frac{2m_f^2}{\Lambda_f^2}\right) - \frac{m_V^2}{\Lambda_f^2}\int_2^\infty d\tau \exp\left(\tau\frac{m_V^2 - m_f^2}{\Lambda_f^2}\right) E_1\left(\tau\frac{m_V^2}{\Lambda_f^2}\right)\right].$$

- In addition to admitting a trivial solution at $m_f = 0$, this equation also has non-trivial solutions that can be computed numerically. In a theory with a single massless vector boson, we get

$$m_f = \frac{g^2}{4\pi^2} m_f E_1\left(\frac{2m_f^2}{\Lambda_f^2}\right).$$

- A solution is obtained when

$$\frac{m_f}{\Lambda_f} = \sqrt{\frac{1}{2} E_1^{-1}\left(\frac{4\pi^2}{g^2}\right)}.$$

Using the electroweak coupling constant g ~0.649, we obtain for leptons

$$\Lambda_f \simeq 4.3 \times 10^{20} m_f.$$

For quarks, we use the strong coupling constant g_s ~ 1.5, and also introduce a color factor 3. Thereafter, we obtain

$$\Lambda_f \simeq 35 m_f.$$

- For a top quark mass m_t = 171.2 GeV, the corresponding energy scale is about $\Lambda_t \sim 6$ TeV.

- In these calculations, Λ_f plays a role that is similar to that of the diagonalized fermion mass matrix in the standard model. The number of undetermined parameters, therefore, is the same as in the standard model: for each fermion a corresponding Λ_f determines its mass.

- Our model permits massive neutrinos. However, as the Λ_f correspond to the diagonal components of a fermion mass matrix, off-diagonal terms are absent, and no flavor mixing takes place. Therefore, self-energy calculations alone are not sufficient to account for observed neutrino oscillations.

• However, in addition to fermion self-energy graphs, another case must be considered. Emission or absorption of a charged vector boson $W^{\pm}$ can be flavor violating, through the off-diagonal components of the CKM matrix. In the standard model, such flavor violating terms are not considered significant, due to the smallness of the corresponding CKM matrix elements. However, in our regularized theory, additional factors $\Lambda_{ff'}$ enter into the picture in a manner similar to the self-energy calculation we just described. These may include terms that correspond to the off-diagonal elements of the neutrino mass matrix, offering a natural explanation for neutrino oscillations without having to introduce new interactions.

8. Schwinger-Dyson Equations and Fermion Masses

- To solve the fermion mass problem, we must consider the integral equation obtained from the Schwinger-Dyson equation (J. S. Schwinger, Proc. Nat. Acad. Sc. 37, 452 (1951); F. J. Dyson, Phys. Rev. 75, 1736 (1949); A. Raya, ArXiv: 0902.1791 [hep-ph])).

$$S_F^{-1}(p) = S_F^{(0)-1}(p) - ig^2 \int \frac{d^4k}{(2\pi)^4} \gamma^\mu S_F(k) \Gamma^\nu(k,p) D_{\mu\nu}(k-p)$$

where g^2 is the coupling, $D_{\mu\nu}$ represents the complete vector particle propagator and Γ^ν the full fermion-vector particle vertex.

- We can derive the Euclidean mass gap equation

$$m_{0f} = \frac{\alpha}{\pi^2} \int \frac{d^3k}{(k-p)^2} \frac{M(k,\Lambda_f)}{k^2 + M^2(k,\Lambda_f)}$$

$$\alpha = g^2/4\pi$$

- This equation can be linearized and we set

$$M^2(k) = M^2(0) = m_f^2$$

which yields the linearized mass gap equation:

$$m_{0f} = \frac{\alpha}{\pi^2} \int \frac{d^3k}{(k-p)^2} \frac{M(k,\Lambda_f)}{k^2 + m_f^2} = 0$$

- $m_{0f} = 0$ is a (trivial) solution of this equation, and would correspond to that derived in perturbation theory. However, we are interested in a non-trivial solution, which can be obtained using analytical and numerical techniques. Dynamical chiral symmetry breaking, QCD confinement and our EW model are crucial features of this equation. This could lead to a fundamental determination of the fermion mass spectrum.

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9. The Running Of Coupling Constants And Unitarity

- The Higgs field resolves the issue of unitarity, by precisely canceling out badly behaved terms in the tree-level amplitude of processes involving longitudinally polarized vector bosons, for instance $W^+_L W^-_L$ -> $W^+_L W^-_L$ or e^+e^- -> $W^+_L W^-_L$. The challenge to any theory that aims to compete with the SM without introducing a Higgs particle is to generate the correct fermion and boson masses on the one hand, and ensure unitary behavior for these types of scattering processes on the other (JWM and V. T. Toth, ArXiv 0812.1994 [hep-ph]).

- Given the way Π^T appears in the vector boson propagator, it is reasonable to make the identification:

$$\Pi^T_{W\,f}(q^2) = m^2_W(q^2). \qquad \Pi^T_{Z\,f}(q^2) = m^2_Z(q^2).$$

- When we rewrite the theory's Lagrangian in terms of massive vector bosons, the Lagrangian picks up a finite mass contribution from the total sum of polarization graphs:

$$\begin{aligned} L_m &= \frac{1}{8}v^2g^2[(W_\mu^1)^2+(W_\mu^2)^2] \\ &\quad +\frac{1}{8}v^2[g^2(W_\mu^3)^2-2gg'W_\mu^3B^\mu+g'^2B_\mu^2] \\ &= \frac{1}{4}g^2v^2W_\mu^+W^{-\mu} \\ &\quad +\frac{1}{8}v^2(W_{3\mu},B_\mu)\begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix}\begin{pmatrix} W^{3\mu} \\ B^\mu \end{pmatrix} \end{aligned}$$

$$m_W=\frac{1}{2}vg, \quad m_Z=\frac{1}{2}v(g^2+g'^2)^{1/2}, \quad m_A=0.$$

- v is the electroweak symmetry breaking scale (which, in the SM, is the vacuum expectation value of the Higgs scalar).

- Consistency requires the running of the constants g and g′. Starting with the W mass, we obtain

$$\frac{g^2(q^2)}{g^2(m_Z^2)}=\frac{\Pi_{Wf}^T(q^2)}{\Pi_{Wf}^T(m_Z^2)}.$$

$$v^2=\frac{4\Pi_{Wf}^T(m_Z^2)}{g^2(m_Z^2)}\simeq(245\ \mathrm{GeV})^2.$$

Using the Z mass we obtain

$$\frac{g^2(q^2)+g'^2(q^2)}{g^2(m_Z^2)+g'^2(m_Z^2)}=\frac{\Pi_{Zf}^T(q^2)}{\Pi_{Zf}^T(m_Z^2)},$$

which establishes the running of $g'(q^2)$.

- These relationships also allow us to calculate the running of the Weinberg angle θ_w, which is defined through the ratio of the coupling constants g and g′ as

$$\cos\theta_w=\frac{\sqrt{g^2+g'^2}}{g}.$$

$$\Pi_{Zf}^T(q^2)=\frac{1}{2}\frac{(g_0^2+g_0'^2)\Lambda_W^2}{(4\pi)^2} \qquad (8$$
$$\times\sum_\psi\{[K_{mm}(q^2)-L_{mm}(q^2)]$$
$$+P_{mm}(q^2)[2\cos^4\theta_w+32\sin^4\theta_w(Q-T^3)^2$$
$$-16\sin^2\theta_w\cos^2\theta_w T^3(Q-T^3)]\},$$

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$$+P_{mm}(q^2)[2\cos^4\theta_w+32\sin^4\theta_w(Q-T^3)^2$$
$$-16\sin^2\theta_w\cos^2\theta_w T^3(Q-T^3)]\},$$

$$\Pi^T_{Wf}(q^2) = \frac{g_0^2 \Lambda_W^2}{(4\pi)^2} \times \sum_{q^L} (K_{m_1 m_2}(q^2) - L_{m_1 m_2}(q^2) + 2P_{m_1 m_2}(q^2))$$

$$K_{m_1 m_2}(q^2) = \int_0^{\frac{1}{2}} d\tau (1-\tau) \left[\exp\left(-\tau \frac{q^2}{\Lambda_W^2} - f_{m_1 m_2} \right) + \exp\left(-\tau \frac{q^2}{\Lambda_W^2} - f_{m_2 m_1} \right) \right], \quad (4)$$

$$P_{m_1 m_2}(q^2) = -\frac{q^2}{\Lambda_W^2} \int_0^{\frac{1}{2}} d\tau \tau (1-\tau) \left[E_1\left(\tau \frac{q^2}{\Lambda_W^2} + f_{m_1 m_2} \right) + E_1\left(\tau \frac{q^2}{\Lambda_W^2} + f_{m_2 m_1} \right) \right], \quad (5)$$

$$L_{m_1 m_2}(q^2) = \int_0^{\frac{1}{2}} d\tau (1-\tau) \left[f_{m_1 m_2} E_1\left(\tau \frac{q^2}{\Lambda_W^2} + f_{m_1 m_2} \right) + f_{m_2 m_1} E_1\left(\tau \frac{q^2}{\Lambda_W^2} + f_{m_2 m_1} \right) \right], \quad (6)$$

$$f_{m_1 m_2} = \frac{m_1^2}{\Lambda_W^2} + \frac{\tau}{1-\tau} \frac{m_2^2}{\Lambda_W^2}.$$

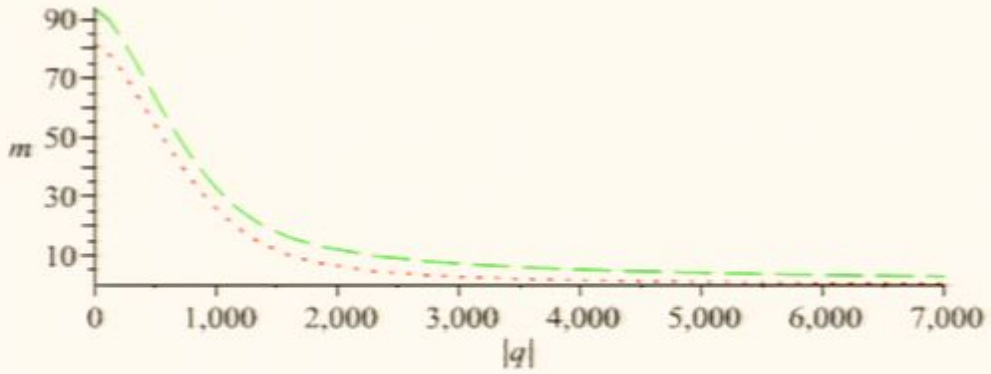


FIG. 1: The running of the W mass (red dotted line) and Z mass (dashed green line) as functions of momentum. Both axes are measured in units of GeV.

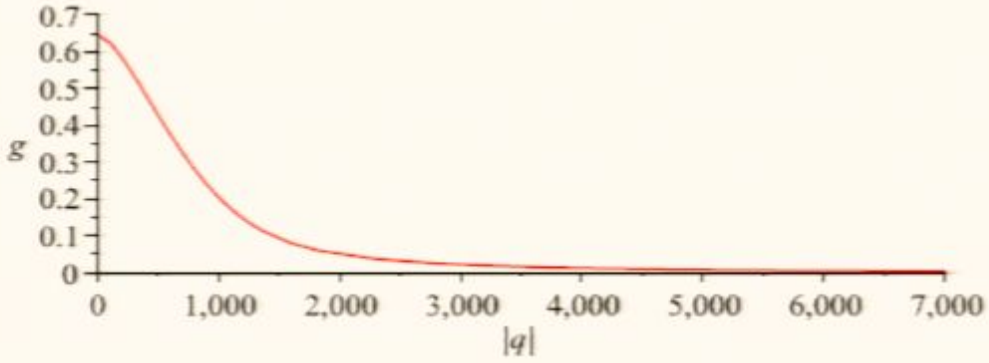


FIG. 2: The running of the electroweak coupling constant g as a function of momentum, measured in GeV.

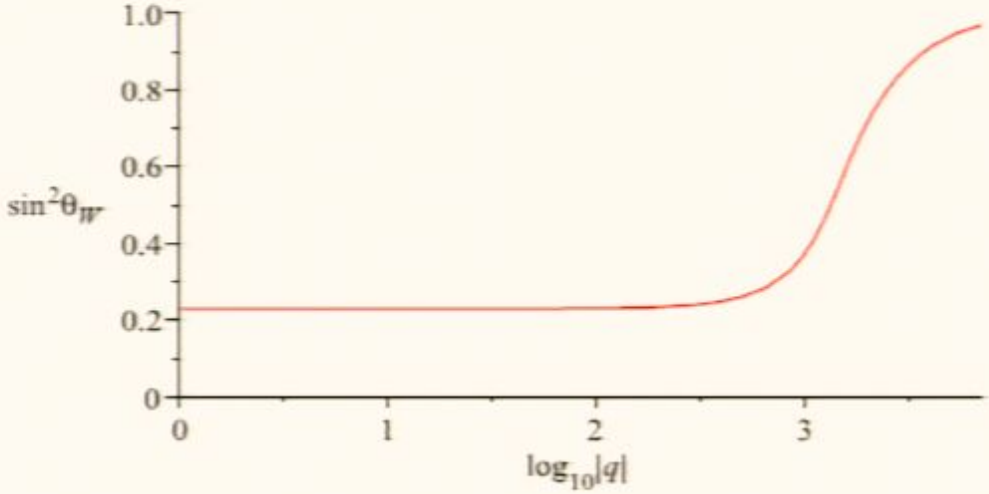
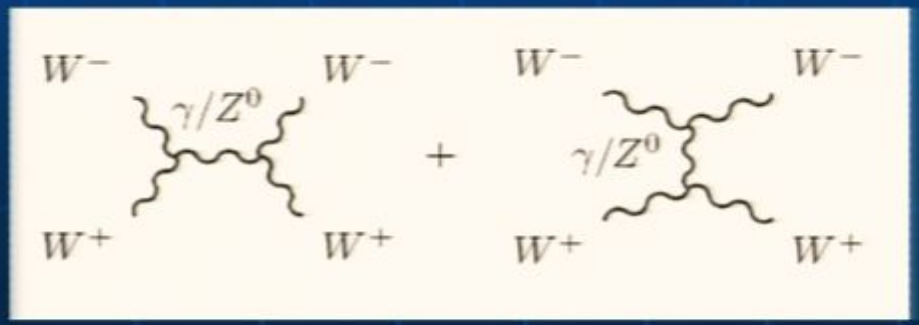
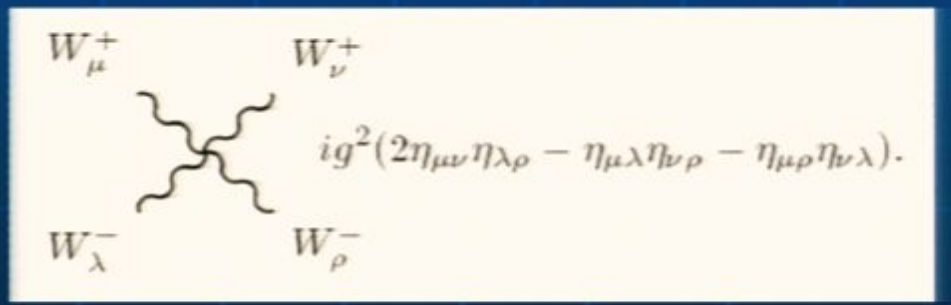


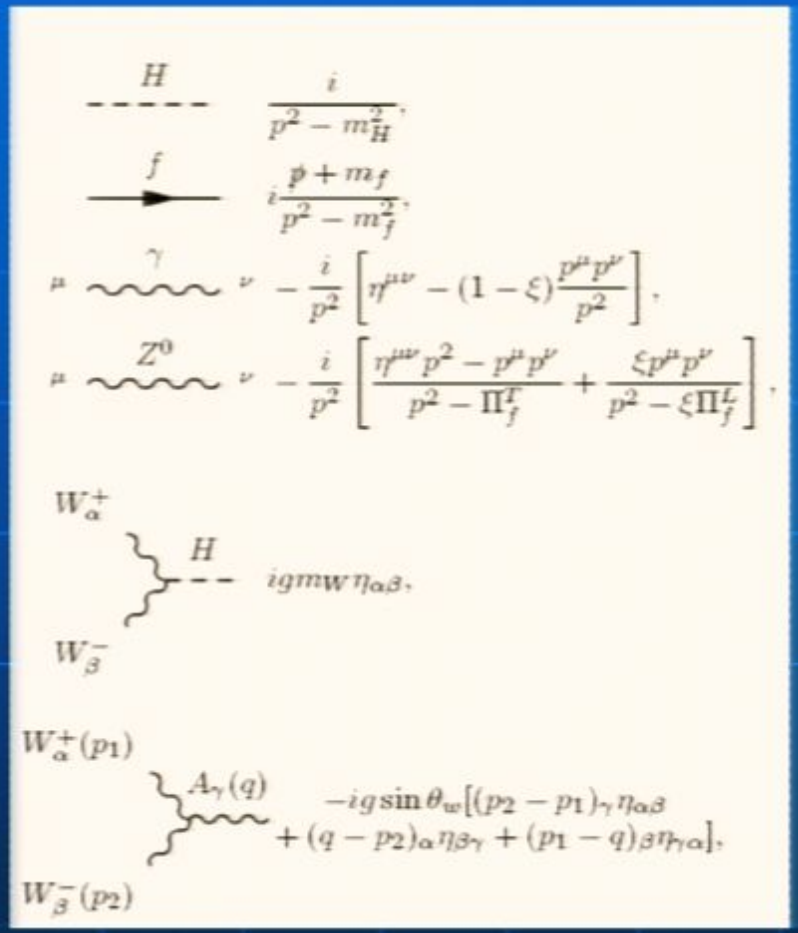
FIG. 3: The running of the Weinberg angle as a function of momentum, measured in GeV.

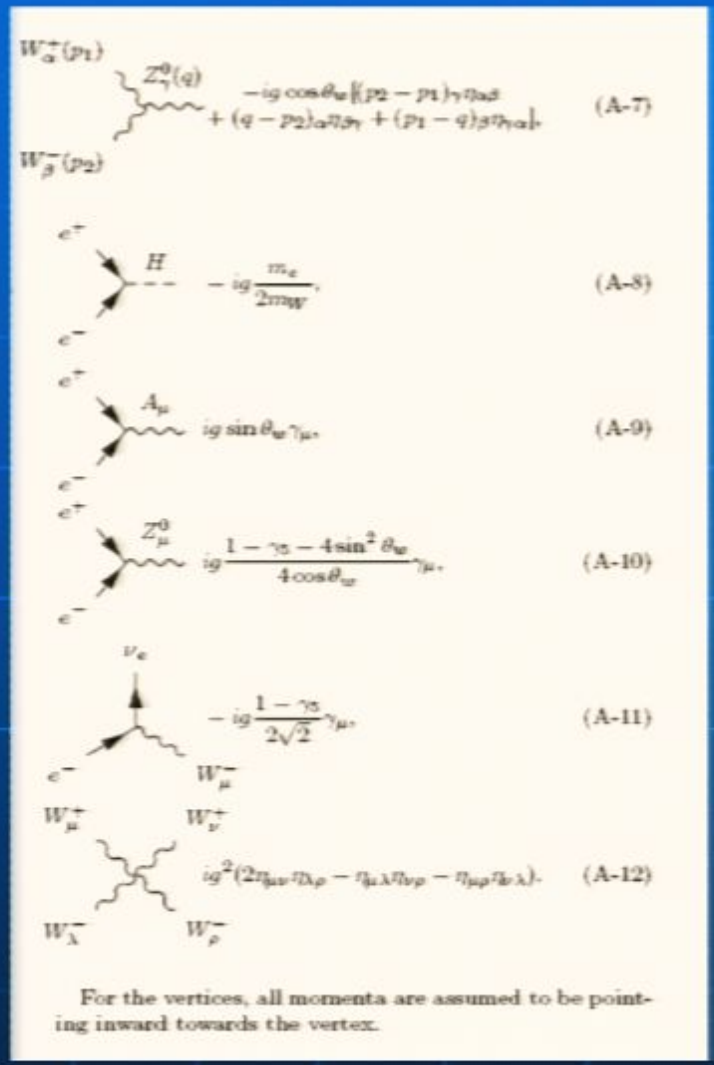
- The scattering of two longitudinally polarized W vector bosons can take place through one of the following processes:





- Feynman Rules Of The Standard Model And The FEW Theory.

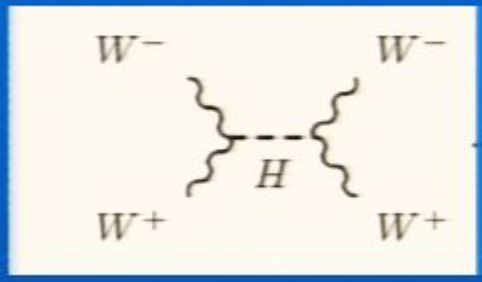




For the vertices, all momenta are assumed to be pointing inward towards the vertex.

- In the high energy limit, the SM with a Higgs yields the matrix element

$$i\mathcal{M}_{\rm SM}(W_L^+W_L^- \to W_L^+W_L^-) = ig^2\left[\frac{\cos^2\theta+3}{4\cos\theta_w^2(1-\cos\theta)} - \frac{m_H^2}{2m_W^2} + \mathcal{O}(s^{-1})\right].$$



- In the Higgless model we get

$$\begin{aligned} i\mathcal{M}_M =& i\mathcal{M}_s + i\mathcal{M}_t + i\mathcal{M}_4 \qquad (38)\\ =& ig^2\left[\frac{(\cos\theta+1)(4m_W^2 - 3\Pi_{Zf}^T\cos^2\theta_w)}{8m_W^4}s + \mathcal{O}(1)\right]. \end{aligned}$$

- In the case of the Higgless FEW theory, no additive cancellation takes place. However, the running of the electroweak coupling constant is such that at high s, g(s)s ~ const., which is sufficient to ensure that unitarity is not violated.

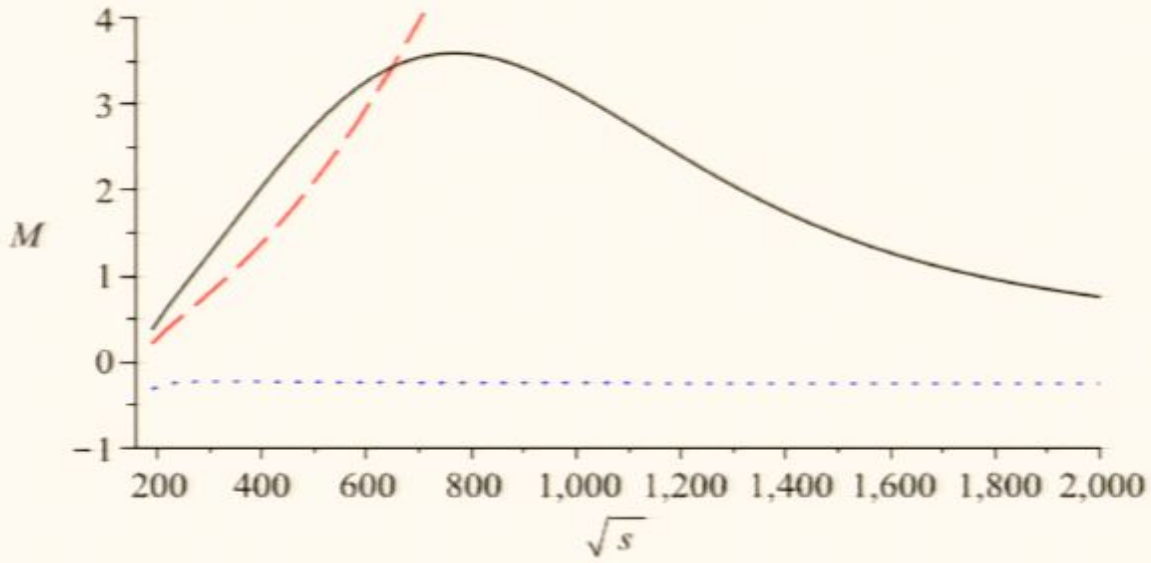
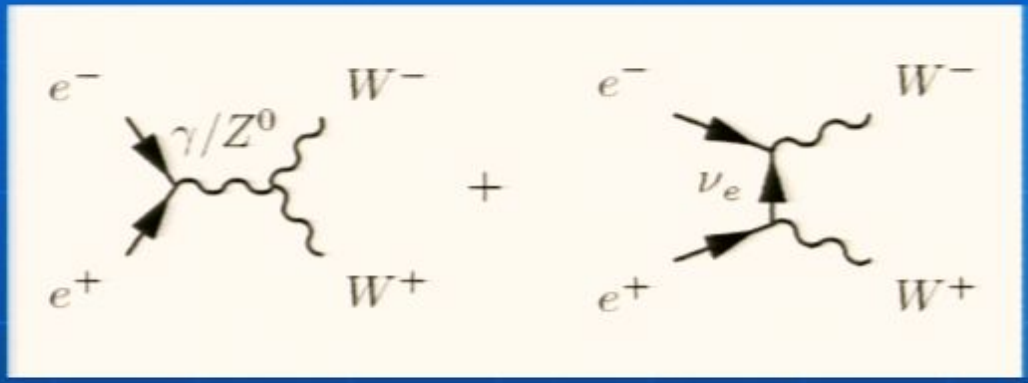


FIG. 4: The tree-level scattering amplitude of longitudinal $W^{\pm}$ bosons at a scattering angle $\theta = \pi/2$. The SM (blue dotted line) predicts an asymptotically constant amplitude at high energy. Without the Higgs particle (red dashed line) the amplitude is divergent. In the FEW theory (black solid line) this divergent amplitude is suppressed by the running of the electroweak coupling constant.

- The production of W^+W^- pairs from electron-positron collisions can take place via one of the following processes:

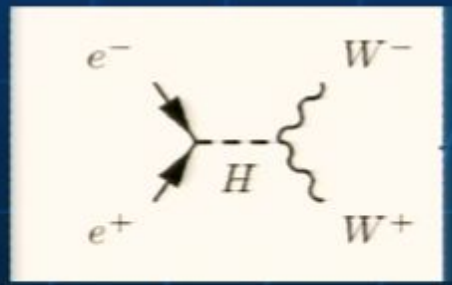


- In the high-energy limit, we get in the Higgless model:

$$i(\mathcal{M}_s + \mathcal{M}_t) = -ig^2 \left[\frac{m_e}{2m_W^2} \sqrt{s} + \mathcal{O}(1) \right].$$

- In the SM with a Higgs:

$$i\mathcal{M}_H = ig^2 \left[\frac{m_e}{2m_W^2} \sqrt{s} + \mathcal{O}(1) \right]$$



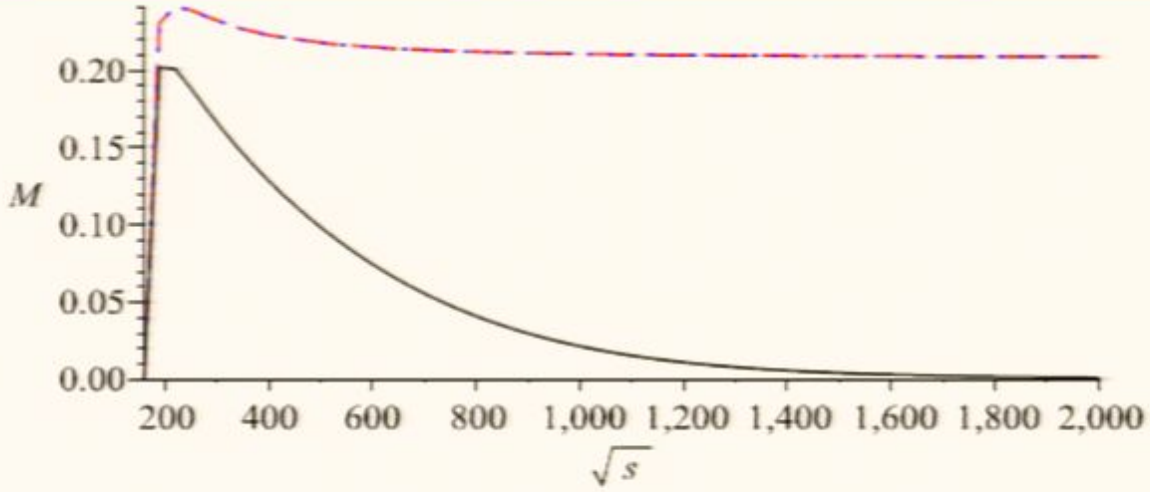
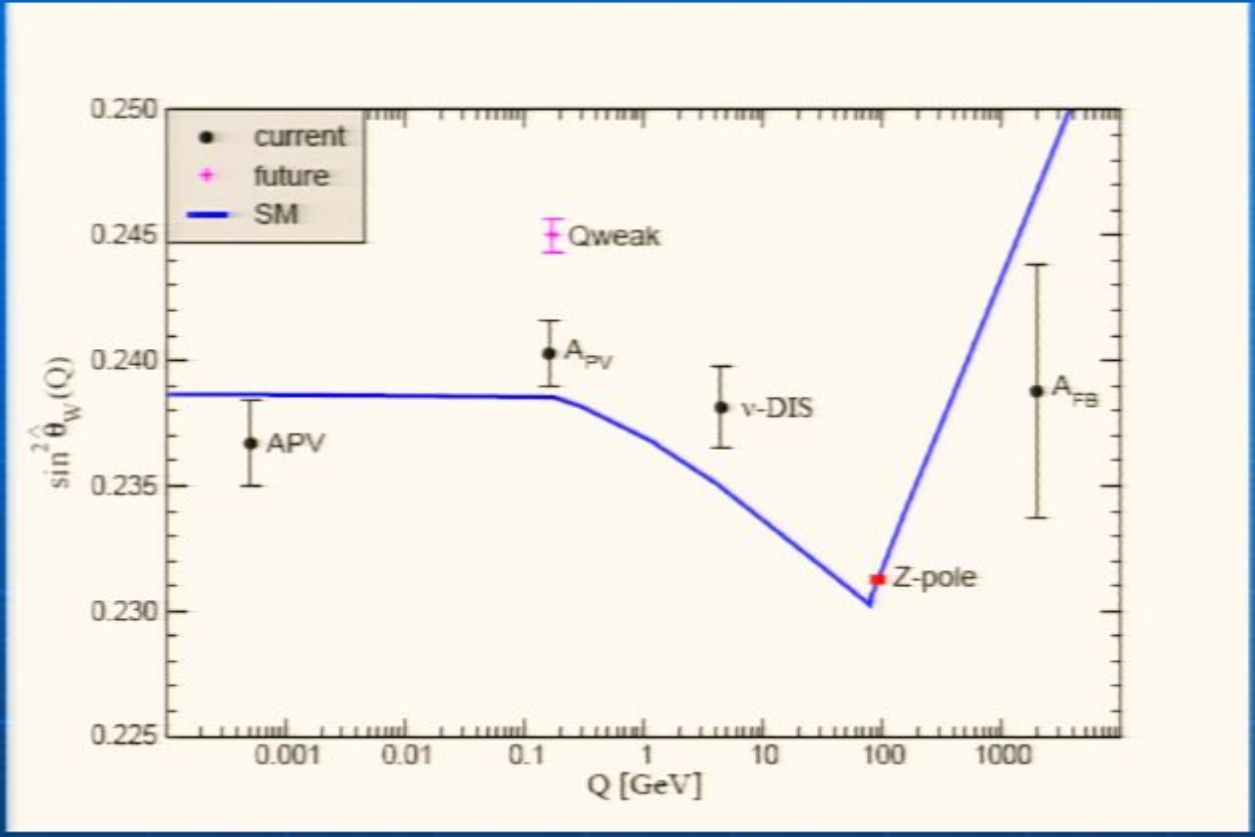
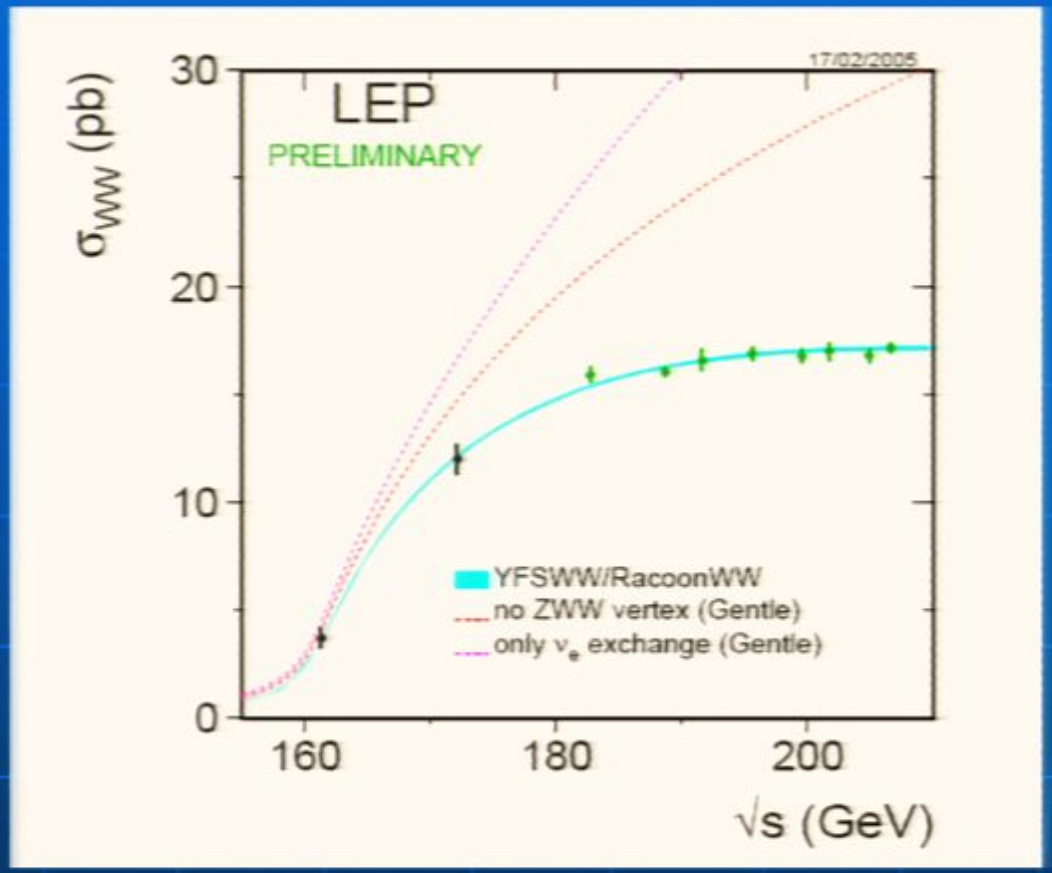


FIG. 5: The scattering amplitude, at a scattering angle of $\theta = \pi/2$, of electrons and positrons annihilating into longitudinally polarized $W^{\pm}$ bosons as a function of the center-of-mass energy $\sqrt{s}$, measured in GeV. The SM result (blue dotted line) is indistinguishable from the SM result that was calculated without the Higgs particle (red dashed line), as due to the smallness of m_e, the divergent term that is proportional to $m_e\sqrt{s}$ does not begin to dominate until much higher energies. Our Higgless theory, however, predicts a significant suppression of the amplitude even at moderate energies.



Running $\sin^2\theta_Z(Q2)$ measured at various scales, compared with the predictions of the SM. The low energy points are from atomic parity violation (APV), the polarized Møller asymmetry (PV) and deep inelastic neutrino scattering. Q_{weak} shows the expected sensitivity of a future polarized e- measurement at Jefferson Lab. Courtesy of the Particle Data Group.



Cross section for e-e+ -> W-W+ compared with the SM expectation. Plot courtesy of the LEP Electroweak Working Group,http://www.cern.ch/LEPEWWG/.

10. Conclusions

• An electroweak model without a Higgs particle that spontaneously breaks $SU_L(2) \times U_Y(1)$ has been developed, based on a finite quantum field theory. We begin with a massless and gauge invariant theory that is UV complete, Poincaré invariant and unitary to all orders of perturbation theory. A fundamental energy scale Λ_W enters into the calculations of the finite Feynman loop diagrams. A path integral is formulated that generates all the Feynman diagrams in the theory. The self-energy boson loop graphs with internal fermions comprised of the observed 12 quarks and leptons have an associated measure in the path integral that is spontaneously broken to generate 3 Nambu-Goldstone scalar modes that give the $W^{\pm}$ and the Z^0 bosons masses, while retaining a zero mass photon. There is no Higgs scalar field particle and no new particles are included in the particle spectrum.

- The $W_L W_L \rightarrow W_L W_L$ and $e^+e^- \rightarrow W^+_L W^-_L$ amplitudes do not violate unitarity at the tree graph level due to the running with energy of the electroweak coupling constants g, g′ and e. This is essential for the physical consistency of the model as is the case in the standard Higgs electroweak model.

- A self-consistent calculation of the energy scale yields $\Lambda_W = 542$ GeV and a prediction of the W mass from the W-boson self-energy diagrams in the symmetry broken phase gives $m_W = 80.05$ GeV, which is accurate to 0.5%.

- The EW cosmological constant problem is solved without fine-tuning.

- The Higgs hierarchy problem is solved without fine-tuning.

- The origin of mass in the universe is due to self-consistent solutions of QFT self-energies – not to a classical scalar Higgs field and Yukawa interactions.

END